Improving Primary Mathematics Education, Teaching and Learning

Research for Development in Resource-Constrained Contexts

Edited by
Mellony Graven
Hamsa Venkat

Palgrave Studies in Excellence and Equity in Global Education

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Editors

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Editors' Preface

The editors of this book, Mellony Graven and Hamsa Venkat, hold the two South African Numeracy Chairs at Rhodes University and Wits University, respectively. These Chairs are the result of partnerships between the state and the private sector, being sponsored by the First Rand Foundation (FRF), with Rand Merchant Bank (RMB), the Anglo American Chairman's fund and the Department of Science and Technology (DST), and are administered by the National Research Foundation (NRF). The Chairs are innovative in several ways—they indicate a partnership between private funders, government funders (DST), national research organisations (NRF) and universities (Rhodes and Wits). Additionally and most importantly, they bring development and research together as equal in a partnership that insists that teacher and research communities work together. The aims of the Chairs are thus twofold and focus on a dialectical relationship between development and research. In development terms, the Chair project teams are focused on improving the quality of numeracy teaching at primary level and improving learner performance in mathematics in primary schools as a result of quality teaching and learning. In research terms, the teams are charged with providing leadership in growing the numeracy research field in ways that point to sustainable solutions to the many challenges faced in South Africa and beyond.

The contributing authors in this book are all part of the Chair project teams at Rhodes University (Grahamstown, Eastern Cape) and Wits University (Johannesburg, Gauteng). The Eastern Cape and Gauteng provinces are different, and so too are the schools that the Chair projects work with. The Eastern Cape is one of the poorest provinces in the country, with among the lowest education performance across a range of national measures. It is also a largely rural province, with 3 predominant languages: isiXhosa, English and Afrikaans. Gauteng is the wealthiest of South Africa's provinces, and its schools have among the highest performance results in education in the country, and thus some relative advantage in the national context in spite of poor performance in comparison with regional and international contexts. Its population is almost entirely urban, and multiple languages are spoken in schools. The combined research across these two diverse contexts points to the way in which combining research and development enables not only one to meet ethical imperatives of responding to the needs of the contexts within which one is working but also richer research that speaks to new avenues of possibility for interventions in resource-constrained contexts. The first two chapters in Part I expand on this. The chapters in Part II explore ways in which assessment of numeracy and its growth, as a key aspect of intervention for improved progression, can be implemented in ways that are responsive to resource-constrained contexts and the various challenges present in such contexts. The chapters in Part III explore language issues in primary mathematics teaching, learning and assessment with a particular view to reconceptualizing language as a resource rather than language as a problem. In South Africa, as in the case of many colonised developing countries, the language of learning and teaching mostly differs from learners' home languages, adding to the challenges of achieving quality and equity in mathematics education. Part IV focuses on the critical aspect of numeracy teacher support and explores ways in which this can be strengthened to enable increased equity and quality within the education system and in mathematics education particularly.

In each chapter, authors share the way in which their work contributes to the broader Chair aims and the field of numeracy education. In the past, numeracy and primary mathematics education has been neglected as the focus of research and intervention into 'the crisis' tended to focus on 'fixing' high-stakes exit-level grade 12 mathematics examination results. Pleasingly, public and government sentiments are increasingly realising that interventions focused on the Further Education and Training (FET grades 10–12) are much too late as the majority of learners are lost to mathematics by the end of grade 9 (as indicated by the 2014 mathematics national average of 11% for the Annual National Assessments [ANAs]). The extreme nature of the mathematics crisis in South Africa provides a powerful context for innovative research aimed at addressing inequity in mathematics education.

Below is a brief outline of the contributions of the various parts and chapters in the book.

Part I: Interventions for Quality and Equity in Primary Education

Chapter 1: Intervening in the Learning and Teaching of Numeracy in Contexts of Poverty

In this introductory chapter, Jill Adler provides a broader commentary on the Mathematics Education and Numeracy Chairs initiative in South Africa. She locates her discussion in the context of primary mathematics as an under-researched field in South Africa, while noting too the ways in which South African research in this area speaks to and informs broader contexts of Research and Development (R&D) in developing contexts marked by scarce and inequitable distribution of resources. Furthermore, she engages with the question of what the 'local' research presented in this book, focused as it is on teaching and learning in 'schools for the poor' in South Africa, might offer the global community.

Chapter 2: Advocating Linked Research and Development in the Primary Mathematics Education Landscape in Contexts of Poverty

Mellony Graven and Hamsa Venkat elaborate, In this Chapter. 2, on the specific rationales for a focus on numeracy, as well as the urgent need for interventions that are both located in, and build up, a research base that can support well-theorised change. In this chapter, they address some of the limitations that have been noted as emanating from university-based research that is frequently criticized for its distance from 'on-the-ground' realities, and NGO development activities that have failed to build a rigorous and cumulative knowledge and theory base, and where effective, have produced largely small-scale, localized and non-reproducible gains. Aspects relating to primary mathematics development in South African and international contexts marked by disadvantage are also introduced and discussed in the two chapters in this part, with concluding comments noting an advocacy for the emergent, linked research and development model as an ethical necessity in under resourced contexts.

Part II: Assessing Numeracy Progression in Resource-Constrained Contexts

Chapter 3: Using Assessments to Explore Early and Later Performance in Mathematics

In this chapter, Marie Weitz and Hamsa Venkat investigate the relationships between performance on the grade 1 and grade 5 ANAs. This performance is then compared to learner performance on an alternative research informed test that assesses many of the competences described as important in the literature. Working with a matched sample of learners from ten government primary schools in Gauteng (five township schools and five suburban schools), all serving predominantly historically disadvantaged learners, their findings point to better predictive potential for the alternative test. Interestingly, they also find differences in the patterns of predictive power between the township and suburban schools. They conclude by drawing out the implications of their findings for improved national numeracy assessment of learners.

Chapter 4: Researching the Nature of Early Numeracy Progression in After-School Clubs

In this chapter, Debbie Stott shares insights of grade 3 students developing mathematical proficiency through their participation in two after-school mathematics clubs. The introduction of after-school mathematics clubs as fun, explorative and engaging mathematical spaces, free from the usual classroom constraints of curriculum compliance and management of large classes, was a key development initiative of the Rhodes University Chair project. Drawing on Wright et al.'s numeracy progression model (the Learning Framework in Number) and Kilpatrick et al.'s strands of mathematical proficiency, she illuminates the way in which learners transformed their mathematical competence, their learning dispositions and their relationship with mathematics.

Chapter 5: Adapting Aspects of the Mathematics Recovery Program for Use in Various South African Contexts

Wright et al.'s Mathematics Recovery (MR) program is extensively utilised across international contexts, and it has been widely used in South African contexts by research students across the two Chair projects. In this chapter, Debbie Stott, Zanele Mofu and Siviwe Ndongeni illuminate the way in which key aspects and activities of the MR program required adaptation for use in each of their three local contexts. Their chapter explores both the usefulness and the challenges of using MR tools for both analysis and development of learner mathematical proficiency. Through combining their research findings, they provide three cases as examples of adaptations and extensions that emerged in their studies.

Chapter 6: Adapting Mathematics Recovery Individual Assessments for Group Administration in Resource-Constrained Contexts

A key aspect of the MR program involves administering assessment interviews to individual learners in order to find the level from which one should begin 'recovery' work. Within poor, under-resourced South African contexts, it is the majority of learners rather than a few who require recovery and it was thus considered unlikely that teachers would find the time to conduct such individual assessments. Thus, Anelia Wasserman explored the possibility for group interview assessments based on the MR program ideas. In this chapter, she shares the insights into her MR action research intervention with 23 learners in one grade 4 class in a township school in the Eastern Cape. Her experiences and adaptations to the individual assessment format point to useable possibilities for teachers as well as constraints in administering interview-based assessments with groups of learners.

Part III: Understanding Language Issues in Primary Mathematics Teaching, Learning and Assessment

Chapter 7: Using an Adapted Model of Reciprocal Teaching to Help Children Unpack Their Word Sums

In this chapter, Melissa Spira and Sally-Ann Robertson address, from a teacher perspective, the challenge of supporting English Language

Learners (ELLs) in making sense of 'word sums'. They share the experiences of Spira's trialling a reciprocal teaching approach as a way of enhancing her grade 3 learners' abilities to make sense of word sums. They argue that while this approach, initially developed as a strategy to help strengthen children's reading literacy skills, has been taken up by members of the mathematics education community, little has been done in relation to younger learners, most particularly those who are learning mathematics in a language that is not their mother tongue. This is the case for the majority of South African learners and for the learners in the class in which this reciprocal approach to teaching 'word sums' is explored through action research. In this way, the chapter contributes to ways in which reciprocal teaching could be used as a strategy for strengthening learners' meaningful engagement with 'word sums' in linguistically diverse classroom contexts.

Chapter 8: Language in Early Number Learning in South Africa: Linking Transparency and Explicitness

In this chapter, Manono Mdluli analyses the ways in which language can be used in primary mathematics teaching to support children in learning about decimal structure by focusing on the pattern and structure of number names in their home language. Using data excerpts from a grade 3 Sepedi-medium classroom, she points to the need for explicit attention to the structuring of number present in the Sepedi language within teaching. In this way, she shares how explicit focus on language can be a resource for learning and, in so doing, theorizes language for mathematics teaching in terms of explicitness and transparency. Her analysis of excerpts of one grade 3 teacher teaching the base ten structure of number in Sepedi as the medium of instruction points to similar opportunities for teaching learners base ten structure through the many other official South African languages.

Chapter 9: Learners Exemplifying for Themselves: Grade 2's Telling Additive Relations Stories

In this chapter, Nicky Roberts describes the outcomes of an intervention in which young learners were supported to exemplify additive relations situations through development of their own stories. The aim was to support children to imagine these situations and to express these situations in story-telling activities. In the chapter, she shows how English language learners can, through a narrative approach, be supported to simultaneously develop their linguistic and mathematical repertoires. Her findings point to the strength of this approach for supporting mathematical sense-making especially in the early primary years in multilingual classrooms.

Chapter 10: Do the Annual National Assessments in Mathematics Unfairly Assess English Language Competence at the Expense of Mathematical Competence?

In this chapter, Lucy Sibanda investigates the nature of the linguistic complexity of the grade 4 Department of Education (DoE) ANAs and expands on how learners (with a poor command of the language of the assessments) experience them. Her comparison of first language isiXhosa learners' performance on selected ANA items with a high linguistic complexity index with their performance on the same items when linguistic mediation (including translation) is provided points to significant improvement in performance of several learners. In this way, she reveals the linguistic bias of the ANAs in favour of first language English speakers and raises questions about their validity. The chapter argues for linguistic complexity checks to be conducted on all ANA items and that this is especially critical for grade 4 level assessments when the majority of South African learners have only recently transitioned from learning (and being assessed) in their home language to learning and being assessed in English.

Part IV: Supporting Change in Primary Mathematics Teaching

In this part, the focus is on studies that theorize and intervene to support changes in primary mathematics teaching.

Chapter 11: Changing Teaching Through a Resources Approach

In this chapter, Hamsa Venkat and Mellony Graven engage with ways in which resources can be used as levers for change in primary mathematics teacher development. They draw on Adler's earlier work on resources, which argued that particularly in contexts of poverty, where resources are scarce, resources become important for effecting change in teaching practices. They explore work within their own projects, and others in the South African landscape in terms of the resources that are attended to across the categories of 'human', 'material' and 'cultural' resources. They use these categories to contrast resources in focus in larger-scale interventions (predominantly material and cultural) with the more in-depth and labour-intensive focus on human resources in their smaller-scale initiatives.

Chapter 12: From Theory to Practice: Challenges in Adopting Pedagogies of Mathematizing in South Africa

In this chapter, Thulelah Thakane, Herman Tshesane and Mike Askew explore specific tensions arising when implementing interventions based on the principles and resources arising from Realistic Mathematics Education (RME) and Cognitively Guided Instruction (CGI) in South African classrooms. Their experiences identify tacit assumptions that underpin mathematizing approaches in RME and CGI and raise issues about the applicability of these assumptions for informing the use of similar approaches in the context of South Africa. They conclude by noting the potential for working with a mathematizing approach in developing nations like South Africa. However, some of the expectations taken for granted in the countries in which these pedagogies emerged need to be made more explicit and then interrogated and adapted for use elsewhere.

Chapter 13: Characterizing Responsive Primary Mathematics Teaching in the South African Context

In this chapter, Lawan Abdulhamid shares classroom interaction data of four teachers to reveal emergent categories for characterizing responsive teaching in classroom contexts marked by gaps in teachers' mathematical knowledge and collective choral practices that limit evaluation of individual learner contributions. These categories emerge from a follow-up study focused on analysis of four teachers' practices following their participation in a one-year in-service primary mathematics for teaching course within the Wits Chair project. This program focused on the development of mathematics knowledge for teaching that included emphasis on responsive teaching. The categories of responsive primary mathematics teaching that emerge in this chapter offer 'home-grown' rather than 'imported' descriptions of local teaching practices with potential for building the kinds of responsive in-the-moment decision making that are important for supporting mathematical learning. He argues that awareness of possible shifts towards responsive teaching within prevalent teaching practices, such as choral responses, is important as it represents openings for moves away from deficit characterizations based on absences, to characterizations directed towards improvement.

Chapter 14: Key Aspects of Communities of Practice that Enable Primary Maths Teacher Learning

In this chapter, Peter Pausigere shares stories of primary maths teacher learning transformation enabled by their participation in the primary maths in-service teacher development program of the Rhodes University Chair project. He illuminates how grade R-6 teachers' different mathematical histories and appropriation of affordances through participation in the Numeracy Inquiry Community of Leader Educators (NICLE) program enabled strengthened mathematics teacher identities. His findings point to reinvigoration of mathematical identities for those with stronger mathematical histories and remediation and activation of mathematical identities for those with weaker or negative mathematical histories. Additionally, he shares the key affordances within NICLE that interviewed teachers identified as enablers of these strengthened mathematical identities, namely, opportunities for: deepening learning of numeracy-domain concepts, changing classroom practices and embracing a supportive and participatory ethos in NICLE.

Reflections

In concluding the book, final reflections on the contributions within and across the chapters are provided by Mike Askew, drawing on his wide experience as a leading international expert in primary mathematics research and development. Here he foregrounds links between the research areas that feature in this volume and the international field, and looks at ways in which the conversation might be taken forward both nationally and internationally in the service of primary mathematics development.

Mellony Graven
Hamsa Venkat

Series Introduction

The series *Palgrave Studies in Excellence and Equity in Global Education* is a bold new initiative for the transnational study of education. The linking of excellence and equity in this timely series is intentional. It is only at a first and indeed, a cursory, glance that the two concepts will appear in any way disparate. A more perceptive view will acknowledge the potentiality in considering excellence and equity in dynamic relation to one another. There are two significant reasons why this latter understanding ought to prevail. First, in the view of many researchers, teachers, policymakers and parents, excellence and equity, very far from being incompatible, remain dual, even inseparable themes in education today. Second, there is a pressing need for scholars to extend and broaden the various debates and issues that surround excellence and equity in a way that clearly focuses on the various ways education systems around the globe have conceived and responded to them. This being the case, it is unfortunate that, as yet, there have been few sustained attempts within a single series to critically examine the way in which excellence and equity both complement and also conflict with one another.

This series is, therefore, designed to serve an important educative function. Specifically, it has a crucial role to play in enabling students, lecturers, researchers and policymakers to develop crucial and critical knowledge regarding the concepts of excellence and equity, and to learn how these play out within a range of different contexts. Thus, it is intended that this multinational series will make a major contribution to the broader international and national debates surrounding excellence and equity. A particular feature of the series is that the authors/editors of each volume will illustrate in their various ways how excellence and equity are broadly conceived within their specific region or nation, through fields of inquiry and methodologies as diverse as history, sociology, critical pedagogy, critical theory, feminist studies, ethnicity studies, policy studies and/or political studies, to name but a few of the approaches currently being explored around the globe in the twenty-first century. In turn, this inclusive approach will challenge readers to confront the issue of what the future may hold for the particular site or location of inquiry selected by each volume in the series.

Moreover, the above approaches will enable rigorous reinterpretations of diverse educational contexts such as curriculum, pedagogy, leadership and policy as well as extending across various contested sites such as early childhood education, elementary-primary schooling, secondary schooling, or the tertiary sector. For instance, authors, editors and contributors to the series might choose to analyse in some depth the various ways in which the concepts of excellence and equity have been conceived in the past, conceptualised in the present and how they might be addressed in the future.

Regardless of the method or approach adopted by the scholars involved in writing for the series, however, there is general agreement that the series should seek to clarify for both specialist *and* general readers, the development and rationale behind current policy pronouncements in a manner that is both scholarly and accessible. Readers will thus be able to appreciate the tensions and challenges involved in implementing both excellence and equity within public education systems. They will also be able to identify broad links between their own specific national context and other national contexts. In seeking to achieve and sustain logical coherence, the series will be giving a specific educational expression to the approaches pioneered by a number of transnational studies that have attempted with considerable success in recent years to explore the ways in which past, present and future events and debates have been shaped by processes and relationships that transcend national borders (Curthoys & Lake, 2005).

Professors Mellony Graven and Hamsa Venkat are the editors of this timely third volume in this series, which is entitled *Research for Development in Primary Mathematics in Resource-Constrained Contexts.* Significantly, both editors are based in South Africa and hold chairs in numeracy education. Their leadership in research and development projects within the South African context has led to this enlightening account of both the progress of mathematics teaching in South Africa and the issues confronting primary mathematics teachers in that country. As the contributors to this volume reveal, the post-1990s massification of schooling needs to be read in the context of a number of major issues. These issues include continuing resource constraint, large class sizes, limited access to print materials, poverty and health, school readiness, gaps in teacher knowledge and in teacher supply.

There are several reasons why mathematics teachers, everywhere, need to be aware of what is happening in South African classrooms today. The country provides an excellent lens in which to explore the ways in which researchers and educators are undertaking the complex task of solving the problems associated with raising mathematics standards while also addressing inequities of provision in resource-constrained contexts. Within South Africa today, there is also an intense will for political redress. Hence, research funding from both public and private sectors is currently being prioritised for research and development. The research-led initiatives, upon which this volume is based, represent one important response to this imperative.

Hence, all the contributions to this excellent volume in their various ways directly address the themes of excellence and equity in global contexts which are central to this series. Teachers of mathematics in many countries as well as those in South Africa will see synergies in the complex challenges relating to teaching mathematics in a language which is not the home language of the students. Indeed, several chapters raise the key issue of whether or not assessments in mathematics unfairly assess English language competence at the expense of mathematical competence. The overall message of this book is the crucial importance of effective interventions in such resource-constraint contexts.

Together, the contributions to the book from a number of local researchers offer inspiration; but they also raise concerns. They inspire hope from the great strides achieved in relation to primary school access. Yet that hope is carefully balanced with a critical interrogation of a number of key equity issues relating to which students are able to achieve and who has access to quality teaching. Moreover, as the authors caution, policy borrowing from more affluent contexts always needs to be tempered with an intimate understanding of the local conditions and context. The task is indeed urgent, but if this volume is any guide, South Africa is well on track to finding sustainable solutions to the enduring problem of enhancing mathematical performance.

Roger Openshaw
Margaret Walshaw

Contents

Author Bios

Jill Adler holds the FRF Mathematics Education Chair at the University of the Witwatersrand, which focuses on research and development in secondary mathematics education. Jill has spearheaded several large-scale teacher development projects, the most recent, within the Chair ambit, begun in 2009, is called the Wits Maths Connect Secondary project. She is a Visiting Professor of Mathematics Education at King's College London, UK. She is the 2012 recipient of the Academy of Science of South Africa (ASSAf) Gold Medal for Science in the Service of Society, and the 2015 Freudenthal Award.

Mike Askew holds a Distinguished Scholar position at the University of the Witwatersrand, having previously been a Professor at Monash University, Melbourne, and King's College, London. In 2006/2007, he was distinguished visiting scholar to the 'Math in the City' project, City College, New York. Mike has directed many research projects, including the influential 'Effective Teachers of Numeracy in Primary Schools', and was deputy director of the five-year Leverhulme Numeracy Research Programme, examining teaching, learning and progression of students from age 5 to 11. The findings from such research have influenced teaching, research and policy in England and internationally.

Mellony Graven holds the South African Chair of Numeracy Education at Rhodes University and is now in her second five-year term of the Chair. Her research team includes master's, doctoral and post-doctoral students searching for ways forward to the challenges in numeracy education. She is the past president of the Southern African Association of Mathematics Science and Technology Education and founding editor of the journal *Learning and Teaching Mathematics*. The Chair project includes several community programmes that focus on in-service primary teacher development, 'family maths' events, after-school mathematics clubs and math camps, aimed at strengthening mathematics learning dispositions and improving mathematical proficiency.

Zanele Mofu is currently a Foundation Phase Mathematics Curriculum Planner in the Eastern Cape Department of Education. Prior to that she was a primary school mathematics teacher and a part-time tutor at Nelson Mandela Metropolitan University. She has a master's in Mathematics Education and is currently doing her doctorate with the South African Numeracy Chair Project at Rhodes University. Her research interest is on teacher learning through participating in an after-school maths club community of practice as a teacher development intervention programme. She has a regional primary mathematics conference in Kenya and has presented papers at AMESA conferences provincially and nationally.

Siviwe Ndongeni has been teaching since 1987 and is currently a Senior Phase mathematics teacher at a school in the Eastern Cape province, South Africa. Her master's in Mathematics Education study focused on grade 4 learners using the Wright et al. (2006) Learning Framework in Number (LFIN) with a specific focus on multiplicative reasoning. She is the Head of the Department for Foundation Phase at her school and she also teaches music.

Peter Pausigere is a post-doctoral fellow in the South African Numeracy Chair Project at Rhodes University. He completed his doctoral thesis in 2014 and since then has disseminated his findings through several peer-reviewed conference and accredited journal publications. His interests are in the areas of primary maths teacher learning, teacher identity, teaching and learning, numeracy workbook analysis and refugee education. His work theoretically draws from communities of practice, sociocultural and educational sociology. Most recently, his research has extended to focus on mathematical access for poor primary learners. He is a former Zimbabwe high school teacher.

Manono Poo is a lecturer at the Wits School of Education in the Foundation Phase division. She is passionate about teaching young children and teacher development in the areas of primary mathematics education. Her research interest lies in the field of primary mathematics teaching. In particular, she is interested in the teaching of early number and the use of language to support the shift from everyday language to the formal language of mathematics. She has co-authored two publications with colleagues in the division in the *South African Journal of Childhood Education*.

Nicky Roberts was a PhD student at the University of the Witwatersrand, supervised by Prof Hamsa Venkat. Her PhD thesis *Telling and illustrating additive relations stories* extended a design experiment embarked upon for her master's thesis *Telling and illustrating stories of parity* supervised by Dr Andreas Stylianides, University of Cambridge. Her research interests encompass early mathematics learning with a focus on the integration of mathematical thinking into rich language learning using a narrative approach; the use of technologies (particularly m-learning) in mathematics; and inclusive mathematical thinking. She is currently a post-doctoral fellow at the University of Johannesburg.

Sally-Ann Robertson worked initially as a primary school teacher, but is now a senior lecturer in the Education Faculty of Rhodes University (Grahamstown, South Africa), involved in teacher education at both the pre- and in-service level. She teaches courses in Sociology of Education, Multicultural Education and English Second Language Teaching. She has written a number of English Second Language textbooks for schools in South Africa, Namibia and Botswana. Her current research focuses on the interface between literacy and numeracy development, and, in particular, on how primary school mathematics teachers use classroom talk to mediate their learners' learning of mathematics.

Lucy Sibanda recently completed her PhD in Education at Rhodes University in the SANCP. In her study, she investigated the linguistic challenges of the grade 4 Mathematics ANAs and how teachers and learners experienced these challenges. She has also conducted research in language used in primary Science and English textbooks. She has presented and published her findings in regional conference proceedings and journals. Her current interest is exploring the linguistic challenges and prospects in South African primary mathematics classrooms.

Melissa Spira is currently completing her master's in Education with the SANCP at Rhodes University. She worked as a Foundation Phase teacher for a number of years in multicultural contexts where many of her learners were English second language speakers. As a trained special needs educator, she was also involved in a variety of programmes to support learners, parents and teachers. Her work as a teacher led her to pursue further studies in the fields of English Language Teaching and Mathematics Education, and she is now in the process of completing her master's degree. She is currently employed as a teacher educator on the initial teacher education programmes at Port Elizabeth's Varsity College.

Debbie Stott recently completed her PhD within the SANCP at Rhodes University. She has been involved in education across primary teaching, university teaching and corporate teaching and training. Her doctoral study investigated learners' numeracy progression and the role of mediation in the context of two after-school mathematics clubs. She coordinates and runs clubs for the after-school Maths Club project for the South African Numeracy Chair project at Rhodes University. She won the 2014 Rhodes University Community Engagement 'Student Researcher of the Year' for her research and development work in the after-school clubs. She is also involved in the SANC Project in-service teacher development programmes and in the supervision of SANCP post-graduate students.

Thulelah Takane has a background in Electrical Engineering with 5 years working experience, and a certificate in Marketing Management. She completed a Post Graduate Certificate in Education at the University of Cape Town in 2010. She proceeded into master's study and completed her degree at Wits while working in the Wits Maths Connect Primary (WMC-P) project in primary mathematics education research and development in 2013. She is currently a full-time PhD fellow working in the WMC-P project, with specific interest in Foundation Phase Mathematics teaching, learning and teacher education. She is also involved in post-graduate teaching and research supervision at Wits University.

Herman Tshesane completed a secondary mathematics Post Graduate Certificate in Education (PGCE) with distinction at Wits in 2008. After teaching secondary-level Mathematics for 3 years, he registered for and completed a full-time BSc Honours degree with distinction during 2012 while working on the Wits Maths Connect-Primary (WMC-P) project in primary mathematics education research and development. He then proceeded into full-time master's study during 2013, and is currently a full-time doctoral student at Wits University.

Hamsa Venkat holds the position of the South African Numeracy Chair at Wits—and is now into a second five-year term leading a linked research and development project—the Wits Maths Connect-Primary project—focused on primary mathematics teaching and learning in government schools in South Africa. This project has focused particularly on supporting teaching for number sense and sense-making more broadly, and primary teachers' mathematical knowledge for teaching. She has worked extensively in mathematics education and mathematics teacher education across primary and secondary phases in England and South Africa.

Anelia Wasserman holds a BA (Hons), HDE and BEd (Hons) Educational Psychology Cum Laude from the University of Stellenbosch. She taught extensively in the UK and South Africa at primary and high schools in both the private and public setting. She completed her master's research within the SANCP at Rhodes University in 2015, graduating with her MEd Cum Laude in 2016. Her research focused on mathematics assessment and recovery in a grade 4 class at a township school in the Eastern Cape. She recently moved to Nova Scotia in Canada.

Marie Weitz holds a position as a mathematics lecturer at the University of the Witwatersrand. She finished her MSc in Maths Education in 2014. She taught in several high schools which includes suburban and township schools. In 2008 she started to teach at the University of the Witwatersrand where she still is. She is also busy with a PhD in primary Mathematics in the Wits Maths Connect Primary Project. She is passionate about mathematics teaching and learning at all levels.

Lawan Abdulhamid completed his PhD at Wits University within the Wits Maths Connect Chair Project supervised by Prof Hamsa Venkat. His doctoral study focused on shifts relating to the nature and extent of responsive teaching in the South African context. The work was a follow-up study of teachers' participation in an in-service teaching development course on primary mathematics knowledge from the perspective of teaching. Prior to this, Lawan obtained both his honors and master degrees at Abubakar Tafawa Balewa University (ATBU), Bauchi – Nigeria. He is a recipient of Aminu Dorayi and T. O. Bamigboye award for best graduating student in Science Education. Lawan has been a lecturer at ATBU since 2005.

List of Abbreviations

ANA	Annual National Assessments
CAPS	Curriculum and Assessment Policy Statement
CPV	Conceptual Place Value
DBE	Department of Basic Education
DoE	Department of Education
EAS	Early Arithmetic Strategies
MR	Mathematics Recovery
LFIN	Learning Framework in Number
NICLE	Numeracy Inquiry Community of Leader Educators
SANC	South African Numeracy Chair
SANCP	South African Numeracy Chair Project
WMC-P	Wits Maths Connect-Primary (Project)

List of Figures

List of Tables

PART I

Interventions for Quality and Equity in Primary Education

CHAPTER 1

Intervening in the Learning and Teaching of Numeracy in Contexts of Poverty

Jill Adler

Introduction: The Local and the Global

This book reports on a range of research studies focused on numeracy practices in contemporary South Africa—a country and educational context marked by enduring poverty coupled with deeply entrenched social and economic inequalities, a country and educational context flushed with expectation and hope for a better future for all learners particularly in the initial post-apartheid years. At this point in our history, 99% of learners of compulsory school age (7–15) are in school (Taylor, van der Berg, & Mabogoane, 2013), a situation vastly different from apartheid-era education. At the same time, however, the country is confronting woefully poor learner performance in mathematics across grades, including the very early years of schooling, as the chapters in this book will reveal. Moreover, there is a widening gap as learners move up school grades, between a successful minority and the poor-performing majority (Spaull & Kotze, 2015), and concomitantly, the phenomenon of a relatively small number of "schools for the rich" and a large number of "schools for the poor" (Shalem & Hoadley, 2009). As Shalem and Hoadley cogently argue, the conditions

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of teachers' work across these polarised socio-economic conditions differ in substantial ways. With this comes the question: What might the "local" research presented in this book, focused as it is on teaching and learning in "schools for the poor" in South Africa, offer the global community?

As already suggested, South African school education, in general, and mathematics in particular, is indicative of a system with large numbers of learners in school, but simultaneously being failed by the system (Adler & Pillay, 2017 p. 10). In their reflection on the Millennium Development Goals for education in the developing world, the Conference of Commonwealth Education Ministers, 2012 (www.cedol.org), described this as "education for all, learning for some". They acknowledged that while many more learners in the developing world are now in school, only some are actually learning. Simply put, learners enter the institution (i.e. there is formal or "institutional" access to schooling), but their access to valued knowledge (their "epistemic" access) is restricted (Morrow, 2007).

Poor-quality education is a multifaceted problem, and not unique to South Africa. We also know that poverty and inequality impact educational outcomes, and indeed that socio-economic status is the strongest predictor of educational success in school (e.g. Coleman et al., 1966; Hoadley, 2010). However, recent studies of quality within schools have argued that "achievement in countries with very low per capita incomes is more sensitive to the availability of school resources" (e.g. Gamoran & Long, 2006, p. 1). As Adler and Pillay (op cit) argue, "social justice imperatives thus demand that we investigate factors related to what happens in schools and how practices might be changed in order to mediate greater education success of poor learners" (p. 15).

What might such investigations do? What practices endure and need to change? What does it take to rise to the challenge of "quality" in the millennium development goals? Specifically, what does it mean to deal with the reality of the consequences of apartheid education in the field of numeracy, and how then might such endeavours speak to the wider field of mathematics education?

The chapters in this book are outcomes of the research-linked development work in two relatively large numeracy projects in South Africa—known collectively as the South African Numeracy Chairs Initiative—an initiative that does indeed rise to the challenge. These projects are unique in their time, place and agendas, but it is precisely through these that they come to speak to concerns beyond them.

The South African Numeracy Chairs Initiative

As part of a wider initiative that included four Chairs in Mathematics Education, two Numeracy Chairs were appointed in 2011, each in a different university. The remit of these Chairs is to mount research and development projects with ten primary schools in one selected district so as to (1) improve the quality of teaching and learning of mathematics in those schools and, through that, learner performance (the development work); (2) research sustainable solutions linked to those projects; and (3) provide leadership in the numeracy research field, particularly in South Africa where this has been a neglected area (the research work). The universities in which the Numeracy Chairs are located are in two different regions of the country, with different levels of material resources, different languages spoken and differing levels of access to and fluency in English, which, despite national policy that endorses 11 official languages, is the preferred language of learning across many schools. The projects further span urban and rural contexts, requiring tailored responses to meet diverse needs in schools across this wide range of conditions. As the chapters across this book will show, it is this combined research across contexts that points to the way in which research-linked development not only enables one to meet ethical imperatives of responding to the needs of the contexts within which one is working, but also enables richer research that speaks to new avenues of possibility for interventions in resource-constrained contexts.

Across the work of both Chairs is the important and additional agenda to disrupt the dominant deficit discourses that have come to define teaching and learning in poor and under-resourced schools, and the continued description of South African education as in “crisis”. The lament that most of the research in mathematics education is carried out in “schools for the rich” (see, e.g. Skovsmose, 2011) has material consequences for the research done by both Chairs, in that methods and instruments (e.g. numeracy tests on progression; analytic frameworks for describing quality teaching) have had to be adapted or constructed anew. What exists tends to not disaggregate performance and practice sufficiently, and so if used, produce overwhelming negative stories of teaching and learning in these schools. Changing this conversation is necessary, and this book provides important messages in what and how this can be done, on how to make progress, how to look for possibilities and opportunities. It is precisely the location of the two Chairs in a particular moment of time in South African educational history that has enabled them to offer a complex and rich story of the teaching and learning of numeracy where issues of access, equity, identity, quality, knowledge and practice, and language are interwoven. All of these issues weave their way through the content of the chapters of this book.

The Chairs' work is discussed and theorised in the next chapter, and so not detailed any further here. It combines with this introductory chapter to form the first part of the book, and set the context for engaging with the remaining chapters.

The Book Chapters and Their Accumulating Relevance and Significance

The chapters in Parts II, III and IV of the book will tell their own stories. Here I provide overview comments that I hope will confirm that they do accumulate into a relevant and significant contribution to the field of primary mathematics education research.

Part II chapters focus on progression in numeracy. The work reported here has drawn on research from elsewhere (e.g. Bob Wright and his colleagues' work from Australia) and then carried out research informed adaptations and adjustments to instruments and methodologies related to the larger scale of issues relating to low base levels of numeracy that point to what learners in South Africa know, how they know what they do, and how these capabilities grow and can be stimulated to grow through careful adapted intervention models. In an international context of accountability and performativity, building knowledge on progression for learners across contexts is of tremendous value, and particularly significant when pressure from national governments, and South Africa is no exception here, is on relatively instant improvements in performance.

Part III chapters focus on issues of language and learning, so critical in a multilingual context like South Africa, where, as noted, English remains, indeed becomes increasingly hegemonic, as the language of access to social and economic goods. Teaching and learning primary mathematics in English is a serious challenge. Getting to grips with the realities that shape this work is of increasing value across the world as the levels of displaced peoples rise, creating multilingual learning communities in schools across a wide range of country contexts.

Part IV chapters focus on teacher support where the work of the Chairs has confronted, squarely, the importance of mathematical knowledge for teaching (in which I include both subject and pedagogic content knowledge), and what this means for different levels of primary teaching. The research studies reported in this part share the theoretical resources, methodologies and frameworks they have drawn on, and how these have been used and developed to explore numeracy teacher knowledge and learning, as well as the interconnectedness of this with identity and practice. As with the chapters in other parts, the chapters here report interventions leading to improvements in numeracy teaching and learning in South Africa. Of course, it is well known that the issue of primary teachers' mathematical knowledge, identity and practice is a worldwide concern, and in the past decade in particular it has been an area of intense study in a wide range of countries. The research reported in these chapters not only builds on such work but also adds a crucial dimension through tools developed in response to challenging empirical settings.

Overall, the chapters across all parts of the book provide rich accounts of what and how the conditions in contexts of enduring poverty and inequality give rise to opportunities that can be harnessed to work with the teaching and learning of numeracy. This is all the more remarkable in that this work developed at a time in South Africa when concern with early learning in school was on literacy to the neglect of numeracy. In a review of research in mathematics education in South Africa 2000–2006, research on teaching and learning primary mathematics was negligible in comparison with work done at the secondary level (Venkat, Adler, Rollnick, Setati, & Vhurumuku, 2009).

Having recently completed a follow-on review of mathematics education research in the country from 2007–2015, we found a notable increase in published research on mathematics (Adler & Alshwaik, in press). That the work of the two Chairs has been influential in this, thus establishing this field of work in the country, is evident, in that the majority of the publications in national journals, and almost all that were published in high-status international journals, have come from the work of the Chairs. This Numeracy Chairs Initiative in South Africa evidences how investment in research-linked development can stimulate new and needed areas of research in our field.

It is not only in South Africa that primary mathematics historically has had a back seat in mathematics education research. The International Commission for Mathematics Instruction (ICMI) has initiated and completed 21 *"ICMI studies" where an international group of scholars collaborate to produce* an account of the state of research internationally on particular selected areas of focus. The first one that has been focused on or dedicated to primary-level mathematics learning and teaching has only recently been completed, and the report of this work will only be published this year, that is, 2016 (Bartolini-Bussi & Sun, forthcoming).

Improving Primary Mathematics Education, Teaching and Learning: Research for Development in Resource-Constrained Contexts thus adds to the growing attention to teaching and learning primary mathematics. Its specific empirical focus is a major contribution as it initiates the crucial process of research with and on the conditions in which most children in the world live and learn.

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CHAPTER 2

Advocating Linked Research and Development in the Primary Mathematics Education Landscape in Contexts of Poverty

Mellony Graven and Hamsa Venkat

In this chapter, we theorise the primary mathematics education landscape in contexts of poverty in four ways. Firstly, we provide an overview of the South African context, detailing challenges, innovations and contributions of prior and current research and development initiatives. Secondly, we locate the South African context in the broader international landscape of countries continuing to deal with resource constraints for primary mathematics. Thirdly, we provide the rationale for the need to focus on numeracy education and the importance of intervention in the early stages of schooling for enabling quality and equity in mathematics education. Finally, we detail and comment on the emergence, in the South African terrain, of an intervention model that blends research and development—focused on

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addressing early numeracy learning, and we also analyse the ways in which they shift more traditional approaches associated with research located in higher education institutions and development located in the NGO sector. Our position is one of advocacy for the emergent, linked research and development model as an ethical imperative and for quality research.

Key issues in the international mathematics education landscape that play out with particular salience in the South African terrain relate to access and progression in early mathematical learning; inequities of provision and their relationships with performance; language and multilingual classrooms; and the prevalence of teaching and learning practices that foreground ritual versus explorative participation (after sfard & Lavie, 2005). Given that the political will for redress is strong in post-apartheid South Africa, and research funding for work at the interface of research and development for the early years is being prioritised across private and government sponsors (e.g. FRF, Anglo American Chairman's Fund, NRF, DST and DHET), we argue that South Africa provides a particularly salient context for reflecting on innovative ways of addressing numeracy challenges in resource-constrained contexts.

Overview of the South African Context of Primary Mathematics Education

Two education systems operate in South Africa. The first is a well-functioning system that compares favourably in terms of learner performance on international measures but caters predominantly for the wealthy minority. The second system is largely dysfunctional, and the performance of learners in this system consistently positions South Africa at the bottom of comparative regional (e.g. SACMEQ studies; see Spaull, 2013; Spaull & Kotze, 2015) and international studies of mathematical performance (e.g. TIMSS; see Reddy, 2006; Reddy, Zuze, Visser, Winnaar, & Juan, 2015). The latter system caters for the vast majority of South African learners who live in relative poverty, in a national context marked by high-income inequality. In this respect, South Africa provides an 'extreme case' of low mathematical performance in a context of high poverty for the majority of learners despite the relative, but inequitably distributed, wealth of the country (SA has the second largest economy on the African continent).

Thus, despite years of post-apartheid political will and rhetoric aimed at redressing educational inequality (and indeed successes in the achievement of universal access to primary schooling), South Africa continues to be one among the countries with high levels of economic inequality and educational inequality (NPC, 2011) and educational inequality in the world (Fleisch, 2008). In terms of mathematical performance, analysis of the TIMSS study indicated extreme levels of performance differences between the rich and poor and, furthermore, that these gaps, rather than improving, worsened over time, seen in the results from our participation in TIMSS 1999 to TIMSS 2003 (Reddy, 2006).

The research contained in this book is conducted predominantly with schools, teachers and learners who participate in the latter system of education. As such, these teachers and learners, included in this research, battle with the many factors that have been attributed to South Africa's poor mathematical performance, namely: high levels of learner and teacher absenteeism; limited teaching time; high levels of poverty, HIV/AIDS, foetal alcohol syndrome; lack of teacher content knowledge; incoherent presentation of concepts; unfamiliarity with the language of instruction; teacher-centred practices accompanied by passive learning dispositions; lack of resources and basic infrastructure, among others (Carnoy et al., 2011; Fleisch, 2008; Graven, 2014; Reddy et al., 2015; Venkat & Naidoo, 2012; Venkat & Spaull, 2015). Across the research projects discussed in the chapters in this book, there is an explicit focus on addressing inequity in mathematics education through researching how these challenges play out in the mathematics classroom and searching for sustainable ways to address these challenges.

Locating South African Mathematics Education in the International Terrain

While South African learners face many challenges similar to those of learners across the world, such as teacher content knowledge, challenges of appropriation of mathematical language and predominantly procedural methods of teaching that can work against conceptual and connected mathematical understanding (see Barwell, Barton, & Setati, 2007; Ma, 1999; Senk et al., 2012), and the addition of a multitude of other factors intersect in complex ways, resulting in South Africa's performance being consistently low. The most recent report on our participation in TIMSS (Reddy et al., 2015) indicates that while in 2011 there were minor improvements from an earlier very low base of TIMSS results (1999 and 2013), three quarters of Grade 9 learners had not acquired 'even the minimum set of mathematical and science skills' (p. 5). Thus, despite more than 20 years of post-apartheid attempts at educational redress, our performance continues to be extremely low. Reddy et al. (2015, p. 6) note in relation to TIMSS results:

> The predominant response is that very little has improved in the South African education landscape since 1994, and that resources are going to waste. The simple ranking of countries from top to bottom performers generally lends support to this perspective. As part of this narrative, South Africa's expenditure on education is contrasted with that of top-ranking developing countries that appear to be making more rapid progress. It is argued that South Africa has abandoned any educational aspirations and is effectively in a race to the bottom of virtually every educational ranking.

Furthermore, inequality in performance between the rich and poor persists as indicated in the 2011 results:

> When broken down according to school type, patterns of achievement are particularly revealing. Roughly 65 per cent of learners attending independent schools, 45 per cent of learners at public fee-paying schools and 15 per cent of learners at public no-fee schools achieved mathematics and science scores that were above the minimum level of competency. (Reddy et al., 2015, p. 37)

Language is noted as a key factor influencing South Africa's consistently poor performance on comparative studies such as TIMSS. The challenges that most learners face in respect of learning in a language that is not their mother tongue is shared with several post-colonial developing countries. In such countries the language of economic power, government and higher education is often that of the colonising country, whether French (e.g. Ivory Coast or Côte d'Ivoire), Portuguese (e.g. Mozambique), Spanish (e.g. Mexico) or English (as in the case of South Africa or India). The challenges of learning, and particularly learning mathematics which has its own complex language, in a language in which learners are not fluent, is widely acknowledged locally and internationally (see for example, Barwell et al., 2007; Moschkovich, 2007; Setati, 2008; Setati, Chitera, & Essian, 2009; Setati Phakeng & Moschkovich, 2013). In this respect, South Africa and many other colonised countries have introduced language policies that encourage mother tongue instruction, especially for the early years of schooling (DBE LiEP 1997 policy, see DBE, 1997).

Table 2.1 Percentage of learners using English as LoLT (Grades 1–12) (2007)

Grade	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
%	21.8	23.8	27.7	79.1	81.1	81.6	80.6	80.9	80	81.2	82	81.4

Data derived from DBE (2010, p. 16)

Such policy suggestions are not however always adhered to. Despite extensive research on the importance of mother tongue instruction in South Africa, English is the preferred language of learning and teaching (LoLT), followed by Afrikaans, despite their connections to the colonial and apartheid era. Thus the Department of Basic Education statistics (DBE, 2010) reveals that although only 17.2% of learners are actual native speakers of English or Afrikaans, 91.4% of learners have either English or Afrikaans as their LoLT (Robertson & Graven, 2015). The majority of Black parents therefore choose English or Afrikaans as the preferred medium of instruction for their children partly because of the access it provides to other 'goods.' Thus, as Setati (2008) notes, epistemological access is sacrificed for access to social and economic 'goods.' Table 2.1, compiled by Robertson and Graven (2015), shows how the preference for English as the LoLT jumps by over 50 from Grade 3 (the end of the Foundation Phase) to Grade 4 (the first year of the Intermediate Phase).

Why the Focus on Early Numeracy Learning

The effects on 'epistemological access' (Setati, 2008) of the sudden switch from learning in mother tongue to learning in English, which happens for the majority of South African learners when they move from Grade 3 to Grade 4, likely contribute to the drop in results of the Department's Annual National Assessment in mathematics. (In 2014, there was a drop of almost 20% from an average of 56% in Grade 3 to 37% in Grade 4.) By Grade 9, the national average for mathematics in 2014 was 11% (DBE, 2014). These results point to a critical need for developing strong numeracy foundations in the early years of schooling, coupled with increased attention to language support in the LoLT in the Intermediate Phase.

As can be seen from the data above, the 'crisis' in mathematics education begins in the early years of schooling (the Foundation Phase) and becomes exacerbated for poorer learners in Grade 4 as the vast majority of these learners transition from learning in mother tongue to learning in

English. The above realities make it particularly important for research and development projects to focus their work on these early years. As Spaull and Kotze (2015) note, the later one intervenes in a learner's schooling, the costlier it is, and Wright, Martland, Stafford, and Stanger (2006) note in the international literature that early gaps between higher- and lower-attaining learners become wider without early, structured remediation.

Additionally, a range of research points to the early years of learning being particularly influential on educational learning trajectories, thus supporting the need for early intervention (Atweh, Bose, Graven, Subramanian, & Venkat, 2014). For example, Field (2010), in his report The Foundation Years: Preventing Poor Children Becoming Poor Adults, argued that there is 'overwhelming evidence that children's life-chances are most heavily predicated on their development in the first five years of life' (p. 5). Furthermore, research points to early intervention being particularly important for bridging the gap between the advantaged and disadvantaged (Burger, 2010).

Another influence is that Mathematics is a hierarchical and progressive subject that requires concepts to be built on existing knowledge. However, within the South African context a wide range of research found that learners fail to progress beyond one-to-one counting methods even well into the Intermediate Phase (e.g. Schollar, 2008) and that in most primary classrooms there is the predominance of concrete over abstract methods of working (Hoadley, 2012). Such practices are underlain by an absence of coherent development of number sense and efficient arithmetic strategies (Venkat & Naidoo, 2012). The effect of this absence of number sense in the Intermediate Phase is that learners apply taught algorithms without consideration of the logic of the answer or the underlying place value of the digits being manipulated (Graven, Venkat, Westaway, & Tshesane, 2013). Thus, using the vertical addition algorithm, learners quite comfortably suggest that 88 + 12 is equal to 910. When asked to do it another way, learners might resort to either finger counting or drawing tally lines and counting all. Given the prevalence of concrete methods of calculation among learners in the Foundation Phase (and beyond), it is unsurprising that learner ANA results drop by almost 20% from grade 3 to grade 4, as in grade 4 such concrete methods are no longer viable for the higher number ranges.

Lack of mathematical progression, while somewhat extreme in South Africa, is not a uniquely South African problem. Research evidence from medium- to low-income countries points to concerns relating to the prevalence of 'flat' learning profiles across primary grades in numeracy in contexts of poverty (Pritchett & Beatty, 2012). Atweh et al. (2014) argue that while research points to divergent ways to respond to this lack of progression, many studies across both developed and developing contexts point to the effectiveness of teaching approaches foregrounding learner-centredness (Lambert & McCombs, 1998), cooperative learning (Moss & Beatty, 2006) and the teaching of mathematics in exploratory and connected ways (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997; Ma, 1999) rather than in transmissive ways. However, there have also been cautionary notes about 'policy-borrowing' from the developed world on issues such as learner-centredness, if attention is not given to beliefs, practices and conditions on the ground in developing countries with resource constraints.

Pritchett and Beatty (2012) also noted that 'overambitious curricula' in low- and medium-income countries, can result in teaching outpacing learning, causing 'more and more students [to be] left behind early and stay behind forever' (p. 13). Indeed, the review report of the implementation of the first post-apartheid curriculum introduced in 1997 indicated that the largely superficial take-up of learner-centred pedagogies and pacing created a particular problem for highly progressive subjects such as mathematics and science. The 1997 curriculum, the so-called Curriculum 2005, did not provide grade-specific indicators of progression and rather suggested learning outcomes that should be achieved by the end of each band of education (e.g. the Foundation Phase band (Gr 1–3), the Intermediate Phase band (Gr 4–6) and the Senior Phase (Gr 7–9)). The absence of clearly stated guidelines for mathematical progression from grade to grade was particularly problematic and has subsequently been provided in later curriculum revisions (i.e. the Curriculum and Assessment Policy (CAPS) DBE, 2011).

A significant body of evidence concerned with low performance and performance differentials in early numeracy tends to argue for numeracy teaching in the early years of schooling to follow a highly structured learning curriculum with a clear progressive learning trajectory that builds each grade's learning outcomes on those established in earlier grades. Similarly, remediation of learning in the early years requires highly structured intervention, while working with much more individually diagnostic approaches (e.g. see Wright et al.'s (2006) Maths Recovery approach). Central to this approach is the development of teacher understandings of progression in early numeracy. This should be done alongside the provision of teaching and assessment activities and resources that are geared towards achieving the early number progression regarded as necessary to achieve mathematical access in subsequent years of schooling. Wright et al.'s (2006) mathematics recovery (MR) programme has been widely implemented and researched across several contexts to remediate numeracy understanding of learners who are performing way below their grade level. This programme is highly structured, based on research of the developmental levels learners' need to progress through in order to develop numeracy foundations. While this programme is implemented individually with learners, and thus would need to be adapted for contexts where individual recovery is unlikely given large class sizes and the scope of early motor skills and language backlogs, it does point to the need for carefully planned and pre-structured activities for remediation that progressively develop learner understanding from where the learner is at. The MR programme, as indicated in the outline of the chapters in this book, has been widely used for both research and intervention purposes across the projects reported on here.

While much of the 'serial innovation' in developing country contexts points to disappointing results (Reddy, 2006), it is worth noting that Sztajn (2003) also points to absence of support for teachers in addressing reform across contexts. This concurs with similar concerns in the South African context (e.g. Reddy, 2006) as reasons for poor national performance in TIMSS. Such studies point to the importance of a structured, developmentally progressive framework as the basis of both teacher development and teaching for numeracy learning and numeracy remediation of foundational concepts. While a range of models is available, they all have similar levels of developmental progress that need to be carefully considered in assessment and teaching of learners and thus need to be fully understood by teachers.

The Emergence of Blended Research and Intervention Models

While poverty, which affects the majority of South African learners, is defined in terms of deficit of access to basic services and goods, we are cautious to not work with a deficit model of teaching and learning. Across our two Chair's project work, we adopt a proactive stance that investigates what is possible within our context in order to reject educational deficit discourses that often perpetuate a sense of hopelessness and lack of agency for change. This is important as research suggests that teachers often adjust their teaching to their perceptions of students' achievement levels. While this may appear to be appropriate, it can restrict the opportunity to learn for low-achieving students (which, in the case of South Africa, is the majority of learners). This is of particular concern when it involves groups of students from certain social, cultural or language backgrounds. Sztajn (2003) noted the tendency of using rote teaching for low SES students and problem solving with high SES students. Hoadley (2007), Carnoy et al. (2011) and Hoadley and Ensor (2009) in South Africa report on similar findings. Luke (1999) warned that the 'dumbing down' of the curriculum for low-achieving students excludes them from developing high-order thinking and intellectual quality work. It also diminishes their opportunity to learn content needed at higher levels of schooling.

This backdrop of the 'extreme' nature of the numeracy challenges in South Africa, coupled with a proactive intention to find solutions, gave rise to an innovative model of academic Chairs that must combine development with research imperatives where both focus on the search for sustainable ways forward to the many challenges faced in numeracy education. The remainder of this chapter outlines briefly this model and argues for the importance of such models, especially in contexts where research must speak with urgency to practice and vice versa.

The model of the Numeracy Chairs differs from those of other Chairs within the South African Research Chairs Initiative, in that they are jointly funded by the private institutions (such as First Rand Foundation and Anglo American Chairman's fund) and the Department of Science and Technology, while being administered by the National Research Foundation and have the dual imperative of research and development. Thus they are tasked with both:

- making a difference in the schools, teachers, learners and communities that they work with and
- researching ways forward to the many challenges we face in numeracy education.

Thus, while our research work investigates the challenges facing primary maths teachers and learners, our development brief is to focus on what is possible within this context of challenge. In this way our Chairs are explicitly tasked with bridging the traditional divide existing between research and community/society issues.

This model is based on the premise that separating Research and Development work is problematic and that more powerful work is possible if both of these constantly inform each other. So while intervention projects often have little time or money to research the complexity of the field they work in—and university-based research seldom has the ethical imperative to respond to the 'problems' it uncovers or to change the deficit picture regularly painted of our schools (which can often lead to a self-fulfilling prophecy)—our Chairs must attend to both.

And so we explore the research-development dialectic in ways that are problem-instigated and solutions-driven where we partner with schools and communities in mutually beneficial ways so that both partners contribute to moving the crisis in and the field of numeracy education forward. This model is thus based on two key premises. The first is that separating research and development is problematic, and second, research and development initiatives can be powerfully blended to be mutually beneficial.

On a final note, we are also particularly pleased to focus on early numeracy education as it is often too late to intervene in Grade 10 to 12 when, for most, failure is entrenched and subject choices away from mathematics are already made. Our experience, as revealed across the chapters in this book, concur with what Wright et al. (2006) say—that young learners are particularly susceptible to benefitting from early interventions.

The chapters that follow emerge from the research and development work of postgraduate students (master's and PhD students) across the two Chairs at Wits and Rhodes. The two Chair projects ran longitudinal in-service programmes as well as a range of classroom-based and after-school interventions. In many cases, these projects (and the classrooms of participating teachers) form the empirical field for the research discussed in the chapters. The intervention projects across the Chairs will therefore be discussed in the various chapters. What we hope is illuminated is the way in which the research has enabled strengthened development work and vice versa.

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PART II

Assessing Numeracy Progression in Resource Constrained Contexts

CHAPTER 3

Using Assessments to Explore Early and Later Performance in Mathematics

Marié Weitz and Hamsa Venkat

Introduction

Literature shows evidence in mathematics education that a set of early mathematical competences are particularly powerful predictors of later mathematical learning. Numerous studies have argued that these early mathematical skills can predict later mathematical performance (Aubrey, Dahl, & Dogfrey, 2006; Aunola, Leskinin, Lerkkanen, & Nurmi, 2004; Claessens & Engel, 2013). Thus, developing these early mathematical skills would be useful for supporting future achievement. Many researchers express this development as vital, as clearly seen from the vast amount of literature that has been published regarding the developmental dynamics of early mathematical skills (Ensor et al., 2009; Wright, Martland, Stafford, & Stanger, 2010).

In this chapter, the predictive power of early assessments in mathematics in South Africa is explored. In the South African landscape, the large-scale systemic Annual National Assessment (ANA) is a relatively recent introduction, having only been run at scale since 2011. The Government introduced the ANA to monitor and improve the levels of educational outcomes in the schooling system with aims encompassing the following: (1) to expose teachers to better assessment practices, (2) to make it easier

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for districts to identify schools requiring most assistance, (3) to encourage schools to celebrate outstanding performance, and (4) to empower parents with important information about their children's performance (DBE, 2011). Of interest in relation to the literature that pointed to the predictive power is that ANA performance patterns in South Africa show that while Grade 1 and Grade 2 children do relatively well in the ANAs, with national mean scores of 57% in 2013 in Grade 1, average scores decrease through the Intermediate Phase, with Grade 6 mean scores of only 27% in 2013. This suggests that high performance in the early grades' ANAs may not be a particularly good predictor of higher-level mathematics performance in the later years. It could be the case, though, that relatively strong correlations exist between earlier and later ANA performance in spite of the lower profile of scores by Grade 6, but current analyses do not tell us if this is the case. Uncertainties about the predictive usefulness of the early grades' ANAs lead to questions about (1) the early number competences that are described as useful in the literature for predicting later mathematics performance, and (2) the extent to which the ANA assesses these skills.

The theoretical hypothesis underlying our analysis is based on the work of Anna Sfard. According to Sfard (2008), *objectification* is a procedure through which a noun starts being used for a process, leading to the naming of the process as an object; for example, responding to a question about the number of objects in two sets of four and three objects with 'There are seven'is the final step of a procedure that usually begins with counting processes that compress over time to acting on the four and/or the three as objects (Sfard, 2008).

Several studies in other countries have considered the relationship between early mathematics performance and mathematics performance in later years. In this chapter, we analyse baseline assessment data collected within the broader Wits Maths Connect Primary project from 2011, based on a sample of early Grade 2 learners from 10 government primary schools (five township and five suburban schools) in one district on the Grade 1 ANA and Wright et al.'s Learning Framework in Number (LFIN) oral interview-based tests. These baseline analyses are related to the mathematical performance of learners present in these schools from the same cohort three years later, that is, in 2014, in Grade 5 in the Grade 5 ANA of 2014. We could only track the approximately 50% of learners who, after three years, were still in the same school, reflecting high learner mobility rates. The schools in the Wits Maths Connect Primary project are all urban primary schools, five suburban and five township schools, with these categories reflecting different historical (and ongoing) patterns of socio-economic advantage (suburban) and disadvantage (township). We were therefore also interested in whether there were differences between the predictive powers of the early tests across these two groups.

Literature Review

Many longitudinal studies have been carried out on the relationship between mathematical performance across several years (Geary, Hoard, Nugent, & Bailey, 2013; Jimerson, Egeland, & Teo, 1999; Manfra, Dinehart, & Senbiante, 2012; Stevenson & Newman, 1986). More specifically, studies have investigated relationships between mathematical performance in the early years and later mathematical performance. A common conclusion across these studies, as summarized by Clements and Sarama (2009, p. 214) is that

> the effect of initial mathematics performance is unusually strong and notably persistent.

Literature does not only point out the importance of early mathematical performance but also the rate of growth. The rate of growth of mathematical expertise is higher among those with more advanced, rather than more basic, early mathematical skills (Aunola et al., 2004). In other words, the gap between early higher achievers and the lower achievers, grows over time (Wright, Martland, & Stafford, 2006).

In this literature review, I consider the findings of longitudinal studies in this area. Within these studies, number knowledge and number sophistication, seen in how children use their mathematical knowledge to respond to tasks and instructions, are seen as particularly important. Alongside this sophistication of response, the early assessments of children's performance in this literature base often look at other psychological and background-level aspects (such as learner attitudes, gender, ethnic group, etc.). My interest is specifically on mathematical trajectories in a South African context of low mathematical performance by Grade 5, and thus, I restrict my attention to the mathematical aspects only. Within each study reviewed, I focus on the early number aspects that were tested and found to be powerful predictors of subsequent mathematical performance, as well as the context in terms of country and ages of learners at the time of testing.

Manfra et al. (2012) researched associations between counting ability in preschool and mathematical performance in 1st grade of 3247 ethnically diverse, low-income children in America. Counting is broadly seen as the first skill children need to master (Ensor et al., 2009; Gelman & Gallistel, 1986; Sfard, 2008; Wright et al., 2006). An important principle here is the understanding of the ordered number system (Clements & Sarama, 2009; Gelman & Gallistel, 1986; McIntosh, Reys, & Reys, 1992; Torbeyns et al., 2002). Clements and Sarama (2009) note that the Hindu-Arabic numeral system is based on two ideas. Firstly, we have only ten digits, namely: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Secondly, all numbers can be presented by a combination of these ten digits and by using the ten digits in different places—that is, the concept of place value. In the counting process where the numbers are organized, repeatable and stable, there are two processes at work simultaneously. The first process is where the child has to differentiate between the items that have been counted, and the ones that still need to be counted. The second process entails presenting separate items one at a time. These two processes must happen simultaneously, connecting counted items with the number words (Gelman & Gallistel, 1986). Children initially operate on some artefact form that represents the real objects before they can operate with abstract numbers when they solve problems (Anghileri, 2006; Haylock & Cockburn, 2008).

Manfra et al. (2012) focussed on the importance of counting and the difference between reciting and counting. *Reciting* is seen as the repetition of the number words sequence without anything actually being counted. They use the word *counting* for the understanding of one-to-one correspondence and for keeping track of what was already counted and what still needs to be counted (Gelman & Gallistel, 1986). They note the specific importance of counting thus:

> Counting on the other hand, requires more advanced cognitive understanding compared to reciting, and is more likely to act as a foundation for later number competence (2012, p. 103).

Manfra et al.'s (2012) investigations were driven by wanting to know what mathematical skills low-income children in the USA required before entering elementary school that would provide them with the basis to achieve high academic performance in Grade 2. The test they use to measure the capability to count was the LAP-D assessment, where every question is developmentally more difficult than the previous one. This test is thus a 'progressively developmental assessment' (p. 105). This overlaps with Wright et al.'s LFIN tests, where the format is such that a wrong answer leads to a less advanced question and a right answer leads to a more advanced question (2006). Early questions in Manfra et al.'s tests were focused on reciting within a number range of 1–10 and then the second part was focused on the counting of up to 10 objects. Thereafter followed reciting in the number range 1–20 and then counting up to 20 objects. Analysis of children's responses led to the allocation of a category ranging from 0–4, which was then compared with the child's mathematical performance in Grade 2, based on classroom tests administered during the whole Grade 2 year. Their evidence showed that low-income children who showed a more advanced counting ability in preschool were more likely to have a higher mathematics performance in elementary school. They also found that learners who could count up to ten, and recite up to 20, showed better performance in later years than those who could count up to 10 but not recite up to 20.

Manfra et al.'s (2012) results suggest that it is important to consider both reciting- and counting-based items in early assessment. Counting can be seen as the bringing together of both an organized linguistic system and a system that is based on the one-to-one principle involving the tracking of concrete objects. Over time, concrete objects are not needed as the connection to quantity becomes internalized, and children can operate directly on numbers (Sfard, 2008). When children can do this, they see numbers as objects. Gray (2008) suggests that numbers simultaneously exist as 'procepts': 'as a process and as a concept, both of which are represented by the same symbol'. Compound entities such as 4 + 3 can also be viewed in these two ways: as a counting-based operation with the numbers shown (process) or as an expression of a holistic entity (concept). The concept emerges as an objectified fact from operational counting process over time. The emergence of this mental object requires the compression of counting processes into objects. When a child has compressed counting processes into objects, the child has a reified understanding of number. Reification refers to processual ways of working becoming compressed into objectified working (Sfard, 1992). These ideas are elaborated further in the theory section later in this proposal.

Geary et al. (2013) tested children in the USA during their kindergarten years and then again when they were adolescents. Their focus was to find out how early mathematical knowledge can account for performance based on a 'functional numeracy test' that they used to measure young adults' employability. The study covered a period of six years. The Functional Numeracy test was administered in the children's grade 7 year. The kindergarten assessments consisted of multiple assessments, which included intelligence, working memory, mathematical cognition, achievement, and in-class attentive behaviour. The mathematical cognition assessment is a mathematical test and as it relates well to this study, it will be focused on in more detail.

In this test, counting was separated into two aspects: *Counting Competence* and *Number System Knowledge*. Counting Competence was investigated at two levels in the context of addition problems: simple and complex addition counting. 14 simple addition items were based on single-digit addition [including integers 2–9] and six complex addition questions were based on adding single- and two-digit numbers [16+7, 3+18, 9+15, 17+4]. They classified learners' responses based on the kinds of counting strategies that were used:

- *Sum* (or *triple count*) was used to describe responses where a child, for example, if asked to solve 4+5, 'counts all', meaning a count from one to four and then from one to five and then from one to nine (Anghileri, 2006);
- *Max* (*start with the smaller number and count on with the second larger number*) was used to describe responses involving counting on from four to nine for 4+5;
- *Min* (*start with the larger number and count on with the smaller number*) was used to describe responses involving counting on from the larger number (five in the case of 4 + 5) and thus involves awareness of commutativity of addition.

A triple count from one on a child's fingers indicates a processual understanding of number. Counting on from four to nine indicates that the child has reified four but not yet five (Sfard, 2008; Wright et al., 2006). Producing the correct answer straight away provides indications that the child has an objectified understanding of the numbers four, five, and nine. When a child starts to take shortcuts—for example, does not count from one, she starts doing operations/procedures on *numbers*. According to Wright et al. (2006), taking shortcuts is the route towards using more sophisticated strategies, and the Stages of Early Arithmetical Learning

(SEAL) aspect of the LFIN tests measures particularly for the extent of this sophistication. The child thus moves/progresses from working with number represented as concrete operations (operational thinking) into calculation procedures in which objectified numbers are operated upon (structural thinking). This kind of classification of learner responses suggests that Geary et al. are interested in measuring aspects related to objectification.

According to Geary et al. (2013), Number System Knowledge consists of six variables, namely:

1. Understanding the relative magnitude of numbers
2. Ordering of numbers
3. Addition retrieval: this indicates the correct retrieval of answers of number combinations
4. Complex addition decomposition: this indicates fluency of correct usage of decomposition and shows the ability to compose and decompose numbers into smaller and larger quantities and to use this knowledge to solve arithmetical problems
5. Number line accuracy: this indicates accuracy in the placement of numerals on a number line
6. Number sets fluency: this indicates a child's ability to spot relations between cards with domino-like rectangles with different combinations of numerals and objects. Learners need to do quick addition by 'seeing the sum', thus providing a check of the subitizing skills (perceiving small number quantities immediately without counting) that have been described as important predictors of later mathematical performance (Geary et al., 2013). The task is to circle rectangles that contain collections of objects, Arabic numerals or a combination of both that match a given target number.

For Geary et al. (2013), *Number System Knowledge* is a more developed understanding than counting on its own. Their finding was that Functional Numeracy is predicted by children's school entry Number System Knowledge. This elaborates the findings of Manfra et al. (2012) where advanced counting skills of pre-schoolers predicted their mathematical performance in Grade 2. I note again here that the variables pointed to here rely on competence in working with a reified sense of early number.

Theoretical Orientation

Many researchers agree on the importance of this developmental process that is needed when children learn mathematics (Ensor et al., 2009; Fleisch, 2008; Gelman & Gallistel, 1986; Schollar, 2008; Wright, Ellemor-Collins, & Tabor, 2012), but describe the progression differently.

Sfard (2008) describes it as a shift from number chanting as ritual through counting processes to objectification of number. The main theoretical ground of sophistication of strategies and the move to logical-mathematical knowledge is the process of objectification of number (Sfard, 2008). According to Sfard, objectification is a discursive process of double elimination, which frees people from both the extension in time and from human agency that are related to the ability to use a discursive object by according a noun to present a long process. Sfard (1991) presents evidence of 'the dual nature of mathematics' (1991, p. 1) and refers to reification in her later work:

> Structurality of word use, the opposite of processuality, is the result of reification (2008, p. 145)

Sfard describes processual discourse as relating to processes and actions, rather than relating to reified objects. In the context of early number, the action of counting objects to solve addition and subtraction problems can relate to processuality. Structurality, in contrast, is described as follows:

> 'Seeing a mathematical entity (like a number) as an object means being capable of referring to it as if it was a real thing', that is, it having a static structure, existing somewhere in space and time. It also means being able to recognize the idea at a glance 'and to manipulate it as a whole, without going into details' (1991, p. 4).

It is important here to emphasize that reification relies on processual working with number preceding structural working with number. Objects cannot be 'given' to learners. Objects are the outcomes of processes. This means that the processual way of working with number is an important start point, but that staying at the level of processes and concrete counting is problematic given its inefficiencies, leading to the need to progress to structural thinking. Structural thinking (with 'thinking' viewed as 'Intrapersonal Communication' in line with Sfard, 2008) enables children to communicate more economically by trading an extensive description of actions with a single sentence (Sfard, 2008). Reified understandings allow moves from working with numbers as concrete objects to reified objects with openings to explain the reified objects in concrete terms. This 'short' way to communicate also increases the flexibility and the applicability of expressions. Sfard (2008) provides examples of the consequences of non-reified discourses relating to number where the child cannot yet use the numbers as adjectives:

> ... if you count the marbles in this box, you end up with the word "five" (p. 53).

For a child at this level, the sentence 3 + 4 = 7 lacks meaning because the numbers 3 and 4 and 7 do not function as nouns to them. Sfard argues that the equality 3 + 4 = 7 is then about the relation between counting to 3, and 4, and 7. Sfard (p. 53) states that the lengthy description of 3 + 4 = 7 will fall along the following lines:

> If I have a set so that whenever I count its elements, I stop at the word three, and I have yet another set such that whenever I count its elements I stop at the word four, and if I put these two sets together, then,
> if I count the elements of the new set, I will always stop at seven (p. 53).

Without reification, children are often unable to solve number problems with fluency and efficiency. Reification helps us to cope with new problems in terms of our experiences and '...gives us tools to plan for the future' (Sfard, 2008).

The second part of objectification is *alienation*. Sfard argues that '[o]nce reified, the alleged products of the mind's actions may undergo the final objectification by being fully dissociated, or alienated, by the actor' (2008, p. 50). She argues that the example of 2 + 3 = 5 eliminates the human subject and is considered as a discursive construct:

> Once reified and put into impersonal sentences, the numbers appear to have a life of its own (Sfard, 2008, p. 50).

Alienation for Sfard is seen as a discursive activity as learners work in natural settings on mathematical tasks. Given that this study is based on task-based interviews (where some questions at least are set in alienated terms), the focus of this study is predominantly on reification progress towards more sophisticated strategies.

In early number, mental calculations usually present questions in alienated forms. Broadly speaking, if we look at numeracy and children's understanding of number, we can say processual thinking relates to the concrete counting of objects and structural thinking relates to working with numbers and number relation statements as discursive objects that exists without (or with fewer) counting-based actions.

Considering this theory, it becomes clear that reliance on concrete counting-based methods becomes unwieldy as number ranges increase. However, linking structural thinking to predecessor operational working with objects remains necessary.

Several researchers have noted the reliance of South African children on concrete counting. There is evidence of a lack of shift from concrete counting-based strategies to more abstract calculation-based strategies (Ensor et al., 2009; Schollar, 2008). Concrete counting-based strategies refer to actions where the learner cannot find the answer to a mathematical problem without counting concrete objects. In contrast, abstract calculation-based strategies involve strategies where the child does not need concrete objects to find the answer, but can instead use mental calculations in which numbers have been transformed into abstract objects upon which operations can then be carried out. Ensor et al. (ibid.) argue that many South African learners stay dependent on concrete counting to solve problems, while noting that the curriculum requires children to have an abstract concept of numbers by the end of the Foundation Phase. Using this theory is important in the South African landscape as the reliance on concrete counting has been widely documented (Ensor et al., 2009; Fleisch, 2008).

Research Context

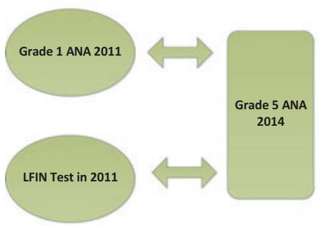
At the end of 2014, we collected the Grade 5 ANA marks of the children that we interviewed in 2011 that were still in the ten schools. I could find a sample of 92 learners that were still in the schools out of the 204 learners that we had ANA 2011 and LFIN 2011 results for. We matched the ANA 2014 marks with both of their 2011 assessment marks and compared them quantitatively. This high 'dropout' rate between Grades 2 and 5 relates to mobility, grade retainment and dropout, and has been noted in some other South African studies. Townsend and colleagues (Townsend, Madhavan, Tollman, Garenne, & Kahn, 2002) studied the residential patterns and educational attainment in rural areas in South Africa. They argued that living arrangements in Africa have been characterized by spatial mobility, a scenario in which temporary labour needs frequently produced the establishment or the disintegration of local communities. In this labour scenario, children were often moved around from relative to relative because of labour-driven migration of parents. In their study, they found that while most children were in school by the age of 11, most of them lagged behind the prescribed grade level and there were great variations in the number of years that they had attended school.

A range of other findings relating to the low proportions of children still in the ten schools after four years was found in the literature. A longitudinal study by Christine Liddell and Gordon Rea on primary school progress in a rural area in South Africa found that a quarter of the cohort (150 children in a rural area) had been retained at least once between the grades 2 and 4. They found that only 39% moved smoothly to grade 7 (2001). Liddell and Rae (2001) found that 36% of the cohort they started their studies with had left their original school by grade 7. Motala, Dieltiens, and Sayed (2009) add to these findings by noting that grade repetition was the greatest predictor of dropping out of school in their study. Many of these studies also comment that while it is known that substantial proportions of children left study schools, there is a lack of information on whether they dropped out or went to another school. Data on grade retention is usually more easily accessible, but datasets on these issues at the level of individual learners rather than at aggregate levels remain relatively piecemeal.

In this analysis, we compare performance on the two tests from 2011 (Grade 1 ANA and the LFIN test) with performance on a test from 2014 (Grade 5 ANA). Our interest began with questions about whether both tests would act as equally good predictors. Figure 3.1 shows the correlations that were tested (including separate aspects of the LFIN test), using Pearson's product–moment correlation statistic. We also want to know whether one of the aspects of the LFIN test is a better predictor than another.

Data were cleaned to separate out the learner sample for which we had matched 2011 and 2014 assessment performance data (n = 92 in comparison to the 204 learners for whom we had complete performance information on both 2011 assessments). We also distinguished between the learners from township (n = 57) and suburban schools (n = 35) in the

Relations between the tests in 2011 and the ANA of 2014	Relations between the ANA of 2014 and the different aspects of the LFIN test in 2011



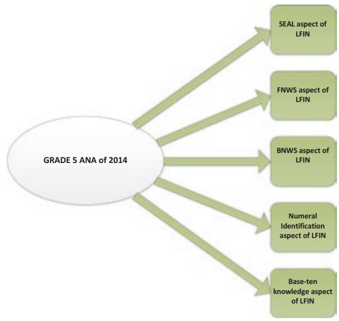


Fig. 3.1 Data collection

matched dataset, given the broader evidence of differences in learning outcomes between high socio-economic status schools and schools in lower socio-economic contexts (Spaull & Kotze, 2015). In the township schools (all of which are 'no-fee' schools), a range of South African home languages are used as the language of teaching in the Foundation Phase grades (Grades R-3) before transferring to English as the medium of teaching in Grade 4. This contrasts with the suburban schools (all of which are fee-paying) where 'straight for English' policies operate.

Findings

We begin our presentation of findings with some box plots (created using SPSS) showing overall profiles of performance on the three assessments for the matched cohort, and comparing these profiles for the township and suburban school learner subgroups (see Fig. 3.2).

The plots in Fig. 3.2 indicate a better match in the percentage score profile between the LFIN test of 2011 and the ANA 2014, with a much higher profile of scores on the G1 ANA 2011 than on either of the other two assessments. Median scores on all three assessments and the profile of the inter-quartile ranges (IQR) of all the tests are higher for the suburban schools than the township schools, with the biggest differences seen on the ANA 2014 performance profile. While significant correlations existed

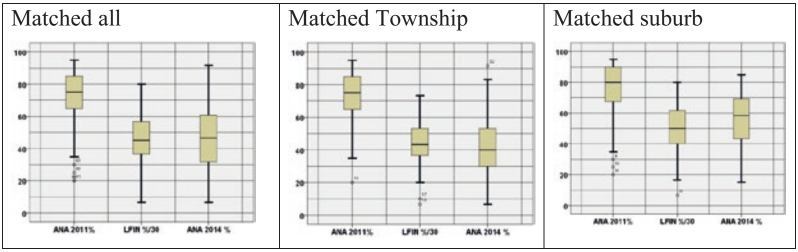


Fig. 3.2 Box plots findings

Table 3.1 Comparision of correlations for township and suburban school groups

	ANA 2014	
	MT	MS
ANA 2011	0.372	0.586
LFIN 2011	0.419	0.658
2011 SEAL Stage aspect	0.311	0.647
2011 Numeral Identification aspect	0.448	0.431
2011 Backward number word sequence aspect	0.371	0.640

MT: Matched township (n = 57)
MS: Matched suburban (n = 35)

between both tests in 2011 and the Grade 5 ANA of 2014, the correlation between the LFIN test of 2011 and the ANA of 2015 was greater (r = 0,531) than the correlation between the ANA of 2011 and the ANA of 2014 (r = 0,450). This suggests that the LFIN of 2011 is a better predictor of Grade 5 ANA of 2014 performance than the ANA of 2011, which agrees with the literature, reviewed earlier on the predictive power related to objectification measures in early numeracy assessment.

Of interest within this ‘global’ finding, though, were differences in the correlations between township and suburban schools, with key results (all significant at the <0.01 level) summarized in Table 3.1, and discussed below.

Discussion

Across the MT and MS subgroups, the relationship between the LFIN of 2011 and the ANA 2014 is stronger than the relationship between ANA 2011 and ANA 2014, pointing to the LFIN test being a better predictor of later ANA performance across both settings. Of interest, though, are how different these correlations are across the two settings, with both LFIN 2011 and ANA 2011 showing substantially lower correlations in the township school group than in the suburban group. This result is important particularly in relation to the relatively low correlation seen between the SEAL Stage and ANA 2014 in the matched township group. As noted earlier, the SEAL stage measures for extent of objectification within children's ways of handling number, and a broad swath of theory has noted that this objectification is central to progression into the abstract ways of working with number that are seen as necessary for moving to higher number ranges and into algebraic thinking. We are uncertain at this stage on possible reasons for the large difference in correlations between 2011 SEAL Stage and ANA 2014 for the MT and MS groups. The 2011 LFIN tests were administered with home language translators in the township schools, which meant that the tests took a little longer to administer in these settings—which may have had an effect with young children. Nevertheless, the theoretical position appears to hold with differential strength across township and suburban schools, leading to questions about the nature of early mathematical learning in these settings that might figure within this difference.

Within the test, there were further aspects showing differential correlations. Notably, the highest correlations for the MT group were at the level of an LFIN aspect—Numeral Identification, which showed a stronger relationship with 2014 ANA performance than the overall 2011 LFIN or 2011 ANA tests. In contrast, the suburban group results behaved more 'predictably' in the sense that the profile of correlations here related better to the theoretical framing.

A range of broader evidence shows differences in learner performance between wealthier and poorer settings in South Africa. Hoadley (2007) has also identified differential 'specialization' in pedagogy across poorer and wealthier schools. Our tracking of results between 2011 and 2014 suggests that for township learners, early number performance is not as good a predictor of later performance in comparison to suburban schools. We speculate that differences in the pedagogies that have been noted in review studies (Hoadley, 2012) may have meant more limited access to specialized meaning for township learners in the early years of mathematics teaching, leading to more 'under-interpretation' of their early number competence in relation to the suburban learners, but this issue needs more investigation.

Two key implications follow from our findings. Firstly, at the policy level, it would appear important to ensure that early years' ANAs incorporate attention to measuring for extent of objectification. We appreciate that it is not possible, on a national systemic scale, to administer the kinds of assessments developed by Wright et al. (2006), but there are examples of items and item formats in the international literature that incorporate this attention. One way is by incorporating items that allocate marks for extent of sophistication, in addition to or instead of marking for the production of correct answers. Another option to push for the importance of sophistication is to include time-limited formats for some questions, or the inclusion of a mental mathematics component, thereby providing a feedback loop into pedagogy of the value of developing reified working with number. The second implication is to understand, on the research side, what is leading to such different correlation values for the SEAL component between the township and suburban schools. This finding adds to Ensor et al.'s (2009) footnote that there was very limited evidence on the trajectories of early number learning in South Africa, leading to their leaning of development trajectories from the developed world. With this data set and the broader doctoral study that it is located within, we hope to understand better the nature of these trajectories in the South African context.

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CHAPTER 4

Researching the Nature of Early Numeracy Progression in After-School Clubs

Debbie Stott

Introduction and Research Context

Learner-focused activities within the South African Numeracy Chair (SANC) project foreground the importance of numeracy as well as creating a "maths is fun" ethos in schools. Learner after-school maths clubs are a key aspect of the project and provide a direct learner-focused intervention. The SANC project clubs are defined as an extracurricular activity focused on developing a supportive learning community where learners' active mathematical participation, engagement and sense making are the focus. Individual, pair and small group interactions with mentors are the dominant practices with few whole class interactions. Activities in the clubs are designed for both the recovery and extension of learners (Graven, 2011; Graven & Stott, 2012).

Working from a broad Vygotskian perspective of learning and development, this study had a dual focus and investigated how Grade 3 learners' mathematical proficiency progressed (or not) whilst participating in after-school maths clubs over the course of a year, and explored how the mediation offered in the clubs enabled or constrained the emergence of zones of proximal development (ZPD) and thus learning for the learners.

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This chapter is based on data gathered with regard to the first part of the research aim in two after-school clubs.

In running after-school clubs since 2011, I have seen many instances where learners are not working efficiently or fluently because they are constrained by their lack of basic foundational mathematical knowledge. For example, it has been noted that the majority of the learners are weak on number bonds. The bonds of 5 and 10 are not recalled automatically and learners rely on counting by ones or their fingers to work out, for example, the answers to 2 + 8 or 2 + 98. Their grasp of basic number sense and basic bonds continues to be severely limited and few learners seem to have other strategies to solve problems. This means that, although they are largely accurate, they tend to be slow at arriving at an answer to a particular problem.

The project as a whole, therefore, works towards improving numeracy proficiency among learners, and its notion of mathematical (or numeracy) proficiency is based on the definition given by Kilpatrick, Swafford & Findell (2001). This definition comprises five intertwined and interrelated strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Hence, the aim is for learners in clubs to develop mathematical proficiency in each of these five strands.

While Kilpatrick et al. (2001) provide a powerful conceptualisation of mathematical proficiency, the elaboration of each of the strands (with the inevitable overlaps that emerge from the interrelationships) does not support the evaluation of learner *progress* in mathematical proficiency. Indeed, a fully conceptualised and idealised form of mathematical proficiency requires all five strands to be present and for each of them to be fully present in relation to their elaborated definitions. However, while the fully elaborated notion of mathematical proficiency is something we work towards in our project, we need to be able to assess different levels of proficiency in different strands across different points in time so that we can assess learner numeracy progression over time. Assessing learner progress in mathematical proficiency requires one to be able to assess the extent to which a learner may or may not have mastered a particular aspect of mathematics and a particular strand or interrelationship of strands at different points in time. Thus, to consider that a student might be procedurally fluent or not is less useful than to gauge the extent to which they are mastering the fluency and to find ways to move them along typical learning pathways. A way of tracking and seeing how learners actually develop towards this fully elaborated notion of mathematical proficiency was therefore both useful and necessary for my research clubs.

The Learning Framework in Number (LFIN) developed by Wright and his colleagues (Wright, Martland & Stafford, 2006) provides a valuable way of doing this. As part of the Maths Recovery (MR) programme, this framework has been used to research and document progress in number learning in the first three years of schooling. The MR programme has been used extensively by school systems in several countries, including Australia, Canada, USA, UK and Ireland (Wright, Martland & Stafford, 2006). Whilst the programme has been used and tested in these other countries, it has not yet been implemented in a South African classroom context, although research in South Africa on the programme is beginning to grow. See, for example, Weitz (2012), Mofu (2013) and Ndongeni (2013). Due to the nature of most South African classrooms, it is not always possible to use such frameworks in a one-to-one scenario and the power of the framework is never realised in our local context. Several ongoing research projects within our project team are working with the LFIN and aim to illuminate how the framework could work in South African classrooms.

In my study, I worked with a version of the LFIN that combines elements from the works of Wright and colleagues in 2006 and 2012. For the purposes of this chapter, these aspects of the LFIN are:

A. Number words and numerals (including forward and backward sequences)
B. Structuring numbers 1 to 20
C. Conceptual place value knowledge (ability to reason in terms of tens and ones)
D. Addition and subtraction 1 to 100 (strategies for counting and solving simple addition and subtraction tasks)
E. Early multiplication and division (Wright, Ellemor-Collins & Tabor, 2012; Wright et al., 2006)

Each of the key aspects of the LFIN is elaborated into a progression of up to six levels or stages, with each level/stage having characteristics (Wright et al., 2006). By way of example, these are detailed below (Tables 4.1 and 4.2) for Conceptual Place Value and Early Arithmetic Strategies. It should be noted that, for Wright and colleagues (2012),

Table 4.1 Conceptual Place Value

ASPECT C: Conceptual Place Value (CPV)		
Level number	*Level descriptor*	*Characteristics*
1	Initial concepts of 10 (ten as a count)	Not able to see ten as a unit composed of ten ones. The child solves tens and ones tasks using **a counting-on or counting-back strategy**. One 10 and 10 ones do not exist for the learner at the same time
2	Intermediate concepts of 10 (ten as a unit)	**Able to see ten as a unit composed of ten ones**. The child uses incrementing and decrementing by tens, rather than counting-on-by-one to solve uncovering board task. The child cannot solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences
3	Facile concepts of 10 (tens and ones)	**Tens and ones are flexibly regrouped**. Ten is a unit that can be repeatedly constructed in place of 10 individual ones. Child is able to solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences by adding and/or subtracting units of tens and ones

Table 4.2 Early Arithmetic Strategies

ASPECT D: Counting as a problem-solving process (Early Arithmetic Strategies)		
Stage number	*Stage descriptor*	*Characteristics (representing increasing levels of sophistication)*
0	Emergent counting	**Cannot count visible items**. The child might not know the number words or might not coordinate the number words with the items
1	Perceptual counting	Can count only **visible** items starting from 1. Including seeing, hearing and feeling
2	Figurative counting	Can count **concealed items** but the learner will "count all" rather than "count on"
3	Initial number sequence	Initial number sequence: The child can **count on** rather than counting from one, to solve + or missing addends. May use the **counting down** to solve removed items (count-back-from)
4	Intermediate number sequence	**Count-down-to** to solve missing subtrahend (e.g. 17–3 as 16, 15 and 14 as an answer. The child is able to use a more **efficient** way to count down from and count down to strategies (count-back-to)
5	Facile number sequence	Uses of range of **non-count-by one strategies**. These strategies such as compensation, using a known result, adding to 10. Commutativity, subtraction as the inverse of addition, awareness of the 10 in a teen.

Conceptual Place Value is distinguished from conventional place value knowledge, in that it involves flexibly incrementing and decrementing numbers by ones, tens and hundreds and is foundational for use in mental strategies involving multi-digit numbers. In this way, it can be seen that progress in this aspect has an effect on progress in other aspects such as Early Arithmetic Strategies and structuring numbers to 20 and beyond.

For my study, the LFIN framework provided a powerful tool for profiling each individual club learner's mathematical proficiency across the range of key aspects over the research period. Profiling of learners' mathematical proficiency also formed the basis for planning club activities and mediatory interventions that are tailored to each learner's current levels of proficiency and strategies. Additionally, the LFIN and the principles of MR provided the basis for the orally administered numeracy instrument (described below) as well as a way of structuring and reporting on data from these instruments as I do in this chapter.

Theoretical Framing

This research was grounded within a broad Vygotskian approach to learning and development. This assumes that learning is an active construction of knowledge through social interactions with others. The central principle of development in a Vygotskian perspective is the "increasing ability of children to control and direct their own behaviour" (Vygotsky, 1978 p. 126), whereby control shifts from the environment/context to the individual. This shift is made possible by the development of new mental processes by use of signs and tools side by side with collaboration or assistance from others. Through mediation and the use of cultural tools, the child can move from being dependent on others and on concrete everyday experiences to being able to remember, internalise and use those experiences independently. Wright, Martland, Stafford, & Stanger (2006) work is based on constructivist principles that learning mathematics is an active process, that children construct their own mathematical knowledge and that they develop mathematical concepts as they engage in sense-making, mathematical activity. Additionally, the MR programme is based on sense-making and mathematical activity and normally takes place alongside a teacher or other adult. In this way, learners are not working on their own discovering knowledge per se but are assisted by a more knowledgeable other. This view coheres with the broader approach taken in my study.

Methodological Approach

The findings discussed in this chapter are based on a longitudinal, qualitative multi-site case study of 11 Grade 3 learners aged between 8 and 10 voluntarily participating in two after-school maths clubs, which ran for 28 sessions in the year 2012. The participating learners had a range of mathematical proficiencies.

As mentioned, the primary data collection instrument for these findings was a diagnostic instrument used to collect data on learners' progress with regard to mathematical proficiency. The instrument combined elements from the work of Askew, Brown, Rhodes, Johnson, and William (1997) and one-to-one interview from the MR programme (Wright, Martland, & Stanger, 2006).

The interview instrument consisted of 24 tasks in total. Individual questions were grouped together to form a full picture for a particular aspect of early numeracy, for example, conceptual place value. Together, Tables 4.3 and 4.4 give details of how the interview instrument was structured. Table 4.3 details each individual task, its order in the interview, which aspect of the LFIN is addressed and the origin of the question (either Wright, Martland, & Stafford, 2006 or Askew et al., 1997). Table 4.4 illustrates the spread of tasks across the LFIN aspects. I provide this information as I also generated summative scores from these interviews as I describe later. The interview was thus out of 88 marks, with a spread across the five LFIN aspects.

Space does not allow me to include the full interview script, but I have included two items below as examples. Task 9 (Fig. 4.1) involves the placing of successive strips of dots in each part of the question. Task 10 (Fig. 4.2) invites the learner to add/subtract 10 from a given number.

An interview script was printed per learner on which the interviewer made notes about learners' answers and the methods used to answer questions. Interviewers also noted speed of answering and anything the learner may have said whilst working on a task or question. Interviews lasted for 45 to 60 minutes and took place during school hours. Questions were asked orally in English and learners were provided with paper and pencils if needed. Resources (such as counters) were provided as specified by each task.

Analysis Approach

Using the notes written on the learner interview scripts, and guidance from the relevant sections of Wright, Martland and Stafford's (2006) book, the interview data was used to profile each learner onto the

Table 4.3 One-to-one interview: Summary of instrument structure

Task no.	*Task number and description*	*No of subparts in task*	*Aspect of LFIN addressed*	*Origin of question*
Interview part 1				
1	Numeral Identification	10	*Aspect B: Number words and numerals*	Wright et al.
2	Number line representation	2		Askew et al.
3	Forward number word sequences	3		Wright et al.
4	Backward number word sequences	3		
5	Number word before	6		
6	Number word after	6		
7	Sequencing numerals	2		
8	Number combinations	2	*Aspect A: Structuring nos. 1 to 20*	
9	Counting in incrementing 10s	5	*Aspect C: Conceptual place value*	
10	Addition/subtraction with 10s	4		Askew et al.
11	Addition/subtraction with 100s	4		
Interview part 2				
12	Horizontal +/− sentences	4	*Aspect D: Early arithmetic strategies*	Wright et al.
13	Word problems	3		Askew et al.
14	Number stories	1		
15	Non-counting by 1s	6		Wright et al.
16	More number combinations	6	*Aspect A: Structuring nos. 1 to 20*	
Interview part 3				
17	Visible items in an array	3	*Aspect E: Early multiplication and division*	Wright et al.
18	(Subitising)	2		Askew et al.
19		2		
20	Equal grouping of visible	3		Wright et al.
21	items	2		
22		3		
23	Times tables	4		Askew et al.
24	½ and ¼ of a collection	2		
	Number of Wright et al. questions	63 in 15 tasks		
	Number of Askew et al. questions	25 in 9 tasks		
	Total number of questions	**88 in 24 tasks**		

Table 4.4 One-to-one interview: Summary of tasks, questions and marks in each LFIN aspect

LFIN Aspect ➔	*Structuring nos. 1 to 20 Aspect A*	*No words and Numerals Aspect B*	*Conceptual Place Value Aspect C*	*Early Arithmetic Strategies Aspect D*	*Early Multiplication & Division Aspect E*
Task numbers	8 and 16	1 to 7	9, 10 and 11	12 to 15	17 to 24
No. of **tasks** for each aspect (24 in total)	2	7	3	4	8
No. of **questions** for each aspect (88 in total)	8	32	13	14	21
No. of marks for each aspect (88 in total)	8	32	13	14	21

[Use pink strip cards. Show strip (a) then add others for steps b to e. Ask] *How many dots are there altogether?*

	Note Answer & How Answered	Correct?
a. The 'four dot' strip		
b. Add a 'ten dot' strip to the right		
c. Add another 10 to make 24		
d. Add another 20 to make 44		
e. Add another 30 to make 74		

Fig. 4.1 Interview task 9—counting with incrementing tens (see Wright, Martland & Stafford, 2006, p. 167)

[Ask orally]	Note Given Answer	Correct
Add 10 to 92		
Add 10 to 294		
Take 10 away from 50		
Take 10 away from 700		

Fig. 4.2 Interview task 10—adding/subtracting tens (Derived from Askew et al., 1997)

levels/stages of the LFIN. As the interviews were carried out twice, I was able to compare and contrast the two profiles to see possible change over time.

Although the creators of the MR programme never intended for the assessment interview to result in a score, I generated scores from the interview data (simply by marking a question as right or wrong) for each learner. For my study, the generation of these scores for the LFIN was a way to supplement the qualitative individual LFIN profile data that is normally generated. Elsewhere I have argued that scores such as these could provide a useful and practical contribution to the South African research context using Wright, Martland, & Stafford (2006) MR programme (Stott & Graven, 2013). This aspect of my findings is discussed in further detail in Chap. 5.

I also drew on Kilpatrick et al.'s (2001) strands of mathematical proficiency. The summarised indicators for these three strands are shown below. *Strategic competence* and *productive disposition* were not included because although the interview included some tasks that are number stories, these tasks did not reveal enough about strategic competence to enable analysis of learners' proficiency in this respect. The productive disposition strand was not a specific focus for my study and was not assessed in the data collection instruments.[1]

- *Conceptual understanding:* the ability to use multiple representations, estimating, making connections and links and understanding properties of number systems (i.e. number sense).
- *Procedural fluency:* the ability to solve a problem without referring to tables and other aids, using efficient ways to add, subtract, multiply and divide mentally and on paper, understanding when it is appropriate to use procedures or not (as not all calculating situations are alike). In addition, I looked at "elements of fluency" which includes knowing basic facts and knowing basic methods (Askew, 2012a, p. 55).
- *Adaptive reasoning:* being able to give informal explanations and justifications for one's work, intuitive reasoning based on pattern, thinking logically about the relationships amongst concepts and determining whether the use of a particular procedure is appropriate or not for the context, that is, being able to adapt the problem-solving approach to the context.

Findings

Using the data generated by the interviews, I present the findings as summative data (in tables) and as descriptive narratives.

Although I am not attributing learner progress specifically to the clubs, using the summative scores, we can see that the learners did make progress throughout the year (Table 4.5). Overall, there was an average percentage point increase of 7.8% from March to November for club one and 8.5% for the four learners interviewed in club two. Individually, there were substantial changes for four learners: Anathi,[2] Thembela, Akhona and Chebe. Thembela and Akhona both showed the greatest positive progression in each club (18.2 and 14.8%, respectively) across *all* aspects between March and November.

Looking at the score and percentage point increases for different LFIN aspects (Table 4.6), we can see more specifically where progress has been made over time. For example, in club one, Anathi, Thembela and Zintle showed large increases in Conceptual Place Value (Aspect C) ranging from 23 to 31 percentage points over time. Similarly, large gains were shown for three of the four learners in club two. Looking at the actual scores the

Table 4.5 Learner one-to-one interview overall percentage score change from March to November for both clubs

Learner	*Score Out of 88 marks*	*% Score*	*Score Out of 88 marks*	*% Score*	*Score Change*	*% Change*
Anathi	78	88.6	87	98.9	9	+10.2
Cebisa	81	92.0	84	95.5	3	+3.4
Kholeka	85	96.6	88	100.0	3	+3.4
Nate	77	87.5	83	94.3	6	+6.8
Zac	81	92.0	86	97.7	5	+5.7
Thembela	71	80.7	87	98.9	16	+18.2
Zintle	77	88.6	84	95.5	7	+6.8
Club One Averages	79	89.4	86	97.2	7	+7.8
Aphiwe	75	85.2	79	89.8	4	+4.5
Akhona	71	80.7	84	95.5	13	+14.8
Chebe	72	81.8	81	92.0	9	+10.2
Kuhle	83	94.3	87	98.9	4	+4.5
Club Two Averages	75	85.5	83	94.0	7.5	+8.5

Table 4.6 Learner one-to-one interview percentage change—March to November per LFIN aspect for both clubs

Learner	*A* Total: 8 marks		*B* Total: 32 marks		*C* Total: 13 marks		*D* Total: 14 marks		*E* Total: 20 marks	
	From/to	*% change*	*From/to*	*% change*	*From/to*	*% change*	*From/to*	*% change*	*From/to*	*% change*
Anathi	FM	0	29/32	9	10/13	23	12/13	7	19/21	10
Cebisa	FM	0	30/32	6	11/13	15	13/11	–14	19/20	5
Kholeka	FM	0	FM	0	12/13	8	FM	0	19/21	10
Nate	7/8	–13	29/31	6	11/13	15	12/11	–7	17/21	19
Zac	FM	0	31/32	3	12/13	8	12/14	14	18/19	4
Thembela	FM	0	26/32	19	10/13	23	10/13	21	17/21	14
Zintle	FM	0	30/29	–3	9/13	31	12/14	14	18/20	9
Club One Averages		–2		6		18		5		10
Aphiwe	7/8	13	29/30	3	9/12	23	13/10	–21	17/19	9
Akhona	7/8	13	26/32	19	10/13	23	9/11	14	19/20	5
Chebe	FM	0	29/31	6	10/12	15	11/11	0	14/19	23
Kuhle	FM	0	FM	0	12/13	8	12/14	14	19/20	5
Club Two Averages		6		7		17		2		10

LFIN ASPECT KEY:
A: Structuring Numbers 1 to 20
B: Number Words and Numerals
C: Conceptual Place Value
D: Early Arithmetic Strategies
E: Early Multiplication and Division
FM = Full marks in both interviews

learners in club one achieved in this aspect, all learners achieved a full 13 marks in the second interview.

For Aspect B, the scores show that a number of learners (Anathi, Cebisa, Zac, Thembela and Akhona) achieved a full 32 marks in the second interview.

It is worth noting the results for Aspect A in Table 4.6. Eight of the 11 show a zero percentage point increase or full marks. This is due to their obtaining correct answers for these questions in the pre- and post-interviews. This does not mean that they did not make further progress in this regard, simply that they correctly answered the questions in both the interviews.

In the descriptive narrative below, Table 4.7 summarises one club learner's (Thembela) progress over time. Thembela was a regular attendee at the Elmtree Prep club. In the first half of the year, she sought my attention and chatted to others (about non-mathematical events) more than she participated in the mathematics activities of the club. In the second half of the year, she engaged more actively and in a focused way with the activities, worked well when asked to work in pairs and discussed mathematical ideas with me and other club participants. Thembela was an "intermediate"-type learner, in that she could already structure early number well, but she made progress relating to Conceptual Place Value which was likely to have an effect on her Early Arithmetic Strategies (see later discussion). The letter followed by a number in square brackets indicates the LFIN aspect and the highest level or stage for that aspect. The two numbers separated by a slash indicate that the learner's position falls between two LFIN levels (i.e. a borderline position). The ➔ indicates movement from one LFIN level to another over time (i.e. from March to November) for the learner.

In looking at Thembela's progress through LFIN aspect levels, she progressed in three Aspects: B, C and D. With the exception of level B she moved only one level for each aspect on the LFIN. In her March interview for Aspects B she was unable to identify 1025, ½ and number line positions and she said "10 hundred" for 100 in the forward number sequence 93 to 112; thus she was placed at level 3 for this aspect. She answered all

Table 4.7 Thembela LFIN profile for March and November interviews

	A *[6]*	*B* *[4]*	*C* *[6]*	*D* *[5]*	*E* *[5]*
Mar – Nov LFIN Level Changes	4➔4	3➔5	2➔3	2/3➔3	1/2➔1/2

these correctly in her November interview, and thus progressed to level 5. For Aspect C in her first interview, she added 10 each time a 10-dot strip was laid down (*Task 9*) and she struggled with *add 10 to 294*, *take 10 away from 700* and *add 100 to 932*. Thus she was placed at level 2. In her November interview, she answered all these correctly and worked them out mentally. For *Task 9*, she did the same as her first interview except for the final question where she verbalised her final answer as $7 \times 10 + 4$. By using a known fact, she moved to level three for this aspect. For Aspect D, her progress was slight (from level 2, borderline 3, to 3). She saw the patterns in *Task 15* and used them to solve the questions in November. Otherwise, she used the same strategies that she did in the first interview.

Pulling it all together using scores, levels and strand indictors, we can see her progress in mathematical proficiency. In the March interview, Thembela's scores for all LFIN aspects (except structuring numbers 1 to 20) were below the club average and ranged from 71% to 85%. However, results from the November interview showed that her results were above or the same as the club averages for all LFIN aspects, achieving 100% in all but early arithmetic strategies. The percentage change from the March interview to the November interview showed that Thembela achieved the highest percentage point change of all club learners for number words and numerals and for Early Arithmetic Strategies. Thembela's progress in Early Arithmetic Strategies is notable as she was below the club average for this aspect in March. Her results also showed substantial progress for Conceptual Place Value and Early Multiplication and Division. While Thembela scored lowest in the March interview, she made the greatest progress in scores in the November interview.

Her progress in LFIN levels was also interesting. She moved one level in *Conceptual Place Value* and *Early Arithmetic Strategies* and a minimum of two levels in *Number Words and Numerals*. So, although she can already structure early number well (Aspects A and B), this progress revealed a number of additional insights. Firstly, although her methods were getting more sophisticated as seen from her progress in Conceptual Place Value and Early Arithmetic Strategies, she still had room for progress further through the Early Arithmetic Strategies levels to stage 5. Wright et al. (2012) point out a relationship between Conceptual Place Value and Early Arithmetic Strategies, in that many non-counting by one's strategies involve switching between incrementing and decrementing by tens and ones. Thus, being able to do this flexibly is crucial to the development of the more facile strategies mentioned in Table 4.5. Thus, in future we would hope to see Thembela using her developed Conceptual Place Value knowledge to progress further towards using facile strategies.

Secondly, in her second interview, she demonstrated elements of procedural fluency by working out answers mentally. Her ability to locate numbers on the number line and to see the patterns in numbers revealed that she was developing some level of conceptual understanding, and by verbalising her thinking, she was beginning to reason adaptively. In the time between the two interviews, Thembela appeared to have made progress in her ability to work accurately by using mental processes where appropriate and in consolidating her understanding of various concepts, thus showing that she was developing several elements of mathematical proficiency.

Discussion

As illustrated, the combination of the broad summative scores and the detailed LFIN profiles gave me the opportunity to tell richer stories about club learners' progress over time. Additionally, the scores provided a broad picture of each club's progress as a whole and allowed comparison across the two case study clubs. The examples above also show the emerging development of three strands of mathematical proficiency (conceptual understanding, procedural fluency and adaptive reasoning) in club learners through this analysis.

Conclusion

This chapter has provided insights with regard to the relationship between the LIFN, the strands of mathematical proficiency and the scoring system I developed for the interview in determining club learners' mathematical proficiency progress. By finding occurrences and absences of indicators for conceptual understanding, procedural fluency and adaptive reasoning from Kilpatrick et al.'s (2001) work, I wove the broader notions of these strands of mathematical proficiency into the analysis of the interviews. This reveals how the strands connect across the LFIN aspects and allows for a richer description to emerge in the presentation and analysis of the data. As stated previously, the LFIN stages and levels represent increasing sophistication and efficiency for each aspect, thus providing typical learning pathways for each learner's mathematical learning in specific early number concepts such as Conceptual Place Value. By overlaying these stages and levels with mathematical proficiency indicators for conceptual understanding, procedural fluency and adaptive reasoning, a more connected picture shows how the LFIN aspects were connected by the broader mathematical strands of proficiency and how deeper mathematical thinking develops. Thus, the strands of mathematical proficiency form an umbrella over the LFIN to pull everything together. The LFIN therefore provides us with typical learning pathways for mathematical learning in specific early number concepts whilst the five strands offer a broader notion of developing mathematical proficiency over time and beyond early number. This idea is illustrated in Fig. 4.3, foregrounding the three strands featured in this analysis.

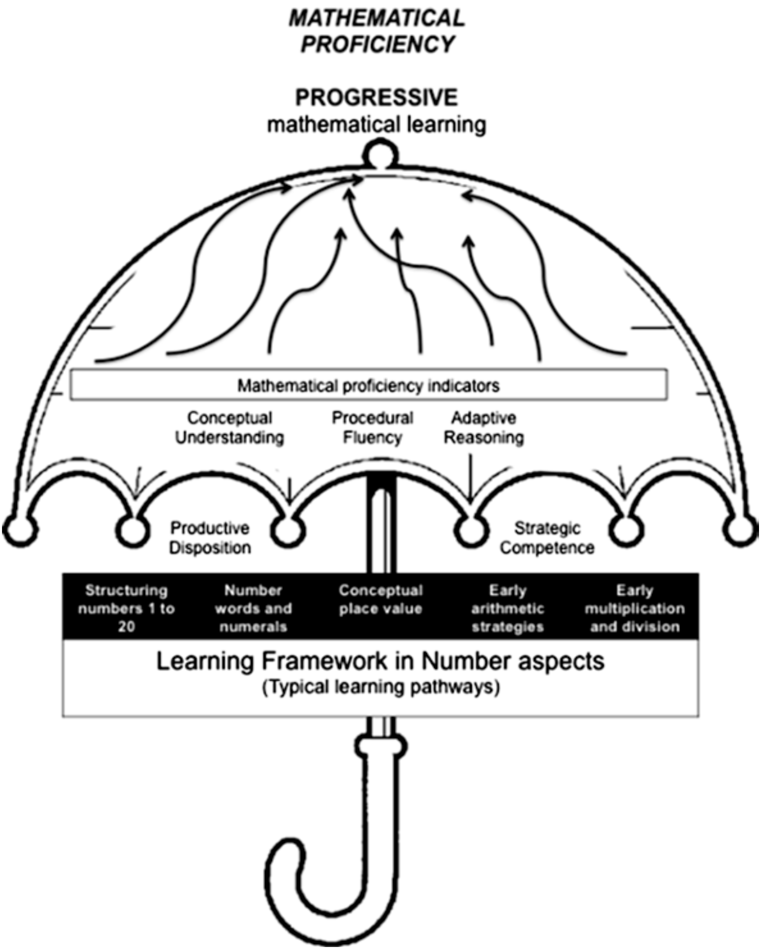

Fig. 4.3 The relationship between three strands of mathematical proficiency and the LFIN

In my study and in the running of the clubs over the research period, using a combination of scores (both raw and percentage-based) and the methods learners used to answer interview questions proved invaluable for both planning purposes and the analysis of progress made over time.

Notes

1. Productive disposition is examined in other research studies carried out by the SANC project. See, for example, Graven, Hewana and Stott (2013) and Graven (2012).
2. Learner names have been changed for confidentiality purposes.

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CHAPTER 5

Adapting Aspects of the Mathematics Recovery Programme for Use in Various South African Contexts

Debbie Stott, Zanele Mofu, and Siviwe Ndongeni

Introduction

Mathematics education in South Africa is widely acknowledged to be 'in crisis' (e.g. Fleisch, 2008) and attention is increasingly diverted from only addressing the problems in the Further Education and Training band (FET) to having them addressed in the early foundation years of learning. The Foundations for Learning Campaign (Department of Education, 2008) was introduced by the DBE in 2008 in order to bring a specific focus to improving reading, writing and numeracy in South African learners. One feature of this campaign has been the introduction of systemic

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assessments in the form of the Annual National Assessments (ANAs) written in numeracy/mathematics and language in grades 1–6 and grade 9 in all government schools.

The 2014 Annual National Assessment results (Department of Basic Education, 2014) reflect South Africa's poor performance in mathematics. The 2014 results show that nationally, grade 3 results have increased by 15% since 2012, grade 4 results have stayed the same and grade 9 results have decreased down to 11%. There is an increase in the number of learners achieving 50% or more for both grades 3 and 6. However, only 3% of grade 9 learners achieve over 50% in the assessment.

The results of these assessments over the past three years confirm that the majority of learners do not have basic numeracy skills and that with each progressive year of schooling more and more learners lag behind meeting the basic requirements for their grade level (Schollar, 2008). So for example a grade 4 learner asked to solve 243 × 59 in class or in an ANA cannot participate if they are still at the level of drawing three groups of 9 in order to find 3 × 9. In this sense, their performance on this ANA question would tell the teacher little about the level of multiplicative reasoning that the learner does have and whether remediation should begin with focusing on grade 1, 2 or 3 work (Graven & Venkat, 2014).

The early years of schooling are a crucial period for fostering the basic skills and love for numeracy (Department of Basic Education (DOE), 2011). For learners to be successful in later mathematics activities and to use mathematics effectively in life, they must have a sound understanding of elementary mathematics concepts, and to develop a positive attitude towards the learning of mathematics, and the belief that an understanding of mathematics is attainable (DOE, 2011). Early intervention requires a teacher to play a vital role in the learner's development, and the intervention needs to be carefully planned to cater for learners from different backgrounds.

A wide range of research points to the need for coherence and progression in the teaching of mathematics (Askew, Venkat, & Mathews, 2012; Schollar, 2008). However, teachers are unlikely to identify useful resources or generate resources with carefully inlaid progression without a solid understanding of the level at which learners are operating and the various levels through which learners must progress in order for foundational numeracy proficiency to be sufficiently in place in order to progress through the mathematics required in the later primary grades. The notion of early intervention in numeracy can be problematic for educators if they are unable to diagnose what the source of the learner's difficulties is.

As mentioned in the paragraphs above, the ANAs as they currently stand do not provide teachers with information about where remediation needs to take place, they simply tell teachers what learners do not know. Therefore, teachers need diagnostic tools to identify the specific problems that they are experiencing with learners and tools which can be used to profile learner strengths and weaknesses, tools that will target learner's misconceptions and less sophisticated strategies. In this respect, Wright, Martland, Stafford, & Stanger (2006) developed the Mathematics Recovery (MR) programme as a resource to support learner development in early number concepts. Specifically, the MR programme includes three tools: (1) one-to-one learner assessment interviews (including interview tasks and profiling schedules), (2) a learning progression model for early number learning, which is described later, and (3) a teaching framework and associated resources for advancing learners in the progression model.

Across each of our research projects, we have found Wright, Martland, and Stafford (2006) progression model of mathematical progress in their early Learning Framework in Number (LFIN) to be particularly useful. We have used this framework for analysis of learner levels of mathematical understanding in order to design learning activities. We have used the other tools from the MR programme for data collection, club interventions and teacher development. In this chapter, we discuss the ways in which our three research projects, in the context of primary after-school mathematics clubs, drew on and adapted tools from the MR programme in order to illuminate both the usefulness and contextual adaptations of this programme for both analysis and developmental purposes in informing teaching practice in numeracy in resource-constrained Southern African contexts. Specifically, we focus on how we have adapted the assessments interviews and the way the data from interviews has been extended to work with the learning progression model (LFIN).

Our Empirical Fields

The South African Numeracy Chair (SANC) project is tasked with researching sustainable ways forward to the many challenges faced in primary mathematics education in South Africa. One development initiative piloted in 2011 and rolled out in 2012 is that of after-school mathematics clubs. Within the SANC project, the clubs serve two purposes: firstly, they are a place where the project team can directly influence what happens with learners and secondly, they provide researchers in the project with an empirical research field where they can observe and interact directly with the learners.

These clubs are conceptualised as informal learning spaces focused on developing a supportive learning community where learners can develop their mathematical proficiency, make sense of their mathematics and where they can engage and participate actively in mathematical activities. Individual, pair and small group interactions with mentors are the dominant practices with few whole class interactions. The clubs were intentionally designed to contrast more formal aspects observed in the classrooms of the SANC project participating schools (Graven, 2011; Graven & Stott, 2012). The three authors are part of the SANC project and have carried out master's and doctoral developmental research in clubs. All ran clubs in 2012–2013. Details of the research are available in their theses and other publications (see e.g. Mofu, 2013; Ndongeni, 2013; Stott, 2014).

Theoretical Framework, Methodology and Analytic Tools

Across all of our research, we have taken a broad socio-cultural perspective in relation to interpreting learner understanding and progression. This assumes that learning is an active construction of knowledge through social interactions with others. Wright et al.'s (2006) work is based on the principles that learning mathematics is an active process, each child constructs their own mathematical knowledge and that they develop mathematical concepts as they engage in sense-making, mathematical activity. Their MR programme is based on sense-making and mathematical activity and normally takes place alongside a teacher or other adult. In this way, learners are not working on their own discovering knowledge per se but are assisted by a more knowledgeable other. This view coheres with those taken by each author in their individual studies.

The three research studies detailed here were qualitative and drew on the case study research design. We used the structured one-to-one interviews from the MR programme (Wright, Martland, & Stafford, 2006) as the means to gather data for the research reported on here. Each author used different aspects of the LFIN as an analysis tool for their study. The five aspects of the LFIN are as follows:

1. Structuring numbers 1–20
2. Number words and numerals (including forward and backward sequences)

3. Conceptual place value knowledge (ability to reason in terms of tens and ones)
4. Early arithmetic strategies (strategies for counting and solving simple addition and subtraction tasks)
5. Early multiplication and division
   (Wright, Ellemor-Collins, & Tabor, 2012; Wright, Martland & Stafford, 2006)

Each of the key aspects of the LFIN are elaborated into a progression of up to six levels or stages with each model describing the characteristics of the levels or stages (Wright, Martland, & Stafford, 2006). Detailed in Table 5.1 are the stages for Early Multiplication and Division as used by

Table 5.1 LFIN progress model for early multiplication and division strategies (Wright, Martland, Stafford, & Stanger, 2006, p. 14)

Level number	*Level descriptor*	*Characteristics (representing increasing levels of sophistication)*
0	Initial grouping and perceptual counting (Forming equal groups)	Able to model or share by dealing in equal groups but not able to see the group as composite units; count each item by ones.
1	Intermediate composite units (Perceptual multiples)	Able to model equal groups and counts using rhythmic, skip or double counting; counts by ones the number of equal groups and the number of items in each group at the same time only if the items are visible.
2	Abstract composite units (Figurative units)	Able to model and counts without visible items, i.e. the learner can calculate composites when they are screened, where they no longer rely on counting by ones. The child may not see the overall pattern of composites such and "3, 4 times".
3	Repeated addition and subtraction	Coordinates composite units in repeated addition and subtraction. Uses a composite unit a specific number of times as a unit, e.g. 3 + 3 + 3 + 3; may not fully coordinate two composite units.
4	Multiplication and division as operations	Two composite units are coordinated abstractly, e.g. "3 groups of 4 makes 12"; "3 by 4" as an array
5	Known multiplication and division facts strategies	Recalls or derives easily, known multiplication and division facts; flexibly uses multiplication and division as an inverse relationship, is able to explain and represent the composite structure in a range of contexts.

Mofu and Ndongeni. Stott used all five aspects of the LFIN for analysis as described in Chap. 4 (this volume).

Our Adaptations to the MR Programme Tools

In this section, we describe how aspects of the MR programme were used and adapted in our three studies.

Stott

In her doctoral study, Stott (2014) used and adapted two aspects of the programme: the one-to-one assessment interview and profiling of learner levels onto the LFIN using data derived from the interviews. The structure of the standard one-to-one interview from the MR programme was adapted to include some additional tasks used by Askew, Brown, Rhodes et al. in their 1997 study. These extra tasks added additional representations, word problems, mental maths and fraction problems to the interview. For more information about the detail and structure of the adapted interview see Stott (Chap. 4. this volume).

Stott's principal adaption was the way in which the interview data was used in her study. As part of the MR programme, the one-to-one assessment is not intended to result in a numeric score (2003). However, Stott argued (2014) that such scoring can be useful in relation to working with groups of learners. In her clubs, she needed to balance the needs and progress of the whole group with those of the individual learners. This quantifiable data helped her to zoom out from individual detail and see the broad picture for a club as a whole and where overall areas of strengths and weakness lay.

She generated *three* types of scores: first, an *overall score* (or mark) for each individual interview expressed as a percentage; second, a *percentage score for each aspect of the LFIN*; and finally, she used these generated scores to *aggregate* across the club as a whole and across more than one club by working out averages, counts and so on. These different types of data gave a rich picture of where each club learner was in their early number learning. Results presented using these scores can be found in Stott (Chap. 4, this volume).

Generating this quantifiable data was achieved by using a Boolean system of scoring, thus entering a '1' for an accurate answer and a '0' for an inaccurate one into a standard spreadsheet. Totals and percentages were created for each learner and additionally, for each different LFIN aspect by grouping questions together. By working with percentages, she was able to usefully aggregate these scores in order to make comparisons across more than one club using tables and graphs. These types of comparisons across the whole club or sets of clubs are not easily noted from the aspect stages or levels detailed within the LFIN itself, as each set of stages or levels is profoundly different and one would not be comparing like with like (Stott & Graven, 2013b).

Her possibility of generating *quantifiable* data from the interview scores, particularly of different LFIN aspects, offers some insight into how such a scoring system could work if the interviews were administered to larger groups of learners and is possibly an accessible way for teachers to deal with the data generated from assessments where there are large class numbers. Her study also showed that using a combination of scores and the methods learners used to answer questions was useful for both planning of club activities and for analysis of progress made by learners through various levels or stages of the LFIN framework. Again, this may be beneficial if the MR programme is to be used for diagnostic assessment and recovery interventions with larger groups of learners.

Mofu

Mofu's (2013) experience in her classroom confirmed that learners experienced difficulties with multiplication. She observed that when working with multiplication, her grade 5 learners were still counting visible objects in ones. Some learners, when performing multiplication tasks, draw circles or small lines for counting and some just added the numbers. Mofu attended a local Early Childhood Development conference in 2012 where a discussion with Bob Wright took place regarding what it would mean to use the MR Programme in a group context rather than on a one-to-one basis. It was mentioned that in the South African context, due to lack of resources, it is not always feasible to use the programme as an individually focussed intervention the way it has been used successfully in other countries. Bob Wright responded that research looking at using it in a group setting would be valuable. Hence, it was Mofu's aim to explore the use of the MR programme with a group of learners with the hope that this study could point to the possibility of using the MR programme in whole class situations and open up further avenues for research. Further she hoped to use results from the study to inform her own class teaching and teaching in her school and to find ways to support primary school teachers at large in developing the strategies to teach and remediate multiplication reasoning.

Mofu collected video recorded one-to-one oral interviews with the learners. A sample of six grade 4 learners were purposively selected to participate in an after-school intervention programme aimed at supporting and remediating multiplicative reasoning. Mofu used all the MR programme tools in her study. The LFIN was to profile the learners using pre- and post-intervention interview data and to determine their levels of multiplicative reasoning. She also used the teaching framework of the MR programme to plan the intervention activities. The teaching intervention was informed by results of the diagnostic pre-assessment interview and ongoing assessment in each session. She selected from the bank of teaching procedures and resources from the MR programme. As a way to investigate using the resources in a different way than that intended by the original MR programme, the intervention was done as a whole group for one hour once a week over the period of four weeks.

Mofu's initial analysis of the assessment interviews was of learner progress in terms of the LFIN levels as guided by the MR programme. The LFIN levels shown in Table 5.1 were used as guidelines for profiling of learners. The data was analysed and discussed qualitatively to get a detailed picture of how the learners had progressed across the five levels on the basis of strategies used to solve multiplication and division tasks in the pre- and post-interviews. Table 5.2 gives an overall picture of how the learners in her study progressed in terms of the LFIN levels from the pre-(March 2013) to the post-(April 2013) assessment.

Given the relatively short intervention in this study, progress made from one level to another was one of the most important results for Mofu. Her data showed that in the pre-assessment, learners were counting in ones (positioning them at level 1) and relying on using constrained methods to solve multiplication tasks. After the intervention, the post-

Table 5.2 Learners' overall progress in LFIN levels over time from pre- to post-assessment (Mofu, 2013 p. 49)

	Learner A		*Learner B*		*Learner C*		*Learner D*		*Learner E*	
	PRE	*POST*	*PRE*	*POST*	*PRE*	*POST*	*PRE*	*POST*	*PRE*	*POST*
LEVELS	2	3	2	3	1	3	3	4	4	5

assessment showed that constrained methods were less obvious and learners were able to count in equal groups and use more efficient and fluent methods to solve the multiplication tasks. The rate of progression in Mofu's study was far greater than she expected; all learners progressed at least one LFIN level.

The results from this initial profiling prompted Mofu to consider further the nature of learner progress. Thus, one adaptation that she made was the development of a spectrum of multiplicative proficiency. She drew on the efficiency spectrum for *procedural fluency* developed by Stott and Graven (2013a). The strategies used by the learners in Mofu's case study confirmed the notions of efficiency and fluency she saw in the analysis of the interview and showed an overlap of learner strategies with regard to multiplicative strategies. The learners displayed a range of responses from restricted/constrained multiplicative fluency towards elaborated and fully flexible fluency. This resonated with her sense that learner's multiplicative proficiency needed to captured in its own right. Thus, Mofu adapted Stott and Graven's (2013a) spectrum for procedural fluency spectrum into multiplicative spectrums (shown in Fig. 5.1) for each learner to further understand their individual progress, particularly in the methods used to solve multiplication problems.

The spectrum provided a visual way of seeing learner progress. Progress was evident when learners moved to the middle or upper end of the spectrum, which indicated increased fluency, flexibility and efficiency in multiplicative thinking. Figure 5.2 shows the positions of each learner according to the methods each used on the spectrum for the pre- and post-assessments across the seven interview tasks starting with constrained (I-Inefficient) on the left, fluent (IE) methods in the middle and flexible methods (E-Efficient) on the right. The values are the number of tasks

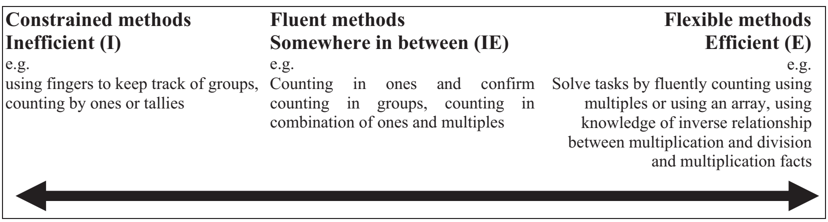

Fig. 5.1 Spectrum of multiplicative proficiency (Mofu, 2013)

	I Constrained	IE Fluent	E Flexible fluency
LEARNER A			
PRE	5	1	1
POST	2	0	5
LEARNER B			
PRE	4	3	0
POST	2	0	5
LEARNER C			
PRE	6	1	0
POST	2	2	3
LEARNER D			
PRE	3	1	3
POST	0	1	6
LEARNER E			
PRE	1	2	4
POST	0	0	7

Fig. 5.2 Summary of multiplicative spectrum methods for all learners across 7 interview tasks (Mofu, 2013 p. 61)

where the learners showed the usage of different methods. So for example, Learner A progressed from using mostly constrained methods in the pre-interview (in 5 questions) to more flexible methods in the post-interview (in 5 questions).

Shifts in learner responses over time are evident. In the pre-assessment, most of the learners used constrained methods except Learner E, who was split between using constrained and flexible methods. Learners A, B and C seemed to rely predominantly on using constrained methods, while Learners B and C did not demonstrate use of flexible methods at all. The post-assessment indicated an overall improvement in multiplicative proficiency for all the learners. There was decreased use of constrained methods and an increase in fluent and flexibly fluency. Although Learners A and B still used some constrained methods in the post-assessment, their biggest shift is to using more flexible methods. It was clear that for all learners during the period of intervention, flexibility and fluency in methods used increased.

Mofu argues that these spectrums have a number of potential uses in the local context. Not only do they assist with visualising learner progress, but also they are potentially a quicker way for educators to map/profile learner progress after administering some kind of diagnostic assessment such as the one from the MR programme.

Mofu found that the use of the tools from the MR programme made it possible for the learners in her case study to progress in terms of multiplicative reasoning. The LFIN specifically highlighted that, as teachers we need to understand the levels that the learners are operating at so as to assist them in their learning trajectory. A key aim of Mofu's research was to explore the extent to which the MR programme could be used to support learners in developing multiplicative reasoning and proficiency. As a teacher she learnt the importance of providing learning tasks that allow collaboration with peers as well as having access to concrete materials like arrays for multiplication and division. She found that the MR programme offered rich learning activities for teachers to use in interventions. Mofu also saw the usefulness of learning as an educator from the administration of the interview, seeing it as a useful developmental tool.

Mofu stated that the key disadvantage of the LFIN was that it was labour intensive and time consuming to administer for more than a few learners. The assessment interviews took approximately one and a half hours for each learner. Additional time was spent profiling learners onto LFIN levels. Thus while Mofu recommended that teachers conduct interviews with a range of their learners in order to gain in-depth insight into learner levels and difficulties in multiplicative reasoning, it is not feasible to assess all learners in a class in this way and possible adaptations such as the spectrums presented here would need to be explored in the classroom. This constraint notwithstanding, the implementation of the multiplication part of the MR programme to a group of learners holds potential to work in classrooms. In her new role as Foundation Phase Mathematics Curriculum Planner, in the Eastern Cape, Mofu has subsequently conducted many fruitful workshops in this regard with Foundation Phase teachers in the Eastern Cape. In addition, doctoral research currently being undertaken by Ndongeni is investigating the use of whole class assessments and the use of the spectrums as a way to profile whole classes of learners.

Ndongeni

We turn now to the adaptations of the MR programme tools resulting from Ndongeni's (2013) research. The aim of her study was to better understand the relationship between multiplicative conceptual understanding and productive disposition in order to explore ways to strengthen these and thus shift teaching for this purpose. Having noticed over a period of years that the grade 7s in her school still relied on unitary counting and written tallies when dealing with multiplication and division problems she focused on the relationship between conceptual understanding and productive disposition (Kilpatrick, Swafford & Findell, 2001) in the context of multiplication.

In the study a purposively selected sample of six grade 4 learners was used: two high, two average and two low performers as indicated by performance on an initial basic assessment of multiplication. Ndongeni also used all of the MR programme tools, including the same one-to-one assessment interviews as Mofu, for exploring the nature of students' conceptual understanding of multiplication in her study. Additionally, Ndongeni designed an instrument for assessing learners' productive disposition (shown below) in multiplication based on Graven's (2012) more general productive disposition instrument. Ndongeni's instrument is shown in Fig. 5.3.

Her study drew on the early multiplication and division LFIN aspect to establish learner levels of conceptual understanding in multiplication and learners' methods and performance were analysed in terms of the LFIN model characteristics. Her broader notion of conceptual understanding was theoretically informed by Kilpatrick et al.'s (2001) five-stranded framework of mathematical proficiency. For the analysis of the mathemati-

1. Multiplication is ______________________________
2. Give some examples of multiplication problems you know ______________________
3. Put a circle around yourself 1 ☺ ☺ ☺ ☺ ☺ ☺ ☺ 9

   Why did you put yourself there? ______________________________
4. Why does Kuhle struggle with multiplication? ______________________
5. Why is Sine good at multiplication? ______________________________
6. What do you do when you get a multiplication problem that you do not know the answer to?

   ____
7. Which do you enjoy more, addition or multiplication problems? ____________________

   Why?______________________________

Fig. 5.3 Adapted productive disposition instrument (Ndongeni, 2013, p. 47) cal dispositions, the similarities between the elements of productive disposition as suggested by Kilpatrick et al. (2001) and those suggested by Carr and Claxton (2002) were combined. The learners' responses from the adapted productive disposition instrument were analysed in terms of the extent to which the indicators shown in Table 5.3 were visible.

By linking the data from the productive disposition instrument and the data from the one-to-one interviews, Ndongeni adapted the way the data from the assessment interview was used to profile the learners. As mentioned, her analysis included the levels allocated to learner on the LFIN. This enabled comparison between each learner's assessed level and their own *perceived* performance level.

Table 5.3 Elements of productive disposition (Ndongeni, 2013, p. 49)

Elements of productive dispositions (Kilpatrick et al., 2001; Carr & Claxton, 2002)	*Linking question(s) from the adapted productive disposition instrument (Fig. 5.3)*	*Exemplars from learners' answers*
Sense making – resourcefulness and playfulness	Q1: Multiplication is … Q2: Give some examples of multiplication problems you know Q7: Which do you enjoy more, addition or multiplication problems?	27 × 10 = 270
Seeing multiplication as useful and worthwhile	Q2: Give some examples of multiplication problems you know	"It helps to do mathematics"
Seeing oneself as an effective learner and doer of multiplication	Q3: Put a circle around yourself 1 ☺ ☺ ☺ ☺ ☺ ☺ ☺ 9 Why did you put yourself there? Q4: Why does Kuhle struggle with multiplication? Q5: Why is Sine good at multiplication? Q6: What do you do when you get a multiplication problem that you do not know the answer to?	Placed herself at 5 and said "I'm not good in multiplication"
Belief that steady effort pays off - resilience	Q6: What do you do when you get a multiplication problem that you do not know the answer to?	"I take a paper and count until I get it" "I take it home for help"

Question 3 from the productive disposition instrument: *Rate yourself between 1 and 9. Why did you put yourself there*? - was intended to investigate how the learners viewed themselves solving multiplication problems, giving a reason why they viewed themselves that way and was related to the productive disposition indicator of seeing oneself as an effective learner and doer of mathematics. The second column in Table 5.4 shows each learner's LFIN level as a result of profiling the learner from the one-to-one interview. Learner responses to the first part of the disposition question are denoted by a rating number between 1 and 9, while the learner's written responses from the second part of the question are denoted by a (°) in the relevant column.

As seen in Table 5.4, four of the six learners placed themselves at 9 (the highest rating), indicating that they saw themselves as good at multiplication and they confidently said that they could count or calculate multiplication. Viwe was the only learner who rated himself as 1 indicating that he did not know multiplication. Sindy rated herself as 5 but added that she was not good at multiplication. The ratings given by the learners above reveal their dispositions towards multiplication in terms of their belief in their ability to do it.

Comparing the LFIN level of conceptual understanding and the learners own dispositions towards multiplication, it was noticeable that Andile rated himself as a 9 while his assessed level of understanding multiplication was at the lowest level of one (i.e. perceptual counting by ones). Within the context of this study, it seems that Andile associated multiplicative competence with giving answers for every question even if they were

Table 5.4 Selection of results combining LFIN levels and productive disposition questions with regard to seeing oneself as an effective learner and doer of multiplication (Ndongeni, 2013 p. 74)

Learner	*LFIN level allocated to the learner*	*Rating scale from question 3 (see Fig. 5.3)*			
		Negative/no confidence	*Less confident*	*Confident*	*Very confident*
Andile	1				9°
Viwe	1–2	1°			
Nako	2				9°
Anda	2–3				9°
Lulu	3–4				9°
Sindy	5		5°		

incorrect. During the intervention period, he was noted as the learner who depended on the teacher when he had problems, not making an effort to solve them himself. Nako also associated effort with competence. She admitted that 'I take a paper and count until I get the answer'. This resilience and steady effort did not however automatically translate to competency as she was allocated a level 2 (perceptual counting) in conceptual understanding of multiplication. Viwe was the only learner whose dispositions and conceptual understanding were aligned. He placed himself at 1 on the disposition instrument and when asked which operation he preferred between addition and multiplication, he stated that he liked addition more. This aligned with his assessed level of borderline level between 1 and 2 on the LFIN (indicating that he was at a perceptual level). Sindy rated herself at 5, which indicated that she perceived herself as an average learner of multiplication, but the interview data showed her to be the most proficient of all the learners in multiplication at level 5 (the highest LFIN level). Tirosh, Tsamir, Levenson, Tabach, and Barkai (2013) cite a range of research where young learners incorrectly associate effort with competency and note that children who rate themselves highly but perform badly may not recognise the need to expend effort in order to learn and achieve something new.

Andile presented himself as a confident learner with a strong belief in his one self-efficacy and steady effort towards multiplication. However, the results from the assessment interview reveal that he lacked more efficient ways to solve multiplication problems. His case suggested that there can be a gap between conceptual understanding and a learner's own sense of self-efficacy. Sindy was not confident in her knowledge of multiplication even while she exhibited aspects of sense-making in the interview.

What was noticeable was that although most of the participants in Ndongeni's study had strong belief of their one self-efficacy towards their knowledge of multiplication, it did not necessarily translate to high conceptual understanding of multiplication. Hewana and Graven (2015) note that in the absence of sense making, confidence in one's mathematical ability can be problematic. For learners to see sense in any area of mathematics, it is essential that they can see the connections between mathematical ideas and topics (Askew, Brown, Rhodes, Johnson, & William, 1997), have experience of coherent and progressive lessons (Venkat & Adler, 2012) as ways to develop their understanding of a topic. This would provide a basis to enable them to evaluate their own proficiency. This therefore means that learners need more opportunities to develop sense making in mathematics in general to develop in turn their broader understanding and own confidence.

The extensions made by Ndongeni to the LFIN profiling raise some interesting questions that warrant further examination particularly with regard to their implications for the classroom.

Concluding Remarks

This chapter reported on the adaptations each of us made to assessment and profiling components of the MR programme. The findings across these studies point to the usefulness of the programme tools for assessing and profiling learner levels of understanding and for planning subsequent interventions in small group situations. The after-school clubs have been a powerful space for exploring the use of the MR programme in the local South African context. However, all three studies worked with small numbers of learners. The results from our combined studies suggest that it would be difficult for teachers to use the tools from MR programme as they stand to determine *whole class* mathematical proficiency and many questions and challenges are evident in this regard. We therefore suggest that further research is required in the South African context on how to administer assessment interviews to groups and classes and how to profile learners on their mathematical learning pathways in less time-consuming ways. In particular, the new additions of scores and spectrums to the learner profiling toolbox may possibly be useful for other research projects and for use in the classroom.

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PART III

Understanding Language Issues in Primary Mathematics Teaching, Learning and Assessment

CHAPTER 6

Adapting Mathematics Recovery Individual Assessments for Group Administration in Resource-Constrained Contexts

Anelia Wasserman

Introduction and Context

This chapter reports on the findings of my research, which was based on an action research intervention focused on recovery of early arithmetic strategies with one Grade 4 class of learners in a township school in the Eastern Cape. The research aimed to understand the possibilities and constraints of the implementation of an assessment and recovery programme adapted from the widely implemented Mathematics Recovery (MR) programme of Wright et al. (Early numeracy: Assessment for teaching & intervention, 2006). The primary adaptation made to the MR programme involved administering the individual interview assessments and one-on-one intervention for groups of learners. Within the context of the many low socio-economic status (SES), under-resourced schools in South Africa where the majority, rather than a select few, require recovery, group-based assessment and recovery sessions were deemed necessary. This chapter focuses on the way in which Wright et al.'s (Early numeracy: Assessment for teaching & intervention, 2006) Conceptual Place Value (CPV) interview was adapted and implemented for

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group assessment for learners in one Grade 4 class. In the chapter, I share the findings and insights that emerged from the action research process of using these assessments with the 23 Grade 4 learners in in the class

Prior to my study, a numeracy baseline assessment was done with this class. This instrument was adapted by SANC from the Brombacher & Associates' US AID test. It consists of 20 tasks assessing the four basic operations by means of five problems for each of the four operations with progressively bigger numbers. It was noted that most learners, sometimes successfully and sometimes unsuccessfully, relied on methods of finger counting or tally counting no matter the size of the numbers being added or subtracted. This predominance of concrete methods, and failure of many students to abstract from concrete representations, has been identified as a significant contributor to poor mathematical achievements of students in South African schools (Ensor et al., 2009, p. 8). This is underlined by Schollar (2008, p. 6) stating that "79.5% of grade 5 and 60.3% of grade 7 children still rely on simple unit counting to solve problems". According to Schollar, the majority of South African learners are "not developing any kind of understanding of the base-10 number system and the associated critical understanding of place value. They cannot (…) manipulate numbers (…) and cannot use the skills upon which all more complex calculations depend (2008, p. 6)".

The results of numerous assessments including the Department of Education's Annual National Assessments (ANAs) point to a crisis in primary mathematics education where intermediate phase learners are generally operating several grade levels below the grade they are in. On average, learners are already 1.8 years behind the benchmark by Grade 3; this grows to 2.8 years behind the benchmark by Grade 9, making effective remediation at this higher grade improbable (Spaull, 2013, p. 6). A large drop in mathematics performance is also seen in the ANA results in Grade 4 learners (the first grade of the transition from foundation phase to intermediate phase). Although reliable grade comparisons cannot be made, a fall-off between phases is noted between the 53% average scored by Grade 3s in 2013, for example, and the 37% in the following year in Grade 4 (DBE, 2013; 2014). Furthermore in Reddy et al.'s recent analysis of the South African Trends in International Maths and Science Survey (TIMMS) performance over the past 20 years, they found—in relation to Grade 9 learners—that "three quarters of South African learners had not acquired even the minimum set of mathematical or science skills by Grade 9" and scored below the international "low" benchmark score (2015, p. 5).

Because individual recovery is an unrealistic luxury in most South African schools, I decided to investigate and then share possible ways of adapting the Wright, Martland, and Stafford (2006) and Wright, Ellemor-Collins, and Tabor (2012) programme for a group/classroom situation in which the majority of the learners require remediation of conceptual place value and early number strategies.

From this research, questions for a broader study emerged:

1. How might Wright et al.'s (2006, 2012) individual interview for assessing conceptual place value and early arithmetic strategies be adapted and implemented with groups of Grade 4 South African learners? How effective is this adapted framework in assessing learners' levels of mathematical knowledge?
2. How might Wright et al.'s individually administered MR programme be adapted for remediation of conceptual place value and early arithmetic strategies in the context of working with learners in groups within a South African classroom context where the majority of learners require remediation? What advantages/difficulties emerge from the adaptation of the recovery programme for use in groups?

In this chapter, I focus on the methodological adaptation of the CPV interview assessment.

Theoretical Framing and Analytic Tools

Von Glasersfeld's theory of cognitive constructivism forms the basic orientation of MR (Wright, 2003). Because I chose to work with the MR programme, and since it coheres with my own learning assumptions, the constructivist framework underpinned my research. The discursive nature of socio-constructivism implies that the teacher and learner co-construct knowledge and should, however, also include learner–learner interaction (Adams, 2007) and thus in adapting the assessment recording methods and MR activities for groups, I considered aspects of learner–learner interactions.

Wright et al.'s (2006) interview-based assessment is based on profiling learners against progressive levels of competence in the Learning Framework in Number (LFIN). This is what Adams (2007, p. 252) would refer to as "assessment for learning" instead of "assessment of learning". The individual LFIN profile obtained from the interview provides rich information regarding a learner's current early number competence. A levelled profile is formed to describe the current knowledge and most advanced numerical strategies (Wright et al., 2006). Stott (2014, p. 114) combines the key aspects of the LFIN from Wright et al.'s, 2006 and 2012 works as follows:

A. Structuring numbers 1–20
B. Number words and numerals (including forward and backward sequences)
C. Conceptual place value knowledge (ability to reason in terms of tens and ones)
D. Strategies for early arithmetical learning (strategies for counting and solving simple addition and subtraction tasks from 1–100)
E. Early multiplication and division

Each of the aspects is subdivided into a progression of three to six additional stages or levels. Because of the results of the SANC baseline four operations assessment and work done with the class prior to the study, I chose to focus on conceptual place value knowledge (CPV) and strategies for early arithmetical learning (SEAL). Due to the scope of this chapter, I focus on the adaptation of CPV:

CPV encompasses instructional sequences that develop knowledge of the structure of multi-digit numbers, as a foundation for mental computation. The main instructional sequence involves flexibly incrementing and decrementing by ones and tens, and later hundreds and thousands, in the context of base-ten materials (Wright, Ellemor-Collins, & Lewis, 2007, p. 848).

The MR model for the development of CPV levels can be summarized by the levels described in Table 6.1.

MR provides various assessment tasks and instructional activities to develop conceptual place value knowledge as summarized in Table 6.2.

Table 6.1 Development of CPV levels (Wright et al., 2006, p. 22)

Level 1	Initial concept of ten	Does not see ten as a unit of ten ones. Solves tasks using a counting-on and counting-back strategy by counting in ones.
Level 2	Intermediate concept of ten	Sees ten as a unit composed of ten ones. Needs representations of units of ten (like open hands or hidden ten strips). Cannot solve addition and subtraction tasks involving tens and ones when presented as written number sentences.
Level 3	Facile concept of ten	Tens and ones are flexibly regrouped without using materials or representations. Can solve written number sentences.

Table 6.2 CPV instructional dimensions (Wright et al., 2012, pp. 80–83)

Dimension A	Extending the range of numbers	When counting on or backwards in tens, begin in the range of 1–100. Extend to 200. Introduce hundreds materials and extend the range to 1000 Extend across 1000 and 1100 Later extend to 2000 and beyond
Dimension B	Making the increments and decrements more complex	Make increments and decrements of multiple tens or hundreds. Switch from increments and decrements of tens, to increments and decrements of ones or hundreds. Make increments and decrements of combinations of ones, tens and hundreds. Later tasks can involve determining unknown increments and decrements.
Dimension C	Distancing the setting	Materials are visible Materials are screened, but increments and decrements are shown briefly. Materials are screened, but increments and decrements are verbally posed. The first number is given as a numeral and increments and decrements are posed verbally.

Sample and Methods

While visiting a township school with a friend who was supporting teachers in the area with language education, I came across a Grade 4 class without a teacher. Although it was originally planned to focus my research on a group of six learners, the reality of a class of 23 learners without a teacher compelled me to work with the class as a whole and not exclude any learners. With the permission of the principal, parents and Department of Education, I started a weekly programme with the class as a whole. I soon decided that learners will benefit from more individualized attention and consequently divided the class into four groups based on their performance on the baseline assessment. After the first set of LFIN interviews, I changed the groups to mixed ability groups in line with the research recommendations of Boaler (2009).

Applying the MR programme in a group context entailed a process of adapting, implementing, reflecting and readapting of material and strategies and therefore my methodology essentially embraced many action research principles. Data from across various sources were triangulated (Koshy, 2005) to check for coherence and possible disconnects. This included the SANC baseline assessment for Grade 3, Wright et al.'s (2006) MR assessment for CPV and early arithmetic strategies (SEAL) (this was conducted before the eight recovery sessions as well as thereafter), a LFIN profile summary page for each learner, my research journal, observation sheets of learners' behaviour and strategies used compiled throughout the assessment and recovery phases, video recordings and photographs, written examples of learners' work done in class before and during the study period, the class's 2014 ANA results and informal interviews with the principal and teachers.

Because isiXhosa is the home language of all the learners in this class and English was introduced as the language of learning and teaching only at the beginning of the Grade 4 year (their current grade at the time of the study), most learners' understanding and use of English was limited. All interview questions were therefore posed in both English and isiXhosa, and with my limited understanding of isiXhosa I was able to use some "code switching" (Setati, 2005, p. 462), the switching between languages. The learners were thus encouraged to simultaneously develop proficiency in English and mathematics.

Administration of CPV Interviews with Groups of Learners

The original Wright et al. (2006) assessments are interview based for individual use. Adaptation for a group therefore meant finding ways of assessing various aspects of CPV (and SEAL) with groups of learners. This process is described below.

At first, I started to conduct the interview assessments with groups of six learners seated around a table with upright beer boxes as dividers. The behavioural patterns of learners and logistical issues during the first few interview sessions necessitated reflection and various adjustments to this initial set-up. The final interview setup entailed four learners, two seated on either side of me at an oblong shaped table with me sitting at the rounded head. Seats were numbered from 1–4 and colour coded in four different colours with coloured cards attached to the table. All questions were posed verbally, but some answers had to be written down by learners on individual answer scripts. Learners also had sheets of cardboard to cover their answers as they were writing to avoid copying.

I wrote down every learner's name next to his/her number on my response schedule. All answers, strategies and observations regarding a specific learner were then recorded in the column corresponding with his/her number. Since some assessment items required each learner in the group to be given a different question/task (as discussed below) I had to carefully administer the materials. I printed the different questions/tasks for every learner on the same cardboard colour as the number at the specific seat. I also highlighted the corresponding column on my response schedule in the same colour. Next to the columns were tick boxes with possible strategies/comments to minimize writing.

This meticulous organization of the assessment set-up eased administering the individual questions, saved time, limited writing and enabled me to focus on learners and not on administering the materials used. Examples are highlighted to provide clarity.

Addition Tasks/Incrementing in Tens (CPV Question 1)
(adapted to being printed on individual learner question sheets rather than demonstrated by the interviewer)

During the Wright et al. (2006) individual interview, a learner is asked to say how many dots are on a ten strip. The interviewer then keeps adding extra ten dot strips (i.e. to increment by ten each time) each time pausing to ask "Now how many dots?" to determine how the learners calculate the number of dots. This indicates whether the learner regards the ten strips both as a unit of ten and as a composite of ones.

Because of the fact that learners could hear each other's counting and answers during the group assessment interview format, I printed the incrementing ten strips on answer sheets for the learners. I deliberately used blank dots (instead of the solid ones used by Wright et al., 2006) and pencils. If they were counting in ones, they used the pencils to count the different dots and consequently left a mark on the blank dot. It was assumed that further blank dots (without marks) indicated counting in tens as shown in the example in Fig. 6.1.

Based on my observations during the interview and from the learners' answer sheets, I was able to record how learners counted the dots. Thus

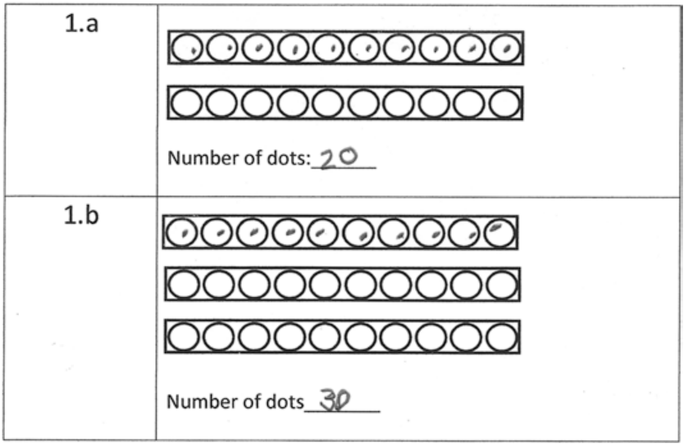


Fig. 6.1 An example of an answering sheet for incrementing tens (CPV, Question 1)

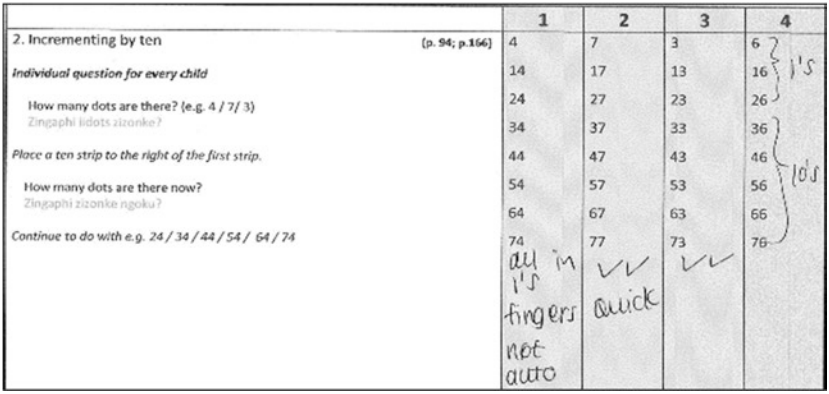
from the above, and for example my observation of a learner called Vuyo (a pseudonym), I could tell that he counted the first strip of every question in ones and the consecutive strips as a unit of ten as shown in Fig. 6.1. This was confirmed by analysis of the video recording.

Incrementing by Tens Off the Decade Tasks (Question 2)

During the Wright et al. (2006) individual interview, a four strip (a strip with only four dots) is placed in front of a learner. Ten strips are added with the increments resulting in 4, 14, 24, etc. In order to adapt this to be done for each learner in the group, I addressed each learner by turn (individually and orally) with every learner starting with a different number of dots (e.g. 7, 17, 27, etc. or 3, 13, 23, etc.). Figure 6.2 shows an example of the notation on my response schedule for one group of learners. Incorrect responses were captured by writing down the numbers offered by learners.

Uncovering Tasks (CPV Question 3) (Given simultaneously to the whole group)

For the two questions involving uncovering tasks (3a and 3b), two A3-sized boards with columns of ten dots and columns with less than ten



	1	2	3	4
2. Incrementing by ten (p. 94; p.166)	4	7	3	6 } 1's
Individual question for every child	14	17	13	16 } 1's
How many dots are there? (e.g. 4 / 7/ 3) Zingaphi iidots zizonke?	24	27	23	26 } 1's
	34	37	33	36 } 10's
Place a ten strip to the right of the first strip.	44	47	43	46 } 10's
How many dots are there now? Zingaphi zizonke ngoku?	54	57	53	56 } 10's
	64	67	63	66 } 10's
Continue to do with e.g. 24 / 34 / 44 / 54 / 64 / 74	74	77	73	76 } 10's
	all in 1's fingers not auto	✓✓ Quick	✓✓	

Fig. 6.2 The notation of incrementing by ten (Question 2)

were used. Increasingly complex ways of incrementing by tens, ones and combinations of tens and ones were uncovered by moving screens (the movement of the screens is indicated by the arrows (→) as seen below). Learners wrote down the total after every increment on their answer sheets next to the number I gave them orally before every move of the screen. Strategies used were noted on the assessment schedule with tick boxes labelled "count by ones", "count by tens", "count ones first" and "other". Where needed specific learners were asked to clarify the strategy used and that was written down as well. Figure 6.3 shows an example of the notation on the response schedule for all four learners for CPV Question 3a (uncovering tasks):

There was no evidence of learners influencing each other's responses in the case of questions answered on the answer sheets.

Horizontal Sentences Tasks (Question 4)

The Wright et al. (2006) individual interview tasks involve learners presented with written two-digit addition problems and written two-digit subtraction problems. They are then asked how they can figure out the answer. When an answer is given, they are asked whether they have another way to solve it.

During the group assessment, learners were given individual written problems (once again printed on colours to correspond with their num-

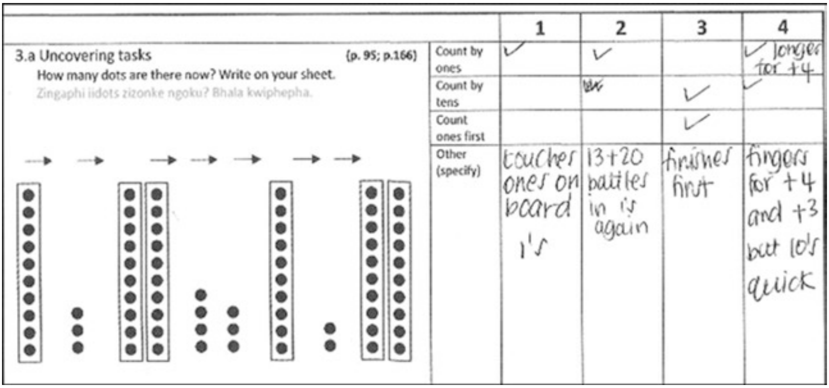


Fig. 6.3 An example from my response schedule for uncovering tasks (3a) for four learners

bers to make administration easier). Care was taken to ensure that the differential number problems for each of the four learners were similar to the ones used by Wright et al. (2006). For example, 42 + 23/33 + 25 (no regrouping) and 27 + 36/38 + 23 (regrouping). Strategies were noted on the assessment schedule with tick boxes.

I took in the paper and pencils after Question 3 was completed because mental strategies are assessed and I wanted to prevent the use of pen and paper strategies like reverting to tally counting or vertical addition and subtraction. This task group was the most time consuming and the most difficult for learners to complete. To prevent learners from getting restless and disruptive while others were solving their individual problems, I handed out the colour coded written tasks to all the learners at the same time and got feedback individually. Because the learners had different response times, I could often get feedback from individual learners while others were still working on their questions. On the response schedule, I recorded the order in which learners were able to give feedback and indicated whether a learner answered relatively quickly or whether he/she needed more time than the others. The recording of the order of answers/feedback given also helped me to monitor the influence that one learner's answer could have on an answer given later by a different learner. Figure 6.4 shows an example of the notation on the response schedule for all four learners.

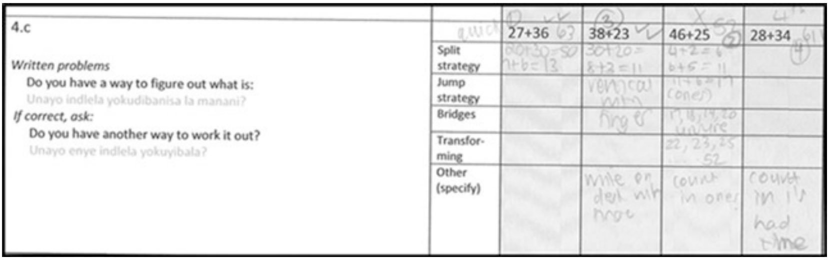


Fig. 6.4 Notation on the response schedule for addition tasks

Reflections on the Influence of the Group Set-up on the Assessment Interview

All challenges associated with the group interview assessment format could unfortunately not be eliminated. Some learners were getting restless and disruptive while waiting for others to answer individualized questions. Learners could also hear each other's answers and in some cases a learner would laugh at another. Fortunately in general I found that the learners' behaviour and self-control were better during the second round of interviews.

The average duration of the first round of CPV interviews per four learners was 36 minutes compared to 31 minutes on average for the second round. In my journal and reflection, I attributed the shorter interview times for the second round to the following factors:

- Changes were made to the initial interview set-up
- I was more familiar with the interview material
- The learners were more relaxed and needed less prompting
- The learners were more familiar with the interview material
- The learners could solve various problems faster than before
- Learners were accustomed to explaining their strategies
- Learners were more familiar with the vocabulary used
- I was familiar with more of the isiXhosa words used as part of their answers
- It was not necessary to repeat all instructions in isiXhosa for all groups

Thus from this it would seem a key aspect of running these assessments in groups is enculturating the learners into the forms of questioning and answering in these types of activities and for the assessor to develop quick ways of interpreting and recording what they are seeing. Apart from the obvious advantage of time, an element of competition seemingly enhanced learning experiences during interviews. For example, during the first CPV interviews, learners had to write down the total number of dots on ten strips as mentioned above. A few learners started out by counting in ones. Others in the same group were counting the number of strips in tens. The moment they shouted “finished!” the others realized that there was an easier way of doing things and figured out that they should count in tens too. The competition element similarly emerged during the second round of interviews when learners, now familiar with ten dot strips, did not count in ones or tens at all and merely wrote down 20 and added ten to the previous answer every time to be able to finish first.

Learning opportunities were sometimes embedded within the CPV interview. For example, Themba described his strategy for **14 + 10** (Question 4a) as:

4 + 0 = 4
1 + 1 = 2
24 (the correct answer)

He did not “echo” the quantity underlying the digits (Graven, Venkat, Westaway, & Tshesane, 2013, p. 138).

For the next question (4b) Zola did the following:

42 + 23
20 + 40 = 60
2 + 3 = 5
60 + 5 = 65

I purposefully repeated Zola’s correct phrasing of place value (i.e. “twenty plus forty” rather than “two plus four”). Themba was listening to her explanation and then began phrasing his calculations as follows:

33 + 25
30 + 20 = 50
3 + 5 = 8
50 + 8 = 58

Thus in subsequent calculations he noted the quantity underlying the tens and ones and used this in the phrasing of his method thus showing learning opportunities that were embedded in the group setting.

Conclusions

From my exploratory attempt at adapting the assessments for a group setting, I realised that mindful planning of both the assessment set-up and assessment material is of the utmost importance. Apart from the fact that this is time conserving and limits writing, it enables the teacher/researcher to watch learners carefully to make observations and interpret responses. Observations could be enriched by careful analysis of the video recordings of the assessments. It is also valuable for the teacher/researcher to be familiar with the test material before the assessment. This helped me to be more relaxed and to be able to focus on the learners and not the manipulation of materials or the instructions needed to be given.

In order for group assessments to be successful, it could be valuable to enculturate learners in the assessment situation. This must be done in a discerning way without compromising the validity of the assessment. Such prior enculturation could include:

- Establishing a relationship between the learners and teacher/researcher
- Giving learners opportunities to learn how to explain the strategies they used in various mathematical contexts
- Introducing the assessment vocabulary in the classroom context—particularly in the case of learners being assessed in their second language
- Instilling appropriate assessment behaviour (e.g. not shouting out answers, waiting for your turn, not laughing at other’s answers and even staying in your seat)

Fortunately in my case, some challenges associated with a group interview set-up could be seen as learning opportunities and could motivate learner performance. Although the group assessment might not have the same value as the individual interviews, the time economical nature thereof could be useful in the resource limited South African context.

While the paper only focused on the CPV adaptation, the SEAL interviews were similarly adapted and administered in the broader study (Wasserman, 2015) and a LFIN profile could be drawn up for every learner.

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CHAPTER 7

Using an Adapted Model of Reciprocal Teaching to Help Children Unpack Their Word Sums

Melissa Spira and Sally-Ann Robertson

Introduction

The ability to comprehend written text in mathematics is recognised as a struggle for many learners both internationally and locally (Department of Basic Education, 2012; Martiniello, 2008). This recognition has contributed to increased awareness of the centrality of language in the teaching and learning of mathematics (Morgan, 2006; Schleppegrell, 2007), and has, in turn, drawn attention to the compounding effects of having to learn mathematics through a second language (L2). In terms of the latter, Southern African researchers have contributed significantly to the international field (e.g. Adler, 1998; Barwell, Setati, Lim, & Nkambule, 2012; Setati, 2005; Setati & Barwell, 2006; Setati Phakeng, 2014; Setati Phakeng & Moschkovich, 2013).

This chapter emerges from the first author's broader research study in which she explores the use of reciprocal teaching as a possible intervention for her Year 3 learners' comprehension difficulties in mathemat-

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ics. Reciprocal teaching originated within the language teaching domain, specifically in relation to enhancing reading comprehension (Palinscar & Brown, 1984). Along with others who have trialled some of its strategies in their mathematics classrooms (*inter alia*, van Garderen, 2004; Meyer, 2014; Quirk, 2010; Reilly, Parsons, & Bortolot, 2009), we believe that this highly scaffolded approach to fostering comprehension can, with appropriate adaptation, provide a potentially powerful means of tackling some of the language challenges young mathematics learners face, most especially those learning their mathematics through a second, or additional, language and those who come from lower socio-economic status (SES) home circumstances where the kinds of literacy and numeracy practices valued in the classroom may have received little attention.

Language as an Issue in South Africa

Before looking at aspects of the language/mathematics interface in relation to word sums, language issues in South Africa's current education landscape of South Africa require brief explication.

In principle, the Language in Education Policy (LiEP) promulgated by South Africa's first post-apartheid Education Ministry promotes a policy of additive multilingualism (Department of Education, 1997). In terms of its most recent curriculum revision, South Africa's Department of Basic Education (DBE) strongly advocates the principle of mother tongue instruction for at least the first three years of formal schooling (Foundation Phase), with a gradual phasing in of additional languages as children progress to the Intermediate Phase (Years 4 and beyond) (DBE, 2011). The rationale here is that a child's dominant language constitutes the most effective linguistic tool for building a strong foundation in early literacy and numeracy.

In practice, however, much of what happens at the chalkface is closer to a subtractive, and essentially monolingual teaching and learning situation. Because English is such a powerful language, and because individual schools, through their School Governing Bodies, have the right to decide what a school's language of learning and teaching (LoLT) should be, preference is overwhelmingly in favour of English. Consequently, by Year 4 more than 70% of South African learners are learning through the medium of English, even though it is the native language for only about 7% of them (DBE, 2010, pp. 12, 16).

The fact that such high numbers of learners are learning in their additional language is identified as a significant contributing factor in South African learners' repeatedly poor performance on national and international literacy and numeracy tests (DBE, 2012; Zimmerman & Smit, 2014). Although linking poor academic achievement to learning in an additional language is an oversimplification of the problem (Reddy, 2006; Setati, Chitera, & Anthony, 2009), research shows that problems of epistemological access in mathematics are undeniably compounded by the learning of mathematics in a language in which learners are not fluent (Setati et al., 2009).

Findings from South Africa's participation in the 2006 Progress in International Reading Literacy Study (PIRLS) in which, for the first time, South African learners' reading literacy levels were externally benchmarked against an international standard, highlighted the significant reading comprehension difficulties experienced by many South African children. Only 13% of the country's Year 4 and 22% of Year 5 learners reached the Low International Benchmark of 400 points, South Africa's Year 4 children achieved the lowest overall scoring in PIRLS (Howie et al., 2008). Findings from the country's participation in PIRLS 2011 subsequently suggested a correlation between reading literacy and mathematics achievement (Howie, Van Staden, Tshele, Dowse, & Zimmerman, 2012; Martin & Mullis, 2013).

All of the foregoing points to the need for teachers (irrespective of their subject area responsibilities) to give concerted, explicit and ongoing attention to the challenges learners face at the language/learning interface.

The Language of Mathematics

There has been growing interest in the contribution of semiotics to the field of mathematics education. Halliday (1993, p. 93) makes the observation that "when children learn language, they are not simply engaging in one kind of learning among many; rather, they are learning the foundation of learning itself". For Halliday, "the distinctive characteristic of human learning is that it is a process of making meaning—a semiotic process" (1993, p. 93).

Mathematics uses multiple semiotic systems to make meaning. These, in combination, constitute a mathematical register (Halliday, 1978). They include natural language, symbols, visual displays such as graphs and diagrams, and technical vocabulary. Some of the technical vocabulary of mathematics is especially dense in meaning (e.g. "right-angle"); and some of it is borrowed from everyday language, and given new meaning (e.g. "even" and "odd", and—indeed—"right"). This creates the potential for ambiguity and/or misunderstanding.

Mathematical language is also syntactically dense. As Schleppegrell explains, mathematical language's "grammatical patterning includes the use of long, dense noun phrases … [which] participate in constructing complex meaning relationships in the problems students have to solve" (2007, p. 143). And, as she further notes, "just knowing mathematical words such as *more*, *less*, and *as many as*, for example, is not enough; students also need to learn the language patterns associated with these words and how they construct concepts in mathematics" (2007, p. 143).

In all, the multisemiotic nature of mathematics language (O'Halloran, 2000) and the interaction between semiotic systems contribute enormously to the comprehension difficulties faced by learners (Duval, 2006), perhaps most acutely in the case of those learners who may already be struggling with the LoLT, either because it is not their native language or because they come from homes where the discourse may differ significantly from what they encounter in the classroom.

Language and "Making Meaning" of Word Sums

A troubling insight emerging from both the 2006 and 2011 PIRLS findings is that many South African learners struggle to apply their thinking and reasoning skills in making meaning of classroom texts (Zimmerman & Smit, 2014), yet these are the very skills they need in order to successfully unpack and solve word sums. Our own experiences echo this finding. We have observed that the challenge for many children lies not so much in the mathematics of a sum or in the language of mathematics. It lies in having the skills needed to unpack the language in order to *get to* the mathematics. So, for example, few of the first author's Year 3 learners experience difficulty in solving the following calculation (whether algorithmically or otherwise):

43 + 52 + 27 = ____

When such a relatively simple calculation is embedded in words, however, the cognitive demands of the task change. The task often moves beyond requiring predominantly procedural action to one that requires interpretation in order to identify the appropriate operation or procedure to begin with. Her learners now need to take a more analytical, conceptual approach so as to identify what calculation applies. Many of the children struggle to work out what the steps are that they need to take in reaching a solution.

By way of illustration, the above number sentence was converted (by the first author) into the following word sum:

> If a butterfly lays 43 eggs on one leaf, 52 on a second leaf and 27 on a third leaf, how many eggs will it lay in total?

Here, the demand mathematically was that the children identify the operation required to solve the problem. To be able to do so, however, they needed to unpack the problem linguistically, which—in some cases—proved to be a more challenging task, even though, as Palinscar and Brown would no doubt point out, this word sum example, with its "focus on content knowledge that to a large extent [learners] already possess[ed]" (1984, p. 119), could be viewed as a largely "considerate text" within mathematics teaching and learning. Skilled problem solving, on the other hand, requires that learners move beyond applying "superficial strategies" to standard word problems (Jiménez & Verschaffel, 2014, p. 100). Instead, one would want to assess their problem-solving abilities by getting them to solve non-standard word problems. Similarly, with regard to reading, proficient readers, argue Palinscar and Brown, are able to "come to grips with a variety of inconsiderate texts", namely the sorts of texts that "creatively violate the accepted structure" (1984, p. 119). It is here though that many weaker readers and problems solvers may come unstuck as they then confront the dual challenge of accessing the text of a word sum as well as its mathematical intent.

It is the view of the authors that any programme aimed at genuinely helping young mathematicians develop their problem-solving abilities must move beyond training them simply to apply superficial strategies. We believe that if teachers step in and help children prematurely by simplifying a task, this short-circuits the intended cognitive challenge of the task. Perhaps, this happens because teachers over-empathise when they see their learners struggle. Or it may be because teachers sometimes underestimate what learners may actually be capable of given optimal levels of scaffolding. It is exhilarating when—after some initial struggle—a learner finds he or she is able to solve a problem independently. We share a brief insight into such a breakthrough "Aha" moment with one of the first author's Year 3 learners in our "Setting things in motion" section.

Our goal for the rest of this chapter is to try to illuminate how we might scaffold young learners' ability to cope with the linguistic demands of the word sums they encounter by building up their capacity to work in more analytical and conceptual ways through the application of a reciprocal teaching approach in mathematics lessons. The word sum examples used, including the one above, come from lessons taught by the first author. In this context, she frequently devises her own word sums based on whatever theme is currently being explored in the other curriculum areas. Given this cross-curricular coverage of topics, the learners are thus already familiar with much of the vocabulary and conceptual background before re-encountering it in their mathematics lessons.

Some Principles of Reciprocal Teaching

Reciprocal teaching is designed to introduce learners to the sorts of cognitive strategies used by proficient readers (Carrell, Gajdusek, & Wise, 2001). It involves four activities: summarising, questioning, clarifying and predicting (Palinscar & Brown, 1984), realised through the application of the following five-step process:

- clarifying the purpose of a particular reading task (both the explicit and implicit task demands);
- focusing attention on the major content of a text and avoiding trivial detail that may distract attention from this content;
- critically evaluating content for its internal consistency, and its compatibility with prior knowledge and common sense;
- ongoing monitoring to detect comprehension breakdowns; and
- drawing and testing inferences (including interpretations, predictions and conclusions).

A reciprocal teaching approach uses scaffolded instruction (Wood, Bruner, & Ross, 1976) and dialogue that is responsive to learners' needs. It also requires a gradual transfer of responsibility. Drawing on Wertsch's Vygotskian work, Palinscar (1986, p. 75) described such a process as one in which learners worked by "posing and responding to their own questions" having internalised "the dialogue they … experienced in the initial stages of problem solving when … collaborating with a more experienced individual". Palinscar went on to explain that teachers need initially to "provide explanation coupled with modelling", but then gradually "fade out the modelling, and function more in the role of coach providing corrective feedback and encouragement" (1986, p. 78).

The Aims and Design for the Present Study

The study from which this chapter emerges is a piece of action research in the first author's Year 3 classroom with supervision support of the second author. The primary school at which she teaches is in a fairly large metropolitan area of the Eastern Cape Province of South Africa. It is a dual-medium government school (English and Afrikaans) with 4 classes per grade (3 English/1 Afrikaans), testimony to the greater demand within the school for an English LoLT. While the school serves children from mainly artisan backgrounds, it is nonetheless classified as a relatively affluent school in the broader South African context. It is a Quintile 5 school. Schools in South Africa are classified into five quintiles based on the relative affluence of a school's catchment area. Using census data, catchment areas are assessed in terms of the income, unemployment rate and level of education of the communities different schools serve, with Quintile 1 representing the poorest schools and Quintile 5 the most affluent schools (Kanjee & Chudgar, 2009). The school's monthly fees are R70,000, which, although a lot less than many other Quintile 5 schools, represents a significant amount for several of the children's families.

There are 33 children in the first author's Year 3 class, 10 of whom are native users of the classroom LoLT (English). IsiXhosa is the home language for 15 of the children, Afrikaans for 7 of them and 1 child is a Zimbabwean immigrant and speaks Shona at home. In effect then, the majority of the children participating in the study are learning their mathematics in their additional language, and not all of them are yet proficient in this language. There is also a fairly wide mathematical and reading literacy performance range in the class, with some children coping comfortably at, or even a little above, grade-appropriate levels and others operating significantly below. Ten of the 33 children in the class have, in fact, been diagnosed by educational psychologists as having mild-to-moderate learning problems. A colleague of the first author has suggested that, based on her experience in more advantaged schools, there is, on average, probably a one-year lag in the reading proficiency levels of the children in this class. So, while some children cope relatively well with both of the main components of reading: the bottom-up word (and/or phrase) recognition skills and the top-down comprehension and interpretation skills, others are still working mainly on the bottom-up "mechanics" of reading (the decoding of text).

The study emerged from the first author's desire to address the diverse learning needs that exist in her classroom through "a form of disciplined, rigorous enquiry" whereby she might better "understand, improve and reform [her own] practice" (Hopkins, in Cohen, Manion, & Morrison, 2011, p. 345). Whilst it may often be easier for a teacher to use more traditional instructional strategies such as *showing and telling* and *teacher explanations* (Anghileri, 2006, p. 39), more creative solutions are needed to overcome the language and learning barriers currently experienced by many children. Towards this end, the first author resolved to explore the use of reciprocal teaching as an intervention strategy for comprehension difficulties in her mathematics lessons.

In their design of a reciprocal teaching approach, Palinscar and Brown (1984) gave close consideration to the skills displayed by good readers. In relation to top-down/bottom-up reading skills, good readers are generally those who have almost fully automatised their bottom-up (decoding) skills, and who are therefore in a position to devote the bulk of their mental focus to top-down, interpretation of text. They will be *reading for meaning*, precisely the aspect fore grounded in Palinscar and Brown's approach.

In the design of the present study, it was felt that to make such a strategy work for mathematical problem solving, equally serious consideration needed to be given to the skills displayed by those adept at solving mathematical word problems. Polya's guidelines for problem solving (1973/1945) appeared to fit well here. Indeed, significant overlap was found between the skills emphasised in reciprocal teaching and those Polya identified as necessary for understanding mathematical problems.

Adaptations based on some combination of the strategies of Palinscar and Brown and of Polya were made to create a simplified model, accessible to younger learners. The model is outlined in Table 7.1 below. It centres around an "investigator" who is required to "identify", "inquire", develop an "image", and "implement", on his or her way to solving a mathematical problem. It was felt that the use of the "i" alliteration, together with visual cues, would make the model both meaningful and memorable for Year 3 learners (see also Fig. 7.1).

As the action research project is still in the implementation phase, full results are not yet available. In the following section, however, we outline some of the features the first author has thus far put in place for her learners, and share two early episodes from the unfolding process.

Table 7.1 Reciprocal teaching and problem-solving strategies: The adapted model

Palinscar & Brown's reciprocal teaching strategy	*Polya's problem-solving process*		*Adapted model*
Questioning	Understand the problem	What information is available? What is not needed?	Identify (Separate the important from the unimportant)
Summarising		Can you state the problem in your own words?	In other words …
Clarifying		Do you understand all the words? Does it make sense?	Inquire
Predicting	Devise a plan and carry it out	Draw a picture/diagram/graph	Imaging (Create a visual image)
		Carry out the plan.	Implement

Fig. 7.1 Model of investigative strategies (Adapted from Palinscar, 1986; Palinscar & Brown, 1984)

Setting Things in Motion

Work began with the children being introduced to the analogy of their approaching word sums, or as they know them, "story problems", as investigators or "detectives". Learners responded to this idea with great enthusiasm. They were then introduced to the five "i" strategies they would be applying in their investigations (Fig. 7.1). Although each of these strategies was explained and modelled separately, an end goal is that the children come to understand the iterative nature of the investigation process.

An essential and distinctive feature for mathematics is the ability to visualise the problem, an aspect clearly evident in Polya's principles. Emphasis in the opening lesson was therefore given to "imaging". Learners were presented with the following problem:

> The worker bee makes three trips in the morning and collects 55 mg of pollen in total. If he collected 18 mg of pollen on the first trip and 16 mg of pollen on the second trip, how much did he collect on the last trip?

Learners were encouraged to create an image in their minds of what was happening in the story problem. They were then asked to transfer it to paper. Whilst the children appeared to understand what was being asked of them, some found it difficult to create an image on paper that represented the most important information and/or relationships within the story problem. The children's drawings revealed some understanding of the problem, but the children's current capacity for representing mathematical processing as images was clearly limited. Their drawings were largely "artistic" and not suited to the task of mathematical problem solving.

In reflecting on the drawings, the first author had initially assumed that her learners were simply struggling to understand the story problem, but an additional dynamic was revealed, namely, the extent to which some children struggled to transform their mental images into an external two-dimensional representation The reason for this might lie in the distinction that Duval (2006, p. 111) makes between *treating* representation and *converting* representations. "Treatments" and "conversions" refer to the transformation of representations, whether from one register to another (conversion) or within a single register (treatment). Duval argues that converting between registers is especially complex because "any change of register first requires recognition of the same represented object between two representations whose contents have very often nothing in common" (2006, p. 112). The first author hypothesises that when transferring a mental picture into a potentially usable mathematical image on paper, learners are being required to make a "conversion" *between* registers rather than a "treatment" *within* a register. This may explain some of the difficulty her learners experienced. Whether such imaging conversion is ill-suited to solving word problems or whether it simply indicates that the learners need further support and opportunities to practice developing this capacity is something requiring our further investigation.

Although the mental images learners created were inaccessible to us, their drawings did provide some insight into elements of their thinking, which allowed the first author to move on to the second of her lessons. Here the focus was on "identifying" important information in the problem of the bee collecting pollen, and looking at ways to transform the information into a mathematical diagram (Gerofsky, in Jiménez & Verschaffel, 2014, p. 95) in the hope of clarifying the problem.

In terms of Palinscar and Brown's reciprocal teaching method (1984), this would be done by giving learners the opportunity to summarise text. This has been incorporated into our model under the heading "In other words…". Learners struggled initially with this. Many began by simply repeating the contents of the problem. Others envisaged scenarios that could not be supported by the story. Below is the transcript which shows how the children were coaxed towards separating important from unimportant information to achieve a satisfactory summary in their own words of the key points of the problem.

Teacher: Now I want to ask you a question. If I asked you to take that story and put it in your own words, who thinks they can do that for me?

Learner 5: What? (Learners look uncertain.)

Teacher: Put it in your own words. Who can do that? So, let's imagine…give me your story…give me the stories back again. The story's gone, disappeared. Close your book. Now I say to you…alright…you've all read the story, what was it about? And what do we want to find out? You don't have to tell me exactly what was said. I just want to know if you understood. Alright, who's brave enough to tell me what it's all about?

Learners: Nervous laughter.

Learner 1: It was about a bee collecting pollen.

Learner 3: It was about a bee making trips to collect pollen in the morning and… *Learner stops.*

Teacher: What do you think the problem was?

Learner 5: I think he doesn't know how much pollen to collect in the last trip.

Learner 3: He was lazy to come to find how many he collected in the morning.

Learner 4: Um…uh…there was 50 in total, but he only collected 16 and 18 and he didn't know which one and how much he must get to make 55.

Teacher: OK, so that's our investigation! That's our problem we want to solve! We know how much he collected on the first trip. We know how much he collected the second time he went, but we don't know how much he collected on the last trip. OK, whose picture helped them to figure out how much he collected on the last trip? Who had a really good picture that they'd like to show us? That helped them. Who thinks they know how much he collected on the last trip?

One learner was eventually able to produce his own summary, even though he had earlier struggled with the "imaging" step. As his visual representation of the problem (Fig. 7.2) demonstrates, he chose an artistic rather than representational path.

The struggle the child experienced here may reflect a combination of the challenge of isolating the essence of the problem and that of repre-

Fig. 7.2 Learner 5's initial drawing

senting this in diagrammatic form. This seems to confirm the difficulty involved in converting between representations, which Duval (2006) argues is the "true challenge of mathematics education" (p. 128). Such "uncertainty", as Wood (in Bliss, Askew, & Macrae, 1996) argued, is perfectly normal:

When we find ourselves needing to act in a very unfamiliar situation, uncertainty is high and our capacity to attend to and remember objects, features and events within the situation is limited. Without help in organising their attention and activity, children may be overwhelmed by uncertainty (p. 42).

With further scaffolded discussion, the Year 3 learners eventually decided that the solution to their pollen-collection word problem lay in using subtraction, and the problem was solved. Further brief discussion then followed regarding the suitability of the drawing one of the learners had made as a step towards solving the problem. Figure 7.3 is the first author's slight refinement of the child's drawing.

This drawing was then used to give learners the opportunity to see how such a diagrammatic representation might also be used to *check* their solution. The discussion revolved around the understanding that addition and subtraction are inverse operations. Below is the extract from the lesson transcript. It shows how, initially, learners were very uncertain, and attempted to simply repeat the process already followed. Rather than "short-circuiting" the task by rescuing the children from their uncertainty, however, the first author continued prompting, probing and rephrasing. Eventually, one learner had an "Aha" moment. He was able to make the connection between addition and subtraction.

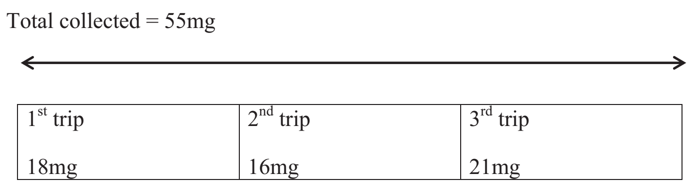

Fig. 7.3 Learner 4's representation refined by teacher

Teacher: So look at, look at it this way. If we drew a block like this. Here's our first trip, here's our second trip, here's our third trip. All together, we collected 55 mg. On the first one we collect 18, then 16, then 21. Who's got a way, can think of a way to check if the 21 is correct? Who can? Now think carefully. This drawing helps us to see if our answer is correct.
Learner 7: We can do a maths sum to check.
Teacher: OK, what kind of maths sum?
Learner 7: Like, like 18, 55 minus 18 gives you 37… *(Learner stops.)*
Teacher: We've done that already and that's how we got to 21. But what can we do to check that the 21 is correct? Think carefully. Look at the picture. On the first trip we got 18, second trip 16 and third trip 21. What can we do to check that the 21 is right?
Learner 3: Minus
Teacher: Minus what? *(No answer)*
Learner 5: You must double-check.
Teacher: Yes, but how? What must we do?
Learner 5: Do everything over.
Teacher: I actually want you to do something different. Look at the picture. Let this picture help you.
Learner 1: We can go over it again.
Teacher: That's what (learner 5) said, but I want you to do something different to check.
Learner 2: Uh, I think I've got… (Learner does an addition sum on page. He writes 18 + 16 = 37, but stops short of adding the 21.)
Teacher: But that equals 37. How much did the bee actually collect?
Learners: 55
Teacher: So let's look at our picture again. We know this amount is correct (trip 1). We know this amount is correct (trip 2). How do we check if this amount is correct?
Learner 5: *(His hand has been up for a short while already. He is very excited.)* You put all the numbers together. If you add them, you get 55.

As the above lesson extract illustrates, there is enormous value in a teacher spending more time helping learners to, in Woods' words, "organise their attention" (cited in Bliss et al., 1996, p. 42), rather than prematurely stepping in and thereby short-circuiting the cognitive challenge set before them. For Learner 5, at least, and possibly others, an important and extremely self-affirming connection had been made!

Closing Comments

The broader action research case study from which the discussion for this chapter has come is still in its initial stages. The work done by the first author has thus far been largely at the explanation and modelling stage, with substantial work still required with regard to the coaching stage leading up to gradual release of responsibility into the hands of the children themselves. However, early indications are that the use of an adapted model of reciprocal teaching is already bearing interesting—indeed exciting—fruit. It has brought a new and enlivening dynamic into play in the classroom. It appears to be having a re-energising effect on the children as, together with their teacher, they seek out more robust (and transferable) strategies for making better mathematical meaning of their word sums.

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CHAPTER 8

Language in Early Number Learning in South Africa: Linking Transparency and Explicitness

Manono Poo

Language as Problem or Language as Resource?

While international literature points, in the main, to benefits of early mathematical learning occurring in children's home languages (Gacheche, 2010), South African evidence continues to show lower performance in mathematics for children learning mathematics in their home languages in comparison to those learning mathematics through the medium of the historically 'white' languages of English or Afrikaans (Department of Education, 2007). The former evidence base has figured within a language in education policy that promotes the use of home languages as the medium of instruction in Foundation Phase (Grades 1 to 3, learners' predominantly aged 6–8), with a move to English occurring in the majority of schools in Grade 4. The latter evidence continues to figure in the viewing of home languages as a 'problem' rather than as a 'resource' for learning (Howie, 2003).

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A small body of writing, though, has taken up the need to work with language as a resource for mathematical learning. This writing on the role of language within mathematics learning has drawn on Lave and Wenger's (1991) pointing to the need for the 'transparent' function of resources in a practice. The notion of transparent function is related, in Lave and Wenger's work, to two subfunctions: visibility and invisibility, which relate to 'the way in which using artifacts and understanding their significance interact to become one learning process' (p. 103). While Lave and Wenger's work has tended to focus on resources as physical artefacts used within communities of practice, Adler (1999) applied Lave and Wenger's notion of transparency to language as a mediating resource within mathematics learning. She noted that, at different times in multilingual classrooms, language could be either more explicitly, or more implicitly, the focus of attention. When language is the explicit focus of attention, it takes on the visibility function, with emphasis on the language itself. When language is used implicitly with the focus on mathematical ideas, it takes on the invisibility function.

Adler's attention in the aforementioned article is on teaching and the dilemma presented for teachers in the need for a balance between visibility and invisibility of language as a resource. She states this dilemma in the following terms:

> The horns of this dilemma are, on the one side, that explicit mathematics language teaching, in which teachers attend to pupils' verbal expressions as a public resource for class teaching, appears to be a primary condition for access to mathematics, particularly for pupils whose main language is not the language of instruction. On the other side, however, there is always the possibility in explicit language teaching of focusing too much on what is said and how it is said. (Adler, 1999, p. 48)

But there is another kind of explicitness that is not considered in this formulation, that relates more intrinsically to the ways in which linguistic forms explicate or mirror mathematical structures. A significant international literature base, detailed later in this chapter, has noted associations between the extent of 'explicitness' in language and children's facility and fluency with learning the number words and connecting them both to an underlying decimal place value structure and then to symbolic number representations. My attention in this chapter is to the extent of explicitness of the place value structure seen in Sepedi, one of the South African home languages, and also to the ways in which teaching draws explicit attention to this structure. The inclusion of attention to explicitness in language, and transparency in language teaching, leads to possibilities for analysis at three levels, rather than the two that are present within the notion of transparency as defined by Lave and Wenger (1991):

- explicitness of the language in relation to mathematical structure
- explicit attention to verbal expressions in teaching (visibility)
- implicit use of language with attention on mathematical structure

Linguistic explicitness and transparency thus form the key theoretical lenses that are applied in this chapter. My focus in bringing these two notions into conversation with each other is to explore the openings provided in language and in pedagogy for appropriating awareness of place value structures.

In the literature overview that follows, I first outline the findings of writing that has noted that the extent of explicitness of place value structure appears to affect the ease with which young children can learn the number word sequence and its place value structure. Second, I review writing that has made use of the notion of transparency of language within mathematics teaching arrangements. This leads into my presentation and analysis of an excerpt of classroom interaction drawn from a Grade 3 Sepedi-medium mathematics classroom. The analysis points to the finding that there are opportunities for learning about place value structure across all three levels of the framework presented above, but that teaching does not necessarily draw on all three levels in optimal ways. In the concluding section, I comment on the ways in which awareness of the three levels helps me to think about teaching and learning development through presenting some hypothetical responses to selected excerpts of classroom interaction presented previously, which may help draw attention to the underlying place value structure more efficiently.

Linguistic Explicitness of Place Value Structure

Number-related concepts figure prominently in early years' mathematical learning, encompassing the ways in which counting skills are linked to culturally distinct numeration systems (Nunes & Bryant, 1996). A significant international literature base has noted associations between the extent of 'explicitness' in language and children's facility and fluency with learning the number words and connecting these number words both to an underlying decimal place value structure and then to symbolic number representations. Nunes and Bryant (1996) note that learning number names in Japanese rests on knowledge of the words representing the units from 1–9, and the word for 10. Beyond 10, all further number names are built through combinations of these ten number names in logical ways that reflect a decimal base structure. For example, the word for 10 is ‘ju’, the word for 4 is ‘shi’ and the word for 2 is ‘ni’. The word for 14 in Japanese is therefore ‘jushi’—literally ‘ten-four’, and the word for 24 is ‘nijushi’—literally ‘two tens-four’. Nunes and Bryant’s comment on the explicitness of place value in the Japanese, and some other, numeration systems is that some languages provide

> much stronger cues to the counting of units of different size and to their additive composition. It is possible that the experience of counting with a regular system helps children to understand the properties of a base-ten system and makes it easier for them to do so than it is for children who have to deal with more capricious systems (pp. 60–61)

English number names are more ‘capricious’ in this sense. The words ‘eleven’ and ‘twelve’ provide no explicit cues that they comprised of a ten and one or a ten and two. The use of the suffix ‘-teen’ for the remaining numbers to 19 does not link directly to the idea of additive composing of a units number to a ten with the extent of explicitness and efficiency that the Japanese number names do. Further, the remaining multiples of tens numbers to a 100 are also capricious in their formulation rather than directly logical compositions of a small set of earlier words, and the suffix ‘-ty’ now stands for the ten rather than the ‘-teen’ that children will have come across in the 13–19 range. This means that substantially more number names have to be rote learnt in English as there is much less logical underlying structure. Fuson (1988a) points out further that these irregularities make transitions to symbolic number forms more complex, because words like ‘seventeen’ sound more like ‘seven ten’, which can be misinterpreted as ‘70’ in symbolic form rather than ‘17’, with ‘seventy’ sounding very familiar but written symbolically as ‘70’ (p. 34). A range of empirical studies comparing young children’s counting skills in different numeration systems have noted that while in English, children have to learn the number names to 20 by rote, notions of structure are already in the foreground to a much greater extent in the 1–20 number range in languages where this structure is more explicitly regular (e.g. Miller & Stigler, 1987).

The classroom interaction excerpt that I present in this chapter occurred in a Sepedi-medium classroom. Sepedi number names in the 1–30 range are given in Table 8.1, with this structure maintaining its regularity up to 99:

The number names from 1 to 10 in Sepedi, as in English and Japanese, are arbitrary, and have to be rote learned. Beyond 10 though, a regular structure is established with tens and units demarcated in a highly predictable, patterned structure. In this structure, 11 is 'lesome tee'—which can be literally translated as 'one ten one'—and 12 is 'lesome pedi' or 'one ten two', and so on. The word for 20 is 'masome pedi', which translates literally in sequence to 'tens two'. The prefix 'ma' before 'pedi' marks that there are a 'plural' number of tens, with the subsequent 'pedi' marking that there are two tens. This patterning continues across the other 'tens' number names, which allows for the construction or reconstruction of these number names based on knowing the first ten number names, rather than increasing the burden on memory for these number names as is the case in English. An arbitrary new term has to be remembered for '100'—'lekgolo', but subsequently, the patterned structure comes into play again. Thus, 236 is 'makgolo pedi masome tharo tshela'—literally 'hundreds two tens three six'. The construction of number names in Sepedi therefore overtly reflects the place value of numerals in terms of the number of hundreds/tens/units used to form the symbolic number.

While there are parallels here with the explicitness of place value in the Japanese language structure pointed to earlier, the differences in the language are also of interest; for example, the fact that the number of tens/hundreds/etc. is conventionally positioned *after* the place value, rather than *before*. Lack of awareness of this convention can lead to confusion about whether the 1–9 number names relate to the preceding or following positional place value. This convention of stating the number of hundreds/tens/etc. after the place value also leads to potential dissonance between oral and written symbolic form. In Japanese, there is consistency between the ways in which number names are read and symbolically represented. For instance, 10 is 'ju' and 4 is 'shi' and 14 is 'jushi'. In Sepedi, there is no consistency between the ways in which number names are read

Table 8.1 Number names in Sepedi

1	2	3	4	5	6	7	8	9	10
tee	pedi	tharo	nne	hlano	tshela	supa	seswai	senyane	lesome
11	12	13	14	15	16	17	18	19	20
lesome tee	lesome pedi	lesome tharo	lesome nne	lesome hlano	lesome tshela	lesome supa	lesome seswai	lesome senyane	masome pedi
21	22	23	24	25	26	27	28	29	30
masome pedi tee	masome pedi pedi	masome pedi tharo	masome pedi nne	masome pedi hlano	masome pedi tshela	masome pedi supa	masome pedi seswai	masome pedi senyane	masome tharo

and symbolically represented. For instance, number 65 is verbally stated as 'tens six five', but symbolically represented as 65. This symbolic form 'ignores' the 'tens' that is said.

Also worth noting is that the oral language history of most South African languages means that the written word names shown above, using the Latin alphabet characters, result in long written word names for two- and three-digit numbers. In summary, the literature suggests that the morphological construction of number names in languages such as Sepedi can support learners to understand the place value-based composition of numbers.

Fuson and Briars (1990) have emphasized the importance of verbal support for learning the early number names and the structure and conventions of numeration systems. This evidence leads into the theoretical position taken in this chapter that while language as a resource can support the development of place value understandings in its own right, the conventional aspects of language usually require explicit teaching in order to draw visible attention to the conventions in ways that then allow for more invisible seeing through the language to the underlying mathematical structure.

Language as a Transparent Resource in Mathematics Learning

Resources play an important role in supporting learners to access knowledge and gain understanding of how a practice works. The work of Lave and Wenger (1991) is instrumental in understanding the transparent function of resources in a practice. These authors argue that for newcomers to become full members of a community of practice, they need access to resources central to the practice. Access to, and engagement with, resources can vary depending on how a resource is used in a community of practice. Lave and Wenger propose that resources have a transparent nature involving two functions of being visible or invisible (described above). While Lave and Wenger did not refer to language as a resource, Adler (1999) extended the notion of resources to include human and cultural resources such as language. In this chapter, language is understood as a transparent resource that teachers can draw on to support learning in their practice of teaching. As introduced earlier, language as a resource is transparent when the interaction with this resource allows learners to focus on the language of mathematics as well as the mathematics embedded in a task. Language can be a visible resource when it is explicitly used and is clearly seen by all. Language can also assume an invisible function when it becomes implicit and its use does not interfere with children's mathematical learning. Adler's findings pointed to dangers of imbalance due to explicit language teaching with limited attention to mathematics. In such contexts, the use of language as a resource becomes problematic because explicit language teaching disrupts mathematical learning. She argues that when the explicit language teaching interferes with mathematics, teachers need to make critical decisions to shift focus from using language explicitly to focusing on the mathematical task at hand. In this context, the use of language as a resource assumes the invisibility function. Adler warns that it often becomes a challenge for teachers to judge when the shift from using language explicitly to using this resource implicitly is required. The episode below is used to illuminate how a Grade 3 Sepedi mathematics teacher uses language as a resource to support children to learn early number and how this use requires both explicitness and transparency of language.

Data and Analysis

In this chapter, I present and analyse an excerpt from teacher Mpho's Sepedi-medium Grade 3 mathematics classroom using the three aspects identified from the chapter's theoretical framing. In the analysis of the excerpt, I attend, in various sections, to what teacher and learner utterances suggest about the explicitness of support offered in the language, as well as the transparency of language as a resource seen in the teaching. Teacher Mpho's classroom comprised 42 children. The classroom was located in a township school in Johannesburg. Though the majority of children in this class (31) spoke Sepedi at home, at least 11 of the children in this classroom spoke Sesotho, Setswana or IsiZulu as their home language, exemplifying the multilingual learning contexts that are common in many South African urban settings (Evans & Cleghorn, 2014). In this excerpt, the teacher wanted children to add 10 to 542 and to later subtract 10 from the answer. Children offered incorrect number names in Sepedi as responses. In the analysis that follows the data extracts, I associate the children's move towards correct answers to affordances provided in the explicitness of the language and to transparency-related uses of language by the teacher.

Excerpts and Analysis

Sepedi	English
Teacher: Re tswela pele. (Points to 542 on the table she had drawn on the board). Ke nomoro mang e?	**Teacher**: We are moving to the next one. (Points to 542 on the table she had drawn on the board). What number is this?

	+10	–10
542		

Sepedi	English
Children: Makgolo hlano masome nne pedi. (Children respond in choral rhythm)	**Children**: Hundreds five tens four two. (542). (Children respond in choral rhythm)

Several children were able to ‘say’ the three-digit number written in symbolic form on the board. This suggests appropriation of the Sepedi number names and conventions.

Teacher: Makgolo hlano masome nne pedi. Re ilo hlakantsha le lesome. Hlakantsha lesome mo nomorong e. Beya tee mo (points under the 4 in 542). Re baya lefela mo (points under the 2 in 542). (No response). E tlo ba eng? Hlakantsha le lesome. **Child:** Masome hlano nne tee	**Teacher:** Hundreds five tens four two. We are going to add ten to this number. We put a one here (points under the 4 in 542). We put a zero here (points under the 2 in 542). (No response). What will it be? We add one ten. **Child**: (Literally) Tens five four one.

There is no explicit attention to the language here; the teacher’s focus is on producing the answer to the ‘add ten’ task that she has set. This points to the ‘invisibility’ functions of language as a resource. This child’s response shows that in this additive situation, neither the explicit place value structure of the language nor the conventions associated with Sepedi number names have been taken up in this case, and the teacher’s task-setting actions have not focused explicitly on linguistic resources either.

Teacher: Heeee! Masome hlano nne teye! (Teacher looks surprised.) Ga o nale le rona. Makgolo hlano masome nne pedi hlakantsha le lesome. E tlo re fa eng? Child: Makgolo hlano masome nne pedi supa lesome	**Teacher:** Heeee! Tens five four one. (Teacher looks surprised.) You are not with us. Hundreds five tens four two plus ten. What will it give us? Child: hundreds five tens four two seven ten

Once again, the language assumes invisibility function, as the language and conventions associated with number names and place value are not in focus.

Teacher: (Calling on another child to give his response), E tla re fa eng? **Child**: Makgolo pedi nne (204)	**Teacher:** (Calling on another child), What will we get? **Child**: hundreds two four (204)

Language continues to assume the invisibility function in the next excerpt that is excluded here. In this instance, the child offered 204 as a response. The child's response is a recognizable number here, unlike the last offer, but still incorrect.

Teacher: Re bua ka makgolo hlano hlakanya le lesome. Re fe karabo. (She calls a few children but gets no response) **Child:** Tshela tee **Teacher**: No, tshela tee, tshela teye (The teacher calls another child to respond) **Child**: Makgolo hlano masome hlano tee (551) **Teacher**: Bona! Boeletsa gape! **Child**: Makgolo hlano masome hlano tee (551) **Teacher**: (Just stares at the child and the child changes his answer) **Child**: Makgolo hlano masome hlano pedi. (552) **Teacher**: Go nale tee mo? (Pointing at 552) **Children**: Ke pedi	**Teacher:** We are talking about hundreds five plus ten. Give us the answer. (She calls a few children but gets no response) **Child**: Six one **Teacher**: No, six one, six one (The teacher calls another child to respond) **Child**: Hundreds five tens five one (551) **Teacher**: Look. Can you repeat? **Child:** Hundreds five tens five one (551) **Teacher**: (Just stares at the child and the child changes his answer) **Child**: Hundreds five tens five two (552) **Teacher**: Is there one here? Go nale tee mo? (Pointing at 552) **Children**: It is a two

In the context of more incorrect answers, the invisibility function of language continues to be at the fore, but at the end of this I-R segment, a correct answer is offered. A possibility here is that the explicit place value structure within the language has helped the last child to produce the correct answer. The child offered a recognizable number (552) as a response though the response is still incorrect.

Teacher: Re tswelapele. Re thomile mo go makgolo hlano masome hlano pedi. (Writes 552 on the board). Ye,ke karabo y arena, hne! Re ntsha lesome. Re ntsha lesome. Tlogelang abacus. Le seka la tshwara abacus. Lebelelang kwa pele. Ntshang lesome bjale. (She called on a few children to answer but they kept quiet). Re ntsha lesome mo. (pointing to the number symbol 552 on the board). Ntsha lesome. Karabo ke eng? (Children continued to look at the teacher). Re ntsha lesome **Child**: Makgolo hlano masome hlano tee. (551) **Teacher**: (Calls on another child to offer a response). Re ntsha lesome go tswa mo makgolo hlano masome hlano pedi. Re ntsha lesome. **Child**: Makgolo hlano masome nne--- **Teacher**: Good people, re boela morago. Ga re ntsha re ya morago. (points to 542 on the table she had drawn on the board). **Child:** Makgolo hlano masome nne pedi. **Teacher**: Makgolo hlano masome nne pedi. Karaba ya tshwana. Bjwale re hlakanya le lekgolo. Karabo ke eng? Lebelelang kwa pele. Lebelelang kwa pele (She urges children to look at the chalkboard) Child: Makgolo tshela masome nne pedi Teacher: Makgolo hlano masome nne pedi. Karaba ya tshwana	**Teacher:** We are moving now. We started at hundreds five tens five two (writes 552 on the board). This is our answer, hne! We take away ten. We take away ten. Do not touch your abacus. Leave the abacus. Face the front. Now we take away ten. (She called on a few children to answer but they kept quiet). We are taking away ten here. (pointing to the number symbol 552 on the board) Take away ten. What is the answer? (Children continued to look at the teacher). We are taking away ten **Child**: hundreds five tens five one. (551) **Teacher:** (Called on another child to offer a response). We are taking away ten from hundreds five tens five two. We are taking away ten. **Child**: Hundreds five tens four **Teacher:** Good people, we are moving back now. When we take away we move back. (Points to 542 on the table she had drawn on the board). **Child**: hundreds five tens four two **Teacher**: hundreds five tens four two. The answer is the same. Now face the front. Face the front. (She urged children to look at the chalkboard) **Child**: hundreds five tens four two **Teacher**: hundreds five tens four two. Our answer is the same

In this section of the task, the teacher wanted learners to take way ten from 552 which was symbolically written on the board. The instruction is repeated several times and pointing is used to show the number been referred to. Language continues to assume the invisibility function where this resource is used to focus children's attention on getting the answer. A child offers a recognizable number as a response though the response is incorrect. Language is not used to probe the incorrect response. Here language is used to guide children on the mathematical action the task requires. A child offers the correct response. The correct response is repeated. The teacher comments that the answer is the same. Language is not used to explain why the answer is correct or incorrect and why the answer is the same. Language continues to assume the invisibility function.

Teacher: Bjwale re hlakantsha le lekgolo. Karabo ke eng? Lebelelang kwa pele. Lebelelang kwa pele (She urged children to look at the chalkboard) **Child**: Makgolo tshela masome nne pedi **Teacher**: Makgolo tshela masome nne pedi. O hlakantshitse le eng? (points to the board where she wrote 542 moving her finger under this number and wrote 100 underneath 542 100 O hlakantshitse 5 le 1, 4 le lefela, 2 le lefela go bona eng? Children: Pedi **Teacher:** Re a thoma, re ntsha lekgolo mo. (Points to 642). Karabo ke eng? **Child**: Makgolo hlano masome nne pedi. **Teacher:** Makgolo hlano masome nne pedi. (Writes 542 on the side on the board not on the table she originally made). La ntlhaloganya nha? Re tswela pele. (Points to 326). Children: Yes	**Teacher:** Now we add a hundred. What is the answer? Look at the front, look at the front. (She urged children to look at the chalkboard) **Child:** hundreds six tens four two. **Teacher**: hundreds six tens four two. What did he add? (Points to the board where she wrote 542 and wrote 100 underneath 542. 100 She added 5 and 1, 4 and zero and 2 and zero to get what? Children: two **Teacher:** We start and take away a hundred here. (Points to 642). What is the answer? **Child**: hundreds five tens four two Teacher: hundreds five tens four two (Writes 542 on the side on the board not on the table she originally made). Do you understand me now, we are moving on (points to 326). Children: Yes

The task requires children to add a hundred to 542. A child offers a correct response. The teacher repeats the response and asks how the child added the two numbers. She then offers her own explanation of how she thinks the child arrived at the answer. Children are required to take away 100 from 642. A correct response is offered. The teacher repeats the response and writes the answer on the board. Language is not used to find out how the child arrived at the answer nor is language used to explain why the answer is correct or incorrect.

Concluding Comments

Several issues of interest arise in the analytical commentary presented above. First, while several children appear able to say the number names of given three-digit numbers, this competence does not broadly extend to the context of adding ten to the given number. In relation to transparency, and in contrast to Adler's (1999) findings, there is exclusive attention to the invisible function of language—the need to produce the answer when adding ten is at the centre of focus. Within this invisible function, there is limited attention to representations in the teacher's offers, beyond suggesting where the '1' and '0' should be written when adding these two numbers together. The language she uses: 'Put the one under the four and the zero under the two' reduces the value of the number ten to one and zero. In pointing attention to the 'digits' rather than the place value/quantity representations, an aspect that is explicit in the Sepedi number names is not brought into play—either visibly or invisibly. Incorrect answers are rejected, but the violations of Sepedi number name conventions in these offers are not pointed to or explained.

In spite of the absence of teacher utterances making use of the visible function of the Sepedi language of number names, some learners are able to offer the correct answer, suggesting that they have achieved some appropriation of the explicit structuring available in the language.

While the emphasis in teacher working in this excerpt is on the invisibility function of language rather than on the visibility function that featured in Adler's (1999) study, the concerns and argument she makes about the need for both functions continue to hold. In many ways, given the explicitness of place value in the language structure, achieving this balance would seem to be even more important and easier to capitalize on.

In terms of Foundation Phase mathematics teacher development, this analysis suggests the need to support teachers with using number name language and place value ideas with more attention to the visibility function. This theoretical framing and analysis points to possibilities for using Dienes block-type representations to provide and connect with physical and/or diagrammatic representations of number.

Adding ten with this kind of representation would allow a physical addition to be mapped back to a linguistic transformation, with the 'tens four' needing to shift to a 'tens five' and other components remaining unchanged. Here, the explicitness of place value structure in Sepedi is recruited and bolstered by a balance between visibility and invisibility of language in the teaching. My sense is that broader opportunities for learning about place value structures can be enabled through this kind of approach.

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CHAPTER 9

Learners Exemplifying for Themselves: Grade 2's Telling Additive Relations Stories

Nicky Roberts

Introduction

Problems relating to lack of sense making in South African primary mathematics classrooms are well documented (Fleisch, 2008; Hoadley, 2012; Schollar, 2008). The complexity of the South African language context is considered to be compounding this and particularly as evident in poor attainment in word problems (Ensor et al., 2002; Schollar, 2001; Sepeng, 2014)

Poor attainment in solving additive relations word problems was found to be a particular problem for Foundation Phase learners (Grades 1–3) in the disadvantaged 'township' school in the Western Cape in South Africa with which I had a working relationship. This was evident in the analysis of Annual National Assessment (ANA) data and confirmed in basic number sense assessment developed by Brombacher and Associates (Brombacher & Associates,, 2015), which are based on the Early Grade Mathematics Assessment (EGMA) developed by RTI International. The majority of learners in this school would be what Brown (2005) would define as 'English Language Learners' (ELLs), learners learning mathematics in

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English when English is not their main language. Poor attainment in word problems by the majority of learners suggested that this was a teaching problem, giving rise to questions about how such ELLs might be better supported in making sense of word problems.

This small exploratory study is set within a broader design experiment that focused on a 'narrative approach' (described in more detail below, summarised in Fig. 9.1) to teaching additive relations word problems. Three intervention cycles were conducted over 10 consecutive mathematics lessons with 3 Foundation phase classes, all in the same focal school. In each of the intervention cycles, I was a teacher-researcher, leading the teaching with the normal classroom teacher present as an observer. This chapter reports on the last lesson in the third cycle intervention where a class of 28 Grade 2 learners were engaged in a 'storytelling task'.

The design of this task drew on theoretical approaches to mathematics learning and teaching relating to example spaces and the mathematical domain of additive relations.

I used Watson and Mason's (2005) concept of an *example space*, where any example in mathematics is seen as a member of a set of examples, as 'examples are usually not isolated; rather they are perceived as instances

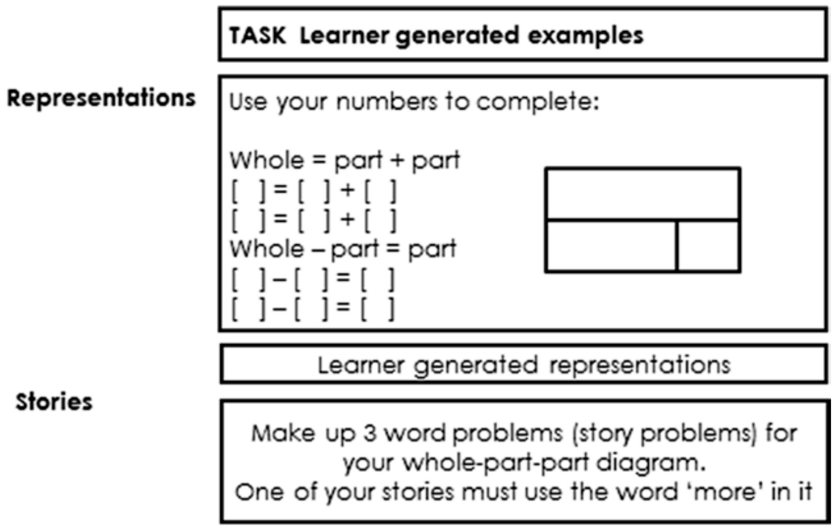


Fig. 9.1 Storytelling task (cycle 3)

or classes of potential examples' (p. 51). Watson and Mason distinguish several kinds of example spaces: a *local personal example space* which may be triggered by current tasks in the environment or recent experiences; a *personal potential example space* from which a local personal example space is drawn and consists of one person's past experience; a *conventional example space* which is understood by mathematicians, displayed in textbooks and into which a teacher hopes to induct her students; and a *collective example space* located in a particular classroom at a particular time (2005, p. 76).

I followed Mason's distinction between 'task' and 'activity': 'The purpose of a task is to initiate activity by a learner. In an activity learners construct and act upon objects whether physical, mental or symbolic that pertain to the mathematical topic' (2004, p. 4). Distinguishing tasks from activities allows attention to be paid separately to the design of a task (teaching side), and to the learner activity (learning side) resulting from the task.

Taking example spaces as worthy of teacher and learner attention, it followed then that initiating activity requiring learners to reflect on their own personal example spaces was a valuable undertaking. To do so may involve the teacher initiating tasks where the learner activity is the 'generation of examples of questions, techniques, actions, notations and mathematical objects' (Watson & Mason, 2005, p. 24). The phrase 'learners generating examples' captures the focus on engaging learners in a process of exemplification for a particular topic or concept. In this chapter, I approach such learner activity from the teaching side. I seek to demonstrate how the collective example space can be used as a means for reflecting on learning of the whole class which can then inform next teaching moves.

The Study Purpose

In the context of a Grade 2 class, the research involved supporting ELL to expand their example space for additive relations word problems, and asked the following questions:

- Were the majority of learners able to complete the storytelling task as expected?
- What does mapping the stories of the collective example space against the conventional example space suggest for future teacher action?

- When examining the three stories generated by an individual learner, what may be inferred about the dimensions of possible variation within their personal example space?

The storytelling task was set and used as a diagnostic tool where learner activity was used to inform next moves in teaching.

Theoretical Framework

I now briefly describe the theoretical features of my approach to mathematics learning and additive relations word problems relevant to the storytelling task.

Theoretical Feature 1: Example Spaces Are Objects Worthy of Attention

The term 'examples' is used in 'a very broad way to stand for anything from which a learner might generalise' (Watson & Mason, 2005, p. 3). This definition makes explicit that to be an example such an object must be an example of something representing a broader collection of similar objects. So, while a task is 'anything a learner might be asked to do' (in mathematics), an example is an object which has an explicit purpose: it is anything from which a learner might generalise (Mason, 2004, p. 4). The definition of 'task' focuses on the action (behaviour) which the learner is expected to perform, while the definition of 'example' focuses on the requirement to generalise. Watson and Mason (2005) also classify different types of examples according to their use in a particular context, referring to 'reference examples' as standard cases that are widely applicable and can be linked to several concepts and results (Watson & Mason, 2005, p. 64).

Theoretical Feature 2: Learners Are Able to Generate Examples for Themselves

Following Mason (2007), my view on learning mathematics sees mathematics as being part of human activity where our innate powers to discriminate, select and generalise are used naturally in our engagement with our world. These powers include imagining and expressing, focusing and de-focusing (discerning), specialising and generalising, conjecturing and convincing, classifying and characterising (Mason, 2007).

The value in such tasks stems from an orienting framework, where attending to thinking is in focus. Learners are expected to *solve* word problems in standardised assessments, but Watson and Mason (2005, p. 24) claim that 'making up examples that need particular techniques to solve them can focus learners on the mathematical structures that relate to those techniques' (p. 23).

For Watson & Mason (2005), gaining insight into learners' personal potential example space may be attained using various strategies for prompting learners to generate examples. Their list of strategies includes 13 related options. I have chosen to focus on only two of these options for this study, namely, 'make up an example' and 'make up an example with some constraints', and the wordings of the tasks for each of these I typically used in the intervention were 'Make up an example of a word problem or tell a story' or 'Make up an example of a word problem that needs the number sentence 8 – 5 = …' or 'Tell a story that uses the numbers 8 and 5 and includes the words "more" and "than" in it'.

Watson and Mason (2005) see learners' generating examples as populating a set of exemplars which can then be the focus of attention. In other words, the set of examples become the object of study, rather than each member of the set being considered in isolation. Consideration is given to what remains invariant across the personally generated examples, and what changes. Attention is also paid to how the variation is constrained, in terms both of the implicit 'dimensions of possible variation' and the related 'range of permissible change'. In this way a shift from specialising (make a particular example) to generalising (think about the set of generated examples and their characteristics) is facilitated. A fundamental belief within this approach to learning mathematics is that 'learners are able to exemplify for themselves and that to do so contributes to their learning' (Watson & Mason, 2005).

To relate the idea of learners' generating examples to additive relations at Foundation Phase level, I offer this illustrative example of a task: 'Think of a word problem where we would need to use the calculation: 8 – 5 = …'. The word problem that first comes to mind is likely to have some personal association with the subtraction calculation, perhaps one remembered from school or recently worked on. Generating the first instance of an example is usually easy. Asking for another example, and another, and perhaps another example, and then reflecting on what is remaining invariant while other elements are changed, helps to reveal your personal potential example space of word problems requiring '8 – 5 = ….' calculations. As mathematics is recognised to be a social activity, comparing personal example spaces with others in a community of mathematics practice further enriches this process.

It should be noted that Watson and Mason's work in mathematics classroom has been predominantly at secondary, tertiary and teacher education levels. As such in the present study their theoretical framing of learners' generating examples (Theoretical Feature 2 above) was approached from the teaching side. As the teacher-researcher, I realised that it was unlikely that within a short intervention period Grade 2 learners would be able to apply these techniques to their own personal example spaces. Rather, I focused on the teacher side of how reflecting on the learner activity on exemplifying word problems from the whole class could be used to inform my teaching.

Situating this theory in its local context is especially important, as, prior to the intervention, the belief that learners are able to exemplify for themselves was not shared by colleagues working with me, in this and similar primary schools. A number of colleagues expressed the view that the ELLs they taught would not be able to exemplify word problems in English. At times, it seemed their English-language skills were seen to be a barrier to exemplification; at others, concerns relating to learners' ill-discipline and/or 'special needs' status were identified as factors mitigating against this approach to mathematics learning. Hence the viability of expecting learners to generate their own examples was being researched in this particular local context.

Theoretical Feature 3: The Conventional Additive Relations Word Problems Example Space Includes Change-, Collection- and Compare-Type Problems, All of Which Share a General Whole-Part-Part Structure

Learning in this study was considered in terms of expansions in the personal example spaces of learners for additive relation word problems. I used the following problem types, defined in the mathematics literature as specifying the conventional example space (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Clements & Sarama, 2009):

Change-increase problems (*type 1*) refer to word problems where there is an action of joining that increases the number in a set. For example, 'I have 5 apples. I get 3 more apples. How many apples do I have now?'

Change-decrease problems (*type 2*) refer to word problems where there is an action of separating which decreases the number in a set. For example, 'I have 8 apples. I eat 3 of them. How many are left?'

Collections problems (*type 3*) refer to word problems where two parts make a whole but there is no action. The situation is static. For example, 'I have 8 apples. 3 are red. The rest are green. How many are green?'

Compare problems (*type 4*) are those where the numbers of objects in two disjoined sets are compared. For example, 'I have 8 apples. You have 3 apples. How many more apples do I have than you?'

Subtypes are defined in Table 9.1 within these four categories of problem types which consider the position of the unknown value in any problem (Clements & Sarama, 2009, p. 62):

Within particular problem types, the difficulty of the word problems depends on the position of the unknown in problem situation. For example, within the change (increase) category, *result-unknown* problems such as 'I have 8 sweets. I get 3 more. How many do I have now?' have been found to be easier than *start-unknown* problems such as 'I have some sweets, I get 3 more now. Now I have 11. How many did I have to start with?' However, it is acknowledged that variations in the wording of the problems, and in the situations they depict, can influence the difficulty of a particular word problem (Carpenter et al., 1999).

Another finding drawn from the additive relations literature offers a generalisation across these word problem types (Fig. 9.2), as all additive relations have a general whole-part-part structure which can be expressed

Table 9.1 Subtypes of additive relations word problems

Problem type	*General number sentence*	*Subtype 1*	*Subtype 2*	*Subtype 3*
Change increase	Start + change = result	Start unknown	Change unknown	Result unknown
Change decrease	Start – change = result	Start unknown	Change unknown	Result unknown
Collection	Part + part = whole	Whole unknown	Part unknown	
Compare	Referent ± difference = whole	Referent unknown	Difference unknown	Whole unknown

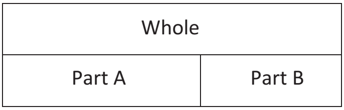


Fig. 9.2 General structure of additive relations (whole-part-part diagram)

Part A + Part B = Whole

Part B + Part A = Whole

Whole – Part A = Part B

Whole – Part B = Part A

Fig. 9.3 Family of equivalent number sentences for any additive relation

using a whole-part diagram and a related family of equivalent number sentences (Fig. 9.3) (Anghileri, 2000).

In the sequence of tasks used in the intervention, the whole-part-part structure was introduced, and then reference word problem examples were selected, sequenced and carefully designed as tasks which engaged the learners in problem solving, explaining and posing.

Theoretical Feature 4: A Narrative Approach to Additive Relations Word Problems Was Adopted

Teaching in this design experiment made use of a 'narrative approach' where expanding example spaces with regard to additive relations word problems was the overarching learning objective. For more details on a narrative approach, see Roberts and Stylianidess (2013). While the efficacy of this narrative approach is examined in the broader design experiment, for the purpose of this study it serves only as background to the teaching context. In the intervention design, I introduced different versions of change-decrease and compare problems figured as reference examples, with the whole-part-part structure and family of equivalent number sentences used as common features to both of them. I had assessed the change-increase problem type and found that most learners could solve these problems. The collection problem type was included in the intervention design, not as a reference example, but rather as a task to be completed independently by learners to assess transfer (the ability to apply the common whole-part structure to a previously unseen problem type).

Methods

As noted earlier, this chapter draws together mathematics literature (relating to examples spaces and learners' generating examples, as well as the conventional example space of additive relations word problems) with an empirical base of 28 Grade 2 learners telling additive relations stories in the last lessons of the third intervention cycle of a design experiment.

Cycle 3 was conducted in the first half of the Grade 2 academic year in April 2014. The class comprised 18 boys and 10 girls of mixed ability. Only two learners had English as their main language. There were 13 learners (47 %) who had repeated Grade 1 or Grade 2 in previous years, 7 of whom had been assessed as having substantial special educational needs[1] resulting in applications for specialist school placements.[2]

The empirical data in this study was gathered via video recording of lessons. Ethical approval (based on the principles of voluntary and informed consent for research participation for the study) was obtained from the University of Witwatersrand, the Western Cape Provincial Department of Education, and the school's principal, the class teachers, the learners and their parents.

The storytelling task comprised three elements: *Element 1*—the whole-part-part diagram; *Element 2*—a family of related number sentence; and *Element 3*—the three stories to be told by learners. Learners were expected to use a number triple to draw a whole-part-part diagram and write the related family of number sentences. The storytelling aspect used 'Make up an example with some constraints'. Two constraints were imposed: learners were given a number triple with which to work and were expected to use this same triple in all of their stories; learners were expected to use the word 'more' in one of their stories. This task was individualised, as each learner in the class worked with a unique number triple provided either by the teacher or self-selected.

As the 'Make up a story' instruction (*Element 3*) made reference to the whole-part-part diagram (*Element 1*), whether learners were connecting their whole-part-part diagram to their stories by using the same number triple in both was of interest. Examining *Element 1*, the 28 learners were first categorised into learners who were able to draw a correct whole-part-part diagram (25 learners) and those who were not able to do this (3 learners). A correct whole-part-part diagram meant that they drew a whole-part-part diagram, positioned the biggest number as the whole with two smaller numbers as the parts, thereby satisfying the relationship 'whole = part + part'.

The 25 learners with correct whole-part-part diagrams were then categorised into learners who had written at least one coherent story using the number triple from the whole-part-part diagram (22 learners), and those who had not (3 learners). A coherent story was defined as a story that made use of an additive relation where the same number triple used in the whole-part-part diagram satisfied the relationship 'whole = part + part'. Some learners generated stories where all three numbers in the number triple were used to express an additive relation in a narrative form such as 'I have 5 stickers. I found 3 stickers. I have 8 stickers' (for the number triple 8-5-3). In most cases, learners used two numbers from the number triple to express an additive relation and pose a question in order to find the third number. For example, 'I have 9 apples. I eat 2. How many I have?'[3] (for the number triple 9-7-2). Both versions of the word problem story—either with or without a question—were included for analysis. Across the 22 learners, 57 coherent stories were generated, and it was this set of 57 stories that constituted the empirical data for the study.

Finally, the learner activity on the storytelling component of this task was considered for each individual learner. In this regard, the extent to which a learner was generating stories that were all within the same problem type (or the same subtype) or spanned across problem types was of interest. It was conjectured that learners who could bring to mind different types of word problems within their three stories may have a wider range of reference examples in their personal example space than learners who generated all three of their stories using the same problem type.

Results

To demonstrate how the collective example space can be used as a means for reflecting on learning of the whole class, I present an analysis of learner work on the task.

Were the majority of learners able to complete the task as expected? The majority of learners (22) were able to draw a correct whole-part-part diagram and generate at least one coherent word problem for the additive relation in this diagram. Amongst these 22 learners, 15 generated three coherent stories, 4 generated two, and 3 generated one. The task design was judged to be appropriate for this class as the majority of learners completed it as expected.

What does mapping the stories in the collective example space against the conventional example space suggest for future teacher action? The 57 connected and coherent stories were mapped as follows to the conventional problem types (Table 9.2):

Each connected and coherent story was then mapped further to the relevant subclasses. Tables 9.3, 9.4, 9.5 and 9.6 present the frequency of stories for each subtype. Up to four typical examples of the learner activity are included.

Table 9.2 Types of stories

Problem type	*Frequency*
Change (increase)	16
Change (decrease)	32
Collection	6
Compare	3
Total	57

Table 9.3 Subtypes for change-increase stories

Change increase	*Frequency*	*Examples*
Result unknown	3	I have 1 flower. I pick 4 more. How many do I have? I have 7 frogs I found 3 more frogs How many frogs do I have altogether? I have 7 lino[a] and my mom give me 2 how many I have?
Change unknown	10	I have 2 cats. My mom bring more. I have 6 cats now. How many cats my mom bring? I have 6 lions how many more lions do I nid to get 10 lions? I have 2 car my dad bring more I have 6 car now. How many car my Dad bring? I have 10 pencils my mommy gave more pencil I have 15 pencil. How many more do I have?
Change-increase situation but no question posed	3	I have 5 stickers. I found 3 stickers. I have 8 stickers. I have 6 books my mom bring 1 more. Do I have [incomplete] I have 6 books I need to have 4 more books

[a]Child possibly mis-spelt 'lion' as 'lino'

Table 9.4 Subclasses for change-decrease stories

Change decrease	*Frequency*	*Examples*
Result unknown	22	I have 9 snakes 5 ran away How many Do I have? I have 9 apples. I eat 2. How many I have? I have 15 ducks. I gave 10 ducks to my friend. How may ducks do I have? I have 4 butterflies 3 butterflies fly a Way how many do I have?
Result unknown, but 'more' is incorrectly inserted into the question	8	I have 6 bows 3 bows get lost. How many more bows do I have now? I have 10 bols. 8 balls bounced away. How many more do I have. I have 10 puzzle. 8 puzzle get lost. How many more do I have? I have 13 sweets. I eat 3 sweets. How many more sweets are left?
Change unknown	1	I have 13 friends. Some of my friends ran away. I have 3 friends left. How many more friends ran away?
Change-decrease situation but difficulty with posing question	1	I have 10 dog. 5 dog run away. dogs 5 left? Are left? How many are left?

Table 9.5 Collection stories

Collection	*Frequency*	*Examples*
Part unknown (contrasting two states)	2	I have 9 chalk. 5 brake. How many do I have? I have 14 pens 10 are red sum are Blue How many are Blue?
Part unknown (1 part missing)	4	I have 6 marbles and 4 marbles are missing. How many marbles are left? I have 6 bols and 4 bols were missing. How many are left? I have 18 tops. 8 are missing. How many do I have left? I have 6 cars. 4 cars were missing. How mane cars ore left?

Table 9.6 Compare stories

Compare	*Frequency*	*Examples*
Compare (difference unknown)	3	How many more than is 10 than 5? I have 9 pen and my friend have 7. How many more do I have Than my [incomplete] I have 16 pencils and my friend have 10 pencils. How many pencils do my friend need?

For the change (increase) problem type, examples of both result-unknown and change-unknown problems featured in the collective example space. No start-unknown problems were told, which suggested that a next teaching move would be to include change-increase (start-unknown) problems in problem-solving tasks. Difficulties with posing questions in English were evident as there were examples of change (increase) stories which did not include a question, or where there was evidence of difficulties when posing a question. This suggested that more tasks which supported learners to pose questions in English would be required.

The *change (decrease) result-unknown* problem type dominated the collective example space. Only one learner told a story of the *change (decrease) change-unknown* problem type. This suggests that teaching moves focused on *change (decrease) change-unknown* and *change (decrease) start-unknown* problems are required. Examining the stories of this problem type, it was clear that some learners attempted to fulfil the task constraint of using the word 'more' by inappropriately inserting the term into the change-decrease situation. This suggested that learners were not yet secure with using 'more' to denote an action of increasing in change-increase situations and to denote a comparison in compare situations. How to use 'more' appropriately required additional teacher intervention.

There were very few stories of the collection problem type. This problem type is defined as relating to a static situation. However, in classifying the stories, it was clear that there is potential overlap between 'change' problems and 'collection' problems. The change is expected to take place over time, and the duration of what is considered a reasonable time lapse is brought into question. The statement 'Something is missing from a set' describes a static situation in the present. However, if something is missing now, it may have been taken or lost or removed in the past. The significance of the change action in relation to a time frame was not obvious from definitions of the problem types. It was in considering whether to classify ‘I have 6 cars. 4 cars were missing. How many cars are left?’ as a change problem, or as a collection problem, that this aspect arose. For the purpose of this analysis, when an element of a set was described as missing, this was considered to be a statement relating to a static situation, and two attributes—missing and present—were being contrasted; as such, this was classified as a collection problem. These ‘missing part’ problems were separated into a subclass of their own within the collection stories.

There were very few collection stories generated in the collective example space (these had not figured as teacher-initiated reference examples in the intervention), suggesting that collection problems should be included in future teaching plans. All of the collection problems were of the part unknown subtype, and so teaching that supported engagement with collection (whole unknown) problems was suggested.

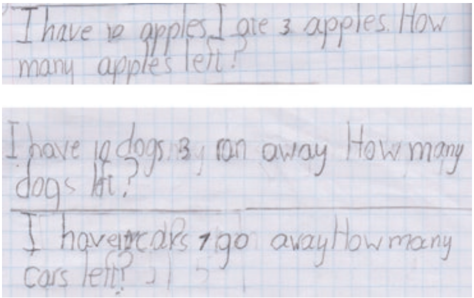
There were very few compare problem types, all of which were difference-unknown problem subtype. This suggested that additional time and experience working with compare-type problems were required.

When examining the three stories generated by an individual learner, what may be inferred about the dimensions of possible variation with their personal example space? While three stories are clearly not the full extent of a learners’ personal example space, creating all three examples within the same problem type may suggest that the chosen problem type is dominant for this learner (and that greater exposure to other reference examples may be necessary).

To demonstrate how the personal example space for each learner was inferred from their work on this storytelling task, I offer the following two vignettes of learner work: Retabile and Lydon (pseudonyms) (see Figs. 9.4 and 9.5).

Vignette 1: Retabile’s stories Retabile generated three stories for the same number triple (10-7-3), but adopted the same change (decrease) situation in all three of them:

As directed, Retabile kept the numbers invariant. She varied the characters in her story and the verbs relating to removal (apples being eaten, dogs running away and cars going away). Her question was kept invariant with the structure ‘How many “characters” left?’ She did not follow the instruction to make use of the word ‘more’ in one of her stories. It seems that for Retabile the change-decrease problem type was a key reference example. The word ‘problem type’ as defined in the conventional example space did not yet seem to be a possible dimension of variation

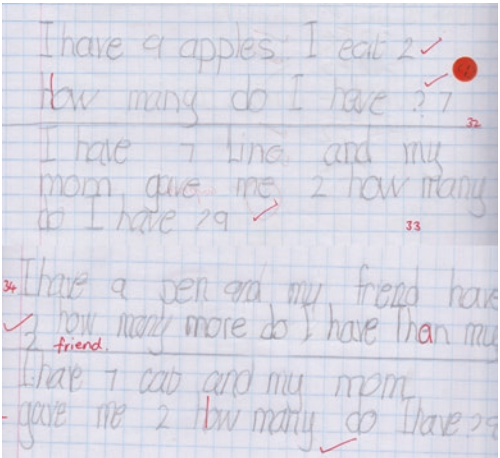


I have 10 apples. I ate 3 apples. How many apples left?

I have 10 dogs. 3 ran away. How many dogs left?

I have 10 cars. 7 go away. How many cars left?

Fig. 9.4 Retabile telling stories for 10-3-7



I have 9 apples. I eat 2. How many do I have?

I have 7 lino my mom give me 2. how many do I have?

I have 9 pen   and my friend have 7. How many more do I have Then my [incomplete]

I have 7 cat and my mom gave me 2 how many do I have?

Fig. 9.5 Lydon telling stories for 9-7-2

for her. She did vary the numbers slightly from the second story to the third, which may suggest awareness of the relationship that if 10 – 3 = 7, then 10 – 7 = 3.

Vignette 2: Lydon's stories

Lydon's three stories included change (increase) word problems alongside change (decrease) stories. Lydon worked with the number triple 9-7-2 and brought to mind two stories of change. The first related to apples being eaten, and the second to 'lino' (possible 'lion') being given.

His third story was in the compare category (although this was not completed) where he had 9 pens and his friend had 7. Lydon's fourth story was a repeat of his second story which was of the change (increase) type; however, he changed 'lino' to cat. When presented with a number triple, Lydon could bring to mind two different change contexts and position this alongside a compare situation. Lydon seems to have a simultaneous awareness of the two change situations (change increase and change decrease). The dimension of possible variation relating to problem types in Lydon's personal example space seemed to include change (increase), change (decrease) and compare problem types.

Concluding Remarks

Taking example spaces as objects worthy of attention (Theoretical Feature 1), it was possible to map the learners' activity against the conventional example space used to plan and reflect on task design. Some suggestions of possible dimensions of possible variation with the personal example space of a young learner could be inferred from three examples. These, however, remained at the level of conjecture as three examples is not sufficient to make secure judgements.

The examples of learner activity presented in this chapter demonstrate that the majority of Grade 2 ELLs (22 of the 28) in this class were able to generate examples for themselves (Theoretical Feature 2). For six of the learners, this task was not yet in reach, and difficulties with correctly completing a whole-part-part diagram and/or connecting additive relations stories to this were evident. The success, however, of 22 of these learners suggests that there may be potential in assigning such tasks to other ELLs of this age group.

The delineation of the conventional example space in relation to change, collection and compare word problems as well as the common whole-part-part structure (Theoretical Feature 3) was used centrally in the design and analysis of the intervention, the storytelling task, and this analysis. Two potential theoretical refinements emerge from the analysis of the learner data above: Potential overlaps between change situations and compare situations were evident when time as a continuum between a static (present) situation and situations in the past or future was brought into focus; and 'missing part' problems emerged as a potential subcategory of the collection-part-unknown problem type.

Discussing the mapping of the collective example space to the conventional example space pointed towards three possible next teaching moves.

Firstly, the six learners who could not yet engage in the task clearly needed additional support on additive relations and the common whole-part-part structure. Secondly, only a minority of learners brought collection and compare problems to mind, suggesting that more time engaging with solving these problem types may be beneficial so that they come to be shared reference examples. Thirdly, more attention needs to be paid to varying the position of the unknown in all of the problem types.

The learner activity in this study revealed two areas of difficulty for ELLs: appropriate use of the term 'more', and question posing. In terms of the former, there are two possible uses of the term 'more'—one to refer to an action in change (increase) situations and another for comparison in compare situations. To allow for flexible use of the term 'more', introducing the comparative language of 'more than' and focusing attention on the problem situations which invoke one-to-one matching actions may better support learning. In terms of the latter, these ELL learners seem to require additional support to fluently pose questions in English.

A further practical contribution of this study is that its storytelling task is explicitly positioned as a potential diagnostic tool which teachers can use to support the planning of next moves in their teaching of additive relations word problems. What is also clear more broadly though is that ELL learners in township school settings are able, with suitable pedagogic support of a narrative approach (Theoretical Feature 4), to create stories that link with given mathematical relationships and, through this, gain integrated access to mathematics and language skills. In the process, these stories provide teachers with access to their learners' current ways of thinking about additive relations. The data shared in this chapter suggests that supporting teachers to work with stories in early number learning may well be useful in further enhancing mathematical learning in the complex multilingual contexts that are the norm in many South African classrooms.

Notes

1. These learners were on waiting lists for transfer to specialist special needs schools. The special educational needs included ADHD, developmental delays as a result of foetal alcohol syndrome, as well as pre-natal exposure to crystal meth (Tik). The special needs are beyond the scope of this chapter, but mentioned as part of brief description of the local context.
2. Applications were made for two learners to immediately transfer to Blouvlei Special Education Facility (where there was a waiting list),

and statements were developed for five learners which enabled them to be placed on waiting lists for the School of Skills following primary schooling.

3. All examples are transcribed directly from what learners wrote. They have not been edited for spelling or grammar.

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CHAPTER 10

Do the Annual National Assessments in Mathematics Unfairly Assess English Language Competence at the Expense of Mathematical Competence?

Lucy Sibanda

Introduction

The study is situated within the South African literacy and numeracy context, where international studies such as TIMSS (Reddy et al., 2015), regional studies such as SACMEQ (see Spaull, 2011; Taylor, 2009), and Annual National Assessments (ANAs) (DBE, 2012, 2013) reveal the underperformance of learners in mathematics, confirming Fleisch's (2008) contention that primary education is in crisis especially in reading, language and mathematics. Additionally, the South African education landscape manifests a growing inequality in performance. While South Africa's 2003 TIMSS performance (which was the lowest of 50 countries) was a cause for concern, equally concerning was South Africa having the largest variation in scores with learners in African schools achieving half the scores of historically White schools. Furthermore, mathematics scores

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for African schools decreased significantly from TIMSS 1999 to TIMSS 2003 (Reddy, 2006), which was not the case for non-African schools, testimony to increasing inequality in mathematics performance.

The Department of Basic Education (2011) attributes South African learners' poor performance in numeracy benchmark tests to inadequate language capabilities. In South Africa, most learners learn mathematics in English, a language that is not their Home Language (HL). Further to that, most learners who perform poorly in high school mathematics rarely use English at home or come from homes where English is rarely used (Reddy et al., 2015) and is only heard, read, spoken and written in a formal school context (Setati & Adler, 2000, p. 251).

In 2011, the Department of Basic Education introduced ANAs to assess learners at Grades 1–6 and Grade 9 in all government schools as part of Foundations for Learning Campaign. The ANAs aim to expose teachers to better assessment practices, help districts to identify schools most needful of assistance, and inform parents about their children's performance (DBE, 2011). The pivotal role ANAs play requires that they inspire confidence in their fairness and validity as measures of learners' competence and performance. The test items need to be pegged at the right levels.

In the Foundation Phase (FP) (where use of HL in classrooms is encouraged in national language policy), ANAs are provided in learners' HLs as requested by schools, but in the Intermediate Phase (IP) (where the language policy demands a switch to either English or Afrikaans), ANAs are set in English or Afrikaans. The DBE website states that

> The tests are administered in all the eleven official languages in the FP and in the two languages of teaching and learning in the IP and Senior Phase. Necessary adaptations are effected for learners who experience various kinds of learning disabilities to ensure that every learner has the opportunity to demonstrate what they know and can do in the assessment. (DBE, 2014)

The point about learning disabilities and necessary adaptations is noteworthy. While multilingualism of most South African learners is an asset, the extent to which learners are disadvantaged by being assessed in English rather than in their HL is the focus of this study.

Results of the ANAs for the past three years are disconcerting. The 2012 and 2013 reports for the ANAs (DBE, 2012, 2013) reveal that learners performed poorly in mathematics across grades. In the FP, learners performed better but as they proceeded to IP, the levels of achievement decreased significantly. The 2013 Mathematics ANA results show a decrease from an average of 60% in Grade 1 to 14% in Grade 9. Important, however, is the large drop that occurs in results from Grade 3 to Grade 4 (from an average of 53% to 37% for 2013) (DBE, 2013). This begs the question of the role language plays in this slump in performance from the FP to IP. The teaching of First Additional Language (FAL) from Grade 1 was made compulsory in 2012 by the Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011). However, the 2014 Grade 3 and 4 learners (who participated in my study) did their Grade 1 and 2 under the National Curriculum Statement (NCS) dispensation when teaching in the FAL was not compulsory. If learners used isiXhosa (or other South African languages) as the language of learning and teaching (LoLT) in Grade 1 and 2 and started learning English as FAL only in Grade 3, then it would be naive to expect them to have already acquired the basic vocabulary and proficiency in English to warrant its use as LoLT in Grade 4.

The linguistic demands of the change in the LoLT that occurs for the majority of South African learners in Grade 4 (see Robertson & Graven, 2015) makes the analysis of the linguistic complexity of the Grade 4 ANAs imperative. In South Africa, Grade 4 is a critical stage where many learners experience four significant transitions from the FP. The first transition already identified is from using isiXhosa (in the Eastern Cape where the study is being done) to using English as LoLT. The second transition is from reading mostly narrative, story-like texts whose language closely approximates ordinary language of everyday social interaction in the FP, to reading expository texts with content-dense vocabulary (Chall, Jacobs, & Baldwin, 1990). The third transition is from 'learning to read' to 'reading to learn' (DoE, 2008) where, in the FP, learners are trying to develop the skill and art of reading but, when they come to Grade 4, they are expected to read different content subjects and learn from what they read. The mechanics of reading, which underpin learning to read, are supposedly developed in the HL in the FP and used in English in Grade 4 to access information from texts. The fourth transition is from more concrete thinking in the FP to more abstract thinking in the IP. Mathematical abstraction is particularly critical for progress in the IP.

Theoretical Perspective, Methods and Analytic Tools

The premise for the study is that language is central in the learning of mathematics. The study is framed by a sociocultural view of language and learning, with Vygotsky's (1976) influential work informing the theory of language and learning.

According to Bergqvist, Dyrvold and Osterholm (2012), mathematics is linguistic in nature because it has mathematical words, symbols, sentences and grammatical structures. These linguistic features describe mathematical concepts in a way which transcends everyday language. For Halliday (1975, p. 65), mathematics register is 'a set of meanings that belong to the language of mathematics and that a language must express if it is used for mathematical purposes.' According to Halliday (1993), the difficulty of mathematics also lies in the grammar of the language used and not just with the vocabulary. He identifies long phrases in questions, complex sentences, syntactic ambiguity, special mathematical expressions, lexical density, among others, as having tremendous effect on the performance of English as second language learners. The grammatical density of sentences engendered by the linguistic features described above present linguistic challenges that confound young learners.

If mathematical language has been found to be complex even for English HL speakers learning mathematics in English (Halliday, 1989), the level of complexity for second language learners can only be higher. According to Schleppegrell (2007, p. 140), 'Learning the language of a new discipline is part of learning the new discipline; in fact, the language and learning cannot be separated.' The implication is that weak language and mathematical foundations combine to effect overall underperformance. Abedi (2006) notes that when assessments have complex language, this negatively affects the performance of learners and the performance gap between English language learners (ELLs) and HL speakers of English is increased. For Abedi, assessments where the linguistic component engenders unwarranted complexity to the mathematical component are unfair and invalid. Abedi (2006) sees standardized achievement tests that take no consideration of the language proficiency of learners as invalid portrayal of what learners really know.

This chapter employs the linguistic complexity checklist by Shaftel, Belton-Kocher, Glasnapp and Poggio (2006) and a formula drawn from the linguistic complexity checklist by Vale (2013) as analytic tools for the content analysis of the 2013 Grade 4 mathematics test items. Shaftel et al. (2006) investigated the influence of the language characteristics of mathematics assessments given to Grade 4, 7 and 10 English language learners

(ELLs) in the USA. They analysed individual word problem test items in a multiple-choice format. The learners' performance was determined by the item difficulty and learner ability to correctly answer the question. Items were coded according to their linguistic complexity, taking into consideration the 'total number of words, sentences, and clauses in each item; syntactic features such as complex verbs, passive voice, pronoun use and vocabulary in terms of both mathematics vocabulary and ambiguous words' (Shaftel et al., 2006, p. 111).

Results from Shaftel et al.'s (2006) study revealed that the mathematical and linguistic features of the test items measured had an impact on learner performance, 'with a moderate-to-large effect at Grade 4, a medium effect at Grade 7, and a smaller effect at Grade 10' (p. 120). At Grade 4, prepositions, ambiguous words, complex verbs (verbs with three or more words), pronouns and mathematics vocabulary showed unique effects on item difficulty. The greater the number of linguistic elements per item, the more difficult the item. For this study, test items are defined as each item for which a learner got some marks.

The Linguistic Complexity Checklist Index (LCI) was developed from the linguistic complexity checklist by Vale (2013) meant to analyse the linguistic complexity of items at the basic level, word level, sentence level and paragraph level. It was represented by Shaftel et al. (2006) as follows:

A. Basic level: Number of words in an item
B. Word level: Items of seven words or more; Relative pronouns (e.g. that, whom, whose); Slang/ambiguous/multiple meaning or idiomatic words (e.g. change, set); Homophones (e.g. two/too, prize/price); Homonyms (e.g. there, their, they're); Specific mathematics vocabulary (e.g. pentagon, symmetry)
C. Sentence level: Prepositional phrases (e.g. beginning with, from, by, at); Infinitive verb phrases (to make, to sell); Pronouns (e.g. his, her, they); Passive voice (were sold, were rounded off); Complex verbs of three words or more (e.g. could have been); Complex sentences (e.g. with subject and predicate); Conditional constructions (e.g. if…then); Comparative constructions (e.g. less than, greater than)
D. Paragraph level: References to specific cultural events (e.g. picnic, funeral)

The Linguistic Complexity Index (LCI) is then calculated as:

$$\text{LCI} = (\text{Number of words} + \text{Sum B} + \text{Sum C} + \text{Sum D}) \div \text{Number of sentences}$$

This analysis enabled selection of ANA items with the greatest LCI and an analysis of learner performance across these high LCI items in three case study classrooms of isiXhosa HL learners for task-based interviews with learners. Thus selected learners' performance and interviews are reported in this chapter.

Methods of Data Collection

For the broader study, three Grade 4 case study classes from two schools in Grahamstown were selected to assess learners' understanding of mathematics ANA items. All the Grade 4 learners in each class, who were willing to participate and whose parents had agreed to their participation, constituted the sample.

Firstly, learners' ANA answer scripts for 2013 were analysed to establish their performance across questions and classes and to establish the difficulties they experienced in responding to the ANAs. Questions that posed the greatest challenge were identified (failed by more than 50% of the learners), and the nature of these difficulties was explored in subsequent learner interviews with mediation through translation or rephrasing. Analysis of the LCI on 2013 ANA questions revealed there were only 10 (out of 38) items with an LCI of less than 10. Some items with low LCI also had low performance (e.g. questions 12.1, 12.2 and 14). Learners also performed well in some of the high LCI items, (e.g. questions 16.1 and 17). Using both learners' results and the results from the LCI analysis across questions, 15 items had high LCI and low learner performance and so merited follow-up in the task-based interviews, which is the chapter's focus.

Having determined the ANA items and learner responses to these, a purposive sample of 26 learners (9 from each of the three classes—3 high performers, 3 middle and 3 low performers) was selected for task-based interviews (one learner from class B was not available for interview). These learners were interviewed using the Newman's error analysis interview questions. These probing questions reveal possible learner difficulties with reading, comprehension, transformation, process skills or encoding as they solve word problems. Probing and prompts enable learners to progress through the problem-solving stages in order to assess possible difficulties in the later stages of problem solving.

This chapter focuses on a subsample of both ANA interview items (5 out of 15) and learners (12 out of 26). The items were selected as those that were linguistically challenging as indicated by each item's LCI score above 20 and those in which learners performed particularly poorly (that is, most of the learners were unable to successfully answer the questions).

The 12 learners who were selected from the interview sample of 26 learners were those who showed the greatest improvement in the interviews and linguistically mediated interview questions. (That is, those learners who improved in at least four of the five selected questions.)

The ANA Test Items with the Highest LCI

The analysis of the linguistic features of the 2013 ANAs revealed that for each question analysed, a number of language features occurred and some appeared more frequently than others. These were arranged from the most frequently used to the least frequently used thus: words with seven or more letters (88), homophones (72), prepositional phrases (55), specific mathematical vocabulary (41), pronouns (26), ambiguous words (11), infinitive phrases (9), complex verbs/passive voice (3), conditional constructions (1) and references to cultural events (0). As noted before, the greater the total number of linguistic features, the more difficult the question was.

Table 10.1 shows, in descending order of complexity, the questions in the 2013 ANAs which had an LCI of 20 or more. The summary is in terms of the 11 word- and syntactic-level features that were the focus of analysis. The LCI formula, which is (Number of words + Sum B + Sum C + Sum D) ÷ Number of sentences, was applied.

The five ANA items in Table 10.1 that featured in learner interviews as a result of widespread poor learner performance on these items are given below (in the order of their appearance in the ANAs). Some items with a high LCI were not included in learner interviews because learners performed

Table 10.1 Summary of complexity levels of nine questions with an LCI of more than 20

Question	*No. of features present out of 11 types of features*	*Aggregate no. of features*	*Linguistic complexity index*
9.1	7	64	32
16.1	7	32	32
9.2	6	29	29
17	6	29	29
8	8	28	28
18.3	7	27	27
1.3	5	24	24
18.1	5	45	22.5
15.2	6	21	21

relatively well on these. So, for example, item 16.1, which has an LCI of 32, involved choosing from the words hexagon, pentagon, quadrilateral and triangle to name two drawn shapes (a triangle and a quadrilateral). This is a familiar naming activity prevalent in learner workbooks. Thus, despite the high LCI, most learners did not experience difficulty with it.

9 Look at the departures board at the airport and answer the questions that follow.

Departures		
Destination	Time	Flight Number
Mossel Bay	07:45	SAA 769
Knysna	10:20	BA 172
Johannesburg	20:00	SAA 372

9.1 Write down the flight number of a flight which will depart for its destination before midday.

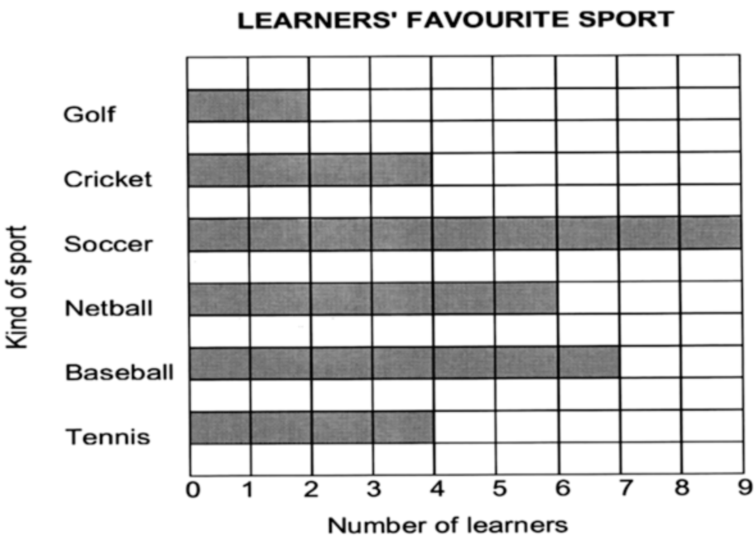
____

9.2 Write down the flight number of a flight which will depart for its destination after midday.

____

15.2 Colour in ¾ of a fraction strip in the fraction wall.

18. This bar graph shows the most popular kind of sport amongst the learners in Grade 4.



18.1 Complete the tally table.

Kind of sport	Tally marks
Golf	
Baseball	
Tennis	

18.3 What is the difference between the number of learners who prefer soccer to cricket?

Table 10.2 shows the 12 sampled learners' comparative performance between the written ANA format and the mediated oral interview format on the five selected ANA questions (with an LCI of 20 or more and in which learners performed poorly). Performance in interviews was aided by linguistic mediation through rewording of the question, explanation or translation of words or misunderstood questions during the task-based interviews. An 'X' indicates a learner's inability to solve the problem and a 'tick' (✓) indicates their ability to solve the problem with linguistic mediation. The learners are identified by their classes A, B or C and by their number in the initial sample selection.

The table shows that several learners, from each class, who could not successfully solve any of the five problems during the ANA test, could solve the same problems in the interviews with mediation as they could get over their 'breakdown' points and move on to the next stage of problem

Table 10.2 Learners' comparative performance in five 2013 ANAs written and task-based interview tasks with an LCI of more than 20

Learners	*Written performance*					*Interview performance*				
	9.1	*9.2*	*15.2*	*18.1*	*18.3*	*9.1*	*9.2*	*15.2*	*18.1*	*18.3*
A27	✓	X	X	✓	X	✓	✓	✓	✓	✓
A5	X	X	X	✓	X	✓	✓	✓	✓	✓
A23	X	X	X	X	X	✓	✓	✓	✓	✓
A28	X	X	X	X	X	✓	✓	✓	✓	✓
B13	X	X	X	X	X	✓	✓	✓	✓	✓
B11	✓	X	X	X	X	✓	✓	✓	✓	✓
B2	X	X	X	X	X	✓	✓	✓	✓	✓
C12	X	X	✓	X	X	✓	✓	✓	✓	✓
C33	X	X	X	X	X	✓	✓	✓	✓	✓
C21	X	X	X	X	X	✓	✓	✓	✓	✓
C24	X	X	✓	X	X	✓	✓	✓	✓	✓
C8	X	X	X	X	X	✓	✓	✓	X	✓

solving. See, for example, A23, A28, B2, B13, C21, C3 and C8. During the task-based interview, learners were asked to read the question and identify words they did not understand which would be explained to them so that they could proceed to tell what the question required them to do. Where they could not establish the demands of the question, these were explained and learners asked to solve the problem. Mediatory prompts were used at learners' 'breakdown points' in order to allow the learners to proceed on to demonstrate the subsequent skill.

Examples of a short extract from a learner interview with a boy named Tino (pseudonym) is given to illustrate the challenges of questions with a high LCI. Tino (B13), as all the other learners interviewed, is a first language isiXhosa-speaking learner. He had not managed any of the five interviewed questions in the written ANAs (as indicated in Table 10.2).

What follows is a brief extract showing linguistic mediation in the task-based interview on question 9, followed by Tino's managing to correctly answer the question.

Example 1: Learner B13 named Tino (pseudonym)

26	Researcher	……….Now, can you read the question?
27	Tino	Write down the flight number of the flight which would depet (*could not read the word*)
28	Researcher	Depart. Depart, you remember what it means?
29	Tino	(No response)
30	Researcher	It means 'Leave a place'
	Tino	Leave (*continues reading*) depart for it di… (*pauses, can't read the word)*
31	Researcher	Destination. It means where it is going
32	Tino	*(continues reading)* destination before myd …
33	Researcher	Midday
35	Researcher	Uhuh. Midday. Do you know what midday means?
36	Tino	*(No response)*
37	Researcher	Midday is the middle of the day, twelve o'clock, before we get to the afternoon or before we get to one o'clock. So now can you try and write down the flight number of the flight which will depart for its destination before midday? Do you now understand what the question wants you to do?
38	Tino	Aah. No, I don't understand the question.
39	Translator	Uthi bala pantsi iflight number ezihamba zifike pamko 12 imidday. Bhala pantsi iflight number. *(It says write down the flight number of a flight which will leave before midday. Write down the flight number)*
40	Tino	Ndibale inumber eyi one na? *(Should I write one number?)*
41	Translator	Ungayibhala ibe yi one ukuba uyafuna. (*You can write one if you want)*
42	Tino	*(writes the answer SAA769)*
43	Researcher	OK. Good.

Tino struggled to understand what the question required him to do until it was translated into isiXhosa, at which point she easily wrote the appropriate flight code that corresponded with the flight. From this, however, we can't determine whether Tino would have managed the question if it had been written in isiXhosa and thus given without verbal mediation.

The brief excerpt, which is typical of the 12 learners who improved on all five interview questions with mediation, illuminate the difficulties learners experience as they try to make sense of linguistically challenging assessment items in order to demonstrate their mathematical competence.

Conclusion

The study was designed to explore the linguistic complexity of the Grade 4 mathematics ANAs in English for ELLs. For the majority of the Grade 4 learners in the study, English had been the LoLT for mathematics for approximately six months only before they wrote the ANAs.

The study notes that, while most learners had difficulties with both the linguistic and mathematical demands of the test items, there are some learners who managed the mathematical demands when linguistic mediation was provided. Given this, it is essential that test designers work as carefully as possible to minimize language complexity of test items, especially in the early grades of learning in English. Where learners cannot understand the demands of the questions, teachers should be allowed to offer linguistic mediation if the tests are not to unfairly disadvantage the majority of learners as ELLs. While developing mathematics language is a critical part of mathematics learning, and I do not want to promote 'ghetto-izing' of mathematics language for ELLs, in these early stages of learning where learners have had little exposure to English, it is essential that the language of assessments is simplified to enable access to questions. Teachers should be sensitized to the kinds of mediation provided in the task-based interviews, that is, explaining, exemplifying and translating questions, which they can employ to learners who struggle to understand. Perhaps ELLs require the same attention as those learners disadvantaged by learning disabilities to 'ensure that every learner has the opportunity to demonstrate what they know and can do in the assessment' (DBE, 2014).

Henning and Dampier (2012) argue that there is need for such research especially in South Africa where the majority of learners grapple with learning in a second language. Such research will also have implications for other countries where learners write assessments in L2 or L3.

Graven and Venkatakrishnann (2013) indicated that while teachers supported the introduction of the ANAs, because they are standardized and provide guidance on what is expected, they noted widespread concern for the complexity of the language used.

It is hoped that the empirical findings of the linguistic challenges of the ANAs will inform educationists, especially those involved in the design of national assessments in these IP years, on the nature of language challenges that learners face when writing the mathematics ANAs, and the need to accord greater attention to the accessibility of the language for ELLs.

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PART IV

Supporting Change in Primary Mathematics Teaching

CHAPTER 11

Changing Teaching Through a Resources Approach

Hamsa Venkat and Mellony Graven

Introduction

Adler (2000), in a widely cited and taken-up article, highlights the importance of resources for effecting changes in teaching and in education more broadly, and particularly so in contexts of poverty where resources are often scarce and/or inequitably distributed. Adler looked at resources in terms of two broad categories—the 'basic resources' required systemically for the maintenance of schooling (e.g. buildings, teacher–pupil ratios, teacher recruitment qualifications) and other resources that are, or can be, used within teaching in classrooms, and in schooling more broadly. Her theorization of resources, with particular interest in the resources in use beyond the basic resource category, focused on three key aspects:

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1. human resources (including teachers' knowledge bases, their orientations to knowledge, collegiality in schools)
2. material resources (including school equipment such as a photocopier, classroom equipment such as a chalkboard or a computer and mathematical equipment such as manipulatives and textbooks)
3. cultural resources (including language and the structuring of time)

In Adler's (2000) article, inner details of theorization focused particularly on the material and cultural resource categories, but in a subsequent revisiting of this work, Adler (2012) extended attention to knowledge resources within the human resources category, reporting on their development of attention to secondary mathematics teachers' knowledge-in-use considered in terms of the 'grounds' to which teachers appealed in their classroom discourses around mathematical objects. In this theorization, knowledge resources were considered in terms of mathematical grounds (empirical or procedural/conventional); experience-related grounds (professional or everyday); and curricular grounds (assessment or textbook-related). In Adler and colleagues' broader Quantum project work, this theorization, with sub-aspects linked strongly to the curricular reform currents of that time, led to understandings of what came to be constituted as mathematics in classrooms and in mathematics teacher education in ways that showed both concordance and dissonance with the broader policy terrain.

Our focus in this chapter is on primary mathematics teacher education and teaching development more broadly. We are interested in resources in terms of potential 'levers' for development of primary mathematics teaching, and therefore, Adler's work located in the South African context is of particular salience. However, there are also differences between the secondary and primary mathematics landscape in terms of resources that bring the need to reconsider the sub-aspects that figure within the resources available for use across basic infrastructure and classroom resources-in-use. A key aspect here relates to the fact that in the early primary years at least, all teachers are involved in mathematics teaching, while at secondary level, mathematics teaching is linked only to mathematics department personnel. This has substantive implications for the numbers of teachers in the system and the costs of longitudinal attention to mathematics teaching development via a resources perspective.

We begin with an overview of this broader landscape in terms of both shifts in provision and what have been commonly described as 'challenges' in both the South African and other developing country contexts, redescribed in terms of Adler's three key resource-in-use categories. This allows us to configure these categories in ways that have salience for thinking about approaches that we, and others, have used within primary mathematics teaching development activities that treat these resources as levers for change. Description and analysis of three key interventions in South Africa thus forms the central thrust of this chapter. Our analysis leads into a consideration of approaches that show promise, the resources that have been leveraged within these approaches, and what remain as key gaps and tensions within the press for primary mathematics teaching development. A discussion of gaps and tensions concludes this chapter, raising issues that we argue require further attention.

South African Resource Provision and Challenges

In the decade and a half that has elapsed since Adler's first (2000) article on resources, some features of education in South Africa have made significant leaps forward. Access to primary-level education is near universal, and completion rates for primary schooling are near universal (UNDP, 2013). In spite of these successes, some key issues relating to human, material and cultural resources continue to be raised as ongoing and widespread problems at primary level (ZDM, 2014).

Human Resources

In the human resources category, the most widely voiced concern relates to the nature and level of primary teachers' mathematical knowledge. Evidence from multiple studies points to gaps relating to the curricular content to be taught among Intermediate Phase teachers (Grades 4–6) (Carnoy et al., 2008; Taylor, 2011; Venkat & Spaull, 2015). In the early primary years, knowledge resources are compromised by issues relating to the ways in which mathematics is understood—disconnected (Venkat & Naidoo, 2012), with limited progression (Ensor et al., 2009), with confusion between givens and unknowns relating to answers that are simply 'presented' rather than being 'derived' (Venkat, 2013). Given wide acceptance that a mathematical subject knowledge base is both critically necessary (although not sufficient) for high-quality teaching, strengthened by evidence showing that 'mathematical knowledge for teaching' comprised, in part, by a specialized knowledge base relating to how mathematics needs to be known for the purposes of teaching that can be related to teaching quality (Hill et al., 2008)—there are strong reasons for taking primary teachers' mathematical knowledge as a key resource lever within teacher development.

The introduction of the Advanced Certificate of Education Diploma courses initiative for in-service teachers as an up-skilling option was a key South African initiative aimed at improving teachers' mathematical knowledge base. However, concerns about variable quality and lack of standardization of content (Council for Higher Education, 2010) led to the scrapping of this option. Further, attention to Numeracy and Mathematics remains limited within the research base focused on early childhood education, restricting capacity for research-informed development of the 'mathematics for teaching'-related knowledge base (Venkat & Graven, 2013).

A number of other issues have secondarily been raised as challenges that can be linked to human resources. The number of teachers training for Foundation Phase teaching, and further the very small numbers with African languages competence in a national context where 'home language' teaching is encouraged for this age group, have been noted as systemic problems that need to be urgently addressed (Green, Adendorff, & Mathebula, 2014). Low teacher morale has also been highlighted as a problem that feeds into poor professionalism, high turnover and problems with recruitment in schools (Graven, 2012; NEEDU, 2013).

Material Resources

The situation relating to systemic material resources has changed substantially in the last decade at several levels. Following widespread critiques of the relatively loose outcomes specifications in Curriculum 2005 (the first curriculum reform of the post-apartheid era), subsequent curriculum changes have moved to increasing specification of content, pacing and sequence. In the most recent round of curriculum change, the primary-level Curriculum and Assessment Policy Statements (DBE, 2011a, 2011b) include weekly schedules outlining content coverage, sequencing and pacing. This format follows research identifying slow pacing as a key factor underlying the widely reported poor curricular coverage (Reeves & Muller, 2005). Further support for coverage has come via the recent provision of national department distributed grade specific mathematics workbooks for all primary school pupils (e.g. DBE, 2015). In this provision, we see clear evidence of material resources (with their term-level specificity) viewed as a key policy lever for changing the slow pacing of teaching in order to achieve better curriculum coverage.

Interim policies such as the Foundations for Learning programme (DoE, 2008) also introduced some sets of manipulative resources such as 100 squares and abaci, while also implementing annual national assessments as a way of both enforcing and monitoring the improved curriculum coverage that was being sought. While resources such as 100 squares, number lines and abaci have been widely written about as important in supporting moves to more sophisticated ways of working with number, it is important to heed to the concerns raised by Adler (2000), Ensor et al. (2009) and Venkat and Askew (2012) about apparatus featuring as ends rather than means, and unstructured use of structured resources, feeding into what Ensor et al. (2009) describe as 'dissipating pedagogic text in time, to the detriment of learners' progress' (p. 26).

Cultural Resources

Adler focuses particularly on language as a key cultural resource in teaching, while also noting that the structuring of time in schools, and norms repacing, also relate to this category. Mdluli (Chap. 8, this volume) presents an analysis that discusses the extent of use of language as a transparent resource in early primary years of teaching. We therefore focus here more on the issue of structuring of time. Critiques of slow pacing and progression towards more sophisticated counting and calculation strategies have been noted in prior writing in South Africa (Schollar, 2008), mirroring findings in other developing country contexts (Pritchett & Beatty, 2012). Ensor et al. (2009) focus explicitly on the 'specialization of time' in lessons through their development of the notion of semantic density, where they attend to the extent to which:

> teaching of numeracy entails the purposeful deployment of time in order to deepen learners' awareness of numbers and the number system (p. 14).

Thus, changing norms about use of time and expectations of progression represents a key cultural resource lever within teacher education with potential for changing pedagogic practice. A focus on progression in relation to time is key; however, teachers also need to have resources that enable the remediation of foundational concepts not mastered by learners in earlier grades. The need to manage remediation while still 'covering'

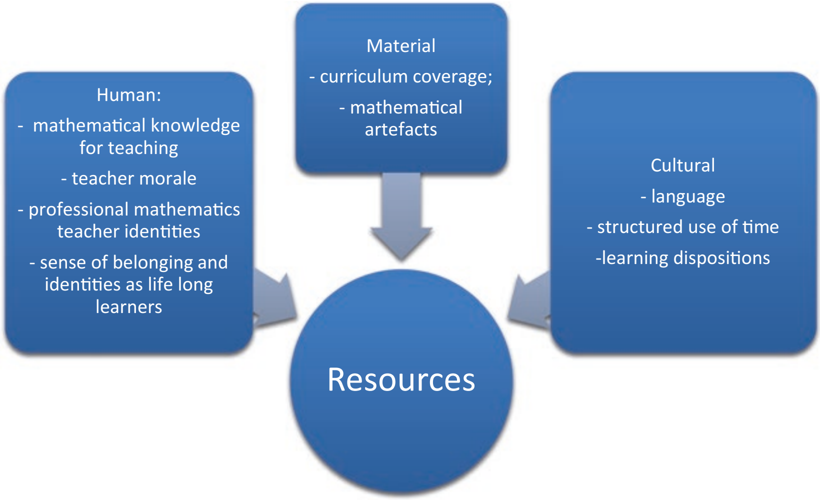

Fig. 11.1 Potential resource levers for primary mathematics teacher development in South Africa

the curriculum in specified timeframes requires skilful planning of lesson sequences. Furthermore, we would argue that supporting teachers to develop the agency to defend the logic of allocation of time on revisiting earlier grade concepts to district officials is a further important aspect of teacher development in this context (Graven, 2016).

Taken together, this discussion of reforms and issues on the ground leads to the development of a model of resources-in-use at primary mathematics teacher development level configured in the ways described in Fig. 11.1:

Resources as Levers: Approaches to Primary Mathematics Teaching Development

In this section, we provide details on and some analysis of primary mathematics teacher education initiatives, in our projects, and more broadly, focusing specifically on aspects that address some combination of the 'ground-related' identification of resources as potential levers shown in Fig. 11.1. We hold the South African Numeracy Chairs at the University of the Witwatersrand (Gauteng) and Rhodes University (Eastern Cape), respectively. These Chairs differ from other research Chairs as they merge developmental and research objectives in establishing partnerships with teachers, schools and learners across a range of intervention projects.

Both of our projects include longitudinal in-service mathematical knowledge for teaching initiatives. In the Wits Maths Connect Primary project, this initiative has worked with groups of teachers in each of the ten project schools in the context of an annual '20-day course' (run each year with a different group of teachers), based on 16 contact days spread in two-day blocks across the year. Development and research associated with this course has centrally focused on shifting the nature of primary teachers' ways of working with mathematics, aiming to build connections and coherence in teachers' work with examples, representations and explanations. This approach follows the evidence outlined earlier of problems with connections identified in our baseline observations. In analytical terms, this has led to the development (with Jill Adler) of the notion of 'mathematical discourse in instruction'. Analyses have shown gains at the level of conceptual knowledge in all three iterations of the course that have currently been conducted, underlain by what the international literature would argue, are important expansions in representational repertoires (Venkat, 2015). Teachers' feedback at the end of three iterations of the course most commonly mentions key representations that they have found useful (e.g. area models for multiplication, 'chunking' number-based repeated subtraction approaches to division, and double number line models for ratio), and an awareness of the need to provide reasons for mathematical actions within problem-solving:

> I now understand the rationale of using certain formulae I took as God given

In a context where concerns have been raised about the paucity of indicators relating to what is being added within mathematics teacher education programmes, in South Africa (DBE, 2013), the linked research and development agenda of the Wits Chair project has worked with mathematical knowledge-related measures focused on specialized content knowledge and mathematical modes of inquiry as important resource levers with the potential for impact on mathematics teaching in primary schools. Emerging qualitative analyses of classroom teaching suggest potential for positive impact on pedagogies in terms of connections (Askew, 2015) and responsiveness (Abdulhamid, Chap. 13, this volume).

In addition to discussion and development of mathematical models as important artefacts within the 20-day course, broader work in the Foundation Phase within the Lesson Starters project has focused on providing all classrooms with key 'structured' number artefacts—for example, bead strings to 20 with beads coloured in groups of 5, and bead strings and number lines to 100 structured in 10s. This provision has worked in line with teacher development workshops and in-class support to build in use of resources in ways that work towards more sophisticated handling of number, underlain by improved number sense. This work follows the research identification noted above of both material resource shortages, but also human resource limitations in terms of the use of structured resources for progression.

Findings emerging from the Joint Education Trust's study (Bowie, 2014) of the selections of mathematical content and orientations within five Intermediate Phase pre-service teacher education programmes in South Africa point to substantial differences in the extent, nature and level of the mathematics and mathematics teaching methodologies that are offered. This difference points to a lack of consensus around what constitute useful directions of change within primary mathematics teaching, and the underlying knowledge bases that can support these changes.

Within Graven's Eastern Cape project, the Numeracy Inquiry Community of Leader Educators (NICLE) programme has also involved partnering with teachers across 12 schools to form the NICLE which is theoretically informed by Wenger's (1998) theory of learning within a community of practice. Within this theoretical perspective, learning and identity are inseparable and are aspects of the same phenomenon (Lave & Wenger, 1991). The NICLE community began in 2011 and has been constituted by over 40 teachers, principals/deputy principals, SANCP research students and district workers each year. While some members have left through movement to other schools or retirements and some new members have joined along the way, the majority of members in this community have participated across the five years (from 2011 to date).

Within this perspective learning for the community involves providing access to high-quality resources related to the joint enterprise of finding ways forward to the challenges of numeracy education and improving mathematics learning in the schools. These high-quality resources include physical resources such as providing teachers with research-informed sets of key teaching resources such as place value cards, tens frames, laminated money sets, dice, cards, plastic clocks, and so on. Teachers were also provided with sets of homework books for all their learners aimed at developing in learners the habit of working independently on homework and thus developing independent hard working dispositions. Homework books also allowed learners to work at differential paces, and they would receive the next book in the series following completion of a book.

More importantly, however, the key resource of this community is access to active participation in and engagement with mathematical knowledge for teaching stimulated by interaction in activities presented by a wide range of mathematics education specialists. These 'specialists' are people invited to lead NICLE workshops and include SANCP members, NICLE teachers, local teacher researchers, mathematics teacher educators from across South African institutions and internationally renowned mathematics educators (such as John Mason). One NICLE teacher commented as follows on his experience of interacting with these 'specialists' (quoted verbatim here):

> people come specialists with different ideas with sessions and it demonstrates it is not just a talk show it is active participation, that keeps it lively and the interaction is not, it is on a level ground where the lecturer is on the same path as you. The interaction is not where you have to just be absorbing theoretical knowledge of someone else.

In line with Wenger's theory, there was a genuine sense of an establishment of a community in which fellow teachers in NICLE become a key resource for sharing and learning within NICLE sessions as well as beyond as teachers shared experiences (and resources) across the range of schools.

At the end of each year, NICLE participants complete a questionnaire which gathers information of teacher experiences of NICLE and what resources and aspects of NICLE they value and how these have influenced their practices. Teacher responses indicate take-up of a range of 'resources' offered through NICLE participation, including, importantly, given the concerns we have flagged in the South African landscape, ideas on how to use time and language more effectively within their teaching:

> Lesson by Prudence [Retired principal and NICLE member in 2011 – she brought her class as a demonstration of teaching measurement] was a good demonstration of how much time a teacher should spend teaching mathematical language. I didn't know what to use or what activity to use to teach measurement but today I know what to use.

Trajectories towards roles in primary mathematics development activities informed by research participation have also been effected in both projects. The quote below follows feedback on a session given by a member of the SANCP research team. When she began her master's studies, she was a teacher in a local school but then moved to become subject specialist in the Department of Education.

> To get an exposure is a very good thing so that you learn different styles and strategies of tackling challenges. I have learnt many concepts that can unpack multiplication. Nice to see a sequential way on which to introduce and develop concept of multiplication.

In the context of concerns about human capacity for mathematically focused primary mathematics teacher development and research at the human resource level, given the scale of need that we outlined earlier, these trajectories are also critical.

Teacher comments from questionnaires at the end of 2013 pointed to the take-up of a range of resources, including physical ones (use of place value or 'Flard cards'), increasing learner participation and activity in class and also the structuring of time:

> My mental maths sessions improved a lot. Learners were struggling with number sense, but NICLE gave me the opportunity to introduce Flard cards.
>
> They loved the homework book but did not want to only complete the given pages. Therefore I have allowed those to complete as many pages as they want to.
>
> Every learner participates actively + willing to try and they correct each other's mistakes in a positive way.
>
> Lots more learner participation. Lots more learning by doing and experiencing.
>
> The periods are better structured to include: Mental Maths; Activities and Assessment (Various Methods).

While the above teacher comments point to the valuing and take-up of a range of resources teachers accessed through NICLE participation, more in-depth journaling and interviewing of a sample of nine teachers in doctoral fellow Pausigere's (2014) thesis indicated strong shifts in teachers' professional ways of being and increased investment in teaching as a profession. This resonated with earlier research by Graven that pointed to the way in which teacher membership of the professional development communities enabled major transformations in their professional identities as well as redirecting career trajectories that were out of teaching to establishing a career path within teaching (Graven, 2012, 2014). Analysis from the in-depth study of a sample of nine teachers (Pausigere & Graven, 2014) explored the learning stories of these teachers and found that two key metaphors emerged in these learning stories (stelos), namely activation and reinvigoration, through participation in NICLE. Teachers with a history of mathematical competence reinvigorated their mathematical identities through participation in the NICLE with some of the teachers' identities outcropping into a wider range of mathematical and maths education practices. The identities of teachers with weak mathematical histories (stunted by negative school mathematical experiences) became remediated, and new mathematical identities were activated through participation in the NICLE Communities of Practice. In this approach to teacher development, issues of low morale—noted as widespread in the profession—are both addressed and studied as an important resource lever for change.

The work of the teacher development programmes across the two Chair projects contrasts with many of the more short-term, and even 'once off' teacher development sessions provided to teachers which have historically tended to focus on dissemination of information. Graven (2012) has argued that, not only are such approaches educationally outdated (based on transmission assumptions of learning), but are increasingly inappropriate in the so-called information society where information is easily accessed via the Internet. As Wenger highlights, in such an information society, issues of identification and negotiability become even more important (Wenger, 1998). Creating in-service spaces that foreground enabling the resource of identification and negotiability is a key to the SANC projects.

In the broader landscape, a number of other initiatives have worked with combinations of the resources-based policy levers we have identified. In Gauteng province, the larger-scale Gauteng Primary Literacy and Mathematics Strategy (GPLMS), working with approximately 800 primary schools, has developed a model centred on the provision of scripted lesson plans. Initial implementation incorporated mentors supporting the take-up and use of these lesson plans, who offered both in-class demonstration lessons and feedback to teachers following lesson observation. This initiative, like the national policy initiatives related to the moves to more prescriptive detail on curriculum sequencing, is focused on improving curriculum coverage. Scripted lessons, with monitoring of adherence to the time lines for coverage, follow the evidence detailed earlier of poor curricular coverage in the primary grades (Fleisch & Schoer, 2014). Early results indicate some successes in terms of Foundation Phase learner performance via the provision of these material resources, but progress at Intermediate Phase remains more elusive, as seen in ANA performance trends (DBE, 2014). Complicating the claims are implementation studies in the GPLMS context that indicate more independent learner work occurring in the contexts of more 'partial' rather than 'full' implementation of the lesson plans (Essien, Venkat, Takane, & Tshesane, 2015).

An important contrast is highlighted here between the approaches in the broader policy landscape via curricular reform such as CAPS and in the GPLMS and the approaches we have analysed in our projects. In the former, the emphasis is on material resources working through into cultural resources, and there is less teacher autonomy in the prescription of content coverage, sequencing and pacing, but this takes the burden off teachers in terms of preparation. In contrast, in the Wits and Rhodes Chair projects, the emphasis is on human resources, and in working longitudinally with teachers, supporting and changing their use of cultural resources. The latter approaches are both more expensive, and more in-depth, and facilitated by the mid-level scale rather than large-scale level of our work (with between ten and fifteen schools). Our approaches are underlain by an ethos of working with teachers for change rather than on change for teachers.

In the landscape, our sense is that both the larger-scale approaches focused on material and cultural resources, and the medium-scale approaches focused on human and cultural resources are needed, and can work usefully to support each other, but that tensions between these need to be attended to with teachers in order to avoid overload. This relates to evidence that, while there is evidence of improved curricular coverage and pacing in the context of CAPS, increasingly high stakes assessment and GPLMS lesson plans—indicating shifts in cultural resources relating to norms of structuring time. There is, however, also evidence that this kind of pressure for coverage and outcomes may work against the kind of morale building and the development of teacher agency to adapt their teaching in ways that are sensitive and responsive to the needs of their learners that have been found to be so critically important in Graven's project to build the professional identities required for constructive working with mathematics (Graven, 2012; Pausigere & Graven, 2014).

Implications

In summary, the larger-scale (national and provincial) initiatives have tended to focus their attention on material resources via the provision of curriculum content specification and coverage-focused artefacts. In the process of their take-up and use, there is evidence of shifts in cultural resources related to use of time in primary mathematics classrooms. Currently, it is smaller-scale rather than more systemic interventions that have focused attention on human resources as levers in terms of knowledge and/or morale. At one level, this is entirely understandable given that international evidence points to both of these levers requiring more longitudinal, in-depth and interpersonal investment to function well (e.g. Joyce & Showers, 2002). However, given the strength of evidence stating that the disciplinary knowledge base and professionalism are fundamental elements for developing quality of teaching, the lack of broader attention to these aspects in larger-scale interventions is problematic. Probably, more worrisome than this is the sense of a lack of shared understandings of what this professionalism and knowledge base look like, and of what content and approaches within teacher education activities might lead towards desired goals. In this context, recent moves towards setting up forums in which groups involved in primary mathematics teacher education can meet to share and discuss their curricular selections and approaches are important. They represent opportunities for developing the kinds of shared language for discussing primary mathematics teacher development that are already common in more developed country contexts. Importantly, they also provide spaces where development attuned to local needs can be addressed. Further, and important to the creation of communities around early mathematical learning, is the recent formation of South African Research Association of Early Childhood Education community, with associated conference and development activities. The Chair project groups have, and continue to contribute to this community with linked research and development focus on mathematics.

Our sense is that framing these discussions around our reworked notion of resources as potential levers for change can usefully contribute to this discussion in ways that are linked to both the international mathematics teacher education literature, and conditions and affordances in the South African policy and socio-cultural context.

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CHAPTER 12

From Theory to Practice: Challenges in Adopting Pedagogies of Mathematising in South Africa

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Introduction

This chapter is located within primary mathematics in South Africa and contributes to the literature on the issues around teaching for understanding that in turn is based in a theoretical position placing 'mathematizing' at the heart of learning. We report upon two intervention studies: one in a Grade 2 IsiZulu classroom (7–8-year-olds) and one in a Grade 4 English classroom. Each intervention broadly drew resources developed on principles adapted from the North American Cognitively Guided Instruction (CGI) and Dutch Realistic Mathematics Education (RME) programmes of research as realised by a series of Big Books of problems written by Askew (2004). Both interventions were based on teacher–pupil

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discussions of realisable (realistic) additive situations as the starting point for, we hoped, the emergence of learners' informal models and solutions which could then form the base for the use of artefacts, in particular the empty number line (ENL), as models or tools for finding solutions and, ultimately, for the guided re-invention of more formal mathematics. As we argue in this chapter, having these expectations realised was not as straightforward as we anticipated.

The literature on the issues in learning and teaching in South African primary mathematics classrooms highlights the difficulties experienced in moving from a dominant 'transmission' style of pedagogy, based in rote memorisation, to a pedagogy that emphasises problem solving and mathematising (Barnes, 2004). The majority of such studies are, however, written from an 'etic' position: that is, the actions and understandings of the teachers are accounted for from the outsider perspective of a researcher, who is not the teacher. In the case of both interventions discussed here, the teacher was also the researcher, hence enabling an 'emic' (insider) account of the challenges in adopting such pedagogy.

We do this by examining critical incidents in which the way lessons played out confronted what the research literature had led us, the teacher/researchers (Thulelah and Herman), to expect in terms of learner responses to the situations which we presented to them. In both cases, we tell the story about assumptions from literature and then discuss points of disjunction in these assumptions. We then talk about how we addressed these breakdowns and the outcomes thereof.

The significance of what we report is that while we consider CGI/RME approaches to be broadly appropriate in South African classrooms, there is a need to identify and question assumptions implicitly embedded in these approaches regarding learner responses when these approaches are relocated to a different context. We argue that both the studies reported here show that changes to teaching and learning are possible, but that there are issues that teachers face in attempting to bring about such changes in the context of a developing nation. Such issues include the dominance, in South Africa, of teacher-centred pedagogies, the familiarity of the approaches and models that teachers are expected to implement, and the background of lack of encouragement for learners to think critically and make sense of their mathematical working. These issues must be addressed if change is to be effective.

Background Theory

Cognitively Guided Instruction and Problem Types

Carpenter, Fennema, Franke and Levi (1999) found that to understand how children think about addition and subtraction it is important to consider differences among problems giving rise to these operations and learners' responses to a range of problems. As a result, they devised a classification of problems that frames an understanding of the evolution of the strategies children use for solving such problems. According to this framing, there is 'a relationship between strategies and problem types and the levels at which strategies may be used' (Carpenter et al., 1999, p. 30). This relationship 'provides a structure for selecting [addition and subtraction] problems for instruction [and assessment] and interpreting how children solve them' (ibid., p. 7). In addition to the classification of problems, Carpenter et al. (1999) classified various strategies in relation to these problems, strategies that vary across and within problem types. Strategies varying across problem types include direct modelling (DM), counting (C) and use of number facts (NF). Strategies varying within DM are Join All, Joining To, Matching, Trial and error; within C are Counting On From First, Count On From Larger, Count On To, Count Down From, Count Down To; within NF are Recalled Facts and Derived Facts (adapted from Carpenter et al.). The assumption is that when children are offered a word problem, they are able to create some model of the situation. Children are then assumed also to be able to progress from the use of informal models and strategies to more sophisticated models and strategies. In a similar fashion, the RME approach refers to this progression as 'horizontal mathematization' and then 'vertical mathematization' (see below).

It is upon Carpenter et al.'s (1999) classification that Askew (2004) derived his framework for the Big Books resources which in turn formed the basis for the intervention studies reported in this chapter. Askew (2004), in preparing the materials, chose to compress Carpenter et al.'s (1999) four classes of *Join, Separate, Part-Part-Whole* and *Compare* into the three classes of *Change (treating join and separate as linked), Combine (part-part-whole)* and *Compare.*

Askew's three classes of problems can be further broken down into three types, also in line with the CGI work: *result unknown*, where the whole is what is not known (5 + 3 = [ ]); *change unknown*, where the addend or subtrahend is what is sought (5 + [ ] = 8); *start unknown*, where the initial value is what is sought ([ ] + 3 = 8).

Realistic Mathematics Education (RME) and Guided Reinvention *Through Progressive Mathematisation*

While CGI provides an overarching framework, via Askew's Big Books, for dealing specifically with teaching additive relations in our studies, we found that the work of the *Realistic Mathematics Education* (RME) researchers provided a complementary framework in terms of thinking further about pedagogy.

To make learners' experience of mathematics as organic as possible, Freudenthal argued that learners' ultimate understanding of formal mathematics needed to stay 'rooted in' their understanding of experientially real everyday-life phenomena (reported in Gravemeijer & Doorman, 1999). Consequently, he worked against what he called an 'anti-didactical inversion' where the end results of the work of mathematicians are taken as the starting point for mathematics education' (ibid., p. 116). By upholding learners' own constructions as central to the human activity of mathematics, Freudenthal advocated for an approach that would see learners 'come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible' (ibid., p. 116), as opposed to seeing themselves as receivers of ready-made mathematics. The notion of *guided re-invention* through progressive mathematisation, therefore, advances the belief that the character of the process of learning mathematics should be that of cognitive growth, as opposed to a stacking up pieces of atomised knowledge. The adjective 'progressive' is meant to connect with the gradual process by which formal mathematics emerges from the mathematical activity of the learners. Approaching learning through mathematising is, however, relatively alien in the context of South Africa where the starting point is, more often than not, the formal mathematics.

Mathematising was defined by Freudenthal (1971) as the process of organising subject matter, and two ways of mathematising are described as 'horizontal' and 'vertical'. Barnes (2004) paraphrases Treffers (1987) in defining *horizontal mathematisation* as 'when learners use their informal strategies to describe and solve a contextual problem', and *vertical mathematisation* as occurring 'when the learners' informal strategies lead them to solve the problem using mathematical language or to find a suitable algorithm' (p. 50).

In both interventions, we were interested to explore learners responses to each of these moves: could they make sense of a narrative problem situation and turn this into an informal model of the situation (make a horizontal mathematising move). And could they then make the vertical mathematising move of creating a more formal mathematical model. In each case, the models produced can be thought of as emergent in the sense that they arise out of and build on learners informal productions, rather than being directly introduced into the learning space by the teacher.

Traditional Methods and Emergent Models

The literature on RME makes a distinction between two broad categories of procedures for solving addition and subtraction problems; namely: *columnwise* and *non-columnwise* processing of numbers (Beishuizen, 1993). The former type is a traditional method, commonly referred to as the *column addition/subtraction* method, and is widely taught in South Africa as the principal means of calculating. Non-columnwise processing is more linked to emergent models and to the development of mental calculation, which broadly can be further divided into the two subcategories of *sequential* and *decomposition* procedures known as the *1010* and the *N10* strategies, respectively.

The 1010 procedure is also referred to in the literature as the *split method* because when operating on the numbers both operands are partitioned into tens and units to be processed separately, and then recomposed to arrive at the sum or difference. For example, adding, say, 45 and 23 would mean splitting each number into its tens and units, adding these separately and then recombining: $45 + 23 = 40 + 5 + 20 + 3 = 40 + 20 + 5 + 3 = 60 + 8 = 68$. Tabor (2008) suggests the following schematisation to illustrate the splitting strategy (Fig. 12.1).

The *split method* is an explicitly taught and assessed calculation method in the Foundation Phase which covers Grades 1–3 in South African schools. On the other hand is the N10 procedure which is also known as the *jump method* because the tens and units of the second operand are added to or subtracted from the first operand which is kept whole (Beishuizen, 1993; Gravemeijer, 1994; Tabor, 2008). And so for the same question above, a learner could begin with 45, add 20 to it to get 65, and then add 3 to arrive at a solution of 68. The jump method is more efficient to use mentally, but it depends on familiarity with the ENL.

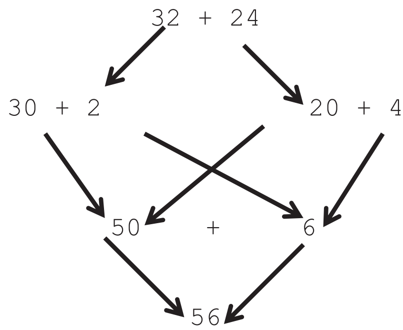

Fig. 12.1 The split method for 32 + 24 (Adapted from Tabor, 2008)

As noted above, common to both CGI and RME is this centrality of learners' informal, emergent models of problem situations. Within RME models are interpreted broadly as vehicles to elicit and support the progression from an informal understanding that is closely connected to the 'real' or imagined problem context to the formal understanding of mathematical systems (Van den Heuvel-Panhuizen, 2003). To be deemed emergent thus a model needs to support progression from thinking about acting in the situation being modelled to thinking about mathematical relations—in other words, to support both horizontal and vertical mathematising.

One such model is the *empty number line* developed and successfully used for over three decades in the instructional design of RME based at the Freudenthal Institute. Its graphic nature makes it an image that represents the logical structure of numbers which relates closely to the counting sequence (Anghileri, 2006) and supports the development of efficient mental methods. As a result, the ENL is a well-theorised choice for modelling the addition and subtraction of quantities as the number range increases. By establishing opposing directions for the operations of addition and subtraction, the *empty number line* model represents this inverse relationship graphically and so makes it visually available for appropriation into learners' knowledge structures. Hence, both interventions investigated the emergence and use of the ENL.

The Interventions

The intervention studies reported upon here were carried out in two different government primary schools in Gauteng province. Both studies incorporated a broadly qualitative case study approach with some quantitative analysis. In each study, the intention of the qualitative data was to explore in-depth the mathematising processes of the learners in solving word problems. The aim of the quantitative aspect of the studies was to look at overall performance before and after the intervention. Both studies found post-test gains. It is, however, beyond the scope of this chapter to report on these quantitative results. The focus here is on insights arising from the qualitative data. The ‘Big Book of Word Problems’ was used in each study to provide word problems that made sense to children, were close to their lived experience, and were imaginable, given the structure of different themes through which situations are presented in the book.

Here we describe the overall framework for the interventions. Below, in discussing incidents from each intervention, we briefly outline where each intervention had to adapt slightly this overall framework.

The Big Book of Word Problems (BBWP) provides 28 structured word problems categorised in semantic problem structure themes and the contexts of the problems chosen in the expectation that learners are able to make sense of them in an everyday sense, if not a mathematical one (although originally written for England’s schools). In line with the RME approach, all the BBWP problems that learners had to solve were adapted and set in contexts that were sufficiently ‘real’ to South African learners in Grade 2 or 4. The tasks were presented as word problems in writing, and each lesson had a different theme while preserving the same basic form so that learners could quickly get used to ‘what is expected of them in the lesson and they can then concentrate on thinking about the mathematics’ (Askew, 2004, p. 6). Each intervention lesson was based around four stages to each lesson:

- Solving the Big Book problems
  Learners are introduced to the context/theme for the problems and to then work in pairs to solve each of the three problems.
- Linking up the problems
  A whole-class discussion focused on bringing out quantitative relationship underlying each of the three problems just solved, in order to draw learners’ attention to the ‘common mathematical structure underlying the problems’ (Askew, 2004, p. 7).
- Follow-up problems
  Individual activity sheets affording learners the opportunity to work more independently on the ideas just discussed in class.

- Wrap-up
  Reconvening into a whole-class discussion for further exploration of the problems in the worksheet and consolidation of the key ideas.

Issue 1: The Challenge of Horizontal Mathematising (Thulelah's Voice)

Context
The Big Book approach was trialled in an IsiZulu Grade 2 classroom (n = ±40 learners). I conducted lessons from the 'Big Book of Word Problems' twice weekly across one term. Pre- and post-tests were administered before and after the intervention, respectively. A pilot lesson helped determine if there should be any changes before the intervention continued. This meant that the language was adapted slightly to suit the context of the children. The order of the lesson structure was also changed and learners' lack of fluency in finding answers meant I decided to provide additional fluency worksheets which the learners worked on outside the intervention lessons.

Assumptions That Influenced the Intervention Approach
My approach to the intervention was based on an assumption from the international literature (Carpenter et al., 1999) that once children are presented with different types of word problems, they will voluntarily directly model the situation to solve the problem. Once they have directly modelled, the role of the teacher is then to guide them to progress to more sophisticated ways of solving the problem. As noted earlier, RME refers to this as 'guided re-invention' (Barnes, 2004). This direct modelling, in RME terms, is referred to as horizontal mathematisation since the organising activity is connected to the situation. Based on this assumption then, the aim of the intervention was to promote the ENL as a more sophisticated model (Gravemeijer & Stephan, 2002), where the ENL assists the progression from horizontal to vertical mathematisation (TT). However, as the intervention unfolded, there was a breakdown that confronted my assumptions in this regard.

Breakdown
This breakdown was reflected through the learners' work both in the classroom and in their classwork books as they attempted to model problem

situations. First, when the learners were asked to come up with their own stories, they offered non-mathematical word problems. Second, when asked to solve problems using the empty line I had assumed that since the learners were able to use number sentence models to solve problems, they should smoothly shift towards using a model such as an ENL. They could not, however, do this without me intervening, and when they did eventually use the ENL, they did this in a way that suggested they saw connections between the model (the ENL) and the problem situation. For example, as shown in Fig. 12.2, the learner was asked to work out the problem:

'There were 8 people on the bus. Thirteen (13) more people got on. How many people are on the bus now?' They used a number sentence to solve it and got a correct answer.

But when learners had to use an ENL to solve the same problem, they could not apply the model in a way that could help get the correct answer (see Fig. 12.3).

As the intervention was unfolding in lesson 4, I moved on to introducing *'Join - Change Unknown Problems'*. More issues emerged with regard to the use of the ENL (see Fig. 12.4).

The question for number '5' was: '*Mandisa had 11 pencils. Aphelele gave her some more pencils. Now Mandisa has 16 pencils. How many pencils*

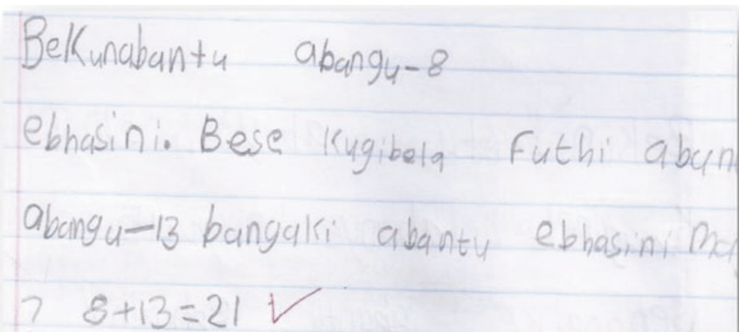

Fig. 12.2 Lindiwe's work solving a problem using a number sentence

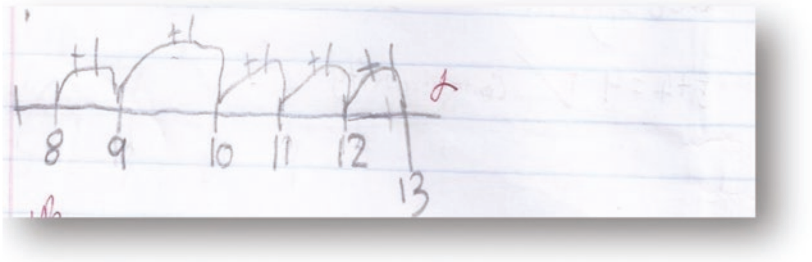

Fig. 12.3 Lindiwe's work reflecting incorrect answer on an ENL

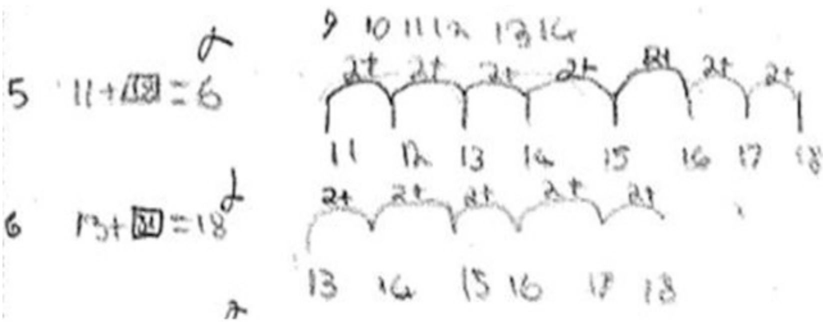

Fig. 12.4 Errors emerging from Sipho's work

did Aphelele give to Mandisa? And the question for number '6' was*: Musa had 13 pencils. Liza gave him some more pencils. Now Musa has 18 pencils. How many pencils did Liza give to Musa?* In both these examples there is evidence of incorrect use of models and strategies which resulted in incorrect answers.

Adaptations

The errors illustrated above led me to decide to ask the children to use pictures in any way that made sense to them and that depicted the situation, rather than rush to the number line. Since horizontal mathematising involves organising across informal situations and more formal mathematics, it thus requires sense-making of the situations as a basis for direct modelling, so I decided to take the learners a step back, and asked them to

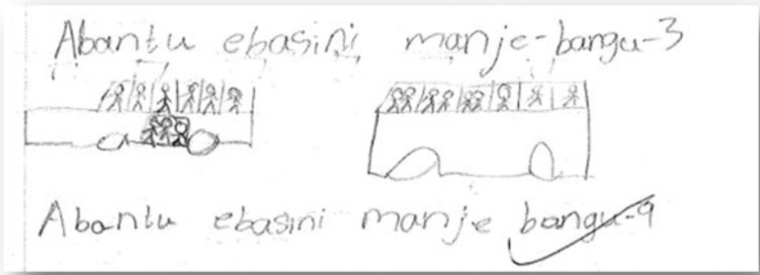

Fig. 12.5 Lizo's work in lesson 7

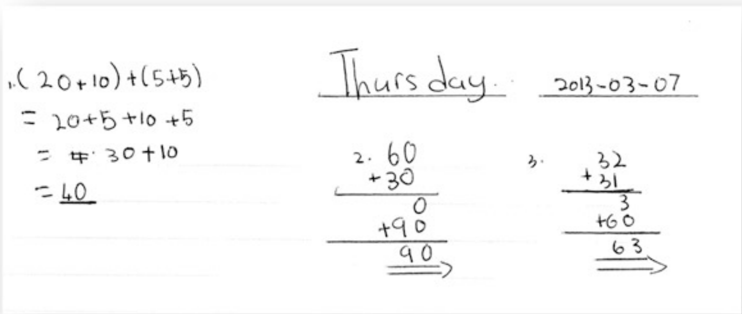

Fig. 12.6 Columnwise and non-columnwise processing: Tamara's working in lesson 1

solve the word problems in a way that made sense to them, which could include drawing pictures. From this point onwards, until the end of the intervention, the learners drew pictures in their own ways, rather than in guided ways, to try to make sense of the situation through their own personal organising activity (see Fig. 12.5).

The word problem posed here was: '*There were 6 people in the bus. Three (3) more people got on. How many people are now in the bus?*' The work example in Fig. 12.6 reflects horizontal mathematisation, in that the learner used pictures (informal mathematics) to solve the problem which formed a basis for formal mathematics. This episode played a major role in the promoting of sense-making of the situations by the learners themselves.

Outcomes

The post-test results led me to conclude that the learners were then able to make sense of the situations and were even able to differentiate between the different problem contexts. For example, in the pre-test, the learners used either tally marks or circle scribbles in random heaps when solving the word problems. However, in the post-test, when solving a word problem about a bus and people, they drew pictures of people; when solving a problem about pencils, they drew pencils or lines; and when they were solving a word problem about balls, they drew circles. This was a consistent pattern in the post-test for most learners.

In relation to RME and CGI, these excerpts alerted me to the assumption in the literature that children, given realistic scenarios, *will* naturally and voluntarily make sense of them and present them in horizontal models that reflect different degrees of informal thinking. My sense of the progress of the early lessons in the intervention sequence was that this was not the case. Rather, I had to change my approach to focus on the sense-making of the situations as reflected through the children's presentations of the model.

Issue 2: The Challenge of Vertical Mathematising (Herman's Voice)

Context

This intervention took place in a suburban school serving a historically disadvantaged population where the language of learning and teaching is English across all the grades. The focal class had 42 learners, with matched initial and post-test data for 40 of these learners. The intervention was broadly carried out in line with the description above.

Assumption

Whenever my Grade 4 learners had to do the class activity for a given intervention lesson, they would ask if they needed to show their working using the ENL or whether they could use any other method. In each of the six lessons, I read this as a question about compliance. It soon—perhaps not soon enough—dawned on me that my interpretation of this question was an incorrect one as only one learner voluntarily used the ENL in their solutions to the post-test. This came as a shock to me given that I only made this discovery of learners' reluctance to use the ENL after the intervention had been completed, after they had taken the post-test. In retrospect, however, while my eye had been on the progress learners were making with the use of the ENL, there were hints pointing to this reluctance, which I should have picked up along the way, and these I detail below.

The Lessons

Learners' workings in the first lesson revealed the currency of both *columnwise* and *non-columnwise* calculating. In the latter case, *decomposition* (1010) was the procedure of choice, while *splitting* (N10) was barely seen. An interesting finding was the use of a modified columnwise process that incorporated the splitting of the addends into units and tens, as seen in Fig. 12.6.

The predominance of this *columnwise-decomposition* procedure pointed to its explicit promotion by the class teacher in the weeks preceding the intervention. If the value of the ENL was to be realised, however, learners had to be familiarised with the *splitting method* (N10) as a non-columnwise procedure associated with the successful use of jumping on the ENL. Consequently, this formed part of the engagement in the second lesson where the ENL was introduced for the first time and subsequently to which it was promoted.

My reflection on the second lesson revealed three categories of learners: those who were *clinging to the use of the columnwise* procedure (7 of 40), those who were *leaning on the columnwise* (25 of 40) procedure, and those who were *moving into using the ENL* (8 of 40). In the first group were learners for whom the ENL model had yet to emerge. Instead they used the column method exclusively. Of the seven learners in this category in the lesson on change increase problems, only two were able to tackle the change decrease variation question successfully. In the second group were learners who—in tackling the same question—first used the column method and then used the ENL. For this group of learners, the column method seemed to function as a means to setting up the number line. Of the 40 learners in the class, 25 were in this category. In the third category were the eight learners who were beginning to experiment with the ENL without overtly first relying on the column method. Of these learners, two were able to handle the *change decrease* question accurately, but only one attempted and correctly answered the *change decrease* with the use of the ENL. Albeit that the partial sums are not annotated, the direction and collective value of the jumps were correct (see Fig. 12.7).

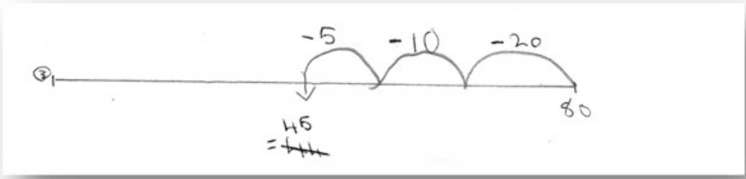

Fig. 12.7 Sheba's tackling of the change decrease problem (80 – 35)

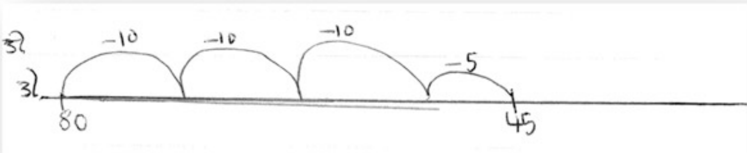

Fig. 12.8 Attempting a change decrease problem: Tamara's working in lesson 2

With 80 % of the learners preferring to use the column method, I should have had a sense of the amount of engagement with the ENL that I needed to put in place for learners to buy into the usefulness of the ENL. With me still assuming that learners were already well familiar with the structure of the number line, I did not see the surprise awaiting me in the post-test.

Critical Incident: Assumption Thwarted

In subsequently reflecting on the second lesson, it became clear that one of my big assumptions had been thwarted: that my Grade 4 learners would be familiar with the structure of the number line. As it turned out, it became clear that they had had little exposure to the number line. It appeared that some were not at all familiar with its structure (see Fig. 12.8).

Following this observation, I thought it would suffice to provide learners with a couple of worksheets for practice at the beginning of the third lesson wherein the setting up of the model was done for them (in that the

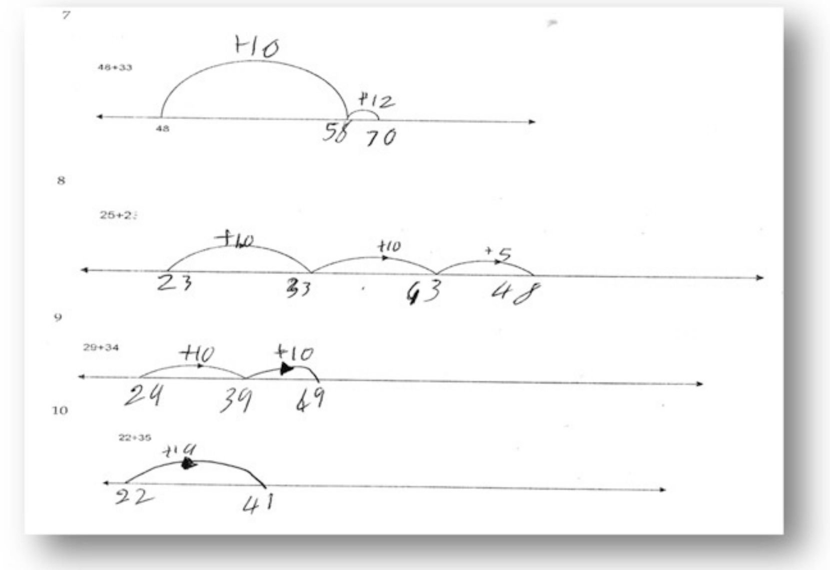

Fig. 12.9 Attempting the worksheet: Kamo's working in the third lesson

jumps were drawn on the number lines): all they would need to do was to annotate the values of the jumps and the partial sums. As exemplified in Fig. 12.9, at this formative stage, many learners struggled.

Following the observation that certain fluencies required for working with the number line were not in place for these learners, I realised that more time would need to be invested into getting them to a point where they would confidently and fluently work with the ENL. Following this realisation, and with the idea of having learners compress their working by reducing the number of jumps, the fourth lesson began with an exercise designed to remind them of the 'friendliness' of 5 and 10.

The message here was for learners to compress the several jumps of ten that they would have to make (as the number range increased) into one jump of a multiple of ten. Alongside this reminder, learners were guided into (among other calculating strategies) the use of *compensation* for addends close to a multiple of ten. For instance, in tackling (41 + 39), Sheba began at 41, made

Task for 4B is 41 + 39 = ; and Task for 4C is 54 + 49 =

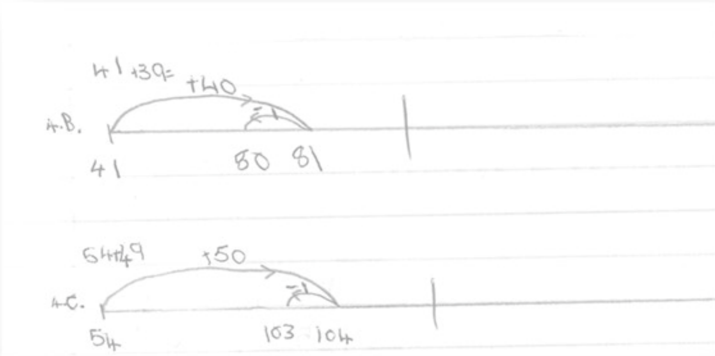

Fig. 12.10 Sheba's working in lesson four: multiples of ten and compensation

one big forward jump of 40 to land on 81, and then made a small backward jump of one to land on 80. Similarly for (54 + 49) (see Fig. 12.10).

The last two lessons proceeded in a similar fashion, with my learners being inducted into more and more efficient ways of working with the ENL prior to engaging with the theme for the day. As already mentioned, however, despite explicit attention to the strategies that would help them work more fluently with ENLs, only one of my learners voluntarily used the ENL in the post-test. This called for a further delayed post-test—where the use of the ENL was made mandatory. Ironically, while this prescription did little to alter learners' dispositions towards the ENL, their performance in the delayed post-test improved over that in the post-test. While learners did not voluntarily use the ENL model in the post-test, the slight improvements in the post-test translated into larger gains in the delayed post-test when the use of the ENL compulsory.

This improvement in results from post-test to delayed post-test suggests the value of the ENL model for shifting learners' strategies beyond counting, with the proviso, of course, that the necessary ground work to familiarise learners with the number line model as an object is already at an advanced stage.

Discussion and Conclusions

Through our examination of the critical incidents presented here, we argue that there is a need to identify and question assumptions implicitly embedded in the CGI/RME approaches regarding learner responses, arising from assumptions that are challenged when working in a developing nation context.

Discussions from both the Grade 2 and Grade 4 interventions reveal how common it is for us, the teachers to have preconceptions based on factors such as theoretical backgrounds or comfort in teaching styles. Important to note is how difficult it was to let go of expectations, even when being thwarted during teaching. There is, however, a lack of suggestions from literature as to how to go about dealing with such challenges. RME promotes guided re-invention, but the literature is neither adequately explicit what this might look like in practice nor clear about the progression from use of concrete models and strategies to the use of more sophisticated and abstract models and strategies. One implication here for teachers is that although these theories suggest the possibility for guided re-invention and progression, the context in which teaching and learning takes place is an important consideration. It is also important that teachers be willing to let go of their expectations when these are not beneficial to their learners. In this instance, we needed to seek more effective ways of helping children's sense-making.

Although the ultimate goal in mathematising is to guide learners to progress from horizontal mathematising to vertical mathematising, it is important for the shift from one level to another to be progressive. In instances where there is no evident progressive shifts, teachers should be able to go a 'step back' and make sure that there is sense-making of the mathematics which can form a basis for translation into direct modelling and eventually vertical mathematisation. This would be true to the RME philosophy that views mathematics as something that should be constituted by the learners themselves, and so it makes sense to 'step back' whenever the need arises so as to connect with the point at which the learners are active in the processes of constitution.

Ultimately teachers need to bear in mind that the constitution of a new mathematical reality through the use of emergent models is a process in which 'the model actually is shaped as a series of signs, in which each new sign comes to signify activity with a previous sign in a chain of signification' (Gravemeijer, 1999, p. 155). As a result, it is worth taking to heart the advice for the teacher to be open to adapting and elaborating upon their initial instructional design as and when the need arises, so that what is learned from preceding activities informs what is to follow.

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CHAPTER 13

Characterizing Responsive Primary Mathematics Teaching in the South African Context

Lawan Abdulhamid

Research Context

In the South African context, concern about low learner performance in mathematics at all levels has led to increasing attention to the nature of teachers' mathematics knowledge and pedagogy in mathematics classrooms. Gaps in the mathematical knowledge base of primary teachers in South Africa are frequently reported (National Education Evaluation & Development Unit [NEEDU], 2013; Taylor & Taylor, 2013; Venkat & Spaull, 2015), with small-scale studies revealing incidences of limited opportunities for learners to understand mathematics in coherent ways (Venkat & Naidoo, 2012). Limited understanding of progression has also been pointed to in studies, noting the ongoing use of, and sometimes a 'pulling back' into, concrete counting approaches to working with numbers instead of moving forward into more efficient, abstract strategies (Ensor et al., 2009).

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Classroom evaluation practices in primary schools in South Africa provide particularly fertile ground for examining the nature of mathematical knowledge for teaching. Hoadley's (2006) study, driven by sociological concerns about differential access to knowledge for poorer and wealthier children, noted the prevalence, in working class schools, of teaching characterized by an absence of evaluative criteria (Hoadley, 2006). Hoadley described this practice in the following terms:

> The teacher engages in other work in her space and is not seen to look at what the learners are doing. She makes no comment on the work as it proceeds. No action is taken to ascertain what the learners are doing. (p. 23)

The consequence of this practice is a situation in which learners may well remain unaware of the extent to which their offers and narratives are 'endorsable' from a mathematical perspective. Importantly, Hoadley has noted that this absence of evaluative criteria represents a feature that has not been described as common in the wealthier country contexts in which the theoretical notions of evaluative criteria were initially developed. In these wealthier contexts, attention has been given to weaker and stronger framing of evaluative criteria, rather than an absence of evaluation (Bernstein, 1990). This particularity leads to a motivation for studying primary mathematics teaching development in relation to the kinds of 'in-the-moment' responses offered by South African teachers.

Broader issues and policies in the South African terrain also feed into the ways in which teacher responses are configured. Highly procedural orientations (Ally & Christiansen, 2013) coupled with selections of low cognitive demand tasks have been noted (Carnoy, Chisholm, & Chilisa, 2012). Chorusing practices, involving collective chanting of answers, have raised concern in relation to the lack of openings for individuation of learning and evaluation thereof (Hoadley, 2012). Conversely, Venkat and Naidoo (2012) also point to a lack of move of individual offers into the collective classroom space in primary mathematics pedagogy. Concerns about curriculum coverage and pacing in primary mathematics pedagogy (e.g. Reeves & Muller, 2005) led to calls for, and subsequently, moves toward, much more tightly prescribed national curriculum specifications. Thus, currently, national mathematics curricula specify content coverage, sequencing and pacing at weekly levels (DBE, 2011), with provincial-level interventions providing teachers with scripted lessons at the daily level (GDE, 2011). The press for coverage and standardized pacing further works against openings for responsive teaching.

Situating the Study in the International Context

A body of recent international research writing (see Chick & Stacey, 2013; Huckstep, 1999; Mason & Davis, 2013; Mason & Spence, 1999; Rowland & Zazkis, 2013; Turner & Rowland, 2011; Watson & Mason, 2005) testifies to specific and current ongoing interest in the ways in which teachers' mathematical knowledge is brought into play in the context of 'in-the-moment' teacher–students interactions in mathematics classrooms. Much of this body of work tends to work from a 'base' in which some evaluation of learner working is a given, in marked contrast to the South African context briefly described above.

In the international research base, teachers' 'in-the-moment' responses to learners' offers thus tend to be analyzed in relation to *what* opportunities for learning they open up, rather than initially, for *whether* learners' offers are acknowledged or evaluated at all. Thus, Ball, Hill, and Bass (2005) provide examples relating to sizing up the extent of generality of an offered procedure, and responding with appropriate follow-up questions or tasks, as instances of what it means to teach responsively. This contrast led to the need for a more grounded approach to characterizing the situations in which responses to learner offers were given, and then analyzing the nature of these responses. These situations form the central empirical base across lesson sequences drawn from four primary teachers' classroom practices focused on aspects related to additive relations in my broader doctoral study.

In this chapter, I report on teacher 'elaborations' as a metaphoric lens to examine responsive teaching, as a practical basis for developing responsive teaching actions, and as a means to 'bring into dialogue' some of the ways of thinking about supporting responsive teaching noted in the international literature with the specificities of the South African context. By 'elaboration,' I mean a form of explaining of mathematical ideas/concepts offered by a teacher in response to learners' inputs in the mathematics classroom.

Theoretical Assumptions

The constellation of factors noted above, on the one hand, makes seeing what the international literature describes as responsive teaching relatively unlikely in the South African context. On the other hand though, it also makes shifts toward responsive teaching from a low base important to understand within teacher development. In this section, international theorizations of what responsive teaching involves are summarized, as they provide guides for aspects to look for in empirical data. The ways in which these aspects occur in the South African landscape are then exemplified later in the chapter, with commentary on why these exemplifications are both important developmentally, and yet contrastive from international literature and theory.

Attention to learners' contributions within lessons has historically formed a key part of constructivist views of learning. While constructivism as a theory of learning has been widely recruited to understand learning and learners, there have also been calls for pedagogy to fit with learners' ways of learning mathematics within constructivist perspectives (Bauersfeld, 1995; Cobb, Yackel, & Wood, 1993). Wood, Cobb, and Yackel (1993) emphasize that

> …teachers must … construct a form of practice that fits with their students' ways of learning mathematics. This is the fundamental challenge that faces mathematics teacher educators. We have to reconstruct what it means to know and do mathematics in school and thus what it means to teach mathematics (p. x)

In response to this call, Simon (1995) advocates a theoretical model for reconstructing mathematics pedagogy that aligns with both sociological and cognitive constructivist perspectives. Central to Simon's model is the 'creative tension between the teacher's goals with regard to student learning and his responsibility to be sensitive and responsive to the mathematical thinking of the learners' (p. 114). This viewpoint provides insights into the possibilities for teaching informed by constructivist perspective, with this teaching seen as improvisational in order to be responsive to emergent learning.

An assumption drawn from the literature and theory underpinning this study is that responsive teaching is creative and fundamentally improvisational because if teaching is not responsive to emergent learning (i.e. entirely directed by the teacher), then learners cannot co-construct their own knowledge (Borko & Livingston, 1989; Erickson, 1982; Sawyer, 2004; Simon, 1995). Teachers need to constantly listen to learners' contributions and evaluate these in ways that create opportunities for learners to co-construct new knowledge. Sawyer (1999) argues that all creativity is an emergent process that involves a social group of individuals engaged in complex, unpredictable interactions. To better conceptualize teaching as an art of creativity, Sawyer uses the metaphor of teaching as *disciplined improvisation*. This is indicative of the collaborative and emergent nature of effective mathematics teaching. In Fig. 13.1, I depict the complex

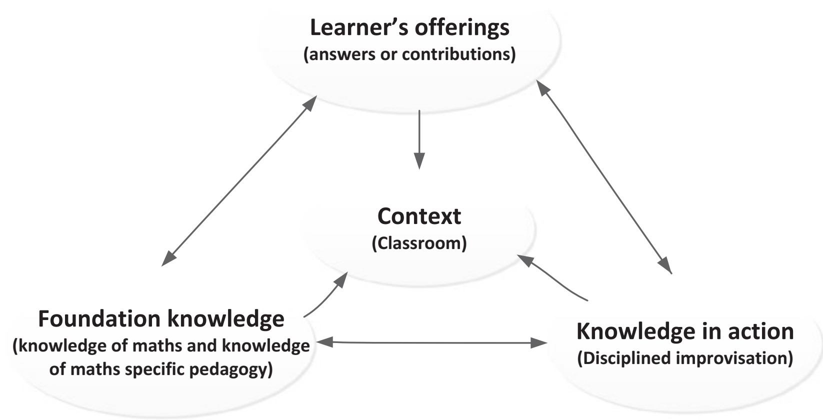

Fig. 13.1 Domain of teachers' knowledge in the context of responsive teaching

interrelationship between what Rowland, Turner, Thwaites, and Huckstep (2009) describe as teachers' 'foundation knowledge'—the teacher's theoretical knowledge and understanding of mathematics *per se* and knowledge of mathematics specific pedagogy; and knowledge in action shaping each other in responses to learner offerings in the mathematics classroom.

Effective teaching, in Sawyer's (2004) view, takes learners' offerings into account because it has to be responsive to the levels of different learners' thinking if they are to construct new knowledge. According to Sawyer (2004):

> …conceiving teaching as improvisational emphasizes the interactional and responsive creativity of a teacher working together with a unique group of students. In particular, effective classroom discussion is improvisational, because the flow of the class is unpredictable and emerges from the actions of all participants, both teacher and students (p. 13).

This emergence requires that the teacher quickly and improvisationally translates her own mathematical knowledge into a form that is responsive to the learner's level of knowledge in-the-moment. This aspect of mathematics knowledge is termed by Rowland et al. as contingency knowledge in their formulation of the knowledge quartet framework (Rowland, 2005; Rowland, Huckstep, & Thwaites, 2005; Rowland et al., 2009). Mason described this form of knowing as 'knowing-to-act in the moment' (see Mason & Johnston-Wilder, 2004; Mason & Spence, 1999) or the ability to think on one's feet as a reflective practitioner (Schon, 1987). Lampert and Ball (1999, p. 39) recommended that 'teachers be prepared for the unpredictable,' because they will have to 'figure out what is right practice in the situation' and cannot entirely depend on experts' advice on what to do.

While conditions in the South African context suggest that teachers' mathematical knowledge base is frequently weak, the sampling of teachers for the broader study worked with relatively favorable conditions by selecting teachers who had performed at higher levels on mathematics and mathematics for teaching assessments in the Wits Maths Connect Primary (WMC-P) project's in-service 'primary mathematics for teaching' course. All four teachers achieved at above 60% in the 2012 course's mathematics post-test, in the context of a cohort mean of 56% ($n=33$). This sampling allowed me to see possibilities for responsive teaching in the South African context in 'bottom-up' rather than 'top-down' ways, thus heeding the cautions that have been widely noted in the international literature about the dangers of uncritical policy and framework borrowing (Ensor & Hoadley, 2004).

A key aspect of adapting international theorizations to the South African conditions relates to my reading of contingency or 'in-the-moment' situations that offer grounds for responsive teaching. This includes teacher response to all learner offers (correct or incorrect answers and insights) during instruction. Some of these responses would be viewed as 'predictable' and amenable to planning in the international literature, and thus, may not be considered as contingent responses in Rowland et al.'s terms. I am reading it in this way because of the absence of evaluation that has been highlighted in the South African context described above.

Methodology

This study took place as a follow-up to the 2012 yearlong 20-day in-service WMC-P project's primary maths knowledge for teaching course, in which 33 teachers, drawn from the project's ten partner primary schools, participated. Four teachers with relatively strong foundation knowledge were purposively selected for the follow-up study. Data sources for the broader doctoral study consisted of two cycles of lesson observations and subsequent Video-Stimulated Recall interviews with reflections guided by the structure of Rowland et al.'s 'knowledge quartet'. A total of 18 lessons from the four teachers were video-recorded in 2013 and 2014.

I took a grounded theory approach (Glaser & Strauss, 1967) to data analysis of the lesson enactments. The use of this approach was considered for two reasons: first, the context outlined in the opening sections meant that existing theories developed in the global North provided limited purchase; and second, Rowland et al.'s (2005) development of codes constituting their initial knowledge quartet categories had been developed through this approach.

In using this approach, I first identified incidents where learners provided a mathematically incorrect offer to a problem, and/or where teachers' provided mathematically orientated responses to learner offerings in the lessons. Through inductive processes of constant comparison across the 18 lessons, and clustering for similarities, two broad categories emerged at an early stage: the teacher choosing to either provide no elaboration (PNE), or provide elaboration (PE). Providing no elaboration involved either ignoring the learner offer, or acknowledging an offer, but then moving on with the lesson. Instances of the provision of elaborations could be categorized into the following kinds of responses:

- response to a learner's incorrect answer
- response to what the teacher viewed as an inefficient solution action
- response to an individual learner's insight by projecting insight to the collective classroom space (collectivization)
- response to a whole class chorus offer by assessing individual learners' awareness of this offer (individuation)

In the broader study, these categories emerged through an inductive analysis process, with evidence too, of change over time in the nature and extent of responsive teaching (Abdulhamid, 2016). In this chapter, my focus is on illustrating these categories of contingent response, noting the background to the incident, verbatim evidence in the form of transcript excerpts, followed by interpretation of the incident and its consequences in relation to responsive teaching.

Responsive Teaching Categories

Response to a Learner's Incorrect Answer

Incorrect answers are an intrinsic part of all learning situations. While often viewed as 'inconvenient' within teaching, there is broad agreement in the literature that errors and misconceptions are a natural stage in knowledge construction and thus inevitable (Askew & William, 1995; Vosniadou & Verschaffel, 2004). Askew and William (1995) argue that it is impossible to teach mathematics in a way that can avoid learner's incorrect answers and misconceptions, and that it is the responsibility of the teacher to uncover and deal with learners' errors in the classroom. Some suggestions were offered by Koshy (2000) on how teachers can use learners' incorrect answers as teachable moments to support their learning. These include: awareness to the sensitivities of children; and the need to build a safe learning environment with openings for learning from incorrect answers.

Below, I present an example of a teacher's response to an incorrect answer drawn from Thandi's (all names are pseudonyms) classroom. Thandi taught Grade 3 learners in a disadvantaged 'township' setting. This incident, at the beginning of the lesson, was part of the oral mental starter activity and lasted for about five minutes. She had begun the lesson by asking learners to count in tens from 10–200, which her class had done without difficulty. She then asked learners to count forward again in tens from 33. Learners offered recurring incorrect counting sequences. My focus below is on Thandi's responses in this context of incorrect learners' offers.

Excerpt 1

9 T: Right! I want you to count again in tens forward again. Now, I want you to count from thirty-three; let's go. Thirty-three

10 C: Thirty-three, forty-three, fifty-three, [etc, in correct sequence], one hundred and three, one hundred and thirty-three, one hundred and forty-three (*all learners counting in chorus*)

11 T: (*teacher claps hands to stop the counting*). I want us to start at ninety-three, let's go: ninety-three, one hundred and three, one hundred and (*counting alongside with the class, teacher stops here waiting for the learners to complete*)

12 C: ninety-three, one hundred and three, one hundred and thirty-three (*all learners counting in chorus, still counted 133 after 103*)

13 T: (*teacher claps hands to stop counting*). Let's go one hundred and three

14 C: one hundred and three, one hundred and thirty-three (*softly some learners call out 113*)
15 T: Remember we are counting in 10s, one hundred and three plus ten?
16 L: one hundred and thirteen
17 T: Yes, let's start again ninety-three
18 C: ninety-three, one hundred and three, one hundred and thirteen, one hundred and twenty-three, one hundred and thirty-three, one hundred and forty-three … (*all learners counting in chorus, teacher counting along with the learners and stops at 103 while learners carry on correctly*)

While the counting sequence from 33 to 103 had run smoothly, learners struggled with counting in tens at 103, evident in the recurring incorrect offers. The teacher's awareness of the specific location of this problem is inferred retrospectively (as Sawyer notes as the case in improvisational situations) from her responses. With learners saying 133 after 103, Thandi is thus confronted with an 'in-the-moment' situation, where she has choices to make: whether to ignore the error, or to respond to it, and if the latter, then how to respond.

Stopping the counting sequence and asking learners to count again from 93 suggests Thandi's awareness of the learners' incorrect offer, as does her counting alongside the learners and stopping at 103, where the incorrect offer was given. Learners repeated the same incorrect offer by calling out 133 after 103. Thandi stopped the counting sequence again and asked learners to count from 103. The same incorrect counting sequence was repeated. At this moment, Thandi *juxtaposes* 'counting in tens' with 'plus ten' on the repeat offer of incorrect answer as seen in line 15. I interpreted Thandi's response as a form of elaboration that is constituted by '*verbal re-framing*' through her provision of an alternative, but equivalent, verbal representation for counting in tens. In this way, the idea of 'counting in tens' is related to 'plus ten,' thereby elaborating the meanings and operational processes that can be associated with counting in tens. However, it is also interesting that the 'general' idea of counting in tens is linked with a 'local' instruction to use the operation of adding 10–103, that is, a specific instance rather than a general instruction.

This verbal re-framing can also be understood from the systemic functional linguistics perspective of the idea of cohesive ties (Halliday & Hassan, 1985). Their notion of co-classification—presenting another instance of the same object—and referring to the two objects as the same thing, also connects and extends understandings. Therefore, the learners' incorrect offer was taken up here as a teachable moment, with Thandi linking the idea of 'plus 10' with counting in tens to get to the next number.

In the South African context, this kind of contingent response is important in the face of evidence of lack of evaluation (Hoadley, 2006), and evidence too of 'repetition' of the same instruction in the face of incorrect answers (Venkat & Naidoo, 2012), rather than the kind of elaboration seen above.

Response to What the Teacher Viewed as an Inefficient Solution Action

Elementary mathematics curricula worldwide advocate the need for increasing sophistication through the move from less to more efficient strategies and representations. Responding to inefficient learners' strategies or representations is a necessary step to support understanding of the connectedness of mathematical ideas, and a step that has been noted as limited in the South African context (Ensor et al., 2009).

I draw on an example of a teacher response to what she viewed as an inefficient solution action (inferred retrospectively again from the response) from Sam's Grade 4 classroom. This incident focused on a set of mental addition problems. Sam had pasted five addition problems on the chalkboard (with 5 further subtraction problems as well that were dealt with later) as shown in Fig. 13.2.

In the setup of the task, Sam made explicit to the learners that she wanted them to quickly work out the task while observing *something* that they were going to discuss upon the completion of the task. After learners had completed all ten problems in their mental maths notebooks, Sam got all ten answers written on the board from learner inputs. She then turned back to the class and asked, '*Who can show me how you were working out*

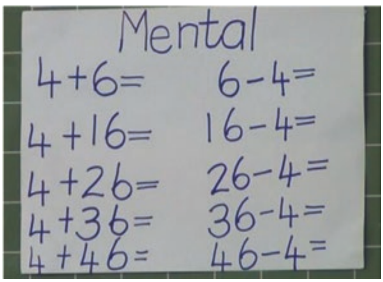

Fig. 13.2 Ten mental addition and subtraction problems

the sums?' Four learners were invited one after the other to show how they worked out the addition problems. The first learner (L1) drew 26 tally marks with repeated counting and re-counting of these tallies, and wrote a '+' sign followed by a symbol of 4 and 30 as the answer. Accepting this offer and asking for '*someone who did it in a different way*,' Sam invited another learner (L2) to respond to the sum 4 + 36. L2 wrote down 4 then 36 below and worked out the sum using column method. Once again, Sam accepted this offer, and asked for another alternative. One learner (L3) shouted, '*I am counting with hands*' and upon invitation, demonstrated starting from 4 and counting on 46 in ones orally (she said 5, 6, 7, 8, 9, 10 … 50). Sam pointed to another learner (L4), who she noted had been the first one to finish and asked L4 to show how she had worked out 4 + 46. L4 started with the bigger number, 46 and opening up 4 fingers one at a time, said, '47, 48, 49, 50'. Sam responded in the following ways:

Excerpt 2

155 T: She just said forty-six. She didn't write all these (*points to the tallies)*, this one was longer, and this one was longer (*points to column addition*). Did you hear [L3's]?
156 L: Yes
157 T: So, which one is the quickest of those four girls?
158 C: Mpumi (*referring to L4*)
159 T: Mpumi isn't it? Mpumi took the bigger number and added the four and she was the first one to finish. Did you notice that?
160 C: Yes

The addition example sequence presented by Sam provides openings for using the patterned construction to build connected derived facts—that is, using the fact that 4 + 6 is equal to 10, to work out 4 + 26 for example, is 10 more than 20, which is 30; and 4 + 46 is 10 more than 40, which is 50, and so on. However, learners' offers showed treatments of the problems as individual sums and also, in *inefficient* ways. Sam's response in lines 157–159 draws explicit attention to this inefficiency by pointing to 'the quickest' strategy as a criterion she values, and providing a partially generalized narrative in relation to the strategy offered by L4: 'Mpumi took the bigger number and added the four and she was the first one to finish. Did you notice that?' I coded Sam's response here as a form of elaboration that is constituted by '*focus on more efficient strategy*'—and is important in the context of evidence of lack of move from concrete to abstract strategies in teaching (Ensor et al., 2009). However, the openings provided to build efficiency by linking between the addition examples were not exploited here, with this feature noted in earlier South African work at primary (Venkat & Naidoo, 2012) and secondary levels (Adler & Venkat, 2014).

Response Projecting Individual Learner's Insight to the Collective Classroom Space

Previous research in the South African context suggests limited projecting of individual responses to the collective classroom space (Venkat & Naidoo, 2012). Earlier research notes that these issues go unattended even when teachers have noticed them. These 'responding' moves to attend to an individual learner by projecting and developing with whole class is crucial to broadening opportunities for learning (Brown & Wragg, 1993; Rowland et al., 2009). Thus, the quality of such responses is determined by teacher's knowledge of mathematics, knowledge of mathematics pedagogy and ability to draw upon this knowledge in an improvisational way to build on the learner's insight.

To exemplify this type of response, I return to Thandi's Grade 3 classroom, where she asked one learner, Solly, to work out a sum 9 + 163 on the board and requested him to work with the whole class. Solly drew an empty number line and wrote down 163 at a mark toward the start of the line. He then turned to face the whole class and asked them what to do next. One learner, Ntuli, offered, 'plus ten,' requiring Solly to make a forward jump of ten. Solly rejected this offer. Thandi interrupted:

460 T: Maybe she is right. Why do you say no? Let's see what she has to say. Let her do the rest of the sum; then we will understand her – why she says plus ten. Do what she asks you to do?

Solly carried on with the offer given by Ntuli following the teacher's interruption. He made a forward jump of 10 and landed on 173. Solly asked the class what to do next. Learners raised their hands to respond, but Thandi interrupted again and asked learners to put down their hands to allow Ntuli to complete the sum. Ntuli asked Solly to minus one by making a backward jump of one. Sipho made the backward jump and landed on 172 as the answer as shown in Fig. 13.3.

In this incident, Thandi created greater interactions among learners in the classroom. Thandi didn't allow Solly to work out the problem alone on the chalkboard; she insisted on his working with the whole class. Therefore, Solly acted like the teacher facilitating the working out of the

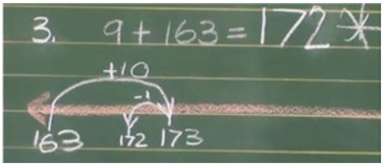

Fig. 13.3 Ntuli's representation for the solution of the sum '9 + 163'

problem on the board. Solly asked questions, imitating the teacher's action by pointing to an individual to respond. When Solly rejected Ntuli's suggestion to add ten, Thandi asked Solly to work out the problem on the board following Ntuli's instructions. Thandi's response at this point both commended Ntuli for her offer, while also providing some rationale for the solution action in the following terms:

Excerpt 3

486 T: The sum says nine plus one hundred and sixty three. So, since we said we start with a bigger number, we start with one hundred and sixty three and we are supposed to add nine. So our sum stands like that (Teacher writes '163 + 9=' on the board) started at one hundred and sixty-three, it says add nine (Teacher points at the number line) but Ntuli says we should add ten do you see that?
487 Class: Yes
488 T: Then we got to what number?
489 Class:One hundred and seventy three
490 T: One hundred and seventy-three and Ntuli added ten instead of nine so did she add more or less than nine?
491 Class: More
492 T: More by how many?
493 Class: One
494 T: By one so she went to subtract
495 Class: One
496 T: isn't it?
497 Class: Yes

I interpreted Thandi's response to Ntuli's solution action as a form of elaboration that is constituted by '*projecting individual learner's insight to the whole class*'. This is evident in the way Thandi quickly and improvisationally responded to the learner's compensation strategy of addition.

In the South African context, this kind of response is important in the face of lack of opportunities for learners to co-construct their own knowledge through exploratory talk among teacher and learners (Mercer, 1995). The opportunities inherent in this kind of response are captured well in Walshaw and Anthony's (2008) comprehensive review of literature around what effective mathematics teachers actually do to deal with classroom discourse. These authors identified creating a classroom space that encourages sharing of insightful learner's ideas as an important domain for building opportunities for learning.

Response Individuating Whole Class Chorus Offers

As noted earlier, classroom instruction involving extensive whole class recitation work, with little or no evaluation of individual learner's understanding has been noted as prevalent in South Africa (Hoadley, 2012). In contrast to this situation is an example from Bongi's Grade 4 classroom where she asked learners to count in twos starting from seven. Learners counted 7, 9, 11, 13, and so on. When they got to 39, Sam asked learners to stop, and she proceeded in the following way:

Excerpt 4

81 T: Let's stop. What is the next number? Yes, Realogile (teacher points to a leaner)
82 L: Forty-one
83 T: Forty-one. What will be the next number? (Points to another learner)
84 L: Forty-three
85 T: Forty-three, what will be the next number? (Points to another learner)
86 L: Forty-five
87 T: Forty-five, and our finishing number? (Points to another learner)
88 L: Forty-seven
89 T: Forty-seven, who can tell me, did you see any pattern? Tell me
90 L: They are odd numbers
91 T: Yes, it's only odd numbers we are counting, but we are counting in twos, isn't it!
92 Class: Yes

The task enactment here moves from oral class chant to assessing individuals' understanding of the counting sequence. Bongi does this assessing several times with different individual learners as seen in excerpt 4. There are two important features here: first, checking whether individual learners can produce the focal counting sequence rather than 'hiding' within the whole class chant; second, Bongi, in line 91, also elicits and elaborates more general rules for the generation of the sequence, that is, that they are all 'odd numbers' (offered by learners) and that odd number sequences involve 'counting in twos' (Bongi's elaboration). Thus, while Bongi is checking with individual learners, she pays attention to the ways in which the pattern of numbers is being generated. I interpreted Bongi's response as a form of elaboration that constituted '*individuating chorus offer*'. This kind of response also offers potential for evaluating individual understandings in ways that counter norms described as prevalent in South African primary mathematics classrooms.

Implications for Teachers' Classroom Practices

I conclude this chapter by delineating the implications of the findings of the study for teachers' classroom practices in the South African context. The four responsive categories developed all present a move forward on the absence of evaluative criteria noted in the literature, and are thus epistemically important from the perspective of mathematical learning. Writing in the educational development terrain, Schweisfurth (2011) has pointed out that the promotion of learner-centered education as a desired feature of teaching from developed country contexts into developing contexts in the global south has been an enterprise 'riddled with stories of failure' (p. 425). Her suggestion for moving forward is to search for 'compromise' solutions focused on 'stages of implementation' toward desired practices.

My sense is that the categories of responsive primary mathematics teaching developed in this chapter following teachers' participation in an in-service teacher development program present some of these starting point stages. They offer 'home-grown' rather than 'imported' descriptions of pedagogies with potential for building the kinds of responsive in-the-moment decision making that are widely described as important in the mathematics education literature as important for supporting emergent mathematical learning. Being aware of these kinds of shifts is also particularly important within teacher education for supporting moves toward responsive teaching. Close attention to the nature of teacher responses thus represents openings for moves away from deficit characterizations based on absences, to staging point characterizations directed toward improvement.

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CHAPTER 14

Key Aspects of Communities of Practice That Enable Primary Maths Teacher Learning

Peter Pausigere

Introduction

Describing why communities of practice (CoPs) are essential for primary maths teacher learning, this chapter dialectically explains the nature of primary maths teacher transformations in relation to their mathematical identities and histories. The process of describing primary maths teachers' learning relative to their mathematical histories and through metaphors developed from our earlier work (Pausigere & Graven, 2014). This chapter goes further and explores the resulting participation semantics and metaphors in connection to three identified CoP enablers (numeracy concepts, classroom and community ethos and practices). These enablers were articulated by teachers in relation to their participation in the primary maths in-service CoP called the NICLE. Thus, teachers with a history of mathematical competence *reinvigorated* their mathematical identities whilst identities of teachers with weak mathematical histories were *remediated* and *activated*. Thus, all the sampled NICLE educators fore-

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grounded taking up numeracy-*domain* concepts and classroom teaching *practices* and embracing the supportive participatory ethos through participation in NICLE. Therefore, this study's contribution relates to linking teacher evolving mathematical identities to key CoP affordances that enable teacher learning.

The primary maths teacher learning transformation semantics and mechanisms and the teacher participation affordances are informed by the situative-participationist theoretical framework (Lave, 1996; Lave & Wenger, 1991; Sfard & Prusak, 2005; Wenger, 1998) and the CoPs concept (Wenger, McDermott, & Sydner, 2002). This study accentuates the sociocultural perspective that learning and identity formation are intertwined and highlights the CoP affordances elements of *domain*, *community* and *practice*. Exploring empirical data using sociocultural theoretical perspectives, this chapter expounds that participation in primary maths CoPs enables primary maths teachers to fashion their identities and take-up key numeracy-domain concepts, classroom teaching practices and take-on NICLE CoP's friendly, professional and respectful ethos.

Contextual Background to the Study

The NICLE primary mathematics teachers' professional development programme forms the empirical field of research to this study. This in-service teacher professional development initiative has been designed and conceptually framed as both a CoP (Wenger, 1998) and a community of inquiry (CoI) (Jaworski, 2006) approach to teacher development. The NICLE initiative focuses on numeracy (primary maths) teacher development within the foundation and the intermediate phases' critical transitions Grades 3–4 in 12 core participating primary schools in the greater Grahamstown area, in the Eastern Cape province of South Africa. The primary maths teacher development programme has been in existence for five years, since 2011, and currently has 43 regularly attending teachers, including principals and deputy principals, who attend NICLE seminars and inquiry sessions.

The establishment of NICLE teacher development programme falls within the strategic developmental research vision of the South African Numeracy Chair project aimed at searching for sustainable ways forward in navigating the many challenges of mathematics education in South Africa. Generally, South African primary maths education has been noted as facing acute problems with reports of teachers' poor conceptual and pedagogical knowledge (Fleisch, 2008).

Given the local national crisis context, attempts are being harnessed to provide for professional development models that results in quality and effective teaching and learning of primary maths. To this effect, Adler (2000) has illustrated that the social practice theory is a useful and powerful framework for the continuous professional development of maths teachers. Locally (Adler, 2000; Graven, 2004) and internationally (e.g. Jaworski, 2006), there have been maths teacher professional development programmes underpinned by the CoP framework. Literature indicates that maths teacher professional learning communities results in successful teacher participation and have great potential worth exploring further. Thus, this research interrogates both the nature of teacher learning and activities, relations and forms of participation affordances within a CoP-informed teacher professional development programme.

The Situative-Participartionists Theoretical Framing

This study is theoretically informed by the situative-participationists framework; that is, the social practice learning theory and the CoPs concept (Lave, 1993, 1996; Lave & Wenger, 1991; Wenger, 1998; Wenger et al., 2002). Under the situative perspective, learning within a CoP is a dual process transforming knowledge and "who we are"—it is a process of becoming involving the construction of identities of participation and entailing "new ways of knowing" (Lave & Wenger, 1991; Wenger, 1998). This sociocultural-participationist perspective of connecting identity formation and learning is central in understanding the processes of teacher learning and identity formation within NICLE.

Lave's (1996) notion of *telos* and Sfard and Prusak's (2005) definition of identities as stories will enable the study to interrogate and focus on teacher learning mechanisms and the processes of primary maths identity formation within NICLE. To operationalise identity and make it an analytical tool for educational research, Sfard and Prusak (2005, p. 14) described identity "as a set of *reifying, endorsable* and *significant* stories about a person". Reification involves the discursive activity of portraying our experiences abstractly. Reifying qualities come with the use of verbs such as "*be, have* or *can* and their derivatives" and also with adverbs that "stress repetitiveness of actions" such as *"always, never, usually* and so forth" (Sfard &

Prusak, 2005, p. 16). An endorsable story "faithfully reflects the state of affairs" as objectively portrayed by the "identity-builder". According to Sfard and Prusak (2005, p. 17), the most "significant stories are often those that imply one's memberships in, or exclusion from, various communities". Sfard and Prusak therefore set specific criteria on the type of teacher narratives to focus on in investigating the research participants' nature of learning and their evolving identities. Thus from the teacher interactive interviews, I glean those utterances and learning stories that are *reifying*, *endorsable* and *significant* and these will inform the discussion and key arguments in this study.

Also useful for interrogating and discussing the "process of coming to know" within NICLE (Matos, 2009, p. 171) is Lave's (1996, p. 156) notion of *telos* which concerns "changes implied in notions of learning". Lave's idea of *telos* is acknowledged in maths teacher education for being an important analytical tool for describing the learning process within maths CoPs (e.g. Matos, 2009). Lave (1996, p. 156) defined *telos* as the "direction of movement or change of learning (*not* the same as goal directed activity)". She went on to elaborate that *telos* encourages "a focus on the trajectories of learners as they change". Such an assertion serves to show that learning within a CoP is dually influenced by the community in as much as it is an individual initiative or motivation. It is important to point out that I reconstruct and combine Sfard and Prusak's (2005) identity-as-*story* construct with the notion of *telos* (Lave, 1996) to propose a new term for learning stories—*stelos*.[1] Sfard & Prusak's operationalised notion of identity and Lave's (1996) concept of *telos* provide the study with analytical and linguistic tools to explore and describe how numeracy teacher learning identities evolve through participation in a primary maths teacher learning community.

Wenger et al.'s (2002) three fundamental elements of a CoP, namely, *domain*, *community* and *practice*, *resonated strongly with teacher utterances gathered through interviews and journals in explaining the* NICLE learning affordances. Insights from Wenger et al. (2002) allow the study to relate the primary maths teachers' transformation process and the CoP teacher learning opportunities. These three elements are thus used as a structuring device for investigating the activities, relations and forms of participation within the teacher learning community which enabled the evolving strengthened primary maths teacher identities.

The *domain* according to Wenger et al. (2002, p. 31) is the "*raison d'être*" that "brings people together and guides their learning".

Members of a CoP domain may share a profession or discipline (e.g. history teachers), have the same job or role, or deal with the same clients. In the same way, primary maths teachers are members of the NICLE's domain—with the NICLE's domain being primary maths education. Wenger et al. (2002, p. 34) defines *community* as a "group of people who interact, learn together, build relationships, and in the process develop a sense of belonging and mutual commitment". CoP members participate in issues important to their domain and interact regularly to develop a shared understanding of their domain and an approach to their practice. A community also consists of a "field of experts" that recognises and validates innovations in a community. The element of *practice* encompasses the body of shared knowledge, resources and a set of socially defined ways of doing things efficiently, which enables participants to learn the craft and become practitioners (Wenger et al., 2002). Relating this aspect to NICLE, one can say the in-service CoP's practices are the primary maths teaching and learning practices. Theoretically, the three fundamentals of a CoP relate to the notion of identity and identity transformation.

The study employs these three concepts as broad categories encompassing the learning affordances or enablers described by teachers in NICLE. Also important for this study and in making connections between the teacher learning processes and the educed NICLE affordances is Wenger et al.'s (2002, p. 44) suggestions that the three elements can be used to interrogate different aspects that participants "draw" from a community. Using Sfard and Prusak's (2005) educationally operationalised identity construct and Lave's (1996) notion of *telos* and connecting these to the CoP fundamental elements (Wenger et al., 2002), the study unpacks the primary maths teachers learning processes to the different affordances aspects that the teachers reportedly drew through participation in NICLE. Thus, the semantics used to describe NICLE participation and identity formation experiences are further explored to enlighten the teacher learning affordances taken up and embraced by the sampled teachers.

Qualitative Educational Interpretive Approach

The broader longitudinal doctoral research (Pausigere, 2014) from which this chapter arises, used a qualitative educational interpretive approach which employs basic qualitative methods to make interpretations within the educational field. The interpretive approach has an anti-positivist orientation, emphasising that knowledge and understanding can be obtained through the subject's lived experiences and interpreting and giving meaning from a subjective perspective (Merriam, 2001). This approach provides an opportunity to gain deep insights and interpret the participant's experiences. Typical of qualitative interpretative approaches, this study involved prolonged fieldwork that gathered data across a two-year period, from March 2011 to December 2012, using four basic data collection strategies of interactive interviews, participant observations, reflective journals and document collection and analysis.[2] Employing the qualitative educational interpretive methodology would enrich and enhance my description and interrogation of the nature of teacher learning and participation affordances within the primary maths professional development programme.

The research study sample consisted of eight primary teachers drawn from NICLE and selected through a combination of purposive and stratified sampling strategies. I intentionally selected teachers who actively participated and frequently attended NICLE sessions and additionally those teachers who were willing to be part of the broader research study. Using the stratified sampling method and with the intentions of providing a relatively representative sample of the general population of primary teachers in South Africa, the selected teachers were from four different categories of schools in the local education system. Two are from a *Farm school*, which has multi-grade classes; two are from an *African township school*; two are from historically *coloured school*s and the other two are from an *ex-model C* preparatory school, in a formerly white area. I also chose my sample to be representative of a range of grades across NICLE teachers. Thus in this sample of teachers, two are intermediate phase male teachers, (Calvin and Robert), with the former only teaching Grades 5 and 6 maths classes at his school at which he is the deputy principal. One of the participants is an all-subjects multi-grade teacher of Grades 4—5 (Everton). Of the five Foundation phase teachers, two are Grade R teachers (Mary and Edna) with the other three teachers teaching Grade 3 classes (Ruth, Melania & Pamela). Notably, all the foundation phase teachers in the sample are female. This is also the case for the larger group of NICLE teachers. For this reason, my sample has more female than male teachers. All the teachers' names are pseudonyms.

Discussion

In this part of the chapter, I analyse the key data themes on teacher learning identity transformation and participation affordances informed by the sociocultural theoretical orientations. The study research findings show how six of the sampled eight teachers with a history of maths competence—"positively valued maths identities", *reinvigorated* their maths identities, whilst two of the sampled teachers with identities that previously shied away from maths have their "devalued or negatively valued identities" (Lave, 1993, p.77) maths identities *remediated*. Both educators emphasised their teacher learning identity transformation as entailing understanding and appropriating numeracy-*domain* concepts and numeracy teaching and learning *practices* into their classrooms and embracing CoP ethos through participating in the primary maths CoP.

The negative or positive maths teacher identities arise from the teachers' stories about their school maths learning and/or classroom maths teaching experiences. Further, maths education studies and the school context also strengthened the mathematical identities of teachers with valued maths identities. Ruth, Melania, Robert, Calvin, Edna and Everton had positively valued maths identities, whilst Mary and Pamela maths learning, teaching and life maths experiences expressed negative valuation of being and doing maths. To provide rich, thick description and in-depth qualitative data on how differently teachers with valued and devalued maths identities learn, I selected the most illustrative direct quotes from *some* of the teachers' interview utterances. The full data set is available from the author's doctoral thesis.

All the teacher utterances relating to their historical maths identities (in the second column of Table 14.1) meet the endorsability and significant criteria as has been explained before. Here, I briefly explain, for illustrative purposes, how Ruth's, Robert's and Mary's first statements indicate reification and reifying effects. Ruth's utterance that she has a "very good number sense", Robert's description of himself as "an authority in the intermediate phase mathematics" and Mary's articulation that she "never had a very good foundation for mathematics" illustrate the concept of reification, since these utterances show certain historically permanent qualities (Sfard & Prusak, 2005) of Ruth's, Robert's and Mary's relationship with maths. Their statements include use of verbs such as *have*, *I*, *am*, *you*, *never* and *an*, which have reifying qualities. These have been *italicised* in the quotes in the table.

Table 14.1 Teachers' articulated mathematical identities, verbs and phrases and the emerging metaphors describing their participation and learning experiences

Teachers' pseudonyms	*Articulated mathematical identity*	*Teachers' verbs/phrases describing their NICLE "reinvigoration" participation and learning experiences*
	Valued maths identity (+)	
Ruth	+ "*I* think *I have* got a very good number sense *you* know inherently" "When you look at the content that we were dealing with (in NICLE) I don't think that was new to me, no…I did do matric maths"	"So it has *supported* what can sometimes become tedious. It has *freshened* it up… may be it has *awakened* that interest again and it has *supported* my, developing my own understanding, thinking about the process and steps of teaching" "I think I will remember *the inspiration* and the energy…when I say energy I mean *energy for the subject the enthusiasm*…" "…I suppose it has *broadened* my involvement in it again. Yes I suppose it also *freshens* up my understanding and extensity as well"
Melania	+"I enjoy maths and hopefully my enthusiasm for maths spills over to the children…" "I spend a lot of time with maths because I enjoy it, so I try to carry over that element of enjoyment…"	"…just every now and then in your teaching career you need to have some *bit of energy* and you need *to be refreshed* you need to be *revitalised* and I think that's what it has been for me it just sort of *refreshed* me given me new ideas, *boosted my enthusiasm* again…." "…it just *sharpened my awareness* again …definitely *it saved my teaching* because I have communicated more with the children about how they are doing things…" "And I said *one tends to get into a rut* and it just helps you to *bring new and exciting ideas* into your classroom"

Robert	+"*I am an* authority in the intermediate phase mathematics" "I have pulled myself from history… teacher towards becoming a more maths-orientated teacher"	"I think *refreshment* is always welcome because I think sometimes you become so confident that you start to just *stagnate* and you don't entertain yourself with new ideas…" "…my understanding of mathematics was *stimulated* and it was *supported* because the little I had I felt it was *starting to grow…*" "So NICLE I would say *supports*. It *illuminates* or *makes it come alive*…so I think NICLE did *support* me in my studies and in my teaching practice"
Everton	+"I really enjoy my mathematics teaching." "Well, I prefer to teach mathematics than virtually anything else, except languages. But all those subjects I enjoy, but mathematics I particularly enjoy"	"… *reinforcing* what I have, what I already do put into practice. Its learning, it's kind of like *support system* for a lot of things I do" "…that *broadened* my understanding of mathematics and how to get it across to children" "*A light shining in on my maths teaching*…giving me more *insigh*t on my maths teaching. It's been *a point of growth* for me as a teacher"
Edna	+"I love numbers… I have a love of numbers" "My strong point in the classroom is also maths. I know I am not supposed to do it but I tend to spend a lot more time with maths than the others"	"…the only way that I will be able to help the learners around me is by to be *better equipped* and to take every opportunity that coming my way, to be *able to grow in that field…*" "I *enriched myself* in order to give a better [lesson]…to carry it over better to the learners"
Calvin	+ "I like teaching maths, in fact I wouldn't like to teach another subject" "At least I'm comfortable with content. I know…I know intersen. I know the content of mathematics. I can teach anything from grade 4 to 6…I enjoy teaching maths"	"…*my sense of mathematics are being tested*, and I am not talking of content based maths only but classroom management"

(*continued*)

Table 14.1 (continued)

Teachers' pseudonyms	*Articulated mathematical identity* *Valued maths identity (+)*	*Teachers' verbs/phrases describing their NICLE "reinvigoration" participation and learning experiences*
	Devalued maths identity (–) – stunted	**Teachers' verbs/phrases describing NICLE's "remediating and activating" participation experiences**
Pamela	– "Because I, before I have attended NICLE I had problems…the problem is when I am teaching it to the kids, I was not sure whether I was doing it in the right form, but after NICLE now I can say I know…" "it (NICLE) has taken some (mathematical) fears that I did have"	"We are having specialists [in NICLE] *they do address our fears.* Sometimes as mathematical teachers *we do have fears* of how we are going to do this in my class. But with NICLE they will invite somebody and *that somebody will address your fears…*" "I am glad that I am part of this program. *It has changed my thinking*, it has also *helped me to love mathematics.* It has *taken some fears that I did have*". "Meeting experts in mathematics. You see *the fears that we sometimes have…* sharing with other teachers from other schools".
Mary	– "*I never* had a very good foundation for mathematics especially in High school so *I* kind of lost, in sense *I* have lost interest" "…the maths for me would be more of a challenge because of the whole mind-set I grew up with. Being told you can't do maths and girls… don't have to worry about maths"	"I often feel very stupid compared to those other teachers…but I feel like *I am still growing*" "Only really this year do *I really regard myself as a numeracy teacher*, if you asked me last year I would have said, *oh no I can't do Maths*, but *this year I can see potential to actually call myself a numeracy teacher*". "…it is helping us develop professionally…*developing my self-confidence with mathematics* and I love the sharing when I do my group work with things that come up, um, you know with the other teachers, sharing experiences"

The table summarises the teachers' articulated mathematical identities and links these to the verbs and phrases describing their participation and learning experiences from which emerge the synonyms and metaphors that dialectically explain the nature of primary maths teacher learning within an in-service CoP.

Reinvigoration

The term "reinvigoration" emerged from the verbs and phrases used by the six teachers with positive maths identities to describe their participation experiences in NICLE. The verbs and phrases in the third column of Table 14.1 can be substituted for or are synonyms of reinvigoration and arise from the teachers' NICLE learning stories. I coded the teachers' responses to their experiences in NICLE into three categories (*freshened, refreshed* and *stimulated; reinforcing* and *enriching;* and *phrases relating to growth*) all relating to "reinvigoration". The actual verbs used and the arising categories are in the same semantic field as the umbrella term *reinvigoration,* which describes the nature of participation in NICLE of teachers whom I regarded as having positively "valued" primary maths identities.

The term "reinvigoration" used by the teachers (independently of each other) to describe their NICLE participation experiences were not part of the NICLE discourse, neither did I employ this term during the interviews. *Reinvigoration* is thus the synonym-cum-metaphor that holistically captures all the teachers' verbs and phrases and in unison describes their participation and learning experiences in NICLE. Because of limited space, the table does not show the full range of verbs and phrases uttered by the teachers in the broader study, it presents the most outstanding and illustrative examples of the teacher utterances relating to "reinvigoration".

Remediation and Activation

In the study sample, I had two teachers—Mary and Pamela, whose identities before their participation in NICLE indicated weak or negative mathematical histories. Mary's and Pamela's negative mathematical histories had arisen through their unpleasant personal school maths learning, during their learning experiences in maths classes and through their own maths teaching classroom experiences. Furthermore, Mary's learning story reveals that gender stereotyping (that women cannot do maths) had negatively influenced her maths identity. Lave (1993, p. 77) gives terminology applicable to Pamela's and Mary's maths fears utterances. According to her, the statement that "we don't know real maths" is associated with "devalued or negatively valued identities". Both Mary and Pamela's stelos reveal that their school maths learning, maths class teaching or their life experiences *stunted* the emergence of positive maths identities. *Stunt* refers to life, school or career experiences or expectations that create trajectories that shy away from maths. The third column in Table 14.1 shows the distribution of the two teachers' verbs or phrases (utterances) relating to "remediation" and "activation".

Both Mary's and Pamela's mathematical challenges and fears were *remediated* through their participation in the NICLE community. The concept of remediation though emanating from empirical data is akin to the term "reconstruction" used by Lave (1993, p. 73). Reconstruction occurs as participants exorcise negative identities and gradually interpret and construct a community identity through life stories (Lave, 1993). Thus, both Mary and Pamela exorcised their past negatively valued mathematical identities through participation in NICLE and in the process *remediating* such identities and *activating* more positive maths identities. This shift to new and transformed mathematical identities was through engaging in the primary maths teacher learning community.

Relating the Transformation of Teacher Mathematical Identities to the Appropriation of NICLE Teacher Learning Affordances

Studies that have recontextualised the notion of CoP in maths teachers' professional development (Adler, 2000; Graven, 2004; Matos, 2009) confirm the empirical findings which show that participation in the primary maths CoPs professional development programme led to change and transformation of the teachers' identities, understanding and knowledge. However, there have been no studies linking or relating teacher evolving mathematical identities to CoP affordances and enablers that teachers take from teacher learning communities. Thus, the second half of this discussion explains the transforming teachers' mathematical identities in terms of the sampled participants' understanding, knowledge and practices. Both empirical data and data from Wenger et al. (2002) help illustrate what teachers appropriate from the primary maths learning community. The research findings focuses on the *affordances* relating to activities, relations and forms of participation that teachers experienced during their participation in NICLE and not the physical and printed resources given to teachers—as some of these materials were primarily intended for the learners, for example homework exercise books. Thus, discussing and analysing NICLE qualitative data from the teacher interviews and journal entries reveals that the sampled teachers mostly took-up primary maths domain concepts, numeracy classroom teaching practices and embraced NICLE CoP relations and participation ethos.

Half of the NICLE sessions reportedly "enjoyed" by the teachers were presented by invited numeracy education specialists, with several NICLE sessions, emphasising the benefits of learning numeracy in a fun and exciting way through alphabet maths games, dice and playing card games. Affordances were also noted in relation to access to watching NICLE teacher live classroom demonstrations and videos of classroom practice. The numeracy games, sessions by guest primary maths experts and lesson demonstration and videos were said by all the sampled teachers to enhance their teaching strategies and in the process promoting learner-engaging, creative and exciting mathematical learning approaches, thus allowing their pedagogical practices to evolve. Changing classroom practices were also reported in relation to the word problem-solving, the Singaporean and maths multilingual approaches. The rows marked *practice* in Table 14.2 show the distribution of specific classroom teaching practices indicated to have been take-up by the sampled teachers. The section marked *domain* shows the key specific numeracy concepts, mainly mental maths, the zero concept, number bonds and the four basic operations understood and appropriated by the teachers from the primary maths teacher learning community activities. Besides enabling teacher learning of key primary maths *domain* concepts and numeracy teaching and learning classroom *practices,* participation in NICLE also provided access to *community* relations and forms of participation which supported teacher learning.

Access to overlapping maths CoPs, NICLE's friendly and respectful regard for teachers as professionals, active practical hands-on experiences, engaging with primary maths experts and the encouraging of school teacher collegiality were the NICLE *community* ethos explained as enabling and promoting teacher learning. These teacher learning participation practices and relations—enablers—were reportedly promoted in the NICLE community and between community members. The full range of NICLE *community* relations and forms of participation encouraged in the primary maths teacher learning community are in the columns marked community in the table. Thus, the reinvigoration and activation or the transformation of teacher learning identities occurred in the NICLE community when teachers reported improved understanding and taking up of key numeracy-*domain* concepts, maths classroom teaching and learning *practices* and the taking on of the NICLE *community*'s friendly and mutual participation ethos.

Table 14.2 NICLE activities, relations and forms of participation that enabled teacher learning and what individual teachers indicated appropriating

NICLE teacher learning affordances & enablers	*Fundamental CoP element(s) it relates to*	*What individual teachers indicated taking from NICLE*							
		Calvin	*Robert*	*Melania*	*Edna*	*Ruth*	*Everton*	*Pamela*	*Mary*
NICLE numeracy games	Domain	–mental maths –number bonds	–mental maths –four basic operations	–counting, addition	–four basic operations	–mental maths	_	_	_
	Practice	–learner participation	–makes maths learning fun	–makes maths learning enjoyable –improved maths presentation	makes maths learning fun	–makes maths learning fun –alternative ways of teaching	–enhanced maths teaching strategies	–provides opportunities for learner engagement	–makes maths learning fun
Invited numeracy education specialist guest speakers	Domain	–understanding the zero concept –number sense	_	_	–zero concept –developing number sense concept	–understanding the zero concept	_	–understanding numerous fraction figures	–understanding the zero concept

	Practice	–teaching fractional mental maths – Singapore maths approach –using RUCSAC approach – constantly teaching fractions –importance of language in maths	–	–using the RUCSAC approach to solve word problems –improved the teaching and representation of fractions	–using RUCSAC approach to solve word problems – importance of language when teaching maths –improved teaching strategies	–teaching fractional mental maths –opportunity to engage learners	–Singapore maths approach –preparing baseline maths tests –constantly teach fractions –learner–engaging practices	–using the RUCSAC approach to solve word problems –improved the teaching of fractions	–using the RUCSAC approach to solve word problems –the importance of language in teaching maths
	Community	–enabled access to overlapping maths CoPs – reinforced school teacher collegiality	–access to overlapping maths CoPs	–provided access to overlapping maths CoPs	–provided access to overlapping maths CoPs	–	–	–enabled access to overlapping maths CoPs	– access to overlapping CoPs –increased relationships amongst teachers in their schools
Live demonstration lessons and video clips	Practice	–learner-engaging practices –importance of language in maths	–	–	–	– teaching for conceptual understanding	–creative teaching approaches	–learner-engaging teaching strategies	– learner-engaging teaching strategies –inclusive teaching

(*continued*)

Table 14.2 (continued)

NICLE teacher learning affordances & enablers		*Fundamental CoP element(s) it relates to*	*What individual teachers indicated taking from NICLE*							
			Calvin	*Robert*	*Melania*	*Edna*	*Ruth*	*Everton*	*Pamela*	*Mary*
Relations	teachers regarded as professionals & respected	Community	–	–	–mutually respectful relations	–	–regarded teachers as professionals	–mutually respectful relations	–regarding teachers as professionals –encouraging divergent views	–accommodate different views, & strategies –fostered critical thinking amongst participants
	Friendly relations	Sense of Community	–promoted camaraderie in the community	–	–friendly positive experiences	–	–	–promoted friendly relations	–	–promoted friendly relations
Forms of participation	Interactions with numeracy experts	Community	–engage with experts in primary maths	–	–	–engaging directly with experts	–	–	–provide access to specialists	–interacting and learning from guest speakers
	Sharing classroom experiences and practices	Practice	–sharing classroom practices and experiences	–sharing classroom practices –classroom dilemmas	–	–sharing ideas on classroom resources	–sharing classroom experiences	–sharing classroom practices with other teachers	–sharing ideas on teaching resources – classroom experiences	–sharing of maths teaching experiences
	Active participation	Community	–	–active participation and egalitarian relationship	–	–active participation & hands–on experiences	–	–practical activities and experiences	–	–active participation and practical demonstrations

Concluding Remarks

This study explains the processes of primary maths teacher learning and participation experiences in NICLE using the metaphors *reinvigoration* and *remediation* and *activation* and relating these semantics to the teachers' mathematical identities and histories. This transformation and change resulted in strong mathematical identities that emphasised taking up key numeracy-domain concepts and improved primary maths teaching and learning practices. Teacher learning was also said to be enabled in friendly, respectful CoPs that regarded teachers as professionals who could share their classroom experiences and engage in maths overlapping communities. The study's research findings illustrate that maths CoP-based professional development models provide a compelling framework for the continuous professional development of primary maths teachers. Whilst teacher CoPs have great potential in enabling teacher learning, Wenger et al.'s (2002) theoretical elements and empirical data point to the need to pay attention to issues of subject matter (or content knowledge) domain, maths teaching practices and CoP relations and participation ethos in in-service teacher programmes. Foregrounding the fundamental concepts of the maths domain, improving teaching practices and promoting CoP participation ethos should inform the conceptualisation of primary maths teacher learning communities and be fully explored as key drivers enabling successful teacher learning in the twenty-first century. This productive shift in both the content focus and the nature of primary maths teacher CoPs should inform the design and millennium outlook of district-mandated in-service teacher workshops.

Notes

1. This term was initially introduced in our earlier work (see Pausigere & Graven, 2014).
2. The PhD data set consisted of 26 observed NICLE sessions and seminars compiled in a fieldwork notebook and several audio-recorded sessions, 16 interactive interviews with the eight selected teachers with each teacher interviewed twice during the two-year period of the data gathering exercise. The teacher interviews were taped and fully transcribed. The main data also consisted of eight copies of teacher reflective journals, and I also gathered and analysed NICLE hand-outs given to teacher, documents and reports compiled by the Chair. National official policy documents were also analysed but these are not part of this chapter.

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CHAPTER 15

Continuing the Conversation: Reflections on Five Years of Primary Numeracy Research in South Africa

Mike Askew

The chapters in this volume are written largely by early career researchers, many of whom draw on studies carried out toward the award of a PhD. Encouraging is the evidence provided here of these researchers not being content merely to build on research conducted in other parts of the world, much of such research coming from contexts that resemble what Graven and Venkat (Chap. 2, this volume) refer to as the 'functioning system' in South Africa—the education accessed by the wealthy minority. In learning from, building on and adapting theories and findings from research carried out in more favorable circumstances, the researchers here do not take these as 'givens,' which somehow have to take on board by schools in South Africa's second system—the 'dysfunctional' one (to use Graven and Venkat's term) that serves the majority of learners living in relative poverty. Not only has history shown that such adoptionist approaches are doomed to failure but also that such approaches can be ethically and politically misguided. No, solutions to the problems of math-

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ematics education in South Africa have to address and grow out of work in South Africa, a stance strongly embraced by the writers here. Even if only in their early years of a career as a researcher (which I sincerely hope those who are go on to pursue) their writing shows that they are not afraid to take a stance with regard to the experts from elsewhere, to be constructively critical and to develop the research conversations into which they are entering in ways that are firmly grounded in the realities of primary mathematics in South African classrooms.

These conversations with research from outside South Africa, with the work of people like Wright and colleagues (2010), Askew and colleagues (1997), Wenger (1998) to name but three, perhaps can now be complemented through growing conversations within the community of researchers writing here, not necessarily to reach consensus on what might be the best ways forward but to deepen and enrich the different perspectives that inevitably are represented in these chapters. And there are possibilities for conversations with policymakers that I also want to point toward. I explore these possible conversations through looking at three themes: the curriculum, the role of diagnostic assessment, the individual and the collective, and the role of problem solving.

The Curriculum

Although posited many years ago, Robitaille and Dirks' (1982) analysis of the curriculum into three aspects—the intended, the implemented and the attained—still has purchase in examining what is happening in teaching and learning mathematics generally, and more particularly in the context under consideration here, in particular in looking at approaches taken to improve teaching and learning at the systemic policy level and at the more local level. Several of the authors point to the fact that systemic policy initiatives have attended in the main to the intended and attained curriculum—it would seem on the assumption that get policy around these right and the implemented curriculum will take care of itself. Although not the main focus of the SANC initiatives, several of the research initiatives suggest that there are problems with the current intended curriculum and measures of the attained curriculum.

With regard to the intended curriculum, we learn here of the CAPS level of detail in terms of weekly curriculum coverage and district-level practices that focus on compliance with coverage (with, in Gauteng at least, the provision of daily lesson plans providing yet more direction on curriculum coverage). Venkat and Graven (Chap. 11, this volume) argue that a material resource in the form of the CAPS documents is a policy lever for better pacing and coverage. Are pace and coverage as set out in CAPS cultural norms or human (knowledge) norms, to use Adler's (2010) terms? It would seems that the pacing and coverage norms at the policy level are cultural—a set of expectations set down to which teachers have to adhere. Yet the attained curriculum, at least as measured by the ANAs (however flawed they may be) and the research reported here and elsewhere indicates that either (or both) the intended or implemented curriculum are not working. With an elaborate mechanism for setting up the CAPS specification, it is all too easy for policymakers to lay the blame for the low attainment at the feet of those responsible for the implemented curriculum—the teachers. The findings presented here do, however, suggest that the questions of whether or not CAPS has 'got it right' in terms of pace and coverage need to be considered. In the context of teachers being encouraged to focus on coverage (possibly at the cost of understanding), we see here the evidence of learners slipping behind. This policy drive for coverage appears to have emerged as a result of what was a slow pace of the curriculum with learners not getting access to the mathematics to which they are entitled. But teaching cannot proceed at a pace that ignores the reality of what learners are actually learning. It has to be, as Abdulhamid (Chap. 13, this volume) points out, responsive to the needs to learners, not simply a delivery of content as directed by policy.

The key issue comes down to access and progression—learners have a right, once they are in school, of access to the mathematics curriculum and to be able to demonstrate progression through that curriculum. Coverage addresses the access, and pace of the progression, but the locus of control over each of these is different—the teacher can be in control of coverage, as this is a matter of lesson design, but progression is as much determined by the learner's response to the coverage as to what is taught. As Griffin puts it 'teaching takes place in time, learning over time' (1989) but a policy that conflates coverage and pace appears to have resulted in a situation where access is outstripping progression—there is little point in providing access to more content if what has been covered is not well embedded. There is much food for thought for policymakers here in considering the implemented curriculum.

Several authors then raise questions about the measures in place for judging the attained curriculum, the ANAs. A strong theme running through these studies is the question of the validity and reliability of these national assessments. Weitz and Venkat (Chap. 3, this volume) present strong evidence that the Foundation Phase ANAs do not have strong predictive power of results in Intermediate Phase. Sibanda's (Chap. 10, this volume) findings point to issues over the complexity of the language used in the ANAs, issues that confound whether or not it is mathematics or language proficiency that is being assessed. Such findings raise serious questions as to whether or not the ANAs are achieving their stated aims, which include the intention to introduce teachers to 'better' assessment practices and identify schools in need. Again policymakers would do well to heed these findings. Whatever the stated aims of the ANAs, such system-wide assessment is really focused on checking the 'health' of the system and there are jurisdictions around the world, many of them with high attainment, that do such check-ups without testing every pupil, but instead assess a representative sample of schools and thus provides a snapshot of the overall profile of attainment that schools not in the sample can use as a benchmark to consider their own performance.

In contrast to the systemic drivers addressing the intended curriculum (CAPS) and the attained curriculum (ANAs), the work of the Numeracy Chairs is firmly focused on the implemented curriculum, on what happens 'inside the black box' of the classroom, in Wiliam and Black's terms (2006), and it is to these classroom level, implemented curriculum, issues that I now turn.

The Role of Diagnostic Assessment

Many of the researches reported on indicate the insights into learner understandings and consequent needs for instruction that can be gained from careful diagnostic assessment (Weitz & Venkat; Stott; Stott, Mofu & Ndongeni; Wasserman). These findings on diagnostic assessments seem to share a common assumption that such assessments work best when all learners are assessed. This is a common assumption, based, I suspect on the widely held view in many nations that education has to meet the needs of individuals, and so diagnosing those needs is the first step to meeting them. Ausubel is famously quoted as saying 'If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly' (Ausubel, 1968, p. 18).[1] Consequently, the writings on diagnostic assessment point to the difficulty, with large classes, of assessing learners individually, with one remedy to this being to carry out group testing. But perhaps we need to question the underlying assumption that diagnostic assessment is most useful when all pupils are assessed.

Some evidence does question whether or not this individualistic approach is the best position to take. In France, for example, some years ago a policy move was made to provide teachers with detailed information about their individual pupils' profiles of learning at the beginning of the school year (traditionally it came at the end when it was too late to make any use of). It turned out that this was too much information for teachers to process and actually was not helpful to them, and the old system was reverted to. In a project Tamara Bibby, Margaret Brown and I carried out at King's College in London (1997), teachers carried out detailed diagnostic assessments of six representative pupils. As well as the profiles of these six learners providing the teachers with an indication of what the range of understanding across the class might be, an important other finding was that the teachers developed an awareness of the style of questioning involved in diagnostic assessment interviews, a style that they could then use within their normal class teaching. In other words, the knowledge of learners gained through diagnostic assessment may actually be less important than teachers' understanding what good diagnosis involves and how to carry it out.

In the studies reported on here, the diagnosis was carried out by the researchers—one direction that this research might turn toward is to work with teachers on how to carry out diagnostic assessment, not in the expectation of them being able to continue to do detailed assessments but as a 'Trojan horse' to change subsequent teaching. Sound knowledge of how learners develop mathematical understandings, with teaching knowledge attained through involvement in detailed diagnosis, can further help ground pedagogic decisions. Abdulhamid makes a strong case for teaching that needs to be responsive and sensitive, yet suggests that being responsive to learners' needs is a relatively rare classroom occurrence. The question is raised as to which might come first—working with teachers on diagnostic assessment, in the expectation that what is revealed by such assessments requires acting on in the classroom or working on professional development that encourages responsive teaching with the expectation that this then raises teachers' awareness of the need for diagnostic assessment.

In the context of teachers being encouraged to focus on coverage but the evidence of learners not keeping up, then the appeal of programs like 'mathematics recovery' is apparent, through the need to help learners to 'catch up.' Mathematics recovery (MR) was preceded by reading recovery (RR), and it is worth looking at some of the principles of that original program which started in New Zealand. The most pertinent principle was that RR was not designed to make up for poor or ineffective initial teaching. New Zealand schools could only get funding to have an RR teacher if, by and large, their results in reading were already good—RR was then an intervention for learners who were falling behind their peers despite being in classes where the majority were succeeding in learning to read. In contrast, schools where the results in reading were below average were provided not with RR but with teacher professional development in effective teaching of reading.

In the South African context of many schools having very low standards of attainment in mathematics then, a 'recovery' approach may be appropriate in the short term, but the question I have is whether or not this is the best 'mindset' to have going forward? As the chapters here demonstrate, the research based in MR is providing valuable insights into learners' understanding, and most importantly, revealing what learners can do rather than focusing on what they cannot do. Where now to take these findings? There is, however, little attention in the chapters based in MR on what the implications of the findings might be back in the classroom. Can the lessons learned from these detailed studies of individual learners be scaled up to practices that can be applied at the class level?

The Individual and the Collective

Perhaps behind assumptions of diagnostic assessment for all is the assumption that good teaching is about meeting the needs of each individual—an assumption that is hard enough to put into practice with a class of 25 learners, let alone a class of 40 or more. But some writers are beginning to question this implicit assumption of starting with and trying to meet the needs of the individual. Davis and Simmt, coming from a perspective on learning based in complexity theory, make the strong claim that the 'move toward understanding the collective as a cognizing agent (as opposed to a collection of cognizing agents) presents some important advantages' (Davis & Simmt, 2003, 144). The shift from seeing a class as a collection of individuals to a collective where the whole is greater than the sum of the parts has, I think, profound implications for how we think about teaching and the role of the teaching, particularly in a context of large classes and also where the breadth of mathematical and linguistic expertise possessed by the collective (not just the teacher) could be a resource for all to tap into.

In the South African context of large classes, exploring the implications of such a stance may be fruitful. But of course this is not be confused with the issue of coverage—deciding on the goal for a class is not simply a matter of taking learners through a lockstep lesson. But as a mindset for thinking about lesson design, teaching and learning, it might reveal insights and practices that are more productive than adaptations of the 'individual needs' stance. And it points to the possibility of conversations across those studies looking at learners and those looking at teachers.

Graven and Venkat (Chap. 2, this volume) acknowledge the central role of Wenger's theory in the work of the Chairs with teachers, and Pausigere (Chap. 14, this volume) addresses the issue of teachers' mathematical identities and how, though engaging in supportive and participatory communities of practice, teachers' mathematical identities changed, including through their sense of being better knowers of mathematics and by changing classroom practices. Conversations might be had about how this theoretical perspective may also have implications for classrooms. Complexity theory suggests that the differences between working with teachers' knowledge and working with learners' knowledge are a question of scale rather than being essentially different. How might the insights form working with each of these groups—teachers and learners—be scaled up or down?

The Role of Problem Solving

Three chapters explicitly deal with the challenges of teaching learners to solve mathematical word problems, taking subtly different stances on how this might best be brought about. Spira and Robertson (Chap. 7, this volume), drawing on the work of Palinscar and Brown on reciprocal teaching (1984), adopt what might be called a deductive reasoning approach whereby each problem is treated as, metaphorically, a mini-detective story with the role of the learner to use the information provided within the 'story' and the knowledge that they bring of the story context to deduce what mathematics use in finding a solution. Roberts and Takane, Tshesane and Askew (Chap. 12, this volume) work with a model from Askew's Big Books approach that designs problems through careful selection of numbers and situations so that the initial deductive problem solving will be within reach. From that starting point of working on individual problems, the pedagogic emphasis shifts to take a more analogical approach, based on the premise that expert problem solvers build up, in Watson and Mason's terms (2005), personal example spaces, which are then used as 'canonical' problem types against which new problems can be matched to identify the type of mathematics to which it might yield.

These subtly different theoretical positions suggest different implications for teaching. Thus, Thakane, Tshesane and Askew argue for providing tasks based around example sets of problems that have a common underlying structure—learner activity is not simply to solve each individual problem but to look for commonalities across the problems. Roberts through eliciting from learners' examples of problems that comprise part of their personal example spaces concludes that the rarity of learners producing collection type problems indicates a need for such problem types to be attended to in further teaching. Spira and Robertson present an adapted model for reciprocal teaching that shows promise in helping learners treat word problems as opportunities for meaning making. This is not to suggest, of course, that these different positions and approaches are mutually exclusive, but that it would be interesting and fruitful to examine what is common across the approaches, what is different and whether some synthesis of ideas could lead to even stronger pedagogic models.

In their discussion of Palinscar and Brown's model of reciprocal teaching, Spira and Robertson remind us of the top-down/bottom-up skills needed in becoming a good reader, bottom-up skills being those of decoding the written text and top-down skills involving interpreting the text. We could argue similarly for a top-down/bottom-up view of learning mathematics, the bottom-up skills including effectively and efficiently carrying out basic arithmetic, the top-down skills including making sense of problems mathematically and setting up appropriate mathematical models. Thus, Roberts (Chap. 9, this volume) argues for a narrative approach to early number work, and there are international studies that would support this. Within mathematics education, the work of Carpenter and Fennema (1997) and the different root types of problems pre-date the work of Lakoff and Núñez (2000) and their argument that mathematics arises from bodily actions. Similarly, the Freudenthal work based in the philosophy of mathematics as a human activity, arising out of mathematizing everyday experiences would point to the importance of basing teaching in contexts that learners can, initially at least, informally make sense of. Yet the CAPS documentation rather presents mathematics, and number in particular, as a collection of decontextualized skills to be 'mastered' and then applied to problems—here again conversation with a policymaker could move this forward.

Although the debate within mathematics about which comes first—basic skills or meaning making—is not as fierce as the phonics—versus—whole books literacy teaching debate, there is room for a conversation across those researchers focusing on understanding and fluency in operating with and on quantities and numbers and those looking at problem solving and meaning making. As Nunes and Bryant note (2009) despite the seminal works mentioned about, there is as yet scant evidence for the 'start with contexts' being more successful than the 'basics first' approach and there is 'a need for experimental and longitudinal studies designed to investigate the progress that students make when teaching starts from formalizations rather than from students' informal knowledge and the long-term consequences of this approach to teaching students about relations' (p. 6). The researchers here look well positioned to begin to explore this need.

Writing elsewhere, Nunes and Bryant, together with Sylva and Barros (2009) also claim that

> Mathematical reasoning, even more so than children's knowledge of arithmetic, is important for children's later achievement in mathematics. (p. 1)

This suggests that findings such as those exemplified by Weitz and Venkat could be further researched—does the LFIN sufficiently address reasoning skills? If so, which ones, and are they themselves predictors of later attainment? If not, how might such assessments be developed to include such items?

Ensor and colleagues (2009) report on the limited evidence of trajectories of learning number in the context of South Africa. Could the sum of the separate researches reported on here be greater than the individual findings? Wright and colleagues' model (2010), for example, focuses primarily on 'pure' number skills. As Stott (Chap. 4, this volume) points out, this model might usefully be extended to encompass elements of the five mathematical proficiencies. The studies looking at problem solving also have some beginning insights into what a trajectory of learning in number that goes beyond calculating skills might look like. And whether working with problems or pure arithmetic, the thorny issue of language of instruction continues to need attention. As Mdluli (Chap. 8, this volume) strongly shows through her research, the language that is used both to teach mathematics and to express it is an under-used resource, remaining invisible (in the sense of explicitly drawing learners attention to it) in the mathematics lessons she has studied. If, as the mathematics educator Gattengo puts it 'only awareness is educable' (1987), then part of good mathematics pedagogy must surely involve drawing learners' awareness to the language of mathematics. I hope that the conversation continues to develop between these researchers about the relationship between arithmetic, problem solving and language, because, as Graven and Venkat (Chap. 2, this volume) point out, 'there is a need for a "structured" developmentally progressive framework to support teacher development, teaching and remediation' (progressive in the sense of a clear progression, rather than progressive in the sense of 'progressive education').

Conclusion

While there is plenty of research from more developed nations on what is considered good or effective practice, that usually takes many things for granted, not least of which is smaller class sizes, reasonably well-resourced rooms and, often, monolingual learners. The findings presented here suggest there is evidence for beginning to think about what 'good practice' in the South African context might look like, building on what is already happening in classrooms and taking the reality of those classrooms into account. Such findings provide a useful corrective to much of the local research that focuses on what is currently 'wrong' with the teaching.

In her 2016 keynote address to the SAARMSTE annual conference, Jill Adler, in surveying the field of Southern African research in mathematics education, noted that local research into primary mathematics was still relatively limited. On the basis of the writing here, it looks primed to grow and make a strong contribution to the field, nationally and internationally.

Note

1. Fine words, from the introduction to the book! One wonders how many readers went on to actually examine the details of how this might be implemented is alas unknown.

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Index

Note: Page numbers followed by "n" refer to notes.

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