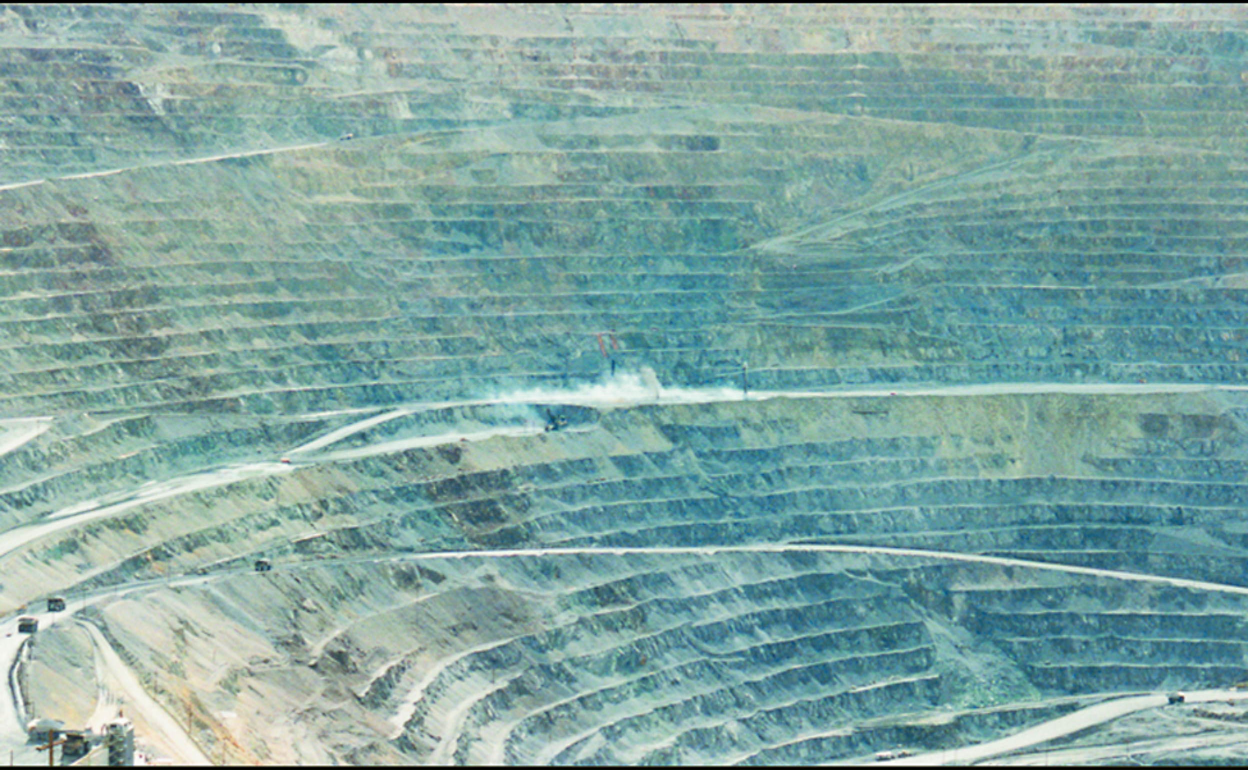

William G. Pariseau

Design Analysis in Rock Mechanics

Solutions Manual

Solutions Manual to Design Analysis in Rock Mechanics

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Solutions Manual to Design Analysis in Rock Mechanics

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Contents

1 Introduction

Basics

1. Identify the three major categories of equations needed to calculate rock mass motion. Give examples.

   (a) Physical laws
   e.g. conservation (balance) of linear momentum

   $$F = \dot{P}$$

   where
   F is resultant of external forces
   $\dot{P}$ is time rate of linear momentum.

   (b) Kinematics (geometry of motion)
   e.g. $a = \dot{v}$ where a and v are acceleration and velocity
   e.g. $\varepsilon_{xx} = \dfrac{\partial u}{\partial x}$ where ε_{xx} is normal strain in the x-direction and u is x-direction displacement.

   (c) Material laws
   e.g. $\sigma = E\varepsilon$ is the one-dimensional form of Hooke's law of linear elasticity.
   σ and ε are stress and corresponding strain, E is young's modulus.

2. Given: $\rho =$ mass density
   Explain why or why not

   $$\dot{M} = \int_V \dot{\rho}\, \mathrm{d}V$$

   where $M =$ mass of body with volume V.

   Solution:

   $$\rho = \frac{\mathrm{d}m}{\mathrm{d}V} \text{ by definition}$$

   where $\mathrm{d}m$ is a mass element with volume $\mathrm{d}V$.
   Conservation of mass requires $\dot{M} = 0$ and

   $$\dot{M} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_B \mathrm{d}m\right)$$

   $$\therefore 0 = \int_V \frac{\mathrm{d}}{\mathrm{d}t}(\rho\, \mathrm{d}V)$$

   $$0 = \int_V (\dot{\rho}\, \mathrm{d}V + \rho\, \mathrm{d}\dot{V})$$

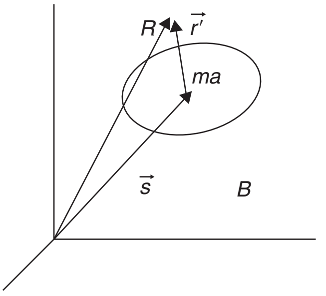
Now $d\dot{V}$ is not generally zero because motion may cause dV to change. Hence

$$\dot{M} \neq \int \dot{\rho}\, dV \quad (\text{Since } \dot{M} = 0)$$

3. Two types of forces in mechanics are contact forces and body forces. Water pressure is a contact force and weight is a body force.

4. Show that the center of mass of a boulder in flight travels along its original trajectory even if it disintegrates.

Solution:



By definition, the mass center position is s and

$$Ms = \int_B r\, dm$$

where $\vec{r} = \vec{s} + \vec{r}'$ and $\vec{r}'$ is position relative to the mass center.

Now $\dot{M}s + M\dot{s} = M\dot{s}$ since $\dot{M} = 0$

$$\therefore M\dot{s} = \int_B v\, dm \quad \text{where } v = \dot{r} \text{ is velocity}$$

Hence $M\ddot{s} = \int_B a\, dm$ where $a = \dot{v}$ is acceleration

But according to Newton's second law

$$F = \dot{P}$$
$$= \frac{d}{dt}\int_B v\, dm$$
$$F = \int_V a\, dm$$
$$\therefore \underline{\underline{F = M\ddot{s}}}$$

which shows that the mass center moves like a mass particle regardless of law the mass of the body is distributed.

5. Given: Static factor of safety as $FS = \dfrac{R}{D}$
where R = resisting force
D = driving force
Show: $FS < 1 \Rightarrow a > 0$.

Solution:

$$F = ma$$

that is $D - R = ma$ since $F = D - R$ (sum – algebraic – of external forces)

Hence $D(1 - FS) = ma$
$\therefore$ if $FS < 1$, $a > 0$

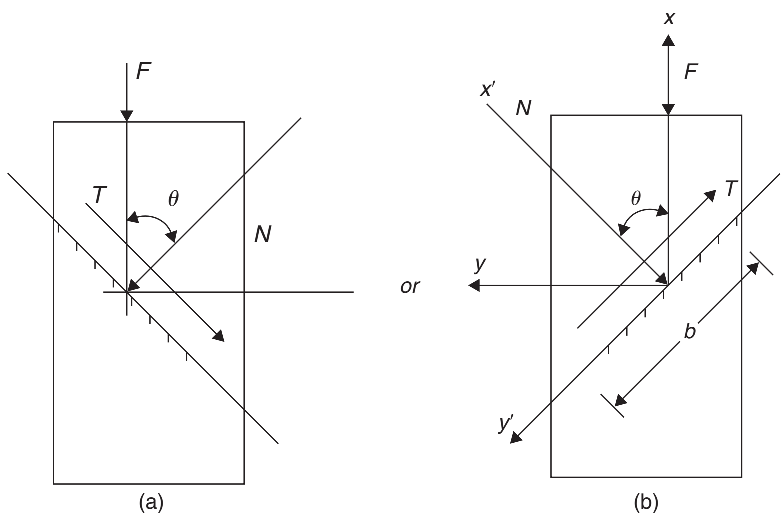
Note: If $FS > 1$, $a < 0$ but when the body considered is at rest, $a = 0$ because of equilibrium (if in motion, the considered body will slow down, decelerate).

6. Given: Sandstone cylinder
 $D =$ diameter
 $h =$ length (height)
 $\frac{L}{D} = 2$
 $F =$ vertical (axial) load

 Find:
 (a) Normal and tangential forces N, T acting on an inclined plane (angle θ to the vertical).
 (b) Average normal and shear stresses σ, τ on the inclined plane in terms of the average vertical stress $\sigma_v = F/A$.

 Note: Area of an ellipse is πab where a, b are major & minor semi-axes.

 Solution:



Considering the second alternative

$$N = F\cos\theta, T = -F\sin\theta$$

where the minus arises from the axes chosen in (b)

$$A' = \pi ab$$
$$a = \frac{D}{2} \text{ by inspection}$$

$$b = \frac{\frac{D}{2}}{\cos\theta}$$

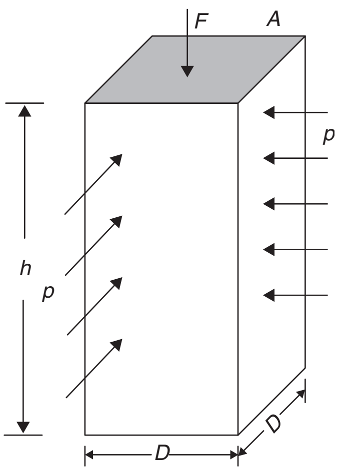
$$\sigma = \frac{N}{A'}, \ \tau = \frac{-F\sin\theta\cos\theta}{\frac{\pi D^2}{4}}$$

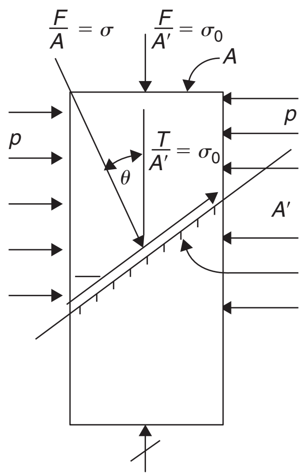
$$\sigma = \frac{F\cos^2\theta}{\frac{\pi D^2}{4}}, \ F = \sigma_v A \ \& \ A = \frac{\pi D^2}{4}$$

$$\sigma = \frac{\sigma_v \frac{\pi D^2}{4}\cos^2\theta}{\frac{\pi D^2}{4}}, \quad \tau = \frac{-\sigma_v \frac{\pi D^2}{4}\sin\theta\cos\theta}{\frac{\pi D^2}{4}}$$

$$\underline{\underline{\sigma = \sigma_v\cos^2\theta}} \ \& \ \underline{\underline{\tau = -\sigma_v\sin\theta\cos\theta}} \qquad \longleftarrow$$

7. Given: Rectangular prism

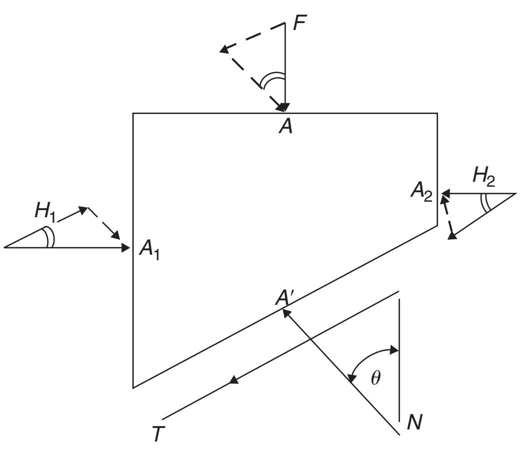




Find: N, T, σ, τ.

Solution:

Free body diagram



$\Sigma F(\text{normal direction}) = 0$
$0 = N\ F\cos\theta - H_1 \sin\theta + H_2 \sin\theta$

$$\begin{aligned}\sigma &= \frac{N}{A'}\\ &= \frac{F}{A'}\cos\theta - \frac{(H_1 - H_2)\sin\theta}{A'}\\ &= \frac{\sigma_v A\cos\theta}{\frac{A}{\cos\theta}} - \frac{p(A_1 - A_2)\sin\theta}{\frac{A}{\cos\theta}},\ A_1 - A_2 = \frac{A'}{\sin\theta}\end{aligned}$$

$$\underline{\underline{\sigma = \sigma_v \cos^2\theta - p\sin^2\theta}}$$

$$\underline{\underline{N = (\sigma_v \cos^2\theta - p\sin^2\theta)\left(\frac{D^2}{\cos\theta}\right)}} \qquad \xleftarrow{\sigma, N}$$

$\Sigma F(\text{tangential direction}) = 0$

$$\begin{aligned}0 &= T + F\sin\theta + H_2\cos\theta - H_1\cos\theta\\ \therefore T &= -F\sin\theta + (H_1 - H_2)\cos\theta\\ \tau &= \frac{T}{A'}\\ &= \frac{-\sigma_v A\sin\theta}{\frac{A}{\cos\theta}} + \frac{p(A_1 - A_2)\cos\theta}{A'} : A_1 - A_2 = \frac{A'}{\sin\theta}\\ &= -\sigma_v \sin\theta\cos\theta + p\cos\theta\sin\theta\end{aligned}$$

$$\underline{\underline{\tau = -(\sigma_v - p)\sin\theta\cos\theta}} \qquad \xleftarrow{\tau}$$

$$T = \tau A'$$

$$\underline{\underline{T = [-(\sigma_v - p)\sin\theta\cos\theta]\left(\frac{D^2}{\cos\theta}\right)}} \qquad \xleftarrow{T}$$

8. The equation

$$Ms = \int_V r\,\mathrm{d}m$$

that defines the mass center of a material body is purely geometrical and is thus a kinematic type of relationship.

9. The equation

$$\dot{\sigma} = \frac{\partial\sigma}{\partial t}$$

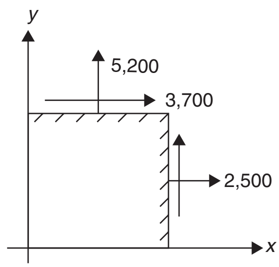
simply defines a stress rate, says nothing about physical or mechanical behavior and thus is a kinematic relation relating to geometry of motion.

10. Darcy's law is a material law because it relates dependent variables (v, h) and thus says something about material behavior. Recall: the independent variables are position and time.

Review of Stress

11. Given: 2D stress state with tension (+)
$\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}700$ and units in psi.

Find: σ_1, σ_3 and direction.



Solution:

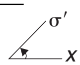
From notes

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{2{,}500+5{,}200}{2} \pm \left[\left(\frac{2{,}500-5{,}200}{2}\right)^2 + (3{,}700)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = 3{,}850 \pm [3{,}939]$$

$$\left.\begin{matrix}\underline{\underline{\sigma_1 = 7{,}789\text{ psi}}}\\ \underline{\underline{\sigma_3 = -89\text{ psi}}}\\ \underline{\underline{\alpha = +55^\circ}}\end{matrix}\right\}$$



$$\tan 2\alpha = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx}-\sigma_{yy})}$$

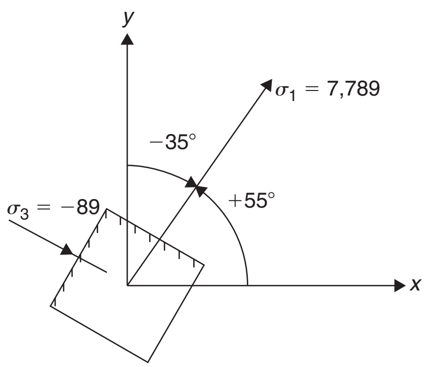
$$= \frac{3{,}700}{\frac{1}{2}(2{,}500-5{,}000)}$$

$$\tan 2\alpha^* = -2.7407$$

$$2\alpha^* = -70$$

$$\alpha^* = -35^\circ$$

$$\text{or}\quad \alpha^* = +55^\circ$$



12. Given: Problem 11 data
$\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}200$ psi
(tension+)
Find: τ_{max}.

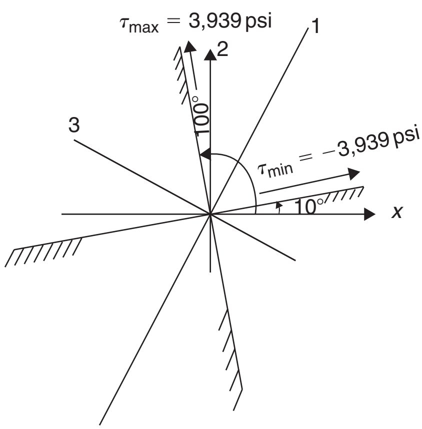
Solution:

From notes

$$\tau_{max} = \left[\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \left[\left(\frac{2{,}500-5{,}200}{2}\right)^2 + (3{,}200)^2\right]^{1/2}$$

$$\underline{\underline{\tau_{max} = 3{,}939\text{ psi}}}$$

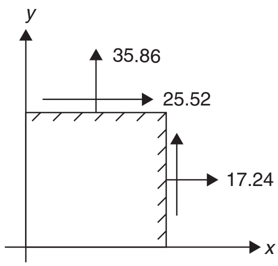


$$
\begin{aligned}
\tan 2\alpha^{**} &= \frac{-\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}{\tau_{xy}} \\
&= \frac{-\left(\frac{2{,}500 - 5{,}200}{2}\right)}{3{,}700} \\
\tan 2\alpha^{**} &= 0.36486 \\
2\alpha^{**} &= 20 \\
\alpha^{**} &= 10^\circ \\
\text{or } \alpha^{**} &= 100^\circ
\end{aligned}
$$

13. Given: 2D stress state with tension (+)
$\sigma_{xx} = 17.24$ Mpa $\quad \sigma_{yy} = 35.86 \quad \tau_{xy} = 25.52$
Find: $\sigma_1, \sigma_3, \alpha$.

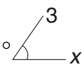
Solution:

Formulas:
$$
\left.\begin{matrix}\sigma_1 \\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}
$$
$$
= \frac{17.74 + 35.86}{2} \pm \left[\left(\frac{17.24 - 35.86}{2}\right)^2 + (25.52)^2\right]^{1/2}
$$

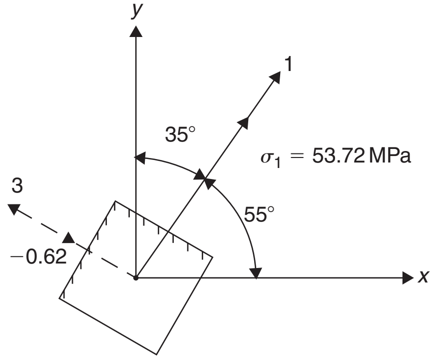


$$
\left.\begin{matrix}\sigma_1 \\ \sigma_3\end{matrix}\right\} = 26.55 \pm [27.17]
$$
$$
\therefore \left.\begin{matrix}\underline{\underline{\sigma_1 = 53.72 \text{ MPa}}} \\ \underline{\underline{\sigma_3 = -0.62 \text{ MPa}}}\end{matrix}\right\} \qquad \xleftarrow{\sigma_1, \sigma_3}
$$

$\underline{\underline{+55^\circ}}$



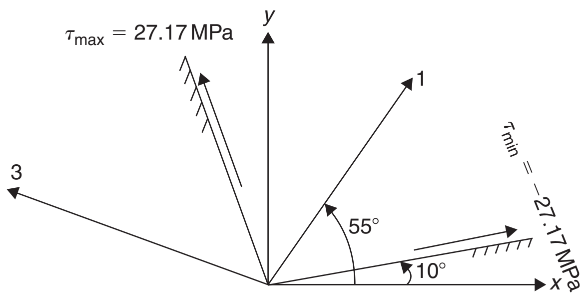
$$
\begin{aligned}
\tan 2\alpha &= \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})} \\
&= \frac{(2)(25.52)}{17.24 - 35.86} \\
\tan 2\alpha &= -2.741 \\
2\alpha &= -70.0^\circ, 110^\circ \\
\alpha &= -35^\circ, +55^\circ
\end{aligned}
$$



14. Given: Problem 13 data:
$\sigma_{xx} = 17.24$ MPa $\quad \sigma_{yy} = 35.86 \quad \tau_{xy} = 25.52$
Find: $\tau_{\max}, \alpha^{**}$.

Formulas

$$\tau_{\max} = \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$
$$= \left[\left(\frac{17.24 - 35.86}{2}\right)^2 + (25.52)^2\right]^{1/2}$$
$$\underline{\underline{\tau_{\max} = 27.17\,\text{MPa}}}$$



$$\tan 2\alpha^{**} = \frac{-\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}{\tau_{xy}}$$
$$= \frac{-\frac{1}{2}(17.24 - 35.86)}{25.52}$$
$$\tan 2\alpha^{**} = 0.3648$$
$$2\alpha^{**} = 20.0°, 200°$$
$$\underline{\underline{\alpha^{**} = 10°, 100°}}$$

15. Given: $\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}700$ psi and
$\tau_{xz} = 0 \quad \tau_{yz} = 0 \quad \sigma_{zz} = 400$ psi
Find: $\sigma_1, \sigma_2, \sigma_3$.

Solution:

Assume tension is positive. The plane that σ_{zz} acts on is shear-free and therefore is a principal plane. Thus, σ_{zz} is a principal stress. Also (from prob. 11) in the xy-plane

$$\sigma_1 = 7{,}789 \ \& \ \sigma_3 = -89\,\text{psi}$$
$$\therefore \underline{\underline{\sigma_1 = 7{,}789, \ \ \sigma_2 = 4{,}000(\sigma_{zz}) \ \& \ \sigma_3 = -89\,\text{psi}}} \qquad \longleftarrow$$

16. Given: Problem 15 data:

Find: The maximum shear stress and orientation of associated plane.

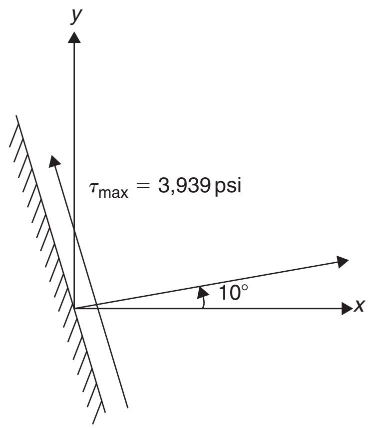
Solution:

Since $\tau_{xz} = \tau_{yz} = 0$, it seems

$\underline{\underline{\tau_{max} = 3{,}939 \text{ psi}}}: \left[= \frac{1}{2}(\sigma_1 - \sigma_3)\right]$

$\underline{\underline{\alpha^{**} = 10°}}$ $\quad \therefore$ must see in x-y view

(from problem 12)



17. Given: $\sigma_{xx} = 17.24$ MPa $\quad \sigma_{yy} = 35.86$ MPa $\quad \sigma_{zz} = 27.59$ MPa $\quad \tau_{xy} = 25.52$
$\tau_{yz} = 0 \quad \tau_{zx} = 0$
Find: $\sigma_1, \sigma_2, \sigma_3$.

Solution:

Assume tension is positive. The plane that σ_{zz} acts on is a shear-free plane ($\tau_{yz} = \tau_{zx} = 0$) and therefore is a principal plane. Thus, σ_{zz} is a principal stress.

From problem 13,

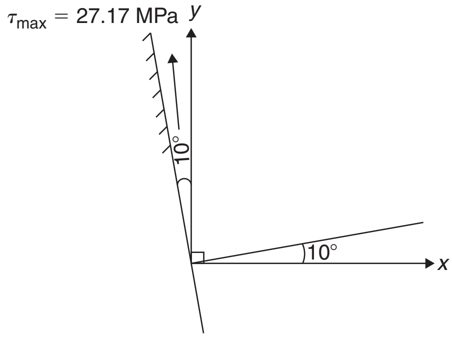
$$\left.\begin{aligned}\sigma_1 &= 53.72 \text{ MPa}\\ \sigma_3 &= -0.62 \text{ MPa}\end{aligned}\right\} \text{ in } x\text{-}y \text{ plane}$$

$$\sigma_2 = \sigma_{zz} = 27.59 \text{ MPa}$$

$$\therefore \underline{\underline{\sigma_1 = 53.72, \sigma_2 = 27.59, \sigma_3 = -0.62 \text{ MPa}}} \qquad \longleftarrow$$

18. Given: Problem 17 data:
Find: Maximum shear stress and orientation

Solution:



Because $\quad \tau_{yz} = \tau_{zx} = 0$

and $\quad \tau_{max} = \dfrac{1}{2}(\sigma_1 - \sigma_3)$

$\underline{\underline{\tau_{max} = 27.17 \text{ MPa}}}$

$\underline{\underline{\alpha^{**} = 10°}} \qquad \longleftarrow$

(from problem14)

19. Given: $\sigma_{xx} = 5{,}200 \quad \sigma_{yy} = 2{,}500 \quad \tau_{xy} = -3{,}700$ psi
with tension(+)
Find: σ_1, σ_3, α.

Solution:

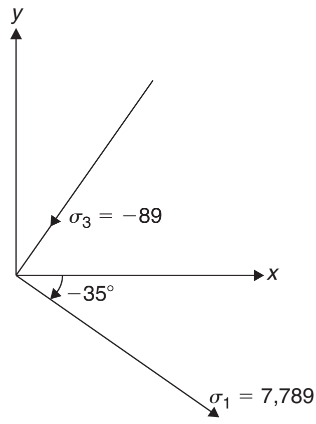
$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \frac{5{,}200+2{,}500}{2} \pm \left[\left(\frac{5{,}200-2{,}500}{2}\right)^2 + (-3{,}700)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = 3{,}850 \pm 3{,}939$$

$$\therefore\ \underline{\underline{\sigma_1 = 7{,}789\text{ psi}}}$$
$$\underline{\underline{\sigma_3 = -89\text{ psi}}}$$

$$\tan 2\alpha^* = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx}-\sigma_{yy})}$$
$$= \frac{-3{,}700}{\frac{1}{2}(5{,}200-2{,}500)}$$
$$\tan 2\alpha^* = -2.7407$$
$$2\alpha^* = -70^\circ$$
$$\alpha^* = -35^\circ$$
$$\text{or } \underline{\underline{\alpha^* = +55^\circ}}$$



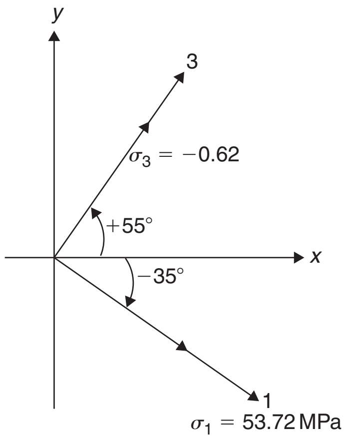
20. Given: 2D stress state
$\sigma_{xx} = 35.86 \quad \sigma_{yy} = 17.24 \quad \tau_{xy} = -25.52$
in MPa (tension +)
Find: σ_1, σ_3, α, sketch.

Solution:

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \frac{35.86+17.24}{2} \pm \left[\left(\frac{35.86-17.24}{2}\right)^2 + (-25.52)^2\right]^{1/2}$$

$$= 26.53 \pm 27.17$$



$$\left.\begin{array}{l}\underline{\underline{\sigma_1 = 53.72}}\\ \underline{\underline{\sigma_3 = -0.62}}\end{array}\right\} \qquad \longleftarrow \sigma_1, \sigma_3 \text{ MPa}$$

$$\begin{aligned}\tan 2\alpha^* &= \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xy} - \sigma_{yy})}\\ &= \frac{-25.52}{\frac{1}{2}(35.86 - 17.24)}\\ \tan 2\alpha^* &= -2.7411\\ 2\alpha^* &= -70°, 110°\\ \underline{\underline{\alpha^*}} &\underline{\underline{= -35°,\ +55°}}\end{aligned}$$

21. Given: $\sigma_{xx} = 3{,}000$ $\sigma_{yy} = 2{,}000$ $\sigma_{zz} = 4{,}000$ $\tau_{xy} = 0$ $\tau_{zx} = 0$ $\tau_{yz} = 2{,}500$ where units are psi & compression (+)

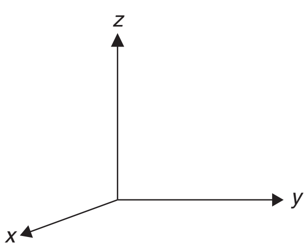
Find: σ_1, σ_2, σ_3 & directions.

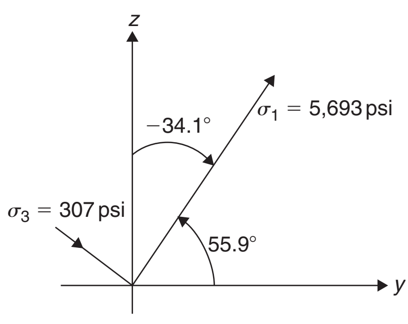
Solution:

By inspection, the *x*-direction shear stresses are zero and so the *yz*-plane is a principal plane (*x*-direction is principal). In the *yz* plane:

$$\left.\begin{array}{l}\sigma_1\\ \sigma_3\end{array}\right\} = \frac{\sigma_{yy} + \sigma_{zz}}{2} \pm \left[\left(\frac{\sigma_{yy} - \sigma_{zz}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$\left.\begin{array}{l}\sigma_1\\ \sigma_3\end{array}\right\} = 3{,}000 \pm 2{,}693$$





SKETCH

$$\therefore \underline{\underline{\sigma_1 = 5{,}693 \text{ psi}}}$$

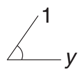
$$\underline{\underline{\sigma_3 = 307 \text{ psi}}} \qquad \longleftarrow \sigma_1, \sigma_3$$

$$\sigma_{xx} = 3{,}000 \quad \therefore \sigma_{xx} = \sigma_2$$

$$\underline{\underline{\sigma_2 = 3{,}000 \text{ psi}}} \qquad \longleftarrow \sigma_2$$

(σ_1 is inclined 55.9° to the y-axis as shown in sketch)

$$\begin{aligned}
\tan 2\alpha^* &= \frac{\tau_{yz}}{\frac{1}{2}(\sigma_{yy} - \sigma_{zz})} \\
&= \frac{2{,}500}{\frac{1}{2}(2{,}000 - 4{,}000)} \\
\tan 2\alpha^* &= -2.5 \\
2\alpha^* &= -68.2^\circ \\
\alpha^* &= -34.1^\circ \\
\text{or} \qquad \alpha^* &= +55.9^\circ
\end{aligned}$$

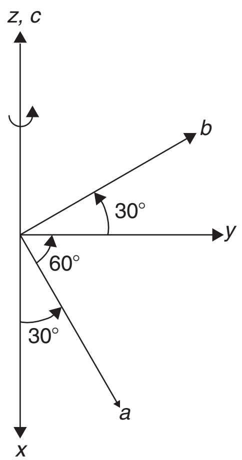


22. Given: $\sigma_{xx} = 3{,}000$ $\sigma_{yy} = 2{,}000$ $\sigma_{zz} = 4{,}000$ $\tau_{xy} = 0$ $\tau_{zx} = 0$ $\tau_{yz} = 2{,}500$ psi with compression (+)

Find: σ_{aa}, σ_{bb}, σ_{cc}, τ_{ab}, τ_{bc}, τ_{ca}, where (abc) is rotated about the z-axis 30° CCW.

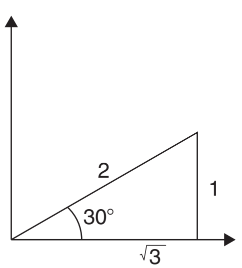
Solution:

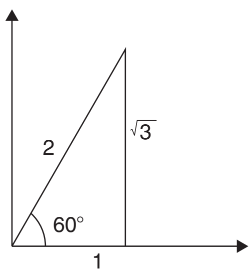
(1) Establish table of direction cosines
(2) Apply Formula $\sigma(abc) = R\sigma(xyz)R^t$

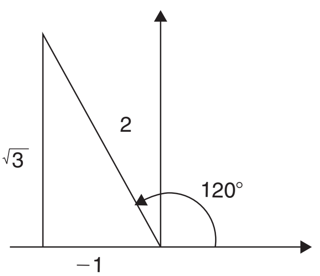


NEW \ OLD	x	y	z
a	cos 30°	cos 60°	cos 90°
b	cos 120°	cos 30°	cos 90°
c	cos 90°	cos 90°	cos 0°

$$\therefore [R] = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$







(more generally for z-axis rotation)
$c = \cos\theta \qquad s = \sin\theta \qquad \theta = \text{rotation angle}$

$$\sigma(abc) = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (c\sigma_{xx} + s\tau_{xy}) & (-s\sigma_{xx} + c\tau_{xy}) & \tau_{xz} \\ (c\tau_{xy} + s\sigma_{yy}) & (-s\tau_{xy} + c\sigma_{yy}) & \tau_{yz} \\ (c\tau_{xz} + s\tau_{yz}) & (-s\tau_{xz} + c\tau_{yz}) & \sigma_{zz} \end{bmatrix}$$

$$= \begin{bmatrix} (c^2\sigma_{xx} + s^2\sigma_{yy} + 2sc\tau_{xy}) & [-sc\sigma_{xx} + sc\sigma_{yy} + (c^2 - s^2)\tau_{xy}] & (c\tau_{xz} + s\tau_{yz}) \\ [-sc\sigma_{xx} + sc\sigma_{yy} + (c^2 - s^2)\tau_{xy}] & (+s^2\sigma_{xx} + c^2\sigma_{yy} - 2cs\tau_{xy}) & (-s\tau_{xz} + c\tau_{yz}) \\ (c\tau_{xz} + s\tau_{yz}) & (-s\tau_{xy} + c\tau_{yz}) & \sigma_{zz} \end{bmatrix}$$

$$\text{since:} \quad c^2 + s^2 = 1 \quad \& \quad c^2 = \frac{1}{2}(1 + c2)$$

$$c^2 - s^2 = c2 \quad \& \quad s^2 = \frac{1}{2}(1 - c2)$$

$$\therefore \quad s2 = 2sc$$

$$\text{2D part} \begin{cases} \sigma_{aa} = \dfrac{\sigma_{xx} + \sigma_{yy}}{2} + \left(\dfrac{\sigma_{xx} - \sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \\ \sigma_{bb} = \dfrac{\sigma_{xx} + \sigma_{yy}}{2} - \left(\dfrac{\sigma_{xx} - \sigma_{yy}}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \\ \underline{\underline{\sigma_{ab} = \tau_{ab} = \tau_{ba} = -\left(\dfrac{\sigma_{xx} - \sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}} \end{cases}$$

$$\left.\begin{aligned} &\sigma_{cc} = \sigma_{zz} \\ &\tau_{bc} = \tau_{cb} = -\tau_{xz}\sin\theta + \tau_{yz}\cos\theta \\ &\underline{\underline{\tau_{ac} = \tau_{ca} = \tau_{xz}\cos\theta + \tau_{yz}\sin\theta}} \end{aligned}\right\} \text{3D part}$$

Plug in: $\theta = 30^\circ$ etc.

$$\left.\begin{matrix} \sigma_{aa} \\ \sigma_{bb} \end{matrix}\right\} = \frac{3{,}000 + 2{,}000}{2} \pm \frac{3{,}000 - 2{,}000}{2} \cos\ 60^\circ \pm 0.\sin 60^\circ$$

$$\underline{\left.\begin{matrix} \sigma_{aa} \\ \sigma_{bb} \end{matrix}\right\} = 2{,}500 \pm 250}$$

$$\underline{\sigma_{cc} = 4{,}000}$$

$$\tau_{ab} = -\left(\frac{3{,}000 - 2{,}000}{2}\right)\sin 60^\circ + 0.\cos 60^\circ$$

$$\underline{\tau_{ab} = -433}$$

$$\tau_{bc} = -0.\sin 30^\circ + 2{,}500\cos 30^\circ$$

$$\underline{\tau_{bc} = 2{,}165}$$

$$\tau_{ac} = 0.\cos(30^\circ) + 2{,}500\sin(30^\circ)$$

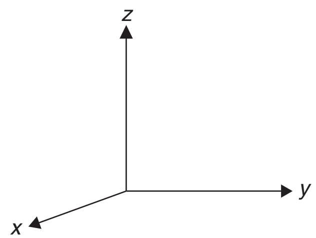
$$\underline{\tau_{ac} = 1{,}250}$$

$$\left.\begin{aligned}\sigma_{aa} &= 2{,}750\text{ psi}\\ \sigma_{bb} &= 2{,}250\text{ psi}\\ \sigma_{cc} &= 4{,}000\text{ psi}\\ \tau_{ab} &= -433\text{ psi}\\ \tau_{bc} &= 2{,}165\text{ psi}\\ \tau_{ca} &= 1{,}250\text{ psi}\end{aligned}\right\} \qquad \longleftarrow \sigma(abc)$$

23. Given: $\sigma_{xx} = 20.69$ $\sigma_{yy} = 13.79$ $\tau_{xy} = 0$ $\sigma_{zz} = 27.59$ $\tau_{zx} = 0$ $\tau_{yz} = 17.24$ in MPa and compression is positive.

Find: σ_1, σ_2, σ_3 and orientations, sketch.

Solution:



The x-direction shear stresses, τ_{xy} and τ_{xz} are zero, $\therefore$ σ_{xx} is a principal stress, x- is a principal direction and the yz-plane is a principal plane.

In the yz-plane

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{yy} + \sigma_{zz}}{2} \pm \left[\left(\frac{\sigma_{yy} - \sigma_{zz}}{2}\right)^2 + (\tau_{yz})^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{13.79 + 27.59}{2} \pm \left[\left(\frac{13.79 - 27.59}{2}\right)^2 + (17.24)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = 20.69 \pm 18.57$$

$\sigma_1 = 39.26$ Mpa

$\sigma_3 = 2.12$ Mpa

$\sigma_2 = \sigma_{xx} = 20.69$ Mpa

$\alpha^* = +55°$,

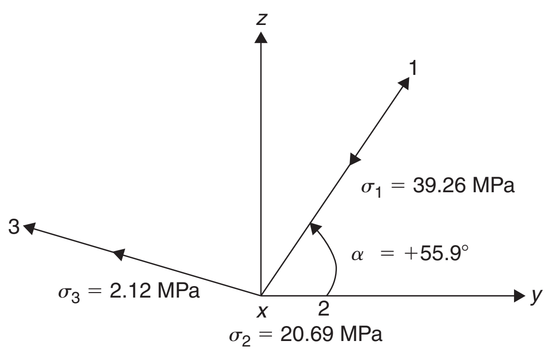
$$\tan 2\alpha^* = \frac{\tau_{yz}}{\frac{1}{2}(\sigma_{yy} - \sigma_{zz})} = \frac{17.24}{\frac{1}{2}(13.79 - 27.59)}$$

$$\tan 2\alpha^* = -2.499$$

$$2\alpha^* = -68.2°, 111.8°$$

$$\alpha^* = -34.1°, 55.9°$$

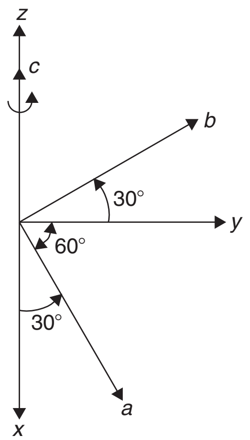
$$\alpha^* = 55.9° \qquad \longleftarrow$$



24. Given: Stress state in problem 23,
Find: Stress state ($\sigma_{aa} \ldots \tau_{ca}$) relative to ($abc$) rotated 30° CCW about z.

Solution:

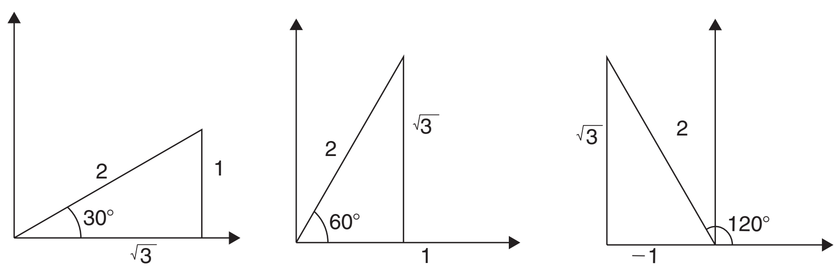
(1) Establish axes and direction cosines.



NEW \ OLD	x	y	z
a	cos 30°	cos 60°	cos 90°
b	cos 120°	cos 30°	cos 90°
c	cos 90°	cos 90°	cos 0°

∴ rotation matric [R] is

$$[R] = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(2) apply formula $\sigma(abc) = [R]\ \sigma(xyz)[R]^t$ (more generally for z-axis rotation) $c = \cos\theta$, $s = \sin\theta$, $\theta =$ rotation angle

$$\sigma(abc) = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (c\sigma_{xx} + s\tau_{xy}) & (-s\sigma_{xx} + c\tau_{xy}) & \tau_{xz} \\ (c\tau_{xy} + s\sigma_{yy}) & (-s\tau_{xy} + c\sigma_{yy}) & \tau_{yz} \\ (c\tau_{xz} + s\tau_{yz}) & (-s\tau_{xz} + c\tau_{yz}) & \sigma_{zz} \end{bmatrix}$$

$$= \begin{bmatrix} (c^2\sigma_{xx} + s^2\sigma_{yy} + 2sc\tau_{xy}) & [-sc\sigma_{xx} + sc\sigma_{yy} + (c^2 - s^2)\tau_{xy}] & (c\tau_{xz} + s\tau_{yz}) \\ [-sc\sigma_{xx} + sc\sigma_{yy} + (c^2 - s^2)\tau_{xy}] & (+s^2\sigma_{xx} + c^2\sigma_{yy} - 2cs\tau_{xy}) & (-s\tau_{xz} + c\tau_{yz}) \\ (c\tau_{xz} + s\tau_{yz}) & (-s\tau_{xy} + c\tau_{yz}) & \sigma_{zz} \end{bmatrix}$$

Since: $c^2+s^2=1$ & $c^2-s^2=c2$, $c^2=\frac{1}{2}(1+c2)$, $s^2=\frac{1}{2}(1-c2)$, & $s2=2sc$

$$\text{2D part}\left\{\begin{aligned}
\sigma_{aa} &= \frac{\sigma_{xx}+\sigma_{yy}}{2}+\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta+\tau_{xy}\sin 2\theta \\
\sigma_{bb} &= \frac{\sigma_{xx}+\sigma_{yy}}{2}-\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta-\tau_{xy}\sin 2\theta \\
\sigma_{ab} &= \tau_{ab}=\tau_{ba}=-\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta+\tau_{xy}\cos 2\theta
\end{aligned}\right.$$

$$\left.\begin{aligned}
\sigma_{cc} &= \sigma_{zz} \\
\tau_{bc} &= \tau_{cb}=-\tau_{xz}\sin\theta+\tau_{yz}\cos\theta \\
\tau_{ac} &= \tau_{ca}=\tau_{xz}\cos\theta+\tau_{yz}\sin\theta
\end{aligned}\right\}\text{3D part}$$

Plug in: $\theta=30^\circ$ etc.

$$\left.\begin{matrix}\sigma_{aa}\\ \sigma_{bb}\end{matrix}\right\}=\frac{20.69+13.79}{2}\pm\frac{20.69-13.79}{2}\cos 60^\circ\pm 0.\sin 60^\circ$$

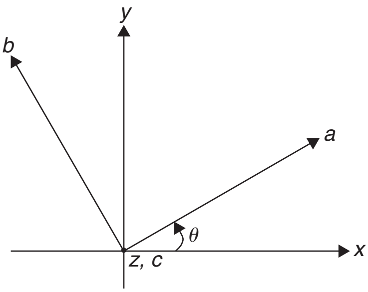
$$\left.\begin{matrix}\sigma_{aa}\\ \sigma_{bb}\end{matrix}\right\}=17.24\pm 1.73$$

$$\underline{\sigma_{aa}=18.97\,\text{Mpa}}$$
$$\underline{\sigma_{bb}=15.51}$$
$$\underline{\sigma_{cc}=27.59}$$
$$\tau_{ab}=-\left(\frac{20.69-13.79}{2}\right)\sin 60^\circ+0.\cos 60^\circ$$
$$\underline{\tau_{ab}=-2.99\,\text{Mpa}}$$
$$\tau_{bc}=-0.\sin 30^\circ+17.24\cos 30^\circ$$
$$\underline{\tau_{bc}=14.93\,\text{Mpa}}$$
$$\tau_{ac}=0.\cos 30^\circ+17.24\sin 30^\circ$$
$$\underline{\tau_{ac}=8.62\,\text{Mpa}}$$

Summary:

$$\left.\begin{aligned}
\sigma_{aa} &= 18.97\,\text{Mpa} \\
\sigma_{bb} &= 15.51\,\text{Mpa} \\
\sigma_{cc} &= 27.59\,\text{Mpa} \\
\tau_{ab} &= -2.99\,\text{Mpa} \\
\tau_{bc} &= 14.93\,\text{Mpa} \\
\tau_{ca} &= 8.62\,\text{Mpa}
\end{aligned}\right\}\qquad \xleftarrow{\sigma(abc)}$$

25. Given: 2D rotation about an axis



Find: Invariate $\dfrac{\sigma_{xx}+\sigma_{yy}}{2}=\sigma_m$

Solution:

$$\left.\begin{matrix}\sigma_{aa}\\ \sigma_{bb}\end{matrix}\right\}=\frac{\sigma_{xx}+\sigma_{yy}}{2}\pm\frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta\pm\tau_{xy}\sin 2\theta$$

$$\therefore\ \sigma_m=\frac{\sigma_{xx}+\sigma_{yy}}{2}=\frac{\sigma_{aa}+\sigma_{bb}}{2}$$

and since orientation of a, b is arbitrary σ_m is invariant w.r.t rotation (2D).

26. Given: 2D axis rotation

Find: $\tau_m=\left[\frac{1}{2}(\sigma_{xx}-\sigma_{yy})^2+\tau_{xy}^2\right]^{1/2}$ invariant

Solution:

From rotation formula.

(1) $\dfrac{\sigma_{aa}-\sigma_{bb}}{2}=\left(\dfrac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta+\tau_{xy}\sin 2\theta$

(2) $\tau_{ab}=-\left(\dfrac{\sigma_{xx}-\sigma_{yy}}{z}\right)\sin 2\theta+\tau_{xy}\cos 2\theta$

Square (1) and (2) then add to obtain

(3) $\left(\dfrac{\sigma_{aa}-\sigma_{bb}}{2}\right)^2+(\tau_{ab})^2=\left(\dfrac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+(\tau_{xy})^2$

Using $\cos^2 2\theta+\sin^2 2\theta=1$ take square root of (3) to obtain
$\tau_m(a,b)=\tau_m(x,y)$ is invariant

Note: $\tau_m=\frac{1}{2}(\sigma_1-\sigma_3)=+\left[\dfrac{(\sigma_{xx}-\sigma_{yy})^2}{2}+(\tau_{xy})^2\right]^{1/2}$

An alternative is to look at the 2D stress state as a 2×2 array $\begin{bmatrix}\sigma_{xx} & \tau_{xy}\\ \tau_{xy} & \sigma_{yy}\end{bmatrix}$

Form the characteristic equation then look at principal invariant combinations to obtain results. [$\lambda^2-\lambda(\sigma_{xx}+\sigma_{yy})+(\sigma_{xx}\sigma_{yy}-\tau_{xy}^2)=0$ is the characteristic equation].

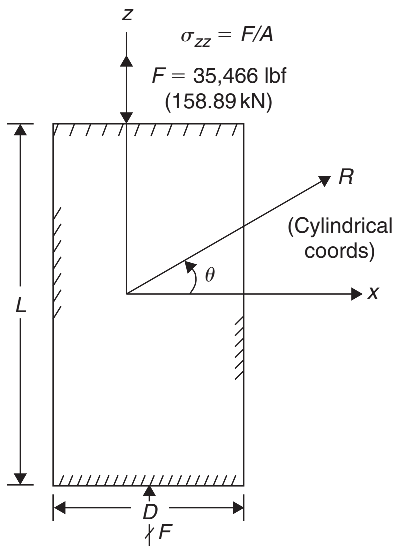
27. Given: NX-core $2\frac{1}{8}$ in. (5.40 cm) diameter, $L/D=2.0$ axial load $F=35{,}466$ lbf (158.89 kN)

Find:

(1) Stress state relative to $(r\,\theta z)$
(2) Stress relative to (abc) rotated to 60° dip.

(3) directly calculate σ_{cc} and compare with rotation equation result
(4) directly calculate τ_{ac} and compare.

Solution:



$$A = \frac{\pi D^2}{4}$$

$$= \frac{(\pi)(2.125)^2}{4} \quad \left[\frac{\pi(5.40)^2}{4}\right]$$

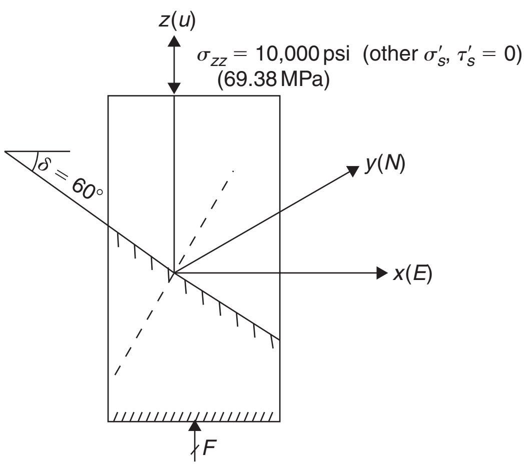
$$A = 3.5466\,\text{in.}^2 \quad (22.90\,\text{cm}^2)$$

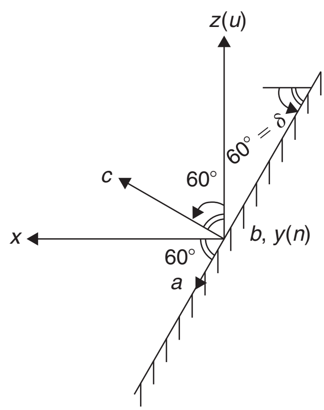
$$\sigma_{zz} = \frac{35{,}466}{3.5466} \quad \left(\frac{158.89}{2.290}\right)$$

$$\underline{\underline{\sigma_{zz} = 10{,}000\,\text{psi}}} \quad \underline{\underline{(69.38\,\text{Mpa})}}$$

$$\left.\begin{array}{l}\underline{\underline{\sigma_{rr} = \sigma_{\theta\theta} = 0}}\\ \underline{\underline{\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = 0}}\end{array}\right\} \leftarrow \text{(1)} \quad \sigma(r\theta z)$$

by inspection, vertical loading.





OLD / NEW	x	y	z
a	cos 60°	cos 90°	cos 150°
b	cos 90°	cos 0°	cos 90°
c	cos 30°	cos 90°	cos 60°

$$[R] = \begin{bmatrix} c & 0 & -s \\ 0 & 1 & 0 \\ s & 0 & c \end{bmatrix} \qquad \begin{array}{l} c = \cos 60^\circ = \dfrac{1}{2} \\[2ex] s = \sin 60^\circ = \dfrac{\sqrt{3}}{2} \end{array}$$

$$[R] = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$\sigma(xyz) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

$$\sigma(abc) = R\sigma(xyz)R^t$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 10^4 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -\frac{10^4\sqrt{3}}{2} \\ 0 & 0 & 0 \\ 0 & 0 & \frac{10^4}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$\sigma(abc) = \begin{bmatrix} +\frac{10^4 \cdot 3}{4} & 0 & -\frac{10^4\sqrt{3}}{4} \\ 0 & 0 & 0 \\ -\frac{10^4\sqrt{3}}{4} & 0 & \frac{10^4}{4} \end{bmatrix} \begin{bmatrix} \frac{3}{4}69.38 & 0 & -\frac{\sqrt{3}}{4}69.38 \\ 0 & 0 & 0 \\ -\frac{\sqrt{3}}{4}69.38 & 0 & \frac{69.38}{4} \end{bmatrix}$$

$$\left.\begin{aligned} \sigma_{aa} &= \left(\frac{3}{4}\right) 10^4 \text{ psi} \\ \sigma_{bb} &= 0 \text{ psi} \\ \sigma_{cc} &= \frac{10^4}{4} \text{ psi} \\ \tau_{ab} &= 0 \text{ psi} \\ \tau_{ac} &= \left(-\frac{\sqrt{3}}{4}\right) 10^4 \text{ psi} \\ \tau_{bc} &= 0 \text{ psi} \end{aligned}\right\} \left\{\begin{matrix} \left(\frac{3}{4}\right) 69.38 \text{ MPa} \\ 0 \\ \left(\frac{1}{4}\right) 69.38 \text{ MPa} \\ 0 \\ \left(-\frac{\sqrt{3}}{4}\right) 69.38 \text{ MPa} \\ 0 \end{matrix}\right\} \xleftarrow{\sigma(abc)} \quad \text{check: } \left.\begin{matrix} z \to c \\ x \to a \\ y \to b \end{matrix}\right\}$$

$$\left.\begin{matrix} \sigma_{cc} \\ \sigma_{aa} \end{matrix}\right\} = \frac{\sigma_{zz} + \sigma_{xx}}{2} \pm \frac{\sigma_{zz} - \sigma_{xy}}{2} \cos 2\theta \pm \tau_{xz} \sin 2\theta$$

$$\tau_{ca} = -\left(\frac{\sigma_{zz} - \sigma_{xx}}{2}\right) \sin 2\theta + \tau_{xz} \cos 2\theta$$

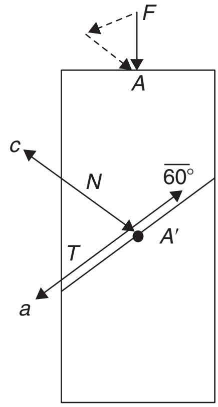
$$\left.\begin{matrix}\sigma_{cc}\\ \sigma_{aa}\end{matrix}\right\} = \frac{10{,}000}{2} \pm \frac{10{,}000}{2}\left(-\frac{1}{2}\right) \pm 0$$

$$\left.\begin{matrix}\sigma_{cc}\\ \sigma_{aa}\end{matrix}\right\} = 10{,}000\left(\frac{1}{2} \pm \frac{1}{4}\right): \text{checks}$$

$$\tau_{ac} = \frac{-10{,}000}{2}\left(\frac{\sqrt{3}}{2}\right): \text{checks}$$

$$\sigma_{bb} = \sigma_{yy} = 0: \text{checks}$$

From Problem 7.



$$\begin{aligned}
\tau_{ab} &= \tau_{bc} = 0\\
\sigma_z &= F/A\\
N &= F\cos 60^\circ\\
T &= -F\sin 60^\circ\\
A &= \frac{\pi D^2}{4}\\
A' &= \frac{A}{\cos\delta}\\
\delta &= 60^\circ\\
\sigma_{cc} &= \sigma\\
\sigma &= N/A'\\
&= \frac{\sigma_z A\cos^2\delta}{A}\\
\sigma &= 10^4\left(\frac{1}{4}\right)\text{psi} \quad \therefore \quad \underline{\underline{\sigma_{cc} = \left(\frac{1}{4}\right)10^4\text{psi} \quad \text{checks}}} \qquad \leftarrow (3)\\
&= 69.38\left(\frac{1}{4}\right)\text{MPa} \quad \underline{\underline{\sigma_{cc} = \left(\frac{1}{4}\right)69.38\text{ MPa}}}
\end{aligned}$$

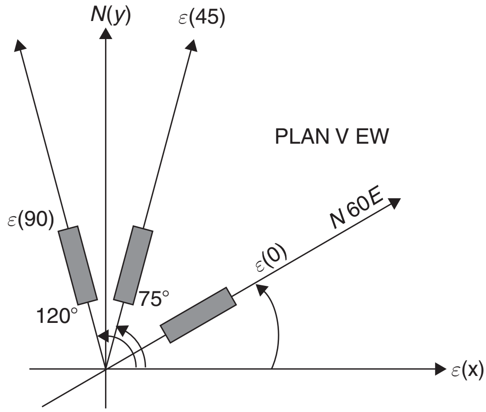
(4)

$$\begin{aligned}
\tau_{ac} &= \tau\\
\tau &= \frac{T}{A'}\\
&= \frac{-\sigma_z A\sin 60^\circ\cos 60^\circ}{A}\\
\tau &= -10^4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right), -69.38\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)\\
\therefore \tau_{ac} &= \underline{\underline{-\left(\frac{\sqrt{3}}{4}\right)10^4\text{psi} \quad \text{checks}}} \qquad \leftarrow (4)\\
&= \underline{\underline{-\left(\frac{\sqrt{3}}{4}\right)69.38\text{ MPa}}}
\end{aligned}$$

Review of Strain and Elasticity

28. Given: Strain rosette data, quarry floor
$\varepsilon(0) = -1{,}480\,\mu\text{in/in.}$ $\varepsilon(45) = -300\,\mu\text{in/in.}$ $\varepsilon(90) = -2{,}760\,\mu\text{in/in.}$
$E = 12.7(10^6)$ psi $v = 0.27$
tension (+)

Find:
(1) $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}$ μin/in.
(2) $\sigma_{zz}, \tau_{yz}, \tau_{yx}$ (psi)
(3) $\sigma_{xx}, \sigma_{yy}, \tau_{xy}$ (psi)
(4) ε_{zz} (μin/in.)
(5) $\varepsilon_{yz}, \varepsilon_{zx}$
(6) $\sigma_1, \sigma_2, \sigma_3$ (directions)
(7) $\sigma_1, \sigma_2, \sigma_3$ (magnitudes)
(8) $\varepsilon_1, \varepsilon_2, \varepsilon_3$ (magnitudes)
(9) $\varepsilon_1, \varepsilon_2, \varepsilon_3$ (directions)
(10) ε_v (volume change/unit) if relieved by overcovering.



Solution:

Equations of transformation – strain
Alt.1

$$\varepsilon_a = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\cos 2\theta_a + \varepsilon_{xy}\sin 2\theta_a$$
$$\varepsilon_b = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2}\cos 2\theta_b + \varepsilon_{xy}\sin 2\theta_b$$
$$\varepsilon_c = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2}\cos 2\theta_c + \varepsilon_{xy}\sin 2\theta_c$$
$$\theta_a = 30^\circ \quad \theta_b = 75^\circ \quad \theta_c = 120^\circ$$

Alt.2 Find relative to (*abc*) then rotate to *xyz*

$$\varepsilon(0) = -1{,}480\,\mu\text{in/in.} = \varepsilon_a$$
$$\varepsilon(45) = -300\,\mu\text{in/in.} = \frac{\varepsilon_a + \varepsilon_b}{2} + \varepsilon_{ab}$$
$$\varepsilon(90) = -2{,}760\,\mu\text{in/in.} = \varepsilon_b$$

$$\therefore \varepsilon_{ab} = \varepsilon(45) - \frac{(\varepsilon_a + \varepsilon_b)}{2}$$
$$= -300 - \frac{1}{2}(-1{,}480 - 2{,}760)$$
$$\underline{\underline{\varepsilon_{ab} = +1{,}820\,\mu\text{in/in.}}}$$

(transformation – rotation of axes to *x*-*y* from *a*-*b*)

$$\left.\begin{matrix}\varepsilon_{xy}\\ \varepsilon_{yy}\end{matrix}\right\} = \frac{\varepsilon_a+\varepsilon_b}{2} \pm \frac{\varepsilon_a-\varepsilon_b}{2}\cos[2(-30)] \pm \varepsilon_{ab}\sin[2(-30)]$$

$$= (-2{,}120) \pm (640)\left(\frac{1}{2}\right) \pm (1{,}820)\left(\frac{-\sqrt{3}}{2}\right)$$

$$\left.\begin{matrix}\varepsilon_{xy}\\ \varepsilon_{yy}\end{matrix}\right\} = -2{,}120 \pm 320 \mp 1{,}576$$

$$\left.\begin{matrix}\varepsilon_{xy}\\ \varepsilon_{yy}\end{matrix}\right\} = \begin{matrix}-3,376\ \mu\text{in/in.}\\ -864\ \mu\text{in/in.}\end{matrix}$$

$$\varepsilon_{xy} = -\left(\frac{\varepsilon_a-\varepsilon_b}{2}\right)\sin[2(-30)] + \varepsilon_{ab}\cos[2(-30)]$$

$$= -640\left(-\frac{\sqrt{3}}{2}\right) + 1{,}820\left(\frac{1}{2}\right)$$

$$\varepsilon_{xy} = 1{,}464\ \text{min/in.}$$

$$\begin{aligned}\varepsilon_{xx} &= -3{,}376\\ \varepsilon_{yy} &= -864\ \mu\text{in/in.}\\ \varepsilon_{xy} &= +1{,}464\\ \varepsilon_{zz} &= +1{,}568\end{aligned}$$

← (1)

By inspection:

$$\sigma_{zz} = \tau_{zx} = \tau_{zy} = 0$$

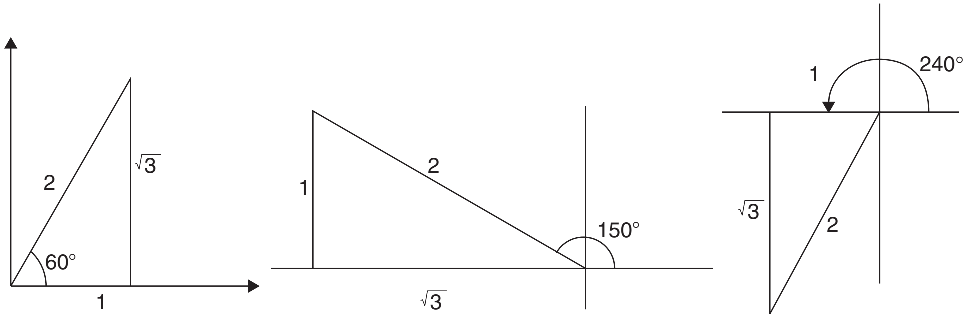
(stress free surface)

← (2)

Alt.1

$$\begin{aligned}\varepsilon_a &= S + D\cos 2(30°) + T\sin 2(30°) \quad S = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}\\ \varepsilon_b &= S + D\cos 2(75°) + T\sin 2(75°) \quad D = \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\\ \varepsilon_c &= S + D\cos 2(120°) + T\sin 2(120°) \quad T = \varepsilon_{xy}\end{aligned}$$

$$\{\varepsilon\} = \begin{bmatrix}1 & \frac{1}{2} & \frac{\sqrt{3}}{2}\\ 1 & \frac{-\sqrt{3}}{2} & \frac{1}{2}\\ 1 & \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\begin{Bmatrix}S\\ D\\ T\end{Bmatrix}$$



$$\begin{vmatrix} 1 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & \frac{-\sqrt{3}}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{-\sqrt{3}}{2} \end{vmatrix} = (1)\left(\frac{3}{4}+\frac{1}{4}\right) - (1)\left(-\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right) + (1)\left(\frac{1}{4}+\frac{3}{4}\right)$$

$$\therefore \Delta = 2$$

$$S = \begin{vmatrix} \varepsilon_a & S_a & D_a \\ \varepsilon_b & S_b & D_b \\ \varepsilon_c & S_c & D_c \end{vmatrix} \Big/ \Delta$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} \varepsilon_a & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \varepsilon_b & \frac{-\sqrt{3}}{2} & \frac{1}{2} \\ \varepsilon_c & \frac{-1}{2} & \frac{-\sqrt{3}}{2} \end{vmatrix}$$

$$S = \frac{\varepsilon_a + \varepsilon_c}{2}$$

$$S = \frac{-1{,}480 - 2{,}760}{2}$$

$$S = -2{,}120\ \mu\text{in/in.}$$

$$D = \left(\frac{1}{2}\right) \begin{vmatrix} 1 & \varepsilon_a & \frac{\sqrt{3}}{2} \\ 1 & \varepsilon_b & \frac{1}{2} \\ 1 & \varepsilon_c & -\frac{\sqrt{3}}{2} \end{vmatrix}$$

$$= \left(\frac{1}{2}\right)\left[-\varepsilon_a\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}\right) + \varepsilon_b\left(-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right) - \varepsilon_c\left(+\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\right]$$

$$D = \frac{1}{4}[-1{,}480(\sqrt{3}+1) + 300(2\sqrt{3}) + 2{,}760(1-\sqrt{3})]$$
$$D = \frac{1}{4}[-4{,}043 + 1{,}039 - 2{,}020]$$
$$\underline{\underline{D = -1{,}256\ \mu\text{in/in.}}}$$

$$\varepsilon_{xx} = \mathrm{S} + \mathrm{D}$$
$$= -2{,}120 - 1{,}256$$
$$\underline{\underline{\varepsilon_{xx} = 3{,}376\ \mu\text{in/in.}}}$$

$$\varepsilon_{yy} = S - D$$
$$\varepsilon_{yy} = -2{,}120 + 1{,}256$$
$$\underline{\underline{\varepsilon_{yy} = -864\ \mu\text{in/in.}}}$$

$$T = \frac{1}{2}\begin{vmatrix} 1 & \frac{1}{2} & \varepsilon_a \\ 1 & -\frac{\sqrt{3}}{2} & \varepsilon_b \\ 1 & -\frac{1}{2} & \varepsilon_c \end{vmatrix}$$
$$= \left(\frac{1}{2}\right)\left[\varepsilon_a\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right) - \varepsilon_b\left(-\frac{1}{2}-\frac{1}{2}\right) + \varepsilon_c\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\right]$$
$$= \frac{1}{4}[-1{,}480(\sqrt{3}-1) - (-300)(-2) - 2{,}760(-1-\sqrt{3})]$$
$$= \frac{1}{4}[-1{,}083 - 600 + 7{,}541]$$
$$\underline{\underline{T = +1{,}464\ \mu\text{in/in.}}}$$

$$\varepsilon_{xy} = 1{,}464\ \mu\text{in/in.}$$
$$[\gamma_{xy} = 2{,}928\ \mu\text{in/in.}]$$

$$\varepsilon_{xx} = -3{,}376\ \mu\text{in/in.}$$
$$\varepsilon_{yy} = -864\ \mu\text{in/in.} \qquad \longleftarrow (1)$$
$$\underline{\underline{\varepsilon_{xy} = 1{,}464\ \mu\text{in/in.}}}$$

(3) Hooke's law

$$\begin{cases} E\varepsilon_{xx} = \sigma_{xx} - v\sigma_{yy} - v\sigma_{zz} \ \&\ 2\varepsilon_{xy} = \gamma_{xy} = \frac{1}{G}\tau_{xy} \\ E\varepsilon_{yy} = \sigma_{yy} - v\sigma_{zz} - v\sigma_{xx} \ \&\ 2\varepsilon_{yz} = \gamma_{yz} = \frac{1}{G}\tau_{yz} \\ E\varepsilon_{zz} = \sigma_{zz} - v\sigma_{yy} - v\sigma_{xx} \ \&\ 2\varepsilon_{zx} = \gamma_{zx} = \frac{1}{G}\tau_{zx} \end{cases}$$

$$\therefore\ \sigma_{zz} = 0\ \&\ \tau_{yz} = \tau_{zx} = 0$$

$$\sigma_{xx} = \frac{E}{1-v^2}(\varepsilon_{xx} + v\varepsilon_{yy})$$

$$\sigma_{yy} = \frac{E}{1-v^2}(\varepsilon_{yy} + v\varepsilon_{xx})$$

$$\tau_{xy} = G(2\varepsilon_{xy})$$

$$\sigma_{xx} = \frac{12.7(10^6)}{1-(0.27)^2}[-3{,}376(10^{-6}) + 0.27(-864)]$$

$$= 13.7(-3{,}376 - 233)$$

$$\underline{\sigma_{xx} = -49{,}438\ \text{psi}}$$

$$\sigma_{yy} = 13.7[-864 + 0.27(-3{,}376)]$$

$$\underline{\sigma_{yy} = -24{,}325\ \text{psi}}$$

$$\tau_{xy} = G(2\varepsilon_{xy})$$

$$= \frac{12.7(10^6)(2)(1{,}464)}{2(1+0.27)}$$

$$\underline{\tau_{xy} = 14{,}640\ \text{psi}}$$

$$\sigma_{xx} = -49{,}438\ \text{psi}$$

$$\sigma_{yy} = 24{,}325\ \text{psi} \qquad \longleftarrow (3)$$

$$\underline{\underline{\tau_{xy} = +14{,}640\ \text{psi}}}$$

(4) $E\varepsilon_{zz} = \sigma_{zz} - v\sigma_{xx} - v\sigma_{yy}$
$\sigma_{zz} = 0$: by inspection, fraction free surface

$$\varepsilon_{zz} = \frac{-0.27}{12.7(10^6)}(-49{,}438 - 24{,}325)$$

$$\varepsilon_{zz} = +1{,}568(10^{-6})$$

$$\underline{\underline{\varepsilon_{zz} = +1{,}568\ \mu\text{in/in}}} \qquad \longleftarrow (4)$$

(5) $\tau_{yz} = \tau_{zx} = 0$: by inspection

$$\gamma_{yz} = \left(\frac{1}{G}\right)\tau_{yz},\ \gamma_{xz} = \left(\frac{1}{G}\right)\tau_{xz}$$

$$\underline{\underline{\varepsilon_{yz} = \frac{1}{2}\gamma_{yz} = 0}}$$

$$\underline{\underline{\varepsilon_{xy} = \frac{1}{2}\gamma_{yz} = 0}}$$

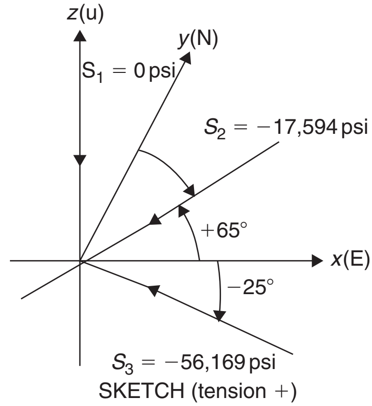
(6) By inspection: The z-direction is a principal direction. In the x-y plane

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})} = \frac{14{,}640}{\frac{1}{2}[-49{,}438 - (-24{,}325)]}$$

$$\tan 2\theta = -1.1659$$

$$2\theta = -49.4°$$

$$\underline{\underline{\theta = -24.7°}}$$



(7) $S_2 = 0$.

$$\left.\begin{matrix} S_1 \\ S_3 \end{matrix}\right\} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= -\frac{49{,}438 - 24{,}325}{2} \pm \left[\left(\frac{-49{,}438 + 24{,}325}{2}\right)^2 + (14{,}640)^2\right]^{1/2}$$

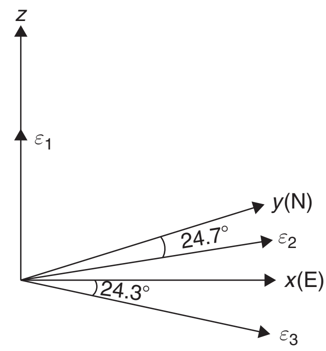
$$\left.\begin{matrix} S_1 \\ S_3 \end{matrix}\right\} = -36{,}882 \pm 19{,}287$$

$$\underline{\underline{S_1 = -56{,}169 \text{ psi}}}$$

$$\underline{\underline{S_3 = -17{,}594 \text{ psi}}}$$

(7) ←

(a)



(8) The principal strains coincide in direction with the principal stresses when the material is isotropic.

$S_1 = \sigma_1 = 0 \quad S_3 = \sigma_3 = -56{,}169$ psi

$S_2 = \sigma_2 = -17{,}594$ psi

Hooke's Law

$E\varepsilon_1 = \sigma_1 - v\sigma_2 - v\sigma_3$

$(12.7)\ 10^6 \varepsilon_1 = 0 - 0.27(-17{,}594 - 56{,}169)$

$\underline{\underline{\varepsilon_1 = +1{,}568\ \mu\text{in/in}}}$

(Parallel to z-dir, $= \varepsilon_{zz}$) check,

$E\varepsilon_2 = \sigma_2 - v\sigma_3 - v\sigma_1$

$12.7(10^6)\varepsilon_2 = -17{,}594 - 0.27(-56{,}169)$

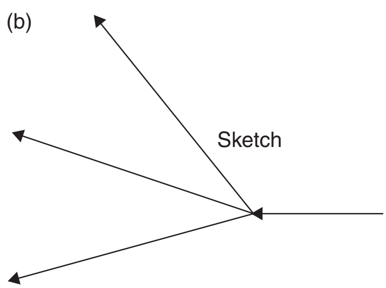
$\underline{\underline{\varepsilon_2 = -19{,}121\ \mu\text{in/in.}}}$

$E\varepsilon_3 = \sigma_3 - v\sigma_1 - v\sigma_2$

$(10^6)\ 12.7\ \varepsilon_3 = -56{,}169 - 0.27(-17{,}594)$

$\underline{\underline{\varepsilon_3 = -40{,}487\ \mu\text{in/in.}}}$

(b)



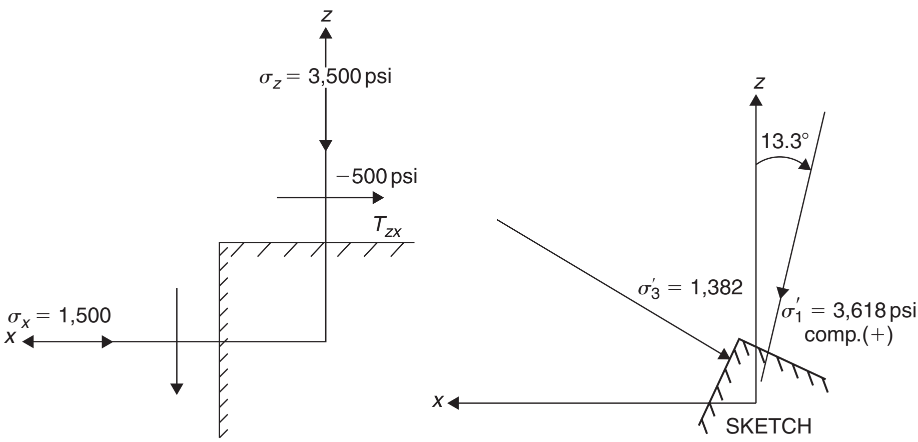
(ε_v will be equal but opposite during over covering

$$\begin{aligned}\varepsilon_v &= \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \\ &= +(40{,}487 + 19{,}121 - 1{,}568)\mu\text{in/in.} \\ \varepsilon_v &= 58{,}040\,\mu\text{in/in. (expansion)}\end{aligned}$$ ←

29. Given: $\sigma_{xx} = 1{,}500$, $\sigma_{yy} = -2{,}000$, $\sigma_{zz} = 3{,}500$
$\tau_{xy} = 600$, $\tau_{yz} = -300$, $\tau_{zx} = -500$
Find: σ_1', σ_3' (zx-plane) of sketch.

Solution:

[comp.(+) units = psi]



$$\begin{aligned}\tan 2\theta &= \frac{\tau_{zx}}{\frac{1}{2}(\sigma_{zz} - \sigma_{xx})} \\ &= \frac{-500}{\frac{1}{2}(3{,}500 - 1{,}500)}\end{aligned}$$

$$\begin{aligned}\tan 2\theta &= -0.5 \\ 2\theta &= -26.6^\circ \\ \theta &= -13.3^\circ\end{aligned}$$

$$\left.\begin{matrix}\sigma_1' \\ \sigma_3'\end{matrix}\right\} = \frac{\sigma_{zz} + \sigma_{xx}}{2} \pm \left[\left(\frac{\sigma_{zz} - \sigma_{xx}}{2}\right)^2 + (\tau_{zx})^2\right]^{1/2}$$

$$= \frac{3{,}500 + 1{,}500}{2} \pm \left[\left(\frac{3{,}500 - 1{,}500}{2}\right)^2 + (-500)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1' \\ \sigma_3'\end{matrix}\right\} = 2{,}500 \pm 1{,}118$$

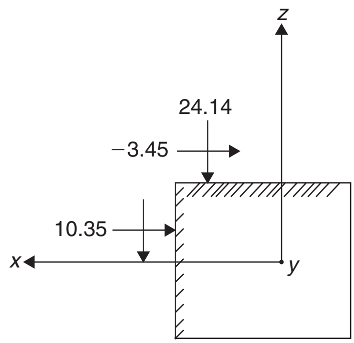
$$\sigma_1' = 3{,}618 \text{ psi}$$

$$\underline{\underline{\sigma_3' = 1{,}382 \text{ psi}}}$$

30. Given (in MPa): $\sigma_{xx} = 10.35$ $\sigma_{yy} = -13.79$ $\sigma_{zz} = 24.14$ $\tau_{xy} = 4.14$ $\tau_{yz} = -2.07$ $\tau_{zx} = -3.45$

where $x =$ east $y =$ north $z =$ up
Compression is (+), units are MPa

Find: Secondary principal stresses in the zx-plane. σ_1', σ_3', α



Solution:

$$\tan 2\alpha' = \frac{\tau_{zx}}{\frac{1}{2}(\sigma_{zz} - \sigma_{xx})} = \frac{3.45}{\frac{1}{2}(24.14 - 10.35)}$$

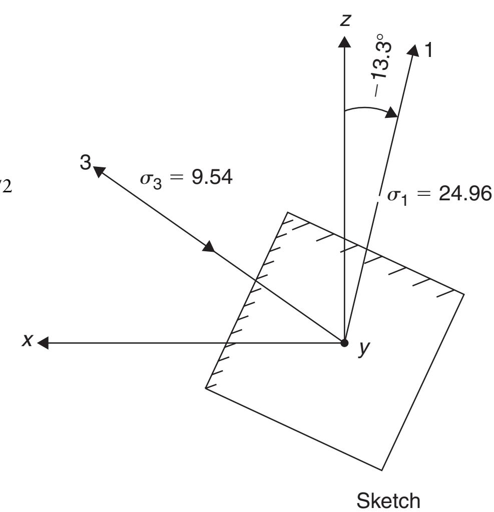
$$\tan 2\alpha' = -0.5004$$

$$2\alpha' = -26.6°, 153.4°$$

$$\underline{\underline{\alpha' = -13.3°}}, 76.7°$$

$$\left.\begin{matrix}\sigma_1' \\ \sigma_3'\end{matrix}\right\} = \frac{\sigma_{zz} + \sigma_{xx}}{2} \pm \left[\left(\frac{\sigma_{zz} - \sigma_{xx}}{2}\right)^2 + (\tau_{zx})^2\right]^{1/2}$$

$$= \frac{24.14 + 10.35}{2} \pm \left[\left(\frac{24.14 - 10.35}{2}\right)^2 + (-3.45)^2\right]^{1/2}$$



$$\left.\begin{matrix}\sigma_1' \\ \sigma_3'\end{matrix}\right\} = 17.25 \pm 7.71$$

$$\underline{\underline{\sigma_1' = 24.96 \text{ MPa}}}$$

$$\underline{\underline{\sigma_3' = 9.54 \text{ MPa},}}$$

$$\underline{\underline{\alpha' = -13.3°}}$$

31. Given: $\varepsilon_{xx} = 2{,}000$ $\varepsilon_{yy} = 3{,}000$ $\varepsilon_{zz} = 4{,}000$ $\gamma_{xy} = -200$ $\gamma_{yz} = 300$ $\gamma_{zx} = 225$ in μin/in. and compression is (+) x = east, y = north, z = up $E = 5(10^6)$ psi, $G = 2(10^6)$ psi

Find: The stresses (σ_{xx}, σ_{yy}, σ_{zz}, τ_{xy}, τ_{yz}, τ_{zx})

Solution:

Hooke's law

$$\sigma_{xx} = \frac{E}{(1+v)(1-2v)}((1-v)\varepsilon_{xx} + v\varepsilon_{yy} + v\varepsilon_{zz})$$

$\sigma_{yy} = \cdots$ similar, permute subscripts
$\sigma_{zz} = \cdots$
$\tau_{xy} = G\gamma_{xy}$ 　　 Note:

$\tau_{yz} = \cdots$ 　　 $v = \dfrac{E}{2G} - 1$

$\tau_{zz} = \cdots$ 　　 $v = \dfrac{5}{4} - 1$

$\sigma_{xx} =$ 　　 $\therefore \underline{v = 0.25}$

$$\frac{E}{(1+v)(1-2v)} = \frac{5(10^6)}{(1.25)[1-2(0.25)]} = \underline{8.0(10^6)\text{ psi}}$$

$\sigma_{xx} = 8(10^6)[(1-0.25)2{,}000 + 0.25(3{,}000) + 0.25(4{,}500)]10^{-6} = (8)(3{,}375)$
$\underline{\underline{\sigma_{xx} = 27{,}000\text{ psi}}}$

$\sigma_{yy} = 8(10^6)(10^{-6})[0.75(3,000) + 0.25(4{,}500 + 200)]$
$\underline{\underline{\sigma_{yy} = 31{,}000\text{ psi}}}$

$\sigma_{zz} = 8(10^6)(10^{-6})[0.75(4{,}500) + 0.25(2{,}000 + 3{,}000)]$
$\underline{\underline{\sigma_{zz} = 37{,}000\text{ psi}}}$

	Summary:
$\tau_{xy} = 2(10^6)(-200)(10^{-6})$	$\sigma_{xx} = 27{,}000$ psi
$\underline{\tau_{xy} = -400\text{ psi}}$	$\sigma_{yy} = 31{,}000$
$\tau_{yz} = 2(10^6)(300)(10^{-6})$	$\sigma_{zz} = 37{,}000$
$\underline{\tau_{yz} = 600\text{ psi}}$	$\tau_{xz} = -400$
$\tau_{zx} = 2(10^6)(225)(10^{-6})$	$\tau_{yy} = +600$
$\underline{\tau_{zz} = 450\text{ psi}}$	$\underline{\underline{\tau_{zx} = +450}}$

32. Given: Cylindrical test cylinder, confining pressure = 3,000 psi, axial compression = 3,000 psi
$\sigma_{zz} = 3{,}000$, $\sigma_{rr} = 3{,}000$, $\sigma_{\theta\theta} = 3{,}000$ 　 $\tau_{rz} = 0$, $\tau_{z\theta} = 0$, $\tau_{\theta r} = 0$
$E = 2.4(10^6)$ psi 　 $v = 0.20$ 　 isotropic

Find:

(1) $\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \gamma_{rz}, \gamma_{z\theta}, \gamma_{\theta r}$
(2) σ_{zz} for $\varepsilon_{zz} = 0$
(3) Strain energy and density

Solution:

Hooke's law

$$\text{(1)}\quad \varepsilon_{zz} = \frac{1}{E}(\sigma_{zz} - v\sigma_{\theta\theta} - v\sigma_{rr}), \text{etc.}$$

$$r_{r\theta} = \frac{1}{G}\tau_{r\theta}, \text{etc.}$$

$$\varepsilon_{zz} = \left(\frac{10^{-6}}{2.4}\right)(1-2v)(3{,}000)$$

$$\underline{\underline{\varepsilon_{rr} = \varepsilon_{\theta\theta} = \varepsilon_{zz} = 750\,\mu\text{in/in.}}} \qquad \longleftarrow \text{(1)}$$

$$\underline{\underline{\gamma_{r\theta} = \gamma_{\theta z} = \gamma_{zr} = 0}}$$

$$\text{(2)}\quad \varepsilon_{zz} = 0 = \frac{1}{E}(\sigma_{zz} - v\sigma_{rr} - v\sigma_{\theta\theta})$$

$$\sigma_{zz} = 0.25(3{,}000 + 3{,}000)$$

$$\underline{\underline{\sigma_{zz} = 1{,}500 \text{ psi}}} \qquad \longleftarrow \text{(2)}$$

(3) Strain energy W, strain energy density ω (per unit volume) one form:

$$\omega = \frac{1}{2E}(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2) - \frac{V}{E}(\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx}) + \frac{1}{2G}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)$$

another form:

$$\omega = \frac{1}{2}(\sigma_{rr}\varepsilon_{rr} + \sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{zz}\varepsilon_{zz} + \tau_{r\theta}\gamma_{r\theta} + \tau_{\theta z}\gamma_{\theta z} + \tau_{zr}\gamma_{zr})$$

$$= \frac{1}{2}[3{,}000 \cdot 750(10^{-6})(3) + 0. + 0. + 0.]$$

$$\underline{\underline{\omega = 3.375(\text{lbf-in./in}^3)}} \qquad \longleftarrow \text{Strain energy per unit volume (density)}$$

$$W = \omega V$$

$$= 3.375\left(\frac{\pi}{4}\right)(2.125)^2(4.25)$$

$$\underline{\underline{W = 50.9\,\text{lbf-in.}}} \qquad \longleftarrow \text{(Strain energy)}$$

33. Given: $\varepsilon_{xx} = 2{,}000 \quad \varepsilon_{yy} = 3{,}000 \quad \varepsilon_{zz} = 4{,}500 \; \gamma_{xy} = -200 \quad \gamma_{yz} = 300 \quad \gamma_{zx} = 225$

x = east, y = north, z = up units are micrometers per meter, $E = 34.48$ GPa, $G = 13.79$ GPa

Find: Stresses for a linear, homogeneous, isotropic, elastic response.

Solution:

By Hooke's law

$$\begin{cases} \sigma_{xx} = \dfrac{E}{(1+v)(1-2v)}\left[(1-v)\varepsilon_{xx} + v\varepsilon_{yy} + v\varepsilon_{zz}\right] \\ \sigma_{yy} = \cdots \\ \sigma_{zz} = \cdots \\ \tau_{xy} = G\,\gamma_{xy} \\ \tau_{yz} = \cdots \\ \tau_{zx} = \cdots \end{cases}$$

with
$$v = \frac{E}{2G} - 1$$
$$v = \frac{34.48}{2(13.79)} - 1$$
$$\underline{\underline{v = 0.25}}$$

and

$$\frac{E}{(1+v)(1-2v)} = \frac{34.48}{(1+0.25)(1-0.5)} = \underline{\underline{55.17\text{ GPa.}}}$$

then
$$\sigma_{xx} = (55.17)(10^9)[(1-0.25)2{,}000 + 0.25(3{,}000) + 0.25(4{,}500)]10^{-6}$$
$$= 55.17(10^9)[3{,}375]10^{-6}$$
$$\underline{\underline{\sigma_{xx} = 186.2\text{ MPa}}}$$

$$\sigma_{yy} = 55.17(10^9)[0.75(3{,}000) + 0.25(4{,}500 + 200)]$$
$$\underline{\underline{\sigma_{yy} = 213.8\text{ MPa}}}$$

$$\sigma_{zz} = 55.17(10^9)[0.75(4{,}500) + 0.25(2{,}000 + 300)]$$
$$\underline{\underline{\sigma_{zz} = 255.2\text{ MPa}}}$$

$$\tau_{xy} = (13.79)(10^9)(10^{-6})(-200)$$
$$\underline{\underline{\tau_{xy} = -2.76\text{ MPa}}}$$

$$\tau_{yz} = 13.79(10^9)(10^{-6})300$$
$$\underline{\underline{\tau_{yz} = 4.14\text{ MPa}}}$$

$\tau_{zx} = 13.79(10^9)(10^{-6})(225)$
$\underline{\underline{\tau_{zx} = 3.10\,\text{MPa}}}$

34. Given: Cylindrical coordinates
$p = 20.69$ MPa $= \sigma_{rr} = \sigma_{\theta\theta}$ $\quad \sigma_{zz} = 20.69$ MPa $\quad \tau_{rz} = \tau_{z\theta} = \tau_{r\theta} = 0$
Find:
(1) $\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \gamma_{rz}, \gamma_{z\theta}, \gamma_{\theta r}$
(2) σ_{zz} for $\varepsilon_{zz} = 0$
(3) strain energy and density
NX-core $\quad L/D = 2 \quad E = 16.55\,\text{GPa} \quad v = 0.2 \quad G = ?$

Solution:

(1) Hooke's law

$$\begin{aligned}
\gamma_{rz} &= \frac{1}{G}\tau_{rz}, \text{ etc.} \\
\therefore \tau_{rz} &= \tau_{z\theta} = \tau_{\theta z} = 0 \quad \text{implies :} \\
&\underline{\underline{\gamma_{rz} = \gamma_{z\theta} = \gamma_{\theta r} = 0}} \qquad \leftarrow (1)\text{a} \\
\varepsilon_{rr} &= \frac{1}{E}\sigma_{rr} - \frac{v}{E}\sigma_{\theta\theta} - \frac{v}{E}\sigma_{zz} \\
\varepsilon_{\theta\theta} &= \cdots \\
\varepsilon_{zz} &= \cdots \\
\varepsilon_{rr} &= \varepsilon_{\theta\theta} = \varepsilon_{zz} = \left(\frac{1-2v}{E}\right)(20.69\,\text{MPa}) \\
&= \frac{1-0.4}{16.55(10^9)}(20.69)10^6 \\
&\underline{\underline{\varepsilon_{rr} = 750(10^{-6})\text{meter/meters}}}
\end{aligned}$$

$$\begin{aligned}
(2)\quad \varepsilon_{zz} &= 0 \;\therefore\; \sigma_{zz} = v(\sigma_{rr} + \sigma_{\theta\theta}) \\
&= 0.25(20.69 + 20.69) \\
&\underline{\underline{\sigma_{zz} = 10.35\,\text{MPa}}}
\end{aligned}$$

(3) Strain energy W, density $\omega = \dfrac{W}{V}$
One form:

$$\omega = \frac{1}{2E}(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2) - \frac{v}{E}(\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx}) + \frac{1}{2G}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)$$

Second form:

$$\omega = \frac{1}{2}(\sigma_{nn}\varepsilon_{nn} + \sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{zz}\varepsilon_{zz} + \tau_{r\theta}\gamma_{r\theta} + \tau_{\theta z}\gamma_{\theta z} + \tau_{zr}\gamma_{zr})$$

$$= \frac{1}{2}[(20.69)(10^6)(750)(10^{-6})(3) + 0 + 0 + 0]$$

$$\underline{\underline{\omega = 2.33(10^4)\ \text{N-m/m}^3}}$$

$$W = \omega V$$

$$= 2.33(10^4)\left(\frac{\pi}{4}\right)(5.4)^2(10^{-4})(10.8)(10^{-2})$$

$$\underline{\underline{W = 5.76\ \text{N-m}}}$$

35. Show that under complete lateral restraint that under gravity load
(1) $S_v = \gamma_z$
(2) $S_h = S_H = \left(\dfrac{v}{1-v}\right) S_v$
where γ = Specific weight, z = depth, v = Poisson's ratio, homogeneous, isotropic rock.

Solution:

Complete lateral restraint means no horizontal strain, displacement. From Hooke's law

$$E\varepsilon_{xx} = 0 = \sigma_{xx} - v\sigma_{yy} - v\sigma_{zz}$$
$$E\varepsilon_{yy} = 0 = \sigma_{yy} - v\sigma_{zz} - v\sigma_{xx}$$

Eliminate σ_{zz} to obtain $\sigma_{xx} = \sigma_{yy}$, then back substitute to obtain

$$\sigma_{xx} = \sigma_{yy} = \frac{v}{1-v}\sigma_{zz}$$

where: x = east y = north z = up (vertical)

$$\therefore \sigma_h = \sigma_H = \frac{v}{1-v}\sigma_v \qquad \longleftarrow (2)$$
$$\underline{\underline{(xx) \quad (yy) \quad (zz)}}$$

From stress equations of equilibrium

$$\frac{\partial \sigma_{xx}}{\partial x} = \frac{\partial \sigma_{yy}}{\partial y} = 0 \ \&\ \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$$

$$\therefore \quad \frac{\partial \sigma_{zz}}{\partial z} + \gamma = 0 \quad \text{and}$$

$$\sigma_{zz} = -\gamma z + \text{constant}$$

at $z = 0$ surface $\sigma_{zz} = 0$

$\therefore$ Constant $= 0$

$\sigma_{zz} = -\gamma_z$

If z is depth (opposite of + vertical up), and compression is +, then

$\sigma_{zz} = \gamma_z$

i.e. $\underline{\underline{S_v = \gamma_z}}$ ← (1)

36. Given: NX-core $L/D = 2$, $E = 10(10^6)$ psi $v = 0.35$ $\varepsilon_{\text{axial}}$ (failure) $= 0.1\%$. uniaxial compression

Find:

(1) Axial load and stress at failure

(2) Relative displacement between ends at failure.

Solution:

Hooke's law

$E\varepsilon_{zz} = \sigma_{zz} - v\sigma_{rr} - v\sigma_{\theta\theta}$

Unconfined: $\sigma_{rr} = \sigma_{\theta\theta} = 0$

$$\sigma_{zz} = 0.1(-10)^{-2}(10)(10^6)$$

$$\underline{\underline{\sigma_{zz} = 10{,}000\,\text{psi}}} \quad \leftarrow \text{failure load (1a)}$$

$$F = A\sigma_{zz}$$

$$= \frac{\pi}{4}(2.125)^2(10^4)$$

$$\underline{\underline{F = 35{,}466\,\text{lbf.}}} \quad \leftarrow \text{failure load (1b)}$$

$$\varepsilon_{zz} = \frac{\partial\omega}{\partial z}$$

$$\omega = \int_{\text{Top}}^{\text{Bottom}} \varepsilon_{zz}\,dz$$

$$= 0.1(10^{-2})(4.25\,\text{in})$$

$$\omega_{\text{rel}} = \underline{\underline{0.00425\,\text{in.}}} \quad \leftarrow \text{rel. disp. ends. at failure (2)}$$

37. Given: NX-core, $L/D = 2$, $E = 68.97$ GPa, $v = 0.35$ $\varepsilon_f = 0.1$.

Find:

(1) Axial load (kN) and stress (MPa) at failure

(2) Relative end displacement (m).

Solution:

Hooke's law

$E\varepsilon_{zz} = \sigma_{zz} - v\sigma_{rr} - v\sigma_{\theta\theta}$

Unconfined $\therefore \sigma_{rr} = \sigma_{\theta\theta} = 0$

$$\sigma_{zz} = (0.001)(68.97)10^9$$
$$\underline{\underline{\sigma_{zz} = 68.97\,\text{Mpa}}}$$
$$F = \sigma_{zz} A$$
$$= 68.97(10^3)\text{kN}\left(\frac{\pi}{4}\right)(5.40)^2(10^{-4})$$
$$\underline{\underline{F = 158\ \text{kN}}}$$

$$\varepsilon_{zz} = \frac{\partial \omega}{\partial z} \text{ by definition}$$
$$\omega = \int_0^{10.8\,\text{cm}} \varepsilon_{zz}\, dz$$
$$= (0.001)(10.8\,\text{cm})(10)$$
$$\underline{\underline{\omega = 0.108\,\text{cm}}}$$

38. Given: Gravity loading in flat compression is positive $\gamma_{\text{ave}} =$ *average* specific weight to any depth, $v =$ Poisson's ratio, $z =$ depth, water bearing S_s 250 ft thick at 1,300 ft pressure at S_s top is 80 psi

Find: Total and effective stresses at center sandstone.

Solution:

$$S_v = \gamma_{\text{ave}} z$$
$$S_v = \gamma_{\text{ave}}\left(1{,}300 + \frac{250}{2}\right)$$

$$\left.\begin{aligned} &\underline{\underline{S_v = 1{,}425\gamma_{\text{ave}}/144}} \\ &\underline{\underline{S_h = S_H = \left(\frac{v}{1-v}\right) 1{,}425\ \gamma_{\text{ave}}/144}} \end{aligned}\right\} \longleftarrow \text{total stresses at } S_s \text{ center (1a) (in psi)}$$

Effective stress:

$$S'_v = S_v - p$$
$$S'_h = S'_H = S_h - p = S_H - p$$
$$p = 80\,\text{psi} + \gamma_w \frac{250}{2}$$
$$= 80 + (62.4)\frac{(125)}{144}$$
$$p = 135\,\text{psi}$$

$$\left.\begin{aligned} S'_v &= 1{,}425\left(\frac{\gamma_{\text{ave}}}{144}\right) - 135 \\ S'_h = S'_H &= \left(\frac{v}{1-v}\right) 1{,}425\left(\frac{\gamma_{\text{ave}}}{144}\right) - 135 \end{aligned}\right\} \leftarrow \text{psi effectives stresses (1b)}$$

39. Given: Gravity loading in flat strata compression positive γ_{ave} = average specific weight to any depth, v = Poisson's ratio, z = depth, water bearing sandstone 76.2 m thick at $z = 396$ m at $z = 396$ m
$p(\text{top}) = 552$ kPa

Find: Effective and total stresses at center depth of sandstone.

Solution:

$$S_v = \gamma_{\text{ave}} z$$
$$= \gamma_{\text{ave}}\left(396 + \frac{76.2}{12}\right)$$

Total stress: $\underline{\underline{S_v = 434\gamma_{\text{ave}}}}$

$$S_v(\text{kPa}) \quad \begin{cases} \gamma_{\text{ave}}(\text{kN/m}^3) \\ S_v(\text{kN/m}^2) \end{cases}$$

Under complete lateral constraint via Hooke's law

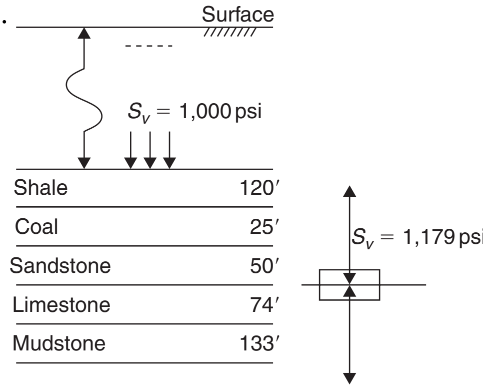
$$S_h = S_H = \frac{v}{1-v} S_v$$
$$\underline{\underline{S_h = S_H = \frac{v}{1-v} 434 \ \gamma_{\text{ave}}(\text{kPa})}}$$

Effective stress:

$$S'_v = S_v - p \qquad \text{by definition}$$
$$S'_h = S'_H = S_H - p = S_h - p$$
$$p = 552\,\text{kPa} + \gamma_w\left(\frac{76.2}{2}\right)$$
$$\gamma_w = \text{sp. wt. water} = 9.87\,\text{kN/m}^3$$

$$p = 552 + 376$$
$\underline{\underline{p = 928\,\text{kPa}}}$ at center of S_s.
$$\underline{\underline{S'_v = 434\gamma_{\text{ave}} - 928\,(\text{kPa})}}$$
$$\underline{\underline{S'_v = S'_H = \frac{v}{1-v} 434\gamma_{\text{ave}} - 928\,(\text{kPa})}}$$

40. Given: Gravity loading and properties table, complete lateral restraint
Find: $S_h = S_H$ (S_s bottom) & (L_m top).



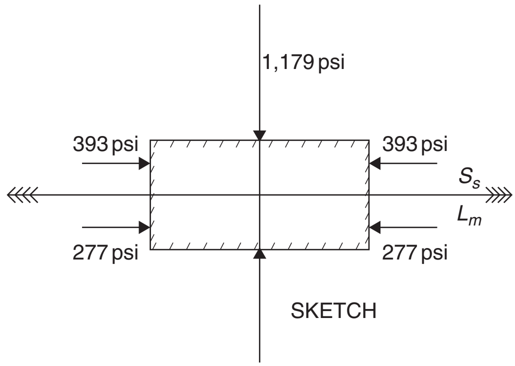
Solution:

$$S_h = S_H = \frac{v}{1-v} S_v$$

$$S_v = 1{,}000\text{ psi} + \left(\frac{120}{144}\right)134 + \left(\frac{25}{144}\right)95 + \left(\frac{178}{144}\right)30$$

$$= 1{,}000 + 112 + 16 + 51$$

$$\underline{\underline{S_v = 1{,}179\text{ psi}}}$$



$$S_h(S_s) = \frac{v}{1-v} S_v$$

$$v = \frac{E}{2G} - 1$$

$$v(S_s) = \frac{3.83}{2(1.53)} - 1 = \underline{\underline{0.25}}$$

$$v(L_m) = \frac{5.72}{2(2.41)} - 1 = \underline{\underline{0.19}}$$

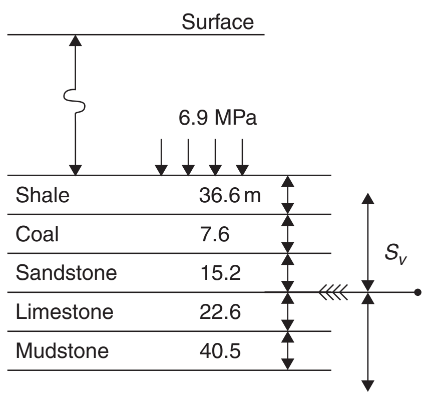
$$S_h(S_s) = \frac{0.25}{1-0.25}(1{,}179)$$

$$S_h(S_s) = \underline{\underline{393\text{ psi}}}$$ ← Bottom of S_s

$$S_h(L_m) = \frac{0.19}{1-0.19}(1{,}179)$$

$$S_h(L_m) = \underline{\underline{277\text{ psi}}}$$ ← top of L_m

41. Given: Table data, compression (+)
Find: $S_h = S_H$ at bottom of sandstone and at top of limestone.



Solution:

$$S_v = 6.9\text{ MPa} + (36.6)(2.15)(9.87) + (7.6)(1.52)(9.87) + (15.2)(2.37)(9.87)$$

$$= 6.9 + 0.78 + 0.11 + 0.36$$

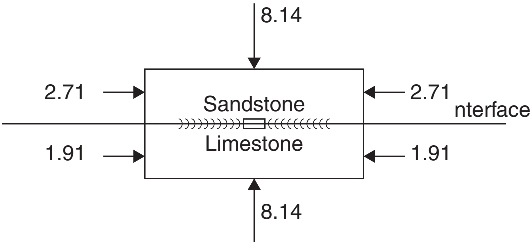
$$\underline{\underline{S_v = 8.14\text{ MPa}}}$$

$$S_h = S_H = \frac{v}{1-v} S_v$$

$$S_h = S_H(S_s) = \frac{0.25}{1-0.25}(8.14)$$
$$\underline{\underline{S_h = S_H(S_s) = 2.71\text{ MPa}}}$$
$$S_h = S_H(L_m) = \frac{0.19}{1-0.19}(8.14)$$
$$\underline{\underline{S_h = S_H(L_m) = 1.91\text{ MPa}}}$$

Note:

$$v = \frac{E}{2G} - 1$$
$$v(S_s) = \frac{26.4}{26(10.6)} - 1$$
$$\underline{v(S_s) = 0.25}$$
$$v(L_m) = \frac{39.4}{2(16.6)} - 1$$
$$\underline{v(L_m) = 0.19}$$



42. Given: stresses of 500 m (1,440 ft)
$S_v = 25$ kPa/m (1.1 psi/ft) $S_H = S_h = 3S_v$
Find: S_h, S_H parts caused by gravity and by other forces.

Solution:

(1) Assume complete lateral constraint for gravity loading
(2) Also assume a reasonable poisson's ratio, v, say, $v = 0.25$

then $S_h = S_H(\text{gravity}) = \frac{v}{1-v}S_v$

$$S_h = S_H(g) = \frac{1}{3}S_v$$
$$S_v = (500)(25),\ 1{,}640(1.1)$$
$$\underline{\underline{S_v = 12.5\text{ MPa, }(1{,}804\text{ psi})}} \longleftarrow$$

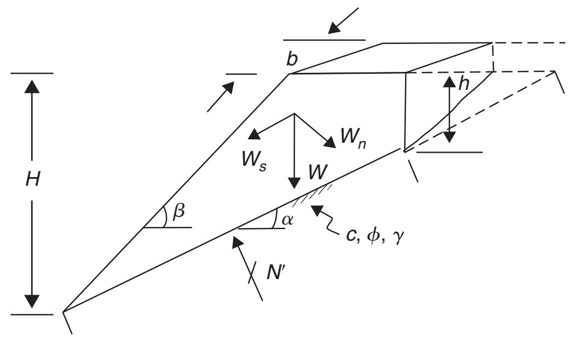
$$\underline{\underline{(\text{gravity})\ S_H = S_h = 4.17\text{ MPa }(601\text{ psi})}} \longleftarrow$$

$$S_h = S_H = 3S_v = 37.5\text{ MPa}(5{,}412\text{ psi})$$
$$(\text{not gravity})S_h = S_H = 37.5 - 4.17$$
$$54.2 - 601$$
$$S_h = S_H(\text{not gravity}) = \left.\begin{array}{l}\underline{\underline{33.3\text{ MPa}}}\\ \underline{\underline{(4{,}811\text{ psi})}}\end{array}\right\} \longleftarrow$$

2 Slope Stability

Planar Block Slides

1. Given: Planar block slide
   Find: Algebraic *FS*.



Solution:

By definition

$$FS = \frac{R}{D}$$

$R =$ resisting forces: Assume Mohr–Coulomb
$D =$ driving forces
$W =$ weight: $W = W_1 - W_2$

By inspection:

$$W = \frac{\gamma H^2 b}{2}(\text{cotan}\,\alpha - \text{cotan}\,\beta) - \frac{\gamma h^2 b}{2}\text{cotan}\,\alpha$$

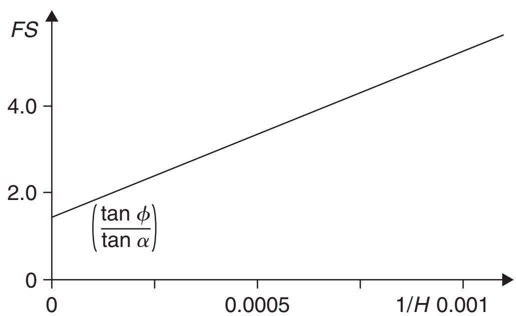
$$R = W_n \tan\phi + cA$$

Note: $\Sigma F_n = 0 \therefore 0 = W_n - N'$ & $N' = W_n$

Also

$$A = b\left(\frac{H}{\sin\alpha} - \frac{h}{\sin\alpha}\right)$$

And $W_n = W\cos\alpha$



SCHEMAT C PLOT (*FS* vs H^{-1})
Note: almost linear in 1/*H*

$$\therefore R = W\cos\alpha\tan\phi + c\text{A}$$

$$D = W\sin\alpha$$

$$FS = \frac{W\cos\alpha\tan\phi + c\text{A}}{W\sin\alpha}$$

$$FS = \frac{\tan\phi}{\tan\alpha} + \frac{cb\left(\frac{H}{\sin\alpha} - \frac{h}{\sin\alpha}\right)}{(\sin\alpha)\left[\left(\frac{\gamma H^2 b}{2}\right)(\cot\alpha - \cot\beta) - \frac{\gamma h^2 b}{2}\text{cotan}\,\alpha\right]}$$

$$FS = \frac{\tan\phi}{\tan\alpha} + \frac{2c\left(1-\frac{h}{H}\right)}{(\sin\alpha)^2\left[(\gamma H)\left[(\cot\alpha-\cot\beta)-\frac{h^2}{H^2}\text{cotan}\,\alpha\right]\right]}$$

$$FS = \frac{\tan\phi}{\tan\alpha} + \frac{2c\left(1-\frac{h}{H}\right)}{\gamma H(\sin\alpha)^2\left[\left(1-\left(\frac{h}{H}\right)^2\right)\text{cotan}\,\alpha-\cot\beta\right]} \qquad \longleftarrow$$

2. Given: Problem 1 data
   Find: Max β (algebraically).

   Solution:

   Solve FS for β then set $\frac{df}{d\beta} = 0$
   Find β^* stationary pts
   Find relative maximum
   Find absolute maximum (end pts) or note by inspection that as β increases the FS decreases so β will be max when FS is min, that is, when $FS = 1$, then

$$[\gamma H(\sin\alpha)^2]\left[\left(1-\left(\frac{h}{H}\right)^2\right)\text{cotan}\,\alpha-\text{cotan}\,\beta\right] = \frac{2c\left(1-\frac{h}{H}\right)}{\left(1.0-\frac{\tan\phi}{\tan\alpha}\right)}$$

$$\text{cotan}\,\beta_{\max} = \left(1-\left(\frac{h}{H}\right)^2\right)\text{cotan}\,\alpha - \frac{2c\left(1-\frac{h}{H}\right)}{\gamma H(\sin\alpha)^2\left(1.0-\frac{\tan\phi}{\tan\alpha}\right)}$$

   Note: $\frac{\pi}{2} \geq \beta > \alpha$ (physical constraints) & if no tension crack ($h = 0$)

$$\text{cotan}\,\beta_{\max} = \text{cotan}\,\alpha - \frac{2c}{\gamma H(\sin\alpha)^2\left(1-\frac{\tan\phi}{\tan\alpha}\right)}$$

3. Given: Problem 1, data allow for water (below bottom of tension crack)
   Find: FS (with water).

   Solution:

   (previous) $FS = \frac{W\cos\alpha\tan\phi + cA}{W\sin\alpha}$

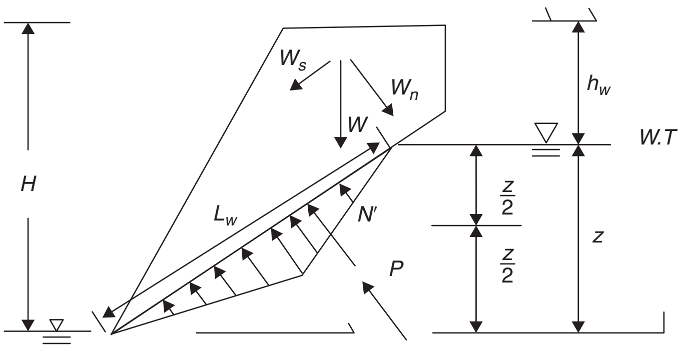
   where $W = \frac{\gamma H^2 b}{2}(\text{cotan}\,\alpha - \text{cotan}\,\beta) - \frac{\gamma h^2 b}{2}\text{cotan}\,\alpha$

   and $A = \frac{b}{\sin\alpha}(H-h)$

Now

$$\underline{\underline{FS = \frac{N' \tan\phi + cA}{W \sin\alpha}}} \qquad (3) \longleftarrow$$

where $\Sigma F_n = 0$ requires $N' = W_n - P$ with $P =$ water force



as before $W_n = W \cos(\alpha)$
but $P = ?$

$$P = \bar{p}A_w, \quad A_w = bL_w$$

$$\bar{p} = \frac{p_{\max}}{2}(\text{rectangle})$$

$p_{\max} = \gamma_w\, z/2$ (a linear increase of water pressure with depth below water table is assumed)

$$\frac{z}{2} = \left(\frac{1}{2}\right)(H - h_w)$$

where $h_w =$ water table depth below crest

$$L_w = \frac{z}{\sin\alpha}$$

$$P = \left(\frac{\gamma_w}{2}\right)\left(\frac{z}{2}\right)(b)(L_w)$$

$$P = \frac{\gamma_w b z^2}{4\sin\alpha}$$

Hence:

$$\underline{\underline{FS = \frac{(W \cos\alpha - P)\tan\phi + cA}{W \sin\alpha}}} \qquad (4) \longleftarrow$$

where:

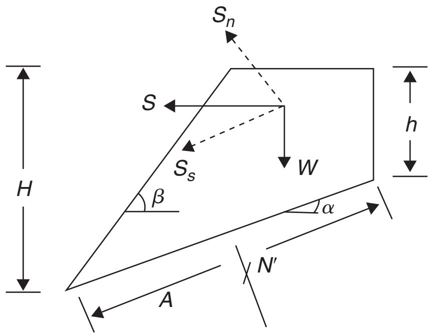
$$P = \frac{\gamma_w b z^2}{4\sin\alpha}; \quad z = H - h_w$$

$$A = \frac{b}{\sin\alpha}(H - h); \quad h = \text{tension crack depth}$$

$$W = \frac{\gamma H^2 b}{2}(\operatorname{cotan}\alpha - \operatorname{cotan}\beta) - \frac{\gamma h^2 b}{2}\operatorname{cotan}\alpha$$

4. Given: Problem data
Find: *FS* for seismic load.

Solution:



By definition $S = \frac{W}{g} a_s$

where a_s = seismic acceleration
i.e. $a_s = a_0 g$ where a_0 = seismic coefficient
e.g. $a_0 = 0.15$

$\therefore S = a_0 W$

$S_n = a_0 W \sin\alpha$

$S_s = a_0 W \cos\alpha$

$N' = W_n - S_n$

$R = N' \tan\phi + cA$

$D = W_s + S_s$

$$FS = \frac{(W_n - S_n)\tan\phi + cA}{W_s + S_s} \qquad (5)$$

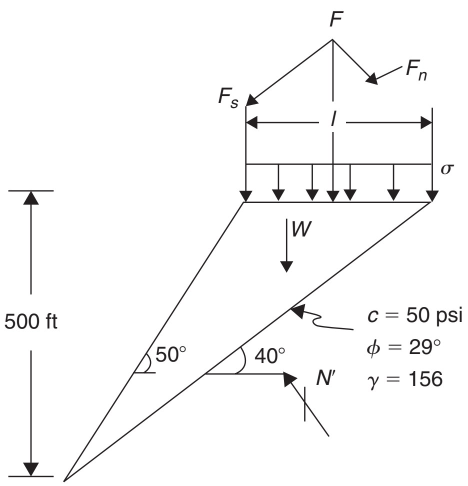
where $W_s = W\sin\alpha$,
$S_s = a_0 W \cos\alpha$
$W_n = W\cos\alpha$,
$S_n = a_0 W \sin\alpha$

$$A = \frac{b}{\sin\alpha}(H - h)$$

$$W = \frac{\gamma b^2 H^2}{2}(\text{cotan}\,\alpha - \text{cotan}\,\beta)$$

5. Given: Data in Fig. 5 and a uniform surcharge σ with $FS = 1.1$ and $b = 25'$
Find: σ.

Solution:



$$FS = \frac{N'\tan\phi + cA + F_n \tan\phi}{W_s + F_s}$$

$F = \sigma b l$
$F_n = F\cos\alpha$
$F_s = F\sin\alpha$
then

$$(FS)F_s - F_n\tan\phi = N'\tan\phi + cA - FS(W_s)$$

$$F[(\sin\alpha)FS - \cos\alpha\tan\phi] = N'\tan\phi + cA - \left(\frac{FS}{W_s}\right)$$

$W = \gamma V$

$$V = \frac{bH^2}{2}(\text{cotan}\,\alpha - \text{cotan}\,\beta)$$
$$= \frac{(25)(500)^2}{2}(\text{cotan}\,40^\circ - \text{cotan}\,50^\circ)$$
$$V = (3.125)10^6(0.35265)$$
$$V = 1.102(10^6)\text{ ft}^3$$

$$W = (156)(1.02)(10^6)$$
$$\underline{W = 1.719(10^8)\text{ lbf}}$$

$$N' = W\cos\alpha$$
$$N' = 1.719(10^8)\cos 40^\circ$$
$$\underline{N' = 1.317(10^8)}$$
$$\underline{N'\tan\phi = 7.3001(10^7)\text{ lbf}}$$

$$A = \frac{(25)(500)}{\sin 40}$$
$$cA = \frac{(50)(144)(25)(500)}{\sin 40}$$
$$\underline{cA = 1.4002(10^8)\text{ lbf.}}$$

$$FS(W_s) = \left(\frac{1.1}{1.719}\right)10^8(\sin 40)$$
$$\underline{FS(W_s) = 1.2156(10^8)}$$

$$N'\tan\phi + cA - FS(W_s) = 7.3001(10^7) + 1.4002(10^8) - 1.2156(10^8)$$

$$F = \frac{[9.146(10^7)\text{lbf}]}{[(\sin 40)(1.1) - \cos 40\tan 29]}$$
$$= \frac{9.146(10^7)}{0.2824}$$
$$\therefore \underline{F = 3.238(10^8)\text{ lbf}}$$
$$\sigma = \frac{3.238(10^8)}{(25)(500)(\cot\alpha - \cot\beta)}$$
$$\underline{\underline{\sigma = 7.346(10^4)\text{ psf}}} \quad \longleftarrow$$
$$\underline{\underline{\sigma = 510\text{ psi}}}$$

FS without surcharge.

$$FS = \frac{[N'\tan\phi + cA]}{W_s}$$
$$= \frac{1.317(10^8)\tan 29^\circ + 1.4002(10^8)}{1.719(10^8)\sin 40^\circ}$$
$$\underline{\underline{FS = 1.93(1.928)}} \quad \longleftarrow$$

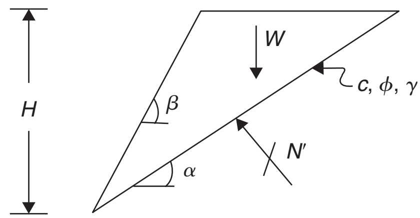
6. Given: Fig. 5 planar block slide and *no* surcharge
Find: H_{max}.

Solution:

H_{max} occurs when FS is min i.e., when

$$1 = \frac{R}{D} \tag{1}$$

$$\begin{aligned} R &= N' \tan\phi + cA \\ D &= W_s \\ W_s &= W \sin\alpha \\ W_b &= W \cos\alpha = N' \\ A &= b\frac{H}{\sin\alpha} \\ W &= \frac{\gamma b H^2}{2}(\cotan\alpha - \cotan\beta) \end{aligned}$$



Solving (1)

$$1 = \frac{\tan\phi}{\tan\alpha} + \frac{Rb\left(\frac{H}{\sin\alpha}\right)}{\frac{\gamma b H^2}{2}(\cotan\alpha - \cotan\beta)\sin\alpha}$$

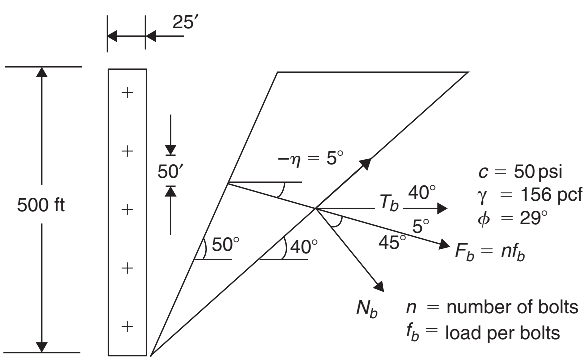
$$H_{max} = \left[\frac{2c}{\gamma(\sin\alpha)^2(\cotan\alpha - \cotan\beta)}\right]\left[\frac{1}{1-\frac{\tan\phi}{\tan\alpha}}\right]$$

$$H_{max} = \frac{(2)(50)(144)}{(156)(\sin 40)^2(\cotan 40 - \cotan 50)\left(1-\frac{\tan 29}{\tan 40}\right)}$$

$$\underline{\underline{H_{max} = 1{,}867\ \text{ft}}} \longleftarrow$$

7. Given: Sketch data and cable bolts
Bolt spacing vertical = 50 ft
Bolt spacing horizontal = 25 ft
Bolts: 12 strand type 270
Bolt angle $\eta = -5°$
Bolt tension = 60% ultimate
Find: FS, ΔF.

Solution:



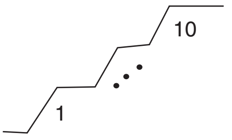
f_b: using Table A1.1 270 k ultimate strength = 495,600 lbf for 12 strands.

$$\underline{f_b = 4.956(10^5)\text{ lbf.}}$$

$$n = \frac{H}{(\text{vert. space})}$$

$$n = \frac{500}{50}$$

$$\underline{n = 10 \text{ holes (benches)}}$$



$$\underline{F_b = (10)(4.956)10^5\text{ lbf}}$$

$$N_b = F_b \cos 45^\circ$$

$$\underline{\underline{N_b = (10)^6(4.956)\frac{1}{\sqrt{2}}}}$$

$$T_b = F_b \sin 45^\circ$$

$$\underline{\underline{T_b = (10^6)(4.956)\left(\frac{1}{\sqrt{2}}\right)}}$$

$$FS = \frac{N' \tan\phi + cA + (N_b \tan\phi + T_b)(0.6)}{W \sin\alpha} (60\% \text{ mobilized})$$

$$FS = FS^\circ + \frac{(N_b \tan\phi + T_b)(0.6)}{W \sin\alpha}$$

$$= 1.928 + \frac{(0.6)\left[(10^6)\left(\frac{1}{\sqrt{2}}\right)(4.956)\tan 29^\circ + 10^6\left(\frac{1}{\sqrt{2}}\right)\right]}{(\sin 40)\, 1.719(10^8)}$$

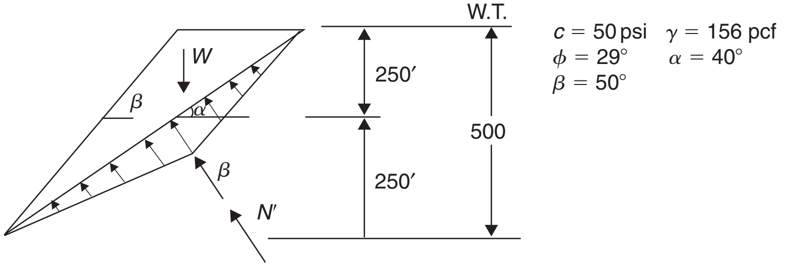
$$FS = 1.928 + \frac{5.447(10^6)}{1.105(10^8)} 0.6$$

$$FS = 1.928 + (0.049)(0.6)$$

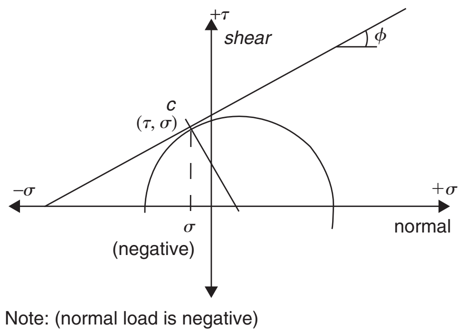
$$\underline{\underline{FS = 1.958, \Delta F = 0.30}}$$

8. Given: Planar block slide data and water table at crest
   Find: *FS*.

   Solution:



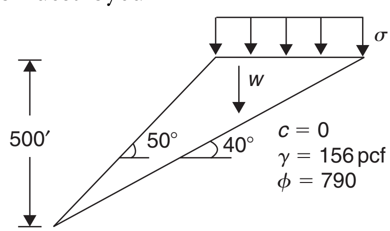
$$FS = \frac{R}{D}$$
$$R = N' \tan\phi + cA$$
$$D = W_s$$
$$N' = W_n - p$$
$$W = \frac{\gamma b H^2}{2}(\cot\alpha - \cot\beta)$$



$$\left.\begin{aligned} W_n &= 1.317(10^8)\text{lbf} \\ W_s &= 1.105(10^8)\text{lbf} \\ cA &= 1.400(10^8)\text{lbf} \end{aligned}\right\} \left(\begin{array}{c}\text{prior}\\ \text{calculations}\end{array}\right)$$

$$P = \bar{p} A_w$$
$$A_w = \frac{bH}{\sin\alpha}$$
$$\bar{p} = \frac{p_{\max}}{2}$$
$$p_{\max} = \frac{\gamma_w z}{2}$$
$$\frac{z}{2} = 250'$$
$$P = (67.4)\left(\frac{250}{2}\right)(25)\left(\frac{500}{\sin 40}\right)$$
$$P = 1.5168(10^8)$$
$$R = N' \tan\phi + cA$$
$$R = (1.317 - 1.517)10^8 \tan 29^\circ + 1.400(10^8)$$
$$R = -1.109(10^7) + 1.4(10^8)$$
$$D = 1.105(10^8)$$
$$FS = \frac{1.289(10^8)}{1.105(10^8)}$$
$$\underline{\underline{FS = 1.167}}$$

9. Given: Planar block slide in sketch with cohesion destroyed
Find: acceleration a



Solution:

Assume $\sigma = 0$ (no surcharge)
then $FS = \dfrac{R}{D}$
But also for the slide mass center
$$F = ma$$
$$m = \frac{W}{g}$$
$$F = D - R$$
$$\therefore D(1 - FS) = \left(\frac{W}{g}\right)a$$

$\therefore a > 0$ if $FS < 1$

$$FS = \frac{N' \tan\phi}{W_s}$$

$$FS = \frac{W \cos\alpha \tan\phi}{W \sin\alpha}$$

$$FS = \frac{\tan\phi}{\tan\alpha}$$

$$FS = \frac{\tan 29}{\tan 40}$$

$$FS = 0.66$$

acceleration – yes downhill = parallel to slide surface
$FS = 0.66$

then $D(1 - 0.66) = \frac{W}{g} a$

$D = W \sin\alpha$

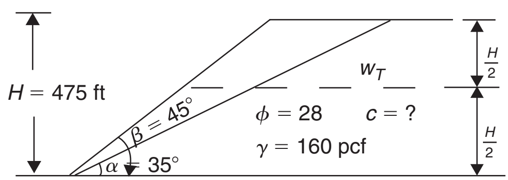
$\therefore a = g \sin\alpha(1 - 0.66)$

$a = 32.2(\sin 40)(0.34)$

$\underline{\underline{a = 7.04 \text{ ft/s}^2}}$

← tangential direction
(parallel to failure surface)

10. Given: Planar block slide
Find: α for $FS = 1.5$.



Solution:

(Free body)

$\Sigma_n F = 0 \qquad W_n - N' - P = 0$

$N' = W_n - P$

$W_n = W \cos\alpha$

$W_s = W \sin\alpha$

$W = \frac{\gamma b H^2}{2}(\cot\alpha - \cot\beta)$

$= (150)(1)\left(\frac{475}{2}\right)^2(\cot 35 - \cot 45)$

$= 7.728$

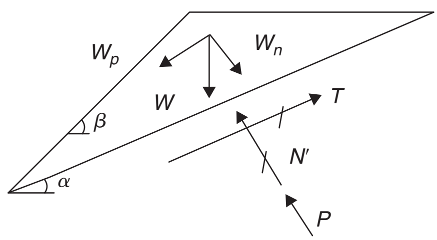
$W = 7.252(10^6)$ lbf

$W_n = 7.252(10^6) \cos 35$

$\underline{W_n = 5.941(10^6) \text{ lbf}}$

$W_s = 7.252(10^6) \sin 35$

$\underline{W_s = 4.156(10^6) \text{ lbf}}$



Water force must check for water head if $\tan\beta > 2\tan\alpha$, then $h_w = \frac{z}{2}$ else

$$h_w = \frac{z}{2}\left(-1 + \frac{\tan\beta}{\tan\alpha}\right)$$

check

$\tan 45° > 2 \tan 35°$

$(1) > 2(0.700)$

no.

$$\therefore h_w = \left(\frac{475}{2}\right)\left(\frac{1}{2}\right)\left(-1 + \frac{\tan 45}{\tan 35}\right)$$

$$\underline{\underline{h_w = 50.84 \text{ ft}}}$$

$$p_{\max} = (62.4)(50.84)$$

$$\bar{p} = \frac{p_{\max}}{2} = \frac{(6.24)(50.84)}{2}$$

$$\underline{\bar{p} = 1.586(10^3) \text{ psf}}$$

$$P = \overline{p}A$$

$$= 1.586(10^3)(1)\left(\frac{H_w}{\sin\alpha}\right)$$

$$= 1.586(10^3)(1)\left(\frac{\frac{475}{2}}{\sin 35}\right)$$

$$\underline{\underline{P = 6.567(10^5) \text{ lbf}}}$$

$$N' = 5.941(10^6) - 0.6567(10^6)$$

$$\underline{\underline{N' = 5.284(10^6)}}$$

$$FS = \frac{N' \tan\phi + cbL}{W_s}$$

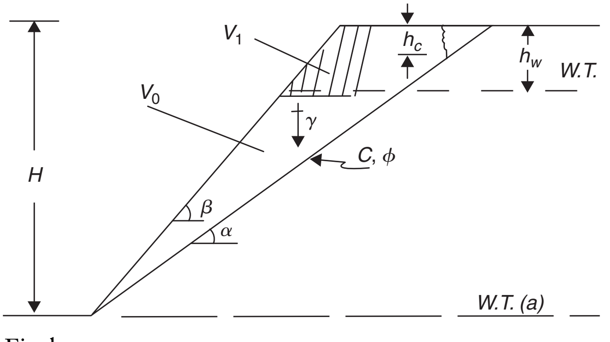
$$cbL = (1.5)(4.156)(10^6) - \tan 28(5.284)10^6$$

$$= 3.424(10^6)$$

$$c = \frac{3.424(10^6)}{\frac{(1)(475)}{\sin 35}}$$

$$\underline{\underline{c = 4.135(10^3) \text{ psf}}}$$

11. Given: Planar block slide in sketch:



Find:

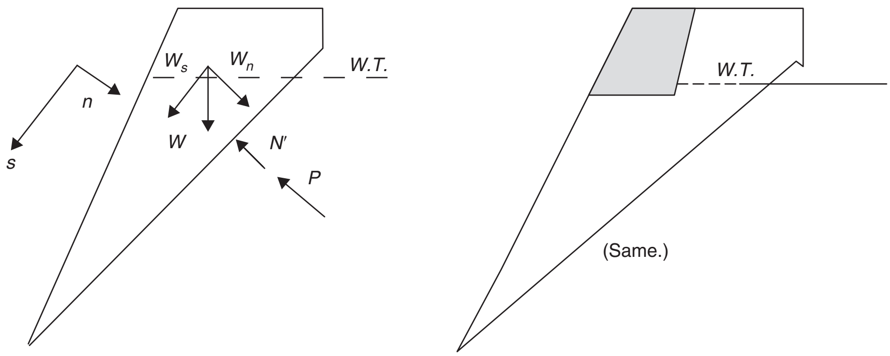
(a) Formula for *FS* with relieving bench V_1

(b) Formula for *FS* with toe berm

Solution:

By definition: $FS = \dfrac{R}{D}$

$R =$ resisting forces
$D =$ driving forces



(a) (b)

Assume M-C criterion

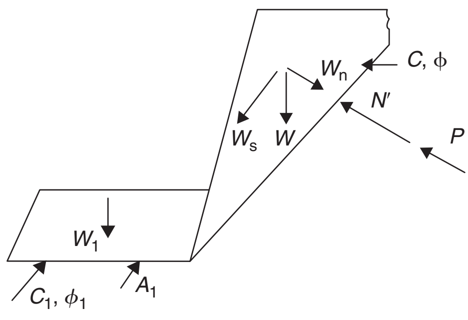
$$FS_a = \frac{N' \tan\phi + C}{W \sin\alpha} \qquad FS_a = \frac{N' \tan\phi + C}{W \sin\alpha}$$

$$\therefore FS_a = \frac{\tan\phi}{\tan\alpha} + \frac{C}{\gamma V_0 \sin\alpha} \qquad FS_b = \frac{\tan\phi}{\tan\alpha} + \frac{C}{\gamma (V_0 - V_1) \sin\alpha}$$

$C = cA,$
$A =$ area of failure surface
$N' = (W \cos\alpha - P)$
$W = \gamma V_0$

$$\therefore FS_a = \frac{(W\cos\alpha - P)\tan\phi + C}{W \sin\alpha}$$

But $P = 0$ when W.T. is below to toe. The first terms are the same, but the second term in FS_b is greater because $V_0 - V_1$ is less then V_0 is FS.



$$FS = \frac{(N' \tan\phi + C) + W_1 \tan\phi + C_1}{W_s}$$

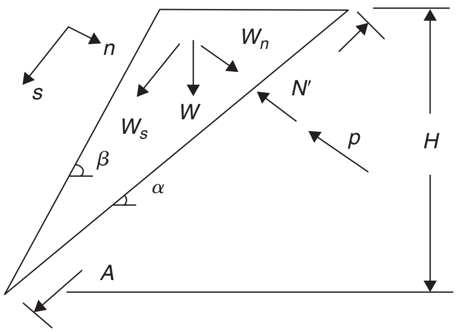
$$FS(\textit{with berm}) = FS(\textit{without berm}) + \frac{W_1 \tan\phi_1 + c_1 A_1}{W_s} > FS(\textit{without berm})$$

The added resistance $W_1 \tan\phi_1 + C_1$, comes without added driving force and thus increases the FS.

12. Given: sketch, data
No tension crack, no benches, $\alpha = 29°$, $\gamma = 156$ pcf, $\beta = 50°$, persistence $= 0.87$
$c_r = 64{,}800$ psf $\quad c_j = 1{,}620$ psf
$\phi_r = 32°$ $\quad \phi_j = 25°$

Find:

(a) $H_{\max}$ when WT at crest
(b) $H_{\max}$ when WT at toe



Solution:

$$FS = \frac{R}{D}$$
$$D = W \sin\alpha$$
$$R = N' \tan\phi + C$$
$$N' = (W_n - p)$$
$$C = cA$$
$$W_n = W\cos\alpha$$

(b) When depressurized $P = 0$

$$W = \frac{\gamma H^2}{2}(\cot\alpha - \cot\beta)$$
$$A = \frac{H}{\sin\alpha}$$
$$FS = \frac{W\cos\alpha\tan\phi}{W\sin\alpha} + \frac{\frac{cH}{\sin\alpha}}{W\sin\alpha}$$
$$FS = \frac{\tan\phi}{\tan\alpha} + \frac{\frac{cH}{\sin\alpha}}{\frac{\gamma H^2}{2}(\text{cotan}\,\alpha - \text{cotan}\,\beta)\sin\alpha}$$
$$\therefore \left(FS - \frac{\tan\phi}{\tan\alpha}\right)\left(\frac{\gamma}{2}\right)\left(\frac{\text{cotan}\,\alpha - \text{cotan}\,\beta}{\frac{c}{\sin^2\alpha}}\right) = \frac{1}{H}$$

By inspection $\frac{1}{H}$ is minimum when FS is minimum

$\therefore H_{\max}$ occurs of $FS = 1$.

need rock mass c, ϕ

$c = (1 - p)c_r + pc_j$
$\quad = (1 - 0.87)64{,}800 + 0.87(1{,}620)$
$c = 8{,}424 + 1{,}409$
$\underline{\underline{c = 9{,}833 \text{ psf}}}$

$\tan\phi = (1 - p)\tan\phi_r + p\tan\phi_j$
$\quad = (1 - 0.87)\tan 32° + 0.27\tan 25°$
$\tan\phi = 0.08123 + 0.4057$
$\tan\phi = 0.4829$
$\underline{\underline{\phi = 25.96 \approx 26°}}$

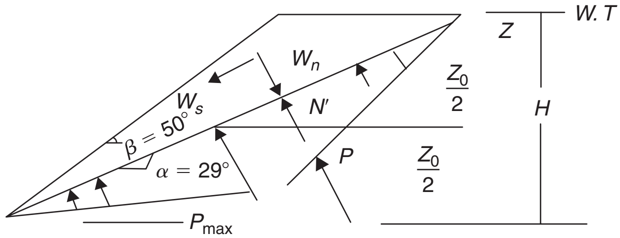
$$\frac{1}{H} = \left(1 - \frac{0.4829}{0.5543}\right)\left(\frac{156}{2}\right)\left(\frac{0.96495}{\frac{9833}{(0.4848)^2}}\right)$$

$$= (0.12157)(78)(0.2307)(10^{-4})$$

$$\frac{1}{H} = 2.188(10^{-4})$$

$$\underline{\underline{H = 4.571\text{ ft}}}$$

$\leftarrow$ $H_{\max} dry$



Water force:

if $\tan\beta > 2\tan\alpha$ then use $\dfrac{z_0}{2}$

$\tan(50) > 2\tan 29°$

$1.1918 > 2(0.5543) = 1.1086$

o.k.

$$P = 2\left(\gamma_w \frac{z_0}{2}\right)\left(\frac{1}{2}\right) L_w$$

$$= \gamma_w \frac{H}{2}\frac{\frac{H}{2}}{\sin\alpha}$$

$$P = \frac{\gamma_w H^2}{4\sin\alpha}$$

$$FS = \frac{(W\cos\alpha - p)\tan\phi + \frac{cH}{\sin\alpha}}{W\sin\alpha}$$

$$= \frac{\tan\phi}{\tan\alpha} + \frac{\frac{c}{\sin\alpha}}{\left(\frac{\gamma}{2}\right) H(\text{cotan}\,\alpha - \text{cotan}\,\beta)\sin\alpha}$$

$$- \frac{\frac{\gamma_w H^2}{4\sin\alpha}\tan\phi}{\left(\frac{\gamma}{2H^2}\right)(\text{cotan}\,\alpha - \text{cotan}\,\beta)\sin\alpha}$$

$$K = \frac{\gamma_w \tan\phi}{\left(\frac{\gamma}{2}\right)(\sin^2\alpha)(\text{cotan}\,\alpha - \text{cotan}\,\beta)}$$

$$= \frac{\left(\frac{1}{4}\right)(62.4)\tan 26°}{\left(\frac{156}{2}\right)(\sin^2 29°)(\text{cotan}\,29 - \text{cotan}\,50)}$$

$$K = 0.4301$$

$$\underline{\underline{\left[FS - \frac{\tan\phi}{\tan\alpha} + K\right]\left[\left(\frac{\gamma}{2}\right)\frac{\text{cotan}\,\alpha - \text{cotan}\,\beta}{\frac{c}{\sin^2\alpha}}\right] = \frac{1}{H}}}$$

$\leftarrow$ formula

$$[0.5502]\left[\left(\frac{156}{2}\right)\frac{\text{cotan}\,29-\text{cotan}\,50}{(9{,}833)\left(\frac{1}{\sin^2 29}\right)}\right]=\frac{1}{H}$$

$$9.899(10^{-4})=\frac{1}{H}$$

$$\underline{\underline{H=1{,}010\text{ ft}}} \qquad \leftarrow H_{max}\text{wet}$$

13. Given: Planar block slide

$\gamma = 158$ pcf

$\phi_n = 38°$ $\quad \phi_j = 27°$
$c_r = 1{,}000$ psi $\quad c_j = 15$ psi } Mohr–Coulomb

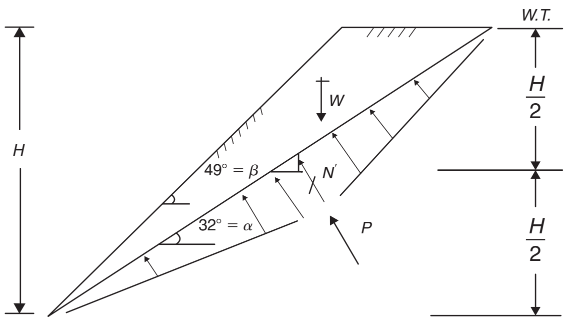
$A_j/A = 0.93$

bench height = 55 ft

b = breadth

Find: H_{max}.

Solution:



H_{max} occurs when $FS = 1$

$$FS=\frac{R}{D}$$

$R = W_n' \tan\phi + cLb$

$D = W_s$

$\therefore W_s = W_n \tan\phi + cLb$

$W \sin\alpha = (W\cos\alpha - P)\tan\phi + cLb$

$$\begin{aligned} W &= \frac{\gamma H^2}{2}(\text{cotan}\,\alpha-\text{cotan}\,\beta)\\ &= (158)\left(\frac{1}{2}\right)H^2(\text{cotan}\,32-\text{cotan}\,49)\\ \underline{W} &\underline{= 57.75\,H^2} \end{aligned}$$

$$57.75\,H^2\sin 32° = 57.75\,H^2\cos 32\tan\phi - P\tan\phi + \frac{cH(1)}{\sin 32}$$

$$\underline{\underline{30.60 = 48.98\tan\phi + \frac{c}{H}1.887 - \frac{P\tan\phi}{H^2}}}$$

need, ϕ, c for rock mass

$$\begin{aligned} c &= (1-0.93)c_r + 0.93c_j\\ &= 0.07(1{,}000) + 0.93(10.0)\\ \underline{c} &\underline{= 79.3\text{ psi}\underline{(11{,}719\text{ psf})}} \end{aligned}$$

$$\tan\phi = 0.07\tan 38^\circ + 0.93\tan 27^\circ$$
$$\underline{\underline{\tan\phi = 0.5286}} \quad \phi = 27.9^\circ$$

return

$$30.60 = 48.98(0.5286) + \frac{(11{,}419)(1.887)}{H} - \frac{P}{H^2}(0.5286)$$

need: $P = \bar{p}Lb$

$$= \left(\frac{\gamma_w H}{4}\right)\left(\frac{H}{\sin\alpha}\right)(1)$$
$$\frac{P}{H^2} = \frac{(62.4)}{4}\frac{H^2}{\sin\alpha}$$

$$30.60 = 25.89 + \frac{1}{H}(22{,}812) - \frac{62.4}{4\sin 32^\circ}(0.5286)$$

$$\frac{1}{H} = \frac{30.60 - 25.89 + 15.56}{21{,}549}$$

$$\underline{\underline{H = 1{,}065\text{ ft.}}}$$

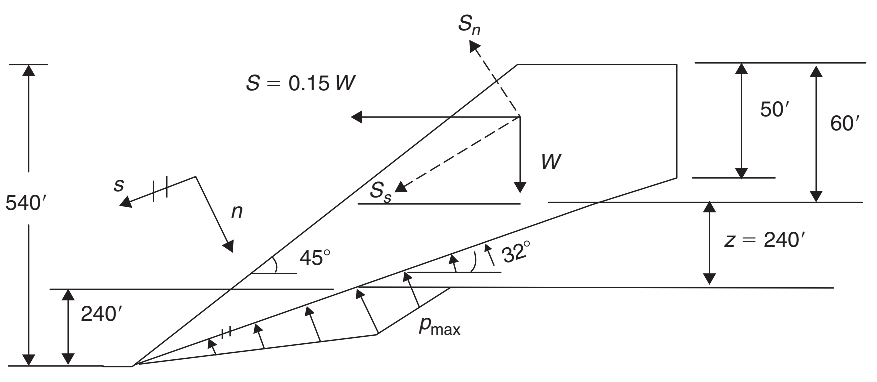
14. Given: Planar block slide data

$FS(\text{min}) = 1.05$

Bench height = 60′

Persistance = 79% $\left(\frac{A_j}{A}\right)$

Wt = $1.351(10^7)$ lbf per ft of thickness.



$\phi_r = 33^\circ$ $\quad c_r = 2{,}870$ psi $\quad \phi_j = 28^\circ$ $\quad c_j = 10.0$ psi $\quad \gamma = 158$ pcf

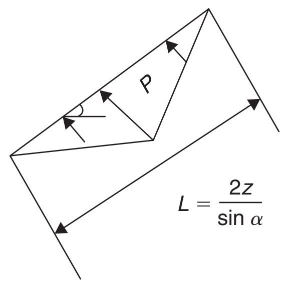
Find: If $FS = 1.05$ possible.

Solution:

Assume water distribution as $p = \gamma_w z$ where z is 1/2 distance (vertical) to toe

$$z = \frac{(540 - 60)}{2} \qquad p_{\max} = \frac{(62.4)(240)}{144}$$

$$\underline{z = 240\text{ ft},} \qquad \underline{p_{\max} = 104\text{ psi}}$$



$$\overline{P} = \frac{p_{\max}}{2}$$
$$\overline{P} = 52\text{ psi}$$
$$P = \overline{p}A$$
$$= \overline{p}(1)\left(\frac{480}{\sin(32)}\right)(144)$$
$$\underline{P = 6.783(10^6)\text{ lbf}}$$

$$W_n = W\cos\alpha$$
$$W_n = 1.351(10^7)\cos 32$$
$$\underline{W_n = 1.146(10^7)\text{ lbf}}$$

$$W_s = 1.351(10^7)\sin 32$$
$$\underline{W_s = 7.159(10^6)\text{ lbf}}\text{ (no seismic force)}$$

$$W_n' = W_n - P$$
$$= 1.146(10^7) - 6.783(10^6)$$
$$\underline{W_n' = 4.6745(10^8)\text{ lsf}}\text{ (no seismic force)}$$

$$W_n' = W_n - P - S_s$$

$$S_n = S\sin\alpha \qquad S_s = S\cos\alpha$$
$$= 0.15\ W\sin 32 \qquad = 0.15\ W\cos\alpha$$
$$= 0.15\ W_s \qquad = 0.15\ W_n$$
$$S_n = 0.15\,(7.159)(10^6) \qquad = 0.15\,(1.146)10^7$$
$$\underline{S_n = 1.074(10^6)\text{ lbf}} \qquad \underline{S_s = 1.719\,(10^6)\text{ lbf}}$$

$$W_n' = 4.6745 - 1{,}074(10^6)$$
$$\underline{W_n' = 3{,}600(10^6)\text{ lbf} \quad \text{with seismic force}}$$

need: c, ϕ

$$c = 0.79\,c_j + 0.21\,c_r$$
$$c = (0.79)(10) + (0.21)(2870)$$
$$c = 7.9 + 603$$
$$\underline{c = 610.9\text{ psi}}$$

$$\tan\phi = (0.79)\tan 28^\circ + (0.21)\tan 33^\circ$$
$$\tan\phi = 0.556$$
$$\underline{\phi = 29.1^\circ}$$

$$FS = \frac{W_n'\tan\phi + cA}{W_s + S_s},$$

$$= \frac{3.60(10^6)\tan 29.1 + (611)(144)(92.5)(1)}{7.159(10^6) + (1.719)(10^6)}$$

Note:

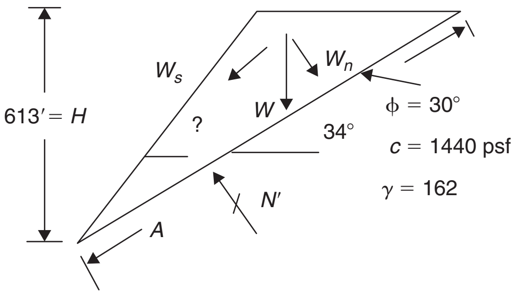
$$L = \frac{540 - 50}{\sin 32}$$

$$= \frac{2.004(10^6) + 8.139(10^7)}{8.878(10^6)}$$

$$= \frac{81.72(10^6)}{8.878(10^6)}$$

$$\underline{\underline{FS = 9.39}} \qquad \text{Yes.}$$

15. Given: Planar block slide Mohr–Coulomb failure
Find: β_{max} (dry).



Solution:

$$F_S = \frac{R}{D}$$

$$= \frac{N' \tan\phi + cA}{W_s}$$

$$N' = W_n$$

$$W_n = W\cos\alpha$$

$$A = \frac{H}{\sin\alpha} \qquad \text{(1 ft thick)}$$

$$W_s = W\sin\alpha$$

$$\therefore FS = \frac{\tan\phi}{\tan\alpha} + \frac{cH}{(\sin\alpha)^2 W}$$

$$\text{But } W = \frac{\gamma H^2}{2}(1)(\cot\alpha - \cot\beta)$$

So FS is min when β is max.

$$FS_{min} = 1.0$$

$$\therefore 1 = \frac{\tan\phi}{\tan\alpha} + \frac{cH}{\sin^2(\alpha)W}$$

$$1 = \frac{\tan 30^\circ}{\tan 34^\circ} + \frac{(1{,}440\text{ psf})(613\text{ ft})(1\text{ ft})}{\sin^2(34^\circ)\, W}$$

$$1 = 0.856 + \frac{2.823(10^6)}{W}$$

$$W = \frac{2.823(10^6)}{0.144}$$

$$\underline{\underline{W = 19.598(10^6)\text{lbf}}}$$

$$W = \frac{\gamma H^2}{2}(\cot\alpha - \cot\beta)$$

$$\cot\alpha - \cot\beta = \frac{(2)(19.598)(10^6)}{(162)(613)^2}$$

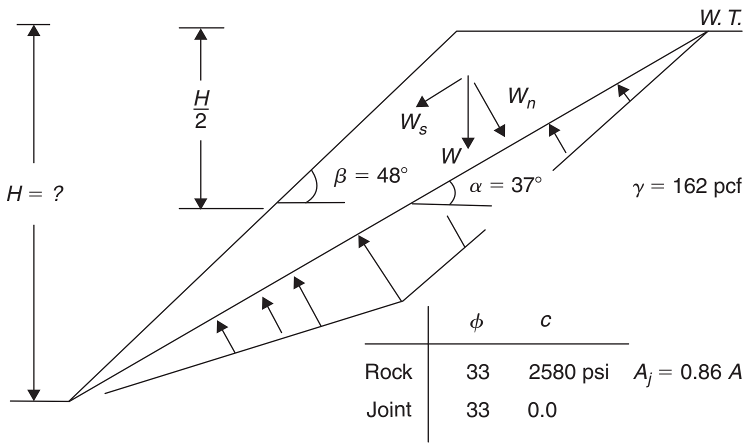
$$\cot\alpha - \cot\beta = 0.64387$$

$$\cot\beta = \cot 34 - 0.64357$$

$$\cot\beta = 0.83869$$

$$\underline{\underline{\beta_{max} = 50^\circ}} \qquad \xleftarrow{\beta_{max}}$$

16. Given: Slope, sketch, no seismic load, etc.



Find: H at $FS = 1.15$.

Solution:

$$
\begin{aligned}
FS &= \frac{F_R}{F_D} \\
F_D &= W_s \\
&= W \sin\alpha \\
F_D &= \frac{\gamma H^2}{2}(b)(\cot\alpha - \cot\beta)\sin(\alpha_j) \\
&= H^2\frac{(162)(1)}{2}(\cot 37 - \cot 48)\sin(37) \\
F_D &= H^2(81)(0.4266)\sin(37^\circ) \\
&\underline{\underline{F_D = 20.80\,H^2}} \\
F_R &= W_n' \tan\phi + cL \\
W_n' &= W_n - P \\
W_n &= W\cos\alpha \\
&= H^2(81)(0.4266)\cos(37) \\
&\underline{\underline{W_n = 27.60\,H^2}}
\end{aligned}
$$

$$
\begin{aligned}
P &= \bar{p}A \qquad \bar{p} = \frac{p_{\max}}{2},\ A = Lb \\
&= \left(\gamma_w \frac{H}{2}\right)\left(\frac{1}{2}\right)(1)\left(\frac{H}{\sin\alpha}\right) \\
&= \left(\frac{62.4}{4}\right)H^2\frac{1}{\sin\ 37^\circ} \\
&\underline{\underline{P = 25.92\,H^2}}
\end{aligned}
$$

$$cL = \left(c_r \frac{A_n}{A} + c_j \frac{A_j}{A}\right)\left(\frac{H}{\sin\alpha}\right)$$

$$= (0.14)(2{,}580)(144)\left(\frac{H}{\sin 37^\circ}\right)$$

$$\underline{\underline{cL = 8.643(10^4)H}}$$

$$\therefore \frac{F_R}{F_D} = \frac{(27.60\,H^2 - 25.92\,H^2)\tan(33) + 8.643(10^4)H}{20.80\,H^2}$$

$$\frac{F_R}{F_D} = 0.0525 + \frac{4.155(10^3)}{H}$$

But $F_S = \dfrac{F_R}{F_D}$

$$1.15 = 0.0525 + \frac{4.155(10^3)}{H}$$

$$\frac{1}{H} = 2.641(10^{-4})$$

$$\underline{\underline{H = 3{,}786\text{ ft}}} \qquad \longleftarrow \text{depth at } FS = 1.15$$

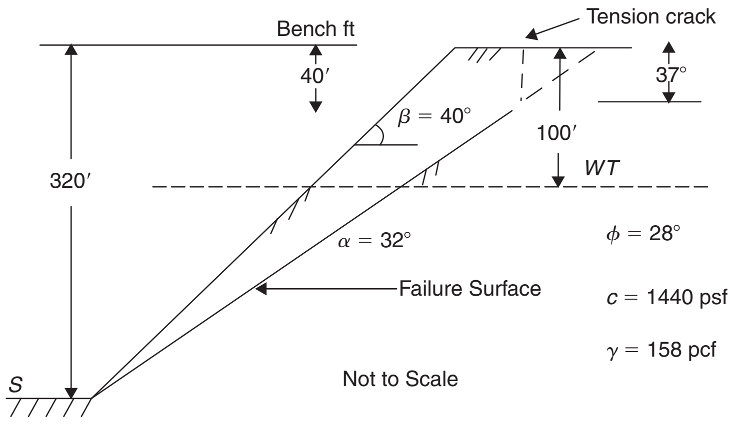
17. Given: Sketch of the potential slope failure shown in the sketch,

Find:

(a) Factor-of safety of a cable bolted slope when the water table is drawn down 100 ft
Bench height = 40′ (vertical bolt spacing)
Horizontal bolt spacing = 20′
Bolt angle = 5° down
Bolt loading = 700 kips/hole

(b) Factor of safety of the same slope but without bolts when the water table is drawn below the toe

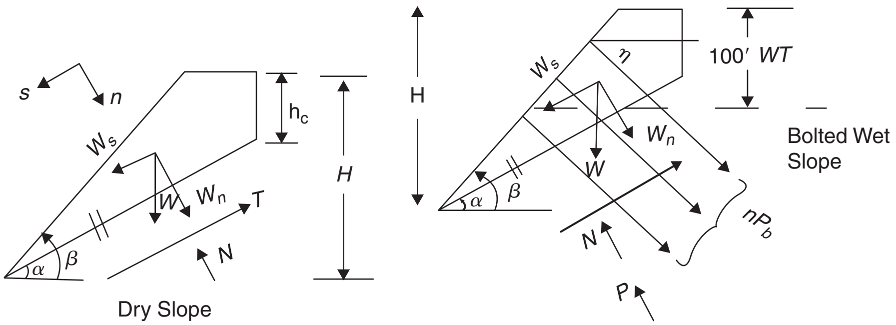
(c) Reasons for preferring one over the other.



Sketch for problem with given data.

Solution:

Free body diagram.



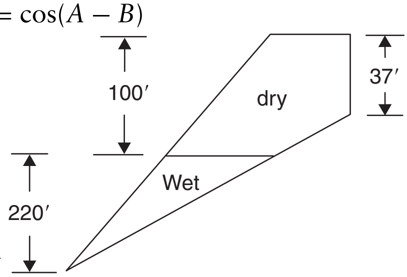
(a) Factor of safety of a bolted slope with water table

$$\eta = \text{bolting angle} = -5^\circ$$
$$n = \text{number of holes per row}$$
$$= \frac{320'}{40'} = 8$$
$$\Sigma F_n = 0$$
$$N = W\cos(\alpha) - P_s + n\,P_b\sin(\alpha-\eta)$$
$$T = N\tan(\phi) + c\left[\frac{H-h_c}{\sin(\alpha)}\right]b$$
$$R = T + nP_b\cos(\alpha-\eta)$$
$$D = W\sin(\alpha)$$

$$FS = \frac{R}{D}$$
$$= \frac{[W\cos(\alpha) - P_s + nP_b\sin(\alpha-\eta)]\tan(\phi) + c\left[\frac{H-h_c}{\sin(\alpha)}\right]b + n\,P_b\cos(\alpha-\eta)}{W\sin(\alpha)}$$
$$FS = \frac{\tan(\phi)}{\tan(\alpha)} - \frac{P_s\tan(\phi)}{W\sin(\alpha)} + \frac{c(H-h_c)b}{W\sin^2(\alpha)} + \frac{nP_b[\sin(\alpha-\eta)\tan\phi + \cos(\alpha-\eta)]}{W\sin\alpha}$$
$$= \frac{\tan(\phi)}{\tan(\alpha)} - \frac{P_s\tan(\phi)}{W\sin(\alpha)} + \frac{c(H-h_c)b}{W\sin^2(\alpha)} + \frac{nP_b\cos(\alpha-\eta-\phi)}{W\sin\alpha\cos\phi}$$

using—$\sin(A)\sin(B) + \cos(A)\cos(B) = \cos(A-B)$

$FS = FS_{\text{wet}}\text{ unbolted} + \Delta FS\text{ bolt}$



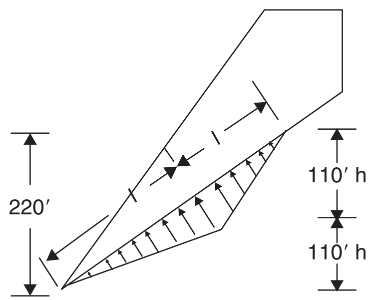
Assume $e = 18\%$

$$G = \frac{\gamma_{\text{dry}}}{\gamma_w}(1+e)$$
$$G = \frac{158}{62.4}(1.18) = 2.99 \rightarrow \text{grain SG gravity}$$

$$\gamma_{\text{wet}} = \left(\frac{2.99+0.18}{1.18}\right)62.4 = \left(\frac{G+Se}{1+e}\right)\gamma_w$$
$$= 168 \text{ pcf}$$
$$W_{\text{wet}} = \gamma_{\text{dry}} V_{\text{dry}} + (\gamma_{\text{wet}} - \gamma_{\text{dry}}) V_{\text{wet}}$$
$$V_{\text{wet}} = \frac{(H_w)^2}{2}[\text{cotan}(\alpha) - \text{cotan}(\beta)]b$$
$$= \frac{(220)^2}{2}[\text{cotan}(32°) - \text{cotan}(40°)]1$$
$$= 9{,}888 \text{ ft}^3$$

$$V_{\text{dry}} = \frac{H^2}{2}[\text{cotan}(\alpha) - \text{cotan}(\beta)]b - \left[\frac{1}{2}h_c^2 \text{cotan}(30°)\right]$$
$$= \frac{320^2}{2}[\text{cotan}(32°) - \text{cotan}(40°)]1 - \left[\frac{37^2}{2}\text{cotan}(32°)\right]$$
$$= 1{,}9824 \text{ ft}^3$$
$$W_{\text{wet}} = (158)1{,}9824(20) + (168 - 158)9{,}888(20)$$
$$= 64{,}621{,}440 \text{ lbs}$$

$$P_s = 2\int_o^l Pb\,\mathrm{d}l$$
$$= 2\int_o^l b\gamma_w l \sin(\alpha)\mathrm{d}l$$
$$= 2\frac{\gamma_w h l b}{2} = \gamma_w hlb$$
$$= (62.4)(110)\left[\frac{110}{\sin(32°)}\right](20) = 28{,}496{,}420 \text{ lbf}$$



$$FS = \frac{\tan(28°)}{\tan(32°)} - \frac{(28{,}496{,}420)\tan(28°)}{(64{,}621{,}440)\sin(32°)} + \frac{1{,}440(320-37)(20)}{(64{,}621{,}440)\sin^2(32°)}$$
$$+ \frac{(8)(700\times 10^3)\cos(32°+5°-28°)}{(64{,}621{,}440)\sin(32°)\cos(28°)}$$
$$= 0.851 - 0.442 + 0.449 + 0.183$$
$$FS = 1.041$$

← *FS* wet, bolted slope

(b) Factor of safety of a dry slope.

$$W = \gamma_{\text{dry}} V_{\text{dry}}$$
$$= (158)19{,}824(20) = 6{,}26{,}43{,}840 \text{ lbs}$$

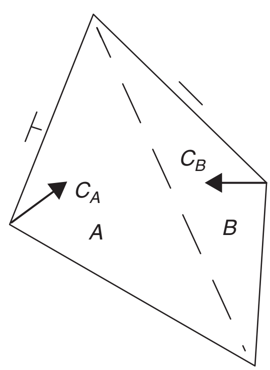
$$\begin{aligned}
\Sigma F_n &= 0 \\
N &= W\cos(\alpha) \\
T &= N\tan\phi + c(H-h_c)\frac{b}{\sin(\alpha)} \\
R &= T \\
&= [W\cos(\alpha)]\tan(\phi) + c(H-h_c)\frac{b}{\sin(\alpha)} \\
D &= W\sin(\alpha)
\end{aligned}$$

$$\begin{aligned}
FS_{\text{dry}} &= \frac{R}{D} = \frac{W\cos(\alpha)+\tan(\phi)+c(H-h_c)\frac{b}{\sin(\alpha)}}{W\sin\alpha} \\
&= \frac{\tan(\phi)}{\tan(\alpha)} + \frac{c(H-h_c)b}{W\sin^2(\alpha)} \\
&= 0.851 + \frac{1440\,(320-37)\,(20)}{6{,}26{,}43{,}840\sin^2(32)} \\
&= 0.851 + 0.463
\end{aligned}$$

$\underline{\underline{FS = 1.314}}$ ← *FS* dry, unbolted

Wedge Failures

18. Given: Wedge data



A: $\alpha = 0°$ $\delta = 60°$
B: $\alpha = 90°$ $\delta = 60°$

Find: normals C_A, C_B.

Solution:

	x	y	z
C	$\sin\delta\sin\alpha$	$\sin\delta\cos\alpha$	$\cos\delta$
A:	$\left(\frac{\sqrt{3}}{2}\right)(0)$	$\left(\frac{\sqrt{3}}{2}\right)(1)$	$\left(\frac{1}{2}\right)$
B:	$\left(\frac{\sqrt{3}}{2}\right)(1)$	$\left(\frac{\sqrt{3}}{2}\right)(0)$	$\left(\frac{1}{2}\right)$

$$\therefore \left.\begin{aligned} \vec{C}_A &= \left(0, \tfrac{\sqrt{3}}{2}, \tfrac{1}{2}\right) \\ \vec{C}_B &= \left(\tfrac{\sqrt{3}}{2}, 0, \tfrac{1}{2}\right) \end{aligned}\right\} \text{direction cosines}$$

$$\left.\begin{aligned} &\vec{C}_A: (90°, 30°, 60°) \\ &\vec{C}_B: (30°, 90°, 60°) \end{aligned}\right\} \text{direction angles}$$

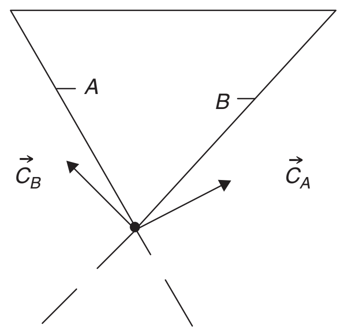
19. Given: The wedge from problem 18,

$c_A = \left(0, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$c_B = \left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$

Find: Dip direction and dip of the line of intersection $\vec{s}$.

Solution:



$\vec{s} = \vec{c}_A \times \vec{c}_B$	form "determinant"		
	i	j	h
c_A:	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
c_B:	$\frac{\sqrt{3}}{2}$	0	$\frac{1}{2}$

$$\vec{s} = \left(\frac{\sqrt{3}}{4}\ \frac{\sqrt{3}}{4} - \frac{3}{4}\right) \qquad \text{dir. numbers}$$

$$|s| = \left[\left(\frac{\sqrt{3}}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2 + \left(-\frac{3}{4}\right)^2\right]^{1/2}$$

$$|s| = \frac{\sqrt{15}}{\sqrt{16}} = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{\sqrt{5}}{1}\right)$$

direction cosines:

	S_x	S_y	S_z
$\vec{S}$	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{\sqrt{3}}{\sqrt{5}}$

$\alpha_s = \underline{\underline{45°}}$

$\delta_s = \underline{\underline{50.8°}}$

$\alpha_s, \delta \longleftarrow$

$$\tan\alpha_s = \frac{S_x}{S_y}$$

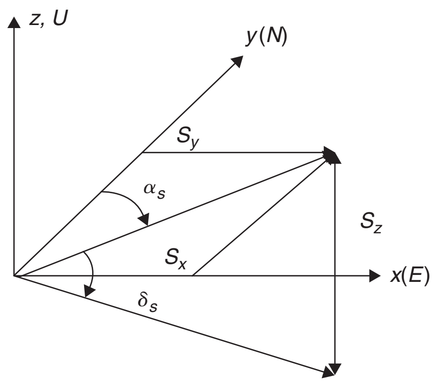
$$\tan\alpha_s = \frac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{5}}}$$

$$\tan\alpha_s = 1$$

$$\therefore \underline{\underline{\alpha_s = 45°}}$$

$$\tan\delta_s = \frac{-S_3}{[S_x^2 + S_y^2]^{1/2}}$$

$$\tan\delta_s = \frac{+\frac{\sqrt{3}}{\sqrt{5}}}{\sqrt{\frac{2}{5}}} = +\frac{\sqrt{3}}{\sqrt{2}} \quad \therefore \delta_s = \underline{\underline{50.8°}}$$

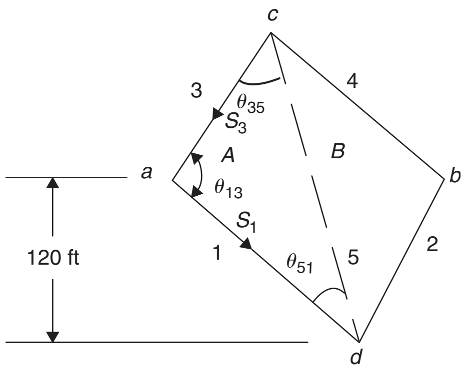


20. Given: Table 2.7 data and $\overline{ad} = 120$ ft

Find: Joint plane areas A_A, B_A (no tension crack).

Solution:

Vectors S_1, S_3 along lines 1 and 2 can be found from intersections of (1) Joint plane A and Face F, and (2) Joint Plane A and Upland U, since normals to F and

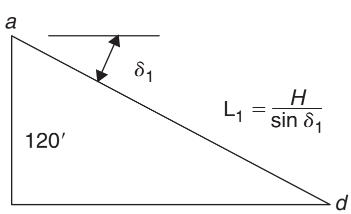


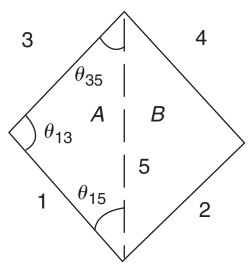
U can be found from their dip directions and dips. The normal to A is known from problem 18.

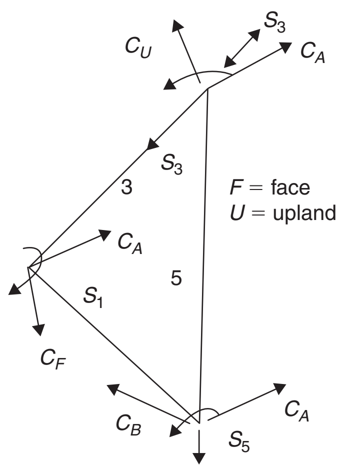
- The angle θ_{13} can be found from $S_1 \cdot S_3 = |S_1||S_3| \cos\theta_{13}$ $|S_1| = 1$ $|S_3| = 1$ by normalization
- Similarly angles θ_{35} & θ_{51} can be found
- The distance L_1 can be found from the dip of line 1 (S_1) and $H = 120'$.
- The distance L_3 can be found.

From the sine law

$$\frac{L_3}{\sin\theta_{15}} = \frac{L_1}{\sin\theta_{35}} = \frac{L_5}{\sin\theta_{13}}$$







Need vectors S_1, S_3, S_5 that are formed by planes of intersection e.g. $S_5 = \vec{C}_A \times \vec{C}_B$ (as in problems 18 & 19) Need direction cosines of C_A, C_F, C_U, C_B

	C: $\sin\delta\sin\alpha$	$\sin\delta\cos\alpha$	$\cos\delta$	
(problem 18)	$C_A = 0$	0.8660	0.5000	$\lvert C_A\rvert = 1$
(Problem 18)	$C_B = 0.8660$	0	0.5000	$\lvert C_B\rvert = 1$
	C_F sin 85 sin 45	sin 85 cos 45	cos 85	
	$C_F = 0.7044$	0.7044	0.08716	$\lvert C_F\rvert = 1$
	$C_U = 0.06163$	0.06163	0.9962	$\lvert C_U\rvert = 1$

CROSS PRODUCTS

$C_A \times C_B =$	0.4330	0.4330	−0.7500	$\lvert S_5\rvert = 0.9682$
	(0.4472)	(0.4472)	(−0.7746)	normalized
$C_A \times C_F =$	−0.2767	0.3522	−0.6100	$\lvert S_1\rvert = 0.7568$
	(−0.3657)	(0.4654)	(−0.8060)	normalized
$C_A \times C_Z =$	0.8321	0.0308	−0.0534	$\lvert S_3\rvert = 0.8344$
	(0.9973)	(0.03692)	(−0.06400)	normalized

dir. cos

	x	y	z
S_5	0.4472	0.4472	−0.7746
S_1	−0.3657	0.4654	−0.8060
S_3	0.9973	0.03692	−0.06400

DOT PRODUCTS

$$S_{13} = |S_1||S_3|\cos\theta_{13} = S_{1x}S_{3x} + S_{1y}S_{3y} + S_{1z}S_{3z}$$

$S_{13} = -0.2959 \quad \therefore \theta_{13} = 107^\circ$ (or 73°alt. sol.)

$S_{15} = 0.6689 \quad \therefore \underline{\theta_{15} = 48.0^\circ}$ [Look of direction of S_3]

$S_{35} = 0.5102 \quad \therefore \underline{\theta_{35} = 59.3^\circ}$

Note: $\theta_{15} + \theta_{35} + \theta_{13} = 180^\circ \; \therefore \underline{\theta_{13} = 73^\circ}$

Sine law for length

$$\frac{L_1}{\sin\theta_{35}} = \frac{L_3}{\sin\theta_{15}}$$

$$L_1 = \frac{H}{\sin\delta_1}$$

need dip of L_1, δ_1

$$\tan\delta_1 = \frac{-S_{1z}}{[S_{1x}^2 + S_{1y}^2]}$$

$$= \frac{-(-0.8060)}{[0.3657^2 + 0.4634^2]^{1/2}}$$

$$\tan\delta_1 = 1.3617$$

$$\therefore\ \delta_1 = 53.70^\circ$$

$$L_1 = \frac{120}{\sin 53.7^\circ}$$

$$\underline{\underline{L_1 = 148.9\,\text{ft}}}$$

$$L_3 = (148.9)\frac{(\sin 48)}{\sin(59.3)}$$

$$\underline{\underline{L_3 = 128.7\,\text{ft}}}$$

$$A_A = \tfrac{1}{2} L_1 L_3 |S_1||S_3| \sin\theta_{13}$$

$$= \left(\tfrac{1}{2}\right)(148.9)(128.7)\sin 73^\circ$$

$$\underline{\underline{A_A = 9{,}161\,\text{sq ft.}}} \qquad \overset{A_A}{\longleftarrow}$$

by symmetry of this problem

$$\underline{\underline{B_A = 9{,}161\,\text{sq ft}}} \qquad \overset{B_A}{\longleftarrow}$$

Computer check using WEDGE (course download) $\underline{\underline{A_A = B_A = 9{,}161\,\text{sq ft o.k.}}}$

20. (Alternative)
Given: Data in Table where the vertical distance between $\underline{a}$ and $\underline{d}$ is 120 ft,
Find: Areas of A and B (without a tension crack)

Solution:

This is a lengthy calculation best done with the aid of a computer program:
Using WEDGE from course downloads

$$A(\text{area}) = B(\text{area}) = \underline{\underline{9{,}160\,\text{ft}^2}} \qquad \longleftarrow$$

Using SWEDGE from ROCSCIENCE (same results)
Given: Data in Table & vertical distance $\overline{ad} = 120$ ft, $\gamma = 158$ pcf water below toe slope tension crack offset = 90 ft
Find: A, B areas and FS_{dry} and volume.

Solution:

Using WEDGE course download.

$\underline{\underline{A = B = 8{,}072\,\text{ft}^2}}$

$\underline{\underline{FS_{\text{dry}} = 1.28}}$

$\underline{\underline{\text{Volume} = 11{,}683\,\text{yds}^3.}}$

Same results using SWEDGE from ROCSCIENCE.

21. (no tension crack)

WEDGE	DIP DIRECTION DEGREES	DIP ANGLE DEGREES
PLANE A	0.0	60.0
PLANE B	90.0	60.0
LINE OF INTERSECTION	45.0	50.8
SLOPE FACE	45.0	85.0
UPLAND	45.0	5.0
TENSION CRACK	45.0	75.0
EXTERNAL LOAD	0.0	0.0

WEDGE HEIGHT (LEFT SIDE)	120.0 FT.
LENGTH PLANE A TRACE (UPLAND)	128.8 FT.
TENSION CRACK OFFSET (PLANE A TRACE)	128.8 FT.
EXTERNAL LOAD MAGNITUDE	0.0 KIPS
ROCK UNIT WEIGHT	158.0 PCF.

	FRICTION ANGLE DEGREES	COHESION PSF.
PLANE A	32.0	1080.0
PLANE B	37.0	1640.0

	AREA SQ. FT.	WATER FORCE KIPS
PLANE A	9160.1	0.0
PLANE B	9160.1	0.0
TENSION CRACK	0.0	0.0

WEDGE VOLUME = 3,27,142.6 CU. FT. = 12116.4 CU. YD.
WEDGE WEIGHT = 51,688.5 KIPS

FACTOR OF SAFETY = 1.33

Checks Table 18.

WEDGE	DIP DIRECTION DEGREES	DIP ANGLE DEGREES
PLANE A	0.0	60.0
PLANE B	90.0	60.0
LINE OF INTERSECTION	45.0	50.8
SLOPE FACE	45.0	85.0
UPLAND	45.0	5.0
TENSION CRACK	45.0	75.0
EXTERNAL LOAD	0.0	0.0

WEDGE HEIGHT (LEFT SIDE)	120.0 FT.
LENGTH PLANE A TRACE (UPLAND)	128.8 FT.

TENSION CRACK OFFSET (PLANE A TRACE)	90.0 FT.
EXTERNAL LOAD MAGNITUDE	0.0 KIPS
ROCK UNIT WEIGHT	158.0 PCF.

	FRICTION ANGLE DEGREES	COHESION PSF.
PLANE A	32.0	1080.0
PLANE B	37.0	1640.0

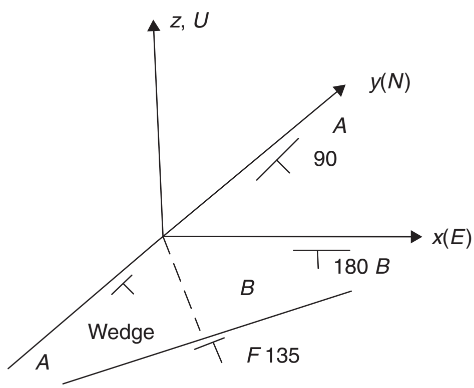
	AREA SQ. FT.	WATER FORCE KIPS
PLANE A	8071.9	0.0
PLANE B	8071.9	0.0
TENSION CRACK	1312.0	0.0

WEDGE VOLUME = 315429.3 CU. FT. = 11682.6 CU. YD.
WEDGE WEIGHT = 49837.8 KIPS

FACTOR OF SAFETY = 1.28 ← *F.S.*

22. Given: Wedge data

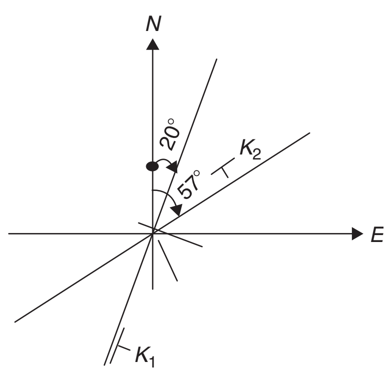
Wedge forms as shown in sketch. Line of intersection has dip direction of 135° and is in the same direction as the face dip direction. When the face dips ±90 to line of intersection, then kinematic failure is impossible. Thus, the range of concern is for face dip directions. (45°, 225°).



23. Given: Wedge data

K_1 joints $\alpha = 110°$ $\delta = 38°$
K_2 joints $\alpha = 147°$ $\delta = 42°$
n_1: (0.5785, −0.2106, 0.7880)
n_2: $(\sin\delta\sin\alpha, \sin\delta\cos\alpha, \cos\delta)$
sin(42) sin(147) sin(42) cos(147) cos(42)

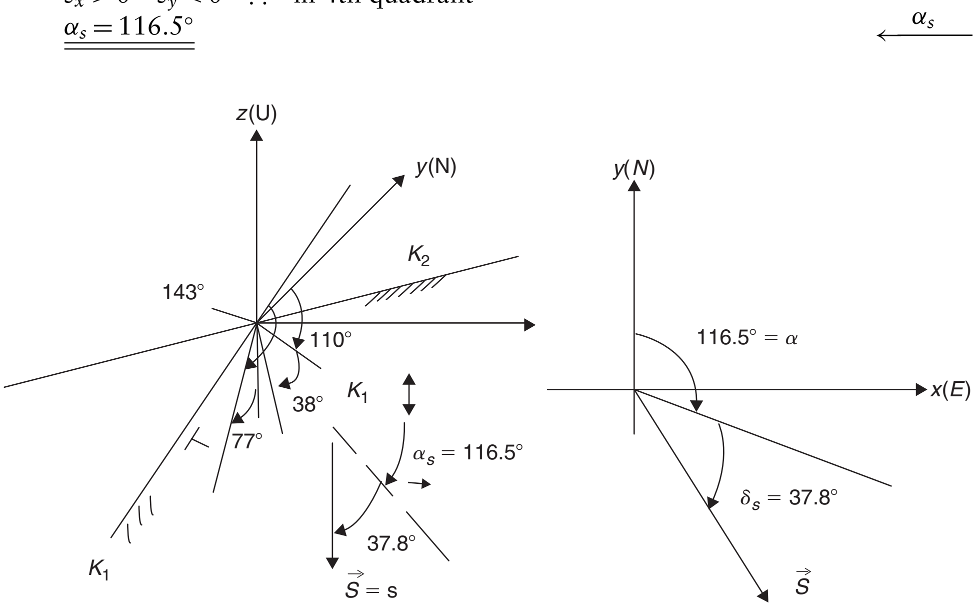
n_2: (0.3644, −0.5662, 0.7431)
$\vec{n}_1 \times \vec{n}_2 = \vec{S}(0.2857 \quad -0.1427 \quad -0.2479)$
$|s| = [(0.285)^2 + (-0.1427)^2 + (-0.2479)^2]^{1/2}$
$|s| = 0.4043$
dir. cos : $\vec{s} = (0.7067, -0.3530, -0.6132)$



$$\tan\alpha_s = \frac{s_x}{s_y} = \frac{0.7067}{-0.3530}$$
$$\tan\alpha_s = -0.2002$$

$\cos\delta' = -0.6132$
$\delta' = 127.8°$
$\delta = \delta' - 90°$
$\underline{\underline{\delta_s = 37.8°}}$ ← δ

$\alpha_s = -63.5°$ or $116.5°$
$s_x > 0 \quad s_y < 0 \quad \therefore$ in 4th quadrant
$\alpha_s = 116.5°$



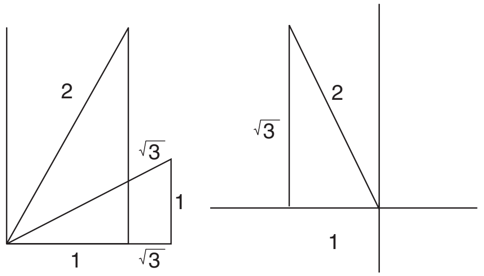
24. Given: Wedge data

Find:
(a) dip direction and dip of lines of intersection of A & B
(b) Range of azimuths that are safe
(c) Length of line formed by A & F (Face)

Solution:

(a) Need normals to A & B
Notes:

	$C_x = \sin\delta\sin\alpha$	$C_y = \cos\delta\cos\alpha$	$C_z = \cos\delta$
	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
B:	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)$	$\frac{1}{2}$
S:	$\left[\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)+\frac{\sqrt{3}}{4}\frac{1}{2}\right]$	$-\left(\frac{\sqrt{3}}{4}\frac{1}{2}-\frac{3}{4}\frac{1}{2}\right)$	$\left[\frac{\sqrt{3}}{4}\left(-\frac{\sqrt{3}}{4}\right)-\frac{3}{4}\cdot\frac{3}{4}\right]$
S:	$\frac{3+\sqrt{3}}{8}$	$\frac{3-\sqrt{3}}{8}$	$-\left(\frac{9+3}{16}\right)$
S:	0.9915	0.1585	−0.7500



$$\sin\alpha = \frac{0.5915}{(0.5915^2 + 0.1585^2)^{1/2}} = \frac{S_x}{(S_x^2 + S_y^2)^{1/2}}$$

$$\sin\alpha = 0.9659$$

$$\underline{\underline{\alpha = 75^\circ}} \qquad \longleftarrow \alpha_{AB}$$

$$\sin\delta = \frac{-S_z}{(S_x^2 + S_y^2 + S_z^2)^{1/2}}$$

$$= \frac{0.75}{(0.5915^2 + 0.1585^2 + 0.75^2)^{1/2}}$$

$$\sin\delta = 0.7746$$

$$\underline{\underline{\delta = 50.8^\circ}} \qquad \longleftarrow \delta_{AB}$$

(b) The face dip direction is 75° if a line of intersection does not penetrate the face, then sliding cannot occur. Thus 75° ± 90° defines range of dip direction that are unsafe (−15°, +165°)
∴ safe $\underline{\underline{(165^\circ, 345^\circ)}}$ $\longleftarrow$ safe α_s'

(c)

	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
F:	sin 85 sin 75	sin 85 cos 75	cos 85
F:	0.9623	0.2578	0.0872
S_{AF}:	0.0654 − 0.1289	−(0.03776 − 0.48115)	0.1116 − 0.7217
S_{AF}:	−0.0635	0.44339	−0.6101

$$\sin\alpha = \frac{-0.0635}{[(-0.0635)^2 + (0.44339)^2]^{1/2}}$$

$$\sin\alpha = -0.1418$$

$$\underline{\underline{\alpha = -8.2^\circ}} \text{ quadrant!}$$

$$\sin\delta = \frac{-(-0.6101)}{(0.0635^2 + 0.4433^2 + 0.6101^2)^{1/2}}$$

$$\sin\delta = 0.8062$$

$$\underline{\underline{\delta = 53.7^\circ}}$$

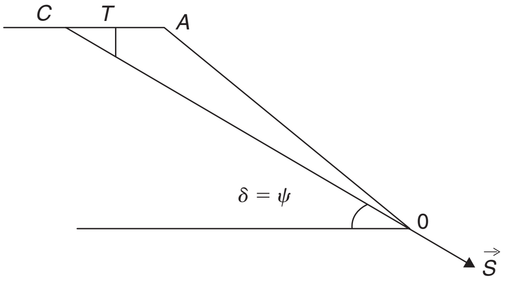
$$L = \frac{H}{\sin\delta}$$
$$= \frac{85}{0.8062}$$
$$\underline{\underline{L = 105.5\ \text{ft}}}$$

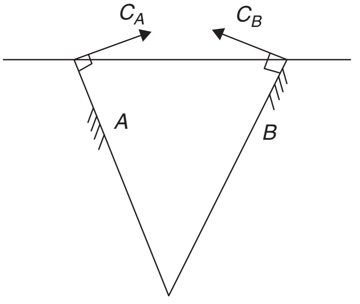
25. Given: Wedge slide data and required $FS = 1.10$

Find: If can obtain $FS = 1.10$.

Solution:

Try quick screening for sliding down line of intersection assuming zero cohesion and lowest $\phi(28°)$





Direction cosines (from T.2)

	x	y	z
$\vec{C}_A$	0.7501	0.4321	0.5000
$\vec{C}_B$	−0.4337	−0.7501	0.5000

$$\vec{S} = \vec{C}_A \times \vec{C}_B = (S_x, S_y, S_z)$$
$$= (0.4321)(0.5000) - (-0.7501)(0.5000)$$
$$- (0.7501)(0.5000) + (-0.4333)(0.5000)$$
$$+ (0.7701)(-0.7501) - (-0.4337)(0.4321)$$
$$\vec{S} = (0.5911, -0.5911, +0.3753)$$
$$\tan\alpha = \frac{0.5911}{(-0.5911)}$$
$$\alpha = 135° \quad \text{dip direction}$$
$$\tan\delta = \frac{S_z}{\sqrt{S_x^2 + S_y^2}}$$
$$= \frac{0.3753}{[(0.5911)^2 + (0.5911)^2]^{1/2}}$$
$$\tan\alpha = 0.44895$$
$$\underline{\underline{\delta = 24.2°}}$$

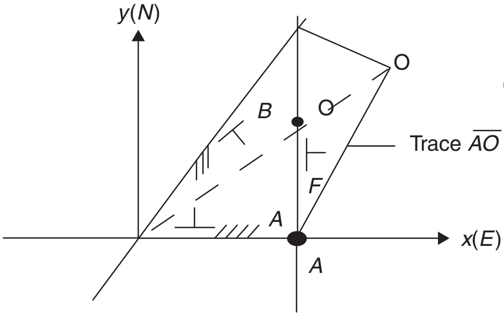
$$FS(\text{dry}, C=0,\ \phi=\phi_{\min}) = \frac{\tan\phi}{\tan\delta} = \frac{\tan 28}{\tan 24.2}$$

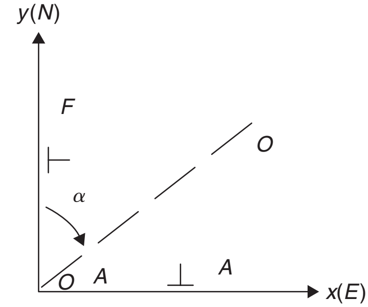
$$\underline{\underline{FS = 1.18}} \qquad \xleftarrow[(>1.10)]{yes}$$

(added cohesion and added friction will increase this *FS*)

26. Given: Wedge data in table, sketch
Find: dip, dip direction, length of $\overline{AO}$

Solution:





Note:

	x	y	z
C_a	$\sin\alpha_a \sin\delta_a$	$\cos\alpha_a \sin\delta_a$	$\cos\delta_a$
C_b	$\sin\alpha_f \sin\delta_f$	$\cos\alpha_f \sin\delta_f$	$\cos\delta_f$

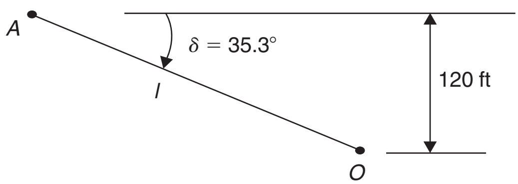
$\alpha_a = 0°$ $\quad \delta_a = 45°$
$\alpha_f = 90°$ $\quad \delta_f = 45°$

S	S_x	S_y	S_z
C_a	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
C_b	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$
S_s	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

$$\tan\alpha_{AO} = \frac{S_x}{S_y} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1 : \underline{\underline{\alpha_{AO} = 45°}} \qquad \xleftarrow{\alpha = 45°}$$

$$\tan\delta_{AO} = \frac{S_z}{\sqrt{S_x^2 + S_y^2}} = \frac{-\frac{1}{2}}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}} = -\frac{1}{\sqrt{2}} : \underline{\underline{\delta_{AO} = -35.3°}} \qquad \xleftarrow{\delta = 35.3°}$$

length



$l \sin 35.5° = 120'$
$\underline{\underline{l = 208 \text{ ft}}}$

length ←

27. Given: Wedge data

Find:

(a) Dip direction and dip of lines of intersection of A & B
(b) Range of azimuths that are safe
(c) Length of line formed by A & F (face)

Solution:

(a) Need normals to A & B

Notes:

$C_x = \sin\delta \sin\alpha \qquad C_y = \sin\delta\cos\alpha \qquad C_z = \cos\delta$

	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
B:	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)$	$\frac{1}{2}$
S:	$\left[\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)+\frac{\sqrt{3}}{4}\frac{1}{2}\right]$	$-\left(\frac{\sqrt{3}}{4}\frac{1}{2}-\frac{3}{4}\frac{1}{2}\right)$	$\left[\frac{\sqrt{3}}{4}\left(-\frac{\sqrt{3}}{4}\right)-\frac{3}{4}\cdot\frac{3}{4}\right]$
S:	$\frac{3+\sqrt{3}}{8}$	$\frac{3-\sqrt{3}}{8}$	$-\left(\frac{9+3}{16}\right)$
S:	0.5915	0.1585	−0.7500

$$\sin\alpha = \frac{0.5915}{(0.5915^2 + 0.1585^2)^{1/2}} = \frac{S_x}{(S_x^2 + S_y^2)^{1/2}}$$

$\sin\alpha = 0.9659$

$\underline{\underline{\alpha = 75°}}$

α_{AB} ←

$$\sin\delta = \frac{-S_z}{(S_x^2 + S_y^2 + S_z^2)^{1/2}}$$
$$= \frac{0.75}{(0.5915^2 + 0.1585^2 + 0.75^2)^{1/2}}$$
$$\sin\delta = 0.7746$$
$$\underline{\underline{\delta = 50.8°}}$$

δ_{AB} ←

(b) The face dip direction is 75° if a line of intersection does not penetrate the face, then sliding cannot occur. Thus 75° ± 90° defines range of dip direction that are unsafe (−15°, +165°)

∴ safe $\underline{\underline{(165°, 345°)}}$

Safe α'_s ←

(c)

	x	y	z
A:	$\frac{\sqrt{3}}{2}\frac{1}{2}$	$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
F:	sin 85 sin 75	sin 85 cos 75	cos 85
F:	0.9623	0.2578	0.0872
S_{AF}:	0.0654 – 0.1289	–(0.03776 – 0.48115)	0.1116 – 0.7217
S_{AF}:	−0.0635	0.44339	−0.6101

$$\sin\alpha = \frac{-0.0635}{[(-0.0635)^2 + (0.4433)^2]^{1/2}}$$
$$\sin\alpha = -0.1418$$
$$\underline{\underline{\alpha = -8.2°}} \qquad \text{quadrant!}$$

$$\sin\delta = \frac{-(-0.6101)}{(0.0635^2 + 0.4433^2 + 0.6101^2)^{1/2}}$$
$$\sin\delta = 0.8062$$
$$\underline{\underline{\delta = 53.7°}}$$

$$L = \frac{H}{\sin\delta}$$
$$= \frac{25.9}{0.8062}$$
$$\underline{\underline{L = 32.13\,\text{m}}}$$

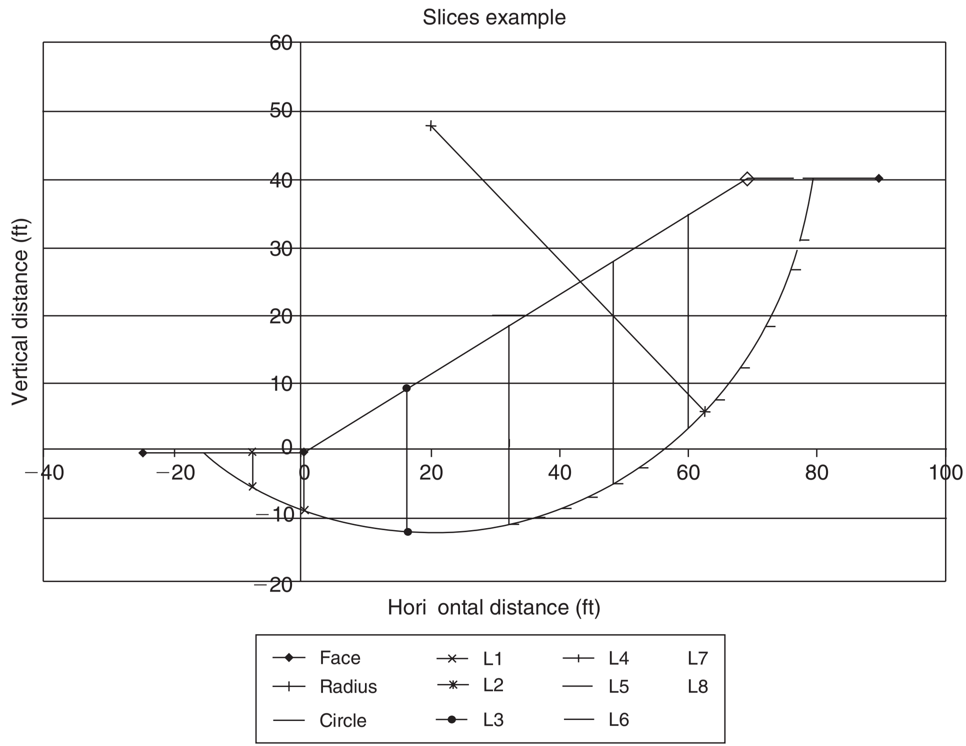
28.

Slices Problem

30 deg slope, dips below toe

$R = 60$ ft | $b = 1$ ft | spwt = 100 pcf

Slice	coordinates x(ft)	h(ft)	h(ave)ft	vol(ft^3)	wt(lbf)	δy	δx	chord slope angle = deg	rads	$W_n =$	$W_s =$	water P(slice)	$\phi =$ deg $N' \tan(\phi)$	$L =$ ft	C = psf $cLb =$ lbf
	−16	0.00													
1			2.53	20.26	2,026	−5.07	8.00	−32.34	−0.56451	1,712	−1,084	1,497	70	9.47	6,818
	−8	5.07										0			
2			6.82	54.54	5,454	−3.50	8.00	−23.64	−0.41268	4,996	−2,187	3,715	416	8.73	6,288
	0	8.57										0			
3			14.84	237.38	23,738	−3.30	16.00	−11.65	−0.20328	23,249	−4,792	15,124	2,640	16.34	11,762
	16	21.10										0			
4			25.18	402.94	40,294	1.08	16.00	3.86	0.06732	40,202	2,711	25,200	4,874	16.04	11,546
	32	29.26										0			
5			31.02	496.33	49,633	5.72	16.00	19.68	0.34344	46,735	16,713	32,892	4,498	16.99	12,234
	48	32.78										0			
6			32.07	384.85	38,485	8.34	12.00	34.81	0.60762	31,596	21,972	29,250	762	14.62	10,524
	60	31.36										0			
7			28.79	267.21	26,721	10.49	9.28	48.51	0.84674	17,701	20,017	25,171	−2,427	14.01	10,086
	69.28	26.23										0			
8			18.04	144.35	14,435	16.37	8.00	63.95	1.11614	6,339	12,968	20,510	−4,605	18.22	13,116
	77.28	9.86										0			
9			4.93	10.80	1,080	9.90	2.19	77.53	1.35316	233	1,054	3,120	−938	10.14	7,303
	79.47	0.00													
									Totals	1,72,764	75,435 dry =		56,134		
				Total =	2,01,865						−8,064 wet =		13,261		89,677
									$FS =$	1.471 wet			WET	DRY	
										2.040 dry		MR =	6.6601E+06	9.2325E+06	
										dry	wet	MD =	4.5261E+06	4.5261E+06	
								RocSci:	Simplified Bishop =	2.318	1.58				
								“Slide”	Janbu =	2.000	1.425				


Slices example
Vertical distance (ft)
60
50
40
30
20
10
0
−10
−20
−40
−20
0
20
40
60
80
100
Hori ontal distance (ft)
Face
Radius
Circle
L1
L2
L3
L4
L5
L6
L7
L8

29.

SLICES														
A part W.T. 1														
	SCALE	1″ = 40′	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_S	W_N	P	$W_{N'}$	TANϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	25,958.4	−11,695.8	0.726543	0	0		0
2	1.211			220,960	57	185,312.6	120,343.4	161,304	−40,960.6	0.649408	0	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110											
TOT-KJ	0.885		110	155,760										
3	3.407			6,41,108.8	38	394,706	505,200.6	349,752	155,448.6	0	0	2,300	59	135,700
B-YOU	1.051		110											
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG-MED	0.609		125	121,800										
4	7.654			1,435,600	13	322,939.7	1398,806	652,579.2	746,226.5	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD-LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		952,697.9		Total friction =		0		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	0.503937							

$FS = 0.504$

SLICES														
B part W.T.2														
	SCALE =	1″ = 40′	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_s	W_N	P	$W_{N'}$	TAN ϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	0	14,262.58	0.726543	10,362.37	0		0
2	1.211			220,060	57	185,312.6	120,343.4	0	120,343.4	0.649408	78,151.94	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110											
TOT-KJ	0.885		110	155,760										
3	3.407			641,108.8	38	394,706	505,200.6	1,03,084.8	402,115.8	0	0	2,300	59	135,700
B-YOU	1.051		110											
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG–MED	0.609		125	121,800										
4	7.654			1,435,600	13	322,939.7	1,398,806	4,71,744	92,7061.7	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD-LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		952,697.9		Total friction =		88,514.31		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	0.596846							

$FS = 0.597$

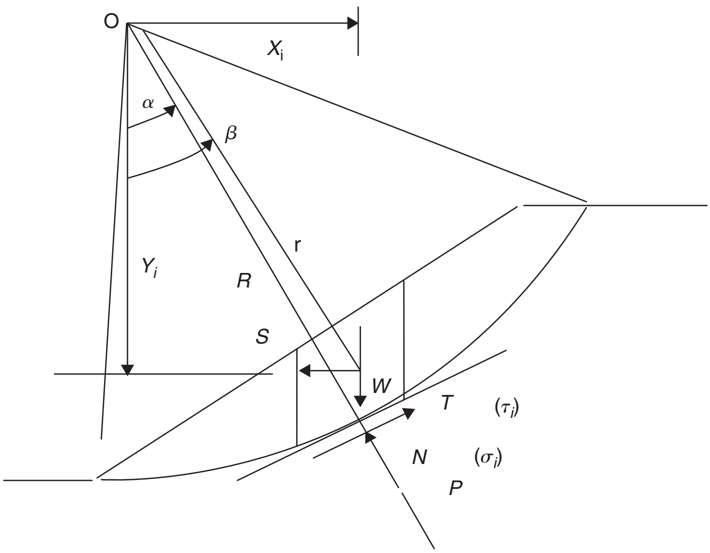
SLICES														
C part W.T. 2 with 140c60 ft relieving bench (crest)														
	SCALE =	1″ = 40′	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_s	W_N	P	$W_{N'}$	TAN ϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	0	14,262.58	0.726543	10,362.37	0		0
2	1.211			207,936	57	174,389.8	113,250.1	0	113,250.1	0.649408	73,545.45	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110	13,024										
TOT-KJ	0.885		110	155,760										
3	3.407			456,132.8	38	280,823.4	359,437.6	103,084.8	256,352.8	0	0	2,300	59	135,700
B-YOU	1.051		110	184,976										
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG-MED	0.609		125	121,800										
4	7.654			907,600	13	2041,65.6	884,338.3	471,744	412,594.3	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD-LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		709,118.3		Total friction =		83907.82		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	0.795365							

$FS = 0.795$

D part W.T. 1 with 140c60 ft relieving bench (crest)														
	SCALE =	1″ = 40′	1 FT THICK											
	(SQ IN)	(SQ FT)	(PCF)	(LBF)										
SEC	AREAS	AREAS	SPWT		ANGLE	W_s	W_N	P	$W_{N'}$	TAN ϕ	R	c	L	cL
1	0.294		110	51,744	74	49,739.53	14,262.58	25,958.4	−11,695.8	0.726543	0	0		0
2	1.211			207,936	57	174,389.8	113,250.1	161,304	−48,053.9	0.649408	0	0		0
KJ-OLD	0.326		125	65,200										
A-YOU	0.074		110	13,024										
TOT-KJ	0.885		110	155,760										
3	3.407			456,132.8	38	280,823.4	359,437.6	349,752	9,685.551	0	0	2,300	59	135,700
B-YOU	1.051		110	184,976										
B″-YOU	1.6788		110	295,468.8										
B′-OLD	1.1192		125	223,840										
JG-MED	0.609		125	121,800										
4	7.654			907,600	13	204,165.6	884,338.3	652,579.2	231,759.1	0	0	2,800	83	232,400
C-YOU	3		110	528,000										
C′-OLD	2		125	400,000										
C″-MED	1.929		125	385,800										
GD LEAN	0.725		105	121,800										
5	7.654				0							700	160	112,000
D-YOU			110	Total driving =		709,118.3		Total friction =		0		Total cohesion =		480,100
D′-OLD			125											
D″-MED			125											
LEAN			105			$FS = R/D =$	1.05	$L =$	377.8203	ft				

$L = 378$ ft

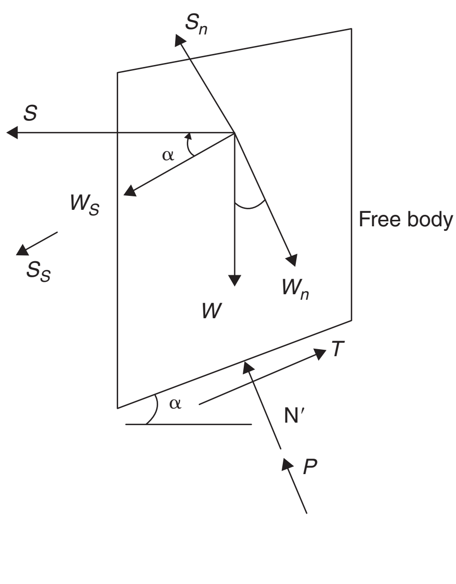
30. Given: Slices analysis with S loads



Show: $$FS = \frac{R\Sigma_1^n(N_i' \tan\phi + C_i)}{R\Sigma_1^n(W_i \sin\alpha_i + S_i \frac{r_i}{R}\cos\alpha_i)}$$

when $N_i' = W_i \cos\alpha_i - P_i - S_i \sin\alpha_i$.

Solution:



Overall equilibrium

$$\Sigma_1^n(W_i x_i + S_i y_i) = \Sigma_1^n \tau_i A_i R$$

$$\Sigma[W_i R \sin(\alpha_i) + S_i R_i \cos(\beta_i)] = \Sigma\tau_i \max \frac{A_i R}{fs_i}$$

- assume $fs_i = fs_j = fs = FS$
- and MC strength so

$$\tau_i \max = \sigma_i' \tan\phi_i + c_i$$

$$\therefore\ \Sigma(W_i R \sin\alpha_i + S_i R_i \cos\beta_i) = \frac{\Sigma(\sigma_i' + \tan\phi_i + c_i)A_i R}{FS}$$

$$\therefore FS = \frac{\Sigma_i^n(N_i' \tan\phi_i + C_i)R}{\Sigma_i^n\left[W_i \sin\alpha_i + S_i\left(\frac{r_i}{R}\right)\cos\beta_i\right]}$$

when $N_i' = \sigma_i' A_i$ & $C_i = c_i \cdot A_i$

- Normal force equilibrium
- Neglect side forces

$$\Sigma_n F = 0$$

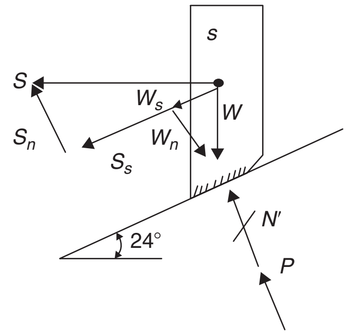
$$0 = W_n - N' - P - S_n$$

$$\underline{\underline{N_i' = W_i \cos\alpha_i - P_i - S_i \sin\alpha_i}}\ \text{(each slice } i\text{)}$$

31. Given: Figure and table data
$R = 300\,\text{ft}$ $H = 120\,\text{ft}$ $\gamma = 95\,\text{psf}$ $C = 367\,\text{psf}$ $\phi = 16°$ $\beta = 29°$

Find:

(1) Seismic force on slice 7, $a_s = 0.15$
(2) Water force on bottom of 7
(3) *FS* of 7
(4) *FS* slip surface, dry & no seismic force.



Solution:

(1) $S = ma$

$$= \frac{W}{g} 0.15\,g$$

$$S = 0.15\,W$$

$$S = 0.15(10.29)(10^5)$$

$$\underline{\underline{S = 1.544(10^5)\,\text{lbf.}}}$$

← (1)

(2) $P = \bar{p}A \quad \bar{p} = rH$

$$= (62.4)(82)(75)(1)$$

$$\underline{\underline{P = 3.838(10^5)\,\text{lbf}}}$$

← (2)

(3) $FS(7) = \dfrac{R(7)}{D(7)}$

$$D = W_s + S_s$$

$$= (10.29)(10^5)\sin 24° + 1.544(10^5)\cos 34°$$

$$\underline{D = 5.596(10^5)\,\text{lbf}}$$

$$R = N'\tan\phi + cL$$

$$N' = W_n - P - S_s$$

$$= 10.29(10^5)\cos 24 - 3.838(10^5) - 1.544(10^5)\sin 24$$

$$\underline{N' = 4.934(10^5)\,\text{lbf.}}$$

$$FS(7) = \frac{4.934(10^5)\tan 16° + (367)(75)(1)}{5.596(10^5)}$$

$$\underline{\underline{FS(7) = 0.302}}$$

← (3)

Note: (Slopes 1–5 are neg.)

(4)

$$FS = \frac{\Sigma M_R}{\Sigma M_D} \quad \text{(dry, no seismic force)}$$

$$= \frac{\sum_1^{10} W_r \tan\phi + \sum_1^{10} C + \sum_1^{5} W_s}{\sum_6^{10} W_s}$$

$$= \frac{52.67(10^5)\tan 16^\circ + (367)(540) + 5.97(10^5)}{12.93(10^5)}$$

$$\underline{\underline{FS = 1.78}} \qquad \xleftarrow{(4)}$$

32. This amounts to part 4 of 31.

$$\underline{\underline{FS = 1.78}}$$

$$FS = \frac{\Sigma M_R}{\Sigma M_D}$$

$$FS = \frac{\sum_1^{10} R(W\cos\alpha\tan\phi + cL) + \sum_1^5 R\,W\sin\alpha}{\Sigma_6^{10} R\,W\sin\alpha}$$

$\therefore$ as in 31(4).

33. Given: Circular are failure data in Fig. 2.33 and Table 2.14.
Find: $FS(2)$, $FS(7)$, show total FS.

Solution:

$FS = \Sigma M_R/\Sigma M_D$, $R =$ radius of slips circle
Slice 2

$$FS = \frac{R[(W_n' \tan\phi) + C]}{RW_s}$$

$$P = \gamma_w WL(1)$$

$$= (62.4)(1)(52)(30)$$

$$\underline{P = 9.734(10^4)\,\text{lbf.}}$$

$$C = cL(1)$$

$$= (367)(30)(1)$$

$$\underline{C = 1.101(10^4)\,\text{lbf}}$$

normal equilibrium:

$$W_n' = W\cos\alpha - P$$

$$= 27.78(10^4)(\cos 28^\circ) - 9.734(10^4)$$

$$W_n' = 24.49(10^4) - 9.734(10^4)$$

$$\underline{W_n' = 14.76(10^4)\,\text{lbf}}$$

$$\underline{W_s = W\sin\alpha \quad \text{But is resisting!}}$$

$$\therefore FS = \frac{[(W_n'\tan\phi + C)R + RW_s]}{0}$$

$$\underline{\underline{FS \to \infty \text{ Slice 2}}} \qquad \xleftarrow{\text{Slice 2}}$$

Slice 7

$$P = (62.4)(1)(75)(82)$$
$$\underline{P = 3.838(10^5)\text{lbf}}$$
$$W_n' = 10.29(10^5)\cos(24) - 3.838(10^5)$$
$$\underline{W_n' = 5.563(10^5)\text{ lbf}}$$

$$C = (367)(75)(1)$$
$$\underline{C = 2.753(10^4)\text{ lbf}}$$

$$W_s = 10.29(10^5)\sin 24$$
$$W_s = 4.185(10^5)\text{ lbf}$$

$$\begin{aligned} M_R &= R(W_n' \tan\phi + C) \\ &= R(5.563(10^5)\tan 16° + 0.275(10^5)) \end{aligned}$$
$$\underline{M_R = R(1.870)10^5\text{ lbf}}$$

$$M_D = RW_n$$
$$\underline{M_D = R(4.185)(10^5)}$$

$$FS = \frac{R(1.870)(10^5)}{R(4.185)(10^5)}$$
$$\underline{FS = 0.447}$$

← Slice 7

Algebraically

$$FS = \frac{\Sigma M_R}{\Sigma M_D}$$
$$FS = \frac{\sum_{i=1}^{10} R(W_n' \tan\phi + C)_i + \sum_{i=1}^{5} R(W_s)_i}{\sum_{i=6}^{10} R(W_s)_i}$$

34. Given: Circular failure in sketch
Find: Expression for safety factor

Solution:

By definition

$$FS = \frac{M_R}{M_D} = \left(\frac{\text{Moments resistance}}{\text{Moments driving}}\right)$$
$$M_R = \sum_{\text{slices}} (\text{forces})(\text{lever arm})_R$$
$$M_D = \sum_{\text{slices}} (\text{forces})(\text{lever arm})_D$$

lever arm = R
since circular failure

$$M_R = R\sum_{i=1}^{5}(N\tan\phi + C)_i + \sum_{i=1}^{2} RT_i$$

$$M_D = R\sum_{i=1}^{5} T$$

T = Tangential component of weight
N = Normal component of weight
α = inclination of slice bottom from horizontal
$N = W\cos(\alpha)$
$T = W\sin(\alpha)$

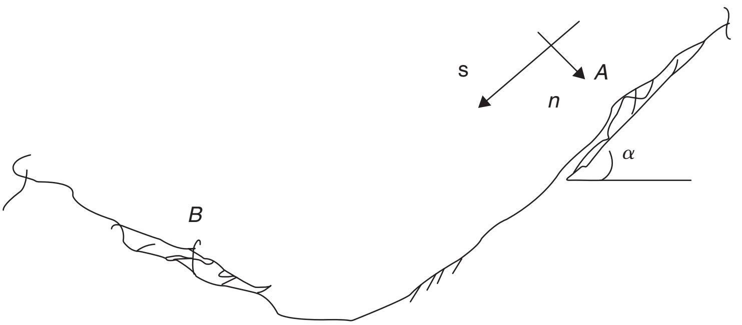
Note: No water table shown, assume dry

$$FS = \frac{\sum_{i=1}^{5} R(N\tan\phi + C)_i + \sum_{i=1}^{2} RT_i}{\sum_{i=3}^{5} RT_i}$$

where $cA = C$ and A = area of slice base

Dynamics, Toppling

35. Given: Slope situation



Find: a, v, s, of mass center.

Solution:

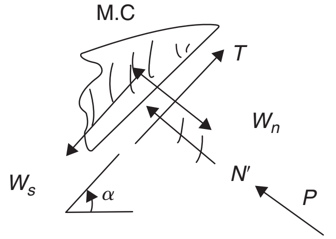
Mass center obeys
F = $m\ddot{s}$ where s = down hill distance
$\dot{s}$ = velocity
$\ddot{s}$ = acc. of mass center

- Choose origin of coordinates n, s at mass center starting point A

At time $t = 0$, $s = \dot{s} = \ddot{s} = 0$

- Downhill forces = D, Resisting up hill forces = F

So $F = D - R$
D = downhill component of weight
Free body of slide mass
$D = W \sin\alpha$
$N = W \cos\alpha$
$P = 0$ dry assumption



- Frictional resistance

$F_f = N' \tan\phi$

$$\therefore W\sin\alpha - W\cos\alpha\tan\phi = \frac{W}{g}\ddot{s}$$

$$W\sin\alpha - W\cos\alpha\tan\phi = \frac{W}{g}\ddot{s}$$

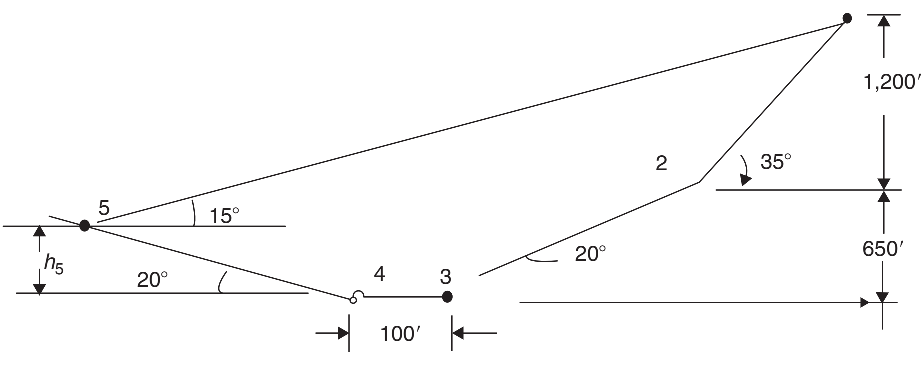
$$\left.\begin{array}{ll}\ddot{s} = g\left[\dfrac{\sin(\alpha-\phi)}{\cos\phi}\right] & (\alpha \ge \phi) \\ \text{(If } \phi > \alpha \text{ then no sliding occurs and } \ddot{s} = 0) & (\alpha \le \phi)\end{array}\right\} \longleftarrow$$

At constant slope

$$\dot{s} = \ddot{s}t + \dot{s}(0)$$

$$\left.\begin{array}{ll}\therefore \dot{s} = g\left[\dfrac{\sin(\alpha-\phi)}{\cos\phi}\right]t & (\alpha \ge 0) \\ \therefore s = g\left[\dfrac{\sin(\alpha-\phi)}{\cos\phi}\right]\dfrac{t^2}{2} & (\alpha \ge 0)\end{array}\right\}$$

36. Given: Profile $\phi = 15°$



Find: h_5

Solution:

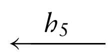
(1) Draw ϕ-line
(2) From notes on dynamics of sliding, $0 = Wh - Wd \tan\phi$

$$\begin{aligned}
\tan\phi &= \frac{h}{d}\\
h &= 1{,}200 + 650 - h_5\\
d &= d_1 + d_2 + 100 + d_5\\
\tan 35^\circ &= \frac{1{,}200}{d_1}\\
\tan 70^\circ &= \frac{650}{d_2}\\
\tan 70^\circ &= \frac{h_5}{d_5}\\
\therefore d &= \frac{1{,}200}{\tan 35} + \frac{65^\circ}{\tan 20} + 100 + \frac{h_5}{\tan 20}
\end{aligned}$$

$$\tan 15^\circ = \frac{(1{,}200 + 650 - h_5)}{\left(\frac{1{,}200}{\tan 35} + \frac{65^\circ}{\tan 20} + 100 + \frac{h_5}{\tan 20}\right)}$$

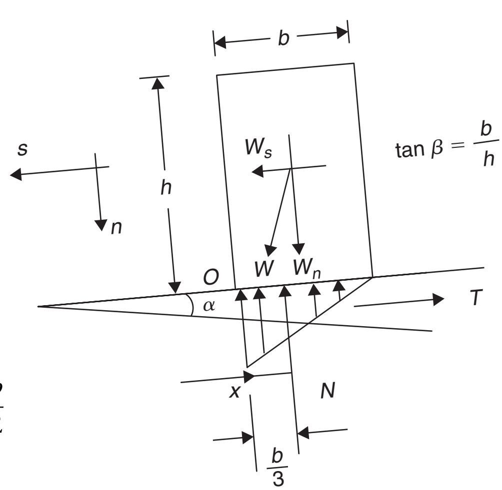
$$(0.260)\left(1{,}713.0 + 1{,}786 + 100 + \frac{h_5}{\tan 20}\right) = 1{,}850 - h_5$$

$$\begin{aligned}
964 + \frac{0.268\,h_5}{\tan 20} &= 1{,}850 - h_5\\
1{,}738\,h_5 &= 886\\
\underline{\underline{h_5 = 510}}
\end{aligned}$$



37. Given: Topping rock block

Show: $\tan(\alpha) < \frac{1}{3}\tan\beta$ for stability against toppling.



Solution:

Equilibrium requires

$$\begin{aligned}
\Sigma_n F &= W \qquad W\cos\alpha = N\\
\Sigma_s F &= 0 \qquad W\sin\alpha = T\\
\Sigma_o M &= 0 \qquad 0 = W_s\frac{h}{2} + Nx - N\frac{b}{2}
\end{aligned}$$

where $x =$ distance from 0 to N

$$\therefore\ W \sin\alpha \frac{h}{2} + W \cos\alpha \left(x - \frac{b}{2} \right) = 0$$

$$\tan\alpha \frac{h}{2} = \frac{b}{2} - x$$

$$x = \frac{b}{2} - \frac{h}{2} \tan\alpha$$

but with a triangular stress distribution

$$x \geq \frac{b}{3}$$

$$\therefore\ \frac{b}{2} - \frac{h}{2} \tan\alpha \geq \frac{b}{3}$$

$$\frac{b}{6} \geq \frac{h}{2} \tan\alpha, \quad \frac{b}{h} = \tan\beta$$

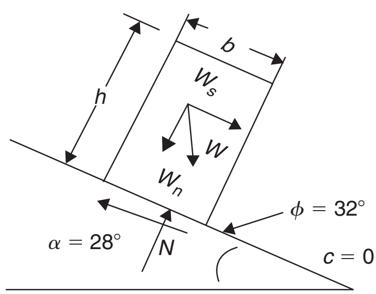
$$\therefore\ \underline{\underline{\tan\alpha \leq \frac{1}{3} \tan\beta}}$$

← (for stability w.r.t toppling)

38. Given: Rock block on slope

Find: Dimension necessary to prevent toppling.

To prevent toppling, must be in equilibrium



$\Sigma F = 0$: $N = W_n = W \cos\alpha$

$T = W_s = W \sin\alpha$

$$\Sigma M = 0 \quad Nx - T\frac{h}{2} = 0$$

$$\therefore\ x = \left(\frac{h}{2}\right)\left(\frac{T}{N}\right)$$

$$x = \left(\frac{h}{2}\right)(\tan\alpha) \ \ \&\ \ x < \frac{b}{2}$$

$$\therefore\ \left(\frac{h}{2}\right) \tan\alpha \leqq \frac{b}{2}$$

$$\frac{b}{h} \geqq \tan\alpha$$

$$b \geqq h \tan 28°$$

$$\underline{\underline{b \geqq h(0.5317)}}$$

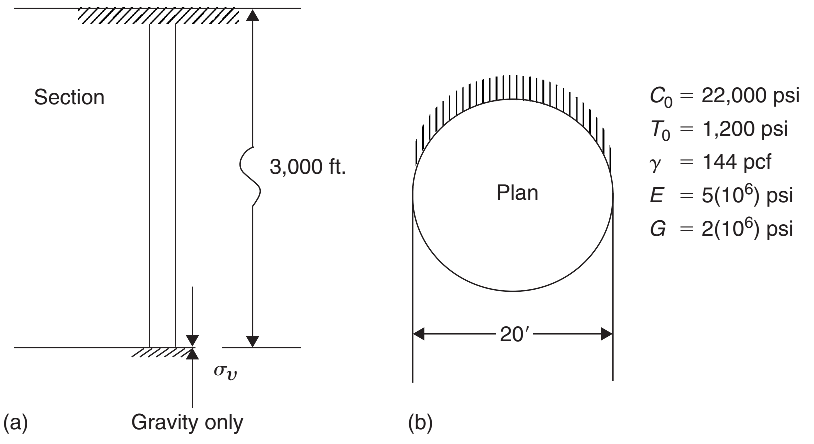
<u>square base ∴ b' into page $= b$.</u>

<u>e.g. if $h = 10'$ $b = b' \geqq 5.317$ ft.</u>

3 Shafts

Single, Naturally Supported Shafts

1. Given: (as a shaft)



Find: FS_c, FS_t.

Solution:

By definition:

$$FS_c = \frac{C_0}{\sigma_c}; \quad FS_t = \frac{T_0}{\sigma_t}$$

Stress concentration concept:

$$\sigma_c = K_c \sigma_1^0; \quad \sigma_t = K_t \sigma_1^0$$

where: σ_1^0 = major principal stress before mining
Gravity only

$$\sigma_v = \gamma h$$

$$\sigma_v(\max) = (144)\frac{3{,}000}{144}$$

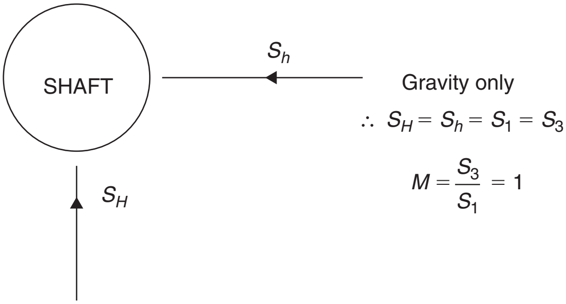
$$\underline{\underline{\sigma_v = 3{,}000 \text{ psi}}}$$

(1) In vertical section, neglecting shaft bottom effect:

$$FS_c = \frac{22{,}000}{3{,}000}$$

$$\underline{\underline{FS_c = 7.33}} \qquad \longleftarrow FS_c\text{(vertical)}$$

In plan view:



with complete lateral restraint, gravity only

$$S_h = S_H = \frac{v}{1-v} S_v$$

$$v = \frac{E}{2G} - 1$$

$$v = \frac{5(10^6)}{2(10^6)2} - 1$$

$$v = 0.25$$

$$S_h = S_H = \frac{1}{3} S_v$$

$$\underline{\underline{\therefore\ S_h = S_H = 1{,}000 \text{ psi}}}$$

$$FS_c = \frac{C_0}{K_c S_1} \quad \text{(text for } K_c\text{)}$$

$$= \frac{22{,}000}{(2)(1{,}000)}$$

$$\underline{\underline{FS_c = 11.0}} \qquad \longleftarrow$$

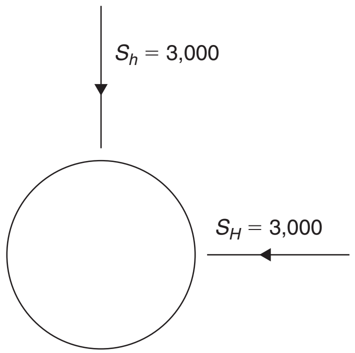
no tension

$$\underline{\underline{\therefore\ FS_t \text{ is not a factor}}} \qquad \longleftarrow$$

2. Given: Problem 1 data – shaft
   But $\sigma_h = \sigma_v$
   i.e., $S_h = S_H = S_v = 3{,}000$
   Find: FS_c, FS_t.

   Solution:

   In plan view



$$M = \frac{3{,}000}{3{,}000} = 1$$

$K_c = 2$ & K_t not a factor

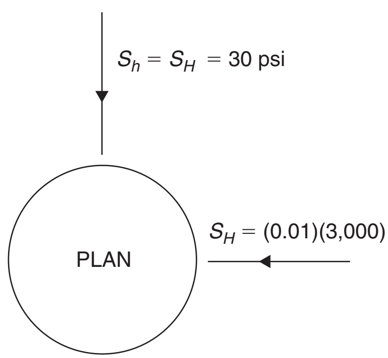
$$FS_c = \frac{22{,}000}{(2)(3{,}000)}$$

$\underline{\underline{FS_c = 3.67}}$

$\underline{\underline{FS_t \text{ not a factor}}}$

3. Given: Problem 1 conditions but $S_h = S_H = 0.01\ S_v$.
   Find: FS_c, FS_t.

   Solution:



$M = 1$

$K_c = 2$ & K_t not a factor

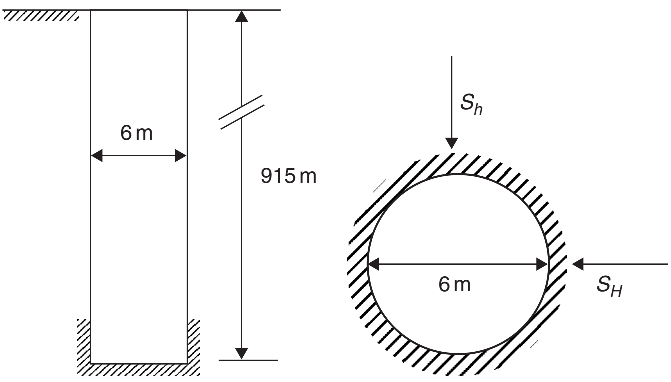
$$FS_c = \frac{22{,}000}{(2)(30)}$$

$\underline{\underline{FS_c = 361}}$

$\underline{\underline{FS_t \text{ not a factor}}}$

4. Given: Shaft 6 m diameter
   $C_0 = 152$ MPa $T_0 = 8.3$ MPa $\gamma = 23$ kN/m^3
   $E = 34.5$ GPa $G = 13.8$ GPa depth = 915 m
   gravity load only
   Find: FS_c, FS_t.

Solution:



(a) Vertical view (b) Hori ontal view

By definition:

$$FS_c = \frac{C_0}{\sigma_c}, \quad FS_t = \frac{T_0}{\sigma_t}$$

$$\sigma_c = K_c\sigma_1^0, \quad \sigma_t = K_t\sigma_1^0$$

$$\sigma_1^0 = \text{reference stress in view considered}$$

(a) $K_c = 1, \quad \sigma_1^0 = \gamma h = S_v$

$$= (23)(915)$$

$$\underline{\underline{\sigma_1^0 = 21.05\,\text{MPa}}}$$

$$FS_c = \frac{152}{21.05}$$

$$\underline{\underline{FS_c = 7.22 \text{ (vertical view)}}}$$

gravity only $S_h = S_H \quad \therefore \quad \text{M} = \dfrac{S_3}{S_1} = 1$

$\& \quad K_c = 2,\ K_t - \text{n.a.}$

$$S_h = S_H = \frac{v}{1-v}S_v$$

$$v = \frac{E}{2G} - 1$$

$$= \frac{34.5}{2(13.8)} - 1$$

$$\underline{v = 0.25}$$

$$S_h = S_H = \left(\frac{1}{3}\right)(21.05)$$

$$\underline{S_h = S_H = 7.02\,\text{MPa}}$$

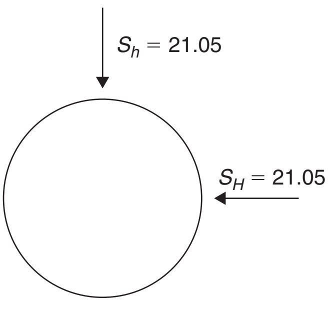
$$FS_c = \frac{152}{(2)(7.02)}$$

$$\underline{\underline{FS_c = 10.8}}\ \text{(plan view)}$$

no tension $\therefore$ $\underline{\underline{FS_t \text{ not a factor}}}$

5. Given: Problem 4 data and $\sigma_h = \sigma_v$ i.e. $\sigma_h = \sigma_H = \sigma_v = 21.05$ MPa
   Find: FS_c, FS_t.

   Solution:



$$M = \frac{S_3}{S_1} = 1$$

$$K_c = 2$$

K_t not a factor

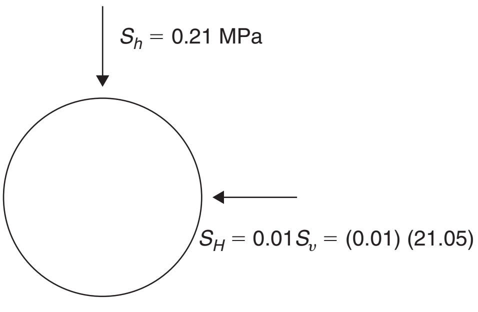
$$FS_c = \frac{152}{2(21.05)}$$

$$\underline{\underline{FS_c = 3.61}}$$

$\underline{\underline{FS_t \text{ not a factor}}}$ ←

6. Given: Problem 4 data and $\sigma_h = \sigma_H = 0.01\,\sigma_v$
   Find: If failure possible FS_c, $FS_t = ?$

   Solution:



$M = 1$ $K_c = 2$ K_t = no tension

$$FS_c = \frac{152}{(2)(0.21)}$$

$$\underline{\underline{FS_c = 361}}$$

$\underline{\underline{FS_t \text{ not a factor}}}$ ←

7. Given: (a) $M = \frac{1}{3}$ (b) $M = 0$ for $\frac{W_0}{H_0} = 2.0$

Find: Best of elliptical, ovaloid or rectangular openings.

Solution:

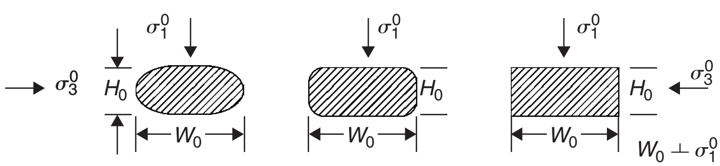
"Best" means highest *FS's* or equivalently lowest stress concentration factors. (Bull. 587 p.11, 12, 14 or text)

$W_0/H_0 = 2$	Fig. 4c		Fig. 5c		Fig. 6d	
Shape	Ellipse		Ovaloid		Rectangle	
	Tens.	Comp.	Tens.	Comp.	Tens.	Comp.
$M = 1/3$	−0.3	4.7	−0.4	3.2	−0.1	4.0
$M = 0$	−1.0	5.0	−0.9	3.4	−0.8	4.0

(a) for $M = \frac{1}{3}$ Ovaloid is best in compression
Rectangle is best in tension

(b) for $M = 0$ Ovaloid is best in compression, still
Rectangle is best in tension, still

Since rock is often much stronger in compression, than in tension one may choose a rectangle. However, the choice is now really site-specific.

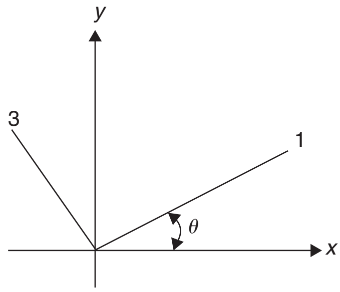


8. Given: 10 × 20 ft rectangular shaft sunk vertically where
$\sigma_{xx} = 1{,}141$ $\sigma_{yy} = 2{,}059$ $\sigma_{zz} = 1{,}600$
$\tau_{xy} = 221$ $\tau_{yz} = 0$ $\tau_{zx} = 0$
$x =$ east, $y =$ north, $z =$ up, compression (+)
$E = 4.5(10^6)$psi, $v = 0.20$, $C_0 = 15{,}000$ psi, $T_0 = 900$ psi

Find:
(a) Most favorable orientation
(b) FS_t, FS_c
(c) Sketch peak stress.

Solution:

"The most favorable orientation is with the long axis (20 ft) parallel to S_1" rule of thumb.



$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})} = \frac{221}{\frac{1}{2}(1{,}141 - 2{,}059)}$$

$$\tan 2\theta = -0.4815$$

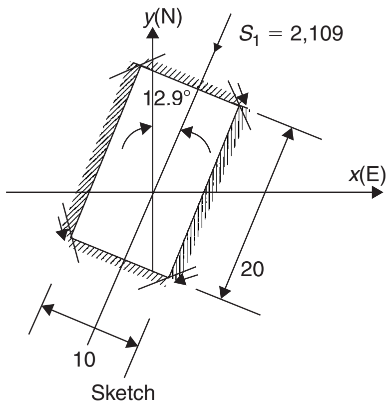
$\underline{\underline{\theta = -12.9°}}$ (cw from y axis) ← (a)

$$\left.\begin{matrix} S_1 \\ S_3 \end{matrix}\right\} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \pm \left\{\left[\frac{1}{2}(\sigma_{xx} - \sigma_{yy})\right]^2 + (\tau_{xy})^2\right\}^{1/2}$$

$$\left.\begin{matrix} S_1 \\ S_3 \end{matrix}\right\} = \frac{1}{2}(1{,}141 + 2{,}059) \pm \left\{\left[\frac{1}{2}(1{,}141 - 2{,}059)\right]^2 + (221)^2\right\}^{1/2}$$

$$= 1{,}600 \pm 509$$

$$\left.\begin{matrix} S_1 \\ S_3 \end{matrix}\right\} = \begin{matrix} \underline{\underline{2{,}109 \text{ psi}}} \\ \underline{\underline{1{,}091 \text{ psi}}} \end{matrix}$$



← (a), (b)

$$M = \frac{S_3}{S_1} \qquad \text{(Table 3.2a, p.108)}$$

$$= \frac{1{,}091}{2{,}109} \qquad M \sim 0.5, \frac{1}{2}$$

$$\underline{\underline{M = 0.517}} \qquad K = 0.5, \frac{1}{2}$$

$$K_t = 0.0, \qquad K_c = 4.57+$$

Peak stress concentration occurs at corners

No tension

$\underline{\underline{FS_t = \text{not a factor,}}}$ $\quad FS_c = \dfrac{C_0}{\sigma_c}$

$$FS_c = \frac{15{,}000}{(4.57)(2{,}109)}$$

$\underline{\underline{FS_c = 1.56}}$ ← (b)

9. Given: 3 × 6 m shaft – rectangular sunk vertically to where

$\sigma_{xx} = 7.9, \quad \sigma_{yy} = 14.2, \quad \sigma_{zz} = 11.0$

$\tau_{xy} = 1.5, \quad \tau_{yz} = 0, \quad \tau_{zx} = 0$ MPa

x = east, $\quad y$ = north, $\quad z$ = up, $\quad E = 31.0$ GPa

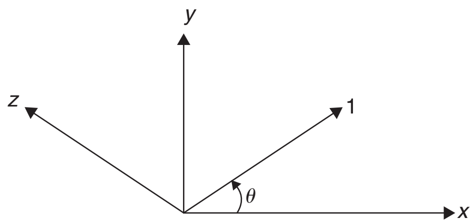
$\upsilon = 0.20, \quad C_0 = 103$ MPa, $\quad T_0 = 6.2$ MPa

Find:
(a) Most favorable orientation
(b) FS_c, FS_t in this orientation
(c) Sketch peak stress.

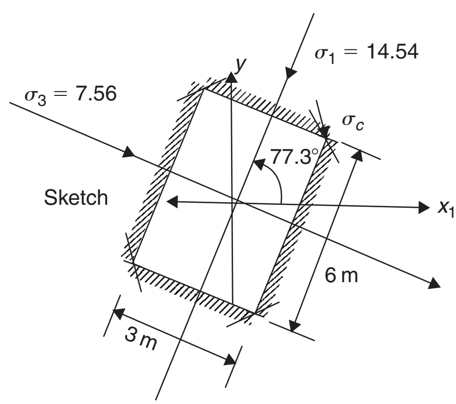
Solution:

Rule: "… Most favorable orientation is with long axis parallel to the major compression".

$$\tan 2\theta = \frac{1.5}{\frac{1}{2}(7.9 - 14.2)}$$
$$\tan 2\theta = -0.4762$$
$$2\theta = -25.5°, 154.5°$$
$$\theta = -12.7°, 77.3°$$
$$\underline{\underline{\theta = 77.3°}}$$



$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$
$$= \left(\frac{1}{2}\right)(7.9 + 14.2) \pm \left[\left(\frac{7.9 - 14.2}{2}\right)^2 + (1.51)^2\right]^{1/2}$$
$$= 11.05 \pm 3.49$$
$$\underline{\underline{\sigma_1 = 14.54\,\text{MPa}}}$$
$$\underline{\underline{\sigma_3 = 7.56\,\text{MPa}}}$$



$$M = \frac{S_3}{S_1} \qquad \text{(Table 3.2a)}$$
$$= \frac{7.56}{14.54} \qquad M \sim 0.5, \frac{1}{2}$$
$$\underline{M = 0.52} \qquad K = 0.5, \frac{1}{2}$$
$$K_t = 0, \qquad \underline{K_c = 4.57}$$

- no tension
- peak compression occurs at corners

$\underline{\underline{FS_t \text{ not a factor}}}$ ⟵

$$FS_c = \frac{103}{(4.57)(14.54)}$$

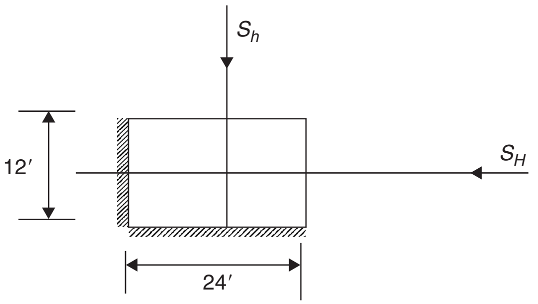
$\underline{\underline{FS_c = 1.55}}$ ⟵

10. Given: Rectangular shaft 12 × 24 ft at 3,000 ft where
$S_v = 1.2h$
$S_h = 120 + 0.5h$: Premining (psi)
$S_H = 3{,}240 + 0.3h$ h = depth (ft)

Find:
(a) Best orientation,
(b) At all depths?

Solution:

If $h = 0$, then $S_H = \sigma_1^0$, $S_h = \sigma_3^0$ and best orientation is with the long dimension parallel to S_H.



(a) ←

For same orientation at depth

$$
\begin{aligned}
S_h &< S_H \\
120 + 0.5h &< 3{,}240 + 0.3h \\
0.2h &< 3{,}120 \\
h &< 15{,}600
\end{aligned}
$$

∴ Best at all depths of shaft (3,000 ft) (b) ←

11. Given: 3.66 × 7.32 m rectangular shaft at a depth of 914 m where:
$S_v = 27.24$
$S_h = 826 + 11.3h$
$S_H = 22{,}345 + 6.8h$
h = depth in meters
Stresses in kPa

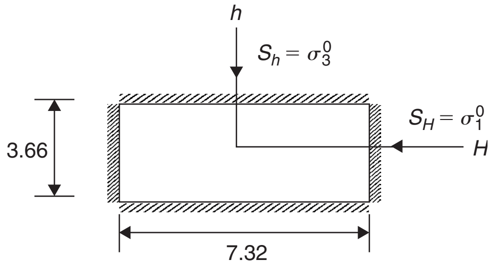
Find:
(a) Best orientation
(b) Best orientation at every depth

Solution:

At $h = 0$ (depth)

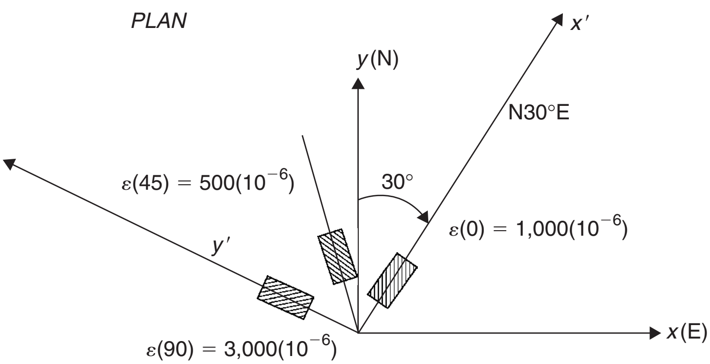
"long axis parallel to σ_1"
same orientation for:

$$S_h < S_H$$
$$826 + 11.3h < 22{,}345 + 6.8h$$
$$4.5h < 21{,}519$$
$$\underline{h < 4{,}782 \text{ m}}$$



$\therefore$ Best at all depths (914 m)

12. Given: Strain gauge data on a flat surface, over-cored



$E = 2.4(10^6)$ psi, $v = 0.20$, tension (+)
Find: State of strain, & stress.

Solution:

Normal strain rotation of axis

$$\varepsilon = \frac{\varepsilon_x + \varepsilon_y}{z} + \frac{\varepsilon_x - \varepsilon_y}{z}\cos 2\theta + \varepsilon_{xy}\sin 2\theta$$

w.r.t $x'y'z'$

$$\varepsilon_{x'x'} = \varepsilon(0)$$
$$\varepsilon_{y'y'} = \varepsilon(90)$$
$$\varepsilon(45) = \frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} + \frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2}\cos 2.45 + \varepsilon_{x'y'}$$
$$\therefore\; 2\varepsilon_{x'y'} = 2\varepsilon(45) - [\varepsilon(0) + \varepsilon(90)]$$

$$\left.\begin{aligned} \varepsilon_{x'x'} &= 1{,}000(10^{-6}) \\ \varepsilon_{y'y'} &= 3{,}000(10^{-6}) \\ 2\varepsilon_{x'y'} &= -3{,}000(10^{-6}) \\ \varepsilon_{x'y'} &= -1{,}500(10^{-6}) \end{aligned}\right\} \text{w.r.t } x'y'z'$$

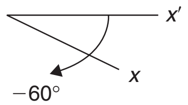
w.r.t xyz – rotation of axis

$$\left.\begin{matrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{matrix}\right\} = \frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} \pm \frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2}\cos 2\theta \pm \varepsilon_{xy}\sin 2\theta$$

$$\varepsilon_{xy} = -\left(\frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2}\right)\sin 2\theta + \varepsilon_{xy}\cos 2\theta$$

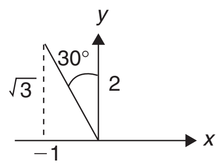
$\theta = -60°$

$2\theta = 120°$



$$\cos 2\theta = -\frac{1}{2}$$

$$\sin 2\theta = -\frac{\sqrt{3}}{2}$$



$$\frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} = \frac{1{,}000 + 3{,}000}{2}10^{-6}$$

$$\frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} = 2{,}000(10^{-6})$$

$$\frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2} = \frac{1{,}000 - 3{,}000}{2}10^{-6}$$

$$\frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2} = -1{,}000(10^{-6})$$

$$\left.\begin{matrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{matrix}\right\} = 2{,}000(10^{-6}) \pm (-1{,}000)(10^{-6})\left(-\frac{1}{2}\right) \pm (-1{,}500)(10^{-6})\left(-\frac{\sqrt{3}}{2}\right)$$

$$\left.\begin{matrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{matrix}\right\} = \begin{matrix}3{,}799\ (10^{-6})\\ 201\ (10^{-6})\end{matrix}$$

$$\varepsilon_{xy} = -(-1{,}000)10^{-6}\left(-\frac{\sqrt{3}}{2}\right) + (-1{,}500)(10^{-6})\left(-\frac{1}{2}\right)$$

$$\varepsilon_{xy} = -116(10^{-6})$$

The surface is shear – stress free & normal stress free

$\tau_{zx} = \tau_{zy} = 0 = \sigma_{zz}$: from Hooke's law

$$G\gamma_{zx} = \tau_{zx} \ \&\ G\gamma_{zy} = \tau_{zy}$$

$$\gamma_{zx} = \gamma_{zy} = 0$$

Also

$$\tau_{xy} = G(2)(-116)10^{-6}$$

$$G = \frac{E}{2(1+0)}$$

$$G = \frac{2.4(10^6)}{2(1+0.2)}$$

$$G = 1.0\,(10^6)\,\text{psi} \quad \gamma_{xy} = 2\varepsilon_{xy}$$

$$\tau_{xy} = +(2)1.0(10^6)(-116)10^{-6}$$

$$\underline{\underline{\tau_{xy} = -232\,\text{psi}}}$$

$$\begin{cases} E\varepsilon_{xx} = \sigma_{xx} - \nu\sigma_{yy} & : \sigma_{zz} = 0 \\ E\varepsilon_{yy} = -\nu\sigma_{xx} + \sigma_{yy} & : \sigma_{zz} = 0 \end{cases}$$

or

$$\begin{cases} \sigma_x = \left(\dfrac{E}{1-\nu^2}\right)(\varepsilon_{xx} + \nu\varepsilon_{yy}) \\ \sigma_y = \left(\dfrac{E}{1-\nu^2}\right)(\nu\varepsilon_{xx} + \varepsilon_{yy}) \end{cases}$$

$$\sigma_{xx} = \left[\frac{(2.4)(10^6)}{1-0.04}\right][3{,}799 + 0.2(201)]\,10^{-6}$$

$$\underline{\underline{\sigma_{xx} = 9{,}598\,\text{psi}}}$$

$$\sigma_{yy} = \left[\frac{2.4(10^6)}{1-0.04}\right][0.2(3{,}799) + 201]\,10^{-6}$$

$$\underline{\underline{\sigma_{yy} = 2{,}402\,\text{psi}}}$$

$$\varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy})$$

$$\varepsilon_{zz} = \left[\frac{-0.2}{2.4(10^6)}\right](9{,}598 + 2{,}402)$$

$$\underline{\underline{\varepsilon_{zz} = -1{,}000(10^{-6})}}$$

(a) Summary

Strain:		Stress:	
$\varepsilon_{xy} = 3{,}799(10^{-6})$	Compression(+)	$\sigma_{xx} = 9{,}598$ psi	Compression(+)
$\varepsilon_{yy} = 201(10^{-6})$		$\sigma_{yy} = 2{,}402$ psi	
$\varepsilon_{zz} = -1{,}000(10^{-6})$		$\sigma_{zz} = 0$	
$\varepsilon_{xz} = -116(10^{-6})$		$\tau_{xy} = -232$ psi	
$\varepsilon_{yz} = 0$		$\tau_{yz} = 0$	
$\varepsilon_{zx} = 0$		$\tau_{zx} = 0$	

(b) Given: State of stress rectangular shaft $10' \times 20'$

Find:

(a) Best orientation

(b) Strengths for $FS_c = 2$, $FS_t = 4$.

Solution:

Best orientation is long axis
Parallel to σ_1^0 (compression t)

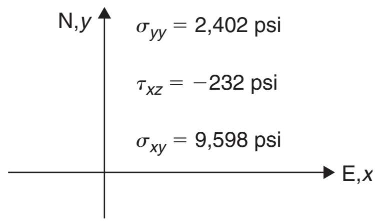
$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}$$

$$= \frac{-232}{\frac{1}{2}(9{,}598 - 2{,}402)}$$

$$\tan 2\theta = -0.064$$

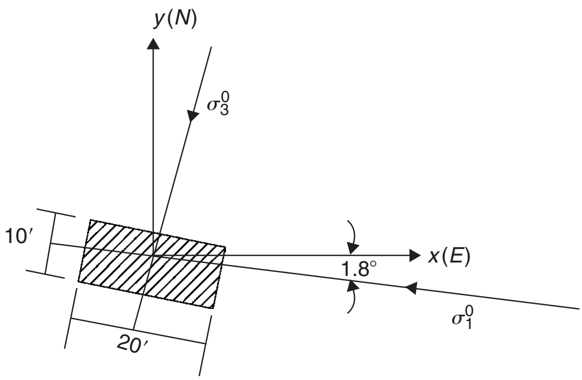
$$2\theta = -3.76^\circ$$

$$\underline{\theta = -1.89^\circ}$$



N91.8°E

Orientation (b)



$$\left.\begin{matrix}\sigma_1^0 \\ \sigma_3^0\end{matrix}\right\} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \frac{9{,}598 + 2{,}402}{2} \pm \left[\left(\frac{9{,}598 - 2{,}402}{2}\right)^2 + (-232)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1^0 \\ \sigma_3^0\end{matrix}\right\} = 6{,}000 + 3{,}605$$

$$\sigma_1^0 = 9{,}605 \text{ psi}$$

$$\underline{\underline{\sigma_3^0 = 2{,}395 \text{ psi}}}$$

$$M = \frac{\sigma_3^0}{\sigma_1^0}$$

$$M = \frac{2{,}395}{9{,}605}$$

$$\underline{\underline{M = 0.249}}$$

For: $M = 0.249, \dfrac{W_0}{H_0} = \dfrac{10}{20}$, Bull. 587 p.14 Fig. 6b [T3.2*a* text]

	$M = \frac{1}{5}$	$M = \frac{1}{3}$	$k = \frac{1}{2}$
K_{min}	−0.41	−0.14	
K_{max}	3.70	4.05	

$$(M = 0)\ 4.0 < K_c < 4.0+ \qquad \left(M = \frac{1}{3}\right)$$
$$K_c \approx 4.0$$
$$(M = 0)\ -0.8 < K_t < -0.1 \qquad \left(M = \frac{1}{3}\right)$$
$$K_t \approx -0.1$$

$$FS_t = \frac{T_0}{K_t\sigma_1^0},\ T_0 = (4)(0.1)(9{,}605), \quad T_0 = 3{,}843\text{ psi (low)}$$ ← tension

$$FS_t = \frac{C_0}{K_c\sigma_1^0}$$
$$C_0 = (2)(4.0)(9{,}605)$$
$$C_0 = 76{,}840\text{ psi}$$ ← (high) compression

13. Given: Problem 12 data plus gravity stress
$\gamma = 28\text{ kN/m}^3, FS_c = 2,\ h = 1{,}234\text{ m}, FS_t = 4$
Find: Strengths needed at depth.

Solution:

At depth

$$\sigma_{zz}(\text{gravity}) = \gamma h$$
$$= (28\text{ kN/m}^3)(1{,}234)$$
$$\sigma_{vg} = 34.6\text{ MPa}$$

Estimated horizontal gravity stress as

$$\sigma_{hg} = \frac{v}{1-v}\sigma_{vg}$$
$$= \frac{0.2}{1-0.2}34.6$$
$$\sigma_{hg} = 8.65\text{ Mpa} = \sigma_{Hg}$$

(From Problem 12)

$$S_1 = \sigma_{hg} + S_1^0 \qquad S_3 = \sigma_{hg} + S_3^0$$
$$= 34.6 + \frac{9{,}605}{145} \qquad = 8.65 + \frac{2{,}395}{145}$$
$$\underline{\underline{S_1 = 100.8\text{MPa}}} \qquad \underline{\underline{S_3 = 25.2\text{ MPa}}}$$

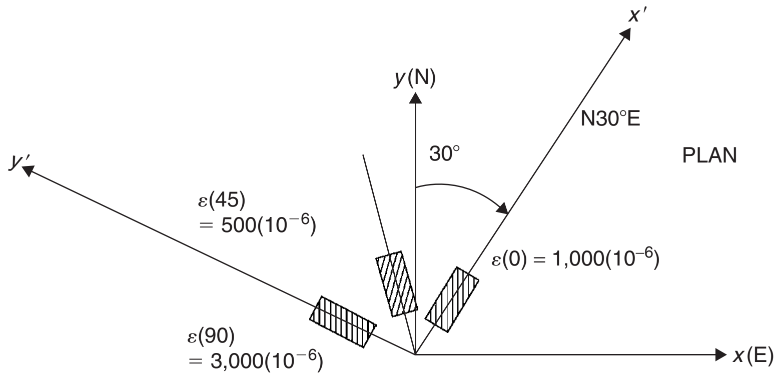
$$M = \frac{S_3}{S_1}$$
$$\underline{\underline{M = \frac{1}{4}}}$$

$\therefore K_t$ and K_c are nearly the same as in Problem 7, say $K_t = -0.1$, $K_c = 4.0$

$$C_0 = (FS_c)(K_c)(S_1^0) \qquad T_0 = (FS_t)(K_t)(S_1^0)$$
$$= (2)(4)(100.8) \qquad = (4)(0.1)(100.8)$$
$$\underline{\underline{C_0 = 806\text{ MPa}}} \qquad \underline{\underline{T_0 = 40.3\text{ MPa}}}$$
(≈117,000 psi!) (≈ 5,850 psi)

The high stress requires high strength, probably greater than laboratory strengths.

14. Given: Strain gauge data on a flat surface, over-cored



$E = 16.55$ GPa, $v = 0.20$, tension (+)
Find: State of strain & stress.

Solution:

Normal strain rotation of axis

$$\varepsilon = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \varepsilon_{xy}\sin 2\theta$$

w.r.t $x'y'z'$

$$\begin{aligned}
\varepsilon_{x'x'} &= \varepsilon(0)\\
\varepsilon_{y'y'} &= \varepsilon(90)\\
\varepsilon(45) &= \frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} + \frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2}\cos 2.45 + \varepsilon_{x'y'}\\
\therefore 2\varepsilon_{x'y'} &= 2\varepsilon(45) - [\varepsilon(0) + \varepsilon(90)]
\end{aligned}$$

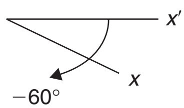
$$\left.\begin{aligned}
\varepsilon_{x'x'} &= 1{,}000(10^{-6})\\
\varepsilon_{y'y'} &= 3{,}000(10^{-6})\\
2\varepsilon_{x'y'} &= -3{,}000(10^{-6})\\
\varepsilon_{x'y'} &= -1{,}500(10^{-6})
\end{aligned}\right\}\text{w.r.t } x'y'z'$$

w.r.t xyz – rotation of axis

$$\left.\begin{matrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{matrix}\right\} = \frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} \pm \frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2}\cos 2\theta \pm \varepsilon_{xy}\sin 2\theta$$

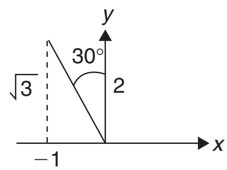
$$\varepsilon_{xy} = -\left(\frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2}\right)\sin 2\theta + \varepsilon_{xy}\cos 2\theta$$

$\theta = -60°$



$$\cos 2\theta = -\frac{1}{2}$$

$$\sin 2\theta = -\frac{\sqrt{3}}{2}$$



$$\frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} = \frac{1{,}000 + 3{,}000}{2}10^{-6}$$

$$\frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} = 2{,}000(10^{-6})$$

$$\frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} = \frac{1{,}000 - 3{,}000}{2}10^{-6}$$

$$\frac{\varepsilon_{x'x'} + \varepsilon_{y'y'}}{2} = -1{,}000(10^{-6})$$

$$\left.\begin{matrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{matrix}\right\} = 2{,}000(10^{-6}) \pm (-1{,}000)(10^{-6})\left(-\frac{1}{2}\right) \pm (-1{,}500)(10^{-6})\left(-\frac{\sqrt{3}}{2}\right)$$

$$\left.\begin{matrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{matrix}\right\} = \begin{matrix}3,799(10^{-6})\\ 201(10^{-6})\end{matrix}$$

$$\varepsilon_{xy} = -(-1{,}000)10^{-6}\left(\frac{-\sqrt{3}}{2}\right) + (-1{,}500)(10^{-6})\left(-\frac{1}{2}\right)$$

$$\varepsilon_{xy} = -116(10^{-6})$$

The surface is shear–stress free & normal stress free
$\underline{\underline{\tau_{zx} = \tau_{zy} = 0 = \sigma_{zz}}}$ $\therefore$ from Hooke's law

$$G\gamma_{zx} = \tau_{zx} \ \& \ G\gamma_{zy} = \tau_{zy}$$

$$\underline{\underline{\gamma_{zx} = \gamma_{zy} = 0}}$$

Also

$$\tau_{xy} = G(2)(-116)10^{-6}$$

$$G = \frac{E}{2(1+v)}$$

$$G = \frac{16.55}{2(1+0.2)}$$

$$\therefore G = 6.895\,\text{Gpa}$$

$$\tau_{xy} = (6.895)10^{9}(2)(-116)10^{-6}$$

$$\underline{\underline{\tau_{xy} = -1.60\,\text{Mpa}}}$$

$$\sigma_{xx} = \left(\frac{E}{1-v^2}\right)\left(\varepsilon_{xx} + v\varepsilon_{yy}\right) : \sigma_{zz} = 0$$

$$\sigma_{yy} = \left(\frac{E}{1-v^2}\right)\left(\varepsilon_{yy} + v\varepsilon_{xx}\right) : \sigma_{zz} = 0$$

$$\sigma_{xx} = \left(\frac{16.55}{1-0.04}\right)[3{,}799 + 0.2(201)]10^{9}10^{-6}$$

$$\underline{\underline{\sigma_{xx} = 66.2\,\text{Mpa}}}$$

$$\sigma_{yy} = \left(\frac{16.55}{1-0.04}\right)[201 + 0.2(3{,}799)]$$

$$\underline{\underline{\sigma_{yy} = 16.56\,\text{Mpa}}}$$

$$\varepsilon_{zz} = \frac{-v}{E}(\sigma_{xx} + \sigma_{yy})$$

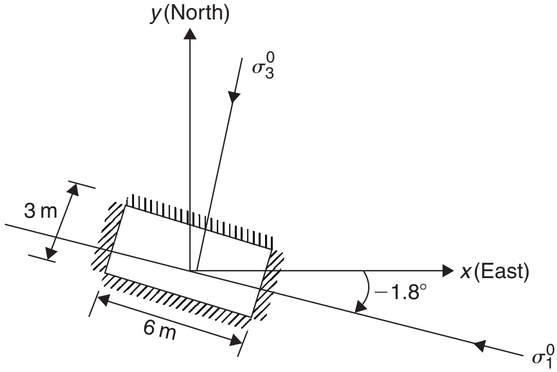
$$= \left[\frac{-0.2}{16.55(10^{9})}\right](66.2 + 16.56)10^{6}$$

$$\underline{\underline{\varepsilon_{zz} = -1{,}000(10^{-6})}}$$

(a) Summary

$\varepsilon_{xx} = 3{,}799(10^{-6})$	$\sigma_{xx} = 66.2\,\text{Mpa}$
$\varepsilon_{yy} = 201(10^{-6})$	$\sigma_{yy} = 16.56$
$\varepsilon_{zz} = -1{,}000(10^{-6})$	$\sigma_{zz} = 0$
$\varepsilon_{xy} = -116(10^{-6})$	$\tau_{xy} = -1.60$
$\varepsilon_{yz} = 0$	$\tau_{yz} = 0$
$\varepsilon_{zx} = 0$	$\tau_{zx} = 0$

(b) Best orientation is with long axis parallel to σ_1^0, rectangular shaft 3×6 m.



$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})} = \frac{-1.60}{\frac{1}{2}(66.2 - 16.56)}$$

$$\tan 2\theta = -0.0644$$

$$2\theta = -3.7^\circ, 176.3^\circ$$

$$\theta = -1.8^\circ, 88.2^\circ$$

orientation is N91.8°E

(c) Strengths for $FS_c = 2$, $FS_t = 4$

$$\left.\begin{matrix}\sigma_1^0\\ \sigma_3^0\end{matrix}\right\} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \left(\tau_{xy}\right)^2\right]^{1/2}$$

$$= \frac{1}{2}(66.2 + 16.56) \pm \left[\left(\frac{66.2 - 16.56}{2}\right)^2 + (-1.60)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1^0\\ \sigma_3^0\end{matrix}\right\} = 41.38 \pm 24.87$$

$$\sigma_1^0 = 66.25 \text{ MPa}$$

$$\sigma_3^0 = 16.51 \text{ MPa}$$

$$M = \frac{16.51}{66.25}$$

$$M = 0.25$$

Text (Table 3.2a)

	$M = \frac{1}{5}$	$M = \frac{1}{3}$	$k = \frac{1}{2}$
$K_{\min}$	−0.41	−0.14	
$K_{\max}$	3.70	4.05	

estimate:
$K_c \sim 4.0$
$K_t \sim 0.2$

$$FS_c = \frac{C_0}{K_c \sigma_1^0}, \qquad FS_t = \frac{T_0}{K_t \sigma_1^0}$$

$$C_0 = (2)(4.0)(66.25), \qquad T_0 = (4)(0.2)(66.25)$$

$$C_0 = 530 \text{ MPa}, \qquad T_0 = 53 \text{ MPa}$$

⟵

15. Given: Problem data plus gravity stress
$\gamma = 28\,\text{kN/m}^3$, $h = 1{,}234\,\text{m}$, $FS_c = 2, FS_t = 4$
Find: Strengths needed at depth.

Solution:

At depth

$$\begin{aligned} \sigma_{zz}(\text{gravity}) &= \gamma h \\ &= \left(28\,\text{kN/m}^3\right)(1{,}234) \\ \sigma_{vg} &= 34.6\,\text{MPa} \end{aligned}$$

Estimate horizontal gravity stress as

$$\begin{aligned} \sigma_{hg} &= \frac{v}{1-v} = \sigma_{vg} \\ &= \frac{0.2}{1-0.2}34.6 \\ \sigma_{hg} &= 8.65\text{ MPa} = \sigma_{Hg} \end{aligned}$$

(From Problem 14)

$$\begin{aligned} S_1 &= \sigma_{vg} + S_1^0 \\ &= 34.6 + \frac{9{,}605}{145} \\ S_1 &= 100.8\,\text{MPa} \end{aligned} \qquad \begin{aligned} S_3 &= \sigma_{hg} + S_3^0 \\ &= 8.65 + \frac{2{,}396}{145} \\ S_3 &= 25.2\,\text{MPa} \end{aligned}$$

$$M = \frac{S_3}{S_1}$$

$$M = \frac{1}{4}$$

$\therefore K_t$ and K_c are nearly the same as in Problem 7 say $K_t = -0.1, K_c = 4.0$

$$\begin{aligned} C_0 &= (FS_c)(K_c)(S_1^0) \\ &= (2)(4)(100.8) \\ C_0 &= 806\,\text{MPa} \end{aligned} \qquad \begin{aligned} T_0 &= (FS_t)(K_t)(S_1^0) \\ &= (4)(0.1)(100.8) \\ T_0 &= 40.3\text{ MPa} \end{aligned}$$

(≈117,000 psi!) (≈5,850 psi)

The high stress requires high strength, probably greater than laboratory strengths.

16. Given: Rectangular shaft

$C_0 = 23{,}700$ psi

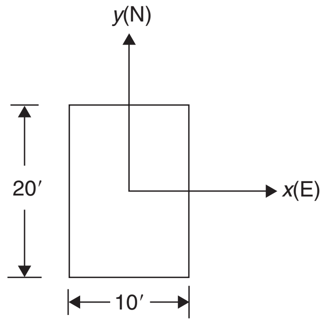
$T_0 = 1{,}450$ psi

$S_E = 350 + 0.2\,h$

$S_N = 420 + 0.35\,h$

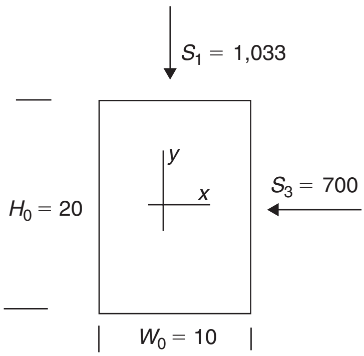
$S_v = 1.12\,h$

deeper to 1,800 ft



Find: FS_c, FS_t at 1750′

Solution:



$$FS_c = \frac{C_0}{\sigma_c}, \qquad FS_t = \frac{T_0}{\sigma_c}$$

$$\sigma_c = K_c S_1 \qquad \sigma_t = K_t S_1$$

$$S_N = 420 + 0.35(1{,}750)$$

$$\underline{S_N = 1{,}033 \text{ psi}}$$

$$S_E = 350 + 0.2(1{,}750)$$

$$\underline{S_E = 700 \text{ psi}}$$

$$S_N = S_1$$

$$M = \frac{700}{1{,}033}$$

$$\underline{M = 0.678}$$

$$\frac{W_0}{H_0} = \frac{10}{20}$$

$$\underline{\frac{W_0}{H_0} = 0.5}$$

Table 3.2 Text, estimate between

	$M = \frac{1}{2}$	$M = \frac{3}{4}$	
K_{max}	4.57	5.48	$K_c \approx 4.9$
K_{min}	0.19	0.69	(no tension)

$$FS_c = \frac{23{,}700}{4.9(1{,}033)} \qquad FS_t = \underline{\underline{\text{not a factor (no tension)}}} \quad \longleftarrow$$

$$\underline{\underline{FS_c = 4.68}}$$

$$FS_v = \frac{23{,}700}{(1.12)(1{,}750)} = \underline{\underline{12.1}}$$

17. Given: 3D hydrostatic stress state

Find: Favorable shape ellipse, rectangle, ovaloid with semi-axes ratio of 2? (USBM Bull 587)

Solution:

		Ellipse	Rectangle	Ovaloid
	K_c	4.0	4.6	3.2
$M = 1$				
	K_t	—	—	—

Note: Hydrostatic $\therefore M = 1$

Ovaloid is best lowest K_c no-tension

(Note: Ovaloid has well-rounded corners, max radius, in fact)

18. Given : 3×6 m rectangular shaft depth = 533 m to 579 m from 290 m

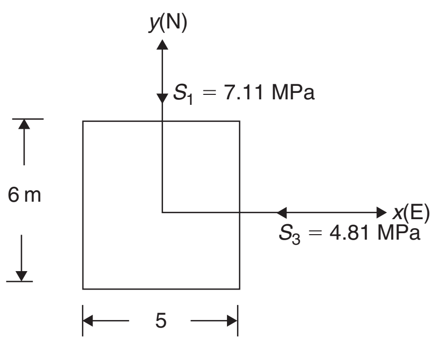
$$C_0 = 164\,\text{MPa}$$
$$T_0 = 10.2$$
$$E = 36.5\,\text{GPa}$$
$$v = 0.27$$
$$\gamma = 26.6\,\text{kN/m}^3$$

$$S_E = 2{,}414 + 4.5h \quad (x)$$
$$S_N = 2{,}897 + 7.9h \quad (y)$$
$$S_v = 25.3h \quad (z)$$

Find: FS_c, FS_t, FS_v for Shaft of 533 m.

Solution:

$$FS_c = \frac{C_0}{\sigma_c} \quad FS_t = \frac{C_0}{\sigma_t} \quad FS_v = \frac{C_0}{S_v}$$



$$\sigma_c = K_c S_v$$
$$\sigma_t = K_t S_v$$
$$S_v = 25.3(533)$$
$$\underline{S_v = 13.49\,\text{MPa}}$$
$$S_E = 2{,}414 + 4.5(533)$$
$$\underline{S_E = 4.81\,\text{MPa}}$$
$$S_N = 2{,}897 + 7.9(533) \qquad M = \frac{S_3}{S_1} = \frac{4.81}{7.11}$$
$$\underline{S_N = 7.11\,\text{MPa}} \qquad \underline{M = 0.68}$$

Table 3.2a:

	$M=\frac{1}{2}$	$M=\frac{3}{4}$
K_{max}	4.57	5.48
K_{min}	0.19	0.69

estimate $K_c \sim 4.9$
K_t not a factor

$$FS_c = \frac{164}{(4.9)(7.11)} \qquad FS_v = \frac{164}{13.49} = 12.2$$

$\underline{\underline{FS_c = 4.71}}$

$\underline{\underline{FS_t \text{ not a factor}}}$

$\underline{\underline{FS_v = 12.2}}$

19. Given: 3D hydrostatic stress state
Find: Favorable shape ellipse, rectangle, ovaloid with semi-axes ratio of 2?
(USBM Bull 587, text for ellipse, rectangle)

Solution:

		Ellipse	Rectangle	Ovaloid
	K_c	4.0	4.6	3.2
$M=1$:				
	K_t	—	—	—

Note: Hydrostatic $\therefore M=1$
$\underline{\underline{\text{Ovaloid is best lowest } K_c\text{, no-tension}}}$

(Note: Ovaloid has well-rounded corners, max r, in fact)

20. Given: Rectangular shaft $18' \times 24'$ at $4800'$

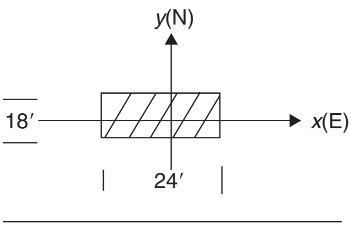
$$\begin{aligned}
\sigma_{xx} &= 750 + 0.5h \\
\sigma_{yy} &= 800 + 0.2h \\
\sigma_{zz} &= 1.1h \\
E &= 6.2(10^6)\,\text{psi}, v = 0.33 \\
c &= 5{,}600\,\text{psi}, \phi = 52 \\
\gamma &= 158\text{ pcf}
\end{aligned}$$

Find:
(a) Best orientation
(b) FS_t, FS_c

Solution:

$$\begin{aligned}
&\text{Near surface } \sigma_{yy} = \sigma_1 \text{ in plan view}\\
&\qquad\qquad \sigma_{xx} = \sigma_3\\
&\text{at depth: } \sigma_{yy} > \sigma_{xx}?\\
&800 + 0.2\,h > 250 + 0.5h\\
&\qquad 550 > 0.3h\\
&\quad 1{,}833 > h\\
&\therefore \text{ at depth} > 1{,}833 \;\; \sigma_{xx} = \sigma_1\\
&\qquad\qquad\qquad\qquad \sigma_{yy} = \sigma_z
\end{aligned}$$

Since stresses are greater with depth & strength constant, plan for depth. Best orientation is with long shaft axis parallel to σ_1, that is, *East* at depth greater than 1,833 ft.



21. Given: Problem 20 data at 4,800 ft

$\sigma_1 = 250 + 0.5(4{,}800) = 2{,}650$ psi
$\sigma_3 = 800 + 0.2(4{,}800) = 1{,}760$ psi
$M = \dfrac{\sigma_3^0}{\sigma_1^0} = \dfrac{1{,}760}{2{,}650} = 0.66$

Bulletin 587 or text Table 3.2

$$\frac{W_0}{H_0} = \frac{18}{24} = 0.75\,(W_0 \perp \text{to}\, \sigma_1 = \sigma_{xx})$$

	K_t	K_c	
$M = 1$	0 (0)	3.8 (4.6)	$\frac{W_0}{H_0} = 1.0$
$M = \frac{1}{3}$	−0.3 (−0.2)	3.2 (3.1)	$\frac{W_0}{H_0} = 6.5$

for $M = 0.66 \quad \frac{W_0}{H_0} = 0.75$

estimate $\underline{\underline{K_t = 0.1 \quad K_c = 3.8}}$

$$\left.\begin{matrix} C_0 \\ T_0 \end{matrix}\right\} = \frac{2c \cos\phi}{1 \mp \sin\phi} = \frac{2(5{,}600)\cos 52}{1 \mp \sin 52}$$

$$\underline{\underline{\left.\begin{matrix} C_0 \\ T_0 \end{matrix}\right\} = \begin{matrix} 32{,}527 \\ 3{,}857\ \text{psi} \end{matrix}}}$$

$$FS_t = \frac{3{,}857}{(0.1)(2{,}650)} = \underline{\underline{14.6}} \quad \textit{high FS's}$$

$$FS_c = \frac{32{,}527}{(3.8)(2{,}650)} = \underline{\underline{3.23}} \qquad \longleftarrow \text{No support needed}$$

But consider

$$FS_c = \frac{32{,}527}{(4{,}800)(1.1)} = \underline{\underline{6.16}} \ \text{(o.k. vertical)}$$

22. Given:

$\sigma_{xx} = 2{,}155 \qquad + = \text{Compression}$

$\sigma_{yy} = 3{,}045 \qquad x = \text{East}$

$\sigma_{zz} = 4{,}200 \qquad y = \text{North}$

$\tau_{yx} = -1{,}222 \qquad z = \text{Up}$

$\tau_{xz} = 0 = \tau_{yz}$

Find: $\sigma_1, \sigma_2, \sigma_3$, and directions, sketch.

Solution:

By inspection, the x-y plane with normal z is shear-free and therefore a principal plane. Thus z is a principal direction and $\sigma_{zz} = \sigma_2$ (for now)

In the x-y plane

$$\left.\begin{matrix} \sigma_1 \\ \sigma_3 \end{matrix}\right\} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

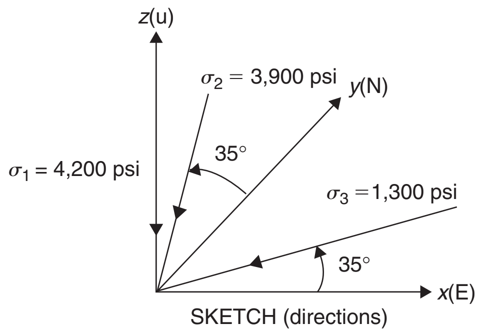
$$= \frac{2{,}155 + 3{,}045}{2} \pm \left[\left(\frac{2{,}155 - 3{,}045}{2}\right)^2 + (-1{,}222)^2\right]^{1/2}$$

$$\left.\begin{matrix} \sigma_1 \\ \sigma_3 \end{matrix}\right\} = 2{,}600 \pm 1{,}300$$

$$\left.\begin{matrix} \sigma_1 \\ \sigma_3 \end{matrix}\right\} = \begin{matrix} 3{,}900 \\ 1{,}300 \end{matrix}\ \text{psi}$$

$$\text{reording}\begin{cases} \sigma_1 = 4{,}200\ \text{psi} & \text{(Vertical)} \\ \sigma_2 = 3{,}900\ \text{N 35 W} & \text{(Horizontal)} \\ \underline{\underline{\sigma_3 = 1{,}300\ \text{N 55 E} \quad \text{(Horizontal)}}} \end{cases} \qquad \longleftarrow \sigma_1, \sigma_2, \sigma_3$$

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} + \sigma_{yy})}$$
$$= \frac{1{,}222}{\frac{1}{2}(2{,}155 - 3{,}045)}$$
$$\tan 2\theta = 2.746$$
$$2\theta = 70^\circ$$
$$\underline{\underline{\theta = 35^\circ}}$$
$+$ ccw from y-axis

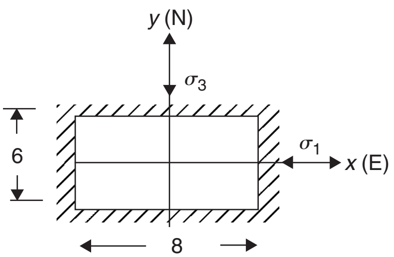


23. Given : 6 × 8 m rectangular shaft at a depth of 1,500 m

$\sigma_{xx} = 1{,}724 + 11.3h$ (E)(kPa, m) = units
$\sigma_{yy} = 5{,}517 + 4.5h$ (N)
$\sigma_{zz} = 2.49h$ (U)
$E = 42.8$ GPa, $\nu = 0.33$
$c = 38.6$ MPa, $\phi = 52^\circ$
$\gamma = 25.0\ \mathrm{kN/m^3}$

Find:
(1) Best orientation
(2) FS_c, FS_v, FS_t.

Solution:
Near surface: $\sigma_{yy} = \sigma_1$
$\sim(h = 0)$ $\sigma_{xx} = \sigma_3$
but strength high, stress low at depth : $\sigma_{xx} = 18.67$ MPa ($h = 1{,}500$ m)
$\sigma_{yy} = 12.27$
∴ Best orientation is with long (8 m) axis parallel to – x(E)



(favorable orientation)

24. Given: Problem 22 data, at 1,500 m , $\sigma_3 = 12.27$ Mpa, $\sigma_1 = 18.67$ Mpa

$$M = \frac{\sigma_3}{\sigma_1}$$
$$= \frac{12.27}{18.67}$$
$$\underline{M = 0.66}$$

$\frac{W_0}{H_0} = \frac{18}{24} = 0.75$

i.e. $k = 0.75$

Table 3.2.

	$M = 0.5$	$M = 0.66$ (interpolate)	$M = 0.75$	
K_{max}	4.57 4.39	5.15 4.77	5.48 4.97	$\left(k = \frac{1}{2},\ k = 1\right)$
K_{min}	0.19 −0.02	– –	0.69 0.40	(tension na)

$$M = 0.66 \quad k = \frac{1}{2} \qquad K_{max} = 5.15 \qquad k = \frac{3}{4}$$

$$k = 1 \qquad K_{max} = 4.77 : \text{interpolate}$$

$$\left.\begin{array}{l} M = 0.66 \\ k = \frac{3}{4} \end{array}\right\} \begin{array}{l} \underline{K_{max} = 4.96} \\ \underline{\text{tension absent}} \end{array}$$

$$\left.\begin{array}{l} C_0 \\ T_0 \end{array}\right\} = \frac{2c\cos\phi}{1 \mp 2\sin\phi} = \frac{(2)(38.6)\cos 52^\circ}{1 \mp \sin 52^\circ} = \left\{\begin{array}{l} 224 \text{ MPa} \\ 26.5 \text{ MPa} \end{array}\right.$$

$$FS_c = \frac{224}{(4.96)(18.67)} = \underline{\underline{2.42}} \qquad \longleftarrow FS_c$$

$$\underline{\underline{FS_t(\text{n.a})}}$$

$$FS_v = \frac{224}{(24.9)(1{,}500)} = \underline{\underline{6.00}} \qquad \longleftarrow FS_v$$

(no support needed)

25. Given: $\sigma_{xx} = 14.9$ MPa$\sigma_{yy} = 21.0\sigma_{zz} = 29.0$ (+ compression) $\tau_{xy} = -8.4$ $\tau_{yz} = 0$ $\tau_{zx} = 0$

Shaft: 4 × 8 m (rectangle, ellipse, ovaloid)

Find:

(1) $\sigma_1, \sigma_2, \sigma_3$ and directions

(2) Select best shape.

x = east y = north z = up

Solution:

By inspection, the z-direction is a principal direction

$$\sigma_{zz} = \sigma_2 = 29.0 \text{ MPa (for the moment)}$$

In x-y plane

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \left(\tau_{xy}\right)^2\right]^{1/2}$$

$$= \left(\frac{1}{2}\right)(14.9 + 21.0) \pm \left[\left(\frac{14.9 - 21.0}{2}\right)^2 + (-8.4)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \begin{matrix}26.89\,\text{MPa (for now)}\\ \underline{9.01\quad}\end{matrix}$$

$$\therefore \sigma_1 = \sigma_{zz} = 29.0 \quad \text{(Vertical)}$$

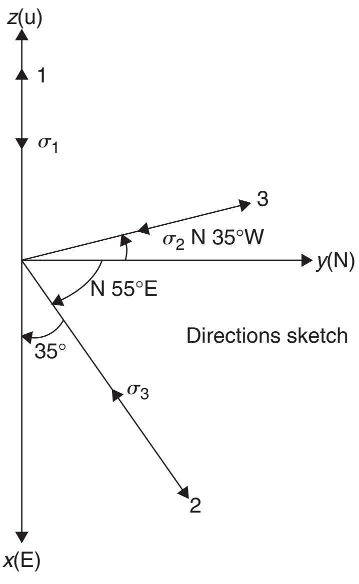
$$\left.\begin{matrix}\sigma_2 = 26.89\\ \sigma_3 = 9.01\end{matrix}\right\} \text{(Horizontal)} \qquad \leftarrow \sigma_1, \sigma_2, \sigma_3$$

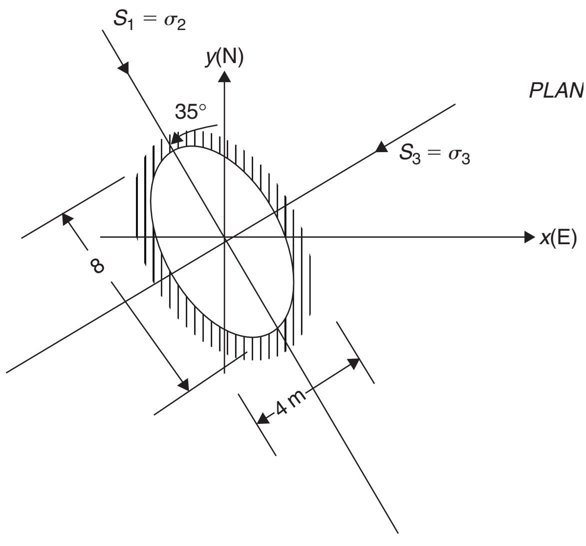
$$\tan 2\theta = \frac{-8.4}{\frac{1}{2}(14.9 - 21.0)}$$

$$\tan 2\theta = 2,754$$

$$2\theta = 70°, 250°$$

$$\theta = 35°, 125°$$





in plan view $\begin{matrix}S_1 = \sigma_2\ \text{N35°W}\\ S_3 = \sigma_3\ \text{N55°E}\end{matrix}$

In this orientation with

$$k = \frac{1}{2} \quad M = \frac{9.01}{26.89} = 0.335 = \frac{1}{3}$$

Ellipse $K_c = 1.67$ $K_t = 0$ (Table 3. 2a)
Rectangle $K_c = 4.05$ $K_t = -0.14$
Ovaloid (rectangle with *very* rounded corners would have K_c, K_t smaller.
$\therefore$ ellipse is best.

26. Given: Vertical shaft
$C_0 = 21{,}500$ psi $E = 6.25(10^6)$ psi $\gamma = 144$ pcf $T_0 = 1{,}530$ psi
$G = 2.5(10^6)$ psi

$$\left.\begin{aligned} S_E &= 200 + 0.3h \\ S_N &= 600 + 0.9h \\ S_v &= 1.14 \end{aligned}\right\} \text{preshaft stress (psi)} \quad h \text{ in ft.}$$

Find:
(a) Most favorable shape ellipse, rectangle ovaloid when shaft is 14,478 ft
(b) FS_t, FS_c at 2,800 ft

Solution:

Plan view, minimize stress concentration (best alignment)

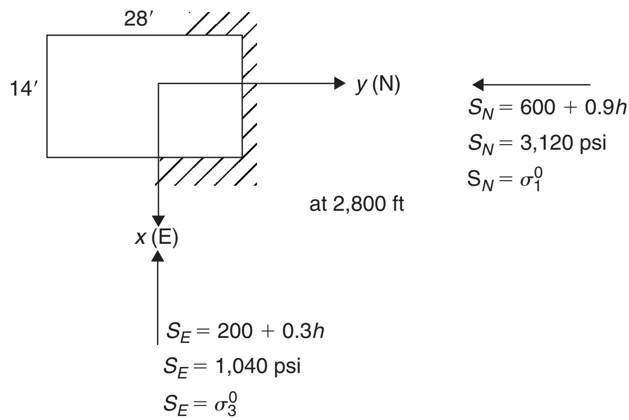
$$\frac{W_0}{H_0} = \frac{14}{28}$$

$$\frac{W_0}{H_0} = 0.5$$

$$M = \frac{\sigma_3^0}{\sigma_1^0}$$

$$M = \frac{1{,}040}{3{,}120}$$

$$M = \frac{1}{3}$$



	K_c	K_t		
Ovaloid	2.65	–	Fig 5	
Ellipse		1.65	–	Fig 4, p 11 USBM Bull. 587
Rectangle	3.10	−0.25	Fig 6	

Ellipse is best in compression and tension

(minimum K_c, no K_t)

$$FS_c = \frac{C_0}{\sigma_0}$$

$$= \frac{21{,}500}{(1.65)(3{,}120)}$$

$$FS_c = 4.18$$

FS_t n.a. (no tension)

27. Given: Vertical shaft 4.5 × 9.0 m
$C_0 = 148.3$ MPa $E = 43.10$ GPa $\gamma = 22.78\ \text{kN/m}^3$ $T_0 = 10.6$ $G = 17.24$ GPa
$S_E = 1{,}379 + 6.8\,h$: x
$S_N = 4{,}138 + 20.4\,h$: y
$S_v = 24{,}94$ (kPa, m) : z
Find:
(a) Best shape (rectangle, ellipse, ovaloid)
(b) *FS*'s at 854 m.

Solution:

Best orientation is with long axis parallel to S_1 in plan view

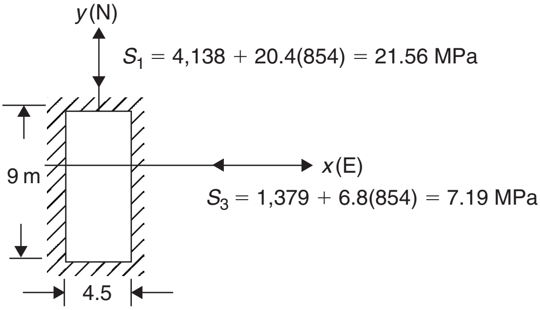
$$k = \frac{1}{2} = \frac{W_0}{H_0} = \frac{4.5}{9.0}$$

$$M = \frac{7.19}{21.56} = \frac{1}{3} = M$$

Table 3.2b: $M = \frac{1}{3}$ $k = \frac{1}{2}$

Ellipse: $K_{max} = 1.67 = K_c$
$\underline{K_{min} = -K_t = \text{nil}}$

Rectangle: $K_{max} = 4.05 = K_c$
$\underline{K_{min} = -0.14 = K_t}$



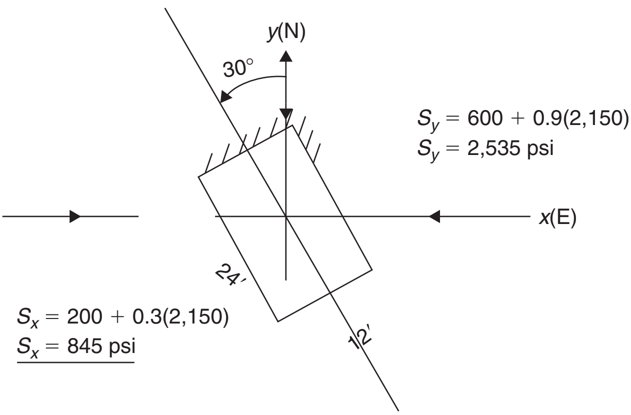
Ovaloid (rounded corners; less than rectangle *K*'s)
∴ Ellipse is best, least K_c, K_t
FS_t not a factor

← *FS*'s

$$FS_c = \frac{148.3}{(1.67)(21.56)} = 4.12 = FS_c$$

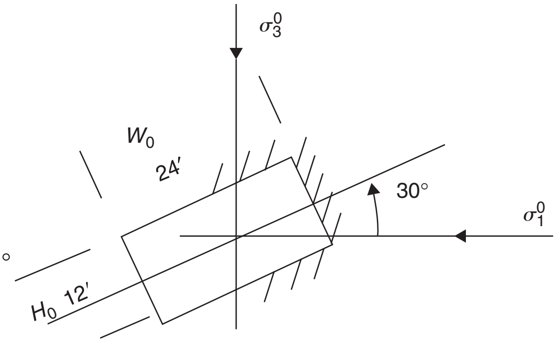
28. Given: Rectangular shaft 12 × 24 ft @ N30W, depth = 2,150 ft.
Find: *FS*'s

Solution:



$S_y > S_x$
$\therefore S_y = \sigma_1^0 \quad S_x = \sigma_3^0$ (Fig. 7, p 15, Bull 587)

$$\frac{W_0}{H_0} = \frac{24}{12} \qquad M = \frac{\sigma_3^0}{\sigma_1^0}$$
$$\frac{W_0}{H_0} = 2.0 \qquad M = \frac{845}{2,535}$$
$$\delta = 30^\circ \qquad M = \frac{1}{3}$$



$K_c = 4.85$ between 22.5 & 45°
4.7 5.2
$K_t =$ (no tension)

$$FS_c = \frac{C_0}{\sigma_c} = \frac{21,500}{4.85(2,535)}$$
$FS_c = 1.75$, FS_t (n.a)

29. Given: Rectangular shaft 12 × 24 ft from 3,000 to 4,800 ft stress state:
$S_{xx} = 2,000 + 1.1d$
$S_{yy} = 50 + 0.9d$
$S_{zz} = 1.15d$
$\tau_{xy} = -350 - 0.3d$
$\tau_{yz} = \tau_{yx} = 0$

Find:
(a) Best orientation of a single shaft
(b) Sketch peak stress concentration
(c) Estimate peak stress.

Solution:

"Best orientation is with long axis parallel to major compression"
$\therefore$ Find σ_1, σ_3 at great depth, i.e., 3,850 ft

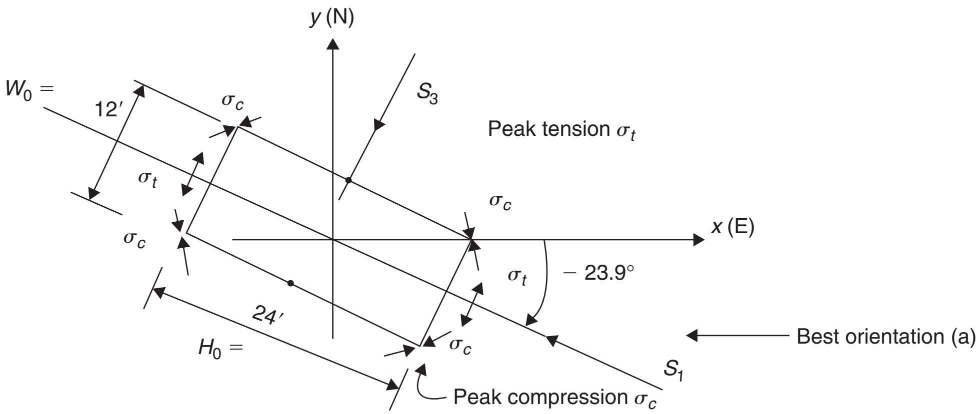
$S_{xx} = 2,000 + (1.1)(3,850)$
$S_{xx} = 6,235$ psi

$S_{yy} = 50 + 0.9(3,850)$
$S_{yy} = 3,515$ psi

$S_{zz} = 1.15(3,850)$
$S_{zz} = 4,428$ psi

$\tau_{xy} = -3.50 - 0.3(3,850)$
$\tau_{xy} = -1,505$ psi

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(S_{xy} - S_{yy})}$$
$$= \frac{-1{,}505}{\frac{1}{2}(6{,}235 - 3{,}515)}$$
$$\tan 2\theta = -1.1066$$
$$2\theta = -47.9^\circ$$
$$\underline{\underline{\theta = -23.9^\circ}}$$



Need S_1, S_3 for stress concession

$$\left.\begin{matrix} S_1 \\ S_3 \end{matrix}\right\} = \frac{S_x + S_y}{2} \pm \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + (\tau_{xy})^2}$$
$$= \frac{6{,}235 + 3{,}515}{2} \pm \sqrt{\left(\frac{6{,}235 - 3{,}515}{2}\right)^2 + (6{,}505)^2}$$
$$= 4{,}875 \pm 2{,}028$$
$$\underline{S_1 = 6{,}904}$$
$$\underline{S_3 = 2{,}847}$$
$$M = \frac{S_3}{S_1}$$
$$M = \frac{2{,}847}{6{,}904}$$
$$\underline{\underline{M = 0.412}} \;\&\; \underline{\underline{\frac{W_0}{H_0} = 0.5}}$$

Handout Bull. 587 pg. 14 Fig. 6b

at	$M = \frac{1}{3}$	$K_t = -0.2$	$K_c = 3.1$
at	$M = 1$	no tension; 0	$K_c = 4.7$
at	$M = 0.412$	$K_t = -0.18$	$K_c = 3.29$

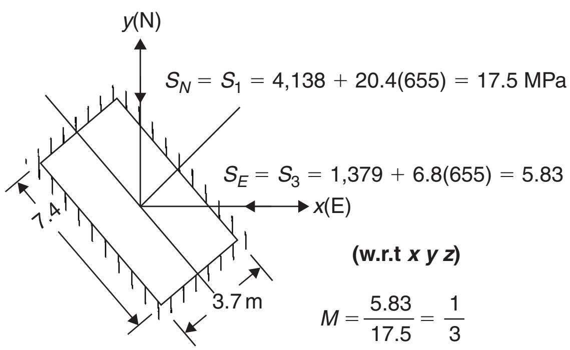
Peak tension occurs of mid-span along short dimension and peak compression occurs at the corners. (b)

$$\sigma_t = K_t\, \sigma_1^0$$
$$= (+0.18)(6{,}904)$$
$$\sigma_t = +1{,}234 \text{ tension (psi)} \quad \text{(c)}$$

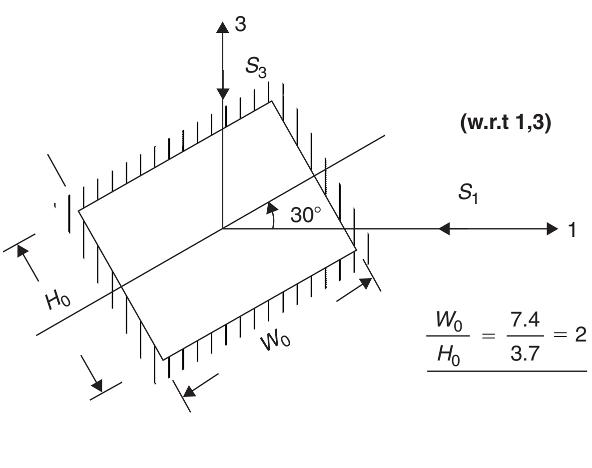
$$\sigma_c = K_c\, \sigma_1^0$$
$$\sigma_c = 3.29\,(6{,}904)$$
$$\sigma_c = 22{,}714 \text{ (psi)} \quad \text{(c)}$$

30. Given:
$C_0 = 148.3$ MPa
$T_0 = 10.55$ MPa
$E = 43.10$ GPa
$G = 17.24$ GPa
$\gamma = 22.8$ kN/m^3
$S_E = 1{,}379 + 6.8h$
$S_N = 4{,}138 + 20.4h$
$S_v = 24.9h$ (kPa,m)
3.7 + 7.4 m rectangular shaft
long axis is N30W

Find: *FS*'s at 655 m.



Solution:



w.r.t USBM Bull. 587 p. 15, Fig. 7. (For inclined openings)
Table 3 (B. 587)

$$M = \frac{1}{3} \quad \frac{W_0}{H_0} = 2$$

Estimate at 22.5° $K_c = 4.7$ K_t – nil
45.0° $K_c = 5.2$ K_t – nil
30.0° $K_c = 4.86$ K_t – nil

$\underline{\underline{FS_t \text{ not a factor}}}$

$$FS_c = \frac{148.3}{4.86(17.5)} = \underline{\underline{1.74}}$$

← *FS*'s

$$FS_c = \frac{148.3}{24.9(655)(11)^{-3}} = \underline{\underline{9.09}}$$

31. Given: 3.7 × 7.4 rectangular shaft to 1,175 m.
$S_{xx} = 13{,}793 + 24.9d$
$S_{yy} = 345 + 20.4d$
$S_{zz} = 26.0d$
$\tau_{xy} = -2{,}414 - 4.8d$
$\tau_{yz} = 0 = \tau_{zx}$

Find: Best orientation, peak stress sites, K_c, K_t.

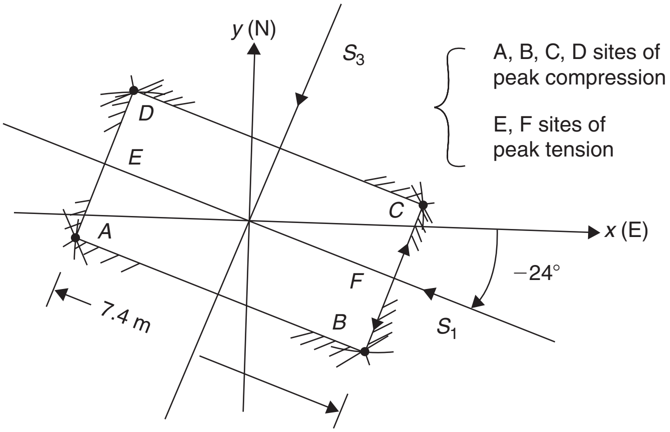
Solution:

"Best orientation is with long axis parallel to S_1" at 1,175 m

$$\begin{aligned} S_{xx} &= 13{,}793 + 24.9(1{,}175) = \underline{43.05\text{ MPa}} \\ S_{yy} &= 345 + 20.4(1{,}175) = \underline{24.32\text{ MPa}} \\ S_{zz} &= 26.0(1{,}175) = \underline{30.55\text{ MPa}} \\ \tau_{xy} &= -2{,}414 - 6.8(1{,}175) = \underline{-10.40\text{ MPa}} \end{aligned}$$

z is a principal direction, by inspection

$$\begin{aligned} \tan 2\theta &= \frac{\tau_{xy}}{\frac{1}{2}(S_{xx} - S_{yy})} = \frac{-10.40}{\frac{1}{2}(43.05 - 2{,}632)} = -1.1105 \\ 2\theta &= -48°, 132° \\ &\underline{\underline{\theta = -24°, +66°}} \end{aligned}$$



$$\left.\begin{matrix} S_1 \\ S_3 \end{matrix}\right\} = \frac{1}{2}(S_{xx} + S_{yy}) \pm \left[\left(\frac{S_{xx} - S_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \frac{1}{2}(43.05 + 24.32) \pm \left[\left(\frac{43.05 - 24.32}{2}\right)^2 + (-10.40)^2\right]^{1/2}$$

$$= 33.69 \pm 14.0$$

$$S_1 = 47.69\,\text{MPa}$$

$$\underline{\underline{S_3 = 19.69}}$$

$$M = \frac{S_3}{S_1} = \frac{19.69}{47.69} = \underline{\underline{0.412 = M}}$$

$$\underline{\underline{\frac{W_0}{H_0} = k = \frac{1}{2} = 0.5}}$$

Table 3.2.

$M = \frac{1}{3}$	$M = \frac{1}{2}$	$k = \frac{1}{2}$
K_{max}	4.05	4.57
K_{min}	−0.14	−0.20

at $M = 0.412$ $\underline{K_{max} = 4.31}$ (interpolate)
$\underline{K_{min} = -0.17}$

$$\sigma_c = K_c\sigma_1$$

$$= (4.31)(43.05)$$

$$\underline{\underline{\sigma_c = 185.5\,\text{MPa}}} \qquad \longleftarrow \sigma_c$$

$$\sigma_t = K_t\sigma_1$$

$$= -0.17(43.05)$$

$$\underline{\underline{\sigma_t = -7.31\,\text{MPa}}} \qquad \longleftarrow \sigma_t$$

32. Given: Vertical circular shaft, rock and concrete properties.
Find: Unlined wall *FS*.
Solution:

Circular shaft, gravity stress ∴ horizontal stresses are equal
$S_1 = S_3$ in plan view and $M = 1$
$K_c = 2, \quad K_t = 0$ (no tension)
Handout Bull 516 pg. 10, Fig. 3. Text $M = 1$, $K = 2$,

$$\sigma_c = K_c S_1$$

S_1 estimate: gravity alone

$$\sigma_h = \sigma_H = \left(\frac{v}{1-v}\right)\sigma_v, \quad \sigma_v = \gamma H$$

$$\sigma_h = \left(\frac{\frac{1}{4}}{1-\frac{1}{4}}\right)\left(\frac{156}{144}\right)(2{,}780)$$

$$\sigma_h = 1{,}004 \text{ psi}$$

$$\therefore \sigma_c = (2)(1{,}004)$$

$$\sigma_c = 2{,}008 \text{ psi}$$

$$FS_c = \frac{C_0}{\sigma_c} = \frac{6{,}750}{2{,}008}$$

$$\underline{\underline{FS_c = 3.36 \text{ (in Plan)}}}$$

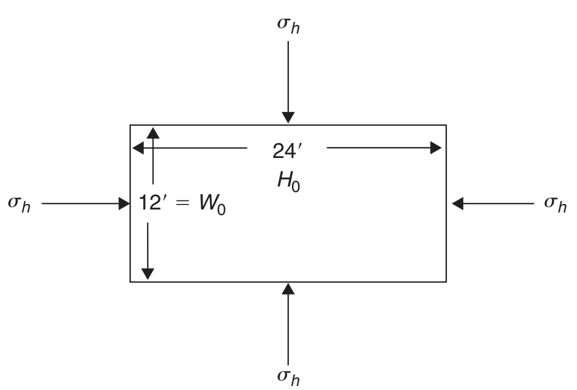
In vertical section:

$$FS_c = \frac{6{,}750}{\sigma_v} = \frac{6{,}750}{\left(\frac{156}{144} \times 2{,}780\right)} \qquad \text{(a)} \longleftarrow$$

$$\underline{\underline{FS_c = 2.24 \text{ (in section)}}}$$

33. Given: Three compartment rectangular shaft 12 × 24 ft, depth = 3,000 ft, σ^0 from gravity loading only.
$C_0 = 8{,}000$ psi, $\quad T_0 = 750$ psi
$E = 4.5 \times 10^4$ psi, $\quad v = 0.25$
$\gamma = 162$ pcf

Find:
(1) FS_t & FS_c at the unlined shaft wall.
(2) Optimum orientation of shaft.



Solution:

$$\frac{W_0}{H_0} = \frac{1}{2}, \text{ under gravity loading}$$

$$\sigma_h = \left[\frac{v}{1-v}\right]\sigma_v$$

where:

$$\sigma_v = \gamma H = \left(\frac{16{,}216}{\text{ft}^3}\right)(3{,}000\text{ ft})\left(\frac{1\text{ ft}^2}{144\text{ in}^2}\right)$$

$$\underline{\sigma_v = 3{,}375\text{ psi}}$$

$$\sigma_h = \left[\frac{0.25}{1-0.25}\right]3{,}375\text{ psi}$$

$$\underline{\sigma_h = 1{,}125\text{ psi}}$$

Vertical element:

$$FS_v = \frac{C_0}{\sigma_v} = \frac{8{,}000\text{ psi}}{3{,}375\text{ psi}}$$

$$\underline{\underline{FS_v = 2.37 \;\therefore\; \text{Stable in vertical compression}}} \qquad \xleftarrow{FS_v}$$

Horizontal FS's:

$$FS_c = \frac{C_0}{K_c\sigma_h}$$

Using USBM Bulliten 587: Alternatively Table 3.2

$M = 1$, $K = \dfrac{1}{2}$

$K_c = 6.46$

$K_t = \text{nil}$

@ $M = \dfrac{\sigma_h}{\sigma_h} = 1$; $\dfrac{W_0}{H_0} = \dfrac{1}{2}$

$K_c = 4.75$

$K_t = 0$

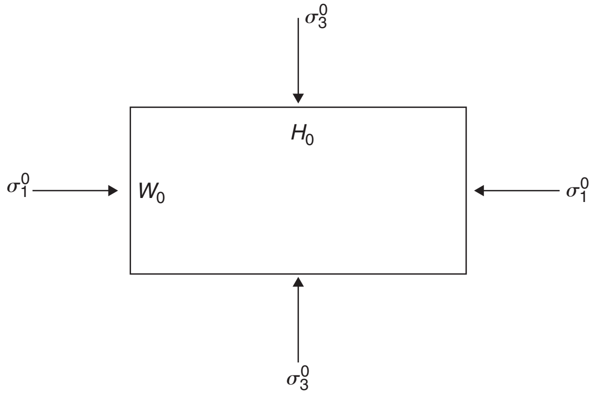
$$FS_c = \frac{8{,}000\text{ psi}}{(4.75)(1{,}125\text{ psi})} \qquad FS_c = \frac{8{,}000}{6.46(1{,}125)}$$

$$\underline{\underline{FS_c = 1.50 \;\therefore\; \text{Stable in horizontal compression or } (FS_c = 1.11)}} \qquad \xleftarrow{FS_c}$$

$$FS_t = \frac{T_0}{K_t\sigma_h}$$

$$\underline{\underline{FS_t = \infty \;\therefore\; \text{Stable in tension}}} \qquad \xleftarrow{FS_c}$$

Because of gravity loading, horizontal stresses are equal in all directions. Therefore there is not a single optimum orientation. If the horizontal stresses were not equal in all directions. The shortest span of the shaft (W_0) should be aligned with the greatest horizontal stress as shown below, i.e., long axis parallel to major compression.



34. Given: Actual stresses are tectonic

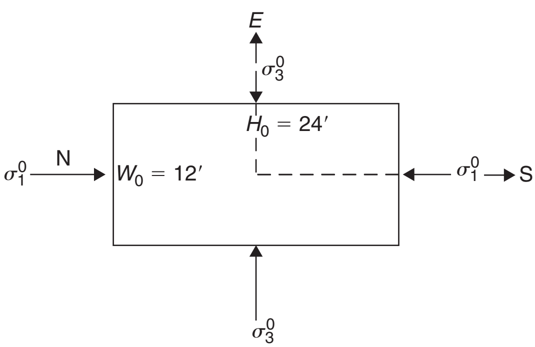
$\sigma^0_{EW} = \sigma_h + 1{,}250$ psi

$\sigma^0_{NS} = \sigma_h + 2{,}500$ psi

Find:

(1) Orient shaft in optimum directions

(2) Calculate *FS*'s.



Solution:

$$\sigma_h = 1{,}125 \text{ psi}$$
$$\sigma_1^0 = \sigma^0_{\text{NS}} = 3{,}625 \text{ psi}$$
$$\sigma_3^0 = \sigma^0_{\text{EW}} = 2{,}375 \text{ psi}$$

Vertical:

FS_v is the same as in Problem 33

$$\underline{\underline{FS_v = 2.37}}$$

$\longleftarrow FS_v$

Horizontal:

$$\frac{W_0}{H_0} = \frac{12'}{24'} = \frac{1}{2}$$
$$M = \frac{\sigma_3^0}{\sigma_1^0} = \frac{2{,}375 \text{ psi}}{3{,}625 \text{ psi}}$$
$$\underline{M = 0.655}$$

Using USBM Bulliten 587:
Estimate K_c & K_t from graph (low)

$K_c = 3.75$

$K_t = 0$

$\underline{\underline{FS_t = \infty}}$

$$FS_c = \frac{8{,}000 \text{ psi}}{(3.75)(3{,}625)\text{psi}}$$

$\underline{\underline{FS_c = 0.59}}$

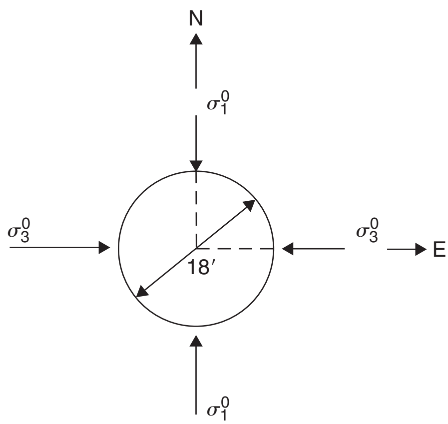
using Table 3.2 = Text
$K_c = 5.1$
$K_t = \text{nil}$

$\underline{\underline{FS_t}}$ (n.a.) ← FS_t

$\underline{\underline{FS_c = 0.43}}$ (will need support)

← FS_c

35. Given: Stress field from Problem 34
USE CIRCULAR SHAFT D = 18 ft



Find: FS's

Solution:

$$\sigma_1^0 = 3{,}625 \text{ psi}$$
$$\sigma_3^0 = 2{,}375 \text{ psi}$$

Vertical:
FS_v is same
$FS_v = 2.37$ ← FS_v

Horizontal:

$$M = \frac{\sigma_3^0}{\sigma_1^0}$$

$\underline{M = 0.66}$

Using USBM Bulliten 587:

estimate K_c & K_t @ $M = 0.66$
$K_c = 2.25$
$K_t = 0$

$\underline{\underline{FS_t = \infty}}$

$$FS_c = \frac{8{,}000 \text{ psi}}{(2.25)(3{,}625 \text{ psi})}$$

$\underline{\underline{FS_c = 0.98}}$

Text formula

$K_{min} = 1 + M - 2(1 - M) = 1 + 0.66 - 2(1 - 0.66)$
$K_{max} = 1 + M + 2(1 - M) = 1 + 0.66 + 2(1 - 0.66)$
$\underline{K_{min} = 0.98}$ ∴ no tension
$\underline{K_{max} = 2.34}$

FS_t (n.a) ← FS_t

$$FS_c = \frac{8{,}000}{(2.34)(3{,}625)}$$

$\underline{\underline{FS_c = 0.94}}$ ← FS_c

36. Given: In-situ stress field for Problem 34 & 35 changes between 3,000 & 3,500 ft to one described by:
$\sigma_v = 1.125h$
$\sigma_H = 3,500 + 0.33h$
$\sigma_h = 3,500 + 0.33h$

σ_v; σ_H & σ_h are principal stresses. (h is in feet, σ in psi)

Find: Unlined FS's @ 4,500 ft.

Solution:

@ 4500 ft:

$$\sigma_v^0 = (1.25)(4{,}500)\text{ psi}$$
$$\underline{\sigma_v^0 = 5{,}063\text{ psi}}$$
$$\sigma_H = \sigma_h = 3{,}500 + 0.33(4{,}500)$$
$$\underline{\sigma_h^0 = 4{,}985\text{ psi}}$$

Vertical:

$$FS_v = \frac{C_0}{\sigma_v^0} = \frac{8{,}000\text{ psi}}{5{,}063\text{ psi}}$$
$$\underline{\underline{FS_v = 1.58}}$$

← FS_v

Assume Circular Shaft
Horizontal:

$$M = \frac{\sigma_H^0}{\sigma_h^0} = 1$$

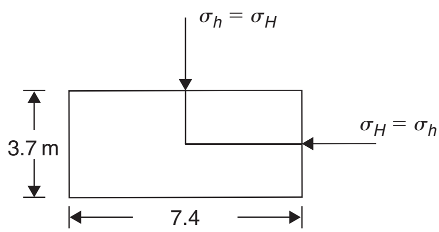
From USBM Bulliten 587: Text formula

$K_c = 2$ $\quad$ $K_c = 2$
$K_t = 0$ $\quad$ $K_t = \text{nil}$ } same

$\underline{\underline{FS_t = \infty}}$ ← FS_t

$$FS_c = \frac{8{,}000\text{ psi}}{2(4{,}985)}$$
$\underline{\underline{FS_c = 0.80}}$ ← FS_c

37. Given: 3 compartment shaft 3.7 × 7.4 m
vertical depth = 914 m, gravity only
$C_0 = 55.17$ MPa $T_0 = 5.17$ MPa
$E = 31.0$ GPa $v = 0.25$
$\gamma = 25.6\text{ kN/m}^3$
Find: FS_c, FS_t, best orientation.
Solution:



Gravity implies equal horizontal stress
∴ <u>there is no one best orientation</u> ← orientation

$FS_c = \dfrac{C_0}{S_v}$

estimate: $S_v = \gamma H$

$= (25.6)(914)$

$\underline{S_v = 23.4\text{ MPa}}$

$\underline{\underline{FS_v}} = \dfrac{55.17}{23.4} = \underline{\underline{2.36}}$ ← FS_v

$FS_v = \dfrac{C_0}{\sigma_c}$, $\qquad FS_t = \dfrac{T_0}{\sigma_t}$

$\sigma_c = K_c \sigma_1^0 \qquad \sigma_t = K_t \sigma_1^0$

$\sigma_h = \sigma_H = \sigma_1^0 \text{ (plan view)} = \sigma_3^0 = \left(\dfrac{v}{1-v}\right) S_v = \dfrac{0.25}{1-0.25}(23.4)$

$\underline{\sigma_h = \sigma_H = 7.8\text{ MPa}}$

$M = 1$ ($K = \frac{1}{2}$, $h = 2$) hydrostatic case

Table 2 $\quad K_{\max} = 6.46 \quad K_{\min} = 0.52$ ∴ no tension

$\underline{\underline{FS_c}} = \dfrac{55.17}{6.46(7.8)} = \underline{\underline{1.14}}$

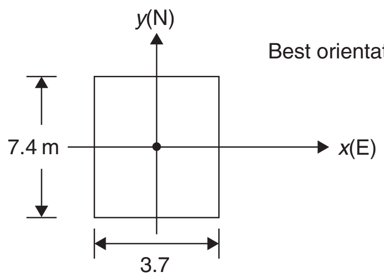
$\underline{\underline{FS_t}}$ (n.a.) ← FS_c, FS_t

38. Given: Previous Problem 37, data and

$\sigma_{EW} = \sigma_h + 8.62\text{ MPa}$

$\sigma_{NS} = \sigma_H + (2)(8.62)\text{ MPa}$

Find:
(a) Best orientation
(b) *FS*'s.



Best orientation

← Orientation

Solution:

$$\sigma_{EW} = 7.8 + 8.62 = \underline{16.42\text{ MPa}}$$
$$\sigma_{NS} = 7.8 + 17.24 = \underline{25.04}$$

FS_v same as in 37 $\quad \underline{\underline{FS_v = 2.36}}$ ← FS_v

In favorable orientation: $K = \dfrac{1}{2} \quad M = 0.66$

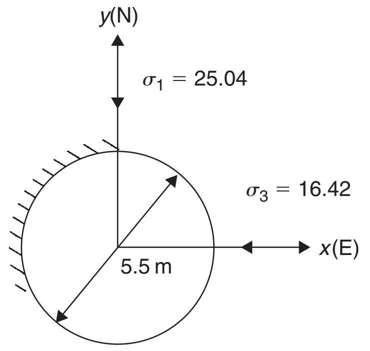
Using Table 3.2 text interpolating, estimating

$K_c = 5.1 \quad \& \quad K_t = \text{nil}$

$\underline{\underline{FS_c}} = \dfrac{55.17}{(5.1)(25.04)} = \underline{\underline{0.43\text{ (low!)}}}$ ← FS_c

$\underline{\underline{FS_t\text{ n.a.}}}$ ← FS_t

39. Given: Stress field of Problem 38
$\sigma_v = 23.4$
$\sigma_{EW} = 16.42$
$\sigma_{NS} = 25.04$ Mpa
Circular shaft, 5.5 m diameter
Find: Shaft wall *FS*'s.



Solution:

S_v is same, C_0 same, $\therefore$ $\underline{\underline{FS_v = 2.36}}$
$M = 0.66$ (as before)
Text formula

$K_{min} = (1 + M) - 2(1 - M) = 0.98$
$K_{max} = (1 + M) + 2(1 - M) = 2.34$

$\therefore$ no tension $\quad \underline{\underline{FS_t \text{ n.a.}}}$

$$FS_c = \frac{55.17}{(2.34)(25.04)}$$

$\underline{\underline{FS_c = 0.94}}$

40. Given: Stress field in Problems 38 & 39 changes to
$\sigma_v = 25.45h$
$\sigma_H = 24{,}138 + 7.47h$
$\sigma_h = 24{,}138 + 7.47h$
(kPa, m)

Find: *FS*'s at 1,372 m (circular shaft).

Solution:

(a) $$FS_v = \frac{55.17}{25.45(1{,}372)(10^{-3})}$$

$\underline{\underline{FS_v = 1.58}}$ $\quad \leftarrow FS_v$

(b) In plan view $\sigma_H = \sigma_h$ $\therefore$ $M = 1$, $K = 2$, no tension (Text formula)

$$FS_c = \frac{55.17}{(2)(\sigma_H)}$$

$\sigma_H = 24{,}138 + 7.47\ (1{,}372)$

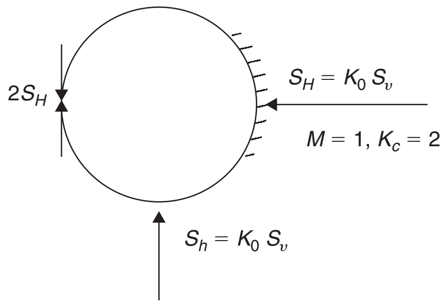
$\sigma_H = 34.4$ MPa

$$\therefore \underline{\underline{FS_c}} = \frac{55.17}{(2)(34.4)} = \underline{\underline{0.80}}$$ $\quad \leftarrow FS_c$

$\underline{\underline{FS_t \text{ n.a.}}}$ $\quad \leftarrow FS_t$

41. Given: and gravity preshaft stress field
Find: If rock safety factor = 3.0, $C_0 = ?$

Solution:



$$FS_c = \frac{C_0}{\sigma_c}$$

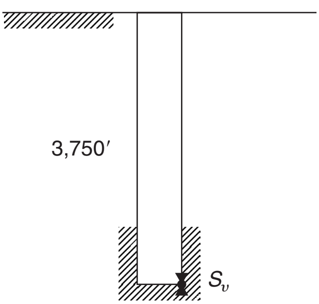
$$C_0 = \sigma_c(3.0)$$

In plan view
$S_v = \gamma h$,
estimate at 1 psi/ft
S_v (max) = 3,750 psi
estimate $K_0 = \frac{1}{3}$

$$\sigma_c = (2)\left(\frac{1}{3}\right)(S_v)$$

$$\underline{\underline{C_0 = \left(\frac{2}{3}\right)(3.0)(S_v)}} : \text{(less)}$$

In vertical section:



$$FS = \frac{C_0}{S_v}$$

$$\underline{\underline{C_0 = (3.0)(S_v)}}$$

$$C_0 = (3.0)(3,750)$$

Vertical rules

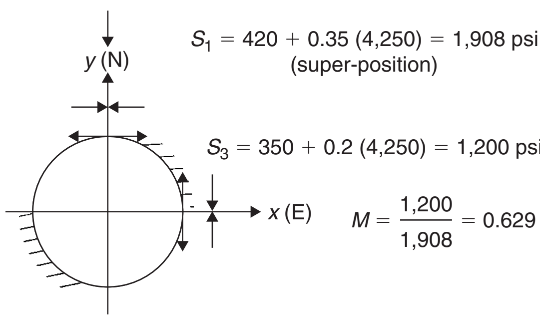
$$\underline{\underline{C_0 = 11,250 \text{ psi}}}$$

(77.6 MPa)

$\leftarrow C_0$

Supported Shafts, Liners, Bolts, Rings

42. Given: Vertical circular shaft, I.D. = 19 ft
Find:
(a) FS_c, FS_t unlined at 4,250 ft depth
(b) Liner thickness t for 90 psi water pressure $f_0' = 3,500$ psi (concrete).



Solution:

-1, 3 uniaxial superposition

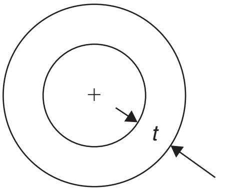
$$\sigma_N = -1(908) + 3(1{,}200) = 1{,}692\text{ psi} \quad -S_N + 3S_E$$
$$\sigma_E = -1(1{,}200) + (1{,}908) = 4{,}524 \quad -S_E + 3S_N$$
$$\underline{\underline{FS_c = \frac{23{,}700}{4{,}524} = 5.24}}$$
$$\underline{\underline{FS_v = \frac{23{,}700}{(1.12)(4{,}250)} = \underline{4.98}}}$$
$$\underline{\underline{FS_t : \text{no tension, not a factor}}}$$

← (a) FS_c, FS_t

(b) $FS_c = \dfrac{C_0}{\sigma_c}$

Max. allowable $\sigma_c = \dfrac{C_0}{FS_c} = 3{,}500$ psi

But also $\sigma_c = \dfrac{2P_b}{1 - \left(\frac{a}{b}\right)^2}$



$$\therefore 3{,}500 = \frac{(2)(90)}{1 - \left(\frac{a}{b}\right)^2}$$

$$3{,}500 = \frac{(2)(90)}{1 - \left(\frac{9.5}{b}\right)^2}$$
$$1 - \left(\frac{9.5}{b}\right)^2 = \frac{180}{3{,}500}$$
$$\left(\frac{9.5}{b}\right)^2 = 1 - \frac{180}{3{,}500}$$
$$\frac{1}{b^2} = 1.05(10^{-2})$$
$$\underline{b = 9.754\text{ ft}}$$
$$t = b - a$$
$$= 9.754 - 9.50$$
$$\underline{\underline{t = 0.254\text{ ft } (3.05\text{ in.})}}$$

← t

43. Given: Liner in Problem 42.
Find: ΔD at failure.

Solution:

$$\Delta D = 2u|_{r=a} \quad \text{(twice radial displacement)}$$

$$= (2)\left(\frac{1+v}{E}\right)\left[(1-2v)\left(\frac{P_b a}{1-\left(\frac{a}{b}\right)^2}\right) + \frac{P_b a}{1-\left(\frac{a}{b}\right)^2}\right]$$

$$\Delta D = (2)\frac{(1+0.25)}{4.75(10^6)}(1.0-0.5+1.0)\left[\frac{P_b a}{1-\left(\frac{a}{b}\right)^2}\right]$$

at failure: $\sigma_c = C_0$

$$\frac{2P_b}{1-\left(\frac{a}{b}\right)^2} = 5{,}740\text{ psi}$$

$$\therefore \Delta D = \frac{(2)(1.25)}{4.75(10^6)}(0.75)(2)\left(\frac{P_b}{1-\left(\frac{a}{b}\right)^2}\right)a$$

$$= \frac{(2.5)(0.75)(5{,}740)(9.5)(12)}{4.75(10^6)}$$

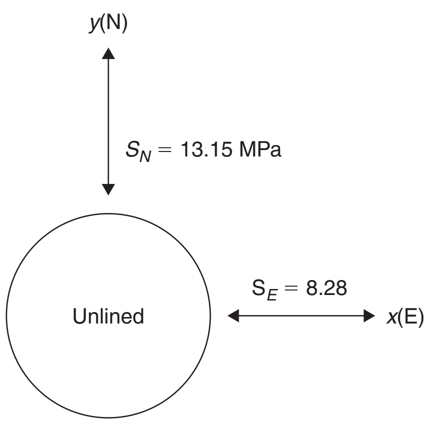
$$\underline{\underline{\Delta D = 0.258\text{ in.}}} \longleftarrow$$

44. Given: Vertical, circular shaft 5.8 m I.D.

[Rock]
$C_0 = 163.5$ MPa, $T_0 = 10.21$
$E = 36.48$ GPa, $v = 0.27$
$\gamma = 25.63$ kN/m^3

[Stress]
$S_E = 2{,}414 + 4.53h \quad : x$
$S_N = 2{,}897 + 7.92h \quad : y \quad$ (kPa, m)
$S_v = 25.3h \quad : z$

[Concrete]
$C_0 = 39.6$ MPa, $T_0 = 2.93$
$E = 32.76$ GPa, $v = 0.25$



Find:
(1) *FS*'s unlined at 1,295 m
(2) Liner thickness h, $P = 0.621$ MPa, $f'_c = 24.14$ MPa (concrete).

Solution:

$\sigma_N = 2{,}897 + 7.92(1{,}295)$

$\underline{\sigma_N = 13.15\text{ MPa}}$

$\sigma_E = 2{,}414 + 4.53(1{,}295)$

$\underline{\sigma_E = 8.28\text{ MPa}}$

Text formula:

$K_{\min} = 1 + M - 2(1-M)$

$K_{\max} = 1 + M + 2(1-M)$

$$M = \frac{8.28}{13.15} \qquad \underline{K_{min}} = 1 + 0.63 - 2(1 - 0.63) = \underline{0.89}$$

$$\underline{M = 0.63} \qquad \underline{K_{max}} = 1 + 0.63 + 2(1 - 0.63) = \underline{2.37}$$

$$FS_c = \frac{C_0}{\sigma_c} \qquad FS_t = \frac{T_0}{\sigma_t} \qquad FS_v = \frac{C_0}{\sigma_v}$$

$$\sigma_c = K_c \sigma_1^0 \qquad \sigma_t = K_t \sigma_1^0 \qquad = \frac{163.5}{(25.3)(1{,}295)(10^{-3})}$$

$$FS_c = \frac{163.5}{(7.37)(13.15)} \qquad K_t \text{ nil } \therefore \text{tension n.a.}$$

$$\underline{\underline{FS_c = 5.25 \qquad FS_t(\text{n.a.}) \qquad FS_v = 4.99}} \qquad \leftarrow FS\text{'s}$$

Max allow C_0 in concrete = 3,500

$$\text{but also } \sigma_{max} = \frac{2p}{1 - \left(\frac{a}{b}\right)^2}$$

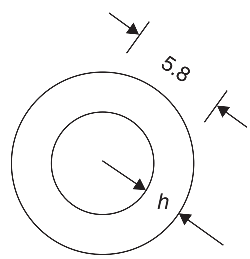
$$(24.14) = \frac{(2)(0.621)}{1 - \left(\frac{2.9}{b}\right)^2} : a = \frac{I.D.}{2}$$

$$\therefore 1 - \left(\frac{2.9}{b}\right)^2 = 0.05145$$

$$\left(\frac{2.9}{b}\right)^2 = 0.9485$$

$$\frac{2.9}{b} = 0.9739$$

$$\underline{b = 2.978\,\text{m}}$$



$$h = b - a$$

$$= 2.978 - 2.900$$

$$h = 0.078\,\text{mm}$$

$$\underline{\underline{h = 7.8\,\text{cm}}} \qquad \leftarrow h$$

45. Given: Previous problem data
Find: ΔD_i at failure (change in inside diameter).

$D_i = 5.8$ mm
$E = 36.48$ GPa $v = 0.27$ [rock]
$E = 32.76$ GPa $v = 0.75$ [concrete]

Solution:

$$\Delta D = 2u|_{r=a}$$

Text – derivation

$$\Delta D_i = 2u|_{r=a} = 2\left(\frac{1+v}{E}\right)\frac{[(1-2v)P_b a + P_b a]}{\left[1-\left(\frac{a}{b}\right)^2\right]}$$

$$\Delta D_i = 2u|_{r=a} = \frac{(1+0.25)}{32.76\,\text{GPa}}(1-0.51+1)\frac{a_2 P_b}{1-\left(\frac{a}{b}\right)^2};\ a = 2.9\,\text{m}$$

$$\text{at failure}: \sigma_c = C_0 = \frac{2P_b}{1-\left(\frac{a}{b}\right)^2}$$

$$\underline{C_0 = 39.6\,\text{MPa}}$$

$$\therefore \Delta D_i = 0.00657\,\text{m}$$

$$\underline{\underline{0.657\,\text{cm}}}$$

← ΔD_i *at* liner failure

46. Vertical, circular shaft

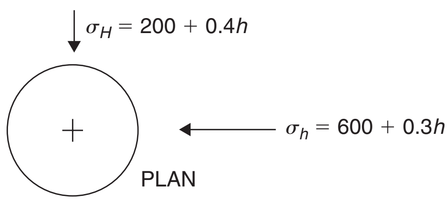
$D = 26$ ft (inside)
depth $= 3{,}200$ ft
$P_w = 210$ psi

Find: Unlined *FS*'s, t for liner (not needed).

Solution:

$h = 3{,}200$ ft
$\sigma_H = 200 + 0.4(3{,}200)$
$\sigma_H = 1{,}480$ psi

$\sigma_h = 600 + 0.3(3{,}200)$
$\sigma_h = 1{,}560$ psi



$\underline{\therefore \sigma_h = \sigma_E = \sigma_1\ \&\ \sigma_H = \sigma_N = \sigma_3\ \text{(plan)}}$

$$M = \frac{\sigma_3^0}{\sigma_1^0} = \frac{1{,}480}{1{,}560}$$

$$\underline{M = 0.9491} \approx 1$$

$\therefore K_c \cong 2.0 \quad K_t \sim \text{nil}$

$$FS_c = \frac{C_0}{\sigma_c} = \frac{27{,}400}{(2.0)(1{,}560)}$$

$$\underline{\underline{FS_c = 8.78\ \text{(plan)}}}$$

$$FS_t = \frac{T_0}{\sigma_t}$$

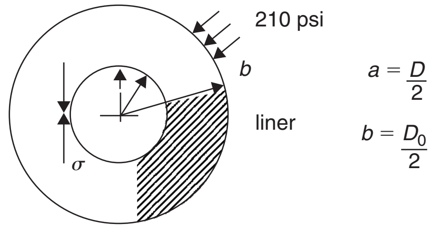
$\underline{\underline{FS_t \text{ – not a factor}}}$

← FS_t

$$\text{Vertical } FS_v = \frac{C_0}{S_v}$$
$$= \frac{27{,}460}{(3{,}200)(1.12)}$$
$$\underline{\underline{FS_v = 7.65 \text{ (vertical)}}}$$

$\leftarrow$ 1/3 FS_c

(b) Liner thickness for $p_w = 210$ psi and for $\dfrac{C_0(\text{concrete})}{FS_{\text{concrete}}} = 3{,}500$ psi (max allow stress)
$\sigma_{\max} = 3{,}500$ psi,



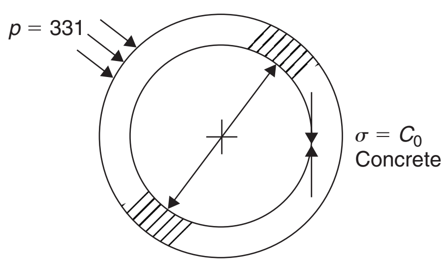
But also: $\sigma = \dfrac{2p}{1 - \left(\frac{a}{b}\right)^2}$

$$\therefore 3{,}500 = \frac{(2)(210)}{1 - \left(\frac{a}{b}\right)^2}$$
$$3{,}500\left[1 - \left(\frac{a}{b}\right)^2\right] = 420$$
$$1 - \left(\frac{a}{b}\right)^2 = 0.12$$
$$\left(\frac{a}{b}\right)^2 = 0.88$$
$$a = \sqrt{0.88}\, b$$
$$b = \frac{1}{\sqrt{0.88}}\left(\frac{26}{2}\right)$$
$$b = 13.86 \text{ ft}$$
$$b - a = 13.86 - 13.00$$
$$\therefore t = b - a$$
$$t = 0.86 \text{ ft}$$
$$\underline{\underline{t = 10.3 \text{ inches}}}$$

$\leftarrow$

47. Given: Impending liner failure
Find: Inside diameter change.
Solution:

Pressure at failure $\sigma = C_0$



$$C_0 = \frac{2p}{1 - \left(\frac{a}{b}\right)^2}$$
$$p = \frac{C_0}{2}\left[1 - \left(\frac{a}{b}\right)^2\right]$$
$$= \left(\frac{5{,}500}{2}\right)\left[1 - \left(\frac{13}{13.86}\right)^2\right]$$
$$\underline{\underline{p_{\text{fail}} = 331 \text{ psi}}}$$

$\Delta D = 2u|_{r=a} = (2)(\text{radial displacement of wall})$

$$u = \left(\frac{1+v}{E_2}\right)\left[(1-2v)\left(\frac{p}{1-\left(\frac{a}{b}\right)^2}a\right) + \frac{p}{1-\frac{a^2}{b^2}}a\right]$$

$$2u = \left(\frac{1+v}{E}\right)(1.5)\frac{2pa}{1-\left(\frac{a}{b}\right)^2} \quad \text{: concrete}$$

$$= \left[\frac{1+0.25}{5.5(10^6)}\right](1.5)(5{,}500)(13)(12)$$

$$2u = 0.293\text{ in.}$$

$$\therefore \underline{\underline{\Delta D = 0.293\text{ in.}}} \qquad \text{(at failure)}$$

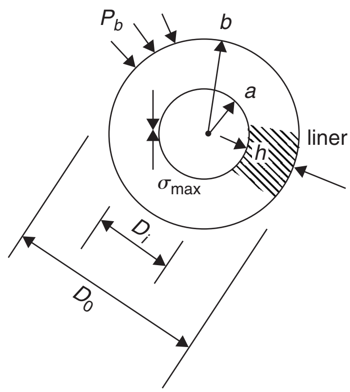
48. Given: Shaft and concrete data
concrete

$C_0 = 37.93\text{ MPa}, T_0 = 3.79$

$E = 37.93\text{ GPa}, v = 0.25$

$P_w = 1.45\text{ MPa}$

$D_i = 8\text{ m}$



Find: Liner thickness
Note: $F_c' = 24.14\text{ MPa}$

Solution:

$$\underset{(\text{allow})}{\sigma_{max}} = \frac{2P_b}{1-\left(\frac{a}{b}\right)^2}$$

$$24.14 = \frac{(2)(1.45)}{1-\left(\frac{a}{b}\right)^2}$$

$$\therefore \left(\frac{a}{b}\right)^2 = 0.88$$

$$\frac{a}{b} = 0.938$$

$$\therefore b = \frac{\frac{8}{2}}{0.938}$$

$$\underline{b = 4.264\text{ m}}$$

$$h = b - a$$

$$= 4.264 - 4$$

$$\underline{h = 0.264\text{ m}}$$

$$\underline{\underline{h = 26.4\text{ cm}}} \qquad h$$

49. Given: Problem 48 data

Find: ΔD_i at failure (cm)

Solution:

Find failure load

$$C_0 = \frac{2P}{1-\left(\frac{a}{b}\right)^2}$$

$$\underline{\underline{C_0 = 37.93\,\text{MPa}}} \qquad \longleftarrow$$

$$\Delta D_i = 2u|_{r=a} = (2)\left(\frac{1+v}{E}\right)[(1-2v)+1]\frac{aP_b}{1-\left(\frac{a}{b}\right)^2}$$

(from text derivation)

$$\Delta D_i = \left(\frac{1+0.25}{37.93\,\text{GPa}}\right)[1-(0.25)(2)+1](4)(37.93)$$

$$\Delta D_i = 0.0075\,\text{m}$$

$$\underline{\underline{\Delta D_i = 0.75\,\text{cm}}} \qquad \longleftarrow \Delta D_i \text{ at failure}$$

50. Given: Vertical, concrete, circular shaft

$p_w = 123$ psi

$D_0 = 22'$

$H = 2{,}890'$

$E = 35(10^6)$, $v = 0.30$

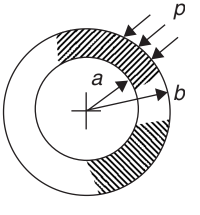
$C_0 = 4{,}500$ psi, $T_0 = 450$ psi

$\gamma = 152$ pcf

$FS = 3.85$

Find: h, D.

Solution:



$$FS = \frac{C_0}{\sigma}$$

$$\sigma = \frac{4{,}500}{3.85} = 1{,}169\text{ psi}$$

But also

$$\sigma = \frac{2P}{1-\left(\frac{a}{b}\right)^2}$$

$$1-\left(\frac{a}{b}\right)^2 = \frac{(2)(123)}{1{,}169} = 0.2105$$

$$\therefore \frac{a}{b} = 0.889$$

$$a = \left(\frac{22}{2}\right)(0.889)$$

$$\underline{a = 9.77\text{ ft}}$$

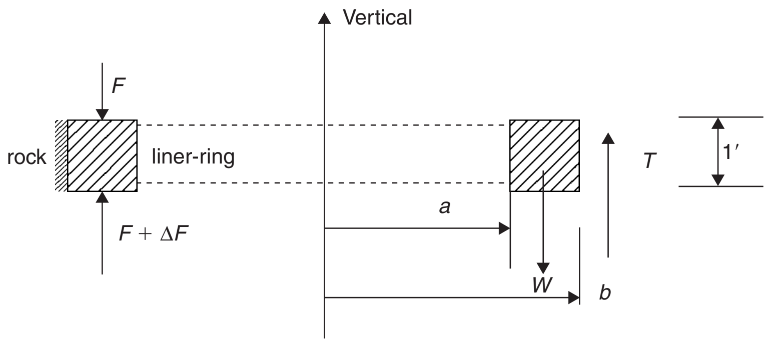
$$t = b - a = 11.0 - 9.775$$

$$t = 1.23'\ \underline{\underline{(14.8\text{ in.})}} \quad \longleftarrow h$$

$$D = 22 - 2(1.23)$$

$$\underline{\underline{D_i = 19.54\text{ ft}}} \quad \longleftarrow D$$

51. Show why shaft liner weight not important
Equilibrium analysis:



$$\Sigma F_v = 0$$

$$0 = \Delta F + T - W$$

$$0 = \Delta F + T - \gamma\pi(b^2 - a^2)(1)$$

if $\Delta F = 0$ then T supports weight of a one ft ring one ft thick, typical e.g.

$$T = (144)(\pi)(1)\left[\left(\frac{21}{2}\right)^2 - \left(\frac{20}{2}\right)^2\right], A = (3.1415)(21)(1)(144)$$

$$\underline{\frac{T}{A} = 0.5\text{ psi}}$$

Thus a slight bond strength between concrete and rock is sufficient for support of weight.

52. Given: Large shaft, vertical 32 ft finished diameter needed, depth 1,270 ft, water pressure = 240 psi

Concrete:
$C_0 = 3{,}500$ psi $E = 5.6(10^6)$ psi
$v = 0.27$ $FS = 2.5$

Find: (a) t.

Solution:

$$FS = \frac{C_0}{\sigma_c},\ \sigma_c = \frac{2p}{\left[1-\left(\frac{a}{b}\right)^2\right]},\ a = \frac{32}{2}$$

$$\frac{(2)(240)}{1-\left(\frac{a}{b}\right)^2} = \frac{3{,}500}{2.5}$$

$$1-\left(\frac{a}{b}\right)^2 = 0.343$$

$$\left(\frac{a}{b}\right) = 0.8106$$

$$b = \frac{16}{0.8106}$$

$$b = 19.74\ \text{ft}$$

$$t = b - a$$

$$= 19.74 - 16.0$$

$$\underline{\underline{t = 3.74\ \text{ft}}} \quad \longleftarrow t$$

Find: (b) ΔD at liner failure.

$$u = \left(\frac{1-2v}{2G}\right)\left[\frac{-Pa}{1-\left(\frac{a}{b}\right)^2}\right] + \frac{1}{2G}\left[\frac{-Pa}{1-\left(\frac{a}{b}\right)^2}\right]$$

$$= \frac{-Pa}{1-\left(\frac{a}{b}\right)^2}\left(\frac{1-2v}{2G} + \frac{1}{2G}\right)$$

$$= \frac{-2Pa}{1-\left(\frac{a}{b}\right)^2}\left(\frac{1-v}{G}\right)$$

Note: $\dfrac{2P}{1-\left(\frac{a}{b}\right)^2} = \sigma_\theta|_{r=a} = 3{,}500\ \text{psi}, G = \dfrac{E}{2(1+v)} = 2.21(10^6)$

$$= (3{,}500)(16)(12)\left[\frac{1-0.27}{2.21(10^6)}\right]$$

$$u = -0.222\ \text{in.}$$

$$2u = \underline{\underline{\Delta D = 0.444\ \text{in.}}} \quad \longleftarrow \Delta D$$

53. Given: Vertical circular shaft, rock and concrete properties

Find:

(a) (*FS*'s unlined not needed)

(b) <u>Liner thickness.</u>

Solution:

Circular shaft, gravity stress
$\therefore$ horizontal stresses are equal
$S_1 = S_3$ in plan view and $M = 1$
$K_c = 2$, $K_t = 0$ (no tension) (hydrostatic case)
$\sigma_c = K_c S_1$
S_1 estimate: gravity alone

$$\sigma_h = \sigma_H = \frac{v}{1-v}\sigma_v \quad \& \quad \sigma_v = \gamma H$$
$$\sigma_h = \left(\frac{\frac{1}{4}}{1-\frac{1}{4}}\right)\left(\frac{156}{144}\right)(2{,}780)$$
$$\sigma_h = 1{,}004\text{ psi}$$
$$\therefore \sigma_c = (2)(1{,}004)$$
$$\sigma_c = 2{,}008\text{ psi}$$
$$FS_c = \frac{C_0}{\sigma_c} = \frac{6{,}750}{2{,}008}$$
$$\underline{\underline{FS_c = 3.36\text{ (in plan)}}}$$

In vertical section:

$$\left.\begin{aligned} FS_c &= \frac{6{,}750}{\sigma_v} \\ &= \frac{6{,}750}{\left(\frac{156}{144}\right)(2{,}780)} \\ FS_c &= 2.24\text{ (in section)} \end{aligned}\right\} \qquad \longleftarrow \text{(a)}$$

(b) Liner thickness? $P_w = 210$ psi with allowable stress at 3,500 psi

$$\sigma_{\text{allow}} = \frac{C_0}{FS}$$
$$FS = \frac{C_0}{\sigma}$$
$$\sigma = \frac{2P}{1-\left(\frac{a}{b}\right)^2}$$
$$\therefore 3{,}500 = \frac{(2)(210)}{1-\left(\frac{18}{2b}\right)^2}$$
$$1-\left(\frac{18}{2b}\right)^2 = \frac{(2)(210)}{3{,}500}$$

$$\left(\frac{18}{2b}\right)^2 = 1 - 0.12$$

$$\frac{a}{b} = 0.938$$

$$b = 9.594$$

$$t = b - a$$

$$= 9.594 - 9.0$$

$$\underline{\underline{t = 0.594\,\text{ft} \quad (7.13\,\text{in.})}}$$

← (b)

54. Given: Large, vertical, concrete liner shaft $D_i = 9.75\,\text{m}$, depth = 387 m, $p_w = 1.66\,\text{MPa}$

Concrete: $C_0 = 24.2\,\text{MPa}$, $E = 38.62\,\text{GPa}$, $v = 0.27$ $FS_c = 2.5$ required

Find:
(a) h-liner
(b) ΔD_i at failure.

Solution:

$$FS_c = \frac{C_0}{\sigma_c}$$

$$\& \ \sigma_c = \frac{2p_w}{1 - \left(\frac{a}{b}\right)^2}$$

$$\therefore (2.5) = \frac{24.2\,\text{MPa}}{(2)(1.66\,\text{MPa})}\left(1 - \frac{a}{b}\right)^2$$

i.e. $$\left(1 - \frac{a}{b}\right)^2 = 0.343$$

$$\therefore \frac{a}{b} = 0.8106$$

$$b = \frac{\frac{9.75}{2}}{0.8106}$$

$$\underline{b = 6.01\,\text{m}}$$

$$h = 6.01 - 4.875$$

$$\underline{\underline{h = 1.125\,\text{m}}}$$

(112.5 cm)

← h

$$\Delta D_i = 2u|_{r=a} \ \text{(at failure)}$$

$$= \frac{(2)\,(1+v)}{E}[(1-2v)+1]\frac{aP_w}{1-\left(\frac{a}{b}\right)^2}$$

$$= \frac{(2)(1+0.27)}{38.62\,\text{GPa}}(2 - 0.54)\,(4.875)\,(24.2)\ \text{MPa}$$

$$\Delta D_i = 0.0113\,\text{m}$$

i.e. $$\underline{\underline{\Delta D_i = 1.13\,\text{cm}}}$$

←

55. Given: Vertical, circular shaft depth = 847 m, $p_w = 1.45$ MPa, $D_i = 5.5$ m, C_0 (concrete) = 24.14 MPa

Find: h (liner thickness).

Solution:

$$\sigma_{\max} = \frac{2p_w}{1-\left(\frac{a}{b}\right)^2}$$

$$(24.14) = \frac{(2)(1.45)}{1-\left(\frac{a}{b}\right)^2}$$

$$\therefore 1-\left(\frac{a}{b}\right)^2 = 0.120$$

$$\frac{a}{b} = 0.938$$

$$b = \frac{2.75}{0.938}$$

$$b = 2.932\,\text{m}$$

$$h = 2.932 - 2.75$$

$$h = 0.182\,\text{m}$$

$$\underline{\underline{h = 18.2\,\text{cm}}}$$

← h

56. Given: Concrete liner below 3,500 ft

Liner: Concrete
1 ft thick

$E = 5 \times 10^6$ psi, $v = 0.20$

$\gamma = 156$ pcf — MC failure criterion

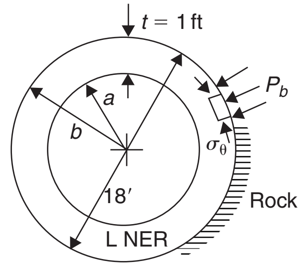
$C_0 = 4{,}500$ psi, $T_0 = 450$ psi

Assume: Circular shaft $D = 18$ ft.

Find: What uniform radial stress would just cause liner to fail?

Solution:

Using MC Criteria:



$$\tau_{\max} = \frac{\sigma_\theta - \sigma_r}{2} = \frac{\sigma_\theta + \sigma_r}{2}\sin\phi + c\cos\phi$$

$$\tau_{\max} = \sigma_m \sin\phi + c\cos\phi$$

$$\frac{\sigma_\theta + \sigma_r}{2} = \frac{P_b b^2}{b^2 - a^2}$$

$$\frac{\sigma_\theta - \sigma_r}{2} = \frac{(P_b - P_a)b^2 a^2}{r^2(b^2 - a^2)}$$

Failure occurs when

$$\frac{P_b b^2 a^2}{r^2(b^2 - a^2)} = \sin\phi \frac{P_b b^2}{b^2 - a^2} + c\cos\phi$$

@ $r = a$ the stress must be less then 4,500 psi and at $r = a$; $\sigma_r = 0$; $P_a = 0$

$$\sigma_\theta = \frac{2P_b b^2}{b^2 - a^2}$$

$$FS = \frac{C_0}{\sigma_\theta}$$

@ failure $FS = 1$
Solving for P_b:

$$P_b = \frac{(\sigma_\theta)(b^2 - a^2)}{2b^2}$$

$$P_b = \left(\frac{C_0}{FS}\right)\left(\frac{b^2 - a^2}{2b^2}\right)$$

$$= \left(\frac{4,500\text{ psi}}{1}\right)\left[\frac{(9\text{ ft})^2 - (8\text{ ft})^2}{2(9\text{ ft})^2}\right]$$

$$\underline{\underline{P_b = 472\text{ psi}}}$$

P_b ←

57. Given: Data from previous problem.
Find: Reduction in diameter of the liner when liner first fails.

Solution:

From previous problem:
@ failure $FS = 1$
$\sigma_\theta = C_0$
$\underline{\underline{P_b = 472\text{ psi}}}$

$$u = \frac{1 - 2v}{2G}\left(\frac{P_a a^2 - P_b b^2}{b^2 - a^2}\right) r + \frac{1}{2G}\left(\frac{P_a - P_b}{b^2 - a^2}\right)\frac{b^2 a^2}{r}$$

$$P_a = 0$$

$$\therefore u = \frac{1 - 2v}{2G}\left(\frac{-P_b b^2}{b^2 - a^2}\right) r + \frac{1}{2G}\left(\frac{-P_b}{b^2 - a^2}\right)\frac{b^2 a^2}{r}$$

For concrete displacement:

$$G = \frac{E}{2(1 + v)}$$

$$= \frac{5 \times 10^6\text{ psi}}{2(1 + 0.20)}$$

$$\underline{G = 2.08 \times 10^6\text{ psi}}$$

Calculate change in inside diameter:
@ $r = a = 8$ ft $\quad b = 9$ ft

$$u = \frac{1-2(0.2)}{2(2.08\times 10^{+6}\text{ psi})}\left[\frac{-(472\text{ psi})(9\text{ft})^2}{(9^2-8^2)\text{ft}^2}\right]8\text{ ft} + \frac{1}{2(2.08\times 10^{6}\text{ psi})}\left[\frac{-472\text{ psi}}{(9^2-8^2)\text{ft}^2}\right]\frac{(9\text{ ft})^2(8\text{ ft})^2}{(8\text{ ft})}$$

$$2u = 1.382(10^{-2})\text{ ft}$$

$$\underline{\underline{\Delta D_i = 0.17\text{ in.}}} \qquad \longleftarrow \Delta D_i$$

58. Given: Problem 56 data

Find: Radial displacement of the interface between the liner & shaft wall when line first fails. $u|_{r=b} = ?$

Solution:

$$u = \frac{1-2v}{2(G)}\left[\frac{-P_b b^2}{b^2-a^2}\right]r + \frac{1}{2G}\left[\frac{-P_b}{b^2-a^2}\right]\frac{b^2a^2}{r}$$

Assuming: The rock must displace the same amount as the outside diameter of the liner. @ $r = b = 9$ ft $\quad a = 8$ ft

$$u = \frac{1-2(0.2)}{2(2.08\times 10^{+6}\text{ psi})}\left[\frac{-472\text{ psi}(9\text{ ft})^2}{(9\text{ ft})^2-(8\text{ ft})^2}\right]9\text{ ft} + \frac{1}{2(2.08\times 10^{6}\text{ psi})}\left[-\frac{472\text{ psi}}{(9\text{ ft})^2-(8\text{ ft})^2}\right]\frac{(9\text{ ft})^2(8\text{ ft})^2}{9\text{ ft}}$$

$$u|_{r=b} = \underline{\underline{0.081\text{ inches}}} \qquad \longleftarrow u$$

59. Given: H_2O Pressure = 80 psi @ 4,500 ft, $FS = 2.5$ minimum

Find: Liner thickness.

Solution:

Allowable stress:

$$FS_c = \frac{C_0}{\sigma_\theta}$$

$$\sigma_\theta = \frac{4{,}500\text{ psi}}{2.5}$$

$$\underline{\sigma_\theta = 1{,}800\text{ psi}}$$

And:

$$\sigma_\theta = \frac{P_b b^2}{b^2-a^2} + \frac{P_b a^2 b^2}{r^2(b^2-a^2)}$$

Stress is maximum @ $r = a$

$$\sigma_\theta = \frac{P_b b^2}{b^2 - a^2} + \frac{P_b a^2 b^2}{a^2(b^2 - a^2)}$$
$$\sigma_\theta = \frac{P_b b^2 + P_b b^2}{b^2 - a^2}$$
$$\sigma_\theta = \frac{2P_b b^2}{b^2 - a^2}$$

Solving for a^2 and a, then t

$$(b^2 - a^2)\sigma_\theta = 2\,P_b b^2$$
$$-a^2 = \frac{2P_b b^2}{\sigma_\theta} - b^2$$
$$a^2 = b^2 - \frac{2P_b b^2}{\sigma_\theta}$$
$$a = \left[(9\text{ ft})^2 - \frac{2(80\text{ psi})(9\text{ ft})^2}{1,800\text{ psi}}\right]^{1/2}$$
$$\underline{a = 8.59\text{ ft}}$$
$$t = b - a$$
$$= 9\text{ ft} - 8.59\text{ ft}$$
$$t = 0.41\text{ ft} = 4.92\text{ in.}$$

$\therefore$ make a liner $t = 5$ in except minimum is 1 ft. ← t

60. Given: Steel liner used for Problem 59 $C_0 = T_0 = 36{,}000$ psi
Find: Thickness of steel.

Solution:

$$FS_c = \frac{C_0}{\sigma_\theta}$$
$$\sigma_\theta = \frac{36{,}000\text{ psi}}{2.5}$$
$$\sigma_\theta = 14,400\text{ psi} \quad \text{(allowable)}$$

And:

$$a = \left[b^2 - \frac{2P_b b^2}{\sigma_\theta}\right]^{1/2}$$
$$= \left[(9\text{ ft})^2 - \frac{2(80\text{ psi})(9\text{ ft})^2}{14{,}400\text{ psi}}\right]$$
$$a = 8.95\text{ ft}$$

$$t = b - a$$
$$= 9.0\,\text{ft} - 9.95\,\text{ft}$$
$$\underline{\underline{t = 0.05\,\text{ft} = 0.6\,\text{in.}}}$$

$\longleftarrow t$

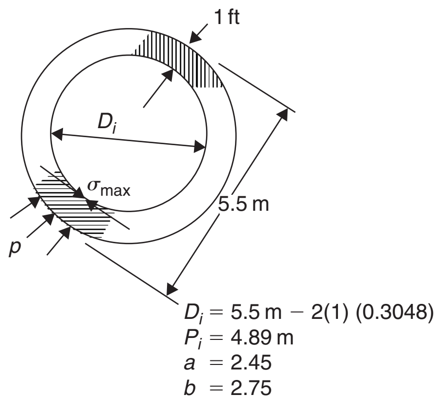
61. Given: Circular concrete shaft liner $D_0 = 5.5$ m, $h = 1$ ft going below 1,067 m
$C_0 = 31.0$ MPa, $T_0 = 3.10$ MPa $E = 34.48$ GPa,
$v = 0.20$, $\gamma = 24.7\,\text{kN/m}^3$ $\sigma_v = 25.45h$,
$\sigma_H = 24,138 + 7.47h$, $\sigma_h = 24,138 + 7.47h$
Find: p at failure.

Solution:

$$\sigma_{\max} = \frac{2P}{1 - \left(\frac{a}{b}\right)^2}$$

$$a = \frac{D_i}{2}$$

$$b = \frac{D_0}{2}$$



at failure $\sigma_{\max} = C_0$

$$31.0 = \frac{2p}{\left[1 - \left(\frac{2.45}{2.75}\right)^2\right]}$$

$$\underline{\underline{p = 3.25\,\text{MPa}}}$$

$\longleftarrow$

62. Given: Problem 61 data
Find: ΔD_i of failure.

Solution:

$$\Delta D_i = 2u|_{r=a} = \left(\frac{1+v}{E}\right)[(1-2v)+1]\frac{2aP}{1-\left(\frac{a}{b}\right)^2}$$

$$\text{But } \frac{2P}{1-\left(\frac{a}{b}\right)^2} = C_0 \text{ at failure}$$

$$\therefore \quad \Delta D_i = \left(\frac{1+0.2}{34.48\,\text{GPa}}\right)(1 - 0.4 + 1)(2.45)(31.0\,\text{MPa})$$

$$\Delta D_i = 4.23(10^{-3})\,\text{m}$$

$$\underline{\underline{\Delta D_i = 0.423\,\text{cm}}}$$

$\longleftarrow$

63. Given: Problem 61 data
Find: Rock-liner radial displacement.

Solution:

$$u|_{r=b} = \left(\frac{1+v}{E}\right)\left[(1-2v)+\left(\frac{a}{b}\right)^2\right]\left[\frac{bP}{1-\left(\frac{a}{b}\right)^2}\right]$$

$$= \left(\frac{1+0.2}{34.48\text{ GPa}}\right)\left[1-0.4+\left(\frac{2.45}{2.75}\right)^2\right]2.75\left(\frac{31.0}{2}\text{ MPa}\right)$$

$$u = 2.068(10^{-3})\text{ m} \qquad \underline{\underline{u = 0.207\text{ cm}}}$$ ←

64. Given: Problem 61 data
$p_w = 0.552$ MPa
$d = 1{,}372$ m
FS(liner) = 2.5
Minimum thickness = 0.3 m

Find: Liner thickness.

Solution:

$$FS = \frac{C_0}{\sigma_c}$$

$$\sigma_c = \frac{2p}{1-\left(\frac{a}{b}\right)^2}$$

$$\therefore 2.5 = (31.0)\frac{\left[1-\left(\frac{a}{b}\right)^2\right]}{(2)(0.552)}$$

$$1-\left(\frac{a}{b}\right)^2 = 8.903(10^{-2})$$

$$\therefore \left(\frac{a}{b}\right) = 0.9545$$

$$a = 2.75(0.9545)$$

$$a = 2.62\text{ m}$$

$$h = b - a$$

$$\underline{h = 0.13\text{ m}}$$

but minimum is 0.30 m
$\therefore \underline{\underline{h = 0.30\text{ m}}}$ ←

65. Given: Problem 64 data and a steel liner $C_0 = 248$ MPa
Find: Liner thickness h.

Solution:

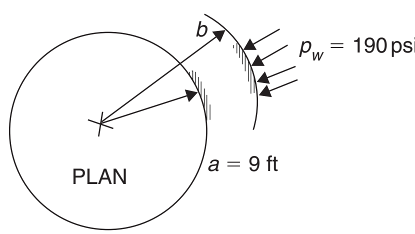
$$FS = \frac{C_0}{\sigma_c}$$
$$\sigma_c = \frac{248}{2.5}$$
$$\sigma_c = 99.2\,\text{MPa}$$

but also

$$\sigma_c = \frac{2p}{\left[1-\left(\frac{a}{b}\right)^2\right]}$$
$$\frac{1}{1-\left(\frac{a}{b}\right)^2} = \frac{99.2}{(2)(0.552)}$$
$$\therefore \left(\frac{a}{b}\right)^2 = 0.989$$
$$a = 2.75(0.9944)$$
$$a = 2.735\,\text{m}$$

$h = 2.75 - 2.735$
$h = 1.535(10^{-2})\text{m}$, $\underline{\underline{h = 1.54\,\text{cm}}}$ ⟵

66. Given: Circular concrete shaft liner
$E = 3.4(10^6)$ psi, $v = 0.25$
$C_0 = 3{,}500$ psi, $T_0 = 350$ psi
$p_w = 190$ psi at $z = 3{,}750$ ft.
$D = 18$ ft, $FS = 2.5$
Find: h (thickness), D_0 (outside diameter).



Solution:

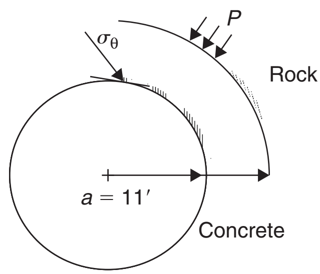
$$\text{Require } FS_c = \frac{C_0}{\sigma_c}$$
$$\sigma_c = \frac{3{,}500}{2.5}$$
$$\sigma_c = 1{,}400\,\text{psi} \quad \text{(max. stress, inside)}$$
$$\text{But also } \sigma_c = \frac{2p}{1-\left(\frac{a}{b}\right)^2}$$
$$\left(\frac{a}{b}\right)^2 = 1 - \frac{2p}{\sigma_c}$$
$$= 1 - \frac{(2)(190)}{1{,}400}$$
$$\therefore \left(\frac{a}{b}\right)^2 = \underline{\underline{0.729}}$$
$$\frac{a}{b} = 0.854$$

$$b = \frac{9}{0.854}$$
$$= \frac{9}{0.854}$$
$$\underline{\underline{b = 10.54\text{ ft}}}$$

$t = b - a$
$= 10.54 - 9.0$
$\underline{\underline{t = 1.54\text{ ft (18.5 in.)}}}$ ← t

$D_0 = 18 + 2t$
$\underline{D_0 = 21.1\text{ ft}}$

67. Given: Circular concrete shaft liner
$E = 3.4(10^6)$ psi, $v = 0.25$
$C_0 = 3{,}500$ psi, $T_0 = 350$ psi
$D = 22$ ft, $h = 1$ ft
Find: Contact pressure between liner & rock on verge of failure?



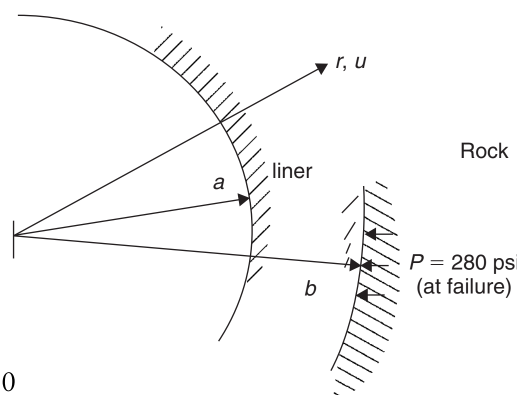
Solution:

At failure

$$\sigma_\theta|_{r=a} = C_0 = \frac{2P}{1-\left(\frac{a}{b}\right)^2}$$
$$\therefore P = \frac{C_0\left[1-\left(\frac{a}{b}\right)^2\right]}{2}$$
$$= \frac{(3{,}500)\left[1-\left(\frac{11}{12}\right)^2\right]}{2}$$
$$P = \underline{\underline{280\text{ psi}}}$$

← contact pressure P

68. Given: Problem 67 conditions
Find: Change in inside shaft liner diameter from no load condition.



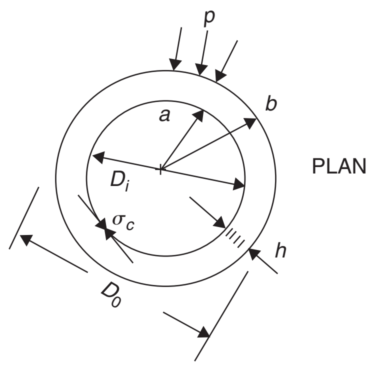
Solution:

$\Delta D = 2u \quad u =$ radial displacement

$$u|_{r=a} = -\frac{2(1-v^2)b^2 a}{E(b^2-a^2)}P_b$$
$$= \frac{-2(1-0.04)(11)(12)}{3.4(10^6\text{ psi})\left[1-\left(\frac{11}{12}\right)^2\right]}280$$
$$u = -0.131\text{ in.}$$
$$\Delta D = -2(0.131)$$
$$\underline{\underline{\Delta D = -0.262\text{ in.}}}$$

← decrease in inside diameter from no-load to failure load

69. Given: Circular concrete shaft liner
$E = 23.45$ GPa, $v = 0.25$
$C_0 = 24.1$ MPa, $T_0 = 2.41$ MPa
$P_w = 1.31$ MPa at 1,143 m
$D_i = 5.5$ m, $FS = 25$
Find: D_0.



Solution:

$$FS = \frac{C_0}{\sigma_c} \quad \text{by definition}$$

$$\sigma_c = \frac{24.1}{2.5}$$

$$\underline{\sigma_c = 9.64\text{ MPa}}$$

$$\text{But also } \sigma_c = \frac{2p}{\left[1 - \left(\frac{a}{b}\right)^2\right]}$$

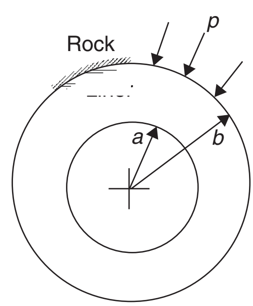
$$1 - \left(\frac{a}{b}\right)^2 = \frac{(2)(1.31)}{9.64}$$

$$\therefore b = \frac{2.75}{0.853}$$

$$\underline{b = 3.22\text{ m}}$$

$h = b - a$
$h = 3.22 - 2.75$
$\underline{\underline{h = 0.473\text{ m} \qquad h = 47.3\text{ cm}}}$ ⟵
$D_0 = 2b \qquad \underline{\underline{D_0 = 6.44\text{ m}}}$ ⟵

70. Given: Vertical, circular concrete shaft liner
$E = 23.45$ GPa $\quad v = 0.25$
$C_0 = 24.1$ MPa $\quad T_0 = 2.41$ MPa
$D_i = 6.7$ m $\quad h = 0.3$ m at failure
Find: Contact pressure p_0.



$$a = \frac{6.7}{2} = 3.35\text{ m}$$
$$b = 3.35 + h$$
$$b = 3.65\text{ m}$$

Solution:

At failure

$$\sigma_{max} = C_0 = \frac{2p}{1 - \left(\frac{a}{b}\right)^2}$$

$$p = \frac{\left[1 - \left(\frac{3.35}{3.65}\right)^2\right](24.1)}{2}$$

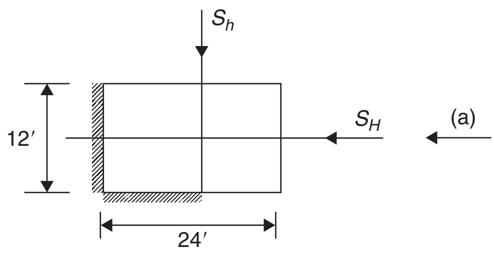
$$\underline{\underline{p = 1.90\text{ MPa}}}$$

71. Given: Problem 70 data where p of failure is 1.90 MPa
$a = 3.35, b = 3.75$
$C_0 = 24.1$ MPa, $E = 23.45$ GPa, $v = 0.25$
Find: ΔD_i at failure (mm).
Solution:

$$\Delta D_i = 2u|_{r=a} \text{ text formula}$$
$$2u|_{r=a} = \left(\frac{1+v}{E}\right)(1-2v+1)\frac{a2P}{1-\left(\frac{a}{b}\right)^2}$$
$$= \frac{(1+0.2)}{23.45\text{ GPa}}(1-0.5+1)(3.35)(24.1\text{ MPa})$$
$$\Delta D_i = 6.197(10^{-3})\text{ m} = \underline{\underline{6.197\text{ mm}}} \quad \longleftarrow$$

Multiple Shafts

72. Given: Rectangular shaft 12 × 24 ft at 3,000 ft where $S_v = 1.2h$
$S_h = 120 + 0.5h$: premining (psi)
$h = \text{depth(ft)}$
$S_H = 3{,}250 + 0.3h$



Find:
(a) Best orientation
(b) Orientation at depth
(c) Best orientation & location of two identical shafts.

Solution:

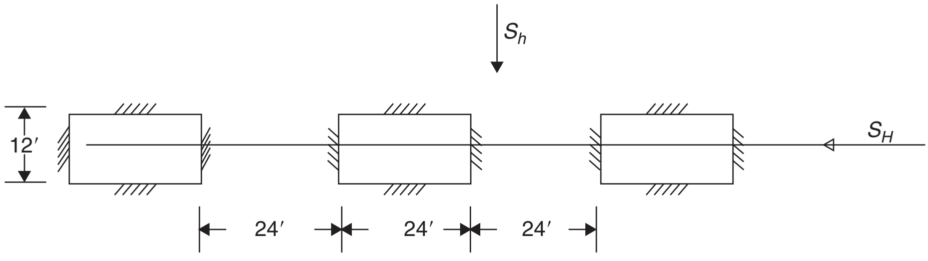
If $h = 0$, then $S_H = \sigma_1^0, S_h = \sigma_3^0$ and best orientation is with the long dimension parallel to S_H.

For same orientation at depth

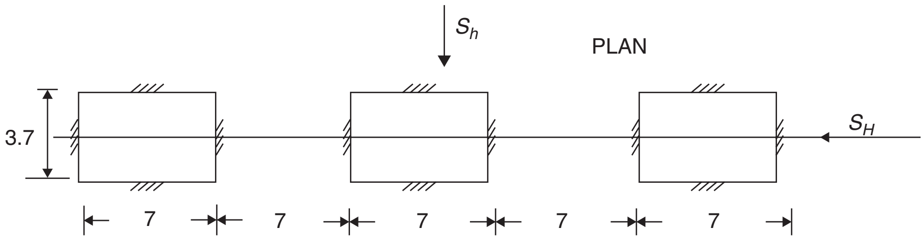
$$S_h < S_H$$
$$120 + 0.5h < 3{,}240 + 0.3h$$
$$0.2h < 3{,}120$$
$$h < 15{,}600$$

∴ best at all depths of shaft (3,000 ft) $\longleftarrow$ (b)

(c) • best orientation for several shafts in a row is for the row axis to be parallel to the major compression
• to minimize, practically, stress concentration from interaction shafts should be separated at least "1-D"



(c) Best row orientation



Note: Rule: Openings separated by "1-D" are essentially isolated (D = long dimension)

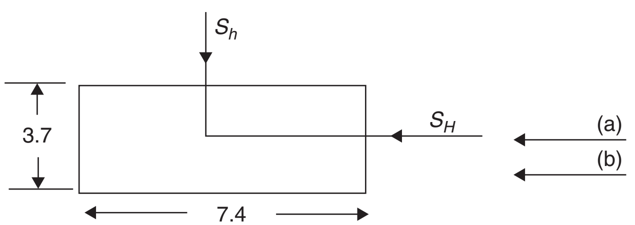
73. Given: 3.2 × 7.4 m rectangular shaft depth to 914 m
$S_v = 27.2\,h$
$S_h = 828 + 11.3h$
$S_H = 22{,}345 + 6.8h$
(kPa, m)

Find:
(a) best orientation
(b) best orientation all depths
(c) best position and orientation of a second and third shafts

Solution:

Rule: Best orientation of a single opening is with long axis parallel to σ_1^0
Rule: Best row orientation is with row axis parallel to σ_1^0

(a) at surface $S_h = 828$ kPa, $S_H = 22{,}345$
$\therefore$ long axis should be parallel to S_H

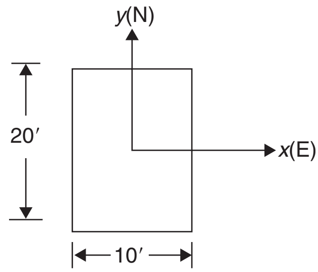


(b) at depth

$$S_H > S_h$$
$$22{,}345 + 6.8\,h > 829 + 11.3\,h$$
$$21{,}516 > 4.5\,h$$

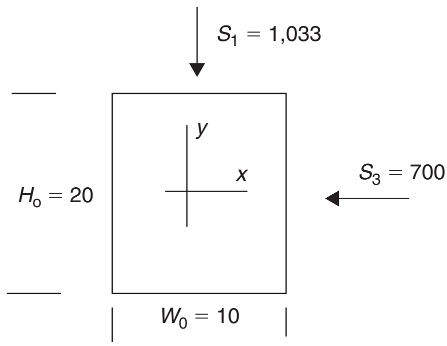
for $h < 4{,}781$ m best
orientation at all depths

74. Given: Rectangular shaft
$C_0 = 23{,}700$ psi $\quad T_0 = 1{,}480$ psi
$S_E = 350 + 0.24\,\text{h} \quad S_N = 420 + 0.354\,\text{h}$
$S_v = 1.124\,\text{h}$
Find: Favorable orientation of two shafts.



Solution:

$$FS_c = \frac{C_0}{\sigma_c}, \qquad FS_t = \frac{T_0}{\sigma_t}$$
$$\sigma_c = K_c\,S_1 \qquad \sigma_t = K_t\,S_1$$
$$S_N = 420 + 0.35(1{,}750)$$
$$S_N = 1{,}033 \text{ psi}$$
$$S_E = 350 + 0.2(1{,}750)$$
$$S_E = 700 \text{ psi}$$
$$\therefore S_N = S_1$$
$$M = \frac{700}{1{,}033}$$
$$M = 0.678$$
$$\frac{W_0}{H_0} = \frac{10}{20}$$
$$\frac{W_0}{H_0} = 0.5\left(k = \frac{1}{2}\right)$$

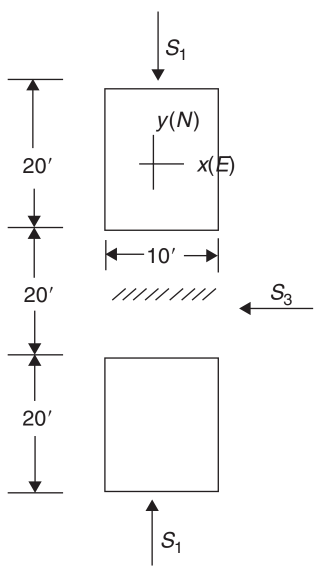


Text Table 3.2.

Estimate between	$M = \frac{1}{2}$	& $M = \frac{3}{4}$	at $M = \frac{2}{3}$
K_c	4.57	5.48	$K_c = 5.2$
K_t	+0.19	+0.69	$K_t =$ (n.a.)

$$FS_c = \frac{23{,}700}{(1{,}033)} \qquad FS_t - \text{not a factor (no tension)}$$

$$FS_c = 4.40 \qquad FS_v = \frac{23{,}700}{(1.12)(1{,}750)} = 12.1$$



2nd identical 10′ × 20′ shaft
Show: most favorable orientation & location.

Solution:

Orientation: long axis parallel to S_1
Orientation: row axis parallel to S_1
Use 1-D rule for separation.

75. Given:

$\sigma_{xx} = 2{,}155$ psi (+) compression
$\sigma_{yy} = 3{,}045$ $x =$ East, $y =$ North, $z =$ up
$\sigma_{zz} = 4{,}200$
$\tau_{yx} = -1{,}222$
$\tau_{xz} = 0$
$\tau_{yz} = 0.$

Find:
(a) σ_1, σ_2, σ_3 and directions, sketch.
(b) Best shape
(c) 2nd shaft location.

Solution:

By inspection, the x-y plane with normal z is shear-free and therefore a principal plane. Thus z is a principal direction and $\sigma_{zz} = \sigma_2$ (for now)
in the x-y plane

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \frac{2{,}155+3{,}045}{2} \pm \left[\left(\frac{2{,}155-3{,}045}{2}\right)^2 + (-1{,}222)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = 2{,}600 \pm 1{,}300$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \begin{matrix}3{,}900\\ 1{,}300\end{matrix}\ \text{psi}$$

reordering
$$\begin{cases}\sigma_1 = 4{,}200\ \text{psi} & \text{(vertical)}\\ \sigma_2 = 3{,}900\ \text{N35W} & \text{(horizontal)}\\ \sigma_3 = 1{,}300\ \text{N55E} & \text{(horizontal)}\end{cases}$$

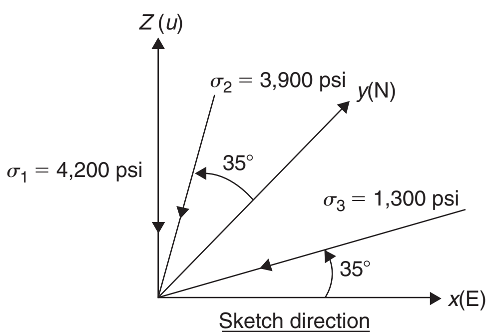
← $\sigma_1\ \sigma_2\ \sigma_3$

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx}-\sigma_{yy})}$$

$$= \frac{-1{,}222}{\frac{1}{2}(2{,}155-3{,}045)}$$

$$\tan 2\theta = 2.746$$
$$2\theta = 70^\circ$$
$$\theta = 35^\circ$$

+ CCW from y-axis



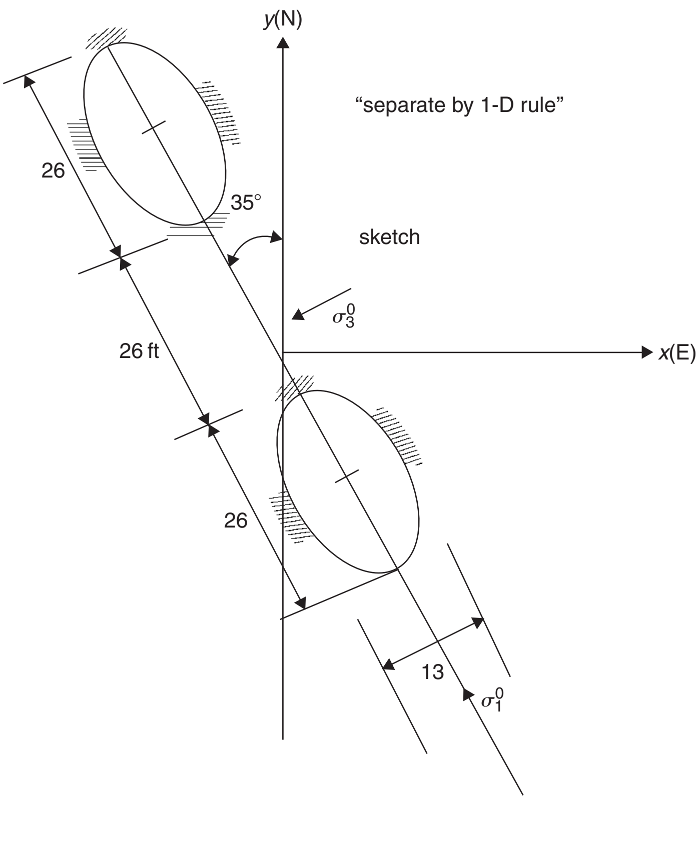
Sketch direction

(b) 13 × 26 ft shaft, rectangle, ellipse, ovaloid shape?
In most favorable position

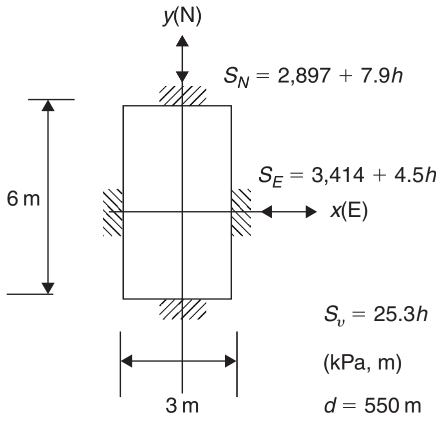
$$\frac{W_0}{H_0} = \frac{13}{26} = \frac{1}{2} \text{ and } \underline{k = \frac{1}{2}(\text{text})}$$
$$M = \frac{\sigma_3^0}{\sigma_1^0} = \frac{1{,}300}{3{,}900} = \underline{\frac{1}{3} = M}$$

Text, Table 3.2a

Ellipse $K_{max} = 1.67$ $K_t = \text{n.a.}$
Rectangle $K_{max} = 4.05$ $K_t = -0.14$
Ovaloid (well-rounded corners, rectangle ∴ K_c, K_t less)
Choose – ellipse ← (b) best shape
"Separate by 1-D rule"



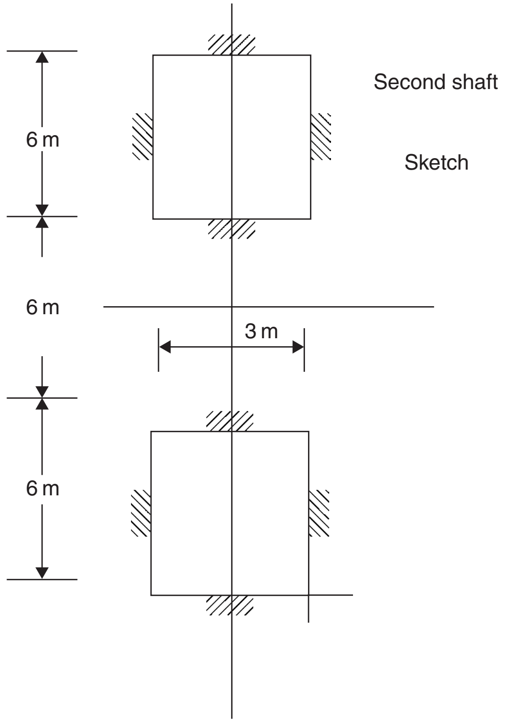
76. Given: $C_0 = 163.5\,\text{MPa}$, $T_0 = 10.2\,\text{MPa}$
$E = 36.48\,\text{GPa}$, $v = 0.27$, $\gamma = 25.6\,\text{kN/m}^3$



Find: Favorable orientation of second shaft and position.

Solution:

Rule: Row axis parallel to σ_1^0. Rule: Separate by "1-D"



$$S_N = 2{,}897 + 7.9(550) = 7.24\,\text{MPA} = \sigma_1^0$$
$$S_E = 3{,}414 + 4.5(550) = 5.89\,\text{MPA} = \sigma_3^0$$
$\therefore$ shaft is in favorable orientation

77. Given: Shaft: 4×8 m.
$\sigma_{xx} = 14.86$ MPa, $\tau_{xy} = -8.43$
$\sigma_{yy} = 21.00$, $\tau_{yz} = 0$
$\sigma_{zz} = 29.00$, $\tau_{zy} = 0$

Find:
(a) $\sigma_1 \sigma_2 \sigma_3$, directions
(b) Best shape (rectangle, ellipse, ovaloid), orientation
(c) Second shaft location, orientation.

Solution:

Need σ_1^0, σ_3^0 in x-y plane (z-dir. is prin)

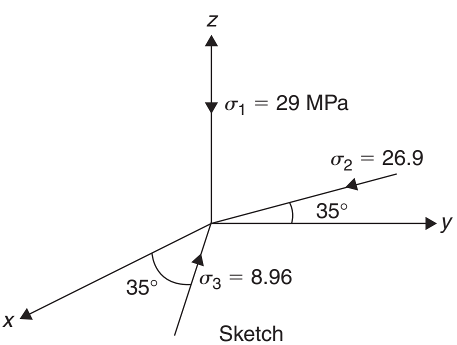
$$\left.\begin{matrix}\sigma_1^0 \\ \sigma_3^0\end{matrix}\right\} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \pm \left[\left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \frac{1}{2}(14.86 \pm 21.0) \pm \left[\left(\frac{14.86 - 21.0}{2}\right)^2 + (-8.43)^2\right]^{1/2}$$

$$= 17.93 \pm 8.97$$

$$\underline{\sigma_1^0 = 26.9\text{ MPa}}, \quad \underline{\sigma_3^0 = 8.96\text{ MPa}}$$

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}$$
$$= \frac{-8.43}{\frac{1}{2}(14.86 - 21.0)}$$
$$\tan 2\theta = 2.746$$
$$2\theta = 70^\circ, 25^\circ$$
$$\underline{\theta = 35^\circ, 125^\circ}$$



at 4×8 m $\quad \dfrac{W_0}{H_0} = \dfrac{1}{2}$ and $k = \dfrac{1}{2}$
in favorable orientation

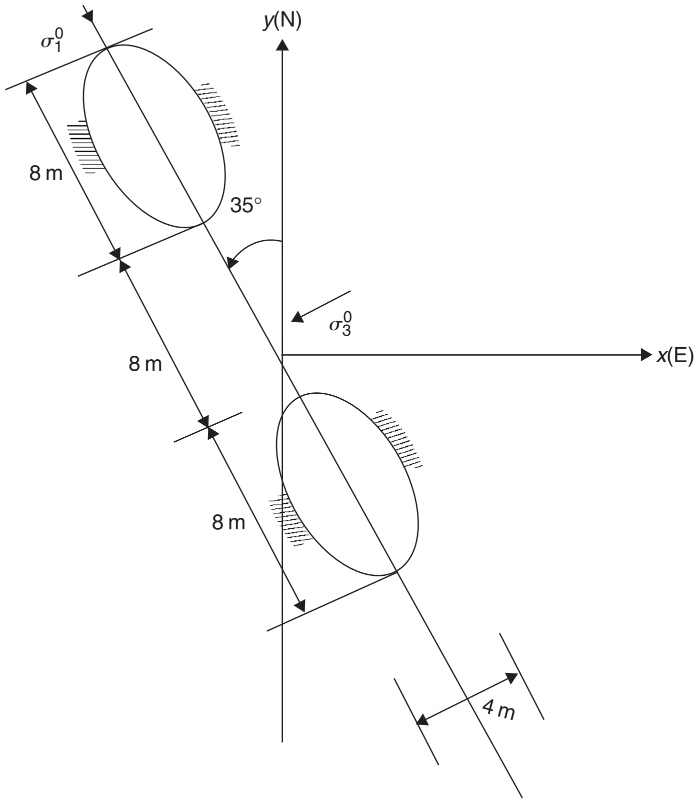
$$M = \frac{\sigma_3^0}{\sigma_1^0} \text{ in plan view}$$
$$= \frac{8.96}{2.69}$$
$$\underline{M = \frac{1}{3}}$$

Table 3.2a text

Ellipse	$K_c = 1.67$	$K_t =$ n.a.
Rectangle	$K_c = 4.05$	$K_t = -0.14$

Ovaloid – rectangular with rounded corners, less than rectangle here.

Ellipse is best



SKETCH
"Separate by 1-D"

4 Tunnels

Naturally Supported Tunnels

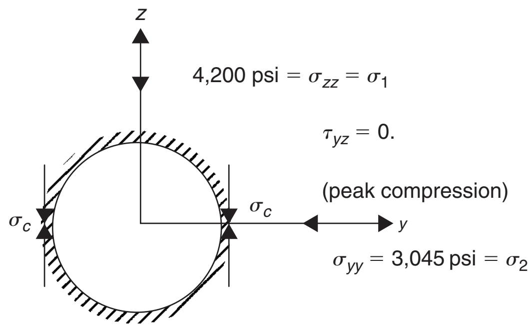
1. Given: Stress state
   Find: K_c, K_t for circular tunnel driven due east.

   Solution:

   $$M = \frac{\sigma_3}{\sigma_1} = \frac{3{,}045}{4{,}200} = 0.725$$

   $\underline{\underline{K_t \text{ is nil}}}$

   $\underline{\underline{K_c = 2.28}}$



   [Text formula]

   $$\text{Note: } K_{\min} = (1+M) - 2(1-M) = (1+0.725) - 2(1-0.725) = 1.175$$
   $$K_{\max} = (1+M) + 2(1-M) = (1+0.725) + 2(1-0.725) = 2.275$$

2. Given: Rectangular "tunnel" 10 × 20 ft wide driven due north 5,000 ft
   Find: max. width

   Solution:

   $$FS_c = \frac{C_c}{K_c S_1}$$

   $$\text{at failure } 1 = \frac{23{,}700}{K_c(1{,}960)}$$

   $K_c = 12.09$

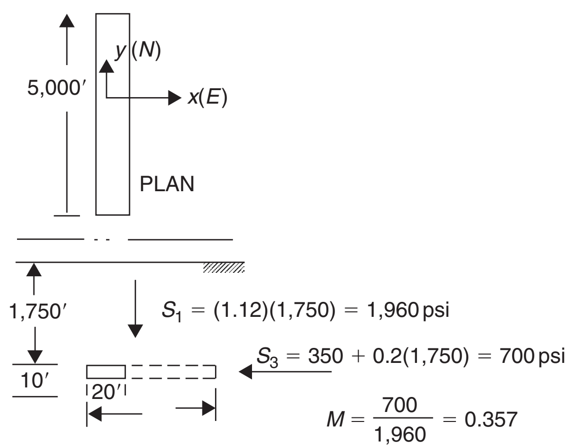
   Text Chapter 3 Fig. 9d.

   $$K_{\max} = 0.69 + 1.27k + 0.68\left(\frac{1}{k}\right) + 3.83\,M$$

   for k: $K_{\max} = 12.09$

   $$\therefore \frac{W_0}{H_0} = 7.8 \quad \underline{\underline{k = 7.8}}$$

   $\underline{\underline{W_0 = 78.0 \text{ ft width at failure.}}}$

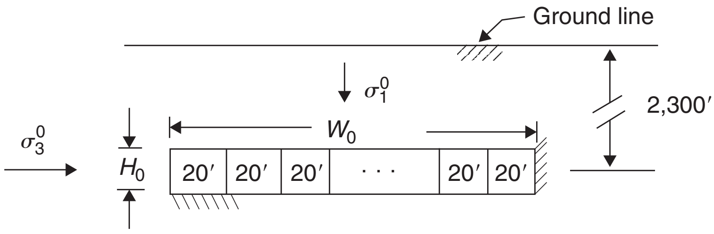


W_0 max

3. Given: Tabular ore body depth = 2,300 ft mined by 20 ft slices, 5,000 ft long $E = 5.7(10^6)$ psi, $\nu = 0.25$ $C_0 = 25{,}300$ psi, $T_0 = 2{,}600$ psi $FS_c = 2.2$, $FS_t = 4.4$ gravity stress field

   Find: Slices @ 20′

   Solution:



Practical height of 15′, mining can proceed until $FS_c = 2.2$ is reached, $FS_t = -1$ assumed not to change significantly with mining, check tension safety.

$$FS_t = 4.4$$
$$4.4 \stackrel{?}{=} \frac{T_0}{K_t \sigma_1^0} \text{ : assume 1 psi/ft depth}$$
$$4.4 \stackrel{?}{=} \frac{2{,}600}{(1)(2{,}300)} \text{(not equal)}$$

∴ Cannot meet tensile safety criterion at start, will need support.

Compression check (with roof support)

$$FS_c = \frac{C_0}{K_c \sigma_1^0}$$
$$K_c = \frac{(25{,}300)}{(2.2)(2{,}300)}$$
$$K_c(\max) = 5.0$$

need $\frac{W_0}{H_0}$, M

Gravity field : $S_H = \frac{\nu}{1-\nu} S_v$

where $S_H = \sigma_3^0 : S_v = \sigma_1^0$

$$M = \frac{\nu}{1-\nu} = \frac{\frac{1}{4}}{1-\frac{1}{4}}$$
$$M = \frac{1}{3}$$

Table 3.2 Text

$$k = \frac{W_0}{H_0} = 4 \quad M = \frac{1}{3} \quad K_c = 7.37$$

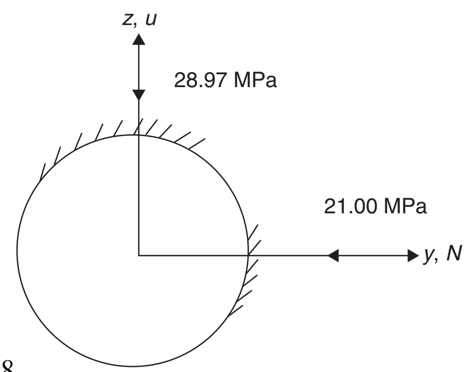
$$k = \frac{W_0}{H_0} = 3 \quad M = \frac{1}{3} \quad K_c = 6.25 \quad \text{(estimate)}$$
$$k = \frac{W_0}{H_0} = 2 \quad M = \frac{1}{3} \quad K_c = 5.15$$

Note: Also could find graphically

$$W_0 \cong (2)(15)$$
$$W_0 \cong 30(\text{ft})$$
$$n = \frac{W_0}{20}$$
$$n = \frac{30}{20}$$
$$\underline{\underline{n = 1 \quad \text{slice (round down)}}} \quad \longleftarrow$$

at 2 slices, corners fail in compression

4. Given: $\sigma_{xx} = 14.86$ MPa $\sigma_{yy} = 21.00$ $\sigma_{zz} = 28.97$ $\tau_{xy} = -8.43$ $\tau_{xz} = \tau_{yz} = 0$
Circular tunnel driven due East
Find: K_c, K_t



Solution:

$$K_c = 1 + M + 2(1 - M)$$
$$K_t = 1 + M - 2(1 - M) \text{ (if negative)}$$
$$M = \frac{21}{28.97} = \underline{0.724}$$

$$\underline{\underline{K_c}} = (1 + 0.724) + 2(1 - 0.724) = \underline{\underline{2.28}} \quad \longleftarrow$$
$$K_t = (1 + 0.724) - 2(1 - 0.724) = 1.17 > 0 \quad \longleftarrow$$
$$\therefore \underline{\underline{K_t \text{ n.a}}}$$

5. Given: Rectangular opening 3 × 6 m wide

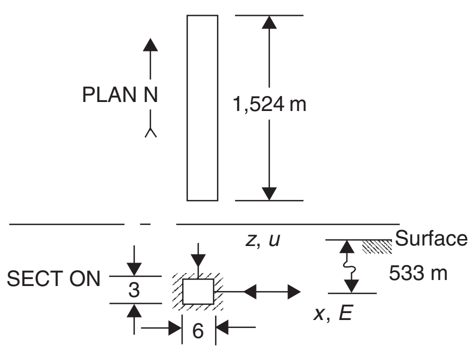
$C_0 = 174.4$ Mpa

$S_z = 25.34(533) = \underline{13.51 \text{ MPa}}$

$S_x = 2{,}414 + 4.53(533) = \underline{4.83 \text{ MPa}}$

$$M = \frac{S_x}{S_z} = \frac{4.83}{13.51} = \underline{\underline{0.357}}$$

Find: Maximum width.



Solution:

$$FS_c = \frac{C_0}{\sigma_c}$$
$$\sigma_c = K_c \sigma_1^0$$

$\therefore$ at failure

$$\underline{K_c} = \frac{174.4}{13.51} = \underline{12.91}$$

Text, Chapter 3, Fig. 9d

$$K_c = 0.69 + 1.27k + 0.68\left(\frac{1}{k}\right) + 3.83\,M$$
$$\therefore 12.91 = 2.06 + 1.27k + 0.68\left(\frac{1}{k}\right)$$

$1.27k^2 - 10.85k + 0.68 = 0$

$\therefore$ $\underline{\underline{k = 7.83}}$ (physically realistic root of quadratic)

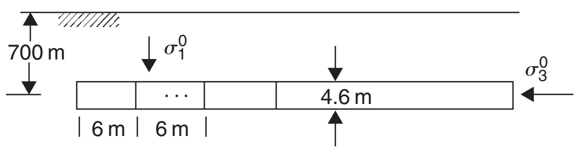
$$\frac{W_0}{H_0} = 7.83, \; H_0 = 3\,\text{m}$$
$$\therefore \; \underline{\underline{W_0 = 23.5\,\text{m}}} \qquad \longleftarrow$$

6. Given: Tabular excavation 4.6 m high & 700 m deep mined by 6 m wide slices 1520 m long. $C_0 = 174.4$ MPa $T_0 = 17.9$ MPa

   Premining stress = gravity only, $E = 39.3$ GPa $v = 0.25$ $FS_c = 2.2$ $FS_t = 4.4$ required

   Find: Excavation wide, number of slices.

Solution:



$$F_t = \frac{T_0}{K_t \sigma_1^0}$$

estimate: $K_t \cong -1$(no more)

estimate: $\sigma_1^0 = \gamma h \cong (25.0)(700)$

$\sigma_1^0 = 17.5$ MPa

$$4.4 \stackrel{?}{=} \frac{(17.9)}{(1)(17.5)}$$

<u>Can't achieve tensile FS_t.</u>

$\therefore$ $\underline{\underline{\text{Need support at outset}}}$

$$FS_c = \frac{C_0}{\sigma_c}$$
$$\sigma_c = K_c \sigma_1^0$$
$$2.2 = \frac{174.4}{K_c(17.5)}$$
$$\underline{\underline{K_c = 4.53(\text{max})}}$$

(gravity only)

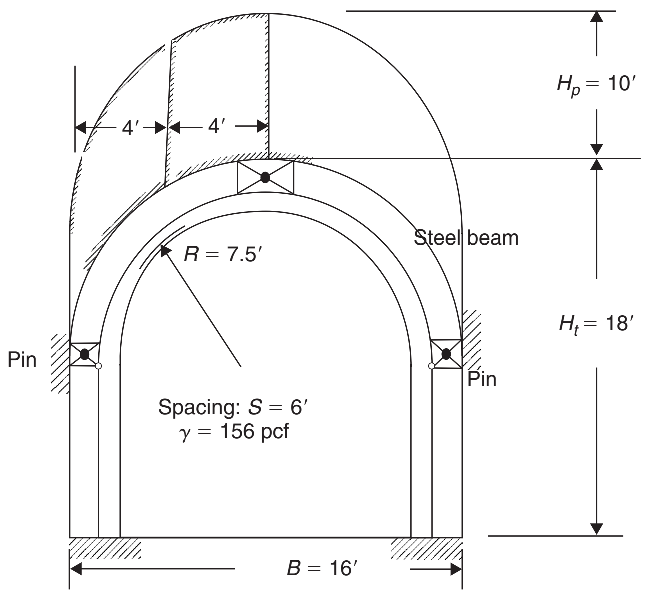
$$M = \frac{\sigma_3^0}{\sigma_1^0} = \frac{\nu}{1-\nu} = \frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$\underline{\underline{M = \frac{1}{3}}}$$

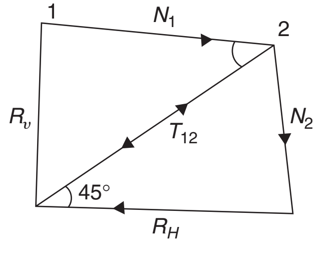
Table 2 Text $M = 1/4$
$k = 2$ $K_{max} = 5.15$
$k = 1$ $K_{max} = 4.06$
$\therefore 1 < k < 2$ Possible
Hence only one slice can be made (the first slice) ←

Supported Tunnels

7. Given:
   Find:
   (a) T, M
   (b) A of $A36$ steel
   (c) $S = 30$ in^3, σ_b.



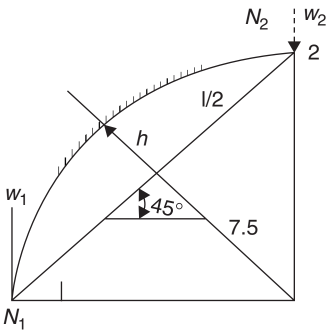
Solution:



(a) Equate $W/2$ to T ignoring rock arch weight transmitted to abutments or do an approximate force polygon.

small angles suggest $N_2 \approx W_2$ & $W_2 = (W/4)$
Note: Also overall equilibrium suggests
$R_v = W/2$, (then $N_2 \approx R_v \approx W/2$).

Then
$N_2 \approx T_{12} \sin(45°)$ $R_v = T_{12} \sin 45°$



i.e. $T_{12} \approx \frac{\sqrt{2}}{1}\left(\frac{W}{2}\right) = 0.71\left(\frac{W}{2}\right)$

A compromise between $W/2$ and $0.71W/2$ is reasonably

$$\underline{\underline{T_{12} = 0.85\frac{W}{2}}}$$

$$\frac{W}{2} = \gamma B H_p \frac{S}{2}$$
$$= \frac{(156)(16)(10)(6)}{2}$$
$$\frac{W}{2} = 7.488(10^4)\text{ lbf}$$

$$\therefore T_{12} = 0.85(7.488)10^4$$
$$\underline{\underline{T_{12} = 63{,}648\text{ lbf}}} \qquad \leftarrow T$$

$M = Th$ (pin-Connected)

$$h = R - \sqrt{R^2 - \left(\frac{l}{2}\right)^2} \;\&\; \frac{l}{2} = \left(\frac{1}{2}\right)\left(\sqrt{2}R\right) = \frac{l}{2} = \left[\left(\frac{1}{2}\right)\sqrt{2}(7.5)\right] = 5.303\text{ ft}$$
$$h = 7.5 - [7.5^2 - (5.303)^2]^{1/2}$$
$$\underline{\underline{h = 2.197\text{ ft}}}$$

$$M = (63{,}648)(2.197)$$
$$\underline{\underline{M = 1.405(10^5)\text{ ft-lbf}}} \qquad \leftarrow M$$

(b) Area to resist thrust, minimum area at $FS = 1.0$

$$T = \sigma_Y A$$
$$A = \frac{63{,}648}{36{,}000}$$
$$\underline{\underline{A = 1.763\text{ square inches}}} \qquad \leftarrow \text{Area A36}$$

(c)

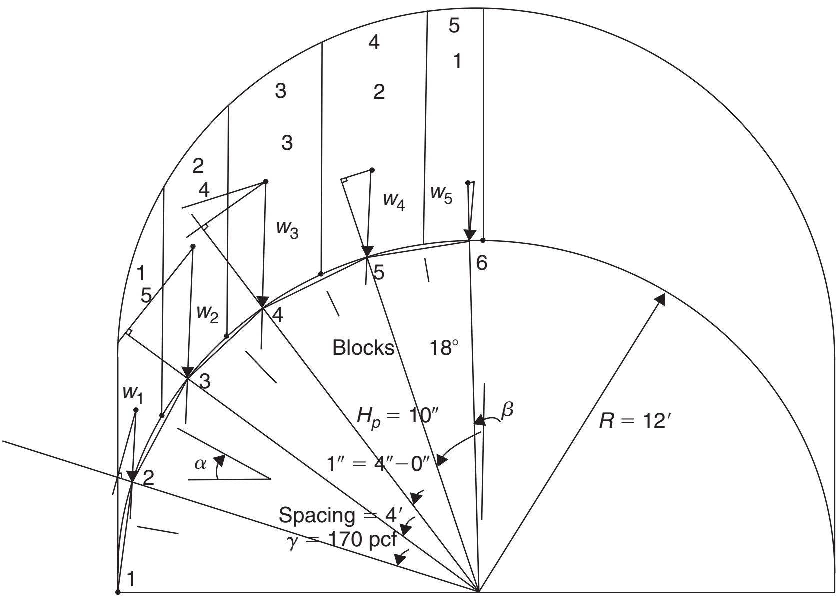
$$\sigma_b = \frac{M}{S}$$
$$= \frac{1.405(10^5)\text{lbf-ft}\,(12)(\text{in/ft})}{30(\text{in.}^3)}$$
$$\underline{\underline{\sigma_b = 56{,}200\text{ psi}}} \qquad \leftarrow \sigma_b$$

8. Given:
   - 2-piece continuous steel set
   - 12 ft radius, 13 ft leg
   - 8″ blocks, spaced ≈50 in.
   - rock load $H_p = 10$ ft.
   - try 1 in. of steel (web) per 3 ft of tunnel
   - 4 ft set spacing
   - rock sp. wt. $\gamma = 170$ pcf

   Find:
   1. Scale diagram of tunnel of rib
   2. Show blocks
   3. Show rock arch to scale
   4. Use R_v for leg reaction
   5. Show chords between blocks
   6. Do force polygon
   7. Find max moment
   8. Find max steel stress
   9. Estimate steel safety factor.

   Solution:

   (similar to example in handout)
   - some calculational details follow using same number of blocks but uniformly spaced



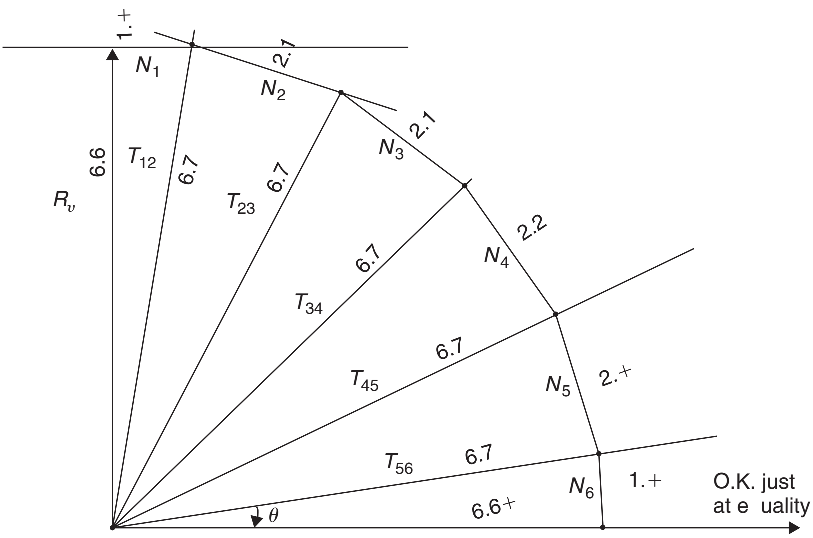
$$s = (12)\left(\frac{18}{180}\right)\pi(12) = 45.2''$$

Block	(w) Width	($\gamma w\, SH_p$) Weight	(angle)	$W_n = W_{\cos\beta}$
1	1.5′	1.02(10^4) lbf	2°	1.02 (10^4) lbf
2	2.7	1.43(10^4)	18°	1.36(10^4)
3	3.2	2.18(10^4)	36°	1.76(10^4)
4	3.4	2.31(10^4)	54°	1.36(10^4)
5	1.8	1.22(10^4)	72°	0.38(10^4)
	Σ12.0′	Σ8.16(10^4)		
checks:	(12.0)	(8.16)10^4		

α-angle	
12:	9°
23:	27°
34:	45°
45:	63°
56:	81°

Block 1/2 weight $\frac{W}{2} = 8.16(10^4)$ lbf

$$\text{Estimate: } R_v = 0.8\,\frac{W}{2}$$
$$R_v = 65.3(10^4)\text{ lbf}$$



$1'' = 20{,}000$ lbf
$\underline{\underline{T_{max} = 6.7(10^4)\text{ lbf}}}$

(almost uniform with equal blocking point spacing)

$$h = R - \left[R^2 - \left(\frac{l}{2}\right)^2\right]^{1/2}$$
$$= 12 - \left[(12)^2 - \left(\frac{45.2}{2(12)}\right)^2\right]^{1/2}$$
$$h = 42.0 - 11.85$$
$$\underline{\underline{h = 0.149\text{ ft}}}$$

$M = 0.85\,Th$
Rule of thumb: 1″ steel per 3 ft of tunnel
∴ 8″ steel (web depth)

AISC HB p. 22
try WF $8 \times 5\frac{1}{4}$ @ 20 lbf/ft

$\underline{\underline{A = 5.88\text{ in.}^2\ \ S = 17.0\text{ in.}^3}}$

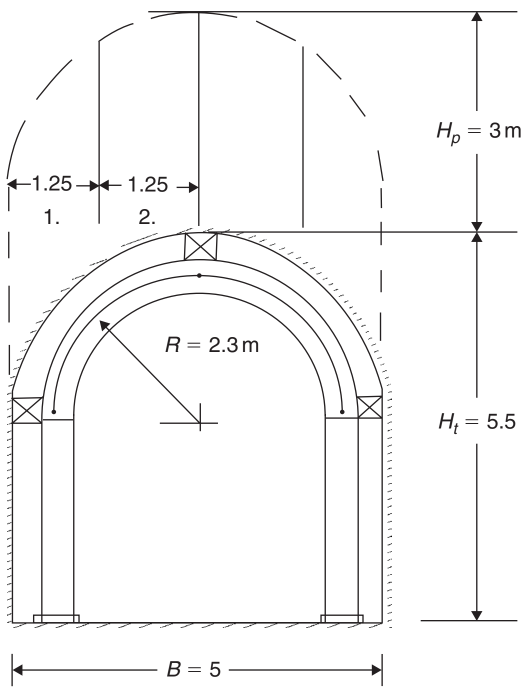
Then
$$\sigma = \frac{T}{A} + \frac{0.85\,\mathrm{m}}{S}$$
$$= \frac{6.7(10^4)}{5.88} + \frac{0.85(6.7)(10^4)(0.149)(12)}{17.0}$$
$$= 1.14(10^4)\ \mathrm{psi} + 0.599(10^4)\ \mathrm{psi}$$
$$\underline{\underline{\sigma = 1.74(10^4)\ \mathrm{psi}}}$$

A36 Steel

$$FS = \frac{36{,}000}{17{,}400}$$
$$\underline{\underline{FS = 2.07}}$$

$$M_{\max} = 0.85\,Th$$
$$= (0.85)(6.7)(10^4)(0.149)(12)$$
$$\underline{\underline{M_{\max} = 10.2(10^4)\ \text{ft-lbf}}}$$

9. Given: 2-piece, pin-connected steel arch.



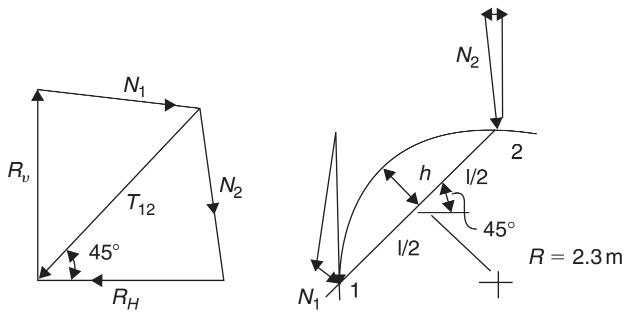
$S = (\text{spacing}) = 2\,\mathrm{m}, \quad \gamma = 25.0\,\mathrm{kN/m^3}$

Find:

(a) T, M

(b) A_s (250 MPa steel)

(c) σ_b ($\tilde{S} = 500\,\mathrm{cm^3}$ section modulus)



$\therefore N_2 \approx T_{12} \sin 45°$

$$T_{12} = \left(\frac{W}{2}\right)\frac{1}{\sqrt{2}} = 0.71\left(\frac{W}{2}\right)$$

But $T_{12} \approx \left(\frac{W}{2}\right)$

small angles: $N_2 \approx W_2$
equilibrium suggests: $\frac{W}{2} \cong R_v$
also suggests $T = R_v$

$\therefore$ compromise and use

$$\underline{\underline{T_{12} = 0.85\left(\frac{W}{2}\right)}}$$

$$\frac{W}{2} = \gamma B H_p \frac{S}{2}$$
$$= (25\,\text{kN/m}^3)(5.0)(3.0)\left(\frac{2.0}{2}\right)$$
$$\underline{\underline{\frac{W}{2} = 375\,\text{kN}}}$$

$$\text{then } T_{12} = 0.85(375)$$
$$\underline{\underline{T_{12} = 319\,\text{kN}}}$$

$\longleftarrow T$

$M = Th$ (pin-connected)

$$h = R - \left[R^2 - \left(\frac{l}{2}\right)^2\right]^{1/2}; \qquad \frac{l}{2} = R \sin 45°$$
$$= 2.3 - [(2.3)^2 - (1.63)^2]^{1/2} \qquad = \frac{1}{\sqrt{2}}2.3$$
$$= 2.3 - 1.62 \qquad \frac{l}{2} = 1.626\,\text{m}$$
$$\underline{\underline{h = 0.677\,\text{m}}}$$

$$M = (319\,\text{kN})(0.677\,\text{m})$$
$$\underline{\underline{M = 216\,\text{kN-m}}}$$

$\longleftarrow M$

at $FS = 1.0$
$T = \sigma_\tau\, A_s$

$$A = \frac{319\,\text{kN}}{250(10^3)\,\text{kN/m}^2}$$

$$A = 1.28(10^{-3})\,\text{m}^2$$

$$\underline{\underline{A = 12.76\,\text{cm}^2}} \qquad \xleftarrow{A_s}$$

$$\sigma_b = \frac{M}{s}$$

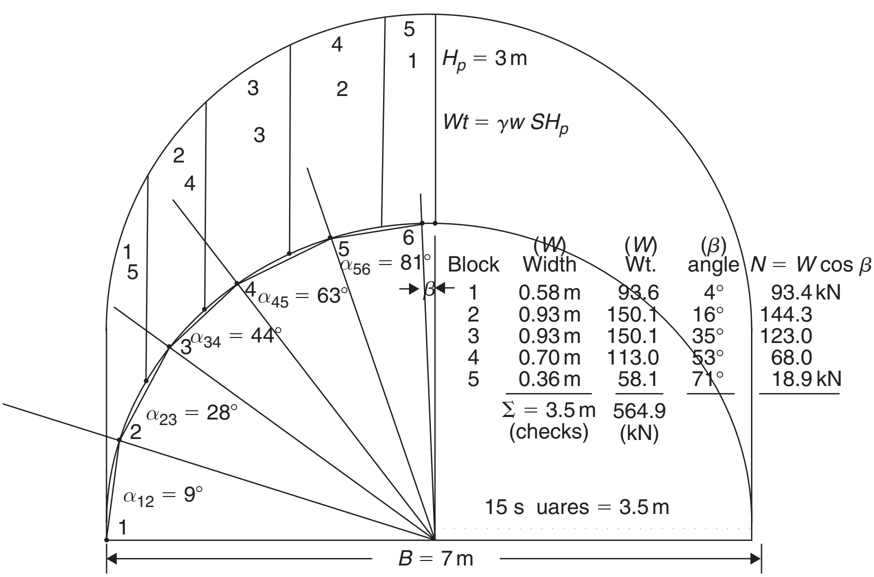
$$= \frac{216\,\text{kN-m}}{500\,\text{cm}^3}$$

$$\underline{\underline{\sigma_b = 432\,\text{MPa}}} \qquad \xleftarrow{\sigma_b}$$

10. Given: 2-piece continuous steel set 3.5 m radius, 4 m leg, $7.5\,\text{m} = H_t$, $7\,\text{m} = B$ 2.5 cm for each 1 m of width (steel/estimate), $S = 1.2$ m 20 cm blocks, spaced = 127 cm, $H_p = 3$ m, $\gamma = 26.9\,\text{kN/m}^3$

Find:

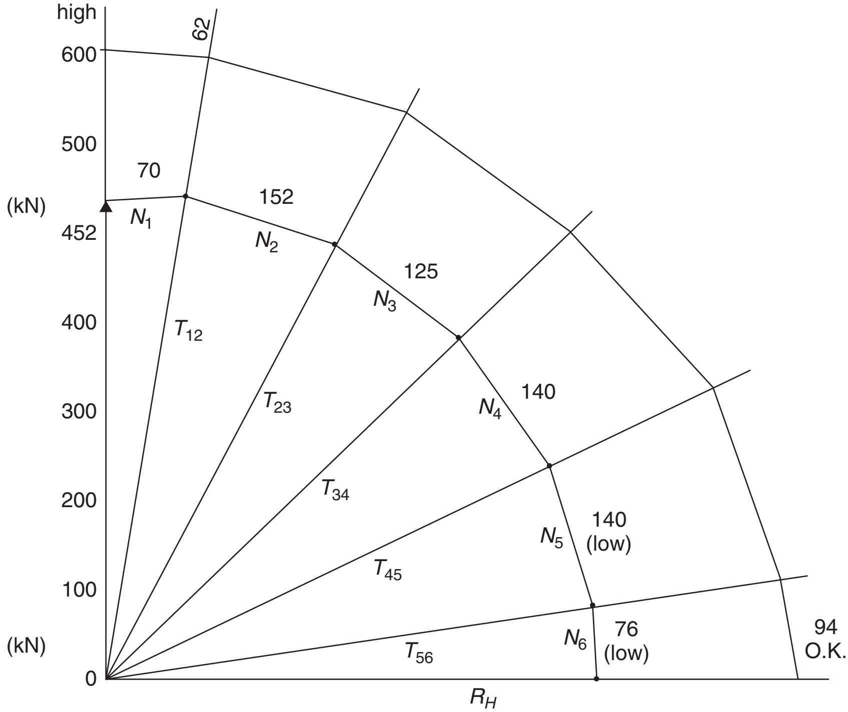
(1) Neat sketch
(2) Block positions
(3) Rock arch and forces
(4) Use R_v for leg reaction
(5) Show cords
(6) Do force polygon
(7) Find maximum moment
(8) Find steel stress
(9) Find steel *FS*.



Block 1/2 weight = 564.9 kN

estimate $R_v = 0.8\,\dfrac{W}{2}$

$\therefore R_v = 452\,\text{kN}$



$T_{max} = 620\,\text{kN}$ (almost uniform)

$$h = R - \left[R^2 - \left(\frac{l}{2}\right)^2\right]^{1/2}; \qquad \frac{l}{2} = \left(\frac{4.75}{2}\right)(0.233)$$

$$= 3.5 - \left[(3.5)^2 - \left(\frac{l}{2}\right)^2\right]^{1/2} \qquad \frac{l}{2} = 0.554\,\text{m}$$

$h = 0.0441\,\text{m}$

$$M = 0.85\,Th$$

$$= 0.85(620\,\text{kN})(0.0441)$$

$$M = 23.21\,\text{kN-m} \qquad \longleftarrow$$

Try 2.5 cm steel for 1 m width.
∴ 17.5 cm web depth (≈7 m steel)
Try 1-beam 7 × 35/8 @ 4.43 lbf/ft
AISC Hand book p 28 $A = 4.43\,\text{in.}^2$
$S = 10.4\,\text{in.}^3$

$$\sigma = \frac{T}{A} + \frac{M}{S}$$
$$= \frac{620\,\text{kN}}{(4.43)(2.54)^2} + \frac{23.21\,\text{kN-m}(10^2)}{(10.4)(2.54)^3}$$
$$= (21.7\,\text{kN/cm}^2) + (13.6\,\text{lbn/cm}^2)$$
$$\sigma = 35.3\,\text{kN/cm}^2,\ \underline{353\,\text{MPa}}$$

Steel @ 250 MPa $\therefore$ σ too high

Try $8 \times 5\frac{1}{4}$ $A = 5.88\,\text{in.}^2$ $S = 17.0\,\text{m}^3$

Repeating: $\underline{\underline{\sigma = 247\,\text{MPa} < 250\,\text{MPa}}}$

Note: $\underline{\underline{FS \text{ near } 1.01}}$ which is low. Need a larger beam as a practical matter.

11. Given: $AT_A = (C_0/T_0)P_b t l_r$ spacing $= S$ & Mohr–Coulomb rock, 45° point load cone angle.

Find: Equivalence to "Bischoff and Smart", reference under Chapter 4 list in text.

Solution:

$C_0 =$ Unconfined compressive strength
$T_0 =$ tensile strength
$P_b =$ bolting pressure
$t =$ rock arch (supporting) thickness
$l_r =$ "set" spacing
B & S handout (reference)
$\Delta T_A = q\dfrac{\sigma_b A_b}{S^2}\,(L - S)$: per ft. of tunnel

$$q = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$
$$\sigma_b = \text{bolting stress}$$
$$A_b = \text{bolt area}$$
$$S = \text{set spacing}$$
$$h = \text{bolt length}$$

Also: $t = L - S$: arch thickness
$F_b = \sigma_b A_b$: bolt force
$P_b = \dfrac{F_b}{S^2}$: bolt pressure & square pattern
$S = l_r$ with these identifications and equivalences B & S becomes $\Delta T_A = qP_b t l_r$ after multiplying by l_r to obtain total ΔT_A between sets (from ΔT_A per foot of tunnel). Must now show that for MC rock,

$$q = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) = \frac{C_0}{T_0}$$

For MC $\sin\phi = \dfrac{C_0 - T_0}{C_0 + T_0}$, & $\dfrac{C_0}{T_0} = \dfrac{1+\sin\phi}{1-\sin\phi}$

Substituting $\underline{\underline{2\mu = \dfrac{\pi}{2} - \phi}}$

$$\frac{C_0}{T_0} = \frac{1+\cos^2\mu}{1-\cos^2\mu}$$

Double angle formulas

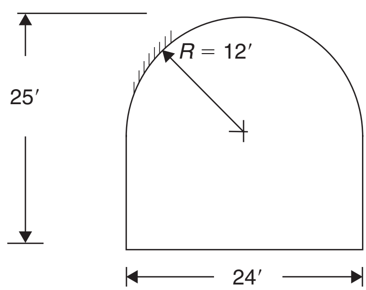
$$\cos^2\mu = \frac{1+\cos^2\mu}{2}$$
$$\sin^2\mu = \frac{1-\cos^2\mu}{2}$$

gives $\dfrac{C_0}{T_0} = \cot^2(\mu) = \tan^2\left(\dfrac{\pi}{2} - \mu\right)$

Hence $\underline{\underline{\dfrac{C_0}{T_0} = \tan^2\left(\dfrac{\pi}{4} + \dfrac{\phi}{2}\right)}}$

12. Given: Problem 8. Steel set data and results
Find: Equivalent bolting pattern using 1 inch diameter steel @ 60,000 psi

Solution:



From Problem 8.
$\underline{\underline{T = 6.7(10^4)\text{ lbf}}}$
$\therefore \underline{\underline{\Delta T_A = 6.7(10^4)\text{ lbf}}}$
Handouts *B* & *S* article

$\Delta T_A = \left(\dfrac{C_0}{T_0}\right)\dfrac{F_b}{S^2}(L-S)l_r$
$t = L - S$
$L =$ bolt length
$S =$ set spacing
$P_b = \dfrac{F_b}{A_s}$ bolting pressure
$\dfrac{C_0}{T_0} =$ ratio of unconfined compressive to tensile strength.

- Assume $\dfrac{C_0}{T_0} = 10$, So that $\phi = 55°$: $\left[\sin\phi\dfrac{\left(\frac{C_0}{T_0}\right) - 1}{\left(\frac{C_0}{T_0}\right) + 1}\right]$

reasonable for rock

$$P_b = \frac{\sigma_b A_b}{S^2}$$
$$= (60{,}000)\left(\frac{\pi}{4}\right)(l)^2$$
$$P_b = \frac{4.71(10)^4}{S^2}$$

Set spacing was 4′
$T = \Delta T_A$

$$6.7(10^4) = (10)(4.71)(10^4)(4)\left(\frac{L-S}{S^2}\right)$$

$$\frac{L-S}{S^2} = 0.0338(55^\circ), \quad L = 0.0338S^2 + S$$

$$\frac{C_0}{T_0} = 10.0\ \phi = 55^\circ$$

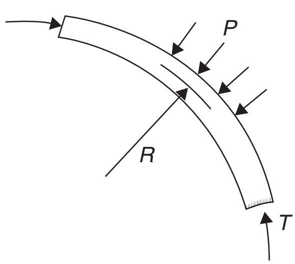
$$\frac{C_0}{T_0} = 3.85\ \phi = 36^\circ$$

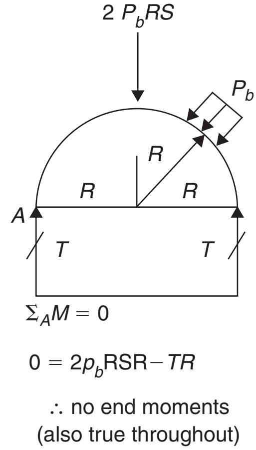
S	3′	4′	5	6	7
L(55)	3.3′	4.6′	5.9′	7.3′	8.7′
L(36)	2.4′	3.1′	3.8′	4.5′	5.3′

A 7 ft spacing, using 9 ft long bolts seems reasonable. (A better plan could be done with more details about the rock mass).

13. Given: Yieldable steel arches in squeezing ground.
$H_p = 15$ ft.
Find: (a) A_s, (b) proportion for bending.

Solution:





Previous derivation:
$T = pRS$
$p = \gamma H_p$
estimate 1 psi/ft
$p = 15$ psi
$T_{max} = \sigma_Y A_s$ & $\sigma_Y \approx 36{,}000$ psi

$$A_s = \frac{(15)(RS)(144)}{36{,}000}$$

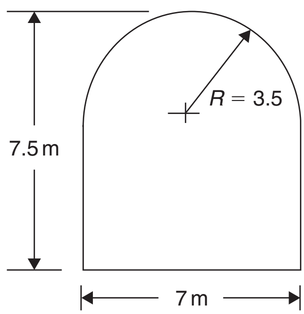
$$A_s = 0.06\,RS$$

← A_s

No moment is present because of the uniformly, distributed load, therefore no area is needed to resist bending stress.

bending area = 0 ←

14. Given: Problem 10 data use 2.5 cm bolts @ 410 MPa
Find: Bolt length spacing for Problem 10 equivalent



Solution:

From Problem 10 $\Delta T_A = 565{,}620$ kN say, 595 kN
From Problem 11: $\Delta T_A = \left(\frac{C_0}{T_0}\right) P_b t l_r$
where
$t = L - S$
$L =$ length

$S =$ spacing
$P_b = F_b/S^2$ (square pattern)
$l_r = 2\,\text{m}$
set spacing was 2 m

$$F_b = \sigma_b A_b$$

Assume reasonable $\underline{\dfrac{C_0}{T_0} = 10}$

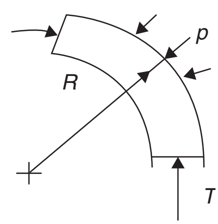
$$F_b = (410)\,\text{MPa}\left(\frac{\pi}{4}\right)(2.5\,\text{cm})^2(10^{-4})$$

$$\underline{F_b = 201\,\text{kN}}$$

$$595(10^3) = (10)\frac{(201)10^3}{S^2}(L-S)(2), \quad \frac{L-S}{S^2} = 0.148$$

if $S = 2\,\text{m}$ then $L = 2.59\,\text{m}$
$S = 1.5$ $L = 1.83\,\text{m}$
$S = 1\,\text{m}$ $L = 1.15\,\text{m}$
(say) $\underline{\underline{S = 1.5\,\text{m}\ L = 1.83\,\text{m}}}$ ← (square pattern)

15. Given: Yieldable steel arches $H_p = 4.6\,\text{m}$ squeezing ground
Estimate: Steel area A_s
Solution:

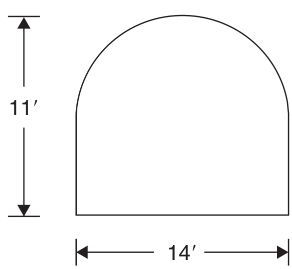


$T = pRS$
$p = \gamma H_p$
$\gamma \cong 25\,\text{kN/m}^3$

$$T = 25(10^3)(4.6)RS$$
$$T = (115)(RS) : (\text{kN})$$
$$A_s\sigma_\tau = T \quad \text{est.} : \sigma_\tau = 250\,\text{MPa}$$
$$A_s = \frac{(115)(RS)\text{kN}}{750(10^3)\,\text{kN}}$$
$$\underline{A_s = 0.46\,RS}$$

← $R =$ radius
$S =$ (spacing)

16. Given – arched tunnel, wet, depth = 1970′.
"moderately blocky and seamy" T4.2 text:



H_p: $0.25(14) = 3.5'$
H_p: $0.35(11 + 14) = 8.75$
$\overline{H}_p = 6.125'$
$\underline{\underline{\text{Use } H_p = 6'}}$

$W = \gamma H_p BS$ assume $\gamma = 162$ pcf

$$W/B \text{ (per ft of width)} = \frac{(162)(6)(S)(14)}{14}$$

$$\begin{aligned} S &= 4' \quad W/B = 3{,}888\ \text{lb/ft} \\ S &= 6' \quad = 5{,}832\ \text{lb/ft} \\ S &= 8' \quad = 7{,}776\ \text{lb/ft} \end{aligned}$$

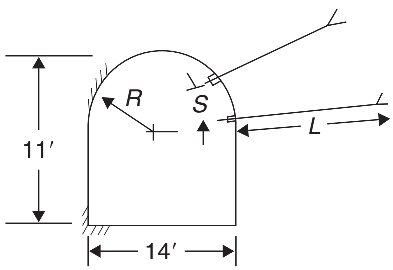
T.I. 14′ 5″ × 5″ Stanchion 16 lb/ft.
(Handout) 8′ spacing
max. block point spacing = 40″ ←

(b) Handle high side pressure by full circle rings.

(c) $T \approx \frac{W}{2}$

$$T = \frac{(162)(14)(6)(8)}{2} = \underline{\underline{54{,}432\ \text{lbf}}}$$ ←

17. Given: Problem 16.
Find: Equivalent bolting.
Solution:



$\Delta T_A = T_s$

$\underline{T_s \cong 54{,}432\ \text{lbf}}/8 = \left\{ \begin{array}{l} 6.80(10^3) \\ \text{per ft. of} \\ \text{tunnel length} \end{array} \right.$

Also: $\Delta T_A = q\dfrac{\sigma_b A_b}{S^2}(L - S)$

$$q = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

$$\sin\phi = \frac{C_0 - T_0}{C_0 + T_0} = \frac{23{,}700 - 1{,}480}{23{,}700 + 1{,}480}$$

$$\sin\phi = 0.8825$$

$$\underline{\underline{\phi = 61.9^\circ}}$$

$$q = \tan^2\left(45 + \frac{61.9}{2}\right)$$

$\underline{\underline{q = 16.0}}$ Note: $q = \dfrac{C_0}{T_0}$, estimate between 10 – 20 try 1″diameter @ 60,000 psi

$$F_b = \frac{\pi}{4}(1)^2(60{,}000)$$

$$\underline{\underline{F_b = 4{,}712(10^4)\text{lbf}}}$$

Try $L \cong \frac{1}{3}W$

Say $L = 6$ ft

$$\Delta T_A = \frac{(16)(4.71)(10^4)(6-5)}{S^2}$$

$$S^2(0.68)(10^4) = 452(10^4) - 75.4(10^4)S$$

$$S^2 + \frac{75.4}{0.68}S - \frac{452}{0.68} = 0$$

$$S = -111 + \frac{[(1111)^2 + 665(4)]^{1/2}}{2}$$

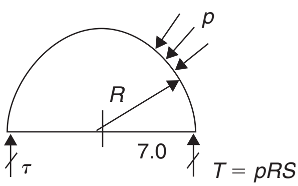
$$S = \frac{11.4}{2}\text{ft}$$

$S = 5.7$ ft

Square pattern $S = 5.7$ ft
1″diam, 60,000 psi steel
Length = 6 ft, ←

18. Given: H_p from 16.
Find: Yieldable arch area.

Solution:



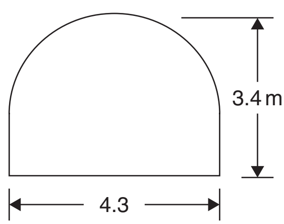
$T = pRS$ and $T = \sigma_s A_s$

$$A_s = \frac{pRS}{\sigma_s}, \quad p = \gamma H_p$$

$$= \frac{(62)(575)(7)(8)}{36,000} A36$$

$A_s = 1.45$ sq in. ← A_s

19. Given: Arched tunnel, "moderately blocking & seamy" wet ground
From: Table 3 "Commercial shearing & stamping"
or text Table 4.2



$H_p = 0.25(4.3)$ to $0.35(4.3 + 3.4)$

$H_p = 1.08$ to 2.70 m

$H_p = 1.89$ m

$W = \gamma S H_p B$ assume $\gamma = 25$ kN/m^3

$= (25)(1.5)(1.89)(4.3)$, try $S = 1.5$ m, $S = 2.0$ m

$W = 305$ kN

$(S = 1.5)$

$W = 406$ kN

$(S = 2.0)$

$$W/B = 70.9\,\text{kN/m} \quad (4{,}858\,\text{lbf/ft})$$
$$(S = 1.5)$$
$$W/B = 94.5\,\text{kN/m} \quad (6{,}473\,\text{lbf/ft})$$
$$(S = 2.01)$$
$$W/B = 118.1\,\text{kN/m} \quad (8{,}091\,\text{lbf/ft})$$
$$(S = 2.5)$$

Table 1 "C&S" handout

(a) A "Light Beam" 6″ × 4″ @ 16 lbf/ft would do when spaced 2.5 m and maximum blocking point spacing of 40″

6″ × 4″ (depth × flange width) = 15.2 × 10.2 cm

16 lbf/ft = 234 N/m

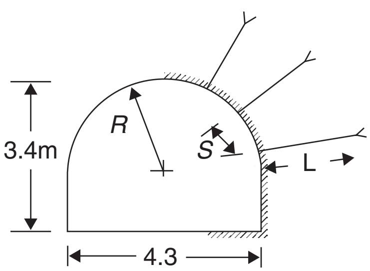
40″ spacing = 102 cm ⟵

(b) side pressure when heavy requires full circle ribs ⟵

(c) $\underline{T} \propto \frac{W}{2} = \frac{\gamma BSH_p}{2} = (25\,\text{kN/m}^3)(4.3)(2.5)\left(\frac{1.89}{2}\right) = \underline{254\,\text{kN}}$ ⟵

20. Given: Problem 19

Find: Equivalent bolting pattern



Solution:

$$\Delta T_A = T_s = 254\,\text{kN}/2.5\,\text{m}$$
$$T_s = 102\,\text{kN/m}(\text{kN per m of tunnel length})$$

But also: $T_s = \left(\frac{C_0}{T_0}\right)\frac{\sigma_b A_s}{S^2}(L - S)$: (square pattern)

estimate: $\frac{C_0}{T_0}$ between 10 − 20, say, 15

steel estimate: $\sigma_\tau = 414\,\text{MPa}$

try 2.54 cm bolt diameter

then $\sigma_b A_s = (414\,\text{MPa})\left(\frac{\pi}{4}\right)(2.54)^2(10^{-4})$

$\sigma_b A_s \cong 210\,\text{kN}$

$102\,\text{kN} = (15)(210\,\text{kN})(L - S)/S^2$

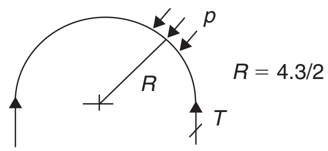
try $\underline{L = 2\,\text{m, then } S = 1.88\,\text{m}}$ ⟵

(other L, S combinations possible in this design)

21. Given: Problem 19 data

Find: Yieldable arch area

Solution:



$T = pRS$, $S =$ set spacing

$p = \gamma H_p$

$= (25\,\text{kN/m}^2)(1.89\,\text{m})$

$\underline{p = 47.25}\,\text{kPa}$

$\therefore T = (47.25)(2.15)(2.5)$

$\underline{T = 254\,\text{kN}}$

But also $T = \sigma_Y\ A_s$, $\sigma_Y = 250\,\text{MPa}$

$$\frac{1}{A_s} = 250(10^3)\,\text{kPa}/254\,\text{kN}$$

$A_s = 1.02(10^{-3})\,\text{m}^2$

$\underline{\underline{A_s = 10.2\,\text{cm}^2}}$ ←

22. Given: arched tunnel 18′ × 21′, $d = 970$ ft "moderately, blocking and seamy" wet

Find:

(a) Steel sets

(b) How to handle high side pressure.

Solution:

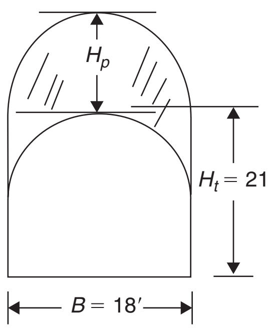


Table 4.2, Rock mass = Type 4

$H_p = 0.25\text{B to } 0.35(B + H_t)$

$= 0.25(18) \text{ to } 0.35(18 + 21)$

$H_p = 4.5 \text{ to } 13.65$

$H_p(\text{ave}) = 9.08'$

use $\underline{\underline{H_p = 9\,\text{ft}}}$ (wet)

$\gamma = 159$ pcf (given) or (estimate)

$$\frac{W}{B} = \frac{\gamma B H_p S}{B}$$

$$\therefore \frac{W}{B} = \frac{(159)(9)(S)}{4} \text{(load per ft. of width)}$$

try $S = 5$ ft

$$\frac{W}{B}\ \left.\begin{matrix}4\\5\\6\end{matrix}\right\} = \begin{matrix}5{,}724\\7{,}155\\8{,}586\end{matrix} \qquad \text{lbf/ft}$$

Handout T.1 continuous ribs, B = 18′ @5′ *spacing*, <u>a 5″ × 5″ H-beam (7190) @ 18.9 lbs/ft (depth) × (width) (max blocking pt spacing = 44″)</u>

(a) ←

(b) go to full circle for high side pressure.

23. Given: $H_p = 9$ ft from 3
Find: Yieldable arch area.

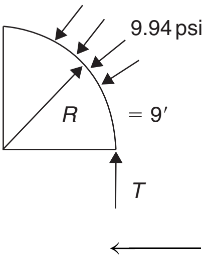
Solution:

$$p = \gamma h = \frac{(159)(9)}{144} = \underline{9.94\,\text{psi}}$$

$T = pRS$

$= (9.94)(144)(9)(5)$ try $S = 5'$

$\underline{\underline{T = 64{,}395\,\text{lbf}}}$



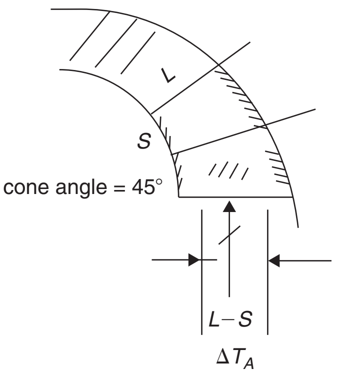
But $T = \sigma_Y A$, $\sigma_Y = 36{,}000$ psi (given)

$$\therefore A = \frac{64{,}395}{36{,}000}$$

$\underline{\underline{A = 1.79\,\text{sq in}}}$

24. Given: Problem 22 conditions
Find: Equivalent bolting pattern.

Solution:



$$T = 64{,}395\,\text{lbf}$$

$$\Delta T_A = \left(\frac{C_0}{T_0}\right)(p)(t)(s)$$

$$64{,}395 = \left(\frac{27{,}400}{1{,}840}\right)(994\,\text{psi})(L - S)(S)(144)$$

$\underline{\underline{4{,}350 = (LS - S^2)144,}}$ (give $L - S$)

$$p = \text{bolting pressure} = \frac{F_b}{S^2}$$

$F_b = \sigma_b A_b$

$\underline{\underline{pS^2 = \sigma_b A_b}}$ & $\underline{\underline{A_b = \frac{\pi D^2 b}{4}}}$

then try $5'$ spacing $10'$ spacing

$$L = \frac{4{,}350}{(144)5} + \left(\frac{25}{5}\right), \quad L = \frac{4{,}350}{144(10)} + \frac{100}{10}$$

$L = 11$ ft $L = 13.0$

try $4'$ spacing

$$L = \frac{4{,}350}{(144)(4)} + 4 = 11.6\,\text{ft}$$

$\underline{\underline{\text{Use 11 ft bolt on 5 ft centers}}}$

$(9.9)(144)(5)^2 = 60{,}000\,A_b$

$A_b = 0.594\ \text{in.}^2$

$D^2 = \left(\frac{4}{\pi}\right)(0.594)$

$\underline{\underline{D = 0.87\ \text{in.}}}$ ←

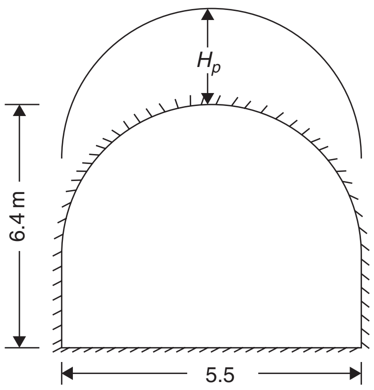
$$\left.\begin{array}{r} \text{strength} = 60{,}000\ \text{psi} \\ \text{length} = 11\ \text{ft} \\ \underline{\underline{\text{spacing} = 5\ \text{ft}}} \end{array}\right\}$$ ← all

25. Given: Arched tunnel in wet ground "moderately blocky and seamy"
$d = 297\ \text{m}$ $B = 5.5\ \text{m}$ $H_t = 6.4\ \text{m}$

Find:

(a) Steel set design, web depth, flange width weight/foot, spacing, blocking point spacing.
(b) How to handle heavy side pressure.

Table 3 handout, Text T4.2



$H_p = 0.25\,B$ to $0.35(H_t + B)$
$= 0.25(5.5)$ to $0.35(6.4 + 5.5)$
$H_p = 1.375$ to 4.17

Use $\underline{H_p = 2.8\ \text{m}}$. Estimate $\gamma = 25\ \text{kN/m}^3$

then $\frac{W}{B} = \frac{\gamma B S H_p}{B}$

$= \frac{(25)(5.5)(2.8)(S)}{(5.5)} = 70S$

$\frac{W}{B} = 105\ \text{kN/m}\ (S = 1.5)$, $\frac{W}{B} = 140\ \text{kN/m}\ (S = 2.0)$,

$\frac{W}{B} = 175\ \text{kN/m}\ (S = 2.5)$

at 2 m spacing $W/B = 9{,}589\ \text{lbf/ft}$ of width $B = 18.0\ \text{ft}$

Table 1 from "C & S" on 8″ × 51/4″ wide flange beam @ 20 lbf/ft would do with max. block pts @ 44″

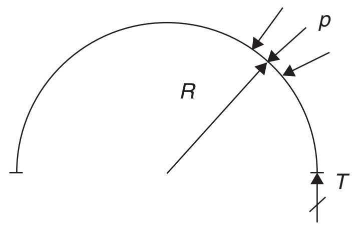
$$\left.\begin{array}{l} \underline{\text{web depth} = 20.2\ \text{cm}} \\ \underline{\text{flange width} = 13.3\ \text{cm}} \\ \underline{\text{wt/pt} = 292\ \text{N/m}} \\ \underline{\text{block pts max. spacing} = 112\ \text{cm}} \\ \underline{\text{set spacing} = 2\ \text{m}} \end{array}\right\}$$ ← (a)

(b) Heavy side pressure handled by full circle ribs.

26. Given: Same rock pressure as in Problem 25
$H_p = 2.8\,\text{m}$
Find: Yieldable arch steel area A_s.

Solution:

Formula: $T = pRS$
$p = \gamma H_p$
estimate $\gamma = 25\,\text{kN/m}^3$
$p = (25)(2.8)$
$\underline{p = 70\,\text{kPa}}$



S = spacing = 2 m
$R = \frac{B}{2} = 2.75\,\text{m}$

$T = (70\,\text{kN/m}^2)(2.75)(2)$
also $\underline{T = 385\,\text{kN}}$

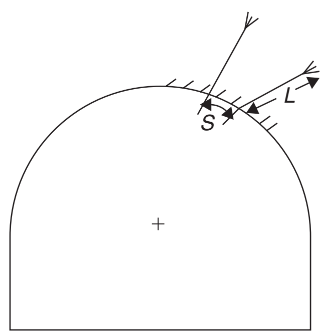
$T = \sigma_Y A_s$ estimate $\sigma_Y = 250\,\text{MPa}$

$$A_s = \frac{385\,\text{kN}}{250(10^3)\,\text{kN/m}^2}$$

$A_s = 1.54(10^{-3})\,\text{m}^2$
$\underline{\underline{A_s = 15.4\,\text{cm}^2}}$ ← A_s

27. Given: Problem 25 conditions
Find: Equivalent bolting pattern.

Solution:



S = bolt spacing L = bolt length

Estimate $T = \dfrac{W}{2}$
$T = \left(\frac{1}{2}\right)(140\,\text{kN/m})5.5$
$\underline{T = 385\,\text{kN}}$

Check:
$\gamma = 25\,\text{kN/m}^3$
$S = 2.0\,\text{m}$
$B = 3.5\,\text{m}$
$H_p = 2.8\,\text{m}$

$W = \gamma SBH_p$
$W = 770\,\text{kN}$
$W/2 = 385\,\text{kN}$ (checks)

formula $\Delta T_A = \left(\dfrac{C_0}{T_0}\right)\left(\dfrac{F_b}{S^2}\right)(L - S)$: per m of tunnel

$$T_s = \frac{T}{2} = 192.5 \text{ per m of tunnel}$$

$$\therefore 192.5\,\text{kN/m} = (15)\left(\frac{F_b}{S^2}\right)(L - S)$$

where C_0/T_0 ranges between 10–20, say 15.

try 2.5 cm diameter both @ 415 MPa σ_Y

then $F_b = \sigma_Y A_b$

$$= \left(\frac{\pi}{4}\right)(2.5)^2(10^{-4})415\ \text{MPa}$$

$\underline{\underline{F_b = 204\ \text{kN}}}$

$$192.5\ \text{kN} = (15)(204\ \text{kN})\left(\frac{L-S}{S^2}\right)$$

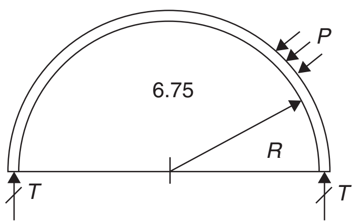
$$192.5 = 3{,}060\left(\frac{L-S}{S^2}\right)$$

if $L = 3$ m then $S = 2.58$ m

bolt length = 3 m

spacing = 2.58 m (square pattern) ←

28. Given: Yieldable arches semi-circle
$p = 20$ psi $R = 6.75$ ft $H = 2{,}350$ ft
A 36
Find: A_s, S, $FS = ?$



Solution:

$T = pRS$ & $T = \sigma_{\text{allow}} A_s$

$$\sigma_{\text{allow}} = \frac{\sigma_\tau}{FS_s}$$

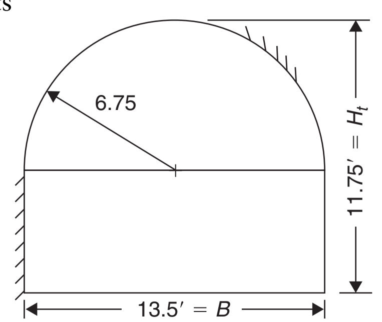
Yieldable arches, assume $\underline{\underline{FS = 1.1}}$

$$\sigma_{\text{allow}} = \frac{36{,}000}{1.1} = 32{,}730\ \text{psi}$$

$\therefore 32{,}730\ A_s = (20)(6.75)(5)(144)$ $\underline{\underline{S = 5\ \text{ft}}}$ ←

$\underline{\underline{A_s = 2.97\ \text{sq in.}}}$

29. Given: Problem 28 conditions but fixed steel sets and $H_t = 11.75$ ft
Find: Steel size, spacing



Solution:

$$\text{at 20 psi} \sim \left(\frac{150}{144}\right)(H_p)$$

$$H_p = \frac{(144)(20)}{(150)} = \underline{\underline{19.2\ \text{ft.}}}$$

$W = \gamma H_p BS, \quad \underline{\underline{S = 5\text{ ft.}}}$ ⟵

$$W' \text{ (per foot of tunnel width)} = \frac{W}{B}$$

$$W' = \frac{(152)(19.2)(13.5)(5)}{13.5}$$
$$\underline{\underline{W' = 13{,}511\text{ lbf/ft}}}$$

Table 1. Ribs
tunnel width 13.5′ use 14′

Closest is $\underline{\underline{6'' \times 6''\text{ ft beam}}}$ @ 25 ⟵
(for 13,270 lbf/ft)
max. blocking $\underline{\underline{\text{point spacing} = 40''}}$ ⟵

30. Given: Problem 29, steel sets, grade 60 (60,000 psi) steel

$E = 4.9(10^6)$, $v = 0.18$ $C_0 = 7{,}500$ psi, $T_0 = 750$ psi $\gamma = 148$ pcf, 1″ diameter Steel
Find: Equivalent bolting pattern

Solution:

$$\Delta T_A = \frac{C_0}{T_0}\frac{\sigma_b A_b}{S^2}(L - S) = T$$

assuming square pattern S c'-c', L long.

$$\text{Estimate } T = \frac{W}{2}\frac{(0.85)}{S} \quad \text{(per ft)}$$
$$= \left(\frac{0.85}{2}\right)\left(\frac{\gamma B H_p S}{S}\right)$$
$$= \left(\frac{0.85}{2}\right)(148)(13.5)(19.2)$$
$$\underline{\underline{T = 16{,}304\text{ lbf/}}}\text{ft of length}$$

$$\Delta T_A = \left(\frac{7{,}500}{750}\right)(60{,}000)\left(\frac{\pi}{4}\right)(1)^2\left(\frac{L - S}{S^2}\right) = 4.712(10^5)\left(\frac{L - S}{S^2}\right)$$
$$4.712(10^5)\left(\frac{L - S}{S^2}\right) = 16{,}304$$
$$\frac{L - S}{S^2} = 0.0346$$

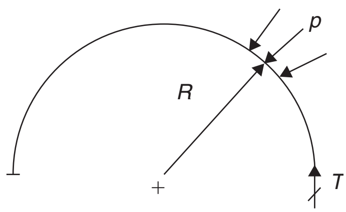
$S = 5$ $L = 5.9'$
$S = 10$ $L = 13.5'$ $\underline{\underline{6'\text{ bolts on }5'\text{ centers}}}$ ⟵
$S = 3'$ $L = 3.3'$

31. Given: Semi-circular yieldable arches $R = 2$ m, depth $= 716$ m $\quad p = 138$ kPa
σ_Y (steel) $= 250$ MPa

Find:
(a) reasonable FS
(b) set spacing.

Solution:
$T = pRS$ & $T = \tilde{\sigma}_Y A_s$

where $\tilde{\sigma}_Y =$ max allowable $= \sigma_Y / FS$
$FS = 1.1$ for yieldable arches (slip before yield)



$$\tilde{\sigma}_Y = \frac{250}{1.1} = 227\,\text{MPa}$$

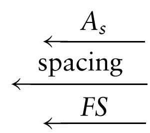
try $T = (138\,\text{kPa})(2)(2) : 2\,\text{m} = 5$
then $T = 552$ kN

$$A_s = \frac{552\,\text{kN}}{227\,\text{MPa}}$$

$A_s = 0.00243\,\text{m}^2, \quad \underline{\underline{24.3\,\text{cm}^2}}$

$\underline{\underline{S = 2\,\text{m}}}$

$\underline{\underline{FS = 1.1}}$



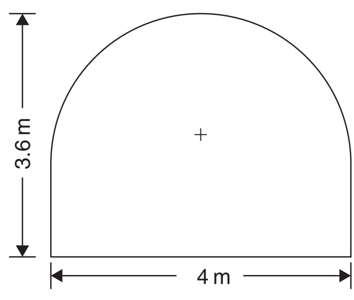
32. Given: Problem 31 data

Find: Consider alternative support when entry height $H_z = 36$ m in form of steel sets (fixed).

Solution:

$p = 138$ kPa (from Problem 31)

$p = \gamma H_p$ est. $\gamma = 25\,\text{kN/m}^3$



$$H_p = \frac{138}{25}$$

$\underline{\underline{H_p = 5.52\,\text{m}}}$

$W = \gamma S B H_p, \quad$ try $S = 1.5$ m

$W = (25)(1.5)(4)(5.52)$

$\underline{\underline{W = 828\,\text{kN}}}$

$$\frac{W}{B} = \frac{828}{4}$$

$$\frac{W}{B} = 207\,\text{kN per meter of tunnel width}$$

(14,178 lbf/ft) @ (13.1 ft wide)

{ Off Table in "C & S" Use 14 ft Column
{ Off Column, try $S = 1.0\,\text{m}$

then $\dfrac{W}{B} = 9{,}452\,\text{lbf/ft}$

a 6″ × 6″ H-beam @ 20 lb/ft would do

Web depth = 15.2 cm
flange width = 15.2 cm
Wt/ft = 292 N/m
Spacing = 1 m. ⟵

33. Given: Problem 32 data

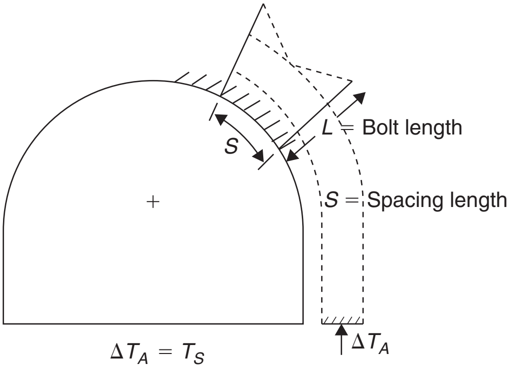
$E = 33.8\,\text{GPa}, v = 0.18$

$C_0 = 51.7\,\text{MPa},\ T_0 = 5.17\,\text{MPa}$

$\gamma = 23.4\,\text{kN/m}^3, \sigma_Y = 410\,\text{MPa}$

One-inch diameter (2.54 cm)

Find: Bolt spacings (in-row, between-row) & bolt length.



Solution:

$$\Delta T_A = \left(\frac{C_0}{T_0}\right)\left(\frac{F_b}{S^2}\right)(L - S)\text{: on a square pattern}$$

$$\frac{C_0}{T_0} = 10$$

$$F_b = (410)\,\text{MPa}\left(\frac{\pi}{4}\right)(2.54)^2\,(10^{-4}) = 208\,\text{kN}$$

$$T_s = \frac{W}{2S}\text{: estimate}$$

$$= \frac{828}{2(1.5)}$$

$$T_s = 276\,\text{kN per m of tunnel length}$$

$$\therefore 276\,\text{kN} = (10)(208)\left(\frac{L - S}{S^2}\right)$$

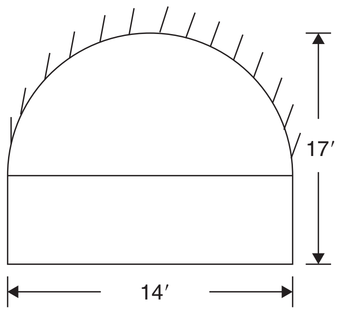
try $L = 2\,\text{m}$ then $276S^2 + 20{,}805S - (2)(2{,}080) = 0$

$S = 1.64\,\text{m}$ ⟵

bolt length = 2 m

bolt spacing = 1.64 m (Square pattern, would do)

34. Given: Arched tunnel steel sets 2,830 ft depth



$C_0 = 14{,}300$ psi, $T_0 = 1{,}430$ psi
$E = 4.25(10^6)$ psi, $G = 1.8(10^6)$ psi
$\gamma = 156$ pcf
"moderately blocky and seamy," wet
$B = 14$ ft $H_t = 17'$
Find: steel sets.

Solution:

Table 3 for H_p, Table 4.2 text
estimate: $H_p = 0.3(B + H_t)$
$H_p = 0.3(14 + 17)$
$H_p = 9.3$ ft

$$\frac{W}{B} = \frac{\gamma B H_p S}{B}, \text{ try } S = 6'$$
$$= \frac{(156)(14)(9.3)(6)}{14}$$
$$\frac{W}{B} = 8{,}705 \text{ lbf/ft of width}$$

Table 1 Handout @ 14′ Spacing = 6 ft
5″ × 5″ H-beam @ 18.9 lb/ft will do
with max blocking pt spacing @ 40″ ⟵

35. Given: Problem 34 data
Find: Equivalent bolting pattern
Solution:

Same thrust capacity

$$T = \Delta T$$
$$T_{\text{steel}} = \sigma_s A_s$$

σ_s = allowable stress
$\sigma_s = 27{,}000$ psi, say

$$\gamma L A = W$$
$$A = \frac{18.9(\text{lb/ft})(1\text{ ft})(144)}{1\text{ ft}\left(\frac{490\text{ lb}}{\text{ft}^3}\right)}$$
$$A = 5.55 \text{ sq in.}$$

$$\therefore T_{\text{steel}} = (27{,}000)(5.55)$$
$$T_{\text{steel}} = 1.5(10^5)\text{ lbf}$$

$$\Delta T = \left(\frac{C_0}{T_0}\right) pS(L - S)$$

$$p = \gamma H_p, \quad \frac{C_0}{T_0} = 10$$
$$= (156)(9.3)$$
$$p = 1.45(10^3)\ \text{lb/ft}^2$$
$$S = ?$$
$$L = \frac{1}{2}(14)$$
$$\Delta T = (10)(1{,}450)(7S - S^2)$$
$$S(7 - S) = \frac{1.5(10^5)}{(10)(1{,}450)}$$
$$S(7 - S) = 1.04$$
$$S^2 - 7S + 1.04 = 0$$
$$S = \frac{7 \pm \sqrt{7^2 - 4(1.04)}}{2}$$
$$S = 4.88\ \text{ft, say}$$
$$\underline{\underline{S = 5\ \text{ft}}}$$

$$F_b = pA_r\text{: bolt force}$$
$$= (1, 450)(5)(5)$$
$$F_b = 3.625(10^4)\ \text{lbf}$$

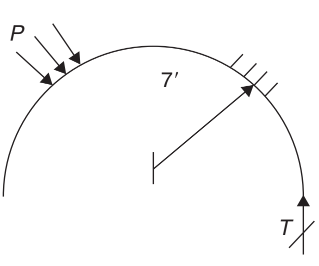
But $F_b = A_b\, \sigma_b$, $\sigma_b = \underline{\underline{60{,}000\ \text{psi}}}$

$$A_b = \frac{3.625(10^4)}{6(10^4)}$$
$$A_b = 0.604\ \text{in.}^2$$

$$D_b = \frac{A_b 4}{\pi}$$
$$D_b = 0.769\ \text{in.}^2$$

∴ Need 1″ bolts, 5 ft c-c, 7 ft long @ 60,000 psi ⟵

36. Given: Problem 7 data and yieldable arches
Find: Yieldable arches support.



Solution:

$$T = pRS, \quad p = \gamma H_b$$
$$= (156)(9.3)(7)(6), \quad \text{try } S = 6'$$
$$T = 6.09(10^4)\ \text{lbf}$$
$$T = \sigma_s A_s$$

$$A_s = \frac{6.09(10^4)}{3.6(10^4)}$$

$$\underline{A_s = 1.69\,\text{sq. in.}}$$

steel strength 36,000 psi ← (9)

set spacing @ 6′

37. Given: An arched back tunnel where depth = 863 m

$C_0 = 98.6$ MPa, $T_0 = 9.86$ MPa

$E = 29.3$ GPa, $\nu = ?$, $G = 12.4$ GPa

$\gamma = 24.7\,\text{kN/m}^3$, $B = 4.3$ m, $H_t = 5.2$ m,

"Moderately blocky & Seamy" wet ground

Find: Steel

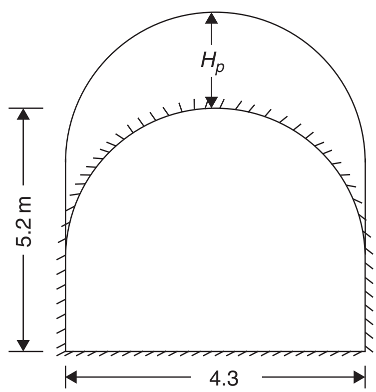
Solution:

Table 3 "C & S"

$$H_p = 0.25B \text{ to } 0.35(B + H_t)$$

$$H_p = 0.25(4.3) \text{ to } 0.35(4.3 + 5.2)$$

i.e. $H_p = 1.08$ to 3.33 m



Use $\underline{H_p = 2.20}$

$W = \gamma B H_p S$, try $S = 2$ m

$= (24.7)(4.3)(2.2)(2)$

then $\underline{W = 467\,\text{kN}}$

$$\frac{W}{B} = \frac{467}{4.3} = \underline{109\,\text{kN/m}} \text{ of tunnel width}$$

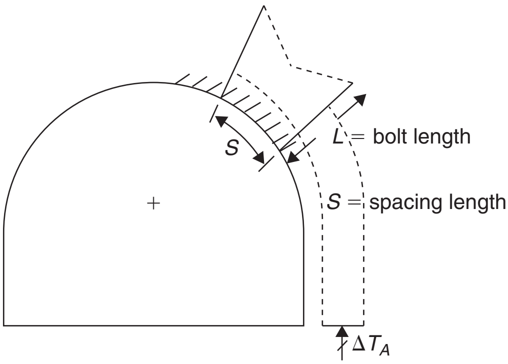
(7,444 lbf/ft), (14.2 ft wide)

Tables "C & S" a 5″ × 5″ H-beam @ 16 lbf/ft will do with blocking pts. @ 40″ max

spacing = 2 m
web depth = 12.7 cm
∴ flange width = 12.7 cm
weight = 234 kN/m
max block pt. spacing = 102 cm. ←

38. Given: Problem 37 data
Find: Square pattern bolt equivalent.

Solution:



$$\text{formula } \Delta T_A = \left(\frac{C_0}{T_0}\right)\left(\frac{F_b}{S^2}\right)(L - S) \quad \text{per m of tunnel length}$$

$$\text{Also } T_s = \left(\frac{W}{2}\right)\left(\frac{1}{S}\right) : \text{ fixed steel set thrust}$$

$$= (467\,\text{kN})\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\underline{T_s = 117\,\text{kN}}$$

$$\underline{\frac{C_0}{T_0} = 10}$$

$F_b = \sigma_Y A_s$ estimate $\sigma_Y = 415$ MPa try 2.54 cm diameter

$$= (415)\left(\frac{\pi}{4}\right)(2.54)^2(10^{-4})$$

$$\underline{F_b = 210\,\text{kN}}$$

$$\therefore 117 = (10)(210)\left(\frac{L - S}{S^2}\right)$$

try $L = 2.5$ m
& $S = 2.2$ m

other combinations possible
2.54 cm diameter bolts
415 MPa σ_Y
2.5 m long
2.2 m spacing square

← bolting

39. Given: Problem 37 data

Find: Yieldable arch support.

Solution:

$T = pRS$ & $T = \sigma_Y A_s$

use mild steel $\sigma_Y = 250$ MPa

estimate: $p = \gamma H_p$

$= (24.7)(2.20)$

$p = 54.3$ kPa

$(250\text{ MPa})(A_s) = (54.3\text{ kPa})(2.15)(S)$

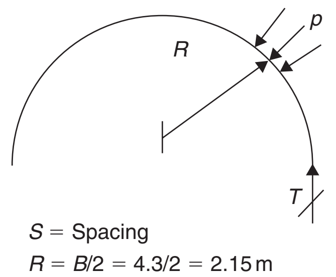
try $S = 2.5$ m then

$A_s = 1.168(10^{-3})\text{ m}^2$

$A_s = 11.8\text{ cm}^2$

$S = 2.5$ m

$\sigma_Y = 250$ MPa



yieldable arches

40. Given: Tunnel, support data.

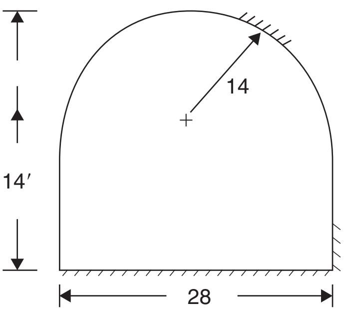
dry, "moderately blocking & seamy"

Find:

(a) Steel sets

(b) T same for yieldable arches

(c) p_b if bolted



Solution:

Use handout for selection (a) Table 3 or 4.2, "moderately blocky & seamy"

$$H_p = 0.25B \text{ to } 0.35(B + H_t)$$
$$= 0.25(28) \text{ to } 0.35(28 + 28)$$
$$H_p = 7\text{ ft to } 19.6\text{ ft}$$
$$H_p(\text{ave}) \approx 13.3\text{ ft}$$

Dry, so use 1/2

$$\therefore H_p = 6.7\text{ ft, say}$$

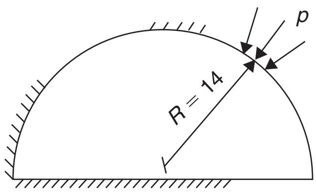
try 11 ft spacing, then assuming $\gamma = 156$ pcf

$$W = \gamma B H_p S$$
$$= (156)(28)(6.7)(4)$$
$$W = 1.171 \times 10^6\text{lbf}$$

$$W' = \frac{W}{B}$$
$$= \frac{1.171(10^5)}{28}$$
$$W' = 4{,}180\text{ lbf/ft of tunnel width}$$

T.1. Handout for continuous ribs.

Smallest in table is 6 × 6 H beam @ 20 lb/ft corresponding to 5,780 lbf/ft of width, should increase spacing but this is the procedure for arches.

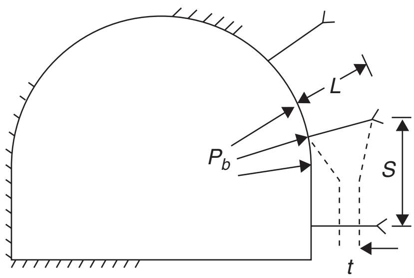


$$
\begin{aligned}
T &= pRS \\
T &= (p)(14)(4) \\
\text{but } p &= \gamma H_p \\
p &= (156)(6.7\,\text{ft}) \\
T &= (156)(6.7)(14)(4) \\
\therefore \underline{\underline{T}} &\underline{\underline{= 58{,}531\,\text{lbf}}}
\end{aligned}
$$

(b) ←

(c) for bolts

$$
\begin{aligned}
\Delta T_A &= \left(\frac{C_0}{T_0}\right) P_b t l_r \\
\Delta T_A &= (10)(P_b)(t)(4) \\
\text{but } \Delta T_A &= 58{,}531\,\text{lbf} \\
p_b &= \frac{58{,}531}{(10)(4)(10)} \\
p_b &= 146\,\text{psf} \\
&\approx (1\,\text{psi})
\end{aligned}
$$



(which is low because of the relatively close set spacing compared with rock head H_p.)

$t = L - S$

$S = l_r$ (set spacing assume square)

$\dfrac{C_0}{T_0} = 10$: assume for rock

$L = ?$: select

estimate $L = 12$

then $t = 14 - 4$

$\therefore t = 10$ ft.

41. Given: An arched back tunnel 8.5 m × 8.5 m is dry moderately blocky and seamy ground.

Find:

(a) Fixed steel sets

(b) Yieldable arches alternative

(c) Bolting alternative.

Solution:

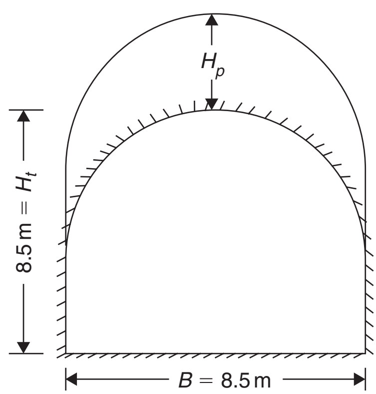


Table 3 "C & S", Table 4.2 text

$$
\begin{aligned}
H_p &= 0.25(B) \text{ to } 0.35(B + H_t) \\
&= 0.25(8.5) \text{ to } 0.35(8.5 + 8.5) \\
H_p &= 2.71\,\text{m to } 5.95\,\text{m}
\end{aligned}
$$

H_p (average) = 4.33 m
DRY! ∴ use 1/2
$\underline{\underline{H_p = 2.17\,\text{m}}}$

estimate $\gamma = 25\,\text{kN/m}^3$
try 2 m spacing

$$W = \gamma SBH_p$$
$$W = (25)(2)(8.5)(2.17)$$
$$\underline{W = 922\,\text{kN}}$$
$$\underline{\frac{W}{B} = 108\,\text{kN/m of tunnel width}}$$

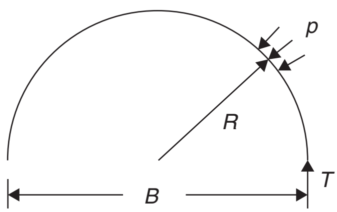
Table 1 "C & S"

(7,397 lb/ft) @ (28 ft wide) an 8″ × 6 1/2″ wide flange @ 24 lb/ft will do with 54 in. (max. blocking point spacing)

spacing = 2 m
web depth = 20.3 cm
flange width = 16.5 cm
weight = 350 kN/m

(b) Yieldable arches

$$R = \frac{B}{2} = \frac{8.5}{2} = \underline{4.25\,\text{m}}$$
$$p = \gamma H_p$$
$$p \cong (25)(2.17)$$
$$p = \underline{54.3\,\text{GPa}}$$
$$T = pRS \quad \& \quad T = \sigma_Y A_s \quad \text{try } S = 2\,\text{m}$$
$$T = (54.3)(4.25)(2)$$
$$\underline{\underline{T = 462\,\text{kN} \quad \text{at 2 m spacing}}}$$



T(b) ←

(c) for bolts

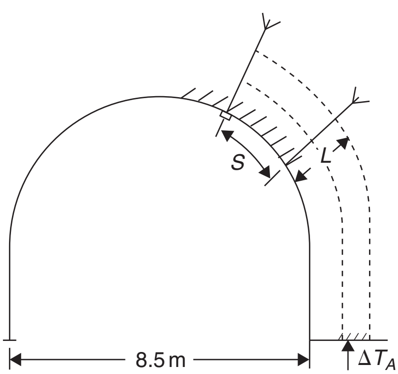
equivalent $\Delta T_A = T_s$

$$\text{and } \Delta T_A = \left(\frac{C_0}{T_0}\right)\left(\frac{F_b}{S_2}\right)(L - S)$$

$$\text{estimate } \frac{C_0}{T_0} = 10$$

given $S = 2\,\text{m}$

$$p = \frac{F_b}{S^2}$$



$$T_0 = \frac{T}{S} = \frac{462}{2} = 231\,\text{kN/m}$$

$231\,\text{kN/m} = (10)(p)(L-2)$
$\therefore$ if $L = 3$ m then $p = 23.1\,\text{kN/m}^2$

$$\therefore \text{at } \left.\begin{aligned} L &= 3 \\ S &= 2 \end{aligned}\right\} \begin{aligned} &\underline{\underline{P = 23.1\,\text{kPa}}} \\ &(\text{low}) \approx 3.4\,\text{psi} \end{aligned}$$

(c) ←

Rock Mass Classification Schemes, RQD

42. (a) The objective of a rock mass classification scheme is to allow for estimation of rock support system requirement. The main components are (1) rock strength (2) joint strength orientation and spacing (3) water pressure.

RQD is important because it correlates with the values output from the scheme and thus allows for comparisons. Main differences are in numerical weighting. RMR ranges to 100%; Q ranges over orders of magnitude, but RMR and Q correlate well. Q does include a stress factor that RMR does not.

(b)

2.4	5.3	8.2	14.2	7.7	25.3	29.3	34.3	36.9	47.5	54.8
2.9	2.9	6.0	3.5	7.6	4.0	5.0	2.6	10.6	7.3	

Note: $\Sigma l > 3.5 = 49.2 \quad \Sigma l = 60.0 \quad RQD = \dfrac{49.2}{60} = 0.82 \quad 82\%$ ← RQD

$$\left(\frac{L}{D}\right)_1 = \frac{4''}{2.125} = \left(\frac{L}{D}\right)_2 \qquad L_2 = \left(\frac{4}{2.125}\right)(1.875) = 3.5''$$

43. (a) The main objectives of rock classification schemes are to determine support requirements for tunnels. RQD is important because it indicates the intensity or spacing of joints or fractures. The main features are: the intact rock strength, joint strength and water,

The differences are slight, RMR and Q are well-correlated.

(b) 1.5 m of NQ-core ($D = 1.875$ in.) $= 4.75$ cm

$$NX\text{:} \left(\frac{L}{D}\right)_1 NQ = \left(\frac{L}{D}\right)_2 \text{ [correction for core size]}$$

$$\left(\frac{4.0}{2.125}\right) = \frac{L_2}{1.875} \qquad \therefore L_2 = 3.53\,\text{in}$$

<u>or $L_2 = 9.0$ cm.</u>

<u>if $L > L_2$ keep for RQD</u>

$0.0 > 8.4$

$8.4 > 9.9$

$18.3 > 15.2$

$33.5 > 6.4$

$39.9 > 26.9$
$66.8 > 10.2$
$77.0 > 13.7$
$90.7 > 4.8$
$95.5 > 15.0$
$110.5 > 23.6$
$134.1 > 3.6$
$137.7 > 12.3$
$\underline{150.0}$

$\Sigma \approx 150, \Sigma = 126.6$

$$RQD = \left(\frac{126.6}{150}\right) 100$$
$$\underline{RQD = 84.4\%} \quad \longleftarrow$$

44. (a) Objective is primarily to determine and quantity need for tunnel support
Main components are:
    rock strength
    joint strength
    water
Main difficulties are:
    method of assessing components.

(b) Large, stiff sets will limit tunnel wall displacements to a tiny amount, provided they hold; flexible light-weight support is used when ground squeezes and cannot be hold.

5 Entries in Stratified Ground

Naturally Supported Roof

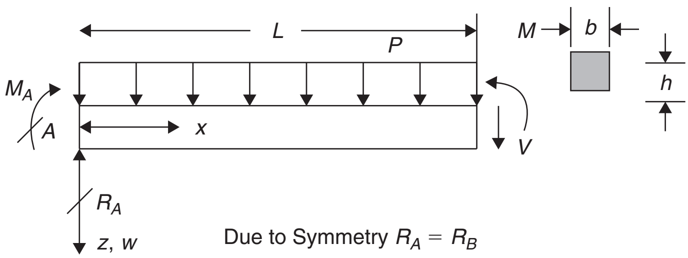
1. Given: 'Weightless' beam, built in ends Thickness h, Breadth b, Span L, Uniform load P.

   Sign convention tension positive. x axis left, z axis down, y axis out of page, CCW moment positive.

   Downward shear force is positive.

   Find: An expression for shear force V, Bending Moment M, Sag w
   Plot shear & Moment diagrams.

   Solution:



$$\sum F_z = 0$$

$$0 = 2R - PbL$$

$$\therefore R_A - R_B = \frac{PbL}{2}$$

$$\sum F_z = 0$$

$$0 = R_A - Pbx - V$$

$$\underline{\underline{V = \frac{PbL}{2} - Pbx.}} \longleftarrow$$

$$\sum_A M = 0$$

$$0 = M_A + \frac{Pbx^2}{2} + V_x - M$$

$$M = M_A + \frac{Pbx^2}{2} + \frac{PbL}{2}x - Pbx^2$$

$$M = M_A + Pb\left(\frac{Lx}{2} - \frac{x^2}{2}\right)$$

Find: M_A

$$\text{known } \frac{d^2w}{dx^2} = \frac{-M}{EI}$$
$$= \left(-\frac{1}{EI}\right)\left[M_A + P_b\left(\frac{Lx}{2} - \frac{x^2}{2}\right)\right]$$
$$\frac{dw}{dx} = \left(-\frac{1}{EI}\right)\left[M_A x + P_b\left(\frac{Lx^2}{4} - \frac{x^3}{6}\right)\right] + C_1$$

Boundary condition (BC)

$$\left.\frac{dw}{dx}\right|_{x=0} = 0 \quad \therefore C_1 = 0 \text{ \{Note: “Built-in” means no end plane rotation\}}$$

$$w = \left(\frac{-1}{EI}\right)\left[M_A\frac{x^2}{2} + Pb\left(\frac{Lx^3}{12} - \frac{x^4}{24}\right)\right] + C_2$$

$$\text{BC: } w|_{x=0}^{=0} \quad \therefore\ C_2 = 0$$

$$\text{BC: } w|_{x=L}^{=0} \quad 0 = \left(-\frac{1}{EI}\right)\left[M_A\frac{L^2}{2} + Pb\left(\frac{L^4}{12} - \frac{L^4}{24}\right)\right]$$

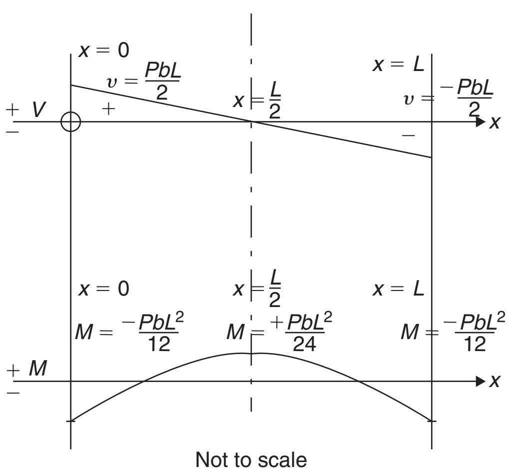
$$\underline{M_A = -\frac{PbL^2}{12}}$$

$$\underline{\underline{M = Pb\left(\frac{-L^2}{12} + \frac{Lx}{2} - \frac{x^2}{2}\right)}} \longleftarrow$$

$$w = \left(\frac{Pb}{EI}\right)\left(\frac{L^2x^2}{24} - \frac{Lx^3}{12} + \frac{x^4}{24}\right) \longleftarrow$$

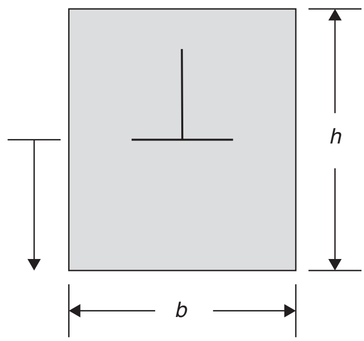
$$w|_{x=L/2} = w_{\max}$$

$$\underline{\underline{w_{\max} = \frac{PbL^4}{384EI}}}$$



2. Given: Roof beam with built in ends.
Show: Maximum tension σ_t occurs at the top of the beam at the beam ends and is given by the expression $[(1/2)(PL^2)]/h^2$

Solution:



$$\sigma = \frac{Mz}{I};$$
$$I = \frac{bh^3}{12} \quad \text{(rectangular section)}$$
$$z = -\frac{h}{2} \quad \text{top of beam.}$$
$$M = Pb\left(-\frac{L^2}{12} + \frac{Lx}{2} - \frac{x^2}{2}\right)$$
$$\frac{dM}{dx} = \frac{PbL}{2} - Pbx = 0 \quad x = \frac{L}{2} \text{ is a point of relative max/min, } M = \frac{PbL^2}{24}$$

Examine end points:

$$\text{at } x = 0;\ \ M = Pb\left(\frac{-L^2}{12}\right)$$
$$\text{at } x = L;\ \ M = Pb\left(\frac{-L^2}{12}\right)$$
$$\Rightarrow \sigma_{\max} = \frac{\left(\frac{-PbL^2}{12}\right)\left(\frac{-h}{2}\right)}{\left(\frac{bh^3}{12}\right)} = \frac{\left(\frac{1}{2}\right)(PL^2)}{h^2}$$
$$\underline{\underline{\sigma_t = \frac{PL^2}{2h^2}}} \quad \longleftarrow$$

3. Given: Roof beam with built in ends (rectangular section).
Show: Maximum sag $w_{\max}$ occurs at $L/2$ and is given by the expression $\left(\frac{1}{32}\right)\left(\frac{PL^4}{Eh^3}\right)$.

Solution:

From Problem 1

$$w = \left(\frac{Pb}{EI}\right)\left(\frac{L^2x^2}{24} - \frac{Lx^3}{12} + \frac{x^4}{24}\right)$$
$$\frac{dw}{dx} = \left(\frac{L^2x}{12} - \frac{Lx^2}{4} + \frac{x^3}{6}\right)$$
$$\left.\frac{dw}{dx}\right|_{x=\frac{L}{2}} = \frac{L^3}{24} - \frac{L^3}{16} + \frac{L^3}{48}$$
$$= L^3\left(\frac{1}{24} - \frac{1}{16} + \frac{1}{48}\right) = 0$$

w_{max} at $\dfrac{L}{2}$

$w = \dfrac{L}{2}$, $w_{max} = \frac{PbL^4}{384EI}$

but $I = \dfrac{bh^3}{12}$

$$w_{max} = \frac{PbL^4}{384E\left(\frac{bh^3}{12}\right)}$$

$$w_{max} = \frac{PL^4}{32Eh^3}. \longleftarrow$$

4. Given: $F_t = \dfrac{T_0}{\sigma_t}$

   Find: L_{max} for built-in ends case.

   Solution:

   From Problem 2 $\sigma_t = \dfrac{PL^2}{2h^2}$

$$F_t = \frac{T_0}{\left(\frac{PL^2}{2h^2}\right)} = \frac{2h^2 T_0}{PL^2}$$

$$\therefore \quad L = \left(\frac{2h^2 T_0}{PF_t}\right)^{1/2} \longleftarrow$$

   L_{max} at F_t minimum $= F_t = 1$

$$L_{max} = \left(\frac{2h^2 T_0}{P}\right)^{1/2} \longleftarrow$$

5. Given: $H_0 = 12'$, $W_0 = 22' = L$, $H = 950'$, $h = 2.25'$
   $E = 3.7 \times 10^6$ psi, $\nu = 0.28$ $\gamma = 134$ pcf $T_0 = 690$ psi

   Find: σ_t, w_{max}, F_t for SS, BI.

   Solution:

   (a) SS;

$$\sigma_t = \frac{3}{4}\frac{PL^2}{h^2}; \quad P = \gamma h$$

$$= \left(\frac{3}{4}\right)(134)(2.25)\left(\frac{22}{2.25}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{\text{lbf}}{\text{ft}^3}\right)(\text{ft})\left(\frac{\text{ft}}{\text{ft}}\right)^2$$

$$= \frac{\text{lbf}}{\text{ft}^2} \times \frac{1}{144} = \frac{\text{lbf}}{\text{in}^2} = \text{psi}$$

$$= 21618.67 \times \frac{1}{144} = 150.13 \text{ psi}$$

$$\underline{\sigma_t = 150 \text{ psi}}$$

$$w_{\max} = \left(\frac{5}{32}\right)\left(\frac{PL^4}{Eh^3}\right)$$

$$= \left(\frac{5}{32}\right)\left(\frac{134}{144}\right)(2.25)\frac{(22)^4}{(2.25)^3}(12)\left(\frac{1}{3.7 \times 10^{+6}}\right)$$

$$\underline{w_{\max} = 0.0218 \text{ in}}$$

$$F_t = \frac{T_0}{\sigma_t} = \frac{690}{150} = 4.6$$

$$\underline{\underline{F_t = 4.6}} \qquad \longleftarrow$$

(b) BI;

$$\sigma_t = \frac{1}{2}\frac{PL^2}{h^2}$$

$$= \left(\frac{1}{2}\right)\left(\frac{134}{144}\right)(2.25)\left(\frac{22}{2.25}\right)^2$$

$$\underline{\sigma_t = 100 \text{ psi}} \qquad \longleftarrow$$

$$w_{\max}(\text{BI}) = \frac{1}{5} w_{\max}(\text{SS})$$

$$= \frac{0.0218}{5}$$

$$w_{\max}(\text{BI}) = 0.0043 \qquad \longleftarrow$$

$$F_t = \frac{T_0}{\sigma_t} = \frac{690}{100} = 6.9$$

$$\underline{\underline{F_t = 6.9}} \qquad \longleftarrow$$

6. Given: High horizontal stress σ_h reduces bending $\sigma_b = 0$. σ_t from Problem 5a, SS case

   Find: σ_h.

   Solution:

$$\sigma_b = \sigma_h + \sigma_t$$

$$\text{Given } \sigma_b = 0.$$

$$\therefore \sigma_h = -\sigma_t$$

$$\sigma_h = -150 \text{ psi (compression)}$$

Tension + sign convention. $\longleftarrow$

7. Given: Entry: 3.66 m high × 6.71 m wide, depth = 290 m, immediate roof layer separation, $h = 0.69$ m, $E = 25.52$ GPa, $\nu = 0.28$, $\gamma = 21.2$ kN/m^3, $T_0 = 4.7$ MPa

   Find: $\sigma_t, w_{\max}, FS_t$
   (a) simply supported,
   (b) built-in.

   Solution:

   (a) SS

$$\sigma_t = \left(\frac{3}{4}\right)\frac{PL^2}{h^2}\ ;\ P = \gamma h$$

$$= \left(\frac{3}{4}\right)(21.2\,\text{kN/m}^3)\frac{(6.71\,\text{m})^2}{(0.69\,\text{m})^2} \qquad \longleftarrow \sigma_t$$

$$\underline{\underline{\sigma_t = 1.04\ \text{Mpa}}}$$

$$FS_t = \frac{T_0}{\sigma_t}$$

$$= \frac{4.7\,\text{MPa}}{1.04} \qquad \longleftarrow FS_1$$

$$\underline{\underline{FS_t = 4.5}}$$

$$w_{\max} = \left(\frac{5}{32}\right)\left(\frac{PL^4}{Eh^3}\right)$$

$$= \frac{\left(\frac{5}{32}\right)(21.2\ \text{kN/m}^3)(0.69\,\text{m})(6.71)^4(100)}{(25.52\,\text{GPa})(0.69)^3}$$

$$\underline{\underline{w_{\max} = 0.055\,\text{cm}}} \qquad \longleftarrow w_{\max}$$

   (b) BI

$$\sigma_t = \left(\frac{1}{2}\right)P\frac{L^2}{h^2}$$

$$= \left(\frac{1}{2}\right)(21.2\,\text{kN/m}^3)(0.69)\left(\frac{6.71}{0.69}\right)^2 \qquad \longleftarrow \sigma_t$$

$$\underline{\underline{\sigma_t = 0.692\,\text{MPa}}}$$

$$FS_t = \frac{T_0}{\sigma_t}$$

$$= \frac{4.7\,\text{MPa}}{0.692\,\text{MPa}}$$

$$\underline{\underline{FS_t = 6.79}} \qquad \longleftarrow FS_t$$

$$w_{\max}(\text{BI}) = \left(\frac{1}{32}\right)\frac{PL^4}{Eh^3}$$

$$= \left(\frac{1}{5}\right) w_{\max}(\text{SS})$$

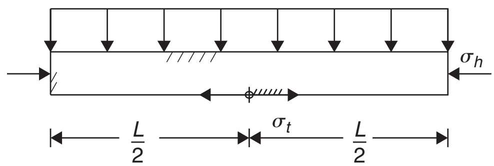
$$w_{\max}(\text{BI}) = \left(\frac{1}{5}\right)(0.055)$$

$$\underline{\underline{w_{\max}(\text{BI}) = 0.011\,\text{cm}}}$$

$\leftarrow w_{\max}$

8. Given: Problem 7 data, $\sigma_t = 1.04$ MPa

Find: σ_h to reduce tension to 0.



Solution:

Need:

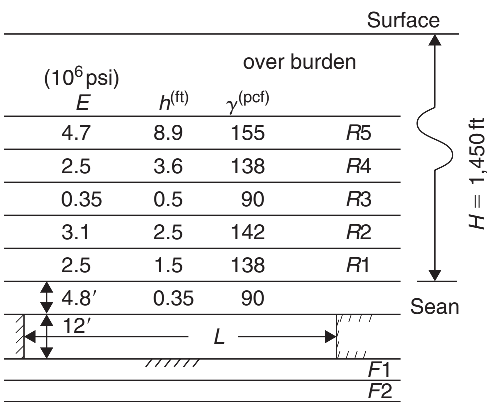
$$0 = \sigma_t + \sigma_h$$

$$\underline{\underline{\therefore \sigma_h = -1.04\,\text{MPa [compression } (-)]}}$$

$\leftarrow \sigma_h$

9. Given: Geologic column & properties, depth = 1450 ft, dip = 8°, mining height 12 ft.

Find: Bed separations.



Solution:

Need $P_i(n)$'s

$$P_1(1) = \gamma_1 h_1 = \left(\frac{90}{144}\right)(4.8) = \underline{\underline{3.00\,\text{psi}}}$$

$$P_1(2) = E_1 h_1^3\left(\frac{\gamma_1 h_1 + \gamma_2 h_2}{E_1 h_1^3 + E_2 h_2^3}\right)$$

$$= \frac{(0.35)(10^6)(4.8)^3}{144}\left[\frac{(90)(4.8) + 138(1.5)}{0.35(10^6)4.8^3 + 2.5(10^6)(1.5)^3}\right]$$

$$= (0.2689)\frac{639}{47.1}$$

$$\underline{\underline{P_1(2) = 3.65\ \text{psi}}}$$

$$P_1(3) = (0.2689)\left(\frac{639 + \gamma_3 h_3}{47.1 + E_3 h_3^3}\right)$$

$$= (0.2689)\left(\frac{639 + 142(2.5)}{47.1 + 3.1(2.5)^3}\right)$$

$$P_1(3) = 0.2689\left(\frac{994}{95.5}\right)$$

$$P_1(3) = 2.80\ \text{psi}$$

$$2.80 = P_1(3) < P_1(2) = 3.64\ \text{psi}$$

$\therefore$ Beds separate between shale ($R1$) and layered sandstone ($R2$)

Restart analysis with $R2$ as #1 layer

$$P_1'(1) = \gamma_1 h_1$$

$$= \left(\frac{42}{144}\right)(2.57)$$

$$\underline{\underline{P_1'(1) = 2.47\ \text{psi}}}$$

$$P_1'(2) = (3.1)(10^6)(2.5)^3 \frac{142(2.5) + 90(0.5)}{3.1(10^6)2.5^3 + 0.35(0.5)^3 10^6}$$

$$= \frac{(3.1)(2.5)^3}{144}\left(\frac{400}{48.5}\right)$$

$$\underline{\underline{P_1'(2) = 400\ \text{psf}\ \ 2.77\ \text{psi}}}$$

$$P_1'(3) = \left(\frac{48.4}{144}\right)\frac{400 + 138(3.6)}{48.5 + 2.5(3.6)^3}$$

$$P_1'(3) = \left(\frac{48.4}{144}\right)\frac{897}{165}$$

$$P_1'(3) = 1.83\ \text{psi}$$

$P_1'(3) < P_1'(2)$ $\therefore$ Beds separate between $R3$ & $R4$ ←

Restart analysis with R4 as #1 layer

$$P_1''(1) = \gamma_4 h_4$$

$$= \left(\frac{138}{144}\right)(3.6)$$

$$\underline{P_1''(1) = 3.45\ \text{psi}}$$

$$P'_1(2) = \frac{2.5(10^6)(3.6)^3}{144} \frac{(138)(3.6) + 155(8.9)}{2.5(10^6)(3.6)^3 + 4.7(8.9)^3 10^6}$$

$$= (0.81)\frac{1{,}876}{3{,}430}$$

$$P''_1(2) = 0.44\,\text{psi} \therefore \underline{\underline{\text{Beds separate between } R4 \text{ \& } R5}}$$ ⟵

10. Given: Problem 9 conditions and all layers must be safe i.e. $= FS \geq 1$.
Find: $L_{max}(\text{min})$, maximum roof span with $FS \geq 1.0$.

Solution:

$$FS_t(i) = \frac{T_0(i)}{\sigma_t(i)}$$

will assume SS (simply supported ends) so that $\sigma_t(i) = \dfrac{3}{4}P_i\dfrac{L^2}{h_i^2}$

Least $FS_t(i)$ occurs when $\dfrac{T_0(i)h_i^2}{p_i}$ is least.

- For first two roof layers (coal & R_1)

$$\underline{\underline{P_1(2) = 3.64\,\text{psi}}}$$

$$P_2(2) = P_1(2)\left(\frac{E_2h_2^3}{E_1h_1^3}\right)$$

$$= (3.64)\frac{(2.5)(10^6)(1.5)^3}{(0.35)(10^6)(4.8)^3}$$

$$\underline{\underline{P_2(2) = 0.79\,\text{psi}}}$$

$$\text{Coal: } \frac{T_0(1)h_1^2}{P_1(2)} = \frac{(25.0)(4.8)^2}{(3.64)} = \underline{\underline{1.582(10^2)\,\text{ft}^2}}$$

$$\text{Shale } R1\text{: } \frac{T_0(2)h_2^2}{P_2(2)} = \frac{(400)(1.5)^2}{(0.79)} = \underline{\underline{1.389(10^3)\,\text{ft}^2}}$$

$$P'_1(2) = 4.00\,\text{psi}$$

$$P'_2(2) = \frac{0.35(10^6)(0.5)^3}{3.1(10^6)(2.5)^3}(4.00)$$

$$P'_2(2) = 0.036\,\text{psi}$$

$$\text{Laminated SS } R2\text{: } \frac{T_0h^2}{P'_1(2)} = \frac{(620)(2.5)^2}{4.00} = \underline{\underline{969\,\text{ft}^2}}$$

$$\text{Coal } R3\text{: } \frac{T_0h^2}{P'_2(2)} = \frac{(2.5)(0.5)^2}{0.036} = \underline{\underline{174\,\text{ft}^2}}$$

$$\text{Shale } R4\text{: } \frac{T_0h^2}{P''_1(1)} = \frac{(400)(3.6)^2}{3.45} = \underline{\underline{1{,}503\,\text{ft}^2}}$$

$$\text{SS } R5 : \frac{T_0 h^2}{P_1''(1)} = \frac{(10{,}300)(8.9)^2}{\left(\frac{155}{144}\right)(8.9)} = \underline{\underline{8.516(10^4)\,\text{ft}^2}}$$

$$FS_{\min} = \frac{T_0 h^2}{\frac{3}{4} P_i L^2} = \frac{4}{3}\frac{1}{L^2}\left(\frac{T_0 h}{P_i}\right)_{\min}$$

$$\left(\frac{T_0 h}{P_i}\right)_{\min} \text{ occurs in the roof coal}$$

$$L^2 = \frac{4}{3}\left(\frac{T_0 h^2}{P_i}\right)\left(\frac{1}{FS}\right) \quad FS = 1 \text{ for max}$$

$$= \left(\frac{4}{3}\right)(158)$$

$$\therefore \underline{\underline{L = 14.5\,\text{ft}}} \qquad \longleftarrow L_{\max}$$

Bolted Roof

12. Given: Sag $w_{\max}$ from 5(b) (BI), assume bolt tension σ_b = Uniformly distributed acting upwards to reduce the sag $w_{\max}$ to zero.

    Find: $\sigma_b|_{w_{\max}=0}$

    Solution:

$$w_{\max}(\text{BI}) = \frac{1}{32}\frac{PL^4}{Eh^3}$$

$$\text{For } w_{\max} = 0;\ P = 0.$$

$$P = \gamma h - \sigma_b = 0$$

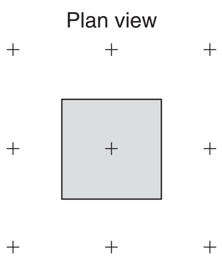
$$\sigma_b = \gamma h = \left(\frac{135}{144}\right)(2.25)$$

$$\underline{\underline{\sigma_b = 2.11\,\text{psi}}} \qquad \longleftarrow$$

13. Given: Bolt force F_b = 16,400 lbf spaced on 5′ centers, γ = 158 pcf

    Find: $\sigma_b, h_{eq.}$

    Solution:



$$A = (5)(5) = 25\,\text{ft}^2$$

$$\sigma_b = \frac{F}{A} = \frac{16{,}400}{25}\frac{1}{144} = 4.56\,\text{psi}$$

$$\underline{\underline{\sigma_b = 4.56\,\text{psi}}} \qquad \longleftarrow$$

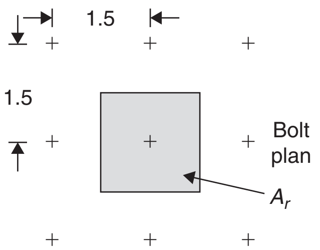
$$\sigma_b = \gamma h_{eq.}$$

$$h_{eq} = \frac{\sigma_b}{\gamma} = \frac{4.56(144)}{158}$$

$$\underline{\underline{h_{eq} = 4.16\,\text{ft}}} \qquad \longleftarrow$$

14. Given: Rock bolt load = 73.5 kN bolts spaced 1.5 m c-c
Find: Bolting pressure, equivalent layer h.

Solution:



$F_b = P_b A_r$

A_r = bolt area of influence

$$P_b = \frac{73.5\,(\text{kN})}{(1.5)(1.5)}$$

$\underline{\underline{P_b = 32.7\,\text{kPa}}}$ ← P_b

$P_b = \gamma h$ assume $\gamma = 23.5\,\text{kN/m}^3$

$$h = \frac{32.7\,\text{kPa}}{23.5\,\text{kPa/m}}$$

$h = 1.392\,\text{m}$

$\underline{\underline{h = 139\,\text{cm}}}$ ← h

15. Given: Sketch. Stratified mine roof data and 5/8″ diameter bolts on 4′ centers or 3/4″ diameter bolts on 5′ centers, High Strength, rib spacing is half.
Find:
(a) bolt safety factors from dead weight approach.
(b) best plan: 5/8″ @ 4′ or 3/4″ @ 5′

Solution:

Dead weight approach; for equilibrium weight of rock = Bolt Load, γ (volume) = F_b

By inspection of the sketch, the shale beds need support. The thick sandstone (5.) will be used as an anchor stratum. Note that the decreasing thicknesses of the shales which have the same modulus implies no bed separation within the shales, but rather at the 4–5 interface.

Volume $= l^2 \sum_{i=1}^{4} h_i$
where l = bolt spacing
Note: For safety Bolt strength > Bolt load.
@ 4′ centers

$$V = \frac{(4)^2(7 + 8 + 10 + 12)}{12}$$

$\underline{V = 49.33\,\text{ft}^3}$

@ 5′ centers

$$V = (5)^2\left(\frac{37}{12}\right)$$

$\underline{V = 77.08\,\text{ft}^3}$

@ 4′ centers

$$W = (135)(49.33)$$
$$\underline{W = 6{,}660\,\text{lbs}}$$

@ 5′ centers

$$W = (135)(77.08)$$
$$\underline{W = 10{,}406\,\text{lbs}}$$

Bolt forces:

Strength −5/8″ bolt, High Strength, Grade 55: $F_y = 12{,}400$ lbs.

−3/4″ bolt, High strength, Grade 55: $F_y = 18{,}400$ lbs.

(from Text, Table 3.4, pg 124)
tension to, say, 70% of yield point
@ 4′ centers

$$F_b = (0.7)(12{,}400)$$
$$\underline{F_b = 8{,}680\,\text{lbs}}$$

@ 5′ centers

$$F_b = 0.7(18{,}400)$$
$$\underline{F_b = 12{,}880\,\text{lbs}}$$

Both plans are certainly adequate since at 70% yield, the bolt forces exceed the dead weight load.
Check: Bolt safety factors w.r.t. yield pt.
@ 4′ centers

$$FS_b = \frac{12{,}400}{6{,}660}$$
$$\underline{\underline{FS_b = 1.86}}$$

@ 5′ centers

$$FS_b = \frac{18{,}400}{10{,}406}$$
$$\underline{\underline{FS_b = 1.77}}$$

∴ Both plans adequate and technically close. 3/4″ on 5′ centers would likely be preferable because the fewer bolts installed would more than offset added individual bolt losts.

16. Given: A suspension action by 3/4″ bolts on 5′ centers, bolt tension is 75% of yield load. Sag is reduced 40%.

    Find: Reduction in roof tension.

Solution:

For simply supported ends

$$w(\max) = \frac{5}{32}\frac{PL^4}{Eh^3}$$
$$\sigma(\max) = \frac{3}{4}\frac{PL^2}{h^2}$$
$$\text{Let } \sigma_w/ = \text{stress with bolts}$$
$$w_w/ = \text{sag with bolts}$$
$$\sigma_{w_0}/ = \text{stress without bolts}$$
$$w_{w_0}/ = \text{sag without bolts}$$

$$\text{Then } W_{w/}/w_{w_0/} = \frac{P_{w/}}{P_{w_0}}$$
$$\& \ \sigma_{w/}/\sigma_{w_0} = \frac{P_{w/}}{P_{w_0}}$$
$$\therefore \frac{\sigma_{w/}}{\sigma_{w_0/}} = \frac{w_{w/}}{w_{w_0/}} = 0.4$$

← 40% reduction

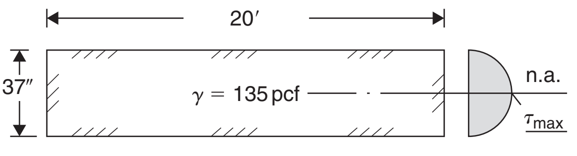
17. Given: Problem 16 data
Find: Safety factor ratio $w/$ to $w_0/$

Solution:

$$FS = \frac{T_0}{\sigma}$$
$$\frac{FS_{w/}}{FS_{w_0/}} = \frac{\sigma_{w_0/}}{\sigma_{w/}}$$
$$\text{But } \frac{\sigma_{w_0}}{\sigma_w} = \frac{1}{0.4}$$
$$\therefore \frac{FS_{w/}}{FS_{w_0/}} = 2.5$$

18. Given: Clamped shale beds from problem 1 that act as a single beam, friction coefficient between shales $\mu = 0.35$
Find:
(a) Maximum shear in monolithic beam (clamped unit)
(b) Bolting pressure estimate needed for clamping.

Solution:

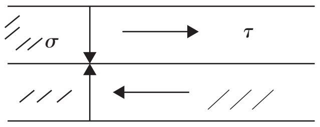


Beam analysis assuming simply supported ends:

$$\tau_{\max} = \frac{3}{4}\frac{PL}{h}; P = \gamma h$$
$$= \left(\frac{3}{4}\right)\left[\frac{(135)\left(\frac{37}{12}\right)(20)}{\left(\frac{37}{12}\right)(144)}\right]$$
$$\underline{\underline{\tau_{\max} = 14.06\,\text{psi}}}$$

← $\tau_{\max}$ in fully clamped unit

(occurs on neutral axis = n.a.)
Bolting pressure?



$$\tau_{\text{friction}} = \mu\sigma_b$$
$$\sigma_b = \frac{\tau}{\mu}$$
$$= \frac{14}{0.35}$$
$$\underline{\underline{\sigma_b = 40.2\,\text{psi}}}$$

← bolting pressure required

(high, but not beyond reason, clamping would mobilize beam building mechanism).

19. Given: Roof truss in the sketch at spacing S and tension T,
Find: Equivalent distributed load in the vertical direction.

Solution:

Equilibrium of truss corner:

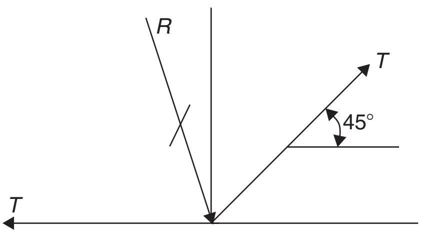
$\Sigma F = 0 = T + R_h - T\cos 45$

hor. dir

$\Sigma F = 0 = T\sin 45 - R_v$
vert. dir

$\therefore R_v = T\sin 45$

$\therefore$ Total vertical Truss Load $= 2R_v$



$$T_v = 2R_v$$
$$T_v = 2T\sin 45$$

The truss vertical force is distributed over the area

$$A = LS$$

where $L =$ the roof span.

The bolting pressure is then

$$P_b = \frac{T_v}{A}$$

$$\underline{\underline{P_b = \frac{\sqrt{2}T}{LS}}} \quad \xleftarrow[\text{bolting pressure}]{\text{equivalent}}$$

20. Given: Laminated shale truss in the sketch.
Find: S and T such that $P_{w/} = 0.65\ P_{w_0/}$

Solution:

Apparent unit weight formula
$h\gamma_a = P$ (i.e. $\gamma_a = P/h$)

$$\text{try1: } P(1,n) = \frac{E_1 h_1^3(\Sigma_1^a h_i\gamma_i + P_g - P_b)}{\Sigma_1^n E_j h_j^3}$$

Uniform thickness @ 4″ same E's, γ's

$$P(1,n) = \frac{h^3(\gamma\Sigma h_j - P_b)}{\Sigma_j^n h_j^3}$$

$$= \left(\frac{h^3}{h^3}\right)\frac{[\gamma(6') - P_b]}{\left(\frac{72}{4}\right)}$$

$$= \frac{(6)(130)}{18} = \frac{P_b}{18}$$

Since layers are identical:

$$\underline{\underline{P(i,n) = 43.3 - \frac{P_b}{18}}}$$

$$\gamma(i,n) = \frac{43.3}{\left(\frac{4}{12}\right)}$$

$$\underline{\underline{\gamma(i,n) = 130\,\text{pcf}}} \quad \text{check}$$

(identical strata – just touch)

$$\gamma_a(w/) = 0.65\ \gamma_a(w_{0/})$$
$$= 0.65(130)$$
$$\gamma_a(w/) = 84.5\,\text{pcf}$$

$$P(i,n)w/ = \gamma_a(w/)h$$
$$= (84.5)\left(\frac{4}{12}\right)$$
$$\underline{P(i,n)w/ = 28.2\ \text{psf}}$$
$$P(i,n)w/ = 43 - \frac{P_b}{18}$$
$$\frac{P_b}{18} = 43.0 - 28.2$$
$$\underline{\underline{P_b = 266.4\ \text{psf}}}$$
$$\underline{P_b = 1.85\ \text{psi}}$$

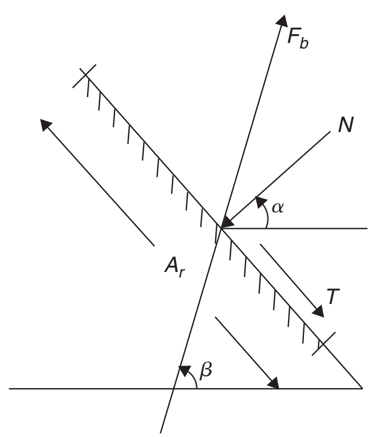
From Problem 6.

$$P_b = \frac{\sqrt{2}T}{LS}$$
$$\therefore \frac{T}{S} = \left(\frac{L}{\sqrt{2}}\right)(266.4),\ L = 20'.$$
$$\frac{T}{S} = \underline{\underline{3{,}768\ \text{lbs/ft of entry.}}}$$

Try: $\underline{\underline{S = 4' \quad T = 15{,}070\ \text{lbs}}}$
$\underline{\underline{S = 8' \quad T = 30{,}140\ \text{lbs}}}$

Note: Table 3.4, p 124: Grade 55, high strength $-\frac{3}{4}''$ diam.
Yield load = 18,400 lbs
So we are in range.

21. Given: Sketch of "bolting a joint".
Find: Optimum bolting angle.



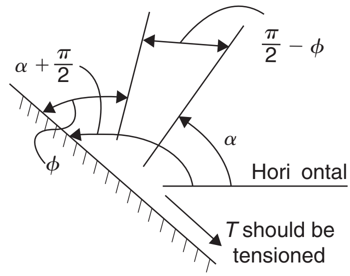
Solution:

Optimum means greatest increase in *FS*.
By definition:
FS = strength/stress
With respect to shear on the joint
$\tau = \sigma \tan\phi + C$, strength rock
Must add resistance from bolting τ_b multiply by area to convert stresses to forces

$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}}$$

Resistance: $R = T_f + T_b$
(uphill)

$T_f = N \tan\phi + K + T_n \tan\phi$
$T_b = F_b \sin(\beta - \alpha)$
Driving $= T_d$



$$FS = \frac{N\tan\phi + K + T_n\tan\phi + F_b\sin(\beta-\alpha)}{T}$$

$$FS = \frac{N\tan\phi + K}{T} + \frac{F_b[\tan\phi\cos(\beta-\alpha) + \sin(\beta-\alpha)]}{T}$$

Optimum β Maximize FS

$$FS = FS^\circ + \frac{F_b\sin(\beta-\alpha+\phi)}{T\cos\phi}$$

This occurs when $\sin(\beta - \alpha + \phi) = 1$

$$\text{i.e. } \underline{\underline{\beta_{\text{opt}} = \frac{\pi}{2} + \alpha - \phi}}$$

Note: The bolt should be tensioned by impending slip on the joint.

22. Given: Joint, bolt sketch & Problem 21.
$c_j = 0$ $\phi_j = 35°$ $F_b = 12{,}000$ lbs $A_r = 4$ sq ft. $\sigma = 648$ psi (horizontal) $\alpha = 30°$
Find: $FS_{w0/}$, $FS_{w/}$

Solution:

$$FS_{w0/} = \frac{N\tan\phi + K}{|T|}$$

$$N = \sigma A_r, \quad T = \tau A_r$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$\sigma = \frac{648}{2} + \frac{648}{2}\cos 60$$

$$\underline{\underline{\sigma = 486 \text{ psi}}}$$

$$\tau = -\frac{(\sigma_x - \sigma_y)}{2}\sin\alpha + \tau_{xy}\cos 2\alpha$$

$$\tau = -\frac{648}{2}\sin 60$$

$$\underline{\underline{\tau = -281 \text{ psi}}}$$

$$FS_{w0/} = \frac{(486)(4)(144)\tan 35° + 0}{|-(281)(4)(144)|}$$

$$\underline{\underline{FS_{w0/} = 1.21}} \quad \longleftarrow \text{unbolted joint } FS$$

Bolting at optimum β:

$$FS_{w/} = FS_{w_0} + \frac{Fb}{|T|\cos\phi}$$
$$= 1.21 + \frac{12{,}000}{(281)(4)(144)\cos 35}$$
$$= 1.21 + 0.09$$
$$\underline{\underline{FS_{w/} = 1.30}}$$

bolted joint FS

23. Given: Problem 22 with an untensioned, passive bolt, 7/8″ diameter grouted in the hole

Find: Shear stress in the bolt.

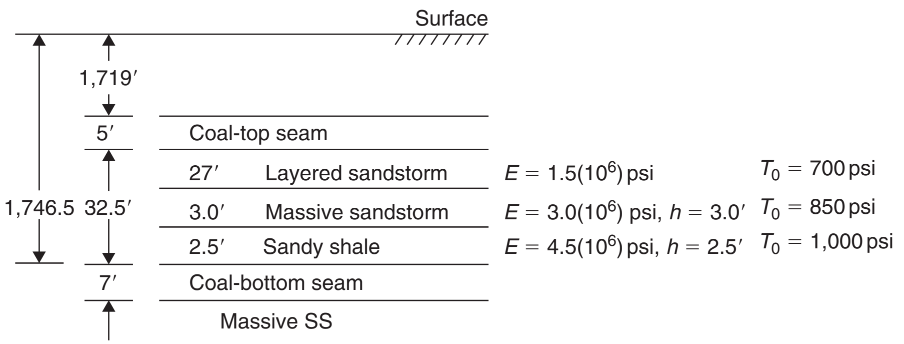
Solution:

Since the $FS_{w0/}$ bolts is greater than 1, no motion occurs and the grouted bolt is not loaded. Stress is zero.

$$\underline{\underline{\tau_b = 0\ \text{psi}}}$$

shear stress in bolt

24. Given: 2-seam coal mine, mining full height, 24-ft wide entries, stratigraphic column, rock properties.



Find:
(1) lower seam Roof1 FS_t (shale)
(2) lower seam Roof2 FS_t (sand)
(3) bolting plan (length, diameter, steel, spacing)

Solution:

Look for bed separation

$$P_1(1) = \gamma_1 h_1$$
$$= (170)(2.5)$$
$$\underline{\underline{P_1(1) = 425\ \text{psf}}}$$

$$P_1(2) = E_1 h_1^3 \left(\frac{\gamma_1 h_1 + \gamma_2 h_2}{E_1 h_1^3 + E_2 h_2^3} \right)$$
$$= (4.5)(10^6)(2.5)^3 \left[\frac{(170)(2.5) + 148.2(3)}{4.5(10^6)(2.5)^3 + 3.0(10^6)(3.0)^3} \right]$$
$$= (70.31) \left(\frac{869}{70.31 + 81.0} \right)$$
$$\underline{\underline{P_1(2) = 404\,\text{psf}}}$$

$P_1(2) < P_1(1) \therefore$ beds separate

$$\underline{\underline{P_1(1) = 425\,\text{psf}}} \qquad \longleftarrow$$

assume simply supported ends

$$\sigma_t(\text{SS}) = \frac{3}{4} P \frac{L^2}{h^2}$$
$$\sigma_t = \left(\frac{3}{4}\right) \left(\frac{425}{144}\right) \left(\frac{24}{2.5}\right)^2$$
$$\underline{\underline{\sigma_t = 204\,\text{psi}}}$$
$$FS_t(1) = \frac{T_0}{\sigma_t}$$
$$FS_t(1) = \frac{1{,}000}{204}$$

(1) $\underline{\underline{FS_t(1) = 4.90}}$ $\qquad \overleftarrow{FS_t(1)}$

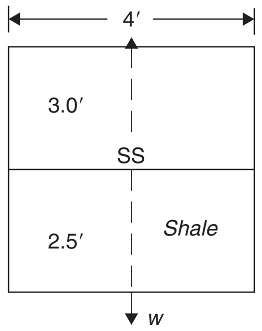
By inspection, the second roof layer separates from the first & $P = \gamma h$

$$\therefore P_2(1) = (148.2)(3.0)$$
$$P_2(1) = 446\,\text{psf}$$
$$\sigma_t(2) = \left(\frac{3}{4}\right) \left(\frac{446}{144}\right) \left(\frac{2.4}{3.0}\right)^2$$
$$\sigma_t(2) = 148.6\,\text{psi}$$
$$FS_t(2) = \frac{T_0(2)}{\sigma_t(2)}$$
$$= \frac{850}{148.6}$$

(2) $\underline{\underline{FS_t(2) = 5.72}}$ $\qquad \overleftarrow{FS_t(2)}$

(3) Bolting plan- An anchor stratum is present in the layered sandstone,

$$\text{Bolt length} = h_1 + h_2 + 0.5\,\text{ft (anchorage)}$$
$$L_b = 2.3 + 3.0 + 0.5$$
$$\underline{\underline{L_b = 6.0\,\text{ft}}} \qquad \longleftarrow$$

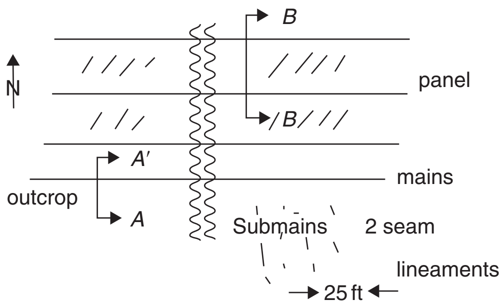


Try 5/8″ grade 55 bolts on 4′ centers @12,400?
Rock block

$$W_n = \gamma_r A_r (h_1 + h_2)$$
$$= (4)(4)[(170)(2.5) + (148.2)(3.0)]$$
$$\underline{W_n = 13{,}914 \text{ lbs}}$$

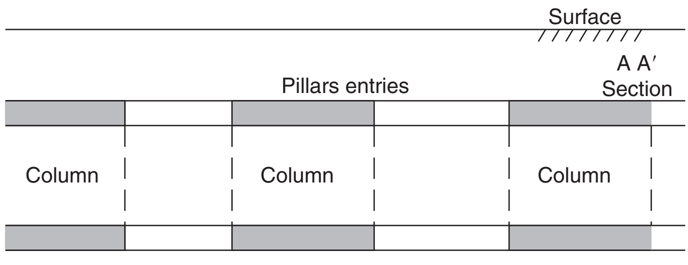
Use $\frac{3}{4}''$ grade 55, 6-ft bolts on 4 ft centers ←

25. Given: Mains, sub-mains, panel entries
Show:
(a) Favorable alignment of mains
(b) Panel entries
(c) w.r.t joints bearing NS, steeply dipping

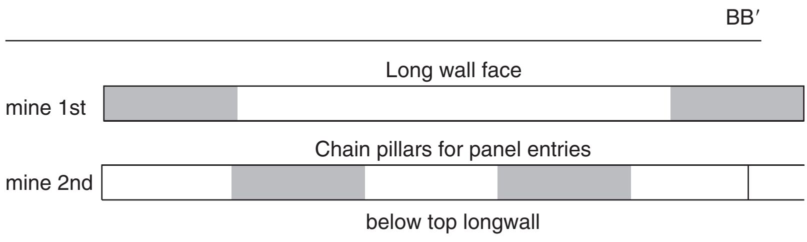


Solution:

(a) "Columnize the mains"

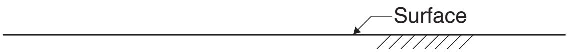


(b) "Stagger the chains"

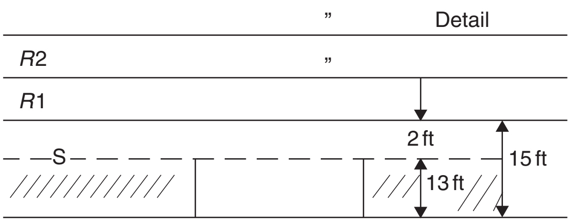


(c) Orientation is o.k. because
   (1) Joints are across mains and do not disrupt roof and are therefore conducive to long-life!
   (2) In panels, joints aid caving

26. Given: Stratigraphic column, mining height = 13′, depth = 1,450′



	$E(10^6)$ psi	ν	C_0(psi)	T_0(psi)	γ(pcf)	h(ft)
*R*5	12.6	0.19	12,700	1,270	160	21.5
*R*4	12.7	0.21	14,200	1,340	158	15.2
*R*3	12.6	0.19	12,700	1,270	156	6.0
*R*2	6.3	0.31	8,450	760	146	1.5
*R*1	5.4	0.23	7,950	610	152	2.0
S	6.75	0.27	3,750	425	92	15.0
*F*1	12.3	0.18	11,500	1,050	155	10.3
*F*2	8.1	0.33	13,700	1,240	160	17.0



Find: $FS(1)$, $FS(2)$, bolting plan.

Solution:

$$FS = \frac{T_0}{\sigma_t}$$

assume simply supported ends

$$\sigma_t = \frac{3}{4} P \left(\frac{L}{4}\right)^2$$

P from bed separation analysis

$$P(1,1) = \gamma_1 h_1$$
$$= (92)(2) \quad \underline{\underline{1 = \text{“coal”, seam is the immediate roof}}}$$
$$\underline{P(1,1) = 184 \text{ psf}}$$
$$P(1,1) = E_1 h_1^3 \frac{\gamma_1 h_1 + \gamma_2 h_2}{E_1 h_1^3 + E_2 h_2^3}$$
$$= (0.75)(10^6)(2)^3 \frac{184 + (152)(2.0)}{0.75(10^6)(2)^3 + 5.4(10^6)(2)^3}$$

$$
\begin{aligned}
P(1,2) &= (6.0)\frac{184+304}{6.0+43.2} \\
&= \frac{(6.0)(488)}{49.2} \\
\underline{P(1,2)} &\underline{= 59.5\text{ psf}}
\end{aligned}
$$

$\underline{P(1,2) < P(1,1) \therefore \text{seam roof separates}}$
restart for next cluster

$$
\begin{aligned}
P(1,1) &= \gamma(R1)h(R1) \\
\underline{P(1,1)} &\underline{= (152)(2)} \\
P(1,1) &= 304\text{ psf} \\
P(1,2) &= (5.4)(10^6)(2)^3\frac{(152)(2)+146(1.5)}{(5.4)(10^6)(2)^3+4.3(10^6)(1.5)^3} \\
\underline{P(1,2)} &\underline{= 339\text{ psf}} > P(1,1) \\
P(1,3) &= (5.4)(2)^3\frac{523+(156)(6)}{66.7+12.6(6)^3} \\
&= 43.2\frac{1{,}459}{2{,}788} \\
P(1,3) &= 22.6\text{ psf}
\end{aligned}
$$

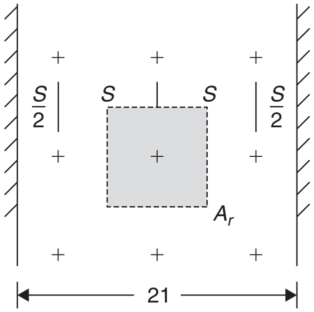
$\therefore$ separation between $R2$ & $R3$
Separation of seam from $R1$, $P(1,1) = 184$ psf
Separation of $R2$ from $R3$, $P(1,2) = 339$ psf

$$
\begin{aligned}
FS\text{ (immediate roof)} &= \frac{T_0}{\sigma_t} \\
\sigma_t &= \left(\frac{3}{4}\right)(184)\left(\frac{L^2}{2^2}\right) \quad L = ? \\
\sigma_t &= 34.5\,L^2\text{ psf} \\
FS &= \frac{(425)(\text{psi})(144)}{34.5\,L^2} \\
\underline{\underline{FS = (\text{roof})}} &\underline{\underline{= \frac{1{,}774}{L^2}}} \quad L = \text{ft}
\end{aligned}
$$

But entry width = 21′

$$
\begin{aligned}
\therefore\ FS &= \frac{1{,}774}{(21)^2} \\
\underline{\underline{FS}} &\underline{\underline{= 4.02}}
\end{aligned}
$$

← immediate proof



$$FS\text{ (next)} = \frac{(610)(144)}{\sigma_t}$$

$$\sigma_t = \left(\frac{3}{4}\right)(339)\left(\frac{21}{2}\right)^3$$

$$\sigma_t = 28{,}031\text{ psf}$$

$$FS = \frac{87{,}840}{28{,}031}$$

$$\underline{\underline{FS = 3.13}}$$

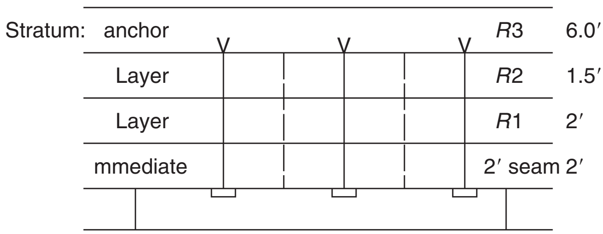
layer above ←

bolting pattern dead weight

$$W = F_b$$

$$L_b = 2 + 2 + 1.5 + 0.5\text{(anchor)}$$

$$L_b = 6'$$



$$[\gamma(S)(2) + \gamma(R1)(2) + \gamma(R2)(1.5)]S^2 = F_b$$

$$[(92)(2) + (152)(2) + (146)(1.5)]S^2 = F_b$$

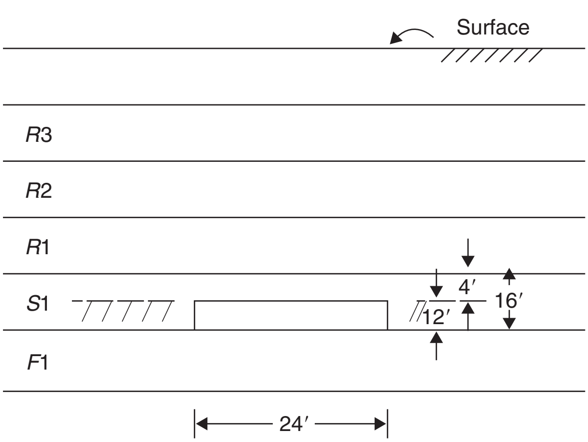
$$707S^2 = F_b$$

$S = 4'$ $F_b = 11{,}131$ lbf $T_y = 14{,}841$ lbf
$S = 5'$ $F_b = 17{,}475$ lbf $T_y = 23{,}5667$ lbf

But also $F_b = T_b = (3/4)T_y$, i.e. $T_y = (4/3)F_b$

$\frac{3}{4}''$ grade 55 bolts, 6′ long, spaced on 4′ centers ($T_y = 18{,}400$ lbf) will do (from Table 3.4)

27. Given:



Find:
(1) Immediate roof FS

Solution:

$$FS = \frac{T_0}{\sigma_t} \quad T_0 = 250\,\text{psi (not mined full thickness)}$$

$\sigma_t \propto P(1,n)$, $n=?$ test for bed separation

$$\begin{aligned} P(1,1) &= \gamma_1 h \\ &= (95)(4) \\ \underline{\underline{P(1,1)}} &\underline{\underline{= 380\,\text{psf}}} \end{aligned}$$

$$\begin{aligned} P(1,2) &= \frac{E_1 h_1^3(\gamma_1 h_1 + \gamma_2 h_2)}{E_1 h_1^3 + E_2 h_2^3} \\ &= \frac{0.35(10^6)(4)^3[(95)(4) + (136)(2)]}{0.35(10^6)(4)^3 + (4)^3(10^6)(2)^3} \\ P(1,2) &= \frac{652}{1 + \frac{4.1}{0.35}\frac{(2)^3}{(4)^3}} \\ \underline{\underline{P(1,2)}} &\underline{\underline{= 265\,\text{psf}}} \end{aligned}$$

$P(1,2) < P(1,1)$
$\therefore$ beds separate
$\therefore n = 1$
assume SS bean ends

$$\begin{aligned} \sigma_t &= \frac{3}{4}P\left(\frac{L}{h}\right)^2 \\ &= \left(\frac{3}{4}\right)\left(\frac{380}{144}\right)\left(\frac{24}{4}\right)^2 \\ \sigma_t &= 71.25\,\text{psi} \\ FS_t &= \frac{250}{71} \\ \underline{\underline{FS_t}} &\underline{\underline{= 3.5}} \end{aligned}$$

← 1–1 FS_t

(2) Find P_b to reduce sag to zero

Solution:

$w_{\max}\, \alpha\, P$

$$P = 0 \text{ will reduce } w_{\max} \text{ to zero}$$

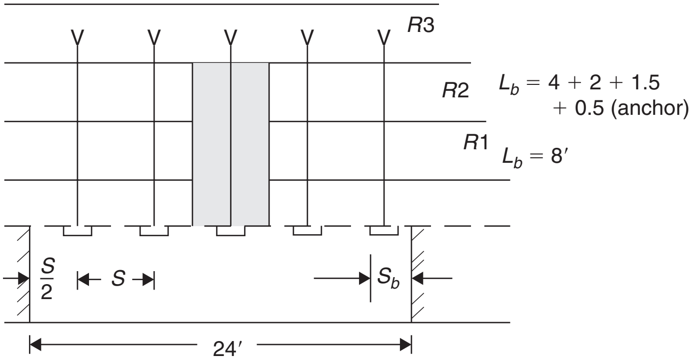
$$P(1,n) = \frac{E_1 h_1^3 \left(\sum_1^n \gamma_i h_i - P_b\right)}{\sum E_i h_i^3}$$

$$\therefore\ P(1,n) = 0 \quad \text{when} \quad \sum_1^n \gamma_i h_i - P_b = 0$$

$$P_b = \sum_{1}^{n} \gamma_i h_i \quad \& \ n \text{ includes } S1, R1, R2 \text{ i.e. } n = 3$$
$$= (95)(4) + 136(2) + 142(1.5)$$
$$P_b = \underline{\underline{865 \text{ psf } (6.0 \text{ psi})}}$$

← 1–2 P_b

(3) layout pf. anchored bolting pattern based on dead weight load



$W = F_b$ for equilibrium

$$(\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3) S^2 = F_b (S = \text{spacing square pattern assumed})$$
$$\text{try } 4'\text{c-c} \quad (865)(16) = 13{,}840 \text{ lbf}$$
$$5'\text{c-c} \quad (865)(25) = 21{,}625 \text{ lbf}$$

5″/8 Grade 75 bolt has yield load of 17,000 will do on 4′ c-c.
Grade 75
4 ft c-c (square)
5″/8 diameter
8 ft length

← bolts

(4) FS_t after bolting = ?

$$FS_t = \frac{T_0}{\sigma_t}$$

since bolting supports dead weight and reduces sag to zero, $\sigma_t = 0$ & then $\underline{\underline{FS_t \rightarrow \infty}}$

← 1–4

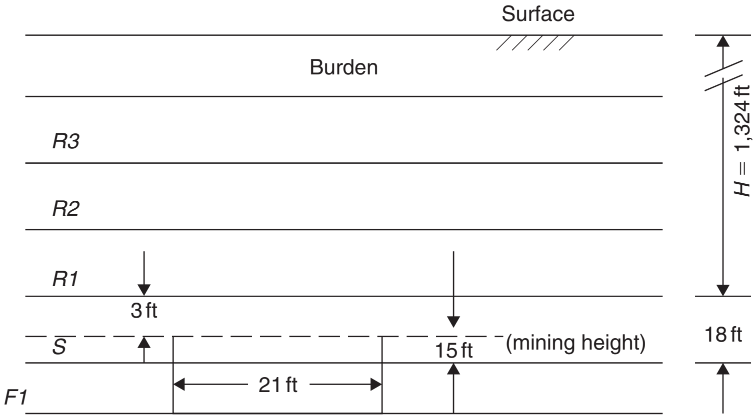
(5) Before bolting, bed separation takes place between roof seam and R1

$$\text{so } P = \gamma_1 h_1 + P_g$$
$$\sigma_t = \tfrac{3}{4} P \left(\frac{L}{b}\right)^2$$
$$\text{so } FS_t = \frac{T_0}{\sigma_t}$$

would be reduced by P_g

← 1–5

28. Strata data, rock properties,



Find:
(1) Immediate roof safety factor.

Solution:

Immediate roof is in coal 3 ft thick

$$P = \gamma h$$
$$P(1,1) = \frac{95(3)}{44}$$
$$\underline{P(1,1) = 1.98\text{ psi}}$$

possible bed separation

$$P(i,j) = E_i h_i^3 \left(\Sigma_j \gamma_j h_j\right) / \Sigma_j E_j h_j^3$$
$$P(1,2) = \frac{(0.35)(3.0)^3(95)(3) + \frac{135(2.0)}{144}}{0.35(3)^3 + 3.1(2)^3}$$
$$= \frac{(9.45)}{144}\left(\frac{550}{34.3}\right)$$
$$\underline{P(1,2) = 1.06\text{ psi}}$$

$P(1, 2) < P(1, 1)$, therefore beds separate

$$FS_t = \frac{T_0}{\sigma_t}$$

assume simply supported ends

$$\therefore \sigma_t = \frac{3}{4} P \left(\frac{L}{h}\right)^2$$
$$= \left(\frac{3}{4}\right)(1.98)\left(\frac{21}{3}\right)^2$$
$$\underline{\sigma_t = 72.8\text{ psi}}$$

$$FS_t = \frac{150}{72.8}$$
$$\underline{\underline{FS_t = 2.06}} \qquad \longleftarrow FS_t \quad (1)$$

Find: P_b needed to reduce sag to zero of all roof layers (seam, $R1$ & $R2$).

Solution:

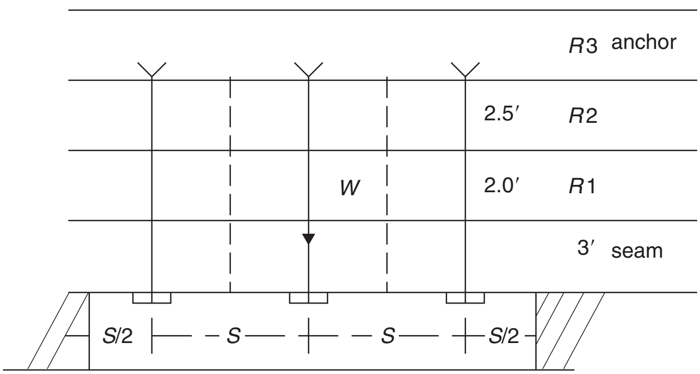
$$P(1,3) = \frac{E_1 h_1^3 \left(\sum_1^3 \gamma_j h_j - P_b\right)}{\sum_1^3 E_j h_j^3}$$

and sag: $w_{\max} \, \alpha \, P$
if $w_{\max} = 0$ then $P = 0$

$$\therefore P_b = \sum_1^3 \gamma_j h_j \text{ (i.e. "dead weight")}$$
$$= \frac{(95)(3)}{144} + \frac{2.0(135)}{144} + \frac{2.5(142)}{144}$$
$$\underline{\underline{P_b = 6.32\,\text{psi}}} \qquad \longleftarrow P_b \ (2)$$

Find: Bolting pattern based on dead weight.

Solution:



assume square pattern S = Spacing

$$\text{then } W = \left(\sum_1^3 \gamma_j h_j\right) S^2$$
$$\text{and } F_b = W$$
$$P_b S^2 = W$$
$$F_b = P_b S^2$$
$$F_b = (6.32\,\text{psi})(S)^2(144)$$

$$\text{But also} \begin{cases} S = 4 \quad F_b = (910)(S)^2 \\ F_b = 14{,}560\,\text{lbf} \end{cases}$$

$$\begin{cases} S = 5 \\ F = 22{,}750\,\text{lbf} \end{cases}$$

Table 3.4 text

Grade 55	3/4″	$F_y = 18{,}700$
Grade 75	5/8″	$F_y = 17{,}000$
	3/4″	$F_y = 25{,}100$

Possible 3/4″ Grade 55 on 4′ center
5/8″ Grade 75 on 4′ center
3/4″ Grade 75 on 5′ center ← (3)

bolt length = 3.0 + 2.5 + 2.0 + 0.5 (center)
$h_b = 8$ ft.

bolt safety factor when tensioned to 2/3, 3/4 σ_y, $FS_b = 1.5$ to 1.33

Find: Immediate roof rock safety factor

$$FS_t = \frac{T_0}{\sigma_t}$$

since sag is reduced to zero $\sigma_t \to 0$ and $FS_t \to \infty$ (theoretically) ← (4)

Find: Gas pressure effect

Solution:

Gas P_g contributes directly to roof pressure through the formula

$$P = \frac{E_i h_i^3 (\sum \gamma_i h_j - P_b + P_g)}{\sum E_j h_j^3} \qquad \leftarrow (5)$$

$$FS_t = \frac{T_0}{P} \text{ for } R1 \quad P(R1) = \gamma h + P_g$$

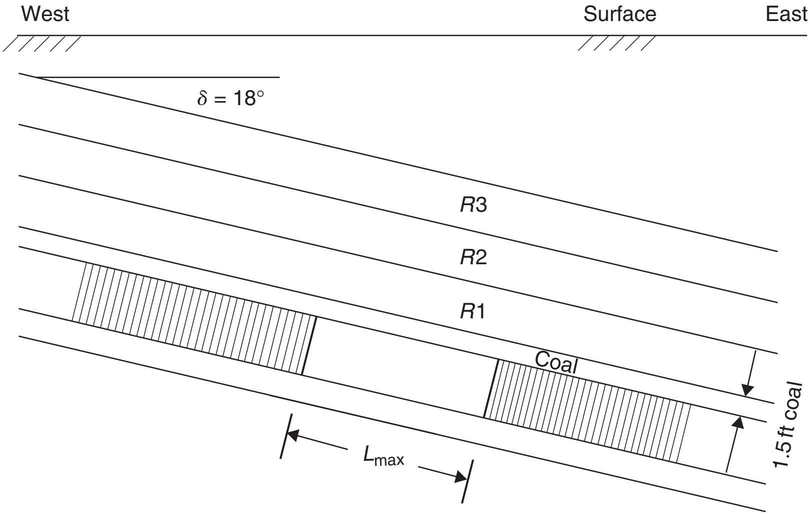
29. Given:

- Coal mine at 1750 ft depth
- Tensile safety factor of 4.0 required in the immediate roof
- $FS_t = 3.0$ for next two stratum
- Rock and joint data in tables
- $\gamma = 156$ pcf (overburden)
- Mining height from floor is 15 ft leaving 1.5 ft of coal in the roof.

Find: max safe roof span $L_{\max}$

Solution:

Test for bed separation



(not corrected for dip yet)

$$\text{coal only } P(1,1) = \gamma h = (90)(1.5) = 135 \text{ psf}$$

$$\text{coal} + R1\ P(1,2) = E_1 h_1^3 \left(\frac{\gamma_1 h_1 + \gamma_2 h_2}{E_1 h_1^3 + E_2 h_2^3} \right)$$

$$= \frac{0.35(10^6)(1.5)^3[(90)(1.5) + 138(1.3)]}{0.35(10^6)(1.5)^3 + 3.62(10^6)(1.3)^3}$$

$$= (1.181)\left[\frac{179}{9.13}\right]$$

$$\underline{P(1,2) = 23.2 \text{ psf}}$$

∴ bed separation occurs between coal & $R1$
$FS = 4.0$: immediate roof – coal
$4.0 = T_0/\sigma_t$ assume simply supported ends, so

$$\sigma_t = \frac{3}{4} P \left(\frac{L}{h}\right)^2$$

where $P = \gamma' h$, $\gamma' = \gamma \cos\delta$

$$\sigma_\tau = \left(\frac{3}{4}\right)\left(\frac{86}{1}\right)\left(\frac{L^2}{1.5}\right)$$

$$\sigma_t = 42.8\, L^2$$

$$4.0 = (310)(144)/42.8\, L^2$$

$$L = \left[\frac{(310)(144)}{(4)(42.8)}\right]^{1/2}$$

$$\underline{\underline{L = 16.1\,\text{ft}}} \qquad \longleftarrow \; L_{\max}\text{: coal constraint}$$

Next roof layer

$$\begin{aligned} P'(1,1) &= \underline{(138)(1.3) = 179\,\text{psf}} \\ P'(1,2) &= \frac{3.62(1.3)^3 10^6[(138)(1.3) + 142(2.7)]}{3.62(1.3)^3 10^6 + 4.16(2.7)^3} \\ &= (7.95)\frac{(563)}{89.8} \\ P'(1,2) &= \underline{49.8\,\text{psf}} \end{aligned}$$

$\therefore$ Shale ($R1$) separates from sandy shale ($R2$)
$FS = 3.0$: for next 2 roof layers

$$3.0 = \frac{T_0}{\sigma_t} \quad \sigma_t = \left(\frac{3}{4}\right)(138)\left(\frac{L^2}{1.3}\right)\cos 18^\circ$$

$$\sigma_t = 75.7\,L^2$$

$$\therefore 3.0 = \frac{(650)(144)}{75.7\,L^2}$$

$$L = \left[\frac{(650)(144)}{(3)(75.7)}\right]^{1/2}$$

Shale: $\underline{\underline{L = 20.3\,\text{ft}}}$ (which is greater than $L_{\max}$ allowed by immediate roof)
By inspection, the sandy shale will separate (sag more) than the overlying mudstone because both E & h are less.

$$\text{Sandy shale:} \quad L = \left[\frac{(72.0)(144)}{\left(\frac{3}{4}\right)\left(\frac{135}{2.7}\right)(3)}\right]^{1/2}$$

$$\underline{\underline{L = 30.4\,\text{ft}}} \quad \text{(which is greater yet)}$$

$$\underline{\underline{\therefore L_{\max} = 16.1\,\text{ft}}} \qquad \longleftarrow \; \frac{L_{\max}}{\text{(Given the constraints cited)}}$$

30. Given: Problem 29 conditions and 15 ft wide entries.
Find:
(a) Maximum sag
(b) Failure sag

Solution:

(a) Assume simply supported ends, then

$$
\begin{aligned}
w_{\max} &= \frac{5PL^4}{384\,EI},\ I = bh^3/12, P = \gamma' h, b = 1\,\text{ft}\\
w_{\max} &= \left(\frac{5}{32}\right)\left(\frac{\gamma' L^4}{Eh^2(1)}\right)\\
&= \left(\frac{5}{32}\right)\left(\frac{86}{0.35}\right)\left(\frac{15}{1}\right)^4\left(\frac{1}{1.5}\right)^2\left(\frac{1}{10^6}\right)\left(\frac{1}{144}\right)\\
w_{\max} &= 5.97\,(10^{-3})\,\text{ft.} \quad (0.072\,\text{in.})
\end{aligned}
$$

$\xleftarrow[\text{(sag)}]{\text{(a)}}$

(b) At failure $\sigma_t = T_0, \quad FS_t = 1.0$

$$
\begin{aligned}
\frac{3}{4}\frac{\gamma' L^2}{h} &= T_0\\
\frac{\gamma' L^4}{h^2} &= \frac{\frac{4}{3}T_0 L^2}{h}\\
&= \left(\frac{4}{3}\right)\left[\frac{(310)(144)(15)^2}{1.5}\right]\\
\frac{\gamma' L^4}{h^2} &= 8.93(10^6)\frac{\text{lbf}}{\text{ft}}
\end{aligned}
$$

$$
\begin{aligned}
w_{\max} &= \left(\frac{5}{32}\right)\left(\frac{1}{E}\right)\left(\frac{\gamma' L^4}{h^2}\right)\\
&= \left(\frac{5}{32}\right)\left(\frac{1}{0.35}\right)\left(\frac{1}{10^6}\right)(8.93)10^6\left(\frac{1}{144}\right)\\
w_{\max} &= 2.77(10^{-2})\text{ft } (0.332\text{in.})
\end{aligned}
$$

$\xleftarrow[\text{failure sag}]{\text{(b)}}$

31. Given: Problems 29/30 data, dead weight approach to bolting using point-anchored mechanical bolts, and 20 ft wide entries.

Find: Bolting plan.

Solution:

- Need an anchor stratum
  - The mudstone appears thick and strong and within reach
  - Try bolds that are $1.5 + 1.3 + 2.7 + 0.5$ ft long (allowing 0.5 ft for anchorage) i.e. 6 ft bolts, dead weight & square pattern S′c-c:

$$
\begin{aligned}
W &= [(1.5)(90) + 1.3(138) + 2.7(142)]S^2\cos(18^\circ)\\
W &= 664\,S^2\,\text{lbf.}
\end{aligned}
$$

@ 4′c-c W = 10,621 lbf
T3.4: a 5/6″ diameter High Strength bolt has an $F_y = 12{,}400$ lbf
Tension to 2/3 F_y, is less than W

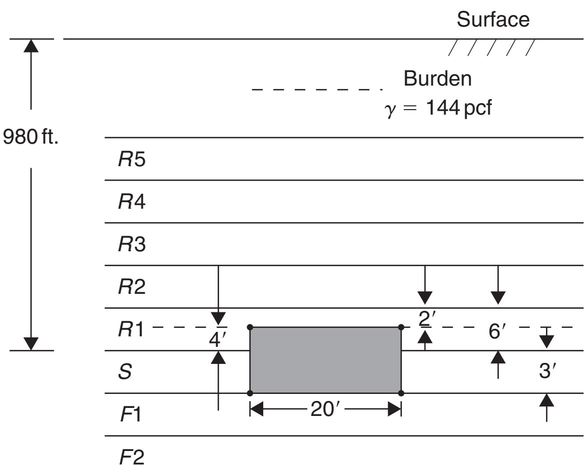
$$\text{i.e. } W = \frac{2}{3}F_y$$

$$\underline{\underline{F_y = \frac{3}{2}W = 16{,}747\ \text{lbf.}}}$$

@ 67% tension
a 5/8″ bolt with Extra High Strength (Grade 75) will do with $F_y = 17{,}000$ lbf

bolt length = 6′
diameter = 5/8″
spacing = 4′c-c/square
steel grade = Grade 75
bolt tension = 11,333 lbf.
(2/3F_y) ⟵

32. Given: Sketch and data for low coal room and pillar mining.



Find:
(a) FS of $R1$
(b) FS of $R2$
(c) bolting plan.

Solution:

By definition: $FS_t = \frac{T_0}{\sigma_t}$
Assuming simply supported ends: $\sigma_t = (3/4)P(L/h)^2$, need P from bed separation analysis.

$$P(1) = \gamma_1 h_1 = \left(\frac{156}{144}\right)(2)$$
$$P(1) = \underline{\underline{2.17\,\text{psi}}}$$

$$P(1,2) = E_1 h_1^3 \frac{(\gamma_1 h_1 + \gamma_2 h_2)}{E_1 h_1^3 + E_2 h_2^3} = 12.6(10^6)(2)^3 \frac{[(156)(2) + 146(1.5)]}{12.6(10^6)2^3 + 6.3(1.5)^3 10^6}$$
$$P(1,2) = \frac{(100.8)}{144}\frac{(531)}{100.8 + 21.0}$$
$$\underline{\underline{P(1,2) = 3.04\,\text{psi}}}$$

$$P(1,3) = \frac{(100.8)(531 + 152(1))}{(144)122.1 + 5.4(1.0)^3}$$
$$= \frac{(100.8)(683)}{144(122.1)}$$
$$\underline{\underline{P(1,3) = 3.75\,\text{psi}}}$$

$$P(1,4) = \frac{100.8}{144}\frac{[6.83 + 158(15.2)]}{(122.1) + 12.7(15.2)^3}$$
$$= \frac{(100.8)}{144}\frac{3{,}084}{(4{,}472)}$$
$$\underline{\underline{P(1,4) = 0.048\,\text{psi}}}$$

$P(1,4) < P(1,3)$ $\therefore$ beds separate between $R3$ and $R4$

$$P(1,3) = 3.75\,\text{psi}$$

$$\sigma_t(1) = \frac{3}{4}(3.75)\left(\frac{20}{2}\right)^2$$
$$\underline{\underline{\sigma_t(1) = 281\,\text{psi}}}$$

$$FS(R1) = \frac{1{,}270}{310}$$
$$\underline{\underline{FS(R1) = 4.52}}$$

← FS(R1)

$$FS(R2) = \frac{760}{\sigma_t(2)}$$

$$P(2,3) = E_2 h_2^3 \frac{\sum_1^3 \gamma_i h_i}{\sum_1^3 E_i h_i^3} = \frac{E_2 h_2^3}{E_1 h_1^3}\left(\frac{E_1 h_1^3 \sum_1^3 \gamma_i h_i}{\sum_1^3 E_i h_i^3}\right)$$
$$= \frac{E_2 h_2^3}{E_1 h_1^3} P(1,3) = \frac{6.3(10^6)(1.5)^3}{12.6(10^6)(2)^3}(3.75)$$
$$P(2,3) = 0.79\,\text{psi}$$

$$\sigma_t(2) = \left(\frac{3}{4}\right)(0.79)\left(\frac{20}{1.5}\right)^2$$

$$\sigma_t(2) = 105\text{ psi}$$

$$FS(\text{R2}) = \frac{760}{105}$$

$$\underline{\underline{FS(\text{R2}) = 7.24}}$$ ← FS(R2)

(c) bold plan – dead wt. load

Wt = Bolt tension

By inspection, and in consideration of bed separation, $R4$ is a candidate anchor stratum.

$Wt = [(152)(1.0) + (146)(1.5) + (156)(2)](4)(4)$

where a 4′c-c square plan is assumed

$Wt = 10{,}928$ lbf.

Try a 5/8″ diameter High Strength (Grade 55) bolt with yield load of 12,400 lbf

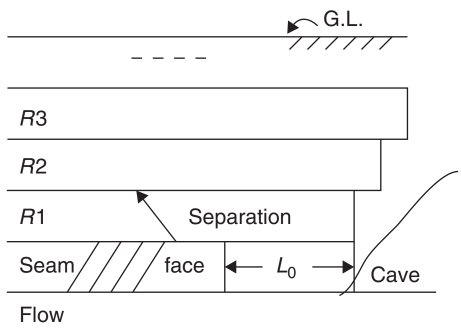
will be $10{,}928/12{,}400 = 88\%$ tensioned

Plan { 4′c-c square
5/8″diameter High strength (55)
5 ft long (allow 0.5 ft for anchorage) } ← (c)

33. Given: Sketch of longwall roof beam data

- Neglect support effects
- Cantilever roof model
- Bed separation $R1$ & $R2$

Find: X face advance to failure of $R1$.



Solution:

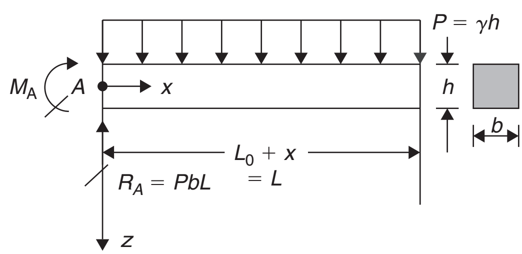
Shear and moment diagrams model as uniformly distributed load

$\Sigma F_z = 0 \quad R_A = PbL$

$\Sigma F_z = 0$

$$0 = R_A - V - Pbx$$

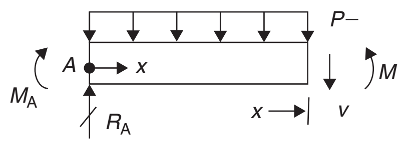
$$\underline{\underline{V = Pb(L - x)}}$$



$\Sigma M_A = 0$

$$0 = M_A + Vx + \frac{Pbx^2}{2} - M$$

$$\underline{\underline{M = M_A + Pb(L - x)x + \frac{Pbx^2}{2}}}$$



$$\frac{d^2w}{dx^2} = \frac{M}{EI}$$

$$EI\frac{dw}{dx} = M_A x + PbLx - \frac{Pbx^3}{3} + \frac{Pbx^3}{6} + C_1$$

at $x=0$ $\frac{dw}{dx}=0$ $\therefore C_1 = 0$

$$EIw = \frac{M_A x^2}{2} + \frac{PbLx^2}{2} - \frac{Pbx^4}{12} + \frac{Pbx^4}{24} + C_2$$

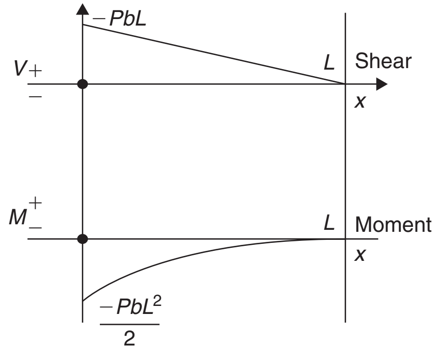
at $x=0$ $w=0 \therefore C_2 = 0$

Also: overall moment equilibrium requires

$$\Sigma M_A = 0$$

$$0 = M_A + PbL\left(\frac{L}{2}\right)$$

$$\therefore M_A = -\frac{PbL^2}{2}$$



The moment diagram shows

$M_{max} = -\frac{PbL^2}{2}$ at $x=0$

$$\sigma = \frac{M_c}{I}(\text{max tension})$$

$$= -\frac{PbL^2(-C)}{\frac{2bh^3}{12}}$$

$$\sigma = 6P\left(\frac{V^2}{h^3}\right)\left(\frac{h}{2}\right)$$

- $\sigma_t = 3P\left(\frac{L}{h}\right)^2$ ← on top of $R1$ at the face
- $FS_t = \frac{T_0}{\sigma_t}$

at failure $FS_t = 1$

$3P\frac{L^2}{h^2} = T_0$, therefore $L = \sqrt{\frac{T_0 h^2}{3P}}$

$$X = L - L_0$$

$$X = \sqrt{\frac{T_0 h^2}{3P}} - L_0$$

← face advance before $R1$ cantilever fails

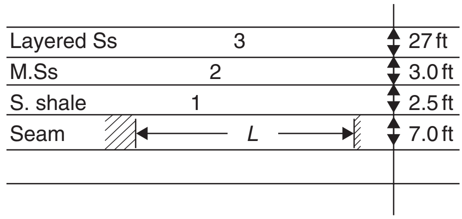
34. Given: Sketch and table data.

Find:

(a) Maximum possible L, lower seam

(b) L at $FS_t = 4$

(c) FS_t of M.Ss. roof layer



Solution:

Need strata loads

Find: Bed separation

$$P(1,1) = \gamma_1 h_1 = \frac{(170)(2.5)}{(144)}$$

$$\underline{P(1,1) = 2.95\text{ psi}}$$

$$P(1,2) = \frac{E_1 h_1^3(\gamma_1 h_1 + \gamma_2 h_2)}{E_1 h_1^3 + E_2 h_2^3} = \frac{(4.5)(10^6)(2.5)^3[(170)(2.5) + (148.2)(3)]}{(144)[(4.5)(10^6)(2.5)^3 + (3.0)(10^6)(3.0)^3]}$$

$$P(1,2) = \frac{(70.31)}{(144)}\frac{(870)}{(151.3)}$$

$$\underline{P(1,2) = 2.81\text{ psi}}$$

$\therefore$ beds separate $P(1,2) < P(1,1)$

* Also bed 2 (M.SS) separates from bed's by inspection, very thick 27′

(a) $FS = \dfrac{T_0}{\sigma_t}$

assume simply supported ends, then

$$\sigma_t = \frac{3}{4}P\left(\frac{L}{h}\right)^2$$

$$\sigma_t = \left(\frac{3}{4}\right)(2.95)\left[\frac{L^2}{(2.5)^2}\right]$$

$$\therefore \left(\frac{3}{4}\right)(2.93)\frac{L^2}{(2.5)^2} = \frac{1{,}000}{1}$$

Note: max L at $FS = 1$

$$L^2 = 2.83(10^3)$$

$$L_{max} = \underline{\underline{53.1\text{ ft}}}$$

$\begin{pmatrix} 65.0 \\ \text{B.I.} \end{pmatrix} L_{max}$ (a) ←

$L(\text{BI}) = \sqrt{\frac{3}{2}}L(\text{SS})$

if $FS = 4$, then

(b) $L^2 = \left(\frac{1{,}000}{4}\right)\frac{(2.5)^2}{2.95}\left(\frac{4}{3}\right)$

$\underline{\underline{L = 26.5\text{ ft.}}}$ ← L (b)

(c) $L = 53.1$

$$\sigma_t = \left(\frac{3}{4}\right)(P)\left(\frac{53.1}{3}\right)^2;\ P = \frac{(148.2)(3)}{144} = 3.09\text{ psi}$$

$$\sigma_t = 725\text{ psi}$$

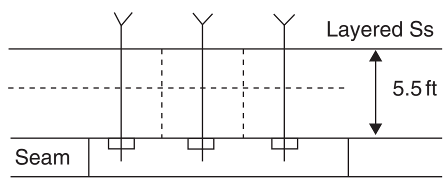
$$FS = \frac{T_0}{\sigma_t}$$

$$FS = \frac{850}{725}$$

$$\underline{\underline{FS = 1.17(\text{SS})}}$$ ← FS(M.Ss) (c)

(1.75: B.I.)

35. Given: Sketch & data $W_0 = 28$ ft lower seam
Find: Using dead weight load approach, a bolting plan



Solution:

(i) use the thick layered sandstone as an anchor stratum
(ii) try 5/8″ Grade 55 bolts on 4 ft center s

$$W_R = \gamma Vol.$$
$$= (4)(4)[(170)(2.5) + (148.2)(3)]$$
$$\underline{\underline{W_R = 13{,}914\text{ lbf.}}}$$

$$F_b = \frac{2}{3}F_y = W_R \quad \left(\text{try tension at } \frac{2}{3}F_y\right)$$

$$F_y = \left(\frac{3}{2}\right)(13{,}914)$$

$$\underline{\underline{F_y = 20{,}870\text{ lbf.}}}$$

According to Table 3.4 text
<u>3/4″ extra high strength bolts (Grade 75) have $F_y = 25{,}100$ lbf,</u>
<u>on 4 ft centers, 6 ft long (1/2 ft anchor)</u> ← ans.

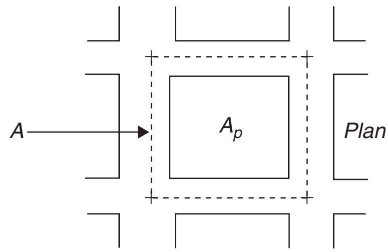
6 Pillars in Stratified Ground

1. Given: Room and pillar mine
   depth = 1,150 ft
   $\gamma_{ave} = 159$ pcf
   $C_0 = 13,700$ psi (lab core, $L/D = 2$)
   $T_0 = 1,250$ psi

   Find:
   (a) Maximum *safe* extraction ratio (no size effects) and $FS = 1.75$.
   (b) Evaluate:

   Solution:

   By definition: $R = A_m/A = 1 - (A_p/A)$
   where $A = A_m + A_p$
   Equilibrium of overburden block and pillar:



$$W = F_p$$
$$\gamma AH = S_p A_p$$

where H = depth S_p = pillar stress,

$$\therefore \frac{A_p}{A} = \frac{\gamma H}{S_p}$$

Then $$R = 1 - \frac{\gamma H}{S_p}$$

But S_p is limited by the pillar safety factor, i.e.

$$FS = \frac{C_p}{S_p}$$

so $$S_p = \frac{C_0}{FS}$$

since $C_p = C_0$ when no size effect exists
Hence,

$$R(\max \textit{safe}) = 1 - \frac{(\gamma H)(FS)}{C_0} \quad \longleftarrow \text{(a)}$$

with $FS = 1.75$, $H = 1,180$ ft, $\gamma = 159$ pcf

$$C_0 = 13{,}200\ \text{psi}$$

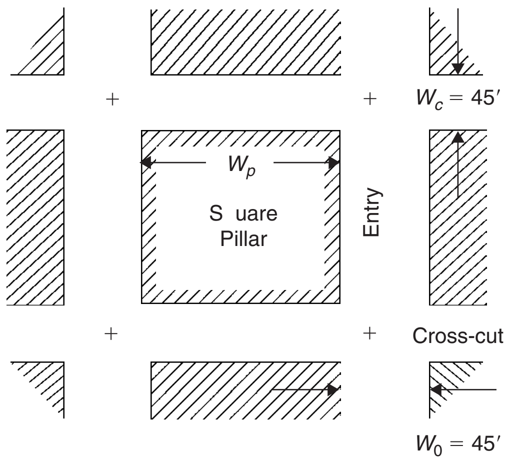
$$R = 1 - \left(\frac{159}{144}\right)\left(\frac{1{,}180}{13{,}700}\right)(1.75)$$

$$\underline{\underline{R(\text{max } safe) = 0.834(83.4\%)}} \qquad \xleftarrow{\text{(b)}}$$

2. Given: Problem 1 data, square pillar, entries and crosscuts 45 ft wide.
   Find:
   (a) Pillar size.

   Solution:

   From Problem 1.



$$1 - \frac{A_p}{A} = R$$

with

$$R = 0.834$$

$$\frac{A_p}{A} = 0.166$$

$$A_p = W_p^2$$

$$A = (W_p + W_0)^2$$

$$\left(\frac{W_p}{W_p + W_0}\right)^2 = 0.166$$

$$1 + \frac{W_0}{W_p} = \sqrt{\frac{1}{0.166}}$$

$$\therefore \frac{W_0}{W_p} = 1.45$$

$$W_p = \frac{45}{1.45}$$

$$\underline{\underline{W_p = 30.9\ \text{ft}}} \qquad \xleftarrow{\text{(a)}}$$

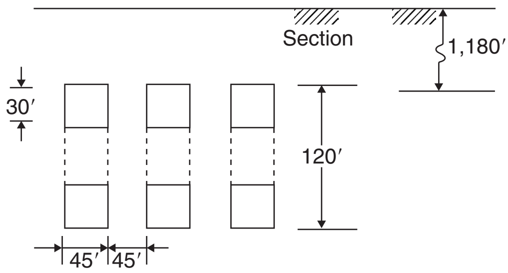
3. Given: Problems 1 and 2 data, $45' \times 45'$ pillars, $45'$ entries and crosscuts & size effects:

$$C_p = C_1\left(0.78 + 0.22\,\frac{W_p}{H_p}\right)$$

   mining height changes from $30'$ to $120'$

   Find:
   (a) Pillar *FS*.
   (b) Pillar *FS* without size effect.

Solution:



$$FS = \frac{C_p}{S_p}$$

$$C_p = C_1[0.78 + 0.22\,(W_p/H_p)]$$

$$C_2 = 13{,}700\text{ psi} \quad L/D = 2$$

lab data

$$C_1 = \frac{13{,}700}{\left[0.78 + 0.22\left(\frac{1}{2}\right)\right]}$$

$$\underline{\underline{C_1 = 15{,}393\text{ psi}}}$$

$$S_p = \frac{S_v}{1-R}$$

$$1 - R = \frac{A_p}{A}$$

$$= \frac{(45)(45)}{(45+45)^2}$$

$$\therefore \underline{\underline{1 - R = 0.25}}$$

$$S_p = \frac{\left(\frac{159}{144}\right)(1{,}180)}{0.25}$$

$$\therefore \underline{\underline{S_p = 5{,}212\text{ psi}}}$$

$$C_p = C_1\left[0.78 + 0.22\left(\frac{45}{120}\right)\right]$$

$$C_p = 15{,}393(0.8625)$$

$$\underline{\underline{C_p = 13{,}376\text{ psi}}}$$

finally

$$FS = \frac{13{,}376}{5{,}212}$$

$$\underline{\underline{FS = 2.55}} \quad \longleftarrow \text{(a)}$$

No size effect:

$$FS = \frac{13{,}700}{5{,}212}$$

$$\underline{\underline{FS = 2.63}} \quad \longleftarrow \text{(b)}$$

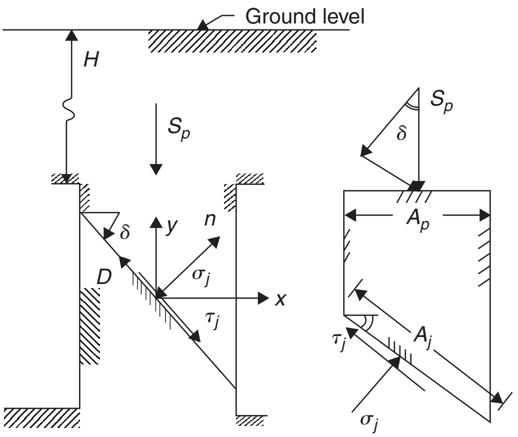
4. Given: Entries and crosscuts 45 ft wide, square pillars 45 ft wide, lab core (L/D = 2) $C_0 = 13{,}700$ psi $T_0 = 1{,}250$ psi $\gamma = 159$ pcf $H = 1{,}180$ ft

joint set present:
$\delta = 68°$ (dip) $\alpha = 54°$ (dip direction)

Find:

(a) Joint stress shear strength to just prevent slip.
(b) Cohesionless joint, Mohr–Coulomb strength, ϕ to prevent slips.
(c) Frictionless joint, c to prevent slip.

Solution:



From Problem 3: $S_p = 5{,}212$ psi
To just prevent shear slip: $\tau_j(\text{stress}) = \tau(\text{strength})$
$\tau_j = (\text{stress}) = T_j(\text{force})/A_j$
$\Sigma F_s = 0 = \tau_j A_j - (S_p A_p) \sin\delta$
$\tau_j = S_p = A_p/A_j \sin\delta$ But: $A_p = A_j \cos\delta$
$\therefore \underline{\tau_j = S_p \cos\delta \sin\delta}$

$$\tau_j = (5{,}212)\cos(68)\sin(68)$$
$$\underline{\underline{\tau_j = 1{,}810 \text{ psi}}} \qquad \longleftarrow \text{(a)}$$

(b) $\tau_j(\text{strength}) = \sigma_j \tan\phi_j + c_j$ $\therefore$ (Mohr–Coulomb criterion) at slip: $T_j(\text{stress}) = T_j(\text{strength})$

$\underline{1{,}810 = \sigma_j \tan\phi_j + 0}$: (cohesionless)

$$\Sigma F_n = 0 = \sigma_j A_j = S_p A_p \cos\delta$$
$$\therefore \sigma_j = S_p \cos^2\delta$$
$$= 5{,}212 \cos^2(68)$$
$$\underline{\sigma_j = 731 \text{ psi}}$$

$$\therefore \tan\phi_j = \frac{1{,}810}{731}$$
$$\tan\phi_j = 2.475$$
$$\underline{\underline{\phi_j = 68^\circ}} \qquad \longleftarrow \text{(checks) (b)}$$

(c) if $\phi_j = 0$ then

$$1{,}810 = 0 + c_j$$
$$\underline{\underline{c_j = 1{,}810\,\text{psi}}} \qquad \longleftarrow \text{(c)}$$

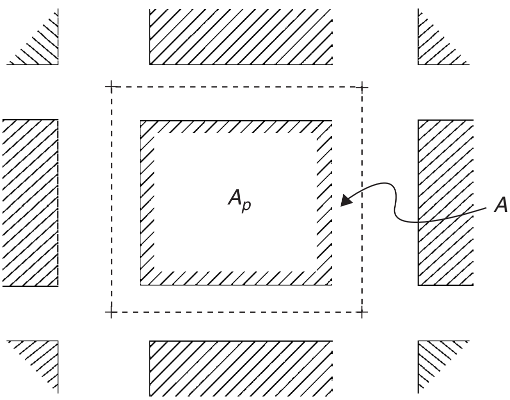
5. Given: Room and pillar mine, 360 m depth, $\gamma = 25.2\,\text{kN/m}^3$, $C_0 = 94.5$ Mpa $T_0 = 8.63$ MPa

Find:
(a) *Safe* extraction ratio ($FS = 1.75$, no size effect).
(b) Evaluate.

Solution:

By definition $R = \dfrac{A_m}{A} = 1 - \dfrac{A_p}{A}$,
$A = A_p + A_m$
Equilibrium of overburden block above A
$W = F_p$
$\underline{\gamma HA = S_p A_p}$
H = depth
S_p = average pillar stress
By definition

$$FS_p = \frac{C_p}{S_p}$$



no size effect $\therefore C_p = C_0$

Hence

$$FS_p = \frac{C_0}{S_p}$$

Also by definition: $$R = 1 - \frac{A_p}{A}$$

$$\gamma H = S_p(1-R)$$
$$FS_p = \frac{C_0(1-R)}{\gamma H}$$

(a) Hence $\underline{\underline{R = 1 - \dfrac{(FS_p)(\gamma H)}{C_0}}}$ $\qquad \longleftarrow$ max *safe* R

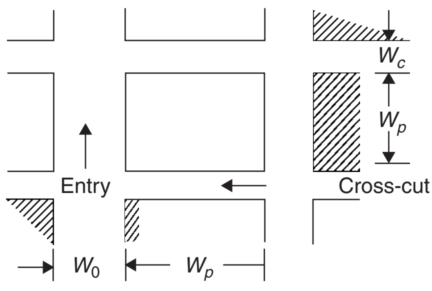
(b) $R = 1 - \dfrac{(1.75)(25.2\,\text{kN/m}^3)(360\,\text{m})}{94.5(10^3)\,\text{kPa}}$

$R = 0.832 \therefore \underline{\underline{R = 83.2\%}}$ ← $R_{max}(safe)$

6. Given: Problem 5 data, square pillars with entries and crosscuts 13.7 m wide
   Find: Pillar size.

   Solution:

   From Problem 5 $\underline{R_{max} = 0.832}$
   $W_0 = W_c = 13.7$ (given)
   $W_p = L_p$ (square)
   By geometry



$$A_p = W_p^2$$
$$A = (W_p + W_0)^2$$
$$\therefore 1 - \frac{A_p}{A} = 1 - \left(\frac{W_p}{W_p + W_0}\right)^2 = R$$

$$\left(\frac{W_p}{W_p + W_0}\right)^2 = 1 - 0.832$$
$$\left(\frac{W_p}{W_p + W_0}\right)^2 = 0.168, \quad W_0 = 13.7$$

$$\underline{\underline{W_p = 9.52\,\text{m}}}$$ ← $W_P = L_P$

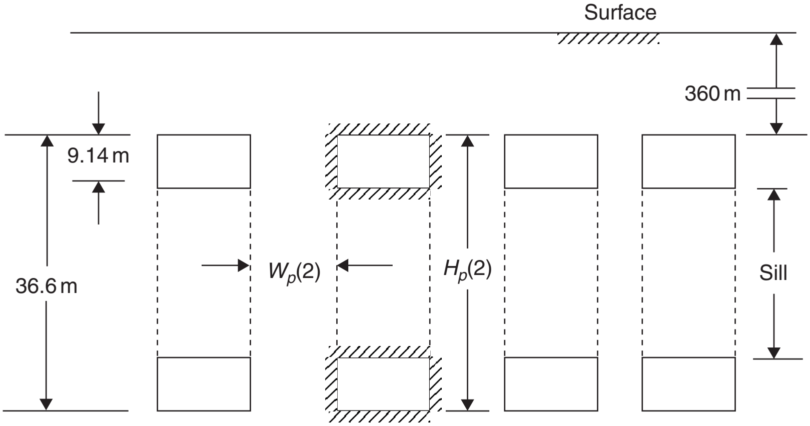
7. Given: Problems 5 and 6 data, pillars are 13.7 × 13.7 m (square) with size effect $C_p = C_1(0.78 + 0.22\,W_p/H_p)$ $H_p(1) = 9.14$ $H_p(2) = 36.6\,\text{m}$
   Find:
   (a) $FS_p(2)$.
   (b) $FS_p(2)$ with no size effect.

   Solution:

$$FS_p = \frac{C_p}{S_p}$$
$$C_p = C_1\left[0.78 + \left(\frac{W_p}{H_p}\right)0.22\right]$$

lab data: $C_0 = 94.5$ MPa at $H_p = 2$

$$\therefore 94.5 = C_1\left[0.78 + \left(\tfrac{1}{2}\right)(0.22)\right]$$
$$\underline{\underline{C_1 = 106.2\,\text{MPa}}}$$



$$C_p(2) = (106.2)\left[0.78 + 0.22\left(\frac{13.7}{36.6}\right)\right]$$

$$\underline{\underline{C_p(2) = 91.6\,\text{MPa}}}$$

$$S_p(2) = \frac{\gamma H}{(1-R)}$$

$$S_p(2) = (25.2\,\text{kN/m}^3)\frac{360}{0.25}$$

Note:

$$1 - R = \frac{A_p}{A}$$

$$= \frac{(13.7)^2}{(13.7+13.7)^2}$$

$$\underline{1 - R = 0.75}$$

$$(R = 0.75)$$

$$\underline{\underline{S_p(2) = 36.29\,\text{Mpa}}}$$

$$FS_p = \frac{91.6}{36.29}$$

$$\underline{\underline{FS_p = 2.52}} \quad \leftarrow \text{with size effect}$$

$$FS_p = \frac{94.5}{36.29}$$

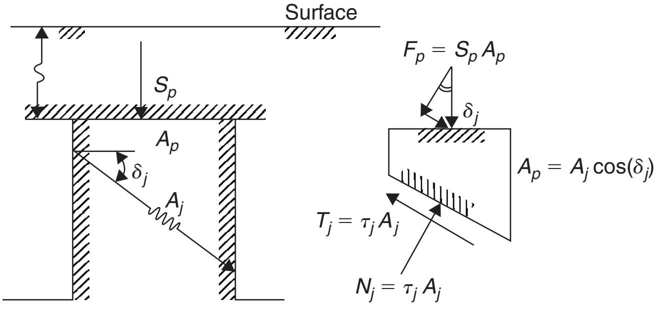
$$\underline{\underline{FS_p = 2.60}} \quad \leftarrow \text{no size effect}$$

(tall pillar reduces strength)

8. Given: Entries and crosscuts 13.7 m wide, square pillars 13.7 m wide, lab core (L/D = 2), $C_0 = 94.5$, $T_0 = 8.63$ MPa, depth = 360 m, $\gamma = 25.2\,\text{kN/m}^3$ (Problems 5 and 7 data) joints:
$\delta = 68°$ (dip) $\alpha = 54°$ (dip direction)

Find:
(a) joint shear strength at $FS_j = 1$.
(b) cohesionless joint, ϕ needed.
(c) frictionless joint, c needed.

Solution:



To prevent slip: $FS_j = \tau_j(\text{strength})/\tau_j(\text{stress})$ must have $FS_j \geqq 1$

(1) at limit $\tau_j(\text{stress}) = \tau_j(\text{strength})$

(2) MC-criterion $\tau_j(\text{strength}) = \sigma_j \tan\phi_j + c_j$

$$\text{Equilibrium:} \quad T_j = S_p A_p \sin(\delta_j)$$

$$\tau_j = S_p \frac{A_p}{A_j} \sin\delta_j$$

$$\therefore \underline{\tau_j = S_p \cos\delta_j \sin\delta}.$$

from Problem 7 $S_p = 36.29$ MPa

$$\therefore \tau_j = 36.29(\cos 68)(\sin 68)$$

$$\underline{\underline{\tau_j = 12.6\text{ MPa}}} \qquad \leftarrow \text{(a)}$$

(b) if $c = 0$

$\tau_j = \sigma_j \tan\phi_j$

Equilibrium:

$$\sigma_j A_j = S_p A_p \cos\delta_j$$

$$\sigma_j = (36.29)(\cos 68)(\cos 68)$$

$$\sigma_z = 5.09\text{ MPa}$$

$$\therefore \tan\phi_j = \frac{12.6}{5.09} = 2.474$$

$$\therefore \underline{\underline{\phi_j = 68^\circ}} \qquad \leftarrow (c_j = 0) \quad \phi_j$$

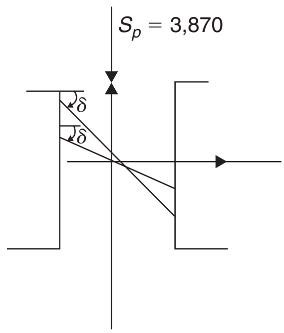
(c) $\phi = 0$ then

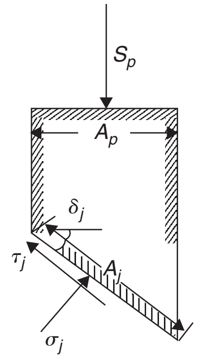
$$c = \tau_j = \underline{\underline{12.6\text{ MPa}}} \qquad \leftarrow (\phi_j = 0) \quad c_j$$

9. Given: Average vertical stress = 3,870 psi (26.7 MPa), room and pillar mine, joint set: $c_j = 130$ psi (0.9 MPa), $\phi_j = 28°$

Find: Range of dips prone to slip.

Solution:





Slip: If $\tau_j(\text{stress}) \geqq \tau(\text{strength})$ then slip may occur.

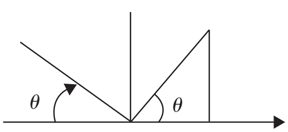
$$\tau_j(\text{stress}) = S_p \cos\delta \sin\delta$$
$$\sigma_j(\text{stress}) = S_p \cos^2\delta$$
$$\tau_j(\text{strength}) = \sigma_j \tan\phi_j + c_j$$

$$S_p \cos\delta \sin\delta \geqq S_p \cos^2\delta \tan\phi_j + c_j$$
$$\frac{S_p}{2}\sin(2\delta) \geq \frac{S_p}{2}\tan\phi_j(1+\cos^2\delta) + c_j$$
$$\sin(2\delta)\cos(\phi_j) \geq \sin(\phi_j)\cos(2\delta) + \sin\phi_j + \frac{2c_j\cos\phi_j}{S_p}$$
$$\underline{\underline{\sin(2\delta - \phi_j) \geqq \sin\phi_j + \frac{2\,c_j\cos\phi_j}{S_p}}}$$

$$\sin(2\delta - \phi_j) \geqq \sin(28°) + \frac{2(130)\cos(28)}{3{,}870}$$
$$\sin(2\delta - \phi_j) \geqq 0.528$$

Note: $\sin(\pi - \theta) = \sin(\theta)$

$$2\delta - \phi_j \geq \sin^{-1}(0.528)$$
$$\delta \geq \frac{1}{2}(28° + 32°)$$
$$\underline{\underline{\delta \geq 30°}} \quad \text{(1st solution)}$$



$$180 - (2\delta - \phi_j) \geq 32°$$
$$\frac{180 + 28 - 32}{2} \geq \delta$$
$$\underline{\underline{88° \geq \delta}} \quad \text{(2nd Solution)}$$

$\therefore$ unsafe for $\underline{\underline{30 \leq \delta \leq 88°}}$ ⟵

10. Given: Coal Seam 1,200 ft deep, dip = 15°, entries and crosscuts 20 ft wide, $C_0 = 2{,}000$ psi, $T_0 = 350$ psi, gravity only premining stress state, Mohr–Coulomb failure *FS* wrt compression of 1.5 required.

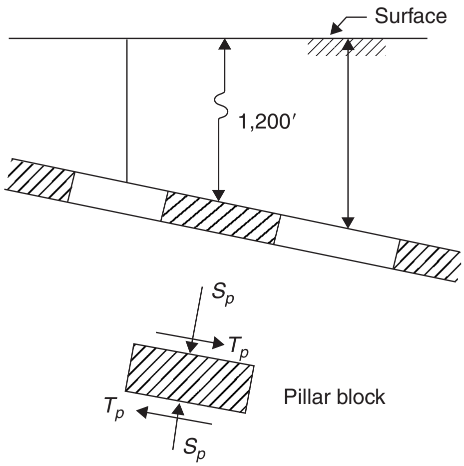
Find:

(a) *R* extraction ratio

(b) FS_s (wrt to shear).

Note: No size effect

Solution:



By definition:

$$R = \frac{A_m}{A} = 1 - \frac{A_p}{A} \quad (1)$$

$$FS_c = \frac{C_0}{S_p} \quad (2)$$

Extraction Ratio:

$$S_p = \frac{S_n}{1-R} \quad (3)$$

$$T_p = \frac{T_n}{1-R}$$

$$1 - R = \frac{S_n}{S_p}$$

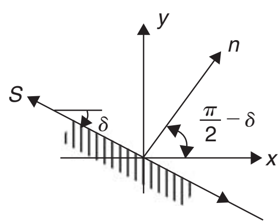
$$R = 1 - \frac{S_n}{S_p}$$

$$\therefore R = 1 - \frac{S_n(FS_c)}{C_0}$$

$$S_n = \frac{S_x + S_y}{2} + \left(\frac{S_x - S_y}{2}\right)\cos 2\theta$$

$$S_y = S_v$$

$$S_x = \left(\frac{\nu}{1-\nu}\right) S_v = S_n$$



assuming complete lateral restraint, gravity only S_v @ 1 psi/ft:

$$S_v = 1{,}200 \text{ psi}$$

$$\nu = 0.25 \quad \text{(assume)}$$

then

$$S_h = \frac{1}{3} S_v$$

$$S_h = 400 \text{ psi}$$

$$S_n = \frac{1{,}200 + 400}{2} + \left(\frac{-1{,}200 + 400}{2}\right)\cos[(2)(90 - 15)]$$

$$= 800 + (-400)(-0.8667)$$

$$S_n = 1{,}147 \text{ psi}$$

$$\therefore\ R = 1 - \frac{(1{,}147)(1.5)}{2{,}000}$$
$$= 1 - 0.860$$
$$\underline{\underline{R = 0.14\ (14\%)}} \qquad \longleftarrow \text{(a)}$$

$FS_s = \tau(\text{strength})/\tau(\text{stress})$
$\tau(\text{stress}) = T_n/(1 - \text{R})$
$\tau(\text{strength}) = \sigma \tan\phi + C$
Note: $\sigma = S_P$

$$\text{M-C criterion}\begin{cases} \sin\phi = \dfrac{\frac{C_0}{T_0} - 1}{\frac{C_0}{T_0} + 1} \\ C_0 = \dfrac{2c\cos\phi}{1 - \sin\phi}, \quad c = \dfrac{C_0}{2}\dfrac{(1 - \sin\phi)}{\cos\phi} \end{cases}$$

$$\sin\phi = \frac{\frac{2{,}000}{350} - 1}{\frac{2{,}000}{350} + 1}, \quad c = \left(\frac{2{,}000}{2}\right)\left[\frac{1 - \sin(44.6)}{\cos(44.6)}\right]$$
$$\sin\phi = 0.702, \qquad \underline{\underline{c = 418\text{ psi}}}$$
$$\underline{\underline{\phi = 44.6^\circ}}$$

$$T_n = -\left(\frac{S_x - S_y}{2}\right)\sin\left[2\left(\frac{\pi}{2} - \delta\right)\right]$$
$$= -\left(\frac{400 - 1{,}200}{2}\right)\sin(150)$$
$$\underline{\underline{T_n = 200\text{ psi}}}$$

$$\tau(\text{stress}) = \frac{200}{1 - 0.14}$$
$$\underline{\underline{\tau(\text{stress}) = 233\text{ psi}}} \qquad \longleftarrow$$

$$\tau(\text{strength}) = S_p\tan\phi + c$$
$$= \left(\frac{1{,}147}{1 - 0.14}\right)\tan(44.6^\circ) + 418$$
$$= 1{,}315 + 418$$
$$\underline{\underline{\tau(\text{strength}) = 1{,}733\text{ psi}}}$$

$$FS_s = \frac{1{,}733}{233}$$
$$\underline{\underline{FS_s = 7.44}} \qquad \longleftarrow \text{(b)}$$

11. Given: R & P trona mine, $H = 1{,}560$ ft, $W_p = 30$ ft, $L_P = 60$ ft, $H_P = 12$ ft, $W_0 = W_c = 24$ ft, Core ($L/D = 2$), $C_0 = 6{,}740$ psi, $T_0 = ?$, C.V. = 38% $FS = 1.67$ required.

    Find: $R_{\max}$ (allowable).

    Solution:

$$FS = \frac{C_p}{S_p}; \quad S_p = \frac{S_n}{1-R}$$

$$1 - R = \frac{S_n(FS)}{C_p}$$

assume 1 psi/1 ft of depth $S_n = 1{,}560$ psi

$$C_p = C_1[0.78 + 0.22(W_p/H_p)]$$

$$6{,}740 = C_1\left[0.78 + 0.22\left(\frac{1}{2}\right)\right]$$

$$C_1 = 7{,}573 \text{ psi}$$

$$C_p = 7{,}573\left[0.78 + 0.22\left(\frac{30}{12}\right)\right]$$

$$C_p = 10{,}072 \text{ psi}$$

$$\therefore 1 - R = \frac{(1{,}560)(1.67)}{10{,}072}$$

$$R = 0.741(74.1\%) \quad \leftarrow R_{\max}(\text{allow})$$

12. Given: Steeply dipping vein, $C_0 = 25{,}000$ psi, $T_0 = 2{,}850$ psi, no size effect

    Find: R_{max} (possible).

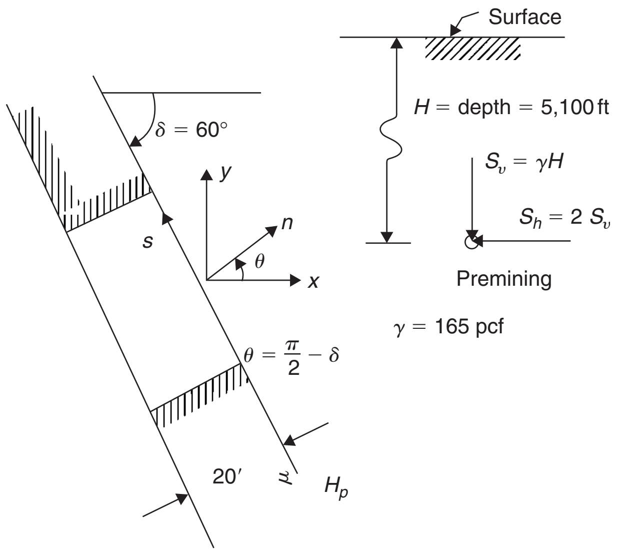
    Solution:

$$FS_c = \frac{C_p}{S_p}$$

$$S_p = \frac{S_n}{1-R}$$

$$FS_c = \frac{C_p(1-R)}{S_n}; \quad C_p = C_0$$

$$1 - R = \frac{S_n(FS_c)}{C_0}$$

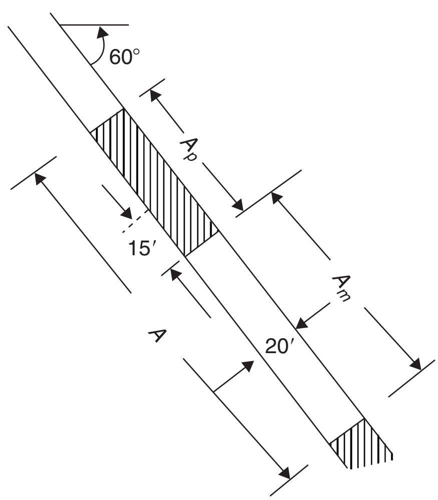


$$S_n = \frac{S_x + S_y}{2} + \frac{S_x - S_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{\gamma H}{2}(2+1) + \frac{\gamma H}{2}(2-1)\cos(2.30) + 0.\sin(60)$$
$$= \left(\frac{3}{2}\right)\left(\frac{165}{144}\right)(5{,}100) + \left(\frac{165}{144}\right)\left(\frac{5{,}100}{2}\right)\left(\frac{1}{2}\right) + 0$$
$$\underline{S_n = 10{,}277 \text{ psi}}$$

$FS = 1.0$ for max R

$$1 - R = \frac{(10{,}277)(1.0)}{25{,}000}$$
$$\therefore \underline{\underline{R_{max} = 0.591 (59.1\%)}} \longleftarrow$$

13. Given: Problem 12 data
Stope advance of 15 ft (beyond R_{max})
Find: If failure stable or not? Explain.



Solution:

At $R_{max} = 59.1\%$

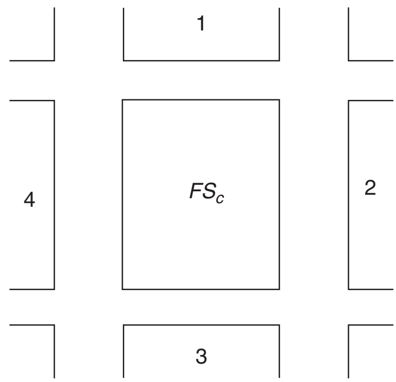
$$R = \frac{A_m}{A}$$

Assume: $A = 100$ ft, then $A_m = 59.1$ ft and $A_p = 41.9$ ft, so $W_p/H_p = 41.9/20 > 1.0$

(o.k – controlled)
After face advance of 15′, $W_p/H_p = 26.9/20 > 1.0$
(close)
After second face advance of 15′, $W_p/H_p = 11.9/20 < 1.0$
(over)
As a guide, fast failure threatens when $W_p/H_p < 1.0$, so until the $W_p/H_p < 1.0$ i.e. $W_p < 20$ is reached, fast failure, that is, bursting is not threatening.

14. Given: Large, regular array of pillars on a regular grid with FS_c.

Find:
(a) FS_c needed to avoid failure of nearest neighbors.
(b) FS_c needed if load shared equally.



Solution:

(a) $FS_c = \dfrac{C_p}{S_p}$

If pillar fails, then nearest neighboring pillars take up load, i.e. 1/4 S_p

Therefore $FS'_c = \dfrac{C_p}{(5/4)(S_p)} = \dfrac{4}{5}FS_c$

So $\underline{\underline{FS_c \geqq \dfrac{5}{4}}}$: (if $FS'_c \geqq 1.0$) ← (a) (nearest neighbors)

(b) All neighbors = 8 $\therefore S'_p = S_p + \dfrac{1}{8}S_p$

$FS'_c = \dfrac{C_p}{\frac{9}{8}S_p} = \dfrac{8}{9}FS_c$

$\underline{\underline{FS_c \geqq \dfrac{9}{8}}}$: (if $FS'_c > 1.0$) ← (b) (equal sharing)

15. Given: Room and pillar mine, depth $H = 980'$, overburden $\gamma = 156$ pcf, rock properties: laboratory data $(L/D = 2)$, $C_0 = 12{,}400$ psi, $T_0 = 1{,}050$ psi

Find:
(a) Expression for maximum extraction ratio R_{max}
(b) Evaluate.

Solution:

(a) By definition $R = \dfrac{A_m}{A} = 1 - \dfrac{A_p}{A}$

A_m = area mined, A_p = pillar area, A = total area

$$\begin{aligned} F_c &= \frac{C_0}{S_p} \\ S_p &= \frac{S_v}{1-R} \\ S_v &= \gamma H \\ \therefore\ F_c &= \frac{C_0(1-R)}{\gamma H} \end{aligned}$$

hence $R = 1 - \dfrac{(F_c)(\gamma H)}{C_0}$

By inspection: R is max when F_c is min

(a) $\underline{\underline{R_{\max} = 1 - \dfrac{\gamma H}{C_0}}}$ ← (a)

(b) $R_{\max} = 1 - \dfrac{\left(\frac{156}{144}\right)(980)}{12{,}400}$

$R_{\max} = 1 - 0.0856$

$\underline{\underline{R_{\max} = 0.914(91.4\%)}}$ ← (b)

16. Given: Problem 15 information and square pillars with no size effect at $F_c = 2$
Find: Pillar size W_p.

Solution:

$$\begin{aligned} R &= 1 - \frac{(F_c)(\gamma H)}{C_0} \\ &= 1 - \frac{(2)\left(\frac{156}{144}\right)(980)}{12{,}400} \\ R &= 1 - 0.1712 \\ &\underline{\underline{R = 0.829}} \end{aligned}$$

But also $R = 1 - \dfrac{A_p}{A}$

$$(1-R) = \frac{(W_p)^2}{(W_0 + W_p)^2}$$

substitute: $r^2 = 1 - R$

then $\dfrac{W_p}{W_0 + W_p} = r$

or
$$W_p = \frac{W_0 r}{1-r}$$
$$= \frac{(30)(0.1712)^{1/2}}{1-(0.1712)^{1/2}}$$
$$W_p = \frac{(30)(0.4138)}{0.5862}$$
$$\underline{\underline{W_p = 21.2\ \text{ft}}} \longleftarrow$$

17. Given: Data from problems 15 and 16 and $30' \times 30'$ pillars, a size effect: $C_p = C_1\left(0.78 + 0.22\ W_p/H_p\right)$, and a pillar height change from $25'$ to $90'$.

Find: F_c.

Solution:

$$W_p = 30'$$
$$H_p = 90'$$
$$C_1 = \frac{C_2}{\left[0.78 + 0.22\left(\frac{1}{2}\right)\right]}$$
$$C_1 = \frac{12{,}400}{0.78 + 0.11}$$
$$C_1 = 13{,}933\ \text{psi}$$
$$C_p = (13{,}933)\left[0.78 + 0.22\left(\frac{30}{90}\right)\right]$$
$$\underline{C_p = 11{,}889\ \text{psi}}$$
$$S_p = \frac{S_v}{1-R}$$

$$1 - R = \frac{A_p}{A}$$
$$= \frac{(30)(30)}{(30+30)^2} \qquad \text{Note: (rooms are } 30')$$
$$1 - R = 0.25$$

$$S_p = \left(\frac{156}{144}\right)\left(\frac{980}{0.25}\right)$$
$$\underline{S_p = 4{,}247\ \text{psi}}$$
$$F_c = \frac{C_p}{S_p}$$
$$= \frac{11{,}889}{4{,}247}$$
$$\underline{\underline{F_c = 2.8}} \longleftarrow \text{with size effect } F_c$$

Note: without size effect (use C_0 at $L/D = 2$)

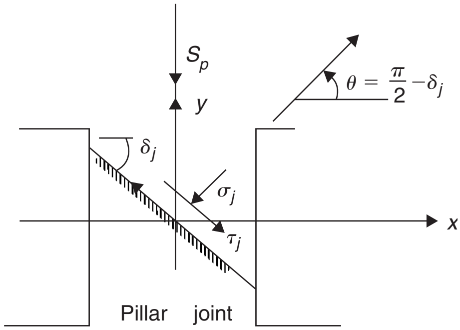
$$F_c = \frac{12{,}400}{4{,}247}$$

$$\underline{\underline{F_c = 2.92}}$$

← without size effect F_c

18. Given: 30′ entries and crosscuts 30′, square pillars, 980 ft depth, 156 pcf, joints: dip = 60°, dip dir = N45E

Find:
(a) shear strength needed to just prevent slip.



Solution:

$$\sigma_j = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_j = -\left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

(from equations of transformation)

$\sigma_{xx} = 0 \quad \tau_{xy} = 0 \quad \sigma_{yy} = S_p$: in pillar

$$\tau_j = \frac{S_p}{2}\sin 2\theta; \quad 2\theta = \pi - 2\delta_j$$

$$\tau_j = \frac{S_p}{2}\sin 2\delta_j$$

$$S_p = \frac{S_v}{1 - R}$$

$$1 - R = \frac{A_p}{A} = \frac{(30)(30)}{(30 + 30)^2}$$

$$\underline{1 - R = 0.25}$$

$$S_p = \left(\frac{156}{144}\right)\left(\frac{980}{0.25}\right)$$

$$\underline{S_p = 4{,}247 \text{ psi}}$$

$$\tau_j = \left(\frac{4{,}247}{2}\right)[\sin(2)(60)]$$

$$\underline{\underline{\tau_j = 1{,}839 \text{ psi}}}$$

← shear strength to prevent slip

$$\sigma_j = \frac{S_p}{2} - \frac{S_p}{2}\cos 2(\pi - 2\delta_j)$$
$$\sigma_j = \frac{S_p}{2} + \frac{S_p}{2}\cos 2\delta_j$$
$$= \frac{4{,}247}{2}[1 + \cos(120)]$$
$$\underline{\underline{\sigma_j = 1{,}062\ \text{psi}}}$$

if $c = 0$ and M-C slip applies to the joints then

$$\tau_j = \sigma_j \tan\phi_j$$
$$\tan\phi_j = \frac{1{,}839}{1{,}062}$$
$$\tan\phi_j = 1.732$$
$$\underline{\underline{\phi_j = 60^\circ}} \qquad \longleftarrow \phi_j(c = 0)$$

if $\phi = 0$ and M-C slip applies then

$$\tau_j = c_j$$
$$\underline{\underline{c_j = 1{,}839\ \text{psi}}} \qquad \longleftarrow c_j(\phi_j = 0)$$

19. Given: Room and pillar mine, 300 m depth, $\gamma = 25\ \text{kN/m}^3$, core: $(L/D) = 2$, $C_0 = 86$ MPa, $T_0 = 7$ MPa

Find:
(a) Formula for max safe extraction ratio
(b) Evaluate.

Solution:

(a) By definition:

$$R = \frac{A_m}{A} = 1 - \frac{A_p}{A}$$
$$A = A_m + A_p$$
$$FS_p = \frac{C_p}{S_p}$$
$$S_p = \frac{S_v}{1 - R}, \quad C_p = C_0 \quad (\text{no size effects})$$
$$\therefore FS_p = \frac{C_0(1 - R)}{S_v}, \quad S_v = \gamma H$$
$$\underline{\underline{R = 1 - (FS_p)\frac{(\gamma H)}{C_0}}} \qquad \longleftarrow \frac{R_{\max}\textit{-safe}}{FS_p = 1 \text{ for } R_{\max}\ (\text{possible})}$$

(b) $R = 1 - \dfrac{(1)(300)(25\,\text{kN/m}^3)}{86\,\text{MPa}}$

where R_{max} is at $FS_p = 1$

$$R_{max} = 1 - 0.0872$$
$$\underline{\underline{R_{max} = 0.913,\ 91.3\%}}$$

$\longleftarrow R_{max}$

20. Given: Problem 19 data, square pillars, entry width = cross cut width = 9 m, $FS = 2$, no size effects.
Find: Pillar size.

Solution:

(1)
$$R = 1 - \frac{(FS_p)(\gamma H)}{C_0}$$
$$R = 1 - \frac{(2)(25)(300)}{86(10)^3}$$
$$\underline{R = 0.826}$$

(2)
$$R = 1 - \frac{A_p}{A}$$
$$0.826 = 1 - \frac{(W_p)^2}{(W_p + 9)^2}$$
$$\therefore \left(\frac{W_p}{W_p + 9}\right) = 0.417\ \&$$
$$\underline{\underline{W_p = 6.44\,\text{m}}}$$

$\longleftarrow W_p$

21. Given: Problems 19, 20 data, pillars 9 × 9 m, $C_p = C_1[0.78 + 0.22\,(W_p/H_p)]$
$H_p(1) = 8\,\text{m}$, $H_p(2) = 27\,\text{m}$
Find: $FS_p(2)$ with size effect and without.

Solution:

$$FS_p = \frac{C_p}{S_p}$$
$$C_0\left(\frac{L}{D} = 2\right) = 86\,\text{MPa}$$
$$\therefore 86 = C_1\left[0.78 + 0.22\left(\frac{1}{2}\right)\right]$$
$$\underline{C_1 = 96.6\,\text{MPa}}$$
$$C_p(2) = 96.6\left[0.78 + 0.22\left(\frac{9}{27}\right)\right]$$

$$\underline{\underline{C_p(2) = 82.4\,\text{MPa}}}$$

$$S_p = \frac{\gamma H}{1-R}$$

$$1-R = \frac{(9)^2}{(9+9)^2} \quad : W_0 = W_c = 9\,\text{m}$$

$$\underline{1-R = 0.25}$$

$$FS_p = \frac{82.4(0.25)}{(25)(300)(10^{-3})}$$

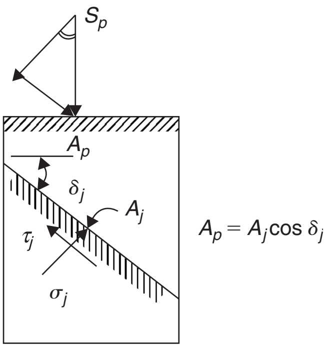
$$\underline{\underline{FS_p = 2.75}} \qquad \leftarrow \underline{\text{with size effect}}$$

$$FS_p = \frac{86(0.25)}{(25)(300)(10^{-3})}$$

$$\underline{\underline{FS_p = 2.87}} \qquad \leftarrow \underline{\text{without size effect}}$$

22. Given: Data from Problems 21 and 19, square pillars 9 m wide, joint data: $\delta = 60°$ (dip), $\alpha = 45°$ (dip direction)
cross-cuts = entry widths = 9 m
depth = 300 m, $1 - R = 0.25$

Find:
(a) shear strength to just prevent slip.
(b) ϕ_j needed when $c_j = 0$.
(c) c_j needed when $\phi_j = 0$.



Solution:

Equilibrium

$$\sigma_j A_j = S_p A_p \cos \delta_j$$

$$\therefore \underline{\sigma_j = S_p (\cos \delta_j)^2}$$

$$\tau_j A_j = S_p A_p \sin \delta_j$$

$$\therefore \underline{\tau_j = S_p \cos \delta_j \sin \delta_j}$$

at slip $\tau_j(\text{strength}) = \tau_j(\text{stress})$

$$S_p = \frac{\gamma H}{1-R}$$

$$= \frac{(25\,\text{kN/m}^3)(300\,\text{m})}{0.25}$$

$$\therefore \underline{S_p = 30\,\text{MPa}}$$

Note:

$$\left.\begin{aligned} 1 - R &= \frac{A_p}{A} \\ &= \frac{(9)^2}{[2(9)]^2} \\ 1 - R &= \frac{1}{4} \end{aligned}\right\}$$

$$\tau_j = (30)(\cos 60)\sin(60)$$

$$\underline{\underline{\tau_j = 13.0\ \text{MPa}}}$$

(a) τ_j ←

(b) $\tau_j = \sigma_j \tan\phi_j + c_j$: strength if $c_j = 0$
then $\tan\phi_j = \tau_j/\sigma_j$

$$\begin{aligned} \sigma_j &= S_p \cos^2\delta_j \\ &= 30\cos^2 60 \\ \sigma_j &= 7.5\ \text{MPa} \\ \tan\phi_j &= \frac{13.0}{7.5} \\ &= 1.732 \\ \underline{\underline{\phi_j}} &\underline{\underline{= 60^\circ}} \end{aligned}$$

(b) ϕ_j ←

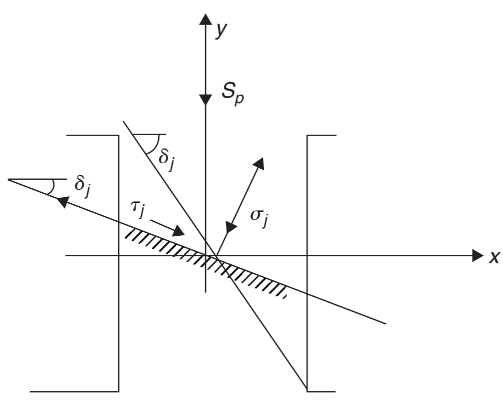
(c) if $\phi_j = 0$ then
$\underline{\underline{c_j = \tau_j = 13.0\ \text{MPa}}}$

(c) c_j ←

23. Given: Pillar stress of 4,247 psi, Mohr–coulomb joint slip criteria
$\phi = 35^\circ$, $c = 600$ psi
Find: Range of unsafe dips.

Solution:

Slip occurs if τ_j(stress) > τ_j(strength) from equations of transformation



$$\begin{aligned} \tau_j(\text{stress}) &= \frac{S_p}{2}\sin 2\delta_j \\ \sigma_j(\text{stress}) &= \frac{S_p}{2}(1 + \cos 2\delta_j) \\ \tau_j(\text{strength}) &= \sigma_j \tan\phi_j + C_j \end{aligned}$$

safe if:

$$\begin{aligned} \frac{S_p}{2}\sin 2\delta_j &< \frac{S_p}{2}(1 + \cos 2\delta_j)\tan\phi_j + c_j \\ \frac{S_p}{2}(\sin 2\delta_j - \cos 2\delta_j \tan\phi_j) &< \frac{S_p}{2}\tan\phi_j + c_j \\ S_p \sin(2\delta_j - \phi_j) &< S_p \sin\phi_j + 2c_j\cos\phi_j \end{aligned}$$

$$\sin(2\delta_j - \phi_j) < \sin\phi_j + \frac{2c_j \cos\phi_j}{S_p}$$
$$< \sin 35^\circ + \frac{(2)(600)(\cos 35)}{4{,}247}$$

$$\sin(2\delta_j - \phi_j) < 0.5736 + 0.2315$$
$$\sin(2\delta_j - \phi_j) < 0.8050$$
$$\text{i.e.}\quad 2\delta_j - \phi_j < 53.6$$
$$2\delta_j < 53.6 + 35$$
$$\underline{\delta_j < 44^\circ}$$

2nd solution using $\sin(\pi - \theta) = \sin\theta$:

$$\sin[\pi - (2\delta_j - \phi_j)] < 0.8050$$
$$180 - 2\delta_j + \phi_j < 44$$
$$180 + 35 - 44 < 2\delta_j$$
$$\underline{\underline{\delta_j > 86^\circ}}$$
$$\therefore\ \underline{\underline{\text{slip if}\quad 44^\circ < \delta_j < 86^\circ}}$$

← range of unsafe dips

check: δ_j at 44°

$$\tau_j = \frac{S_p}{2}\sin 2\delta_j \qquad \underline{\text{e.g. } \delta_j \text{ at } 40^\circ},\quad \underline{\text{at } 88^\circ},\quad \underline{\text{at } 80^\circ}$$
$$\tau_j(\text{stress}) = \frac{4{,}247}{2}\sin 2\delta_j$$
$$\underline{\tau_j(\text{stress}) = 2{,}122\sin 2\delta_j} \qquad \tau_j = 2{,}091\text{ psi},\quad 148\text{ psi},\quad 726\text{ psi}$$

$$\sigma_j(\text{stress}) = \frac{S_p}{2}(1 + \cos 2\delta_j)$$
$$\sigma_j(\text{stress}) = \frac{4{,}247}{2}(1 + \cos 2\delta_j)$$
$$\sigma_j = 2{,}197(1 + \cos 2\delta_j),\quad \sigma_j = 2{,}492,\qquad 5.2,\qquad 128$$

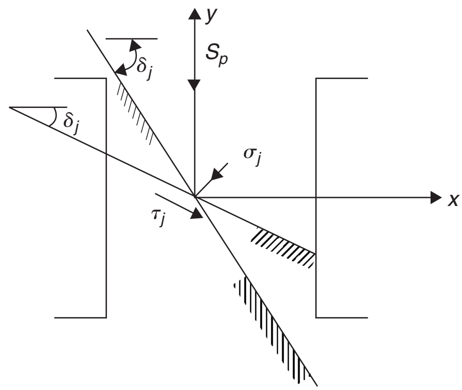
$$\tau_{j(\text{strength})} = \sigma_j \tan\phi + c_j$$
$$= (2{,}197)\tan 35^\circ + 600$$
$$\underline{\tau_{\text{strength}} = 2{,}138}$$
$$\tau_{\text{strength}} = 2{,}138,\quad 2{,}345,\quad 602,\quad 690\text{ psi}$$
$$(\text{safe})\quad (\text{safe})\quad (\text{safe})\quad (\text{unsafe})$$

24. Given: Pillar stress = 29 MPa

Joints: $\delta_j = \text{N}45^\circ\text{W}$
$\delta_j = ?$

MC: $\tau_j = \sigma_j \tan\phi_j + c_j$

$c_j = 4\,\text{MPa}, \phi_j = 35°$



Find: Range of unsafe joint dips δ_j.

Solution:

Slip occurs if τ_j(stress) > τ_j(strength), rotation of axis wrt to $x-y$

$$\tau_j = \frac{S_p}{2}\sin 2\delta_j \;:\text{(stress)}$$
$$\sigma_j = \frac{S_p}{2}(1+\cos 2\delta_j)$$
$$\tau_j = \sigma_j \tan\phi_j + c_j \;:\text{(strength)}$$

Safe if
$$\frac{S_p}{2}\sin 2\delta_j < \frac{S_p}{2}(1+\cos 2\delta_j)\tan\phi_j + c_j$$
$$S_p \sin(2\delta_j - \phi_j) < S_p \sin\phi_j + 2c_j$$
$$\sin(2\delta_j - \phi_j) < \sin\phi_j + \frac{2c_j}{S_p}$$
$$< \sin(35°) + \frac{2(4\,\text{MPa})}{29\,\text{MPa}}$$
$$\sin(2\delta_j - \phi_j) < 0.8494$$
$$2\delta_j - \phi_j < 58.2°$$
$$\underline{\underline{\delta_j < 46.6°}}$$

Second solution:
$\sin(\pi - \theta) = \sin\theta$
$\therefore \pi - 2\delta_j + \phi_j < 46.6$
$84.2 < \delta_j$
$\therefore$ unsafe
$\underline{\underline{46.6° < \delta_j < 84.2°}}$ ← unsafe dip range

25. Given: Cross-cuts 18 ft wide on 120′ centers $L_p = 3W_p$.
Find: Minimum pillar size.

Solution:

"Minimum pillar size occurs at minimum safety factor"

$$FS = \frac{C_0}{S_p}$$
$$S_p = \frac{S_v A}{A_p} \quad \& \quad R = 1 - \frac{A_p}{A}$$
$$FS = \frac{C_0(1-R)}{S_v}$$

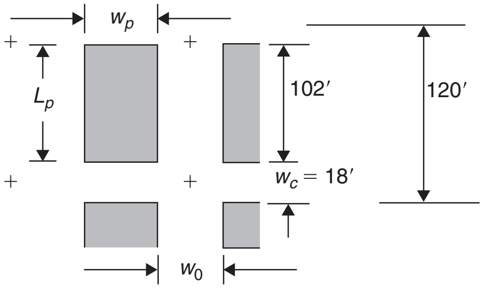
"*FS* is least when *R* is greatest that is when A_p is least"

$$\therefore S_v = C_0(1 - R)$$
$$S_v = (1\text{psi/ft})(1{,}754\,\text{ft}) \text{ assuming 1 psi/ft, consider deepest seam}$$

or

$$S_v = (144)(1{,}694) + (148.2)(1{,}714 - 1{,}694) + (75)(5) + (22)(149.5) + (3)(148.2) + (7)(170)$$
$$S_v = 2.521(10^5)\,\text{psf} = \underline{1{,}751\,\text{psi}}$$

$$\therefore 1 - R = \frac{1{,}751}{3{,}500}$$
$$\underline{\underline{1 - R = 0.50}}$$



But

$$1 - R = \frac{A_p}{A} = \frac{L_p W_p}{(W_p + W_0)(W_c + L_p)}$$

$\underline{\underline{\text{But } L_p = 3W_p}}$

$$0.50 = \frac{3W_p^2}{(W_p + 24)(18 + 3W_p)}$$

$$3W_p^2 + (24)(18) + (72 + 18)W_p = 6.0\,W_p^2$$
$$3.0\,W_p^2 - 90W_p - 432 = 0$$
$$W_p = \frac{-(-90) \pm [(-90)^2 - (4)(3.0)(-432)]^{1/2}}{(2)(3.0)}$$
$$W_p = \frac{90 \pm 115}{6.0}$$
$$W_p = 34.2\,\text{ft}$$
$$L_p = (3)(34.2)$$
$$\underline{\underline{L_p = 102.6\,\text{ft},\ W_p = 34.2\,\text{ft}}}$$

← pillar size

Check:

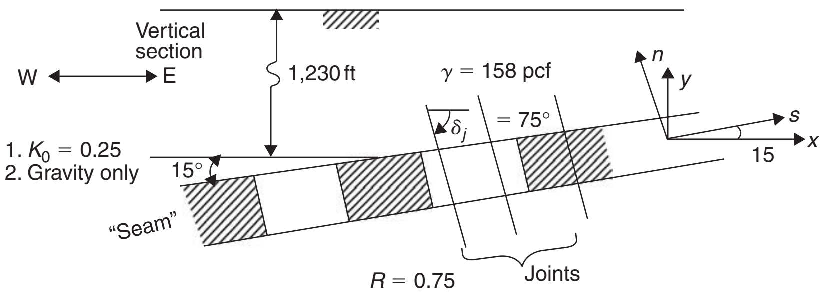
$$1 - R = \frac{A_p}{A} = \frac{(102.6)(34.2)}{(24 + 34.2)(18 + 102.6)}$$
$$\underline{1 - R = 0.50}$$

Note:

$$L_p + 18 = 120$$
$$\therefore \underline{\underline{L_p = 102\text{ ft}}}$$
$$W_p = \frac{102}{3}$$
$$\underline{\underline{W_p = 34\text{ ft}}}$$

← essentially the same at $R_{\max}$, $FS_{\max}$,

26. Given: Hardrock room of pillar data.



Find: If pillars stable?

Solution:

Stable if $FS > 1$, $FS_p > 1$, $FS_j > 1$
Need S_p, T_p where

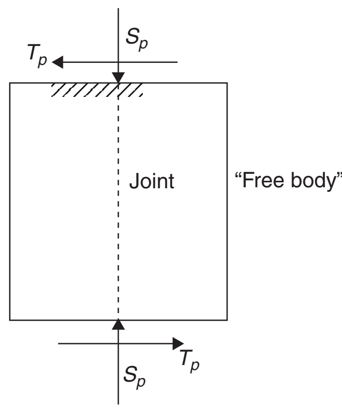
$$S_p = \frac{S_n}{1-R}, \quad T_p = \frac{T_s}{1-R}$$
$$S_p = \frac{S_n}{1-0.75}, \quad T_p = \frac{T_s}{1-0.75}$$
$$\therefore \underline{S_p = 4S_n}, \quad \underline{T_p = 4T_s}$$

$$S_n = \frac{S_x + S_y}{2} + \frac{S_x - S_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$T_s = -\frac{(S_x - S_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\theta = 15^\circ, \quad \tau_{xy} = 0, \quad S_y = \gamma h, \quad S_x = \frac{1}{4}S_y$$

$$S_y = (1{,}230)\left(\frac{158}{144}\right)$$
$$S_y = 1{,}349.6 \approx \underline{1{,}350\text{ psi}}$$
$$S_x = \left(\frac{1}{4}\right)(1{,}349.6) \approx \underline{337\text{ psi}}$$



$$C_0 = \frac{2c\cos\phi}{1-\sin\phi}$$
$$c = \frac{C_0(1-\sin\phi)}{2\cos\phi}$$
$$= \frac{(15{,}000)(1-0.8182)}{(2)(0.5749)}$$
$$\underline{\underline{c = 2{,}372\text{ psi}}}$$

$$S_n = \frac{337 + 1{,}350}{2} - \frac{337 - 1{,}350}{2}\cos 30^\circ$$
$$S_n = 843.7 - (-438.5)$$
$$\underline{\underline{S_n = 1{,}282\ \text{psi}}}$$

$$T_s = -\left(\frac{337 - 1{,}350}{2}\right)\sin 30$$
$$\underline{\underline{T_s = 253\ \text{psi}}}$$

$$S_p = 4(1{,}282)$$
$$\underline{\underline{S_p = 5{,}128\ \text{psi}}}$$

$$T_p = 4(253)$$
$$\underline{\underline{T_p = 1{,}012\ \text{psi}}}$$

Text, P. 294, eqn. 29

$$\underline{\underline{FS_p = \frac{(1-R)\left[\frac{2c\cos\phi}{1-\sin(\phi-\beta)}\right]}{(S_n^2 + T_n^2)^{1/2}}}}$$
$$= (1-0.75)\frac{\left[\frac{(2)(2{,}372)(\cos 54.9)}{1-\sin(54.9-11.2^\circ)}\right]}{(S_n^2 + T_n^2)^{1/2}}$$
$$= (0.25)\frac{\left(\frac{2{,}728}{0.3087}\right)}{(1{,}282^2 + 253^2)^{1/2}}$$
$$= \frac{(0.25)(2{,}728)}{(0.3087)(1.3067)}$$
$$\underline{\underline{FS_p = 1.70}} \quad \therefore \text{stable}$$
$$FS_j = ?$$

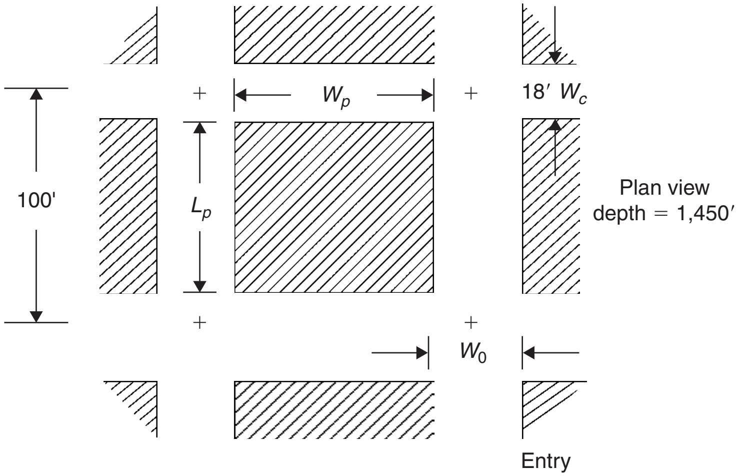
$$\sin\phi = \frac{C_0 - T_0}{C_0 + T_0}$$
$$= \frac{15{,}000 - 1{,}500}{15{,}000 + 1{,}500}$$
$$\sin\phi = 6.8182$$
$$\underline{\underline{\phi = 54.9^\circ}}$$

$$\tan\beta = \frac{\frac{T_p}{2}}{\frac{S_p}{2}}$$
$$= \frac{\frac{1{,}012}{2}}{\frac{5{,}128}{2}}$$
$$\tan\beta = 0.1973$$
$$\beta = 11.2^\circ$$

Note:
Joints are "vertical" and essentially unloaded, therefore "*safe*".

27. Given: Problem 1 data, cross cuts 18′ wide,
100′ c-c, $3W_p = L_p$
Find:
(a) R(max, *safe*).
(b) L_p, W_p when $FS = 1.5$.

Solution:



$$L_p = 3W_p$$

$$FS = \frac{C_p}{S_p}$$

$$C_p = \text{(no size effects, assume)}$$

$$\underline{C_p = 3{,}750} \text{ psi} \quad \text{(Table)}$$

$$S_p = \frac{S_v}{1-R}$$

$$S_v = \gamma h \approx 1{,}450 \text{ psi @ 1 psi/ft depth}$$

$$FS = \frac{3{,}750}{1{,}450}(1-R)$$

for "max. *safe*" R assume $FS = 2$ to 4 @ $FS = 2.0$

$$1 - R = \frac{(2)(1{,}450)}{3{,}750}$$

$$1 - R = 0.773$$

$$\underline{\underline{R = 0.227\ (22.7\%)}} \quad \longleftarrow \text{at max } safe\ R$$

(b) L_p, W_p ? @ $FS = 1.5$

$$1 - R = \frac{(1.5)(1{,}450)}{(3{,}750)}$$

$$1 - R = 0.580$$

$$\underline{R = 0.420\ (42\%)}$$

But also $$FS = \frac{C_p(1-R)}{S_v} \text{ o.k.}$$

And $$1 - R = \frac{A_p}{A} = \frac{L_p W_p}{(L_p + W_c)(W_p + W_0)}$$

With W_c on 100′ centers

$$100 = L_p + W_c$$
$$L_p = 100 - 18$$

$$\left.\begin{aligned} &\underline{\underline{L_p = 82 \text{ ft}}} \\ &W_p = \frac{L_p}{3} \\ &\underline{\underline{W_p = 27\tfrac{1}{3} \text{ ft}}} \end{aligned}\right\} \qquad \longleftarrow L_p, W_p$$

Note: $$\frac{(82)(27\frac{1}{3})}{(82+18)(27\frac{1}{3} + W_0)} = 0.42$$

$$\frac{22.39}{27\frac{1}{3} + W_0} = 0.42$$

$$\underline{\underline{W_0 = 25.97 \text{ ft.}}}$$

28. Given: Dipping seam 15°W
Dipping 75°E

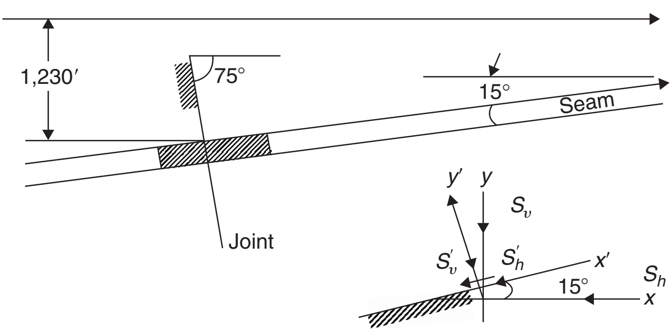
$$\text{depth} = 1230 \text{ ft}$$
$$\bar{\gamma} = 158 \text{ pcf}$$

	C_0	T_0	c	ϕ
Rock	15,000 psi	1,500		
Joint			20 psi	28 deg

$$S_h = \frac{1}{4} S_v \text{ gravity field.}$$

Find:
(a) premining σ, τ.
(b) post-mining S_p, T_p.
(c) $R_{(\text{max,safe})}$ $FS = 1.5$ no size effects on C_p.
(d) τ(joint) for $FS_j = 1.5$.

Solution:



$$\sigma = \frac{S_x + S_y}{2} - \left(\frac{S_x - S_y}{2}\right)\cos 2.15 - \tau_{xy}\sin 2.15$$

$$\tau = -\left(\frac{S_x - S_y}{2}\right)\sin 2.15 + \tau_{xy}\cos 2.15$$

$$S_v = S_y = \left(\frac{158}{144}\right)1{,}230$$

$$\underline{S_y = 1{,}350\text{ psi}}$$

$$S_h = \left(\frac{1}{4}\right)S_v = S_x = \left(\frac{1}{4}\right)(1{,}350)$$

$$\underline{S_x = 337\text{ psi}}$$

$$\sigma = \left(\frac{337 + 1{,}350}{2}\right) - \left(\frac{337 - 1{,}350}{2}\right)\cos 30^\circ - 0$$

$$\sigma = 843 - (-506)(0.868)$$

$$\left.\begin{array}{l} \underline{\underline{\sigma = 1{,}281\text{ psi}}}\ (S_n) \\ \tau = -\dfrac{(337 - 1{,}350)}{2}\sin(30^\circ) \\ \underline{\underline{\tau = 253\text{ psi}}}(T_s) \end{array}\right\} \longleftarrow \text{(a) pre-mining seam normal \& seam shear stresses}$$

(b) $S_p, T_p,\ S_n = \sigma,\ T_s = \tau$

$$\left.\underline{\underline{S_p = \frac{S_n}{1-R},\ T_p = \frac{T_s}{1-R}}}\right\}\text{post-mining} \qquad \longleftarrow \text{(b)}$$

Post mining:

formula, $$\underline{\underline{S_p = \frac{S_n}{1-R},\ T_p = \frac{T_s}{1-R}}}$$

$S_n = \sigma,\ T_s = \tau$
(all that can be said without R)

(c) $R_{\max}$ at $FS = 1.5$ (no size effects)
Text. 294, Eq. (29)

$$FS_p = \frac{(1-R)\left[\frac{2c\cos\phi}{1-\sin(\phi-\beta)}\right]}{[(S_n)^2 + (T_n)^2]^{1/2}} \tag{29}$$

where $$\tan\beta = \frac{\frac{T_p}{2}}{\frac{S_p}{2}}$$

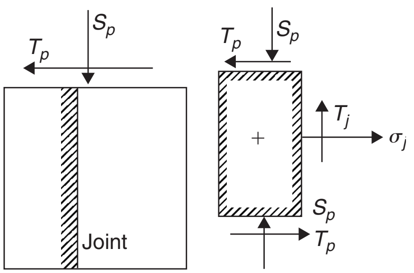
from given

$$c = \left(\frac{C_0}{2}\right)\frac{(1-\sin\phi)}{\cos\phi}$$
$$\sin\phi = \frac{C_0 - T_0}{C_0 + T_0}$$
$$\sin\phi = \frac{15{,}000 - 1{,}500}{15{,}000 + 1{,}500}$$
$$\sin\phi = 0.818^2$$
$$\underline{\underline{\phi = 54.9^\circ}}$$

$$c = \left(\frac{15{,}000}{2}\right)\left(\frac{1-0.8182}{\cos 54.9}\right)$$
$$\underline{\underline{c = 2{,}322 \text{ psi}}}$$

$$\tan\beta = \frac{\frac{T_p}{2}}{\frac{S_p}{2}} = \frac{\frac{T_s}{1-R}}{S_h(1-R)}$$
$$\tan\beta = \frac{253}{1{,}281}$$
$$\tan\beta = 0.1975$$
$$\therefore \underline{\underline{\beta = 11.1^\circ}}$$

$$\therefore 1.5 = (1-R)\frac{\left[\frac{(2)(2{,}372)\cos 54.8}{1-\sin(54.9-11.1)}\right]}{[(1{,}281)^2 + (253)^2]^{1/2}}$$



$$1.5 = (1-R)\frac{8{,}835}{1{,}306}$$
$$\therefore 1 - R = \frac{1.50}{6{,}766}$$
$$1 - R = 0.222$$
$$\underline{\underline{R_{max} = 77.8\%}} \qquad \longleftarrow \text{(c)}$$

Equilibrium indicates joint is not loaded
$\therefore$ any joint strength will do ⟵ (d)

29. Given: Table data, and $W_c = W_0$ (equal crosscut and entry widths) $L_p = 3W_p$ (pillar length = 3 pillar width).
Find:
(1) R_{max} at $FS = 1.4$ no size effect.
Solution:

$$FS = \frac{C_p}{S_p} : C_p = 3{,}000 \text{ psi}$$
$$S_p = \frac{S_v}{1-R} : S_v = \gamma_H$$

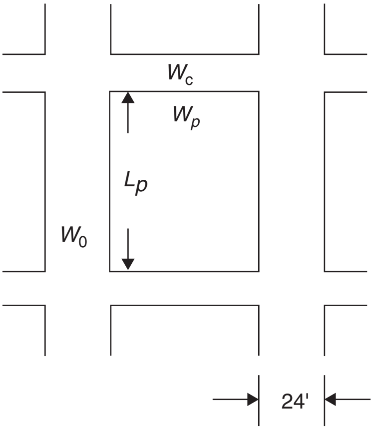
estimate S_v at 1 psi/ft
$S_v = 1{,}670$ psi

$$1.4 = \frac{3{,}000}{\left(\frac{1{,}670}{1-R}\right)}$$
$$1 - R = \frac{(1.4)(1{,}670)}{3{,}000}$$
$$1 - R = 0.779$$
$$\underline{\underline{R_{\max} = 0.221}} \quad 22.1\%$$

$\longleftarrow R_{\max}$

(2) L_p, W_p

Solution:



$$1 - R = \frac{A_p}{A}$$
$$= \frac{L_p W_p}{(W_0 + W_p)(W_0 + L_p)}$$
$$1 - R = \frac{3W_p^2}{(W_0 + W_p)(W_0 + 3W_p)}$$
$$= \frac{3}{\left(\frac{W_0}{W_p} + 1\right)\left(\frac{W_0}{W_p} + 3\right)}$$

$$\left(\frac{W_0}{W_p}\right)^2 + 4\left(\frac{W_0}{W_p}\right) + 3 = \frac{3}{0.779}$$
$$\left(\frac{W_0}{W_p}\right) = \frac{-4 \pm [4^2 + 4(0.851)]^2}{2}$$
$$\frac{W_0}{W_p} = 0.203$$
$$W_p = \frac{24}{0.203}$$
$$\underline{\underline{W_p = 118.5 \text{ ft}}}$$

$\longleftarrow W_p$

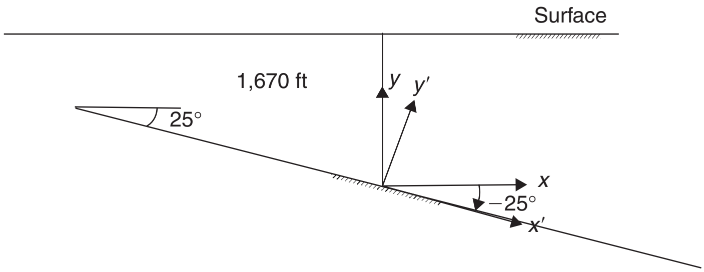
$$L_p = 3W_p$$
$$\underline{\underline{L_p = 355.5 \text{ ft}}}$$

$\longleftarrow L_p$

(3) S_p, T_p
Strata dip = 25°, depth = 1,1670 ft, $R = 20\%$

Solution:

$$S_p = \frac{S_n}{1-R}, \quad T_p = \frac{T_s}{1-R}$$



Equations of rotation

$$S_y = S_n = \frac{S_x + S_y}{2} - \frac{S_x - S_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$T_{xy} = T_p = -\left(\frac{S_x - S_y}{2}\right)\sin 2\theta + \tau_{xy}\cos\theta$$
$$\theta = -25°, \quad S_x = S_h, \quad S_y = S_v, \quad \underline{S_v = 1\text{ psi/ft} = 1{,}670\text{ psi},}$$

estimate $S_h = \dfrac{\nu}{1-\nu}S_v$, assume $\nu = 0.2$, $S_h = \dfrac{1}{4}S_v$
$\tau_{xy} = 0$ (gravity only) $\underline{S_h = 417\text{ psi}}$

$$S_n = \frac{417 + 1{,}670}{2} - \frac{417 - 1{,}670}{2}\cos(-50°) + 0$$
$$S_n = 1{,}044 - (-403)$$

$$S_n = 1{,}446\text{ psi}$$
$$S_p = \frac{S_n}{1-R} = \frac{1{,}446}{1-0.2}$$
$$\underline{\underline{S_p = 1{,}808\text{ psi}}}$$

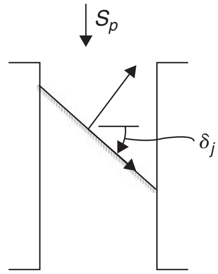
$\overleftarrow{\quad S_p \quad}$

$$T_s = -\left(\frac{417 - 1{,}670}{2}\right)\sin(-50°) + 0$$
$$T_s = -480\text{ psi}$$
$$T_p = \frac{-480}{1-R}$$
$$\underline{\underline{T_p = -600\text{ psi}}}$$

$\overleftarrow{\quad T_p \quad}$

(4) Flat seam $R = 20\%$, $c_j = 0$, $\phi_j = 28°$
Find: Joint dip slip range.

Solution:



slip if $|\tau_j(\text{stress})| > \tau_j(\text{strength})$
rotation

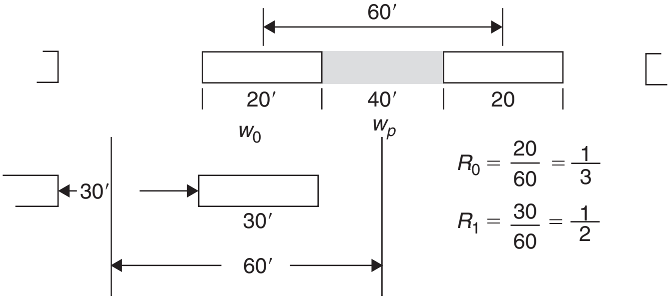
$$\sigma_j = \frac{S_p}{2} - \frac{S_p}{2}\cos 2\delta_j$$
$$\tau_j = \frac{S_p}{2}\sin(-2\delta_j)$$
$$S_p = \frac{S_v}{1-R}$$
$$S_p = \frac{1{,}620}{1-0.2}$$
$$\underline{\underline{S_p = 2{,}088 \text{ psi}}}$$

$\tau_j(\text{strength}) = \sigma_j \tan\phi_j + c_j$

$$FS_j = \frac{\tau_j(\text{strength})}{\tau_j(\text{stress})} = \frac{\sigma_j \tan\phi_j}{\tau_j} = \frac{\frac{S_p}{2}(1-\cos 2\delta_j)\tan\phi_j}{\frac{S_p}{2}\sin 2\delta_j} = \frac{\cos^2\delta_j \tan\phi_j}{\sin\delta\cos\delta_j}$$
$$FS_j = \frac{\tan\phi_j}{\tan\delta_j}$$

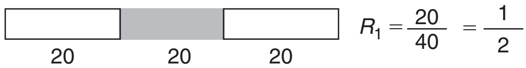
slip where $FS_j < 1$
i.e. $\therefore$ $\underline{\underline{\text{where } \delta_j > \phi_j}}$ ← Frictional slip

30. Proposed increase in R to 50% from 30% by:
(1) Increase W_0/W_p to 1.0 from 0.5 by entry width increase from, 20′ to 30′



(2) Increase W_0/W_p to 1, from 0.5 while maintaining W_p at 40′

(3) Increase W_0/W_p to 1, from 0.5 keeping $W_0 = 20$, decreasing W_p



$$\text{criterion:} \quad FS_c = \frac{C_0}{S_p}, \quad S_p = \frac{S_v}{1 - R}$$

$\therefore$ in all cases FS_c is the same final result.

criterion: $FS_t = \dfrac{T_0}{\sigma_t}$

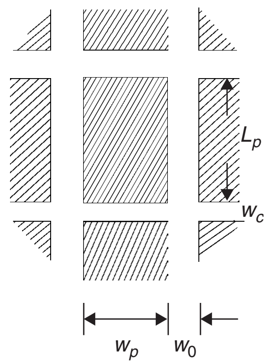
FS_t depends inversely on $\sigma_t \propto L^2$

$\therefore$ keep L small to avoid large σ_t and smaller FS_t

Thus (3) is best.

31. Given: Strata data & table data with $L_p = 2W_p$

Find: R with $FS = 1.8$, no size effects.



Solution:

$$1 - R = \frac{A_p}{A}$$

and also $\quad FS = \dfrac{C_0}{S_p}$

where $\quad S_p = \dfrac{S_v}{1 - R}$

$$FS = \frac{C_0(1 - R)}{S_v}$$

$$1 - R = \frac{(1.8)(1{,}394)}{3{,}000}$$

$$\therefore 1 - R = 0.794$$

$$R = \underline{\underline{20.6\% = 0.206}} \qquad \longleftarrow (1)$$

Find: Pillar dimension, crosscut spacing, entry spacing.

Solution:

$$1 - R = \frac{L_p W_p}{(L_p + W_0)(W_p + W_0)}$$

$$= \frac{3W_p^2}{(3W_p + 21)(W_p + 21)}$$

$$0.794 = \frac{3}{\left(3 + \frac{21}{W_p}\right)\left(1 + \frac{21}{W_p}\right)}$$

$$\therefore \left(\frac{21}{W_p} + 1\right)\left(\frac{21}{W_p} + 3\right) - \frac{5}{0.794} = 0$$

$$\left(\frac{21}{W_p}\right)^2 + 4\left(\frac{21}{W_p}\right) + 3 - \frac{3}{0.794} = 0$$

$$\therefore \left(\frac{21}{W_p}\right) = \frac{-4 \pm \sqrt{4^2 + 4(0.778)}}{2}$$

$$= \frac{-4 \pm 4.372}{2}$$

$$\frac{21}{W_p} = 0.186$$

$$W_p = \frac{21}{0.186}$$

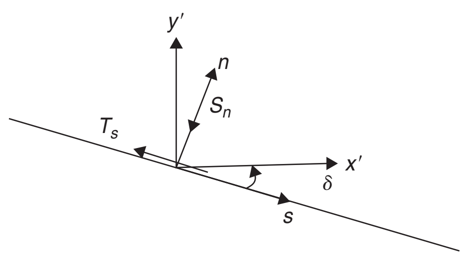
$$\left.\begin{array}{l} \underline{\underline{W_p = 113\,\text{ft}}} \\ L_p = 2W_p \\ \underline{\underline{L_p = 226\,\text{ft}}} \end{array}\right\} \longleftarrow \text{pillar size}$$

$$\text{entry spacing} = 113 + 21 = \underline{\underline{134\,\text{ft}}} \quad \longleftarrow \text{c-c entries (2)}$$

$$\text{cross spacing} = 226 + 21 = \underline{\underline{247\,\text{ft}}} \quad \text{c-c crosscuts}$$

Find: S_p, T_p $\quad R = 20\%$ $\quad \delta = 30^\circ$.

Solution:



$$S_p = \frac{S_n}{1-R}, \quad T_p = \frac{T_s}{1-R}$$

$$S_n = \frac{S_x + S_y}{2} - \frac{S_x - S_y}{2}\cos 2\delta$$

$$T_s = -\left(\frac{S_x - S_y}{2}\right)\sin 2\delta$$

$$S_y = S_v = \gamma h = 1{,}324\,\text{psi}$$

$$S_x = S_n = \frac{\nu}{1-\nu}S_v = \frac{0.3}{1-0.3}S_v = \frac{0.3}{0.7}1{,}324 = 575\,\text{psi}$$

$$S_n = \frac{1{,}324 + 575}{2} - \frac{575 - 1{,}324}{2}\cos 60^\circ$$

$$T_s = -\left(\frac{575 - 1{,}324}{2}\right)\sin 60^\circ$$

$$S_n = 959 + 187 = 1{,}146\,\text{psi} \quad \underline{S_n = 1{,}146\,\text{psi}}$$

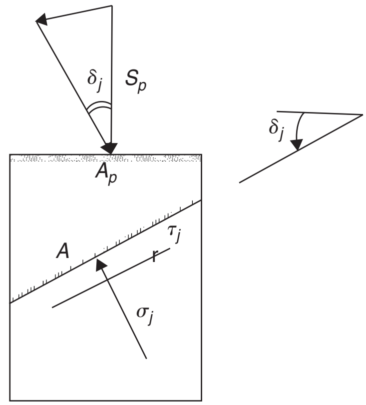
$$T_n = +(324) \quad \underline{T_n = 324\,\text{psi}}$$

$$S_p = \frac{1{,}146}{(1-0.2)} \qquad \underline{\underline{S_p = 1{,}433 \text{ psi}}}$$
$$T_p = \frac{324}{(1-0.2)} \qquad \underline{\underline{T_p = 405 \text{ psi}}}$$

← (3)

Find: Range of joint dips that may slip.

Solution:



$$FS_j = \frac{\tau_j(\text{strength})}{\tau_j(\text{stress})}$$

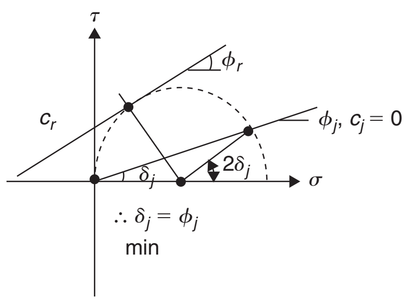
MC: $\tau_j(\text{strength}) = \sigma_j \tan \phi_j + C_j$
Need: σ_j, τ_j

Equilibrium: $\sigma_j = \dfrac{S_p A_p}{A} \cos \delta$

$\tau_j = \dfrac{S_p A_p}{A} \sin \delta$

Geometry: $A_p = A \cos \delta$

$\therefore \underline{\sigma_j = S_p \cos^2 \delta}$
$\underline{\tau_j = S_p \sin \delta \cos \delta}$



$$FS_j = \frac{S_p \cos^2 \delta \tan \phi_j + 0}{S_p \sin \delta \cos \delta}$$

$$FS_j = \frac{\tan \phi_j}{\tan \delta_j}$$

If $FS_j < 1$ then slip

i.e. $$\frac{\tan \phi_j}{\tan \delta_j} < 1$$

or $\tan \delta_j > \tan \phi_j$
Hence, if $\underline{\underline{\delta_j > \phi_j \quad \text{then slip}}}$

← δ_j

32. Given: Mining plan:
B-Level 102 ft, $d = 950$ ft
A-Level 18 ft, $d = 921$ ft
no size effect, ignore joints.

Find:
(a) $R_{\max}$.
(b) show layout, explain.
(c) cite *FS* recommended.

Solution:

(a) criterion

$$FS = \frac{C_p}{S_p}$$
$$C_p = C_0(\text{given})$$
$$S_p = \frac{S_v}{1-R}(\text{flat})$$
$$S_v = 1.1d(\text{given})$$
$$\therefore FS = \frac{C_0(1-R)}{1.1d}$$
$$R = 1 - \frac{(FS)(1.1)d}{C_0}$$

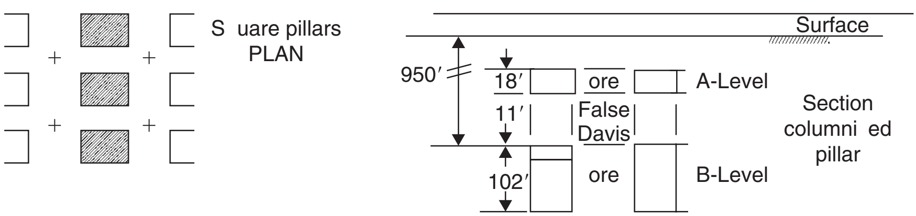
$\therefore R_{\max}$ occurs at $FS_{\min}$ & $FS_{\min} = 1.0$

$$R_{\max} = 1 - \frac{(1.1)(950)}{18{,}270} = 1 - \frac{(1.1)(950)}{5{,}210}$$

pillars in "ore" $C_0 = 18{,}270$ psi, $C_0 = 5{,}210$ psi False Davis

$\underline{\underline{R_{\max} = 0.943}} = \underline{\underline{0.\ 80}}$ ← (a)

(b) Layout, multi-level/rule columnize mains



Suggest $FS = 2.0$ (compression, minimum) because of tall pillars.

33. Given: A-Level roof rock in Bonneterre dolomite and Davis shale

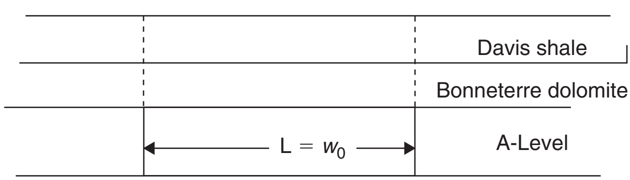
Find:

(a) Max. roof span, (opening, width)

(b) Assume square pillar, find W_p (wrt to ore strength)

(c) Recommended FS for A-Level Roof.

Solution:



Bonneterre: $E = 6.75(10^6)$ psi $\quad h = 6$ ft

Davis: $E = 5.35(10^6)$ psi $\quad h = 150$ ft
Bed separation of Bonneterre (proof)

$$P(1,1) = \frac{166.3(6)}{144}$$
$$\underline{\underline{P(1,1) = 6.93 \text{ psf}}}$$

$$P(1,2) = \frac{6.75(10^6)(6)^3[166.3(6) + 161.3(150)]}{144[6.75(10^6)6^3 + 5.35(10^6)(150)^3]}$$
$$\underline{\underline{P(1,2) = 2.03 \text{ psf},}} \; 0.014 \text{ psi}$$

Beam analyses, assume simply supported

$$\sigma_t = \frac{3}{4}P\frac{L^2}{h^2}$$
$$FS = \frac{T_0}{\sigma_t} = \frac{4T_0h^2}{3PL^2}$$

max. space at min $FS = 1.0$

$$L^2 = \frac{4}{3}\frac{T_0h^2}{P}$$
$$= \frac{\left(\frac{4}{3}\right)(935)(6)^2}{6.93}$$
$$\therefore \underline{\underline{L_{\max} = 80.5 \text{ ft}}} \qquad \longleftarrow \text{(a) max width}$$

(b) W_p(min), square

$$R = 1 - \frac{(FS)(1.1d)}{C_0}$$

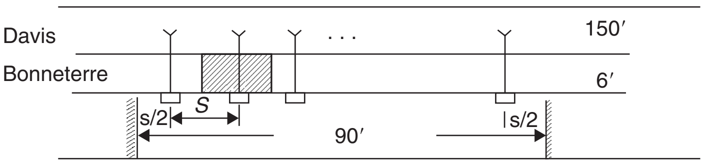
also

$$R = 0.943 \text{ (ore)}$$
$$R = \frac{A_m}{A} = 1 - \frac{A_p}{A}$$
$$1 - R = \left(\frac{W_p}{W_0 + W_p}\right)^2$$
$$\sqrt{0.057} = \frac{W_p}{W_0 + W_p}$$
$$W_p = \frac{(0.239)(80.5)}{1 - 0.239}$$
$$\underline{\underline{W_p = 25.3 \text{ ft} \quad (\min W_p)}} \qquad \longleftarrow \text{(b) min } W_p$$

(c) I would use an $\underline{FS_t = 4.0 \text{ for the roof}}$, no less, because of tensile failure mode.

34. Given: Roof in Problem 32 – A level, 90 ft rooms

Find: Determine a bolting plan solution.



Solution:

Assume: Square pattern spacing S, anchor in Davis shale, allow 1 ft for anchorage, so $\underline{\underline{L_p = 1 + 6 = 7\text{ ft}}}$

Use dead weight approach, bolt force must be at least equal to block weight $W = \gamma S^2 h$

trials: $W = (166.3)(6)(S^2)$ **bolts, text Table 3.4, page 124.**

$$W(5\times 5) = 24{,}945\text{ lbf} \qquad \frac{3}{4}'' \text{ EHS, Grade 75}$$
$$W(4\times 4) = 15{,}965 \qquad \frac{3}{4}'' \text{ HS, Grade 55}$$
$$W(6\times 6) = 35{,}921 \qquad \frac{7}{8}'' \text{ EHS, Grade 75}$$

Select: $\frac{7}{8}''$ diameter, grade 75 bolts, on a 6 × 6 patterns 7 ft long ← (a)

(b) W_p, FS_p?

Given: 90 ft $W_0 = W_c$

Find: W_p, L_p, FS_p

Solution:

Recommended $FS_p = 2.0$ square pillars cross-cuts, have same width as rooms.

ore:

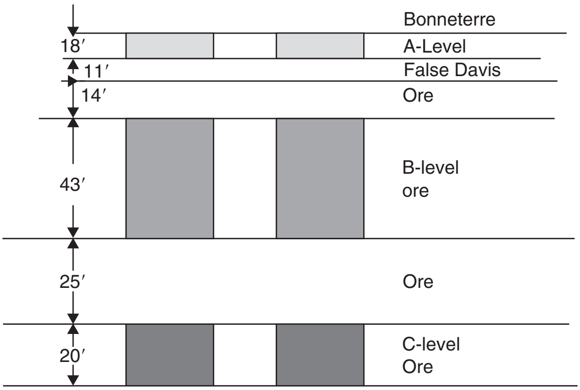
$$1 - R = \frac{(20)(1.1)(950)}{18{,}270}$$
$$1 - R = 0.114$$
$$\underline{\underline{R = 0.886}}$$

also

$$1 - R = \left(\frac{W_p}{W_0 + W_p}\right)^2$$
$$W_p = \frac{0.338(90)}{1 - 0.338}$$
$$\underline{\underline{W_p = 45.95\text{ ft}}}$$

← W_p

35. Given: New mining plan (Problem 32)



Find: L_{max} B-Level.

Solution:

Roof is in ore and has false Davis above, E, h:
False Davis: $3.79(10^6)$ psi, 11 ft
ore: $8.75(10^6)$ psi, 14 ft
ore is thicker and stiffer than the False Davis, so

$$\therefore P(1,2) = \frac{8.75(14)^3(10^6)[218.7(14) + 152(11)]}{(144)[8.75(14)^3(10^6) + 3.79(10^6)(11)^3]}$$

$$P(1,2) = 27.4 \text{ psi}$$

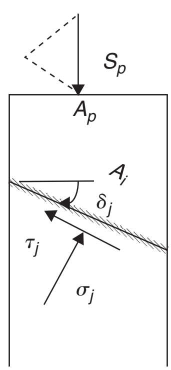
$$\left(\sigma_t = \frac{3}{4}\frac{pL^2}{h^2}, FS = \frac{T_0}{\sigma_t}\right)$$

$$L^2 = \frac{\left(\frac{4}{3}\right)(1{,}000)(14)^2}{27.4}$$

$$\therefore L_{max} = 98 \text{ ft} \qquad \leftarrow L_{max}(\text{B-Level})$$

36. Given: Pillars sized at R_{max} to core,
Find: Joint set 2 *FS*.

Solution:



Set 2

$$A_p = A_j \cos \delta_j$$
$$\delta_j = 30°$$
$$c_j = 10 \text{ psi}$$
$$\phi_j = 30°$$

$$FS_j = \frac{\tau_j(\text{strength})}{\tau_j(\text{stress})}$$

$\tau_j(\text{strength}) = \sigma_j \tan \delta_j + C_j$

Equilibrium

$$\sigma_j A_j = S_p A_p \cos\delta \qquad S_p = \frac{(1.1)(950)}{1-0.943} = 18{,}333\text{ psi}$$

$$\sigma_j = (18{,}333)(\cos 30)^2$$

$$\underline{\sigma_j = 13{,}375\text{ psi}}$$

$$\tau_j = S_p A_p \sin\delta$$

$$= (18{,}333)(\sin 30)(\cos 30)$$

$$\underline{\tau_j = 7{,}938\text{ psi}}$$

$$FS_j = \frac{(13{,}375)\tan(30°) + (10)}{7{,}938}$$

$$\underline{\underline{FS_j = 1.001}}$$

← joint will almost slip

37. Given: Multi-level room and pillar metal mine, table data, geologic column, mining plan.

Find:
(a) $R_{\max}$ with no size effect.
(b) Sketch plan and sections.
(c) Recommend *FS*.

Solution:

$$FS = \frac{C_p}{S_p},$$

$$S_p = \frac{S_v}{1-R}$$

$$\therefore FS = \frac{C_0(1-R)}{S_v}$$

$$R = 1 - \frac{(FS)(S_v)}{C_0}$$

$$\therefore R_{\max} \quad \text{at } FS_{\min}, \quad FS_{\min} = 1$$

$$R_{\max} = 1 - \frac{S_v}{C_0}$$

$S_v = 25d$ (kPa) $\quad d = 290$ m (951 ft)
$S_v = (25)(290)$ $\quad$ (deep)
$S_v = 7.25$ MPa pillars are in ore, $C_0 = 126$ MPa pillars include False Davis, $C_0 = 35.9$ MPa

$$\therefore R_{\max} = 1 - \frac{7.25}{126}$$

$$R_{\max} = 94.2\% \qquad (0.942)$$

$$\underline{\underline{\text{(ore pillars)}}}$$

←

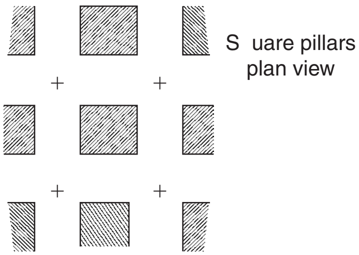
(a) $R_{max} = 0.800 \quad (80\%)$ $\leftarrow R_{max}$

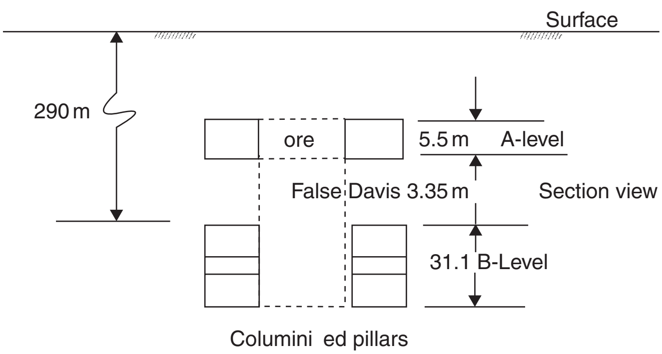
(b) layout

(c) $FS = 2$ recommended for tall pillars formed.

$$R_{max} = 1 - \frac{7.25}{35.9}$$

(in False Davis)





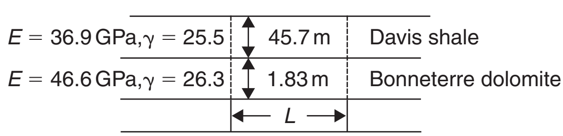
38. Given: Problem 37 data A-level roof rock

Find:

(a) Max. opening width L.

(b) Minimum pillar width W_p (square).

(c) Recommend FS for roof.



Solution:

(a) consider bed separation

$$P(1,1) = 26.3(1.83) = 48.1\,\text{kPa}$$

$$P(1,2) = 46.6\,\text{GPa}(1.83)^3 \frac{[26.3(1.83) + 25.5(45.7)]}{[46.6(1.83)^3 + 36.9(45.7)^3]}$$

$$P(1,2) \cong 0.1\,\text{kPa}$$

$\therefore$ Bed separation occurs with beam action, assume simply supported ends, then

$$\sigma_t = \left(\frac{3}{4}\right) P \left(\frac{L}{h}\right)^2$$

$$\text{At } FS_t = 1,\ \sigma_t = T_0,$$

$$6.45(10^3) = \left(\frac{3}{4}\right)(48.1)(L^2)\left(\frac{1}{1.83}\right)^2$$

$$L^2 = 598.8$$

$$\underline{\underline{L = 24.5\,\text{m}}} \qquad \longleftarrow \text{max roof span (a)}$$

(b) pillars in ore $R = 0.943$ (gives min. W_p)
pillars in $F.D.$ $R = 0.800$

$$1 - R = \frac{W_p^2}{(W_p + W_0)^2}$$

$$0.057 = \left(\frac{W_p}{W_p + 24.5}\right)^2$$

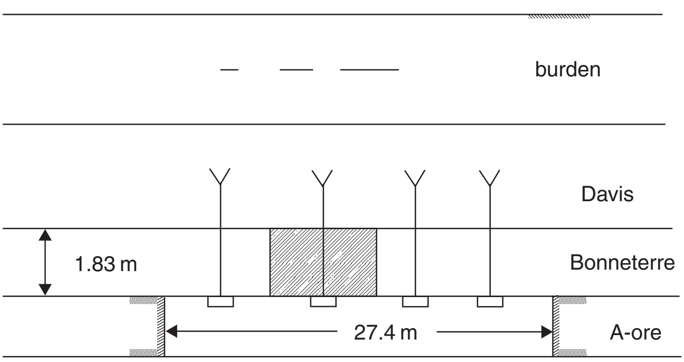
$$\frac{W_p}{W_p + 24.5} = 0.2388$$

$$\underline{\underline{W_p = 7.68\,\text{m}}} \qquad \longleftarrow \text{min } W_p \text{ (b)}$$

(c) I suggest $\underline{\underline{FS_t = 4.0}}$ for roof tension ⟵ (c)

39. Given: Problem 38 data, & rooms 27.4 m wide.
Find:
(a) Bolting plan.
(b) Pillar size & safety factor.

Solution:



assume a square pattern allow for 15.7 cm anchorage length

$$\therefore \underline{\underline{L_b = 2\,\text{m}}} \qquad \longleftarrow L_b$$

assume dead weight support

$$\text{block wt.} \quad W = \gamma S^2 h = F_b \text{ (minimum)}$$
$$W = (26.3)(1.83)S^2$$

Bolts (Table-Text)

$$S = 1.5\,\text{m} \quad W = 108\,\text{kN} \quad \tfrac{7}{8}'' \quad 55\text{Grade}$$
$$S = 2.0\,\text{m} \quad W = 193\,\text{kN} \quad 1\tfrac{1}{2}'' \quad 60 \text{ Grade}$$
$$S = 2.5\,\text{m} \quad W = 300\,\text{kN} \quad 1\tfrac{1}{2}'' \quad 100 \text{ Grade}$$

Try $\frac{7}{8}''$ 55 grade bolts, 1.5 m c-c, 2 m long ⟵

(c) $W_p = 27.4$ m use $FS = 2.0$ for pillars (ore)

$$1 - R = \left(\frac{W_p}{W_p + W_0}\right)^2 \quad : \text{square}$$

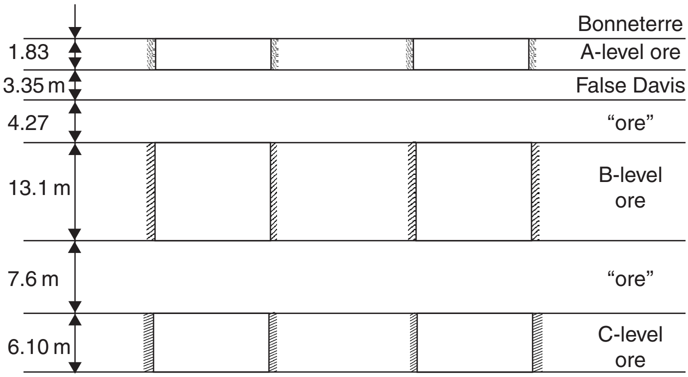
and

$$FS = \frac{C_0(1-R)}{\gamma H}$$

$$(2) = \frac{(126)(1-R)}{(25)(290)}$$
$$1 - R = 0.115$$
$$0.115 = \left(\frac{W_p}{W_p + 27.4}\right)^2$$
$$\frac{W_p}{W_p + 27.4} = 0.339$$
$$W_p = 14.1\,\text{m}$$

⟵ pillar W_p

40. Given: New mining plan (Problem 37)



Find: L_{max} for B-level.

Solution:

	E(GPa)	h(m)
False Davis	26.1	3.35
Ore	60.3	4.27

ore is thicker & stiffer than F.D.

$$\therefore P(1,2) = (60.3)(4.27)^3\left[\frac{35.6(4.27)+24.0(3.35)}{60.3(4.27)^3+26.1(3.35)^3}\right]$$

$$\underline{P(1,2) = 192.2\,\text{kPa}}$$

Assume simply supported ends, so

$$\sigma_t = \frac{3}{4}p\left(\frac{L}{h}\right)^2, \qquad p = 192.2\,\text{GPa} \quad h = 4.27\,\text{m}$$

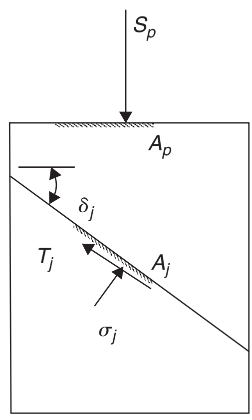
$$L^2 = \frac{(6.94)(10^3)\left(\frac{4}{3}\right)(4.27)^2}{192.2} \qquad FS_t = 1(\max L)$$

$$\underline{\underline{L = 29.6\,\text{m}}} \qquad \longleftarrow L_{max}$$

41. Given: Problem 37 data pillars at max R joint data
Find: If joints in pillar safe (set 2).

Solution:

Set 2



$$A_p = A_j \cos\delta_j$$
$$\delta_j = 30°$$
$$c_j = 0.069\,\text{MPa}$$
$$\phi_j = 30°$$

Criterion: $FS_j = \dfrac{\tau_j(\text{strength})}{\tau_j(\text{stress})}$

$$\tau_j(\text{strength}) = \sigma_j \tan\phi_j + c_j$$

Equilibrium:

$$\sigma_j A_j = S_p A_p \cos\delta_j \qquad S_p = \frac{(25)(290)}{1-0.943}$$

$$\sigma_j = (127.2)\cos^2 30$$

$$\underline{\sigma_j = 95.4\,\text{MPa}} \qquad \underline{S_p = 127.2\,\text{MPa}}$$

$$T_j = S_p \frac{A_p}{A_j} \sin \delta_j$$

$$T_j = (127.2) \cos 30 \sin 30$$

$$\underline{T_j = 55.1\,\text{MPa}}$$

$$FS_j = \frac{(95.4)\tan(30) + 0.069}{55.1}$$

$$\underline{\underline{FS_j = 1.001}}$$

← very near slip

42. Given: data tables, convert units and $FS = 1.75$ wrt $\tau_{\max}$, required for pillars. Find: R (extraction ratio).

- without joints
- no size effect

Solution:

By definition:

$$FS = \frac{\tau_{\max}(\text{strength})}{\tau_{\max}(\text{actual})}$$

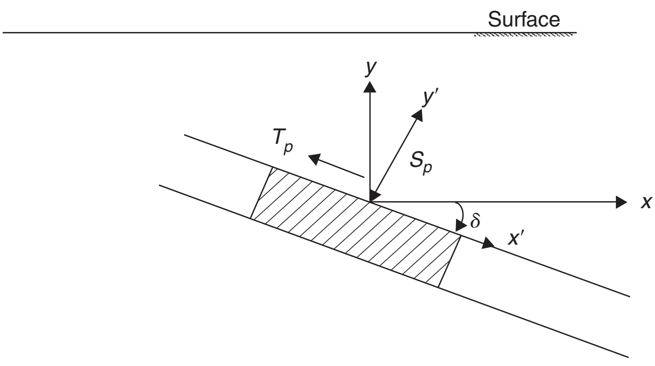
$\tau_{\max}(\text{strength}) = \sigma_m \sin\phi + c\cos\phi$

assuming failure, $\tau_{\max} = (1/2)(\sigma_1 - \sigma_3)$

after mining:

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{x'x'} + \sigma_{y'y'}}{2} \pm \left[\left(\frac{\sigma_{x'x'} - \sigma_{y'y'}}{2}\right)^2 + (\tau_{x'y'})^2\right]^{1/2}$$

$$\sigma_{x'x'} = 0 \qquad \sigma_{y'y'} = S_p \qquad \tau_{x'y'} = T_p$$



extraction ratio formulas

$$S_p = \frac{S_n}{(1-R)}, \quad T_p = \frac{T_s}{(1-R)}$$

where S_n, T_s are premining stresses in $x' - y'$ reference

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{0 + S_n}{2(1-R)} \pm \left\{\left[\frac{-S_n}{2(1-R)}\right]^2 + \left(\frac{T_s}{1-R}\right)^2\right\}^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{1}{1-R}\left\{\frac{S_n}{2} \pm \left[\left(\frac{S_n}{2}\right)^2 + (T_s)^2\right]^{1/2}\right\}$$

$$S_n = \frac{S_x + S_y}{2} - \left(\frac{S_x - S_y}{2}\right)\cos(-2\delta) + T_{xy}\sin(-2\delta)$$

$$S_x = S_h = 600 + 0.25(1{,}750)$$

$$\underline{\underline{S_x = 1{,}038 \text{ psi}}}$$

$$S_y = S_v = 1.05(1{,}750)$$

$$\underline{\underline{S_y = 1{,}838 \text{ psi}}}$$

$$\underline{\underline{T_{xy} = 0.0 \quad \& \quad \delta = 18^\circ}}$$

$$\therefore S_n = \frac{1{,}038 + 1{,}838}{2} - \left(\frac{1{,}038 - 1{,}838}{2}\right)\cos(-36^\circ) + 0.\sin(-36^\circ)$$

$$S_n = 1{,}438 + 324$$

$$\underline{\underline{S_n = 1{,}761 \text{ psi}}}$$

$$T_s = -\left(\frac{S_x - S_y}{2}\right)\sin 2\delta + T_{xy}\cos 2\delta$$

$$= -\left(\frac{1{,}038 - 1{,}838}{2}\right)\sin(-36^\circ) + 0.\cos(-36^\circ)$$

$$\underline{\underline{T_s = -235 \text{ psi}}}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \left(\frac{1}{1-R}\right)\left\{\frac{1{,}761}{2} \pm \left[\left(\frac{1{,}761}{2}\right)^2 + (-235)^2\right]^{1/2}\right\}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \left(\frac{1}{1-R}\right)[881 \pm 911]$$

$$\sigma_1 = \frac{1{,}792}{1-R}$$

$$\sigma_3 = \frac{-30}{1-R}(!)$$

$$\tau_{\max} = \left(\frac{1}{1-R}\right)\left[\frac{1{,}792-(-30)}{2}\right]$$

$$\underline{\underline{\tau_{\max} = \frac{911}{1-R}\text{ psi}}} \qquad \text{(stress)}$$

Need: c, for $\tau_{\max}$(strength)

$$\sin\phi = \frac{C_0 - T_0}{C_0 + T_0} \qquad c = \frac{C_0}{2}\left(\frac{1-\sin\phi}{\cos\phi}\right)$$

$$= \frac{3{,}400-310}{3{,}400+310} \qquad = \left(\frac{3{,}400}{2}\right)\left(\frac{1-\sin 56}{\cos 56}\right)$$

$$\sin\phi = 0.833 \qquad \underline{\underline{c = 508\text{ psi}}}$$

$$\underline{\underline{\phi = 56^\circ}}$$

$$\begin{aligned}\tau_{\max} &= \sigma_m \sin\phi + C\cos\phi\\ &= \left(\frac{\sigma_1+\sigma_3}{2}\right)\sin 56 + 508\cos 56\\ &= \frac{1{,}792-30}{2(1-R)}\sin 56 + 508\cos 56\\ \tau_{\max} &= \frac{730}{1-R} + 284\text{ psi}\end{aligned}$$

$$\begin{aligned}FS &= \frac{\tau_{\max}(\text{strength})}{\tau_{\max}(\text{stress})}\\ 1.75 &= \frac{\left(\frac{730}{1-R}\right)+284}{\left(\frac{911}{1-R}\right)}\\ (1.75)(911) &= 730 + 284(1-R)\\ 1-R &= 3.04\\ R &= 1-3.04\\ &\underline{R = -2.04 \text{ !not possible}}\end{aligned}$$

Note:

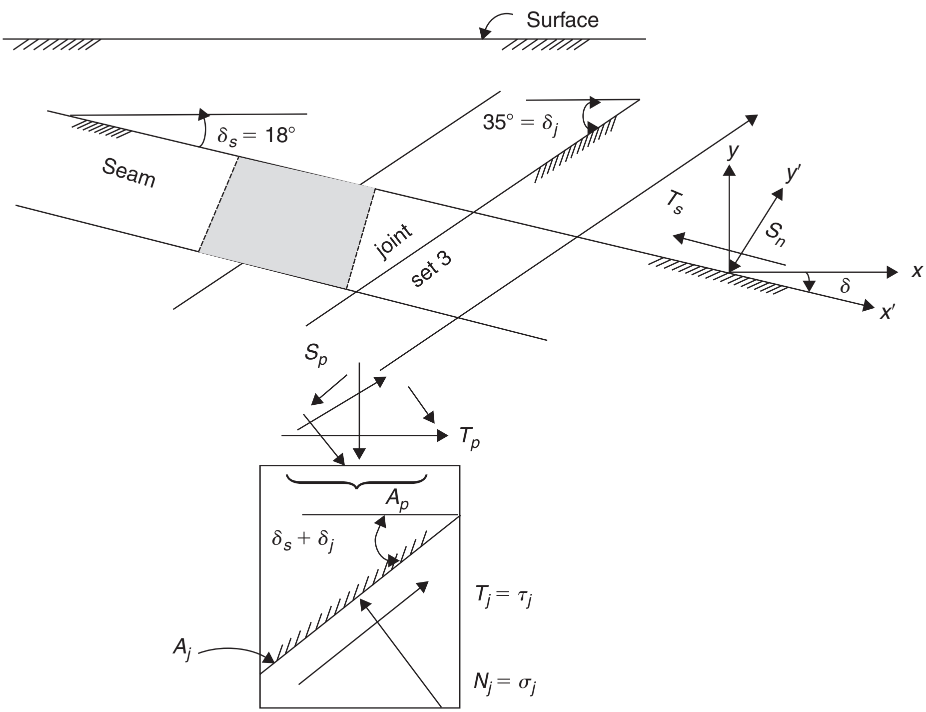
$$\begin{aligned}(FS_{\max})911 &= 730 + (1-R)284\\ FS_{\max} &= 0.801 + (1-R)0.312 \geqq 1.0\\ R &= 0\\ FS_{\max} &= 1.11\\ 1-R \geq \frac{0.179}{0.322} &= 0.638\\ R \leq 1-0.638 &= 0.362\end{aligned}$$

$R = 0.20 \quad FS_{max} = 1.05$
$R = 0.40 \quad FS_{max} = 0.988$
$R = 0.362 \quad FS_{max} = 1.000$

At $\underline{FS = 1.0}$ $\underline{\underline{R_{max} = 0.362(36.2\%)}}$

43. Given: test data, etc. and $FS_j = 1.75$
Find: R_{max} (considering joint set 3 only).

Solution:
$FS_j = \dfrac{\tau_j(\text{strength})}{\tau_j(\text{stress})}$
$\tau_j(\text{strength}) = \sigma_j \tan\phi_j + c_j$
$\tau_j(\text{stress}) = ?$
Need: σ_j, T_j in pillar after mining



Need: S_p, T_p extraction ratio formulas

$$S_p = \frac{S_n}{1-R} \quad T_p = \frac{T_s}{1-R}$$

Need: S_n, T_s (premining)

$$S_n = \frac{S_x + S_y}{2} - \left(\frac{S_x - S_y}{2}\right)\cos(-2\delta_s) - T_{xy}\sin(-2\delta_s)$$

$$S_x = S_h = 600 + 0.25\,\mathrm{d}, \quad \delta = \delta_s = 18^\circ$$

$$S_h = 600 + 0.25(1{,}750), \quad T_{xy} = 0$$

$$\underline{\underline{S_h = 1{,}038\ \text{psi}}}$$

$$S_y = S_n = 1.05d$$

$$= 1.05(1{,}750)$$

$$\underline{\underline{S_n = 1{,}838\ \text{psi}}}$$

$$S_n = \frac{1{,}038 + 1{,}838}{2} - \left(\frac{1{,}038 - 1{,}838}{2}\right)\cos(-2\cdot 18^\circ) - 0$$

$$= 1{,}438 - (-400)(0{,}809)$$

$$\underline{\underline{S_n = 1{,}761\ \text{psi}}}$$

$$T_s = -\left(\frac{S_x - S_y}{2}\right)\sin(-2\cdot 18) + 0$$

$$\underline{\underline{T_s = -235\ \text{psi}}}$$

$$\underline{\underline{S_p = \frac{1{,}761}{1-R}}}$$

$$\underline{\underline{T_p = \frac{235}{1-R}}} \quad \text{(reverse direction for sign change)}$$

Joint – Pillar block equilibrium $\delta = \delta_s + \delta_j = 18 + 35 = 53^\circ$

$$\sum F_n = 0$$

$$0 = N_j, A_j - S_p A_p \cos\delta - T_p A_p \sin\delta$$

$$A_p = A_j \cos\delta$$

$$N_j = S_p \cos^2\delta + T_p \cos\delta \sin\delta$$

$$N_j(1-R) = S_n \cos^2\delta + T_n \cos\delta \sin\delta$$

$$= 1{,}761 \cos^2 53 + 235 \cos 53 \sin 53$$

$$N_j(1-R) = 638 + 113$$

$$\therefore \underline{\underline{N_j = \left(\frac{751}{1-R}\right)}}$$

$$\sum F_s = 0$$

$$0 = T_j A_j - S_p A_p \sin\delta + T_p A_p \cos\delta$$

$$T_j = S_p \cos\delta \sin\delta - T_p \cos^2\delta$$

$$T_j(1-R) = 1{,}761\cos\delta\,\sin\delta - 235\,\cos^2\delta$$
$$= 1{,}761\cos 53\sin 53 - 235\cos^2 53$$
$$= 846 - 85$$
$$T_j(1-R) = 761$$
$$\therefore T_j = \frac{761}{1-R}$$

$$FS_j = \frac{\tau_j(\text{strength})}{\tau_j(\text{stress})}$$
$$= \sigma_j \tan\phi_j + \frac{c_j}{\tau_j}$$
$$= \frac{\left(\frac{751}{1-R}\tan\phi_j + c_j\right)}{\left(\frac{761}{1-R}\right)}$$
$$FS_j = \frac{\left(\frac{751}{1-R}\tan 25^\circ + 20\right)}{\left(\frac{761}{1-R}\right)}$$
$$(761)(1.75) = 751\tan 75^\circ + 20(1-R)$$
$$1 - R = 49.1$$
$$R = -48.1$$

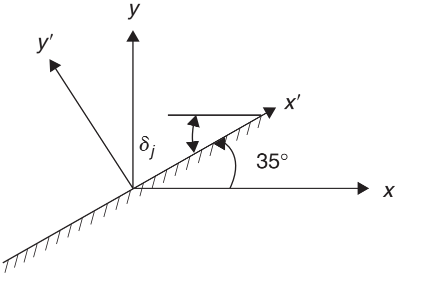
Thus cannot meet requirement $FS_j = 1.75$ ⟵

However, try $FS_j \geq 1.0$ then $761 \geq 751\tan 25^\circ + 20(1-R)$

$$1 - R \geq 2.05$$
$$R \leq -1.05$$

Cannot achieve $FS_j = 1.0$ ⟵
at any extraction, relieves confining pressure and allows joint slip.
Thus must consider premining reinforcement ⟵

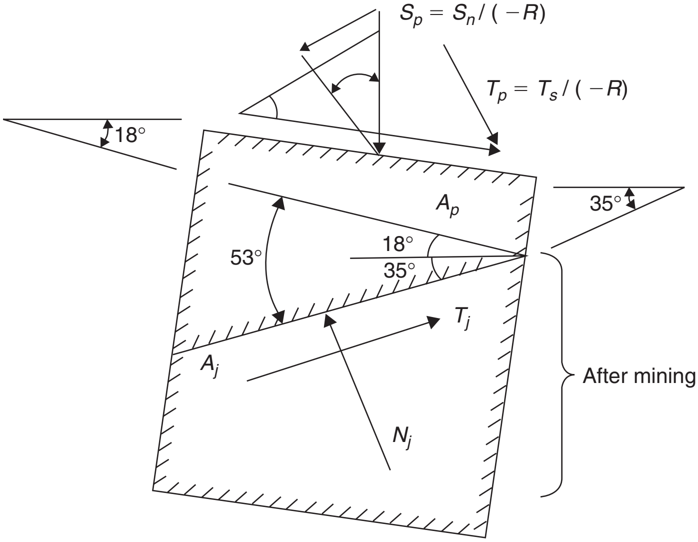
Note: Before mining



$$\sigma_j = \left(\frac{S_x + S_y}{2}\right) - \left(\frac{S_x - S_y}{2}\right)\cos(2\cdot 35) - T_{xy}\sin(+2\cdot 35)$$
$$\sigma_j = 1{,}438 - (-400)0.342 - 0$$
$$\sigma_j = 1{,}575 \text{ psi}$$

$$\tau_j = -\left(\frac{S_x - S_y}{2}\right)\sin(+2\cdot 35)$$
$$= -(-400)(0.940)$$
$$\tau_j = 376 \text{ psi}$$

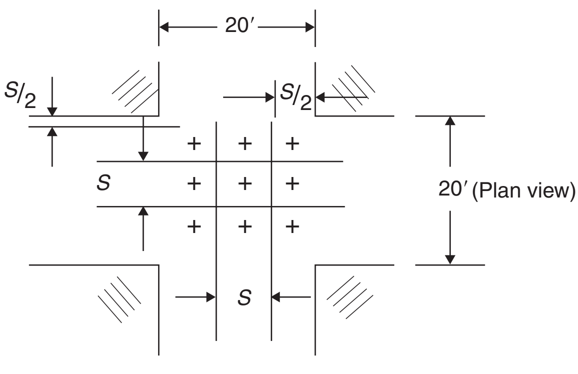
$$FS_j = \frac{\sigma_j \tan\phi_j + c_j}{\tau_j}$$
$$= \frac{1{,}575 \tan 25^\circ + 20}{376}$$
$$\underline{\underline{FS_j = 2.00 \text{ before mining}}}$$



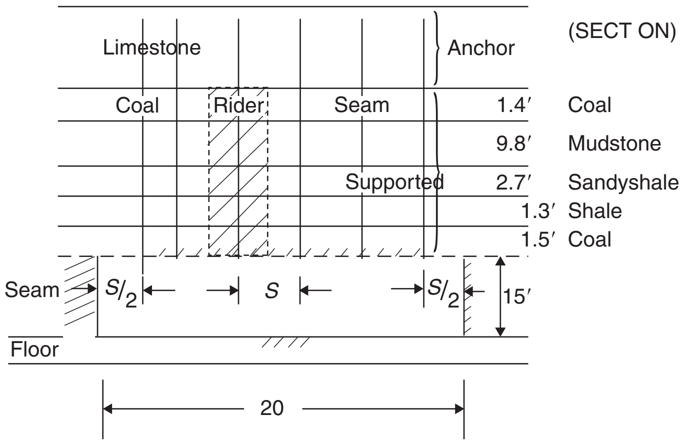
44. Given: Caving to the coal "rider" seam at entry-crosscut intersection 20 ft wide, each. Bond strength = 750 psi, 5/8″ cable bolts

Find: Cable bolting plan to defend against caving.

Solution:



try square pattern S′ C-C with S/2 distance from rib, so number of bolts per row is = 20/S



Dead weight for all beds below limestone:

$$W = [1.5(90) + 1.3(138) + 142(2.7) + 9.8(153) + 93(1.4)]S^2$$

$$\underline{\underline{W = 2{,}206\,S^2}}$$

Bolt force $\underline{\underline{F_b = nf_b}}$ $n =$ bolts per hole

$f_b = 56{,}000$ lbf/bolt

Allowable bolt force

$\underline{\underline{F'_b = \frac{F_b}{FS_b}}}$ $FS_b =$ bolt safety factor

Formula

$$W = F'_b$$

$$2{,}206\,S^2 = \frac{n56{,}000}{FS_b}$$

$$S^2 = \frac{n}{FS_b}25.4$$

try $FS_b = 1.5$ and $n = 2$, then

$$S^2 = \left(\frac{2}{1.5}\right)(25.4)$$

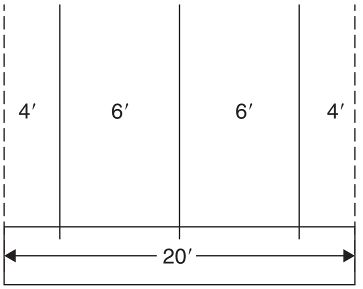
$$S = 5.8'$$

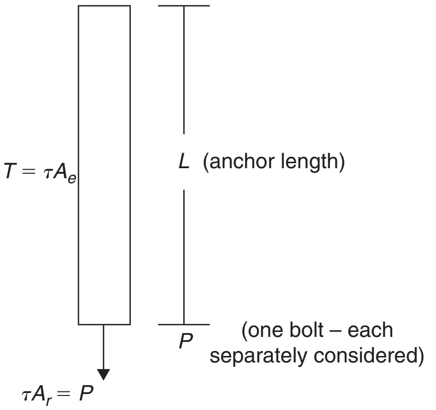
try $S = 5'$ then $FS_b = 2.0$ $(n = 2)$

try $S = 8'$ then $FS_b = 0.79$ $(n = 2)$

try $S = 6'$ then $FS_b = 1.4$ $(n = 2)$

Use $S = 6'$, $n = 2$, $5/8''$ bolt, 56,000 lbf capacity,
Space 4′ from ribs, 6′ between, so there are 3 bolts per row & bolt length = 22 ft.





Bolt length:

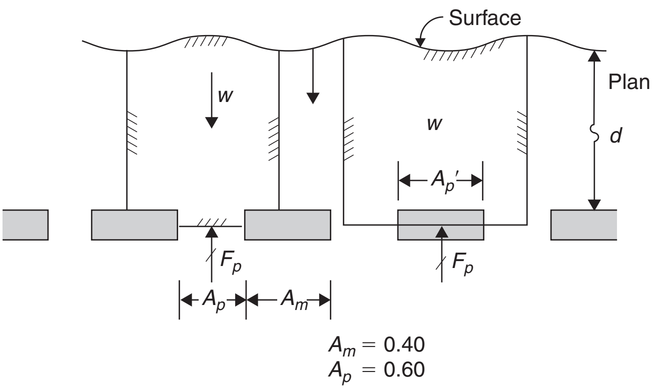
$$(750)(\pi)\left(\frac{5}{8}\right)L = 56{,}000\,\text{lbf}$$

$L = 38.0$ in. at pull out & breakage
if $\tau_{\text{allows}} = \tau(\text{strength})/FS_\tau$
and $FS_\tau = 1.4$ (Same as direct pull FS_b)
then
$L = (38.0)/(1.4)$
$L = 53.2$ in. (easily anchored in limestone)
bolt length $= (53.2/17) + 1.4 + 9.8 + 2.7 + 1.3 + 1.5$, bolt length $= \underline{\underline{21.2'}}$

45. Given: Grouted fill data, $C_0 = 3{,}400$ psi, $E = 100$ psi, $R = 40\%$, old pillars gone

Find: Settlement.

Solution:



Originally: $W = F_p$
Now rooms filled and become pillars

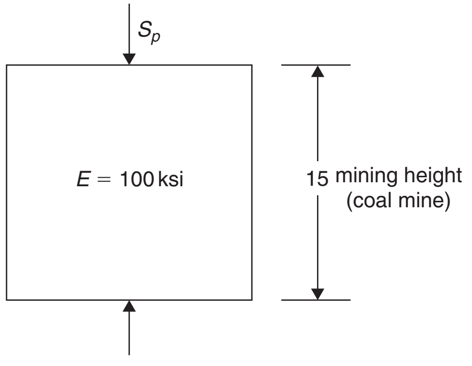
Equilibrium:

$$W = F_p'$$
$$\gamma A d = S_p' A_p'$$

But

$$A_p' = A_m$$
$$\therefore S_p' = \frac{\gamma A d}{A_m} : \frac{A_m}{A} = R$$
$$S_p' = \frac{\gamma d}{R}$$
$$S_p' = \frac{\gamma d}{0.4} = \frac{S_v}{0.4}$$
$$= \frac{1{,}750(1.05)}{0.4}$$
$$\underline{\underline{S_p' = 4{,}594 \text{ psi}}}$$

Note: $C_0 = 3{,}400$ psi!



Hooke's law:

$$E\varepsilon_v = S_p - \nu S_h' - \nu S_H' = E\frac{\Delta L}{L}$$

Assume horizontal stresses are nil after mining, drainage implied

$$\Delta L = \frac{S_p L}{E}$$
$$= \frac{(4{,}594)(15)(12)}{10^5}$$
$$\underline{\underline{\Delta L = 8.3 \text{ ln. } (0.7 \text{ ft})}}$$

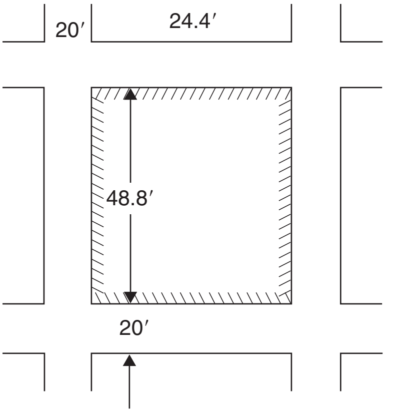
With horizontal confinement by adjacent old pillar

$$\Delta L = \frac{[S_p - \nu(S_h' + S_H')]L}{E}$$

which would make ΔL less.

46. Given: data for strata, table, and $L_p = 2W_p$, $W_0 = W_0$

Find: R_{max}, W_p at $FS = 1.5$, Cross-cut Spacing:



Solution:

$$R = \frac{A_m}{A_p + A_m}$$

$$\& \; \underline{1 - R = 1 - \frac{A_p}{A}}$$

Also

$$FS_c = \frac{C_p}{S_p}$$

$$S_p = \frac{S_v}{1 - R}$$

$$\therefore FS_c = \frac{C_P(1 - R)}{S_v} \quad \text{at } FS = 1.0 \text{ for } R_{max}$$

$$1 - R = \frac{(980)(1.0)}{3{,}750}$$

$$1 - R = 0.26$$

$$\underline{\underline{R_{max} = 0.74}} \qquad \text{(a)} \longleftarrow$$

$$1 - R = \frac{(980)(1.5)}{3{,}750} \quad \text{at } FS = 1.5$$

$$\underline{1 - R = 0.39}$$

$$\underline{\underline{R = 0.61}}$$

But

$$R = 1 - \frac{A_p}{A}$$

$$= 1 - \frac{L_p W_p}{(L_p + W_0)(W_p + W_0)}$$

$$R = 1 - \frac{2W_p^2}{(2W_p + W_0)(W_p + W_0)}$$

$$1 - R = \frac{2}{\left(2 + \frac{W_0}{W_p}\right)\left(1 + \frac{W_0}{W_p}\right)}$$

$$\frac{2}{1 - R} = 2 + \frac{3W_0}{W_p} + \left(\frac{W_0}{W_p}\right)^2$$

$$\left(\frac{W_0}{W_p}\right)^2 + 3\frac{W_0}{W_p} + 2 - \frac{2}{1 - R} = 0$$

$1 - R = 0.39$

$$\left(\frac{W_0}{W_p}\right)^2 + 3\frac{W_0}{W_p} + 2 - \frac{2}{0.39} = 0$$

$$\frac{W_0}{W_p} = -3 \pm \frac{\left[9 - 4\left(2 - \frac{2}{0.39}\right)\right]^{1/2}}{2}$$

$$= \frac{-3 \pm 4.64}{2}$$

$$\frac{W_0}{W_p} = 0.819$$

$$W_p = \frac{W_0}{0.819}$$

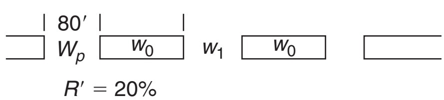
$$= \frac{20}{0.819}$$

$$\underline{\underline{W_p = 24.4\,\text{ft}}} \qquad \longleftarrow W_p(\text{b})$$

$$\text{Checks } 0.61 = 1 - \frac{(24.4)^2(2)}{[(2)(24.4) + 20](24.4 + 20)}$$

$$\text{Crosscut Spacing} = (W_c + W_p)\underline{\underline{= 68.8\,\text{ft}}} \qquad \longleftarrow \text{Spacing (c)}$$

47. Given: Choice of increasing extraction ratio (2D view)
Now:



By definition

$$R' = \frac{W_0}{(W_0 + W_p)}$$

$$0.2 = \frac{1}{1 + \frac{W_p}{W_0}}$$

$$\underline{\underline{W_0 = 20\,\text{ft}}}$$

$$\text{Check } 0.2 \geqq \frac{20}{20 + 80} \text{ O.K.}$$

(1) decrease pillar width W_0 constant
(2) Increase W_0, W_p constant
(3) both.

Criterion: FS_c for pillar;

$$FS_c = \frac{C_p}{S_p}$$
$$S_p = \frac{S_v}{1-R}$$

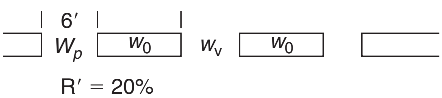
Since R is same at 33% no reason to choose

$$FS_t = \frac{T_0}{\sigma_t}$$
$$\sigma_t = \frac{3}{4}P\left(\frac{L}{h}\right)^2$$

where $L = W_0$ Increasing opening width will reduce roof span safety,
Chose plan (1) to keep W_0 same (other plans increase W_0)

48. Given: Choice of increasing extraction ratio (2D View)

now:



By definition

$$R' = \frac{W_0}{W_0 + W_p}$$
$$0.2 = \frac{1}{1 + \frac{W_p}{W_0}}$$
$$W_0 =$$

Check $0.2 \geqq \dfrac{6.0}{6.0 + 24.0}$ O.K.

(1) decrease pillar width W_0 constant
(2) Increase W_0, W_P constant
(3) both.

Criterion: FS_c for Pillars;

$$FS_c = \frac{C_p}{S_p}$$
$$S_p = \frac{S_v}{1-R}$$

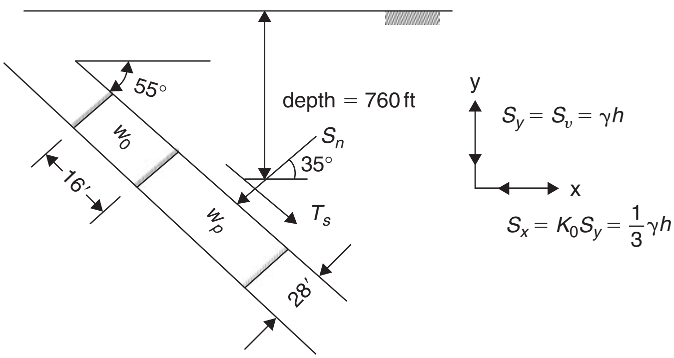
Since R is same 33%, no reason to choose

$$FS_t = \frac{T_0}{\sigma_t}$$

$$\sigma_t = \frac{3}{4}P\left(\frac{L}{h}\right)^2$$

where $L = W_0$ increasing opening width will reduce roof span safety,
Chose plan (1) to keep W_0 same (other plans increase W_0)

49. Given: Steeply dipping anthracite mine



(assume $K_0 = \frac{1}{3}$ based on gravity alone.)
∴ 2D-view
Pillar $C = C_1$ (0.78 + 0.22 D/L)
$C_1 = 5{,}750$ psi
M-C $c = 1{,}210$ psi $\phi = 35°$
Required: $FS = 2.75$
Find: W_p, R.

Solution:

$$FS = \frac{C_p}{S_p}, \quad S_p = \frac{S_n}{1-R}$$

$$FS = \frac{C_p(1-R)}{S_n}$$

$$1 - R = (S_n)\frac{(FS)}{C_p}$$

$$\begin{cases} S_n = \dfrac{S_x + S_y}{2} + \dfrac{S_x - S_y}{2}\cos 2\theta + T_{xy}\sin 2\theta \\ T_s = -\left(\dfrac{S_x - S_y}{2}\right)\sin 2\theta + T_{xy}\cos 2\theta \end{cases}$$

$$S_n = \frac{1}{2}(S_y)\left(\frac{1}{3}+1\right) + \left(\frac{1}{3}-1\right)\frac{S_y}{2}\cos 2\cdot 35$$

$$S_n = \frac{2}{3}(760) + \left(-\frac{1}{3}\right)(760)0.34$$

$$\underline{\underline{S_n = 420\text{ psi}}}$$

$$T_s = +\frac{1}{3}(760)\sin 2\cdot 35$$

$$\underline{\underline{T_s = 238\text{ psi}}}$$

Assume, neglect size effect, then
(assume $L = D$)
$C_p = 5{,}250$ psi

$$1 - R = \frac{\left(\frac{2.75}{420}\right)}{5{,}250}$$

$$1 - R = 0.22$$

$$\underline{\underline{R = 0.78}} \quad \longleftarrow \frac{\text{Max } \textit{Safe } R}{\text{(no size effect)}}$$

$$R = \frac{W_0}{W_0 + W_p}$$

$$W_p = \frac{W_0}{R} - W_0 = 16\left(\frac{1-R}{R}\right) = 16\left(\frac{0.22}{0.78}\right)$$

$$\underline{\underline{W_p = 4.5\text{ ft}}} \quad \longleftarrow W_p\text{(no Size effect)}$$

This suggests pillars are relatively high (28 ft) and thus will show only a small size effect that will reduce R somewhat with size effect (2D view).

$$FS = \frac{C_p(1-R)}{S_n}$$

$$C_p = C_1\left(0.78 + \frac{0.22 W_p}{H_p}\right)$$

$$1 - R = \frac{(S_n)(FS)}{C_1}\left(0.78 + \frac{0.22\, W_p}{H_p}\right)$$

$$\left(1 - \frac{W_0}{W_0 + W_p}\right)\left(C_1\, 0.78 + \frac{C_1\, 0.22}{H_p} W_p\right) = \frac{S_n}{FS}$$

$$W_p\left(C_1\, 0.78 + \frac{C_1\, 0.22\, W_p}{H_p}\right) = (S_n)(FS)(W_0 + W_p)$$

$$W_p^2\left(\frac{C_1\, 0.22}{H_p}\right) + W_p[C_1\, 0.78 - FS(S_n)] - S_n(FS)W_0 = 0$$

$$W_p^2 a + W_p b + C = 0$$
$$a = \frac{(5{,}250)(0.22)}{28} = 41.25$$
$$b = (5{,}250(0.78) - 2.75(420)) = 2{,}940$$
$$c = (420)(2.75)(16) = 18{,}480$$

$$W_p = -\frac{2{,}940 \pm [(2{,}940)^2 + (4)(41.25)(18{,}480)]^{1/2}}{(2)(41.25)}$$
$$W_p = -35.6 \pm 41.4$$
$$\underline{\underline{W_p = 5.8\text{ ft}}} \quad \longleftarrow W_p\text{(With Size effect)}$$

Note if no size effect, then $0.22 \to 0$, $0.78 \to 1$
Coefficient of $W_p^2 \to 0$, i.e., $a \to 0$, $b \to C_1 - (FS)(S_n)$, $c \to$ same

$$W_p = \frac{(FS)(S_n)W_0}{C - (FS)(S_n)}$$
$$= \frac{(2.75)(420)(16)}{5{,}250 - (2.75)(420)}$$
$$\underline{\underline{W_p = 4.5\text{ ft}}}\text{ checks} \quad \longleftarrow \text{no Size effect}$$

50. Given: Room and pillar limestone mine, depth = 845 ft, Joints: $c = 630$ psi, $\phi = 40°$, $H_p = 95$ ft, $C_0 = 21{,}500$ psi (rock), $\gamma = 156$ pcf, $FS = 2.5$ required
Find: If can meet FS required.

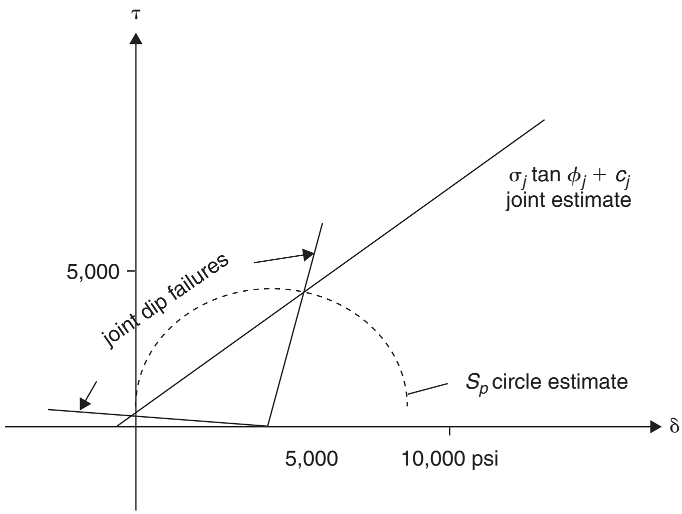
Solution:

For pillar rock

$$S_p = \frac{S_v}{1 - R}$$
$$FS = \frac{C_p}{S_p}$$
$$FS = \frac{C_p(1 - R)}{S_v}$$
$$1 - R = \frac{(845)(2.5)}{21{,}500}$$
$$1 - R = 0.098$$
$$\underline{\underline{R = 90.2\%}}$$

$$S_p = \frac{(845)}{0.098}$$
$$\underline{\underline{S_p = 8{,}622\text{ psi}}} \quad \longleftarrow \text{(rock pillars okay)}$$

Estimate of range of joint dip failures if any



Joints may fail at adverse dips.

51. Given: Room of pillar limestone mine
$H_p = 29$ m, $h = 258$ m (depth)
Rock: $E = 77.9$ GPa, $v = 0.20$
$C_0 = 148$ MPa, $T_0 = 11.6$ MPa
$\gamma = 24.7$ kN/m^3
Joints: $c = 4.34$ MPa, $\phi = 40°$
Find: If $FS = 2.5$ for pillars possible.

Solution:

$$FS = \frac{C_p}{S_p}, \quad S_p = \frac{S_v}{1-R}$$

(a) Rock $1 - R = \dfrac{(2.5)(24.7\,\text{kN/m}^3)(258)}{148\,\text{MPa}}$

$1 - R = 0.108$

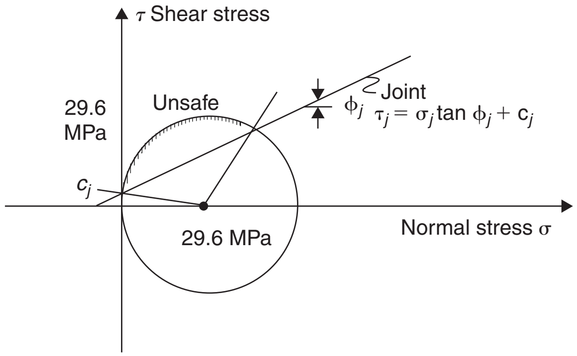
$\underline{\underline{R = 0.89}}$ ← O.K. (rock) (a)

$S_p = \dfrac{(24.7)(258)}{0.108}$

$\underline{\underline{S_p = 59.2\,\text{MPa}}}$

$C_0 = 148$ MPa

(b) Joint



Joints will fail at adverse dip. ← (b)

52. Given: Data, lower seam mining
Find: Maximum extraction ratio.

Solution:

$$FS = \frac{C_p}{S_p}$$

$$S_p = \frac{S_c}{1-R}, \quad S_v = \overline{\gamma}H$$

$$\therefore FS = \frac{C_p(1-R)}{S_v}$$

R is max at min $FS(=1)$

$$\therefore 1 - R = \frac{S_v}{C_p}, \quad S_v \text{ at } 1\ \text{psi/ft}$$

$$= \frac{1{,}747}{3{,}500}$$

$$1 - R = 0.499$$

$$R_{\max} = 0.501$$

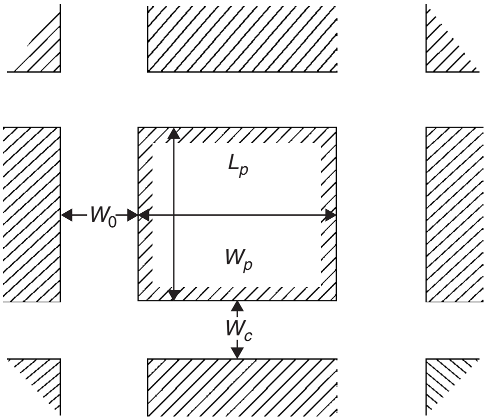
← $R_{\max}$

53. Given data:

$FS = 1.5$ wrt C_p
$W_0 = W_c = 18$ ft
$W_p = \frac{1}{2}L_p, \quad L_p = 2\ W_p$
no size effect

Find:
(a) $W_p,\ L_p$
(b) entry and crosscut spacing.

Solution:



(1) $FS = \dfrac{C_p(1-R)}{S_v}$

(2) $1-R = \dfrac{A_p}{A}$

$$1-R = \frac{(1.5)(1{,}747)}{3{,}500}$$

$$\therefore \underline{1-R = 0.75}$$

$$\frac{A_p}{A} = \frac{W_p L_p}{(W_0+W_p)(W_0+L_p)}$$

$$\therefore 0.75 = \frac{2W_p^2}{(18+W_p)(18+2W_p)}$$

$$0.75 = \frac{2}{\left(\frac{18}{W_p}+1\right)\left(\frac{18}{W_p}+2\right)}$$

$$\left(\frac{18}{W_p}\right)^2 + 3\left(\frac{18}{W_p}\right) + 2 = \frac{8}{3}$$

$$\frac{18}{W_p} = \frac{-3 \pm \sqrt{(3)^2 + 4\left(\frac{2}{3}\right)}}{2}$$

$$\frac{18}{W_p} = \frac{-3 \pm 3.42}{2}$$

$$\frac{18}{W_p} = 0.208$$

$$\left.\begin{array}{l}\underline{\underline{W_P = 86.6\ \text{ft}}}\\ \underline{\underline{L_p = 173.2\ \text{ft}}}\end{array}\right\} \longleftarrow \text{(a)}$$

Check $\dfrac{A_p}{A} = \dfrac{(86.6)(173.2)}{(18+86.6)(18+173.2)}$

$\dfrac{A_p}{A} = 0.75$ checks

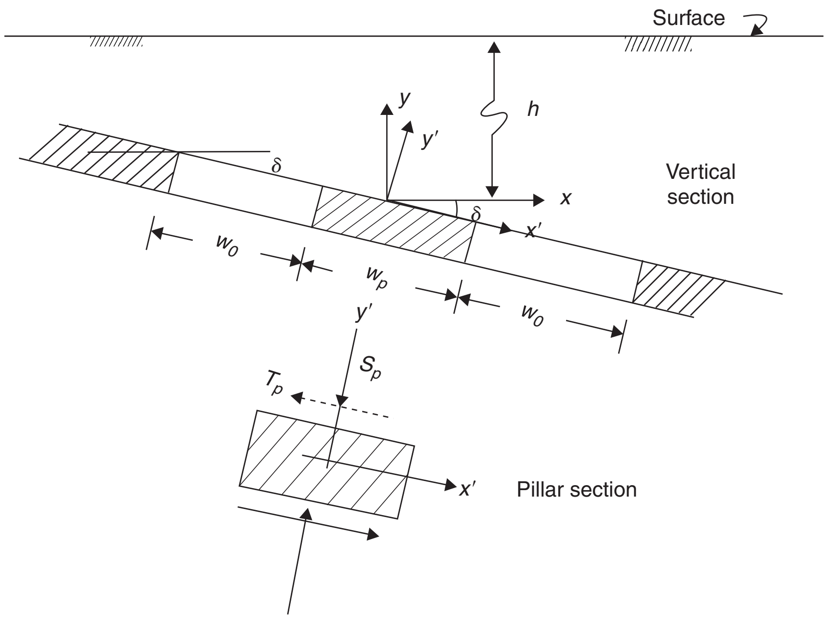
Entry spacing $= W_p + W_0$
$= 86.6 + 18$
Entry spacing $= 104.6$ ft

Crosscut spacing $= W_c + L_p$
$= 18 + 173.2$
Crosscut spacing $= 191.2$ ft

54. Given tabular ore body: strike = due North
dip = 22° East, gravity premining stress, depth = 2,590 ft, lane and pillar (2D) mining rooms:
$W_0 = 52$ ft, $C_0 = 18{,}500$ psi, $T_0 = 1{,}650$ psi, $E = 11\ (10^6)$ psi, $\nu = 0.19$
FS wrt $\tau_{max} = 1.5$ required
(no size effect on strength)

Find:
(a) extraction ratio R.
(b) pillar width.

Solution:



By definition:

$$FS = \frac{\tau_{max}(\text{strength})}{\tau_{max}(\text{stress})}$$

and $\tau_{max} = \dfrac{1}{2}(\sigma_1 - \sigma_3)$

after mining: $\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \dfrac{\sigma_{x'x'} + \sigma_{y'y'}}{2} \pm \sqrt{\left(\frac{\sigma_{x'x'} - \sigma_{y'y'}}{2}\right) + \tau_{x'y'}^2}$

After mining in pillar coordinates (x', y')

$$\begin{aligned}\sigma_{x'x'} &= 0\\ \sigma_{y'y'} &= S_p\\ \tau_{x'y'} &= T_p\end{aligned}$$

Extraction ratio formulas:

$$S_p = \frac{S_n}{1-R}$$

$$T_p = \frac{T_s}{1-R}$$

Transformation formulas from $(x,\ y)$ to $(x',\ y')$

$$S_n = S_{y'y'} = \frac{S_x + S_y}{2} - \left(\frac{S_x - S_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \left(\frac{S_h + S_v}{2}\right) - \left(\frac{S_h - S_v}{2}\right)\cos(-2\cdot\delta) + 0.\sin(2\delta)$$

$$\delta = 22^\circ$$

$$S_v = \gamma h$$

$$S_h = \frac{\nu}{1-\nu} s_v \text{ : assuming complete lateral restraint under gravity only}$$

$$\underline{S_v = 2{,}590 \text{ psi @ } 144 \text{ pcf}}$$

$$S_h = \frac{0.19}{1-0.19}(2{,}540)$$

$$\underline{S_h = 608 \text{ psi}}$$

$$S_n = \frac{2{,}590 + 608}{2} - \left(\frac{-2{,}590 + 608}{2}\right)\cos(-44^\circ) + 0$$

$$S_n = 1{,}599 + 713$$

$$\underline{\underline{S_n = 2{,}312 \text{ psi}}}$$

$$T_s = -\left(\frac{S_x - S_y}{2}\right)\sin(2\cdot\delta) + 0$$

$$= -\left[\left(\frac{-2{,}590 + 608}{2}\right)\sin(-44^\circ)\right]$$

$$\underline{\underline{T_s = -688 \text{ psi}}}$$

$$\sigma_{x'y'} = 0 \qquad \sigma_{y'y'} = S_p \qquad \tau_{x'y'} = T_p$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{0 + S_p}{2} \pm \left[\left(\frac{0 - S_p}{2}\right)^2 + T_p^2\right]^{1/2}$$

$$= \frac{1}{2}\left(\frac{S_n}{1-R}\right) \pm \left[\left(\frac{S_n}{2}\right)^2\left(\frac{1}{1-R}\right)^2 + \frac{T_s^2}{(1-R)^2}\right]^{1/2}$$

$$= \left(\frac{1}{1-R}\right)\left[\frac{S_n}{2} \pm \left(\frac{S_n}{4} + T_s^2\right)^{1/2}\right]$$

$$= \frac{1}{1-R}\left\{\frac{2{,}312}{2} \pm \left[\frac{(2{,}312)^2}{4} + (688)^2\right]^{1/2}\right\}$$

$$\left.\begin{array}{l}\sigma_1\\ \sigma_3\end{array}\right\} = \left(\frac{1}{1-R}\right)(1{,}156 \pm 1{,}345)$$

$$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_3)$$

$$\tau_{\max} = \left(\frac{1}{1-R}\right)\left(\frac{1}{2}\right)(1{,}345 + 1{,}345)$$

$$\underline{\underline{\tau_{\max} = \left(\frac{1{,}345}{1-R}\right)}} : \underline{\underline{\text{stress}}}$$

$$\tau_{\max}(\text{strength}) = \sigma_m \sin\phi + c\cos\phi$$

$$\sin\phi = \frac{C_0 + T_0}{C_0 + T_0}$$

$$= \frac{18{,}500 - 1{,}650}{18{,}500 + 1{,}650}$$

$$\underline{\underline{\sin\phi = 0.8362, \quad \phi = 57^\circ}}$$

$$c = \frac{1}{2}C_0(1 - \sin\phi)$$

$$= \left(\frac{1}{2}\right)(18{,}500)\frac{(1 - 0.8362)}{\cos(57^\circ)}$$

$$\underline{\underline{c = 2{,}740 \text{ psi}}}$$

$$\sigma_m = \frac{\sigma_1 + \sigma_3}{2} = \frac{2S_n}{2(1-R)} \cdot \frac{1}{2}$$

$$\underline{\underline{\sigma_m = \frac{2{,}312}{2(1-R)} = \frac{1{,}156}{1-R} \text{ psi}}}$$

$$\tau_{\max}(\text{strength}) = \frac{1{,}156}{1-R}\sin(57^\circ) + (2{,}740)\cos(57^\circ)$$

$$\underline{\underline{\tau_{\max}(\text{strength}) = \frac{969}{1-R} + 1{,}492 \text{ psi}}}$$

$$FS = \frac{\tau_m(\text{strength})}{\tau_m(\text{stress})}$$

$$(1.5) = \frac{\left(\frac{969}{1-R} + 1{,}492\right)}{\left(\frac{1{,}345}{1-R}\right)}$$

$$\therefore 1.5 = \frac{969}{1{,}345} + \frac{1{,}492}{1{,}345}(1 - R)$$
$$1 - R = 0.703$$
$$\underline{\underline{R = 0.297}} \qquad \longleftarrow \text{(a)}$$

$$R = \frac{A_m}{A_{\text{Total}}}$$
$$R = R = \frac{W_0}{W_0 + W_p}$$
$$W_p = \frac{W_0}{R} - W_0$$
$$= \frac{52}{0.297} - 52$$
$$\underline{\underline{W_p = 122\,\text{ft}}} \qquad \longleftarrow W_p\text{(b)}$$

55. Given: Data from problem 1 and joints – strike due north dip 45° East $\phi_j = 18°$
Find: c_j for $FS_j = 1.5$ wrt τ.

Solution:

From sketch and equilibrium requirements:

$$\sum F_n = 0$$
$$0 = N_j A_j - S_p A_p \cos\delta + T_p A_p \sin\delta$$

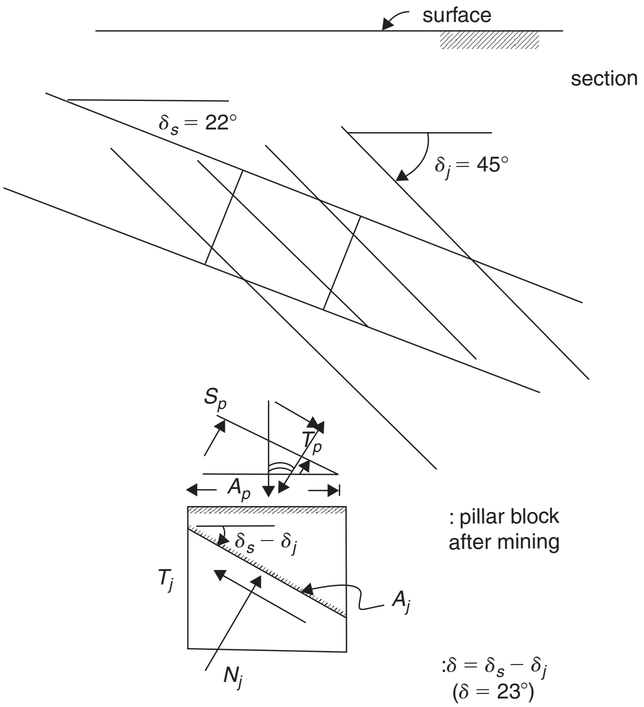
$$A_p = A_j \cos\delta$$
$$\underline{\underline{N_j = S_p \cos^2\delta - T_p \cos\delta \sin\delta}}$$

$$\sum F_s = 0$$
$$0 = T_j A_j - S_p A_p \sin\delta - T_p A_p \cos\delta$$
$$\underline{T_j = S_p \cos\delta \sin\delta + T_p \cos^2\delta}$$

$S_p = 2{,}312$ psi $\quad T_p = +688$ psi (per diagram)

$$N_j = (2{,}312)\cos^2(23°) - (688)(\cos 23)\sin(23)$$
$$\underline{N_j = 1{,}716\text{ psi}}$$

$$T_j = (2{,}312)\cos 23 \sin 23 + 688 \cos^2 23$$
$$\underline{T_j = 1{,}414\text{ psi}}$$



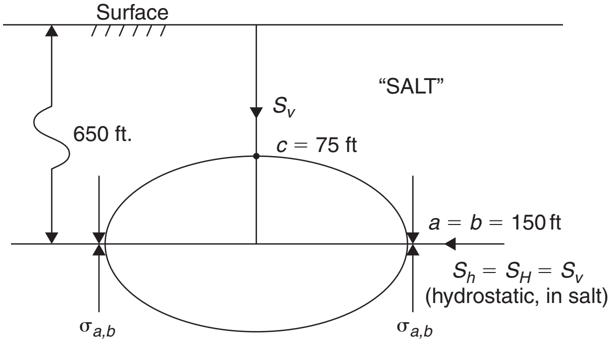
$$
\begin{aligned}
FS_j &= \frac{(\sigma_j \tan \phi_j + c_j)}{\tau_j} \\
c_j &= FS_j \tau_j - \sigma_j \tan \phi_j \\
&= (1.5)(1{,}414) - 1{,}716 \tan 18° \\
c_j &= 1{,}564 \text{ psi}
\end{aligned}
$$

$\longleftarrow (c_j)$

7 Three-Dimensional Excavations

3D Caverns

1. Given: oblate spheroid and:



Find: Peak stresses, location.

Solution:

Text – Fig. 7.6, page 353 – Stress concentration (at a, b)

$$K_c = 1.1784(a/c) + 0.2464$$
$$K_t \text{(n.a.)}$$
$$\therefore \underline{K_c = 2.60}$$

estimate $\underline{S_v = 650 \text{ psi}}$ of 1 psi/ft

$$\sigma_{a,b} = (2.60)(650) \quad \text{(peak compression)}$$
$$\underline{\underline{\sigma_{a,b} = 1{,}692 \text{ psi}}} \quad \text{(no tension)}$$

←

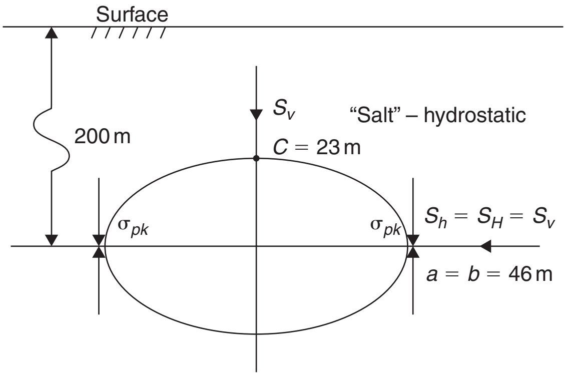
2. Given: An oblate spheroid
$a = b = 46$ m $c = 23$ m depth $= 200$ m

Find: Peak stresses, locations.

Solution:

Text – Fig. 7.6, page 353 regression eqs.

$$\underline{\underline{K_t - \text{not a factor}}}$$
$$K_c = 1.1784(a/c) + 0.2464$$
$$\therefore \underline{\underline{K_c = 2.60}}$$



$$2.60 = \frac{\sigma_{pk}}{S_v}, \quad S_v = rH$$

estimate:

$$\begin{aligned} S_v &= (25)(200) = 5.0\,\text{Mpa} \\ \sigma_{pk} &= (2.60)(5.0) \\ \underline{\underline{\sigma_{pk}}} &\underline{\underline{= 13.0\,\text{Mpa}}} \\ &\text{(no tension)} \end{aligned}$$

compression ←

3. Given: Salt cavern, sphere
$D = 150$ ft, $E = 5(10^6)$ psi, $G = 2(10^6)$ psi, $d = 1{,}450$ ft
$S_v = S_h = S_H$(hydrostatic), $C_0 = 13{,}200$ psi, $T_0 = 1{,}230$ psi
Find: FS_c.

Solution:

$$\begin{aligned} FS_c &= \frac{C_0}{\sigma_c} \\ \sigma_c &= K_c \sigma_1^0 \end{aligned}$$

estimate $\sigma_1^0 = \gamma d = S_v = (1\text{ psi/ft})\ (1{,}450)$
$\underline{\sigma_1^0 = S_v = 1{,}450\text{ psi}}$

$$v = \frac{E}{2G} - 1 = \frac{5.0}{2(2.0)} - 1$$
$$\underline{v = 0.25}$$

Ex. 7.1 text, also Table 7.1 data, page 347.

$$\begin{aligned} &\underline{K_c = 1.5} \\ \sigma_c &= (1.5)(1,450) \\ \sigma_c &= 2{,}175 \text{ psi} \end{aligned}$$

$$FS_c = \frac{13{,}200}{2{,}175}$$
$$\underline{\underline{FS_c = 6.07}} \quad \longleftarrow$$

4. Given: Spherical cavity in salt, hydrostatic stress
   $d = 442$, $\gamma = 22.8\ \text{kN/m}^3$, $C_0 = 91.0\ \text{MPa}$, $T_0 = 8.5\ \text{MPa}$
   Find: FS_c, FS_t.

   Solution:

$$FS_c = \frac{C_0}{\sigma_c} \quad FS_t = \frac{T_0}{\sigma_t}$$

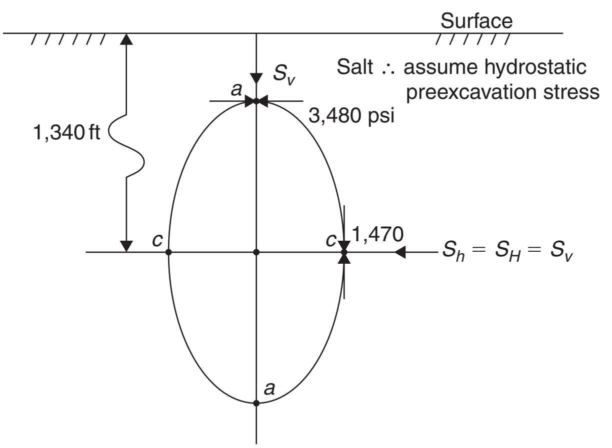
   hydrostatic stress $= S_v = \gamma H$
   $\underline{S_v} = (22.8)(442) = \underline{10.07\ \text{Mpa}}$

   $\sigma_c = K_c S_v \quad \sigma_t = K_t S_v$
   Text Ex. 7.1 & Table 7.1, page 347: Sphere

$$K_c = 1.5 \qquad K_t \text{ (nil)}$$
$$\sigma_c = 1.5(10.07)$$
$$\sigma_c = 15.1\ \text{MPa} \qquad \sigma_t \text{ (nil)}$$
$$FS_c = \frac{91.0}{15.1}$$
$$\underline{\underline{FS_c = 6.02}} \quad \longleftarrow$$
$$\underline{\underline{FS_t = \text{n.a.}}}$$

5. Given: Salt cavern prolate spheroid 100 ft high 50 ft wide @ 1,340 ft
   Find:
   (a) required salt strength C_0
   (b) show stresses in vertical section.

   Solution:



$$FS_c = \frac{C_0}{\sigma_c}, \ \sigma_c = K_c S_H$$
$$C_0 = \sigma_c$$
$$= (2.60)(1{,}340)$$
$$\underline{\underline{C_0 = 3{,}480\ \text{psi}}}$$

Note: Treat as oblate, then use Fig. 7.6 data

$$K_c = 1.1784(a/c) + 0.2464$$
$$\underline{K_c = 2.60}$$
$$S_v = (1\ \text{psi/ft})(1{,}340)$$
$$\underline{S_v = 1{,}340}$$

estimate

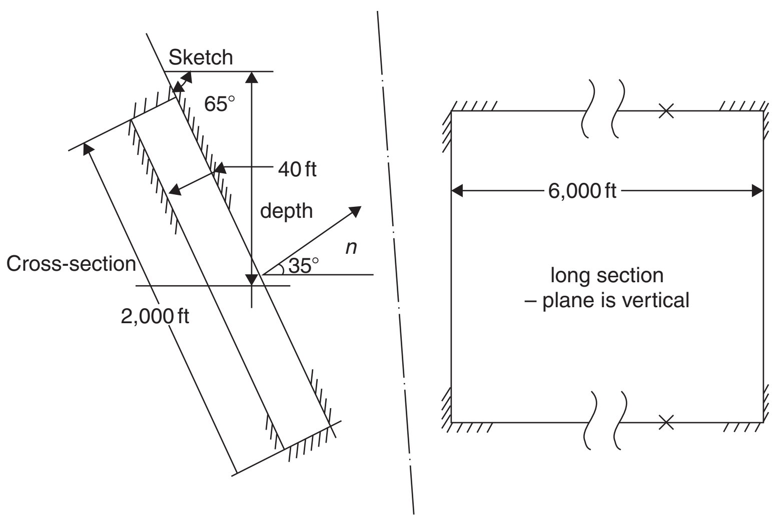
$$K_c'(a + c) \text{ from Fig.}$$
$$K_c' \sim 1.1$$

$$\therefore \sigma_c' = (1.1)(1{,}340)$$
$$\underline{\underline{\sigma_c' = 1{,}470\ \text{psi}}}$$
$$\underline{\underline{C_0 = 3{,}480\ \text{psi}}} \quad \longleftarrow$$

6. Given: Tabular excavation, 65° dip, 40 ft thick, 6,000 ft on strike, 2,000 ft down dip

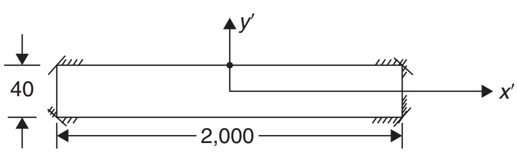
Find: Peak stresses & location.

Solution:



Because the mined region is more than twice as long as it is wide, it may be seen as a "tunnel" – like excavation.

aspect ratio: $\dfrac{W_0}{H_0} = \dfrac{2{,}000}{40}$
$= 50$



From text, p. 107, stress concentration is $\propto \dfrac{W_0}{H_0} = k$

$K_c \approx 65$ or very high when k is high $\approx 2 + 1.27\,k$

Estimate: $M = \dfrac{1}{3} = K_0$

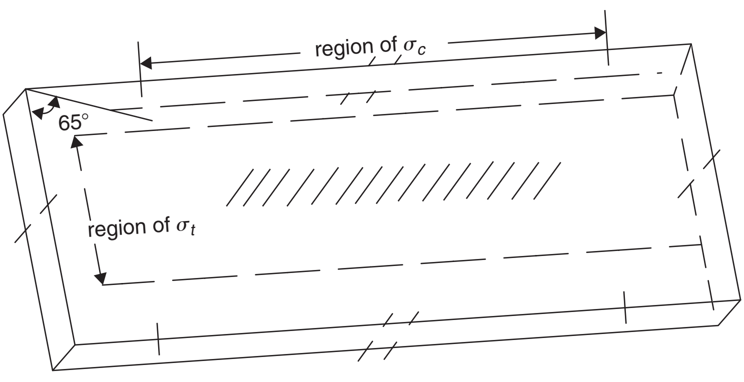
$$\text{average depth} = (1000)(\sin 65°)$$
$$\bar{d} = 906\text{ ft}$$

$$S_n = \frac{S_x + S_y}{2} + \left(\frac{S_x - S_y}{2}\right)\cos 2\theta + S_{xy}\sin 2\theta$$
$$= 906\left[\left(\frac{\left(\frac{1}{3}\right)+1}{2}\right) + \left(\frac{\left(\frac{1}{3}\right)-1}{2}\right)\cos 2.25\right] + 0$$
$$S_n = 605 + (-195)$$
$$\bar{S}_n = 410\text{ psi}$$

$$\sigma_c = K_c\bar{S}_n$$
$$= 65(410)$$
$$\sigma_c \cong 26{,}000\text{ psi} \quad \leftarrow \text{peak compression}$$

For $M = 1/3$ estimate $K_t = 0.2$

$$\sigma_t = 0.2(410)$$
$$\sigma_t \cong -80\text{ psi} \quad \leftarrow \text{peak tension}$$

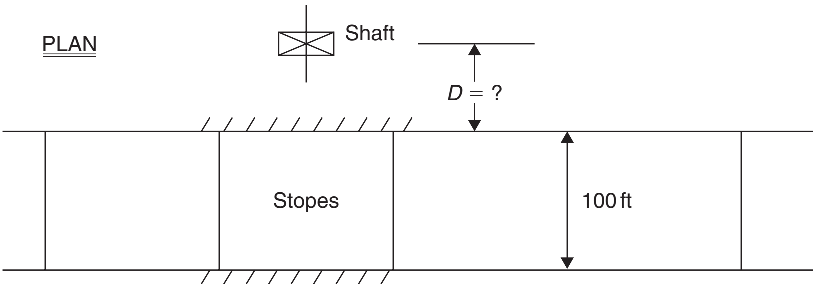


The peak compression is located on the *long* edges of the opening away from the ends.
The peak tension is located along the *intermediate* edges of the opening.

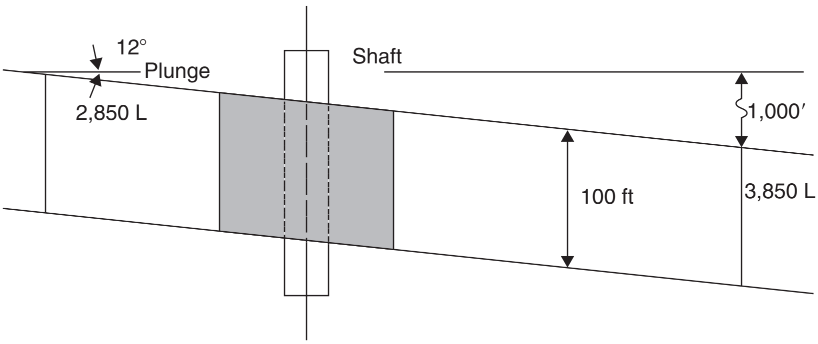
7. Given: 100 ft stopes, vertical walls, 12° plunge.
   Find: Safe shaft distance D.

Solution:

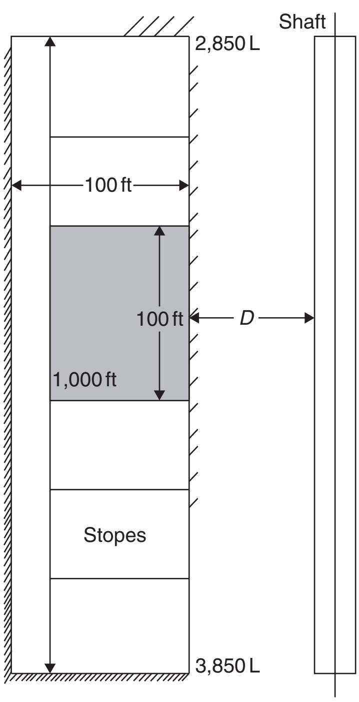
Sketches:



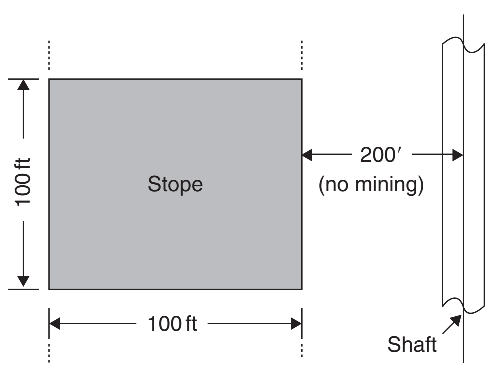
SECTION 1.



SECTION 2.



The mining region, stopes, are tunnel-like as they progress, up and down the plunge of the fold. 100 ft is given as a typical dimensions.

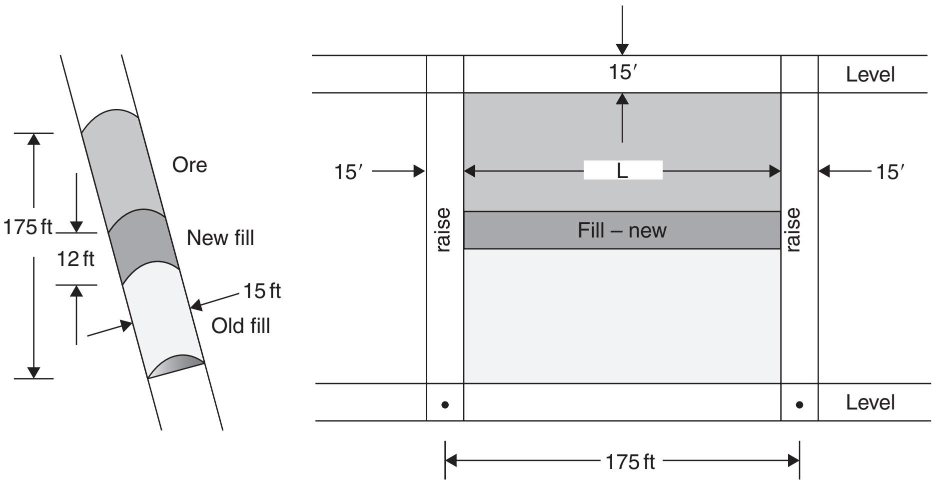


Because of the importance of the shaft, separating stopes and shaft should be more than 1-D to be certain to the beyond the zone of stope influence and stress concentration, say "2D", i.e., Distance = 200 ft ←

Back fill

8. Given: Cut and fill stope, 15 ft wide, level & raises @ 175 ft c-c
Fill: $\gamma = 100$ pcf, $n = 35\%$, 12 ft per lift

Find:
(a) Tons of solid in fill
(b) Gallons water.



(a) Cross-section (b) Long-section

- estimate drift and raises at 15 ft wide, so $L = 175 - 15 = 160$ ft
- no dip is given, so will use 15 ft for fill width also
- fill volume $V_{total} = (15)(12)(160) = \underline{\underline{2.88\ (10^4)\ \text{ft}^3}}$

$$V_v = Vn = V_w \text{ (water volume)}$$
$$= (2.88)10^4(0.35)$$
$$\underline{\underline{V_w = (1.01)(10^4)\ \text{ft}^3}}$$

231 Cu in./gal = 0.134 ft^3/gal
gals = (1.01)(10^4)/0.134
gals = $\underline{\underline{7.54\ (10^4)\ \text{gal. } H_2O}}$ ← (b)

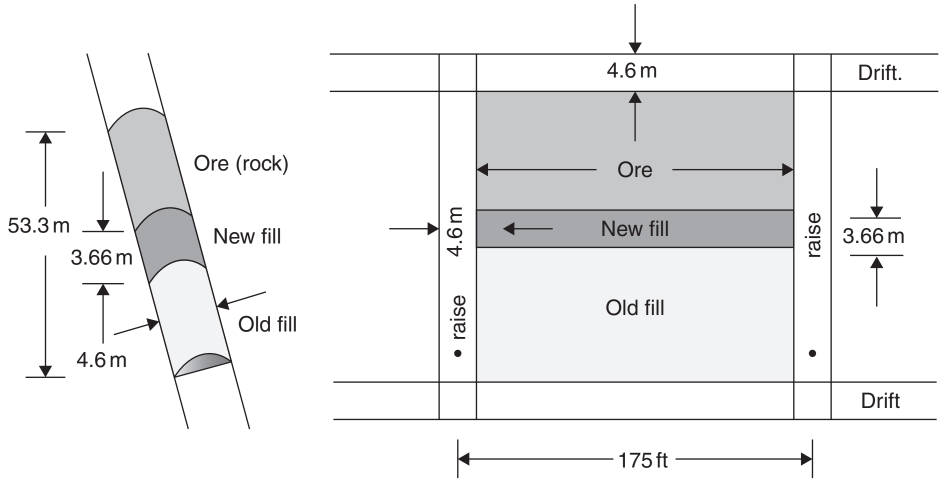
$W_{H_2O} = (62.4\ \text{lb/ft}^3)(1.01)10^4(\text{ft}^3)$
$\underline{W_{H_2O} = 6.29\ (10^5)\ \text{lbs}}$

$$W_{total} = \gamma V = (100)(2.80)10^4 = \underline{2.8(10^6)\ \text{lbs}}$$
$$W_{solids} = W_t - W_w = 2.8(10^5) - 6.29(10^5)$$

$$W_s = 21.7(10^5)\text{ lbs}$$
$$\underline{\underline{W_s = 1.09(10^3)\text{ tons}}} \qquad \leftarrow \text{(a) } W_s$$

9. Given: A cut and fill stope 4.6 m wide level & raises @ 53.3 m fill 3.66 m high $\gamma = 15.8\text{ kN/m}^3$, $n = 35\%$ (porosity).

   Find: Tons of fill liters, of water in fill.



(a) Cross-section (b) Long-section

- $L = 53.3 - 4.6$
  $L = 48.7\text{ m}$
- no dip given, assume a reasonable 4.6 m width of fill

- Volume: $V = (48.7)(4.6)(3.66)$
  $\underline{V = 820\text{ m}^3}$

- Void volume: $V_v = nV = 0.35\ (820)$
  $\underline{V_v = 287\text{ m}^3}$

$$\therefore \underline{\underline{V_v = 287(10^3)\text{ liters}}} \qquad \leftarrow H_2O$$

$$V_s = V - V_v$$
$$= 820 - 287$$
$$\underline{V_s = 533\text{ m}^3}$$
$$W_s = W - W_w$$
$$= (15.86\text{ kN/m}^3)(820) - 287(9.86\text{ kN/m}^3)$$

$$\underline{W_s = 10.17\text{ MN}}$$
$$\underline{\underline{(1.02(10^3)\text{ tonnes})}} \qquad \leftarrow \text{wt}$$
$$(\sim 2240\text{ lbs.})$$

10. Given: Void ratio $e = \dfrac{V_v}{V_s}$

Find: $e(n)$: $n =$ porosity

Solution:

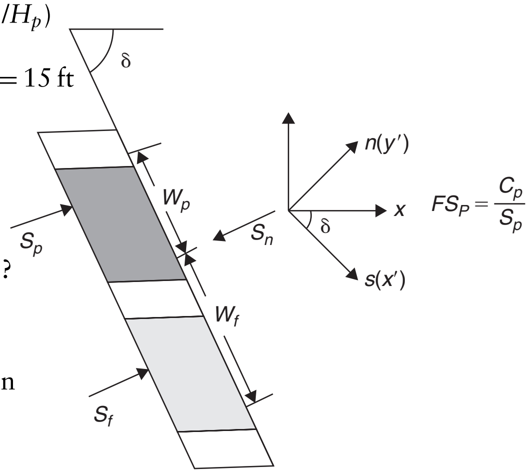
$$
\begin{aligned}
e &= \frac{V_v}{V_s} \\
&= \frac{V_v}{V - V_v} \\
&= \frac{\frac{V_v}{V}}{1 - \frac{V_v}{V}} \\
e &= \frac{n}{1-n} \qquad \longleftarrow e(n)
\end{aligned}
$$

Also: $n = \dfrac{e}{1+e}$ $\longleftarrow$

11/12. Given: Narrow vein cut and fill stope, over hand, rock modulus E_r and fill modulus E_f with uniform closure.

Find:
(a) Safety factor for crown pillar,
(b)[convert units]

$E_f = 25$ ksi, $E_r = 1{,}000$ ksi
C_p (rock) $= C_1(0.78 + 0.22\ W_p/H_p)$
$C_1 = 28{,}000$ psi
$H_P =$ Measured across the dip $= 15$ ft
$W_p =$ pillar width, on the dip
Level $=$ raise interval $= 175$ ft.
dip $=$ vertical
$S_v = 1.1$ d
$S_h = S_H = 2S_v$
H_p(@ failure, pillar "width")=?



Solution:

- Equilibrium in normal direction (reference sketch)

$$A_pS_p + A_fS_f = S_nA \tag{1}$$

where $A = A_p + A_f$

- Uniform closure, 1-D Hookes' law $\varepsilon = \dfrac{1}{E}\sigma$

$$\varepsilon_f = \frac{S_f}{E_f} \ \&\ \varepsilon_p = \frac{S_p}{E_p} \tag{2}$$

since $\varepsilon_f = \varepsilon_p$

$$S_f = \frac{E_f}{E_p} S_p$$

and from (1)

$$A_p S_p = S_n A + A_f \left(\frac{E_f}{E_p}\right) S_p$$

$$\therefore (1 - R) S_p = S_n + \left(\frac{E_f}{E_p}\right) R S_p$$

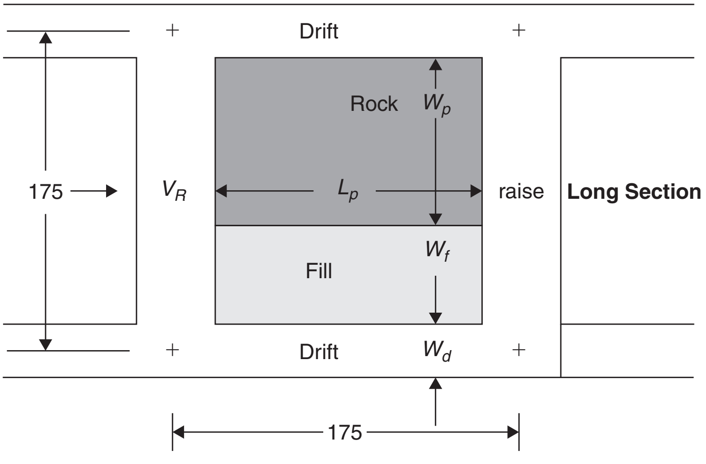
$$S_p = \left[\frac{S_n}{1 - R\left(1 - \frac{E_F}{E_p}\right)}\right]$$

$$FS_p = \left[\frac{C_1\left(0.78 + 0.22\ \frac{W_p}{H_p}\right)}{S_n}\right]\left[1 - R\left(1 - \frac{E_f}{E_p}\right)\right] \quad \longleftarrow$$

$$\text{where } S_n = \left(\frac{S_h + S_v}{2}\right) + \left(\frac{S_h - S_v}{2}\right)\cos(-2\delta)$$

$$R = 1 - \frac{W_p L_p}{(W_p + W_d)(L_p + W_r)}$$

$$\begin{cases} W_d = \text{drift width} = \text{vein width} = H_p \\ W_n = \text{raise width} = W_d = H_p \end{cases}$$



- at failure $FS_p = 1.0$
- at $d = 5{,}000$ ft, $S_v = 5{,}500$ psi & $S_h = 2S_v = S_H = 11{,}000$ psi
- vein is vertical, $\therefore S_h = S_n = S_H = \underline{11{,}000 \text{ psi}}$

- estimate drifts and raises are full vein width (H_p)

$$\underline{\underline{W_d = W_r = 15\,\text{ft}\ \&\ L_p = 175 - 15 = 160\,\text{ft}}}$$

- $E_f/E_r = \underline{\underline{25(10^3)/10^3(10^3) = 0.025}}$

$$\therefore (11{,}000\,\text{psi}) = (28{,}000)\left[0.78 + 0.22\frac{W_p}{15}\right][1 - R(1 - 0.025)]$$

$$\left(\frac{11}{28}\right) = \left[0.78 + \frac{0.22}{15}W_p\right][1 - R(1 - 0.025)]$$

$$\underline{0.3929 = [0.7800 + 0.01467W_p][1 - 0.975R]}$$

$$1 - R = A_p/A$$

$$R = 1 - \frac{W_p(175 - 15)}{(175)^2}$$

$$R = 1 - W_p(0.005225)$$

$$\underline{\underline{0.975\ R = 0.975 - 0.005094\ W_p}}$$

$$0.3929 = [0.7800 + 0.01467\ W_p][1 - 0.975 + 0.005094\ W_p]$$

$$77.12 = [0.7800 + 0.01467\ W_p][4.908 + W_p]$$

$$0 = 0.01467\ W_p^2 + 0.7947\ W_p - 73.29$$

$$0 = W_p^2 + 54.17\ W_p - 4{,}997$$

$$W_p = -\frac{54.17}{2} \pm \frac{1}{2}[(54.17)^2 + 4(4{,}997)]^{1/2}$$

$$= -27.08 \pm 75.7$$

$$\underline{\underline{W_p = 48.62\,\text{ft.}}} \qquad \longleftarrow W_p$$

Note: No size effect	(without fill)
$\frac{11}{28} = 1 - 0.975\ R$	$\frac{11}{28} = 1 - R$
$\underline{\underline{R = 0.6227}}$	$(R = 0.6071)$
	Fill helps some

$$1 - R = 0.3773 = \frac{W_p 160}{(175)^2}$$

$$W_p = 72.2\,\text{ft.} \quad (W_p > H_p)$$

with size effect

$$1 - R = \frac{(48.62)(160)}{(175)^2}$$

$$\underline{\underline{R = 0.746}}$$

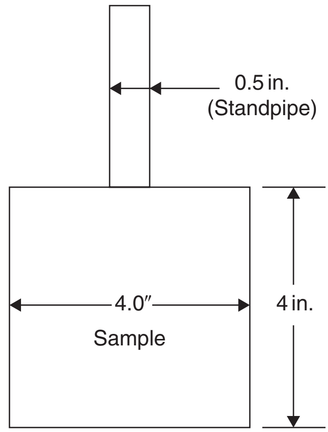
i.e. ($W_p > H_p$) increases strength and allows higher extraction.

13. Given: Laboratory test data falling head permeameter

Data: Time (min)	0	1.9	4.22	7.22	11.4
Head (in.)	36.0	30.0	24.0	18.0	12.0
k	–	0.361	0.361	0.361	0.361 (in./hr)

← k

Find: An estimate of hydraulic conductivity (in./hour).



Solution:

Apply Darcy's law:

$$-a\frac{\mathrm{d}h}{\mathrm{d}t} = k\frac{h}{L}A$$

$$\frac{\mathrm{d}h}{h} = -\frac{Ak}{La}\mathrm{d}t$$

$$\ln h = -\frac{Ak}{aL}t + \text{constant}$$

at $t = 0$, $h = k_0$ $\therefore$ $\ln h_0 = \text{constant}$

$$\ln(h_0/h) = \frac{Ak}{aL}t$$

$$k = \frac{aL}{At}\ln\left(\frac{h_0}{h}\right)$$

$$k = \frac{\frac{\pi}{4}(0.5)^2(4)}{\left(\frac{\pi}{4}\right)(4)^2 t}\ln\left(\frac{36.0}{h}\right)$$

$$\underset{(\text{in./hr})}{h} = \frac{0.0625}{(1/60)t}\ln\left(\frac{36.0}{h}\right)$$ Tabulated above:

14. Given: Falling head permeameter data

Head (in.)	35.8	29.9	16.2	10.8	7.3	4.9
Time (sec)	0	45	90	135	180	225
k	–	1	2	3	4	5

Find: h (hydraulic conductivity) and whether satisfactory/fill.

Solution:

$$\frac{-a\mathrm{d}h}{\mathrm{d}t} = \frac{kAh}{L} \qquad \text{(Darcy's law)}$$

$$\frac{\mathrm{d}h}{h} = \frac{-kA}{aL}\mathrm{d}t$$

$$\ln h = \frac{-kA}{aL}t + \text{const.}$$

at $h = h_0$ $\quad t = 0$

$$\underline{\underline{k = \frac{aL}{At}\ln\frac{h_0}{h}}} = \frac{\left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2(8)(3600\text{s/hr})\ln\left(\frac{h_0}{h}\right)}{\left(\frac{\pi}{4}\right)(4)^2 t(\text{ sec })}$$

$$k = \frac{450}{t}\ln\frac{36}{h}$$

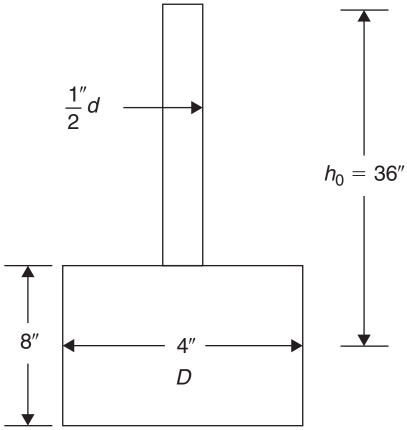
$$k_1 = \frac{450}{45}\ln\frac{36}{29.9} = 1.857\,\text{in/hr (data } x\text{ ?)}$$

$$k_2 = \frac{450}{90}\ln\frac{36}{16.2} = 3.99\,\text{in/hr}$$

$$k_3 = \frac{450}{135}\ln\frac{36}{10.8} = 4.013\,\text{in/hr}$$

$$k_4 = \frac{450}{180}\ln\frac{36}{7.3} = 3.99\,\text{in/hr}$$

$$k_5 = \frac{450}{225}\ln\frac{36}{4.9} = 3.99\,\text{in/hr}$$



All tests near 4 in./hr or 100 mm/hr, (first test low, however) fill o.k. ←

15. Given: Falling head permeameter data.
Find: h (hydraulic conductivity).

Solution:

$$k = \frac{aL}{At}\ln\left(\frac{h_0}{h}\right)$$

$$k = \frac{\left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2(4)}{\left(\frac{\pi}{4}\right)(4)^2 t}\ln\left(\frac{h_0}{h}\right)60(\text{min/hr})$$

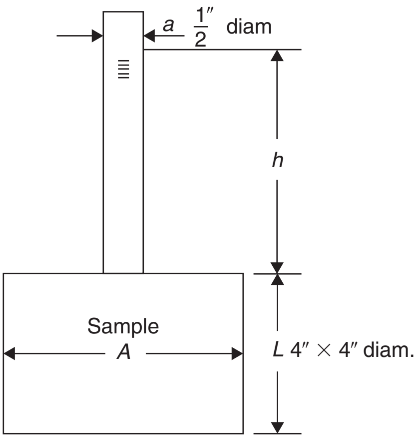
$$k = \frac{3.75}{t}\ln\left(\frac{h_0}{h}\right) : (\text{in./hr})$$

$$k_1 = \frac{3.75}{1.9}\ln\frac{36}{30} = 0.36\,\text{in./hr}$$

$$k_2 = \frac{3.75}{4.22}\ln\frac{36}{24} = 0.36\,\text{in./hr}$$

$$k_3 = \frac{3.75}{7.22}\ln\frac{36}{18} = 0.36\,\text{in./hr}$$

$$k_4 = \frac{3.75}{11.4}\ln\frac{36}{12} = 0.36\,\text{in./hr}$$



(very consistent data!) $\underline{\underline{k = 0.36\,\text{in./hr}}}$ ←

Cable Bolting

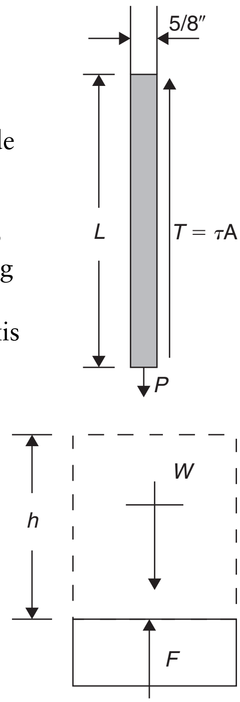
16. Given:
Large mechanized cut and fill slope

- 5/8″ diameter, per-hole cable bolt
- Ultimate cable strength −56,000 lbf
- Bond strength between grout and cable steel – 700 psi
- Shear failure in steel grout interface expected.

Find:

(a) How many inches of hole length are required to provide anchorage capacity shear equal to the bolt tensile strength.
(b) Develop a formula that relates the thickness h of a slab that could be supported in the stope back to the spacing S of the bolts assumed n bolts per hole.
(c) Plot the formula with h on the x axis and S on the y axis for slab thickness ranging from 0 to 10 ft use $n = 1, 2, 3, 4$ as a parameter.
(d) If the spacing is 10 ft what slab thickness could be supported with two bolts per hole.

Free body diagram



Solution:

(a) $\tau = 700$ psi

$A = \pi DL$

$\Sigma F_V = 0 = P - T$

$P = \tau\pi DL$

$$L = \frac{P}{\tau\pi D} = \frac{56{,}000}{(700)(3.14)\left(\frac{5}{8}\right)}$$

$\underline{L = 40.74\text{ in}}$ ← (a)

(b) Using dead weight load

$$W = F$$

$$W = \gamma S^2 h$$

$$F = \frac{nP}{FS_b}$$

$$\underline{h = \frac{nP}{\gamma S^2 (FS_b)}}$$

← (b)

(c) $$S = \left[\frac{nP}{\gamma (FS_b) h}\right]^{1/2}$$

Assume

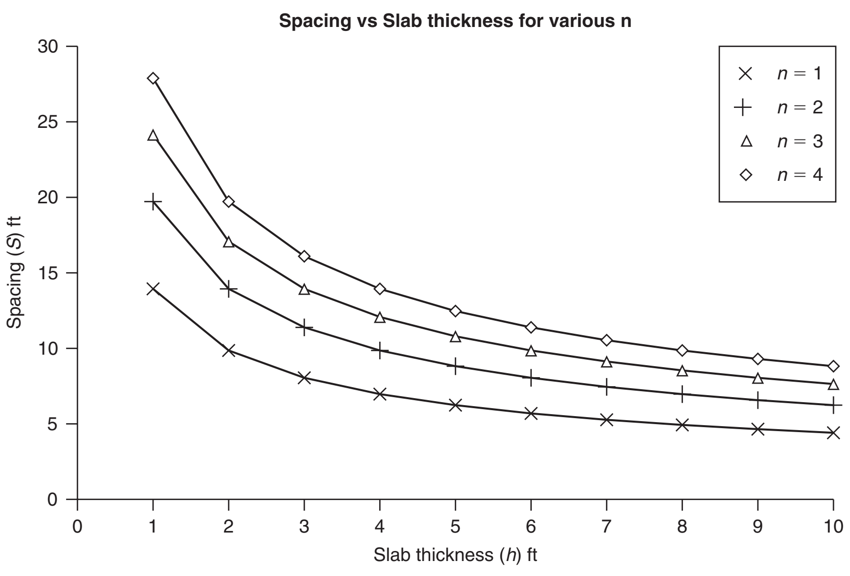
$$\gamma = 144 \text{ pcf}$$
$$FS_b = 2$$
$$h = 0 - 10 \text{ ft}$$
$$n = 1, 2, 3, 4$$

See graph ← (c)

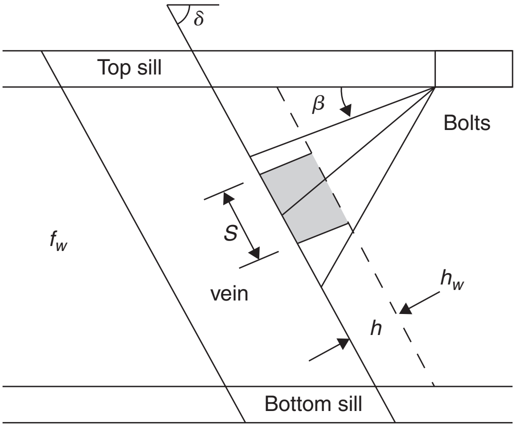
(d) @ $S = 10$ ft; $n = 2$; $FS_b = 2$

$$h = \frac{(2)56{,}000}{(144)(100)(2)}$$
$$\underline{h = 3.9 \text{ ft}}$$ ← (d)

	$n=1$	$n=2$	$n=3$	$n=4$
H	S	S	S	S
1	13.94	19.72	24.15	27.89
2	9.86	13.94	17.08	19.72
3	8.05	11.39	13.94	16.10
4	6.97	9.86	12.08	13.94
5	6.24	8.82	10.80	12.47
6	5.69	8.05	9.86	11.39
7	5.27	7.45	9.13	10.54
8	4.93	6.97	8.54	9.86
9	4.65	6.57	8.05	9.30
10	4.41	6.24	7.64	8.82



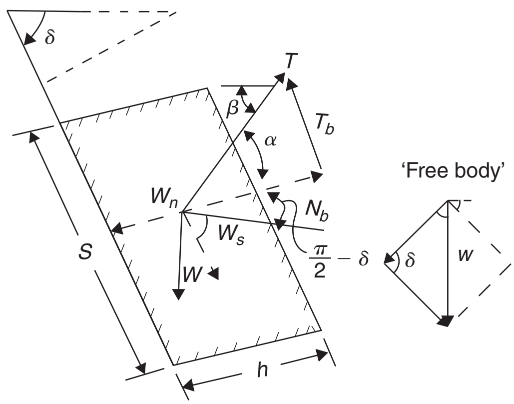
17. Given:



Find: Equilibrium requirement, normal direction.

Solution:

$$\sum_n F = 0$$
$$0 = W_n - N_b$$
$$W_n = W\cos\delta,$$
$$N_b = T\cos\alpha$$
$$\beta = \alpha + \frac{\pi}{2} - \delta$$
$$\alpha = \beta + \delta - \frac{\pi}{2}$$
$$\cos\alpha = \sin(\beta+\delta)$$
$$\therefore W\cos\delta = T_{\sin}(\beta+\delta)$$



where $W = \gamma S^2 h$ (square pattern)
Hence: $\underline{\underline{\gamma S^2 h\cos\delta = nf_b\sin(\beta+\delta)\ \&\ T = nf_b}}$

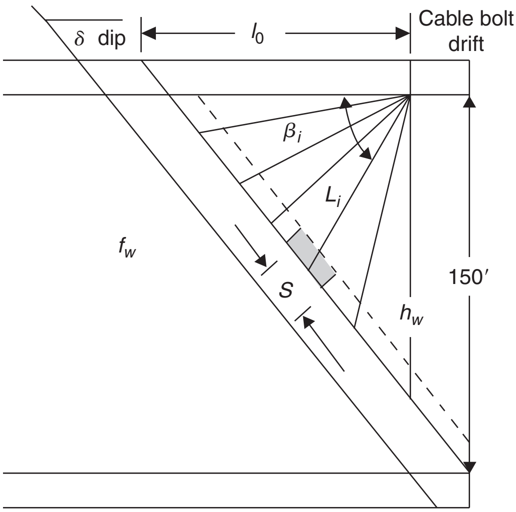
$$\begin{cases} \gamma = \text{specific weight of rock} \\ S = \text{spacing(square pattern)} \\ h = \text{slab thickness} \\ \delta = \text{vein dip} \\ n = \text{number of bolts per hole} \\ f_b = \text{force per bolt} \\ \beta = \text{bolting angle} \end{cases}$$

18. Given: Cable bolt drift, 50 ft wide vein, 65 deg. dip, 10 ft max spacing, 2 –25-ton bolts per hole, level interval = 150 ft

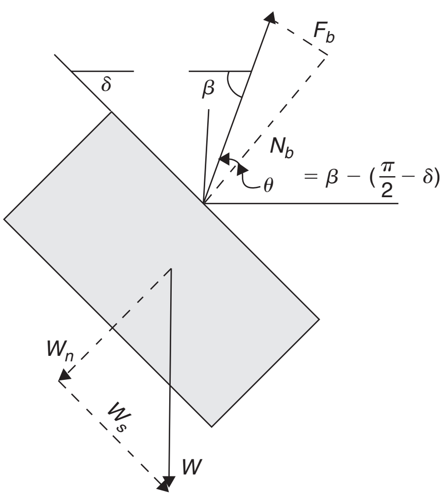
Find: Bolt pattern, angles and length.

Solution:

Sketch



For the *i*-th bolt and block



Equilibrium in the normal direction across the dip

$$F_b \cos\left(\beta + \delta - \frac{\pi}{2}\right) = W \cos(\delta)$$

within allowable bolting force

$$F_b = \frac{T_b}{FS_b}$$

T_b = bolt strength

FS_b = bolt safety factor

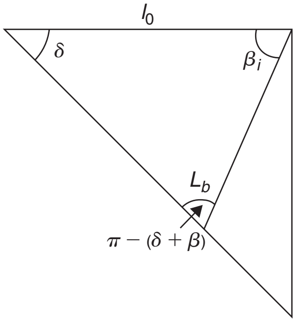
$$\text{Thus } FS_b = \frac{T_b \cos\left(\beta + \delta - \frac{\pi}{2}\right)}{\gamma S^2 h \cos\delta} \longleftarrow$$

where $W = \gamma S^2 h$

S = Spacing on a square pattern

h = slab thickness

Bolt length, angle? Assume no bolt goes past vertical

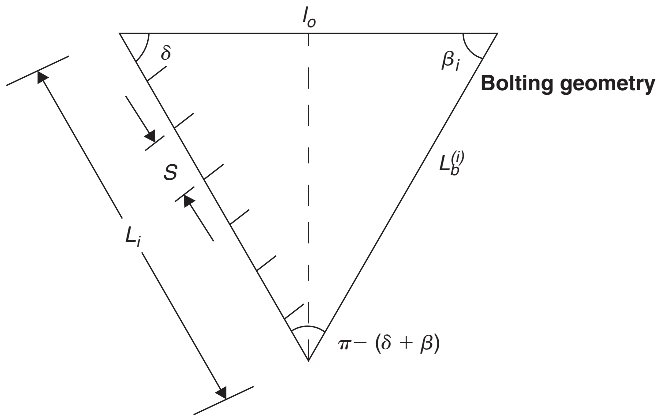


$$150' = L_0$$

$$\tan\delta = \frac{L_0}{l_0}$$

$$\frac{l_0}{\sin(\pi - \delta - \beta)} = \frac{L_b}{\sin\delta}$$

$$\left.\begin{aligned} L_b^{(i)} &= \frac{(\sin\delta)(l_0)}{\sin(\pi - \delta - \beta_i)} \\ l_0 &= \frac{L_0}{\tan\delta} \end{aligned}\right\} \text{fixes bolt length for given } \beta_i$$



(1) $L_i \sin\delta = L_b(i)\sin\beta_i$

(2) $L_i \cos\delta + L_b(i)\cos\beta_i = l_0$

$L_b(i)\cos\beta_i = l_0 - L_i\cos\delta$

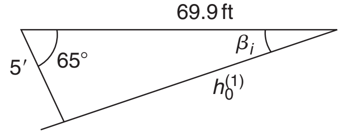
$$\tan\beta_i = \frac{L_i\sin\delta}{l_0 - L_i\cos\delta}$$

$$\tan\beta_i = \frac{\frac{S}{2}\sin\delta}{l_0 - \frac{S}{2}\cos\delta}, \; l_0 = \frac{L_0}{\tan\delta}$$

$$= \frac{\left(\frac{16}{2}\right)\sin 65^\circ}{\frac{150}{\tan 65} - \frac{10}{2}\cos 65^\circ}$$

$$\tan\beta_i = 0.0668$$

$$\beta_i = 3.82^\circ$$



$$\text{Note: } L_i(\max) \cong 166 = \frac{156}{\sin 65^\circ}$$

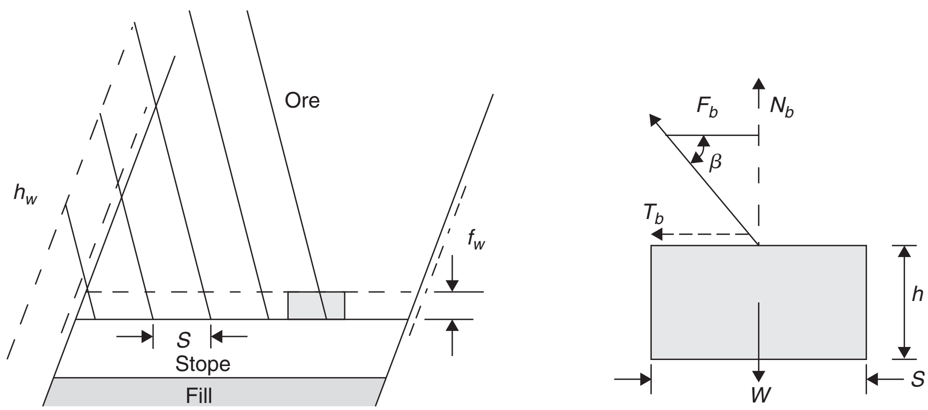
$$\underline{\underline{i \cong 16 \text{ bolts}}}$$

19. Given: Cable bolted medium width cut and fill stope, $\delta = 65°$, width = 50 ft, level interval = 200 ft, 5/8-in., 25-ton cables, one per hole.

Find:

(a) formula
(b) specify pattern

Solution:



use dead weight approach, than for equilibrium:

$$\Sigma F_n = 0$$
$$0 = N_b - W$$
$$0 = F_b \sin\beta - W$$

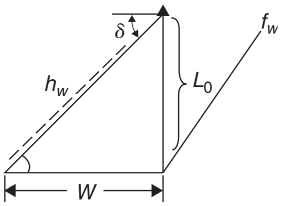
$$F_b = \text{allowable bolt tension} = \frac{\text{bolt strength}}{FS_b}$$

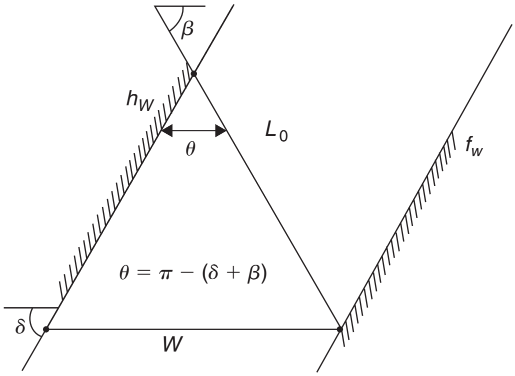
$$\therefore \frac{nS_b}{FS_b} \sin\beta = \gamma S^2 h$$

$$\therefore FS_b = \left(\frac{nS_b \sin\beta}{\gamma S^2 h}\right)$$

n = bolts per hole
S_b = 25-ton = 50,000 lbf
β = bolting angle from horizontal
γ = rock specific weight
S = spacing (assumed square)
h = slab thickness imagined in the back

- Bolt length must provide for secure anchorage of block next to hanging wall. Say, use 1 ft per inch of hole diameter $L_a = 2\frac{1}{4}$ ft using 2¼ inch diameter bolt holes.
- For maximum FS_b, β should be vertical. This would lead to a bolt length in ore of $L_0 = W \tan\delta$ where W = vein width (horizontal not true)





$$\cot\delta = \frac{W}{L_0}$$
$$L_0 = W\tan\delta$$

- Bolt length $\underline{\underline{L_b = L_a + L_0}}$ (vertical)

When, $\beta < \frac{\pi}{2}$, then

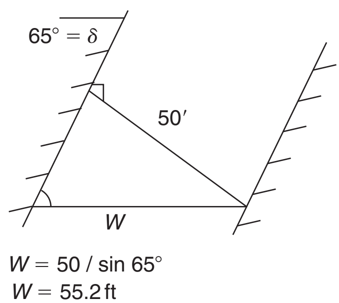
$$W = \frac{L_0}{\sin\delta}\sin(\pi - \alpha - \beta)$$

- $L_0 = \dfrac{W\sin\delta}{\sin(\pi - \alpha - \beta)}$

Bolt length $\underline{\underline{L_b = L_a + L_0}}$ angle bolted

(b) Specify bolting pattern
try on 8 × 8 ft square pattern if vertical, then at $FS_b = 1$, assuming $\gamma = 156$ pcf
$50{,}000 = (156)(8)^2(h)$
$h = 5$ ft
which is a reasonable slab thickness.
If $FS_b = 1.3$, then

$$h = \left(\frac{50{,}000}{1.3}\right)\left(\frac{1}{(156)(8^2)}\right)$$
$$\underline{h = 3.85 \text{ ft}}$$



$$L_0 = W\tan\delta$$
$$= (55.2)(\tan 65)$$
$$L_0 = 118 \text{ ft.}$$
$$L_a = 2.25 \text{ ft.}$$
$$\therefore L_b = 120 \text{ ft.}$$

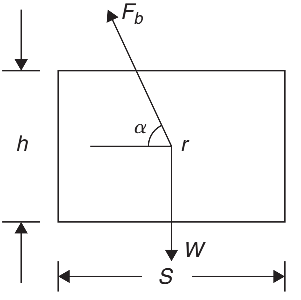
Spacing = 8 ft square pattern
Bolting angle = vertical one bolt per hole
5/8″-diameter, 2.5-ton bolts maximum length = 120 ft.

20. Given:

$$S^2 = \frac{NUV}{T(SF)\sin^3\alpha}$$

Find:
Identify terms, derive
Handout:
S = spacing
N = number in a hole
U = tensile strength
V = "specific weight" $\left(\frac{\text{ft}^3}{\text{ton}}\right)$
T = slab thickness
SF = safety factor
α = bolting angle from horizontal
Bolt block

Equilibrium (vertical)



$$W = F_b \cos\alpha$$

$$\gamma h S^2 = F_b \sin\alpha$$

$$\text{But also } F_b = \frac{\text{Bolt strength}}{FS \text{ (safety factor)}}$$

$$F_b = \frac{F_Y}{FS}$$

$$\therefore \gamma h S^2 = \frac{F_T}{FS}\sin\alpha$$

$$S^2 = \frac{F_Y \sin\alpha}{(FS)(\gamma h)}$$

Translation:

$$S = S$$

$$T = h$$

$$SF = FS$$

$$V = \frac{1}{\gamma}$$

$$N_u = F_T$$

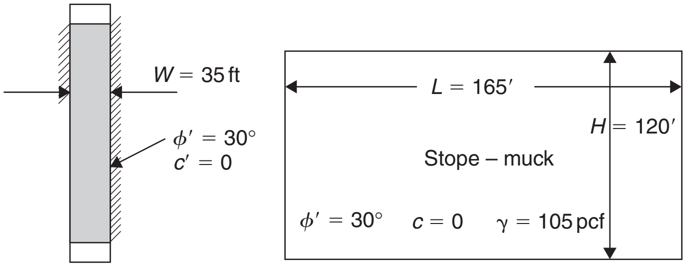
$$S^2 = \frac{(NV)(V)\sin\alpha}{(SF)(T)}$$

[given formula is incorrect]

8 Subsidence

Chimney Caving

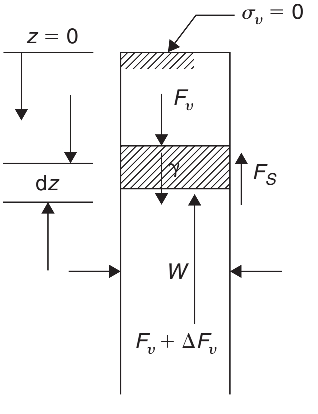
1. Given: Shrinkage stope, vertical.



Find:
(a) σ_v – formula per unit of strike length L
(b) σ_v, σ_h, τ at stope bottom (dry)
(c) σ_v, σ_h, τ where water filled.

Solution:

Slice equilibrium
$\Sigma F_v = 0$
$\Delta F_v - W + F_s = 0$
$\underline{A\Delta\sigma_v - \gamma A \, dz + \tau C \, dz = 0}$



A = area of slice normal to F_v
C = circumference
σ_v = average vertical stress
τ = side wall shear stress
$\tau = \mu\sigma_h$ friction side slip

$k = \dfrac{\sigma_h}{\sigma_v}$ estimate from Mohr–Coulomb condition

$$\frac{d\sigma_v}{dz} - \gamma + \frac{C\mu k}{A}\sigma_v = 0$$

But $A = WL$ & $C = 2L$ (no end walls, muck on muck)

$$\therefore \frac{C}{A} = \frac{2}{W}$$

$$\underline{\underline{C_1 = \frac{2\mu k}{W}}}$$

after integration

$$\underline{\underline{\sigma_v = \frac{\gamma}{C_1}[1 - \exp(-C_1 z)]}} \qquad \xleftarrow{\text{(a)}}$$

where $\sigma_v = 0$ at $z = 0$

$$\text{(b)}\quad C_1 = \frac{2\tan(37^\circ)\left(\frac{1}{3}\right)}{35} \qquad k = \frac{1-\sin\phi}{1+\sin\phi}$$

$$C_1 = 0.01435 \qquad k = \frac{1-\sin 30^\circ}{1+\sin 30^\circ}$$

$$\frac{\gamma}{C_1} = \frac{105}{0.01435} \qquad k = \frac{1}{3}$$

$$\underline{\frac{\gamma}{C_1} = 7.315(10^3)}$$

$$\sigma_v = 7.315(10^3)\{1 - \exp[-0.01435(120)]\}$$

$$\left.\begin{aligned}
&\underline{\underline{\sigma_v = 6{,}008 \text{ psf } (41.7 \text{ psi})}}\\
&\underline{\underline{\sigma_h = \frac{1}{3}\sigma_v = (13.9 \text{ psi})}}\\
&\tau = \mu\sigma_h\\
&\tau = \tan(37)(13.9)\\
&\underline{\underline{\tau = 10.5 \text{ psi}}}
\end{aligned}\right\} \qquad \xleftarrow{\sigma_v, \sigma_h, \tau \text{ at bottom} \quad \text{(b)}}$$

Rederive with water equilibrium:

$$\frac{d\sigma_v}{dz} - \gamma + C_1\sigma'_v = 0 \quad \text{since } \tau = \mu\sigma'_h = \mu k\sigma'_v$$

where σ'_v, σ'_h, are effective stresses

e.g. $\sigma'_v = \sigma_v - p_f$

where $p_f =$ water pressure

i.e., $p_f = \gamma_w z$

also since saturated, $\gamma = \gamma_{sat}$

$$\text{then} \quad \frac{d(\sigma'_v + \gamma_w z)}{dz} - \gamma_{sat} + C_1\sigma'_v = 0$$

$$\therefore \frac{d\sigma'_v}{dz} + C_1\sigma'_v = \gamma_{sat} - \gamma_w$$

$$\& \ \sigma'_v = \frac{\gamma_{sat} - \gamma_w}{C_1}[1 - \exp(-C_1 z)]$$

where $\sigma'_v = \sigma_v = p_f = 0$ at $z = 0$

i.e. water to the top.

$$\left.\begin{aligned}
&\therefore\ \underline{\underline{\sigma_v = \left(\frac{\gamma_{sat} - \gamma_w}{C_1}\right)[1 - \exp(-C_1 z)] + \gamma_w z}}\\
&\underline{\underline{\sigma_h = k\sigma'_v + \gamma_w z, \quad \sigma_h = k\sigma'_v}}\\
&\underline{\underline{\tau = \mu\sigma'_h}}
\end{aligned}\right\} \qquad \longleftarrow \text{"wet" formulas (water at the top of the muck)}$$

$$
\begin{aligned}
\text{at } z &= 120' \\
\gamma_w &= 62.4\,\text{pcf} \\
\gamma_{sat} &= 105\,\text{pcf (same as before assuming saturation)} \\
C_1 &= 0.01435 \\
\sigma_v' &= \left(\frac{105-62.4}{0.01435}\right)\{1-\exp[(-0.01435)(120)]\} \\
\sigma_v' &= (2.969)(10^3)(0.8213)
\end{aligned}
$$

$$
\left.
\begin{aligned}
&\underline{\underline{\sigma_v' = 2{,}438\,\text{psf}\ (16.93\,\text{psi})}} \\
&p_f = (62.4)(120) \\
&\underline{\underline{p_f = 7{,}488\,\text{psf}\ (52.0\,\text{psi})}} \\
&\underline{\underline{\sigma_v = \sigma_v' + p_f = (68.9\,\text{psi})}} \\
&\underline{\underline{\sigma_h' = \left(\tfrac{1}{3}\right)(\sigma_v') = (5.64\,\text{psi})}} \\
&\underline{\underline{\sigma_h = \sigma_h' + p_f = (57.6\,\text{psi})}} \\
&\tau = \mu\sigma_h' \\
&\quad = \tan(37)(5.64) \\
&\underline{\underline{\tau = (4.25\,\text{psi})}}
\end{aligned}
\right\}
$$

← wet stresses (c)

Note (total stress)	dry (psi)	wet (psi)	
σ_v	41.7	68.9	(+65.2%)
σ_h	13.9	57.6	
τ	10.5	4.25	(−59.5%)

per foot of strike length?

$$
\begin{aligned}
W &= \gamma WLH \\
\sigma_v A &= WL\sigma_v \\
F_p' &= \int_0^H \tau C\,dz \\
\tau &= \mu\sigma_h' \\
\sigma_h' &= h\sigma_v' \\
F_p' &= \int_0^H \mu k C\sigma_v'\,dz \\
&= (\mu k C)\int_0^H \left(\frac{\gamma'}{C_1}\right)(1-e^{-C_1 z})dz
\end{aligned}
$$

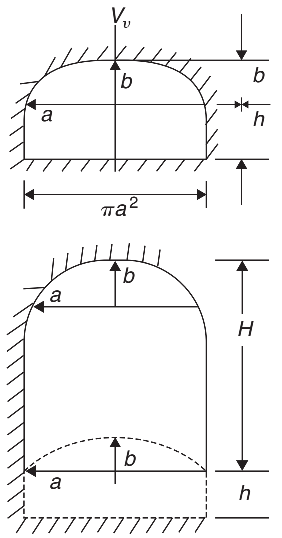
$$
\begin{aligned}
C_1 &= \frac{\mu k c}{A} \\
&= (\gamma' A)\left(z + \frac{e^{-C_1 z}}{C_1}\right)\Bigg|_0^H \\
&= \gamma' A\left(H + \frac{e^{-C_1 H}}{C_1} - e^0\right) \\
F'_s &= \underbrace{\gamma' A H}_{wt} - \underbrace{\frac{\gamma' A}{C_1}(1 - e^{-C_1 H})}_{\sigma'_v A}
\end{aligned}
$$

But

$$
\begin{aligned}
W &= \sigma_v A + F'_s \\
W &= (\sigma'_v + \gamma_f)A + \gamma' A H - \sigma'_v A \\
&= \gamma_f A + \gamma' A H \\
&= \gamma_w A H + (\gamma_{\text{sat}} - \gamma_w) A H \\
W &= \gamma_{\text{sat}} A H \quad \text{checks!}
\end{aligned}
$$

2. Given: Chimney caving figure
   Find: Derive a formula for H.

   Solution:



   Bulking porosity

$$B = \frac{V_v}{V_T}; \quad V_T = V_v + V_s$$

V_v = Volume of voids
V_s = Volume of solids
V_v = Initial cut volume
$V_v = \pi a^2 h + \frac{2}{3}\pi a^2 b$
$V_s = \pi a^2 (H - b)$

$$B = \frac{\pi a^2\left(h + \frac{2}{3b}\right)}{\pi a^2\left[\left(h + \frac{2}{3b}\right) + (H - b)\right]}$$

$$B = \frac{h + \frac{2}{3b}}{H + \frac{2}{3b} + h - b}$$

solving for H

$$H = \frac{(h + (2/3)b)(1 - B)}{b} + b$$

$\leftarrow H$

3. Given: Chimney caving

$$
\begin{aligned}
a &= 105 \text{ ft} \\
h &= 14 \text{ ft} \\
h + b &= 35 \text{ ft} \Rightarrow b = 21 \text{ ft} \\
H &= 1{,}150 \text{ ft}
\end{aligned}
$$

Find: Bulking porosity.

Solution:

$$B = \frac{h + \frac{2}{3b}}{H + \frac{2}{3b} + h - b} \quad \text{(from Problem 2)}$$

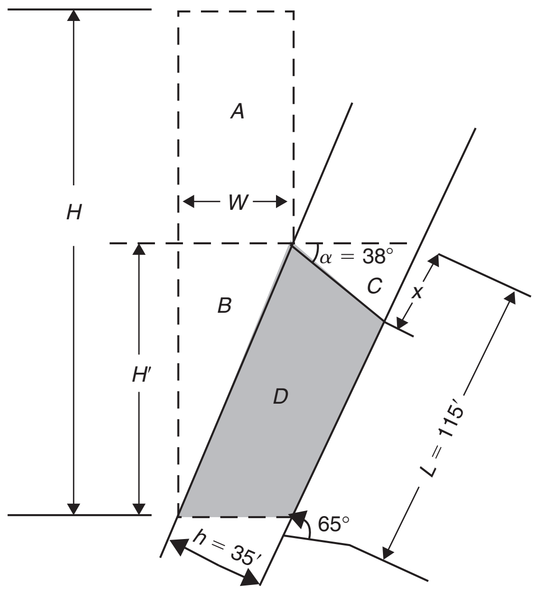
$$B = \frac{14 + \frac{2}{3}(21)}{1{,}150 + \frac{2}{3}(21) + 14 - 21}$$

$$= 0.0242$$

$$\underline{\underline{B = 2.4\%}} \qquad \longleftarrow$$

4. Given: Inclined stope, figure $L = 115$ ft
$h = 35$ ft
$B = 0.072$
stope depth $H_s = 4{,}650$
strike length 105 ft and 205 ft
Find: Cave height H for 105 & 205 ft strike.

Solution:



$$B = \frac{V_v}{V_T} = \frac{V_v}{V_v + V_s}$$

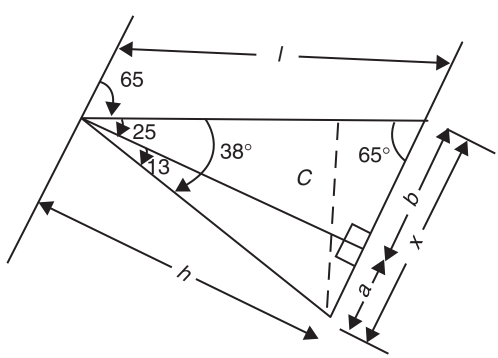
$$V_s = A + B$$

$$V_v = D - C$$

$$V_s = (H - H')W + \frac{1}{2}H'W$$

$$\tan(13) = \frac{a}{h}, a = h\tan(13)$$

$$\tan(25) = \frac{b}{h}, b = h\tan(25)$$



$$x = a + b = h[\tan(13) + \tan(25)]$$
$$x = 35[\tan(13) + \tan(25)]$$
$$x = 24.4\ \text{ft}$$

$$l = \frac{h}{\sin(65)} = \frac{35}{\sin(65)} = 38.6\ \text{ft}$$

$$\begin{aligned}\text{Area } C &= \frac{1}{2} x l \sin(65)\\ &= \frac{1}{2}(24.4)(38.6)\sin(65)\\ C &= 427\ \text{ft}^2\end{aligned}$$

$$\begin{aligned}\text{Area } D &= hL = (35)115 = 4{,}025\ \text{ft}^2\\ \Rightarrow V_v &= 4{,}025 - 427\\ &\underline{V_v = 3{,}598\ \text{ft}^2}\end{aligned}$$

$$\begin{aligned}H' &= L\sin(65) = 115[\sin(65)] = 104\ \text{ft}\\ W &= L\cos(65) = 115[\cos(65)] = 49\ \text{ft}\end{aligned}$$

$$\begin{aligned}V_s &= (H - 104)49 + \frac{1}{2}(104)(49)\\ &= 49H - 5{,}096 + 2{,}548\\ V_s &= 49H - 2{,}548\end{aligned}$$

$$\begin{aligned}\Rightarrow B &= \frac{3{,}598}{3{,}598 + 49H - 2{,}548}\\ 0.072 &= \frac{3{,}598}{1{,}050 + 49H}\\ 49H &= \frac{3{,}598}{0.072} - 1{,}050\\ H &= \frac{\left(\frac{3{,}598}{0.072}\right) - 1{,}050}{49}\\ &\underline{H = 998\ \text{ft}}\end{aligned}$$

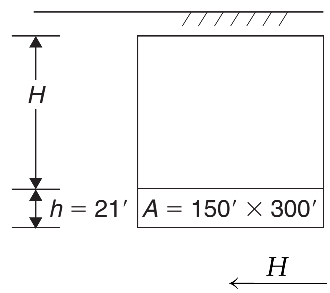
←

<u>H is independent of strike length. Same in both cases.</u>

5. Given: Panel Block Caving
   Depth 4,250 ft
   Panel dimension 150 ft × 300 ft
   Initial undercut = 21 ft
   Bulking porosity = 0.37

   Find: Cave height possible before must be drawn.

Solution: $B = \dfrac{V_v}{V_v + V_s}$



$$\begin{aligned} V_v &= (150)(300)(21) = 945{,}000\,\text{ft}^3 \\ V_s &= (150)(300)(H) = 45{,}000\,\text{H}\,\text{ft}^3 \end{aligned}$$

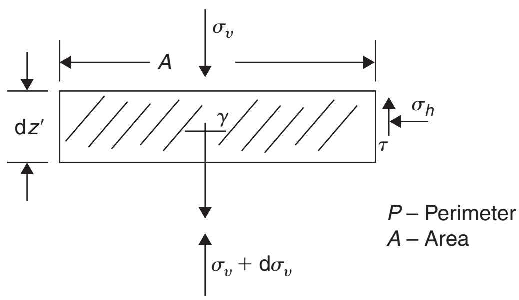
$$\begin{aligned} 0.37 &= \frac{945{,}000}{945{,}000 + 45{,}000\,H} \\ H &= \frac{945{,}000(1 - 0.37)}{45{,}000(0.37)} \\ &\underline{H = 35.8\,\text{ft}} \end{aligned}$$

6. Given: Block caving Problem 5.
$\gamma_{\text{BULK}} = 105$ pcf
$\phi' = 23°$ (muck on rock)
$\phi = 33°$ (muck on muck)
$\sigma_h/\sigma_v = 0.32$
Find:
(a) σ_v at bottom of cave as $f(z)$
(b) Plot σ_v versus depth
(c) σ_v, σ_h, τ when cave is 300′, 600′.

Solution:

$$\begin{aligned} \tau &= \mu\sigma_h; \quad \mu = \tan\phi' \\ \sigma_h &= k\sigma_v; \quad k = \frac{1 - \sin\phi}{1 + \sin\phi} \end{aligned}$$

Equilibrium analysis:



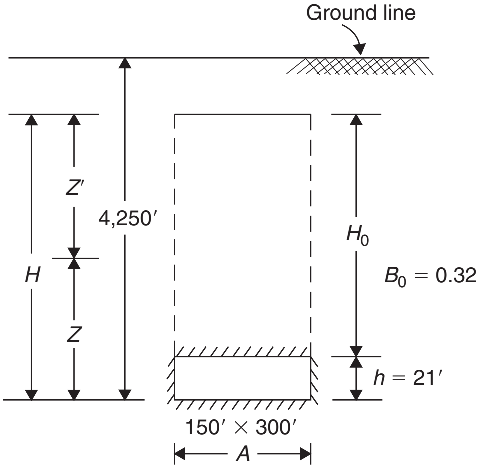
$$\begin{aligned} d\sigma_v A - \gamma A\,dz' + \tau P\,dz' &= 0 \\ d\sigma_v - \gamma\,dz' + \frac{\sigma_v \mu k P}{A} dz' &= 0 \end{aligned}$$

Integrating

$$\sigma_v = \left(\frac{\gamma A}{\mu k P}\right)\left(1 - e^{\frac{-\mu k P}{A} z'}\right)$$

as a function of z; $z = H - z'H =$ cave height & $z' = H - z$

$$\Rightarrow \underline{\sigma_v = \left(\frac{\gamma A}{\mu k P}\right)\left(1 - e^{-\frac{\mu k P}{A}(H - z)}\right)} \qquad \text{(a)}$$



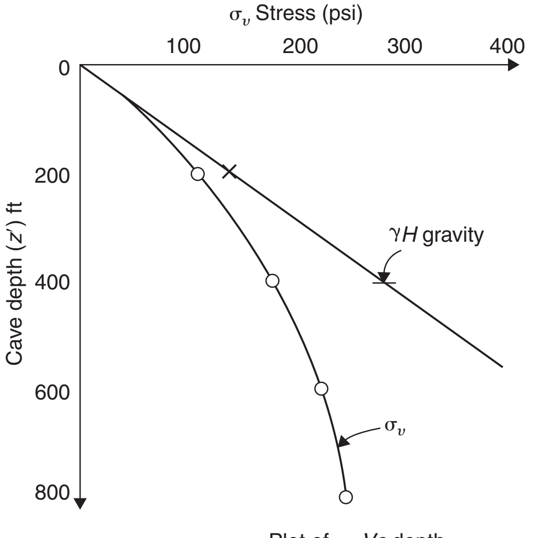
(b) $P = 2(150) + 2(300)\,(\text{ft}) = 900\,\text{ft}$
$\mu = \tan(23) = 0.4245$
$A = (150 \times 300)(\text{ft}^2) = 45{,}000\,\text{ft}^2$
$\gamma = 105\,\text{pcf}$

$$k = \frac{1 - \sin(33)}{1 + \sin(33)} = 0.2948$$

$$\frac{\mu k P}{A} = \frac{(0.4245)(900)(0.2948)}{45{,}000}$$

$$\frac{\mu k P}{A} = 2.50285 \times 10^{-3}$$

$$\frac{\gamma A}{\mu k P} = 4.1952 \times 10^{4}$$



Plot of σ_v *Vs* depth

$@z' = 0\text{ ft} \quad \sigma_v = 0\text{ psi} \quad \gamma z' = 0\text{ psi}$

$@z' = 200\text{ ft} \quad \sigma_v = 115\text{ psi} \quad \gamma z' = 146\text{ psi}$

$@z' = 400\text{ ft} \quad \sigma_v = 184\text{ psi} \quad \gamma z' = 292\text{ psi}$

$@z' = 600\text{ ft} \quad \sigma_v = 226\text{ psi}$

$@z' = 800\text{ ft} \quad \sigma_v = 252\text{ psi}$

$$\sigma_v = \frac{4.1952 \times 10^4}{144}[1 - e^{-2.50285\times10^{-3}(400)}]$$

(c) $@\ z' = 300$ ft.

$$\sigma_v = \frac{4.1952 \times 10^4}{144}[1 - e^{-2.50285\times10^{-3}(300)}]$$

$\underline{\sigma_v = 154\text{ psi}}$ ← $\sigma_v\ @\ z' = 300\text{ ft}$

$$\sigma_h = k\sigma_v = (0.2948)154$$

$\underline{\sigma_h = 45\text{ psi}}$ ← $\sigma_h\ @\ z' = 300\text{ ft}$

$$\tau = \mu\sigma_h = (0.4245)(45)$$

$\underline{\tau = 19.1\text{ psi}}$ ← $\tau\ @\ z' = 300\text{ ft}$

$@\ z' = 600$ ft

$$\sigma_v = \frac{4.1952 \times 10^4}{144}[1 - e^{-2.50285\times10^{-3}(600)}]$$

$\underline{\sigma_v = 226\text{ psi}}$ ← $\sigma_v\ @\ z' = 600\text{ ft}$

$$\sigma_h = (0.2948)(226)$$

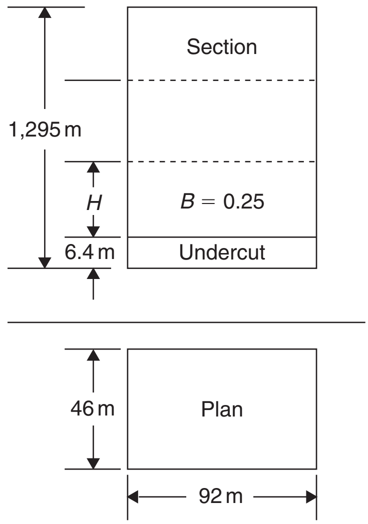
$\underline{\sigma_h = 67\text{ psi}}$ ← $\sigma_h\ @\ z' = 600\text{ ft}$

$$\tau = \mu\sigma_h = (0.4245)(67)$$

$\underline{\tau = 28\text{ psi}}$ ← $\tau\ @\ z' = 600\text{ ft}$

7. Given: Block caving panel.
   B (initial) = 0.25
   $S.G.$ (solid) = 2.60
   $S.C.$ (caved) = 1.73
   ϕ (muck on rock) = 23°
   ϕ (muck) = 33°
   $S_h \backslash S_v = 0.32$

Find:
(1) Initial cave height
(2) σ_v, σ_h, τ when $H = 183$ m
(3) B when cave reaches surface.



Solution:

(1) By definition

$$B = \frac{V_v}{V_v + V_s}$$
$$V_v = (6.4)(46)(92)$$
$$V_s = (H)(46)(92)$$
$$\therefore 0.25 = \frac{6.4}{H + 6.4}$$
$$H = \frac{6.4}{0.25} - 6.4$$
$$\underline{\underline{H = 19.2\text{ m}}}$$

(1) ←

(2) $z = 183$ m

Notes: $\sigma_v = \dfrac{\gamma}{C_1}(1 - e^{-C_1 z})$

$$C_1 = \frac{\mu k C}{A}$$
$$= \tan(23)(0.32) + \left[\frac{(3)(92+46)}{(92)(46)}\right]$$
$$\underline{C_1 = 8.859(10^{-3})}$$

$$\gamma = (S.G.)(\gamma_w)\text{: } \gamma_w(\text{N/m}^3)\text{: water} = 9.806(10^3)\text{N/m}^3$$
$$= \frac{(1.73)(62.4)}{6.366(10^{-3})}$$
$$\underline{\gamma = 1.696(10^4)\text{N/m}^3}$$

$$\sigma_v = \frac{1.696(10^4)}{8.859(10^{-3})}[1 - e^{-8.859(10^{-3})(183)}]$$
$$\sigma_v = 1.9144(10^6)(1 - 0.198)$$
$$\sigma_v = 1.535(10^6)\text{N/m}^2$$
$$\underline{\underline{\sigma_v = 1.535\text{ MPa}}}$$

$$\sigma_h = 0.32\sigma_v$$
$$= 0.32(1.535)$$
$$\underline{\underline{\sigma_h = 0.492\text{ MPa}}}$$
$$\tau = \tan(23)\sigma_h$$
$$= \tan(23)0.492$$
$$\underline{\underline{\tau = 0.209\text{ MPa}}}$$

(2) ←

(3) B when cave reaches surface.
W (in chimney bulk) $= \gamma V$

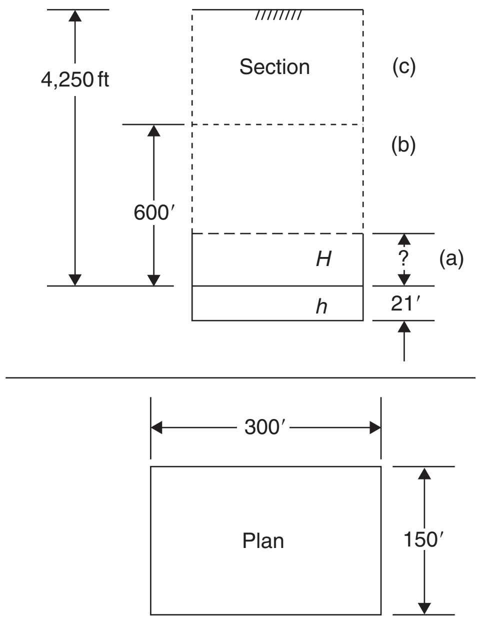
$$\begin{aligned}
\text{But also } W &= \gamma_s V_s \\
W &= \gamma_s (V - V_v) \\
\therefore\ \gamma_{\text{bulk}}\ V &= \gamma_{\text{solid}} (V - V_v) \\
\frac{\gamma_b}{\gamma_s} &= 1 - \frac{V_v}{V} \\
\text{However } B &= \frac{V_v}{V} \\
\therefore\ B &= 1 - \frac{\gamma_b}{\gamma_s} \\
B &= 1 - \frac{1.73}{2.60} \\
\underline{\underline{B}} &\underline{\underline{= 0.335}}
\end{aligned}$$

(3) ←

8. Given: Block caving data.

- initial $B = 0.25$
- $\gamma_{\text{bank}} = 162$ pcf
- $\gamma_{\text{bulk}} = 108$ pcf
- $\phi_{\text{muck}} = 33°$
- $\phi_{\text{muck/rock}} = 23°$
- $k = \dfrac{\sigma_h}{\sigma_v} = 0.32$

Find:
(a) initial height before pull necessary
(b) σ_v, τ, σ_h @ $z = 600$ ft
(c) B when surface breached.



Solution:

$$\begin{aligned}
\text{(a)}\quad B &= \frac{V_v}{V} \\
B &= \frac{Ah}{Ah + HA} \\
H &= \frac{h}{B} - h \\
&= \frac{21}{6.25} - 21 \\
\underline{\underline{H}} &\underline{\underline{= 63 \text{ ft.}}}
\end{aligned}$$

(a) ←

$$\begin{aligned}
\text{(b)}\quad z &= 600 \text{ ft} \\
\sigma_v &= \frac{\gamma}{C_1}(1 - e^{-C_1 z})
\end{aligned}$$

Notes:

$$C_1 = \frac{\mu k C}{A}$$

$$C_1 = \tan(23)(0.32)\frac{(600+300)}{(300)(150)}$$

$$C_1 = 2.717(10^{-3})$$

$$\sigma_v = \frac{108}{2.717(10^{-3})}(1 - e^{-(2.717)10^{-3}(600)})$$

$$= [\quad][1 - 0.1959]$$

$$\underline{\underline{\sigma_v = 3.196(10^4)\ \text{psf} \quad (222.1\ \text{psi})}}$$

$$\sigma_h = k\sigma_v = (0.32)3.196(10^4)$$

$$\underline{\underline{\sigma_h = 1.0228(10^4)\ \text{psf} \quad (71.03\ \text{psi})}}$$

$$\tau = \mu\sigma_h$$

$$\tau = \tan(23)[1.0228(10^4)]$$

$$\underline{\underline{\tau = 4.342(10^3)\ \text{psf} \quad (30.15\ \text{psi})}}$$

← σ_v, σ_u, τ @600 ft.

(c) B at surface breach
W (in chimney) $= \gamma V$
$= (108)(300)(150)(4{,}250 + 21)$
$W = 2.076(10^{10})$ lbf. (not needed)
This corresponds to a solid weight of

$$W = \gamma_s V_s = \gamma_s(V - V_v)$$

$$W = (162)(V - V_v)$$

i.e. $\gamma V = \gamma_s V_s$

$$(108)(V) = (162)(V - V_v)$$

$$(108) = (62)\left(1 - \frac{V_v}{V}\right)$$

But $B = \dfrac{V_v}{V}$ by definition

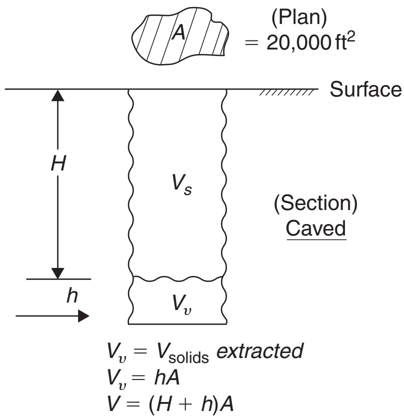
Hence: $1 - \dfrac{108}{162} = B$

$$\underline{\underline{B = 0.333 = \left(\frac{1}{3}\right)}}$$

← (B)

9. Given: Block caving
$B = 0.015 \quad D = 1{,}570\ \text{ft} \quad A = 20{,}000\ \text{ft}^2$
Find: ore extracted before cave reaches surface.

Solution:



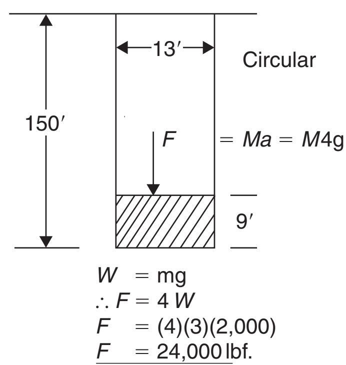
$$H + h = 1{,}570\text{ ft}$$
$$B = \frac{V_v}{V}$$
$$B = \frac{hA}{(H+h)A}$$

$$(20{,}000)0.015(1{,}570) = hA$$

$\underline{\underline{Ah = 4.71(10^5)\text{ ft}^3}}$ ← Volume of solids extracted

$(h = 23.6\text{ ft})$ ← extraction height

10. Given: Ore pass muck $\gamma = 105$ pcf
$\phi = 38°$
$\mu = 0.57$
$k = 0.33$
$a = 4$ g (muck dump)
$W = 3$ tons (6,000 lbs)
$H = 150$ ft
$h = 9$ ft
diameter = 13 ft
Find: Increase in σ_v due to muck drop.



Solution:

Formula for equilibrium with surcharge:

$$\sigma_v = \frac{\gamma A}{\mu k C}\left(1 - e^{-\frac{\mu k C}{A}z}\right) + \sigma_v^0 e^{\frac{-\mu k C}{A}z}$$

$$\Delta\sigma_v = \sigma_v^0 e^{\frac{-\mu k C}{A}z}$$

$$= \left[\frac{24{,}000}{\pi\frac{(13)^2}{4}}\right]\exp\left\{-\left(\frac{(0.57)(0.33)\pi 13}{\frac{\pi(13)^2}{4}}\right)9\right\}$$

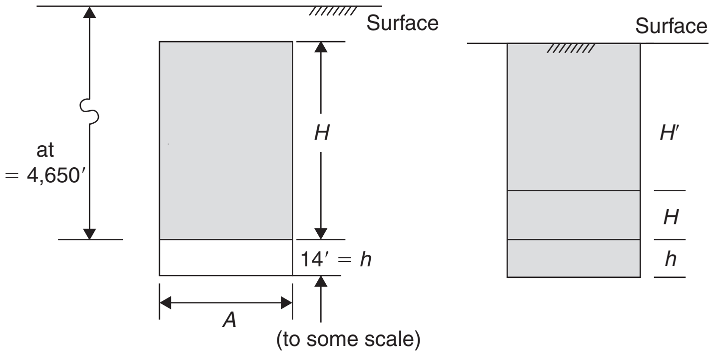
$$= (180.8)\exp(-0.521)$$

$$\underline{\underline{\Delta\sigma_v = 107.4\text{ lbf/ft}^2}} \quad \longleftarrow$$

11. Given: Rock and joint data, depth = 4,650 ft, panel cave 250 × 600 ft in plan, 14 ft high, bulking porosity $B = 0.075$, 12 ft³/ton & 50,000 tons per day.

Find:
(a) Cave zone height
(b) Additional draw to reach surface

Solution:



$$
\begin{aligned}
B &= \frac{V_v}{V_s + V_v} \\
&= \frac{Ah}{Ah + AH} \\
B &= \frac{h}{h + H}
\end{aligned}
$$

$$
\begin{aligned}
H &= \frac{h}{B} - h \\
&= \frac{14}{0.075} - 14 \\
\underline{\underline{H}} &\underline{\underline{= 172.7\text{ ft}}}
\end{aligned}
$$

← (a)

(b) When caving extends to surface?

$$
B = \frac{V_v'}{V_v' + V_s'}, \quad V_v' + V_s' = (H' + H + h)A = V'
$$

$$
\begin{aligned}
V_v' &= V_v + \Delta V_s \\
V_s' &= V_s - \Delta V_s
\end{aligned}
$$

$$
B = \frac{\Delta V_v}{\Delta V_v + \Delta V_s}, \quad \Delta V_v = \Delta V_s
$$

$$
\begin{aligned}
V_v' &= B(H' + H + h)A \\
&= 0.075(4{,}650 + 14)(250)(600) \\
\underline{V_v'} &\underline{= 5.247(10^7)\text{ ft}^3}, \quad V_v' - V_v = \Delta V_v \quad H' = 335.8\text{ ft.}
\end{aligned}
$$

$$\Delta V_v = 5.247(10^7) - (14)(250)(600)$$
$$\Delta V_v = 5.04(10^7)\,\text{ft}^3$$
$$\Delta V_v = \Delta V_s \quad \text{(new void volume comes from solid removed)}$$

$$\text{tons solid} = 5.04\frac{(10^7)}{12}$$
$$\text{tons} = 4.20(10^6)\text{tons}$$
$$\text{time} = \frac{\text{tons}}{\text{tons/day}}$$
$$= \frac{4.20(10^6)}{5(10^4)}$$
$$\underline{\underline{\text{days} = 84.0 \text{ days}}}$$

time elasped from undercut to sink hole appearance at surface.

Alternative:

To cave $H'A$ ft^3 at (50,000)(12) ft^3/day

$$\Delta t = \frac{(4{,}650 - 173)(250)(600)}{5(10^4)12}(0.075)$$
$$\underline{\underline{\Delta t = 84.0 \text{ days}}}$$

← void space rate & solid draw rate (b) (checks)

Combination Support

12. Given: Paper by F. Kendorski (Chapter 4 reference)

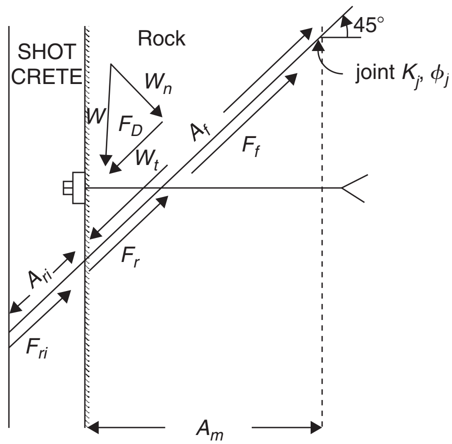
$$F_D = F_f + F_r + F_{ri}$$
$$(48{,}400)(0.707) = (10)(144)(5)(1.414) + (48{,}000)(0.707)(0.21) + (0.5)(0.60)(60{,}000) + 126(144)A_{ri}$$

Find: Explain meaning of each number, where obtained, physical meaning, are they correct?

Solution:

F_D = driving force "down" the joint = W_t
F_f = joint shear resistance
F_r = shear resistance of the reinforcement, bolt
F_{ri} = shotcrete wall shear resistance

$$F_D = Wt = \lambda\frac{W}{2}\sin 45°$$
$$\lambda = \text{abutment load factor}$$



Drift support design

$$\frac{W}{2} = \text{one-half of back arch weight}$$

$$\lambda = 2 \ \& \ W = \gamma B H_p \text{ per foot of drift}$$

$$H_p = 1.0(B + H_t) \quad \text{Drift size } 12' \times 12'$$

$$= 1.0(12 + 12); \quad \gamma = 168 \text{ pcf}$$

$$\underline{H_p = 24 \text{ ft}}$$

$$W = (168)(12)(24)$$

$$\underline{W = 48{,}384 \text{ lb/ft}}$$

$$\lambda \frac{W}{2} = 2\frac{(48{,}384)}{2} \simeq 48{,}400 \text{ lb/ft}$$

$$\sin(45) = \frac{1}{\sqrt{2}}$$

$$\underline{F_D = (48{,}400)(0.707) \quad \text{LHS} \quad \text{ok}}$$

$$F_f = A_j(\sigma_j \tan\phi_j + K_j)$$

bolt length = 5′

$$A_j = \frac{A_m}{\cos 45}$$

$$A_j = (5)(\sqrt{2})(\text{ft}^2 \text{ per foot of drift})$$

$$K_j = 10 \text{ psi}; \quad \phi_j = 12^\circ$$

$$K_j = 10(144) \text{ pcf}$$

$$A_j\sigma_j = W_n(\text{per foot of drift})$$

$$= W\cos(45)$$

$$A_j\sigma_j = (48{,}400)(0.707) \text{ lbs/ft}$$

$$F_f = (48{,}400)(0.707)(\tan(12)) + A_jK_j$$

$$F_f = (48{,}400)(0.707)(0.213) + (5)(\sqrt{2})(10)(144)$$

$$\leftarrow \text{joint friction} \rightarrow\leftarrow \text{joint cohesion} \rightarrow$$

$$F_r = \tau_b A_b$$

$$\tau_b = \text{bolt shear stress.}$$

$$A_b = \text{bolt area}$$

$$T_0 = 60{,}000 \text{ psi (steel strength)}$$

$$D_b = \frac{7}{8} \text{ in} \quad A_b = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 (\text{ln}^2)$$

$$A_b = 0.601\,\text{ln}^2$$
$$T_b = \tfrac{1}{2}T_0$$
$\underline{F_r = (0.5)(0.601)(60{,}000)}$ ← bolt shear resistance for thin shotcrete wall in shear

$$F_{ri} = 2\sqrt{fc'}A_{ri}$$
$$f_c' = 4{,}000\text{ psi}$$
$$F_{ri} = 2\sqrt{4{,}000}\ A_{ri}$$

A_{ri} in ln^2

$$\underline{F_{ri} = (126.5)(144)A_{ri}} \qquad \longleftarrow$$

F_{ri} is the shotcrete shear resistance.

13/14. Given: Grizzly drifts 9 ft high by 12 ft wide below the undercut level, 1st supported with bolts and wire mesh
Bolts: 3/4″ diameter, bolts 55 grade
$L_b = 6$ ft, grouted full length
Concrete: $C_0 = 5{,}500$ psi
Rock: $\gamma = 180\,\text{lb/ft}^3$ $\quad \phi = 42°$ $\quad c = 3{,}450$ psi
Joints: 3 sets of orthogonal joints.
$27° < \phi_j < 38°$
$7 < c_j < 70$ psi
Fracture persistence: 87% Intact rock.
13%
FS rock – 1.5
FS concrete – 1.4
$\lambda = 185\% = 1.85$
Find: Liner thickness needed.

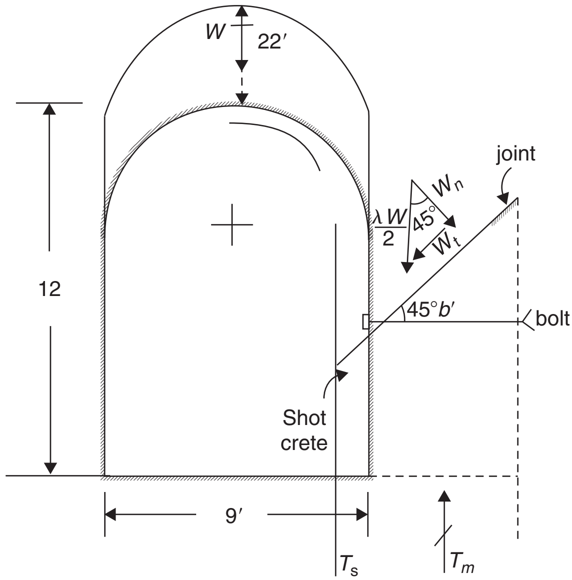
Solution:

Using the F. Kendorski article

$$h_p = 1.0(b + h_t)$$
$$= 1.0(9 + 12)$$
$$\underline{h_p = 21\text{ ft}}$$

$$W = (180\,\text{lb/ft}^3)(9\text{ ft})(21\text{ ft})(1\text{ ft})$$
$$\underline{= 34{,}020\text{ lbf}}$$

$$c_m = (0.87)(\bar{c}_j) + (0.13)(c_r)$$
$$= 0.87\left(\frac{7+70}{2}\right)(\text{psi}) + (0.13)(3{,}450)(\text{psi})$$
$$\underline{c_m = 481\text{ psi}}$$



Grizzly drift

$$\tan\phi_m = (0.87)\tan\overline{\phi}_j + (0.13)\tan\phi_r$$

$$\tan\phi_m = (0.87)\tan\left(\frac{27+38}{2}\right) + (0.13)\tan(42°)$$

$$\rightarrow \underline{\phi_m = 33.9°}$$

$$c_m = \frac{2c_m \sin\phi_m}{1-\sin\phi_m}$$

$$= \frac{2(481)\sin(33.9)}{1-\sin(33.9)}$$

$$c_m = 1{,}213 \text{ psi}$$

Equilibrium

$$\frac{W}{2} = T_s + T_m$$

$$T_s = \frac{34{,}020}{2}\text{ lbf} - (1{,}213\text{ psi})\left(144\frac{\text{ln}^2}{\text{ft}^2}\right)(6\text{ ft})$$

$$T_s = -1{,}031{,}022 \text{ lbf/foot of drift}$$

$\therefore$ No support needed for back arch

$$F_D = F_f + F_r + F_{ri}$$

$$F_D = W_t\ @\ \lambda = 1.85$$

$$= (1.85)(34{,}020)\left(\frac{1}{\sqrt{2}}\right)$$

$$\underline{F_D = 44{,}503 \text{ lbf/ft}}$$
$$F_f = A_j \sigma_j \tan \phi_j + A_j C_j$$

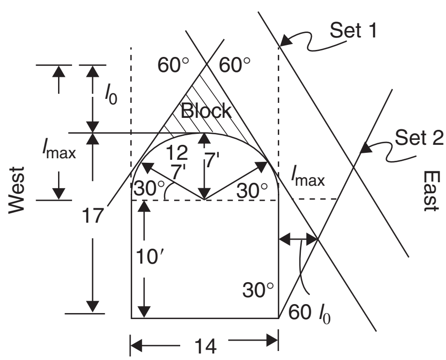
use low joint properties for a conservative estimate.

$$\begin{aligned}
F_f &= (44{,}503)\tan(27) + (b)(\sqrt{2})(7)(144) \\
F_f &= 31{,}228 \text{ lbf/ft} \\
F_r &= \tau_b A_b \\
&= \tfrac{1}{2}(18{,}400 \text{ lbf})/\text{ft} \quad \text{(Table 3.4, page 124)} \\
F_r &= 9{,}200 \text{ lbf/ft} \\
F_{ri} &= \sqrt{fc'}\, A_{ri} \\
&= 2(4{,}500)^{1/2} A_{ri} \\
F_{ri} &= 134\, A_{ri} \\
44{,}503 &= 31{,}228 + 9{,}200 + 134\, A_{ri} \\
&\underline{A_{ri} = 30.4 \text{ In}^2/\text{ft}} \\
t &= \frac{A_{ri}}{12} \\
&\underline{\underline{t = 2.5 \text{ inches}}}
\end{aligned}$$

$\leftarrow t$

15. Given:

Find: If conventional bolting sufficient.



Solution:

Main consideration is bolt length

$$\underline{\underline{\text{Rib:}}} \quad \frac{l_{max}}{10} = \tan 30°$$
$$l_{max} = (10)\tan 30°$$
$$\underline{\underline{l_{max} = 5.8 \text{ ft}}} \quad \text{(rib)}$$

$L_0 < l_{max}$ ∴ Conventional (rib) bolts sufficient

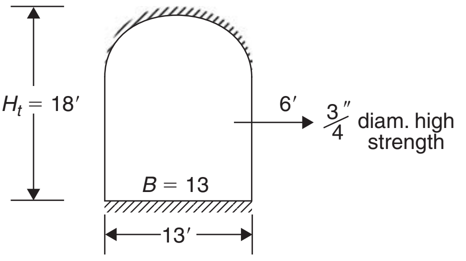
$$\underline{\underline{\text{Back:}}} \quad \frac{l_{max}}{7} = \tan 60°$$
$$l_{max} = (7)\tan 60°$$
$$\underline{l_{max} = 12.1 \text{ ft}} \quad \text{(back)}$$

this is long and a high estimate

$$l_0 = l_{max} - 7.0$$
$$\underline{\underline{l_0 = 5.1 \text{ ft}}}$$

<u>Yes, conventional length bolts should also work in the back.</u> $\leftarrow$ yes

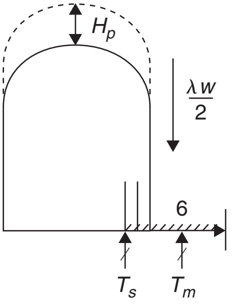
16/17. [convert units for 17] Given: Caving operation
joint persistence = 0.88
Rock: $C_0 = 12{,}750$ psi, $\phi_r = 29°$
$\gamma = 157$ pcf
Joints:
$c_j = 75$ psi $\phi_j = 20°$
depth = 3,750 ft
$\lambda = 2.00$ (100% increase)
$FS = 1.5$



Find: Combination support with FS's = 1.5

Solution:

Equilibrium



$$\frac{\lambda W}{2} = T_s + T_m$$
$$W = \gamma H_p BS$$
$$B = 13'$$
$$H_p = 1.0(B + H_t)$$
$$H_p = 1.0(13 + 18)$$
$$H_p = 31'$$
$$S = 1' \text{ (per ft of tunnel)}$$
$$W = (157)(31)(13)(1)$$
$$\underline{W = 6.327(10^4) \text{ lbf}}$$

$$\frac{(2)(6.327)(10^4)}{2} = T_s + T_m$$
$$T_m = C_0 A$$
$$C_0 = pC_{0j} + (1 - p)C_{0m}$$
$$c_{0j} = \frac{2c_j \cos\phi_j}{1 - \sin\phi_j}$$
$$= \frac{(2)(75)\cos 20°}{1 - \sin 20°}$$
$$\underline{\underline{c_{0j} = 214 \text{ psi}}}$$

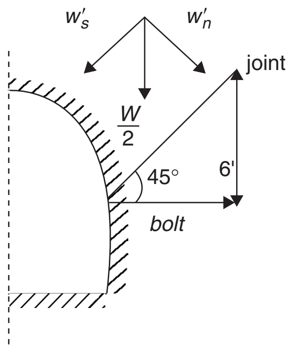
$$C_{0m} = 0.88(214) + (1 - 0.88)(12{,}750)$$
$$\underline{C_{0m} = 1{,}718 \text{ psi}}$$

$$\therefore\ T_s = 6.327(10^4) - (1{,}718)(144)(6)$$
$$T_s = 6.327(10^4) - 1.484(10^6)$$
$$T_s = -1.42(10^6)$$

∴ No external support needed for equilibrium ($FS > 1.5$ clearly)

Joint failure:

Equilibrium:
$F_D = F_t + F_r$ (if shotcrete not needed)



$$\left(\frac{\lambda W}{2}\right)(\sin 45°) = \underline{6.327(10^7)\left(\frac{1}{\sqrt{2}}\right)} = \underline{4.474(10^7)}$$

$$F_f = N \not{c} \tan\phi_j + C_j$$
$$= 6.327(10^4)\left(\frac{1}{\sqrt{2}}\right)\tan 20° + (75)(144)(6)(\sqrt{2})$$
$$F_f = 1.628(10^4) + 9.164(10^4)$$
$$\underline{F_f = 10.79(10^5)\text{lbf.}}$$

$\therefore$ even without bolt, joint is safe and has $FS > 1.5$.

$$F_r' = \frac{1}{2}\text{bolt strength} = \frac{1}{2}(18{,}400) = 9{,}200\,\text{lbf.}$$

(This is *not* per ft of tunnel)

18. Given: Block caving underground data, drifts $16' \times 16'$, semicircular backs, at 3,000 – 5,000 ft ramped down.

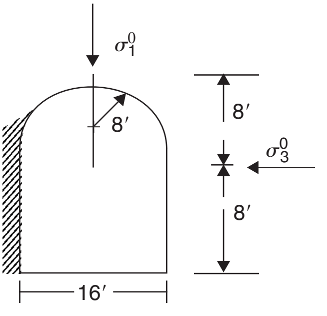
Find:

(a) FS_c, FS_t no support
(b) combination support
$FS = 1.8$ rock mass,
Bolts @ $\frac{1}{2}(16') = 8'$

Solution:

Assume σ_c at sharp corner and treat as rectangle with $W_0/H_0 = 2$, i.e. σ_1^0 = vertical gravity only, S_v assume $M = 1/3$ then from USBM Bulletin 587 or text Table 3.2, approximately:

$$K_c = 4.0 \qquad K_t = -0.1$$
$$\sigma_c = K_c\sigma_1^0 \qquad \sigma_t = K_t\sigma_1^0$$



at 5,000 ft, assume 1 psi/ft or better
$\sigma_1^0 = (156/144)(5{,}000) = \underline{\underline{5{,}418\text{ psi}}}$

$$\sigma_c = (4)(5{,}418) \qquad \sigma_t = (0.1)(5{,}418)$$
$$\underline{\underline{\sigma_c = 21{,}667\text{ psi}}} \qquad \underline{\underline{\sigma_t = 542\text{ psi}}}$$
$$FS_c = \frac{C_0}{\sigma_c} \qquad FS_t = \frac{T_0}{\sigma_t}$$

$$C_0 = \frac{2c\cos\phi}{1 \mp \sin\phi} = \frac{(2)(332)\cos 28.7}{1 \mp \sin 28.7}$$

$$\underline{\underline{C_0 = 1{,}120\ \text{psi}}}$$

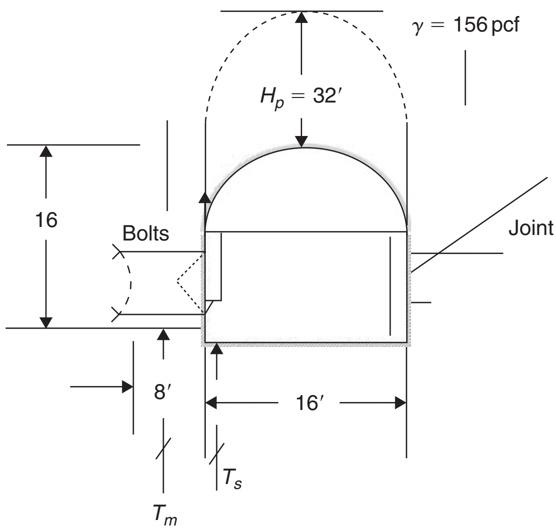
From previous problem:
$C = 332$ psi
$\phi = 28.7°$
$\underline{\underline{T_0 = 393\ \text{psi}}}$

$$FS_c = \frac{1{,}120}{21{,}667} \quad FS_t = \frac{397}{542}$$

$$\underline{\underline{FS_c = 0.0517 \quad FS_t = 0.732}}$$

← FS
naturally supported (will need support)

(b)



$$FS = \frac{C_0 A}{\sigma A}$$

$$\therefore \sigma = \frac{C_0}{FS}$$

$$\Sigma F_v = 0 \qquad : (\lambda = 2, \text{caving})$$

$$> \frac{W}{2} = T_m + T_s \qquad : (\text{per ft of drift})$$

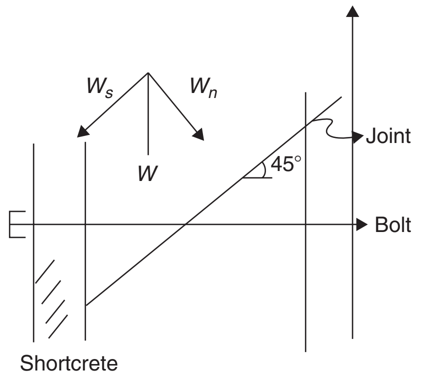
$$(156)(16)(32)(1) = \frac{(8)(1{,}120)(144)(1)}{1.8} + T_s$$

$$T_s = 7.987(10^4) - 7.168(10^5)$$

since $T_s < 0$, no support needs for rock arch.
Check: Shotcrete, bolt, joint slip

$$\underset{\text{(joint)}}{W_f} + \underset{\text{(bolt)}}{W_n} + \underset{\text{(shortcrete)}}{W_{ri}} \qquad \text{Equilibrium.}$$

Joint only?



$$W_D = W \sin 45°$$
$$W_f = W_n \tan\phi' + cA$$

$$A = (1)(L_j)$$
$$= (1)\left(\frac{H}{\sin 45°}\right)$$
$$A = (1)(8)(\sqrt{2})$$

$$W_f = W \sin 45° \tan 27° + (17)(144)(8)\sqrt{2}$$
$$= \frac{W}{\sqrt{2}} \tan 27° + 2{,}770(10)$$
$$W \geqq W \tan 27° + (17)(144)(8)2 \quad W = (156)(32)(16)$$
$$1 \geq 0.510 + \frac{(17)(144)(8)(2)}{(156)(32)(16)}$$
$$1 \geqq 0.510 + 0.490 = 0.999$$

∴ Barely in equilibrium, bolts will get $FS > 1.0$ will still need shotcrete to get FS higher – probably.

Note: Very low FS for natural supported rock mass from stress concentration but using rock and load of rock mass gives no support needed. Support needed only to avoid slip on joints. Conflicting views (arch load is small independent of depth and thus not reliable for deep opening!)

19/20. Given: Caving operation joint persistence is 88%
[Units convert for 20]
$C_0 = 12{,}750$ psi
$\phi = 29°$
$\gamma = 157$ pcf
depth = 3,750 ft
$c_j = 75$ psi
$\phi_j = 20°$

Development drift, 13 × 18 ft, $\lambda = 100\%$

- rock mass is not self supporting with a safety factor of 3.0
- rock bolts, shotcrete and wire mesh
- shotcrete compressive strength 3,800 psi
- required $FS = 1.5$

Find: Design combination support system

- bolt size
- length
- steel grade

– spacing
– shotcrete thickness

Solution:

Need cohesion of the intact rock

$$\sin(\phi) = \frac{C_0 - T_0}{C_0 + T_0}; \quad \text{Assuming } M - C \text{ criterion}$$

$$(C_0 + T_0)\sin(\phi_r) = C_0 - T_0$$

$$\Rightarrow \quad T_0(1 + \sin(\phi_r) = C_0 - (\sin\phi_r)C_0$$

$$\Rightarrow \quad T_0 = C_0\left(\frac{1 - \sin(\phi_r)}{1 + \sin(\phi)_r}\right)$$

$$= 12{,}750\left(\frac{1 - \sin(29)}{1 + \sin(29)}\right)$$

$$T_0 = 4{,}424$$

$$c_r = \left(\frac{1}{2}\right)\sqrt{C_0 T_0} = \left(\frac{1}{2}\right)[12{,}750(4{,}424)]^{1/2}$$

$$\underline{c_r = 3{,}755 \text{ psi}}$$

$$c = c_f + R(c_r - c_f)$$

$$= 75 + (0.12)(3{,}755 - 75)$$

$$\underline{c = 517 \text{ psi}}$$

$$\tan\phi = \tan\phi_f + R(\tan\phi_r - \tan\phi_f)$$

$$= \tan(20) + 0.12[\tan(29) - \tan(20)]$$

$$\tan\phi = 0.5315$$

$$\underline{\phi = 21^\circ}$$

$$\sigma_1 = 2c\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) + \sigma_3\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

$$\sigma_1 = 2(517)\tan\left(\frac{\pi}{4} + \frac{21}{2}\right) + 0 = 1{,}504 \text{ psi}$$

$$\sigma_m = \frac{\sigma_1}{FS_m}$$

$$= \frac{1{,}504}{3}$$

$$\underline{\sigma_m = 501 \text{ psi}}$$

$$h_p = 1(b + h_f)$$

$$= 13 + 18$$

$$\underline{h_p = 31 \text{ ft}}$$

Assume a bolt length of 6 ft.

To satisfy equilibrium. 1/2 weight to each sides

$$\frac{\lambda \gamma h_p BS}{2} = \sigma_m A_m + \sigma_s A_s; \quad \lambda = 200\%$$
$$\frac{2(157)(31)(6)}{2} = (501)(6)(144) + \sigma_s A_s$$
$$\sigma_s A_s = 29{,}202 - 4{,}32{,}864$$
$$\underline{\sigma_s A_s = -4{,}03{,}662 \text{ lbs.}}$$

No artificial support is necessary during abutment loading.
Checking shear loading during abutment loading of 100% on a through going fault unfavorably oriented at 45°.

$$F_D = F_f + F_r + F_r;$$
$$F_D = \lambda \frac{W_t}{2} \sin(45); \quad \lambda = 200\%$$
$$W_t = (157)(13)(31)(1)$$
$$= 63{,}271 \text{ lbf.}$$
$$F_D = (2)\left(\frac{1}{2}\right)(63{,}271)\left(\frac{1}{\sqrt{2}}\right)$$
$$\underline{F_D = 44{,}739 \text{ lbf.}}$$
$$F_f = A_j \sigma_j \tan \phi_j + A_j C_j; \quad A_j \sigma_j = W_n$$
$$= (44{,}739) \tan(20) + (6)(\sqrt{2})(75)(144)$$
$$\underline{F_f = 1{,}07{,}923 \text{ lbf}}$$

$$F_r = \tau_b A_b$$
$$T_D = 60{,}000 \text{ psi}$$
$$D_b = \frac{7}{8} \text{ in}$$
$$A_b = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.601 \text{ in}^2$$
$$\tau_b = \frac{1}{2} T_0 = 30{,}000 \text{ psi}$$
$$F_r = (30{,}000)(0.601) = 18{,}030 \text{ lbf}$$
$$\underline{F_r = 18{,}030 \text{ lbf}}$$

The joint cohesion is high and the factor of safety mobilized by joint friction and cohesion alone is

$$\frac{F_f}{F_b} = \frac{1{,}07{,}923}{44{,}739} = 2.41$$

No artificial support is required.

Subsidence Troughs

21. Given: A longwall panel with trough subsidence and UK conditions
    $W = 820'$ face length, panel width
    $h = 1{,}300'$ deep
    $m = 16.5'$ mining height, seam thickness
    $L = 6{,}200'$ panel length

    Find:
    1. S_{max}
    2. $S_{max}/w = 520'$
    3. Surface fully developed subsidence profiles.
    4. Surface strain profiles, $E+$, $E-$, e
    5. Barrier pillar width W_p
    6. Required time t @ 7,600 T/shift; 2 shifts/day; 250 days/year
    7. Face advance rate v_f (ft/day)
    8. Max subsidence and trough width after mining 4 identical panels adjacent to the 1st panel
    9. Max subsidence and trough width after mining identical panel 80′ below the 1st panel
    10. Utah conditions, subsidence factor = 0.65, angle of draw = 28°, estimate S_{max}, trough width, for 1 panel, 5 panels (adjacent)
    11. Max tensile and compressive strains 1 panel, Utah conditions.

    Solution:

    1. S_{max}

$$\text{Depth in meters} = \frac{1{,}300}{3.281} = 396\,\text{m}$$

$$\text{Width in meters} = \frac{820}{3.281} = 250\,\text{m}$$

$$\frac{W}{H} = \frac{820}{1{,}300} = \frac{250}{396}$$

$$\frac{W}{H} = 0.63$$

From Figure 3 in (SEH) Subsidence Engineers hand book – UK, text, Figure 8.15 or eqn.

$$\frac{S_{max}}{m} \approx 0.6.$$

$$S_{max} = (0.6)(16.5)$$

$$\underline{S_{max} = 9.9'(3.0\,\text{m})} \qquad \longleftarrow \underline{S_{max}}$$

2. $\dfrac{S}{W} = 520' = 158\,\text{m}$

$$\frac{L}{h} = \frac{158}{396} = 0.399$$

From Figure 8.16 text or SEH figure 4

$$\frac{s}{S} = 0.36$$
$$\Rightarrow s = (0.36)9.9$$
$$\underline{s = 3.6', (1.1\,\text{m})}$$

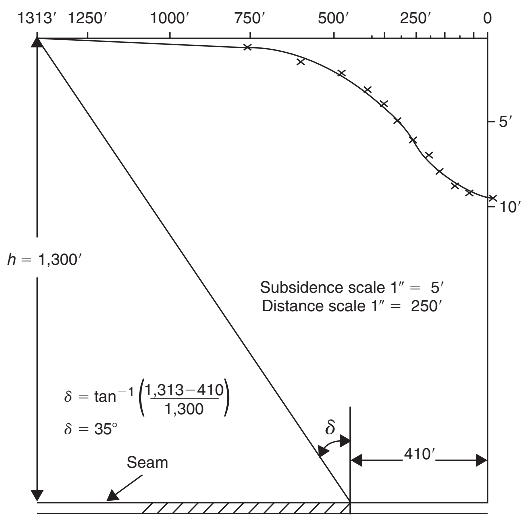
$\longleftarrow S$

3. Surface subsidence profile

$$\frac{W}{h} = \frac{820}{1{,}300} = 0.63 \simeq 0.62$$

Assume horizontal seam, need only half profile
From Table 1 SHE or Table 8.1 text interpolating.

s/S_{max}	0	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	1.00
l/h	1.01	0.59	0.47	0.37	0.31	0.27	0.23	0.19	0.16	0.13	0.09	0.06	0
S(ft)	0	0.50	0.99	1.98	2.97	3.96	4.95	5.94	6.93	7.92	8.91	9.41	9.90
l(ft)	1,313	767	611	481	403	351	299	247	208	169	117	78	0



i.e., vertical to horizontal scale ratio is 5:250

4. Surface strain profile
From Table 5 in SEH or text Table 8.3, page 445

$$\frac{W}{h} = \frac{820}{1{,}300} = 0.631$$
$$+E = \frac{x - 0.8}{0.65 - 0.8} = \frac{0.631 - 0.5}{0.8 - 0.5}$$
$$+E = 0.73\,\frac{S}{h}$$
$$-E = \frac{x - 1.35}{0.7 - 1.35} = \frac{0.631 - 0.5}{0.8 - 0.5}$$

$$-E = 1.07\,\frac{S}{h}$$

$$G = \frac{x - 3.35}{2.8 - 3.35} = \frac{0.631 - 0.5}{0.8 - 0.5}$$

$$G = 3.11\,\frac{S}{h}$$

From Figure 14, SEH or text figure 8.21

$$\frac{-e}{-E} = 0.89 \quad @\ \frac{W}{h} = 0.631$$

$$-e = (0.89)(1.09)\,\frac{S}{h}$$

$$= 0.97\,\frac{S}{h}$$

$$\text{since } \frac{S}{h} = \frac{S_{max}}{h}$$

$$= \frac{9.9}{1{,}300} = 7.62 \times 10^{-3}$$

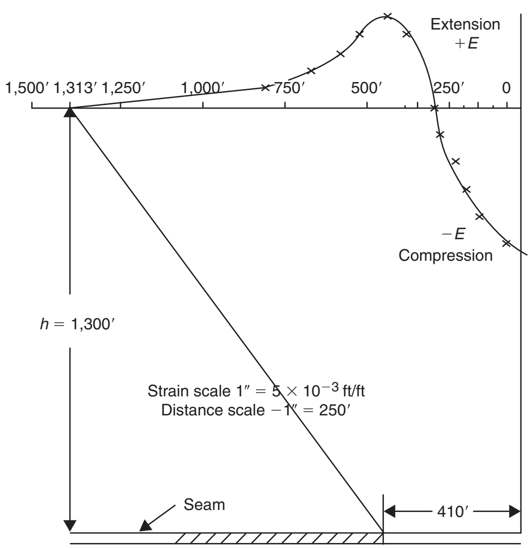
$$\underline{+E = 5.563 \times 10^{-3}}\ \text{ft/ft}$$

$$\underline{-E = 8.153 \times 10^{-3}}\ \text{ft/ft} \qquad \leftarrow +E, -E, -e$$

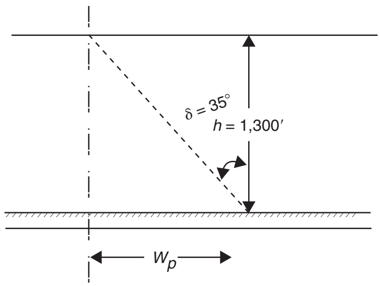
$$\underline{-e = 7.391 \times 10^{-3}}\ \text{ft/ft}$$

Now using Table 4 SEH or text Table 8.2 and $W/H = 0.62$, interpolating

	Extension $(+E)$							Compression $(-E)$					
$\frac{e}{E}$	0.0	0.2	0.4	0.6	0.8	1.0	0.8	0	0.2	0.4	0.6	0.8	1.00
$\frac{l}{h}$	1.01	0.63	0.52	0.45	0.41	0.34	0.30	0.23	0.21	0.18	0.15	0.12	0.05
$e \times 10^{-3}$	0	1.13	2.25	3.38	4.51	5.64	4.51	0	1.66	3.32	4.98	6.64	8.30
l (ft)	1,313	819	676	585	533	442	390	299	273	234	195	156	65



5. Barrier Pillar width



$$W_p = h \tan\delta$$
$$\Rightarrow W_p = 1,300\,[\tan(35°)]$$
$$\underline{W_p = 910'}$$

W_p ⟵

6. Total panel volume = (6,200)(820)(16.5)
= 83,88,600 ft^3

@ 7,600 Tons/Shift; 2 shifts/day; 250 days/year; Assuming 20 ft^3/ton for coal i.e. $\gamma = 100\,\text{lbf/ft}^3$

Volume moved in a year = (7,600)(2)(250)(20)
= 7,60,00,000 ft^3/year

to move 83.886×10^6 ft^3 it will take 1.1 years. ⟵

7. Cubic feet moved per day = (7,600)(2)(20)
$= 304 \times 10^3$ ft^3

$$\text{Face advance/day} = \frac{\text{cubic ft/day mined}}{\text{Face Area}}$$
$$= 304 \times 10^3/(820)(16.5) = 22.5 \text{ ft}$$

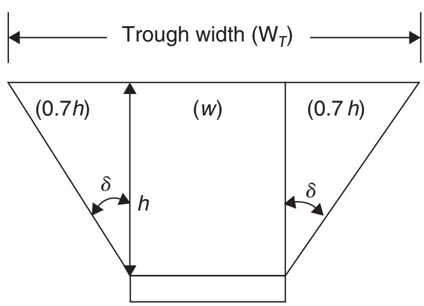
Average face advance/day = 22.5 ft ⟵

8. Maximum subsidence after mining 4-adjacent panels

Now the face width = (5)(820) = 4,100′

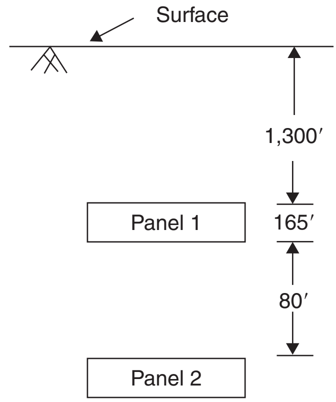
Using Figure 3 SEH – P-9

$$\text{Face width} = \frac{4,100}{3.281} = 1,250 \text{ m}$$
$$\text{Depth} = 396 \text{ m}$$
$$\Rightarrow \frac{S_{max}}{m} = 0.9$$
$$\Rightarrow \underline{S_{max} = (0.9)(16.5) = 14.9'}$$ ⟵



$$\Rightarrow \delta = 35°$$
$$\Rightarrow W_T = 2(910) + 4,100'$$
$$\underline{W_T = 5,920'}$$ ⟵

9. Calculate subsidence for each panel then use superposition principle
Panel 1 $S_{max} = 9.9'$, from 1.



Subsidence due to Panel 2

$h = 1{,}397' = 426$ m
$W = 820' = 250$ m

$$\frac{S_{max}}{m} = 0.55$$

$S_{max} = (0.55)(165) = 9.1'$

$S_{max}(\text{Total}) = \underline{9.9 + 9.1 = 19' = 5.8\text{ m}}$ ⟵

10. Assuming Utah conditions

$$\frac{S_{max}}{m} = 0.65; \quad \delta = 28° \quad \text{@ critical width}$$

Single panel

$$\begin{aligned} W_c &= 2h\tan(28) \\ &= 2(1{,}300)\tan(28) \\ W_c &= 1{,}382 \text{ ft} \end{aligned}$$

A single panel is only 820′ i.e. sub-critical
If sub-critical width.

$$\begin{aligned} S_{max}(\text{Utah}) &= \frac{0.65}{0.9}\left(\frac{S_{max}(\text{UK})}{m}\right)m \\ &= \frac{0.65(0.6)(16.5)}{0.9} \\ \underline{S_{max}(\text{Utah})} &\underline{= 7.16\text{ ft}, \ (2.18\text{ m})} \end{aligned}$$ ⟵

$$\begin{aligned} \text{Trough width}(W_T) &= 2(\tan 28)(1{,}300) + 820 \\ &= 2{,}202' \\ \underline{W_T} &\underline{= 671.3\text{ m}} \end{aligned}$$ ⟵

Four Adjacent panels.

Panel width $= (5)(820) = 4{,}100' \Rightarrow$ supercritical

$$\begin{aligned} S_{max} &= (0.65)(16.5) \\ \underline{S_{max}} &\underline{= 10.73' = (3.3\text{ m})} \end{aligned}$$ ⟵

$$\begin{aligned} W_T &= 2[\tan(28)](1{,}300) + 5(820) \\ \underline{W_T} &\underline{= 5{,}482'\ (1{,}671\text{ m})} \end{aligned}$$ ⟵

11. For Utah conditions, single panel.

$$\frac{S_{max}}{h} = \frac{7.16}{1{,}300} = 5.508 \times 10^{-3}\ \text{ft/ft}$$

$$\Rightarrow +E = (0.73)(5.508 \times 10^{-3})$$

$$\underline{+E = 4.021 \times 10^{-3}}\ \text{ft/ft}$$

$$-E = (1.09)(5.508) \times 10^{-3}$$

$$\underline{-E = 6.004 \times 10^{-3}}\ \text{ft/ft}$$

$$\text{for } \frac{W}{h} = 0.62$$

Max tensile strain = 6.108×10^{-3} ft/ft
Max Compressive strain = 8.997×10^{-3} ft/ft ←

22. Given: Long wall panel.
1,900 m long
250 m Face length
396 m deep
5.0 m thick
UK conditions apply

Find:

(1) S_{max} for a Single panel
Fig. 3. SEH or text Figure 8.15, $S/m = 0.60$
$\therefore S_{max} = (5)(0.60)$
$\underline{\underline{S_{max} = 3.00\ \text{m.}}}$ ← (1)

(2) S_{max} at a face advance of 158 m
Fig. 4. SHE, text Figure 8.16

$$\frac{l}{h} = \frac{158}{396}$$

$$\frac{l}{h} = 0.399$$

$$\frac{s}{S} = 0.38$$

$$S = (0.38)(3.00)$$

$\underline{\underline{S_{max} = 1.14\ \text{m}}}$ ← (2)

(3) Peak strains $E+$, $E-$, e, SHE Table 5, text Table 8.3

$$\frac{W}{h} = \frac{250}{396}$$

$$\underline{\frac{W}{h} = 0.631}$$

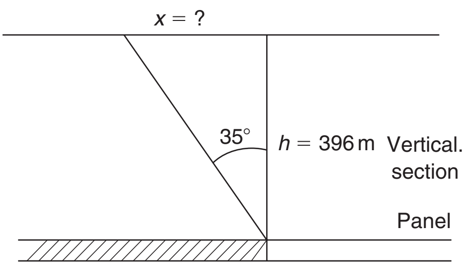
W/h	$E+$	$E-$
0.5	0.80	$1.35\,S/h$
0.631	$0.735\,S/h$	$1.066\,S/h$
6.8	$0.65\,S/h$	$0.7\,S/h$

$\longrightarrow$ (interpolation)

$$\frac{S}{h} = \frac{3.0}{396}$$
$$\frac{S}{h} = 7.576(10^{-3})$$
$$E+ = 0.735(7.576)(10^{-3}) = 5.57(10^{-3}) \text{ tension}$$
$$E- = 1.066(7.576)(10^{-3}) = 8.076(10^{-3}) \text{ compression}$$
$$E = 0.9E- = 0.9(8.076)10^{-3} = \underline{\underline{7.268(10^{-3})\text{compression}}}$$

$\longleftarrow$ (3)

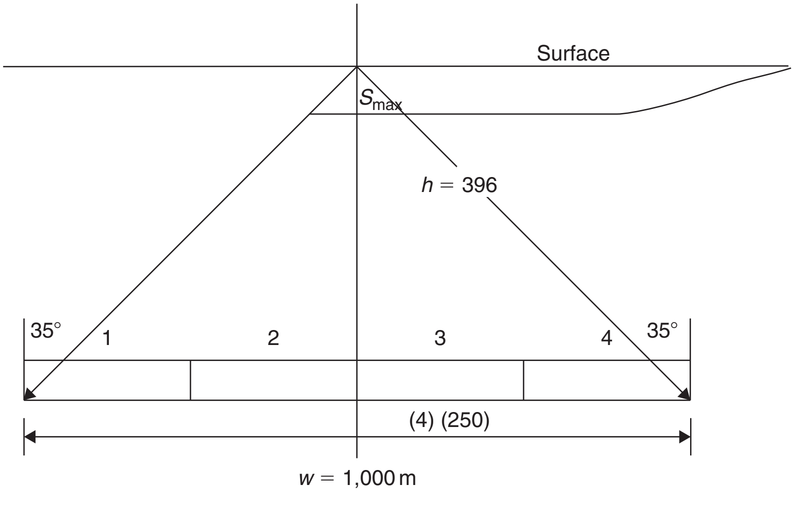
(4) W_B



$$\tan 35° = 0.7$$
$$0.7 = \frac{z}{396}$$
$$z = W_B = \underline{\underline{277\text{ m}}}$$

$\longleftarrow$ (4)

(5) 4 Panels, S_{max}



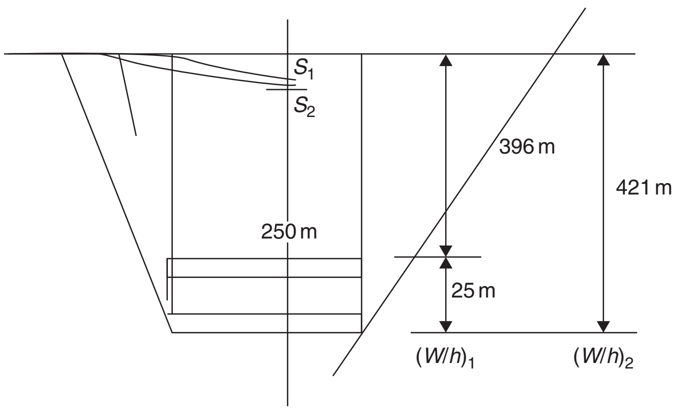
$$\frac{w}{h} = \frac{1{,}000}{396}$$
$$\frac{w}{h} = 2.525$$

But critical width $W_c = h \tan \delta$

$$W_c = (396)(0.7)(2)$$
$$W_c = 554\,\text{m}$$
$$\therefore\ S_{max} = 0.9\,\text{m}$$
$$S_{max} = 0.9(5.0)$$
$$\underline{\underline{S_{max} = 4.5\,\text{m.}}}$$

← (5)

(6) Two panels S_{max} W-trough



$$S_{max} = S_1(\text{max}) + S_2(\text{max})$$
$$= 0.6\,\text{m} + 0.6\,\text{m}$$
$$S_{max} = (2)(0.6)(5.0)$$
$$\underline{\underline{S_{max} = 6.0\,\text{m}}}$$

← (6)

$$W = 2\,h \tan \delta + 250$$
$$= (2)(421)(0.7) + 250$$
$$\underline{\underline{W = 839\,\text{m}}}$$

← (6)

(7) Utah conditions

$$0.65 = \text{Subsidence factor}$$
$$28^\circ = \text{angle of draw}$$

$$\text{UK: } S/M = 0.9$$
$$\text{UT: } S/M = 0.65$$
$$m = \frac{0.9}{S(\text{OK})} = \frac{0.65}{S_{(\text{UT})}}$$
$$S_{(\text{UT})} = \left(\frac{0.65}{0.90}\right)(3.0)$$
$$\underline{\underline{S_{(\text{UT})} = 2.17\,\text{m.}}} \quad \longleftarrow (7)$$

$$W = 2h \tan\delta(\text{UT}) + 250\,\text{m}$$
$$= (2)(396)\tan 28^\circ + 250\,\text{m}$$
$$\underline{\underline{W = 671\,\text{m}}} \quad \longleftarrow (7)$$

23. Given: Long wall – trough data, depth = 1,300 ft, thickness = 16.5 ft (full height mining), length = 6,200 ft, face = 820 ft

Find:

(a) S_{max}

face width = (820)(0.3048) = 250 m
depth = (1,300)(0.3048) = 396 m
Fig. 3 SEH, text figure 8.15 S/m = 0.60

$$S_{max} = (0.60)(16.5) = \underline{\underline{9.9\,\text{ft}}} \quad \longleftarrow (a)$$
$$(3.02\,\text{m})$$

Fig. 11 SEH, text Figure 8.16

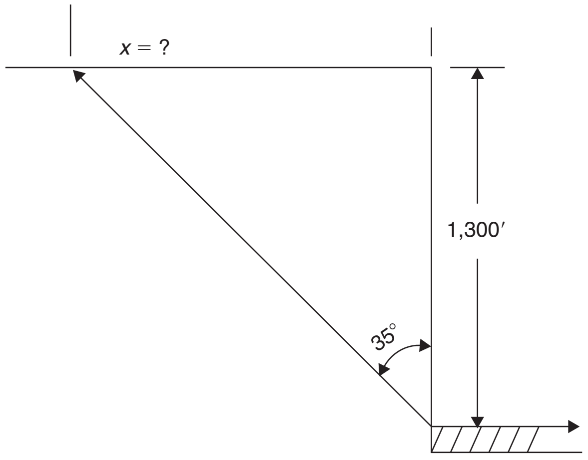
$$\frac{l}{h} = \frac{520}{1{,}300} = 0.40$$
$$\frac{s}{S} = 0.38$$
$$s = (0.38)(9.9) = \frac{3.76\,\text{ft}}{(1.15\,\text{m})} \quad \longleftarrow (b)$$

Table 5 SHE, text Table 8.3

$$\frac{W}{h} = \frac{820}{1{,}300} = 0.631$$

Between:

	$+E$	$-E$
0.5	0.8 S/h	1.35 S/h
0.631		
0.8	0.65 S/h	0.7 S/h
0.631	0.735 S/h	1.066 S/h



$$\frac{S}{h} = \frac{9.9}{1{,}300}$$

$$\underline{\underline{E+ = 5.60(10^{-3})\text{ tension}}}$$

$$\underline{\underline{E- = 8.12(10^{-3})\text{ compression}}} \qquad \leftarrow \text{(c)}$$

$$e = 7.30(10^{-3})\text{ compression}$$

$$\frac{W}{h} = 0.631 \qquad \frac{-e}{-E} = 0.9$$

$$-e = (0.9)(8.12)(10^{-3})$$

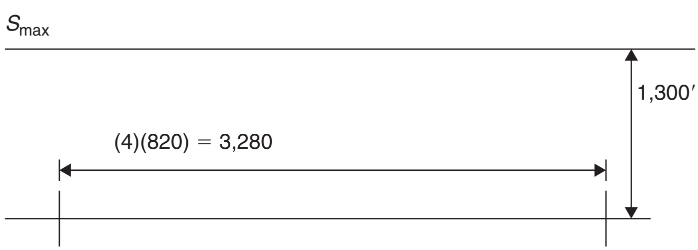
$$\underline{-e = 7.307(10^{-3})}$$

$$\tan 35^\circ = \frac{x}{1{,}300}$$

$$x = (0.7)(1{,}300)$$

$$\underline{\underline{x = 910\text{ ft}}} \qquad \leftarrow \text{(d)}$$

5. S_{max}



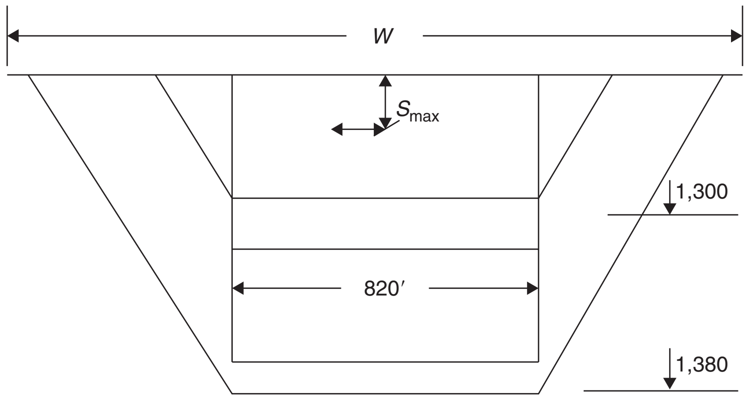
width is greater than 1.4 (1,300)

$$\therefore \frac{S_{max}}{m} = 0.9$$

$$S_{max} = (0.9)(16.5) = \underline{\underline{14.9\ ft}}$$ (5a)
$$(4.54\ m)$$

Trough width $3{,}280 + (2)(910) = \underline{\underline{5{,}100\ ft}}$ (5b)

6. S_{max}



Subsidence at 1,380 depth is nearly the same as at 1,300

$$S_{max} = 2S_{max}$$
$$= (2)(9.9)$$
$$\underline{\underline{S_{max} = 19.8\ ft}}$$ (6a)

Trough width = ?

$$W = 820 + (2)(0.7)(1{,}380)$$
$$\underline{\underline{W = 2{,}752\ ft}}$$ (6b)

7. UT condition
   UK $= S_{max/m} = 0.90$ unlimited
   UT $= S_{max/m} = 0.65$
   $\therefore \text{UT}(S) = \left(\frac{0.65}{0.90}\right) \text{UK}(S)$

   Note

$$m = \frac{S_{max}(\text{UK})}{0.90} = \frac{S_{max}(\text{UT})}{0.65}$$
$$\text{UT}(S_{max}) = \left(\frac{0.65}{0.90}\right)(9.9) = \underline{\underline{7.15\ ft}}$$ (7a)

through:

$$W = 820 + (2)\tan 28^\circ(1{,}300)$$
$$W(\text{UT}) = \underline{\underline{2{,}204\ ft}}$$ (7b)

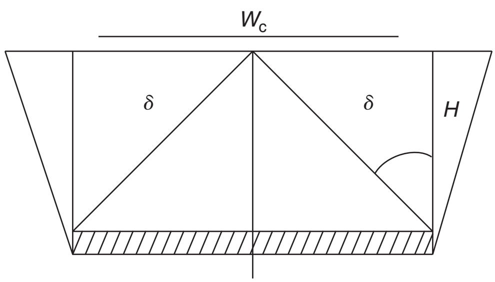
24. Given: Longwall panel, UK conditions.
$M = 5$ m (thick, mined)
$H = 400$ m (depth)
Flat
300 m wide
3,000 m long
3,000 t/shift, 2 shifts/day = 16,000 t/day.
@ 5 days per week = 80,000 t/week.

Find:

(1) $S_{\max}$: $\dfrac{S}{m} = 0.68$

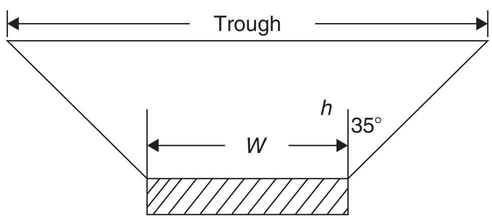
$$S = (5)(0.68)$$
$$\underline{\underline{S_{\max} = 3.40\text{ m}}}$$
← $S_{\max}$

(2) Critical area



$$A_c = W_c^2$$
$$W_c = 1.4H$$
$$= (1.4)(400)$$
$$\underline{W_c = 560\text{ m}}$$
$$A_c = (560)^2$$
$$\underline{\underline{A_c = 3.136(10^5)\text{m}^2}}$$
← A_c (2)

(3)



$$\text{trough } W' = W + 1.4H$$
$$= (300) + (1.4)(400)$$
$$\underline{\underline{\text{Width} = 860\text{ m}}}$$
← Trough width (3)

(4) $E+$ $\dfrac{W}{H} = \dfrac{300}{400}$

$$\frac{W}{H} = 0.75$$

(Table 5)

0.5	0.8 *S*/*h*
0.75	0.675
0.8	0.65 *S*/*h*

$$\frac{0.25}{0.3} = \frac{x}{0.15}$$
$$x = 0.125$$
$$\therefore\ E(+) = 0.675\frac{S}{h}$$
$$= 0.675\left(\frac{3.4}{400}\right)$$
$$\underline{\underline{E(+) = 5.738(10^{-3})}} \qquad \overleftarrow{E(+)}$$

(5) max E−

(Table 5)

0.5	0.135 *S/h*
0.75	0.606
0.8	0.7 *S/h*

$$\frac{0.25}{0.30} = \frac{x}{0.565}$$
$$x = 0.471$$
$$E(-) = 0.606\left(\frac{3.4}{400}\right)$$
$$\underline{\underline{E(-) = 5.1496(10^{-3})}} \qquad \overleftarrow{E(-)}$$

(6) Severity of change: depends on structure length at $5 + (10^{-3})$ tensile strain will be severe for 25 m long structure.

(7) *e* at center of subside profile.

$$\frac{W}{H} = \frac{300}{400} = 0.75$$
$$\frac{-e}{E} = 0.76$$
$$-e = (0.76)(5.15)10^{-3}$$
$$\underline{\underline{-e = 3.91(10^{-3})}} \qquad \overleftarrow{-e}$$

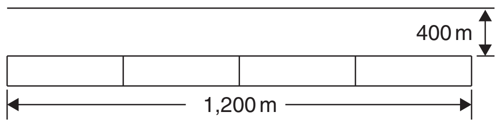
(8) $S_{\max}$ at face = 200 m

$$\frac{s}{S}\alpha\frac{L}{H} = \frac{200}{400}$$
$$\frac{L}{H} = 0.5$$
$$\frac{s}{S} = 0.5$$
$$s = (0.5)(3.4)$$
$$\underline{\underline{s = 1.7\,\text{m}}} \qquad \overleftarrow{s\ @\ 200\,\text{m}}$$

(9) time required for mining
rate = 16,000 t/day,
Volume = (5)(300)(3,000) = $4.5(10^6)m^3$ @$35.3\ ft^3/m^3$
Assume: $\gamma = 100$ pcf
$w_t = (4.5)(10^6)(35.3)(100)/2,000$ lb/s
$w_t = 7.95(10^6)$tons
days = $7.95(10^6)/16(10^3)$
days = 497 ← Working days

(10) S_{max} for 4 panels

$W = 4(300)$
$W = 1,200$
$W = 400$
$\frac{W}{H} = 3$

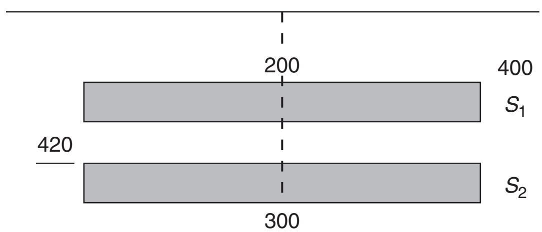


Supercritical

$$S_{max} = 0.9\ m$$
$$= 0.9(5)$$
$$S_{max} = 4.5\ m$$

← S_{max}(4 panels)

(11) Two panels



$S_1 = 3.4$ m from(11)
$S_2 = (0.65)$ m
$S_2 = 0.65(5)$
$S_2 = 3.25$ m

Superposition: $S_1 + S_2 = S$
$S = 6.65$ m ← S_{max}(2 panels)

25. Given: Subsidence trough information, depth = 1,500 ft, 750 ft face length, 7,500 ft panel length, mining height 15 ft, Utah conditions

Find:
(a) S_{max} single panel
(b) critical width
(c) S_{max} six panels adjacent
(d) $E+$ single panel
(e) $E-$ six panels adjacent.

Solution:

Utah condition assume $S_{max}/m = 0.67$ and $\delta = 28°$ then reduction factors from U.K. condition will be

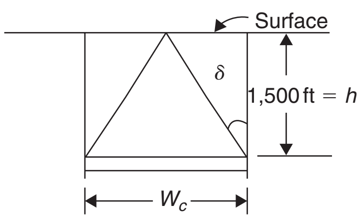
$$\frac{1}{S_f} = \frac{0.90}{0.67}$$
$$\underline{\underline{S_f = 0.74}}$$

(a) depth = 1,500 ft = 457 m
Width = 750 ft = 229 m

$$\frac{S}{m} = 0.45 \text{ U.K}$$
$$\frac{S}{m}(\text{Utah}) = 0.45\ S_f$$
$$\frac{S}{m} = 0.45(0.74)$$
$$\frac{S}{m} = 0.333$$
$$S = 0.333(15)$$
$$\underline{\underline{S = 5.0\text{ ft}}}$$

← S_{max} single (R) panel

(b) Critical width



$$\frac{W_c}{2} = h\tan\delta$$
$$W_c = 2(1{,}500)\tan 28°$$
$$\underline{\underline{W_c = 1{,}595\text{ ft.}}}$$

← (b)

(c) 6 panel adjacent

$$W = 6(750)$$
$$W = 4{,}500\text{ ft}$$
$$L = 7{,}500\text{ ft}$$

Area is Super critical i.e., $W > W_c$ (wide and long)

$$\therefore\ S = S_{max}$$
$$S_{max} = \left(\frac{S_{max}}{m}\right)m$$
$$= 0.67(15)$$
$$\underline{\underline{S_{max} = 10.0\text{ ft}}}$$

← (c)

(d) $E+$ single panel, SEH, Table 5

W/h (width/depth ratio) $= \dfrac{750}{1{,}500} = 0.5$

$$+E = 0.8\,S/h$$
$$= (0.8)(5\text{ ft})/1{,}500\text{ ft}$$
$$\underline{\underline{+E = 2{,}667\text{ min/in}}} \quad (2.67)(10^{-3})$$

(d) ←

(e) $E-$ Six panels

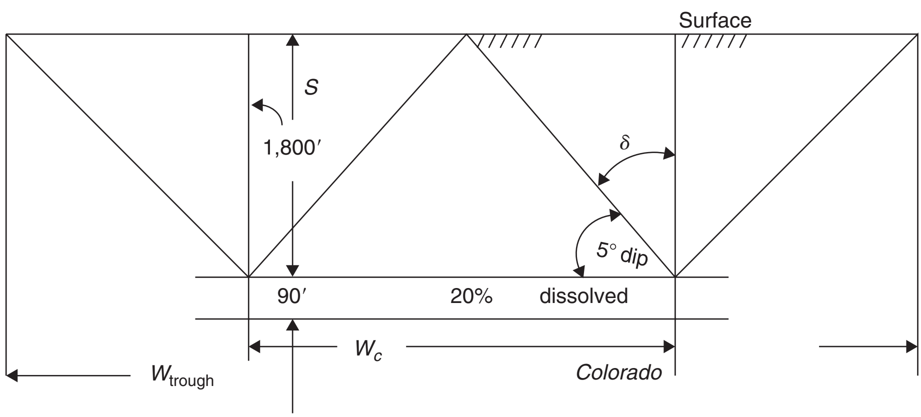
$$W = 6(750) = 4{,}500\text{ ft}$$
$$d = 1{,}500\text{ ft}$$
$$\frac{W}{d} = \frac{4{,}500}{1500}$$
$$\frac{W}{h} = 3.0$$

Tops out at $0.51\,S/h$ (table 8.3 text)

$$E(-) = 0.51\,S/h : \text{supercritical}$$
$$= (0.51)(10.0/1{,}500)$$
$$\underline{\underline{E(-) = -3{,}400\,\mu\text{in/in.} \quad 3{,}400(10^{-3})}}$$

(e) ←

26/27. Given: Solution mining–nahcolite [units conversion]



Find:
(1) Subsidence factor, angle of draw
(2) S_{max}
(3) W_c (critical width)
(4) trough width at W_c.

Solution:

For western U.S. Colorado, use Utah conditions, say

$$\underline{\underline{\frac{S_{max}}{m} = 0.67, \quad \delta = 28^\circ}}$$

(1) ←

$$S_{\max} = 0.67\,\text{m}$$
$$m = 20\% \text{ of } 90\,\text{ft}$$
$$m = 18\,\text{ft}$$
$$S_{\max} = 0.67(18)$$
$$\underline{\underline{S_{\max} = 12.06\,\text{ft}}} \qquad \longleftarrow (2)$$

$$W_c = 2h \tan\delta$$
$$= (2)(1{,}800)\tan 28^\circ$$
$$\underline{\underline{W_c = 1{,}914\,\text{ft}}} \qquad \longleftarrow (3)$$

$$W_{trough} = W_c + 2h\tan\delta$$
$$= 2W_c$$
$$\underline{\underline{W_{trough} = 3{,}828\,\text{ft.}}} \qquad \longleftarrow (4)$$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beautiful book
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group, we are free, independent,
and not caring MAD people.