Solutions Manual for

Fluid Mechanics: Fundamentals and Applications

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CHAPTER 8
FLOW IN PIPES

Laminar and Turbulent Flow

8-1C

Solution We are to discuss why pipes are usually circular in cross section.

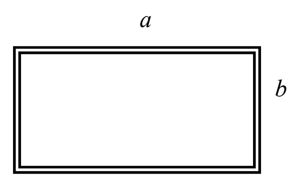
Analysis Liquids are usually transported in circular pipes because pipes with a circular cross section **can withstand large pressure differences between the inside and the outside without undergoing any significant distortion**.

Discussion Piping for gases at low pressure are often non-circular (e.g., air conditioning and heating ducts in buildings).

8-2C

Solution We are to define and discuss Reynolds number for pipe and duct flow.

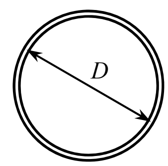
Analysis *Reynolds number* is the **ratio of the inertial forces to viscous forces**, and it serves as a criterion for determining the flow regime. At *large* Reynolds numbers, for example, the flow is turbulent since the inertia forces are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. It is defined as follows:



(*a*) For flow in a circular tube of inner diameter *D*: $\boxed{\mathrm{Re} = \frac{VD}{\nu}}$

(*b*) For flow in a rectangular duct of cross-section $a \times b$: $\boxed{\mathrm{Re} = \frac{VD_h}{\nu}}$

where $D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$ is the hydraulic diameter.



Discussion Since pipe flows become fully developed far enough downstream, diameter is the appropriate length scale for the Reynolds number. In boundary layer flows, however, the boundary layer grows continually downstream, and therefore downstream distance is a more appropriate length scale.

8-3C

Solution We are to compare the Reynolds number in air and water.

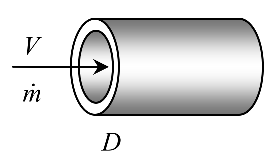
Analysis Reynolds number is inversely proportional to kinematic viscosity, which is much smaller for water than for air (at 25°C, ν_{air} = 1.562×10^{-5} m^2/s and $\nu_{water} = \mu/\rho$ = 0.891×10^{-3}/997 = 8.9×10^{-7} m^2/s). Therefore, noting that Re = VD/ν, the **Reynolds number is higher for motion in water for the same diameter and speed**.

Discussion Of course, it is not possible to walk as fast in water as in air – try it!

8-4C

Solution We are to express the Reynolds number for a circular pipe in terms of mass flow rate.

Analysis Reynolds number for flow in a circular tube of diameter D is expressed as



$$\mathrm{Re} = \frac{VD}{\nu} \quad \text{where} \quad V = V_{avg} = \frac{\dot m}{\rho A_c} = \frac{\dot m}{\rho\left(\pi D^2/4\right)} = \frac{4\dot m}{\rho \pi D^2} \quad \text{and} \quad \nu = \frac{\mu}{\rho}$$

Substituting,

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{4\dot m D}{\rho \pi D^2 (\mu/\rho)} = \frac{4\dot m}{\pi D \mu}. \text{ Thus, } \boxed{\mathrm{Re} = \frac{4\dot m}{\pi D\mu}}$$

Discussion This result holds only for circular pipes.

8-5C

Solution We are to compare the pumping requirement for water and oil.

Analysis **Engine oil requires a larger pump** because of its much larger viscosity.

Discussion The density of oil is actually 10 to 15% smaller than that of water, and this makes the pumping requirement smaller for oil than water. However, the viscosity of oil is orders of magnitude larger than that of water, and is therefore the dominant factor in this comparison.

8-6C

Solution We are to discuss the Reynolds number for transition from laminar to turbulent flow.

Analysis The generally accepted value of the Reynolds number above which the flow in a smooth pipe is turbulent is **4000**. **In the range 2300 < Re < 4000, the flow is typically transitional between laminar and turbulent**.

Discussion In actual practice, pipe flow may become turbulent at Re lower or higher than this value.

8-7C

Solution We are to compare pipe flow in air and water.

Analysis Reynolds number is inversely proportional to kinematic viscosity, which is much smaller for water than for air (at 25°C, $\nu_{air} = 1.562\times10^{-5}$ m^2/s and $\nu_{water} = \mu/\rho = 0.891\times10^{-3}/997 = 8.9\times10^{-7}$ m^2/s). Therefore, for the same diameter and speed, the Reynolds number will be higher for water flow, and thus **the flow is more likely to be turbulent for water**.

Discussion The actual viscosity (dynamic viscosity) μ is larger for water than for air, but the density of water is so much greater than that of air that the kinematic viscosity of water ends up being smaller than that of air.

8-8C

Solution We are to define and discuss hydraulic diameter.

Analysis For flow through non-circular tubes, the Reynolds number and the friction factor are based on the *hydraulic diameter* D_h defined as $\boxed{D_h = \frac{4A_c}{p}}$ where A_c is the cross-sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it **reduces to ordinary diameter *D* for circular tubes** since $\boxed{D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D}$.

Discussion Hydraulic diameter is a useful tool for dealing with non-circular pipes (e.g., air conditioning and heating ducts in buildings).

8-9C

Solution We are to define and discuss hydrodynamic entry length.

Analysis The **region from the tube inlet to the point at which the boundary layer merges at the centerline** is called the *hydrodynamic entrance region*, and the length of this region is called *hydrodynamic entry length*. **The entry length is much longer in laminar flow than it is in turbulent flow**. But at very low Reynolds numbers, L_h is very small (e.g., $L_h = 1.2D$ at Re = 20).

Discussion The entry length increases with increasing Reynolds number, but there is a significant change in entry length when the flow changes from laminar to turbulent.

8-10C

Solution We are to compare the wall shear stress at the inlet and outlet of a pipe.

Analysis The wall shear stress τ_w is **highest at the tube inlet where the thickness of the boundary layer is nearly zero, and decreases gradually to the fully developed value**. The same is true for turbulent flow.

Discussion We are assuming that the entrance is well-rounded so that the inlet flow is nearly uniform.

8-11C

Solution We are to discuss the effect of surface roughness on pressure drop in pipe flow.

Analysis In turbulent flow, tubes with rough surfaces have much higher friction factors than the tubes with smooth surfaces, and thus **surface roughness leads to a much larger pressure drop in turbulent pipe flow**. In the case of laminar flow, the effect of surface roughness on the friction factor and pressure drop is negligible.

Discussion The effect of roughness on pressure drop is significant for turbulent flow, as seen in the Moody chart.

Fully Developed Flow in Pipes

8-12C

Solution We are to discuss how the wall shear stress varies along the flow direction in a pipe.

Analysis The wall shear stress τ_w **remains constant along the flow direction in the fully developed region** in both laminar and turbulent flow.

Discussion However, in the entrance region, τ_w starts out large, and decreases until the flow becomes fully developed.

8-13C

Solution We are to discuss the fluid property responsible for development of a velocity boundary layer.

Analysis The fluid **viscosity** is responsible for the development of the velocity boundary layer.

Discussion You can think of it this way: As the flow moves downstream, more and more of it gets slowed down near the wall due to friction, which is due to viscosity in the fluid.

8-14C

Solution We are to discuss the velocity profile in fully developed pipe flow.

Analysis In the fully developed region of flow in a circular pipe, the velocity profile **does *not* change** in the flow direction.

Discussion This is, in fact, the definition of fully developed – namely, the velocity profile remains of constant shape.

8-15C

Solution We are to discuss the relationship between friction factor and pressure loss in pipe flow.

Analysis The f**riction factor for flow in a tube is proportional to the pressure loss**. Since the pressure loss along the flow is directly related to the power requirements of the pump to maintain flow, the **friction factor is also proportional to the power requirements to overcome friction**. The applicable relations are

$$\boxed{\dot{W}_{\text{pump}} = \frac{\dot{m}\Delta P_L}{\rho}} \quad \text{and} \quad \boxed{\dot{W}_{\text{pump}} = \frac{\dot{m}\Delta P_L}{\rho}}$$

Discussion This type of pressure loss due to friction is an *irreversible* loss. Hence, it is always positive (positive being defined as a pressure drop down the pipe). A negative pressure loss would violate the second law of thermodynamics.

8-16C

Solution We are to discuss the value of shear stress at the center of a pipe.

Analysis **The shear stress at the center of a circular tube during fully developed laminar flow is zero** since the shear stress is proportional to the velocity gradient, which is zero at the tube center.

Discussion This result is due to the axisymmetry of the velocity profile.

8-17C

Solution We are to discuss whether the maximum shear stress in a turbulent pipe flow occurs at the wall.

Analysis **Yes, the shear stress at the surface of a tube during fully developed turbulent flow is maximum** since the shear stress is proportional to the velocity gradient, which is maximum at the tube surface.

Discussion This result is also true for laminar flow.

8-18C

Solution We are to discuss the change in head loss when the pipe length is doubled.

Analysis In fully developed flow in a circular pipe with negligible entrance effects, if the length of the pipe is doubled, **the head loss also doubles** (the head loss is proportional to pipe length in the fully developed region of flow).

Discussion If entrance lengths are not negligible, the head loss in the longer pipe would be less than twice that of the shorter pipe, since the shear stress is larger in the entrance region than in the fully developed region.

8-19C

Solution We are to examine a claim about volume flow rate in laminar pipe flow.

Analysis **Yes**, the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2. This works for fully developed laminar pipe flow in round pipes since $\dot{V} = V_{\text{avg}} A_c = (V_{\max}/2)A_c$.

Discussion This is *not* true for turbulent flow, so one must be careful that the flow is laminar before trusting this measurement. It is also *not* true if the pipe is not round, even if the flow is fully developed and laminar.

8-20C

Solution We are to examine a claim about volume flow rate in laminar pipe flow.

Analysis **No**, the average velocity in a circular pipe in fully developed laminar flow *cannot* be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). The average velocity is $V_{max}/2$, but the velocity at $R/2$ is

$$V(R/2) = V_{max}\left(1 - \frac{r^2}{R^2}\right)_{r=R/2} = \frac{3V_{max}}{4},$$ which is much larger than $V_{max}/2$.

Discussion There is, of course, a radial location in the pipe at which the local velocity *is* equal to the average velocity. Can you find that location?

8-21C

Solution We are to compare the head loss when the pipe diameter is halved.

Analysis In fully developed laminar flow in a circular pipe, the head loss is given by

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = \frac{64}{\mathrm{Re}}\frac{L}{D}\frac{V^2}{2g} = \frac{64}{V\,D/\nu}\frac{L}{D}\frac{V^2}{2g} = \frac{64\nu}{D}\frac{L}{D}\frac{V}{2g}$$

The average velocity can be expressed in terms of the flow rate as $V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4}$. Substituting,

$$h_L = \frac{64\nu}{D^2}\frac{L}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right) = \frac{64\nu}{D^2}\frac{4L\dot{V}}{2g\pi D^2} = \frac{128\nu L\dot{V}}{g\pi D^4}$$

Therefore, at constant flow rate and pipe length, the head loss is inversely proportional to the 4th power of diameter, and thus **reducing the pipe diameter by half increases the head loss by a factor of 16**.

Discussion This is a very significant increase in head loss, and shows why larger diameter tubes lead to much smaller pumping power requirements.

8-22C

Solution We are to discuss why the friction factor is higher in turbulent pipe flow compared to laminar pipe flow.

Analysis In turbulent flow, it is the **turbulent eddies due to enhanced mixing** that cause the friction factor to be larger. This turbulent mixing leads to a much larger wall shear stress, which translates into larger friction factor.

Discussion Another way to think of it is that the turbulent eddies cause the turbulent velocity profile to be much *fuller* (closer to uniform flow) than the laminar velocity profile.

8-23C

Solution We are to define and discuss turbulent viscosity.

Analysis Turbulent viscosity μ_t is caused by turbulent eddies, and it **accounts for momentum transport by turbulent eddies**. It is expressed as $\boxed{\tau_t = -\rho\overline{u'v'} = \mu_t\frac{\partial\bar{u}}{\partial y}}$ where $\bar{u}$ is the mean value of velocity in the flow direction and u' and u' are the fluctuating components of velocity.

Discussion Turbulent viscosity is a derived, or non-physical quantity. Unlike the viscosity, it is *not* a property of the fluid; rather, it is a property of the *flow*.

8-24C

Solution We are to discuss the dimensions of a constant in a head loss expression.

Analysis We compare the dimensions of the two sides of the equation $h_L = 0.0826\, fL \dfrac{\dot{V}^2}{D^5}$. Using curly brackets to mean "the dimensions of", we have $\{L\} = \{0.0826\} \cdot \{1\}\{L\} \cdot \{L^3 t^{-1}\}^2 \cdot \{L^{-5}\}$, and the dimensions of the constant are thus $\{0.0826\} = \{L^{-1} t^2\}$. Therefore, **the constant 0.0826 is *not* dimensionless**. **This is *not* a dimensionally homogeneous equation**, and it cannot be used in any consistent set of units.

Discussion Engineers often create dimensionally inhomogeneous equations like this. While they are useful for practicing engineers, they are valid only when the proper units are used for each variable, and this can occasionally lead to mistakes. For this reason, the present authors do not encourage their use.

8-25C

Solution We are to discuss the change in head loss due to a decrease in viscosity by a factor of two.

Analysis In fully developed laminar flow in a circular pipe, the pressure loss and the head loss are given by

$$\Delta P_L = \frac{32\mu LV}{D^2} \quad \text{and} \quad h_L = \frac{\Delta P_L}{\rho g} = \frac{32\mu LV}{\rho g D^2}$$

When the flow rate and thus the average velocity are held constant, the head loss becomes proportional to viscosity. Therefore, **the head loss is reduced by half** when the viscosity of the fluid is reduced by half.

Discussion This result is *not* valid for turbulent flow – only for laminar flow. It is also not valid for laminar flow in situations where the entrance length effects are not negligible.

8-26C

Solution We are to discuss the relationship between head loss and pressure drop in pipe flow.

Analysis The head loss is related to pressure loss by $\boxed{h_L = \Delta P_L / \rho g}$. For a given fluid, the head loss can be converted to pressure loss by multiplying the head loss by the acceleration of gravity and the density of the fluid. Thus, for constant density, **head loss and pressure drop are linearly proportional to each other**.

Discussion This result is true for both laminar and turbulent pipe flow.

8-27C

Solution We are to discuss if the friction factor is zero for laminar pipe flow with a perfectly smooth surface.

Analysis During laminar flow of air in a circular pipe with perfectly smooth surfaces, **the friction factor is *not* zero** because of the no-slip boundary condition, which must hold even for perfectly smooth surfaces.

Discussion If we compare the friction factor for rough and smooth surfaces, roughness has no effect on friction factor for fully developed laminar pipe flow unless the roughness height is very large. For turbulent pipe flow, however, roughness very strongly impacts the friction factor.

8-28C

Solution We are to explain why friction factor is independent of Re at very large Re.

Analysis At very large Reynolds numbers, the flow is fully rough and the friction factor is independent of the Reynolds number. This is because **the thickness of viscous sublayer decreases with increasing Reynolds number, and it be comes so thin that the surface roughness protrudes into the flow**. The viscous effects in this case are produced in the main flow primarily by the protruding roughness elements, and the contribution of the viscous sublayer is negligible.

Discussion This effect is clearly seen in the Moody chart – at large Re, the curves flatten out horizontally.

8-29E

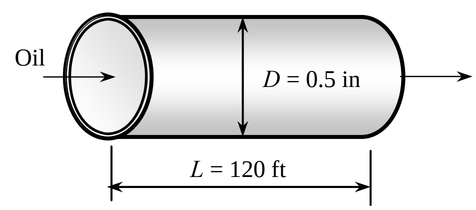
Solution The pressure readings across a pipe are given. The flow rates are to be determined for three different orientations of horizontal, uphill, and downhill flow.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is laminar (to be verified). **4** The pipe involves no components such as bends, valves, and connectors. **5** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of oil are given to be ρ = 56.8 lbm/ft^3 and μ = 0.0278 lbm/ft·s, respectively.

Analysis The pressure drop across the pipe and the cross-sectional area of the pipe are

$$\Delta P = P_1 - P_2 = 120 - 14 = 106 \text{ psi}$$
$$A_c = \pi D^2/4 = \pi(0.5/12 \text{ ft})^2/4 = 0.001364 \text{ ft}^2$$



(*a*) The flow rate for all three cases can be determined from

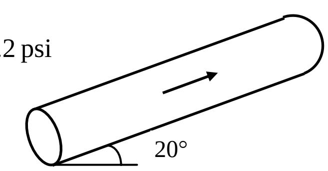
$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} = \frac{(106 \text{ psi})\pi(0.5/12 \text{ ft})^4}{128(0.0278 \text{ lbm/ft}\cdot\text{s})(120 \text{ ft})}\left(\frac{144 \text{ lbf/ft}^2}{1 \text{ psi}}\right)\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = \mathbf{0.0109\ ft^3/s}$$

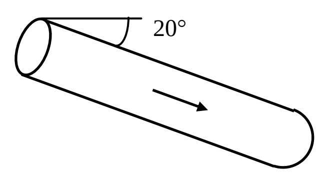
(*b*) For uphill flow with an inclination of 20°, we have θ = +20°, and

$$\rho g L \sin\theta = (56.8 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(120 \text{ ft})\sin 20°\left(\frac{1 \text{ psi}}{144 \text{ lbf/ft}^2}\right)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right) = 16.2 \text{ psi}$$



$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} =$$
$$= \frac{(106 - 16.2 \text{ psi})\pi(0.5/12 \text{ ft})^4}{128(0.0278 \text{ lbm/ft}\cdot\text{s})(120 \text{ ft})}\left(\frac{144 \text{ lbf/ft}^2}{1 \text{ psi}}\right)\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = \mathbf{0.00923\ ft^3/s}$$

(*c*) For downhill flow with an inclination of 20°, we have θ = -20°, and



$$\dot{V}_{\text{downhill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$
$$= \frac{[106 - (-16.2) \text{ psi}]\pi(0.5/12 \text{ ft})^4}{128(0.0278 \text{ lbm/ft}\cdot\text{s})(120 \text{ ft})}\left(\frac{144 \text{ lbf/ft}^2}{1 \text{ psi}}\right)\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right) = \mathbf{0.0126\ ft^3/s}$$

The flow rate is the highest for downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$V = \frac{\dot{V}}{A_c} = \frac{0.0126 \text{ ft}^3/\text{s}}{0.001364 \text{ ft}^2} = 9.24 \text{ ft/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(56.8 \text{ lbm/ft}^3)(9.24 \text{ ft/s})(0.5/12 \text{ ft})}{0.0278 \text{ lbm/ft}\cdot\text{s}} = 787$$

which is less than 2300. Therefore, the flow is **laminar** for all three cases, and the analysis above is valid.

Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. As can be seen from the calculated rates above, gravity opposes uphill flow, but helps downhill flow. Gravity has no effect on the flow rate in the horizontal case. Downhill flow can occur even in the absence of an applied pressure difference.

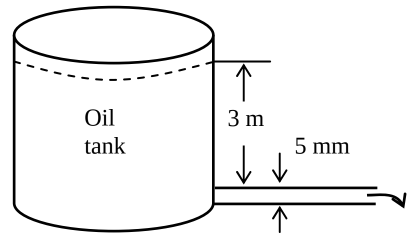
8-30

Solution Oil is being discharged by a horizontal pipe from a storage tank open to the atmosphere. The flow rate of oil through the pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The entrance and exit loses are negligible. **4** The flow is laminar (to be verified). **5** The pipe involves no components such as bends, valves, and connectors. **6** The piping section involves no work devices such as pumps and turbines.

Properties The density and kinematic viscosity of oil are given to be $\rho = 850$ kg/m^3 and $\nu = 0.00062$ m^2/s, respectively. The dynamic viscosity is calculated to be

$$\mu = \rho\nu = (850\ \text{kg/m}^3)(0.00062\ \text{m}^2/s) = 0.527\ \text{kg/m}\cdot\text{s}$$



Analysis The pressure at the bottom of the tank is

$$P_{1,\text{gage}} = \rho gh$$

$$= (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 25.02\ \text{kN/m}^2$$

Disregarding inlet and outlet losses, the pressure drop across the pipe is

$$\Delta P = P_1 - P_2 = P_1 - P_{\text{atm}} = P_{1,\text{gage}} = 25.02\ \text{kN/m}^2 = 25.02\ \text{kPa}$$

The flow rate through a horizontal pipe in laminar flow is determined from

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} = \frac{(25.02\ \text{kN/m}^2)\pi(0.005\ \text{m})^4}{128(0.527\ \text{kg/m}\cdot\text{s})(40\ \text{m})}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) = 1.821\times10^{-8}\ \text{m}^3/\text{s} \cong \mathbf{1.82\times10^{-8}\ m^3/s}$$

The average fluid velocity and the Reynolds number in this case are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{1.821\times10^{-8}\ \text{m}^3/\text{s}}{\pi(0.005\ \text{m})^2/4} = 9.27\times10^{-4}\ \text{m/s}$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(850\ \text{kg/m}^3)(9.27\times10^{-4}\ \text{m/s})(0.005\ \text{m})}{0.527\ \text{kg/m}\cdot\text{s}} = 0.0075$$

which is less than 2300. Therefore, the flow is *laminar* and the analysis above is valid.

Discussion The flow rate will be even less when the inlet and outlet losses are considered, especially when the inlet is not well-rounded.

8-31

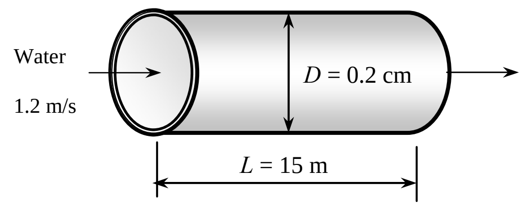
Solution The average flow velocity in a pipe is given. The pressure drop, the head loss, and the pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m·s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number of the flow is

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \mathrm{kg/m^3})(1.2\ \mathrm{m/s})(2\times10^{-3}\ \mathrm{m})}{1.307\times10^{-3}\ \mathrm{kg/m\cdot s}} = 1836$$



which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{1836} = 0.0349$$

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0349\frac{15\ \mathrm{m}}{0.002\ \mathrm{m}}\frac{(999.7\ \mathrm{kg/m^3})(1.2\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{kN}}{1000\ \mathrm{kg\cdot m/s}}\right)\left(\frac{1\ \mathrm{kPa}}{1\ \mathrm{kN/m^2}}\right) = \mathbf{188\ kPa}$$

(*b*) The head loss in the pipe is determined from

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.0349\frac{15\ \mathrm{m}}{0.002\ \mathrm{m}}\frac{(1.2\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = \mathbf{19.2\ m}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\dot{V} = VA_c = V(\pi D^2/4) = (1.2\ \mathrm{m/s})[\pi(0.002\ \mathrm{m})^2/4] = 3.77\times10^{-6}\ \mathrm{m^3}/s$$

$$\dot{W}_{\mathrm{pump}} = \dot{V}\Delta P = (3.77\times10^{-6}\ \mathrm{m^3}/s)(188\ \mathrm{kPa})\left(\frac{1000\ \mathrm{W}}{1\ \mathrm{kPa\cdot m^3/s}}\right) = \mathbf{0.71\ W}$$

Therefore, power input in the amount of 0.71 W is needed to overcome the frictional losses in the flow due to viscosity.

Discussion If the flow were instead *turbulent*, the pumping power would be much greater since the head loss in the pipe would be much greater.

8-32

Solution The flow rate through a specified water pipe is given. The pressure drop, the head loss, and the pumping power requirements are to be determined.

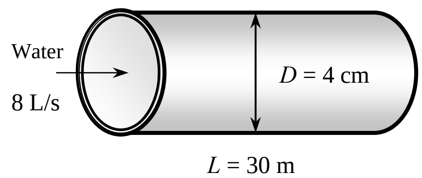
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s, respectively. The roughness of stainless steel is 0.002 mm.

Analysis First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.008\ \text{m}^3/s}{\pi (0.04\ \text{m})^2/4} = 6.366\ \text{m}/s$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.1\ \text{kg/m}^3)(6.366\ \text{m/s})(0.04\ \text{m})}{1.138\times 10^{-3}\ \text{kg/m}\cdot\text{s}} = 2.236\times 10^5$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D = \frac{2\times 10^{-6}\ \text{m}}{0.04\ \text{m}} = 5\times 10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{5\times 10^{-5}}{3.7} + \frac{2.51}{2.236\times 10^5\sqrt{f}}\right)$$

It gives f = 0.01573. Then the pressure drop, head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.01573\frac{30\ \text{m}}{0.04\ \text{m}}\frac{(999.1\ \text{kg/m}^3)(6.366\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{239\ \ kPa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.01573\frac{30\ \text{m}}{0.04\ \text{m}}\frac{(6.366\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{24.4\ m}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.008\ \text{m}^3/s)(239\ \text{kPa})\left(\frac{1\ \text{kW}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{1.91\,kW}$$

Therefore, useful power input in the amount of 1.91 kW is needed to overcome the frictional losses in the pipe.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.0155, which is sufficiently close to 0.0157. Also, the friction factor corresponding to ε = 0 in this case is 0.0153, which indicates that stainless steel pipes in this case can be assumed to be smooth with an error of about 2%. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

8-33E

Solution The flow rate and the head loss in an air duct is given. The minimum diameter of the duct is to be determined.

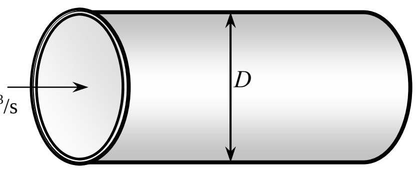
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The duct involves no components such as bends, valves, and connectors. **4** Air is an ideal gas. **5** The duct is smooth since it is made of plastic, $\varepsilon \approx 0$. **6** The flow is turbulent (to be verified).

Properties The density, dynamic viscosity, and kinematic viscosity of air at 100°F are ρ = 0.07088 lbm/ft^3, μ = 0.04615 lbm/ft·h, and ν = 0.6512 ft^2/s = 1.809×10^{-4} ft^2/s.

Analysis The average velocity, Reynolds number, friction factor, and the head loss relations can be expressed as (D is in ft, V is in ft/s, Re and f are dimensionless)

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{12\ \text{ft}^3/s}{\pi D^2/4}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{VD}{1.809\times 10^{-4}\ \text{ft}^2/s}$$



$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 50 = f\frac{L}{D}\frac{V^2}{2g} = f\frac{400\ \text{ft}}{D}\frac{V^2}{2(32.2\ \text{ft/s}^2)}$$

This is a set of 4 equations in 4 unknowns, and solving them with an equation solver gives

D = **0.88 ft**, f = 0.0181, V = 19.8 ft/s, and Re = 96,040

Therefore, the diameter of the duct should be more than 0.88 ft if the head loss is not to exceed 50 ft. Note that Re > 4000, and thus the turbulent flow assumption is verified.

The diameter can also be determined directly from the third Swamee-Jain formula to be

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

$$= 0.66\left[0 + (0.180\times 10^{-3}\ \text{ft}^2/s)(12\ \text{ft}^3/s)^{9.4}\left(\frac{400\ \text{ft}}{(32.2\ \text{ft/s}^2)(50\ \text{ft})}\right)^{5.2}\right]^{0.04}$$

$$= 0.89\ \text{ft}$$

Discussion Note that the difference between the two results is less than 2%. Therefore, the simple Swamee-Jain relation can be used with confidence.

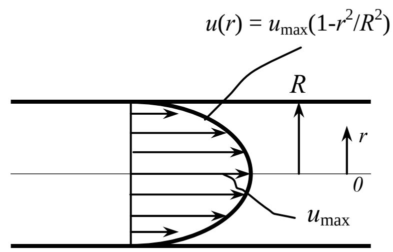
8-34

Solution In fully developed laminar flow in a circular pipe, the velocity at $r = R/2$ is measured. The velocity at the center of the pipe ($r = 0$) is to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



where $u_{\max}$ is the maximum velocity which occurs at pipe center, $r = 0$. At $r = R/2$,

$$u(R/2) = u_{\max}\left(1 - \frac{(R/2)^2}{R^2}\right) = u_{\max}\left(1 - \frac{1}{4}\right) = \frac{3u_{\max}}{4}$$

Solving for $u_{\max}$ and substituting,

$$u_{max} = \frac{4u(R/2)}{3} = \frac{4(6 \text{ m/s})}{3} = \mathbf{8.00\ m/s}$$

which is the velocity at the pipe center.

Discussion The relationship used here is valid only for fully developed laminar flow. The result would be much different if the flow were turbulent.

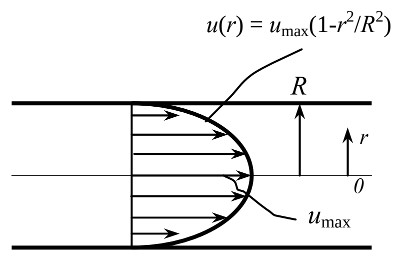
8-35

Solution The velocity profile in fully developed laminar flow in a circular pipe is given. The average and maximum velocities as well as the flow rate are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 4(1 - r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be $\boldsymbol{u_{max}}$ **= 4.00 m/s**. Then the average velocity and volume flow rate become

$$V_{avg} = \frac{u_{max}}{2} = \frac{4 \text{ m/s}}{2} = \mathbf{2.00\ m/s}$$

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi R^2) = (2 \text{ m/s})[\pi(0.02 \text{ m})^2] = \mathbf{0.00251\ m^3/s}$$

Discussion A unique feature of fully developed laminar pipe flow is that the maximum velocity is exactly twice the average velocity. This is *not* the case for turbulent pipe flow, since the velocity profile is much fuller.

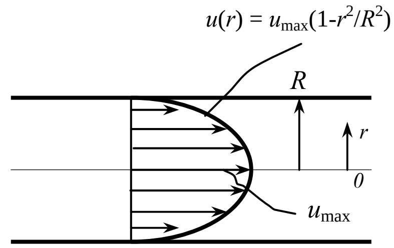
8-36

Solution The velocity profile in fully developed laminar flow in a circular pipe is given. The average and maximum velocities as well as the flow rate are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$u(r) = u_{\max}\left(1-\frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 4(1-r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be $u_{\max}$ **= 4.00 m/s**. Then the average velocity and volume flow rate become

$$V_{avg} = \frac{u_{max}}{2} = \frac{4\text{ m/s}}{2} = \mathbf{2.00\ m/s}$$

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi R^2) = (2\text{ m/s})[\pi(0.07\text{ m})^2] = \mathbf{0.0308\ m^3/s}$$

Discussion Compared to the previous problem, the average velocity remains the same since the maximum velocity (at the centerline) remains the same, but the volume flow rate increases as the diameter increases.

8-37

Solution Air enters the constant spacing between the glass cover and the plate of a solar collector. The pressure drop of air in the collector is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The roughness effects are negligible, and thus the inner surfaces are considered to be smooth, ε ≈ 0. **4** Air is an ideal gas. **5** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and 45° are ρ = 1.109 kg/m^3, μ = 1.941×10^{-5} kg/m⋅s, and ν = 1.750×10^{-5} m^2/s.

Analysis Mass flow rate, cross-sectional area, hydraulic diameter, average velocity, and the Reynolds number are

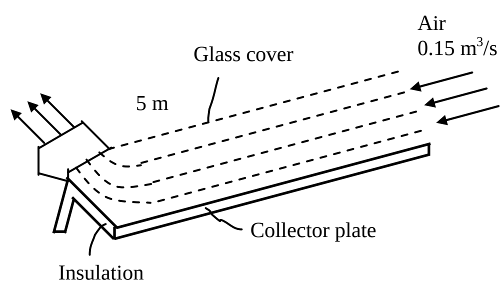
$$\dot{m} = \rho\dot{V} = (1.11\text{ kg/m}^3)(0.15\text{ m}^3/\text{s}) = 0.1665\text{ kg/s}$$

$$A_c = a\times b = (1\text{ m})(0.03\text{ m}) = 0.03\text{ m}^2$$

$$D_h = \frac{4A_c}{p} = \frac{4(0.03\text{ m}^2)}{2(1+0.03)\text{ m}} = 0.05825\text{ m}$$

$$V = \frac{\dot{V}}{A_c} = \frac{0.15\text{ m}^3/s}{0.03\text{ m}^2} = 5\text{ m}/s$$

$$\text{Re} = \frac{VD_h}{\upsilon} = \frac{(5\text{ m/s})(0.05825\text{ m})}{1.750\times10^{-5}\text{ m}^2/\text{s}} = 1.664\times10^4$$



Since Re is greater than 4000, the flow is turbulent. The friction factor corresponding to this Reynolds number for a smooth flow section (ε/*D* = 0) can be obtained from the Moody chart. But to avoid reading error, we use the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0+\frac{2.51}{16{,}640\sqrt{f}}\right)$$

which gives f = 0.0271. Then the pressure drop becomes

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0271\frac{5\text{ m}}{0.05825\text{ m}}\frac{(1.11\text{ kg/m}^3)(5\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ Pa}}{1\text{ N/m}^2}\right) = \mathbf{32.3\ Pa}$$

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.0270, which is sufficiently close to 0.0271.

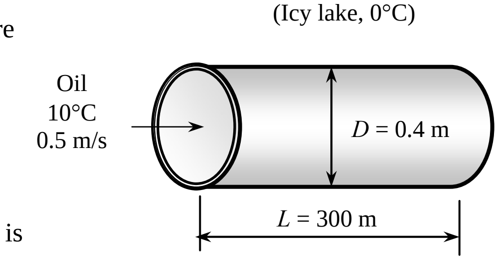
8-38

Solution Oil flows through a pipeline that passes through icy waters of a lake. The pumping power needed to overcome pressure losses is to be determined.

Assumptions The flow is steady and incompressible. **2** The flow section considered is away from the entrance, and thus the flow is fully developed. **3** The roughness effects are negligible, and thus the inner surfaces are considered to be smooth, $\varepsilon \approx 0$.

Properties The properties of oil are given to be ρ = 894 kg/m^3 and μ = 2.33 kg/m·s.

Analysis The volume flow rate and the Reynolds number in this case are



$$\dot{V} = VA_c = V\frac{\pi D^2}{4} = (0.5\text{ m/s})\frac{\pi(0.4\text{ m})^2}{4} = 0.0628\text{ m}^3/\text{s}$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(894\text{ kg/m}^3)(0.5\text{ m/s})(0.4\text{ m})}{2.33\text{ kg/m}\cdot\text{s}} = 76.7$$

which is less than 2300. Therefore, the flow is laminar, and the friction factor is

$$f = \frac{64}{\text{Re}} = \frac{64}{76.7} = 0.834$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.834\frac{300\text{ m}}{0.4\text{ m}}\frac{(894\text{ kg/m}^3)(0.5\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) = 69.9\text{ kPa}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.0628\text{ m}^3/\text{s})(69.9\text{ kPa})\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{4.39\ kW}$$

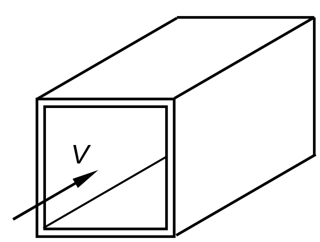
Discussion The power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

8-39

Solution Laminar flow through a square channel is considered. The change in the head loss is to be determined when the average velocity is doubled.

Assumptions **1** The flow remains laminar at all times. **2** The entrance effects are negligible, and thus the flow is fully developed.

Analysis The friction factor for fully developed laminar flow in a square channel is



$$f = \frac{56.92}{\text{Re}} \quad \text{where} \quad \text{Re} = \frac{\rho VD}{\mu}$$

Then the head loss for laminar flow can be expressed as

$$h_{L,1} = f\frac{L}{D}\frac{V^2}{2g} = \frac{56.92}{\text{Re}}\frac{L}{D}\frac{V^2}{2g} = \frac{56.92\mu}{\rho VD}\frac{L}{D}\frac{V^2}{2g} = 28.46V\frac{\mu L}{\rho g D^2}$$

which shows that the head loss is proportional to the average velocity. Therefore, **the head loss doubles when the average velocity is doubled**. This can also be shown as

$$h_{L,2} = 28.46V_2\frac{\mu L}{\rho g D^2} = 28.46(2V)\frac{\mu L}{\rho g D^2} = 2\left(28.46V\frac{\mu L}{\rho g D^2}\right) = 2h_{L,1}$$

Discussion The conclusion above is also valid for laminar flow in channels of different cross-sections.

8-40

Solution Turbulent flow through a smooth pipe is considered. The change in the head loss is to be determined when the average velocity is doubled.

Assumptions **1** The flow remains turbulent at all times. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The inner surface of the pipe is smooth.

Analysis The friction factor for the turbulent flow in smooth pipes is given as

$$f = 0.184\,\mathrm{Re}^{-0.2} \quad \text{where} \quad \mathrm{Re} = \frac{\rho V D}{\mu}$$

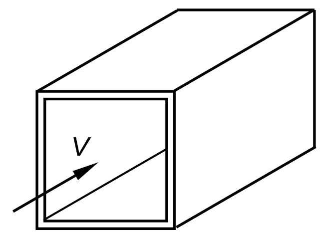
Then the head loss of the fluid for turbulent flow can be expressed as

$$h_{L,1} = f\frac{L}{D}\frac{V^2}{2g} = 0.184\,\mathrm{Re}^{-0.2}\frac{L}{D}\frac{V^2}{2g} = 0.184\left(\frac{\rho V D}{\mu}\right)^{-0.2}\frac{L}{D}\frac{V^2}{2g} = 0.184\left(\frac{\rho D}{\mu}\right)^{-0.2}\frac{L}{D}\frac{V^{1.8}}{2g}$$

which shows that the head loss is proportional to the 1.8$^{\text{th}}$ power of the average velocity. Therefore, the head loss increases by a factor of $2^{1.8} = 3.48$ when the average velocity is doubled. This can also be shown as

$$h_{L,2} = 0.184\left(\frac{\rho D}{\mu}\right)^{-0.2}\frac{L}{D}\frac{V_2^{1.8}}{2g} = 0.184\left(\frac{\rho D}{\mu}\right)^{-0.2}\frac{L}{D}\frac{(2V)^{1.8}}{2g}$$

$$= 2^{1.8}\left[0.184\left(\frac{\rho D}{\mu}\right)^{-0.2}\frac{L}{D}\frac{V^{1.8}}{2g}\right] = 2^{1.8}h_{L,1} = 3.48h_{L,1}$$



For fully rough flow in a rough pipe, the friction factor is independent of the Reynolds number and thus the flow velocity. Therefore, **the head loss increases by a factor of 4** in this case since

$$h_{L,1} = f\frac{L}{D}\frac{V^2}{2g}$$

and thus the **head loss is proportional to the square of the average velocity** when *f, L,* and *D* are constant.

Discussion Most flows in practice are in the fully rough regime, and thus the head loss is generally assumed to be proportional to the square of the average velocity for all kinds of turbulent flow. Note that we use diameter *D* here in place of hydraulic diameter D_h. For a square duct, it turns out that $D_h = D$, so this is a valid approximation.

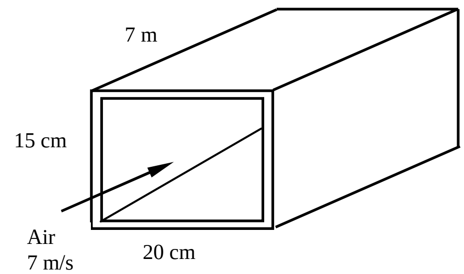
8-41

Solution Air enters a rectangular duct. The fan power needed to overcome the pressure losses is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines

Properties The properties of air at 1 atm and 35°C are ρ = 1.145 kg/m^3, μ = 1.895×10^{-5} kg/m·s, and ν = 1.655×10^{-5} m^2/s. The roughness of commercial steel surfaces is ε = 0.000045 m.

Analysis The hydraulic diameter, the volume flow rate, and the Reynolds number in this case are



$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{4(0.15\text{ m})(0.20\text{ m})}{2(0.15+0.20)\text{ m}} = 0.1714\text{ m}$$

$$\dot{V} = VA_c = V(a\times b) = (7\text{ m/s})(0.15\times 0.20\text{ m}^2) = 0.21\text{ m}^3\text{/s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(1.145\text{ kg/m}^3)(7\text{ m/s})(0.1714\text{ m})}{1.895\times 10^{-5}\text{ kg/m}\cdot\text{s}} = 72{,}490$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D_h = \frac{4.5\times 10^{-5}\text{ m}}{0.1714\text{ m}} = 2.625\times 10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.625\times 10^{-4}}{3.7} + \frac{2.51}{72{,}490\sqrt{f}}\right)$$

It gives f = 0.02034. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.02034\frac{7\text{ m}}{0.1714\text{ m}}\frac{(1.145\text{ kg/m}^3)(7\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ Pa}}{1\text{ N/m}^2}\right) = 23.3\text{ Pa}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = \left(0.21\text{ m}^3\text{/s}\right)\left(23.3\text{ Pa}\right)\left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{4.90\ W}$$

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.02005, which is sufficiently close to 0.02034. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency.

8-42E

Solution Water passes through copper tubes at a specified rate. The pumping power required per ft length to maintain flow is to be determined.

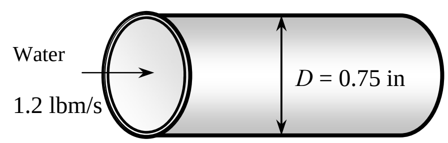
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water at 60°F are ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft·h = 7.536×10^{-4} lbm/ft·s. The roughness of copper tubing is 5×10^{-6} ft.

Analysis First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V=\frac{\dot{m}}{\rho A_c}=\frac{\dot{m}}{\rho(\pi D^2/4)}=\frac{1.2\text{ lbm}/s}{(62.36\text{ lbm/ft}^3)[\pi(0.75/12\text{ ft})^2/4]}=6.272\text{ ft/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(62.36\text{ lbm/ft}^3)(6.272\text{ ft/s})(0.75/12\text{ ft})}{7.536\times10^{-4}\text{ lbm/ft}\cdot\text{s}}=32{,}440$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{5\times10^{-6}\text{ ft}}{0.75/12\text{ ft}}=8\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\ \rightarrow\ \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{8\times10^{-5}}{3.7}+\frac{2.51}{32{,}440\sqrt{f}}\right)$$

It gives f = 0.02328. Then the pressure drop and the required power input become

$$\Delta P=\Delta P_L=f\frac{L}{D}\frac{\rho V^2}{2}=0.02328\frac{1\text{ ft}}{0.75/12\text{ ft}}\frac{(62.36\text{ lbm/ft}^3)(6.272\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}}\right)=14.2\text{ lbf/ft}^2$$

$$\dot{W}_{\text{pump}}=\dot{V}\Delta P=\frac{\dot{m}\Delta P}{\rho}=\frac{(1.2\text{ lbm/s})(14.2\text{ lbf/ft}^2)}{62.36\text{ lbm/ft}^3}\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right)=\mathbf{0.37\text{ W}}\text{ (per ft length)}$$

Therefore, useful power input in the amount of 0.37 W is needed per ft of tube length to overcome the frictional losses in the pipe.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.02305, which is sufficiently close to 0.02328. Also, the friction factor corresponding to ε = 0 in this case is 0.02306, which indicates that copper pipes can be assumed to be smooth with a negligible error. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

8-43

Solution The pressure of oil in a pipe which discharges into the atmosphere is measured at a certain location. The flow rates are to be determined for 3 different orientations.

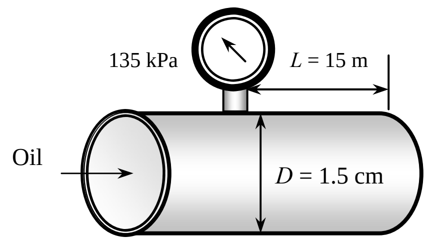
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is laminar (to be verified). **4** The pipe involves no components such as bends, valves, and connectors. **5** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of oil are given to be ρ = 876 kg/m^3 and μ = 0.24 kg/m·s.

Analysis The pressure drop across the pipe and the cross-sectional area are

$$\Delta P = P_1 - P_2 = 135 - 88 = 47 \text{ kPa}$$

$$A_c = \pi D^2/4 = \pi(0.015 \text{ m})^2/4 = 1.767\times10^{-4} \text{ m}^2$$



(*a*) The flow rate for all three cases can be determined from,

$$\dot V = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\dot V_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} = \frac{(47 \text{ kPa})\pi(0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right) = \mathbf{1.62\times10^{-5} \ m^3/s}$$

(*b*) For uphill flow with an inclination of 8°, we have θ = +8°, and

$$\dot V_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[(47{,}000 \text{ Pa} - (876 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(15 \text{ m})\sin 8°]\pi(0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{1.00\times10^{-5} \ m^3/s}$$

(*c*) For downhill flow with an inclination of 8°, we have θ = -8°, and

$$\dot V_{\text{downhill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[(47{,}000 \text{ Pa} - (876 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(15 \text{ m})\sin(-8°)]\pi(0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{2.24\times10^{-5} \ m^3/s}$$

The flow rate is the highest for downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$V = \frac{\dot V}{A_c} = \frac{2.24\times10^{-5} \text{.m}^3\text{/s}}{1.767\times10^{-4} \text{ m}^2} = 0.127 \text{ m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(876 \text{ kg/m}^3)(0.127 \text{ m/s})(0.015 \text{ m})}{0.24 \text{ kg/m}\cdot\text{s}} = 7.0$$

which is less than 2300. Therefore, the flow is **laminar** for all three cases, and the analysis above is valid.

Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. As can be seen from the calculated rates above, gravity opposes uphill flow, but helps downhill flow. Gravity has no effect on the flow rate in the horizontal case.

8-44

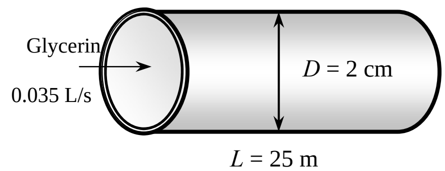
Solution Glycerin is flowing through a horizontal pipe which discharges into the atmosphere at a specified flow rate. The absolute pressure at a specified location in the pipe, and the angle θ that the pipe must be inclined downwards for the pressure in the entire pipe to be atmospheric pressure are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is laminar (to be verified). **4** The pipe involves no components such as bends, valves, and connectors. **5** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of glycerin at 40°C are given to be ρ = 1252 kg/m^3 and μ = 0.27 kg/m·s.

Analysis (*a*) The flow rate for horizontal or inclined pipe can be determined from

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L} \qquad (1)$$



where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} \qquad (2)$$

Solving for ΔP and substituting,

$$\Delta P = \frac{128\mu L \dot{V}_{\text{horiz}}}{\pi D^4} = \frac{128(0.27\text{ kg/m}\cdot\text{s})(25\text{ m})(0.035\times10^{-3}\text{ m}^3/\text{s})}{\pi(0.02\text{ m})^4}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 60.2\text{ kN/m}^2 = 60.2\text{ kPa}$$

Then the pressure 25 m before the pipe exit becomes

$$\Delta P = P_1 - P_2 \quad \rightarrow \quad P_1 = P_2 + \Delta P = 100 + 60.2 = \mathbf{160.2\ kPa}$$

(*b*) When the flow is gravity driven downhill with an inclination θ, and the pressure in the entire pipe is constant at the atmospheric pressure, the hydrostatic pressure rise with depth is equal to pressure drop along the pipe due to frictional effects. Setting $\Delta P = P_1 - P_2 = 0$ in Eq. (1) and substituting, θ is determined to be

$$\dot{V}_{\text{downhill}} = \frac{\rho g \sin\theta \pi D^4}{128\mu}$$

$$0.035\times10^{-3}\text{ m}^3/\text{s} = \frac{-(1252\text{ kg/m}^3)(9.81\text{ m/s}^2)\sin\theta\pi(0.02\text{ m})^4}{128(0.27\text{ kg/m}\cdot\text{s})} \quad \rightarrow \quad \theta = \mathbf{-11.3°}$$

Therefore, the pipe must be inclined 11.3° downwards from the horizontal to maintain flow in the pipe at the same rate.

Verification: The average fluid velocity and the Reynolds number in this case are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.035\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.02\text{ m})^2/4} = 0.111\text{ m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(1252\text{ kg/m}^3)(0.111\text{ m/s})(0.02\text{ m})}{0.27\text{ kg/m}\cdot\text{s}} = 10.3$$

which is less than 2300. Therefore, the flow is **laminar**, as assumed, and the analysis above is valid.

Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. Gravity has no effect on the flow rate in the horizontal case, but it governs the flow alone when there is no pressure difference across the pipe.

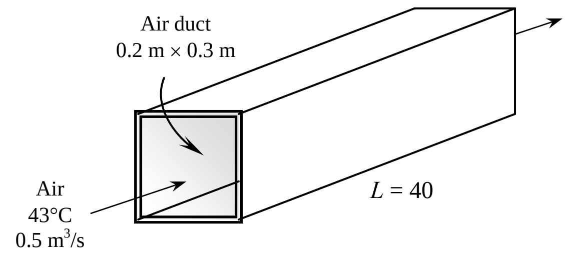
8-45

Solution Air in a heating system is distributed through a rectangular duct made of commercial steel at a specified rate. The pressure drop and head loss through a section of the duct are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines.

Properties The roughness of commercial steel surfaces is ε = 0.000045 m. The dynamic viscosity of air at 40°C is μ = 1.918×10^{-5} kg/m·s, and it is independent of pressure. The density of air listed in that table is for 1 atm. The density at 105 kPa and 315 K can be determined from the ideal gas relation to be

$$\rho = \frac{P}{RT} = \frac{105\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg.K})(40+273\text{ K})} = 1.169\text{ kg/m}^3$$



Analysis The hydraulic diameter, average velocity, and Reynolds number are

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{4(0.3\text{ m})(0.20\text{ m})}{2(0.3+0.20)\text{ m}} = 0.24\text{ m}$$

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{a\times b} = \frac{0.5\text{ m}^3/\text{s}}{(0.3\text{ m})(0.2\text{ m})} = 8.333\text{ m/s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(1.169\text{ kg/m}^3)(8.333\text{ m/s})(0.24\text{ m})}{1.918\times10^{-5}\text{ kg/m}\cdot\text{s}} = 121{,}900$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the duct is

$$\varepsilon / D_h = \frac{4.5\times10^{-5}\text{ m}}{0.24\text{ m}} = 1.875\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{1.875\times10^{-4}}{3.7} + \frac{2.51}{121{,}900\sqrt{f}}\right)$$

It gives f = 0.01833. Then the pressure drop in the duct and the head loss become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.01833\frac{40\text{ m}}{0.24\text{ m}}\frac{(1.169\text{ kg/m}^3)(8.333\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 124\text{ N/m}^2 = \mathbf{124\text{ Pa}}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.01833\frac{40\text{ m}}{0.24\text{ m}}\frac{(8.333\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{10.8\text{ m}}$$

Discussion The required pumping power in this case is

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.5\text{ m}^3/\text{s})(124\text{ Pa})\left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3/\text{s}}\right) = 62\text{ W}$$

Therefore, 62 W of mechanical power needs to be imparted to the fluid. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency. Also, the friction factor could be determined easily from the explicit Haaland relation. It would give f = 0.0181, which is sufficiently close to 0.0183.

8-46

Solution Glycerin is flowing through a smooth pipe with a specified average velocity. The pressure drop per 10 m of the pipe is to be determined.

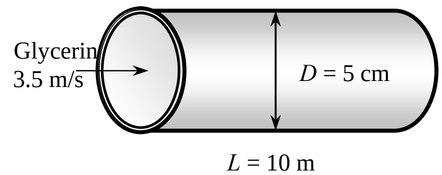
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of glycerin at 40°C are given to be ρ = 1252 kg/m^3 and μ = 0.27 kg/m·s, respectively.

Analysis The volume flow rate and the Reynolds number are

$$\dot{V} = VA_c = V(\pi D^2/4) = (3.5\text{ m/s})[\pi(0.05\text{ m})^2/4] = 0.006872\text{ m}^3\text{/s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(1252\text{ kg/m}^3)(3.5\text{ m/s})(0.05\text{ m})}{0.27\text{ kg/m}\cdot\text{s}} = 811.5$$



which is less than 2300. Therefore, the flow is laminar, and the friction factor for this circular pipe is

$$f = \frac{64}{\text{Re}} = \frac{64}{811.5} = 0.07887$$

Then the pressure drop in the pipe becomes

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.07887\frac{10\text{ m}}{0.05\text{ m}}\frac{(1252\text{ kg/m}^3)(3.5\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) = \mathbf{121\text{ kPa}}$$

Discussion The required pumping power in this case is

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.006872\text{ m}^3\text{/s})(121\text{ kPa})\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{0.83\text{ kW}}$$

Therefore, 0.83 kW of mechanical power needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

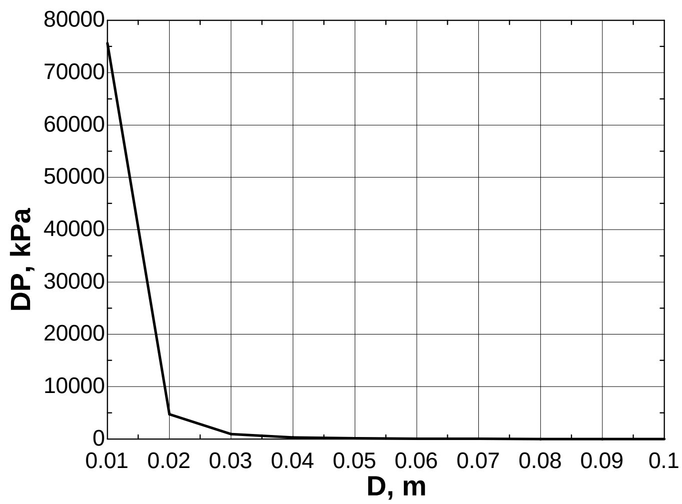
8-47

Solution In the previous problem, the effect of the pipe diameter on the pressure drop for the same constant flow rate is to be investigated by varying the pipe diameter from 1 cm to 10 cm in increments of 1 cm.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
g=9.81
Vdot=3.5*pi*(0.05)^2/4
Ac=pi*D^2/4
rho= 1252
nu=mu/rho
mu=0.27
L= 10
V=Vdot/Ac
"Reynolds number"
Re=V*D/nu
f=64/Re
DP=f*(L/D)*rho*V^2/2000 "kPa"
W=Vdot*DP "kW"
```

D, m	ΔP, kPa	*V*, m/s	Re
0.01	75600	87.5	4057
0.02	4725	21.88	2029
0.03	933.3	9.722	1352
0.04	295.3	5.469	1014
0.05	121	3.5	811.5
0.06	58.33	2.431	676.2
0.07	31.49	1.786	579.6
0.08	18.46	1.367	507.2
0.09	11.52	1.08	450.8
0.1	7.56	0.875	405.7



Discussion The pressure drop decays quite rapidly with increasing diameter – by several orders of magnitude, in fact. We conclude that larger diameter pipes are better when pressure drop is of concern. Of course, bigger pipes cost more and take up more space, so there is typically an optimum pipe size that is a compromise between cost and practicality.

8-48E

Solution Air is flowing through a square duct made of commercial steel at a specified rate. The pressure drop and head loss per ft of duct are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines.

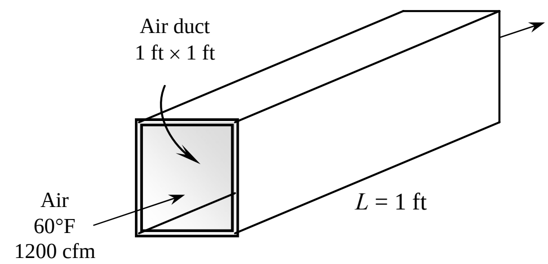
Properties The density and dynamic viscosity of air at 1 atm and 60°F are ρ = 0.07633 lbm/ft^3, μ = 0.04365 lbm/ft·h, and ν = 0.5718 ft^2/s = 1.588×10^{-4} ft^2/s. The roughness of commercial steel surfaces is ε = 0.00015 ft.

Analysis The hydraulic diameter, the average velocity, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 1\,\text{ft}$$

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{a^2} = \frac{1200\ \text{ft}^3/\text{min}}{(1\,\text{ft})^2} = 1200\ \text{ft/min} = 20\ \text{ft/s}$$

$$\text{Re} = \frac{VD_h}{\nu} = \frac{(20\ \text{ft/s})(1\,\text{ft})}{1.588\times10^{-4}\ \text{ft}^2/\text{s}} = 1.259\times10^5$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the duct is

$$\varepsilon / D_h = \frac{0.00015\,\text{ft}}{1\,\text{ft}} = 1.5\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \;\rightarrow\; \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{125{,}900\sqrt{f}}\right)$$

It gives f = 0.0180. Then the pressure drop in the duct and the head loss become

$$\Delta P = \Delta P_L = f\,\frac{L}{D}\frac{\rho V^2}{2} = 0.0180\,\frac{1\,\text{ft}}{1\,\text{ft}}\,\frac{(0.07633\,\text{lbm/ft}^3)(20\ \text{ft/s})^2}{2}\left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}}\right) = \mathbf{8.53\times10^{-3}\ lbf/ft^2}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\,\frac{L}{D}\frac{V^2}{2g} = 0.0180\,\frac{1\,\text{ft}}{1\,\text{ft}}\,\frac{(20\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = \mathbf{0.112\ ft}$$

Discussion The required pumping power in this case is

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (1200/60\ \text{ft}^3/\text{s})(8.53\times10^{-3}\ \text{lbf/ft}^2)\left(\frac{1\,\text{W}}{0.737\,\text{lbf}\cdot\text{ft/s}}\right) = 0.231\,\text{W (per ft length)}$$

Therefore, 0.231 W of mechanical power needs to be imparted to the fluid per ft length of the duct. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency. Also, the friction factor could be determined easily from the explicit Haaland relation. It would give f = 0.0178, which is sufficiently close to 0.0180.

8-49

Solution Liquid ammonia is flowing through a copper tube at a specified mass flow rate. The pressure drop, the head loss, and the pumping power required to overcome the frictional losses in the tube are to be determined.

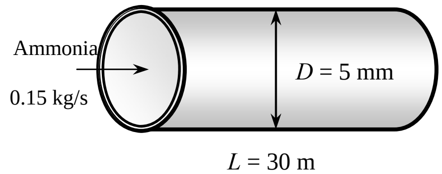
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of liquid ammonia at -20°C are ρ = 665.1 kg/m^3 and μ = 2.361×10^{-4} kg/m·s. The roughness of copper tubing is 1.5×10^{-6} m.

Analysis First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{0.15\text{ kg/s}}{(665.1\text{ kg/m}^3)[\pi(0.005\text{ m})^2/4]} = 11.49\text{ m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(665.1\text{ kg/m}^3)(11.49\text{ m/s})(0.005\text{ m})}{2.361\times10^{-4}\text{ kg/m}\cdot\text{s}} = 1.618\times10^5$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{1.5\times10^{-6}\text{ m}}{0.005\text{ m}} = 3\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{3\times10^{-4}}{3.7} + \frac{2.51}{1.618\times10^5\sqrt{f}}\right)$$

It gives f = 0.01819. Then the pressure drop, the head loss, and the useful pumping power required become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2}$$

$$= 0.01819\frac{30\text{ m}}{0.005\text{ m}}\frac{(665.1\text{ kg/m}^3)(11.49\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) = 4792\text{ kPa} \cong \mathbf{4790\text{ kPa}}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.01819\frac{30\text{ m}}{0.005\text{ m}}\frac{(11.49\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{734\text{ m}}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.15\text{ kg/s})(4792\text{ kPa})}{665.1\text{ kg/m}^3}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{1.08\text{ kW}}$$

Therefore, useful power input in the amount of 1.08 kW is needed to overcome the frictional losses in the tube.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.0180, which is sufficiently close to 0.0182. The friction factor corresponding to ε = 0 in this case is 0.0163, which is about 10% lower. Therefore, the copper tubes in this case are nearly "smooth".

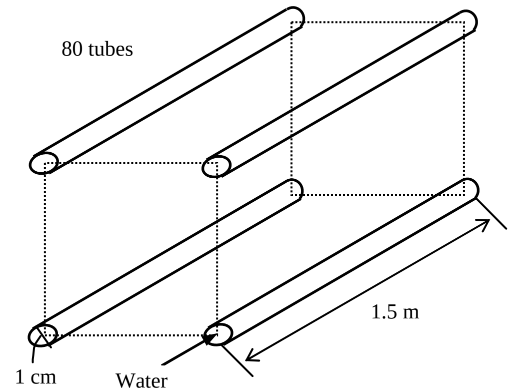
Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

8-50 **[*Also solved using EES on enclosed DVD*]**

Solution Water is flowing through a brass tube bank of a heat exchanger at a specified flow rate. The pressure drop and the pumping power required are to be determined. Also, the percent reduction in the flow rate of water through the tubes is to be determined after scale build-up on the inner surfaces of the tubes.

Assumptions **1** The flow is steady, horizontal, and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (this is a questionable assumption since the tubes are short, and it will be verified). **3** The inlet, exit, and header losses are negligible, and the tubes involve no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water at 20°C are ρ = 983.3 kg/m^3 and μ = 0.467×10^{-3} kg/m·s, respectively. The roughness of brass tubing is 1.5×10^{-6} m.



Analysis First we calculate the average velocity and the Reynolds number to determine the flow regime:

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{N_{\text{tune}}(\pi D^2/4)} = \frac{0.015\ \text{m}^3/\text{s}}{80[\pi(0.01\ \text{m})^2/4]} = 2.387\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(983.3\ \text{kg/m}^3)(2.387\ \text{m/s})(0.01\ \text{m})}{0.467\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 50{,}270$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{1.5\times10^{-6}\ \text{m}}{0.01\ \text{m}} = 1.5\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{50{,}270\sqrt{f}}\right)$$

It gives f = 0.0214. Then the pressure drop, the head loss, and the useful pumping power required become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0214\frac{1.5\ \text{m}}{0.01\ \text{m}}\frac{(983.3\ \text{kg/m}^3)(2.387\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{8.99\ kPa}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.015\ \text{m}^3/\text{s})(8.99\ \text{kPa})\left(\frac{1\ \text{kW}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.135\ kW}$$

Therefore, useful power input in the amount of 0.135 kW is needed to overcome the frictional losses in the tube. The hydrodynamic entry length in this case is

$$L_{\text{h,turbulent}} \approx 10D = 10(0.01\ \text{m}) = 0.1\ \text{m}$$

which is much less than 1.5 m. Therefore, the assumption of fully developed flow is valid. (The effect of the entry region is to increase the friction factor, and thus the pressure drop and pumping power).

After scale buildup: When 1-mm thick scale builds up on the inner surfaces (and thus the diameter is reduced to 0.8 cm from 1 cm) with an equivalent roughness of 0.4 mm, and the useful power input is fixed at 0.135 kW, the problem can be formulated as follows (note that the flow rate and thus the average velocity are unknown in this case):

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{N_{\text{tune}}(\pi D^2/4)} \rightarrow V = \frac{\dot{V}}{80[\pi(0.008\ \text{m})^2/4]} \qquad (1)$$

$$\text{Re} = \frac{\rho V D}{\mu} \rightarrow \text{Re} = \frac{(983.3\ \text{kg/m}^3)V(0.008\ \text{m})}{0.467\times10^{-3}\ \text{kg/m}\cdot\text{s}} \qquad (2)$$

$$\varepsilon/D = \frac{0.0004\ \text{m}}{0.008\ \text{m}} = 0.05$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.05}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad (3)$$

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2} \rightarrow \Delta P = f\frac{1.5\text{ m}}{0.008\text{ m}}\frac{(983.3\text{ kg/m}^3)V^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) \qquad (4)$$

$$\dot{W}_{\text{pump}} = 0.135\text{ kW} \rightarrow \dot{V}\Delta P = 0.135 \qquad (5)$$

Solving this system of 5 equations in 5 unknown (f, Re, V, ΔP, and $\dot{V}$) using an equation solver (or a trial-and-error approach, by assuming a velocity value) gives

f = 0.0723, Re = 28,870, V = 1.714 m/s, ΔP = 19.6 kPa, and $\dot{V}$ = 0.00689 m^3/s = 6.89 L/s

Then the percent reduction in the flow rate becomes

$$\text{Reduction ratio} = \frac{\dot{V}_{\text{clean}} - \dot{V}_{\text{dirty}}}{\dot{V}_{\text{clean}}} = \frac{15-6.89}{15} = 0.54 = \mathbf{54\%}$$

Therefore, for the same pump input, the flow rate will be reduced to less than half of the original flow rate when the pipes were new and clean.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

Minor Losses

8-51C

Solution We are to define minor loss and minor loss coefficient.

Analysis The **head losses associated with the flow of a fluid through fittings, valves, bends, elbows, tees, inlets, exits, enlargements, contractions, etc.** are called *minor losses*, and are expressed in terms of the *minor loss coefficient* as

$$K_L = \frac{h_L}{V^2/(2g)}$$

Discussion Basically, any irreversible loss that is not due to friction in long, straight sections of pipe is a minor loss.

8-52C

Solution We are to define equivalent length and its relationship to the minor loss coefficient.

Analysis *Equivalent length* is **the length of a straight pipe which would give the same head loss as the minor loss component**. It is related to the minor loss coefficient by

$$L_{\text{equiv}} = \frac{D}{f}K_L$$

Discussion Equivalent length is not as universal as minor loss coefficient because it depends on the roughness and Reynolds number of the equivalent straight section of pipe.

8-53C

Solution We are to discuss the effect of rounding a pipe inlet.

Analysis The effect of rounding of a pipe inlet on the loss coefficient is **(*c*) very significant**.

Discussion In fact, the minor loss coefficient changes from 0.8 for a reentrant pipe inlet to about 0.03 for a well-rounded pipe inlet – quite a significant improvement.

8-54C

Solution We are to discuss the effect of rounding on a pipe outlet.

Analysis The effect of rounding of a pipe exit on the loss coefficient is **(*a*) negligible**.

Discussion At any pipe outlet, all the kinetic energy is wasted, and the minor loss coefficient is equal to α, which is about 1.05 for fully developed turbulent pipe flow. Rounding of the outlet does not help.

8-55C

Solution We are to compare the minor losses of a gradual expansion and a gradual contraction.

Analysis **A gradual expansion, in general, has a greater minor loss coefficient than a gradual contraction** in pipe flow. This is due to the adverse pressure gradient in the boundary layer, which may lead to flow separation.

Discussion Note, however, that pressure is "recovered" in a gradual expansion. In other words, the pressure rises in the direction of flow. Such a device is called a *diffuser*.

8-56C

Solution We are to discuss ways to reduce the head loss in a pipe flow with bends.

Analysis Another way of reducing the head loss associated with turns is to **install turning vanes inside the elbows**.

Discussion There are many other possible answers, such as: **reduce the inside wall roughness of the pipe**, **use a larger diameter pipe**, **shorten the length of pipe as much as possible**, etc.

8-57C

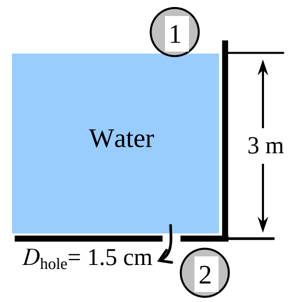
Solution We are to compare two different ways to reduce the minor loss in pipe bends.

Analysis The loss coefficient is lower for flow through a 90° miter elbow with well-designed vanes ($K_L \approx 0.2$) than it is for flow through a smooth curved bend ($K_L \approx 0.9$). Therefore, **using miter elbows with vanes results in a greater reduction in pumping power requirements**.

Discussion Both values are for threaded elbows. The loss coefficients for flanged elbows are much lower.

8-58

Solution Water is to be withdrawn from a water reservoir by drilling a hole at the bottom surface. The flow rate of water through the hole is to be determined for the well-rounded and sharp-edged entrance cases.



Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **3** The effect of the kinetic energy correction factor is disregarded, and thus $\alpha = 1$.

Analysis The loss coefficient is $K_L = 0.5$ for the sharp-edged entrance, and $K_L = 0.03$ for the well-rounded entrance. We take point 1 at the free surface of the reservoir and point 2 at the exit of the hole. We also take the reference level at the exit of the hole ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is zero ($V_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L\dfrac{V^2}{2g}$. Substituting and solving for V_2 gives

$$z_1=\alpha_2\frac{V_2^2}{2g}+K_L\frac{V_2^2}{2g} \quad\rightarrow\quad 2gz_1=V_2^2(\alpha_2+K_L) \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{\alpha_2+K_L}}=\sqrt{\frac{2gz_1}{1+K_L}}$$

since $\alpha_2 = 1$. Note that in the special case of $K_L = 0$, it reduces to the Toricelli equation $V_2=\sqrt{2gz_1}$, as expected. Then the volume flow rate becomes

$$\dot V = A_cV_2=\frac{\pi D_{hole}^2}{4}\sqrt{\frac{2gz_1}{1+K_L}}$$

Substituting the numerical values, the flow rate for both cases are determined to be

Well-rounded entrance: $\dot V=\dfrac{\pi D_{hole}^2}{4}\sqrt{\dfrac{2gz_1}{1+K_L}}=\dfrac{\pi(0.015\text{ m})^2}{4}\sqrt{\dfrac{2(9.81\text{ m/s}^2)(3\text{ m})}{1+0.03}}=\mathbf{1.34\times10^{-3}\ m^3/s}$

Sharp-edged entrance: $\dot V=\dfrac{\pi D_{hole}^2}{4}\sqrt{\dfrac{2gz_1}{1+K_L}}=\dfrac{\pi(0.015\text{m})^2}{4}\sqrt{\dfrac{2(9.81\text{ m/s}^2)(3\text{ m})}{1+0.5}}=\mathbf{1.11\times10^{-3}\ m^3/s}$

Discussion The flow rate in the case of frictionless flow ($K_L = 0$) is 1.36×10^{-3} m^3/s. Note that the frictional losses cause the flow rate to decrease by 1.5% for well-rounded entrance, and 18.5% for the sharp-edged entrance.

8-59

Solution Water is discharged from a water reservoir through a circular hole of diameter D at the side wall at a vertical distance H from the free surface. A relation for the "equivalent diameter" of the sharp-edged hole for use in frictionless flow relations is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **3** The effect of the kinetic energy correction factor is disregarded, and thus $\alpha = 1$.

Analysis The loss coefficient is $K_L = 0.5$ for the sharp-edged entrance, and $K_L = 0$ for the "frictionless" flow. We take point 1 at the free surface of the reservoir and point 2 at the exit of the hole, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is zero ($V_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbin e, e}}+h_L \quad\rightarrow\quad H=\alpha_2\frac{V_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L\dfrac{V^2}{2g}$. Substituting and solving for V_2 gives

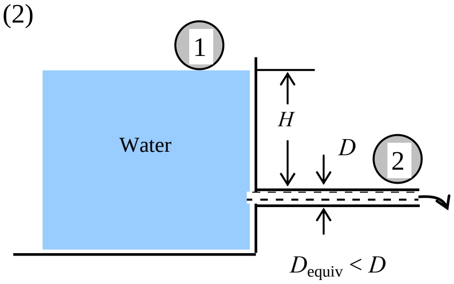
$$H=\alpha_2\frac{V_2^2}{2g}+K_L\frac{V_2^2}{2g} \quad\rightarrow\quad 2gH=V_2^2(\alpha_2+K_L) \quad\rightarrow\quad V_2=\sqrt{\frac{2gH}{\alpha_2+K_L}}=\sqrt{\frac{2gH}{1+K_L}}$$

since $\alpha_2 = 1$. Then the volume flow rate becomes

$$\dot V = A_c V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}} \qquad (1)$$

Note that in the special case of $K_L = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $V_{2,\text{frictionless}} = \sqrt{2gH}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

$$\dot V = A_{c,\text{equiv}}V_{2,\text{frictionless}} = \frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH} \qquad (2)$$



Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH}=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}}$$

which gives

$$D_{\text{equiv}}=\frac{D}{(1+K_L)^{1/4}}=\frac{D}{(1+0.5)^{1/4}}=\mathbf{0.904\,D}$$

Discussion Note that the effect of frictional losses of a sharp-edged entrance is to reduce the diameter by about 10%. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot V \propto D_{\text{equiv}}^2 = (0.904D)^2 = 0.82D^2$. Therefore, the flow rate through a sharp-edged entrance is about 18% less compared to the frictionless entrance case.

8-60

Solution Water is discharged from a water reservoir through a circular hole of diameter D at the side wall at a vertical distance H from the free surface. A relation for the "equivalent diameter" of the slightly rounded hole for use in frictionless flow relations is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **3** The effect of the kinetic energy correction factor is disregarded, and thus $\alpha = 1$.

Analysis The loss coefficient is $K_L = 0.12$ for the slightly rounded entrance, and $K_L = 0$ for the "frictionless" flow.

We take point 1 at the free surface of the reservoir and point 2 at the exit of the hole, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is zero ($V_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbin e, e}}+h_L \quad\rightarrow\quad H=\alpha_2\frac{V_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L\dfrac{V^2}{2g}$. Substituting and solving for V_2 gives

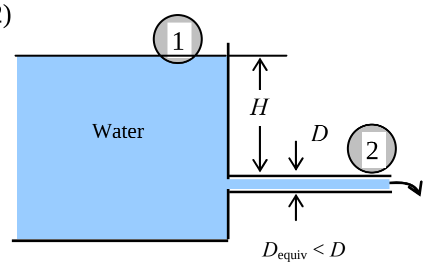
$$H=\alpha_2\frac{V_2^2}{2g}+K_L\frac{V_2^2}{2g} \quad\rightarrow\quad 2gH=V_2^2(\alpha_2+K_L) \quad\rightarrow\quad V_2=\sqrt{\frac{2gH}{\alpha_2+K_L}}=\sqrt{\frac{2gH}{1+K_L}}$$

since $\alpha_2 = 1$. Then the volume flow rate becomes

$$\dot{V}=A_cV_2=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}} \qquad (1)$$

Note that in the special case of $K_L = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $V_{2,\text{frictionless}}=\sqrt{2gz_1}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

$$\dot{V}=A_{c,\text{equiv}}V_{2,\text{frictionless}}=\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH} \qquad (2)$$



Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4}\sqrt{2gH}=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{1+K_L}}$$

which gives

$$D_{\text{equiv}}=\frac{D}{(1+K_L)^{1/4}}=\frac{D}{(1+0.12)^{1/4}}=\mathbf{0.972\,D}$$

Discussion Note that the effect of frictional losses of a slightly rounded entrance is to reduce the diameter by about 3%. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot{V}\propto D_{\text{equiv}}^2=(0.972D)^2=0.945D^2$. Therefore, the flow rate through a slightly rounded entrance is about 5% less compared to the frictionless entrance case.

8-61

Solution A horizontal water pipe has an abrupt expansion. The water velocity and pressure in the smaller diameter pipe are given. The pressure after the expansion and the error that would have occurred if the Bernoulli Equation had been used are to be determined.

Assumptions **1** The flow is steady, horizontal, and incompressible. **2** The flow at both the inlet and the outlet is fully developed and turbulent with kinetic energy corrections factors of $\alpha_1 = \alpha_2 = 1.06$ (given).

Properties We take the density of water to be $\rho = 1000$ kg/m^3.

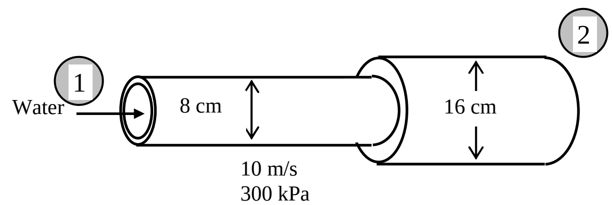
Analysis Noting that ρ = const. (incompressible flow), the downstream velocity of water is

$$\dot m_1 = \dot m_2 \;\rightarrow\; \rho V_1 A_1 = \rho V_2 A_2 \;\rightarrow\; V_2 = \frac{A_1}{A_2}V_1 = \frac{\pi D_1^2/4}{\pi D_2^2/4}V_1 = \frac{D_1^2}{D_2^2}V_1 = \frac{(0.08\text{ m})^2}{(0.16\text{ m})^2}(10\text{ m/s}) = 2.5\text{ m/s}$$

The loss coefficient for sudden expansion and the head loss can be calculated from

$$K_L = \left(1 - \frac{A_{\text{small}}}{A_{\text{large}}}\right)^2 = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 = \left(1 - \frac{0.08^2}{0.16^2}\right)^2 = 0.5625$$

$$h_L = K_L\frac{V_1^2}{2g} = (0.5625)\frac{(10\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 2.87\text{ m}$$



Noting that $z_1 = z_2$ and there are no pumps or turbines involved, the energy equation for the expansion section can be expressed in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad\rightarrow\quad \frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + h_L$$

Solving for P_2 and substituting,

$$P_2 = P_1 + \rho\left\{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2} - gh_L\right\}$$

$$= (300\text{ kPa}) + (1000\text{ kg/m}^3)\left\{\frac{1.06(10\text{ m/s})^2 - 1.06(2.5\text{ m/s})^2}{2} - (9.81\text{ m/s}^2)(2.87\text{ m})\right\}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)$$

$$= \mathbf{322\ \ kPa}$$

Therefore, despite the head (and pressure) loss, the pressure increases from 300 kPa to 321 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the velocity is decreased.

When the head loss is disregarded, the downstream pressure is determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad\rightarrow\quad \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \quad\rightarrow\quad P_1 = P_1 + \rho\frac{V_1^2 - V_2^2}{2}$$

Substituting,

$$P_2 = (300\text{ kPa}) + (1000\text{ kg/m}^3)\frac{(10\text{ m/s})^2 - (2.5\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) = 347\text{ kPa}$$

Therefore, the error in the Bernoulli equation is Error = $P_{2,\text{ Bernoulli}} - P_2$ = 347 – 322 = **25.0 kPa**

Note that the use of the Bernoulli equation results in an error of (347 – 322) / 322 = 0.078 or 7.8%.

Discussion It is common knowledge that higher pressure upstream is necessary to cause flow, and it may come as a surprise that the downstream pressure has *increased* after the abrupt expansion, despite the loss. This is because the sum of the three Bernoulli terms which comprise the total head, consisting of pressure head, velocity head, and elevation head, namely $[P/\rho g + ½V^2/g + z]$, drives the flow. With a geometric flow expansion, initially higher velocity head is converted to downstream pressure head, and this increase outweighs the non-convertible and non-recoverable head loss term.

Piping Systems and Pump Selection

8-62C

Solution We are to compare the flow rate and pressure drop in two pipes of different diameters in series.

Analysis For a piping system that involves two pipes of different diameters (but of identical length, material, and roughness) connected in *series*, (*a*) **the flow rate through both pipes is the same** and (*b*) **the pressure drop through the smaller diameter pipe is larger**.

Discussion The wall shear stress on the smaller pipe is larger, friction factor *f* is larger, and thus the head loss is higher.

8-63C

Solution We are to compare the flow rate and pressure drop in two pipes of different diameters in parallel.

Analysis For a piping system that involves two pipes of different diameters (but of identical length, material, and roughness) connected in *parallel*, (*a*) **the flow rate through the larger diameter pipe is larger** and (*b*) **the pressure drop through both pipes is the same**.

Discussion Since the two pipes separate from each other but then later re-join, the pressure drop between the two junctions must be the same, regardless of which pipe segment is under consideration.

8-64C

Solution We are to compare the pressure drop of two different-length pipes in parallel.

Analysis **The pressure drop through both pipes is the *same*** since the pressure at a point has a single value, and the inlet and exits of these the pipes connected in parallel coincide.

Discussion The length, diameter, roughness, and number and type of minor losses are all irrelevant – for any two pipes in parallel, both have the same pressure drop.

8-65C

Solution We are to discuss whether the required pump head is equal to the elevation difference when irreversible head losses are negligible.

Analysis **Yes**, when the head loss is negligible, the required pump head is equal to the elevation difference between the free surfaces of the two reservoirs.

Discussion A pump in a piping system may: (1) raise the fluid's elevation, and/or (2) increase the fluid's kinetic energy, and/or (3) increase the fluid's pressure, and/or (4) overcome irreversible losses. In this case, (2), (3), and (4) are zero or negligible; thus only (1) remains.

8-66C

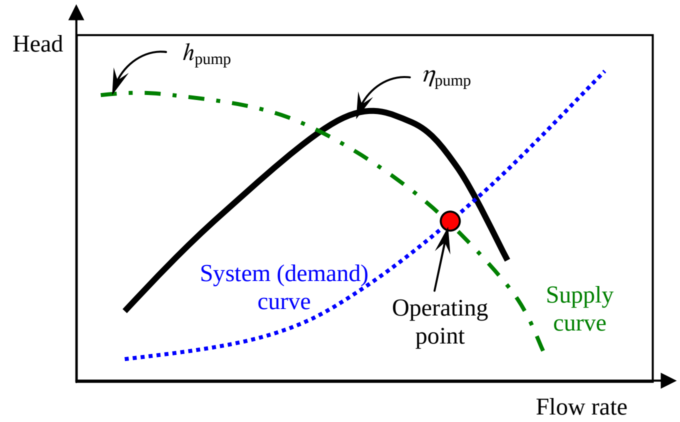
Solution We are to explain how the operating point of a pipe/pump system is determined.

Analysis The pump installed in a piping system **operates at the point where the *system curve* and the *characteristic curve* intersect**. This **point of intersection** is called the *operating point*.

Discussion The volume flow rate "automatically" adjusts itself to reach the operating point.

8-67C

Solution We are to draw a pump head versus flow rate chart and identify several parameters.



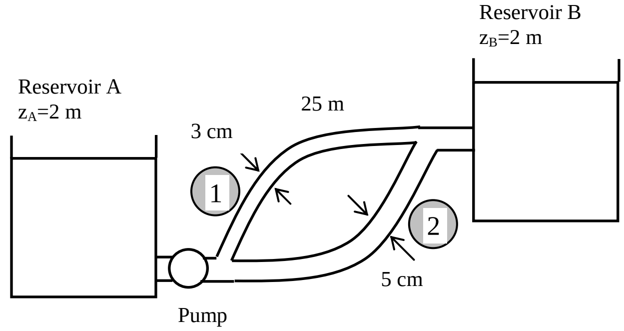
Analysis The **plot of the head loss versus the flow rate** is called the *system curve*. The experimentally determined **pump head and pump efficiency versus the flow rate curves** are called *characteristic curves*. **The pump installed in a piping system operates at the point where the *system curve* and the *characteristic curve* intersect**. This point of intersection is called the *operating point*.

Discussion By matching the system (demand) curve and the .

8-68 [*Also solved using EES on enclosed DVD*]

Solution The pumping power input to a piping system with two parallel pipes between two reservoirs is given. The flow rates are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible. **5** The flows through both pipes are turbulent (to be verified).



Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s. Plastic pipes are smooth, and their roughness is zero, ε = 0.

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Therefore, we would normally use a trial-and-error approach here. However, nowadays the equation solvers such as EES are widely available, and thus below we will simply set up the equations to be solved by an equation solver. The head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect,in}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump-motor}}} \quad \rightarrow \quad 7000\text{ W} = \frac{(998\text{ kg/m}^3)\dot{V}(9.81\text{ m/s}^2)h_{\text{pump, u}}}{0.68} \qquad (1)$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{\text{atm}}$) and that the fluid velocities at both points are zero ($V_A = V_B$ =0), the energy equation for a control volume between these two points simplifies to

$$\frac{P_A}{\rho g} + \alpha_A \frac{V_A^2}{2g} + z_A + h_{\text{pump, u}} = \frac{P_B}{\rho g} + \alpha_B \frac{V_B^2}{2g} + z_B + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{pump, u}} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump, u}} = (9-2) + h_L \qquad (2)$$

where

$$h_L = h_{L,1} = h_{L,2} \qquad (3)\,(4)$$

We designate the 3-cm diameter pipe by 1 and the 5-cm diameter pipe by 2. The average velocity, Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi(0.03\text{m})^2/4} \qquad (5)$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi(0.05\text{m})^2/4} \qquad (6)$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\text{ kg/m}^3)V_1(0.03\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot s} \qquad (7)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\text{ kg/m}^3)V_2(0.05\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot s} \qquad (8)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (9)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (10$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1 \frac{25\text{ m}}{0.03\text{ m}} \frac{V_1^2}{2(9.81\text{ m/s}^2)} \qquad (11)$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2 \frac{25\text{ m}}{0.05\text{ m}} \frac{V_2^2}{2(9.81\text{ m/s}^2)} \qquad (12)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \qquad (13)$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$\dot{V} = \mathbf{0.0183\ m^3/s}$, $\dot{V}_1 = \mathbf{0.0037\ m^3/s}$, $\dot{V}_2 = \mathbf{0.0146\ m^3/s}$,

V_1 = 5.30 m/s, V_2 = 7.42 m/s, $h_L = h_{L,1} = h_{L,2} = 19.5$ m, $h_{\text{pump,u}}$ = 26.5 m

Re_1 = 158,300, Re_2 = 369,700, f_1 = 0.0164, f_2 = 0.0139

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using an iterative approach, but it will be very time consuming. Equation solvers such as EES are invaluable for this kind of problems.

8-69E

Solution The flow rate through a piping system connecting two reservoirs is given. The elevation of the source is to be determined.

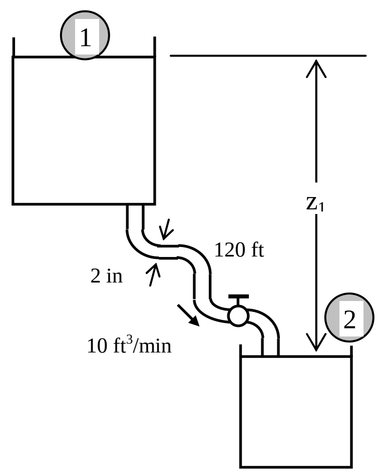
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** There are no pumps or turbines in the piping system.

Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.360 lbm/ft·h = 6.556×10^{-4} lbm/ft·s. The roughness of cast iron pipe is ε = 0.00085 ft.

Analysis The piping system involves 120 ft of 2-in diameter piping, a well-rounded entrance (K_L = 0.03), 4 standard flanged elbows (K_L = 0.3 each), a fully open gate valve (K_L = 0.2), and a sharp-edged exit (K_L = 1.0). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), the fluid velocities at both points are zero ($V_1 = V_2 = 0$), the free surface of the lower reservoir is the reference level (z_2 = 0), and that there is no pump or turbine ($h_{pump,u} = h_{turbine}$ = 0), the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=h_L$$

where $h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}$



since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{10/60\ \text{ft}^3/\text{s}}{\pi(2/12\ \text{ft})^2/4}=7.64\ \text{ft/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(62.3\ \text{lbm/ft}^3)(7.64\ \text{ft/s})(2/12\ \text{ft})}{1.307\times10^{-3}\ \text{lbm/ft}\cdot\text{s}}=60{,}700$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00085\ \text{ft}}{2/12\ \text{ft}}=0.0051$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\quad\rightarrow\quad\frac{1}{\sqrt f}=-2.0\log\left(\frac{0.0051}{3.7}+\frac{2.51}{60{,}700\sqrt f}\right)$$

It gives f = 0.0320. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+4K_{L,\text{elbow}}+K_{L,\text{valve}}+K_{L,\text{exit}}=0.03+4\times0.3+0.2+1.0=2.43$$

Then the total head loss and the elevation of the source become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left((0.0320)\frac{120\ \text{ft}}{2/12\ \text{ft}}+2.43\right)\frac{(7.64\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}=23.1\ \text{ft}$$

$$z_1=h_L=\mathbf{23.1\ ft}$$

Therefore, the free surface of the first reservoir must be 23.1 ft above the free surface of the lower reservoir to ensure water flow between the two reservoirs at the specified rate.

Discussion Note that fL/D = 23.0 in this case, which is almost 10 folds of the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in an error of about 10%.

8-70

Solution A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere. The initial velocity from the tank and the time required to empty the tank are to be determined.

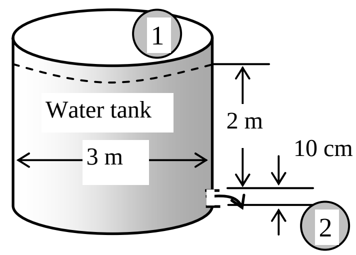
Assumptions **1** The flow is uniform and incompressible. **2** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **3** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the orifice. We also take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad z_1 = \alpha_2 \frac{V_2^2}{2g} + h_L$$

where the head loss is expressed as $h_L = K_L \dfrac{V^2}{2g}$. Substituting and solving for V_2 gives



$$z_1 = \alpha_2 \frac{V_2^2}{2g} + K_L \frac{V_2^2}{2g} \quad \rightarrow \quad 2gz_1 = V_2^2 \left(\alpha_2 + K_L\right) \quad \rightarrow \quad V_2 = \sqrt{\frac{2gz_1}{\alpha_2 + K_L}}$$

where $\alpha_2 = 1$. Noting that initially $z_1 = 2$ m, the initial velocity is determined to be

$$V_2 = \sqrt{\frac{2gz_1}{1 + K_L}} = \sqrt{\frac{2(9.81\ \text{m/s}^2)(2\ \text{m})}{1 + 0.5}} = \mathbf{5.11\ m/s}$$

The average discharge velocity through the orifice at any given time, in general, can be expressed as

$$V_2 = \sqrt{\frac{2gz}{1 + K_L}}$$

where z is the water height relative to the center of the orifice at that time.

(*b*) We denote the diameter of the orifice by D, and the diameter of the tank by D_0. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the orifice area,

$$\dot{V} = A_{\text{orifice}} V_2 = \frac{\pi D^2}{4} \sqrt{\frac{2gz}{1 + K_L}}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV = \dot{V} dt = \frac{\pi D^2}{4} \sqrt{\frac{2gz}{1 + K_L}} dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\text{tank}}(-dz) = -\frac{\pi D_0^2}{4} dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4} \sqrt{\frac{2gz}{1 + K_L}} dt = -\frac{\pi D_0^2}{4} dz \quad \rightarrow \quad dt = -\frac{D_0^2}{D^2} \sqrt{\frac{1 + K_L}{2gz}} dz \quad \rightarrow \quad dt = -\frac{D_0^2}{D^2} \sqrt{\frac{1 + K_L}{2g}} z^{-1/2} dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2} \sqrt{\frac{1 + K_L}{2g}} \int_{z=z_1}^{0} z^{-1/2} dz \quad \rightarrow \quad t_f = -\frac{D_0^2}{D^2} \sqrt{\frac{1 + K_L}{2g}} \left. \frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right|_{z_1}^{0} = \frac{2D_0^2}{D^2} \sqrt{\frac{1 + K_L}{2g}} z_1^{1/2}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+K_L)}{g}} = \frac{(3\,\mathrm{m})^2}{(0.1\,\mathrm{m})^2}\sqrt{\frac{2(2\,\mathrm{m})(1+0.5)}{9.81\,\mathrm{m/s^2}}} = \mathbf{704\ s} = \mathbf{11.7\ min}$$

Discussion The effect of the loss coefficient K_L on the draining time can be assessed by setting it equal to zero in the draining time relation. It gives

$$t_{f,\text{zero loss}} = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1}{g}} = \frac{(3\,\mathrm{m})^2}{(0.1\,\mathrm{m})^2}\sqrt{\frac{2(2\,\mathrm{m})}{9.81\,\mathrm{m/s^2}}} = 575\,\mathrm{s} = 9.6\ \mathrm{min}$$

Note that the loss coefficient causes the draining time of the tank to increase by (11.7- 9.6)/11.7 = 0.18 or 18%, which is quite **significant**. Therefore, the loss coefficient should always be considered in draining processes.

8-71

Solution A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe. The initial velocity from the tank and the time required to empty the tank are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes). **5** The effect of the kinetic energy correction factor is negligible, so we set $\alpha = 1$.

Properties The loss coefficient is K_L = 0.5 for a sharp-edged entrance. The friction factor of the pipe is given to be 0.015.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the pipe (z_2 = 0), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

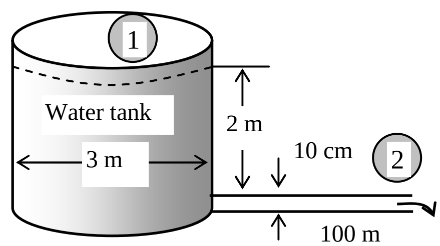
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. Substituting and solving for V_2 gives

$$z_1=\alpha_2\frac{V_2^2}{2g}+\left(f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{\alpha_2+fL/D+K_L}}$$



where α_2 = 1. Noting that initially z_1 = 2 m, the initial velocity is determined to be

$$V_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}=\sqrt{\frac{2(9.81\text{ m/s}^2)(2\text{ m})}{1+0.015(100\text{ m})/(0.1\text{ m})+0.5}}=\mathbf{1.54\ m/s}$$

The average discharge velocity at any given time, in general, can be expressed as

$$V_2=\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

where z is the water height relative to the center of the orifice at that time.

(*b*) We denote the diameter of the pipe by D, and the diameter of the tank by D_o. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V}=A_{pipe}V_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$dV=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}\,dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV=A_{\tan k}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt=-\frac{\pi D_0^2}{4}dz \;\rightarrow\; dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2gz}}dz=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \;\rightarrow t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D+K_L)}{g}}=\frac{(3\,\text{m})^2}{(0.1\,\text{m})^2}\sqrt{\frac{2(2\,\text{m})[1+(0.015)(100\,\text{m})/(0.1\,\text{m})+0.5]}{9.81\,\text{m/s}^2}}=2334\,\text{s}=\mathbf{38.9\ min}$$

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only 11.7 min. Therefore, the pipe in this case increases the draining time by more than 3 folds.

8-72

Solution A water tank open to the atmosphere is initially filled with water. A sharp-edged orifice at the bottom drains to the atmosphere through a long pipe equipped with a pump. For a specified initial velocity, the required useful pumping power and the time required to empty the tank are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant. **5** The effect of the kinetic energy correction factor is negligible, so we set $\alpha = 1$.

Properties The loss coefficient is K_L = 0.5 for a sharp-edged entrance. The friction factor of the pipe is given to be 0.015. The density of water at 30°C is $\rho = 996$ kg/m^3.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the pipe. We take the reference level at the centerline of the orifice ($z_2 = 0$), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\to\quad z_1+h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. Substituting and noting that the initial discharge velocity is 4 m/s, the required useful pumping head and power are determined to be

$$\dot m=\rho A_c V_2=\rho(\pi D^2/4)V_2=(996\text{ kg/m}^3)[\pi(0.1\text{ m})^2/4](4\text{ m/s})=31.3\text{ kg/m}^3$$

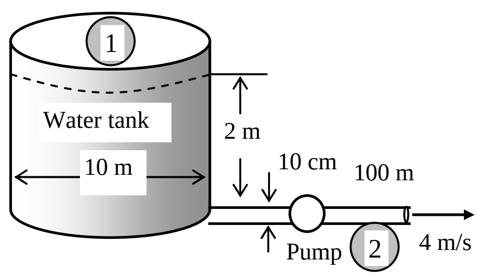
$$h_{\text{pump, u}}=\left(1+f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g}-z_1=\left(1+(0.015)\frac{100\text{ m}}{0.1\text{ m}}+0.5\right)\frac{(4\text{ m/s})^2}{2(9.81\text{ m/s}^2)}-(2\text{ m})=11.46\text{ m}$$

$$\dot W_{\text{pump, u}}=\dot V\Delta P=\dot m g h_{\text{pump, u}}=(31.3\text{ kg/s})(9.81\text{ m/s}^2)(11.46\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)=\mathbf{3.52\ kW}$$

Therefore, the pump must supply 3.52 kW of mechanical energy to water. Note that the shaft power of the pump must be greater than this to account for the pump inefficiency.

(*b*) When the discharge velocity remains constant, the flow rate of water becomes

$$\dot V=A_c V_2=(\pi D^2/4)V_2=[\pi(0.1\text{ m})^2/4](4\text{ m/s})=0.03142\text{ m}^3/\text{s}$$



The volume of water in the tank is

$$V=A_{\text{tank}}z_1=(\pi D_0^2/4)z_1=[\pi(3\text{ m})^2/4](2\text{ m})=14.14\text{ m}^3$$

Then the discharge time becomes

$$\Delta t=\frac{V}{\dot V}=\frac{14.14\text{ m}^3}{0.03142\text{ m}^3/\text{s}}=450\text{ s}=\mathbf{7.5\ min}$$

Discussion

1 Note that the pump reduces the discharging time from 38.9 min to 7.5 min. The assumption of constant discharge velocity can be justified on the basis of the pump head being much larger than the elevation head (therefore, the pump will dominate the discharging process). The answer obtained assumes that the elevation head remains constant at 2 m (rather than decreasing to zero eventually), and thus it under predicts the actual discharge time. By an exact analysis, it can be shown that when the effect of the decrease in elevation is considered, the discharge time becomes 468 s = 7.8 min. This is demonstrated below.

2 The required pump head (of water) is 11.46 m, which is more than 10.3 m of water column which corresponds to the atmospheric pressure at sea level. If the pump exit is at 1 atm, then the absolute pressure at pump inlet must be negative (= -1.16 m or – 11.4 kPa), which is impossible. Therefore, the system cannot work if the pump is installed near the pipe exit, and cavitation will occur long before the pipe exit where the pressure drops to 4.2 kPa and thus the pump must be installed close to the pipe entrance. A detailed analysis is given below.

Demonstration 1 for Prob. 8-72 (extra) (the effect of drop in water level on discharge time)

Noting that the water height z in the tank is variable, the average discharge velocity through the pipe at any given time, in general, can be expressed as

$$h_{\text{pump, u}} = \left(1 + f\frac{L}{D} + K_L\right)\frac{V_2^2}{2g} - z \quad \rightarrow \quad V_2 = \sqrt{\frac{2g(z + h_{\text{pump, u}})}{1 + fL/D + K_L}}$$

where z is the water height relative to the center of the orifice at that time. We denote the diameter of the pipe by D, and the diameter of the tank by D_0. The flow rate of water from the tank can be obtained by multiplying the discharge velocity by the cross-sectional area of the pipe,

$$\dot{V} = A_{pipe}V_2 = \frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump, u}})}{1 + fL/D + K_L}}$$

Then the amount of water that flows through the orifice during a differential time interval dt is

$$dV = \dot{V}dt = \frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump, u}})}{1 + fL/D + K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the tank,

$$dV = A_{\tan k}(-dz) = -\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the tank during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2g(z + h_{\text{pump,u}})}{1 + fL/D + K_L}}dt = -\frac{\pi D_0^2}{4}dz \quad \rightarrow dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}(z + h_{\text{pump,u}})^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained tank) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}\int_{z=z_1}^{0}(z + h_{\text{pump, u}})^{-1/2}dz$$

Performing the integration gives

$$t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1 + fL/D + K_L}{2g}}\left.\frac{(z + h_{\text{pump}})^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{D_0^2}{D^2}\left(\sqrt{\frac{2(z_1 + h_{\text{pump}})(1 + fL/D + K_L)}{g}} - \sqrt{\frac{2h_{\text{pump}}(1 + fL/D + K_L)}{g}}\right)$$

Substituting the values given, the draining time is determined to be

$$t_f = \frac{(3\text{ m})^2}{(0.1\text{ m})^2}\left(\sqrt{\frac{2(2 + 11.46\text{ m})[1 + 0.015 \times 100/0.1 + 0.5]}{9.81\text{ m/s}^2}} - \sqrt{\frac{2(11.46\text{ m})[1 + 0.015 \times 100/0.1 + 0.5]}{9.81\text{ m/s}^2}}\right)$$
$$= 468\text{ s} = \mathbf{7.8\text{ min}}$$

Demonstration 2 for Prob. 8-72 (on cavitation)

We take the pump as the control volume, with point 1 at the inlet and point 2 at the exit. We assume the pump inlet and outlet diameters to be the same and the elevation difference between the pump inlet and the exit to be negligible. Then we have $z_1 \cong z_2$ and $V_1 \cong V_2$. The pump is located near the pipe exit, and thus the pump exit pressure is equal to the pressure at the pipe exit, which is the atmospheric pressure, $P_2 = P_{\text{atm}}$. Also, the can take $h_L = 0$ since the frictional effects and loses in the pump are accounted for by the pump efficiency. Then the energy equation for the pump (in terms of heads) reduces to

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad \frac{P_{1,\text{abs}}}{\rho g} + h_{\text{pump, u}} = \frac{P_{\text{atm}}}{\rho g}$$

Solving for P_1 and substituting,

$$P_{1,\text{abs}} = P_{\text{atm}} - \rho g h_{\text{pump, u}}$$
$$= (101.3\ \text{kPa}) - (996\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(11.46\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{-10.7\ kPa}$$

which is impossible (absolute pressure cannot be negative). The technical answer to the question is that cavitation **will occur** since the pressure drops below the vapor pressure of 4.246 kPa. The practical answer is that the question is invalid (void) since the system will not work anyway. Therefore, we conclude that the pump must be located **near the beginning**, not the end of the pipe. Note that when doing a cavitation analysis, we must work with the absolute pressures. (If the system were installed as indicated, a water velocity of V = 4 m/s could not be established regardless of how much pump power were applied. This is because the atmospheric air and water elevation heads alone are not sufficient to drive such flow, with the pump restoring pressure after the flow.)

To determine the furthest distance from the tank the pump can be located without allowing cavitation, we assume the pump is located at a distance L^* from the exit, and choose the pump and the discharge portion of the pipe (from the pump to the exit) as the system, and write the energy equation. The energy equation this time will be as above, except that h_L (the pipe losses) must be considered and the pressure at 1 (pipe inlet) is the cavitation pressure, P_1 = 4.246 kPa:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad \frac{P_{1,\text{abs}}}{\rho g} + h_{\text{pump, u}} = \frac{P_{\text{atm}}}{\rho g} + f\frac{L^*}{D}\frac{V^2}{2g}$$

or
$$f\frac{L^*}{D}\frac{V^2}{2g} = \frac{P_{1,\text{abs}} - P_{\text{atm}}}{\rho g} + h_{\text{pump, u}}$$

Substituting the given values and solving for L^* gives

$$(0.015)\frac{L^*}{0.1\ \text{m}}\frac{(4\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \frac{(4.246 - 101.3)\ \text{kN/m}^2}{(996\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right) + (11.46\ \text{m}) \quad \rightarrow \quad L^* = 12.5\ \text{m}$$

Therefore, the pump must be at least 12.5 m from the pipe exit to avoid cavitation at the pump inlet (this is where the lowest pressure occurs in the piping system, and where the cavitation is most likely to occur).

Cavitation onset places an upper limit to the length of the pipe on the suction side. A pipe slightly longer would become vapor bound, and the pump could not pull the suction necessary to sustain the flow. Even if the pipe on the suction side were slightly shorter than 100 – 12.5 = 87.5 m, cavitation can still occur in the pump since the liquid in the pump is usually accelerated at the expense of pressure, and cavitation in the pump could erode and destroy the pump.

Also, over time, scale and other buildup inside the pipe can and will increase the pipe roughness, increasing the friction factor f, and therefore the losses. Buildup also decreases the pipe diameter, which increases pressure drop. Therefore, flow conditions and system performance may change (generally decrease) as the system ages. A new system that marginally misses cavitation may degrade to where cavitation becomes a problem. Proper design avoids these problems, or where cavitation cannot be avoided for some reason, it can at least be anticipated.

8-73

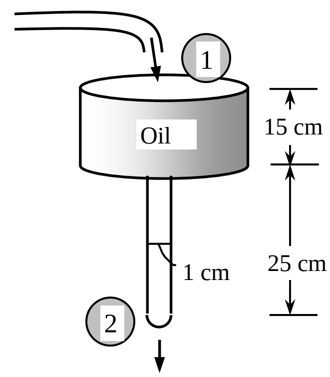
Solution Oil is flowing through a vertical glass funnel which is always maintained full. The flow rate of oil through the funnel and the funnel effectiveness are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (to be verified). **3** The frictional loses in the cylindrical reservoir are negligible since its diameter is very large and thus the oil velocity is very low.

Properties The density and viscosity of oil at 20°C are ρ = 888.1 kg/m^3 and μ = 0.8374 kg/m·s.

Analysis We take point 1 at the free surface of the oil in the cylindrical reservoir, and point 2 at the exit of the funnel pipe which is also taken as the reference level (z_2 = 0). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is negligible ($V_1 \cong 0$). For the ideal case of "frictionless flow," the exit velocity is determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad V_2=V_{2,\max}=\sqrt{2gz_1}$$



Substituting,

$$V_{2,\max}=\sqrt{2gz_1}=\sqrt{2(9.81\text{ m/s}^2)(0.40\text{ m})}=2.801\text{ m/s}$$

This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the maximum flow rate and the Reynolds number become

$$\dot{V}_{\max}=V_{2,\max}A_2=V_{2,\max}(\pi D_2^2/4)$$
$$=(2.801\text{ m/s})[\pi(0.01\text{ m})^2/4]=2.20\times10^{-4}\text{ m}^3\text{/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(888.1\text{ kg/m}^3)(2.801\text{ m/s})(0.01\text{ m})}{0.8374\text{ kg/m}\cdot\text{s}}=29.71$$

which is less than 2300. Therefore, the flow is laminar, as postulated. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length in this case is

$$L_h=0.05\,\text{Re}\,D=0.05\times29.71\times(0.01\text{ m})=0.015\text{ m}$$

which is much less than the 0.25 m pipe length. Therefore, the entrance effects can be neglected as postulated.

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from the appropriate relation with θ = -90° since the flow is downwards in the vertical direction,

$$\dot{V}=\frac{(\Delta P-\rho gL\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P=P_{\text{pipe inlet}}-P_{\text{pipe exit}}=(P_{atm}+\rho gh_{\text{cylinder}})-P_{atm}=\rho gh_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{\text{pipe}}$, and sin θ = sin (-90°) = -1. Substituting, the flow rate is determined to be

$$\dot{V}=\frac{\rho g(h_{\text{cylinder}}+h_{\text{pipe}})\pi D^4}{128\mu L}=\frac{(888.1\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.15+0.25\text{ m})\pi(0.01\text{ m})^4}{128(0.8374\text{ kg/m}\cdot\text{s})(0.25\text{ m})}=\mathbf{4.09\times10^{-6}\ m^3/s}$$

Then the "funnel effectiveness" becomes

$$\text{Eff}=\frac{\dot{V}}{\dot{V}_{\max}}=\frac{4.09\times10^{-6}\text{ m}^3\text{/s}}{2.20\times10^{-4}\text{ m}^3\text{/s}}=0.0186 \quad \text{or} \quad \mathbf{1.86\%}$$

Discussion Note that the flow is driven by gravity alone, and the actual flow rate is a small fraction of the flow rate that would have occurred if the flow were frictionless.

8-74

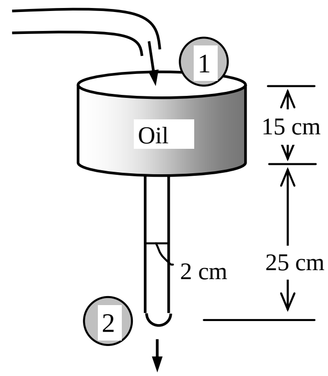
Solution Oil is flowing through a vertical glass funnel which is always maintained full. The flow rate of oil through the funnel and the funnel effectiveness are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed (to be verified). **3** The frictional loses in the cylindrical reservoir are negligible since its diameter is very large and thus the oil velocity is very low.

Properties The density and viscosity of oil at 20°C are ρ = 888.1 kg/m^3 and μ = 0.8374 kg/m·s.

Analysis We take point 1 at the free surface of the oil in the cylindrical reservoir, and point 2 at the exit of the funnel pipe, which is also taken as the reference level (z_2 = 0). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is negligible ($V_1 \cong 0$). For the ideal case of "frictionless flow," the exit velocity is determined from the Bernoulli equation to be

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad V_2=V_{2,\max}=\sqrt{2gz_1}$$



(*a*) **Case 1**: *Pipe length remains constant at 25 cm, but the pipe diameter is doubled* to D_2 = 2 cm:

Substitution gives

$$V_{2,\max}=\sqrt{2gz_1}=\sqrt{2(9.81\text{ m/s}^2)(0.40\text{ m})}=2.801\text{ m/s}$$

This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the maximum flow rate and the Reynolds number become

$$\dot{V}_{\max}=V_{2,\max}A_2=V_{2,\max}(\pi D_2^2/4)=(2.801\text{ m/s})[\pi(0.02\text{ m})^2/4]$$
$$=8.80\times10^{-4}\text{ m}^3/\text{s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(888.1\text{ kg/m}^3)(2.801\text{ m/s})(0.02\text{ m})}{0.8374\text{ kg/m}\cdot\text{s}}=59.41$$

which is less than 2300. Therefore, the flow is laminar. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length is

$$L_h=0.05\,\text{Re}\,D=0.05\times59.41\times(0.02\text{ m})=0.059\text{ m}$$

which is considerably less than the 0.25 m pipe length. Therefore, the entrance effects can be neglected (with reservation).

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from the appropriate relation with θ = -90° since the flow is downwards in the vertical direction,

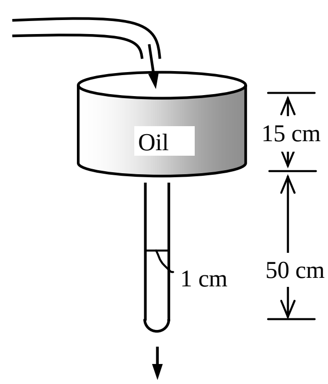
$$\dot{V}=\frac{(\Delta P-\rho gL\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P=P_{\text{pipe inlet}}-P_{\text{pipe exit}}=(P_{atm}+\rho gh_{cylinder})-P_{atm}=\rho gh_{cylinder}$ is the pressure difference across the pipe, $L=h_{pipe}$, and sin θ = sin (-90°) = -1. Substituting, the flow rate is determined to be

$$\dot{V}=\frac{\rho g(h_{cylinder}+h_{pipe})\pi D^4}{128\mu L}=\frac{(888.1\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.15+0.25\text{ m})\pi(0.02\text{ m})^4}{128(0.8374\text{ kg/m}\cdot\text{s})(0.25\text{ m})}=\mathbf{6.54\times10^{-5}\ m^3/s}$$

Then the "funnel effectiveness" becomes

$$\text{Eff}=\frac{\dot{V}}{\dot{V}_{\max}}=\frac{0.654\times10^{-4}\text{ m}^3/\text{s}}{8.80\times10^{-4}\text{ m}^3/\text{s}}=0.074 \quad \text{or} \quad \mathbf{7.4\%}$$



(*b*) **Case 2**: *Pipe diameter remains constant at 1 cm, but the pipe length is doubled to L = 50 cm*:

Substitution gives

$$V_{2,\max}=\sqrt{2gz_1}=\sqrt{2(9.81\text{ m/s}^2)(0.65\text{ m})}=3.571\text{ m/s}$$

This is the flow velocity for the frictionless case, and thus it is the maximum flow velocity. Then the maximum flow rate and the Reynolds number become

$$\dot{V}_{\max} = V_{2,\max} A_2 = V_{2,\max}(\pi D_2^2/4) = (3.571\text{ m/s})[\pi(0.01\text{ m})^2/4]$$
$$= 2.805\times10^{-4}\ \text{m}^3/s$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(888.1\text{ kg/m}^3)(3.571\text{ m/s})(0.01\text{ m})}{0.8374\text{ kg/m}\cdot\text{s}} = 37.87$$

which is less than 2300. Therefore, the flow is laminar. (Note that in the actual case the velocity and thus the Reynolds number will be even smaller, verifying the flow is always laminar). The entry length is

$$L_h = 0.05\,\text{Re}\,D = 0.05\times37.87\times(0.01\text{ m}) = 0.019\text{ m}$$

which is much less than the 0.50 m pipe length. Therefore, the entrance effects can be neglected.

Noting that the flow through the pipe is laminar and can be assumed to be fully developed, the flow rate can be determined from the appropriate relation with θ = -90° since the flow is downwards in the vertical direction,

$$\dot{V} = \frac{(\Delta P - \rho g L\sin\theta)\pi D^4}{128\mu L}$$

where $\Delta P = P_{\text{pipe inlet}} - P_{\text{pipe exit}} = (P_{\text{atm}} + \rho g h_{\text{cylinder}}) - P_{\text{atm}} = \rho g h_{\text{cylinder}}$ is the pressure difference across the pipe, $L = h_{\text{pipe}}$, and sin θ = sin (-90°) = -1. Substituting, the flow rate is determined to be

$$\dot{V} = \frac{\rho g(h_{\text{cylinder}} + h_{\text{pipe}})\pi D^4}{128\mu L} = \frac{(888.1\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.15+0.50\text{ m})\pi(0.01\text{ m})^4}{128(0.8374\text{ kg/m}\cdot\text{s})(0.50\text{ m})} = \mathbf{3.32\times10^{-6}\ m^3/s}$$

Then the “funnel effectiveness” becomes

$$\text{Eff} = \frac{\dot{V}}{\dot{V}_{\max}} = \frac{3.32\times10^{-6}\ \text{m}^3/\text{s}}{2.805\times10^{-4}\ \text{m}^3/\text{s}} = 0.0118 \quad \text{or} \quad \mathbf{1.18\%}$$

Discussion Note that the funnel effectiveness increases as the pipe diameter is increased, and decreases as the pipe length is increased. This is because the frictional losses are proportional to the length but inversely proportional to the diameter of the flow sections.

8-75

Solution Water is drained from a large reservoir through two pipes connected in series. The discharge rate of water from the reservoir is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pipes are horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **5** The pipes involve no components such as bends, valves, and other connectors. **6** The piping section involves no work devices such as pumps and turbines. **7** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **8** The water level in the reservoir remains constant. **9** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s, respectively. The loss coefficient is K_L = 0.5 for a sharp-edged entrance, and it is 0.46 for the sudden contraction, corresponding to $d^2/D^2 = 4^2/10^2 = 0.16$. The pipes are made of plastic and thus they are smooth, $\varepsilon = 0$.

Analysis We take point 1 at the free surface of the reservoir, and point 2 at the exit of the pipe, which is also taken to be the reference level (z_2 = 0). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), the fluid level in the reservoir is constant (V_1 = 0), and that there are no work devices such as pumps and turbines, the energy equation for a control volume between these two points (in terms of heads) simplifies to

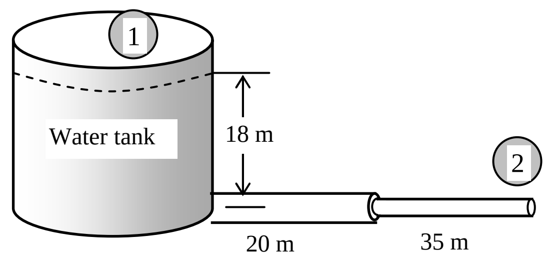
$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad z_1 = \alpha_2 \frac{V_2^2}{2g} + h_L$$

where α_2 = 1. Substituting,

$$18\text{ m} = \frac{V_2^2}{2(9.81\text{m/s}^2)} + h_L \qquad (1)$$

where

$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \sum \left(f\frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$



Note that the diameters of the two pipes, and thus the flow velocities through them are different. Denoting the first pipe by 1 and the second pipe by 2, and using conservation of mass, the velocity in the first pipe can be expressed in terms of V_2 as

$$\dot m_1 = \dot m_2 \quad \rightarrow \quad \rho V_1 A_1 = \rho V_2 A_2 \quad \rightarrow \quad V_1 = \frac{A_2}{A_1} V_2 = \frac{D_2^2}{D_1^2} V_2 = \frac{(4\text{ cm})^2}{(10\text{ cm})^2} V_2 \quad \rightarrow \quad V_1 = 0.16 V_2 \qquad (2)$$

Then the head loss can be expressed as

$$h_L = \left(f_1 \frac{L_1}{D_1} + K_{L,\text{entrance}} \right) \frac{V_1^2}{2g} + \left(f_2 \frac{L_2}{D_2} + K_{L,\text{contraction}} \right) \frac{V_2^2}{2g}$$

or

$$h_L = \left(f_1 \frac{20\text{ m}}{0.10\text{ m}} + 0.5 \right) \frac{V_1^2}{2(9.81\text{m/s}^2)} + \left(f_2 \frac{35\text{ m}}{0.04\text{ m}} + 0.46 \right) \frac{V_2^2}{2(9.81\text{m/s}^2)} \qquad (3)$$

The flow rate, the Reynolds number, and the friction factor are expressed as

$$\dot V = V_2 A_2 = V_2 (\pi D_2^2 / 4) \quad \rightarrow \quad \dot V = V_2 [\pi (0.04\text{ m})^2 / 4] \qquad (4)$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(999.1\text{ kg/m}^3)\mathbf{V}_1(0.10\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(999.1\text{ kg/m}^3)\mathbf{V}_2(0.04\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon / D_1}{3.7} + \frac{2.51}{\text{Re}_1 \sqrt{f_1}} \right) \quad \rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_1 \sqrt{f_1}} \right) \qquad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon / D_2}{3.7} + \frac{2.51}{\text{Re}_2 \sqrt{f_2}} \right) \quad \rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0\log\left(0 + \frac{2.51}{\text{Re}_2 \sqrt{f_2}} \right) \qquad (8)$$

This is a system of 8 equations in 8 unknowns, and their simultaneous solution by an equation solver gives

$$\dot{V} = \mathbf{0.00595\ m^3/s},\ V_1 = 0.757\text{ m/s},\ V_2 = 4.73\text{ m/s},\ h_L = h_{L1} + h_{L2} = 0.13 + 16.73 = 16.86\text{ m},$$

$$\text{Re}_1 = 66{,}500,\quad \text{Re}_2 = 166{,}200,\quad f_1 = 0.0196,\quad f_2 = 0.0162$$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is valid.

Discussion This problem can also be solved by using an iterative approach by assuming an exit velocity, but it will be very time consuming. Equation solvers such as EES are invaluable for this kind of problems.

8-76E

Solution The flow rate through a piping system between a river and a storage tank is given. The power input to the pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The elevation difference between the free surfaces of the tank and the river remains constant. **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

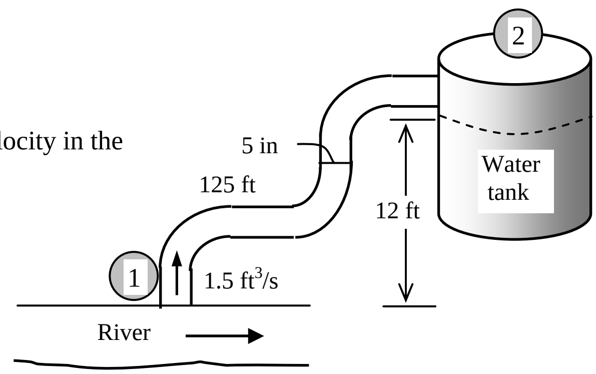
Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.360 lbm/ft·h = 6.556×10^{-4} lbm/ft·s. The roughness of galvanized iron pipe is ε = 0.0005 ft.

Analysis The piping system involves 125 ft of 5-in diameter piping, an entrance with negligible loses, 3 standard flanged 90° smooth elbows (K_L = 0.3 each), and a sharp-edged exit (K_L = 1.0). We choose points 1 and 2 at the free surfaces of the river and the tank, respectively. We note that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), and the fluid velocity is 6 ft/s at point 1 and zero at point 2 (V_1 = 6 ft/s and V_2 =0). We take the free surface of the river as the reference level (z_1 = 0). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \alpha_1\frac{V_1^2}{2g}+h_{\text{pump, u}}=z_2+h_L$$

where α_1 = 1 and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$



since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{1.5\ \text{ft}^3/\text{s}}{\pi(5/12\ \text{ft})^2/4}=11.0\ \text{ft/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(62.3\ \text{lbm/ft}^3)(11.0\ \text{ft/s})(5/12\ \text{ft})}{6.556\times10^{-4}\ \text{lbm/ft}\cdot\text{s}}=435{,}500$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.0005\ \text{ft}}{5/12\ \text{ft}}=0.0012$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\quad\rightarrow\quad\frac{1}{\sqrt f}=-2.0\log\left(\frac{0.0012}{3.7}+\frac{2.51}{435{,}500\sqrt f}\right)$$

It gives f = 0.0211. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{exit}}=0+3\times0.3+1.0=1.9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left((0.0211)\frac{125\ \text{ft}}{5/12\ \text{ft}}+1.90\right)\frac{(11.0\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}=15.5\ \text{ft}$$

The useful pump head input and the required power input to the pump are

$$h_{\text{pump, u}}=z_2+h_L-\frac{V_1^2}{2g}=12+15.5-\frac{(6\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}=26.9\ \text{ft}$$

$$\dot W_{\text{pump}}=\frac{\dot W_{\text{pump, u}}}{\eta_{\text{pump}}}=\frac{\dot V\rho g h_{\text{pump, u}}}{\eta_{\text{pump}}}$$

$$=\frac{(1.5\ \text{ft}^3/\text{s})(62.30\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(26.9\ \text{ft})}{0.70}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{kW}}{737\ \text{lbf}\cdot\text{ft/s}}\right)=\mathbf{4.87\ kW}$$

Therefore, 4.87 kW of electric power must be supplied to the pump.

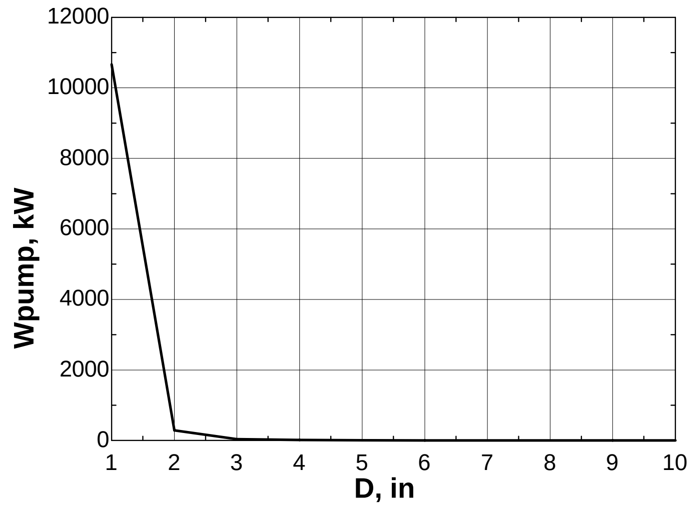
Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.0211, which is identical to the calculated value. The friction coefficient would drop to 0.0135 if smooth pipes were used. Note that fL/D = 6.3 in this case, which is about 3 times the total minor loss coefficient of 1.9. Therefore, the frictional losses in the pipe dominate the minor losses, but the minor losses are still significant.

8-77

Solution In the previous problem, the effect of the pipe diameter on pumping power for the same constant flow rate is to be investigated by varying the pipe diameter from 1 in to 10 in in increments of 1 in.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
g=32.2
L= 125
D=Dinch/12
z2= 12
rho=62.30
nu=mu/rho
mu=0.0006556
eff=0.70
Re=V2*D/nu
A=pi*(D^2)/4
V2=Vdot/A
Vdot= 1.5
V1=6
eps1=0.0005
rf1=eps1/D
1/sqrt(f1)=-2*log10(rf1/3.7+2.51/(Re*sqrt(f1)))
KL= 1.9
HL=(f1*(L/D)+KL)*(V2^2/(2*g))
hpump=z2+HL-V1^2/(2*32.2)
Wpump=(Vdot*rho*hpump)/eff/737
```



D, in	W_{pump}, kW	V, ft/s	Re
1	2.178E+06	275.02	10667.48
2	1.089E+06	68.75	289.54
3	7.260E+05	30.56	38.15
4	5.445E+05	17.19	10.55
5	4.356E+05	11.00	4.88
6	3.630E+05	7.64	3.22
7	3.111E+05	5.61	2.62
8	2.722E+05	4.30	2.36
9	2.420E+05	3.40	2.24
10	2.178E+05	2.75	2.17

Discussion We see that the required pump power decreases very rapidly as pipe diameter increases. This is due to the significant decrease in irreversible head loss in larger diameter pipes.

8-78

Solution A solar heated water tank is to be used for showers using gravity driven flow. For a specified flow rate, the elevation of the water level in the tank relative to showerhead is to be determined.

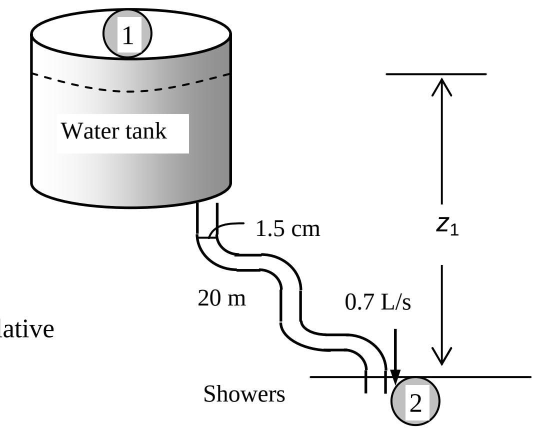
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The elevation difference between the free surface of water in the tank and the shower head remains constant. **5** There are no pumps or turbines in the piping system. **6** The losses at the entrance and at the showerhead are said to be negligible. **7** The water tank is open to the atmosphere. **8** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 40°C are ρ = 992.1 kg/m^3 and μ = 0.653×10^{-3} kg/m⋅s, respectively. The loss coefficient is K_L = 0.5 for a sharp-edged entrance. The roughness of galvanized iron pipe is ε = 0.00015 m.

Analysis The piping system involves 20 m of 1.5-cm diameter piping, an entrance with negligible loss, 4 miter bends (90°) without vanes (K_L = 1.1 each), and a wide open globe valve (K_L = 10). We choose point 1 at the free surface of water in the tank, and point 2 at the shower exit, which is also taken to be the reference level (z_2 = 0). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), and V_1 = 0. Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$



since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.0007\ \text{m}^3/\text{s}}{\pi(0.015\ \text{m})^2/4}=3.961\ \text{m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(992.1\ \text{kg/m}^3)(3.961\ \text{m/s})(0.015\ \text{m})}{0.653\times10^{-3}\ \text{kg/m}\cdot\text{s}}=90{,}270$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00015\ \text{m}}{0.015\ \text{m}}=0.01$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\ \rightarrow\ \frac{1}{\sqrt f}=-2.0\log\left(\frac{0.0051}{3.7}+\frac{2.51}{90{,}270\sqrt f}\right)$$

It gives f = 0.03857. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+4K_{L,\text{elbow}}+K_{L,\text{valve}}+K_{L,\text{exit}}=0+4\times1.1+10+0=14.4$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the total head loss and the elevation of the source become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.03857)\frac{20\ \text{m}}{0.015\ \text{m}}+14.4\right)\frac{(3.961\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=52.6\ \text{m}$$

$$z_1=\alpha_2\frac{V_2^2}{2g}+h_L=(1)\frac{(3.961\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}+52.6\ \text{m}=\mathbf{53.4\ m}$$

since α_2 = 1. Therefore, the free surface of the tank must be 53.4 m above the shower exit to ensure water flow at the specified rate.

Discussion We neglected the minor loss associated with the shower head. In reality, this loss is most likely significant.

8-79

Solution The flow rate through a piping system connecting two water reservoirs with the same water level is given. The absolute pressure in the pressurized reservoir is to be determined.

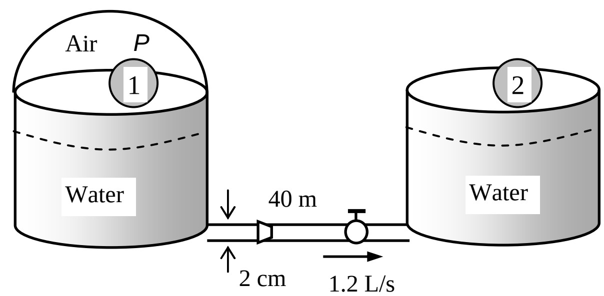
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** There are no pumps or turbines in the piping system.

Properties The density and dynamic viscosity of water at 10°C are ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m·s. The loss coefficient is K_L = 0.5 for a sharp-edged entrance, K_L = 2 for swing check valve, K_L = 0.2 for the fully open gate valve, and K_L = 1 for the exit. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis We choose points 1 and 2 at the free surfaces of the two reservoirs. We note that the fluid velocities at both points are zero ($V_1 = V_2$ =0), the fluid at point 2 is open to the atmosphere (and thus $P_2 = P_{\text{atm}}$), both points are at the same level ($z_1 = z_2$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \;\rightarrow\; \frac{P_1}{\rho g}=\frac{P_{\text{atm}}}{\rho g}+h_L \;\rightarrow\; P_1=P_{\text{atm}}+\rho g h_L$$

where $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$



since the diameter of the piping system is constant. The average flow velocity and the Reynolds number are

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.0012\ \text{m}^3/\text{s}}{\pi(0.02\ \text{m})^2/4}=3.82\ \text{m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(999.7\ \text{kg/m}^3)(3.82\ \text{m/s})(0.02\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}}=58{,}400$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\ \text{m}}{0.02\ \text{m}}=0.013$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right) \;\rightarrow\; \frac{1}{\sqrt f}=-2.0\log\left(\frac{0.013}{3.7}+\frac{2.51}{58{,}400\sqrt f}\right)$$

It gives f = 0.0424. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}}+K_{L,\text{check valve}}+K_{L,\text{gate valve}}+K_{L,\text{exit}}=0.5+2+0.2+1=3.7$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.0424)\frac{40\ \text{m}}{0.02\ \text{m}}+3.7\right)\frac{(3.82\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=65.8\ \text{m}$$

Substituting,

$$P_1=P_{\text{atm}}+\rho g h_L=(88\ \text{kPa})+(999.7\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(65.8\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)=\mathbf{734\ kPa}$$

Discussion The absolute pressure above the first reservoir must be 734 kPa, which is quite high. Note that the minor losses in this case are negligible (about 4% of total losses). Also, the friction factor could be determined easily from the explicit Haaland relation (it gives the same result, 0.0424). The friction coefficient would drop to 0.0202 if smooth pipes were used.

8-80

Solution A tanker is to be filled with fuel oil from an underground reservoir using a plastic hose. The required power input to the pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Fuel oil level remains constant. **5** Reservoir is open to the atmosphere.

Properties The density and dynamic viscosity of fuel oil are given to be ρ = 920 kg/m^3 and μ = 0.045 kg/m·s. The loss coefficient is K_L = 0.12 for a slightly-rounded entrance and K_L = 0.3 for a 90° smooth bend (flanged). The plastic pipe is smooth and thus ε = 0. The kinetic energy correction factor at hose discharge is given to be α = 1.05.

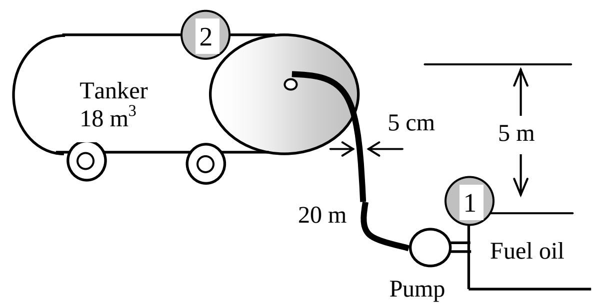
Analysis We choose point 1 at the free surface of oil in the reservoir and point 2 at the exit of the hose in the tanker. We note the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and the fluid velocity at point 1 is zero (V_1 = 0). We take the free surface of the reservoir as the reference level (z_1 = 0). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\to\quad h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+z_2+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the piping system is constant. The flow rate is determined from the requirement that the tanker must be filled in 30 min,



$$\dot V=\frac{V_{\text{tanker}}}{\Delta t}=\frac{18\text{ m}^3}{(30\times 60\text{ s})}=0.01\text{ m}^3/\text{s}$$

Then the average velocity in the pipe and the Reynolds number become

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.01\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4}=5.093\text{ m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(920\text{ kg/m}^3)(5.093\text{ m/s})(0.05\text{ m})}{0.045\text{ kg/m}\cdot\text{s}}=5206$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right) \quad\to\quad \frac{1}{\sqrt f}=-2.0\log\left(0+\frac{2.51}{5206\sqrt f}\right)$$

It gives f = 0.0370. The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+2K_{L,\text{bend}}=0.12+2\times 0.3=0.72$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system (in this case it is not). Then the total head loss, the useful pump head, and the required pumping power become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.0370)\frac{20\text{ m}}{0.05\text{ m}}+0.72\right)\frac{(5.093\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=20.5\text{ m}$$

$$h_{\text{pump, u}}=\frac{V_2^2}{2g}+z_2+h_L=1.05\frac{(5.093\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+5\text{ m}+20.5\text{ m}=26.9\text{ m}$$

$$\dot W_{\text{pump}}=\frac{\dot V\rho g h_{\text{pump, u}}}{\eta_{\text{pump}}}=\frac{(0.01\text{ m}^3/\text{s})(920\text{ kg/m}^3)(9.81\text{ m/s}^2)(26.9\text{ m})}{0.82}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)=\mathbf{2.96\text{ kW}}$$

Discussion Note that the minor losses in this case are negligible (0.72/15.52 = 0.046 or about 5% of total losses). Also, the friction factor could be determined easily from the Haaland relation (it gives 0.0372).

8-81

Solution Two pipes of identical length and material are connected in parallel. The diameter of one of the pipes is twice the diameter of the other. The ratio of the flow rates in the two pipes is to be determined

Assumptions **1** The flow is steady and incompressible. **2** The friction factor is given to be the same for both pipes. **3** The minor losses are negligible.

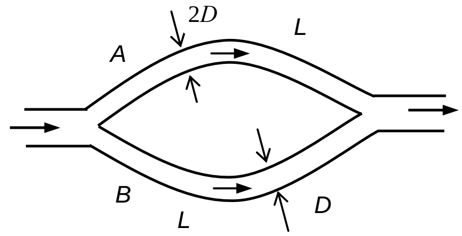
Analysis When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{A_c}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2 = 8f\frac{L}{D}\frac{1}{g}\frac{\dot{V}^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{\dot{V}^2}{D^5}$$

Solving for the flow rate gives

$$\dot{V} = \sqrt{\frac{\pi^2 h_L g}{8fL}}D^{2.5} = kD^{2.5} \qquad (k = \text{constant of proportionality})$$

When the pipe length, friction factor, and the head loss is constant, which is the case here for parallel connection, the flow rate becomes proportional to the 2.5th power of diameter. Therefore, when the diameter is doubled, the flow rate will increase by a factor of $2^{2.5} = 5.66$ since



If $\qquad \dot{V}_A = kD_A^{2.5}$

Then $\qquad \dot{V}_B = kD_B^{2.5} = k(2D_A)^{2.5} = 2^{2.5}kD_A^{2.5} = 2^{2.5}\dot{V}_A = 5.66\dot{V}_A$

Therefore, the ratio of the flow rates in the two pipes is **5.66**.

Discussion The relationship of flow rate to pipe diameter is not linear or even quadratic.

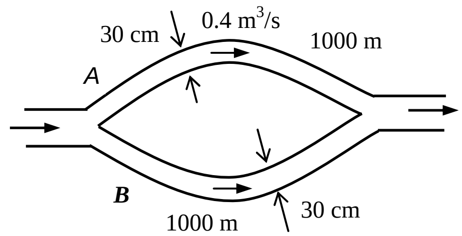
8-82

Solution Cast iron piping of a water distribution system involves a parallel section with identical diameters but different lengths. The flow rate through one of the pipes is given, and the flow rate through the other pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number (to be verified).

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis The average velocity in pipe *A* is



$$V_A = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.4\text{ m}^3/\text{s}}{\pi(0.30\text{ m})^2/4} = 5.659\text{ m/s}$$

When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D is

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

Writing this for both pipes and setting them equal to each other, and noting that $D_A = D_B$ (given) and $f_A = f_B$ (to be verified) gives

$$f_A\frac{L_A}{D_A}\frac{V_A^2}{2g} = f_B\frac{L_B}{D_B}\frac{V_B^2}{2g} \quad\rightarrow\quad V_B = V_A\sqrt{\frac{L_A}{L_B}} = (5.659\text{ m/s})\sqrt{\frac{1000\text{ m}}{3000\text{ m}}} = 3.267\text{ m/s}$$

Then the flow rate in pipe *B* becomes

$$\dot{V}_B = A_c V_B = [\pi D^2/4]V_B = [\pi(0.30\text{ m})^2/4](3.267\text{ m/s}) = \mathbf{0.231\ m^3/s}$$

***Proof** that flow is fully turbulent and thus friction factor is independent of Reynolds number:*

The velocity in pipe *B* is lower. Therefore, if the flow is fully turbulent in pipe *B*, then it is also fully turbulent in pipe *A*. The Reynolds number in pipe *B* is

$$\text{Re}_B = \frac{\rho V_B D}{\mu} = \frac{(999.1\text{ kg/m}^3)(3.267\text{ m/s})(0.30\text{ m})}{1.138\times10^{-3}\text{ kg/m}\cdot\text{s}} = 0.860\times10^6$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00026\text{ m}}{0.30\text{ m}} = 0.00087$$

From Moody's chart, we observe that for a relative roughness of 0.00087, the flow is fully turbulent for Reynolds number greater than about 10^6. Therefore, the flow in both pipes is fully turbulent, and thus the assumption that the friction factor is the same for both pipes is valid.

Discussion Note that the flow rate in pipe *B* is less than the flow rate in pipe *A* because of the larger losses due to the larger length.

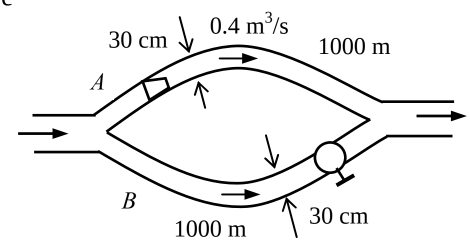
8-83

Solution Cast iron piping of a water distribution system involves a parallel section with identical diameters but different lengths and different valves. The flow rate through one of the pipes is given, and the flow rate through the other pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses other than those for the valves are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis For pipe *A*, the average velocity and the Reynolds number are



$$V_A = \frac{\dot{V}_A}{A_c} = \frac{\dot{V}_A}{\pi D^2/4} = \frac{0.4\ \text{m}^3/\text{s}}{\pi(0.30\ \text{m})^2/4} = 5.659\ \text{m/s}$$

$$\text{Re}_A = \frac{\rho V_A D}{\mu} = \frac{(999.1\ \text{kg/m}^3)(5.659\ \text{m/s})(0.30\ \text{m})}{1.138\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1.49\times10^6$$

The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00026\ \text{m}}{0.30\ \text{m}} = 8.667\times10^{-4}$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{8.667\times10^{-4}}{3.7}+\frac{2.51}{1.49\times10^6\sqrt{f}}\right)$$

It gives f = 0.0192. Then the total head loss in pipe *A* becomes

$$h_{L,A} = \left(f\frac{L_A}{D}+K_L\right)\frac{V_A^2}{2g} = \left((0.0192)\frac{1000\ \text{m}}{0.30\ \text{m}}+2.1\right)\frac{(5.659\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 107.9\ \text{m}$$

When two pipes are parallel in a piping system, the head loss for each pipe must be same. Therefore, the head loss for pipe *B* must also be 107.9 m. Then the average velocity in pipe *B* and the flow rate become

$$h_{L,B} = \left(f\frac{L_B}{D}+K_L\right)\frac{V_B^2}{2g} \quad\rightarrow\quad 107.9\ \text{m} = \left((0.0192)\frac{3000\ \text{m}}{0.30\ \text{m}}+10\right)\frac{V_B^2}{2(9.81\ \text{m/s}^2)} \quad\rightarrow\quad V_B = 3.24\ \text{m/s}$$

$$\dot{V}_B = A_c V_B = [\pi D^2/4]V_B = [\pi(0.30\ \text{m})^2/4](3.24\ \text{m/s}) = \mathbf{0.229\ m^3/s}$$

Discussion Note that the flow rate in pipe *B* decreases slightly (from 0.231 to 0.229 m^3/s) due to the larger minor loss in that pipe. Also, minor losses constitute just a few percent of the total loss, and they can be neglected if great accuracy is not required.

8-84

Solution Geothermal water is supplied to a city through stainless steel pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

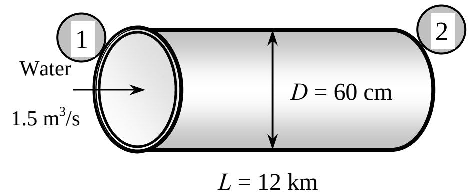
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are $\rho = 950.6$ kg/m^3, $\mu = 0.255\times10^{-3}$ kg/m·s, and $C_p = 4.229$ kJ/kg·°C. The roughness of stainless steel pipes is 2×10^{-6} m.

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The average velocity and the Reynolds number are



$$V=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{1.5\text{ m}^3\text{/s}}{\pi(0.60\text{ m})^2/4}=5.305\text{ m/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})(0.60\text{ m})}{0.255\times10^{-3}\text{ kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{2\times10^{-6}\text{ m}}{0.60\text{ m}}=3.33\times10^{-6}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\quad\rightarrow\quad\frac{1}{\sqrt f}=-2.0\log\left(\frac{3.33\times10^{-6}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt f}\right)$$

It gives $f = 0.00829$. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=\Delta P_L=f\frac{L}{D}\frac{\rho V^2}{2}=0.00829\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=2218\text{ kPa}$$

$$h_L=\frac{\Delta P_L}{\rho g}=f\frac{L}{D}\frac{V^2}{2g}=(0.00829)\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(5.305\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=238\text{ m}$$

$$\dot W_{\text{electric, in}}=\frac{\dot W_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot V\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\text{ m}^3\text{/s})(2218\text{ kPa})}{0.74}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3\text{/s}}\right)=4496\text{ kW}\cong\mathbf{4500\text{ kW}}$$

Therefore, the pumps will consume 4496 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 2218 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount}=\dot W_{\text{elect,in}}\Delta t=(4496\text{ kW})(24\text{ h/day})=107{,}900\text{ kWh/day}$$

$$\text{Cost}=\text{Amount}\times\text{Unit cost}=(107{,}900\text{ kWh/day})(\$0.06\text{/kWh})=\$6474\text{/day}\cong\mathbf{\$6470/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 4496 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{mech}} = \rho \dot{V} c_p \Delta T \;\rightarrow\; \Delta T = \frac{\eta_{\text{pump-motor}} \dot{W}_{\text{elect,in}}}{\rho \dot{V} c_p} = \frac{0.74 \times (4496\ \text{kJ/s})}{(950.6\ \text{kg/m}^3)(1.5\ \text{m}^3/\text{s})(4.229\ \text{kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{0.55^\circ C}$$

Therefore, the temperature of water will rise at least 0.55°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

8-85

Solution Geothermal water is supplied to a city through cast iron pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

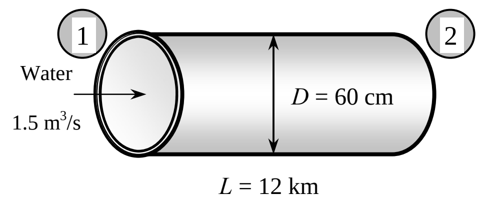
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, μ = 0.255×10^{-3} kg/m⋅s, and C_p = 4.229 kJ/kg⋅°C. The roughness of cast iron pipes is 0.00026 m.

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The average velocity and the Reynolds number are



$$V=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{1.5\text{ m}^3/\text{s}}{\pi(0.60\text{ m})^2/4}=5.305\text{ m/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})(0.60\text{ m})}{0.255\times10^{-3}\text{ kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\text{ m}}{0.60\text{ m}}=4.33\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\ \rightarrow\ \frac{1}{\sqrt f}=-2.0\log\left(\frac{4.33\times10^{-4}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt f}\right)$$

It gives f = 0.0162. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=\Delta P_L=f\frac{L}{D}\frac{\rho V^2}{2}=0.0162\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=4334\text{ kPa}$$

$$h_L=\frac{\Delta P_L}{\rho g}=f\frac{L}{D}\frac{V^2}{2g}=(0.0162)\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(5.305\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=465\text{ m}$$

$$\dot W_{\text{elect, in}}=\frac{\dot W_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot V\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\text{ m}^3/\text{s})(4334\text{ kPa})}{0.74}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=8785\text{ kW}\cong\mathbf{8790\text{ kW}}$$

Therefore, the pumps will consume 8785 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 4334 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount}=\dot W_{\text{elect,in}}\Delta t=(8785\text{ kW})(24\text{ h/day})=210{,}800\text{ kWh/day}$$

$$\text{Cost}=\text{Amount}\times\text{Unit cost}=(210{,}800\text{ kWh/day})(\$0.06/\text{kWh})=\$12{,}650/\text{day}\cong\mathbf{\$12{,}700/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 8785 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{mech}} = \rho\dot{V}c_p\Delta T \;\rightarrow\; \Delta T = \frac{\eta_{\text{pump-motor}}\dot{W}_{\text{elect,in}}}{\rho\dot{V}c_p} = \frac{0.74\times(8785\ \text{kJ/s})}{(950.6\ \text{kg/m}^3)(1.5\ \text{m}^3/\text{s})(4.229\ \text{kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{1.1^\circ C}$$

Therefore, the temperature of water will rise at least 1.1°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

8-86E

Solution The air discharge rate of a clothes drier with no ducts is given. The flow rate when duct work is attached is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects in the duct are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **4** The losses at the vent and its proximity are negligible. **5** The effect of the kinetic energy correction factor on discharge stream is negligible, $\alpha = 1$.

Properties The density of air at 1 atm and 120°F is $\rho = 0.06843$ lbm/ft^3. The roughness of galvanized iron pipe is $\varepsilon = 0.0005$ ft. The loss coefficient is $K_L \approx 0$ for a well-rounded entrance with negligible loss, $K_L = 0.3$ for a flanged 90° smooth bend, and $K_L = 1.0$ for an exit. The friction factor of the duct is given to be 0.019.

Analysis To determine the useful fan power input, we choose point 1 inside the drier sufficiently far from the vent, and point 2 at the exit on the same horizontal level so that $z_1 = z_2$ and $P_1 = P_2$, and the flow velocity at point 1 is negligible ($V_1 = 0$) since it is far from the inlet of the fan. Also, the frictional piping losses between 1 and 2 are negligible, and the only loss involved is due to fan inefficiency. Then the energy equation for a control volume between 1 and 2 reduces to

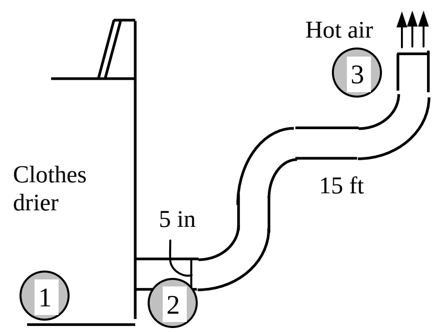
$$\dot{m}\left(\frac{P_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1\right)+\dot{W}_{\text{fan}}=\dot{m}\left(\frac{P_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \quad\rightarrow\quad \dot{W}_{\text{fan, u}}=\dot{m}\frac{V_2^2}{2} \qquad (1)$$

since $\alpha = 1$ and $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, fan}} + \dot{E}_{\text{mech loss, piping}}$ and $\dot{W}_{\text{fan, u}} = \dot{W}_{\text{fan}} - \dot{E}_{\text{mech loss, fan}}$.

The average velocity is $V_2 = \dfrac{\dot{V}}{A_c} = \dfrac{\dot{V}}{\pi D^2/4} = \dfrac{1.2 \text{ ft}^3/\text{s}}{\pi(5/12 \text{ ft})^2/4} = 8.80 \text{ ft/s}$

Now we attach the ductwork, and take point 3 to be at the duct exit so that the duct is included in the control volume. The energy equation for this control volume simplifies to

$$\dot{W}_{\text{fan, u}} = \dot{m}\frac{V_3^2}{2}+\dot{m}gh_L \qquad (2)$$



Combining (1) and (2),

$$\rho\dot{V}_2\frac{V_2^2}{2}=\rho\dot{V}_3\frac{V_3^2}{2}+\rho\dot{V}_3 gh_L \quad\rightarrow\quad \dot{V}_2\frac{V_2^2}{2}=\dot{V}_3\frac{V_3^2}{2}+\dot{V}_3 gh_L \qquad (3)$$

where

$$V_3=\frac{\dot{V}_3}{A_c}=\frac{\dot{V}_3}{\pi D^2/4}=\frac{\dot{V}_3 \text{ ft}^3/\text{s}}{\pi(5/12 \text{ ft})^2/4}=7.33\dot{V}_3 \text{ ft/s}$$

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_3^2}{2g}=\left(0.019\frac{15 \text{ ft}}{5/12 \text{ ft}}+3\times 0.3+1\right)\frac{V_3^2}{2g}=2.58\frac{V_3^2}{2g}$$

Substituting into Eq. (3),

$$\dot{V}_2\frac{V_2^2}{2}=\dot{V}_3\frac{V_3^2}{2}+\dot{V}_3 g\times 2.58\frac{V_3^2}{2g}=\dot{V}_3\frac{(7.33\dot{V}_3)^2}{2}+\dot{V}_3\times 2.58\frac{(7.33\dot{V}_3)^2}{2}=96.2\dot{V}_3^3$$

Solving for $\dot{V}_3$ and substituting the numerical values gives

$$\dot{V}_3=\left(\dot{V}_2\frac{V_2^2}{2\times 96.2}\right)^{1/3}=\left(1.2\frac{8.80^2}{2\times 96.2}\right)^{1/3}=\mathbf{0.78\ ft^3/s}$$

Discussion Note that the flow rate decreased considerably for the same fan power input, as expected. We could also solve this problem by solving for the useful fan power first,

$$\dot{W}_{\text{fan, u}}=\rho\dot{V}_2\frac{V_2^2}{2}=(0.06843 \text{ lbm/ft}^3)(1.2 \text{ ft}^3/\text{s})\frac{(8.80 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1 \text{ W}}{0.737 \text{ lbf}\cdot\text{ft/s}}\right)=0.13 \text{ W}$$

Therefore, the fan supplies 0.13 W of useful mechanical power when the drier is running.

8-87

Solution Hot water in a water tank is circulated through a loop made of cast iron pipes at a specified average velocity. The required power input for the recirculating pump is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Minor losses other than those for elbows and valves are negligible.

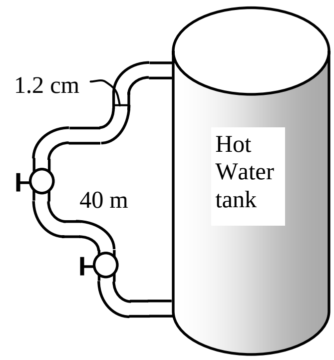
Properties The density and dynamic viscosity of water at 60°C are ρ = 983.3 kg/m^3, μ = 0.467×10^{-3} kg/m·s. The roughness of cast iron pipes is 0.00026 m. The loss coefficient is K_L = 0.9 for a threaded 90° smooth bend and K_L = 0.2 for a fully open gate valve.

Analysis Since the water circulates continually and undergoes a cycle, we can take the entire recirculating system as the control volume, and choose points 1 and 2 at any location at the same point. Then the properties (pressure, elevation, and velocity) at 1 and 2 will be identical, and the energy equation will simplify to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

where

$$h_L=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$



since the diameter of the piping system is constant. Therefore, the pumping power is to be used to overcome the head losses in the flow. The flow rate and the Reynolds number are

$$\dot V=VA_c=V(\pi D^2/4)=(2.5\text{ m/s})[\pi(0.012\text{ m})^2/4]=2.83\times10^{-4}\text{ m}^3\text{/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(983.3\text{ kg/m}^3)(2.5\text{ m/s})(0.012\text{ m})}{0.467\times10^{-3}\text{ kg/m}\cdot\text{s}}=63{,}200$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\text{ m}}{0.012\text{ m}}=0.0217$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\ \rightarrow\ \frac{1}{\sqrt f}=-2.0\log\left(\frac{0.0217}{3.7}+\frac{2.51}{63200\sqrt f}\right)$$

It gives f = 0.05075. Then the total head loss, pressure drop, and the required pumping power input become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.05075)\frac{40\text{ m}}{0.012\text{ m}}+6\times0.9+2\times0.2\right)\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=55.8\text{ m}$$

$$\Delta P=\Delta P_L=\rho g h_L=(983.3\text{ kg/m}^3)(9.81\text{ m/s}^2)(55.8\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=538\text{ kPa}$$

$$\dot W_{\text{elect}}=\frac{\dot W_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot V\Delta P}{\eta_{\text{pump-motor}}}=\frac{(2.83\times10^{-4}\text{ m}^3\text{/s})(538\text{ kPa})}{0.70}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3\text{/s}}\right)=\mathbf{0.217\ kW}$$

Therefore, **the required power input of the recirculating pump is 217 W**.

Discussion It can be shown that the required pumping power input for the recirculating pump is 0.210 kW when the minor losses are not considered. Therefore, the minor losses can be neglected in this case without a major loss in accuracy.

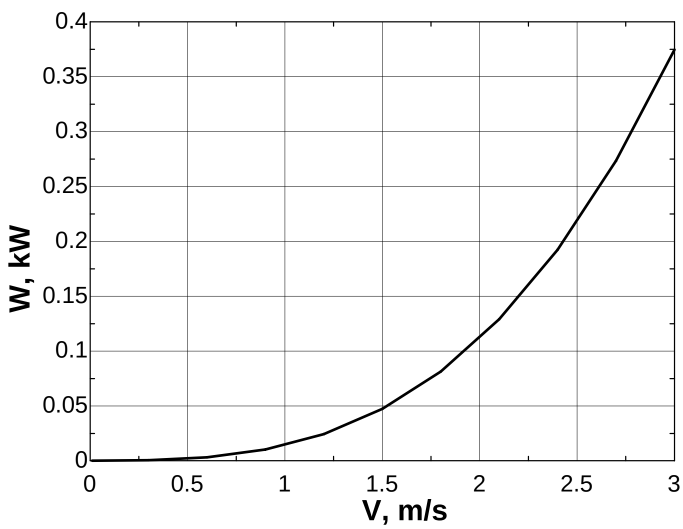
8-88

Solution In the previous problem, the effect of average flow velocity on the power input to the recirculating pump for the same constant flow rate is to be investigated by varying the velocity from 0 to 3 m/s in increments of 0.3 m/s.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
g=9.81
rho=983.3
nu=mu/rho
mu=0.000467
D=0.012
L=40
KL=6*0.9+2*0.2
Eff=0.7
Ac=pi*D^2/4
Vdot=V*Ac
eps=0.00026
rf=eps/D
"Reynolds number"
Re=V*D/nu
1/sqrt(f)=-2*log10(rf/3.7+2.51/(Re*sqrt(f)))
DP=(f*L/D+KL)*rho*V^2/2000 "kPa"
W=Vdot*DP/Eff "kW"
HL=(f*L/D+KL)*(V^2/(2*g))
```

V, m/s	W_{pump}, kW	ΔP_L, kPa	Re
0.0	0.0000	0.0	0
0.3	0.0004	8.3	7580
0.6	0.0031	32.0	15160
0.9	0.0103	71.0	22740
1.2	0.0243	125.3	30320
1.5	0.0472	195.0	37900
1.8	0.0814	279.9	45480
2.1	0.1290	380.1	53060
2.4	0.1922	495.7	60640
2.7	0.2733	626.6	68220
3.0	0.3746	772.8	75800



Discussion As you might have suspected, the required power does not increase linearly with average velocity. Rather, the relationship is nearly quadratic. A larger diameter pipe would cut reduce the required pumping power considerably.

8-89

Solution Hot water in a water tank is circulated through a loop made of plastic pipes at a specified average velocity. The required power input for the recirculating pump is to be determined.

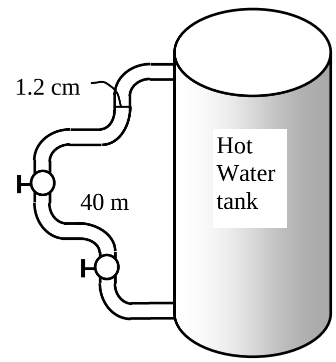
Assumptions **1** The flow is steady and incompressible. **2** The flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Minor losses other than those for elbows and valves are negligible.

Properties The density and viscosity of water at 60°C are ρ = 983.3 kg/m^3, μ = 0.467×10^{-3} kg/m·s. Plastic pipes are smooth, and thus their roughness is very close to zero, ε = 0. The loss coefficient is K_L = 0.9 for a threaded 90° smooth bend and K_L = 0.2 for a fully open gate valve.

Analysis Since the water circulates continually and undergoes a cycle, we can take the entire recirculating system as the control volume, and choose points 1 and 2 at any location at the same point. Then the properties (pressure, elevation, and velocity) at 1 and 2 will be identical, and the energy equation will simplify to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

where



$$h_L=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the piping system is constant. Therefore, the pumping power is to be used to overcome the head losses in the flow. The flow rate and the Reynolds number are

$$\dot{V}=VA_c=V(\pi D^2/4)=(2.5\text{ m/s})[\pi(0.012\text{ m})^2/4]=2.827\times10^{-4}\text{ m}^3\text{/s}$$

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{(983.3\text{ kg/m}^3)(2.5\text{ m/s})(0.012\text{ m})}{0.467\times10^{-3}\text{ kg/m}\cdot\text{s}}=63{,}200$$

which is greater than 4000. Therefore, the flow is turbulent. The friction factor corresponding to the relative roughness of zero and this Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{63200\sqrt{f}}\right)$$

It gives f = 0.0198. Then the total head loss, pressure drop, and the required pumping power input become

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}=\left((0.0198)\frac{40\text{ m}}{0.012\text{ m}}+6\times0.9+2\times0.2\right)\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=22.9\text{ m}$$

$$\Delta P=\rho gh_L=(983.3\text{ kg/m}^3)(9.81\text{ m/s}^2)(22.9\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=221\text{ kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(2.827\times10^{-4}\text{ m}^3\text{/s})(221\text{ kPa})}{0.70}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3\text{/s}}\right)=\mathbf{0.0893\text{ kW}}$$

Therefore, **the required power input of the recirculating pump is 89.3 W**.

Discussion It can be shown that the required pumping power input for the recirculating pump is 82.1 W when the minor losses are not considered. Therefore, the minor losses can be neglected in this case without a major loss in accuracy. Compared to the cast iron pipes of the previous problem, the plastic pipes reduced the required power by more than 50%, from 217 to 89.3 W. Furthermore, plastic pipes are lighter and easier to install, and they don't rust.

Flow Rate and Velocity Measurements

8-90C

Solution We are to discus the primary considerations when choosing a flowmeter.

Analysis The primary considerations when selecting a *flowmeter* are **cost, size, pressure drop, capacity, accuracy, and reliability**.

Discussion As with just about everything you purchase, you usually get what you pay for.

8-91C

Solution We are to explain how a Pitot-static tube works and discuss its application.

Analysis A *Pitot-static tube* **measures the difference between the stagnation and static pressure**, which is the *dynamic pressure*, which is related to flow velocity by $\boxed{V = \sqrt{2(P_1 - P_2)/\rho}}$. Once the average flow velocity is determined, the flow rate is calculated from $\dot{V} = VA_c$. The Pitot tube is inexpensive, highly reliable since it has no moving parts, it has very small pressure drop, and its accuracy (which is about 3%) is acceptable for most engineering applications.

Discussion The term "Pitot tube" or "Pitot probe" is often used in place of "Pitot-static probe". Technically, however, a Pitot probe measures only stagnation pressure, while a Pitot-static probe measures both stagnation *and* static pressures.

8-92C

Solution We are to discuss the operation of obstruction flowmeters.

Analysis An *obstruction flowmeter* measures the flow rate through a pipe by **constricting the flow, and measuring the decrease in pressure due to the increase in velocity at (or downstream of) the constriction site**. The flow rate for obstruction flowmeters is expressed as $\boxed{\dot{V} = A_o C_0 \sqrt{2(P_1 - P_2)/[\rho(1-\beta^4)]}}$ where $A_0 = \pi d^2/4$ is the cross-sectional area of the obstruction and $\beta = d/D$ is the ratio of obstruction diameter to the pipe diameter. Of the three types of obstruction flow meters, the orifice meter is the cheapest, smallest, and least accurate, and it causes the greatest head loss. The Venturi meter is the most expensive, the largest, the most accurate, and it causes the smallest head loss. The nozzle meter is between the orifice and Venturi meters in all aspects.

Discussion As diameter ratio β decreases, the pressure drop across the flowmeter increases, leading to a larger minor head loss associated with the flowmeter, but increasing the sensitivity of the measurement.

8-93C

Solution We are to discuss the operation of positive displacement flowmeters.

Analysis A **positive displacement flowmeter** operates by **trapping a certain amount of incoming fluid, displacing it to the discharge side of the meter, and counting the number of such discharge-recharge cycles to determine the total amount of fluid displaced**. Positive displacement flowmeters are commonly used to meter gasoline, water, and natural gas because they are simple, reliable, inexpensive, and highly accurate even when the flow is unsteady.

Discussion In applications such as a gasoline meter, it is not the flow *rate* that is measured, but the flow *volume*.

8-94C

Solution We are to discuss the operation of a turbine flowmeter.

Analysis A *turbine flowmeter* consists of a **cylindrical flow section that houses a turbine that is free to rotate, and a sensor that generates a pulse each time a marked point on the turbine passes by to determine the rate of rotation**. Turbine flowmeters are relatively inexpensive, give highly accurate results (as accurate as 0.25%) over a wide range of flow rates, and cause a very small head loss.

Discussion Turbine flowmeters must be calibrated so that a reading of the rpm of the turbine is translated into average velocity in the pipe or volume flow rate through the pipe.

8-95C

Solution We are to discuss the operation of rotameters.

Analysis A *variable-area flowmeter* (or *rotameter*) consists of **a tapered conical transparent tube made of glass or plastic with a float inside that is free to move**. As fluid flows through the tapered tube, the float rises within the tube to a location where the float weight, drag force, and buoyancy force balance each other. Variable-area flowmeters are very simple devices with no moving parts except for the float (but even the float remains stationary during steady operation), and thus they are very reliable. They are also very inexpensive, and they cause a relatively small head loss.

Discussion There are also some disadvantages. For example, they must be mounted vertically, and most of them require a visual reading, and so cannot be automated or connected to a computer system.

8-96C

Solution We are to compare thermal and laser Doppler anemometers.

Analysis A *thermal anemometer* involves a **very small electrically heated sensor (hot wire) which loses heat to the fluid, and the flow velocity is related to the electric current needed to maintain the sensor at a constant temperature**. The flow velocity is determined by measuring the voltage applied or the electric current passing through the sensor. A *laser Doppler anemometer (LDA)* does not have a sensor that intrudes into flow. Instead, it **uses two laser beams that intersect at the point where the flow velocity is to be measured, and it makes use of the frequency shift (the Doppler effect) due to fluid flow to measure velocity**.

Discussion Both of these devices measure the flow velocity at a point in the flow. Of the two, the hot wire system is much less expensive and has higher frequency resolution, but may interfere with the flow being measured.

8-97C

Solution We are to compare LDV and PIV.

Analysis *Laser Doppler velocimetry (LDV)* **measures velocity at a point**, but *particle image velocimetry (PIV)* **provides velocity values simultaneously throughout an entire cross-section and thus it is a whole-field technique**. PIV combines the accuracy of LDV with the capability of flow visualization, and provides instantaneous flow field mapping. Both methods are non-intrusive, and both utilize laser light beams.

Discussion In both cases, optical access is required – a hot-wire system does not require optical access, but, like the LDV system, measures velocity only at a single point.

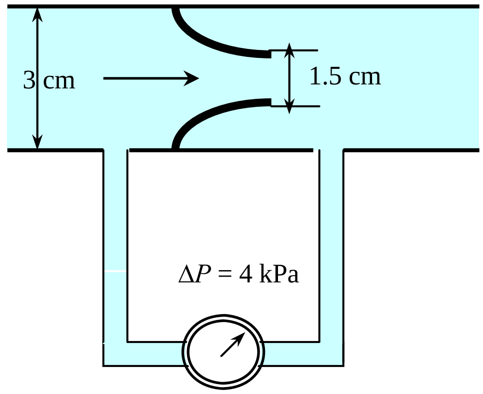
8-98

Solution The flow rate of ammonia is to be measured with flow nozzle equipped with a differential pressure gage. For a given pressure drop, the flow rate and the average flow velocity are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is $C_d = 0.96$.

Properties The density and dynamic viscosity of ammonia are given to be $\rho = 624.6$ kg/m^3 and $\mu = 1.697\times10^{-4}$ kg/m·s, respectively.

Analysis The diameter ratio and the throat area of the meter are



$$\beta = d / D = 1.5/3 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(0.015\text{ m})^2/4 = 1.767\times10^{-4}\text{ m}^2$$

Noting that $\Delta P = 4$ kPa $= 4000$ N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (1.767\times10^{-4}\text{ m}^2)(0.96)\sqrt{\frac{2\times4000\text{ N/m}^2}{(624.6\text{ kg/m}^3)((1-0.50^4)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}$$

$$= \mathbf{0.627\times10^{-3}\ m^3/s}$$

which is equivalent to 0.627 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.627\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.03\text{ m})^2/4} = \mathbf{0.887\ m/s}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(624.6\text{ kg/m}^3)(0.887\text{ m/s})(0.03\text{ m})}{1.697\times10^{-4}\text{ kg/m}\cdot\text{s}} = 9.79\times10^4$$

Substituting the β and Re values into the orifice discharge coefficient relation gives

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(9.79\times10^4)^{0.5}} = 0.983$$

which is about 2% different than the assumed value of 0.96. Using this refined value of C_d, the flow rate becomes 0.642 L/s, which differs from our original result by only 2.4%. If the problem is solved using an equation solver such as EES, then the problem can be formulated using the curve-fit formula for C_d (which depends on Reynolds number), and all equations can be solved simultaneously by letting the equation solver perform the iterations as necessary.

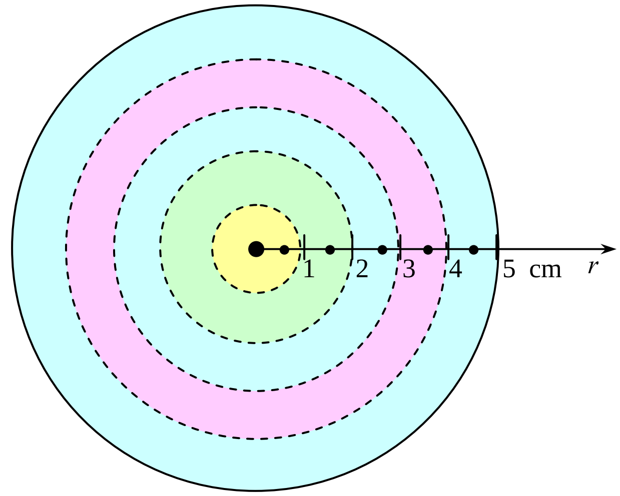
8-99

Solution The flow rate of water through a circular pipe is to be determined by measuring the water velocity at several locations along a cross-section. For a given set of measurements, the flow rate is to be determined.

Assumptions The points of measurements are sufficiently close so that the variation of velocity between points can be assumed to be linear.

Analysis The velocity measurements are given to be

R, cm	*V*, m/s
0	6.4
1	6.1
2	5.2
3	4.4
4	2.0
5	0.0



The divide the cross-section of the pipe into 1-cm thick annual regions, as shown in the figure. Using midpoint velocity values for each section, the flow rate is determined to be

$$\dot{V} = \int_{A_c} V dA_c \cong \sum V\pi(r_{out}^2 - r_{in}^2)$$

$$= \pi\left(\frac{6.4+6.1}{2}\right)(0.01^2 - 0) + \pi\left(\frac{6.1+5.2}{2}\right)(0.02^2 - 0.01^2) + \pi\left(\frac{5.2+4.4}{2}\right)(0.03^2 - 0.02^2)$$

$$+\pi\left(\frac{4.4+2.0}{2}\right)(0.04^2 - 0.02^2) + \pi\left(\frac{2.0+0}{2}\right)(0.05^2 - 0.04^2)$$

$$= \mathbf{0.0297\ m^3/s}$$

Discussion We can also solve this problem by curve-fitting the given data using a second-degree polynomial, and then performing the integration.

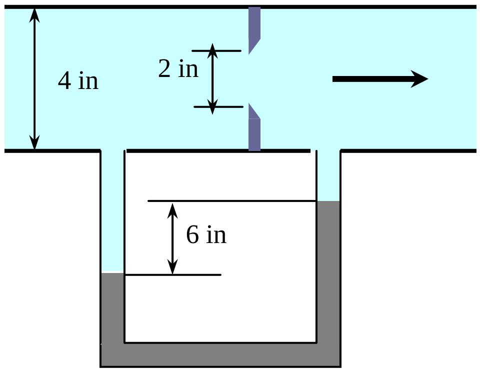
8-100E

Solution The flow rate of water is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate, the average velocity, and the head loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is C_d = 0.61.

Properties The density and dynamic viscosity of water are given to be ρ = 62.36 kg/m^3 and μ = 7.536×10^{-4} lbm/ft·s, respectively. We take the density of mercury to be 847 lbm/ft^3.

Analysis The diameter ratio and the throat area of the orifice are



$$\beta = d/D = 2/4 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(2/12\text{ ft})^2/4 = 0.02182\text{ ft}^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{f}})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1-P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}}-\rho_{\text{f}})gh}{\rho_{\text{f}}(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}}/\rho_{\text{f}}-1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (0.02182\text{ ft}^2)(0.61)\sqrt{\frac{2(847/62.36-1)(32.2\text{ ft/s}^2)(6/12\text{ ft})}{1-0.50^4}} = \mathbf{0.277\ ft^3/s}$$

The average velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.277\text{ ft}^3/\text{s}}{\pi(4/12\text{ ft})^2/4} = \mathbf{3.17\ ft/s}$$

The percent pressure (or head) loss for orifice meters is given in Fig. 8-59 for β = 0.5 to be 74%. Therefore, noting that the density of mercury is 13.6 times that of water,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.74(0.50\text{ ft Hg}) = \mathbf{0.37\ ft\ Hg} = \mathbf{5.03\ ft\ H_2O}$$

The head loss between the two measurement sections can also be estimated from the energy equation. Since $z_1 = z_2$, the head form of the energy equation simplifies to

$$h_L \approx \frac{P_1-P_2}{\rho_f g} - \frac{V_2^2-V_1^2}{2g} = \frac{\rho_{Hg} g h_{Hg}}{\rho_f g} - \frac{\left[(D/d)^4-1\right]V_1^2}{2g}$$

$$= \text{SG}_{Hg} h_{Hg} - \frac{\left[(D/d)^4-1\right]V_1^2}{2g} = 13.6(0.50\text{ ft}) - \frac{\left[2^4-1\right](3.17\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = \mathbf{4.46\ ft\ H_2O}$$

This head loss, though a reasonable estimate, is lower than the exact one calculated above because it does not take into account irreversible losses downstream of the pressure taps, where the flow is still "recovering," and is not yet fully developed.

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(62.36\text{ kg/m}^3)(3.17\text{ ft/s})(4/12\text{ ft})}{7.536\times10^{-4}\text{ lbm/ft}\cdot\text{s}} = 8.744\times10^4$$

Substituting β and Re values into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

gives C_d = 0.606, which is very close to the assumed value of 0.61. Using this refined value of C_d, the flow rate becomes 0.275 ft^3/s, which differs from our original result by less than 1%. Therefore, it is convenient to analyze orifice meters using the recommended value of C_d = 0.61 for the discharge coefficient, and then to verify the assumed value.

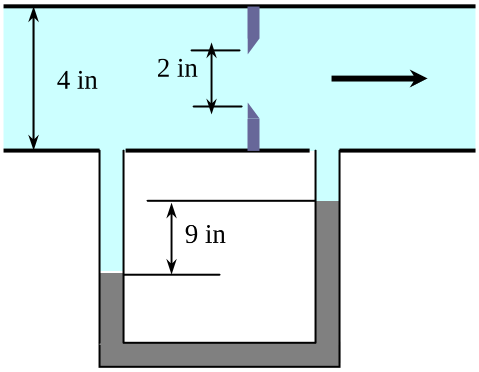
8-101E

Solution The flow rate of water is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate, the average velocity, and the head loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is C_d = 0.61.

Properties The density and dynamic viscosity of water are given to be ρ = 62.36 kg/m^3 and $\mu = 7.536\times10^{-4}$ lbm/ft·s, respectively. We take the density of mercury to be 847 lbm/ft^3.

Analysis The diameter ratio and the throat area of the orifice are



$$\beta = d/D = 2/4 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(2/12\text{ ft})^2/4 = 0.02182\text{ ft}^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{f}})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}} - \rho_{\text{f}})gh}{\rho_{\text{f}}(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}}/\rho_{\text{f}} - 1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (0.02182\text{ ft}^2)(0.61)\sqrt{\frac{2(847/62.36 - 1)(32.2\text{ ft/s}^2)(9/12\text{ ft})}{1 - 0.50^4}} = \mathbf{0.339\ ft^3/s}$$

The average velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.339\text{ ft}^3/\text{s}}{\pi(4/12\text{ ft})^2/4} = \mathbf{3.88\ ft/s}$$

The percent pressure (or head) loss for orifice meters is given in Fig. 8-59 for β = 0.5 to be 74%. Therefore, noting that the density of mercury is 13.6 times that of water,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.74(0.75\text{ ft Hg}) = \mathbf{0.555\ ft\ Hg} = \mathbf{7.55\ ft\ H2O}$$

The head loss between the two measurement sections can also be estimated from the energy equation. Since $z_1 = z_2$, the head form of the energy equation simplifies to

$$h_L \approx \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = \frac{\rho_{Hg} g h_{Hg}}{\rho_f g} - \frac{\left[(D/d)^4 - 1\right]V_1^2}{2g}$$

$$= \text{SG}_{Hg} h_{Hg} - \frac{\left[(D/d)^4 - 1\right]V_1^2}{2g} = 13.6(0.75\text{ ft}) - \frac{\left[2^4 - 1\right](3.88\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = \mathbf{6.69\ ft\ H_2O}$$

This head loss, though a reasonable estimate, is lower than the exact one calculated above because it does not take into account irreversible losses downstream of the pressure taps, where the flow is still "recovering," and is not yet fully developed.

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(62.36\text{ kg/m}^3)(3.88\text{ ft/s})(4/12\text{ ft})}{7.536\times10^{-4}\text{ lbm/ft}\cdot\text{s}} = 1.070\times10^5$$

Substituting β and Re values into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

gives C_d = 0.605, which is very close to the assumed value of 0.61. Using this refined value of C_d, the flow rate becomes 0.336 ft^3/s, which differs from our original result by less than 1%. Therefore, it is convenient to analyze orifice meters using the recommended value of C_d = 0.61 for the discharge coefficient, and then to verify the assumed value.

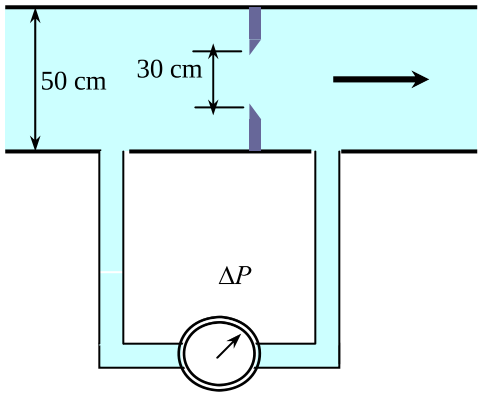
8-102

Solution The flow rate of water is measured with an orifice meter. The pressure difference indicated by the orifice meter and the head loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is C_d = 0.61.

Properties The density and dynamic viscosity of water are given to be ρ = 998 kg/m^3 and μ = 1.002×10^{-4} kg/m·s, respectively.

Analysis The diameter ratio and the throat area of the orifice are



$$\beta = d / D = 30 / 50 = 0.60$$

$$A_0 = \pi d^2 / 4 = \pi (0.30 \text{ m})^2 / 4 = 0.07069 \text{ m}^2$$

For a pressure drop of $\Delta P = P_1 - P_2$ across the orifice plate, the flow rate is expressed as

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

Substituting,

$$0.25 \text{ m}^3/s = (0.07069 \text{ m}^2)(0.61)\sqrt{\frac{2\Delta P}{(998 \text{ kg/m}^3)((1-0.60^4)}}$$

which gives the pressure drop across the orifice plate to be

$$\Delta P = 14{,}600 \text{ kg} \cdot \text{m/s}^2 = \mathbf{14.6 \text{ kPa}}$$

It corresponds to a water column height of

$$h_w = \frac{\Delta P}{\rho_w g} = \frac{14{,}600 \text{ kg} \cdot \text{m/s}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 1.49 \text{ m}$$

The percent pressure (or head) loss for orifice meters is given in Fig. 8-59 for β = 0.6 to be 64%. Therefore,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.64(1.49 \text{ m}) = \mathbf{0.95 \text{ m } H_2O}$$

The head loss between the two measurement sections can also be estimated from the energy equation. Since $z_1 = z_2$, the head form of the energy equation simplifies to

$$h_L \approx \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = h_w - \frac{\left[(D/d)^4 - 1\right]V_1^2}{2g} = 1.49 \text{ m} - \frac{\left[(50/30)^4 - 1\right](1.27 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \mathbf{0.940 \text{ m } H_2O}$$

where $V_1 = \dfrac{\dot{V}}{A_c} = \dfrac{\dot{V}}{\pi D^2/4} = \dfrac{0.250 \text{ m}^3/\text{s}}{\pi(0.50 \text{ m})^2/4} = 1.27 \text{ m/s}$

This head loss, though a reasonable estimate, is lower than the exact one calculated above because it does not take into account irreversible losses downstream of the pressure taps, where the flow is still "recovering," and is not yet fully developed.

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(998 \text{ kg/m}^3)(1.27 \text{ m/s})(0.50 \text{ m})}{1.002 \times 10^{-3} \text{ kg/m} \cdot \text{s}} = 6.32 \times 10^5$$

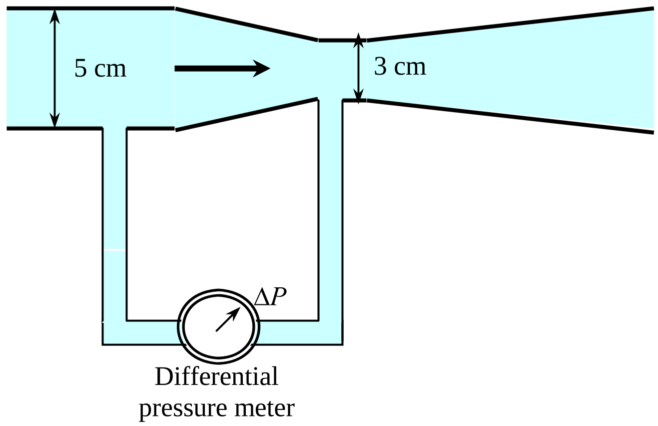
Substituting β and Re values into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

gives C_d = 0.605, which is very close to the assumed value of 0.61.

8-103

Solution A Venturi meter equipped with a differential pressure gage is used to measure to flow rate of water through a horizontal pipe. For a given pressure drop, the volume flow rate of water and the average velocity through the pipe are to be determined.



Assumptions The flow is steady and incompressible.

Properties The density of water is given to be ρ = 999.1 kg/m^3. The discharge coefficient of Venturi meter is given to be C_d = 0.98.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 3/5 = 0.60$$

$$A_0 = \pi d^2 / 4 = \pi(0.03\,\text{m})^2 / 4 = 7.069\times10^{-4}\ \text{m}^2$$

Noting that ΔP = 5 kPa = 5000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (7.069\times10^{-4}\ \text{m}^2)(0.98)\sqrt{\frac{2\times5000\ \text{N/m}^2}{(999.1\,\text{kg/m}^3)((1-0.60^4)}\left(\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right)}$$

$$= \mathbf{0.00235\ m^3/s}$$

which is equivalent to 2.35 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.00235\,\text{m}^3/\text{s}}{\pi(0.05\,\text{m})^2/4} = \mathbf{1.20\ m/s}$$

Discussion Note that the flow rate is proportional to the square root of pressure difference across the Venturi meter.

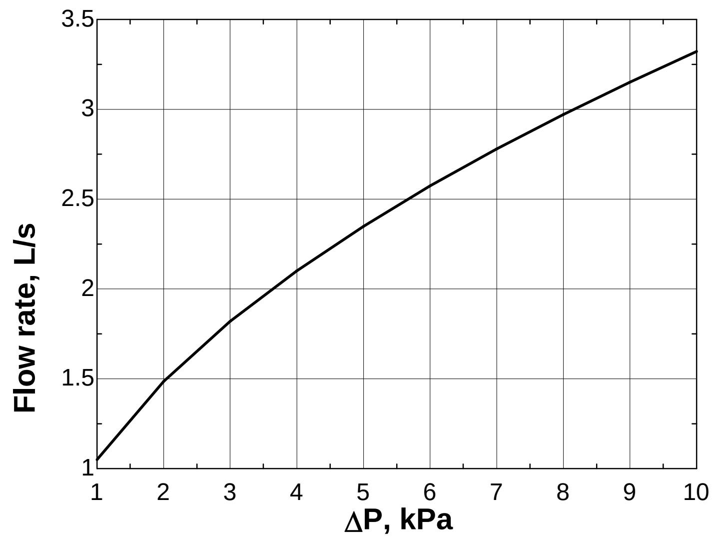
8-104

Solution The previous problem is reconsidered. The variation of flow rate as the pressure drop varies from 1 kPa to 10 kPa at intervals of 1 kPa is to be investigated, and the results are to be plotted.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
rho=999.1 "kg/m3"
D=0.05 "m"
d0=0.03 "m"
beta=d0/D
A0=pi*d0^2/4
Cd=0.98
Vol=A0*Cd*SQRT(2*DeltaP*1000/(rho*(1-beta^4)))*1000 "L/s"
```

Pressure Drop ΔP, kPa	Flow rate L/s
1	1.05
2	1.49
3	1.82
4	2.10
5	2.35
6	2.57
7	2.78
8	2.97
9	3.15
10	3.32

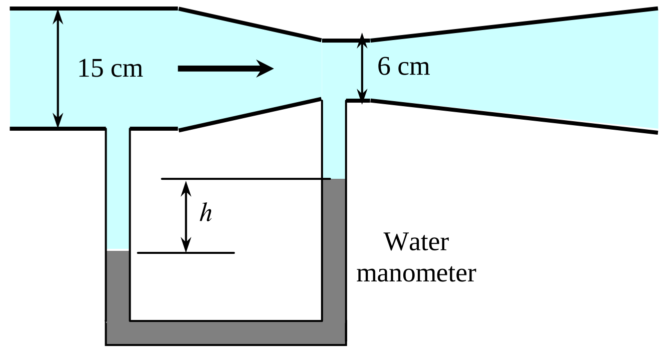


Discussion This type of plot can be thought of as a calibration plot for the flowmeter, although a real calibration plot would use actual experimental data rather than data from equations. It would be interesting to compare the above plot to experimental data to see how close the predictions are.

8-105

Solution A Venturi meter equipped with a water manometer is used to measure to flow rate of air through a duct. For a specified maximum differential height for the manometer, the maximum mass flow rate of air that can be measured is to be determined.

Assumptions The flow is steady and incompressible.



Properties The density of air is given to be ρ_{air} = 1.204 kg/m^3. We take the density of water to be ρ_w = 1000 kg/m^3. The discharge coefficient of Venturi meter is given to be C_d = 0.98.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d/D = 6/15 = 0.40$$

$$A_0 = \pi d^2/4 = \pi(0.06\text{ m})^2/4 = 0.002827\text{ m}^2$$

The pressure drop across the Venturi meter can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_w - \rho_f)gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_w - \rho_f)gh}{\rho_f(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_w/\rho_{air} - 1)gh}{1-\beta^4}}$$

Substituting and using h = 0.40 m, the maximum volume flow rate is determined to be

$$\dot{V} = (0.002827\text{ m}^2)(0.98)\sqrt{\frac{2(1000/1.204 - 1)(9.81\text{ m/s}^2)(0.40\text{ m})}{1-0.40^4}} = 0.2265\text{ m}^3/s$$

Then the maximum mass flow rate this Venturi meter can measure is

$$\dot{m} = \rho\dot{V} = (1.204\text{ kg/m}^3)(0.2265\text{ m}^3\text{/s}) = \mathbf{0.273\ kg/s}$$

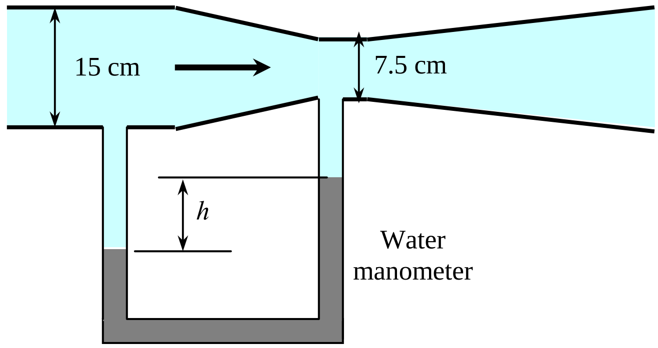
Also, the average flow velocity in the duct is

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2265\text{ m}^3/\text{s}}{\pi(0.15\text{ m})^2/4} = 12.8\text{ m/s}$$

Discussion Note that the maximum available differential height limits the flow rates that can be measured with a manometer.

8-106

Solution A Venturi meter equipped with a water manometer is used to measure to flow rate of air through a duct. For a specified maximum differential height for the manometer, the maximum mass flow rate of air that can be measured is to be determined.



Assumptions The flow is steady and incompressible.

Properties The density of air is given to be ρ_{air} = 1.204 kg/m^3. We take the density of water to be ρ_w = 1000 kg/m^3. The discharge coefficient of Venturi meter is given to be C_d = 0.98.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d/D = 7.5/15 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(0.075\text{ m})^2/4 = 0.004418\text{ m}^2$$

The pressure drop across the Venturi meter can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_w - \rho_f)gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_w - \rho_f)gh}{\rho_f(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_w/\rho_{air} - 1)gh}{1-\beta^4}}$$

Substituting and using h = 0.40 m, the maximum volume flow rate is determined to be

$$\dot{V} = (0.004418\text{ m}^2)(0.98)\sqrt{\frac{2(1000/1.204 - 1)(9.81\text{ m/s}^2)(0.40\text{ m})}{1 - 0.50^4}} = 0.3608\text{ m}^3/s$$

Then the maximum mass flow rate this Venturi meter can measure is

$$\dot{m} = \rho\dot{V} = (1.204\text{ kg/m}^3)(0.3608\text{ m}^3\text{/s}) = \mathbf{0.434\ kg/s}$$

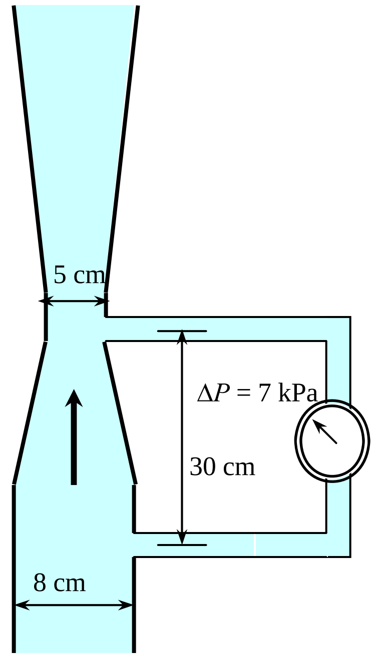
Also, the average flow velocity in the duct is

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.3608\text{ m}^3/\text{s}}{\pi(0.15\text{ m})^2/4} = 20.4\text{ m/s}$$

Discussion Note that the maximum available differential height limits the flow rates that can be measured with a manometer.

8-107

Solution A Venturi meter equipped with a differential pressure gage is used to measure the flow rate of liquid propane through a vertical pipe. For a given pressure drop, the volume flow rate is to be determined.



Assumptions The flow is steady and incompressible.

Properties The density of propane is given to be $\rho = 514.7$ kg/m^3. The discharge coefficient of Venturi meter is given to be $C_d = 0.98$.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 5/8 = 0.625$$

$$A_0 = \pi d^2 / 4 = \pi (0.05\text{ m})^2 / 4 = 0.001963\text{ m}^2$$

Noting that $\Delta P = 7$ kPa = 7000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (0.001963\text{ m}^2)(0.98)\sqrt{\frac{2 \times 7000\text{ N/m}^2}{(514.7\text{ kg/m}^3)((1-0.625^4)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}$$

$$= \mathbf{0.0109\ m^3/s}$$

which is equivalent to 10.9 L/s. Also, the average flow velocity in the pipe is

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.0109\text{ m}^3/\text{s}}{\pi (0.08\text{ m})^2/4} = 2.17\text{ m/s}$$

Discussion Note that the elevation difference between the locations of the two probes does not enter the analysis since the pressure gage measures the pressure differential at a specified location. When there is no flow through the Venturi meter, for example, the pressure gage would read zero.

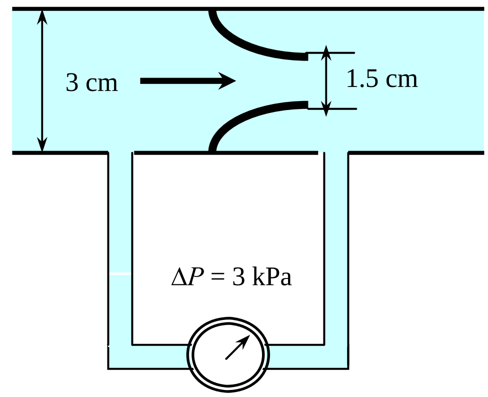
8-108

Solution The flow rate of water is to be measured with flow nozzle equipped with a differential pressure gage. For a given pressure drop, the flow rate, the average flow velocity, and head loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is $C_d = 0.96$.

Properties The density and dynamic viscosity of water are given to be ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m·s, respectively.

Analysis The diameter ratio and the throat area of the meter are



$$\beta = d / D = 1.5/3 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(0.015\text{ m})^2/4 = 1.767\times10^{-4}\text{ m}^2$$

Noting that ΔP = 4 kPa = 4000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (1.767\times10^{-4}\text{ m}^2)(0.96)\sqrt{\frac{2\times3000\text{ N/m}^2}{(999.7\text{ kg/m}^3)((1-0.50^4)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}$$

$$= \mathbf{0.429\times10^{-3}\ m^3/s}$$

which is equivalent to 0.429 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.429\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.03\text{ m})^2/4} = \mathbf{0.607\ m/s}$$

The water column height corresponding to a pressure drop of 3 kPa is

$$h_w = \frac{\Delta P}{\rho_w g} = \frac{3000\text{ kg}\cdot\text{m/s}^2}{(999.7\text{ kg/m}^3)(9.81\text{ m/s}^2)} = 0.306\text{ m}$$

The percent pressure (or head) loss for nozzle meters is given in Fig. 8-59 for β = 0.5 to be 62%. Therefore,

$$h_L = \text{(Permanent loss fraction)(Total head loss)} = 0.62(0.306\text{ m}) = \mathbf{0.19\ m\ H_2O}$$

The head loss between the two measurement sections can be determined from the energy equation, which simplifies to (for $z_1 = z_2$)

$$h_L = \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = h_w - \frac{[(D/d)^4 - 1]V_1^2}{2g} = 0.306\text{ m} - \frac{[(3/1.5)^4 - 1](0.607\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.024\text{ m H2O}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\text{ kg/m}^3)(0.607\text{ m/s})(0.03\text{ m})}{1.307\times10^{-3}\text{ kg/m}\cdot\text{s}} = 1.39\times10^4$$

Substituting the β and Re values into the orifice discharge coefficient relation gives

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(1.39\times10^4)^{0.5}} = 0.958$$

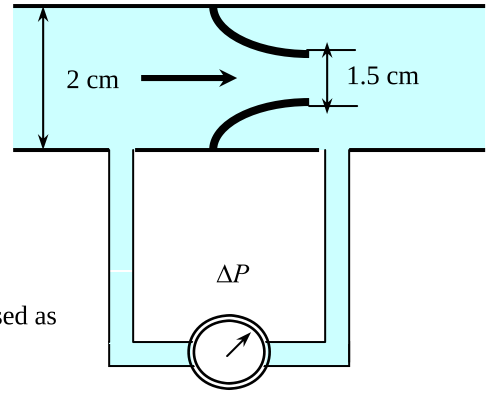
which is practically identical to the assumed value of 0.96.

8-109

Solution A kerosene tank is filled with a hose equipped with a nozzle meter. For a specified filling time, the pressure difference indicated by the nozzle meter is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is $C_d = 0.96$.

Properties The density of kerosene is given to be $\rho = 820\ \text{kg/m}^3$.



Analysis The diameter ratio and the throat area of the meter are

$$\beta = d/D = 1.5/2 = 0.75$$

$$A_0 = \pi d^2/4 = \pi(0.015\ \text{m})^2/4 = 1.767\times10^{-4}\ \text{m}^2$$

To fill a 16-L tank in 20 s, the flow rate must be

$$\dot{V} = \frac{V_{\text{tank}}}{\Delta t} = \frac{16\ \text{L}}{20\ \text{s}} = 0.8\ \text{L/s}$$

For a pressure drop of $\Delta P = P_1 - P_2$ across the meter, the flow rate is expressed as

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

Substituting,

$$0.0008\ \text{m}^3/s = (1.767\times10^{-4}\ \text{m}^2)(0.96)\sqrt{\frac{2\Delta P}{(820\ \text{kg/m}^3)((1-0.75^4)}}$$

which gives the pressure drop across the meter to be

$$\Delta P = 6230\ \text{kg}\cdot\text{m/s}^2 = \mathbf{6.23\ kPa}$$

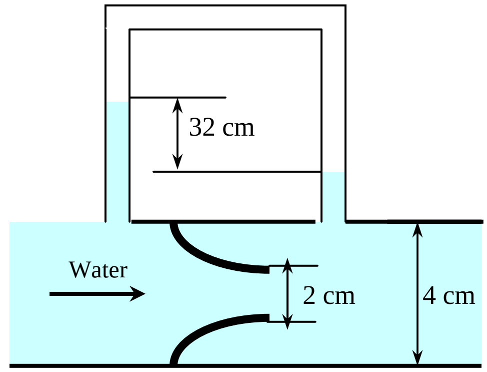
Discussion Note that the flow rate is proportional to the square root of pressure difference across the nozzle meter.

8-110

Solution The flow rate of water is to be measured with flow nozzle equipped with an inverted air-water manometer. For a given differential height, the flow rate and head loss caused by the nozzle meter are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the nozzle meter is C_d = 0.96.

Properties The density and dynamic viscosity of water are given to be ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s, respectively.



Analysis The diameter ratio and the throat area of the meter are

$$\beta = d/D = 2/4 = 0.50$$

$$A_0 = \pi d^2/4 = \pi(0.02\text{ m})^2/4 = 3.142\times10^{-4}\text{ m}^2$$

Noting that ΔP = 4 kPa = 4000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d\sqrt{\frac{2(P_1-P_2)}{\rho(1-\beta^4)}} = A_o C_d\sqrt{\frac{2\rho_w gh}{\rho_w(1-\beta^4)}} = A_o C_d\sqrt{\frac{2gh}{1-\beta^4}}$$

$$= (3.142\times10^{-4}\text{ m}^2)(0.96)\sqrt{\frac{2(9.81\text{ m/s}^2)(0.32\text{ m})}{1-0.50^4}}$$

$$= \mathbf{0.781\times10^{-3}\ m^3/s}$$

which is equivalent to 0.781 L/s. The average flow velocity in the pipe is

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.781\times10^{-3}\text{ m}^3/\text{s}}{\pi(0.04\text{ m})^2/4} = 0.621\text{ m/s}$$

The percent pressure (or head) loss for nozzle meters is given in Fig. 8-59 for β = 0.5 to be 62%. Therefore,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.62(0.32\text{ m}) = \mathbf{0.20\ m\ H_2O}$$

The head loss between the two measurement sections can be determined from the energy equation, which simplifies to (z_1 =z_2)

$$h_L = \frac{P_1-P_2}{\rho_f g} - \frac{V_2^2-V_1^2}{2g} = h_w - \frac{\left[(D/d)^4-1\right]V_1^2}{2g} = 0.32\text{ m} - \frac{\left[(4/2)^4-1\right](0.621\text{ m/s})^2}{2\left(9.81\text{ m/s}^2\right)} = 0.025\text{ m H}_2\text{O}$$

Discussion The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(998\text{ kg/m}^3)(0.621\text{ m/s})(0.04\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}} = 2.47\times10^4$$

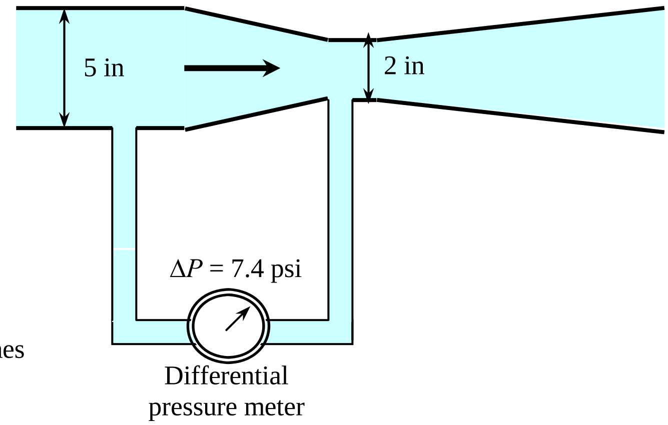
Substituting the β and Re values into the orifice discharge coefficient relation gives

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(2.47\times10^4)^{0.5}} = 0.968$$

which is almost identical to the assumed value of 0.96.

8-111E

Solution A Venturi meter equipped with a differential pressure meter is used to measure to flow rate of refrigerant-134a through a horizontal pipe. For a measured pressure drop, the volume flow rate is to be determined.



Assumptions The flow is steady and incompressible.

Properties The density of R-134a is given to be ρ = 83.31 lbm/ft^3. The discharge coefficient of Venturi meter is given to be C_d = 0.98.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d/D = 2/5 = 0.40$$

$$A_0 = \pi d^2/4 = \pi(2/12\text{ ft})^2/4 = 0.02182\text{ ft}^2$$

Noting that ΔP = 7.4 psi = 7.4×144 lbf/ft^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (0.02182\text{ ft}^2)(0.98)\sqrt{\frac{2\times 7.4\times 144\text{ lbf/ft}^2}{(83.31\text{ lbm/ft}^3)((1-0.40^4)}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)}$$

$$= \mathbf{0.622\ ft^3/s}$$

Also, the average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.622\text{ ft}^3/\text{s}}{\pi(5/12\text{ ft})^2/4} = \mathbf{4.56\ ft/s}$$

Discussion Note that the flow rate is proportional to the square root of pressure difference across the Venturi meter.

Review Problems

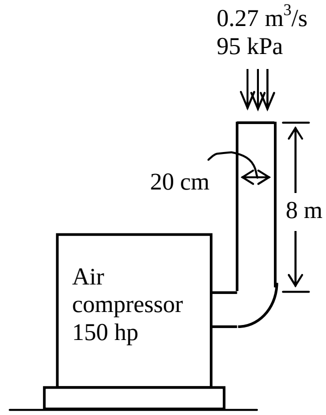
8-112

Solution A compressor takes in air at a specified rate at the outdoor conditions. The useful power used by the compressor to overcome the frictional losses in the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors, and thus minor losses are negligible. **5** The flow section involves no work devices such as fans or turbines.

Properties The properties of air at 1 atm = 101.3 kPa and 15°C are ρ_0 = 1.225 kg/m^3 and μ = 1.802×10^{-5} kg/m⋅s. The roughness of galvanized iron surfaces is ε = 0.00015 m. The dynamic viscosity is independent of pressure, but density of an ideal gas is proportional to pressure. The density of air at 95 kPa is

$\rho = (P/P_0)\rho_0 = (95/101.3)(1.225\text{ kg/m}^3) = 1.149\text{ kg/m}^3$.



Analysis The average velocity and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.27\text{ m}^3/\text{s}}{\pi(0.20\text{ m})^2/4} = 8.594\text{ m/s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(1.149\text{ kg/m}^3)(8.594\text{ m/s})(0.20\text{ m})}{1.802\times10^{-5}\text{ kg/m}\cdot\text{s}} = 1.096\times10^5$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{1.5\times10^{-4}\text{ m}}{0.20\text{ m}} = 7.5\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{7.5\times10^{-4}}{3.7} + \frac{2.51}{1.096\times10^5\sqrt{f}}\right)$$

It gives f = 0.02109. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.02109\frac{8\text{ m}}{0.20\text{ m}}\frac{(1.149\text{ kg/m}^3)(8.594\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ Pa}}{1\text{ N/m}^2}\right) = 35.8\text{ Pa}$$

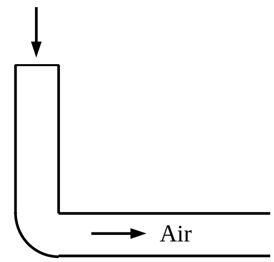
$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.27\text{ m}^3/\text{s})(35.8\text{ Pa})\left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{9.66\ W}$$

Discussion Note hat the pressure drop in the duct and the power needed to overcome it is very small (relative to 150 hp), and can be disregarded. The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.02086, which is very close to the Colebrook value. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency (but probably no more than 20 W).

8-113

Solution Air enters the underwater section of a circular duct. The fan power needed to overcome the flow resistance in this section of the duct is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** Air is an ideal gas. **4** The duct involves no components such as bends, valves, and connectors. **5** The flow section involves no work devices such as fans or turbines. **6** The pressure of air is 1 atm.



Properties The properties of air at 1 atm and 15°C are ρ_0 = 1.225 kg/m^3 and μ = 1.802×10^{-5} kg/m⋅s. The roughness of stainless steel pipes is ε = 0.000005 m.

Analysis The volume flow rate and the Reynolds number are

$$\dot{V} = VA_c = V(\pi D^2/4) = (3\text{ m/s})[\pi(0.20\text{ m})^2/4] = 0.0942\text{ m}^3/\text{s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(1.225\text{ kg/m}^3)(3\text{ m/s})(0.20\text{ m})}{1.802\times10^{-5}\text{ kg/m}\cdot\text{s}} = 4.079\times10^4$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{5\times10^{-6}\text{ m}}{0.20\text{ m}} = 2.5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.5\times10^{-5}}{3.7} + \frac{2.51}{4.079\times10^4\sqrt{f}}\right)$$

It gives f = 0.02195. Then the pressure drop in the duct and the required pumping power become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.02195\frac{15\text{ m}}{0.2\text{ m}}\frac{(1.225\text{ kg/m}^3)(3\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ Pa}}{1\text{ N/m}^2}\right) = 9.07\text{ Pa}$$

$$\dot{W}_{\text{electric}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(0.0942\text{ m}^3/\text{s})(9.07\text{ Pa})}{0.62}\left(\frac{1\text{ W}}{1\text{ Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{1.4\ W}$$

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.02175, which is sufficiently close to 0.02195. Assuming the pipe to be smooth would give 0.02187 for the friction factor, which is almost identical to the f value obtained from the Colebrook relation. Therefore, the duct can be treated as being smooth with negligible error.

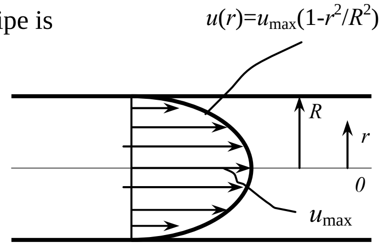
8-114

Solution The velocity profile in fully developed laminar flow in a circular pipe is given. The radius of the pipe, the average velocity, and the maximum velocity are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by $u(r) = 6(1 - 0.01r^2)$. Comparing the two relations above gives the pipe radius, the maximum velocity, and the average velocity to be

$$R^2 = \frac{1}{100} \quad\rightarrow\quad R = \mathbf{0.10\ m}$$

$$u_{\max} = \mathbf{6\ m/s} \quad\rightarrow\quad V_{avg} = \frac{u_{\max}}{2} = \frac{6\text{ m/s}}{2} = \mathbf{3\ m/s}$$

Discussion In fully developed laminar pipe flow, average velocity is exactly half of maximum (centerline) velocity.

8-115E

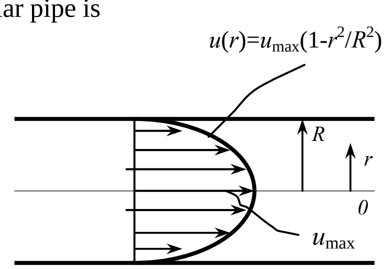
Solution The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions **1** The flow is steady, laminar, and fully developed. **2** The pipe is horizontal.

Properties The density and dynamic viscosity of water at 40°F are ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft·h = 1.039×10^{-3} lbm/ft·s, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 0.8(1 - 625r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, and the average velocity to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04 \text{ ft}$$

$$u_{\max} = 0.8 \text{ ft/s}$$

$$V = V_{avg} = \frac{u_{\max}}{2} = \frac{0.8 \text{ ft/s}}{2} = 0.4 \text{ ft/s}$$

Then the volume flow rate and the pressure drop become

$$\dot{V} = VA_c = V(\pi R^2) = (0.4 \text{ ft/s})[\pi(0.04 \text{ ft})^2] = \mathbf{0.00201\ ft^3/s}$$

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} \quad \rightarrow \quad 0.00201 \text{ ft}^3\text{/s} = \frac{(\Delta P)\pi(0.08 \text{ ft})^4}{128(1.039\times10^{-3} \text{ lbm/ft}\cdot\text{s})(80 \text{ ft})}\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right)$$

It gives

$$\Delta P = 5.16 \text{ lbf/ft}^2 = \mathbf{0.0358\ psi}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (0.00201 \text{ ft}^3\text{/s})(5.16 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.014\ W}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(0.4 \text{ ft/s})(0.08 \text{ ft})}{1.039\times10^{-3} \text{ lbm/ft}\cdot\text{s}} = 1922$$

which is less than 2300. Therefore, the flow is laminar.

Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

8-116E

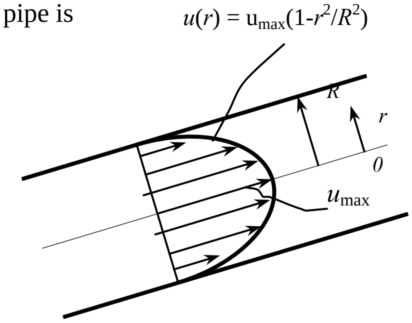
Solution The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Properties The density and dynamic viscosity of water at 40°F are ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft·h = 1.039×10^{-3} lbm/ft·s, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 0.8(1 - 625r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, the average velocity, and the volume flow rate to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04\text{ ft}$$

$$u_{\max} = 0.8\text{ ft/s}$$

$$V = V_{avg} = \frac{u_{\max}}{2} = \frac{0.8\text{ ft/s}}{2} = 0.4\text{ ft/s}$$

$$\dot{V} = VA_c = V(\pi R^2) = (0.4\text{ ft/s})[\pi(0.04\text{ ft})^2] = \mathbf{0.00201\ ft^3/s}$$

For uphill flow with an inclination of 12°, we have θ = +12°, and

$$\rho gL\sin\theta = (62.42\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(80\text{ ft})\sin 12°\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 1038\text{ lbf/ft}^2$$

$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho gL\sin\theta)\pi D^4}{128\mu L} \rightarrow 0.00201\text{ ft}^3\text{/s} = \frac{(\Delta P - 1038)\pi(0.08\text{ ft})^4}{128(1.039\times10^{-3}\text{ lbm/ft}\cdot\text{s})(80\text{ ft})}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)$$

It gives

$$\Delta P = 1043\text{ lbf/ft}^2 = 7.24\text{ psi}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (0.00201\text{ ft}^3\text{/s})(1043\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{2.84\ W}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(62.42\text{ lbm/ft}^3)(0.4\text{ ft/s})(0.08\text{ ft})}{1.039\times10^{-3}\text{ lbm/ft}\cdot\text{s}} = 1922$$

which is less than 2300. Therefore, the flow is laminar.

Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

8-117

Solution Water is discharged from a water reservoir through a circular pipe of diameter D at the side wall at a vertical distance H from the free surface with a reentrant section. A relation for the "equivalent diameter" of the reentrant pipe for use in relations for frictionless flow through a hole is to be obtained.

Assumptions **1** The flow is steady and incompressible. **2** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **3** The water level in the reservoir remains constant. **4** The pipe is horizontal. **5** The entrance effects are negligible, and thus the flow is fully developed and the friction factor f is constant. **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The loss coefficient is $K_L = 0.8$ for the reentrant section, and $K_L = 0$ for the "frictionless" flow.

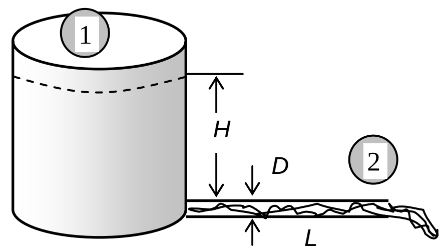
Analysis We take point 1 at the free surface of the reservoir and point 2 at the exit of the pipe, which is also taken as the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the reservoir is zero ($V_1 = 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad H = \alpha_2 \frac{V_2^2}{2g} + h_L$$

where $\alpha_2 = 1$ and

$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D} + \sum K_L \right) \frac{V_2^2}{2g} = \left(f\frac{L}{D} + K_L \right) \frac{V_2^2}{2g}$$

since the diameter of the pipe is constant. Substituting and solving for V_2 gives



$$H = \frac{V_2^2}{2g} + \left(f\frac{L}{D} + K_L \right) \frac{V_2^2}{2g} \quad \rightarrow \quad V_2 = \sqrt{\frac{2gH}{1 + fL/D + K_L}}$$

Then the volume flow rate becomes

$$\dot{V} = A_c V_2 = \frac{\pi D^2}{4} \sqrt{\frac{2gH}{1 + fL/D + K_L}} \qquad (1)$$

Note that in the special case of $K_L = 0$ and $f = 0$ (frictionless flow), the velocity relation reduces to the Toricelli equation, $V_{2,\text{frictionless}} = \sqrt{2gz_1}$. The flow rate in this case through a hole of D_e (equivalent diameter) is

$$\dot{V} = A_{c,\text{equiv}} V_{2,\text{frictionless}} = \frac{\pi D_{\text{equiv}}^2}{4} \sqrt{2gH} \qquad (2)$$

Setting Eqs. (1) and (2) equal to each other gives the desired relation for the equivalent diameter,

$$\frac{\pi D_{\text{equiv}}^2}{4} \sqrt{2gH} = \frac{\pi D^2}{4} \sqrt{\frac{2gH}{1 + fL/D + K_L}}$$

which gives

$$D_{\text{equiv}} = \frac{D}{(1 + fL/D + K_L)^{1/4}} = \frac{D}{(1 + 0.018 \times 10/0.04 + 0.8)^{1/4}} = \mathbf{0.63}\boldsymbol{D} = \mathbf{0.025\ m}$$

Discussion Note that the effect of frictional losses of a pipe with a reentrant section is to reduce the diameter by about 40% in this case. Also, noting that the flow rate is proportional to the square of the diameter, we have $\dot{V} \propto D_{\text{equiv}}^2 = (0.63D)^2 = 0.40D^2$. Therefore, the flow rate through a sharp-edged entrance is about two-thirds less compared to the frictionless flow case.

8-118

Solution A water tank open to the atmosphere is initially filled with water. The tank is drained to the atmosphere through a 90° horizontal bend of negligible length. The flow rate is to be determined for the cases of the bend being a flanged smooth bend and a miter bend without vanes.

Assumptions **1** The flow is steady and incompressible. **2** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **3** The water level in the tank remains constant. **4** The length of the bend and thus the frictional loss associated with its length is negligible. **5** The entrance is well-rounded, and the entrance loss is negligible.

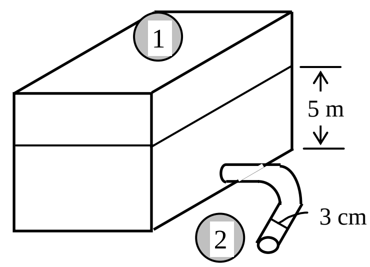
Properties The loss coefficient is $K_L = 0.3$ for a flanged smooth bend and $K_L = 1.1$ for a miter bend without vanes.

Analysis (*a*) We take point 1 at the free surface of the tank, and point 2 at the exit of the bend, which is also taken as the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where the head loss is expressed as $h_L = K_L\dfrac{V^2}{2g}$. Substituting and solving for V_2 gives

$$z_1=\alpha_2\frac{V_2^2}{2g}+K_L\frac{V_2^2}{2g} \quad\rightarrow\quad 2gz_1=V_2^2(\alpha_2+K_L) \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{\alpha_2+K_L}}$$



Then the flow rate becomes

$$\dot V = A_{\text{pipe}}V_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{\alpha_2+K_L}}$$

(*a*) Case 1 **Flanged smooth bend (K_L = 0.3):**

$$\dot V = A_c V_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{\alpha_2+K_L}}=\frac{\pi(0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1.05+0.3}}=\mathbf{0.00603\ m^3/s=6.03\ L/s}$$

(*b*) Case 2 **Miter bend without vanes (K_L = 1.1):**

$$\dot V = A_c V_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{\alpha_2+K_L}}=\frac{\pi(0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1.05+1.1}}=\mathbf{0.00478\ m^3/s=4.78\ L/s}$$

Discussion Note that the type of bend used has a significant effect on the flow rate, and a conscious effort should be made when selecting components in a piping system. If the effect of the kinetic energy correction factor is neglected, $\alpha_2 = 1$ and the flow rates become

(*a*) Case 1 (K_L = 0.3): $$\dot V = A_c V_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{1+K_L}}=\frac{\pi(0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1+0.3}}=0.00614\ \text{m}^3\text{/s}$$

(*b*) Case 2 (K_L = 1.1): $$\dot V = A_c V_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz_1}{1+K_L}}=\frac{\pi(0.03\text{ m})^2}{4}\sqrt{\frac{2(9.81\text{ m/s}^2)(5\text{ m})}{1+1.1}}=0.00483\ \text{m}^3\text{/s}$$

Therefore, the effect of the kinetic energy correction factor is (6.14 - 6.03)/6.03 = 1.8% and (4.83 - 4.78)/4.83 = 1.0%, which is negligible.

8-119 **[*Also solved using EES on enclosed DVD*]**

Solution The piping system of a geothermal district heating system is being designed. The pipe diameter that will optimize the initial system cost and the energy cost is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses, the only significant energy loss arises from pipe friction. **4** The piping system is horizontal. **5** The properties of geothermal water are the same as fresh water. **6** The friction factor is constant at the given value. **7** The interest rate, the inflation rate, and the salvage value of the system are all zero. **8** The flow rate through the system remains constant.

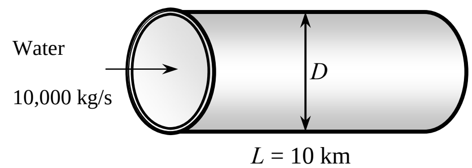
Properties We take the density of water to be $\rho = 1000$ kg/m^3. The friction factor is given to be $f = 0.015$.

Analysis The system operates in a loop, and thus we can take any point in the system as points 1 and 2 (the same point), and thus $z_1 = z_2$, $V_1 = V_2$, and $P_1 = P_2$. Then the energy equation for this piping system simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_1\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2}$$



The flow rate of geothermal water is

$$\dot{V}=\frac{\dot{m}}{\rho}=\frac{10{,}000\text{ kg/s}}{1000\text{ kg/m}^3}=10\text{ m}^3\text{/s}$$

To expose the dependence of pressure drop on diameter, we express it in terms of the flow rate as

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2}=f\frac{L}{D}\frac{\rho}{2}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2=f\frac{16L}{D}\frac{\rho\dot{V}^2}{2\pi^2D^4}=f\frac{8L}{D^5}\frac{\rho\dot{V}^2}{\pi^2}$$

Then the required pumping power can be expressed as

$$\dot{W}_{\text{pump}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{\dot{V}}{\eta_{\text{pump-motor}}}f\frac{8L}{D^5}\frac{\rho\dot{V}^2}{\pi^2}=f\frac{8L}{D^5}\frac{\rho\dot{V}^3}{\eta_{\text{pump-motor}}\pi^2}$$

Note that the pumping power requirement is proportional to f and L, consistent with our intuitive expectation. Perhaps not so obvious is that power is proportional to the *cube* of flow rate. The fact that the *power is inversely proportional to pipe diameter D to the fifth power* averages that a slight increase in pipe diameter will manifest as a tremendous reduction in power dissipation due to friction in a long pipeline. Substituting the given values and expressing the diameter D in meters,

$$\dot{W}_{\text{pump}}=(0.015)\frac{8(10{,}000\text{ m})}{D^5\text{ m}^5}\frac{(1000\text{ kg/m}^3)(10\text{ m}^3\text{/s})^3}{\pi^2(0.80)}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right)=\frac{1.52\times10^5}{D^5}\text{ kW}$$

The number of hours in one year are 24×365 = 8760 h. Then the total amount of electric power used and its cost per year are

$$E_{\text{pump}}=\dot{W}_{\text{pump}}\Delta t=\frac{1.52\times10^5}{D^5}(8760\text{ h})=\frac{1.332\times10^9}{D^5}\text{ kWh/yr}$$

$$\text{Energy cost}=E_{\text{pump}}\times\text{Unit cost}=\left(\frac{1.332\times10^9}{D^5}\text{ kWh/y}\right)(\$0.06\text{/kWh})=\frac{7.99\times10^7}{D^5}\text{ \$/yr}$$

The installation cost of the system with a 30-year lifetime is given to be Cost = $\$10^6 D^2$ where D is in meters. The annual cost of the system is then 1/30$^{\text{th}}$ of it, which is

$$\text{System cost}=\frac{\text{Total cost}}{\text{Life time}}=\frac{\$10^6D^2}{30\text{ yr}}=\$3.33\times10^4D^2\text{ (per year)}$$

Then the total annual cost of the system (installation + operating) becomes

$$\text{Total cost} = \text{Energy cost} + \text{System cost} = \frac{7.99\times10^7}{D^5} + 3.33\times10^4 D^2 \quad \$/\text{yr}$$

The optimum pipe diameter is the value that minimizes this total, and it is determined by taking the derivative of the total cost with respect to D and setting it equal to zero,

$$\frac{\partial(\text{Total cost})}{\partial D} = -5\times\frac{7.99\times10^7}{D^6} + 2\times3.33\times10^4 D = 0$$

Simplifying gives $D^7 = 5998$ whose solution is

D = **3.5 m**

This is the optimum pipe diameter that minimizes the total cost of the system under stated assumptions. A larger diameter pipe will increase the system cost more than it decreases the energy cost, and a smaller diameter pipe will increase the system cost more than it decreases the energy cost.

Discussion The assumptions of zero interest and zero inflation are not realistic, and an actual economic analysis must consider these factors as they have a major effect on the pipe diameter. This is done by considering the time value of money, and expressing all the costs at the same time. Pipe purchase is a present cost, and energy expenditures are future annual costs spread out over the project lifetime. Thus, to provide consistent dollar comparisons between initial and future costs, all future energy costs must be expressed as a single present lump sum to reflect the time-value of money. Then we can compare pipe and energy costs on a consistent basis. Economists call the necessary factor the “Annuity Present Value Factor”, F. If interest rate is 10% per year with n = 30 years, then F = 9.427. Thus, if power costs $1,000,000/year for the next 30 years, then the present value of those future payments is $9,427,000 (and not $30,000,000!) if money is worth 10%. Alternatively, if you must pay $1,000,000 every year for 30 years, and you can today invest $9,437,000 at 10%, then you can meet 30 years of payments at the end of each year. The energy cost in this case can be determined by dividing the energy cost expression developed above by 9.427. This will result in a pipe diameter of D = 2.5 m. In an actual design, we also need to calculate the average flow velocity and the pressure head to make sure that they are reasonable. For a pipe diameter of 2.5 m, for example, the average flow velocity is 1.47 m/s and the pump pressure head is 5.6 m.

8-120

Solution Water is drained from a large reservoir through two pipes connected in series at a specified rate using a pump. The required pumping head and the minimum pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pipes are horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The flow is turbulent so that the tabulated value of the loss coefficients can be used. **5** The pipes involve no components such as bends, valves, and other connectors that cause additional minor losses. **6** The reservoir is open to the atmosphere so that the pressure is atmospheric pressure at the free surface. **7** The water level in the reservoir remains constant. **8** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 15°C are $\rho = 999.1$ kg/m^3 and $\mu = 1.138\times10^{-3}$ kg/m·s. The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance. The roughness of cast iron pipes is $\varepsilon = 0.00026$ m.

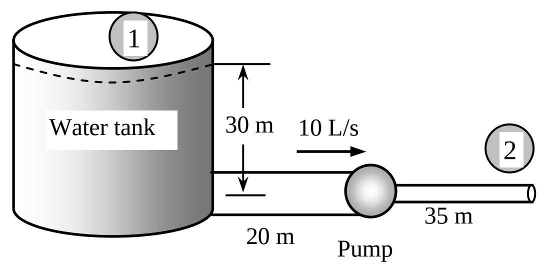
Analysis We take point 1 at the free surface of the tank, and point 2 and the reference level at the centerline of the pipe ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1+h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\sum\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}$$

and the summation is over two pipes. Noting that the two pipes are connected in series and thus the flow rate through each of them is the same, the head loss for each pipe is determined as follows (we designate the first pipe by 1 and the second one by 2):



Pipe 1: $$V_1=\frac{\dot V}{A_{c1}}=\frac{\dot V}{\pi D_1^2/4}=\frac{0.018\ \text{m}^3/\text{s}}{\pi(0.06\ \text{m})^2/4}=6.366\ \text{m/s}$$

$$\text{Re}_1=\frac{\rho V_1 D_1}{\mu}=\frac{(999.1\ \text{kg/m}^3)(6.366\ \text{m/s})(0.06\ \text{m})}{1.138\times10^{-3}\ \text{kg/m}\cdot\text{s}}=335{,}300$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_1=\frac{0.00026\ \text{m}}{0.06\ \text{m}}=0.00433$$

The friction factor corresponding to this relative roughness and the Reynolds number is, from the Colebrook equation,

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{\varepsilon/D_1}{3.7}+\frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)\ \rightarrow\ \frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{0.00433}{3.7}+\frac{2.51}{335{,}300\sqrt{f_1}}\right)$$

It gives $f_1 = 0.02941$. The only minor loss is the entrance loss, which is $K_L = 0.5$. Then the total head loss of the first pipe becomes

$$h_{L1}=\left(f_1\frac{L_1}{D_1}+\sum K_L\right)\frac{V_1^2}{2g}=\left((0.02941)\frac{20\ \text{m}}{0.06\ \text{m}}+0.5\right)\frac{(6.366\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=21.3\ \text{m}$$

Pipe 2: $$V_2=\frac{\dot V}{A_{c2}}=\frac{\dot V}{\pi D_2^2/4}=\frac{0.018\text{m}^3/\text{s}}{\pi(0.04\ \text{m})^2/4}=14.32\ \text{m/s}$$

$$\text{Re}_2=\frac{\rho V_2 D_2}{\mu}=\frac{(999.1\ \text{kg/m}^3)(14.32\ \text{m/s})(0.04\ \text{m})}{1.138\times10^{-3}\ \text{kg/m}\cdot\text{s}}=502{,}900$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_2=\frac{0.00026\ \text{m}}{0.04\ \text{m}}=0.0065$$

The friction factor corresponding to this relative roughness and the Reynolds number is, from the Colebrook equation,

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon / D_2}{3.7} + \frac{2.51}{\mathrm{Re}_2\sqrt{f_2}}\right) \rightarrow \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{0.0065}{3.7} + \frac{2.51}{502{,}900\sqrt{f_2}}\right)$$

It gives f_2 = 0.03309. The second pipe involves no minor losses. Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the head loss for the second pipe becomes

$$h_{L2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = (0.03309)\frac{35\text{ m}}{0.04\text{ m}}\frac{(14.32\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 302.6\text{ m}$$

The total head loss for two pipes connected in series is the sum of the head losses of the two pipes,

$$h_L = h_{L,\text{total}} = h_{L1} + h_{L2} = 21.3 + 302.6 = 323.9\text{ m}$$

Then the pumping head and the minimum pumping power required (the pumping power in the absence of any inefficiencies of the pump and the motor, which is equivalent to the useful pumping power) become

$$h_{\text{pump, u}} = \alpha_2 \frac{V_2^2}{2g} + h_L - z_1 = (1)\frac{(14.32\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 323.9 - 30 = 304.4\text{ m}$$

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = \rho\dot{V}gh_{\text{pump, u}}$$

$$= (999.1\text{ kg/m}^3)(0.018\text{ m}^3\text{/s})(9.81\text{ m/s}^2)(304.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{53.7\ kW}$$

Therefore, the pump must supply a minimum of 53.7 kW of useful mechanical energy to water.

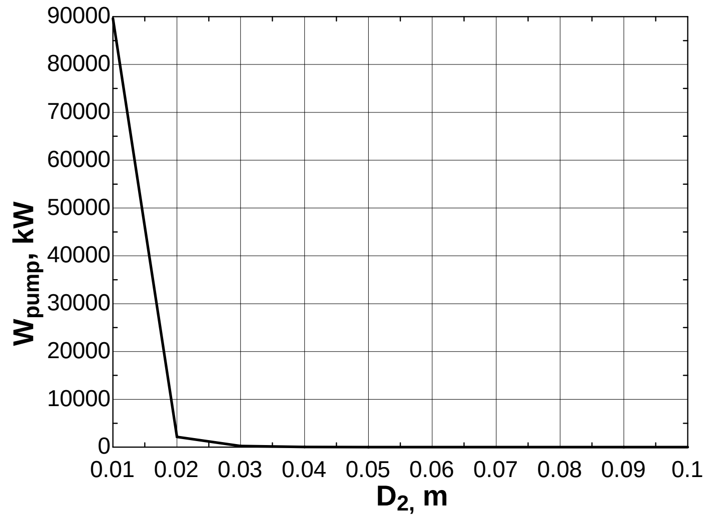
Discussion Note that the shaft power of the pump must be greater than this to account for the pump inefficiency, and the electrical power supplied must be even greater to account for the motor inefficiency.

8-121

Solution In the previous problem, the effect of second pipe diameter on required pumping head for the same flow rate is to be investigated by varying the pipe diameter from 1 cm to 10 cm in increments of 1 cm.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
rho=999.1
mu=0.001138
nu=mu/rho
Vdot=0.018 "m3/s"
g=9.81 "m/s2"
z1=30 "m"
L1=20 "m"
D1=0.06 "m"
Ac1=pi*D1^2/4
Re1=V1*D1/nu
V1=Vdot/Ac1
eps1=0.00026
rf1=eps1/D1
1/sqrt(f1)=-2*log10(rf1/3.7+2.51/(Re1*sqrt(f1)))
KL1=0.5
HL1=(f1*L1/D1+KL1)*V1^2/(2*g)
L2=35
Re2=V2*D2/nu
V2=Vdot/(pi*D2^2/4)
eps2=0.00026
rf2=eps2/D2
1/sqrt(f2)=-2*log10(rf2/3.7+2.51/(Re2*sqrt(f2)))
HL2=f2*(L2/D2)*V2^2/(2*g)
HL=HL1+HL2
hpump=V2^2/(2*g)+HL-z1
Wpump=rho*Vdot*g*hpump/1000 "kW"
```



D_2, m	W_{pump}, kW	h_{L2}, m	Re
0.01	89632.5	505391.6	2.012E+06
0.02	2174.7	12168.0	1.006E+06
0.03	250.8	1397.1	6.707E+05
0.04	53.7	302.8	5.030E+05
0.05	15.6	92.8	4.024E+05
0.06	5.1	35.4	3.353E+05
0.07	1.4	15.7	2.874E+05
0.08	-0.0	7.8	2.515E+05
0.09	-0.7	4.2	2.236E+05
0.10	-1.1	2.4	2.012E+05

Discussion Clearly, the power decreases quite rapidly with increasing diameter. This is not surprising, since the irreversible frictional head loss (major head loss) decreases significantly with increasing pipe diameter.

8-122

Solution Two pipes of identical diameter and material are connected in parallel. The length of one of the pipes is twice the length of the other. The ratio of the flow rates in the two pipes is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully turbulent in both pipes and thus the friction factor is independent of the Reynolds number (it is the same for both pipes since they have the same material and diameter). **3** The minor losses are negligible.

Analysis When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{A_c}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2 = 8f\frac{L}{D}\frac{1}{g}\frac{\dot{V}^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{\dot{V}^2}{D^5}$$

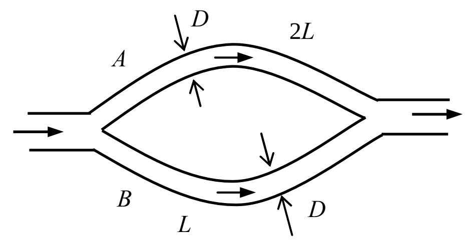
Solving for the flow rate gives

$$\dot{V} = \sqrt{\frac{\pi^2 h_L g D^5}{8fL}} = \frac{k}{\sqrt{L}} \qquad (k \text{ is a constant})$$

When the pipe diameter, friction factor, and the head loss is constant, which is the case here for parallel connection, the flow rate becomes inversely proportional to the square root of length L. Therefore, when the length is doubled, the flow rate will decrease by a factor of $2^{0.5} = 1.41$ since

If $$\dot{V}_A = \frac{k}{\sqrt{L_A}}$$

Then $$\dot{V}_B = \frac{k}{\sqrt{L_B}} = \frac{k}{\sqrt{2L_A}} = \frac{k}{\sqrt{2}\sqrt{L_A}} = \frac{\dot{V}_A}{\sqrt{2}} = 0.707\dot{V}_A$$



Therefore, the ratio of the flow rates in the two pipes is **0.707**.

Discussion Even though one pipe is twice as long as the other, the volume flow rate in the shorter pipe is *not* twice as much – the relationship is nonlinear.

8-123 **[*Also solved using EES on enclosed DVD*]**

Solution A pipeline that transports oil at a specified rate branches out into two parallel pipes made of commercial steel that reconnects downstream. The flow rates through each of the parallel pipes are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Entrance effects are negligible, and thus the flow is fully developed. **3** Minor losses are disregarded. **4** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of oil at 40°C are ρ = 876 kg/m^3 and μ = 0.2177 kg/m⋅s. The roughness of commercial steel pipes is ε = 0.000045 m.

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Below we will set up the equations to be solved by an equation solver. The head loss in two parallel branches must be the same, and the total flow rate must the sum of the flow rates in the parallel branches. Therefore,

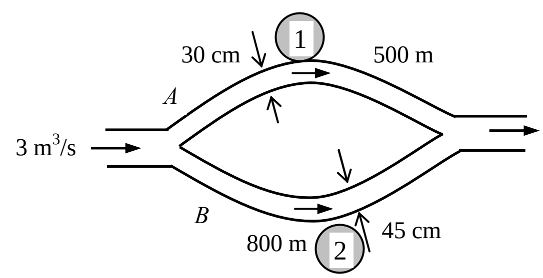
$$h_{L,1} = h_{L,2} \qquad (1)$$

$$\dot V = \dot V_1 + \dot V_2 \;\rightarrow\; \dot V_1 + \dot V_2 = 3 \qquad (2)$$

We designate the 30-cm diameter pipe by 1 and the 45-cm diameter pipe by 2. The average velocity, the relative roughness, the Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$V_1 = \frac{\dot V_1}{A_{c,1}} = \frac{\dot V_1}{\pi D_1^2/4} \;\rightarrow\; V_1 = \frac{\dot V_1}{\pi(0.30\text{m})^2/4} \qquad (3)$$

$$V_2 = \frac{\dot V_2}{A_{c,2}} = \frac{\dot V_2}{\pi D_2^2/4} \;\rightarrow\; V_2 = \frac{\dot V_2}{\pi(0.45\text{m})^2/4} \qquad (4)$$



$$\text{rf}_1 = \frac{\varepsilon_1}{D_1} = \frac{4.5\times10^{-5}\text{ m}}{0.30\text{ m}} = 1.5\times10^{-4}$$

$$\text{rf}_2 = \frac{\varepsilon_2}{D_2} = \frac{4.5\times10^{-5}\text{ m}}{0.45\text{ m}} = 1\times10^{-4}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \;\rightarrow\; \text{Re}_1 = \frac{(876\text{ kg/m}^3)\mathbf{V}_1(0.30\text{ m})}{0.2177\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \;\rightarrow\; \text{Re}_2 = \frac{(876\text{ kg/m}^3)\mathbf{V}_2(0.45\text{ m})}{0.2177\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \quad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{1\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \quad (8)$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} \;\rightarrow\; h_{L,1} = f_1\frac{500\text{ m}}{0.30\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \qquad (9)$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} \;\rightarrow\; h_{L,2} = f_2\frac{800\text{ m}}{0.45\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \qquad (10)$$

This is a system of 10 equations in 10 unknowns, and solving them simultaneously by an equation solver gives

$\dot V_1 = \mathbf{0.91\ m^3/s}$, $\dot V_2 = \mathbf{2.09\ m^3/s}$,

V_1 = 12.9 m/s, V_2 = 13.1 m/s, $h_{L,1} = h_{L,2} = 392$ m

Re_1 = 15,540, Re_2 = 23,800, f_1 = 0.02785, f_2 = 0.02505

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using an iterative approach, but it would be very time consuming. Equation solvers such as EES are invaluable for theses kinds of problems.

8-124

Solution The piping of a district heating system that transports hot water at a specified rate branches out into two parallel pipes made of commercial steel that reconnects downstream. The flow rates through each of the parallel pipes are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are given to be negligible. **4** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 100°C are ρ = 957.9 kg/m^3 and μ = 0.282×10^{-3} kg/m·s. The roughness of commercial steel pipes is ε = 0.000045 m.

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Below we will set up the equations to be solved by an equation solver. The head loss in two parallel branches must be the same, and the total flow rate must the sum of the flow rates in the parallel branches. Therefore,

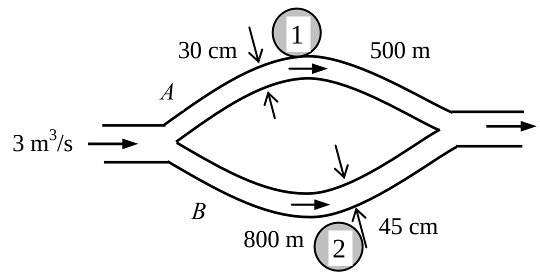
$$h_{L,1} = h_{L,2} \qquad (1)$$

$$\dot V = \dot V_1 + \dot V_2 \;\rightarrow\; \dot V_1 + \dot V_2 = 3 \qquad (2)$$

We designate the 30-cm diameter pipe by 1 and the 45-cm diameter pipe by 2. The average velocity, the relative roughness, the Reynolds number, friction factor, and the head loss in each pipe are expressed as

$$V_1 = \frac{\dot V_1}{A_{c,1}} = \frac{\dot V_1}{\pi D_1^2/4} \;\rightarrow\; V_1 = \frac{\dot V_1}{\pi (0.30\text{m})^2/4} \qquad (3)$$

$$V_2 = \frac{\dot V_2}{A_{c,2}} = \frac{\dot V_2}{\pi D_2^2/4} \;\rightarrow\; V_2 = \frac{\dot V_2}{\pi (0.45\text{m})^2/4} \qquad (4)$$



$$\text{rf}_1 = \frac{\varepsilon_1}{D_1} = \frac{4.5\times10^{-5}\text{ m}}{0.30\text{ m}} = 1.5\times10^{-4}$$

$$\text{rf}_2 = \frac{\varepsilon_2}{D_2} = \frac{4.5\times10^{-5}\text{ m}}{0.45\text{ m}} = 1\times10^{-4}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \;\rightarrow\; \text{Re}_1 = \frac{(957.9\text{ kg/m}^3)V_1(0.30\text{ m})}{0.282\times10^{-3}\text{ kg/m}\cdot s} \qquad (5)$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \;\rightarrow\; \text{Re}_2 = \frac{(957.9\text{ kg/m}^3)V_2(0.45\text{ m})}{0.282\times10^{-3}\text{ kg/m}\cdot s} \qquad (6)$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{1.5\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \qquad (7)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \;\rightarrow\; \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{1\times10^{-4}}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right) \qquad (8)$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} \;\rightarrow\; h_{L,1} = f_1\frac{500\text{ m}}{0.30\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)} \qquad (9)$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} \;\rightarrow\; h_{L,2} = f_2\frac{800\text{ m}}{0.45\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)} \qquad (10)$$

This is a system of 10 equations in 10 unknowns, and their simultaneous solution by an equation solver gives

$\dot V_1 = \mathbf{0.919\ m^3/s}$, $\dot V_2 = \mathbf{2.08\ m^3/s}$,

V_1 = 13.0 m/s, V_2 = 13.1 m/s, $h_{L,1} = h_{L,2} = 187$ m

Re_1 = 1.324×10^7, Re_2 = 2.00×10^7, f_1 = 0.0131, f_2 = 0.0121

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Discussion This problem can also be solved by using a trial-and-error approach, but it will be very time consuming. Equation solvers such as EES are invaluable for these kinds of problems.

8-125E

Solution A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main. For a specified flow rate, the minimum diameter of the piping system is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The pressure at the water main remains constant. **4** There are no dynamic pressure effects at the pipe-water main connection, and the pressure at the pipe entrance is 60 psia. **5** Elevation difference between the pipe and the fountain is negligible ($z_2 = z_1$) . **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.360 lbm/ft·h = 6.556×10^{-4} lbm/ft·s. The roughness of cast iron pipe is ε = 0.00085 ft. The minor loss coefficient is K_L = 0.5 for a sharp-edged entrance, K_L = 1.1 for a 90° miter bend without vanes, K_L = 0.2 for a fully open gate valve, and K_L = 5 for an angle valve.

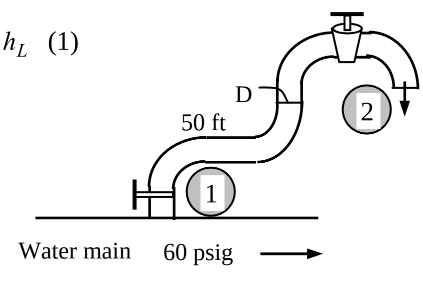
Analysis We choose point 1 in the water main near the entrance where the pressure is 60 psig and the velocity in the pipe to be low. We also take point 1 as the reference level. We take point 2 at the exit of the water fountain where the pressure is the atmospheric pressure ($P_2 = P_{atm}$) and the velocity is the discharge velocity. The energy equation for a control volume between these two points is

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where α_2 = 1 and $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$

since the diameter of the piping system is constant. Then the energy equation becomes

$$\frac{60\text{ psi}}{(62.3\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=\frac{V_2^2}{2(32.2\text{ ft/s}^2)}+h_L \quad (1)$$



The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4} \quad\rightarrow\quad V_2=\frac{20/60\text{ gal/s}}{\pi D^2/4}\left(\frac{0.1337\text{ ft}^3}{1\text{ gal}}\right) \quad (2)$$

$$\text{Re}=\frac{\rho V_2 D}{\mu} \quad\rightarrow\quad \text{Re}=\frac{(62.3\text{ lbm/ft}^3)V_2 D}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}} \quad (3)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad\rightarrow\quad \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{0.00085/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \quad (4)$$

The sum of the loss coefficients is

$$\sum K_L=K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{gate valve}}+K_{L,\text{angle valve}}=0.5+3\times1.1+0.2+5=9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g} \quad\rightarrow\quad h_L=\left(f\frac{50\text{ ft}}{D}+9\right)\frac{V_2^2}{2(32.2\text{ ft/s}^2)} \quad (5)$$

These are 5 equations in the 5 unknowns of V_2, h_L, D, Re, and f, and solving them simultaneously using an equation solver such as EES gives

V_2 = 14.3 ft/s, h_L = 135.5 ft, D = 0.0630 ft = **0.76 in**, Re = 85,540, and f = 0.04263

Therefore, the diameter of the pipe must be at least 0.76 in (or roughly 3/4 in).

Discussion The pipe diameter can also be determined approximately by using the Swamee and Jain relation. It would give D = 0.73 in, which is within 5% of the result obtained above.

8-126

Solution A water fountain is to be installed at a remote location by attaching a cast iron pipe directly to a water main. For a specified flow rate, the minimum diameter of the piping system is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The pressure at the water main remains constant. **4** There are no dynamic pressure effects at the pipe-water main connection, and the pressure at the pipe entrance is 60 psia. **5** Elevation difference between the pipe and the fountain is negligible ($z_2 = z_1$) . **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.360 lbm/ft·h = 6.556×10^{-4} lbm/ft·s. The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$. The minor loss coefficient is $K_L = 0.5$ for a sharp-edged entrance, $K_L = 1.1$ for a 90° miter bend without vanes, $K_L = 0.2$ for a fully open gate valve, and $K_L = 5$ for an angle valve.

Analysis We choose point 1 in the water main near the entrance where the pressure is 60 psig and the velocity in the pipe to be low. We also take point 1 as the reference level. We take point 2 at the exit of the water fountain where the pressure is the atmospheric pressure ($P_2 = P_{atm}$) and the velocity is the discharge velocity. The energy equation for a control volume between these two points is

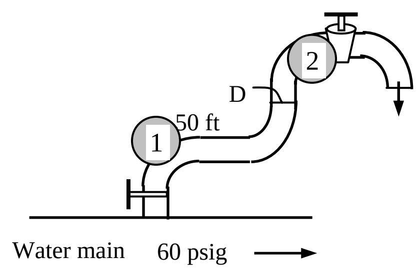
$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$

since the diameter of the piping system is constant. Then the energy equation becomes

$$\frac{60 \text{ psi}}{(62.3\,\text{lbm/ft}^3)(32.2\text{ ft/s}^2)}\left(\frac{144\text{ lbf/ft}^2}{1\text{ psi}}\right)\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)=\frac{V_2^2}{2(32.2\text{ ft/s}^2)}+h_L \quad (1)$$

The average velocity in the pipe and the Reynolds number are



$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4} \quad\rightarrow\quad V_2=\frac{20/60\text{ gal/s}}{\pi D^2/4}\left(\frac{0.1337\text{ ft}^3}{1\text{ gal}}\right) \quad (2)$$

$$\text{Re}=\frac{\rho V_2 D}{\mu} \quad\rightarrow\quad \text{Re}=\frac{(62.3\text{ lbm/ft}^3)V_2 D}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}} \quad (3)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right) \quad\rightarrow\quad \frac{1}{\sqrt f}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt f}\right) \quad (4)$$

The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}}+3K_{L,\text{elbow}}+K_{L,\text{gate valve}}+K_{L,\text{angle valve}}=0.5+3\times1.1+0.2+5=9$$

Then the total head loss becomes

$$h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g} \quad\rightarrow\quad h_L=\left(f\frac{50\text{ ft}}{D}+9\right)\frac{V_2^2}{2(32.2\text{ ft/s}^2)} \quad (5)$$

These are 5 equations in the 5 unknowns of V_2, h_L, D, Re, and f, and solving them simultaneously using an equation solver such as EES gives

V_2 = 18.4 ft/s, h_L = 133.4 ft, D = 0.05549 ft = **0.67 in**, Re = 97,170, and f = 0.0181

Therefore, the diameter of the pipe must be at least 0.67 in.

Discussion The pipe diameter can also be determined approximately by using the Swamee and Jain relation. It would give D = 0.62 in, which is within 7% of the result obtained above.

8-127

Solution In a hydroelectric power plant, the flow rate of water, the available elevation head, and the combined turbine-generator efficiency are given. The electric power output of the plant is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The minor losses are given to be negligible. **4** The water level in the reservoir remains constant.

Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s. The roughness of cast iron pipes is ε = 0.00026 m.

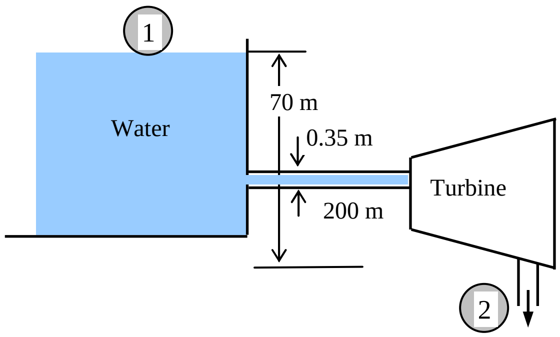
Analysis We take point 1 at the free surface of the reservoir, and point 2 and the reference level at the free surface of the water leaving the turbine site (z_2 = 0). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are very low ($V_1 \cong V_2 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{turbine, e}}=z_1-h_L$$

The average velocity, Reynolds number, friction factor, and head loss in the pipe are

$$V=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.8\ \text{m}^3/\text{s}}{\pi(0.35\ \text{m})^2/4}=8.315\ \text{m/s}$$

$$\text{Re}=\frac{\rho V D_h}{\mu}=\frac{(998\ \text{kg/m}^3)(8.513\ \text{m/s})(0.35\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}=2.899\times10^6$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D_h=\frac{0.00026\ \text{m}}{0.35\ \text{m}}=7.43\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right)\quad\rightarrow\quad\frac{1}{\sqrt f}=-2.0\log\left(\frac{7.43\times10^{-4}}{3.7}+\frac{2.51}{2.899\times10^6\sqrt f}\right)$$

It gives f = 0.0184. When the minor losses are negligible, the head loss in the pipe and the available turbine head are determined to be

$$h_L=f\frac{L}{D}\frac{V^2}{2g}=0.0184\frac{200\ \text{m}}{0.35\ \text{m}}\frac{(8.315\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=37.05\ \text{m}$$

$$h_{\text{turbine, e}}=z_1-h_L=70-37.05=32.95\ \text{m}$$

Then the extracted power from water and the actual power output of the turbine become

$$\dot W_{\text{turbine, e}}=\dot m g h_{\text{turbine, e}}=\rho\dot V g h_{\text{turbine, e}}$$

$$=(998\ \text{kg/m}^3)(0.8\ \text{m}^3/\text{s})(9.81\ \text{m/s}^2)(32.95\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right)=258\ \text{kW}$$

$$\dot W_{\text{turbine-gen}}=\eta_{\text{turbine-gen}}\dot W_{\text{turbine, e}}=(0.84)(258\ \text{kW})=\mathbf{217\ kW}$$

Discussion Note that a perfect turbine-generator would generate 258 kW of electricity from this resource. The power generated by the actual unit is only 217 kW because of the inefficiencies of the turbine and the generator. Also note that more than half of the elevation head is lost in piping due to pipe friction.

8-128

Solution In a hydroelectric power plant, the flow rate of water, the available elevation head, and the combined turbine-generator efficiency are given. The percent increase in the electric power output of the plant is to be determined when the pipe diameter is tripled.

Assumptions **1** The flow is steady and incompressible. **2** Entrance effects are negligible, and thus the flow is fully developed and friction factor is constant. **3** Minor losses are negligible. **4** Water level is constant.

Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s. The roughness of cast iron pipes is ε = 0.00026 m.

Analysis We take point 1 at the free surface of the reservoir, and point 2 and the reference level at the free surface of the water leaving the turbine site (z_2 = 0). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are very low ($V_1 \cong V_2 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\to\quad h_{\text{turbine, e}}=z_1-h_L$$

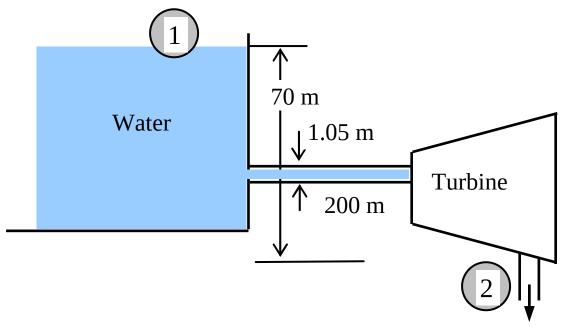
The average velocity, Reynolds number, friction factor, and head loss in the pipe for both cases (pipe diameter being 0.35 m and 1.05 m) are

$$V_1=\frac{\dot V}{A_{c1}}=\frac{\dot V}{\pi D_1^2/4}=\frac{0.8\ \text{m}^3/\text{s}}{\pi(0.35\ \text{m})^2/4}=8.315\ \text{m/s},$$

$$V_2=\frac{\dot V}{A_{c2}}=\frac{\dot V}{\pi D_2^2/4}=\frac{0.8\ \text{m}^3/\text{s}}{\pi(1.05\ \text{m})^2/4}=0.9239\ \text{m/s}$$

$$\text{Re}_1=\frac{\rho V_1 D_1}{\mu}=\frac{(998\ \text{kg/m}^3)(8.315\ \text{m/s})(0.35\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}=2.899\times10^6$$

$$\text{Re}_2=\frac{\rho V_2 D_1}{\mu}=\frac{(998\ \text{kg/m}^3)(0.9239\ \text{m/s})(1.05\ \text{m})}{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}=0.9662\times10^6$$



which are greater than 4000. Therefore, the flow is turbulent for both cases. The relative roughness of the pipe is

$$\varepsilon/D_1=\frac{0.00026\ \text{m}}{0.35\ \text{m}}=7.43\times10^{-4} \quad\text{and}\quad \varepsilon/D_2=\frac{0.00026\ \text{m}}{1.05\ \text{m}}=2.476\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left(\frac{7.43\times10^{-4}}{3.7}+\frac{2.51}{2.899\times10^6\sqrt{f_1}}\right) \quad\text{and}\quad \frac{1}{\sqrt{f_2}}=-2.0\log\left(\frac{2.476\times10^{-4}}{3.7}+\frac{2.51}{0.9662\times10^6\sqrt{f_2}}\right)$$

The friction factors are determined to be f_1 = 0.01842 and f_2 = 0.01520. When the minor losses are negligible, the head losses in the pipes and the head extracted by the turbine are determined to be

$$h_{L1}=f_1\frac{L}{D_1}\frac{V_1^2}{2g}=0.01842\frac{200\ \text{m}}{0.35\ \text{m}}\frac{(8.315\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=37.09\ \text{m},\quad h_{\text{turbine,1}}=z_1-h_{L1}=70-37.09=32.91\ \text{m}$$

$$h_{L2}=f_2\frac{L}{D_2}\frac{V_2^2}{2g}=0.0152\frac{200\ \text{m}}{1.05\ \text{m}}\frac{(0.9239\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}=0.13\ \text{m},\quad h_{\text{turbine,2}}=z_2-h_{L2}=70-0.13=69.87\ \text{m}$$

The available or actual power output is proportional to the turbine head. Therefore, the increase in the power output when the diameter is doubled becomes

$$\text{Increase in power output}=\frac{h_{\text{turbine,2}}-h_{\text{turbine,1}}}{h_{\text{turbine,1}}}=\frac{69.87-32.91}{32.91}=\mathbf{1.12\ \ or\ \ 112\%}$$

Discussion Note that the power generation of the turbine more than doubles when the pipe diameter is tripled at the same flow rate and elevation.

8-129E

Solution The drinking water needs of an office are met by siphoning water through a plastic hose inserted into a large water bottle. The time it takes to fill a glass when the bottle is first opened and when it is empty are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The on/off switch is fully open during filing. **4** The water level in the bottle remains nearly constant during filling. **5** The flow is turbulent (to be verified). **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.360 lbm/ft·h = 6.556×10^{-4} lbm/ft·s. The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$. The total minor loss coefficient is given to be 2.8.

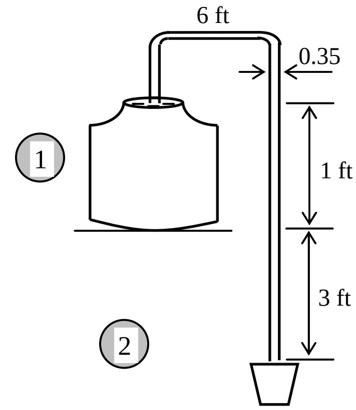
Analysis We take point 1 to be at the free surface of water in the bottle, and point 2 at the exit of the hose., which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and $h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g} \;\rightarrow\; h_L=\left(f\frac{6\text{ ft}}{0.35/12\text{ ft}}+2.8\right)\frac{V_2^2}{2(32.2\text{ ft/s}^2)}$ (1)

since the diameter of the piping system is constant. Then the energy equation becomes

$$z_1=(1)\frac{V_2^2}{2(32.2\text{ ft/s}^2)}+h_L \qquad (2)$$



The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4} \;\rightarrow\; V_2=\frac{\dot V\text{ ft}^3\text{/s}}{\pi(0.35/12\text{ ft})^2/4} \qquad (3)$$

$$\text{Re}=\frac{\rho V_2 D}{\mu} \;\rightarrow\; \text{Re}=\frac{(62.3\text{ lbm/ft}^3)V_2(0.35/12\text{ ft})}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}} \qquad (4)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right) \;\rightarrow\; \frac{1}{\sqrt f}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt f}\right) \qquad (5)$$

Finally, the filling time of the glass is

$$\Delta t=\frac{V_{\text{glass}}}{\dot V}=\frac{0.00835\text{ ft}^3}{\dot V\text{ ft}^3\text{/s}} \qquad (6)$$

These are 6 equations in the 6 unknowns of V_2, $\dot V$, h_L, Re, f, and Δt, and solving them simultaneously using an equation solver such as EES with the appropriate z_1 value gives

Case (*a*): The bottle is full and thus z_1 = 3+1 = 4 ft:

V_2 = 5.185 ft/s, h_L = 3.58 ft, $\dot V$ = 0.00346 ft^3/s, Re = 14,370, f = 0.02811, and Δt = **2.4 s**

Case (*b*): The bottle is almost empty and thus z_1 = 3 ft:

V_2 =4.436 ft/s, h_L = 2.69 ft, $\dot V$ = 0.00296 ft^3/s, Re = 12,290, f = 0.02926, and Δt = **2.8 s**

Note that the flow is turbulent for both cases since Re > 4000.

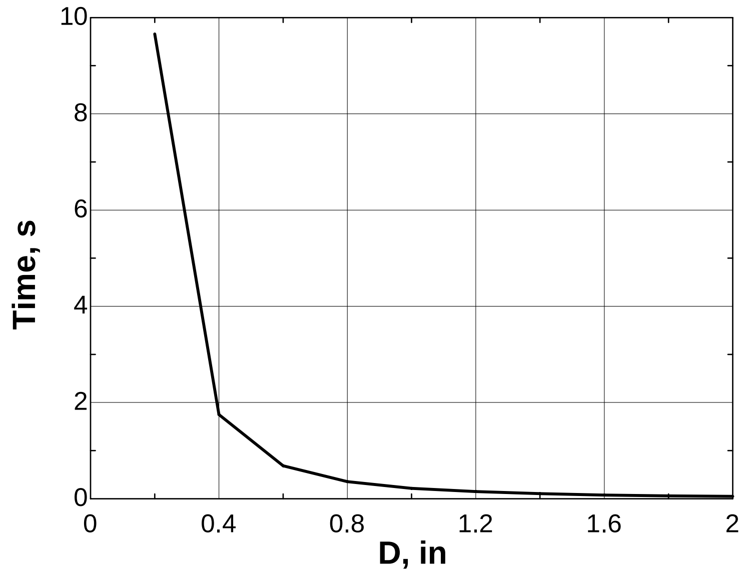
Discussion The filling time of the glass increases as the water level in the bottle drops, as expected.

8-130E

Solution In the previous problem, the effect of the hose diameter on the time required to fill a glass when the bottle is full is to be investigated by varying the pipe diameter from 0.2 to 2 in. in increments of 0.2 in.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
rho=62.3
mu=2.36/3600
nu=mu/rho
g=32.2
z1=4
Volume=0.00835
D=Din/12
Ac=pi*D^2/4
L=6
KL=2.8
eps=0
rf=eps/D
V=Vdot/Ac
"Reynolds number"
Re=V*D/nu
1/sqrt(f)=-2*log10(rf/3.7+2.51/(Re*sqrt(f)))
HL=(f*L/D+KL)*(V^2/(2*g))
z1=V^2/(2*g)+HL
Time=Volume/Vdot
```



D, in	*Time,* s	h_L, ft	Re
0.2	9.66	3.76	6273
0.4	1.75	3.54	17309
0.6	0.68	3.40	29627
0.8	0.36	3.30	42401
1.0	0.22	3.24	55366
1.2	0.15	3.20	68418
1.4	0.11	3.16	81513
1.6	0.08	3.13	94628
1.8	0.06	3.11	107752
2.0	0.05	3.10	120880

Discussion The required time decreases considerably as the tube diameter increases. This is because the irreversible frictional head loss (major loss) in the tube decreases greatly as tube diameter increases. In addition, the minor loss is proportional to V^2. Thus, as tube diameter increases, V decreases, and even the minor losses decrease.

8-131E

Solution The drinking water needs of an office are met by siphoning water through a plastic hose inserted into a large water bottle. The time it takes to fill a glass when the bottle is first opened is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed and the friction factor is constant for the entire pipe. **3** The on/off switch is fully open during filing. **4** The water level in the bottle remains constant during filling. **5** The flow is turbulent (to be verified). **6** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 70°F are ρ = 62.30 lbm/ft^3 and μ = 2.360 lbm/ft·h = 6.556×10^{-4} lbm/ft·s. The plastic pipes are considered to be smooth, and thus their roughness is $\varepsilon = 0$. The total minor loss coefficient is given to be 2.8 during filling.

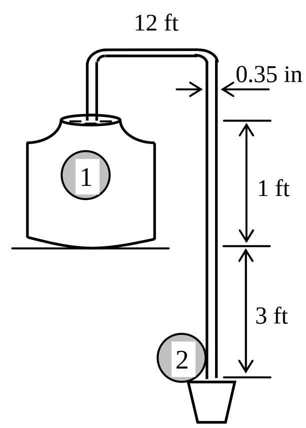
Analysis We take point 1 to be at the free surface of water in the bottle, and point 2 at the exit of the hose, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and $h_L=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g} \;\rightarrow\; h_L=\left(f\frac{12\text{ ft}}{0.35/12\text{ ft}}+2.8\right)\frac{V_2^2}{2(32.2\text{ ft/s}^2)}$ (1)

since the diameter of the piping system is constant. Then the energy equation becomes

$$z_1=(1)\frac{V_2^2}{2(32.2\text{ ft/s}^2)}+h_L \qquad (2)$$



The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4} \;\rightarrow\; V_2=\frac{\dot V\text{ ft}^3\text{/s}}{\pi(0.35/12\text{ ft})^2/4} \qquad (3)$$

$$\text{Re}=\frac{\rho V_2 D}{\mu} \;\rightarrow\; \text{Re}=\frac{(62.3\text{ lbm/ft}^3)V_2(0.35/12\text{ ft})}{1.307\times10^{-3}\text{ lbm/ft}\cdot\text{s}} \qquad (4)$$

The friction factor can be determined from the Colebrook equation,

$$\frac{1}{\sqrt f}=-2.0\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{2.51}{\text{Re}\sqrt f}\right) \;\rightarrow\; \frac{1}{\sqrt f}=-2.0\log\left(0+\frac{2.51}{\text{Re}\sqrt f}\right) \qquad (5)$$

Finally, the filling time of the glass is

$$\Delta t=\frac{V_{\text{glass}}}{\dot V}=\frac{0.00835\text{ ft}^3}{\dot V\text{ ft}^3\text{/s}} \qquad (6)$$

These are 6 equations in the 6 unknowns of V_2, $\dot V$, h_L, Re, f, and Δt, and solving them simultaneously using an equation solver such as EES with the appropriate z_1 value gives

Case (*a*): The bottle is full and thus z_1 = 3+1 = 4 ft:

V_2 = 3.99 ft/s, h_L = 3.75 ft, $\dot V$ = 0.002667 ft^3/s, Re = 11,060, f = 0.03007, and Δt = **3.1 s**

Case (*b*): The bottle is almost empty and thus z_1 = 3 ft:

V_2 = 3.40 ft/s, h_L = 2.82 ft, $\dot V$ = 0.002272 ft^3/s, Re = 9426, f = 0.03137, and Δt = **3.7 s**

Note that the flow is turbulent for both cases since Re > 4000.

Discussion The filling times in Prob. 8-129E were 2.4 s and 2.8 s, respectively. Therefore, doubling the tube length increases the filling time by 0.7 s when the bottle is full, and by 0.9 s when it is empty.

8-132

Solution A water pipe has an abrupt expansion from diameter D_1 to D_2. It is to be shown that the loss coefficient is $K_L = (1 - D_1^2 / D_2^2)^2$, and K_L and P_2 are to be calculated.

Assumptions **1** The flow is steady and incompressible. **2** The pressure is uniform at the cross-section where expansion occurs, and is equal to the upstream pressure P_1. **3** The flow section is horizontal (or the elevation difference across the expansion section is negligible). **4** The flow is turbulent, and the effects of kinetic energy and momentum-flux correction factors are negligible, $\beta \approx 1$ and $\alpha \approx 1$.

Properties We take the density of water to be $\rho = 1000$ kg/m^3.

Analysis We designate the cross-section where expansion occurs by x. We choose cross-section 1 in the smaller diameter pipe shortly before x, and section 2 in the larger diameter pipe shortly after x. We take the region occupied by the fluid between cross-sections 1 and 2 as the control volume, with an inlet at 1 and exit at 2. The velocity, pressure, and cross-sectional area are V_1, P_1, and A_1 at cross-section 1, and V_2, P_2, and A_2 at cross-section 2. We assume the pressure along the cross-section x to be P_1 so that $P_x = P_1$. Then the continuity, momentum, and energy equations applied to the control volume become

(1) *Continuity*: $\dot m_1 = \dot m_2 \rightarrow \rho V_1 A_1 = \rho V_2 A_2 \rightarrow V_2 = \dfrac{A_1}{A_2} V_1$ (1)

(2) *Momentum*: $\sum \vec F = \sum_{\text{out}} \beta \dot m V - \sum_{\text{in}} \beta \dot m V \rightarrow P_1 A_1 + P_1 (A_x - A_1) - P_2 A_2 = \dot m (V_2 - V_1)$

But $P_1 A_1 + P_1 (A_x - A_1) = P_1 A_x = P_1 A_2$

$$\dot m (V_2 - V_1) = \rho A_2 V_2 (V_2 - V_1) = \rho A_2 \frac{A_1}{A_2} V_1 \left(\frac{A_1}{A_2} V_1 - V_1 \right) = \rho A_2 \frac{A_1}{A_2} \left(\frac{A_1}{A_2} - 1 \right) V_1^2$$

Therefore, $P_1 A_2 - P_2 A_2 = \rho A_2 \dfrac{A_1}{A_2} \left(\dfrac{A_1}{A_2} - 1 \right) V_1^2 \rightarrow \dfrac{P_1 - P_2}{\rho} = \dfrac{A_1}{A_2} \left(\dfrac{A_1}{A_2} - 1 \right) V_1^2$ (2)

(3) *Energy*: $\dfrac{P_1}{\rho g} + \alpha_1 \dfrac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \dfrac{P_2}{\rho g} + \alpha_2 \dfrac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \rightarrow h_L = \dfrac{P_1 - P_2}{\rho g} + \dfrac{V_1^2 - V_2^2}{2g}$ (3)

Substituting Eqs. (1) and (2) and $h_L = K_L \dfrac{V_1^2}{2g}$ into Eq. (3) gives

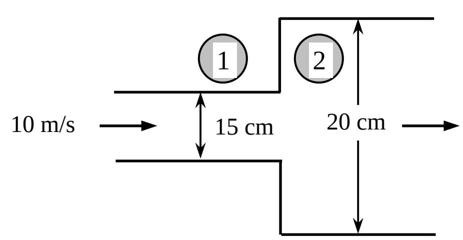
$$K_L \frac{V_1^2}{2g} = \frac{A_1}{A_2} \left(\frac{A_1}{A_2} - 1 \right) \frac{V_1^2}{g} + \frac{V_1^2 - (A_1^2 / A_2^2) V_1^2}{2g} \rightarrow K_L = \frac{2A_1}{A_2} \left(\frac{A_1}{A_2} - 1 \right) + \left(1 - \frac{A_1^2}{A_2^2} \right)$$

Simplifying and substituting $A = \pi D^2/4$ gives the desired relation and its value,

$$K_L = \left(1 - \frac{A_1}{A_1} \right)^2 = \left(1 - \frac{\pi D_1^2 / 4}{\pi D_2^2 / 4} \right)^2 = \left(1 - \frac{D_1^2}{D_2^2} \right)^2 = \left(1 - \frac{(0.15\text{ m})^2}{(0.20\text{ m})^2} \right)^2 = 0.1914$$

Also, $h_L = K_L \dfrac{V_1^2}{2g} = (0.1914) \dfrac{(10\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.9756\text{ m}$

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{D_1^2}{D_2^2} V_1 = \frac{(0.15\text{ m})^2}{(0.20\text{ m})^2} (10\text{ m/s}) = 5.625\text{ m/s}$$



Solving for P_2 from Eq. (3) and substituting,

$$P_2 = P_1 + \rho \{ (V_1^2 - V_2^2)/2 - g h_L \}$$

$$= (120\text{ kPa}) + (1000\text{ kg/m}^3) \left\{ \frac{(10\text{ m/s})^2 - (5.625\text{ m/s})^2}{2} - (9.81\text{ m/s}^2)(0.9756\text{ m}) \right\} \left(\frac{1\text{ kPa} \cdot \text{m}^2}{1000\text{ kg} \cdot \text{m/s}^2} \right) = \mathbf{145\text{ kPa}}$$

Note that the pressure increases by 25 kPa after the expansion due to the conversion of dynamic pressure to static pressure when the velocity is decreased. Also, $K_L \cong 1$ (actually, $K_L = \alpha$) when $D_2 >> D_1$ (discharging into a reservoir).

Discussion At a discharge into a large reservoir, *all* the kinetic energy is wasted as heat.

8-133

Solution A swimming pool is initially filled with water. A pipe with a well-rounded entrance at the bottom drains the pool to the atmosphere. The initial rate of discharge from the pool and the time required to empty the pool completely are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The entrance effects are negligible, and thus the flow is fully developed. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes during flow). **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

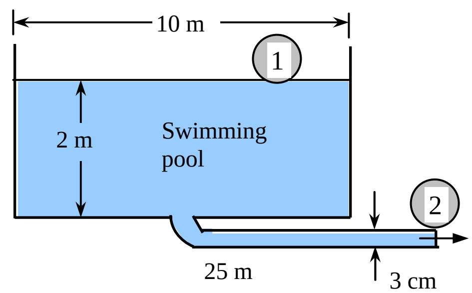
Properties The density and dynamic viscosity of water at 20°C are ρ = 998 kg/m^3 and μ = 1.002×10^{-3} kg/m·s. The friction factor of the pipe is given to be 0.022. Plastic pipes are considered to be smooth, and their surface roughness is ε = 0.

Analysis We take point 1 at the free surface of the pool, and point 2 and the reference level at the exit of the pipe (z_2 = 0), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$



since the minor losses are negligible. Substituting and solving for V_2 gives

$$z_1=\alpha_2\frac{V_2^2}{2g}+\left(f\frac{L}{D}\right)\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{\alpha_2+fL/D}}$$

Noting that α_2 = 1 and initially z_1 = 2 m, the initial velocity and flow rate are determined to be

$$V_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D}}=\sqrt{\frac{2(9.81\text{ m/s}^2)(2\text{ m})}{1+0.022(25\text{ m})/(0.03\text{ m})}}=1.425\text{ m/s}$$

$$\dot{V}_{initial}=V_{2,i}A_c=V_{2,i}(\pi D^2/4)=(1.425\text{ m/s})[\pi(0.03\text{ m})^2/4]=1.01\times10^{-3}\text{ m}^3\text{/s}=1.01\text{ L/s}$$

The average discharge velocity at any given time, in general, can be expressed as

$$V_2=\sqrt{\frac{2gz}{1+fL/D}}$$

where z is the water height relative to the center of the orifice at that time.

We denote the diameter of the pipe by D, and the diameter of the pool by D_o. The flow rate of water from the pool can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V}=A_cV_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}$$

Then the amount of water that flows through the pipe during a differential time interval dt is

$$d\mathcal{V}=\dot{V}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$d\mathcal{V}=A_{c,\tan k}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D}}dt=-\frac{\pi D_0^2}{4}dz \;\rightarrow\; dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2gz}}dz=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f}dt=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}\int_{z=z_1}^{0}z^{-1/2}dz \;\rightarrow\; t_f=-\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0}=\frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f=\frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D)}{g}}=\frac{(10\text{ m})^2}{(0.03\text{ m})^2}\sqrt{\frac{2(2\text{ m})[1+(0.022)(25\text{ m})/(0.03\text{ m})]}{9.81\text{ m/s}^2}}=312{,}000\text{ s}=\mathbf{86.7\ h}$$

Checking: For plastic pipes, the surface roughness and thus the roughness factor is zero. The Reynolds number at the beginning of draining process is

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(998\text{ kg/m}^3)(1.425\text{ m/s})(0.03\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}}=42{,}580$$

which is greater than 4000. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \;\rightarrow\; \frac{1}{\sqrt{f}}=-2.0\log\left(0+\frac{2.51}{42{,}570\sqrt{f}}\right)$$

It gives $f = 0.022$. Therefore, the given value of 0.022 is accurate.

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only about 18 h. Therefore, the pipe in this case increases the draining time by about a factor of 5.

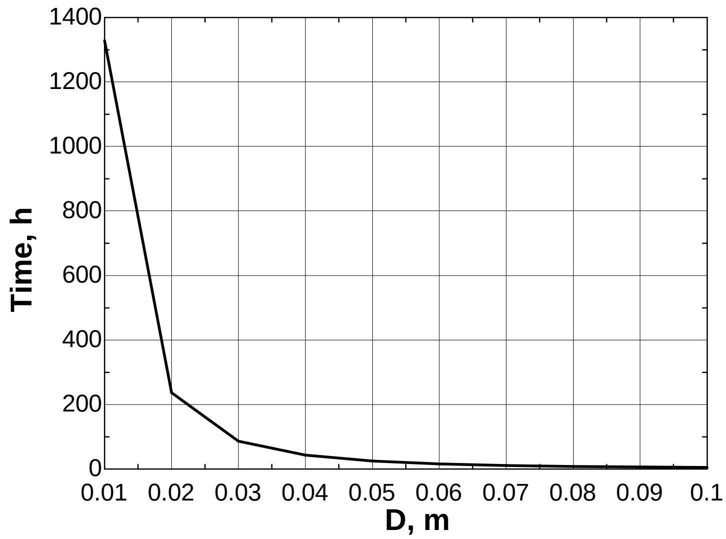
8-134

Solution In the previous problem, the effect of the discharge pipe diameter on the time required to empty the pool completely is to be investigated by varying the pipe diameter from 1 cm to 10 cm in increments of 1 cm.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
rho=998
mu=0.001002
g=9.81
Dtank= 10
Ac=pi*D^2/4
L=25
f=0.022
z1=2
V=(2*g*z1/(1+f*L/D))^0.5
Vdot=V*Ac
Time=(Dtank/D)^2*(2*z1*(1+f*L/D)/g)^0.5/3600
```

D, m	*Time,* h	V_{initial}, m/s	Re
0.01	1327.4	0.84	8337
0.02	236.7	1.17	23374
0.03	86.7	1.42	42569
0.04	42.6	1.63	64982
0.05	24.6	1.81	90055
0.06	15.7	1.96	117406
0.07	10.8	2.10	146750
0.08	7.8	2.23	177866
0.09	5.8	2.35	210572
0.10	4.5	2.46	244721



Discussion The required drain time decreases quite rapidly as pipe diameter is increased.

8-135

Solution A swimming pool is initially filled with water. A pipe with a sharp-edged entrance at the bottom drains the pool to the atmosphere. The initial rate of discharge from the pool and the time required to empty the pool completely are to be determined.

Assumptions **1** The flow is uniform and incompressible. **2** The draining pipe is horizontal. **3** The flow is turbulent so that the tabulated value of the loss coefficient can be used. **4** The friction factor remains constant (in reality, it changes since the flow velocity and thus the Reynolds number changes during flow). **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s. The loss coefficient for the sharp-edged entrance is $K_L = 0.5$. Plastic pipes are considered to be smooth, and their surface roughness is $\varepsilon = 0$.

Analysis We take point 1 at the free surface of the pool, and point 2 and the reference level at the exit of the pipe (z_2 = 0), and take the positive direction of z to be upwards. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the energy equation for a control volume between these two points (in terms of heads) simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $\alpha_2 = 1$ and

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}=\left(f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g}$$

since the diameter of the piping system is constant. Substituting and solving for V_2 gives

$$z_1=\frac{V_2^2}{2g}+\left(f\frac{L}{D}+K_L\right)\frac{V_2^2}{2g} \quad\rightarrow\quad V_2=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}$$

Noting that initially $z_1 = 2$ m, the initial velocity and flow rate are determined to be

$$V_{2,i}=\sqrt{\frac{2gz_1}{1+fL/D+K_L}}=\sqrt{\frac{2(9.81\text{ m/s}^2)(2\text{ m})}{1+0.022(25\text{ m})/(0.03\text{ m})+0.5}}=1.407\text{ m/s}$$

$$\dot{V}_{initial}=V_{2,i}A_c=V_{2,i}(\pi D^2/4)=(1.407\text{ m/s})[\pi(0.03\text{ m})^2/4]=9.94\times10^{-4}\text{ m}^3/\text{s}=0.994\text{ L/s}$$

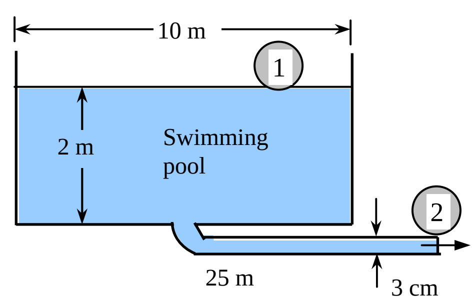
The average discharge velocity at any given time, in general, can be expressed as

$$V_2=\sqrt{\frac{2gz}{1+fL/D+K_L}}$$

where z is the water height relative to the center of the orifice at that time.

We denote the diameter of the pipe by D, and the diameter of the pool by D_o. The flow rate of water from the pool can be obtained by multiplying the discharge velocity by the pipe cross-sectional area,

$$\dot{V}=A_cV_2=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}$$



Then the amount of water that flows through the pipe during a differential time interval dt is

$$d\mathcal{V}=\dot{\mathcal{V}}dt=\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt \qquad (1)$$

which, from conservation of mass, must be equal to the decrease in the volume of water in the pool,

$$d\mathcal{V}=A_{c,\tan k}(-dz)=-\frac{\pi D_0^2}{4}dz \qquad (2)$$

where dz is the change in the water level in the pool during dt. (Note that dz is a negative quantity since the positive direction of z is upwards. Therefore, we used $-dz$ to get a positive quantity for the amount of water discharged). Setting Eqs. (1) and (2) equal to each other and rearranging,

$$\frac{\pi D^2}{4}\sqrt{\frac{2gz}{1+fL/D+K_L}}dt = -\frac{\pi D_0^2}{4}dz \rightarrow dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2gz}}dz = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z^{-\frac{1}{2}}dz$$

The last relation can be integrated easily since the variables are separated. Letting t_f be the discharge time and integrating it from $t = 0$ when $z = z_1$ to $t = t_f$ when $z = 0$ (completely drained pool) gives

$$\int_{t=0}^{t_f} dt = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\int_{z=z_1}^{0} z^{-1/2}dz \rightarrow t_f = -\frac{D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}\left.\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right|_{z_1}^{0} = \frac{2D_0^2}{D^2}\sqrt{\frac{1+fL/D+K_L}{2g}}z_1^{\frac{1}{2}}$$

Simplifying and substituting the values given, the draining time is determined to be

$$t_f = \frac{D_0^2}{D^2}\sqrt{\frac{2z_1(1+fL/D+K_L)}{g}} = \frac{(10\text{ m})^2}{(0.03\text{ m})^2}\sqrt{\frac{2(2\text{ m})[1+(0.022)(25\text{ m})/(0.03\text{ m})+0.5]}{9.81\text{ m/s}^2}} = 316{,}000\text{ s} = \mathbf{87.8\ h}$$

This is a change of (87.8-86.7)/86.7 = 0.013 or 1.3%. Therefore, the minor loss in this case is **truly minor**.

Checking: For plastic pipes, the surface roughness and thus the roughness factor is zero. The Reynolds number at the beginning of draining process is

$$\text{Re} = \frac{\rho V_2 D}{\mu} = \frac{(998\text{ kg/m}^3)(1.407\text{ m/s})(0.03\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}} = 42{,}030$$

which is greater than 4000. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0+\frac{2.51}{42{,}030\sqrt{f}}\right)$$

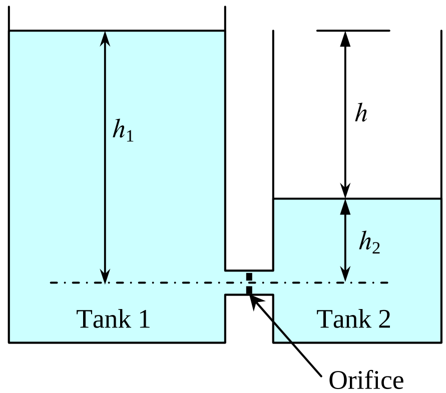
It gives $f = 0.022$. Therefore, the given value of 0.022 is accurate.

Discussion It can be shown by setting $L = 0$ that the draining time without the pipe is only about 24 h. Therefore, the pipe in this case increases the draining time more than 3 folds.

8-136

Solution A system that concsists of two interconnected cylindrical tanks is used to determine the discharge coefficient of a short 5-mm diameter orifice. For given initial fluid heights and discharge time, the discharge coefficient of the orifice is to be determined.

Assumptions **1** The fluid is incompressible. **2** The entire systems, including the connecting flow section, is horizontal. **3** The discharge coefficient remains constant (in reality, it may change since the flow velocity and thus the Reynolds number changes during flow). **4** Losses other than the ones associated with flow through the orifice are negligible. **5** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.



Analysis We take point 1 at the free surface of water in Tank 1, and point 0 at the exit of the orifice. We take the centerline of the orifice as the reference level ($z_1 = h_1$ and $z_0 = 0$). Noting that the fluid at point 1is open to the atmosphere (and thus $P_1 = P_{atm}$ and $P_0 = P_{atm} + \rho g h_2$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the *Bernoulli equation* between these two points gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_0}{\rho g} + \frac{V_0^2}{2g} + z_0 \quad \rightarrow \quad \frac{P_{atm}}{\rho g} + h_1 = \frac{P_{atm} + \rho g h_2}{\rho g} + \frac{V_0^2}{2g} \quad \rightarrow \quad V_0 = \sqrt{2g(h_1 - h_2)} = \sqrt{2gh}$$

where $h = h_1 - h_2$ is the vertical distance between the water levels in the two tanks at any time t. Note that h_1, h_2, h, and V_0 are all variable (h_1 decreases while h_2 and h increase during discharge.

Noting that the fluid is a liquid (ρ = constant) and keeping the conservation of mass in mind and the definition of the discharge coefficient C_d, the flow rate through the orifice can be expressed as

$$\dot{V} = C_d V_o A_o = -A_1 \frac{dh_1}{dt} = A_2 \frac{dh_2}{dt} \quad \longrightarrow \quad dh_2 = -\frac{A_1}{A_2} dh_1$$

Also, $\quad h = h_1 - h_2 \longrightarrow dh = dh_1 - dh_2 \longrightarrow dh_1 = dh_2 + dh \quad$ (Note that $dh < 0$, $dh_1 < 0$, and $dh_2 > 0$)

Combining the two equations above, $\quad dh_1 = \dfrac{dh}{1 + A_1/A_2}$

Then, $\quad \dot{V} = C_d V_o A_o = -A_1 \dfrac{dh_1}{dt} \longrightarrow C_d A_o \sqrt{2gh} = -A_1 \dfrac{1}{1 + A_1/A_2} \dfrac{dh}{dt}$

which can be rearranged as $\quad -dt = \dfrac{A_1 A_2}{A_1 + A_2} \dfrac{1}{C_d A_o \sqrt{2g}} \dfrac{dh}{\sqrt{h}}$

Integrating $\quad -\displaystyle\int_0^t dt = \frac{A_1 A_2}{A_1 + A_2} \frac{1}{C_d A_o \sqrt{2g}} \int_{h_1}^{h} \frac{dh}{\sqrt{h}}$

Performing the integration $\quad t = -\dfrac{A_1 A_2}{A_1 + A_2} \dfrac{2}{C_d A_o \sqrt{2g}} \left[\sqrt{h} - \sqrt{h_1}\right]$

Solving for C_d $\quad C_d = \dfrac{2(\sqrt{h_1} - \sqrt{h})}{(A_0 / A_2 + A_0 / A_1) t \sqrt{2g}}$

Fluid flow stops when the liquid levels in the two tanks become equal (and thus $h = 0$). Substituign the given values, the discharge coefficient is determined to be

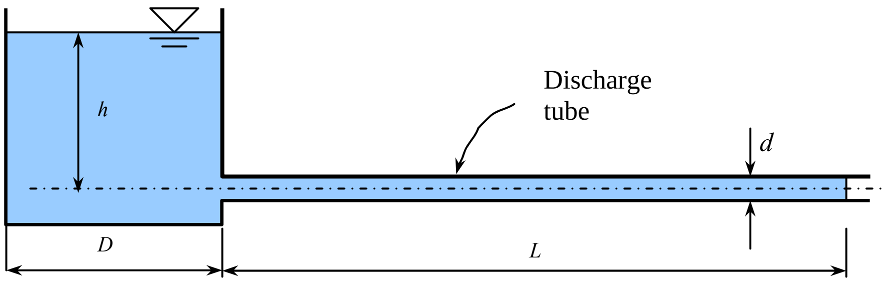
$$\frac{A_0}{A_2} + \frac{A_0}{A_1} = \left(\frac{D_0}{D_2}\right)^2 + \left(\frac{D_0}{D_1}\right)^2 = \left(\frac{0.5 \text{ cm}}{30 \text{ cm}}\right)^2 + \left(\frac{0.5 \text{ cm}}{12 \text{ cm}}\right)^2 = 0.002014 \,,$$

$$C_d = \frac{2\sqrt{0.5 \text{ m}}}{(0.002014)(170 \text{ s})\sqrt{2 \times 9.81 \text{ m/s}^2}} = \mathbf{0.933}$$

Discussion We could add the minor losses at the pipe inlet and outlet without much extra effort.

8-137

Solution A highly viscous liquid discharges from a large container through a small diameter tube in laminar flow. A relation is to be obtained for the variation of fluid depth in the tank with time.



Assumptions **1** The fluid is incompressible. **2** The discharge tube is horizontal, and the flow is laminar. **3** Entrance effects and the velocity heads are negligible.

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the exit of the pipe. We take the centerline of the pipe as the reference level ($z_1 = h$ and $z_2 = 0$). Noting that the fluid at both points 1 and 2 are open to the atmosphere (and thus $P_1 = P_2 = P_{\text{atm}}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$) and the velocity heads are negligible, the energy equation for a control volume between these two points gives

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L \quad\to\quad \frac{P_{atm}}{\rho g}+h=\frac{P_{atm}}{\rho g}+h_L \quad\to\quad h_L=h \qquad (1)$$

where h is the liquid height in the tank at any time t. The total head loss through the pipe consists of major losses in the pipe since the minor losses are negligible. Also, the entrance effects are negligible and thus the friction factor for the entire tube is constant at the fully developed value. Noting that $f = \text{Re}/64$ for fully developed laminar flow in a circular pipe of diameter d, the head loss can be expressed as

$$h_L = f\frac{L}{d}\frac{V^2}{2g}=\frac{64}{\text{Re}}\frac{L}{d}\frac{V^2}{2g}=\frac{64}{Vd/\nu}\frac{L}{d}\frac{V^2}{2g}=\frac{64\nu L}{d^2}\frac{V}{2g} \qquad (2)$$

The average velocity can be expressed in terms of the flow rate as $V=\dfrac{\dot V}{A_c}=\dfrac{\dot V}{\pi d^2/4}$. Substituting into (2),

$$h_L=\frac{64\nu L}{d^2}\frac{1}{2g}\left(\frac{\dot V}{\pi d^2/4}\right)=\frac{64\nu L}{d^2}\frac{4\dot V}{2g\pi d^2}=\frac{128\nu L\dot V}{g\pi d^4} \qquad (3)$$

Combining Eqs. (1) and (3): $$h=\frac{128\nu L\dot V}{g\pi d^4} \qquad (4)$$

Noting that the liquid height h in the tank decreases during flow, the flow rate can also be expressed in terms of the rate of change of liquid height in the tank as

$$\dot V=-A_{\text{tank}}\frac{dh}{dt}=-\frac{\pi D^2}{4}\frac{dh}{dt} \qquad (5)$$

Substituting Eq. (5) into (4): $$h=-\frac{128\nu L}{g\pi d^4}\frac{\pi D^2}{4}\frac{dh}{dt}=-\frac{32\nu LD^2}{gd^4}\frac{dh}{dt} \qquad (6)$$

To separate variables, it can be rearranged as $$dt=-\frac{32\nu LD^2}{gd^4}\frac{dh}{h}$$

Integrating from $t = 0$ (at which $h = H$) to $t = t$ (at which $h = h$) gives

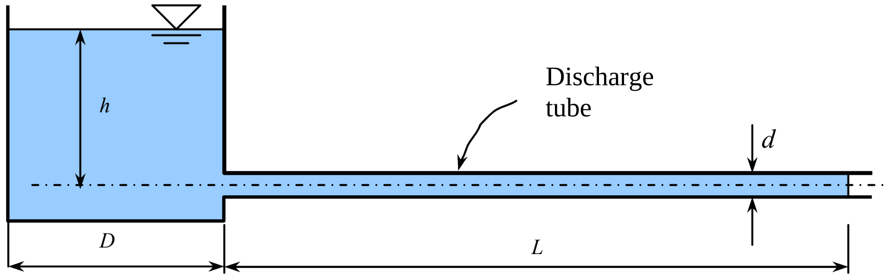
$$\boxed{t=\frac{32\nu LD^2}{gd^4}\ln(H/h)}$$

which is the desired relation for the variation of fluid depth h in the tank with time t.

Discussion If the entrance effects and the outlet kinetic energy were included in the analysis, the time would be slower.

8-138

Solution Using the setup described in the previous problem, the viscosity of an oil is to be determined for a given set of data.



Assumptions **1** The oil is incompressible. **2** The discharge tube is horizontal, and the flow is laminar. **3** Entrance effects and the inlet and the exit velocity heads are negligible.

Analysis The variation of fluid depth h in the tank with time t was determined in the previous problem to be

$$t = \frac{32\nu LD^2}{gd^4}\ln(H/h)$$

Solving for ν and substituting the given values, the kinematic viscosity of the oil is determined to be

$$\nu = \frac{gd^4}{32LD^2\ln(H/h)}t = \frac{(9.81\,\text{m/s}^2)(0.006\,\text{m})^4}{32(0.65\,\text{m})(0.63\,\text{m})^2\ln(0.4/0.36)}(2842\,\text{s}) = \mathbf{4.15\times10^{-5}\ m^2/s}$$

Discussion Note that the entrance effects are not considered, and the velocity heads are disregarded. Also, the value of the viscosity strongly depends on temperature, and thus the oil temperature should be maintained constant during the test.

Design and Essay Problems

8-139 to 8-142

Solution Students' essays and designs should be unique and will differ from each other.