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Elmakias

New Computational Methods in Power System Reliability

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David Elmakias (Ed.)

New Computational Methods in Power System Reliability

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Editor-in-chief
Prof. Janusz Kacprzyk
Systems Research Institute
Polish Academy of Sciences
ul. Newelska 6
01-447 Warsaw
Poland
E-mail: kacprzyk@ibspan.waw.pl

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Dr. David Elmakias
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Planning, Development & Technology Division
1 Netiv Ha'or St.
New Office Building
Haifa, 31000, Israel

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In memoriam of my lovely son Sharon Menachem Elmakias (Oct. 9.1981-Jan.9.2005)

"Sharon, I wanted to thank you.
Thank you for being with me in joyful times as well as sad.
Thank you for having such a big heart I am able to be a better person.
Thank you for respecting me even if you did not agree with me.
Thank you for worrying about me sleeping proper hours and having home leave.
Thank you for sacrificing your own comforts for the sake of others.
Thank you on behalf of tens of soldiers whom you helped.
Thank you for supporting me in times of trouble.
Thank you for making me smiling even when everything looked grey.
Thank you for being dependable and trusting all the time.
Thank you for forgiving and not angry every time I made mistakes.
Thank you for standing at my side every step of the way.
Thank you for letting me understand the meaning of the word "perfect".
Thank you for your contribution at complex and dangerous times.
Thank you for your infinite help with good word on everything.
Thank you for saying "yes" more than you sad "no".
Thank you for showing peace and self security.
Thank you for never ending ideas, concepts and improvements.
Thank you for proving what humility is.
Thank you for teaching me what the true friendship is".

Shai Siman Tov. From letter to Sharon's family.

Preface

Power system reliability is in the focus of intensive study due to its critical role in providing energy supply to the modern society. The basic methods of power system reliability assessment and improvement were presented in the following comprehensive books: Endrenyi J (1979) Reliability modeling in electric power systems, John Wiley & Sons, New York; Billinton R, Allan R, Salvaderi L (1991) Applied Reliability Assessment in Electric Power Systems. IEEE Press; Billinton R, Li W (1994) Reliability Assessment of Electrical Power Systems Using Monte Carlo Methods, Springer, New York; Billinton R. and Allan R (1996) Reliability evaluation of power systems. Plenum Press, Boston; Brown R (2002) Electric Power Distribution Reliability. Marcel Dekker, New York; Pansini A (2004) Transmission Line Reliability and Security. Marcel Dekker, New York. This book is not aimed at providing the overview of the state of the art in power system reliability. On the contrary, it describes application of some new specific techniques: universal generating function method and its combination with Monte Carlo simulation and with random processes methods, Semi-Markov and Markov reward models and genetic algorithm. The book can be considered as complementary to power system reliability textbooks. It is suitable for different types of readers. It primarily addresses practising reliability engineers and researchers who have an interest in reliability and performability analysis of power systems. It can also be used as a textbook for senior undergraduate or graduate courses in electrical engineering.

The book is divided into five chapters.

Chapter 1 presents a brief general description of power system structure. It describes basic concepts of reliability based planning and introduces the multi-state systems as an object of study.

Chapter 2 describes the basic mathematical tools used in this book for reliability analysis and optimization.

Chapter 3 presents several reliability assessment and optimization problems arising in power generation systems on power plant and power system levels.

Chapter 4 is devoted to reliability assessment of elements of transmission and transformation systems. It considers substation reliability evaluation based on overload capability of the transformers, reliability analysis of substation configurations and high voltage protections.

Chapter 5 considers methods of reliability enhancement in distribution systems and presents optimal sectionalizing and optimal insulation methodologies.

The book was written by the following authors: Dr. D. Elmakias (sections 1, 2.1, 3, 4, 5), Dr. H. Ben Haim (sections 1, 4.3), Eng. Sh. Frant (sections 1.3.2, 3.2), Dr. V. Gurevich (sections 2.3, 3.2), Dr. D. Laredo (section 4.3), Dr. G. Levitin (sections 1.5, 2.2, 2.3.3, 2.4, 3.4, 3.5, 5), Dr. A. Lisnianski (sections 2.1, 3.3, 4.3), Eng. Sh. Masal Tov (section 1.4, 5.1), Eng. D. Vishan (sections 4.1, 4.2). Sections 4.1 and 4.2 are based on joint research of Dr. Elmakias and Prof. Braunshtein. I thank the team of my co-authors for fruitful collaboration. I also thank Ms. R. Kalman and Mrs. T. Tarakanov for technical support.

I would like to express my sincere appreciation to Professor Janusz Kacprzyk, from Systems Research Institute (Polish Academy of Sciences), Editor-in-Chief of the Springer Series of Studies in Computational Intelligence, for providing me with the chance to include this book in the series.

It was a pleasure working with the Springer-Verlag editor Dr. Thomas Ditzinger.

Haifa, Israel David Elmakias

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1 Basic Notions of Power System Reliability

1.1 Introduction

Before describing the basic structure of power systems system in Section 1.2., let us survey some of the aspects which have made the field of power system reliability scientifically so challenging and interesting. A power system is a typical example of systems requiring high reliability due to both high system infrastructure investment and failure cost.

1.1.1 Three basic sub-systems

The power system is functionally and technically divided into four sub-systems of which only three are generally substantially improved by suitable sophisticated reliability tools:
- The generation system (power stations).
- The transmission system (high voltage lines and switching stations).
- The distribution systems (medium voltage lines and equipments)
- The supply networks (low voltage lines and equipments) which are not truly designed and developed as a system and, as a result, are much less influenced by applying the reliability analysis.

According to this functional division, a specific family of reliability indices was historically defined for each sector, and therefore, different kits of reliability methodology tools were developed and applied.

However, prior to any reliability study of a part of the system or of a specific function of the system it should be remembered that for many issues, the power system should be studied as an electrical bounded system (i.e. stability studies, reactive power analysis etc.).

The generation system is aimed at producing the energy that will meet the demand including the self consumed energy of the system. Therefore reliability indices are defined to characterize the "adequacy" of the system (i.e. time during which the system is able to meet all the demand). Power Stations which are the components of this system are therefore mainly measured by their available power capacity at each hour of the day. As far as adequacy is concerned, the generation system may be studied stand-alone as power stations are much less reliable than their connection to the

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system and the system network. Reliability study of this system is complicated due to the fact that this study strongly interacts with other aspects such as financial, economical and environmental.

The transmission system is aimed at the transport of the energy to the distribution networks. Not only energy adequacy indices are defined for the transmission system as a whole (i.e. Average Interruption Time which is an energy related index normalized to time dimension), but also security indices (i.e. probability of a blackout event) as the network has also to sustain transient phenomena. The transmission system is often referred to as "the system" in respect to the fact that its analysis includes modeling of the included generation system. In comparison to the generation system, reliability analysis of the transmission system is much more complex because it includes many different kinds of components (lines, switching equipments, transformers, control and protection equipments), and needs to be electrically simulated to evaluate the reliability state of the network and its components.

Both the ***distribution system and the supply network*** are consumers oriented. The purpose is to keep the consumers connected to the sources of energy through the networks without interruption irrespective of whether the consumer is actually consuming or not. All the indices are connected to the frequency and length of supply interruptions,
The supply network is generally driven by the consumer: physical extensions are triggered by consumer demand, failures are best located through consumer complains and except for special client, the way of connection is standardized. Network extension is therefore "passively" evaluated by a socio-econometrics model and its reliability essentially depends on its components performance.

On the contrary, there are still techno-economical issues in the distribution system for which reliability performance tradeoff may be solved through system analysis. These analyses may be improved and made more valuable by means of an elaborated reliability model and software.

1.1.2 Fields of significant added-value of reliability tools

The reliability theory and risk analysis are used in most areas electrical power companies deal with: conceptual design, equipment acquisition, operation decision, maintenance and spare part policies, assessment of system and sub-system state and performance; for example:

– Setting planning criteria in order to choose the required level of redundancy in the whole system or for power station and substation configuration.
– Evaluation of the dependency between the level of reliability and the level of investment.
– Setting and conducting RAMS demonstration test.
– Fixing hot and stand-by level of redundancy for different operation modes.
– Setting the spare parts policy for strategic equipment.
– Evaluation of the reliability level of the generation, transmission and distribution systems and main sub-systems.
– Evaluation of the ageing of equipment and the economical justification for its replacement.
– Evaluation of the level of damages. As a result of failures, the structural changes accomplished in recent years in utility companies around the world in order to introduce competition in this business require utilities to operate the system close to the maximum equipment capacity. For example, the utilities may overload the transformers and/or lines, which increases the level of risk and affects system reliability. Those changes therefore increase the need for evaluating the reliability and risk that utilities assume in such operating conditions.

Reliability tools are easily combined with economic or engineering analysis to support decision making as they suit the random and uncertain characteristics of most of the factors that affect power system performances.

1.1.3 Concepts and methodologies

In order to evaluate the reliability level of any system or plant, the following types of indices are used in reliability theory (Descamps and Feltin 1996):
– Probability (such as availability or reliability).
– Average length of time. For example, the average time between adjoining failures, average duration of a failure.
– Expectancy (such as the average expected number of days per year when the peak load demand is not met).
– Probabilities of failures.
– Frequencies of failures.
– Severity (the expected yearly total unsupplied energy etc.)

In addition to those indices there are other indices which are specific for power systems, such as probability that system load will exceed available generating capacity, which is referred to as LOLP (Loss of Load Probability) in the literature or LOLH which is the expected number of days during the year on which load will be higher than available generation capacity (is obtained based on LOLP).

In calculating reliability indices in a power system, except the complexity resulting from the systems themselves and their inter-relations, the random nature of load introduces another complication to the analytical models. This is true especially for the transmission and distribution systems, in which loading capacity of lines and transformers depends on temperature, which in its turn depends on the load.

Two approaches exist in the application of reliability indices to the planning of power systems.

In the first one, the system is planned such that the reliability level, measured by one or more indices, is not less than a pre determined value. From the alternatives meeting the required reliability criterion, the less expensive one is selected. This method is very common in the planning of the generation system, in which the reliability is usually measured according to LOLH (for example, LOLH should not exceed 0.1 day/year). The second method is based on the selection of the least expensive alternative, meeting the technical requirements, wherein the desired reliability level is given. Regarding the methods for calculating reliability indices, two methods are used:

– An analytical method based on probability theory in the development of analytical models.
– The Monte Carlo simulation (Descamps and Feltin 1996).

The greatest advantage of the analytical method is that much less computation time is required compared to the Monte Carlo method.

Calculation of reliability indices of a plant or a system requires relating to following four aspects:

1. Knowledge of the structure of the system and its components, functions of the system, the physical laws under which the system operates in order to perform its tasks, without exceeding the capacity of the system's components. The task of a power system is to supply the energy at the required quality as measured by voltages and frequency in the system, within the range of permitted technical parameters of the equipment (frequency, voltage, current and the level of short circuit currents, etc.) under various operating conditions of unavailability of system components as a result of failure or maintenance. The ability of the system to cope with failures such as various short circuits that might occur in it while maintaining its integrity during the transition stages from one

situation to another needs also to be evaluated during the steady-state condition as well as during the transient.

The level of voltages in the system and the currents in the various regions are calculated using load flow equations. The differential equations are used for analysis of the system's ability to cope with transient phenomena (such as system stability under short circuit conditions).

2. Using the models for calculating reliability indices that reflect the system performance and its functioning under various operating conditions that can occur as a result of reduction of redundancy due to the outage of one or more elements because of failure or maintenance. The models should include analysis of transients from one state to another.
3. The kind of failures that may occur in the power system elements or subsystems (e.g. high voltage lines, substations etc.) making them unavailable for the time needed to repair. Some events in power systems can cause unavailability of a single element (for example short circuit in one line), some events lead to common cause failures of several elements (for example lightning that hits a double-circuit line).
4. Impact of failures on the system: does the system, after removal of one or more components from operation still perform its tasks and transits to the new condition without loss of stability.

Further in this chapter we will describe the basic structure of the power system, review the kinds of failures and their impact on various system components and describe the reliability criteria that characterize the system's performance. We will discuss the main planning criteria on which the system reliability analysis is based, i.e. the system's reliability and risk.

1.2 Structure of power system

Under the traditional approach, the task of the power system is to provide the electrical energy to customers at a level of reliability and quality as defined by the customers' demands.

The quality of electrical power is measured according to the technical parameters of frequency and voltage. The level of reliability is measured by the reliability criteria that will be described later.

The level of reliability is determined by the level of redundancy (e.g. reserves in the system).

The structural changes in the electrical power systems in the world, the opening of the markets to competition, massive introduction of "green" energy and the introduction of load management methods imposed additional technical requirements on the power system. All this in order to enable trading in electrical power and to fulfill green energy requirements aimed at reducing the impact of energy usage on our planet.

In addition, those changes caused the introduction of new, modern generation facilities such as wind turbines, solar energy and more. Some of these facilities are connected to the electrical grid, sometimes closer to the consumers.

In this chapter we will describe the basic structure of the power system and the task of each one of the systems comprising it. This will help to understand the reliability criteria and how they are evaluated.

In principle, as it was mentioned above, the power system is composed of three basic sub-systems: the generating system, the transmission system and the distribution system.

Those sub-systems are interconnected in such a way that the energy generated in the generating system is transmitted via the transmission and distribution grids to the consumers, at the quality and reliability levels demanded by the consumers.

The structure of each of the sub-systems and their technical characteristics (transformers capacity, level of voltages, phase scheme, etc.) are determined by techno–economic considerations. The level of redundancy in each of the systems determines the level of reliability that the consumers get and influences the price of electricity. From the electricity standpoint, it is actually one system, built to supply the energy according to the various physical laws. The system must constantly balance the power generated in the power stations with the demand created by the consumers (including losses).

In order to understand the relations between the above mentioned subsystems and the basic structure of the power system we will start with discussing the structure step by step, until obtaining the complete picture.

The generating system is composed of power stations that generate electrical power by conversion of energy. In most conventional power stations fired by fossil fuels (coal, fuel oil, etc.), thermal energy is converted into electrical energy. Such power stations naturally are mechanical systems operating under high temperatures and pressures and therefore require a lot of maintenance and the number of failures in them is relatively high compared to the other parts of the power system (for example the transmission lines). As a result the power generating plants are out of service for a relatively high percentage of the time due to failures and planned maintenance.

Experience shows that conventional power stations, which today constitute the majority of power plants, are out of operation between 10% and 15% of the time each year for maintenance or as a result of failure in their components. Therefore, if the power stations should operate separately to supply power only in their region (as presented in Fig. 1.1, in which separate power supply of three regions is shown) a high reserve level (number of power stations) is needed for each region in order to supply the energy at the required level of reliability. Otherwise the number of hours of unsupplied power is high.

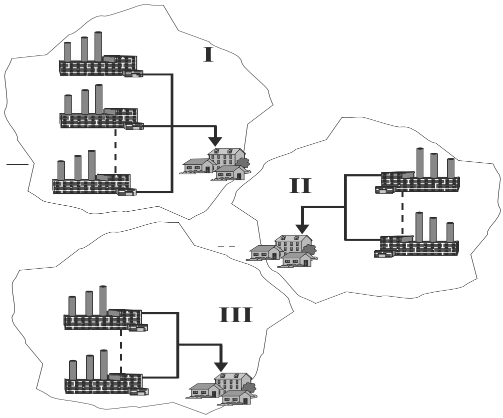

Fig. 1.1. Three regions supplied by separated generating systems

Interconnecting the power stations of all three regions by means of the electricity grid (as presented in Fig. 1.2) enables mutual backup of all the power stations. This enables reducing the total investment in the generation system, and, moreover, it enables optimal operation of the generating system and, as a result, a reduction of the cost of electricity.

This grid which enables electrical connection between the power stations constitutes the transmission system. This system is composed of elements (lines, transformers, buses etc.) which have a much higher degree of reliability compared to the power stations. In this way a high level of reliability of the whole system (generation and transmission) is obtained.

Another task of this system is to transfer power from the generation sources to the consumption centers.

Hence the transmission system has an important task in the electricity market. It enables mutual backup of power stations, optimal operation of the generating system, transfer of power from the power stations to the consumption area, enables trading of electricity in deregulated markets. The electrical connection enables synchronization of all the generating units together and a high inertia system is obtained which makes it possible to keep a stable frequency and constant voltage, i.e. improved quality of electricity.

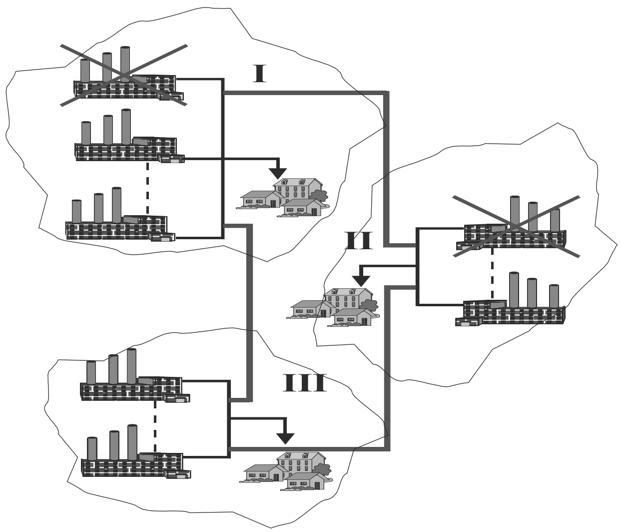

Fig. 1.2. Three regions supplied by generating systems interconnected by a transmission system

In order to enable transmission of large amounts of power, this system is usually operated at high voltages, starting at 110 kV and up to 750 kV and more. The choice of voltage is made according to techno-economic considerations and depends, among others, on the powers transmitted through this system and on the distances over which this power must be transmitted. The summary of the tasks of this system is presented in Fig. 1.3.

The power generated in the power stations is transmitted to the switching stations where wholesale delivery of electric energy is made to the substations. The task of those sub-stations is to act as kind of regional junctions which receive the energy from the main transmission system at high voltage, process it by means of transformers to the distribution voltages (intermediate voltages such as 22 kV, 13 kV, etc.) and distribute the power among feeder lines at intermediate voltage penetrating to immediate proximity to each consumer center, large or small.

Distribution transformers are installed in the distribution lines, supplying the electrical power to the consumers at low voltage (110 V, 400 V) in order to guarantee a level of safety close to electrical plants.

A schematic presentation of the power system can be seen in Fig. 1.4.

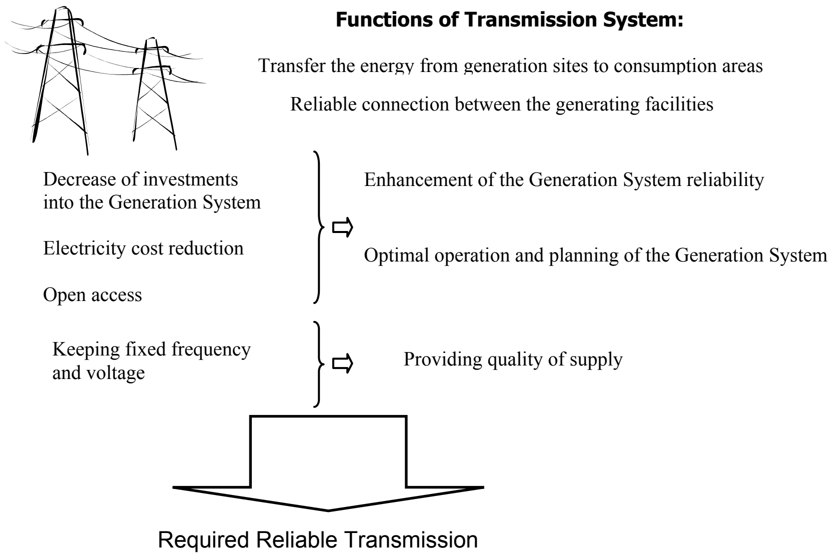

Fig. 1.3. The tasks of the transmission system

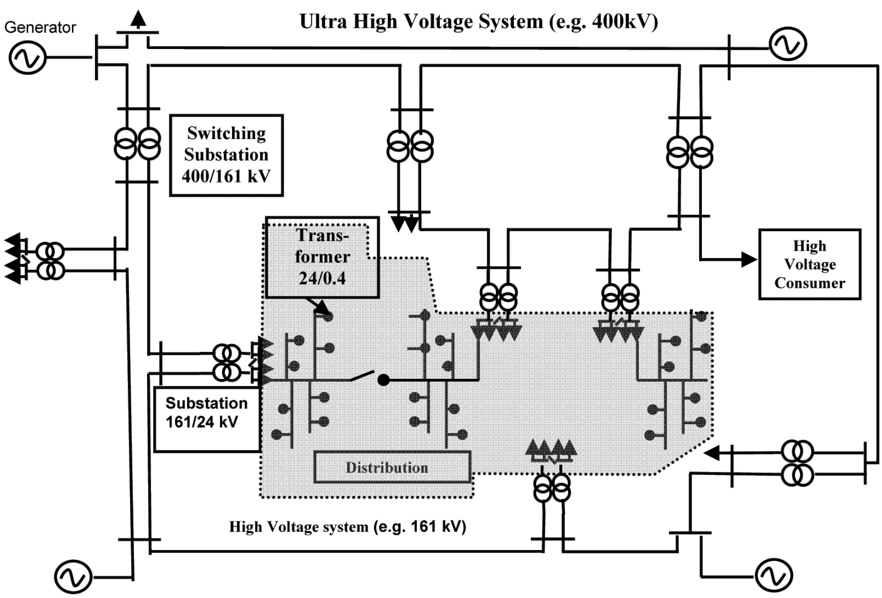

Fig. 1.4. Schematic structure of the power system

The transmission system can be composed of several subsystems with different voltage levels according to the development of the power system. In the above figure, there are two voltages: 161 kV and 400 kV. The electrical connection between the two voltages, in order to obtain one system, is done by means of transformers or auto-transformers.

In addition to the basic components composing the electrical supply system there are many additional elements which provide protection in the event of failures (short circuits, line damages, etc.) and removal from exploitation of elements (lines, transformers) for planned maintenance and its safe execution by operation and maintenance personnel. Those elements include: circuit breakers, disconnectors, voltage and current transformers, capacitor banks, reactors etc. The trading in electricity also requires means for measuring energy installed in various places, as well as communication equipment for the transfer of data and control etc. so that in practice each electrical junction in Fig. 1.4 forms an electrical plant by itself. Practically the coming lines, transformers, generators and the equipment needed for protection, operation and maintenance are connected to buses.

As an example, 3 types of possible bus configurations forming one of the junctions in the system are presented in Figs. 1.5-1.7.

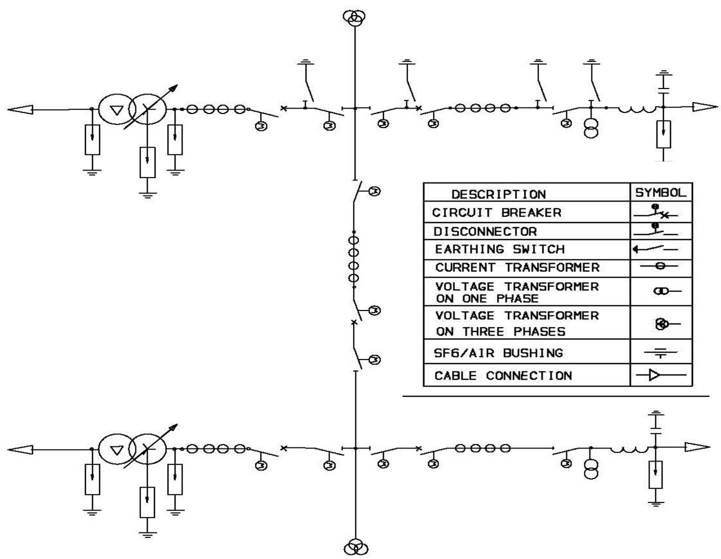

Fig. 1.5. Single bus feeding configuration

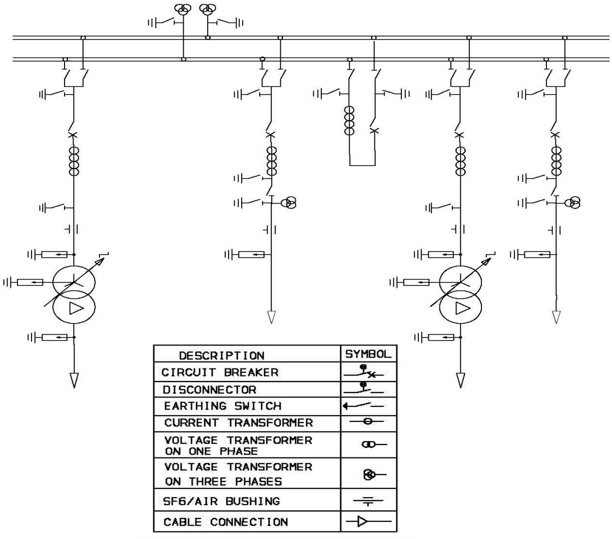

Fig. 1.6. Double bus feeding configuration

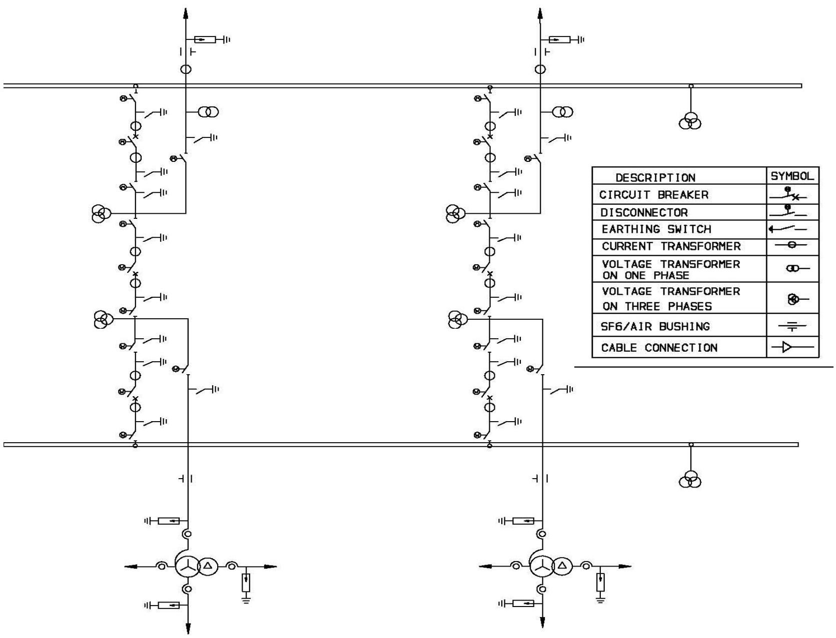

Fig. 1.7. One-and-a-half breaker bus feeding configuration

In practice choosing a bus scheme is affected by reliability considerations, required by the relevant junction. Calculating the reliability indices for those schemes is discussed in Chapter 4.

1.3 Reliability based planning in power systems

As we have seen, the power system is composed of many elements that are exposed to failures and require removal from operation for the purpose of maintenance. Since it is not possible to prevent failures in the electrical system, prohibitively high investments are needed to make the system completely fault tolerant. Therefore the planning criteria are selected so that an acceptable reliability level for the consumers will be obtained, while limiting the level of investment in order to obtain a reasonable electricity price. These criteria practically define the reasonable level of risk for unsupplied electricity.

In a regulated environment, the planning criteria are defined so that the reliability level for the consumers should be relatively high and yet limiting the investments in the supply system. The obtained reliability level of the electricity supply is acceptable for most of the consumers.

Even in deregulation of the market and interconnection environment, one has to compare the actual system to a reference one.

These are the implicit conclusions of most studies done after blackout events of 2003. The small end consumer has no power in the market and his needs are not represented. The market is still not ideal and the deviation from the planning criteria may be used as one of the indicators of its imperfection.

As mentioned above, the interconnection is aimed at improving system performance given the same investment level. However, it does not substitute the national responsibility. It makes it more difficult to assess the performance of the national system, but the need for a reference level still exists.

Therefore, planning criteria are still necessary even if their implementation depends on the structure of the energy economy.

According to the structure of the system and its complexity two major criteria are accepted:

1. Probability criteria (especially accepted in the generation and distribution systems).
2. Deterministic criteria (especially accepted in the transmission systems).

The threshold value of the criteria may be chosen according to past experience or by performing a comprehensive optimization when a price is attributed to unreliability of supply (for example measured as the cost of unsupplied energy). A threshold value can also be set according to a predetermined level of risk as will be shown later.

In the probability criteria method a threshold value is set for the level of reliability the system needs to provide. This threshold value may be determined

- By judgment based on experience;
- By establishing relations with other indices for which criteria are already available;
- By determining the optimal value of the reliability index through cost-benefit analysis.

In the cost-benefit analysis management decision involving reliability considerations are based on the desire to find an optimal balance between the gains achieved from higher reliability and the cost of providing it. A diagram describing the conditions in general is shown in Fig. 1.8.

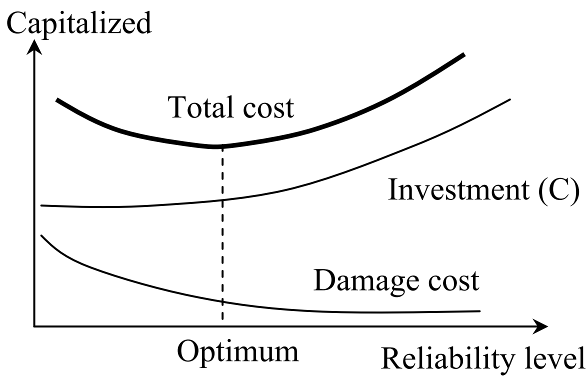

Fig. 1.8. Cost-Benefits analysis

If all relevant factors were known, this method could be used to determine reliability standards. This method can be applied to a generation system, as will be discussed later.

In the deterministic criteria method the various operational conditions with which the system has to cope and supply electricity without damaging the equipment of the system's customers' are pre-determined. This method is very common in the transmission and distribution system. The most frequently used deterministic criterion is N-1 meaning that the system must continue to operate if any single system component is out of operation.

In order to evaluate the system reliability, it should be analyzed in two conditions: steady state and transient. In the steady state one estimates whether the system is suitable for the supply of demand at the required quality level without endangering the equipment (this, for example, in the transmission system, checking currents in all the system branches, the voltages in all the buses, and the expected short circuit currents). This test is done by solving the equation in the steady state without regard to how the system reaches this state or how it exits it. The various system states are defined by the components in operation and by the level of demand in the system.

The second stage of the analysis is related to the transient phenomenon when the system transits from one state to another. Here one asks whether the system will pass the transient phenomenon smoothly or the loss of stability will cause load shedding or even regional or national blackouts. The evaluation of the transit phenomena is performed by means of differential equations describing the physical process in transition from one steady state to another.

Based on the above power system reliability can be described by adequacy and security (the exact definition of these aspects will be given later). Another aspect of reliability is system integrity, the ability to maintain interconnected operations. Integrity is violated if uncontrolled separation occurs in the presence of severe disturbances.

Further we will refer to reliability indices and accepted planning criteria for the planning of the generation system, the transmission system and the distribution system. We will then try to set threshold values for risk levels in each one of the systems.

1.3.1 Effect of failures on power system

The level of redundancy of any subsystem is determined taking into account the possible damage caused by failures and the probability of these failures. In Fig. 1.9 one can see different locations of possible failures. The consequences of the failures depend on which system components are affected.

For example, failures in a low voltage network (location 1) lead to unsupplied energy of a small number of customers. The duration of these failures is relatively short (several hours).

Failures in a substation transformer (locations 2 and 4) or in a high voltage line feeding the substation (location 3) can affect a much greater number of customers. Therefore, good system planning should provide possibility of system reconfiguration without power supply interruption.

Such planning should meet the so called N-1 criterion (that will be discussed later).

Failures in the transmission system (location 5) can cause very dangerous transient processes that may lead to the entire system becoming unstable and even to local or wide area blackouts.

To understand this phenomenon, consider two interconnected subsystems (Fig. 1.10). The subsystems are connected by an extra high voltage (400 KV) line with great transmission capability and by a high voltage (161 KV) line with much lower transmission capability. Assume that the load flow from subsystem A to subsystem B is 1,400 MW (1,000 MW through 400 KV line and 400 KW through 161 KV lines). In the case of common failure in a 400 KV line the interruption of both its circuits leads to transfer of the load to the 161 KV line that cannot transmit such flow. As a result, the protective relays disconnect both lines causing total separation of the subsystems, which leads to surplus generation in subsystem A and deficit of power in subsystem B. The load imbalance causes an increase of frequency in subsystem A and reduction of frequency in subsystem B that can result in load shedding or even total blackout. This cascade effect causes a security problem in the power systems.

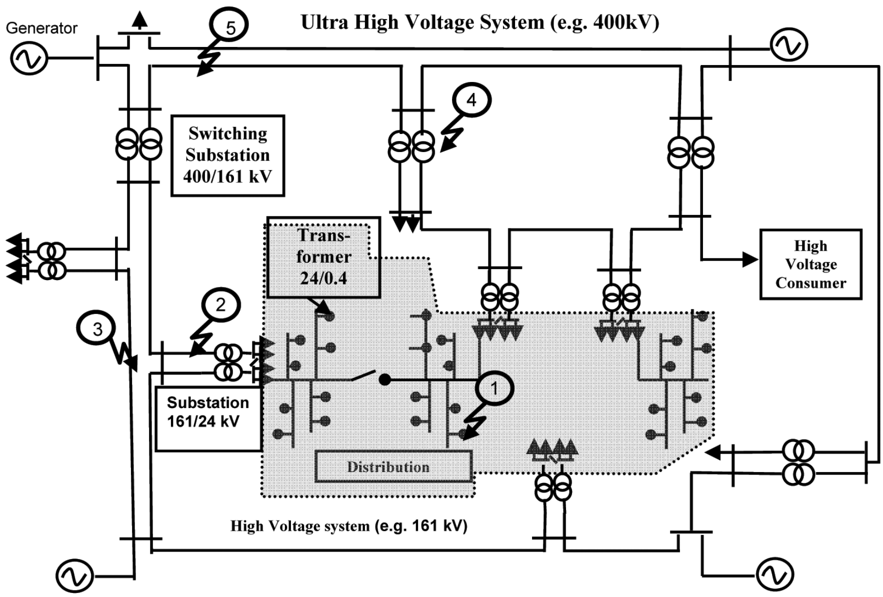

Fig. 1.9. Effect of failures in different parts of a power system

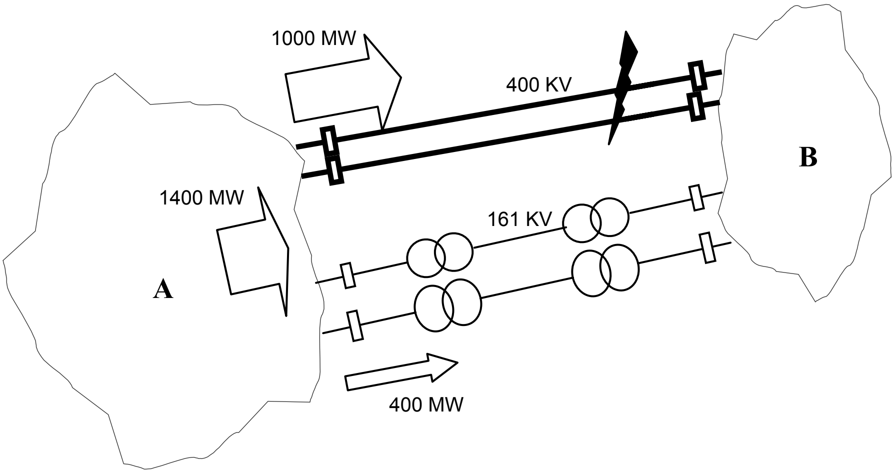

Fig. 1.10. Example of two interconnected subsystems

1.3.2 Planning criteria

Generation expansion planning

The main issues in generation expansion planning are when and how much generation capacity needs to be installed, as well as what kind of technology it should use (i.e. hydro, coal, gas turbines, combined cycle plants etc.).

In order to achieve its goal, generation planning integrates the following three analyses: reliability analysis, electricity production simulation and investment cost analysis.

A commonly accepted way to do this is by performing the generation expansion optimization process using a long-range planning model. Usually, when using long-range planning models, an optimization is performed to achieve the least expensive plan that is to minimize the cumulated and discounted expenses to produce electricity throughout a sufficiently long planning period. While the total cost of the plan is used as optimization criteria, other very important issues as reliability and environmental requirements are involved in this process as constraints. Fig. 1.11 describes this commonly accepted optimization process.

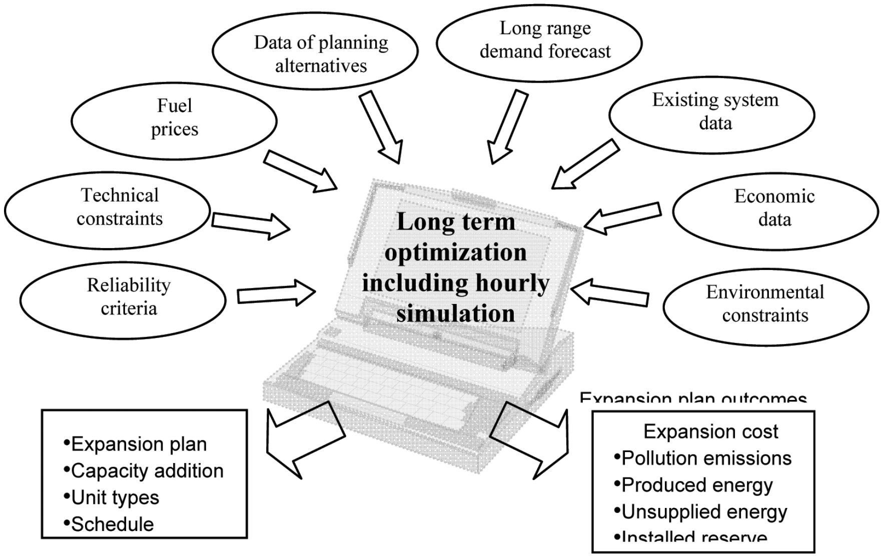

Fig. 1.11. Generation expansion planning optimization

As the system planning process is performed for a relatively long time period, there is a high degree of uncertainty regarding various parameters, such as demand growth rate, fuel prices forecast, future generating technologies etc. The above mentioned emphasizes the need to use a decision tree or risk analysis theory in evaluating generation business decisions in an uncertain environment. In general, the methodology for generation expansion planning under uncertainty conditions includes the following steps:

- Definition of future scenarios space considering uncertainty in electricity demand forecast, fuel prices, technical and economic parameters of generating units;
- Determination of optimal solution for each scenario;
- Detailed simulation of the system operation for each scenario;
- Determination of the robust (scenario independent) expansion plan.

Schematically this planning process under uncertainty conditions looks as in Fig. 1.12.

Existing long-range planning models are usually based on the convolution approach to model random outages. However, this approach will not lead to a correct value of a variable, which depends on chronological operation of the generation system. For example, this approach enables to impose a limited fuel constraint only on the expected fuel consumption, while a true solution requires imposing the fuel constraint on each random sample of fuel consumption reflecting various forced outage scenarios, and only then taking the mean value.

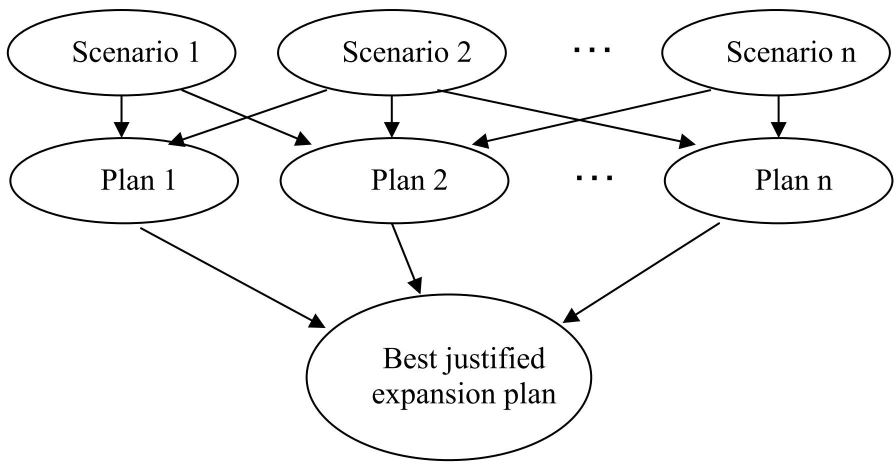

Fig. 1.12. Generation expansion planning under uncertainty conditions

Therefore, along with commonly known long-range planning models based on load duration curves and dynamic programming optimization method, the following models can be used for generation system planning purposes:

1. A chronological model for detailed simulation of power system operation accounting for various peculiarities and constraints related to specific requirements of utility's operation policy. This model, based on Monte Carlo simulation, performs optimization of unit commitment and economic dispatch.
2. A model for optimization of generating units maintenance scheduling. This model provides a detailed simulation of specific requirements and constraints related to performance and scheduling of maintenance works.
3. A model for detailed evaluation of reliability indices based on probabilistic simulation of full and partial failures of generating units using probabilities of transitions between different availability states. This model should also simulate postponed maintenance states.

Reliability criteria for generation expansion planning

Establishment of proper reliability criteria for optimization of the system development is one of the most complicated problems in generation planning.

Here we outline a practical approach for establishing the proper reliability criteria for long term generation expansion planning.

Reliability of electricity supply strongly affects all three major sectors of national economy: industrial, commercial and residential. For this reason, from a socio-economic point of view, it is justified to define the optimal reliability level for generation system planning as a point when marginal benefits to society from improvement of reliability is equal to marginal cost for utilities to enable the improvement.

Generation system planning requires the simulation of system operation, which strongly depends on random demand and availability of generating units. Therefore most of the computer programs intended for long-term generation planning enable calculation of the probabilistic reliability indices. The commonly accepted reliability criteria for generation planning are based on probabilistic indices, such as loss of load probability (LOLP), hourly loss of load expectation (HLOLE), expected unsupplied energy (EUE), etc.

The approach to establishing a proper reliability criterion for generation expansion planning can be as follows:

Stage 1. Model I: long-term generation system optimization, based on gas turbines only, without any requirements regarding reliability and considering a given cost of unsupplied energy (using the EGEAS software).

Stage 2. Applying simulation Model II for calibration (improving precision) of reliability indices obtained by the long range planning of Model I, according to the reliability values that resulted from a detailed simulation of the system reliability performance. The calibration is made for a certain number of years during the study period, based on the generation plan obtained in Stage 1. (Detailed description of Model II is given further).

Stage 3. Same as Stage 1, but with the unsupplied energy cost multiplied by the calibration factor calculated in Stage 2. The upper bound of expected loss of load hours is established based on the resulting optimal expansion plan.

Stage 4. Checking if further improvement of reliability (over the optimum) can be achieved by insignificant expenses and by formulating the final recommendation upon the most suitable reliability criteria for long-term generation expansion planning.

The above mentioned stages are explained further in more details.

Stage 1. Optimization of the generation system with gas turbines. Since one has to define the reliability criterion, namely Hourly Loss of Load Expectation (HLOLE), the influence of advancing the incorporation of efficient (low production cost) generating units on the optimization process should be neutralized. For this purpose, in Stage 1 the optimization is performed with industrial gas turbines as the only planning alternative for generation system expansion.

In the case of isolated systems, gas turbines are the lowest installation cost alternative but on the other hand - the highest production cost alternative. Therefore, the turbines are incorporated in the expansion plan only when it is justified due to increasing unsupplied energy cost. In interconnected systems long-term electricity purchase agreements may sometimes become cheaper than installation of new gas turbines. In this case such an alternative may be considered in the evaluation of the reliability criteria.

Stage 2. Calibration of Model I results according to unsupplied energy values obtained by Model II. The calculation of loss of load hours and unsupplied energy in the models used for long range planning of the generation system is not accurate, since these models do not take into consideration the following most important aspects:

- Probabilities of transitions between various availability states of generating units;
- Postponed maintenance;
- Unit commitment.

Postponed maintenance is maintenance due to failures of the generating units which can be repaired whenever convenient (during low demand periods, such as nights or weekends), without reducing the units' availability during high demand periods.

Unit commitment policy is the policy regarding start-up and shut-down time of the units, taking into consideration the spinning reserve requirements and other limitations defined by the operation policy.

In order to make a calibration of Model I, reliability indices were calculated by Model II for 6 different years of the study period based on the expansion plan obtained in Stage 1.

For example, assuming an energy cost of 3 \$/kWh, the annual unsupplied energy values (in GWh) which were obtained for a sample system (see Example 1.1) by Model I and by Model II, are shown in Table 1.1. Based on these values, a calibration factor (ratio between the values of unsupplied energy obtained by Model II and Model I) is K_{EUE}=3.8 (least squares approximation).

Table 1.1. Expected unsupplied energy

Year	Model I [GWh]	Model II [GWh]
2007	4.33	14.59
2008	4.90	17.63
2010	4.21	16.40
2012	5.23	18.43
2014	5.75	23.56
2015	7.37	32.92

Stage 3 repeats stage 1 with unsupplied energy cost corrected according to stage 2. The unsupplied energy cost UEC used for running the Model I is multiplied by the calibration factor between Model I and Model II which was calculated in stage 2, which gives the adjusted unsupplied energy cost AUEC.

$$AUEC = K_{EUE}\, UEC \ (\$/kWh).$$

The adjusted unsupplied energy cost accounting for the calibration factor becomes: AUEC=3.8·3=11.4 $/kWh. This cost of the unsupplied energy was further used as input for Model I and then another optimization using a single alternative (industrial gas turbines as shown in stage 1) was performed. The corrected annual loss of load hours upper bound obtained as a result of the optimization became 3 hours per year.

From a socio-economic point of view this is the optimal value of the upper bound of annual loss of load hours for the generation expansion planning, when assuming an average unsupplied energy cost to society of 3$/kWh.

Stage 4. Correction of the reliability criteria for long range planning of the generation system. Usually, in practice the decision makers and planners prefer to be on the safe side (a small overexpansion is always preferred over a deficit) and check the consequences of further improvement in reliability (over the optimum) if it can be achieved by insignificant expenses. For this reason the planners go further and perform additional long term optimizations with a whole set of planning alternatives and run by run reducing the upper bound of annual loss of load hours. The optimal criterion, which was evaluated in stage 3, serves as the initial point for these calculations. The use of a whole set of alternatives is required in this case in order to assess the real implications of increased reliability requirements (over the optimum) on long-term electricity generation cost.

It should be mentioned that the calibration factor calculated in stage 2 is not suitable for a system which was developed based on the whole set of planning alternatives (including coal, nuclear or combined cycle plants with different availability data). Therefore, in order to increase the degree of precision, it is recommended to evaluate the calibration factor also for the system including the whole set of alternatives, exactly as it was done in stage 2.

Example 1.1

Application of the above mentioned method is demonstrated on a sample power system composed of 4,800 MW of coal units, 1,600 MW of gas or oil #6 fired steam units, 2800 MW of gas fired combined cycle plants and 2,000 MW of gas or oil #2 fired gas turbines. It was assumed that during the three coming years additional three gas fired combined cycle plants projects of 360 MW each (which are already included in the expansion plan) will be fulfilled.

The load curves (8,760 hourly values) were based on a forecast beginning with a peak load of 10,200 MW and energy consumption of 44,300 MWh in year 2007 and assuming an average annual growth rate of 2.6% per year until year 2030. The set of considered planning alternatives is presented in Table 1.2. Interest rate of 7% per year was assumed in all calculations. The obtained results are shown in Fig. 1.13.

The optimal upper bounds of annual loss of load hours (obtained in stage 3) lie on the x-axis and are 3, 1.6 and 1 hours for unsupplied energy costs of 3, 6 and 9 $/kWh respectively.

Table 1.2. Considered planning alternatives for generation system

Planning Alternative	Capacity [MW]	Inst. Cost [$/kW]	Heat Rate [Kcal/kWh]	Planned Maintenance [days/year]	EFOR [%]	Fix. O&M Cost [$ /kWy]	Var. O&M Cost [$ /MWh]
Coal	2x550	1500	2117	50	8	33.0	0.7
GTCC	350	650	1815	18	7	13.0	1.5
GT	2x120	400	2605	12	11	6.8	0.5

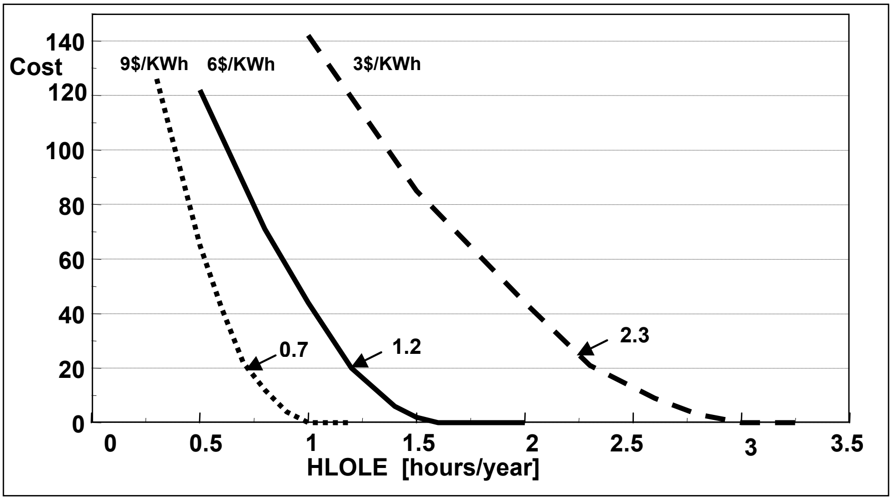

Fig. 1.13. Additional cost of generation expansion plans due to increased reliability requirements

Between this optimum and the “knee” point, tightening up the reliability requirements will be associated with relatively small additional expenses. An additional decrease of the loss of load hours’ upper bound (over the “knee”) will require a significant increase in investments. Therefore, usually the planners and decision makers prefer to make use of the “knee” value as the upper bound of annual loss of load hours for long-term generation planning.

Since it is rather difficult to assess the exact cost of unsupplied energy, the system planners consider an approximate assessment, provided it is based on statistical processing of large scale surveys over a long period of time and covering various types of consumers. Using the value of 6 $/kWh as the average unsupplied energy cost one obtains 1.2 loss-of-load hours per year at most as the reliability criterion adopted for the generation expansion planning (the “knee” point on the 2nd curve).

It is known that the cost of unsupplied energy rises in accordance to the consumers’ growing dependence on electrical energy. Naturally, the unsupplied energy cost in the future should be higher than in the present, and consequently, it is recommended to make the reliability criteria more stringent for planning of the power system in the more distant future (for example 0.7 instead of 1.2 loss of load hours, that corresponds to 9 instead of 6 $ per unsupplied kWh).

The approach, which was described here for the evaluation of proper reliability criteria for generation planning is not the only way to do that. For example, some utilities or reliability controlling institutions analyze the previous performance of their system during the 5 or 10 last years, perform the simulation of the system using their long-term planning models and, based on the reliability indices values obtained for the past, they decide upon the reliability criteria which will be used for the future.

It should be mentioned that the above mentioned calculations of socially-optimal reliability criteria depend on the cost of capital in a given power market serving a given economy. The discount rate, used by the system planners in their programs, may differ considerably from country to country. Usually, the cost of capital is lower for a regulated monopoly, facing a lower degree of risk, than in a deregulated market, opened to free competition. Therefore, for the same cost of unsupplied energy to society, the higher cost of capital (i.e. more expensive capacity installations) in a competitive market may lead to a less stringent reliability criterion, and as a consequence – to a lower reserve margin. Nevertheless, the decision upon a proper reliability level is very important for the long-term sustainability of any power market.

With regard to reliability it would be very risky and incorrect to rely only on the market forces. Even in a competitive market the relevant bodies (transmission system operator, government, regulator) supervising the functioning of the power system should determine the reliability criteria and constantly supervise the adequacy of the system development in order to meet the required reliability criteria.

After making a decision upon the desirable reliability level in a competitive market, the regulation rules may be adjusted to enable the implementation of the required capacity development, while preserving the equilibrium conditions in the power market. The following regulation means can be used without introducing severe deviations in conditions of competition:

- Determination of the proper operating reserve
- Determination of the price cap
- Requirements regarding the extent of the long term bilateral contracts
- Promotion of demand-side management to ensure a certain degree of elasticity of demand.

If all this does not lead to the required extent of investments, a more aggressive intervention in the market, such as establishment of capacity market or even publication of tenders by the transmission system operator or another governmental institution, may be used to ensure the achievement of the desired reliability level.

Reliability criteria for transmission system planning

The probabilistic models for power systems are largely implemented in the generation system. The corresponding computation algorithms are relatively easy to implement. Therefore the reserves (the level of redundancies) in the generation system are determined on the basis of indices derived by procedures which most utility companies regard as standard. With a transmission system, on the other hand, the calculation is much more complicated.

Firstly, the typical transmission system today is composed of hundreds of components. Secondly, the fundamental laws of electric circuits (Kirchoff's laws) during transient phenomena and during steady state must be satisfied. In order to take into consideration all the possible events when calculating the reliability of the system, an enormous amount of calculations is needed. This is the reason why most utility companies throughout the world use deterministic criteria for transmission system planning. However, during the last decades one can also observe a development of probabilistic models for transmission system evaluations.

The reliability calculation models of a transmission system may be classified into two groups. The first one includes probabilistic models for the basic components of the transmission system, such as overhead lines, switching stations. The second group includes models for calculating reliability indices for composite systems (generation and transmission), in order to assess a composite system's adequacy.

System operation under various states is simulated by means of load flow calculations in direct current (DC) approximation.

In order to test and establish the planning criteria for the transmission system we will further relate to the following important aspects of the transmission system:

- Classification of the failures on the transmission system and their impact.
- Classification of failures on transmission lines.
- Classification of the reliability criteria for transmission system planning.

Classification of the failures and their impact. With regard to the transmission system, the influence of the failures can be described by two basic terms: adequacy and security

- Adequacy is the capability of the system to fulfill the customers' requirements from the technical point of view, in the range of the technical values (frequency, load and current) allowed by the system's equipment. This must occur in all operational situations (including forced and planned outages), in a permanent state.

– Security is the capability of the system to withstand transition phenomena which occur during failures. For example, the capability of the system to keep its integrity in order to preserve the synchronization during transition phenomena and thus to prevent uncontrolled division in several sub-systems during the failure.

The above terms are of great importance in getting a correct and clear picture of the reliability of supply. As shown in Fig. 1.14 two systems with similar adequacy indexes and entirely different security indices can exist. The upper plot describes the case when the system withstands the transient phenomenon and transits to another state in which part of the load is not supplied until the end of the restoration process. The lower plot corresponds to the case when the system cannot maintain its integrity after the failure and becomes unstable which causes much greater unsupplied energy than in the first case.

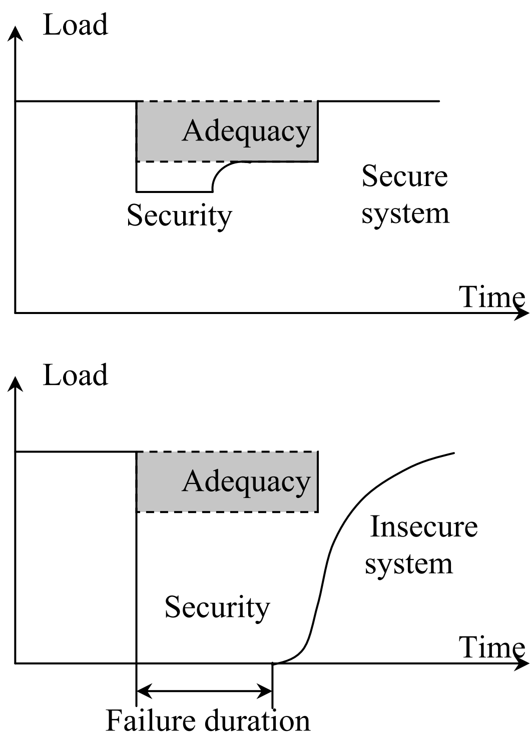

Fig. 1.14. Adequacy and security of power system

The difference between security indexes is a result of two essential factors:

- Different load shedding policy;
- Different behavior of the system during transition phenomena.

The failures can be classified according to their impact on the adequacy and security, as follows:

1. Impact on adequacy.

- Impact on the quality of electricity (for instance, low quality of electricity supply due to forced load shedding).
- Impact on the electricity supply (inability to supply some of the customers due to lack of generating, transmission or transformation facilities).

2. Impact on security.

- Impact on the system itself causing a total lack of control on synchronization which may lead to a general blackout or to irreversible damage to equipment.
- Failures that cause a sudden interruption of supply of electrical energy (may affect from a few customers to millions of customers).

Disconnections which occur as a result of failures in the transmission system are different in their extent, duration and frequency of appearance from those occurring in the distribution system. Most of the events leading to disconnections occur in the distribution system, however in these cases the number of customers affected is relatively small; on the other hand, events that occur in the transmission system are less frequent, but they affect a larger number of customers, therefore they have a greater socio-economical impact. Moreover, the public reaction to these events is stronger.

Disconnections leading to general blackout occur as a result of a sequence of events and not an isolated one. These events relate to:

- A system operating at a low redundancy (weak system);
- Maintenance;
- Events leading to a simultaneous outage of several elements;
- Chain reaction resulting from a failure in the protection system;
- Human errors, etc.

The public is quite tolerant in cases of local failures, as long as they are not very frequent and do not last for a long period of time; therefore in such cases the public image of the electric company suffers no damage. However, the public is much less tolerant to partial or total blackouts, especially if they are not caused by natural calamities (storms for instance) which can be easily explained and accepted.

Most failures of the transmission system occur on the lines. Previous experience shows that such events on the lines can cause a chain reaction which might lead to a partial or in some cases a total blackout, as illustrated in Fig. 1.15.

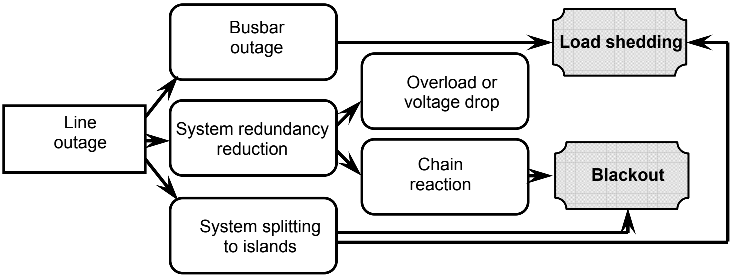

Fig. 1.15. Schematic description of the failure process on a transmission line, resulting in blackout

As mentioned in the previous chapter, a failure occurring on a transmission line may cause a partial or a total blackout, namely a system failure and loss of control over the system's synchronization. In a system based on double circuit lines the failures can he classified as being of two types:

1. Failures causing outage of one of the two circuits on the common pole.
2. Failures causing simultaneous outage of both circuits on the pole (common-cause outage).

Moreover, there are failures whose outcome is outage of a number of circuits passing through the same corridor.

Table 1.3 shows an example of classification of failures according to their causes.

The influence of such failures depends, among other causes, on the stage of development of the system. If the transmission system is in its advanced stages of development, then the system becomes a meshed network having greater tolerance to such events. However, systems in their first stages of development are more vulnerable and even an isolated event may cause a general blackout. The severity of these events (from the point of view of system planning) depends on the impact of the event and on the probability of its occurrence. The reliability criterion should be chosen so that the transmission system can withstand events with the following characteristics:

– Possibility of a severe development of the event which might lead to a total blackout.
– Probability of the occurrence of the event.

Table 1.3. Main failures of very high voltage circuits

Causes of outage	Single circuit outage	Double circuit outage	Common outage of different lines
Disconnection of phase conductor	*	*	
Disconnection of ground wire			*
Struck tower or faulted line		*	*
Pollution	*		
Faulted insulator string	*	*	
Lightning strokes		*	*
Fire		*	*
Human error	*	*	
Fault in the switching substation	*	*	
Protective system operation	*	*	*
Trees	*	*	
Burnt connecting bridge	*	*	
Birds	*		

Based on the experience the components of the transmission system have a high level of reliability (10 to 100 times lower unavailability than power plants). Therefore the probability of outage of more than two independent components is very low. Therefore in the deterministic criterion one should check the system under forced outage of one or two components.

Deterministic criteria. The general procedure for application of deterministic criteria may be described as follows (CIGRE 1999):

– Select one or several base cases to test the system capability. The base cases may differ by load conditions, by generation dispatch and by network configuration corresponding to various planned and forced outage conditions.
– Subject each base case to a series of generation and transmission incidents (e.g. forced outage of transmission line caused short circuits).
– Check that the capability of the system is related to compliance with security and adequacy standards in static and in transient conditions.
– In particular voltage in static conditions must not deviate more than 5% of the nominal value; line and transformer loadings must be below the maximum value corresponding to thermal capacities; breaking capacity of the circuit breakers must be over the maximum failure currents; sys-

tem must be stable after the phase short circuits disconnected by protection first stage.

The most widespread deterministic criteria can be grouped in two classes known as N-1 and N-2 criteria, according to the number of network components out of service (one or two, respectively) (CIGRE 1987, CIGRE 2003). According to N-1 criteria the transmission system should not deviate from the allowed technical values (frequency, voltage, loading of lines and transformers, etc.) in case of outage of a single system component (line, cable, and transformer).

According to N-2 criterion the system should tolerate the simulated outage of two system components (for example one network component and one generating unit).

The N-2 criterion may be used when a security problem arises (for example the loss of both lines in a double circuit line).

Deterministic and probabilistic criteria. The typical transmission systems are composed of hundreds of components. In order to take into consideration all the possible events when calculating the reliability of the system enormous calculation resources are needed. This is the reason why most electric companies throughout the world use deterministic criteria for transmission system planning. The deterministic criteria presume considering a relatively small number of operational scenarios, defined at the beginning of the planning process by the planner. These scenarios are considered problematic for the system, but since they might occur, the planner should provide system tolerance to them. The rest of events in the system are defined as less problematic or not probable. The advantage of this method is a relatively small number of chosen events. It is therefore possible to test the system's behavior accurately, under the given scenarios.

The probabilistic approach is based on considering a large number of operational situations which enable the calculation of unsupplied energy and economic indices. Finally, the approach results in various possible alternatives. In fact, the probabilistic criteria are an expansion of the deterministic criteria: instead of choosing a certain number of operational situations based on the planner's reasoning, one chooses by means of the probabilistic approach (e.g. Monte Carlo simulation) a large number of situations. Finally, average indexes weighing the various events according to the probability of their occurrence are obtained. The advantage of the probabilistic criteria vs. the deterministic criteria is in its ability to obtain an overall description of the system, free of the subjective planner's reasoning (the planner can omit significant events which are liable to occur in a complex system). However, since this approach deals with a great number of events, their description is not as deep as in the deterministic approach. Fig. 1.16 presents the deterministic and probabilistic approaches to system planning using the N-1 criterion and illustrates the differences between these approaches.

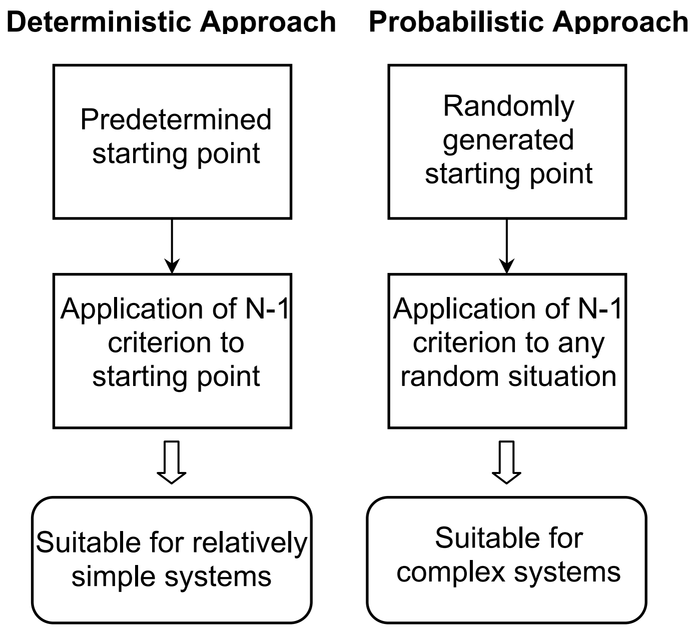

Fig. 1.16. Deterministic and probabilistic approaches to system planning using the N-1 criterion

The application of the probabilistic approach is usually based on a Monte Carlo simulation (Noferi and Paris 1972, Dodu and Merlin 1979, Chao and Li 2004, Chao et al. 2004, Leite da Silva et al. 1999, Leite da Silva et al. 2002). The simulation uses a large number of starting points (when all N units of equipment are available) and performing on each situation the N-1 criterion. Fig. 1.17 presents a flowchart of the program. The Monte Carlo simulation procedure is used to generate system states, taking into account generation and transmission equipment availability. The system operation is simulated by means of load flow calculations in DC approximation (Dodu and Merlin 1975). For each studied state, the model adjusts the generation output to the power load demand, taking into account the network capacities. The optimal adjustment is obtained as a solution of the program $\min \sum (C_i \cdot g_i + C_{UE} \cdot r_i)$ subject to the following constraints: $\boldsymbol{B} \cdot \boldsymbol{v} = \boldsymbol{L} - \boldsymbol{g} - \boldsymbol{r}$ (DC Load Flow); $g_{\min} < g < g_{\max}$ (generation constraints); $0 < \boldsymbol{r} < \boldsymbol{L}$ (load shedding constraints); $-f_{\max} < f < f_{\max}$ (transmission constraints). Here C_i is energy cost for generator *I*, g_i is output of generator *I*, C_{UE} is unsupplied energy cost, r_i is curtailed load in bus *I*, *B* is susceptance matrix of transmission, $\boldsymbol{v}$ is vector of node voltage angle, $\boldsymbol{L}$ is the demand vector, $\boldsymbol{g}$ is the generation vector, $\boldsymbol{r}$ is the load curtailment vector, $g_{\min}$ is the minimal generation bounds, $g_{\max}$ is the maximal generation bounds, $f_{\max}$ is the power flow limits. For the N-1 simulation, the same objective function is used, but the transmission constraints are included. For detailed description of the calculation method the reader is referred to (Cunha et al. 1985).

A simulation of a situation consists of generating a random sampling of availability states of generation units, transmission lines and HV transformers. Using available units in order of merit, the model implements a generation plan to meet the power demand, without taking the transmission system into account in this first step. If the total available generation output fails to meet the demand, load shedding is used to balance generation and demand. Once generation and demand is known on each node, the model operates a load flow computation with the available transmission equipment. Depending on the computation results, two situations appear:

- If the transmission system is not overloaded, the simulation ends without any generation and shortage costs due to the network;
- If some transmission elements are overloaded, the model modifies the generation outputs, using the available more expensive units. If necessary, the model performs load shedding to avoid violation of any network constraint. In this case, generation and shortage costs arise. These costs should be minimized.

The simulation is ended with N-1 analysis from which an additional generation cost can result, due to a contingent outage of every network component. For each simulated situation during the N-l analysis, every single network component is assumed to be unavailable. The generation plan is modified in a preventive way (including possible load shedding) to avoid any uncontrollable constraint violation on the transmission system after a line outage: this is represented by limiting the line overload to a certain level (typically the 20% or 30% limitations) after N-1 contingencies. The ultimate generation plan resulting from this analysis satisfies both the N and N-1 constraints.

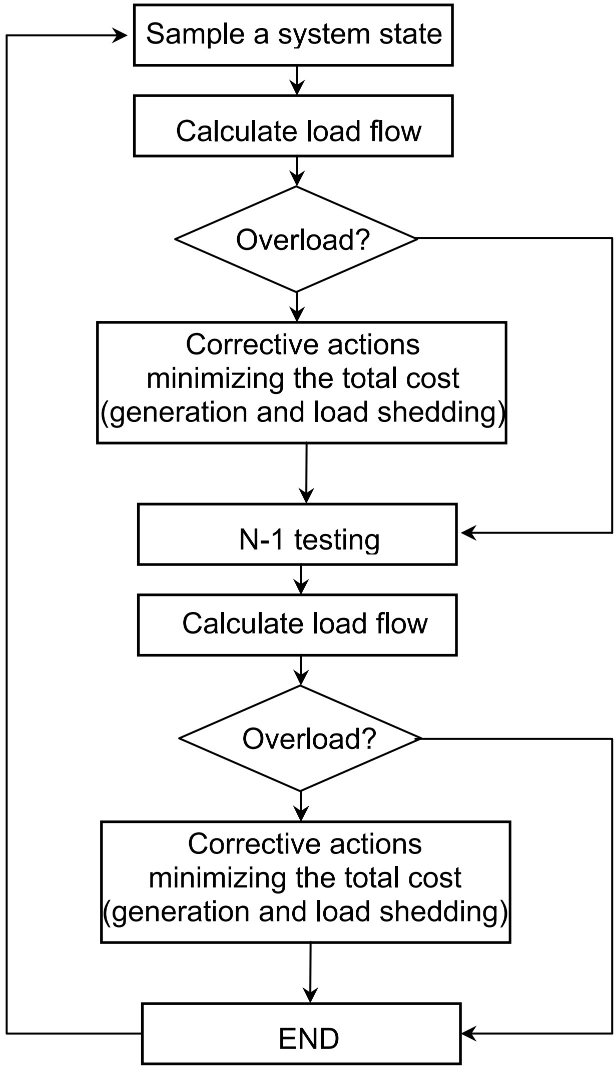

Fig. 1.17. Flowchart of Monte Carlo simulation approach based on N-1 criterion

Considering all the results of the various availability states, the following average values are calculated for the studied load demand situation:

– Loss of load due to generation;
– Probability of not meeting demand;
– Loss of load due to the network failure;
– Losses;

– Frequency of constraint violation in every network component (or risk of overload);
– Marginal range of capacity increase for the network components which have been overloaded during the simulation.

Finally, several points of the load duration curve may be considered in order to perform an annual evaluation.

Example 1.2

Consider the application of the presented simulation methodology to the three area IEEE reliability test system (Grigg et al. 1999) at peak load. The load of area B is increased by 30%, whereas that the load of area A is decreased by 30%. In addition, the fuel costs of the generating units in area B are increased by 30%. The cost of unsupplied energy is 6000$. 50000 samples were run for the N analysis and 5000 samples were run for the N-1 analysis. The system load is 8550 MW, the nominal system generating capacity is 10215 MW, the mean generating capacity is 9539.38 MW.

The following adequacy related test results for one hour peak period were obtained:
– LOLP in the existing system: 0.018;
– Unsupplied energy in generating system: 3.527 MWh/h;
– Unsupplied energy in transmission system (sample system): 0.982 MWh/h;
– Operating cost without network constraints: 155,276.58 $/h;
– Operating cost for sample system: C_N=157,157.30 $/h;
– Operating cost for N-1 testing : C_{N-1}=159,566.27 $/h.

The difference between C_N and C_{N-1} is equal to the additional operating cost needed to meet the N-1 criterion.

The results of sensitivity analysis for "N" and "N-1" tests are presented in Table 1.4 and Table 1.5 respectively.

Table 1.4. Simulation results for sample (N) test

Busbar names		Number of circuit	X $/MW
Alder	Baffin	1	-147.19
Carew	Carrel	1	-93.70
Basov	Bates	1	-44.65
Balch	Balzac	1	-21.89

In these tables X is the index that represents the increase of system cost per 1 MW of additional transmission line capacity ($/MW). For example, by increasing the capacity of line between Alder and Baffin by 1 MW (Table 1.5) one achieves the reduction of fuel and unsupplied energy cost by 301.42 $. The tables give the preference order for lines reinforcement.

Table 1.5. Simulation results for N-1 test

Busbar names		Number of circuit	X $/MW
Alder	Baffin	1	-301.42
Basov	Bates	1	-201.22
Austen	Bates	1	-199.50
Bajer	Banks	1	-155.29
Balch	Banks	1	-34.96
Clark	Bloch	1	-20.86
Balch	Balzac	1	-20.81

1.4 Risk analysis in power system planning

There exist quite a lot of analytical tools to assess the security of power systems. However, few studies were done to define a coherent criterion to support decision making and enable a more clear understanding of why supplementary investments are required. This section considers a methodology which may help system planners or managers to define a coherent level of security at which his specific power system has to be planned so that it suits other planning criteria or market derived adequacy level.

Risk index definition

Listed below are the necessary steps for performing risk analysis of a power system according to classical risk analysis methodology:

1. Risk identification – definition of the System States that are perceived as System Emergency
2. Risk extents (range): analysis of local, regional or nationwide emergency (blackout).
3. Risk scenarios: identification of the events which may lead to system emergency.
4. Risk probability evaluation: quantification of a specific risk (state and extent).

Prior or in parallel to these steps one has to define some risk index in order to be able to focus the risk analysis.

As described before, the defined index should enable calibration of security criteria to other planning criteria or reliability level as derived from the market.

The first problem is to assess a so called “tolerated risk level” R_0 which results, for a given system, from the planning criteria. Assuming that the System is acting in accordance with planning criteria, R_0 is indeed a measure of the inherent risk level that is implicitly tolerated by the system planners.

This risk level might therefore be different for generation and for transmission which have different planning criteria and might also be specific for each country.

Another possible way to calibrate the risk level is to assess the present system adequacy level in accordance with the market requirements, providing that this level is indeed acceptable. Planning criteria will further refer also to the market derived level.

The second problem is to assess such an index consequently to the deviation from the planning criteria. A scenario leading to such a deviation is, for instance, asset project schedule deviation. Let's name that risk level, R_1, the “deviation risk level”.

An immediate candidate to be a risk index is the ratio K between the deviation risk level, R_1 and the tolerated risk level, R_0: $K= R_1/R_0$.

This index can be implemented in the following way. Up to K_1 (for instance K_1=2), one may consider that the risk is still can be tolerated and will be naturally eliminated without any special action in the future realization of the planned system development. From K greater than K_1 but lower than K_2 (for instance K_2=7), the risk may be tolerated for a short period from one to two years but the system asset planner has to follow up the system development to be sure that the adequate resources are invested in order to eliminate the gap. The system operator may add operation limitation to reduce that risk in a specific location and/or situation.

From K greater than K_2 but lower than K_3 (for instance K_3=20), the system asset planner in coordination with the system operator has to find temporary solutions to lower the risk till it can be definitely eliminated by the original planning realization. One should also carefully check if any major action has to be taken in order to control the situation.

Above K_3, one has to regard the system as being in real danger and to mobilize the relevant decision makers in order to rapidly take care of the situation.

This index may also be used to characterize over-invested systems.

The planning criteria which have been used in a regulated environment were based on decades of worldwide experience in planning and development of the power systems. The planning criteria provide a tolerable risk level for unsupplied energy to be used as reference for any further insight in the system reliability analysis.

What should be the "tolerated" level of this probability so that the system might be regarded as a "secure system"?

The threshold criterion is usually given by the number of events that can cause the blackout (at least three consecutive events) and by the probability of such events to occur. For instance with regard to airplane failures, the probability should be less than 10^{-7}.

We will further evaluate the threshold probability of an event leading the system to blackout, as the result of existing planning criteria of the electric system. This evaluation will be based on widely used probabilistic planning criteria and evaluation of the total damage (unsupplied energy cost).

Regarding the LOLP index used in the generation system planning it should be noted that

- The LOLP index only measures the lack of adequacy of the generation system to the required load.
- This index changes in time and its maximal value only appears before a generating unit is added, as opposed to the security problem which may still exist even after adding units.

For example, consider a system with the planning criteria obtained as a result of the result of economical optimization, whereas the cost of unsupplied energy for the economy was evaluated as 6$/kWh. The obtained LOLP index is

$$LOLE = 1.2hr/yr \text{ that is } LOLP_G = \frac{1.2}{8760} \cong 1.4 \times 10^{-4}$$

If we presume that this index causes unsupplied energy UE_G as a result of lack of generation then the threshold lower bound of a blackout probability (the tolerated risk R_0) results from

$$R_0 = LOLP_G \times \left(\frac{UE_G}{UE_B} \right)^{\alpha}$$

where UE_B is the unsupplied energy expected from a blackout event, and α is an amplification factor reflecting the fact that the blackout is much more critical than local lack of generation. This factor may be derived from the evaluated cost of blackout compared to local unsupplied energy.

As a result of the planning criteria, the expected average unsupplied energy span in one year by the generation system is estimated by the system simulation as approximately UE_G = 200 MWh.

Based on historical events, the unsupplied energy consequent to a typical blackout can be evaluated by the outage of half the system load centers for six hours:

$$UE_B \cong 4000MW \times 6h = 24000MWh\ .$$

Therefore for α=1 we get

$$R_0 = 1.4 \times 10^{-4} \times \frac{200}{24000} \cong 1.1 \times 10^{-6}\ .$$

This means that one should choose such a system structure that ensures that the probability of the blackout is, at least, lower than 10^{-6} which means that frequency of the total system blackout should be less than one per 120 years. For α=2 $R_0 = 10^{-8}$, which means that frequency of the total system blackout should be less than one per 14,000 years.

1.5 Introduction to multi-state systems

Power system is a typical example of a system that cannot be analyzed by means of classical binary state reliability theory that presumes only two possible system states: full operation state and total failure state. The available generating power and transmission capacity in complex power systems can change gradually depending on the availability of numerous elements composing the system. It is therefore important to consider the power system and its subsystems as complex objects that can be in different states, characterized by different levels of ability to meet technical and economic requirements. Instead of probability of successful operation one has to consider probabilistic distribution of indices characterizing the system functioning. The following chapters present the introduction to the theory of multi-state systems that is used for solving many problems of reliability evaluation and optimization in power system planning and management.

1.5.1 Main definitions and models

Basic concepts of multi-state systems

All technical systems are planned to perform their intended tasks in a given environment. Some systems can perform their tasks with various distinguished levels of efficiency, usually referred to as *performance rates*. A system that can have a finite number of performance rates is called a Multi-state System (MSS). Usually an MSS is composed of elements that in their turn can be multi-state.

Actually, a binary system is the simplest case of an MSS having two distinguished states (perfect functioning and complete failure).

There are many different situations in which a system should be considered to be an MSS. Any system consisting of different units that have a cumulative effect on the entire system performance has to be considered as an MSS. Indeed, the performance rate of such a system depends on the availability of its units, as different numbers of the available units can provide different levels of task performance.

The simplest example of such a situation is the well-known k-out-of-n system. These systems consist of n identical binary units and can have n+1 states, depending on the number of available units. The system performance rate is assumed to be proportional to the number of available units and the performance rates corresponding to more than k–1 available units are acceptable. When the contributions of different units to the cumulative system performance rate are different, the number of possible MSS states grows dramatically, as different combinations of k available units can provide different performance rates for the entire system. Such systems cannot be analyzed using the technique developed for the binary case.

The performance rate of elements composing a system can also vary as a result of their deterioration (fatigue, partial failures) or because of variable ambient conditions. Element failures can lead to the degradation of the entire MSS performance.

The performance rates of the elements can range from perfect functioning up to complete failure. Failures that lead to decrease in the element performance are called partial failures. After partial failure, elements continue to operate at reduced performance rates, and after complete failure the elements are totally unable to perform their tasks.

Example 1.3

In a power supply system consisting of generating and transmitting facilities, each generating unit can function at different levels of capacity. Generating units are complex assemblies of many parts. The failures of different parts may lead to situations in which the generating unit continues to operate, but at a reduced capacity. This can occur during the outages of several auxiliaries, such as pulverizers, water pumps, fans, *etc.* For example, Billinton and Allan (1996) describe a three-state 50 MW generating unit. The performance rates (generating capacity) corresponding to these states and probabilities of the states are presented in Table 1.6.

Table1.6. Capacity distribution of 50 MW generator

Number of state	Generating capacity (MW)	State probability
1	50	0.960
2	30	0.033
3	0	0.007

The theory of MSSs was developed since the mid 1970s, when the studies of Murchland (1975), El-Neveihi *et al.* (1978), Barlow and Wu (1978), Ross (1979) appeared. These studies formulated the basic concepts of MSS reliability. Griffith (1980), Natvig (1982) and Hudson and Kapur (1982) subsequently generalized the results obtained in the earlier studies. Natvig (1984), El-Neveihi and Prochan (1984), Reinshke and Ushakov (1988) summarized the achievements attained until the mid 1980s. The review of the modern state of the art in MSS reliability can be found in (Lisnianski and Levitin 2003).

Generic multi-state system model

In order to analyze MSS behavior one has to know the characteristics of its elements. Any system element j can have k_j different states corresponding to the performance rates, represented by the set $\mathbf{g}_j=\{g_{j0}, g_{j1},\ldots, g_{jk_j-1}\}$, where g_{ji} is the performance rate of element j in the state i, $i\in\{0, 1, \ldots, k_j-1\}$.

The performance rate G_j of element j at any time instant is a random variable that takes its values from $\mathbf{g}_j$: $G_j\in\mathbf{g}_j$.

The probabilities associated with the different states (performance rates) of the system element j can be represented by the set

$$\boldsymbol{p}_j = \{p_{j0}, p_{j1}, ..., p_{jk_j-1}\} \quad (1.1)$$

where

$$p_{ji} = \Pr\{G_j = g_{ji}\} \quad (1.2)$$

As in the case of binary systems, the state probabilities of the MSS elements can be interpreted as the state probabilities during a fixed mission time, the state probabilities at a specified time, or the availabilities (in the case of binary elements). The system reliability measure corresponds to the reliability measures used to express the state probabilities of elements.

Note that, since the element states compose the complete group of mutually exclusive events (meaning that the element can always be in one and only in one of k_j states) $\sum_{i=0}^{k_j-1} p_{ji} = 1$.

Expression (1.2) defines the p.m.f. for a discrete random variable G_j. The collection of pairs g_{ji}, p_{ji}, $i = 0, 1,\ldots, k_j-1$, completely determines the probability distribution of performance (PD) of the element j.

Observe that the behavior of binary elements (elements with only total failures) can also be represented by performance distribution. Indeed, consider a binary element i with a nominal performance (performance rate corresponding to a fully operable state) g^* and the probability that the element is in the fully operable state p. Assuming that the performance rate of the element in a state of complete failure is zero, one obtains its PD as follows: $\boldsymbol{g}_i=\{0, g^*\}$, $\boldsymbol{p}_i=\{1-p, p\}$.

The PDs can be represented graphically in the form of cumulative curves. In this representation, each value of performance x corresponds to the probability that the element provides a performance rate that is no less than this level: $\Pr\{G_j \geq x\}$.

For comparison, the graphs representing the PD of the binary element i and the element j with five different states are presented in Fig. 1.18. Observe that the cumulative discrete PD is always a decreasing stepwise function.

When the MSS consists of n elements, its performance rates are unambiguously determined by the performance rates of these elements. At each moment, the system elements have certain performance rates corresponding to their states. The state of the entire system is determined by the states of its elements. Assume that the entire system has K different states and that g_i is the entire system performance rate in state i ($i \in \{0, ..., K-1\}$). The MSS performance rate is a random variable that takes values from the set $\{g_1, \ldots, g_{K-1}\}$.

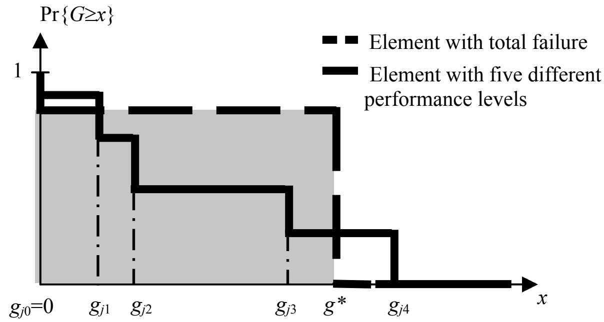

Fig. 1.18. Cumulative performance curves of multi-state elements

Let $L^n = \{g_{10},...,g_{1k_1-1}\} \times \{g_{20},...,g_{2k_2-1}\} \times ... \times \{g_{n0},...,g_{nk_n-1}\}$ be the space of possible combinations of performance rates for all of the system elements and $M=\{g_0, \dots, g_{K-1}\}$ be the space of possible values of the performance rate for the entire system. The transform $\phi(\mathrm{G}_1,\dots,\mathrm{G}_\mathrm{n})$: $L^n \to M$, which maps the space of the elements' performance rates into the space of system's performance rates, is named the system structure function. Note that the MSS structure function is an extension of a binary structure function. The only difference is in the definition of the state spaces: the binary structure function is mapped $\{0,1\}^n \to \{0,1\}$, whereas in the MSS one deals with much more complex spaces.

The set of random element performances $\{G_1, \dots, G_n\}$ plays the same role in an MSS that the element state vector plays in binary systems.

Now we can define a generic model of the MSS. This model includes the p.m.f. of performances for all of the system elements and system structure function:

$$\boldsymbol{g_j}, \boldsymbol{p_j}, 1 \le j \le n \tag{1.3}$$

$$\phi(G_1,\dots,G_n) \tag{1.4}$$

It should be noted that this simple MSS model, while being satisfactory for many applications, fails to describe some important characteristics of MSSs, such as mean time to failure, mean number of failures during the operation period, *etc.* Analysis of these characteristics requires application of a random process approach and is beyond the scope of this book.

It does not matter how the structure function is defined. It can be represented in a table, in analytical form, or be described as an algorithm for unambiguously determining the system performance G for any given set $\{G_1, \dots, G_n\}$.

Acceptability function

The MSS behavior is characterized by its evolution in the space of states. The entire set of possible system states can be divided into two disjoint subsets corresponding to acceptable and unacceptable system functioning. The system entrance into the subset of unacceptable states constitutes a failure. The MSS reliability can be defined as its ability to remain in the acceptable states during the operation period.

Since the system functioning is characterized by its output performance G, the state acceptability depends on the value of this index. In some cases this dependency can be expressed by the binary acceptability function $F(G)$ that takes a value of 1 if and only if the MSS functioning is acceptable. This takes place when the efficiency of the system functioning is completely determined by its internal state (for example, only the states where a network preserves its connectivity are acceptable). In such cases, a particular set of MSS states is of interest to the customer. Usually, the unacceptable states (corresponding to $F(G) = 0$) are interpreted as system failure states, which, when reached, imply that the system should be repaired or discarded. The set of acceptable states can also be defined when the system functionality level is of interest at a particular point in time (such as at the end of the warranty period).

Much more frequently, the system state acceptability depends on the relation between the MSS performance and the desired level of this performance (demand) that is determined outside of the system. When the demand is variable, the MSS operation period T is often partitioned into M intervals T_m ($1 \le m \le M$) and a constant demand level w_m is assigned to each interval m. In this case the demand W can be represented by a random variable that can take discrete values from the set $\boldsymbol{w}=\{w_1,...,w_M\}$. The p.m.f. of the variable demand can be represented (in analogy with the p.m.f. of MSS performance) by two vectors ($\boldsymbol{w}$, $\boldsymbol{q}$), where $\boldsymbol{q}=\{q_1, ..., q_M\}$ is the vector of probabilities of corresponding demand levels $q_j = \Pr\{W = w_j\}$. The desired relation between the system performance and the demand can also be expressed by the acceptability function $F(G, W)$. The acceptable system states correspond to $F(G, W) = 1$ and the unacceptable states correspond to $F(G, W) = 0$. The last equation defines the MSS failure criterion.

Example 1.4

An on-load tap changer control system is aimed at maintaining the voltage in the electric power distribution system between $u_{\min}$ and $u_{\max}$. The exit of the system voltage outside this range constitutes the system's failure. The system's output performance is the controlled voltage $G = U$ that can vary discretely. The acceptability function can be expressed as

$$F(G)=1(G \le u_{\max}) \times 1(G \ge u_{\min})$$

or

$$F(G)=1(|2G-(u_{\min}+u_{\max})| \le u_{\max}-u_{\min})$$

Example 1.5

A power generation system should supply the customers with variable demand W. If the cumulative power of the available generating units is much greater than the demand (usually at night) then some units can be disconnected and transferred to a standby state. If the cumulative power of all of the available units is not enough to meet the demand (either because of a sharp increase in demand or due to the outage of some of the units) then the system fails. The system's performance is the cumulative available power G, which should exceed the random demand W. In this case the acceptability function takes the form

$$F(G,W) = 1(G>W)$$

This type of acceptability function is used in many practical cases when the MSS performance should exceed the demand.

1.5.2 Simplest types of multi-state system

According to the generic model (1.3) and (1.4), one can define different types of MSS by determining the performance distribution of its elements and defining the system's structure function. It is possible to invent an infinite number of different structure functions in order to obtain different models of MSS. The question is whether or not the MSS model can be applied to real technical systems. This section presents different application-inspired MSS models that are most commonly used in reliability engineering.

Series structure

The series connection of system elements represents a case where a total failure of any individual element causes an overall system failure. In the binary system the series connection has a purely logical sense. The topology of the physical connections among elements represented by a series reliability block diagram can differ, as can their allocation along the system's functioning process. The essential property of the binary series system is that it can operate only when all its elements are fully available.

When an MSS is considered and the system performance characteristics are of interest, the series connection usually has a "more physical" sense. Indeed, assuming that MSS elements are connected in a series means that some processes proceed stage by stage along a line of elements. The process intensity depends on the performance rates of the elements. Observe that the MSS definition of the series connection should preserve its main property: the total failure of any element (corresponding to its performance rate equal to zero) causes the total failure of the entire system (system performance rate equal to zero).

One can distinguish several types of series MSS, depending on the type of performance and the physical nature of the interconnection among the elements.

First, consider a system that uses the capacity (productivity or throughput) of its elements as the performance measure. The operation of these systems is associated with some media flow continuously passing through the elements. Examples of these types of system are power systems, energy or materials continuous transmission systems, continuous production systems, *etc.* The element with the minimal transmission capacity becomes the bottleneck of the system (Barlow and Wu 1978). Therefore, the system capacity is equal to the capacity of its "weakest" element. If the capacity of this element is equal to zero (total failure), then the entire system capacity is also zero.

Example 1.6

An example of the flow transmission (capacity-based) series system is a power station coal transportation unit (Fig. 1.19) that continuously supplies the system of boilers and consists of five basic elements:

1. Primary feeder, which loads the coal from the bin to the primary conveyor.
2. Set of primary conveyors, which transport the coal to the stacker-reclaimer.
3. Stacker-reclaimer, which lifts the coal up to the secondary conveyor level.
4. Secondary feeder, which loads the set of secondary conveyors.
5. Set of secondary conveyors, which supplies the burner feeding system of the boilers.

The amount of coal supplied to the boilers at each time unit proceeds consecutively through each element. The feeders and the stacker-reclaimer can have two states: working with nominal throughput and total failure.

The throughput of the sets of conveyors (primary and secondary) can vary depending on the availability of individual two-state conveyors. It can easily be seen that the throughput of the entire system is determined as the throughput of its elements having minimal transmission capacity. The system reliability is defined as its ability to supply a given amount of coal (demand) during a specified operation time.

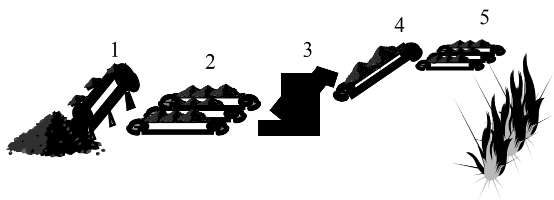

Fig. 1.19. Power station coal transportation system

Another category in the series systems is a task processing system, for which the performance measure is characterized by an operation time (processing speed). This category may include control systems, information or data processing systems, manufacturing systems with constrained operation time, *etc.* The operation of these systems is associated with consecutive discrete actions performed by the ordered line of elements. The total system operation time is equal to the sum of the operation times of all of its elements. When one measures the element (system) performance in terms of processing speed (reciprocal to the operation time), the total failure corresponds to a performance rate of zero. If at least one system element is in a state of total failure, then the entire system also fails completely. Indeed, the total failure of the element corresponds to its processing speed equal to zero, which is equivalent to an infinite operation time. In this case, the operation time of the entire system is also infinite.

Example 1.7

An example of the task processing series system is a manipulator control system (Fig. 1.20) consisting of:

1. Visual image processor.
2. Multi-channel data transmission subsystem, which transmits the data from the image processor to main processing unit.
3. Main multi-processor unit, which generates control signals for manipulator actuators.

4. Manipulator.

The system performance is measured by the speed of its response to the events occurring. This speed is determined by the sum of the times needed for each element to perform its task (from initial detection of the event to the completion of the manipulator actuators performance). The time of data transmission also depends on the availability of channels, and the time of data processing depends on the availability of the processors as well as on the complexity of the image. The system reliability is defined as its ability to react within a specified time during an operation period.

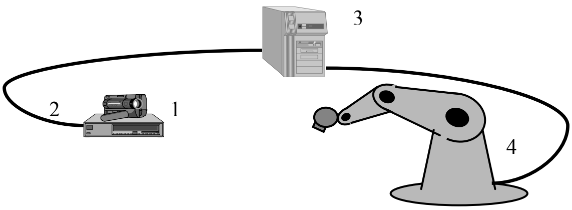

Fig. 1.20. Example of task processing series system

Parallel structure

The parallel connection of system elements represents a case where a system fails if and only if all of its elements fail. Two basic models of parallel systems are distinguished in binary reliability analysis. The first one is based on the assumption that all of the elements are active and work sharing. The second one represents a situation where only one element is operating at a time (active or standby redundancy without work sharing).

An MSS with a parallel structure inherits the essential property of the binary parallel system so that the total failure of the entire system occurs only when all of its elements are in total failure states. The assumption that MSS elements are connected in parallel means that some tasks can be performed by any one of the elements. The intensity of the task accomplishment depends on the performance rate of available elements.

For an MSS with work sharing, the entire system performance rate is usually equal to the sum of the performance rates of the parallel elements for both flow transmission and task processing systems. Indeed, the total flow through the former type of system is equal to the sum of flows through its parallel elements. In the latter type of MSS, the system processing speed depends on the rules of the work sharing. The most effective rule providing the minimal possible time of work completion shares the work among the elements in proportion to their processing speed. In this case, the processing speed of the parallel system is equal to the sum of the processing speeds of all of the elements.

Example 1.8

Consider a system of several parallel coal conveyors supplying the same system of boilers (Fig. 1.21A) or a multi-processor control unit (Fig. 1.21B), assuming that the performance rates of the elements in both systems can vary. In the first case the amount of coal supplied is equal to the sum of the amounts supplied by each one of the conveyors. In the second case the unit processing speed is equal to the sum of the processing speeds of all of its processors.

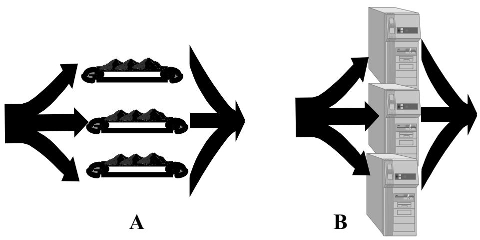

Fig. 1.21. Examples of parallel systems with work sharing. (A: flow transmission system; B: task processing system)

In an MSS without work sharing the system performance rate depends on the discipline of the elements' activation. Unlike binary systems, where all the elements have the same performance rate, the choice of an active element from the set of different ones affects the MSS performance. The most common policy in both flow transmission and task processing MSSs is to use an available element with the greatest possible performance rate. In this case, the system performance rate is equal to the maximal performance rate of the available parallel elements (Barlow and Wu 1978).

Example 1.9

Consider a system with several generators and commutation equipment allowing only one generator to be connected to the electrical network (Fig. 1.22A). If the system task is to provide the maximal possible power supply, then it keeps the most powerful generator from the set of those available in operation. The remainder of the generators can be either in an active state (hot redundancy), which means that they are rotating but are not connected to the network, or in a passive state (cold redundancy), where they do not rotate.

Another example is a multi-channel data transmission system (Fig. 1.22B). When a message is sent simultaneously through all the channels, it reaches a receiver by the fastest channel and the transmission speeds of the rest of the channels do not matter.

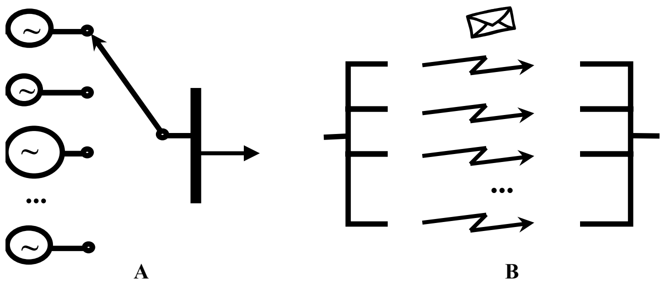

Fig. 1.22. Examples of parallel systems without work sharing. (A: flow transmission system; B: task processing system)

A hybrid combination of series and parallel structures results in series-parallel systems. The performance rates of these structures can be obtained by the consecutive evaluation of the performance rates of pure series or parallel subsystems and then considering these subsystems as single equivalent elements.

1.5.3 Measures of multi-state system performance

To characterize MSS behavior numerically from a reliability and performance point of view, one has to determine the MSS performance measures. Some of the measures are based on a consideration of the system's evolution in the time domain. In this case, the relation between the system's output performance and the demand represented by the two corresponding stochastic processes must be studied. This study is not within the scope of this book since the UGF technique allows one to determine only the measures based on performance distributions.

When a system is considered in the given time instant or in a steady state (when its output performance distribution does not depend on time) its behavior is determined by its performance rate represented as a random variable G. Consider several measures of system output performance that can characterize any system state.

The first natural measure of a system's performance is its output performance rate G. This measure can be obtained by applying the system structure function over the performance rates of the system's elements. Each specific system state j is characterized by the associated system performance rate $G = g_j$, which determines the system's behavior in the given state but does not reflect the acceptability of the state from the customer's point of view.

In order to represent the system state acceptability, we can use the acceptability function $F(G)$ or $F(G,W)$ defined in Section #.1.3. The acceptability function divides the entire set of possible system states into two disjoint subsets (acceptable and unacceptable states). Therefore, if the system's behavior is represented by an acceptability function, the system as a whole can be considered to be a binary one.

In many practical cases it is not enough to know whether the state is acceptable or not. The damage caused by an unacceptable state can be a function of the system's performance rate deviation from a demand. Usually, the one-sided performance deviation (performance deviation from a demand when the demand is not met) is of interest. For example, the cumulative generating capacity of available electric generators should exceed the demand. In this case the possible performance deviation (performance deficiency) takes the form

$$D^{-}(G,W) = \max(W - G, 0) \tag{1.5}$$

When the system's performance should not exceed demand (for example, the time needed to complete the assembling task in an assembly line should be less than a maximum allowable value in order to maintain the desired productivity), the performance redundancy is used as a measure of the performance deviation:

$$D^{+}(G,W) = \max(G - W, 0) \tag{1.6}$$

Fig. 1.23 shows an example of the behavior of the MSS performance and the demand as the realizations of the discrete stochastic processes and the corresponding realizations of the measures of the system's output performance.

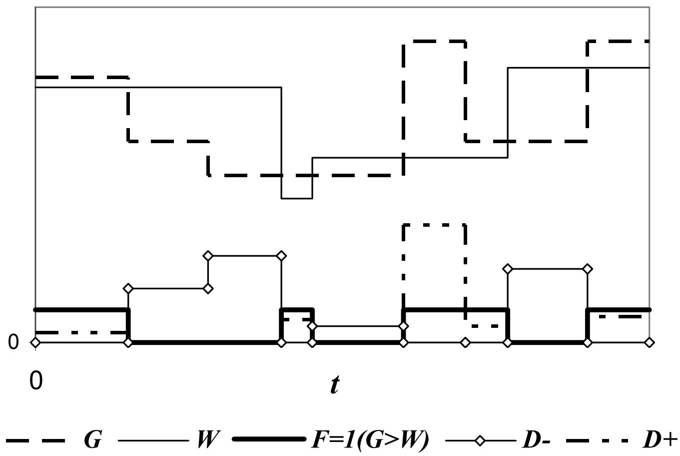

Fig. 1.23. Example of a realization of the measures of system output performance

The expected system acceptability $E(F(G,W))$ determines the system reliability or availability (the probability that the MSS is in one of the acceptable states: $\Pr\{F(G,W)=1\}$). Depending on the meaning of the system and element state probabilities, it can be interpreted as $R(t)$, the MSS reliability at a specified time t, as $R(T)$, the MSS reliability during a fixed mission time T (for unrepairable systems), or as instantaneous (point) availability $A(t)$ or steady-state availability A (for repairable systems).

The expected system performance deviation $E(D^{-}(G,W))$ or $E(D^{+}(G,W))$ can be interpreted as Δ_t, the expected instantaneous performance deviation at instant t, or as a mean steady-state performance deviation Δ.

In some cases we need to know the conditional expected performance of the MSS. This measure represents the mean performance of the MSS given that it is in acceptable states. In order to determine the conditional expected performance $\tilde{\varepsilon}$ we define the auxiliary function as $\tilde{G}(G,W)=GF(G,W)$. The measure $\tilde{\varepsilon}$ can be determined as follows:

$$\tilde{\varepsilon}=E(\tilde{G})/\Pr\{F(G,W)=1\}=E(GF(G,W))/E(F(G,W)) \quad (1.7)$$

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2 Basic Tools and Techniques

2.1 Modern random processes methods

2.1.1 General concepts

The purpose of this chapter is to describe basic concepts of applying a random process approach to power system reliability assessment. Here, we do not present the basics of the measure-theoretic framework that are necessary to those who are not familiar with the general theory of stochastic processes. Readers who need this fundamental framework and more detailed stochastic processes presentation can find it in the following literature: (Kallenberg 1997), (Karlin and Taylor 1981), (Ross 1983). For reliability engineers and analysts, the books of Trivedi (2002), Aven and Jensen (1999) and Lisnianski and Levitin (2003) are especially recommended.

A stochastic or random process is, essentially, a set of random variables, where the variables are ordered in a given sequence. For example, the daily maximum temperatures at a weather station form a sequence of random variables, and this ordered sequence can be considered as a stochastic process. Another example is the sequence formed by the continuously changing number of people waiting in a queue at ticket window of a railway station.

More formally, the sequence of random variables in a process can be denoted by $X(t)$ where t is the index of the process. In this book, we deal with stochastic processes where t represents time.

A random variable X can be considered as the rule for assigning to every outcome ς of an experiment the number $X(\varsigma)$. A stochastic process is a rule for assigning to every ς the function $X(t,\varsigma)$. Thus, a stochastic process is a family of time functions depending on the parameter ς or, equivalently, a function of t and ς. The domain of ς is the set of all the possible experimental outcomes and the domain of t is a set of non-negative real numbers.

For example, a trajectory of an airplane's flight from point A to point B at given height H will be a stochastic process. Each flight can be considered as an experimental outcome ς and each flight will have its own tra-

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jectory $X(t, \varsigma)$ that characterize for this case a height of the flight as a function of time. This trajectory will be different from trajectories of other flights because of the influence of many random factors (such as wind, temperature, pressure etc.). In Fig. 2.1 one can see three different trajectories for three flights that can be treated as three different realizations of the stochastic process. It should be noticed that the cut of this stochastic process at any time t_1 will present the random variable with mean H. In power systems such parameters as generating capacity, voltage, and frequency may be considered as stochastic processes.

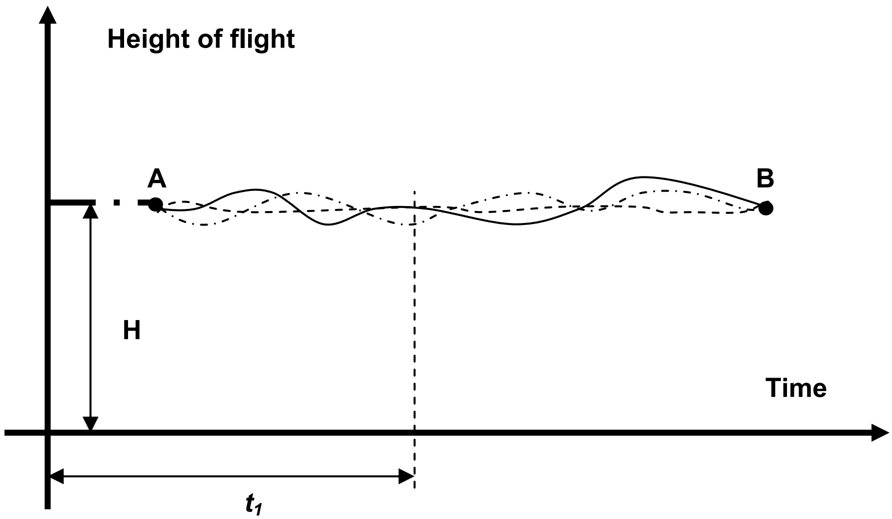


Fig. 2.1. Trajectories of three flights

The time may be discrete or continuous. A discrete time may have a finite or infinite number of values; continuous time obviously has only an infinite number of values. The values taken by the random variables constitute the state-space. This state-space, in its turn, may be discrete or continuous. Therefore, stochastic processes may be classified into four categories according to whether their state-spaces and time are continuous or discrete. A process with discrete state-space is usually called a chain.

The stochastic process $X(t, \varsigma)$ has the following interpretations:

1. It is a family of functions $X(t, \varsigma)$ where t and ς are variables.
2. It is a single time function or a realization (sample) of the given process if t is a variable and ς is fixed.
3. It is a random variable equal to the state of the given process at time t when t is fixed and ς is variable.

4. It is a number if t and ς are fixed.

One can use the notation $X(t)$ to represent a stochastic process omitting, as in the case of random variables, its dependence on ς.

First, consider a process with a discrete set of indices $\{t_i\}$ and a discrete state-space $\{x_k\}$. The indices are ordered so that $t_i<t_{i+1}$ for any i (no such arrangement is necessary for the set $\{x_k\}$). The process is defined by some rules indicating how the distribution of any random variable $X_n=X(t_n)$ depends on the values assumed by all previous variables X_i $(1 \le i \le n\text{-}1)$. These rules actually define conditional probabilities

$$Pr\{X_n=x_n \mid (X_1=x_1)\cap(X_2=x_2)\cap...\cap(X_{n-1}=x_{n-1})\} \quad (2.1)$$

and provide the distribution of X_n in terms of what can be called the “past history” of the process.

In general, a different distribution of X_n is obtained for every realization of the process in the time domain (t_1,t_{n-1}), with a given set $x_1,...,x_{n-1}$ representing a single realization. The dependence of the distribution of X_n on past history can be illustrated by the daily temperature example. Assume that neither the date, nor the season, is known about day t_n and the only information available is the maximum temperature readings for a number of preceding days. The expected temperature distribution for day t_n is then based on this information and is also greatly dependent on it. These readings are characteristic of the season since different temperature probabilities can be expected in the summer and in the winter.

Of course, it is also possible that the distribution of X_n does not depend on past history. Process with this property is called the Markov process. In the Markov process, the probabilities of the random variable at time t_n depend on the value of the random variable at t_{n-1} but not on the realization of the process prior to t_{n-1}. In other words, the state probabilities at a future instant, given the present state of the process, do not depend on the states occupied in the past. Therefore, this process is also called “memoryless”.

In power system reliability analysis, stochastic processes with a continuum set $\{t_i\}$ and a discrete state-space $\{x_k\}$ are widely used. In order to illustrate this type of stochastic processes we define here two important processes that will be used in the future: point and renewal processes.

A point process is a set of random points t_i on the time axis. For each point process one can associate a stochastic process $X(t)$ equal to the number of points t_i in the interval $(0,t)$. In reliability theory point processes are widely used to describe the appearance of events in time (e.g., failures, terminations of repair, etc.)

An example of the point processes is the so-called Poisson process. The Poisson process is usually introduced using the Poisson points. These points are associated with certain events and the number $N(t_1,t_2)$ of the points in an interval (t_1, t_2) of length $t=t_2-t_1$ is a Poisson random variable with parameter λt, where λ is the mean occurrence rate of the events:

$$\Pr\{N(t_1,t_2)=k\} = \frac{e^{-\lambda t}(\lambda t)^k}{k!} . \tag{2.2}$$

If the intervals (t_1,t_2) and (t_3,t_4) are not overlapping, then the random variables $N(t_1,t_2)$ and $N(t_3,t_4)$ are independent. Using the points t_i, one can form the stochastic process $X(t) = N(0,t)$.

The Poisson process plays a special role in reliability analysis, comparable to the role of the normal distribution in probability theory. Many real physical situations can be successfully described with the help of Poisson processes.

A well-known type of point process is the so-called renewal process. This process can be described as a sequence of events, the intervals between which are independent and identically distributed random variables. In reliability theory, this kind of mathematical model is used to describe the flow of failures in time.

To every point process t_i one can associate a sequence of random variables y_n such that $y_1=t_1$, $y_2=t_2-t_1, \ldots, y_n=t_n-t_{n-1}$ where t_1 is the first random point to the right of the origin. This sequence is called a renewal process. An example is the life history of the items that are replaced as soon as they fail. In this case, y_i is the total time the i-th item is in operation and t_i is the time of its failure.

One can see a correspondence among the following three processes:

- a point process t_i,
- a discrete-state stochastic process $X(t)$ increasing (or decreasing) by 1 at the points t_i,
- a renewal process consisting of the random variables y_i such that $t_n = y_1 + \ldots + y_n$.

A generalization of this type of process is the so-called alternating renewal process. This process consists of two types of independent and identically distributed random variables alternating with each other in turn. This type of process is convenient for the description of reparable systems. For such systems, periods of successful operation alternate with periods of idle time.

In this chapter, the power system reliability models will be consequently studied based on the Markov processes, the Markov rewards processes and semi-Markov processes. The Markov processes are widely used for reliability analysis because the number of failures in arbitrary time interval in many practical cases can be described as a Poisson process and the time up to the failure and repair time are often exponentially distributed. It will be shown how by using the Markov processes theory power system reliability measures can be determined. It will be also shown how such power system reliability measures as the mean time up to the failure, mean number of failures in a time interval, mean sojourn time in a set of unacceptable states can be found by using the Markov reward models. In practice, basic assumptions about exponential distributions of times between failures and repair times often do not hold. In this case, the more complicated mathematical technique named the semi-Markov processes may be applied.

2.1.2 Markov models

Main definitions and properties

A discrete-state continuous-time stochastic process $X(t) \in \{1,2,...\}$ is called a Markov chain if for $t_1 < t_2 < ... < t_{n-1} < t_n$, its conditional probability mass function satisfies the relation

$$\Pr\{X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1},..., X(t_2) = x_2, X(t_1) = x_1\} =$$

$$= \Pr\{X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}\}\ . \tag{2.3}$$

Introducing the notations t=t_{n-1} and t_n=t_{n-1}+ Δt the expression (2.3) simplifies to:

$$\Pr\{X(t + \Delta t) = i \mid X(t) = j\} = \pi_{ji}(t, \Delta t)\ . \tag{2.4}$$

These conditional probabilities are called *transition probabilities*. If the probabilities $\pi_{ji}(t, \Delta t)$ do not depend on *t*, but only on the time difference Δt, the Markov process is said to be *homogeneous*. Note that $\pi_{jj}(t, \Delta t)_{jj}$is the probability that no change in the state will occur in a time interval of length Δt given that the process is in state *j* at the beginning of the interval.

One can define for each *j* a nonnegative continuous function $a_j(t)$:

$$a_j(t) = \lim_{\Delta t \to 0} \frac{\pi_{jj}(t,0) - \pi_{jj}(t, t + \Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{1 - \pi_{jj}(t, t + \Delta t)}{\Delta t} \tag{2.5}$$

and for each *j* and $i \neq j$ a nonnegative continuous function $a_{ji}(t)$:

$$a_{ji}(t) = \lim_{\Delta t \to 0} \frac{\pi_{ji}(t,0) - \pi_{ji}(t, t + \Delta t)}{- \Delta t} = \lim_{\Delta t \to 0} \frac{\pi_{ji}(t, t + \Delta t)}{\Delta t}\ . \tag{2.6}$$

The function $a_{ji}(t)$ is called the *transition intensity* from state *i* to state *j* at time *t*. For the homogeneous Markov processes, the transition intensities do not depend on *t* and therefore are constant.

If the process is in state *j* at a given moment, in the next Δt time interval there is either a transition from *j* to some state *i* or the process remains at *j*. Therefore

$$\pi_{jj}(\Delta t) + \sum_{i \neq j} \pi_{ji}(\Delta t) = 1. \tag{2.7}$$

Designating $a_{jj} = -a_j$ and combining (2.7) with (2.5) one obtains

$$a_{jj} = -a_j = \lim_{\Delta t \to 0} -\frac{1}{\Delta t} \sum_{i \neq j} \pi_{ji}(\Delta t) = -\sum_{i \neq j} a_{ji}. \tag{2.8}$$

Let $p_i(t)$ be the state probabilities of $X(t)$ at time *t*:

$$p_i(t) = \Pr\{X(t) = i\}, j=1, \ldots, k;\ t \geq 0. \tag{2.9}$$

Therefore, expression (2.9) defines the probability mass function (pmf) of $X(t)$.

Since at any given time the process must be in one of *k* states,

$$\sum_{i=1}^{k} p_i(t) = 1 \tag{2.10}$$

for any $t \geq 0$,

The states probabilities at instant $t + \Delta t$ can be expressed based on states probabilities at instant *t* by using the following equations:

$$p_j(t + \Delta t) = p_j(t)[1 - \sum_{i \neq j} a_{ji} dt] + \sum_{i \neq j} p_i(t) a_{ij} dt,\ i, j = 1, \ldots, k\ . \tag{2.11}$$

Equation (2.11) can be obtained by using the following considerations. The process can achieve the state *j* at instant $t + \Delta t$ by two ways.

1. The process may already be in the state *j* at instant *t* and doesn't leave this state up to the instant $t + \Delta t$. These events have probabilities $p_j(t)$ and $1 - \sum_{i \neq j} a_{ji} \Delta t$ respectively.
2. At instant *t* the process may be in one of the states $i \neq j$ and during time Δt transits from state *i* to state *j*. These events have probabilities $p_i(t)$ and $a_{ij}\Delta t$ respectively. These probabilities should be multiplied and

summarized for all $i \neq j$ because the process can achieve state j from any state i.

Now one can rewrite (2.11) by using (2.8) and obtain the following

$$p_j(t+\Delta t) = p_j(t)[1 + a_{jj}\Delta t] + \sum_{i \neq j} p_i(t) a_{ij} \Delta t\,, \tag{2.12}$$

or

$$p_j(t+\Delta t) - p_j(t) = \sum_{i=1}^{k} p_i(t) a_{ij} \Delta t\,. \tag{2.13}$$

After dividing both sides of equation (2.13) by Δt and passing to limit $\Delta t \to 0$, we get

$$\frac{dp_j(t)}{dt} = \sum_{i=1}^{k} p_i(t) a_{ij}\,,\ j=1,2,\ \ldots,\ k. \tag{2.14}$$

The system of differential equations (2.14) is used for finding the state probabilities $p_j(t), j=1,\ldots,k$ for the homogeneous Markov process when the initial conditions are given

$$p_j(t) = \alpha_j,\quad j = 1,\ldots,k\,, \tag{2.15}$$

More mathematical details about (2.14) may be found in (Trivedi 2002) or in (Ross 1993).

Equation (2.14) defines the following rule: time-derivative of $p_j(t)$ for any arbitrary state j equals the sum of the probabilities of the states that have transitions to the state j multiplied by the corresponding transition intensities minus the probability of state j multiplied by the sum of the intensities of all transitions from the state j.

Introducing the row-vector $\boldsymbol{p}$(t)=$[p_1(t), p_2(t),\ldots, p_k(t)]$ and the transition intensity matrix $\boldsymbol{a}$

$$\boldsymbol{a} = \begin{vmatrix} a_{11} & a_{12}\cdots & a_{1K} \\ a_{21} & a_{22}\cdots & a_{2K} \\ & \cdots & \\ a_{K1} & a_{K2}\cdots & a_{KK} \end{vmatrix} \tag{2.16}$$

in which the diagonal elements are defined as a_{jj}=-a_j we can rewrite the system (2.14) in matrix notation

$$\frac{d\boldsymbol{p}(t)}{dt} = \boldsymbol{p}(t)\boldsymbol{a}. \tag{2.17}$$

Note that the sum of the matrix elements in each row equals 0: $\sum_{j=1}^{K} a_{ij} = 0$ for each i: $1 \le i \le K$.

When the system state transitions are caused by failures and repairs of its elements, the corresponding transition intensities are expressed by the element's failure and repair rates.

The element's failure rate $\lambda(t)$ is the instantaneous conditional density of the probability of failure of an initially operational element at time t given that the element has not failed up to time t. Briefly, one can say that $\lambda(t)$ is the time-to-failure conditional probability density function (pdf). It expresses a hazard of failure in time instant t under a condition where there was no failure up to time t. The failure rate of an element at time t is defined as

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left[\frac{F(t+\Delta t) - F(t)}{R(t)}\right] = \frac{f(t)}{F(t)}, \tag{2.18}$$

where $F(t)$ is the cdf of the time to failure of the element, $f(t)$ is pdf of the time to failure of the element, $R(t) = 1 - F(t)$ is the reliability function of the element.

For homogeneous Markov processes the failure rate doesn't depend on t and can be expressed as

$$\lambda = MTTF^{-1}, \tag{2.19}$$

where *MTTF* is mean time to failure. Similarly, the repair rate $\mu(t)$ is the time-to-repair conditional pdf. For homogeneous Markov processes a repair rate does not depend on t and can be expressed as

$$\mu = MTTR^{-1}, \tag{2.20}$$

where *MTTR* is the mean time to repair.

In many applications, the long-run (final) or steady state probabilities $\lim_{t\to\infty} p_i(t)$ are of interest for the repairable element. If the long-run state probabilities exist, the process is called *ergodic*. For the final state probabilities, the computations become simpler. The set of differential equations (2.14) is reduced to a set of k algebraic linear equations because for the constant probabilities all time-derivatives $\frac{dp_i(t)}{dt}$, i=1,...,k are equal to zero.

Let the final states probabilities $p_i = \lim_{t\to\infty} p_i(t)$ exist. For this case in steady state, all derivatives of states probabilities in the right side of (2.14) will be zeroes. So, in order to find the long run probabilities the following system of algebraic linear equations should be solved

$$0 = \sum_{i=1}^{k} p_i(t) a_{ij}\ , \qquad j=1,2,\ \dots,\ k. \tag{2.21}$$

The k equations in (2.21) are not linearly independent (the determinant of the system is zero). An additional independent equation can be provided by the simple fact that the sum of the state probabilities is equal to 1 at any time:

$$\sum_{i=1}^{k} p_i = 1\,. \tag{2.22}$$

The frequency f_i of state i, is defined as the expected number of arrivals into this state per unit time. Usually the concept of frequency is associated with the long-term (steady-state) behavior of the process. In order to relate the frequency, probability and mean time of staying in state i, we consider the system evolution in the state space as consisting of two alternating periods – the stays in i and the stays outside i. Thus, the process is represented by two states. Designate the mean duration of the stays in state i as $\overline{T}_i$ and that of the stays outside i, $\overline{T}_{oi}$. The mean cycle time, $\overline{T}_{ci}$ is then:

$$\overline{T}_{ci} = \overline{T}_i + \overline{T}_{oi}\ . \tag{2.23}$$

From the definition of the state frequency it follows that, in the long run, f_i equals the reciprocal of the mean cycle time

$$f_i = \frac{1}{\overline{T}_{ci}}\,. \tag{2.24}$$

Multiplying by $\overline{T}_i$, both two sides of equation (2.23) one gets

$$\overline{T}_i f_i = \frac{\overline{T}_i}{\overline{T}_{ci}} = p_i\,. \tag{2.25}$$

Therefore,

$$f_i = \frac{p_i}{\overline{T}_i}. \tag{2.26}$$

This is a fundamental equation, which provides the relation between the three state parameters in the steady state.

Unconditional random value T_i is minimal from all random values T_{ij} that characterize the conditional random time of staying in state i, if the transition will be performed from state i to any state $j \neq i$

$$T_i = \min\{T_{i1},...,T_{ij}\}. \tag{2.27}$$

All conditional times T_{ij} are distributed exponentially with the following cumulative distribution functions $F_{ij}(T_{ij} \le t) = 1 - e^{-a_{ij}t}$. All transitions from state i are independent and, therefore, the cumulative distribution function of unconditional time T_i of staying in state i can be computed as the follows

$$\begin{aligned} F_i(T_i \le t) &= 1 - \Pr\{T_i > t\} = 1 - \prod_{j \neq i} \Pr\{T_{ij} > t\} = \\ &= 1 - \prod_{j \neq i}[1 - F_{ij}(T_{ij} \le t)] = 1 - \prod_{j \neq i} e^{-a_{ij}t} = 1 - e^{-\sum_{j \neq i} a_{ij}t}. \end{aligned} \tag{2.28}$$

It means that unconditional time T_i is distributed exponentially with parameter $a_i = \sum_j a_{ij}$, and the mean time of staying in state i is the following

$$\overline{T}_i = \frac{1}{\sum_{j \neq i} a_{ij}}. \tag{2.29}$$

Substituting $\overline{T}_i$ in the expression (2.26) we finally get

$$f_i = p_i \sum_{j \neq i} a_{ij}. \tag{2.30}$$

Example 2.1

Consider a power generating unit that has k=4 possible performance levels (generating capacities): g_4=100 MW, g_3=80 MW, g_2=50 MW and g_1=0 MW.

The unit has the following failure rates

$$\lambda_{4,3} = 2\,year^{-1}, \lambda_{3,2} = 1\,year^{-1}, \lambda_{2,1} = 0.7\,year^{-1},$$

$$\lambda_{3,1} = 0.4 year^{-1}\ ,\ \lambda_{4,2} = 0.3 year^{-1},\ \lambda_{4,1} = 0.1 year^{-1},$$

and the following repair rates

$\mu_{3,4} = 100 year^{-1}$, $\mu_{2,3} = 80 year^{-1}$, $\mu_{1,2} = 50 year^{-1}$ (for minor repairs),

$\mu_{1,4} = 32 year^{-1}$, $\mu_{1,3} = 40 year^{-1}$, $\mu_{2,4} = 45 year^{-1}$ (for major repairs).

The state-space diagram for the unit is presented in Fig. 2.2. The initial state is state 4.

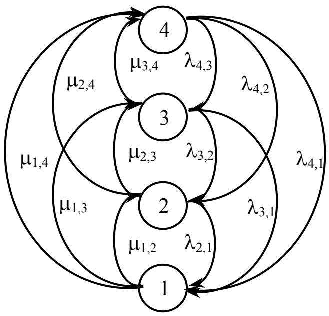


Fig. 2.2. State-space diagrams for four-state unit

In order to find the state probabilities the following system of differential equations needs to be solved (see (2.14))

$$\begin{cases} \dfrac{dp_4(t)}{dt} = -(\lambda_{4,3} + \lambda_{4,2} + \lambda_{4,1})p_4(t) + \mu_{3,4}p_3(t) + \mu_{2,4}p_2(t) + \mu_{1,4}p_1(t) \\ \dfrac{dp_3(t)}{dt} = \lambda_{4,3}p_4(t) - (\lambda_{3,2} + \lambda_{3,1} + \mu_{3,4})p_3(t) + \mu_{1,3}p_1(t) + \mu_{2,3}p_2(t) \\ \dfrac{dp_2(t)}{dt} = \lambda_{4,2}p_4(t) + \lambda_{3,2}p_3(t) - (\lambda_{2,1} + \mu_{2,3} + \mu_{2,4})p_2(t) + \mu_{1,2}p_1(t) \\ \dfrac{dp_1(t)}{dt} = \lambda_{4,1}p_4(t) + \lambda_{3,1}p_3(t) + \lambda_{2,1}p_2(t) - (\mu_{1,2} + \mu_{1,3} + \mu_{1,4})p_1(t) \end{cases}$$

with the initial conditions $p_4(t) = 1$, $p_3(t) = p_2(t) = p_1(t) = 0$.

The state probabilities obtained by solving this system are presented in Fig. 2.3.

The unit instantaneous availability can be obtained for different constant demand levels w

$$A_3(t) = p_4(t), \text{ for } g_3 < w \le g_4\ ;$$

$$A_2(t) = p_4(t) + p_3(t), \text{ for } g_2 < w \le g_3\ ;$$

$$A_1(t) = p_4(t) + p_3(t) + p_2(t) = 1 - p_1(t), \text{ for } g_1 < w \le g_2$$

The unit's mean instantaneous capacity at time t is

$$E_t = \sum_{k=1}^{4} g_k p_k(t) = 100 p_4(t) + 80 p_3(t) + 50 p_2(t) + 0 p_1(t),$$

and the mean instantaneous capacity deficiency (for constant demand w=60 MW) is

$$D_t = \sum_{k=1}^{4} p_k(t) \max(w - g_k, 0) = 10 p_2(t) + 60 p_1(t).$$

These indices, as functions of time, are presented in Fig. 2.4.

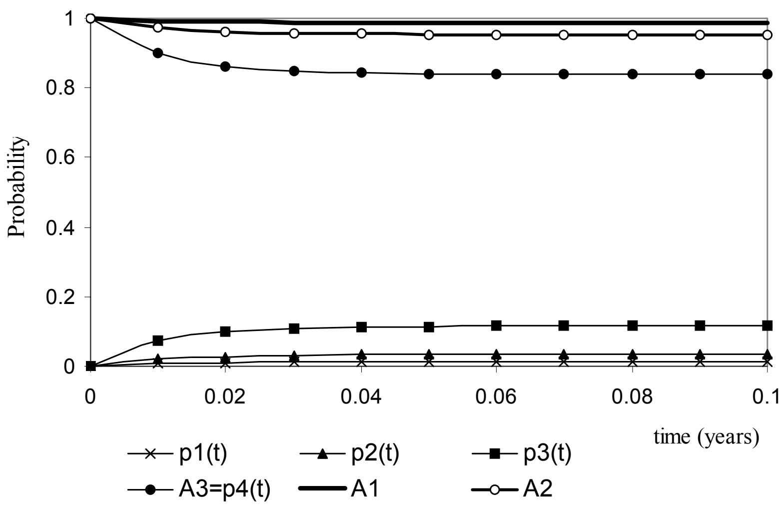


Fig. 2.3. State probabilities and instantaneous availability of the four-state element

The final state or steady state probabilities can be found by solving the system of linear algebraic equations (2.21) in which one of the equations is replaced by the equation (2.22). In our example, the system takes the form

$$\begin{cases} (\lambda_{4,3} + \lambda_{4,2} + \lambda_{4,1}) p_4 = \mu_{3,4} p_3 + \mu_{2,4} p_2 + \mu_{1,4} p_1 \\ (\lambda_{3,2} + \lambda_{3,1} + \mu_{3,4}) p_3 = \lambda_{4,3} p_4 + \mu_{2,3} p_2 + \mu_{1,3} p_1 \\ (\lambda_{2,1} + \mu_{2,3} + \mu_{2,4}) p_2 = \lambda_{4,2} p_4 + \lambda_{3,2} p_3 + \mu_{1,2} p_1 \\ p_1 + p_2 + p_3 + p_4 = 1 \end{cases}$$

Solving this system, we obtain the final state probabilities:

$$p_1 = \frac{\mu_{1,4}(b_2c_3 - b_3c_2) + \mu_{1,2}(a_2b_3 - a_3b_2) + \mu_{1,3}(a_3c_2 - a_2c_3)}{a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3},$$

$$p_2 = \frac{\mu_{2,3}(a_1c_3 - a_3c_1) + \mu_{2,4}(b_3c_1 - b_1c_3) + (\lambda_{2,1} + \mu_{2,3} + \mu_{2,4})(a_1b_3 - a_3b_1)}{a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3},$$

$$p_3 = \frac{\lambda_{3,2}(a_1b_2 - a_2b_1) + (\lambda_{3,2} + \lambda_{3,1} + \mu_{3,4})(a_1c_2 - a_2c_1) + \mu_{3,4}(b_1c_2 - b_2c_1)}{a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_1b_3c_2 - a_2b_1c_3},$$

$$p_4 = 1 - p_1 - p_2 - p_3,$$

where

$$a_1 = \mu_{1,4} - \mu_{2,4}, \quad a_2 = \mu_{1,4} - \mu_{3,4}, \quad a_3 = \mu_{1,4} + \lambda_{4,3} + \lambda_{4,2} + \lambda_{4,1},$$
$$b_1 = \mu_{1,3} - \mu_{2,3}, \quad b_2 = \mu_{1,3} + \lambda_{3,2} + \lambda_{3,1} + \mu_{3,4}, \quad b_3 = \mu_{1,3} - \lambda_{4,3},$$
$$c_1 = \mu_{1,2} + \lambda_{2,1} + \mu_{2,3} + \mu_{2,4}, \quad c_2 = \mu_{1,2} - \lambda_{3,2}, \quad c_3 = \mu_{1,2} - \lambda_{4,2}.$$

Corresponding state frequencies are obtained according to (2.30):

$$f_1 = p_1(\mu_{1,2} + \mu_{1,3} + \mu_{1,4}),$$
$$f_2 = p_2(\mu_{2,4} + \mu_{2,3} + \lambda_{2,1}),$$
$$f_3 = p_3(\mu_{3,4} + \lambda_{3,2} + \lambda_{3,1}),$$
$$f_4 = p_4(\lambda_{4,3} + \lambda_{4,2} + \lambda_{4,1}).$$

The steady state availability of the element for constant demand w=60 MW is

$$A = p_4 + p_3,$$

the mean steady state capacity is

$$E_\infty = \sum_{k=1}^{4} g_k p_k = 100p_4 + 80p_3 + 50p_2 + 0p_1,$$

and the mean steady state capacity deficiency is

$$D_\infty = \sum_{k=1}^{4} p_k \max(w - g_k, 0) = 10p_2 + 60p_1.$$

As can be seen in Fig. 2.3 and Fig. 2.4, the steady state values of the state probabilities are achieved during a short time. After 0.07 years, the process becomes stationary. Due to this consideration, only the final solution is important in many practical cases. This is especially so for elements with a relatively long lifetime. This is the case in our example if the unit lifetime is at least several years. However, if one deals with highly responsible components and takes into account even small energy losses at the beginning of the process, the analysis based on a system of differential equations should be performed.

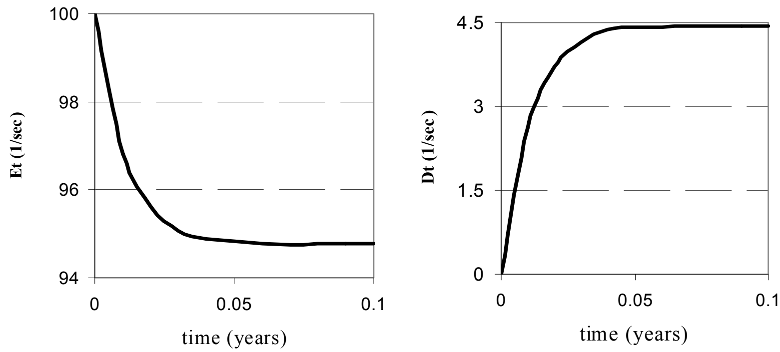


Fig. 2.4. Instantaneous mean capacity and capacity deficiency of the four-state generating unit

Example 2.2

This example presents the development of Markov model for typical generation unit based on historical failure data (Goldner and Lisnianski 2006).

The model representing a commercial 360-megawatt coal fired generation unit that was designed to incorporate the major unit's derated/outage states derived from the analysis of the historical failure data (unit's failures in the period from 1985 to 2003). The methodology of the data acquisition is based on NERC-GADS Data Reporting Instruction (North American Electric Reliability Council, 2003, *Generating Availability Data System. Data Reporting Instructions.* Princeton, NJ.)

The total of 1498 recorded events are classified as planned or unplanned outages, scheduled or forced deratings (each derating is reported independently and relates to capacity reduction from a nominal level). Based on the data analysis ten space states were identified as most significant. Fig. 2.5 presents the distribution of the events as a function of available capacity remaining after the event.

Table 2.1 provides a summary of failure statistics and reliability parameters for the unit. Each event that leads to capacity derating is treated as a failure. For example, there were 111 events (failures) that lead to capacity derating from nominal capacity E_n=360 MW down to capacity levels ranging from 290 MW up to 310 MW. An average derated capacity level for this kind of failure is G_8=303 MW. The mean time up to this kind of failures is $MTTF_{10,8}$=1,454 hours. Mean time up to unit repair and return to the state with nominal generating capacity is $MTTR_{8,10}$ = 6.7 hours.

It is assumed that the time to repair (TTR) and time to failure (TTF) are distributed exponentially. So, MTTF and MTTR define the corresponding transition intensities according to expressions (2.19) and (2.20).

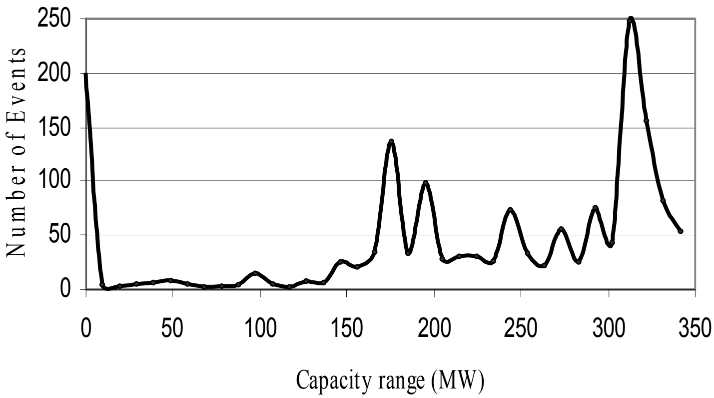


Fig. 2.5. Distribution of all types of failure events

The state-space diagram for the coal fired generation unit is presented in Fig. 2.6. According to Table 2.1, the diagram has 10 different states, ranging from state 10 with nominal generating capacity of 360 MW up to state 1 (complete failure with a capacity of 0 MW).

Table 2.1. Summary of failure parameters

Level number i	Average capacity level G_i (MW)	Capacity range (MW)	Number of failures	$MTTR_{i,10}$ (hr)	$MTTF_{10,i}$ (hr)
1	0	0	189	93.5	749
2	124	(0, 170]	134	4.3	1,219
3	181	(170, 190]	160	7.7	1,022
4	204	(190, 220]	147	6.4	1,120
5	233	(220, 240]	52	3.9	3.187
6	255	(240, 270]	120	7.8	1,389
7	282	(270, 290]	75	6.0	2,221
8	303	(290, 310]	111	6.7	1,464
9	328	(310, 360)	510	6.9	311

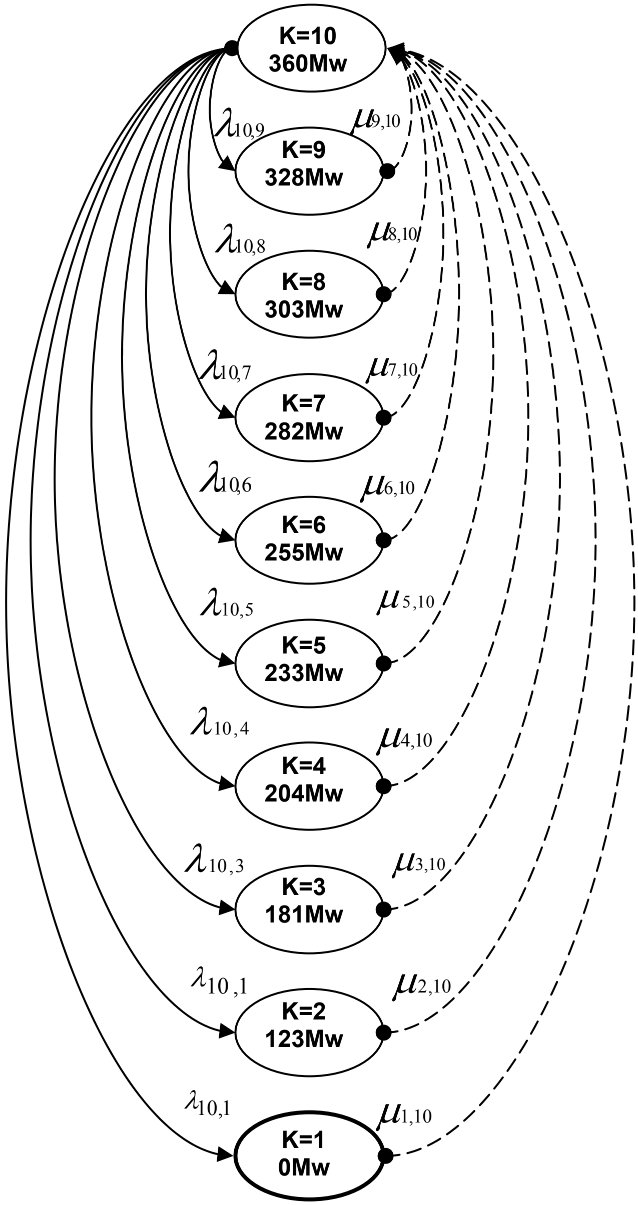


Fig. 2.6. State-space diagram for the coal fired generation unit

We designate an intensity of transition from state 10 (with nominal generating capacity) to any derated state *i*=1,2,…,9 as $\lambda_{10,i} = 1/MTTF_{10,i}$. In the same way we designate an intensity of transition from state i to state 10 as $\mu_{i,10} = 1/MTTR_{i,10}$.

According to the state-space diagram in Fig. 2.6 and transition parameters from Table 2.1, the corresponding system of differential equations for the state probabilities $p_k(t)$, k=1,….10 takes the form:

$$\frac{dp_k(t)}{dt} = -\mu_{k,10} * p_k(t) + \lambda_{10,k} * p_{10}(t) \text{ for } k=1,\dots,9$$

$$\frac{dp_{10}(t)}{dt} = \sum_{k=1}^{9} \mu_{k,10} * p_k(t) - \sum_{k=1}^{10} \lambda_{10,k} * p_{10}(t)$$

Solving this system under initial conditions $p_{10}(0)=1$; $p_9(0)=\ldots=p_1(0)=0$ one obtains the state probabilities.

For example, for t=1month=744 hours: $p_{10}(744)=0.854$, $p_9(744)=0.020$, $p_8(744)=0.004$.

The element instantaneous availability can be obtained for different constant demand levels as the sum of probabilities of acceptable states, where the generating capacity is greater than or equal to the demand level. For the demand lower than or equal to capacity level in state k we obtain:

$$A_k(t) = \sum_{e=k}^{10} p_e(t)$$

The corresponding graphs $A_k(t)$ for k=9, k=7, and k=8 are presented in Fig. 2.7.

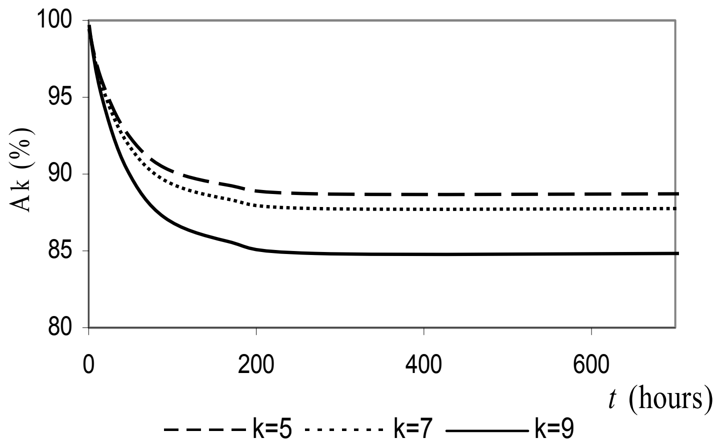


Fig. 2.7. Availability of generation unit for different demand levels

The unit mean instantaneous performance at time t is

$$E(t) = \sum_{k=1}^{10} G_k P_k(t)$$

The graph of the function $E(t)$ (in percents of the nominal unit capacity) is presented in Fig. 2.8.

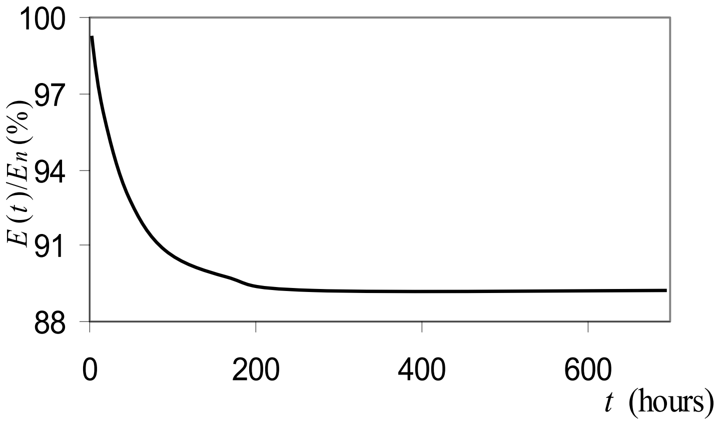


Fig. 2.8. Instantaneous mean generating capacity of generation unit

Based on the derated Markov state we obtained the real availability and mean performance indices for typical coal fired unit. This model allows analyst to compare performances indices for different generation units based on their failure history data, which is important for the units' availability improvement and for system planning support.

2.1.3 Markov reward models

Basic definition and model description

In the preceding subchapters, it was shown how some important reliability indices can be found by using the Markov technique. Here we consider additional indices such as states frequencies and the mean number of system failures during an operating period. We demonstrate the method for their computation, which is based on the general Markov reward model that was primarily introduced by Howard (1962) and then was essentially extended in (Mine and Osaki 1970) and many other research works.

This model considers the continuous-time Markov chain with a set of states $\{1,\ldots,K\}$ and transition intensity matrix $\boldsymbol{a}=|a_{ij}|$, $i,j=1,\ldots,K$. It is suggested that if the process stays in any state i during the time unit, a certain cost r_{ii} should be paid. It is also suggested that each time that the process transits from state i to state j a cost r_{ij} should be paid. These costs r_{ii} and r_{ij} are called *rewards* (the reward may also be negative when it characterizes losses or penalties). The Markov process with the rewards associated with its states or/and transitions is called the Markov process with rewards. For these processes, an additional matrix $\boldsymbol{r}=|r_{ij}|$, $i,j=1,\ldots,K$ of rewards is determined. If all rewards are zeroes, the process reduces to the ordinary Markov process.

Note that the rewards r_{ii} and r_{ij} have different dimensions. For example, if r_{ij} is measured in cost units, the reward r_{ii} is measured in cost units per time unit. The value that is of interest is the total expected reward accumulated up to time instant t under specified initial conditions.

Let $V_i(t)$ be the total expected reward accumulated up to time t, given the initial state of the process at time instant t=0 is state i. According to Howard, the following system of differential equations must be solved under specified initial conditions in order to find the total expected rewards:

$$\frac{dV_i(t)}{dt} = r_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{K} a_{ij} r_{ij} + \sum_{j=1}^{K} a_{ij} V_j(t)\,,\ i=1,\ldots,K. \tag{2.31}$$

The system (2.31) can be obtained in the following manner. Assume that at time instant t=0 the process is in the state i. During the time increment Δt, the process can remain in this state or transit to some other state j. If it remains in state i during time Δt, the expected reward accumulated during this time is $r_{ii}\Delta t$. Since at the beginning of the time interval $[\Delta t, \Delta t + t]$ the process is still in state i, the expected reward during this interval is $V_i(t)$ and the expected reward during the entire interval $[0, \Delta t + t]$ is $V_i(\Delta t + t) = r_{ii}\Delta t + V_i(t)$. The probability that the process will remain in state i during the time interval Δt equals to 1 minus the probability that it will transit to any other state $j \neq i$ during this interval:

$$\pi_{ii}(0, \Delta t) = 1 - \sum_{\substack{j=1 \\ j\neq i}}^{K} a_{ij}\Delta t = 1 + a_{ii}\Delta t\,. \tag{2.32}$$

On the other hand, during time Δt the process can transit to some other state $j \neq i$ with the probability $\pi_{ij}(0, \Delta t) = a_{ij}\Delta t$. In this case the expected reward accumulated during the time interval $[0, \Delta t]$ is r_{ij}. At the beginning of the time interval $[\Delta t, \Delta t + t]$ the process is in the state j. Therefore, the expected reward during this interval is $V_j(t)$ and the expected reward during the interval $[0, \Delta t+t]$ is $V_i(\Delta t+t)=r_{ij}+V_j(t)$.

In order to obtain the total expected reward one must summarize the products of rewards and corresponding probabilities for all of the states. Thus, for the small Δt one has

$$V_i(\Delta t+t) \approx (1+a_{ii}\Delta t)[r_{ii}\Delta t + V_i(t)] + \sum_{\substack{j=1 \\ j\neq i}}^{K} a_{ij}\Delta t[r_{ij} + V_j(t)], \tag{2.33}$$

for $i=1,\ldots,K$.

Neglecting the terms with an order greater than Δt one can rewrite the last expression as follows:

$$\frac{V_i(\Delta t+t)-V_i(t)}{\Delta t} = r_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{K} a_{ij}r_{ij} + \sum_{j=1}^{K} a_{ij}V_j(t), \text{ for } i=1,\ldots,K. \tag{2.34}$$

Passing to the limit in this equation gives (2.31).

Defining the vector-column of total expected rewards $\boldsymbol{V}(t)$ with components $V_1(t),\ldots,V_K(t)$ and vector-column $\boldsymbol{u}$ with components

$$u_i = r_{ii} + \sum_{\substack{j\neq i \\ j=1}}^{K} a_{ij}r_{ij}, \; i=1,\ldots,K \tag{2. 35}$$

one obtains the equation (2.31) in matrix notation:

$$\frac{d}{dt}\boldsymbol{V(t)}=\boldsymbol{u}+\boldsymbol{aV(t)}. \tag{2.36}$$

Usually the system (2.31) should be solved under initial conditions $V_i(0)=0$, $i=1,\ldots,K$.

In order to find the long-run (steady state) solution of (2.31) the following system of algebraic equations must be solved

$$\boldsymbol{0}=\boldsymbol{u}+\boldsymbol{aV(t)}, \tag{2.37}$$

where $\boldsymbol{0}$ is vector-column with zero components.

Example 2.3

As an example of straightforward application of the method we consider a power generator with the nominal capacity $L=10^5$ KW where the generator only has complete failures with a failure rate of $\lambda = 1\ year^{-1}$. The unsupplied energy penalty is c_p=3 \$ per KW*hour. After the generator failure, a repair is performed with a repair rate of $\mu = 200\ year^{-1}$. The mean cost of repair is $c_r = 50000$ \$.

The problem is to evaluate a total expected cost C_T associated with the unreliability of the generator during the time interval [0, T].

The state-space diagram for the power generating unit is presented in Fig. 2.9. It has only two states: perfect functioning with a nominal generating capacity (state 2) and complete failure where the unit generating capacity is zero (state 1). The transitions from state 2 to state 1 are associated with failures and have intensity λ. If the generator is in state 1, the penalty cost c_pL should be paid for each time unit (hour). Hence, the reward r_{11} associated with state 1 is r_{11}=c_pL.

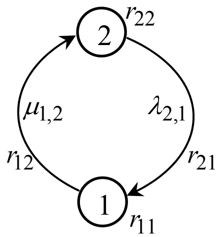


Fig. 2.9. Markov reward model for power generating unit

The transitions from state 1 to state 2 are associated with repairs and have an intensity of μ. The repair cost is c_r, therefore the reward associated with the transition from state 1 to state 2 is r_{12}=c_r.

There are no rewards associated with the transition from state 2 to state 1 and with remaining in the state 2: $r_{22} = r_{21} = 0$.

The reward matrix takes the form

$$\boldsymbol{r} = |\, r_{ij} \,| = \begin{vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{vmatrix} = \begin{vmatrix} c_p L & c_r \\ 0 & 0 \end{vmatrix},$$

and the transition intensity matrix takes the form

$$\boldsymbol{a} = |\, a_{ij} \,| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} -\mu & \mu \\ \lambda & -\lambda \end{vmatrix}.$$

Using (2.31) the following system of differential equations can be written in order to find the expected total rewards $V_1(t)$ and $V_2(t)$:

$$\begin{cases} \dfrac{dV_1(t)}{dt} = c_p L + \mu c_r - \mu V_1(t) + \mu V_2(t) \\ \dfrac{dV_2(t)}{dt} = \lambda V_1(t) - \lambda V_2(t) \end{cases} .$$

The total expected cost in time interval [0, t] associated with the unreliability of the generator is equal to the expected reward $V_2(t)$ accumulated up to time t, given the initial state of the process at time instant t=0 is state 2.

Using the Laplace-Stieltjes transform under the initial conditions $V_1(0) = V_2(0) = 0$, we transform the system of differential equations to the following system of linear algebraic equations

$$\begin{cases} s v_1(s) = \dfrac{c_p L + \mu c_r}{s} - \mu v_1(s) + \mu v_2(s) \\ s v_2(s) = \lambda v_1(s) - \lambda v_2(s) \end{cases}$$

Where $v_k(s)$ is the Laplace-Stieltjes transform of a function $V_k(t)$.

The solution of this system is

$$v_2(s) = \frac{\lambda c_p L + \lambda \mu c_r}{s^2 (s + \lambda + \mu)} .$$

After applying the inverse Laplace-Stieltjes transform we obtain

$$V_2(t) = L^{-1}\{v_2(s)\} = \frac{\lambda c_p L + \lambda \mu c_r}{(\mu + \lambda)^2} [e^{-(\lambda+\mu)t} + (\mu + \lambda)t - 1] .$$

The total expected cost C_T during the operation time T is

$$C_T = V_2(T) = \frac{\lambda c_p L + \lambda \mu c_r}{(\mu + \lambda)^2} [e^{-(\lambda+\mu)T} + (\mu + \lambda)T - 1] .$$

For relatively large T the term $e^{-(\lambda+\mu)T}$ can be neglected and the following approximation can be used

$$C_T \approx \frac{\lambda (c_p L + \mu c_r)}{\mu + \lambda} T .$$

Therefore, for large T, the total expected reward is a linear function of time and the coefficient

$$c_{un} = \frac{\lambda(c_p L + \mu c_r)}{\mu + \lambda}$$

defines the annual expected cost associated with generating unit unreliability. For the data given in the example, $c_{un} = 13.14 \cdot 10^6$ \$/year.

Computation of power system reliability measures by using Markov reward models

In its general form the Markov reward model was intended to provide economical and financial calculations. However, it was shown by Volik et al. (1988) that some important reliability measures could be found by the corresponding determination of the rewards in matrix $\boldsymbol{r}$. The method was extended by Lisnianski (2007a) to systems with variable demand levels. Here we apply this method for computation of such important power system reliability measures as the mean number of failures and failure states frequencies, mean time to failure and expected energy not supplied to consumers.

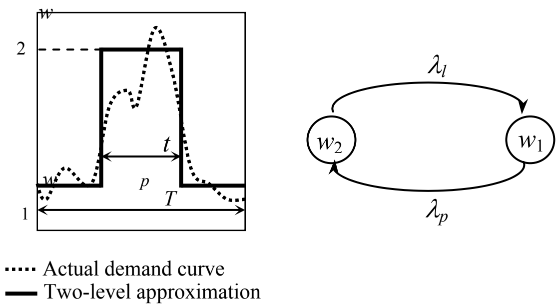
In the previous section, the power system was considered to have constant demand. In practice, it is often not so. A power system can fall into a set of unacceptable states in two ways: either through capacity derating because of failures or through an increase in demand (load).

For example, consider the demand variation that is typical for power systems. Usually demand can be represented by a daily demand curve. This curve is cyclic in nature with a maximum level (peak) during the day and a minimum level at night (Endrenyi 1979), (Billinton and Allan 1996). In the simplest and most frequently used model, the cyclic demand variation can be approximated by a two-level demand curve as shown in Fig. 2.10A.

In this model, the demand is represented as a continuous time Markov chain with two states: $\boldsymbol{w}=\{w_1, w_2\}$ (Fig. 2.10B). When the cycle time T_c and the mean duration of the peak t_p are known (usually T_c=24 hours), the transition intensities of the model can be obtained as

$$\lambda_p = \frac{1}{T_c - t_p} \quad , \; \lambda_l = \frac{1}{t_p}. \tag{2.38}$$

In the further extension of the variable demand model the demand process can be approximated by defining a set of discrete values $\{w_1, w_2, \ldots, w_m\}$ representing different possible demand levels and determining the transition intensities between each pair of demand levels (usually derived from the demand statistics). The realization of the stochastic process of the demand for a specified period and the corresponding state-space diagram are shown in Fig. 2.11. b_{ij} is the transition intensity from demand level w_i to demand level w_j.



A **B**

Fig. 2.10. Two-level demand model (A. Approximation of actual demand curve; B. State-space diagram)

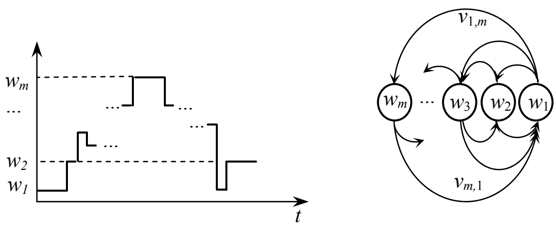


Fig. 2.11. Discrete variable demand (A. Realization of stochastic demand process, B. Space-state diagram).

So, for a general case we assume that demand $W(t)$ is also a random process that can take on discrete values from the set $\boldsymbol{w}=\{w_1,\dots,w_M\}$. The desired relation between the power system capacity and the demand at any time instant t can be expressed by the acceptability function $\Phi(G(t),W(t))$. The acceptable system states correspond to $\Phi(G(t),W(t))\geq 0$ and the unacceptable states correspond to $\Phi(G(t),W(t))<0$. The last inequality defines the system failure criterion. Usually in power systems, the system generating capacity should be equal to or exceed the demand. Therefore, in such cases the acceptability function takes on the following form:

$$\Phi(G(t),W(t))=G(t)-W(t), \tag{2.39}$$

and the criterion of state acceptability can be expressed as

$$\Phi(G(t), W(t)) = G(t) - W(t) \geq 0 . \tag{2.40}$$

Below we present a general method, which proved to be very useful for the computation of power system reliability measures when output capacity and demand (load) are independent discrete-state continuous-time Markov processes.

Combined capacity-demand model

Power system output generating capacity is represented by a stochastic process $G(t)$ that is described as a continuous-time Markov chain Ch1 with K different possible states $g_1, ..., g_K$ and corresponding transition intensities matrix $\boldsymbol{a}=\left|a_{ij}\right|$, $i,j=1,2, ..., K$. Therefore, Ch1 is a mathematical model for generating capacity stochastic process $G(t)$. It is graphically represented in Fig. 2.12, where system output capacities for each state are presented inside the ellipses and the state number is presented near the corresponding ellipse. Transition intensities are presented near the arcs connecting the corresponding states. The state with the largest performance g_K is the best state and all the states are ordered according to their capacity, so that $g_K > g_{K-1} > ... > g_1$.

The demand process $W(t)$ is also modeled as a continuous-time Markov chain Ch2 with m different possible states $w_1, ..., w_m$ and corresponding constant transition intensities with the matrix $\boldsymbol{b}=\left|b_{ij}\right|$, $i,j=1,2,...,m$. Ch2 is a mathematical model for the demand stochastic process $W(t)$ and it is graphically presented in Fig. 2.13. The demand levels for each state are presented inside the ellipses. As in the previous case, the state number is presented near the corresponding ellipse and transition intensities are presented near the corresponding arcs (connecting corresponding states). The state m is the state with the largest demand and all states are ordered according to their demand levels, so that $w_m > w_{m-1} > ... > w_1$.

The capacity and demand models can be combined based on the independence of events in these two models. The probabilities of transitions in each model are not affected by the events that occur in another one. The state-space diagram for the combined m-state demand model and K-state output capacity model is shown in Fig. 2.14. Each state in the diagram is labeled by two indices indicating the demand level $w \in \{w_1,..., w_m\}$ and the element performance rate $g \in \{g_1, g_2,..., g_K\}$.

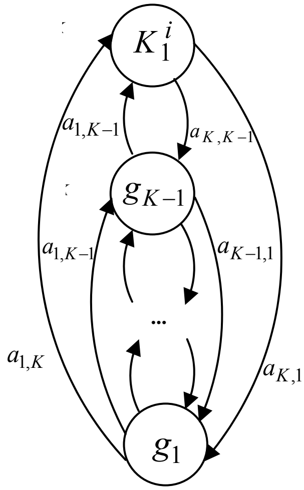


Fig. 2.12. Markov model for power system output performance

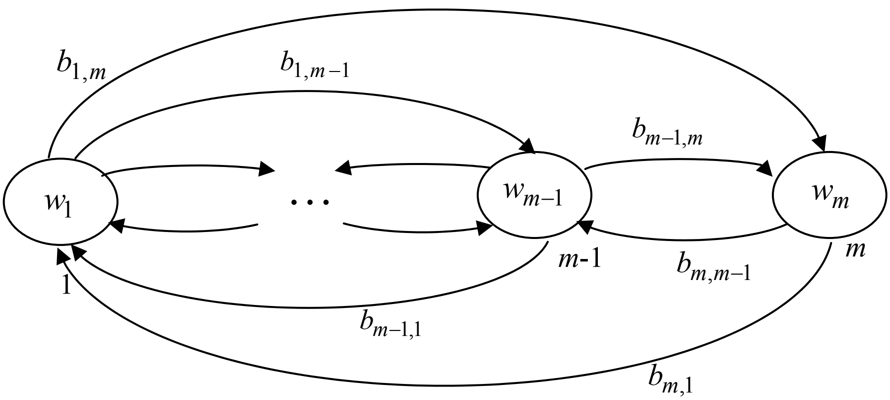


Fig. 2.13. Markov model for power system demand

These indices for each state are presented in the lower part of the corresponding ellipse. The combined model is considered to have *mK* states. Each state corresponds to a unique combination of demand levels w_i and element performance g_j and is numbered according to the following rule:

$$z=(i-1)K+j, \tag{2.41}$$

where *z* is a state number in the combined capacity-demand model, *n*=1, … , *mK*; *I* is a number of demand level, *i*=1,…*m*; *j* is a number of MSS output performance level, *j*=1,…*K*.

In order to designate that state z in a combined performance-demand model corresponds to demand level w_i and performance g_j we use the form $z \sim \{w_i, \ g_j\}$.

In Fig. 2.14 the number of each state is shown in the upper part of the corresponding ellipse.

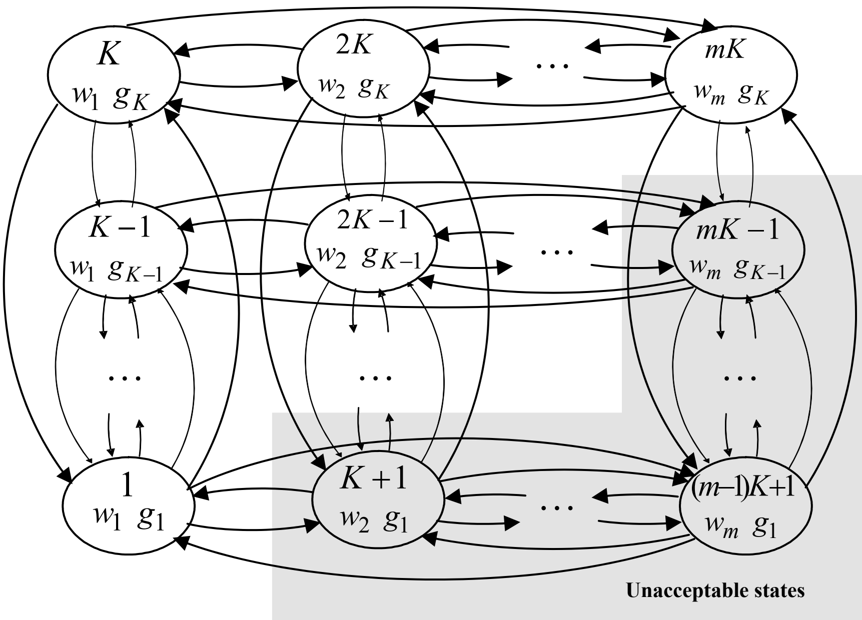


Fig. 2.14. Combined capacity–demand model

In addition to transitions between the states with different capacities, there are transitions between the states with the same capacities, but with different demand levels. All intensities of horizontal transitions are defined by transition intensities $b_{i,j}$, i,j=1,…m of the Markov demand model Ch2, and all intensities of vertical transitions are defined by transition intensities $a_{i,j}$, i,j=1,…K of the performance model Ch1. All other (diagonal) transitions are forbidden. We designate the transition intensities matrix for combined capacity-demand model as $\boldsymbol{c}=\left|c_{ij}\right|$, where i,j=1,2,…,mK .

Thus, the algorithm of the combined capacity-demand model building based on separate capacity and demand models Ch1 and Ch2 can be presented by the following steps.

1. The state-space diagram of combined capacity-demand model is shown in Fig. 2.14, where the nodes represent power system states and the arcs represent corresponding transitions.
2. The graph consists of mK nodes that should be ordered in K rows and m columns.
3. Each state (node) should be numbered according to the rule (2.41).
4. All intensities c_{z_1,z_2} of horizontal transitions from state z_1 (corresponding to demand w_i and capacity g_j) to state z_2 (corresponding to demand w_s and the same capcity g_j according to the rule (2.41)) are defined by demand transition intensities matrix $\boldsymbol{b}$,

$$c_{z_1,z_2} = b_{i,s}, \tag{2.42}$$

   where $z_1 \sim \{w_i, \ g_j\}$, $z_2 \sim \{w_s, \ g_j\}$, $i,s = \overline{1,m}$, $j = \overline{1,K}$.
5. All intensities of vertical transitions from state z_1 (corresponding to demand w_i and performance g_j) to state z_3 (corresponding to the same demand w_i and capacity g_t according to the rule (2.41) are defined by the capacity transition intensities matrix $\boldsymbol{a}$,

$$c_{z_1,z_3} = a_{j,t}, \tag{2.43}$$

   where $z_1 \sim \{w_i, \ g_j\}$, $z_3 \sim \{w_i, \ g_t\}$, $i = \overline{1,m}$, $j,t = \overline{1,K}$.
6. All diagonal transitions are forbidden so that the corresponding transitions' intensities in matrix $\boldsymbol{c}$ are zeroed.

Rewards determination for combined capacity-demand model

In the previous section we have built the combined capacity-demand model and therefore, defined its transition intensities matrix $\boldsymbol{c}$ based on matrixes $\boldsymbol{a}$ and $\boldsymbol{b}$ for capacity and demand processes. In order to find reliability measures for a power system the specific reward matrix $\boldsymbol{r}$ should be defined for each measure. At first, all states in the model can be subdivided by two different sets – the set of acceptable states and the set of unacceptable states, according to the value of the acceptability function. The example of such model is presented in the Fig. 2.15. The transition intensity for each transition is determined by the matrix $\boldsymbol{c}$. For each state i determined reward r_{ii} during time unit, and for each transition from state i to state j the reward r_{ij} associated with this transition is determined. Rewards corresponding to states are written inside of states cycles. Rewards corresponding to transitions are written after corresponding transitions intensities.

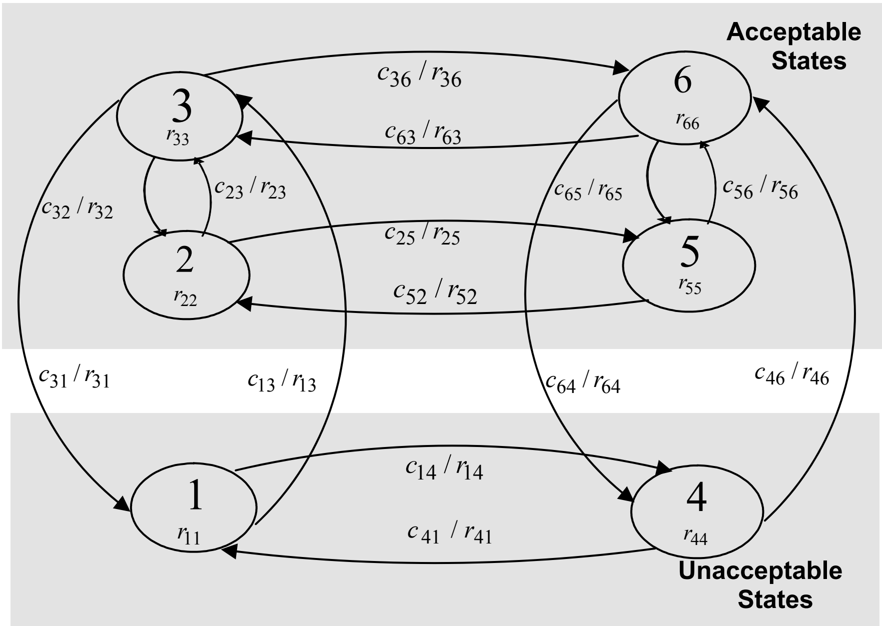


Fig. 2.15. Combined capacity–demand model with rewards

The power system average availability $\overline{A}(T)$ is defined as a mean fraction of time when the system resides in the set of acceptable states during the time interval [0, *T*],

$$\overline{A}(T) = \frac{1}{T}\int_0^T A(t)dt\,. \tag{2.44}$$

In order to prevent misunderstanding, it should be noticed that there is another measure that is closely related to the system availability - the power system instantaneous (point) availability $A(t)$. It is the probability that the system at instant t>0 is in one of the acceptable states:

$$A(t) = \Pr\{\Phi(G(t), W(t)) \geq 0\}\,. \tag{2.45}$$

As was shown in the previous section, $A(t)$ can be found by solving the differential equations (2.14) and summarizing probabilities corresponding to all acceptable states.

To assess average availability $\overline{A}(T)$ for a power system the Markov reward model can be used. The rewards in matrix $\boldsymbol{r}$ for the combined capacity-demand model can be determined in the following manner:

- The rewards associated with all acceptable states should be defined as 1.
- The rewards associated with all unacceptable states should be zeroed as well as all the rewards associated with the transitions.

For the example in Fig. 2.11 this means that $r_{22}=r_{33}=r_{55}=r_{66}=1$. All other rewards are zeroes.

The mean reward $V_i(T)$ accumulated during interval [0, T] defines a part of time that the power system will be in the set of acceptable states in the case where state i is the initial state. This reward should be found as a solution of the general system (2.31). That for the combined capacity – demand model would be the following:

$$\frac{dV_i(t)}{dt} = r_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{mK} c_{ij} r_{ij} + \sum_{j=1}^{mK} c_{ij} V_j(t), \; i=1,...,mK. \qquad (2.46)$$

After solving the (2.46) and finding $V_i(t)$, power system average availability can be obtained for every different initial state $i = 1,..., K$:

$$\overline{A}_i(T) = \frac{V_i(T)}{T}. \qquad (2.47)$$

Usually the state K (with greatest capacity level and minimum demand) is determined as an initial state.

Mean number $N_{fi}(T)$ of power system failures during the time interval [*0, T*], *if state i is the initial state.* This measure can be treated as a mean number of power system entrances into the set of unacceptable states during the time interval [0, T]. For its computation, the rewards associated with each transition from the set of acceptable states to the set of unacceptable states should be defined as 1. All other rewards should be zeroed.

For the example in Fig. 2.15 it means that $r_{31}=r_{21}=r_{64}=r_{54}=1$. All other rewards are zeroes.

In this case the mean accumulated reward $V_i(T)$, obtained by solving (2.31) provides the mean number of entrances into the unacceptable area during the time interval [*0, T*]:

$$N_{fi}(T) = V_i(T). \qquad (2.48)$$

When the mean number of system failures is computed, the corresponding *frequency of failures or frequency of entrances into the set of unacceptable states* can be found

$$f_{fi}(T) = \frac{1}{N_{fi}(T)} \tag{2.49}$$

Expected Energy Not Supplied (EENS) to consumers can be defined as mean capacity deficiency accumulated within interval [0, T]. The rewards for any state number $z=(i\text{-}1)K+j$, in a combined model, where $w_j\text{-}g_i>0$, should be defined as $r_{nn}=w_j\text{-}g_i$. All other rewards should be zeroed. Therefore, the mean reward $V_i(T)$ accumulated during the time interval [0, T], if the state i is in the initial state, defines the mean accumulated capacity deficiency or EENS:

$$EENS_i = V_i(T) = E\{\int_0^T (W(t) - G(t))dt\}\ . \tag{2.50}$$

Mean Time To Failure (MTTF) is the mean time up to the instant when the system enters the subset of unacceptable states for the first time. For its computation the combined performance-demand model should be transformed - all transitions that return the power system from an unacceptable states should be forbidden, as in this case all unacceptable states should be treated as absorbing states. For the example in Fig. 2.15 it means that $c_{13}=c_{46}=0$.

In order to assess MTTF for a power system, the rewards in matrix ***r*** for the transformed performance-demand model should be determined as follows:

- The rewards associated with all acceptable states should be defined as 1.
- The reward associated with unacceptable (absorbing) states should be zeroed as well as all rewards associated with transitions.

For the example in Fig. 2.15 it means that $r_{22}=r_{33}=r_{55}=r_{66}=1$. All other rewards are zeroes.

In this case, the mean accumulated reward $V_i(t)$ defines the mean time accumulated up to the first entrance into the subset of unacceptable states (MTTF), if the state i is the initial state.

Probability of power system failure during the time interval [*0, T*]. The combined capacity-demand model should be transformed as in the previous case – all unacceptable states should be treated as absorbing states and, therefore, all transitions that return the system from unacceptable states should be forbidden. As in the previous case for the example in Fig. 2.15, $c_{13}=c_{46}=0$.

Rewards associated with all transitions to the absorbing state should be defined as 1. All other rewards should be zeroed. For the example in Fig. 2.15 it means that $r_{22}=r_{33}=r_{55}=r_{66}=1$. All other rewards are zeroes.

The mean accumulated reward $V_i(T)$ in this case defines the probability of system failure during the time interval [0, T], if the state i is the initial state. Therefore, the power system reliability function can be obtained as:

$$R_i(T) = 1 - V_i(T), \text{ where } i=1,\ldots,K. \tag{2.51}$$

Example 2.4

Consider reliability evaluation for a power system, the output generating capacity of which is represented by a continuous time Markov chain with 3 states. Corresponding capacity levels for states 1, 2, 3 are $g_1 = 0,\ g_2 = 70,\ g_3 = 100$ respectively and the transition intensities matrix is such as the following:

$$\boldsymbol{a} = |a_{ij}| = \begin{vmatrix} -500 & 0 & 500 \\ 0 & -1000 & 1000 \\ 1 & 10 & -11 \end{vmatrix}.$$

All intensities a_{ij} are represented in such units as 1/year.

The corresponding capacity model Ch1 is graphically shown in Fig. 2.17A.

The demand for the power system is also represented by a continuous time Markov chain with three possible levels w_1=0, w_2=60, w_3=90. This demand is graphically shown in Fig. 2.16.

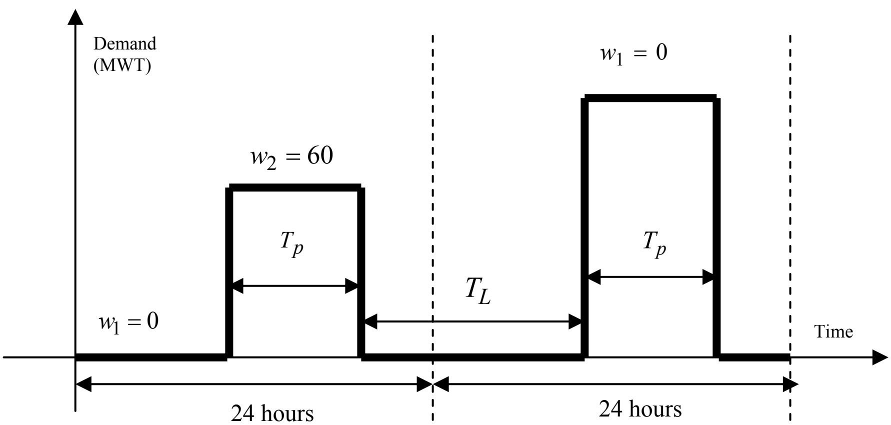


Fig. 2.16. Daily demand for the power system

Daily peaks w_2 and w_3 occur twice a week and five times a week respectively and the mean duration of the daily peak is T_p=8 hours. Mean duration of low demand level $w_1 = 0$ is defined as $T_L = 24 - 8 = 16$ hours.

According to the approach presented in (Endrenyi 1979) that is justified for a power system, peak duration and low level duration are assumed to be exponentially distributed random values.

Markov demand model Ch2 is shown in Fig. 2.13B. States 1, 2 and 3 represent corresponding demand levels w_1, w_2 and w_3. Transition intensities are such as follows:

$$b_{21} = b_{31} = \frac{1}{T_p} = \frac{1}{8} \quad hours^{-1} = 1110 \quad years^{-1},$$

$$b_{12} = \frac{2}{7}\frac{1}{T_L} = \frac{2}{7}\frac{1}{16} = 0.0179 \quad hours^{-1} = 156 \quad years^{-1},$$

$$b_{13} = \frac{5}{7}\frac{1}{T_L} = \frac{5}{7}\frac{1}{16} = 0.0446 \quad hours^{-1} = 391 \quad years^{-1}.$$

There are no transitions between states 2 and 3, therefore

$$b_{23} = b32 = 0.$$

Taking into account the sum of elements in each row of the matrix to be zero, we can find the diagonal elements in the matrix.

Therefore, a transition intensities matrix $\boldsymbol{b}$ for the demand takes the form:

$$\boldsymbol{b} = \left| b_{ij} \right| = \begin{vmatrix} -547 & 156 & 391 \\ 1110 & -1110 & 0 \\ 1110 & 0 & -1110 \end{vmatrix}.$$

All intensities b_{ij} are also represented in 1/year.

The acceptability function is given: $\Phi(G(t), W(t)) = G(t) - W(t)$. Therefore, a failure is treated as an entrance in the state where the acceptability function is negative or $G(t) < W(t)$.

By using the suggested method we find the *mean number $N_f(T)$ of system failures during the time interval* $[0,T]$, if the state with maximal generating capacity and minimal demand level is given as the initial state.

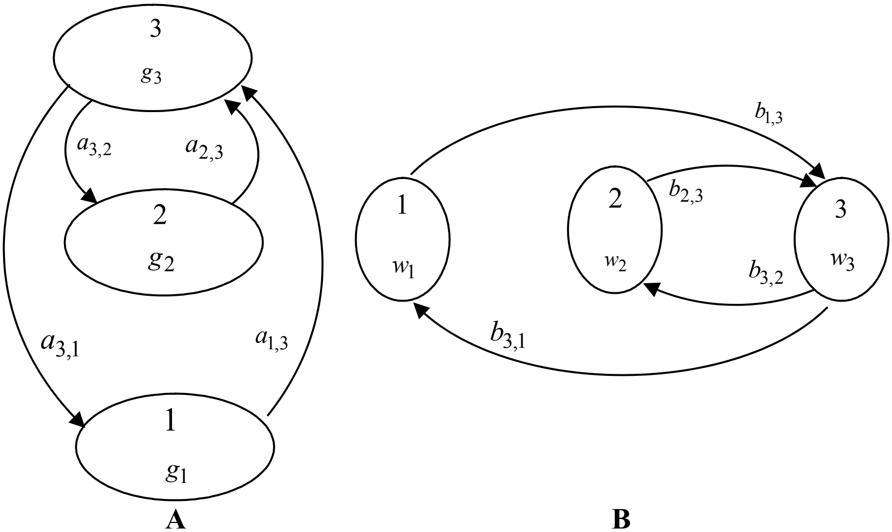


Fig. 2.17. Output capacity model (A) and demand model (B)

First, the combined capacity-demand model should be built according to the algorithm presented above.

The model consists of *mK*=3*3=9 states (nodes) that should be ordered in *K*=3 rows and *m*=3 columns.

Each state should be numbered according to the rule (2.41).

All intensities of horizontal transitions from state $z_1 \sim \{w_i,\ g_j\}$ to state $z_2 \sim \{w_s,\ g_j\}$, $i,s=\overline{1,3}$, $j=\overline{1,3}$ are defined by demand transition intensities matrix ***b***,

$$c_{z_1 z_2} = b_{i,s}\,.$$

All intensities of vertical transitions from state $z_1 \sim \{w_i,\ g_j\}$ to state $z_3 \sim \{w_i,\ g_t\}$, $i=\overline{1,3}$, $j,t=\overline{1,3}$ are defined by the capacity transition intensities matrix ***a***,

$$c_{z_1 z_3} = a_{j,t}\,.$$

All diagonal transitions are forbidden; therefore, the corresponding transition intensities in matrix ***c*** are zeroed.

The state-space diagram for the *combined capacity-demand Markov* model for this example is shown in Fig. 2.18.

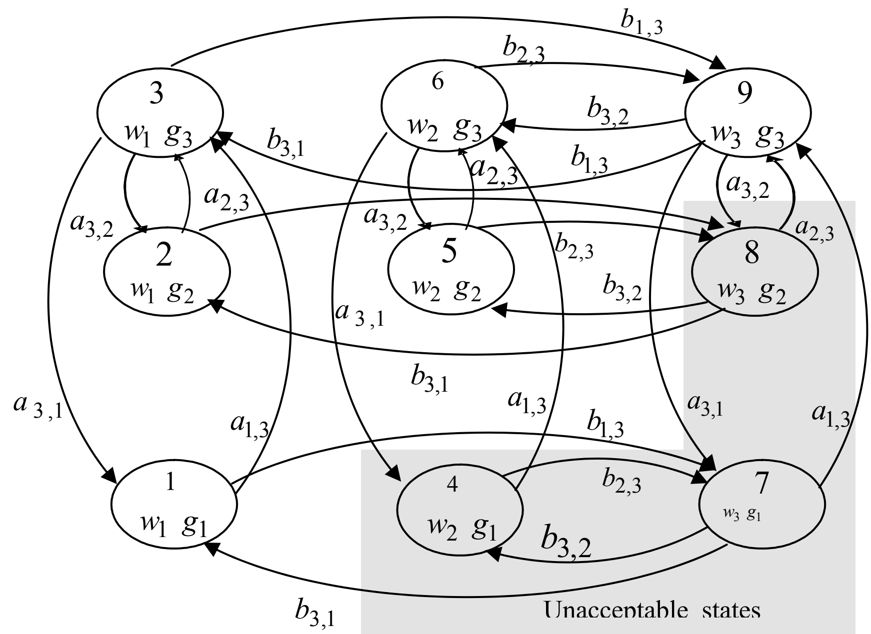


Fig. 2.18. Combined capacity-demand model

Corresponding transition intensity matrix $\boldsymbol{c}$ for the combined capacity-demand model can be written as follows:

$$\boldsymbol{c}=\left|c_{ij}\right|=\begin{vmatrix} x_1 & 0 & a_{13} & b_{12} & 0 & 0 & b_{13} & 0 & 0 \\ 0 & x_2 & a_{23} & 0 & b_{12} & 0 & 0 & b_{13} & 0 \\ a_{31} & a_{32} & x_3 & 0 & 0 & b_{12} & 0 & 0 & b_{13} \\ b_{21} & 0 & 0 & x_4 & 0 & a_{13} & 0 & 0 & 0 \\ 0 & b_{21} & 0 & 0 & x_5 & a_{23} & 0 & 0 & 0 \\ 0 & 0 & b_{21} & a_{31} & a_{32} & x_6 & 0 & 0 & 0 \\ b_{31} & 0 & 0 & 0 & 0 & 0 & x_7 & 0 & a_{13} \\ 0 & b_{31} & 0 & 0 & 0 & 0 & 0 & x_8 & a_{23} \\ 0 & 0 & b_{31} & 0 & 0 & 0 & a_{31} & a_{32} & x_9 \end{vmatrix}$$

where $x_1=-a_{13}-b_{12}-b_{13}$, $x_2=-a_{23}-b_{12}-b_{13}$, $x_3=-a_{31}-a_{32}-b_{12}-b_{13}$, $x_4=-a_{13}-b_{21}$, $x_5=-a_{23}-b_{21}$, $x_6=-a_{31}-a_{32}-b_{21}$, $x_7=-a_{13}-b_{31}$, $x_8=-a_{23}-b_{31}$, $x_9=-a_{31}-a_{32}-b_{31}$.

The state with the maximal performance $g_3=100$ and the minimal demand $w_1=0$ (the state 3) is given as the initial state. In states 2, 5, 8 the MSS performance is 70, in states 3, 6, 9 it is 100, and in states 1, 4, 7 it is 0. In states 4, 7 and 8 the MSS performance is lower than the demand. These states are unacceptable and have a performance deficiency:

$D_4 = w_2 - g_1 = -60$, $D_7 = w_3 - g_1 = -90$ and $D_8 = w_3 - g_2 = -70$ respectively. States 1, 2, 3, 5, 6, 9 constitute the set of acceptable states.

In order to find the mean number of failures the reward matrix should be defined according to the suggested method. Each reward associated with transition from the set of acceptable states to the set of unacceptable states should be defined as 1. All other rewards should be zeroed. Therefore, in reward matrix $r_{14}=r_{17}=r_{28}=r_{98}=r_{97}=1$ and all other rewards are zeroes.

So, the reward matrix $\boldsymbol{r}$ is obtained:

$$\boldsymbol{r} = \left|r_{ij}\right| = \begin{vmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix}.$$

By solving the system of differential equations (2.46) under the initial conditions $V_i(t)=0$, $i=1,\ldots,9$ all expected rewards $V_i(t)$, $i=1,\ldots,9$ can be found as functions of time t.

The state $K=3$, in which the system has a maximum capacity level and a minimum demand, is given as the initial state. Then, according to expression (2.48) the value $V_3(T)$ is treated as the mean numbers of system entrances into the area of unacceptable states or the mean number of power system failures during the time interval $[0, T]$. The function $N_{f3}(t) = V_3(t)$ is graphically presented in Fig. 2.19, where $N_{f3}(t)$ is the mean number of system failures in the case, when state 3 is an initial state.

The function $N_{f1}(t) = V_1(t)$ characterizes a mean number of system failures in the case where state 1 is given as the initial state. It is also presented in this figure. As shown $N_{f3}(t) < N_{f1}(t)$ because state 1 is "closer" to the set of unacceptable states – it has the direct transition to the set in the unacceptable area and state 3 does not. Therefore, at the beginning of the process the system's entrance into the set of unacceptable states is more likely from state 1 than from state 3. Fig. 2.19 graphically represents a number of power system failures for a short period – only 15 days. However, after this short period the function $N_{f3}(t)$ will be a linear function.

The reliability evaluation is usually performed over an extended period (years). For example, for one year we obtain $N_{f3}(T=1 \quad year) \approx 3.5$.

According to (2.49) the frequency of the power system failures can be obtained:

$$f_{f3} = \frac{1}{N_{f3}} = 0.286 \quad year^{-1}.$$

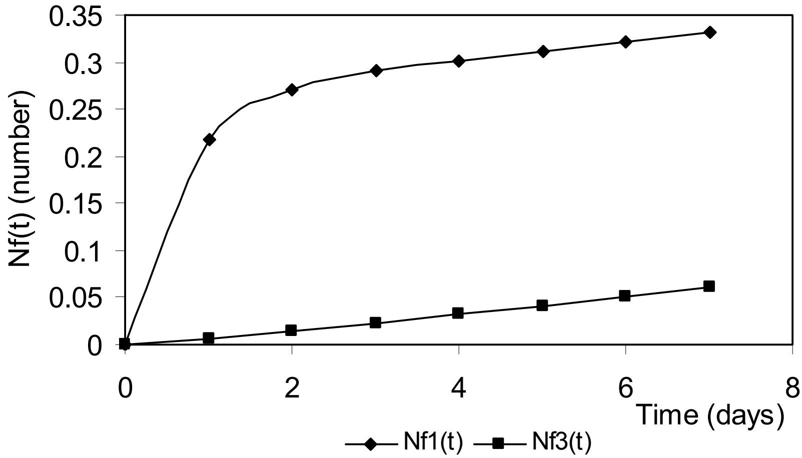


Fig. 2.19. Mean number of the generator entrances to the set of unacceptable states

2.1.4 Semi-Markov models

As was mentioned above, a discrete-state, continuous-time stochastic process can only be represented as a continuous-time Markov chain when the transition time between any states is distributed exponentially. This fact seriously restricts the application of the Markov chain model to real world problems. One of the ways to investigate processes with arbitrarily distributed sojourn times is to use a semi-Markov processes model. The main advantage of semi-Markov model is that it allows non-exponential distributions for transitions between states and generalizes several kinds of stochastic processes. Since in many real cases the lifetime and repair times are not exponential, this is very important.

The semi-Markov processes were introduced almost simultaneously by Levy (1954) and Smith (1955). At the same time, Takacs (1955) introduced essentially the same type of processes and applied them to some problems in counter theory. The foundations of the theory of semi-Markov processes can be found in (Cinlar 1975), (Gihman and Skorohod 1974), (Korolyuk and Swishchuk 1995), (Silverstov 1980). For the readers interested in the field of semi-Markov processes applications to the reliability theory and performability analysis, the following books may be especially recommended: (Limnios and Oprisan 2000), (Kovalenko et al. 1997), (Lisnianski and Levitin 2003), (Sahner et al. 1996).

The general theory of semi-Markov processes is quite complex. Here we study some aspects of reliability evaluation based on using semi-Markov processes that do not involve very complex computations. In many real world problems, relatively simple computation procedures allow engineer to assess reliability for power system with arbitrary transition times without Monte-Carlo simulation. This especially relates to power system steady state behavior.

Definition of semi-Markov process

In order to define a semi-Markov process, consider a system that at any time instant $t \geq 0$ can be in one of possible states $g_1, g_2, \ldots, g_K$. The system behavior is defined by the discrete state continuous time stochastic performance process $G(t) \in \{g_1, g_2, \ldots, g_K\}$. We assume that the initial state i of the system and one-step transition probabilities are given:
$G(0) = g_i$, $i \in \{1,\ldots,K\}$,

$$\pi_{jk} = P\{G(t_m) = g_k \mid G(t_{m-1}) = g_j\},\ j,k \in \{1,\ldots,K\}. \tag{2.52}$$

Here π_{jk} is the probability that the system will transit from state j with performance rate g_j to state k with performance rate g_k. Probabilities $\pi_{jk}, j,k \in \{1,\ldots,K\}$ define the one-step transition probability matrix $\boldsymbol{\pi} = |\pi_{jk}|$ for the discrete time Markov chain $G(t_m)$, where transitions from one state to another may happen only at discrete time moments $t_1, t_2, \ldots, t_{m-1}, t_m, \ldots$.

To each $\pi_{jk} \neq 0$ a random variable corresponds T^*_{jk} with the cumulative distribution function

$$F^*_{jk}(t) = F^*_{jk}(T^*_{jk} \leq t) \tag{2.53}$$

and probability density function $f^*_{jk}(t)$. This random variable is called a conditional sojourn time in the state j and characterizes the system sojourn time in the state j under condition that the system transits from state j to state k.

The graphical interpretation of possible realization of the considered process is shown in Fig. 2.20. At the initial time instant $G(0)=g_i$. The process transits to state j (with performance rate g_j) from the initial state i with probability π_{ij} . Therefore, if the next state is state j, the process remains in state i during random time T^*_{ij} with cdf $F^*_{ij}(t)$. When the process transits to state j, the probability of the transition from this state to any state k is π_{jk} . If the system transits from state j to state k, it remains in state j during random time T^*_{jk} with cdf $F^*_{jk}(t)$ up to the transition to state k.

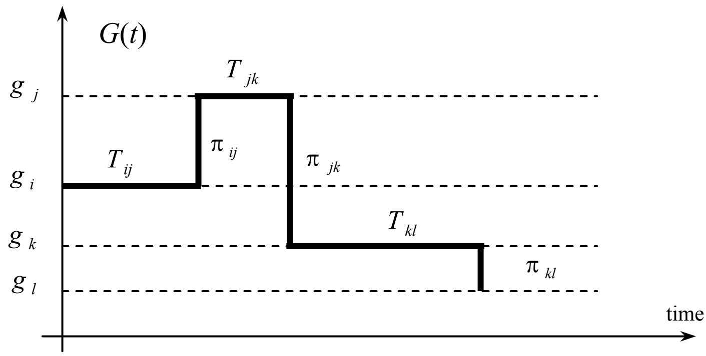


Fig. 2.20. Semi-Markov performance process

This process can be continued over an arbitrary period T. Each time the next state and the corresponding sojourn time in the current state must be chosen independently of the previous history of the process. The described performance stochastic process $G(t)$ is called a semi-Markov process.

In order to define the semi-Markov process one has to define the initial state of the process and the matrices: $\boldsymbol{\pi} = |\pi_{jk}|$ and $\boldsymbol{F}^*(t)=|F^*_{ij}(t)|$ for $i, j \in \{1,...,K\}$. The discrete time Markov chain $G(t_m)$ with one-step transition probabilities π_{jk} , $j,k \in \{1,...,K\}$ is called an imbedded Markov chain.

Note that the process in which the arbitrary distributed times between transitions are ignored and only time instants of transitions are of interest is a homogeneous discrete time Markov chain. However, in a general case, if one takes into account the sojourn times in different states, the process does not have Markov properties. (It remains the Markov process only if all the sojourn times are distributed exponentially). Therefore, the process can be considered a Markov process only at time instants of transitions. This explains why the process was named semi-Markov.

The most general definition of the semi-Markov process is based on kernel matrix $\boldsymbol{Q}(t)$. Each element $Q_{ij}(t)$ of this matrix determines the probability that transition from state i to state j occurs during time interval [0, t]. Using kernel matrix, one-step transition probabilities for embedded Markov chain can be obtained as:

$$\pi_{ij} = \lim_{t \to \infty} Q_{ij}(t) \qquad (2.54)$$

and the cdf $F^*_{ij}(t)$ of conditional sojourn time in the state i can be obtained as

$$F^*_{ij}(t) = \frac{1}{\pi_{ij}} Q_{ij}(t). \qquad (2.55)$$

Based on the kernel matrix the cdf $F_i(t)$ of unconditional sojourn time T_i in any state i can be defined:

$$F_i(t) = \sum_{j=1}^{K} Q_{ij}(t) = \sum_{j=1}^{K} \pi_{ij} F^*_{ij}(t). \qquad (2.56)$$

Hence, for probability density function (pdf) of unconditional sojourn time in the state i with performance rate g_i, we can write

$$f_i(t) = \frac{d}{dt} F_i(t) = \sum_{j=1}^{K} \pi_{ij} f^*_{ij}(t). \qquad (2.57)$$

Based on (2.58), the mean unconditional sojourn time in the state i can be obtained as

$$\overline{T}_i = \int_0^{\infty} t f_i(t) dt = \sum_{j=1}^{K} \pi_{ij} \overline{T}^*_{ij}, \qquad (2.58)$$

where $\overline{T}^*_{ij}$ is the mean conditional sojourn time in the state i given that the system transits from state i to state j.

The kernel matrix $\boldsymbol{Q}(t)$ and the initial state completely define the stochastic behavior of semi-Markov process.

In practice, when MSS reliability is studied, in order to find the kernel matrix for a semi-Markov process, one can use the following considerations (Lisnianski and Yeager 2000). Transitions between different states are usually executed as consequences of such events as failures, repairs, inspections, etc. For every type of event, the cdf of time between them is known. The transition is realized according the event that occurs first in a competition among the events.

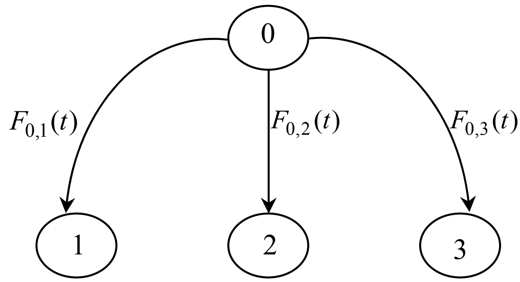


Fig. 2.21. State-space diagram of simplest semi-Markov system

In Fig. 2.21, one can see a state space diagram for the simplest semi-Markov system with three possible transitions from the initial state 0. If the event of type 1 is the first one, the system transits to state 1. The time between events of type 1 is random variable $T_{0,1}$ distributed according to cdf $F_{0,1}(t)$. If the event of type 2 occurs earlier than other events, the system transits from the state 0 to state 2. The random variable $T_{0,2}$ that defines the time between events of type 2 is distributed according to cdf $F_{0,2}(t)$. At last, if the event of type 3 occurs first, the system transits from state 0 to state 3. The time between events of type 3 is random variable $T_{0,3}$ distributed according to cdf $F_{0,3}(t)$. The probability $Q_{01}(t)$ that the system transits from state 0 to state 1 up to time t (the initial time $t=0$) may be determined as the probability that under condition $T_{0,1} \le t$, the random variable $T_{0,1}$ is less than variables $T_{0,2}$ and $T_{0,3}$. Hence, we have

$$Q_{01}(t) = \Pr\{(T_{0,1} \le t)\,\&\,(T_{0,2} > t)\,\&\,(T_{0,3} > t)\} =$$
$$= \int_0^t dF_{0,1}(u) \int_t^\infty dF_{0,2}(u) \int_t^\infty dF_{0,3}(u) = \int_0^t [1 - F_{0,2}(u)][1 - F_{0,3}(u)]dF_{0,1}(u)\,. \tag{2.59}$$

In the same way we obtain

$$Q_{02}(t) = \int_0^t [1 - F_{0,1}(u)][1 - F_{0,3}(u)]dF_{0,2}(u)\,, \tag{2.60}$$

$$Q_{03}(t) = \int_0^t [1 - F_{0,1}(u)][1 - F_{0,2}(u)]dF_{0,3}(u)\,. \tag{2.61}$$

For the semi-Markov process that describes the system with the state space diagram presented in Fig. 2.21, we have the following kernel matrix

$$\boldsymbol{Q}(t)=\begin{vmatrix} 0 & Q_{01}(t) & Q_{02}(t) & Q_{03}(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}. \tag{2.62}$$

Expressions (2.60)-(2.62) can be easily generalized to the arbitrary number of possible transitions from the initial state 0.

In order to demonstrate the technique of kernel matrix computation we consider the following example. Assume that, for the simplest system with state space diagram shown in Fig. 2.21, two random variables $T_{0,1}$ and $T_{0,2}$ are exponentially distributed with cdf $F_{0,1}(t)=1-e^{-\lambda_{0,1}t}$ and $F_{0,2}(t)=1-e^{-\lambda_{0,2}t}$ respectively and the third random variable $T_{0,3}$ has the following cdf

$$F_{0,3}(t)=\begin{cases} 0 \text{ if } t<T_c, \\ 1 \text{ if } t \ge T_c, \end{cases} \tag{2.63}$$

(which corresponds to the arrival of events with constant period T_c).

Using (2.60)-(2.62) we obtain

$$Q_{01}(t)=\begin{cases} \dfrac{\lambda_{0,1}}{\lambda_{0,1}+\lambda_{0,2}}[1-e^{-(\lambda_{0,1}+\lambda_{0,2})t}], \text{ if } t<T_c \\ \dfrac{\lambda_{0,1}}{\lambda_{0,1}+\lambda_{0,2}}[1-e^{-(\lambda_{0,1}+\lambda_{0,2})T_c}], \text{ if } t \ge T_c \end{cases} \tag{2.64}$$

$$Q_{02}(t)=\begin{cases} \dfrac{\lambda_{0,2}}{\lambda_{0,1}+\lambda_{0,2}}[1-e^{-(\lambda_{0,1}+\lambda_{0,2})t}], \text{ if } t<T_c \\ \dfrac{\lambda_{0,2}}{\lambda_{0,1}+\lambda_{0,2}}[1-e^{-(\lambda_{0,1}+\lambda_{0,2})T_c}], \text{ if } t \ge T_c \end{cases} \tag{2.65}$$

$$Q_{03}(t)=\begin{cases} 0, \text{ if } t<T_c, \\ e^{-(\lambda_{0,1}+\lambda_{0,2})T_c}, \text{ if } t \ge T_c. \end{cases} \tag{2.66}$$

According to (2.56), unconditional sojourn time T_0 in state 0 is distributed as follows:

$$F_1(t)=\begin{cases} 1-e^{-(\lambda_{0,1}+\lambda_{0,2})t} \text{ if } t<T_c, \\ 1 \text{ if } t \ge T_c. \end{cases} \tag{2.67}$$

One-step transition probabilities for the embedded Markov chain are defined according to (2.54):

$$\pi_{01} = \frac{\lambda_{0,1}}{\lambda_{0,1} + \lambda_{0,2}}[1 - e^{-(\lambda_{0,1}+\lambda_{0,2})T_c}], \tag{2.68}$$

$$\pi_{02} = \frac{\lambda_{0,2}}{\lambda_{0,1} + \lambda_{0,2}}[1 - e^{-(\lambda_{0,1}+\lambda_{0,2})T_c}], \tag{2.69}$$

$$\pi_{03} = e^{-(\lambda_{0,1}+\lambda_{0,2})T_c}. \tag{2.70}$$

According to (2.55), we obtain the cdf of conditional sojourn times

$$F^*_{01}(t) = \begin{cases} \dfrac{1 - e^{-(\lambda_{0,1}+\lambda_{0,2})t}}{1 - e^{-(\lambda_{0,1}+\lambda_{0,2})T_C}}, & \text{if } t < T_c \\ 1, & \text{if } t \ge T_c \end{cases} \tag{2.71}$$

$$F^*_{02}(t) = \begin{cases} \dfrac{1 - e^{-(\lambda_{0,1}+\lambda_{0,2})t}}{1 - e^{-(\lambda_{0,1}+\lambda_{0,2})T_C}}, & \text{if } t < T_c \\ 1, & \text{if } t \ge T_c \end{cases} \tag{2.72}$$

$$F^*_{03}(t) = \begin{cases} 0 & \text{if } t < T_c, \\ 1 & \text{if } t \ge T_c. \end{cases} \tag{2.73}$$

Evaluation of reliability indices based on semi-Markov processes

In order to find the MSS reliability indices, the system state space diagram should be built as it was done in previous sections for Markov processes. The only difference is that, in the case of the semi-Markov model, the transition times may be distributed arbitrarily. Based on transition time distributions $F_{i,j}(t)$, the kernel matrix $\boldsymbol{Q}(t)$ should be defined according to the method presented in the previous section.

The main problem of semi-Markov processes analysis is to find the state probabilities. Let $\theta_{ij}(t)$ be the probability that the process that starts in initial state i at instant t=0 is in state j at instant t. It was shown that probabilities $\theta_{ij}(t)$, $i,j \in \{1, \ldots, K\}$ can be found from the solution of the following system of integral equations:

$$\theta_{ij}(t) = \delta_{ij}[1 - F_i(t)] + \sum_{k=1}^{K} \int_0^t q_{ik}(\tau)\theta_{kj}(t-\tau)d\tau \,, \tag{2.74}$$

where

$$q_{ik}(\tau) = \frac{dQ_{ik}(\tau)}{d\tau} \,, \tag{2.75}$$

$$F_i(t) = \sum_{j=1}^{K} Q_{ij}(t) \,, \tag{2.76}$$

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases} \tag{2.77}$$

The system of linear integral equations (2.74) is the main system in the theory of semi-Markov processes. By solving this system, one can find all the probabilities $\theta_{ij}(t)$, $i, j \in \{1,...,K\}$ for the semi-Markov process with a given kernel matrix $|Q_{ij}(t)|$ and given initial state.

Based on the probabilities $\theta_{ij}(t)$, $i, j \in \{1,...,K\}$, important reliability indices can easily be found. Suppose that system states are ordered according to their performance rates $g_K \geq g_{K-1} \geq ... \geq g_2 \geq g_1$ and demand $g_m \geq w > g_{m-1}$ is constant. State K with performance rate g_K is the initial state. In this case system instantaneous availability is treated as the probability that a system starting at instant t=0 from the state K will be at instant $t \geq 0$ in any state $g_K, \dots ,g_m$. Hence, we obtain

$$A(t, w) = \sum_{j=m}^{K} \theta_{Ki}(t) \,. \tag{2.78}$$

The mean system instantaneous output performance and the mean instantaneous performance deficiency can be obtained, respectively, as

$$E_t = \sum_{i=1}^{K} g_i \theta_{Ki}(t) \tag{2.79}$$

and

$$D_t(w) = \sum_{i=1}^{m-1} (w - g_i)\theta_{Ki}(t)\, 1(w > g_i). \tag{2.80}$$

In the general case, the system of integral equations (2.74) can be solved only by numerical methods. For some of the simplest cases the method of Laplace-Stieltjes transform can be applied in order to derive an analytical solution of the system. As was done for Markov models, we designate a Laplace-Stieltjes transform of function $f(x)$ as

$$\tilde{f}(s) = L\{f(x)\} = \int_0^t e^{-sx} f(x)dx\,. \tag{2.81}$$

Applying Laplace-Stieltjes transform to both sides of (2.74) we obtain

$$\tilde{\theta}_{ij}(s) = \delta_{ij}\tilde{\Psi}_i(s) + \sum_{k=1}^{K} \pi_{ik} \tilde{f}_{ik}(s)\tilde{\theta}_{kj}(s)\,, \quad 1 \le i, j \le K\,, \tag{2.82}$$

where $\tilde{\Psi}_i(s)$ is Laplace-Stieltjes transform of the function

$$\Psi_i(t) = 1 - F_i(t) = \int_t^{\infty} f_i(t)dt = \Pr\{T_i > t\} \tag{2.83}$$

and, therefore,

$$\tilde{\Psi}_i(s) = \frac{1}{s}[1 - \tilde{f}_i(s)]\,. \tag{2.84}$$

The system of algebraic equations (2.82) defines Laplace-Stieltjes transform of probabilities $\theta_{ij}(t)$, $i, j \in \{1,...,K\}$ as a function of main parameters of a semi-Markov process.

By solving this system, one can also find steady state probabilities. The detailed investigation is out of scope of this book and we only give here the resulting formulae for computation of steady state probabilities. Steady state probabilities $\theta_{ij} = \lim_{t\to\infty} \theta_{ij}(t)$ (if they exist) do not depend on the initial state of the process *i* and for their designation, one can use only one index: θ_j. It is proven that

$$\theta_j = \frac{p_j \overline{T}_j}{\sum_{j=1}^{K} p_j \overline{T}_j}\,, \tag{2.85}$$

where p_j, j=1,…,K are steady state probabilities of the embedded Markov chain. These probabilities are the solutions of the following system of algebraic equations

$$\begin{cases} p_j = \sum_{i=1}^{K} p_i \pi_{ij}\ ,\ \ j = 1,...,K, \\ \sum_{i=1}^{K} p_i = 1 \end{cases} \tag{2.86}$$

Note that the first K equations in (2.86) are linearly dependant and we cannot solve the system without the last equation $\sum_{i=1}^{K} p_i = 1$.

In order to find the reliability function, the additional semi-Markov model should be built in analogy with the corresponding Markov models: all states corresponding to the performance rates lower than constant demand w should be united in one absorbing state with the number 0. All transitions that return the system from this absorbing state should be forbidden. The reliability function is obtained from this new model as $R(w,t) = \theta_{K0}(t)$.

Example 2.5

Consider an electric generator that has four possible performance (generating capacity) levels g_4=100 MW, g_3=70 MW, g_2=50 MW and g_1=0. The constant demand is w=60 MW. The best state with performance rate g_4=100 MW is the initial state.

Only minor failures and minor repairs are possible. Times to failures are distributed exponentially with following parameters

$$\lambda_{4,3} = 10^{-3}(\text{hours}^{-1}),\ \ \lambda_{3,2} = 5*10^{-4}(\text{hours}^{-1}),\ \ \lambda_{2,1} = 2*10^{-4}(\text{hours}^{-1})\ .$$

Hence, times to failures $T_{4,3}$, $T_{3,2}$, $T_{2,1}$ are random variables distributed according to the corresponding cdf:

$$F_{4,3}(t) = 1 - e^{-\lambda_{4,3}t},\ \ F_{3,2}(t) = 1 - e^{-\lambda_{3,2}t},\ \ F_{2,1}(t) = 1 - e^{-\lambda_{2,1}t}\ .$$

Repair times are normally distributed. $T_{3,4}$ has mean time to repair $\overline{T}_{3,4} = 240$ hours and standard deviation $\sigma_{3,4}$=16 hours, $T_{2,3}$ has a mean time to repair $\overline{T}_{2,3} = 480$ hours and standard deviation $\sigma_{2,3} = 48$ hours, $T_{1,2}$ has a mean time to repair $\overline{T}_{1,2} = 720$ hours and standard deviation $\sigma_{1,2} = 120$ hours. Hence, the cdf of random variables $T_{3,4}$, $T_{2,3}$ and $T_{1,2}$ are respectively

$$F_{3,4}(t) = \frac{1}{\sqrt{2\pi\sigma_{3,4}^2}} \int_0^t \exp[-\frac{(u - \overline{T}_{3,4})}{2\sigma_{3,4}}]du\ ,$$

$$F_{2,3}(t)=\frac{1}{\sqrt{2\pi\sigma_{2,3}^2}}\int_0^t \exp[-\frac{(u-\overline{T}_{2,3})}{2\sigma_{2,3}}]du\ ,$$

$$F_{1,2}(t)=\frac{1}{\sqrt{2\pi\sigma_{1,2}^2}}\int_0^t \exp[-\frac{(u-\overline{T}_{1,2})}{2\sigma_{1,2}}]du\ .$$

We have to find the generator reliability function, steady state availability, mean steady state performance (generating capacity), and mean steady state performance deficiency.

The evolution of the generator in the state space is shown in Fig. 2.22A.

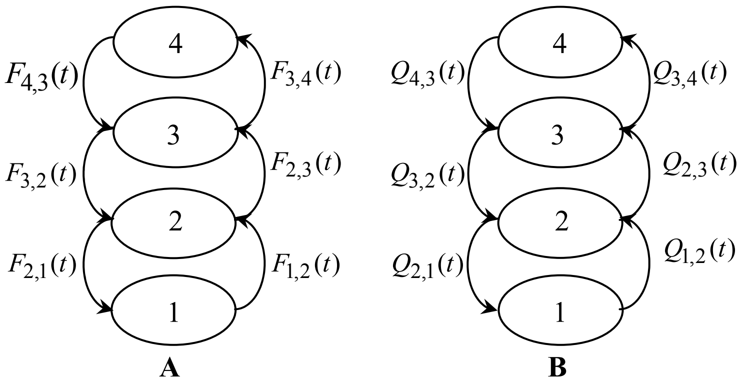


Fig. 2.22. Behavior of the electric generator. A. Evolution in the state-space. B. Semi-Markov model

Based on (2.59)-(2.61), we obtain the kernel matrix $\boldsymbol{Q}(t)=\left|Q_{ij}(t)\right|$, i,j=1,2,3,4:

$$\boldsymbol{Q}(t)=\begin{vmatrix} 0 & Q_{12}(t) & 0 & 0 \\ Q_{21}(t) & 0 & Q_{23}(t) & 0 \\ 0 & Q_{32}(t) & 0 & Q_{34}(t) \\ 0 & 0 & Q_{43}(t) & 0 \end{vmatrix}$$

in which

$$Q_{12}(t)=F_{1,2}(t)\,,\ Q_{21}(t)=\int_0^t[1-F_{2,3}(t)]dF_{2,1}(t)\,,$$

$$Q_{2,3}(t)=\int_0^t[1-F_{2,1}(t)]dF_{2,3}(t),\ Q_{3,2}(t)=\int_0^t[1-F_{3,4}(t)]dF_{3,2}(t)\,,$$

$$Q_{3,4}(t)=\int_0^t [1-F_{3,2}(t)]dF_{3,4}(t)\,,\, Q_{4,3}(t)=F_{4,3}(t)\,.$$

The corresponding semi-Markov process is presented in Fig. 2.22B.

Based on the kernel matrix, the cdf of unconditional sojourn times in states 1, 2, 3, and 4 can be written according to (2.56) as

$$F_1(t)=Q_{12}(t)\,,\, F_2(t)=Q_{12}(t)+Q_{23}(t)\,,$$

$$F_3(t)=Q_{32}(t)+Q_{34}(t)\,,\, F_4(t)=Q_{43}(t)\,.$$

According to (2.57) and (2.58) we have the following mean unconditional sojourn times

$$\overline{T}_1=720\text{ hours},\ \overline{T}_2=457\text{ hours},\ \overline{T}_3=226\text{ hours},\ \overline{T}_4=1000\text{ hours}.$$

Using (2.54) we obtain one-step probabilities for the embedded Markov chain

$$\pi_{12}=F_{1,2}(\infty)=1,\ \pi_{21}=\int_0^\infty [1-F_{2,3}(t)]dF_{2,1}(t)\,,$$

$$\pi_{23}=\int_0^\infty [1-F_{2,1}(t)]dF_{2,3}(t),\ \pi_{32}=\int_0^\infty [1-F_{3,4}(t)]dF_{3,2}(t)\,,$$

$$\pi_{34}=\int_0^\infty [1-F_{3,2}(t)]dF_{3,4}(t)\,,\ \pi_{43}=F_{4,3}(\infty)=1\,.$$

Calculating the integrals numerically, we obtain the following one-step probability matrix for the embedded Markov chain:

$$\boldsymbol{\pi}=\lim_{t\to\infty}\boldsymbol{Q}(t)=\begin{vmatrix}0 & \pi_{12} & 0 & 0\\ \pi_{21} & 0 & \pi_{23} & 0\\ 0 & \pi_{32} & 0 & \pi_{34}\\ 0 & 0 & \pi_{43} & 0\end{vmatrix}=\begin{vmatrix}0 & 1 & 0 & 0\\ 0.0910 & 0 & 0.9090 & 0\\ 0 & 0.1131 & 0 & 0.8869\\ 0 & 0 & 1 & 0\end{vmatrix}.$$

In order to find steady state probabilities p_j, j=1,2,3,4 for the embedded Markov chain, we have to solve the system of algebraic equations (2.86) that takes the form

$$\begin{cases} p_1 = \pi_{21} p_2 \\ p_2 = \pi_{12} p_1 + \pi_{32} p_3 \\ p_3 = \pi_{23} p_2 + \pi_{43} p_4 \\ p_4 = \pi_{34} p_3 \\ p_1 + p_2 + p_3 + p_4 = 1 \end{cases}$$

By solving this system we obtain

$$p_1 = 0.0056,\ p_2 = 0.0615,\ p_3 = 0.4944,\ p_4 = 0.4385.$$

Now using (2.61) we obtain the steady state probabilities

$$\theta_1 = \frac{p_1 \overline{T}_1}{\sum_{j=1}^{4} p_j \overline{T}_j} = 0.0069,\ \theta_2 = \frac{p_2 \overline{T}_2}{\sum_{j=1}^{4} p_j \overline{T}_j} = 0.0484,$$

$$\theta_3 = \frac{p_3 \overline{T}_3}{\sum_{j=1}^{4} p_j \overline{T}_j} = 0.1919,\ \theta_4 = \frac{p_4 \overline{T}_4}{\sum_{j=1}^{4} p_j \overline{T}_j} = 0.7528.$$

The steady state availability of the generator for the given constant demand is

$$A(w) = \theta_3 + \theta_4 = 0.9447.$$

According to (2.79), we obtain mean steady state performance

$$E_\infty = \sum_{k=1}^{4} g_k \theta_k = 91.13 \text{ MW}.$$

and according to (2.80), we obtain mean steady state performance deficiency

$$D_\infty = (w - g_2)\theta_2 + (w - g_1)\theta_1 = 0.50 \text{ MW}.$$

In order to find the reliability function for the given constant demand w=60 MW, we unite states 1 and 2 into one absorbing state 0. The modified graphical representation of the system evolution in the state space for this case is shown in Fig. 2.23A. In Fig. 2.23B, the state space diagram for the corresponding semi-Markov process is shown.

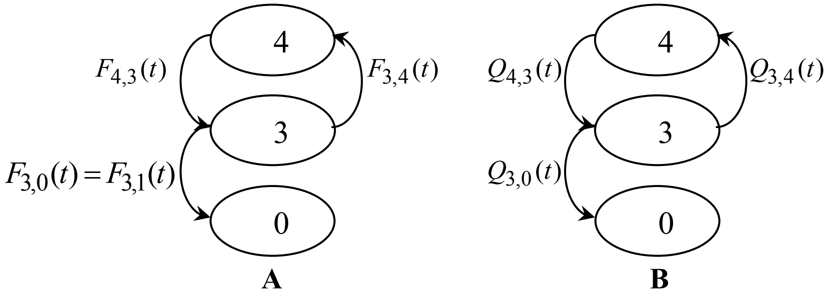


Fig. 2.23. Diagrams for evaluating reliability function of generator. A. Evolution in modified state-space. B. Semi-Markov model.

As in the previous case, we define the kernel matrix for the corresponding semi-Markov process based on expressions (2.59)-(2.61):

$$\boldsymbol{Q}(t)=\begin{vmatrix} 0 & 0 & 0 \\ Q_{30}(t) & 0 & Q_{34}(t) \\ 0 & Q_{43}(t) & 0 \end{vmatrix},$$

where

$$Q_{30}(t)=\int_0^t[1-F_{3,4}(t)]dF_{3,1}(t),\quad Q_{3,4}(t)=\int_0^t[1-F_{3,1}(t)]dF_{3,4}(t),\ Q_{43}(t)=F_{4,3}(t).$$

The reliability function for constant demand w=60 MW is defined as

$$R(w,t)=\theta_{40}(t).$$

According to (2.74), the following system of integral equation can be written in order to find the probability $\theta_{40}(t)$

$$\begin{cases} \theta_{40}(t)=\int_0^t q_{43}(\tau)\theta_{30}(t-\tau)d\tau \\ \theta_{30}(t)=\int_0^t q_{34}(\tau)\theta_{40}(t-\tau)d\tau+\int_0^t q_{30}(\tau)\theta_{00}(t-\tau)d\tau. \\ \theta_{00}(t)=1 \end{cases}$$

The reliability function obtained by solving this system numerically is presented in Fig. 2.24.

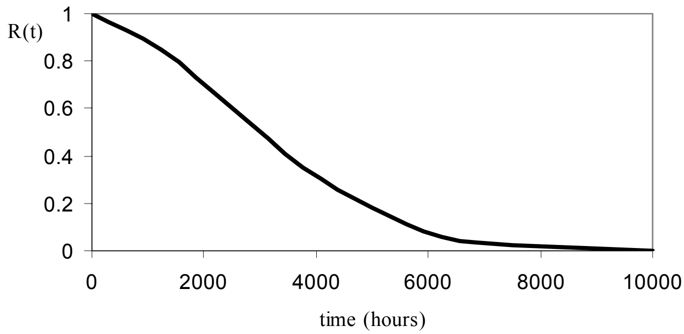


Fig. 2.24. Reliability function of generator

2.2 The Universal Generating Function Method

The recently emerged universal generating function (UGF) technique allows one to find the entire MSS performance distribution based on the performance distributions of its elements by using algebraic procedures. This technique (also called the method of generalized generating sequences) generalizes the technique that is based on using a well-known ordinary generating function. The basic ideas of the method were introduced by Professor I. Ushakov in the mid 1980s (Ushakov 1986, Ushakov 1987). Since then, the method has been considerably expanded (Lisnianski and Levitin 2003, Levitin 2005).

The UGF approach is straightforward. It is based on intuitively simple recursive procedures and provides a systematic method for the system states' enumeration that can replace extremely complicated combinatorial algorithms used for enumerating the possible states in some special types of systems (such as consecutive systems or networks).

The UGF approach is effective. Combined with simplification techniques, it allows the system's performance distribution to be obtained in a short time. The computational burden is the crucial factor when one solves optimization problems where the performance measures have to be evaluated for a great number of possible solutions along the search process. This makes using the traditional methods in reliability optimization problematic. Contrary to that, the UGF technique is fast enough to be implemented in optimization procedures.

The UGF approach is universal. An analyst can use the same recursive procedures for systems with a different physical nature of performance and different types of element interaction.

2.2.1 Moment-generating function and z-transform

Consider a discrete random variable X that can take on a finite number of possible values. The probabilistic distribution of this variable can be represented by the finite vector $\boldsymbol{x} = (x_0, \dots, x_k)$ consisting of the possible values of X and the finite vector $\boldsymbol{p}$ consisting of the corresponding probabilities $p_i = \Pr\{X = x_i\}$. The mapping $x_i \rightarrow p_i$ is usually called the *probability mass function* (pmf).

X must take one of the values x_i. Therefore

$$\sum_{i=0}^{k} p_i = 1 \tag{2.87}$$

Example 2.6

Suppose that one performs k independent trials and that each trial can result either in a success (with probability π) or in a failure (with probability $1-\pi$). Let random variable X represent the number of successes that occur in k trials. Such a variable is called a binomial random variable. The pmf of X takes the form

$$x_i = i,\ \ p_i = \binom{k}{i} \pi^i (1-\pi)^{k-i},\ \ 0 \le i \le k$$

According to the binomial theorem it can be seen that

$$\sum_{i=0}^{k} p_i = \sum_{i=0}^{k} \binom{k}{i} \pi^i (1-\pi)^{k-i} = [\pi + (1-\pi)]^k = 1$$

The expected value of X is defined as a weighted average of the possible values that X can take on, where each value is weighted by the probability that X assumes that value:

$$E(X) = \sum_{i=0}^{k} x_i p_i \tag{2.88}$$

Example 2.7

The expected value of a binomial random variable is

$$E(X) = \sum_{i=0}^{k} x_i p_i = \sum_{i=0}^{k} i \binom{k}{i} \pi^i (1-\pi)^{k-i}$$

$$= k\pi \sum_{i=0}^{k-1} \binom{k-1}{i} \pi^i (1-\pi)^{k-i-1} = k\pi[\pi + (1-\pi)]^{k-1} = k\pi$$

The moment-generating function $m(t)$ of the discrete random variable X with pmf $\boldsymbol{x}, \boldsymbol{p}$ is defined for all values of t by

$$m(t) = E(e^{tX}) = \sum_{i=0}^{k} e^{tx_i} p_i \tag{2.89}$$

The function $m(t)$ is called the moment-generating function because all of the moments of random variable X can be obtained by successively differentiating $m(t)$. For example:

$$m'(t) = \frac{\mathrm{d}}{\mathrm{d}t}(\sum_{i=0}^{k} e^{tx_i} p_i) = \sum_{i=0}^{k} x_i e^{tx_i} p_i . \tag{2.90}$$

Hence

$$m'(0) = \sum_{i=0}^{k} x_i p_i = E(X) . \tag{2.91}$$

Then

$$m''(t) = \frac{\mathrm{d}}{\mathrm{d}t}(m'(t)) = \frac{\mathrm{d}}{\mathrm{d}t}(\sum_{i=0}^{k} x_i e^{tx_i} p_i) = \sum_{i=0}^{k} x_i^2 e^{tx_i} p_i \tag{2.92}$$

and

$$m''(0) = \sum_{i=0}^{k} x_i^2 p_i = E(X^2) \tag{2.93}$$

The n-th derivative of $m(t)$ is equal to $E(X^n)$at $t = 0$.

Example 2.8

The moment-generating function of the binomial distribution takes the form

$$m(t) = E(e^{tX}) = \sum_{i=0}^{k} e^{ti} \binom{k}{i} \pi^i (1-\pi)^{k-i}$$

$$= \sum_{i=0}^{k} (\pi \ e^t)^i \binom{k}{i} (1-\pi)^{k-i} = (\pi \ e^t + 1 - \pi)^k$$

Hence

$$m'(t) = k(\pi \ e^t + 1 - \pi)^{k-1} \pi e^t \quad \text{and} \quad E(X) = m'(0) = k\pi .$$

The moment-generating function of a random variable uniquely determines its pmf. This means that a one-to-one correspondence exists between the pmf and the moment-generating function.

The following important property of moment-generating function is of special interest for us. The moment-generating function of the sum of the independent random variables is the product of the individual moment-generating functions of these variables. Let $m_X(t)$ and $m_Y(t)$ be the moment-generating functions of random variables X and Y respectively. The pmf of the random variables are represented by the vectors

$$\boldsymbol{x} = (x_0, \ldots, x_{k_X}), \quad \boldsymbol{p}_X = (p_{X0}, \ldots, p_{Xk_X}) \tag{2.94}$$

and

$$\boldsymbol{y} = (y_0, \ldots, y_{k_Y}), \quad \boldsymbol{p}_Y = (p_{Y0}, \ldots, p_{Yk_Y}) \tag{2.95}$$

respectively. Then $m_{X+Y}(t)$, the moment-generating function of $X+Y$, is obtained as

$$m_{X+Y}(t) = m_X(t) m_Y(t) = \sum_{i=0}^{k_X} e^{tx_i} p_{X_i} \sum_{j=0}^{k_Y} e^{ty_i} p_{Y_j}$$

$$= \sum_{i=0}^{k_X} \sum_{j=0}^{k_Y} e^{tx_i} e^{ty_j} p_{X_i} p_{Y_j} = \sum_{i=0}^{k_X} \sum_{j=0}^{k_Y} e^{t(x_i + y_j)} p_{X_i} p_{Y_j} \tag{2.96}$$

The resulting moment-generating function $m_{X+Y}(t)$ relates the probabilities of all the possible combinations of realizations $X = x_i$, $Y = y_j$, for any i and j, with the values that the random function $X + Y$ takes on for these combinations.

In general, for n independent discrete random variables $X_1, \ldots, X_n$

$$m_{\sum_{i=1}^{n} X_i}(t) = \prod_{i=1}^{n} m_{X_i}(t) \tag{2.97}$$

By replacing the function e^t by the variable z in Eq. (2.89) we obtain another function related to random variable X that uniquely determines its pmf:

$$\omega(z)=E(z^X)=\sum_{i=0}^{k} z^{x_i} p_i \tag{2.98}$$

This function is usually called the z-transform of discrete random variable X. The z-transform preserves some basic properties of the moment-generating functions. The first derivative of $\omega(z)$ is equal to $E(X)$ at $z = 1$. Indeed:

$$\omega'(z) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=0}^{k} z^{x_i} p_i\right) = \sum_{i=0}^{k} x_i z^{x_i - 1} p_i \tag{2.99}$$

Hence

$$\omega'(1) = \sum_{i=0}^{k} x_i p_i = E(X) \tag{2.100}$$

The z-transform of the sum of independent random variables is the product of the individual z-transforms of these variables:

$$\begin{aligned} \omega_{X+Y}(z) &= \omega_X(z)\omega_Y(z) = \sum_{i=0}^{k_X} z^{x_i} p_{X_i} \sum_{j=0}^{k_Y} z^{y_i} p_{Y_j} \\ &= \sum_{i=0}^{k_X} \sum_{j=0}^{k_Y} z^{x_i} z^{y_j} p_{X_i} p_{Y_j} = \sum_{i=0}^{k_X} \sum_{j=0}^{k_Y} z^{(x_i + y_j)} p_{X_i} p_{Y_j} \end{aligned} \tag{2.101}$$

and in general

$$\omega_{\sum_{i=1}^{n} X_i} = \prod_{i=1}^{n} \omega_{X_i}(z) \tag{2.102}$$

The reader wishing to learn more about the generating function and z-transform is referred to the books (Grimmett and Stirzaker 1992) and (Ross 2000).

Example 2.9

Suppose that one performs k independent trials and each trial can result either in a success (with probability π) or in a failure (with probability $1-\pi$). Let random variable X_j represent the number of successes that occur in the jth trial.

The pmf of any variable X_j ($1 \le j \le k$) is

$$\Pr\{X_j = 1\} = \pi, \ \Pr\{X_j = 0\} = 1 - \pi.$$

The corresponding z-transform takes the form

$$\omega_{Xj}(z) = \pi z^1 + (1-\pi)z^0$$

The random number of successes that occur in k trials is equal to the sum of the numbers of successes in each trial

$$X = \sum_{j=1}^{k} X_j$$

Therefore, the corresponding z-transform can be obtained as

$$\omega_X(z) = \prod_{j=1}^{n} \omega_{X_j}(z) = [\pi \ z + (1-\pi)z^0]^k$$

$$= \sum_{i=0}^{k} \binom{k}{i} z^i \pi^i (1-\pi)^{k-i} = \sum_{i=0}^{k} z^i \binom{k}{i} \pi^i (1-\pi)^{k-i}$$

This z-transform corresponds to the binomial pmf:

$$x_i = i, \ p_i = \binom{k}{i} \pi^i (1-\pi)^{k-i}, \ 0 \le i \le k$$

2.2.2 Mathematical fundamentals of the Universal Generating Function

Definition of the Universal Generating Function

Consider n independent discrete random variables $X_1, \ldots, X_n$ and assume that each variable X_i has a pmf represented by the vectors $\boldsymbol{x}_i$, $\boldsymbol{p}_i$. In order to evaluate the pmf of an arbitrary function $f(X_1, \ldots, X_n)$, one has to evaluate the vector $\boldsymbol{y}$ of all of the possible values of this function and the vector $\boldsymbol{q}$ of probabilities that the function takes these values.

Each possible value of function f corresponds to a combination of the values of its arguments $X_1, \ldots, X_n$. The total number of possible combinations is

$$K = \prod_{i=1}^{n} (k_i + 1) \tag{2.103}$$

where $k_i + 1$ is the number of different realizations of random variable X_i. Since all of the n variables are statistically independent, the probability of each unique combination is equal to the product of the probabilities of the realizations of arguments composing this combination.

The probability of the jth combination of the realizations of the variables can be obtained as

$$q_j = \prod_{i=1}^{n} p_{ij_i} \tag{2.104}$$

and the corresponding value of the function can be obtained as

$$f_j = f(x_{1j_1}, \ ..., \ x_{nj_n}) \tag{2.105}$$

Some different combinations may produce the same values of the function. All of the combinations are mutually exclusive. Therefore, the probability that the function takes on some value is equal to the sum of probabilities of the combinations producing this value. Let A_h be a set of combinations producing the value f_h. If the total number of different realizations of the function $f(X_1, \ldots, X_n)$ is H, then the pmf of the function is

$$\boldsymbol{y} = (f_h : 1 \le h \le H), \quad \boldsymbol{q} = (\sum_{(x_{1j_1},\ldots,x_{nj_n}) \in A_h} \prod_{i=1}^{n} p_{ij_i} : 1 \le h \le H) \tag{2.106}$$

Example 2.10

Consider two random variables X_1 and X_2 with pmf $\boldsymbol{x_1} = (1, 4)$, $\boldsymbol{p_1} = (0.6, 0.4)$ and $\boldsymbol{x_2} = (0.5, 1, 2)$, $\boldsymbol{p_2} = (0.1, 0.6, 0.3)$. In order to obtain the pmf of the function $Y = X_1^{X_2}$ we have to consider all of the possible combinations of the values taken by the variables. These combinations are presented in Table 2.2.

The values of the function Y corresponding to different combinations of realizations of its random arguments and the probabilities of these combinations can be presented in the form

$$\boldsymbol{y} = (1, 2, 1, 4, 1, 16),\ \ \boldsymbol{q} = (0.06, 0.04, 0.36, 0.24, 0.18, 0.12)$$

Table 2.2. pmf of the function of two variables

No of combination	Combination probability	Value of X_1	Value of X_2	Value of Y
1	0.6×0.1 = 0.06	1	0.5	1
2	0.4×0.1 = 0.04	4	0.5	2
3	0.6×0.6 = 0.36	1	1	1
4	0.4×0.6 = 0.24	4	1	4
5	0.6×0.3 = 0.18	1	2	1
6	0.4×0.3 = 0.12	4	2	16

Note that some different combinations produce the same values of the function Y. Since all of the combinations are mutually exclusive, we can obtain the probability that the function takes some value as being the sum of the probabilities of different combinations of the values of its arguments that produce this value:

$$\Pr\{Y = 1\} = \Pr\{X_1 = 1,\ X_2 = 0.5\} + \Pr\{X_1 = 1,\ X_2 = 1\}$$

$$+ \Pr\{X_1 = 1,\ X_2 = 2\} = 0.06 + 0.36 + 0.18 = 0.6$$

The pmf of the function Y is

$$\boldsymbol{y} = (1, 2, 4, 16),\ \ \boldsymbol{q} = (0.\ 6, 0.04, 0.24, 0.12)$$

The z-transform of each random variable X_i represents its pmf $(x_{i0},\ ..., x_{ik_i}),\ (p_{i0},\ ..., p_{ik_i})$ in the polynomial form

$$\sum_{j=0}^{k_i} p_{ij} z^{x_{ij}} \tag{2.107}$$

According to (2.102), the product of the z-transform polynomials corresponding to the variables $X_1, \ldots, X_n$ determines the pmf of the sum of these variables.

In a similar way one can obtain the z-transform representing the pmf of the arbitrary function f by replacing the product of the polynomials by a more general composition operator $\otimes_f$ over z-transform representations of pmf of n independent variables:

$$\underset{f}{\otimes}\Big(\sum_{j_i=0}^{k_i} p_{ij_i} z^{x_{ij_i}}\Big) = \sum_{j_1=0}^{k_1}\ \sum_{j_2=0}^{k_2}\dots\sum_{j_n=0}^{k_n}\Big(\prod_{i=0}^{n} p_{ij_i} z^{f(x_{ij_1},\dots,x_{nj_n})}\Big) \tag{2.108}$$

The technique based on using z-transform and composition operators $\otimes_f$ is named the universal z-transform or universal (moment) generating function (UGF) technique. In the context of this technique, the z-transform of a random variable for which the operator $\otimes_f$ is defined is referred to as its u-function. We refer to the u-function of variable X_i as $u_j(z)$, and to the u-function of the function $f(X_1, \dots, X_n)$ as $U(z)$. According to this notation

$$U(z) = \underset{f}{\otimes}(u_1(z),\ u_2(z),\ \dots,\ u_n(z)) \tag{2.109}$$

where $u_i(z)$ takes the form (2.107) and $U(z)$ takes the form (2.108). For functions of two arguments, two interchangeable notations can be used:

$$U(z) = \underset{f}{\otimes}(u_1(z),\ u_2(z)) = u_1(z)\underset{f}{\otimes}u_2(z) \tag{2.110}$$

Despite the fact that the u-function resembles a polynomial, it is not a polynomial because:

- Its coefficients and exponents are not necessarily scalar variables, but can be other mathematical objects (*e.g.* vectors);
- Operators defined over the u-functions can differ from the operator of the polynomial product (unlike the ordinary z-transform, where only the product of polynomials is defined).

When the u-function $U(z)$ represents the pmf of a random function $f(X_1,\dots, X_n)$, the expected value of this function can be obtained (as an analogy with the regular z-transform) as the first derivative of $U(z)$ at $z = 1$.

In general, the u-functions can be used not just for representing the pmf of random variables. In the following chapters, we also use other interpretations. However, in any interpretation the coefficients of the terms in the u-function represent the probabilistic characteristics of some object or state encoded by the exponent in these terms.

The u-functions inherit the essential property of the regular polynomials: they allow for collecting like terms. Indeed, if a u-function representing the pmf of a random variable X contains the terms $p_h z^{x_h}$ and $p_m z^{x_m}$ for which $x_h = x_m$, the two terms can be replaced by a single term $(p_h + p_m)z^{x_m}$, since in this case $\Pr\{X = x_h\} = \Pr\{X = x_m\} = p_h+p_m$.

Example 2.11

Consider the pmf of the function Y from Example 2.10, obtained from Table 2.2. The u-function corresponding to this pmf takes the form:

$$U(z) = 0.06z^1 + 0.04z^2 + 0.36z^1 + 0.24z^4 + 0.18z^1 + 0.12z^{16}.$$

By collecting the like terms in this u-function we obtain:

$$U(z) = 0.6z^1 + 0.04z^2 + 0.24z^4 + 0.12z^{16},$$

which corresponds to the final pmf obtained in Example 2.10.

The expected value of Y can be obtained as

$$E(Y) = U'(1) = 0.6\times1 + 0.04\times2 + 0.24\times4 + 0.12\times16 = 3.56$$

The described technique of determining the pmf of functions is based on an enumerative approach. This approach is extremely resource consuming. Indeed, the resulting u-function $U(z)$ contains K terms (see Eq. (2.103)), which requires excessive storage space. In order to obtain $U(z)$ one has to perform $(n\text{-}1)K$ procedures of probabilities multiplication and K procedures of function evaluation. Fortunately, many functions used in reliability engineering produce the same values for different combinations of the values of their arguments. The combination of recursive determination of the functions with simplification techniques based on the like terms collection allows one to considerably reduce the computational burden associated with evaluating the pmf of complex functions.

Example 2.12

Consider the function

$$Y = f(X_1, \ldots, X_5) = (\max(X_1, X_2) + \min(X_3, X_4))\, X_5$$

of five independent random variables $X_1, \ldots, X_5$. The probability mass functions of these variables are determined by pairs of vectors $\boldsymbol{x}_i$, $\boldsymbol{p}_i$ $(0 \le i \le 5)$ and are presented in Table 2.3.

These pmf can be represented in the form of u-functions as follows:

$$u_1(z) = p_{10}z^{x_{10}} + p_{11}z^{x_{11}} + p_{12}z^{x_{12}} = 0.6z^5 + 0.3z^8 + 0.1z^{12};$$

$$u_2(z) = p_{20}z^{x_{20}} + p_{21}z^{x_{21}} = 0.7z^8 + 0.3z^{10};$$

$$u_3(z) = p_{30}z^{x_{30}} + p_{31}z^{x_{31}} = 0.6z^0 + 0.4z^1;$$

$$u_4(z) = p_{40}z^{x_{40}} + p_{41}z^{x_{41}} + p_{42}z^{x_{42}} = 0.1z^0 + 0.5z^8 + 0.4z^{10};$$

$$u_5(z) = p_{50}z^{x_{50}} + p_{51}z^{x_{51}} = 0.5z^1 + 0.5z^{1.5}.$$

Using the straightforward approach one can obtain the pmf of the random variable Y applying the operator (2.108) over these u-functions. Since $k_1 + 1 = 3$, $k_2 + 1 = 2$, $k_3 + 1 = 2$, $k_4 + 1 = 3$, $k_5 + 1 = 2$, the total number of term multiplication procedures that one has to perform using this equation is 3×2×2×3×2 = 72.

Table 2.3. pmf of random variables

X_1	p_1	0.6	0.3	0.1
	x_1	5	8	12
X_2	p_2	0.7	0.3	-
	x_2	8	10	-
X_3	p_3	0.6	0.4	-
	x_3	0	1	-
X_4	p_4	0.1	0.5	0.4
	x_4	0	8	10
X_5	p_5	0.5	0.5	-
	x_5	1	1.5	-

Now let us introduce three auxiliary random variables X_6, X_7 and X_8, and define the same function recursively:

$$X_6 = \max\{X_1, X_2\};$$

$$X_7 = \min\{X_3, X_4\};$$

$$X_8 = X_6 + X_7;$$

$$Y = X_8 X_5.$$

We can obtain the pmf of variable Y using composition operators over pairs of u-functions as follows:

$$u_6(z) = u_1(z) \underset{\max}{\otimes} u_1(z) = (0.6z^5+0.3z^8+0.1z^{12}) \underset{\max}{\otimes} (0.7z^8+0.3z^{10})$$

$$=0.42z^{\max\{5,8\}}+0.21z^{\max\{8,8\}}+0.07z^{\max\{12,8\}}+0.18z^{\max\{5,10\}}+0.09z^{\max\{8,10\}}+0.03z^{\max\{12,10\}} = 0.63z^8 + 0.27z^{10} + 0.1z^{12};$$

$$u_7(z) = u_3(z) \underset{\min}{\otimes} u_4(z)=(0.6z^0+0.4z^2) \underset{\min}{\otimes} (0.1z^0+0.5z^3+0.4z^5)$$

$$=0.06z^{\min\{0,0\}}+0.04z^{\min\{2,0\}}+0.3z^{\min\{0,3\}}+0.2z^{\min\{2,3\}}$$

$$+0.24z^{\min\{0,5\}}+0.16z^{\min\{2,5\}}=0.64z^0+0.36z^2;$$

$$u_8(z)=u_6(z) \underset{+}{\otimes} u_7(z)=(0.63z^8+0.27z^{10}+0.1z^{12}) \underset{+}{\otimes} (0.64z^0+0.36z^2)$$

$$=0.4032z^{8+0}+0.1728z^{10+0}+0.064z^{12+0}+0.2268z^{8+2}+0.0972z^{10+2}$$

$$+0.036z^{12+2}=0.4032z^8+0.3996z^{10}+0.1612z^{12}+0.036z^{14};$$

$$U(z) = u_8(z) \underset{\times}{\otimes} u_5(z)$$

$$= (0.4032z^8+0.3996z^{10}+0.1612z^{12}+0.036z^{14})(0.5z^1+0.5z^{1.5})$$

$$= 0.2016z^{8\times1}+0.1998z^{10\times1}+0.0806z^{12\times1}+0.018z^{14\times1}+0.2016z^{8\times1.5}$$

$$+0.1998z^{10\times1.5}+0.0806z^{12\times1.5}+0.018z^{14\times1.5}=0.2016z^8+0.1998z^{10}$$

$$+0.2822z^{12}+0.018z^{14}+0.1998z^{15}+0.0806z^{18}+0.018z^{21}.$$

The final u-function $U(z)$ represents the pmf of Y, which takes the form

$$\boldsymbol{y} = (8, 10, 12, 14, 15, 18, 21)$$

$$\boldsymbol{q} = (0.2016, 0.1998, 0.2822, 0.018, 0.1998, 0.0806, 0.018).$$

Note that during the recursive derivation of this pmf we used only 26 term multiplication procedures. This considerable computational complexity reduction is possible because of the like term collection in intermediate u-functions.

The problem of system reliability analysis usually includes evaluation of the pmf of some random values characterizing the system's behavior. These values can be very complex functions of a large number of random variables. The explicit derivation of such functions is an extremely complicated task. Fortunately, the UGF method for many types of system allows one to obtain the system u-function recursively. This property of the UGF method is based on the associative property of many functions used in reliability engineering. The recursive approach presumes obtaining u-functions of subsystems containing several basic elements and then treating the subsystem as a single element with the u-function obtained when computing the u-function of a higher-level subsystem. Combining the recursive approach with the simplification technique reduces the number of terms in the intermediate u-functions and provides a drastic reduction of the computational burden.

Properties of composition operators

The properties of composition operator $\otimes_f$ strictly depend on the properties of the function $f(X_1, ..., X_n)$. Since the procedure of the multiplication of the probabilities in this operator is commutative and associative, the entire operator can also possess these properties if the function possesses them.

If

$$f(X_1, X_2, \dots, X_n) = f(f(X_1, X_2, \dots, X_{n-1}), X_n), \tag{2.111}$$

then

$$\begin{aligned} U(z) &= \underset{f}{\otimes}(u_1(z), u_2(z), \ ..., \ u_n(z)) \\ &= \underset{f}{\otimes}(\underset{f}{\otimes}(u_1(z), u_2(z), \ ..., \ u_{n-1}(z)), \ u_n(z)). \end{aligned} \tag{2.112}$$

Therefore, one can obtain the u-function $U(z)$ assigning $U_1(z) = u_1(z)$ and applying operator $\underset{f}{\otimes}$ consecutively:

$$U_j(z) = \underset{f}{\otimes} (U_{j-1}(z), u_j(z)) \text{ for } 2 \le j \le n, \tag{2.113}$$

such that finally $U(z) = U_n(z)$.

If the function f possesses the associative property

$$f(X_1, \dots, X_j, X_{j+1}, \dots, X_n) = f(f(X_1, \dots, X_j), f(X_{j+1}, \dots, X_n)) \tag{2.114}$$

for any j, then the $\underset{f}{\otimes}$ operator also possesses this property:

$$\begin{aligned} &\underset{f}{\otimes}(u_1(z), \ ..., \ u_n(z)) \\ &= \underset{f}{\otimes}(\underset{f}{\otimes}(u_1(z), \ ..., \ u_{j-1}(z)), \underset{f}{\otimes}(u_j(z), \ ..., \ u_n(z)) \end{aligned} \tag{2.115}$$

If, in addition to the property (2.112), the function f is also commutative:

$$f(X_1, \dots, X_j, X_{j+1}, \dots, X_n) = f(X_1, \dots, X_{j+1}, X_j, \dots, X_n) \tag{2.116}$$

then for any j, which provides the commutative property for the $\underset{f}{\otimes}$ operator:

$$\underset{f}{\otimes}(u_1(z),...,u_j(z),u_{j+1}(z),...,u_n(z))$$
$$=\underset{f}{\otimes}(u_1(z),...,u_{j+1}(z),u_j(z),...,u_n(z)) \quad (2.117)$$

the order of arguments in the function $f(X_1, ..., X_n)$ is inessential and the u-function $U(z)$ can be obtained using recursive procedures (2.111) and (2.113) over any permutation of u-functions of random arguments $X_1,\ldots,X_n$.

If a function takes the recursive form

$$f(f_1(X_1, \ldots, X_j),\ f_2(X_{j+1}, \ldots, X_h),\ \ldots,\ f_m(X_l, \ldots, X_n)) \quad (2.118)$$

then the corresponding u-function $U(z)$ can also be obtained recursively:

$$\underset{f}{\otimes}(\underset{f_1}{\otimes}(u_1(z),...,u_j(z)),\ \underset{f_2}{\otimes}(u_{j+1}(z),...,u_h(z)),...,\ \underset{f_m}{\otimes}(u_l(z),...,u_n(z)). \quad (2.119)$$

Example 2.13

Consider the variables X_1, X_2, X_3 with pmf presented in Table 2.2. The u-functions of these variables are:

$$u_1(z) = 0.6z^5 + 0.3z^8 + 0.1z^{12};$$

$$u_2(z) = 0.7z^8 + 0.3z^{10};$$

$$u_3(z) = 0.6z^0 + 0.4z^1.$$

The function Y=min(X_1,X_2,X_3) possesses both commutative and associative properties. Therefore

$$\min(\min(X_1,X_2),X_3)=\min(\min(X_2,X_1),X_3)=\min(\min(X_1, X_3), X_2)$$

$$=\min(\min(X_3, X_1), X_2)=\min(\min(X_2, X_3), X_1)$$

$$=\min(\min(X_3, X_2), X_1).$$

The u-function of Y can be obtained using the recursive procedure

$$u_4(z)=u_1(z)\underset{\min}{\otimes}u_2(z)=(0.6z^5+0.3z^8+0.1z^{12})\underset{\min}{\otimes}(0.7z^8+0.3z^{10})$$

$$=0.42z^{\min\{5,8\}}+0.21z^{\min\{8,8\}}+0.07z^{\min\{12,8\}}$$

$$+0.18z^{\min\{5,10\}}+0.09z^{\min\{8,10\}}+0.03z^{\min\{12,10\}}=0.6z^5+0.37z^8+0.03z^{10};$$

$$U(z)=u_4(z)\underset{\min}{\otimes}u_3(z)\ (0.6z^5+0.37z^8+0.03z^{10})\underset{\min}{\otimes}(0.6z^0+0.4z^1)$$

$$= 0.36z^{\min\{5,0\}}+0.222z^{\min\{8,0\}}+0.018z^{\min\{12,0\}}$$

$$+ 0.24z^{\min\{5,1\}}+0.148z^{\min\{8,1\}}+0.012z^{\min\{12,1\}}=0.6z^0+0.4z^1.$$

The same u-function can also be obtained using another recursive procedure

$$u_4(z)=u_1(z) \underset{\min}{\otimes} u_3(z)=(0.6z^5+0.3z^8+0.1z^{12}) \underset{\min}{\otimes} (0.6z^0+0.4z^1)$$

$$= 0.36z^{\min\{5,0\}}+0.18z^{\min\{8,0\}}+0.06z^{\min\{12,0\}}$$

$$+0.24z^{\min\{5,1\}}+0.12z^{\min\{8,1\}}+0.04z^{\min\{12,1\}}=0.6z^0+0.4z^1;$$

$$U(z)=u_3(z) \underset{\min}{\otimes} u_2(z)=(0.6z^0+0.4z^1) \underset{\min}{\otimes} (0.7z^8+0.3z^{10})$$

$$=0.42z^{\min\{0,8\}}+0.28z^{\min\{1,8\}}+0.18z^{\min\{0,10\}}+0.12z^{\min\{1,10\}}=0.6z^0+0.4z^1$$

Note that while both recursive procedures produce the same u-function, their computational complexity differs. In the first case, 12 term multiplication operations have been performed; in the second case, only 10 operations have been performed.

Consider a random variable X with pmf represented by u-function $u_X(z) = \sum_{j=0}^{k} p_j z^{x_j}$. In order to obtain the u-function representing the pmf of function $f(X, c)$ of the variable X and a constant c one can apply the following simplified operator:

$$U(z) = u_X(z) \underset{f}{\otimes} c = (\sum_{j=0}^{k} p_j z^{x_j}) \underset{f}{\otimes} c = \sum_{j=0}^{k} p_j z^{f(x_j,c)} \qquad (2.120)$$

This can be easily proved if we represent the constant c as the random variable C that can take the value of c with a probability of 1. The u-function of such a variable takes the form

$$u_c(z) = z^c. \qquad (2.121)$$

Applying the operator $\otimes_f$ over the two u-functions $u_X(z)$ and $u_c(z)$ we obtain Eq. (2.120).

2.2.3 Obtaining the system reliability and performance indices using the UGF

Having the pmf of the random MSS output performance G and the pmf of the demand W in the form of u-functions $U_{\text{MSS}}(z)$ and $u_w(z)$, one can obtain the u-functions representing the pmf of the random functions $F(G,W)$,

$\tilde{G}(G,W)$, $D^-(G,W)$ or $D^+(G,W)$ (see Section 1.5.3) using the corresponding composition operators over $U_{\text{MSS}}(z)$ and $u_w(z)$:

$$U_F(z) = U_{\text{MSS}}(z) \underset{F}{\otimes} u_w(z) \tag{2.122}$$

$$U_{\tilde{G}}(z) = U_{\text{MSS}}(z) \underset{\tilde{G}}{\otimes} u_w(z) \tag{2.123}$$

$$U_D(z) = U_{\text{MSS}}(z) \underset{D}{\otimes} u_w(z) \tag{2.124}$$

Since the expected values of the functions G, F, D and $\tilde{G}$ are equal to the derivatives of the corresponding u-functions $U_{\text{MSS}}(z)$, $U_F(z)$, $U_D(z)$ and $U_{\tilde{G}}(z)$ at $z = 1$, the MSS performance measures can now be obtained as

$$E(G) = U'_{\text{MSS}}(1) \tag{2.125}$$

$$E(F(G,W)) = U'_F(1) \tag{2.126}$$

$$E(D(G,W)) = U'_D(1) \tag{2.127}$$

$$E(\tilde{G}(G,F)) / E(F(G,W)) = U'_{\tilde{G}}(1) / U'_F(1) \tag{2.128}$$

Example 2.14

Consider two power system generators with a nominal capacity of 100 MW as two separate MSSs. In the first generator, some types of failure require its capacity G_1 to be reduced to 60 MW and other types lead to a complete outage. In the second generator, some types of failure require its capacity G_2 to be reduced to 80 MW, others lead to a capacity reduction to 40 MW, and others lead to a complete outage. The generators are repairable and each of their states has a steady-state probability.

Both generators should meet a variable two-level demand W. The high level (day) demand is 50 MW and has the probability 0.6; the low level (night) demand is 30 MW and has the probability 0.4.

The capacity and demand can be presented as a fraction of the nominal generator capacity. There are three possible relative capacity levels that characterize the performance of the first generator:

$$g_{10} = 0.0,\ \ g_{11} = 60/100 = 0.6,\ \ g_{12} = 100/100 = 1.0$$

and four relative capacity levels that characterize the performance of the second generator:

$$g_{20} = 0.0,\ g_{21} = 40/100 = 0.4,\ g_{22} = 80/100 = 0.8,\ g_{23} = 100/100 = 1.0$$

Assume that the corresponding steady-state probabilities are

$$p_{10} = 0.1,\ p_{11} = 0.6,\ p_{12} = 0.3$$

for the first generator and

$$p_{20} = 0.05,\ p_{21} = 0.35,\ p_{22} = 0.3,\ p_{23} = 0.3$$

for the second generator and that the demand distribution is

$$w_1 = 50/100 = 0.5,\ w_2 = 30/100 = 0.3,\ q_1 = 0.6,\ q_2 = 0.4$$

The u-functions representing the capacity distribution of the generators (the pmf of random variables G_1 and G_2) take the form

$$U_1(z) = 0.1z^0+0.6z^{0.6}+0.3z^1,\quad U_2(z) = 0.05z^0+0.35z^{0.4}+0.3z^{0.8}+0.3z^1$$

and the u-function representing the demand distribution takes the form

$$u_w(z) = 0.6z^{0.5}+0.4z^{0.3}.$$

The mean steady-state performance (capacity) of the generators can be obtained directly from these u-functions:

$$\varepsilon_1 = E(G_1) = U'_1(1) = 0.1\times 0 + 0.6\times 0.6 + 0.3\times 1.0 = 0.66$$

which means 66% of the nominal generating capacity for the first generator, and

$$\varepsilon_2 = E(G_2) = U'_2(1) = 0.05\times 0 + 0.35\times 0.4 + 0.3\times 0.8 + 0.3\times 1.0 = 0.68$$

which means 68% of the nominal generating capacity for the second generator.

The available generation capacity should be no less than the demand. Therefore, the system acceptability function takes the form

$$F(G,W) = 1(G \ge W)$$

and the system performance deficiency takes the form

$$D^-(G,W) = \max(W - G, 0)$$

The u-functions corresponding to the pmf of the acceptability function are obtained using the composition operator $\underset{F}{\otimes}$:

$$U_{F1}(z) = U_1(z) \underset{F}{\otimes} u_w(z) = (0.1z^0 + 0.6z^{0.6} + 0.3z^1) \underset{F}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.06z^{1(0 \ge 0.5)} + 0.36z^{1(0.6 \ge 0.5)} + 0.18z^{1(1 \ge 0.5)} + 0.04z^{1(0 \ge 0.3)}$$
$$+ 0.24z^{1(0.6 \ge 0.3)} + 0.12z^{1(1 \ge 0.3)} = 0.06z^0 + 0.36z^1 + 0.18z^1 + 0.04z^0$$
$$+ 0.24z^1 + 0.12z^1 = 0.9z^1 + 0.1z^0$$

$$U_{F2}(z) = U_2(z) \underset{F}{\otimes} u_w(z)$$
$$= (0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1) \underset{F}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.03z^0 + 0.21z^0 + 0.18z^1 + 0.18z^1 + 0.02z^0 + 0.14z^1$$
$$+ 0.12z^1 + 0.12z^1 = 0.74z^1 + 0.26z^0$$

The system availability (expected acceptability) is

$$A_1 = E(1(G_1 \ge W) = U'_{F1}(1) = 0.9$$
$$A_2 = E(1(G_2 \ge W) = U'_{F2}(1) = 0.74$$

The u-functions corresponding to the pmf of the performance deficiency function are obtained using the composition operator $\underset{D}{\otimes}$:

$$U_{D1}(z) = U_1(z) \underset{D}{\otimes} u_w(z)$$
$$= (0.1z^0 + 0.6z^{0.6} + 0.3z^1) \underset{D}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.06z^{\max(0.5-0,0)} + 0.36z^{\max(0.5-0.6,0)} + 0.18z^{\max(0.5-1,0)}$$
$$+ 0.04z^{\max(0.3-0,0)} + 0.24z^{\max(0.3-0.6,0)} + 0.12z^{\max(0.3-1,0)}$$
$$= 0.06z^{0.5} + 0.36z^0 + 0.18z^0 + 0.04z^{0.3} + 0.24z^0 + 0.12z^0$$
$$= 0.06z^{0.5} + 0.04z^{0.3} + 0.9z^0$$

$$U_{D2}(z) = U_2(z) \underset{D}{\otimes} u_w(z)$$
$$= (0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1) \underset{D}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.03z^{0.5} + 0.21z^{0.1} + 0.18z^0 + 0.18z^0 + 0.02z^{0.3} + 0.14z^0$$
$$+ 0.12z^0 + 0.12z^0 = 0.03z^{0.5} + 0.21z^{0.1} + 0.02z^{0.3} + 0.74z^0$$

The expected performance deficiency is

$$\Delta_1 = E(\max(W - G_1, 0)) = U'_{D1}(1)$$
$$= 0.06 \times 0.5 + 0.04 \times 0.3 + 0.9 \times 0 = 0.042$$

$$\Delta_2 = E(\max(W - G_2, 0)) = U'_{D2}(1)$$
$$= 0.03 \times 0.5 + 0.21 \times 0.1 + 0.02 \times 0.3 + 0.74 \times 0 = 0.042$$

In this case, Δ may be interpreted as expected electrical power unsupplied to consumers. The absolute value of this unsupplied demand is 4.2 MW for both generators. Multiplying this index by T, the system operating time considered, one can obtain the expected unsupplied energy.

Note that since the performance measures obtained have different natures they cannot be used interchangeable. For instance, in the present example the first generator performs better than the second one when availability is considered ($A_1 > A_2$), the second generator performs better than the first one when the expected capacity is considered ($\varepsilon_1 < \varepsilon_2$), and both generators have the same expected unsupplied demand ($\Delta_1 = \Delta_2$).

Now we determine the conditional expected system performance. The u-functions corresponding to the pmf of the function $\tilde{G}$ are obtained using the composition operator $\underset{\tilde{G}}{\otimes} = \underset{GF}{\otimes}$:

$$U_{\tilde{G}1}(z) = U_1(z) \underset{\tilde{G}}{\otimes} u_w(z) = (0.1z^0 + 0.6z^{0.6} + 0.3z^1) \underset{GF}{\otimes} (1z^{0.5} + 0.4z^{0.3})$$
$$= 0.06z^{0 \times 1(0 \geq 0.5)} + 0.36z^{0.6 \times 1(0.6 \geq 0.5)} + 0.18z^{1 \times 1(1 \geq 0.5)} + 0.04z^{0 \times 1(0 \geq 0.3)}$$
$$+ 0.24z^{0.6 \times 1(0.6 \geq 0.3)} + 0.12z^{1 \times 1(1 \geq 0.3)} = 0.06z^0 + 0.36z^{0.6} + 0.18z^1$$
$$+ 0.04z^0 + 0.24z^{0.6} + 0.12z^1 = 0.3z^1 + 0.6z^{0.6} + 0.1z^0$$

$$U_{\tilde{G}2}(z) = U_2(z) \underset{\tilde{G}}{\otimes} u_w(z)$$
$$= (0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1) \underset{G \cdot F}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.03z^0 + 0.21z^0 + 0.18z^{0.8} + 0.18z^1 + 0.02z^0 + 0.14z^{0.4}$$
$$+ 0.12z^{0.8} + 0.12z^1 = 0.3z^1 + 0.3z^{0.8} + 0.14z^{0.4} + 0.26z^0$$

The system conditional expected performance is

$$\tilde{\varepsilon}_1 = U'_{\tilde{G}1}(1) / U'_{F1}(1) = (0.3 \times 1 + 0.6 \times 0.6 + 0.1 \times 0) / 0.9$$
$$= 0.66 / 0.9 = 0.733$$

$$\tilde{\varepsilon}_2 = U'_{\tilde{G}2}(z)/U'_{F2}(1) = (0.3\times 1 + 0.3\times 0.8 + 0.14\times 0.4 + 0.26\times 0)/0.74$$
$$= 0.596/0.74 = 0.805$$

This means that generators 1 and 2 when they meet the variable demand have average capacities 73.3 MW and 80.5 MW respectively.

Note that the acceptability function $F(G,W)$ is a binary one. Therefore, if the demand is constant ($W \equiv w$), operator $U_F(z) = U_{\text{MSS}}(z) \otimes_F w$ produces a u-function in which all of the terms corresponding to the acceptable states will have the exponent 1 and all of the terms corresponding to the unacceptable states will have the exponent 0. It can easily be seen that $U'_F(1)$ is equal to the sum of the coefficients of the terms with exponents 1. Therefore, instead of obtaining the u-function of $F(G, w)$ and calculating its derivative at z=1 for determining the system's reliability (availability), one can calculate the sum of the terms in $U_{\text{MSS}}(z)$ that correspond to the acceptable states. Introducing an operator $\delta_w(U_{\text{MSS}}(z))$ that produces the sum of the coefficients of those terms in $U_{\text{MSS}}(z)$ that have exponents g satisfying the condition $F(g,w)$= 1 (and correspond to the states acceptable for the demand level w) we obtain the following simple expression for the system's expected acceptability:

$$E(F(G,w)) = \delta_w(U_{\text{MSS}}(z)) \tag{2.129}$$

When the demand is variable, the system reliability can also be obtained as

$$\sum_{i=1}^{M} \Pr(W = w_i)E(F(G,w_i)) = \sum_{i=1}^{M} q_i E(F(G,w_i)) = \sum_{i=1}^{M} q_i \delta_{w_i}(U_{\text{MSS}}(z)) \tag{2.130}$$

Example 2.15

Consider the two power system generators presented in the previous example and obtain the system availability directly from the u-functions $U_{\text{MSS}}(z)$ using Eq. (2.130). Since, in this example, $F(G,W) = 1(G \geq W)$, the operator $\delta_w(U(z))$ sums up the coefficients of the terms having exponents not less than w in the u-function $U(z)$:
For the first generator with $U_1(z) = 0.1z^0 + 0.6z^{0.6} + 0.3z^1$

$$E(F(G_1,w_1)) = \delta_{0.5}(0.1z^0 + 0.6z^{0.6} + 0.3z^1) = 0.6 + 0.3 = 0.9$$

$$E(F(G_1, w_2)) = \delta_{0.3}(0.1z^0 + 0.6z^{0.6} + 0.3z^1) = 0.6 + 0.3 = 0.9$$

$$A_1 = q_1 E(F(G_1, w_1)) + q_2 E(F(G_1, w_2)) = 0.6 \times 0.9 + 0.4 \times 0.9 = 0.9$$

For the second generator with $U_2(z) = 0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1$

$$E(F(G_2, 0.5)) = \delta_{0.5}(0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1) = 0.3+0.3=0.6$$

$$E(F(G_2, 0.3)) = \delta_{0.3}(0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1)$$
$$= 0.35 + 0.3 + 0.3 = 0.95$$

$$A_1 = q_1 E(F(G_2, w_1)) + q_2 E(F(G_2, w_2)) = 0.6 \times 0.6 + 0.4 \times 0.95 = 0.74$$

2.2.4 UGF in analysis of series-parallel multi-state systems

Reliability block diagram method

Having a generic model of an MSS in the form (1.3) and (1.4) we can obtain the measures of MSS reliability and performance by applying the following steps:

1. Represent the pmf of the random performance of each system element j, in the form of the u-function

$$u_j(z) = \sum_{i=0}^{k_j - 1} p_{ji} z^{g_{ji}}, \quad 1 \le j \le n \tag{2.131}$$

2. Obtain the u-function of the entire system (representing the pmf of the random variable G) applying the composition operator that uses the system structure function.

3. Obtain the u-functions representing the random functions F, $\tilde{G}$ and D using operators (2.122)-(2.124).

4. Obtain the system reliability measures by calculating the values of the derivatives of the corresponding u-functions at z=1 and applying Equations. (2.125)-(2.128).

While steps 1 and 3 are rather trivial, step 2 may involve complicated computations. Indeed, the derivation of a system structure function for various types of system is usually a difficult task.

As shown in Section 2.2.1, representing the functions in the recursive form is beneficial from both the derivation clarity and computation simplicity viewpoints. In many cases, the structure function of the entire MSS can be represented as the composition of the structure functions corresponding to some subsets of the system elements (MSS subsystems). The *u*-functions of the subsystems can be obtained separately and the subsystems can be further treated as single equivalent elements with the performance pmf represented by these *u*-functions.

The method for distinguishing recurrent subsystems and replacing them with single equivalent elements is based on a graphical representation of the system structure and is referred to as the reliability block diagram method. This approach is usually applied to systems with a complex series-parallel configuration.

While the structure function of a binary series-parallel system is unambiguously determined by its configuration (represented by the reliability block diagram), the structure function of a series-parallel MSS also depends on the physical meaning of the system and of the elements' performance and on the nature of the interaction among the elements.

Series Systems. In the flow transmission MSS, where performance is defined as capacity or productivity, the total capacity of a subsystem containing n independent elements connected in series is equal to the capacity of a bottleneck element (the element with least performance). Therefore, the structure function for such a subsystem takes the form

$$\phi_{\text{ser}}(G_1,...,G_n) = \min\{G_1,...,G_n\} \tag{2.132}$$

In the task processing MSS, where the performance is defined as the processing speed (or operation time), each system element has its own operation time and the system's total task completion time is restricted. The entire system typically has a time resource that is larger than the time needed to perform the system's total task. However, unavailability or deteriorated performance of the system elements may cause time delays, which in turn would cause the system's total task performance time to be unsatisfactory. The definition of the structure function for task processing systems depends on the discipline of the elements' interaction in the system.

When the system operation is associated with consecutive discrete actions performed by the ordered line of elements, each element starts its operation after the previous one has completed its operation. Assume that the random performances G_j of each element j is characterized by its processing speed. The random processing time T_j of any system element j is defined as $T_j = 1/G_j$. The total time of task completion for the entire system is

$$T = \sum_{j=1}^{n} T_j = \sum_{j=1}^{n} G_j^{-1} \tag{2.133}$$

The entire system processing speed is therefore

$$G = 1/T = (\sum_{j=1}^{n} G_j^{-1})^{-1} \tag{2.134}$$

Note that if for any j G_j=0 the equation cannot be used, but it is obvious that in this case G=0. Therefore, one can define the structure function for the series task processing system as

$$\phi_{\text{ser}}(G_1,...,G_n) = \times(G_1,...,G_n) = \begin{cases} 1/\sum_{j=1}^{n} G_j^{-1} & \text{if} \quad \prod_{j=1}^{n} G_j \neq 0 \\ 0 & \text{if} \quad \prod_{j=1}^{n} G_j = 0 \end{cases} \tag{2.135}$$

One can see that the structure functions presented above are associative and commutative (*i.e.* meet conditions (2.114) and (2.116)). Therefore, the u-functions for any series system of described types can be obtained recursively by consecutively determining the u-functions of arbitrary subsets of the elements. For example, the u-function of a system consisting of four elements connected in a series can be determined in the following ways:

$$\begin{aligned} &[(u_1(z) \underset{\phi_{\text{ser}}}{\otimes} u_2(z)) \underset{\phi_{\text{ser}}}{\otimes} u_3(z)] \underset{\phi_{\text{ser}}}{\otimes} u_4(z) \\ &= (u_1(z) \underset{\phi_{\text{ser}}}{\otimes} u_2(z)) \underset{\phi_{\text{ser}}}{\otimes} (u_3(z) \underset{\phi_{\text{ser}}}{\otimes} u_4(z)) \end{aligned} \tag{2.136}$$

and by any permutation of the elements' u-functions in this expression.

Example 2.16

Consider a system consisting of n elements with the total failures connected in series. Each element j has only two states: operational with a nominal performance of g_{j1} and failure with a performance of zero. The probability of the operational state is p_{j1}. The u-function of such an element is presented by the following expression:

$$u_j(z) = (1 - p_{j1})z^0 + p_{j1}z^{g_{j1}}, \quad j = 1, \ldots, n$$

In order to find the u-function for the entire MSS, the corresponding $\otimes_{\phi_{\text{ser}}}$ operators should be applied. For the MSS with the structure function (2.132) the system u-function takes the form

$$U(z) = \underset{\min}{\otimes}(u_1(z),...,u_n(z)) = (1 - \prod_{j=1}^{n} p_{j1})z^0 + \prod_{j=1}^{n} p_{j1}z^{\min\{g_{11},...,g_{n1}\}}$$

For the MSS with the structure function (2.135) the system u-function takes the form

$$U(z) = \underset{\times}{\otimes}\{u_1(z),...,u_n(z)\} = (1-\prod_{j=1}^{n} p_{j1})z^0 + \prod_{j=1}^{n} p_{j1} z^{(\sum_{j=1}^{n} g_{j1}^{-1})^{-1}}$$

Since the failure of each single element causes the failure of the entire system, the MSS can have only two states: one with the performance level of zero (failure of at least one element) and one with the performance level $\hat{g} = \min\{g_{11},..., g_{n1}\}$ for the flow transmission MSS and $\hat{g} = 1/\sum_{j=1}^{n} g_{j1}^{-1}$ for the task processing MSS.

The measures of the system performance $A(w) = \Pr\{G \geq w\}$, $\Delta^{-}(w) = E(\max(w-G,0))$ and $\varepsilon = E(G)$ are presented in the Table 2.4.

Table 2.4. Measures of MSS performance

w	$A(w)$	$\Delta^{-}(w)$	ε
$w > \hat{g}$	0	$w(1-\prod_{j=1}^{n} p_{j1}) + (w-\hat{g})\prod_{j=1}^{n} p_{j1} = w - \hat{g}\prod_{j=1}^{n} p_{j1}$	$\hat{g}\prod_{j=1}^{n} p_{j1}$
$0 < w \leq \hat{g}$	$\prod_{j=1}^{n} p_{j1}$	$w(1-\prod_{j=1}^{n} p_{j1})$	

The u-function of a subsystem containing n identical elements ($p_{j1}=p$, $g_{j1}=g$ for any j) takes the form

$$(1-p^n)z^0 + p^n z^g \tag{2.137}$$

for the system with the structure function (2.132) and takes the form

$$(1-p^n)z^0 + p^n z^{g/n} \tag{2.138}$$

for the system with the structure function (2.135).

Parallel Systems. In the flow transmission MSS, in which the flow can be dispersed and transferred by parallel channels simultaneously (which provides the work sharing), the total capacity of a subsystem containing n independent elements connected in parallel is equal to the sum of the capacities of the individual elements. Therefore, the structure function for such a subsystem takes the form

$$\phi_{\text{par}}(G_1,...,G_n) = +(G_1,...,G_n) = \sum_{j=1}^{n} G_j. \qquad (2.139)$$

In some cases, only one channel out of n can be chosen for the flow transmission (no flow dispersion is allowed). This happens when the transmission is associated with the consumption of certain limited resources that does not allow simultaneous use of more than one channel. The most effective way for such a system to function is by choosing the channel with the greatest transmission capacity from the set of available channels. In this case, the structure function takes the form

$$\phi_{\text{par}}(G_1,...,G_n) = \max\{G_1,...,G_n\}. \qquad (2.140)$$

In the task processing MSS, the definition of the structure function depends on the nature of the elements' interaction within the system.

First consider a system without work sharing in which the parallel elements act in a competitive manner. If the system contains n parallel elements, then all the elements begin to execute the same task simultaneously. The task is assumed to be completed by the system when it is completed by at least one of its elements. The entire system processing time is defined by the minimum element processing time and the entire system processing speed is defined by the maximum element processing speed. Therefore, the system structure function coincides with (2.140).

Now consider a system of n parallel elements with work sharing for which the following assumptions are made:

1. The work x to be performed can be divided among the system elements in any proportion.
2. The time required to make a decision about the optimal work sharing is negligible, the decision is made before the task execution and is based on the information about the elements state during the instant the demand for the task executing arrives.
3. The probability of the elements failure during any task execution is negligible.

The elements start performing the work simultaneously, sharing its total amount x in such a manner that element j has to perform x_j portion of the work and $x = \sum_{j=1}^{n} x_j$. The time of the work processed by element j is x_j/G_j. The system processing time is defined as the time during which the last portion of work is completed: $T = \max_{1 \le j \le n}\{x_j / G_j\}$. The minimal time of the entire work completion can be achieved if the elements share the work in proportion to their processing speed G_j: $x_j = xG_j / \sum_{k=1}^{n} G_k$.

The system processing time T in this case is equal to $x/\sum_{k=1}^{n} G_k$ and its total processing speed G is equal to the sum of the processing speeds of its elements. Therefore, the structure function of such a system coincides with the structure function (2.139).

One can see that the structure functions presented also meet conditions (2.114) and (2.116). Therefore, the u-functions for any parallel system of described types can be obtained recursively by the consecutive determination of u-functions of arbitrary subsets of the elements.

Example 2.17

Consider a system consisting of two elements with total failures connected in parallel. The elements have nominal performance g_{11} and g_{21} ($g_{11}<g_{21}$) and the probability of operational state p_{11} and p_{21} respectively. The performances in the failed states are $g_{10} = g_{20} =0$. The u-function for the entire MSS is

$$U(z) = u_1(z) \underset{\phi_{\text{par}}}{\otimes} u_2(z)$$

$$= [(1-p_{11})z^0 + p_{11}z^{g_{11}}] \underset{\phi_{\text{par}}}{\otimes} [(1-p_{21})z^0 + p_{21}z^{g_{21}}]$$

which for structure function (2.139) takes the form

$$U(z) = (1-p_{11})(1-p_{21})z^0 + p_{11}(1-p_{21})z^{g_{11}} + p_{21}(1-p_{11})z^{g_{21}} + p_{11}p_{21}z^{g_{11}+g_{21}}$$

and for structure function (2.140) takes the form

$$U(z) = (1-p_{11})(1-p_{21})z^0 + p_{11}(1-p_{21})z^{g_{11}} + p_{21}(1-p_{11})z^{g_{21}} + p_{11}p_{21}z^{\max(g_{11},g_{21})} = (1-p_{11})(1-p_{21})z^0 + p_{11}(1-p_{21})z^{g_{11}} + p_{21}z^{g_{21}}.$$

The measures of the system output performance for MSSs of both types are presented in Tables 2.5 and 2.6.

Table 2.5. Measures of MSS performance for system with structure function (2.139)

w	$A(w)$	$\Delta^-(w)$	ε
$w>g_{11}+g_{21}$	0	$w-p_{11}g_{11}-p_{21}g_{21}$	
$g_{21}<w\le g_{11}+g_{21}$	$p_{11}p_{21}$	$g_{11}p_{11}(p_{21}-1)+g_{21}p_{21}(p_{11}-1)+w(1-p_{11}p_{21})$	
$g_{11}<w\le g_{21}$	p_{21}	$(1-p_{21})(w-g_{11}p_{11})$	$p_{11}g_{11}+p_{21}g_{21}$
$0<w\le g_{11}$	$p_{11}+p_{21}-p_{11}p_{21}$	$(1-p_{11})(1-p_{21})w$	

Table 2.6. Measures of MSS performance for system with structure function (2.140)

w	$A(w)$	$\Delta^{-}(w)$	ε
$w>g_{21}$	0	$w-p_{11}g_{11}-p_{21}g_{21}+p_{11}p_{21}g_{11}$	
$g_{11}<w\le g_{21}$	p_{21}	$(1-p_{21})(w-g_{11}p_{11})$	$p_{11}(1-p_{21})g_{11}+p_{21}g_{21}$
$0<w\le g_{11}$	$p_{11}+p_{21}-p_{11}p_{21}$	$(1-p_{11})(1-p_{21})w$	

The u-function of a subsystem containing n identical parallel elements ($p_{j1} = p$, $g_{j1} = g$ for any j) can be obtained by applying the operator $\otimes_{\phi_{\text{par}}}(u(z),....,u(z))$ over n identical u-functions $u(z)$ of an individual element. The u-function of this subsystem takes the form

$$\sum_{k=0}^{n} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} z^{kg} \tag{2.141}$$

for the structure function (2.139) and

$$(1-p)^n z^0 + (1-(1-p)^n) z^g \tag{2.142}$$

for the structure function (2.140).

Series-Parallel Systems. The structure functions of complex series-parallel systems can always be represented as compositions of the structure functions of statistically independent subsystems containing only elements connected in a series or in parallel. Therefore, in order to obtain the u-function of a series-parallel system one has to apply the composition operators recursively in order to obtain u-functions of the intermediate pure series or pure parallel structures.

The following algorithm realizes this approach:

1. Find the pure parallel and pure series subsystems in the MSS.
2. Obtain u-functions of these subsystems using the corresponding $\otimes_{\phi_{\text{ser}}}$ and $\otimes_{\phi_{\text{par}}}$ operators.
3. Replace the subsystems with single elements having the u-function obtained for the given subsystem.
4. If the MSS contains more then one element return to step 1.

The resulting u-function represents the performance distribution of the entire system.

The choice of the structure functions used for series and parallel subsystems depends on the type of system. Table 2.7 presents the possible combinations of structure functions corresponding to the different types of MSS.

Table 2.7. Structure functions for a purely series and for purely parallel subsystems

No of MSS type	Description of MSS	Structure function for series elements (ϕ_{ser})	Structure function for parallel elements (ϕ_{par})
1	Flow transmission MSS with flow dispersion	(2.132)	(2.139)
2	Flow transmission MSS without flow dispersion	(2.132)	(2.140)
3	Task processing MSS with work sharing	(2.135)	(2.139)
4	Task processing MSS without work sharing	(2.135)	(2.140)

2.2.5 Combination of random processes methods and the UGF technique

In many cases, the state probability distributions of system elements are unknown whereas the state transition rates (failure and repair rates) can easily be evaluated from history data or mathematical models. The Markov process theory allows analyst to obtain probability of any system state at any time solving a system of differential equations.

The main difficulty of the random processes methods application to the MSS reliability evaluation is the "dimension damnation". Indeed, the number of differential equations in the system that should be solved using the Markov approach is equal to the total number of MSS states (product of numbers of states of all of the system elements). This number can be very large even for a relatively small MSS. Even though the modern software tools provide solutions for high-order systems of differential equations, building the state-space diagram and deriving the corresponding system of differential equations is a difficult non-formalized process that may cause numerous mistakes.

The UGF-based reliability block diagram technique can be used for reducing the dimension of system of equations obtained by the random process method. The main idea of the approach lies in solving the separated smaller systems of equations for each MSS element and then combining the solutions using the UGF technique in order to obtain the dynamic behavior of the entire system. The approach not only separates the equations but also reduces the total number of equations to be solved. The approach was introduced in (Lisnianski & Levitin 2003) and extended in (Lisnianski 2004) and in (Lisnianski 2007b). The basic steps of the approach are as follows:

1. Build the random process Markov model for each MSS element (considering only state transitions within this element). Obtain two sets $\boldsymbol{g_j}=\{g_{j1},g_{j2},\dots,g_{jk_j}\}$ and $\boldsymbol{p_j}(t)=\{p_{j1}(t),p_{j2}(t),\dots,p_{jk_j}(t)\}$ for each element j ($1\le j\le n$) by solving the system of k_j ordinary differential equations. Note that instead of solving one high-order system of $\prod_{j=1}^{n} k_j$ equations one has to solve n low-order systems with the total number of equations $\sum_{j=1}^{n} k_j$.
2. Having the sets $\boldsymbol{g_j}$ and $\boldsymbol{p_j}(t)$ for each element j define u-function of this element in the form $u_j(z)=p_{j1}(t)z^{g_{j1}}+p_{j2}(t)z^{g_{j2}}+\dots+p_{jk_j}(t)z^{g_{jk_j}}$.
3. Using the generalized RBD method, obtain the resulting u-function for the entire MSS.
4. Obtain the u-functions representing the random functions F , $\tilde{G}$ and D using operators (2.122)-(2.124).
5. Obtain the system reliability measures by calculating the values of the derivatives of the corresponding u-functions at z=1 and applying Eqs. (2.125)-(2.128).

Example 2.18

Consider a flow transmission system (Fig. 2.25) consisting of three pipes. The oil flow is transmitted from point C to point E. The pipes' performance is measured by their transmission capacity (ton per minute). Elements 1 and 2 are binary. A state of total failure for both elements corresponds to a transmission capacity of 0 and the operational state corresponds to the capacities of the elements 1.5 and 2 ton per minute respectively so that $G_1(\mathrm{t})\in\{0,1.5\}$, $G_2(\mathrm{t})\in\{0,2\}$. Element 3 can be in one of three states: a state of total failure corresponding to a capacity of 0, a state of partial failure corresponding to a capacity of 1.8 tons per minute and a fully operational state with a capacity of 4 tons per minute so that $G_3(\mathrm{t})\in\{0,1.8,4\}$. The demand is constant: θ^*=1.0 ton per minute.

The system output performance rate $V(t)$ is defined as the maximum flow that can be transmitted between nodes A and B: $V(t)=\min\{G_1(t)+G_2(t),G_3(t)\}$.

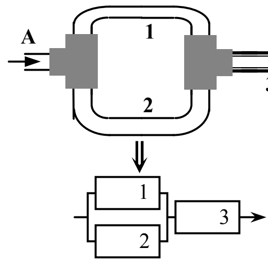


Fig. 2.25. Simple flow transmission MSS

The state-space diagrams of the system elements are shown in Fig. 2.26.

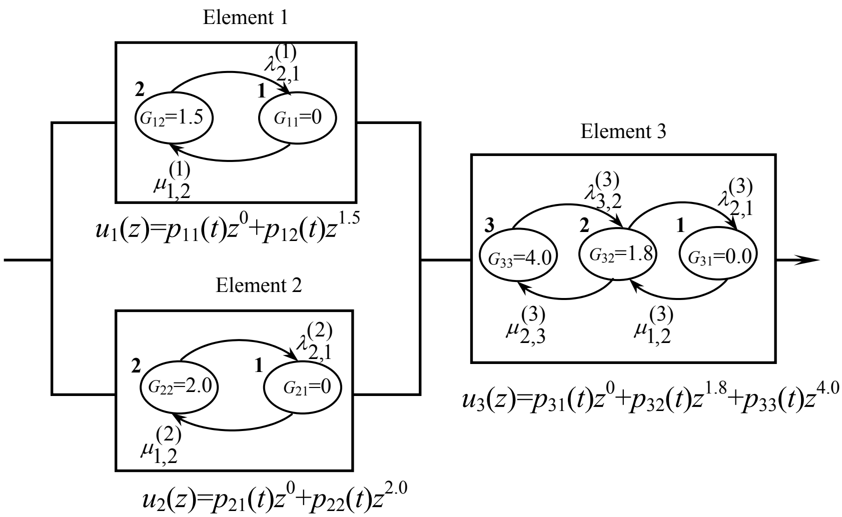


Fig. 2.26. State-space diagrams and u-functions of system elements

The failure rates and repair rates corresponding to these two elements are

$$\lambda_{2,1}^{(1)} = 7\,year^{-1}, \mu_{1,2}^{(1)} = 100\,year^{-1} \text{ for element 1,}$$

$$\lambda_{2,1}^{(2)} = 10 year^{-1}, \mu_{1,2}^{(2)} = 80 year^{-1} \text{ for element 2.}$$

Element 3 is a multi-state element with only minor failures and minor repairs. The failure rates and repair rates corresponding to element 3 are

$$\lambda_{3,2}^{(3)} = 10 year^{-1}, \lambda_{3,1}^{(3)} = 0, \lambda_{2,1}^{(3)} = 7 year^{-1},$$
$$\mu_{1,3}^{(3)} = 0, \mu_{1,2}^{(3)} = 120 year^{-1}, \mu_{2,3}^{(3)} = 110 year^{-1}.$$

According to the classical Markov approach, one has to enumerate all the system states corresponding to different combinations of all possible states of system elements (characterized by their performance levels). The total number of different system states is $K=k_1k_2k_3=2*2*3=12$. The state-space diagram of the system is shown in Fig. 2.27 (in this diagram the vector of element performances for each state and the corresponding system performance are shown respectively in the upper and lower parts of the ellipses).

Then the state transition analysis should be performed for all pairs of system states. For example, for the state number 2 where states of the elements are $\{g_{11},g_{22},g_{33}\}=\{2,4,2\}$ the transitions to states 1, 5 and 6 exist with the intensities $\mu_{1,2}^{(1)}, \lambda_{2,1}^{(2)}, \lambda_{3,2}^{(3)}$ respectively.

The corresponding system of differential equations for the state probabilities $p_i(t)$, $2 \le i \le 12$ takes the form:

$$\frac{dp_1(t)}{dt} = -(\lambda_{2,1}^{(1)} + \lambda_{2,1}^{(2)} + \lambda_{3,2}^{(3)})p_1(t) + \mu_{1,2}^{(1)}p_2(t) + \mu_{1,2}^{(2)}p_3(t) + \mu_{2,3}^{(3)}p_4(t),$$

$$\frac{dp_2(t)}{dt} = \lambda_{2,1}^{(1)}p_1(t) - (\mu_{1,2}^{(1)} + \lambda_{2,1}^{(2)} + \lambda_{3,2}^{(3)})p_2(t) + \mu_{1,2}^{(2)}p_5(t) + \mu_{2,3}^{(3)}p_6(t),$$

$$\frac{dp_3(t)}{dt} = \lambda_{2,1}^{(2)}p_1(t) - (\mu_{1,2}^{(2)} + \lambda_{2,1}^{(1)} + \lambda_{3,2}^{(3)})p_3(t) + \mu_{1,2}^{(1)}p_5(t) + \mu_{2,3}^{(3)}p_7(t),$$

$$\frac{dp_4(t)}{dt} = \lambda_{2,3}^{(3)}p_1(t) - (\mu_{2,3}^{(3)} + \lambda_{2,1}^{(1)} + \lambda_{2,1}^{(2)} + \lambda_{2,1}^{(3)})p_4(t) + \mu_{1,2}^{(1)}p_6(t) +$$
$$\mu_{1,2}^{(2)}p_7(t) + \mu_{1,2}^{(3)}p_8(t),$$

$$\frac{dp_5(t)}{dt} = \lambda_{2,1}^{(2)}p_2(t) + \lambda_{2,1}^{(1)}p_3(t) - (\mu_{1,2}^{(2)} + \mu_{1,2}^{(1)} + \lambda_{3,2}^{(3)})p_5(t) + \mu_{2,3}^{(3)}p_9(t),$$

$$\frac{dp_6(t)}{dt} = \lambda_{3,2}^{(3)} p_2(t) + \lambda_{2,1}^{(1)} p_4(t) - (\mu_{2,3}^{(3)} + \mu_{1,2}^{(1)} + \lambda_{2,1}^{(2)} + \lambda_{2,1}^{(3)}) p_6(t) + \mu_{1,2}^{(2)} p_9(t) + \mu_{1,2}^{(3)} p_{10}(t),$$

$$\frac{dp_7(t)}{dt} = \lambda_{3,2}^{(3)} p_3(t) + \lambda_{2,1}^{(1)} p_4(t) - (\mu_{2,3}^{(3)} + \mu_{1,2}^{(2)} + \lambda_{1,2}^{(1)} + \lambda_{2,1}^{(3)}) p_7(t) + \mu_{1,2}^{(1)} p_9(t) + \mu_{1,2}^{(3)} p_{11}(t),$$

$$\frac{dp_8(t)}{dt} = \lambda_{2,1}^{(3)} p_4(t) - (\mu_{1,2}^{(3)} + \lambda_{2,1}^{(1)} + \lambda_{2,1}^{(2)}) p_8(t) + \mu_{1,2}^{(1)} p_{10}(t) + \mu_{1,2}^{(2)} p_{11}(t),$$

$$\frac{dp_9(t)}{dt} = \lambda_{3,2}^{(3)} p_5(t) + \lambda_{2,1}^{(2)} p_6(t) + \lambda_{2,1}^{(1)} p_7(t) - (\mu_{2,3}^{(3)} + \mu_{1,2}^{(2)} + \mu_{1,2}^{(1)} + \lambda_{2,1}^{(3)}) p_9(t) + \mu_{1,2}^{(1)} p_{10}(t) + \mu_{1,2}^{(3)} p_{12}(t),$$

$$\frac{dp_{10}(t)}{dt} = \lambda_{2,1}^{(3)} p_6(t) + \lambda_{2,1}^{(1)} p_8(t) - (\mu_{1,2}^{(3)} + \mu_{1,2}^{(1)} + \lambda_{2,1}^{(2)}) p_{10}(t) + \mu_{1,2}^{(2)} p_{12}(t),$$

$$\frac{dp_{11}(t)}{dt} = \lambda_{2,1}^{(3)} p_7(t) + \lambda_{2,1}^{(2)} p_8(t) - (\mu_{1,2}^{(3)} + \mu_{1,2}^{(2)} + \lambda_{2,1}^{(1)}) p_{11}(t) + \mu_{1,2}^{(1)} p_{12}(t),$$

$$\frac{dp_{12}(t)}{dt} = \lambda_{2,1}^{(3)} p_9(t) + \lambda_{2,1}^{(2)} p_{10}(t) + \lambda_{2,1}^{(1)} p_{11}(t) - (\mu_{1,2}^{(3)} + \mu_{1,2}^{(2)} + \mu_{1,2}^{(1)}) p_{12}(t).$$

Solving this system with the initial conditions $p_1(0) = 1$, $p_i(0) = 0$ for $2 \le i \le 12$ one obtains the probability of each state at time t.

According to Fig. 2.27, in different states MSS has the following performance rates: in the state 1 v_1=3.5, in the state 2 v_2=2.0, in the states 4 and 6 $v_4 = v_6$=1.8, in the states 3 and 7 $v_3 = v_7$=1.5, in the states 5, 8, 9, 10, 11 and 12 $v_5 = v_8 = v_9 = v_{10} = v_{11} = v_{12} = 0$. Therefore,

$\Pr\{V=3.5\} = p_1(t)$, $\Pr\{V=2.0\} = p_2(t)$, $\Pr\{V=1.8\} = p_4(t) + p_6(t)$,

$\Pr\{V=0\} = p_5(t) + p_8(t) + p_9(t) + p_{10}(t) + p_{11}(t) + p_{12}(t)$.

For the constant demand level $\theta^*=1$ one obtains the MSS instantaneous availability as a sum of states probabilities where the MSS output performance is greater than or equal to 1. The states 1, 2, 3, 4, 6 and 7 are acceptable. Hence $A(t) = p_1(t) + p_2(t) + p_3(t) + p_4(t) + p_6(t) + p_7(t)$.

The MSS instantaneous expected performance is $W(t)=\sum_{i=1}^{12} p_i(t)v_i$.

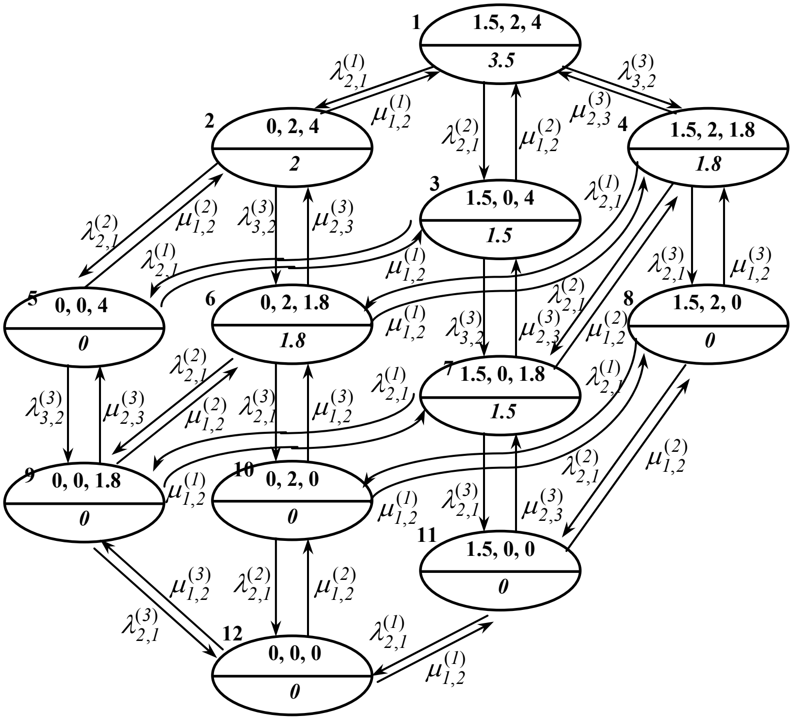


Fig. 2.27. State-space diagram for the entire system

Solving the system of 12 differential equations is quite complicated task that can be solved only numerically. Applying the combination of Markov and UGF technique one can drastically simplify the calculations and even obtain an analytical solution for the reliability and expected performance of the given system. One should proceed as follows:

1. According to the Markov method build the following systems of differential equations for each element separately (using the state-space diagrams presented in Fig. 2.28):

   For element 1:

$$\begin{cases} dp_{11}(t)/dt = -\mu_{1,2}^{(1)} p_{11}(t) + \lambda_{2,1}^{(1)} p_{12}(t) \\ dp_{12}(t)/dt = -\lambda_{2,1}^{(1)} p_{12}(t) + \mu_{1,2}^{(1)} p_{11}(t) \end{cases}$$

Initial conditions are: $p_{12}(0)=1$, $p_{11}(0)=0$.

For element 2:

$$\begin{cases} dp_{21}(t)/dt = -\mu_{1,2}^{(2)} p_{21}(t) + \lambda_{2,1}^{(2)} p_{22}(t) \\ dp_{22}(t)/dt = -\lambda_{2,1}^{(2)} p_{22}(t) + \mu_{1,2}^{(2)} p_{21}(t) \end{cases}$$

Initial conditions are: $p_{21}(0)=1$, $p_{22}(0)=0$.

For element 3:

$$\begin{cases} dp_{31}(t)/dt = -\mu_{1,2}^{(3)} p_{31}(t) + \lambda_{2,1}^{(3)} p_{32}(t) \\ dp_{32}(t)/dt = \lambda_{3,2}^{(3)} p_{33}(t) - (\lambda_{2,1}^{(3)} + \mu_{2,3}^{(3)}) p_{32}(t) + \mu_{1,2}^{(3)} p_{31}(t) \\ dp_{33}(t)/dt = -\lambda_{3,2}^{(3)} p_{33}(t) + \mu_{2,3}^{(3)} p_{32}(t) \end{cases}$$

Initial conditions are: $p_{31}(0)=p_{32}(0)=0$, $p_{33}(0)=1$.

After solving the three separate systems of differential equations under the given initial conditions, we get the following expressions for state probabilities:

For element 1:

$$p_{11}(t) = \frac{\lambda_{2,1}^{(1)}}{\mu_{1,2}^{(1)} + \lambda_{2,1}^{(1)}} - \frac{\lambda_{2,1}^{(1)}}{\mu_{1,2}^{(1)} + \lambda_{2,1}^{(1)}} e^{-(\lambda_{2,1}^{(1)} + \mu_{1,2}^{(1)})t},$$

$$p_{12}(t) = \frac{\mu_{1,2}^{(1)}}{\mu_{1,2}^{(1)} + \lambda_{2,1}^{(1)}} + \frac{\lambda_{2,1}^{(1)}}{\mu_{1,2}^{(1)} + \lambda_{2,1}^{(1)}} e^{-(\lambda_{2,1}^{(1)} + \mu_{1,2}^{(1)})t},$$

For element 2:

$$p_{21}(t) = \frac{\lambda_{2,1}^{(2)}}{\mu_{1,2}^{(2)} + \lambda_{2,1}^{(2)}} - \frac{\lambda_{2,1}^{(2)}}{\mu_{1,2}^{(2)} + \lambda_{2,1}^{(2)}} e^{-(\lambda_{2,1}^{(2)} + \mu_{1,2}^{(2)})t},$$

$$p_{22}(t) = \frac{\mu_{1,2}^{(2)}}{\mu_{1,2}^{(2)} + \lambda_{2,1}^{(2)}} + \frac{\lambda_{2,1}^{(2)}}{\mu_{1,2}^{(2)} + \lambda_{2,1}^{(2)}} e^{-(\lambda_{2,1}^{(2)} + \mu_{1,2}^{(2)})t},$$

For element 3:

$$p_{31}(t) = A_1 e^{\alpha t} + A_2 e^{\beta t} + A_3,$$

$$p_{32}(t) = B_1 e^{\alpha t} + B_2 e^{\beta t} + B_3 ,$$

$$p_{33}(t) = C_1 e^{\alpha t} + C_2 e^{\beta t} + C_3 ,$$

where

$$\alpha = -\eta/2 + \sqrt{\eta^2/4 - \zeta}, \quad \beta = -\eta/2 - \sqrt{\eta^2/4 - \zeta},$$

$$A_1 = \frac{\lambda_{2,1}^{(3)} \lambda_{3,2}^{(3)}}{\alpha(\alpha-\beta)}, \quad A_2 = \frac{\lambda_{2,1}^{(3)} \lambda_{3,2}^{(3)}}{\beta(\beta-\alpha)}, \quad A_3 = \frac{\lambda_{2,1}^{(3)} \lambda_{3,2}^{(3)}}{\zeta},$$

$$B_1 = \frac{(\mu_{1,2}^{(3)} + \alpha)\lambda_{3,2}^{(3)}}{\alpha(\alpha-\beta)}, \quad B_2 = \frac{(\mu_{1,2}^{(3)} + \alpha)\lambda_{3,2}^{(3)}}{\beta(\beta-\alpha)}, \quad B_3 = \frac{\mu_{1,2}^{(3)} \lambda_{3,2}^{(3)}}{\zeta},$$

$$C_1 = \frac{(\mu_{1,2}^{(3)} + \alpha)\lambda_{3,2}^{(3)} \mu_{2,3}^{(3)}}{\alpha(\alpha-\beta)(\alpha + \lambda_{3,2}^{(3)})}, \quad C_2 = \frac{(\mu_{1,2}^{(3)} + \beta)\lambda_{3,2}^{(3)} \mu_{2,3}^{(3)}}{\beta(\beta-\alpha)(\beta + \lambda_{3,2}^{(3)})},$$

$$C_3 = \frac{\mu_{1,2}^{(3)} \mu_{2,3}^{(3)} (\beta + \lambda_{3,2}^{(3)} (\lambda_{3,2}^{(3)} - \alpha))}{\alpha\beta(\alpha + \lambda_{3,2}^{(3)})(\beta + \lambda_{3,2}^{(3)})},$$

$$\eta = \lambda_{2,1}^{(3)} + \lambda_{3,2}^{(3)} + \mu_{1,2}^{(3)} + \mu_{2,3}^{(3)}, \quad \zeta = \lambda_{2,1}^{(3)} \lambda_{3,2}^{(3)} + \mu_{1,2}^{(3)} \mu_{2,3}^{(3)} + \mu_{1,2}^{(3)} \lambda_{3,2}^{(3)}.$$

After determining the state probabilities for each element, we obtain the following performance distributions:

for element 1: $\boldsymbol{g}_1 = \{g_{11}, g_{12}\} = \{0, 1.5\}$, $\boldsymbol{p}_1(t) = \{p_{11}(t), p_{12}(t)\}$;

for element 2: $\boldsymbol{g}_2 = \{g_{21}, g_{22}\} = \{0, 2.0\}$, $\boldsymbol{p}_2(t) = \{p_{21}(t), p_{22}(t)\}$;

for element 3: $\boldsymbol{g}_3 = \{g_{31}, g_{32}, g_{33}\} = \{0, 1.8, 4.0\}$,

$\boldsymbol{p}_3(t) = \{p_{31}(t), p_{32}(t), p_{33}(t)\}$.

2. Having the sets $\boldsymbol{g}_j$, $\boldsymbol{p}_j(t)$ for j=1,2,3 obtained in the first step we can define the u-functions of the individual elements as:

$$u_1(z)=p_{11}(t)\,z^{g_{11}}+p_{12}(t)\,z^{g_{12}}=p_{11}(t)z^0+p_{12}(t)z^{1.5}.$$

$$u_2(z)=p_{21}(t)\,z^{g_{21}}+p_{22}(t)\,z^{g_{22}}=p_{21}(t)z^0+p_{22}(t)z^2.$$

$$u_3(z)=p_{31}(t)\,z^{g_{31}}+p_{32}(t)\,z^{g_{32}}+p_{33}(t)\,z^{g_{33}}=p_{31}(t)z^0+p_{32}(t)z^{1.8}+p_{33}(t)z^4.$$

3. Using the composition operators for flow transmission MSS we obtain the resulting u-function for the entire series-parallel MSS $U(z)=[u1(z)\underset{+}{\otimes}u2(z)]\underset{\min}{\otimes}u3(z)$ by the following recursive procedure:

$$u_1(z)\underset{+}{\otimes}u_2(z))=[p_{11}(t)z^0+p_{12}(t)z^{1.5}]\underset{+}{\otimes}[p_{21}(t)z^0+p_{22}(t)z^2]$$

$$=p_{11}(t)p_{21}(t)z^0+p_{12}(t)p_{21}(t)z^{1.5}+p_{11}(t)p_{22}(t)z^2+p_{12}(t)p_{22}(t)z^{3.5}.$$

$$U(z)=u_3(z)\underset{\min}{\otimes}[u_1(z)\underset{+}{\otimes}u_2(z)]$$

$$=[p_{31}(t)z^0+p_{32}(t)z^{1.8}+p_{33}(t)z^4]\underset{\min}{\otimes}p_{11}(t)p_{21}(t)z^0+p_{12}(t)p_{21}(t)z^{1.5}+p_{11}(t)p_{22}(t)z^2$$

$$+p_{12}(t)p_{22}(t)z^{3.5})=p_{31}(t)p_{11}(t)p_{21}(t)z^0+p_{31}(t)p_{12}(t)p_{21}(t)z^0+p_{31}(t)p_{11}(t)p_{22}(t)z^0$$

$$+p_{31}(t)p_{12}(t)p_{22}(t)z^0+p_{32}(t)p_{11}(t)p_{21}(t)z^0+p_{32}(t)p_{12}(t)p_{21}(t)z^{1.5}+p_{32}(t)p_{11}(t)p_{22}(t)z^{1.8}$$

$$+p_{32}(t)p_{12}(t)p_{22}(t)z^{1.8}+p_{33}(t)p_{11}(t)p_{21}(t)z^0+p_{33}(t)p_{12}(t)p_{21}(t)z^{1.5}+p_{33}(t)p_{11}(t)p_{22}(t)z^2$$

$$+p_{33}(t)p_{12}(t)p_{22}(t)z^{3.5}.$$

Taking into account that

$$p_{31}(t)+p_{32}(t)+p_{33}(t)=1,\ p_{21}(t)+p_{22}(t)=1 \text{ and } p_{11}(t)+p_{12}(t)=1,$$

we obtain the u-function that determines the performance distribution $\boldsymbol{v}$, $\boldsymbol{q}(t)$ of the entire MSS in the following form $U(z)=\sum_{i=1}^{5} q_i(t)z^{v_i}$ where

$v_1=0,$ $q_1(t)=p_{11}(t)p_{21}(t)+p_{31}(t)p_{12}(t)+p_{31}(t)p_{11}(t)p_{22}(t),$

$v_2=1.5$ tons/min, $q_2(t)=p_{12}(t)p_{21}(t)[p_{32}(t)+p_{33}(t)],$

$v_3=1.8$ tons/min, $q_3(t)=p_{32}(t)p_{22}(t),$

$v_4=2.0$ tons/min, $q_4(t)=p_{33}(t)p_{11}(t)p_{22}(t),$

$v_5=3.5$ tons/min, $q_5(t)=p_{33}(t)p_{12}(t)p_{22}(t).$

4. Based on the entire MSS u-function $U(z)$ we obtain the MSS reliability indices:

The instantaneous MSS availability for different demand levels θ* takes the form

$A(t) = q_2(t) + q_3(t) + q_4(t) + q_5(t)$ for $0<\theta^*\le 1.5$;

$A(t) = q_3(t) + q_4(t) + q_5(t)$ for $1.5<\theta^*\le 1.8$;

$A(t) = q_4(t) + q_5(t)$ for $1.8<\theta^*\le 2$;

$A(t) = q_5(t)$ for $2<\theta^*\le 3.5$;

$A(t) = 0$ for $3.5<\theta^*$.

The instantaneous expected performance at any instant $t>0$ is

$$W(t) = \sum_{i=1}^{5} q_i(t)v_i = 1.5q_2(t)+1.8q_3(t)+2q_4(t)+3.5q_5(t).$$

The obtained functions $W(t)$ and $A(t)$ for $\theta^*=1$ are presented in Fig. 2.28.

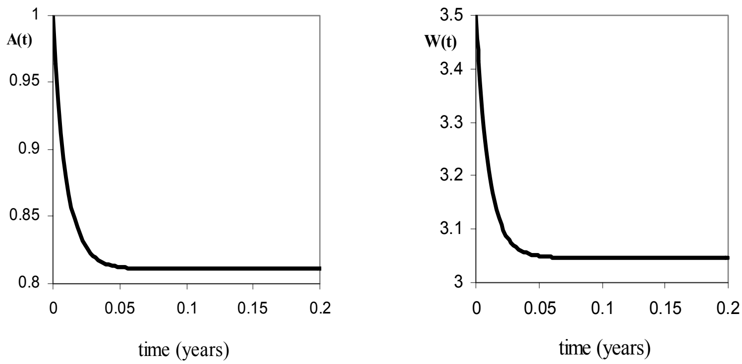


Fig. 2.28. System availability for $\theta^*=1$ and instantaneous expected performance

2.3 Monte Carlo simulation

Monte Carlo simulation (MCS) is applied for problems involving random variables with known or assumed probability distributions. This simulation process uses a particular set of values of random variables generated in accordance with the corresponding probability distributions in each simulation. The process is repeated using different sets of values of the random variables. The MCS results are presented in the histogram form which is especially useful for further statistical assessment. MCS process is actually *deterministic* for every given set of random numbers, generated in advance from prescribed probability distributions.

As such, simulation can be performed either analytically or numerically. With the advent of computers, numerically performed simulation has become a much more practical tool, widely applied to study the system performance for engineering purposes. The simulation process allows for estimating a specific performance with a given set of the system parameters. The sensitivity of the system performance to variation in the system parameters may be examined through repeated simulations. Simulation is also used to check alternative designs and determine the optimal one.

Monte Carlo methods can be applied to large and complex systems. They are generally used as a very effective mean when analytical solution methods are not available or too complex. For high accuracy, Monte Carlo method requires a large number of samplings.

2.3.1 Generation of random numbers

Generation of the random variables in accordance with the respective prescribed probability distributions is the significant part of the MCS process. This can be performed for each variable by prior generating a uniformly distributed random number between 0 and 1. The basis for this is as follows.

Suppose X is a random variable with distribution function $F(x)$. Then, at a given probability $F(x) = u$, the value of x is

$$x = F^{-1}(u) \tag{2.143}$$

Now suppose that u is a specific value of the standard uniformly distributed variable U, with a uniform density function ranging from 0 to 1. Then the corresponding value x of the variable X will have a probability $F(x)$. Therefore, if $(u_1, u_2, \ldots u_n)$ is a set of values from U, the corresponding set of values $(x_1, x_2, \ldots, x_n)$ obtained through Eq. (2.143) will model the desired distribution function $F(x)$. The relationship between u and x may be seen graphically in Fig. 2.29.

Uniformly distributed random numbers between 0 and 1 provide a basis for obtaining random numbers with a general probability distribution. In reality, random numbers generated by any systematic procedure, can be duplicated exactly, thus constituting a deterministic set. They are only “pseudo random” rather then random numbers. Note that the generated pseudo random numbers are cyclic; that is, they are repeated with a given, though rather large period. The computers are now commonly enhanced with an extremely fast built-in random number generator.

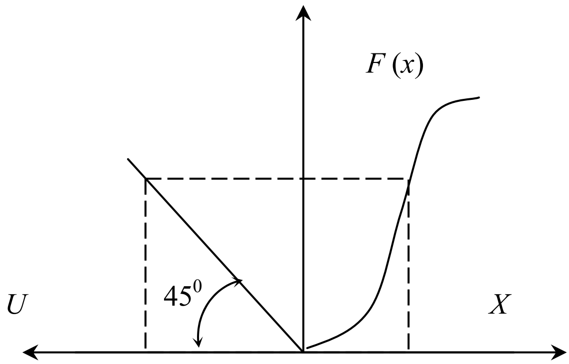


Fig. 2.29. Relation between *u* and *x*

Example 2.19

This example demonstrates using the MCS for computing the expected unsupplied energy (EUE) for the demand of 1000 MW in a generating system, which consists of 9 interconnected operating units with parameters presented in Table 2.8 where Forced Outage Range (FOR) is a probability that unit is not available. For simplicity, we assume that any unit i can have only two states: either perfect functioning with nominal capacity c_i or total failure with capacity 0 (which corresponds to the limiting values of the admissible capacity interval: $C_i \in \{0, c_i\}$):

Table 2.8. Parameters of the generating system

Unit	Nominal capacity c_i	FOR
1	200	0.2
2	200	0.2
3	200	0.2
4	150	0.1
5	150	0.1
6	100	0.15
7	100	0.15
8	100	0.15
9	100	0.15

Using the MCS we performed a set of 10 samplings each containing a randomly generated numbers $p_{i,j}$, $0 \le p_{i,j} \le 1$ (for units i=1,…,9 and samples

j=1,…,10) that define the probability of full availability for unit i (i =1,…,9) as follows

$$C = 0, \text{ when } p_{i,j} < \text{FOR}_i \text{ and } C_i = c_i \text{ otherwise} \qquad (2.144)$$

With this in view, the EUE is calculated as

$$EUE = \max(0, L - \sum_{i=1}^{9} C_i), \qquad (2.145)$$

where L is a supposed demand system value at the calculation moment (1000Mw).

Random numbers for 10 samplings have been generated as presented in Table 2.9.

Table 2.9. Ten randomly generated samples

Unit	Sample									
	1	2	3	4	5	6	7	8	9	10
1	0.57	0.47	0.03	0.93	0.28	0.62	0.52	0.09	0.99	0.33
2	0.01	0.13	0.59	0.38	0.72	0.07	0.19	0.64	0.43	0.78
3	0.12	0.80	0.25	0.71	0.17	0.18	0.85	0.31	0.77	0.22
4	0.90	0.24	0.92	0.40	0.94	0.95	0.30	0.98	0.10	1.00
5	0.34	0.69	0.70	0.49	0.61	0.04	0.74	0.75	0.54	0.67
6	0.23	0.02	0.37	0.27	0.05	0.29	0.08	0.42	0.32	0.11
7	0.68	0.91	0.81	0.60	0.83	0.73	0.97	0.87	0.65	0.89
8	0.79	0.35	0.48	0.15	0.50	0.84	0.41	0.53	0.21	0.55
9	0.45	0.58	0.14	0.82	0.39	0.51	0.63	0.20	0.88	0.44

The units' available capacities, obtained from these randomly generated numbers in accordance with (2.144) are presented in Table 2.10.

The expected EUE is obtained as the average value over 10 samples:

EUE = (100+250)/10 = 35 MWh.

The obtained estimation is pretty close to the theoretical assessment.

Repetition of such calculations for a successive set of moments (taken, for example, with an hourly resolution) estimates the system performance over a study period. The larger is the number of simulations the better is the statistical accuracy of the system assessment.

Table 2.10. Available capacity of units

Unit	Available capacity									
	1	2	3	4	5	6	7	8	9	10
1	200	200	0	200	200	200	200	0	200	200
2	0	0	200	200	200	0	0	200	200	200
3	0	200	200	200	0	0	200	200	200	200
4	150	150	150	150	150	150	150	150	0	150
5	150	150	150	150	150	0	150	150	150	150
6	100	0	100	100	0	100	0	100	100	0
7	100	100	100	100	100	100	100	100	100	100
8	100	100	100	100	100	100	100	100	100	100
9	100	100	0	100	100	100	100	100	100	100
Total	900	1000	1000	1300	1000	750	1000	1100	1150	1200
Eue	100	0	0	0	0	250	0	0	0	0

2.3.2 Continuous random variables

As indicated above, random numbers with a prescribed distribution can be generated using Equation. (2.143) once the standard uniformly distributed random numbers have been obtained. The so-called inverse transform method performs generation of random numbers due to Equation (2.143). It is especially effective when the inverse function $F^{-1}(u)$ can be expressed in a closed form as explained below

Example 2.20

Consider the exponential distribution

$$F(x) = 1 - e^{-\lambda x}$$

The inverse function is

$$x = F^{-1}(u) = -\frac{1}{\lambda}\ln(1-u)$$

After generating the standard uniformly distributed random numbers u_i for i = 1,2,..., the corresponding exponentially distributed random numbers are obtained, according to Eq. (2.1), as

$$x_i = -\frac{1}{\lambda}\ln(1-u_i)$$

Since $(1 - u_i)$ is also uniformly distributed, the required random numbers may also be generated as

$$x_i = -\frac{1}{\lambda}\ln u_i ; \;\; i = 1,2,...$$

Example 2.21

Let Y = max{X1, …,Xn}, where Xi represent independent and identically distributed random variables with distribution function *F*(*x*). The distribution function of *Y* is

$$G(x) = [F(x)]^n$$

Then, from

$$G(x) = u$$

It follows that

$$F(x) = u^{1/n}$$

Thus,

$$x = F^{-1}(u^{1/n})$$

In order to obtain a value *x* of the random variable *Y*, we first generate a value u for the standard uniformly distributed variable *U* and then compute $F^{-1}(u^{1/n})$.

The MCS schemes briefly sketched here can be further developed and tailored to specific features of the considered problem. The detailed description of the MCS technique can be found in (Gentle 2003, Robert 2005).

2.3.3 Combining the Monte Carlo simulation and UGF for estimating reliability confidence bounds of multi-state systems

Whenever no economic or time constraints exist, an effective approach to obtain the true value of system reliability would be to test an infinite number of systems in real life situations until failure occurs. Unfortunately, system and component testing is limited to tight economic budgets and schedules. Thus, it is often unrealistic and infeasible to perform extensive system testing both at the component and system level. A more efficient approach to estimate reliability of complex systems is to understand the relationships between component and system level interactions.

In general, the evaluation of MSS reliability depends on two key factors: reliability data associated to each component and an entire MSS reliability evaluation technique. With respect to the first factor, component testing is usually limited due to budget and schedule constraints. That is, data regarding component reliability is scarce and thus, component and MSS reliability should be regarded as an estimate of the true reliability value based on this limited data. Previous research in reliability (Coit 1997, Coit and Jin 2001, Ramirez-Marquez et al. 2004, Ramirez-Marquez and Jiang 2006) showed that uncertainty at the component level propagates to the system level and significantly affects accuracy of system reliability estimation.

Unfortunately, current methods for the estimation of system reliability and associated uncertainty are restricted to the case where the system and the components follow a binary behavior. Currently, for different MSS, reliability and associated uncertainty estimates cannot be approximated. It is thus of interest to quantify the uncertainty associated with the reliability estimation of these systems so that, similar to the binary case, one can identify, measure, and prioritize risks to improve reliability and safety (Coit 1997).

This section presents a method for the estimation of multi-state system reliability confidence bounds based on component reliability and uncertainty data (Ramirez-Marquez and Levitin 2007). The proposed method is based on a structured approach that generates an α-level confidence interval (CI). The method can be applied only to multi-state systems consisting of two-state components that can either work with nominal performance or totally fail.

The general problem addressed in this section is related to the accuracy of the reliability estimate for MSS applications. Currently, simulation approaches based on component reliability and uncertainty data can only be used. These approaches are time consuming and usually underestimate the true value of system reliability. Here we use the UGF to generate an estimate of MSBC reliability that is based on component reliability estimates. The estimate is then used to develop a (1-α)% confidence interval.

First, we provide a method for quantifying the uncertainty in terms of variance associated to the reliability estimate of general MSS. The second objective provides statistical inference methods that can be used to make an accurate estimation of MSS reliability based on component level reliability data.

Different studies indicate that system structure and component reliability estimates, with its associated uncertainty, contribute to the propagation to system level uncertainty. In general multi-state and binary reliability applications, a common approach to obtain these estimates is to obtain binomial test data (Wright and Bierbaum 2002, Stamatelatos 2001) with variance being the preferred measure of uncertainty.

The proposed approach works under the assumption that a specific number of units of component type j, n_j, are tested for t hours. After completion of the test, the state (working at a nominal performance or failed) of each unit, can be regarded as an independent Bernoulli trial with parameter $r_j(t)$, where the index j is associated with the type of component being tested. An unbiased estimate for $r_j(t)$, and its associated variance can be determined from the binomial distribution as

$$\hat{r}_j(t) = 1 - \frac{f_j(t)}{n_j}, \tag{2.146}$$

and

$$v(\hat{r}_j(t)) = \frac{\hat{r}_j(t)(1-\hat{r}_j(t))}{n_j - 1}, \tag{2.147}$$

respectively, where $f_j(t)$ defines the number of failures observed during the duration of the test.

For binary and capacitated systems with a single demand, closed form expressions for quantifying the uncertainty associated with the reliability estimate of any system can be obtained as long as component reliability is available (Ramirez-Marquez and Jiang 2005). However, for the multi-state case, developing closed form expressions is dependent on obtaining the variance of each individual term associated with the loss of load probability equation. Computing the exact values associated with the covariance terms is computationally burdensome and no closed form expression is currently at hand. To overcome, this we use the UGF method, to approximate the system reliability variance estimate and provide MSS reliability bounds. The method assumes that component test data is readily available.

The UGF method utilizes the information generated from the test data or by MCS of components' performance (estimates of components' reliability) to immediately provide an estimate of the system reliability without the need to further simulate the behavior of the test data. For any combination of component reliability estimates, this method calculates the entire MSS performance distribution using the presented reliability block diagram technique and obtains the system reliability estimate.

Two alternatives to generate bounds for multi-state system reliability at a desired confidence level α are described below.

Confidence Bound 1. This bound is intended to provide an intuitive or practical explanation about the distribution of system failures. It assumes that the number of successes during N tests of a multi-state system follows a Binomial distribution with parameter $R(t)$. The rationale behind this assumption is that, if completed N systems were subject to test, and the numbers of failures were recorded at time t, then it is intuitive to assume that the true value of reliability of the system, $R(t)$, could be estimated via $\hat{R}(t)$, obtained by combination of MCS and UGF method. If the number of tests N is large enough, the confidence bounds can be obtained through the s-normal approximation of $R(t)$ with mean $\hat{R}(t)$, and variance $\frac{\hat{R}(t)(1-\hat{R}(t))}{N}$.

The two sided confidence interval of $R(t)$ at a (1-α)% level is given as

$$\hat{R}(t)-z_{\alpha/2}\sqrt{\frac{\hat{R}(t)(1-\hat{R}(t))}{N}}+\frac{1}{N}\le R(t)\le \hat{R}(t)+z_{\alpha/2}\sqrt{\frac{\hat{R}(t)(1-\hat{R}(t))}{N}}-\frac{1}{N}, \quad (2.148)$$

where

$$N=\begin{cases}\frac{\hat{\mathrm{R}}(1-\hat{\mathrm{R}})}{v(\hat{\mathrm{R}})}, & \text{if the number of testing units is not the same for every component, as given by (Jin and Coit 2001)}\\ n_j, & \text{otherwise}\end{cases}$$

Similarly, the s-normal approximation can be employed to generate lower, and upper confidence bounds:
(1-α)% Confidence Lower Bound

$$\hat{R}(t)-z_{\alpha}\sqrt{\frac{\hat{R}(t)(1-\hat{R}(t))}{N}}+\frac{1}{N}\le R(t) \quad (2.149)$$

(1-α)% Confidence Upper Bound

$$R(t)\le \hat{R}(t)+z_{\alpha}\sqrt{\frac{\hat{R}(t)(1-\hat{R}(t))}{N}}-\frac{1}{N} \quad (2.150)$$

Confidence Bound 2. This bound is based on the assumption that the distribution for the system reliability $R(t)$ follows an unknown discrete distribution that can be well approximated by a Gaussian distribution. The mean of this unknown distribution is $\hat{R}$, while the variance is $\frac{\hat{R}(t)(1-\hat{R}(t))}{N}$.

However, this alternative uses Wilson's score method based on the component reliability estimates. Similar to the case of Bound 1, $\hat{R}(t)$ can be obtained by combination of MCS and UGF method.

These assumptions allow for the computation of a (1-α)% level two-sided CI for $R(t)$, given as:

$$\frac{2N\hat{R}(t)+\left(z_{\alpha/2}\right)^2-z_{\alpha/2}\sqrt{\left(z_{\alpha/2}\right)^2+4N\hat{R}(t)\left(1-\hat{R}(t)\right)}}{2\left(N+\left(z_{\alpha/2}\right)^2\right)}+\frac{1}{N}\leq R(t), \qquad (2.151)$$

$$\frac{2N\hat{R}(t)+\left(z_{\alpha/2}\right)^2+z_{\alpha/2}\sqrt{\left(z_{\alpha/2}\right)^2+4N\hat{R}(t)\left(1-\hat{R}(t)\right)}}{2\left(N+\left(z_{\alpha/2}\right)^2\right)}-\frac{1}{N}\geq R(t), \qquad (2.152)$$

where N is given as in confidence bound 1.

The lower and upper confidence bounds can also be constructed:

(1-α)% Confidence Lower Bound

$$\frac{2N\hat{R}(t)+\left(z_{\alpha}\right)^2-z_{\alpha}\sqrt{\left(z_{\alpha}\right)^2+4N\hat{R}(t)\left(1-\hat{R}(t)\right)}}{2\left(N+\left(z_{\alpha}\right)^2\right)}+\frac{1}{N}\leq R(t), \qquad (2.153)$$

(1-α)% Confidence Upper Bound

$$R(t)\leq\frac{2N\hat{R}(t)+\left(z_{\alpha}\right)^2+z_{\alpha}\sqrt{\left(z_{\alpha}\right)^2+4N\hat{R}(t)\left(1-\hat{R}(t)\right)}}{2\left(N+\left(z_{\alpha}\right)^2\right)}-\frac{1}{N}. \qquad (2.154)$$

Example 2.22

Consider a series-parallel system shown in Fig. 2.30. Table 2.11 introduces the reliability data for each component and its associated nominal performance while Table 2.12 presents the demand distribution. The results of a single test run when considering a data sample size of 50 simulations per component are presented in Table 2.13. Table 2.14 presents MSS reliability lower bounds, corresponding to each of the bounding methods, obtained from the simulated component test data and the UGF approach. This table also includes the coverage associated to the level α=0.1 against each of the bounds actual coverage.

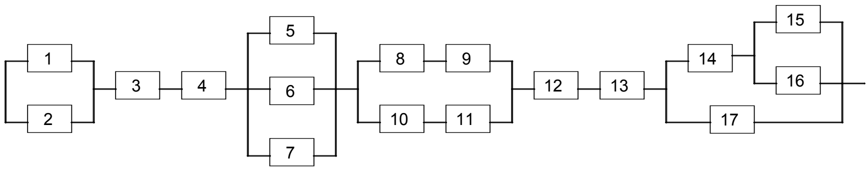


Fig. 2.30. Series-parallel system

Table 2.11. Component Data for Series-Parallel System

Component Type	Nominal Reliability	Nominal performance	Component Type	Nominal Reliability	Nominal performance
1	0.91	7	9	0.95	5
2	0.91	6	10	0.92	7
3	0.96	12	11	0.95	7
4	0.98	15	12	0.96	14
5	0.8	4	13	0.99	15
6	0.8	5	14	0.9	7
7	0.8	7	15	0.83	4
8	0.92	6	16	0.83	4
			17	0.88	9

Table 2.12. Demand distribution

Demand	4	5	6	7	9	11	12
Probability	1/7	1/7	1/7	1/7	1/7	1/7	1/7

To test the accuracy of the proposed approaches the binomial component data were obtained by Monte Carlo simulation. This simulation was based on nominal component reliability values and done for one thousand repetitions of a test of 50 units for each component type. For each repetition, the estimate of MSS reliability was obtained using the UGF approach and by pure MCS (in which the entire system failures were counted). Table 2.15 illustrates 20 random estimates out of the 1,000 estimates of the MSS reliability obtained by the combined MCS and UGF approach and pure MCS approach. For this case, it is evident that the UGF provides a more precise estimation of the MSS reliability without the need of extensive simulation for evaluating the system reliability for any given combination of reliabilities of its components.

Table 2.13. MSS test run data

Component Type	No of units	Components Failed	Reliability Estimate
1	50	8	0.84
2	50	5	0.9
3	50	3	0.94
4	50	2	0.96
5	50	15	0.7
6	50	4	0.92
7	50	11	0.78
8	50	5	0.9
9	50	5	0.9
10	50	4	0.92
11	50	2	0.96
12	50	2	0.96
13	50	0	1
14	50	6	0.88
15	50	12	0.76
16	50	3	0.94
17	50	6	0.88
MCS+UGF MSS Reliability	0.5326	Pure MCS Reliability	0.4743

Table 2.14. Reliability bounds for test run data

	Bound 1	Bound 2
Reliability Bound	0.4622	0.4822
Actual α Coverage	0.065	0.111

Table 2.15. Comparison of MSS Reliability Estimate obtained by combined MCS and UGF method and pure MCS

Repetition	MCS+UGF Reliability Estimate	Pure MCS Reliability Estimate	Repetition	MCS+UGF Reliability Estimate	Pure MCS Reliability Estimate
1	0.5665	0.4914	11	0.4779	0.4543
2	0.6192	0.5886	12	0.5649	0.5057
3	0.6452	0.5629	13	0.6173	0.5829
4	0.5225	0.5029	14	0.5898	0.5143
5	0.5587	0.4943	15	0.6083	0.5457
6	0.6029	0.5229	16	0.6318	0.5629
7	0.5565	0.5000	17	0.5485	0.4657
8	0.6108	0.5343	18	0.6060	0.4971
9	0.5740	0.4914	19	0.4956	0.4143
10	0.6031	0.5457	20	0.5742	0.4629

As illustrated by Table 2.14 Bound 2 provides a much better coverage when compared against the desired α level. For this case, the actual coverage aligns almost exactly with the desired α level. That is, for the 90% bound desired, Bound 2 provides coverage of 88.9%.

2.4 Introduction to Genetic Algorithms

An abundance of optimization methods have been used to solve various reliability optimization problems. The algorithms applied are either heuristics or exact procedures based mainly on modifications of dynamic programming and nonlinear programming. Most of these methods are strongly problem oriented. This means that, since they are designed for solving certain optimization problems, they cannot be easily adapted for solving other problems. In recent years, many studies on reliability optimization use a universal optimization approach based on metaheuristics. These metaheuristics hardly depend on the specific nature of the problem that is solved and, therefore, can be easily applied to solve a wide range of optimization problems. The metaheuristics are based on artificial reasoning rather than on classical mathematical programming. Their important advantage is that they do not require any information about the objective function besides its values corresponding to the points visited in the solution space. All metaheuristics use the idea of randomness when performing a search, but they also use past knowledge in order to direct the search. Such search algorithms are known as randomized search techniques.

Genetic algorithms (GA's) are one of the most widely used metaheuristics. They were inspired by the optimization procedure that exists in nature, the biological phenomenon of evolution. A GA maintains a population of different solutions allowing them to mate, produce offspring, mutate, and fight for survival. The principle of survival of the fittest ensures the population's drive towards optimization. The GA's have become the popular universal tool for solving various optimization problems, as they have the following advantages:

- they can be easily implemented and adapted;
- they usually converge rapidly on solutions of good quality;
- they can easily handle constrained optimization problems;
- they produce variety of good quality solutions simultaneously, which is important in the decision-making process.

The GA concept was developed by John Holland at the University of Michigan and first described in his book (Holland 1975). Holland was impressed by the ease with which biological organisms could perform tasks, which eluded even the most powerful computers. He also noted that very few artificial systems have the most remarkable characteristics of biological systems: robustness and flexibility. Unlike technical systems, biological ones have methods for self-guidance, self-repair and reproducing these features. Holland's biologically inspired approach to optimization is based on the following analogies:

- As in nature, where there are many organisms, there are many possible solutions to a given problem.
- As in nature, where an organism contains many genes defining its properties, each solution is defined by many interacting variables (parameters).
- As in nature, where groups of organisms live together in a population and some organisms in the population are more fit than others, a group of possible solutions can be stored together in computer memory and some of them are closer to the optimum than others.
- As in nature, where organisms that are more fit have more chances of mating and having offspring, solutions that are closer to the optimum can be selected more often to combine their parameters to form new solutions.
- As in nature, where organisms produced by good parents are more likely to be better adapted than the average organism because they received good genes, offspring of good solutions are more likely to be better than a random guess, since they are composed of better parameters.

- As in nature, where survival of the fittest ensures that the successful traits continue to get passed along to subsequent generations, and are refined as the population evolves, the survival-of-the-fittest rule ensures that the composition of the parameters corresponding to the best guesses continually get refined.

GA's maintain a population of individual solutions, each one represented by a finite string of symbols, known as the *genome*, encoding a possible solution within a given problem space. This space, referred to as the *search space*, comprises all of the possible solutions to the problem at hand. Generally speaking, a GA is applied to spaces, which are too large to be searched exhaustively.

GA's exploit the idea of the survival of the fittest and an interbreeding population to create a novel and innovative search strategy. They iteratively create new populations from the old ones by ranking the strings and interbreeding the fittest to create new strings, which are (hopefully) closer to the optimum solution for the problem at hand. In each generation, a GA creates a set of strings from pieces of the previous strings, occasionally adding random new data to keep the population from stagnating. The result is a search strategy that is tailored for vast, complex, multimodal search spaces.

The idea of survival of the fittest is of great importance to genetic algorithms. GA's use what is termed as the fitness function in order to select the fittest string to be used to create new, and conceivably better, populations of strings. The fitness function takes a string and assigns it a relative fitness value. The method by which it does this and the nature of the fitness value do not matter. The only thing that the fitness function must do is rank the strings in some way by producing their fitness values. These values are then used to select the fittest strings.

GA's use the idea of randomness when performing a search. However, it must be clearly understood that the GA's are not simply random search algorithms. Random search algorithms can be inherently inefficient due to the directionless nature of their search. GA's are not directionless. They utilize knowledge from previous generations of strings in order to construct new strings that will approach the optimal solution. GA's are a form of a randomized search, and the way that the strings are chosen and combined comprise a stochastic process.

The essential differences between GA's and other forms of optimization, according to (Goldberg 1989), are as follows.

GA's usually use a coded form of the solution parameters rather than their actual values. Solution encoding in a form of strings of symbols (an analogy to chromosomes containing genes) provides the possibility of crossover and mutation. The symbolic alphabet that was used was initially binary, due to certain computational advantages purported in (Goldberg 1989). This has been extended to include character-based encodings, integer and real-valued encodings, and tree representations (Michalewicz 1996).

GA's do not just use a single point on the problem space, rather they use a set, or population, of points (solutions) to conduct a search. This gives the GA's the power to search noisy spaces littered with local optimum points. Instead of relying on a single point to search through the space, GA's look at many different areas of the problem space at once, and use all of this information as a guide.

GA's use only payoff information to guide them through the problem space. Many search techniques need a range of information to guide themselves. For example, gradient methods require derivatives. The only information a GA needs to continue searching for the optimum is some measure of fitness about a point in the space.

GA's are probabilistic in nature, not deterministic. This is a direct result of the randomization techniques used by GA's.

GA's are inherently parallel. Herein lies one of their most powerful features. GA's, by their nature, are very parallel, dealing with a large number of solutions simultaneously. Using schemata theory, Holland has estimated that a GA, processing n strings at each generation, in reality processes n^3 useful substrings.

Two of the most common GA implementations are "generational" and "steady state", although recently the steady-state technique has received increased attention (Kinnear 1993). This interest is partly attributed to the fact that steady-state techniques can offer a substantial reduction in the memory requirements of a system: the technique abolishes the need to maintain more than one population during the evolutionary process, which is necessary in the generational GA. In this way, genetic systems have greater portability for a variety of computer environments because of the reduced memory overhead. Another reason for the increased interest in steady-state techniques is that, in many cases, a steady-state GA has been shown to be more effective than a generational GA (Syswerda 1991), (Vavak and Fogarty 1996). This improved performance can be attributed to factors such as the diversity of the population and the immediate availability of superior individuals.

A comprehensive description of a generational GA can be found in (Goldberg 1989). Here, we present the structure of a steady-state GA.

2.4.1 Structure of steady-state Genetic Algorithms

The steady-state GA (see Fig. 2.31) proceeds as follows (Whitley 1989) an initial population of solutions, generated randomly or heuristically. Within this population, new solutions are obtained during the genetic cycle by using the *crossover* operator. This operator produces an offspring from a randomly selected pair of parent solutions (the parent solutions are selected with a probability proportional to their relative fitness), facilitating the inheritance of some basic properties from the parents to the offspring. The newly obtained offspring undergoes *mutation* with the probability p_{mut}.

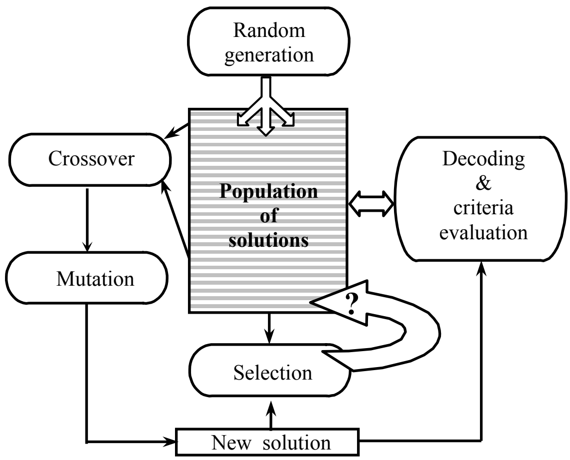


Fig. 2.31. Structure of a steady-state GA

Each new solution is decoded and its objective function (fitness) values are estimated. These values, which are a measure of quality, are used to compare different solutions. The comparison is accomplished by a selection procedure that determines which solution is better: the newly obtained solution or the worst solution in the population. The better solution joins the population, while the other is discarded. If the population contains equivalent solutions following selection, then redundancies are eliminated and the population size decreases as a result.

A genetic cycle terminates when N_{rep} new solutions are produced or when the number of solutions in the population reaches a specified level. Then, new randomly constructed solutions are generated to replenish the shrunken population, and a new genetic cycle begins. The whole GA is terminated when its termination condition is satisfied. This condition can be specified in the same way as in a generational GA. The following is the steady-state GA in pseudo-code format.

```
begin STEADY STATE GA
 Initialize population Π
 Evaluate population Π {compute fitness values}
 while GA termination criterion is not satisfied do
       {GENETIC CYCLE}
    while genetic cycle termination criterion is not satisfied do
              Select at random Parent Solutions S1, S2 from Π
              Crossover: (S1, S2) → SO {offspring}
              Mutate offspring SO → S*O with probability pmut
              Evaluate S*O
              Replace SW {the worst solution in Π with S*O } if S*O is
              better than SW
              Eliminate identical solutions in Π
    end while
    Replenish Π with new randomly generated solutions
 end while
end GA
```

Example 2.23

In this example we present several initial stages of a steady-state GA, that maximizes the function of six integer variables $x_1, \ldots, x_6$ taking the form

$$f(x_1,\ldots,x_6) = 1000\ [(x_1-3.4)^2+(x_2-1.8)^2+(x_3-7.7)^2$$
$$+(x_4-3.1)^2+(x_5-2.8)^2+(x_6-8.8)^2]^{-1}$$

The variables can take values from 1 to 9. The initial population, consisting of five solutions ordered according to their fitness (value of function f), is:

No.	x_1	x_2	x_3	x_4	x_5	x_6	$f(x_1,\ldots,x_6)$
1	4	2	4	1	2	5	297.8
2	3	7	7	7	2	7	213.8
3	7	5	3	5	3	9	204.2
4	2	7	4	2	1	4	142.5
5	8	2	3	1	1	4	135.2

Using the random generator that produces the numbers of the solutions, the GA chooses the first and third strings, *i.e.* (4 2 4 1 2 5) and (7 5 3 5 3 9) respectively. From these strings, it produces a new one by applying a crossover procedure that takes the three first numbers from the better parent string and the last three numbers from the inferior parent string. The resulting string is (4 2 4 5 3 9). The fitness of this new solution is $f(x_1, ..., x_6) = 562.4$. The new solution enters the population, replacing the one with the lowest fitness. The new population is now

No.	x_1	x_2	x_3	x_4	x_5	x_6	$f(x_1,...,x_6)$
1	4	2	4	5	3	9	562.4
2	4	2	4	1	2	5	297.8
3	3	7	7	7	2	7	213.8
4	7	5	3	5	3	9	204.2
5	2	7	4	2	1	4	142.5

Choosing at random the third and fourth strings, (3 7 7 7 2 7) and (7 5 3 5 3 9) respectively, the GA produces the new string (3 7 7 5 3 9) using the crossover operator. This string undergoes a mutation that changes one of its numbers by one (here, the fourth element of the string changes from 5 to 4). The resulting string (3 7 7 4 3 9) has a fitness of $f(x_1, ..., x_6) = 349.9$. This solution is better than the inferior one in the population; therefore, the new solution replaces the inferior one. Now the population takes the form

No.	x_1	x_2	x_3	x_4	x_5	x_6	$f(x_1,...,x_6)$
1	4	2	4	5	3	9	562.4
2	3	7	7	4	3	9	349.9
3	4	2	4	1	2	5	297.8
4	3	7	7	7	2	7	213.8
5	7	5	3	5	3	9	204.2

A new solution (4 2 4 4 3 9) is obtained by the crossover operator over the randomly chosen first and second solutions, *i.e.* (4 2 4 5 3 9) and (3 7 7 4 3 9) respectively. After the mutation this solution takes the form (4 2 4 5 3 9) and has the fitness $f(x_1,..., x_6) = 1165.5$. The population obtained after the new solution joins it is

No.	x_1	x_2	x_3	x_4	x_5	x_6	$f(x_1,...,x_6)$
1	4	2	5	4	3	9	1165.5
2	4	2	4	5	3	9	562.4
3	3	7	7	4	3	9	349.9
4	4	2	4	1	2	5	297.8
5	3	7	7	7	2	7	213.8

Note that the mutation procedure is not applied to all the solutions obtained by the crossover. This procedure is used with some pre-specified probability p_{mut}. In our example, only the second and the third newly obtained solutions underwent the mutation.

The actual GA's operate with much larger populations and produce thousands of new solutions using the crossover and mutation procedures. The steady-state GA with a population size of 100 obtained the optimal solution for the problem presented after producing about 3000 new solutions. Note that the total number of possible solutions is $9^6 = 531441$. The GA managed to find the optimal solution by exploring less than 0.6% of the entire solution space.

Both types of GA are based on the crossover and mutation procedures, which depend strongly on the solution encoding technique. These procedures should preserve the feasibility of the solutions and provide the inheritance of their essential properties.

2.4.2 Adaptation of Genetic Algorithms to specific optimization problems

There are three basic steps in applying a GA to a specific problem.

In the first step, one defines the solution representation (encoding in a form of a string of symbols) and determines the decoding procedure, which evaluates the fitness of the solution represented by the arbitrary string.

In the second step, one has to adapt the crossover and mutation procedures to the given representation in order to provide feasibility for the new solutions produced by these procedures as well as inheriting the basic properties of the parent solutions by their offspring.

In the third step, one has to choose the basic GA parameters, such as the population size, the mutation probability, the crossover probability (generational GA) or the number of crossovers per genetic cycle (in the steady-state GA), and formulate the termination condition in order to provide the greatest possible GA efficiency (convergence speed).

The strings representing GA solutions are randomly generated by the population generation procedure, modified by the crossover and mutation procedures, and decoded by the fitness evaluation procedure. Therefore, the solution representation in the GA should meet the following requirements:

- It should be easily generated (the sophisticated complex solution generation procedures reduce the GA speed).
- It should be as compact as possible (using very long strings requires excessive computational resources and slows the GA convergence).

- It should be unambiguous (i.e. different solutions should be represented by different strings).
- It should represent feasible solutions (if not any randomly generated string represents a feasible solution, then the feasibility should be provided by simple string transformation).
- It should provide feasibility inheritance of new solutions obtained from feasible ones by the crossover and mutation operators.

The field of reliability optimization includes the problems of finding optimal parameters, optimal allocation and assignment of different elements into a system, and optimal sequencing of the elements. Many of these problems are combinatorial by their nature. The most suitable symbol alphabet for this class of problems is integer numbers. The finite string of integer numbers can be easily generated and stored. The random generator produces integer numbers for each element of the string in a specified range. This range should be the same for each element in order to make the string generation procedure simple and fast. If for some reason different string elements should belong to different ranges, then the string should be transformed to provide solution feasibility.

In the following sections, we show how integer strings can be interpreted for solving different kinds of optimization problems.

Parameter determination problems

When the problem lies in determining a vector of H parameters (x_1, x_2, ..., x_H) that maximizes an objective function $f(x_1, x_2, ..., x_H)$ one always has to specify the ranges of the parameter variation:

$$x_j^{\min} \le x_j \le x_j^{\max} \text{ for } 1 \le j \le H \tag{2.155}$$

In order to facilitate the search in the solution space determined by inequalities (2.155), integer strings $\boldsymbol{a} = (a_1\ a_2\ ...\ a_H)$ should be generated with elements ranging from 0 to N and the values of parameters should be obtained for each string as

$$x_j = x_j^{\min} + a_j(x_j^{\max} - x_j^{\min})/N. \tag{2.156}$$

Note that the space of the integer strings just approximately maps the space of the real-valued parameters. The number N determines the precision of the search. The search resolution for the j-th parameter is $(x_j^{\max} - x_j^{\min})/N$. Therefore the increase of N provides a more precise search. On the other hand, the size of the search space of integer strings grows drastically with the increase of N, which slows the GA convergence. A reasonable compromise can be found by using a multistage GA search.

In this method, a moderate value of N is chosen and the GA is run to obtain a “crude” solution. Then the ranges of all the parameters are corrected to accomplish the search in a small vicinity of the vector of parameters obtained and the GA is started again. The desired search precision can be obtained by a few iterations.

Example 2.24

Consider a problem in which one has to minimize a function of seven parameters. Assume that following a preliminary decision the ranges of the possible variations of the parameters are different.

Let the random generator provide the generation of integer numbers in the range of 0 - 100 (N = 100). The random integer string and the corresponding values of the parameters obtained according to (2.156) are presented in Table 2.16.

Table 2.16. Example of parameters encoding

No. of variable	1	2	3	4	5	6	7
$x_j^{\min}$	0.0	0.0	1.0	1.0	1.0	0.0	0.0
$x_j^{\max}$	3.0	3.0	5.0	5.0	5.0	5.0	5.0
Random integer string	21	4	0	100	72	98	0
Decoded variable	0.63	0.12	1.0	5.0	3.88	4.9	0.0

Partition and Allocation Problems

The partition problem can be considered as a problem of allocating Y items belonging to a set Φ in K mutually disjoint subsets Φ_i, *i.e.* such that

$$\bigcup_{i=1}^{K} \Phi_i = \Phi, \;\; \Phi_i \bigcap \Phi_j = \varnothing, \; i \neq j \tag{2.157}$$

Each set can contain from 0 to Y items. The partition of the set Φ can be represented by the Y-length string $\boldsymbol{a} = (a_1\ a_2\ \ldots\ a_{Y-1}\ a_Y)$ in which a_j is a number of the set to which item j belongs. Note that, in the strings representing feasible solutions of the partition problem, each element can take a value in the range $(1, K)$.

Now consider a more complicated allocation problem in which the number of items is not specified. Assume that there are H types of different items with an unlimited number of items for each type h. The number of items of each type allocated in each subset can vary. To represent an allocation of the variable number of items in K subsets one can use the following string encoding $\boldsymbol{a} = (a_{11}\ a_{12}\ \ldots a_{1K}\ a_{21}\ a_{22}\ \ldots\ a_{2K} \ldots\ a_{H1}\ a_{H2} \ldots\ a_{HK})$, in which a_{ij} corresponds to the number of items of type i belonging to subset j. Observe that the different subsets can contain identical elements.

Example 2.25

Consider the problem of allocating items of three different types in two disjoint subsets. In this problem, H=3 and K=2. Any possible allocation can be represented by an integer string using the encoding described above. For example, the string (2 1 0 1 1 1) encodes the solution in which two type 1 items are allocated in the first subset and one in the second subset, one item of type 2 is allocated in the second subset, one item of type 3 is allocated in each of the two subsets.

When K = 1, one has an assignment problem in which a number of different items should be chosen from a list containing an unlimited number of items of K different types. Any solution of the assignment problem can be represented by the string $\boldsymbol{a} = (a_1\ a_2\ ...\ a_K)$, in which a_j corresponds to the number of chosen items of type j.

The range of variance of string elements for both allocation and assignment problems can be specified based on the preliminary estimation of the characteristics of the optimal solution (maximal possible number of elements of the same type included into the single subset). The greater the range, the greater the solution space to be explored (note that the minimal possible value of the string element is always zero in order to provide the possibility of not choosing any element of the given type to the given subset). In many practical applications, the total number of items belonging to each subset is also limited. In this case, any string representing a solution in which this constraint is not met should be transformed in the following way:

$$a_{ij}^* = \begin{cases} \left\lfloor a_{ij} N_j / \sum_{h=1}^{H} a_{hj} \right\rfloor, & \text{if } N_j < \sum_{h=1}^{H} a_{hj} \\ a_{ij}\ , & \text{otherwise} \end{cases} \quad \text{for } 1 \le i \le H, 1 \le j \le K \tag{2.158}$$

where N_j is the maximal allowed number of items in subset j.

Example 2.26

Consider the case in which the items of three types should be allocated into two subsets. Assume that it is prohibited to allocate more than five items of each type to the same subset. The GA should produce strings with elements ranging from 0 to 5. An example of such a string is (4 2 5 1 0 2).

Assume that for some reason the total numbers of items in the first and in the second subsets are restricted to seven and six respectively. In order to obtain a feasible solution, one has to apply the transform (2.158) in which $N_1 = 7$, $N_2 = 6$:

$$\sum_{h=1}^{3} a_{h1} = 4+5+0=9,\ \sum_{h=1}^{3} a_{h2} = 2+1+2=5.$$

The string elements take the values

$$a_{11} = \lfloor 4\times 7/9 \rfloor = 3,\ a_{21} = \lfloor 5\times 7/9 \rfloor = 3,\ a_{31} = \lfloor 0\times 7/9 \rfloor = 0$$

$$a_{12} = \lfloor 2\times 6/5 \rfloor = 2,\ a_{22} = \lfloor 1\times 6/5 \rfloor = 1,\ a_{32} = \lfloor 2\times 6/5 \rfloor = 2$$

After the transformation, one obtains the following string: (3 2 3 1 0 2).

When the number of item types and subsets is large, the solution representation described above results in an enormous growth of the length of the string. Besides, to represent a reasonable solution (especially when the number of items belonging to each subset is limited), such a string should contain a large fraction of zeros because only a few items should be included in each subset. This redundancy causes an increase in the need of computational resources and lowers the efficiency of the GA. To reduce the redundancy of the solution representation, each inclusion of m items of type h into subset k is represented by a triplet (m h k). In order to preserve the constant length of the strings, one has to specify in advance a maximal reasonable number of such inclusions I. The string representing up to I inclusions takes the form (m_1 h_1 k_1 m_2 h_2 k_2 ... m_I h_I k_I). The range of string elements should be (0, max{M, H, K}), where M is the maximal possible number of elements of the same type included into a single subset. An arbitrary string generated in this range can still produce infeasible solutions. In order to provide the feasibility, one has to apply the transform $a_j^* = \text{mod}_{x+1}\, a_j$, where x is equal to M, H and K for the string elements corresponding to m, h and k respectively. If one of the elements of the triplet is equal to zero, then this means that no inclusion is made.

For example, the string (*3 1* 2 1 2 *3 2 1* 1 2 2 *2 3 2*) represents the same allocation as string (3 2 3 1 0 2) in Example 2.26. Note that the permutation of triplets, as well as an addition or reduction of triplets containing zeros, does not change the solution. For example, the string (4 0 1 *2 3 2* 2 1 2 *3 1 1* 1 2 2 *3 2 1*) also represents the same allocation as that of the previous string.

2.4.3 Determination of solution fitness

Having a solution represented in the GA by an integer string $\boldsymbol{a}$ one then has to estimate the quality of this solution (or, in terms of the evolution process, the fitness of the individual). The GA seeks solutions with the greatest possible fitness. Therefore, the fitness should be defined in such a way that its greatest values correspond to the best solutions.

For example, when optimizing the system reliability R (which is a function of some of the parameters represented by $\boldsymbol{a}$) one can define the solution fitness equal to this index, since one wants to maximize it. On the contrary, when minimizing the system cost C, one has to define the solution fitness as $M - C$, where M is a constant number. In this case, the maximal solution fitness corresponds to its minimal cost.

In the majority of optimization problems, the optimal solution should satisfy some constraints. There are three different approaches to handling the constraints in GA (Michalewicz 1996). One of these uses penalty functions as an adjustment to the fitness function; two other approaches use "decoder" or "repair" algorithms to avoid building illegal solutions or repair them respectively. The "decoder" and "repair" approaches suffer from the disadvantage of being tailored to the specific problems and thus are not sufficiently general to handle a variety of problems. On the other hand, the penalty approach based on generating potential solutions without considering the constraints and on decreasing the fitness of solutions, violating the constraints, is suitable for problems with a relatively small number of constraints. For heavily constrained problems, the penalty approach causes the GA to spend most of its time evaluating solutions violating the constraints. Fortunately, the reliability optimization problems usually deal with few constraints.

Using the penalty approach one transforms a constrained problem into an unconstrained one by associating a penalty with all constraint violations. The penalty is incorporated into the fitness function. Thus, the original problem of maximizing a function $f(\boldsymbol{a})$ is transformed into the maximization of the function

$$f(\boldsymbol{a}) - \sum_{j=1}^{J} \pi_j \eta_j \tag{2.159}$$

where J is the total number of constraints, π_j is a penalty coefficient related to the j-th constraint ($j = 1, \ldots, J$) and η_j is a measure of the constraint violation. Note that the penalty coefficient should be chosen in such a way as to allow the solution with the smallest value of $f(\boldsymbol{a})$ that meets all of the constraints to have a fitness greater than the solution with the greatest value of $f(\boldsymbol{a})$ but violating at least one constraint.

Consider, for example, a typical problem of maximizing the system reliability subject to cost constraint: $R(\boldsymbol{a}) \rightarrow$ max subject to $C(\boldsymbol{a}) \le C^*$.

The system cost and reliability are functions of parameters encoded by a string $\boldsymbol{a}$: $C(\boldsymbol{a})$ and $R(\boldsymbol{a})$ respectively. The system cost should not be greater than C^*. The fitness of any solution $\boldsymbol{a}$ can be defined as

$$M+R(\boldsymbol{a})-\pi\eta(C^*, \boldsymbol{a})$$

where

$$\eta(C^*, \boldsymbol{a})=(1+C(\boldsymbol{a})-C^*)1(C(\boldsymbol{a})>C^*) \tag{2.160}$$

The coefficient π should be greater than one. In this case the fitness of any solution violating the constraint is smaller than M (the smallest violation of the constraint $C(\boldsymbol{a}) \le C^*$ produces a penalty greater than π) while the fitness of any solution meeting the constraint is greater than M. In order to keep the fitness of the solutions positive, one can choose $M>\pi\ (1+C_{\max}-C^*)$, where $C_{\max}$ is the maximal possible system cost.

Another typical optimization problem is minimizing the system cost subject to the reliability constraint: $C(\boldsymbol{a}) \rightarrow$ min subject to $R(\boldsymbol{a}) \ge R^*$.

The fitness of any solution $\boldsymbol{a}$ of this problem can be defined as

$$M-C(\boldsymbol{a})-\pi\eta(R^*,\boldsymbol{a})$$

where

$$\eta(A^*, \boldsymbol{a})=(1+R^*-R(\boldsymbol{a}))1(R(\boldsymbol{a})<R^*) \tag{2.161}$$

The coefficient π should be greater than $C_{\max}$. In this case, the fitness of any solution violating the constraint is smaller than $M - C_{\max}$ whereas the fitness of any solution meeting the constraint is greater than $M - C_{\max}$. In order to keep the fitness of the solutions positive, one can choose $M>C_{\max} + 2\pi$.

2.4.4 Basic Genetic Algorithm procedures and parameters

The crossover procedures create a new solution as the offspring of a pair of existing ones (parent solutions). The offspring should inherit some useful properties of both parents in order to facilitate their propagation throughout the population. The mutation procedure is applied to the offspring solution. It introduces slight changes into the solution encoding string by modifying some of the string elements. Both of these procedures should be developed in such a way as to provide the feasibility of the offspring solutions given that parent solutions are feasible.

When applied to parameter determination, partition, and assignment problems, the solution feasibility means that the values of all of the string elements belong to a specified range. The most commonly used crossover procedures for these problems generate offspring in which every position is occupied by a corresponding element from one of the parents. This property of the offspring solution provides its feasibility. For example, in the *uniform crossover* each string element is copied either from the first or second parent string with equal probability.

The commonly used mutation procedure changes the value of a randomly selected string element by 1 (increasing or decreasing this value with equal probability). If after the mutation the element is out of the specified range, it takes the minimal or maximal allowed value.

When applied to the sequencing problems, the crossover and mutation operators should produce the offspring that preserve the form of permutations. This means that the offspring string should contain all of the elements that appear in the initial strings and each element should appear in the offspring only once. Any omission or duplication of the element constitutes an error. For example, in the *fragment crossover* operator all of the elements from the first parent string are copied to the same positions of the offspring. Then, all of the elements belonging to a randomly chosen set of adjacent positions in the offspring are reallocated within this set in the order that they appear in the second parent string. It can be seen that this operator provides the feasibility of the permutation solutions.

The widely used mutation procedure that preserves the permutation feasibility swaps two string elements initially located in two randomly chosen positions.

There are no general rules in order to choose the values of basic GA parameters for solving specific optimization problems. The best way to determine the proper combination of these values is by experimental comparison between GA's with different parameters.

A detailed description of a variety of different crossover and mutation operators and recommendations concerning the choice of GA parameters can be found in the GA literature.

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3 Reliability of Generation Systems

The generated electric energy, which is transferred and distributed to the customers, should be of good quality (taking the voltage and frequency into account) and have a sufficient reliability level. Operating availabilities of the generation system plays an important role in meeting these demands. In past years, a high level of power system reliability has been assured by careful planning and operating practices based on experience and engineering judgment. The results were satisfactory as long as power systems were relatively small and simple. With the increasing size and diversity of power systems, the problems became much more complicated. This necessitated developing new methods for determination of planning and operation criteria, based on more sophisticated techniques and the reliability theory.

Using detailed mathematical models and the probabilistic approach, new methods have been developed in the last decade. They permit assessing the generation system reserve which is required to provide a predetermined reliability level.

The reliability indices should be determined for the entire system, i.e., the generation system, the transmission and the distribution networks. The complicated nature of a power system makes this problem very difficult. Consequently, most of the reliability studies have dealt with only one subsystem, assuming that there are no reliability problems with the rest of the system. In actual practice, this assumption is especially true for the generation part of the entire system. In this case the required generating capacity reserve for a given load can be determined together with techno-economical aspects (Allan and Billinton 1992). Today the planner has sufficient tools to evaluate the generation system reliability in terms of probabilistic reliability indices, such as the loss of load probability.

The system indices have been similarly assessed through the transmission network However, such solutions are not accurate enough to be used for the design of the transmission network (unlike the generation system). This is due to the strong dependency between the generation and transmission systems. Therefore, the correct approach is to study the composite reliability of both systems.

In the current chapter, the reliability indices of the generation system are studied. With them the required generation reserve can be determined under the assumption that the other parts of the power system are fully reliable. The reliability of the generation system can be characterized by the existing redundancy level, which is influenced by several factors such

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as, for example: the required load level or the reliability of the individual generating units. Clearly, the higher the reliability level of the individual units, the higher will be the reliability level of the whole generation system.

Generally speaking, the reliability level of any system is determined by the risk that under certain conditions the system will not operate satisfactorily due to failures in the system elements. In the generation system these indices can be determined by a relatively simple approach, namely by comparison of the required load to the generation capacity in the probabilistic manner.

As in an entire system, its generation part reliability is described with the aid of two types of such risk indices. The first type includes the risk relating to the static condition of the system, i.e., the system behavior at steady state. The second type includes the risk for the dynamic conditions, i.e., the system behavior under transient phenomena. Thus, the static indices do not relate to the transient phenomenon starting at the failure occurrence and ending at the new steady state condition. If, for example, the required reserve is studied, one should differentiate between operational reserve (which includes the spinning or hot reserve and a quick start of gas turbines) and the installed reserve capacity. The operational reserve relates to the dynamic condition of the system, i.e., to the system operation, while the installed reserve relates to the system design which determines the number and size of the units and the time of their operation in response to increasing electricity demand.

The static and dynamic system conditions can be illustrated by the following example. Assume that one of the large generating units is out of operation due to a failure, when the load level is, say, at peak. At that moment, the system enters a transient condition, where the controls of frequency react immediately. If the frequency rate drops or the frequency itself reaches certain predetermined values, a load shedding procedure (i.e., disconnection of certain consumers) takes place. Such a procedure may take a few seconds. At that point, the dispatcher intervenes by, for example, "throwing in" of gas turbines, which takes minutes to start operating. Later, if required, hot units can be committed in the system. This may take hours. Finally, the power system will reach a new steady state, where there may still be unsupplied consumers due to shortage in generating capacity.

Two distinct states characterize the above procedure, namely:

1. The dynamic state, which starts at the moment of failure and continues throughout the procedure of adding generating units,
2. The static state, which is the new steady state.

The reliability indices for the generation system at steady state are discussed first. The installed reserve, which is required to provide the system operation at a predetermined risk level, can be studied by means of the abovementioned indices. In this case it is commonly accepted to define the installed reserve as the difference between the total installed capacity of all generating units and the annual peak load.

This is an important tool for the planner for determination of the number of generating units, their size and commission dates into the system in order to meet the expected load demand.

The risk level can be calculated in a similar way even for the dynamic state, when a part of the required load may be non-supplied during the time interval between the failure occurrence and the new steady state. The results of this study are of great importance to the system operator in order to define *a priori* the immediate operational reserve required for a known daily load at a predetermined risk level. The design of the generation system in the past was based on deterministic reliability criteria. In this approach, the reserve margin (RM) is defined as the difference between the total available generation capacity G_A, and the annual peak load L_p:

$$\text{RM}=\frac{G_A - L_p}{L_p}100\% . \tag{3.1}$$

This index is *not* probabilistic. It has been used until new models based on probabilistic reliability indices were developed. As for the reserve, it was determined by accumulated praxis. It should be noted that the RM index does not take into account the type of the generating units (thermal, gas turbines, etc.), their sizes and their reliability levels. Thus, two systems with markedly different reliability levels may have the same RM value. As an example, consider two systems with the same annual peak load value and the same RM. The first system is composed of large generating units and the second of smaller units, say 50% of the large units. It is obvious that the second system is characterized by higher reliability level. The reason for that is that the probability for one large unit outage is higher than the probability for a simultaneous outage of two small units.

Another disadvantage of the deterministic indices is that they ignore the *stochastic* nature of factors influencing the reliability level, like, for example, the available generation capacity or the predicted load level. As a result, the deterministic indices overestimate the system reliability in contrast to a more sound probabilistic approach.

Accurate reliability evaluation should take into account the unit commitment and given operational constraints. The combination of a thorough mathematical analysis with computer-based computations gives not only more realistic indices values by simulating the unit commitment through randomly generated availabilities of the units but also allows for working out the cost-saving reserve management during system operation. The enhanced features of the modern approach are detailed below.

3.1 Reliability indices of generation systems

The calculation of reliability indices for the steady state, required for the planning of the generation system is based nowadays on two analytical methods. These methods enable to determine the required reserve for the generation system, taking into account the predicted load behavior and a predetermined reliability level. The first method is the loss of load probability, which means the probability of the load being larger than the available generation. The second method enables, besides the determination of the probability, to calculate the frequency and duration where the load is larger than the available generation. In this method, the frequency and duration, besides the probability, are studied. It is called the Frequency and Duration Method. The main difference between these two methods lies in the load models and the reliability parameters of generating units.

Both methods mentioned above are widely used, though the first one is more "popular". It should be mentioned that there is a third method for calculation of reliability indices, based on Monte Carlo simulation. All these methods are presented and discussed in this chapter.

3.1.1 Calculating reliability indices using the loss of load probability method

In order to calculate the reliability indices of the generation system, it is assumed that the rest of the power system (transformation, transmission, distribution, etc.) is of the highest reliability level. This means that any unavailability of power supply to the consumers is due to the generation system only. In other words, there are no constraints in transferring electric power via the transmission and transformation systems to the load for any given condition of the generation system. Such constraints may be, for example, overloading of transmission lines, voltage drop problems, etc. The problem under discussion is thus illustrated in Fig. 3.1

In the steady state operation of the system there should be a balance between the total operating generation capacity G and the load L: $G = L$. In the case of a failure in the generation system, the load may exceed the available generation: $G<L$. The failure condition (relation between the

generation and the load corresponding to the failure states) can be represented by a load loss function LLF:

$$\mathrm{LLF}(G, L) = 1(G < L) = \begin{cases} 1, & \text{if } G < L \\ 0, & \text{if } G \geq L. \end{cases} \quad (3.2)$$

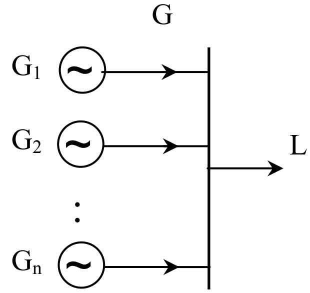

Fig. 3.1. The simple model of the generation system and the load (G_1,..., G_n are nominal generating capacities of n generating units, L is the load at a certain time)

The LLF produces a binary value for any combination of G and L. The expected value of the random binary LLF is equal to the probability that LLF=1 or, in other words, to the probability of the generation system inability to supply the load. This index is usually referred to as LOLP (Loss of Load Probability):

$$\mathrm{LOLP}(G, L) = E\{\mathrm{LLF}(G, L)\} = \Pr\{\mathrm{LLF}(G, L) = 1\} = \Pr\{G < L\}. \quad (3.3)$$

The total available generation capacity G usually takes on discrete values and, therefore can be represented by a pmf g_j, p_j=Pr$\{G$=$g_j\}$ for $1 \leq j \leq n$. The discretized load curve is also represented by its pmf l_i, q_i=Pr$\{L$=$l_i\}$ for $1 \leq i \leq k$. Having the pmf of the two random variables one can obtain the pmf of their function LLF(G,L). For this purpose the pmf of G and L should be represented by u-functions

$$U_G(z) = \sum_{j=1}^{n} p_j z^{g_j} \quad (3.4)$$

and

$$U_L(z) = \sum_{i=1}^{k} q_i z^{l_i}. \quad (3.5)$$

Applying the composition operator $\underset{LLF}{\otimes}$ over $U_G(z)$ and $U_L(z)$ one obtains the u-function representing the pmf of LLF(G,L):

$$U_{LLF}(z) = U_G(z) \underset{LLF}{\otimes} U_L(z) = (\sum_{j=1}^{n} p_j z^{g_j}) \underset{LLF}{\otimes} (\sum_{i=1}^{k} q_i z^{l_i})$$
$$= \sum_{j=1}^{n}\sum_{i=1}^{k} p_j q_i z^{1(g_j, l_i)}. \tag{3.6}$$

The expected LLF(G,L) or the LOLP can now be obtained as

$$\text{LOLP} = E(\text{LLF}) = U'_{\text{LLF}}(1) = \sum_{j=1}^{n}\sum_{i=1}^{k} p_j q_i \text{LLF}(g_j, l_i). \tag{3.7}$$

The Loss of Load Expectation (LOLE) represents the expected time (in hours or days) where the load is larger than the available generation. Assume that the operation time T is divided into S equal intervals $\delta t = T/S$ (time units). In each interval s the load takes on a random value of L (it is assumed that during the interval the load does not vary). The random time when the constant load L is not supplied in the interval s is equal to $\tau_s = \delta t \cdot LLF(G,L)$. This variable can take on either value of δt or 0. The random time when the variable load is not supplied during the entire operation period T can be obtained as

$$\sum_{s=1}^{S} \tau_s = \sum_{s=1}^{S} \delta t \cdot LLF(G,L) = \sum_{s=1}^{S} \frac{T}{S} \cdot LLF(G,L) = \frac{T}{S} \sum_{s=1}^{S} LLF(G,L). \tag{3.8}$$

The expectation of this time is

$$LOLE = E\left(\sum_{s=1}^{S} \frac{T}{S} \cdot LLF(G,L)\right) = \frac{T}{S} \cdot S \cdot E(LLF(G,L)) = T \cdot \text{LOLP}. \tag{3.9}$$

Another important index named Expected Load Not Supplied (ELNS) can be obtained by introducing the following unsupplied load function

$$\text{UL}(G,L) = \max\{L-G, 0\}. \tag{3.10}$$

The u-function representing the pmf of UL(G,L) can be obtained using the $\underset{UL}{\otimes}$ operator over $U_G(z)$ and $U_L(z)$

$$U_{UL}(z) = U_G(z) \underset{UL}{\otimes} U_L(z) = (\sum_{j=1}^{n} p_j z^{g_j}) \underset{UL}{\otimes} (\sum_{i=1}^{k} q_i z^{l_i})$$
$$= \sum_{j=1}^{n} \sum_{i=1}^{k} p_j q_i z^{\max\{l_i - g_j, 0\}}. \quad (3.11)$$

The Expected Unsupplied Load (EUL) is the expected value of the function UL(G,L). It can be obtained as

$$\text{EUL} = E(\text{UL}) = U'_{\text{UL}}(1) = \sum_{j=1}^{n} \sum_{i=1}^{k} p_j q_i \text{UL}(g_j, l_i)$$
$$= \sum_{j=1}^{n} \sum_{i=1}^{k} p_j q_i \max\{l_j - g_i, 0\}. \quad (3.12)$$

The Expected Unsupplied Energy (EUE) is obtained as the product of the EUL and the loss of load expectation:

$$\text{EUE} = \text{EUL} \cdot LOLE. \quad (3.13)$$

Example 3.1

Consider two separate generating systems (power plants) with a nominal capacity of 100 MW. In the first system, some types of failure require its capacity G_1 to be reduced to 60 MW and other types lead to a complete outage. In the second generator, some types of failure require its capacity G_2 to be reduced to 80 MW, others lead to a capacity reduction to 40 MW, and the rest lead to a complete outage. The generators are repairable and each of their states has a steady-state probability.

Both generators should supply a variable two-level load L. The high level (day) load is 50 MW and has the probability 0.6; the low level (night) load is 30 MW and has the probability 0.4.

The capacity and the load can be presented as a fraction of the nominal generator capacity. There are three possible relative capacity levels that characterize the performance of the first generating system:

$g_{10} = 0.0$, $g_{11} = 60/100 = 0.6$, $g_{12} = 100/100 = 1.0$, and four relative capacity levels that characterize the performance of the second generating system:

$g_{20} = 0.0$, $g_{21} = 40/100 = 0.4$, $g_{22} = 80/100 = 0.8$, $g_{23} = 100/100 = 1.0$
Assume that the corresponding steady-state probabilities are

$p_{10} = 0.1$, $p_{11} = 0.6$, $p_{12} = 0.3$
for the first generating system and

$p_{20} = 0.05$, $p_{21} = 0.35$, $p_{22} = 0.3$, $p_{23} = 0.3$

for the second generating system and that the load distribution is
$l_1 = 50/100 = 0.5$, $l_2 = 30/100 = 0.3$, $q_1 = 0.6$, $q_2 = 0.4$.

The probabilistic distributions of the generation capacities and the load are presented in Table 3.1.

Table 3.1. Distributions of the generation capacities and the load

		State 1	State 2	State 3	State 4
First generating	g	1.0	0.6	0	-
system	p	0.3	0.6	0.1	-
Second generating	g	1.0	0.8	0.4	0.0
system	p	0.3	0.3	0.35	0.05
Load	l	0.5	0.3	-	-
	q	0.6	0.4	-	-

The u-functions representing the capacity distribution of the generating systems (the pmf of random variables G_1 and G_2) take the form
$U_{G1}(z) = 0.1z^0+0.6z^{0.6}+0.3z^1$, $U_{G2}(z) = 0.05z^0+0.35z^{0.4}+0.3z^{0.8}+0.3z^1$
and the u-function representing the load distribution takes the form
$U_L(z) = 0.6z^{0.5}+0.4z^{0.3}$

The expected steady-state capacity of the generating systems can be obtained directly from these u-functions:

$$E(G_1) = U'_{G1}(1) = 0.1\times 0 + 0.6\times 0.6 + 0.3\times 1.0 = 0.66$$

which means 66% of the nominal generation capacity for the first generating system, and

$$E(G_2) = U'_{G2}(1) = 0.05\times 0 + 0.35\times 0.4 + 0.3\times 0.8 + 0.3\times 1.0 = 0.68$$

which means 68% of the nominal generation capacity for the second generating system.

The u-functions corresponding to the pmf of the LLF are obtained using the composition operator:

$$U_{LLF1}(z) = U_{G1}(z) \underset{LLF}{\otimes} U_L(z)$$

$$= (0.1z^0 + 0.6z^{0.6} + 0.3z^1) \underset{LLF}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$

$$= 0.06z^{1(0<0.5)} + 0.36z^{1(0.6<0.5)} + 0.18z^{1(1<0.5)} + 0.04z^{1(0<0.3)}$$

$$+ 0.24z^{1(0.6<0.3)} + 0.12z^{1(1<0.3)} = 0.06z^1 + 0.36z^0 + 0.18z^0 + 0.04z^1$$

$$+ 0.24z^0 + 0.12z^0 = 0.9z^0 + 0.1z^1$$

$$U_{LLF2}(z) = U_{G2}(z) \underset{LLF}{\otimes} U_L(z)$$
$$= (0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1) \underset{LLF}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.03z^1 + 0.21z^1 + 0.18z^0 + 0.18z^0 + 0.02z^1 + 0.14z^0$$
$$+ 0.12z^0 + 0.12z^0 = 0.74z^0 + 0.26z^1$$

The system LOLP (expected LLF) is

$$LOLP_1 = U'_{LLF1}(1) = 0.1$$

$$LOLP_2 = U'_{LLF2}(1) = 0.26$$

The u-functions corresponding to the pmf of the UL(G,L) function are obtained using the composition operator:

$$U_{UL1}(z) = U_{G1}(z) \underset{UL}{\otimes} U_L(z)$$
$$= (0.1z^0 + 0.6z^{0.6} + 0.3z^1) \underset{UL}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.06z^{\max(0.5-0,0)} + 0.36z^{\max(0.5-0.6,0)} + 0.18z^{\max(0.5-1,0)}$$
$$+ 0.04z^{\max(0.3-0,0)} + 0.24z^{\max(0.3-0.6,0)} + 0.12z^{\max(0.3-1,0)}$$
$$= 0.06z^{0.5} + 0.36z^0 + 0.18z^0 + 0.04z^{0.3} + 0.24z^0 + 0.12z^0$$
$$= 0.06z^{0.5} + 0.04z^{0.3} + 0.9z^0$$

$$U_{UL2}(z) = U_{G2}(z) \underset{UL}{\otimes} U_L(z)$$
$$= (0.05z^0 + 0.35z^{0.4} + 0.3z^{0.8} + 0.3z^1) \underset{UL}{\otimes} (0.6z^{0.5} + 0.4z^{0.3})$$
$$= 0.03z^{0.5} + 0.21z^{0.1} + 0.18z^0 + 0.18z^0 + 0.02z^{0.3} + 0.14z^0$$
$$+ 0.12z^0 + 0.12z^0 = 0.03z^{0.5} + 0.21z^{0.1} + 0.02z^{0.3} + 0.74z^0$$

The Expected Unsupplied Load is

$$EUL_1 = U'_{UL1}(1) = 0.06 \times 0.5 + 0.04 \times 0.3 + 0.9 \times 0 = 0.042$$
$$EUL_2 = U'_{UL2}(1) = 0.03 \times 0.5 + 0.21 \times 0.1 + 0.02 \times 0.3 + 0.74 \times 0 = 0.042$$

The absolute value of the EUL is 0.042·100 = 4.2 MW for both generating systems. Multiplying this index by T, the system operating time considered, one can obtain the EUE.

In complex power generation systems consisting of different generating units each unit besides its total failure state (with zero capacity) and full operation state (with nominal capacity) can have several intermediate states due to failures and/or maintenance and inspection actions leading to partial unit incapacitation (Billinton and Li 1992). The unit capacity in the intermediate states is less then its nominal capacity, but greater than zero. Therefore each generating unit should be considered as a multi-state element with number of states greater than two.

The probability of each state can be obtained based on failure statistics and maintenance schedule. Having the possible generating capacity levels and the corresponding probabilities for each unit one obtains the pmf of its random capacity. For the generating system consisting of different generating units with determined capacity distributions one can easily obtain the total capacity distribution (pmf) using the universal generating function method.

Consider a generation system consisting of n different generating units. Since the total system capacity is equal to the sum of capacities of the generating units:

$$G = \sum_{j=1}^{n} G_n \, , \tag{3.14}$$

One can obtain the u-function representing the pmf of the entire system capacity $U_G(z)$ applying the following operators over u-functions $U_{Gj}(z)$ representing the pmf of the units' capacities:

$$U_G(z) = U_{G1}(z) \underset{+}{\otimes} U_{G2}(z) \underset{+}{\otimes} \ldots \underset{+}{\otimes} U_{Gn}(z). \tag{3.15}$$

Example 3.2

Consider a generation system consisting of three generating units with variable random capacity (the capacity variations are caused by failures and maintenance/inspection actions). The system should supply a variable two-level load L. The high level (day) load is 260 MW and has the probability 0.6; the low level (night) load is 180 MW and has the probability 0.4. The distributions of unit capacities and of the load are presented in Table 3.2.

The u-functions representing the capacity distribution of the generating units (the pmf of random variables G_1, $G_2$and G_3) take the form

$U_{G1}(z) = 0.02z^0+0.98z^{50}$, $U_{G2}(z) = 0.02z^0+0.02z^{50}+0.96z^{100}$,

$U_{G3}(z) = 0.03z^0+0.01z^{50}+0.02z^{100}+0.94z^{150}$.

Table 3.2. Distributions of the unit generation capacities and the load

		State 1	State 2	State 3	State 4
First generating unit	g (MW)	50	0	-	-
	p	0.98	0.02	-	-
Second generating unit	g (MW)	100	50	0	-
	p	0.96	0.02	0.02	-
Third generating unit	g (MW)	150	100	50	0
	p	0.94	0.02	0.01	0.03
Load	l (MW)	240	110	-	-
	q	0.6	0.4	-	-

The u-function representing the capacity distribution of the entire generation system (the pmf of random variable G) can be obtained as

$$U_G(z)=U_{G1}(z)\underset{+}{\otimes}U_{G2}(z)\underset{+}{\otimes}U_{G3}(z)=(0.02z^0+0.98z^{50})$$

$$\underset{+}{\otimes}(0.02z^0+0.02z^{50}+0.96z^{100})\underset{+}{\otimes}(0.03z^0+0.01z^{50}+0.02z^{100}+0.94z^{150})$$

$$=(0.02z^0+0.98z^{50})(0.02z^0+0.02z^{50}+0.96z^{100})(0.03z^0+0.01z^{50}+0.02z^{100}$$

$$+0.94z^{150})=10^{-4}(0.12z^0+6.04z^{50}+13.72z^{100}+293.88z^{150}+289.84z^{200}$$

$$+552.88z^{250}+8843.52z^{300}).$$

The expected steady-state capacity of the generation system can be obtained directly from the u-function representing the pmf of the system capacity:

$$E(G)=U'_G(1)=10^{-4}(0.12\times0+6.04\times50+13.72\times100+293.88\times150$$

$$+289.84\times200+552.88\times250+8843.52\times300)=289.5$$

The u-function corresponding to the pmf of the LLF are obtained using the composition operator:

$$U_{LLF1}(z)=U_G(z)\underset{LLF}{\otimes}U_L(z)$$

$$=10^{-4}(0.12z^0+6.04z^{50}+13.72z^{100}+293.88z^{150}+289.84z^{200}$$

$$+552.88z^{250}+8843.52z^{300})\underset{LLF}{\otimes}(0.6z^{240}+0.4z^{110})$$

$$=0.6\cdot10^{-4}(0.12z^{1(0<240)}+6.04z^{1(50<240)}+13.72z^{1(100<240)}+293.88z^{1(150<240)}$$

$$+289.84z^{1(200<240)}+552.88z^{1(250<240)}+8843.52z^{1(300<240)})$$

$$+0.4\cdot10^{-4}(0.12z^{1(0<110)}+6.04z^{1(50<110)}+13.72z^{1(100<110)}+293.88z^{1(150<110)}$$

$$+289.84z^{1(200<110)}+552.88z^{1(250<110)}+8843.52z^{1(300<110)})$$

$$= 0.6\cdot 10^{-4}(0.12z^1 + 6.04z^1 + 13.72z^1 + 293.88z^1 + 289.84z^1$$
$$+ 552.88z^0 + 8843.52z^{0)}) + 0.4\cdot 10^{-4}(0.12z^1 + 6.04z^1 + 13.72z^1$$
$$+ 293.88z^0 + 289.84z^0 + 552.88z^0 + 8843.52z^0)$$
$$= 0.6\cdot 10^{-4}(603.6z^1 + 9396.4z^0) + 0.4\cdot 10^{-4}(19.88z^1 + 9980.12z^0)$$
$$= 10^{-4}(370.112z^1 + 9629.888z^0).$$

The system LOLP (expected LLF) is

$$LOLP = U'_{LLF}(1) = 0.0370$$

The u-function corresponding to the pmf of the unsupplied load function is obtained using the composition operator:

$$U_{LLF}(z) = U_G(z) \underset{UL}{\otimes} U_L(z)$$
$$= 10^{-4}(0.12z^0 + 6.04z^{50} + 13.72z^{100} + 293.88z^{150} + 289.84z^{200}$$
$$+ 552.88z^{250} + 8843.52z^{300}) \underset{UL}{\otimes} (0.6z^{240} + 0.4z^{110})$$
$$= 0.6\cdot 10^{-4}(0.12z^{240} + 6.04z^{190} + 13.72z^{140} + 293.88z^{90}$$
$$+ 289.84z^{40} + 552.88z^0 + 8843.52z^0) + 0.4\cdot 10^{-4}(0.12z^{110} + 6.04z^{60}$$
$$+ 13.72z^{10} + 293.88z^0 + 289.84z^0 + 552.88z^0 + 8843.52z^0).$$

The Expected Unsupplied Load is

$$EUL = U'_{UL}(1) = 0.6\cdot 10^{-4}(0.12\times 240 + 6.04\times 190 + 13.72\times 140$$
$$+ 293.88\times 90 + 289.84\times 40) + 0.4\cdot 10^{-4}(0.12\times 110 + 6.04\times 60$$
$$+ 13.72\times 10) = 2.489\ \text{MW}.$$

3.2 Evaluating generation system reliability with respect to unit commitment and operating constraints

There is a growing need within electric utility system operation to refine approaches to assessing reliability parameters as loss of load probability (LOLP) and expected unsupplied energy (EUE). These parameters are greatly affected by the system operation policy requirements such as spinning reserve requirements, the minimum unit in operation times, the minimum unit out of operation times and unit starting times**.** Account of those requirements renders the methods like those presented in (Baleriaux et al. 1967) and (Booth 1972) convolution impossible. In our opinion Monte-Carlo simulation of the generating units' availability is preferable for calculating the reliability parameters when considering the system operation policy. An important fact is that uncertain properties of a generating unit are dependent on its current state. Therefore the use of Markov transition matrices in approximation of electric utility system operation is more adequate than using the partial forced outage probabilities. Obviously, both unit commitment plans and forced outages of generating units have a direct effect on the value of available system capacity.

Therefore the main objectives of the presented method are:

- to suggest a Monte-Carlo based reliability evaluation model taking into consideration the system operation policy and constraints and using the transition probabilities;
- to measure the sensitivity of reliability indices with respect to variations in generating units parameters;
- to demonstrate the utilization of the model in assessing the effect of the system operation policy on the system reliability when using real utility provided data;
- to develop the model that can be used for long range planning purposes.

The suggested RMUCM (Reliability Modeling with Unit Commitment by Monte-Carlo) algorithm based on the Monte-Carlo simulation provides a computationally effective tool for system reliability indices calculation and sensivity analysis with respect to system operation policy and generating units' parameters. The reminder of this section details the approach first sketched in (Gurevich et al. 1995).

The existing tools for evaluation of the generation system operation reliability are basically different with respect to the method of representing the system load levels. For example, models such as EGEAS (Fleck and Charny 1984), Prosceen (Ashworth 1992), WASP (Jenkins and Joy 1974), Maintenance Scheduling Optimization Program (Gurevich et al. 1992) rely on the load duration curve. In turn, models such as POWRSYM (Babb 1983), BENCHMARK and PROMOD make use of chronological hourly load. Both approaches have advantages and limitations. Load duration curve representation is usually used in long range planning models where a rough evaluation of the system reliability is performed for various combinations of planning alternatives. In this case enormously long computation time prevents the possibility of increasing the accuracy of calculations. For the same reason the load duration curves are used in many maintenance scheduling algorithms.

In turn, the short term planning models, performing chronological detailed probabilistic simulation of power system operation, are usually based on traditional Forced Outage Rates (FOR) of the generating units, thus suffering from the lack of information about transitional probabilities between different availability states of a generating unit.

Obviously, the Monte-Carlo based approaches enable much better simulation of the real operation of a power system, thus leading to a much more precise evaluation of reliability indices (Billinton and Li 1994). Nevertheless, the limitation of these methods is the considerable computational effort (proportional to the number of samplings) depending on the desired accuracy of estimation.

The approach presented here separates the Monte-Carlo sampling process from the system reliability evaluation. First, the Monte-Carlo sampling process is performed for some predefined number of time periods and generating units. Thereafter, the desired reliability calculations (including the system operation simulation) require only a short computational time, as the major computational effort had already been made at the first stage. Such an approach allows performing the sensitivity analysis of the results regarding possible variations in system operation policy and generating units' parameters.

The reliability evaluation problem consists of the assessment of generation capacity adequacy, taking into account the equipment failures, the reduced capacity states and operation requirements and constraints.

The period of time for generation system reliability evaluation (the study period) can be subdivided into T small time intervals during which the availability state of the generation system is unchanged. The generation system is composed of N units. Each unit can be found in one out of a set of possible states. Four types of unit states are considered:

1. in operation with operating capacity equal to nominal;
2. in operation with maintenance reduced capacity (or out of operation);
3. in operation with forced outage reduced capacity (or out of operation);
4. out of operation as a result of a maintenance schedule performed prior to the study period.

Let the state of the generation system at the instant t (t = 1, ..., T) be represented by the vector $x(t) = (x_1(t), x_2(t,...,x_N(t))$, where $x_i(t)$ is the state of unit i. The trajectory of all the states of generation system during the study period is denoted by X and is further called the scenario of generation system reliability evaluation. The set of all possible scenarios x comprises the scenario space X.

For every scenario $x \in X$ there is an associated probability $P(x)$. Finally, let $LOLP(x)$ and $EUE(x)$ represent loss of load probability and expected unsupplied energy for a given scenario x. We have

$$LOLP(x) = \sum_{t=1}^{T} LOLP(x(t)), \tag{3.16}$$

and

$$EUE(x) = \sum_{t=1}^{T} EUE(x(t))\,. \tag{3.17}$$

Here $EUE(x(t))$ represents the energy not supplied at the instant t for scenario x and

$$LOLP(x(t)) = \begin{cases} 1 & if\ EUE(x(t)) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.18}$$

The expected value of loss of load probability is calculated as:

$$LOLP = \sum_{x \in X} LOLP(x) * P(x)\,, \tag{3.19}$$

and the expected value of unsupplied energy is calculated as :

$$EUE = \sum_{x \in X} EUE(x) * P(x)\,, \tag{3.20}$$

In the Monte-Carlo method it is presumed that each scenario has the same probability. If $\overline{LOLP}$ is the expected value of $LOLP$, and $\overline{EUE}$ is the expected value of EUE then these values are estimated as

$$\overline{LOLP} = \frac{1}{M} \sum_{m=1}^{M} LOLP(x^m) \tag{3.21}$$

and

$$\overline{EUE} = \frac{1}{M} \sum_{m=1}^{M} EUE(x^m)\,, \tag{3.22}$$

where $\overline{LOLP}$ is the estimate of loss of load probability, $\overline{EUE}$ is the estimate of expected unsupplied energy, M is number of samplings, x^m is m^{th} sampled value.

It is important to observe that $\overline{LOLP}$ and $\overline{EUE}$ are not the real expected values of $LOLP$ and EUE but their estimates. As x, $LOLP(x)$ and $EUE(x)$ are random variables; the estimators $\overline{LOLP}$ and $\overline{EUE}$ which are the averages of M samples of random variables $LOLP(x)$ and $EUE(x)$, are also random variables. The uncertainties about the estimates are given by the variance of the estimators:

$$V(\overline{LOLP}) = V(LOLP) / M \tag{3.23}$$

and

$$V(\overline{EUE}) = V(EUE) / M, \tag{3.24}$$

where variances $V(LOLP)$ and $V(EUE)$ are estimated as

$$V(LOLP) = \frac{1}{M} \sum_{m=1}^{M} (LOLP(x^m) - \overline{LOLP})^2 \tag{3.25}$$

and

$$V(EUE) = \frac{1}{M} \sum_{m=1}^{M} (EUE(x^m) - \overline{EUE})^2 \tag{3.26}$$

These expressions indicate that the uncertainty of the estimates depends on the variances of *LOLP* and *EUE* and is inversely proportional to the number of samplings. This uncertainty is often represented as a relative uncertainty or coefficient of variation:

$$\beta_{LOLP} = \sqrt{\overline{LOLP}} / \overline{LOLP} \tag{3.27}$$

and

$$\beta_{EUE} = \sqrt{\overline{EUE}} / \overline{EUE} \tag{3.28}$$

The Monte-Carlo based reliability evaluation scheme can then be summarized in the following steps:

1. initialize the number of samples $M = 0$;
2. obtain a scenario $x^m \in X$ by probability simulation, update M;
3. calculate the reliability parameters *LOLP* and *EUE* for the sampled scenario x^m;
4. estimate the expected values of the reliability indices $\overline{LOLP}$ and $\overline{EUE}$;
5. calculate the uncertainty of the estimates;
6. if the uncertainty is acceptable, stop the simulation process. Otherwise return to step 2.

The expressions defining the coefficients of variation can be used to estimate the required number of samplings for a given accuracy β. Substituting, for $V(\overline{LOLP})$ and $V(\overline{EUE})$, their expressions we obtain

$$\beta_{LOLP} = \sqrt{V(LOLP)/M} / \overline{LOLP} \tag{3.29}$$

and

$$\beta_{EUE} = \sqrt{V(EUE)/M} / \overline{EUE} \tag{3.30}$$

Rewriting these expressions with respect to M gives

$$M = max\left\{\frac{V(LOLP)}{(\beta * \overline{LOLP})^2}, \frac{V(EUE)}{(\beta * \overline{EUE})^2}\right\} \tag{3.31}$$

This expression illustrates some advantages and limitations of the Monte-Carlo method:

- Computational effort is not much affected by the system size: for this reason Monte-Carlo methods are very suitable for handling the complex process, such as generation system operation simulation;
- Computational effort is affected by the variance of the estimated reliability parameters;
- Computational effort is substantially affected by the required accuracy.

3.2.1 Monte-Carlo sampling of the system availability states

Scenario of the generation system capacity availability consists of *N* scenarios of all generating facilities. Each scenario of a generating unit is obtained by probability simulation. There is a planned maintenance outage schedule for each study period. The maintenance schedule is considered as an input for probability simulation of generation system operation and it is the same for all the samples. At the end of the maintenance period the generating unit available capacity is set equal to nominal.

The set *S* of unit's availability states contains three subsets:

- S_1 consists of only one state when the unit's available capacity is equal to nominal;
- S_2 consists of the states with the unit's available capacity reduced (or at full outage) as a result of a failure;
- S_3 consists of the states when the unit's available capacity is reduced (or totally zeroed) as a result of a maintenance which was planned prior to the unit commitment fulfillment.

There are many small maintenance actions (with durations less than 48 hours) which are not planned in advance. The necessity of those actions has emerged during the generation system operation. It is possible to postpone such actions and perform it during the convenient hours (night hours as a rule). Therefore the twenty-four hours are subdivided into the two time subperiods: night PT_1 and day PT_2. The statistical analysis shows that those maintenance periods are distributed uniformly during each subperiod.

The sets S_2 and S_3 consist of K' states. Each state is defined by a capacity level from *0* to nominal (not including nominal as it belongs to S_1). Thus, a generating unit can be put in one of the $K=2*K'+1$ states. For the probability simulation of generation system operation it is assumed that transition from one availability state to another may only depend on the assumed current state and belong to the corresponding time subperiod PT_τ. In such a way a generating unit state transitions can be modeled by Markov process.

Let us denote the probability of transition of unit *n* to state *j* from state *i* in time subperiod PT_τ by $p_{n\tau ij}$. For each unit *n* there are two transition matrices $P_{n\tau}$ corresponding to the subperiod PT_τ:

$$P_{n\tau} = \begin{pmatrix} p_{n\tau 11} & p_{n\tau 12} & \cdots & p_{n\tau 1K} \\ p_{n\tau 21} & p_{n\tau 22} & \cdots & p_{n\tau 2K} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n\tau K1} & p_{n\tau K2} & \cdots & p_{n\tau KK} \end{pmatrix} \tag{3.32}$$

These matrices are known as the stochastic transitional probability matrices. It should be noted that the sum of all the probabilities in each row of the matrix must equal 1 since each row *i* represents all the possible transitions to the other states in a particular instant of time given that it is in the state *i*.

For a class of Markov chain possessing the ergodic property, the state probabilities will converge to a vector of steady-state probabilities

$$p_{n\tau} = (p_{n\tau 1}\ p_{n\tau 2}\ \ \dots\ \ p_{n\tau K}) \tag{3.33}$$

such that

$$p_{n\tau} * P_{n\tau} = p_{n\tau}. \tag{3.34}$$

This matrix equation together with

$$\sum_{k=1}^{K} p_{n\tau k} = 1 \tag{3.35}$$

determine the vector $p_{n\tau}$.

To sample a generating unit state in the initial instant of time a value of *x* is sampled using the [0,1] uniform distribution. Thereafter, the index $k(x)$ is found to satisfy the inequality

$$\sum_{k=1}^{k(x)-1} p_{n1k} < x \le \sum_{k=1}^{k(x)} p_{n1k} \tag{3.36}$$

(it is assumed that the initial instant of time coincides with the beginning of twenty-four hours daily period, and consequently belongs to the first time subperiod PT_1). The initial state k_0 of the generating unit is equal to $k(x)$ To sample a generating unit state in a certain instant of time the same sampling procedure as in the initial instant is performed first to determine the value x. Next the index $k(x)$ is found to satisfy the inequality

$$\sum_{k=1}^{k(x)-1} p_{n\tau k_s k} < x \le \sum_{k=1}^{k(x)} p_{n\tau k_s k} \tag{3.37}$$

in the corresponding subperiod PT_τ for the current state k_s. The next unit state k_n is equal to $k(x)$. Execution of the sampling procedure for each instant of time during the study period results in the unit availability scenario.

The set T_n of instants at which $k_n \neq k_s$ and the available unit capacity has changed divides the study period into time intervals. The state of the unit, regarding the unit commitment during each interval, does not change. Set T_n contains the set $\overline{T_n}$ of instants at which $k_n \in S_2$ (the instants of failures) and the available unit capacity decreased. The set T_n divides the study period into time intervals so that the unit state can vary during these intervals but these variations are known beforehand and, therefore, it is possible to take them into account in unit commitment procedure.

Joining up all generating unit availability scenarios leads to the entire generation system availability scenario. The set

$$\overline{T} = \bigcup_{n=1}^{N} \overline{T_n} \tag{3.38}$$

divides the study period into time intervals so that the variations of the system state during each time interval are known from the beginning (there are no failures during this interval). Consequently, it is possible to fulfill the unit commitment for such an interval entirely.

Now we have a sequence of the generation system operation scenarios. Each scenario consists of integration of generating unit operation scenarios and subdivision of the study period into the time intervals without failures.

3.2.2 Implementing unit commitment

The purpose of unit commitment is to define, for each unit, the on line intervals during the unit commitment period to meet the load demand and to satisfy the generation system operation policy. The unit commitment period is subdivided into time intervals of the same length. This length is equal to the minimum time in order to start up an emergency unit (as a rule a jet turbine). Load is represented by peak loads at these time intervals.

The spinning reserve requirements are given in percent of load demand or in MW and may vary during the study period. It should be noted that sometimes it is impossible to satisfy the spinning-reserve requirements because of the generating units capacity shortages. These constraints can be written in the form

$$\sum_{j \in U_t} OC_{jt} \geq LOAD_t + SR_t \tag{3.39}$$

where j is index of unit, t is time instant, OC_{jt} is available capacity of unit j at instant t, $LOAD_t$ is load demand at instant t, SR_t is spinning reserve requirement at instant t, U_t is set of committed units at instant t.

The generation system operating constraints include:

- unit minimum uptime requirement according to which a generating unit is started up only if it is needed for more than a specified minimum number of hours of continuous operation;
- unit minimum down time rule. which does not permit a generating unit to be started up if it was out of operation less than a specified minimum number of hours;
- unit minimum start time, which specifies the minimum number of hours needed for bringing the unit from down to operating condition;
- unit must-run rule, which requires a specified generating unit to be operated all the time when it is available.

To perform the unit commitment the units are put into the priority queue and brought into operation according to their position in the queue. Must-run units (units that can never be shut down) are put first in the priority queue. Other units are ordered according to increasing production cost, decreasing minimum up/down time and increasing start up cost. Peaking units with high production cost and short minimum up/down time are located last in the priority queue.

To describe the unit commitment procedure the following notation is introduced:

T_{MDN}^{j} unit's minimum down time,

T_{MUP}^{j} unit's minimum up time,

T_{ST}^{j} unit's minimum start time,

SL_{jt} unfulfilled system demand faced at the instant t,

T duration of the unit commitment horizon,

τ_0 time during which unit j has been on-line (if $\tau_0>0$) or off-line (if $\tau_0<0$),

τ_s time from switching-on the unit j if it has been initially off-line ($\tau_0 < 0$) or 0 if the unit continues to be switched-off.

For each generating unit which is available and does not have the must-run status the first free-decision instant t_0 is defined as follows:

$$t_0 = \begin{cases} \max\{T^j_{MUP} - \tau_0 + 1, 1\} & \text{if } \tau_0 > 0 \\ \max\{T^j_{ST} - \tau_s + 1, 1\} & \text{if } \tau_0 < 0 \text{ and } \tau_s > 0 \\ \max\{T^j_{MDN} + \tau_0 + 1, T^j_{ST} + 1, 1\} & \text{if } \tau_0 < 0 \text{ and } \tau_s = 0 \end{cases} \tag{3.40}$$

If $\tau_0 > 0$ then unit j is on line in interval $[1, t_0)$ and off line otherwise. Based on the commitment priority rule, unit j should be committed at a free-decision instant $t \in [t_0, T]$ if there is unsupplied system demand at this instant ($SL_{jt} > 0$). As a result, each free-decision instant t must be either of the following:

- An on line instant ($SL_{jt} > 0$),
- An unclassified instant ($SL_{jt} \leq 0$) , for which commitment status has not yet been determined

Let I_k, $k = 1,...,K$ represent the closed interval k of consecutive unclassified instants contained in the free decision interval $[t_0, T]$, and let T_k represent the duration of interval I_k. These unclassified intervals are numbered according to their chronological orders. Two adjacent unclassified intervals are separated by an on line interval. Let $I_k = [t_{k1}, t_{k2}]$, then

$$T_k = t_{k2} - t_{k1} + 1. \tag{3.41}$$

Let τ_k represent the duration of interval when unit j was on-line ($\tau_k > 0$) or off-line ($\tau_k < 0$) just before the instant t_{k1}. Based on τ_k, the tentative commitment strategy associated with unclassified interval I_k can be estimated as follows:

- If $0 < \tau_k < T^j_{MUP} - T_k$ then unit will be set on-line during the whole interval I_k;
- If $0 < \tau_k < T^j_{MUP}$ and $\tau_k > T^j_{MUP} - T_k$ then determine $\overline{t_{k1}} = t_{k1} + (T^j_{MUP} - \tau_k)$, $\overline{T_k} = T_k - (T^j_{MUP} - \tau_k)$
  - If $\overline{T_k} < T^j_{MDN}$ then the unit will be set on line during the whole interval I_k;
  - If $\overline{T_k} \geq T^j_{MDN}$ then the unit will be set on line during the interval $[t_{k1}, \overline{t_{k1}})$ and off-line during the interval $[\overline{t_{k1}}, t_{k2}]$

- If $\tau_k \geq T^j_{MUP}$ and $T_k < T^j_{MDN}$ then the unit will be set on-line during the whole interval I_k;
- If $\tau_k \geq T^j_{MUP}$ and $T_k \geq T^j_{MDN}$ then the unit will be set off-line during the whole interval I_k;

If interval I_k is the first one ($k = 1$) and it begins at t_0 then the following rules are added to the commitment strategy:

- If $\tau_s > 0$ and $T_1 \leq T^j_{MUP}$ then the unit will be set on-line during the whole interval I_1 ;
- If $\tau_s > 0$ and $T_1 > T^j_{MUP}$ then determine $\overline{t_{11}} = t_{11} + T^j_{MUP}$, $\overline{T_1} = T_1 - T^j_{MUP}$
  - If $\overline{T_1} < T^j_{MDN}$ then the unit will be set on line during the whole interval I_1 ;
  - If $\overline{T_1} \geq T^j_{MDN}$ then the unit will be set on line during the interval $[t_{11}, \overline{t_{11}})$ and off-line during the interval $[\overline{t_{11}}, t_{12}]$.
  - If $\tau_s = 0$ and $\tau_1 < 0$ then the unit will be set off line during the whole interval I_1.

It should be noted that the commitment strategy associated with the unclassified interval T_k can be determined if τ_k is known. Since τ_k (and τ_s , if k=1) can be determined from j^s unit known commitment status prior to the interval I_k, the commitment strategy associated with the interval I_k can be evaluated. Based on τ_k and the commitment strategy associated with the interval I_1, τ_{k+1} can be easily determined. Thus, the commitment strategy associated with each unclassified interval can be recursively estimated.

If there are not enough units to meet the system demand at each instant, the sum of all the positive parts of differences between the total available generation system capacity and the system demand at each instant during the unit commitment horizon is added to the expected unsupplied energy and the total number of these instants contributes to the loss of load probability. When a failure happens at the last instant of the unit commitment horizon, then capacity drop is subtracted from the committed capacity if it was committed. If the remaining committed capacity does not meet the system demand then their difference is also added to the expected unsupplied energy and the number of loss of load instants is increased by unity .

The model described above provides a means for rapid evaluation of the influence of the system operation rules on reliability indices. It may become an efficient tool for choosing the proper system operation policy supporting the reliability requirements without additional capacities.

Example 3.3

Using a very simple example we show how to model power system operation for obtaining a statistically reliable assessment of the loss-of-load probability and the unserved energy. The random nature of the forced outages is described by the Markov chains combined with the Monte-Carlo simulations of the unit availability.

The generating system is formed of only five units detailed in Table 3.3

Table 3.3. Parameters of generating units

Unit	Nominal Capacity (Mw)	Minimum Capacity (Mw)	Min UpTime (minutes)	Min DownTime (minutes)	Start Time (minutes)
1	500	250	480	360	120
2	200	100	360	240	60
3	300	150	240	60	30
4	100	50	60	30	15
5	20	10	10	0	0

The units are sorted in the ascending order of their power generating cost, that is, the first unit produces the cheapest energy. This sorting defines the merit order for unit commitment.

We also suppose that each unit can be in either one of three different states: full unavailability (i=1), 50% availability (minimum capacity) (i=2) and full availability (i=3). The (3x3) probability transition matrices are presented in Table 3.4, where P_{ni} stands for the probability of the unit transition from the n-th to i-th state every 15 minutes. The diagonal entries of the matrices dominate the off-diagonal ones due to the unit ability of preserving its current state. The transition diagram, corresponding to the first matrix is shown on Fig. 3.2.

Table 3.4. The transution probabilities matrices

	Unit 1			Unit 2			Unit 3		
P_{ni}	i=1	i=2	i=3	i=1	i=2	i=3	i=1	i=2	i=3
n=1	95%	3%	2%	92%	5%	3%	85%	7%	8%
n=2	1%	97%	2%	4%	91%	5%	7%	87%	6%
n=3	2%	4%	94%	6%	4%	90%	6%	10%	84%

Table 3.4. (cont.)

	Unit 4			Unit 5		
P_{ni}	i=1	i=2	i=3	i=1	i=2	i=3
n=1	98%	1%	1%	93%	3%	4%
n=2	2%	97%	1%	4%	91%	5%
n=3	3%	2%	95%	5%	3%	92%

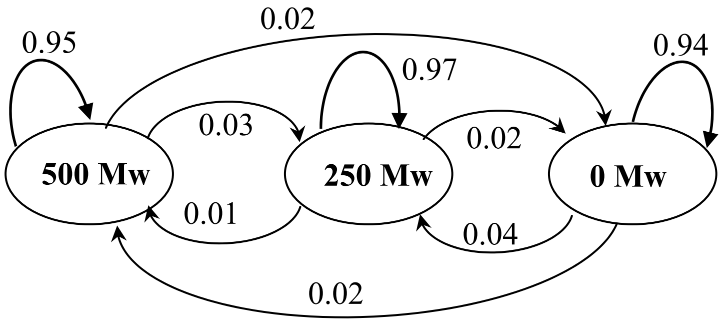

Fig. 3.2. Transition diagram of generating unit

For further simplicity, we assume that the load demand (*LD*) and spinning reserve (*SR*) are constant during 3 hours: *LD* = 900 Mw, *SR* = 50 Mw. The system operation should be organized in such a way as to provide (LD+SR) = 950 Mw of available capacity all the time at minimum cost. The installed capacity of the system (500+200+300+100+20) = 1120 Mw exceeds this requirement, so the unit commitment/optimal dispatch scheduling would be a very simple if there were not unforeseen forced outages. At specific moments they can result in the system failure to provide a required demand. In such a case one should discriminate between impossibility to satisfy the load demand or only the spinning reserve requirement. The first case results in a specific amount of the unserved energy while the latter is only potentially dangerous. Depending on the future system state, it may also lead to the unserved energy at the subsequent moments or show no impact on the consumers. Both cases are detailed in the rest of the paragraph.

Table 3.5 presents randomly generated availability of each unit for the three hours simulation period with the 15-minutes resolution.

Table 3.5. Availability of generating units

Time (Minutes)	Units					
	1	2	3	4	5	Total
0-15	500	200	300	100	20	1120
15-30	500	**100**	300	100	20	1020
30-45	500	**100**	300	100	20	1020
45-60	500	**100**	300	100	20	1020
60-75	500	**100**	300	100	20	1020
75-90	500	200	300	100	20	1120
90-105	500	200	300	100	20	1120
105-120	500	200	300	100	20	1120
120-135	500	200	300	100	20	1120
135-150	500	200	**150**	100	20	970
150-165	500	**100**	**150**	100	20	870
165-180	500	**100**	**150**	100	20	870
180-195	500	200	300	100	20	1120
195-210	500	200	300	100	20	1120
210-225	500	200	300	100	20	1120
225-240	500	200	300	100	20	1120

For reader convenience the partial and full unavailability of the units are boldfaced.

At first glance, the total availability of the units permits to provide LD+SR at each moment. However, in actual practice, the more expensive units (4 and 5 in our example) are shut down, whenever possible, to minimize the energy generation cost. This policy may lead to the SR shortage or, more severely, to the unsupplied energy, when either of the basic units 1-3 is not fully available due to an unexpected failure. Indeed, the unit 4 cannot be started immediately (because of its relatively long start up time) while the quickly started unit 5 has too small capacity for compensating even a partial unavailability of the basic units.

Tables 3.6 and 3.7 display the corresponding dispatch and spinning reserve contributions of the units.

At the first moment all the units are fully available. The generation of the required LD = 900 Mw is provided by nominal loading of the two cheapest units and partial loading (200 Mw) of the third unit. The rest of its available capacity (300-200) = 100 Mw covers the required SR = 50Mw. The expensive units 4 and 5 are not started up at all. At the moment (Time = 15) unit 2 drops down to only 50% of its nominal capacity 100 Mw), so that the total available on-line capacity at this moment is (500+100+300) = 900 Mw. This is sufficient only to satisfy the load demand (900 Mw) with no spinning reserve contribution.

Table 3.6. Dispatch of generating units during 3 hours.

Time	Units					
(Minutes)	1	2	3	4	5	Total
0-15	500	200	200			900
15-30	500	100	290		10	900
30-45	500	100	250	50		900
45-60	500	100	250	50		900
60-75	500	100	250	50		900
75-90	500	200	150	50		900
90-105	500	200	200			900
105-120	500	200	200			900
120-135	500	200	200			900
135-150	500	200	150		20	**870**
150-165	500	100	150	100	20	**870**
165-180	500	100	150	100	20	**870**
180-195	500	200	150	50		900
195-210	500	200	150	50		900
210-225	500	200	200			900
225-240	500	200	200			900

Table 3.7. Spinning reserve contribution of generating units

Time	Units					
(Minutes)	1	2	3	4	5	Total
0-15	0	0	100			50
15-30	0	0	10		10	**20**
30-45	0	0	50	50		100
45-60	0	0	50	50		100
60-75	0	0	50	50		100
75-90	0	0	150	50		200
90-105	0	0	100			100
105-120	0	0	100			100
120-135	0	0	100			100
135-150	0	0	0		0	**0**
150-165	0	0	0	0		**0**
165-180	0	0	0	0		**0**
180-195	0	0	150	50		200
195-210	0	0	150	50		200
210-225	0	0	100			50
225-240	0	0	100			50

Unit 4, though available, cannot be started up immediately due to its relatively long start-up time. For this reason, the only possible reaction is to start up unit 5 with practically zero start-up time at minimum loading level (10Mw) and to reduce loading of unit 3 to 290 Mw. This operation contributes only 20 Mw to the spinning reserve.

The shortage (50-20) = 30Mw of spinning reserve forces the dispatcher to start up unit 4 which becomes available after 15 minutes, which are needed for the start-up, at the moment (Time = 30). Unit 4 is loaded at the minimal level of 50 Mw and unit 3 is loaded at the level of 250 Mw. Spinning reserve contribution will be 100 Mw.

It continues to work up to the moment (Time = 75), when unit 2 restores its full availability. For saving purposes, unit 4 should be shut down. Nevertheless, because of the minimum up time requirement it will continue to be on line for another 15 minutes up to the moment (Time = 90). During this period unit 2 is loaded at the nominal capacity and unit 3 at the level of 150 Mw. Spinning reserve contribution will be 200 Mw.

At the moment (Time = 90) unit 4 is shut down and unit 3 is loaded at the level 200 Mw. Spinning reserve contribution will be 100 Mw.

At the moment (Time = 135), unit 3 possesses only 50% of its availability. Again, as before, unit 4 can not be started up immediately. Instead, the dispatcher performs the quick start of unit 5. This gives totally (500+200+150+20) = 870Mw, with (900-870) = 30Mw unsupplied.

At the moment (Time = 150) unit 4 is started but unit 2 drops to the level of 100 Mw. With no regard to ramp rate requirement let us suppose that the load of unit 4 could be immediately increased to the nominal level. Therefore, the total available capacity is (500+100+150+100+20) = 870Mw, with (900-870) = 30Mw unsupplied up to the moment Time = 180, when units 2 and 3 will be available.

Due to the minimum down requirements unit 4 is not shut down until the moment Time = 210. Thus, the load of unit 2 is increased to the nominal capacity, the load of unit 4 is decreased to the 50Mw, unit 5 is shut down and the load of unit 3 is not changed. Spinning reserve contribution is then 200 Mw.

At the moment Time = 210 unit 4 is shut down and unit 3 is loaded at the level 200 Mw. Spinning reserve contribution is equal to 100 Mw.

In this example we have 45 minutes of loss of load = 0.75LOLH (Time135-180) and the unsupplied energy (30Mw*0.75) = 22.5MWh.

3.3 Optimal reserve management for power generation systems

The purpose of power generating systems is to provide reliable and economical electrical energy to their customers. Back up resource or energy, which is designated as reserve, is commonly used in generating systems in order to provide adequate supply under unexpected system operating conditions such as generating system failures and load growth. A generating system can plan its own reserve and can also share reserve with other generating systems according to their reserve contracts. The reserve structure of a generating system should be determined based on the balance between the required reliability and the cost of reserve. The objective of reserve management for a generating system is to schedule the reserve at the minimum total system reserve cost while maintaining a required level of supply reliability to its customers (Ding, Wang, Lisnianski 2006).

A generating system usually consists of generating units (GUs) with different capacity, cost and availability. Different generating systems are usually connected together through supply network to form a complex generating system (CGS) as shown in Fig. 3.3.

The supply network is designated as transmitting system (TS). The objective of the interconnection is to share the reserve. A GS usually signs the reserve agreements with other GSs to increase its reliability and to reduce the cost of reserve. A CGS can operate in different states or performance levels due to random failures. The performance levels of a GS in the CGS are usually determined by the number of units, capacity and availability of each unit, transmitting system and reserve agreements with other GSs. The reserve management of a generating system in a CGS is, therefore, a complex optimization problem. A CGS and its GSs and TSs are multi-state systems (Lisnianski and Levitin 2003) that can be represented using the corresponding equivalents (Wang and Billinton 2003, Billinton and Wang 1999).

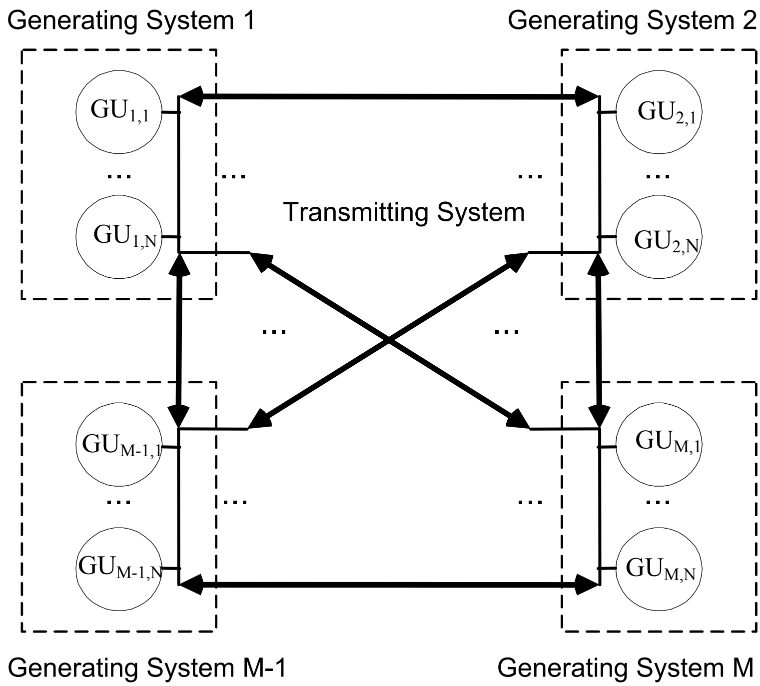

Fig. 3.3. Complex generation system

3.3.1 Multi-state system models

In order to determine the optimal reserve structure for a specific GS in a CGS, this GS, other GSs (serving as reserve providers) and the network between the GS and providers are represented as multi-state elements (multi-state generating system MSGS, multi-state reserve providers MSRPs and multi-state transmission system respectively). The model of a CGS for the reserve optimization is shown in Fig. 3.4.

Assume that MSGS has K_g states and each state i_g is associated with available generating capacity AG_{i_g}. These states can be divided into two sub-sets: the normal states (NS) and the contingency states (CS). For the normal states, AG_{i_g} is greater or equal than a load of the MSGS. For the contingency states, AG_{i_g} is less than a load of the MSGS. We assume that $\{1,2,...,m_g\}\in$ NS, and $\{m_g+1,...,K_g\}\in$ CS.

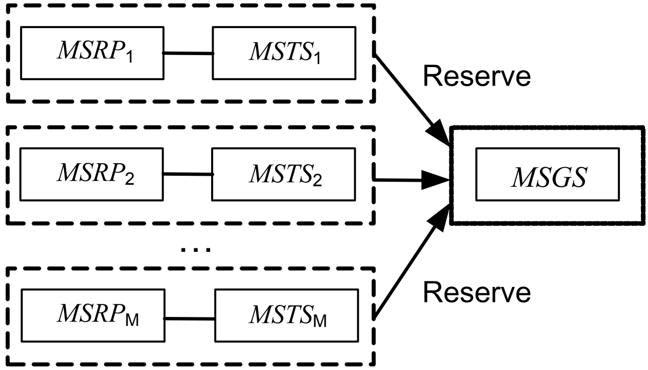

Fig. 3.4. Equivalent Model of a CGS for the MSGS

The performance level of MSRP *m* is characterized by its available generating capacity (AG_{mi_r} that can take K_m^r values in accordance with its states) and available reserve capacity AR_{mi_r} . AR_{mi_r} is determined by AG_{mi_r} , by the contractual reserve capacity CR_m , and by the types of contracts. There are two types of contracts: firm and infirm.

For the firm contract, $AR_{mi_r} = CR_m$ if $AG_{mi_r} \geq CR_m$ and $AR_{mi_r} = AG_{mi_r}$ if $AG_{mi_r} \leq CR_m$ so that

$$AR_{mi_r} = \min\{AG_{mi_r}, CR_m\} \tag{3.42}$$

For the infirm contract, CR_m is not guaranteed. It depends on the load level of MSRP *m* for interval *n* L_m^n . AR_{mi_r} is defined as follows:

$$AR_{mi_r} = \min\{\max(AG_{mi_r} - L_m^n, 0), CR_m\} \tag{3.43}$$

The performance level of MSTS *h* is represented by its available transmission capacity AT_{mi_t} that can take K_m^t values in accordance with its states.

The number of different states for the entire CGS is:

$$K = K_g \prod_{m=1}^{M} K_m^r \prod_{h=1}^{M} K_h^t \tag{3.44}$$

If all the elements in the system are independent, the probability of state *i* for the entire CGS is determined by the state probabilities of MSGS, MSRPs and MSTSs associated with state *i*:

$$p_i = p_{i_g} \prod_{m=1}^{M} p_{mi_r} \prod_{m=1}^{M} p_{mi_t} \tag{3.45}$$

When the CGS stays in state i, the performance level w_i of the MSGS after considering the assistance from the MSRPs can be determined based on AG_{i_g}, AR_{mi_r} and AT_{mi_t}.

The capacity distribution of the MSGS is determined by p_i and w_i. For a CGS which consists of a large number of elements, determining performance level of the MSGS is very resource consuming. The computational burden can be reduced by considering only the states in the *CS* of the MSGS.

The variable load is represented by a load duration curve (LDC), which is the rearrangement of time varying loads in descending order. The LDC is partitioned into different time periods T_n $(n = 1,...,N)$ with constant load level L^n for each time interval T_n.

Two reliability indices used for a MSGS are the MSGS unavailability or the lost of load probability (LOLP) and the expected energy not supplied (EENS):

$$LOLP = \sum_{n=1}^{N} \sum_{i=1}^{K} \Pr\{L^n > w_i\} \cdot T_n / T \tag{3.46}$$

$$EENS = \sum_{n=1}^{N} \sum_{i=1}^{K} \Pr\{L^n > w_i\} \cdot (L^n - w_i) \cdot T_n \tag{3.47}$$

Problem formulation

The reserve contract between the MSGS and MSRP m is determined by the contract type, CR_m, per unit costs of reserve capacity and of the utilized reserve provided by MSRP m (ε_{Cm} and ε_{Um} respectively), and also by the reliability of MSRP m and MSTS m. The objective of reserve management for a MSGS is to determine the optimal reserve structure, which includes the reserve contracts with MSRPs and the utilization order of MSRPs, under the minimum total reserve cost while satisfying the system reliability requirement. The total reserve cost includes the reserve capacity cost *CC* and the expected reserve utilization cost *EC*. The reserve optimization problem can be summarized as:

$$Min \sum_{m=1}^{M}(CC_m + EC_m) \tag{3.48}$$

$$CC_m = \varepsilon_{Cm} CR_m T \tag{3.49}$$

where

$$EC_m = \sum_{n=1}^{N}\sum_{i=1}^{K} p_i \varepsilon_{Um} UR_{mi}^n T_n \,, \tag{3.50}$$

subject to the following constraints.

The reserve constraints:

$$UR_{mi}^n \le CR_m \,, \tag{3.51}$$

$$UR_{mi}^n \le \min\left\{AR_{mi_r}, AT_{mi_t}\right\}; \tag{3.52}$$

The reliability constraints:

$$LOLP \le LOLP^{spec} \,, \tag{3.53}$$

$$EENS \le EENS^{spec} \,, \tag{3.54}$$

where $LOLP^{spec}$ and $EENS^{spec}$ are the required levels of the LOLP and EENS.

The load balance constraint takes the form

$$AG_{i_g} + \sum_{m=1}^{M} UR_{mi}^n = L^n - LL_i^n \,. \tag{3.55}$$

Load curtailment LL_i^n is utilized when the total generating and reserve capacity cannot meet the load of the MSGS.

UGF-based reliability evaluation

For an MSGS with G_N generating units connected in parallel, the *u*-function representing its total generating capacity distribution can be represented using the $\underset{+}{\otimes}$ operator over *u*-functions of G_N generating units:

$$U_{MSGS}(z) = \underset{+}{\otimes}\left[U_{G_1}(z),...,U_{G_N}(z)\right] = \prod_{j=G_1}^{G_N} U_j(z) = \\ \prod_{j=G_1}^{G_N}\left[\left(1-A_j\right)\cdot z^0 + A_j \cdot z^{w_{j,1}}\right] = \sum_{i_g=1}^{K_g} p_{i_g} \cdot z^{AG_{i_g}} \tag{3.56}$$

where A_j and $w_{j,1}$ are the availability and the nominal generating capacity of generating unit *j* respectively.

Notice that the first m_G terms in the $U_{MSGS}(z)$ with the associated states belonging to *NS* can be removed from the *u*-function without changing the results of reliability indices. In order to simplify the algorithm one can remove the first m_G terms from the *u*-function and define it as

$$U_{MSGS}(z) = \sum_{i_g=m_g+1}^{K_g} p_{i_g} \cdot z^{AG_{i_g}}. \tag{3.57}$$

Although all the generators of a MSRP are also connected in parallel, the *u*-functions representing the distribution of the random reserve depend on the type of the reserve contracts. For MSRP *m* with Rm_N generating units which are connected in parallel, the *u*-functions of MSRP *m* for the firm and infirm contracts can be represented using the operators $\underset{firm}{\otimes}$ and $\underset{infirm}{\otimes}$, respectively, over *u*-functions of Rm_N generating units.

For the firm contract

$$U_{MSRP}(z) = \underset{firm}{\otimes}\left[U_{Rm_1}(z),...,U_{Rm_N}(z), CR_m\right] = \\ = \underset{firm}{\otimes}\left[\underset{firm}{\otimes}\left(U_{Rm_1}(z),...,U_{Rm_N}(z)\right), CR_m\right] = \underset{firm}{\otimes}\left[\prod_{j=Rm_1}^{Rm_N} U_j(z), CR_m\right] \\ = \underset{firm}{\otimes}\left\{\prod_{j=Rm_1}^{Rm_N}\left[\left(1-A_j\right)\cdot z^0 + A_j \cdot z^{w_{j,1}}\right], CR_m\right\} \\ = \sum_{i_r=1}^{K_m^r} p_{mi_r} \cdot z^{\min(AG_{mi_r}, CR_m)} = \sum_{i_r=1}^{K_m^r} p_{mi_r} \cdot z^{AR_{mi_r}}. \tag{3.58}$$

For the infirm contract

$$
\begin{aligned}
U_{MSRP}(Z) &= \underset{infirm}{\otimes}\left[U_{Rm_1}(z),\ldots,U_{Rm_N}(z),CR_m,L_m^n\right]= \\
&= \underset{infirm}{\otimes}\left[\underset{infirm}{\otimes}\left(U_{Rm_1}(z),\ldots,U_{Rm_N}(z)\right),CR_m,L_m^n\right] = \underset{infirm}{\otimes}\left[\prod_{j=Rm_1}^{Rm_N} U_j(z),CR_m,L_m^n\right] \\
&= \underset{infirm}{\otimes}\left\{\prod_{j=Rm_1}^{Rm_N}\left[\left(1-A_j\right)\cdot z^0 + A_j\cdot z^{w_{j,1}}\right],CR_m,L_m^n\right\} \\
&= \sum_{n=1}^{N}\sum_{i_r=1}^{K_m^r} p_{mi_r}\cdot z^{\min(\max(AG_{mi_r}-L_m^n,0),\,CR_m)}\cdot T_n/T = \sum_{n=1}^{N}\sum_{i_r=1}^{K_m^r} p_{mi_r}\cdot z^{AR_{mi_r}}\cdot T_n/T
\end{aligned}
\tag{3.59}
$$

For MSTS m with Tm_N transmission lines connecting the MSGS with MSRP m, the u-function representing the distribution of the transmitting capacity can be determined using the $\underset{+}{\otimes}$ operator:

$$
\begin{aligned}
&U_{MSTS}(Z) = \underset{+}{\otimes}\left[U_{Tm_1}(z),\ldots,U_{Tm_N}(z)\right] = \prod_{j=Tm_1}^{Tm_N} U_j(z) \\
&\prod_{j=Tm_1}^{Tm_N}\left[\left(1-A_j\right)\cdot z^0 + A_j\cdot z^{w_{j,1}}\right] = \sum_{i_t=1}^{K_m^t} p_{mi_t}\cdot z^{AT_{mi_t}}
\end{aligned}
\tag{3.60}
$$

where A_j and $w_{j,1}$ are the availability and the nominal capacity of transmission line j.

The MSRP m and MSTS m are connected in series. To calculate the UGF for the series-connected subsystem, the $\underset{\min}{\otimes}$ operator should be used:

$$
\begin{aligned}
&\underset{\min}{\otimes}(U_{MSPR}(z),U_{MSTS}(z)) = \underset{\min}{\otimes}\left[\sum_{i_r=1}^{K_m^r} p_{mi_r}\cdot z^{AR_{mi_r}},\sum_{i_t=1}^{K_m^t} p_{mi_t}\cdot z^{AT_{mi_t}}\right] \\
&= \sum_{i_r=1}^{K_m^r}\sum_{i_t=1}^{K_m^t} p_{mi_r}\cdot p_{mi_t}\cdot z^{\min(AR_{mi_r},AT_{mi_t})}
\end{aligned}
\tag{3.61}
$$

The u-function of the MSGS $U_{MSGS}^{WR}(z)$ with the reserve agreements with MSRPs and the capacity limits of MSTSs can be now obtained as:

$$
\begin{aligned}
&U_{MSGS}^{WR}(z) = \underset{+}{\otimes}\Big\{U_{MSGS}(z),\ldots,\underset{\min}{\otimes}\left[U_{MSRS_m}(z),U_{MSTS_m}(z)\right],\ldots, \\
&\underset{\min}{\otimes}\left[U_{MSRS_M}(z),U_{MSTS_M}(z)\right]\Big\}
\end{aligned}
\tag{3.62}
$$

From $U_{MSGS}^{WR}(Z)$ we can obtain the $LOLP$ using the following equation

$$
LOLP=\sum_{n=1}^{N}\sum_{i=1}^{K}\Pr\left\{L^n > w_i\right\}\cdot T_n/T = \sum_{n=1}^{N}\sum_{i\in CS} p_i\cdot \mathbf{1}(L^n - w_i)\cdot T_n/T
\tag{3.63}
$$

The $EENS$ for a given period can be obtained as

$$EENS = \sum_{n=1}^{N}\sum_{i=1}^{K} \Pr\left\{L^n > w_i\right\} \cdot (L^n - w_i) \cdot T_n$$
$$= \sum_{n=1}^{N}\sum_{i \in CS} \Pr\left\{L^n > w_i\right\} \cdot (L^n - w_i) \cdot T_n = \sum_{n=1}^{N}\sum_{i \in CS} p_i \max\left\{L^n - w_i, 0\right\} \cdot T_n. \quad (3.64)$$

Implementing the Genetic Algorithm

A mixed numerical and binary string of length $\sum_{m=1}^{M} D_m$ is used to encode a solution in the GA, where D_m is a number of binary bits encoding the amount of reserve capacity CR_m. A typical chromosome representing three reserve providers is shown in Fig 3.5.

The first M elements ($a_1 a_{2 \ldots} a_M$) of string represent order of MSRPs utilization in a contingency state (a_j is an order of MSRP j). This substring should be a permutation of integer numbers from 1 to M.

The next $\sum_{m=1}^{M} D_m$ bits represent the contracted reserve capacity CR_m for M MSRPs. Encoding is performed using different number of bits D_m for each contracted reserve amount, depending on the desired accuracy. The decoding procedure obtains the amount of the reserve for each contract as

$$CR_m = CR_m^{\min} + (CR_m^{\max} - CR_m^{\min}) \cdot \frac{k}{2^{D_m} - 1} \quad (3.65)$$

where CR_m takes the values in the interval $[CR_m^{\min}, CR_m^{\max}]$ and k is the decimal number represented by the binary substring.

Using this encoding algorithm, the solutions for obtaining the reserve utilization order are within the feasible space. According to the string represented in Fig. 3.5, MSRP 2 is used first, MSRP 1 is used second, and so on, up to the point the load is met, or the whole available reserve is used in a contingency state.

To let the GA look for the solution with minimal total cost without violating the constraints $LOLP \le LOLP^{spec}$ and $EENS \le EENS^{spec}$, the solution quality (fitness) is defined as

$$F = q \cdot \max(0, LOLP - LOLP^{spec}, EENS - EENS^{spec}) + \sum_{m=1}^{M} (CC_m + EC_m) \quad (3.66)$$

where q is a sufficiently large penalty coefficient.

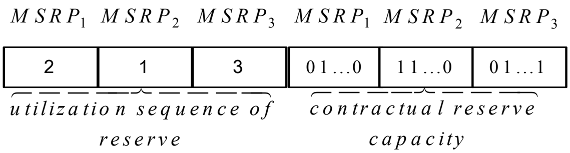

Fig. 3.5. GA chromosome structure

Example 3.4

Consider the restructured IEEE-RTS (IEEE Task Force 1979) consisting of six MSGSs in which the optimal reserve structure should be determined for the specific MSGS, which has reserve agreements with other five MSGSs. The MSGS owns 6×50MW hydro units, 5×12MW oil units, 2×155MW coal units and 2×400MW nuclear units. MSRP1 owns 2×20MW gas turbine units and 2×76MW coal units. MSRP 2 owns 3×100MW oil units. MSRP 3 owns 2×20MW gas turbine units and 2×76MW coal units. MSRP 4 owns 3×197MW oil units. MSRP 5 owns 2×155MW coal units and one 350MW coal unit. The MSRPs are connected to the MSGS through tie lines.

The tie line reliability data (Allan et al. 1986) is used in this paper. We assume that all the tie lines are identical, each with the capacity rating of 300MW and unavailability 0.00130873. MSTSs 1, 2 and 3 each have one tie line. MSTSs 4 and 5 each have two tie lines.

The four-step model of the LDC obtained from hourly loads is used. The peak load of the MSGS is 1253MW. For the MSGS, the load levels L^n and the total time periods T^n of a particular step are shown in Table 3.8. The load level L^n of a particular step is determined by the average value of hourly load in the time period of T^n.

Table 3.8. The MSGS load level and time period

L^n (MW)	549	754	939	1051
T^n (h)	2281	3909	1458	968

All the MSRPs are firm contract providers for the MSGS. The utilization prices for MSRPs are 25$/MWh, 55$/MWh 75$/MWh, 60$/MWh and 20$/MWh, respectively. The reserve capacity prices for MSRPs are 2.5$/MWh, 5.5$/MWh 7.5$/MWh, 6$/MWh and 2$/MWh, respectively. The time frame for the contracts is one year. Two examples with different reliability constraints are investigated in this case.

In example A, the *LOLP* and *EENS* constraints for the MSGS are set as 0.01 and 500MWh/yr respectively. In example B, the reliability constraints are tightened and the *LOLP* and *EENS* for the MSGS are 0.0005 and 50MWh/yr respectively.

Table 3.9 shows the total reserve cost and the reserve capacities between the MSGS and MSRPs for the two examples.

Table 3.9. The total reserve cost and reserve structure for for the first case

	Example A	Example B
Total Reserve Cost (k$)	10197.0	10896.1
CR_{MSRP1} (MW)	53.728	70.355
CR_{MSRP2} (MW)	1.7895	0.0185
CR_{MSRP3} (MW)	0.2981	0.0186
CR_{MSRP4} (MW)	6.4949	0.0524
CR_{MSRP5} (MW)	374.82	419.02

The capacity reserve cost and expected cost of reserve utilization for each MSRP are shown in Table 3.10. It can be seen from Tables 3.9 and 3.10 that the reserve cost has increased dramatically when the reliability requirements were tightened.

Table 3.10. The capacity reserve cost and expected cost of utilized reserve for the first case

	Example A		Example B	
MSRP No.	CC_m (k$)	EC_m (K$)	CC_m (k$)	EC_m (K$)
1	1172.6	24.7	1536.1	18.2
2	86.0	1.02	0.89	0.0013
3	19.5	0.23	1.22	0.0017
4	340.4	4.05	2.74	0.004
5	6549.4	1999.0	7321.2	2015.7

3.4 Determination of the random generating capacity distribution for power plant

In this section we consider the evaluation of reliability indices of a separated power plant. These indices are used for the entire generation system reliability calculation.

The power plant usually consists of several generating units. Each of the generating units comprises a complex system of technical components connected in parallel or in series in reliability block diagram sense. In order to increase the reliability of generating units, redundancy of their auxiliary facilities (feeders, conveyors, pumps, heaters, condensers, heat exchangers, air fans, storage tanks etc.) is provided. Failures of some components may cause a total outage (for example a failure in the boiler) whereas failures of other components cause only a reduction in the power output of the unit (for example a failure in one of the feeder pumps). Besides unexpected failures, planned maintenance of components also affects the unit reliability.

The components are aimed at performing their tasks with a nominal performance rate. They can usually be represented by a two state model. When the component h is available its performance G_h is equal to the nominal performance rate g_h and if the component is not available, its performance equals zero. The pmf of the element performance can be represented by the following u-function

$$u_h(z) = A_h z^{g_h} + (1 - A_h) z^0 = A_h z^{g_h} + \overline{A}_h z^0, \tag{3.67}$$

where A_h is the availability of component h. The availability of a component at steady state is calculated by two statistical parameters: the Mean Time to Failure (MTTF) and the Mean Time to Repair (MTTR).

$$A_h = \frac{MTTF_h}{MTTF_h + MTTR_h} = \frac{\mu_h}{\mu_h + \lambda_h} \tag{3.68}$$

where λ_h and μ_h are the forced outage rate and the repair rate of component h respectively, λ = 1/MTTF, μ = 1 / MTTR.

The unavailability $\overline{A}_h$ which is also called FOR (Forced Outage Rate), is calculated as:

$$\overline{A}_h = \mathrm{P}(G_h = 0) = \mathrm{FOR} = \frac{\lambda_h}{\mu_h + \lambda_h}. \tag{3.69}$$

When component performance is defined as capacity or productivity, the total capacity $G_{\{i,j\}}$ of a subsystem containing two independent components i and j connected in series is equal to the capacity of a bottleneck component (the component with lower performance):

$$G_{\{i,j\}} = \min\{G_i, G_j\}. \qquad (3.70)$$

The u-function representing the pmf of $G_{\{i,j\}}$ can be, therefore, obtained using the operator $\underset{\min}{\otimes}$ over the u-functions representing pmf of G_i and G_j. For example the u-function $U_{1,2}(z)$ representing the total performance of a subsystem consisting of a pair of two-state components with nominal performances g_1 and g_2 and availabilities A_1 and A_2 connected in series takes the form

$$\begin{aligned} U_{1,2}(z) &= u_1(z) \underset{\min}{\otimes} u_2(z) \\ &= (A_1 z^{g_1} + \overline{A}_1 z^0) \underset{\min}{\otimes} (A_2 z^{g_2} + \overline{A}_2 z^0) \\ &= A_1 A_2 z^{\min\{g_1,g_2\}} + \overline{A}_1 A_2 z^0 + A_1 \overline{A}_2 z^0 + \overline{A}_1 \overline{A}_2 z^0 \\ &= A_1 A_2 z^{\min\{g_1,g_2\}} + (1 - A_1 A_2 z^0). \end{aligned} \qquad (3.71)$$

The u-functions for any series system can be obtained recursively by consecutively determining the u-functions of pairs of the components. For example the u-function of a system consisting of four components connected in a series can be determined in the following way:

$$u_1(z) \underset{\min}{\otimes} u_2(z) \underset{\min}{\otimes} u_3(z) \underset{\min}{\otimes} u_4(z) = U_{1,2}(z) \underset{\min}{\otimes} U_{3,4}(z)$$

where

$$U_{1,2}(z) = u_1(z) \underset{\min}{\otimes} u_2(z) \text{ and } U_{3,4}(z) = u_3(z) \underset{\min}{\otimes} u_4(z).$$

The total capacity of a subsystem containing two independent components connected in parallel (able to perform the same task) is equal to the sum of the capacities of the individual components. Therefore, the structure function for such a subsystem takes the form

$$G_{\{i,j\}} = G_i + G_j. \qquad (3.72)$$

The u-function representing the pmf of $G_{\{i,j\}}$ can be, therefore, obtained using the operator $\underset{+}{\otimes}$ over the u-functions representing pmf of G_i and G_j. For example the u-function $U_{1,2}(z)$ representing the total performance of a subsystem consisting of a pair of two-state components with nominal performances g_1 and g_2 and availabilities A_1 and A_2 connected in parallel takes the form

$$\begin{aligned} U_{1,2}(z) &= u_1(z) \underset{+}{\otimes} u_2(z) = (A_1 z^{g_1} + \bar{A}_1 z^0) \underset{+}{\otimes} (A_2 z^{g_2} + \bar{A}_2 z^0) \\ &= A_1 A_2 z^{g_1+g_2} + \bar{A}_1 A_2 z^{g_2} + A_1 \bar{A}_2 z^{g_1} + \bar{A}_1 \bar{A}_2 z^0. \end{aligned} \tag{3.73}$$

As in the case of the series connection, the u-functions for any parallel system can be obtained recursively by consecutively determining the u-functions of pairs of the components.

An example of a typical block diagram for a power unit is presented in Fig. 3.6. In this figure the random capacities of different facilities, subsystems and an entire power unit are determined in accordance with the block diagram.

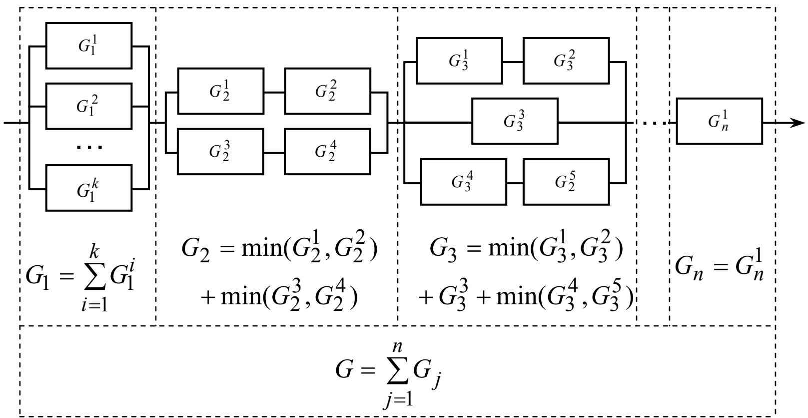

Fig. 3.6. Example of power unit block diagram

The complex series-parallel system can always be represented as composition of the independent subsystems containing only components connected in a series or in parallel. Therefore, in order to obtain the u-function of a series-parallel system one has to apply the composition operators recursively and obtain u-functions of the intermediate pure series or pure parallel structures.

The following algorithm realizes this approach:

1. Find the subsystem consisting of two components connected in parallel or in series in the given reliability block diagram.

2. Obtain u-function of these subsystem using the corresponding $\underset{\min}{\otimes}$ or $\underset{+}{\otimes}$ operators.
3. Replace in the diagram the subsystem with single component having the u-function obtained for the given subsystem.
4. If the reliability block diagram contains more then one element return to step

The resulting u-function represents the capacity distribution of the entire system.

Example 3.5

In order to illustrate the recursive approach (the reliability block diagram method), consider the series-parallel power station coal feeding system presented in Fig. 3.7A. The parameters of the system elements are presented in Table 3.11.

Table 3.11. Parameters of elements of series-parallel system

j	1	2	3	4	5	6	7
	Primary feeder	First-level stacker	Second-level stacker	Third-level stacker	Bypass stacker	Secondary feeder	Bypass conveyor
g_j	5	3	5	4	2	6	3
A_j	0.9	0.8	0.9	0.7	0.6	0.8	0.8

First, one can find a series subsystem consisting of elements with the u-functions $u_2(z)$, $u_3(z)$ and $u_4(z)$. By calculating the u-functions $U_1(z)=u_2(z)\underset{\min}{\otimes}u_3(z)$ and $U_2(z)=U_1(z)\underset{\min}{\otimes}u_4(z)$ and replacing the three elements with a single element with the u-function $U_2(z)$ one obtains a system with the structure presented in Fig. 3.7B. This system contains a purely parallel subsystem consisting of elements with the u-functions $U_2(z)$ and $u_5(z)$, which in their turn can be replaced by a single element with the u-function $U_3(z)=U_2(z)\underset{+}{\otimes}u_5(z)$ (Fig. 3.7C). The structure obtained has three elements connected in a series that can be replaced with a single element having the u-function $U_5(z)$, where $U_4(z)=u_1(z)\underset{\min}{\otimes}U_3(z)$ and $U_5(z)=U_3(z)\underset{\min}{\otimes}u_6(z)$ (Fig. 3.7D). The resulting structure contains two elements connected in parallel. The u-function of this structure representing the pmf of the entire system performance is obtained as $U(z) = U_5(z) \underset{+}{\otimes} u_7(z)$.

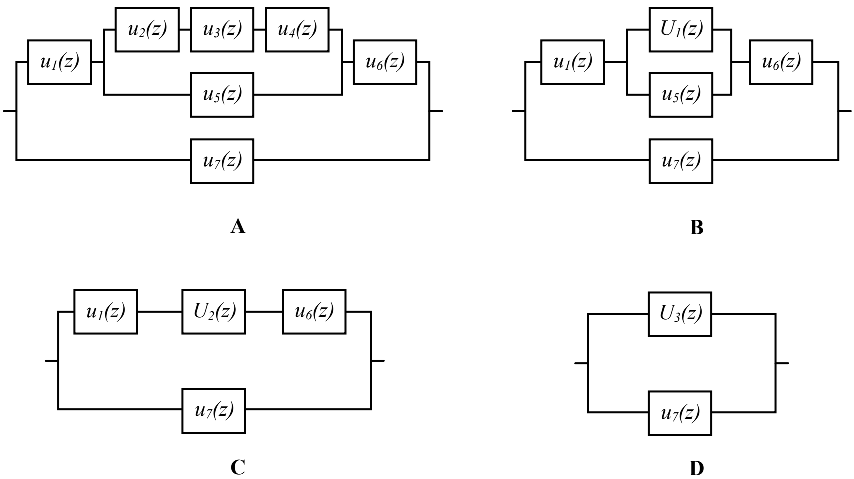

Fig. 3.7. Example of recursive determination of the *u*-function for a series-parallel system

The process of calculating $U(z)$ for the system is as follows:

$U_1(z)=u_2(z) \underset{\min}{\otimes} u_3(z) =(0.8z^3+0.2z^0) \underset{\min}{\otimes} (0.9z^5+0.1z^0)=0.72z^3+0.28z^0$

$U_2(z)= U_1(z) \underset{\min}{\otimes} u_4(z)= (0.72z^3+0.28z^0) \underset{\min}{\otimes} (0.7z^4+0.3z^0)$

$=0.504z^3+0.496z^0$

$U_3(z)=U_2(z) \underset{+}{\otimes} u_5(z)=(0.504z^3+0.496z^0) \underset{+}{\otimes} (0.6z^3+0.4z^0)$

$= 0.3024z^6+0.4992z^3+0.1984z^0$

$U_4(z)=u_1(z) \underset{\min}{\otimes} U_3(z)=(0.9z^5+0.1z^0) \underset{\min}{\otimes} (0.3024z^6+0.4992z^3+0.1984z^0)$

$= 0.27216z^5+0.44928z^3+0.27856z^0$;

$U_5(z) = U_4(z) \underset{\min}{\otimes} u_6(z)=(0.27216z^5+0.44928z^3$

$+0.27856z^0) \underset{\min}{\otimes} (0.8z^6+0.2z^0)= 0.217728z^5+0.359424z^3+0.422848z^0$;

$U(z)=U_5(z) \underset{+}{\otimes} u_7(z)=(0.217728z^5+0.359424z^3+0.422848z^0) \underset{+}{\otimes} (0.8z^3$

$+0.2z^0)=0.1741824z^8+0.2875392z^6+0.0435456z^5$

$+0.4101632z^3+0.0845696z^0$.

Having the system *u*-function that represents its capacity distribution one can easily obtain the system mean capacity $E(G)=U'(1)=4.567$. The system LOLP for different constant load levels l can be obtained by summing the probabilities corresponding to system capacity levels that are not less than the load l in the system capacity pmf represented by the *u*-function $U(z)$:

$$LOLP(l) = 0.0846 \text{ for } 0<l\leq 3$$

$$LOLP(l) = 0.4947 \text{ for } 3<l\leq 5$$

$$LOLP(l) = 0.5383 \text{ for } 5<l\leq 6$$

$$LOLP(l) = 0.8258 \text{ for } 6<l\leq 8$$

$$LOLP(l) = 1 \text{ for } l>8$$

3.5 Power plant reliability optimization problems

Reliability improvement is of critical importance in various kinds of systems; however, any effort for this type of improvement usually requires resources that are limited by technical and/or economical constraints. Two approaches can be distinguished in the reliability optimization problem. The first one is aimed at achieving the greatest possible reliability subject to different constraints (according to (Gnedenko and Ushakov 1995), this problem is named the direct reliability optimization problem), and the second one focuses on minimizing the resources needed for providing a required reliability level (this problem is named the inverse reliability optimization problem.

There are four general methods for improving system reliability:

- a provision of redundancy;
- an optimal adjustment of the system’s parameters, an optimal arrangement of the existing elements or the assignment of interchangeable elements;
- an enhancement of the reliability (availability) and/or performance of the system’s elements;
- a combination of the above-mentioned methods.

Applied to power generation systems, these methods affect two basic system properties: its configuration and the capacity distribution of its elements.

The UGF method that allows for system capacity distribution, and thereby its performance measures to be evaluated based on a fast procedure, opens new possibilities for solving reliability optimization problems. Based on the UGF technique, the system reliability can be obtained as a function of the system structure and the capacity distributions of its elements. Therefore, numerous optimization problems can be formulated in which the optimal composition of all or part of the factors influencing the entire system reliability has to be found subject to different constraints.

3.5.1 Structure optimization problems

In binary systems, providing redundancy means to incorporate several identical parallel elements into a certain functional subsystem (component). The redundancy optimization problem for a system, which may consist of elements with different capacity distributions, is a problem of system structure optimization. Indeed, when solving practical system design problems, the reliability engineer deals with a variety of products existing on the market. Each product is characterized by its capacity distribution, price, *etc.* To find the optimal system structure, one should choose the appropriate versions from a list of available products for each type of equipment, as well as the number of parallel elements of these versions.

In the simplest structure optimization problem, each component can contain only identical elements. This optimization problem is relevant in cases where contracting or maintenance considerations prevent purchasing or using different elements of the same type. In some cases, such a limitation can be undesirable or even unacceptable for two reasons:

- Allowing different versions of the elements to be allocated in the same system component, one obtains a solution providing the desired reliability level at a lower cost than what appears in the solution containing identical parallel elements.
- In practice, when a system needs to be modernized according to new demand levels or new reliability requirements, the designer often has to include additional elements in the existing system. Some system components can contain versions of elements that are unavailable. In this case, some elements with the same functionality but with different parameters should make up the components. Therefore, in the general case, the generating system structure optimization problem should be solved without placing a limitation on the diversity of the versions of the elements.

The above-mentioned problem of optimal single-stage system expansion to enhance its reliability and/or performance is an important extension of the structure optimization problem. In this case, one has to decide which elements should be added to the existing system and to which component they should be added. There is a similar problem with the optimal single-stage replacement of the system elements. Here, one has to decide which elements should be replaced by new ones having better characteristics.

During the system's operation time, the demand and reliability requirements can change. To provide a desired level of system performance, management should develop a multistage expansion plan. For the problem of optimal multistage system expansion, it is important to answer not only the question of what must be included into the system, but also the question of when it must be included into the system.

Optimal structure of systems with identical elements in each component

Problem Formulation. A system consists of N components. Each component is a subsystem that can consist of parallel elements with the same functionality. The interaction between the system components is given by a reliability block diagram. Different versions of elements may be chosen for any given system component, but each component can contain only elements of the same version. For each component i there are B_i element versions available in the market. A capacity distribution $\boldsymbol{g}_i(b)$, $\boldsymbol{p}_i(b)$ and cost $c_i(b)$ can be specified for each version b of element of type i.

The structure of system component i is defined by the number of the element version chosen for this component b_i ($1 \le b_i \le B_i$) and by the number of parallel elements n_i ($1 \le n_i \le n_{\max i}$), where $n_{\max i}$ is the maximum allowed number of parallel elements of type i. The vectors $\boldsymbol{b} = (b_1, \ldots, b_N)$ and $\boldsymbol{n} = (n_1, \ldots, n_N)$ define the entire system structure.

For given $\boldsymbol{b}$ and $\boldsymbol{n}$ the total cost of the system can be calculated as

$$C = \sum_{i=1}^{N} n_i c_i(b_i) \tag{3.74}$$

To take into account price discounting, the element cost can be considered as a function of the number of elements purchased. In this case

$$C = \sum_{i=1}^{N} n_i c_i(n_i, b_i) \tag{3.75}$$

The problem of system structure optimization is formulated in (Levitin et al. 1998) as finding the minimal cost system configuration $\boldsymbol{b}$, $\boldsymbol{n}$ that provides the required level A^* of the system availability A:

$$C(\boldsymbol{b}, \boldsymbol{n}) \to \min \text{ subject to } f(A, A^*) = 1 \tag{3.76}$$

where $f(A,A^*)$ is a function representing the desired relation between A and A^*. If the system should provide a given level of availability, $f(A, A^*)=1$ ($A \ge A^*$).

Implementing the Genetic Algorithm. In order to represent the system structure in the GA one has to use 2N-length integer strings: $\boldsymbol{a}$ = $(a_1,\ldots,a_i,\ldots,a_{2N})$, where for each $i \le N$ $b_i = a_j$ and for $i>N$ $n_{i-N} = a_i$. An arbitrary integer string cannot represent a feasible solution because each b_i should vary within limits $1 \le b_i \le B_i$ and each n_i within limits $1 \le n_i \le n_{\max i}$. To provide solution feasibility, a decoding procedure should first transform each string $\boldsymbol{a}$* to a string $\boldsymbol{a}$ in which

$$a_i = \mathrm{mod}_{B_i}(a_i^*) + 1 \text{ for } i \le N \text{ and } a_i = \mathrm{mod}_{n_{\max i}}(a_i^*) + 1 \text{ for } i > N. \quad (3.77)$$

The solution decoding procedure, based on the UGF technique, performs the following steps:

1. Determines n_i and b_i for each system component from the string $\boldsymbol{a}$.
2. Determines u-functions $u_{ib_i}(z)$ of each version of elements according to their capacity distribution $\boldsymbol{g_i}(b_i)$, $\boldsymbol{p_i}(b_i)$.
3. Determines u-functions of each component i ($1 \le i \le N$) by assigning $U_1(z) = u_{1b_1}(z)$ and applying the recursive equation

$$U_j(z) = U_{j-1}(z) \underset{\phi_{\text{par}}}{\otimes} u_{ib_i}(z) \text{ for } j = 2,\ldots,n_i \quad (3.78)$$

4. Determines the u-function of the entire system $U(z)$ by applying the corresponding composition operators using the reliability block diagram method.
5. Having the system u-function, determines its performance measure as described in the previous chapter.
6. Determines the total system cost using Equation (3.75).
7. Determines the solution's fitness as a function of the system cost and performance measure as

$$M - C(\boldsymbol{a}) - \pi(1+|A-A^*|)(1-f(A,A^*)) \quad (3.79)$$

where π is a penalty coefficient and M is a constant value. The fitness function is penalized when $f(A,A^*) = 0$. The solution with minimal cost and with $f(A,A^*)=1$ provides the maximal possible value of the fitness function.

Example 3.6

Consider a power station coal transportation system consisting of five basic components connected in series (Fig. 3.8):
1. subsystem of primary feeders;
2. subsystem of primary conveyors;
3. subsystem of stackers-reclaimers;
4. subsystem of secondary feeders;
5. subsystem of secondary conveyors.

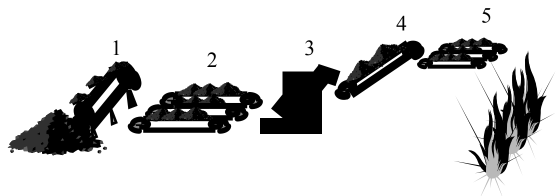

Fig. 3.8. Power station coal transportation system

The system belongs to the type of flow transmission system with flow dispersion, since its main characteristic is the transmission capacity and parallel elements can transmit the coal simultaneously. The system should meet a variable demand W. Its acceptability function is defined as $F(G,W)=1(G\geq W)$. The system should have availability not less than $A^*=0.99$ for the given demand distribution $\boldsymbol{w}$, $\boldsymbol{q}$ presented in Table 3.12.

Table 3.12. Demand distribution

$\boldsymbol{w}$	1.00	0.80	0.50	0.20
$\boldsymbol{q}$	0.48	0.09	0.14	0.29

Each system element is an element with total failure (which means that it can have only two states: functioning with the nominal capacity or total failure, corresponding to a capacity of zero). For each type of equipment, a list of products available on the market exists. Each version of equipment is characterized by its nominal capacity g, availability p, and cost c. The list of available products is presented in Table 3.13. The maximal number of elements in each component should not exceed six.

Table 3.13. Parameters of available system elements

No. of version of system element	Component 1 Primary feeders			Component 2 Primary conveyors			Component 3 Stackers-reclaimers		
	g	*p*	*c*	*g*	*p*	*c*	*g*	*p*	*c*
1	1.20	0.980	0.590	1.00	0.995	0.205	1.00	0.971	7.525
2	1.00	0.977	0.535	0.92	0.996	0.189	0.60	0.973	4.720
3	0.85	0.982	0.470	0.53	0.997	0.091	0.40	0.971	3.590
4	0.85	0.978	0.420	0.28	0.997	0.056	0.20	0.976	2.420
5	0.48	0.983	0.400	0.21	0.998	0.042			
6	0.31	0.920	0.180						
7	0.26	0.984	0.220						

Table 3.13. (Cont.)

No. of version of system element	Component 4 Primary feeders			Component 5 Primary conveyors		
	g	*p*	*c*	*g*	*p*	*c*
1	1.15	0.977	0.180	1.28	0.984	0.986
2	1.00	0.978	0.160	1.00	0.983	0.825
3	0.91	0.978	0.150	0.60	0.987	0.490
4	0.72	0.983	0.121	0.51	0.981	0.475
5	0.72	0.981	0.102			
6	0.72	0.971	0.096			
7	0.55	0.983	0.071			
8	0.25	0.982	0.049			
9	0.25	0.97	0.044			

The total number of components in the problem considered is N = 5. Each integer string containing 10 integer numbers can represent a possible solution. In order to illustrate the string decoding process performed by the GA, consider, for example, the string $\boldsymbol{a}^*$ = (9, 10, 4, 1, 3, 0, 7, 6, 1, 2). From Table 3.13 we have $B_1 = 7$, $B_2 = 5$, $B_3 = 4$, $B_4 = 9$, $B_5 = 4$; $n_{\max i} = 6$ for any component. After transforming the string according to Equation (3.77) we obtain $\boldsymbol{a}$ = (3, 1, 1, 2, 4, 1, 2, 1, 2, 3). This string corresponds to one primary feeder of version 3, two primary conveyors of version 1, one stacker of version 1, two secondary feeders of version 2, and three secondary conveyors of version 4.

According to step 2 of the decoding procedure, the u-functions of the chosen elements are determined as $u_{13}(z) = 0.018z^0+0.982z^{0.85}$ (for the primary feeder), $u_{21}(z) = 0.005z^0+0.995z^{1.00}$ (for the primary conveyors), $u_{31}(z) = 0.029z^0+0.971z^{1.00}$ (for the stacker), $u_{42}(z) = 0.022z^0+0.978z^{1.00}$ (for the secondary feeders) and $u_{54}(z) = 0.019z^0+0.981z^{0.51}$ (for the secondary conveyor).

According to step 3, we determine the u-functions of the five system components using the composition operator $\underset{+}{\otimes}$:

$$U_1(z) = u_{13}(z) = 0.018z^0+0.982z^{0.85}$$

$$U_2(z) = u_{21}(z) \underset{+}{\otimes} u_{21}(z) = (0.005z^0+0.995z^1)^2$$

$$U_3(z) = u_{31}(z) = 0.029z^0+0.971z^{1.00}$$

$$U_4(z) = u_{42}(z) \underset{+}{\otimes} u_{42}(z) = (0.022z^0+0.978z^{1.00})^2$$

$$U_5(z)= u_{54}(z) \underset{+}{\otimes} u_{54}(z) \underset{+}{\otimes} u_{54}(z) = (0.019z^0+0.981z^{0.51})^3$$

The availability of flow transmission system consisting of five components connected in series can be obtained as a product of availabilities of these components:

$$A = \sum_{m=1}^{4} q_m \prod_{i=1}^{5} A_i(w_m),$$

where $A_i(w_m)$ is availability of component I for demand w_m. The values $A_i(w_m)$ for each system component and each demand level are presented in Table 3.14.

The overall system availability for the given demand distribution is A=0.495. The total system cost is

$$C = 0.47+2\times0.205+7.525+2\times0.16+3\times0.475 = 10.15.$$

Table 3.14. Availability of system components for different demand levels

w	$A_1(w_m)$	$A_2(w_m)$	$A_3(w_m)$	$A_4(w_m)$	$A_5(w_m)$
1.0	0.000000	0.999975	0.971000	0.999516	0.998931
0.8	0.982000	0.999975	0.971000	0.999516	0.998931
0.5	0.982000	0.999975	0.971000	0.999516	0.999993
0.2	0.982000	0.999975	0.971000	0.999516	0.999993

Since the given system configuration provides an availability that is less than the desired level A*, the solution fitness function is penalized. The value of the fitness function obtained in accordance with Equation (3.79) for M=50 and π=25 is 50−10.15−25·(1+0.99−0.495) = 2.475.

The best solution obtained by the GA provides the system's availability as A = 0.992. According to this solution, the system should consist of two primary feeders of version 2, two primary conveyors of version 3, three stackers of version 2, three secondary feeders of version 7, and three secondary conveyors of version 4. The total system cost is C = 17.05. Since in this solution $f(A, A^*) = 1$, its fitness is 50–17.05 = 32.95.

Optimal structure of system with different elements in each component

Problem Formulation. The problem definition is similar to the one presented in the previous section. However, in this case different versions and the number of elements may be chosen for any given system component (Levitin et al. 1997).

The structure of the system component i is defined by the numbers of the parallel elements of each version b chosen for this component: n_{ib} ($1 \le b \le B_i$). The vectors $\boldsymbol{n}_i = (n_{i1}, \ldots, n_{iB_i}$, $1 \le i \le N$) define the entire system structure. For a given set of vectors $\{\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_N\}$ the total cost of the system can be calculated as

$$C(n_1, \ldots, n_N) = \sum_{i=1}^{N} \sum_{b=1}^{B_i} n_{ib} c_i(b) \tag{3.80}$$

Having the system structure defined by its components' reliability block diagram and by the set $\{\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_N\}$, one can determine the entire system performance measure $O(\boldsymbol{w}, \boldsymbol{q}, \boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_N)$ for any given demand distribution $\boldsymbol{w}$, $\boldsymbol{q}$. The problem of system structure optimization is formulated as finding the minimal cost system configuration $\{\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_N\}$ that provides the required level A^* of the system availability A:

$$C(\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_N) \to \min \ \text{subject to} \ f(A, A^*) = 1, \tag{3.81}$$

Implementing the Genetic Algorithm. The natural way of encoding the solutions of the problem (3.81) in the GA is by defining a B-length integer string, where B is the total number of versions available:

$$B = \sum_{i=1}^{N} B_i \tag{3.82}$$

Each solution is represented by string $\boldsymbol{a} = (a_1, \ldots, a_j, \ldots, a_B)$, where for each

$$j = \sum_{m=1}^{i-1} B_m + b, \tag{3.83}$$

a_j denotes the number of parallel elements of type i and version b: $n_{ib} = a_j$. One can see that $\boldsymbol{a}$ is a concatenation of substrings representing the vectors $\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_N$.

The solution decoding procedure, based on the UGF technique, performs the following steps:

1. Determines n_{ib} for each system component and each element version from the string $\boldsymbol{a}$.
2. Determines u-functions $u_{ib}(z)$ of each version of elements according to their capacity distribution $\boldsymbol{g}_i(b)$, $\boldsymbol{p}_i(b)$.
3. Determines u-functions of subcomponents containing the identical elements by applying the composition operators $\otimes_{\phi_{\text{par}}}$ over n_{ib} identical u-functions $u_{ib}(z)$.
4. Determines u-functions of each component i ($1 \le i \le N$) by applying the composition operators $\otimes_{\phi_{\text{par}}}$ over the u-functions of all nonempty subcomponents belonging to this component.
5. Determines the u-function of the entire system $U(z)$ by applying the reliability block diagram method.
6. Determines the system performance measure from the system performance pmf represented by the u-function $U(z)$.
7. Determines the total system cost using Equation (3.80).
8. Determines the solution fitness as a function of the system cost and performance measure according to Equation (3.79).

Example 3.7

Consider the coal transportation system described in Example 3.6 and allow each system component to consist of different versions of the elements.

The total number of available versions in the problem considered is B = 7+5+4+9+4 = 29. Each string containing 29 integer numbers can represent a possible solution. In order to illustrate the string decoding process performed by the GA, consider the string

(0, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 1, 0, 0).

This string corresponds to two primary feeders of version 4, one primary feeder of version 7, two primary conveyors of version 3, one stacker of version 1, three secondary feeders of version 7, and one secondary conveyor of version 2.

According to step 2 of the decoding procedure, the u-functions of the chosen elements are determined as $u_{14}(z) = 0.022z^0+0.978z^{0.85}$, $u_{17}(z) = 0.016z^0+0.984z^{0.26}$ (for the primary feeders), $u_{23}(z) = 0.003z^0+0.997z^{0.53}$ (for the primary conveyors), $u_{31}(z) = 0.029z^0+0.971z^{1.00}$ (for the stacker), $u_{47}(z) = 0.017z^0+0.983z^{0.55}$ (for the secondary feeders) and $u_{52}(z) = 0.016z^0+0.984z^{1.28}$ (for the secondary conveyor).

According to steps 3 and 4, we determine the u-functions of the five system components using the composition operator $\underset{+}{\otimes}$:

$$U_1(z) = u_{14}(z) \underset{+}{\otimes} u_{14}(z) \underset{+}{\otimes} u_{17}(z)$$

$$= (0.022z^0+0.978z^{0.85})^2(0.016z^0+0.984z^{0.26})$$

$$U_2(z) = u_{23}(z) \underset{+}{\otimes} u_{23}(z) = (0.003z^0+0.997z^{0.53})^2$$

$$U_3(z) = u_{31}(z) = 0.029z^0+0.971z^{1.00}$$

$$U_4(z) = u_{47}(z) \underset{+}{\otimes} u_{47}(z) \underset{+}{\otimes} u_{47}(z) = (0.017z^0+0.983z^{0.55})^3$$

$$U_5(z) = u_{52}(z) = 0.016z^0+0.984z^{1.28}$$

The entire system availability can be calculated the same way as in the previous example. We obtain A = 0.95 for the given system structure and demand distribution.

The total system cost, according to Equation (3.80), is

$$C = 2\times0.42+0.22+2\times0.091+7.525+3\times0.071+0.825 = 9.805$$

The fitness of the solution is estimated using Equation (3.79), where M = 50 and π = 25. For the desired value of system availability A^* = 0.99 the fitness takes the value

$$50–9.805–25(1+0.99–0.95) = 14.195$$

For the desired value of system availability A^* = 0.95, the fitness takes the value

$$50–9.805 = 40.195$$

The minimal cost solutions obtained by the GA for different desired availability levels A^* are presented in Table 3.15. This table presents the cost, calculated availability, and structure of the minimal cost solutions obtained by the GA. The structure of each system component i is represented by a string of the form $n_{i1}*b_{i1}$, ..., $n_{im}*b_{im}$, where n_{im} is the number of identical elements of version b_{im} belonging to this component.

Consider, for example, the best solution obtained for A^* = 0.99. The minimal cost system configuration that provides the system availability A = 0.992 consists of two primary feeders of version 4, one primary feeder of version 6, two primary conveyors of version 3, two stackers of version 2, one stacker of version 3, three secondary feeders of version 7, and three secondary conveyors of version 4. The cost of this configuration is 15.870, which is 7% less than the cost of the optimal configuration with identical parallel elements obtained in Example 3.6.

Table 3.15. Optimal solution for system structure optimization problem

	A^* = 0.95	A^* = 0.97	A^* = 0.99
System availability A	0.950	0.970	0.992
System cost C	9.805	10.581	15.870
	System structure		
Primary feeders	1*7, 2*4	2*2	2*4, 1*6
Primary conveyors	2*3	6*5	2*3
Stackers-reclaimers	1*1	1*1	2*2, 1*3
Secondary feeders	3*7	6*9	3*7
Secondary conveyors	1*2	3*3	3*4

3.5.2 Optimal single-stage system expansion

In practice, the designer often has to include additional elements in the existing system. It may be necessary, for example, to modernize a system according to new demand levels or new reliability requirements. The problem of minimal cost system expansion is very similar to the problem of system structure optimization (Levitin et al. 1997). The only difference is that each system component already contains some working elements. The initial structure of the system is defined as follows: each component of type i contains B'_i different subcomponents connected in parallel. Each subcomponent j in its turn contains n'_{ij} identical elements, which are also connected in parallel. Each element is characterized by its capacity distribution $\mathbf{g}'_i(j)$, $\mathbf{p}'_i(j)$. The entire initial system structure can, therefore, be defined by a set $\{\mathbf{g}'_i(j), \mathbf{p}'_i(j), n'_{ij}: 1\le i\le N, 1\le j\le B'_i\}$ and by a reliability block diagram representing the interconnection among the components.

The optimal system expansion problem formulation and the GA implementation are the same as in the case of system structure optimization. The only difference is that for the availability evaluation one should take into account u-functions of both the existing elements and the new elements chosen from the list, while the cost of the existing elements should not be taken into account when the system expansion cost is calculated.

Example 3.8

Consider the same coal transportation system that was presented in Example 3.6. The initial structure of this system is given in Table 3.16. Each component contains a single subcomponent consisting of identical elements: B'_i = 1 for $1\le i\le 5$. The existing structure can satisfy the demand presented in Table 3.12 with the availability $A(\mathbf{w},\mathbf{q})$=0.506. In order to increase the system's availability to the level of A^*, the additional elements should be included. These elements should be chosen from the list of available products (Table 3.13).

Table 3.16. Parameters of initial system structure

Component no.	Capacity	Availability	Number of parallel elements
1	0.75	0.988	2
2	0.28	0.997	3
3	0.66	0.972	2
4	0.54	0.983	2
5	0.66	0.981	2

The minimal cost system expansion solutions for different desired values of system availability A^* are presented in Table 3.17.

Table 3.17. Best obtained solutions for system expansion problem

	$A^* = 0.95$	$A^* = 0.97$	$A^* = 0.99$
System availability	0.950	0.971	0.990
Expansion cost	0.630	3.244	4.358
Added elements			
Primary feeders	–	1*6	1*6
Primary conveyors	2*5	1*5, 1*4	1*5
Stackers-reclaimers	–	1*4	1*3
Secondary feeders	1*7	1*7	1*7
Secondary conveyors	1*4	1*4	1*4

Consider, for example, the best solution obtained for $A^* = 0.99$, encoded by the string (0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1). The corresponding minimal cost system expansion plan that provides the system availability $A = 0.990$ presumes the addition of the primary feeder of version 6, the primary conveyor of version 5, the stacker of version 3, the secondary feeder of version 7 and the secondary conveyor of version 4. The cost of this expansion plan is 4.358.

3.5.3 Optimal multistage system expansion

In many cases, when the demand and/or reliability requirements increase with time, the design problem concerns multiple expansions (reinforcements) rather than the construction of complete new systems or their single-stage expansions. While in the problems of system structure optimization or single-stage expansion it is sufficient to ask what should be done and where, in designing the expansion plan an answer is also needed to the question of when.

Using the single-stage system expansion algorithm, one can consider the reinforcement planning problem as a sequence of expansions so that each expansion is a separate problem. However, this may not lead to the optimization of the whole plan, because, in general, partial optimal solutions cannot guarantee an overall optimal solution. Therefore, a more complex method should be developed to solve the system-expansion planning problem in several stages, rather than in a single stage (Levitin 2000).

This section addresses the multistage expansion problem for systems. The study period is divided into several stages. At each stage, the demand distribution is predicted. The additional elements chosen from the list of available products may be included into any system component at any stage to increase the total system availability. The objective is to minimize the sum of costs of investments over the study period while satisfying availability constraints at each stage.

Problem Formulation. As in the single-stage expansion problem, the initial system structure is defined by a reliability block diagram representing the interconnection among system components and by a set $\{\mathbf{g}'_i(j), \mathbf{p}'_i(j), n'_{ij} : 1 \le i \le N, 1 \le j \le B'_i\}$.
In order to provide the desired level of availability, the system should be expanded at different stages of the study period. Each stage y ($1 \le y \le Y$) begins $\tau(y)$ years after the base stage (stage 0) and is characterized by its demand distribution $w(y)$, $q(y)$. The timing of stages can be chosen for practical reasons so that $\tau(y)$ must not be evenly spaced. Increasing the number of stages Y during the planning horizon increases the solution flexibility. Indeed, the addition of intermediate stages may allow investments to be postponed without violating the system availability constraints. On the other hand, the increase of number of stages expands drastically the search space and, therefore, slows the algorithm convergence. The recommended strategy is to obtain a number of solutions for different Y in a reasonable time and to choose the best one.

Different versions and numbers of elements may be chosen for expansion of any given system component. The versions can be chosen from the list of elements available in the market. The expansion of system component i at stage y is defined by the numbers $n_{ib}(y)$ of parallel elements of each version b ($1\le b\le B_i$) included into the component at this stage The set $\boldsymbol{n}_y=\{n_{ib}(y): 1\le i\le N, 1\le b\le B_i\}$ defines the entire system expansion at stage y.

For a given set $\boldsymbol{n}_y$ the total cost of the system expansion at stage y can be calculated in present values as

$$C(y)=\frac{1}{(1+\mathrm{IR})^{\tau(y)}}\sum_{i=1}^{N}\sum_{b=1}^{B_i} n_{ib}(y)c_i(b) \tag{3.84}$$

where IR is the interest rate. The total system expansion cost for the given expansion plan $\{\boldsymbol{n}_1,\ldots,\boldsymbol{n}_Y\}$ is

$$C=\sum_{y=1}^{Y}\frac{1}{(1+\mathrm{IR})^{\tau(y)}}\sum_{i=1}^{N}\sum_{b=1}^{B_i} n_{ib}(y)c_i(b) \tag{3.85}$$

The system structure at stage y (after expansion) is defined by a set of elements composing the initial structure $\boldsymbol{n}' = \{n'_{ij} : 1\le i\le N, 1\le j\le B'_i\}$ and by a set of included elements $\boldsymbol{m}_y = \{m_{ib}(y): 1\le i\le N, 1\le b\le B_i\}$, where

$$m_{ib}(y)=\sum_{k=1}^{y} n_{ib}(k) \tag{3.86}$$

Having the system structure and demand distribution for each stage y, one can evaluate the system performance measures. The problem of system expansion optimization can be formulated as follows:

Find the minimal cost of system expansion plan that provides the required level A^* of the system availability A_y at each stage y:

$$C(\boldsymbol{n}_1,\ldots,\boldsymbol{n}_Y)\rightarrow \min$$

$$\text{subject to } f(A_y(\boldsymbol{n}_1,\ldots,\boldsymbol{n}_Y, \boldsymbol{w}(y), \boldsymbol{q}(y)), A^*)=1 \text{ for } 1\le y\le Y \tag{3.87}$$

where $f(A_y,A^*)$ is a function representing the desired relation between A_y and A^*.

The problem of system structure optimization can be considered as a special case of this formulation. Indeed, if the set $\boldsymbol{n}'$ of the elements composing the system at stage 0 is empty and Y=1, then the problem is reduced to system structure optimization subject to reliability constraints. If $\boldsymbol{n}'$ is empty but $Y\neq 1$, then the problem is that of multistage system structure optimization, which includes both determination of the initial system structure and its expansion plan.

Implementing the Genetic Algorithm. The natural way to represent the expansion plan $\boldsymbol{n}_1,...,\boldsymbol{n}_Y$ is to use a string containing integer numbers corresponding to $n_{ib}(y)$ for each type i, version b_i and stage y. Such a string contains $Y\sum_{i=1}^{N} B_i$ elements, which can result in an enormous growth of the length of the string even for problems with a moderate number of available versions of elements and number of stages. Besides, to represent a reasonable solution, such a string should contain a large percentage of zeros, because only small number of elements should be included into the system at each stage. This redundancy causes an increase in the need for computational resources and lowers efficiency of the GA.

In order to reduce the redundancy of expansion plan representation, each inclusion of n elements of version b into the system at stage y is represented by the triplet $\{n, b, y\}$. The maximum number of possible inclusions into each component (diversity factor) K and the maximum number of identical elements in each inclusion n_{max} are defined as parameters which can be preliminarily specified for each problem. Thus, the first K triplets represent the total expansion plan for the first component, the second K triplets for the second component, and so on. The length of string representing a solution is in this case $3KN$, which can be much smaller than that in the first case.

Consider a string $\boldsymbol{a} = (a_1,...,a_{3KN})$ where each element a_j is generated in the range

$$0 \le a_j \le \max\{n_{\max}, Y, \max_{1\le i\le N} B_i\} \tag{3.88}$$

For each system component i, each of K triplets (a_x,a_{x+1},a_{x+2}), where

$$x = 3K(i-1)+3(j-1) \text{ for } 1\le j\le K \tag{3.89}$$

determines the addition of $\text{mod}_{n_{\max}+1}(a_x)$ elements of version $\text{mod}_{B_i}(a_{x+1})+1$ at stage $\text{mod}_Y(a_{x+2})+1$. This transform provides solution feasibility by mapping each of the variables into a range of its possible values, since $n_{ib}(y)$ varies from 0 to n_{max}, b varies from 1 to B_i and y varies from 1 to Y.

For example, consider a problem with $K = 3$, $Y = 5$, $n_{max} = 4$, where $\max_{1\le i\le N}\{B_i\} = 9$. Substring (7, 3, 2, 0, 2, 6, 1, 7, 0) corresponding to a component for which eight different versions are available, represents addition of $\text{mod}_{4+1}7 = 2$ elements of version $\text{mod}_8 3+1 = 4$ at stage

$mod_5 2+1 = 3$ and $mod_{4+1} 1 = 1$ element of version $mod_8 7+1 = 8$ at stage $mod_5 0+1 = 1$. The second triplet (0, 2, 6) represents addition of zero elements and, therefore, should be discarded. The existence of such "dummy" triplets allows the number of additions to each component to vary from 0 to K, providing flexibility of solution representation.

Having the vector $\boldsymbol{a}$, one can determine the system structure in each stage y using (3.86) and obtain the system capacity distribution using the UGF technique. For this capacity distribution and for the given demand distribution $\boldsymbol{w}(y)$, $\boldsymbol{q}(y)$ one obtains the system availability A_y. The solution fitness for the estimated values of A_y and C calculated using (3.85) should be determined as

$$M - C(\boldsymbol{a}) - \pi \sum_{y=1}^{Y} [(1+|A^* - A_y(\boldsymbol{a})|)(1 - f(A_y(\boldsymbol{a}), A^*)] \tag{3.90}$$

Example 3.9

In this example, we consider the same coal transportation system from Example 3.6. The initial system structure is presented in Table 3.16. Table 3.18 contains the boiler system demand distributions at five different stages and times from the present to the beginning of these future stages.

Because of demand increase, the availability index of the system becomes $A_1(\boldsymbol{w}(1), \boldsymbol{q}(1)) = 0.506$ at the first stage. To provide a desired availability level at all the stages, the system should be expanded. The characteristics of products available in the market for each type of equipment are presented in Table 3.13.

Table 3.18. Demand distributions

Stage no.	$\tau(y)$		Demand distribution			
1	0	$\boldsymbol{w}$	1.00	0.80	0.50	0.20
		$\boldsymbol{q}$	0.48	0.09	0.14	0.29
2	3	$\boldsymbol{w}$	1.20	0.80	0.50	-
		$\boldsymbol{q}$	0.43	0.32	0.25	-
3	6	$\boldsymbol{w}$	1.40	1.20	0.80	0.50
		$\boldsymbol{q}$	0.10	0.39	0.31	0.20
4	9	$\boldsymbol{w}$	1.40	1.20	0.80	-
		$\boldsymbol{q}$	0.41	0.24	0.35	-
5	12	$\boldsymbol{w}$	1.60	1.40	1.00	-
		$\boldsymbol{q}$	0.30	0.45	0.25	-

The best expansion plans obtained by the GA for different values of A^* are presented in Tables 3.19 - 3.21. The interest rate for the problems considered is IR = 0.1. Expansion of each system component i at stage y is presented in the form $n*b$, where n is a number of identical elements of version b to be included into the component at a given stage. The table also contains system availability A_y obtained at each stage, total expansion costs in present values C, and costs of the system expansions at the first stage $C(1)$.

Table 3.19. Best obtained multistage expansion plan for $A^* = 0.95$

Expansion cost	$C = 4.127$, $C(1) = 0.645$				
Stage	1	2	3	4	5
System availability	0.951	0.951	0.963	0.952	0.960
Added elements					
Primary feeders		1*4			
Primary conveyors	2*5		1*5	1*5	
Stacker-reclaimers			1*4		1*2
Secondary feeders	1*7	1*7			
Secondary conveyors	1*3			1*4	

Table 3.20. Best obtained multistage expansion plan for $A^* = 0.97$

Expansion cost	$C = 6.519$, $C(1) = 5.598$				
Stage	1	2	3	4	5
System availability	0.972	0.970	0.987	0.975	0.970
Added elements					
Primary feeders	1*4				1*7
Primary conveyors	3*5			2*5	
Stacker-reclaimers	1*4		1*2		
Secondary feeders	2*7				
Secondary conveyors	1*3				1*4

Table 3.21. Best obtained multistage expansion plan for $A^* = 0.99$

Expansion cost	$C = 7.859$, $C(1) = 5.552$				
Stage	1	2	3	4	5
System availability	0.995	0.991	0.990	0.996	0.994
Added elements					
Primary feeders	1*6	1*6	2*6		
Primary conveyors	1*3		2*4		
Stacker-reclaimers	1*2			1*3	
Secondary feeders	1*7	2*7			
Secondary conveyors	1*3		1*3		

One can compare expansion at the first stage of multistage plans with the single-stage expansion plans presented in Table 3.17. The demand distribution for the single-stage problem is the same as that in the first stage of the multistage problem. The solutions for the first stage of the multistage plan differ from ones that are obtained for single-stage expansion. The comparison shows that a single-stage expansion plan is less expensive than the first stage of a multistage plan, which should consider the effect of the first stage expansion on the further stages.

To demonstrate the advantages offered by incremental expansion, the solutions that satisfy reliability constraints for the final stage demand distribution but require all expansion investments at time zero were obtained (Table 3.22). One can see that incremental expansion provides considerably less expensive solutions.

Table 3.22. Best obtained single-stage expansion plans

	$A^* = 0.95$	$A^* = 0.97$	$A^* = 0.99$
System availability	0.951	0.970	0.990
Expansion cost	8.660	8.895	10.192
	Added elements		
Primary feeders	1*7	1*4	2*6, 1*7
Primary conveyors	4*5	1*3, 2*4	5*5
Stacker-reclaimers	2*3	2*3	1*2, 1*3
Secondary feeders	2*7	2*7	2*7
Secondary conveyors	2*4	2*4	2*4

3.5.4 Optimal multistage system modernization

When the system demand and/or reliability requirements increase over time, multiple system modernization actions are usually performed. Besides the system's multistage expansion the system modernization plan can include modification of the existing equipment and changes in the maintenance policy. Both types of action lead to changes in the capacity distribution of the elements already presented in the system. The multistage modernization plan should consider all these possible alternatives.

Problem Formulation. The initial system structure is defined by a reliability block diagram representing the interconnection among N system components and by a set $\Phi(0) = \{\mathbf{g}'_i(j),\ \mathbf{p}'_i(j),\ n'_{ij}\!: 1\le i\le N,\ 1\le j\le B'_i\}$. Each component i can contain up to E_i different elements connected in parallel. In order to allow number of elements in each component to vary we introduce "dummy" elements of version 0.

In order to provide the desired levels of the system performance O^*_y at different stages y of the study period, the system should be modernized.

Each stage y ($1 \le y \le Y$) begins $\tau(y)$ years after the base stage (stage 0) and is characterized by its demand distribution $\boldsymbol{w}(y)$, $\boldsymbol{q}(y)$. At each stage, three different types of action may be undertaken: modification of some of the system's elements, addition of new elements, and removal of some of the system's elements. In order to manipulate the system structure in a uniform way, we will formulate all of these actions in terms of element replacements:

The modification of element j of component i or changing the procedure of the element maintenance leads to changes in its capacity distribution. We can consider this action as replacement of the element of version b_{ij} with the element of version b^*_{ij}. Parameters of new versions of the element should be specified for each element modification action.

The addition of a new element into the system can be considered as the replacement of the element of the corresponding type i and version 0 with a new element with version b^*_{ij}. For each component i there are B_i element versions available in the market.

The removal of the element of version b_{ij} from the system can be formulated as its replacement with the element of version 0.

Now one can define each system modernization action x by vector

$$\{i_x,\ b_{i_x x},\ b^*_{i_x x},\ c_x\} \tag{3.91}$$

where i_x is the number of the component in which the element of version $b_{i_x x}$ should be replaced with the element of version $b^*_{i_x x}$ and c_x is the cost of the action. Modernization action is feasible if the system component i_x already contains elements of version $b_{i_x x}$. Unfeasible actions should be replaced with "dummy" ones which correspond to $x = 0$. Action $x = 0$ is associated with cost $c_0 = 0$ and implies that no replacements are performed.

The system modernization plan at stage y can be defined by the vector $\boldsymbol{x}_y = (x_{ys},\ 1 \le h \le H)$ where H is the maximal number of possible actions at a single stage. The cost of plan $\boldsymbol{x}_y$ can be calculated in present values as

$$C_y = \frac{1}{(1+\mathrm{IR})^{\tau(y)}} \sum_{h=1}^{H} c_{x_{yh}} \tag{3.92}$$

where IR is the interest rate.

Each modernization plan $\boldsymbol{x}_y$ transforms the set of versions of elements presented in the system at stage $y-1$ into the set of versions of elements presented in the system at stage y: $(\Phi(y-1), \boldsymbol{x}_y) \to \Phi(y)$ for $1 \le y \le Y$, where $\Phi(0)$ is the initial structure of the system considered.

The total system modernization cost for the given modernization plan $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_Y$ is

$$C = \sum_{y=1}^{Y} C_y = \sum_{y=1}^{Y} \frac{1}{(1+\mathrm{IR})^{\tau(y)}} \sum_{h=1}^{H} c_{x_{yh}} \tag{3.93}$$

Having the system structure $\Phi(y)$ and demand distribution $\boldsymbol{w}(y)$, $\boldsymbol{q}(y)$ for each stage y, one can evaluate the system performance measures. The problem of optimal system expansion can be formulated as follows (Levitin and Lisnianski 1999):

Find the minimal cost system modernization plan that provides the required level A^* of the availability A_y at each stage y:

$$C\,(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_Y) \rightarrow \min$$

subject to

$$f(A_y(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_y, \boldsymbol{w}(y), \boldsymbol{q}(y)), A^*) = 1 \text{ for } 1 \le y \le Y. \tag{3.94}$$

where $f(A_y, A^*)$ is a function representing the desired relation between A_y and A^*.

Implementing the Genetic Algorithm. The natural way to represent the multistage modernization plan $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_Y$ is to use a string $\boldsymbol{a}$ containing HY integer numbers corresponding to the numbers of the actions to be undertaken during the study period. In the string $\boldsymbol{a}$, each number a_m for $H(y-1)+1 \le m \le yH$ corresponds to the action number $m-H(y-1)$ undertaken at stage y. All of the numbers should be generated in the range $0 \le a_m \le L$, where L is the total number of available actions. Action with number 0 means "do nothing"; it has no cost and is used in order to allow the total number of actions to vary. One can see that the solution representation string $\boldsymbol{a}$ contains sequentially allocated vectors $\boldsymbol{x}_y$ of the modernization plan at each stage $y = 1,\ldots, Y$.

By having the vector $\boldsymbol{a}$, one can determine the system structure in each stage y by replacing the elements of version $b_{i_x x}$ with elements of version $b^*_{i_x x}$ in accordance with the modernization plan x_y. Using the u-functions of the elements composing the system at each stage y after the corresponding replacements, one obtains the entire system capacity distribution by applying the UGF technique. For this capacity distribution and for the given demand distribution $\boldsymbol{w}(y)$, $\boldsymbol{q}(y)$ one obtains the system availability A_y. The solution fitness for the estimated values of A_y and the total cost C calculated using (3.93) is determined according to Equation (3.90).

Example 3.10

Consider two different optimization problems applied to the coal transportation system from Example 3.6. In the first problem we have to determine the initial system structure as well as its modernization plan. In this case, the initial structure set is empty: $\Phi(0) = \varnothing$. In the second problem we have to find the optimal modernization plan for the system with the initial structure presented in Table 3.23. Table 3.24 contains the demand distributions at five different stages and times from the present to the beginning of these future stages.

Table 3.23. Initial coal transportation system structure

No. of system component	1	2	3	4	5
Versions of elements, present in the component	1, 1	2, 2, 2	3, 3	4, 4	5, 5

The demand distributions have the same maximal load at stages 1-3 and 4-5. Within these periods, variations of the load demand are caused by intensification of boiler use. At stage 4, the installation of a new boiler is planned, which should be supplied by the same coal supply system.

Because of increased demand, the availability index of the system becomes $A_1(\boldsymbol{w}(1), \boldsymbol{q}(1)) = 0.506$ at the first stage. To provide a desired availability level $A^* = 0.95$ at all of the stages, the system should be modernized. The characteristics of the available modernization actions are presented in Table 3.25 in the form (3.91). The action costs c_x comprise both the investment and operational costs.

Table 3.24. Demand distributions

Stage no.	$\tau(y)$		Demand distribution			
1	0	$\boldsymbol{w}$	1.0	0.8	0.5	0.2
		$\boldsymbol{q}$	0.48	0.09	0.14	0.29
2	3	$\boldsymbol{w}$	1.0	0.8	0.5	-
		$\boldsymbol{q}$	0.63	0.12	0.25	-
3	6	$\boldsymbol{w}$	1.0	0.9	0.8	-
		$\boldsymbol{q}$	0.65	0.19	0.16	-
4	9	$\boldsymbol{w}$	2.0	1.6	0.8	-
		$\boldsymbol{q}$	0.41	0.24	0.35	-
5	12	$\boldsymbol{w}$	2.0	1.7	1.2	0.9
		$\boldsymbol{q}$	0.52	0.15	0.13	0.20

Each action leads to the appearance of an element of a certain version either purchased in the market or obtained by modification of some existing element or its maintenance procedure (equipment removal actions are not considered in this example).

Table 3.25. List of available actions

x	i_x	$b_{i_x x}$	$b^*_{i_x x}$	c_x	Description
1	1	0	6	18.0	Addition of new feeder of version 6
2	1	0	7	42.0	Addition of new feeder of version 7
3	1	1	8	5.0	Electric drive system modification aimed at providing feeder speed increase
4	1	1	9	2.0	Installation of engines monitoring system in feeder of version 1
5	1	6	10	1.6	Installation of engines monitoring system in feeder of version 6
6	2	0	11	9.1	Addition of new conveyor of version 11
7	2	0	12	5.6	Addition of new conveyor of version 12
8	2	0	13	4.2	Addition of new conveyor of version 13
9	2	2	14	0.8	Changes in mechanical equipment to provide load increase
10	2	14	15	0.3	Installation of engines monitoring system in conveyor of version 14
11	2	14	16	0.5	Reduction of flat-belt condition inspection (replacement) period in conveyor of version 14
12	2	13	17	0.4	Installation of engines monitoring system in conveyor of version 13
13	3	0	3	567.0	Addition of new stacker of version 3
14	3	0	18	359.0	Addition of new stacker of version 18
15	3	3	19	6.0	Installation of hydraulics monitoring system in stacker of version 3
16	4	0	20	7.1	Addition of new feeder of version 20
17	4	4	21	2.0	Reduction of engines inspection period in feeder of version 4
18	4	20	22	1.8	Reduction of engines inspection period in feeder of version 20
19	5	0	23	47.5	Addition of new conveyor, version 23
20	5	5	24	1.1	Installation of engines monitoring system in conveyor of version 5
21	5	5	25	1.4	Reduction of flat-belt inspection (replacement) period
22	5	25	26	1.1	Action 20 performed after action 21
23	5	24	26	1.4	Action 21 performed after action 20

The list of parameters of different versions of elements is presented in Table 3.26. The table also contains the origins of the versions (P corresponds to versions already present in the system, N corresponds to new elements available on the market, and M corresponds to versions obtained by the modification of existing versions or their maintenance procedures).

Table 3.26. Parameters of available system elements

No. of version	*g*	*p*	Component	Origin
1	0.75	0.998	1	P
2	0.28	0.997	2	P
3	0.66	0.972	3	P
4	0.54	0.983	4	P
5	0.66	0.981	5	P
6	0.31	0.920	1	N
7	0.85	0.978	1	N
8	0.93	0.970	1	M
9	0.75	0.995	1	M
10	0.31	0.985	1	M
11	0.53	0.997	2	N
12	0.28	0.997	2	N
13	0.21	0.988	2	N
14	0.33	0.980	2	M
15	0.33	0.991	2	M
16	0.33	0.992	3	M
17	0.21	0.999	3	M
18	0.40	0.971	3	N
19	0.66	0.980	3	M
20	0.55	0.983	4	N
21	0.54	0.998	4	M
22	0.55	0.995	4	M
23	0.51	0.981	5	N
24	0.66	0.990	5	M
25	0.66	0.988	5	M
26	0.66	0.997	5	M

The best modernization plans obtained by the suggested algorithm for the first and the second problems are presented in Tables 3.27 and 3.28 respectively. The tables contain the list of actions to be undertaken for each stage, the structure of each of the system's components, and the system's availability index (if there are a few identical actions or elements, they are presented in the form $n*x$, where n corresponds to the number of identical actions of type x). For the interest rate IR = 0.1, the total modernization costs for problems 1 and 2 are in the present values C = 1783.23 and C = 542.55 respectively.

Table 3.27. Optimal modernization plan for problem the with $\Phi(0) = \varnothing$

Stage No.	List of actions	A	Structure of system components				
			1	2	3	4	5
1	1, 2*2, 2*6, 13, 14, 3*16, 3*19	0.958	6, 2*7	2*11	3, 18	3*20	3*23
2	15	0.951	6, 2*7	2*11	18, 19	3*20	3*23
3	2, 6, 18	0.950	6, 3*7	3*11	18, 19	2*20, 22	3*23
4	2*6, 2*13, 2*16, 2*19	0.952	6, 3*7	5*11	2*3, 18, 19	4*20, 22	5*23
5	5, 2*15, 18	0.950	10, 3*7	5*11	18, 3*19	3*20, 2*22	5*23

Table 3.28. Optimal modernization plan for problem the with $\Phi(0) \neq \varnothing$

Stage No	List of actions	A	Structure of system components				
			1	2	3	4	5
1	6, 16, 2*21, 2*22	0.950	2*1	3*2, 11	2*3	2*4, 20	2*26
2	4, 2*15	0.951	1, 9	3*2, 11	2*19	2*4, 20	2*26
3	2	0.951	1, 7, 9	3*2, 11	2*19	2*4, 20	2*26
4	2, 2*6, 13, 14, 2*16, 3*19	0.955	1, 2*7, 9	3*2, 3*11	3, 18, 2*19	2*4, 3*20	2*26, 3*23
5	2*9, 15	0.951	1, 2*7, 9	2, 3*11, 2*14	18, 3*19	2*4, 3*20	2*26, 3*23

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4. Reliability Assessment for Elements of Transmission and Transformation Systems

4.1 Calculating reliability indices of substations based on the overload capability of the transformers

The reliability indices for a generating system take into account the load and the available capacity of the system (the total rated power of all available generation units). The analytical models for determining the reliability indices of the generation system are relatively simple. This is true, especially when the load model is represented by the load duration curve i.e., without considering the load variation in time. Moreover, the load duration curve can be represented by the daily peak loads. The load representation by the load duration curve is justified theoretically for studies of power generation. The reason for this is that the load capacity of the generation units is practically close to their rated power.

When transformers are considered, the situation is different. The load capacity of transformers is determined by the hottest spot temperature of the insulation and strongly depends on the daily load curve of the transformer. The temperature has strong influence on the life expectancy of the transformer. Thus, the load duration curve should not be used for the calculation of reliability indices of substations when the real load capacity of the transformers is considered.

For transformers, designed in accordance with IEC standards principles (IEC 60354/1991, IEEE Std. C57. 91-1995 and IEC 60076-7/2005) the so-called “rated” life expectancy is obtained when the transformer is operated so that the hottest point temperature of the insulation is equal to 98°C. Thus two possible operational conditions may be defined:

1. Normal operation of the transformer. In this case, the permissible load level is determined so that the lifetime of the transformer is equal to its “rated” usage period, i.e., the usage period acquired at a constant temperature of 98°C.
2. Transformer operation under emergency conditions. Such a case is, for example, a forced outage of one of the transformers. In this case, the remaining transformers may be overloaded and operating at temperatures above 98°C.

In any case, the load capacity of the transformer is limited in such a way, that the insulation hottest spot temperature should not exceed 140°C

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and the oil temperature in the upper part of the transformer should not increase above 115°C (for transformer design according to IEC standards). This means that if reliability indices are to be determined by a probabilistic approach, they should be calculated in relation to the hottest spot temperature of the insulation.

At normal operation conditions, the hottest spot temperature of the insulation and the rate by which the transformer life time is determined, depend on the load curve and the ambient temperature. In the case of a forced outage of one of the substation transformers, a part of the load is usually transferred to the remaining ones. Sometimes, if necessary, a part of the load is transferred to adjacent substations by switching procedures at the station itself and the distribution network. The temperature of the remaining transformers, which, in this case, determines their load capacity, depends on the following stochastic variables:

- the load curves,
- the time of the failure,
- the forced outage duration,
- the frequency of failures,
- the ambient temperature.

A Monte Carlo method for calculation of the reliability indices, which takes into account the overload capability of the transformer, will be discussed first. Later, an analytical model for determination of reliability indices in relation to the insulation temperature is studied. Some assumptions regarding the load model are made. It should be pointed out, that the statistical calculation of the forced outage rate of the transformer should also incorporate the failures in the various elements of the feeding scheme of the substation. The failures mentioned above are those which cause forced outages of the transformer. When the statistics for determination of the forced outage rates are studied, two extreme cases can be defined. These two cases differ in their forced outage rate and the forced outage duration time (the repair time). The first case is characterized by a long repair time, such as burned out transformer windings, but with a low failure frequency. The second case is characterized by short and medium repair times and a relatively high failure frequency. Failures of the first type are dealt with by keeping reserve transformers for the whole system. In failures of the second type, it is a common practice to overload the remaining transformers, or to transfer loads to adjacent stations. Statistically, the frequency of failures of the first type is also taken into account, but with regard to the forced outage duration time, only the required time for the replacement of the transformer should be used.

Thus, by studying the reliability indices, taking into account the overload of the transformer, one may determine (similar to the case of the generation system) the substation maximum load level which will assure a predetermined reliability level.

Prior to the calculation of the reliability indices based on probabilistic approach, we will discuss the deterministic criterion, used both for planning and operation of transformers.

4.1.1 The deterministic approach to study of the load capability of transformers

In the deterministic approach for design, it is common to use a single contingency criterion. This means that the system is designed to operate even if one of the transformers in the substation is out of use at the peak load period or at any predetermined conditions. According to this approach, the maximum permissible load capacities of the remaining transformers can be determined, so that the insulation temperature will be within acceptable limits. This is shown schematically in Fig. 4.1.

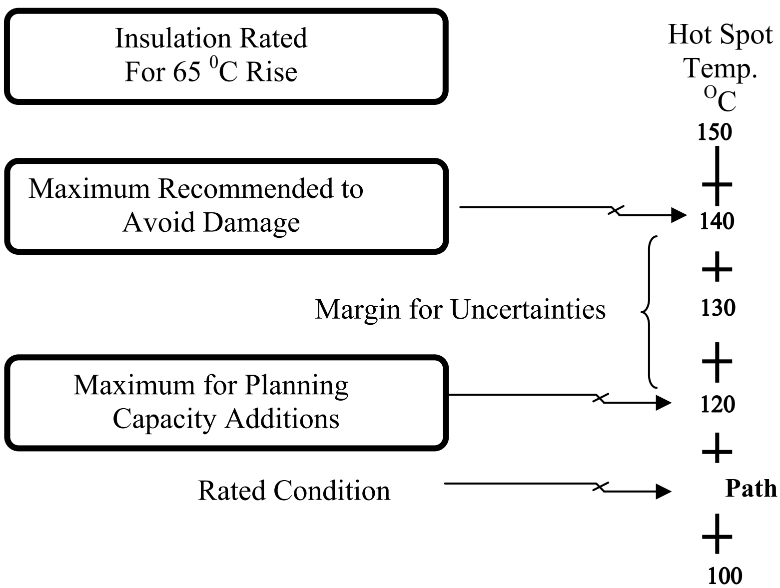

Fig. 4.1. An example for determination of the transformer load capacity using the deterministic approach based on a single contingency criterion

There is a clear distinction between the operational conditions and the planning criterion, i.e., between the permissible load capacity during the operation and the planned load capacity. Usually, the latter is lower in order to leave a safety margin taking into account the uncertainty in prediction of the load capacity.

In any case, the maximum permissible temperature of the insulation should not exceed 140°C. A continued operation of the transformer at loads resulting in higher temperatures may cause irreversible damage to the insulation. For planning purposes, the load capacity, which actually dictates the expansion of the transformation system (an addition of a transformer or a construction of a new substation in the region), is based on a temperature lower than 140°C. It is based (for example, see Fig. 4.1) on 120°C. The temperature difference serves as a safety margin. The disadvantage of this method is that the factors influencing the load capacity are of a stochastic nature and not deterministic.

4.1.2 Calculating reliability indices based on the overload of the transformer using the Monte-Carlo method

As mentioned above, the reliability indices of the substation should be calculated in relation to the insulation temperature. The basic reliability indices used in the power transmission and transformation systems are:

1. failure (outage) frequency, F_D [l/yr];
2. failure duration, T_D [h];
3. failure probability, P_D.

The Monte Carlo simulation in reliability studies is used generally in cases, where the theoretical analysis is complicated. The main disadvantage of the Monte-Carlo method is the relatively long computer time required for the simulation procedure. The simulation procedure should be as close as possible to reality. After running the simulation procedure for a certain time, the reliability indices are studied and assessed. The simulation is thus regarded as a set of practical experiments defined by stochastic contingencies along a time scale according to a certain distribution function.

The simulation procedure is applied to the case of a substation with m transformers connected in parallel, see Fig.4.2. Each transformer has a rated power S_n [VA] and the total power of the substation is S_T [VA].

As mentioned before, in case of a forced outage of a transformer, the load is transferred either to other transformers in the same substation or to another adjacent substation. The first case is discussed here, i.e. the load or a part of it is transferred to the remaining transformers in the same substation. The amount of the transferred load depends on the hottest spot temperatures of the insulation of the remaining transformers, i.e. on their load capacity. If this load capacity is not sufficient, the rest of the load will be considered as unsupplied energy. Such a procedure is shown schematically in Fig. 4.3

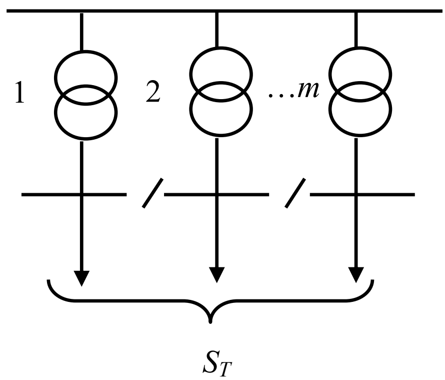

Fig. 4.2. A substation with *m* parallel-connected transformers

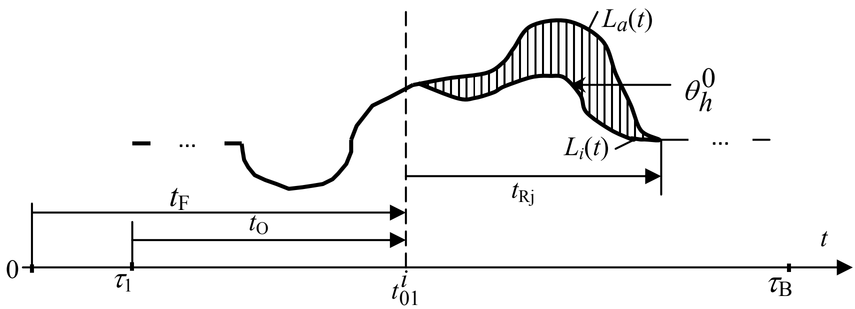

Fig. 4.3. A schematic description of the procedure of single transformer outage in a substation. $L_a(t)$ is the actual substation load prior to the fault, $L_i(t)$ is the maximum available substation load (power) after the outage (the available loading capacity), t^i_{oi} is the moment of the fault occurrence, t_{Rj} is the repair time, θ^0_h is the maximum hottest spot temperatures of the insulation of the remaining transformers under load

On the basis of the abovementioned procedure, the reliability indices are computed by means of a Monte Carlo simulation.

The reliability indices for the generation system have been calculated in relation to the case where the actual load is larger than the available generation capacity. In a similar way, the reliability indices of the substation, in the case in which the available load capacity (the overload capability of the remaining transformers) is smaller than the actual load, can be determined. Since the available load capacity of the remaining transformers depends on the insulation temperature, the reliability indices should be calculated in the case where the hottest spot temperature is higher than a predetermined value. This temperature determines the available capacity of the remaining transformers. In other words, the probability of the contingency, where the remaining transformers load capacity is smaller than the required load, is equal to the contingency probability, where the hottest spot temperatures of the transformers exceed a predetermined value.

The reliability indices for a substation, such as probability, frequency, duration time and the expected average unsupplied energy, are determined by a Monte Carlo simulation.

In order to calculate the reliability indices using the conditional probability law, two contingencies should be defined, namely:

1. k out of m existing transformers fail (event A_k);
2. The hottest spot temperatures of the insulation for the remaining m-k transformers are higher than a predetermined value θ^0_h (event B_k).

Thus, the probability that a load L is higher than the available transformation capacity (considering overloading) of a substation with m transformers L_w can be determined by:

$$P(L > L_0) = P(\theta > \theta_h^0) = \sum_{k=1}^{k=m} P(A_k)P(B_k/A_k) \tag{4.1}$$

where $P(B_k/A_k)$ is the probability that the insulation temperatures of the remaining m-k transformers, during a forced outage of k transformers is higher than θ^0_h.

The probability of the event A_k can be calculated in a similar way to that used for generation units, based on the forced outage rate of the transformer λ_T and the forced outage duration (repair time) $1/\mu_T$. If the installed transformers in the substation are identical (as it happens in most practical cases), $P(A_\mathrm{k})$ can be determined according to the binomial distribution:

$$P(A_k) = \binom{m}{k} \left\{ \frac{\lambda_T}{\lambda_T + \mu_T} \right\}^k \left\{ \frac{\mu_T}{\lambda_T + \mu_T} \right\}^{m-k} \tag{4.2}$$

The conditional probability $P(B_k/A_k)$ is calculated by Monte Carlo simulation. Prior to the detailed calculations, some points should be clarified:

1. The transformers availability $A_T=\mu_T/\lambda_T+\mu_T$ is much grater than the generators' availability and is very close to 1.
2. The number of installed transformers in a substation is much smaller than the number of generation units.
3. The average forced outage duration time of the transformers $1/\mu_T$ is much smaller than the one for the generation units.
4. The critical duration time for the load capacity of the transformers is roughly a few hours (the duration time of the peak load).

Due to these reasons only single failure should be considered. It should be pointed out that the calculation of the transformation capacity by means of the deterministic approach is based on one contingency only. However, the method presented here will be expanded to deal even with two contingencies as well. In order to clarify the method, see Fig. 4.3. Calculating the conditional probability $P(B_k/A_k)$ using the Monte Carlo method is based on the assumption that only one out of m transformers is out of service, i.e. k=1. The main steps of calculating $P(B_k/A_k)$ are as follows:

1. Determine the moment of the failure by means of a random number generator and the distribution function of time to failure, as discussed later.
2. Calculate the initial insulation temperature at the hottest spot, based on the acquired daily load curve up to t^i_{01}. The hottest spot temperature of the insulation θ^0_h, is determined by division of the time axis to intervals as shown in Section 4.1.6. It should be mentioned that for the calculation of θ^0_h, additional parameters, besides the load curve, are required, see Section 4.1.6.
3. The hottest spot temperatures of the insulation of the transformers remaining in operation is determined for a given repair time t_{Rj}, assuming that the load of the faulted transformer is equally divided between the remaining transformers. If the hottest spot temperature of the insulation of the remaining transformers is higher than θ^0_h, the maximum permissible load $L_i(t)$ should be calculated by some iterations. The maximum permissible load is the load for which the predetermined temperature θ^0_h is obtained. Assuming that the permissible load curve $L_i(t)$ is similar in its character to the actual load curve $L^i_a(t)$, which characterizes the substation load under normal conditions, the insulation temperature of the remaining transformers $\theta_h(t)$ is calculated. This temperature depends on the load curve $L_i(t)$, the transformer parameters, the initial transformer temperature and the ambient temperature.
4. The load curve to be used is the annual load curve obtained by measurements of the substation output power.

5. The calculations are repeated N_T times, for each given repair time value t_{Rj}, when the failure moment t^i_{0j} is randomly generated according to the distribution function of time to failure.

Thus, based on a given repair time t_{Rj}, the conditional probability $P[(A_1/B_1)/t_{Rj}]$ can be obtained after running the abovementioned simulation procedure N_T times:

$$P[(A_1/B_1)/t_{R_j}] = \frac{N_j[(\theta_h > \theta_h^0)/t_{R_j}]}{N_T} \tag{4.3}$$

where $N_j[(\theta_h > \theta_h^0)/t_{R_j}]$ is the number of times out of N_T, where $\theta_h > \theta^0_h$ for a given repair time t_{Rj}.

The simulation based calculations are repeated for all possible values of repair time t_{Rj}, for $j = 1,...,q$, determined by the probability density function (pdf) for repair time $f_{tR}(t_{Rj})$ as shown in Fig. 4.4.

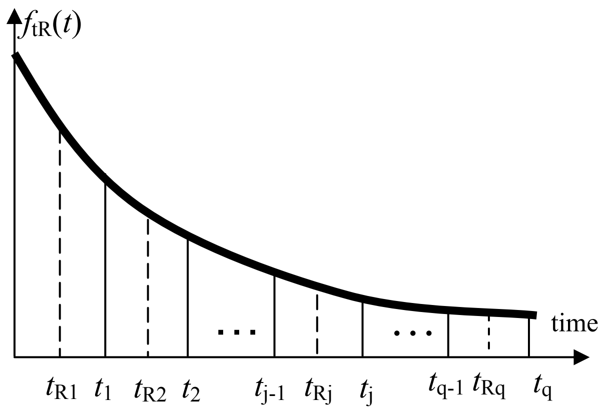

Fig. 4.4. The pdf for repair time t_R.

According to the conditional probability theorem, it is obtained:

$$\begin{aligned} P(A_1/B_1) &= \sum_{j=1}^{q} p[(A_1/B_1)/t_{R_j}]\mathrm{P}(t_R = t_{R_j}) \\ &= \sum_{j=1}^{q} \frac{N_j[(\theta_h > \theta_h^0)/t_{R_j}]}{N_T} P(t_R = t_{R_j}). \end{aligned} \tag{4.4}$$

The probability $P(t_R = t_{Rj})$ is calculated by means of the pdf curve of t_R, shown in Fig. 4.4, where the time axis is divided into q intervals.

If it is assumed that the repair time t_R has an exponential distribution, i.e. $f_{tR}(t) = \mu e^{-\mu t}$, then:

$$P(\mathrm{t}_{\mathrm{R}_j}) = P(\mathrm{t}_{\mathrm{j-1}} \le t_R \le t_j) = \int_{t_{j-1}}^{t_j} \mu e^{-\mu t} dt = e^{-\mu t_{j-1}} - e^{-\mu t_j} \tag{4.5}$$

Based on the assumption that one transformer out of m transformers is out of operation ($k = 1$), one obtains from Equation (4.2)

$$P(A_1) = \left(\frac{\lambda_T}{\lambda_T + \mu_T}\right)\left(\frac{\mu_T}{\lambda_T + \mu_T}\right)^{m-1} m\,. \tag{4.6}$$

Insertion of Equations (4.4) and (4.6) into Equation (4.1) yields

$$P(L > L_0) = P(\theta_h > \theta_h^0) = \left(\frac{\lambda_T}{\lambda_T + \mu_T}\right)\left(\frac{\mu_T}{\lambda_T + \mu_T}\right)^{m-1} m \times \sum_{j=1}^{q} \frac{N_j[(\theta_h > \theta_h^0)/t_{R_j}]}{N_T} P(t_R = t_{R_j}) \tag{4.7}$$

In a way similar to the calculation of the contingency probability $P(\theta_h > \theta^0_h)$, the frequency of the contingency $f(\theta_h > \theta^0_h)$, in which the temperature of the remaining transformers is larger than θ^0_h can be determined. The frequency for the case, when $\theta_h > \theta^0_h$, denoted by $f_j[(\theta_h > \theta^0_h)/t_{Rj}]$ is

$$f_j[(\theta_h > \theta_h^0)/t_{R_j}] = f\,\frac{N_j[(\theta_h > \theta_h^0)/t_{R_j}]}{N_T}, \tag{4.8}$$

where f is the contingency frequency for the case that one transformer is out of operation:

$$f = \left(\frac{\lambda_T}{\lambda_T + \mu_T}\right)\left(\frac{\mu_T}{\lambda_T + \mu_T}\right)^{m-1} m[\mu_T + (m-1)\lambda_T] \tag{4.9}$$

As $\mu_T/\lambda_T + \mu_T \approx 1$ and $\mu_T >> (m\text{-}1)\,\lambda_T$ then:

$$f \approx m\lambda_T \tag{4.10}$$

and thus:

$$f(\theta_h > \theta_h^0) = m\lambda_T \sum_{j=1}^{q} \cdot \frac{N_j[(\theta_h > \theta_h^0)/t_{R_j}]}{N_T} P(t_R = t_{R_j}) \tag{4.11}$$

Determining the time to failure by random number generator (RNG)

The Random Number Generator (RNG) produces random numbers u_0, u_1, ... uniformly distributed between 1 and 0. As it is seen above, the failure time is determined by

$$t_{0_1}^i = F_{t_0}^{-1}(u_i), \tag{4.12}$$

where $F_{t0}(t)$ is the distribution function of t_0, defined as

$$F_{t_0}(\mathrm{t}) = P(t_0 \le t). \tag{4.13}$$

In calculating the conditional probability $P(A_1/B_1)$ using the Monte Carlo method, one determines the probability that the temperature of the remaining transformers (for an outage of one transformer) exceeds a certain given value. Thus, $F_{t0}(t)$ should be calculated as the time-to-failure distribution function, given that the failure occurs within the above mentioned time limits. This is between τ_1 to τ_B, where τ_B is generally one year, see Fig. 4.3

The time-to-failure distribution function is, then

$$P_{t_0}(t) = P(t_0 \le t) = P[(t_F \le t)/(\tau_1 < t_F < \tau_B)] \tag{4.14}$$

where t_F is the time-to-failure stochastic variable, usually exponentially distributed as

$$f_{t_0}(t) = \lambda_T e^{-\lambda_T t}. \tag{4.15}$$

The expression given in Equation (4.14). can be developed by using the conditional probability theorem, thus

$$F_{t_0}(t) = \frac{t_F \le T \cap \tau_1 < t_F < \tau_B}{P(\tau_1 < t_F < \tau_B)} \tag{4.16}$$

and, therefore,

$$F_{t_0}(t) = \frac{P(\tau_1 \le t_F \le t)}{P(\tau_1 \le t_F \le \tau_B)}. \tag{4.17}$$

Assuming that t is exponentially distributed (see Equation (4.15)) we get

$$F_{t_0}(t) = \frac{\int_{\tau_1}^{t} \lambda_T e^{-\lambda_T t'} dt'}{\int_{\tau_1}^{\tau_B} \lambda_T e^{-\lambda_T t'} dt'}, \tag{4.18}$$

which yields, after the integration,

$$F_{t_0}(t) = \frac{1}{1 - e^{-\lambda_T(\tau_B - \tau_1)}}[1 - e^{-\lambda_T(t-\tau_1)}]. \tag{4.19}$$

τ_1 is actually the lower time limit of the calculation period and, therefore, it can be set equal to zero. This means that

$$F_{t_0}(t) = \frac{1 - e^{-\lambda_T t}}{1 - e^{-\lambda_T \tau_B}}. \tag{4.20}$$

The failure time t^i_{01} for each random number u_i obtained by the RNG is, according to Equation (4.12)

$$t^i_{0_1} = F^{-1}_{t_0}(u_i) = -\frac{1}{\lambda_T}\ell n[1 - u_i(1 - e^{-\lambda_T \tau_B})] \tag{4.21}$$

Determining the expected unsupplied energy by the Monte Carlo method

The unsupplied energy probability, as any other stochastic variable, is

$$UE = \sum_{k=1}^{m} P(A_k) \cdot UE_k , \tag{4.22}$$

where $P(A_k)$ is the probability of an outage of k out of m transformers and UE_k is the expected unsupplied energy probability obtained when k transformers are out of operation.

Assuming that at most one transformer out of m is out of operation, we obtain

$$UE = m\left(\frac{\lambda_T}{\lambda_T + \mu_T}\right)\left(\frac{\mu_T}{\lambda_T + \mu_T}\right)^{m-1} UE_1 . \tag{4.23}$$

The expected unsupplied energy probability UE_1 is calculated by means of the Monte Carlo simulation. For a given repair time t_{R_j} and failure time t^i_{01} (determined by a RNG), the expected unsupplied energy is actually the shaded area shown in Fig. 4.3, namely:

$$UE_1\left(t_R = t_{R_j}\ , t_0 = t_{0_1}^i\right) = \int_{t_{0_1}^i}^{t_{R_j}} [L_a^i(t) - L_i(t)]dt\ . \tag{4.24}$$

The expected unsupplied energy probability for a given repair time is given by

$$UE_1(t = t_{R_j}) = \frac{1}{N_T}\sum_{I=1}^{N_T}\int_{t_{0_1}^i}^{t_{R_j}} [L_a^i(t) - L_i(t)]dt \tag{4.25}$$

Thus, the total expected unsupplied energy probability for the case where one transformer is out of operation is

$$UE_1 = \sum_{j=1}^{q}\left[\left(\frac{1}{N_T}\sum_{i=1}^{N}\int_{t_{0_1}^i}^{t_{R_j}} [L_a^i(t) - L_i(t)]dt\right)\cdot P(t_R = t_{R_j})\right]. \tag{4.26}$$

The expected annual unsupplied energy, due to an outage of the transformer, using Equations (4.23) and (4.26) is

$$UE = m\left(\frac{\lambda_T}{\lambda_T + \mu_T}\right)\left(\frac{\mu_T}{\lambda_T + \mu_T}\right)^{m-1}\sum_{j=1}^{q}\left[\left(\frac{1}{N_T}\sum_{i=1}^{N_T}\int_{t_{0_1}^i}^{t_{R_j}} [L_a^i(t) - L_i(t)]dt\right)\cdot P(t_R = t_{R_j})\right] \tag{4.27}$$

4.1.3 Analytical model for calculating reliability Indices for substations based on transformers overloading

As was shown above, the problem is to determine the reliability indices based on the hottest spot temperature of the insulation, which determines the transformation capacity of the remaining transformers. This temperature depends on the following stochastic variables:

- the load curve $L(t)$,
- the ambient temperature $\theta_a(t)$,
- the repair time t_R,
- the failure time t^i_{01},
- the topography of the HV network (which determines the load that can be transferred to adjacent substations and the required switching times).

The calculation of the reliability indices in this case is more complicated than in the case of the generation system. The power generation capacity does not depend on the temperature or any other similar parameter and it has a rather strict limit. Transformers, on the other hand, can be loaded with a few tens percents above their rated capacity. In this case, one should also consider the decrease in their expected lifetime. This should be studied, as well as the reliability indices, while planning and operating substations.

As was discussed above, the problem can be solved by means of a Monte Carlo simulation. A disadvantage of this method is the long time required for computation. This is true, especially, when a dynamical model is involved, i.e. when an increase of the load is incorporated in the calculations. This is the main reason why an analytical model should be studied and developed. In order to do that, the following assumptions are made:

- The ambient temperature is constant and equal to the weighted average temperature throughout the period under discussion (IEC 60354/1991, IEC 60076-7/2005).
- The time to failure is exponentially distributed.
- The time to failure is much larger than the forced outage time or the repair time.
- The time to failure and the repair time are independent variables.
- The forced outage of the transformer does not depend on the load level.
- As the daily peak load duration is a few hours only and as the time to failure is longer than the repair time, the probability that more than one transformer will fail during the abovementioned period is small. Therefore, it is assumed, as it was done in the Monte Carlo simulation, that only one transformer can fail.

The developed model is based on modeling the load and the failure time (required for the determination of the temperatures).

The load model

As was seen above, the generation system reliability calculations are based mainly on two analytical methods, which differ by the load model used. The first method is the calculation of the "Loss of load probability" (LOLP), there the load is given by the load duration curve, see Fig. 4.5.

The second method is the frequency and duration approach, by which, besides the probability, the frequency and duration of load levels higher than the generation capacity, can be determined. The daily load curve is represented in this method by two load levels, namely: the off-peak load L_o, and the peak load L_{Pi} (see Fig. 4.6). It is assumed that the duration of the peak load (where the load is above a predetermined load level L_i) is e_i (Endrenyi et al. 1973).

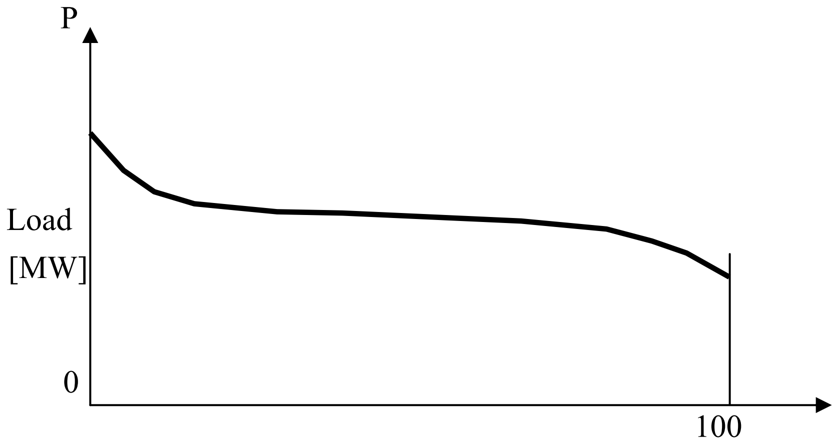

Fig. 4.5. The load duration curve

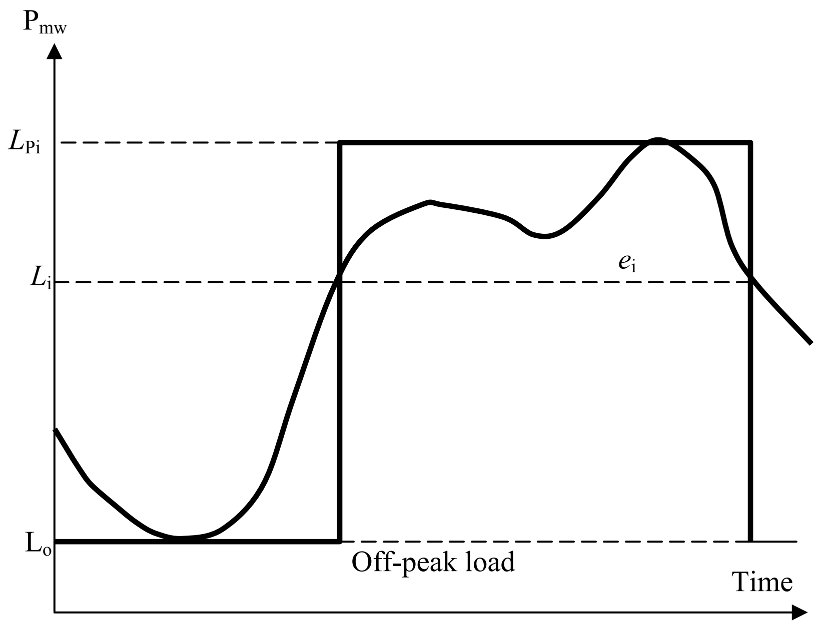

Fig. 4.6. Load model used in frequency and duration approach

In reliability of generation system calculations, the time variation of the load is less important because of two reasons:

1. The load capacity of the generators is dictated by the manufacturers and it does not depend on the time variations of the load;
2. The duration of forced outages usually exceeds one day. Therefore, when generation system is planned, one should provide supply of the peak load, which justifies use of the load duration curve.

Thus, for calculations of reliability of generation system, the load duration curve can be used in load models. As for transformers, where the insulation temperature depends on the load variations in time, a model representing the daily load by a peak and off-peak load (two levels) is required. An approximate analytical method for the choice of the load model parameters (the equivalent peak load, the equivalent off-peak load and the duration of the peak load) is given in (IEEE Std. C57. 91-1995). Based on this method, the actual load curve can be represented by an equivalent load curve, shown in Fig. 4.7.

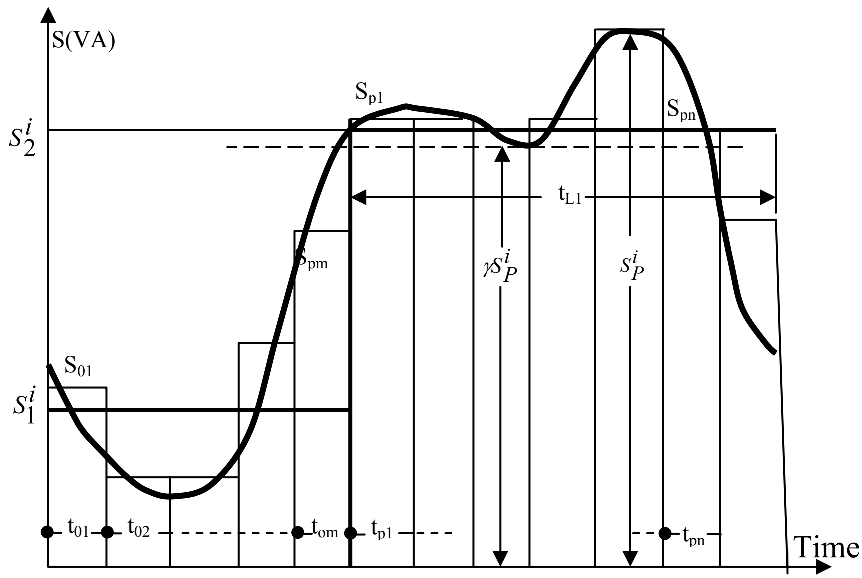

Fig. 4.7. Equivalent load cycle

The equivalent peak load S^i_2 for a load curve of a day i, is the root mean square value of the loads during one interval, when the load is larger than a given value γS^i_p ($\gamma < 1$). S^i_2 is the load, which results in the same heat amount during the same time t_{Li} as the actual load. The equivalent load peak, according to (IEEE Std. C57. 91-1995), is at least 90% of the peak load S^i_p (i.e. $\gamma = 0.9$).

The equivalent loads S^i_1 and S^i_2 for a given load curve *i* are obtained, according to (IEEE Std. C57. 91-1995) as

$$S_1^i = \sqrt{\frac{\sum_{i=1}^{m} S_{0i}^2 \cdot t_{0_i}}{\sum_{i=1}^{m} t_{0_i}}}, \quad S_2^i = \sqrt{\frac{\sum_{i=1}^{n} S_{pi}^2 \cdot t_{p_i}}{\sum_{i=1}^{n} t_{p_i}}} \tag{4.28}$$

where $S_{01},...,S_{0m},S_{p1},\ldots,S_{pm}$ are various load levels in the actual load curve (see Fig. 4.7).

Assuming that the substation load is evenly distributed between the transformers, the equivalent values of the peak and the off-peak loads, in relative units based on the rated power of the transformer S_r can be expressed as

$$k_1^i = \frac{S_1^i}{NS_r} = \frac{S_1^i}{K_R S_{py}} \tag{4.29}$$

$$k_2^i = \frac{S_2^i}{NS_r} = \frac{S_2^i}{K_R S_{py}} \tag{4.30}$$

where *N* is the number of transformers in the substation, S^i_1 and S^i_2 are the equivalent loads in MVA based on the substation power output, S_{py} is the annual peak power in MVA, k_R is the transformation reserve coefficient, i.e. the ratio between the rated power and the annual peak power of the substation in MVA.

After determining the peak and low levels for each daily load curve, the annual load curve (or for any time period) is given and shown in Fig. 4.8.

Here it is assumed that the off-peak load level is constant and equal to the average off-peak load level throughout the period under discussion:

$$\begin{cases} L_i = k_2^i \\ L_0 = \dfrac{1}{n}\sum_{i=1}^{n} k_1^i \end{cases} \tag{4.31}$$

where *n* is the number of days in the period under discussion.

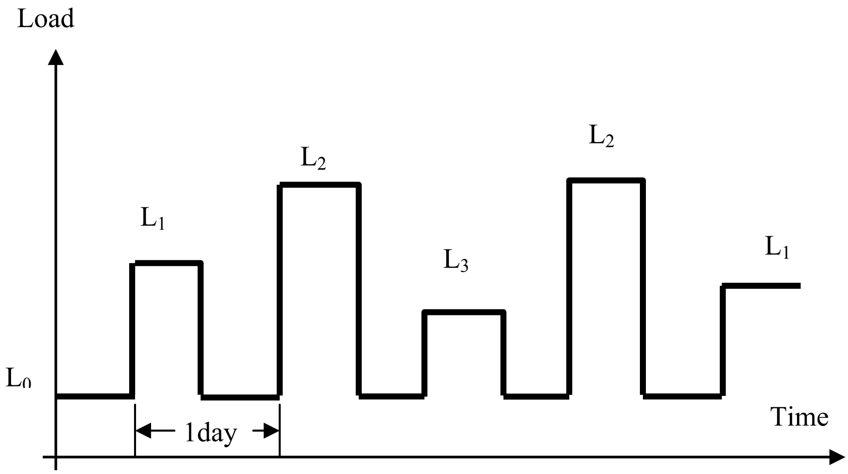

Fig. 4.8. The annual load model

To summarize all said above:

1. The daily load curve is represented by two load levels, the peak load level L_i and the off-peak load level L_0 as shown in Fig. 4.7.
2. The load curve for a period of n days is represented by n daily peak loads L_1, ...,L_n (see Fig. 4.8) which form a random series of n various load levels. The daily off-peak load is constant throughout the n days and equals L_o. As mentioned above, the daily peak load levels appear stochastically.
3. After a peak load L_i comes, a off-peak load L_0 and then another L_i value appear stochastically.
4. The duration of the daily peak load t_L and the duration of the daily off-peak load t_{L0} are stochastic variables.

It should be noted that in analytical models, where the load is represented as shown above, it is assumed that both t_L and t_R are exponentially distributed. Such a distribution is widely accepted. On the other hand, when long term conditions are discussed, such an assumption is not always necessary.

If T_{Lo} is the average duration time of the off-peak load level L_0 and T_L is the average duration time of the peak load level L_i, then $T_L + T_{Lo}$= 24h for the daily load curve.

The description of the analytical model

As was observed, two stochastic procedures are involved in the model, the first one describes the state of the transformer and the second one models the daily load. The state of the transformer is given by a periodical procedure, changing between a state of operation and a state of failure. This procedure is described by two stochastic variables: time to failure t_F and the repair time t_R. The daily load also changes periodically between peak load and off-peak load. It is represented by two stochastic time lengths, the time t_L for the peak load L_i and the time $t_{L\mathrm{o}}$ for the off-peak load L_o.

It is assumed that the two abovementioned procedures are independent, i.e. the time of the failure does not necessarily depend on the load of the transformers. In case of a forced outage of one transformer, the hottest spot temperature may increase above a predetermined value in two cases: when the failure occurs at the peak load and when the failure occurs at off-peak load and the repair time continues into the following peak load. Thus, in these two cases, there is an overlapping between the forced outage time (or repair time t_R) and the peak load time t_L. These cases are shown schematically in Fig. 4.9.

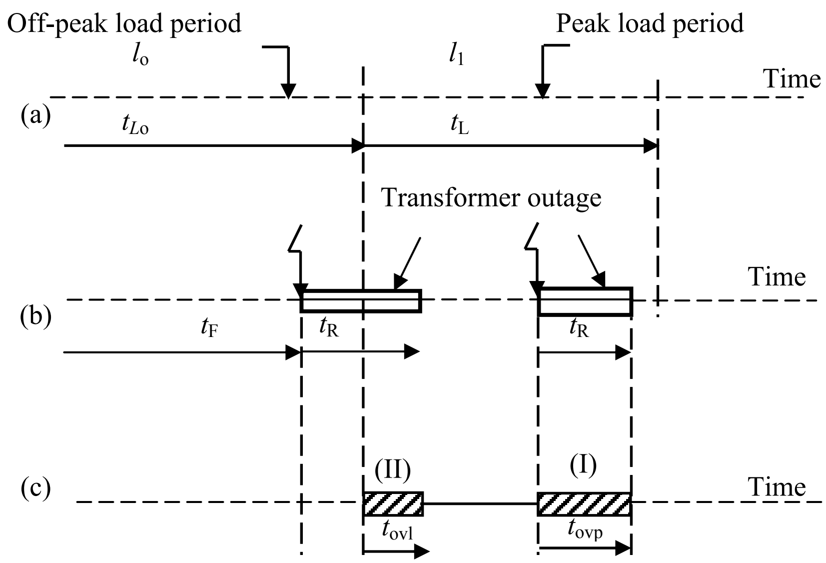

Fig. 4.9. a). The load model: t_L is peak load duration time, t_{LO} is off-peak load duration time. b). The transformer state model: t_F is time to failure, t_R is forced outage or repair time. c) Overlapping time periods

As can be observed the load and transformer state modeling procedures determine the overlapping times t_{ovp} and t_{ovl}, where t_{ovp} is the overlapping time between the peak load L_i and the forced outage time, i.e. a failure occurring at the peak load. t_{ovl} is the overlapping time between the load and the forced outage time for a failure which occurs at off-peak load and the repair time continues into the peak load. As it will be discussed later, the pdf of the overlapping times can be studied by the knowledge of the pdf of the stochastic variables involved t_F, t_R, t_L and t_{Lo}. With the knowledge of the pdf of the overlapping times, the pdf of the temperature can be determined and thus the required reliability indices can be studied. Moreover, for the study of the temperature θ_h a certain temperature value at the occurrence of the failure should be assumed.

In power system reliability studies, it is common to assume exponential distributions resulting in simple models. It should be pointed out that such a distribution is true especially for the time to failure t_F when the equipment used has not reached yet the "aging" process. If another distribution is chosen for the stochastic variables, the solution of the problem is analytically complicated. This is because in this case, the time depending process is of prime interest and not only the steady state condition (as in the generation system). The hottest spot temperature depends on the abovementioned stochastic variables. The stochastic variables t_F, t_R, t_L and t_{L0} are assumed to have exponential distributions**.** The pdf of the stochastic variables is thus given by:

$$\begin{cases} f_{t_F}(t) = \lambda e^{-\lambda t} \\ f_{t_R}(t) = \mu e^{-\mu t} = \dfrac{1}{T_R} e^{-t/T_R} \\ f_{t_L}(t) = \lambda_L e^{-\lambda_L t} = \dfrac{1}{T_L} e^{-t/T_L} \\ f_{t_{L_0}}(t) = \lambda_{L_0} e^{-\lambda_{L_0} t} = \dfrac{1}{T_{L_0}} e^{-t/T_{L_0}} \end{cases} \tag{4.32}$$

where: λ is failure rate [l/year], μ is repair rate [1/year], T_R mean outage duration time [year], T_L mean peak load duration time [year], t_{L0} mean off-peak load duration time [year].

Based on these assumptions, various system states can be described by a Markov procedure as shown in the state transition diagram (Fig. 4.10) that presents all possible operation states of the system, assuming a forced outage of one transformer (normal operation and failure states at various load levels). The various possible transitions are also shown in Fig. 4.10. The total number of states is $2(n + 1)$, where there are $n + 1$ normal operation states and $n + 1$ failure states.

Based on Fig. 4.10 we obtain the following equations, describing the steady-state conditions

$$\begin{cases} state\ 0: -\left[\lambda + \sum_{i=1}^{n} \alpha_i \lambda_{L_0}\right] P_0 + \lambda_L \sum_{i=1}^{n} P_i + \mu P_0' = 0 \\ state\ 0': -\left[\mu + \sum_{i=1}^{n} \alpha_i \lambda_{L_0}\right] P_i' + \lambda P_0 = 0 \\ state\ 1: -(\lambda_L + \lambda) P_1 + \alpha_1 \lambda_{L_0} P_0 + \mu P_1' = 0 \\ state\ 1': -(\lambda_L + \lambda) P_1 + \alpha_1 \lambda_{L_0} P_0' + \lambda P_1 = 0 \\ state\ i: -(\lambda_L + \lambda) P_i + \alpha_i \lambda_{L_0} P_0 + \mu P_i' = 0 \\ state\ i': -(\lambda_L + \lambda) P_i' + \alpha_i \lambda_{L_0} P_0' + \lambda P_i = 0 \\ state\ n: -(\lambda_L + \lambda) P_n + \alpha_n \lambda_{L_0} P_0 + \mu P_n' = 0 \\ state\ n': -(\lambda_L + \lambda) P_n' + \alpha_n \lambda_{L_0} P_0' + \lambda P_n = 0 \end{cases} \tag{4.33}$$

where $P_o,...,P_n$ are normal operation state probabilities, $P'_o,...,P'_n$ are failure state probabilities. (4.33) is a set of homogeneous linear algebraic equations that should be solved with the additional equation resulting from the fact that the system should be in one of the above mentioned states, i.e.:

$$(P_0 + P_0') + (P_1 + P_0') + ... + (P_i + P_1') + ... + (P_n + P_n') = 1 \tag{4.34}$$

By solving the set of equations, the probabilities for the various states take the form

$$\begin{cases} P_0 = \left(\dfrac{\mu}{\lambda + \mu}\right)\left(\dfrac{\lambda_L}{\lambda_L + \lambda_{L_0}}\right) \\ P_0' = \left(\dfrac{\mu}{\lambda + \mu}\right)\left(\dfrac{\lambda_L}{\lambda_L + \lambda_{L_0}}\right) \\ P_i = \left(\dfrac{\mu}{\lambda + \mu}\right)\left(\dfrac{\alpha_i \lambda_{L_0}}{\lambda_L + \lambda_{L_0}}\right) \text{for } i = 1,...,n \\ P_i' = \left(\dfrac{\mu}{\lambda + \mu}\right)\left(\dfrac{\alpha_i \lambda_{L_0}}{\lambda_L + \lambda_{L_0}}\right) \text{for } i = 1,...,n \end{cases} \tag{4.35}$$

It should be noted that the same results could be obtained even without the assumption of exponential distributions. This can be done by using the renewal process theorem, in the case when there exist mean values for the stochastic variables t_F, t_R, t_L and t_{L0}. These values are calculated as

$$\begin{cases} E[t_F] = m_F = \int_0^\infty f_{t_F}(t)dt \\ E[t_R] = m_R = \int_0^\infty f_{t_R}(t)dt \\ E[t_{L_0}] = T_{L_0} = \int_0^\infty t\ f_{t_{L_0}}(t)dt \\ E[t_L] = T_L = \int_0^\infty t\ f_{t_L}(t)dt \end{cases} \tag{4.36}$$

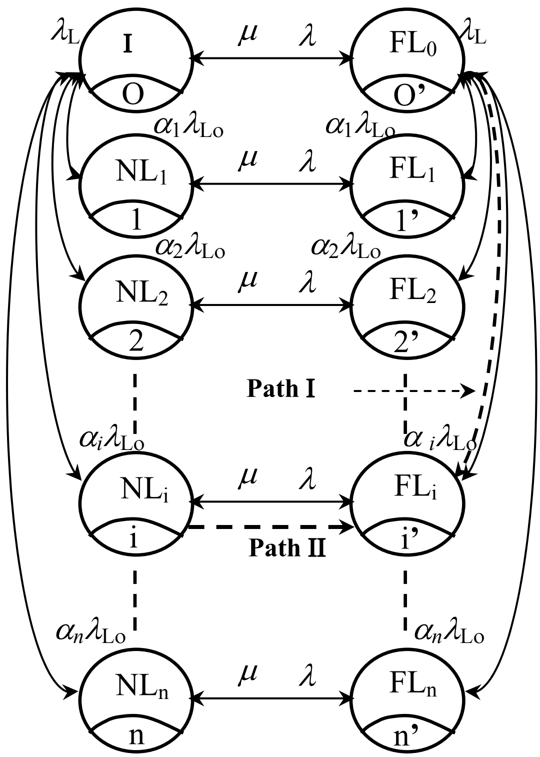

Fig. 4.10. State transition diagram. NL_0: normal system state, off-peak load L_o; NL_i: normal system state, peak load L_i; FL_0: system failure, off-peak load L_o; FL_i: system failure, peak load L_i; N_i number of occurrences of L_i; $\alpha_i = N_i/365$.

Based on the renewal process theorem, see (Shooman 1990), if $t \to \infty$, then

$$\begin{cases} A = \lim\limits_{t\to\infty} A(t) = \dfrac{m_F}{m_F + m_R} = \dfrac{1/\lambda}{1/\lambda + 1/\mu} = \dfrac{\mu}{\lambda + \mu} \\ \overline{A} = 1 - A = \dfrac{\lambda}{\lambda + \mu} \end{cases} \tag{4.37}$$

where A is the probability that the transformer is in normal operation state (availability), $\overline{A}$ is the probability that the transformer is in failure state.

In a similar way, for the load process:

$$\begin{cases} A_{L_0} = \lim\limits_{t\to\infty} A_{L_0}(t) = \dfrac{T_{L0}}{T_{L_0} + T_L} = \dfrac{\lambda_L}{\lambda_L + \lambda_{L_0}}, \\ A_{L_i} = \lim\limits_{t\to\infty} A_{L_i}(t) = \dfrac{\alpha_i T_L}{T_{L_0} + T_L} = \dfrac{\alpha_i \lambda_{L0}}{\lambda_L + \lambda_{L_0}} \quad \text{for } i = 1,2,\dots,n \end{cases} \tag{4.38}$$

Assuming that all processes are independent, the state probabilities are obtained by Equation (4.35).

The state frequencies for a transformer can be calculated by using the Markov model (if the reliability indices for the substation are calculated, the following equations should be multiplied by the number m of transformers in the substation):

$$\begin{cases} f_0 = P_0\left(\lambda + \sum\limits_{i=1}^{n} \alpha_i \lambda_{L0}\right) \\ f_{o'} = P_0'\left(\mu + \sum\limits_{i=1}^{n} \alpha_i \lambda_{L0}\right) \\ f_i = P_i(\lambda + \lambda_L) \quad \text{for } i = 1,\dots,n \\ f_{i'} = P'_i(\mu + \lambda_L) \quad \text{for } i = 1,\dots,n \end{cases} \tag{4.39}$$

As $\sum\limits_{i=1}^{n} \alpha_i = 1$ the frequencies for the states 0 and 0' are

$$\begin{cases} f_0 = P_i(\lambda + \lambda_{L_0}) \\ f_{0'} = P_0'(\mu + \mu_L) \end{cases} \tag{4.40}$$

Insertion of the probabilities obtained in Equation (4.35) into Equation (4.40) yields

$$\begin{cases} f_0 = \left(\dfrac{\mu}{\lambda+\mu}\right)\left(\dfrac{\lambda_L}{\lambda_L+\lambda_{L_0}}\right)\left(\lambda+\lambda_{L_0}\right) \\ f_{o'} = \left(\dfrac{\lambda}{\lambda+\mu}\right)\left(\dfrac{\lambda_L}{\lambda_L+\lambda_{L_0}}\right)\left(\mu+\lambda_{L_0}\right) \\ f_i = \left(\dfrac{\mu}{\lambda+\mu}\right)\left(\dfrac{\lambda_{L0}}{\lambda_L+\lambda_{L_0}}\right)\left(\mu+\lambda_L\right)\alpha_i \\ f_{i'} = \left(\dfrac{\lambda}{\lambda+\mu}\right)\left(\dfrac{\lambda_{L0}}{\lambda_L+\lambda_{L_0}}\right)\left(\mu+\lambda_L\right)\alpha_i \end{cases} \tag{4.41}$$

While studying the reliability indices until now, no attention was paid to the insulation temperature θ_h, which is actually determined by the load capacity of the transformers. In the following discussion both the reliability indices, the probability and the frequency will be studies in relation to the insulation hottest spot temperature.

The problem is to determine the frequencies and probabilities in the cases where the insulation hottest spot temperature θ_h is larger than a predetermined value $\theta^o{}_h$. The situation where $\theta_h > \theta^0{}_h$, can occur in one of two cases

1. A failure occurs at the daily off-peak load level L_0 and continues due to the repair time into the peak load. This transition is shown by path I in Fig. 4.10.
2. A failure occurs at the peak load. This transition is shown by path II in Fig. 4.10.

Thus, the frequency of failures, where θ_h is larger than $\theta^0{}_h$, for the i-th load cycle, can be calculated as

$$F_{D_i}(\theta_h^i > \theta_h^o) = f_i^I \cdot P\left(\theta_{h_L}^i > \theta_h^o\right) + f_i^{II} \cdot P(\theta_{hp}^i > \theta_h^o), \tag{4.42}$$

where $f^I{}_i$ is the frequency of failure occurrence at the off-peak load level of the daily load curve i which continues into the peak load (Fig. 4.10, path I), $f^{II}{}_i$ is the frequency of failure occurrence at the daily peak load i (Fig. 4.10, path II), $\theta^i{}_{hL}$ is the hottest spot temperature of the transformers remaining in operation after a forced outage of a transformer at the off-peak load, which continues due to repair time into the peak load for a load curve i, $\theta^i{}_{hP}$ is the hottest spot temperature of the remaining transformers after a forced outage of a transformer at the peak load i, $P(\theta^i{}_{hL} > \theta^0{}_h)$ is the probability that the temperature of the remaining transformers is larger than $\theta^0{}_h$ after a forced outage of a transformer at the low level of the daily load curve i, $P(\theta^i_{hP}>\theta^0_h)$ is the probability that the temperature of the remaining transformers is larger than θ^0_h after a forced outage of a transformer at the peak level of the daily load curve i.

The frequency f^I_i is obtained by multiplying the probability P'_0 of state 0' by the transition rate from state 0' to state i' (path I). The frequency f^{II}_i is obtained by multiplying the probability P_i of state i by the transition rate from state i to state i' (path II). Thus:

$$\begin{cases} f_i^I = P'_0 \lambda_{L0} \alpha_i \\ f_i^{II} = P_i \lambda \end{cases} \tag{4.43}$$

Inserting the expression for P'_0, and P_i from Equation (4.35) into Equation (4.43) yields

$$\begin{cases} f_i^I = \dfrac{\lambda}{\lambda+\mu} \cdot \dfrac{\lambda_L}{\lambda_L+\lambda_{L0}} \cdot \lambda_{L0}\ \alpha_i \\ f_i^{II} = \dfrac{\mu}{\lambda+\mu} \cdot \dfrac{\lambda_{L0}}{\lambda_L+\lambda_{L0}} \cdot \lambda\ \alpha_i \end{cases} \tag{4.44}$$

If by definition $T_R = 1/\mu$ [hours] and $T_L = 1/\lambda_L$ [hours] then $T_{Lo} = 1/\mu Lo =$ 24 $- T_L$ hours and $\alpha_i = \dfrac{N_i}{365}$. Therefore:

$$\begin{cases} f_i^I = \dfrac{\lambda T_R}{\lambda T_R + 8760} N_i [1/\text{year}] \\ f_i^{II} = \dfrac{\lambda T_R}{\lambda T_R + 8760} N_i [1/\text{year}] \end{cases} \tag{4.45}$$

Equations (4.42) and (4.45) give

$$F_{Di}(\theta_h^i > \theta_h^o) = \frac{\lambda T_R N_i}{\lambda T_R + 8760} \cdot P(\theta_{hL}^i > \theta_h^o) + \frac{\lambda T_L N_i}{\lambda T_R + 8760} \cdot P(\theta_{hp}^i > \theta_h^o)\,. \tag{4.46}$$

Then, for n various load curves F_D is

$$F_D(\theta_h > \theta_h^o) = \frac{\lambda}{\lambda T_R + 8760} \sum_{i=1}^{n} [T_R N_i \cdot P(\theta_{hL}^i > \theta_h^o) + \\ + T_L N_i \cdot P(\theta_{hp}^i > \theta_h^o)] \tag{4.47}$$

The probability can be calculated in a similar way. In general, the relation between the probability P, the time duration T and the frequency F is

$$P = FT. \tag{4.48}$$

One should calculate the probability that the temperatures of the transformer remaining in operation after a forced outage of other transformer will be larger than θ_{oh}. For this calculation a transition diagram shown in Fig. 4.11 is used. For the sake of simplicity, this transition diagram is given for one load level L_i only.

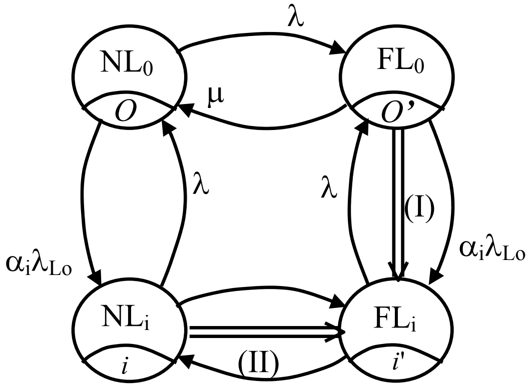

Fig. 4.11. Transition diagram for the state *i*

The temperature of the transformers remaining in operation may exceed the value $\theta^0{}_h$ in one of two cases:

1. a failure occurs at the low level of the load curve and its repair continues into the peak load (path II);
2. a failure occurs at the peak load (path II).

Based on Equation (4.48), the probability $P_i(\theta^i_h > \theta^0_h)$ that the temperature of the remaining transformers exceeds $\theta^0{}_h$ for the load curve i is

$$P_i(\theta^i_h > \theta^o_h) = f_i^I T_{ovl} P(\theta^i_{hL} > \theta^o_h) + f_i^{II} T_{ovp} P(\theta^i_{hp} > \theta^o_h), \tag{4.49}$$

where T_{ovl} is the mean overlapping time for a failure occurring at off-peak load, T_{ovp} is the mean overlapping time for a failure occurring at peak load, $P(\theta^i{}_{hL} > \theta^0{}_h)$ is the probability that the temperature of the remaining transformers exceeds $\theta^0{}_h$ in the case of a failure occurring at off-peak load and continuing into peak load, $P(\theta^i{}_{hp} > \theta^0{}_h)$ is the probability that the temperature of the transformers exceeds θ^0_h in the case of a failure occurring at peak load.

For n various load curves:

$$P(\theta_h > \theta_h^o) = \sum_{i=1}^{n} [f_i^I T_{ovl} P(\theta_{hL}^i > \theta_h^o) + f_i^{II} T_{ovp} P(\theta_{hp}^i > \theta_h^o)] \tag{4.50}$$

4.1.4 Probability $P(\theta_h > \theta_h^0)$

The calculation of the reliability indices in relation to the insulation temperature of the transformer yielded the equations (4.47) and (4.50). However, a further discussion of $P(\theta^i_{hL}>\theta^0_h)$ and $P(\theta^i_{hP}>\theta^0_h)$ is required for the cases of failures occurring at the low and peak levels of the daily load curve *i*.

The above mentioned probabilities are determined based on the pdf of the temperature $f_\theta(\theta)$, which, in its turn, is based on the pdf of t_F, t_R, t_L and t_{LO} and on the existing relation between these variables and the temperature.

At first, the relation between the temperature and the above mentioned stochastic variables t_F, t_R, t_L and t_h is studied. The temperature can be calculated on the basis of well-known equations given in various Standards. It should be pointed out that the temperature depends also on certain technical data of the transformer.

Calculation of the transformer temperature rise under failure conditions

The calculation of the insulation hottest spot temperature θ_h is based on the "Loading guide for oil-immersed transformers" (IEC 60354/1991). Moreover, it is assumed that in the case of a transformer outage, its load is equally divided between other transformers remaining in operation in the same substation. The possibility of transferring loads to adjacent substations is neglected at this stage. Based on these assumptions the load of the remaining transformers is shown in Fig. 4.12. All transformers are assumed to be identical.

The temperatures of the transformers under load during a forced outage of a transformer as a function of t_{ovp} and t_{ovl} are

$$\begin{cases} \theta_{hp}(t_{ovp}) = \alpha_p e^{-t_{ovp}/T} + \beta_p \\ \theta_{hl}(t_{ovp}) = \alpha_0 e^{-t_{ovp}/T} + \beta_0 \end{cases} \tag{4.51}$$

where T is the thermal time constant of the transformer [h], α_p<0 is the difference between the initial oil temperature at the moment of the failure and the temperature at the steady state for a failure occurrence at peak load [^{0}C], β_p is the winding temperature at the hottest spot in steady state for a failure occurrence at peak load [^{0}C], α_0<0 is the difference between the initial oil temperature and the temperature at the steady state for a failure occurrence at off-peak load [^{0}C], β_0 is the winding temperature at the hottest spot in steady state for a failure occurrence at off-peak load [^{0}C]. The values of α_p, α_0, β_p, and β_0 are calculated based on the load diagram and number of transformers as described in Section 4.1.6.

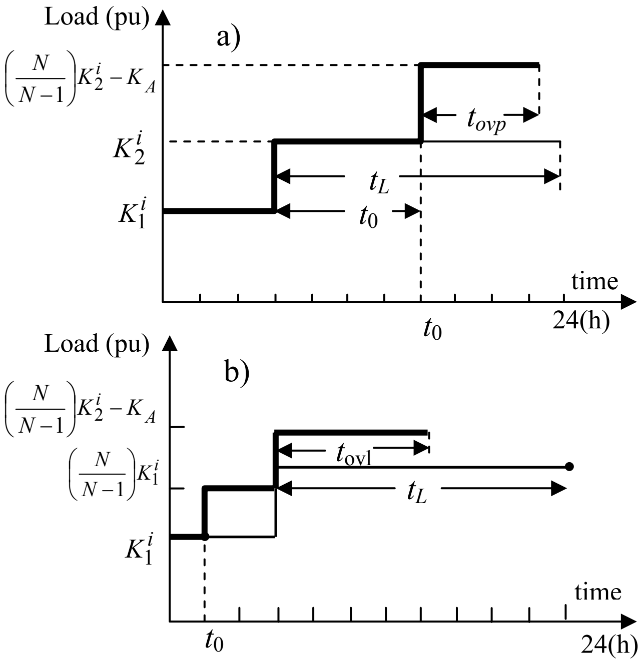

Fig. 4.12. Equivalent load cycle: a) failure occurrence at peak load; b) failure occurrence at off-peak load with continuation into peak load. N is the number of transformers in the substation, K^i_1 is equivalent transformer load at off-peak load of daily load curve i, K^i_2 is equivalent transformer load at peak load of daily load curve i, t_0 is time of failure occurrence, t_{ovp} and t_{ovl} are overlapping times between repair time t_R and the duration time of peak load t_L for a failure occurring at the peak and off-peak loads respectively. K^i_1, K^i_2 are stated in Equations 4.29 and 4.30.

Note that t_0 should be known in order to calculate the above mentioned temperatures. The expectancy of the failure occurrence moment within a certain time interval is discussed later in Section 4.1.5. The results are going to be applied to the problem at hand. As shown in Section 4.1.5, in most cases the failure is due to occur roughly in the middle of the time interval. As the functions $\theta_h(t_{ovp})$ and $\theta_h(t_{ovl})$ are monotonous, see Fig. 413,

$$f_{\theta_h}(\theta_h) = \frac{f_{t_{ov}}(\theta_h)}{\left|\frac{d\theta_h}{dt_{ov}}\right|} \tag{4.52}$$

the pdf of the temperature $f_{\theta h}(\theta_h)$ can be determined by the pdf of the overlapping t_{ov}(t_{ovp} or t_{ovl}) :

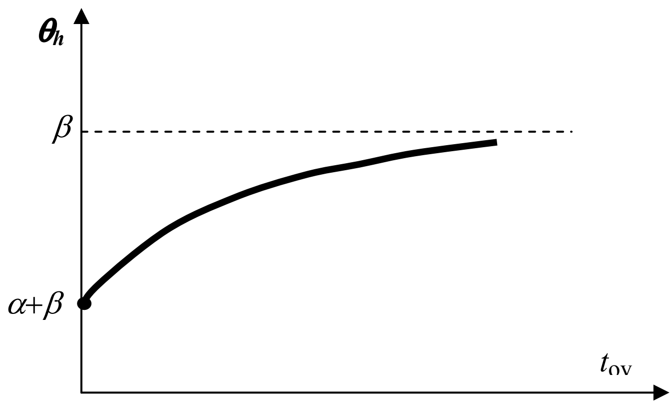

Fig. 4.13. Function $\theta_h = f(t_{ov})$

Calculating the pdf of overlapping times

The temperature of the transformers remaining in operation depends, as it was seen, on two stochastic variables: the time of failure occurrence t_o and the overlapping time between the repair time t_R and the duration time of the equivalent peak load t_L.

Note that t_{ovp} is the overlapping time for a failure occurrence at the peak level of the daily load curve whereas t_{ovl} is the overlapping time for a failure occurring at off-peak load and continuing into peak load. For a daily load curve t_{ovp} and t_{ovl} are (see Figs. 4.14 and 4.15):

$$\begin{cases} t_{ovp} = \min(t_R, t_L - t_0) = \min(t_R, \tau_p) \\ t_{ovl} = \min(t_L, t_R - t_1) = \min(t_L, \tau_0) \end{cases} \tag{4.53}$$

where $\tau_p = t_L - t_0$ and $\tau_0 = t_R - t_1$.

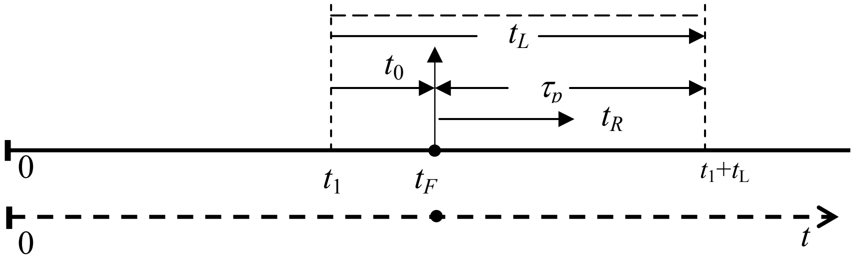

Fig. 4.14. Definition of the stochastic variables in the case of fault occurrence at peak load. t_L is duration time of equivalent peak load, t_o is time of forced outage occurrence throughout the overlapping state measured from the beginning of peak load t_o, t_R is repair time, t_F is time between two forced outages

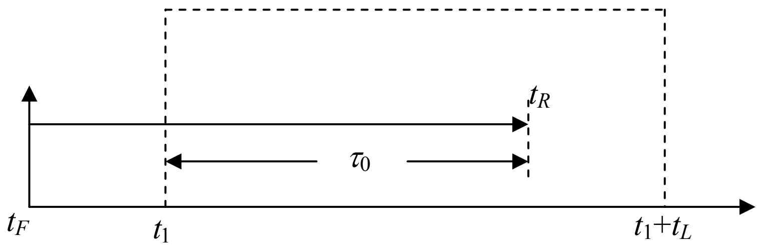

Fig. 4.15. Definition of the stochastic variables in case of fault occurrence at off-peak load

The pdf for the stochastic variables defined by Equation (4.53) are (Endrenyi 1978, Shooman 1990)

$$\begin{cases} f_{t_{ovp}}(t) = f_{t_R}(t)\cdot[1-F_{\tau_p}(t)] + f_{\tau_p}(t)\cdot[1-F_{t_R}(t)] \\ f_{t_{ovl}}(t) = f_{t_L}(t)\cdot[1-F_{\tau_0}(t)] + f_{\tau_0}(t)\cdot[1-F_{t_L}(t)] \end{cases} \tag{4.54}$$

where $f_{t_R}(t), f_{\gamma_p}(t), f_{t_L}(\mathrm{t}), f_{\gamma_0}(t)$ are pdf for the variables t_R, τ_p, t_L and τ_0 respectively, $F_{t_R}(t), F_{\gamma_p}(t)$, $F_{t_L}(t), F_{\gamma_0}(t)$ are distribution functions for t_R, τ_p, t_L and τ_0 respectively.

From the definition of t_{ovp} and t_{ovl} follows that the pdf and the distribution functions of τ_p and τ_0 should be determined first.

If the repair time t_R is assumed to be exponentially distributed, then

$$\begin{cases} f_{t_R}(t) = \mu\ e^{-\mu t} \\ F_{t_R}(t) = 1 - e^{-\mu t} \end{cases} \tag{4.55}$$

The calculation of the distribution function of τ_p should be carried out if two conditions are fulfilled, namely:

1. The system is under normal operation conditions up to t_1 for a failure occurrence at peak load;
2. Overlapping exists between the failure duration time and the peak load.

Thus, the distribution function of time to failure t'_F is studied first, assuming that the system was under normal condition up to t_1. The corresponding mathematical expression is

$$F_{t'_F}(t) = P(t_F \le t/t_F \ge t_1) = \frac{P(t_F \le t \cap t_F \ge t_1)}{P(t_F \ge t_1)} = \frac{P(t_1 \le t_F \le t)}{P(t_F \ge t_1)}. \tag{4.56}$$

For an exponential distribution of time to failure

$$F_{t'_F}(t) = \frac{\int_{t_1}^{t} \lambda\ e^{-\lambda t'}\ dt'}{\int_{t_1}^{\infty} \lambda\ e^{-\lambda t'}\ dt'} = \frac{e^{-\lambda t_1} - e^{-\lambda t}}{e^{-\lambda t_1}} = 1 - e^{-\lambda(t - t_1)} \tag{4.57}$$

and the pdf of t'_F is

$$f_{t'_F}(t) = \frac{dF_{t'_F}(t)}{dt} = \lambda\ e^{-\lambda(t - t_1)} \quad \text{for } t \ge t_1 . \tag{4.58}$$

Thus, if the distribution function of t_F is exponential and the system is in normal operation condition up to t_1, then the distribution function of time to failure starting at t_1 should also be exponential. The distribution function of τ_p can be determined using the definition

$$F_{\tau_p}(t) = P(\tau_p \le t) \tag{4.59}$$

The expression for τ_p (see Equation (4.53)) yields :

$$P(\tau_p \le t) = P(t_L - t_0 \le t). \tag{4.60}$$

Taking into account the existing overlapping, this probability can be expressed by the stochastic variable t'_F:

$$P(t_L - t_0 \le t) = P(t_L + t_1 - t'_F \le t / t_1 \le t'_F \le t_1 + t_L) \tag{4.61}$$

Using the conditional probability theorem, we get

$$P(t_L - t_0 \le t) = \frac{P[(\ t_L - t'_F \le t - t_1\ ) \cap (t_1 \le t'_F \le t_1 + t_L)]}{P(t_1 \le t'_F \le t_1 + t_L)}$$
$$= \frac{P[(t_L - t'_F \le t - t_1) \cap (t_L - t'_F \ge -t_1 \cap t'_F \ge t_1)]}{P(t'_F \ge t_1 \cap t_L - t'_L - t'_F \ge -t_1)} \tag{4.62}$$

Since $P(t'_F \ge t_1) = 1$ (see the distribution function Equation (4.58))

$$P(t_L - t_0 \le t) = \frac{P(-t_1 \le t_L - t'_F \le t - t_1)}{P(t_L - t'_F \ge -t_1)} \tag{4.63}$$

Now we can calculate the distribution function of τ'_p as $\tau'_p = t_L - t'_F$. As these stochastic variables are independent, the pdf of τ'_p can be determined as

$$f_{\tau'_p}(t) = \int_{-\infty}^{\infty} f_{t_L}(t - t') f_{t'_F}(-t') dt' \tag{4.64}$$

where $f_{t_L}(t) = \lambda_L\ e^{-\lambda L}$.

The integration limits are determined by the boundaries of the pdf $f_{tL}(t)$ and $f'_{tF}(t)$:

$$\begin{cases} f_{t_L}(t - t') \ne 0 \ \text{ for } \ t' \le t \\ f_{t_L}(t - t') = 0 \ \text{ for } \ t' > t \end{cases} \tag{4.65}$$

$$\begin{cases} f_{t_F}(-t') \ne 0 \text{ for } \ t' \le -t_1 \\ f_{t_F}(-t') = 0 \text{ for } t' > t_1 \end{cases} \tag{4.66}$$

where for $t > -t_1$, the integration limits are (-∞, -t) and for $t \le -t_1$ the integration limits are (-∞, t).

Insertion of the function $f_{tL}(t\text{-}t')$ and $f_{t'F}(\text{-}t)$ into Equation (4.64) yields

$$f_{\tau'_p}(t) = \begin{cases} \int_{-\infty}^{t_1} \lambda_L\ e^{\lambda_L(t-t')} \cdot \lambda\ e^{-\lambda(-t'-t_1)} dt' \ \text{ for } t \ge -t_1 \\ \int_{-\infty}^{t} \lambda_L\ e^{\lambda_L(t-t')} \cdot \lambda\ e^{-\lambda(-t'-t_1)} dt' \ \text{ for } t < -t_1 \end{cases} \tag{4.67}$$

and after the integration:

$$f_{\tau'_p}(t) = \begin{cases} \dfrac{\lambda\lambda_L}{\lambda + \lambda_L} e^{-\lambda_L(t+t_1)} \ \text{for } t \ge -t_1 \\ \dfrac{\lambda\lambda_L}{\lambda + \lambda_L} e^{\lambda(t+t_1)} \ \text{for } t < -t_1 \end{cases} \tag{4.68}$$

Thus, the distribution function of τ_p is

$$F_{\tau_\rho}(t)=\frac{P\left(-t_1 \le \tau'_\rho \le t-t_1\right)}{P\left(\tau'_\rho \ge -t_1\right)}=\frac{\int_{-t_1}^{t-t_1}\frac{\lambda\lambda_L}{\lambda+\lambda_L}e^{-\lambda_L(t'+t_1)}\,dt'}{\int_{-t_1}^{\infty}\frac{\lambda\lambda_L}{\lambda+\lambda_L}e^{-\lambda_L(t'+t_1)}dt'} \tag{4.69}$$

and after the integration:

$$F_{\tau_\rho}(t)=1-e^{-\lambda_L t}. \tag{4.70}$$

The pdf of τ_p is

$$f_{\tau_\rho}(t)=\frac{dF_{\tau_\rho}(t)}{dt}=\lambda_L\ e^{-\lambda_L t} \tag{4.71}$$

Having the pdf and distribution functions of τ_p and t_R the pdf for the overlapping time t_{ovp} can be obtained using Equation (4.54):

$$\begin{aligned} f_{t_{ovp}}(t) &= f_{t_R}(t)\cdot[1-F_{\tau_\rho}(t)]+f_{\tau_\rho}(t)\cdot[1-F_{t_R}(t)] \\ &= \mu\ e^{-\mu t}[1-\left(1-e^{-\lambda_L t}\right)]+\lambda_L\ e^{-\lambda_L t}[1-\left(1-e^{-\mu t}\right)] \\ &= \left(\mu+\lambda_L\right)e^{-(\mu+\lambda_L)\ t} \end{aligned} \tag{4.72}$$

The mean overlapping time T_{ovp} for a failure occurrence at peak load is

$$T_{ovp}=\int_0^{\infty}t\cdot\left(\mu+\lambda_L\right)e^{-(\mu+\lambda_L)\ t}\ dt=\frac{1}{\mu+\lambda_L} \tag{4.73}$$

Insertion of $\mu=1/T_R$ and $\lambda_L=1/T_L$ yields

$$T_{ovp}=\frac{T_R T_L}{T_R+T_L} \tag{4.74}$$

The pdf of t_{ovl} is determined in a similar way (see Fig. 4.15). Since the overlapping exists from the moment t_1 that the peak load starts, the distribution function of τ_0 can be determined based on t_R as follows

$$F_{\tau_0}(t)=P(\tau_0\le t)=P(t_R-t_1\le t\mid t_R\ge t_1). \tag{4.75}$$

Using the conditional probability theorem we get

$$F_{\tau_0}(t)=\frac{P(t_R\le t+t_1\cap t_R\ge t_1)}{P(t_R\ge t_1)}=\frac{P(t_1\le t_R\le t+t_1)}{P(t_R\ge t_1)}. \tag{4.76}$$

For exponentially distributed t_R

$$F_{\tau_0}(t) = \frac{\int_{t_1}^{t+t_1} \mu\ e^{-\mu t'}\ dt'}{\int_{t_1}^{\infty} \mu\ e^{-\mu t'}\ dt'} \tag{4.77}$$

and after the integration:

$$F_{\mu_0}(t) = 1 - e^{-\mu t}\ . \tag{4.78}$$

Using the expression given by Equation (4.54) we get the pdf of t_{ovl} as

$$\begin{aligned} f_{t_{ovl}}(t) &= \lambda_L\ e^{-\lambda_L t}[1-(1-e^{-\mu t})] + \mu e^{-\mu t}[1-(1-e^{-\mu t})] \\ &= (\mu + \lambda_L)e^{-(\mu+\lambda_L)\ t} \end{aligned} \tag{4.79}$$

The mean overlapping time T_{ovl} for a failure occurring at off-peak load and continuing into peak load is

$$T_{ovl} = \int_0^{\infty} t \cdot f_{t_{ovl}}(t)dt = \frac{1}{\mu + \lambda_L} = \frac{T_R T_L}{T_R + T_L}\ . \tag{4.80}$$

The identical results for T_{ovp} and T_{ovl} are expected as it was assumed that the stochastic variables were exponentially distributed.

The pdf of the temperatures $f_{\theta hp}(\theta)$ and $f_{\theta hl}(\theta)$ can be determined by using the pdf of $f_{tovp}(t)$ and $f_{tovl}(t)$ and θ_h as a function of these overlapping times, see Equation (4.51). As the functions for both overlapping times are identical, we use the same notation t_{ov} for the both cases.

The pdf for t_{ov} is

$$f_{t_{ov}}(t_{ov}) = (\mu + \lambda_L)e^{-(\mu+\lambda_L)t_{ov}} \tag{4.81}$$

Then using Equation (4.51) we get

$$t_{ov} = -T\ \ln\frac{\theta_h - \beta}{\alpha} = \ln\left(\frac{\theta_h - \beta}{\alpha}\right)^{-T}\ . \tag{4.82}$$

Therefore

$$\frac{d\theta_h}{dt_{ov}} = -\frac{\alpha}{T}e^{-t_{ov}/T} \tag{4.83}$$

Insertion of t_{ov} into this equation yields

$$\frac{d\theta_h}{dt_{ov}} = -\frac{\alpha}{T}\exp\left(-\frac{1}{T}\ln\left(\frac{\theta_h \beta}{\alpha}\right)^{-T}\right) = -\frac{\alpha}{T}\exp^{\ln\left(\frac{\theta_h-\beta}{\alpha}\right)} = -\frac{\theta_h-\beta}{T}$$

$$\text{or } \left|\frac{d\theta_h}{dt_{ov}}\right| = \frac{\beta-\theta_h}{T} \tag{4.84}$$

From Equations (4.81) and (4.82) we get

$$f_{t_{ov}}(\theta_h) = (\mu+\lambda_L)e^{-(\mu+\lambda_L)\cdot\ell n\left(\frac{\theta_h-\beta}{\alpha}\right)^{-T}} = (\mu+\lambda_L)\left(\frac{\theta_h-\beta}{\alpha}\right)^{(\mu+\lambda_L)T} \tag{4.85}$$

which together with Equations (4.52) and (4.84) gives

$$f_{\theta_h}(\theta_h) = \frac{(\mu+\lambda_L)\left(\frac{\theta_h-\beta}{\alpha}\right)^{(\mu+\lambda_L)T}}{\left(\frac{\beta-\theta_h}{T}\right)}. \tag{4.86}$$

Using the definition of T_R and T_L we get

$$\mu+\lambda_L = \frac{1}{T_R}+\frac{1}{T_L} = \frac{T_R+T_L}{T_R\cdot T_L} = \frac{1}{T_{ov}} \tag{4.87}$$

where T_{ov} is the mean overlapping time.

As $\alpha < 0$ and $\theta_h < \beta$, then

$$f_{\theta_h}(\theta_h) = \frac{T}{T_{ov}|\alpha|^{T/T_{ov}}}(\beta-\theta_h)^{(T/T_{ov})-1} \quad \text{for } \alpha+\beta \le \theta_h < \beta \tag{4.88}$$

Let ξ be the ratio of the thermal time constant of the transformer to the mean overlapping time $\xi = \frac{T}{T_{ov}}$. Thus

$$f_{\theta_h}(\theta_h) = \frac{\xi}{|\alpha|^{\xi}}(\beta-\theta_h)^{\xi-1} \quad \text{for } \alpha+\beta \le \theta_h < \beta \tag{4.89}$$

It should be noted that this function has the characteristics of a pdf:

$f_{\theta h}(\theta_h) \ge 0$ and

$$\int_{-\infty}^{+\infty} f_{\theta_h}(\theta_h)d\theta_h = \frac{\xi}{|\alpha|^{\xi}}\int_{\alpha+\beta}^{\beta}(\beta-\theta_h)^{\xi-1}d\theta_h = 1. \tag{4.90}$$

We should study the pdf, $f_{\theta h}(\theta_h)$ for three possible values of ξ:

1. $\xi = 1$:

$$f_{\theta_h}(\theta_h) = \frac{1}{|\alpha|^{\xi}} \text{ for } \alpha+\beta \le \theta_h \le \beta \tag{4.91}$$

2. $\xi > 1$:

$$\frac{df_{\theta_h}(\theta_h)}{d\theta_h} < 0 \text{ for } \quad \alpha+\beta \le \theta_h \le \beta$$

$$f_{\theta h}(\beta) = 0 \text{ and } f_{\theta_h}(\alpha+\beta) = \frac{\xi}{|\alpha|} \tag{4.92}$$

3. $\xi < 1$:

$$\frac{df_{h}(\theta_h)}{d\theta_h} > 0 \text{ for } \alpha+\beta \le \theta_h \le \beta$$

$$f_{\theta_h}(\alpha+\beta) = \frac{\xi}{|\alpha|} \quad \text{and} \quad f_{\theta_h}(\beta) \to \infty \tag{4.93}$$

The function $f_{\theta h}(\theta_h)$ is shown in Fig. 4.16.

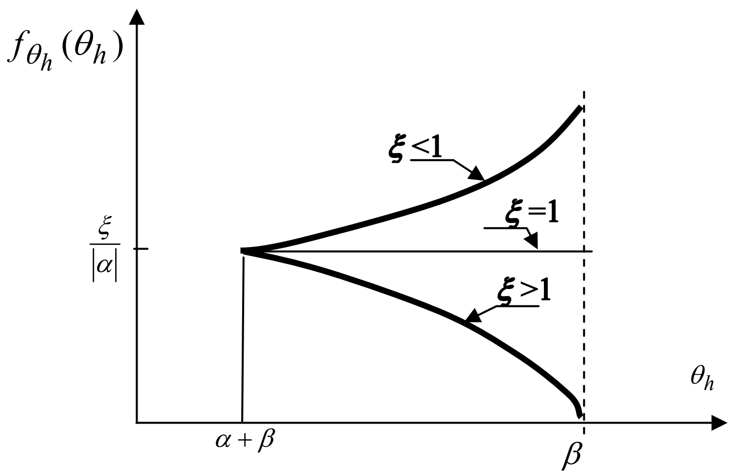

Fig. 4.16. The pdf of the hottest spot temperature under forced outage conditions

The mean temperature at failure conditions $\bar{\theta}_h$ can be calculated as

$$\bar{\theta}_h = \int_{-\infty}^{+\infty} \theta_h f_{\theta_h}(\theta_h) d\theta_h \tag{4.94}$$

Insertion of $f_{\theta h}(\theta_h)$ from Equation (4.89) yields:

$$\begin{aligned} \bar{\theta}_h &= \int_{\alpha+\beta}^{\beta} \theta_h \frac{\xi}{|\alpha|^{\xi}} (\beta - \theta_h)^{\xi-1} \, d\theta_h \\ &= -\frac{\xi}{|\alpha|^{\xi}} \left[\frac{\theta_h (\beta - \theta_h)^{\xi-1}}{\xi} + \frac{1}{\xi} \frac{(\beta - \theta_h)^{\xi+1}}{\xi+1} \right]_{\alpha+\beta}^{\beta} = \frac{\xi}{\xi+1}\alpha + \beta \end{aligned} \tag{4.95}$$

For ξ=0 $\bar{\theta} = \beta$. This means that theoretically if the thermal time constant of the transformer is small and close to zero, the temperature will reach the boundary value β, obtained at steady state. Then for ξ>>1, $\bar{\theta}_h \cong \alpha + \beta$. $\bar{\theta}_h$ as a function of ξ is given in Fig. 4.17.

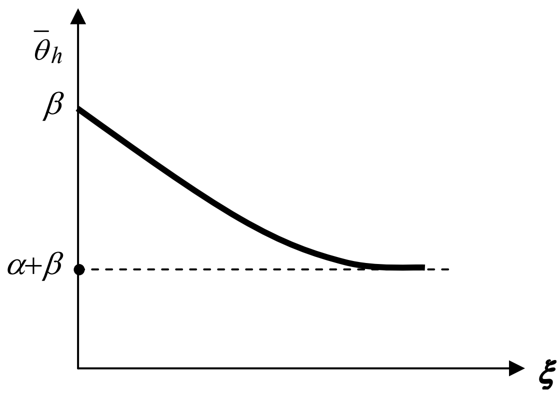

Fig. 4.17. The expected mean hottest spot temperature under forced outage conditions as a function of ξ

With the knowledge of the temperature pdf, the probability, $P(\theta_h > \theta^0_h)$ can be computed as

$$P\left(\theta_h > \theta_h^0\right) = \int_{\theta_h^0}^{\beta} f_{\theta_h}(\theta_h) d\theta_h$$

$$= \frac{1}{|\alpha|^{\xi}} (\beta - \theta_h)^{\xi} \Bigg|_{\theta_{h0}}^{\beta} = \frac{1}{|\alpha|^{\xi}} \left(\beta - \theta_h^0\right)^{\xi} \tag{4.96}$$

and for a specific load curve *i*

$$P\left(\theta_h^i > \theta_h^0\right) = \frac{1}{\left|\alpha^i\right|^{\xi}} \left(\beta^i - \theta_h^0\right)^{\xi} \tag{4.97}$$

Insertion of Equation (4.97) into Equations (4.47) and (4.50) yields the expressions for determining the reliability indices for a forced outage of a transformer:

$$F_D\left(\theta_h > \theta_h^0\right) = \frac{\lambda}{\lambda T_R + 8760} \left[\sum_{i=1}^{n} \left(\begin{array}{l} T_R N_i \cdot \frac{1}{\left|\alpha_h^i\right|^{\xi}} \left(\beta_0^i - \theta_h^0\right)^{\xi} + \\ + T_L N_i \cdot \frac{1}{\left|\alpha_p^i\right|^{\xi}} \left(\beta_p^i - \theta_h^0\right)^{\xi} \end{array} \right) \right] \tag{4.98}$$

and

$$P_D\left(\theta_h > \theta_h^0\right) = \sum_{i=1}^{n} \left[f_i^I T_{ovl} \frac{1}{\left|\alpha_0^i\right|} \left(\beta_0^i - \theta_h^0\right)^{\xi} + f_i^{II} T_{ovp} \frac{1}{\left|\alpha_p^i\right|^{\xi}} \left(\beta_p^i - \theta_h^0\right)^{\xi} \right] \tag{4.99}$$

Calculating pdf of overlap time when the duration of load peak is deterministic

Assuming the time of the load peak is a constant and equals T_L, the cdf $F_{T_L}(t)$ could be expressed by using a step function when (see Fig. 4.18):

$$F_{T_L}(t) = u(t - T_L) \tag{4.100}$$

$$f_{T_L}(t) = \frac{dF_{T_L}(t)}{dt} = \delta(t - T_L) \tag{4.101}$$

where $\delta(t - T_L)$ is the derivative of the step function called “Dirk Function”.

In order to calculate the cdf of t_{ovp}, we refer to Fig. 4.19.

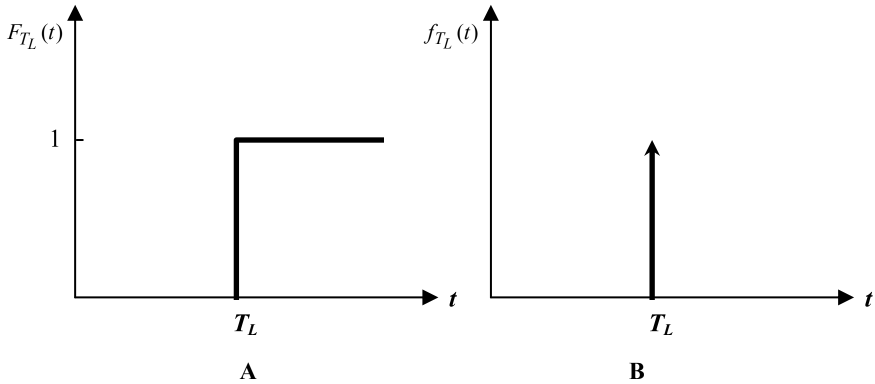

Fig. 4.18. cdf (A) and pdf (B) of a deterministic variable T_L

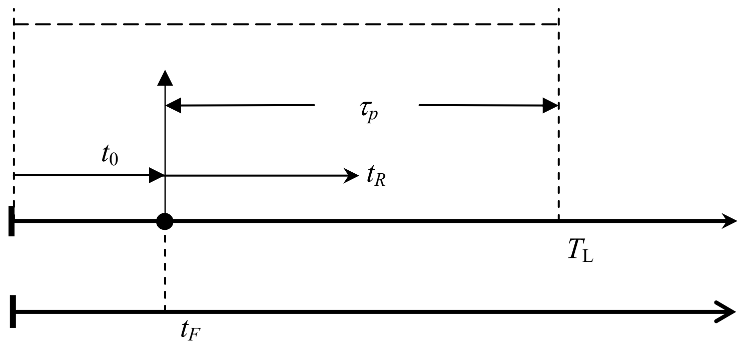

Fig. 4.19. Definition of the random variables in the case of a fault initiation at the peak load (T_L is the deterministic duration of the equivalent peak load).

Similar to the previous case of a fault initiation at the off-peak load cdf of τ can be defined by T_L and by the random variable t_F as follows

$$\begin{aligned} F_\tau(t) &= P(\tau \le t) = P(T_L - t_F \le t / 0 \le t_F \le T_L) \\ &= \frac{P(T_L - t_F \le t \cap t_F \ge 0 \cap T_L - t_F \ge 0)}{P(t_F > 0 \cap T_L - t_F \ge 0)} \\ &= \frac{P(0 \le T_L - t_F \le t \cap t_F \ge 0)}{P(T_L - t_F \ge 0 \cap t_F \ge 0)} \end{aligned} \tag{4.102}$$

Since $P(t_F \ge 0) = 1$, we obtain

$$F_{\tau}(t)=\frac{P(0\le T_L-t_F\le t)}{P(T_L-t_F\ge 0)} \tag{4.103}$$

Since the variables T_L and t_F are independent, the pdf of the $\tau^{'}=T_L-t_F$ increment will be achieved by

$$f_{\tau^{'}}(t)=\int_{-\infty}^{\infty} f_{T_L}(t-t^{'})\cdot f_{t_F}(-t^{'})dt^{'} \tag{4.104}$$

For the sake of convenience, by replace the integration variable t' with $\tau=-t^{'}$ and get

$$f_{\tau^{'}}(t)=\int_{-\infty}^{\infty} f_{T_L}(t+\tau)\cdot f_{t_F}(\tau)d\tau \tag{4.105}$$

The integration limit can be determined according to the limit definition for functions $f_{T_L}(t+\tau), f_{T_F}(\tau)$ as follows

$$f_{t_F}(\tau)=\begin{cases}\lambda e^{-\lambda t} & \tau\ge 0\\ 0 & \tau<0\end{cases} \tag{4.106}$$

$$f_{T_L}(t+\tau)=\delta[\tau-(T_L-t)]\,. \tag{4.107}$$

Condition (4.106) implies that

$$f_{\tau^{'}}(t)=\int_{0}^{\infty} \delta[\tau-(T_L-t)]\cdot\lambda e^{-\lambda\tau}dt \tag{4.108}$$

We will perform integration (4.108) using the following Dirk Function characteristic:

$$\int_{-\infty}^{\infty} g(t)\cdot\delta(t-T)dt=g(t-T) \tag{4.109}$$

For $T_L-t\ge 0$ or $t\le T_L$ the result will be:

$$f_{\tau'}(t) = \int_0^{\infty} \delta[\tau - (T_L - t)] \cdot \lambda e^{-\lambda\tau} d\tau = \lambda e^{-\lambda(T_L - t)} . \tag{4.110}$$

For $T_L - t < 0$ or $t > T_L$ the result is

$$f_{\tau'}(t) = 0 \tag{4.111}$$

Therefore:

$$f_{\tau'}(\tau) = \begin{cases} \lambda e^{-\lambda(T_L - t)} & t \le T_L \\ 0 & t > T_L \end{cases} \tag{4.112}$$

Based on equation (4.103) we obtain the cdf of τ_p as follows

$$F_{\tau_p}(t) = \frac{\int_0^t \lambda e^{-\lambda(T_L - t')} dt'}{\int_0^{T_L} \lambda e^{-\lambda(T_L - t')} dt'} \tag{4.113}$$

After performing the integration the result takes the form

$$F_{\tau_p}(t) = \begin{cases} \dfrac{e^{\lambda t} - 1}{e^{\lambda T_L} - 1} & 0 \le t \le T_L \\ 1 & t > T_L \end{cases} \tag{4.114}$$

The pdf of τ_p is then

$$f_{\tau_p}(t) = \frac{dF_{\tau_p}(t)}{dt} = \begin{cases} \dfrac{\lambda e^{\lambda t}}{e^{\lambda T_L} - 1} & 0 \le t \le T_L \\ 0 & t > T_L \end{cases} \tag{4.115}$$

Usually $\lambda T_L >> 1$, $\lambda t << 1$, ($t < T_L$). Thus, it is possible to approximate the functions $F_{\tau_p}(t), f_{\tau_p}(t)$ as follows

$$f_{\tau_p}(t) \cong \frac{\lambda \cdot 1}{1+\lambda T_L - 1} = \frac{1}{T_L} \qquad 0 \le t \le T_L$$
$$F_{\tau_p}(t) \cong \frac{\lambda t + 1 - 1}{1+\lambda T_L - 1} = \frac{t}{T_L} \qquad 0 \le t \le T_L \tag{4.116}$$

meaning a homogenous function.

Having the pdf and the cdf of τ_p and t_R we can obtain the pdf $f_{t_{ovp}}(t)$ as follows (see equation (4.54))

$$f_{t_{ovp}}(t) = \begin{cases} \mu e^{-\mu t}(1 - \frac{e^{\lambda t}-1}{e^{\lambda T_L}-1} + \frac{\lambda e^{\lambda t}}{e^{\lambda T_L}-1}) \cdot [1-(1-e^{-\mu t})] & 0 \le t \le T_L \\ 0 & t > T_L \end{cases} \tag{4.117}$$

And based on the arranging the terms Equation (4.117) takes the form

$$f_{t_{ovp}}(t) = \begin{cases} \frac{1}{e^{\lambda T_L}-1}[(\lambda-\mu)e^{(\lambda-\mu)t} + \mu e^{\lambda T_L} \cdot e^{-\mu t}] & 0 \le t \le T_L \\ 0 & t > T_L \end{cases} \tag{4.118}$$

And based on the approximating (4.116) we get

$$f_{t_{ovp}}(t) = \begin{cases} \mu e^{-\mu t}(1 - \frac{t}{T_L}) + \frac{1}{T_L} e^{-\mu t} & 0 \le t \le T_L \\ 0 & t > T_L \end{cases} \tag{4.119}$$

As a result, the average overlap time takes the form

$$T_{ovp} = \int_0^{T_L} t[\mu e^{-\mu t}(1 - \frac{t}{T_L}) + \frac{1}{T_L} + e^{-\mu t}]dt \tag{4.120}$$

After performing the integration in (4.120), and substitution of μ with $\frac{1}{T_R}$, we get

$$T_{ovp} = T_R[1 + \frac{T_R}{T_L}(e^{-T_L/T_R} - 1)] \tag{4.121}$$

In order to calculate a fault initiating during the daily off-peak load, and which duration's in regards to repair time lasts until the load peak, we will use Fig. 4.15.

Similar to the exponential distribution function, the distribution function for τ_0 is given by:

$$F_{\tau_0} = 1 - e^{-\mu t}$$
$$f_{\tau_0}(t) = \mu e^{-\mu t} \tag{4.122}$$

By using Equation (4.54) we get the pdf of the overlap time with regards to a fault initiated during the off-peak load according to

$$f_{t_{ovl}}(t) = \mu e^{-\mu t}[1 - u \cdot (t - T_L)] + \delta(t - T_L) \cdot e^{-\mu t} \tag{4.123}$$

where $u(t-T_L), \delta(t-T_L)$ are the pdf and the cdf for the deterministic variable respectively. Meaning

$$f_{t_{ovl}}(t) = \begin{cases} \mu \cdot e^{-\mu t} + e^{-\mu t} \cdot \delta(t - T_L) & t \le T_L \\ 0 & t > T_L \end{cases} \tag{4.124}$$

The average overlap time will be resulted by:

$$T_{ovl} = \int_0^{T_L} t[\mu e^{-\mu t} + e^{-\mu t}\delta(t - T_L)]dt$$
$$= \int_0^{T_L} t\mu e^{-\mu t} dt + \int_0^{T_L} te^{-\mu t}\delta(t - T_L)dt == -e^{-\mu t}(t + \frac{1}{\mu})\Big|_0^{T_L} + T_L e^{-\mu T_L} \tag{4.125}$$

After substituting with the integration limits the result will be:

$$T_{ovl} = T_R[1 - e^{-T_L/T_R}] \tag{4.126}$$

4.1.5 Estimation of instant of failure within a certain time interval

It is often important to estimate the moment of failure occurrence. Such cases are, for example, the study of the temperature of the transformers under fault conditions.

It should be pointed out that the main interest here is to evaluate an expectation of a failure occurrence within a predetermined time interval. The distribution function of the time T_f between two failures is assumed to be exponential, i.e. the probability density function

$$f_{T_f}(t) = \lambda e^{-\lambda t}, \tag{4.127}$$

where λ is the failure rate.

Assume that the study is done for a failure occurring during the peak level of the daily load curve, which duration is also a random variable denoted by T_L. We designate as T_0 the random time from the peak load beginning up to the instant of beginning of failure occurrence under condition that the failure will occur during peak load (see Fig. 4.20).

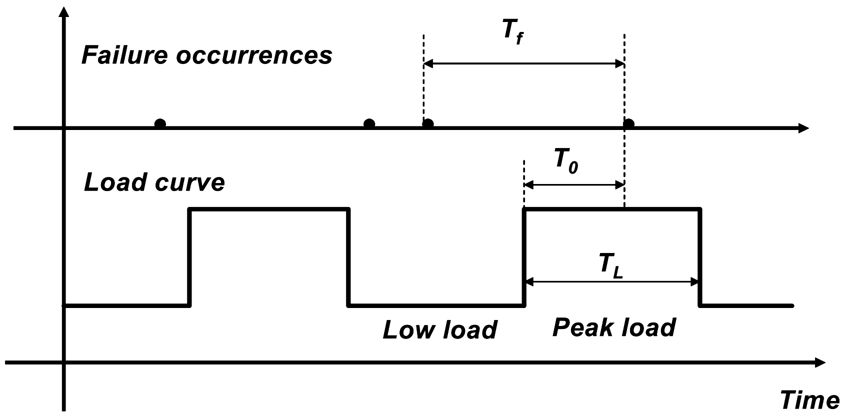

Fig. 4.20. Definitions of the random variables

The problem is to determine the expectation of random variable T_0 for the given condition that the failure occurs within random time interval T_L. The computation of $E(T_0)$ is based on the condition expectation

$$E(T_0) = E[E(T_0 / T_L)], \tag{4.128}$$

where $E(T_0 / T_L)$ is the (conditional) expectation of T_0 under condition that the failure will occur during peak load duration T_L. It can be written

$$E(T_0 / T_L) = \int_{-\infty}^{+\infty} T_0 f_{T_0/T_L}(T_0 / T_L) dT_0, \tag{4.129}$$

where $f_{T_0/T_L}(T_0 / T_L)$ is the conditional pdf. of T_0 for a given value of T_L.

From expressions (4.128) and (4.129) it is obtained:

$$E[E(T_0 / T_L) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} T_0 f_{T_0/T_L}(T_0 / T_L) dT_L dT_0 \, , \tag{4.130}$$

In order to find the solution by using Equation (4.130), the conditional pdf $f_{T_0/T_L}(T_0 / T_L)$ should be studied first. This function is obtained from the distribution function $F_{T_0/T_L}(t)$. Under the assumption that repair time is negligibly small compared to the time between failures, the distribution $F_{T_0/T_L}(t)$ can be defined on the basis of the stochastic variables T_f and T_L, by the following manner:

$$F_{T_0/T_L}(t) = \Pr\{T_f \le t / 0 \le T_f < T_L\} \tag{4.131}$$

This means that $F_{T_0/T_L}(t)$ is equal to the probability that the stochastic variable T_f is equal or smaller than a certain value of t, when the instant of failure occurrence is between 0 and T_L, which is a random variable too.

By using the conditional probability theorem we obtain

$$F_{T_0/T_L}(t) = \frac{\Pr\{T_f \le t \cap 0 \le T_f < T_L\}}{\Pr\{0 \le T_f < T_L\}} \tag{4.132}$$

For $t > T_L$ $\{T_f \le t \cap 0 \le T_f < T_L\} = \{0 \le T_f < T_L\}$, hence

$$F_{T_0/T_L}(t) = \frac{\Pr\{0 \le T_f < T_L\}}{\Pr\{0 \le T_f < T_L\}} = 1 \tag{4.133}$$

For $t \le T_L$ $\{T_f \le t \cap 0 \le T_f < T_L\} = \{0 \le T_f < t\}$, hence

$$F_{T_0/T_L}(t) = \frac{\Pr\{0 \le T_f < t\}}{\Pr\{0 \le T_f < T_L\}} = \frac{\int_0^t f_{T_f}(x)dx}{\int_0^{T_L} f_{T_f}(x)dx}$$
$$= \frac{\int_0^t \lambda e^{-\lambda x} dx}{\int_0^{T_L} \lambda e^{-\lambda x} dx} = \frac{1 - e^{-\lambda t}}{1 - e^{-\lambda T_L}}, \quad for\ T_L \ne 0. \tag{4.134}$$

Therefore, the distribution function $F_{T_0/T_L}(t)$ may be obtained as

$$F_{T_0/T_L}(t)=\begin{Bmatrix}\dfrac{1-e^{-\lambda t}}{1-e^{-\lambda T_L}}, & for\ t\le T_L, \\ 1\ \ for\ t>T_L. & \end{Bmatrix} \tag{4.135}$$

The conditional pdf is obtained by differentiating:

$$f_{T_0/T_L}=\frac{d}{dt}[F_{T_0/T_L}(t)]=\begin{Bmatrix}\dfrac{\lambda e^{-\lambda t}}{1-e^{-\lambda T_L}}, & for\ t\le T_L, \\ 0\ \ for\ t>T_L. & \end{Bmatrix} \tag{4.136}$$

Substitution of Equation (4.136) into Equation (4.129) yields:

$$E(T_0/T_L)=\int_0^{T_L}\frac{te^{-\lambda t}}{1-e^{-\lambda T_L}}dt=\frac{1}{\lambda}[1-\frac{\lambda T_L e^{-\lambda T_L}}{1-e^{-\lambda T_L}}]. \tag{4.137}$$

$E(T_0)$ can now be obtained by using equations (4.128) or (4.130)

$$E(T_0)=\int_{-\infty}^{+\infty}E(T_0/T_L)f_{T_L}(x)dx\,. \tag{4.138}$$

$E(T_0)$ will be calculated for two cases. The first case is when T_L is exponentially distributed the second one when T_L is deterministic.

Case 1. T_L is exponentially distributed. In this case, equation (4.138) takes the form:

$$E(T_0)=\int_0^{\infty}\frac{1}{\lambda}[1-\frac{\lambda te^{-\lambda T_L}}{1-e^{-\lambda T_L}}]\lambda_L e^{-\lambda_L T_L}dT_L\,. \tag{4.139}$$

where $\lambda_L=\dfrac{1}{\overline{T}_L}$ and $\overline{T}_L$ is the mean peak load duration time.

The integral (4.139) has no closed-form mathematical solution. Study the approximation of the following function:

$$g(T_L)=\frac{1}{\lambda}[1-\frac{\lambda te^{-\lambda T_L}}{1-e^{-\lambda T_L}}]=\frac{1}{\lambda}[1-\frac{\lambda T_L}{e^{-\lambda T_L}-1}] \tag{4.140}$$

In most practical cases $\lambda T_L<<1$, thus the exponential expressions can be developed into series:

$$g(T_L) = \frac{1}{\lambda}\left[1 - \frac{\lambda T_L}{e^{-\lambda T_L} - 1}\right] = \frac{1}{\lambda}\left[1 - \frac{\lambda T_L}{1 + \lambda T_L + \frac{\lambda^2 T_L^2}{2!} + \ldots - 1}\right]$$

$$= \frac{1}{\lambda}\left[1 - \frac{\lambda T_L}{\lambda T_L + \frac{\lambda^2 T_L^2}{2!} + \ldots}\right] = \frac{1}{\lambda}\left[1 - \frac{1}{1 + \frac{\lambda T_L}{2!} + \frac{\lambda^2 T_L^2}{3!} \ldots}\right]. \quad (4.141)$$

By neglecting terms of second order and higher, we obtain:

$$g(T_L) \cong \frac{1}{\lambda}\left[1 - \frac{1}{1 + \frac{\lambda T_L}{2}}\right] \cong \frac{1}{\lambda}\left[1 - (1 - \frac{\lambda T_L}{2}\}\right] = \frac{1}{\lambda}\frac{\lambda T_L}{2} = \frac{T_L}{2}. \quad (4.142)$$

The function $g(T_L)$ is not defined at zero and the integration limits are between 0 and ∞. Thus, it should be shown that

1. There is a final limit to the function at zero,
2. The function is bounded by the value $\frac{T_L}{2}$ for the whole integration interval.

First study the limit of the function at zero.

$$\lim_{T_L \to 0} g(T_L) = \lim_{T_L \to \infty}\left[\frac{1}{\lambda}(1 - \frac{\lambda T_L}{e^{\lambda T_L} - 1})\right] = \frac{1}{\lambda}\left[1 - \lim_{T_L \to \infty} \frac{\lambda T_L}{e^{\lambda T_L} - 1}\right]. \quad (4.143)$$

By using the L'Hospital rule, it can be obtained:

$$\lim_{T_L \to 0} g(T_L) = \frac{1}{\lambda}\left[1 - \lim_{T_L \to \infty} \frac{\lambda}{\lambda e^{\lambda T_L}}\right] = \frac{1}{\lambda}(1 - 1) = 0. \quad (4.144)$$

Now study the bounding of the function for the integration interval:

$$g(T_L) = \frac{1}{\lambda}\left[1 - \frac{\lambda T_L}{e^{-\lambda T_L} - 1}\right] = \frac{1}{\lambda}\frac{e^{\lambda T_L} - 1 - \lambda T_L}{e^{\lambda T_L} - 1}. \quad (4.145)$$

Represent the term $e^{\lambda T_L}$ as the series, then:

$$g(T_L) = \frac{1}{\lambda}[\frac{\sum_{n=0}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n - 1 - \lambda T_L}{\sum_{n=0}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n - 1}] = \frac{1}{\lambda}\frac{\sum_{n=2}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}{\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}$$

$$= \frac{1}{\lambda}\frac{\frac{\lambda T_L}{2}\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n + \sum_{n=2}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n - \frac{\lambda T_L}{2}\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}{\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}$$

$$= \frac{1}{\lambda}[\frac{\lambda T_L}{2} + \frac{\sum_{n=2}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n - \frac{\lambda T_L}{2}\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}{\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}$$

$$= \frac{1}{\lambda}\left[\frac{\lambda T_L}{2} + \frac{\sum_{n=3}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n - \frac{\lambda T_L}{2}\sum_{n=2}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}{\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}\right] \tag{4.146}$$

$$= \frac{1}{\lambda}\left[\frac{\lambda T_L}{2} + \left(\sum_{n=3}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n - \sum_{n=2}^{\infty}\frac{\left(\frac{\lambda T_L}{n!}\right)^{n-1}}{\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}\right)\right]$$

$$= \frac{1}{\lambda}\left[\frac{\lambda T_L}{2} + \frac{\sum_{n=3}^{\infty}\left(\frac{1}{n!} - \frac{1}{2(n-1)!}\right)(\lambda T_L)^n}{\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}\right]$$

$$= \frac{1}{\lambda}\left[\frac{\lambda T_L}{2} + \frac{\sum_{n=3}^{\infty}\frac{1}{(n-1)!}\left(\frac{1}{n}-\frac{1}{2}\right)(\lambda T_L)^n}{\sum_{n=1}^{\infty}\left(\frac{\lambda T_L}{n!}\right)^n}\right].$$

This expression can be written as

$$g(T_L) = \frac{T_L}{2} + \frac{1}{\lambda}\psi(T_L), \tag{4.147}$$

where

$$\psi(T_L) = \frac{\sum_{n=3}^{\infty}\frac{1}{(n-1)!}(\frac{1}{n}-\frac{1}{2})(\lambda T_L)^n}{\sum_{n=1}^{\infty}(\frac{\lambda T_L}{n!})^n}. \tag{4.148}$$

For $n \geq 3$, $\psi(T_L) < 0$, as: $\frac{1}{n} - \frac{1}{2} < 0$, for $n \geq 3$ and $\lambda > 0$, $T_L > 0$. Thus:

$$g(T_L) = \frac{T_L}{2} + \frac{1}{\lambda}\psi(T_L) \leq \frac{T_L}{2}. \tag{4.149}$$

The calculations of $g(T_L)$ for various values of T_L and λ were done and the results are represented for a practical value $\lambda = 0.126$ $year^{-1}$, in Table 4.1.

As it can be observed from Table 4.1:

$$g(T_L) \cong \frac{T_L}{2} \tag{4.150}$$

throughout the integration interval.

Substitution of Equation (4.150) into Equation (4.140) and then into Equation (4.139), yields:

$$E(T_0) \cong \int_0^{\infty} \frac{T_L}{2}\lambda_L e^{-\lambda_L T_L}\, dT_L = \frac{T_L}{2}. \tag{4.151}$$

The result given by Equation (4.151) is important for various reliability studies. For example, in the calculations of the temperature during a forced outage of a transformer, it may be assumed that the failure occurs after time $T_0 = \dfrac{T_L}{2}$ from the peak load beginning.

Table 4.1. Function g(TL) for λ=0.126 year^{-1}

T_L	$g(T_L)$	T_L	$g(T_L)$	T_L	$g(T_L)$	T_L	$g(T_L)$
0.5	0.3	12.5	6.3	24.5	12.3	260.0	129.9
1.0	0.5	13.0	6.5	30.0	15.0	270.0	134.9
1.5	0.8	13.5	6.8	40.0	20.0	280.0	139.9
2.0	1.0	14.0	7.0	50.0	25.0	290.0	144.9
2.5	1.3	14.5	7.3	60.0	30.0	300.0	149.9
3.0	1.5	15.0	7.5	70.0	35.0	310.0	154.9
3.5	1.8	15.5	7.8	80.0	40.0	320.0	159.9
4.0	2.0	16.0	8.0	90.0	45.0	330.0	164.9
4.5	2.3	16.5	8.3	100.0	50.0	340.0	169.9
5.0	2.5	17.0	8.5	110.0	55.0	350.0	174.9
5.5	2.8	17.5	8.8	120.0	60.0	360.0	179.8
6.0	3.0	18.0	9.0	130.0	65.0	370.0	184.8
6.5	3.3	18.5	9.3	140.0	70.0	380.0	189.8
7.0	3.5	19.0	9.5	150.0	75.0	390.0	194.8
7.5	3.8	19.5	9.8	160.0	80.0	400.0	199.8
8.0	4.0	20.0	10.0	170.0	85.0	410.0	204.8
8.5	4.3	20.5	10.3	180.0	90.0	420.0	209.8
9.0	4.5	21.0	10.5	190.0	95.0	430.0	214.8
9.5	4.8	21.5	10.8	200.0	999.5	440.0	219.8
10.0	5.0	22.0	11.0	210.0	104.9	450.0	224.8
10.5	5.3	22.5	11.3	220.0	109.9	460.0	229.7
11.0	5.6	23.0	11.5	230.0	114.9	470.0	234.7
11.5	5.8	23.5	11.8	240.0	119.9	480.0	239.7
12.0	6.0	24.0	12.0	250.0	124.9	490.0	244.7
						500.0	249.7

Case 2. T_L is deterministic. The cumulative distribution function of a deterministic variable, such as $\overline{T}_L$ can be expressed by the following step function

$$F_{T_L}(t) = \begin{cases} 1 & if\ t \ge \overline{T}_L, \\ 0 & if\ t < \overline{T}_L. \end{cases} \tag{4.152}$$

The corresponding pdf can be represented as a Dirac function:

$$f_{T_L}(t) = \frac{dF_{T_L}(t)}{dt} = \delta(t - \overline{T}_L)\,. \tag{4.153}$$

In a similar manner to the one described in the previous Paragraph, $E(T_0 / T_L)$ can be written as:

$$E(T_0 / T_L) = g(T_L) = \frac{1}{\lambda}[1 - \frac{\lambda T_L}{e^{\lambda T_L} - 1}]. \tag{4.154}$$

Equations (4.138) and (4.154) yield:

$$\begin{aligned} E(T_0) &= \int_{-\infty}^{+\infty} \frac{1}{\lambda}[1 - \frac{\lambda T_L}{e^{\lambda T_L} - 1}]\delta(T_L - \overline{T}_L) \\ &= g(T_L) = \frac{1}{\lambda}[1 - \frac{\lambda T_L}{e^{\lambda T_L} - 1}] \cong \frac{T_L}{2}. \end{aligned} \tag{4.155}$$

As it is observed, when the instant of failure within a certain time interval should be assessed, it can be assumed that it occurs in the middle of the time interval.

4.1.6 Determining the temperature rise

Transformer insulation deterioration (ageing) primarily depends on the winding hottest spot temperature. This calculation method, which is widely accepted in the literature, relays on standards' recommendations (IEC 60354/1991, IEEE Std. C57. 91-1995 and IEC 60076-7/2005).

The basic assumption which lies in the calculation presented by these references is that the temperature distribution along the winding is linear, as show in Fig. 4.21.

Following (IEC 60354/1991), the winding hot spot temperature θ_h is calculated as

$$\theta_h = \Delta\theta_b + H \cdot \Delta\theta_{wo} + \theta_a \tag{4.156}$$

Where $\Delta\theta_b$ is the top-oil temperature rise over ambient temperature, $\Delta\theta_{wo}$ is average winding temperature rise over average oil temperature, and θ_a is ambient temperature. H is the hot-spot factor. The hot-spot temperature is calculated using H=1.1 (IEC 60354/1991), and according to new IEC standard (IEC 60076-7/2005) H=1.3.

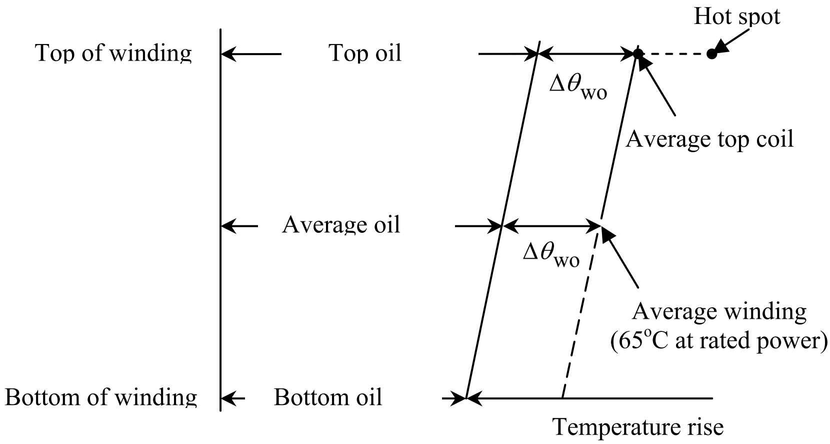

Fig. 4.21. Thermal diagram

The temperature rise indices $\Delta\theta_b$ and $\Delta\theta_{wo}$, under a constant load can be obtained (see IEC 60354/1991) as

$$\Delta\theta_b = \Delta\theta_{br}\left(\frac{1+dK^2}{1+d}\right)^x, \tag{4.157}$$

$$\Delta\theta_{wo} = \Delta\theta_{wor}\ K^y \tag{4.158}$$

Where $\Delta\theta_{br}$ is top-oil temperature rise at a rated power [°C], $\Delta\theta_{wor}$ is winding temperature rise over oil temperature at rated power [°C], d is ratio of load losses at rated current to no-load losses, K is load factor (load current/rated current), x is the exponential power of total losses versus oil temperature rise, y is the exponential power of total losses versus winding temperature rise.

In transformers designed in accordance with the IEC guidelines:

- $\Delta\theta_{br}$=55°C, for free convection;
- $\Delta\theta_{br}$=40°C, for forced convection;
- $\Delta\theta_{wor}$=20°C, for ONAN (oil natural-air natural) ONAF (oil natural-air forced) cooling methods;
- {x = 0.9, y = 1.6}, for ONAN (oil natural-air natural), ONAF (oil natural-air forced), OFAF (oil forced-air forced) cooling methods;
- {x = 1, y = 2}, for and ODAF (oil direct-air forced) cooling methods;

Substituting $\Delta\theta_{br}$ and $\Delta\theta_{wo}$ from (4.157) and (4.158) into (4.156) yields the following expression for the hottest spot under constant load

$$\theta_h = \Delta\theta_{br}\left(\frac{1+dK^2}{1+d}\right)^x + 1.1\Delta\theta_{wor}(K)^y + \theta_a \tag{4.159}$$

Transient state (Fig. 4.22) oil temperature rise above ambient as a function of time, *t*, is calculated according to (IEC 60354/1991):

$$\Delta\theta_{bt} = \Delta\theta_{bo}\ e^{-t/\tau} + \Delta\theta_b\left(1-e^{-t/\tau}\right), \tag{4.160}$$

where $\Delta\theta_{bo}$ is initial oil temperature rise at $t = 0$, $\Delta\theta_b$ is ultimate oil temperature rise corresponding to the load applied, and τ is oil time constant.

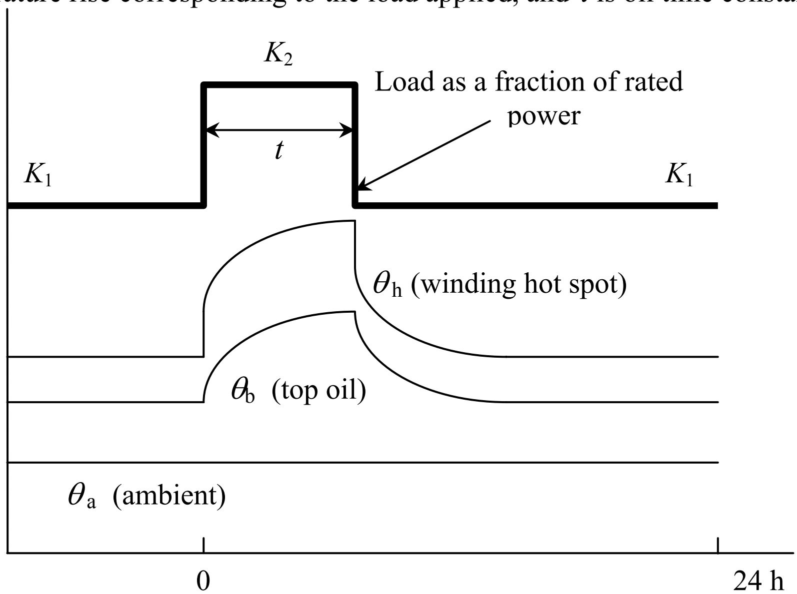

Fig. 4.22. Various temperatures under transient conditions

Since the time constant of the winding is significantly shorter compare to the oil's time constant, the coil's hottest spot temperature during transient is donated by:

$$\theta_h(t) = \Delta\theta_{bo}\ e^{-t/T} + \Delta\theta_b\left(1-e^{-t/T}\right) + H\cdot\Delta\theta_{wor}K^y + \theta_a \tag{4.161}$$

Hot-spot temperature calculation in case of an outage during peak load

After a forced outage of a transformer, the temperature of the remaining transformers in use is calculated based of the diagram, see Fig 4.23.

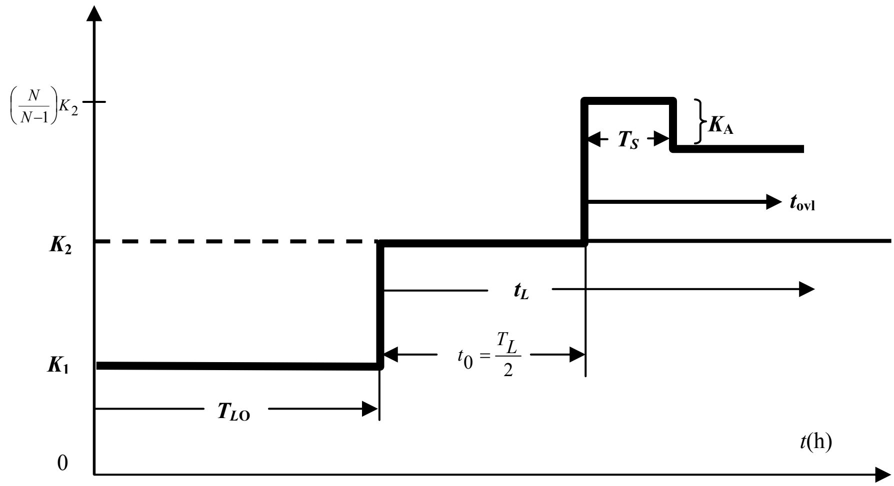

Fig. 4.23. Equivalent load cycle in the case an outage has initiated during high peak load power (K_1 is off-peak peak load power as a fraction of rated power [pu]; K_2 is peak load power as a fraction of rated power [pu]; K_A is load transferred factor to the nearby stations as a fraction of rated power [pu]; N is number of transformers in the station; T_L is peak load duration; T_{Lo} = 24 - T_L is off-peak peak load duration; t_0 = ½ T_L is time of outage occurrence, measured since peak load; t_{ovp} is overlap duration between peak load duration and repair duration.

The initial oil temperature rise, $\Delta\theta_{bo}$, at the beginning of the off-peak load is normally calculated based on to the equivalent load of the peak load. This is according to the load prior to the off-peak load, under the assumption that it is equal to the existing peak load. Thus, the initial oil temperature rise is

$$\Delta\theta_{bo} = \Delta\theta_{br}\left(\frac{1+dK_2^2}{1+d}\right)^x \tag{4.162}$$

The calculation requires defining the steady state oil temperature rise at the different load levels:

$$\Delta\theta_{\infty 1} = \Delta\theta_{br}\left(\frac{1+dK_1^2}{1+d}\right)^x, \tag{4.163}$$

$$\Delta\theta_{\infty 2} = \Delta\theta_{br}\left(\frac{1+dK_2^2}{1+d}\right)^x, \tag{4.164}$$

$$\Delta\theta_{\infty 3} = \Delta\theta_{br} \left[\frac{1 + d\left(\frac{N}{N-1} K_2 \right)^2}{1+d} \right]^x , \qquad (4.165)$$

$$\Delta\theta_{\infty 4} = \Delta\theta_{b} \left[\frac{1 + d\left(\frac{N}{N-1} K_2 - K_A \right)^2}{1+d} \right]^x , \qquad (4.166)$$

According to (4.160) the oil temperature rise at the beginning of the high peak load, $\Delta\theta_{b1}$, is

$$\Delta\theta_{b1} = \Delta\theta_{b0}\ e^{-T_{LO}/T} + \Delta\theta_{\infty 1}\left(1 - e^{-T_{LO}/T}\right). \qquad (4.167)$$

Similarly, the oil temperature rise at the occurrence of the outage, $\Delta\theta_{b2}$, and after partial load transfer (K_A) to nearby stations, $\Delta\theta_{b3}$, is

$$\Delta\theta_{b2} = \Delta\theta_{b1}\ e^{-T_L/2T} + \Delta\theta_{\infty 2}\left(1 - e^{-T_L/2T}\right) \qquad (4.168)$$

and

$$\Delta\theta_{b3} = \Delta\theta_{b2}\ e^{-T_S/2T} + \Delta\theta_{\infty 3}\left(1 - e^{-T_S/2T}\right) \qquad (4.169)$$

accordingly.

The hot spot temperature, as a function of the overlap time, is therefore

for $0 \le t_{\text{ovp}} \le T_S$:

$$\theta_h\left(t_{ovp}\right) = \Delta\theta_{b2} e^{-t_{ovp}/T} + \Delta\theta_{\infty 3}\left(1 - e^{-tovp/T}\right) + H \cdot \Delta\theta_{wor} \left(\frac{N}{N-1} K_2 \right)^y + \theta_a \qquad (4.170)$$

for $t_{\text{ovp}} > T_S$:

$$\theta_h\left(t_{ovp}\right) = \Delta\theta_{b3}\ e^{-\left(t_{ovp} - T_S\right)/T} + \Delta\theta_{\infty 4}\left(1 - e^{-\left(t_{ovp} - T_S\right)/T}\right)$$

$$+H \cdot \Delta\theta_{\mathrm{wor}}\left(\frac{N}{N-1}K_2 - K_A\right)^y + \theta_a \tag{4.171}$$

In the case of $K_A = 0$ the last expression reduces to

$$\theta_h(t_{ovp}) = \Delta\theta_{b2}\, e^{-t_{ovp}/T} + \Delta\theta_{\infty 3}\left(1 - e^{-t_{ovp}/T}\right) + H \cdot \Delta\theta_{wor}\left(\frac{N}{N-1}K_2\right)^y + \theta_a \tag{4.172}$$

Now one can define

$$\begin{cases} \alpha_p \triangleq \Delta\theta_{b2} - \Delta\theta_{\infty 3} \\ \beta_p \triangleq \Delta\theta_{\infty 3} + \mathrm{H} \cdot \Delta\theta_{wor}\left(\frac{N}{N-1}K_2\right)^y + \theta_a, \end{cases} \tag{4.173}$$

where α_p and β_p are constant for any given load diagram and a fixed number of transformers (actually, β_p is the hot spot steady state temperature, in the case that an outage starts during the high peak load and $\alpha_p < 0$ is temperature difference between the initial oil temperature at the time of outage occurrence and the steady state oil temperature during an outage).

Substituting (4.173) into (4.172) yields a simple expression for the hot spot temperature as a function of the overlap duration is

$$\theta_h(t_{ovp}) = \alpha_p e^{-t_{ovp}/T} + \beta_p \tag{4.174}$$

Hot-spot temperature calculation in the case when an outage starts during an off-peak peak load

Similarly to the previous section, after a removing a transformer out of the array, the temperature of the remaining transformers in use is calculated based on the load diagram in Fig. 4.24.

In addition to the oil temperature rise defined in (4.163)-(4.166), we have

$$\Delta\theta_{\infty 5} = \Delta\theta_{br}\left[\frac{1 + d\left(\frac{N}{N-1}K_1\right)^2}{1+d}\right]^x \tag{4.175}$$

$$\Delta\theta_{\infty 6} = \Delta\theta_{br}\left[\frac{1+d\left(\frac{N}{N-1}K_1 - K_A\right)^2}{1+d}\right]^x \tag{4.176}$$

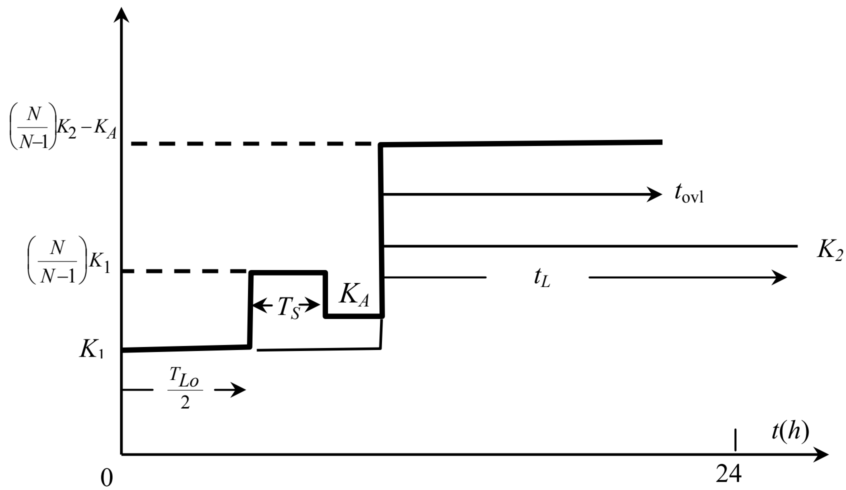

Fig. 4.24. Equivalent load cycle in the case an outage starts during an equivalent low peak load. t_{ovl} is the overlap period between the load duration, t_L, and the repair duration, t_R. All other notations are identical to the notation in Fig. 4.23.

According to (4.160), the oil temperature rise at the beginning of an outage, during a off-peak load, is

$$\Delta\theta_{\infty 4} = \Delta\theta_{bo}\ e^{-T_{L0}/2T} + \Delta\theta_{\infty 1}\left(1 - e^{-T_{LO}/2T}\right) \tag{4.177}$$

Similarly, the oil temperature rise after partial load transfer (K_A) to nearby stations, $\Delta\theta_{b5}$, and at the beginning time of peak load), $\Delta\theta_{b6}$, are

$$\Delta\theta_{b5} = \Delta\theta_{b4}\ e^{-T_S/T} + \Delta\theta_{\infty 5}\left(1 - e^{-T_S/T}\right) \tag{4.178}$$

$$\Delta\theta_{b6} = \Delta\theta_{b5}\ e^{-\left(T_{LO}/2 - T_S\right)/T} + \Delta\theta_{\infty 6}\left[1 - e^{-\left(T_{LO}/2 - T_S\right)/T}\right] \tag{4.179}$$

respectively.

The hot spot temperature, as a function of the overlap time, t_{ovl}, is, therefore,

$$\theta_h(t_{ovr}) = \Delta\theta_{b6}\ e^{-t_{ov1}/T} + \Delta\theta_{\infty 4}\left(1 - e^{-tov1/T}\right) +$$

$$+ \mathrm{H} \cdot \Delta\theta_{wor}\left(\frac{N}{N-1}K_2 - K_A\right)^y + \theta_a \tag{4.180}$$

Let us define α_0 and β_0 as follows

$$\alpha_o \stackrel{\Delta}{=} \Delta\theta_{b6} - \Delta\theta_{\infty 4}\,, \tag{4.181}$$

$$\beta_o \stackrel{\Delta}{=} \Delta\theta_{\infty 4} + H \cdot \Delta\theta_{wor}\left(\frac{N}{N-1}K_2 - K_A\right)^y + \theta_a\,. \tag{4.182}$$

It can be seen that α_0 and β_0 are constants for any given load diagram and a fixed number of transformers. Actually, β_0 is hot spot steady state temperature, in the case that an outage starts during low peak, and α_0 is oil temperature difference between the initial state at the time of swell (the high peak load begins) to the steady state oil temperature during an outage.

Substituting (4.181) and (4.182) into (4.180) yields a simple expression for the hot spot temperature as a function of the overlap duration, t_{ovl},

$$\theta_h(t_{ov1}) = \alpha_o\ e^{-t_{ov1}/T} + \beta_o \tag{4.183}$$

Hot-spot temperature calculation using a Monte-Carlo simulation

In order to calculate the hot-spot temperature of the transformers, which remain in use after the occurrence of an outage, the actual load diagram is replaced by a set of step functions simplification, as show in Fig. 4.25. In practice, time line is divided into equal intervals, Δt = 0.5h, from the moment an outage occurs, t_0, until the repair is done, t_{Rj}, assuming that the load is constant along each interval and equal to the effective load during that time.

The oil temperature rise over ambient temperature at the end of interval j, $\Delta\theta_j$, is obtained by using (4.160)

$$\Delta\theta_j = \Delta\theta_{\infty j}\left(1 - e^{-\Delta t/T}\right) + \Delta\theta_{j-1}\ e^{-\Delta t/T} \qquad j = 1,2,\ldots,q, \tag{4.184}$$

where $\Delta\theta_{\infty \mathrm{j}}$ is oil temperature rise at steady state and constant load S_j, $\Delta\theta_{j\text{-}1}$ is oil temperature rise at the beginning of interval j (or the end of the previous interval, j-1).

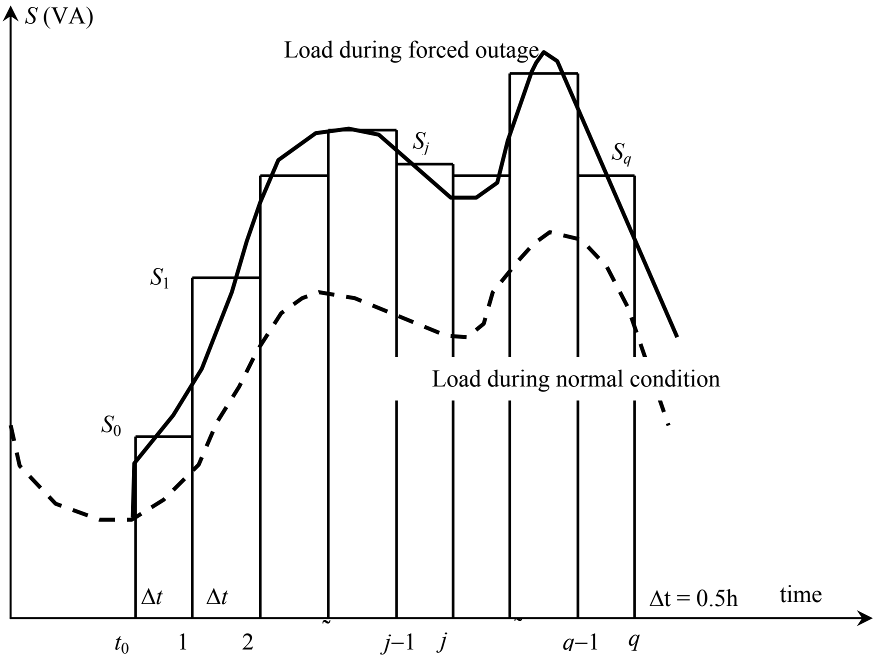

Fig. 4.25. Alternating the load diagram in case of an outage using a step functions curve approximation (q is the number of intervals on the time line; S_j is the constant load level in the interval (j, j-1) during an outage; t_0 is the outage time of occurrence).

An expression for $\Delta\theta_{\infty j}$ is obtained using (4.157)

$$\Delta\theta_{\infty j} = \Delta\theta_{br} \left(\frac{1 + dK_j^2}{1 + d} \right)^x \tag{4.185}$$

where K_j indicates the normalized load based on the transformer rated power. During a transformer outage this factor is modified as follows

$$K_j = \left(\frac{N}{N-1} \right) \frac{S_j}{K_R S_{py}}, \tag{4.186}$$

where K_R is transforming reserve factor, S_{py} is annual peak load, N is number of transformers in the substation.

Since the time interval Δt is constant, substituting $\delta = e^{-\Delta t/T}$ into (4.184) yields

$$\Delta\theta_j = \Delta\theta_{\infty j}\,(1-\delta) + \Delta\theta_{j-1}\delta. \tag{4.187}$$

Applying Equation (4.187) on each interval, combined with the assumption that the initial oil temperature rise (at the time t_0, when the outage occurs) is $\Delta\theta_0$, yields a set of equations

$$\Delta\theta_1 = \Delta\theta_{\infty 1}\ (1-\delta) + \Delta\theta_o \cdot \delta\,,$$

$$\Delta\theta_2 = \Delta\theta_{\infty 2}\ (1-\delta) + \Delta\theta_1\delta = (1-\delta)\left(\Delta\theta_{\infty 2} + \delta\Delta\theta_{\infty 2} + \delta^2\Delta\theta_{\mathrm{o}}\right),$$

$$\dots$$

$$\Delta\theta_j = (1-\delta)\left[\Delta\theta_{\infty j} + \delta\Delta\theta_{\infty_{\,j-1}}\delta^2\Delta\theta_{\infty_{\,j-2}}\dots + \delta^{j-1}\Delta\theta_{\infty 1}\right] + \delta^j\Delta\theta_o\,. \qquad (4.188)$$

This set of equations enables one to calculate the oil temperature rise at the end of each interval, j, in terms of the steady state temperature of all previous intervals and the oil temperature rise at the time of outage.

The initial temperature rise, $\Delta\theta_0$, is determined by the transformers loading prior to the outage occurrence, using the prior daily load diagram. The calculation takes a few iterations to converge, till the temperature at the beginning of the load interval and at the end of the load interval equaled. An example for such a calculation is given in Fig 4.26.

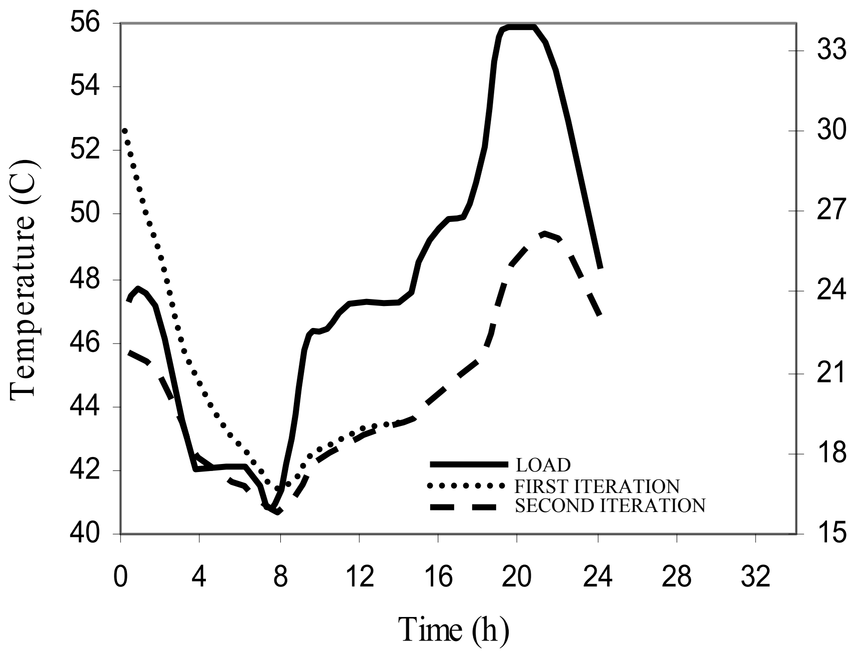

Fig. 4.26. An example of the initial temperature calculation

The hot spot temperature at the end of each interval is calculated by modifying (4.103) as follows:

$$\theta_{h_j} = \Delta\theta_j + H \cdot \Delta\theta_{wor} \left(K_j\right)^y + \theta_a \,. \tag{4.189}$$

Example 4.1

The transformer loading capability is limited primarily by the ageing or deterioration of the insulation, which is a function of time and hot spot temperature. Assume that according to a loading guide the normal rate of deterioration (or "Normal Life Expectancy") takes place at 98°C hot-spot temperature. Under emergency conditions (transformer-forced outage) a higher rate of deterioration is permissible. Emergency overloads are limited to those implying host-spot temperature below 140°C. Transformer operation at hot-spot temperature exceeding 140°C, increases the risk of the dielectric breakdown.

Based on the above, it is important, both from technical and economical points of view, to determine the reliability indices for the event hot-spot temperature greater than 98°C and for the event that hot-spot temperature greater than 140°C. The following numerical example applies to the above considerations.

Consider a transformer with mean outage duration T_R=12 hours; transformer outage rate λ_0=0.126 (1/Yr); hot-spot temperature θ_h= 98°C; maximum hot-spot temperature θ_h^M =140°C. Parameters for calculating the hot-spot temperature are: ratio of load losses at rated power to no-load losses d= 5.5; oil temperature rise at rated power $\Delta\theta_{bz}$=55°C; exponential power of total losses versus top-oil (in tank) temperature rise (oil exponent) x=0.9; exponential power of current versus winding temperature rise (winding exponent) y=1.6; the weighted ambient temperature θ_a=25°C (for summer), θ_a=20°C (for winter); oil air thermal time constant of power transformer at rated power T_L =3 hours. The load parameters are: number of daily load cycles n=365 per year; mean duration of the equivalent peak load (obtained from load curves as mean duration of the load which is higher than 90% of the daily peak load) T=7.648 hours.

The distribution of daily peak loads is given in Table 4.2.

The yearly load curve is represented by 35 typical daily load curves. A few such curves are shown in Fig. 4.27.

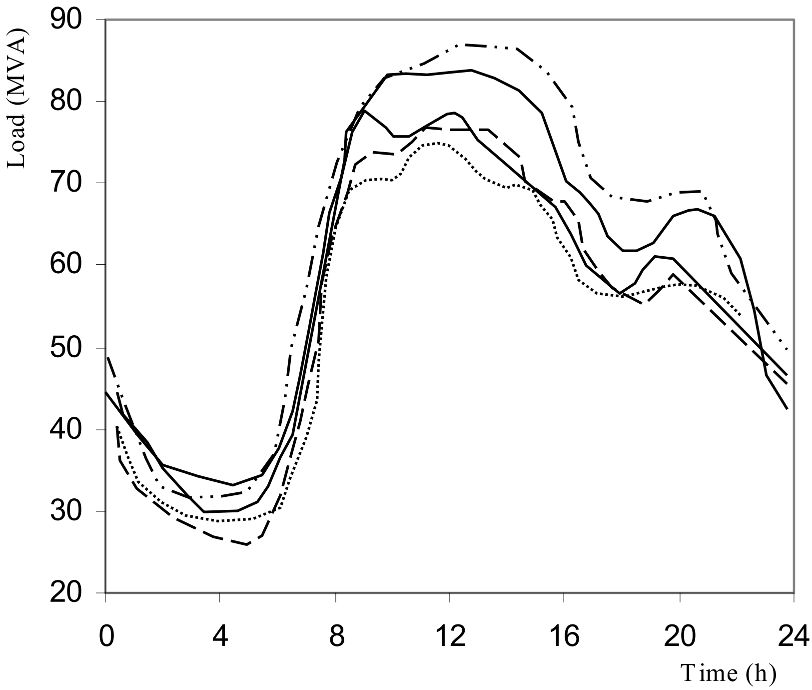

Fig. 4.27. Examples of load curves

The constants for calculating the transformer loss of life (IEEE Std. C57. 91-1995) are: A = -30.834, B = 16054.

The results obtained by software are given in Tables 4.3 - 4.6. Tables 4.3 and 4.4 present the analytical model results, while Tables 4.5 and 4.6 show the Monte Carlo simulation results.

The reserve factor K_R is defined as the ratio of substation name plate rating to the yearly peak load.

The Monte Carlo method has been applied by means of ten simulations. Each simulation had 1000 trials for given outage duration. The average values are the mean values of ten simulations. The maximum and minimum values correspond to the maximum and minimum results of the ten simulations.

Table 4.2. Distribution of daily peak loads

Summer			Winter		
Limits [MW]	Power [MW]	Number	Limits [MW]	Power [MW]	Number
31 - 35	33	5	35 – 39	38	7
36 - 39	39	9	40 – 43	42	9
40 – 42	41	12	44 – 47	45	9
43 – 46	46	6	48 – 51	49	5
47 – 53	53	7	52 – 55	53	5
54 – 58	58	9	56 – 57	57	14
59 – 61	60	14	58 – 59	58	14
62 – 64	64	11	60 – 62	61	12
65 – 67	67	13	63 – 65	65	13
68 – 69	69	12	66 – 68	68	17
70 – 71	70	17	69 – 71	70	14
72 – 73	73	20	72 - 76	74	6
74 – 75	74	13	77 – 79	79	11
76 – 77	76	11	80 – 83	82	11
78 – 79	78	8	84 – 86	85	12
80 – 83	83	9	87 – 90	88	7
84 – 90	86	8	91 – 94	93	7
0 – 0	0	0	95 – 100	100	9

Table 4.7 presents the error of the Monte Carlo simulation, using 1000 trials for various outage durations and reserve factors. The average error for a substation with two transformers is within the range of 3.2-14 percents.

Table 4.3. Results of reliability indices from analytical model (hot-spot temperature) greater than 98°C (H.S.T is hot-spot temperature; N is number of transformers; KR is reserve factor; PD is probability that H.S.T. exceeds 98°C; FD is frequency of the event that H.S.T. exceeds 98^0C [1/Yr]; DL is expected loss of life [%/Yr], calculated according to [1]; TY= 8760*PD [H/Yr]; TD is mean overlapping duration [h] ; TL is mean equivalent peak load duration (TL =7.648 h)

N	Kr	PD	TY	FD	TD	UE	DL
2	0.8	1.012E-04	1.064E+02	1.431E-01	6.195	8.228E+04	3.060E+00
2	0.9	9.950E-05	1.046E+02	1.407E-01	6.195	7.190E+04	7.086E-01
2	1.0	9.568E-05	1.006E+02	1.353E-01	6.195	6.224E+04	2.319E-01
2	1.1	8.875E-05	9.329E+01	1.255E-01	6.195	5.248E+04	9.781E-02
2	1.2	7.687E-05	8.080E+01	1.087E-01	6.195	4.167E+04	4.976E-02
2	1.3	6.254E-05	6.575E+01	8.844E-02	6.195	3.130E+04	2.911E-02
2	1.4	4.780E-05	5.024E+01	6.759E-02	6.195	2.221E+04	1.893E-02
2	1.5	3.516E-05	3.696E+01	4.971E-02	6.195	1.525E+04	1.334E-02
2	1.6	2.190E-05	2.302E+01	3.097E-02	6.195	8.904E+03	1.001E-02
3	0.8	8.975E-05	1.415E+02	1.269E-01	6.195	1.642E+05	3.060E+00
3	0.9	7.391E-05	1.165E+02	1.045E-01	6.195	1.202E+05	7.086E-01
3	1.0	5.550E-05	8.751E+01	7.848E-02	6.195	8.123E+04	2.320E-01
3	1.1	3.722E-05	5.870E+01	5.264E-02	6.195	4.953E+04	9.784E-02
3	1.2	2.038E-05	3.213E+01	2.881E-02	6.195	2.485E+04	4.977E-02
3	1.3	9.054E-05	1.428E+01	1.280E-02	6.195	1.019E+04	2.911E-02
3	1.4	2.381E-05	3.754E+00	3.367E-03	6.195	2.489E+03	1.892E-02
3	1.5	6.201E-05	9.778E-01	8.769E-04	6.195	6.050E+02	1.332E-02
3	1.6	0.000E+00	0.000E+00	0.000E+00	0.000	0.000E+00	9.922E-03
4	0.8	8.975E-05	1.415E+02	1.269E-01	6.195	1.642E+05	3.060E+00
4	0.9	7.391E-05	1.165E+02	1.045E-01	6.195	1.202E+05	7.086E-01
4	1.0	5.550E-05	8.751E+01	7.848E-02	6.195	8.123E+04	2.320E-01
4	1.1	3.722E-05	5.870E+01	5.264E-02	6.195	4.953E+04	9.784E-02
4	1.2	2.038E-05	3.213E+01	2.881E-02	6.195	2.485E+04	4.977E-02
4	1.3	9.054E-05	1.428E+01	1.280E-02	6.195	1.019E+04	2.911E-02
4	1.4	2.381E-05	3.754E+00	3.367E-02	6.195	2.489E+03	1.892E-02
4	1.5	6.201E-05	9.778E-01	8.769E-02	6.195	6.050E+02	1.332E-02
4	1.6	0.000E+00	0.000E+00	0.000E+00	0.000	0.000E+00	9.922E-03

Table 4.4. Results of reliability indices obtained by analytical model for $\theta_h \geq$ 140°C

N	Kr	PD	TY	FD	TD	UE
2	0.8	9.191E-05	9.661E+01	1.300E-01	6.195	7.473E+04
2	0.9	7.737E-05	8.133E+01	1.094E-01	6.195	5.592E+04
2	1.0	5.857E-05	6.157E+01	8.282E-02	6.195	3.810E+04
2	1.1	3.856E-05	4.053E+01	5.452E-02	6.195	2.280E+04
2	1.2	1.958E-05	2.059E+01	2.769E-02	6.195	1.062E+04
2	1.3	6.706E-06	7.050E+00	9.483E-03	6.195	3.356E+03
2	1.4	2.149E-06	2.259E+00	3.038E-03	6.195	9.984E+02
2	1.5	5.553E-07	5.837E-01	7.852E-04	6.195	2.408E+02
2	1.6	0.000E+00	0.000E+00	0.000E+00	0.000	0.000E+00
3	0.8	4.149E-05	6.542E+01	5.867E-02	6.195	7.591E+04
3	0.9	1.824E-05	2.876E+01	2.579E-02	6.195	2.966E+04
3	1.0	5.017E-06	7.910E+00	7.094E-03	6.195	7.342E+03
3	1.1	6.457E-07	1.018E+00	9.130E-04	6.195	8.591E+02
4	0.8	4.149E-05	6.542E+01	5.867E-02	6.195	7.591E+04
4	0.9	1.824E-05	2.876E+01	2.579E-02	6.195	2.966E+04
4	1.0	5.017E-06	7.910E+00	7.094E-03	6.195	7.342E+03
4	1.1	6.457E-07	1.018E+00	9.130E-04	6.195	8.591E+02

Table 4.5. Results of reliability indices obtained by Monte-Carlo simulation for $\theta_h \geq$ 98°C

		FD (1/YR)		
N	KR	Average	Max	Min
2	0.7	1.319E-01	1.321E-01	1.316E-01
2	0.8	1.286E-01	1.293E-01	1.278E-01
2	0.9	1.229E-01	1.241E-01	1.218E-01
2	1	1.151E-01	1.167E-01	1.136E-01
2	1.1	1.053E-01	1.076E-01	1.030E-01
2	1.2	9.271E-02	9.544E-02	8.975E-02
2	1.3	7.598E-02	7.872E-02	7.372E-02
2	1.4	6.102E-02	6.309E-02	5.826E-02
2	1.5	4.331E-02	4.560E-02	4.052E-02

Table 4.6. Results of reliability indices from Monte-Carlo simulation for $\theta_h \geq 140°C$

N	KR	FD (1/YR)
		Average
2	0.8	6.652E-02
2	0.9	4.652E-02
2	1.1	1.442E-02
2	1.2	6.331E-03
2	1.3	1.615E-03
3	0.8	2.003E-02
3	0.9	6.856E-03
3	1	6.836E-04
3	1.1	0.000E+00
3	1.2	0.000E+00
3	1.3	0.000E+00
4	0.8	7.431E-03
4	0.9	0.000E+00
4	1	0.000E+00
4	1.1	0.000E+00
4	1.2	0.000E+00
4	1.3	0.000E+00

The error for substation with 3 or 4 transformers is higher than in Table 4.7, since the probability is lower.

The error is estimated by the following equation:

$$\varepsilon(t_{R_j}) = 200 \cdot \sqrt{\frac{1 - P(\theta_h > \theta_h^0 / t_{R_j})}{n \cdot P(\theta_h > \theta_h^0 / t_R)}}$$

The conditional probability $P(\theta_h > \theta_h^0 / t_R)$ is calculated by Monte Carlo simulation based on repair time distribution function (Elmakis 1986).

A comparison of the results obtained by the analytical model and the Monte-Carlo simulation for the case of substation with two transformers is given in Fig. 4.28.

Table 4.7. Error calculation of the Monte-Carlo simulation for two transformers

N	KR	TR	P' (H.S.T>130 C/TR)	ERROR (%)
2	0.8	2	0.631999969	4.826E+00
2	0.8	6	0.846999884	2.688E+00
2	0.8	10	0.932999969	1.695E+00
2	0.8	14	0.950999975	1.436E+00
2	0.8	18	0.974999905	1.013E+00
2	0.8	22	0.969999909	1.112E+00
2	0.8	26	0.977999926	9.486E-01
2	0.8	30	0.986999989	7.258E-01
2	0.8	34	0.993999958	4.914E-01
2	0.8	38	0.995999932	4.008E-01
2	0.8	42	0.993999958	4.914E-01
2	0.8	26	0.995999932	4.008E-01
2	0.8	50	0.997999907	2.831E-01
2	0.8	54	1	0.000E+00
2	0.8	58	0.997999907	2.831E-01
2	0.8	62	0.997999907	2.831E-01
AVERAGE ERROR=2.528E+00				
2	1.3	2	0.005999999	8.140E+01
2	1.3	6	0.099999994	1.897E+01
2	1.3	10	0.185999989	1.406E+01
2	1.3	14	0.260999978	1.064E+01
2	1.3	18	0.32099998	9.198E+00
2	1.3	22	0.324999988	9.115E+00
2	1.3	26	0.398999989	7.762E+00
2	1.3	30	0.410999954	7.571E+00
2	1.3	34	0.442999959	7.092E+00
2	1.3	38	0.462999999	6.811E+00
2	1.3	42	0.532999992	5.920E+00
2	1.3	26	0.519999981	6.076E+00
2	1.3	50	0.535999894	5.884E+00
2	1.3	54	0.58099997	5.371E+00
2	1.3	58	0.594999909	5.218E+00
2	1.3	62	0.636999965	4.774E+00
AVERAGE ERROR=3.206E+01				

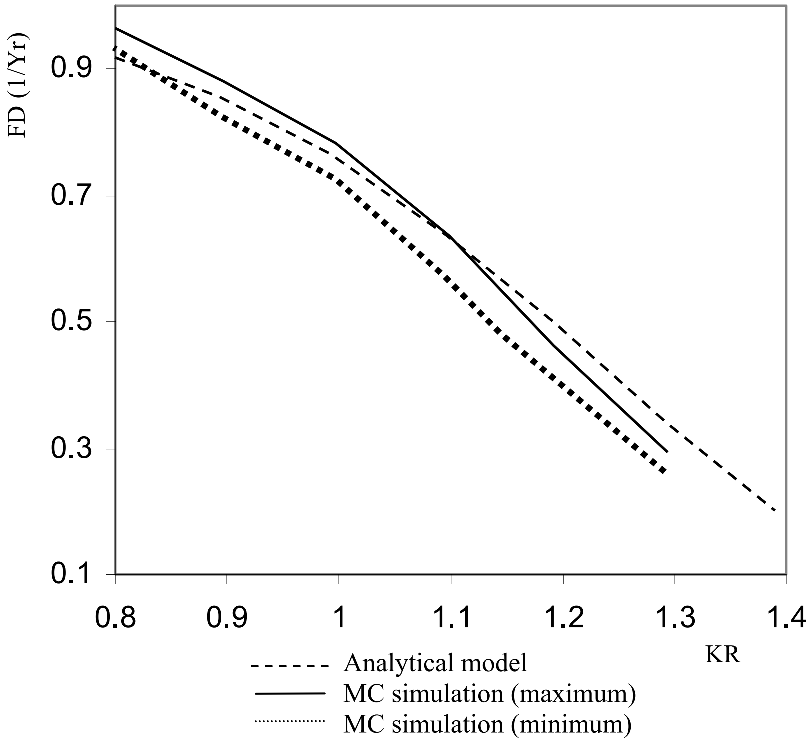

Fig. 4.28. The frequency of events when $\theta_h > 98^0$C as a function of the reserve factor

The substation development data can be calculated through analytical model, for a period of 30 years. The substation starts with 2 transformers and ends with 3 or 4 transformers, based on the area load growth and reserve factor. Table 4.8 contains the analytical model results for the substation development.

This numerical example describes a model used for determination of the reliability indices based on hot-spot temperature of the power transformer by a probabilistic approach, which faith-fully reflects the reality. One can see the good agreement between the analytical model and the Monte Carlo simulation.

The economic loading of the power transformer can be determined, based on the reliability indices derived in this example, by analyzing the loss of life, together with the cost of the transformer and its operation.

Table 4.8. Results of the substation development data based on analytical model (S_p is load peak; T_D is outage time of transformer; L_{LF} is loss of life; U_E is unsupplied energy)

Year	No of transformer	S_p MVA	L_{LF} %/Yr/St	U_E kWh/Yr/St	T_D min/Yr
1	2	51.9	0.01821	3.33E+01	0.57
2	2	55.1	0.01881	7.18E+01	3.45
3	2	58.4	0.02111	1.83E+02	7.77
4	2	61.9	0.02381	4.95E+02	17.89
5	2	65.6	0.02891	1.67E+03	38.97
6	2	69.5	0.03673	3.63E+03	50.21
7	2	73.7	0.05046	6.11E+03	59.11
8	3	78.1	0.02938	2.36E+01	0.23
9	3	82.8	0.03269	3.34E+01	0.16
10	3	87.8	0.04	5.52E+01	0.23
11	3	93.0	0.05391	6.31E+01	0.23
12	3	98.6	0.081	9.16E+01	0.47
13	3	104.5	0.14153	5.90E+02	2.48
14	3	110.8	0.28557	7.45E+02	2.59
15	3	117.4	0.64969	9.26E+02	6.18
16	3	124.5	1.63292	1.55E+03	22.38
17	4	132.0	0.12745	1.59E+02	1.55
18	4	139.9	0.23847	1.08E+03	3.80
19	4	148.3	0.51154	1.34E+03	7.05
20	4	157.2	23.00373	1.93E+03	19.65
21	4	166.6	3.25428	5.26E+03	71.03
22	4	124.6	0.08061	1.07E+02	0.54
23	4	132.1	0.12845	1.71E+02	1.63
24	4	140.0	0.24249	1.06E+03	3.80
25	4	148.4	0.52055	1.34E+03	6.91
26	4	157.4	1.25372	1.91E+03	19.16
27	4	166.8	3.3243	5.53E+03	73.69
28	4	168.0	3.76447	6.19E+03	79.52
29	4	168.0	3.76446	6.17E+03	79.00
30	4	168.0	3.76445	6.25E+03	80.58

4.2 Reliability evaluation and analysis of substation configurations

4.2.1 Problem definition

The system discussed in this chapter consists basically of a transformer substation supplying the load for a given area. The substation is fed by the transmission line and it contains all the components necessary to operate the substation.

The selection of configurations for substations or switching stations is a complex problem, which requires taking into account both technical and economical considerations. By calculating reliability parameters for different substation schemes, one can obtain:

- Information which is helpful in the decision making regarding the selection of substation scheme.
- The rate of expected forced outages of the transformers.

Reliability calculation of substation schemes includes two basic components:

- Determination of a mathematical model for the reliability calculation.
- Failure Mode and Effect Analysis (FMEA).

As for the mathematical model, two analytical methods are usually employed in the reliability calculation of substation schemes.

The first direct method (Endrenyi 1978, Billinton et al. 1996), relies on the laws of probability and is especially easy to calculate when the states are mutually independent.

The second method based on a Homogeneous Discrete State Continuous–transition Markov Process is especially useful when there are interdependent situations, which the system can be in.

In reliability calculations of substation schemes (Endrenyi 1978, Billinton et al. 1996) with each of the above mentioned methods, the following operational states are taken into consideration:

- The occurrence of a failure in one of the components of the scheme, under normal operation of the scheme.
- The occurrence of a failure while performing repair work necessitated by a former failure of some other component of the scheme.
- The occurrence of a failure in one of the scheme components, during maintenance work on some other component of the substation scheme.

Regarding the elements composing the scheme, two main types are to be considered, differing in their failure modes and their effects (Endrenyi 1978).

- Passive elements, such as transformer, disconnector, busbar, overhead line etc.
- Active elements: circuit breakers.

In passive elements the usual failure is ground short circuit or short circuit between phases, caused by various reasons and usually leading to the activation of circuit breakers.

As far as circuit breakers are concerned, three failure modes are to be considered:

- Earth short circuit or short circuit between phases.
- Failure to operate under failure state in the protected area of the circuit breaker.
- False operations of the circuit breaker or of the protection system.

Regarding FMEA, three types of failures have to be considered:

- Failures which do not interrupt the energy supply to the consumers.
- Failures which only require switching operations in order to restore energy supply to the customers. In such a case the duration of the forced outage is the time required for the appropriate switching operations.
- Failures requiring the execution of repair work in order to restore energy supply to the consumers. In this case the forced outage time is the duration of the repair work.

The difficulty in the FMEA is caused by the huge number of states that need to be examined. For instance, in a scheme containing 20 elements one has to examine at least 1,180 states (not including the additional states to be examined as a result of the circuit breakers), resulting from the following operating states: single element failures (repair or switching): $2C_{20}^{1}=40$; two simultaneous failures (2 elements are under repair or one in switching and another under repair or vice versa): $3C_{20}^{2}=570$; failure and maintenance (maintenance and repair, maintenance and switching): $3C_{20}^{2}=570$.

In this chapter we develop:

- Analytical models for the calculation of reliability metrics for different operation modes of the substation scheme, which were mentioned above, including a model for the circuit breakers.
- A computationally efficient FMEA algorithm, based on logic theory.

In any case one should calculate the failure probability, frequency and duration for forced outage of *K* out of *M* existing transformers in the scheme (*K* = 1,…,*M*). For instance, for a station including 3 transformers we should calculate the above metrics for the occurrences of outage of one out of three, two out of three and three out of three transformers.

FMEA will be performed also for substation schemes containing normally Open components, for instance a connection between two busbars.

4.2.2 Analytical models for evaluating the reliability parameters

According to the definition of the problem, the reliability parameters have to be calculated for the following operational states:

- One failure in one of the scheme components.
- Two failures (a failure in one of the components, during the execution of repair work on another component of the scheme).
- A failure and maintenance (a failure in one of the components, during the execution of maintenance work on another component of the scheme).
- Other failures, according to the circuit breaker model.

The development of the analytical models for the calculation of the reliability parameters for the above mentioned operational states is based on the assumption that the mean time between failures, t_F and the duration of the repair, t_R are exponentially distributed, i.e. the probability density functions of t_F and of t_R are given by:

$$f_{t_F}(t) = \lambda e^{-\lambda t} \text{ and } f_{t_R}(t) = \mu e^{-\mu t}, \qquad (4.190)$$

where λ is the failure rate and $T_R = 1/\mu$ is mean repair time.

For the purpose of the calculation, the substation scheme is split into groups of elements, where each group constitutes a functional block. The choice of the elements belonging to a certain functional block is made according to the character of the operation of the substation scheme and according to the protection zones of the protection system. The division should be made so that in case of a forced outage of one of the elements of the functional block, the rest of the block elements should be out of operation (for the purpose of performing switching or repair operations).

Also a failure in one of the components of the functional block will be covered by the same protection system. In any case, a circuit breaker will constitute a separate functional block.

Figs. 4.29 and 4.30 describe the division into functional blocks of one and a half circuit-breaker substation scheme and double busbar substation scheme respectively.

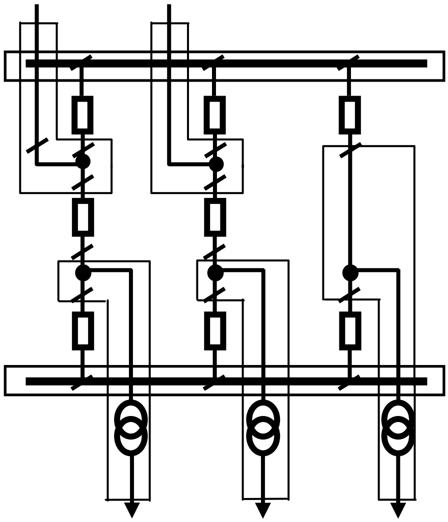

Fig. 4.29. Functional blocks for one and a half circuit-breaker configuration

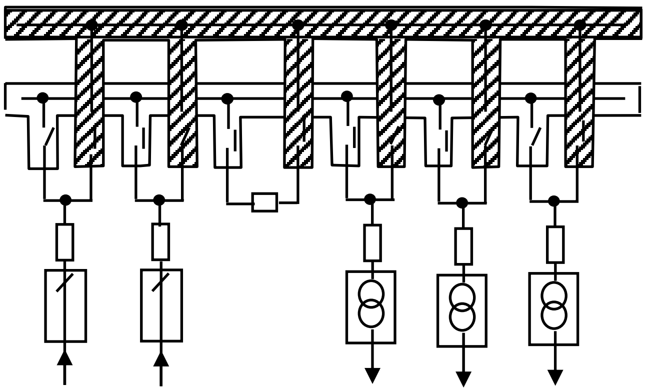

Fig.4. 30. Functional blocks for double busbar configuration

According to the above mentioned division criteria, the group of elements belonging to the functional block *i* can be considered as series elements, i.e. a failure in one of the elements of the functional block causes failure of the entire block. Therefore the rate of forced outages and the rate of equivalent repair times μ_i , λ_i can be calculated as follows:

$$\lambda_i = \sum_{i=1}^{K_i} \lambda_j \quad \text{and} \quad \mu_i \cong \lambda_i / \sum_{i=1}^{K_i} \frac{\lambda_j}{\mu_j}, \tag{4.191}$$

where K_i is number of elements in block *i*, λ_j is failure rate of element *j*, $T_{R_j} = 1/\mu_j$ is mean repair time of element *j*.

Approximate formulas, common in the literature (Endrenyi 1978), for the calculation of the reliability parameters for the various operational states of the substation schemes regarding one failure, two failures, failure and maintenance appear in Table 4.10, assuming that the probability of combination of more than two failures is negligible.

Table 4 .10. Approximate formulas for calculating the reliability parameters

state	contribution to the probability	contribution to failure frequency (1/Yr)	Duration (h)
I_R	$\frac{\lambda_i T_{R_i}}{8760}$	λ_i	T_{R_i}
I_S	$\frac{\lambda_i T_S}{8760}$	λ_i	T_S
$I_R\ J_R$	$\frac{\lambda_i T_{R_i}}{8760} \cdot \frac{\lambda_j T_{R_j}}{8760}$	$\lambda_i \lambda_j \left[\frac{T_{R_i}}{8760} + \frac{T_{R_j}}{8760}\right]$	$\frac{T_{R_i} \cdot T_{R_j}}{T_{R_i} + T_{R_j}}$
$I_R\ J_S$	$\frac{\lambda_i T_{R_i}}{8760} \cdot \frac{\lambda_j T_S}{8760}$	$\lambda_i \lambda_j \left[\frac{T_{R_i}}{8760} + \frac{T_S}{8760}\right]$	$\frac{T_{Ri} \cdot T_S}{T_{R_i} + T_S}$
$I_M\ J_R$	$\frac{\lambda_{M_i} \lambda_j T_{M_i} T_{R_j}}{8760^2 (T_{M_i} + T_{R_j})}$	$\frac{T_{M_i}\ \lambda_{M_i}\ \lambda_j}{8760}$	$\frac{T_{Mi} \cdot T_{Rj}}{T_{M_i} + T_{R_j}}$
$I_M\ J_S$	$\frac{\lambda_{M_i}\ \lambda_j \cdot T_{M_i} \cdot T_S}{8760^2}$	$\frac{T_{M_i}\ \lambda_{M_i}\ \lambda_j}{8760}$	T_S

Here I_R is the state when element *i* is under repair; I_S is the state when element *i* is in switching state; $I_R\ J_R$ is the state when both elements *i, j* are under repair; $I_R\ J_S$ is the state in which element *i* is under repair and element *j* is in switching state; $I_M\ J_R$ is the state in which element *i* is under maintenance and element *i* is under repair; $I_M\ J_S$ is the state when element *i* is under maintenance and element *j* in switching; λ_i is the failure rate of element *i* in (1/Yr); T_{Ri} is the mean repair duration of element *i* in hour; T_{Mi} is the mean maintenance duration of element *i* in hour ; λ_{M_i} is the maintenance frequency of element *i* in (1/Yr).

4.2.3 The circuit breaker model

Most reliability calculation models, including those in use in power systems, are based on the assumption that every one of the components of the relevant system has a single mode of failure. This assumption is usually justified for the passive elements, such as: disconnector, busbar, line etc. As for a circuit breaker, being an active element, there are in principle three failure modes to be considered in reliability calculations:

- Ground short circuit or phase-to-phase short circuit.
- Circuit breaker failure to operate when a failure occurs in one of its protection zones.
- False operation of the circuit breaker.

The above mentioned three failures differ from each other by their effect on the energy supply, as well as by the way of the reliability parameters calculation.

The model considering all the above three mentioned failure modes is presented in Fig. 4.31 (Endrenyi 1978). This figure describes the different states of the circuit breaker and the component protected by this circuit breaker.

In Fig. 4.31 B is the circuit breaker; C is the component protected by the circuit breaker; R is a repair operation; S is a switching operation; λ_1 is the failure rate of the circuit breaker with respect to ground fault or failures between phases; λ_2 is the rate of false operations of the circuit breaker; λ_c is the failure rate of component C; $T_c = 1/\mu_c$ is the mean repair time of component C; $T_B = 1/\mu_B$ is the mean repair time of the circuit breaker; ρ is the circuit breaker failure probability.

The above model does not make the clear distinction between the different states according to the failure modes. This is important with regard to the FMEA. For instance, in any circuit breaker failure state, it is important to know whether this state was caused by a ground fault of the circuit breaker or by a false operation of the circuit breaker. In the first case additional circuit breakers will be activated in order to remove the failure in the circuit breaker itself. In the second case only the circuit breaker itself will be activated. A more elaborate model is described in Fig. 4.32 (Elmakis 1986), where a clear distinction between the different states according to the failure modes is made. This model is the basis for the rest of the calculations.

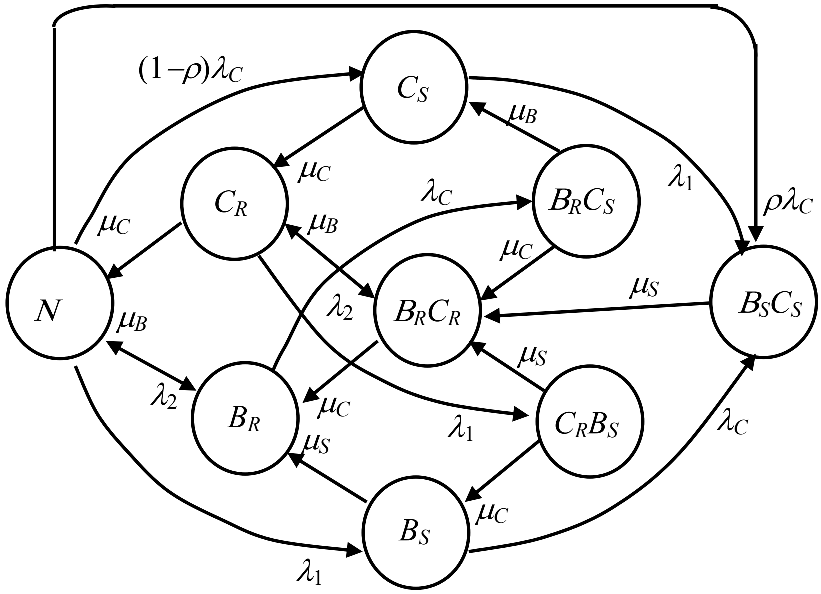

Fig. 4.31. Circuit breaker model including switching after fault

In Fig. 4.31 B^1 is circuit breaker failure state caused by a ground short circuit or phase-to-phase short circuit; B^2 is circuit breaker failure caused by a false operation of the circuit breaker or of the protection system; B^3 is failure state caused by circuit breaker failure in the case of a failure in component C. The other signs and data are the same as in Fig. 4.31.

Assuming that the mean time between failures and repair duration are exponentially distributed the model in Fig. 4.32 can be described according to a homogeneous discrete state continuous-transition Markov Process, represented by the following system of equations:

$$[\lambda_1+\lambda_2+(1-\rho)\lambda_C+\rho\lambda_C]P_1-\mu_C P_2-\mu_B P_8-\mu_B P_5-\mu_B P_3=0$$
$$-[\lambda_2+\lambda_1+\mu_C]P_2+\mu_S P_{12}+\mu_B P_6+\mu_B P_7+\mu_B P_4=0$$
$$-[\mu_B+\lambda_C]P_3+\mu_S P_{10}+\mu_C P_6=0 \quad -[\mu_B+\mu_C]P_4+\mu_S P_9=0$$
$$-[\lambda_C+\mu_B]P_5+\mu_S P_{16}+\mu_C P_7=0$$
$$-[\mu_B+\mu_C]P_6+\mu_S P_{11}+\mu_S P_{13}=0$$

$$-[\mu_C + \mu_B]P_7 + \mu_S P_{15} + \mu_S P_{14} = 0$$
$$-\mu_B P_8 + \mu_C P_4 + 0$$
$$-\mu_S P_9 + \rho\lambda_C P_1 = 0$$
$$-\mu_S P_{10} + \lambda_1 P_1 = 0$$
$$-\mu_S P_{11} + \lambda_C P_3 = 0$$
$$-\mu_S P_{12} + (1-\rho)\lambda_C P_1 = 0$$
$$-\mu_S P_{13} + \lambda_1 P_2 = 0$$
$$-\mu_S P_{14} + \lambda_2 P_2 = 0$$
$$-\mu_S P_{15} + \lambda_C P_5 = 0$$
$$-\mu_S P_{16} + \lambda_2 P_1 = 0 \tag{4.192}$$

where P_i is probability of state i ($1 \le i \le 16$).

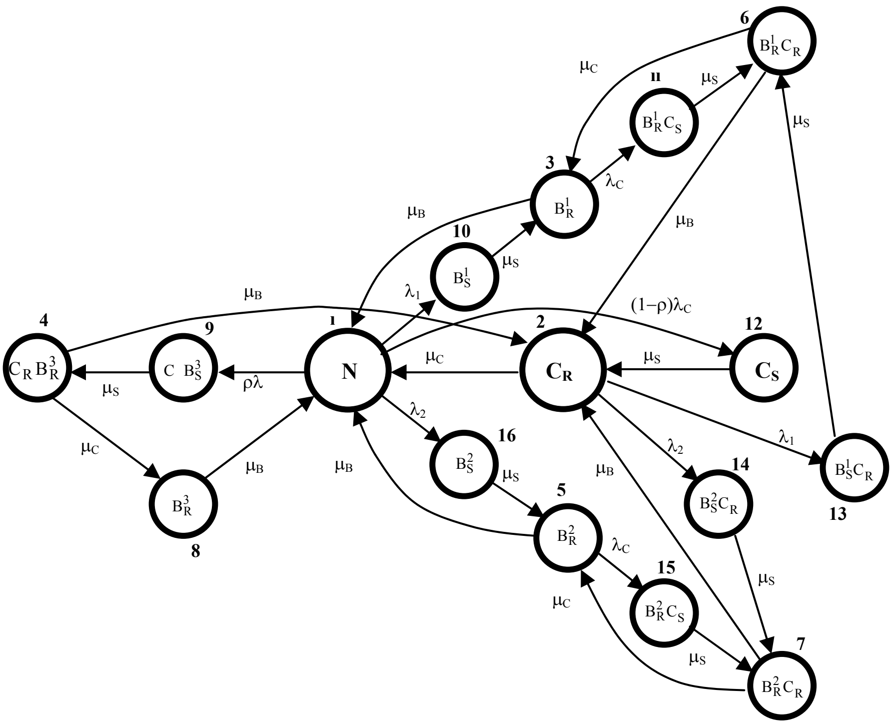

Fig. 4.32. The detailed circuit breaker model

This is a system of homogeneous linear algebraic equations, thus a non trivial solution can be obtained if an additional non homogeneous equation exists. The additional equation will be derived from the fact that the system must be in one of the 16 states, i.e.

$$\sum_{i=1}^{16} P_i = 1 \tag{4.193}$$

The exact solution of the system is:

$$\begin{aligned}
&P_1 = \frac{\mu_B \mu_C (\mu_B + \mu_C)\varepsilon}{\Delta},\\
&P_2 = \frac{\mu_B \mu_C \ (\mu_B + \mu_C)\varepsilon}{\Delta} - \frac{\rho\lambda_C \ \mu_C \mu_B \ (\lambda_C + \mu_B + \mu_C)}{\Delta},\\
&P_3 = \frac{\lambda_1 \ \mu_C (\mu_B + \mu_C) \ \varepsilon - \rho\lambda_C \ \lambda_1 \ \mu_C^2}{\Delta}, \ P_4 = \frac{\rho\lambda_C \ \mu_B \ \mu_C \ \varepsilon}{\Delta},\\
&P_5 = \frac{\lambda_2 \ \mu_C \ (\mu_B + \mu_C)\varepsilon - \rho\lambda_C \ \lambda_2 \ \mu_C^2}{\Delta},\\
&P_6 = \frac{\lambda_C \ \lambda_1 \ (\mu_B + \mu_C)\varepsilon - \rho\lambda_C \ \lambda_1 \ \mu_C (\mu_B + \lambda_C)}{\Delta},\\
&P_7 = \frac{\lambda_C \ \lambda_2 \ (\mu_B + \mu_C)\varepsilon - \rho\lambda_C \ \lambda_2 \ \mu_C (\mu_B + \lambda_C)}{\Delta},\\
&P_8 = \frac{\rho\lambda_C \ \mu_C^2 \varepsilon}{\Delta}, \ P_9 = \frac{\rho\lambda_C}{\mu_S} P_1, \ P_{10} = \frac{\lambda_1}{\mu_S} P_1, \ P_{11} = \frac{\lambda_C}{\mu_S} P_3,\\
&P_{12} = \frac{(1-\rho)\lambda_C}{\mu_S} P_1, \ P_{13} = \frac{\lambda_1}{\mu_S} P_2, \ P_{14} = \frac{\lambda_2}{\mu_S} P_2, \ P_{15} = \frac{\lambda_2}{\mu_S} P_5, \ P_{16} = \frac{\lambda_2}{\mu_S} P_1,
\end{aligned} \tag{4.194}$$

where

$\lambda_B = \lambda_1 + \lambda_2$;

$\varepsilon = \lambda_C + \mu_C + \lambda_B + \mu_B$;

$$\Delta = (\mu_B + \mu_C)\ \varepsilon\ [(\lambda_B + \mu_B)\ (\lambda_C + \mu_C)]$$

$$+\frac{\mu_C}{\mu_B}(\lambda_B + \mu_B) + \frac{\mu_B}{\mu_S}\lambda_1\ (\lambda_C + \mu_C)\ ] + \rho\lambda_C\ \ \mu_C[\lambda_C(\mu_C - \lambda_B)]$$

$$+(\mu_B + \mu_C)\left(\mu_C + \frac{\lambda_B}{\mu_S}\lambda_C + \frac{\lambda_B}{\mu_S}\mu_B\right)];$$

In reality, the following relations exist:

$\lambda_i \ll \mu_i$ for any i; $\rho\lambda_C \ll 1$;

Based on the above, the exact solution can be approximated as follows:

$$P_2 \cong \frac{\lambda_C}{\mu_C} = \lambda_C T_C;\ P_3 \cong \frac{\lambda_1}{\mu_B} = \lambda_1 T_B;$$

$$P_4 \cong \frac{\rho\lambda_C}{\mu_B + \mu_C} = \frac{T_B T_C}{T_B + T_C}\ \ \rho\lambda_C;\ P_5 \cong \frac{\lambda_2}{\mu_B} = \lambda_2 T_B;$$

$$P_6 \cong \frac{\lambda_C}{\mu_C}\ \frac{\lambda_1}{\mu_B} = \lambda_C\lambda_1\ T_C T_B;\ P_7 \cong \frac{\lambda_C\lambda_2}{\mu_C\mu_B} = \lambda_C\lambda_2 T_B T_C;$$

$$P_8 \cong \rho\lambda_C \frac{\mu_C}{\mu_B(\mu_B + \mu_C)} = \rho\lambda_C\left(\frac{T_B}{T_B + T_C}\right)T_B;\ P_9 \cong \frac{\rho\lambda_C}{\mu_S} = \rho\lambda_C \cdot T_S;$$

$$P_{10} \cong \frac{\lambda_1}{\mu_S} = \lambda_1 T_S;\ P_{11} \cong \frac{\lambda_1}{\mu_B}\ \frac{\lambda_C}{\mu_S} = \lambda_1\lambda_C T_B T_S;$$

$$P_{12} \cong (1-\rho)\frac{\lambda_C}{\mu_S} = (1-\rho)\lambda_C T_S;\ P_{13} \cong \frac{\lambda_1\lambda_C}{\mu_S\mu_C} = \lambda_1\lambda_C T_S T_C;$$

$$P_{14} \cong \frac{\lambda_2\lambda_C}{\mu_S\mu_C} = \lambda_2\lambda_C T_S T_C;\ P_{15} \cong \frac{\lambda_C\lambda_2}{\mu_S\mu_B} = \lambda_2\lambda_C T_S T_B;$$

$$P_{16} \cong \frac{\lambda_2}{\mu_S} = \lambda_2 T_S. \tag{4.195}$$

From the FMEA point of view, states 4, 8, 9, 12 are caused by a failure to operate of the circuit breaker, which will cause the activation of further circuit breakers by the protection system.

States 7, 14, 15, 16 are caused by a false operation of the circuit breaker.

The other states are similar to the states described in Table 4.10, as far as the failure effect is concerned.

If element C (or functional block) is protected by N_{BC} circuit breakers, and assuming that the probability of a failure by two circuit breakers or more of the group N_{BC}, protecting element C is negligible, we shall get approximately the same probabilities as those related to the states of failure to operate of one of the circuit breakers (out of N_{BC}), as follows:

$$\text{State 4 } \left(C_R B_R^3\right):\ P_4 \cong \frac{T_B T_C}{T_B + T_C} \cdot \lambda_C \cdot \rho(1-\rho)^{\ N_{BC}-1}, \tag{4.196}$$

$$\text{State 8 } \left(B_R^3\right):\ P_8 \cong \frac{T_B}{T_B + T_C} \cdot T_B \lambda_C \rho(1-\rho)^{N_{BC}-1}, \tag{4.197}$$

$$\text{State 9 } (C_S B_S):\ P_9 \cong T_S \lambda_C\ \rho(1-\rho)^{N_{BC}-1}, \tag{4.198}$$

where ρ is the probability of fail to operate of a circuit breaker; N_{BC} is the number of circuit breakers protecting by component C (or functional block C), $(1-\rho)^{\ N_{BC}-1}$ is the probability that the rest of the circuit breakers $(N_{BC}-1)$ will be operated.

The frequency of entering state i can be calculated according to (Endrenyi 1978):

$$f_i = P_i \sum_{\substack{j=1 \\ j \neq i}}^{n} \rho_{ij} \tag{4.199}$$

where P_i is the probability of state i; $\sum \rho_{ij}$ is the sum of the transition rates from state i to other states.

Based on equation (4.191), one obtains the frequency of the additional states, caused by failure to operate of one of the circuit breakers or as a result of a false operation of the circuit breaker, as follows:

$$\text{State 4 } (C_R B_R^3):\quad f_4 \cong \rho(1-\rho)^{N_{BC}-1} \cdot \lambda_C, \tag{4.200}$$

$$\text{State 8 } \left(B_R^3\right):\quad f_8 \cong \rho(1-\rho) \frac{\lambda_C T_B}{T_S + T_C}, \tag{4.201}$$

$$\text{State 9 } (C_S B_S^3):\quad f_9 \cong \rho(1-\rho)^{N_{BC}-1} \cdot \lambda_C, \tag{4.202}$$

State 5 $\left(B_R^2\right): \ f_5 \cong \lambda_2,$ (4.203)

State 7 $\left(B_R^2 C_R\right): \ f_7 \cong \lambda_C \lambda_2 \ \left(T_B + T_C\right),$ (4.204)

State 15 $\left(B_R^2 C_R\right): \ \left(B_R^2 C_S\right),$ (4.205)

The other state frequencies for the approximate solution are according to Table 4.10.

4.2.4 Failure mode and effect analysis

As far as the analysis of the failures and their effect on the system are concerned, three failure modes are to be considered:

- Failures which do not cause an interruption of the energy supply to the consumers.
- Failures necessitating switching operations only for the restoration of energy supply to the consumers.
- Failures necessitating the carrying out of repair operations for the restoration of energy supply to the consumers.

The contribution for the reliability metrics of failures of the first type shall be considered to be zero, as no interruption to the consumers is caused.

In the other two types of failures there is a contribution to the reliability metrics, while the main difference between the two is the duration of the interruption. In failures of the second type the duration of the interruption will be the time required for the switching operation, T_S, which is usually much shorter than the time required for repair operations.

In the effect analysis of the failures:

- All the failure states have to be classified according to the above mentioned three types of failures.
- Determine for each failure the number of disconnected transformers and the mode of putting them back in operation (by switching or by repair operation).

For this purpose, a logical algorithm is developed, which enables performing computerized FMEA for all failure types.

4.2.5 Logical algorithm for FMEA

As a matter of principle, any substation scheme can be described by incoming feeders and outgoing feeders (transformers), interconnected by components (circuit breakers, disconnectors, busbars etc.), in such a way that several paths exist between each incoming and outgoing feeder. Each such path contains several components (or functional blocks) necessary for the electrical energy flow.

Two types of the paths are to be considered:

Paths which are normally closed in normal operation of the substation scheme and those which are normally open in normal operation of the substation scheme. By numbering the components (or the functional blocks), a path matrix can be built for each outgoing line, where each column in the matrix represents one path.

The elements in this matrix take values 0 or 1. 0 indicates that the element does not belong to the path, 1 indicates that the element belongs to the path.

In this way a path matrix can be built between each incoming feeder and all outgoing feeders, where the number of rows is equal to the number of elements (or functional blocks) and the number of columns is equal to the number of paths.

The building of the path matrices for both the normally closed paths and the normally open paths in normal operation is done by the computer algorithm based on the tree method.

Consider two illustrative examples with 2 incoming feeders and 3 outgoing feeders (transformers). Fig. 4.33 presents a one and a half circuit-breaker scheme which includes 15 functional blocks and contains only normally closed paths in normal operating state. Fig. 4.34 presents a two busbar substation scheme which includes 13 functional blocks and contains both normally closed and normally open paths in normal operating state.

The number of rows and the number of columns in the corresponding path matrix is equal to the number of the elements, where the value of the terms in the matrix, e_{ij}, can take three values: 0, 1 or 2, depending on the connection between elements i and j: $e_{ij}=0$ if no connection exists, $e_{ij}=1$ if the connection exists and is closed, $e_{ij}=2$ if the connection exists and is normally open.

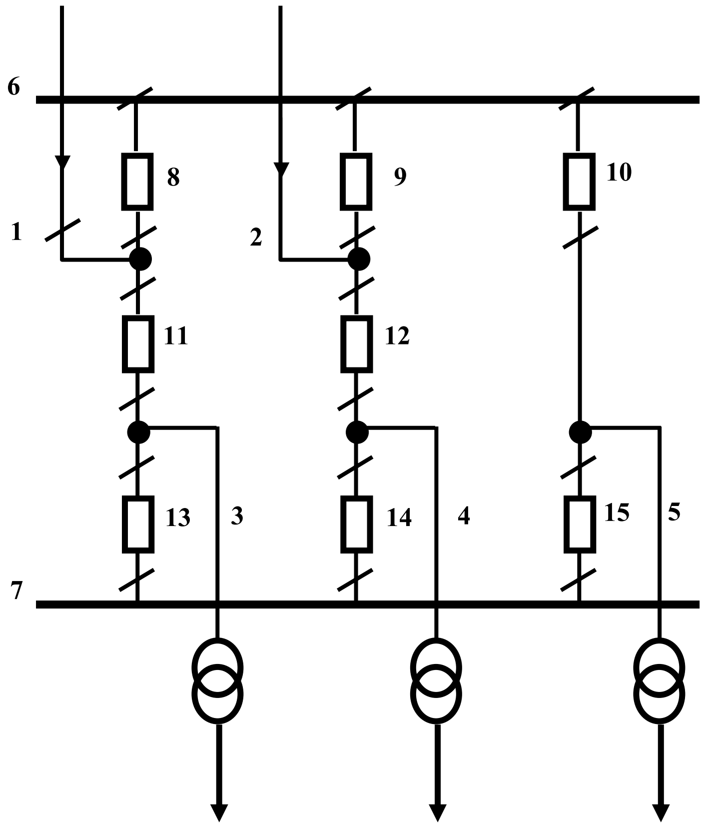

Fig. 4.33. One and a half circuit-breaker scheme (1, 2 are incoming feeders; 3, 4, 5 are outgoing feeders; 6, 7 are busbars; 8 - 15 are circuit breakers)

Based on the above, one can construct the path matrices for the substation schemes described in Figs. 4.33, 4.34, $\boldsymbol{L}_1$, $\boldsymbol{L}_2$, as follows:

$$L1=\begin{vmatrix}
0&0&0&0&0&0&0&1&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&1\\
0&0&0&0&0&0&0&1&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
1&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&0&0&0&0&0&0&0&0&0\\
1&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&1&0&0&0&0&0&0&0&0
\end{vmatrix}$$

$$
L2=\begin{vmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{vmatrix}
$$

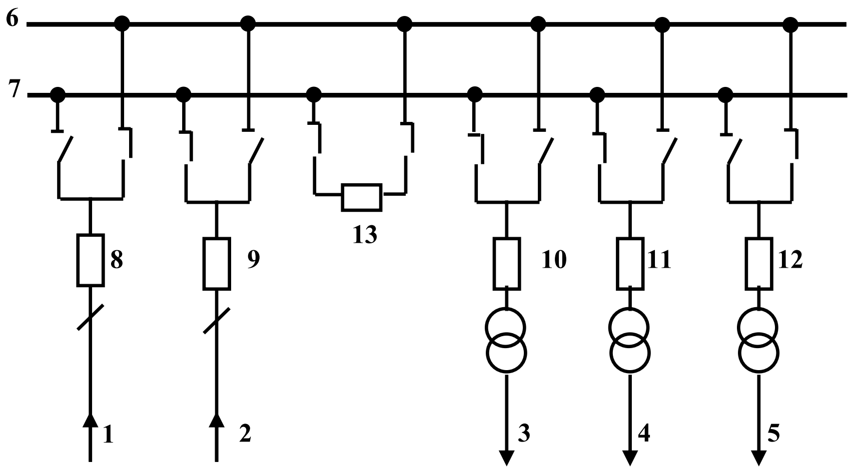

Fig. 4.34. Double busbar substation scheme (1, 2 are incoming feeders; 3, 4, 5 are outgoing feeders; 6, 7 are busbars; 8 - 12 are circuit breakers; 13 is busbar coupler

Based on the connection matrix $\boldsymbol{L}$, describing the various connections between the elements in the substation scheme, the matrices may be determined for the possible paths A_0^i, A_C^i related to the normally closed paths and to the normally open paths between each incoming feeder and each outgoing feeder (transformer), respectively.

The terms in the above matrices, a_{ij} take the value of 1 if the element i belongs to path j or value of 0 otherwise.

The number of rows in the path matrix is equal to the number of elements (or the functional blocks) forming the substation scheme. The number of columns is equal to the number of possible paths between all the incoming feeders and the relevant outgoing line.

The path matrices were built by a software algorithm based on a fault tree model.

The path, matrices for the substation schemes described in Figs. 4.33, 4.34 are as follows:

a) The path matrices for the one and a half circuit-breaker substation scheme (Fig. 4.33):

$$A_C^3 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{vmatrix}$$

$$A_C^4 = \begin{vmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

$$A_C^5 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{vmatrix}$$

where A_C^3, A_C^4, A_C^5 are the path matrices between the incoming feeders 1, 2 and the outgoing feeders 3, 4, 5, respectively.

b) The path matrices for a double busbar substation scheme (Fig. 4.34):

$$A_C^3 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{vmatrix}, \quad A_C^5 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{vmatrix}, \quad A_C^4 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{vmatrix}$$

$$A_O^3 = \begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1
\end{vmatrix}$$

$$A_o^4 = \begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1
\end{vmatrix}$$

where A_C^3, A_C^4, A_C^5 are the path matrices between the incoming feeders 1, 2 and outgoing feeders 3, 4, 5 respectively for the normally closed paths and

A_O^3, A_O^4, A_O^5 are the path matrices between the incoming feeders 1, 2 and outgoing feeders 3, 4, 5 respectively for the normally open paths.

A logical FMEA algorithm for all the operational states occurring during one or two failures is based on the path matrices A_O^i, A_C^i.

Logical algorithm for single failure

One should check for every failure, whether it causes outage of the transformer. Therefore one has to find out whether it is possible to put the failed transformers back into operation by a switching operation or by repair of the element.

The way of putting the failed transformers back into operation can be determined by checking existence of paths connecting the incoming feeders with the relevant outgoing feeder, under switching and repair operational states. The difference between these two operational states lies in the fact that under switching state the circuit breakers protecting the failed functional block is opened.

Furthermore, there is an operational difference between schemes containing only normally closed paths, such as the scheme in Fig. 4.33, and schemes containing also normally open paths, such as the scheme in Fig. 4.34. The substation scheme is divided into functional blocks, according to the protection zones of the relays, so that each functional block j has ${N_B}^j$ circuit breakers, the serial numbers of which are known. The number of the circuit breakers ${N_B}^j$ and their respective serial numbers constitute the input to the algorithm and defines the protection zones. For example, based on the conventional protection systems, Table 4.11 presents data for one and a half circuit-breaker substation scheme (Fig. 4.33) and for double busbar substation scheme (Fig. 4.34).

For example, it can be seen from the table that in the case of a failure in functional block j = 7, in one and a half circuit-breaker scheme, ${N_B}^j = 3$ circuit breakers with serial numbers 13, 14 and 15 are activated. As the normally open paths (if they exist) are not affected by the switching operations, one should refer for switching states only to the path matrices for the normally closed paths A_C^i.

Table 4.11. Definition of the operation of the protection systems

Double busbar substation scheme								One and a half circuit-breaker scheme				
j	$N_B^{\ j}$	numbers of breakers						j	$N_B^{\ j}$	numbers of breakers		
1	1	8						1	2	8	11	
2	1	9						2	2	9	12	
3	1	10						3	2	11	13	
4	1	11						4	2	12	14	
5	1	12						5	2	10	15	
6	4	9	10	11	13			6	3	8	9	10
7	3	8	12	13				7	3	13	14	15
8	2	12	13					8	3	9	10	11
9	3	10	11	13				9	3	8	10	12
10	3	9	11	13				10	3	8	9	15
11	3	9	10	13				11	2	8	13	
12	2	8	13					12	2	9	14	
13	5	8	9	10	11	12		13	3	14	15	11
								14	3	13	15	12
								15	3	13	14	10

For the purpose of the FMEA for outgoing feeder *I*, a switching matrix M_S^i should be constructed from the path matrix A_C^i. Line *j* of matrix M_S^i, which can be described as a logical vector $\vec{m}_j^i$, will be derived from the path matrix A_C^i as:

$$\vec{m}_j^i = OR\cdot\left(A_C^i\left[I_j\right], A_C^i\left[I_{k_j}\right], A_C^i\left[I_{L_j}\right] \ldots\ A_C^i\left[I_{Q_j}\right]\right) \tag{4.206}$$

where $A_C^i\left[I_j\right]$ represents line *j* in the path matrix A_C^i and $A_C^i\left[I_{K_j}\right]$ represents line K_j in the path matrix A_C^i.

Eq. (4.206) represents the logic OR operation, for every path between line *j* (corresponding to the failed element) and lines K_j, L_j,.....Q_j, indicating the circuit breakers which open if a failure occurs in functional block *j*. For instance, for one and a half circuit-breaker substation scheme, the 7th line of the switching matrix for output 3 can be derived as:

$$\vec{m}_7^3 = OR \cdot \left(A_C^3[I_7], A_C^3[I_{13}], A_C^3[I_{14}], A_C^3[I_{15}] \right)$$

i.e. the OR operation has to be performed among rows 7, 13, 14, 15 of matrix A_C.

By performing the OR operation among these lines we obtain:

$$\vec{m}_7^3 = (0,1,1,0,1,1,1)$$

where 1 means that at least the element in which the failure occurred (in this case element 7) or at least one of the opened circuit breakers (in this case 13, 14, 15) is necessary for the relevant outgoing feeder (in this case feeder 3); 0 means that neither the element nor any one of the opened circuit breakers is necessary for the relevant path with respect to the relevant output.

The other lines of the switching matrix M_S^i, are constructed in a similar way.

The switching matrix for outgoing line 3, M_S^3, of the one and a half circuit breaker, presented in Fig. 4.33 is:

$$M_S^3 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{vmatrix}$$

The matrix M_S^i is of size $K \times P$, where *K* is the number of functional blocks, *P* is the number of possible paths between the incoming feeders and output *i*.

A logical switching vector $\vec{S}_i$ (represented by a column) can be obtained from the switching matrix M_S^i:

$$S_i = AND \cdot \left(M_S^i[J_1], M_S^i[J_2], \ldots, M_S^i[J_P] \right) \tag{4.207}$$

where $M_S^i[J_K]$ is column K of the matrix M_S^i .

Eq. (4.207) represents logical AND operations among the columns of the switching matrix M_S^i, i.e. execution of logical AND operation between all the possible paths, with regards to each functional block.

For instance, by executing a logical AND on the switching matrix M_S^3, we obtain:

$$\vec{S}_3 = \begin{vmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

where 1 means that the failed element or the circuit breakers that have opened under the failure are necessary for all the paths and 0 means that they are not necessary, at least for one path (with regard to the normally closed paths).

In a similar way, repair logical vectors will be built for the normally closed paths, $\vec{R}_C^i$, and for the normally open paths, $\vec{R}_0^i$, with regard to the relevant output i.

The vectors $\vec{R}_C^i, \vec{R}_0^i$ can be derived from the path matrices $\vec{A}_C^i, \vec{A}_0^i$, as follows:

$$\vec{R}_C^i = AND(A_C^i[J_1], A_C^i[J_2], ..., A_C^i[J_{P_C}]) \tag{4.208}$$

$$\vec{R}_0^i = AND(A_0^i[J_1], A_0^i[J_2], ..., A_0^i[J_{P_C}]) \tag{4.209}$$

where $A_C^i[J_K]$ is column K of the normally closed paths matrix A_C^i, with regard to output I, $A_0^i[J_K]$ is column K of the normally open path matrix A_0^i, with regard to outgoing feeder *I*, P_C is the possible number of normally closed paths among all the incoming feeders and the relevant outgoing feeder *I*, P_O is the possible number of the normally open paths between all the incoming feeders and the relevant outgoing feeder *i*.

For instance, by performing a logical AND operation on the path matrix A_C^3 with regard to the one and a half circuit-breaker substation scheme (Fig. 4.33), for outgoing feeder 3, $\vec{R}_C^3$ takes the form:

$$\vec{R}_C^3 = \begin{vmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

where "0" means that the element under repair is not necessary for at least one path, 1 means that the element under repair is necessary for all the normally closed paths.

By combining the logical vectors $\vec{R}_C^i, \vec{S}_i$, it is possible to obtain the effect of each failure, i.e. to obtain an indication whether a failure in a certain element causes an energy interruption, and if so, how the restoration of energy to the consumers is performed, whether by switching or by repair operation.

The combination of vectors $\vec{R}_C^i, \vec{S}_i$ will be performed by an algebraic summation as follows:

$$\vec{I}_N^i = \vec{R}_C^i + \vec{S}_i \tag{4.210}$$

Vector $\vec{I}_N^i$ is called the indication vector, with regard to one failure for outgoing feeder *i*.

For example, by adding vectors $\vec{R}_C^3, \vec{S}_3$ obtained for one and a half circuit-breaker substation scheme (Fig. 4.33) with regard to outgoing number 3 $\vec{I}_N^3$ takes the form:

$$\vec{I}_N^3 = \left|\begin{array}{c|c} 0 & - \\ 0 & - \\ 2 & R \\ 0 & - \\ 0 & - \\ 0 & - \\ 0 & - \\ 0 & - \\ 0 & - \\ 0 & - \\ 1 & S_w \\ 0 & - \\ 0 & - \\ 0 & - \\ 0 & - \end{array}\right.$$

where 0 means that the failure in the relevant element does not cause outage of transformer *i* (in this case transformer number 3); 1 means that the failure in the relevant element causes outage of transformer *i* for the switching time T_S only (1 can only be obtained from the switching vector); 2 means that the failure in the relevant element causes outage of transformer *i* for the repair time T_R (2 is composed from 1 appearing in the repair vector and from 1 appearing in the switching vector. This is obvious because if 1 appears in the repair vector it will appear in the switching vector).

For schemes in which normally open paths also exist, the indication will be performed by combining the vectors $\vec{R}_0^i$ and $\vec{I}_N^i$ in a two dimensional matrix (two columns of which represent $\vec{I}_N^i$ and $\vec{R}_0^i$ respectively).

The matrix row (0, 0) means that a failure in the relevant element does not cause an energy interruption to the consumers. The matrix rows (1,0) or (2,0) mean that energy can be restored to the consumers by a switching operation alone and the matrix row (2,1) means that it is necessary to perform repair operations on the failed element in order to restore energy to the consumers.

The indication should be performed for each outgoing feeder and an indication matrix MI with dimension $I_0 \times K$ should be obtained instead of the indication vector, where I_0 is the number of outputs and K is the number of functional blocks.

The terms of indication matrix MI can take value of 0 (no interruption), of 1 (switching) and of 2 (repair).

Intermediate results of the logical algorithm described above with regard to one failure for the one and a half circuit-breaker substation scheme and for the double busbar substation scheme are presented in the following:

The indication matrices MI (result of the software) for one and a half circuit-breaker scheme described in Fig. 4.33 and for the double busbar substation scheme described in Fig. 4.34 respectively takes the form:

one and a half circuit-breaker scheme			
Element	Transformer No		
	3	4	5
1	0	0	0
2	0	0	0
3	2	0	0
4	0	2	0
5	0	0	2
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0	0
10	0	0	1
11	1	0	0
12	0	1	0
13	1	0	0
14	0	1	0
15	0	0	1

and

double busbar scheme			
Element	Transformer No		
	3	4	5
1	0	0	0
2	0	0	0
3	2	0	0
4	0	2	0
5	0	0	2
6	1	1	0
7	0	0	1
8	0	0	1
9	1	1	0
10	2	1	0
11	1	2	0
12	0	0	2
13	1	1	1

Logical algorithm for two failures

For every two elements i and j (of functional blocks), the failure effect should be analyzed for the following operational states (see also Table 4.10):

- $i_R \; j_R$ (two elements are under repair);
- $i_R \; j_S$ (element *I* is under repair, element *J* is in switching state);
- $i_S \; j_R$ (element *I* is in switching state, element *J* is under repair)

As for two failures (in elements *I* and *J*) an operational difference exists between substation schemes containing only normally closed paths, such as the substation scheme in Fig. 4.33, and substation schemes also containing normally open paths, such as the substation scheme in Fig. 4.34.

In schemes containing normally open paths it is, as far as the operation of the protection system is concerned, possible for the operational scheme to be different in case one of the elements is under repair. So that when a repair is carried out on one of the busbars (or on one of the disconnectors belonging to this bar), in the substation scheme described in Fig. 4.34, the other incoming and outgoing feeders will be connected to the other busbar, so that in case of failure in one of the elements it is possible that more circuit breakers will open than under normal operating states.

Therefore the two types of substation schemes should be dealt with separately.

1. Substation schemes containing only normally closed paths:

According to the above mentioned with respect to every two elements *i* and *j*, three states are to be investigated for each output:

$i_R, j_S \;; i_S, j_R \;; i_R, j_R.$

State i_R, j_R:

By performing the logical operation OR between rows *i* and *j* of the matrix A_C^i, a logical vector $\vec{R}_{ij}^i$ is obtained as

$$\vec{R}_{ij}^i = OR(A_C^i[I_i], A_C^i[I_j]) \tag{4.211}$$

The logical operation AND is performed over elements of vector $\vec{R}_{ij}^i$ such that in the obtained index

$$r_{ij}^i = AND\left(\vec{R}_{ij}^i\right) \tag{4.212}$$

$r_{ij}^i = 1$ means that at least one path exists in which not both elements are required. In this case it has to be investigated whether a switching operation has been performed i.e. investigate states i_S, j_R and i_R, j_S.

State i_R, j_S :

In this state a failure occurs in element *j* while a repair operation is being performed on element *i*. For the sake of the analysis, the OR operation is performed between row *i* of the paths matrix, A_C^i and row *j* of the switching matrix M_S and a logical vector $\vec{S}_{ij}^i$ is obtained as

$$\vec{S}_{ij}^i = OR(A_C^i[I_i], M_S^i[I_j]) \tag{4.213}$$

The operation AND is performed over the elements of vector $\vec{S}_{ij}^{i}$ and in the resulting index

$$S_{ij}^{i} = AND\left(\vec{S}_{ij}^{i}\right) \tag{4.214}$$

S_{ij}^{i}=0 means that at least one path was not disconnected by the circuit breakers (of course not by the elements either, which has been investigated in state i_R, j_R). Therefore there is no disconnection with regard to output *i*. $S_{ij}^{i} = 1$ means that the circuit breaker caused disconnection of all paths (not the elements, because $r_{ij}^{i} = 0$ has been obtained before). Therefore switching operations are required in order to restore energy supply to the consumers. So S_{ij}^{i}=0 corresponds to no failure state and S_{ij}^{i}= 1 corresponds to switching state. State i_S, j_R is analyzed in a similar way (by exchanging elements *i* and *j*).

2. Substation schemes in which normally open paths also exist:

As mentioned above, in substation schemes in which normally open paths exist, the substation schemes under an operating state in which one of the elements is under repair can differ from the normal operating state. This happens when the element under repair is a busbar. The difference can be noted only in switching states, as will be shown in the following.

Similar to substation schemes containing only normally closed paths, three operational states: i_S, j_R; i_R, j_S and i_R, j_R are to be analyzed for every two elements i, j.

State i_R, j_R:

The logical operation OR is performed over rows i, j of matrices A_0^i and A_C^i giving

$$\vec{R}_{Cij}^{i} = OR\left(A_C^i[I_i], A_C^i[I_j]\right) \tag{4.215}$$

$$\vec{R}_{0ij}^{i} = OR\left(A_0^i[I_i], A_0^i[I_j]\right) \tag{4.216}$$

On each of the vectors $\vec{R}_{0ij}^{i}, \vec{R}_{Cij}^{i}$, the logical operation AND is performed and a two dimensional vector

$$\vec{Q}_{ij}^{i} = \left[AND\left(\vec{R}_{Cij}^{i}\right), AND\left(\vec{R}_{0ij}^{i}\right)\right] \tag{4.217}$$

is obtained.

The AND operation is performed on the elements of vector $\vec{Q}^i_{ij}$ giving the index

$$T^i_{ij} = AND\left(\vec{Q}^i_{ij}\right) \tag{4.218}$$

in which $T^i_{ij} = 1$ means that at least one of the elements (*i* or *j*) is necessary for all the normally closed paths as well as for all the normally open paths, connecting among all of the incoming feeders and the relevant outgoing feeder *i*. Therefore the indication is that a repair operation is required in order to restore transformer *i*, which went out of use. If $T^i_{ij} = 0$ there are two possibilities:

- A failure in one of the elements, while the other is under repair, does not cause the forced outage of the transformer *i*.
- A failure in one of the elements, while the other is under repair causes outage of the transformer *i*, for the time required for a switching operation only ($i_R\ j_S\ , i_S\ j_R$).

Because of the above mentioned operational difference, when one of the elements under repair is a busbar, the analysis considers two separate cases depending on whether the element under repair is a busbar or not.

Furthermore, it should be noted that in a state in which one of the elements is under repair, it is possible that some of the normally open paths will be closed.

For the purpose of the analysis, further on, three switching matrices are generated, similar to the switching matrix M^i_S, which was built for the normally closed paths:

M^i_{SO} is the switching matrix for the normally open paths, according to the circuit breaker list similar to that of the switching matrix M^i_S.

M^i_{SCB} is the switching matrix for the normally closed paths, according to a additional list of circuit breakers that open if the element under repair is a busbar.

M^i_{SOB} is the switching matrix for the normally open paths, according to an additional list of circuit breakers that open if the element under repair is a busbar.

The value $T_{ij}^{i} = 0$ is obtained from one of the three possible realizations of the components of vector $\vec{Q}_{ij}^{i}$:

$$\vec{Q}_{ij}^{i} = (0,0), \vec{Q}_{ij}^{i} = (0,1), \vec{Q}_{ij}^{i} = (1,0)$$

In order to find out whether it is necessary to perform a switching or not, each one of the three possibilities will be analyzed separately.

1. $\vec{Q}_{ij}^{i} = (1,0)$ means that at least one of the elements (*i* or *j*) is necessary for all the normally closed paths and that there is at least one normally open path in which not both elements are necessary.

For the analysis of state i_R, j_S, the following logical operations are performed:

If i_R is not a busbar, the logical vector $\vec{V}_{ij}^{i}$ is generated as

$$\vec{V}_{ij}^{i} = OR\left(A_0^i[I_i], M_{S0}^{i}[I_j]\right) \tag{4.219}$$

if i_R is a busbar , the logical vector $\vec{V}_{ij}^{i}$ is generated as

$$\vec{V}_{ij}^{i} = OR\left(A_0^i[I_i], M_{S0}^{i}[I_i], M_{SCB}^{i}[I_j], M_{SOB}^{i}[I_j]\right) \tag{4.220}$$

and the index V_{ij}^{i} is obtained by

$$V_{ij}^{i} = AND\left(\vec{V}_{ij}^{i}\right) \tag{4.221}$$

$V_{ij}^{i} = 1$ corresponds to a switching operation; $V_{ij}^{i} = 0$ if the failure does not cause the outage of the transformer *i*.

2. $\vec{Q}_{ij}^{i} = (0,1)$ if at least one of the elements (*i* or *j*) is necessary for all the normally open paths and that there is at least one normally closed path in which not both elements are required.

Similar to the previous case, the analysis will be performed as follows:

$$\vec{V}_{ij}^{i} = OR\left(A_C^i[I_i], M_S^i[I_j]\right), \; V_{ij}^{i} = AND\left(\vec{V}_{ij}^{i}\right) \tag{4.222}$$

if i_R is not a busbar and

$$\vec{V}_{ij}^{i} = OR\left(A_C^i[I_i], M_S^i[I_j], M_{SOB}^{i}[I_j]\right) \tag{4.223}$$

$$V_{ij}^{i} = AND\left(\vec{V}_{ij}^{i}\right) = \begin{cases} 1, \text{switching} \\ 0, \text{no failure} \end{cases} \tag{4.224}$$

if i_R is a busbar.

3. If $Q_{ij} = (0,0)$, the analysis is similar to the one in the previous case.

The same analysis should be performed for the state i_S, j_R.

It should be noted that the analysis in the case of failure and maintenance is similar to the analysis in the case of two failures.

Logical algorithm for the failure states of the circuit breaker

Based on the circuit breaker model described in Fig. 4.32, there are additional failure states caused by two types of failures related to the circuit breaker.

States 4, 8, 9, 12 in Fig. 4.32 are caused by the circuit breaker failure to operate.

States 7, 14, 15, 16 are caused by a false operation of the circuit breaker (or of the protection system).

As far as the failure effects are concerned, the analysis for some of the above failure states is similar to the analysis performed for one failure or for two failures.

The situation is different for state 9 (B_S^3, C_S). The difference is a result of the fact that in this state further circuit breakers, protecting the failed circuit breaker B, open. Based on the definition of the protection zones, each element *j* has N_B^j circuit breakers with serial numbers $L_j, K_j, ..., q_j$ that open in the case of failure of element *j*. If one of the circuit breakers, say L_j fails to operate (and the rest of the circuit breakers $K_j, ..., q_j$ operate normally), further $N_B^{L_j}$ circuit breakers with serial numbers $L_{L_j}, K_{L_j}, ..., q_{L_j}$, protecting the circuit breaker L_j, will operate.

For the purpose of analyzing the failures related to this state, the following logical operations are performed:

$$\vec{S}^{\,i}_{j,L_j} = OR\left(M^{\,i}_{SC}\left[I_j\right], M^{\,i}_{SC}\left[I_{Lj}\right]\right) \tag{4.225}$$

$$S^{\,i}_{j,L_j} = AND\left(\vec{S}^{\,i}_j, L_j\right) = \begin{cases} 1, \text{switching} \\ 0, \text{no failure} \end{cases} \tag{4.226}$$

A similar operation is performed with regard to other circuit breakers $K_{L_j}, ..., q_{L_j}$, protecting component *j*.

4.2.6 SharonSoft Reliability Software

The *SharonSoft* program is a reliability analysis tool of substation layout configuration, based on FMEA event tree

The program runs under Microsoft Windows and is capable of analyzing large substation scheme, like: single and double busbars, one and a half circuit breaker.

Users may feed the protection substation matrix and the failure and maintenance data of the scheme components. The source file is a Microsoft Excel spreadsheets file.

The components are divided into two groups: active and passive components. Lines, cables, transformers, disconnectors are included to the first group and the circuit breaker is included in the second group.

The components data are: component failure rate λ (1/Yr); component repair time T_R (hour); component maintenance rate λ_M (1/Yr); component maintenance duration T_M (hour), (1/Yr); component switching duration T_S (hour).

Fig. 4.35 presents the flow chart of the *SharonSoft* program.

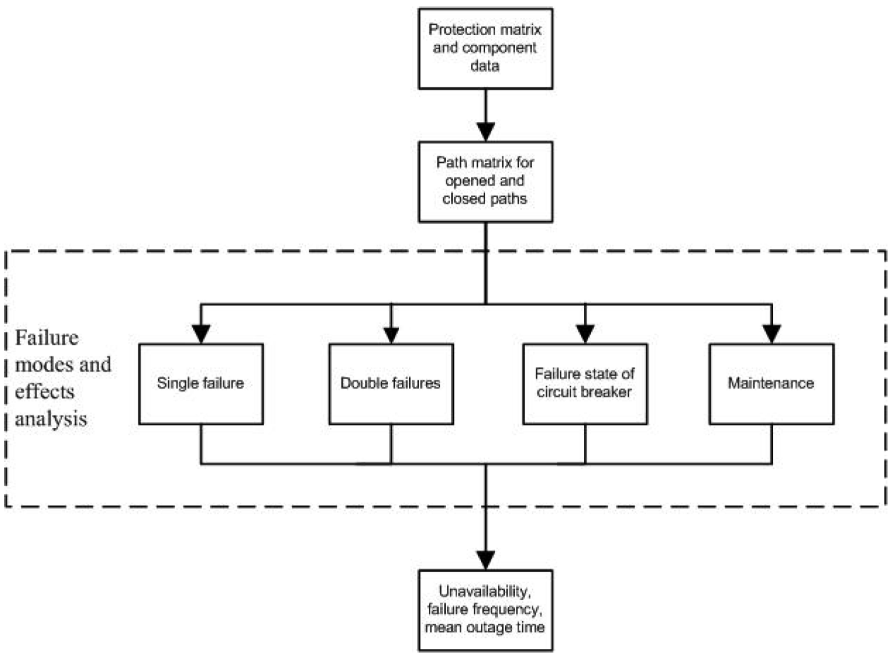

Fig. 4.35. Flow chart of the *SharonSoft* program

Numerical examples

The suggested algorithm performs the FMEA for the following failure states:

- One failure (occurrence of one failure in one of the substation scheme components).
- Two failures (occurrence of a failure in one of the substation scheme components during repair on another component of the substation scheme).
- Failure and maintenance (occurrence of a failure in one of the substation scheme components during maintenance work on another component).
- Failures of the circuit breaker (circuit breaker failure to operate and circuit breaker false operation).

Based on the FMEA, the contribution of the reliability parameters was calculated for each one of the above failure states.

The reliability parameters (the failure frequency, F_T and the failure probability, P_T) for the entire substation scheme are obtained as the sum of the partial contributions of the various failure states (mutually independent). The mean outage time T_D is obtained as P_T/F_T.

Two numerical examples of calculating the reliability parameters, related to one and a half circuit-breaker configuration and to double busbar configuration are presented in the following.

Example 4.2

The one and a half circuit-breaker configuration, which includes two incoming feeders and three outgoing feeders, is presented in Fig. 4.33.

Input data. The input data (the reliability parameters of the substation components) are presented in Table 4.12.

Table 4.12. Reliability parameters of substation components

Component	Failure rate (1/Yr)	Repair time (h)	Maintenance rate (1/Yr)	Maintenance duration (h)
Busbar	$\lambda_{BB}=0.024$	$T_{R_{BB}}=2$	0.4	8
Disconnector	$\lambda_I=0.002$	$T_{R_I}=2$	0.4	8
Circuit breaker	$\lambda_{CB}=0.01$	$T_{R_{CB}}=12$	0.4	8

The above scheme consists of 15 functional blocks. Based on (4.2) and Table 4.12, the equivalent reliability parameters of the functional blocks are calculated from the data of the components composing the blocks, as follows:

$$\lambda_2 = \lambda_1 = 3\lambda_I = 3 \times 0.002 = 0.006 \ \left[\ 1/Yr\ \right]$$

$$T_{R_2} = T_{R_1} = \frac{1}{\mu_1} \cong \frac{3\lambda_I / \mu_I}{\lambda_1} = 2[h]$$

$$\lambda_3 = \lambda_4 = \lambda_5 = 2\lambda_I = 2 \times 0.002 = 0.004 \ \left[\ 1/Yr\ \right]$$

$$T_{R_3} = T_{R_4} = T_{R_5} \cong \frac{2\lambda_I / \mu_I}{\lambda_1} 2[h]$$

$$\lambda_7 = \lambda_6 = 3\lambda_I + \lambda_{BB} = 3 \times 0.002 + 0.024 = 0.03 \ \left[\ 1/Yr\ \right]$$

$$T_{R_7} = T_{R_6} \cong \frac{3\lambda_I / \mu_I + \lambda_{BB} / \mu_{BB}}{\lambda_6} = 2[h] \quad \lambda_k = \lambda_{CB} = 0.1 \ [1/Yr] \text{ and}$$

$$T_{R_k} = T_{R_{CB}} = 12 \ [h] \text{ for } 8 \le k \le 15$$

$$f_{M_k} = 0.4 \ [1/Yr] \text{ and } T_{M_k} = 8 \ [h] \text{ for } 1 \le k \le 7$$

$$f_{M_k} = 0.1 \ [1/Yr] \text{ and } T_{M_k} = 8 \ [h] \text{ for } 8 \le k \le 15$$

The failure rate and the rate of false operations of a circuit breaker are respectively $\rho = 0.05$ and $\lambda_2 = 0.16$ [1/Yr].

The connection matrix and data describing the protection system take the form:

Element connection matrix

15	2	1	2	3	3	4	5							
0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	0	0	0	0	0	0	0

Activated Breakers			
2		8	11
2		9	12
2		11	13
2		12	14
2		10	15
3	8	9	10
3	13	14	15
3	9	10	11
3	8	10	12
3	8	9	15
2		8	13
2		9	14
3	14	15	11
3	13	15	12
3	13	14	15

The first line (before the connection matrix) defines that the scheme consists of 15 functional blocks, 2 incoming feeders (number – 1 and 2) and 3 outgoing feeders (number 3, 4 and 5).

The order of the matrix is 15x15 (according to the number of functional blocks);

If 0 appears in row i and column j, this means that there is no connection between the elements i and j.

The circuit breaker data for each element include the number of the circuit breakers, which operate in case of a failure in the relevant element, and their serial numbers.

For instance, in element number 7, 3 circuit breakers will operate, the serial numbers of which are 9, 10, 11.

Software results for the one and a half circuit-breaker substation scheme. The results obtained by the considered algorithm for the one and a half circuit-breaker configuration are as follows:

The possible path matrices A_C^3, A_C^4, A_C^5 for the incoming feeders 1 and 2 and outgoing feeders 3, 4, 5, respectively take the form:

$$A_C^3: \begin{vmatrix} 1&1&1&1&1&0&0\\ 0&1&0&1&1&1&1\\ 1&1&1&1&1&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&0&1\\ 0&1&1&1&1&0&1\\ 0&1&1&0&1&1&1\\ 0&1&1&1&1&0&0\\ 0&1&0&1&0&0&1\\ 0&0&1&0&1&0&1\\ 1&0&0&1&1&0&0\\ 0&1&0&0&1&1&0\\ 0&1&1&0&0&1&1\\ 0&1&0&0&1&1&0\\ 0&0&1&0&1&0&1 \end{vmatrix} \quad A_C^4: \begin{vmatrix} 1&1&1&1&0&1&0\\ 1&1&0&0&1&1&1\\ 0&1&1&0&0&1&0\\ 1&1&1&1&1&1&1\\ 0&1&0&1&0&0&1\\ 1&1&0&1&0&1&1\\ 0&1&1&1&0&1&1\\ 1&0&0&1&0&1&0\\ 1&1&0&0&0&1&1\\ 0&1&0&1&0&0&1\\ 0&1&1&0&0&1&0\\ 1&1&0&0&1&0&0\\ 0&1&1&0&0&1&0\\ 0&0&1&1&0&1&1\\ 0&1&0&1&0&0&1 \end{vmatrix} \quad A_C^5: \begin{vmatrix} 1&1&1&1&1&0&1&0\\ 0&1&0&1&1&1&1&1\\ 0&1&1&0&1&0&1&0\\ 0&1&0&1&1&0&0&1\\ 1&1&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&0\\ 0&1&1&1&1&0&1&1\\ 1&0&0&1&1&0&1&0\\ 0&1&0&1&0&1&1&0\\ 1&1&0&0&1&1&0&0\\ 0&1&1&0&1&0&1&0\\ 0&1&0&1&1&0&0&1\\ 0&1&1&0&1&0&1&0\\ 0&1&0&1&1&0&0&1\\ 0&0&1&1&0&0&1&1 \end{vmatrix}$$

The failure indication matrix for single failure takes the form:

Element	Transformer No		
	3	4	5
1	0	0	0
2	0	0	0
3	2	0	0
4	0	2	0
5	0	0	2
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0	0
10	0	0	1
11	1	0	0
12	0	1	0
13	1	0	0
14	0	1	0
15	0	0	1

In the above matrix 1, 2 and 0 indicate a switching operation, a repair operation and a case when the failure in the relevant element does not cause the outage of the transformer respectively.

For instance, a failure in element 3 causes forced outage of transformer number 3 for the duration of the repair, whereas a failure in element 12 causes a forced outage of element 4 for the duration of the switching.

Table 4.13 presents FMEA results for two failures.

Table 4.13. Two failures effect analysis for one and a half circuit-breaker configuration (S- Switching, R- Repair, 0 - No outage, 1- Switching outage state, 2- Repair outage state)

Element		1			2			3			4			5			6			7			8		
Outlet		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
	rr				2	2	2	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	0	0
1	sr				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rr							2	0	0	0	2	0	0	0	2	0	0	0	0	2	0	0	0	0
2	sr							0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs							0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
	rr										2	2	0	2	0	2	2	0	0	2	0	0	2	0	0
3	sr										0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs										0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rr													0	2	2	0	2	0	0	2	0	0	2	0
4	sr													0	0	0	0	0	0	0	0	0	0	0	0
	rs													0	0	0	0	0	0	0	0	0	1	0	1
	rr																0	0	2	0	0	2	0	0	2
5	sr																0	0	0	0	0	0	0	0	0
	rs																0	0	0	0	0	0	0	0	0
	rr																			0	0	2	0	0	0
6	sr																			0	0	0	0	0	0
	rs																			0	0	0	0	0	0
	rr																						0	0	0
7	sr																						0	0	0
	rs																						1	0	1

Table 4.13. (cont.)

Element		9			10			11			12			13			14			15		
Outlet		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
	rr	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
1	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	1	1	1	0	0	1	1	0	0	1	1	1	0	0	0	1	1	0	1	0	1
	rr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
2	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	0	0	0	1	1	1	1	0	1	0	1	1	0	0	0	0	0	1	1
	rr	2	0	0	2	0	0	2	0	0	2	0	0	2	0	0	2	0	0	2	0	0
3	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	1	1	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1
	rr	0	2	0	0	2	0	0	2	0	0	2	0	0	2	0	0	2	0	0	2	0
4	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1
	rr	0	0	2	0	0	2	0	0	2	0	0	2	0	0	2	0	0	2	0	0	2
5	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0
	rr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
6	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	0	0	0	1	1	0	0	0	1	0	1	0	1	0	1	1	0	0	0
	rr	0	0	0	0	0	2	2	0	0	0	2	0	0	0	0	0	0	0	0	0	0
7	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1
	rr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	sr	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	0	1	0	0	1
	rs	0	0	0	0	0	1	1	0	0	0	1	0	1	0	0	0	1	0	0	0	1
	rr				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	sr				0	0	0	1	1	1	0	0	0	0	1	1	0	1	0	0	0	1
	rs				0	0	1	1	0	0	0	1	0	1	0	0	0	1	0	0	0	1
	rr							0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
10	sr							0	0	1	0	0	1	0	0	1	0	0	1	0	0	0
	rs							1	0	0	0	1	0	1	0	1	0	1	1	0	0	0
	rr										0	0	0	2	0	0	0	0	0	0	0	0
11	sr										1	0	0	0	0	0	1	0	0	1	0	0
	rs										0	1	0	0	0	0	1	1	0	1	0	1
	rr													0	0	0	0	2	0	0	0	0
12	sr													0	1	0	0	0	0	0	1	0
	rs													1	1	0	0	0	0	0	1	1
	rr																0	0	0	0	0	0
13	sr																1	0	0	1	0	0
	rs																0	1	0	0	0	1
	rr																			0	0	0
14	sr																			0	1	0
	rs																			0	0	1

Tables 4.14, 4.15 and 4.16 present the obtained reliability parameters for the cases of the forced outage of single transformer, two transformers and three transformers, respectively.

Table 4.13 contains the FMEA results for a failure of two elements. The two elements can be: both on repair state (rr), one switching and other repair (sr), one repair and other switching (rs).

Table 4.14. Reliability parameters for forced outage of single transformer (R, r – Repair, S, s – Switching, B –Breaker, C–Component, M – Maintenance)

Type of system failure state		Probability	Frequency [1/year]	Mean duration [hours]
Single	R	2.74E-06	1.20E-02	2.00E+00
Failure	S	1.37E-04	6.00E-01	2.00E+00
Subtotal		1.40E-04	6.12E-01	2.00E+00
Double	RR	6.35E-08	1.20E-04	4.65E+00
Failure	SR	1.08E-07	5.55E-04	1.70E+00
Subtotal		1.71E-07	6.75E-04	2.22E+00
Maintenance	MR	6.61E-07	1.32E-03	4.40E+00
& Failure	MS	2.54E-07	1.11E-03	2.00E+00
Subtotal		9.15E-07	2.43E-03	3.30E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
failures	BrCr	1.91E-05	2.87E-02	5.83E+00
to operate	BsCs	1.63E-05	7.13E-02	2.00E+00
Sub Total		3.54E-05	9.99E-02	3.10E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
false	BrCs	9.51E-08	5.41E-04	1.54E+00
operations	BrCr	2.64E-08	1.35E-04	1.71E+00
Subtotal		1.22E-07	6.76E-04	1.58E+00
TOTAL		1.76E-04	7.16E-01	2.16E+00

Table 4.15. Reliability parameters for forced outage of two transformers

Type of system failure state		Probability	Frequency [1/year]	Mean duration [hours]
Single	R	0.00E+00	0.00E+00	0.00E+00
Failure	S	0.00E+00	0.00E+00	0.00E+00
Subtotal		0.00E+00	0.00E+00	0.00E+00
Double	RR	2.50E-12	2.19E-08	1.00E+00
Failure	SR	2.58E-08	1.35E-04	1.68E+00
Subtotal		2.58E-08	1.35E-04	1.68E+00
Maintenance	MR	1.60E-09	8.77E-06	1.60E+00
& Failure	MS	1.00E-07	4.38E-04	2.00E+00
Subtotal		1.02E-07	4.47E-04	1.99E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
failures	BrCr	0.00E+00	0.00E+00	0.00E+00
to operate	BsCs	6.18E-06	2.71E-02	2.00E+00
Sub Total		6.18E-06	2.71E-02	2.00E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
false	BrCs	1.50E-08	9.50E-05	1.39E+00
operations	BrCr	0.00E+00	0.00E+00	0.00E+00
Subtotal		1.50E-08	9.50E-05	1.39E+00
TOTAL		6.32E-06	2.78E-02	2.00E+00

Symbols 1, 2 and 0 indicate a switching operation, a repair operation and a case when the failure in the relevant element does not cause the outage of the transformer respectively.

For instance, a common failure mode of elements 1 and 2 causes a forced outage of all the transformers for the repair duration. And a failure in element 2 and a switching in element 8 causes a forced outage of all the transformers for the switching duration, etc.

Table 4.16. Reliability parameters for forced outage of three transformers

Type of system failure state		Probability	Frequency [1/year]	Mean duration [hours]
Single	R	0.00E+00	0.00E+00	0.00E+00
Failure	S	0.00E+00	0.00E+00	0.00E+00
Subtotal		0.00E+00	0.00E+00	0.00E+00
Double	RR	1.88E-12	1.64E-08	1.00E+00
Failure	SR	6.38E-09	3.31E-05	1.69E+00
Subtotal		6.38E-09	3.31E-05	1.69E+00
Maintenance	MR	8.01E-10	4.38E-06	1.60E+00
& Failure	MS	3.75E-08	1.64E-04	2.00E+00
Subtotal		3.83E-08	1.69E-04	1.99E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
failures	BrCr	0.00E+00	0.00E+00	0.00E+00
to operate	BsCs	2.06E-06	9.03E-03	2.00E+00
Sub Total		2.06E-06	9.03E-03	2.00E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
false	BrCs	1.00E-08	5.11E-05	1.71E+00
operations	BrCr	0.00E+00	0.00E+00	0.00E+00
Subtotal		1.00E-08	5.11E-05	1.71E+00
TOTAL		2.12E-06	9.28E-03	2.00E+00

Example 4.3

A double busbar configuration, consisting of two incoming feeders and three outgoing feeders is presented in Fig. 4.34.

Input data. Similar to the previous example the reliability parameters for the 13 functional blocks are as follows:

$$\lambda_1 = \lambda_2 = 0.006 \; [\; 1/Yr \;], \qquad \lambda_3 = \lambda_4 = \lambda_5 = 0.004 \; [\; 1/Yr \;]$$

$$\lambda_6 = \lambda_7 = 0.036 \; [\; 1/Yr \;], \; \lambda_k = 0.1 \; [\; 1/Yr \;] \text{ for } 8 \le k \le 13$$

$$T_{R_k} = 2 \; [h], \text{for } 1 \le k \le 7, \; T_{R_k} = 12 \; [h] \text{for } 8 \le k \le 13,$$

$$f_{M_k} = 0.4 \; [1/Yr] \text{for } 1 \le k \le 7, \; f_{M_k} = 0.1 \; [1/Yr] \text{for } 8 \le k \le 13,$$

$$T_{M_k} = 8 \;\; [h] \text{ for } 1 \leq k \leq 13.$$

The connection matrix and the substation scheme data take the form:

Element connection matrix

13	2	1	2	3	3	4	5				6	7
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	2	1	1	1	2	1
0	0	0	0	0	0	0	1	2	2	2	1	1
1	0	0	0	0	2	1	0	0	0	0	0	0
0	1	0	0	0	1	2	0	0	0	0	0	0
0	0	1	0	0	1	2	0	0	0	0	0	0
0	0	0	1	0	1	2	0	0	0	0	0	0
0	0	0	0	1	2	1	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0

Activated Breakers					
1	8				
1	9				
1	10				
1	11				
1	12				
4	9	10	11	13	
3	8	12	13		
2	12	13			
3	10	11	13		
3	9	11	13		
3	9	10	13		
2	8	13			
5	8	9	10	11	12

Additional Activated Breakers for Busbar					
1					8
1					9
1					10
1					11
1					12
2					12
3			8		11
3		9	10		11
2	9		10		8
2			12		8
2			12	12	8
3			11	10	9
5	8	9	10	11	12

In the connection matrix 2 indicates an open connection between the elements. The other data is the same as in the previous example.

Software results for the double busbar configuration. The results obtained by the algorithm for the double busbar substation configuration are as follows:

$$A_C^3 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{vmatrix} \quad A_C^4 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{vmatrix} \quad A_C^5 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$A_O^3 = \begin{vmatrix}
1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1&1 \\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&1&1&1&0&1&1&1&1&1&1&1&1&1&0&1&1&1 \\
0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\
1&1&1&1&1&1&1&1&1&1&0&0&0&1&0&0&0&0 \\
0&1&0&0&0&1&0&0&0&1&1&1&1&1&1&1&1&1 \\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\
0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0 \\
0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&1&0 \\
0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&1
\end{vmatrix}$$

$$A_O^4 = \begin{vmatrix}
1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&1&1&1&0&1&1&1&1&1&1&1&1&1&0&1&1&1 \\
0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\
1&1&1&1&1&1&1&1&1&1&0&0&0&1&0&0&0&0 \\
0&1&0&0&0&1&0&0&0&1&1&1&1&1&1&1&1&1 \\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\
0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&1&0 \\
0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&1
\end{vmatrix}$$

$$A_O^5 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{vmatrix}$$

where A_C^3, A_C^4, A_C^5 are the matrices of possible paths between incoming feeders 1,2 and outgoing feeders 3, 4, 5, respectively for the normally closed paths; A_0^3, A_0^4, A_0^5 are the matrix of possible paths between incoming feeders 1, 2 and outgoing feeders 3, 4, 5, respectively for the normally open paths.

The single failure indication matrix takes the form:

Element	Transformer No		
	3	4	5
1	0	0	0
2	0	0	0
3	2	0	0
4	0	2	0
5	0	0	2
6	1	1	0
7	0	0	1
8	0	0	1
9	1	1	0
10	2	1	0
11	1	2	0
12	0	0	2
13	1	1	1

Table 4.17 presents the FMEA for double failure. The obtained reliability parameters for a forced outage of one, two and three transformers are presented in Tables 4.18, 4.19 and 4.20 respectively.

Table 4.17. Double failure effect analysis for a double busbar configuration

Element		1			2			3			4			5			6			7		
Outlet		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
	rr				2	2	2	2	0	0	0	2	0	0	0	2	0	0	0	0	0	0
1	sr				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs				0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
	rr							2	0	0	0	2	0	0	0	2	0	0	0	0	0	0
2	sr							0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs							0	0	0	0	0	0	0	0	0	1	1	0	1	1	1
	rr										2	2	0	2	0	2	2	0	0	2	0	0
3	sr										0	0	0	0	0	0	0	0	0	0	0	0
	rs										0	0	0	0	0	0	0	1	0	0	0	1
	rr													0	2	2	0	2	0	0	2	0
4	sr													0	0	0	0	0	0	0	0	0
	rs													0	0	0	1	0	0	0	0	1
	rr																0	0	2	0	0	2
5	sr																0	0	0	0	0	0
	rs																1	1	0	0	0	0
	rr																			2	2	2
6	sr																			0	0	0
	rs																			0	0	0

Table 4.17. (cont.)

Element		8			9			10			11			12			13		
Outlet		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
	rr	0	0	0	2	2	2	2	0	0	0	2	0	0	0	2	0	0	0
1	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	1	0	0	0	0	1	1	1	0	1	0	0	0	1	1	1
	rr	2	2	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0
2	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	0	1	1	0	0	1	0	1	0	0	1	1	0	1	1	1
	rr	2	0	0	2	0	0	2	0	0	2	2	0	2	0	2	2	0	0
3	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	1	1
	rr	0	2	0	0	2	0	2	2	0	0	2	0	0	2	2	0	2	0
4	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	1	0	1
	rr	0	0	2	0	0	2	2	0	2	0	2	2	0	0	2	0	0	2
5	sr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	rs	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	1	1	0
	rr	0	0	0	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0
6	sr	1	1	1	1	1	0	0	1	0	1	0	0	1	1	0	1	1	0
	rs	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
	rr	0	0	0	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0
7	sr	0	0	1	1	1	1	0	0	1	0	0	1	0	0	0	0	0	1
	rs	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
	rr				2	2	2	2	0	0	0	2	0	0	0	2	0	0	0
8	sr				0	0	0	0	0	1	0	0	1	0	0	0	0	0	1
	rs				0	0	0	0	1	1	1	0	1	0	0	0	1	1	1
	rr							2	0	0	0	2	0	0	0	2	0	0	0
9	sr							0	1	0	1	0	0	1	1	0	1	1	0
	rs							0	1	0	1	0	0	1	1	0	1	1	1
	rr										2	2	0	2	0	2	2	0	0

Table 4.17. (cont.)

Element		8			9			10			11			12			13		
Outlet		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
10	sr										0	0	0	0	1	0	0	1	0
	rs										0	0	0	0	0	0	0	1	1
	rr													0	2	2	0	2	0
11	sr													1	0	0	1	0	0
	rs													0	0	0	1	0	1
	rr																0	0	2
12	sr																0	0	0
	rs																1	1	0

Table 4.18. Reliability parameters for forced outage of single transformer

Type of system failure state		Probability	Frequency [1/year]	Mean duration [hours]
Single	R	4.14E-04	3.12E-01	1.16E+01
Failure	S	3.11E-05	1.36E-01	2.00E+00
Subtotal		4.45E-04	4.48E-01	8.70E+00
Double	RR	1.78E-07	2.95E-04	5.30E+00
Failure	SR	4.15E-08	2.13E-04	1.71E+00
Subtotal		2.20E-07	5.08E-04	3.79E+00
Maintenance	MR	5.96E-07	1.18E-03	4.43E+00
& Failure	MS	1.34E-07	5.88E-04	2.00E+00
Subtotal		7.30E-07	1.77E-03	3.62E+00
Breaker	Br	3.02E-05	2.21E-02	1.20E+01
failures	BrCr	3.77E-05	5.89E-02	5.61E+00
to operate	BsCs	3.03E-06	1.33E-02	2.00E+00
Sub Total		7.10E-05	9.42E-02	6.60E+00
Breaker	Br	1.10E-04	4.80E-01	2.00E+00
false	BrCs	5.75E-08	3.00E-04	1.68E+00
operations	BrCr	6.01E-08	3.07E-04	1.71E+00
Subtotal		1.10E-04	4.81E-01	2.00E+00
TOTAL		6.26E-04	1.03E+00	5.35E+00

The presented algorithm performs a failure and effect analysis, both for substation schemes with only normally closed paths and for substation schemes including both normally closed and normally open paths.

The algorithm calculates the reliability parameters for forced outage of any number of outgoing feeders (transformers).

The input required for the algorithm is minimal and enables a simple and clear input of data directly from the substation scheme drawing.

The algorithm obtains the rate of transformer forced outages and the mean repair time for any substation configuration. These parameters are required for calculating of the level of transformation reserve.

Table 4.19. Reliability parameters for forced outage of two transformers

Type of system failure state		Probability	Frequency [1/year]	Mean duration [hours]
Single	R	0.00E+00	0.00E+00	0.00E+00
Failure	S	7.67E-05	3.36E-01	2.00E+00
Subtotal		7.67E-05	3.36E-01	2.00E+00
Double	RR	5.71E-08	8.61E-05	5.81E+00
Failure	SR	2.98E-08	1.57E-04	1.66E+00
Subtotal		8.68E-08	2.43E-04	3.13E+00
Maintenance	MR	1.52E-07	2.85E-04	4.68E+00
& Failure	MS	1.42E-07	6.21E-04	2.00E+00
Subtotal		2.94E-07	9.06E-04	2.84E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
failures	BrCr	6.18E-06	9.03E-03	6.00E+00
to operate	BsCs	7.40E-06	3.24E-02	2.00E+00
Sub Total		1.36E-05	4.14E-02	2.87E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
false	BrCs	2.04E-08	1.23E-04	1.46E+00
operations	BrCr	1.62E-08	8.29E-05	1.71E+00
Subtotal		3.66E-08	2.05E-04	1.56E+00
TOTAL		9.07E-05	3.79E-01	2.10E+00

Table 4.20. Reliability parameters for forced outage of three transformers

Type of system failure state		Probability	Frequency [1/year]	Mean duration [hours]
Single	R	0.00E+00	0.00E+00	0.00E+00
Failure	S	2.28E-05	1.00E-01	2.00E+00
Subtotal				
Double	RR	2.28E-05	1.00E-01	2.00E+00
Failure	SR	1.92E-08	2.99E-05	5.62E+00
Subtotal		9.72E-09	5.41E-05	1.57E+00
Maintenance	MR			
& Failure	MS	2.89E-08	8.40E-05	3.02E+00
Subtotal		5.59E-08	1.23E-04	3.97E+00
Breaker	Br	7.84E-08	3.43E-04	2.00E+00
failures	BrCr			
to operate	BsCs	1.34E-07	4.67E-04	2.52E+00
Sub Total		0.00E+00	0.00E+00	0.00E+00
Breaker	Br	0.00E+00	0.00E+00	0.00E+00
false	BrCs	1.06E-05	4.66E-02	2.00E+00
operations	BrCr			
Subtotal		1.06E-05	4.66E-02	2.00E+00
TOTAL		0.00E+00	0.00E+00	0.00E+00

4.3 Reliability analysis of protection systems for high voltage transmission lines

4.3.1 Introduction

Protection systems play a crucial role in power systems. The increasing demand for electric power leads to the complexity of the network and, therefore, to increased search for the improvement of the reliability of the high voltage (161 kV and over) line system protections. Two types of line protection are mainly used in power systems: distance (z-protection) and differential protection ($\Delta - protection$) (CIGRE 1999). The reliability of these two types of line protection was studied in many research works (Johannesson et al. 2003, Anderson and Agarwal 1992, Ward et al. 2004) and their corresponding advantages and disadvantages are widely known. In order to improve protection system reliability, redundancy is often recommended (IEEE 1990, Declerq et al. 2002). Two following configurations of protection systems that use redundancy were suggested for the critical lines (CIGRE 1999, IEEE 1990). The first one is the configuration $\{z, z\}$, where two distance protections are connected in parallel in order to prevent protection system failure. The second one is the configuration $\{z, \Delta\}$, where a distance protection is connected in parallel with differential protection. Redundancy essentially changes system reliability, but concerning protection systems its impact to reliability may be controversial. Some reliability indices may be improved, but some may be worsened. For example, using redundancy in a protection system in order to prevent its missing operation will have a great impact on an increasing number of unwanted operations.

Therefore, reliability analysis for configurations $\{z, \Delta\}$ and $\{z, z\}$ is more complicated than for single protection devices. Such analysis has to be based on specified reliability indices that were introduced for protection systems by CIGRE (1999).

In this section the reliability analysis was performed in order to compare these two widely used configurations of protection system. Our description is based on the work (Lisnianski et al. 2006), where such analysis was primarily presented.

4.3.2 Description of protection system configurations

The configuration with two distance protections

The configuration {*z, z*} is presented in Fig. 4.36. Protected line connects two substations: R and G. Measurement transformers CT (current transformer) and VT (voltage transformer) measure a current and a voltage in the line. These signals from CT and VT are inputs to the main and redundant distance protection devices z' and z'' respectively for each substation R and G.

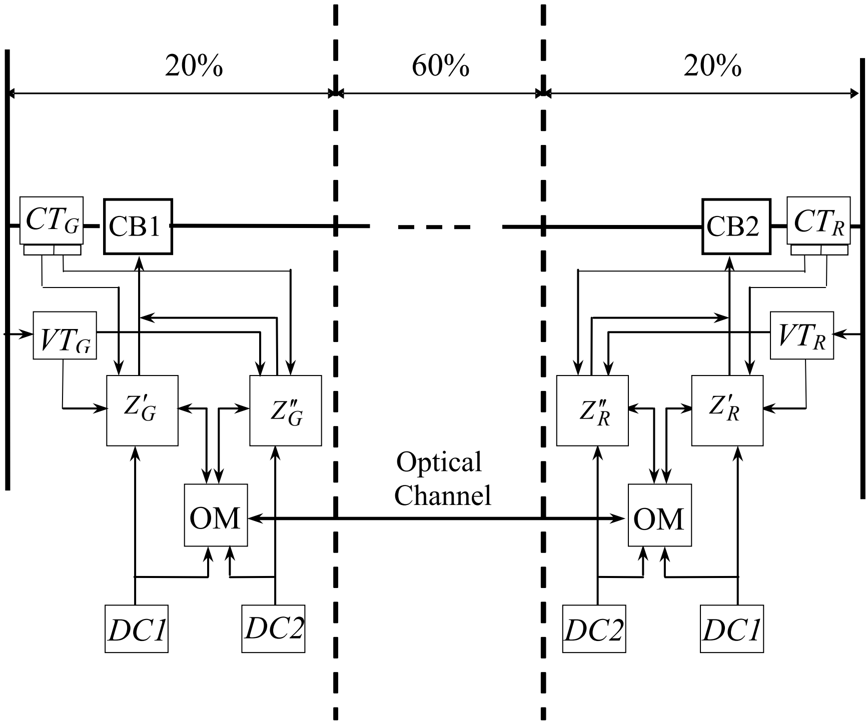

Fig. 4.36. Protection configuration {z,z}

Devices z' and z'' calculate the line impedance. If a short circuit was identified (calculated impedance value is lower than a preliminary determined set point), then the devices send a signal to corresponding circuit breaker (CB) in their substation. CB should be opened in order to isolate a short circuit point. Batteries DC1 and DC2 provide the power supply to all the substation's devices. In addition, by using a communication device

(OM) and optical communication channel the signal reaches protection devices in another substation. These devices, in their turn, send the signal to CB in their substation in order to open it and isolate the short circuit point from the second end of the line. For example, if a short circuit occurs near substation G, (the distance between the short circuit point and substation G is less than 20% of the line length), measuring transformers VT_G, CT_G measure the voltage and current and send these parameters to the protection devices $Z_G^{'}$ and $Z_G^{''}$.

The protection devices calculate the line impedance and compare its value with a set point. If a short circuit is detected (the impedance value is lower than the set point), the devices send corresponding signals to the circuit breaker CB1 to open it at first protection level. The operating at the first protection level is necessary for critical high voltage lines, where two circuit breakers should be opened (in order to isolate the point of short circuit from both sides of the line) during a time less then 100 ms after the short circuit occurred. So, in addition, the signal to open CB2 in substation R at the first protection level should be sent by using devices OM, optical channel and devices $Z_R^{'}$ and $Z_R^{''}$. In such a case, if the communication channel fails, the distance protection devices $Z_R^{'}$ and $Z_R^{''}$ detect a short circuit by themselves and can also open CB2, but only at the second protection level (in which the time of protection response is about 500 ms). In this section we study the protection system reliability for critical high voltage lines where operations according to only first protection level are needed.

The configuration with distance and differential protections

The configuration $\{z, \ \Delta\}$ is presented in Fig.4.37. As in the previous case the protected line connects substations G and R. Current and voltage transformers CT and VT measure a current and a voltage respectively. In order to perform differential protection, the differential protection device Δ should have a current measurement from two ends of the line. Differential protection device Δ_G receives a signal from current transformer CT_G directly and from current transformer CT_R by using the communication optical channel and differential protection device Δ_R and communication module *CM*. Based on these two signals from two current transformers, device Δ_G detects a short circuit in the line and sends the signal to open circuit breaker CB1. In the same way device Δ_R sends a signal to open CB2. At the same time distance protection device Z_G receives signals from current transformer CT_G and voltage transformer VT_G. When device Z_G detects a short circuit, it sends a signal to open CB1. By using the optical channel, device Δ_G and module CM the device Z_G also sends the signal to the device Z_R. This device in its turn sends the signal to open CB2.

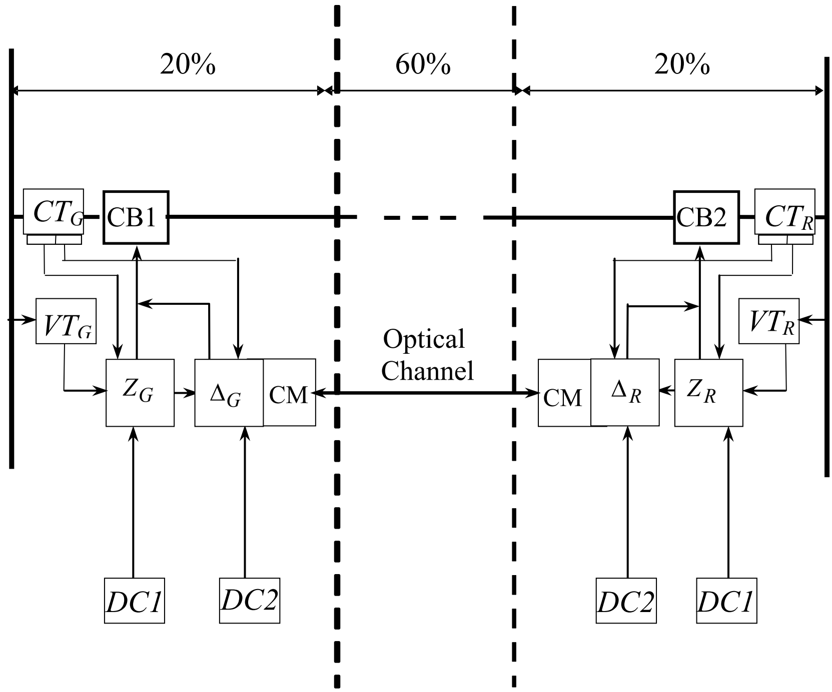

Fig. 4.37. Protection configuration {z,Δ}

Reliability data (MTTF and MTTR) for all modules are presented in Table 4.21.

Table 4.21. Reliability data for modules

Module	Mean Time to Failure (years)	Mean Time to Repair (hours)
VT, CT	715	6
Cbl	710	6
Δ	67	6
DC	43	6
OC	1810	84
OM	12	6
Z	67	6
CM	12	6

According to Table 4.21 the failure rate λ and the repair rate μ for any module can be defined as $\lambda = \frac{1}{MTTF}$ and $\mu = \frac{1}{MTTR}$.

4.3.3 Failure definition and reliability block diagrams for protection systems

According to CIGRE definition (CIGRE 1999), a protection system fails in the following cases.

– It doesn't respond when it should operate. Such failures are called *missing operation*.
– It operates when it should not operate. Such failures are called *unwanted operation*.

Both missing operations and unwanted operations are called *incorrect protection*.

According to (CIGRE 1999, Johannesson et al. 2003) the following index was recommended as a protection system reliability measure:

$$N_{CIGRE} = \frac{N_c}{N_c + N_f + N_u} = \frac{N_{dem} - N_{inop}}{N_{dem}}, \tag{4.227}$$

where N_c is the mean number of correct operations during time *T*, N_f is the mean number of missing operations during time *T*, N_u is the mean number of unwanted operations during time *T*, N_{dem} is the mean number of demands for the protection system to operate during time *T*.

Note, that

$$N_{inop} = N_u + N_f, \tag{4.228}$$

$$N_{dem} = N_c + N_f + N_u. \tag{4.229}$$

Demands for protection system to operate are relatively rare events. The mean time between demands arrivals is usually about 3-4 months for a 161 KV line with a length of 50 km. So, $N_{dem} \approx 3.5$ demands per year per 50 km of line and corresponding demand rate $\lambda_{dem} \approx 3.5 \ year^{-1}$.

A protection system serves a demand very quickly; a mean operation time T_{op} for the first protection level is usually less then 0.1 second. Therefore, most of the time a protection system is waiting for demands to arrive.

Usually engineers try to prevent missing operation of a protection system, because it is more dangerous than unwanted operation. In order to prevent missing operation the following scheme for connection of output contacts of protection devices is used (see Fig. 4.38).

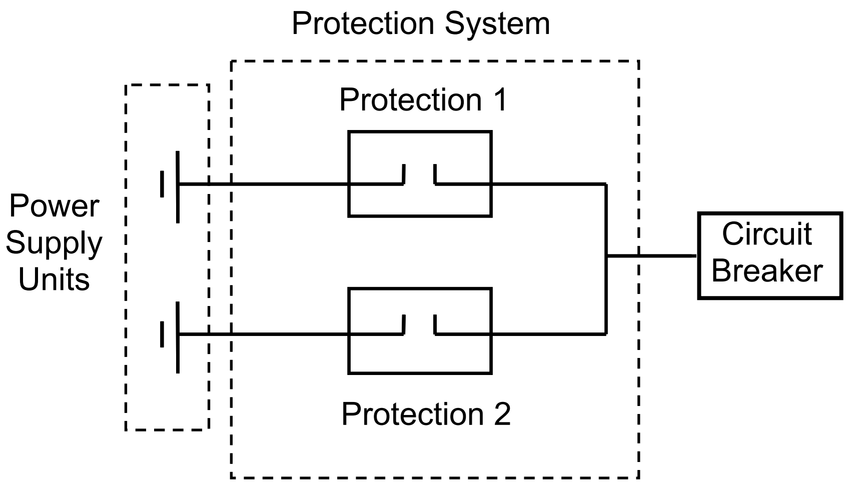

Fig. 4.38. Connection of output contacts

If protection device 1 has a failure of the missing operation type, device 2 operates and opens a circuit breaker. On the other hand, each unwanted operation in either device 1 or 2 will cause an unwanted circuit breaker opening. Therefore, the scheme of protection redundancy shown in Fig. 4.38 prevents missing operation, but it essentially decreases protection system reliability regarding unwanted operation.

Taking into account the importance of missing operation in this section we deal with this most important case and consider the protection system capability to prevent missing operation. Configurations $\{z,\ \Delta\}$ and $\{z,\ z\}$ are therefore compared according to their reliability, where the reliability is treated as the ability of the corresponding protection system to prevent a missing operation. As a reliability measure we can use the following index

$$N_{CIGRE}^{'} = \frac{N_c}{N_c + N_f} = \frac{N_{dem} - N_f}{N_{dem}}, \tag{4.230}$$

or the so-called, missing unreliability function $UR(t)$ which characterizes the probability of protection system missing operation during time t. Such index takes into account only missing operations.

In order to evaluate the reliability indices of the protection systems presented in Fig. 4.36 and Fig. 4.37 reliability block diagrams are built. The reliability block diagram for $\{z, \Delta\}$ protection system is presented in Fig. 4.39.

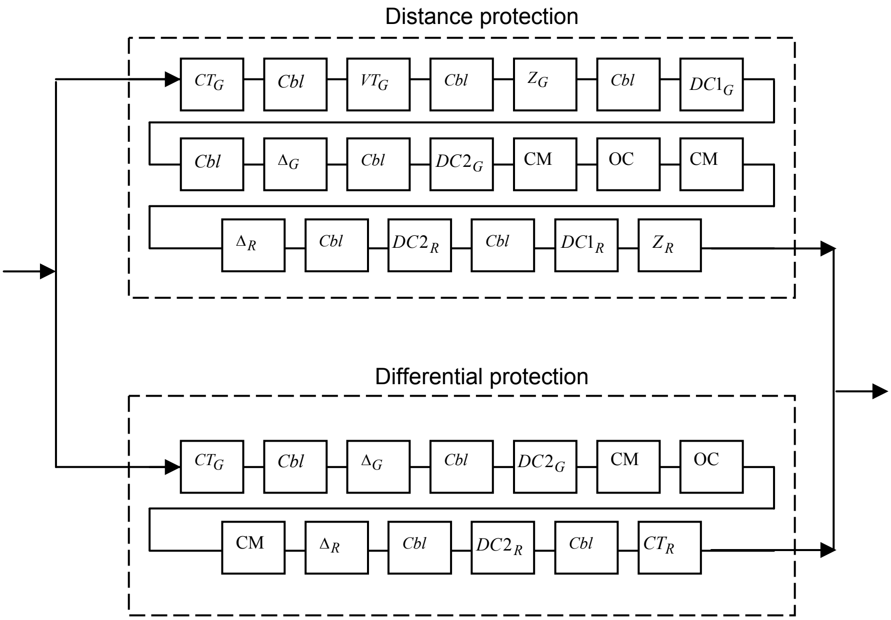

Fig. 4.39. Reliability block diagram for {z,Δ} protection system

It is important to emphasize that the distance protection and differential protection in this configuration are not independent. Distance and differential protections use the same optical channel OC and modules CM. In addition, the differential protection devices Δ_R and Δ_G have to work in order to provide the signal transmission by using optic channel.

By using the logical rule $B1 \& B2 \bigcup B1 \& B3 = B1 \& (B2 \bigcup B3)$ one can transform the reliability block diagram in Fig. 4.39 with dependent elements to an equivalent block diagram presented in Fig. 4.40 with independent elements. As can be seen from Fig. 4.40, united blocks B1, B2, B3 consist of corresponding modules connected in series. For these blocks the corresponding failure rates λ_{B1}, $\lambda_{B2,}$ λ_{B3} are equal to the sum of failure rates of all modules that are included in the block.

So, we obtain

$$\lambda_{B1} = \lambda_{CT} + \lambda_{cbl} + \lambda_{\Delta} + \lambda_{cbl} + \lambda_{DC} + \lambda_{CM} + \lambda_{OC} + \lambda_{CM} + \lambda_{\Delta} + \lambda_{cbl} + \lambda_{DC},$$

$\lambda_{B2} = \lambda_{VT} + \lambda_{cbl} + \lambda_{Z} + \lambda_{cbl} + \lambda_{DC} + \lambda_{cbl} + \lambda_{cbl} + \lambda_{DC} + \lambda_{Z}$,

$\lambda_{B3} = \lambda_{CT} + \lambda_{cbl}$.

Based on the data in Table 4.21 we have

$$\lambda_{B1} = 0.24782 \ \ y^{-1}, \ \ \lambda_{B2} = 0.08201 \ \ y^{-1}, \ \ \lambda_{B3} = 0.00281 \ \ y^{-1}.$$

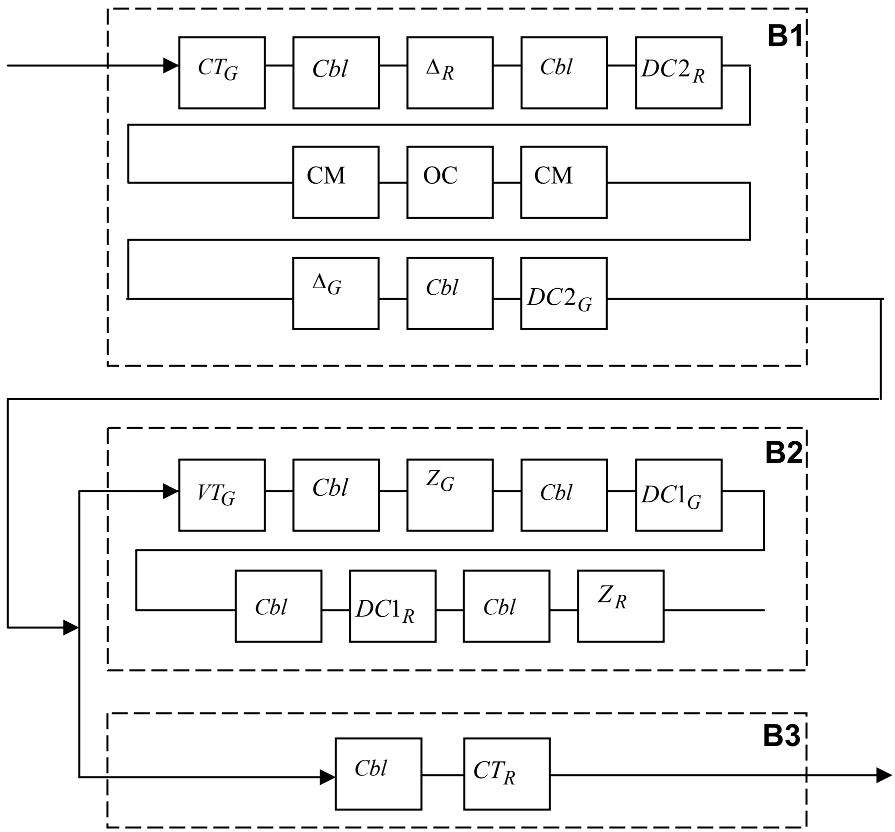

Fig. 4.40. Equivalent reliability block diagram for {z,Δ} protection system with independent elements

Mean time to repair $MTTR_B$ for any block B that consists of n independent elements connected in series can be defined by using the following formulae (Ushakov 1994)

$$MTTR_B = \sum_{i=1}^{n} \frac{\lambda_i}{\sum_{j=1}^{n} \lambda_j} MTTR_i \tag{4.231}$$

Therefore, the repair rate μ_B for block B can be obtained as

$$\mu_B = \frac{1}{MTTR_B} = \frac{1}{\sum_{i=1}^{n} \frac{1}{\mu_i} \frac{\lambda_i}{\sum_{j=1}^{n} \lambda_j}}. \tag{4.232}$$

Using (4.232) we get the following repair rates for blocks B1, B2 and B3:

$$\mu_{B1} = 1419 \;\; y^{-1}, \;\; \mu_{B2} = 1460 \;\; y^{-1}, \;\; \mu_{B3} = 1460 \;\; y^{-1}.$$

A reliability block diagram for $\{z, z\}$ protection system (Fig. 4.36) is presented in the Fig. 4.41. All blocks in this diagram are statistically independent.

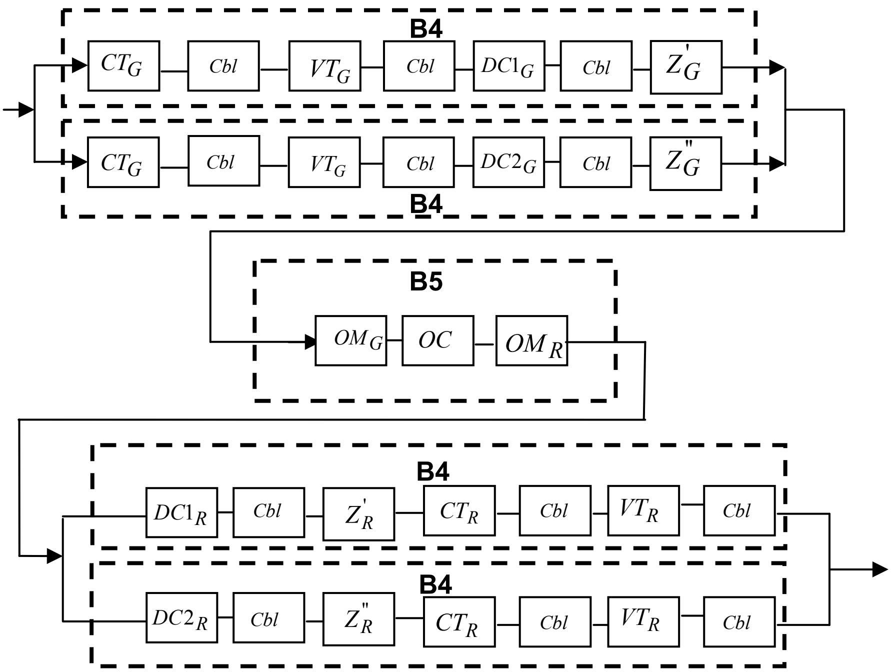

Fig. 4.41. Reliability block diagram for {z,z} protection system

As in the previous case, blocks' failure rates λ_{B4}, $\lambda_{B5,}$ are equalled to the sum of failure rates of all modules that are included in the corresponding block. So, we obtain

$$\lambda_{B4} = \lambda_{CT} + \lambda_{cbl} + \lambda_{VT} + \lambda_{cbl} + \lambda_{DC} + \lambda_{cbl} + \lambda_{Z},$$

$$\lambda_{B5} = \lambda_{OM} + \lambda_{OC} + \lambda_{OM}.$$

Based on the data in Table 4.21 we have

$$\lambda_{B4} = 0.04522 \ y^{-1}, \ \lambda_{B5} = 0.16721 \ y^{-1}.$$

As in the previous case based on (4.232), we obtain the repair rates

$$\mu_{B4} = 1460 \ y^{-1}; \ \mu_{B5} = 1400 \ y^{-1}.$$

4.3.4 Markov models for protection systems reliability assessment

In this section we apply the Markov method that was described in section 2.1. In order to do this the Markov model should be built at the first stage and then a reliability index should be obtained by solving the corresponding system of differential equations.

The Markov model for two different configurations of a line protection system is built under the following assumptions:

1. There are no additional failures during any repair.
2. The failure is detected immediately after its occurrence and repair begins without delay.
3. The probability of more than two independent failures in a protection system during its operating is negligibly small.
4. When the failed module is under repair, redundant modules are available and can operate.

The state space diagram and corresponding transition intensities of the Markov model for reliability assessment of {z, Δ} protection system are presented in the Fig. 4.42.

In state 0 the protection system {z, Δ} is waiting for the demand to operate and all its blocks B1, B2 and B3 are available and ready to operate.

If the demand to operate arrives (short circuit occurs in the line) the system transits to state 3 with intensity rate λ_{dem}. In state 3 the protection system operates as was described in section 4.3.2. We assume that operation time T_{op} is an exponentially distributed random variable with mean $\overline{T}_{op}$.

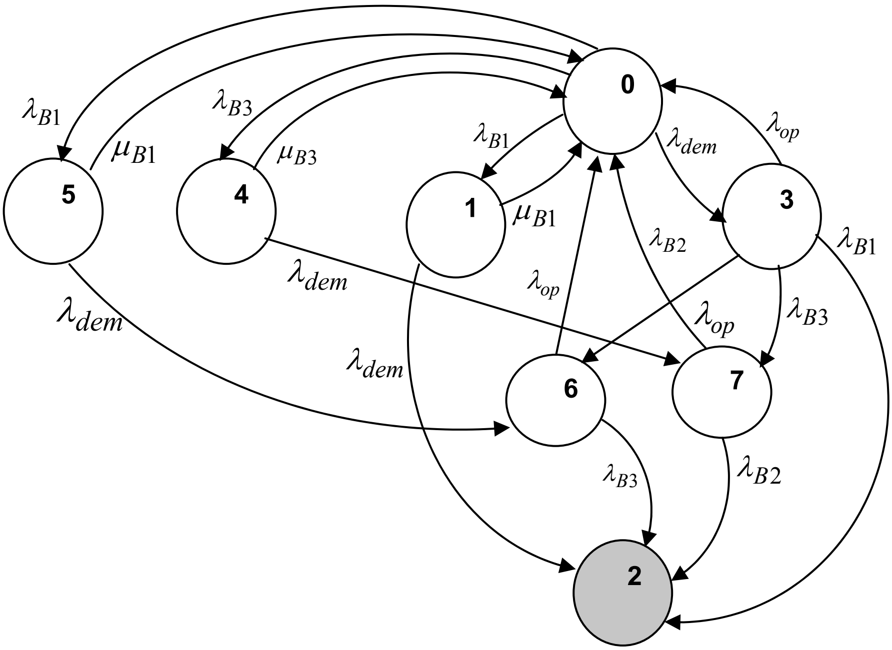

Fig. 4.42. Markov model for$\{z,\Delta\}$ protection system reliability assessment

If the system completes operation without any failure, it returns to state 0 with intensity rate $\lambda_{op} = \frac{1}{\overline{T}_{op}}$ and waits in this state for the next demand.

If during the system operation in state 3 block B1 fails, the system transits from state 3 to state 2 with intensity rate λ_{B1}. State 2 is the state where the protection system cannot operate and satisfy the demand. This state is considered to be the protection system's missing operation state.

If block B2 fails to operate when the system is in state 3, the system transits from state 3 to state 6. In state 6 it continues to operate, because block B2 has redundant block B3. If block B1 or block B3 do not fail during the system operation in state 6, the system finishes its operation and transits from state 6 to state 0 with transition rate λ_{op}. If in state 6 a failure occurs in one of the blocks B3 or B1, the system transits from state 6 to state 2 with intensity rate $\lambda_{B1} + \lambda_{B3}$.

If block B3 fails to operate when the system is in state 3, the system transits from state 3 to state 7. In state 7 it continues to operate, because block B3 has redundant block B2. If either block B1 or block B2 does not fail during the system operation in state 7, the system finishes its operation and transits from state 7 to state 0 with transition rate λ_{op}. If in state 7 a failure occurs either in block B2 or in block B1, the system transits from state 7 to state 2 with transition rate $\lambda_{B1} + \lambda_{B2}$.

If in state 0, when the system waits for the demand to operate, block B1 fails, the system transits from state 0 to state 1 with intensity rate λ_{B1}. The failure is detected immediately and repair begins. When the repair is completed the system transits from state 1 to state 0 with intensity rate μ_{B1}. In state 1 when block B1 is under repair, the protection system is unable to operate, if the demand arrives. So, if the demand arrives in state 1 before completion of the repair, the system transits to state 2 from state 1 with intensity rate λ_{dem}.

If in state 0, when the system waits for the demand to operate, block B2 fails, the system transits from state 0 to state 5 with intensity rate λ_{B2}. In state 5, block B2 is under repair, but the system can operate if a demand arrives because it has the redundant block B3. So, if the demand arrives, the system transits from state 5 to state 6 with intensity rate λ_{dem}. If the repair of block B2 is completed before the demand's arrival, the system transits from state 5 to state 0 with intensity μ_{B2}.

If in state 0 block B3 fails, the system transits from state 0 to state 4 with intensity rate λ_{B3}. In state 4 block B3 is under repair, but the system can operate, if a demand arrives, because it has the redundant block B2. So, if the demand arrives, the system transits from state 4 to state 7 with intensity rate λ_{dem}. If the repair of block B3 is completed before the arrival of the demand, the system transits from state 4 to state 0 with intensity rate μ_{B3}.

Now the system of differential equations (4.233) can be written for finding the state probabilities $p_i(t)$, $i = 0,1,...,7$, for discrete state continuous time Markov process with state space diagram presented in Fig. 4.42.

$$\begin{cases} \frac{dp_0(t)}{dt} = -(\lambda_{dem} + \lambda_{B1} + \lambda_{B2})p_0(t) + \mu_{B1}p_1(t) \\ + \lambda_{op}p_3(t) + \mu_{B3}p_4(t) + +\lambda_{B2}p_5(t) + \lambda_{op}p_6(t) + \lambda_{op}p_7(t) \\ \frac{dp_1(t)}{dt} = \lambda_{B1}p_0(t) - (\mu_{B1} + \lambda_{dem})p_1(t) \\ \frac{dp_2(t)}{dt} = \lambda_{dem}p_1(t) + \lambda_{B1}p_3(t) + (\lambda_{B1} + \lambda_{B3})p_6(t) \\ + (\lambda_{B1} + \lambda_{B2})p_7(t) \\ \frac{dp_3(t)}{dt} = \lambda_{dem}p_0(t) - (\lambda_{op} + \lambda_{B1} + \lambda_{B2} + \lambda_{B3})p_3(t) \\ \frac{dp_4(t)}{dt} = \lambda_{B3}p_0(t) - (\mu_{B3} + \lambda_{dem})p_4(t) \\ \frac{dp_5(t)}{dt} = \lambda_{B2}p_0(t) + \lambda_{B1}p_3(t) - (\mu_{B2} + \lambda_{dem})p_5(t) \\ \frac{dp_6(t)}{dt} = \lambda_{B2}p_3(t) + \lambda_{dem}p_5(t) - (\lambda_{op} + \lambda_{B1} + \lambda_{B3})p_6(t) \\ \frac{dp_7(t)}{dt} = \lambda_{B3}p_3(t) + \lambda_{dem}p_4(t) - (\lambda_{op} + \lambda_{B1} + \lambda_{B2})p_7(t) \end{cases} \tag{4.233}$$

We assume that the process begins from the state 0 at time t=0. Therefore, the initial conditions for the system of differential equations are:

$$p_0(0) = 1, \quad p_1(0) = p_2(0) = ... = p_7(0) = 0\,. \tag{4.234}$$

The probability of missing operation is determined by the probability of the system entrance into state 2 during time t $p_2(t)$. The probability $p_2(t)$ characterizes the unreliability function $UR_{z,\Delta}(t)$ for {z, Δ} protection system

$$UR_{z,\Delta}(t) = p_2(t)\,. \tag{4.235}$$

The state space diagram and corresponding transition intensities of the Markov model for reliability assessment of {z, Δ} protection system are presented in the Fig. 4.43.

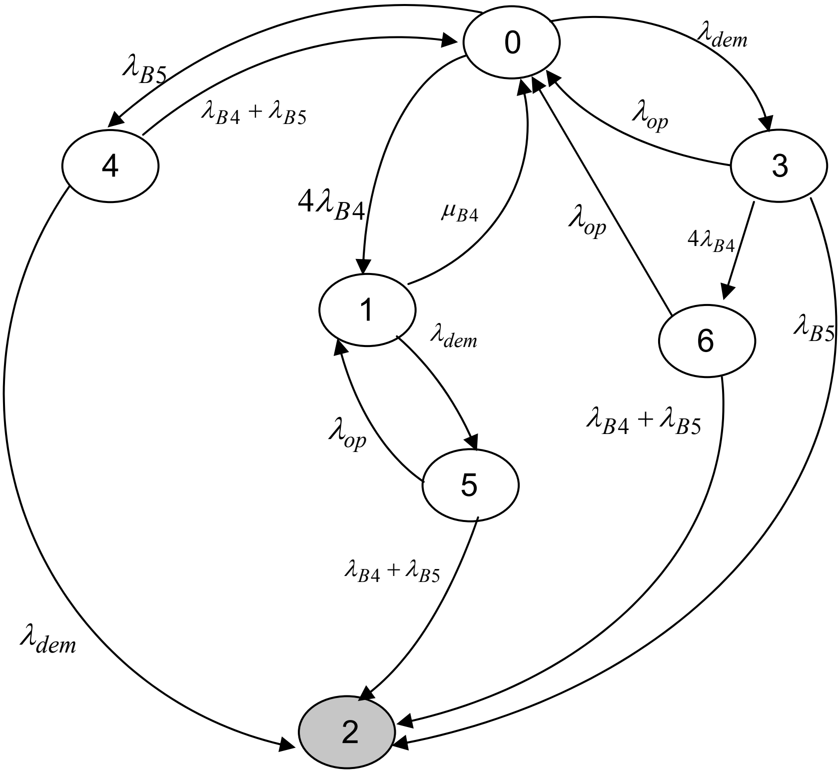

Fig. 4.43. Markov model for{z,z} protection system reliability assessment

In state 0 the protection system {z, z} waits for the demand's arrival and its blocks B4 and B5 are available and ready to operate.

If the demand to operate arrives (short circuit occurs in the line) the system transits from state 0 to state 3 with intensity rate λ_{dem}. In state 3 the protection system operates as was described in section 4.3.2. As in the previous case, we assume that operation time T_{op} is an exponentially distributed random variable with mean $\overline{T}_{op}$. If the system completes operation without any failure, it returns from state 3 to state 0 with intensity rate $\lambda_{op} = \frac{1}{\overline{T}_{op}}$ and waits for the next demand.

If during the system operation in state 3 block B5 fails, the system transits from state 3 to state 2 with intensity rate λ_{B5}. State 2 is the state in which the protection system cannot operate and satisfy demand. As in the previous case, state 2 is the protection system missing operation state.

If any block of the four blocks B4 fails during operating in state 3, the system can continue its operation because there is redundancy for each block B4. So, the system transits from state 3 to state 6 with intensity rate $4\lambda_{B4}$. If no additional failures occur during the system operation, the system transits from state 6 to state 0 with intensity rate λ_{op}. If the failure in redundant block B4 occurs as well as the failure in block B5 (communication channel), then the system transits from state 6 to failure state 2 with intensity rate $\lambda_{B4} + \lambda_{B5}$.

If block B5 fails when the protection system is in state 0, the system transits with intensity rate λ_{B5} from state 0 to state 4 where the repair begins. If the demand to operate arrives before completion of the repair of block 4, the system transits from state 4 to state 2, because without block 5 the system cannot operate. The transition intensity rate is λ_{dem}. If the repair is completed before the demand's arrival, the system transits back from state 4 to state 0 with intensity rate μ_{B5}.

If one of four blocks B4 will fail when the system is in state 0, the system transits from state 0 to state 1 with intensity rate $4\lambda_{B4}$. In state 1 the repair begins. If the repair is completed before the arrival of the demand, the system transits back from state 4 to state 0 with intensity rate μ_{B4}. If the demand arrives before completion of the repair, the system transits from state 1 to state 5 with intensity rate λ_{dem}. In state 5 the system operates. If there are no failures during operating, the system completes its function and transits back to state 1 with intensity rate λ_{op}. If failures occur in state 5 during system operation in block B5 or in block B4 (redundant to block B4 that failed in state 0), the system transits to state 2 with intensity rate $\lambda_{B4} + \lambda_{B5}$.

Now the system of differential equations can be written for finding the state probabilities $p_i(t)$, $i = 0,1,...,6$, for discrete state continuous time Markov process presented in the Fig. 4.43.

$$
\begin{cases}
\dfrac{dp_0(t)}{dt} = -(\lambda_{dem} + 4\lambda_{B4} + \lambda_{B5})p_0(t) + \mu_{B4}p_1(t) + \lambda_{op}p_3(t) + \\
\qquad + \mu_{B5}p_4(t) + \lambda_{op}p_6(t) + \mu_{B5}p_4(t) \\
\dfrac{dp_1(t)}{dt} = 4\lambda_{B1}p_0(t) - (\mu_{B4} + \lambda_{dem})p_1(t) + \lambda_{op}p_5(t) \\
\dfrac{dp_2(t)}{dt} = \lambda_{B5}p_3(t) + \lambda_{dem}p_4(t) + (\lambda_{B4} + \lambda_{B5})p_5(t) + (\lambda_{B4} + \lambda_{B5})p_6(t) \\
\dfrac{dp_3(t)}{dt} = \lambda_{dem}p_0(t) - (\lambda_{op} + 4\lambda_{B4} + \lambda_{B4} + \lambda_{B5})p_3(t) \\
\dfrac{dp_4(t)}{dt} = \lambda_{B5}p_0(t) - (\mu_{B5} + \lambda_{dem})p_4(t) \\
\dfrac{dp_5(t)}{dt} = \lambda_{dem}p_1(t) - (\lambda_{op} + \lambda_{B4} + \lambda_{B5})p_5(t) \\
\dfrac{dp_6(t)}{dt} = 4\lambda_{B4}p_3(t) - (\lambda_{op} + \lambda_{B4} + \lambda_{B5})p_6(t)
\end{cases}
\tag{4.236}
$$

As in the previous case, we assume that the process begins from state 0 at time t=0. Therefore, the initial conditions for the system of differential equations are:

$$p_0(0) = 1,\ \ p_1(0) = p_2(0) = ... = p_6(0) = 0\,. \tag{4.237}$$

The probability of missing operation is determined by the probability of the system entrance into state 2 during time t $p_2(t)$. This probability characterizes the unreliability function $UR_{z,z}(t)$ for {z, z} protection system

$$UR_{z,z}(t) = p_2(t)\,. \tag{4.238}$$

Computation results

The results of unreliability functions calculation for both protection system configurations are presented in Fig. 4.44. Unreliability functions that characterize the probability of missing operations are presented in as functions of time.

It should be noticed that for such small values of missing operation probability $UR_{z,z}(t) < UR_{z,\Delta}(t) < 0.001 << 1$ for $t \in [0,\ 1\ \text{year}]$ the mean number of missing operations during time t and the probability of missing operations are almost the same. So, as can be seen from Fig. 4.44, the mean numbers of missing operations during a one year period (CIGRE reliability index) are 0.0004 and 0.0006 for $\{z,\ z\}$ and $\{z,\ \Delta\}$ configurations respectively.

In order to interpret these results, we consider, for example, the power system in which there are 300 high voltage lines with an average length of 50 km. If the configuration $\{z,\ z\}$ is used in order to protect all these lines in the power system, the expected time between missing operations is about 8.3 years. If the configuration $\{z,\ \Delta\}$ is used in order to protect all these lines, the expected time between missing operations is about 5.6 years.

Therefore, the configuration $\{z,\ z\}$ is preferable from the reliability point of view to the configuration $\{z,\ \Delta\}$, when we consider missing operations. The main reason for this fact is unsuccessful redundancy in the $\{z,\ \Delta\}$ configuration, when the main and redundant channels are not independent and, therefore, are sensitive to common cause failures.

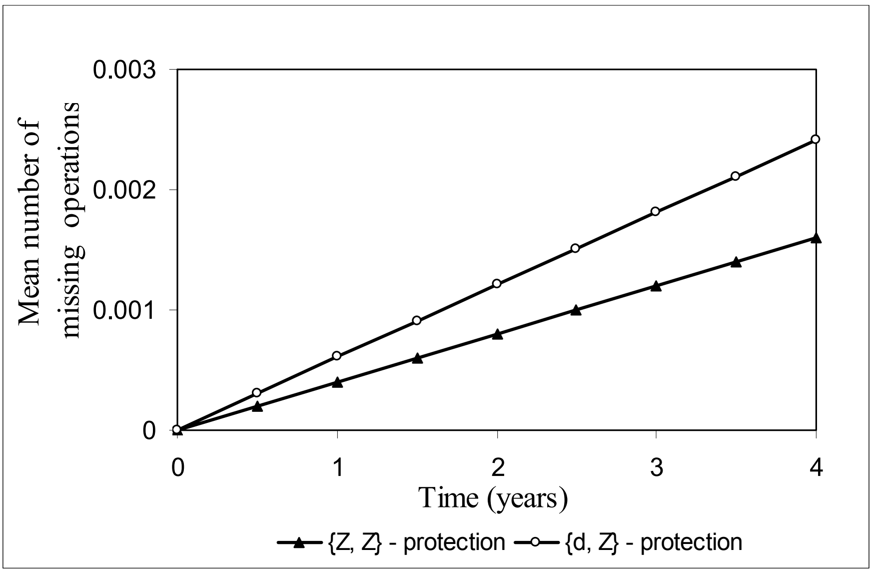

Fig. 4.44. Probability of missing operations for {z,z} and {z,Δ} configurations of line protection system

4.4 Reliability assessment of high voltage transmission lines

To save the land occupied by the transmission lines companies use multiple circuit configurations mounting several circuits (sometimes with different voltage) on the same towers. This affects the line reliability because in such configurations failures of some lines can cause unavailability of other lines during the repair, which results in common cause failures.

This section considers reliability of two possible configurations of two-circuit lines: separate circuits (mounted on different towers) and double circuit (mounted on the same towers). In the latter configuration some failures of one of the circuits can be repaired only when the other one is disconnected for safety reasons. The presented models are based on failure statistics and Markov chain approach.

4.4.1 Single circuit line model

In the single circuit line failures in both circuits are partly interdependent whereas the repairs are independent. Therefore, even when common cause failure occurs it can be repaired simultaneously in both circuits.

Moreover, as statistic analysis of failures shows, some common cause failures are caused by malfunctions of the protection system independently on whether both lines are mounted on the same or different towers in the same corridor. This type of failures (when outage of one line results from failure in the line and outage of another line results from wrong functioning of the protection system) is named common cause switching. The second line is out of functioning for a relatively short period of time, until the necessary switching operations are performed. The first line is out of operation for a longer period of time, until the repair work is completed.

Fig. 4.45 shows a Markov chain diagram for calculating the reliability indexes for two single circuit lines in parallel. In this diagram each state corresponds to a combination of states of two circuits. Each circuit can be in operation state (u), outage state (d) and common cause switching state (s). The transitions between different states in the diagram have the following intensities:

- $\lambda_1 = \lambda_2$ forced outage rate of the independent failures (1/Yr);
- $\lambda_{CS1} = \lambda_{CS2}$ forced outage rate of the common cause switching type of failures (1/Yr);
- λ_{C1} forced outage rate of the common cause type of failures (1/Yr) (equal repair time for failures which can be repaired simultaneously);

– λ_{C2} forced outage rate of the common cause type of failures (different repair times for failures which should be repaired separately);
– $\mu_1 = \mu_2$ repair rates of lines 1 and 2 respectively (1/Yr);
– μ_{C1} repair rate for common cause type of failure (equal repair time) (1/Yr);
– μ_{C2} repair rate for common cause type of failure (different repair time) (1/Yr);
– $\mu_{CS1} = \mu_{CS2}$ repair rate for common cause switching type of failure (1/Yr).

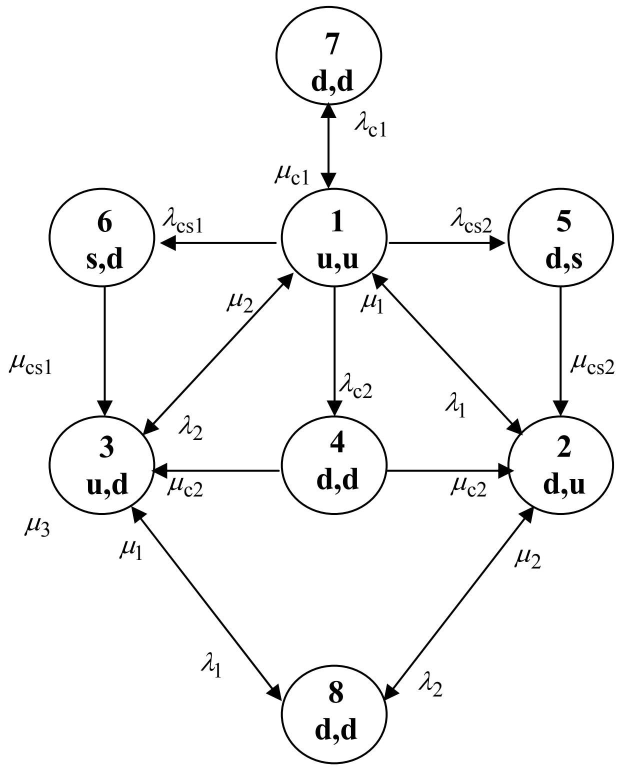

Fig. 4.45. Model for evaluating reliability of single circuit lines

4.4.2 Double circuit line model

As opposed to single circuit lines, common cause type of failures (for instance, damage to a tower) on double circuit lines cannot be repaired separately. There are other common cause type failures which enable repair of each circuit separately and independently. Fig. 4.46 shows a Markov chain diagram for reliability calculation of double circuit lines. In this diagram two additional states exist corresponding to planned outages of one circuit after a failure on the other circuit. The corresponding transition rates are

- $\lambda_3 = \lambda_4$ planned outage rate on one circuit after a failure on the other circuit (1/Yr);
- $\mu_3 = \mu_4$ repair rate of lines 1 and 2 respectively, after a planned outage (1/Yr).

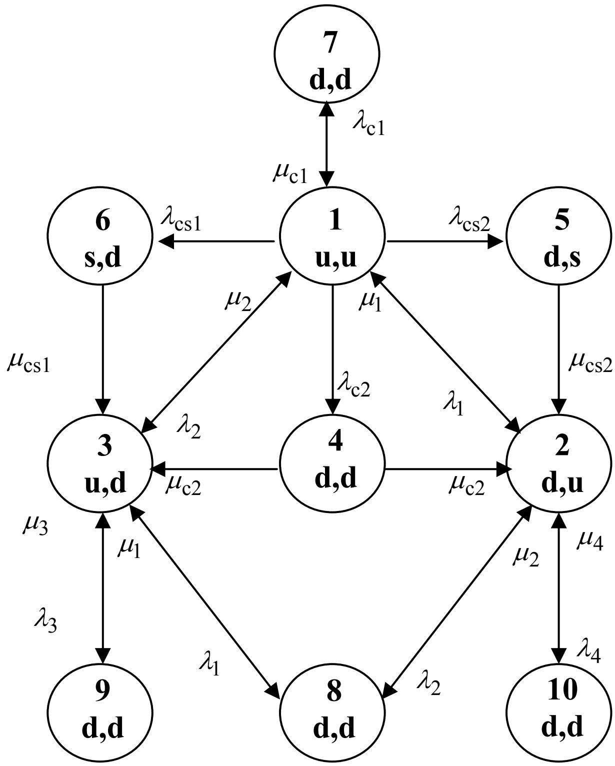

Fig. 4.46. Model for evaluating reliability of double circuit lines

Example 4.4.

Consider 400 kV lines with parameters obtained based on failure data. The parameters (failure and repair rates) are presented in Table 4.22.

The state probabilities obtained for the two type of lines based on the presented Markov models are presented in Table 4.23.

Table. 4.22. Transition rates for 50 km 400 KV lines

Parameter	Rate (1/YR)
$\lambda_1 = \lambda_2$	0.65032
λ_{C1}	0.00256
λ_{C2}	0.00290
$\lambda_{CS1} = \lambda_{CS2}$	0.04200
$\mu_1 = \mu_2$	1498.0
μ_{C1}	5540.0
μ_{C2}	7219.2
$\mu_{CS1} = \mu_{CS2}$	14798.9
$\lambda_3 = \lambda_4$	1.0101
$\mu_3 = \mu_4$	100

Table. 4.23. State probabilities for 50 km single circuit and double circuit 400 KV lines

	State probability	
state	single circuit	double circuit
1	0.99907	0.99905
2	4.63E-04	4.62E-04
3	4.63E-04	4.63E-04
4	2.01E-07	2.01E-07
5	2.84E-06	2.84E-06
6	2.84E-06	2.84E-06
7	4.62E-07	4.62E-07
8	2.01E-07	2.01E-07
9		7.80E-06
10		7.78E-06

According to N-1 criterion the system can tolerate an outage of a single line. Therefore, in order to evaluate system failure probability P_f one has to obtain the overall probability of the system being in one of the states when both lines are out of operation. Since these states are mutually exclusive, the probability of system failure is equal to the sum of probabilities of both lines being out of operation. In the case of single circuit lines the failure probability is $P_f = p_4 + p_7 + p_8 = 0.863 \cdot 10^{-6}$. In the case of double circuit lines the failure probability is $P_f=p_4+p_7+p_8+p_9+p_{10}=16.442\cdot10^{-6}$. It can be seen that the reliability of single circuit lines is 19.0 times greater than the reliability of double circuit lines, which can be explained by the existence of planned outages caused by common cause failures.

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5 Reliability of Distribution Systems

5.1 Methods of reliability enhancement in radial distribution systems

Customer failure statistics show that medium voltage distribution systems make the greatest individual contribution to the unavailability of supply to customer. Hence, in order to significantly improve overall system performance, investment emphasis should be on distribution networks. Considerable effort has been devoted to the reduction of unsupplied energy in distribution networks (Allan and Billinton 1996). The following methods are used for distribution system reliability enhancement:

- Line length reduction by addition of new substations.
- Replacement of regular overhead lines by insulated ones or underground cables.
- Automatic network sectionalizing.
- Automatic network reconfiguration, to provide alternative supply capability.

The first and second methods reduce the total number of failures in the network whereas the third and fourth methods help to reduce the unsupplied energy caused by failures. Automatic sectionalizing and reconfiguration are the most efficient methods of reducing unsupplied energy. However, since the practicalities of protection grading limit the number of sectionalizing switches, these methods may be insufficient. In such cases the whole line or some part of its sections should be insulated. In this chapter an optimization algorithms are suggested which solve two problems:

1. Optimal allocation of sectionalizing switches which minimizes unsupplied energy caused by network failures.
2. Choosing the sections which should be insulated to achieve the greatest unsupplied energy reduction subject to investment cost limitation.

5.2 Optimal sectionalizing in single radial feeder networks

A typical radial distribution system consists of a number of single feeder tree-like networks interconnected through tie-lines. A single radial feeder distribution network consists of a series of lines and underground cables

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connecting each load point to the single supply source (substation). These lines and cables are divided into sections by connection nodes and load points. Each section has its individual failure rate which depends on the type of line construction, section length and environmental conditions. In practice it is normally found that the failure rates are approximately proportional to the length of the section.

A short circuit in any system component causes the main feeder breaker to operate. This leads to supply interruption for all the customers until the repair team arrives and disconnects the faulted section from the network. If there is an automatic sectionalizer somewhere between the energy source and the faulted section, the sectionalizer opens and disconnects a subsystem containing the faulted section. In this case only part of the customers feels interruption, while the rest are normally supplied. In the example presented in Fig. 5.1A, the damage caused by the failure in section i is localized by the sectionalizer installed in section $D(i)$. The subsystem **A** is unsupplied during the time of faulted section search before repair team arrival. When the team reaches the faulted section, it disconnects it from the network. The sectionalizer can now be closed and supply resumed for part of the customers. The subsystem **C** will be disconnected during repair time.

When an interruption occurs, the outage duration for an individual customer depends on how its location is related to the failure and to the sectionaliers. The expected unsupplied energy depends, therefore, on sectionalizers' allocation. Compare, for example, sectionalizer locations in Fig. 51 **A** and **B**. In the first case the failure in section i causes the supply interruption in subsystem A, whereas the failure in section k causes the interruption for all of the customers. Moving the sectionalizer towards the supply source (Fig. 5.1.B) extends the subsystem **A**, however it reduces the set of unsupplied customers in the case of failure in section k.

Since technical and economic factors limit the number of sectionalizers, their optimal allocation allows achieving the greatest possible reliability improvement to the system. In this section a criterion for sectionalizers allocation is used which considers both the cost of unsupplied energy and capital investments into the sectionalizers installation. A mathematical model for UE cost is developed for a single radial feeder distribution system. The model considers the cases when sectionalizers are not fully reliable and when they may cause short circuits in sections where they are installed. A genetic algorithm is then used to achieve the optimal sectionalizers allocation.

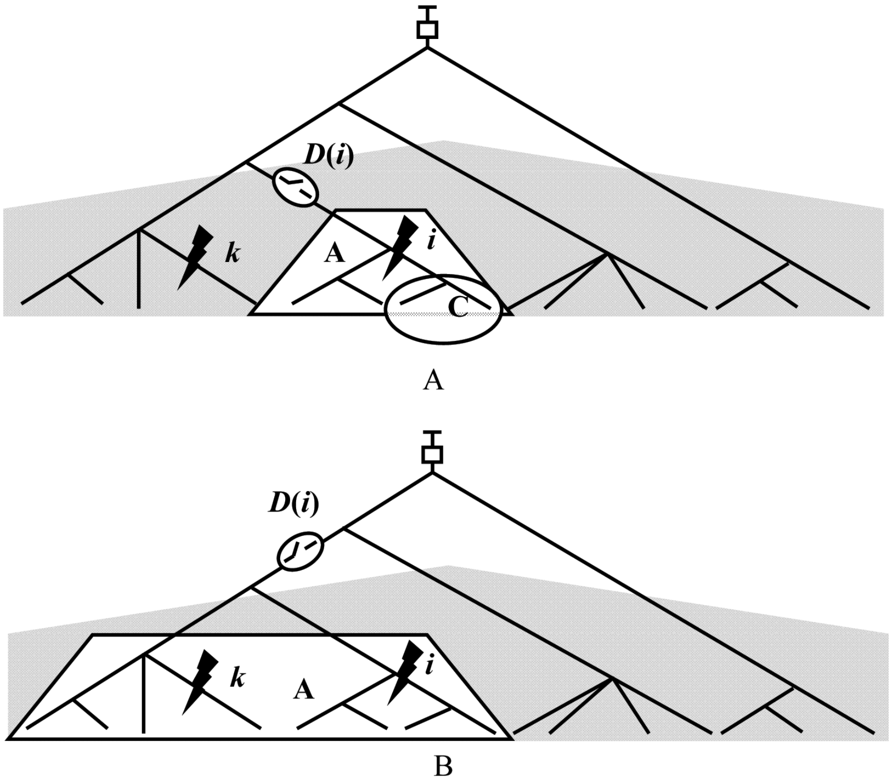

Fig. 5.1. Sectionalizing in single feeder distribution system

5.2.1 Mathematical formulation of the problem

The radial distribution system may be represented by a tree graph G, in which nodes o_i ($0 \le i \le N$) correspond to connection or load points. Since in such tree graphs each edge has its unique end node, the edge (o_i, o_j) may be denoted as j. Edge j corresponds to the distribution system section j. Let $s(i)$ be immediate predecessor of edge i if the graph G contains edge $(o_s(i), o_i)$. One can see that the isolation of a section k also causes isolation of those sections i for which $k=s(i)$.

The set of predecessors for each edge i is defined as $S_i=\{s(i), s(s(i)), s(s(s(i))),\ldots\}$. S_i contains all sections belonging to the path that connects section i with the energy source (section 0). In the example of a simple network presented in Fig. 5.3 $S_{17}=S_{18}=S_{19}=S_{20}=\{6,1,0\}$, $S_{12}=S_{13}=\{9,3,2,1,0\}$ etc.

Let us define the cost Z_k of energy supply interruption caused by disconnection of the arbitrary section k for a period of one hour. For each $k \in L$, where L is a set of loaded sections, that is, sections directly connected to the load transformers (note that for $k \in L$ sections i for which $s(i)=k$ do not exist):

$$Z_k = P_k C_e, \tag{5.1}$$

where P_k average load in node k, C_e is the cost of unsupplied energy (per kWh). For a section k, which does not belong to the set L:

$$Z_k = \sum_{s(i)=k} Z_i \tag{5.2}$$

Using this definition one can determine Z_k for all $0 \le k \le N$ using the following simple procedure:

1. For each k ($0 \le k \le N$) set $Z_k = 0$.
2. For each $i \in L$ set $Z_i = P_i C_e$ and add Z_i to all $Z_j, j \in S_i$.

For the example in Fig. 5.3 $Z_4 = Z_5 + Z_{11}$, $Z_3 = Z_5 + Z_{10} + Z_{11} + Z_{12} + Z_{13}$, $Z_8 = Z_{14} + Z_{15} + Z_{16}$ etc.

It should be noted that the cost of the total system isolation is

$$Z_0 = \sum_{k \in L} Z_k \tag{5.3}$$

Suppose that sectionalizers are already allocated in the radial distribution system. For each section i one can find the closest protected predecessor (CPP), that is, the closest predecessor with sectionalizer installed $D(i) \in \{S_i, i\}$. Actually, $D(i)$ is the number of the section containing the point of isolation until the repair team arrival to the faulted section i. If the fault occurs in a section j with a sectionalizer installed, then $D(j)=j$.

A short circuit in the section i causes the main breaker to isolate the whole system. After some time T_s (detection of the fault and switching) the sectionalizer in section $D(i)$ is opened and the breaker is re-closed. During the repair team arrival time r_i, section $D(i)$ will be disconnected.

Now we can define the cost of energy not supplied due to faults which occurred in section k during a year:

$$c_k = \lambda_k (T_s \sum_{i \in L} Z_i + r_k Z_{D(k)}) = \alpha_k + \beta_k Z_{D(k)} \tag{5.4}$$

where λ_k is the failure rate in section k,

$$\alpha_k = \lambda_k T_s \sum_{i \in L} Z_i = \lambda_k T_s Z_0 \tag{5.5}$$

is the cost of UE caused by the total isolation, and

$$\beta_k Z_{D(k)} = \lambda_k r_k Z_{D(k)} \tag{5.6}$$

is the cost of UE caused by the local isolation.

For the whole distribution system the annual cost of unsupplied energy is

$$C = \sum_{k=1}^{N} c_k = \sum_{k=1}^{N} \alpha_k + \sum_{k=1}^{N} \beta_k Z_{D(k)} \, . \tag{5.7}$$

If the number of sectionalizers is given, our objective is to find the distribution function D, minimizing $C(D)$:

$$D = \arg\left\{ C(D) = \sum_{k=1}^{N} \alpha_k + \sum_{k=1}^{N} \beta_k Z_{D(k)} = \min \right\} . \tag{5.8}$$

As $\sum_{k=1}^{N} \alpha_k$ does not depend on D, only the functional $\sum_{k=1}^{N} \beta_k Z_{D(k)}$ should be minimized. For an arbitrary number of sectionalizers, M, the cost of the sectionalizer units should be considered as well. The objective function in this case is:

$$\tilde{C}(D, M) = \sum_{k=1}^{N} \beta_k Z_{D(k)} + Mh \tag{5.9}$$

where h is the annual cost of a sectionalizer.

Let us now estimate the effect of sectionalizer addition to the section k assuming the initial allocation (D, M) is given. To do this we introduce an auxiliary function $d(i)$ which gives the CPP number for the section i, assuming that the sectionalizer in section i is removed (if in section i sectionalizer was not installed $d(i)=D(i)$). $d(0)=0$ by definition.

The fault in any section j after the new sectionalizer addition ($D(j)=i$) now causes UE cost Z_i per hour, whereas before this addition it caused UE cost $Z_{d(i)}$ per hour. Therefore

$$\Delta \tilde{C}(D, M, i) = (Z_i - Z_{d(i)}) \sum_{D(j)=i} \beta_j + h \, . \tag{5.10}$$

We now have a criterion for sectionalizer addition: the addition to section i is worthwhile if $\Delta \tilde{C} < 0$ that is

$$\frac{h}{\sum\limits_{D(j)=i} \beta_j} + Z_i < Z_{d(i)} . \tag{5.11}$$

For all the load sections $i \in L$,

$$\sum_{D(j)=i} \beta_j = \beta_i \tag{5.12}$$

and the previous expression may be rewritten as

$$\frac{h}{\beta(j)} + Z_i < Z_{d(i)} \tag{5.13}$$

Effect of sectionalizer operation failures

The sectionalizers occasionally fail to operate. If the sectionalizer in section $D(k)$ cannot be opened it is up to the sctionalizer in section $d(D(k))$ to open and isolate the relevant subsystem. Let p be the probability of sectionalizer failure. Following a short-circuit in section k the sectionalizer in section $D(k)$ isolates the subsystem with probability 1-p, the sectionalizer in section $d(D(k))$ with probability p(1-p),etc. The cost of unsupplied energy caused by the failure in section k is

$$\begin{aligned} c_k &= \alpha_k + \beta_k[(1-p)Z_{D(k)} + p(1-p)Z_{d(D(k))} \\ &+ p^2(1-p)Z_{d(d(D(k)))} + ...] \end{aligned} \tag{5.14}$$

Assuming that p is small enough, we can neglect all elements multiplied by p^n for $n > 1$. Doing so, we get

$$c_k = \alpha_k + \beta_k[(1-p)Z_{D(k)} + p(1-p)Z_{d(D(k))}] \tag{5.15}$$

As in (5.9) we can estimate $\tilde{C}(D,M)$:

$$\tilde{C}(D,M) = \sum_{i=1}^{N} \beta_i \left[(1-p)Z_{D(i)} + pZ_{d(D(i))}\right] + Mh. \tag{5.16}$$

To estimate the effect of sectionalizer addition to section i, we have to compare the UE caused by a one hour failure in section j ($D(j) = i$) before the addition,

$$c_{old} = (1\text{-}p)Z_{D(i)} + pZ_{d(D(i))} \tag{5.17}$$

and after the addition,

$$C_{new} = (1 - p)Z_i + pZ_{d(i)}. \tag{5.18}$$

The total cost difference after the sectionalizer addition is

$$\Delta\tilde{C}(D,M,i) = (C_{new} - C_{old}) \sum_{D(j)=i} \beta_j + h \tag{5.19}$$

The condition for sectionalizer addition derived from this expression is

$$\frac{h}{\sum_{D(j)=i} \beta_j} + (1-p)Z_i < (1-2p)Z_{d(i)} + pZ_{d(d(i))} \tag{5.20}$$

Effect of short-circuits caused by sectionalizer failures

The sectionalizer itself may cause the short-circuit in its section. To estimate this effect we introduce the failure rate μ and repair time τ for sectionalizers. The short-circuits caused by sectionalizers lead to an increase in the total isolation time as well as to an increase in the failure rate of the protected sections. Note that in the case of a fault caused by a sectionalizer in section *i*, it can be isolated only by the sectionalizer in section *d*(*i*) (not *D*(*i*) because the sectionalizer in section *i* has failed). The annual cost of losses is now

$$\begin{aligned}\tilde{C}(D,M) &= \sum_{i=1}^{N} \beta_i \left[(1-p)Z_{D(i)} + pZ_{d(D(i))}\right] + \sum_{i=1}^{N} \gamma\left[(1-p)Z_{d(i)} + pZ_{d(d(i))}\right]\delta_i \\ &\quad + M(h + \mu T_s Z_0) \\ &= \sum_{i=1}^{N} \left\{\delta_i\beta_i\left[(1-p)Z_i + pZ_{d(i)}\right] + \varepsilon_i\left[(1-p)Z_{d(i)} + pZ_{d(d(i))}\right]\right\} + M(h + \mu T_s Z_0)\end{aligned} \tag{5.21}$$

where $\gamma = \mu\tau$, $\varepsilon_i = \delta_i\gamma + (1-\delta_i)\beta_i$, δ_i=1 if there is a sectionalizer in section *i* and δ_i=0 otherwise.

It can be shown that the addition of a sectionalizer to section *i* causes the following differences in $\tilde{C}$:

$$\begin{aligned}\Delta\tilde{C}(D,M,i) &= \left[(1-p)Z_i - (1-2p)Z_{d(i)} - pZ_{d(d(i))}\right] \times \sum_{D(j)=i} \beta_j \\ &\quad + \gamma(1-p)Z_{d(i)} + \gamma pZ_{d(d(i))} + h + \mu T_s Z_0.\end{aligned} \tag{5.22}$$

The criterion for sectionalizer addition is therefore

$$h+\mu T_s Z_0+(1-p)Z_i B_i < [B_i-\gamma-p(2B_i-\gamma)]Z_{d(i)}+(B_i-\gamma)pZ_{d(d(i))} \quad (5.23)$$

where

$$B_i=\sum_{D(j)=i}\beta_j \quad (5.24)$$

For the loaded sections $B_i=\beta_i$. The expression then takes the form

$$\Omega_i<\Theta_i Z_{d(i)}+\Psi_i Z_{d(d(i))} \quad (5.25)$$

where

$$\Omega_i=h+\mu T_s Z_0+(1-p)Z_i\beta_i\,, \quad (5.26)$$

$$\Theta_i=\beta_i-\gamma-p(2\beta_i-\gamma), \quad (5.27)$$

$$\Psi_i=(\beta_i-\gamma)p \quad (5.28)$$

do not depend on the sectionalizer allocation *D*.

Assuming $\mu=0$ we get expression (5.20), and assuming $\mu=p=0$ we get expression (5.11), as special cases of the general expression (5.23).

5.2.2 Implementing the Genetic Algorithm

The binary string solution representation is very suitable for the sectionalizer allocation problem. Indeed, each j^{th} bit of *N*-bit length solution string can represent the value of δ_j (if the bit *j* is equal to 1, section *j* contains a sectionalizer). The simplest decoding procedure, using δ_i defined for each section, determines *D*(*i*) for $1\le i\le N$ and computes $\tilde{C}(D,M)=\tilde{C}(\delta)$ following expression (5.21).

The GA seeks for the best binary string which minimizes $\tilde{C}$. To achieve a better GA performance an advanced decoding procedure may be applied. Note that for each given sectionalizer allocation (*D*, *M*) one can check whether addition of a sectionalizer to the section *i* is worthwhile or not using criterion (5.23). For all elements $i\in L$ (sections connected to load nodes) this criterion depends only on the sectionalizer allocation in sections not belonging to the set *L*. Therefore the GA may generate binary strings representing $N-N_L$ unloaded sections (N_L is the number of load nodes), while for each string (solution) sectionalizers may be allocated optimally in all directly loaded sections. It should be mentioned again that Ω_i, Θ_i, and Ψ_i in (5.25) do not depend on allocation D and therefore may be determined before the GA starts. The reduction of the solution representation vector by N_L bits is very important because it allows the search space to be truncated considerably.

The initial input data for the optimization algorithm are: the structure of the radial distribution system which is represented by pairs i, $s(i)$ for $1 \le i \le N$; the average load cost Z_j for each of the N_L loads (in \$ per hour); the failure rates for each section, λ_i, and for sectionalizers, μ :the estimated repair time for each section, r_i, and for sectionalizers, τ; the switching time T_s, the probability of sectionalizer failure, p, and the annual cost of the sectionalizer unit, h.

In its initial stage the algorithm calculates γ and β_k for section k, Z_j for unloaded sections and Ω_i, Θ_i, and Ψ_i for all loaded sections $(i \in L)$. It also defines the length of the binary string representing solutions in the GA as $N - N_L$. After the GA is started, the optimal allocation of sectionalizers in the loaded sections is defined for each solution, and the objective function $\tilde{C}(D,M)$ is estimated.

Example 5.1

A simple distribution system with $N = 25$ and $N_L = 15$ is presented in Fig. 5.2 (Levitin et al. 1995a). The tree graph representation of this system is shown in Fig. 5.3. For the given system, the set of load sections is L={5, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}. Table 5.1 contains $(i, s(i))$ pairs, β_i and Z_i for each section. Note that, for i not belonging to L, Z_i are calculated using (5.2).

Three solutions (D,M) are obtained for different system parameters p and γ $(T_s = 0.03, \tau = 1.0)$. Parameters values as well as the resulting M and $\tilde{C}$ are presented in Table 5.2. The optimal sectionalizer allocation is displayed in Fig 5.2. The sections where sectionalizers should be installed are marked by an arrow and the number of the corresponding solution.

Table 5.1. Distribution system data

i	$s(i)$	Z_i	β_i
1	0	9100	0.04
2	1	6700	0.06
3	2	5000	0.16
4	3	1000	0.24
5	4	300	0.14
6	1	2400	0.195
7	8	2300	0.155
8	0	2900	0.015
9	3	3900	0.06
10	3	100	0.235
11	4	700	0.14
12	9	2300	0.015
13	9	1600	0.12
14	8	600	0.06
15	7	2000	0.08
16	7	300	0.1
17	6	800	0.22
18	6	300	0.1
19	6	600	0.175
20	6	700	0.1
21	24	400	0.135
22	22	1300	0.195
23	24	500	0.195
24	2	1700	0.045
25	22	800	0.055

Table 5.2. Sectionalizer allocation solutions

Solution no.	h ($)	γ (hour)	p	M	$\tilde{C}$ ($)
I	1000	0.02	0	6	14869
II	1000	0.1	0.01	5	19544
III	1000	0.25	0.005	3	27007

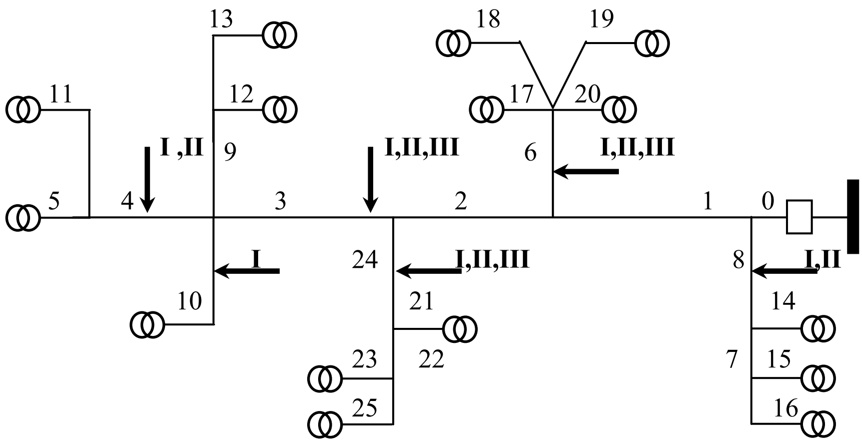

Fig. 5.2. A simple distribution system

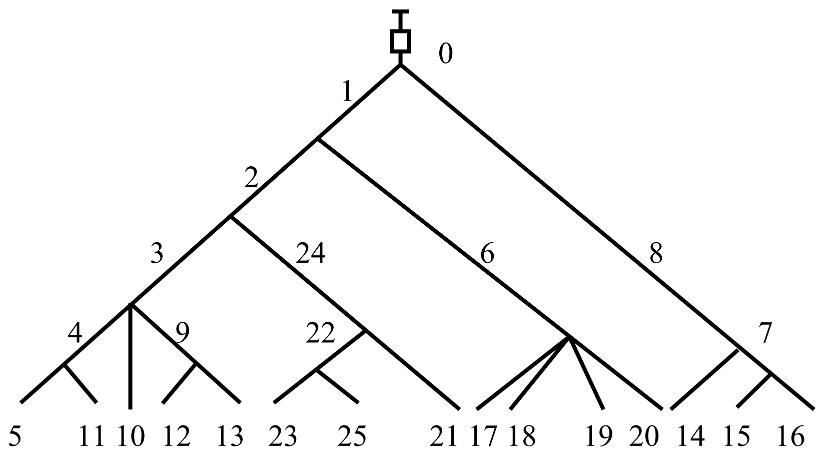

Fig. 5.3. Tree graph representation of the distribution system

5.3 Optimal sectionalizing in interconnected radial networks

A typical radial distribution system consists of a number of single feeder tree-like networks interconnected through tie-lines. Tie-lines contain switches that are usually open and are used in supply restoration or line reconfiguration.

Additional unsupplied energy reduction can be achieved by network reconfiguration. A subsystem that is disconnected from its main source of energy may be alternatively supplied through tie-lines connecting different feeders. The faulted section must be isolated from both sources in order to make alternative supply possible. This can be done by using sectionalizers in network reconfiguration management. In the example in Fig. 5.4, the sectionalizer in section $D(i)$ disconnects the faulted section i from the main source, while the sectionalizer in section k disconnects it from the alternative source. After the tie-line switch is closed only the subsystem **B** is unsupplied until repair team arrival. Note that without alternative supply much greater number of customers (subsystem **A**) remains unsupplied.

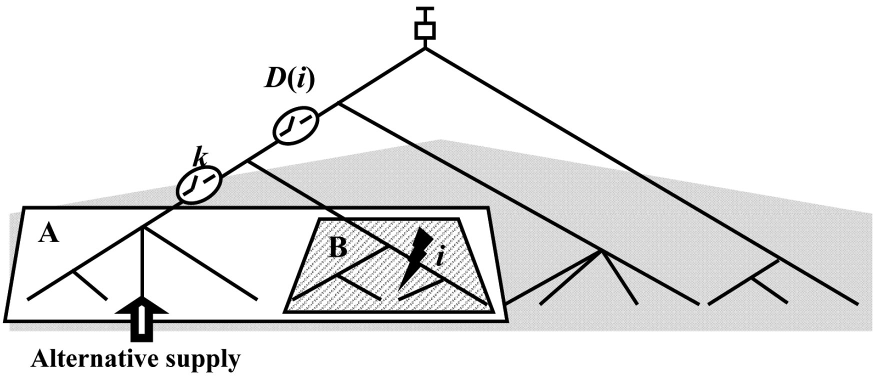

Fig. 5.4. Sectionalizing in radial distribution system with alternative supply

This section presents an algorithm that gives optimal sectionalizer allocation taking into account alternative supply possibilities. Sectionalizers are usually installed when the tie-lines already exist. However, to achieve the greatest effect on system reliability, allocation of tie-lines and sectionalizars should be considered simultaneously, as a single combinatorial problem.

Two types of optimization problem are addressed.

Type 1. Optimal allocation of a specified number of sectionalizers in a radial system with a given allocation of tie-lines. The problem arises when decisions about sectionalizer installation in existing networks are made.

Type 2. Optimal allocation of a specified number of tie-lines and sectionalizers in a given radial system. This definition of the problem is relevant during network planning. It may also be used when decisions about network extensions and made.

For the sake of simplicity we assume that there are no line capacity limitations and sectionalizing switches themselves are 100% reliable. It is also assumed that the probability of simultaneous failure in different section of the network is negligibly small. Therefore, the energy supply can be resumed after the single faulted section is isolated from all the sources.

5.3.1 Effect of alternative supply on the unsupplied energy

If there is no alternative supply in the system, disconnection of any section $j \in S_i$ inevitably causes isolation of section i.

Let us suppose now that an alternative supply tie-line is allocated somewhere downstream of the section m containing a sectionalizer. Consider the failure that causes operation of the closest upstream section m sectionalizer located in section $x = D(s(m))$. This failure should be located downstream of x but not downstream of m. Therefore, after the switch in the section x is opened and the faulted section is disconnected from the main source, the switch in section m may also be opened. This will isolate the faulted section from the alternative source. The tie-line switch may be closed and the supply restored for all sections located downstream of m with total load cost Z_m. The annual reduction of UE cost caused by the faults followed by operation of a sectionalizer located in section m will be

$$\Delta c_m = Z_m \sum_{D(i)=x} \lambda_i r_i \tag{5.29}$$

In the example presented in Fig. 5.5, the tie-line is allocated downstream of sections 3 and 2 containing sectionalizers. For m=3, x=D(s(3))=D(2)=2 and D(i)=2 for $i \in \{2, 12\}$, therefore, $\Delta c_3 = Z_3(\lambda_2 r_2 + \lambda_{12} r_{12})$. For m=2, x=D(s(2))=D(1)=0 and $D(i) = 0$ for $i \in \{1, 6, 13, 14\}$, thus, $\Delta c_2 = Z_2(\lambda_1 r_1 + \lambda_6 r_6 + \lambda_{13} r_{13} + \lambda_{14} r_{14})$.

To calculate the total effect of the alternative supply, the set of sections Q should be defined using the following rule: section i belongs to Q if it contains a sectionalizer and if it lies upstream of a tie-line. Total UE cost reduction for the entire network is

$$\Delta C = \sum_{j \in Q} Z_j \sum_{D(i)=D(s(j))} \lambda_i r_i \tag{5.30}$$

The cost of load isolated by operating the switch located in section m of a network containing alternative supply tie-lines may be calculated as

$$Z_m^*(Q) = Z_m - \sum_{\substack{j \in Q \\ D(s(j))=m}} Z_j \tag{5.31}$$

The total annual unsupplied energy cost for a distribution system with alternative supply may be calculated by substituting Z with Z^* in equation (5.7).

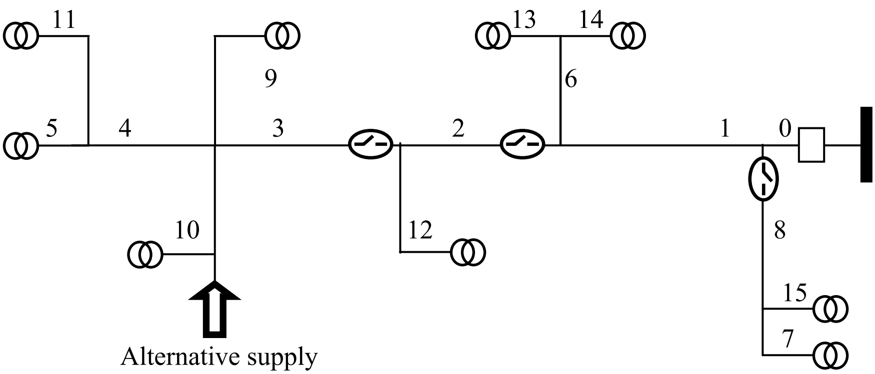

Fig. 5.5. A simple distribution system with single alternative supply tie-line

The following procedure calculates reliability indices for a network with alternative supply for a given allocation of sectionalizers and tie-lines:

Step 1. Calculate average failure time $\lambda_i r_i$ for each network section.

Step 2. Calculate indices Z_i for each section i.

Step 3. Define Q as a set of sections containing sectionalizers and having tie-lines located downstream.

Step 4. For each section i define $Z_i^*=Z_i$.

Step 5. For each $j \in Q$ subtract Z_j from $Z_{D(j)}^*$.

Step 6. Calculate UE cost as

$$C^* = \sum_{i=1}^{K} \lambda_i r_i Z_{D(i)}^* \tag{5.32}$$

The first two steps of the procedure do not depend on the allocation of tie-lines and sectionalizers whereas steps 3-6 should be used for calculating C^* for any given allocation. Steps 4 and 5 define loads disconnected after each fault according to equation (5.31).

5.3.2 Implementing the Genetic Algorithm

As in the case of a single feeder (section 5.2.2) an arbitrary N-length binary string A defines sectionalizer allocation and represents a solution.

To define the allocation of alternative supply tie-lines, solutions are represented by concatenation of binary strings A and B. String B contains S bits, where S is a maximum allowed number of sectionalisers. Each bit b_j in substring B $(1 \le j \le S)$ corresponds to j^{th} nonzero element of substring A. (Note that feasible solutions should contain no more than S nonzero bits). b_j=1 implies that there is a tie-line in the network downstream from section j which contains a sectionalizer. To ensure that the number of tie-lines is no greater than N_t, only the first N_t nonzero bits of B are considered.

Consider, as an example, the binary string (A:B)=(011000010000000:010) which represents the sectionalizer and tie-line allocation in the network presented in Fig. 5.5. The set of nonzero bits in A is A*= $\{a_2, a_3, a_8\}$. The second nonzero bit in B points to a_3 (the second element of A*). This implies that a tie-line is allocated downstream of section 3. It should be noted that B=(010) and B=(110) represent the same solution for given A because allocating a tie-line downstream of section 3 inevitably allocates it downstream of section 2. In contrast, B=(100) implies that a tie-line is allocated downstream of section 2 but not downstream of section 3 (in section 12 in our example).

To solve the sectionalizer allocation problem for a given tie-line configuration (Type 1 problem), the tie-line configuration need not be coded in the solution string. Thus, it is sufficient to use substring A as the representation of the solution.

The only difference between Type I and Type 2 problems is that in the latter the decoding operator must define the numbers of sections connected to tie-lines by decoding the B substring. For Type I problems these numbers are already defined. The final solution representation also differs depending on the problem type. In the first case, it contains only sectionalizer allocation whereas in the second case, information about tie-lines is also included. If substring B points to some section m, this could be interpreted as an indication that the network contains an alternative supply tie-line either in section m or somewhere downstream of this section.

Example 5.2

A medium-scale distribution network was chosen to test the suggested algorithm (Levitin et al. 1995b). The network, which contains 96 sections and 52 load points, is presented in Fig. 5.6.

All installed loads (normal font) and section lengths (italic font) are displayed in the scheme. The total overall load installed is 15.75 MW and the total line length is 40.55 km. The line failure rate is 14 per year per 100 km; the failure rate for the load transformer is 0.007 per year. The restoration time for both lines and transformers is 120 minutes.

The optimal tie-line and sectionalizer allocations for S=4 are displayed in Fig. 5.6. The locations of the switches are distinctly marked for different numbers of tie-lines ($0 \le N_t \le 3$).

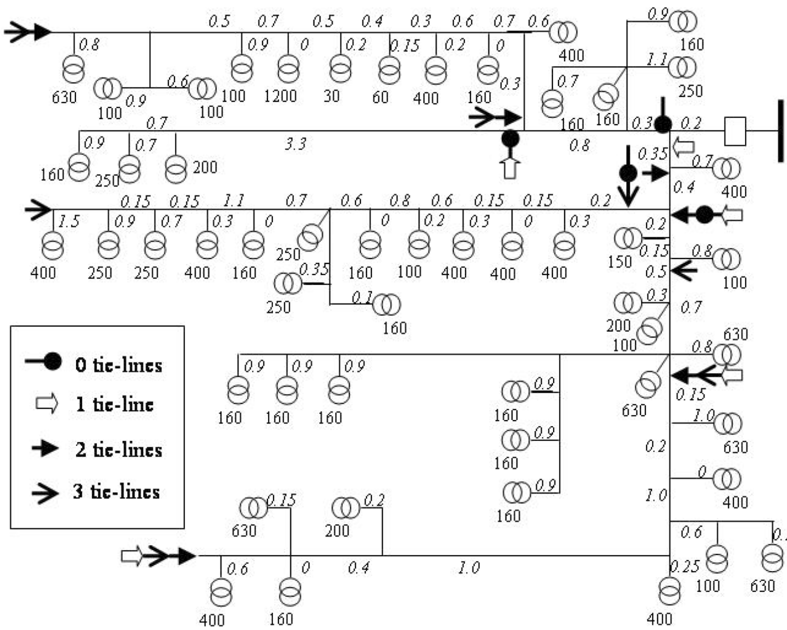

Fig. 5.6. Medium-scale radial distribution network

5.4 Optimal insulation in radial distribution networks

When the reliability enhancement, obtained by the sectionalizing is insufficient the whole line or some part of its sections should be insulated.

The insulation of an individual section reduces its failure rate. To estimate the failure reduction coefficient and insulation cost, the length of the section should be considered as well as technical and environmental conditions and social factors. Generally these indices are available from utility statistics. In this section two types of optimization problem are addressed.

Type 1. Choosing the sections which should be insulated to achieve the greatest unsupplied energy reduction subject to investment cost limitation.

Type 2. Choosing the sections which should be insulated to achieve the desired level of unsupplied energy reduction by the minimum cost.

One can see that in the distribution network with the sectionalizers, the unsupplied energy as well as the number of customers that lose supply as a result of failure in any section depends on the location of this section relative to the sectionalizers and tie-lines. Hence the insulation of each section in the network makes a different individual contribution to the total reliability enhancement of the distribution system.

Let the allocation of all the sectionalizers and alternative supply tie-lines be given. For failures in section i the annual cost of the unsupplied energy caused by the supply interruption during the repair team arrival is (according to 5.32)

$$c_i = \sum_{i=1}^{K} \lambda_i r_i Z^*_{D(i)}, \tag{5.33}$$

where Z^*_m is determined using Eq. (5.31) for any m. After the arrival of the repair team, the faulted section i is disconnected using manual switches, and remains disconnected during the repair. The sectionalizer in section $D(i)$ (or main breaker if $D(i) = 0$) is closed and normal supply is resumed for all customers except those who were supplied through the faulted section. If a tie-line to an alternative source exists downstream of section i, this section can be disconnected manually at both ends and isolated from all energy sources. In this case the load initially supplied through section i can be supplied from the alternative sources.

The cost of load disconnected during the repair in section i can be defined as

$$\widetilde{Z}_i = \begin{cases} Z_i \text{, no tie} - \text{lines downstream of section } i \\ 0, \text{otherwise} \end{cases} \tag{5.34}$$

The annual cost of energy not supplied during repairs in section i can be estimated as $\lambda_i \rho_i c_i^*$, where ρ_i is the repair time for section i.

The total annual cost of the unsupplied energy caused by failures in section i is

$$c_i = \lambda_i (r_i Z^*_{D(i)} + \rho_i \widetilde{Z}_i). \tag{5.35}$$

5.4.1 Effect of section insulation

The insulation of a section reduces its failure rate drastically. The effect of insulation depends on environmental conditions and the type of insulated line (cable) used. Let us introduce a coefficient of failure reduction in section *i*, α_i, which can be estimated from utility statistics for each specific group of sections. The insulation can also affect the repair time of lines. This is taken into account by introducing a repair time variation coefficient β_i.

The total cost of the annual unsupplied energy caused by failures in section *i* after its insulation is

$$\hat{c}_i = \alpha_i \lambda_i (r_i Z^*_{D(i)} + \rho_i \beta_i \widetilde{Z}_i). \tag{5.36}$$

We can now estimate the effect of section insulation in reducing annual unsupplied energy cost:

$$\Delta c_i = c_i - \hat{c}_i = (1-\alpha_i)\lambda_i r_i Z^*_{D(i)} + (1-\alpha_i \beta_i)\lambda_i \rho_i \widetilde{Z}_i. \tag{5.37}$$

In distribution systems without automatic sectionalizing, $D(i)=0$ for each section. To obtain the cost of the unsupplied energy reduction for such a system one has to replace the term $Z^*_{D(i)}$ in Eq. (5.37) with the total network load cost Z_0.

For systems which operate under homogeneous conditions the following assumptions can be made:

- the failure rates are proportional to the section length ($\lambda_i = \lambda$);
- the series failure rates are equal for all sections;
- the arrival and repair times do not depend on the section location and are equal for all sections;
- the effect of insulation does not depend on specific section location.

In this simplified case the unsupplied energy cost reduction can be calculated as

$$\Delta c_i = c_i - \hat{c}_i = (1-\alpha)\vartheta l_i r Z^*_{D(i)} + (1-\alpha\beta)\vartheta l_i \rho \widetilde{Z}_i. \tag{5.38}$$

where ϑ is the series failure rate and l_i is the length of *i*-th section.

In reduced form Eq. (5.38) can be written as

$$\Delta c_i = c_i - \hat{c}_i = l_i(\delta Z^*_{D(i)} + \gamma \widetilde{Z}_i), \tag{5.38}$$

where δ and γ are constant parameters:

$$\delta = (1-\alpha)\vartheta\ r, \quad \gamma = (1-\alpha\beta)\vartheta\ \rho. \tag{5.39}$$

5.4.2 Optimization problem

For each section i the cost of its insulation, h_i, can be estimated. This cost depends on section length, type of insulation, environmental conditions and social factors.

In the type 1 optimization problem the total investment cost which should not be exceeded, H, is given. To achieve the maximal insulation efficiency, the maximum possible reduction of the system unsupplied energy cost has to be obtained at a cost that does not exceed H. The problem is to define a set of sections, X, to be insulated in such a way that

$$\sum_{i\in X}\Delta c_i \to \max, \quad \sum_{i\in X} h_i \le H. \tag{5.40}$$

This problem can easily be formulated in terms of 0-1 integer linear programming as follows.

Find x_i for $0\le i \le N$:

$$\sum_{i\in X}\Delta c_i x_i \to \max, \quad \sum_{i\in X} h_i x_i \le H, \quad x_i \in \{0,1\} \tag{5.41}$$

where $x_i = 1$ implies that section i should be insulated.

In the Type 2 formulation the desired level of unsupplied energy cost reduction, C_{des}> is given. The problem is to define a set of sections, X, to be insulated in such a way that

$$\sum_{i\in X} h_i \to \min, \quad \sum_{i\in X}\Delta c_i \ge C_{des} \tag{5.42}$$

or, in terms of integer linear programming,

$$\sum_{i\in X} h_i x_i \to \min, \quad \sum_{i\in X}\Delta c_i x_i \ge C_{des}, \quad x_i \in \{0,1\}. \tag{5.43}$$

5.4.3 Implementing the Genetic Algorithm

Any solution of the optimal insulation problem can be represented in the genetic algorithm by N-length binary string, representing x_i for $0\le i \le N$. After decoding a binary string one can determine the solution fitness as

$$\sum_{i \in X} \Delta c_i x_i - \Omega \cdot 1(\sum_{i \in X} h_i x_i > H) \quad (5.44)$$

for the Type 1 problem and

$$\Omega \cdot 1(\sum_{i \in X} \Delta c_i x_i \geq C_{des}) - \sum_{i \in X} h_i x_i \quad (5.45)$$

for the Type 2 problem, where Ω is a sufficiently large constant.

In the initial stage of the algorithm, unsupplied load costs Z_i* *and* $\widetilde{Z}_i$ are calculated using the procedure defined in Section 5.3.1 and Eq. (5.34) respectively. The Δc_i indices are calculated for each section using Eq. (5.37) or Eq. (5.38). Having the Δc_i and h_i coefficients, the GA procedure then solves the optimization problem according to formulation (5.41) or (5.43).

Example 5.3

A simple distribution system with 15 load units and 25 sections is presented in Fig. 5.2 (Levitin et al. 1996). Table 5.3 shows system parameters according to section number. The tie-line to an alternative power source is connected to section 25.

Table 5.3. Distribution system data

Section No i	Line type	Load (kW)	Load flow through section	Section length (km)
1	1	0	1490	1.4
2	1	0	1050	0.9
3	1	0	630	0.6
4	1	0	140	2.4
5	1	70	70	1.4
6	1	0	440	1.95
7	2	0	330	1.55
8	1	0	390	1.5
9	1	0	390	1
10	1	100	100	2.35
11	1	70	70	1.4
12	1	230	230	1.5
13	1	160	160	1.2
14	2	60	60	0.6
15	2	200	200	0.8
16	2	130	130	0.1
17	1	80	80	2.2

Table 5.3. (cont.)

Section No i	Line type	Load (kW)	Load flow through section	Section length (km)
18	1	130	130	0.1
19	1	160	160	0.75
20	1	70	70	0.1
21	3	140	140	1.35
22	3	0	280	0.5
23	3	200	200	1.5
24	1	0	420	0.5
25	3	80	80	0.55

The sections are divided into three different types in accordance with specific environmental conditions. The failure rates, repair times, cost and effect of line insulation are different for each type. These indices are presented in Table 5.4.

Table 5.4. Line parameters

Type	ϑ (km^{-1})	r (min)	ρ (min)	h/l ($/km)	α	β
1	0.08	60	120	20000	0.25	1.5
2	0.12	60	150	23000	0.1	1.8
3	0.12	90	150	21000	0.1	2.0

Table 5.5 shows the solutions of Type 1problem which were obtained for an investment limitation of H =100 000 $, for six versions of the allocation of sectionalizers and tie-lines. One can see that partial insulation is most effective in the network without sectionalizing and alternative supply, where the damage caused by each failure is maximal.

Table 5.5. Optimal partial insulation solutions

Solution no.	Sections with sectionalizers	Sections with tie-lines	Total UE reduction (kW)	Total insulation cost	Set of insulated sections
1	-	-	1545.3	99900	2,21,22,23
2	-	25	1471.9	100000	14,16,18,21,22,23
3	24	25	856.2	99850	3,7,9,14,15
4	2	25	885.4	99800	7,16,18,21, 23
5	2,24	25	622.5	99850	3,7,15,16,23
6	2,24	-	1164.1	99850	1,7,14,15,18,20

5.5 Optimal replacement scheduling for distribution system equipment

5.5.1 Formulation of the problem

Optimal replacement scheduling is one of the important problems in the reliability engineering (Gertsbakh 1999). It is also practically important for power system reliability enhancement (Billinton and Allan 1996). The quality and reliability of power system equipment is constantly improved. The old equipment disappears from the market. Therefore when the old equipment installed fails, it can only be replaced by a new type. It is sometimes worthwhile to replace the old type equipment by a new type preventively in order to improve system reliability. Since such preventive replacement is associated with certain expenses, it should be justified by reduction of losses associated with failures occurring in the system. In order to make a decision about preventive replacement one has to compare costs of system failures, replacement costs and costs of new and old equipment maintenance for different replacement options.

In this section we consider the case in which a power system contains a number of identical units. These units fail at a given failure rate. Each failure causes damage associated with energy losses in the power system. The faulty units are replaced by units of a new type that have a lower failure rate than that of the old type units. The old and new units have different costs of preventive surveillance and maintenance.

In general there exist three possible replacement policies. The first one is to replace all old type elements at the same time (at the time when the new type units appear on the market). The second one is a so-called "natural strategy" when every old type element is replaced by the new type element immediately after its failure and no preventive replacements are performed. The third policy is combined one. Following this policy one uses the "natural strategy" till K-1 old-type units are failed and replaced by the new type ones according to natural policy. When the K-th old type element fails, all the remaining old-type units are replaced.

If the initial number of old type units in the system is N, one can consider the first and the second policies as a special cases of the third policy where K=N and K=0 respectively. So, the problem is to find the optimal K, minimizing the expected total cost.

To solve the formulated problem the following assumptions are made:

1. The replacement time is negligibly small, hence replacement is executed instantly.

2. Up-to-failure time is distributed exponentially for both types of system units.
3. The number of units in the system is large enough, so that the following condition is satisfied,

$$\sum_{k=N}^{K+1} \frac{1}{k\lambda_0} << T, \tag{5.46}$$

where T is a planning time horizon and λ_o is failure rate of old type equipment.

5.5.2 Model description

State-space diagram for the system model is presented in Fig. 5.7A. Every state is designated by two symbols: (i,j). The first one is the number of old type units in the system, the second one is the number of new type units in the system.

In the initial state all the units in the system are old type, so the system state is $(N, 0)$. After the first old type unit failure the new type unit will replace the failed element. So, the system transits to the state $(N\text{-}1,1)$. The transition intensity from state $(N, 0)$ to state $(N\text{-}1,1)$ is $N\lambda_0$. The cost (negative reward) associated with the transition is $c_f=c_d+c_r$, because in this case both the replacement expenses c_r and the damage losses c_d arise.

In each state $(N\text{-}j, j)$ there exist two options. If one of the remaining old type units fails before the new type unit had failed (the first option), the new type unit replaces the failed old type unit. So, the system transits from state $(N\text{-}j,j)$ to state $(N\text{-}j\text{-}1,j+1)$ with transition intensity $(N\text{-}j)\lambda_0$ and with associated cost of single unit replacement (including damage cost) c_f. If the new type unit fails before any one of remaining old type units (the second option) the failed unit is immediately replaced with a unit of the same type. So, with intensity $j\lambda_n$ the process transits to the same state $(N\text{-}j,j)$ (remains in its present state). The cost associated with this transition is also c_f. In general case a number of transitions from the state $(N\text{-}j,j)$ to the same state are possible (with intensity $j\lambda_n$, where λ_n is failure rate of new type equipment) before system transition to the state $(N\text{-}j\text{-}1,j+1)$.

The process has the same structure until arrival to state $(K,N\text{-}K)$, where only K old type units remain. At this time instant t_k all the remaining K old type units are to be replaced by new type units. The process immediately transits to the final state $(0,N)$ (with intensity ∞). The cost associated with this transition is determined by the cost of all K new type units, which replace remaining old type units: Kc_p, where c_p is the cost of preventive replacement of single unit.

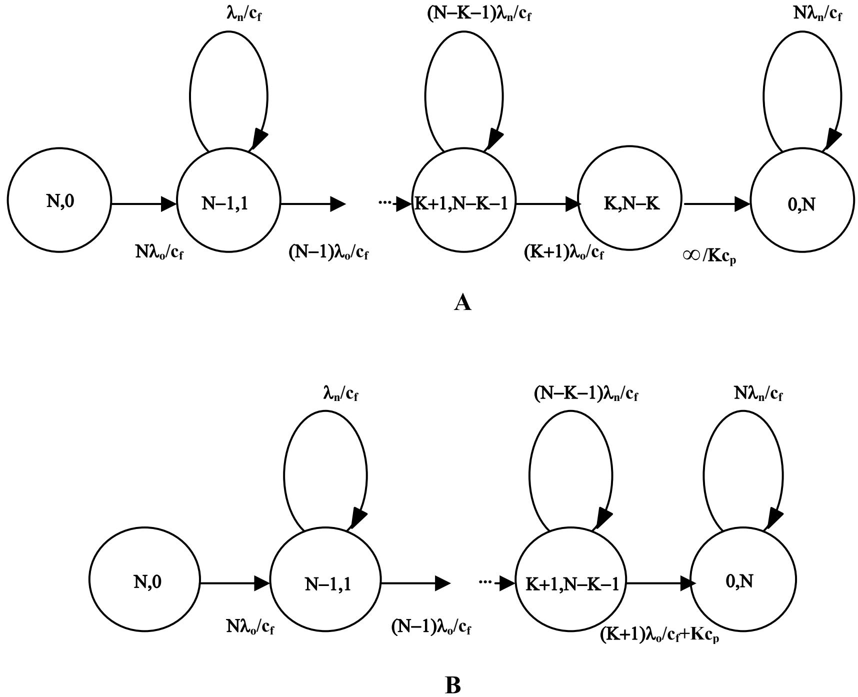

Fig. 5.7. State-space diagram of the system with N elements

After arrival at the final state (0, N) the process continues up to horizon time point T as an ordinary renewal process. In other words, when only the N new type units comprise the system, the failures occur with intensity $N\lambda_n$. The failed units are to be replaced immediately by identical ones and the process returns to the state (0, N). The cost associated with each transition is c_f.

As was mentioned above, all the replacements in the state (K,N-K) are performed instantly (the replacement time is 0), so that one can unite two states (0,N) and (K,N-K). The state (K,N-K) can be removed from the diagram in Fig. 5.7A so that the system transits from the state (K+1,N-K-1) to the final state (0,N) with intensity $(K+1)\lambda_0$ and associated cost (c_f+Kc_p). The final diagram is presented in Fig. 5.7B.

5.5.3 Estimating the total expected cost

Failures and replacement cost

In the model considered the first state of the system is $(N,0)$ and its final state is $(K,N-K)=(0,N)$.

One has to estimate the total expected reward (cost) $C_{tot}(T)$ associated with time interval $[0,T]$, where the system's initial state at instant $t=0$ is $(N,0)$.

First, consider the failure costs before preventive replacement. Since the transition intensity in the state $(N,0)$ is $N\lambda_0$, the expected time of transition from the initial state $(N,0)$ to the state $(N-1,1)$ is $1/(N\lambda_0)$. The number of year in which the expected transition occurs can be obtained as $\lfloor 1/(N\lambda_0)\rfloor$ where $\lfloor x \rfloor$ is a maximal integer not greater than x. Now one can obtain the expected cost of transition from state $(N,0)$ to state $(N-1, 1)$ in present values as follows:

$$c_f(1+IR)^{-\lfloor 1/N\lambda_0 \rfloor}, \tag{5.47}$$

where IR is the interest rate.

According to the general system transition diagram (Fig. 5.7B), the state transition diagram for state $(N-j,j)$ where $1\le j\le N-K-1$ can be represented by Fig. 5.8. In this figure states A and B correspond to states $(N-j,j)$ and $(N-j-1,j+1)$ respectively, transition intensities are

$$\lambda_A=j\lambda_n,\ \lambda_B=(N-j)\lambda_o \tag{5.48}$$

and transition costs are c_f.

For exponentially distributed up-to-failure times the cumulative distribution functions for times in which the system remains in state A before transition to state A or B are respectively

$$F_{AA}(t)=1-e^{-\lambda_A t} \text{ and } F_{AB}(t)=1-e^{-\lambda_B t}. \tag{5.49}$$

According to (Sahner et al. 1996) one can obtain the one-step final transition probabilities as follows:

$$q=\lim_{t\to\infty}\int_0^t [1-F_{AB}(t)]dF_{AA}(t)$$

$$\text{and } p = \lim_{t\to\infty}\int_0^t [1 - F_{AA}(t)]dF_{AB}(t), \qquad (5.50)$$

where q is the probability of transition from state A to the same state A at one step and p is the probability of transition from state A to state B. By substituting $F_{AA}(t)$ and $F_{AB}(t)$ in these expressions with expressions (5.49) one obtains

$$q = \frac{\lambda_A}{\lambda_A + \lambda_B}, \qquad p = \frac{\lambda_B}{\lambda_A + \lambda_B}. \qquad (5.51)$$

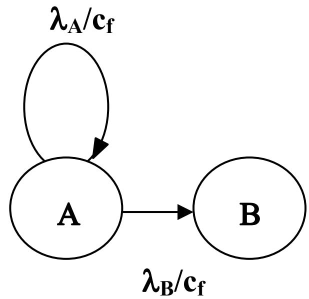

Fig. 5.8. State-space diagram for transitions from a single state

So, in state A we can consider the process as a sequence of Bernoulli trials with random variable X defined as the number of trials required in order to achieve the first success (transition to the state B). The number X is equal to number of transitions from state A to state A itself before the transition from state A to state B. It is distributed according to geometric distribution (Hines and Montgomery 1996):

$$p_m = \Pr\{X = m\} = q^m p = \frac{\lambda_A^m \lambda_B}{(\lambda_A + \lambda_B)^{m+1}}, \quad m=0, 1, \ldots \qquad (5.52)$$

Suppose, that the process presented in Fig. 5.8 begins when time t_A has passed from the beginning of the process (state (N,0)). The expected time of m-th transition from state A to state A is t_A+m/λ_A. The expected cost of this transition is, therefore,

$$c_f(1 + IR)^{-\lfloor t_A + m/\lambda_A \rfloor} \qquad (5.53)$$

The total expected cost associated with transitions from state A to itself could be obtained as

$$c_f \sum_{m=1}^{\infty} p_m \sum_{i=1}^{m} (1+IR)^{-\lfloor t_A + i/\lambda_A \rfloor}. \quad (5.54)$$

The expected time of transition from state A to state B (which does not depend on transitions from A to A) is

$$t_A + 1/\lambda_B. \quad (5.55)$$

The expected cost of this transition (in present values) is

$$c_f (1+IR)^{-\lfloor t_A + 1/\lambda_B \rfloor}. \quad (5.56)$$

Using Eq. (5.48) and (5.55) one can obtain the expected time τ_j of system arrival to arbitrary state $(N\text{-}j,j)$ (state A in the diagram presented in Fig. 5.8):

$$t_A = \tau_j = \sum_{k=0}^{j} \frac{1}{(N-k)\lambda_o}. \quad (5.57)$$

Using (5.48), (5.52) and (5.57) one can rewrite the Eq. (5.54) for the state A=$(N\text{-}j,j)$ as

$$c_{nj} = c_f \sum_{m=1}^{\infty} \frac{(j\lambda_n)^m (N-j)\lambda_0}{(j\lambda_n + (N-j)\lambda_0)^{m+1}} \sum_{i=1}^{m} (1+IR)^{-\left\lfloor \sum_{k=0}^{j} \frac{1}{(N-k)\lambda_0} + \frac{i}{j\lambda_n} \right\rfloor}, \quad (5.58)$$

and Eq. (5.56) for this state as

$$c_{oj} = c_f (1+IR)^{-\left\lfloor \sum_{k=0}^{j} \frac{1}{(N-k)\lambda_0} + \frac{i}{(N-j)\lambda_0} \right\rfloor}. \quad (5.59)$$

By summing up the corresponding expected costs c_{nj} and c_{oj} for all the states $(N\text{-}j,j)$ $(1 \le j \le N\text{-}K\text{-}1)$ one obtains the total failure costs before preventive replacement:

$$C_o(K) = \sum_{j=1}^{N-K-1} c_{oj}, \; C'_n(K) = \sum_{j=1}^{N-K-1} c_{nj}, \quad (5.60)$$

where C_o and C'_n are total costs associated with failures of the old type equipment and of the new type equipment before preventive replacement respectively (in present values).

The estimated time of the system arrival to the state $(K,N\text{-}K)$ is

$$\tau_{N-K} = \sum_{i=K+1}^{N} \frac{1}{i\lambda_o}. \quad (5.61)$$

When the system arrives at this state, the preventive replacement of K old type units is performed immediately. The cost of this action C_p in present values is

$$C_p(K)=Kc_p(1+IR)^{-\left\lfloor \sum_{i=K+1}^{N} \frac{1}{i\lambda_0} \right\rfloor}. \quad (5.62)$$

Consider now the expected cost C''_n associated with new type element failures occurring in the united final state $(K,N-K)=(0,N)$ during the period $(\tau_{N-K} - T)$.

The time that the system remains in its final state $(0,N)$ is $T-\tau_{N-K}$. The failures intensity in the final state is $N\lambda_n$ and the expected time of j-th failure is $\tau_{N-K}+j/(N\lambda_n)$.

The time of the last failure considered should not be greater than the time horizon T. Therefore the number of last considered failure j^* can be obtained from inequality

$$\tau_{N-K}+j^*/(N\lambda_n)\le T \quad (5.63)$$

as

$$j^*=\lfloor N\lambda_n(T-\tau_{N-K})\rfloor. \quad (5.64)$$

Hence, the total expected cost C''_n is:

$$C''_n(K)=c_f\sum_{j=1}^{\lfloor N\lambda_n(T-\tau_{N-K})\rfloor}(1+IR)^{-\left\lfloor \sum_{i=K+1}^{N} \frac{1}{i\lambda_0}+\frac{j}{N\lambda n} \right\rfloor}. \quad (5.65)$$

Maintenance cost

Since the expected time of the j-th old type unit failure is τ_j (determined by Eq. (5.57)), the maintenance period (in years) for this unit is $\lfloor \tau_j \rfloor$. After replacement of the old type unit with a new type one the latter should be maintained during the period from τ_j to T. The expected maintenance cost (in present values) for j-th unit before replacement is $\psi(\tau_j)c_{mo}$, where c_{mo} is an annual cost of single unit maintenance for old type equipment and

$$\psi(x) = \sum_{k=1}^{\lfloor x \rfloor} (1 + IR)^{-k} \ . \tag{5.66}$$

The expected maintenance cost (in present values) for *j*-th unit after replacement is

$$\{\psi(T)\text{-}\psi(\tau_j)\}c_{\text{mn}}, \tag{5.67}$$

where c_{mn} is an annual cost of single unit maintenance for new type equipment.

For all the units that are replaced following failures one can obtain the maintenance cost as

$$\sum_{j=1}^{N-K-1} \{ (\psi(T)c_{mn} + \psi(\tau_j)(c_{mo} - c_{mn})\} \ . \tag{5.68}$$

Since *K* old type units are replaced at the same time $\tau_{N\text{-}K}$, the total maintenance cost for these units is

$$K\{(\psi(T)c_{mn} + \psi(\tau_{N-K})(c_{mo} - c_{mn})\} \ . \tag{5.69}$$

The total maintenance cost $C_m(K)$ can be obtained as sum of Eq. (5.68) and (5.69).

Using Eq. (5.58)-(5.60), (5.62), (5.65)-(5.69) one obtains the total cost as function of decision variable *K* as

$$C_{\text{tot}}(K)=C_{\text{o}}(K)+C'_n(K)+C''_n(K)+C_p(K)+C_m(K). \tag{5.77}$$

Using a simple optimization procedure one can obtain the value of *K* that minimizes $C_{\text{tot}}(K)$.

Example 5.4

Consider a system containing *N*=500 units with parameters presented in Table 5.6. The optimal preventive replacement policy should be based on estimating the total cost as a function of decision variable *K* and damage cost c_{d} associated with each failure.

Table 5.6. Parameters of distribution system with 500 units to be replaced

N	λ_o	λ_n	c_{p} (1000 $)	c_{mo} (1000 $/year)	c_{mn} (1000 $/year)	T (Years)	IR
500	0.01	0.005	12	0.15	0.1	30	0.1

The range of K is determined by the time horizon T. Indeed, for K corresponding to times $\tau_{N-K} > T$ the preventive replacement makes no sense since it should not be performed within the considered time interval. Therefore the minimal possible value of K can be obtained from inequality

$$\tau_{N-K} = \sum_{i=K+1}^{N} \frac{1}{i\lambda_0} < T .$$

For the given set of system parameters this value is equal to 370.

Fig. 5.9 shows the system costs as functions of K for c_d=150,000 \$. One can see that $C_{tot}(K)$ is a decreasing function, which means that the later the preventive replacement is performed the lower is the cost. In this case the preventive replacement is not worthwhile within the defined time horizon.

For greater damage costs the $C_{tot}(K)$ becomes increasing function of K. Fig. 5.10 presents the system costs for c_d=350,000 \$. In this case the preventive replacement should be performed as soon as possible.

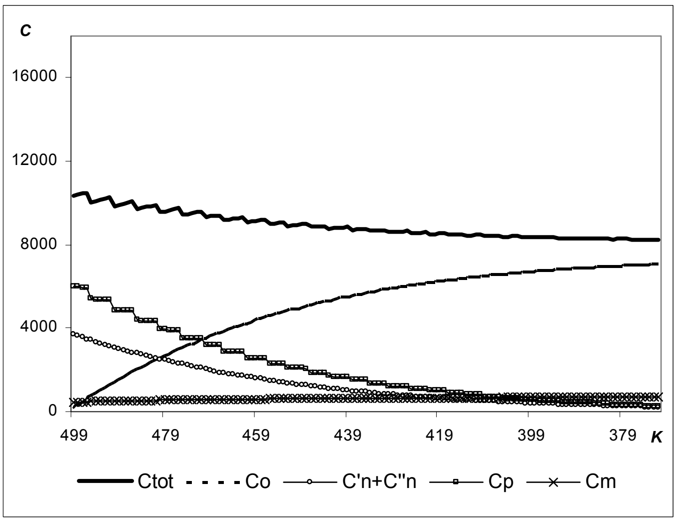

Fig. 5.9. The system costs as functions of K for c_d=150,000 \$.

For some values of damage cost the total cost can be almost independent of the time of preventive replacement. Indeed, as one can see in Fig. 5.11, corresponding to c_d=\$250,000, value of variable K does not affect the total cost $C_{tot}(K)$.

The general conclusion for the presented example is that the preventive replacement should be performed only if the damage cost associated with a single unit failure is greater than $250,000. In this case it should be performed even before the first failure occurs in the system.

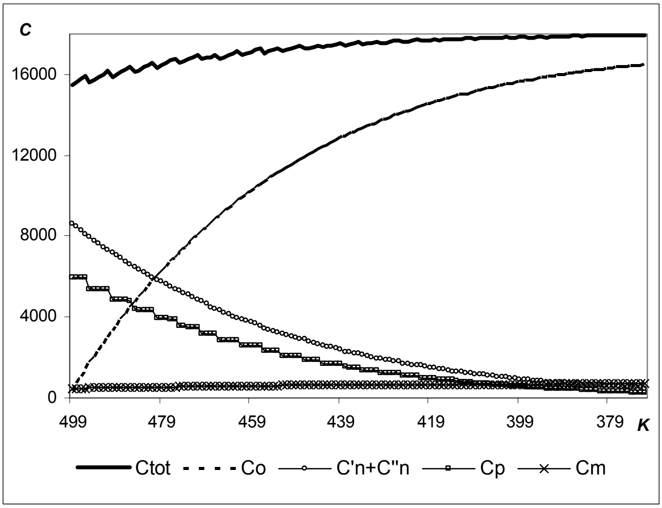

Fig. 5.20. The system costs as functions of K for c_d=$350,000.

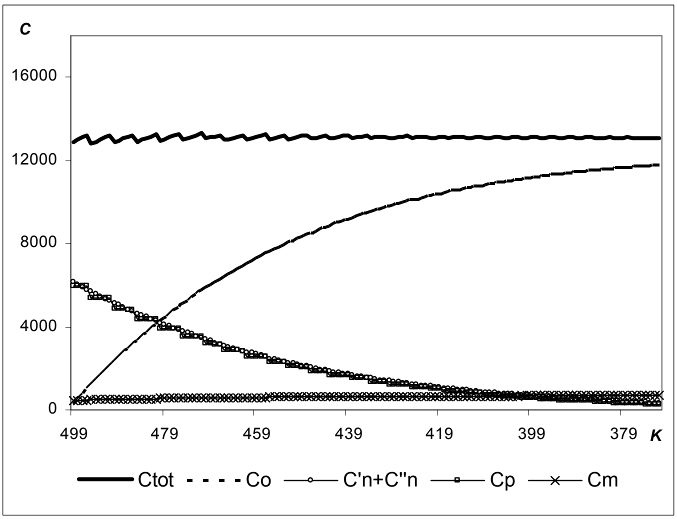

Fig. 5.11. The system costs as functions of K for c_d=$250,000.

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