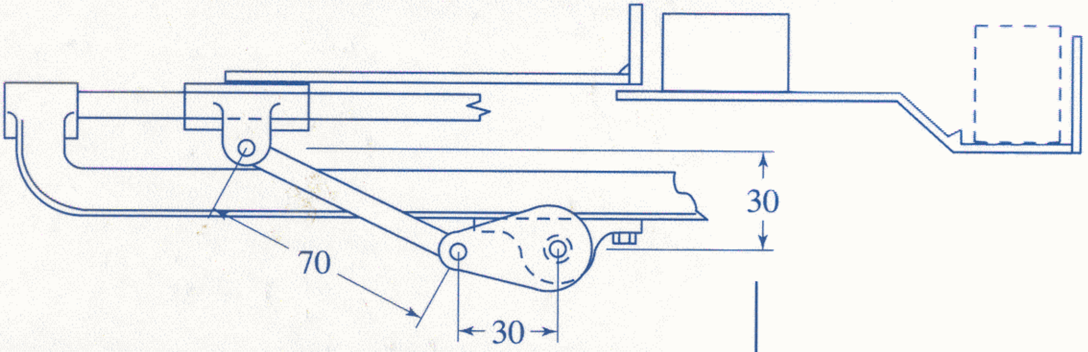
MACHINES AND MECHANISMS



APPLIED KINEMATIC ANALYSIS

DAVID H. MYSZKA

MACHINES and MECHANISMS

Applied Kinematic Analysis

David H. Myszka
University of Dayton

PRENTICE HALL
Upper Saddle River, New Jersey Columbus, Ohio

Library of Congress Cataloging-in-Publication Data
Myszka, David.
Machines and mechanisms : applied kinematics analysis / David Myszka.
p. cm.
ISBN 0-13-597915-3
1. Machinery, Kinematics of. 2. Mechanical movements. I. Title.
TJ175.M97 1999 98-20255
621.8′11—dc21 CIP

Editor: Stephen Helba
Production Coordination: Clarinda Publication Services
Text Designer: Clarinda Publication Services
Cover Designer: Dan Eckel
Design Coordinator: Karrie M. Converse
Production Manager: Deidra Schwartz
Illustrations: Academy Artworks
Marketing Manager: Frank Mortimer, Jr.

This book was set in Sabon by The Clarinda Company, and was printed and bound by R. R. Donnelley & Sons Company. The cover was printed by Phoenix Color Corp.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN: 0-13-597915-3

Reprinted with corrections October, 1999

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall of Canada, Inc., *Toronto*
Prentice-Hall Hispanoamericana, S. A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Prentice-Hall Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

Preface

The objective of this book is to provide the techniques necessary to study the motion of machines. A focus is placed on the application of kinematic theories to real world machinery. It is intended to bridge the gap between a theoretical study of kinematics and the application to practical mechanisms. Students completing a course of study using this book should be able to determine the motion characteristics of a machine. Further, such analysis could be performed on design concepts to optimize the motion of a machine arrangement.

It is expected that students using this book will have a good background in technical drawing, college algebra and trigonometry. Also, knowledge of vectors and mechanics, and use of computer application software, such as spreadsheets, will be useful. However, these concepts have been introduced in the book.

This approach of applying theoretical developments to practical problems is consistent with the philosophy of engineering technology programs. This book is primarily oriented toward mechanical and manufacturing related engineering technology programs. It can be used in either associate or baccalaureate degree programs.

Following are some distinctive features of this book:

1. Pictures and sketches of machinery that contains mechanisms are incorporated throughout the text.
2. The focus is on the application of kinematic theories to practical mechanisms.
3. Both graphical techniques and analytical methods are used in the analysis of mechanisms.
4. A student copy of *Working Model,* a commercially available dynamic software package is obtainable for use with this book. Tutorials and problems that utilize this software are integrated into the book.
5. Suggestions for implementing the graphical techniques on computer aided design (CAD) systems are included.
6. Every chapter concludes with a few case studies. These cases illustrate a mechanism that is used on industrial equipment and challenges the student to discuss the rationale behind the design and suggest improvements.
7. Basic mechanism force analysis methods are introduced.

8. Every major concept is followed by an example problem to illustrate the application of the concept.
9. Every example problem begins with an introduction of a real machine that relies on the mechanism being analyzed.
10. Numerous end-of-chapter problems are consistent with the application approach of the text. Every concept introduced in the chapter has at least one associated practice problem. Most of these problems include the machine that relies on the mechanism being analyzed.
11. Where applicable, end-of-chapter problems are provided that utilize the analytical methods, and are best suited for programmable devices (calculators, spreadsheets, math software, etc.)

I developed this textbook after teaching mechanisms for several semesters and noticing that students do not always see the practical applications of the material. To this end, I have grown quite fond of the case study problems and begin each class with one. The students refer to this as the "mechanism of the day." I found this to be an excellent opportunity to focus attention on operating machinery. Additionally, it promotes dialog and creates a learning community in the classroom.

Finally, the purpose of any textbook is to guide the students through a learning experience in an effective manner. I sincerely hope that this book will fulfill this intention. I welcome all suggestions and comments and can be reached at dmyszka@engr.udayton.edu.

ACKNOWLEDGEMENTS

I thank the reviewers of this text for their comments and suggestions: Walter E. Benulis, Professor Emeritus, Wentworth Institute of Technology; Rich Englund, Penn State at Erie; Bill Jaeger, Lakeshore Technical College; Masoud Olia, Wentworth Institute of Technology; and Gerald Post, Harrisburg Area Community College.

Dave Myszka

Contents

1 Introduction to Mechanisms and Kinematics

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Explain the need for kinematic analysis of mechanisms.
2. Define the basic components that comprise a mechanism.
3. Draw a kinematic diagram from a view of a complex machine.
4. Compute the number of degrees of freedom of a mechanism.
5. Identify a four-bar mechanism and classify it according to its possible motion.
6. Identify a slider-crank mechanism.

1.1 INTRODUCTION

The analysis of mechanisms and machines aims at understanding the relationships between the motions of the machine parts and the forces that produce the motions. The initial problem in the design or analysis of a mechanical system is determining the system's motion. The study of motion is termed *kinematics*. Kinematic analysis is the primary focus of this text.

As a secondary problem, a power source must be selected that will provide sufficient forces to operate the machine. The study of the dynamic forces is termed *kinetics*. This problem is introduced in Chapter 9 of this text. However, the concepts relative to the transmission of forces are posed throughout the text.

1.2 MECHANISMS

Machines are devices used to accomplish work, such as a chain saw to cut wood. A *mechanism* is the heart of a machine. It is the mechanical portion of a machine that has the function of transferring motion and forces from a power source to an output. For the chain saw, the mechanism takes power from a small engine and delivers it to the cutting edge of the chain.

Figure 1.1 illustrates an adjustable height platform that is driven by hydraulic cylinders. Although the entire device could be called a machine, the parts that take the

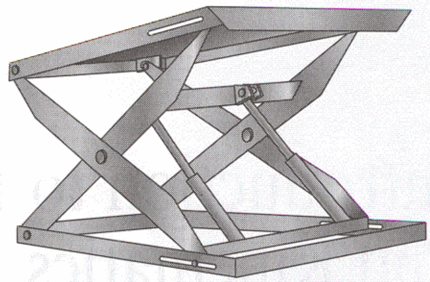


Figure 1.1 Adjustable height platform.

power from the cylinders and drive the raising and lowering of the platform are mechanisms.

A mechanism can be considered rigid parts that are arranged and connected so that they produce the desired motion of the machine. The purpose of the mechanism in Figure 1.1 is to lift the platform and any objects that are placed upon it. Mechanism analysis ensures that the device will exhibit motion that will accomplish the desired purpose of a machine.

1.3 KINEMATICS

Kinematics deals with the way things move. Formally defined, it is the study of the geometry of motion. Kinematic analysis involves determination of position, displacement, rotation, speed, velocity, and acceleration.

To illustrate the importance of such analysis, refer to the device in Figure 1.1. Kinematic analysis provides insight to significant design questions, such as:

What is the significance of the length of the legs that support the platform?
Is it necessary for the support legs to cross and be connected at their midspan, or is it better to arrange them so that they cross closer to the platform?
How far must the cylinder extend to raise the platform 8 inches?

As a second step, dynamic force analysis of the platform could provide insight into another set of pertinent design questions:

What capacity (maximum force) is required of the hydraulic cylinder?
Is the platform free of any tenancy to tip over?
What cross-sectional size of the legs and material is required of the support legs so they don't fail?

A majority of common mechanisms exhibit motion such that the parts move in parallel planes. For the device in Figure 1.1, two identical mechanisms are used on opposite sides of the platform for stability. However, the motion of these mechanisms is strictly in the vertical plane. Therefore, this is called a *planar mechanism*. Planar mechanisms are the primary focus of this book.

1.4 MECHANISM TERMINOLOGY

As stated, mechanisms consist of connected parts. A *linkage* is a mechanism where all parts are connected to form a closed chain. One part is designated the *frame* because it serves as the frame of reference for the motion of all other parts. The frame is typically a part that exhibits no motion. For the mechanism shown in Figure 1.1, the base that sits on the ground would be considered the frame.

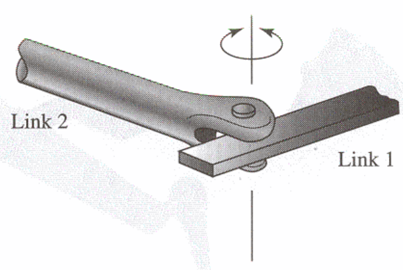
Links are the individual parts of the mechanism. They are considered rigid bodies and are connected with other links to transmit motion and forces. A true rigid body does not change shape during motion. Although a true rigid body does not exist, mechanism components are designed to minimally deform and are considered rigid.

Elastic parts, such as springs, are not rigid and, therefore, are not considered links. They have no effect on the kinematics of a mechanism, and are usually ignored during kinematic analysis. They do supply forces, and must be included during the dynamic force portion of analysis.

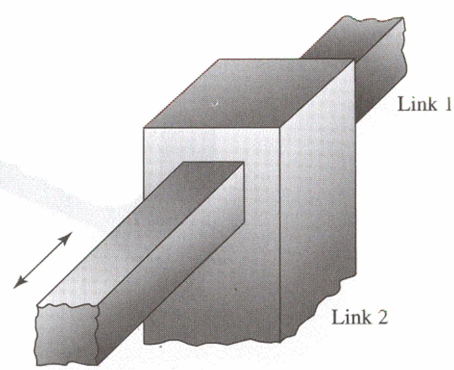
A *joint* is a moveable connection between links and allows relative motion between the links. The two *primary joints* are the revolute and sliding joint. The revolute joint is also called a pin or hinge joint. It allows pure rotation between the two links that it connects. The sliding joint is also called a piston or prism joint. It allows linear sliding between the links that it connects. Figure 1.2 illustrates these two primary joints.

A cam joint is shown in Figure 1.3A. It allows for both rotation and sliding of the two links that it connects. Because of the complex motion allowed, the cam connection is called a *higher order joint*. A gear connection also allows rotation and sliding of the teeth as they mesh. This arrangement is shown in Figure 1.3B. The gear connection is also a higher order joint.

A *simple link* is a rigid body that contains only two joints, which connects it to other links. Figure 1.4A illustrates a simple link. A *crank* is a simple link that is able to complete a full rotation about a fixed center. A *rocker* is a simple link that oscillates through an angle, reversing its direction at certain intervals.

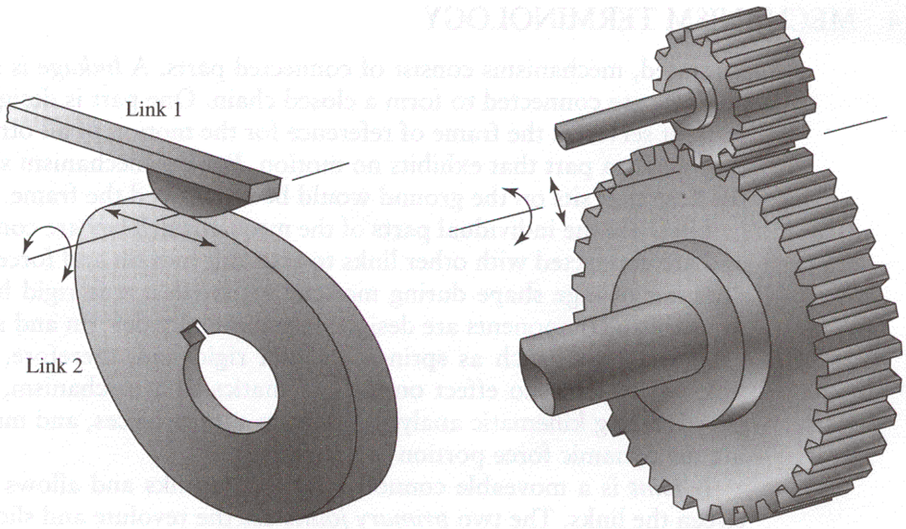


(a) Pin



(b) Sliding

Figure 1.2 Primary joints: (A) Pin and (B) Sliding.

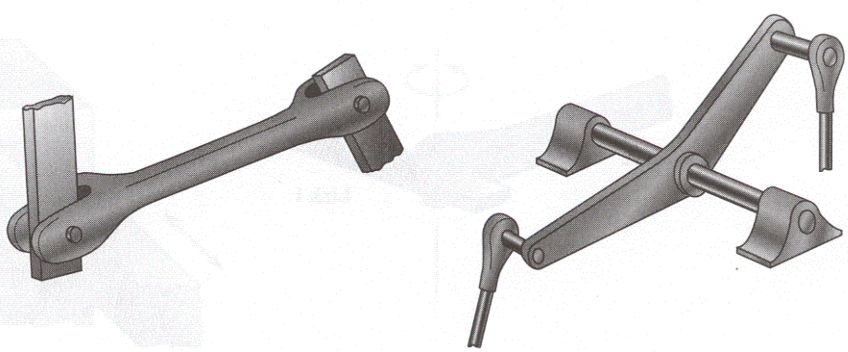


(a) Cam joint (b) Gear joint

Figure 1.3 Higher order joints: (A) Cam joint and (B) Gear joint.

A *complex link* is a rigid body that contains more than two joints. Figure 1.4B illustrates a complex link. A *rocker arm* is a complex link, containing three joints, that is pivoted near its center. A bellcrank is similar to a rocker arm, but is bent in the center. The complex link shown in Figure 1.4B is a bell-crank.

A *point of interest* is a point on a link where the motion is of special interest. Once kinematic analysis is performed, the displacement, velocity, and accelerations of that point are determined.



(a) Simple link (b) Complex link

Figure 1.4 Links: (A) Simple Link and (B) Complex link.

1.5 KINEMATIC DIAGRAMS

In analyzing the motion of a machine, it is often difficult to visualize the movement of the components in a full assembly drawing. Figure 1.5 shows a machine that is used to handle parts on an assembly line. A motor produces rotational power, which drives a mechanism that moves the arms back and forth in a synchronous fashion. As can be seen, an assembly view of the entire machine becomes complex and it is difficult to focus on the motion of the mechanism under consideration.

It is easier to represent the parts in skeleton form so that only the dimensions that influence the motion of the mechanism are shown. These "stripped-down" sketches of mechanisms are often referred to as kinematic diagrams. The purpose of these diagrams are similar to electrical circuit schematic or piping diagrams, in that they represent variables that affect the primary function of the mechanism. Table 1.1 shows typical conventions used in creating kinematic diagrams.

A kinematic diagram should be drawn to a scale proportional to the actual mechanism. For convenient reference, the links are numbered, starting with the frame as link number 1. To avoid confusion, the joints should be lettered.

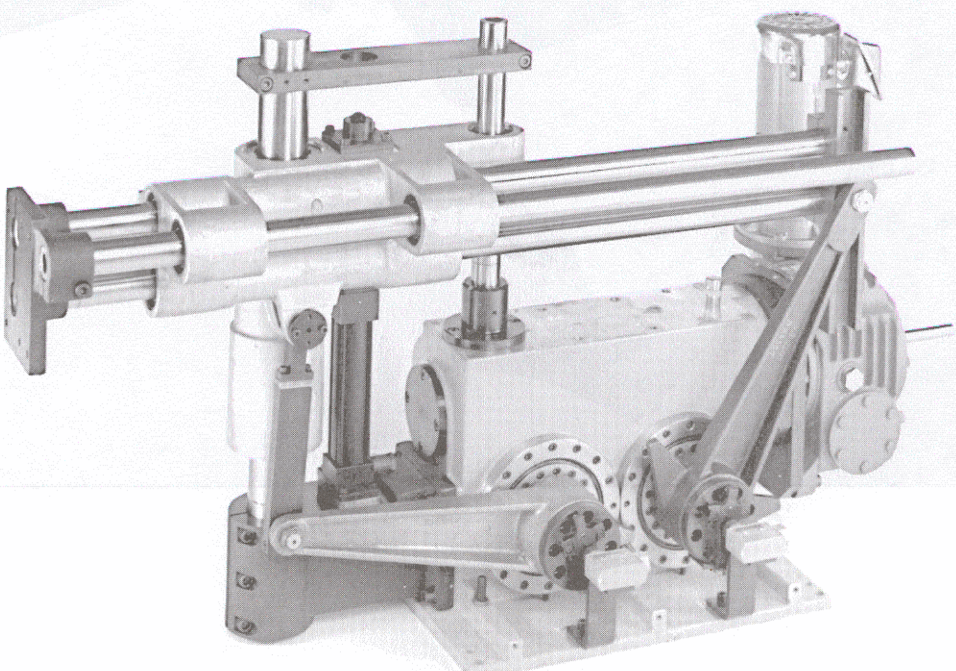


Figure 1.5 Two-armed synchro loader (Courtesy PickOmatic Systems, Ferguson Machine Co.).

TABLE 1.1 Symbols Used in Kinematic Diagrams.

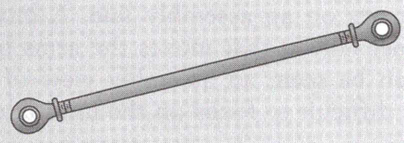
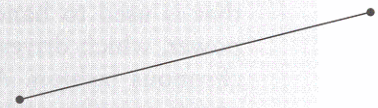
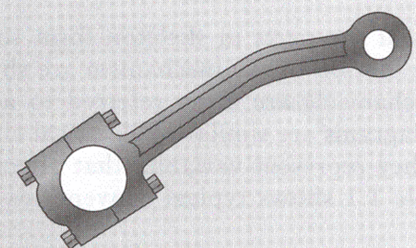
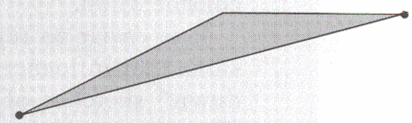
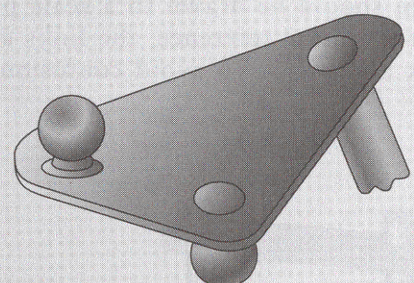
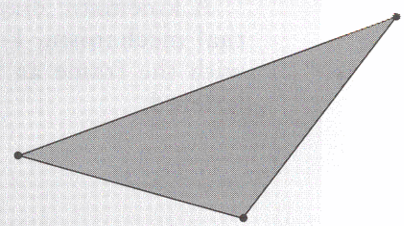
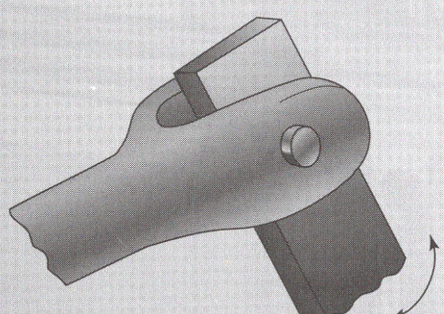
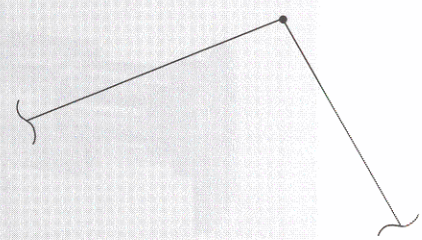
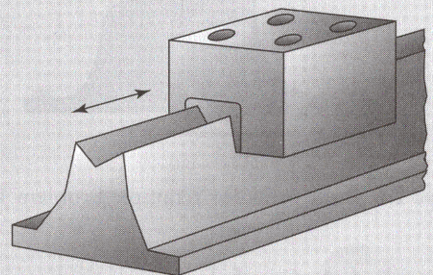
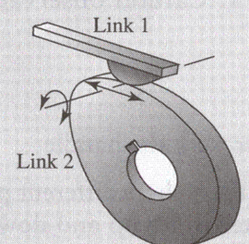
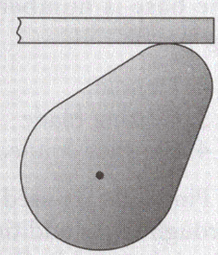
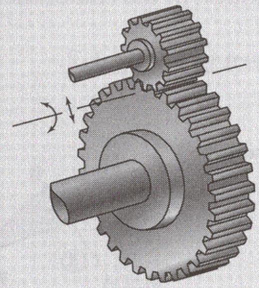
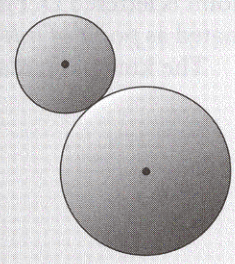
Component	Typical Form	Kinematic Representation
Simple Link		
Simple Link (with point of interest)		
Complex Line		
Pin Joint		

TABLE 1.1 Symbols Used in Kinematic Diagrams—*continued.*

Component	Typical Form	Kinematic Representation
Slider Joint		
Cam Joint		
Gear Joint		

EXAMPLE PROBLEM 1.1

Figure 1.6 shows a shear that is used to cut and trim electronic circuit board laminates. Draw a kinematic diagram.

Solution:

The first step in constructing a kinematic diagram is to decide the part that will be designated as the frame. The motion of all other links will be determined relative to the frame. In some cases, its selection is obvious as the frame is firmly attached to the ground.

In this problem, the large base that is bolted to the table is designated as the frame. The motion of all other links is determined relative to the base.

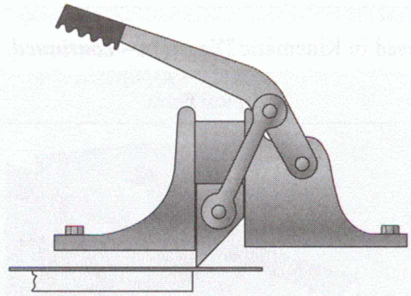


Figure 1.6 Shear press for Example Problem 1.1.

The base is numbered as *link 1*. Careful observation reveals three other moving parts:

2. Handle
3. Cutting blade
4. Bar that connects the cutter with the handle

Pin joints are used to connect these three different parts. These joints are lettered A through C. In addition, the cutter slides up and down, along the base. This sliding joint is lettered D. Finally, the motion of the end of the handle is desired. This is designated as *point of interest* X.

The kinematic diagram is given in Figure 1.7.

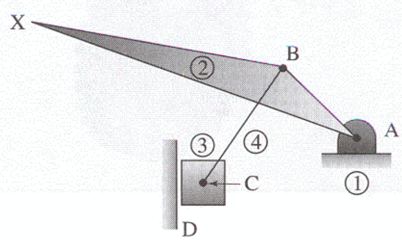


Figure 1.7 Kinematic diagram for Example Problem 1.1.

EXAMPLE PROBLEM 1.2

Figure 1.8 shows a pair of vice grips. Draw a kinematic diagram.

Solution:

The first step is to decide the part that will be designated as the frame. In this problem, no parts are attached to the ground. Therefore, the selection of the frame is rather arbitrary.

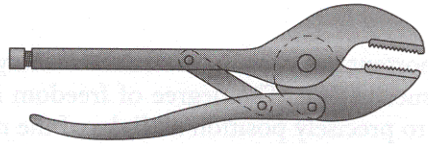


Figure 1.8 Vice grips for Example Problem 1.2.

The top handle is designated as the frame. The motion of all other links is determined relative to the top handle. The top handle is numbered as link 1. Careful observation reveals three other moving parts:

2. Bottom handle
3. Bottom jaw
4. Bar that connects the top and bottom handle

Four-pin joints are used to connect these different parts. These joints are lettered A through D. In addition, the motion of the end of the bottom jaw is desired. This is designated as point of interest X. Finally, the motion of the end of the lower handle is also desired. This is designated as point of interest Y.

The kinematic diagram is given in Figure 1.9.

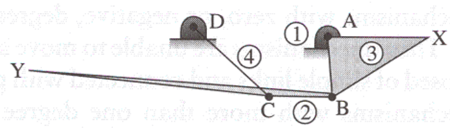


Figure 1.9 Kinematic diagram for Example Problem 1.2.

1.6 KINEMATIC INVERSION

The first step in drawing a kinematic diagram is selecting a member to serve as a fixed link or frame. In some cases, the selection of a frame is arbitrary, as in the vice grips from Example Problem 1.2. As different links are chosen as a frame, the relative motion of the links is not altered. The absolute motion can be drastically different, depending on the link selected as the frame. However, the relative motion is often the desired result.

In Example Problem 1.2, an important result of kinematic analysis is the distance that the handle must be raised in order to open the jaw. This is a question of relative position of the links: the handle and jaw. Because the relative motion of the links does not change with the selection of a frame, the choice of a frame link is not important. The process of selecting alternate links to serve as the fixed link is termed *kinematic inversion.*

1.7 MOBILITY

An important property in mechanism analysis is the number of degrees of freedom of the mechanism. The degree of freedom is the number of independent inputs required to precisely position all links of the mechanism with respect to the ground. It can also be defined as the number of drivers required to operate the mechanism. Examples of mechanism drivers could be manually moving one link to another position, connecting a motor to the shaft of one link, or pushing a piston of a hydraulic cylinder.

The number of degrees of freedom of a mechanism is also called the mobility, and given the symbol "F." When the configuration of a mechanism is completely defined by positioning one link, that system has one degree of freedom. Most machine mechanisms have one degree of freedom. Robotic arms can have three, or more, degrees of freedom.

Degrees of freedom for planar mechanisms joined with common joints can be calculated through *Gruebler's equation:*

(1.1) $$F = \text{degrees of freedom} = 3(n - 1) - 2j_p - j_h$$

where: n = total number of links in a mechanism, j_p = total number of primary joints (pins or sliders), and j_h = total number of higher-order joints(cams or gears).

Mechanisms with one degree of freedom are termed *constrained mechanisms.* As mentioned, most mechanisms used in machines are constrained.

Mechanisms with zero, or negative, degrees of freedom are termed *locked mechanisms.* These mechanisms are unable to move and form a structure. A *truss* is a structure composed of simple links and connected with pin joints and zero degrees of freedom.

Mechanisms with more than one degree of freedom are termed *unconstrained mechanisms.* These mechanisms need more than one driver to precisely operate them. Common unconstrained mechanisms are robotic arms and other "reaching" machines, such as back hoes and cranes. These types of mechanisms are commonly referred to as *open-loop linkages.* Formally defined, an open-loop linkage is when one or more links are connected to only one other link.

EXAMPLE PROBLEM 1.3

Figure 1.10 shows a toggle clamp. Draw a kinematic diagram, using the clamping surface and the handle as points of interest. Also compute the degrees of freedom for the clamp.

Solution:

The component that is bolted to the table is designated as the frame. The motion of all other links is determined relative to this frame. The frame is numbered as link 1. Careful observation reveals three other moving parts:

2. Handle
3. Arm that serves as the clamping surface
4. Bar that connects the clamping arm and handle

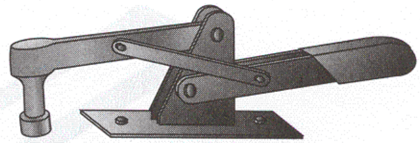


Figure 1.10 Toggle clamp for Example Problem 1.3.

Four-pin joints are used to connect these different parts. These joints are lettered A through D. In addition, the motion of the clamping surface is desired. This is designated as point of interest X. Finally, the motion of the end of the handle is also desired. This is designated as point of interest Y.

The kinematic diagram is detailed in Figure 1.11.

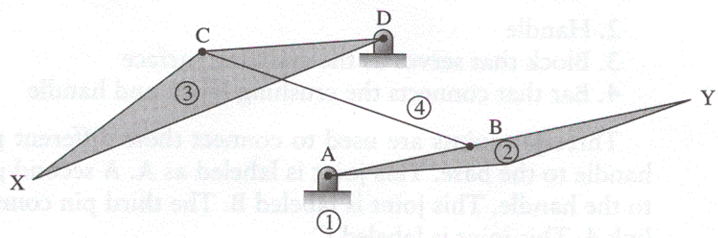


Figure 1.11 Kinematic diagram for Example Problem 1.3.

Calculating the mobility of the mechanism, it is seen that there are 4 links. There are also four-pin joints. Therefore:

$$n = 4,\ j_p = 4 \text{ pins},\ j_h = 0$$

and:

$$F = 3(n - 1) - 2j_p - j_h$$
$$= 3(4 - 1) - 2(4) - 0 = 1$$

With one degree of freedom, the clamp mechanism is constrained. Moving only one link, the handle, precisely positions all other links in the clamp.

EXAMPLE PROBLEM 1.4

Figure 1.12 shows a beverage can crusher used to reduce the size of cans for easier storage prior to recycling. Draw a kinematic diagram, using the end of the handle as a point of interest. Also compute the degrees of freedom for the device.

Solution:

The back portion of the device serves as a base and can be attached to a wall. This component is designated as the frame. The motion of all other links is determined

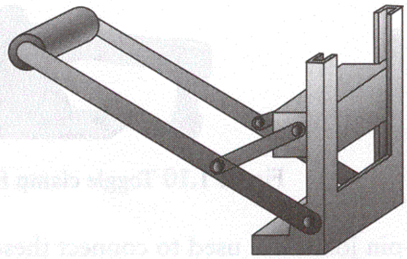


Figure 1.12 Can crusher for Example Problem 1.4.

relative to this frame. The frame is numbered as link 1. Careful observation shows a planar mechanism with three other moving parts:

2. Handle
3. Block that serves as the crushing surface
4. Bar that connects the crushing block and handle

Three-pin joints are used to connect these different parts. One pin connects the handle to the base. This joint is labeled as A. A second pin is used to connect link 4 to the handle. This joint is labeled B. The third pin connects the crushing block and link 4. This joint is labeled C.

The crushing block slides vertically during operation; therefore, a sliding joint connects the crushing block to the base. This joint is labeled D. In addition, the motion of the handle end is desired. This is designated as point of interest X.

The kinematic diagram is given in Figure 1.13.

To calculate the mobility, it was determined that there are four links in this mechanism. There are also three-pin joints and one slider joint. Therefore:

$$n = 4,\ j_p = (3 \text{ pins} + 1 \text{ slider}) = 4,\ j_h = 0$$

and:

$$F = 3(n - 1) - 2j_p - j_h$$
$$= 3(4 - 1) - 2(4) - 0 = 1$$

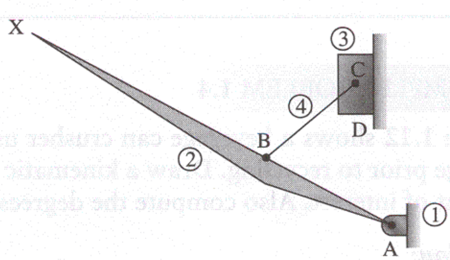


Figure 1.13 Kinematic diagram for Example Problem 1.4.

With one degree of freedom, the can crusher mechanism is constrained. Moving only one link, the handle precisely positions all other links and crushes a beverage can placed under the crushing block.

EXAMPLE PROBLEM 1.5

Figure 1.14 shows another device that can be used to shear material. Draw a kinematic diagram, using the end of the handle and the cutting edge as points of interest. Also, compute the degrees of freedom for the shear press.

Solution:

The base is bolted to a working surface and can be designated as the frame. The motion of all other links is determined relative to this frame. The frame is numbered as link 1. Careful observation reveals two other moving parts:

2. Gear/handle
3. Cutting lever

Two pin joints are used to connect these different parts. One pin connects the cutting lever to the frame. This joint is labeled as A. A second pin is used to connect the gear/handle to the cutting lever. This joint is labeled B.

The gear/handle is also connected to the frame with a gear joint. This joint is labeled C. In addition, the motion of the handle end is desired and is designated as point of interest X. The motion of the cutting surface is also desired and is designated as point of interest Y.

The kinematic diagram is given in Figure 1.15.

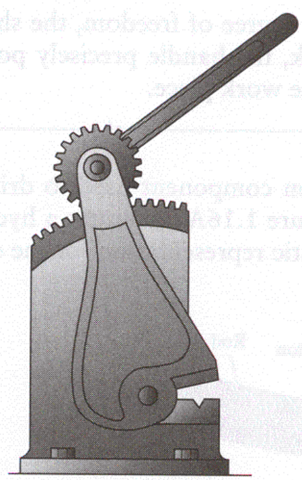


Figure 1.14 Shear press for Example Problem 1.5.

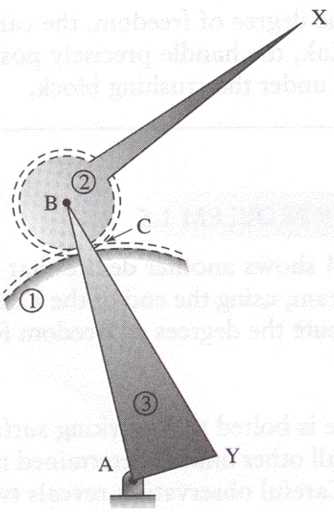


Figure 1.15 Kinematic diagram for Example Problem 1.5.

To calculate the mobility, it was determined that there are three links in this mechanism. There are also two pin joints and one gear joint. Therefore:

$$n = 3 \quad j_p = (2 \text{ pins}) = 2 \quad j_h = (1 \text{ gear connection}) = 1$$

and:

$$\begin{aligned} F &= 3(n - 1) - 2j_p - j_h \\ &= 3(3 - 1) - 2(2) - 1 = 1 \end{aligned}$$

With one degree of freedom, the shear press mechanism is constrained. Moving only one link, the handle precisely positions all other links and brings the cutting edge onto the work piece.

A common component used to drive a mechanism is a hydraulic or pneumatic cylinder. Figure 1.16A illustrates a hydraulic cylinder. Figure 1.16B shows the common kinematic representation for the cylinder unit.

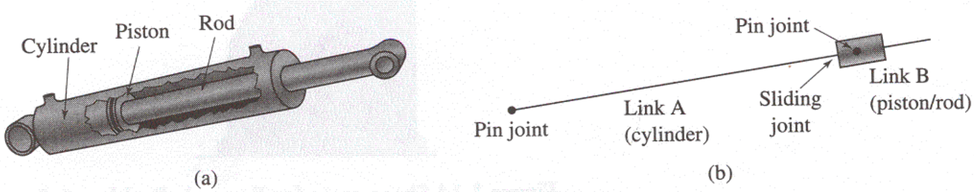


Figure 1.16 Hydraulic cylinder.

The cylinder unit typically contains a rod and piston assembly that slides relative to a cylinder. For kinematic purposes, these are two links (i.e., piston/rod and cylinder), connected with a sliding joint. In addition, the cylinder and rod end usually has provisions for pin joints. The following Example Problem incorporates a cylinder unit.

EXAMPLE PROBLEM 1.6

Figure 1.17 shows an outrigger foot to stabilize a utility truck. Draw a kinematic diagram, using the bottom of the stabilizing foot as a point of interest. Also compute the degrees of freedom.

Solution:

During operation of the outriggers, the utility truck is stationary. Therefore, the truck is designated as the frame. The motion of all other links is determined relative to the truck. The frame is numbered as link 1. Careful observation reveals three other moving parts:

2. Out rigger leg
3. Cylinder
4. Piston/rod

Three pin joints are used to connect these different parts. One connects the outrigger leg with the truck frame. This is labeled as joint A. Another connects the outrigger leg with the cylinder rod and is labeled as joint B. The last pin joint connects the cylinder to the truck frame and is labeled as C.

One sliding joint is present in the cylinder unit. This connects the piston/rod with the cylinder. It is labeled as joint D.

The stabilizer foot is part of link 2 and a point of interest located on the bottom of the foot is labeled as point X. The resulting kinematic diagram is given in Figure 1.18.

To calculate the mobility, it was determined that there are four links in this mechanism, as well as three pin joints and one slider joint. Therefore:

$$n = 4,\ j_p = (3 \text{ pins} + 1 \text{ slider}) = 4,\ j_h = 0$$

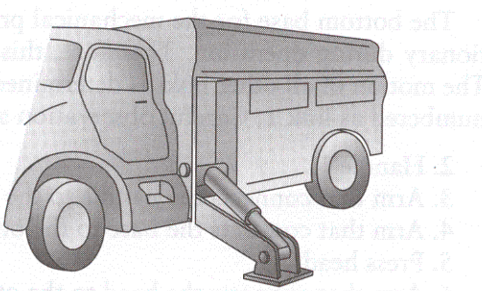


Figure 1.17 Outrigger for Example Problem 1.6.

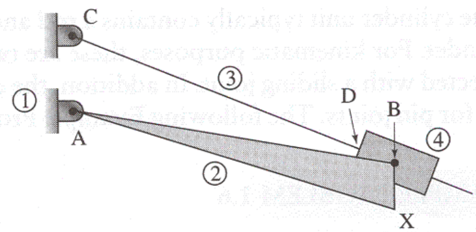


Figure 1.18 Kinematic diagram for Example Problem 1.6.

and:

$$F = 3(n - 1) - 2j_p - j_h$$
$$= 3(4 - 1) - 2(4) - 0 = 1$$

With one degree of freedom, the outrigger mechanism is constrained. Moving only one link, the piston precisely positions all other links in the outrigger, placing the stabilizing foot on the ground.

Some mechanisms have three links that are all connected at a common pin joint. This situation brings some confusion to kinematic modeling. Physically, only one pin may connect all three links. However, by definition, a joint can connect two links.

For kinematic analysis, this configuration must be modeled as two separate joints. One joint connects the first and second links. The second joint then connects the second and third links. Therefore, when three links come together at a common pin, the joint must be modeled as two pins. This scenario is illustrated in the following Example Problem.

EXAMPLE PROBLEM 1.7

Figure 1.19 shows a mechanical press used to exert large forces to insert a small part into a larger one. Draw a kinematic diagram using the end of the handle as a point of interest. Also compute the degrees of freedom.

Solution:

The bottom base for the mechanical press sits on a work bench and remains stationary during operation. Therefore, this bottom base is designated as the frame. The motion of all other links is determined relative to the bottom base. The frame is numbered as link 1. Careful observation shows five other moving parts:

2. Handle
3. Arm that connects the handle to the other arms
4. Arm that connects the base to the other arms
5. Press head
6. Arm that connects the head to the other arms

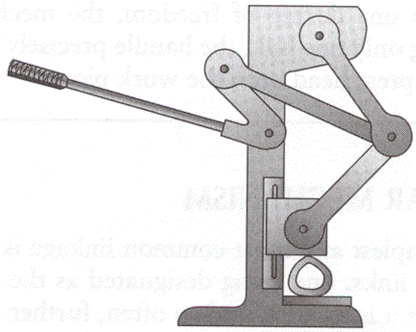


Figure 1.19 Mechanical press for Example Problem 1.7.

Pin joints are used to connect the several different parts. One connects the handle to the base and is labeled as joint A. Another connects link 3 to the handle and is labeled as joint B. Another connects link 4 to the base and is labeled as C. Another connects link 6 to the press head and is labeled as D.

It appears that a pin is used to connect the three arms—links 3, 4 and 6—together. Because three separate links are joined at a common point, this must be modeled as two separate joints. They are labeled as E and F.

A sliding joint connects the press head with the base. This joint is labeled as G. Motion of the end of the handle is desired and is labeled as point of interest X.

The kinematic diagram is given in Figure 1.20.

To calculate the mobility, it was determined that there are six links in this mechanism, as well as six pin joints and one slider joint. Therefore:

$$n = 6,\ j_p = (6 \text{ pins} + 1 \text{ slider}) = 7,\ j_h = 0$$

and:

$$\begin{aligned} F &= 3(n - 1) - 2j_p - j_h \\ &= 3(6 - 1) - 2(7) - 0 = 15 - 14 = 1 \end{aligned}$$

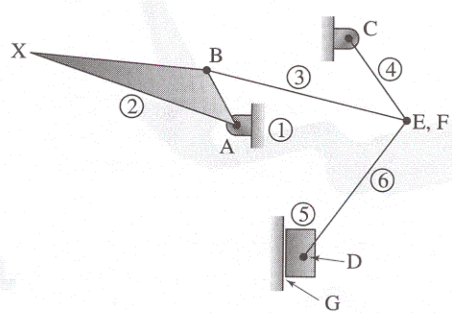


Figure 1.20 Kinematic diagram for Example Problem 1.7.

With one degree of freedom, the mechanical press mechanism is constrained. Moving only one link, the handle precisely positions all other links in the press, sliding the press head onto the work piece.

1.8 THE FOUR BAR MECHANISM

The simplest and most common linkage is the four-bar linkage. It is a combination of four links, one being designated as the frame, and connected by four pin joints. Because it is encountered so often, further exploration is in order.

The mechanism for an automotive rear window wiper system is shown in Figure 1.21A. The kinematic diagram is shown in Figure 1.21B. Notice that this is a four-bar mechanism because it is comprised of four links connected by four pin joints and one link is unable to move.

The mobility of a four-bar mechanism consists of the following:

$$n = 4 \quad j_p = 4 \text{ pins} \quad j_h = 0$$

and:

$$F = 3(n - 1) - 2j_p - j_h$$
$$= 3(4 - 1) - 2(4) - 0 = 1$$

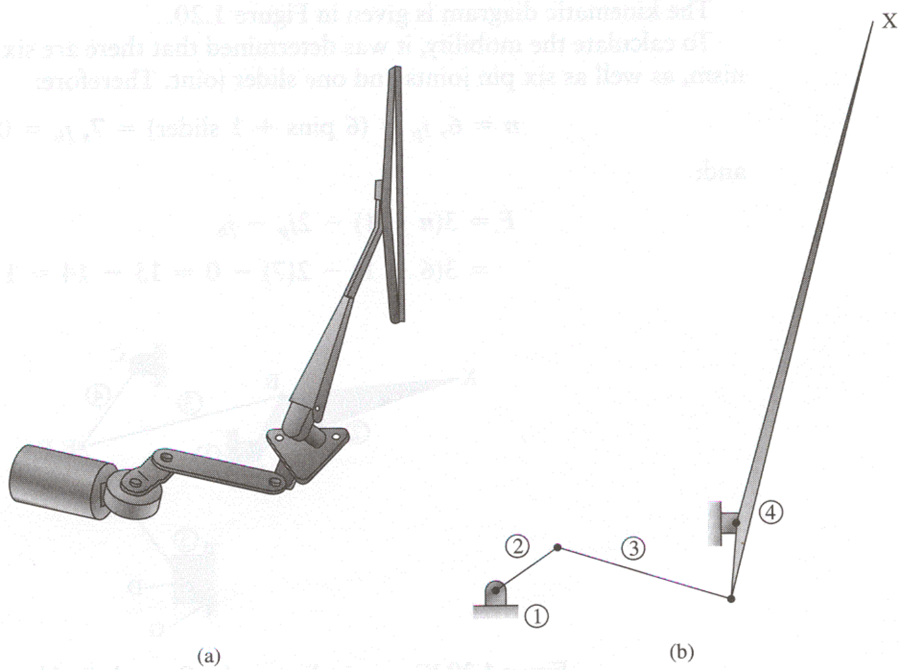


Figure 1.21 Rear-window wiper mechanism.

Because the four-bar mechanism has one degree of freedom, it is constrained or fully operated with one driver. The wiper system in Figure 1.21 is activated by a DC electric motor.

Of course, the link that is unable to move is referred to as the frame. Typically, the pivoted link that is connected to the driver or power source is called the *input link*. The other pivoted link that is attached to the frame is designated the *output link* or *follower*. The *coupler* or *connecting arm* "couples" the motion of the input link to the output link. The following designations are used to describe the length of the four links:

L_1 = Length of the fixed link
L_2 = Length of the shorter pivoting link (typically the input link).
L_3 = Length of the coupler.
L_4 = Length of the longer pivoting link (typically the output link).

Depending on the arrangement and lengths of the links and a four-bar mechanism can be classified as the following three linkage types:

Crank-rocker A crank-rocker has the shortest link of the four-bar mechanism configured adjacent to the frame. When this shortest link continuously rotates, the output link oscillates between limits. Thus, the shortest link is called the *crank*, and the output link is called the *rocker*. The wiper system in Figure 1.21 is designed to be a crank-rocker. As the motor continuously rotates the input link, the output link oscillates or "rocks." The wiper arm and blade are firmly attached to the output link, oscillating the wiper across a windshield. A four-bar mechanism behaves as a crank-rocker when the following conditions exist[6]:

$$L_2 < L_1 \text{ or } L_3 \text{ or } L_4$$
$$\text{and } L_1 < (L_3 + L_4 - L_2)$$
$$\text{and } L_1 > (|L_3 - L_4| + L_2)$$

Note that the $|x|$ notation refers to the absolute value of x.

Crank-crank The crank-crank, or double crank, has the shortest link of the four-bar mechanism configured as the fixed link or frame. When one of the pivoted links rotates continuously, the other pivoted link also rotates continuously. Thus, the two pivoted links, 2 and 4, are able to both rotate through a full revolution. The shorter of the two rotating links is typically the input link. The crank-crank mechanism is also called a drag link mechanism. A four-bar mechanism behaves as a crank-crank when the following condition exists [6]:

$$L_1 < L_2 \text{ or } L_3 \text{ or } L_4$$
$$\text{and } L_1 < (L_3 + L_4 - L_2)$$
$$\text{and } L_1 > (|L_3 - L_4| + L_2)$$

Rocker-rocker The rocker-rocker, or double rocker, has the link opposite the shortest link of the four-bar mechanism configured as the frame. In this configuration, neither link connected to the frame is able to complete a full revolution. Thus, both input and output links are constrained to oscillate between limits, and called rockers. A four-bar mechanism behaves as a rocker-rocker when any of the above conditions do not exist and the following condition is met or [6]:

$$L_3 < L_1 \text{ or } L_2 \text{ or } L_4$$

EXAMPLE PROBLEM 1.8

A nose wheel assembly for a small aircraft is shown in Figure 1.22. Classify the motion of this four-bar mechanism based on the configuration of the links.

Solution:

In an analysis that focuses on the landing gear, the motion of the wheel assembly is determined relative to the body of the aircraft. Therefore, the aircraft body is designated as the frame. Figure 1.23 shows the kinematic diagram for the wheel assembly, numbering, and labeling the links. The tip of the wheel was designated as point of interest X.

The lengths of the links include:

$$L_1 = 32 \text{ in. } L_2 = 12 \text{ in. } L_3 = 30 \text{ in. } L_4 = 26 \text{ in.}$$

The shortest link is adjacent to the frame. Therefore, it is designated L_2. This mechanism can be either a crank-rocker or a rocker-rocker since the frame is not the shortest link. The criteria for the different categories of four-bar mechanisms should be reviewed.

For a crank-rocker:

$$L_2 < L_1 \text{ or } L_3 \text{ or } L_4$$

$$12 < 32 \text{ \{yes\}}$$

$$12 < 30 \text{ \{yes\}}$$

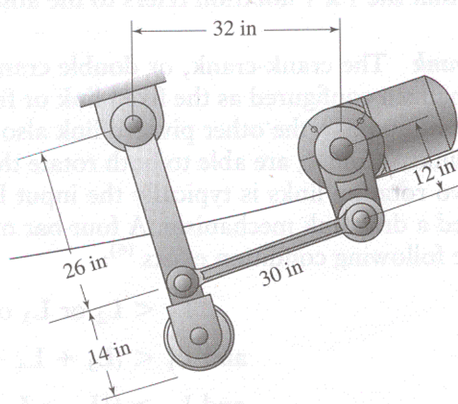


Figure 1.22 Nose wheel assembly for Example Problem 1.8.

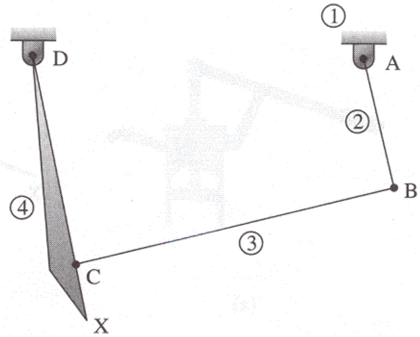


Figure 1.23 Kinematic diagram for Example Problem 1.8.

$$12 < 26 \text{ \{yes\}}$$
$$\text{and } L_1 < (L_3 + L_4 - L_2)$$
$$32 < (30 + 26 - 12) = 44 \text{ \{yes\}}$$
$$\text{and } L_1 > (|L_3 - L_4| + L_2)$$
$$32 > (|30 - 26| + 12) = 16 \text{ \{yes\}}$$

Because all criteria for a crank rocker are valid, the nose wheel assembly *is* a crank-rocker mechanism.

1.9 SLIDER CRANK MECHANISM

Another mechanism that is commonly encountered is a slider crank. This mechanism also consists of a combination of four links, with one being designated as the frame. This mechanism, however, is connected by three pin joints and one sliding joint.

A mechanism that drives a manual water pump is shown in Figure 1.24A. The corresponding kinematic diagram is given in figure 1.24B.

The mobility of a slider crank mechanism is represented by the following:

$$n = 4,\ j_p = (3 \text{ pins} + 1 \text{ sliding}) = 4,\ j_h = 0$$

and:

$$F = 3(n - 1) - 2j_p - j_h$$
$$= 3(4 - 1) - 2(4) - 0 = 1$$

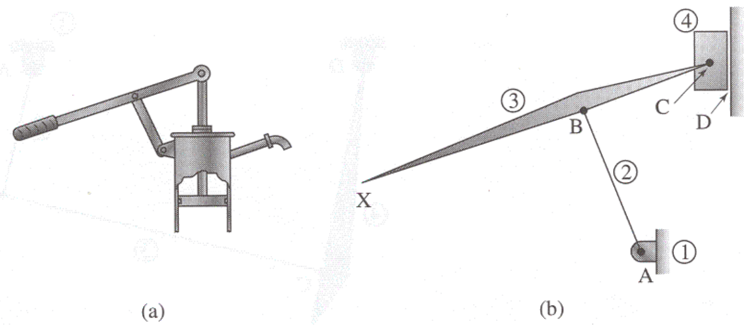


Figure 1.24 Pump mechanism for a manual water pump: (A) Mechanism and (B) Kinematic diagram.

Because the slider crank mechanism has one degree of freedom, it is constrained or fully operated with one driver. The pump in Figure 1.24 is activated manually by pushing on the handle (link 3).

In general, the pivoted link connected to the frame is called the crank. This link is not always capable of completing a full revolution. The link that translates is called the slider. This link is the piston/rod of the pump in Figure 1.24. The coupler or connecting rod "couples" the motion of the crank to the slider.

1.10 TECHNIQUES OF MECHANISM ANALYSIS

Most of the analysis of mechanisms involves geometry. Often, graphical methods are employed because the motion of the mechanism can be clearly visualized.

Graphical solutions involve drawing "scaled" lines at specific angles. One example is the drawing of a kinematic diagram. A graphical solution involves preparing a drawing where all links are shown at a proportional scale to the actual mechanism. The orientation of the links must also be shown at the same angles as on the actual mechanism.

This graphical approach has merits in ease and solution visualization. However, accuracy must be a serious concern to achieve results that are consistent with analytical techniques. For several decades, mechanism analysis was primarily completed using graphical approaches. Even with its popularity, many scorned graphical techniques as being imprecise. However, the development of computer-aided design (CAD) systems has allowed the graphical approach to be applied with precision. This text attempts to illustrate the most common methods used in the practical analysis of mechanisms. Each of these methods is briefly introduced in the following sections.

1.10.1 Traditional Drafting Techniques

Over the past decades, all graphical analysis was performed using traditional drawing techniques. Drafting equipment was used to draw the needed scaled lines at specific angles. The equipment used to perform these analyses included triangles, parallel straight edges, compasses, protractors, and engineering scales. As mentioned, this method was often criticized as being imprecise. However, with proper attention to detail, accurate solutions can be obtained.

It was the rapid adoption of CAD software over the past several years that limited the use of traditional graphical techniques. Even with the lack of industrial application, many still believe that traditional drafting techniques can still be used by students to illustrate the concepts behind graphical mechanism analysis. Of course, these concepts are identical to those used in graphical analysis using a CAD system. But with using traditional drawing techniques, the student can concentrate on the kinematic theories and will not be "bogged down" with learning CAD commands.

1.10.2 CAD Systems

As mentioned, graphical analysis may be performed using traditional drawing procedures or using a CAD system, as is commonly practiced in industry. Any one of the numerous commercially available CAD systems can be used for mechanism analysis. The most popular CAD systems include AutoCAD, Microstation, Unigrahpics, and ProEngineer. Although the commands differ from system to system, all have the capability to draw highly accurate lines at designated lengths and angles. This is exactly the capability required for graphical mechanism analysis.

Besides increased accuracy, another benefit of CAD is that the lines do not need to be scaled to fit on a piece of drawing paper. On the computer, lines are drawn on "virtual" paper that is of infinite size.

This text does not attempt to thoroughly discuss the system-specific commands used to draw the lines, but several of the Example Problems are solved using a CAD system. The main goal of this text is to instill an understanding of the concepts of mechanism analysis. This goal can be realized regardless of the specific CAD system incorporated. Therefore, the student should not be overly concerned with the CAD system used for accomplishing graphical analysis. For that manner, the student should not be concerned whether manual or computer graphics are used to learn mechanism analysis.

1.10.3 Analytical Techniques

Analytical methods can also be used to achieve precise results. Advanced analytical techniques often involve intense mathematical functions, which are beyond the scope of this text and routine mechanism analysis. In addition, the significance of the calculations are often difficult to visualize.

The analytical techniques incorporated in this text, couple the theories of geometry, trigonometry, and graphical mechanism analysis. This approach will achieve accurate solutions, yet the graphical theories allow the solutions to be visualized. This approach does have the pitfall of laborious calculations for more complex mechanisms. Still, a significant portion of this text is dedicated to these analytical techniques.

1.10.4 Computer Methods

As the more accurate analytical solutions are desired for several positions of a mechanism, the number of calculations can become unwieldy. In these situations, the use of computer solutions is appropriate. Computer solutions are also valuable when several design iterations must be analyzed.

A computer approach to mechanism analysis can take several forms:

- *Spreadsheets* are very popular for more routine mechanism problems. An important feature of the spreadsheet is that as a cell containing input data is changed, all other results are updated. This allows design iterations to be completed with ease. As problems become more complex, they can be difficult to manage on a spreadsheet. Nonetheless, spreadsheets are used in problem solution throughout the text.
- *Commercially available dynamic analysis programs,* such as Working Model, ADAMS (Automatic Dynamic Analysis of Mechanical Systems), or I-DEAS (Integrated Design Engineering Analysis Software) are available. Dynamic models of systems can be created from menus of general components. These packages are best suited when kinematic analysis is large component of the job. Chapter 2 is dedicated to dynamic analysis programs.
- *User-written computer programs* in a high-level language, such as FORTRAN, BASIC, or C can be created. The programming language selected must have direct availability to trigonometric and inverse trigonometric functions. Due to the time and effort required to write a special program, they are most effective when a complex, yet not commonly encountered, problem needs to be solved. Some simple algorithms are provided for elementary kinematic analysis in Chapter 9.

PROBLEMS

Problems in Sketching Kinematic Diagrams

1-1. A mechanism is used to open the door of a heat-treating furnace and is shown in Figure P1.1. Draw a kinematic diagram of the mechanism. The end of the handle should be identified as a point of interest.

1-2. A pair of bolt cutters are shown in Figure P1.2. Draw a kinematic diagram of the mechanism, selecting the lower handle as the frame. The end of the upper handle and the cutting surface of the jaws should be identified as points of interest.

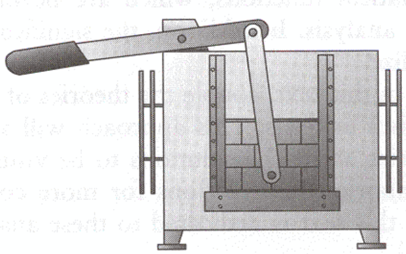


Figure P1.1 Problems 1-1 and 1-26.

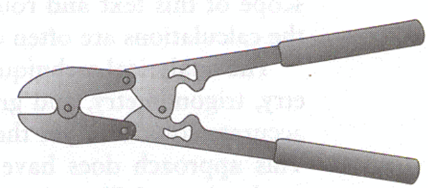


Figure P1.2 Problems 1-2 and 1-27.

1-3. A folding chair that is commonly used in stadiums is shown in Figure P1.3. Draw a kinematic diagram of the folding mechanism.

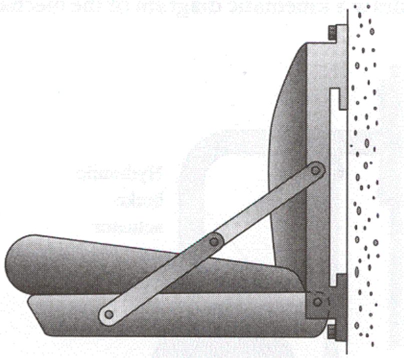


Figure P1.3 Problems 1-3 and 1-28.

1-4. A foot pump that can be used to fill bike tires, toys, etc. is shown in Figure P1.4. Draw a kinematic diagram of the pump mechanism. The foot pad should be identified as a point of interest.

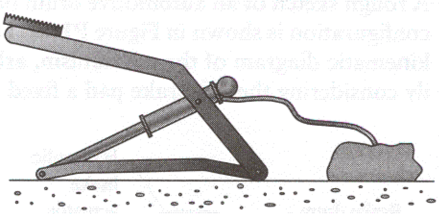


Figure P1.4 Problems 1-4 and 1-29.

1-5. A pair of pliers is shown in Figure P1.5. Draw a kinematic diagram of the mechanism.

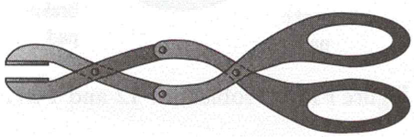


Figure P1.5 Problems 1-5 and 1-30.

1-6. Another configuration for a pair of pliers is shown in Figure P1.6. Draw a kinematic diagram of the mechanism.

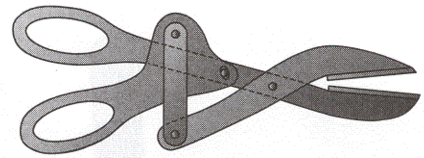


Figure P1.6 Problems 1-6 and 1-31.

1-7. A mechanism for a window is shown in Figure P1.7. Draw a kinematic diagram of the mechanism.

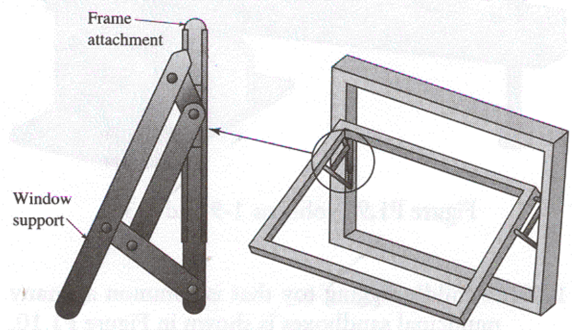


Figure P1.7 Problems 1-7 and 1-32.

1-8. Another mechanism for a window is shown in Figure P1.8. Draw a kinematic diagram of the mechanism.

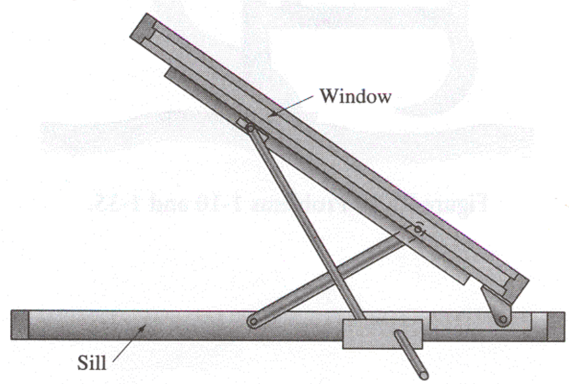


Figure P1.8 Problems 1-8 and 1-33.

1-9. A toggle clamp used for holding a workplace while it is being machined is shown in Figure P1.9. Draw a kinematic diagram of the mechanism.

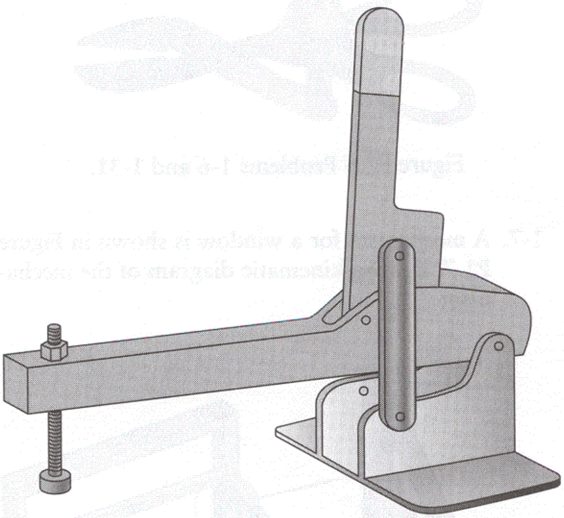


Figure P1.9 Problems 1-9 and 1-34.

1-10. A child's digging toy that is common at many municipal sandboxes is shown in Figure P1.10. Draw a kinematic diagram of the mechanism.

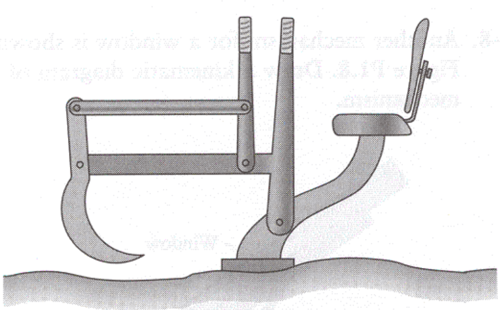


Figure P1.10 Problems 1-10 and 1-35.

1-11. A brake that is used to slow a drive shaft on general industrial equipment is shown in Figure P1.11. Understanding that the drive shaft and disk are not part of the braking mechanism, draw a kinematic diagram of the mechanism.

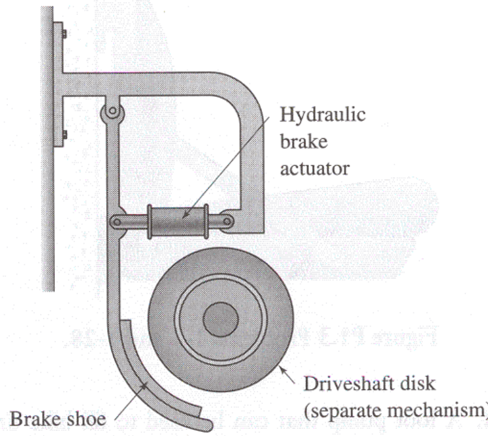


Figure P1.11 Problems 1-11 and 1-36.

1-12. A rough sketch of an automotive drum braking configuration is shown in Figure P1.12. Draw a kinematic diagram of the mechanism, arbitrarily considering the left brake pad a fixed link.

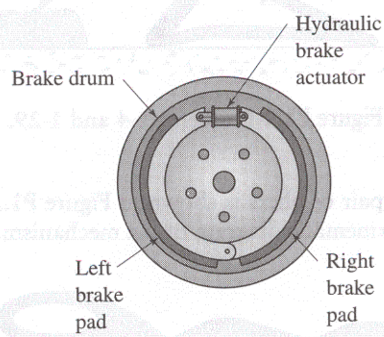


Figure P1.12 Problems 1-12 and 1-37.

1-13. A sketch of a microwave oven carrier used to assist people in wheelchairs is shown in Figure P1.13. Draw a kinematic diagram of the mechanism.

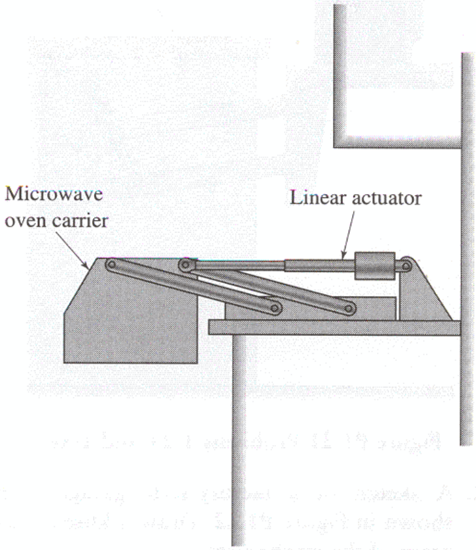


Figure P1.13 Problems 1-13 and 1-38.

1-14. A sketch of a truck used to deliver supplies to passenger jets is shown in Figure P1.14. Draw a kinematic diagram of the mechanism.

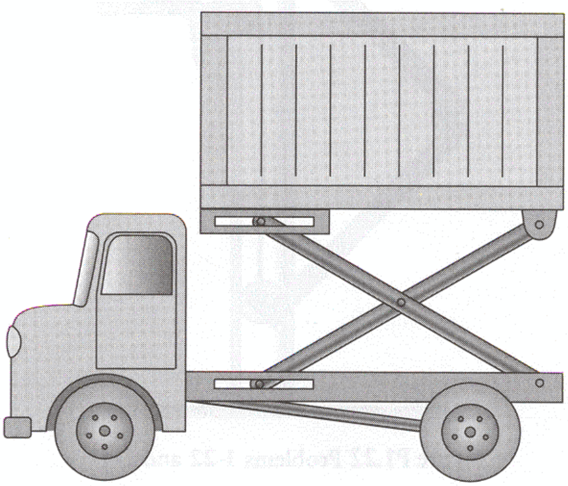


Figure P1.14 Problems 1-14 and 1-39.

1-15. A sketch of a device to move packages from an assembly bench to a conveyor is shown in Figure P1.15. Draw a kinematic diagram of the mechanism.

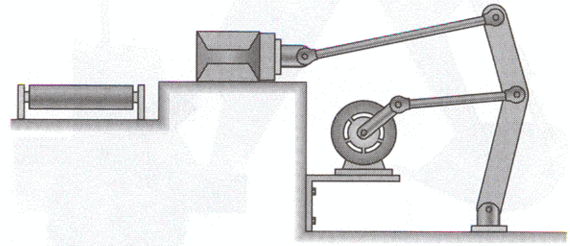


Figure P1.15 Problems 1-15 and 1-40.

1-16. A sketch of a lift platform is shown in Figure P1.16. Draw a kinematic diagram of the mechanism.

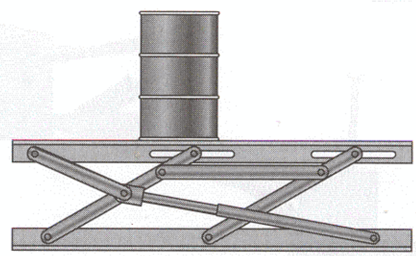


Figure P1.16 Problems 1-16 and 1-41.

1-17. A sketch of a lift platform is shown in Figure P1.17. Draw a kinematic diagram of the mechanism.

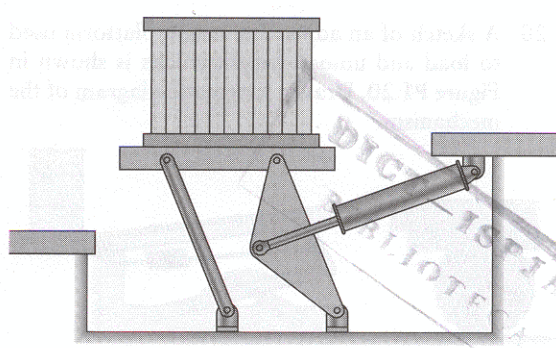


Figure P1.17 Problems 1-17 and 1-42.

1-18. A sketch of a back hoe is shown in Figure P1.18. Draw a kinematic diagram of the mechanism.

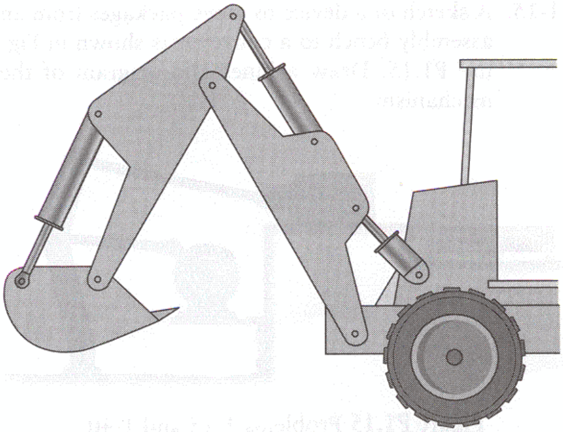


Figure P1.18 Problems 1-18 and 1-43.

1-19. A sketch of a front loader is shown in Figure P1.19. Draw a kinematic diagram of the mechanism.

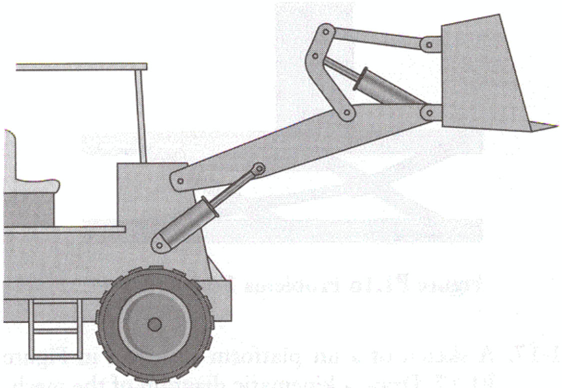


Figure P1.19 Problems 1-19 and 1-44.

1-20. A sketch of an adjustable height platform used to load and unload freight trucks is shown in Figure P1.20. Draw a kinematic diagram of the mechanism.

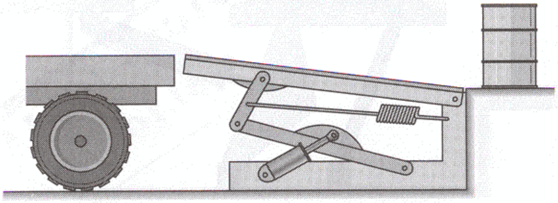


Figure P1.20 Problems 1-20 and 1-45.

1-21. A sketch of a kitchen appliance carrier, used for under-counter storage, is shown in Figure P1.21. Draw a kinematic diagram of the mechanism.

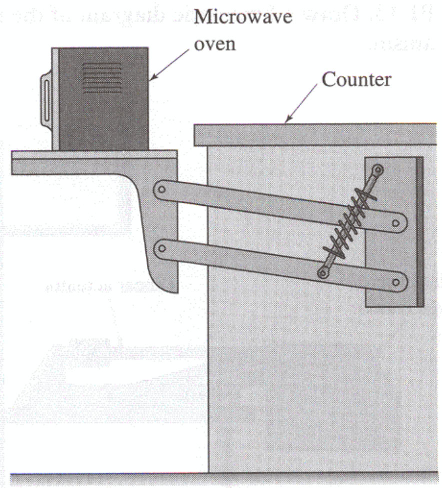


Figure P1.21 Problems 1-21 and 1-46.

1-22. A sketch of a factory-style garage door is shown in Figure P1.22. Draw a kinematic diagram of the mechanism.

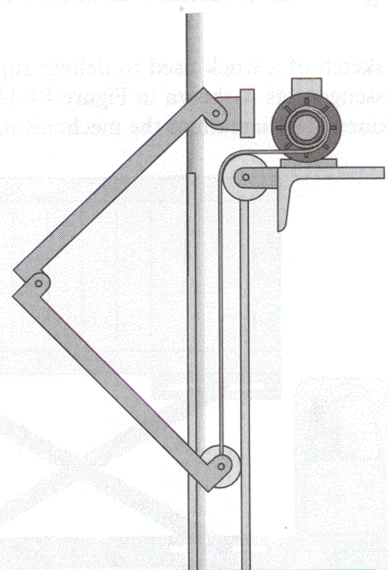


Figure P1.22 Problems 1-22 and 1-47.

1-23. A sketch of a device to close the top flaps of boxes is shown in Figure P1.23. Draw a kinematic diagram of the mechanism.

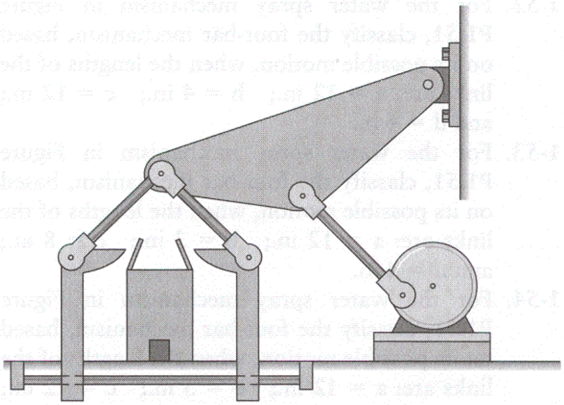


Figure P1.23 Problems 1-23 and 1-48.

1-24. A sketch of an automated factory window opener is shown in Figure P1.24. Draw a kinematic diagram of the mechanism.

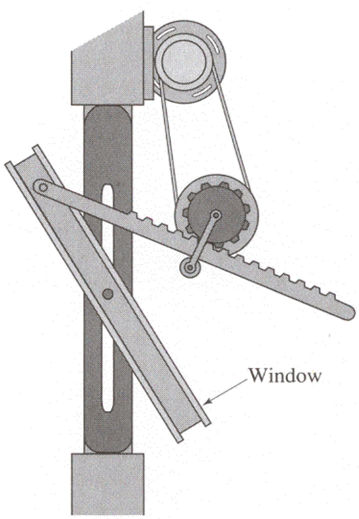


Figure P1.24 Problems 1-24 and 1-49.

1-25. A sketch of a wear test fixture is shown in Figure P1.25. Draw a kinematic diagram of the mechanism.

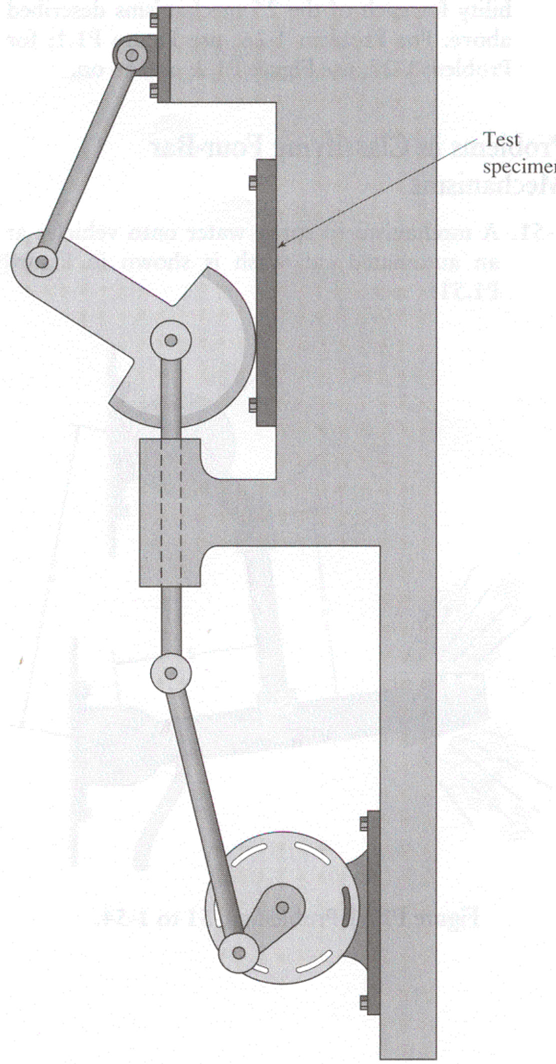


Figure P1.25 Problems 1-25 and 1-50.

Problems in Calculating Mobility

1-26 through 1-50. Specify the number of links, the number and type of joints, and calculate the mobility for each of the 25 mechanisms described above. For Problem 1-26, use Figure P1.1; for Problem 1-27, use Figure P1.2; and so on.

Problems in Classifying Four-Bar Mechanisms

1-51. A mechanism to spray water onto vehicles at an automated car wash is shown in Figure P1.51.

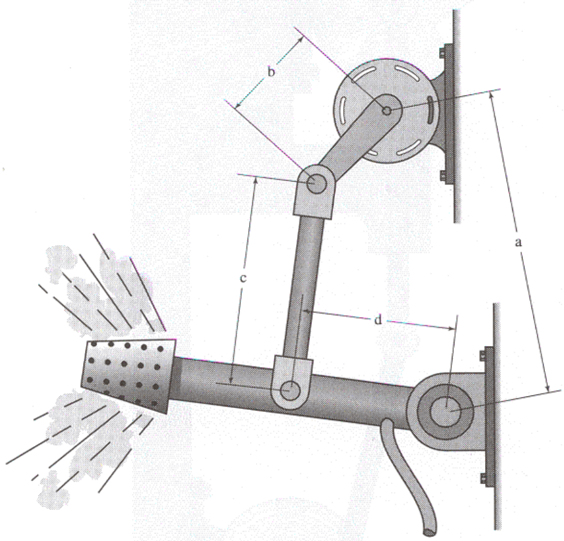


Figure P1.51 Problems 1-51 to 1-54.

Classify the four-bar mechanism, based on its possible motion, when the lengths of the links are: a = 12 in.; b = 2 in.; c = 14 in.; and d = 4 in.

1-52. For the water spray mechanism in Figure P1.51, classify the four-bar mechanism, based on its possible motion, when the lengths of the links are: a = 12 in.; b = 4 in.; c = 12 in.; and d = 4 in.

1-53. For the water spray mechanism in Figure P1.51, classify the four-bar mechanism, based on its possible motion, when the lengths of the links are: a = 12 in.; b = 3 in.; c = 8 in.; and d = 4 in.

1-54. For the water spray mechanism in Figure P1.51, classify the four-bar mechanism, based on its possible motion, when the lengths of the links are: a = 12 in.; b = 3 in.; c = 12 in.; and d = 5 in.

CASE STUDIES

1-1. The mechanism shown in Figure C1.1 has been taken from a feed device for an automated ball-bearing assembly machine. An electric motor is attached to link F as shown. Carefully examine the configuration of the components in the mechanism. Then answer the following leading questions to gain insight into the operation of the mechanism.

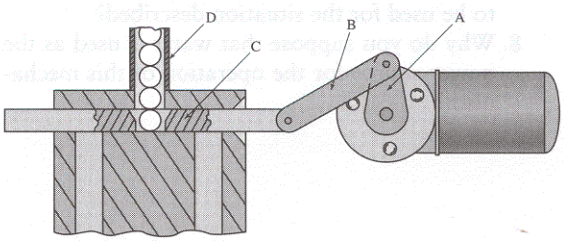


Figure C1.1 (Courtesy Industrial Press, Inc.)

1. As link A rotates clockwise 90°, what will happen to slide C?
2. What happens to the ball trapped in slide C when it is at this position?
3. As link A continues another 90° clockwise, what action occurs?
4. What is the purpose of this device?
5. Why are there chamfers at the entry of slide C?
6. Why do you suppose there is a need for such a device?

1-2. Figure C1.2 shows a mechanism that is typical in the tank of a water closet. Note that flapper C is hollow and filled with trapped air. Carefully examine the configuration of the components in the mechanism. Then answer the following leading questions to gain insight into the operation of the mechanism.

1. As the handle A is rotated counterclockwise, what is the motion of the flapper C?
2. When the flapper C is raised, what effect is seen?
3. When flapper C is lifted, it tends to remain in an upward position for a period of time. What causes this tendency to keep the flapper lifted?

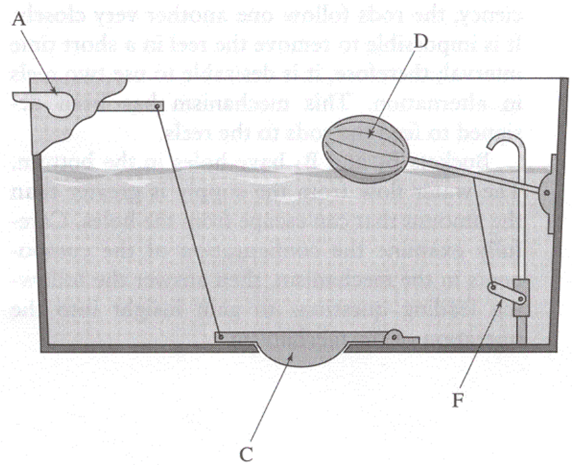


Figure C1.2

4. When will this tendency (to keep the flapper C lifted) cease?
5. What effect will cause item D to move?
6. As item D is moved in a counterclockwise direction, what happens to item F?
7. What does item F control?
8. What is the overall operation of these mechanisms?
9. Why is there a need for this mechanism and a need to store water in this tank?

1-3. Figure C1.3 shows a mechanism that guides newly formed steel rods to a device that rolls them into reels. The rods are hot when formed, and water is used assist in the cooling process. The rods can be up to several thousand feet long and slide at rates up to 25 miles per hour through channel S.

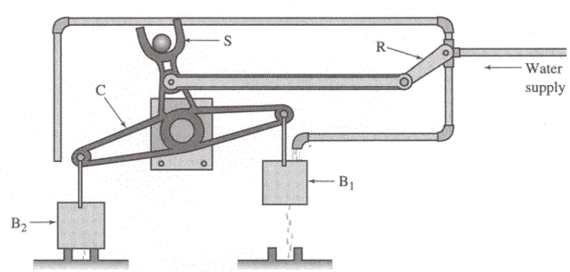


Figure C1.3 (Courtesy Industrial Press, Inc.)

Once the reel is full, the reel with the coiled rod is then removed. In order to obtain high efficiency, the rods follow one another very closely. It is impossible to remove the reel in a short time interval; therefore, it is desirable to use two reels in alternation. This mechanism has been designed to feed the rods to the reels.

Buckets B_1 and B_2 have holes in the bottom. The water flow from the supply is greater than the amount that can escape from the holes. Carefully examine the configuration of the components in the mechanism, then answer the following leading questions to gain insight into the operation of the mechanism.

1. In the shown configuration, what is happening to the level of water in bucket B_1?
2. In the shown configuration, what is happening to the level of water in bucket B_2?
3. What would happen to rocker arm C if bucket B_2 were forced upward?
4. What would happen to rocker arm R if bucket B_2 were forced upward?
5. What does rocker R control?
6. What is continual motion of this device?
7. How does this device allow two separate reels to be used for the situation described?
8. Why do you suppose that water is used as the power source for the operation of this mechanism?

2 Building Computer Models of Mechanisms Using Working Model® Software

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Understand the use of commercially available software for mechanism analysis.
2. Use Working Model® to build kinematic models of mechanisms.
3. Use Working Model® to animate the motion of mechanisms.
4. Use Working Model® determine the kinematic values of a mechanism.

2.1 INTRODUCTION

The rapid development of computers and software has altered the manner in which many engineering tasks are completed. In the study of mechanisms, software packages have been developed that allow a designer to construct computer models of a mechanism. These computer models allow the designer to simulate the mechanism. Software packages can solve kinematic and dynamic equations and determine the motion and force values of the mechanism during operation. In addition, the software can animate the computer model of the mechanism, allowing visualization of the design in action.

This chapter primarily serves as a tutorial for simulating machines and mechanisms using the Working Model® demonstration software, obtainable with the text. While the student may not yet understand the kinematic values generated during the analysis, the visualization of the mechanism can be extremely insightful. The material presented in the next several chapters will allow the student to understand the numerical solutions of the dynamic software.

2.2 COMPUTER SIMULATION OF MECHANISMS

Along with Working Model®, other commercially available dynamic analysis programs such as ADAMS® (Automatic Dynamic Analysis of Mechanical Systems), DADS® (Dynamic Analysis of Dynamic Systems) or I-DEAS® (Integrated Design Engineering Analysis Software) are available. Each computer program will allow creation of a mechanism from menus of general components. The general components include simple links, complex link, pin joints, sliding joints, etc. The mechanism is operated by selecting driver components, such as motors or cylinders, from menus.

The strategy for performing the dynamic analysis with these packages is also very similar. A general procedure for model building and analysis can be summarized as:

1. Define a set of rigid bodies (sizes, weights and inertial properties)
2. Place constraints on the rigid bodies (connecting the rigid bodies with joints)
3. Specify the input motion (define the properties of the driving motor, cylinder, etc.)
4. Run the analysis
5. Review the motion of the links and forces through the mechanism.

Of course, the commands and details will very greatly among the different packages. As mentioned, a demonstration version of Working Model® is obtainable with this text and the details of its use will be further illustrated in the following sections.

2.3 OBTAINING A LIMITED VERSION OF WORKING MODEL 2D

A limited version of Working Model 2D can be readily obtained, at no cost. Of course, a fully functioning version is available for purchase. In the limited version, some features have been disabled, most notably the Save and Print functions. Regardless, this version can provide an excellent introduction to building computer models of mechanisms. The tutorials in the following sections can be completed using this limited version. The software is distributed by Knowledge Revolution, and can be obtained from the following Web site:

http://www.krev.com/

The software can also be obtained by contacting:

Knowledge Revolution
66 Bovet Road, Suite 200
San Mateo, CA 94402
(800) 766-6615

Included with the software are directions for installation and setup. This method of distribution will allow the most recent version of Working Model 2D to be obtained. However, as the Working Model software is updated, the menus and icons may be slightly different from the tutorials that follow. Using some intuition, the student will be able to adapt and successfully complete mechanism simulations.

2.4 USING WORKING MODEL® TO MODEL A FOUR-BAR MECHANISM

Working Model is a popular, commercially available motion simulation package. It rapidly creates a model on a desktop computer that represents a mechanical system and performs dynamic analysis. This section uses Working Model to build a model of a four-bar linkage and run a simulation[11]. It is intended to be a tutorial; that is, it should be followed while actually using Working Model. The student is then encouraged to experiment with the software to perform other analyses.

Step 1: Open Working Model

1. Click on the Working Model program icon to start the program
   Working Model opens and begins running a demonstration dialog.
2. Explore Working Model using the introduction dialog boxes.
   The demonstration dialog boxes provide an overview of Working Model, customer examples, and ordering information.
3. Enter the Working Model application by selecting "Exit to Working Model"
   A tool bar appears on the left of the window and tape controls display along the bottom. The tool bar contains the options necessary to create simulations. The tape controls operate running and viewing simulations.

Step 2: Create the Links

This step creates the three moving links in a four-bar mechanism. The background serves as the fixed, fourth link.

1. Create a new Working Model document by selecting "New" from the "File" menu.
2. Construct the linkage by creating the three non-fixed links. Double click on the rectangle tool on the toolbar.
   The tool turns black, indicating that it can be used multiple times.
3. Using the rectangle tool, sketch out three bodies as shown in Figure 2.1.
   Rectangles are drawn by positioning the mouse at the first corner, click once, then move the mouse to the location of the opposite corner and click again. Rectangles are parametrically defined and their precise sizes will be specified later.
4. Open the "Properties box" and "Geometry box" in the "Window" menu.
   This displays information about the links and allows editing of this information.
5. Use the "Properties box" and "Geometry box" to change the center of the horizontal link to 0,0. Also change the geometry to a width of 8.5 and a height of 0.5.
   The shape and location of the rectangle should change upon data entry.
6. Likewise, use the "Properties box" and "Geometry box" to change the geometry of the long vertical link to a width of 0.5 and a height of 3. Also change the geometry of the short vertical link to a width of 0.5 and a height of 1.5.
   Again, the shape and location of the rectangle should change upon data entry.
7. Close both the "Properties box" and "Geometry box" windows.

Step 3: Place Points of Interest on the Links

This step teaches the usage of the "Object Snap" tool to place points precisely. The "Object Snap" tool highlights commonly used positions, like the center of a side, with an "X." When a point is placed using "Object Snap," the point's position is au-

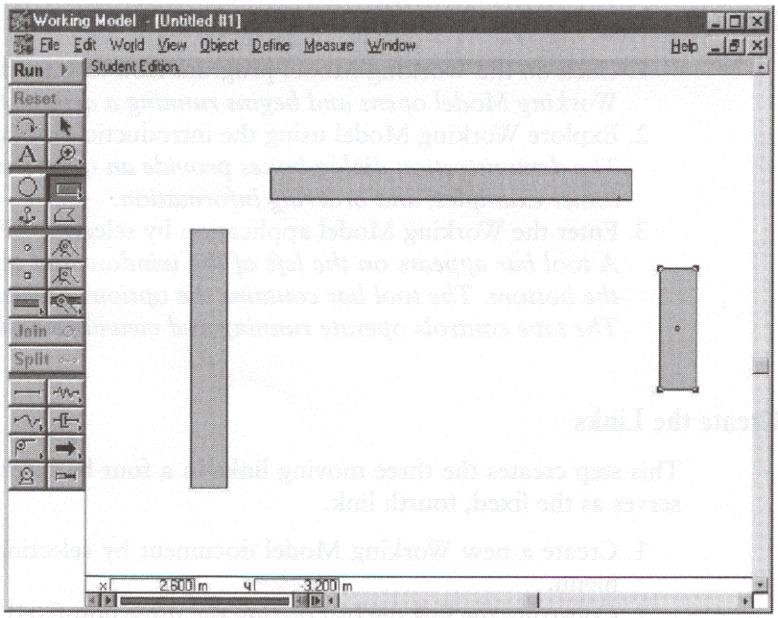


Figure 2.1 Three links sketched using the rectangle tool.

tomatically defined with parametric equations. These equations ensure that the point maintains its relative location even after resizing or other adjustments.

1. Double click on the point tool. The icon is a small circle and is directly below the anchor tool.
   The point tool turns black, indicating that it can be used multiple times without needing to be reselected before each new point is sketched.
2. Move the cursor over one of the links.
   Notice that an "X" appears around the pointer when it is centered on a side, over a corner, or over the center of a rectangle. This feature is called "Object Snap" and highlights the commonly-used parts of a link.
3. Place the cursor over the upper portion of one of the vertical links. When an "X" appears around the pointer (Figure 2.2), click the mouse button.
4. Place additional points as shown in Figure 2.2.
   Make sure that each of these points is placed at a "snap point" as evidenced with the "X" appearing at the pointer.
5. Select the pointer tool. The icon is an arrow pointed up and to the left.
6. Double click on one of the points that were sketched in Steps 3 or 4 to open the "Properties" window.
7. Edit the "Properties" window such that the points on the horizontal link are separated by 8 units. Edit the point on the long vertical link so that it is 1.5 units directly above the center of that link. Edit the point on the short vertical link so that it is 0.5 units directly above the center of that link.

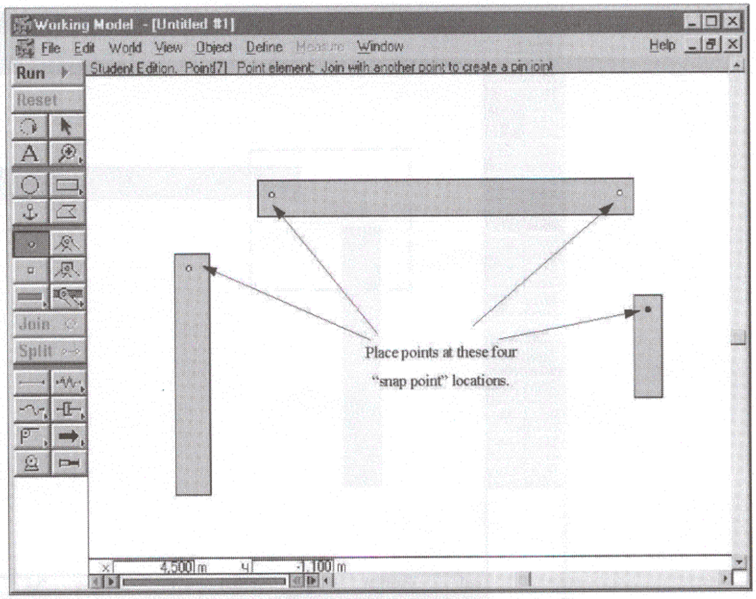


Figure 2.2 Point locations.

Step 4: Connecting the Points to Form Pin Joints

This step joins the points to create pin joints. A pin joint acts as a hinge between two bodies. The SmartEditor prevents joints from breaking during a drag operation.

1. Select the anchor tool.
2. Click on the horizontal link to anchor the link down.
   An anchor is used to tell the SmartEditor not to move this body during construction. After the pin joints have been created, the anchor will be deleted.
3. Select the pointer tool.
4. With the pointer tool selected, click and drag on the background to make a selection box that surrounds the two left points as shown in Figure 2.3. Release the mouse button, and the two points should now be highlighted (darkened).
   This method of selecting objects is called "box select." Any object that is contained completely within the box when the mouse is released is highlighted.
5. Click on the "Join" button in the tool bar which will highlight the object.
   The SmartEditor creates a pin joint between the two points selected, moving the unanchored link into place. The moved link may no longer be vertical. This is fixed in a moment.
6. Perform steps 4 and 5 for the two right points to create another pin joint.
   Once again, the horizontal link remains in this original position, and the SmartEditor moves the vertical link to create the pin joint.

Step 1: Open Working Model

1. Click on the Working Model program icon to start the program
   Working Model opens and begins running a demonstration dialog.
2. Explore Working Model using the introduction dialog boxes.
   The demonstration dialog boxes provide an overview of Working Model, customer examples, and ordering information.
3. Enter the Working Model application by selecting "Exit to Working Model"
   A tool bar appears on the left of the window and tape controls display along the bottom. The tool bar contains the options necessary to create simulations. The tape controls operate running and viewing simulations.

Step 2: Create the Links

This step creates the three moving links in a four-bar mechanism. The background serves as the fixed, fourth link.

1. Create a new Working Model document by selecting "New" from the "File" menu.
2. Construct the linkage by creating the three non-fixed links. Double click on the rectangle tool on the toolbar.
   The tool turns black, indicating that it can be used multiple times.
3. Using the rectangle tool, sketch out three bodies as shown in Figure 2.1.
   Rectangles are drawn by positioning the mouse at the first corner, click once, then move the mouse to the location of the opposite corner and click again. Rectangles are parametrically defined and their precise sizes will be specified later.
4. Open the "Properties box" and "Geometry box" in the "Window" menu.
   This displays information about the links and allows editing of this information.
5. Use the "Properties box" and "Geometry box" to change the center of the horizontal link to 0,0. Also change the geometry to a width of 8.5 and a height of 0.5.
   The shape and location of the rectangle should change upon data entry.
6. Likewise, use the "Properties box" and "Geometry box" to change the geometry of the long vertical link to a width of 0.5 and a height of 3. Also change the geometry of the short vertical link to a width of 0.5 and a height of 1.5.
   Again, the shape and location of the rectangle should change upon data entry.
7. Close both the "Properties box" and "Geometry box" windows.

Step 3: Place Points of Interest on the Links

This step teaches the usage of the "Object Snap" tool to place points precisely. The "Object Snap" tool highlights commonly used positions, like the center of a side, with an "X." When a point is placed using "Object Snap," the point's position is au-

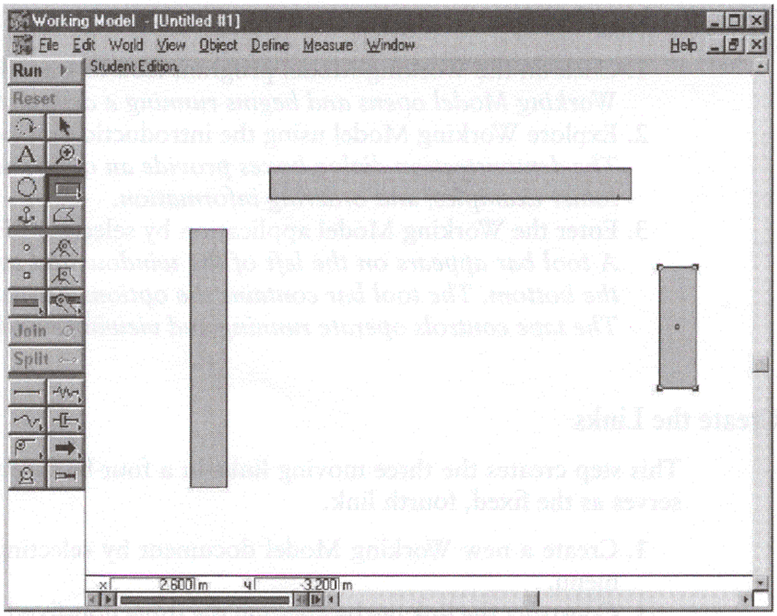


Figure 2.1 Three links sketched using the rectangle tool.

tomatically defined with parametric equations. These equations ensure that the point maintains its relative location even after resizing or other adjustments.

1. Double click on the point tool. The icon is a small circle and is directly below the anchor tool.
   The point tool turns black, indicating that it can be used multiple times without needing to be reselected before each new point is sketched.
2. Move the cursor over one of the links.
   Notice that an "X" appears around the pointer when it is centered on a side, over a corner, or over the center of a rectangle. This feature is called "Object Snap" and highlights the commonly-used parts of a link.
3. Place the cursor over the upper portion of one of the vertical links. When an "X" appears around the pointer (Figure 2.2), click the mouse button.
4. Place additional points as shown in Figure 2.2.
   Make sure that each of these points is placed at a "snap point" as evidenced with the "X" appearing at the pointer.
5. Select the pointer tool. The icon is an arrow pointed up and to the left.
6. Double click on one of the points that were sketched in Steps 3 or 4 to open the "Properties" window.
7. Edit the "Properties" window such that the points on the horizontal link are separated by 8 units. Edit the point on the long vertical link so that it is 1.5 units directly above the center of that link. Edit the point on the short vertical link so that it is 0.5 units directly above the center of that link.

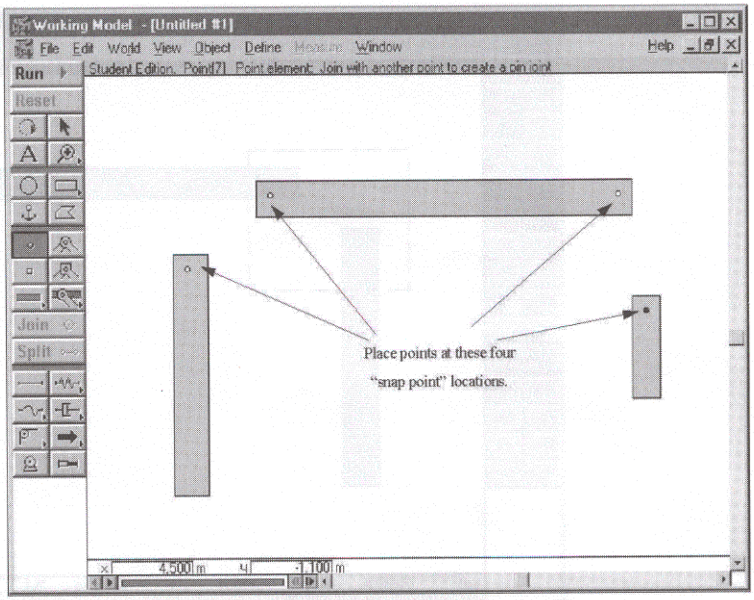


Figure 2.2 Point locations.

Step 4: Connecting the Points to Form Pin Joints

This step joins the points to create pin joints. A pin joint acts as a hinge between two bodies. The SmartEditor prevents joints from breaking during a drag operation.

1. Select the anchor tool.
2. Click on the horizontal link to anchor the link down.
   An anchor is used to tell the SmartEditor not to move this body during construction. After the pin joints have been created, the anchor will be deleted.
3. Select the pointer tool.
4. With the pointer tool selected, click and drag on the background to make a selection box that surrounds the two left points as shown in Figure 2.3. Release the mouse button, and the two points should now be highlighted (darkened).
   This method of selecting objects is called "box select." Any object that is contained completely within the box when the mouse is released is highlighted.
5. Click on the "Join" button in the tool bar which will highlight the object.
   The SmartEditor creates a pin joint between the two points selected, moving the unanchored link into place. The moved link may no longer be vertical. This is fixed in a moment.
6. Perform steps 4 and 5 for the two right points to create another pin joint.
   Once again, the horizontal link remains in this original position, and the SmartEditor moves the vertical link to create the pin joint.

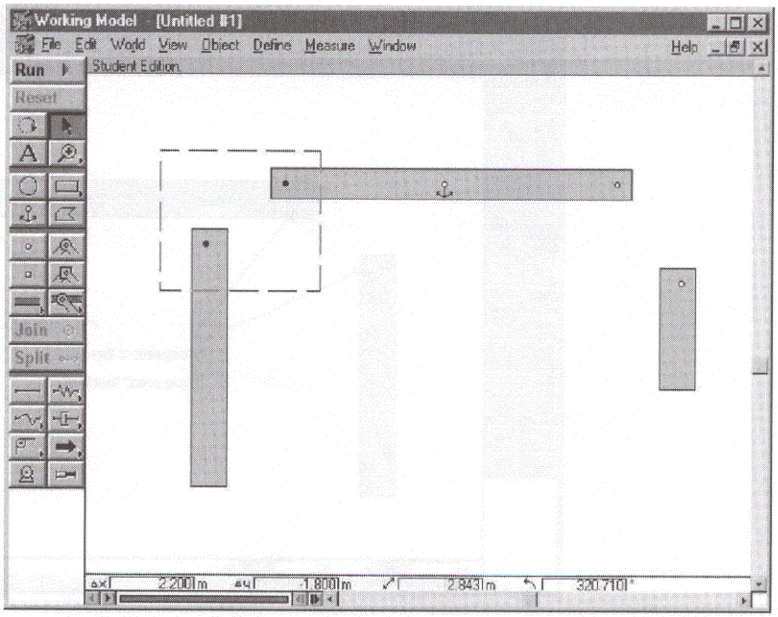


Figure 2.3 Select two points to join as a pin joint.

7. Select the left vertical link by clicking on it with the point tool. *Four black boxes appear around the link indicating that it has been selected. The boxes are called handles and can be dragged to resize an object.*
8. Using the coordinates bar at the bottom of the screen, enter a "0" in the f (rotation) field.
   The coordinates fields at the bottom of the screen are useful to obtain information on Working Model objects. These fields can also be used to edit object information. Changing the rotation to 0° adjusts the bar back to its original, vertical position.
9. If needed, complete Steps 7 and 8 on the right vertical link.
10. Select the anchor used to keep the horizontal link in position during building, and press the delete key to remove it.
    The anchor is no longer needed, and should be removed.
11. Select the "Pin Joint" tool and place another pin joint using the snap point, at point B as indicated in Figure 2.4. The "Pin Joint" tool appears as two links joined by a circle. It is the fifth icon in the right column.
    The "Pin Joint" tool is similar to the point tool used to create the last two pin joints. The pin tool automatically creates two points, attaches them to the bodies beneath the cursor (or the body and the background, as in this case), and creates a join in one seamless step. This pin joint joins the rectangle to the background.
12. Double click on the pin joint to open the "Properties" window. Edit the location of the joint to 1.5 units directly below the center of the long vertical link.

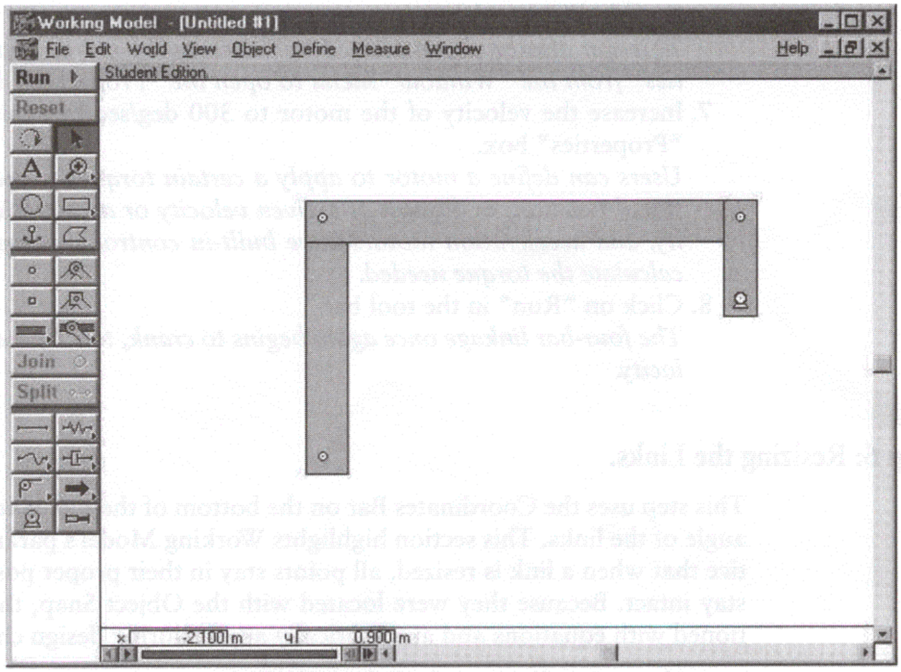


Figure 2.4 Adding the final pin joint and motor to the linkage.

Step 5: Adding a Motor to the Linkage

This step adds the motor to one of the links to drive the linkage.

1. Click on the motor tool in the tool box. This tool appears as a circle, sitting on a base and a point in its center.
   The motor tool becomes shaded, indicating that it has been selected. The cursor should now look like a small motor.
2. Place the cursor over the "snap point" on the right, vertical link at point D. Click the mouse.
   A motor appears on the four-bar linkage as shown in Figure 2.4. Similar to a pin joint, a motor has two attachment points. A motor automatically connects the top two bodies. If only one body were to lay beneath the motor, it would join the body to the background. A motor would then apply a torque between the two bodies to which it is pinned.
3. Double click on the motor to open the "Properties" box. Edit the location of the motor to place it 0.5 units directly below the short vertical link.
4. Click on "Run" in the tool bar.
   The four-bar linkage begins slowly cranking through its range of motion.
5. Click on "Reset" in the tool bar.
   The simulation will reset to frame 0.

6. Double-click on the motor to open the "Properties" box.
   This can also be accomplished by selecting the motor and choosing "Properties" from the "Window" menu to open the "Properties" box.
7. Increase the velocity of the motor to 300 deg/sec by typing this value in the "Properties" box.
   Users can define a motor to apply a certain torque, to move to a given rotational position, or to turn at a given velocity or acceleration. Rotation, velocity, and acceleration motors have built-in control systems that automatically calculate the torque needed.
8. Click on "Run" in the tool bar.
   The four-bar linkage once again begins to crank, this time at a much higher velocity.

Step 6: Resizing the Links.

This step uses the Coordinates Bar on the bottom of the screen to adjust the size and angle of the links. This section highlights Working Model's parametric features. Notice that when a link is resized, all points stay in their proper positions and all joints stay intact. Because they were located with the Object Snap, these points are positioned with equations and automatically adjust during design changes.

1. If not already selected, click on the Pointer tool.
2. Click once on the vertical left-hand link to select it.
3. Enter a slightly larger number in the "h" (height) box of the selected link in the Coordinates Bar at the bottom of the screen.
   The link resizes on the screen. Notice how the Smart Editor automatically resizes, repositions, and rebuilds the model based upon the parametric equations entered for each joint location.
4. Similarly, resize the other links and watch the Smart Editor rebuild the model.
   Different configurations of a model can be investigated using Working Model's parametric features.

Step 7: Measuring a Point's Position

1. Click on "Reset" in the tool bar.
   The simulation stops and resets to frame 0.
2. Select the point tool from the tool bar. It appears as a small, hollow circle.
   The point tool is located immediately to the left of the pin joint tool used earlier.
3. Place the cursor over the horizontal link of the four-bar linkage and press the mouse button.
   A point is attached to the bar. This is a single point, and does not attach the bar to the background. It is simply a "point of interest."
4. When a point is not already selected (darkened), select it by clicking on it.

5. Create a meter to measure the position of this point by choosing "Position" from the "Measure" menu.
   A new meter appears. Position meters default to display digital (numeric) information. A digital meter can be changed to a graph by clicking once on the triangle in the upper left hand corner.
6. Click on "Run" in the tool bar.
   The simulation immediately begins running and measurement information appears in the meter as shown in Figure 2.5. Meter data can be exported as an ASCII file, copied onto the clipboard and pasted into a spreadsheet program for further analysis. In this case, the spreadsheet would receive four columns of information: Time, X, Y and Rotation. One row would appear for each integration time step calculated.
7. Modify the simulation and rerun it.
   Working Model's seamless integration between the editing and running of the dynamics engine allows the user to quickly investigate many different simulation configurations. As an example, modify the mass of the horizontal bar using the "Properties" box, and rerun the simulation. The pin locations can be modified and links resized; then the velocities and forces can be measured. This four-bar linkage can even be investigated in zero gravity by turning off gravity under the "World" menu.

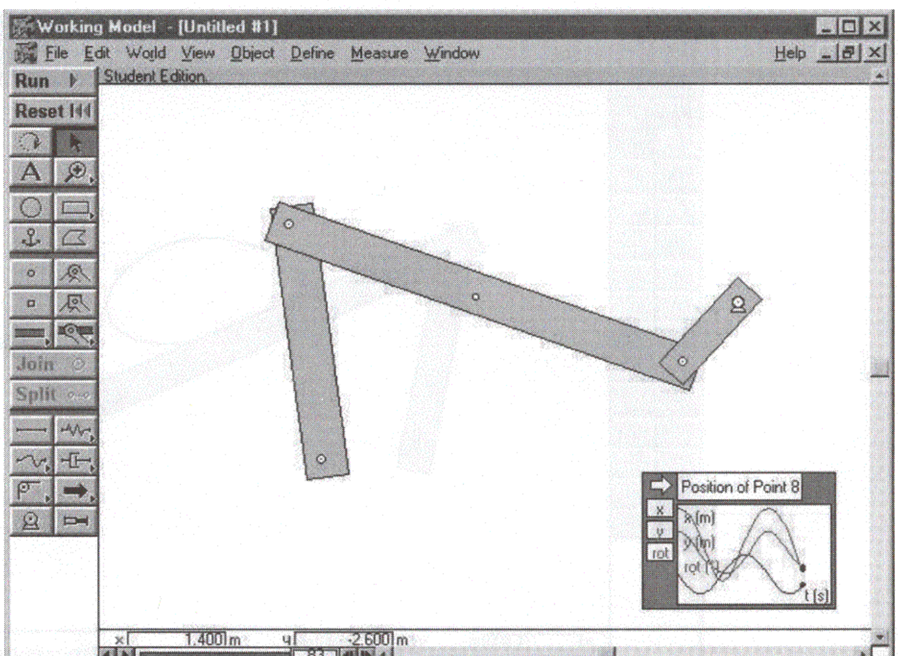


Figure 2.5 Running a simulation with a meter.

Step 8: Tracing the Path of a Point of Interest.

This step creates a trace of the movement of a selected point.

1. Select all objects using the box select method described earlier.
   All elements appear black.
2. Select the appearance option in the "Window" menu.
3. In the appearance window, turn off "Track Center of Mass," "Track Connect," and "Track Outline."
   These features can be turned off by clicking over the appropriate check mark.
4. Click on the background to deselect all objects.
5. Select only the point of interest created in Step 7.
   Only this point should appear black.
6. Select the appearance option in the "Window" menu.
7. In the appearance window, turn on "Track Connect." Make sure only the one point is selected.
   This feature can be turned on by clicking over the appropriate check mark.

Run the simulation. The screen should look like Figure 2.6.

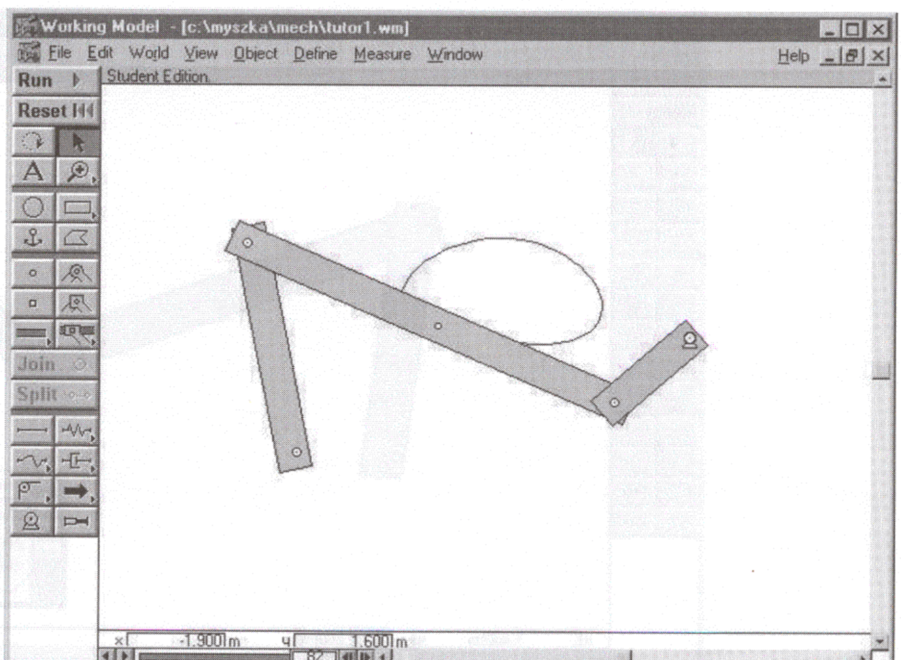


Figure 2.6 Tracing the path of a point.

Applying What has been Learned.

This demonstration illustrates how to create and run simple simulations in Working Model. The student is encouraged to experiment with this simulation, or create an original mechanism. Working Model has an incredible array of features that allows the creation of models to analyze the most complex mechanical devices.

2.5 USING WORKING MODEL® TO MODEL A SLIDER-CRANK MECHANISM

This section serves as a tutorial to create a slider-crank mechanism. It should be followed while actually using Working Model. Again the student is encouraged to experiment with the software to perform other analyses.

Step 1: Open Working Model as in Step 1 of the previous section.

Step 2: Create the Links

This step creates the three moving links in the slider-crank mechanism. Again, the background serves as the fixed, fourth link.

1. Create a new Working Model document by selecting "New" from the "File" menu.
2. Construct the linkage by creating the three non-fixed links. Double click on the rectangle tool in the toolbar.
   The tool turns black, indicating that it can be used multiple times.
3. Using the rectangle tool, sketch out three bodies as shown in Figure 2.7.
   Position the mouse at the first corner, click once, then move the mouse to the location of the opposite corner and click again. Rectangles are parametrically defined and their precise sizes are specified later.

Step 3: Use the Slot Joint to Join the Sliding Link to the Background

1. Click and hold down on the slot joint button. The icon is in the right-hand column above the "Join" button.
2. Select the "keyed slot" icon. The icon appears as a rectangle inside a slot.
3. Move the cursor over the center of the rectangular sliding link and click the mouse button. The screen should look like Figure 2.8.
4. Select the pointer tool.
5. Double click on the slot.
   This opens the "Properties" window for the slot.
6. Change the angle to −45°.
   The incline of the slot changes.

Drag the other links until the screen appears similar to Figure 2.9.

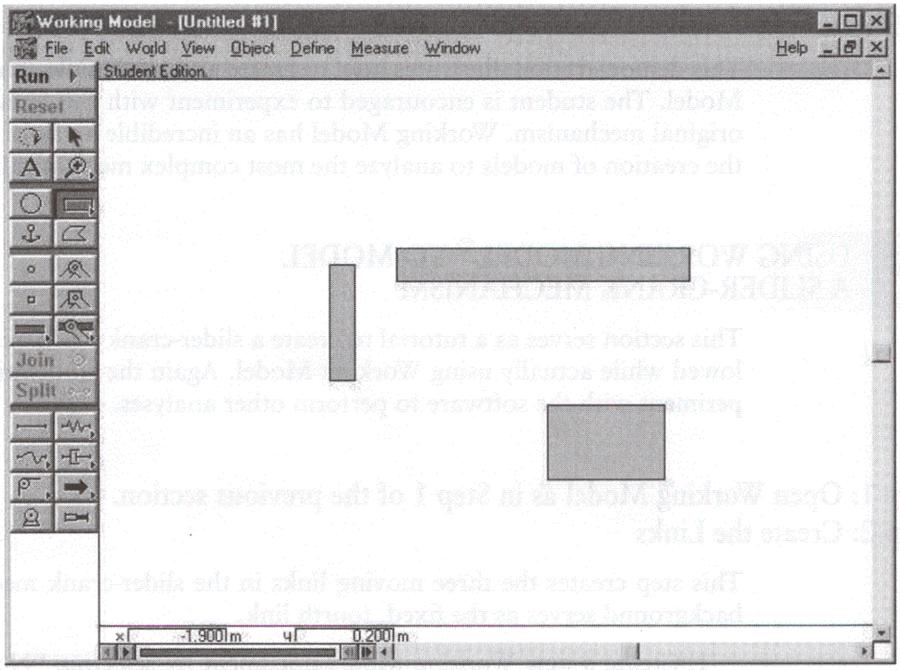


Figure 2.7 Three links sketched using the rectangle tool.

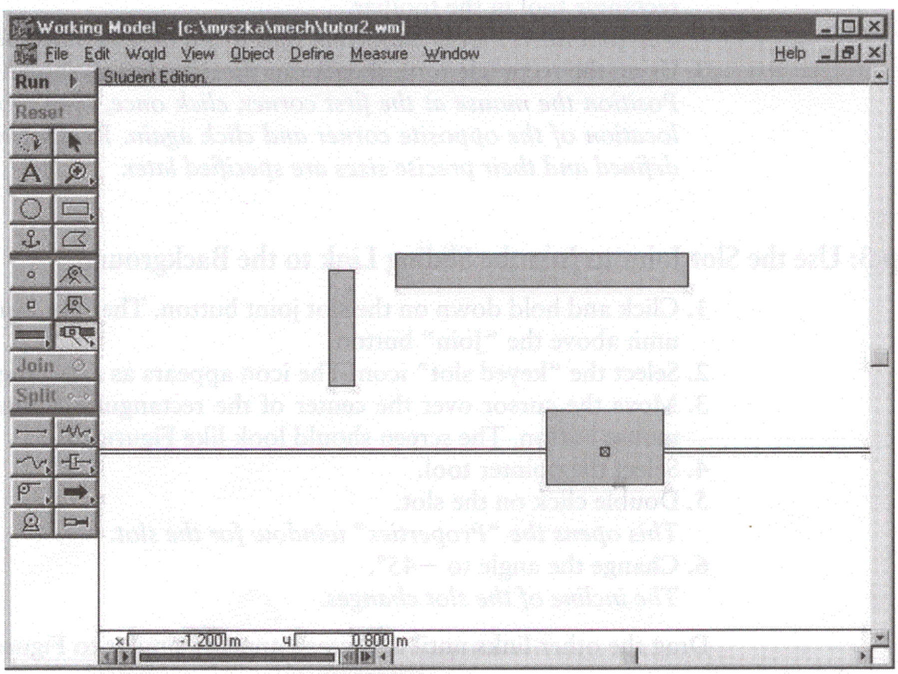


Figure 2.8 Point and slot location.

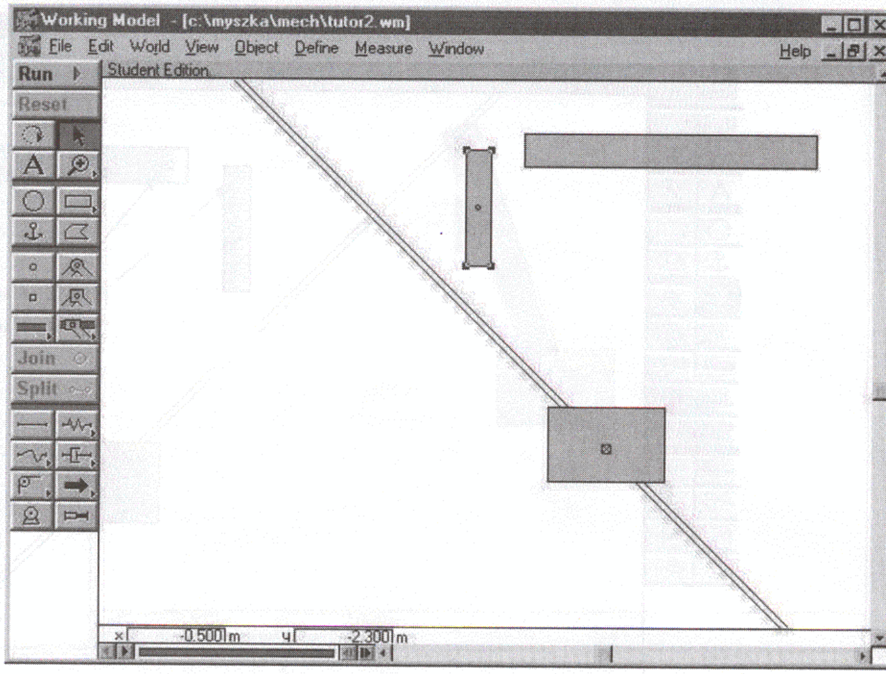


Figure 2.9 Sliding joint.

Step 4: Connecting Other Links to Form Pin Joints

This step creates points, and joins them to create pin joints. A pin joint acts as a hinge between two bodies.

1. Select the anchor tool.
2. Click on the vertical link to anchor the link down.
   An anchor tells the SmartEditor not to move this body during construction. After the pin joints have been created, the anchor will be deleted.
3. Double click on the point tool. The icon is a small circle and is directly below the anchor tool.
   The point tool turns black, indicating that it can be used multiple times without needing to be reselected before each new point is sketched.
4. Place the cursor over the upper portion of one of the vertical links. When an "X" appears around the pointer (Figure 2.10), click the mouse button.
5. Place additional points at the ends of the horizontal link as shown in Figure 2.10.
   Make sure that each of these points is placed at a "snap point" as evidenced with the "X" appearing at the pointer.
6. Place another point at the center of the sliding rectangle.
   This point is used to create a pin joint to the coupler.

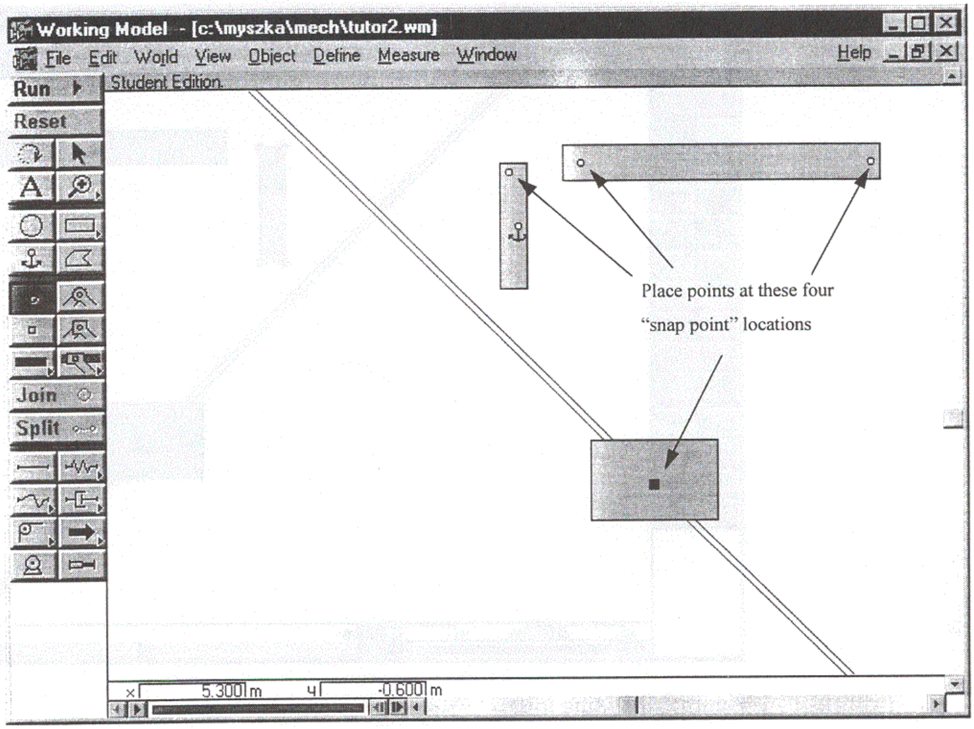


Figure 2.10 Placing points on the other links.

7. Select the pointer tool.
8. With the pointer tool selected, click on one point that will be connected with a pin joint. Then, holding down the Shift key, click on the second point that will form a pin joint. Notice that the two points should now be highlighted (darkened).
9. Click on the "Join" button in the tool bar, which will highlight it.
   The SmartEditor creates a pin joint between the two selected points, moving the unanchored link into place. The moved link may no longer be vertical. This will be fixed in a moment.
10. Perform Steps 8 and 9 for the other two points that will create another pin joint. The screen will appear similar to Figure 2.11.
    Once again, the vertical link remains in this original position, and the SmartEditor moves the vertical link to create the pin joint.
11. Click on the vertical link.
    Four black boxes appear around the link indicating that it has been selected.
12. Select the "Move to front" option in the "Object" window.
    This places the vertical link in front of the connecting link, making the anchor visible.
13. Select the anchor used to keep the horizontal link in position during building, and press the Delete key to remove it.
    The anchor is no longer needed, and should be removed.

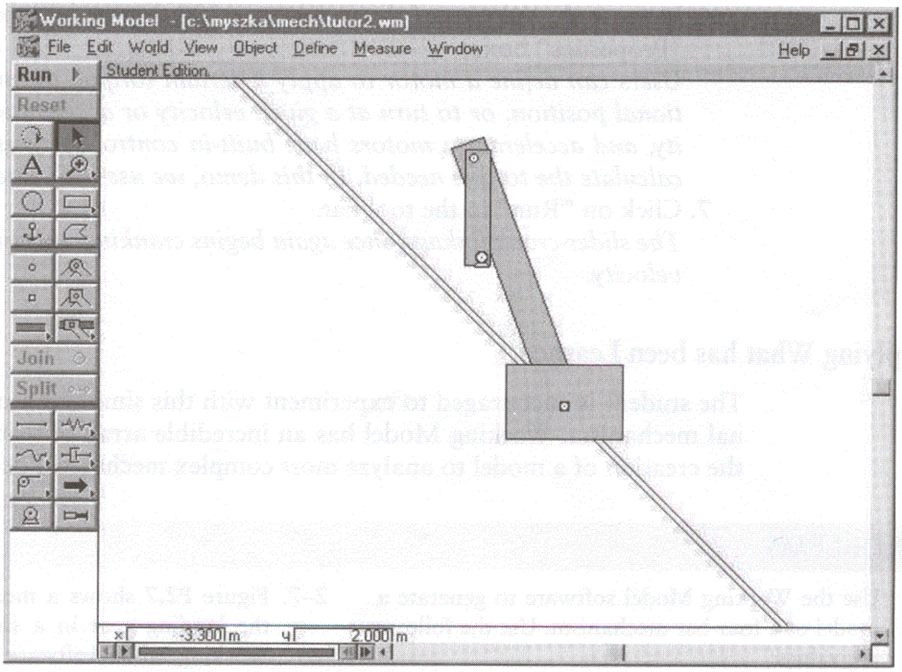


Figure 2.11 Adding the pin joints and motor to the linkage.

Step 5: Adding a Motor to the Linkage

This step adds the motor to one of the links to drive the linkage.

1. Click on the motor tool in the tool box. This tool appears as a circle, sitting on a base with a point in its center.
   The motor tool becomes shaded, indicating that it has been selected. The cursor should now look like a small motor.
2. Place the cursor over the "snap point" on the vertical link. Click the mouse.
   A motor appears on the slider-crank linkage as shown in Figure 2.11. Similar to a pin joint, a motor has two attachment points. A motor automatically connects the top two bodies. If only one body were to lay beneath the motor, the motor would join the body to the background. The motor then applies a torque between the two bodies to which it is pinned.
3. Click on "Run" in the tool bar.
   The slider-crank linkage begins slowly cranking through its range of motion.
4. Click on "Reset" in the tool bar.
   The simulation resets to frame 0.
5. Double-click on the motor to open the "Properties" box.
   This can also be accomplished by selecting the motor and choosing "Properties" from the "Window" menu to open the "Properties" box.

6. Increase the velocity of the motor to −300 deg/sec by typing this value in the "Properties" box.
   Users can define a motor to apply a certain torque, to move to a given rotational position, or to turn at a given velocity or acceleration. Rotation, velocity, and acceleration motors have built-in control systems that automatically calculate the torque needed. In this demo, we use the velocity motor.
7. Click on "Run" in the tool bar.
   The slider-crank linkage once again begins cranking, this time at a much higher velocity.

Applying What has been Learned

The student is encouraged to experiment with this simulation or to create an original mechanism. Working Model has an incredible array of features that allows for the creation of a model to analyze most complex mechanical devices.

PROBLEMS

Use the Working Model software to generate a model of a four-bar mechanism. Use the following values:

2–1. frame = 9 in; crank = 1 in; coupler = 10 in; follower = 3.5 in; crank speed = 200 rad/sec

2–2. frame = 100 mm; crank = 12 mm; coupler = 95 mm; follower = 24 mm; crank speed = 30 rad/sec

2–3. frame = 2 ft.; crank = 0.5 ft; coupler = 2.1 ft; follower = 0.75 ft; crank speed = 25 rpm

Use the Working Model software that was supplied with the text to generate a model of a slider-crank mechanism. Use the following values:

2–4. offset = 0 in; crank = 1.45 in; coupler = 4.5 in; crank speed = 200 rad/sec

2–5. offset = 0 mm; crank = 95 mm; coupler = 350 mm; crank speed = 200 rad/sec

2–6. offset = 50 mm ; crank = 95 mm; coupler = 350 mm; crank speed = 200 rad/sec

2–7. Figure P2.7 shows a mechanism that operates the landing gear in a small airplane. Use the Working Model software to generate a model of this linkage. The motor operates clockwise, at a constant rate of 20 rpm.

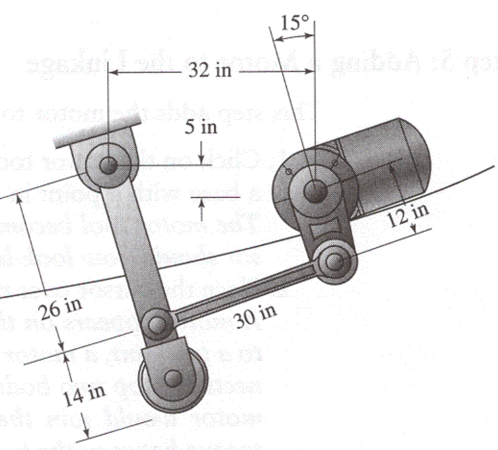


Figure P2.7 Problem 7.

2–8. Figure P2.8 shows a mechanism that operates a coin operated, child's amusement ride. Use the Working Model software to generate a model of this linkage. The motor operates counter clockwise, at a constant rate of 60 rpm.

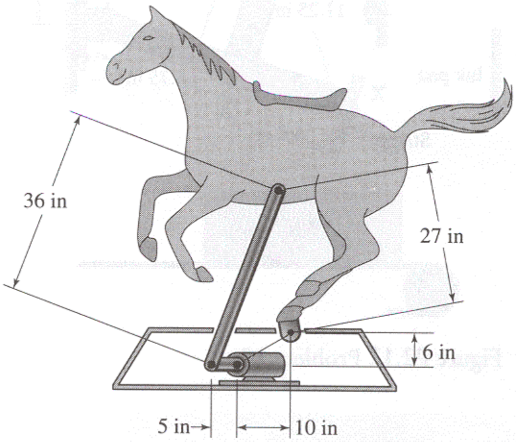


Figure P2.8 Problem 8.

2–9. Figure P2.9 shows a transfer mechanism that lifts crates from one conveyor to another. Use the Working Model software to generate a model of this linkage. The motor operates counter clockwise, at a constant rate of 120 rpm.

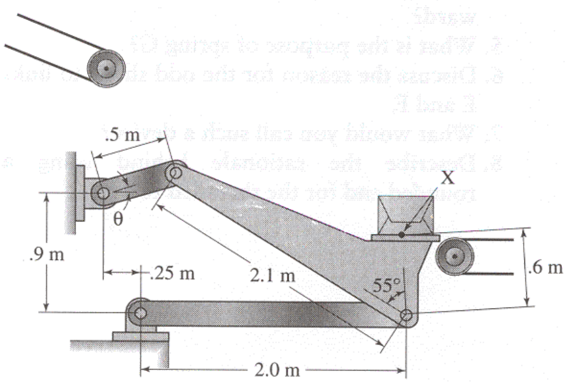


Figure P2.9 Problem 9.

2–10. Figure P2.10 shows a another transfer mechanism that pushes crates from one conveyor to another. Use the Working Model software to generate a model of this linkage. The motor operates clockwise, at a constant rate of 40 rpm.

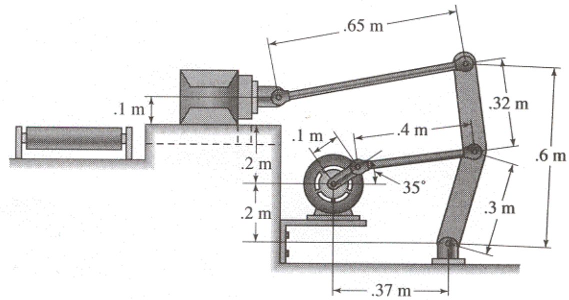


Figure P2.10 Problem 10.

2–11. Figure P2.11 shows a yet another transfer mechanism that lowers crates from one conveyor to another. Use the Working Model software to generate a model of this linkage. The cylinder extends, at a constant rate of 1 fpm.

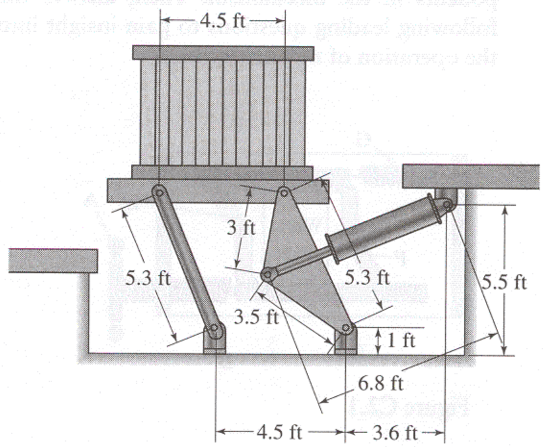


Figure P2.11 Problem 11.

2–12. Figure P2.12 shows a mechanism that applies labels to packages. Use the Working Model software to generate a model of this linkage. The motor operates counter clockwise, at a constant rate of 300 rpm.

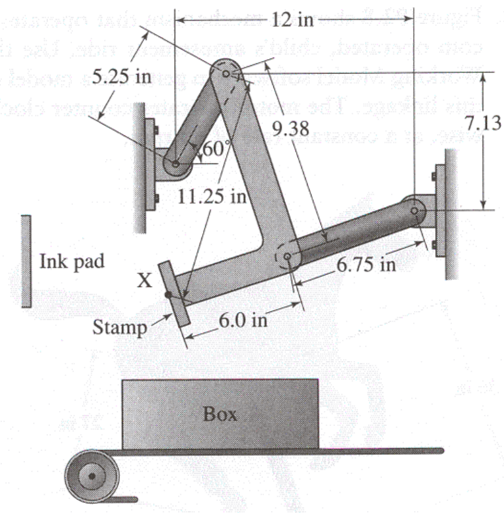


Figure P2.12 Problem 12.

CASE STUDIES

2–1. The mechanism shown in figure C2.1 is a top view of a fixture in a machining operation. Carefully examine the configuration of the components in the mechanism. Then, answer the following leading questions to gain insight into the operation of the mechanism.

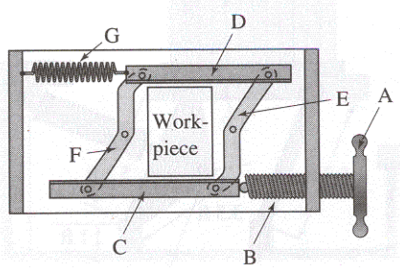


Figure C2.1

1. As the handle A is turned, moving the threaded rod B to the left, describe the motion of grip C.
2. As the handle A is turned, moving the threaded rod B to the left, describe the motion of grip D.
3. What is the purpose of this mechanism?
4. What action would cause link D to move upward?
5. What is the purpose of spring G?
6. Discuss the reason for the odd shape to links E and F.
7. What would you call such a device?
8. Describe the rationale behind using a rounded end for the threaded rod B.

3 Vectors

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Differentiate between a scalar quantity and a vector.
2. Apply the appropriate trigonometry principles to a right triangle.
3. Apply the appropriate trigonometry principles to a general triangle.
4. Determine the resultant of two vectors, using both graphical and analytical methods.
5. Resolve vector quantities into components in the horizontal and vertical directions.
6. Subtract two vectors, using both graphical and analytical methods.
7. Manipulate vector equations.
8. Utilize a vector equation to determine the magnitude of two vectors.

3.1 INTRODUCTION

Mechanism analysis involves manipulating vector quantities. Displacement, velocity, acceleration, and force typically are the desired properties of a mechanism, and are all vectors. Prior to working with mechanisms, a thorough introduction to vectors and vector manipulation is in order. In this chapter, both graphical and analytical solution techniques are presented. Students who have completed a mechanics course may omit this chapter or use it as a reference to review vector manipulation.

3.2 SCALARS AND VECTORS

In the analysis of mechanisms, two types of quantities need to be distinguished. A *scalar* is a quantity that is sufficiently defined by simply stating a magnitude. By saying "a dozen donuts," one describes the quantity of donuts in a box. Because only a number fully defines the amount of donuts in the box, an amount is a scalar quantity. The following are additional examples of scalar quantities: a board is 8 ft long;, a class meets for 50 min; or the temperature is 78° F. Length, time and temperature are all scalar quantities.

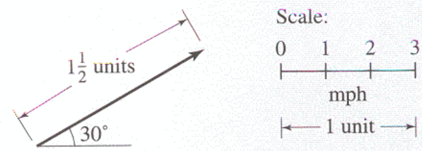


Figure 3.1 A 45 mph velocity vector.

In contrast, a *vector* is not fully defined by stating only a magnitude. Indicating the direction of the quantity is also required. Stating that a golf ball traveled 200 yards does not fully describe its path. Neglecting to express the direction of travel hides the fact that the ball has landed in a lake. Thus, the direction must be included to fully describe such a quantity. Examples of properly stated vectors include "the crate is being pulled to the right with 5 lb or "the train is traveling at a speed of 50 mph in a northerly direction." Displacement, force, and velocity are vector quantities.

The common notation used to describe a vector is with a line segment having an arrowhead placed at one end. With a graphical approach to analysis, the length of the line segment is drawn proportional to the magnitude of the quantity that the vector describes. The direction is defined by the arrowhead and the incline of the line with respect to some reference axis. Figure 3.1 shows a fully defined velocity vector.

3.3 GRAPHICAL VECTOR ANALYSIS

Much of the work involved in the study of mechanisms and analysis of vectors involves geometry. Often graphical methods are employed in such analyses because the motion of a mechanism can be clearly visualized. For more complex mechanisms, analytical calculations involving vectors also become laborious.

A graphical approach to analysis involves drawing scaled lines at specific angles. To achieve results that are consistent with analytical techniques, accuracy must be a major objective. For several decades, accuracy in mechanism analyses was obtained with attention to precision and proper drafting equipment. Even with its popularity, many scorned graphical techniques as being imprecise. However, the development of computer-aided design (CAD), and its accurate geometric constructions, has allowed graphical techniques to be applied with precision.

3.4 DRAFTING TECHNIQUES REQUIRED IN GRAPHICAL VECTOR ANALYSIS

The methods of graphical mechanism and vector analysis are identical, whether using drafting equipment or a CAD package. Although it may be an outdated mode in industrial analyses, drafting can be successfully employed to learn and understand the techniques.

For those using drafting equipment, fine lines and circular arcs are demanded to produce accurate results. Precise linework is needed to accurately determine intersection points. Thus, care must be taken in maintaining sharp drawing equipment.

Accurate measurement is as important as line quality. The length of the lines must be drawn to a precise scale and linear measurements should be made as accurately as possible. Therefore, a proper engineering scale with inches divided into fifty parts is desired. Angular measurements must be equally precise.

Lastly, a wise choice of a drawing scale is also a very important factor. Typically, the larger the construction, the more accurate the measured results are. Drawing precision to 0.05 in. produces less error when the line is 10 in long, as opposed to 1 in. Limits in size do exist in that very large constructions require special equipment. However, an attempt should be made to create constructions as large as possible.

A drawing textbook should be consulted for the details of general drafting techniques and geometric constructions.

3.5 CAD KNOWLEDGE REQUIRED IN GRAPHICAL VECTOR ANALYSIS

As stated, the methods of graphical mechanism and vector analysis are identical, whether using drafting equipment or a CAD package. CAD allows for greater precision. Fortunately, only a limited level of proficiency on a CAD system is required to properly complete graphical vector analysis. Therefore, utilization of a CAD system is preferred, and should not require a large investment on a "learning curve."

As mentioned, the graphical approach of vector analysis involves drawing lines at precise lengths and specific angles. The following list outlines the CAD abilities required for vector analysis. The user should be able to:

- Draw lines at a specified length and angle;
- Insert lines, perpendicular to existing lines;
- Extend existing lines to the intersection of another line;
- Trim lines at the intersection of another line;
- Draw arcs, centered at a specified point, with a specified radius;
- Locate the intersection of two arcs;
- Measure the length of existing lines;
- Measure the included angle between two lines.

Of course, proficiency beyond the items listed above facilitates more efficient analysis. However, familiarity with CAD commands that accomplish these actions is sufficient to accurately complete vector analysis.

3.6 TRIGONOMETRY REQUIRED IN ANALYTICAL VECTOR ANALYSIS

In the analytical analysis of vectors, knowledge of basic trigonometry concepts is required. Trigonometry is the study of the properties of triangles. The first type of triangle examined is the right triangle.

3.6.1 Right Triangle

In performing vector analysis, the use of the basic trigonometric functions is vitally important. The basic trigonometric functions apply only to right triangles. Figure 3.2 illustrates a right triangle with sides denoted as a, b, and c, and interior angles as A, B, and C. Note that angle C is a 90° right angle. Therefore, the triangle is called a right triangle.

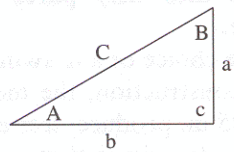


Figure 3.2 The right triangle.

The basic trigonometric relationships are defined as:

(3.1) $$\text{sine}\angle A = \sin\angle A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

(3.2) $$\text{cosine}\angle A = \cos\angle A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

(3.3) $$\text{tangent}\angle A = \tan\angle A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b}$$

These definitions can also be applied to angle B:

$$\sin\angle B = \frac{b}{c}$$

$$\cos\angle B = \frac{a}{c}$$

$$\tan\angle B = \frac{b}{a}$$

The Pythagorean theorem gives the relationship of the three sides of a right triangle. For the triangle shown in Figure 3.2, it is defined as:

(3.4) $$a^2 + b^2 = c^2$$

Finally, the sum of all angles in a triangle is 180°. Knowing that angle C is 90°, the sum of the other two angles must be:

(3.5) $$\angle A + \angle B = 90°$$

EXAMPLE PROBLEM 3.1

Figure 3.3 shows a front loader with cylinder BC in a vertical position. Determine the required length of the cylinder to orient arm AB in the shown configuration.

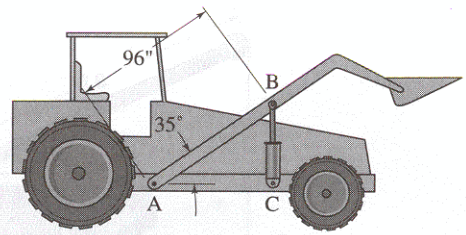


Figure 3.3 Front loader for Example Problem 3.1.

Solution:

The triangle side, BC can be found using Equation 3.1

$$\sin\angle A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin 35° = \frac{BC}{(96\text{ in})}$$

solving:

$$BC = (96\text{ in})\sin 35° = 55.06\text{ in.}$$

Notice that the distance between A and C can similarly be determined using Equation 3.2. Thus:

$$\cos\angle A = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\cos 35° = \frac{AC}{(96\text{ in})}$$

solving:

$$AC = (96\text{ in})\cos 35° = 78.64\text{ in.}$$

EXAMPLE PROBLEM 3.2

Figure 3.4 shows a tow truck with an 8-ft boom, which is inclined at a 25° angle. Determine the horizontal distance that the boom extends from the truck.

Solution:

The horizontal projection of the boom can be determined from Equation 3.2:

$$\cos 25° = \frac{\text{horizontal projection}}{(8\text{ ft})}$$

$$\text{horizontal projection} = (8\text{ ft})\cos 25° = 7.25\text{ ft.}$$

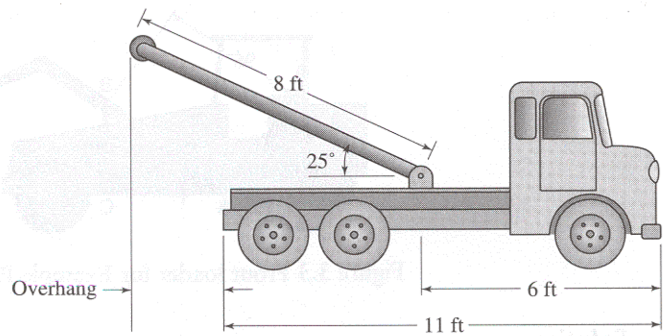


Figure 3.4 Tow truck for Example Problem 3.3.

The horizontal distance from the front end of the truck to the end of the boom is:

$$6 \text{ ft} + 7.25 \text{ ft} = 13.25 \text{ ft}$$

Because the overall length of the truck is 11 ft, the horizontal distance that the boom extends from the truck is:

$$13.25 \text{ ft} - 11 \text{ ft} = 2.25 \text{ ft}.$$

3.6.2 Oblique Triangle

In the previous discussion, the analysis was restricted to right triangles. An approach to general or oblique triangles is also important in the study of mechanisms. Figure 3.5 shows a general triangle. Again, a, b, and c denote the length of the sides and $\angle A$, $\angle B$, and $\angle C$ represent the interior angles.

For this general case, the basic trigonometric functions described in the previous section are not applicable. To analyze the general triangle, the law of sines and the law of cosines have been developed. The *law of sines* can be stated as:

(3.6)
$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$

The *law of cosines* can be stated as:

(3.7)
$$c^2 = a^2 + b^2 - 2ab\cos\angle C$$

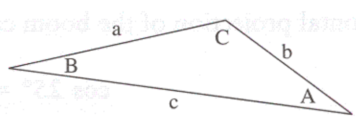


Figure 3.5 The oblique triangle.

In addition, the sum of all interior angles in a general triangle must total 180°. Stated in terms of Figure 3.4 the equation would be:

(3.8) $$\angle A + \angle B + \angle C = 180°$$

Problems involving the solution of a general triangle fall into one of four cases: *Case 1: Given one side (a) and two angles (∠A and ∠B).*

To solve a problem of this nature, Equation 3.8 can be used to find the third angle:

$$\angle C = 180° - \angle A - \angle B$$

The law of sines can be rewritten to find the remaining sides.

$$b = a\left\{\frac{\sin\angle B}{\sin\angle A}\right\}$$

$$c = a\left\{\frac{\sin\angle C}{\sin\angle A}\right\}$$

Case 2: Given two sides (a and b) and the angle opposite to one of the sides (∠A).

To solve a Case 2 problem, the law of sines can be used to find the second angle. Equation 3.6 is rewritten as :

$$\angle B = \sin^{-1}\left\{\left(\frac{b}{a}\right)\sin\angle A\right\}$$

Equation 3.8 can be used to find the third angle:

$$\angle C = 180° - \angle A - \angle B$$

The law of cosines can be used to find the third side. Equation 3.7 is rewritten as:

$$c = \sqrt{\{a^2 + b^2 - 2ab\cos\angle C\}}$$

Case 3: Given two sides (a and b) and the included angle (∠C).

To solve a Case 3 problem, the law of cosines can be used to find the third side:

$$c = \sqrt{\{a^2 + b^2 - 2ab\cos\angle C\}}$$

The law of sines can be used to find a second angle. Equation 3.6 is rewritten as:

$$\angle A = \sin^{-1}\left\{\left(\frac{a}{c}\right)\sin\angle C\right\}$$

Equation 3.8 can be used to find the third angle:

$$\angle B = 180° - \angle A - \angle C$$

Case 4: Given three sides.

To solve a Case 4 problem, the law of cosines can be used to find an angle. Equation 3.7 is rewritten as:

$$\angle C = \cos^{-1}\left\{\frac{(a^2 + b^2 - c^2)}{2ab}\right\}$$

The law of sines can be used to find a second angle. Equation 3.6 is rewritten as:

$$\angle A = \sin^{-1}\left\{\left(\frac{a}{c}\right)\sin\angle C\right\}$$

Equation 3.8 can be used to find the third angle:

$$\angle B = 180° - \angle A - \angle C$$

Once familiarity in solving problems involving general triangles is gained, referring to the specific cases will be unnecessary.

EXAMPLE PROBLEM 3.3

Figure 3.6 shows a front loader. Determine the required length of the cylinder to orient arm AB in the shown configuration.

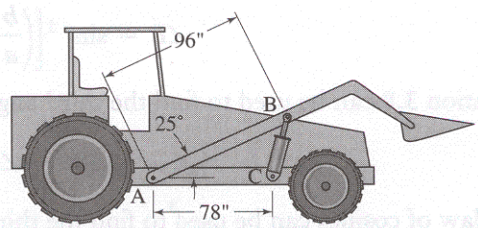


Figure 3.6 Front loader for Example Problem 3.3.

Solution:

By focusing on the triangle created by points A, B, and C, it is apparent that this is a Case 3 problem. The third side can be found by using the law of cosines:

$$\begin{aligned} c &= \sqrt{\{a^2 + b^2 - 2ab\cos\angle C\}} \\ &= \sqrt{\{(78 \text{ in})^2 + (96 \text{ in})^2 - 2(78 \text{ in})(96 \text{ in})\cos 25°\}} \\ &= 41.55 \text{ in.} \end{aligned}$$

Because it was not required to determine the remaining angles, the procedure described for Case 3 problems will not be completed.

EXAMPLE PROBLEM 3.4

Figure 3.7 shows the drive mechanism for an engine system. Determine the crank angle as shown in the figure.

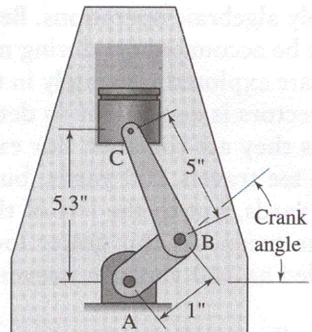


Figure 3.7 Engine linkage for Example Problem 3.4.

Solution:

By focusing on the triangle created by points A, B, and C, it is apparent that this is a Case 4 problem. Angle A can be determined by redefining the variables in the law of cosines:

$$\angle A = \cos^{-1}\left\{\frac{b^2 + c^2 - a^2}{2cb}\right\}$$

$$= \cos^{-1}\left\{\frac{(5.3\text{ in})^2 + (1\text{ in})^2 - (5\text{ in})^2}{2(5.3\text{in})(1\text{ in})}\right\} = 67.3^\circ$$

This angle is defined between side AC (the vertical side) and leg AB. Because the crank angle is defined from a horizontal axis, the crank angle can be determined by the following:

$$\text{Crank angle} = 90^\circ - 67.3^\circ = 22.7^\circ$$

Although not required in this problem, angle C can be determined by:

$$\angle C = \sin^{-1}\left\{\left(\frac{c}{a}\right)\sin\angle A\right\}$$

$$= \sin^{-1}\left\{\left(\frac{1\text{ in}}{5\text{ in}}\right)\sin 67.3^\circ\right\} = 10.6^\circ$$

Finally, angle B can be found by:

$$\angle B = 180^\circ - 67.3^\circ - 10.6^\circ = 102.1^\circ$$

3.7 VECTOR MANIPULATION

Throughout the analysis of mechanisms, vector quantities (e.g., displacement or velocity) must be manipulated in different ways. In a similar manner to scalar quantities, vectors can be added and subtracted. However, unlike scalar quantities, these are not simply algebraic operations. Because it is also required to define a vector, direction must be accounted for during mathematical operations. Vector addition and subtraction are explored separately in the following sections.

Adding vectors is equivalent to determining the combined or net effect of two quantities as they act together. For example, in playing a round of golf, the first shot off the tee travels 200 yards, but veers off to the right. A second shot then travels 120 yards, but to the left of the hole. A third shot of 70 yards places the golfer on the green. As this golfer looks on the score sheet, she notices that the hole is labeled as 310 yards; however, her ball traveled 390 yards (200 + 120 + 70 yards).

As repeatedly stated, the direction of a vector is just as important as the magnitude. During vector addition, 1 + 1 does not always equal 2; it depends on the direction of the individual vectors.

3.8 GRAPHICAL VECTOR ADDITION (+>)

Graphical addition is an operation that determines the net effect of vectors. A graphical approach to vector addition involves drawing the vectors to scale and at the proper orientation. These vectors are then relocated, maintaining the scale and orientation. The tail of the first vector is designated as the origin (point O). The second vector is relocated so that its tail is placed on the tip of the first vector. The process then is repeated for all remaining vectors. This technique is known as the *tip-to-tail* method of vector addition. The name is derived from viewing a completed vector polygon. The tip of one vector runs into the tail of the next.

The combined effect is the vector that extends from the tail of the first vector in the series to the tip of the last vector in the series. Mathematically, an equation can be written that represents the combined effect of vectors:

$$R = A +> B +> C +> D +> \ldots$$

Vector R is a common notation used to represent the resultant of a series of vectors. A *resultant* is a term used to describe the combined effect of vectors. Also note that the symbol +> is used to identify vector addition and to differentiate it from algebraic addition.

It should be noted that vectors follow the commutative law of addition; that is, the order in which the vectors are added does not alter the result. Thus:

$$R = (A +> B +> C) = (C +> B +> A) = (B +> A +> C) = \ldots$$

The process of combining vectors can be completed graphically, using either manual drawing techniques or CAD software. Whatever method is used, the underlying concepts are identical. The following Example Problems illustrate this concept.

EXAMPLE PROBLEM 3.5

Determine the combined effect of velocity vectors *A* and *B*, shown in Figure 3.8.

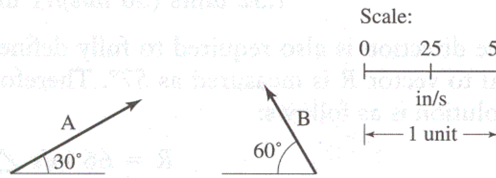


Figure 3.8 Vectors for Example Problem 3.5.

Solution:

The physical length of vector *A* is 1.18 units long. Thus, it represents a velocity of:

$$1.18 \text{ units } (50 \text{ in/s})(1 \text{ unit}) = 59 \text{ in/s}$$

Vector *B* is 0.76 units long. It represents a velocity of:

$$0.76 \text{ units } (50 \text{ in/s})(1 \text{ unit}) = 30 \text{ in/s}$$

To determine the resultant, the vectors must be relocated so that the tail of *B* is located at the tip of *A*. To verify the commutative law, the vectors were redrawn so that the tail of *A* is placed at the tip of *B*. The resultant is the vector drawn from the tail of the first vector, the origin, to the tip of the second vector. Both vector diagrams are shown in Figure 3.9.

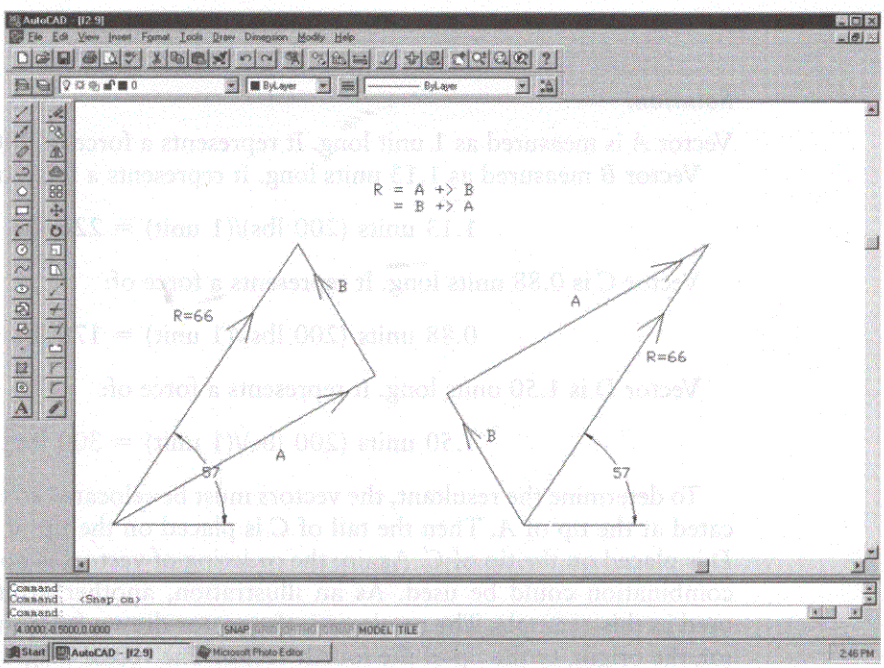


Figure 3.9 The combined effect of vectors *A* & *B* for Example Problem 3.5.

The length vector R is measured as 1.40 units; therefore, the resultant represents a velocity of:

$$1.32 \text{ units } (50 \text{ in/s})/(1 \text{ unit}) = 66 \text{ in/s}$$

The direction is also required to fully define vector R. The angle from the horizontal to vector R is measured as 57°. Therefore, the proper manner of presenting the solution is as follows:

$$R = 66 \text{ in/s } \angle 57°$$

EXAMPLE PROBLEM 3.6

Determine the combined effect of force vectors A, B, C, and D, shown in Figure 3.10.

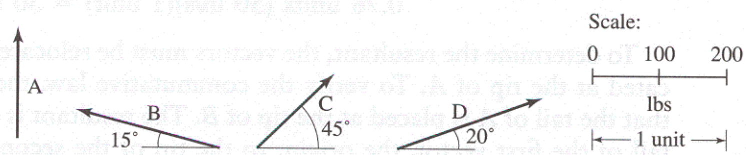


Figure 3.10 Vectors for Example Problem 3.6.

Solution:

Vector A is measured as 1 unit long. It represents a force of 200 lbs.

Vector B measured as 1.13 units long. It represents a force of:

$$1.13 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 226 \text{ lbs}$$

Vector C is 0.88 units long. It represents a force of:

$$0.88 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 176 \text{ lbs}$$

Vector D is 1.50 units long. It represents a force of:

$$1.50 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 300 \text{ lbs}$$

To determine the resultant, the vectors must be relocated so that the tail of B is located at the tip of A. Then the tail of C is placed on the tip of B. Finally, the tail of D is placed on the tip of C. Again, the ordering of vectors is not important, and any combination could be used. As an illustration, another arbitrary combination is used in this example. The resultant is the vector drawn from the tail of the first vector, the origin, to the tip of the fourth vector. The vector diagrams are shown in Figure 3.11.

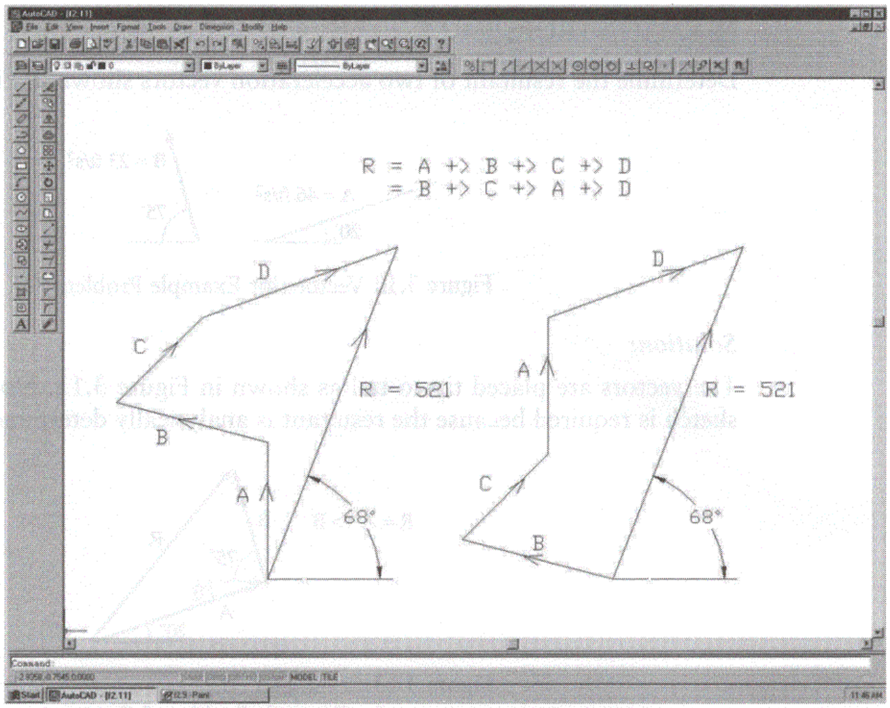


Figure 3.11 The combined effect of vectors *A*, *B*, *C* & *D* for Example Problem 3.6.

The length vector *R* is measured as 2.61 units; therefore, the resultant represents a velocity of:

$$2.61 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 521 \text{ lbs}$$

The direction is also required to fully define the vector *R*. The angle from the horizontal to vector *R* is measured as 68°. Therefore, the proper manner of presenting the solution is as follows:

$$R = 521 \text{ lbs } \angle 68°$$

3.9 ANALYTICAL VECTOR ADDITION (+>): TRIANGLE METHOD

Two analytical methods can be used to determine the net effect of vectors. The first method is best suited when the resultant of only two vectors is required. As with the graphical method, the two vectors to be combined are placed tip-to-tail. The resultant is found by connecting the tail of the first vector to the tip of the second vector. Thus, the resultant forms the third side of a triangle. In general, this is an oblique triangle, and the trigonometric laws described in section 3.6.2 can be applied. The length of the third side and a reference angle must be determined through the law of sines and cosines to fully define the resultant vector. This method can be illustrated through an Example Problem.

EXAMPLE PROBLEM 3.7

Determine the resultant of two acceleration vectors shown in Figure 3.12.

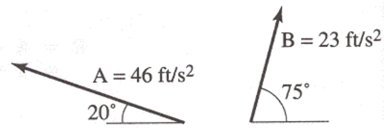


Figure 3.12 Vectors for Example Problem 3.7.

Solution:

The vectors are placed tip-to-tail as shown in Figure 3.13. Note that only a rough sketch is required because the resultant is analytically determined.

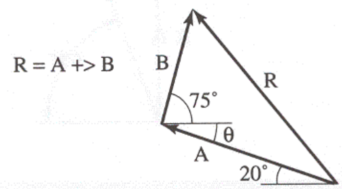


Figure 3.13 Combined effect of vectors *A* & *B* for Example Problem 3.7.

Because the angle between *A* and the horizontal is 20°, the angle labeled "θ" is also 20°. By examining Figure 3.13, the angle between vectors *A* and *B* is 20° + 75° = 95°. Therefore, the problem of determining the resultant of two vectors is actually a general triangle situation described in Section 3.6.2 (Case 3).

By following the procedure outlined for a Case 3 problem, the law of cosines is used to find the magnitude of the resultant:

$$R = \sqrt{\{a^2 + b^2 - 2ab \cos C\}}$$
$$= \sqrt{(46 \text{ ft/s}^2)2 + (23 \text{ ft/s}^2)2 - 2(46 \text{ ft/s}^2)(23 \text{ ft/s}^2)\cos 95°\}} = 53.19 \text{ ft/s}^2$$

The law of sines can be used to find the angle between vectors *A* and *R*:

$$B = \sin^{-1}\left\{\left(\frac{b}{c}\right)\sin C\right\}$$
$$= \sin^{-1}\left\{\frac{(23 \text{ ft/s}^2)}{53.19 \text{ ft/s}^2)}\sin 95°\right\} = 25.5°$$

The angle from the horizontal is 20° + 25.5° = 45.5°. The resultant can be properly written as:

$$R = 53.19 \text{ ft/s}^2 \searrow 45.5°$$
$$\text{or } R = 53.19 \text{ ft/s}^2 \angle 134.5°$$

3.10 COMPONENTS OF A VECTOR

The second method for analytically determining the resultant of vectors is best suited for problems where more than two vectors are to be combined. This method involves resolving vectors into perpendicular components.

Resolution of a vector is the reverse of combining vectors. A single vector can be broken into two separate vectors, along convenient directions. The two vector components have the same effect as the original vector.

In most applications, it is desirable to concentrate on a set of vectors directed vertically and horizontally; therefore, a typical problem involves determining the horizontal and vertical components of a vector. This problem can be solved by using the tip-to-tail approach, but in reverse. To explain the method, a general vector, A, is shown in Figure 3.14.

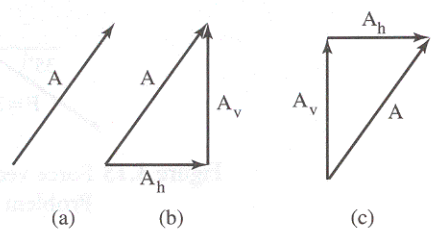


Figure 3.14 Components of a vector.

Two vectors can be drawn tip-to-tail along the horizontal and vertical that have the net effect of the original. The tail of the horizontal vector is placed at the tail of the original and the tip of the vertical vector is placed at the tip of the original vector. This vector resolution into a horizontal component, A_h, and the vertical component, A_v, is shown in Figure 3.14B. Recall that the order of vector addition is not important. Therefore, it is not important whether the horizontal or vertical vector is drawn first. Figure 3.14C illustrates the components of a general vector in the opposite order.

Notice that the magnitude of the components can be found from determining the sides of the triangles shown in Figure 3.14. These triangles are always right triangles, and the methods described in Section 3.3 can be used. The direction of the components are taken from sketching the vectors as in Figure 3.14B or 3.14C. Standard notation consists of defining horizontal vectors directed toward the right as positive. All vertical vectors directed upward are also defined as positive. In this fashion, the direction of the components can be determined from the algebraic sign associated with the component.

An alternative method to determine the rectangular components of a vector is to identify the vector's angle with the positive x-axis of a conventional Cartesian coordinate system. This angle is designated as θ_x. The magnitude of the two components can be computed from the basic trigonometric relations as:

(3.9) $$A_h = A \cos \theta_x$$

(3.10) $$A_v = A \sin \theta_x$$

The importance of this method lies in the fact that the directions of the components are evident from the sign that results from the trigonometric function. That is, a vector that points into the second quadrant of a conventional Cartesian coordinate system has an angle, θ_x, between 90° and 180°. The cosine of such an angle results in a negative value, and the sine results in a positive value. Equations 3.9 and 3.10 imply that the horizontal component is negative (i.e., towards the left in a conventional coordinate system) and the vertical component is positive (i.e., upward in a conventional system).

EXAMPLE PROBLEM 3.8

A force, F, of 3.5 kN is shown in Figure 3.15. Determine the horizontal and vertical components of this force.

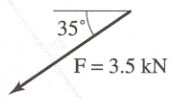


Figure 3.15 Force vector for Example Problem 3.8.

Solution:

The horizontal vector (component) is drawn from the tail of vector F. A vertical vector (component) is drawn from the horizontal vector to the tip of the original force vector. These two components are shown in Figure 3.16.

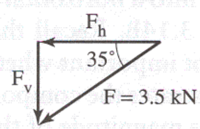


Figure 3.16 Force components for Example Problem 3.8.

Working with the right triangle, an expression for both components can be written using trigonometric functions:

$$\sin 35° = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{F_v}{3.5 \text{ kN}}$$

$$\cos 35° = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{F_h}{3.5 \text{ kN}}$$

Both these expressions can be solved in terms of the magnitude of the desired components:

$$F_h = (3.5\text{ kN})\cos 35° = 2.87\text{ kN} \leftarrow$$
$$= -2.87\text{ kN}$$
$$F_v = (3.5\text{ kN})\sin 35° = 2.0\text{ kN} \downarrow$$
$$= -2.0\text{ kN}$$

An alternative solution is obtained by using Equations 3.9 and 3.10. The angle from the positive x-axis, θ_x to the vector F is 215°. The components are computed as follows:

$$F_h = F\cos\theta_x = (3.5\text{ kN})\cos 215° = -2.87\text{ kN}$$
$$= 2.87\text{ kN} \leftarrow$$
$$F_v = F\sin\theta_x = (3.5\text{ kN})\sin 215° = -2.0\text{ kN}$$
$$= 2.0\text{ kN} \downarrow$$

3.11 ANALYTICAL VECTOR ADDITION (+>): COMPONENT METHOD

The components of a series of vectors can be used to determine the net effect of the vectors. As mentioned, this method is best suited when more than two vectors need to be combined. This method involves resolving each individual vector into horizontal and vertical components. It is standard to use the algebraic sign convention for the components as described above.

All horizontal components may then be added into a single vector component. This component represents the net horizontal effect of the series of vectors. It is worth noting that the component magnitudes can be simply added together because they all lay in the same direction. These components are treated as scalar quantities. A positive or negative sign is used to denote the sense of the component. This concept can be summarized in the following equation:

(3.11) $$R_h = A_h + B_h + C_h + D_h + \ldots$$

Similarly, all vertical components may be added together into a single vector component. This component represents the net vertical effect of the series of vectors:

(3.12) $$R_v = A_v + B_v + C_v + D_v + \ldots$$

The two net components may then be added vectorally into a resultant. Trigonometric relationships can be used to produce the following equations:

(3.13) $$R = \sqrt{R_h^2 + R_v^2}$$

(3.14) $$\theta_x = \tan^{-1}\left(\frac{R_v}{R_h}\right)$$

This resultant is the combined effect of the entire series of vectors. This procedure can be conducted most efficiently when the computations are arranged in a table as demonstrated in the following Example Problem.

EXAMPLE PROBLEM 3.9

Three forces act on a hook as shown in Figure 3.17. Determine the net effect of these forces.

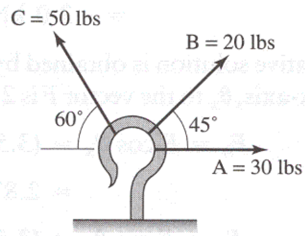


Figure 3.17 Forces for Example Problem 3.9.

Solution:

The horizontal and vertical components of each force are determined by trigonometry and shown in Figure 3.18. Also shown are the vectors rearranged in a tip-to-tail fashion. The components are organized in Table 3.1.

TABLE 3.1 **Vector Components for Example Problem 3.9.**

Vector	Reference Angle θ_x	h-component (lbs) $F_h = F\cos\theta_x$	v-component (lbs) $F_v = F\sin\theta_x$
A	90°	$A_h = (30)\cos 90° = +30$ lbs lbs.	$A_v = (30)\sin 90° = 0$
B	45°	$B_h = (20)\cos 45° = +14.14$ lbs	$B_v = (20)\sin 45° = +14.14$ lbs
C	120°	$C_h = (50)\cos 120° = -25$ lbs.	$C_v = (50)\sin 60° = 43.30$ lbs
		$R_h = 19.14$	$R_v = 57.44$

Notice in Figure 3.18 that adding the magnitudes of the horizontal components is tracking the total "distance" navigated by the vectors in the horizontal direction. The same holds true for adding the magnitudes of the vertical components. This is the logic behind the component method of combining vectors. For this problem, adding the individual horizontal and vertical components gives the components of the resultant as follows:

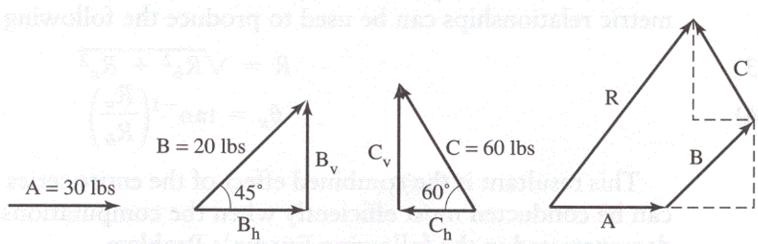


Figure 3.18 Components of vectors in Example Problem 3.9

$$R_h = 19.14 \text{ and } R_v = 57.44$$

The resultant is the vector sum of two perpendicular vectors, as shown in Figure 3.19.

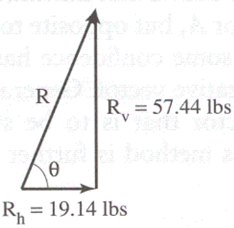


Figure 3.19 Resultant vector for Example Problem 3.9.

The magnitude of the resultant can be found from equation 2.13:

$$R = \sqrt{R_h^2 + R_v^2}$$
$$= \sqrt{(19.14 \text{ lbs})^2 + (57.44 \text{ lbs})^2} = 60.54 \text{ lbs}$$

The angle of the resultant can be found:

$$\tan \theta_x = \left(\frac{R_v}{R_h}\right) = \left(\frac{57.44 \text{ lbs}}{19.14 \text{ lbs}}\right) = 3.00$$

and:

$$\theta_x = \tan^{-1}(3.00) = 71.6°$$

Thus, the resultant of the three forces can be formally stated as:

$$R = 60.54 \text{ lbs } \angle 71.6°$$

3.12 VECTOR SUBTRACTION (−>)

In certain cases, the difference between vector quantities is desired. In these situations, the vectors need to be subtracted. The symbol −> denotes vector subtraction, which differentiates it from algebraic subtraction. Subtracting vectors is accomplished in a similar manner as combining them. In effect, subtraction adds the negative of the vector to be subtracted. The negative of a vector is equal in magnitude, but opposite in direction. Figure 3.20 illustrates a vector A and its negative, →A.

Whether a graphical or analytical method is used, a vector diagram should be drawn to understand the procedure. Consider a general problem where vector B must be subtracted from A, as shown in Figure 3.21A.

This subtraction can be accomplished by first drawing the negative of vector B, −>B. This is shown in Figure 3.21B. Then, vector −>B can be added to vector A, as shown in Figure 3.21C. This subtraction can be stated mathematically as:

$$R = A -> B = A +> (->B)$$

Notice that this statement is identical to the subtraction of scalar quantities through basic algebraic methods.

Figure 3.21D shows that the same result can be obtained by placing the original vector B onto vector A, but opposite to the tip-to-tail orientation. This method is usually preferred after some confidence has been established because it eliminates the need to redraw a negative vector. Generally stated, as vectors are combined in a tip-to-tail format, the vector that is to be subtracted must point towards the vector being subtracted. This method is further explored as the individual solution methods are reviewed.

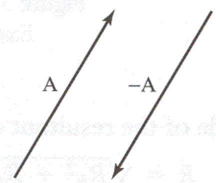


Figure 3.20 Negative vector.

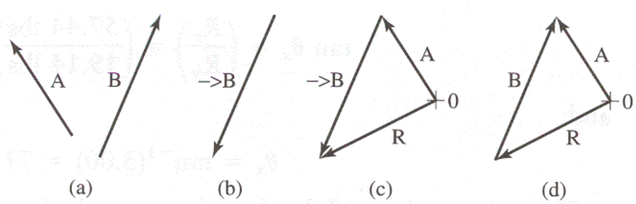


Figure 3.21 Vector subtraction.

3.13 GRAPHICAL VECTOR SUBTRACTION (−>)

As discussed, vector subtraction closely parallels vector addition. To graphically subtract vectors, they are relocated to scale to form a tip-to-tail vector diagram. The vector to be subtracted must be treated in the manner discussed above.

Again, the process of subtracting vectors can be completed graphically, using either manual drawing techniques or CAD software. Whatever method is used, the underlying concepts are identical. The specifics of the process are shown in the following examples.

EXAMPLE PROBLEM 3.10

Determine the result of subtracting the velocity vector B from A, $R = A -> B$, shown in Figure 3.22.

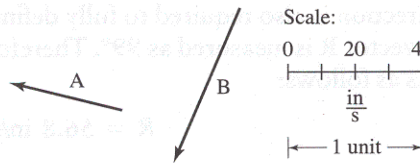


Figure 3.22 Vectors for Example Problem 3.10.

Solution:

First, the length of the vectors are measured, and the scale is used to determine the magnitudes of vectors A and B. Note that in Figure 3.22, 1 drawing unit represents 40 in/s:

$$|A| = 0.80 \text{ units } (40 \text{ in/s})/(1 \text{ unit}) = 32 \text{ in/s}$$

$$|B| = 1.40 \text{ units } (40 \text{ in/s})/(1 \text{ unit}) = 56 \text{ in/s}$$

To determine the result, the vectors are located in the tip-to-tail form, but vector B points towards vector A. Again, this occurs because B is being subtracted (opposite to addition). The vector diagram is shown in Figure 3.23.

The resultant extends from the tail of A, the origin, to the tail of B. The length vector R is measured as 1.42 units; therefore, the resultant represents a velocity of:

$$1.42 \text{ units } (40 \text{ in/s})/(1 \text{ unit}) = 57 \text{ in/s}$$

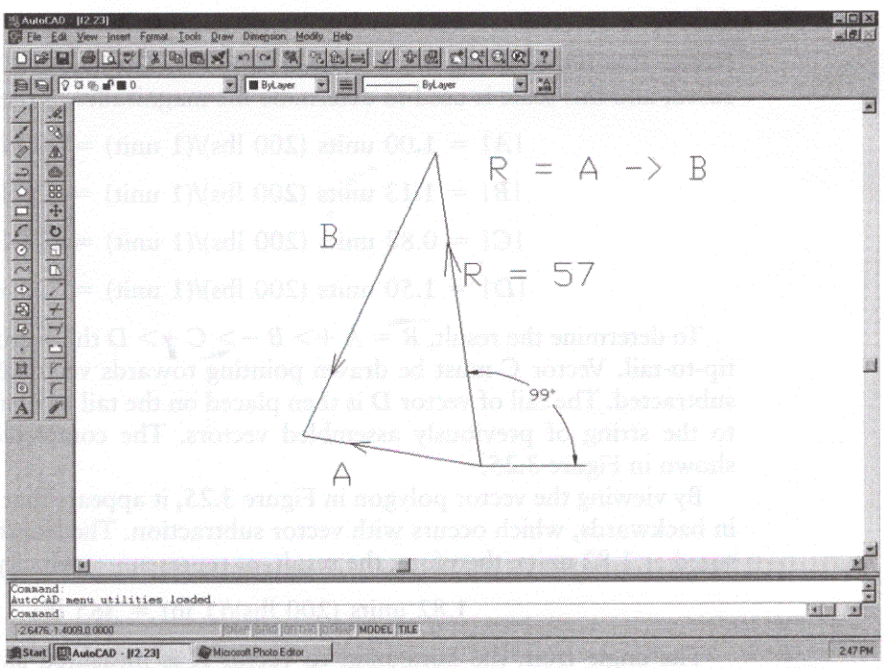


Figure 3.23 $R = A -> B$ for Example Problem 3.10.

The direction is also required to fully define the vector R. The angle from the horizontal to vector R is measured as 99°. Therefore, the proper manner of presenting the solution is as follows:

$$R = 56.8 \text{ in/s} \searrow 81^\circ$$

or

$$R = 56.8 \text{ in/s} \angle 99^\circ$$

EXAMPLE PROBLEM 3.11

Determine the result, $R = A -> B -> C +> D$, of the force vectors shown in Figure 3.24.

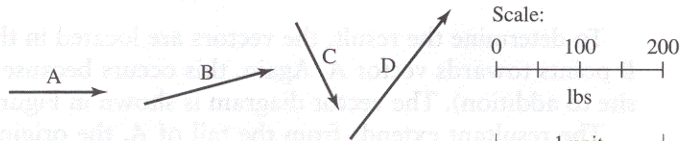


Figure 3.24 Vectors for Example Problem 3.11.

Solution:

Notice that one drawing unit represents 200 lbs. The length of the vectors are measured, and this scale is used to determine the magnitudes of the vectors:

$$|A| = 1.00 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 200 \text{ lbs}$$

$$|B| = 1.13 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 226 \text{ lbs}$$

$$|C| = 0.88 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 176 \text{ lbs}$$

$$|D| = 1.50 \text{ units } (200 \text{ lbs})/(1 \text{ unit}) = 300 \text{ in/s}$$

To determine the result, $R = A +> B -> C +> D$ the vectors must be relocated tip-to-tail. Vector C must be drawn pointing towards vector B because C is being subtracted. The tail of vector D is then placed on the tail of C since D is to be added to the string of previously assembled vectors. The completed vector diagram is shown in Figure 3.25.

By viewing the vector polygon in Figure 3.25, it appears that vector C was placed in backwards, which occurs with vector subtraction. The length of vector R is measured as 1.82 units; therefore, the resultant represents a velocity of:

$$1.82 \text{ units } (200 \text{ lbs})/(1 \text{ in}) = 365 \text{ lbs}$$

The angle from the horizontal to vector R is measured as 81°. Therefore, the proper manner of presenting the solution is as follows:

$$R = 365 \text{ lbs} \angle 81^\circ$$

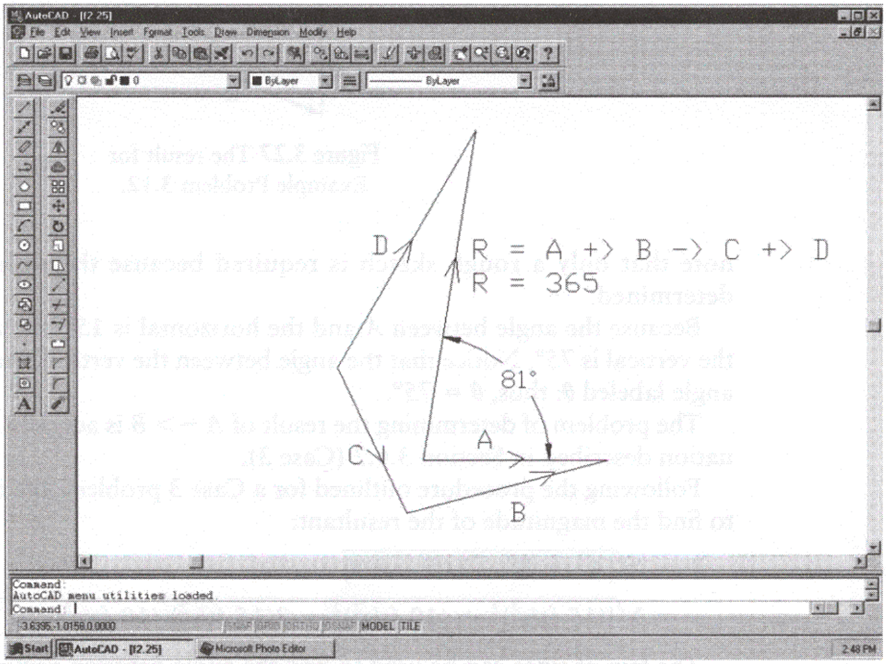


Figure 3.25 Result for Example Problem 3.6.

3.14 ANALYTICAL VECTOR SUBTRACTION (−>): TRIANGLE METHOD

As in analytically adding vectors, the triangle method is best suited for manipulation of only two vectors. A vector diagram should be sketched using the logic as described in the previous section. Then the triangle laws can be used to determine the result of vector subtraction. This method can be visualized through an Example Problem.

EXAMPLE PROBLEM 3.12

Determine the result of the vectors $R = A -> B$ shown in Figure 3.26.

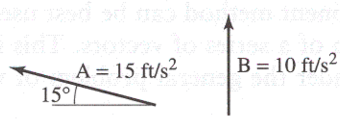


Figure 3.26 Vectors for Example Problem 3.12.

Solution:

The vectors are placed into a vector polygon as shown in Figure 3.27. Again, vector B is placed pointing towards vector A because it is to be subtracted. Also

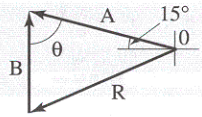


Figure 3.27 The result for Example Problem 3.12.

note that only a rough sketch is required because the resultant is analytically determined.

Because the angle between A and the horizontal is 15°, the angle between A and the vertical is 75°. Notice that the angle between the vertical and A is the same as the angle labeled θ: thus, $\theta = 75°$.

The problem of determining the result of $A -> B$ is actually a general triangle situation described in Section 3.6.2 (Case 3).

Following the procedure outlined for a Case 3 problem, the law of cosines is used to find the magnitude of the resultant:

$$R = \sqrt{\{A^2 + B^2 - 2AB\cos\theta\}}$$

$$= \sqrt{\{(15\ \text{ft/s}^2)^2 + (10\ \text{ft/s}^2)^2 - 2(15\ \text{ft/s}^2)(10\ \text{ft/s}^2)\cos 75°\}} = 15.73\ \text{ft/s}^2$$

The law of sines can be used to find the angle between vectors A and R:

$$B = \sin^{-1}\left\{\left(\frac{B}{R}\right)\sin\theta\right\} \quad B = \sin^{-1}\{(b/c)\sin C\}$$

$$= \sin^{-1}\left\{\frac{(10\ \text{ft/s}^2)}{(15.73\ \text{ft/s}^2)\sin 75°}\right\} = 37.9°$$

From examining Figure 3.27, the angle from the horizontal is $37.9° - 15° = 22.9°$. The resultant can be properly written as:

$$R = 15.73\ \text{ft/s}^2 \angle 22.9°$$

3.15 ANALYTICAL VECTOR SUBTRACTION (−>): COMPONENT METHOD

The component method can be best used to analytically determine the result of the subtraction of a series of vectors. This is done in the exact manner as vector addition. Consider the general problem of vector subtraction defined by the following equation.

$$R = A +> B -> C +> D +> \ldots$$

The horizontal and vertical components of each vector must be determined (see Section 3.10). Also, a sign convention to denote the sense of the component is required. The convention that was used in Section 3.10 granted components that point either to the right or upward a positive algebraic sign.

Because they are scalar quantities, the individual components can be algebraically combined by addition or subtraction. For the general problem stated above, the horizontal and vertical components of the result can be written as follows:

$$R_h = A_h + B_h - C_h + D_h + \dots$$

$$R_v = A_v + B_v - C_v + D_v + \dots$$

Notice the components of C are subtracted from all the other components. This is consistent with the desired vector subtraction. Using Equations 3.13 and 3.14, the two result components may then be combined vectorally into a resultant. This resultant is the result of the vector manipulation of the entire series of vectors. Again, the procedure can be conducted most efficiently when the computations are arranged in a table.

EXAMPLE PROBLEM 3.13

Analytically determine the result $R = A -> B +> C +> D$ for the velocity vectors shown in Figure 3.28.

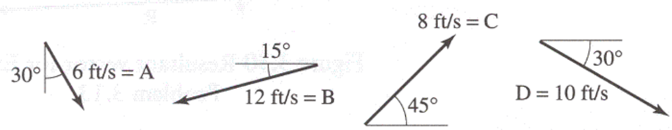


Figure 3.28 Forces for Example Problem 3.13.

Solution:

The horizontal and vertical components of each velocity are determined by trigonometry using Equations 3.9 and 3.10 and shown in Figure 3.29. Also shown are the vectors rearranged in a tip-to-tail fashion, subtracting vector B.

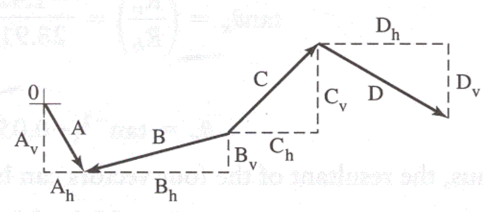


Figure 3.29 Result for Example Problem 3.13.

The values of the component are entered into Table 3.2.

Algebraic manipulation of the individual horizontal and vertical components gives the components of the resultant:

$$R_h = A_h - B_h + C_h + D_h$$

$$= (+3.0) - (-11.59) + (+5.66) + (+8.66) = +28.91 \text{ ft/s}$$

TABLE 3.2 Component Values for Example Problem 3.13.

Vector	Reference Angle θ_x	h-component (ft/s) $V_h = V\cos\theta_x$	v-component (ft/s) $V_v = V\sin\theta_x$
A	300°	+3.00	−5.19
B	195°	−11.59	−3.11
C	45°	+5.66	+5.66
D	330°	+8.66	−5.00

$$R_v = A_v - B_v + C_v + D_v$$

$$= (-5.19) - (-3.11) + (+5.66) + (-5.00) = -1.42 \text{ ft/s}$$

The magnitude and direction of the resultant may be determined by vectorally adding the components (Figure 3.30).

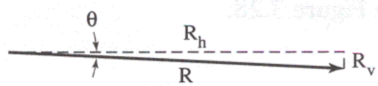


Figure 3.30 Resultant vector for Example Problem 3.13.

The magnitude of the resultant can be found from Equation 3.13:

$$R = \sqrt{R_h^{\,2} + R_v^{\,2}}$$

and:

$$R = \sqrt{(28.91 \text{ ft/s})^2 + (-1.42 \text{ ft/s})^2} = 28.94 \text{ ft/s}$$

The angle of the resultant can be found from the tangent function:

$$\tan\theta_x = \left(\frac{R_v}{R_h}\right) = \frac{-1.42 \text{ ft/s}}{28.91 \text{ ft/s}} = -0.05$$

and:

$$\theta_x = \tan^{-1}(-0.05) = 2.8°$$

Thus, the resultant of the four vectors can be formally stated as:

$$R = 28.94 \text{ ft/s} \angle 2.8°$$

3.16 VECTOR EQUATIONS

As already seen, vector operations can be expressed in equation form. The result of subtracting two vectors, $R = A -> B$, is actually a vector equation. Vector equations can be manipulated in a similar manner to algebraic equations. The terms can be transposed by changing their signs. For example, the equation:

$$A +> B -> C = D$$

can be rearranged as:

$$A +> B = C +> D.$$

The significance of vector equations has also been seen with the introduction of vector addition and subtraction. In addition, vectors can be placed tip-to-tail, and the resultant is a vector that extends from the start of the first vector to the end of the final vector. In the previous sections, the start of the first vector has been called the origin of the vector diagram. Figure 3.31A illustrates the vector diagram for the following:

$$R = A +> B +> C.$$

The equation can be rewritten as:

$$B +> C = R -> A.$$

The vector diagram shown in Figure 3.31B illustrates this form of the equation. Notice that because vector *A* is subtracted from *R*, vector *A* must point toward *R*. Recall that this is the opposite of the tip-to-tail method because subtraction is the opposite of addition.

Notice that as the diagram forms a closed polygon, the magnitude and directions for all vectors are maintained. This verifies that vector equations can be manipulated without altering their meaning. The equation can be rewritten once again as follows (Figure 3.31C):

$$->B +> R = A +> C.$$

As seen from Figure 3.31, a vector equation can be rewritten into several different forms. Although the vector polygons created by the equations have different shapes, the individual vectors remain unaltered. By using this principle, a vector equation can be written from viewing a vector diagram.

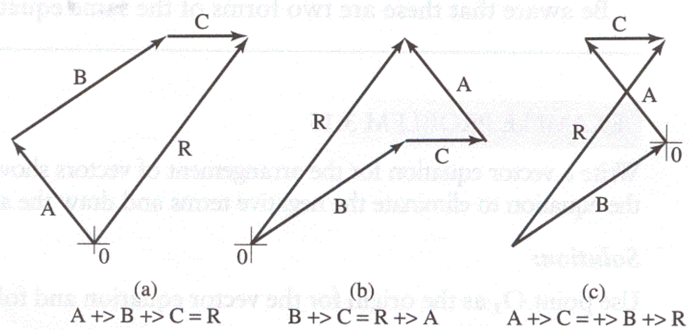


Figure 3.31 Vector equations.

EXAMPLE PROBLEM 3.14

Write a vector equation for the arrangement of vectors shown in Figure 3.32.

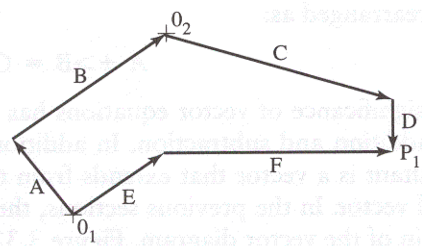


Figure 3.32 Vector diagram for Example Problem 3.14.

Solution:

Use point O_1 as the origin for the vector equation and follow the paths to point P_1:

The upper path states: $A +> B +> C +> D$

The lower path states: $E +> F$

Because they start at a common point and end at a common point, both paths must be vectorally equal. Thus, the following equation can be written:

$$O_1P_1 = A +> B +> C +> D = E +> F$$

Another equation can be written by using point O_2 as the origin and following the paths to point P_1:

The upper path states: $C +> D$

The lower path states: $-> B -> A +> E +> F$

Thus, the equation can be written as follows:

$$O_1P_1 = C +> D = -> A -> B +> E +> F$$

Be aware that these are two forms of the same equation.

EXAMPLE PROBLEM 3.15

Write a vector equation for the arrangement of vectors shown in Figure 3.33. Then rewrite the equation to eliminate the negative terms and draw the associated vector diagram.

Solution:

Use point O_1 as the origin for the vector equation and follow the paths to the point P_1:

The upper path states: $A -> B +> C -> D$

The lower path states: $-> E +> F$

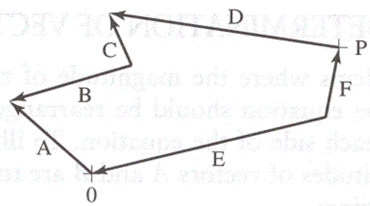


Figure 3.33 Vector diagram for Example Problem 3.15.

Thus, the following equation can be written as:

$$O_1P_1 = A -> B +> C -> D = -> E +> F$$

To eliminate the negative terms, vectors *B*, *D*, and *E* all must be transposed to their respective opposite sides of the equation. This yields the following equation:

$$A +> C +> E = B +> D +> F$$

Note that the order of addition is not significant. Rearranging the vectors into a new diagram is shown in Figure 3.34.

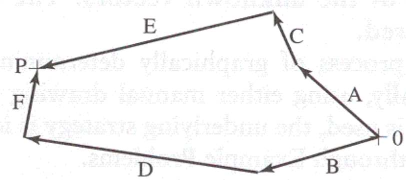


Figure 3.34 Rearranged diagram for Example Problem 3.15.

Familiarity with vector equations should be gained, as they are used extensively in mechanism analysis. For example, determining the acceleration of even simple mechanisms involve vector equations with six or more vectors.

3.17 APPLICATION OF VECTOR EQUATIONS

Each vector in an equation represents two quantities, a magnitude and a direction. Therefore, a vector equation actually represents two constraints: the combination of the vector magnitudes and the directions must be equivalent. Therefore, a vector equation can be used to solve for two unknowns. In the addition and subtraction problems previously discussed, the magnitude and direction of the resultant were determined.

A common problem in mechanism analysis involves knowing the direction of all vectors involved in the analysis, yet the magnitude of two vectors is unknown. As is similar to the addition of vectors, this problem also involves two unknowns. Therefore, one vector equation is sufficient to solve these problems.

3.18 GRAPHICAL DETERMINATION OF VECTOR MAGNITUDES

For problems where the magnitude of two vectors in an equation must be determined, the equation should be rearranged so that one unknown vector is the last term on each side of the equation. To illustrate this point, consider the case where the magnitudes of vectors *A* and *B* are to be found. The vector equation consists of the following:

$$A +> B +> C = D +> E$$

and should be rearranged as:

$$C +> B = D +> E -> A$$

Notice that both vectors with unknown magnitudes, *A* and *B*, are the last terms on both sides of the equation.

To graphically solve this problem, the known vectors on each side of the equation are placed tip-to-tail starting from a common origin. Of course, both sides of the equation must end at the same point.* The intersection of these two lines represent the direction of the unknown vectors and solves the problem. The lines can be measured and scaled to determine the magnitudes of the unknown vectors. The sense of the unknown vector is also discovered.

This process of graphically determining vector magnitudes can be completed graphically, using either manual drawing techniques or CAD software. Whatever method is used, the underlying strategy is identical. The solution strategy can be explained through Example Problems.

EXAMPLE PROBLEM 3.16

A vector equation can be written as:

$$A +> B +> C = D +> E$$

The directions for vectors *A*, *B*, *C*, *D*, and *E* are known, and the magnitudes of vectors *B*, *C*, and *D* are also known (Figure 3.35). Graphically determine the magnitudes of vectors *A* and *E*.

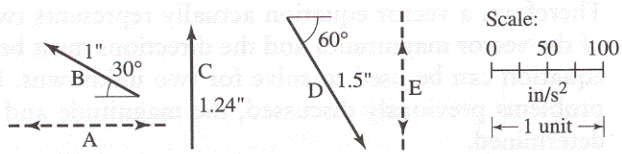


Figure 3.35 Vectors for Example Problem 3.16.

*Therefore, lines at the proper direction should be inserted into the vector polygon

Solution:

First, the equation is rewritten so that the unknown magnitudes appear as the last term on each side of the equation:

$$B +> C +> A = D +> E$$

Using point O as the common origin, vectors B and C can be drawn tip-to-tail. Because it is on the other side of the equation, vector D should be drawn from the origin (Figure 3.36A).

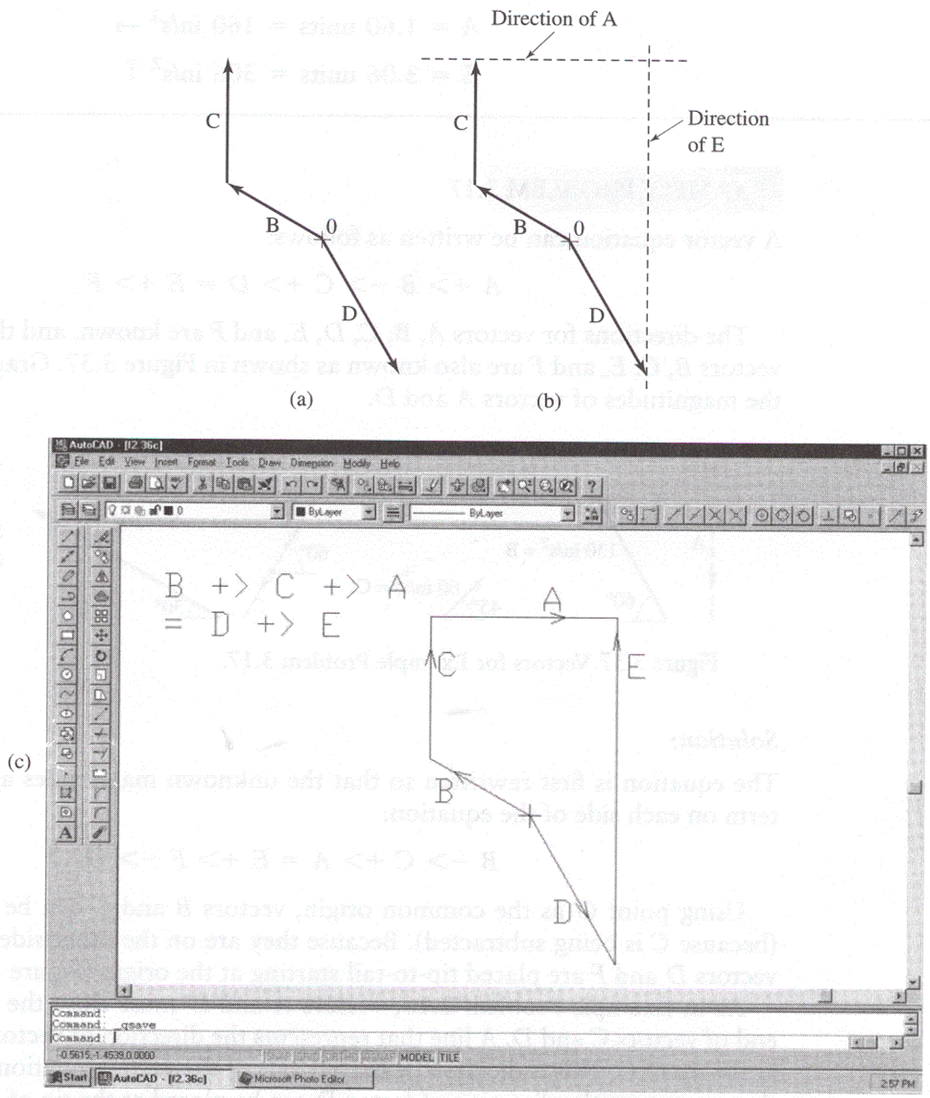


Figure 3.36 Vector diagrams for Example Problem 3.16.

Obviously, vectors A and E close the gap between the end of vectors C and D. A line that represents the direction of vector A can be placed at the tip of C. This is dictated by the left side of the vector equation. Likewise, a line that represents the direction of vector E can be placed at the tip of D (Figure 3.36B).

The point of intersection of the two lines defines both the magnitude and sense of vectors A and E. A complete vector polygon can be drawn as prescribed by a vector equation (Figure 3.36C).

The following equations are obtained by measuring vectors A and E and using the proper scale of one drawing unit representing 100 in/s^2:

$$A = 1.60 \text{ units} = 160 \text{ in/s}^2 \rightarrow$$

$$E = 3.06 \text{ units} = 306 \text{ in/s}^2 \uparrow$$

EXAMPLE PROBLEM 3.17

A vector equation can be written as follows:

$$A +> B -> C +> D = E +> F$$

The directions for vectors A, B, C, D, E, and F are known, and the magnitudes of vectors B, C, E, and F are also known as shown in Figure 3.37. Graphically solve for the magnitudes of vectors A and D.

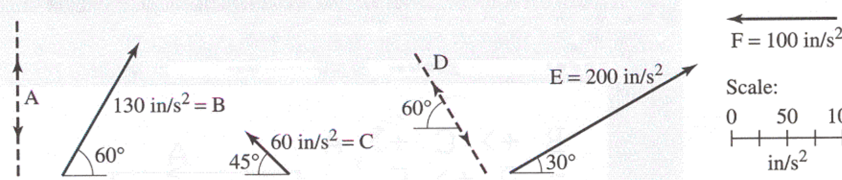


Figure 3.37 Vectors for Example Problem 3.17.

Solution:

The equation is first rewritten so that the unknown magnitudes appear as the last term on each side of the equation:

$$B -> C +> A = E +> F -> D$$

Using point O as the common origin, vectors B and C can be drawn tip-to-tip (because C is being subtracted). Because they are on the other side of the equation, vectors D and F are placed tip-to-tail starting at the origin (Figure 3.38A).

As in Example Problem 2.15, vectors A and D must close the gap between the end of vectors C and D. A line that represents the direction of vector A can be placed at the tip of C. This is dictated by the left side of the vector equation. Likewise, a line that represents the direction of fector D can be placed at the tip of F (Figure 3.36B).

The point of intersection of the two lines defines both the magnitude and sense of vectors A and D. The sense of D is chosen in a direction that is consistent with it being subtracted from the right side of the equation. The complete vector polygon can be drawn as prescribed by the vector equation (Figure 3.38C).

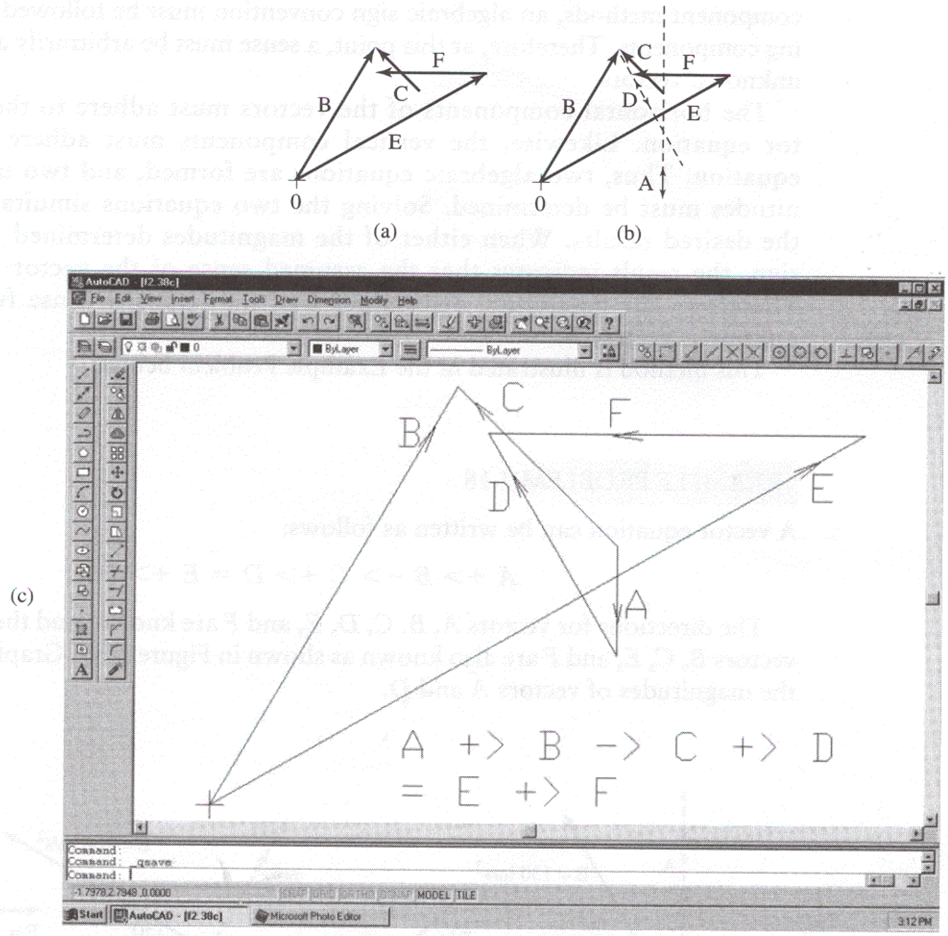


Figure 3.38 Vector diagrams for Example Problem 3.17.

The following equations are obtained by measuring vectors A and D and using the proper scale of one drawing unit representing 100 in/s^2:

$$A = 0.30 \text{ units} = 30 \text{ in/s}^2 \downarrow$$

$$E = 0.68 \text{ iunits} = 68 \text{ in/s}^2 \nwarrow 60°$$

3.19 ANALYTICAL DETERMINATION OF VECTOR MAGNITUDES

An analytical method can also be used to determine the magnitude of two vectors in an equation. In these cases, the horizontal and vertical components of all vectors should be determined as in Section 3.10. Components of the vectors with unknown magnitudes can be written in terms of the unknown quantity. As in the previous component methods, an algebraic sign convention must be followed while computing components. Therefore, at this point, a sense must be arbitrarily assumed for the unknown vectors.

The horizontal components of the vectors must adhere to the original vector equation. Likewise, the vertical components must adhere to the vector equation. Thus, two algebraic equations are formed, and two unknown magnitudes must be determined. Solving the two equations simultaneously yields the desired results. When either of the magnitudes determined has a negative sign, the result indicates that the assumed sense of the vector was incorrect. Therefore, the magnitude determined and the opposite sense fully define the unknown vector.

This method is illustrated in the Example Problem below.

EXAMPLE PROBLEM 3.18

A vector equation can be written as follows:

$$A +> B -> C +> D = E +> F$$

The directions for vectors *A*, *B*, *C*, *D*, *E*, and *F* are known, and the magnitudes of vectors *B*, *C*, *E*, and *F* are also known as shown in Figure 3.39. Graphically solve for the magnitudes of vectors *A* and *D*.

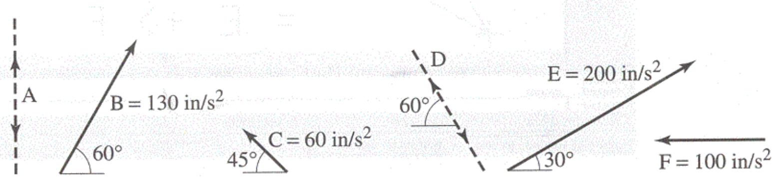


Figure 3.39 Vectors for Example Problem 3.18.

Solution:

The horizontal and vertical components of each force are determined by trigonometry. The sense of the unknown vectors is assumed and the components are found in terms of the unknown quantity. (Assume vector *A* points upward and vector *D* points down and to the right.)

TABLE 3.3 Vector Components for Example Problem 3.18.

Vector	Reference Angle θ_x	h-component (in/s^2) $a_h = a\cos\theta_x$	v-component (in/s^2) $a_v = a\sin\theta_x$
A	90°	0	A
B	60°	65.0	112.6
C	135°	−42.4	42.4
D	300°	.500D	−.866D
E	30°	173.2	100
F	180°	−100	0

The components can be used to generate algebraic equations that are derived from the original vector equation:

$$A +> B -> C +> D = E +> F$$

horizontal components:

$$A_h + B_h - C_h + D_h = E_h + F_h$$

$$(0) + (65.0) - (-42.4) + (+0.500\ \mathrm{D}) = (+173.2) + (-100.0)$$

vertical components:

$$A_v + B_v - C_v + D_v = E_v + F_v$$

$$(+A) + (+112.6) - (42.4) + (-0.866\ D) = (+100.0) + (0)$$

In this case, the horizontal component equation can be solved independently for *D*. In general, both equations are coupled and solved simultaneously. Solve the horizontal component equation to obtain the following:

$$D = -68.4 \text{ in/s}^2$$

Substitute this value into the vertical component equation to obtain:

$$A = -29.4 \text{ in/s}^2$$

Because both values are negative, the original directions assumed for the unknown vectors were incorrect. Therefore, the corrected results are:

$$A = 29.4 \text{ in/s}^2 \downarrow$$

$$D = 68.4 \text{ in/s}^2 \nwarrow 60°$$

PROBLEMS

While manual drafting techniques are instructive for problems that require graphical solution, use of a CAD system is highly recommended.

Working With Triangles

For the triangle shown in Figure P3.1:

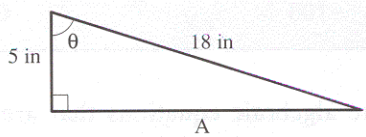


Figure P3.1 Problems 1 and 2.

3–1. Analytically determine the angle θ.

3–2. Analytically determine the length of side A.

For the triangle shown in Figure P3.3.

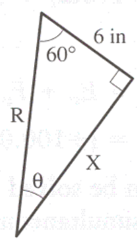


Figure P3.3 Problems 3 and 4:

3–3. Analytically determine the length of side x.

3–4. Calculate the angle θ and the hypotenuse R.

3–5. Calculate the angle θ and the hypotenuse R Figure P3.5.

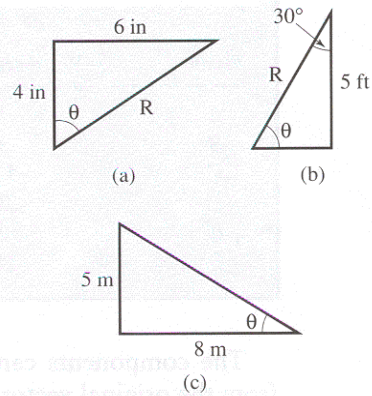


Figure P3.5 Problem 5.

For the folding shelf in Figure P3.6:

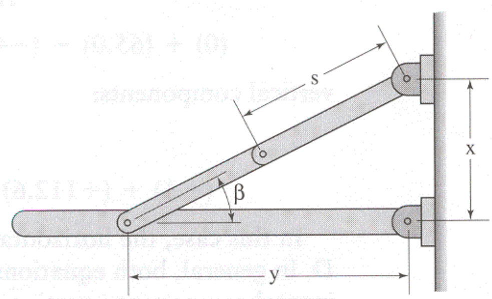


Figure P3.6 Problems 6-9.

3–6. Determine the angle, β, and the length, s, of the two identical support links when $x = 150$ mm and $y = 275$ mm.

3–7. Determine the distance, x, and the length, s, of the two identical support links when $\beta = 35°$ and $s = 10$ in.

3–8. For the folding shelf in Figure P3.6 with $\beta = 35°$ and $s = 10$ in, determine the distances, x and y.

3–9. A roof that has an 8 on 12 pitch slopes upward 8 vertical in for every 12 in of horizontal distance. Determine the angle with the horizontal of such a roof.

For the swing-out window in Figure P3.10:

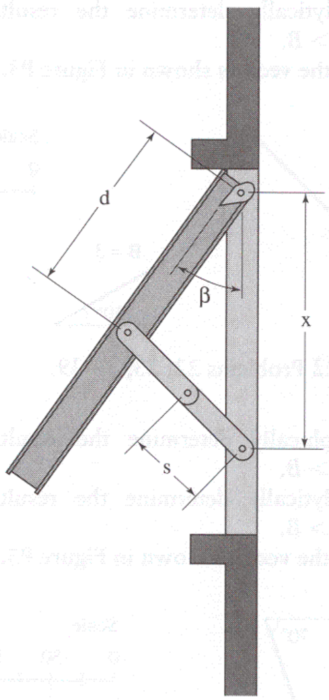


Figure P3.10 Problems 10 and 11.

3–10. Determine the length, s, of the two identical support links when $x = 850$ mm, $d = 500$ mm, and $\beta = 35°$.

3–11. Determine the angle β when $x = 24$ in, $d = 16$ in, and $s = 7$ in.

Figure P3.12 shows a ramp that attaches onto a tractor trailer bed.

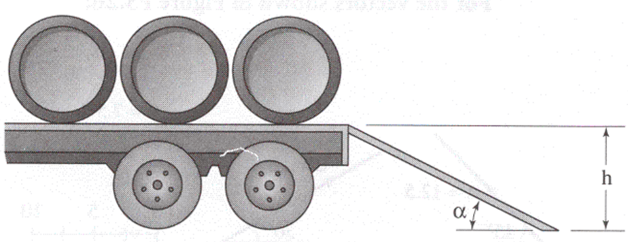


Figure P3.12 Problems 12 and 13.

3–12. If the height, h, of the trailer is 52 in, determine the length of ramp needed to maintain an angle, β, of 30°.

3–13. Determine the angle with the ground when the trailer height is 1.5 m and the ramp is 4 m long.

Figure P3.14 shows a ladder resting on the side of a building.

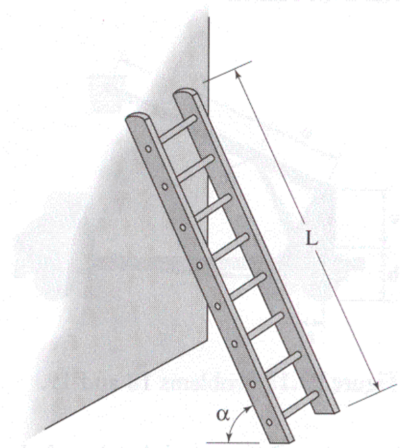


Figure P3.14 Problems 14 and 15.

3–14. When the length of the ladder is 12 ft and the angle with the ground, β, is 70°, determine the vertical distance on the wall where the ladder is resting.

3–15. Determine the angle with the ground when the ladder is 7 m long and rests on the ground 2 m from the wall.

Figure P3.16 shows a belt conveyor used on a farm to lift hay into lofts.

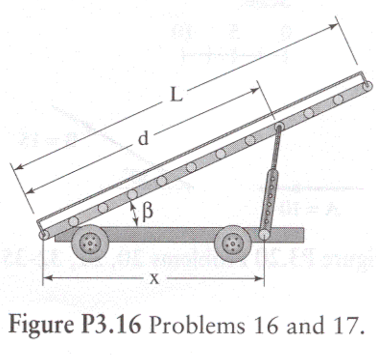


Figure P3.16 Problems 16 and 17.

3–16. Determine the required length of the support rod when the angle is $\beta = 28°$ and the distances are $x = 20$ ft and $d = 16$ ft. Also determine the vertical height of the end of the conveyor when $L = 25$ ft.

3–17. Determine the angle β when a vertical height of 8 m is required at the end of the conveyor and $x = 8$ m, $d = 10$ m, and $L = 13$ m.

Figure P3.18 shows a utility truck equipped with a lift basket.

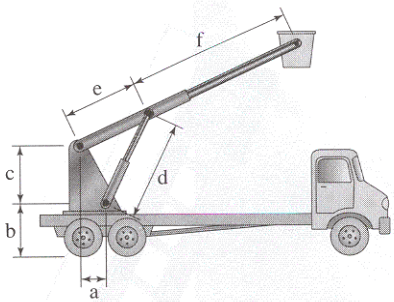


Figure P3.18 Problems 18 and 19.

3–18. Determine the vertical height of the basket when: $a = 24$ in, $b = 36$ in, $c = 30$ in, $d = 60$ in, $e = 6$ ft and $f = 10$ ft.

3–19. Determine the vertical height of the basket when the hydraulic cylinder is shortened to 50 in.

Vector Addition (Graphical and Analytical Problems)

For the vectors shown in Figure P3.20:

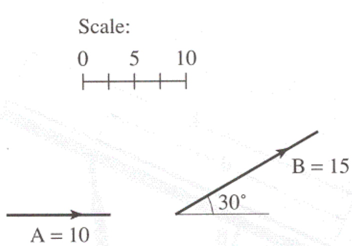


Figure P3.20 Problems 20, 21, 32-35.

3–20. Graphically determine the resultant, $R = A +> B$.

3–21. Analytically determine the resultant, $R = A +> B$.

For the vectors shown in Figure P3.22:

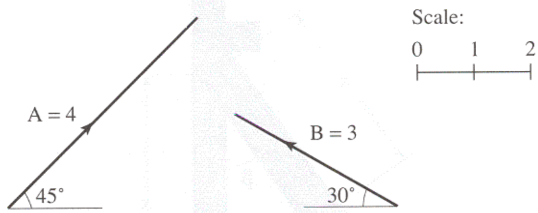


Figure P3.22 Problems 22, 23, 36-39.

3–22. Graphically determine the resultant, $R = A +> B$.

3–23. Analytically determine the resultant, $R = A +> B$.

For the vectors shown in Figure P3.24:

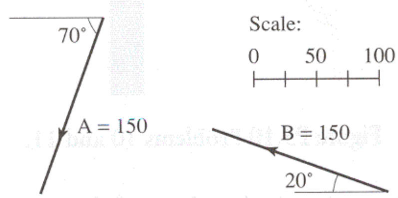


Figure P3.24 Problems 24, 25, 40-43.

3–24. Graphically determine the resultant, $R = A +> B$.

3–25. Analytically determine the resultant, $R = A +> B$.

For the vectors shown in Figure P3.26:

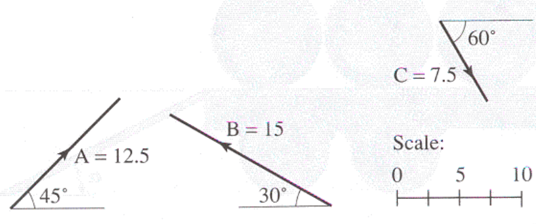


Figure P3.26 Problems 26, 27, 44-47.

3–26. Graphically determine the resultant, $R = A +> B +> C$.

3–27. Analytically determine the resultant, $R = A +> B +> C$.

For the vectors shown in Figure P3.28:

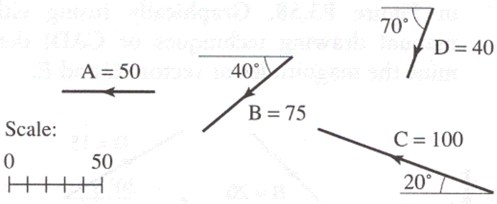


Figure P3.28 Problems 28, 29, 48-51.

3–28. Graphically determine the resultant, $R = A +> B +> C +> D$.

3–29. Analytically determine the resultant, $R = A +> B +> C +> D$.

For the vectors shown in Figure P3.30:

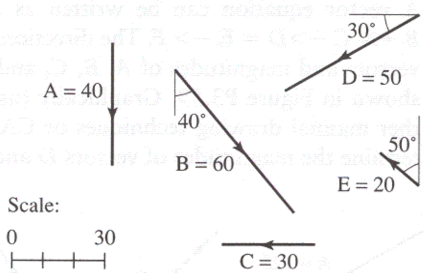


Figure P3.30 Problems 30, 31, 52-55.

3–30. Graphically determine the resultant, $R = A +> B +> C +> D +> E$.

3–31. Analytically determine the resultant, $R = A + > B +> C +> D +> E$.

Vector Subtraction

3–32. For the vectors shown in Figure P3.20, graphically determine the vector, $J = A -> B$.

For the vectors shown in Figure P3.20:

3–33. Graphically determine the vector, $K = B -> A$.

3–34. Analytically determine the vector, $J = A -> B$.

3–35. Analytically determine the vector, $K = B -> A$.

For the vectors shown in Figure P3.22:

3–36. graphically determine the vector, $J = A -> B$.

3–37. Graphically determine the vector, $K = B -> A$.

3–38. Analytically determine the vector, $J = A -> B$.

3–39. Analytically determine the vector, $K = B -> A$.

For the vectors shown in Figure P3.24:

3–40. Graphically (determine the vector, $J = A -> B$.

3–41. Graphically determine the vector, $K = B -> A$.

3–42. Analytically determine the vector, $J = A -> B$.

3–43. Analytically determine the vector, $K = B -> A$.

General Vector Equations (Graphical and Analytical Problems)

For the vectors shown in Figure P3.26:

3–44. Graphically determine the vector, $J = C +> A -> B$.

3–45. Graphically determine the vector, $K = B -> A -> C$.

3–46. Analytically determine the vector, $J = C +> A -> B$.

3–47. Analytically determine the vector, $K = B -> A -> C$.

For the vectors shown in Figure P3.28:

3–48. Graphically determine the vector, $J = C +> A -> B +> D$.

3–49. Graphically determine the vector, $K = B -> D +> A -> C$.

3–50. Analytically determine the vector, $J = C +> A -> B +> D$.

3–51. Analytically determine the vector, $K = B -> D +> A -> C$.

For the vectors shown in Figure P3.30:

3–52. Graphically determine the vector, $J = C +> A -> B +> D -> E$.

3–53. Graphically determine the vector, $K = B -> D +> A -> C +> E$.

3–54. Analytically determine the vector, $J = C +> A -> B +> D -> E$.

3–55. Analytically determine the vector, $K = B -> D +> A -> C +> E$.

3–56. Using the vector diagram in Figure P3.56:

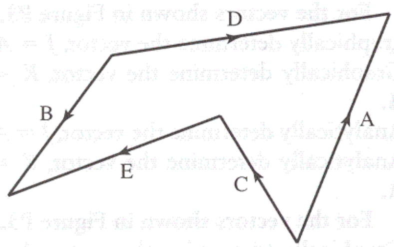


Figure P3.56 Problem 56.

a) Generate an equation that describes the vector diagram.
b) Rewrite the equations to eliminate the negative terms.
c) Scale the vectors and rearrange them according to the equation generated in part b.

3–57. Using the vector diagram in Figure P3.57:

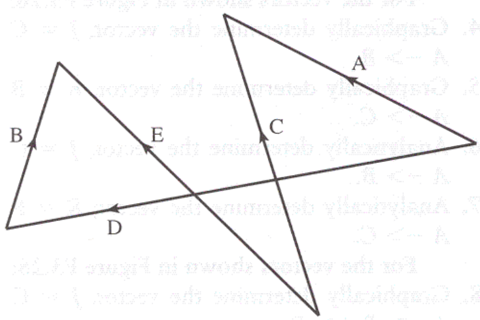


Figure P3.57 Problem 57.

a) Generate an equation that describes the vector diagram.
b) Rewrite the equations to eliminate the negative terms.
c) Scale the vectors and rearrange them according to the equation generated in part b.

Solving For Vector Magnitudes (Graphical and Analytical Problems)

3–58. A vector equation can be written as $A +> B +> C = D +> E$. The directions of all vectors and magnitudes of A, B, and D are shown in Figure P3.58. Graphically (using either manual drawing techniques or CAD) determine the magnitudes of vectors C and E.

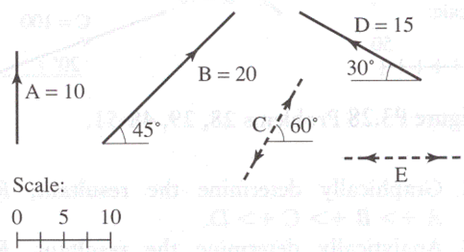


Figure P3.58 Problems 58 and 61.

3–59. A vector equation can be written as $A +> B +> C -> D = E -> F$. The directions of all vectors and magnitudes of A, B, C, and E are shown in Figure P3.59. Graphically (using either manual drawing techniques or CAD) determine the magnitudes of vectors D and F.

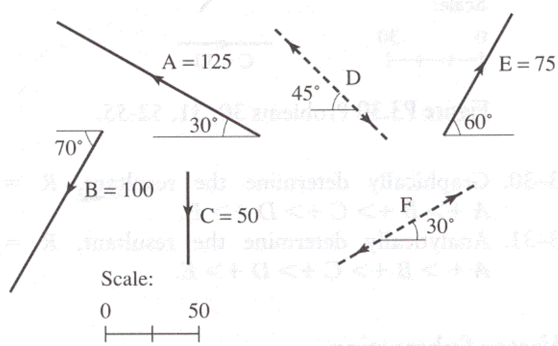


Figure P3.59 Problems 59 and 62.

3–60. A vector equation can be written as $A -> B -> C +> D = -> E +> F$. The directions of all vectors and magnitudes of A, D, E, and F are shown in Figure P3.60. Graphically (using either manual drawing techniques or CAD) determine the magnitudes of vectors B and C.

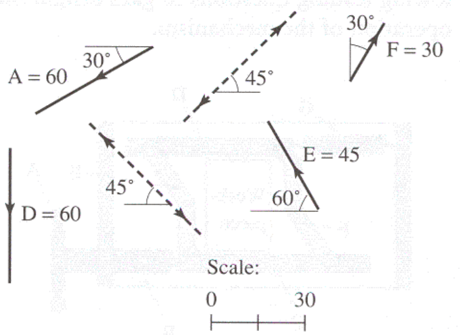


Figure P3.60 Problems 60 and 63.

3–61. Analytically determine vectors C and E from problem 3–58.

3–62. Analytically determine vectors D and F from problem 3–59.

3–63. Analytically determine vectors B and C from problem 3–60.

CASE STUDIES

3–1 Figure C3.1 shows two of many keys from an adding machine that was popular several years ago. End views are also shown to illustrate the configuration at key 1 and 2. Carefully examine the configuration of the components in the mechanism. Then, answer the following leading questions to gain insight into the operation of the mechanism.

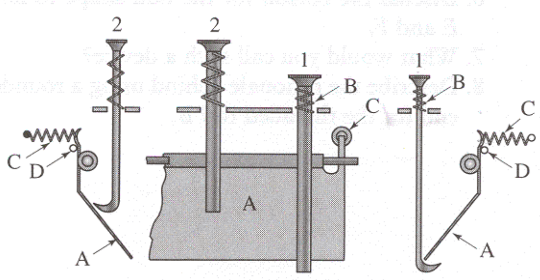


Figure C3.1 (Courtesy, Industrial Press)

1. As key *2* is pressed, what happens to rocker plate *A*?
2. What is the purpose of spring *C*?
3. What is the purpose of spring *B*?
4. As button *2* is pressed, that happens to button *1*?
5. What is the purpose of this device?
6. Because force is a vector, its direction is important. What direction must the force applied by the spring *B* act?
7. What direction must the force applied by spring *C* act?
8. List other machines, other than an adding machine, that could use this device.
9. What is the function of pin *D*?

3–2 An automatic machine that forms steel wire occasionally jams when the raw material is oversized. To prevent serious damage to the machine, it was necessary for the operator to cut off power immediately when the machine became jammed. However, the operator is unable to maintain close watch over the machine to prevent damage. Therefore, the following mechanism has been suggested to solve the problem.

Figure C3.2 shows that gear *C* drives a mating gear (not shown) which operates the wire-forming machine. Driveshaft *A* carries collar *B*, which is keyed to it. Gear *C* has a slip fit onto shaft *A*. Two pins, *G* and *E*, attach links *F* and *D*, respectively, to gear *C*. An additional pin on gear *C* is used to hold the end of spring *H*. Carefully examine the configuration of the components in the mechanism. Then, answer the following leading questions to gain insight into the operation of the mechanism.

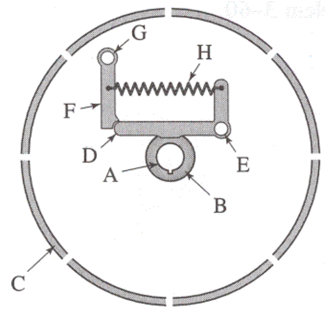


Figure C3.2 (Courtesy, Industrial Press)

1. As driveshaft *A* turns, what is the motion of collar *B*?
2. If gear *C* is not directly attached to collar *B*, how can the motion from the shaft rotate the gear?
3. What happens to the motion of gear *C* if link *D* were forced upward?
4. What action would cause link *D* to move upward?
5. What resistance would link *D* have to moving upward?
6. What is the purpose of this device?
7. What would you call such a device?
8. How does this device aid the automatic wire-forming machine described above?
9. This device must be occasionally "reset." Why and how will that be accomplished?
10. Because force is a vector, its direction is important. What direction must the forces applied by the spring *H* act?
11. List other machines, other than the wire-forming one, that could use this device.

3–3 The mechanism shown in Figure C3.3 is a top view of a fixture in a machining operation. Carefully examine the configuration of the components in the mechanism. Then, answer the following leading questions to gain insight into the operation of the mechanism.

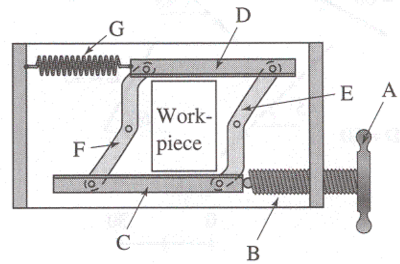


Figure C3.3

1. As handle *A* is turned, moving the threaded rod *B* to the left, describe the motion of grip *C*.
2. As handle *A* is turned, moving the threaded rod *B* to the left, describe the motion of grip *D*.
3. What is the purpose of this mechanism?
4. What action would cause link *D* to move upward?
5. What is the purpose of spring *G*?
6. Discuss the reason for the odd shape to links *E* and *F*.
7. What would you call such a device?
8. Describe the rationale behind using a rounded end for the threaded rod *B*.

4 Position Analysis

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Define position and displacement of a point.
2. Graphically and analytically determine the position of all links in a mechanism as the driver link(s) are displaced.
3. Graphically and analytically determine the limiting positions of a mechanism.
4. Graphically and analytically determine the position of all links for an entire cycle of mechanism motion.
5. Plot a displacement diagram for various points on a mechanism as a function of the motion of other points on the mechanism.

4.1 INTRODUCTION

For many mechanisms, the sole purpose of analysis is to determine the position of all links as the mechanism is moved into another position. The task may be to reposition a clamp into a closed position. Analysis may involve determining the range of motion of automotive windshield wipersPosition analysis is often repeated at several intervals of mechanism movement to determine the position of all links at various phases of the operation cycle. Analysis may involve determining the spatial requirements of a robotic arm. Such position analysis is the focus of this chapter.

4.2 POSITION

The primary purpose in analyzing a mechanism is to study its motion. Motion is the act of changing the position of the linkson a mechanism, and the points on those links. As the position of the links are altered, the mechanism is forced into a different configuration.

The *position* of a point on a mechanism is the spatial location of that point. As the mechanism moves into a new configuration, the point firmly attaches to the mechanism and moves along with it. The position of the point can be visually seen by drawing the point on the mechanism or on its kinematic skeleton. Formally, a point can be located by using coordinates in some reference coordinate system. For most analyses, it is sufficient to show the position of a point on a sketch of the mechanism.

Recall from Chapter 1 that an important property of a mechanism is the mobility or number of degrees of freedom. For linkages with one degree of freedom, the position of one link or point can precisely determine the position of all other links or points. Likewise, for linkages with two degrees of freedom, the position of two links can precisely determine the position of all other links. Therefore, the positions of points and links in a mechanism are not arbitrary and independent.

The degrees of freedom are the number of independent parameters required to specify the position of every link in a mechanism. The independent parameters are the positions of certain "driver" links. Most practical linkages have one degree of freedom. A primary goal of position analysis is to determine the resulting position of the points on a mechanism as a function of the position of some "driver" points.

4.3 DISPLACEMENT

Displacement is the end product of motion. It is a vector that represents the distance between the starting and ending positions of a point. Two types of displacements are considered here: linear and angular.

Linear displacement is the straight line distance between an original position of a point on a mechanism and some later position. It is not influenced by the path or distance that a point travels during motion. Linear displacement is simply the straight line distance between the starting and ending position of a point during a time interval under consideration. The linear displacement of a point, for example point P, is denoted as s_P.

Angular displacement is the angular distance between two positions of a rotating link. By definition, only a link can rotate; any rotation of a single point is ignored. A link may rotate a few degrees or several revolutions, yet the displacement is only the angle between the starting and ending positions. The angular displacement of a link, for example link 3, is denoted as θ_3.

As mentioned above, displacement is a vector. The magnitude of the displacement is the distance between the initial and final positions during an interval. This magnitude is in linear units (i.e., inches, feet, millimeters) for linear displacement, or in rotational units (i.e., degrees, radians, revolutions) for rotational displacement.

Of course, a direction is needed to fully define a vector. For linear displacement, the direction can be identified by an angle from a reference axis to the line that connects the two positions. The sense of the vector is obtained from the linw that connects the initial position and is directed towards the final position. For angular displacement, the direction is stated as whether the displacement is clockwise or counterclockwise.

4.4 POSITION ANALYSIS

A common problem is determining the position of all links in a mechanism, as the driver link(s) are displaced. As stated in Section 4.2, the degrees of freedom of a mechanism determine the number of independent driver links. For the most common mechanisms, those with one degree of freedom, position analysis consists of determining the position of all links as one link is displaced. The positions of all links are called the *configuration of the mechanism.*

Figure 4.1 illustrates this problem. The mechanism shown has four links, as numbered. Recall that the fixed link, or frame, must always be included as a link. The mechanism also has four revolute, or pin, joints.

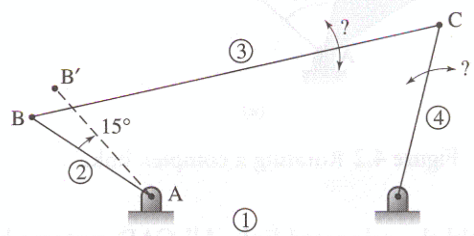


Figure 4.1 Typical position analysis.

From equation 1.1, the degrees of freedom can be calculated as follows:

$$F = 3(4 - 1) - 2(4) = 1$$

With one degree of freedom, moving one link precisely positions all other links in the mechanism. Therefore, a typical position analysis problem involves determining the position of links 3 and 4 in Figure 4.1, as link 2 moves to a specified displacement. In this example, the displacement is angular, $\theta_2 = 15°$ clockwise.

4.5 POSITION: GRAPHICAL ANALYSIS

In placing a mechanism in a new configuration, it is necessary to redraw links in their respective new positions. Links that rotate about fixed centers can be relocated by drawing arcs through the main points on the link, centered at the fixed point. This was illustrated in Figure 4.1 as link 2 is rotated 15° clockwise.

In some analyses, complex links that are fixed to the ground also must be rotated. This can be done using several methods. In most cases the simplest method begins by relocating only one line of the link. The other geometry that describes the link can then be reconstructed, based on the position of the line that has already been relocated.

Figure 4.2 illustrates the problem of rotating a complex link. In Figure 4.2A, line AB of the link is displaced to its desired position, $\Delta\theta = 60°$ clockwise. Notice that the relocated position of point B is designated as B'.

The next step is to determine the position of the relocated point C, which is designated as C'. Because the complex link is rigid and does not change shape during movement, the lengths of lines AC and BC do not change. Therefore, point C' can be located by capturing the lengths of AC and BC and striking arcs from points A and B', respectively (Figure 4.2B).

When graphical displacement analysis is employed on a CAD system, rotating links is not difficult. The lines that comprise the link can be duplicated and rotated

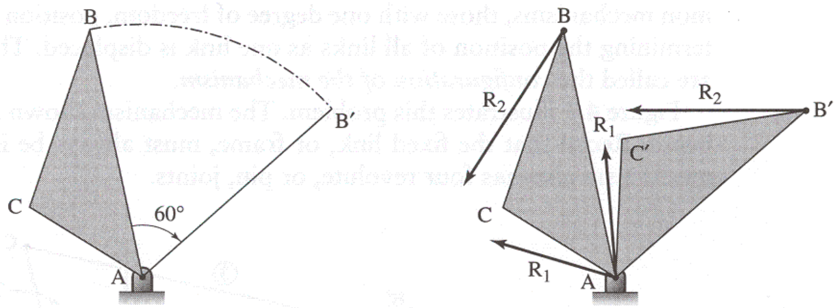


Figure 4.2 Rotating a complex link.

to yield the relocated link. All CAD systems have a command that can easily rotate and copy geometric entities. It is convenient to display the rotated link in an alternate color and place it on a different layer.

Once the driver link is repositioned, the position of all other links must be determined. To accomplish this, the possible paths of all links that are connected to the frame can be constructed. For links that are pinned to the frame, all points on the link can only rotate relative to the frame. Thus, the possible paths of those points are circles, centered at the pin connecting the link to the frame.

Figure 4.3 illustrates a kinematic diagram of a mechanism. Links 2, 4, and 6 are all pinned to the ground. Because points *B*, *C*, and *E* are located on links 2, 4, and 6 respectively, their possible paths can be readily constructed. The possible path of point *B* is a circle centered at point *A*, which is the pin that connects link 2 to the frame. The paths of *C* and *E* can be determined in a similar manner.

The path of a point on a link that is connected to the frame with a slider joint can also be easily determined. All points on this link move in a straight line, parallel to the direction of the sliding surface.

After the paths of all links joined to the frame are constructed, the position of the connecting links can be determined. This is a logical process that stems from the fact that a link is rigid. Rigidity means that the links do not change lengths or shape during motion.

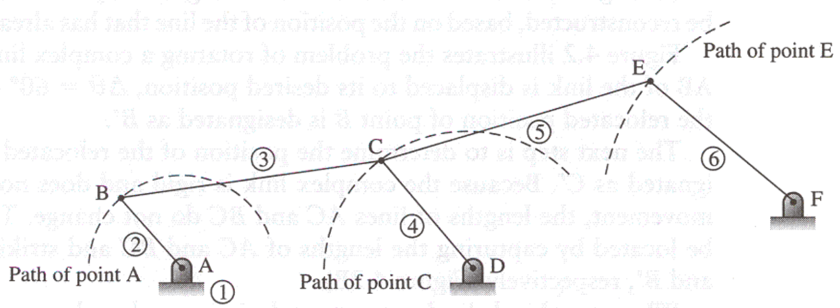


Figure 4.3 Paths of points on a link pinned to the frame.

In Figure 4.1, the positions of links 3 and 4 are desired as link 2 rotates 15° clockwise. Using the procedures described above, link 2 has been relocated to the prescribed location which determined the position of point B'. The possible path of point C has also been constructed (Figure 4.4).

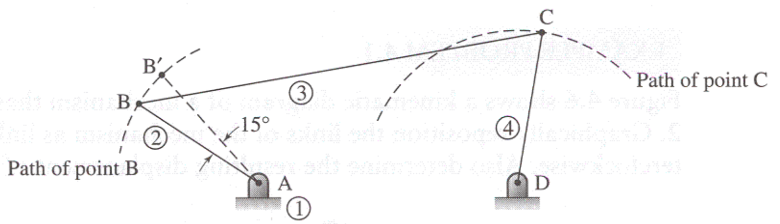


Figure 4.4 Constructing the path of C.

From the rigidity concept, the length of link 3 does not change during motion. Although link 2 has been repositioned, the length between points B and C does not change. To summarize the facts of this position analysis, the following is known:

1. Point B has been moved to B';
2. Point C must always lay on its possible path; and,
3. The length between B and C must stay constant.

From these facts, the new position of link 3 can be constructed. The length of line BC can be captured. Because point B has been moved to B', an arc of length BC is constructed with its center at B'. By sweeping this arc, the feasible path of point C' has been determined. However, point C' must lay on its possible path. Therefore, point C' must be located at the intersection of the two arcs. This process is illustrated in Figure 4.5.

Once C' has been located, the position of links 3 and 4 can be drawn. Thus, the configuration of mechanism, as the driver link was repositioned, has been determined.

The above description describes the logic behind graphical position analysis. The actual solution can be completed using manual drawing techniques (using a protrac-

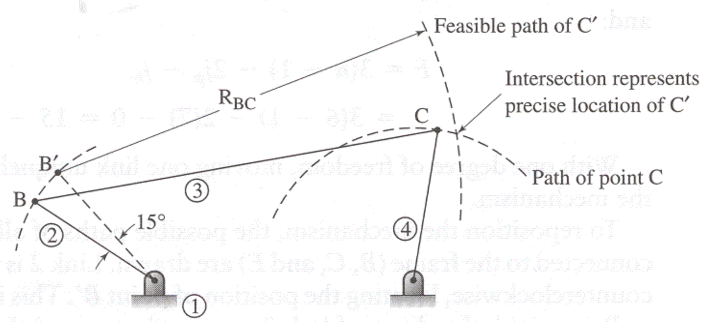


Figure 4.5 Locating the position of C′.

tor and compass) or can be completed on a CAD system (using a rotate and copy command). The logic is identical; however, the CAD solution is not susceptible to the limitations of drafting accuracy. Regardless of the method used, the underlying concepts of graphical position analysis can be further illustrated and expanded through Example Problems below.

EXAMPLE PROBLEM 4.1

Figure 4.6 shows a kinematic diagram of a mechanism that is driven by moving link 2. Graphically reposition the links of the mechanism as link 2 is displaced 30° counterclockwise. Also determine the resulting displacement of point E.

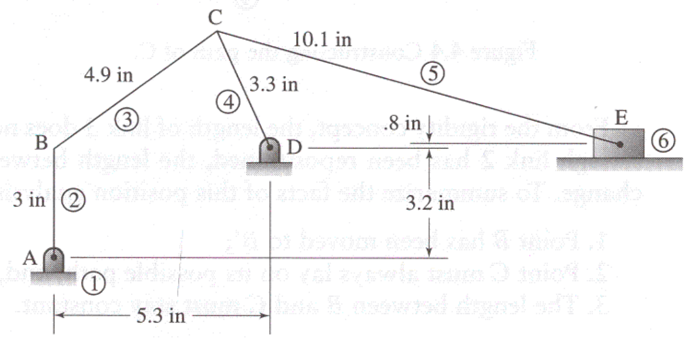


Figure 4.6 Kinematic diagram for Example Problem 4.1.

Solution:

To verify that the mechanism is uniquely positioned by moving one link, its mobility can be calculated . Six links are labeled. Three of these links are connected at point C. Recall from Chapter 1 that this configuration must be counted as two pin joints. Therefore, a total of 6 pin joints is counted. One sliding joint connects links 1 and 6. No gear of cam joints exists:

$$n = 6 \quad j_p = (6 \text{ pins} + 1 \text{ sliding}) = 7 \quad j_h = 0$$

and:

$$F = 3(n - 1) - 2j_p - j_h$$

$$= 3(6 - 1) - 2(7) - 0 = 15 - 14 = 1$$

With one degree of freedom, moving one link uniquely positions all other links of the mechanism.

To reposition the mechanism, the possible paths of all the points on links that are connected to the frame (B, C, and E) are drawn. Link 2 is then graphically rotated 30° counterclockwise, locating the position of point B'. This is shown in Figure 4.7A.

Being rigid, the shape of link 3 cannot change and the distance between points B and C remains constant. Because point B has been moved to B', an arc can be drawn

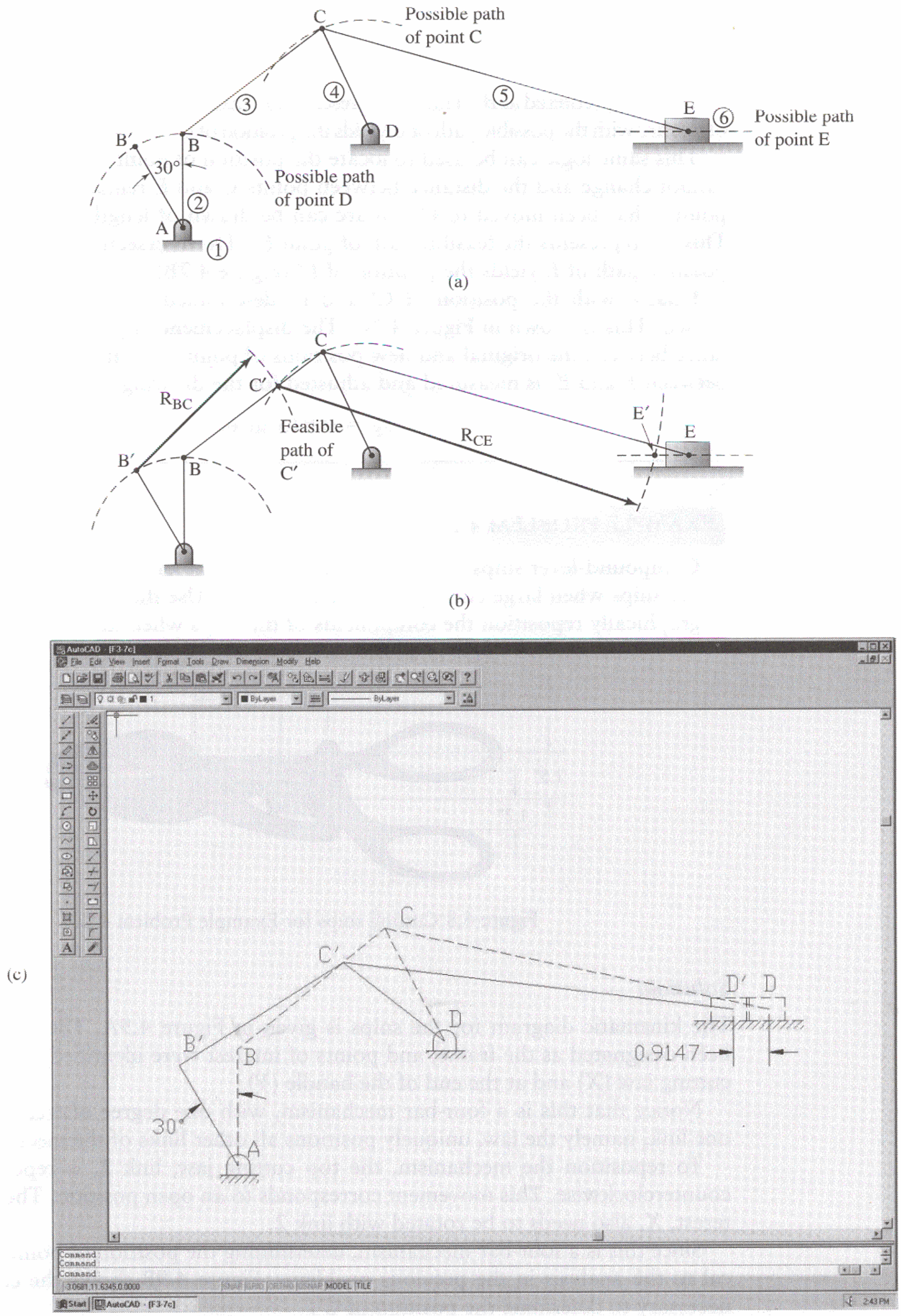


Figure 4.7 Displacement constructions for Example Problem 4.1.

of length BC, centered at B'. This arc represents the feasible path of point C'. The intersection of this arc with the possible path of C yields the position of C'. This is shown in Figure 4.7B.

This same logic can be used to locate the position of point E'. The shape of link 5 cannot change and the distance between points C and E remains constant. Because point C has been moved to C', an arc can be drawn of length CE, centered at C'. This arc represents the feasible path of point E'. The intersection of this arc with the possible path of E yields the position of E' (Figure 4.7B).

Finally, with the position of C' and E' determined, links 3 through 6 can be drawn. This is shown in Figure 4.7C. The displacement of point E is the linear distance between the original and new positions of point, described as E′. The distance between E and E' is measured and adjusted for the drawing scale.

$$s_E = 0.915 \text{ in} \leftarrow$$

EXAMPLE PROBLEM 4.2

Compound-lever snips, as shown in Figure 4.8, are often used in place of regular tinner snips when large cutting forces are required. Use the top handle as the frame to graphically reposition the components of the snips when the jaw is opened 15°.

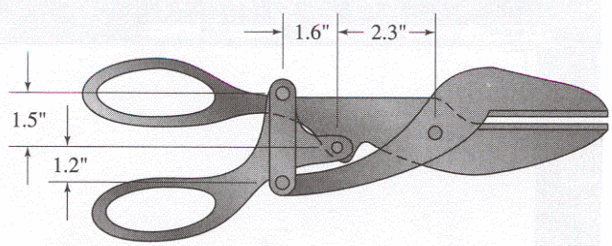


Figure 4.8 Cutting snips for Example Problem 4.2.

Solution:

The kinematic diagram for the snips is given in Figure 4.9A. The top handle has been designated as the frame, and points of interest were identified at the tip of the cutting jaw (X) and at the end of the handle (Y).

Notice that this is a four-bar mechanism, with one degree of freedom. Moving one link, namely the jaw, uniquely positions all other links of the mechanism.

To reposition the mechanism, the top cutting jaw, link 2, is repositioned 15° counterclockwise. This movement corresponds to an open position. The point of interest, X, also needs to be rotated with link 2.

Since this is a four-bar mechanism, determining the position of point C' is identical to the analysis in the previous problems. Figure 4.9B shows the constructions necessary to determine the position of C'.

Finally, the location of point of interest Y must be determined. Link 4 is rigid and its shape is not altered. Because the side $C'D$ has been located, point Y' can be readily found.

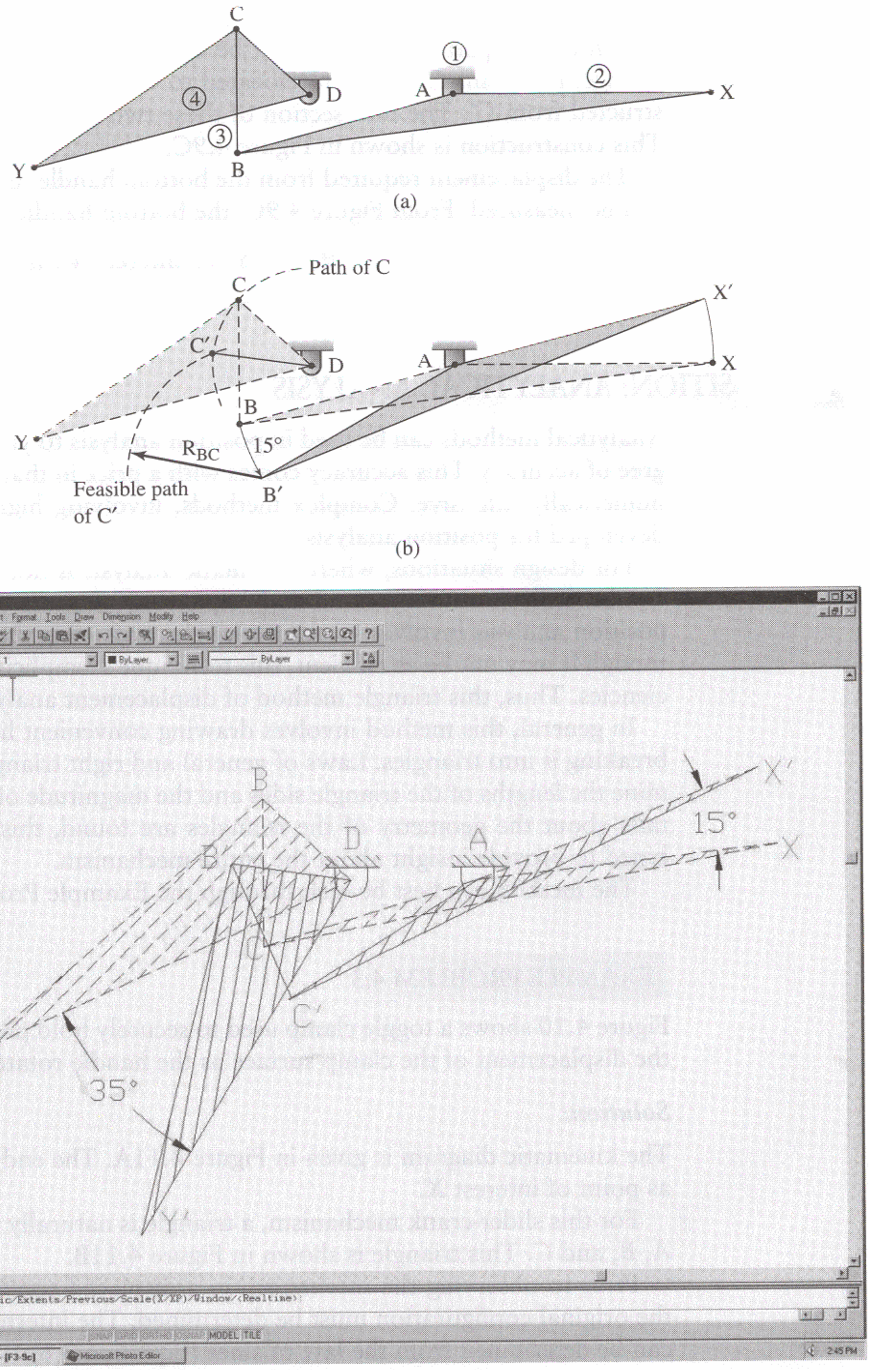


Figure 4.9 Constructions for Example Problem 4.2.

Similar to the procedure used in Figure 4.2, the distance DY does not change; therefore, the path of point Y can be determined. The distance CY also does not change, but point C has been relocated to C'. The feasible path for Y' can be constructed from C'. The intersection of these two paths gives the final location of Y'. This construction is shown in Figure 4.9C.

The displacement required from the bottom handle, in order to open the jaw 15°, can be measured. From Figure 4.9C, the bottom handle, link 4, must be displaced:

$$\theta_4 = 35° \text{ counterclockwise}$$

4.6 POSITION: ANALYTICAL ANALYSIS

Analytical methods can be used in position analysis to yield results with a higher degree of accuracy. This accuracy comes with a price in that the methods often become numerically intensive. Complex methods, involving higher-order math, have been developed for position analysis.

For design situations, where kinematic analysis is not a daily task, such methods can be difficult to understand and implement. A more straightforward method of position analysis involves locating triangles and using the laws of trigonometry. Although it may not be as efficient, this technique's simplicity far outweighs any inefficiencies. Thus, this triangle method of displacement analysis is used in this text.

In general, this method involves drawing convenient lines through a mechanism, breaking it into triangles. Laws of general and right triangles are then used to determine the lengths of the triangle sides and the magnitude of the interior angles. As details about the geometry of the triangles are found, this new information is combined to provide insight about the entire mechanism.

The method can best be seen through the Example Problems below.

EXAMPLE PROBLEM 4.3

Figure 4.10 shows a toggle clamp used to securely hold parts. Analytically determine the displacement of the clamp surface as the handle rotates downward 15°.

Solution:

The kinematic diagram is given in Figure 4.11A. The end of the handle was labeled as point of interest X.

For this slider-crank mechanism, a triangle is naturally formed between pin joints A, B, and C. This triangle is shown in Figure 4.11B.

Prior to observing the mechanism in a displaced configuration, all properties of the original configuration must be determined. The internal angle at joint C, $\angle BCA$ can be determined from the law of sines (Equation 3.6):

$$\frac{\sin\angle BAC}{(BC)} = \frac{\sin\angle BCA}{(AB)}$$

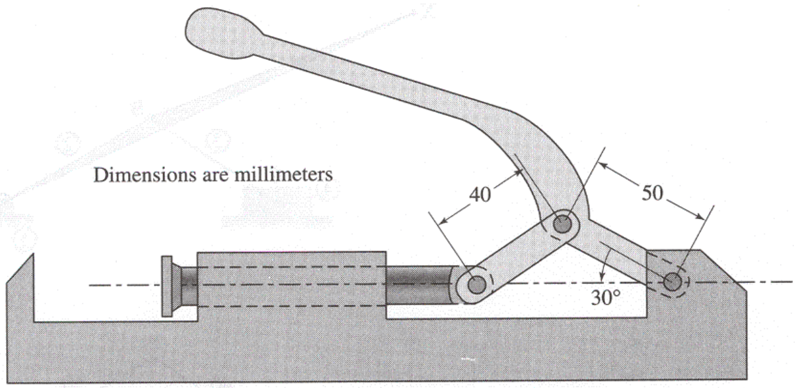


Figure 4.10 Toggle clamp for Example Problem 4.3.

$$\angle BCA = \sin^{-1}\left[\left(\frac{AB}{BC}\right)\sin\angle BAC\right]$$

$$= \sin^{-1}\left[\left(\frac{50 \text{ mm}}{40 \text{ mm}}\right)\sin 30°\right] = 38.68°$$

The interior angle at joint B, $\angle ABC$, can be found because the sum of all interior angles in any triangle must total 180°:

$$\angle ABC = 180° - (30° + 38.68°) = 111.62°$$

The length side AC represents the original position of the slider, and can be determined from the law of cosines, Equation 3.7:

$$AC = \sqrt{AB^2 + BC^2 - 2(AB)(BC)\cos\angle CBA}$$

$$= \sqrt{(50 \text{ mm})^2 + (40 \text{ mm})^2 - 2(50 \text{ mm})(40 \text{ mm})\cos(111.62)}$$

$$= 74.66 \text{ mm}$$

The displaced configuration is shown in Figure 4.11C when the handle is rotated downward 15°. Note that this displacement yields an interior angle at joint A, $\angle C'AB'$, of 15°. The law of sines can be used to find the interior angle at joint C', $\angle B'C'A$:

$$\angle B'C'A = \sin^{-1}\left[\left(\frac{AB'}{B'C'}\right)\sin\angle C'AB'\right]$$

$$= \sin^{-1}\left[\left(\frac{50 \text{ mm}}{40 \text{ mm}}\right)\sin 15°\right] = 18.88°$$

Again, the interior angle at joint B', $\angle AB'C'$, can be found because the sum of all interior angles in any triangle must total 180°:

$$\angle AB'C' = 180° - (15° + 18.88°) = 146.12°$$

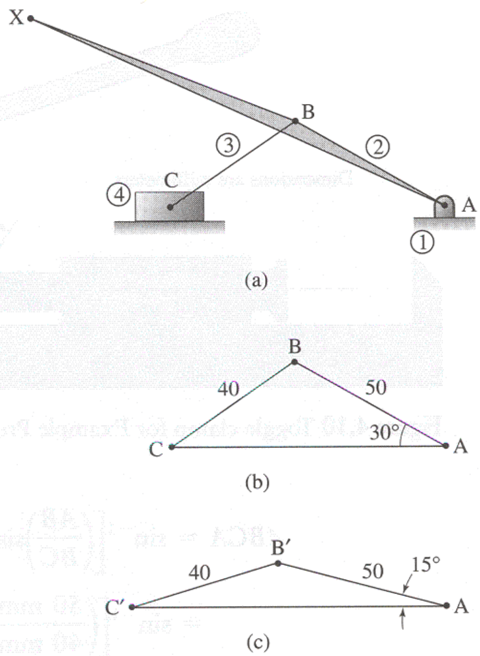


Figure 4.11 Mechanism for Example Problem 4.3.

The length side AC' represents the displaced position of the slider. As before, it can be determined from the law of cosines:

$$AC' = \sqrt{AB'^2 + B'C'^2 - 2(AB')(B'C')\cos\angle}$$
$$= \sqrt{(50 \text{ mm})^2 + (40 \text{ mm})^2 - 2(50 \text{ mm})(40 \text{ mm})\cos(142.12°)}$$
$$= 85.19 \text{ mm}$$

The displacement of point C during this motion can be found as the difference of the triangle sides AC' and AC:

$$s_c = 85.19 - 74.66 = 10.53 \text{ mm} \leftarrow$$

As stated, this clamp mechanism is a slider-crank linkage. Specifically, it is termed an in-line slider-crank mechanism because the line of slider motion extends through the center of the crank rotation. Figure 4.12 illustrates the basic configuration of an in-line slider-crank linkage.

Because this is a common mechanism, the results from the previous problem can be generalized.[10] A typical analysis problem determines the position of the slider (L_4) and the interior joint angles (θ_3 and γ) for known links (L_2 and L_3) at a certain crank angle (θ_2).

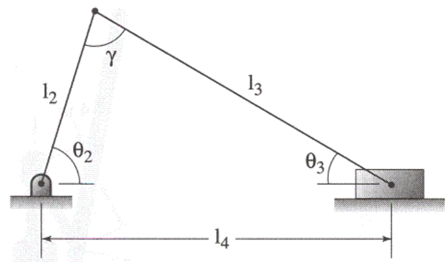


Figure 4.12 In-line slider-crank mechanism.

The equations used in Example Problem 4.3 are summarized in terms of L_2 , L_3 and θ_2:

(4.1) $$\theta_3 = \sin^{-1}\left[\frac{L_2}{L_3}\sin\theta_2\right]$$

(4.2) $$\gamma = 180° - (\theta_2 + \theta_3)$$

(4.3) $$L_4 = \sqrt{L_2{}^2 + L_3{}^2 - 2(L_2)(L_3)\cos\gamma}$$

These equations can be used to determine the position of the links in any mechanism configuration. Recall, however, that these equations are only applicable to an in-line slider-crank mechanism.

EXAMPLE PROBLEM 4.4

Figure 4.13 shows a concept for a hand pump used for increasing oil pressure in a hydraulic line. Analytically determine the displacement of the piston as the handle rotates 15° counterclockwise.

Solution:

The kinematic diagram is given in Figure 4.14A. The end of the handle was labeled as point of interest *X*.

In contrast to the previous problem, this mechanism is an offset slider-crank mechanism. For this type of mechanism, it is convenient to focus on two right triangles. These triangles are shown in Figure 4.14B. Notice that the complementary angle to the 10°, the 80° angle, is also shown.

Prior to observing the mechanism in a displaced configuration, all properties of the original configuration must be determined. Focusing on the lower right triangle, the sides *AD* and *BD* can be determined from the following trigonometric functions:

$$\cos\angle BAD = \frac{AD}{AB}$$

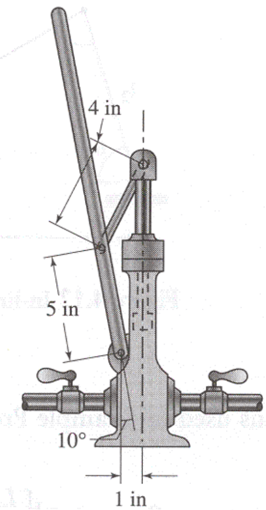


Figure 4.13 Toggle clamp for Example Problem 4.4.

$$AD = (AB)\cos\angle BAD = (5 \text{ in})\cos(80°) = 0.87 \text{ in.}$$

$$\sin\angle BAD = \frac{BD}{AB}$$

$$BD = (AB)\sin\angle BAD = (5 \text{ in})\sin(80°) = 4.92 \text{ in.}$$

By focusing on the top triangle, the length of side CE can be found as the sum of the offset distance and the length of side AD from the lower triangle:

$$CE = \text{Offset} + AD$$

$$= 1.0 + 0.87 = 1.87 \text{ in.}$$

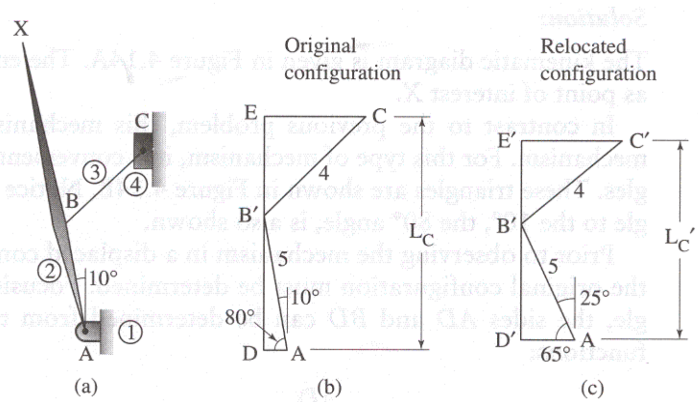


Figure 4.14 Mechanism diagrams for Example Problem 4.4

Use the Pythagorean theorem (Equation 3.4) to determine side BE:

$$CB^2 = BE^2 + CE^2$$

$$BD = \sqrt{BC^2 - CE^2}$$

$$= \sqrt{(4)^2 - (1.87)^2} = 3.54 \text{ in}$$

The original position of the piston, point C, can be determined by summing BD and BE:

$$L_c = BD + BE = 4.92 + 3.54 = 8.46 \text{ in.}$$

Although not required in this problem, the angle that defines the orientation of link 3 is often desired. The internal angle $\angle BCE$ can be determined through the following trigonometric functions:

$$\cos\angle BCE = \frac{CE}{BC}$$

$$\angle BCE = \cos^{-1}\frac{CE}{BC} \quad \angle BCE = \cos^{-1}(CE/BC)$$

$$= \cos^{-1}\left(\frac{1.87 \text{ in}}{4 \text{ in}}\right) = 62.13°$$

The displaced configuration is shown in Figure 4.14C with the handle rotated downward 15°. Note that this displacement yields an angle at joint A of 25°, and its complement 35° is also shown. Focusing on the lower right triangle, the sides AD' and $B'D'$ can be determined from the following trigonometric functions:

$$AD' = (AB')\cos\angle B'AD' = (5 \text{ in})\cos(65°) = 2.11 \text{ in.}$$

$$B'D' = (AB')\sin\angle B'AD' = (5 \text{ in})\sin(65°) = 4.53 \text{ in.}$$

Focusing on the top triangle, the length of side $C'E'$ can be found as the sum of the offset distance and the length of side AD' from the lower triangle:

$$C'E' = \text{Offset} + AD'$$

$$= 1.0 + 2.11 = 3.11 \text{ in.}$$

Side $B'E'$ can then be determined:

$$B'E' = \sqrt{(B'C')^2 - (C'E')^2}$$

$$= \sqrt{(4 \text{ in})^2 - (3.11 \text{ in})^2} = 2.52 \text{ in}$$

The displaced position of the piston can be determined by summing BD and BE:

$$L_{c'} = B'D' + B'E' = 4.53 + 2.52 = 7.05 \text{ in.}$$

The displacement of the piston, point C, during this motion can be found by subtracting the length $L_{c'}$ from L_c:

$$s_C = 8.46 - 7.05 = 1.41 \text{ in} \downarrow$$

As stated, this mechanism is an offset slider-crank mechanism because the line of slider motion does not extend through the center of the crank rotation. Figure 4.15 illustrates the basic configuration of an in-line slider-crank linkage.

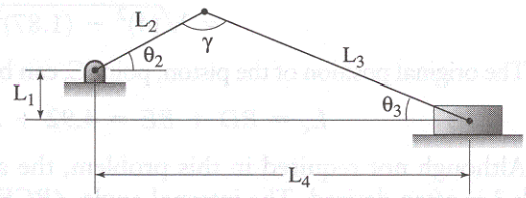


Figure 4.15 Offset slider-crank mechanism.

Because this is also a common mechanism, the results from the previous problem can be generalized. A typical analysis problem consists of determining the position of the slider (L_4) and the interior joint angles (θ_3 and γ) for known links (L_1, L_2, and L_3) at a certain crank angle (θ_2).

The equations used in Example Problem 4.4 are combined and summarized in terms of L_1, L_2, L_3, and θ_2.[10]

(4.4) $$\theta_3 = \sin^{-1}\left[\frac{L_1 + L_2 \sin\theta_2}{L_3}\right]$$

(4.5) $$L_4 = L_2 \cos(\theta_2) + L_3 \cos(\theta_3)$$

(4.6) $$\gamma = 180° - (\theta_2 + \theta_3)$$

These equations can be used to determine the position of the links in any mechanism configuration. Recall, however, that these equations are only applicable to an offset slider-crank mechanism. The equations also apply when the offset distance is in the opposite direction. For these cases, L_1, in the above equations should be substituted as a negative value.

EXAMPLE PROBLEM 4.5

Figure 4.16 shows a toggle clamp used for securing a workpiece during a machining operation. Analytically determine the angle that the handle must be displaced in order to lift the clamp arm 30° clockwise.

Solution:

The kinematic diagram for the clamp is given in Figure 4.17A. The end of the handle was labeled as point of interest X and the clamp nose was identified as point of interest Y.

This mechanism is the common four-bar linkage. In order to more closely analyze the geometry, Figure 4.17B focuses on the kinematic chain *ABCD*. A diagonal is created by connecting *B* and *D*, forming two triangles.

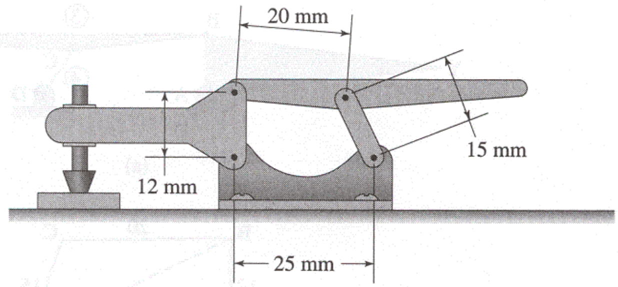


Figure 4.16 Clamp for Example Problem 4.5.

Prior to observing the mechanism in a displaced configuration, all properties of the original configuration must be determined. The diagonal BD can be found by using the lower left triangle ABD and the Pythagorean theorem.

$$BD = \sqrt{(AB)^2 + (AD)^2}$$
$$= \sqrt{(12)^2 + (25)^2} = 27.73 \text{ mm}$$

The internal angles, $\angle ABD$ and $\angle BDA$, can be determined from the following basic trigonometric functions:

$$\sin\angle ABD = \left(\frac{25 \text{ mm}}{27.73 \text{ mm}}\right) = 0.901$$
$$\angle ABD = 63.36°$$
$$\cos\angle BDA = \left(\frac{25 \text{ mm}}{27.73 \text{ mm}}\right) = 0.901$$
$$\angle BDA = \cos^{-1}(0.901) = 25.64°$$

Focusing on the top triangle, the internal angle $\angle BCD$ can be found from the law of cosines, introduced in Equation 3.7:

$$BD^2 = BC^2 + CD^2 - 2(BC)(CD)\cos\angle BCD$$

rewriting:

$$\cos\angle BCD = \frac{BC^2 + CD^2 - BD^2}{2(BC)(CD)}$$
$$= \frac{(20 \text{ mm})^2 + (15 \text{ mm})^2 - (27.73 \text{ mm})^2}{2(20 \text{ mm})\ (15 \text{ mm})} = -0.240$$
$$\angle BCD = \cos^{-1}(-0.240) = 103.9°$$

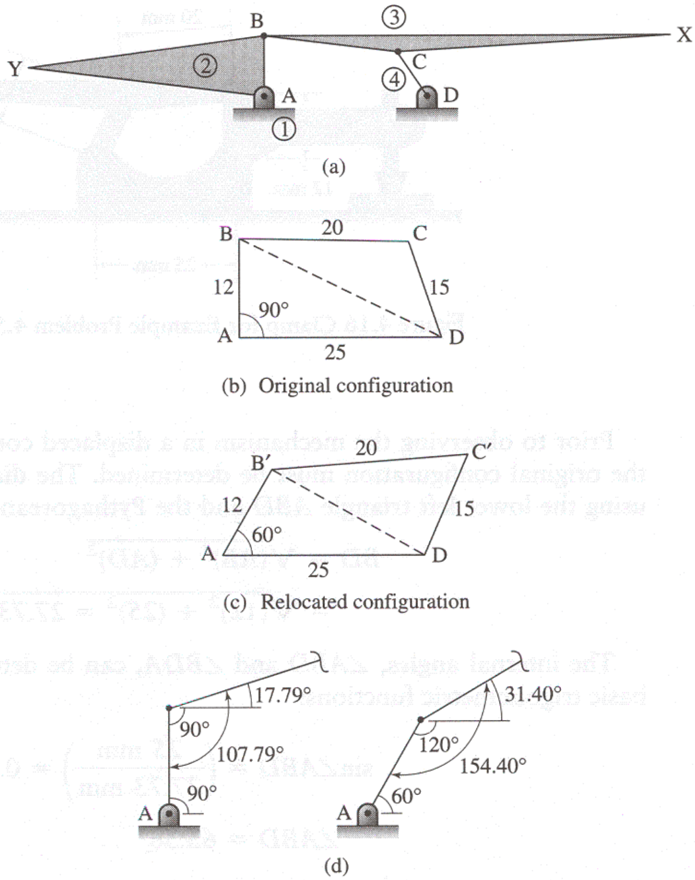


Figure 4.17 Mechanism for Example Problem 4.5.

The internal angles $\angle CBD$ and $\angle BDC$ can be determined from the law of sines:

$$\angle CBD = \sin^{-1}\left[\left(\frac{CD}{BD}\right)\sin\angle BCD\right]$$

$$= \sin^{-1}\left[\frac{(20 \text{ mm})}{(27.73 \text{ mm})}\sin 103.9°\right] = 44.4°$$

The interior angle at $\angle BDC$ can be found because the sum of all interior angles in any triangle must total 180°. Thus:

$$\angle BDC = 180° - (103.9° + 44.4°) = 31.7°$$

The total mechanism angles at joint B (between links 2 and 3) and at joint D (between links 1 and 4) can be determined.

At joint B:

$$\angle ABC = \angle ABD + \angle CBD$$
$$= 63.4° + 44.4° = 107.8°$$

At joint D:

$$\angle CDA = \angle BDC + \angle BDA$$
$$= 31.7° + 25.6° = 57.3°$$

The displaced configuration is shown in Figure 4.17C with the clamp nose, link 2, rotated clockwise 30°. Notice that this leaves the interior angle at joint A, $\angle DAB'$ as 60°. Also, the lower triangle is no longer a right triangle.

The diagonal $B'D$ can be found by using the lower triangle, ΔABD, and the law of cosines:

$$(B'D) = (AB')^2 + (AD)^2 - 2(AB')(AD)\cos\angle B'AD$$
$$= \sqrt{(12\text{ mm})^2 + (25\text{ mm})^2 - 2(12\text{ mm})(25\text{ mm})\cos 60°}$$
$$= 21.66\text{ mm}$$

The internal angle $\angle AB'D$ can be determined from the law of sines:

$$\sin\angle AB'D = \left[\left(\frac{AD}{B'D}\right)\sin\angle B'AD\right]$$

$$= \left[\frac{(25\text{ mm})}{(21.66\text{ mm})}\sin 60°\right] = 0.99$$

$$\angle AB'D = \sin^{-1}(0.99)88.3°$$

The total of the interior angles of any triangle must be 180°. Therefore, angle $\angle B'DA$ can be readily determined:

$$\angle B'DA = 180° - (\angle DAB' - \angle AB'D)$$
$$= 180° - (60° - 88.3°) = 31.7°$$

Focusing on the top triangle, the internal angle $B'C'D$ can be found from the law of cosines:

$$\cos\angle B'C'D = \frac{B'C'^2 + C'D^2 - B'D^2}{2(B'C')(C'D)}$$

$$= \frac{(20\text{ mm})^2 + (15\text{ mm})^2 - (21.66\text{ mm})^2}{2(20\text{ mm})(15\text{ mm})} = 0.26$$

$$\angle B'C'D = \cos^{-1}(0.26) = 74.9°$$

The internal angles $\angle C'B'D$ and $\angle B'DC'$ can be determined from the law of sines:

$$\angle C'B'D = \sin^{-1}\left[\left(\frac{C'D}{B'D}\right)\sin\angle B'C'D\right]$$

$$= \sin^{-1}\left[\frac{(20\text{ mm})}{(21.66\text{ mm})}\sin 74.9°\right] = 63.1°$$

The final interior angle, $\angle B'DC'$, of the upper triangle can be found by the following:

$$\angle B'DC' = 180° - \angle C'B'D - \angle B'C'D$$

$$= 180° - (63.1° - 74.9°) = 42.0°$$

The total mechanism angles at joint B (between links 2 and 3) and at joint D (between links 1 and 4) can be determined by the following:
At joint B:

$$\angle AB'C' = \angle AB'D + \angle C'B'D$$

$$= 88.3° + 63.1° = 151.4°$$

At joint D:

$$\angle C'DA = \angle B'DC' + \angle B'DA$$

$$= 42.0° + 31.7° = 73.7°$$

The angular displacement of the handle, link 3, can be determined by focusing on joint B, as shown in Figure 4.17D. For the original configuration, the angle of link 3, above the horizontal is expressed as:

$$\angle ABC - 90° = 107.79° + 90.0° = 17.79°$$

For the displaced configuration, the angle of link 3, above the horizontal is expressed as:

$$\angle AB'C' - 120° = 151.40° - 120.0° = 31.40°$$

Finally, the angular displacement of link 3 is determined by:

$$\Delta\theta_3 = 31.40° - 17.79° = 13.61°\ CCW$$

The four-bar mechanism is another very common linkage. Therefore, equations that were generated in this previous analysis can be generalized for future use. Figure 4.18 illustrates a general four-bar linkage.

A typical analysis problem consists of determining the interior joint angles (θ_3, θ_4, and γ) for known links (L_1, L_2, L_3, and L_4) at a certain crank angle (θ_2).

The equations used in Example Problem 4.5 can be combined and summarized in terms of L_1, L_2, L_3, L_4, and θ_2 [10]:

(4.7) $$BD = \sqrt{L_1^2 + L_1^2 - 2(L_1)(L_2)\cos(\theta_2)}$$

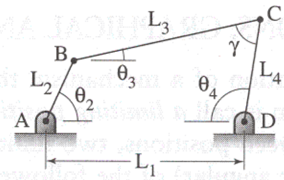


Figure 4.18 The four-bar mechanism.

$$\gamma = \cos^{-1}\left[\frac{L_3^2 + L_4 - BD^2}{2(L_3)(L_4)}\right] \tag{4.8}$$

$$\theta_4 = 180° - \cos^{-1}\left[\frac{L_1^2 + L_2^2 + BD^2}{2(L_1)(BD)}\right] - \cos^{-1}\left[\frac{L_4^2 + L_3^2 + BD^2}{2(L_4)(BD)}\right] \tag{4.9}$$

$$\theta_3 = \theta_4 - \gamma \tag{4.10}$$

These equations can be used to determine the position of the links in any mechanism configuration. The equations are applicable to any four-bar mechanism assembled as shown in Figure 4.18.

For four-bar mechanisms classified as crank-rockers(as described in Section 1.8) there are two regions of possible motion. The mechanism shown in Figure 4.18 operates in the first circuit (Figure 4.19A).

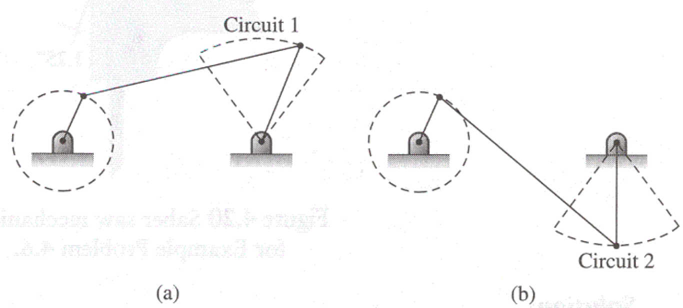


Figure 4.19 Circuits of a four-bar mechanism.

By physically disconnecting joint C, the links can be rotated and reassembled into the configuration shown in Figure 4.19B. As this mechanism is operated, it exhibits motion in the second circuit. Although the motion of the mechanism appears to be different, depending on the circuit of operation, the relative motion between the links does not change. However, the circuit in which the mechanism is assembled must be determined to fully understand the operation of the mechanism.

For four-bar mechanisms operating in the second circuit, equation 4.9 must be slightly altered as follows:

$$\theta_4 = 180° - \cos^{-1}\left[\frac{L_1^2 + L_2^2 + BD^2}{2(L_1)(BD)}\right] + \cos^{-1}\left[\frac{L_4^2 + L_3^2 + BD^2}{2(L_4)(BD)}\right] \tag{4.11}$$

4.7 LIMITING POSITIONS: GRAPHICAL ANALYSIS

The configuration of a mechanism that places one of the follower links in an extreme position is call *a limiting position*. Because most mechanisms have links that oscillate between positions, two limiting positions are common. The displacement (i.e., linear or angular) of the follower link from one extreme position to the other defines the stroke of the follower. A limiting position also gives a useful phase from which other motions can be referenced.

The position of a driver or input link which places a follower link in an extreme position is often desired. The logic used in solving such a problem is identical to the position analysis discussed above.

EXAMPLE PROBLEM 4.6

The mechanism shown in Figure 4.20 is the driving linkage for a reciprocating saber saw. Determine the limiting positions of the mechanism that places the saw blade in its extreme positions.

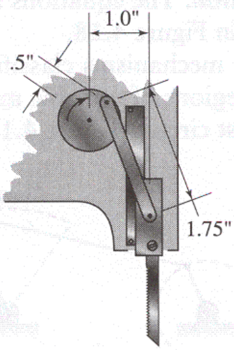


Figure 4.20 Saber saw mechanism for Example Problem 4.6.

Solution:

The kinematic diagram for the reciprocating saw mechanism is given in Figure 4.21A. Notice that this is a slider-crank mechanism as defined in Chapter 1. The slider crank has one degree of freedom.

The saw blade, link 4, reaches its extreme downward position as links 2 and 3 are in line. This configuration provides the maximum distance between points *A* and *C*. To determine this maximum distance, the lengths of links 2 and 3 must be combined. A procedure for graphically accomplishing this is as follows:

1. Draw the possible path of point *C*. Because link 4 is connected to the frame with a sliding joint, the possible path is a vertical line (Figure 4.21B).
2. Draw an arc, the length of link 2 centered at point *B* (Figure 4.21B.
3. Continue the line between the joints on link 3 to intersect the arc. The point of intersection is labeled point *D* in Figure 4.21B.

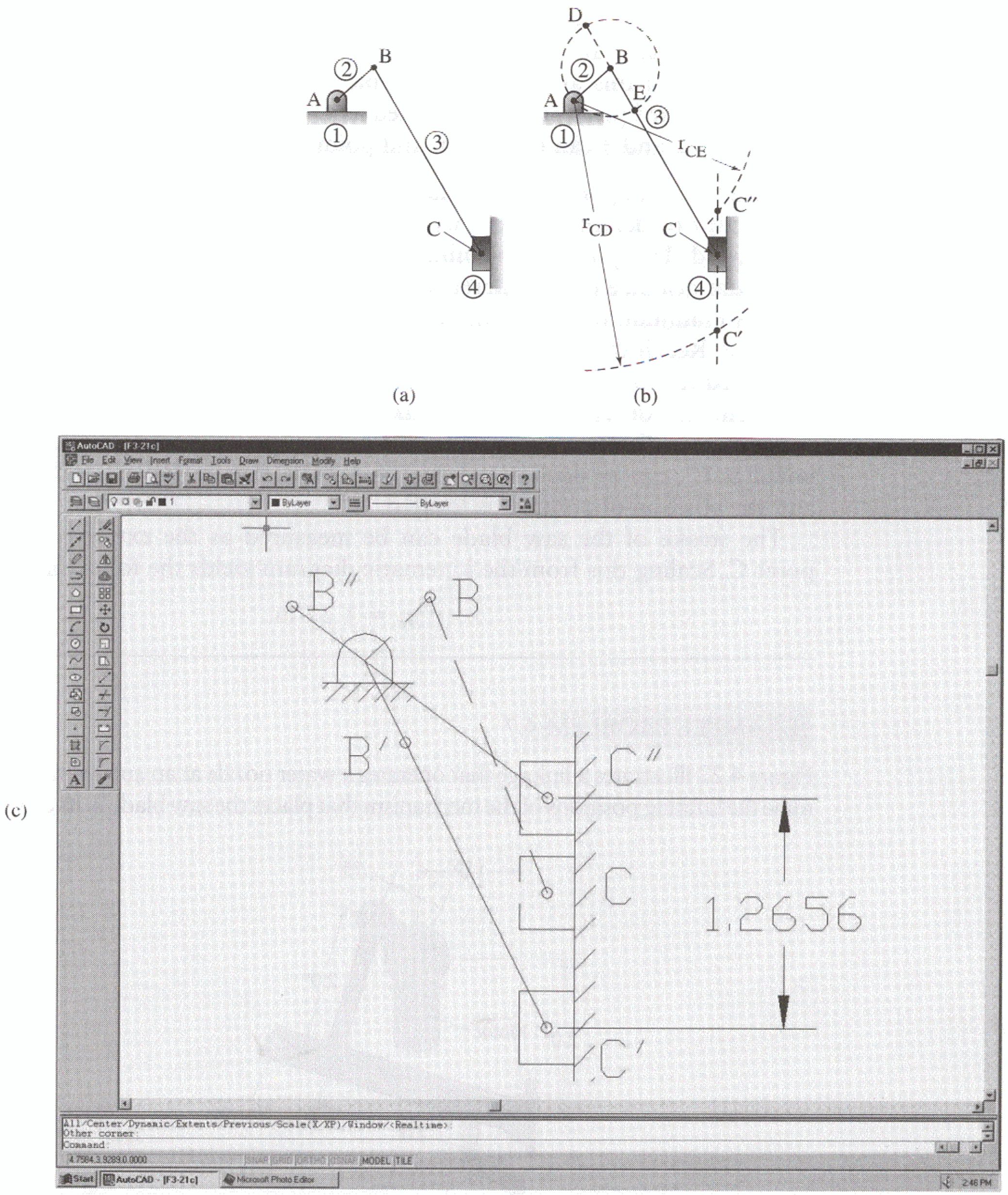


Figure 4.21 Extreme positions for Example Problem 4.6.

4. The length between D and C is the combined length of links 2 and 3. Therefore, an arc can be constructed of this length, centered at point A. The intersection of this arc and the possible path of point C determines the extreme downward position of C, denoted as C'.
5. Links 2 and 3 can be drawn, and point B' can be determined (Figure 4.21C).

Next, the configuration that places the saw blade, link 4, in its extreme upper position must be determined. In this configuration, links 2 and 3 are again in line, but overlapped. This provides the minimum distance between points A and C. Thus, this minimum distance is the difference between the lengths of links 3 and 2.

This minimum distance can be determined in a similar fashion to the maximum distance. Recall that the distance between C and D in Figure 4.21B represents the combined length of links 2 and 3. Similarly, the distance between points C and E represents the difference between links 3 and 2.

Using this distance, the position of point C at its extreme upward position, denoted as C'', can be determined (Figure 4.21B). Finally, links 2 and 3 can be drawn and the position of point B'' is located.

The stroke of the saw blade can be measured as the extreme displacement of point C. Scaling this from the kinematic diagram yields the following result:

$$s_c = 1.27 \text{ in.}$$

EXAMPLE PROBLEM 4.7

Figure 4.22 illustrates a linkage that operates a water nozzle at an automatic car wash. Determine the limiting positions of the mechanism that places the saw blade in its extreme positions.

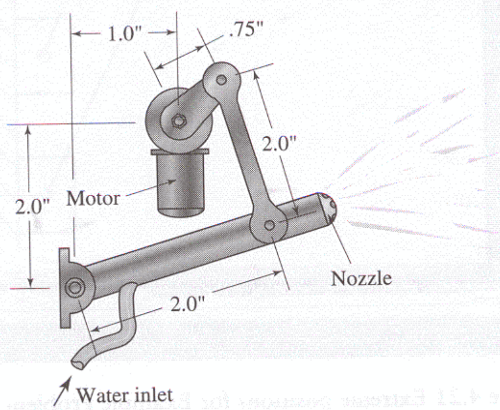


Figure 4.22 Water nozzle linkage for Example Problem 4.7.

Solution:

The kinematic diagram for the water nozzle linkage is given in Figure 4.23A. Notice that this is a four-bar mechanism.

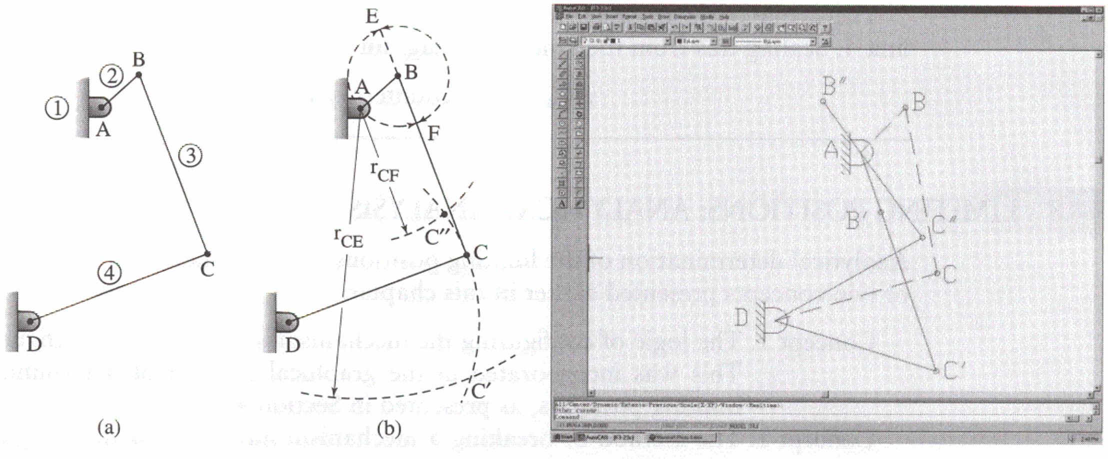


Figure 4.23 Extreme positions for Example Problem 4.7.

The analysis in this example is very similar to the previous problem. The nozzle, link 4, reaches its extreme downward position as links 2 and 3 are in line. This configuration provides the maximum distance between points *A* and *C*. To determine this maximum distance, the lengths of links 2 and 3 must be combined. A procedure for graphically accomplishing this is as follows:

1. Draw the possible path of point *C*. Because link 4 is connected to the frame with a pin joint, the possible path is an arc centered at point *D* (Figure 4.23B).
2. Draw an arc, the length of link 2 centered at point *B* (Figure 4.23B).
3. Continue the line between the joints on link 3 to intersect the arc. The point of intersection is labeled point *E* in Figure 4.23B.
4. The length between *E* and *C* is the combined length of links 2 and 3. Therefore, an arc can be constructed of this length, centered at point *A*. The intersection of this arc and the possible path of point *C* determines the extreme downward position of *C*, denoted as *C′*.
5. Links 2 and 3 can be drawn, and point *B′* can be determined (Figure 4.23C).

Next, the configuration that places the nozzle, link 4, in its extreme upper position must be determined. Similar to the slider crank discussed above, links 2 and 3 are in line, but overlapped. This provides the minimum distance between points *A* and *C*. Thus, this minimum distance is the difference between the lengths of links 3 and 2.

This minimum distance can be determined in a similar fashion to the maximum distance. Recall that the distance between *C* and *E* in Figure 4.23B represents the combined length of links 2 and 3. Similarly, the distance between points *C* and *F* represents the difference between links 3 and 2.

Using this distance, the position of point *C* at its extreme upward position, denoted as *C″*, can be determined. This is shown in Figure 4.23B. Finally, links 2 and 3 can be drawn and the position of point *B′* is located.

The stroke of the nozzle can be measured as the extreme angular displacement of link 4. Scaling this from the kinematic diagram yields the following:

$$\Delta\theta_4 = 47° \text{ counterclockwise}$$

4.8 LIMITING POSITIONS: ANALYTICAL ANALYSIS

Analytical determination of the limiting positions for a mechanism is a combination of two concepts presented earlier in this chapter:

Concept 1. The logic of configuring the mechanism into a limiting configuration. This was incorporated in the graphical method of determining the limiting positions, as presented in Section 4.7.

Concept 2. The method of breaking a mechanism into convenient triangles and using the laws of trigonometry to determine all mechanism angles and lengths, as presented in Section 4.6.

Combining these two concepts to determine the position of all links in a mechanism at a limiting position is illustrated through the Example Problem below.

EXAMPLE PROBLEM 4.8

Figure 4.24 shows a conveyor transfer mechanism. Its function is to feed packages to a shipping station at specific intervals. Analytically determine the extreme positions of the lifting conveyor segment.

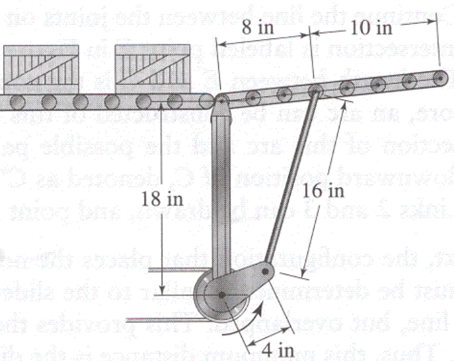


Figure 4.24 Conveyor feed for Example Problem 4.8.

Solution:

The kinematic diagram for the mechanism is given in Figure 4.25A. The end of the conveyor segment is labeled as point of interest X.

This mechanism is another four-bar linkage. As seen in Section 4.7, the follower of a four-bar is in an extreme position when links 2 and 3 are in line with each other.

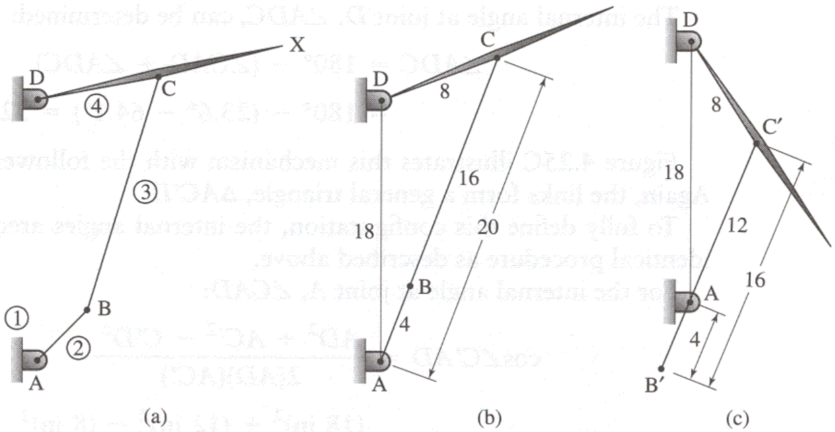


Figure 4.25 Mechanism for Example Problem 4.8.

Figure 4.25B illustrates this mechanism with the follower in its upper position. Notice that the links form a general triangle, ΔACD.

This upper-limiting position is fully defined by determination of the internal angles. The internal angle at joint A, $\angle CAD$, can be found using the law of cosines:

$$\cos\angle CAD = \frac{AD^2 + AC - CD^2}{2(AD)(AC)}$$

$$= \frac{(18 \text{ in})^2 + (20 \text{ in})^2 - (8 \text{ in})^2}{2(18 \text{ in})(20 \text{ in})} = 0.917$$

$$\angle CAD = \cos^{-1}(0.917) = 23.6°$$

The law of sines can be used to find either of the remaining internal angles. The law of sines may present some confusion with angles between 90° and 180° because:

$$\sin\theta = \sin(180° - \theta)$$

When the inverse sine function is used on a calculator, an angle is between 0° and 90°. However, the desired result may be an angle between 90° and 180°. To minimize this confusion, it is recommended to draw the triangles to an approximate scale, and verify numerical results. Also, it is best to use the law of sines with angles that are in the range of 0° to 90°.

Using that approach, the internal angle at joint C, $\angle ACD$, is determined using the law of sines since it is obviously smaller than 90°.

$$\sin\angle ACD = \left(\frac{AD}{DC}\right)\sin\angle CAD$$

$$= \frac{(18 \text{ in})}{(8 \text{ in})}\sin 23.6° = .900$$

$$\angle ACD = \sin^{-1}(0.90) = 64.1°$$

The internal angle at joint D, $\angle ADC$, can be determined:

$$\angle ADC = 180° - (\angle CAD + \angle ADC)$$
$$= 180° - (23.6° - 64.1°) = 92.4°$$

Figure 4.25C illustrates this mechanism with the follower in its lower position. Again, the links form a general triangle, $\Delta AC'D$.

To fully define this configuration, the internal angles aredetermined through an identical procedure as described above.

For the internal angle at joint A, $\angle CAD$:

$$\cos\angle C'AD = \frac{AD^2 + AC'^2 - C'D^2}{2(AD)(AC')}$$
$$= \frac{(18 \text{ in})^2 + (12 \text{ in})^2 - (8 \text{ in})^2}{2(18 \text{ in})(12 \text{ in})} = 0.935$$
$$\angle C'AD = \cos^{-1}(0.935) = 20.7°$$

The internal angle at D is in the range of 0° to 90°. Therefore, for the internal angle at joint D, $\angle AC'D$:

$$\sin\angle ADC' = \left(\frac{AD}{DC'}\right)\sin\angle C'AD$$
$$= \frac{(18 \text{ in})}{(8 \text{ in})}\sin 20.7° = 0.795$$
$$\angle ADC' = \sin^{-1}(0.795) = 52.8°$$

Finally, the internal angle at joint C', $\angle AC'D$, can be determined by the following:

$$\angle AC'D = 180° - (\angle C'AD + \angle ADC')$$
$$= 180° - (20.7° + 52.8°) = 106.4°$$

To summarize, the conveyor segment cycles between 92.4° and 52.8°, as measured upward from the vertical:

$$52.82° < \theta_4 < 92.38°$$

and the stroke is:

$$\Delta\theta_4 = 92.4° - 52.8° = 39.6°$$

4.9 COMPLETE CYCLE: GRAPHICAL POSITION ANALYSIS

The configuration of a mechanism at a particular instant is termed the *phase of the mechanism*. The position analyses discussed in this chapter focus on determining the phase of a mechanism at a given displacement of an input link.

A cycle of a mechanism is the motion from a given phase until all the links of the mechanism assume the original configuration. From an analysis standpoint, the actual determination of an original configuration is not important. Any convenient configuration, can be selected as the original phase.

To complete a full-position analysis, it is common to determine the configuration of the mechanism at several phases of its cycle. The procedure, whether graphical or analytical, is virtually the same as detailed in the previous sections. However, the only adaptation includes repeating these procedures at some interval of input position. The following Example Problems illustrate the position analysis for a full cycle.

EXAMPLE PROBLEM 4.9

Figure 4.26 shows the driving mechanism of hand-held grass shears. The mechanism operates by rotating the large disc as shown. Graphically determine the position of the driving mechanism at several phases of its operating cycle.

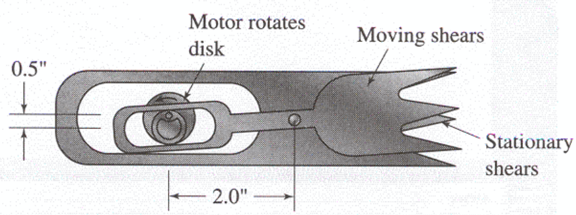


Figure 4.26 Grass shears for Example Problem 4.9.

Solution:

The kinematic diagram is given in Figure 4.27A. The end of the middle cutting blade is labeled as point of interest X.

This mechanism is constrained and is verified by the following equations:

$$n = 4 j_p = (3 \text{ pins} + 1 \text{ sliding}) = 4 \ j_h = 0$$

and:

$$F = 3(n - 1) - 2j_p - j_h$$
$$= 3(4 - 1) - 2(4) - 0 = 1$$

As mentioned, drawing the mechanism in several phases of its cycle is identical to the previous position analysis, but repetitive. While drawing the different phases, the kinematic diagram can become cluttered very quickly. It is highly recommended that different colors or fonts be used to represent each phase of the cycle. When using CAD, it is also beneficial to place each phase on a different layer, which can be rapidly displayed or hidden.

For this problem, the driving link, link 2, is positioned at 45° intervals throughout its cycle. This is shown in Figure 4.27B. In practice, even smaller increments are used depending on the desired level of precision.

(a)

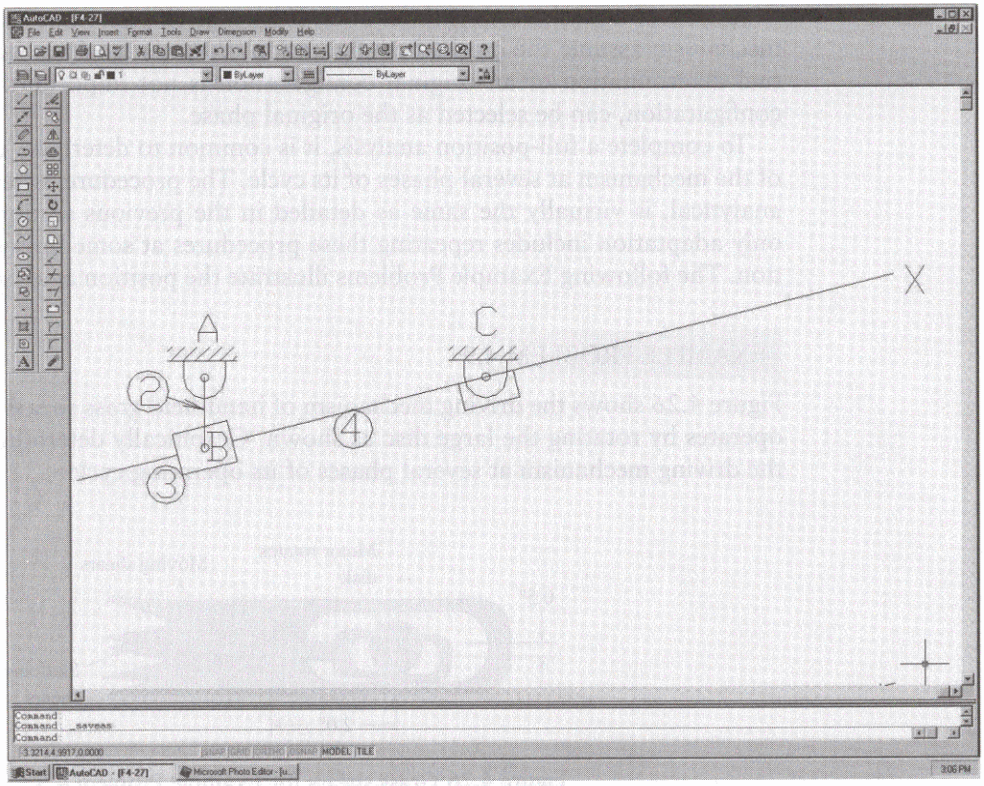


Figure 4.27 Mechanism phases for Example Problem 4.9.

The phases associated with the limiting positions should also be determined. The shear blade reaches its upward-most position when link 4 rotates to the greatest angle possible. This occurs when link 4 is tangent to the circle that represents the possible positions of point B. The point of tangency is denoted as B' and the corresponding position of the blade is denoted as X'. This is shown in Figure 4.15C.

Similarly, the lowest position of the blade occurs when link 4 dips to its lowest angle. Again, this occurs when link 4 is tangent to the circle that represents the possible paths of B. The points related to this lowest configuration are denoted in Figure 4.27C as B'' and X''.

The maximum displacement of link 4 can be scaled from the kinematic diagram:

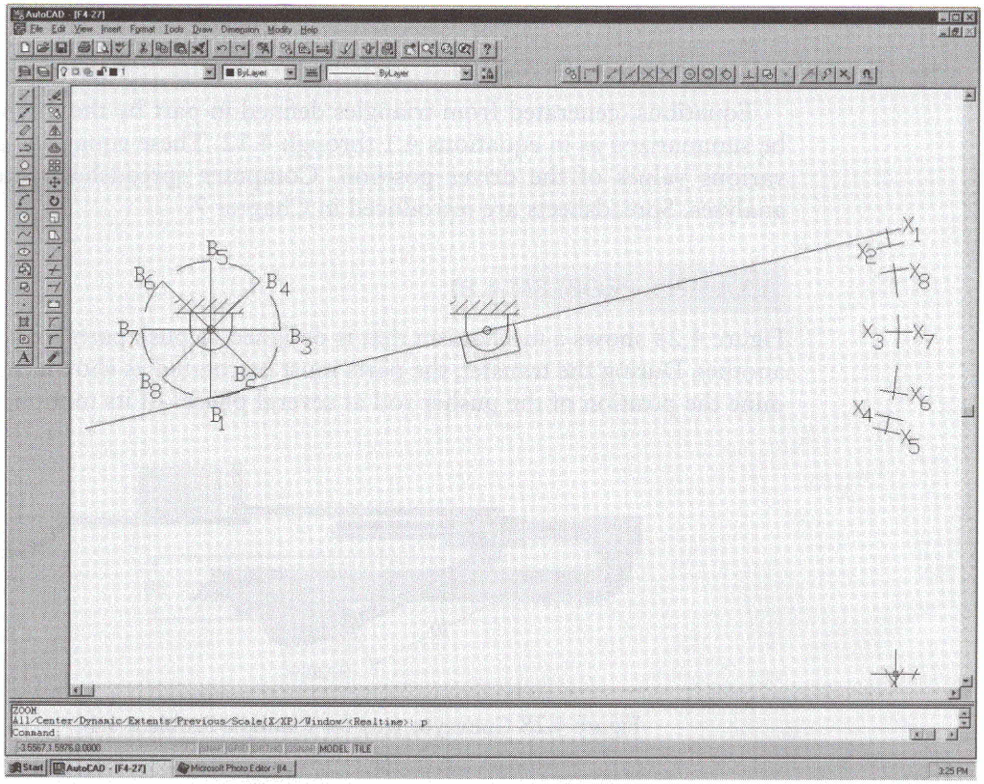
$$\Delta\theta_4 = 17° \text{ clockwise}$$

4.10 COMPLETE CYCLE: ANALYTICAL POSITION ANALYSIS

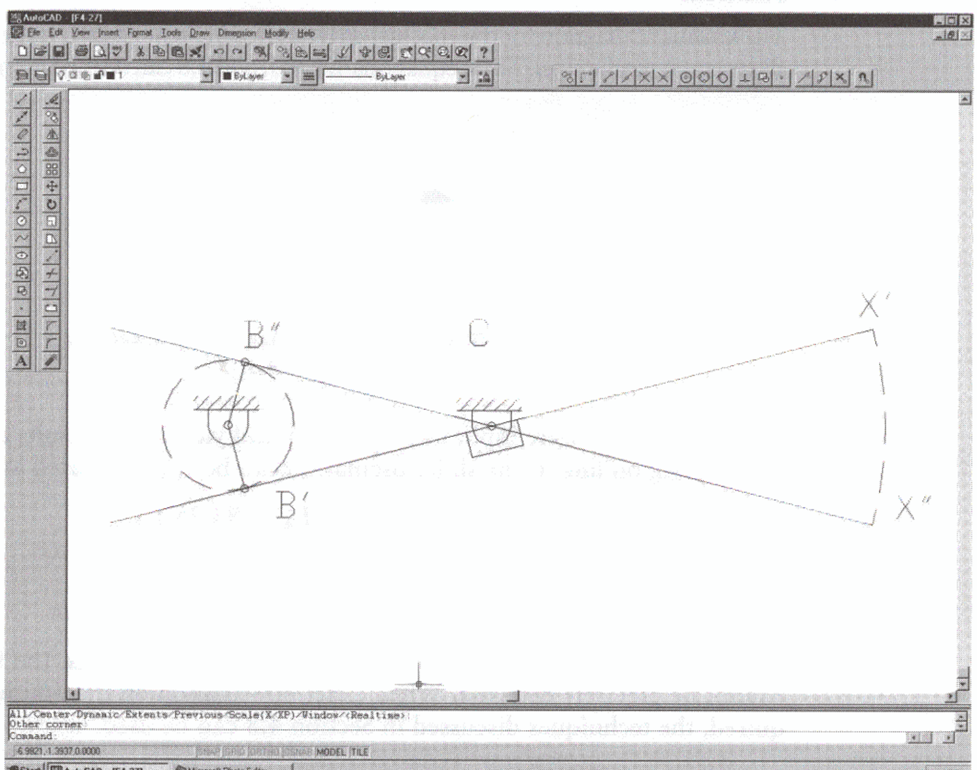
To generate the configuration of a mechanism throughout a cycle, analytical analysis can be repeated to obtain various phases. This can be an extremely repetitious process.

Figure 4.27
continued

(b)



(c)



Equations, generated from triangles defined in part by the mechanism links, can be summarized as in equations 4.1 through 4.12. These equations can be solved for various values of the driver position. Computer spreadsheets are ideal for such analyses. Spreadsheets are introduced in Chapter 7.

EXAMPLE PROBLEM 4.10

Figure 4.28 shows a mechanism that is designed to push parts from one conveyor to another. During the transfer, the parts must be rotated as shown. Analytically determine the position of the pusher rod at several phases of its motion.

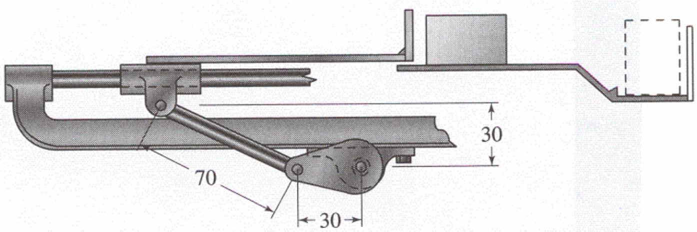


Figure 4.28 Conveyor feed for Example Problem 4.10.

Solution:

The kinematic diagram for this mechanism is shown in Figure 4.29. Notice that it is an offset slider-crank mechanism. Thus, equations 4.4, 4.5, and 4.6 can be used in a full-cycle analysis.

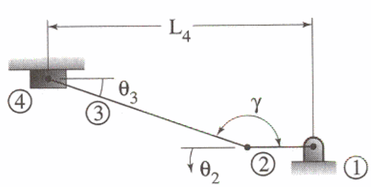


Figure 4.29 Kinematic diagram for Example Problem 4.10.

The equations were input into Table 4.1 and yielded the results as shown. Focusing on link 4, the slider oscillation can be approximated as:

$$26.51 \text{ mm} < L_4 < 93.25 \text{ mm}$$

and the maximum displacement as:

$$s_4 = \Delta L_4 \cong 93.25 - 26.51 = 66.74 \text{ mm}$$

This is only an approximation because, at 15° increments, the limiting position cannot be precisely detected. When exact information on the limiting position is required, the techniques discussed in Section 4.8 can be used.

TABLE 4.1 Position of Pusher Rod for Example Problem 4.10.

θ_2 (deg)	L_4 (mm)	θ_3 (deg)	γ (deg)
0	93.25	25.38	154.62
15	87.92	32.65	132.35
30	79.60	40.01	109.99
45	68.93	47.02	87.98
60	57.03	53.10	66.90
75	45.47	57.41	47.59
90	36.06	59.00	31.00
105	29.94	57.41	17.59
120	27.03	53.10	6.90
135	26.51	47.02	2.02
150	27.64	40.01	349.99
165	29.96	32.65	342.35
180	33.25	5.38	334.62
195	37.40	18.52	326.48
210	42.39	12.37	317.63
225	48.23	7.21	307.79
240	54.88	3.29	296.71
255	62.23	0.84	284.16
270	70.00	0.00	270.00
285	77.76	0.84	254.16
300	84.88	3.29	236.71
315	90.66	7.21	217.79
330	94.35	12.37	197.63
345	95.35	18.52	176.48
360	93.25	25.38	154.62

Observing the information on the joint angles, it appears that the equations for γ led to a discontinuity at the crank angle, θ_2, of 150°. The values are correct and physically correspond to the configuration where link 2 passes under link 3, and γ measures the outside angle. This illustrates the need to verify the information obtained from the equations with the physical mechanism.

4.11 DISPLACEMENT DIAGRAMS

Once a full-cycle position analysis is completed, it is common to plot the displacement of one point corresponding to the displacement of another point. It is most common to plot the displacement of a point on the follower to the displacement of a point on the driver.

Typically, the displacement of the driver is plotted on the x-axis. In the case of a crank, the driver displacement consists of one revolution. The corresponding displacement of the follower is plotted along the y-axis. The displacement plotted may be linear or angular depending on the motion obtained from the specific mechanism.

EXAMPLE PROBLEM 4.11

Figure 4.30 shows the driving mechanism of a reciprocating compressor. Plot a displacement diagram of the piston displacement relative to the crankshaft rotation.

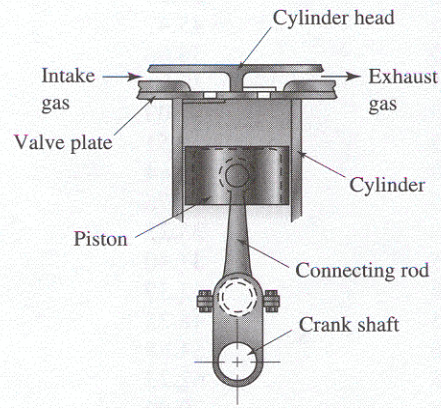


Figure 4.30 Compressor for Example Problem 4.11.

Solution:

After close examination, the compressor mechanism is identified as a slider crank. Recall that this mechanism has one degree of freedom, and can be operated by rotating the crank. The kinematic diagram with the appropriate dimensions is shown in Figure 4.31.

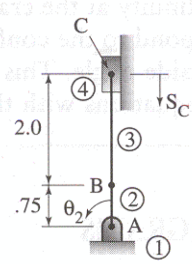


Figure 4.31 Kinematic diagram for Example Problem 4.11.

The figure displays the reference positions from which the displacements of the crank (link 2) and the piston (point C) are measured.

TABLE 4.2 Position of Piston for Example Problem 4.11

Crank (deg)	0	30	60	90	120	150	180
Piston (in)	0	0.14	0.48	0.90	1.23	1.40	1.50
Crank (deg)		210	240	270	300	330	360
Piston (in)		1.40	1.23	0.90	0.48	0.14	0.0

The actual displacements can be determined either analytically or graphically, using the methods presented in the previous sections. For this problem, the displacements were obtained analytically and are tabulated in Table 4.2. The crank displacement (θ_2) is in degrees and the piston displacement (s_c) is in inches.

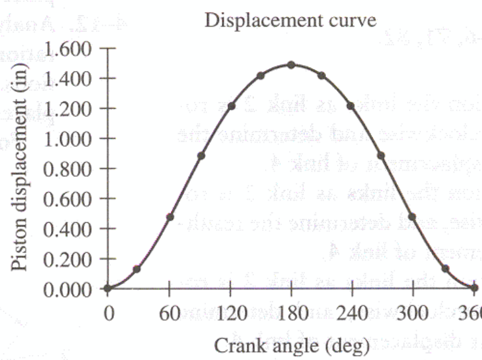


Figure 4.32 Displacement diagram for Example Problem 4.11.

These values are plotted in Figure 4.32 to form a displacement diagram.

4.12 COUPLER CURVES

The motion of the connecting rod or coupler of a four-bar mechanism is rather complex. The path traced by any point on the coupler is termed a *coupler curve.* Two of these paths, namely those traced by the pin connections of the coupler, are either a circle centered at the fixed pivot, or a line parallel to a sliding joint. However, all other points on the coupler trace rather complex curves.

The methods in this chapter can be used to construct the trace of the motion of certain points on a mechanism. Section 4.9 introduces the concept of constructing the configuration at several phases of its cycle. As these phases are constructed, the position of a certain point can be retained. The curve formed by joining the position of this point at several phases of the mechanism forms the trace of that point. If the point resides on a floating link, the resulting trace, or coupler curve, is a complex shape. These traces can be used to determine the spatial requirements of a mechanism.

PROBLEMS

While manual drafting techniques are instructive for problems that require graphical solution, use of a CAD system is highly recommended.

Graphical and Analytical Position Analysis

For the mechanism shown in Figure P4.1:

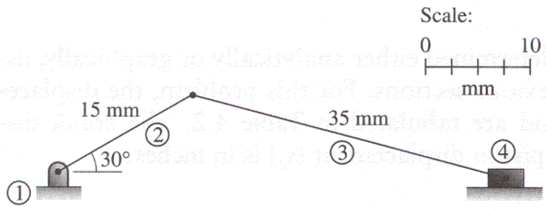


Figure P4.1 Problems 1-6, 71, 82.

4–1. Graphically position the links as link 2 is rotated 90° counterclockwise and determine the resulting linear displacement of link 4.

4–2. Graphically position the links as link 2 is rotated 120° clockwise, and determine the resulting linear displacement of link 4.

4–3. Analytically position the links as link 2 is rotated 120° counterclockwise, and determine the resulting linear displacement of link 4.

4–4. Analytically position the links as link 2 is rotated 90° clockwise, and determine the resulting linear displacement of link 4.

4–5. Graphically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum linear displacement (stroke) of link 4.

4–6. Analytically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum linear displacement (stroke) of link 4.

For the mechanism shown in Figure P4.7:

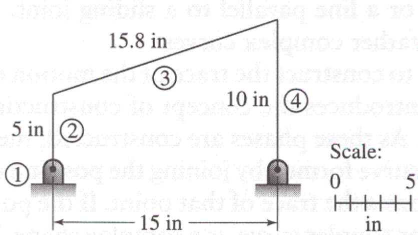


Figure P4.7 Problems 7-12, 71, 83.

4–7. Graphically position the links as link 2 is rotated 120° counterclockwise, and determine the resulting angular displacement of link 4.

4–8. Graphically position the links as link 2 is rotated 60° clockwise, and determine the resulting angular displacement of link 4.

4–9. Analytically position the links as link 2 is rotated 90° counterclockwise, and determine the resulting angular displacement of link 4.

4–10. Analytically position the links as link 2 is rotated 90° clockwise, and determine the resulting angular displacement of link 4.

4–11. Graphically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum angular displacement (stroke) of link 4.

4–12. Analytically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum angular displacement (stroke) of link 4.

For the mechanism shown in Figure P4.13:

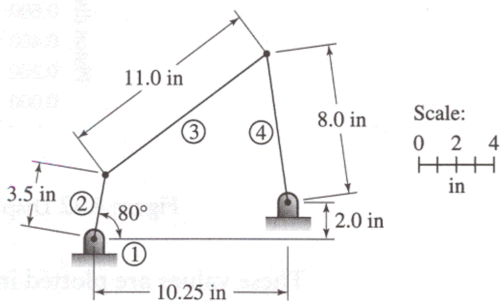


Figure P4.13 Problems 13-18, 72, 84.

4–13. Graphically position the links as link 2 is rotated 90° counterclockwise, and determine the angular displacement of link 4.

4–14. Graphically position the links as link 2 is rotated 30° clockwise, and determine the angular displacement of link 4.

4–15. Analytically position the links as link 2 is rotated 150° counterclockwise, and determine the angular displacement of link 4.

4–16. Analytically position the links as link 2 is rotated 90° counterclockwise, and determine the angular displacement of link 4.

4–17. Graphically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum angular displacement (stroke) of link 4.

4–18. Analytically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum angular displacement (stroke) of link 4.

For the mechanism shown in Figure P4.19:

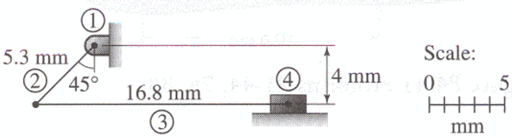


Figure P4.19 Problems 19-26, 73, 85.

4–19. Graphically position the links as link 2 is rotated 120° counterclockwise, and determine the displacement of link 4.

4–20. Graphically position the links as link 2 is rotated 60° clockwise, and determine the displacement of link 4.

4–21. Analytically position the links as link 2 is rotated 90° counterclockwise, and determine the displacement of link 4.

4–22. Analytically position the links as link 2 is rotated 30° clockwise, and determine the displacement of link 4.

4–23. Graphically position the links as link 4 is moved 5 mm to the right, and determine the angular displacement of link 2.

4–24. Analytically position the links as link 4 is moved 4 mm to the left, and determine the angular displacement of link 2.

4–25. Graphically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum displacement (stroke) of link 4.

4–26. Analytically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum displacement (stroke) of link 4.

For the mechanism shown in Figure P4.27:

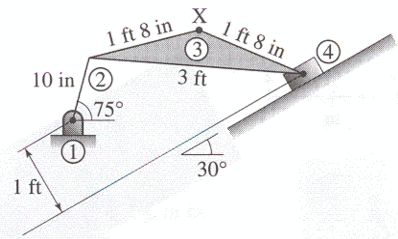


Figure P4.27 Problems 27-34, 74, 86.

4–27. Graphically position the links as link 2 is rotated 90° counterclockwise, and determine the displacement of point *X*.

4–28. Graphically position the links as link 2 is rotated 150° clockwise, and determine the displacement of point *X*.

4–29. Analytically position the links as link 2 is rotated 110° counterclockwise, and determine the displacement of link 4.

4–30. Analytically position the links as link 2 is rotated 70° clockwise, and determine the displacement of link 4.

4–31. Graphically position the links as link 4 is displaced 5 in up the incline, and determine the displacement of point *X*.

4–32. Analytically position the links as link 4 is rotated 7 in down the incline, and determine the displacement of link 4.

4–33. Graphically position the links into the configurations that place link 4 in its limiting positions. Determine the maximum displacement of point X and the stroke of link 4.

4–34. Analytically position the links into the configurations that place link 4 in its limiting positions. Determine the stroke of link 4.

For the dump truck mechanism shown in Figure P4.35:

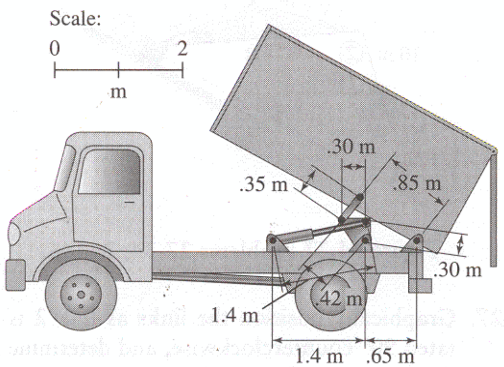


Figure P4.35 Problems 35-40, 75, 87.

4–35. Graphically determine the displacement of the bed as the cylinder is shortened 0.15 m.

4–36. Shown in Figure P4.35 is a dump truck with a bed that is operated by a hydraulic cylinder. Graphically determine the displacement of the dump truck bed as the cylinder is lengthened 0.2 m.

4–37. Shown in Figure P4.35 is a dump truck with a bed that is operated by a hydraulic cylinder. Graphically determine the required displacement of the cylinder when the dump truck bed is raised 15°.

4–38. Shown in Figure P4.35 is a dump truck with a bed that is operated by a hydraulic cylinder. Analytically determine the displacement of the dump truck bed as the cylinder is lengthened 0.1 m.

4–39. Shown in Figure P4.35 is a dump truck with a bed that is operated by a hydraulic cylinder. Analytically determine the displacement of the dump truck bed as the cylinder is shortened 0.1 m.

4–40. Analytically determine the required displacement of the cylinder when the bed is to be lowered 20°.

For the vice grips shown in Figure P4.41:

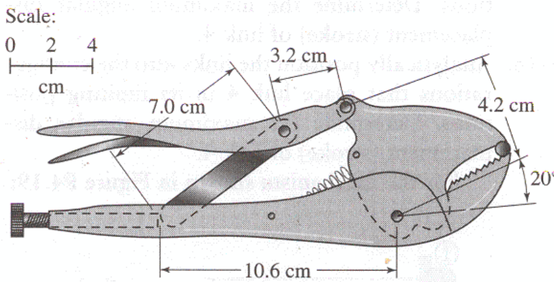


Figure P4.41 Problems 41-44, 76, 88.

4–41. Graphically determine the position of all links, as the top jaw is opened 40° from the position shown, while the lower jaw is held stationary. Determine the angular displacement of the top handle to accomplish this motion.

4–42. Graphically determine the position of all links in the vice grips in Figure P4.41, as the top handle is opened 20° from the position shown, while the lower handle is held stationary. Determine the resulting angular displacement of the top jaw. As the thumb screw in Figure P4.41 is rotated, the effective pivot point of its contact link is moved. During this motion, the spring prohibits the jaws from moving.

4–43. Graphically reposition all links as this effective pivot point is moved 2 cm to the right. Then graphically determine the position of all links in the vice grips as the top jaw is opened 40° from the position shown, while holding the bottom jaw fixed. Determine the angular displacement of the top handle to accomplish this motion.

4–44. As the thumb screw in Figure P4.41 is rotated, the effective pivot point of its contact link is moved. During this motion, the spring prohibits the jaws from moving. Analytically reposition all links as this effective pivot point is moved 2 cm. to the right. Then analytically determine the position of all links in the vice grips as the top handle is opened 20° from the position shown, while holding the bottom handle fixed. Determine the resulting angular displacement of the top jaw.

The heat-treated furnace door is closed when configured with the dimensions shown in Figure P4.45:

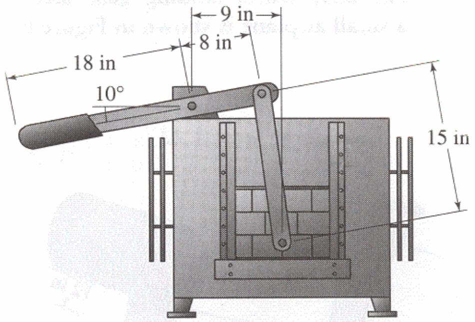


Figure P4.45 Problems 45-48, 77, 89.

4–45. Graphically determine the amount that the door opens as the lever, which is originally set at 10°, is rotated to 40°.

4–46. Graphically determine the amount that the lever needs to be rotated in order to raise the door 3 in.

4–47. Analytically determine the amount that the door opens as the lever, which is originally set at 10°, is rotated to 25°.

4–48. Analytically determine the amount that the lever needs to be rotated in order to raise the door 6 in.

A small, foot-operated air pump is shown in Figure P4.49:

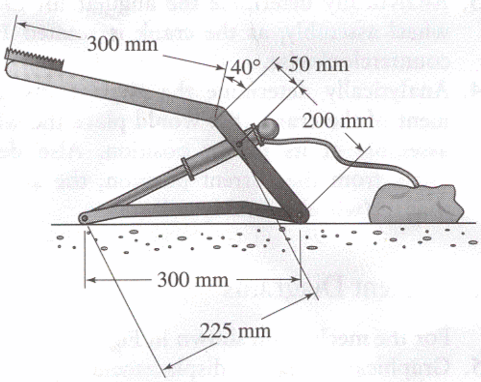


Figure P4.49 Problems 49-52, 78, 90.

4–49. Graphically determine the distance that the air cylinder contracts when the foot pedal is rotated 25° counterclockwise from its current position. Knowing that the diameter of the cylinder is 25 mm, calculate the volume of air that is displaced by the pump, also determine the displacement of point *X*.

4–50. Graphically determine the amount of rotation of the foot pedal to compress the air cylinder to 175 mm, and determine the displacement of point *X*.

4–51. Analytically determine the distance that the air cylinder contracts, when the foot pedal is rotated 20° counterclockwise from its current position. Knowing that the diameter of the cylinder is 25 mm, also calculate the volume of air that is displaced by the pump.

4–52. Analytically determine the amount of rotation of the foot pedal to compress the air cylinder to 130 mm.

A carrier for a microwave oven, to assist people in wheelchairs, is shown in Figure P4.53. The two support links are identical.

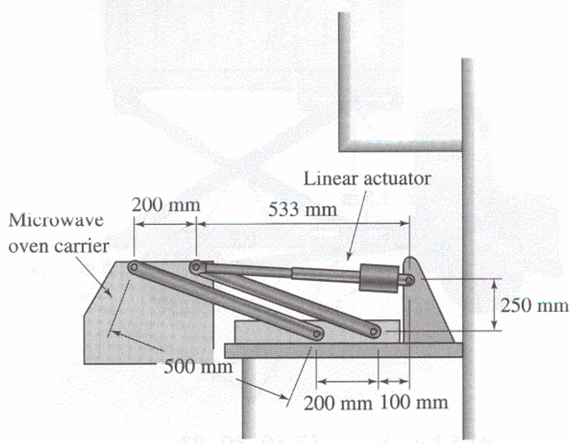


Figure P4.53 Problems 53-56, 79, 91.

4–53. Graphically determine the amount of rotation of the from support link when the linear actuator is retracted to a distance of 400 mm. Also, determine the vertical distance that the carrier is lifted.

4–54. Graphically determine the amount that the linear actuator needs to retract when the front support link is raised 45°, and determine the vertical distance that the carrier is lifted.

4–55. Analytically determine the amount of rotation of the support link when the linear actuator is retracted to a distance of 425 mm, and determine the vertical distance that the carrier is lifted.

4–56. Analytically determine the amount that the linear actuator needs to retract when the front support link is raised 70°, and determine the vertical distance that the carrier is lifted.

A lift truck, used to load and unload supplies into passenger jets is shown in Figure P4.57:

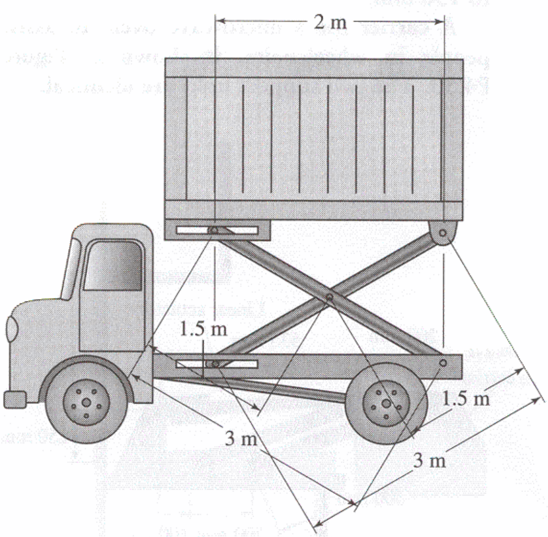


Figure P4.57 Problems 57-60, 80, 92.

4–57. Graphically determine the vertical distance that the truck box lowers, if the bottom sliding pin were moved toward the cab, from the distance of 2.0 m to 2.5 m.

4–58. Graphically determine the distance the lower pins must widen to lower the truck box 0.75 m.

4–59. Analytically determine the vertical distance that the truck box lowers, if the bottom sliding pin were moved from the distance of 2.0 m to 1.5 m.

4–60. Analytically determine the distance the lower pins must be widened to lower the truck box 1.0 m.

The nose wheel landing gear mechanism for a small airplane is shown in Figure P4.61.

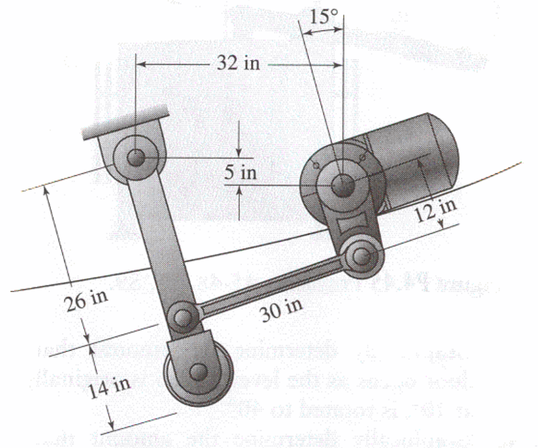


Figure P4.61 Problems 61-64, 81, 93.

4–61. Graphically determine the angular lift of the wheel assembly, as the crank is rotated 60° counterclockwise.

4–62. Graphically determine the angular displacement of the crank that would place the wheel assembly at its highest position. Also determine, from the current position, the amount that the wheel assembly will rotate.

4–63. Analytically determine the angular lift of the wheel assembly, as the crank is rotated 110° counterclockwise.

4–64. Analytically determine the angular displacement of the crank that would place the wheel assembly at its lowest position. Also determine, from the current position, the amount that the wheel assembly will rotate.

Displacement Diagrams

For the mechanism shown in Figure P4.1;

4–65. Graphically create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

4–66. Analytically create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

For the mechanism shown in Figure P4.7:

4–67. Graphically create a displacement diagram of the angular motion of link 4 as link 2 rotates a full cycle.

4–68. Analytically create a displacement diagram of the angular motion of link 4 as link 2 rotates a full cycle.

For the mechanism shown in Figure P4.13:

4–69. Graphically create a displacement diagram of the angular motion of link 4 as link 2 rotates a full cycle.

4–70. Analytically create a displacement diagram of the angular motion of link 4 as link 2 rotates a full cycle.

For the mechanism shown in Figure P4.19:

4–71. Graphically create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

4–72. Analytically create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

For the mechanism shown in Figure P4.27:

4–73. Graphically create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

4–74. Analytically create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

For the dump truck mechanism shown in Figure P4.35:

4–75. Graphically create a displacement diagram of the angular motion of the bed as the cylinder extends from 1.25 m to 1.75 m.

4–76. For the vice grips shown in Figure P4.41, graphically create a displacement diagram of the angular motion of the top handle as the bottom handle is selected as the frame and the top jaw opens to 45° from the current position.

4–77. For the furnace door shown in Figure P4.45, graphically create a displacement diagram of the vertical motion of the door as the handle is rotated downward 45° from the current position.

4–78. For the bicycle pump shown in Figure P4.49, graphically create a displacement diagram of the length of the cylinder as the foot pedal is rotated to downward 30° from the current position.

4–79. For the microwave oven carrier shown in Figure P4.53, graphically create a displacement diagram of the vertical displacement of the carrier as the linear actuator extends from 200 mm to 700 mm.

4–80. For the lifting truck box shown in Figure P4.57, graphically create a displacement diagram of the height of the box as the lower pins are separated from 1.5 m to 2.8 m.

4–81. For the nose wheel mechanism shown in Figure P4.61, graphically create a displacement diagram of the angular displacement of the wheel assembly as the motor crank rocks clockwise 105° from its current position.

Displacement Problems Using Working Model

Use the Working Model© software obtained with the text to create a model and provide the desired output for the following mechanisms.

4–82. Using the mechanism in P4.1, create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

4–83. For the mechanism shown in Figure P4.7, create a displacement diagram of the angular motion of link 4 as link 2 rotates a full cycle.

4–84. For the mechanism shown in Figure P4.13, create a displacement diagram of the angular motion of link 4 as link 2 rotates a full cycle.

4–85. For the mechanism shown in Figure P4.19, create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

4–86. For the mechanism shown in Figure P4.27, create a displacement diagram of the linear motion of link 4 as link 2 rotates a full cycle.

4–87. For the dump truck mechanism shown in Figure P4.35, create a displacement diagram of the angular motion of the bed as the cylinder extends from 1.25m to 1.75m.

4–88. For the vice grips shown in Figure P4.41, create a displacement diagram of the angular motion of the top handle as the bottom handle is selected as the frame and the top jaw opens to 45° from the current position.

4–89. For the furnace door shown in Figure P4.45, create a displacement diagram of the vertical motion of the door as the handle is rotated downward 45° from the current position.

4–90. For the bicycle pump shown in Figure P4.49, create a displacement diagram of the length of the cylinder as the foot pedal is rotated downward 30° from the current position.

4–91. For the microwave oven carrier shown in Figure P4.53, create a displacement diagram of the vertical displacement of the carrier as the linear actuator extends from 200 mm to 700 mm.

4–92. For the lifting truck box shown in Figure P4.57, create a displacement diagram of the height of the box as lower pins are separated from 1.5 m to 2.8 m.

4–93. For the nose wheel mechanism shown in Figure P4.61, create a displacement diagram of the angular displacement of the wheel assembly as the motor crank rocks clockwise 105° from its current position.

CASE STUDIES

4–1. Figure C4.1 shows a mechanism that was designed to impart motion on a machine slide. Carefully examine the configuration of the components in the mechanism, then answer the following leading questions to gain insight into the operation.

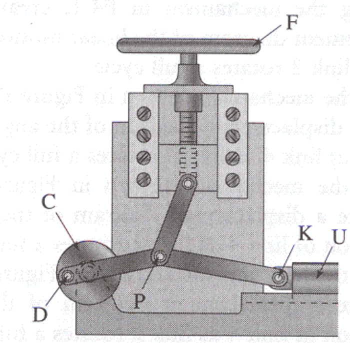


Figure C4.1 (Courtesy, Industrial Press)

1. As wheel *C* is rotated clockwise, what is the motion of pin *D*?
2. What is the motion of pin *P*?
3. What is the motion of pin *K*?
4. What effect does turning the handwheel *F* have on slide *U*?

   Describe the manner in which the motion of the handwheel is transferred to the slide *U*.
5. What effect does turning the handwheel *F* have on the motion of the mechanism? Be sure to comment on all characteristics of the motion.
6. What is the purpose of this device?

4–2. Figure C4.2 presents an interesting materials handling system for advancing small parts onto a feed track. Carefully examine the the configuration of the components in the mechanism, then answer the following leading questions to gain insight into the operation.

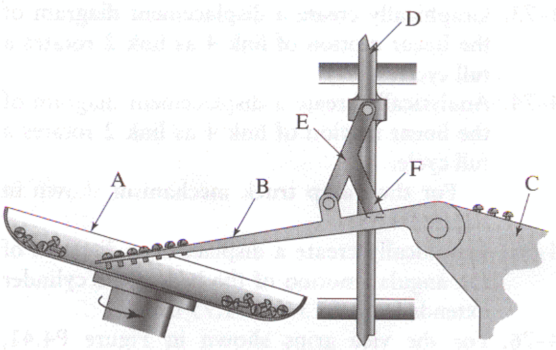


Figure C4.2 (Courtesy, Industrial Press)

1. The small, round-head screw blanks are fed into a threading machine through track *B* and C. How do the screws get from bowl *A* to track *B*?
2. Although not clearly shown, track *B* is of a parallel finger design. Why is a parallel finger arrangement used to carry the screws?
3. As a second mechanism intermittently raises link *D*, what is the motion of track *B*?
4. Determine exactly what connects link *D* and track *B*.
5. What is the purpose of link *E*?

6. As a second mechanism intermittently raises link *D*, what is the motion of the screws?
7. What determines the lowest position that link *D* can travel? Notice that the tips of track *B* do not contact the bottom of bowl *A*.
8. As screws are congested in the outlet track *C*, what happens to finger *F* as link *D* is forced lower?
9. As screws are congested in the outlet track *C*, what happens to the tips of track *B*?
10. What is the purpose of this device, and discuss its special features?
11. What type of mechanism could be operating link *D*?

4–3. Figure C4.3 depicts a production transfer machine that moves clutch housings from one station to another. Platform A supports the housings during the transfer. Carefully examine the the configuration of the components in the mechanism, then answer the following leading questions to gain insight into the operation.

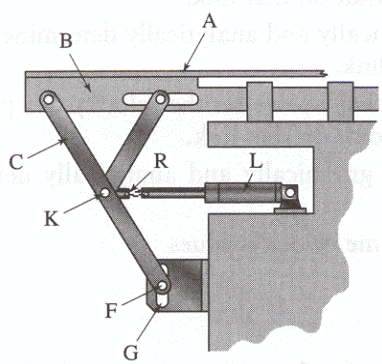


Figure C4.3 (Courtesy, Industrial Press)

1. To what type of motion is bar *B* restricted?
2. What motion does link *C* encounter as air cylinder *L* is shortened?
3. What is the motion of point *K* as air cylinder *L* is shortened?
4. Why does joint *F* need to ride in slot *G*?
5. What is the purpose of this mechanism?
6. What effect does turning the threaded rod end *R* have, thus lengthening the cylinder rod?
7. Knowing that the drawing is one-quarter size, draw a kinematic sketch of this mechanism.
8. Compute the mobility of this mechanism.
9. Graphically reposition all links as the air cylinder is shortened 2 in.

5 Velocity Analysis

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Define linear, rotational, and relative velocities.
2. Convert between linear and angular velocities.
3. Using the relative velocity method, graphically solve for the velocity of a point on a link, knowing the velocity of another point on that link.
4. Using the relative velocity method, graphically and analytically determine the velocity of a point of interest on a floating link.
5. Using the relative velocity method, analytically solve for the velocity of a point on a link, knowing the velocity of another point on that link.
6. Using the instantaneous center method, graphically and analytically determine the velocity of a point.
7. Construct a velocity curve to locate extreme velocity values.

5.1 INTRODUCTION

Velocity analysis involves determining "how fast" certain points on the links of a mechanism are traveling. Velocity is important because it associates the movement of a point on a mechanism with time. Often the timing in a machine is critical.

For instance, the mechanism that "pulls" video film through a movie projector must advance the film at a very precise rate. A mechanism that feeds packing material into a crate must operate in sequence with the conveyor that indexes the crates. A windshield wiper mechanism operating on high speed must sweep the wiper across the glass at least 45 times every minute.

The determination of velocity in a linkage is the purpose of this chapter. Two common analysis procedures are examined: the relative velocity method and the instantaneous center method. Consistent with other chapters in this book, both graphical and analytical techniques are included.

5.2 VELOCITY OF A POINT

Linear velocity of a point is the linear displacement of that point per unit time. Recall that linear displacement of a point, s, is a vector and defined as a change in position of that point. This was introduced in Section 4.3.

As described in Chapter 4, the motion of a point is viewed as translation and is discussed in linear terms. By definition, a point can only have a linear displacement. When the time elapsed during a displacement is measured, the velocity can be determined.

As with displacement, velocity is a vector. The magnitude of velocity is often referred to as "speed." Understanding the direction of linear velocity requires determining the direction in which a point is moving at a specific instant. Figure 5.1 illustrates the velocity of two points on a link. The velocity of points *A* and *B* are denoted as v_A and v_B, respectively. Note that although they are on the same link, both these points can have different linear velocities. Points that are farther from the pivot travel faster. This produces a "whipping" action, that can be felt on several amusement rides.

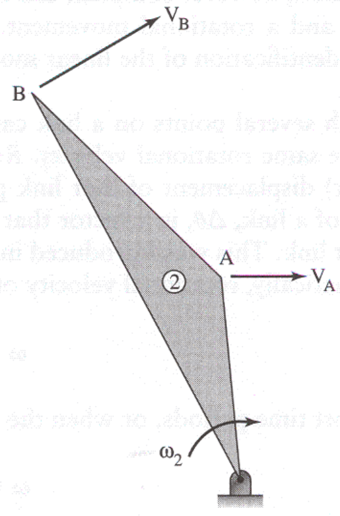


Figure 5.1 Linear velocities of points on a link.

Mathematically, linear velocity of a point is defined as:

$$v = \frac{ds}{dt} \tag{5.1}$$

and for short time periods as:

$$v \cong \frac{\Delta s}{\Delta t} \tag{5.2}$$

Because displacement is a vector, Equation 5.1 states that velocity is also a vector. Therefore, a direction is also required to completely define velocity. Linear velocity of a point is directed along the line of instantaneous motion of the point. From Figure 5.1, v_A is directed along the path that point A is moving at this instant. In casual terms, if point A were to break away from link 2 at this instant, point A would travel in the direction of its linear velocity.

Linear velocity is expressed in the units of length divided by time. In the United States Customary System, the common units used are feet per second (ft/s or fps), feet per minute (ft/s or fpm), or inches per second (in/s or ips). In the International System, the common units used are meters per second (m/s) or millimeters per second (mm/s).

5.3 VELOCITY OF A LINK

As seen in Figure 5.1, several points on a link can have drastically different linear velocities. This is especially true as the link simply rotates about a fixed point, as in Figure 5.1. In general, the motion of a link can be rather complex as it moves and spins.

Any motion, however complex, can be viewed as a combination of a straight line movement and a rotational movement. Fully describing the motion of a link can consist of identification of the linear motion of one point and the rotational motion of the link.

Although several points on a link can have different linear velocities, the entire link has the same rotational velocity. *Rotational velocity* of a link is the rotational (or angular) displacement of that link per unit of time. Recall that rotational displacement of a link, $\Delta\theta$, is a vector that is defined as the angular change in orientation of that link. This was introduced in Section 4.3.

Mathematically, rotational velocity of a link is defined as:

(5.3)
$$\omega = \frac{d\theta}{dt}$$

and for short time periods, or when the velocity can be assumed linear:

(5.4)
$$\omega \cong \frac{\Delta\theta}{\Delta t}$$

Because rotational displacement is a vector, Equation 5.3 states that rotational velocity is also a vector. The direction of rotational velocity is in the direction of rotation. It can be fully described by specifying either the term clockwise or counterclockwise. For example, the link shown in Figure 5.1 has a rotational velocity that is consistent with the linear velocities of the points that are attached to the link. Thus, the link has a clockwise rotational velocity.

Rotational velocity is expressed in the units of angle divided by time. In both the United States Customary System and the International System, the common units used are revolutions per minute (rpm), degrees per second (deg/s), or radians per second (rad/s or rps).

5.4 RELATIONSHIP BETWEEN LINEAR AND ANGULAR VELOCITIES

For a link in pure rotation, the linear velocity of any point attached to the link is related to the angular velocity of the link. This relationship is expressed as:

(5.5) $$v = r\omega$$

where:

v = linear velocity of the point of interest
r = distance from the center of rotation to the point of interest
ω = angular velocity of the rotating link

Linear velocity is always perpendicular to a line that connects the center of the link rotation to the point of interest. Thus, linear velocity of a point on a link in pure rotation is often called the *tangential velocity*. This is because the linear velocity is tangent to the circular path of that point, or perpendicular to the line that connects the point with the pivot.

It is extremely important to remember that the angular velocity, ω, in Equation 5.5 must be expressed as units of radians per unit of time. To be exact, the radian is a dimensionless unit of angular measurement and is often omitted. Linear velocity is expressed in units of length per time and not radians times length per time, as Equation 5.5 would infer.

Often the conversion must be made from the more common unit of revolutions per minute (rpm):

(5.6) $$\omega(\text{rad/min}) = [\omega(\text{rev/min})]\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 2\pi[\omega\ (\text{rev/min})]$$

and:

(5.7) $$\omega(\text{rad/sec}) = [\omega(\text{rev/min})]\left[\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\right] = \frac{2\pi}{60}[\omega(\text{rev/min})]$$

As stated in the previous section, the angular velocity of the link and the linear velocities of points on the link are consistent. That is, the velocities (rotational or linear) are in the direction that the object (link or point) is instantaneously moving. As mentioned, linear velocity is always perpendicular to a line that connects the center of link rotation to the point of interest.

EXAMPLE PROBLEM 5.1

Figure 5.2 illustrates a cam mechanism used to drive the exhaust port of an internal combustion engine. Point B is a point of interest on the rocker plate. At this instant, the cam forces point B upward at 30 mm/s. Determine the rotational velocity of the rocker plate and the velocity of point C.

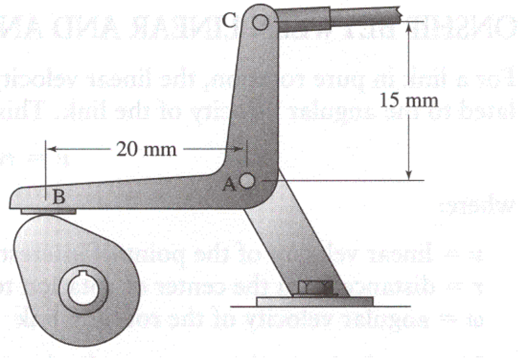


Figure 5.2 Mechanism for Example Problem 5.1.

Solution:

The rocker plate is connected to the frame with a pin joint at point A. The velocity of point B is a vector directed upward with a magnitude of 30 mm/s. Figure 5.3 shows a kinematic diagram.

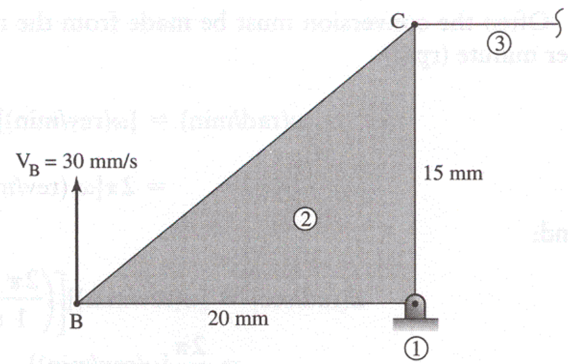


Figure 5.3 Kinematic diagram for Example Problem 5.1.

It should be apparent that as point B travels upward, the rocker plate, link 2, is forced to rotate clockwise. Therefore, as point B has upward linear velocity, the rocker plate must have a clockwise rotational velocity. The magnitude of the rotational velocity is found by rearranging Equation 5.5:

$$\omega_2 = \frac{v_B}{r_b}$$

$$= \frac{(30 \text{ mm/s})}{20 \text{ mm}} = 1.5 \text{ rad/s}$$

This can be converted to rpm by rearranging Equation 5.6:

$$\omega_2\left(\frac{\text{rev}}{\text{min}}\right) = \frac{60}{2\pi}\left[\omega_2\left(\frac{\text{rad}}{\text{sec}}\right)\right]$$

$$= \frac{60}{2\pi}\left[1.5\frac{\text{rad}}{\text{sec}}\right] = 14.3 \text{ rpm}$$

Including the direction:

$$\omega_2 = 1.5 \text{ rad/s, clockwise.}$$

The linear velocity of point C can also be computed from Equation 5.5:

$$v_C = r_c\omega_2$$

$$= (15 \text{ mm})\left(1.5\ \frac{\text{rad}}{\text{sec}}\right) = 22.5\ \frac{\text{mm}}{\text{s}}$$

The direction of the linear velocity of C must be consistent with the rotational velocity of link 2. The velocity also is perpendicular to the line that connects the center of rotation of link 2, point A, to point C. Therefore, the velocity of point C is directed to the right horizontally.

Including the direction:

$$v_C = 22.5 \text{ mm/s} \rightarrow$$

5.5 RELATIVE VELOCITY

The difference between the motion of two points is termed *relative motion.* Consider a situation where two cars travel on the interstate highway. The car in the left lane travels at 65 miles per hour (mph) and the car in the right lane travels at 55 mph. These speeds are measured in relationship to a stationary radar unit. Thus, they are a measurement of *absolute motion.*

Although both are moving forward, it appears to the people in the faster car that the other car is actually moving backwards. That is, the relative motion of the slower car to the faster car is in the opposite direction of the absolute motion. Conversely, it appears to the people in the slower car that the faster car is traveling at 10 mph. That is, the relative velocity of the faster car to the slower car is 10 mph.

Relative velocity is a term used when the velocity of one object is related to that of another reference object, which can also be moving. The following notation distinguishes between absolute and relative velocities:

v_A = absolute velocity of point A
v_B = absolute velocity of point B
$v_{B/A}$ = relative velocity of point B with respect to A
= velocity of point B "as observed" from point A

Relative motion – the difference between the motion of two points – can be written mathematically as:

(5.8) $$v_{B/A} = v_B -> v_A$$

or rearranged as:

(5.9) $$v_B = v_A +> v_{B/A}$$

Note that Equations 5.8 and 5.9 are vector equations. Therefore, in order to use the equations, vector polygons must be prepared in accordance with the equations. The techniques discussed in Section 2.16 must be used in dealing with these equations.

EXAMPLE PROBLEM 5.2

Figure 5.4 shows a cargo lift mechanism for a delivery truck. At this instant, point *A* has a velocity of 12 in/s in the direction shown, and point *B* has a velocity of 20 in/s, also in the direction shown. Determine the rotational velocity of the lower link and the relative velocity of point *B* relative to point *A*.

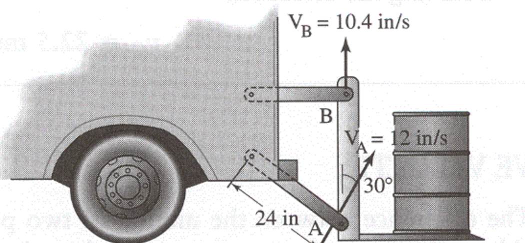


Figure 5.4 Mechanism for Example Problem 5.2.

Solution:

Figure 5.5A shows the kinematic diagram of this mechanism. Notice that it is the familiar four-bar mechanism.

From the kinematic diagram, it should be apparent that as point A travels up and to the right, link 2 rotates counterclockwise. Thus, link 2 has a counterclockwise rotational velocity. The magnitude of the rotational velocity is found by rearranging Equation 5.5 as follows:

$$\omega_2 = \frac{v_a}{r_A}$$

$$= \frac{(12 \text{ in/s})}{(24 \text{ in})} = 0.5 \frac{\text{rad}}{\text{sec}}$$

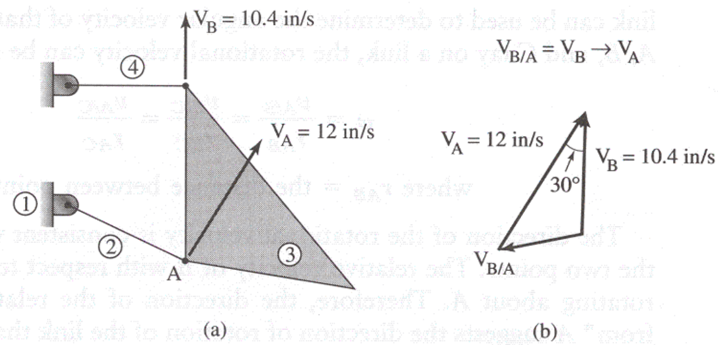


Figure 5.5 Kinematic diagram for Example Problem 5.2.

This can be converted to rpm by rearranging Equation 5.6 as:

$$\omega_2\left(\frac{\text{rev}}{\text{min}}\right) = \frac{60}{2\pi}\left[\omega_2\left(\frac{\text{rad}}{\text{sec}}\right)\right]$$

$$= \frac{60}{2\pi}\left[0.5\,\frac{\text{rad}}{\text{sec}}\right] = 5.8 \text{ rpm}$$

Including the direction:

$$\omega_2 = 5.8 \text{ rpm, counterclockwise.}$$

The relative velocity of B with respect to A can be found from Equation 5.8:

$$v_{B/A} = v_B -> v_A$$

A vector polygon is formed from this Equation and is given in Figure 5.5B. Notice that this is a general triangle. Either a graphical or analytical solution can be used to determine the vector $v_{B/A}$.

Arbitrarily using an analytical method, the velocity $v_{B/A}$ can be found from the law of cosines.

$$v_{B/A} = \sqrt{[v_A{}^2 + v_B{}^2 - 2\,(v_A)(v_B)(\cos 30°)]}$$

$$= \sqrt{[(12\text{in/s})^2 + (10.4\text{in/s})^2 - 2(12\text{in/s})(10.4\text{in/s})(\cos 30°)]} = 6.0\text{in/s}$$

The angle between $v_{B/A}$ and v_B is shown as θ in Figure 5.5B. It can be found by using the law of sines:

$$\theta = \sin^{-1}\left[\frac{(12\text{in/s})}{(6\text{in/s})}\sin 30°\right] = 90°$$

Thus, this vector polygon actually formed a right triangle. The relative velocity of B with respect to A is stated formally as follows:

$$v_{B/A} = 6.0 \text{ in/s} \leftarrow$$

Relative velocity between two points on a link is useful in determining velocity characteristics of the link. Specifically, the relative velocity of any two points on a link can be used to determine the angular velocity of that link. Assuming that points *A, B,* and *C* lay on a link, the rotational velocity can be stated as:

$$\omega = \frac{v_{A/B}}{r_{AB}} = \frac{v_{B/C}}{r_{BC}} = \frac{v_{A/C}}{r_{AC}} \tag{5.10}$$

where r_{AB} = the distance between points *A* and *B*.

The direction of the rotational velocity is consistent with the relative velocity of the two points. The relative velocity of *B* with respect to *A* implies that *B* is seen as rotating about *A*. Therefore, the direction of the relative velocity of *B* "as seen from" *A* suggests the direction of rotation of the link that is shared by points *A* and *B*. Referring to Figure 5.6, when $v_{B/A}$ is directed up and to the left, the rotational velocity of the link is counterclockwise. Conversely, when $v_{B/A}$ is directed down and to the right, the rotational velocity of the link is clockwise.

5.6 GRAPHICAL VELOCITY ANALYSIS: RELATIVE VELOCITY METHOD

Graphical velocity analysis will determine the velocity of mechanism points in a single configuration. It must be understood that the results of this analysis correspond to the current position of the mechanism. As the mechanism moves, the configuration changes, and the velocities also change.

The basis of the relative velocity method of analysis is derived from the following fact:

> Two points that reside on the same link can only have a relative velocity that is in a direction perpendicular to the line that connects the two points.

This fact is an extension of the definition of relative velocity. Figure 5.6 illustrates two points, *A* and *B*, that are on the same link. Recall that $v_{B/A}$ is the velocity of *B* "as observed" from *A*. For an observer at *A*, it appears that *B* is simply rotating around *A*, as long as both *A* and *B* are on the same link. Thus, the velocity of *B* with respect to *A* must be perpendicular to the line that connects *B* to *A*.

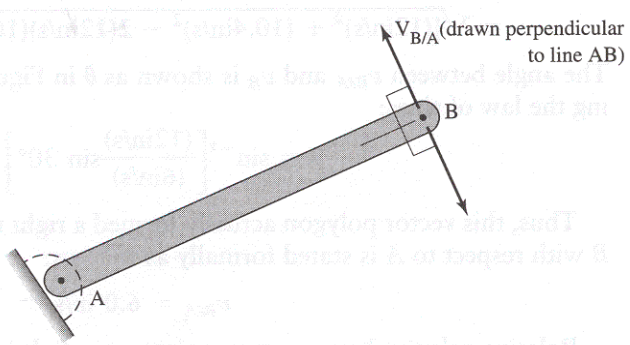


Figure 5.6 Relative velocity of two points on the same link.

With this fact, and vector analysis techniques, the velocity of points on a mechanism can be determined.

5.6.1 Points on Links Limited to Pure Rotation or Pure Translation

The most basic analysis involves the determination of the velocity of a point on a link, that is limited to pure rotation or pure translation, knowing the velocity of another point on the same link. Pin joints are often convenient points of analysis because they reside on two links, where one is typically constrained to pure rotation or pure translation. The solution of this problem can be summarized as:

1. Determine the direction of the unknown velocity by using the constraints imposed by the joint, either pure rotation or pure translation
2. Determine the direction of the relative velocity between the two joints. For two points on the same link, the relative velocity is always perpendicular to the line that connects the points.
3. Use the relative velocity equation below to draw a vector polygon:

$$v_{\text{Unknown point}} = v_{\text{Known point}} +> v_{\text{Unknown point/Known point}}$$

4. Using the methods outlined in Section 2.18, and the vector equation above, determine the magnitudes of:

$$v_{\text{Unknown point}} \;\&\; v_{\text{Unknown point/Known point}}$$

This analysis describes the logic behind graphical velocity analysis. The actual solution can be completed using manual drawing techniques (using a protractor and compass) or can be completed on a CAD system (using a rotate and copy command). The logic is identical; however, the CAD solution is not susceptible to limitations of drafting accuracy. Regardless of the method being practiced, the underlying concepts of graphical position analysis can be further illustrated and expanded through the following Example Problem.

EXAMPLE PROBLEM 5.3

Figure 5.7 shows a lift platform used to raise cargo onto commercial planes. It is operated by an electric motor that rotates spool *A* which either retrieves or releases cable *B*. Cable *B*, in turn, moves pin *C* in its slot. At this instant, the cable pulls pin *C* to the left at a rate of 10 ft/min. Determine the velocity of the center joint, point *D*.

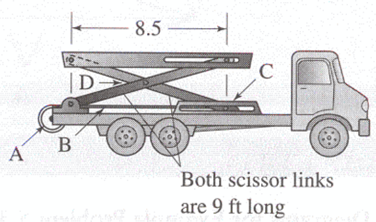


Figure 5.7 Mechanism for Example Problem 5.3.

Solution:

Figure 5.8A shows the kinematic diagram of this mechanism.

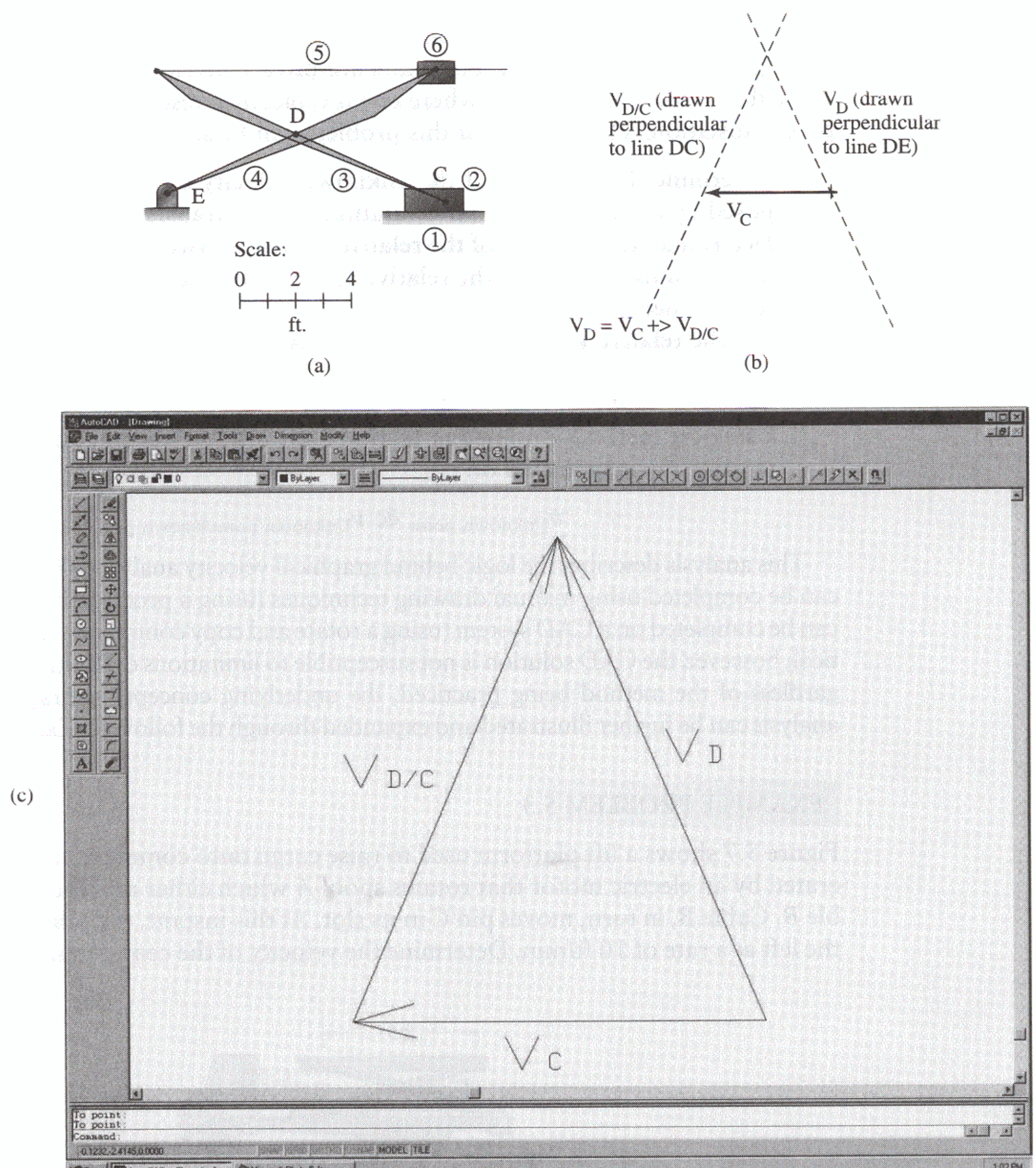


Figure 5.8 Diagrams for Example Problem 5.3.

To fully understand this mechanism, the mobility is computed as follows:

$$n = 6 j_p = (5 \text{ pins} + 2 \text{ sliders}) = 7 \; j_h = 0$$

and:

$$F = 3(n - 1) - 2j_p - j_h$$
$$= 3(6 - 1) - 2(7) - 0 = 1$$

With one degree of freedom, this mechanism is fully operated with one input motion. Of course, this motion is the actuation of the cable to the left at a rate of 10 ft/min.

Because link 4 is fixed to the frame at E, point D is limited to rotation about E; therefore, the velocity of point D must be perpendicular to the line DE.

In addition, both points D and C reside on the same link, namely, link 3. Therefore, the relative velocity of D with respect to C must be perpendicular to line DC.

From the previous two statements, the directions of both velocities v_D and $v_{D/C}$ are known. These velocities can be related using Equation 5.9:

(5.9) $$v_D = v_C +> v_{D/C}$$

In this equation, only the magnitudes of v_D and $v_{D/C}$ are unknown. This is identical to the problems illustrated in Section 3.18. The vector polygon used to solve this problem is shown in Figure 5.8B. The magnitudes can be determined by observing the intersection of the directed lines of v_D and $v_{D/C}$. The completed vector polygon is shown in Figure 5.8C. The magnitudes of the velocities can be scaled, yielding the following equations:

$$v_D = 11.3 \text{ fpm} \searrow 65°$$

$$v_{D/C} = 11.3 \text{ fpm} \angle 65°$$

Additionally, the angular velocities of links 3 and 4 may be of interest. These angular velocities can be determined from Equation 5.10 and knowing the distance between points C-D and D-E:

$$\omega_3 = \frac{v_{D/C}}{r_{CD}}$$

$$= \frac{11.3 \text{ fpm}}{2.2 \text{ ft}} = 5.14 \text{ rad/s}$$

$$\omega_4 = \frac{v_D}{r_{DR}}$$

$$= \frac{11.3 \text{ fpm}}{2.2 \text{ ft}} = 5.14 \text{ rad/s}$$

Convert these results to rpm with the following:

$$\omega\left(\frac{\text{rev}}{\text{min}}\right) = \frac{60}{2\pi}\left[\omega\left(\frac{\text{rad}}{\text{sec}}\right)\right]$$

$$= \frac{60}{2\pi}\left[5.14\left(\frac{\text{rad}}{\text{sec}}\right)\right] = 49.0 \text{ rpm}$$

The direction of the linear velocity of point *D* forces link 4 to rotate counterclockwise. The direction of *D* relative to *C* forces link 3 to rotate clockwise. Formally stating the results:

$$\omega_3 = 5.14 \text{ rad/s, clockwise.}$$

$$\omega_4 = 5.14 \text{ rad/s, counterclockwise.}$$

5.6.2 General Points on a Floating Link

Determining the velocity of general points on a floating link presents a slightly more complicated analysis. A floating link is simply a link that is not limited to pure rotation or pure translation. The difficulty arises in that neither the direction nor magnitude of the unknown velocity is known. This is fundamentally different from the analysis presented in Example Problem 5.3.

To determine the velocity of a point on a floating link, the velocity of two additional points on the link must be already determined. As previously stated, the two points are commonly pin joints constrained to rotation because their velocities are readily obtained. This restriction assumes that the analysis similar to Example Problem 5.3 has been completed.

Figure 5.9 illustrates a link in which the velocity of points *A* and *B* are already determined.

To determine the velocity of point *C*, the following procedure can be followed:

1. Two equations can be written.

$$v_C = v_A +> v_{C/A}$$

$$v_C = v_B +> v_{C/B}$$

Notice that the directions of $v_{C/A}$ and $v_{C/B}$ are perpendicular to lines *CA* and *CB*, respectively.

2. The individual relative velocity equations can be set equal to each other. In this case, this yields the following:

$$v_A +> v_{C/A} = v_B +> v_{C/B}$$

3. The relative velocities can be solved by again using the techniques outlined in Section 3.18. This involves constructing the vector polygon as shown in Figure 5.9B.

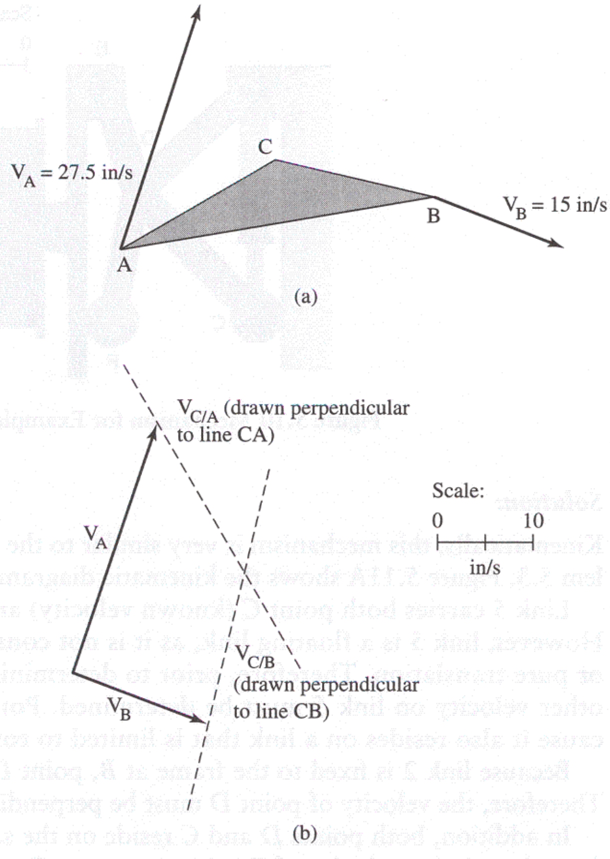


Figure 5.9 Velocity of a point of interest.

4. The relative velocity magnitudes can be measured from the vector polygon.
5. Knowing the relative velocities, the velocity of the point of interest, point *C*, can be determined by using one of the individual equations outlined in step 1. This can be readily found from the original vector polygon as shown in Figure 5.9C.

Again, vector polygons can be constructed using identical logic with either manual drawing techniques or CAD. This logic behind the analysis is illustrated in the following Example Problem.

EXAMPLE PROBLEM 5.4

Figure 5.10 illustrates a mechanism that extends reels of cable from a delivery truck. It is operated by a hydraulic cylinder at *A*. At this instant, the cylinder retracts at a rate of 5 mm/s. Determine the velocity of the top joint, point *E*.

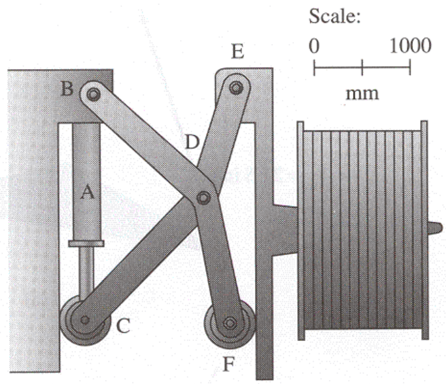


Figure 5.10 Mechanism for Example Problem 5.4.

Solution:

Kinematically, this mechanism is very similar to the one presented in Example Problem 5.3. Figure 5.11A shows the kinematic diagram of this mechanism.

Link 5 carries both point *C* (known velocity) and point *E* (unknown velocity). However, link 5 is a floating link, as it is not constrained to either pure rotation or pure translation. Therefore, prior to determining the velocity of point *E*, one other velocity on link 5 must be determined. Point *D* is a convenient point because it also resides on a link that is limited to rotation.

Because link 2 is fixed to the frame at *B*, point *D* is limited to rotation about *E*. Therefore, the velocity of point *D* must be perpendicular to the line *BD*.

In addition, both points *D* and *C* reside on the same link, namely, link 5. Therefore, the relative velocity of *D* with respect to *C* must be perpendicular to the line *DC*. From the previous two statements, the directions of both velocities v_D and $v_{D/C}$ are known. These velocities can be related using Equation 5.5:

$$v_D = v_C +> v_{D/C}$$

In this equation, only the magnitudes of v_D and $v_{D/C}$ are unknown. This is identical to the problems illustrated in Section 2.18. The vector polygon used to solve this problem is shown in Figure 5.11B. The magnitudes can be determined by observing the intersection of the directed lines, v_D and $v_{D/C}$. The magnitudes of the velocities can be scaled, yielding the following equations:

$$v_{D/C} = 3.5 \text{ mm/s} \searrow 45°$$

$$v_D = 3.5 \text{ mm/s} \angle 45°$$

Now that the velocity of two points on link 5 are fully known, the velocity of point *E* can be determined. Using the relative velocity equations, the velocity of the points involved can be related:

$$v_E = v_C +> v_{E/C} = v_D +> v_{E/D}$$

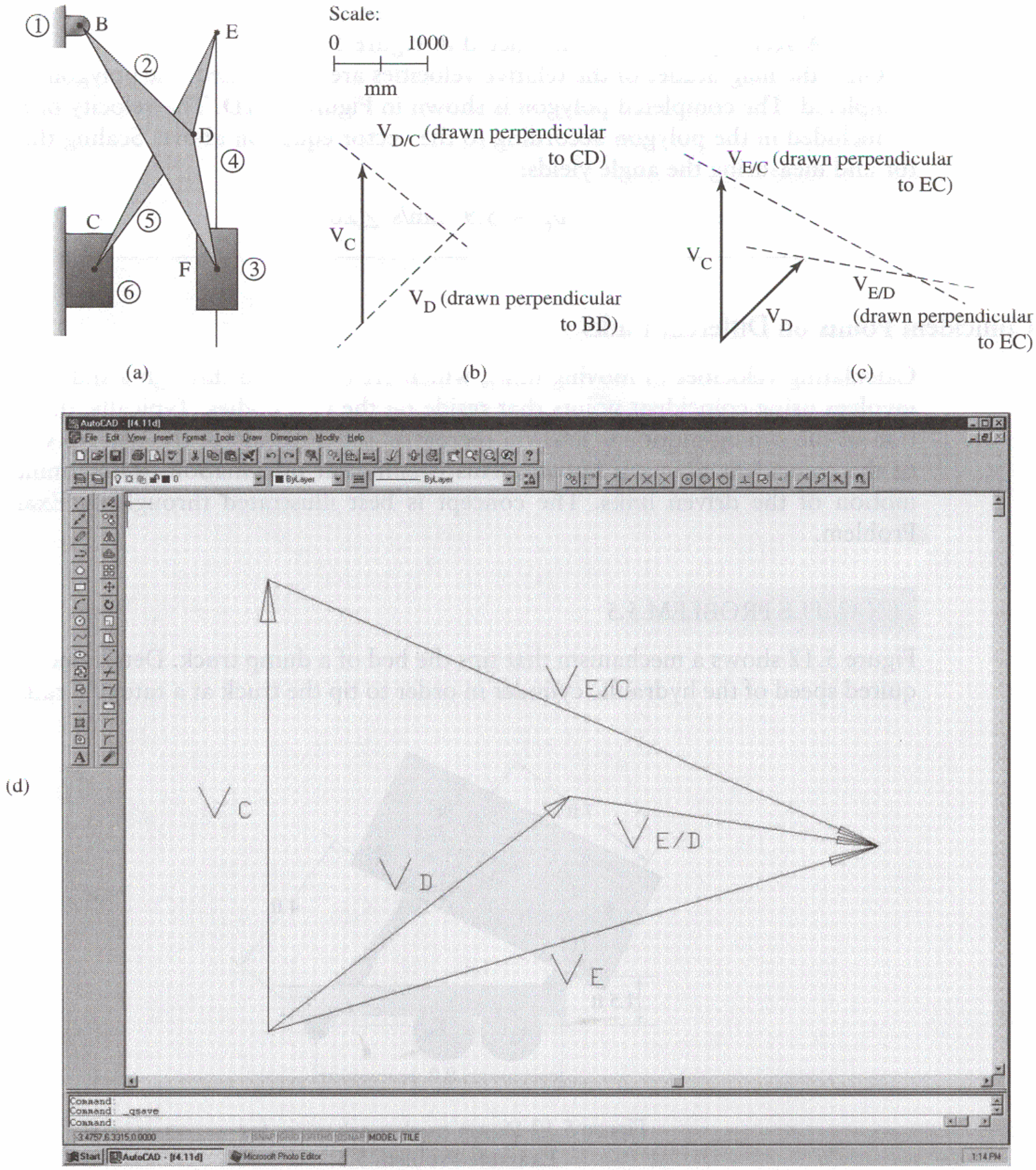


Figure 5.11 Diagrams for Example Problem 5.4.

The velocities of C and D as well as the direction of the relative velocities are known. A vector polygon is constructed in Figure 5.11C.

Once the magnitudes of the relative velocities are determined, the polygon can be completed. The completed polygon is shown in Figure 5.11D. The velocity of E can be included in the polygon according to the vector equation above. Scaling the vector and measuring the angle yields:

$$v_E = 5.9 \text{ mm/s } \angle 20°$$

5.6.3 Coincident Points on Different Links

Calculating velocities of moving links, which are connected through a sliding joint, involves using coincident points that reside on the two bodies. Typically, the direction of the sliding motion is known; therefore, the direction of the relative velocity of the coincident points is known. This is sufficient information to determine the motion of the driven links. The concept is best illustrated through an Example Problem.

EXAMPLE PROBLEM 5.5

Figure 5.12 shows a mechanism that tips the bed of a dump truck. Determine the required speed of the hydraulic cylinder in order to tip the truck at a rate of 5 rad/min.

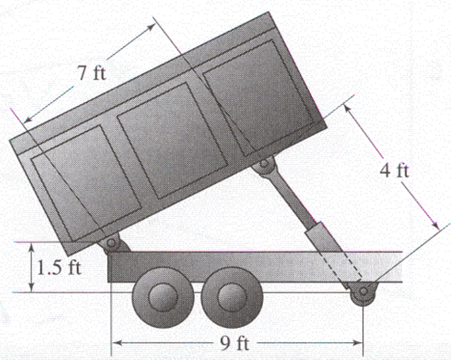
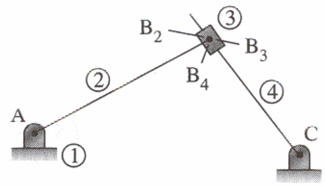


Figure 5.12 Dump truck mechanism for Example Problem 5.5.

Solution:

Kinematically, this mechanism is an inversion to the common slider-crank mechanism. Notice that the sliding joint of the cylinder and rod connects the rotating bodies of the bed and cylinder. Thus, two coincident points are defined and used to solve the problem. Figure 5.13A shows the kinematic diagram of this mechanism.



(a)

(b)

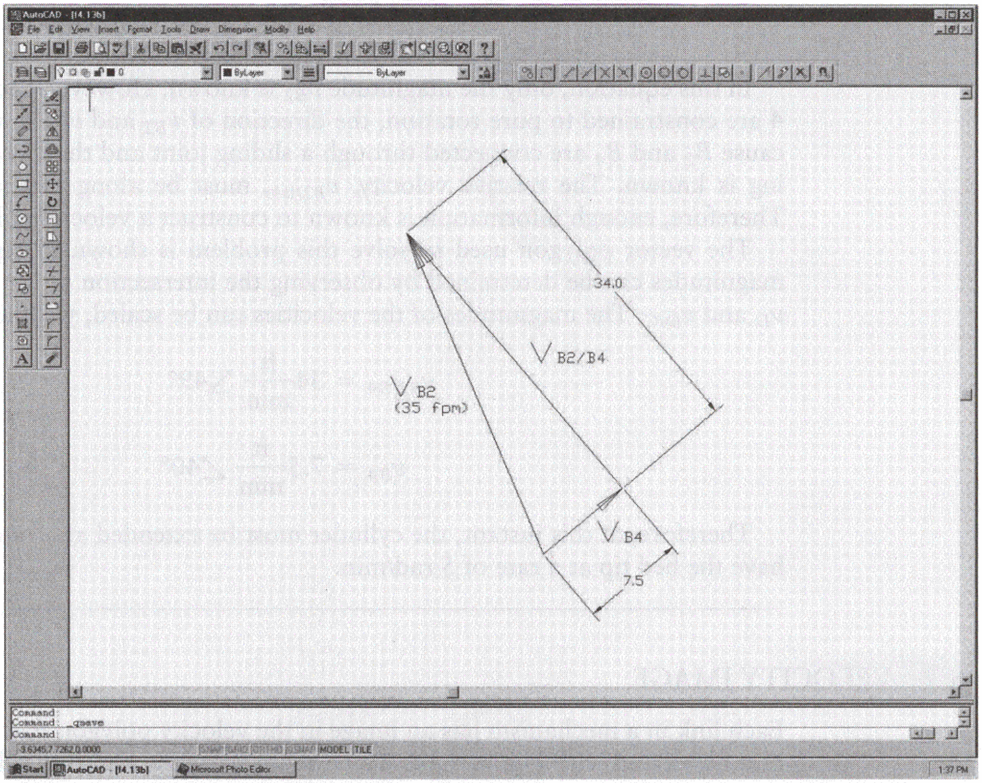


Figure 5.13 Diagrams for Example Problem 5.5.

Link 1 is the bed frame, 2 represents the cylinder, 3 represents the piston/rod, and 4 represents the bed. Notice that the pin joint that connects links 2 and 3 is labeled as point *B*. However, because links 2, 3, and 4 are located at point *B*, the coincident points are distinguished as B_2, B_3, and B_4. The extending motion of the cylinder is a relative motion of point B_2 relative to point B_4. Thus, the problem is to determine the velocity of B_2 relative to B_4, which will cause link 2 to rotate at a rate of 5 rad/min, counterclockwise.

The desired velocity of B_2 can be found with the following:

$$v_{B2} = \omega_2 r_{B2}$$
$$= \left(5\frac{\text{rad}}{\text{min}}\right)(7 \text{ ft}) = 35\ \frac{\text{ft}}{\text{min}}$$

The direction of the velocity of point B_2 is perpendicular to link 2, up and to the left.

The velocities of the coincident points are related though Equation 5.5:

$$v_{B2} = v_{B4} +> v_{B2/B4}$$

In this equation, only the magnitude v_{B2} is known. However, because links 2 and 4 are constrained to pure rotation, the direction of v_{B2} and v_{B4} is known. Also, because B_2 and B_4 are connected through a sliding joint and the direction of the sliding is known. The relative velocity, $v_{B2/B4}$, must be along this sliding direction. Therefore, enough information is known to construct a velocity polygon.

The vector polygon used to solve this problem is shown in Figure 5.13B. The magnitudes can be determined by observing the intersection of the directed lines of v_D and $v_{D/C}$. The magnitudes of the velocities can be scaled, yielding the following:

$$v_{B2/B4} = 34\frac{\text{ft}}{\text{min}} \searrow 49°$$

$$v_{B4} = 7.5\frac{\text{ft}}{\text{min}} \angle 40°$$

Therefore, at this instant, the cylinder must be extended at a rate of 34 ft/min to have the bed tip at a rate of 5 rad/min.

5.7 VELOCITY IMAGE

Each link in a mechanism has an image in the velocity polygon that is useful. This is best illustrated with Figure 5.14, which reproduces the absolute velocities from Ex-

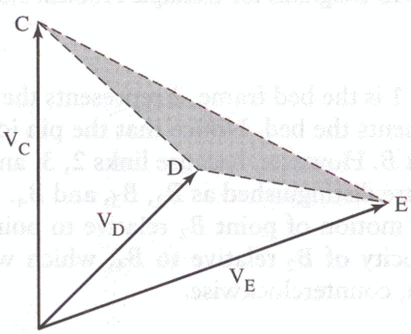


Figure 5.14 Velocity image.

ample Problem 5.4. Notice that a triangle was drawn using the terminus of the three vectors. This triangle is shaped with proportional dimensions to the link itself. The shape in the velocity polygon is termed a *velocity image* of the link.

If this concept of velocity image had been known initially, the solution process could be reduced considerably. The prior technique can be used to determine the velocity of two points on a link. These two points can be used as the base of the velocity image. Any other points from that link can be scaled and included in the velocity polygon. Care must be taken not to allow the shape of the link to be flipped between the kinematic diagram and the velocity polygon.

5.8 ANALYTICAL VELOCITY ANALYSIS: RELATIVE VELOCITY METHOD

Analytical velocity analysis involves exactly the same logic as illustrated in the previous section. The vector polygons are created according to the appropriate relative velocity equations. Because analytical techniques are used, the accuracy of the polygon is not a major concern, although a rough scale allows insight into the solutions. The vector equations can be solved using the analytical techniques presented in Chapter 4.

Analytical solutions are presented in the following Example Problems.

EXAMPLE PROBLEM 5.6

Figure 5.15 shows a primitive well pump that is popular in undeveloped areas. To maximize water flow, the piston should travel upward at a rate of 50mm/s. In the position shown, determine the angular velocity that must be imposed on the handle to achieve the desired piston speed.

Solution:

Kinematic Diagram

Figure 5.16A shows the kinematic diagram of this mechanism. Notice that this is a variation of a slider-crank mechanism, which has one degree of freedom.

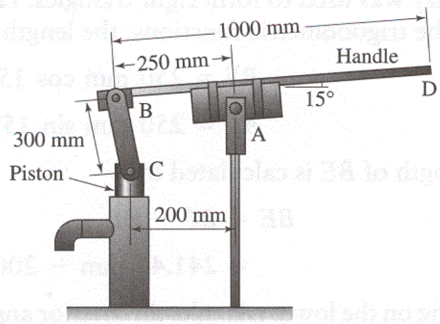


Figure 5.15 Well pump for Example Problem 5.6.

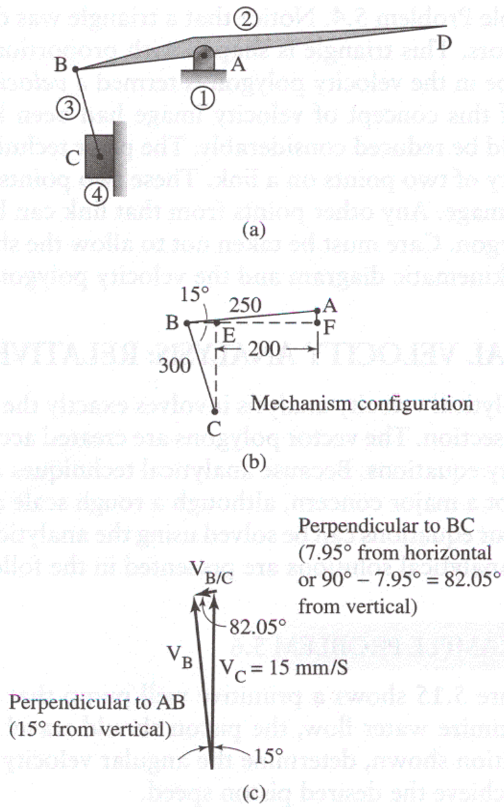


Figure 5.16 Diagrams for Example Problem 5.6.

Link 2 represents the handle. Therefore, the goal of this problem is to determine ω_2.

Geometric Configuration

Figure 5.16B isolates the geometry of the core mechanism links. Notice that this geometry was used to form right triangles. Focusing on the upper triangle *ABF*, and using the trigonometric functions, the length of sides *BF* and *AF* can be determined.

$$BF = 250 \text{ mm} \cos 15° = 241.48 \text{ mm}$$

$$AF = 250 \text{ mm} \sin 15° = 64.70 \text{ mm}$$

The length of *BE* is calculated by:

$$BE = BF - EF$$

$$= 241.48 \text{ mm} - 200 \text{ mm} = 41.48 \text{ mm}$$

Focusing on the lower triangle, the interior angle at C can be found with the following:

$$\angle BCE = \sin^{-1}\left(\frac{41.48}{300}\right) = 7.95°$$

Velocity

To solve for the angular velocity of link 2, the linear velocity of points *B* or *D*, which reside on link 2, must be determined. Link 3 is of special interest because it carries both point *C* (known velocity) and point *B* (unknown velocity).

Because link 2 is fixed to the frame at *A*, point *B* is limited to rotation about *A*. Therefore, the velocity of point *B* must be perpendicular to line *AB*.

In addition, both points *B* and *C* reside on the same link (link 3). Therefore, the relative velocity of *B* with respect to *C* must lay perpendicular to the line *BC*.

From the previous two statements, the directions of both velocities v_B and $v_{B/C}$ are known. Velocity V_B is perpendicular to *AB*, 15° from the vertical. Velocity $V_{B/C}$ is perpendicular to *BC*, 7.95° from the horizonatal, or 90° − 7.95° = 82.05° from the vertical. These velocities can be related using Equation 5.5:

$$v_B = v_C +> v_{B/C}$$

In this equation, only the magnitudes of v_B and $v_{B/C}$ are unknown. The vector polygon that is used to solve this problem is shown in Figure 5.16C. The magnitudes can be determined by solving for the length of the sides (vector magnitudes) of the general triangle.

The remaining interior angle of this vector triangle is:

$$180° - 82.05° - 15° = 82.95°$$

The law of sines is used to determine the vector magnitudes:

$$v_{B/C} = v_C\left(\frac{\sin 15°}{\sin 82.95°}\right)$$

$$= 13.04\ \frac{\text{mm}}{\text{s}} \swarrow 7.95°$$

$$v_B = v_C\left(\frac{\sin 82.05°}{\sin 82.95°}\right)$$

$$= 49.90\frac{\text{mm}}{\text{s}} \searrow 15° \quad = 49.9\frac{\text{mm}}{\text{s}} \searrow 75°$$

Now that the velocity *B* is determined, the angular velocity of link 2 can be solved. Notice that consistent with the direction of v_B, link 2 must rotate clockwise:

$$\omega_2 = \frac{v_B}{r_{AB}}$$

$$= \frac{49.9\text{m/s}}{250\text{ mm}} = 0.20\ \frac{\text{rad}}{\text{sec}}\ \text{clockwise}$$

Convert this result to rpm with the following:

$$\omega\left(\frac{\text{rev}}{\text{min}}\right) = \frac{60}{2\pi}\left[\omega\left(\frac{\text{rad}}{\text{sec}}\right)\right]$$

$$= \frac{60}{2\pi}\left[0.20\ \frac{\text{rad}}{\text{sec}}\right] = 1.9\ \text{rpm clockwise}$$

EXAMPLE PROBLEM 5.7

Figure 5.17 illustrates a roofing material delivery truck conveyor. Heavy roofing materials can be transported on the conveyor to the roof. The conveyor is lifted into place by extending the hydraulic cylinder. At this instant, the cylinder is extending at a rate of 8 fpm (ft/min). Determine the rate that the conveyor is being lifted.

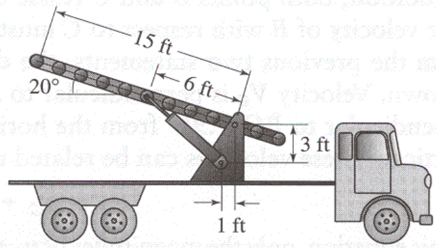


Figure 5.17 Conveyor for Example Problem 5.7.

Solution:

Kinematic Diagram

Figure 5.18A shows the kinematic diagram of this mechanism. Since a sliding joint is used to connect two rotating links, defining coincident points will aid problem solution. Point B_2 is attached to link 2, and point B_4 is attached as a point of reference to link 4.

Link 4 represents the conveyor. Therefore, the goal of this problem is to determine ω_4.

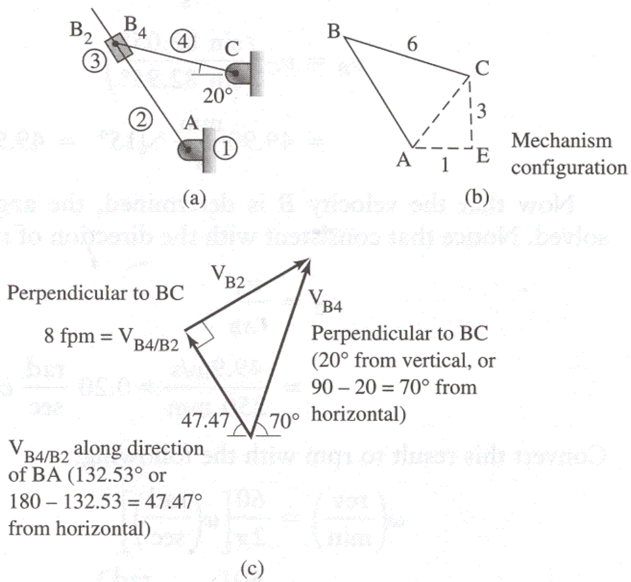


Figure 5.18 Diagrams for Example Problem 5.7.

Geometric Configuration

Figure 5.18B isolates the geometry of the core mechanism links. Notice that this geometry was used to form right triangles. Focusing on the lower triangle ACE yields the following:

$$AC = \sqrt{[AE^2 + CE^2]}$$

$$= [1^2 + 3^2] = 3.16 \text{ ft}$$

$$\angle CAE = \tan^{-1}\left(\frac{CE}{AE}\right) = 71.57°$$

$$\angle ACE = \tan^{-1}\left(\frac{AE}{CE}\right) = 18.43°$$

Because link 4 is inclined at 20° above horizontal, the full angle at C is:

$$\angle BCE = 90° + 20° = 110°$$

then the angle at C in the upper triangle is:

$$\angle ACB = \angle BCE - \angle ACE = 110° + 18.43° = 91.57°$$

The geometry of the upper triangle can be fully determined by:

$$AB = \sqrt{AC^2 + BC^2 - 2(AC)(BC)\cos\angle ABC}$$

$$= \sqrt{(3.16 \text{ ft})^2 + (6 \text{ ft})^2 - 2(3.16 \text{ ft})(6 \text{ ft}) \cos 91.57°} = 6.86 \text{ ft}$$

$$\angle BAC = \sin^{-1}\left\{\left(\frac{6 \text{ ft}}{6.86 \text{ ft}}\right) \sin 91.57°\right\} = 60.96°$$

$$\angle CBA = \sin^{-1}\left\{\left(\frac{3.16 \text{ ft}}{6.86 \text{ ft}}\right) \sin 91.57°\right\} = 27.42°$$

The total included angle at A is:

$$\angle BAE = \angle CAE + \angle BAC = 71.57° + 60.96° = 132.53°$$

Velocity

To solve for the angular velocity of link 2, the linear velocity of point B_2, which resides on link 2, must be determined. The extension of the hydraulic cylinder is given, which represents the velocity of B_4 relative to B_2, $V_{B4/B2}$.

Because link 4 is fixed to the frame at C, point B_4 is limited to rotation about C. Therefore, the velocity of point B_4 must be perpendicular to the line BC.

In addition, link 2 is fixed to the frame at A, and point B_2 is limited to rotation about A. Therefore, the velocity of point B_2 must be perpendicular to the line AB.

From the previous two statements, the directions of both velocities v_{B4} and v_{B2} are known. These velocities can be related using Equation 5.5:

$$v_B = v_{B4/B2} +> v_{B2}$$

In this equation, only the magnitudes of v_{B4} and v_{B2} are unknown. The vector polygon that is used to solve this problem is shown in Figure 5.18C. Notice that these vectors form a right triangle. The magnitudes can be determined by solving for the length of the sides (vector magnitudes) of the right triangle.

The bottom interior angle of this vector triangle is:

$$180° - 70° - 47.47° = 62.53°$$

The velocity of B_2 is found from the following trigonometric functions:

$$v_{B2} = \{v_{B4/B2} \tan 62.53°\}$$
$$= 15.39 \text{ fpm} \angle 42.53°$$

the direction was determined as $90° - 47.47°$.

Now that velocity B is known, the angular velocity of link 2 can be solved. Notice that consistent with the direction of v_B, link 2 must rotate clockwise:

$$\omega_2 = \frac{v_{B2}}{r_{BC}}$$
$$= \frac{15.39 \text{ fpm}}{6 \text{ ft}} = 2.56 \frac{\text{rad}}{\text{sec}} \text{ clockwise}$$

Convert this result to rpm by:

$$\omega\left(\frac{\text{rev}}{\text{min}}\right) = \frac{60}{2\pi}\left[\omega\left(\frac{\text{rad}}{\text{sec}}\right)\right]$$
$$= \frac{60}{2\pi}\left[2.56\left(\frac{\text{rad}}{\text{sec}}\right)\right] = 24.44 \text{ rpm clockwise}$$

5.9 ALGEBRAIC SOLUTIONS FOR COMMON MECHANISMS

For the common slider-crank and four-bar mechanisms, closed-form algebraic solutions have been derived [10]. They are given in the following sections.

5.9.1 Slider-Crank Mechanism

A general slider-crank mechanism was illustrated in Figure 4.21 and is uniquely defined with dimensions L_1, L_2 and L_3. With one degree of freedom, the motion of one link must be specified to drive the other links. Most often the crank is driven. Therefore, knowing θ_2, ω_2, and the position of all the links, from Equations 4.4 and 4.5, the velocities of the other links can be determined. As presented in Chapter 4, the position equations are:

(4.4)
$$\theta_3 = \sin^{-1}\left\{\frac{L_1 + L_2 \sin\theta_2}{L_3}\right\}$$

(4.5) $$L_4 = L_2 \cos(\theta_2) + L_3 \cos(\theta_3)$$

The velocity equations are given as:

(5.11) $$\omega_3 = -\omega_2\left(\frac{L_2 \cos \theta_2}{L_3 \cos \theta_3}\right)$$

(5.12) $$v_4 = -\omega_2 L_2 \sin \theta_2 - \omega_3 L_3 \sin \theta_3$$

5.9.2 Four-Bar Mechanism

A general four-bar mechanism was illustrated in Figure 4.24 and is uniquely defined with dimensions L_1, L_2, L_3, and L_4. With one degree of freedom, the motion of one link must be specified to drive the other links. Most often the crank is driven. Therefore, knowing θ_2, ω_2, and the position of all the links, from Equations 4.7, 4.8, 4.9, and 4.10, the velocities of the other links can be determined. As presented in Chapter 4, the position equations are as follows:

(4.7) $$BD = \sqrt{L_1^2 + L_2^2 - 2(L_1)(L_2)\cos \theta_2}$$

(4.8) $$\gamma = \cos^{-1}\left(\frac{L_3^2 + L_4^2 - BD^2}{2L_3L_4}\right)$$

(4.9) $$\theta_4 = 180° - \cos^{-1}\left(\frac{L_1^2 - L_2^2 + BD^2}{2L_1BD}\right) - \cos^{-1}\left(\frac{L_4^2 - L_3^2 + BD^2}{2L_4BD}\right)$$

(4.10) $$\theta_3 = \theta_4 - \gamma$$

The velocity equations are as follows:

(5.13) $$\omega_3 = \omega_2\left(\frac{L_2 \sin(\theta_4 - \theta_2)}{L_3 \sin \gamma}\right)$$

(5.14) $$\omega_4 = \omega_2\left(\frac{L_2 \sin(\theta_3 - \theta_2)}{L_4 \sin \gamma}\right)$$

5.10 INSTANTANEOUS CENTER OF ROTATION

In determining the velocity of points on a mechanism, the concept of instant centers can be used as an alternative approach to the relative velocity method. This approach is based on the fact that any link, regardless of the complexity of its motion, instantaneously appears to be rotating only about a single point. This imagined pivot point is termed the *instant center* of rotation for the particular link. The instant center for a floating link, link 3, is shown in Figure 5.19.

Using this concept, each link can be analyzed as if it were undergoing pure rotation. An instant center may exist on or off the body, and its position is not fixed in time. As a link moves, its instant center also moves. However, the velocities of different points on a mechanism are also instantaneous. Therefore, this fact does not place a serious restriction on the analysis.

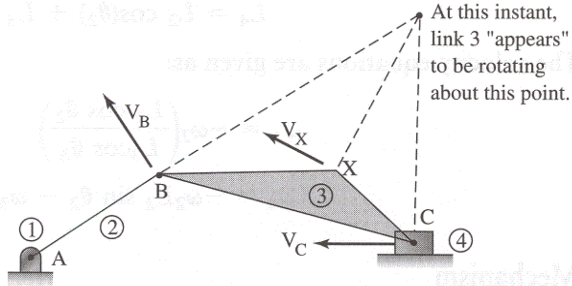


Figure 5.19 Instantaneous center.

This concept also extends to relative motion. That is, the motion of any link, relative to any other link, instantaneously appears to be rotating only about a single point. Again, the imagined pivot point is termed the instant center between the two links. For example, if two links were designated as 1 and 3, the instant center would be the point in which link 3 instantaneously appears to be rotating relative to link 1. This instant center is designated as (13), and verbalized as "one three," not thirteen. Note that the instant center shown in Figure 5.19 is designated as (13). If link 1 were the frame, as is the typical designation, this instant center would describe the absolute motion of link 3. From kinematic inversion, this point is also the center of instantaneous motion of link 1 relative to link 3. Thus, the instant center (13) is the same as (31).

Because every link has an instant center with every other link, each mechanism has several instant centers. The total number of instant centers in a mechanism with n links is:

$$\text{Total Number of Instant Centers} = \frac{n(n-1)}{2} \tag{5.15}$$

5.10.1 Locating Instant Centers

In a typical analysis, it is seldom that every instant center is used. However, the process of locating each center should be understood because every center could conceivably be employed. Some instant centers can be located by simply inspecting a mechanism. These centers are termed *primary centers*. In locating primary centers, the following rules are used:

Rule 1. When two links are connected by a pin joint, the instant center between the two links is at this pivot point.

Rule 2. The instant center for two links in rolling contact with no slipping is located at the point of contact.

Rule 3. The instant center for two links with straight line sliding is at infinity, in a direction perpendicular to the direction of sliding. The velocity of all points on a link, which is constrained to straight sliding relative to another link, is identical and in the direction of sliding. Therefore, it can be imagined that this straight motion is rotation about a point at a great distance because a straight line can be modeled as a portion of a circle with an infinitely large radius. Because velocity is always perpendicular to a line drawn to the pivot, this instant center must be perpendicular to the sliding direction. This center could be considered to be on any line parallel to the sliding direction because the lines meet at infinity.

Rule 4. The instant center for two links having general sliding contact must lay somewhere along the line normal to the direction of sliding contact.

EXAMPLE PROBLEM 5.8

Figure 5.20 illustrates an air compressor mechanism. For this mechanism, locate all the primary instant centers.

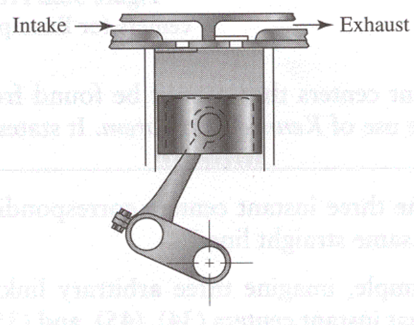


Figure 5.20 Air compressor for Example Problem 5.8.

Solution:

The kinematic diagram for the air compressor is illustrated in Figure 5.21.

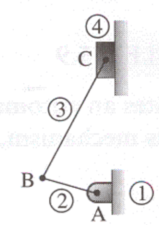


Figure 5.21 Kinematic diagram for Example Problem 5.8.

The four links are numbered on the kinematic diagram. The pin joints are also lettered. The first pin joint, *A*, connects link 1 and link 2. From the first rule for primary instant centers, this joint is the location of instant center (12). Similarly, pin joint *B* is instant center (23) and pin joint *C* is instant center (34).

Because a straight sliding joint occurs between links 4 and 1, this instant center is visualized at infinity, in a direction perpendicular to the sliding direction. Figure 5.22 illustrates the notation used to identify this, along with labeling all other primary instant centers. Recall that this instant center could be on line parallel to this line because it can be considered that the lines intersect at infinity.

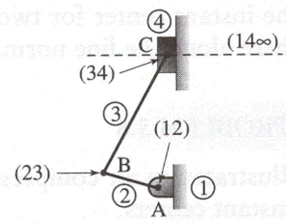


Figure 5.22 Primary instant centers for Example Problem 5.8.

Instant centers that cannot be found from the preceding four rules are located with the use of *Kennedy's theorem*. It states that:

"The three instant centers corresponding with any three bodies all lay on the same straight line."

For example, imagine three arbitrary links, link 3, 4, and 5. Kennedy's theorem states that instant centers (34), (45), and (35) all lay on a straight line. This theorem, along with first locating primary instant centers, can be used to find all other instant centers. Locating the instant centers can be accomplished by either using graphical or analytical methods. Of course, graphical methods include both manual drawing techniques or CAD.

The following Example Problems illustrate the procedure.

EXAMPLE PROBLEM 5.9

Figure 5.23 illustrates an automated, self-locking brace for a platform used on shipping docks. For this mechanism, locate all the instant centers.

TABLE 5.1 Possible Instant Centers in a Mechanism (n=4)

1	2	3	4
12	23	34	
13	24		
14			

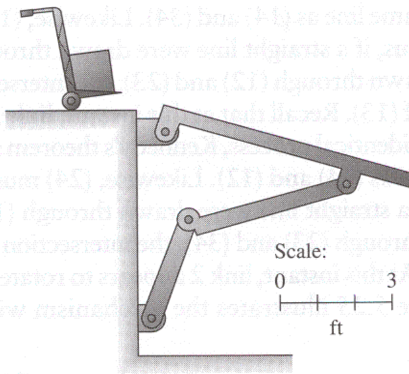


Figure 5.23 Locking brace for Example Problem 5.9.

Solution:

The kinematic diagram for the loading platform is illustrated in Figure 5.24.

The four links are numbered on the kinematic diagram. The pin joints are also lettered. Compute the total number of instant centers, with $n = 4$ links, as follows:

$$\text{Total Number of Instant Centers} = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

Table 5.1 can be used to systematically list all possible instant centers in a mechanism.

The first pin joint, A, connects links 1 and 2. From the first rule for primary instant centers, this joint is the location of instant center (12). Similarly, pin joints B, C, and D are instant centers (23), (34), and (14), respectively.

The remaining combinations that need to be determined are instant centers (13) and (24). By applying Kennedy's theorem, it is known that instant center (13) must be

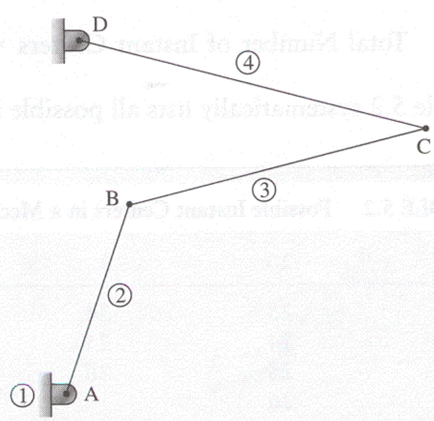


Figure 5.24 Kinematic diagram for Example Problem 5.9.

on the same line as (14) and (34). Likewise, (13) must also be on the same line as (12) and (23). Thus, if a straight line were drawn through (14) and (34) and another straight line were drawn through (12) and (23), the intersection of these lines would determine the location of (13). Recall that at this instant, link 3 appears to be rotating around point (13).

In an identical process, Kennedy's theorem states that instant center (24) must be on the same line as (14) and (12). Likewise, (24) must also be on the same line as (23) and (34). Thus, if a straight line were drawn through (14) and (12) and another straight line were drawn through (23) and (34), the intersection of these lines would determine the location of (24). At this instant, link 2 appears to rotate—relative to link 4—around point (24).

Figure 5.25 illustrates the mechanism with all instant centers located.

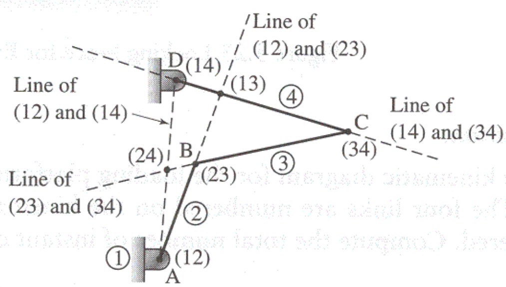


Figure 5.25 Instant centers for Example Problem 5.9.

EXAMPLE PROBLEM 5.10

Figure 5.26 illustrates a rock crusher. For this mechanism, locate all the instant centers.

Solution:

The kinematic diagram for the rock crusher is illustrated in Figure 5.27.

The six links are numbered on the kinematic diagram. The pin joints are also lettered. Compute the total number of instant centers, with $n = 6$ links, as follows:

$$\text{Total Number of Instant Centers} = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

Table 5.2 systematically lists all possible instant centers in a mechanism.

TABLE 5.2 Possible Instant Centers in a Mechanism (n=6)

1	2	3	4	5	6
12	23	34	45	56	
13	24	35	46		
14	25	36			
15	26				
16					

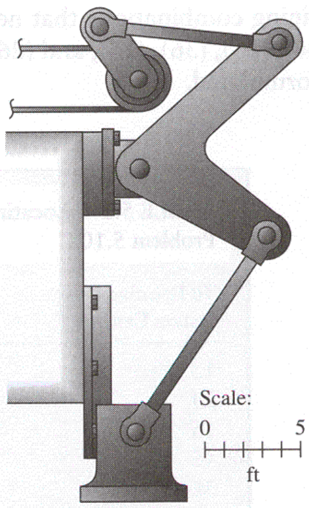


Figure 5.26 Rock crusher for Example Problem 5.10.

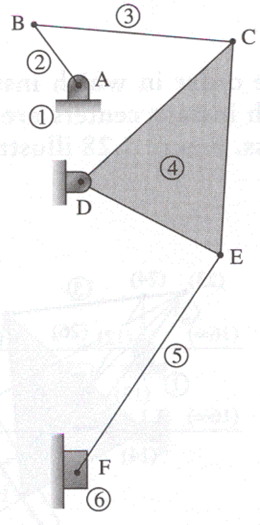


Figure 5.27 Kinematic diagram for Example Problem 5.10.

The first pin joint, *A*, connects links 1 and 2. From the first rule for primary instant centers, this joint is the location of instant center (12). Similarly, pin joints *B-F* are instant centers (23), (34), (14), (45), and (56), respectively.

Because a straight sliding joint exists between links 6 and 1, this instant center (16) visualizes at infinity, in a direction perpendicular to the sliding direction. Recall that this instant center could be on line parallel to this line because the lines meet at infinity.

The remaining combinations that need to be determined are instant centers (13), (24), (35), (46), (25), (36), (15), and (26). By repeatedly applying Kennedy's theorem, Table 5.3 is formulated.

TABLE 5.3 Locating Instant Centers for Example Problem 5.10

To Locating Instant Center	Use Intersecting Lines
13	(12) - (23) and (14) - (34)
24	(12) - (14) and (23) - (34)
15	(16) - (56) and (14) - (45)
46	(14) - (16) and (45) - (56)
36	(13) - (16) and (34) - (46)
26	(12) - (16) and (23) - (36)
35	(56) - (36) and (34) - (45)
25	(24) - (45) and (23) - (35)

Note that the order in which instant centers are found is extremely dependent on which instant centers are already located. This becomes quite an iterative process. Figure 5.28 illustrates the mechanism with all instant centers located.

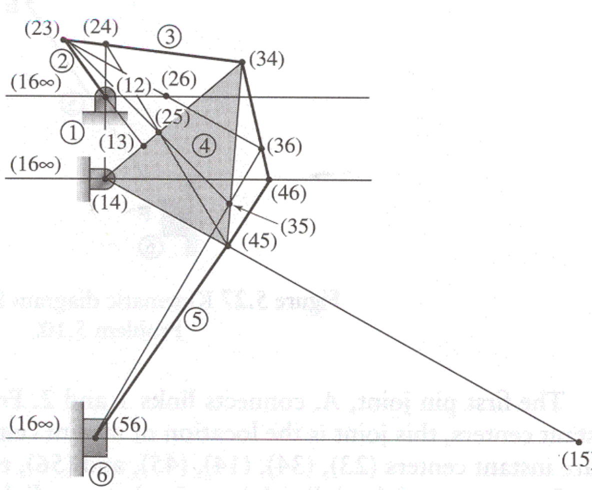


Figure 5.28 Instant centers for Example Problem 5.10.

5.10.2 Graphical Velocity Analysis: Instant Center Method

The instant center method is based on the following three principles:

1. The velocity on a rotating body is proportional to the distance from the pivot point.
2. The instant center that is common to two links can be considered on either link.
3. The absolute velocity of the point, which serves as the common instant center, is the same, no matter which link is considered fixed to that point.

Using these principles, the absolute velocity of any point on the mechanism can be readily obtained through a general method. This method is outlined in the following six steps:

1. Isolate the link with a known velocity (link *A*), the link containing the point for which the velocity is desired (link *B*), and the fixed link (link *C*).
2. Locate the instant center that is common to the link with the known velocity and the fixed link (instant center *AC*).
3. Locate the instant center that is common to the link with the known velocity and the link that contains the point where the velocity is desired (instant center *AB*).
4. Determine the velocity of the instant center (*AB*). This can be done by understanding that the velocity of a point on a link is proportional to the distance from the pivot. The instant center (*AC*) serves as the pivot. The known velocity on link *A* can be proportionally scaled to determine the velocity of the instant center (*AB*).
5. Locate the instant center that is common to the link with the point whose velocity is desired and the link fixed (instant center *BC*).
6. Determine the desired velocity. This can be done by understanding that the velocity on a link is proportional to the distance from the pivot. The instant center (*BC*) serves as this pivot. The velocity of the common instant center (*AB*) can be proportionally scaled to determine the desired velocity.

A graphical technique for proportionally scaling a vector uses a *line of centers, LC*. This is a line drawn from the pivot point of the link to the start of the known vector. A *line of proportion, LP*, must also be constructed. This is a line drawn from the pivot point to the end of the known vector. Figure 5.29A illustrates both the line of centers and the line of proportion. The distance from the pivot to the desired point can be transferred to the line of centers. The magnitude of the proportionally scaled vector is determined as parallel to the known vector, and extending from *LC* to *LP* at the transferred distance. This is also illustrated in Figure 5.29A.

Of course, the magnitude of the velocity is perpendicular to the line that connects the point with unknown velocity to the pivot point. Determining the magnitude and positioning of that vector in the proper direction fully defines the vector. Thus, the vector is graphically proportioned. The result is shown in Figure 5.29B.

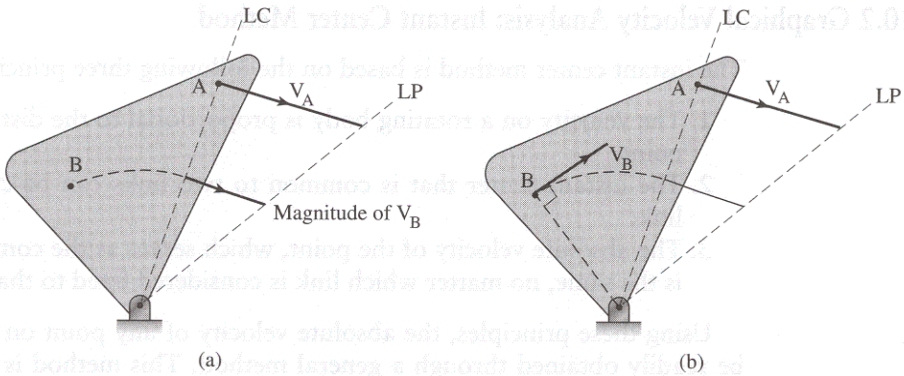


Figure 5.29 Using a line of centers and line of proportion.

This previous description describes the logic behind the instantaneous center method of velocity analysis using graphical techniques. The actual solution can be completed with identical logic whether using manual drawing or CAD. Regardless of the process used, the underlying concepts of a graphical approach to the instantaneous center method of velocity analysis can be illustrated through the following Example Problems.

EXAMPLE PROBLEM 5.11

Figure 5.23 illustrated an automated, self-locking brace for a platform used on shipping docks. Example Problem 5.9 located all instant centers for the mechanism. Determine the angular velocity of link 4, knowing that link 2 is rising at a constant rate of 3 rad/sec.

Solution:

The kinematic diagram, with the instant centers and scale information, is reproduced as Figure 5.30A.

The linear velocity of point B can be determined from the rotational velocity of link 2. Point B has been scaled to be positioned at a distance of 3 ft from the pivot of link 2:

$$v_B = r_B\omega_2 = (3\text{ ft})\left(3\frac{\text{rad}}{\text{sec}}\right) = 9\frac{\text{ft}}{\text{sec}}$$

In a similar fashion, the rotational velocity of link 4 can be found from the linear velocity of the point labeled as C. The general method for using the instant center method can be followed to solve the problem.

1. Isolate the links:
   Link 2 contains the known velocity;
   Link 4 contains the point for which the velocity is desired; and,
   Link 1 is the fixed link.

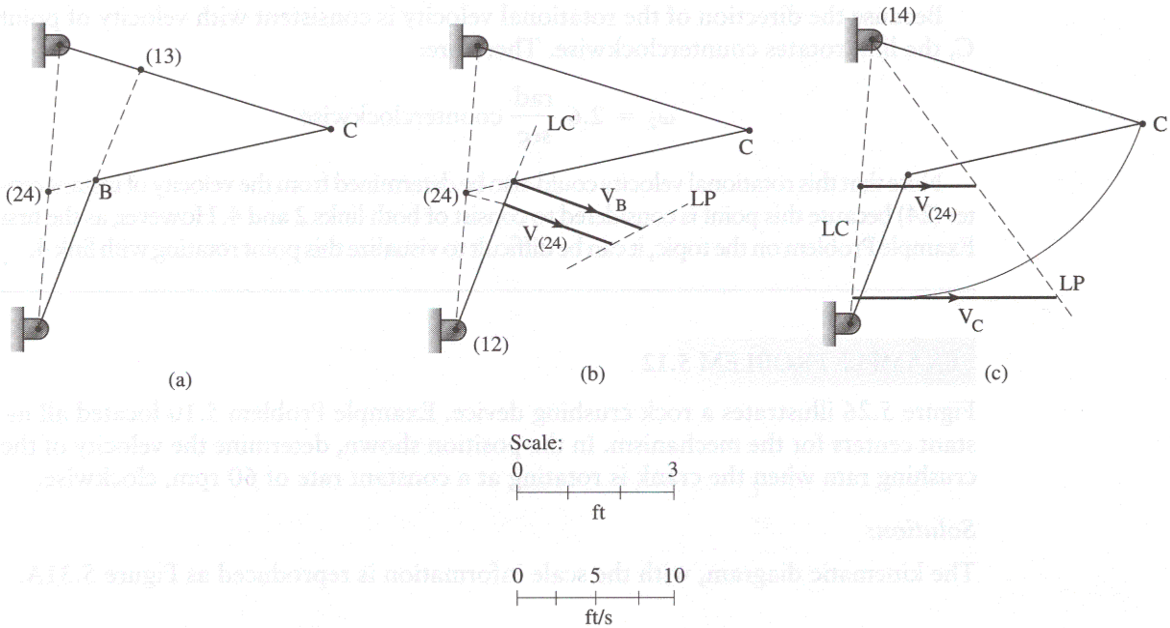


Figure 5.30 Kinematic diagram for Example Problem 5.11.

2. The common instant center between the known and fixed link velocities is (12).
3. The common instant center between the known and unknown link velocities is (24).
4. The velocity of instant center (24) is graphically proportioned from the velocity of point B. Link 2 contains both point B and instant center (24); therefore, the velocity is proportionally scaled relative to instant center (12). This construction is shown in Figure 5.30B. The magnitude of this velocity, $v_{(24)}$, is scaled to 7.4 ft/sec.
5. The common instant center between the unknown and fixed link velocities is (14).
6. The velocity of point C is graphically proportioned from the velocity of instant center (24). Link 4 contains both point C and instant center (24); therefore, the velocity is proportionally scaled relative to instant center (14). This construction is shown in Figure 5.30C. The magnitude of this velocity, v_C, is scaled to 13.8 ft/sec.

Finally, the rotational velocity of link 4 can be found from the velocity of point C. Point C has been scaled to be positioned at a distance of 5.4 ft from the pivot of link 4:

$$\omega_4 = \frac{v_C}{r_C} = \frac{13.8\text{ft/sec}}{5.4\text{ ft}} = 2.6\frac{\text{rad}}{\text{sec}}$$

Because the direction of the rotational velocity is consistent with velocity of point C, the link rotates counterclockwise. Therefore:

$$\omega_2 = 2.6 \frac{\text{rad}}{\text{sec}} \text{ counterclockwise}$$

Note that this rotational velocity could also be determined from the velocity of instant center (24) because this point is considered to consist of both links 2 and 4. However, as the first Example Problem on the topic, it can be difficult to visualize this point rotating with link 4.

EXAMPLE PROBLEM 5.12

Figure 5.26 illustrates a rock crushing device. Example Problem 5.10 located all instant centers for the mechanism. In the position shown, determine the velocity of the crushing ram when the crank is rotating at a constant rate of 60 rpm, clockwise.

Solution:

The kinematic diagram, with the scale information is reproduced as Figure 5.31A.

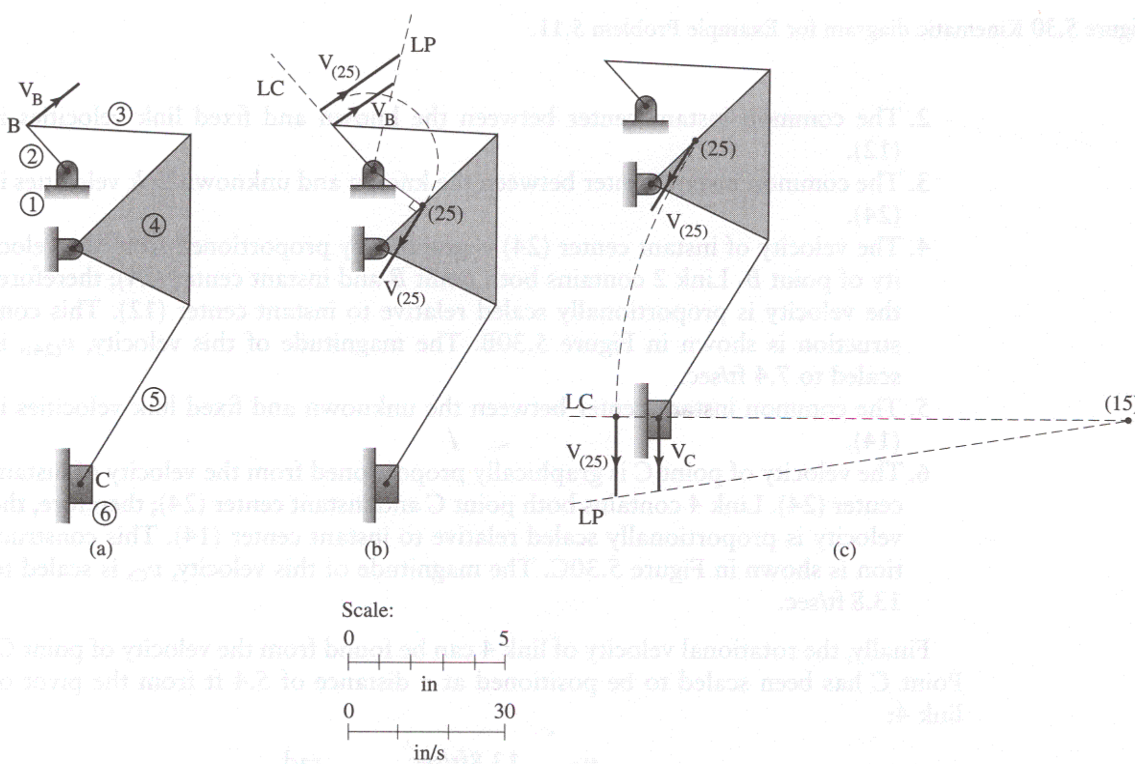


Figure 5.31 Diagrams for Example Problem 5.12.

The linear velocity of point B can be determined from the rotational velocity of link 2. Point B has been scaled to be positioned at a distance of 4.5 in from the pivot of link 2:

$$\omega_2 = 60 \text{ rpm}\left(\frac{2\pi}{60}\right) = 6.28\frac{\text{rad}}{\text{sec}}$$

$$v_B = r_B\omega_2 = (4.5\text{in})\left(6.28\frac{\text{rad}}{\text{sec}}\right) = 28.3\frac{\text{in}}{\text{sec}}$$

The purpose of this problem is to determine the linear velocity of point C. The general method for using the instant center method can be followed to solve the problem.

1. Isolate the links:
   Link 2 contains the known velocity;
   Link 5 (or 6) contains the point for which the velocity is desired; and,
   Link 1 is the fixed link.
2. The common instant center between the known and fixed link velocities is (12).
3. The common instant center between the known and unknown link velocities is (25).
4. The velocity of the instant center (25) is graphically proportioned from the velocity of point B. Link 2 contains both point B and instant center (25); therefore, the velocity is proportionally scaled relative to instant center (12). This construction is shown in Figure 5.31B. The magnitude of this velocity, $v_{(25)}$, is scaled to 34.5 in/sec.
5. The common instant center between the unknown and fixed link velocities is (15).
6. The velocity of point C is graphically proportioned from the velocity of instant center (25). Link 5 contains both point C and instant center (25); therefore, the velocity of instant center (25) is rotated to a line of centers created by point C and instant center (15). The velocity of instant center (25) is used to create a line of proportions. This line of proportions is then used to construct the velocity of C. This construction is shown in Figure 5.31C. The magnitude of this velocity, v_C, is scaled to 32.4 in/s.

Formally stated:

$$v_C = 32.4\frac{\text{in}}{\text{s}}\downarrow$$

5.10.3 Analytical Velocity Analysis: Instant Center Method

The instant center method is virtually unaltered when an analytical approach is used in the solution. The only difference is that the positions of the instant centers must be determined through trigonometry, as opposed to constructing lines and locating the intersection points. This can be a burdensome task; thus, it is common to locate only the instant centers required for the velocity analysis. An analytical approach is illustrated through the following Example Problem.

EXAMPLE PROBLEM 5.13

Figure 5.32 shows a mechanism used in a production line to turnover cartons so that labels can be glued to the bottom of the carton. The driver arm is 15 in long and, at the instant shown, it is inclined at a 60° angle with a clockwise angular velocity of 5 rad/sec. The follower link is 16 in long. The distance between the pins on the carriage is 7 in, and the distance between the mounting bearings is 7.3 in. Determine the angular velocity of the carriage.

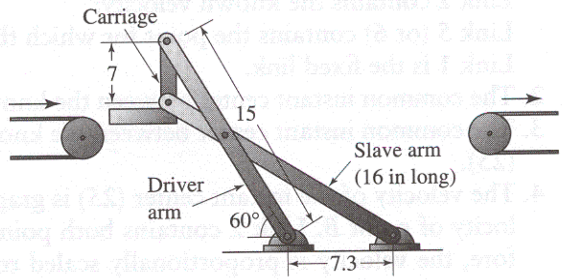


Figure 5.32 Turnover mechanism for Example Problem 5.13.

Solution:

The kinematic diagram is shown in Figure 5.33A. A point of interest, X, was included at the edge of the carriage.

Trigonometry is used to determine the distances and angles inherent with this mechanism's configuration. Triangles used to accomplish this are shown in Figure 5.33B. The distances BM and AM can be determined from triangle ABM:

$$BM = AB \sin(60°) = (15 \text{ in})\sin(60°) = 13.0 \text{ in}$$

$$AM = AB \cos(60°) = (15 \text{ in})\ 15 \cos(60°) = 7.5 \text{ in}$$

Along the vertical BCM:

$$CM = BM - BC = 13.0 - 7.0 = 6.0 \text{ in}$$

The angle ADC and the distance DN can be determined from triangle CDN:

$$\angle ADC = \sin^{-1}\left(\frac{CM}{CD}\right) = \sin^{-1}\left[\left(\frac{6 \text{ in}}{16 \text{ in}}\right)\right] = 22.0°$$

$$DM = CD \cos(22°) = (16 \text{ in})\cos(22°) = 14.8 \text{ in}$$

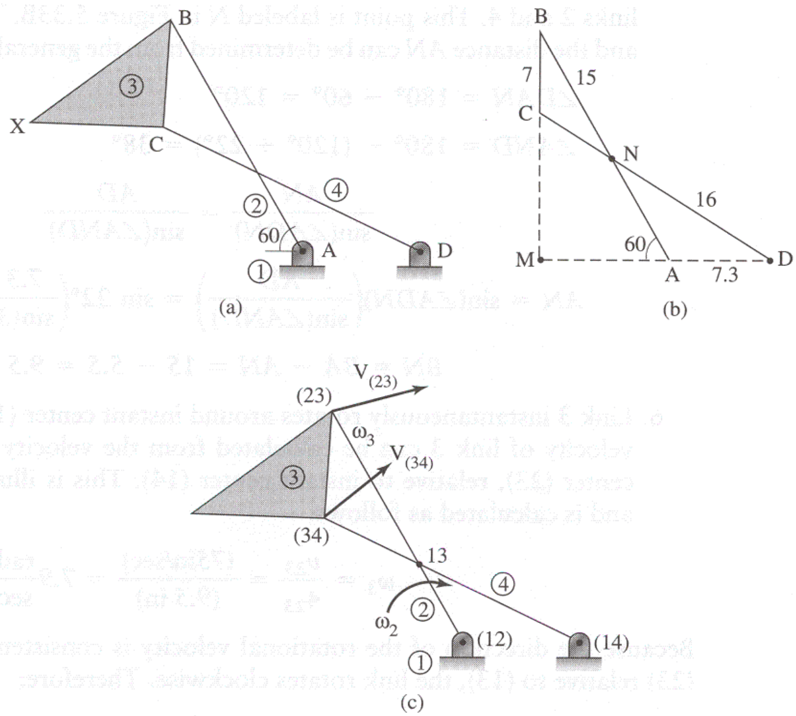


Figure 5.33 Kinematic diagram for Example Problem 5.13.

At this point, the general method for using the instant center method can be followed to solve the problem.

1. Isolate the links:
   Link 2 contains the known velocity;
   Link 3 contains the point for which the velocity is desired; and,
   Link 1 is the fixed link.
2. The common instant center between the known and fixed link velocities is (12). By inspection this instant center is located at point *A*.
3. The common instant center between the known and unknown link velocities is (23). By inspection, this instant center is located at point *B*.
4. The velocity of instant center (23) is simply the velocity of point *B*. This can be determined as:

$$v_B = r_B\omega_2 = (15 \text{ in})\left(5\frac{\text{rad}}{\text{sec}}\right) = 75\frac{\text{in}}{\text{sec}}$$

5. The common instant center between the unknown and fixed link velocities is (13). This instant center is located at the intersection of instant centers (12) - (23) and (14) - (34). By inspection, instant center (34) is located at point *C* and (14) is located at point *D*. Therefore, instant center (13) is located at the intersection of

links 2 and 4. This point is labeled N in Figure 5.33B. The angles DAN, AND, and the distance AN can be determined from the general triangle AND:

$$\angle DAN = 180° - 60° = 120°$$

$$\angle AND = 180° - (120° + 22°) = 38°$$

$$\frac{AN}{\sin(\angle ADN)} = \frac{AD}{\sin(\angle AND)}$$

$$AN = \sin(\angle ADN)\left(\frac{AD}{\sin(\angle AND)}\right) = \sin 22°\left(\frac{7.3\text{ in}}{\sin(38°)}\right) = 5.5\text{ in}$$

$$BN = BA - AN = 15 - 5.5 = 9.5\text{ in}$$

6. Link 3 instantaneously rotates around instant center (13). Thus, the rotational velocity of link 3 can be calculated from the velocity of the common instant center (23), relative to instant center (14). This is illustrated in Figure 5.33C and is calculated as follows:

$$\omega_3 = \frac{v_{23}}{4_{23}} = \frac{(75\text{in/sec})}{(9.5\text{ in})} = 7.9\frac{\text{rad}}{\text{sec}}$$

Because the direction of the rotational velocity is consistent with velocity of point (23) relative to (13), the link rotates clockwise. Therefore:

$$\omega_3 = 7.9\,\frac{\text{rad}}{\text{sec}}\text{ clockwise}$$

5.11 VELOCITY CURVES

The analyses, presented up to this point in the chapter, are used to calculate the velocity of points on a mechanism at a specific instant. Although the results can be useful, they only provide a "snapshot" of the motion. The obvious shortcoming of this analysis is that determination of the extreme conditions is difficult. It is necessary to investigate several positions of the mechanism to discover the critical phases.

It is convenient to trace the velocity of a certain point, or link, as the mechanism moves through its cycle. A *velocity curve* is such a trace. A velocity curve can be generated from a displacement diagram, as described in Section 4.15.

Recall that a displacement diagram plots the movement of a point or link as a function of the movement of an input point or link. The measure of input movement can be readily converted to time. This is particularly common when the driver operates at a constant velocity.

As discussed throughout the chapter, velocity is the time rate of change of displacement. Restating Equation 5.2:

$$\text{linear velocity} = v = \text{change in linear displacement per change in time}$$

$$v \cong \frac{\Delta s}{\Delta t}$$

Restating Equation 5.4:

rotational velocity = w = change in angular displacement per change in time

$$\omega \cong \frac{\Delta\theta}{\Delta t}$$

Often the driver of a mechanism operates at a constant velocity. For example, an input link driven by an electric motor, in steady state, operates at constant velocity. The motor shaft could cause the crank to rotate at 300 rpm, thus providing constant angular velocity.

This constant velocity of the driver link converts the x-axis of a displacement diagram from rotational displacement to time. In linear terms, rearranging Equation 5.2 yields:

(5.16)
$$\Delta t = \frac{\Delta s}{V}$$

In rotational terms, rearranging Equation 5.4 yields:

(5.17)
$$\Delta t = \frac{\Delta\theta}{\omega}$$

Thus, Equation 5.16 and 5.17 can be used to convert the displacement increment of the x-axis to a time increment. This is illustrated with Example Problem 5.14.

EXAMPLE PROBLEM 5.14

A displacement diagram of the piston operating in a compressor was plotted in Example Problem 4.5. This diagram was plotted relative to the crankshaft rotation. Use this data to plot the piston displacement relative to time when the crankshaft drives by an electric motor at 1750 rpm.

Solution:

The main task of this problem is to convert the increment of crankshaft rotation in Figure 4.16 to time. The x-axis increment is 30° and the crankshaft rotates at 1750 rpm. To keep units consistent, the x-axis increment is converted to revolutions:

$$\Delta\theta = 30°\left(\frac{1 \text{ rev}}{360°}\right) = 0.08333 \text{ rev}$$

The time increment for the crank to rotate 0.08333 rev (30°) can be computed from Equation 5.11:

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{(0.0833 \text{ rev})}{(1750 \frac{\text{rev}}{\text{min}})}$$

$$= 0.0000476 \text{ min}$$

$$= (0.0000476 \text{ min})\left(\frac{60 \text{ sec}}{1 \text{ min}}\right) = 0.00285 \text{ sec.}$$

The results of position analysis are reproduced with the time increment inserted, creating Table 5.4, which shows time tabulated in hundredths of a second.

TABLE 5.4 Table of Piston Displacement for Example Problem 5.14

Crank Angle (deg)	Time (.0001 sec)	Piston Position (in)
0	0	0.0
30	2.85	0.14
60	5.70	0.48
90	8.55	0.90
120	11.40	1.23
150	14.25	1.44
180	17.10	1.50
210	19.95	1.44
240	22.80	1.23
270	25.65	0.90
300	28.50	0.48
330	31.35	0.14
360	34.20	0.0

These values are plotted in Figure 5.34 to form a displacement diagram relative to time.

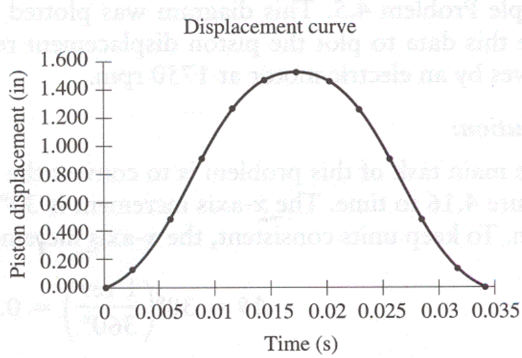


Figure 5.34 Time displacement diagram for Example Problem 5.14.

These displacement diagrams relative to time can be used to generate a velocity curve:

$$\text{velocity} = \frac{d(\text{displacement})}{d(\text{time})}$$

because Differential calculus suggests that the velocity at a particular instant is the slope of the displacement diagram at that instant. The task is to estimate the slope of the displacement diagram at several points.

5.11.1 Graphical Differentiation

The slope at a point can be estimated by sketching a line through the point of interest, tangent to the displacement curve. The slope of the line can be determined by calculating the change in y-value (displacement) divided by the change in x-value (time).

The procedure is illustrated in Figure 5.35. Notice that a line drawn tangent to the displacement diagram at t_1 is horizontal. The slope of this tangent line is zero. Therefore, the magnitude of the velocity at t_1 is zero.

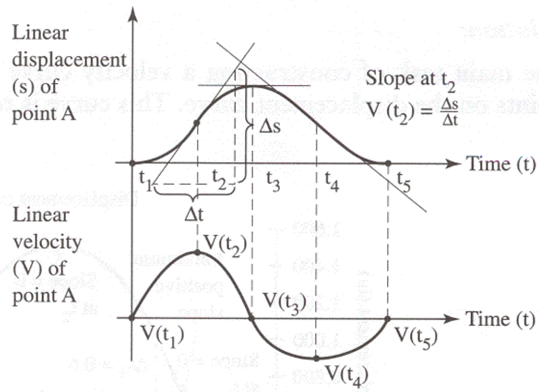


Figure 5.35 Velocity curves.

A line drawn tangent to the displacement diagram at t_2 is slanted upward as shown. The slope of this line can be calculated as the change of displacement divided by the corresponding change in time. Notice that this Δs, Δt triangle was drawn rather large to improve measurement accuracy. The velocity at t_2 is found as $\Delta s/\Delta t$, and is positive due to the upward slant of the tangent line. Also notice that this is the steepest section of the upward portion of the displacement curve. This translates to the greatest positive velocity.

This procedure can be repeated at several locations along the displacement diagram. However, only the velocity extremes and abrupt changes between them are usually desired. Using the notion of differential calculus and slopes, the positions of interest can be visually detected. In general, locations of interest include:

- The steepest portions of the displacement diagram, which correspond to the extreme velocities; and,
- The locations on the displacement diagram with the greatest curvature, which correspond to the abrupt changes of velocities.

As mentioned, the velocity at t_2 is greatest because t_2 is the steepest portion of the displacement diagram. The velocity at t_4 is the greatest velocity in the negative direction because t_4 is the steepest downward portion of the displacement diagram.

Identifying the positions of extreme velocities is invaluable. A complete velocity analysis, as presented in the previous sections of this chapter, can then be performed at these locations. Thus, comprehensive velocity analysis is only performed during important mechanism configurations.

EXAMPLE PROBLEM 5.15

A displacement diagram relative to time was constructed for a compressor mechanism in Example Problem 5.14. Use this data to plot a velocity curve relative to time.

Solution:

The main task of constructing a velocity curve is to determine the slope of many points on the displacement curve. This curve is reprinted as Figure 5.36.

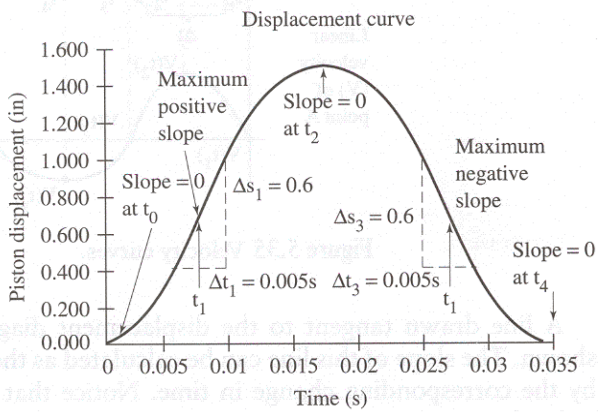


Figure 5.36 Displacement curve for Example Problem 5.15.

From this curve, it is apparent that the curve has a horizontal tangent, or zero slope, at 0, 0.017, and 0.034 sec. Therefore, the velocity of the piston is zero at 0, 0.017, and 0.034 sec. These points are labeled t_0, t_2, and t_4, respectively.

The maximum upward slope appears at 0.008 sec. This point was labeled as t_1. An estimate of the velocity can be made from the values of Δs_1 and Δt_1 read off the graph. The velocity at 0.008 sec is estimated as:

$$v_1 = \frac{0.60 \text{ in}}{0.005 \text{ sec}} = 120 \frac{\text{in}}{\text{sec}}$$

Likewise, the maximum downward slope appears at 0.027 sec. This point was labeled as t_3. Again, an estimate of the velocity can be made from the values of Δs_3 and Δt_3 read off the graph. The velocity at 0.027 sec is estimated as:

$$v_3 = \frac{-0.60 \text{ in}}{0.005 \text{ sec}} = -120 \frac{\text{in}}{\text{sec}}$$

The procedure of determining the slope of the displacement curve can be repeated at other points in time. Compiling the slope and time information, a velocity curve can be constructed (Figure 5.37).

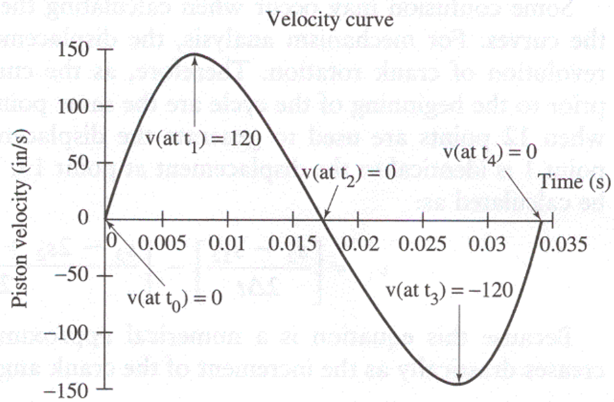


Figure 5.37 Velocity curve for Example Problem 5.15.

5.11.2 Numerical Differentiation

In creating a velocity curve using graphical differentiation, the theories of differential calculus are strictly followed. However, even with careful attention, inaccuracies are commonly encountered when generating tangent curves. Thus, other methods, namely numerical approaches, are often used to determine the derivative of a curve defined by a series of known points. The most popular method of numerically determining the derivative is the Richardson method.[3] It is valid for cases where the increments between the independent variables are equal. This limits the analysis to a constant time interval, which is not typically difficult. The derivative of the displacement-time curve can be numerically approximated from the following equation:

(5.18)
$$v_i = \left[\frac{S_{i+1} - S_{i-1}}{2\Delta t}\right] - \left[\frac{S_{i+2} - 2S_{i+1} + 2_{s-1} - S_{i-2}}{12\Delta t}\right]$$

where:

i = data point index

$$s_i = \text{displacement at data point } i$$

$$\Delta t = t_2 - t_1 = t_3 - t_2 = t_4 - t_3$$

$$t_i = \text{time at data point } i$$

Although the general form may look confusing with the terms i, $i+1$, etc., actual substitution is straightforward. To illustrate the use of this equation, the velocity at the 5th data point can be found by the following equation:

$$v_s = \left[\frac{s_6 - s_4}{2\Delta t}\right] - \left[\frac{s_7 - 2s_6 + 2s_4 - s_3}{12\Delta t}\right]$$

Some confusion may occur when calculating the derivative at the end points of the curves. For mechanism analysis, the displacement diagram repeats with every revolution of crank rotation. Therefore, as the curve is repeated, the data points prior to the beginning of the cycle are the same points at the end of the cycle. Thus, when 12 points are used to generate the displacement curve, the displacement at point 1 is identical to the displacement at point 13. Then the velocity at point 1 can be calculated as:

$$v_1 = \left[\frac{s_2 - s_{12}}{2\Delta t}\right] - \left[\frac{s_3 - 2s_2 + 2s12 - s_{11}}{12\Delta t}\right]$$

Because this equation is a numerical approximation, the associated error decreases drastically as the increment of the crank angle and time are reduced.

EXAMPLE PROBLEM 5.16

A displacement diagram of the piston operating in a compressor was plotted in Example Problem 4.5. This diagram was converted to a displacement curve relative to time in Example Problem 5.14. Use this data to numerically generate a velocity curve.

Solution:

The data from Example Problem 5.14 is expanded by inserting an additional column to include the piston velocity. The time increment is calculated as follows:

$$\Delta t = t_2 - t_1 = (0.00285 - 0.0) = 0.00285 \text{ sec}$$

To illustrate the calculation of the velocities, a few sample calculations are shown:

$$v_2 = \left[\frac{(s_3 - s_1)}{2\Delta t}\right] - \left[\frac{s_4 - 2s_3 + 2s_1 - S_{12}}{12\Delta t}\right]$$

$$= \left[\frac{(0.48 - 0.0)}{2(0.00285)}\right] - \left[\frac{0.90 - 2(0.90) + 2(1.43) - 0.14}{12(0.00285)}\right] = 141.7 \text{ in/sec}$$

$$v_9 = \left[\frac{(s_{10} - s_8)}{\Delta t}\right] - \left[\frac{s_{11} - 2s_{10} + 2s_8 - S_7}{12\Delta t}\right]$$

$$= \left[\frac{(0.90 - 1.43)}{2(0.00285)}\right] - \left[\frac{0.48 - 2(0.90) + 2(1.43) - 1.50}{12(0.00285)}\right] = -96.4 \text{ in/sec}$$

$$v_{12} = \left[\frac{(s_{13} - s_{11})}{2\Delta t}\right] - \left[\frac{s_2 - 2s_{13} + 2s_{11} - S_{10}}{12\Delta t}\right]$$

$$= \left[\frac{(0.0 - 0.48)}{2(0.00285)}\right] - \left[\frac{0.13 - 2(0.0) + 2(0.48) - 0.90}{2(0.00285)}\right] = -90.9 \text{ in/sec}$$

The results can be computed and placed into table form (Table 5.5), and used efficiently to perform these redundant calculations. For those who are unfamiliar with them, tables are introduced in Section 7.1.

TABLE 5.5 Table of Piston Displacement for Example Problem 5.16

Data Point Index	Crank Angle (deg)	Time 0.001sec)	Piston Displacement (in)	Piston Velocity $\left(\frac{\text{in}}{\text{sec}}\right)$
1	0	0.00	0.00	0.00
2	30	2.85	0.14	90.9
3	60	5.70	0.48	141.7
4	90	8.55	0.90	137.4
5	120	11.40	1.23	96.4
6	150	14.25	1.44	46.6
7	180	17.10	1.50	0.00
8	210	19.95	1.44	−46.6
9	240	22.80	1.23	−96.4
10	270	25.65	0.90	−137.4
11	300	28.50	0.48	−141.7
12	330	31.35	0.14	−90.9
13	360	34.20	0.00	0.00

These values are plotted in Figure 5.38 to form a velocity diagram relative to time.

Notice that this curve is still rather rough. For accuracy purposes, it is highly suggested that the crank angle increment be reduced to 10° or 15°. When a spreadsheet is used to generate the velocity data, even smaller increments are advisable to reduce the difficulty of the task.

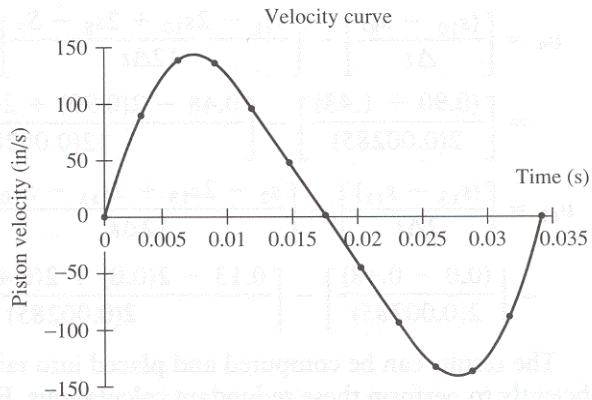


Figure 5.38 Velocity curve for Example Problem 5.16.

PROBLEMS

While manual drafting techniques are instructive for problems that require graphical solution, use of a CAD system is highly recommended.

General Velocity Problems

5–1. A package was moved from one end of a 25-ft horizontal conveyor to the other end in 15 sec. Determine the linear velocity of the conveyor belt.

5–2. A hydraulic cylinder extends at a constant rate of 2 fpm. Determine the time required to transverse the entire stroke of 15 in.

5–3. A gear uniformly rotates 270° in 2 sec. Determine the angular velocity in rpm and rad/s.

5–4. Determine the angular speed (in rpm) of the second-, minute, and hour-hands of a clock.

5–5. The drive roller for a conveyor belt is shown in Figure P5.5. Determine the speed of the roller when the belt operates at 10 fpm.

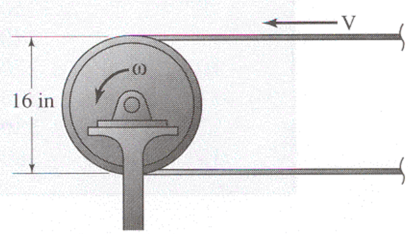


Figure P5.5 Problem 5.

Figure P5.6 shows a kinematic diagram of a four-bar linkage. Graphically determine the velocity of links 2 and 4 and the relative velocity of point B with respect to point A.

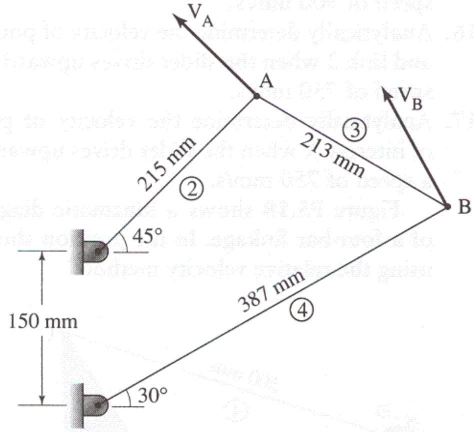


Figure P5.6 Problems 6 and 7.

5–6. Use $V_A = 800 \frac{mm}{s}$ and $V_B = 888 \frac{mm}{s}$

5–7. Use $V_A = 20 \frac{ft}{min}$ and $V_B = 22.2 \frac{ft}{s}$

Figure P5.8 shows a kinematic diagram of a slider-crank linkage. using the following data, graphically determine the velocity of link 2 and the relative velocity of point A with respect to point B.

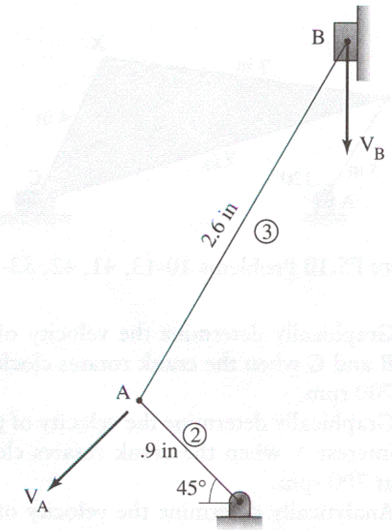


Figure P5.8 Problems 8 and 9.

5–8. Use $V_A = 380 \frac{mm}{s}$ and $V_B = 336.5 \frac{mm}{s}$

5–9. Use $V_A = 20 \frac{ft}{s}$ and $V_B = 17.7 \frac{ft}{s}$

Velocity Analysis Using the Relative Velocity Method

Figure P5.10 shows a kinematic diagram of a slider-crank linkage. In the position shown, using the relative velocity method:

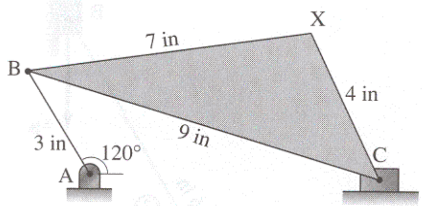


Figure P5.10 Problems 10-13, 41, 42, 53-56.

5–10. Graphically determine the velocity of points *B* and *C* when the crank rotates clockwise at 700 rpm.

5–11. Graphically determine the velocity of point of interest *X* when the crank rotates clockwise at 700 rpm.

5–12. Analytically determine the velocity of points *B* and *C* when the crank is rotating clockwise at 300 rpm.

5–13. Analytically determine the velocity of point of interest *X* when the crank rotates clockwise at 300 rpm.

Figure P5.14 shows a kinematic diagram of a slider-crank linkage. In the position shown, using the relative velocity method:

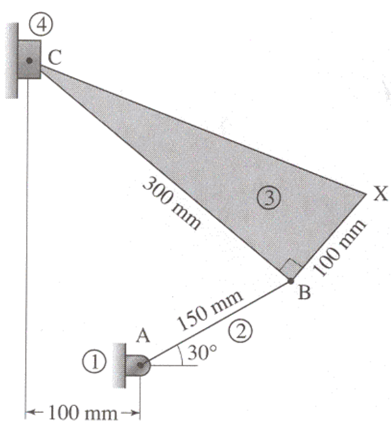


Figure P5.14 Problems 14-17, 43, 44, 57-60.

5–14. Graphically determine the velocity of point *B* and link 2 when the slider drives upward at a speed of 900 mm/s.

5–15. Graphically determine the velocity of point of interest *X* when the slider drives upward at a speed of 900 mm/s.

5–16. Analytically determine the velocity of point *B* and link 2 when the slider drives upward at a speed of 750 mm/s.

5–17. Analytically determine the velocity of point of interest *X* when the slider drives upward at a speed of 750 mm/s.

Figure P5.18 shows a kinematic diagram of a four-bar linkage. In the position shown, using the relative velocity method:

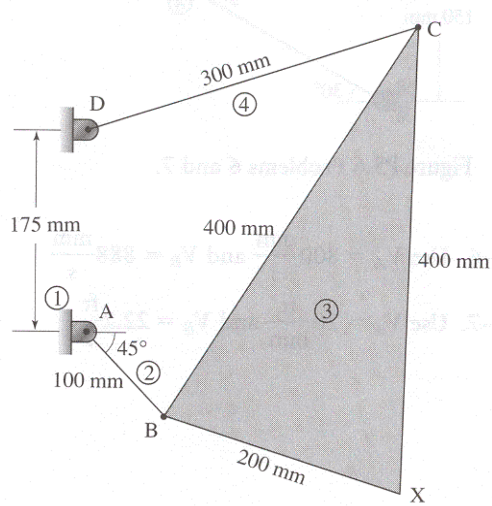


Figure P5.18 Problems 18-21, 45, 46, 61-64.

5–18. Graphically determine the velocity of points *B* and *C*, and link 4 when the 100 mm crank drives counterclockwise at 60 rpm.

5–19. Graphically determine the velocity of point of interest *X* when the 100 mm crank drives counterclockwise at 60 rpm.

5–20. Analytically determine the velocity of points *B* and *C*, and link 4 when the 100 mm crank drives clockwise at 80 rpm.

5–21. For the kinematic diagram of the four-bar mechanism shown in Figure P5.18, using the relative velocity method, analytically determine the velocity of point of interest X when the 100 mm crank drives clockwise at 80 rpm.

Figure P5.22 shows a kinematic diagram of a mechanism. In the position shown, using the relative velocity method:

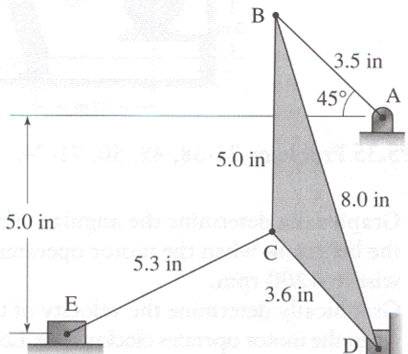


Figure P5.22 Problems 22-25, 47, 48, 65-68.

5–22. Graphically determine the velocity of points B and C when the 3.5 in crank drives clockwise at 300 rpm.

5–23. Graphically determine the velocity of points D and E when the crank drives clockwise at 300 rpm.

5–24. Analytically determine the velocity of points B and C when the crank drives counterclockwise at 960 rpm.

5–25. Graphically determine the velocity of points D and E when the crank drives counterclockwise at 960 rpm.

Figure P5.26 shows a hand-operated metal shear. In the position shown, using the relative velocity method:

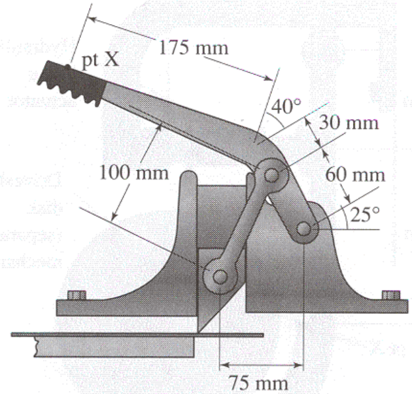


Figure P5.26 Problems 26-30, 67, 68.

5–26. Graphically determine the rotational velocity of the handle when the shear cuts through the metal at a rate of 3 mm/s.

5–27. Graphically determine the velocity of point of interest X when the shear cuts through the metal at a rate of 3 mm/s.

5–28. Analytically determine the velocity of the shear blade when the handle rotates clockwise at a rate of 2 rad/sec.

5–29. Analytically determine the velocity of point of interest X when the handle rotates clockwise at a rate of 2 rad/sec.

5–30. Figure P5.30 shows an industrial braking mechanism. In the position shown, determine the required velocity of the hydraulic actuator when point X must approach the drive-shaft disk at a rate of 0.2 in/sec.

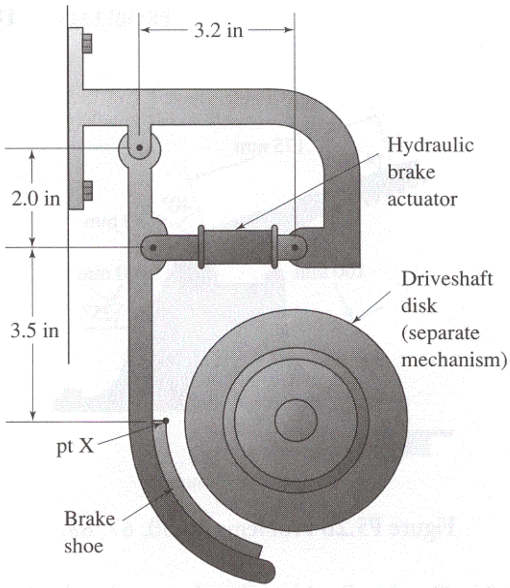


Figure P5.30 Problem 30.

Figure P5.31 shows a foot-operated air pump. In the position shown, using the relative velocity method:

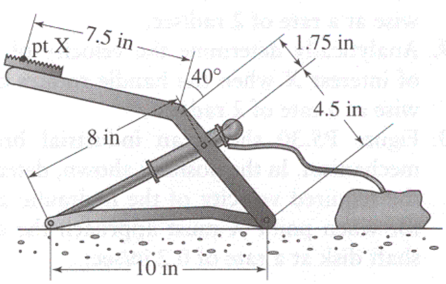


Figure P5.31 Problems 31-34, 69, 70.

5–31. Graphically determine the rotational velocity of the foot petal assembly when the cylinder contracts at a rate of 0.5 in/sec.

5–32. Graphically determine the velocity of point of interest *X* when the cylinder contracts at a rate of 0.5 in/sec.

5–33. Analytically determine the rate of cylinder compression when the rotational velocity of the foot pedal assembly is 1 rad/sec, counterclockwise.

5–34. Analytically determine the velocity of point of interest *X* when the rotational velocity of the foot pedal assembly is 1 rad/sec, counterclockwise.

Figure P5.35 shows a package moving device. In the position shown, using the relative velocity method:

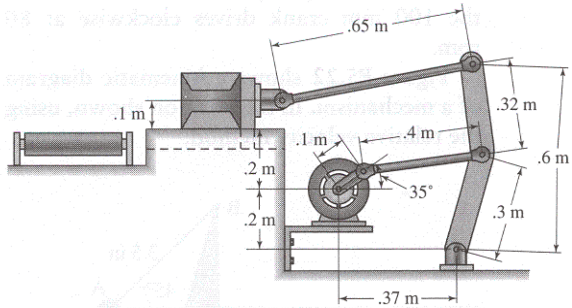


Figure P5.35 Problems 35-38, 49, 50, 71-74.

5–35. Graphically determine the angular velocity of the bellcrank when the motor operates clockwise at 1200 rpm.

5–36. Graphically determine the velocity of the ram when the motor operates clockwise at 1200 rpm.

5–37. Analytically determine the angular velocity of the bellcrank when the motor operates counterclockwise at 900 rpm.

5–38. Determine the velocity of the ram when the motor operates counterclockwise at 900 rpm.

Figure P5.39 shows another package moving device. In the position shown, using the relative velocity method:

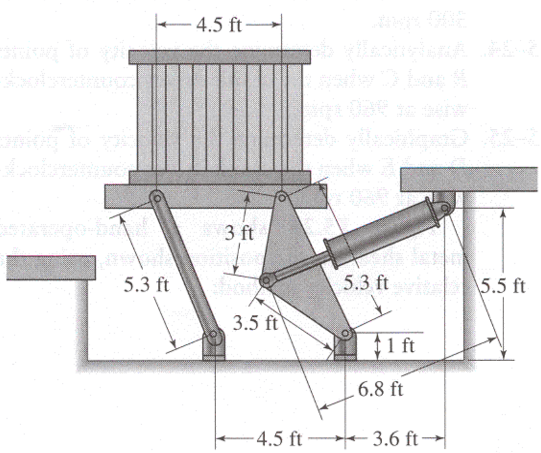


Figure P5.39 Problems 39, 40, 51, 52, 75, 76.

5–39. Graphically determine the velocity platform and package when the hydraulic cylinder extends at a rate of 16 fpm.

5–40. Analytically determine the velocity of the platform and package when the hydraulic cylinder retracts at a rate of 10 fpm.

Locating Instantaneous Centers

Determine the location of all of the instantaneous centers for the following mechanisms.

5–41. The kinematic diagram of the slider-crank mechanism shown in Figure P5.10. Use graphical techniques.

5–42. The kinematic diagram of the slider-crank mechanism shown in Figure P5.10 Use analytical techniques.

5–43. The kinematic diagram of the slider-crank mechanism shown in Figure P5.14 Use graphical techniques.

5–44. The kinematic diagram of the slider-crank mechanism shown in Figure P5.14. Use analytical techniques.

5–45. The kinematic diagram of the four-bar mechanism shown in Figure P5.18. Use graphical techniques.

5–46. The kinematic diagram of the four-bar mechanism shown in Figure P5.18. Use analytical techniques.

5–47. The kinematic diagram of the mechanism shown in Figure P5.22. Use graphical techniques

5–48. The kinematic diagram of the mechanism shown in Figure P5.22. Use analytical techniques.

5–49. The transfer mechanism shown in Figure P5.35. Use graphical techniques.

5–50. The transfer mechanism shown in Figure P5.35. Use analytical techniques.

5–51. The transfer mechanism shown in Figure P5.39. Use graphical techniques.

5–52. The transfer mechanism shown in Figure P5.39. Use analytical techniques.

Velocity Analysis Using the Instantaneous Center Method

For the kinematic diagram of the slider-crank mechanism shown in Figure P5.10, using the instantaneous center method:

5–53. Graphically determine the velocity of points *B* and *C* when the crank rotates counterclockwise at 450 rpm.

5–54. Graphically determine the velocity of point *X* when the crank rotates counterclockwise at 450 rpm.

5–55. Analytically determine the velocity of points *B* and *C* when the crank rotates clockwise at 500 rpm.

5–56. Analytically determine the velocity of point *X* when the crank rotates clockwise at 500 rpm.

For the kinematic diagram of the slider-crank mechanism shown in Figure P5.14, using the instantaneous center method:

5–57. Graphically determine the velocity of point *B* and link 2 when the slider drives downward at a speed of 60 mm/s.

5–58. Graphically determine the velocity of point *X* when the slider drives downward at a speed of 60 mm/s.

5–59. Analytically determine the velocity of point *B* and link 2 when the slider drives upward at a speed of 350 mm/s.

5–60. Analytically determine the velocity of point *X* when the slider drives upward at a speed of 350 mm/s.

For the kinematic diagram of the four-bar mechanism shown in Figure P5.18, using the instantaneous center method:

5–61. Graphically determine the velocity of points *B* and *C*, and link 4 when the crank drives counterclockwise at 60 rpm.

5–62. Graphically determine the velocity of point *X* when the crank drives counterclockwise at 60 rpm.

5–63. Analytically determine the velocity of points *B* and *C*, and link 4 when the crank drives clockwise at 250 rpm.

5–64. Analytically determine the velocity of point *X* when the crank drives clockwise at 250 rpm.

For the kinematic diagram of the mechanism shown in Figure P5.22, using the instantaneous center method:

5–65. Graphically (using either manual drawing techniques or CAD) determine the velocity of points *B*, *C*, *D*, and *E* when the crank drives clockwise at 450 rpm.

5–66. Analytically determine the velocity of points *B*, *C*, *D*, and *E* when the crank drives counterclockwise at 620 rpm.

For the sheet-metal shear mechanism shown in Figure P5.26, using the instantaneous center method:

5–67. Graphically (using either manual drawing techniques or CAD) determine the rotational velocity of the handle, and point *X*, when the shear cuts through the metal at a rate of 32 mm/s.

5–68. Analytically determine the rotational velocity of the handle, and point *X*, when the shear is pulled upward at a rate of 13 mm/s.

For the air pump mechanism shown in Figure P5.31, using the instantaneous center method:

5–69. Graphically determine the rotational velocity of the foot pedal assembly, and point *X*, when the cylinder retracts at a rate of 1.5 in/sec.

5–70. Analytically determine the rotational velocity of the foot pedal assembly, and point *X*, when the cylinder extends at a rate of 2.0 in/sec.

For the package moving device shown in Figure P5.35, using the instantaneous center method:

5–71. Graphically determine the rotational velocity of the bellcrank when the motor operates counterclockwise at 1200 rpm.

5–72. Graphically determine the velocity of the ram and package when the motor operates counterclockwise at 1200 rpm.

5–73. Analytically determine the rotational velocity of the bellcrank when the motor operates clockwise at 530 rpm.

5–74. Analytically determine the velocity ram and package when the motor operates clockwise at 530 rpm.

For the package moving device shown in Figure P5.39, using the instantaneous center method:

5–75. Graphically determine the velocity platform and package when the hydraulic cylinder retracts at a rate of 12 fpm.

5–76. Analytically determine the velocity platform and package when the hydraulic cylinder extends at a rate of 5 fpm.

Velocity Curves

For the mechanism shown in Figure P5.10, construct a velocity diagram for the motion of point C, as the crank rotates:

5–77. At a constant velocity of 20 rad/sec, clockwise. Use graphical techniques.

5–78. At a constant velocity of 30 rad/sec, counterclockwise. Use analytical techniques.

For the mechanism shown in Figure P5.14, graphically construct a velocity diagram for the motion of point C, as the crank rotates:

5–79. At a constant velocity of 60 rad/sec, counterclockwise. Use graphical techniques.

5–80. At a constant velocity of 40 rad/sec, clockwise. Use analytical techniques.

For the mechanism shown in Figure P5.18, construct a velocity diagram for the motion of link 4, as the link 2 rotates:

5–81. At a constant velocity of 60 rpm, clockwise. Use graphical techniques.

5–82. At a constant velocity of 115 rpm, counterclockwise. Use analytical techniques.

5–83. For the mechanism shown in Figure P5.35, graphically construct a velocity diagram for the package and ram, as link 2 rotates at a constant velocity of 600 rpm, clockwise.

Velocity Problems Using Working Model

Use the Working Model software obtainable with the text to create a model and provide the desired output for the following mechanisms.

5–84. Using the mechanism shown in Figure P5.10, create a velocity diagram for the motion of point C as the crank rotates at a constant velocity of 20 rad/sec clockwise.

5–85. Using the mechanism shown in P5.14, create a velocity diagram for the motion of point *C* as the crank rotates at a constant velocity of 60 rad/sec counterclockwise.

5–86. Using the mechanism shown in Figure P5.18, create a velocity diagram for the motion of link 4 as link 2 rotates at a constant velocity of 60 rpm clockwise.

5–87. Using the mechanism shown in Figure P5.35 create a velocity diagram for the package and ram as link 2 rotates at a constant velocity of 600 rpm clockwise.

CASE STUDIES

C5–1 Figure C5.1 illustrates a mechanism that is used to drive a power hack saw. The mechanism is powered with an electric motor shaft, keyed to gear *A*. Carefully examine the configuration in question then answer te following leading questions to gain insight into the operation of the mechanism.

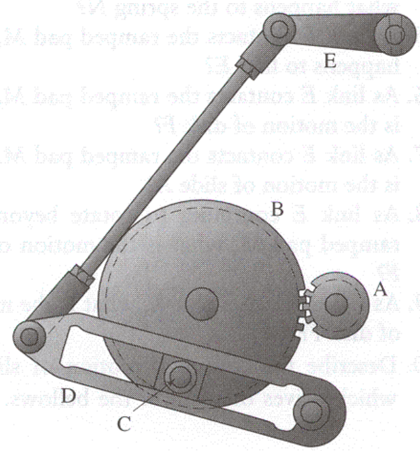


Figure C5.1 (Courtesy, Industrial Press)

1. When gear *A* is forced to rotate counterclockwise, what is the motion of mating gear *B*?
2. When gear *A* is forced to rotate counterclockwise, what is the motion stud pin *C*?
3. When gear *A* is forced to rotate counterclockwise, what is the motion of lever *D*?
4. How does the motion of lever *D* differ from the motion of lever *E*?
5. Determine the position of gear *B* that would place lever *D* at its lower extreme position.
6. Determine the position of gear *B* that would place lever *D* at its upper extreme position.
7. Examine the amount of rotation of gear *B* to raise lever *D* and the amount of rotation to lower the lever.
8. Approximately, what is the difference between the time to raise and the time to lower lever *D*?
9. Comment on the continual motion of lever *E*.

C5–2 Figure C5.2 illustrates the mechanism that drives a table for a special grinding operation. Carefully examine the configuration in question then answer te following leading questions to gain insight into the operation of the mechanism.

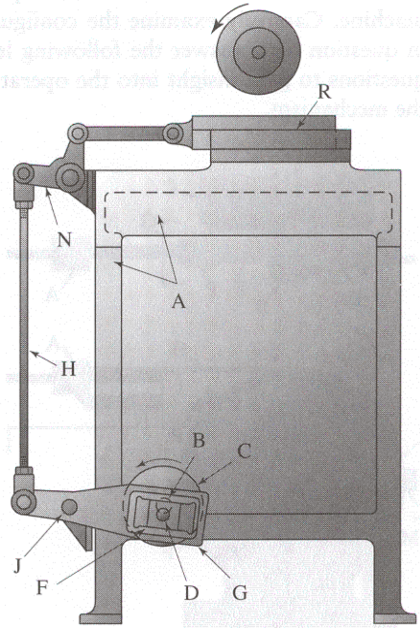


Figure C5.2 (Courtesy, Industrial Press)

1. When wheel *C* is forced to rotate counterclockwise, what is the motion of pin *D*?
2. When wheel *C* is forced to rotate counterclockwise, what is the motion of link *G*?
3. Determine the position of wheel *C* that would place link *G* at its upper extreme position.
4. Determine the position of wheel *C* that would place link *G* at its upper extreme position.
5. Examine the amount of rotation of wheel *C* to raise link *G* and the amount of rotation to lower the link.
6. Approximately, what is the difference between the time to raise and the time to lower link *G*?

7. Comment on the cyclical motion of lever *E*.
8. Describe the motion of table *R*.
9. What is the function of this mechanism?
10. Why are there screw threads on both ends of link *H*?
11. Compute the mobility of this mechanism.

C5–3 Figure C5.3 illustrates the mechanism that drives a bellows for an artificial respiration machine. Carefully examine the configuration in question then answer the following leading questions to gain insight into the operation of the mechanism.

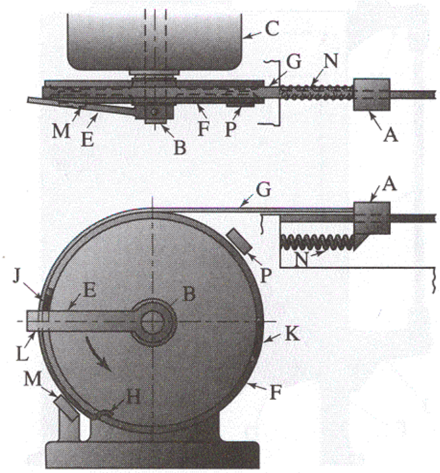


Figure C5.3 (Courtesy, Industrial Press)

1. When link *E* drives continually counterclockwise, and rides slot *J* at the instant shown, what is the motion of disk *F*?
2. When link *E* drives continually counterclockwise, and rides slot *J*, at the instant shown, what is the motion of strap *G*?
3. When link *E* drives continually counterclockwise, and rides slot *J*, at the instant shown, what is the motion of slide *A*?
4. As link *E* approaches the ramped pad *M*, what happens to the spring *N*?
5. As link *E* contacts the ramped pad *M*, what happens to link *E*?
6. As link *E* contacts the ramped pad *M*, what is the motion of disk *F*?
7. As link *E* contacts the ramped pad *M*, what is the motion of slide *A*?
8. As link *E* continues to rotate beyond the ramped pad M, what is the motion of disk *F*?
9. As link *E* catches slot *K*, what is the motion of disk *F*?
10. Describe the continual motion of slide *A*, which drives one end of the bellows.

6 Acceleration Analysis

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Define linear, rotational, normal, tangential, corriolis, and relative accelerations.
2. Using the relative acceleration method, graphically solve for the acceleration of a point on a link, knowing the acceleration of another point on that link.
3. Using the relative acceleration method, graphically determine the acceleration of a point of interest on a floating link.
4. Understand when the Coriolis acceleration is present, and include it in the analysis.
5. Using the relative acceleration method, analytically solve for the acceleration of a point.
6. Using the relative acceleration method, analytically determine the acceleration of a point of interest on a floating link.
7. Construct an acceleration curve to locate extreme acceleration values.

6.1 INTRODUCTION

Acceleration analysis involves determining the amount that certain points on the links of a mechanism are either "speeding up" or "slowing down." The amount of acceleration is a critical property because of the inertial forces associated with it. In the study of forces, Sir Isaac Newton discovered that an inertial force is proportional to the acceleration imposed on a body.

An important part of mechanical design is to ensure that the strength of the links and joints is sufficient to withstand the forces imposed on them. Understanding all forces is important, especially the inertial force. Force analysis is introduced in Chapter 9. However, as a preliminary step, acceleration analysis of a mechanism's links is crucial.

The determination of accelerations in a linkage is the purpose of this chapter. The primary procedure used in this analysis is the relative acceleration velocity method, which utilizes the results of the relative velocity method introduced in Chapter 5. Consistent with other chapters in this book, both graphical and analytical techniques are utilized.

6.2 LINEAR ACCELERATION OF A POINT MOVING IN A STRAIGHT LINE

Linear acceleration of a point is the change of linear velocity of that point per unit of time. Chapter 5 was dedicated to velocity analysis. Because velocity is a vector quantity, and defined with both a magnitude and a direction, a change in either constitutes acceleration.

Consider the case of a point having straight line motion. Such a point is most commonly found on a link that is attached to the frame with a sliding joint. For this case, only the magnitude of the velocity vector can change. The acceleration can be mathematically defined as:

$$a = \frac{dv}{dt} \tag{6.1}$$

However, because:

$$v = \frac{ds}{dt}$$

then:

$$a = \frac{d^2s}{dt^2} \tag{6.2}$$

For short time periods, or when the acceleration can be assumed to be linear, the following relationship can be used:

$$a \cong \frac{\Delta v}{\Delta t} \tag{6.3}$$

Because velocity is a vector, Equation 6.1 states that acceleration is also a vector. The direction of linear acceleration is in the direction of linear movement when the link accelerates. Conversely, when the link decelerates, the direction of linear acceleration is opposite to the direction of linear movement.

Linear acceleration is expressed in the units of velocity (length per time) divided by time, or length per squared time. In the United States Customary System, the common units used are feet per squared second (ft/s^2) or inches per squared second (in/s^2). In the International System, the common units used are meters per squared second (m/s^2) or millimeters per squared second (mm/s^2).

By rewriting Equation 6.3, the velocity change that occurs during a period of constant acceleration is expressed as follows:

$$\Delta v = v_{\text{final}} - v_{\text{initial}} = a\Delta t \tag{6.4}$$

Additionally, the corresponding displacement that occurs during a period of constant acceleration can be written as:

$$s = \frac{1}{2}a\,\Delta t^2 + v_{\text{initial}}\Delta t \tag{6.5}$$

EXAMPLE PROBLEM 6.1

An express elevator used in tall buildings can reach a full speed of 15 mph in 3 sec. Assuming that the elevator experiences constant acceleration, determine the acceleration and the displacement during the 3 sec.

Solution:

Assuming that the acceleration is constant, Equation 6.3 can be accurately used. Because the elevator starts at rest, the equation begins as follows:

$$\Delta v = (15 \text{ mph} - 0) = 15 \text{ mph}$$

$$= \left(\frac{15 \text{ miles}}{\text{hour}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mile}}\right)\left(\frac{1 \text{ hour}}{3600 \text{ seconds}}\right) = 22\frac{\text{ft}}{\text{sec}}$$

The acceleration then is calculated as:

$$a' = \frac{\Delta v}{\Delta t} = \frac{(22\text{ft/sec})}{3 \text{ sec}} = 7.3\frac{\text{ft}}{\text{sec}^2}\uparrow$$

When people accelerate in an elevator, the acceleration often "normalizes" relative to the acceleration due to gravity. The standard acceleration due to gravity (g) on earth is 32.1740 ft/sec^2 or 9.80665 m/sec^2. Therefore, the acceleration of the elevator can be expressed as:

$$a = 7.3\frac{\text{ft}}{\text{sec}^2}\left(\frac{1\text{g}}{32.2\text{ft/s}^2}\right) = 0.22 \text{ g}$$

The displacement can be determined from Equation 6.5:

$$s = \frac{1}{2}a\,\Delta t^2 + v_{\text{initial}}\Delta t = \frac{1}{2}\left(7.3\frac{\text{ft}}{\text{sec}^2}\right)(3 \text{ sec})^2 + (0)(3 \text{ sec})$$

$$= 30.5 \text{ ft}\uparrow \text{ (or roughly 3 floors)}$$

6.3 LINEAR ACCELERATION OF A POINT IN GENERAL MOTION

The velocity of a point, moving in a general fashion, may change in two ways:

1. The magnitude of the velocity can change. This produces an acceleration acting along the path of motion, as presented in the previous section. This acceleration is termed *tangential acceleration.*
2. The direction of the velocity vector can change over time. This occurs as the link, with which the point is associated, undergoes rotational motion. It produces a centrifugal acceleration that acts perpendicular to the direction of the path of motion. This acceleration is termed *normal acceleration.*

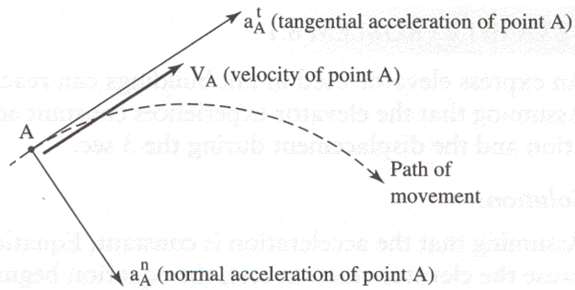


Figure 6.1 Acceleration of point *A*.

Figure 6.1 illustrates point *A* which is accelerating along a curved path. The tangential acceleration of point *A*, $a^t{}_A$, is the linear acceleration along the direction of motion. Note that the vector points in the direction of motion because point *A* is accelerating. If point *A* were decelerating, the vector would point opposite to the direction of movement. Of course, the velocity vector always points in the direction of motion. The magnitude of tangential acceleration can be determined using Equations 6.1, 6.2, or 6.3.

The normal acceleration of point *A*, $a^n{}_A$, is a result of a change in the direction of the velocity vector. It acts along a line that is perpendicular to the direction of movement, and toward the center of curvature of this path. The magnitude of the normal acceleration is presented in Section 6.5.

6.4 ACCELERATION OF A LINK

Recall from Section 5.3 that any motion, however complex, can be viewed as a combination of a straight line movement and a rotational movement. Fully describing the motion of a link can consist of identification of the linear motion of one point and the rotational motion of the link about that point. As with velocity, as several points on a link can have different accelerations, the entire link has the same rotational acceleration.

Rotational acceleration of a link is the rotational (or angular) velocity of that link per unit of time. Mathematically, rotational acceleration of a link is defined as:

(6.6)
$$\alpha = \frac{d\omega}{dt}$$

However, because:

$$\omega = \frac{d\theta}{dt}$$

then:

(6.7)
$$\alpha = \frac{d^2\theta}{dt^2}$$

For short time periods, or when the angular acceleration can be assumed to be linear, the following relationship can be used:

(6.8) $$\alpha \cong \frac{\Delta\omega}{\Delta t}$$

Because rotational velocity is a vector, Equation 6.6 states that rotational acceleration is also a vector. Similarly to the discussion in Section 6.2, the direction of rotational acceleration is in the direction of motion when the rotational velocity increases or the link accelerates. Conversely, the rotational acceleration is in the opposite direction of motion when the rotational velocity decreases, or the link is decreasing. The direction can be easily described as either clockwise or counterclockwise.

Angular acceleration is expressed in the units of angular velocity (angle per time) divided by time, or angle per squared time. In the both the United States Customary System and the International System, the common units used are degrees per squared second (deg/s^2), revolutions per squared second (rev/s^2), or the preferred unit of radians per squared second (rad/s^2).

By rewriting Equation 6.8, the angular velocity change that occurs during a period of constant acceleration is:

(6.9) $$\Delta\omega = \omega_{\text{final}} - \omega_{\text{initial}} = \alpha\Delta t$$

Additionally, the corresponding angular displacement that occurs during a period of constant angular acceleration can be written as:

(6.10) $$\Delta\theta = \frac{1}{2}\alpha\Delta t^2 + \omega_{\text{initial}}\Delta t$$

EXAMPLE PROBLEM 6.2

An electric motor which drives the grinding wheel shown in Figure 6.2 speeds up to 1800 rpm in 2 sec when the power is turned on. Assuming that this speed-up is at a constant rate, determine the rotational acceleration of the grinding wheel. Also determine the number of revolutions that the wheel spins before it is at full speed.

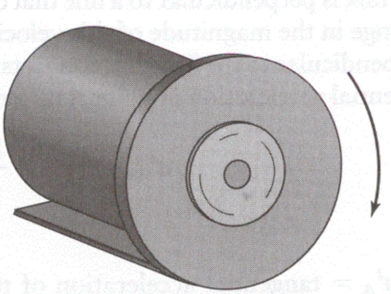


Figure 6.2 Grinding wheel for Example Problem 6.2.

Solution:

Convert the speed of the grinding wheel to rad/sec with the following:

$$1800 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 188.5\frac{\text{rad}}{\text{sec}}$$

With constant acceleration, Equation 6.9 can be used giving:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{(188.5\text{rad/sec} - 0)}{2 \text{ sec}} = 94.2\frac{\text{rad}}{\text{sec}^2}$$

The direction of the acceleration is clockwise, which is in the direction of motion since the grinding wheel is speeding up. The number of revolutions during this speeding up period can be determined through Equation 6.10:

$$\Delta\theta = \frac{1}{2}\alpha\Delta t^2 + \omega_{\text{initial}}\Delta t = \frac{1}{2}\left(94.2\frac{\text{rad}}{\text{sec}^2}\right)(2 \text{ sec})^2 + (0)(2 \text{ sec})$$

$$= 188.4 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 30 \text{ revolutions}$$

6.5 NORMAL AND TANGENTIAL ACCELERATION

As presented in Section 6.3, the velocity of a point moving in a general path can change in two independent ways. The magnitude or the direction of the velocity vector can change over time. Of course, acceleration is the time rate of velocity change. Thus, acceleration is commonly separated into two elements: normal and tangential components. The normal component is created as a result of a change in the direction of the velocity vector. The tangential component is formed as a result of a change in the magnitude of the velocity vector.

For point on a rotating link, little effort is required to determine the direction of these acceleration components. Recall that the instantaneous velocity of a point on a rotating link is perpendicular to a line that connects that point to the center of rotation. Any change in the magnitude of this velocity creates tangential acceleration, which is also perpendicular to the line that connects the point with the center of rotation. Thus, the tangential acceleration of point A on a rotating link 2 can be expressed as:

(6.11) $$a^t{}_A = \frac{dv_a}{dt} = \frac{d(\omega_2 r_A)}{dt} = r_A\alpha$$

where:

$a^t{}_A$ = tangential acceleration of the point of interest, A.

v_A = linear velocity of the point of interest, A.

r_A = distance from the center of rotation to the point of interest

ω_2 = angular velocity of the rotating link, 2

α_2 = angular acceleration of the rotating link, 2

It is extremely important to remember that the angular acceleration, α, in Equation 6.11 must be expressed as units of radians per squared time. Radians per squared second is the most common unit. Similarly to the discussion in Section 6.2, tangential acceleration occurs in the direction of motion when the velocity increases or the point accelerates. Conversely, tangential acceleration occurs in the opposite direction of motion when the velocity decreases or the point decelerates.

Any change in velocity direction creates normal acceleration, which is always directed towards the center of rotation. This occurs because, as the point rotates around a fixed pivot, the velocity vector changes along the curvature of motion. Thus, the change in direction will be normal to this curvature and always is directed toward the fixed pivot. The normal acceleration of point A on rotating link 2 can be expressed as:

(6.12)
$$a^n{}_A = v_A \omega_2 = r\omega_2{}^2 = \frac{v_A{}^2}{r}$$

where in addition to the above notation:

$a^n{}_A$ = normal acceleration of the point of interest, A.

As previously mentioned, acceleration analysis is important because inertial forces result from accelerations. These loads must be determined to ensure that the machine is adequately designed to handle these dynamic loads. Inertial forces are proportional to the total acceleration of a body. The *total acceleration* is the vector resultant of the tangential and normal components, as discussed above. Mathematically, it is expressed as follows:

(6.13)
$$a_A = \sqrt{(a^t{}_A)^2 + (a^n{}_A)^2}$$

EXAMPLE PROBLEM 6.3

The mechanism shown in Figure 6.3 is used in a distribution center to push boxes along a platform and to a loading area. The input link is driven by an electric motor

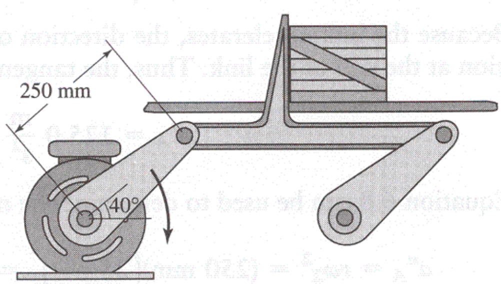


Figure 6.3 Transfer mechanism for Example Problem 6.3.

which, at the instant shown, has a velocity of 25 rad/sec and accelerates at a rate of 500 rad/sec^2. Knowing that the input link has a length of 250 mm, determine the instantaneous acceleration of the end of the input link in the position shown.

Solution:

The kinematic diagram for the transfer mechanism is shown as Figure 6.4A. Notice that it is the familiar four-bar mechanism.

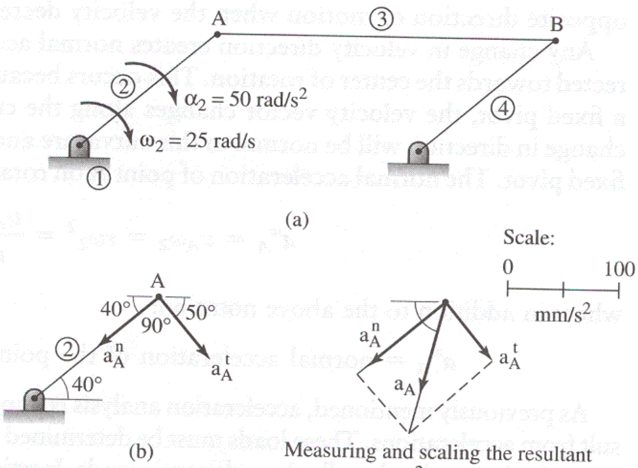


Figure 6.4 Diagrams for Example Problem 6.3.

Because the input link (link 2) is in pure rotation, the acceleration components of the end of the link can be readily obtained. Equation 6.11 can be used to determine the tangential component of acceleration:

$$a^t{}_A = r\alpha_2 = (250\text{ mm})\left(500\frac{\text{rad}}{\text{sec}^2}\right) = 125000\frac{\text{mm}}{\text{s}^2} = 125.0\frac{\text{m}}{\text{s}^2}$$

Because the link accelerates, the direction of the vector is in the direction of the motion at the end of the link. Thus, the tangential acceleration is:

$$a^t{}_A = 125.0\ \frac{\text{m}}{\text{s}^2} \searrow 50°$$

Equation 6.8 can be used to determine the normal component of acceleration:

$$a^n{}_A = r\omega_2{}^2 = (250\text{ mm})\left(25\frac{\text{rad}}{\text{sec}}\right)^2 = 156250\frac{\text{mm}}{\text{s}^2} = 156.25\ \frac{\text{m}}{\text{s}^2}$$

Normal acceleration always occurs toward the center of rotation. Thus, normal acceleration is calculated as follows:

$$a^n{}_A = 156.25\frac{\text{m}}{\text{s}^2} \swarrow\!\!\!\!\nearrow 40°$$

The components of the acceleration are shown in Figure 6.4B. The total acceleration can be found from Equation 6.9:

$$a_A = \sqrt{(a^n{}_A)^2 + (a^t{}_A)^2}$$
$$= \sqrt{(125.0\text{m/s}^2)^2 + (156.25\text{m/s}^2)^2} = 200.10\text{m/s}^2$$

The angle of the total acceleration vector from the normal component can be calculated as:

$$\tan^{-1}\left(\frac{a^t{}_A}{a^n{}_A}\right) = \tan\left(\frac{125.0\frac{\text{m}}{\text{s}^2}}{156.25\frac{\text{m}}{\text{s}^2}}\right) = 38.7°$$

Therefore, the direction of the total acceleration vector from the horizontal axis is as follows:

$$40.0° + 38.7° = 78.7°$$

Formally, the total acceleration can then be written as:

$$a_A = 200.10\frac{\text{mm}}{\text{s}^2} \swarrow\!\!\!\!\nearrow 78.7°$$

The total acceleration can also be determined through a graphical procedure using either CAD or traditional drawing techniques, as explained in Chapter 3. This method is illustrated in Figure 6.4C.

6.6 RELATIVE ACCELERATION

As discussed in detail in Chapter 5, the difference between the motion of two points is termed *relative motion. Relative velocity* is defined as the velocity of one object as observed from another reference object, which is also moving. Likewise, *relative acceleration* is the acceleration of one object as observed from another reference object, which is also moving.

As with velocity, the following notation is used to distinguish between absolute and relative accelerations:

a_A = absolute acceleration (total) of point A

a_B = absolute acceleration (total) of point B

$a_{B/A}$ = relative acceleration (total) of point B with respect to A

= acceleration (total) of point B "as observed" from point A

From Equation 5.9, the relationship between absolute velocity and relative velocity is written as:

$$v_B = v_A +> v_{B/A}$$

Taking time derivative of the entire Equation yields the relationship for absolute and relative accelerations. This can be written mathematically as:

(6.13) $$a_B = a_A +> a_{B/A}$$

For points that are associated with rotating links, it is more convenient to separate the total accelerations in Equation 6.10 into normal and tangential components. Thus, each acceleration is separated into its two components yielding the following:

(6.14) $$a_B^{\,n} +> a_B^{\,t} = a_A^{\,n} +> a_A^{\,t} +> a_{B/A}^{\,n} +> a_{B/A}^{\,t}$$

Note that 6.13 and 6.14 are vector equations and the techniques discussed in Section 3.16 must be used in dealing with these equations.

EXAMPLE PROBLEM 6.4

Figure 6.5 shows a power hacksaw. At this instant, the electric motor rotates counterclockwise and drives the free end of the motor crank (point B) at a velocity of 12 in/s. The crank accelerates at a rate of 37 rad/s^2. The top portion of the hacksaw also moves toward the left with a velocity of 9.8 in/s and accelerates at a rate of 82 in/s^2. Determine the relative acceleration of point C relative to point B.

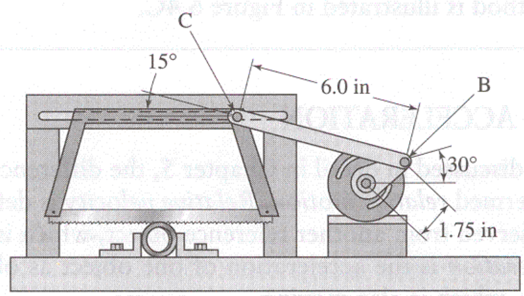


Figure 6.5 Power saw for Example Problem 6.4.

Solution:

Figure 6.6A shows the kinematic diagram of the power hacksaw. Notice that it is the familiar slider-crank mechanism.

From the kinematic diagram, it should be apparent that point B travels up and to the left as link 2 rotates counterclockwise. Because the motor crank (link 2) is in pure rotation, the components of the acceleration at the end of the link can be read-

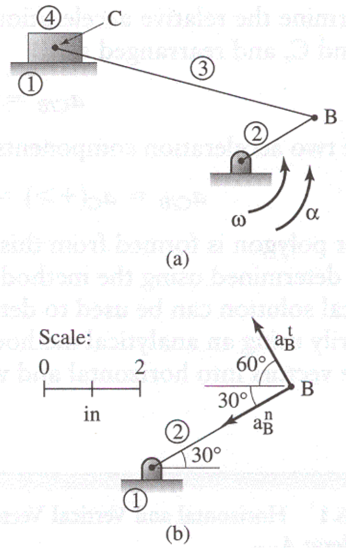


Figure 6.6 Kinematic diagram for Example Problem 6.4.

ily obtained. Equation 6.11 can be used to determine the tangential component of acceleration:

$$a^t{}_B = r\alpha_2 = (1.75 \text{ in})\left(37 \frac{\text{rad}}{\text{sec}^2}\right) = 64.75\frac{\text{in}}{\text{sec}^2}$$

Because the link accelerates, the direction of the vector is in the direction of the motion at the end of the link. Thus, the tangential acceleration is calculated as follows:

$$a^t{}_B = 64.75 \frac{\text{in}}{\text{sec}^2} \nwarrow 60°$$

Equation 6.8 can be used to determine the normal component of acceleration:

$$a^n{}_B = \frac{v_B{}^2}{r_B} = \frac{(12\text{in/s}^2)}{1.75\text{in}} = 82.3\frac{\text{in}}{\text{s}^2}$$

Normal acceleration is always directed toward the center of rotation. Thus, normal acceleration is:

$$a^n{}_B = 82.29 \frac{\text{in}}{\text{sec}^2} \swarrow 30°$$

Link 2 is isolated and the components of this acceleration are shown in Figure 6.6B. The motion of point C is strictly linear. Therefore, no normal acceleration occurs. The total acceleration is given in the problem statement as:

$$a_C = 82 \frac{\text{in}}{\text{s}^2} \leftarrow$$

To determine the relative acceleration, Equation 6.13 can be written in terms of points B and C, and rearranged as:

$$a_{C/B} = a_C -> a_B$$

Because two acceleration components of point B exist, the equation is written as:

$$a_{C/B} = a_C(+>) = a_C -> a_B{}^n -> a_B{}^t$$

A vector polygon is formed from this equation (Figure 6.6C). The unknown vector can be determined using the methods presented in Chapter 3. Either a graphical or analytical solution can be used to determine vector $a_{C/B}$.

Arbitrarily using an analytical method, the acceleration $a_{C/B}$ can be found by separating the vectors into horizontal and vertical components. See Table 6.1

TABLE 6.1 Horizontal and Vertical Vector Components for Accelerat $A_{C/B}$

Vector	Reference Angle (θ_x)	Horizontal Component $a_h = a \cos \theta_x$	Vertical Component $a_v = a \sin \theta_x$
a_c	180°	−82.00	0
$a_B{}^n$	210°	−71.26	−41.15
$a_B{}^t$	120°	−32.83	56.08

Separate algebraic equations can be written for the horizontal and vertical components as follows:

$$a_{C/B} = a_C -> a_B{}^n -> a_B{}^t$$

$$a_{C/B} - (\text{horizontal comp}) = (-82.0) - (-71.27) - (-32.38)$$

$$= +21.35 = 21.35\frac{\text{in}}{\text{sec}^2}$$

$$a_{C/B} - (\text{vertical comp}) = (0) - (-41.15) - (+56.08) = -14.93\frac{\text{in}}{\text{sec}^2}$$

The magnitude of the acceleration can be found by:

$$\text{a}_{C/B} = \sqrt{(\text{a}_{C/B} - \text{horiz comp})^2 + (a_{C/B} - \text{vert comp})^2}$$

$$= \sqrt{(21.35)^2 + (-14.93^2} = 26.05\text{in/s}^2$$

The direction of the vector can be determined by the following:

$$\theta_x = \tan^{-1}\left\{\frac{(a_{c/b} - \text{vert. comp.})}{a_{c/b} - \text{horiz. comp.})}\right\} = \tan^{-1}\left\{\frac{-14.93\text{in/s}^2}{21.35\text{in/s}^2}\right\} = -35°$$

Finally, the relative acceleration of *C* with respect to *B* is:

$$a_{C/B} = 26.05 \frac{\text{in}}{\text{s}^2} \nwarrow 35°$$

6.7 COMPONENTS OF RELATIVE ACCELERATION

The accelerations of a mechanism can be much more easily analyzed when separated into normal and tangential components. For links that are attached directly to the frame, the direction of the acceleration components are obvious as described in the previous section. The normal component is always directed to the center of rotation and the tangential component is perpendicular to the normal component and in the direction that is consistent with either the acceleration or deceleration of the point. Recall that tangential acceleration is in the direction of motion when the point accelerates. Conversely, tangential acceleration is opposite to the direction of motion when the point decelerates.

For points that are on the same link, a link that is not directly attached to the frame, the analysis focuses on the relative accelerations of these points. Figure 6.7 shows such a link that is not directly attached to the frame, thus demonstrating a floating link. The relative acceleration of two points, which reside on that link, is shown. Notice that the normal and tangential components of this acceleration are also shown. Stated again, the relative acceleration of two points is the acceleration of one point as seen from the other reference point.

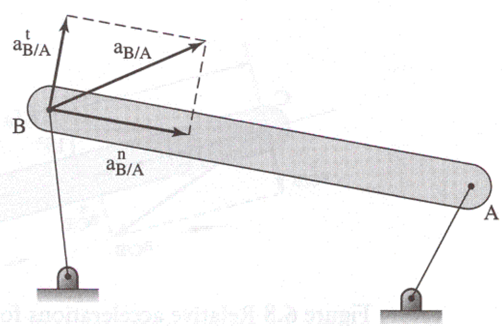


Figure 6.7 Relative normal and tangential accelerations.

As with velocity analysis, relative motion consists of pure relative rotation of the observed point about the reference point. In other terms, the relative motion of *B* with respect to *A* is analyzed as if point *B* is rotating around point *A*. Thus, a normal component of relative acceleration is directed toward the center of relative rotation, or the reference point. The tangential relative acceleration is directed perpendicular to the normal relative acceleration. The magnitude of these components is computed in a similar fashion to the absolute acceleration of points rotating around fixed points:

(6.15) $$a^t_{B/A} = \frac{dv_{B/A}}{dt} = \frac{d(\omega_3 r_{BA})}{dt} = r_{BA}\alpha_3$$

(6.16) $$a_{B/A} = r_{BA}\omega^2 = \frac{v_{B/A}^2}{r_{BA}}$$

The direction of the relative tangential acceleration is consistent with the angular acceleration of the floating link, and vice versa. Referring to Figure 6.7, the relative tangential acceleration shows the tangential acceleration of point B, as it rotates around point A, which infers a clockwise angular accretion of link 3.

EXAMPLE PROBLEM 6.5

For the power hacksaw in Example Problem 6.3, determine the angular acceleration of the 6 in connecting link (link 3).

Solution:

The relative acceleration of C with respect to B was determined as follows:

$$a_{C/B} = 26.05 \frac{\text{in}}{\text{s}^2} \searrow 35°$$

Also note from Figure 6.5 that the connecting link is inclined at a 15° angle. Using this data, the total relative acceleration can be resolved into normal and tangential components. These components are shown in Figure 6.8.

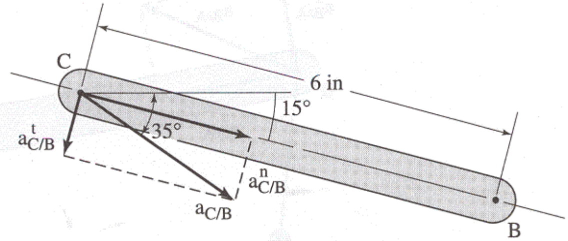


Figure 6.8 Relative accelerations for Example Problem 6.5.

Figure 6.8 illustrates that 20° (35° − 15°) separates the total relative acceleration and the direction of the normal component. Thus, the magnitudes of the relative acceleration components can be analytically determined from the following trigonometric relationships:

$$a^t_{C/B} = a_{C/B}(\sin 20°) = 26.05\frac{\text{in}}{\text{sec}^2}(\sin 20°) = 8.91\frac{\text{in}}{\text{sec}^2}$$

$$a^n_{C/B} = a_{C/B}(\cos 20°) = 26.05\frac{\text{in}}{\text{sec}^2}(\cos 20°) = 24.48\frac{\text{in}}{\text{sec}^2}$$

From Figure 6.8, the tangential component of the relative acceleration infers that the angular acceleration of link 3 is counterclockwise. The magnitude can be determined as follows:

$$\alpha_3 = \frac{a^t_{C/B}}{r_{CB}} = \frac{8.91 \text{in/s}^2}{6 \text{ in}} = 1.49 \frac{\text{rad}}{\text{s}^2}$$

Therefore, the angular acceleration of the connecting link is determined by:

$$\alpha_3 = 1.49 \frac{\text{rad}}{\text{sec}^2} \text{ counterclockwise}$$

6.8 RELATIVE ACCELERATION ANALYSIS: GRAPHICAL METHOD

Acceleration analysis is primarily incorporated to determine the acceleration of several points on a mechanism, in a single configuration. It must be understood that the results of this analysis yield instantaneous motion characteristics and, as the mechanism moves—even an infinitesimal amount—the motion characteristics change. Nonetheless, the instantaneous characteristics are needed, particularly the extreme values. It was emphasized earlier that accelerations impose inertial forces through the links of a mechanism. The resulting stresses must be fully understood to ensure safe operation of a machine.

The strategy for determining the acceleration of a point involves knowing the acceleration of another point on that same link. In addition, the velocity of the desired point and the relative velocity between the two points must be known. This information must be found using the relative velocity methods as described in Chapter 5.

Analysis can proceed around a mechanism by using points that are common to two links. For example, a point that occurs on a joint is common to two links. Therefore, determining the velocity of this point enables one to subsequently determine the velocity of a point on either link it joins. In this manner, the acceleration of any point on a mechanism can be determined by working outward from the input link.

Recall from Equation 6.10 that the relative acceleration equation can be expanded to include the normal and tangential components:

$$a_B{}^n +> a_B{}^t = a_A{}^n +> a_A{}^t +> a_{B/A}{}^n +> a_{B/A}{}^t$$

Assume that the acceleration of point B needs to be determined and the acceleration of point A is already known. In a typical situation, the direction of all six components are known. All normal components are directed toward the center of or relative rotation. All tangential components are perpendicular to the normal components. In addition, the magnitudes of all the normal acceleration vectors can be found from Equations 6.12 or 6.15. Of course, the magnitude of the tangential acceleration of the known point, point A, is also known. Therefore, the analysis determines only the magnitude of the tangential component of the point desired, and the magnitude of the relative tangential component.

Relative acceleration analysis forms a vector problem identical to the general problems presented in Section 3.9. Both graphical and analytical solutions are feasible, as seen in Chapter 3. In many problems, the magnitude of certain terms may be zero, eliminating some of the six vector components in Equation 6.10. For example, when the known point is at a joint that is common to a constant angular velocity link, the point has no tangential acceleration. Another example occurs when a point is common to a link that is restricted to linear motion. This point does not rotate and has no normal acceleration.

As in velocity analysis, the graphical solution of the vector polygons can be completed using manual drawing techniques or on a CAD system. The logic is identical; however, the CAD solution is not susceptible to limitations of drafting accuracy. Regardless of the method being practiced, the underlying concepts of graphical position analysis can be further illustrated and expanded through the Example Problems below.

EXAMPLE PROBLEM 6.6

The mechanism shown in Figure 6.9 is designed to move parts along a conveyor tray then rotate and lower those parts to another conveyor. The driving wheel rotates with a constant angular velocity of 120 rpm. Determine the angular acceleration of the rocker arm that rotates and lowers the parts.

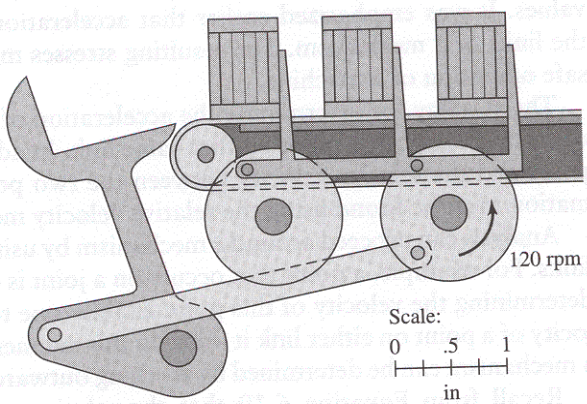
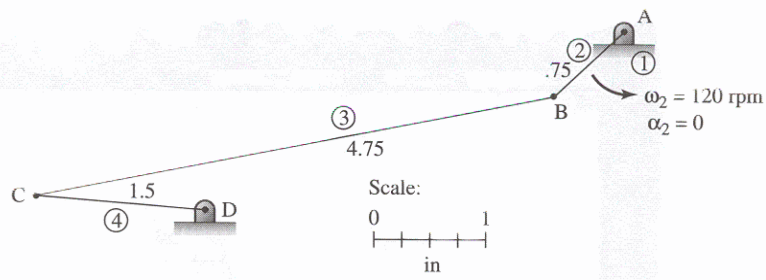


Figure 6.9 Mechanism for Example Problem 6.6.

Solution:

The portion of the mechanism that is under consideration includes the driving wheel, the follower arm, and the link that connects the two. Notice that, once again, this is the common four-bar mechanism. A scaled, kinematic diagram is shown in Figure 6.10A.

The angular acceleration of the rocker (link 4) can be determined from the tangential component of the acceleration of point C. Thus, the crux of the problem is to determine the acceleration of point C. In turn, the acceleration of point C, which also resides on link 3, can be determined from knowing the acceleration of point B. Point B is positioned on both links 2 and 3. Therefore, the acceleration of point B can be determined from knowing the motion of the input link, link 2.



(a)

(b)

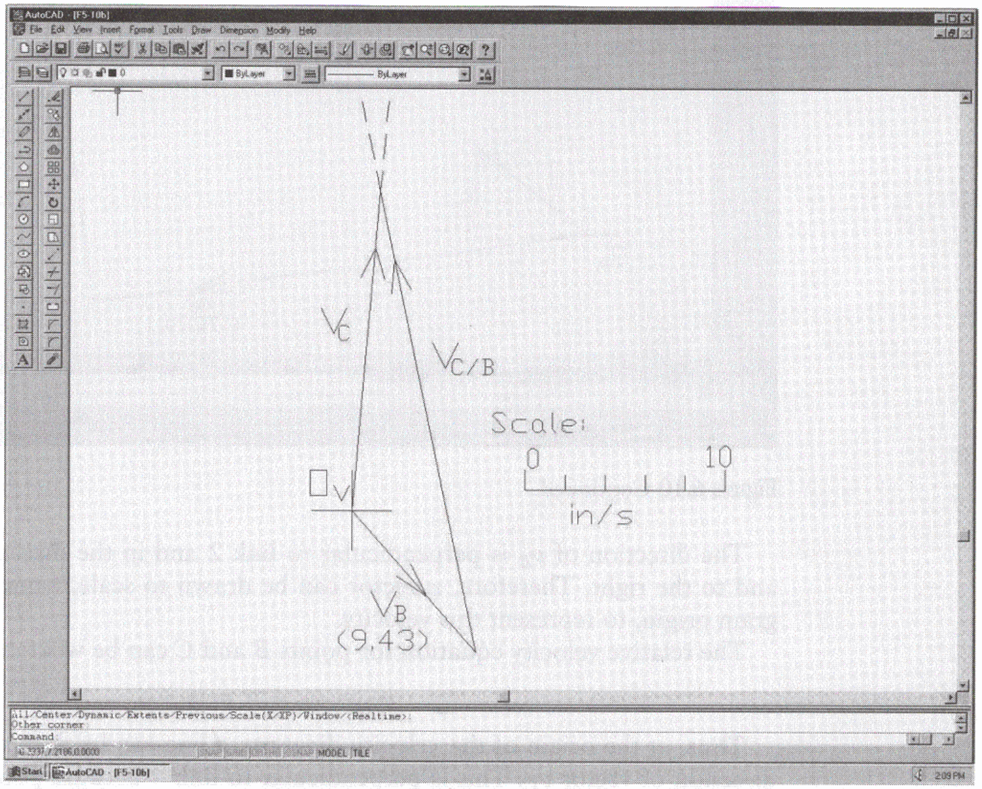


Figure 6.10 Diagrams for Example Problem 6.6.

The first step is to construct a velocity diagram, which includes points B and C. Calculating the magnitude of the velocity of point B can be accomplished with the following:

$$w_2 \frac{\text{rad}}{\text{sec}} = \frac{2\pi}{60}(120 \text{ rpm}) = 12.57 \frac{\text{rad}}{\text{sec}}$$

$$v_B = w_2 r_B = \left(12.57 \frac{\text{rad}}{\text{sec}}\right)(0.75 \text{ in}) = 9.43 \frac{\text{in}}{\text{s}}$$

(c)

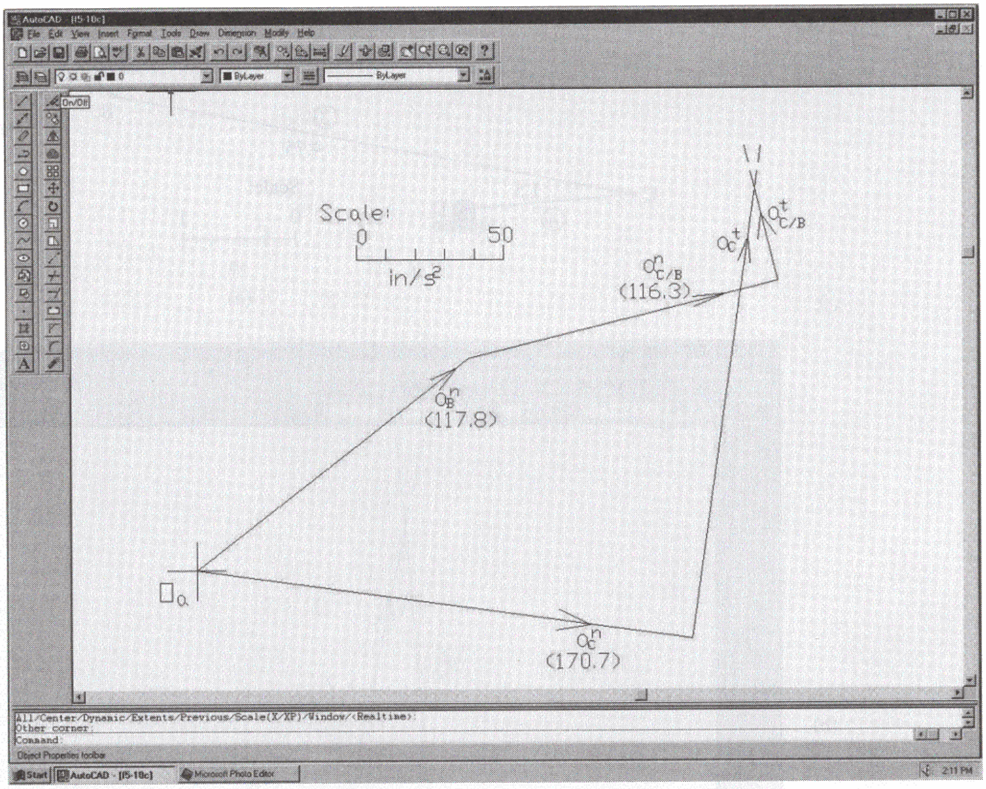


Figure 6.10 Continued.

The direction of v_B is perpendicular to link 2 and in the direction of w_2, down and to the right. Therefore, a vector can be drawn to scale, from the velocity diagram origin, to represent this velocity.

The relative velocity equation for points B and C can be written as:

$$v_C = v_B +\!> v_{C/B}$$

Thus, at the origin of the velocity diagram, a line can be drawn to represent the direction of vector v_C. This is perpendicular to link 4 because point C resides on a link that pivots about a fixed center. At the end of the vector v_B, a line can also be drawn to represent the direction of $v_{C/B}$. As with all relative velocity vectors, the direction is perpendicular to the line that connects points C and B. The intersection of the v_C and $v_{C/B}$ direction lines determines the magnitudes of both vectors. The completed velocity diagram is shown in Figure 6.10B.

Scaling the vector magnitudes from the diagram yields the following:

$$v_C = 16\frac{\text{in}}{\text{s}} \angle 83^\circ$$

$$v_{C/B} = 23.5\frac{\text{in}}{\text{s}} \measuredangle 78^\circ$$

The next step is to construct an acceleration diagram, which includes points *B* and *C*. Calculating the magnitudes of the known accelerations is accomplished by:

$$a^n{}_B = \frac{(v_B)^2}{r_{AB}} = \frac{(9.4\text{in/s})^2}{0.75\text{ in}} = 117.8\frac{\text{in}}{\text{s}^2}$$

(directed toward the center of rotation, point *A*)

$$a^t{}_B = \alpha_2 r_{AB} = (0)(0.75\text{ in}) = 0\frac{\text{in}}{\text{s}^2}$$

$$a^n{}_{C/B} = \frac{(v_{C/B})^2}{r_{CB}} = \frac{(23.5\text{in/s})^2}{4.75\text{ in}} = 116.3\frac{\text{in}}{\text{s}^2}$$

(directed from *C* toward *B*)

$$a^n{}_C = \frac{(v_C)^2}{r_{CD}} = \frac{(16\text{in/s})^2}{1.5\text{ in}} = 170.7\frac{\text{in}}{\text{s}^2}$$

(directed toward the center of rotation, point *D*)

The relative acceleration equation for points *B* and *C* can be written as:

$$a_C{}^n +> a_C{}^t = a_B{}^n +> a_B{}^t +> a_{C/B}{}^n +> a_{C/B}{}^t$$

In forming the acceleration diagram, vector construction arbitrarily starts on the right side of the equation. At the origin of the acceleration diagram, a line can be drawn to represent the vector $a_B{}^n$, which is known. Because it is zero, the magnitude of vector $a_B{}^t$ can be eliminated in the acceleration diagram. Therefore, at the end of vector $a_B{}^n$, another line can be drawn to represent the vector $a_{C/B}{}^n$, which is also known. At the end of this vector, a line can be drawn to represent the direction of vector $a_{C/B}{}^t$. This is perpendicular to the normal component, $a_{C/B}{}^n$ but has an unknown magnitude.

Since $a_C{}^n$ is on the left side of the equation, it will be placed at the origin of the acceleration diagram. A line can be drawn to represent vector $a_C{}^n$, which is known. At the end of this vector, a line can be drawn to represent the direction of vector $a_C{}^t$; however, the vector is unknown. The line is directed perpendicular to the normal component $a_C{}^n$. Finally, the intersection of the $a_C{}^t$ and $a_{C/B}{}^t$ direction lines determines the magnitudes of both vectors. The completed acceleration diagram is shown in Figure 6.9C.

Scaling the vector magnitudes from the diagram yields the following:

$$a_C{}^t = 156\frac{\text{in}}{\text{s}^2}\angle 83°$$

$$a_{C/B}{}^t = 25\frac{\text{in}}{\text{s}^2}\diagdown 75°$$

Finally, the angular acceleration of link 4 can be determined. By observing the direction of the tangential component of the acceleration of point *C*, up and to the right, it is obvious that link 4 accelerates in a clockwise direction. The magnitude of this angular acceleration is computed as:

$$\alpha_4 = \frac{a^t_c}{r_{CD}} = \frac{156 \text{in/s}^2}{1.5 \text{ in}} = 104 \frac{\text{rad}}{\text{s}^2}$$

Therefore, the angular acceleration of the follower link is:

$$\alpha_4 = 104 \frac{\text{rad}}{\text{s}^2} \text{ clockwise}$$

EXAMPLE PROBLEM 6.7

The mechanism shown in Figure 6.11 is a common punch press, designed to perform successive stamping operations. At this instant, the machine has just been powered and is coming up to full speed. The drive shaft rotates clockwise with an angular velocity of 72 rad/sec and accelerates at a rate of 250 rad/sec^2. At the instant shown, determine the acceleration of the stamping die, which will strike the workpiece.

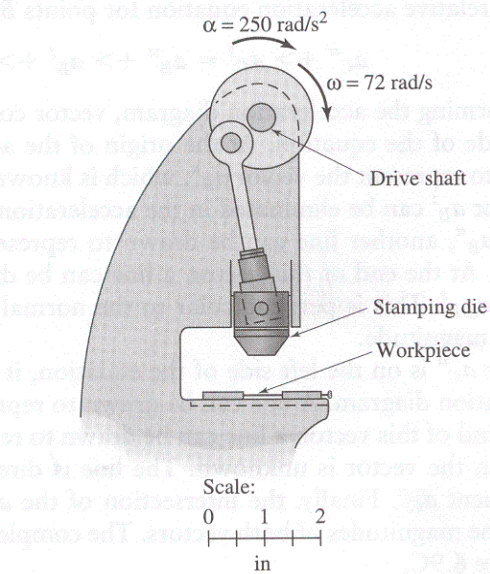
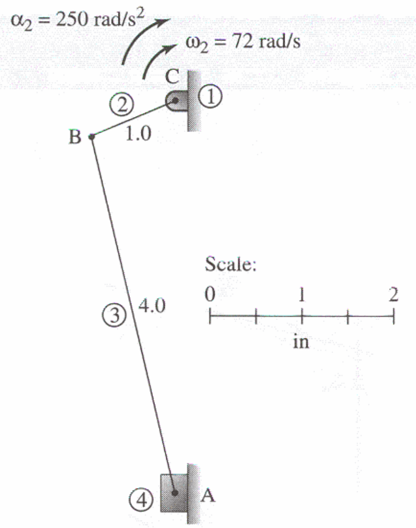


Figure 6.11 Mechanism for Example Problem 6.7.

Solution:

The portion of the mechanism that is under consideration includes the driving wheel, the stamping die, and the link that connects the two. Notice that this is the common slider-crank mechanism. A scaled, kinematic diagram is shown in Figure 6.12A.



(a)

(b)

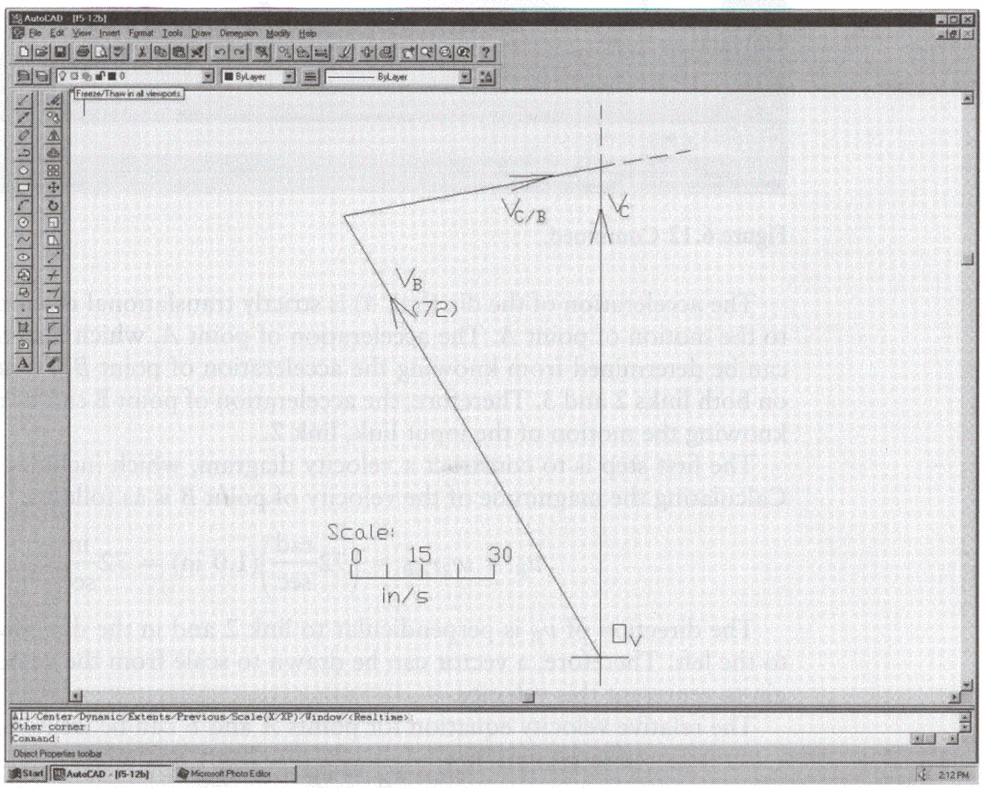


Figure 6.12 Diagrams for Example Problem 6.7.

(c)

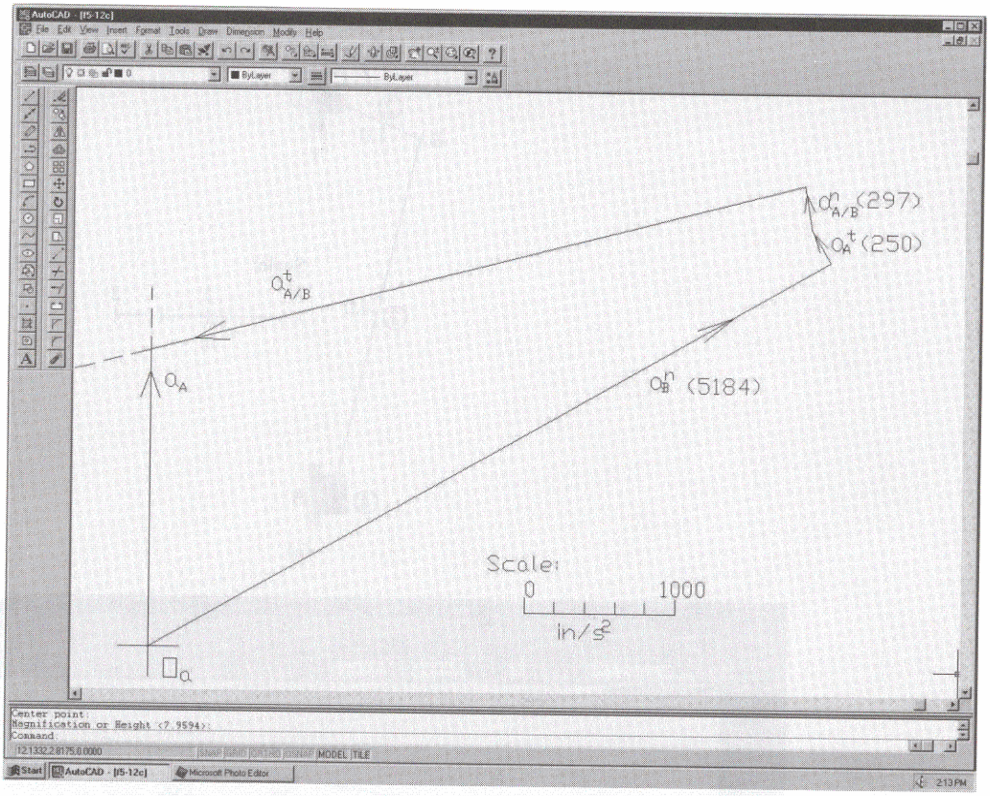


Figure 6.12 Continued.

The acceleration of the die (link 4) is strictly translational motion and is identical to the motion of point *A*. The acceleration of point *A*, which also resides on link 3, can be determined from knowing the acceleration of point *B*. Point *B* is positioned on both links 2 and 3. Therefore, the acceleration of point B can be determined from knowing the motion of the input link, link 2.

The first step is to construct a velocity diagram, which includes points *A* and *B*. Calculating the magnitude of the velocity of point *B* is as follows:

$$v_B = w_2 r_{AB} = \left(72\frac{\text{rad}}{\text{sec}}\right)(1.0 \text{ in}) = 72\frac{\text{in}}{\text{sec}}$$

The direction of v_B is perpendicular to link 2 and in the direction of w_2, up and to the left. Therefore, a vector can be drawn to scale from the velocity diagram origin to represent this velocity.

The relative velocity equation for points *A* and *B* can be written as:

$$v_A = v_B +> v_{A/B}$$

Thus, at the origin of the velocity diagram, a line can be drawn to represent the direction of vector v_A. This is parallel to the sliding surface, since link 4 is constrained to sliding motion. At the end of the vector v_B, a line can also be drawn to represent the direction of $v_{A/B}$. As with all relative velocity vectors, the direction is perpendicular to the line that connects points C and B. The intersection of the v_A and $v_{A/B}$ direction lines determines the magnitudes of both vectors. The completed velocity diagram is shown in Figure 6.12B.

Scaling the vector magnitudes from the diagram is determined as follows:

$$v_A = 69\frac{\text{in}}{\text{s}} \uparrow$$

$$v_{A/B} = 34.5\frac{\text{in}}{\text{s}} \angle 10°$$

The next step is to construct an acceleration diagram that includes points A and B. Calculating the magnitudes of the known accelerations is accomplished by the equations:

$$a_B^n = \frac{(V_B)^2}{r_{BC}} = \frac{(72\text{in/s})^2}{1.0 \text{ in}} = 5184\frac{\text{in}}{\text{s}^2}$$

(directed toward the center of rotation, point C)

$$a^t_B = \alpha_2 r_{AB} = \left(250\frac{\text{rad}}{\text{s}^2}\right)(1.0 \text{ in}) = 250\frac{\text{in}}{\text{s}^2}$$

(directed perpendicular to BC, in the direction of rotational acceleration)

$$a_{A/B}^n = \frac{(V_{A/B})^2}{r_{AB}} = \frac{(34.5\text{in/s})^2}{4.0 \text{ in}} = 297\frac{\text{in}}{\text{s}^2}$$

(directed from A toward B)

Note that point A does not have a normal acceleration because the motion is strictly translational.

The relative acceleration equation for points A and B can be written as:

$$a_A{}^n +> a_A{}^t = a_B{}^n +> a_B{}^t +> a_{A/B}{}^n +> a_{A/B}{}^t$$

In forming the acceleration diagram, vector construction arbitrarily starts on the right side of the equation. At the origin of the acceleration diagram, a line can be drawn to represent the vector $a_B{}^n$, which is known. At the end of this vector, a line can be drawn to represent vector $a_B{}^t$, which is also known. At the end of vector $a_B{}^t$, another line can be drawn to represent vector $a_{A/B}{}^n$, which is also known. At the end of this vector, a line can be drawn to represent the direction of vector $a_{A/B}{}^t$. This is perpendicular to the normal component, $a_{A/B}{}^n$, but has an unknown magnitude.

Since $a^n{}_A$ is on the left side of the equation, it will be placed at the origin of the acceleration diagram. A line can be drawn to represent the direction of vector $a_A{}^t$; however, the magnitude is unknown. The line is directed parallel to the sliding motion of link 4. Finally, the intersection of the $a_A{}^t$ and $a_{A/B}{}^t$ direction lines determines the magnitudes of both vectors. The completed acceleration diagram is shown in Figure 6.12C.

Scaling the vector magnitudes from the diagram is done with the following:

$$a_{A/B}{}^{t} = 4460\frac{\text{in}}{\text{s}^2} \nearrow 10°$$

$$a_A{}^{t} = 1970\frac{\text{in}}{\text{s}^2} \uparrow$$

Formally state the acceleration of point A as:

$$a_A = a_A{}^{t} = 1970\frac{\text{in}}{\text{s}^2} = 164\frac{\text{ft}}{\text{s}^2} \uparrow$$

6.9 RELATIVE ACCELERATION ANALYSIS: ANALYTICAL METHOD

The strategy for analytically determining the acceleration of various points on a mechanism is identical to the method outlined in the previous section. The difference is that once the vector polygons have been formed, they can be solved using the analytical methods introduced in Chapter 3, and incorporated in Chapter 5 and earlier sections in this chapter. Perhaps the most effective manner of presenting the analytical method of acceleration analysis is through an Example Problem.

EXAMPLE PROBLEM 6.8

The mechanism shown in Figure 6.13 is used to feed cartons to a labeling machine and, at the same time, to prevent the stored cartons from moving down. At full speed, the drive shaft rotates clockwise with an angular velocity of 200 rpm. At the instant shown, determine the acceleration of the ram and the angular acceleration of the connecting rod.

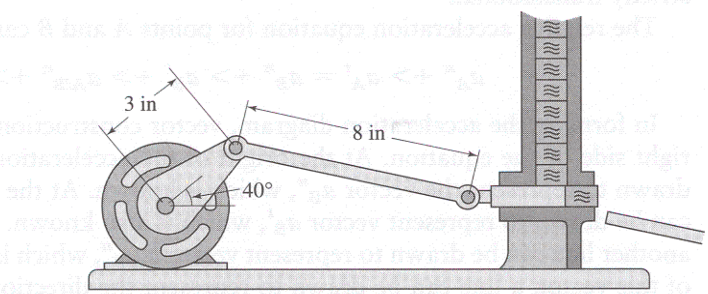


Figure 6.13 Mechanism for Example Problem 6.8.

Solution:

The portion of the mechanism that is under consideration includes the drive crank, the pusher ram, and the link that connects the two. Once again, notice that this is the common slider-crank mechanism. A kinematic diagram is shown in Figure 6.14A.

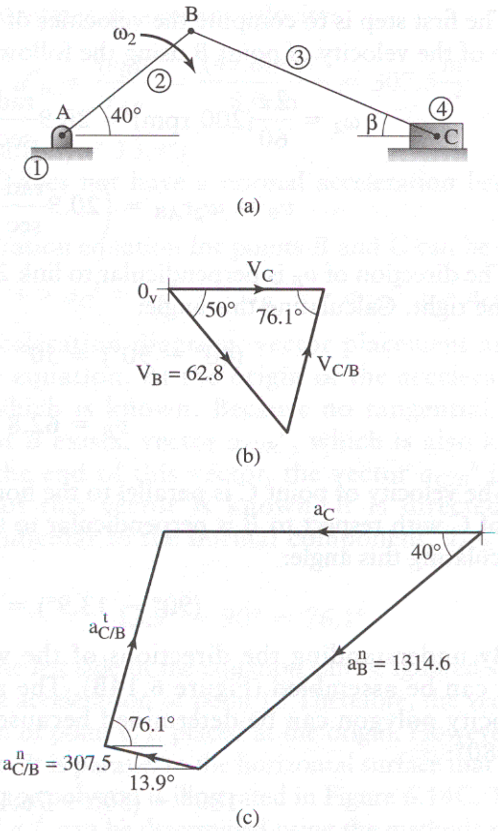


Figure 6.14 Diagrams for Example Problem 6.8.

As in Example Problem 6.6, the acceleration of the ram (link 4) is strictly translational motion and is identical to the motion of point C. The acceleration of point C, which also resides on link 3, can be determined from knowing the acceleration of point *B*. Point *B* is positioned on both links 2 and 3. Therefore, the acceleration of point *B* can be determined from knowing the motion of the input link, link 2.

The angle between link 3 and the horizontal sliding surface of link 4 can be determined from the law of sines:

$$\frac{\sin\beta}{r_{AB}} = \sin\frac{40^\circ}{r_{BC}}$$

$$\sin\beta = r_{AB}\left\{\frac{\sin 40^\circ}{r_{BC}}\right\} = 3 \text{ in}\left\{\frac{\sin 40^\circ}{8 \text{ in}}\right\}$$

Solving:

$$\beta = 13.9^\circ$$

The first step is to compute the velocities of points B and C. Calculate the magnitude of the velocity of point B using the following equation:

$$\omega_2 = \frac{2\pi}{60}(200 \text{ rpm}) = 20.9\frac{\text{rad}}{\text{sec}}$$

$$v_B = w_2 r_{AB} = \left(20.9\frac{\text{rad}}{\text{sec}}\right)(3.0 \text{ in}) = 62.8\frac{\text{in}}{\text{sec}}$$

The direction of v_B is perpendicular to link 2 and in the direction of ω_2, down and to the right. Calculating this angle:

$$(40° - 90°) = 50°$$

$$v_B = 62.8 \frac{\text{in}}{\text{sec}} \searrow 50°$$

The velocity of point C is parallel to the horizontal sliding surface and the velocity of C with respect to B is perpendicular to the link that connects point B and C. Calculating this angle:

$$(90° - 13.9°) = -76.1°$$

By understanding the directions of the vectors of interest, a velocity polygon can be assembled (Figure 6.14B). The magnitude of the third angle in the velocity polygon can be determined because the sum of all angles in a triangle is 180°:

$$180° - (50° + 76.1°) = 53.9°$$

The magnitudes of the velocities can be found from the law of sines.

$$v_C = v_B\left(\frac{\sin 50°}{\sin 76.1°}\right) = 62.8\text{in/s}\left(\frac{\sin 50°}{\sin 76.1°}\right) = 52.3\frac{\text{in}}{\text{s}^2} \rightarrow$$

Solve for the unknown velocities with the following:

$$v_{C/B} = v_B\left(\frac{\sin 50°}{\sin 76.1°}\right) = 62.8\text{in/s}\left(\frac{\sin 50°}{\sin 76.1°}\right) = 49.6\frac{\text{in}}{\text{s}^2} \angle 76.1°$$

The next step is to construct an acceleration diagram that includes points B and C. Calculate the magnitudes of the known accelerations using the following equations:

$$a^n{}_b = \frac{(v_B)^2}{r_{AB}} = \frac{(62.8\text{in/s})}{3.0 \text{ in}} = 1315\frac{\text{in}}{\text{s}^2}$$

(directed toward the center of rotation, point A, $\swarrow$ 40°)

$$a^t{}_B = \alpha_2 r_{AB} = \left(0\frac{\text{rad}}{\text{s}^2}\right)(3.0 \text{ in}) = 0$$

Because the drive link rotates at constant velocity:

$$a^n{}_b = \frac{(v_B)^2}{r_{AB}} = \frac{(49.6\text{in/s})^2}{8.0 \text{ in}} = 307.5\frac{\text{in}}{\text{s}^2}$$

(directed from C toward B, ⁄ 13.9°)

Note that point C does not have a normal acceleration because the motion is strictly translational.

The relative acceleration equation for points B and C can be written as:

$$a_C{}^n +> a_C{}^t = a_B{}^n +> a_B{}^t +> a_{C/B}{}^n +> a_{C/B}{}^t$$

In forming an acceleration diagram, vector placement arbitrarily starts on the right side of the equation. At the origin of the acceleration diagram, vector $a_B{}^n$ is placed, which is known. Because no tangential component of the acceleration of point B exists, vector $a_{C/B}{}^n$, which is also known, is placed at the end of $a_B{}^n$. At the end of this vector, the vector $a_{C/B}{}^t$ is placed; however, only the direction of this vector is known. It is directed perpendicular to $a_{C/B}{}^n$. This is perpendicular to the normal component, $a_{C/B}{}^n$. The angle is calculated as:

$$13.9° - 90° = 76.1°$$

The first term on the left side of the equation can be ignored since there is no normal component of the acceleration of point C. Therefore, the vector representing the tangential acceleration of point C is placed at the origin. However, only the direction of this vector is known. It is parallel to the horizontal surface that link 4 is constrained to slide upon. The vector polygon is illustrated in Figure 6.14C. The unknown vector magnitudes, $a_{C/B}{}^t$ and $a_C{}^t$, can be determined using the methods presented in Chapter 3. First, each vector can be separated into horizontal and vertical components, as shown in Table 6.2.:

Separate algebraic equations can be written for the horizontal and vertical components as:

$$a_C = a_B{}^n +> a_{C/B}{}^n +> a_{C/B}{}^t$$

TABLE 6.2 Acceleration Components for Example Problem 6.8.

Vector	Reference Angle (θ_x)	Horizontal Component $a_h = a \cos \theta_x$	Vertical Component $a_v = a \sin \theta_x$
$a_B{}^n$	220°	−1007	−845
$a^n{}_{C/B}$	166.1°	−299	74
$a^t{}_{C/B}$	76.1°	.240 $a^t{}_{C/B}$	.971 $a^t{}_{C/B}$
a_C	180°	$-a_C$	0

(horizontal comp)

$$+a_C = (-1007.0) + (-298.5) + (+0.240 a_{C/B}{}^t)$$

(vertical comp)

$$0 = (-845.0) + (+73.9) + (+0.971\ a_{C/B}{}^t)$$

The vertical component equation can be solved algebraically to give:

$$a_{C/B}{}^t = 794.1 \frac{\text{in}}{\text{s}^2}$$

This result can then be substituted into the horizontal equation to give:

$$a_C = 1496.1 \frac{\text{in}}{\text{s}^2}$$

Formally stated, the motion of the ram:

$$v_C = 52.3 \frac{\text{in}}{\text{sec}} \rightarrow$$

$$a_c = 1496.1 \frac{\text{in}}{\text{s}^2} \leftarrow$$

Notice that because the acceleration is in the opposite direction of the ram movement and velocity, the ram is decelerating. Finally, to determine the motion of the connecting arm, use the following:

$$\omega_3 = \frac{v_{C/B}}{r_{CB}} = \frac{49.6\text{in/s}}{8\ \text{in}} = 6.2 \frac{\text{rad}}{\text{s}^2}$$

in a direction consistent with the velocity of C relative to B, counterclockwise.

$$\alpha_3 = \frac{a^t{}_{C/B}}{r_{CB}} = \frac{794.1\text{in/s}^2}{8.0\ \text{in}} = 99.3 \frac{\text{rad}}{\text{s}^2}$$

in a direction consistent with the tangential acceleration of C relative to B, counterclockwise.

6.10 ALGEBRAIC SOLUTIONS FOR COMMON MECHANISMS

For the common slider-crank and four-bar mechanisms, closed-form algebraic solutions have been derived.[10] They are given in the following sections.

6.10.1 Slider-Crank Mechanism

A general slider-crank mechanism was illustrated in Figure 4.21 and is uniquely defined with dimensions L_1, L_2, and L_3. With one degree of freedom, the motion of one link must be specified to drive the other links. Most often the crank is driven.

Therefore, knowing θ_2, ω_2, and α_2 along with the position and velocities of all the links, from Equations 4.4, 4.5, 5.11, and 5.12, the accelerations of the other links can be determined. As presented in Chapter 4, the position equations include:

(4.4) $$\theta_3 = \sin^{-1}\left\{\frac{L_1 + L_2 \sin \theta_2}{L_3}\right\}$$

(4.5) $$L_4 = L_2 \cos(\theta_2) + L_3 \cos(\theta_3)$$

As presented in Chapter 5, the velocity equations are:

(5.11) $$\omega_3 = -\omega_2 \frac{L_2 \cos \theta_2}{L_3 \cos \theta_3}$$

(5.12) $$v_4 = -\omega_2 L_2 \sin \theta_2 - \omega_3 L_3 \sin \theta_3$$

The acceleration equations are then given as[10]

(6.17) $$\alpha_3 = \frac{\omega_2^2 L2 \sin \theta_2 + \omega_3^2 L_3 \sin \theta_3 - \alpha_2 L_2 \cos\theta_2}{L_3 \cos \theta_3}$$

(6.18) $$a_4 = -\alpha_2 L_2 \sin \theta_2 - \alpha_3 L_3 \sin \theta_3 - \omega_2{}^2 L_2 \cos \theta_2 - \omega_3{}^2 L_3 \sin \theta_3$$

6.10.2 Four-Bar Mechanism

A general four-bar mechanism was illustrated in Figure 4.24 and is uniquely defined with dimensions L_1, L_2, L_3, and L_4. With one degree of freedom, the motion of one link must be specified to drive the other links. Most often the crank is driven. Therefore, knowing θ_2, ω_2, and α_2 along with the position and velocity of all the links, the accelerations of the other links can be determined from Equations 4.4, 4.8, 4.9, 4.10, 5.13, and 5.14. As presented in Chapter 4, the position equations are:

(4.7) $$BD = \sqrt{L_1{}^2 + L_2{}^2 - 2(L_1)(L_2)\cos(\theta_2)}$$

(4.8) $$\gamma = \cos^{-1}\left\{\frac{L_3{}^2 + L_4{}^2 - BD^2}{2L_3L_4}\right\}$$

(4.9) $$\theta_4 = 180° - \cos^{-1}\left\{\frac{L_1{}^2 - L_2{}^2 + BD^2}{2L_1\, BD}\right\} - \cos^{-1}\left\{\frac{L_4{}^2 - L_3{}^2 + BD^2}{2L_4\, BD}\right\}$$

(4.10) $$\theta_3 = \theta_4 - \gamma$$

As presented in Chapter 5, the velocity equations are:

(5.13) $$\omega_3 = \omega_2 \frac{L_2 \sin(\theta_4 - \theta_2)}{L_3 \sin \gamma}$$

(5.14) $$\omega_4 = \omega_2 \frac{L_2 \sin(\theta_3 - \theta_2)}{L_4 \sin \gamma}$$

The acceleration equations can be presented as[10]:

$$\alpha_3 = \frac{\alpha_2 L_2 \sin(\theta_2 - \theta_4) + \omega_4{}^2 L_2 \cos(\theta_2 - \theta_4) - \omega_4{}^2 L_4 + \omega_3{}^2 L_3 \cos(\theta_4 - \theta_3)}{L_3 \sin(\theta_4 - \theta_3)} \quad (6.19)$$

$$\alpha_4 = \frac{\alpha_2 L_2 \sin(\theta_2 - \theta_3) + \omega_2{}^2 L_2 \cos(\theta_2 - \theta_3) - \omega_3{}^2 L_4 \cos(\theta_4 - \theta_3)}{L_4 \sin(\theta_4 - \theta_3)} \quad (6.20)$$

6.11 ACCELERATION OF A GENERAL POINT ON A FLOATING LINK

Recall that a floating link is not directly connected to the fixed link. Therefore, the motion of a floating link is not limited to only rotation or translation, but a combination of both. In turn, the direction of the motion of points that reside on the floating link are not generally known. Contrast this with the motion of a point on a link that is pinned to the fixed link. The motion of that point must pivot at a fixed distance from the pin connection. Thus, the direction of motion is known.

During the acceleration analyses presented in the preceding sections, the underlying premise of the solution is that the direction of the motion is known. For a general point on a floating link, this is not true. For these cases, two relative acceleration equations must be used and solved simultaneously.

To illustrate the strategy of determining the acceleration of a general point on a floating link, consider the kinematic sketch of the four-bar linkage shown in Figure 6.15.

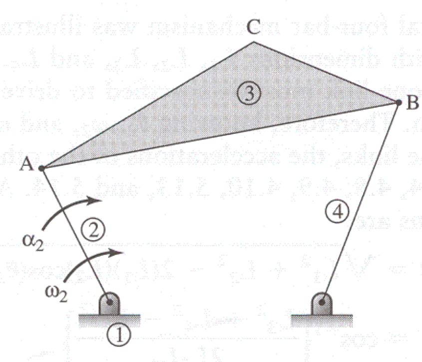


Figure 6.15 Point on a floating link.

Link 3 is a floating link because it is not directly attached to link 1, the fixed link. Because points A and B both reside on the floating link as well as links attached to the fixed link, the acceleration of these points can be readily determined using the methods discussed in the previous two sections. However, point C does not reside on a link that is directly attached to the fixed link. Therefore, the exact path of motion of point C is not obvious. However, two relative acceleration equations can be written as:

$$a_C = a_B{}^n +> a_B{}^t +> a_{C/B}{}^n +> a_{C/B}{}^t$$

$$a_C = a_A{}^n +> a_A{}^t +> a_{C/A}{}^n +> a_{C/A}{}^t$$

In the first equation, both the magnitude and direction of a_C is unknown along with the direction of $a_{C/B}{}^t$. The second equation introduces an additional unknown, namely the direction of $a_{C/A}{}^t$. Overall, there are two vector equations, each with the capability of determining two unknowns. There are also four unknowns. Using the two equations simultaneously, the acceleration of point C can be determined either through a graphical or analytical procedure. The following Example Problems illustrate this method.

EXAMPLE PROBLEM 6.9

The mechanism shown in Figure 6.16 is used to pull movie film through a projector. The mechanism is driven by the drive wheel rotating at 560 rpm. At the instant shown, graphically determine the acceleration of the claw, which engages with the film.

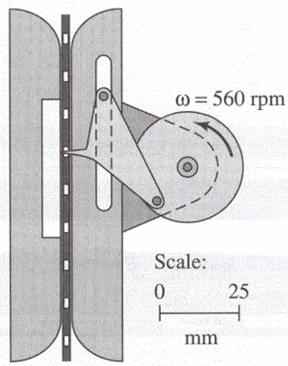


Figure 6.16 Film advance mechanism for Example Problem 6.9.

Solution:

A scaled, kinematic diagram of this mechanism is shown in Figure 6.17A. Notice that this is the basic slider-crank mechanism with a point of interest, point X, located at the claw.

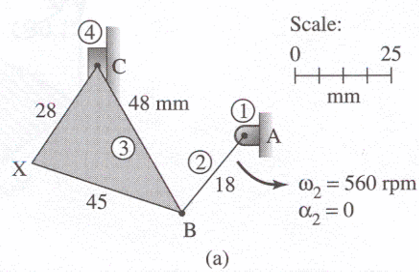
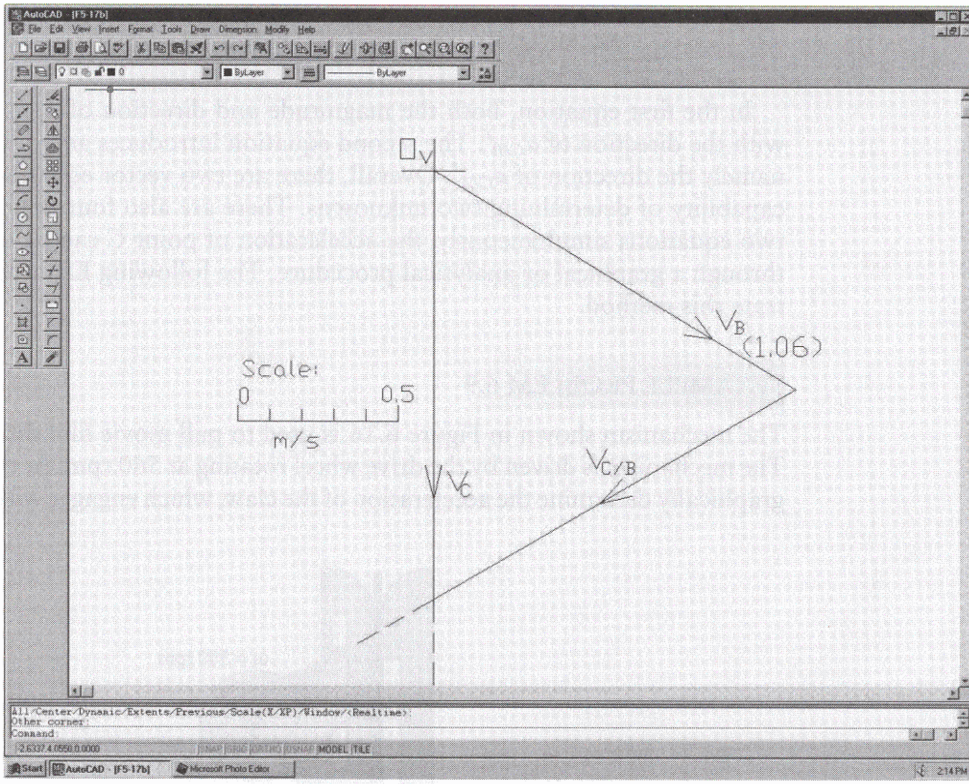


Figure 6.17 Diagrams for Example Problem 6.9.

Figure 6.17
Continued.

(b)



(c)

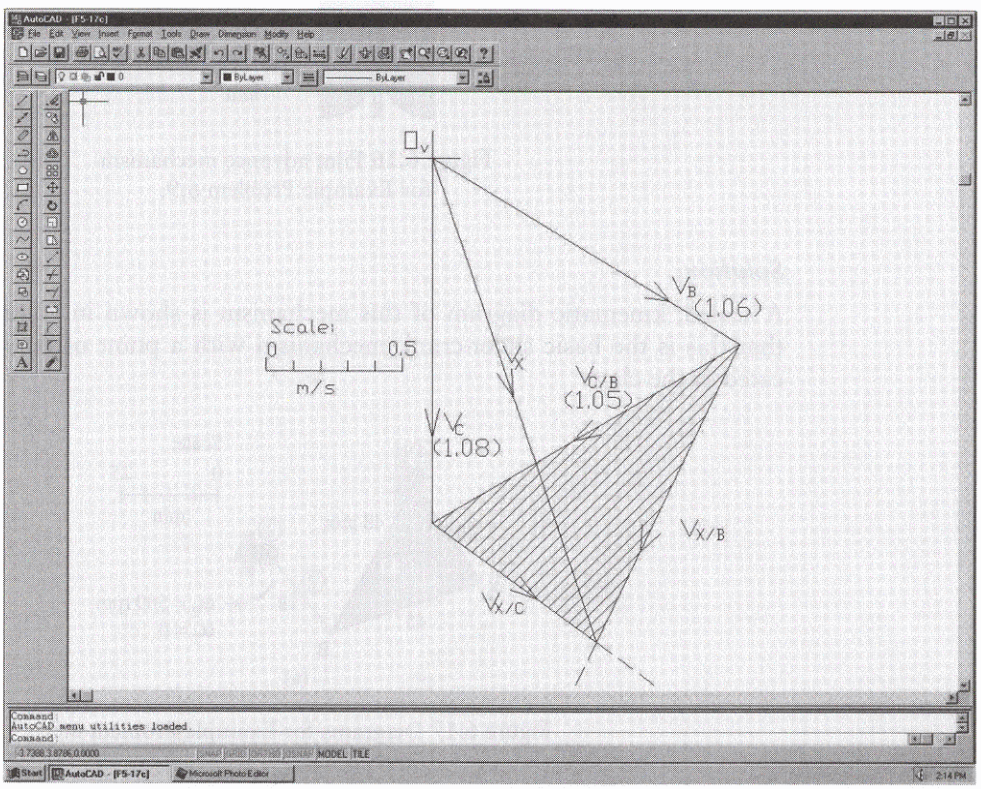
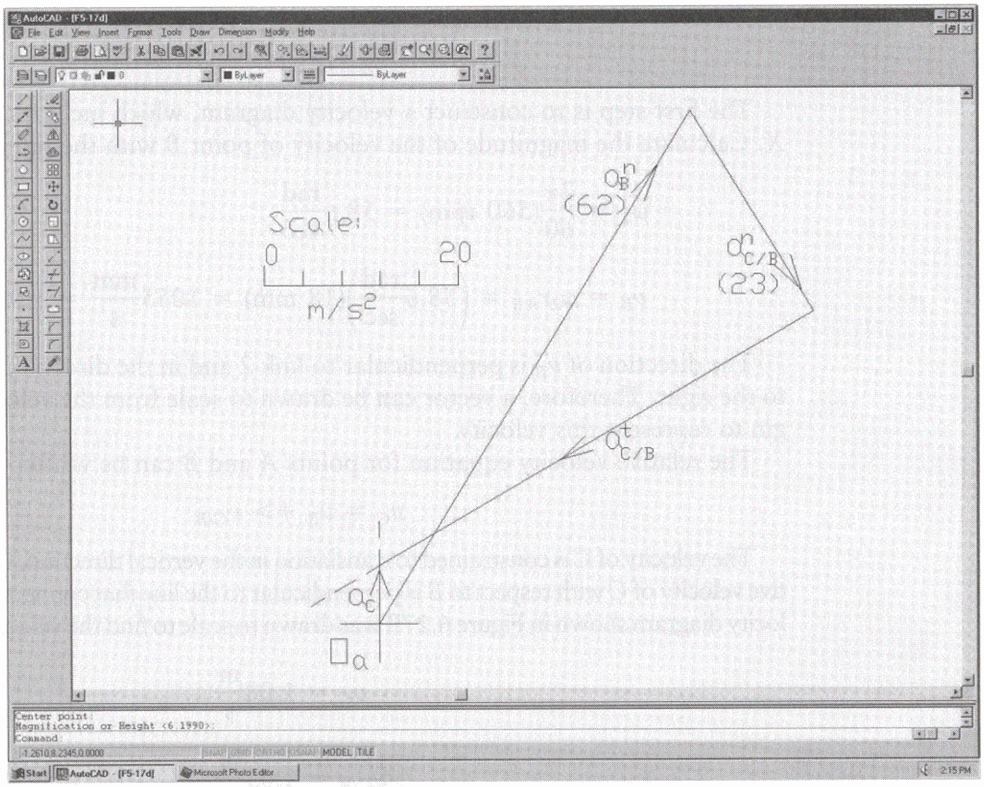
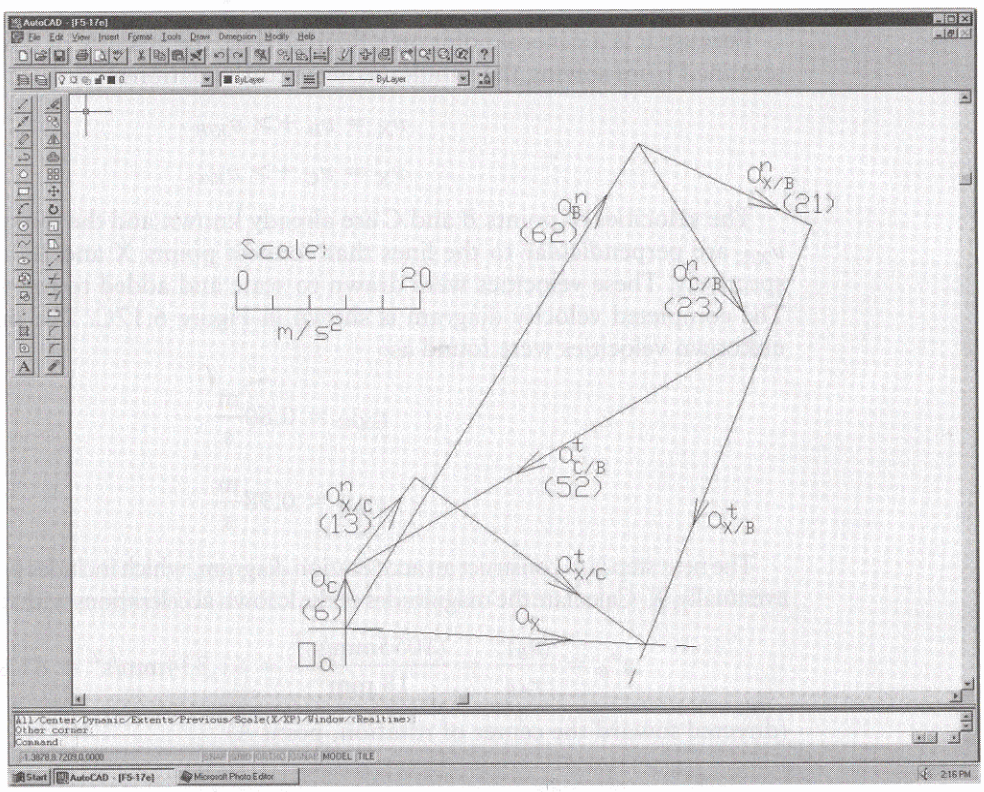


Figure 6.17
Continued.

(d)



(e)



The first step is to construct a velocity diagram, which includes points *B*, *C*, and *X*. Calculate the magnitude of the velocity of point *B* with the following:

$$\omega_2 = \frac{2\pi}{60}(560\text{ rpm}) = 58.6\frac{\text{rad}}{\text{sec}}$$

$$v_B = \omega_2 r_{AB} = \left(58.6\frac{\text{rad}}{\text{sec}}\right)(18\text{ mm}) = 1055\frac{\text{mm}}{\text{s}} = 1.055\frac{\text{m}}{\text{s}}$$

The direction of v_B is perpendicular to link 2 and in the direction of ω_2, down and to the right. Therefore, a vector can be drawn to scale from the velocity diagram origin to represent this velocity.

The relative velocity equation for points *A* and *B* can be written as:

$$v_C = v_B +> v_{C/B}$$

The velocity of *C* is constrained to translation in the vertical direction. Of course, the relative velocity of *C* with respect to *B* is perpendicular to the line that connects *C* and *B*. The velocity diagram shown in Figure 6.17B was drawn to scale to find the velocity magnitudes of:

$$v_C = 1.08\frac{\text{m}}{\text{s}}$$

$$v_{C/B} = 1.05\frac{\text{m}}{\text{s}}$$

Because it is a general point on a floating link, the velocity of point *X* must be determined from solving the simultaneous vector equations:

$$v_X = v_B +> v_{X/B}$$

$$v_X = v_C +> v_{X/C}$$

The velocities of points *B* and *C* are already known and the directions of $v_{X/B}$ and $v_{X/C}$ are perpendicular to the lines that connect points *X* and *B*, and *X* and *C*, respectively. These velocities were drawn to scale and added to the velocity polygon. The completed velocity diagram is shown in Figure 6.17C. The magnitudes of the unknown velocities were found as:

$$v_{X/C} = 0.60\frac{\text{m}}{\text{s}}$$

$$v_{X/B} = 0.98\frac{\text{m}}{\text{s}}$$

The next step is to construct an acceleration diagram, which includes points *A* and *B* and, eventually, *X*. Calculate the magnitudes of the known accelerations with the following:

$$a^n{}_B = \frac{(v_B)^2}{r_{AB}} = \frac{(1055\text{mm/s})^2}{18\text{ mm}} = 61{,}834\text{mm/s}^2 = 61.8\frac{\text{m}}{\text{s}^2}$$

(directed toward the center of rotation, point *A*)

$$a^t_B = 0$$

$$a^n_{C/B} = \frac{(v_{C/B})^2}{r_{CB}} = \frac{(1050\text{mm/s})^2}{48\text{ mm}} = 23\frac{\text{m}}{\text{s}^2}$$

(directed from C toward B)

Note that point C does not have normal acceleration because the motion is strictly translational.

Understanding that there is no $a_B{}^t$ and $a_C{}^n$ components of acceleration, the relative acceleration equation for points B and C can be written as:

$$a_C = a_C{}^t = a_B{}^n +> a_B{}^t +> a_{C/B}{}^n +> a_{C/B}{}^t$$

An acceleration diagram drawn to scale is shown in Figure 6.17D. Scale the vector magnitudes from the diagram by using the following:

$$a_{C/B}{}^t = 52.0\frac{\text{m}}{\text{s}^2} \angle 30°$$

$$a_C = a_C{}^t = 6.0\frac{\text{m}}{\text{s}^2}\uparrow$$

As with velocities, because point X is a general point on a floating link, its acceleration must be determined from solving the simultaneous vector equations:

$$a_X = a_B{}^n +> a_B{}^t +> a_{X/B}{}^n +> a_{X/B}{}^t$$

$$a_X = a_C{}^n +> a_C{}^t +> a_{X/C}{}^n +> a_{X/C}{}^t$$

From the work above, the accelerations $a_B{}^t$ and $a_C{}^n$ have been found to be zero, and $a_B{}^n$ and $a_C{}^t$ are known. In addition, the magnitudes of the relative normal components can be calculated:

$$a^n_{X/B} = \frac{(v_{X/B})^2}{r_{BX}} = \frac{(980\text{mm/s})^2}{45\text{ mm}} = 21.3\frac{\text{m}}{\text{s}^2}$$

(directed from X to B)

$$a^n_{X/C} = \frac{(v_{X/C})^2}{r_{BX}} = \frac{(600\text{mm/s})^2}{28\text{ mm}} = 12.9\frac{\text{m}}{\text{s}^2} \text{ (directed from } X \text{ toward } C\text{)}$$

Again, in a similar fashion to the velocity analysis, the two acceleration equations are superimposed onto the original acceleration polygon. The accelerations were drawn to scale and the completed acceleration diagram is shown in Figure 6.17E. The magnitudes of the unknown accelerations were found by the equations:

$$a_{X/C}{}^t = 31.0\frac{\text{m}}{\text{s}^2} \angle 35°$$

$$a_{X/B}{}^t = 52.0\frac{\text{m}}{\text{s}^2} \angle 68°$$

and finally:

$$a_X = 33.0\frac{\text{m}}{\text{s}^2} \nwarrow 8°$$

6.12 CORIOLIS ACCELERATION

Throughout the preceding analyses, two components of an acceleration vector (i.e., normal and tangential) were thoroughly examined. In certain conditions, a third component of acceleration is encountered. This additional component is known as the *Coriolis component of acceleration* and is present in cases where sliding contact occurs between two rotating links.

Mechanisms used in machines have been known to fail due to the mistaken omission of this Coriolis component. Omitting the Coriolis component understates the acceleration of a link and the associated inertial forces. Therefore, the actual stresses in the machine components can be greater than the design allows and failure may occur. Therefore, every situation must be studied to determine whether a Coriolis acceleration component exists.

Specifically, the Coriolis component is encountered in the relative acceleration of two points when all of the following three conditions are simultaneously present:

1. The two points are coincident, but on different links;
2. The point on one link traces a path on the other link; and,
3. The link that contains the path rotates.

Figure 6.18 illustrates a rear hatch of a mini-van and the related kinematic diagram. Notice that point B can be associated with either link 2, 3, or 4. To clarify the association to a link, point B is referred to as B_2, B_3, and B_4. Up to this point in the chapter, a coincident point on different links had the same acceleration because only pin joints were used to connect two rotating links. In Figure 6.18, both pin and sliding joints are used to connect the two rotating links, links 2 and 4. In this case, the velocities and accelerations of B_2 and B_4 are not the same.

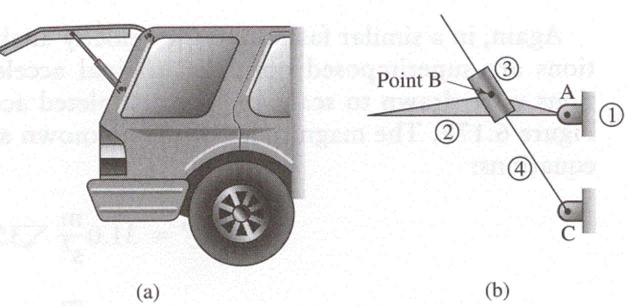


Figure 6.18 Case where Coriolis acceleration is encountered.

Relative motion equations can be used to relate the velocities and accelerations as follows:

$$v_{B4} = v_{B2} +> v_{B4/B2}$$

$$a_{B4} = a_{B2} +> a_{B4/B2}$$

This situation represents a mechanism analysis case where the Coriolis component must be included in the relative acceleration term, $a_{B4/B2}$. Notice that:

- The points are coincident, but not on the same link (condition 1);
- Point B_2 slides along and traces a path on link 4 (condition 2); and,
- The link that contains the path, link 4 rotates (condition 3).

Separating the relative acceleration term into its components yields the following:

$$a_{B4/B2} = a_{B4/B2}{}^{n} +> a_{B4/B2}{}^{t} +> a_{B4/B2}{}^{c}$$

where $a_{B4/B2}{}^{c}$ = the Coriolis component of acceleration

The magnitude of the Coriolis component has been derived [5] as:

(6.21) $$a_{B4/B2}{}^{c} = 2\, v_{B4/B2}\,\omega_4$$

Both the relative linear velocity and the absolute angular velocity can be determined from a thorough velocity analysis of the mechanism. The angular velocity, ω, must be of the link that contains the path of the sliding point. Care must be taken because a common error in calculating the Coriolis component is selecting the wrong angular velocity.

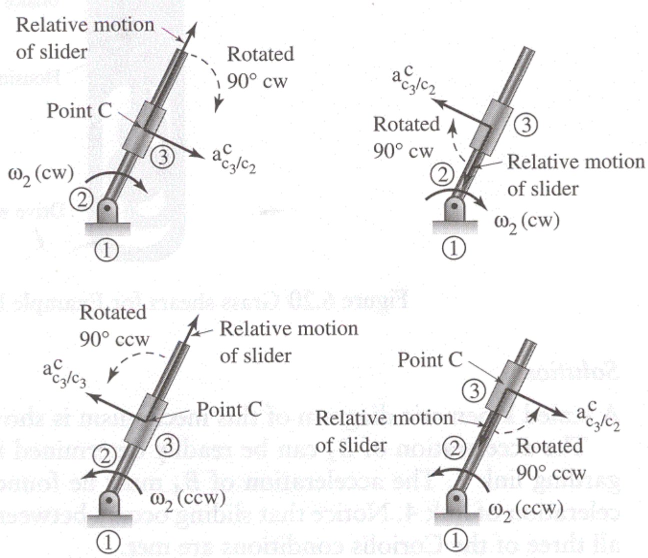


Figure 6.19 Directions of the Coriolis acceleration component.

The direction of the Coriolis component is perpendicular to the relative velocity vector, $v_{B4/B2}$. The sense is obtained by rotating the relative velocity vector such that the head of the vector rotates in the direction of the angular velocity of the path. Thus, when the angular velocity of the path, ω_4, rotates clockwise, the Coriolis direction is obtained by rotating the relative velocity vector, 90° clockwise. Conversely, when the angular velocity of the path, ω_4, rotates counterclockwise, the Coriolis direction is obtained by rotating the relative velocity vector, 90° counterclockwise. Figure 6.19 illustrates the four cases where the direction of the Coriolis component is determined.

Because both the magnitude and direction of the Coriolis component can be readily calculated from the velocity data, no additional unknown quantities are added to the acceleration equation. However, in solving problems, it is more convenient to write the acceleration equation with the point tracing on the left side. The technique for such acceleration analyses is best illustrated through the Example Problem below.

EXAMPLE PROBLEM 6.10

Figure 6.20 illustrates hand-held grass shears,used for trimming areas that are hard to reach with mowers or weed whackers. The drive wheel rotates at 400 rpm. Determine the angular acceleration of the oscillating blades at the instant shown.

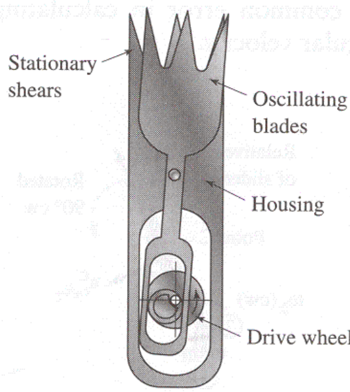


Figure 6.20 Grass shears for Example Problem 6.10.

Solution:

A scaled kinematic diagram of this mechanism is shown in Figure 6.21A.

The acceleration of B_2 can be readily determined from the input information regarding link 2. The acceleration of B_4 must be found to determine the angular acceleration of link 4. Notice that sliding occurs between rotating links (2 and 4); thus, all three of the Coriolis conditions are met.

The first step is to construct a velocity diagram, which includes points B_2 and B_4. The distance between points A and B_2 was scaled as 1.4 in. Calculate the magnitude of the velocity of point B_2 with the following:

$$\omega_2 = \frac{2\pi}{60}(400 \text{ rpm}) = 41.9\frac{\text{rad}}{\text{sec}}$$

$$v_{B2} = \omega_2 r_{AB} = (41.9\frac{\text{rad}}{\text{sec}})(1.4 \text{ in}) = 58.6\frac{\text{in}}{\text{s}}$$

The direction of v_{B2} is perpendicular to link 2 and in the direction of ω_2, down and to the right. Therefore, a vector can be drawn to scale from the velocity diagram origin to represent this velocity.

The relative velocity equation for points B_2 and B_4 can be written as:

$$v_{B2} = v_{B4} +> v_{B2/B4}$$

Because link 4 is pinned to the fixed link, rotating, the velocity of B_4 is perpendicular to the line that connects B_4 with the center of rotation. For this case, the relative velocity of B_2 with respect to B_4 is parallel to link 4 because B_2 slides along link 4. The velocity diagram shown in Figure 6.21B was drawn to scale to find the velocity magnitudes of:

$$v_{B4} = 52.5\frac{\text{in}}{\text{s}} \searrow 15°$$

$$v_{B2/B4} = 23.3\frac{\text{in}}{\text{s}} \swarrow 76°$$

The distance between points C and B_4 was scaled as 3.6 in. Therefore, the angular velocity of link 4 can be calculated as:

$$\omega_4 = \frac{v_{B4}}{r_{CB4}} = \frac{52.5\text{in/s}}{3.6 \text{ in}} = 14.5\frac{\text{rad}}{\text{s}}$$

Because the velocity of B_4 has been found to be directed down and to the right, the angular velocity of link 4 must be counterclockwise.

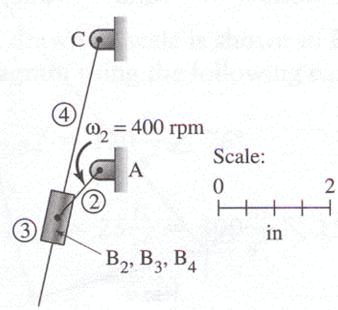
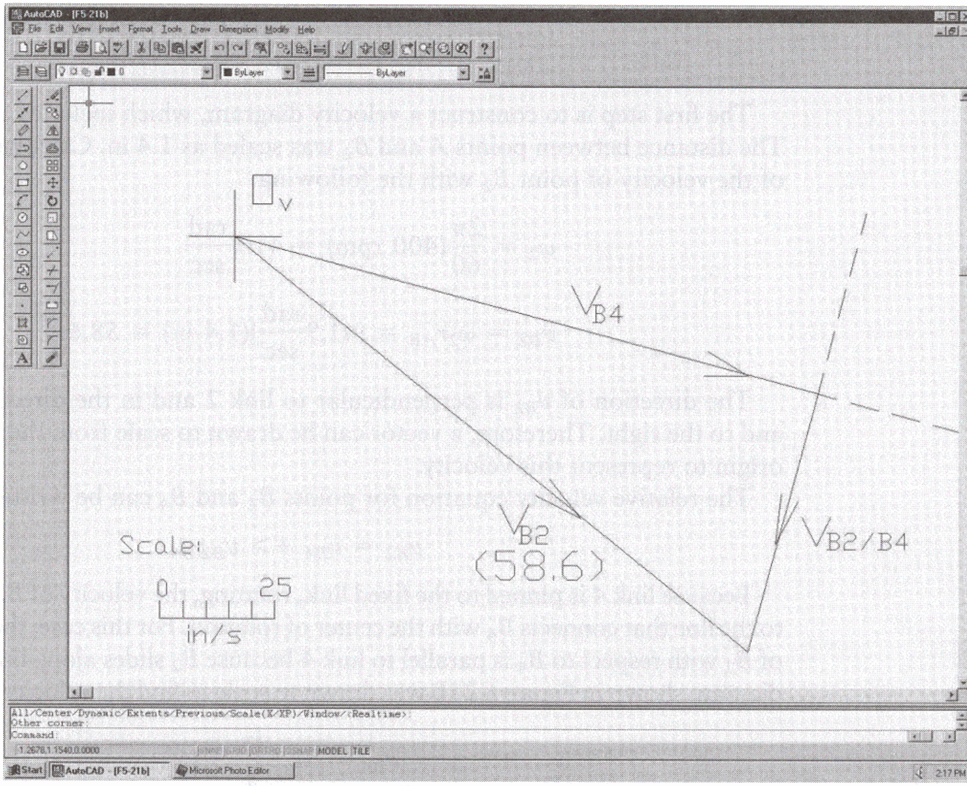


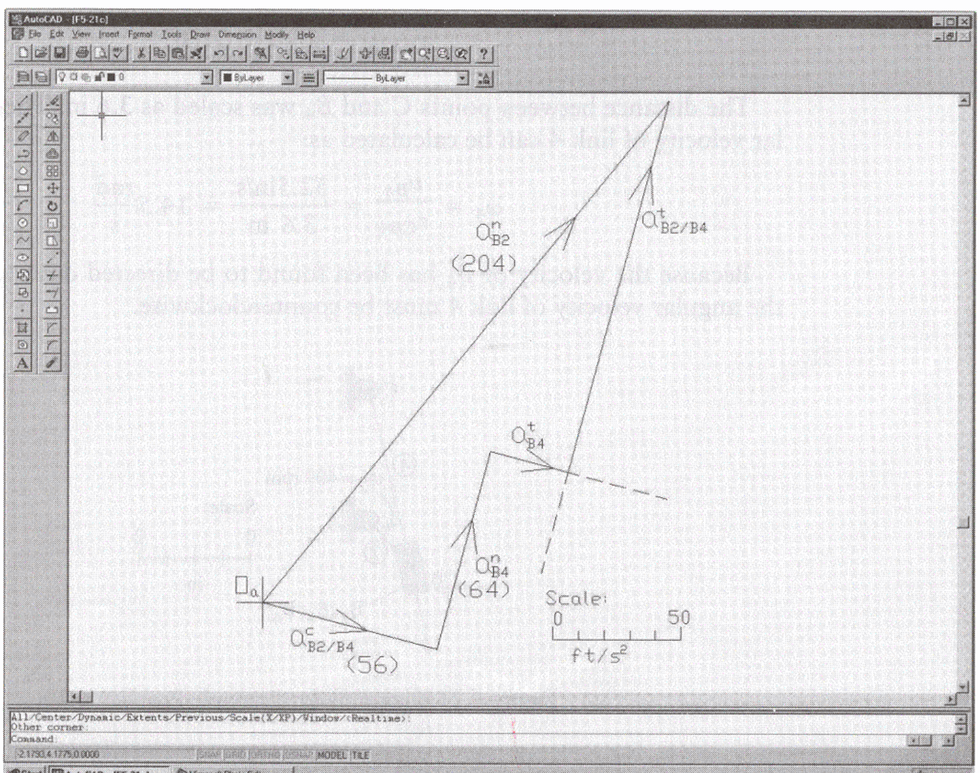
Figure 6.21 Diagrams for Example Problem 6.10.

Figure 6.21
Continued.

(b)



(c)



The next step is to construct an acceleration diagram that includes points B_2 and B_4. As mentioned, it is typically more convenient to write the acceleration equation with the point doing the tracing, B_2, on the left side. Using this guideline, the acceleration equation is written as:

$$a_{B2}^{\ n} +> a_{B2}^{\ t} = a_{B4}^{\ n} +> a_{B4}^{\ t} +> a_{B2/B4}^{\ n} +> a_{B2/B4}^{\ t} +> a_{B2/B4}^{\ c}$$

Calculate the magnitudes of the known accelerations with the following:

$$a_{B2}^{n} = \frac{(v_{B2})^2}{r_{CB2}} = \frac{(58.6\text{in/s})^2}{1.4 \text{ in}} = 2453\frac{\text{in}}{\text{s}^2} = 204\frac{\text{ft}}{\text{s}^2}$$

(directed toward the center of rotation, point A)

$$a^{t}_{B2} = 0 \text{ no rotational acceleration of link 2}$$

$$a_{B4}^{n} = \frac{(v_{B4})^2}{r_{CB4}} = \frac{(52.5\frac{\text{in}}{\text{s}})^2}{3.6 \text{ in}} = 766\frac{\text{in}}{\text{s}^2} = 64\frac{\text{ft}}{\text{s}^2}$$

(directed toward the center of rotation, point C)

$$a^{n}_{B2/B4} = 0$$

since B_2 is sliding on B_4 and the relative motion is purely translational (no relative rotation)

$$a_{B2/B4}^{\ c} = 2(v_{B2/B4})(\omega_4) = 2\left(23.3\frac{\text{in}}{\text{sec}}\right)\left(14.5\frac{\text{rad}}{\text{s}}\right) = 676\frac{\text{in}}{\text{s}^2} = 56\frac{\text{ft}}{\text{s}^2}$$

The direction of the Coriolis component is that of $v_{B2/B4}$, which is parallel to the path of B_2 relative to B_4 (↗), rotated 90° in the direction of ω_4 (ccw). Therefore, the Coriolis component is directed perpendicular to link 4, down and toward the right.

The unknown quantities in the acceleration equation are $a_{B4}^{\ t}$ and $a_{B2/B4}^{\ t}$. Rewrite the acceleration equation, so that each unknown is the last term on both sides of the equation:

$$a_{B2}^{\ n} +> a_{B2}^{\ t} \rightarrow a_{B2/B4}^{\ t} = a_{B2/B4}^{\ n} +> a_{B2/B4}^{\ c} +> a_{B4}^{\ n} +> a_{B4}^{\ t}$$

An acceleration diagram drawn to scale is shown in Figure 6.21C. Scale the vector magnitudes from the diagram using the following equations:

$$a_{B2/B4}^{\ t} = 120\frac{\text{ft}}{\text{s}^2} \angle 76°$$

$$a_{B4}^{\ t} = 25\frac{\text{ft}}{\text{s}^2} = 300\frac{\text{in}}{\text{s}^2} \measuredangle 15°$$

and finally:

$$\alpha_4 = \frac{a^{t}_{B4}}{r_{CB4}} = \frac{300\text{in/s}^2}{3.6 \text{ in}} = 83.3\frac{\text{rad}}{\text{s}^2}$$

Because the tangential acceleration of B_4 was determined to be down and to the right, the corresponding rotational acceleration of link 4 must be counterclockwise; therefore:

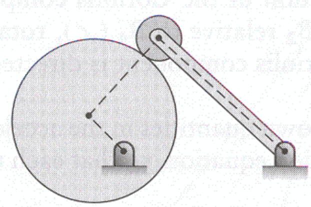
$$\alpha_4 = 83.3 \frac{\text{rad}}{\text{s}^2} \text{ ccw}$$

6.13 EQUIVALENT LINKAGES

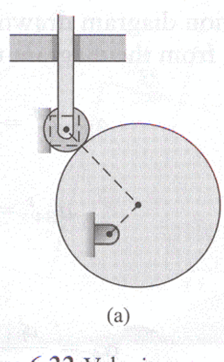
Up to this point in the text, the examples of motion analysis involved only mechanisms with primary joints, that is, pin and sliding joints. Recall from Chapter 1 that a higher order joint, such as a cam or gear joint, involves rolling and sliding motion. Both cams and gears are the focus in later chapters. However, the motion analysis of mechanisms with higher order joints can be performed using the concepts already presented.

Velocity and acceleration analysis of mechanisms that utilize higher order joints is greatly simplified by constructing an equivalent linkage. This method converts the instantaneous configuration of a mechanism to an *equivalent mechanism,* where the links are connected by primary joints. Figure 6.22 illustrates two cam mechanisms that contain rolling and sliding joints. The dotted lines represent the equivalent linkages.

Notice that the coupler of these equivalent linkages is drawn from the respective centers of curvature of the two mating links. For a finite length of time, the two centers of curvature for the two mating links will remain a constant distance apart.



(a)



(a)

Figure 6.22 Velocity curve.

Therefore, a coupler link with two pin joints can be used to replace this higher order, sliding joint. It is important to note that since the center of curvature may change as the mechanism moves. After the equivalent linkage is constructed, the method of motion analysis is identical to the problems previously encountered in this text.

6.14 ACCELERATION CURVES

The analyses, presented up to this point in the chapter, are used to calculate the acceleration of points on a mechanism at a specific instant. Although they are important, the results only provide a snapshot of the motion. The obvious shortcoming of this analysis is that determination of the extreme conditions throughout a cycle is difficult. It is necessary to investigate several positions of the mechanism to discover the critical phases.

As shown with velocity, it is also convenient to trace the acceleration of a certain point, or link, as the mechanism moves through its cycle. Such a trace provides information about critical phases in the cycle. An acceleration curve provides this trace. An acceleration curve plots the acceleration of a point, or link, as a function of time. It can be generated from a velocity curve, which was introduced in Section 5.8.

Recall that a velocity curve plots the velocity of a point or link as a function of time. A velocity curve is generated from a displacement curve, which was introduced in Section 3. Thus, a displacement curve can be used to generate a velocity curve which, in turn, can be used to generate an acceleration curve because:

$$\text{acceleration} = \frac{d(\text{velocity})}{dt}$$

Differential calculus suggests that the acceleration at a particular instant is the slope of the velocity curve at that instant. Since velocity is the time derivative of displacement, acceleration can also be written as:

$$\text{acceleration} = \frac{d^2(\text{displacement})}{dt^2}$$

This equation suggests that acceleration at a particular instant is the curvature of the displacement curve. Admittedly, curvature may not be as convenient to determine as the slope. However, it is easy to visualize the locations of extreme accelerations by locating the regions of sharp curves on the displacement diagram. Although values may be difficult to calculate, the mechanism can be configured to the desired position, then a thorough acceleration analysis can be performed, as presented in the preceding sections.

To determine values for the acceleration curves, it is best to determine the slope at several regions of the velocity curve (see Section 5.8).

6.14.1 Graphical Differentiation

The task is to estimate the slope of the velocity curve diagram at several points. Theslope, at a point, can be graphically estimated by sketching a line through the point of interest, tangent to the velocity curve. The slope of the line can be determined by calculating the change in y-value (velocity) divided by the change in x-value (time).

This procedure can be repeated at several locations along the velocity diagram. However, only the acceleration extremes and abrupt changes between them are usually desired. Using the notion of differential calculus and slopes, the positions of interest can be visually detected, which include:

- The steepest portions of the velocity diagram, which correspond to the extreme accelerations; and,
- The locations on the velocity diagram with the greatest curvature, which correspond to the abrupt changes of accelerations.

It must be noted that errors can easily occur when determining the slope of a curve. These errors are magnified as the slope is determined again, as deriving an acceleration curve from a velocity curve. Therefore, the values obtained for the acceleration diagram should be used cautiously. Identifying the positions of extreme accelerations is invaluable. A complete acceleration analysis, as presented in the previous sections of this chapter, should then be performed at these mechanism orientations to obtain accurate acceleration values. The benefit of the acceleration curve is locating the important mechanism configurations; therefore, comprehensive acceleration analysis can be performed.

EXAMPLE PROBLEM 6.11

A velocity curve was constructed for a compressor mechanism in Example Problem 5.14. Use this data to plot an acceleration curve.

Solution:

The main task of constructing an acceleration curve is to determine the slope of many points on the velocity curve. This velocity curve is reprinted as Figure 6.23.

From this curve, it is apparent that the curve has a horizontal tangent, or zero slope, at 0.007 and 0.027 sec. Therefore, the acceleration of the piston is zero at 0.007 and 0.027 sec. These points are labeled t_1 and t_3, respectively.

The maximum upward slope appears at 0 sec. This point was labeled as t_0. An estimate of the velocity can be made from the values of Δv_0 and Δt_0 read from the graph. Acceleration at 0 sec is estimated as:

$$a_0 = \frac{\Delta v_0}{\Delta t_0} = \frac{80\text{in/s}}{.0025\text{ sec}} = 32000\frac{\text{in}}{\text{s}^2}$$

Likewise, the maximum downward slope appears at 0.017 sec. This point was labeled as t_2. Again, an estimate of the acceleration can be made from the values of Δv_2 and Δt_2 read from the graph. The velocity at 0.017 sec is estimated as:

$$a_2 = \frac{\Delta v_2}{\Delta t_2} = \frac{-70\text{in/s}}{.005\text{ sec}} = -1400\frac{\text{in}}{\text{s}^2}$$

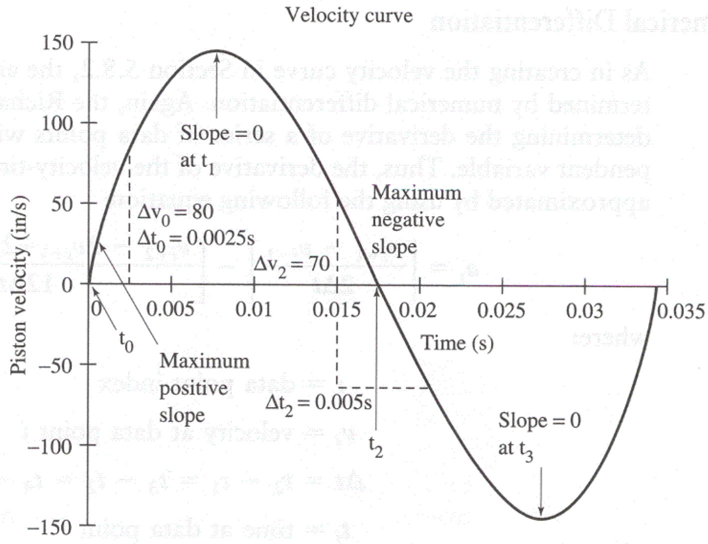


Figure 6.23 Velocity curve.

The procedure for determining the slope of the velocity curve can be repeated at other points in time. By compiling the slope and time information, an acceleration curve can be constructed (Figure 6.24).

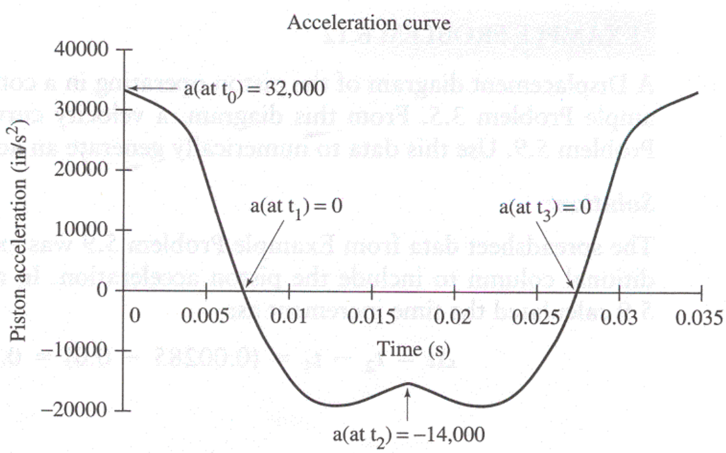


Figure 6.24 Acceleration curve for Example Problem 6.11.

6.14.2 Numerical Differentiation

As in creating the velocity curve in Section 5.8.2, the acceleration curve can be determined by numerical differentiation. Again, the Richardson method[3] is used for determining the derivative of a series of data points with an equally spaced, independent variable. Thus, the derivative of the velocity-time curve can be numerically approximated by using the following equation:

$$a_i = \left[\frac{v_{i+1} - v_{i-1}}{2\Delta t}\right] - \left[\frac{v_{i+2} - 2v_{i+1} + 2v_{i-1} - v_{i-2}}{12\Delta t}\right] \tag{6.22}$$

where:

$$i = \text{data point index}$$

$$v_i = \text{velocity at data point } i$$

$$\Delta t = t_2 - t_1 = t_3 - t_2 = t_4 - t_3$$

$$t_i = \text{time at data point } i$$

The second derivative can be similarly determined through numerical approximations. This allows the acceleration curve to be derived directly from the displacement-time curve. Again, the Richardson method is used to numerically determine the second derivative with the following equation:

$$a_i = \left[\frac{s_{i+1} - 2s_i + s_{i-1}}{\Delta t^2}\right] \tag{6.23}$$

where in addition to the notation above:

$$s_i = \text{displacement at data point } i$$

EXAMPLE PROBLEM 6.12

A Displacement diagram of the piston operating in a compressor was plotted in Example Problem 3.5. From this diagram, a velocity curve was derived in Example Problem 5.9. Use this data to numerically generate an acceleration curve.

Solution:

The spreadsheet data from Example Problem 5.9 was expanded by inserting an additional column to include the piston acceleration. In addition, Example Problem 5.9 calculated the time increment as:

$$\Delta t = t_2 - t_1 = (0.00285 - 0.0) = 0.00285 \text{ sec}$$

To illustrate the calculation of the accelerations, a few sample calculations using Equation 6.8 are shown below:

$$a_2 = \left[\frac{v_3 - v_1}{2\Delta t}\right] - \left[\frac{v_4 - 2v_3 + 2v_i - v_{12}}{12\Delta t}\right]$$

$$= \left[\frac{141.7 - 0.0}{2(.00285)}\right] - \left[\frac{137.5 - 2(141.7) + 2(0.0) - (-90.8)}{12(.00285)}\right]$$

$$= 26473.5\frac{\text{in}}{\text{sec}^2}$$

$$a_9 = \left[\frac{v_{10} - v_8}{2\Delta t}\right] - \left[\frac{v_{11} - 2v_{10} + 2v_8 - v_7}{12\Delta t}\right]$$

$$= \left[\frac{(-141.7) - (-96.4)}{2(.00285)}\right] - \left[\frac{-90.8) - 2(-141.7) + 2(-96.4) - (46.6)}{12(.00285)}\right]$$

$$= -9313.4\frac{\text{in}}{\text{sec}^2}$$

$$a_{12} = \left[\frac{v_{13} - v_{11}}{2\Delta t}\right] - \left[\frac{v_2 - 2v_{13} + 2v_{11} - v_{10}}{12\Delta t}\right]$$

$$= \left[\frac{(0.0) - (-141.7)}{2(.00285)}\right] - \left[\frac{(90.8) - 2(0.0) + 2(-90.7) - (141.7)}{12(.00285)}\right]$$

$$= 26473.5\frac{\text{in}}{\text{sec}^2}$$

The resulting information, with all values calculated, is given in Table 6.3 below. These values are plotted in Figure 6.25 to form an acceleration diagram, relative to time.

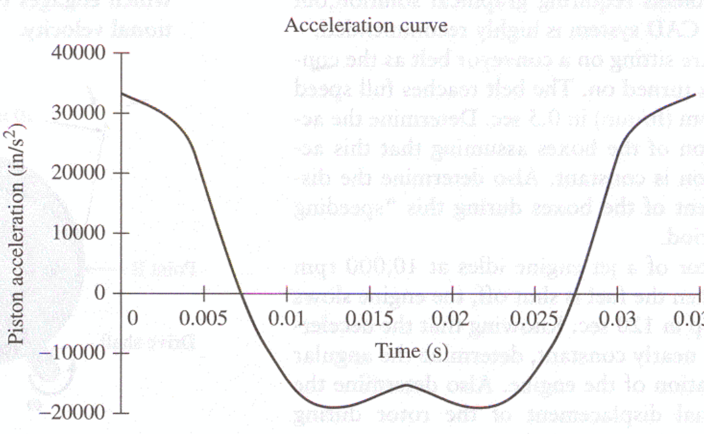


Figure 6.25

TABLE 6.3 Information for Example Problem 6.12

Data Point Index	Crank Angle (deg)	Time (0.001 sec)	Piston Displ. (in)	Piston Velocity $\left(\frac{in}{sec}\right)$	Piston Acceleration $\left(\frac{in}{sec^2}\right)$
1	0	0.00	0.00	0.00	34218.1
2	30	2.85	0.14	90.9	26473.5
3	60	5.70	0.48	141.7	8082.8
4	90	8.55	0.90	137.4	−9313.4
5	120	11.40	1.23	96.4	−17109.1
6	150	14.25	1.40	46.4	−17160.1
7	180	17.10	1.50	0.00	−16165.5
8	210	19.95	1.40	−46.6	−17160.1
9	240	22.80	1.23	−96.4	−17109.1
10	270	25.65	0.90	−137.5	−9313.4
11	300	28.50	0.48	−141.7	8082.8
12	330	31.35	0.14	−90.8	26473.5
13	360	34.20	0.00	0.00	34218.2

Notice that this curve is still rather rough. For accuracy purposes, it is highly suggested that the crank angle increment be reduced to 10° or 15°. When a spreadsheet is used to generate the acceleration data, even smaller increments are advisable and do not make the task any more difficult.

PROBLEMS

Manual drawing techniques can be instructive for problems requiring graphical solution,but using a CAD system is highly recommended.

6–1. Boxes are sitting on a conveyor belt as the conveyor is turned on. The belt reaches full speed of 45 fpm (ft/min) in 0.5 sec. Determine the acceleration of the boxes assuming that this acceleration is constant. Also determine the displacement of the boxes during this "speeding up" period.

6–2. The rotor of a jet engine idles at 10,000 rpm and, when the fuel is shut off, the engine slows to a stop in 120 sec. Knowing that the deceleration is nearly constant, determine the angular deceleration of the engine. Also determine the rotational displacement of the rotor during this "shut down" period.

Figure P6.3 shows a centrifugal clutch, which engages two shafts at a threshold rotational velocity.

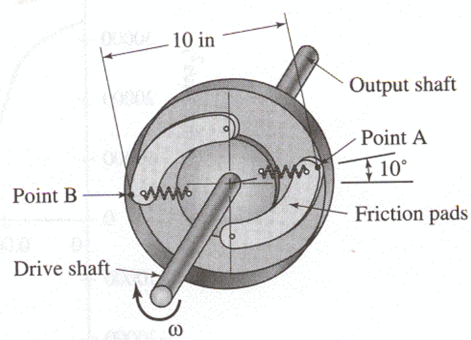


Figure P6.3 Problems 3-5.

6–3. Determine the normal acceleration of point A on the friction pads at the instant shown, while the drive shaft rotates at 300 rpm.

6–4. Determine the total acceleration of the friction pad at the instant shown, while the drive shaft rotates at 300 rpm and is speeding up at a rate of 300 rad/sec^2.

6–5. Determine the total acceleration of the friction pad at the instant shown, while the drive shaft rotates at 200 rpm and is slowing down at a rate of 200 rad/sec^2.

Relative Acceleration Problems

6–6. For the kinematic diagram shown in Figure P6.6, the length of link AB is 10 mm and $\theta = 35°$. Box A moves upward at a velocity of 10 mm/s and accelerates at 5 mm/sec^2. At the same time, the velocity of box B is 7 mm/sec and accelerates at a rate of 25 mm/sec^2. Graphically determine:

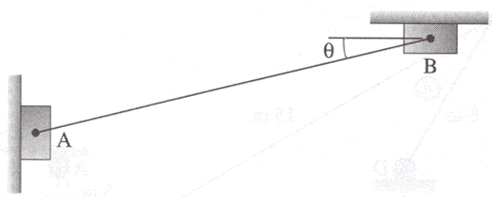


Figure P6.6 Problems 6 and 7.

A. The relative velocity of A with respect to B; and,

B. The relative acceleration of A with respect to B.

6–7. For the kinematic diagram shown in Figure P6.6, the length of link AB is 15 in and $\theta = 40°$. Box A moves upward at a velocity of 50 in/s and decelerates at 125 in/sec^2. At the same time, the velocity of the box B is 42 in/sec and accelerates at a rate of 48.6 in/sec^2. Analytically determine:

a) The relative velocity of A with respect to B; and

b) The relative acceleration of A with respect to B.

6–8. Figure P6.8 shows a device used to automatically open windows commonly found in elevated locations of gymnasiums and factories. At the instant when $\theta = 25°$, the drive nut moves to the right at a velocity of 1 ft/s and accelerates at 1 ft/sec^2. At the same time, the velocity of the shoe is 0.47 ft/sec and accelerates at a rate of 0.91 ft/sec^2. Construct a kinematic diagram and graphically determine:

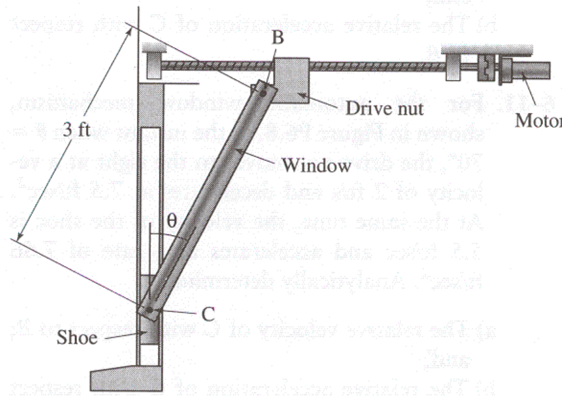


Figure P6.8 Problems 8 and 9.

a) The relative velocity of C with respect to B; and,

b) The relative acceleration of C with respect to B.

6–9. For the automatic window mechanism, shown in Figure P6.8, at the instant when $\theta = 55°$, the drive nut moves to the right at a velocity of 2 ft/s and accelerates at 1 ft/sec^2. At the same time, the velocity of the shoe is 2.95 ft/sec and accelerates at a rate of 8.51 ft/sec^2. Analytically determine:

a) The relative velocity of C with respect to B; and,

b) The relative acceleration of C with respect to B.

6–10. For the automatic window mechanism, shown in Figure P6.8, at the instant when $\theta = 30°$, the drive nut moves to the right at a velocity of 2 ft/s and decelerates at 2 ft/sec^2. At the same time, the velocity of the shoe is 1.15 ft/sec and accelerates at a rate of 0.90 ft/sec^2. Construct a kinematic diagram and graphically determine:

a) The relative velocity of C with respect to B; and,
b) The relative acceleration of C with respect to B.

6–11. For the automatic window mechanism, shown in Figure P6.8, at the instant when $\theta = 70°$, the drive nut moves to the right at a velocity of 2 ft/s and decelerates at 7.5 ft/sec^2. At the same time, the velocity of the shoe is 5.5 ft/sec and accelerates at a rate of 7.66 ft/sec^2. Analytically determine:

a) The relative velocity of C with respect to B; and,
b) The relative acceleration of C with respect to B.

Problems in Acceleration Analysis

For Problems 6–12 through 6–15, use the kinematic diagram shown in Figure P6.12 to determine:

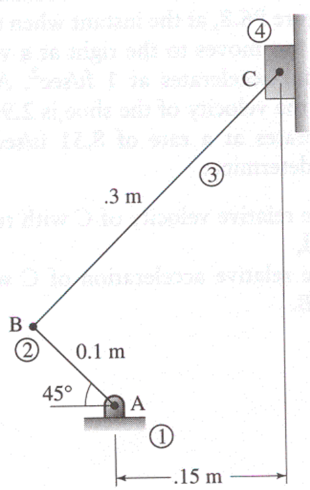


Figure P6.12 Problems 12-15.

A. The velocity of points B and C;
B. The rotational velocity of link 3;
C. The acceleration of points B and C; and,
D. The rotational acceleration of link 3.

6–12. Link 2 rotates clockwise at a constant velocity of 135 $\frac{\text{rad}}{\text{s}}$. Use graphical methods.

6–13. Link 2 rotates counterclockwise at a constant velocity of 45 $\frac{\text{rad}}{\text{s}}$. Use analytical methods.

6–14. Link 2 rotates clockwise at a constant velocity of 60 rad/sec and accelerates at a rate of 1000 rad/sec^2. Graphically (using either manual drawing techniques or CAD) determine:

6–15. Link 2 rotates clockwise at a constant velocity of 75 rad/sec and decelerates at a rate of 500 rad/sec^2. Use analytical methods.

For Problems 6–16 through 6–19, use the kinematic diagram shown in Figure P6.16 to determine:

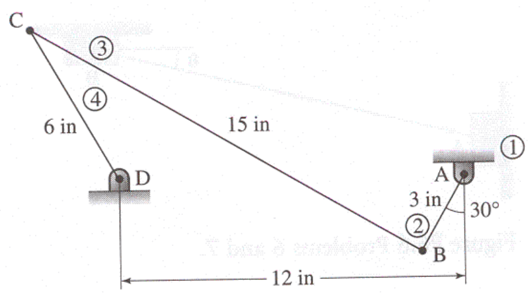


Figure P6.16 Problems 16-19.

a) The velocity of points B and C;
b) The rotational velocity of links 3 and 4;
c) The acceleration of points B and C; and,
d) The rotational acceleration of links 3 and 4.

6–16. Link 2 rotes counter-clockwise at a constant velocity of 10 rad/sec. Use graphical methods.

6–17. Link 2 rotates clockwise at a constant velocity of 65 rad/sec. Use analytical methods.

6–18. Link 2 rotates clockwise at a constant velocity of 25 rad/sec and decelerates at a rate of 300 rad/sec^2. Use graphical methods.

6–19. Link 2 rotates counterclockwise at a constant velocity of 40 rad/sec and accelerates at a rate of 300 rad/sec^2. Use analytical methods.

Figure P6.20 shows the mechanism that drives a needle of a sewing machine. For Problems 6–20 through 6–24, use Figure P6.20 to determine:

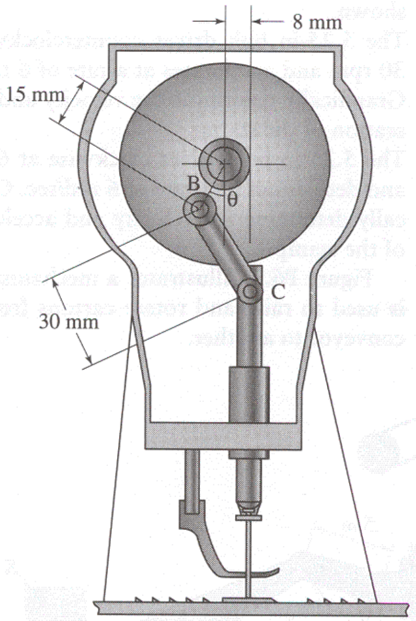


Figure P6.20 Problems 20-24.

A. The linear velocity and the linear acceleration of pin C; and,
B. The rotational velocity and rotational acceleration of the connecting rod, BC.

6–20. At the instant when $\theta = 30°$ and the drive wheel has a constant rotational velocity of 200 rpm, counter-clockwise. Use graphical methods.

6–21. At the instant when $\theta = 30°$, the drive wheel has a constant rotational velocity of 350 rpm, clockwise. Use analytical methods.

a) The linear velocity and the linear acceleration of pin B; and,
b) The rotational velocity of the connecting rod.

6–22. At the instant when $\theta = 120°$, the drive wheel has a clockwise rotational velocity of 50 rad/sec and decelerates at a rate of 900 rad/sec^2. Use graphical methods.

6–23. At the instant when $\theta = 120°$, the drive wheel has a counterclockwise rotational velocity of 20 rad/sec and accelerates at a rate of 400 rad/sec^2. Use analytical methods.

Figure P6.24 shows a coin-operated mechanical horse ride, commonly found at stores. For problems 6–24 through 6–29, construct a kinematic diagram and determine the rotational velocity and the rotational acceleration of the horse.

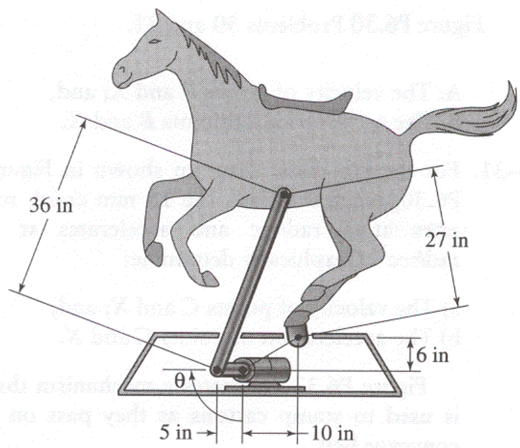


Figure P6.24 Problems 24-29.

6–24. The motor rotates clockwise at a constant 90 rpm and $\theta = 30°$. Use graphical methods.

6–25. The motor rotates clockwise at a constant 75 rpm $\theta = 30°$. Use analytical methods.

6–26. The motor rotates counterclockwise at 60 rpm and accelerates at a rate of 30 rad/sec^2 and $\theta = 45°$. Use graphical methods.

6–27. The motor rotates counterclockwise at 100 rpm and accelerates at a rate of 40 rad/sec^2 and $\theta = 45°$.Use analytical methods.

6–28. The motor rotates clockwise at 120 rpm and decelerates at a rate of 60 rad/sec^2, and $\theta = 60°$. Use graphical methods.

6–29. The motor rotates clockwise at 120 rpm and decelerates at a rate of 3 rad/sec^2 and $\theta = 60°$. Use analytical methods.

Acceleration of Points on a Floating Link

6–30. For the kinematic diagram shown in Figure P6.30, when $\theta = 30°$, point C moves to the right with a velocity of 10 mm/sec and accelerates at a rate of 2 mm/sec^2. Graphically determine:

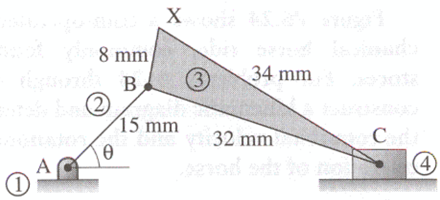


Figure P6.30 Problems 30 and 31.

A. The velocity of points *B* and *X*; and,
B. The acceleration of points *B* and *X*.

6–31. For the kinematic diagram shown in Figure P6.30, when $\theta = 30°$, the 15 mm crank rotates at 5 rad/sec and accelerates at 3 rad/sec^2. Graphically determine:

a) The velocity of points *C* and *X*; and,
b) The acceleration of points *C* and *X*.

Figure P6.32 illustrates a mechanism that is used to stamp cartons as they pass on a conveyor belt.

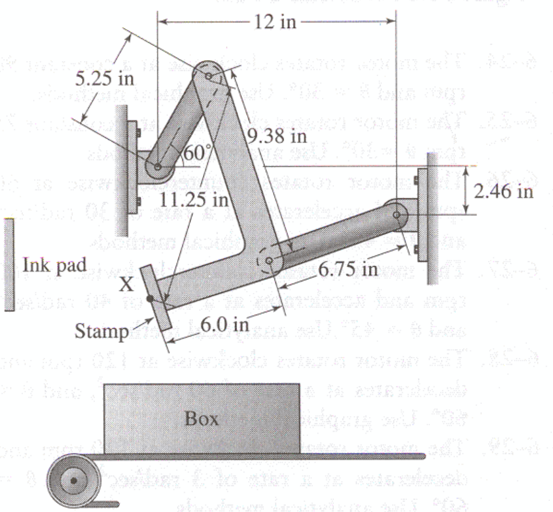


Figure P6.32 Problems 32-35.

6–32. At the instant shown, the 5.25-in link drives clockwise at a constant rate of 20 rpm. Graphically determine the velocity and acceleration of the stamp.

6–33. The 5.25-in link drives clockwise at a constant rate of 20 rpm. Graphically determine the velocity and acceleration of the stamp when the crank rotates 60° from the position shown.

6–34. The 5.25-in link drives counterclockwise at 30 rpm and accelerates at a rate of 6 rad/sec. Graphically determine the velocity and acceleration of the stamp.

6–35. The 5.25-in link drives clockwise at 60 rpm and decelerates at a rate of 6 rad/sec. Graphically determine the velocity and acceleration of the stamp.

Figure P6.36 illustrates a mechanism that is used to raise and rotate cartons from one conveyor to another.

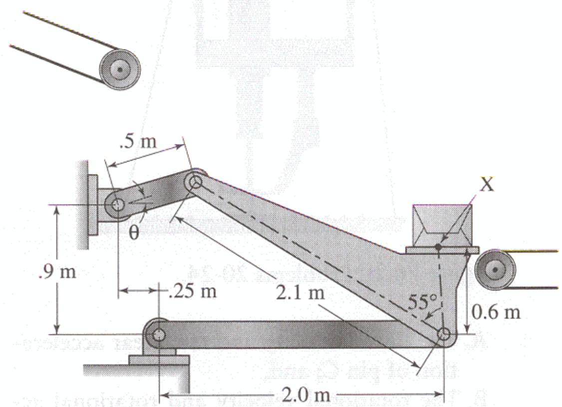


Figure P6.36 Problems 36-39.

6–36. At the instant shown, with $\theta = 20°$, the 0.5-m link drives counterclockwise at a constant rate of 12 rpm. Graphically determine the velocity and acceleration of point X.

6–37. At the instant shown, when $\theta = 30°$, the 0.5-m link drives clockwise at a constant rate of 20 rpm. determine the velocity and acceleration of point X.

6–38. At the instant shown, when $\theta = 20°$, the 0.5-m link drives clockwise at a rate of 30 rpm and accelerates at a rate of 5 rad/sec^2. Graphically determine the velocity and acceleration of point X.

6–39. At the instant shown, when $\theta = 0°$, the 0.5-m link drives counterclockwise at a rate of 18 rpm and decelerates at a rate of 5 rad/sec^2. Graphically determine the velocity and acceleration of point *X*.

Problems Dealing with Coriolis Acceleration

6–40. For the kinematic diagram shown in Figure P6.40, $\theta = 60°$, the rotational velocity of link 2 is 30 rad/sec, clockwise. Slide 3 also moves outward on link 2 at a rate of 15 mm/sec. Determine the Coriolis acceleration of point *B* on link 3 relative to link 2.

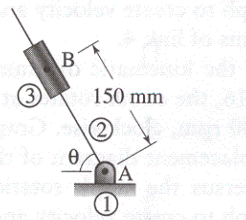


Figure P6.40 Problems 40-42.

6–41. For the kinematic diagram shown in Figure P6.40, $\theta = 45°$, the rotational velocity of link 2 is 30 rad/sec, counterclockwise. Slide 3 also moves outward on link 2 at a rate of 15 mm/sec. Determine the Coriolis acceleration of point *B* on link 3 relative to link 2.

6–42. For the kinematic diagram shown in Figure P6.40, $\theta = 30°$, the rotational velocity of link 2 is 30 rad/sec, clockwise. Slide 3 also moves inward on link 2 at a rate of 15 mm/sec. Determine the Coriolis acceleration of point *B* on link 3 relative to link 2.

For Problems 6–43 through 6–45, use the kinematic diagram shown in Figure P6.43, to determine:

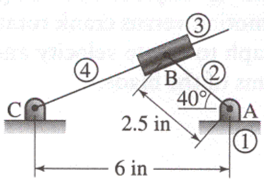


Figure P6.43 Problems 43-45.

A. The rotational velocity of link 4;
B. The sliding velocity of link 3 on link 4; and,
C. The angular acceleration of link 4.

6–43. When the rotationalocity of link 2 is 20 rad/sec counterclockwise. Use graphical methods.

6–44. The rotational velocity of link 2 is 20 rad/sec, counterclockwise. Use analytical methods.

6–45. When the rotational velocity of link 2 is 20 rad/sec, counterclockwise, and accelerates at a rate of 5 rad/sec. Use graphical methods.

Figure P6.46 illustrates the driving mechanism in a saber saw. Graphically determine the acceleration of the saw blade.

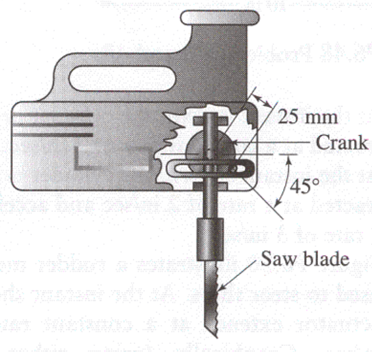


Figure P6.46 Problems 46 and 47.

6–46. At the instant shown, the crank rotates at a constant rate of 300 rpm, clockwise.

6–47. At the instant shown, the crank rotates at a rate of 200 rpm, clockwise, and accelerates at a rate of 45 rad/sec^2.

Figure P6.48 illustrates a bicycle pump mechanism. Graphically determine the rotational velocity and acceleration of the pedal assembly and the velocity and acceleration of point *X*.

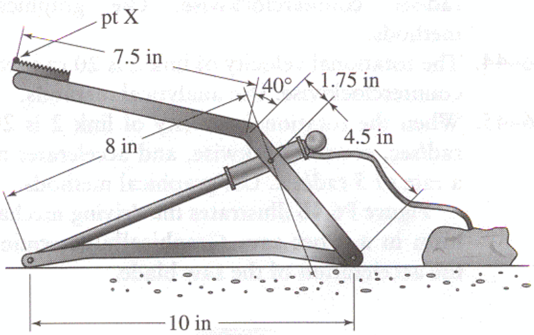


Figure P6.48 Problems 48 and 49.

6–48. At the instant shown, the cylinder is being retracted at a constant rate of 2 in/sec.

6–49. At the instant shown, the cylinder is being retracted at a rate of 2 in/sec and accelerates at a rate of 3 in/sec^2.

6–50. Figure P6.50 illustrates a rudder mechanism used to steer ships. At the instant shown, the actuator extends at a constant rate of 0.1 m/sec. Graphically (using either manual drawing techniques or CAD) determine the rotational velocity and acceleration of the rudder assembly.

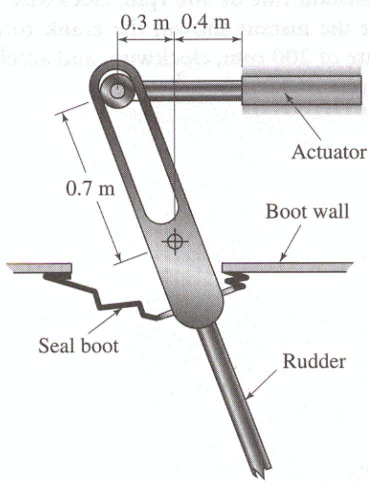


Figure P6.50 Problems 50 and 51.

6–51. For the rudder mechanism in Figure P6.50, the actuator extends at a rate of 0.1 m/sec, and decelerates at a rate of 0.3 m/sec^2. Graphically (using either manual drawing techniques or CAD) determine the rotational velocity and acceleration of the rudder assembly.

Acceleration Diagram Problems

6–52. For the kinematic diagram shown in Figure P6.12, the crank rotates at a constant rate of 600 rpm, counterclockwise. Graphically generate a displacement diagram of the motion of link 4 versus crank rotation. Then use this graph to create velocity and acceleration diagrams of link 4.

6–53. For the kinematic diagram shown in Figure P6.16, the crank rotates at a constant rate of 1800 rpm, clockwise. Graphically generate a displacement diagram of the rotation of link 4 versus the crank rotation. Then use this graph to create velocity and acceleration diagrams of link 4.

6–54. For the sewing machine mechanism shown in Figure P6.20, the crank rotates at a constant rate of 100 rpm, clockwise. Graphically generate a displacement diagram of needle motion versus crank rotation. Then use this graph to create velocity and acceleration diagrams of the needle.

6–55. For the coin-operated horse mechanism shown in Figure P6.24, the crank rotates at a constant rate of 60 rpm, counterclockwise. Graphically generate a displacement diagram of horse rotation versus crank rotation. Then use this graph to create velocity and acceleration diagrams of the horse.

6–56. For the saber saw mechanism shown in Figure P6.46, the crank rotates at a constant rate of 300 rpm, counterclockwise. Graphically generate a displacement diagram of saw blade motion versus crank rotation. Then use this graph to create velocity and acceleration diagrams of the blade.

Accleration Problems Using Working Model

Use the Working Model software obtainable with the text to create a model and provide the desired output for the following mechanisms.

6–57. Using the kinematic diagram in Figure P6.12, create an acceleration diagram of link 4 as the crank is rotating at a constant rate of 600 rpm, counter-clockwise.

6–58. Using the kinematic diagram in Figure P6.16, create an acceleration diagram of link 4 as the crank is rotating at a constant rate of 1800 rpm, clockwise.

6–59. Using the sewing machine mechanism in Figure P6.20, create an acceleration diagram of the needle as the crank is rotating at a constant rate of 100 rpm, clockwise.

6–60. Using the coin operated horse mechanism in Figure P6.24, create an angular acceleration diagram of the horse as the crank rotates at a constant rate of 60 rpm, counter-clockwise.

6–61. Using the saber saw mechanism shown in Figure P6.46, create an acceleration diagram of the saw blade as the crank rotates at a constant rate of 300 rpm, counter-clockwise.

CASE STUDIES

6–1. Figure C6.1 shows a specialty machine that is driven by crank shaft *I*. The top cap *H* on the machine drives another mechanism which is not shown. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

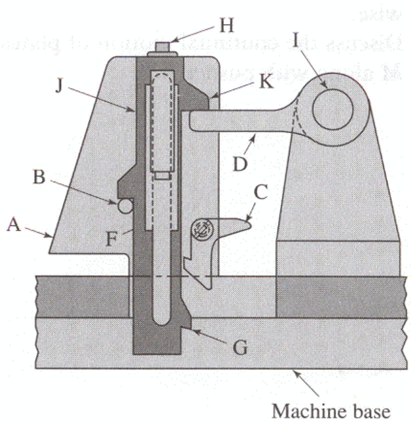


Figure C6.1 (Courtesy, Industrial Press)

1. As crank shaft *I* rotates clockwise, 30° from the position shown, what is the motion of slide *J*?
2. As crank shaft *I* rotates a few more degrees clockwise, what happens to the mechanism?
3. What purpose does item *C* serve?
4. As crank shaft *I* continues to rotate, describe the motion of the slide.
5. What purpose does item *B* serve?
6. Describe the purpose of this mechanism.

6–2. Figure C6.2 shows a machine feeds rivets to an automated assembly machine. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

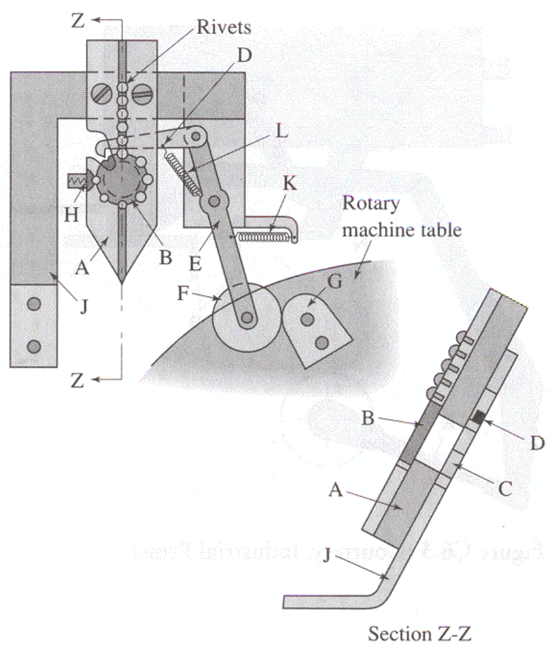


Figure C6.2 (Courtesy, Industrial Press)

1. As the rotating machine table turns counterclockwise, what happens to lever *E*?
2. What purpose does spring *K* serve?
3. As the rotating table turns, what is the motion of item *D*?
4. What purpose does spring *L* serve?
5. What is the general name of the type of connection between items *C* and *D*? Then describe the details of its function.
6. What is the purpose of the components at item *H*?
7. Describe motion and actions that take place during operation of this machine.

6–3. Figure C6.3 shows a specialty machine that accepts partially wrapped cartons from slot *B*. The machine folds the top and bottom wrappers down and moves the carton to another operation. In the position illustrated, a carton is shown as *A* and is being ejected from the machine. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

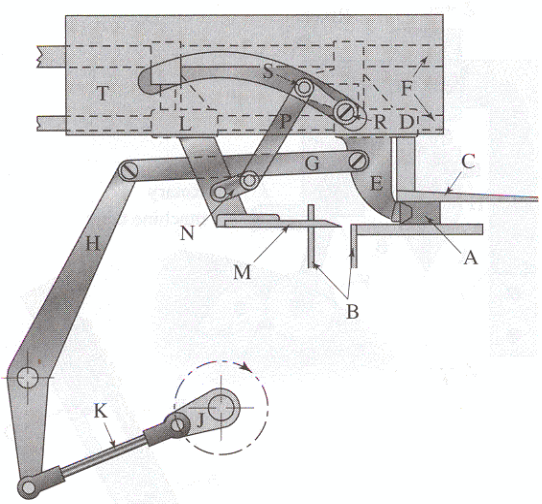


Figure C6.3 (Courtesy, Industrial Press)

1. As link *J* rotates clockwise, 90° from the position shown, what is the motion of bellcrank *H*?
2. As link *J* rotates clockwise, 90° from the position shown, what is the motion of pusher *E* and plate *C*?
3. As link *J* rotates clockwise, 90° from the position shown, what is the motion of pin *S*? (Note that pin *S* is not constrained to ride in the groove.)
4. As link *J* rotates clockwise, 90° from the position shown, what is the motion of guide pin *R*? (Note that pin *R* is constrained to ride in the groove.)
5. As link *J* rotates clockwise, 90° from the position shown, what is the motion of bellcrank *P*?
6. As link *J* rotates clockwise, 90° from the position shown, what is the motion of slide *L* and plate *M*?
7. Why is there a need for a short link *N*? Can't link *P* be directly connected to slide *L*?
8. Comment on the relative spacing between plate *C* and *M* after link *J* rotates 90° clockwise.
9. Discuss the continual motion of plates *C* and *M* along with pusher *E*.

7 Mechanism Design

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Describe mechanism synthesis.
2. Design an in-line slider-crank mechanism.
3. Knowing the desired mechanism time ratio, determine an appropriate imbalance angle.
4. Using graphical methods, design offset slider-crank and crank-rocker mechanisms.
5. Using graphical methods, design a single-pivoted link to move between two prescribed positions.
6. Using graphical methods, design a four-bar mechanism in which the coupler link moves between two prescribed positions.
7. Using graphical methods, design a four-bar mechanism, where the coupler link moves between three prescribed positions.

7.1 INTRODUCTION

Up to this point in the text, an emphasis was placed on the analysis of mechanisms. Methods have been explored that enable determination of displacements, velocities, and accelerations of a specific mechanism. Compared to this analysis, the design of a mechanism has the opposite problem: That is, given the desired motion, a specific mechanism must be determined. *Synthesis* is the term given to describe the process of designing a mechanism that produces a desired output motion for a given input motion.

To design a mechanism, intuition can be used along with analysis methods described in earlier chapters. This usually involves several iterations, especially for inexperienced designers, and can be an inefficient process. However, the iterate-and-analyze method does have merit, especially in problems where synthesis procedures have not or cannot be developed. However, several methods for mechanism synthesis have been developed and can be quite helpful. This chapter serves as an introduction to these methods. Because analytical techniques can become quite complex, the focus is on graphical techniques. As stated throughout the text, employing graphical techniques on a CAD system generally produces accurate results.

7.2 DESIGN OF SLIDER-CRANK MECHANISMS

Many mechanisms require the reciprocating, linear sliding motion of a component. Engines and compressors require a piston to move through a precise distance, called the *stroke,* as a crank continuously rotates. Many other applications require a similar, linear, reciprocating motion. A form of the slider-crank mechanism is used in virtually all these applications.

7.2.1 In-line Slider-Crank Mechanism

An in-line slider-crank mechanism has the crank pivot along the line of the sliding motion of the connecting arm/piston pin. An in-line slider-crank mechanism is illustrated in Figure 7.1A. The *stroke, s,* is defined as the linear distance that the sliding link exhibits between the extreme positions. Because the motion of the crank and connecting arm is symmetric about the sliding axis, the crank angle required to execute a forward stroke is the same as that for the return stroke. For this reason, the in-line slider-crank mechanism is a balanced linkage. Assuming that the crank is driven with a constant velocity source, as an electric motor, the time consumed during a forward stroke is equivalent to the time for the return stroke.

The design of an in-line slider-crank mechanism involves determining the appropriate length of the two links, l_2 and l_3, to achieve the desired stroke, s. As can be seen from Figure 7.1, the stroke of the in-line slider-crank mechanism is twice the length of the crank. Therefore, the length of crank, l_2, must be determined as followed:

(7.1) $$l_2 = \frac{s}{2}$$

The length of the connecting arm, l_3, does not affect the stroke of an in-line slider-crank mechanism. However, a shorter connecting arm yields greater velocity and ac-

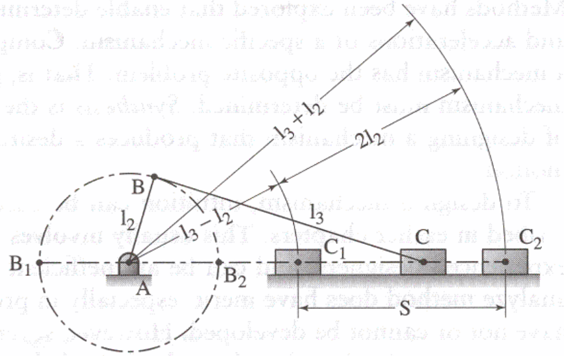


Figure 7.1 In-line slider-crank mechanism.

celeration values. Therefore, the connecting arm length should be made as large as possible. A detailed acceleration analysis (Chapter 6) should be completed to determine the inherent inertial loads. Maximum acceleration always occurs at the end of the stroke.

7.2.2 Offset Slider-Crank Mechanism

The mechanism illustrated in Figure 7.2A is an offset slider-crank mechanism. With an offset slider-crank mechanism, an offset distance is introduced. This offset distance, l_1, is the distance between the crank pivot and the sliding axis. With the presence of an offset, the motion of the crank and connecting arm is no longer symmetric about the sliding axis. Therefore, the crank angle required to execute the forward

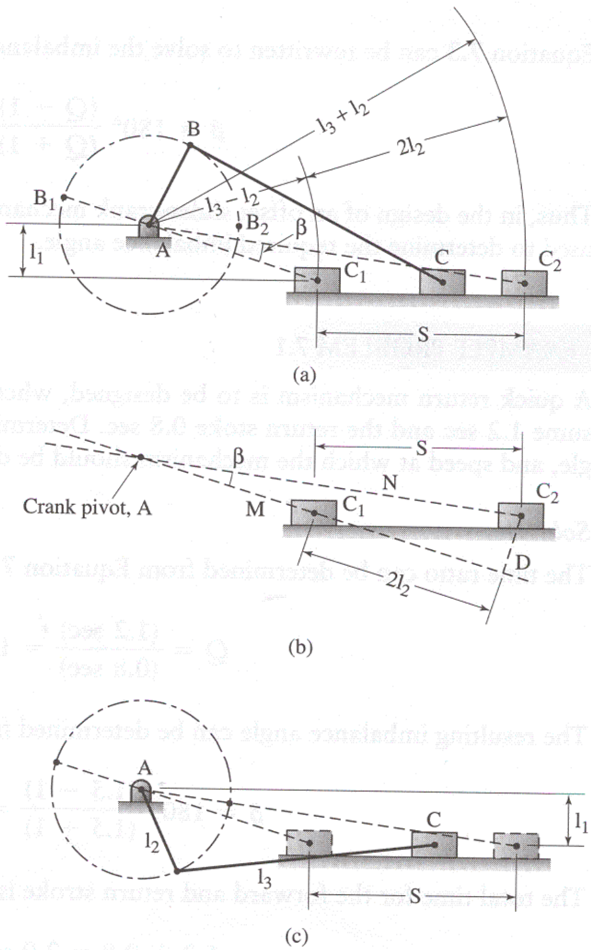


Figure 7.2 Offset slider-crank mechanism.

stroke is different from the crank angle required for the return stroke. An offset slider-crank mechanism provides a quick return when a slower working stroke is needed.

A measure of the quick return action of a mechanism is the *time ratio, Q*, which is defined as follows:

$$Q = \frac{\textit{Time of advance stroke}}{\textit{Time of return stroke}} \tag{7.2}$$

An imbalance angle, β, is also shown in Figure 7.2B. This angle can be related to the time ratio, Q:

$$Q = \frac{180° + \beta}{180° - \beta} \tag{7.3}$$

Equation 7.3 can be rewritten to solve the imbalance angle as follows:

$$\beta = 180° \frac{(Q - 1)}{(Q + 1)} \tag{7.4}$$

Thus, in the design of an offset slider-crank mechanism the desired time ratio can be used to determine the required imbalance angle.

EXAMPLE PROBLEM 7.1

A quick return mechanism is to be designed, where the outward stroke must consume 1.2 sec and the return stoke 0.8 sec. Determine the time ratio, imbalance angle, and speed at which the mechanism should be driven.

Solution:

The time ratio can be determined from Equation 7.2:

$$Q = \frac{(1.2 \text{ sec})}{(0.8 \text{ sec})} = 1.5$$

The resulting imbalance angle can be determined from Equation 7.5:

$$\beta = 180° \frac{(1.5 - 1)}{(1.5 + 1)} = 36°$$

The total time for the forward and return stroke is as follows:

$$1.2 + 0.8 = 2.0 \text{ sec}$$

Because one cycle of machine operation involves both the forward and return stroke, the time for the crank to complete one revolution is also 2.0 sec. The required crank speed is determined from Equation 5.4:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$= \frac{1.0 \text{ rev}}{2.0 \text{ sec}} = 0.5\frac{\text{rev}}{\text{sec}}$$

$$0.5\frac{\text{rev}}{\text{sec}} = \left(\frac{60 \text{ sec}}{1 \text{ min}}\right) = 30\frac{\text{rev}}{\text{min}}$$

In Figure 7.2A, the stroke of an offset slider-crank mechanism is always greater than twice the crank length. As the offset distance increases, the stroke also becomes larger. In Figure 7.2A, the feasible range for the offset distance can be written as:

(7.4) $$l_1 < l_3 - l_2$$

Locating the limiting positions of the sliding link are shown in Figure 7.2A and were discussed in Chapter 4. The design of a slider-crank mechanism involves determining an appropriate offset distance, l_1, and the two links lengths, l_2 and l_3, to achieve the desired stroke, s, and imbalance angle, β. The graphical procedure to synthesize a slider-crank mechanism is as follows:

1. Locate the sliding axis of the pin joint on the sliding link. This joint is labeled as point C in Figure 7.2A.
2. Draw the extreme positions of the sliding link, separated by the stroke, s.
3. At one of the extreme positions, construct *any* line M through the sliding link pin joint. This point is labeled C_1 on Figure 7.2B.
4. At the other extreme position, draw a line N through the sliding link pin joint, labeled C_2 in Figure 7.2B, inclined at an angle β from line M.
5. The intersection of lines M and N define the pivot point for the crank, point A. The offset distance, l_1, can be scaled from the construction (Figure 7.2B).
6. From the construction of the extreme positions, it is known that the arc length between C_1 and D is $2l_2$. Note that this arc is centered at point A. The distance C_1D can be scaled to determine the appropriate crank length, l_2.
7. From the construction of limiting positions, it is known that the length $AC_1 = l_3 - l_2$. The connecting arm length, l_3, is determined as $l_3 = AC_1 + l_2$.

The complete mechanism is shown in Figure 7.2C. The design procedure, implemented with a CAD system, achieves accurate results.

Note that any line N can be drawn though point C_1. Therefore, an infinite number of suitable mechanisms can be designed. In general, the mechanisms that produce the longest connecting arm have lower accelerations, and subsequently lower inertial forces. A detailed acceleration analysis should be completed to determine the inherent inertial loads (Chapter 6). As in the in-line slider-crank mechanism, maximum values of acceleration are obtained at the end of the stroke.

7.3 DESIGN OF CRANK-ROCKER MECHANISMS

A crank-rocker mechanism has also been discussed on several occasions. It is common for many applications where repeated oscillations are required. Figure 7.3A illustrates the geometry of a crank-rocker. Comparable to the stroke of a slider-crank mechanism, the crank-rocker mechanism exhibits a *throw angle,* ϕ (Figure 7.3A). This throw angle is defined as the angle between the extreme positions of the rocker link.

Similar to the offset slider-crank mechanism, a crank-rocker can be used as a quick return mechanism. The time ratio defined in Equations 7.2 and 7.3 equally apply to a crank-rocker. The imbalance angle, β, of a crank-rocker mechanism is also shown in Figure 7.3A.

The limiting positions of a crank-rocker are shown in Figure 7.3A and were discussed extensively in Chapter 3. Note that the radial length between the two extreme positions is twice the crank length. This notion becomes important when designing a crank-rocker mechanism.

The design of a crank-rocker mechanism involves determining appropriate lengths of all four links to achieve the desired throw angle, ϕ, and imbalance angle, β. The graphical procedure to synthesize a crank-rocker mechanism is as follows:

1. Locate the pivot of the rocker link, point D in Figure 7.3B.
2. Choose *any* feasible rocker length, l_4. This length is typically constrained by the spatial allowance for the mechanism.
3. Draw the two positions of the rocker, separated by the throw angle, ϕ.
4. At one of the extreme positions, construct *any* line M through the end of the rocker link. This point is labeled C_2 on Figure 7.3B.
5. At the other extreme position, draw a line N through the end of the rocker link, which is inclined an angle β from line M.
6. The intersection of lines M and N define the pivot point for the crank, point A. The length between the two pivots, l_1, can be scaled from the construction (Figure 7.3C).
7. From the construction of the extreme positions, it is known that the arc length between C_1 and E is $2l_2$. Note that this arc is centered at point A. The distance C_1E can be scaled to determine the appropriate crank length, l_2.
8. From the construction of limiting positions, it is known that the length $AC_1 = l_3 - l_2$. The coupler length, l_3, is determined by the equation:

$$l_3 = AC_1 + l_2$$

The completed mechanism is shown in Figure 7.3C. Note that any line M can be drawn though point C_2. Therefore, an infinite number of suitable mechanisms can be designed. In general, the mechanisms that produce the longest coupler have lower

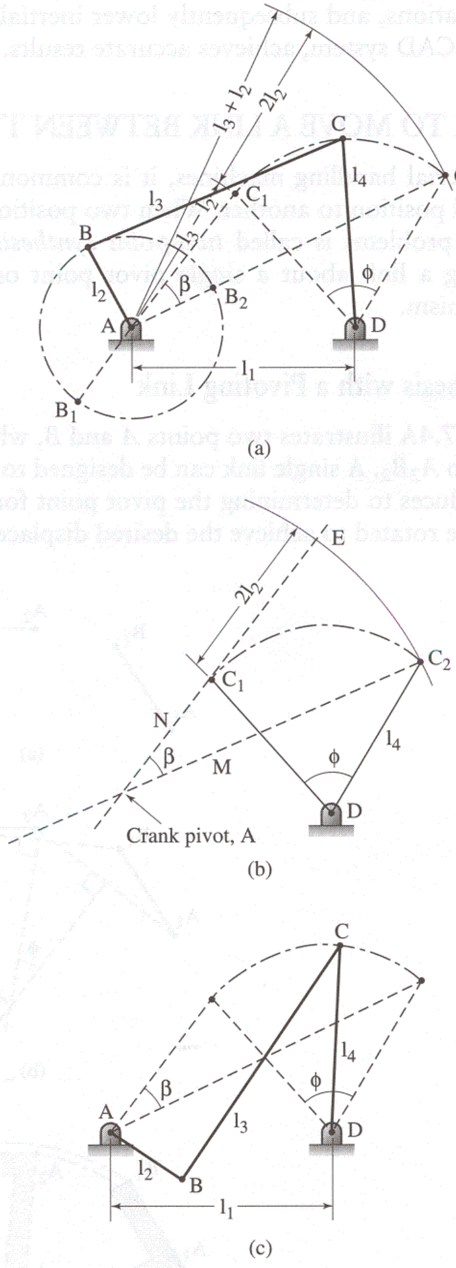


Figure 7.3 Crank-rocker mechanism.

accelerations, and subsequently lower inertial forces. This procedure, implemented with a CAD system, achieves accurate results.

7.4 MECHANISM TO MOVE A LINK BETWEEN TWO POSITIONS

In material handling machines, it is common to have a link that moves from one general position to another. When two positions of a link are specified, this class of design problems is called *two-point synthesis*. This task can be accomplished by rotating a link about a single pivot point or by using the coupler of a four-bar mechanism.

7.4.1 Two-Point Synthesis with a Pivoting Link

Figure 7.4A illustrates two points A and B, which must sit on a link and move from A_1B_1 to A_2B_2. A single link can be designed to produce this displacement. The problem reduces to determining the pivot point for this link, and the angle that the link must be rotated to achieve the desired displacement.

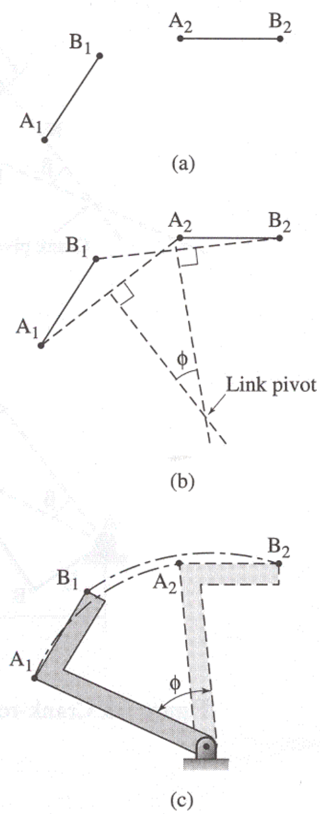


Figure 7.4 Two-point synthesis with a pivoting link.

The graphical procedure to design a pivoting link for two-point synthesis is as follows:

1. Construct two lines that connect A_1A_2 and B_1B_2, respectively.
2. Construct a perpendicular bisector of A_1A_2.
3. Construct a perpendicular bisector of B_1B_2.
4. The intersection of these two perpendicular bisectors is the required location for the link pivot point (Figure 7.4B).
5. The angle between the two perpendicular bisectors is the required angle that the link must be rotated about the link pivot to produce the desired displacement. This angle is labeled "ϕ" in Figure 7.4B.

The completed link is shown in Figure 7.4C.

7.4.2 Two-Point Synthesis of the Coupler of a Four-Bar Mechanism

In an identical problem to the one presented in the preceding section, Figure 7.5A illustrates two points A and B, which must sit on a link and move from A_1B_1 to A_2B_2. Some applications may make a single pivoted link unfeasible, such as when the pivot point of the single link is inaccessible. In these cases, the coupler of a four-bar linkage can be designed to produce the required displacement. Appropriate lengths must

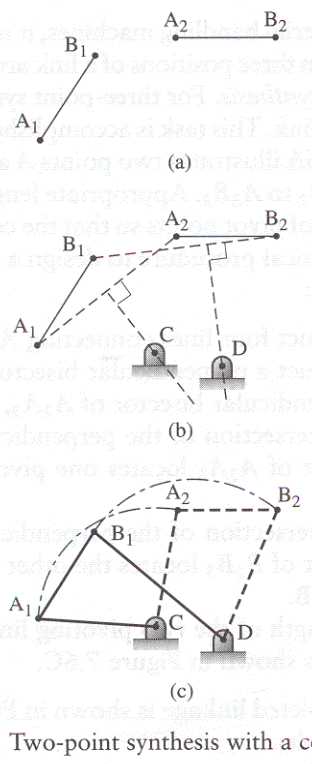


Figure 7.5 Two-point synthesis with a coupler link.

be determined for all four links and the location of pivot points so that the coupler achieves the desired displacement.

The graphical procedure to design a four-bar mechanism for two-point synthesis is as follows:

1. Construct two lines that connect A_1A_2 and B_1B_2, respectively.
2. Construct a perpendicular bisector of A_1A_2.
3. Construct a perpendicular bisector of B_1B_2.
4. The pivot points of the input and output link can be placed anywhere on the perpendicular bisector. These pivot points are shown as C and D in Figure 7.5B.
5. The length of the two pivoting links are determined by scaling lengths A_1C and B_1D (Figure 7.5C).

The completed linkage is shown in Figure 7.4C. Because the pivot points C and D can be placed anywhere along the perpendicular bisectors, an infinite number of mechanisms can be designed to accomplish the desired displacement. Note that longer pivoting links rotate at a smaller angle to move the coupler between the two desired positions. This produces larger transmission angles and reduces the force required to drive the linkage. The CAD system produces accurate results.

7.5 MECHANISM TO MOVE A LINK BETWEEN THREE POSITIONS

In some material handling machines, it is desired to have a link move between three positions. When three positions of a link are specified, this class of design problem is called *three-point synthesis*. For three-point synthesis, it generally is not possible to use a single pivoting link. This task is accomplished with the coupler of a four-bar mechanism.

Figure 7.6A illustrates two points A and B, which must sit on a link and move from A_1B_1 to A_2B_2 to A_3B_3. Appropriate lengths must be determined for all four links and the location of pivot points so that the coupler achieves this desired displacement.

The graphical procedure to design a four-bar mechanism for three point synthesis is as follows:

1. Construct four lines connecting A_1 to A_2, B_1 to B_2, A_2 to A_3, and B_2 to B_3.
2. Construct a perpendicular bisector of A_1A_2, a perpendicular bisector of B_1B_2, a perpendicular bisector of A_2A_3, and a perpendicular bisector of B_2B_3.
3. The intersection of the perpendicular bisector of A_1A_2 and the perpendicular bisector of A_2A_3 locates one pivot point. This is shown as point A in Figure 7.6B.
4. The intersection of the perpendicular bisector of B_1B_2 and the perpendicular bisector of B_2B_3 locates the other pivot point. This is shown as point D in Figure 7.6B.
5. The length of the two pivoting links is determined by scaling lengths A_1C and B_1D, as shown in Figure 7.6C.

The completed linkage is shown in Figure 7.6C. Again, the CAD system produces accurate results.

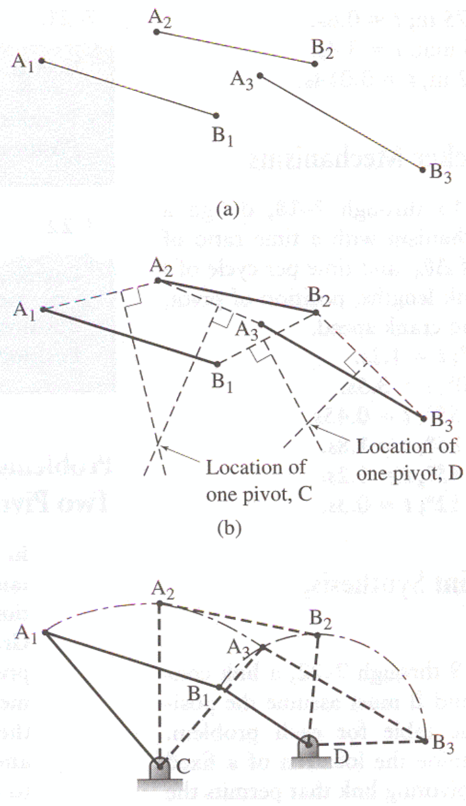


Figure 7.6 Three-point synthesis with a coupler link.

PROBLEMS

Design of Slider Crank Mechanisms

In Problems 7–1 through 7–3, a quick return mechanism is to be designed where the outward stroke consumes t_1 sec and the return stoke t_2 sec. Determine the time ratio, imbalance angle, and speed at which the mechanism should be driven.

7–1. $t_1 = 1.1s$; $t_2 = 0.8s$.
7–2. $t_1 = 0.35s$; $t_2 = 0.20s$.
7–3. $t_1 = 0.041s$; $t_2 = 0.027s$.

For Problems 7–4 through 7–6, a quick return mechanism drives at ω rpm and has an imbalance angle of β. Determine the time to complete the outward and return strokes.

7–4. $\omega = 180$ rpm; $\beta = 25°$.
7–5. $\omega = 75$ rpm; $\beta = 37°$.
7–6. $\omega = 500$ rpm; $\beta = 20°$.

For Problems 7–7 through 7–12, design a slider-crank mechanism with a time ratio of Q, stroke of s in, and time per cycle of t sec. Specify the link lengths, position of pivot, offset distance, and crank speed.

7–7. $Q = 1$; $s = 2$ in; $t = 1.2s$.
7–8. $Q = 1$; $s = 8$ mm; $t = 0.08s$.
7–9. $Q = 1$; $s = 0.9$ mm; $t = 0.4s$.

7–10. $Q = 1.25$; $s = 2.75$ in; $t = 0.6s$.
7–11. $Q = 1.37$; $s = 46$ mm; $t = 3.4s$.
7–12. $Q = 1.15$; $s = 1.2$ in; $t = 0.014s$.

Design of Crank-Rocker Mechanisms

For Problems 7–13 through 7–18, design a crank-rocker mechanism with a time ratio of Q, throw angle of $\Delta\theta_4$, and time per cycle of t sec. Specify the link lengths, position of pivot, offset distance, and crank speed.

7–13. $Q = 1$; $\Delta\theta_4 = 78°$; $t = 1.2s$.
7–14. $Q = 1$; $\Delta\theta_4 = 100°$; $t = 3.5s$.
7–15. $Q = 1.15$; $\Delta\theta_4 = 55°$; $t = 0.45s$.
7–16. $Q = 1.24$; $\Delta\theta_4 = 85°$; $t = 1.8s$.
7–17. $Q = 1.36$; $\Delta\theta_4 = 45°$; $t = 1.2s$.
7–18. $Q = 1.1$; $\Delta\theta_4 = 112°$; $t = 0.3s$.

Problems of Two Point Synthesis, Single Pivot

For Problems 7–19 through 7–22, a link containing points A and B must assume the positions listed in the table for each problem. Graphically determine the location of a fixed pivot for a single pivoting link that permits the motion listed. Also determine the degree that the link must be rotated to move from position 1 to position 2.

7–19.

Coordinates:	A_x (in)	A_y (in)	B_x (in)	B_y (in)
Position 1	0.00	9.00	5.00	9.00
Position 2	6.36	6.36	9.90	2.83

7–20.

Coordinates:	A_x (in)	A_y (in)	B_x (in)	B_y (in)
Position 1	2.28	5.34	6.86	7.38
Position 2	9.74	8.50	12.5	4.34

7–21.

Coordinates:	A_x (mm)	A_y (mm)	B_x (mm)	B_y (mm)
Position 1	−53	41	78	19
Position 2	−36	40	87	−9

7–22.

Coordinates:	A_x (mm)	A_y (mm)	B_x (mm)	B_y (mm)
Position 1	26	47	83	11
Position 2	97	30	151	72

Problems of Two Point Synthesis, Two Pivots

In Problems 7–23 through 7–26, a link containing points A and B is to assume the positions listed in the table for each problem. Graphically, find the location of two fixed pivots and the lengths of all four links of a mechanism with a coupler that will exhibit the motion listed. Also, determine the amount that the pivot links must be rotated to move the coupler from position 1 to position 2.

7–23.

Coordinates:	A_x (in)	A_y (in)	B_x (in)	B_y (in)
Position 1	−0.2	4.8	4.4	3.3
Position 2	−3.1	3.2	1.5	5.0

7–24.

Coordinates:	A_x (in)	A_y (in)	B_x (in)	B_y (in)
Position 1	0.9	4.5	9.0	7.7
Position 2	−1.0	5.6	5.6	11.4

7–25.

Coordinates:	A_x (mm)	A_y (mm)	B_x (mm)	B_y (mm)
Position 1	−40	−60	30	−30
Position 2	−67	−26	8	−42

7–26.

Coordinates:	A_x (mm)	A_y (mm)	B_x (mm)	B_y (mm)
Position 1	−37	−2	−18	1
Position 2	−18	−3	1	−8

Problems of Three Point Synthesis

For Problems 7–27 through 7–30, a link containing points A and B must assume the three positions listed in the table for each problem. Graphically, find the location of two fixed pivots and the lengths of all four links of a mechanism with a coupler that will exhibit the motion listed. Also, determine the amount that the pivot links must be rotated to move the coupler from position 1 to position 2, then from position 2 to position 3.

7–27.

Coordinates:	A_x (in)	A_y (in)	B_x (in)	B_y (in)
Position 1	−1.0	−0.9	5.3	−1.8
Position 2	−2.7	−1.3	3.6	−1.0
Position 3	−4.4	−2.0	1.8	−0.5

7–28.

Coordinates:	A_x (in)	A_y (in)	B_x (in)	B_y (in)
Position 1	−5.5	−0.1	7.9	5.2
Position 2	−2.4	0.5	12.1	1.2
Position 3	−0.6	1.6	13.7	−1.1

7–29.

Coordinates:	A_x (mm)	A_y (mm)	B_x (mm)	B_y (mm)
Position 1	0	40	55	45
Position 2	21	51	72	31
Position 3	39	49	82	15

7–30.

Coordinates:	A_x (mm)	A_y (mm)	B_x (mm)	B_y (mm)
Position 1	43	−76	150	−50
Position 2	3	−52	110	−72
Position 3	−12	−33	91	−69

CASE STUDIES

7–1. Figure C7.1 shows a mechanism that drives a sliding block, I. Block I, in turn, moves the blade of a power hack saw. Carefully examine the configuration of the components in the mechanism. Then answer the following questions to gain insight into the operation of the mechanism.

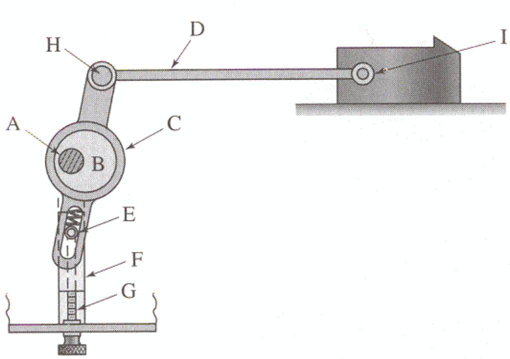


Figure C7.1 (Courtesy, Industrial Press)

1. As shaft A rotates 90°, what is the motion of item B?
2. As shaft A rotates 90°, what is the motion of item C?
3. Why is a slot at roller E?
4. As shaft A rotates 90°, what is the motion of pin H?
5. As shaft A rotates 90°, what is the motion of pin I?
6. Determine the mobility of this mechanism.

7. As thread G rotates to pull roller E downward, how does that alter the motion of link C?
8. As thread G rotates to pull roller E downward, how does that alter the motion of pin H?
9. What is the purpose of this mechanism?

7–2. Figure C7.2 shows a mechanism that also drives a sliding block B. This sliding block, in turn, drives a cutting tool. Carefully examine the configuration of the components in the mechanism. Then answer the following questions to gain insight into the operation of the mechanism.

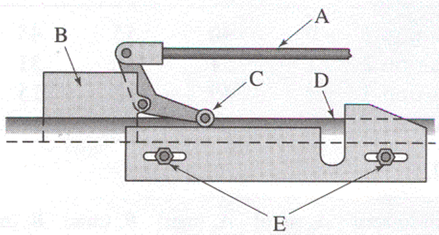


Figure C7.2 (Courtesy, Industrial Press)

1. As rod A drives to the right, what is the motion of sliding block B?
2. Describe the motion of sliding block B as roller C reaches groove D.
3. Describe the motion of sliding block B as rod A drives to the left.
4. Describe the continual motion of sliding block B.
5. What is the purpose of this mechanism?
6. Describe a device that could drive rod A to the left and right.
7. The adjustment slots at E provide what feature to the mechanism?

8 Computer-Aided Mechanism Analysis

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Understand the basics of a general spreadsheet.
2. Understand the strategy for using a general spreadsheet for mechanism analysis.
3. Create computer routines for determining kinematic properties of either four-bar or slider-crank mechanisms.

8.1 INTRODUCTION

Throughout the text, both graphical and analytical techniques of mechanism analysis are introduced. As the more accurate, analytical solutions are desired for several positions of a mechanism, the number of calculations can become unwieldy. In these situations, the use of computer solutions is appropriate. Computer solutions are also valuable when several design iterations must be analyzed. In Chapter 2, Mechanism Simulation, the use of dedicated dynamic analysis software was introduced. This chapter focuses on other forms of computer approaches to mechanism analysis. These other forms include using spreadsheets and creating routines using programming languages.

8.2 SPREADSHEETS

Spreadsheets are very popular in the professional environment for a variety of tasks. Commercially available spreadsheets include Microsoft® Excel, Lotus® 1-2-3, and Borland Quattro Pro®. Although the commands are different for the various spreadsheets, all have numerous built-in functions, ease of plotting results, and the ability to recognize formulas. These analytical features prompted widespread use of spreadsheets for more routine mechanism problems. Spreadsheets have been used in various problem solutions in this text. This section outlines the basics of using spreadsheets. Of course the specific software manuals should be consulted for further details.

A spreadsheet is arranged in a large array of columns and rows. The number of columns and rows varies among the different software products. Column headings

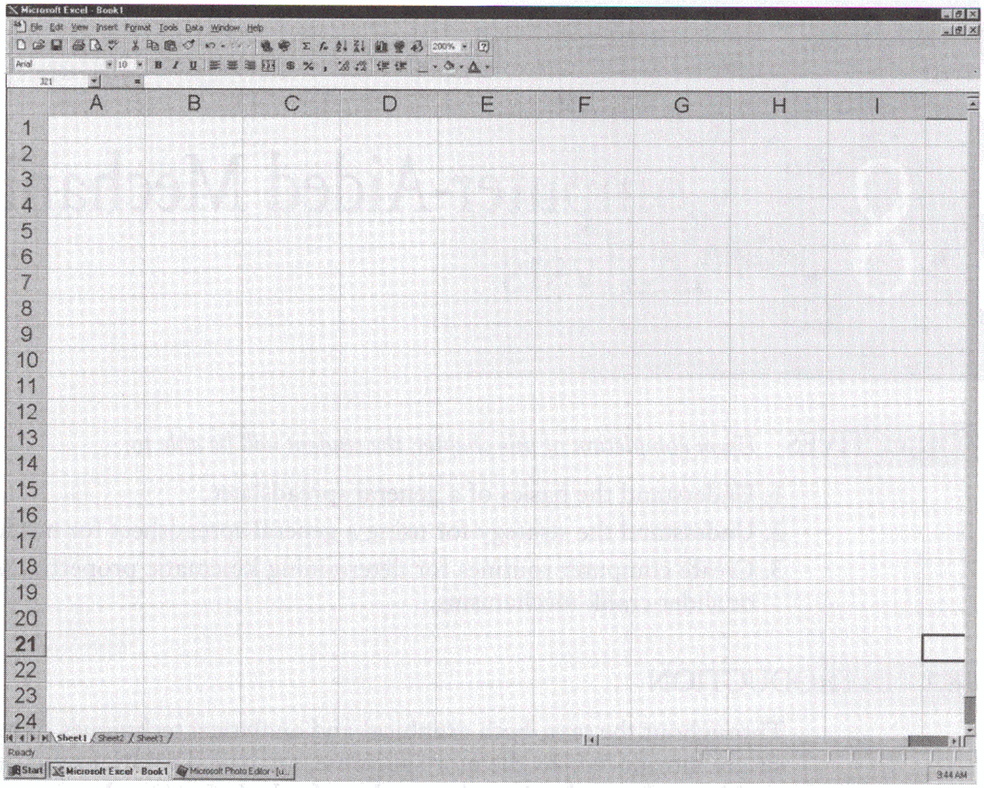


Figure 8.1 General spreadsheet.

are lettered from A to Z , then from AA to AZ, then BA to BZ, and so on. Row headings are numbered 1, 2, 3, etc. The top corner of a general spreadsheet is shown in Figure 8.1. The intersection of a column and a row is called a cell. Each cell is referred to by a cell address, which consists of the column and row that define the cell. Cell D3 is defined by the fourth (D) column and the third row. The cursor can be moved among cells with either the keyboard (arrow keys) or a mouse.

The value of a spreadsheet lies in storing, manipulating, and displaying data contained in a cell. This data commonly consists of either text, numbers, or formulas. The spreadsheet shown in Figure 8.2 has text entered into cells A1, F1, and F2 and numbers entered into cells A2 through A24, G1, and G2.

Although subtle differences may exist in the syntax among the spreadsheet programs, the logic behind creating formulas is identical. The syntax given here is applicable to Microsoft® Excel. The user's manual of another product should be consulted for the details on any differences in syntax.

Entering a formula into a cell begins with an equal sign (=). The actual formula is then constructed using values, operators (+, −, *, /), cell references (e.g., G2), and

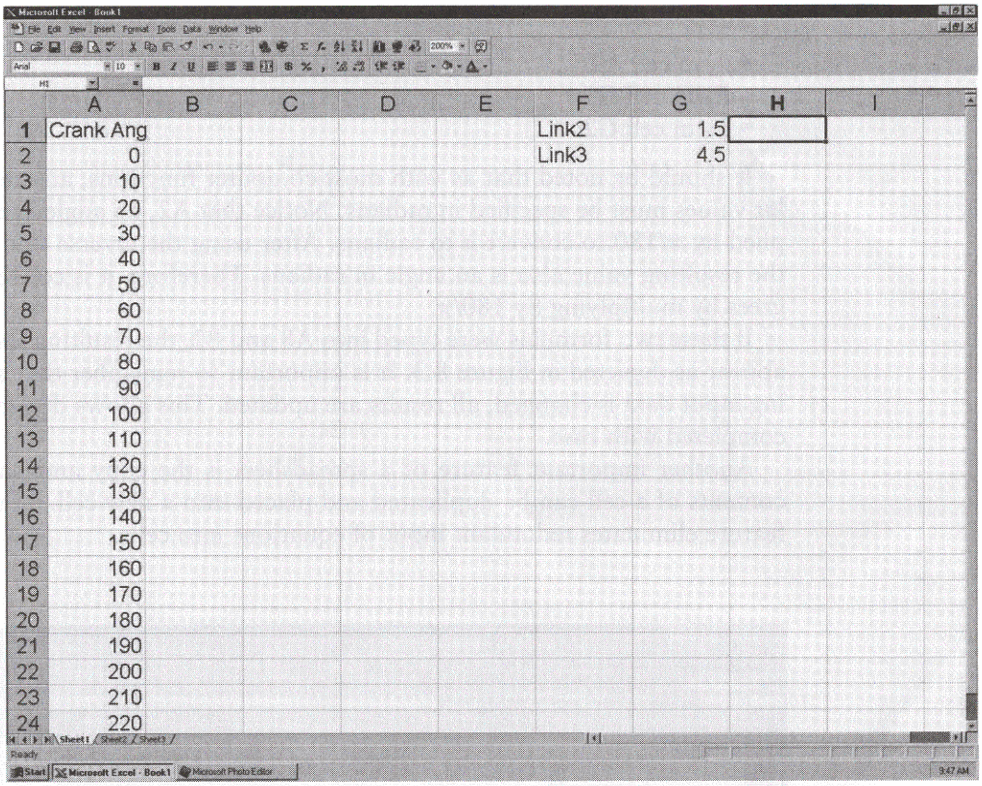


Figure 8.2

functions (e.g., SIN, AVG). Formulas can get rather complex. As an example, a simple formula can be placed in cell A8:

=A7 + 10

Although the actual cell contents would contain this formula, the spreadsheet would visually show the number 60 in cell A8. The calculation would be automatically performed. For another example, the following expression can be inserted into cell B2:

=(ASIN(G1*SIN(A2*3.1416/180)/G2))*180/3.1416

This expression represents the angle between connecting rod and the sliding plane for an in-line, slider-crank mechanism. It was presented as Equation 4.1:

$$\theta_3 = \sin^{-1}\left(\frac{L_2 \sin \theta_2}{L_3}\right) \tag{4.1}$$

The spreadsheet formula assumes that the following values have been entered:

- θ_2 in cell A2
- L_2 in cell G1
- L_3 in cell G2

It should be noted that as with most computer functions, any reference to angular values must be specified in radians. Notice that A2, an angle in degrees, is multiplied by $\pi/180$ to convert it to radians. After using the inverse sine function, ASIN, the resulting value also is an angle in radians. Therefore, it is converted back to degrees by multiplying by $180/\pi$.

If these two formulas were typed into A8 and B2, the resulting spreadsheet would appear as depicted in Figure 8.3. It is important to remember that as a cell containing input data is changed, all results are updated. This allows design iterations to be completed with ease.

Another important feature of a spreadsheet is the copy and paste feature. The contents of a cell can be duplicated and placed into a new cell. The copy and paste feature eliminates redundant input of equations into cells.

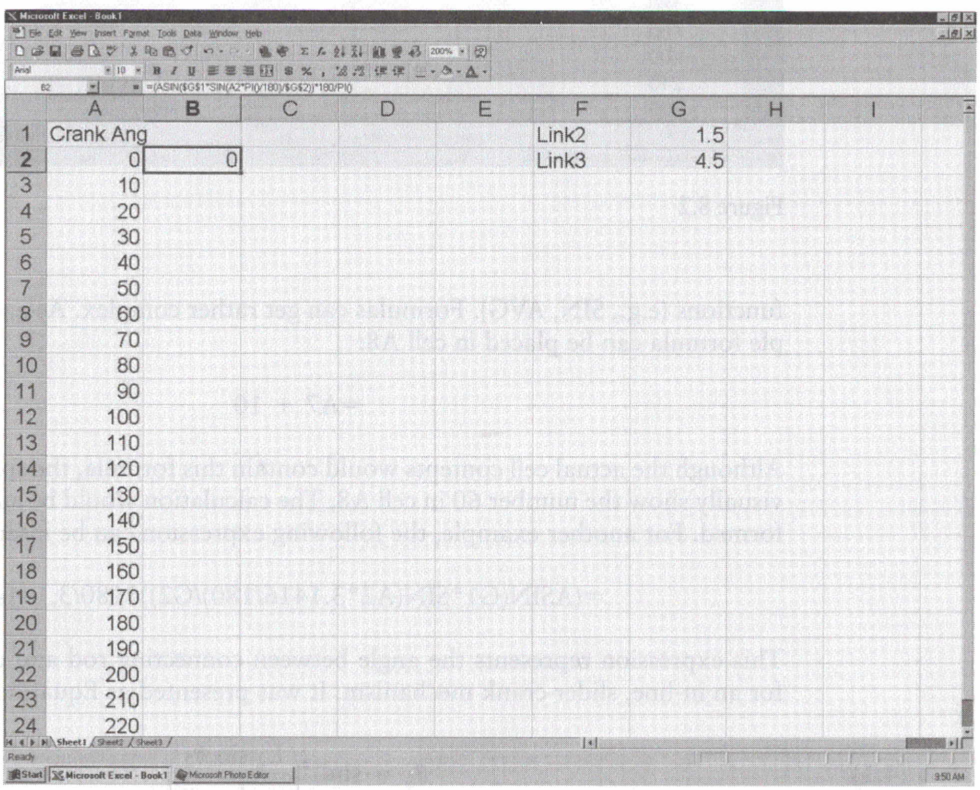


Figure 8.3

Cell references in a formula can be either relative or absolute. Relative references are automatically adjusted when a copy of the cell is placed into a new cell. Consider the following formula—

$$=A7 + 10$$

—into cell A8. The cell reference A7 is a relative reference to the cell directly above the cell that contains the formula, A8. If this equation were copied and placed into cell A9, the new formula would become:

$$=A8 + 10$$

Again, the cell reference A8 is a relative one; therefore, the spreadsheet would automatically adjust the formula.

An absolute address does not automatically adjust the cell reference after using the copy and paste feature. However, to specify an absolute reference, a dollar symbol must be placed prior to the row and column. For example, an absolute reference *to cell G1 must appear as* $G$1.

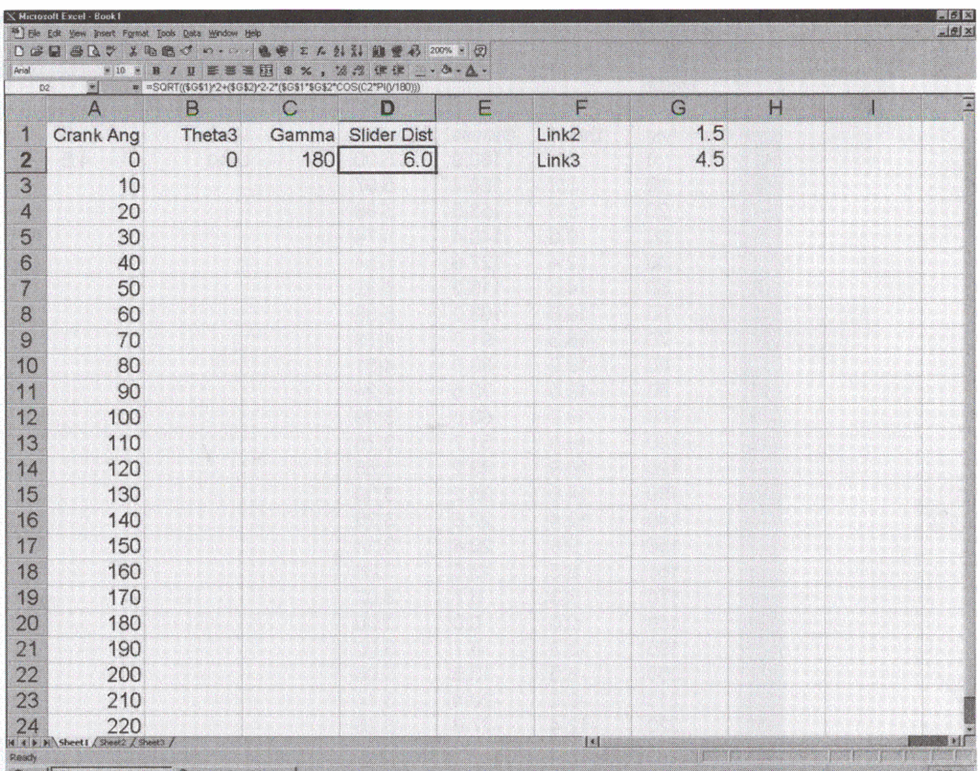


Figure 8.4

Consider the complex formula placed into cell B2. To be most efficient, this formula should be slightly modified to read:

=(ASIN($G$2*SIN(A2*3.1416/180)/$G$1))*180/3.1416

In this manner, only the angle in cell A2 is a relative address. If the formula were copied to cell B3, the new formula would become:

=(ASIN($G$2*SIN(A3*3.1416/180)/$G$1))*180/3.1416

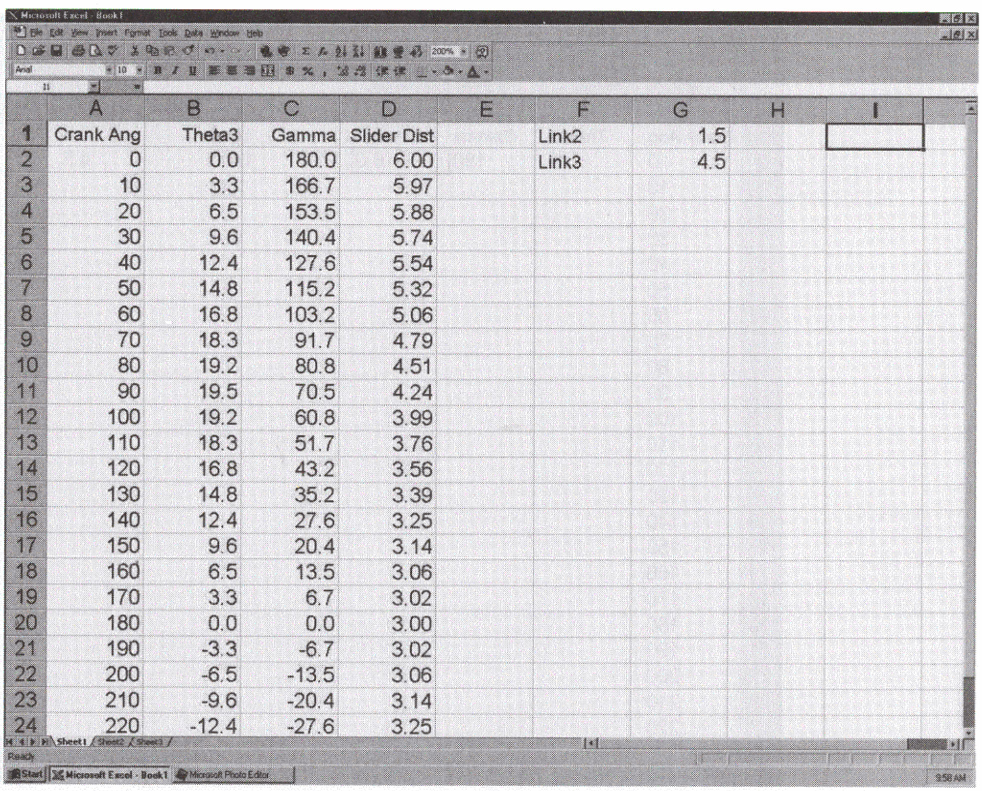
Notice that the address of cell A2 has been automatically adjusted to read "A3." The connecting rod angle is calculated for the crank angle specified in cell A3.

To continue with an analysis of a mechanism, the following formula can be typed into cell C2:

=180 − (A2 + A3)

This formula calculates the interior angle between the crank and connecting rod (Equation 4.6):

(4.2) $$\gamma = 180^\circ - (\theta_2 + \theta_3)$$



	A	B	C	D	E	F	G	H	I
1	Crank Ang	Theta3	Gamma	Slider Dist		Link2	1.5		
2	0	0.0	180.0	6.00		Link3	4.5		
3	10	3.3	166.7	5.97					
4	20	6.5	153.5	5.88					
5	30	9.6	140.4	5.74					
6	40	12.4	127.6	5.54					
7	50	14.8	115.2	5.32					
8	60	16.8	103.2	5.06					
9	70	18.3	91.7	4.79					
10	80	19.2	80.8	4.51					
11	90	19.5	70.5	4.24					
12	100	19.2	60.8	3.99					
13	110	18.3	51.7	3.76					
14	120	16.8	43.2	3.56					
15	130	14.8	35.2	3.39					
16	140	12.4	27.6	3.25					
17	150	9.6	20.4	3.14					
18	160	6.5	13.5	3.06					
19	170	3.3	6.7	3.02					
20	180	0.0	0.0	3.00					
21	190	-3.3	-6.7	3.02					
22	200	-6.5	-13.5	3.06					
23	210	-9.6	-20.4	3.14					
24	220	-12.4	-27.6	3.25					

Figure 8.5

Because the angles are simply added, and a function is not called, a radian equivalent is not required.

Also, the following formula can be typed into cell D2:

```
=SQRT(($G$1)^2+($G$2)^2-($G$1+$G$2*SIN(A2*3.1416/180))^2)
```

This formula calculates the distance from the crank pivot to the slider pin joint (Equation 4.1):

(4.3)
$$L_4 = \sqrt{L_2{}^2 + L_3{}^2 + 2(L_2)(L_3)\cos\gamma}$$

If these two formulas were typed into C2 and D2, and text descriptions were typed into cells B1, C1, and D1, the resulting spreadsheet would appear as depicted in Figure 8.4.

Finally, since much care was taken with using absolute and relative cell addresses in creating the formulas in B2, C2, and D2, they can be copied into the cells down their respective columns. The user's manual should be consulted for the actual steps needed to copy the data into the remaining cells, which is usually a simple two- or three-step procedure. The resulting spreadsheet is shown in Figure 8.5.

EXAMPLE PROBLEM 8.1

Figure 8.6 illustrates a linkage that operates a water nozzle at an automatic car wash. Using a spreadsheet, analytically determine the angular motion of the nozzle throughout the cycle of crank rotation.

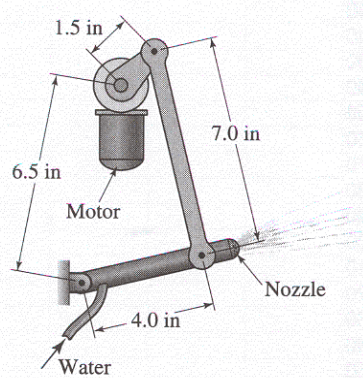


Figure 8.6 Water nozzle linkage for Example Problem 8.1.

Solution:

The nozzle mechanism is a familiar four-bar linkage. Figure 8.7 shows the kinematic representation of this mechanism.

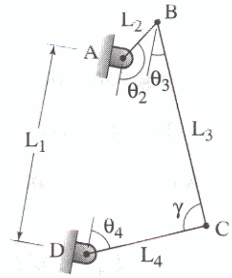
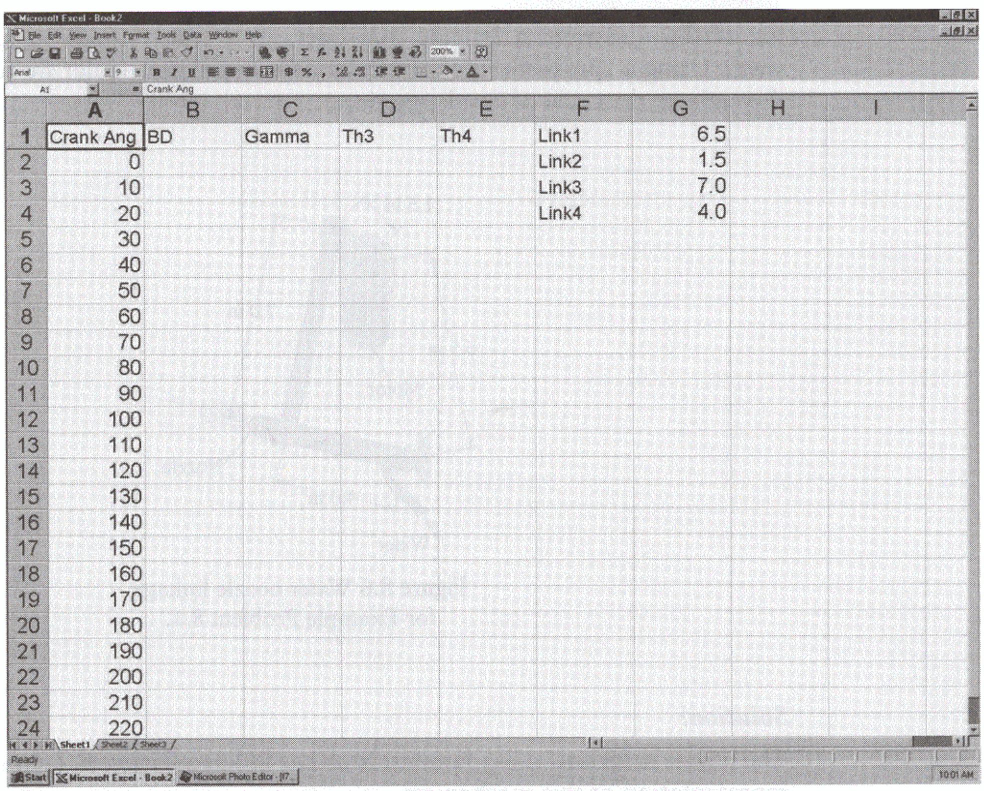


Figure 8.7 Kinematic sketch for Example Problem 8.1.

A spreadsheet for this analysis has been set-up and the upper portion is shown in Figure 8.8.

General equations, which govern the motion of the links of a four bar mechanism, were given in Chapter 4. Equation 4.7 gave the general equation for the diagonal from points B to D, as shown in Figure 8.7:

$$BD = \sqrt{L_1{}^2 + L_2{}^2 - 2(L_1)(L_2)\cos(\theta_2)}$$



	A	B	C	D	E	F	G	H	I
1	Crank Ang	BD	Gamma	Th3	Th4	Link1	6.5		
2	0					Link2	1.5		
3	10					Link3	7.0		
4	20					Link4	4.0		
5	30								
6	40								
7	50								
8	60								
9	70								
10	80								
11	90								
12	100								
13	110								
14	120								
15	130								
16	140								
17	150								
18	160								
19	170								
20	180								
21	190								
22	200								
23	210								
24	220								

Figure 8.8

A spreadsheet version of this equation can be placed and copied down column B. In cell B2, the following formula is inserted:

=SQRT(($G$1^2)+($G$2^2)−2*$G$1*$G$2*COS(A2*3.1416/180))

To facilitate copying the formula, note the use of absolute and relative addresses. Equation 4.8 gave the general equation for the transmission angle, γ, as shown in Figure 8.7:

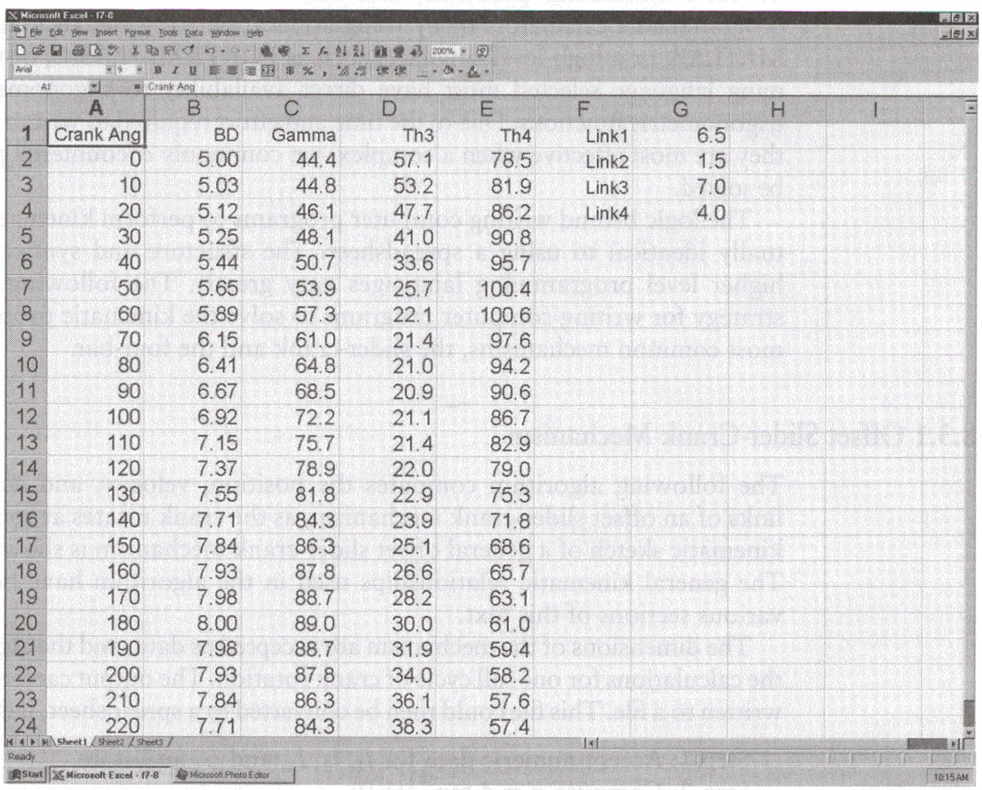
$$\gamma = \cos^{-1}\frac{L_3^{\,2} + L_4^{\,2} - BD^2}{2L_3L_4}$$

A spreadsheet version of this equation can be placed and copied down column C. In cell C2, the following formula is inserted:

=(180/3.1416)*ACOS((($G$3^2)+($G$4^2)−(B2^2))/(2*$G$3*$G$4))

Rewriting Equation 4.9 will give the general equation for the angle of link 4, θ_4, as shown in Figure 8.7:

$$\theta_4 = \sin^{-1}\left\{\frac{L_2 \sin\theta_2}{BD}\right\} + \sin^{-1}\left\{\frac{L_3 \sin\gamma}{BD}\right\}$$



	A	B	C	D	E	F	G	H	I
1	Crank Ang	BD	Gamma	Th3	Th4	Link1	6.5		
2	0	5.00	44.4	57.1	78.5	Link2	1.5		
3	10	5.03	44.8	53.2	81.9	Link3	7.0		
4	20	5.12	46.1	47.7	86.2	Link4	4.0		
5	30	5.25	48.1	41.0	90.8				
6	40	5.44	50.7	33.6	95.7				
7	50	5.65	53.9	25.7	100.4				
8	60	5.89	57.3	22.1	100.6				
9	70	6.15	61.0	21.4	97.6				
10	80	6.41	64.8	21.0	94.2				
11	90	6.67	68.5	20.9	90.6				
12	100	6.92	72.2	21.1	86.7				
13	110	7.15	75.7	21.4	82.9				
14	120	7.37	78.9	22.0	79.0				
15	130	7.55	81.8	22.9	75.3				
16	140	7.71	84.3	23.9	71.8				
17	150	7.84	86.3	25.1	68.6				
18	160	7.93	87.8	26.6	65.7				
19	170	7.98	88.7	28.2	63.1				
20	180	8.00	89.0	30.0	61.0				
21	190	7.98	88.7	31.9	59.4				
22	200	7.93	87.8	34.0	58.2				
23	210	7.84	86.3	36.1	57.6				
24	220	7.71	84.3	38.3	57.4				

Figure 8.9 Exporting data to a spreadsheet.

A spreadsheet version of this equation can be placed and copied down column E. In cell E2, the following formula is inserted:

=(180/3.1416)*(ASIN($G$2*SIN(A2*3.1416/180)/B2)
+ASIN($G$3*SIN(C2*3.1416/180)/B2))

Finally, Equation 4.10 gave the general equation for the angle of link 3, θ_3, as shown in Figure 8.7:

$$\theta_3 = 180° - (\gamma + \theta_4)$$

A spreadsheet version of this equation can be placed and copied down column D. In cell D2, the following formula is inserted:

= 180 − (C2 + E2)

These formulas in cells B2, C2, D2, and E2 can then be copied and pasted in their respective columns. The resulting upper part of the spreadsheet is shown in Figure 8.9.

8.3 USER-WRITTEN COMPUTER PROGRAMS

To solve mechanism problems, user-written computer routines can be written on programmable calculators or by using application software, such as MATHCAD or MATLAB, or a high-level language, such as FORTRAN, BASIC, or C. The programming language selected must have direct availability to trigonometric and inverse trigonometric functions. Due to the time and effort required to write a special program, they are most effective when a complex, yet commonly encountered problem needs to be solved.

The logic behind writing computer programs to perform kinematic analysis is virtually identical to using a spreadsheet. The structure and syntax of the different higher level programming languages vary greatly. The following sections offer a strategy for writing computer programs to solve the kinematic properties of the two most common mechanisms, the slider-crank and the four-bar.

8.3.1 Offset Slider-Crank Mechanism

The following algorithm computes the position, velocity, and acceleration of all links of an offset slider-crank mechanism, as the crank rotates at constant velocity. A kinematic sketch of a general offset slider-crank mechanism is shown in Figure 8.10. The general kinematic relationships used in the algorithm have been presented in various sections of this text.

The dimensions of the mechanism are accepted as data, and the algorithm performs the calculations for one full cycle of crank rotation. The output can be either printed or written to a file. This file could then be converted to a spreadsheet, if desired.

Step 1: Accept numeric data for l_1, l_2, l_3, and ω_2 and store
Step 2: Compute $\pi = 4 \tan^{-1}(1.0)$

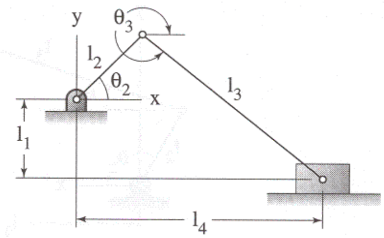


Figure 8.10 Offset slider-crank mechanism.

Step 3: Enter a loop that indexes i from 0 to 360
Step 4: Compute $a = i\ (\pi/180)$
Step 5: Compute $b = l_2 \sin a$
Step 6: Compute $c = l_2 \cos a$
Step 7: Compute $d = -\sin^{-1}\{(l_1 + \mathrm{a})/l_3\}$
Step 8: Compute $\theta_3 = d(180/\pi)$
Step 9: Compute $e = l_2 \sin d$
Step 10: Compute $f = l_2 \cos d$
Step 11: Compute $g = l_3 \sin d$
Step 12: Compute $h = l_3 \cos d$
Step 13: Compute $l_4 = c + f$
Step 14: Compute $\omega_3 = -\omega_2\ (c/f)$
Step 15: Compute $v_4 = -\omega_2(b) - \omega_3(g)$
Step 16: Compute $\alpha_3 = \{b(\omega_2)^2 + g(\omega_3)^2\}/h$
Step 17: Compute $a_4 = -\{g(\alpha_3) + c(\omega_2)^2 + h(\omega_3)^2\}$
Step 18: Print (or write to file) $I, \theta_3, \omega_3, \alpha_3, l_4, v_4, a_4$
Step 19: Control loop to return back to step 3

Recall that computer functions assume that angles are given in radians. Therefore, it is necessary to convert angular input and output as has been done in Steps 4 and 8. This algorithm also works for an in-line, slider-crank mechanism, by specifying $l_1 = 0$ as input.

8.3.2 Four-Bar Mechanism

The following algorithm computes the position, velocity, and acceleration of all links of a four-bar mechanism, as the crank rotates at constant velocity. A kinematic sketch of a general four-bar mechanism is shown in Figure 8.11. Again, the general kinematic relationships used in this algorithm have been presented in various sections of this text.

As in the previous algorithm, the dimensions of the mechanism are accepted as data, and the algorithm performs the calculations for one full cycle of crank rotation. The output can be either printed, or written to a file. This file could then be converted to a spreadsheet, if desired.

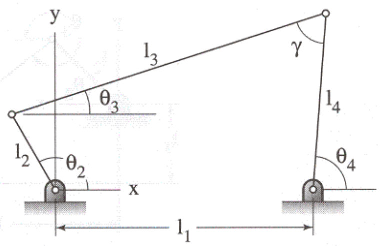


Figure 8.11 Four-bar mechanism.

Step 1: Accept numeric data for l_1, l_2, l_3, l_4 and ω_2 and store
Step 2: Compute $\pi = 4 \tan^{-1}(1.0)$
Step 3: Enter a loop that indexes i from 0 to 360
Step 4: Compute $a = i(\pi/180)$
Step 5: Compute $b = (l_3^2 + l_4^2 - l_1^2 - l_2^2)/(2l_3l_4)$
Step 5: Compute $c = l_1/l_3l_4$
Step 6: Compute $d = l_2 \sin a$
Step 7: Compute $e = l_2 \cos a$
Step 8: Compute $f = \cos^{-1}(b + ce)$
Step 9: Compute $\gamma = f(180/\pi)$
Step 10: Compute $g = \sin f$
Step 11: Compute $h = \cos f$
Step 12: Compute $p = 2 \tan^{-1}\{(-d + l_4g)/(e + l_3 - l_1 - l_4h)\}$
Step 13: Compute $\theta_3 = p(180/\pi)$
Step 14: Compute $q = 2 \tan^{-1}\{(d - l_3g)/(e + l_4 - l_1 - l_3h)\}$
Step 15: Compute $\theta_4 = q(180/\pi)$
Step 16: Compute $\omega_3 = \omega_2 l_2 \sin(q - a)/(l_3g)$
Step 17: Compute $\omega_4 = \omega_2 l_2 \sin(p - a)/(l_4g)$
Step 18: Print (or write to file) $i, \gamma, \theta_3, \omega_3, \theta_4, \omega_4$
Step 19: Control loop to return back to Step 3

Recall that computer functions will assume that angles are given in radians. Therefore it is necessary to convert angular input and output as has been done in Steps 4, 9, 13, and 15. This algorithm will give the solution for a four-bar mechanism in the open configuration. If the mechanism was assembled in the crossed position, this routine could be quickly modified to reflect that configuration. That can be accomplished by changing the plus and minus signs in the numerators of Steps 12 and 14.

PROBLEMS

For Problems 8–1 and 8–2, develop a spreadsheet that can analyze the position of all links in an offset, slider-crank mechanism for crank angles that range from 0 to 360. Keep it flexible so that the length of any link can be quickly altered. Using the listed values, produce a plot of the slider distance versus crank angle.

8–1. offset = 0.5 in; crank = 1.25 in; coupler = 7.0 in.

8–2. offset = 10 mm; crank = 25 mm; coupler = 140 mm.

For Problems 8–3 and 8–4, develop a spreadsheet that can analyze the position of all links in a four-bar mechanism for crank angles that range from 0 to 360. Keep it flexible so that the length of any link can be quickly altered. Using the listed values, produce a plot of the follower angle versus crank angle.

8–3. frame = 750 mm; crank = 50 mm; coupler = 750 mm; follower = 75 mm.

8–4. frame = 14 in; crank = 1 in; coupler = 16 in; follower = 4.0 in.

For Problems 8–5 and 8–6, develop a spreadsheet that can determine the slider position, velocity and acceleration for crank angles that range from 0 to 360. Keep it flexible so that the length of any link can be quickly altered Using the listed values, produce a plot of the slider velocity versus crank angle.

8–5. offset = 1.25 in; crank = 3.25 in; coupler = 17.5 in; crank speed = 20 rad/sec; crank acceleration = 0 rad/sec^2.

8–6. offset = 30 mm; crank = 75 mm; coupler = 420 mm; crank speed = 35 rad/sec; crank acceleration = 100 rad/sec^2.

For Problems 8–7 and 8–8, develop a spreadsheet that can determine the follower position and velocity for crank angles that range from 0 to 360. Keep it flexible so that the length of any link can be quickly altered Using the following values, produce a plot of the follower velocity versus crank angle.

8–7. frame = 9 in; crank = 1 in; coupler = 10 in; follower = 3.5 in; crank speed = 200 rad/sec; crank acceleration = 0 rad/sec^2.

8–8. frame = 360 mm; crank = 40 mm; coupler = 400 mm; follower = 140 mm; crank speed = 6 rad/sec; crank acceleration = 20 rad/sec^2

For Problems 8–9 and 8–10, develop a computer program that can determine the position, velocity and acceleration of all links in a slider-crank mechanism, for crank angles that range from 0 to 360. Keep it flexible so that the length of any link can be quickly altered. Using the listed values, determine the crank angle that produces the maximum slider acceleration.

8–9. offset = 3 in; crank = 7.5 in; coupler = 52.5 in; crank speed = 4 rad/sec; crank acceleration = 0 rad/sec^2.

8–10. offset = 40 mm; crank = 94 mm; coupler = 525 mm; crank speed = 10 rad/sec; crank acceleration = 10 rad/sec^2;

For Problems 8–11 and 8–12, develop a computer program that can determine the position and velocity of all links in a four-bar mechanism, for crank angles that range from 0 to 360. Using the listed values, determine the crank angle that produces the maximum slider acceleration.

8–11. frame = 18 in; crank = 2 in; coupler = 20 in; follower = 7 in; crank speed = 150 rad/sec; crank acceleration = 0 rad/sec^2.

8–12. frame = 60 mm; crank = 18 mm; coupler = 70 mm; follower = 32 mm; crank speed = 360 rad/sec; crank acceleration = 20 rad/sec^2

CASE STUDIES

8–1. The mechanism shown in Figure C8.1 is an elaborate crankshaft and crank for a slider-crank mechanism that is not shown, but connects to link *K*. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into the operation.

1. In the position shown, as slide bar *E* pulls to the left, what is the motion of link *D*?

2. In the position shown, as slide bar *E* pulls to the left, what is the motion of slide block *I*?
3. Pulley *J* is keyed to shaft *A*. As pulley *J* rotates, what is the motion of crank pin *C*?
4. As pulley *J* rotates, what is the motion of slide bar *E*?
5. What effect does moving slide bar *E* to the left have on crank pin *C* and on the motion of the slider-crank mechanism it drives?
6. Sleeve F is keyed to the housing *H*. As pulley *J* drives shaft *A*, what is the motion of sleeve *F*?
7. Sleeve *J* is integrally molded with item *G*. What is item *G*?
8. Sleeve *J* has internal threads at its right end, and sleeve *F* has external threads at its right end. As item *G* rotates, what happens to sleeve *F*?
9. As item *G* rotates, what happens to slide bar *E*?
10. What is the purpose of this mechanism and how does it operate?

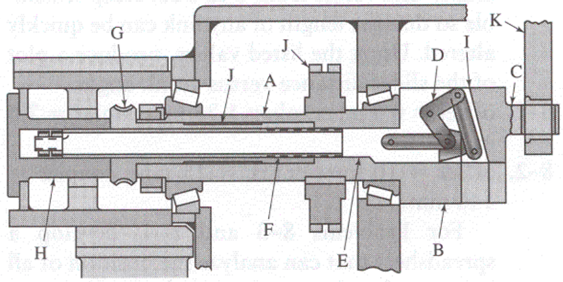


Figure C8.1 (Courtesy, Industrial Press)

9 Dynamic Force Analysis

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Define and identify a force.
2. Calculate the moment of a force.
3. Understand the difference between mass and weight.
4. Calculate the mass moment of inertia of an object by either assuming a similarity to a basic shape, or from the radius of gyration.
5. Transfer the mass moment of inertia to an alternative reference axis.
6. Understand and apply Newton's three laws of motion.
7. Create a free-body diagram of a general machine component.
8. Calculate sliding frictional force, and identify its direction.
9. Identify and use the special conditions for equilibrium of two-force and three-force members.
10. Calculate inertial forces and torques.
11. Use the superposition method to facilitate solution of an analysis with several forces.
12. Graphically determine the forces acting throughout a mechanism.
13. Analytically determine the forces acting throughout a mechanism.

9.1 INTRODUCTION

The general function of any machine is to transmit motion and forces from a power source to perform a task. Consider an escalator used on many commercial buildings: Electrical power is fed into motors, which drive mechanisms that move and fold the stairs. Of course, the task is to safely and efficiently move people up and down multilevel buildings.

Up to this point in the book, the sole focus was on the motion of a machine. This chapter is dedicated to an introduction of machine forces. A critical task in the design of machines is to ensure that the strength of the links and joints is sufficient to withstand the forces imposed on them. As mentioned in the previous chapter, an inertial force results from any accelerations present in a linkage. In many high-speed machines, the inertial forces created by the motion of a machine exceed those required to perform the intended task. Thus, understanding these forces—especially inertia—becomes extremely important.

The determination of forces in a linkage is the purpose of this chapter. Mechanism force analysis utilizes the results of an acceleration analysis as introduced in Chapter 6. Consistent with other chapters in this book, both graphical and analytical techniques are introduced. A basic knowledge of statics is expected, as this chapter quickly reviews the preliminary topics. A textbook dedicated to statics should be consulted for an in-depth coverage.

9.2 FORCES AND TORQUES

A *force* is a vector quantity that represents a pushing or pulling action on a part. Pulling a child in a wagon insinuates that a force is applied to the handle of the wagon. Being a vector, this force is defined by a magnitude and a direction of the pulling action. In the United States Customary System, the common unit for the magnitude of a force is the avoirdupois pound or simply pound (lb). In the International System, the primary unit used is the Newton (N).

Two or more forces that may be applied to a part can be combined to determine the net effect of the force. Combining forces to find a resultant is identical to adding displacement, velocity, or acceleration vectors. Being vector quantities, forces can be manipulated through the methods illustrated in Chapter 2.

A torque or moment is the twisting action produced by a force. Pushing on the handle of a wrench produces an action that tends to rotate a nut on a bolt. Thus, the force causes a twisting action around the center of a bolt. This resulting action is termed a *moment* or *torque*. Figure 9.1 illustrates such a force, causing a twisting action.

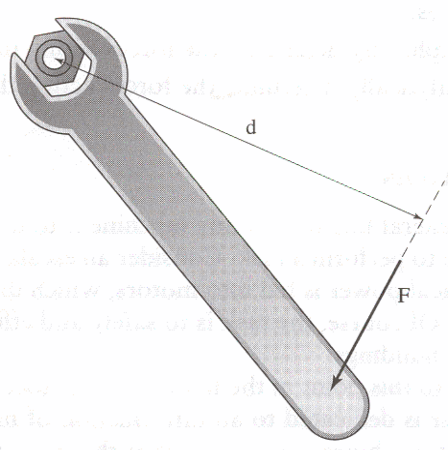


Figure 9.1 The definition of a moment or torque.

A moment is a property that is stated relative to a reference point (or axis when three dimensions are considered). A moment is also a vector; however, its direction is rotational and can be simply designated as clockwise or counterclockwise. The direction is consistent with the twisting direction of the force around the reference point. The twisting action illustrated in Figure 9.1, relative to the nut, is a clockwise moment.

A moment, created by a force, can be calculated as:

(9.1)
$$M_A = F\,d$$

where:

F = Force

d = Perpendicular distance between reference point (point A) and force

A = reference point designation

Moments are expressed in the units of force multiplied by distance. In the United States Customary System, the common units for moments are inch-pound (lb-in) or foot-pound (lb-ft). In the International System, the common units used are Newton-millimeters (N mm) or Newton-meters (N m).

EXAMPLE PROBLEM 9.1

A mechanism to automatically open a door exerts a 37-lb force on the door as shown in Figure 9.2. Determine the moment, relative to the pivot of the door, that this force creates.

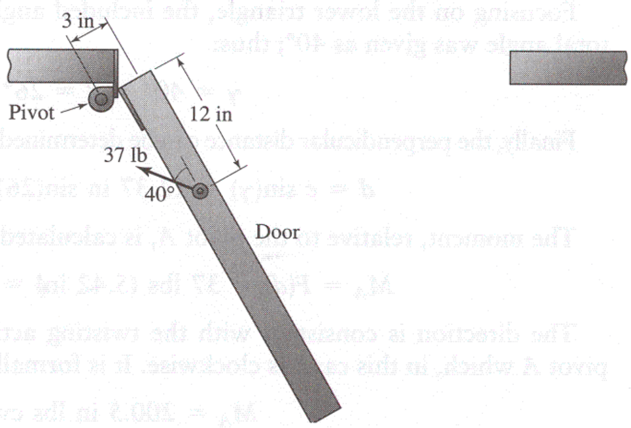


Figure 9.2 Door for Example Problem 9.1.

Solution:

The moment can be computed from Equation 9.1. Although the force is given, the geometry of the door must be examined to determine the perpendicular distance, d. The geometry has been isolated and broken into two triangles in Figure 9.3.

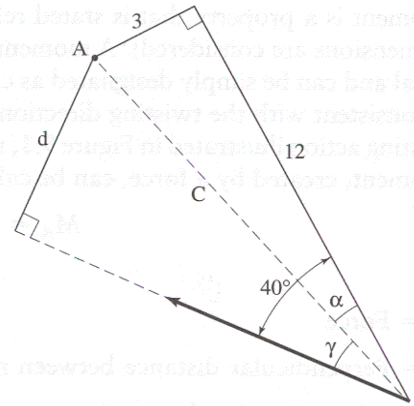


Figure 9.3 Door geometry for Example Problem 9.1.

Notice that both objects are right triangles. The common side to the two triangles, labeled as side c, can be determined with the known data for the upper triangle. From the Pythagorean theorem:

$$c = \sqrt{(12 \text{ in})^2 + (3 \text{ in})^2} = 12.37 \text{ in}$$

The included angle, β, can also be found from the trigonometric relations:

$$\beta = \tan^{-1}\frac{3 \text{ in}}{12 \text{ in}} = 14.0°$$

Focusing on the lower triangle, the included angle, γ, can be found because the total angle was given as 40°; thus:

$$\gamma = 40° - \beta = 26°$$

Finally, the perpendicular distance can be determined from the trigonometric relations:

$$d = c \sin(\gamma) = 12.37 \text{ in} \sin(26) = 5.42 \text{ in}$$

The moment, relative to the pivot A, is calculated from Equation 9.1:

$$M_A = F(d) = 37 \text{ lbs } (5.42 \text{ in}) = 200.5 \text{ in lbs}$$

The direction is consistent with the twisting action of the force relative to the pivot A which, in this case, is clockwise. It is formally stated as follows:

$$M_A = 200.5 \text{ in lbs cw.}$$

9.3 MASS AND WEIGHT

Mass, m, is a measure of the amount of material in an object. Mass can also be described as the resistance of an object to acceleration. It is more difficult to "speed up" an object with a large mass.

The *weight, w,* of an object is a measure of the pull of gravity on it. Thus, weight is a force directed toward the center of the Earth. The *acceleration of gravity, g,* varies depending on the location relative to a gravitational pull. Thus, the weight of an object will vary. Mass, on the other hand, is a quantity that does not change with gravitational pull. As stated, it is used to describe the amount of material in a part.

Weight and mass can be related through Newton's gravitational law:

$$W = m\,g \tag{9.2}$$

In most analyses on Earth, the acceleration of gravity is assumed to be:

$$g = 32.2\frac{\text{ft}}{\text{s}^2} = 386.4\frac{\text{in}}{\text{s}^2} = 9.81\frac{\text{m}}{\text{s}^2} = 9810\frac{\text{mm}}{\text{s}^2}$$

This assumption is applicable to all machines and mechanisms discussed in this book. Of course, in the case of designing machines for use in outer space, a different gravitational pull would exist.

Mass and weight are often confused in the United States Customary System; it is most convenient to use a derived unit for mass which is the slug. This unit directly results from the use of Equation 9.2:

$$\text{slug} = \{\text{lb}/(\frac{\text{ft}}{\text{s}^2})\} = \text{lb s}^2/\text{ft}$$

Occasionally, the pound-mass (lb_m) is also used as a measure of mass. It is the mass that weighs 1 pound on the surface of the Earth. Assuming that the standard value of gravity applies, the pound-mass can be converted to slugs by:

$$1 \text{ slug} = 32.2 \text{ lb}_m$$

Generally stated, any calculation in the United States Customary System should use the unit of slug for mass.

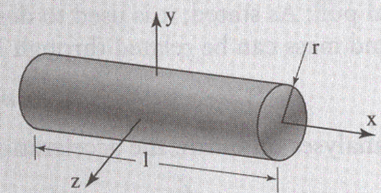
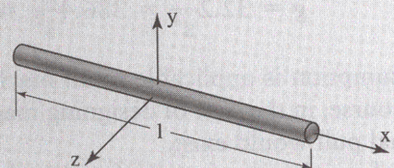
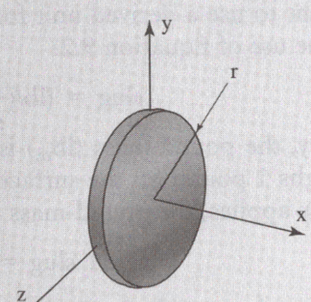
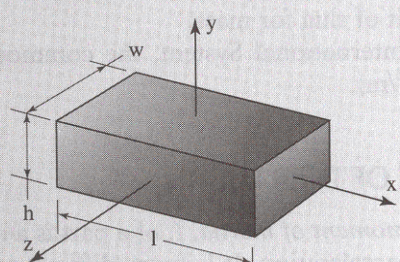
In the International System, the common units used for mass is the kilogram ($\text{kg} = \text{N s}^2/\text{m}$).

9.4 MASS MOMENT OF INERTIA

The *mass moment of inertia, I,* of a part is a measure of the resistance of that part to rotational acceleration. It is more difficult to "speed up" a spinning object with a large mass moment of inertia. Mass moment of inertia, or simply moment of inertia, is dependent on the mass of the object along with the shape and size of that object. In addition, inertia is a property that is stated relative to a reference point (or axis when three dimensions are considered). This reference point is commonly the center of gravity of the part. Table 9.1 gives equations that can be used to compute the mass moment of inertia for common solid shapes of uniform density.

Mass moment of inertia is expressed in the units of mass times squared length. In the United States Customary System, the common units are slug-squared feet (slug ft^2) which converts to pound-feet-squared seconds (lb ft s^2). In the International System, the common units used are kilogram-squared meters (kg m^2).

TABLE 9.1 Mass Moments of Inertia

Shape Name	Rendering	Mass Moment of Inertia
Cylinder		$I_x = \frac{1}{2}[mr^2]$ $I_y = \frac{1}{12}[m(3r^2 + l^2)]$ $I_z = \frac{1}{12}[m(3r^2 + l^2)]$
Slender rod		$I_x = 0$ $I_y = \frac{1}{12}[ml^2]$ $I_z = \frac{1}{12}[ml^2]$
Thin disk		$I_x = \frac{1}{2}[mr^2]$ $I_y = \frac{1}{4}[mr^2]$ $I_z = \frac{1}{4}[mr^2]$
Rectangular Block		$I_x = \frac{1}{12}[m(w^2 + h^2)]$ $I_y = \frac{1}{12}[m(w^2 + l^2)]$ $I_z = \frac{1}{12}[m(h^2 + l^2)]$

The inertia of a part can also be computed from the experimentally determined property *radius of gyration, k*. Conceptually, the radius of gyration is a measure of the size and shape of a part, with respect to an axis. The radius of gyration can be used to compute the mass moment of inertia by:

$$I = m k^2 \tag{9.3}$$

The radius of gyration is expressed in the units of length. In the United States Customary System, the common units are feet (ft) or inches (in). In the International System, the common units used are meters (m) or millimeters (mm).

As stated, the mass moment of inertia is stated relative to an axis. Occasionally, the mass moment of inertia is desired relative to an alternate, parallel axis. A parallel axis transfer equation has been derived [5] to accomplish this task. To transfer the mass moment of inertia from the x axis to a parallel x' axis, the transfer equation is as follows:

(9.4) $$I_{x'} = I_x \pm m\,d^2$$

The value d, in Equation 9.4, is the perpendicular distance between the two axes. Notice that the second term in Equation 9.4 can be either added or subtracted. The term is added when the reference axis is moved away from the center of gravity of the part. Conversely, the term is subtracted when the transfer is toward the center of gravity.

EXAMPLE PROBLEM 9.2

The part in Figure 9.4 has been weighed at 3 lb. Determine the mass moment of inertia of the part, relative to an x-axis at the center of the part and also relative to an x-axis at the end of the part.

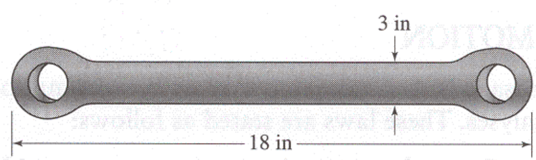


Figure 9.4 Part for Example Problem 9.2.

Solution:

The part weighs 3 lb and it is assumed to be used on the Earth's surface. The mass can be calculated from Equation 9.2:

$$m = \frac{\mathrm{w}}{g} = \frac{3 \text{ lbs}}{32.2 \text{ ft/s}^2} = 0.093 \text{ slugs}$$

Although it has some intricate details, the part can be reasonably assumed to be a solid cylinder. The holes at the end of the part are determined to be small and insignificant in determination of inertia. For the solid cylinder:

$$r = 3 \text{ in} = 0.25 \text{ ft}$$

$$l = 18 \text{ in} = 1.5 \text{ ft}$$

From Table 9.1, the mass moment of inertia relative to the x-axis at the center of the part is as follows:

$$I_x = \frac{1}{12}\{m(3r^2 + l^2)\}$$

$$= \frac{1}{12}\{0.093 \text{ slugs } (3(0.25 \text{ ft})^2 + (1.5 \text{ ft})^2)\}$$

$$= 0.0189 \text{ slug ft}^2 = 0.0189 \text{ lb ft s}^2$$

The mass moment of inertia is also required at the end of the part. The distance of the transfer from the center to the end of the part is:

$$d = \frac{1}{2}(18 \text{ in}) = 9 \text{ in} = 0.75 \text{ ft}$$

Equation 9.4 can be used to transfer the reference axis to the end of the part. Notice that the second term is added because the transfer is away from the center of gravity:

$$I_x = I_{x'} + m\,d^2 = 0.0189 \text{ slug ft}^2 + (0.093 \text{ slug})(0.75 \text{ ft})^2$$
$$= 0.0712 \text{ slug ft}^2 = 0.0712 \text{ lb ft s}^2$$

9.5 LAWS OF MOTION

Sir Isaac Newton developed three laws of motion that serve as the basis of all force analyses. These laws are stated as follows:

1. Every object remains at rest, or moves with constant velocity, unless an unbalanced force acts upon it.
2. A body that has an unbalanced force has:
   - Acceleration that is proportional to the force;
   - Acceleration that is in the direction of the force; and,
   - Acceleration that is inversely proportional to the mass of the object.
3. For every action, there is an equal and opposite reaction.

All of these laws are utilized in the study of mechanisms. All forces that act on a link must be examined. It is widely accepted that the best manner to track these forces is to construct a free-body diagram. A *free-body diagram* is a picture of the isolated part, as if it were floating freely. The part appears to be floating because all the supports and contacts with other parts have been removed. All these supports and contacts are then replaced with equivalent forces. Thus, a free-body diagram of a part shows *all* the forces acting on the part.

Figure 9.5 illustrates a free-body diagram of an isolated link. Notice that this part is designated as link 3. A convenient notation is to label the forces consistent with

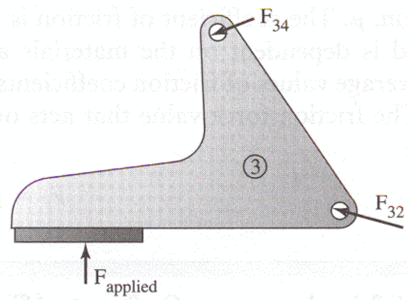


Figure 9.5 Free-body diagram.

the link number that is being acted upon and the link number that is driving the action. Thus, a force designated as F_{34}, is a force on link 3 from the contact of link 4.

9.6 STATIC FORCES

The first law applies to all links that are at rest or moving at constant velocity; thus, the condition is referred to as static equilibrium. For an object to be in static equilibrium, the combination or resultant of all forces must result in zero. In addition, the vector sum of all moments, about some arbitrary point, must result in zero. Mathematically, the first law can be summarized as:

$$\Sigma F = 0 \tag{9.5}$$

$$\Sigma M = 0 \tag{9.6}$$

The symbol, Σ, represents the vectoral summation. Therefore, Equation 9.5 can be written as:

$$F_1 +> F_2 +> F_3 +> \ldots +> F_N = 0$$

The methods for vector manipulation that were introduced in Chapter 3 can be used with these vector equations to solve for vector quantities, either graphically or analytically.

9.6.1 Sliding Contact Force

A contact force, as a result of a sliding joint, always acts perpendicular to the surface in contact. This contact force is commonly referred to as a *normal force, N,* because it acts perpendicular to the surfaces in contact.

When friction cannot be neglected in machine analysis, an additional force, *friction force, F_f,* is observed. Friction always acts to impede motion. Therefore, a friction force acts onto a sliding link, perpendicular to the normal force, and in a direction opposite to the motion.

For a stationary object, friction works to prevent motion until the maximum attainable friction has been reached. This maximum value is a function of a *coefficient* *of friction*, μ. The coefficient of friction is a property that is determined experimentally and is dependent on the materials and surface conditions of the contacting links. Average values of friction coefficients for common materials are given in Table 9.2.[2] The friction force value that acts on sliding components in motion is calculated as:

$$F_f = \mu N \tag{9.7}$$

TABLE 9.2 Approximate Coefficients of Sliding Friction

		Dry	Lubricated
Hard steel	On hard steel	0.45	0.08
	On babbitt	0.35	0.15
Mild steel	On mild steel	0.60	0.12
	On bronze	0.34	0.17
	On brass	0.44	—
	On copper lead	0.36	0.15
	On cast iron	0.23	0.13
	On lead	0.95	0.30
	On aluminum	0.50	—
	On laminated plastic	0.35	0.05
	On teflon		0.04
Cast iron	On cast iron	0.15	0.07
	On bronze	0.22	0.07
	On brass	0.30	—
	On copper	0.29	—
	On zinc	0.21	—
Aluminum	On aluminum	1.40	—

As mentioned, for moving objects, the friction force acts opposite to the direction of the relative sliding motion.

9.6.2 Analysis of a Two-Force Member

In order for any object to be in equilibrium when subjected to only two forces, those two forces must:

1. Have the same magnitude;
2. Act along the same line; and,
3. Are opposite in sense.

Because the two forces must act along the same line, the only line that can satisfy this constraint is the line that extends between the points where the forces are applied. Thus, a link with only two forces simply exhibits either tension or compression.

This fact can be extremely useful in force analysis. When the locations of the forces are known, the direction of the forces are defined. When the magnitude and sense of a single force are known, the other force's magnitude and sense can be immediately determined. Thus, the analysis of a two-force member is simple.

9.6.3 Analysis of a Three-Force Member

In order for any object to be in equilibrium when subjected to only three forces, the following must occur:

1. Resultant of all three forces must be zero; and,
2. Directional lines of the three forces intersect at the same point.

The first and second conditions guarantee that Equations 9.5 and 9.6 are satisfied, respectively.

A common analysis involves a three-force member, where one force is completely known and only the directions of the other two forces are known. Using the vector methods presented in Chapter 2, and incorporated throughout the book, the magnitude of the remaining two forces can be determined.

EXAMPLE PROBLEM 9.3

A novelty nut cracker is shown in Figure 9.6. When a force of 5 lb is applied to the top handle as shown and the mechanism does not move (static), draw a free-body diagram for each link. For this analysis, the weight of each link can be considered negligible.

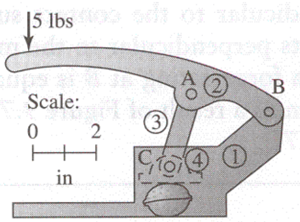


Figure 9.6 Nut cracker for Example Problem 9.3.

Solution:

Notice that link 3 (AC) is a simple link, in that it contains only two pin joints. In addition, no other force acts on this link. Thus, it is a two-force member and the forces acting on the link must be equal and along the line that connects the two pins. The free-body diagram for link 3 is shown as Figure 9.7A. As stated previously, F_{32} is a force applied to link 3 as a result of contact from link 2.

Link 2 (AB) is also a simple link, which contains only two pin joints; however, an additional force is applied to the handle. Thus, this link is a three-force member

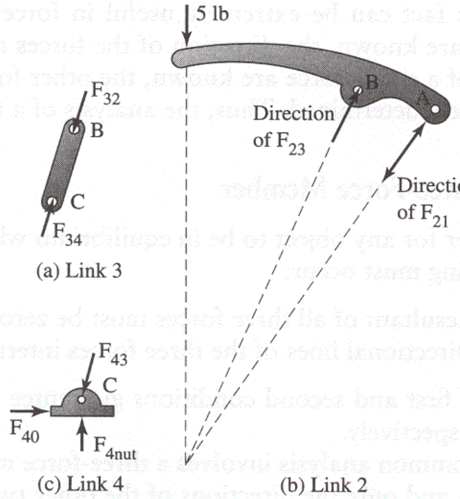


Figure 9.7 Free-body diagrams for Example Problem 9.3.

and the line of action of all three forces must meet at a concurrent point. Newton's third law stipulates that a force acting at A is equal and opposite to F_{32}. Thus, the direction of F_{23} is known as a result of Figure 9.7A. Because the direction of the applied force is also known, the concurrent point can be used to determine the direction of the force from link 1. The free-body diagram for link 2 is shown in Figure 9.7B.

Link 4 has sliding contact with link 1. Neglecting friction, this contact force acts perpendicular to the contact surface. The contact force from the nut itself similarly acts perpendicular to the mating surface. Also, Newton's third law stipulates that a force acting at B is equal and opposite to F_{34}. Thus, the direction of F_{43} is known as a result of Figure 9.7A. The free-body diagram for link 4 is shown in Figure 9.7C.

EXAMPLE PROBLEM 9.4

For the novelty nut cracker shown in Figure 9.6 and the free-body diagrams constructed in Example Problem 9.3, graphically solve for all contact forces.

Solution:

Focusing on link 2, a three-force member, the direction of all three forces and the magnitude of one force are known. This data is sufficient to use the vector Equation, 9.5, to determine the magnitude of the unknown vectors:

$$\Sigma F = 0$$

$$F_{applied} +> F_{23} +> F_{21} = 0$$

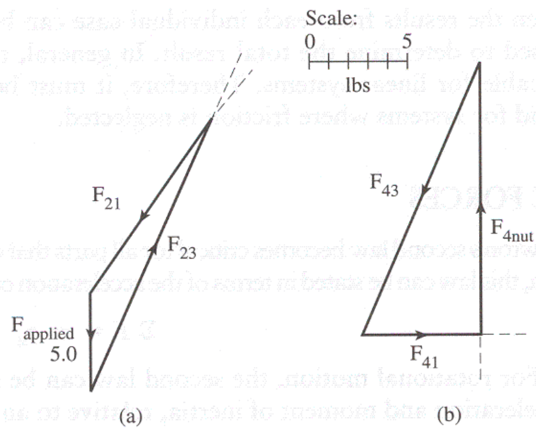


Figure 9.8 Force polygons for Example Problem 9.4.

The resulting force polygon is shown as Figure 9.8A. The unknown vectors were scaled using the following:

$$11.0 \text{ lbs} \angle 56°$$

$$15.4 \text{ lbs} \angle 66°$$

Focusing on link 3, a two-force member, the forces are immediately determined. These forces are directly influenced by F_{23}; thus:

$$15.4 \text{ lbs} \angle 66°$$

$$15.4 \text{ lbs} \angle 66°$$

Focusing on link 4, a three-force member, the direction of all three forces and the magnitude of one force, F_{43}, are known. Again, this data is sufficient to use the vector Equation, 9.6, to determine the magnitude of the unknown vectors:

$$\Sigma F = 0$$

$$F_{4nut} +> F_{34} +> F_{41} = 0$$

The resulting force polygon is shown in Figure 9.7B. The unknown vectors were scaled as:

$$F_{4nut} = 14.1 \text{ lbs} \uparrow$$

$$F_{41} = 6.0 \text{ lbs} \rightarrow$$

9.6.4 Superposition of Forces

The principle of force superposition states that the net effect of several forces on a system is the combination of the effects of each individual force. Therefore, a mechanism with several forces can be analyzed by concentrating on one force at a time.

Then the results from each individual case can be vectorally combined or superimposed to determine the total result. In general, the concept of superposition is applicable for linear systems. Therefore, it must be emphasized that this principle is valid for systems where friction is neglected.

9.7 DYNAMIC FORCES

Newton's second law becomes critical for all parts that experience acceleration. For linear motion, this law can be stated in terms of the acceleration of the link's center of gravity, a_g; thus:

(9.8) $$\Sigma F = m\, a_g$$

For rotational motion, the second law can be summarized in terms of rotational acceleration and moment of inertia, relative to an axis through the center of gravity:

(9.9) $$\Sigma M_g = I_g\, \alpha$$

Again, the subscript "g" refers to the reference point at the link's center of gravity.

9.7.1 d'Alembert's Principle

Equation 9.8 can be rewritten as:

(9.10) $$\Sigma F - m\, a_g = 0$$

Defining an *inertial force*, $F_g^{\,i}$, at the center of gravity of the link, Equation 9.10 can be rewritten again:

(9.11) $$\Sigma F - F_g^{\,i} = 0$$

where:

(9.12) $$F_g^{\,i} = m\, a_g$$

This inertial force is simply a vector in the direction of the acceleration, with a magnitude of mass multiplied by acceleration. The concept of rewriting Equation 9.10 in the form of Equation 9.11 is known as d'Alembert's principle. It allows for analysis of accelerating links, using the same methods incorporated in a static analysis.

In a similar fashion, an *inertial torque*, $T_g^{\,i}$, can be defined at the center of gravity of the link:

(9.13) $$\Sigma T - T_g^{\,i} = 0$$

where:

(9.14) $$T_g^{\,i} = I_g\, \alpha_g$$

9.7.2 Equivalent, Offset Inertia Force

The acceleration of a link, undergoing general plane motion, can be characterized by the linear acceleration of the center of gravity, and the rotational acceleration of the entire link. These properties, a_g and α, appear in Equations 9.9 and 9.10, respectively. The previous section introduced the concept of an inertial force acting at the center of gravity, which accounts for the force produced by linear acceleration. An *inertial torque*, $T_g^{\,i}$, can also be defined to account for the torque produced by rotational acceleration.

The inertial force can be relocated a distance away from the center of gravity, producing a torque about the center of gravity, which is equivalent to the inertial torque. Doing this eliminates the inertial torque, but still accounts for its action. The two equivalent force and torque systems are illustrated in Figure 9.9A,B. The distance that the inertial force moves, d_g, depends on the magnitude of the inertial torque:

$$\Sigma\, M_g \text{ (due to inertial torque)} = \Sigma\, M_g \text{ (due to moved inertial force)}$$

$$I_g\, a = (m\, a_g) d_g$$

therefore:

(9.15)
$$d_g = \frac{I_g\, \alpha}{m\, a_g}$$

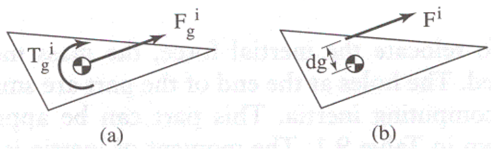


Figure 9.9 Relocating the inertial force.

An important issue in relocating the inertial force is the direction of the move. The offset inertial force must cause the same twisting action as the original inertial torque. In Figure 9.9A, the inertial clockwise torque is consistent with the direction of angular acceleration. Figure 9.9B shows the relocated inertial force which is offset from the center of gravity. The twisting effect, relative to the center of gravity, is also clockwise. This consistency in twisting effects is extremely important. Thus, caution must taken when relocating the inertial force.

EXAMPLE PROBLEM 9.5

The link shown in Figure 9.10 is isolated from its mechanism. The state of acceleration can be characterized by the linear acceleration of the center of gravity (cg) and the rotational acceleration of the link as shown. The link also has a mass of 12 kg. Determine the magnitude of the inertia force, and offset the force from the cg, such that it also compensates for the inertial torque.

Solution:

The inertial force acts in the direction of the acceleration of the cg. It can be calculated from Equation 9.10:

$$F_g^{\,i} = m\, a_g = (12 \text{ kg})(2.5 \text{ m/s}^2) = 30 \text{ N}$$

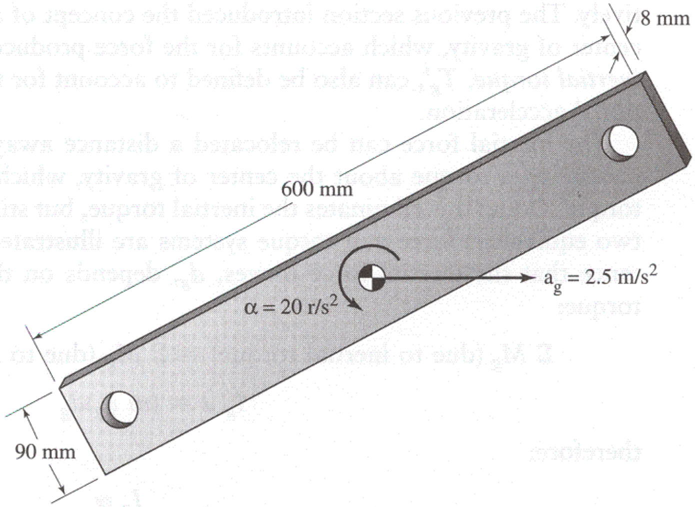


Figure 9.10 Link for Example Problem 9.5.

To relocate the inertial force, the mass moment of inertia must first be determined. The holes at the end of the part are small and will will have a minimal effect on computing inertia. This part can be approximated as a rectangular prism as shown in Table 9.1. The moment of inertia is required relative to the axis which is normal to the 600 × 90 face. Thus, the equation for the y axis is used with the following:

$$a = 0.09 \text{ m}, \quad b = 0.008 \text{ m}, \quad \text{and} \quad 1 = 0.60 \text{ m:}$$

$$I_y = \frac{1}{12}\{m(a^2 + 1^2)\}$$

$$= \frac{1}{12}\{12 \text{ kg}[(0.09 \text{ m})^2 + (0.60 \text{ m})^2]\} = 0.37 \text{ kg m}^2$$

An inertial torque occurs in the direction of the angular acceleration. It can be calculated from Equation 9.12:

$$T_g^{\,i} = I\,\alpha = (0.37 \text{ kg m}^2)(20 \text{ rad/s}^2) = 7.40 \text{ N m}$$

Figure 9.11A illustrates the link with the inertial forces.

Equation 9.15 can be used to calculate the distance that the inertial force must be offset to compensate for the inertial torque:

$$d_g = \frac{Ig\,\alpha}{ma_g}$$

$$= \frac{(0.37 \text{ kg m}^2)(20 \text{ rad/s}^2)}{(12 \text{ kg})\,(2.5 \text{ m/s}^2)} = 0.25 \text{ m}$$

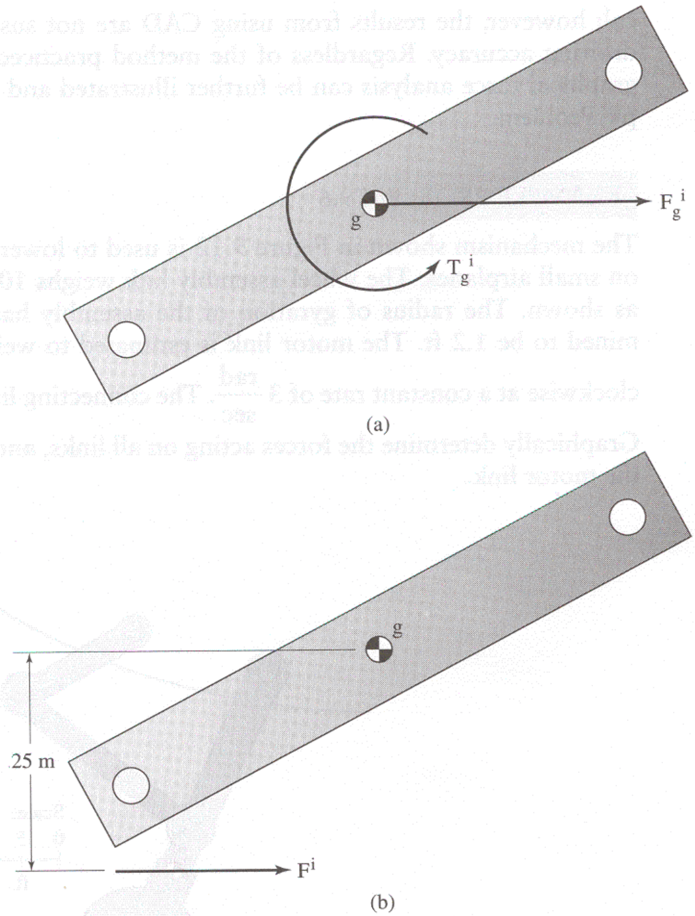


Figure 9.11 Inertial loads for Example Problem 9.5.

The offset must locate the inertial force such that it creates the same twisting action around the cg as the inertial torque. Recall that the inertial torque is the same direction as the rotational acceleration. Therefore, for this problem, the inertial force must be lowered to create the same twisting action. This resolved force is illustrated in Figure 9.11B.

9.8 DYNAMIC FORCES IN MECHANISMS: GRAPHICAL ANALYSIS

The underlying principles behind a dynamic force analysis were developed in previous sections. An example problem is presented to illustrate the combination of all these principles and to determine the dynamic forces acting in a mechanism.

As with all other graphical methods, force analysis can be completed using manual drawing techniques or a CAD system. The strategy behind the solution is identical; however, the results from using CAD are not susceptible to the limitations of drafting accuracy. Regardless of the method practiced, the underlying concepts of graphical force analysis can be further illustrated and expanded through an Example Problem.

EXAMPLE PROBLEM 9.6

The mechanism shown in Figure 9.12 is used to lower and retract the landing gear on small airplanes. The wheel assembly link weighs 100 lb, with a center of gravity as shown. The radius of gyration of the assembly has been experimentally determined to be 1.2 ft. The motor link is estimated to weigh 5 lb and rotates counterclockwise at a constant rate of 3 $\frac{\text{rad}}{\text{sec}}$. The connecting link is estimated to weigh 3 lb. Graphically determine the forces acting on all links, and the torque required to drive the motor link.

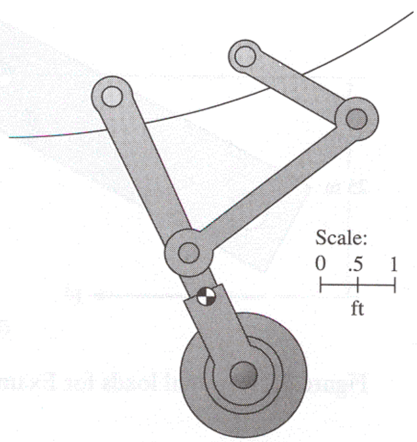


Figure 9.12 Landing gear for Example Problem 9.6.

Solution:

This mechanism is the common four-bar linkage. A kinematic diagram is given in Figure 9.13A.

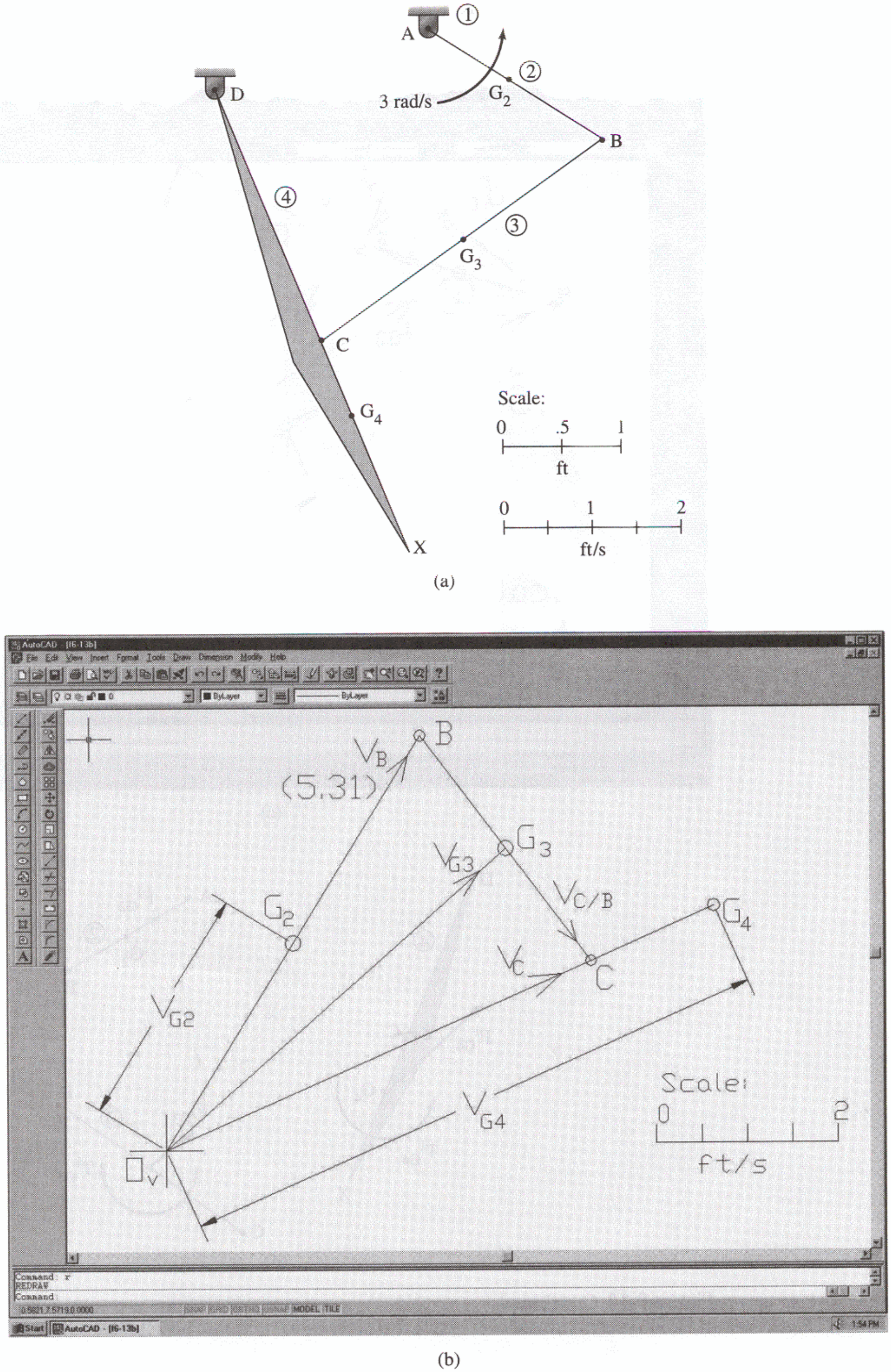
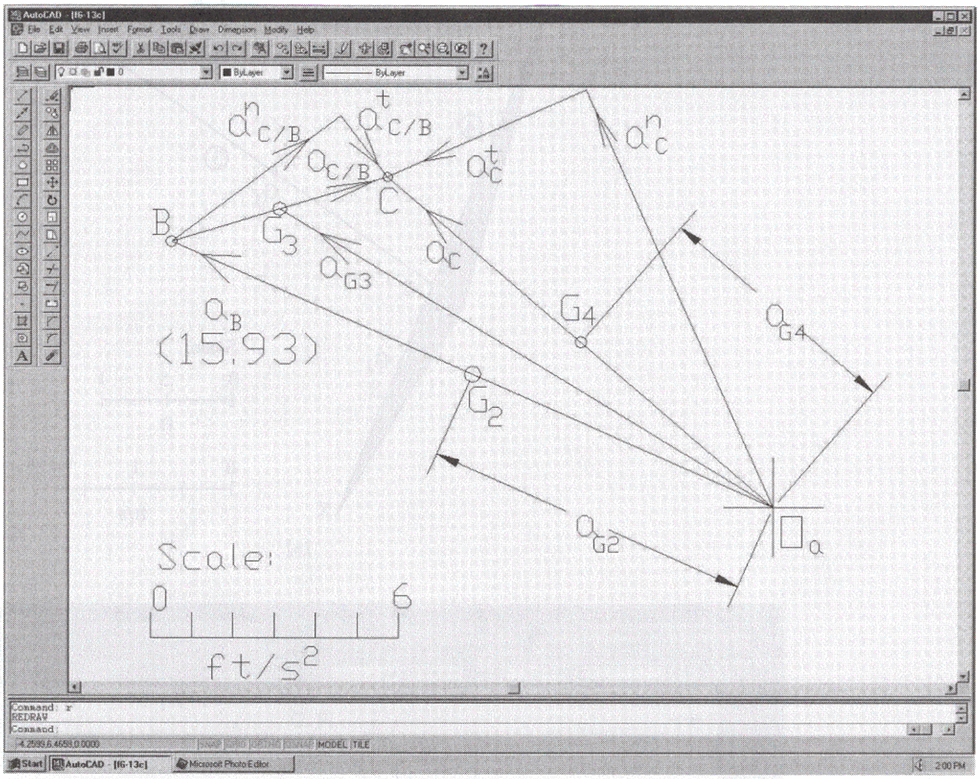
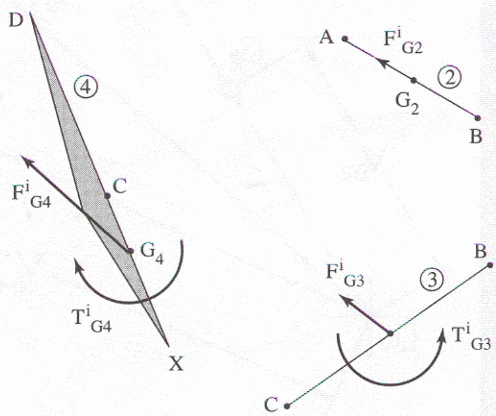


Figure 9.13 Diagrams for Example Problem 9.6.



(c)



(d)

Figure 9.13 Continued.

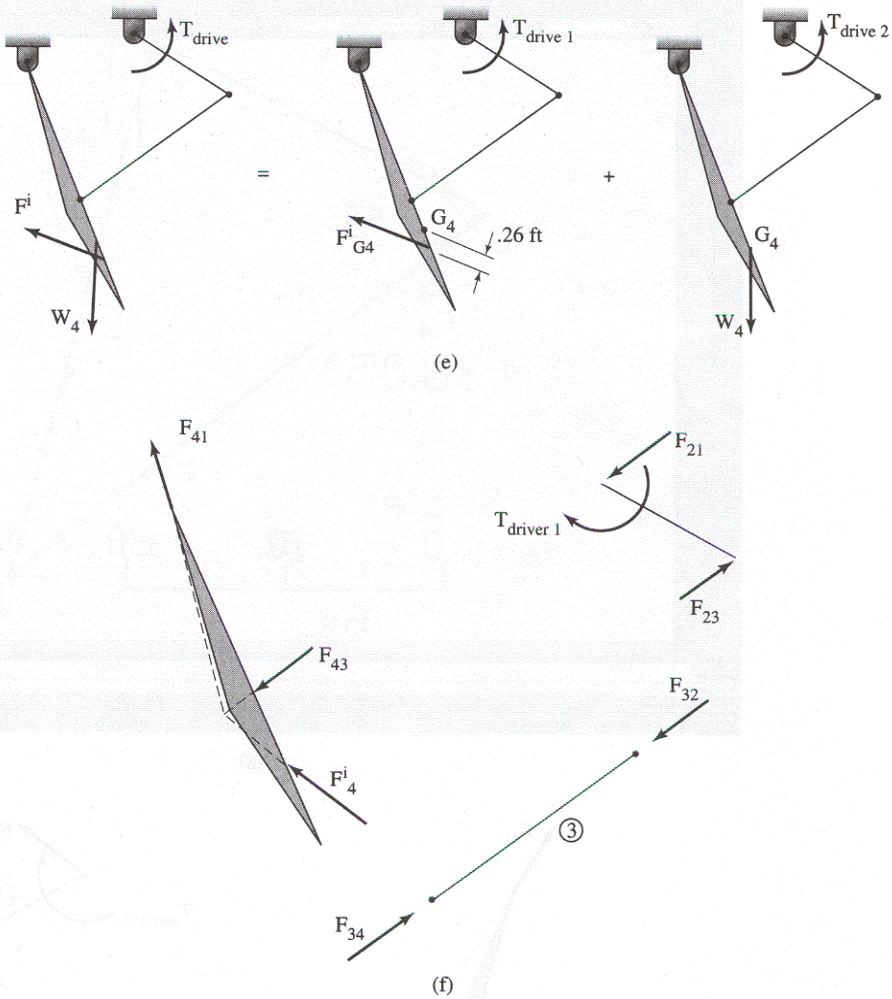
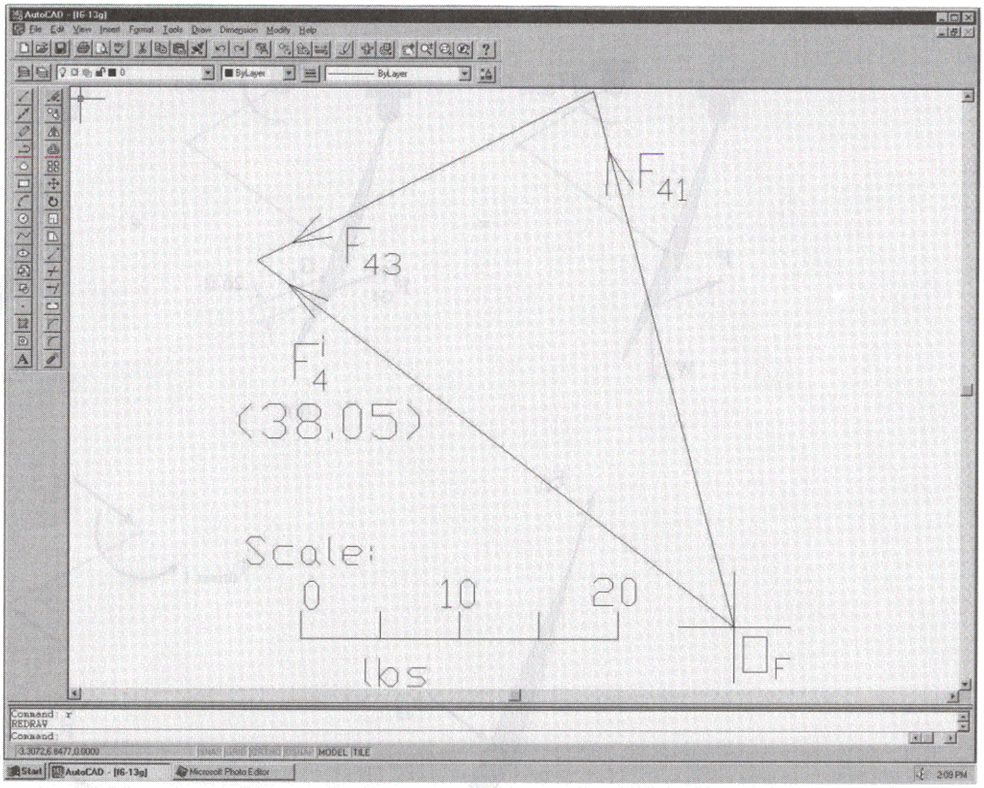
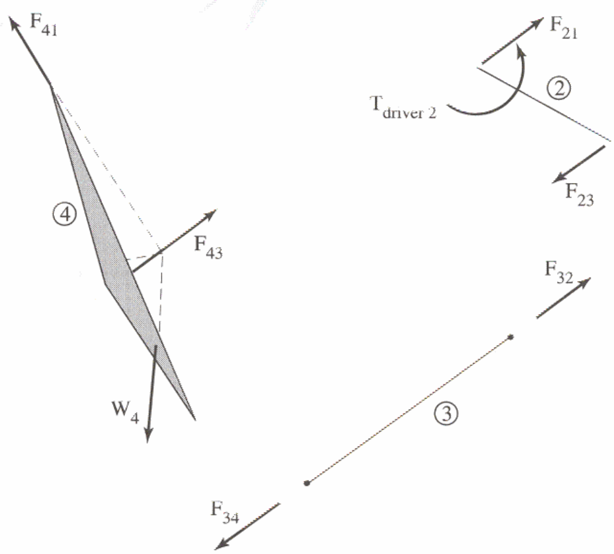


Figure 9.13 Continued.

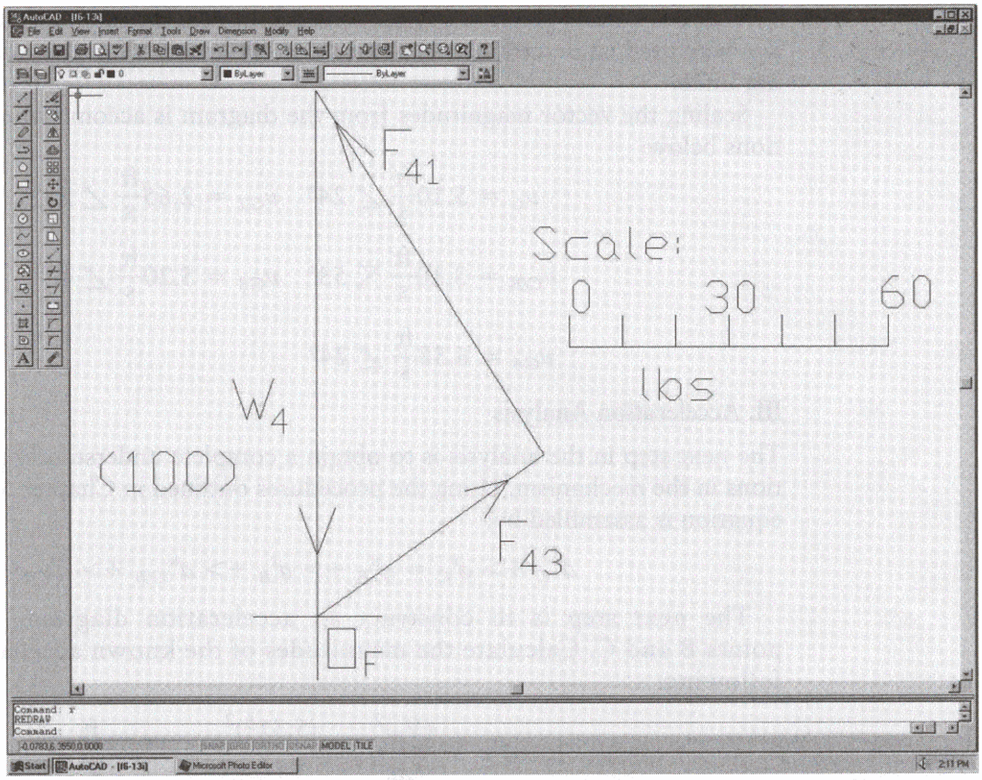


(g)



(h)

Figure 9.13 Continued.



(i)

Figure 9.13 Continued.

I. Geometric Analysis

he link lengths are scaled from the diagram as follows:

$$r_{AB} = 1.77 \text{ ft} \quad r_{CD} = 2.33 \text{ ft} \quad r_{C/B} = 3.0 \text{ ft}$$

II. Velocity Analysis

The next step in the analysis is to develop a complete understanding of the velocities in the mechanism. The velocity of point B can be computed using the following:

$$v_B = \omega_2 r_B = \left(3 \frac{\text{rad}}{\text{sec}}\right)(1.77 \text{ ft}) = 5.31 \frac{\text{ft}}{\text{s}}$$

Using the procedures outlined in Chapter 4, a velocity equation is used and a velocity polygon is constructed:

$$v_C = v_B +> v_{C/B}$$

This polygon is given in Figure 9.13B. Notice that the notion of a velocity image was used to determine the velocity of the center of gravity of the three moving links.

Scaling the vector magnitudes from the diagram is accomplished with the equations below:

$$v_C = 5.10\frac{\text{ft}}{\text{s}} \angle 24° \quad v_{G2} = 2.65\frac{\text{ft}}{\text{s}} \angle 58°$$

$$v_{C/B} = 3.10\frac{\text{ft}}{\text{s}} \measuredangle 53° \quad v_{G3} = 5.20\frac{\text{ft}}{\text{s}} \angle 42°$$

$$v_{G4} = 6.58\frac{\text{ft}}{\text{s}} \angle 24°$$

III. Acceleration Analysis

The next step in the analysis is to obtain a complete understanding of the accelerations in the mechanism. Using the procedures outlined in Chapter 5, an acceleration equation is assembled by:

$$a^n{}_C +> a^t{}_C = a^n{}_B +> a^t{}_B +> a^n{}_{C/B} +> a^t{}_{C/B}$$

The next step is to construct an acceleration diagram, which includes points B and C. Calculate the magnitudes of the known accelerations with the following:

$$a^n{}_B = \frac{(V_B)^2}{r_{AB}} = \frac{(5.31\frac{\text{ft}}{\text{s}})^2}{1.77\text{ ft}} = 15.93\frac{\text{ft}}{\text{s}^2}$$

(directed toward the center of rotation, point A)

$$a^t{}_B = \alpha_2 \quad r_{AB} = (0)(1.77\text{ ft}) = 0\ \frac{\text{in}}{\text{s}^2}$$

$$a^n{}_{C/B} = \frac{(v_{C/B})^2}{r_{CB}} = \frac{(3.10\frac{\text{ft}}{\text{s}})^2}{3\text{ ft}} = 3.20\frac{\text{ft}}{\text{s}^2}$$

(directed from C toward B)

$$a^n{}_C = \frac{(v_C)^2}{r_{CD}} = \frac{(5.10\frac{\text{ft}}{\text{s}})^2}{2.33\text{ ft}} = 11.16\frac{\text{ft}}{\text{s}^2}$$

(directed toward the center of rotation, point D)

The acceleration polygon is constructed and shown in Figure 9.13C. Notice that the notion of an acceleration image was used to determine the acceleration of the center of gravity of the three moving links.

Scale the vector magnitudes from the diagram, as detailed in the equations below:

$$a_{G2} = 7.96\frac{\text{ft}}{\text{s}^2} \measuredangle 24° \quad a_{G3} = 14.00\frac{\text{ft}}{\text{s}^2} \measuredangle 36°$$

$$a_{G4} = 12.25\frac{\text{ft}}{\text{s}^2} \measuredangle 40°$$

The angular accelerations of the links can then be determined by:

$$\alpha = \frac{a_B{}^t}{r_{AB}} = \frac{0}{1.77 \text{ ft}} = 0.0$$

$$\alpha_3 = \frac{a^t{}_{C/B}}{r_{CB}} = \frac{(0.75\frac{\text{ft}}{\text{s}^2})}{3 \text{ ft}} \text{ ft} = 0.92\frac{\text{rad}}{\text{s}^2}$$

$$\alpha_4 = \frac{a_C{}^t}{r_{CD}} = \frac{(5.25\frac{\text{ft}}{\text{s}^2})}{2.33 \text{ ft}} = 12.25\frac{\text{rad}}{\text{s}^2} \text{ cw}$$

IV. Mass and Inertia

Focusing on the inertial loads, the mass of each link is calculated as:

$$m_2 = \frac{W_2}{g} = \frac{5 \text{ lbs}}{32.2\frac{\text{ft}}{\text{s}^2}} = 0.155 \text{ slugs}$$

$$m_3 = \frac{W_3}{g} = \frac{3 \text{ lbs}}{32.2\frac{\text{ft}}{\text{s}^2}} = 0.093 \text{ slugs}$$

$$m_4 = \frac{W_4}{g} = \frac{100 \text{ lbs}}{32.2\frac{\text{ft}}{\text{s}^2}} = 3.106 \text{ slugs}$$

Although they include some intricate details for calculating mass moment of inertia, links 2 and 3 are assumed to be slender rods:

$$I_{G2} = \frac{1}{12}(m\, l^2)$$

$$= \frac{1}{12}\{0.155 \text{ slugs}(1.77 \text{ ft})^2\}$$

$$= 0.040 \text{ slug ft}^2 = 0.040 \text{ lb ft s}^2$$

$$I_{G3} = \frac{1}{12}(m\, l^2)$$

$$= \frac{1}{12}\{0.093 \text{ slugs}(3.0 \text{ ft})^2\}$$

$$= 0.069 \text{ slug ft}^2 = 0.069 \text{ lb ft s}^2$$

The mass moment of inertia of link 4 can be calculated from the radius of gyration. Use Equation 9.4:

$$I_{G4} = m\, k^2$$

$$= (3.106 \text{ slugs})(1.2 \text{ ft})^2$$

$$= 4.473 \text{ slug ft}^2 = 4.473 \text{ lb ft s}^2$$

V. Inertial Loads

Calculate the inertial loads from Equations 9.10 and 9.11:

Link 2:

$$F_{g2}{}^{i} = m\, a_{g2} = (0.155 \text{ slugs})(7.96 \text{ ft/s}^2) = 1.24 \text{ lbs}$$

$$T_{g2}{}^{i} = I_{g2}\, \alpha_2 = (0.040 \text{ lb ft s}^2)(0 \text{ rad/s}^2) = 0.0 \text{ lb-ft}$$

Link 3:

$$F_{g3}{}^{i} = m\, a_{g3} = (0.093 \text{ slugs})(14.00 \text{ ft/s}^2) = 1.30 \text{ lbs}$$

$$T_{g3}{}^{i} = I_{g3}\, \alpha_3 = (0.0469 \text{ lb ft s}^2)(0.92 \text{ rad/s}^2) = 0.06 \text{ lb-ft ccw}$$

Link 4:

$$F_{g4}{}^{i} = m\, a_{g4} = (3.106 \text{ slugs})(12.25 \text{ ft/s}^2) = 38.05 \text{ lbs}$$

$$T_{g4}{}^{i} = I_{g4}\, \alpha_4 = (4.473 \text{ lb ft s}^2)(2.25 \text{ rad/s}^2) = 10.06 \text{ lb-ft cw}$$

The application of these inertial loads on the respective links is illustrated in Figure 9.13D. These inertial forces can be offset from the center of gravity and compensate for the inertial torque. The offset distance for each link is as follows:

Link 2:

$$d_{g2} = \frac{I_{g2}\, \alpha_2}{m_2\, a_{g2}} = 0$$

Link 3:

$$d_{g3} = \frac{I_{g3}\, \alpha_3}{m_3\, a_{g3}}$$

$$= \frac{(0.047 \text{ lb ft s}^2)\,(0.92\frac{\text{rad}}{\text{s}^2})}{(0.093 \text{ slugs})\,(14.00\frac{\text{ft}}{\text{s}^2})}$$

$$= 0.03 \text{ ft } \{\text{right and upward}\}$$

Link 4:

$$d_{g4} = \frac{I_{g4}\, \alpha_4}{m_4\, a_{g4}}$$

$$= \frac{(4.473 \text{ lb ft s}^2)(2.25\frac{\text{rad}}{\text{s}^2})}{(3.106 \text{ slugs})(12.25\frac{\text{ft}}{\text{s}^2})}$$

$$= 0.26 \text{ ft } \{\text{down and to the right}\}$$

VI. Dynamic Force Analysis

By observing the forces on this mechanism, it appears that the magnitude of the weight and inertia of the wheel assembly is significantly greater than the weight and inertia of the other links. Therefore, loads on links 2 and 3 are neglected. The method of superposition can be used to break this problem into three sub-problems as shown in Figure 9.13E.

Focusing on Sub-Problem 1 The associated free-body diagrams are shown in Figure 9.13F. Link 3 is a two-force member. Thus, the direction of the forces acting on it are along the link and the magnitudes are equal. Link 4 is a three-force member and, because the directions of the inertial load and F_{43} are known, the coincident theory reveals the direction of the third force. This data is sufficient to use vector Equation 9.6 to determine the magnitude of the unknown vectors:

$$\Sigma F = ma_g$$

$$\Sigma F - F^i_4 = 0$$

$$F_{41} +> F_{43} -> F^i_4 = 0$$

The resulting force polygon is shown as Figure 9.11G. The unknown vectors were scaled as:

$$F_{41} = 40.0 \text{ lbs} \searrow 75°$$

$$F_{43} = 27.0 \text{ lbs} \nearrow 38°$$

Thus, the forces acting on link 3 are immediately determined by:

$$F_{34} = 27.0 \text{ lbs} \swarrow 38°$$

$$F_{32} = 27.0 \text{ lbs} \nearrow 38°$$

The free-body diagram of link 2 illustrates that the force F_{23} must be counteracted by the driving torque. The perpendicular distance, d, from the force F_{23} to the pivot A is scaled as:

$$d = 1.65 \text{ ft}$$

from the equilibrium equation:

$$\Sigma M = 0$$

$$F_{23}\,(d) - T_{\text{driver1}} = 0$$

$$T_{\text{driver1}} = F_{23}\,(d) = (27.0 \text{ lbs})(1.65 \text{ ft}) = 44.6 \text{ ft lbs cw}$$

Focusing on Sub-Problem 2 The associated free-body diagrams are shown as Figure 9.11H. As in sub-problem 1, link 3, is a two-force member. Thus, the direction of the forces acting on it are along the link and the magnitudes are equal. Link 4 is again a three-force member and, because the directions of the inertial load and F_{43} are known, the coincident theory reveals the direction of the third force. This data is sufficient to use vector Equation, 9.6 to determine the magnitude of the unknown vectors:

$$\Sigma F = 0$$

$$F_{41} +> F_{43} +> W_4 = 0$$

The resulting force polygon is shown in Figure 9.13I. The unknown vectors were scaled as:

$$F_{41} = 81.3 \text{ lbs} \nearrow 38°$$

$$F_{43} = 52.5 \text{ lbs} \angle 38°$$

Thus, the forces acting on link 3 are immediately determined by:

$$F_{34} = 52.5 \text{ lbs} \nearrow 38°$$

$$F_{32} = 52.5 \text{ lbs} \angle 38°$$

The free-body diagram of link 2 illustrates that force F_{23} must be counteracted by the driving torque. The perpendicular distance, d, from the force F_{23} to the pivot A is scaled as:

$$d = 1.65 \text{ ft}$$

from the equilibrium equation:

$$\Sigma M = 0$$

$$F_{23}\,(d) - T_{driver2} = 0$$

$$T_{driver2} = F_{23}(d) = (52.5 \text{ lbs})(1.65 \text{ ft}) = 86.7 \text{ ft lbs ccw}$$

Finally, the forces from both sub-problems can be vectorally combined to determine the net effect using the following equations:

$$\{F_{41}\}_{net} = \{F_{41}\}_{sub\ problem\ 1} +> \{F_{41}\}_{sub\ problem\ 2}$$

$$\{F_{43}\}_{net} = \{F_{43}\}_{sub\ problem\ 1} +> \{F_{43}\}_{sub\ problem\ 2}$$

$$\{T_{driver}\}_{net} = \{T_{driver}\}_{sub\ problem\ 1} +> \{T_{driver}\}_{sub\ problem\ 2}$$

The vector additions are shown in Figure 9.11J, and the net forces are scaled as:

$$\{F_{41}\}_{net} = 120.5 \text{ lbs} \searrow 64°$$

$$\{F_{43}\}_{net} = 25.5 \text{ lbs} \angle 38°$$

$$\{T_{driver}\}_{net} = 42.1 \text{ lb-ft ccw}$$

Recall that the inertial loads of links 2 and 3 were assumed to be negligible. If these loads and any others would need to be considered, the force analysis would be partitioned into several other sub-problems. Although such an analysis would be rather redundant, the overall procedure would be exactly the same.

9.9 DYNAMIC FORCES IN MECHANISMS: ANALYTICAL ANALYSIS

The underlying principles behind dynamic force analysis were developed in previous sections. The only difference in an analytical approach is that the equations of equilibrium can be broken into orthogonal directions and solved algebraically. Of course, this method was used when analytically solving other vector equations. However, in force analysis, this algebraic approach can handle all free-body diagrams, and the force superposition method is no longer needed. An Example Problem, using an analytical approach, is presented to illustrate the combination of all these principles and to determine the dynamic forces acting in a mechanism.

EXAMPLE PROBLEM 9.7

The mechanism shown in Figure 9.14 is used to drive a small, automated leather punch at a rate of 20 punches per minute. The punch assembly has a mass of 1.2 kg and has an upward force of 16 N applied to it. The crank and connecting arm have masses of 0.35 and 0.75, respectively. The crank is also designed to rotate counterclockwise. The coefficient between the punch assembly and the guides is considerable and can be assumed to be 0.15. In the position shown, determine the torque required to drive the punch and the forces at the joints.

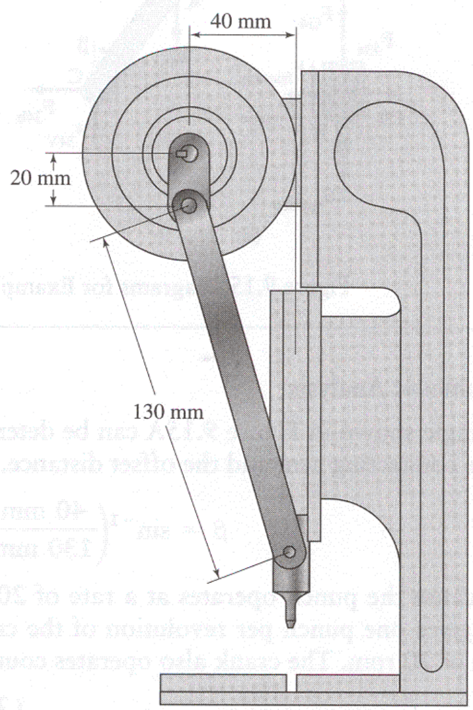


Figure 9.14 Leather punch for Example Problem 9.7.

Solution:

This mechanism is the common slider-crank linkage. A kinematic diagram is given in Figure 9.15A.

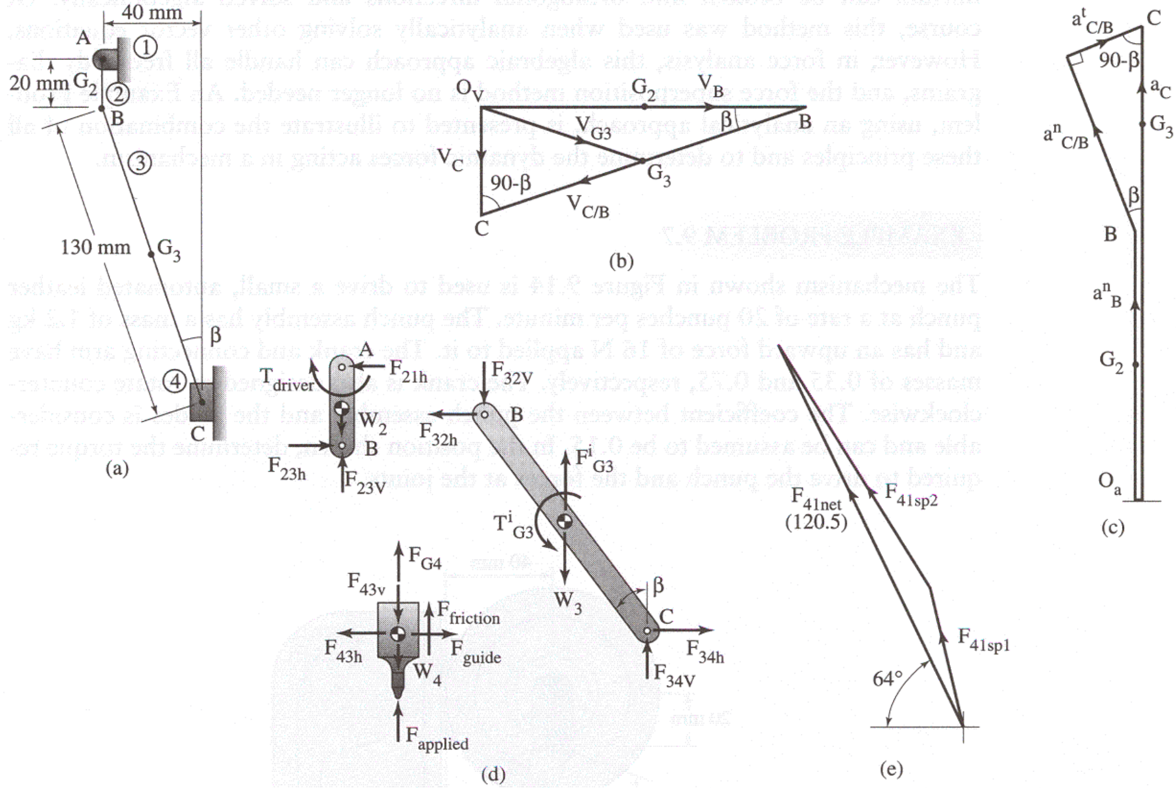


Figure 9.15 Diagrams for Example Problem 9.7.

I. Geometric Analysis

The angle shown in Figure 9.15A can be determined from the right triangle formed by the connecting arm and the offset distance. Thus:

$$\beta = \sin^{-1}\left(\frac{40 \text{ mm}}{130 \text{ mm}}\right) = 17.9°$$

Because the punch operates at a rate of 20 punches per minute and the mechanism gives one punch per revolution of the crank, the crank must have a constant speed of 20 rpm. The crank also operates counterclockwise. Therefore:

$$\omega_2 = 20 \text{ rpm} = (20 \text{ rpm})\left\{\frac{2\pi}{60}\right\} = 2.09\frac{\text{rad}}{\text{sec}} \text{ ccw}$$

II. Velocity Analysis

The first step in analysis is to obtain a complete understanding of the velocities in the mechanism. The velocity of point B can be computed with the following:

$$v_B = \omega_2 \, r_B = \left(2.09 \frac{\text{rad}}{\text{sec}}\right)(20 \text{ mm}) = 41.8 \frac{\text{mm}}{\text{s}} \rightarrow$$

Using the procedures outlined in Chapter 4, a velocity equation is used and a velocity polygon is constructed (Figure 9.15B):

$$v_C = v_B +> v_{C/B}$$

Notice that because link 3 inclines at an angle from the vertical, the relative velocity $v_{C/B}$ inclines at an angle of $90 - \beta$ from the vertical. This occurs because the relative velocity between two points on the same link is perpendicular to the line that connects those points.

The velocity of the joints can be found from trigonometry:

$$v_C = \frac{v_B}{\tan(90 - \beta)}$$

$$= \frac{41.8 \frac{\text{mm}}{\text{s}}}{\tan\,(90 - 17.9)} = 13.5 \text{ mm/s} \downarrow$$

$$v_{C/B} = \frac{v_B}{\sin(90 - \beta)} = \frac{41.8 \frac{\text{mm}}{\text{s}}}{\sin(90 - 17.9)}$$

$$= 43.9 \frac{\text{mm}}{\text{s}} \angle 17.9$$

The velocity image can be used to determine the velocity of the center of gravity of links 2 and 3:

$$v_{G2} = \frac{v_B}{2} = 20.9 \frac{\text{mm}}{\text{s}} \rightarrow$$

$$v_{G3} = \sqrt{(v_B)^2 + \left(\frac{v_{C/B}}{2}\right)^2 - 2(v_c)\left(\frac{v_{C/B}}{2}\right)\cos(\beta)}$$

$$= \sqrt{(41.8)^2 + \left(\frac{43.9}{2}\right)^2 - 2(41.8)\left(\frac{43.9}{2}\right)\cos(17.9)} = 21.9 \frac{\text{mm}}{\text{s}}$$

at an angle from the horizontal of:

$$\sin^{-1}\left\{\frac{\frac{V_{CB}}{2}}{V_{G3}} \sin\beta\right\}$$

$$= \sin^{-1}\left\{\frac{\frac{42.9}{2}}{21.9} \sin 17.9\right\} = 17.5°$$

formally:

$$v_{G3} = 21.9\frac{\text{mm}}{\text{s}} \nwarrow 17.5$$

III. Acceleration Analysis

The next step in the analysis is to obtain a complete understanding of the accelerations in the mechanism. Using the procedures outlined in Chapter 5, an acceleration equation is assembled as follows:

$$a^n{}_C +> a^t{}_C = a^n{}_B +> a^t{}_B +> a^n{}_{C/B} +> a^t{}_{C/B}$$

The next step is to construct an acceleration diagram, which includes points *B* and C. Calculate the magnitudes of the known accelerations using the equations:

$$a^n{}_B = \frac{(v_B)^2}{r_{AB}} = \frac{(41.8\frac{\text{mm}}{\text{s}})^2}{20 \text{ mm}} = 87.4\frac{\text{mm}}{\text{s}^2}$$

(directed toward the center of rotation, ↑)

$$a^t{}_B = \alpha_2 \, r_{AB} = (0)(20 \text{ mm}) = 0\frac{\text{mm}}{\text{s}^2}$$

$$a^n{}_{C/B} = \frac{(v_{C/B})^2}{r_{CB}} = \frac{(43.9\frac{\text{mm}}{\text{s}})^2}{130 \text{ mm}} = 14.8\frac{\text{mm}}{\text{s}^2}$$

(directed from *C* toward *B*, ↑ 72.1°)

$$a^n{}_C = 0 \text{ (linear sliding motion)}$$

The acceleration polygon is constructed and shown in Figure 9.15C. The unknown accelerations can be analytically determined. See Table 9.3.

TABLE 9.3 Aceleration Components for Example Problem 9.7.

Vector	Reference Angle (θ_x)	Horizontal Component $a_h = a \cos\theta_x$	Vertical Component $a_v = a \sin\theta_x$
a_C	90°	0	a_C
$a_B{}^n$	90°	0	87.4
$a^n{}_{C/B}$	107.9°	−4.5	14.1
$a^t{}_{C/B}$	72.1°	0.31 $a^t{}_{C/B}$	0.91 $a^t{}_{C/B}$

Separate algebraic equations can be written for the horizontal and vertical components:

$$a_C = a^n{}_B +> a^n{}_{C/B} +> a^t{}_{C/B}$$

horizontal components:

$$0 = 0 + (-4.5) + (+0.31\, a_{C/B}{}^{t})$$

solving:

$$a_{C/B}{}^{t} = \frac{4.5}{0.31} = 14.5\frac{\text{mm}}{\text{s}^2}$$

vertical components:

$$a_C = (87.4) + (14.1) + (0.95\, a^{t}{}_{C/B})$$

solving:

$$a_C = (87.4) + (14.1) + [(0.95(14.5)] = 115.3\frac{\text{mm}}{\text{s}^2}$$

The notion of an acceleration image can be used to determine the acceleration of the center of gravity of the three moving links.

After reviewing the acceleration polygon, vector magnitudes can be obtained algebraically:

$$a_{G2} = \frac{a^{n}{}_{B}}{2} \text{ \{half between } O_A \text{ and } B\}$$

$$= \frac{(87.4)}{2} = 43.7\frac{\text{mm}}{\text{s}^2} \uparrow$$

$$a_{G3} = \frac{(a_C + a^{n}{}_{B})}{2} \text{ \{half between } B \text{ and } C\}$$

$$= \frac{(115.3 + 87.4)}{2} = 101.4\frac{\text{mm}}{\text{s}^2} \uparrow$$

$$a_{G4} = a_C \text{ \{link 4 is pure linear motion\}}$$

$$= 115.3\frac{\text{mm}}{\text{s}^2} \uparrow$$

The angular accelerations of the links can be determined by:

$$\alpha_2 = 0.0$$

$$\alpha_3 = \frac{a^{t}{}_{C/B}}{r_{CB}} = \frac{14.5\frac{\text{mm}}{\text{s}^2}}{130\text{ mm}} = 0.11\frac{\text{rad}}{\text{s}^2}\text{ ccw}$$

$$\alpha_4 = 0.0$$

VI. Mass and Inertia

By focusing on the inertial loads, the mass and weight of each link are given as:

$$m_2 = 0.35\text{ kg } W_2 = 0.35\text{ kg}\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 3.43\text{ N}$$

$$m_3 = 0.75 \text{ kg} \quad W_3 = 0.75 \text{ kg}\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 7.35 \text{ N}$$

$$m_4 = 1.2 \text{ kg} \quad W_4 = 1.2 \text{ kg}\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 11.76 \text{ N}$$

Although they include some intricate details for calculating mass moment of inertia, links 2 and 3 are assumed to be slender rods:

$$I_{G2} = \frac{1}{12}(m\, l^2)$$

$$= \frac{1}{12}\{0.35 \text{ kg}(20 \text{ mm})^2\} = 11.7 \text{ kg mm}^2$$

$$I_{G3} = \frac{1}{12}(m\, l^2)$$

$$= \frac{1}{12}\{0.75 \text{ kg}(130 \text{ mm})^2\} = 1056.3 \text{ kg mm}^2$$

Because link 4 undergoes pure linear motion, the mass moment of inertia is irrelevant.

V. Inertial Loads

Calculate the inertial loads from Equations 9.10 and 9.11:

Link 2:

$$F_{G2}{}^{i} = m_2\, a_{G2} = (0.35 \text{ kg})\left(43.7\frac{\text{mm}}{\text{s}^2}\right)$$

$$= 15.3 \text{ kg}\frac{\text{mm}}{\text{s}^2} = 0.0153 \text{ N} \uparrow$$

$$T_{G2}{}^{i} = I_{G2}\, \alpha_2 = (11.7 \text{ kg mm}^2)\left(0\, \frac{\text{rad}}{\text{s}^2}\right) = 0.0 \text{ Nmm}$$

Link 3:

$$F_{G3}{}^{i} = m_3\, a_{G3} = (0.75 \text{ kg})\left(101.4\frac{\text{mm}}{\text{s}^2}\right)$$

$$= 76.1 \text{ kg mm/s}^2 = 0.0761 \text{ N} \uparrow$$

$$T_{G3}{}^{i} = I_{G3}\, \alpha_3 = (1056.3 \text{ kg mm}^2)\left(0.11\frac{\text{rad}}{\text{s}^2}\right)$$

$$= 116.2 \text{ kg}\frac{\text{mm}^2}{\text{s}^2} = 0.1162 \text{ Nmm ccw}$$

Link 4:

$$F_{G4}{}^{i} = m_4\, a_{G4} = (1.2\text{ kg})\left(115.3\frac{\text{mm}}{\text{s}^2}\right)$$

$$= 138.4\text{ kg}\frac{\text{mm}}{\text{s}^2} = 0.1384\text{ N}\uparrow$$

VI. Dynamic Force Analysis

Free-body diagrams of all three moving links are shown in Figure 9.15E. Notice that the unknown joint forces are broken into vertical and horizontal components. Newton's third law states that for every action, there is an equal and opposite reaction. Therefore, the direction of the pin forces are shown appropriately and the magnitudes are equal:

$$F_{23h} = F_{32h}\ F_{23v} = F_{32v}\ F_{34h} = F_{43h}\ F_{34v} = F_{43v}$$

By focusing on the crank (*link* 2), equilibrium equations can be written as:

- $\Sigma F -> F^{i}_{2} = 0$

horizontal direction:

(9.4.1) $$-F_{21h} + F_{23h} = 0$$

vertical direction:

$$-F_{21v} + F_{23v} - W_2 - (+F^{i}_{G2}) = 0$$

(9.4.2) $$F_{21v} = -F_{23v} + W_2 + (+F^{i}_{G2}) = 3.45 - F_{23v}$$

- $\Sigma M -> T^{i}_{2} = 0$ {using point A as reference}

(9.4.3) $$F_{23h}(r_{AB}) - T_{\text{driver}} = 0$$

By focusing on the connecting arm (*link* 3), equilibrium equations can be written as:

- $\Sigma F -> F^{i}_{3} = 0$

horizontal direction:

$$+F_{32h} - F_{34h} = 0$$

(9.4.4) $$F_{32h} = F_{34h}$$

vertical direction:

(9.4.5) $$-F_{32v} - W_3 + F_{34v} - (+F^{i}_{G3}) = 0$$

- $\Sigma M -> T^{i}_{3} = 0$ {using point G_3 as reference}

(9.4.6) $$-F_{32h}(r_{BC}/2)(\cos\beta) + F_{32v}(r_{BC}/2)(\sin\beta) - F_{34h}(r_{BC}/2)(\cos\beta) + F_{34v}(r_{BC}/2)(\sin\beta) - (+\,T^{i}_{G3}\,) = 0$$

By focusing on the punch assembly *(link 4)*, equilibrium equations can be written as:

- $\Sigma F \rightarrow F^i_4 = 0$

horizontal direction:

$$+F_{43h} - F_{guide} = 0$$

(9.4.7) $$F_{43h} = F_{guide}$$

vertical direction:

(9.4.8) $$-F_{43v} - W_4 + F_{applied} + F_{friction} - (+F^i_{G4}) = 0$$

Because the line of action of all forces acting on link 4 converge at the same point, a moment equation of equilibrium is not applicable. In addition, the friction associated with the punch guide acts against the direction of motion (upward) and can be denoted as:

$$F_{friction} = \mu\, F_{guide}$$

Therefore, Equation 9.4.5 becomes:

$$F_{43v} = -W_4 + F_{applied} + F_{friction} - (+F^i_{G4})$$

$$= -11.76 + 15 + 0.15F_{guide} - 0.14$$

(9.4.9) $$= 0.15F_{guide} + 3.10$$

The task is to solve the eight preceding simultaneous equations for the eight unknown forces. Equation 9.4.9 can be substituted into 9.4.5 and rewritten as:

$$F_{32v} = -W_3 + F_{34v} - (+F^i_{G3})$$

$$= -W_3 + (0.15F_{guide} + 3.10) - (+F^i_{G3})$$

$$= -7.35 + 0.15F_{guide} + 3.10 - 0.08$$

(9.4.10) $$= 0.15F_{guide} - 4.33$$

Equation 9.4.7 can be substituted into 9.4.4:

(9.4.11) $$F_{32h} = F_{34h} = F_{guide}$$

Equations 9.4.7, 9.4.9, 9.4.10, and 9.4.11 can be substituted into 9.4.6:

$$F_{32h}\,(r_{BC}/2)(\cos\beta) + F_{32v}\,(r_{BC}/2)(\sin\beta) + F_{34h}\,(r_{BC}/2)(\cos\beta) + F_{34v}(r_{BC}/2)(\sin\beta) - (+\,T^i_{G3}) = 0$$

$$F_{guide}\,(r_{BC}/2)(\cos\beta) + (0.15F_{guide} - 4.33)\,(r_{BC}/2)(\sin\beta) + F_{guide}(r_{BC}/2)(\cos\beta) + (0.15F_{guide} + 3.10)\,(r_{BC}/2)(\sin\beta) - (+\,T^i_{G3}) = 0$$

$$F_{guide}(130/2)(\cos 17.9) + (0.15F_{guide} - 4.33)(130/2)(\sin 17.9)$$

$$F_{guide}(130/2)(\cos 17.9) + (0.15F_{guide} + 3.10)\,(130/2)(\sin 17.9) - (+\,0.12) = 0$$

solving:

$$F_{guide} = 0.19 \text{ N} \rightarrow$$

Substituting back to the previous equations:

$$F_{34h} = 0.19 \text{ N} \rightarrow F_{32h} = 0.19 \text{ N} \leftarrow$$

$$F_{34h} = 3.13 \text{ N} \uparrow F_{32v} = -4.30 \text{ N} (\downarrow)$$

$$F_{friction} = 0.03 \text{ N} \rightarrow F_{21h} = 0.19 \text{ N} \leftarrow$$

$$F_{21v} = 7.75 \text{ N} \downarrow T_{driver} = 3.80 \text{ N mm cw}$$

Any value, which was algebraically solved and contained a negative sign, reveals that the directions of those forces were incorrectly assumed when preparing the free-body diagrams. Therefore, the force, F_{32v}, is actually acting upward; following the same logic, F_{23v} is acting downward.

PROBLEMS

Although manual drawing methods can be instructive for problems requiring graphical solution, use of a CAD system is highly recommended.

Three forces are shown in Figure P9.1. For Problems 9–1 through 9–6,determine the resultant of the forces.

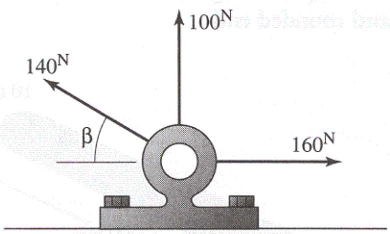


Figure P9.1 Problems 1-6.

9–1. $\beta = 25°$. Use graphical methods.
9–2. $\beta = 25°$. Use analytical methods.
9–3. $\beta = 65°$. Use graphical methods.
9–4. $\beta = 65°$. Use analytical methods.
9–5. $\beta = 105°$. Use graphical methods.
9–6. $\beta = 105°$. Use analytical methods.

A force that is applied to a box wrench is shown in Figure P9.7. For Problems 9–7 through 9–9, determine the moment—relative to the center of the nut—that is created by this force.

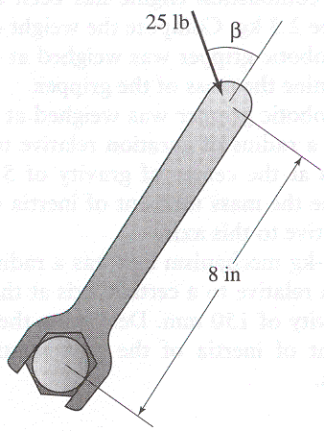


Figure P9.7 Problems 7-9.

9–7. $\beta = 90°$.
9–8. $\beta = 75°$.
9–9. $\beta = 110°$.

A force that is applied to a control lever is shown in Figure P9.10. For Problems 9–10 through 9–12, determine the moment relative to the pivot block that is created by this force.

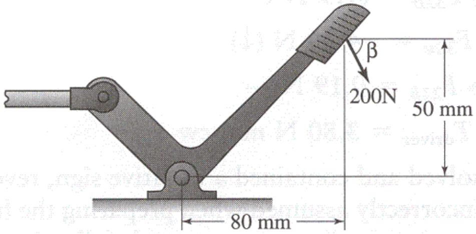


Figure P9.10 Problems 10-12.

9–10. $\beta = 0°$
9–11. $\beta = 60°$.
9–12. $\beta = 130°$.

Problems with Mass and Mass Moment of Inertia

9–13. The mass of a connecting rod from an internal combustion engine has been determined to be 2.3 kg. Compute the weight of the rod.

9–14. A robotic gripper was weighed at 4.5 lb. Determine the mass of the gripper.

9–15. A robotic gripper was weighed at 4.5 lb and has a radius of gyration relative to a certain axis at the center of gravity of 5 in. Determine the mass moment of inertia of the part relative to this axis.

9–16. A 6-kg mechanism link has a radius of gyration relative to a certain axis at the center of gravity of 150 mm. Determine the mass moment of inertia of the part relative to this axis.

9–17. The part shown in Figure P9.17 has been weighed at 6 lb. Determine the mass moment of inertia of the part relative to an x-axis at the center of the part and also relative to an x-axis at the end of the part. Neglect the effect of the holes.

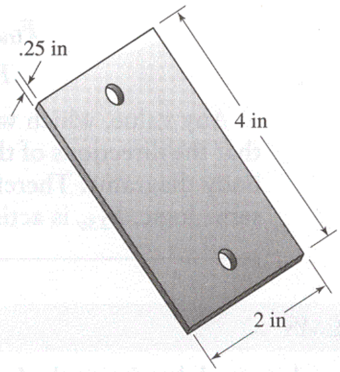


Figure P9.17 Problem 17.

9–18. For the 10-kg part shown in Figure P9.18, determine the mass moment of inertia of the part relative to an x-axis at the center of the part and also relative to an x-axis at the end of the part. Neglect the effect of the holes and rounded ends.

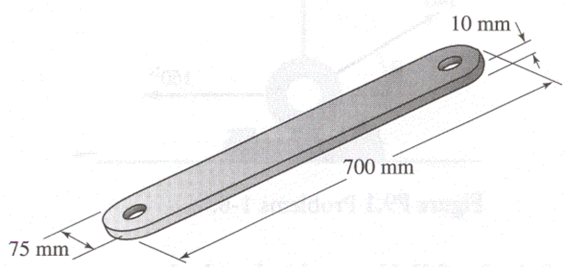


Figure P9.18 Problems 18.

A force that is applied to a control lever is shown in Figure P9.10. For Problems 9–10 through 9–12, determine the moment relative to the pivot block that is created by this force.

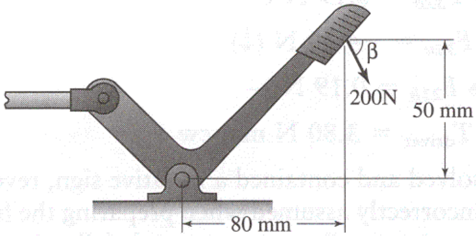


Figure P9.10 Problems 10-12.

9–10. $\beta = 0°$

9–11. $\beta = 60°$.

9–12. $\beta = 130°$.

Problems with Mass and Mass Moment of Inertia

9–13. The mass of a connecting rod from an internal combustion engine has been determined to be 2.3 kg. Compute the weight of the rod.

9–14. A robotic gripper was weighed at 4.5 lb. Determine the mass of the gripper.

9–15. A robotic gripper was weighed at 4.5 lb and has a radius of gyration relative to a certain axis at the center of gravity of 5 in. Determine the mass moment of inertia of the part relative to this axis.

9–16. A 6-kg mechanism link has a radius of gyration relative to a certain axis at the center of gravity of 150 mm. Determine the mass moment of inertia of the part relative to this axis.

9–17. The part shown in Figure P9.17 has been weighed at 6 lb. Determine the mass moment of inertia of the part relative to an x-axis at the center of the part and also relative to an x-axis at the end of the part. Neglect the effect of the holes.

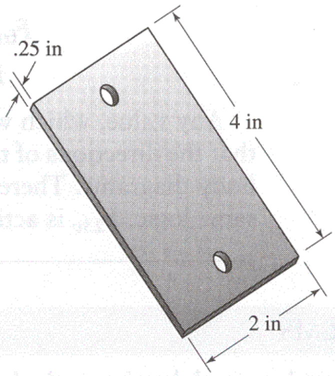


Figure P9.17 Problem 17.

9–18. For the 10-kg part shown in Figure P9.18, determine the mass moment of inertia of the part relative to an x-axis at the center of the part and also relative to an x-axis at the end of the part. Neglect the effect of the holes and rounded ends.

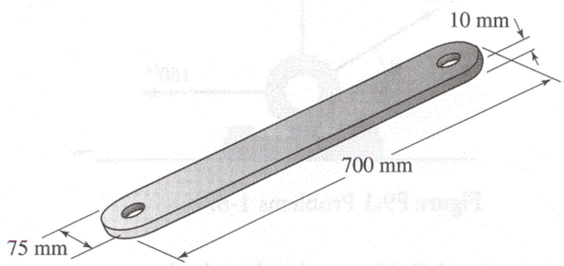


Figure P9.18 Problems 18.

Generating Free Body Diagrams

9–19. Figure P9.19 shows a overhead lift device. When a 600-lb force is suspended from the top boom, while the mechanism is stationary, draw a free-body diagram for each link. The top boom weighs 80 lb and the weight of the cylinder is negligible.

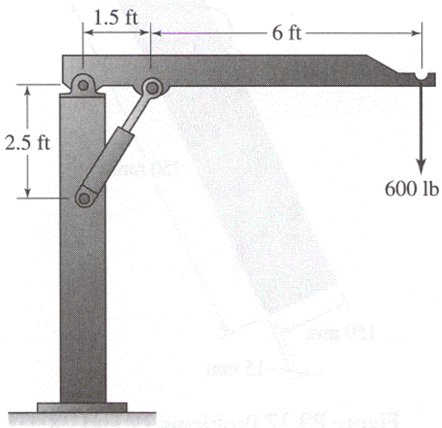


Figure P9.19 Problems 19 and 22.

9–20. Figure P9.20 shows a mechanism that raises packages in a transfer mechanism. When a 100-N package sits on the horizontal link, while the mechanism is stationary, draw a free-body diagram for each link. The weights of the links are negligible.

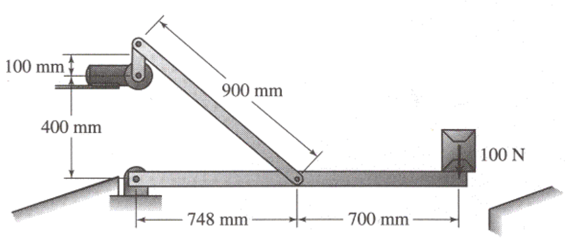


Figure P9.20 Problems 20 and 23.

9–21. Figure P9.21 shows a mechanism that is used to shear thin gauge sheet metal. When a 200-N force is applied as shown, draw a free-body diagram for each link. The weights of the links are negligible.

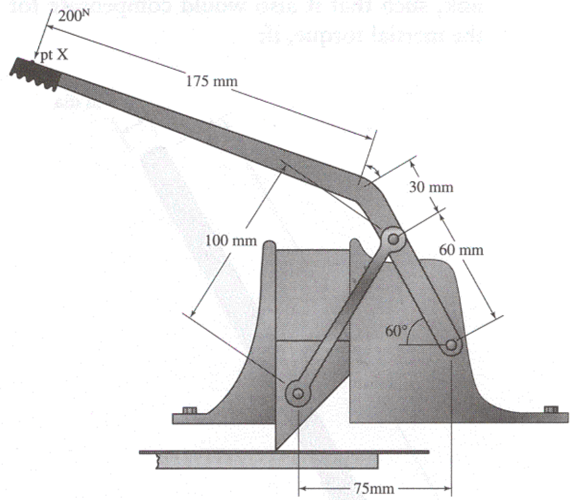


Figure P9.21 Problems 21 and 24.

Problems with Static Machine Forces

9–22. For the mechanism described in Problem 9–19, determine the force in the hydraulic cylinder.

9–23. For the mechanism described in Problem 9–20, determine the torque required from the motor.

9–24. For the mechanism described in Problem 9–21, determine the force that is applied to the sheet metal.

Problems with Inertial Forces

Figure P9.25 shows a link that weighs 4 lbs and is rotating clockwise at 20 rad/sec. For Problems 9–25 and 9–26, determine the magnitude of the inertial force and locate it on the link, such that it also would compensate for the inertial torque, if:

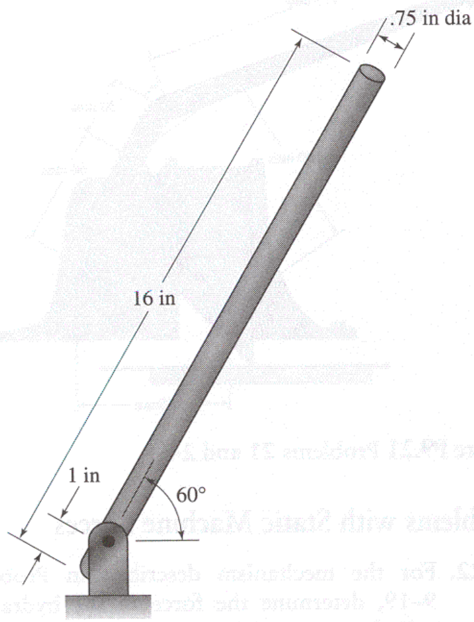


Figure P9.25 Problems 25 and 26.

9–25. The link was accelerating at $60 \frac{\text{rad}}{\text{s}^2}$.

9–26. The link decelerates at $60 \frac{\text{rad}}{\text{s}^2}$.

Figure P9.27 shows a 10 kg link that rotates counterclockwise at 15 rad/sec. Determine the magnitude of the inertial force and locate it on the link, such that it also compensates for the inertial torque, if:

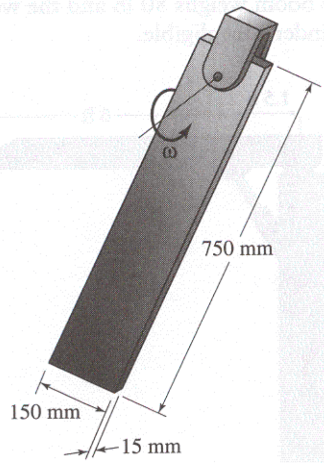


Figure P9.27 Problems 27 and 28.

9–27. The link accelerates at $40 \frac{\text{rad}}{\text{s}^2}$.

9–28. The link decelerates at $40 \frac{\text{rad}}{\text{s}^2}$.

9–29. Figure P9.29 shows a slider-crank mechanism. Link 2 rotates clockwise at a constant 2 rad/sec. The weight of link 2 is negligible, link 3 is 3 lb, and link 4 is 2 lb. The radius of gyration of link 3 relative to the center of gravity is 3 in. For $\beta = 45°$, graphically determine the following:

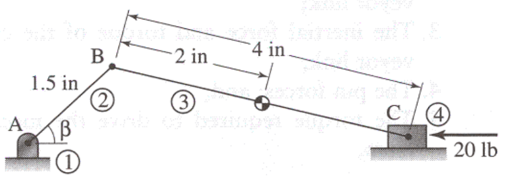


Figure P9.29 Problems 29-32.

1. The linear acceleration of link 4 and the center of gravity of link 3;
2. The angular acceleration of link 3;
3. The inertial force and torque of the coupler link;
4. The pin forces at *B* and *C*; and,
5. The torque to drive the mechanism in this position.

9–30. Solve Problem 9–29 using an analytical method.

9–31. Repeat Problem 9–29 with $\beta = 120°$.

9–32. Solve Problem 9–29, with $\beta = 120°$ and using an analytical method.

9–33. Figure P9.33 shows a four-bar mechanism. Link 2 rotates counterclockwise at a constant 10 rad/sec. The weight of link 2 and 3 are negligible, and link 4 is 17 kg. The radius of gyration of link 4 relative to the center of gravity is 45 mm. For $\beta = 45°$, graphically determine the following:

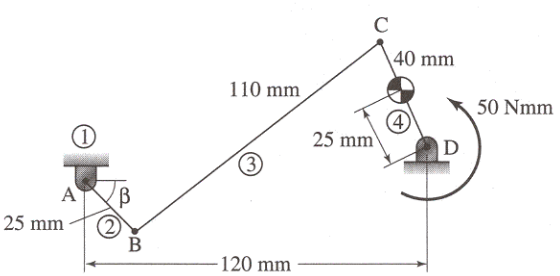


Figure P9.33 Problems 33-36.

1. The linear acceleration of the center of gravity of link 4;
2. The angular acceleration of link 4;
3. The inertial force and torque of link 4;
4. The pin forces at *B* and *C*; and,
5. The torque to drive the mechanism in this position.

9–34. Solve Problem 9–33 using an analytical method.

9–35. Repeat Problem 9-33 with $\beta = 90°$.

9–36. Solve Problem 9-33 with $\beta = 90°$ and using an analytical method.

9–37. Figure P9.37 shows a small hydraulic jack. At this instant, a 10-lb force is applied to the handle. This causes the 3.5 in link to rotate clockwise at a constant rate of 5 rad/sec. The weight of link 2 and 3 are negligible, and link 4 is 1.5 lb. Graphically determine the following:

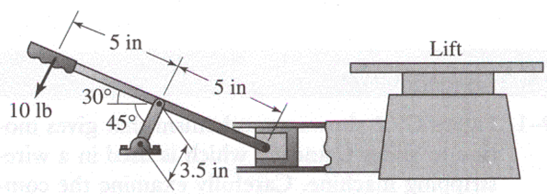


Figure P9.37 Problems 37 and 38.

1. The linear acceleration of the piston;
2. The inertial force of link 4;
3. The pin forces; and,
4. The force developed on the piston due to the hydraulic fluid.

9–38. Solve Problem 9-37 using an analytical method.

9–39. Figure P9.39 shows a mechanism for a transfer conveyor. The driving link rotates counterclockwise at a constant rate of 25 rpm. The box weighs 50 lb as shown. The weight of the driving link and the coupler are negligible. The weight of the conveyor link is 28 lb and the center of gravity is at its midspan.

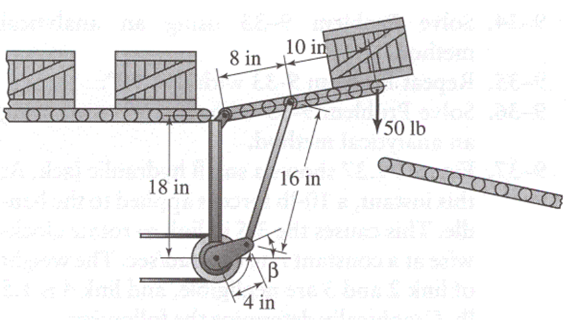


Figure P9.39 Problems 39-42.

The radius of gyration of the conveyor link relative to the center of gravity is 26 in. For $\beta = 30°$, graphically (using manual drawing techniques or CAD) determine the following:

1. The linear acceleration of the center of gravity of the conveyor linkl;
2. The rotational acceleration of the conveyor link;
3. The inertial force and torque of the conveyor link;
4. The pin forces; and,
5. The torque required to drive the mechanism.

9–40. Solve Problem 9–39 using an analytical method.

9–41. Repeat Problem 9–39 with $\beta = 100°$.

9–42. Solve Problem 9–39 with $\beta = 100°$ and using an analytical method.

CASE STUDIES

9–1. Figure C9.1 shows a mechanism that gives motion to slides *C* and *D*, which is used in a wire-stripping machine. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

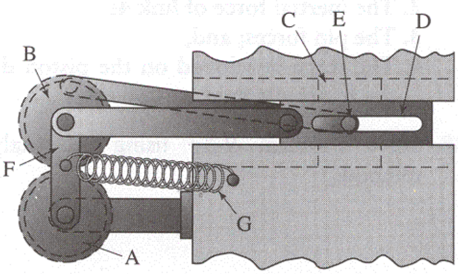


Figure C9.1 (Courtesy, Industrial Press)

1. As gear *A* rotates clockwise, describe the motion of gear *B*.
2. As gear *A* rotates clockwise, what is the immediate motion of slide *C*?
3. Discuss the action that takes place as pin *E* reaches the end of the slot.
4. Discuss precisely the continual motion of slides *C* and *D*.
5. Discuss how this motion could possibly be used in a wire-stripping machine.
6. What is the purpose of spring *G*?
7. How would the mechanism change if a "stiffer" spring were installed?

9–2. Figure C9.2 shows a mechanism used to actuate the reclining feature on some passenger airline seats. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

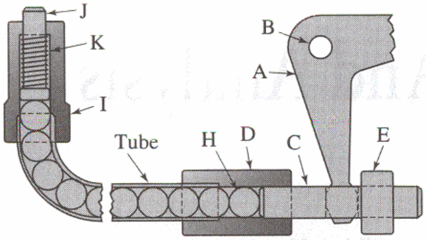


Figure C9.2 (Courtesy, Industrial Press)

1. As lever *A* is rotated, what type of motion does item *C* exhibit?
2. What type of connection does item *A* and *C* have?
3. What type of motion does ball *H* have?
4. What type of motion does plunger *J* have?
5. What is the purpose of spring *K*?
6. What is the purpose of item *E*?
7. What is the purpose of this mechanism?
8. Compare this mechanism to another mechanical concept.

10 Cams: Design And Analysis

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Identify the different types of cams and cam followers.
2. Create a follower displacement diagram from prescribed follower motion criteria.
3. Understand the benefits of different follower motion schemes.
4. Use equations to construct cam follower displacement diagrams.
5. Geometrically construct cam follower displacement diagrams.
6. Graphically and analytically construct disk cam profiles with several types of followers.
7. Graphically and analytically construct cylindrical cam profiles.

10.1 INTRODUCTION

A cam is a common mechanism element, which drives a mating component known as a follower. From a functional viewpoint, a cam and follower arrangement is very similar to the linkages discussed throughout this book. The cam accepts an input motion similar to a crank and imparts a resultant motion to a follower.

The unique feature of a cam is that it can impart a very distinct motion to its follower. In fact, cams can be used to obtain unusual or irregular motion that would be difficult to obtain from other linkages. Because its motion can be prescribed, the cam is well suited for applications where distinct displacements and timing are paramount. Cams are often used in factory automation equipment because they can sequence displacements in a cost efficient manner.

Figure 10.1 illustrates one of the most common cam applications, namely the valve train of an automotive engine. In this application, an oblong-shaped cam is machined on a shaft. This cam shaft is driven by the engine. As the cam rotates, a rocker arm drags on its oblong surface. The rocker arm, in turn, imparts a linear, reciprocating motion to a valve stem. The motion of the valve must be such that the exhaust pathway is closed during a distinct portion of the combustion cycle, and open during another distinct portion. Thus, the application is perfect for a cam because timing and motion must be precisely sequenced.

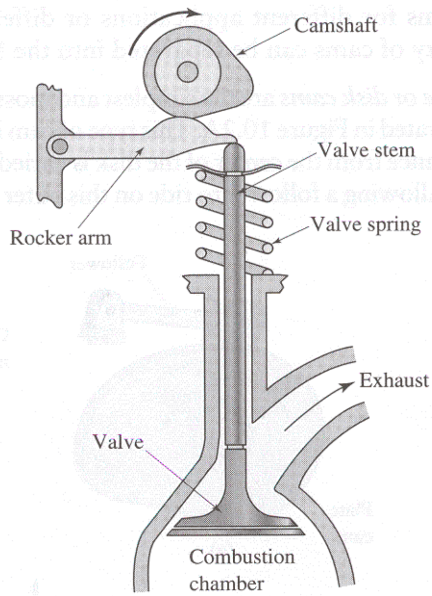


Figure 10.1 Engine valve train.

Notice a spring is used around the valve stem. The rocker arm follower needs to maintain contact with the cam surface to achieve the desired motion. Thus, in most cam applications, the follower is forced against the cam surface through some mechanical means. Springs are very popular for this purpose. In cases where the follower is in the vertical plane, the weight of the follower may be sufficient to maintain contact. The important point is that contact between the cam and the follower must be sustained.

A cam can be designed to impart a prescribed motion to the follower. In contrast, the linkages described previously in this text are analyzed to determine their motion. Achieving an exact, desired motion often involves trial-and-error design. This is done by analyzing an original linkage configuration. By observing the resulting motion, modifications are made to the linkage until the desired motion is achieved. This process of designing a mechanism to achieve a prescribed motion is termed *kinematic synthesis*.

Although advanced kinematic techniques allow for direct synthesis of possible linkages, the trial-and-error design methods are more commonly used in practice. However, synthesis of cams is a straightforward process and is the primary focus of this chapter.

10.2 TYPES OF CAMS

A great variety of cams are available from companies that specialize in design and manufacture. The manufacturers may classify cams into subcategories and market the cams for different applications or different configurations. However, the great majority of cams can be separated into the following three general types:

Plate or disk cams are the simplest and most common type of cam. A plate cam is illustrated in Figure 10.2A. This type of cam is formed on a disk or plate. The radial distance from the center of the disk is varied throughout the circumference of the cam. Allowing a follower to ride on this outer edge gives the follower a radial motion.

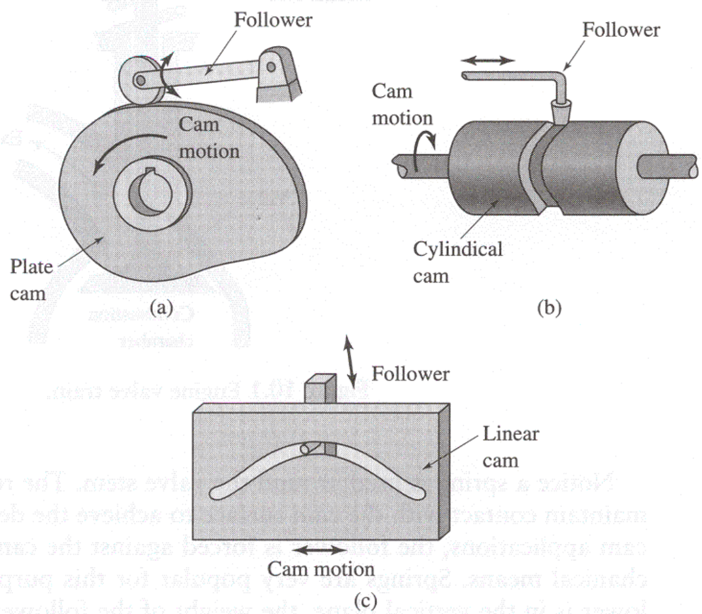


Figure 10.2 Cam types.

A *cylindrical or drum cam* is illustrated in Figure 10.2B. This type of cam is formed on a cylinder. A groove is cut into the cylinder, which varies along the axis of rotation. Attaching a follower that rides in the groove gives the follower motion along the axis of rotation.

A *linear cam* is illustrated in Figure 10.2C. This type of cam is formed on a translated block. A groove is cut into the block with a distance that varies from the plane of translation. Attaching a follower that rides in the groove gives the follower motion perpendicular to the plane of translation.

As mentioned, plate cams are the most common type of cam. Once the underlying theory is understood, it also is equally applicable to other types of cams.

10.3 TYPES OF FOLLOWERS

Followers are classified by their motion, shape, and position. The details of these classifications are discussed below.

Follower Motion

Follower motion can be separated into the following two categories:

Translating followers are constrained to motion in a straight line (Figure 10.3A).

Swinging arm or pivoted followers are constrained to rotational motion (Figure 10.3B).

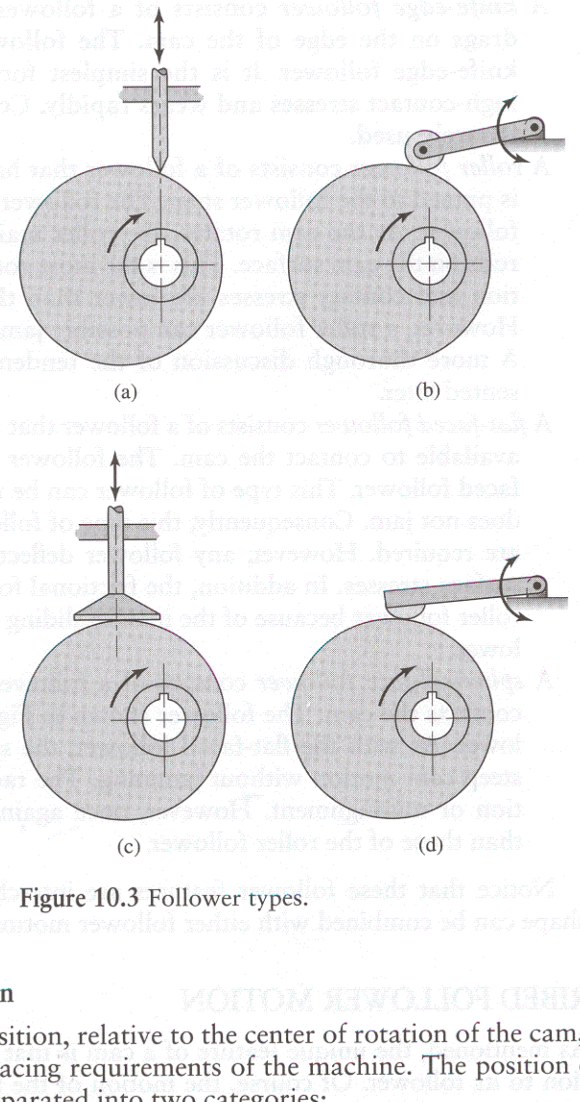


Figure 10.3 Follower types.

Follower Position

The follower position, relative to the center of rotation of the cam, is typically influenced by any spacing requirements of the machine. The position of translating followers can be separated into two categories:

An *in-line follower* exhibits straight line motion, such that the line of translation extends through the center of rotation of the cam (Figure 10.3A).

An *offset follower* exhibits straight line motion, such that the line of the motion is offset from the center of rotation of the cam (Figure 10.3C).

In the case of pivoted followers, there is no need to distinguish between in-line and offset followers because they exhibit identical kinematics.

Follower Shape

Finally, the follower shape can be separated into the following four categories:

A *knife-edge follower* consists of a follower that is formed to a point and drags on the edge of the cam. The follower shown in Figure 10.3A is a knife-edge follower. It is the simplest form but the sharp edge produces high-contact stresses and wears rapidly. Consequently, this type of follower is rarely used.

A *roller follower* consists of a follower that has a separate part, the roller, that is pinned to the follower stem. The follower shown in Figure 10.3B is a roller follower. As the cam rotates, the roller maintains contact with the cam and rolls on the cam surface. This is the most commonly used follower as the friction and contact stresses are lower than those for the knife-edge follower. However, a roller follower can possibly jam during steep cam displacements. A more thorough discussion of the tendency for a follower to jam is presented later.

A *flat-faced follower* consists of a follower that is formed with a large, flat surface available to contact the cam. The follower shown in Figure 10.3C is a flat-faced follower. This type of follower can be used with a steep cam motion but does not jam. Consequently, this type of follower is used when quick motions are required. However, any follower deflection or misalignment causes high surface stresses. In addition, the frictional forces are greater than those of the roller follower because of the intense sliding contact between the cam and follower.

A *spherical-face follower* consists of a follower formed with a radius face that contacts the cam. The follower shown in Figure 10.3D is a spherical-face follower. As with the flat-faced follower, the spherical-face can be used with a steep cam motion without jamming. The radius face compensates for deflection or misalignment. However, once again the frictional forces are greater than those of the roller follower.

Notice that these follower features are interchangeable. That is, any follower shape can be combined with either follower motion or position.

10.4 PRESCRIBED FOLLOWER MOTION

As mentioned, the unique feature of a cam is that it can impart a very distinct motion to its follower. Of course, the motion of the follower depends on the task desired and can be prescribed to exacting detail.

For example, suppose a follower is used to drive pick-up fingers on a paper handling machine. Prescribing the desired follower involves separating the motion into segments, and defining the action that must take place during the segments. To illustrate this process, assume that the pickup fingers must:

1. Remain closed (extreme upward follower position) for 0.003 sec.
2. It must open to a distance of 0.25 in, from the closed position, in 0.001 sec.
3. It must remain in this open position for 0.002 sec.
4. It must move to the closed position in 0.001 sec.

Thus, by listing the exact requirements of the fingers, the motion of the follower has been prescribed.

It is convenient to plot the prescribed follower motion versus time. Actually, the follower motion can be expressed in terms of angular cam displacement rather than time. A plot of follower displacement versus time, or cam angular displacement is termed a *follower displacement diagram.* This diagram is indispensable in that the follower motion and kinematics can be explored without regard to the shape of the cam itself. The vertical axis of this diagram displays the linear follower displacement, expressed in inches or millimeters. The horizontal axis displays time, measured in seconds or minutes, or angular cam displacements, measured in degrees or fractions of a revolution. This diagram is usually constructed to scale and along with follower kinematic analysis, it is extremely useful in determining cam shape.

For kinematic analysis, the follower displacement versus time curve is preferred. To assist in the task of designing a cam shape, the follower displacement versus cam angle curve is desired. Relating the cam rotation and time is a straightforward process using the theory presented in Chapter 5. Equation 5.4 gave the following:

(4.4) $$\omega = \frac{\Delta\theta}{\Delta t}$$

When the cam is assumed to rotate at a constant velocity, which incorporates the overwhelming majority of applications, time can be related to angular displacement and vice versa .The angular displacement of the cam that corresponds with each motion segment can be determined as:

(10.1) $$\Delta\theta = \omega\Delta t$$

The required speed of the cam is determined by observing the time consumed during one cycle. Thus:

$$\omega_{cam} = \frac{\Delta\theta}{\Delta t}$$

setting:

$$\Delta\theta = 1 \text{ cam revolution}$$

$$\Delta t = \text{time consumed in 1 cycle}$$

(10.2) $$\omega_{cam} = \frac{1 \text{ rev}}{\Delta t_{\text{for 1 cycle}}}$$

The period of cam rotation where there is no follower motion is termed a *dwell.* The details of motion during the follower raising and lowering sequences are primarily dictated by the task that needs to be accomplished and dynamic considerations. Because large forces are associated with large accelerations, there is a benefit to minimizing acceleration.

EXAMPLE PROBLEM 10.1

A cam is to be used for a platform that will repeatedly lift boxes from a lower conveyor to an upper conveyor. This machine is shown in Figure 10.4. Plot a displacement diagram and determine the required speed of the cam when the follower motion sequence is as follows:

1. Rise 2 in. in 1.2 sec
2. Dwell for 0.3 sec
3. Fall 1 in. in 0.9 sec
4. Dwell 0.6 sec
5. Fall 1 in. in 0.9 sec

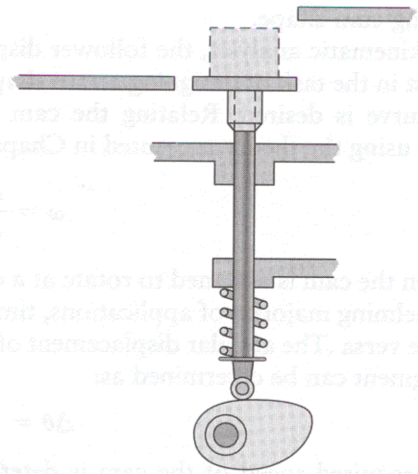


Figure 10.4 Cam system for Example Problem 10.1.

Solution:

The total time to complete the full cycle is needed to determine the required speed of the cam.

$$\Delta t_{\text{for 1 cycle}} = (1.2 + 0.3 + 0.9 + 0.6 + 0.9)\ \text{sec} = 3.9\ \text{sec}$$

Then from Equation 10.2:

$$\omega_{\text{cam}} = \frac{1\ \text{rev}}{\Delta t_{\text{for 1 cycle}}}$$

$$= \frac{1 \text{ rev}}{3.9 \text{ sec}} = 0.256\frac{\text{rev}}{\text{sec}}$$

$$= 0.256\frac{\text{rev}}{\text{sec}}\left(\frac{60\text{sec}}{1 \text{ min}}\right) = 15.38 \text{ rpm}$$

The angular increment of the cam consumed by each follower motion sequence is determined by Equation 10.1:

$$\Delta\theta_1 = (\omega_{cam})\ (\Delta t) = \left(0.256\frac{\text{rev}}{\text{sec}}\right)(1.2 \text{ sec}) = 0.307 \text{ rev}$$

$$\Delta\theta_1 = 0.307 \text{ rev } (360° / 1 \text{ rev}) = 110.5°$$

$$\Delta\theta_2 = \left(0.256\frac{\text{rev}}{\text{sec}}\right)(0.3 \text{ sec}) = 0.077 \text{ rev} = 27.6°$$

$$\Delta\theta_3 = \left(0.256\frac{\text{rev}}{\text{sec}}\right)(0.9 \text{ sec}) = 0.230 \text{ rev} = 82.9°$$

$$\Delta\theta_4 = \left(0.256\frac{\text{rev}}{\text{sec}}\right)(0.6 \text{ sec}) = 0.154 \text{ rev} = 55.3°$$

$$\Delta\theta_5 = \left(0.256\frac{\text{rev}}{\text{sec}}\right)(0.9 \text{ sec}) = 0.230 \text{ rev} = 82.9°$$

The resulting displacement diagram is shown in Figure 10.5. Notice that a curved displacement profile was constructed during the rise and fall sequences. Dynamic considerations dictate the actual shape of the rise and fall sections.

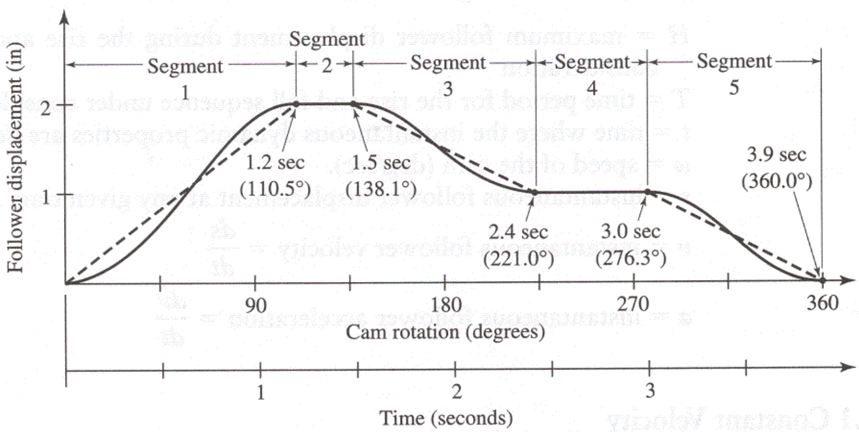


Figure 10.5 Displacement diagram for Example Problem 10.1.

10.5 FOLLOWER MOTION SCHEMES

In designing a cam, the objective is to identify a suitable profile. Through this process, the primary interest is to ensure that the follower achieves the desired displacements. Of course, these displacements are outlined in the displacement diagram. The shape of the cam is merely a means to obtain this motion.

In the discussion of the previous section, the follower motion during rise and fall sequences was not fully identified. It was mentioned that the dynamic characteristics of the follower are important. Large accelerations cause large forces and, consequently, high stresses. Rapidly changing accelerations cause vibration and, consequently, noise. Due to these fundamental dynamic principles, the rise and fall portions of a cam displacement diagram are of vital importance.

For slow-moving cams, high accelerations are not a factor. Therefore, the cam is designed to merely yield the given displacements at the specified instant. The manner in which the follower arrives at the given point is irrelevant. In these cases, the cam is manufactured in the most convenient manner, as long as the given displacement is achieved. A plate cam can be simply composed of a combination of circular arcs and straight lines, which can be readily manufactured.

For high-speed applications, it is not enough to provide a given displacement at a specified instant. The dynamic characteristics of the follower during the rise and fall sequences must be studied in considerable detail in order to minimize the forces and vibrations.

A wide variety of motion schemes have been thoroughly analyzed, all with the sole purpose of defining the motion of the follower during a rise or fall sequence. The objective of these schemes is to produce the movement with smooth accelerations.

In studying the dynamic characteristics of the follower for the different motion schemes, the following nomenclature is used:

H = maximum follower displacement during the rise and fall sequence under consideration
T = time period for the rise and fall sequence under consideration
t = time where the instantaneous dynamic properties are defined
ω = speed of the cam (deg/sec).
s = instantaneous follower displacement at any given time
v = instantaneous follower velocity $= \dfrac{ds}{dt}$
a = instantaneous follower acceleration $= \dfrac{dv}{dt}$

10.5.1 Constant Velocity

The simplest follower motion during a rise or fall sequence is constant velocity. Constant velocity motion is characterized with a straight line displacement diagram because velocity is uniform. The dynamic characteristics of a constant velocity rise are listed in Table 10.1:

TABLE 10.1 Dynamic Characteristics of a Constant Velocity Rise

	Rise	Fall
Displacement:	$s = \frac{Ht}{T}$	$s = \frac{H}{T(1 - t)}$
Velocity:	$v = \frac{H}{T}$	$v = -\frac{H}{T}$
Acceleration:	$a = 0$	$a = 0$

Although the notion of zero acceleration is appealing, the ends of this motion scheme cause problems. Theoretically, the instantaneous jump from any constant value of velocity to another constant value of velocity results in an infinite acceleration. Because mass is always involved in machines, this theoretically results in an infinite force. Actually, an instantaneous change in velocity is impossible due to the flexibility in machine members. Nevertheless, any shock is serious and must be kept to a minimum. Therefore, this motion in its pure form is impractical except for low-speed applications.

A constant velocity displacement diagram, along with velocity and acceleration curves, is shown in Figure 10.6.

TABLE 10.2 Dynamic Characteristics of Constant Acceleration Rise

	Rise	Fall
(For $0 < t < 0.5\,T$)		
Displacement	$s = 2H\left(\frac{t}{T}\right)^2$	$s = H - 2H\left(\frac{t}{T}\right)^2$
Velocity	$v = \frac{4Ht}{T^2}$	$v = -\frac{4Ht}{T^2}$
Acceleration	$a = \frac{4H}{T^2}$	$a = -\frac{4H}{T^2}$
(For $0.5T < t < T$)		
Displacement	$s = H - 2H\left[1 - \frac{t}{T}\right]^2$	$s = 2H\left[1 - \frac{t}{T}\right]^2$
Velocity	$v = \frac{4H}{T^2}\left[1 - \frac{t}{T}\right]$	$v = -\frac{4H}{T^2}\left[1 - \frac{t}{T}\right]$
Acceleration	$a = -\frac{4H}{T^2}$	$a = \frac{4H}{T^2}$

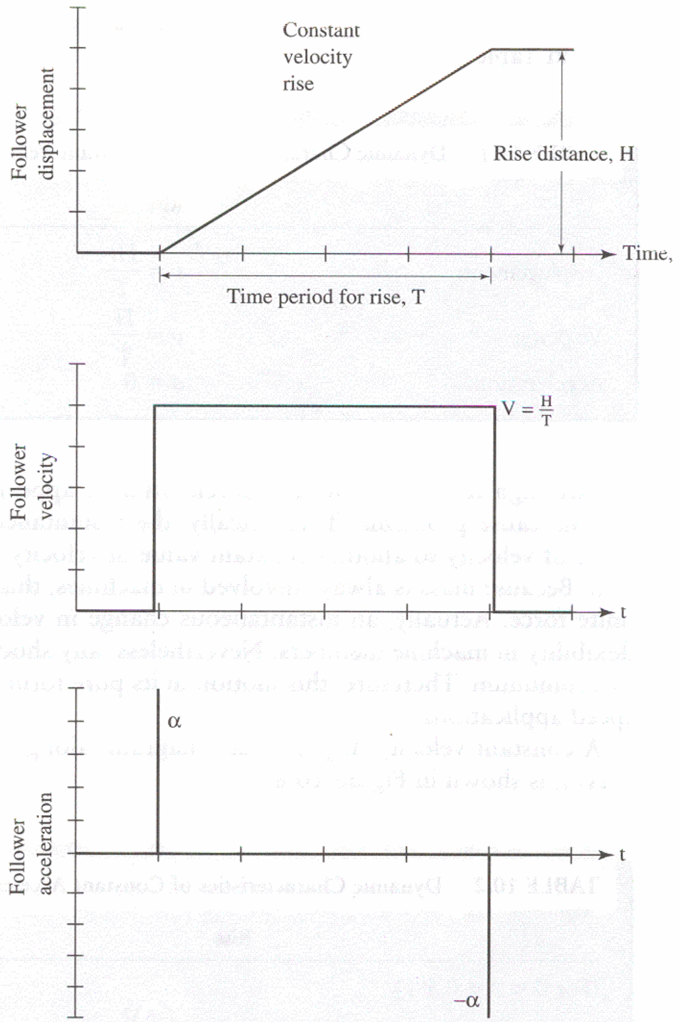


Figure 10.6 Constant velocity motion curves.

10.5.2 Constant Acceleration

Constant acceleration motion during a rise or fall sequence produces the smallest possible values of acceleration for a given rise and time interval. The displacement diagram for a rise or fall interval is divided into two halves, one of constant acceleration and the other of constant deceleration. The shapes of each half of the displacement diagram are mirror-image parabolas. The dynamic characteristics of a constant acceleration rise are listed in Table 10.2:

This motion scheme, also known as parabolic or gravity motion, has constant positive and negative accelerations. However, it has an abrupt change of acceleration at the end of the motion and at the transition point between acceleration and deceleration halves. These abrupt changes result in abrupt changes in inertial forces which typically cause undesirable vibrations. Therefore, this motion in its pure form is uncommon except for low-speed applications.

A constant acceleration displacement diagram, along with velocity and acceleration curves, is shown in Figure 10.7.

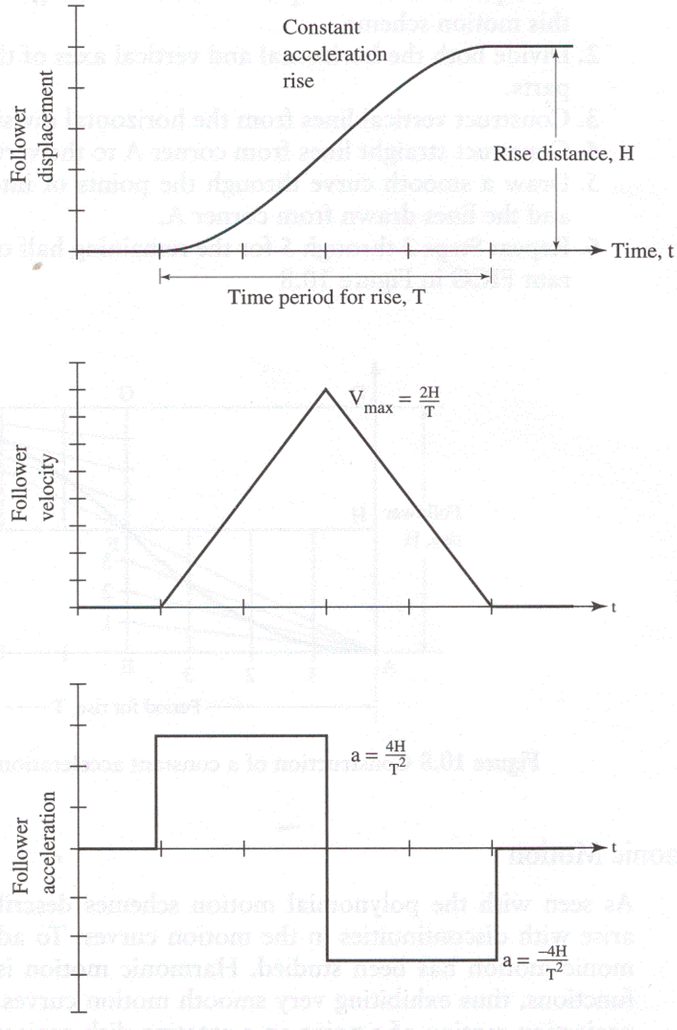


Figure 10.7 Constant acceleration motion curves.

A scaled displacement diagram is required to construct the actual cam profile. The equations presented above can be used in conjunction with a spreadsheet, or other equation plotting package, to complete this diagram. Although this analytical method is precise, care must be taken to plot the diagram to scale.

Geometric construction of a displacement diagram is an alternative method to generate a displacement diagram to scale. Such a construction using the constant acceleration motion scheme can be accomplished by referring to Figure 10.8 and using the following procedure:

1. Divide the follower rise (or fall) sequence into two halves. From Figure 10.8, AE represents the time period and EF the magnitude of rise for the first half of this motion scheme.
2. Divide both the horizontal and vertical axes of the quadrant AEFH into equal parts.
3. Construct vertical lines from the horizontal divisions.
4. Construct straight lines from corner A to the vertical divisions.
5. Draw a smooth curve through the points of intersection of the vertical lines and the lines drawn from corner A.
6. Repeat Steps 2 through 5 for the remaining half of the curve as shown in quadrant FICG in Figure 10.8

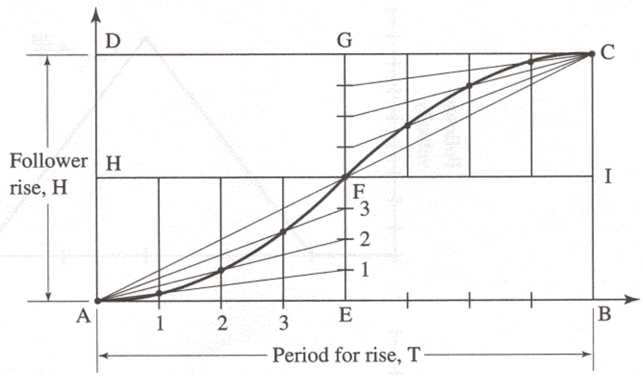


Figure 10.8 Construction of a constant acceleration displacement diagram.

10.5.3 Harmonic Motion

As seen with the polynomial motion schemes described above, inertial problems arise with discontinuities in the motion curves. To address that shortcoming, harmonic motion has been studied. Harmonic motion is derived from trigonometric functions, thus exhibiting very smooth motion curves. In a physical sense, it is the projection motion of a point on a rotating disk projected to a straight line. The dynamic characteristics of a harmonic rise are listed in Table 10.3.

This motion scheme is a definite improvement of the previous curves. It has a smooth, continuous acceleration. However, it has a sudden change of acceleration at the ends of the motion. Again, this sudden change can be objectionable at higher speeds.

A harmonic displacement diagram, along with velocity and acceleration curves are shown in Figure 10.9.

TABLE 10.3 Dynamic Characteristics of a Harmonic Motion

	Rise	Fall
Displacement	$s = 0.5H\left[1 - \cos\left(\frac{\pi t}{T}\right)\right]$	$s = 0.5H\left[1 + \cos\left(\frac{\pi t}{T}\right)\right]$
Velocity	$v = \frac{0.5H\pi}{T}\left[\sin\left(\frac{\pi t}{T}\right)\right]$	$v = -\frac{0.5\pi H}{T}\left[\sin\left(\frac{\pi t}{T}\right)\right]$
Acceleration	$a = \frac{0.5H\pi^2}{T^2}\left[\cos\left(\frac{\pi t}{T}\right)\right]$	$a = -\frac{0.5H\pi^2}{T^2}\left[\cos\left(\frac{\pi t}{T}\right)\right]$

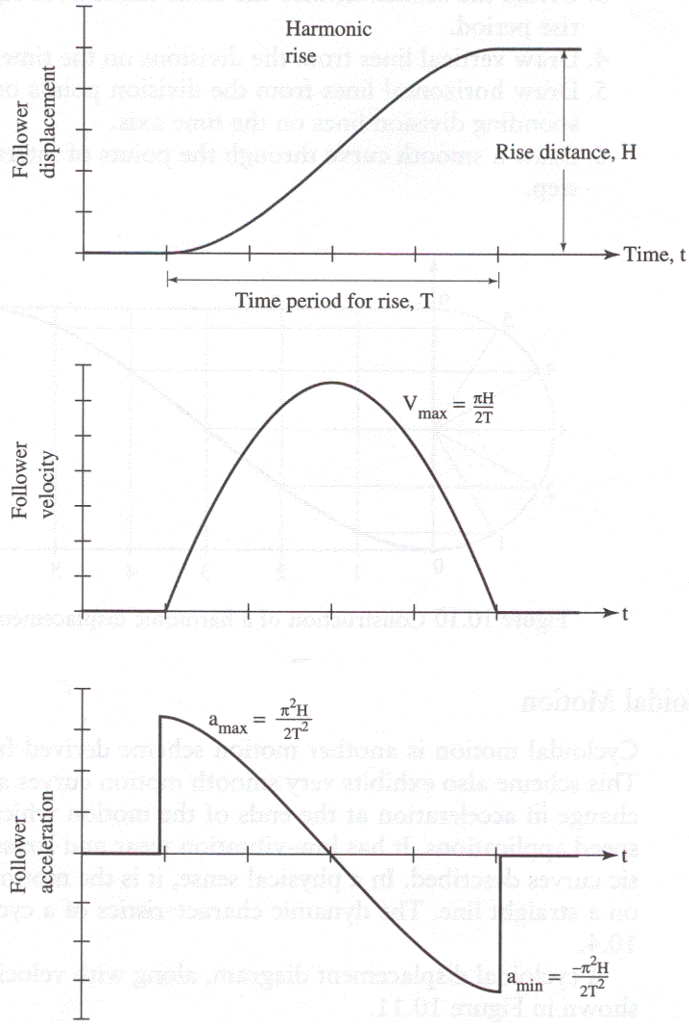


Figure 10.9 Harmonic motion curves.

As with other schemes, a scaled displacement diagram is required to construct the actual cam profile. The equations presented above can be used in conjunction with a spreadsheet or other equation plotting package to complete this diagram. Although this analytical method is precise, care must be taken to plot the diagram to scale.

Geometric construction of a displacement diagram is an alternative method to generate a displacement diagram to scale. Such a construction using the harmonic motion scheme can be accomplished by referring to Figure 10.10 and using the following procedure:

1. Construct a semicircle, having a diameter equal to the amount of rise (or fall) desired.
2. Divide the rise time period into incremental divisions.
3. Divide the semicircle into the same number of equal divisions of the follower rise period.
4. Draw vertical lines from the divisions on the time axis.
5. Draw horizontal lines from the division points on the semicircle to the corresponding division lines on the time axis.
6. Draw a smooth curve through the points of intersection found in the previous step.

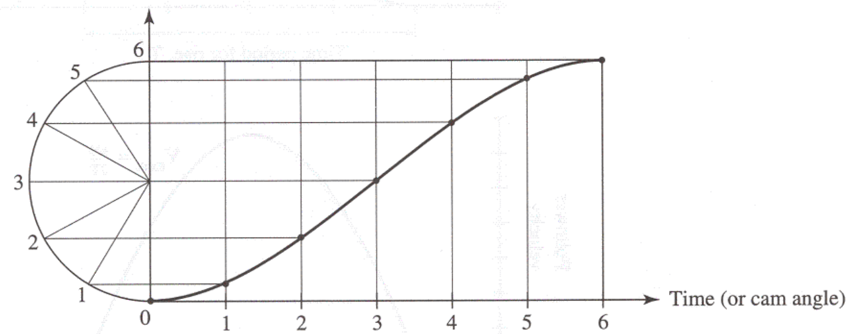


Figure 10.10 Construction of a harmonic displacement diagram.

10.5.4 Cycloidal Motion

Cycloidal motion is another motion scheme derived from trigonometric functions. This scheme also exhibits very smooth motion curves and does not have the sudden change in acceleration at the ends of the motion which makes it popular for high-speed applications. It has low-vibration wear and stress characteristics of all the basic curves described. In a physical sense, it is the motion of a point on a disk rolling on a straight line. The dynamic characteristics of a cycloidal rise are listed in Table 10.4.

A cycloidal displacement diagram, along with velocity and acceleration curves, is shown in Figure 10.11.

As before, a scaled displacement diagram is required to construct the actual cam profile. The equations presented above can be used in conjunction with a spread-

TABLE 10.4 Dynamic Characteristics of a Cycloidal Rise

	Rise	Fall
Displacement	$s = H\left[\frac{t}{T} - \frac{1}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\right]$	$s = H\left[1 - \left(\frac{t}{T}\right) + \frac{1}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\right]$
Velocity	$v = \frac{H}{T}\left[1 - \cos\left(\frac{2\pi t}{T}\right)\right]$	$v = -\frac{H}{T}\left[1 - \cos\left(\frac{2\pi t}{T}\right)\right]$
Acceleration	$a = \frac{2\pi H}{T^2}\left[\sin\left(\frac{2\pi t}{T}\right)\right]$	$a = \frac{-2\pi H}{T^2}\left[\sin\left(\frac{2\pi t}{T}\right)\right]$

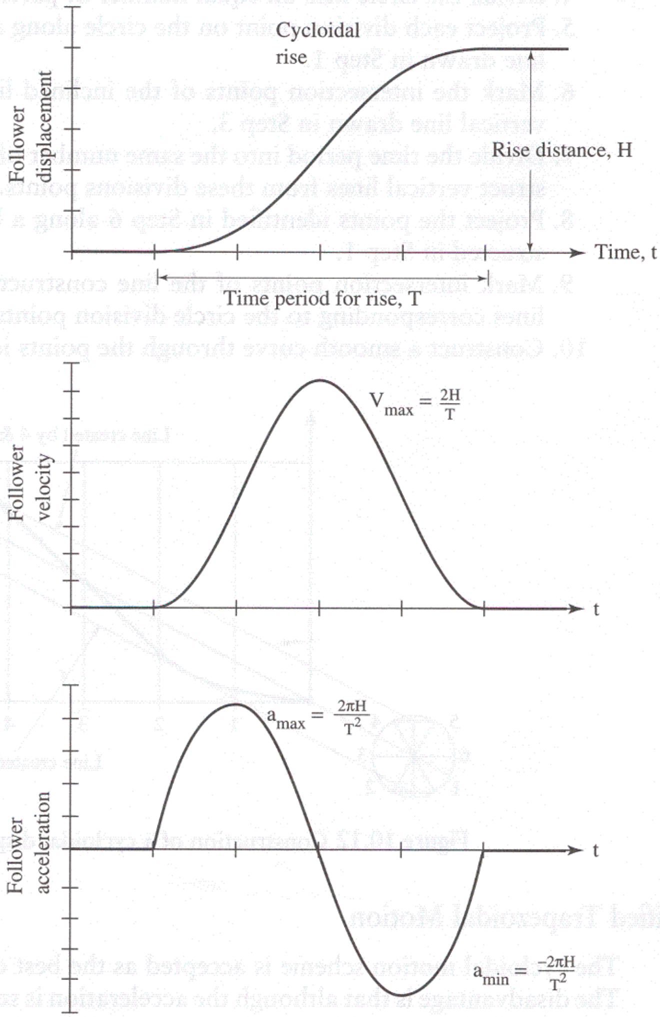


Figure 10.11 Cycloidal motion curves.

sheet or other equation plotting package to complete this diagram. Although this analytical method is precise, care must be taken to plot the diagram to scale.

Geometric construction of a displacement diagram is an alternative method to generating a displacement diagram to scale. Such a construction using the cycloidal motion scheme can be accomplished by referring to Figure 10.12 and using the following procedure:

1. On a displacement diagram grid, draw a line from the beginning point of the rise (or fall) to the final point. This line is drawn from *A* to *C* on Figure 10.12.
2. Extend the line drawn in the previous step and draw a circle, with radius $r = \frac{H}{2\pi}$ centered anywhere on that line.
3. Construct a vertical line through the center of the circle.
4. Divide the circle into an equal number of parts.
5. Project each division point on the circle along an inclined line, parallel to the line drawn in Step 1.
6. Mark the intersection points of the inclined lines drawn in Step 5 with the vertical line drawn in Step 3.
7. Divide the time period into the same number of equal parts as the circle. Construct vertical lines from these divisions points.
8. Project the points identified in Step 6 along a line parallel with the line constructed in Step 1.
9. Mark intersection points of the line constructed in Step 8 with the vertical lines corresponding to the circle division points that created the inclined line.
10. Construct a smooth curve through the points identified in Step 9.

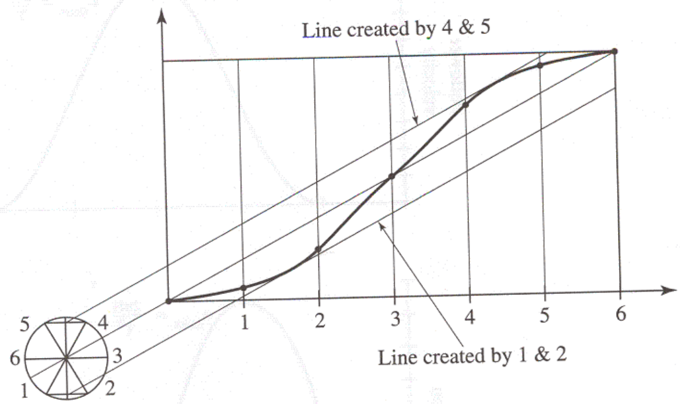


Figure 10.12 Construction of a cycloidal displacement diagram.

10.5.5 Modified Trapezoidal Motion

The cycloidal motion scheme is accepted as the best of the basic schemes discussed. The disadvantage is that although the acceleration is smooth, the extreme magnitudes are high. Many other motion schemes have been studied. Again, the primary goal of a scheme is to optimize the dynamic characteristics, thus minimizing acceleration without any sudden changes. The most popular motion scheme for high-speed applications is modified trapezoidal motion. Modified trapezoidal motion is a combination trigonometric and polynomial function. The acceleration appears as two trapezoids: one positive during acceleration and the other negative during deceleration. To facilitate smooth curves, the corners of the trapezoid are rounded off (Table 10.5).

A modified trapezoidal displacement diagram, along with velocity and acceleration curves, is shown in Figure 10.13.

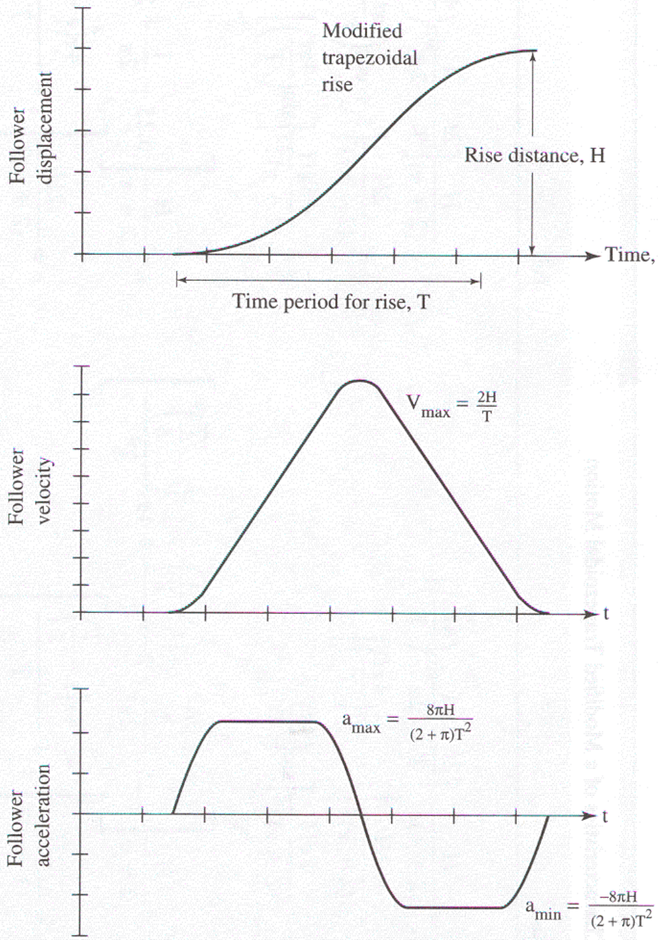


Figure 10.13 Modified trapezoidal motion curves.

As before, a scaled displacement diagram is required to construct the actual cam profile. The equations presented above can be used in conjunction with a spreadsheet or other equation plotting package to complete this diagram. Although this

TABLE 10.5 Dynamic Characteristics of a Modified Trapezoidal Motion

	Rise	Fall
$(0 < t < T/8)$		
Displacement	$s = \frac{H}{(2+\pi)} = \left[\frac{2t}{T} - \frac{1}{2\pi}\sin\left(\frac{4\pi t}{T}\right)\right]$	$s = \frac{H}{(2+\pi)}\left[\frac{2t}{T} + \frac{1}{2\pi}\sin\left(\frac{4\pi t}{T}\right)\right]$
Velocity	$v = \frac{2H}{(2+\pi)T}\left[1 - \cos\left(\frac{4\pi t}{T}\right)\right]$	$v = \frac{2H}{(2+\pi)T}\left[1 + \cos\left(\frac{4\pi t}{T}\right)\right]$
Acceleration	$a = \frac{8\pi H}{(2+\pi)T^2}\left[\sin\left(\frac{4\pi t}{T}\right)\right]$	$a = \frac{-8\pi H}{(2+\pi)T^2}\left[\sin\left(\frac{4\pi t}{T}\right)\right]$
$(T/8 < t < 3T/8)$		
Displacement	$s = \frac{H}{2+\pi}\left[0.25 - \frac{1}{2\pi} + 2\frac{\left(t - \frac{T}{8}\right)}{T} + 4\pi\frac{\left(t - \frac{T}{8}\right)^2}{T^2}\right]$	$s = \frac{H}{2+\pi}\left[0.25 - \frac{1}{2\pi} + 2\frac{\left(t - \frac{T}{8}\right)}{T} - 4\pi\frac{\left(t - \frac{T}{8}\right)^2}{T^2}\right]$
Velocity	$v = \frac{2H}{(2+\pi)T}\left[1 + \frac{4\pi\left(t - \frac{T}{8}\right)}{T}\right]$	$v = \frac{2H}{(2+\pi)T}\left[1 - \frac{4\pi\left(t - \frac{T}{8}\right)}{T}\right]$
Acceleration	$a = \frac{8\pi H}{(2+\pi)T^2}$	$a = \frac{-8\pi H}{(2+\pi)T^2}$
$(3T/8 < t < 5T/8)$		
Displacement	$s = \frac{H}{2+\pi}\left[\frac{-\pi}{2} + \frac{2(1+\pi)t}{T} - \frac{1}{2\pi}\sin\left(\frac{4\pi\left(t - \frac{T}{4}\right)}{T}\right)\right]$	$s = \frac{H}{2+\pi}\left[\frac{-\pi}{2} + \frac{2(1+\pi)t}{T} + \frac{1}{2\pi}\sin\left(\frac{4\pi\left(t - \frac{T}{4}\right)}{T}\right)\right]$
Velocity	$v = \frac{2H}{(2+\pi)T}\left[1 + \pi - \cos\left(\frac{4\pi\left(t - \frac{T}{4}\right)}{T}\right)\right]$	$v = \frac{2H}{(2+\pi)T}\left[1 + \pi + \cos\left(\frac{4\pi\left(t - \frac{T}{4}\right)}{T}\right)\right]$

Acceleration	$a = \frac{8\pi H}{(2+\pi)T^2}\sin\left(\frac{4\pi\left(t-\frac{T}{4}\right)}{T}\right)$	$a = \frac{-8\pi H}{(2+\pi)T^2}\sin\left(\frac{4\pi\left(t-\frac{T}{4}\right)}{T}\right)$
$(5T/8 < t < 7T/8)$		
Displacement	$s = \frac{H}{2+\pi}\left[1.75+\pi+\frac{1}{2\pi}+\frac{2}{T\left(t-\frac{7}{8}T\right)}-\frac{4\pi}{T^2\left(t-\frac{7}{8}T\right)^2}\right]$	$s = \frac{H}{2+\pi}\left[1.75+\pi-\frac{1}{2\pi}+\frac{2}{T\left(t-\frac{7}{8}T\right)}+\frac{4\pi}{T^2\left(t-\frac{7}{8}T\right)^2}\right]$
Velocity	$v = \frac{2H}{(2+\pi)T}\left[1-\frac{4\pi}{T}\left(t-\frac{7}{8}T\right)\right]$	$v = \frac{2H}{(2+\pi)T}\left[1+\frac{4\pi}{T}\left(t-\frac{7}{8}T\right)\right]$
Acceleration	$a = \frac{-8\pi H}{(2+\pi)T^2}$	$a = \frac{8\pi H}{(2+\pi)T^2}$
$(7T/8 < t < T)$		
Displacement	$s = \frac{H}{(2+\pi)}\left[1.5+\pi+\frac{2\left(t-\frac{6}{8}T\right)}{T}+\frac{1}{2\pi}\sin\left(\frac{4\pi\left(t-\frac{6}{8}T\right)}{T}\right)\right]$	$s = \frac{H}{(2+\pi)}\left[1.5+\pi+\frac{2\left(t-\frac{6}{8}T\right)}{T}-\frac{1}{2\pi}\sin\left(\frac{4\pi\left(t-\frac{6}{8}T\right)}{T}\right)\right]$
Velocity	$v = \frac{2H}{(2+\pi)T}\left[1+\cos\left(\frac{4\pi\left(t-\frac{6}{8}T\right)}{T}\right)\right]$	$v = \frac{2H}{(2+\pi)T}\left[1-\cos\left(\frac{4\pi\left(t-\frac{6}{8}T\right)}{T}\right)\right]$
Acceleration	$a = \frac{-8\pi H}{(2+\pi)T^2}\sin\left(\frac{4\pi\left(t-\frac{6}{8}T\right)}{T}\right)$	$a = \frac{8\pi H}{(2+\pi)T^2}\sin\left(\frac{4\pi\left(t-\frac{6}{8}T\right)}{T}\right)$

analytical method is precise, care must be taken to plot the diagram to scale. Geometric construction techniques have not been developed for the modified trapezoidal motion scheme.

EXAMPLE PROBLEM 10.2

A cam is to be designed for an automated part loader as shown in Figure 10.14. Using the motion equations, construct a chart that tabulates follower displacement versus time and cam rotation. Also plot this data when the prescribed motion for this application is as follows.

1. Rise 50 mm in 1.5 sec using the constant velocity motion scheme.
2. Return in 2.0 sec using the cycloidal motion scheme.
3. Dwell for 0.75 sec.
4. Repeat the sequence.

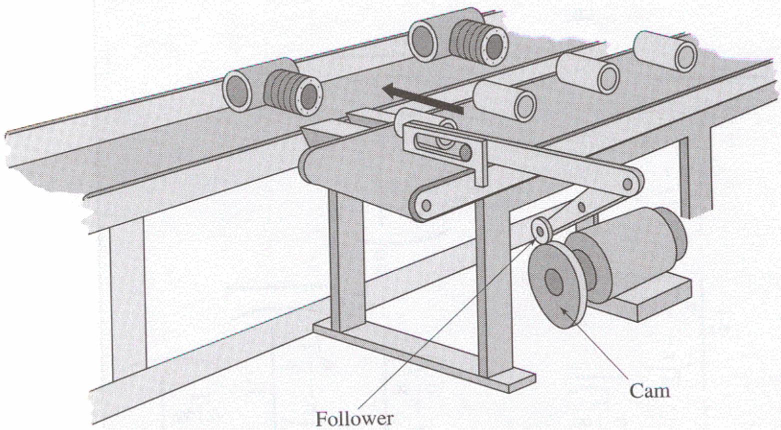


Figure 10.14 Part loader for Example Problem 10.2.

Solution:

The total time to complete the full cycle is needed to determine the required speed of the cam.

$$\Delta t_{\text{for 1 cycle}} = (1.5 + 0.75 + 2.0) \text{ sec} = 4.25 \text{ sec}$$

Then from Equation 10.2:

$$\omega_{\text{cam}} = \frac{1 \text{ rev}}{\Delta t_{\text{for 1 cycle}}}$$

$$= \frac{1 \text{ rev}}{4.25 \text{ sec}} = 0.235 \frac{\text{rev}}{\text{sec}} = 14.12 \text{ rpm}$$

The angular increment of the cam consumed by each follower motion sequence is determined by Equation 10.1:

$$\Delta\theta_1 = \omega_{cam}\Delta t = \left(0.235\frac{\text{rev}}{\text{sec}}\right)(1.5\text{ sec}) = 0.353\text{ rev} = 127.0°$$

$$\Delta\theta_2 = \left(0.235\frac{\text{rev}}{\text{sec}}\right)(2.0\text{ sec}) = 0.470\text{ rev} = 169.3°$$

$$\Delta\theta_3 = \left(0.235\frac{\text{rev}}{\text{sec}}\right)(0.75\text{ sec}) = 0.177\text{ rev} = 63.7°$$

For the first motion, a constant velocity rise, the displacement equation is given as:

$$s = \frac{Ht}{T}$$

This first motion sequence has $H = 50$ mm and $T = 1.5$ sec. For the second motion, a cycloidal fall, the displacement equation is given as:

$$s = H\left[1 - \left(\frac{t}{T}\right) + \frac{1}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\right]$$

This second motion sequence has $H = 50$ mm and $T = 2.0$ sec. Of course, the last motion sequence is a dwell, where $s =$ constant. This dwell occurs at the retracted follower position, thus: $s = 0$.

TABLE 10.6 Follower Displacements for Example Problems 10.2

Time (sec)	Cam Ang. (deg)	Follower Displ. (mm)
0.00	0.0	0.00
0.25	21.2	8.33
0.50	42.4	16.67
0.75	63.5	25.00
1.00	84.7	33.33
1.25	105.9	41.67
1.50	127.1	50.00
1.75	148.2	49.38
2.00	169.4	45.46
2.25	190.6	36.88
2.50	211.8	25.00
2.75	232.9	13.12
3.00	254.1	4.54
3.25	275.3	0.00
3.75	317.6	0.00
4.25	360.0	0.00

These above equations were substituted into a spreadsheet (Table 10.6). This data is used to produce the plot in Figure 10.15.

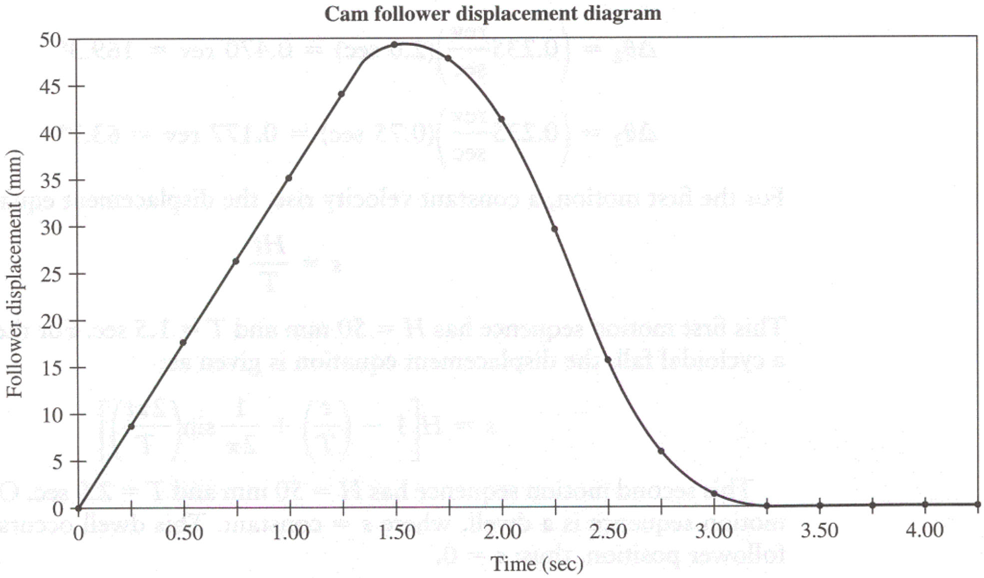


Figure 10.15 Follower displacement diagram for Example Problem 10.2.

EXAMPLE PROBLEM 10.3

For the application presented in Example Problem 10.2, graphically construct a follower displacement diagram.

Solution:

Using the data from Example Problem 10.2, the displacement diagram shown in Figure 10.16 can be constructed. Note that the circle used to construct the cycloidal fall has a radius of:

$$r = \frac{H}{2\pi} = \frac{(50\text{ mm})}{2\pi} = 7.96\text{ mm}$$

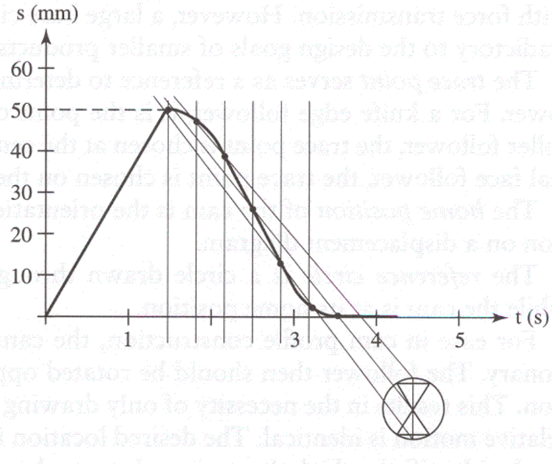


Figure 10.16 Follower displacement diagram for Example Problem 10.3.

10.6 GRAPHICAL DISK CAM PROFILE DESIGN

Once the desired motion of a cam and follower has been defined through a displacement diagram, the actual shape of the cam can be designed. The shape of the cam depends on the size of the cam along with the configuration of the follower.

Prior to designing the profile of a disk cam, some geometric features must be defined. The following features are illustrated in Figure 10.17.

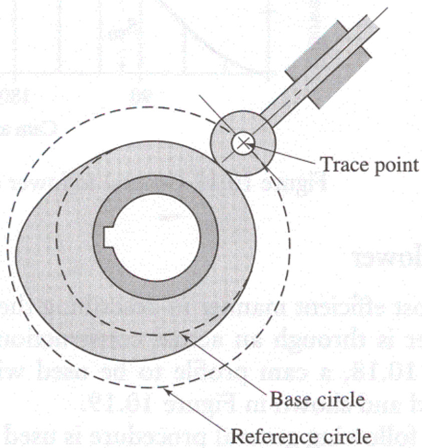


Figure 10.17 Cam nomenclature.

The *base circle* is the smallest circle centered on the cam rotation axis, and tangent to the cam surface. The size of the base circle is typically dictated by the spatial restrictions of the application. In general, a large base circle causes fewer problems with force transmission. However, a large base circle and, hence, a large cam is contradictory to the design goals of smaller products.

The *trace point* serves as a reference to determine the effective location of the follower. For a knife edge follower, it is the point of cam and follower contact. For a roller follower, the trace point is chosen at the center of the roller. For a flat or spherical face follower, the trace point is chosen on the contact surface of the follower.

The *home position* of the cam is the orientation that corresponds to the 0° position on a displacement diagram.

The *reference circle* is a circle drawn through the trace point of the follower, while the cam is at its home position.

For ease in cam profile construction, the cam should be imagined as being stationary. The follower then should be rotated opposite to the direction of cam rotation. This results in the necessity of only drawing the cam profile once. However, the relative motion is identical. The desired location for several positions of the follower can be identified, which determines the actual cam shape.

The specific procedures for different follower arrangements are illustrated in the following sections. The general displacement diagram shown in Figure 10.18 illustrates all constructions. Notice that specific displacements have been identified from the rise and fall portions. These prescribed displacements are translated to the cam profile.

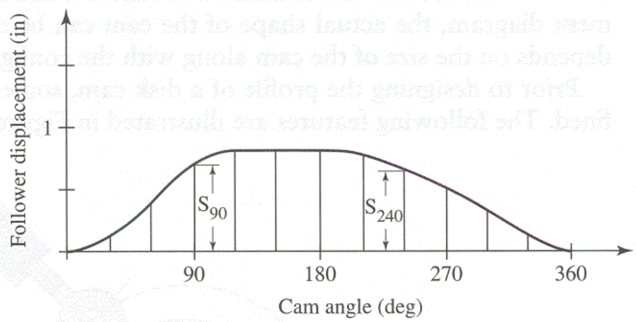


Figure 10.18 General follower displacement diagram.

10.6.1 Knife Edge Follower

The most efficient manner in describing the construction of a cam with a knife edge follower is through an actual construction. Using the displacement diagram from Figure 10.18, a cam profile to be used with a knife edge follower has been constructed and shown in Figure 10.19.

The following general procedure is used to graphically construct such a profile.

1. Draw the base circle, where the size is a function of the spatial constraints of the application.
2. Draw the follower in the home position.
3. Draw radial lines from the center of the cam, corresponding to the cam angles of the reference displacements on the displacement diagram. Recall that the follower rotates in a direction opposite to the cam rotation.

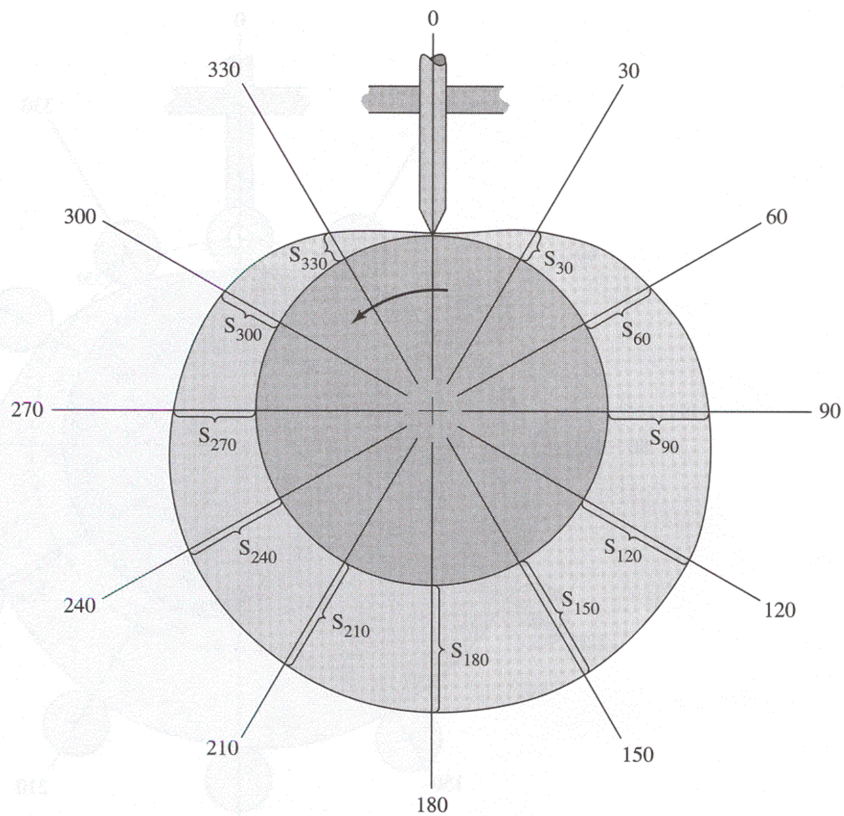


Figure 10.19 Cam profile design—knife edge follower

4. Transfer the displacements from the displacement diagram to the radial lines. Measure these displacements from the base circle.
5. Draw a smooth curve through these prescribed displacements.
6. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.

10.6.2 In-line Roller Follower

Again, the most efficient manner for describing the construction of a cam with an in-line roller follower is through an actual construction. Using the displacement diagram from Figure 10.18, a cam profile to be used with an in-line roller follower has been constructed and shown in Figure 10.20. The following general procedure is used to construct such a profile.

1. Draw the base circle, where the size is a function of the spatial constraints of the application.

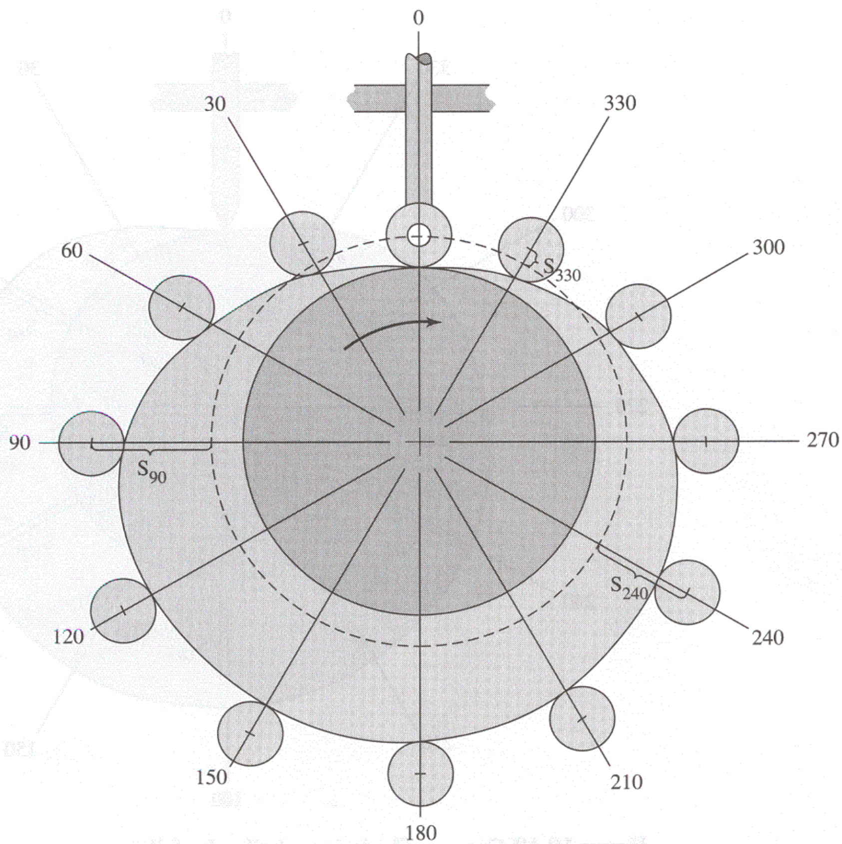


Figure 10.20 Cam profile design—in-line roller follower.

2. Draw the follower in the home position, tangent to the base circle.
3. Draw radial lines from the center of the cam, corresponding to the cam angles of the reference displacements on the displacement diagram. Recall that the follower rotates in a direction opposite to the cam rotation.
4. Identify the trace point at the home position. For a roller follower, this is the point at the center of the roller.
5. Draw the reference circle through the trace point at its home position.
6. Transfer the displacements from the displacement diagram to the radial lines. Measure these displacements from the reference circle.
7. Draw the roller outline, centered at the prescribed displacements identified in the previous step.
8. Draw a smooth curve tangent to the roller at these prescribed displacements.
9. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.

10.6.3 Offset Roller Follower

The most efficient manner for describing the construction of a cam with an offset roller follower is through an actual construction. Using the displacement diagram from Figure 10.18, a cam profile to be used with an offset roller follower has been constructed and shown in Figure 10.21. The following general procedure is used to construct such a profile. The procedure is identical whether using manual drawing techniques or a CAD system.

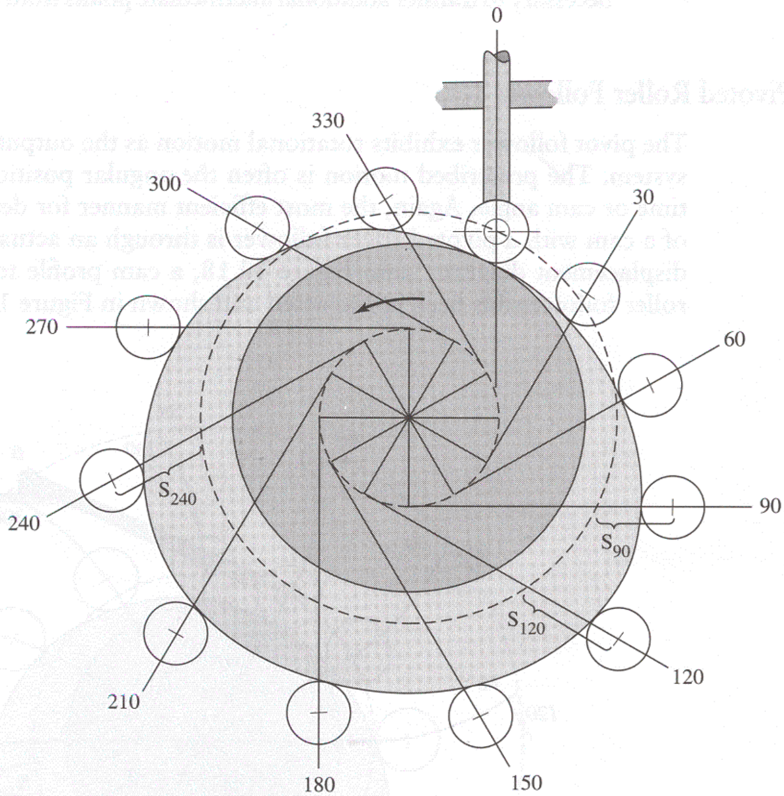


Figure 10.21 Cam profile design—offset roller follower.

1. Draw the base circle, where the size is a function of the spatial constraints of the application.
2. Draw the follower in the home position, tangent to the base circle.
3. Identify the trace point at the home position. For a roller follower, this is the point that is at the center of the roller.
4. Draw the reference circle through the trace point at its home position.
5. Draw an offset circle tangent to the follower center line.

6. Draw lines tangent to the offset circle, corresponding to the reference cam angles on the displacement diagram. Recall that the follower rotates in a direction opposite to the cam rotation.
7. Transfer the displacements from the displacement diagram to the offset lines. Measure these displacements from the reference circle.
8. Draw the roller outline, centered at the prescribed displacements identified in the previous step.
9. Draw a smooth curve tangent to the roller at these prescribed displacements.
10. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.

10.6.4 Pivoted Roller Follower

The pivot follower exhibits rotational motion as the output of the cam and follower system. The prescribed motion is often the angular position of the follower versus time or cam angle. Again, the most efficient manner for describing the construction of a cam with a pivoted roller follower is through an actual construction. Using the displacement diagram from Figure 10.18, a cam profile to be used with a pivoted roller follower has been constructed and shown in Figure 10.22.

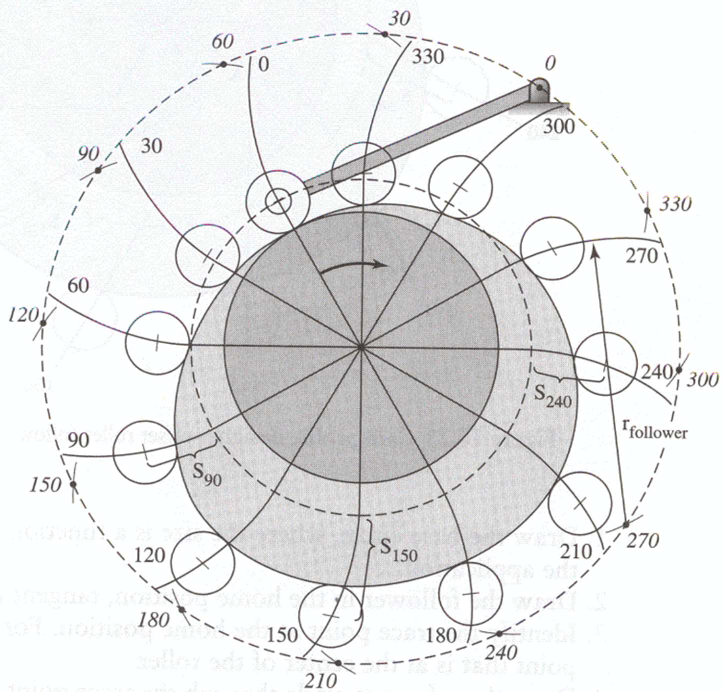


Figure 10.22 Cam profile design—pivoted roller follower.

The following general procedure is used to graphically construct such a profile. The procedure is identical whether using manual drawing techniques or a CAD system.

1. Draw the base circle, where the size is a function of the spatial constraints of the application.
2. Draw the follower in the home position, tangent to the base circle.
3. Identify the trace point at the home position. For a roller follower, this is the point at the center of the roller.
4. Draw the reference circle through the trace point at its home position.
5. Draw radial lines from the center of the cam to the reference circle, corresponding to the reference cam angles on the displacement diagram. Recall that the follower rotates in a direction opposite to the cam rotation.
6. Draw a pivot circle through the center of the follower pivot.
7. Locate pivot points around the pivot circle corresponding to the reference cam angles. These points are a distance equal to the length of the follower arm from the reference circle at the reference cam angles.
8. From each pivot point, draw an arc with a radius equal to the length of the follower arm outward from the reference circle.
9. Transfer the displacements from the displacement diagram to the pivot arcs drawn in the previous step. Recall that these displacements are angular and must be measured from the follower home position.
10. Draw the roller outline centered at the prescribed displacements identified in the previous step.
11. Draw a smooth curve tangent to the roller at these prescribed displacements.
12. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.

10.6.5 Translating Flat-Faced Follower

The most efficient manner for describing the construction of a cam with an flat-faced follower is through an actual construction. Using the displacement diagram from Figure 10.18, a cam profile to be used with a translating flat-faced follower has been constructed and shown in Figure 10.23.

The following general procedure is used to graphically construct such a profile. The procedure is identical whether using manual drawing techniques or a CAD system.

1. Draw the base circle, where the size is a function of the spatial constraints of the application. Recall that for this type of follower, the base circle also serves as the reference circle.
2. Draw the follower in the home position, tangent to the base circle.
3. Draw radial lines from the center of the cam, corresponding to the reference cam angles on the displacement diagram. Recall that the follower rotates in a direction opposite to the cam rotation.

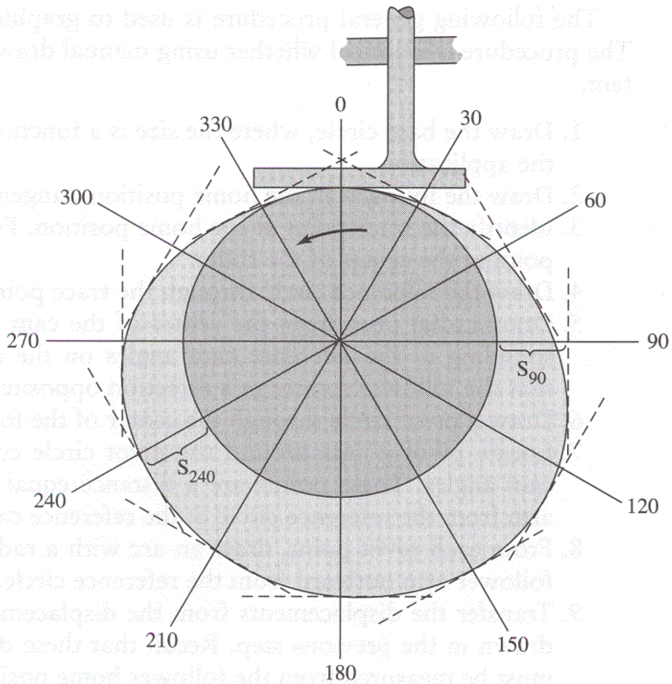


Figure 10.23 Cam profile design—flat-faced follower.

4. Transfer the displacements from the displacement diagram to the radial lines, measured from the base circle.
5. Draw the flat-faced outline by constructing a line perpendicular to the radial lines at the prescribed displacements.
6. Draw a smooth curve tangent to the flat-faced outlines.
7. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.

10.7 PRESSURE ANGLE

Because a force is always transmitted perpendicular to surfaces in contact, the cam does not always push the follower in the direction of its motion. As discussed in the previous section, the curvature of the cam affects the position between the follower centerline and the actual contact point.

The force required to push the follower depends on the application of the cam system. However, the contact force between the cam and follower can be much greater, depending on the location of the contact point. Actually, only one component of the contact force generates the follower motion The other force component is undesirable as it generates a side thrust and must be absorbed by the follower bearings and other components.

The *pressure angle,* δ, is used to distinguish between the two components of the contact force. The pressure angle can be defined as the angle between the direction where the follower is traveling and the direction of the cam contact force. More precisely, it is the angle between the path of the follower motion and the line perpendicular to the cam profile at the point of follower contact. Each point on the cam surface has a pressure angle. The pressure angle is illustrated in Figure 10.24.

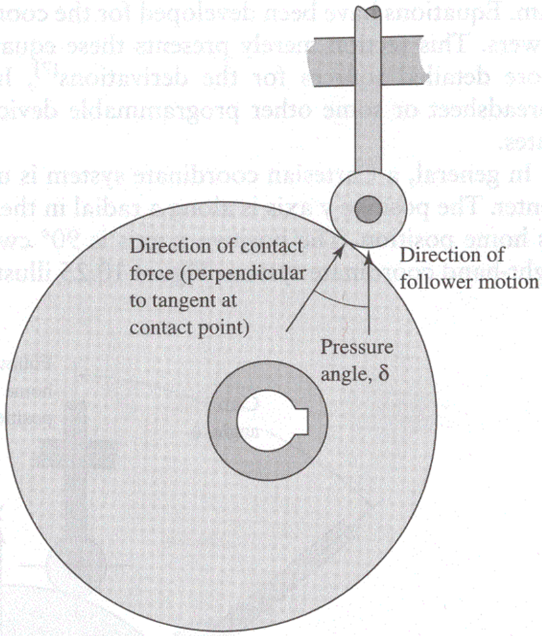


Figure 10.24 Pressure angle.

After graphically constructing a cam profile, the magnitude of the pressure angle can be visualized by observing the location of the contact point in relation to the follower centerline. The regions where the cam profile exhibits the greatest curvature should be isolated, since these locations will have the largest pressure angles. Approximate measurements of the pressure angles in this region should be obtained. In general the pressure angle should be kept as small as possible and should not exceed 30°. The magnitude of the pressure angle can be decreased by:

1. Increasing the size of the base circle.
2. Decreasing the magnitude of follower displacement.
3. Increasing the angle of cam rotation prescribed for the follower rise or fall.
4. Modifying the amount of follower offset.
5. Modifying the follower motion scheme.

10.8 ANALYTICAL CAM PROFILE DESIGN

The previous section illustrated the method to design a cam profile graphically. Depending on the precision required for the application, this method can produce sufficiently accurate profiles. Of course, the accuracy is increased when the construction is accomplished on a CAD system, and small cam angle increments are used.

However, in some situations, the need for highly accurate cams is required. It is desirable to be able to analytically determine the coordinates of points on the cam surface as well as coordinates of a milling cutter that will be used to manufacture the cam. Equations have been developed for the coordinates of the different types of followers. This section merely presents these equations, and the reader is referred to more detailed sources for the derivations[7]. Incorporating the equations into a spreadsheet or some other programmable device can quickly generate the coordinates.

In general, a Cartesian coordinate system is used so that the origin is at the cam center. The positive y axis is along a radial in the direction of the follower motion in its home position. The positive x axis is 90° cw from the y axis, consistent with a right-hand coordinate system. Figure 10.25 illustrates this coordinate system.

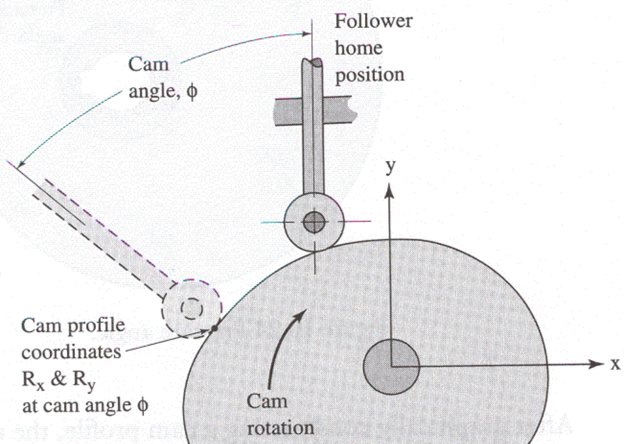


Figure 10.25 Cam profile coordinate system.

10.8.1 Knife Edge Follower

The x and y coordinates of the cam profile are given as:

$$R_x = (R_b + s_\phi)\sin\phi \tag{10.3}$$

$$R_y = (R_b + s_\phi)\cos\phi \tag{10.4}$$

where the following notation is used:

R_x = x coordinate of cam surface profile

R_y = y coordinate of cam surface profile

R_b = Base circle radius

ϕ = Cam rotation angle measured against the direction of cam rotation from the home position

s_ϕ = Follower displacement at cam angle ϕ

The x and y coordinates of the milling cutter are given as:

(10.5) $$C_x = (R_c + R_b + s_\phi)\sin\phi$$

(10.6) $$C_y = (R_c + R_b + s_\phi)\cos\phi$$

where the following additional notation is used:

C_x = x coordinate of cutter center

C_y = y coordinate of cutter center

R_c = Mill cutter radius

TABLE 10.7 Cam Profile Coordinates for Example Problem 10.4

Time (sec)	Cam Ang. (deg)	Follower Displ. (mm)	R_x (mm)	R_x (mm)
0.00	0.0	0.00	0.00	100.00
0.25	21.2	8.33	39.17	101.00
0.50	42.4	16.67	78.67	86.16
0.75	63.5	25.00	111.87	55.77
1.00	84.7	33.33	132.76	12.32
1.25	105.9	41.67	136.25	−38.81
1.50	127.1	50.00	119.64	−90.48
1.65	139.8	47.57	95.25	−112.71
1.80	152.5	34.68	62.19	−119.46
1.95	165.2	15.32	29.46	−111.49
2.10	177.9	2.43	3.75	−102.36
2.25	190.6	0.00	−18.40	−98.29
2.75	232.9	0.00	−79.76	−60.32
3.25	275.3	0.00	−99.57	9.24
3.75	317.6	0.00	−67.43	73.85
4.25	360.0	0.00	0.00	100.00

EXAMPLE PROBLEM 10.4

For the application stated in Example Problem 10.2, analytically determine the cam profile coordinates when a knife edge follower is incorporated. Because of size constraints of the machine, a cam with a base circle diameter of 200 mm must be used. The cam is to rotate counterclockwise.

Solution:

The base circle radius is half of the base circle diameter; thus:

$$R_c = 100 \text{ mm}$$

Fitting Equations 10.3 and 10.4 into a spreadsheet yields the results in Table 10.7. The coordinates are then plotted in Figure 10.26.

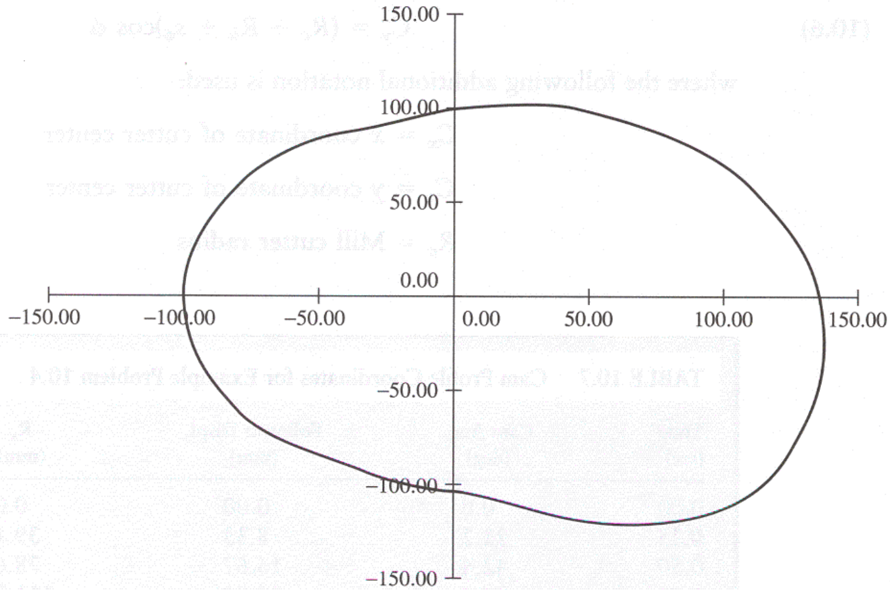


Figure 10.26 Cam profile for Example Problem 10.4.

10.8.2 In-Line Roller Follower

In general, a roller follower is complicated by the fact that the cam contact point is not in-line with the roller center. The angle between the follower centerline and the cam contact point varies with curvature of the cam profile. The instantaneous angle can be computed as:

$$\alpha = \tan^{-1}\frac{V_\phi}{\omega}\frac{R_f + R_b + S_\phi}{(R_f + R_b + S_\phi)^2} \tag{10.7}$$

For an in-line roller follower, this angle is also the pressure angle. In addition to the notation used in Section 10.8.1, the following terms are defined as:

R_f = the radius of the roller follower.

v_ϕ = the instantaneous velocity of the cam follower

ω = the rotational speed of the cam in radians per time

Then, the x and y coordinates of the cam profile can be given as:

(10.8) $$R_x = -\,[R_f + R_b + s_\phi\,]\sin\phi + R_f\sin(\phi - \alpha)$$

(10.9) $$R_y = [R_f + R_b + s_\phi\,]\cos\phi - R_f\cos(\phi - \alpha)$$

The x and y coordinates of the milling cutter are given as:

(10.10) $$C_x = -\,[R_f + R_b + s_\phi]\sin\phi + [R_c - R_f]\sin(\phi - \alpha)$$

(10.11) $$C_y = [R_f + R_b + s_\phi]\cos\phi - [R_c - R_f]\cos(\phi - \alpha)$$

EXAMPLE PROBLEM 10.5

Two cams are used to drive a gripper, of a mechanical part handler. The two cams can generate independent horizontal and vertical motions to the gripper. Such machines can relocate parts in a similar fashion to a robot at a fraction of the cost. The part handler is shown in Figure 10.27.

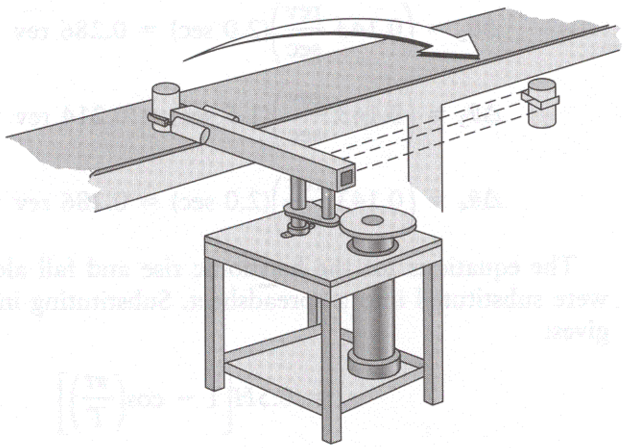


Figure 10.27 Part handling machine for Example Problem 10.5.

The prescribed motion for one of the cam followers is as follows:

1. Rise 1.5 in. in 1.5 sec using the harmonic motion scheme.
2. Dwell for 2 sec.
3. Return in 1.5 sec using the cycloidal motion scheme.

4. Dwell for 2 sec.
5. Repeat the sequence.

An in-line roller follower with a radius of 0.5 in, is used on a cam with a base circle radius of 3.5 in. Tabulate the follower motion and specify the coordinates of the cam profile.

Solution:

The total time to complete the full cycle is needed to determine the required speed of the cam:

$$\Delta t_{\text{for 1 cycle}} = (1.5 + 2.0 + 1.5 + 2.0)\text{sec} = 7.0 \text{ sec}$$

Then from Equation 10.2:

$$\omega_{\text{cam}} = \frac{1 \text{ rev}}{\Delta t_{\text{for 1 cycle}}}$$

$$= \frac{1 \text{ rev}}{7.0 \text{ sec}} = 0.143\frac{\text{rev}}{\text{sec}} = 0.899\frac{\text{rad}}{\text{sec}} = 8.57 \text{ rpm}$$

The angular increment of the cam consumed by each follower motion sequence is determined by Equation 10.1:

$$\Delta\theta_1 = (\omega_{\text{cam}})(\Delta t) = \left(0.143 \frac{\text{rev}}{\text{sec}}\right)(1.5 \text{ sec}) = 0.214 \text{ rev} = 77.2°$$

$$\Delta\theta_2 = \left(0.143 \frac{\text{rev}}{\text{sec}}\right)(2.0 \text{ sec}) = 0.286 \text{ rev} = 102.8°$$

$$\Delta\theta_3 = \left(0.143 \frac{\text{rev}}{\text{sec}}\right)(1.5 \text{ sec}) = 0.214 \text{ rev} = 77.2°$$

$$\Delta\theta_4 = \left(0.143 \frac{\text{rev}}{\text{sec}}\right)(2.0 \text{ sec}) = 0.286 \text{ rev} = 102.8°$$

The equations for the harmonic rise and fall along with the profile generation were substituted into a spreadsheet. Substituting into the harmonic rise equations gives:

$$s_\phi = 0.5H\left[1 - \cos\left(\frac{\pi t}{T}\right)\right]$$

$$= 0.5(1.5 \text{ in})\left[1 - \cos\left(\frac{\pi t}{1.5 \text{ sec}}\right)\right]$$

$$v_\phi = \frac{0.5\pi\omega H}{T}\left[\sin\left(\frac{\pi t}{T}\right)\right]$$

$$s_\phi = \frac{0.5\pi(0.89\frac{\text{rad}}{\text{sec}})(1.5 \text{ in})}{(1.5 \text{ sec})}\left[\sin\left(\frac{\pi t}{1.5 \text{ sec}}\right)\right]$$

Substituting into the harmonic fall equations gives:

$$s_\phi = 0.5H\left[1 + \cos\left(\frac{\pi t}{T}\right)\right]$$

$$= 0.5(1.5\text{ in})\left[1 + \cos\left(\frac{\pi t}{1.5\text{ sec}}\right)\right]$$

$$v_\phi = \frac{-0.5\pi\omega H}{T}\left[\sin\left(\frac{\pi t}{T}\right)\right]$$

$$= \frac{-0.5\pi(0.89\frac{\text{rad}}{\text{sec}})(1.5\text{ in})}{(1.5\text{ sec})}\left[\sin\left(\frac{\pi t}{1.5\text{ sec}}\right)\right]$$

The profile coordinate equation substitutions include:

$$\alpha = \tan^{-1}\left\{\frac{v_\phi(R_f + R_b + s_\phi)}{\omega(R_f + R_b + s_\phi)^2}\right\}$$

$$= \tan^{-1}\left\{\frac{v_\phi\, 0.5 + 3.5 + s_\phi}{.89\frac{\text{rad}}{\text{sec}}(0.5 + 3.5 + s_\phi)^2}\right\}$$

TABLE 10.8 Cam Profile Coordinates for Example Problem 10.5

Time (sec)	Cam Ang. (deg)	Follower Displ. (in)	Follower Vel (in/s)	Rx (in)	Ry (in)
0.00	0	0.00	0.00	0.000	3.500
0.35	18	0.19	1.05	−1.632	3.618
0.70	36	0.67	1.56	−3.002	3.350
1.05	54	1.19	1.27	−4.249	2.554
1.40	72	1.48	0.33	−4.836	1.369
1.75	90	1.50	0.00	−5.000	0.000
2.10	108	1.50	0.00	−4.755	−1.545
2.45	126	1.50	0.00	−4.045	−2.939
2.80	144	1.50	0.00	−2.939	−4.045
3.15	162	1.50	0.00	−1.545	−4.755
3.50	180	1.50	0.00	0.000	−5.000
3.85	198	1.31	−1.05	1.164	−4.895
4.20	216	0.83	−1.56	2.349	−4.012
4.55	234	0.31	−1.27	3.045	−2.769
4.90	252	0.02	−0.33	3.330	−1.342
5.25	270	0.00	0.00	3.500	0.000
5.60	288	0.00	0.00	3.329	1.082
5.95	306	0.00	0.00	2.832	2.057
6.30	324	0.00	0.00	2.057	2.832
6.65	342	0.00	0.00	1.082	3.329
7.00	360	0.00	0.00	0.000	3.500

$$Rx = -[R_f R_b + s_\phi]\sin\phi + R_f \sin(\phi - \alpha)$$
$$= -[0.5 + 3.5 + s_\phi]\sin\phi + 0.5\sin(\phi - \alpha)$$
$$R_y = [R_f + R_b + s_\phi]\cos\phi - R_f\cos(\phi - \alpha)$$
$$= [0.5 + 3.5 + s_\phi]\cos\phi - 0.5\cos(\phi - \alpha)$$

Thus, the spreadsheet gives the results listed in Table 10.8.

A plot of the profile coordinates can be used to visualize the cam (Figure 10.28).

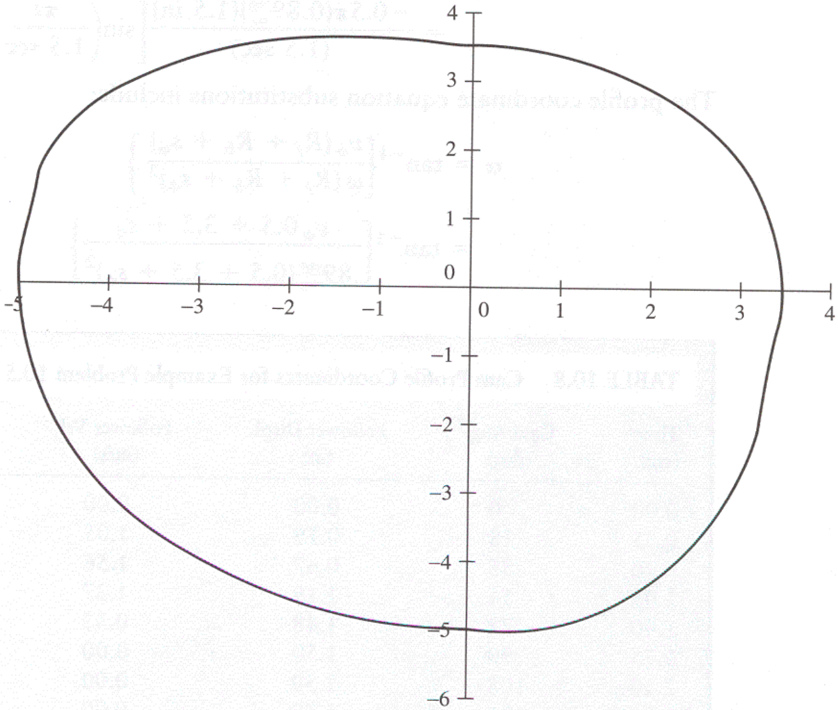


Figure 10.28 Cam profile for Example Problem 10.5

10.8.3 Offset Roller Follower

An offset roller follower is further complicated by the fact that the follower motion is not in-line with the cam contact point which, in turn, is not in-line with the roller center. Thus, the profile equations become a bit more complex. The angle between the follower centerline and the cam contact point again varies with curvature of the cam profile and can be computed as:

$$\alpha = \tan^{-1}\left\{\left(\frac{v_\phi}{\omega}\frac{R_f + R_b + s_\phi}{e^2 + (R_f + R_b + s_\phi)^2}\right) - \left(\frac{e\, v_\phi}{\omega}\right)\right\} \tag{10.12}$$

And the pressure angle, *d*, can be calculated as:

$$\delta = \alpha - \tan^{-1}\left(\frac{e}{R_f + R_b + s_\phi}\right) \quad (10.13)$$

where, in addition to the notation used above, the following term is defined:

e = the offset distance of the follower motion from the center of the cam

Then, the x and y coordinates of the cam profile can be given as:

$$R_x = -(e)\cos\phi - [R_f + R_b + s_\phi]\sin\phi + R_f\sin(\phi - \alpha) \quad (10.14)$$

$$R_y = -(e)\sin f\phi + [R_f + R_b + s_\phi]\cos\phi - R_f\cos(\phi - \alpha) \quad (10.15)$$

The x and y coordinates of the milling cutter are given as:

$$C_x = -(e)\cos\phi - [R_f + R_b + s_\phi]\sin\phi + [R_c - R_f]\sin(\phi - \alpha) \quad (10.16)$$

$$C_y = -(e)\sin\phi + [R_f + R_b + s_\phi]\cos\phi - [R_c - R_f]\cos(\phi - \alpha) \quad (10.17)$$

10.8.4 Pivoted Roller Follower

The analytical construction of an pivoted roller follower is similar to the offset translating follower. The main difference is that the motion of the follower is rotational and the prescribed motion is usually the angular position of the follower versus time or cam angle. Of course, the angular motion of the follower can be used to determine the linear motion of the trace point:

$$s_\phi = R_L\,\zeta_\phi \quad (10.18)$$

$$v_\phi = R_L\,\omega_\phi \quad (10.19)$$

$$a_\phi{}^t = R_L\,\alpha_\phi \quad (10.20)$$

The following notation is used:

R_L = length of the follower pivot link

ζ_ϕ = angular position of the follower pivot link

ω_ϕ = angular velocity of the follower pivot link

α_ϕ = angular acceleration of the follower pivot link

Again, the angle between the follower centerline and the cam contact point varies with the curvature of the cam profile and can be computed as:

$$\alpha = \tan^{-1}\left\{\frac{v_\phi}{\omega}\,\frac{1}{(R_f + R_b + s_\phi) - \frac{v_\phi}{\omega}\cos\gamma}\right\} \quad (10.21)$$

In addition to the notation used above, the following terms are defined as:

$$\gamma = \cos^{-1}\left\{\left(\frac{(R_f + R_b + s_\phi)^2 + R_L{}^2 + m^2}{2(R_f + R_b + s_\phi)(R_L)}\right)\right\}$$

The coordinate system that is used to generate the cam coordinates has the origin at the cam center. The positive x-axis is along the radius to the follower pivot center. The positive y-axis is in the general direction of the roller follower, 90° counterclockwise from the x-axis. This is consistent with a right-hand coordinate system. Then the x and y coordinates of the cam profile can be given as:

(10.22) $$R_x = \{[R_f + R_b + s_\phi] + R_f \cos \alpha\}\cos \phi$$

(10.23) $$R_y = [R_f + R_b + s_\phi]\cos \phi - R_f \cos(\phi - \alpha)$$

The x and y coordinates of the milling cutter are given as:

(10.24) $$C_x = -(e)\cos \phi - [R_f + R_b + s_\phi]\sin \phi + [R_c - R_f]\sin(\phi - \alpha)$$

(10.25) $$C_y = -(e)\sin \phi + [R_f + R_b + s_\phi]\cos \phi - [R_c - R_f]\cos(\phi - \alpha)$$

10.8.5 Translating Flat-Faced Follower

The analytical construction of an translating flat-faced follower can also exhibit a contact point that is not in-line with the cam centerline. The angle between the follower centerline and the cam contact point again varies with the curvature of the cam profile and can be computed as:

(10.26) $$\alpha = \tan^{-1}\left\{\frac{v_\phi}{\omega}\frac{1}{(R_b + s_\phi)}\right\}$$

Then the x and y coordinates of the cam profile can be given as:

(10.27) $$R_x = [(R_b + s_\phi)/\cos \alpha]\cos(\phi + \alpha)$$

(10.28) $$R_y = [(R_b + s_\phi)/\cos \alpha]\sin(\phi + \alpha)$$

The x and y coordinates of the milling cutter are given as:

(10.29) $$C_x = \left[\frac{R_b + s_\phi + R_c}{\cos \gamma}\right]\cos(\phi + \gamma)$$

(10.30) $$C_y = \left[\frac{R_b + s_\phi + R_c}{\cos \gamma}\right]\sin(\phi + \gamma)$$

where:

(10.31) $$\gamma = \tan^{-1}\left\{\frac{R_b + s_\phi \tan(\alpha)}{R_c + R_b + s_\phi}\right\}$$

10.9 CYLINDRICAL CAMS

While disk cams are the most common type of cam, cylindrical cams are also widely used. As presented in Section 10.2 and illustrated in Figure 10.2B, a cylindrical cam consists of a groove, wrapped around a cylinder. A cylindrical cam is a positive motion cam, in that the follower is constrained in a groove and an external member is not needed to maintain contact between the follower and the cam. There are many applications in which it is necessary for the cam to exert a positive control of the follower during the rise or fall sequences.

Often a tapered roller follower is used as shown in Figure 10.2B. It is used because the top edge of the groove travels at a higher speed than the bottom portion. Thus, the taper can compensate for the difference and prevent any slipping and skidding action of the roller. When a cylindrical roller is used, it is advisable to use a narrow width to minimize the velocity difference across the face of the roller.

In general, calculation and layout procedures are similar to those for the disk cam. The following sections discuss profile generation techniques for a cylindrical cam with a translating follower. The profile generation for other types of followers are similar.

10.8.1 Graphical Cylindrical Cam Profile

The most efficient manner for describing the construction of a cylindrical cam is through an actual construction. Using the displacement diagram from Figure 10.18, a cylindrical cam profile has been constructed and shown in Figure 10.29. The following general procedure is used to construct such a profile:

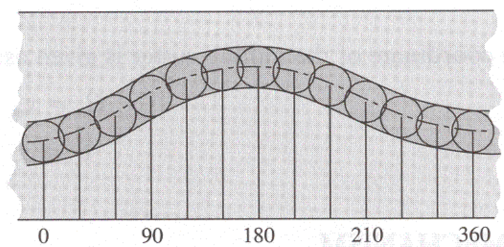


Figure 10.29 Cylindrical cam profile design.

1. Draw a straight line equal to the circumference of the cylindrical cam.
2. Divide this line into sections corresponding to the reference cam angles on the displacement diagram.
3. Transfer the displacements from the displacement diagram to the lines corresponding to the reference cam angles.
4. Draw the roller follower at the prescribed displacements.
5. Draw a smooth curve tangent to the roller outlines.
6. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.

10.8.2 Analytical Cylindrical Cam Profile

Because a cylindrical cam is wrapped around a cylinder, a cylindrical coordinate system is used to define the groove profile. The angular coordinate, θ, is the angle around the cam and the z-axis is the axial position on the cam. The angle between the follower centerline and the cam contact point varies with curvature of the groove profile and can be computed as:

$$\alpha = \tan^{-1}\left(\frac{v_\phi}{\omega}\right) \tag{10.32}$$

The notation used is the same as the preceding sections. For a translating follower, this angle is also the pressure angle. Similar to disk cams, the pressure angle should be kept to a minimum and not exceed 30°.

The z coordinate of the upper groove profile can be given as:

$$R_z = s_\phi + R_\phi \cos\alpha \tag{10.33}$$

$$\omega = \phi - \tan^{-1}\frac{R_f \cos\alpha}{R_b} \tag{10.34}$$

Here, R_b is the diameter of the cylindrical cam.

The z coordinate of the lower groove profile can be given as:

$$R_z = s_\phi - R_f \cos\alpha \tag{10.35}$$

$$\omega = \phi + \tan^{-1}\frac{R_f \cos\alpha}{R_b} \tag{10.36}$$

The coordinate of the milling cutter is given as:

$$C_z = s_\phi \tag{10.37}$$

$$\theta = \phi \tag{10.38}$$

10.9 GENEVA MECHANISM

A geneva mechanism is a unique design that produces a repeated indexing motion from constant rotational motion. Because of this motion, the geneva mechanism is commonly classified with cams. A four-station, geneva mechanism is illustrated in Figure 10.30.

The geneva mechanism consists of a driving roller and a geneva wheel. The geneva wheel consists of a disc with several radial slots and is fastened to an output shaft. The driving roller is fastened to an arm which, in turn, is fastened to the input shaft. The arm is usually attached to a locking disk that prevents the wheel from rotating when the driving roller is not engaged in a slot. The locking disk fits into a cut-out on the wheel.

The motion of the geneva mechanism is characterized by the roller entering a slot in the wheel, indexing the wheel. When the roller exits the slot, the wheel locks into position until the roller enters the next slot. In Figure 10.30A, the roller rotates clockwise and is just about to enter the geneva wheel. In Figure 10.30B, the roller has entered the slot and has turned the wheel counterclockwise. Notice that the locking disk has moved away from the wheel, allowing it to rotate.

When designing a wheel, it is important that the roller enters the slot tangentially. Otherwise, impact loads are created and the mechanism will perform poorly at high

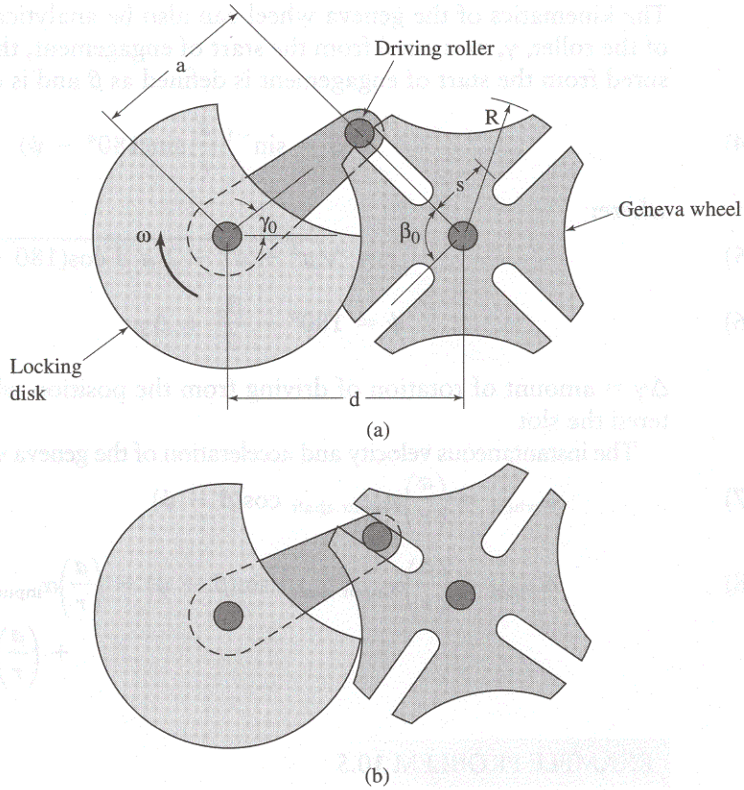


Figure 10.30 Geneva mechanism.

speeds or loads. Because of this constraint, the following geometric relationships are derived.[7] Refer to Figure 10.30 for definitions of the geometric properties:

(10.39)
$$\beta_o = \frac{360°}{n}$$

where: n = the number of stations in the geneva wheel

(10.40)
$$\gamma_o = \frac{180° - \beta_o}{2}$$

(10.41)
$$a = d \sin\left(\frac{\beta_o}{2}\right)$$

(10.42)
$$R = d \cos\left(\frac{\beta_o}{2}\right)$$

(10.43)
$$S < d - a$$

The kinematics of the geneva wheel can also be analytically determined. The angle of the roller, γ, measured from the start of engagement, the angle of the wheel, measured from the start of engagement is defined as β and is calculated as:

(10.44) $$\beta = \sin^{-1}\frac{a}{r}\sin(180° - \psi)$$

where:

(10.45) $$r = \sqrt{a^2 + d^2 - 2\,a\,d\cos(180 - \psi}$$

(10.46) $$\psi = 180° - \frac{\beta_o}{2} + \Delta\gamma$$

$\Delta\gamma$ = amount of rotation of driving from the position where the roller has just entered the slot

The instantaneous velocity and acceleration of the geneva wheel has been found [3] by:

(10.47) $$\omega_{\text{wheel}} = \left(\frac{a}{r}\right)\omega_{\text{input shaft}}\cos(\beta - \psi)$$

(10.48) $$\alpha_{\text{wheel}} = \left(\frac{a}{r}\right)\omega_{\text{input shaft}}^{\;2}\sin(\beta - \psi) - \left(\frac{a}{r}\right)\alpha_{\text{input shaft}}\cos(\beta - \psi) + \left(\frac{a}{r}\right)^2\omega_{\text{input shaft}}^{\;2}\sin(2\beta - 2\psi)$$

EXAMPLE PROBLEM 10.5

A geneva mechanism has been designed with six stations, as shown in Figure 10.31. The distance between the driving and driven shafts is 80 mm. The driving arm rotates at a constant rate of 80 rpm, clockwise. Determine the angular velocity and acceleration of the wheel when the driving arm has rotated 15° from the position where the roller has just entered the slot.

Solution:

Equations 10.39 through 10.43 can be used to calculate the geometric properties of this geneva mechanism:

$$\beta_o = \frac{360°}{n} = \frac{360°}{6} = 60°$$

$$\gamma_o = \frac{180° - \beta_o}{2} = \frac{180° - 60°}{2} = 60°$$

$$a = d\sin\left(\frac{\beta_o}{2}\right) = (80\text{ mm})\sin\left(\frac{60°}{2}\right) = 40\text{ mm}$$

$$R = d\cos\left(\frac{\beta_o}{2}\right) = (80\text{ mm})\cos\left(\frac{60°}{2}\right) = 69.3\text{ mm}$$

$$S < d - a = 80 - 40 = 40\text{ mm}$$

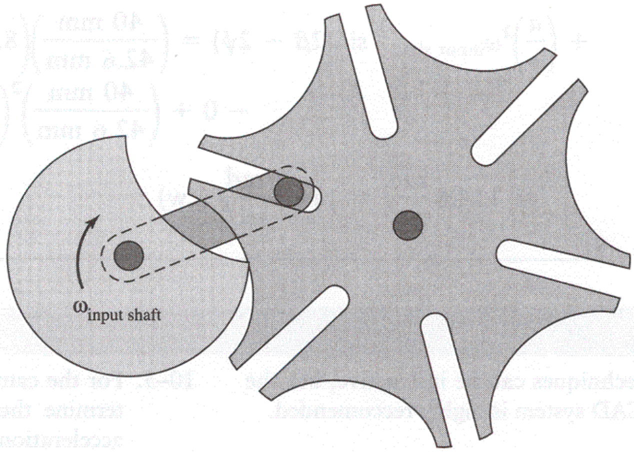


Figure 10.31 Geneva mechanism for Example Problem 10.5.

Equations 10.44 through 10.48 can be used to determine the kinematic relationships when the driving arm has rotated 15° from the position where the roller has just entered the slot.

$$\Delta\gamma = 15°$$

$$\psi = 180° - \frac{\beta_o}{2} + \Delta\gamma = 180° - \frac{60°}{2} + 15° = 165°$$

$$r = \sqrt{a^2 + d^2 - 2\,a\,d\cos(180 - \psi}$$

$$= \sqrt{(40\text{ mm})^2 + (80\text{ mm})^2 - 2(40\text{ mm})(80\text{ mm})\cos(15°)} = 42.6\text{ mm}$$

$$\beta = \sin^{-1}\left[\frac{a}{r}\sin\psi\right] = \frac{40\text{ mm}}{42.6\text{ mm}}\sin 15° = 14°$$

$$\omega_{\text{input shaft}} = 80\text{ rpm} = 80\left(\frac{2\pi}{60}\right) = 10.4\text{ rad/sec}$$

$$\omega_{\text{wheel}} = \left(\frac{a}{r}\right)\omega_{\text{input shaft}}\cos(\beta - \psi)$$

$$= \left(\frac{40\text{ mm}}{42.6\text{ mm}}\right)\left(8.4\frac{\text{rad}}{\text{sec}}\right)\cos(14° - 165°) = -6.9\frac{\text{rad}}{\text{sec}}$$

$$= 65.9\text{ rpm (ccw)}$$

$$\alpha_{\text{input shaft}} = 0(\text{constant angular velocity of input shaft})$$

$$\alpha_{\text{wheel}} = \left(\frac{a}{r}\right)\omega_{\text{input shaft}}{}^2\sin(\beta - \psi) - \left(\frac{a}{r}\right)\alpha_{\text{input shaft}}\cos(\beta - \psi)$$

$$+\left(\frac{a}{r}\right)^2 \omega_{\text{input shaft}}{}^2 \sin(2\beta - 2\psi) = \left(\frac{40\text{ mm}}{42.6\text{ mm}}\right)\left(8.4\frac{\text{rad}}{\text{sec}}\right)^2 \sin(14° - 165°)$$

$$- 0 + \left(\frac{40\text{ mm}}{42.6\text{ mm}}\right)^2\left(8.4\frac{\text{rad}}{\text{sec}}\right)^2 \sin[2(14°) - 2(165°)]$$

$$= 114.8\frac{\text{rad}}{\text{sec}^2} = 114.8\frac{\text{rad}}{\text{sec}^2}\text{ (cw)}$$

PROBLEMS

Manuel techniques can be instructive, but the use of a CAD system is highly reccomended.

Displacement Diagram Problems

10–1. A cam drive is used for a shaker platform. This platform is used to test the shipping worthiness of packaged items. The cam follower motion sequence must be:

1. Rise outward 1 in. with constant acceleration in 0.7 sec
2. Dwell for 0.2 sec
3. Fall with constant acceleration in 0.5 sec

Determine the speed of the cam and graphically plot a displacement diagram.

10–2. For the cam in Problem 10–1, analytically determine the maximum follower velocity and acceleration.

10–3. For the cam in Problem 10–1, use a spreadsheet to plot the motion curves.

10–4. A cam drive is used for a mechanism that feeds papers into a printing press. The cam follower motion sequence must be:

1. Rise outward 1 in. with constant acceleration in 1.7 sec.
2. Dwell for 0.8 sec.
3. Fall 0.5 in. with constant acceleration in 0.8 sec.
4. Dwell for 0.3 sec.
5. Fall 0.5 in. with constant acceleration in 0.8 sec.

Determine the speed of the cam and graphically plot a displacement diagram.

10–5. For the cam in Problem 10–4, analytically determine the maximum follower velocity and acceleration.

10–6. For the cam in Problem 10–4, use a spreadsheet to plot the motion curves.

10–7. A cam drive is used for a mechanism that drives a automated assembly machine. The cam follower motion sequence must be:

1. Rise outward 13 mm with constant velocity in 3 sec.
2. Dwell for 3 sec.
3. Fall 5 mm with constant acceleration in 2 sec.
4. Dwell for 3 sec.
5. Fall with constant acceleration in 2 sec.

Determine the speed of the cam and graphically plot a displacement diagram.

10–8. For the cam in Problem 10–7, analytically determine the maximum follower velocity and acceleration.

10–9. For the cam in Problem 10–1, use a spreadsheet to plot the motion curves.

10–10. A cam drive is used for a mechanism that tests the durability of oven doors. The cam follower motion sequence must be:

1. Rise outward 2 in. with harmonic motion in 1 sec.
2. Dwell for 0.5 sec.
3. Fall with harmonic motion in 1 sec.
4. Dwell for 1 sec.

Determine the speed of the cam and graphically plot a displacement diagram.

10–11. For the cam in Problem 10–10, analytically determine the maximum follower velocity and acceleration.

10–12. For the cam in Problem 10–10, use a spreadsheet to plot the motion curves.

10–13. A cam drive is used for a mechanism that moves a tool in an automated screw machining process. The cam follower motion sequence must be:

1. Rise outward 24 mm with harmonic motion in 0.2 sec.
2. Dwell for 0.3 sec.
3. Fall 10 mm with harmonic motion in 0.3 sec.
4. Dwell for 0.2 sec.
5. Fall 14 mm with harmonic motion in 0.2 sec.

Determine the speed of the cam and graphically plot a displacement diagram.

10–14. For the cam in Problem 10–13, analytically determine the maximum follower velocity and acceleration.

10–15. For the cam in Problem 10–13, use a spreadsheet to plot the motion curves.

10–16. A cam drive is used for a mechanism that packs stuffing into shipping boxes The cam follower motion sequence must be:

1. Rise outward 1 in. with cycloidal motion in 1.5 sec
2. Fall with cycloidal motion in 1.0 sec.
3. Dwell for 0.5 sec.

Determine the speed of the cam and graphically plot a displacement diagram.

10–17. For the cam in Problem 10–16, analytically determine the maximum follower velocity and acceleration.

10–18. For the cam in Problem 10–16, use a spreadsheet to plot the motion curves.

10–19. A cam drive is used for a mechanism incorporated in a shoe sewing machine. The cam follower motion sequence must be:

1. Rise outward 0.5 in. with cycloidal motion in 0.7 sec
2. Dwell for 0.2 sec.
3. Fall 0.25 in. with cycloidal motion in 0.5 sec.
4. Dwell for 0.2 sec.
5. Fall with cycloidal motion in 0.5 sec.

Determine the speed of the cam and graphically plot a displacement diagram.

10–20. For the cam in Problem 10–19, analytically determine the maximum follower velocity and acceleration.

10–21. For the cam in Problem 10–19, use a spreadsheet to plot the motion curves.

10–22. A cam drive is used for a mechanism that feeds parts into a machining set-up. The cam follower motion sequence must be:

1. Rise outward 0.5 in. with modified trapazoidal motion in 0.7 sec
2. Dwell for 0.2 sec.
3. Fall with modified trapazoidal motion in 0.5 sec.

Analytically determine the maximum follower velocity and acceleration.

10–23. For the cam in Problem 10–22, use a spreadsheet to plot the motion curves.

10–24. A cam drive is used for a mechanism that distributes ink in a printing press. The cam follower motion sequence must be:

1. Rise outward 3 mm with modified trapazoidal motion in 0.5 sec.
2. Dwell for 0.2 sec.
3. Rise 2 mm with modified trapazoidal motion in 0.4 sec.
4. Dwell for 0.2 sec.
5. Fall with modified trapazoidal motion in 0.6 sec.

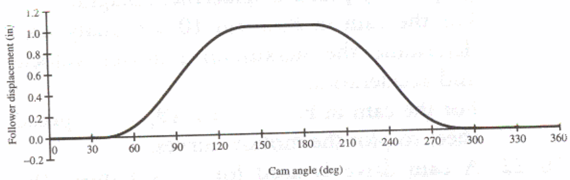
Analytically, determine the maximum follower velocity and acceleration.

Disk Cam Profile Design Problems

For problems requiring analytical solution, the use of a spreadsheet is highly recommended.

10–25. For the cam in Problem 10–24, use a spreadsheet to plot the motion curves.

10–26. A disk cam must provide the displacement shown in Figure P10.26 to a reciprocating knife edge follower. The cam must have a base circle of 3.0 in and rotate clockwise. Graphically construct the profile.



Cam angle (deg)	Follower Displ. (in)
0	0.000
10	0.000
20	0.000
30	0.000
40	0.004
50	0.029
60	0.091
70	0.196
80	0.337
90	0.500
100	0.663
110	0.804
120	0.909

Cam angle (deg)	Follower Displ. (in)
130	0.971
140	0.996
150	1.000
160	1.000
170	1.000
180	1.000
190	0.996
200	0.971
210	0.909
220	0.804
230	0.663
240	0.500

Cam angle (deg)	Follower Displ. (in)
250	0.337
260	0.196
270	0.091
280	0.029
290	0.004
300	0.000
310	0.000
320	0.000
330	0.000
340	0.000
350	0.000
360	0.000

Figure P10.26 Problems 26-40.

10–27. For the cam described in Problem 10–26, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle.

10–28. A disk cam must provide the displacement shown in Figure P10.26 for a reciprocating knife edge follower. The cam must have a base circle of 2.0 in and rotate counterclockwise. Graphically construct the profile.

10–29. A disk cam must provide the displacement shown in Figure P10.26 for a reciprocating in-line roller follower. The roller diameter is 1.0 in and the cam must have a base circle of 3.0 in and rotate clockwise. Graphically construct the profile and estimate the largest pressure angle.

10–30. For the cam described in Problem 10–29, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

10–31. A disk cam must provide the displacement shown in Figure P10.26 for a reciprocating in-line roller follower. The roller diameter is 0.75 in and the cam must have a base circle of 2.0 in and rotate counterclockwise. Graphically construct the profile and estimate the largest pressure angle.

10–32. For the cam described in Problem 10–31, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

10–33. A disk cam must provide the displacement shown in Figure P10.26 for a reciprocating offset roller follower. The follower is positioned in the vertical plane, contacting the top of the cam. The offset distance is 0.75 in. to the left of the cam center. The roller diameter is 1.0 in and the cam must have a base circle of 3.0 in and rotate clockwise. Graphically construct the profile and estimate the largest pressure angle.

10–34. For the cam described in Problem 10–33, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

10–35. A disk cam must provide the displacement shown in Figure P10.26 for a reciprocating offset roller follower. The follower is positioned in the vertical plane, contacting the top of the cam. The offset distance is 0.5 in. to the right of the cam center. The roller diameter is 0.75 in and the cam must have a base circle of 2.0 in and rotate counterclockwise. Graphically construct the profile and estimate the largest pressure angle.

10–36. For the cam described in Problem 10–35, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

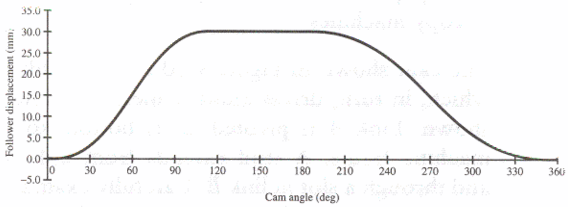
10–37. A disk cam must provide the displacement shown in Figure P10.26 for a reciprocating flat-faced follower. The cam must have a base circle of 5.0 in and rotate clockwise. Graphically construct the profile and estimate the largest pressure angle.

10–38. For the cam described in Problem 10–37, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

10–39. A disk cam must provide the displacement shown in Figure P10.26 for a reciprocating flat-faced follower. The cam must have a base circle of 6.0 in and rotate counterclockwise. Graphically construct the profile and estimate the largest pressure angle.

10–40. For the cam described in Problem 10–39, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

10–41. A disk cam must provide the displacement shown in Figure P10.41 for a pivoted roller follower. The length of the follower is 4 in and pivots 5 in from the cam rotation axis. The roller diameter is 1 in. The cam must have a base circle of 3.0 in and rotate clockwise. Graphically construct the profile and estimate the largest pressure angle.



Cam angle (deg)	Follower Displ. (mm)
0	0.000
10	0.113
20	0.865
30	2.725
40	5.865
50	10.113
60	15.000
70	19.887
80	24.135
90	27.275
100	29.135
110	29.887
120	30.000

Cam angle (deg)	Follower Displ. (mm)
130	30.000
140	30.000
150	30.000
160	30.000
170	30.000
180	30.000
190	29.966
200	29.736
210	29.135
220	28.035
230	26.369
240	24.135

Cam angle (deg)	Follower Displ. (mm)
250	21.402
260	18.300
270	15.000
280	11.700
290	8.598
300	5.865
310	3.631
320	1.965
330	0.865
340	0.264
350	0.034
360	0.000

Figure P10.41 Problems 41-48.

10–42. For the cam described in Problem 10–41, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

10–43. A disk cam must provide the displacement shown in Figure P10.42 for a pivoted roller follower. The length of the follower is 3 in and pivots 3.5 in from the cam rotation axis. The roller diameter is 0.75 in. The cam must have a base circle of 2.0 in and rotate counterclockwise. Graphically construct the profile and estimate the largest pressure angle.

10–44. For the cam described in Problem 10–43, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

Cylindrical Cam Design Problems

10–45. A cylindrical cam must provide the displacement shown in Figure P10.26 for a reciprocating roller follower. The roller diameter is 1.0 in; the cylinder diameter is 5 in and rotates clockwise. Graphically construct the profile and estimate the largest pressure angle.

10–46. For the cam described in Problem 10–46, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

10–47. A cylindrical cam must provide the displacement shown in Figure P10.26 for a reciprocating roller follower. The roller diameter in 30 mm; the cylinder diameter is 150 mm and rotates clockwise. Graphically construct the profile and estimate the largest pressure angle.

10–48. For the cam described in Problem 10–47, analytically generate a chart of the profile coordinates for every 30° of cam rotation angle. Also calculate the largest pressure angle.

Geneva Mechanism Problems

10–49. A geneva mechanism has been designed with four stations. The distance between the driving and driven shafts is 3 in. The driving arm rotates at a constant rate of 60 rpm, counterclockwise. Determine the angular velocity and acceleration of the wheel when the driving arm has rotated 25° from the position where the roller has just entered the slot.

10–50. A geneva mechanism has been designed with five stations. The distance between the driving and driven shafts is 60 mm. The driving arm rotates at a constant rate of 70 rpm, clockwise. Determine the angular velocity and acceleration of the wheel when the driving arm has rotated 20° from the position where the roller has just entered the slot.

10–51. A geneva mechanism has been designed with six stations. The distance between the driving and driven shafts is 4 in. The driving arm rotates at a constant rate of 90 rpm, counterclockwise. Determine the angular velocity and acceleration of the wheel when the driving arm has rotated 25° from the position where the roller has just entered the slot.

CASE STUDIES

10–1. The cam shown in Figure C10.1 is used to feed papers to a printing press. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

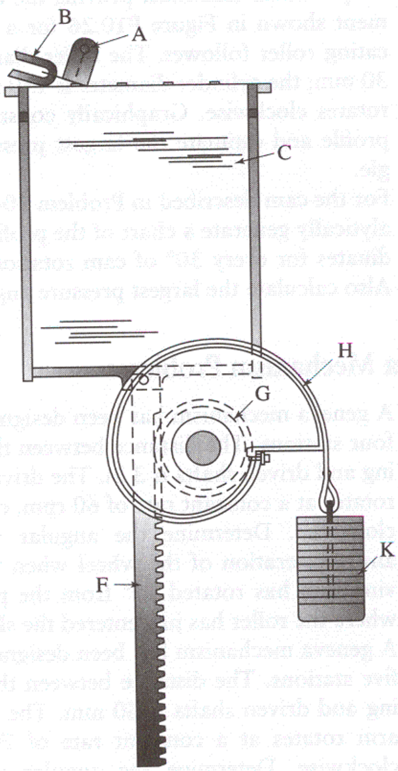


Figure C10.1 (Courtesy, Industrial Press)

1. As shaft *G* is forced to turn clockwise, determine the motion of item *F*.
2. What is the name of the connection between items *F* and *G*?
3. What causes the stack of papers, sitting on item *J*, to remain at a level where a mechanism at *B* can grab them?
4. Why does the radius on item *H* change?
5. What feature allows any rotation of item *H* to be transferred to item *G*?
6. Describe the mechanism that performs the same function as this cam for smaller stacks of papers used in computer printers and copy machines.

10–2. The cam shown in Figure C10.2 drives link *J* which, in turn, drives another mechanism not shown. Link *A* is pivoted at its bottom to a machine frame. A stud extends from link *A* and through a slot in link *B*. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

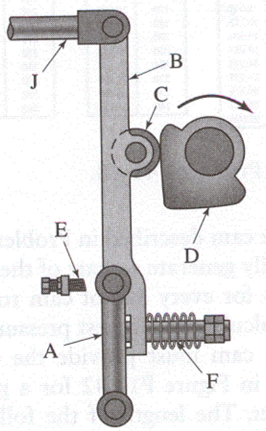


Figure C10.2 (Courtesy, Industrial Press)

1. As cam *D* rotates clockwise, describe the motion of link *B*.
2. What type of cam is *D*?
3. What type of follower is *C*?
4. What type of component is item *F*?
5. Describe the action of item *F*.
6. What type of component is item *E*?
7. Describe the function of item *E*.

8. Describe the cyclical motion of item *B*.
9. What changes would occur to the motion of *B* if item *E* were moved?
10. What changes would occur to the motion of *B* if the stud at *F* were shortened?

10–3. The machine shown in Figure C10.3 stamps and forms steel parts. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

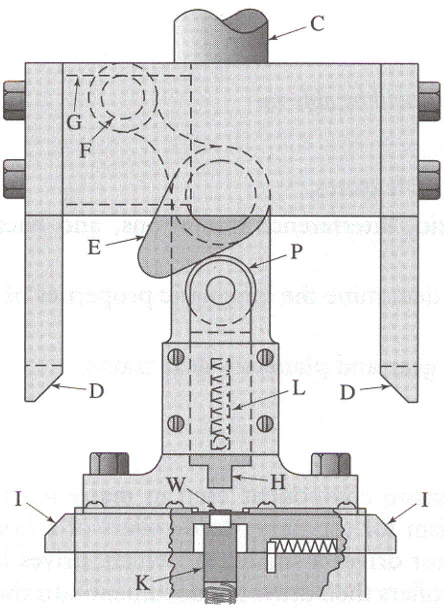


Figure C10.3 (Courtesy, Industrial Press)

1. As rod *C* starts to slide downward, what is the motion of cam *E*?
2. What is the motion of plunger *H*?
3. What happens to a strip of sheet metal clamped at *W*?
4. As rod *C* continues to turn, what is the motion of the plunger?
5. What is the motion of the slides *I*?
6. What happens to the steel strip at *W*?
7. As rod *C* starts to slide upward, what is the motion of plunger *H*?
8. What is the purpose of this mechanism?
9. Why are springs contacting slides *I*?
10. Why is a spring supporting item *K*?
11. What type of mechanism could drive rod *C*?

10–4. A machine is shown in Figure C10.4. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

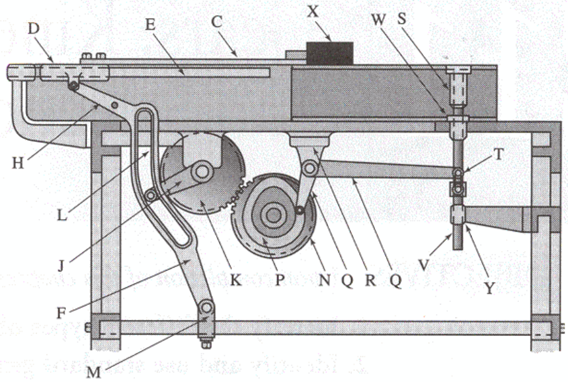


Figure C10.4 (Courtesy, Industrial Press)

1. As gear *K* rotates clockwise, describe the motion of link *F*.
2. Discuss the specifics of the cyclical motion of link *F*.
3. As gear *K* rotates clockwise, describe the motion of slide *D*.
4. As gear *K* rotates clockwise, describe the motion of gear *N*.
5. As gear *K* rotates clockwise, describe the motion of link *Q*.
6. What type of component is item *P* called?
7. Describe the motion to which item *V* is constrained.
8. Discuss exactly the manner in which link *Q* is attached to item *V*.
9. Discuss the cyclical motion of the entire machine.
10. State a need for such a machine.

11 Gears: Kinematic Analysis and Selection

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Identify the different types of gears.
2. Identify and use standard gear geometric features.
3. Calculate center distance, contact ratio, interference limitations, and backlash variations.
4. Calculate and use the velocity ratio to determine the kinematic properties of mating gears.
5. Determine the kinematic properties of gear and planetary gear trains.

11.1 INTRODUCTION

Gears are an extremely common mechanism component used in many machines. Figure 11.1 illustrates the drive mechanism for the paper feed rollers of a fax machine. In this application, an electric motor drives a small gear which, drives larger gears that turn the feed rollers. The feed rollers then draw the document into the machine's scanning device.

In general, the function of a gear is to transmit motion from one rotating shaft to another. In the case of the feed drive of Figure 11.1, the motion of the motor must be transmitted to the shafts carrying the rollers. In addition to simply transmitting the motion, gears are often used to increase or reduce speed, or change the direction of motion from one shaft to the other.

It is extremely common for the output of mechanical power sources, such as electric motors and engines, to be rotating at much greater speeds than the application requires. The fax machine requires that the rollers feed the document through the machine at a rate compatible with the scanning device. However, typical electric motors rotate at greater speeds than are needed at the rollers. Therefore, the speed of the motor must be reduced as it is transmitted to the feed roller shafts. Also the upper rollers must rotate in the direction opposite to that of the lower rollers. Thus, gears are a natural choice for this application.

Figure 11.2A illustrates two mating spur gears designed to transmit motion between their respective shafts. Figure 11.2B shows two friction rollers or disks that

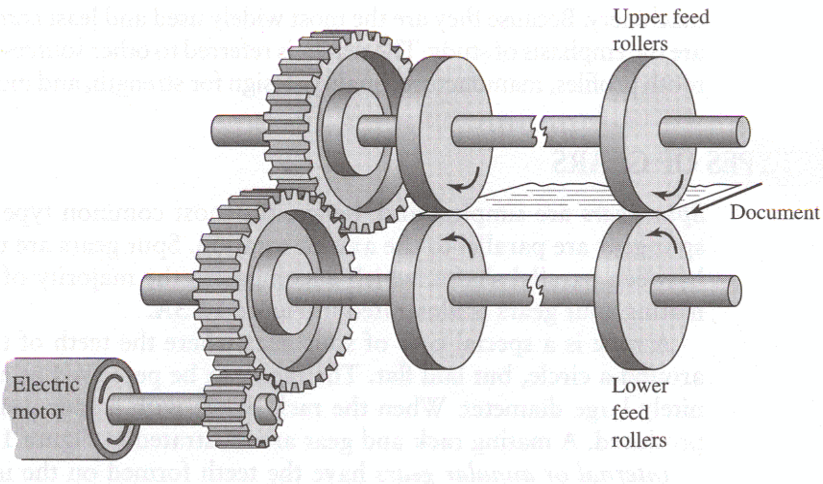
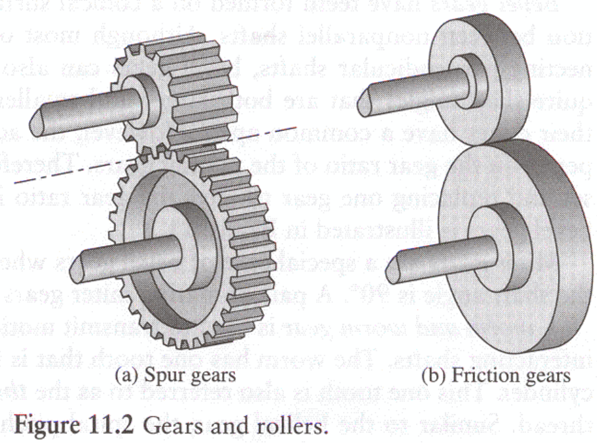


Figure 11.1 Feed rollers for a fax machine.

are also designed to transmit motion between the shafts. Such disks are obviously less costly than complex gear configurations. However, the disks rely on friction to transmit forces that may accompany the motion. Because many applications require the transmission of power (both motion and forces), smooth disk surfaces may not be able to generate sufficient frictional forces and, thus, will slip under larger loads.

To remedy the possibility of slipping, a gear is formed such that the smooth surfaces of the disks are replaced by teeth. The teeth provide a positive engagement and eliminate slipping. From a kinematic viewpoint, the gear pair in Figure 11.2A would replace the disks of 11.2B because the effective diameters are identical.

The principles of general gearing and the associated kinematic relations are presented in this chapter. The focus of this book is on the analysis and design of mechanisms that are necessary to provide the motion required of machinery. Consistent with this mission, the focus of this chapter is on the selection of standard gears used in



(a) Spur gears (b) Friction gears

Figure 11.2 Gears and rollers.

machinery. Because they are the most widely used and least complicated gear, spur gears are the emphasis of study. The reader is referred to other sources for further detail on gear tooth profiles, manufacture, quality, design for strength, and more complex gears.[5]

11.2 TYPES OF GEARS

Spur gears are simplest and, hence, the most common type of gear. The teeth of a spur gear are parallel to the axis of rotation. Spur gears are used to transmit motion between parallel shafts, which encompasses the majority of applications. A pair of mating spur gears is illustrated in Figure 11.3A.

A *rack* is a special case of spur gear where the teeth of the rack are not formed around a circle, but laid flat. The rack can be perceived as a spur gear with an infinitely large diameter. When the rack mates with a spur gear, translating motion is produced. A mating rack and gear are illustrated in Figure 11.3B.

Internal or annular gears have the teeth formed on the inner surface of a circle. When mating with a spur gear, the internal gear has the advantage of reducing the distance between the gear centers for a given speed variation. An internal gear mating with a traditional spur gear is illustrated in Figure 11.3C.

Helical gears are similar to, and can be used in the same applications as, spur gears. The difference is that the teeth of a helical gear are inclined to the axis of rotation. The angle of inclination is termed the *helix angle*, φ. This angle provides a more gradual engagement of the teeth during meshing, and produces less impact and noise. Because of this smoother action, helical gears are preferred in high-speed applications. However, the helix angle produces thrust forces and bending couples, which are not generated in spur gears. A pair of mating helical gears is illustrated in Figure 11.3D.

Herringbone gears are used in the same applications as spur gears and helical gears. In fact, they are also referred to as double helical gears. The herringbone gear appears as two opposite hand helical gears butted against each other. This complex configuration counterbalances the thrust force of a helical gear. A herringbone gear is shown in Figure 11.3E.

Bevel gears have teeth formed on a conical surface and are used to transmit motion between nonparallel shafts. Although most of their applications involve connecting perpendicular shafts, bevel gears can also be used in applications that require shaft angles that are both larger and smaller than 90°. As bevel gears mesh, their cones have a common apex. However, the actual cone angle of each gear depends on the gear ratio of the mating gears. Therefore, bevel gears are designed as a set and replacing one gear to alter the gear ratio is not possible. A pair of mating bevel gears is illustrated in Figure 11.3F.

Miter gears are a special case of bevel gears where the gears are of equal size and the shaft angle is 90°. A pair of mating miter gears is illustrated in Figure 11.3G.

A *worm and worm gear* is used to transmit motion between nonparallel and noninteracting shafts. The worm has one tooth that is formed in a spiral around a pitch cylinder. This one tooth is also referred to as the *thread* because it resembles a screw thread. Similar to the helical gear, the spiral pitch of the worm generates an axial

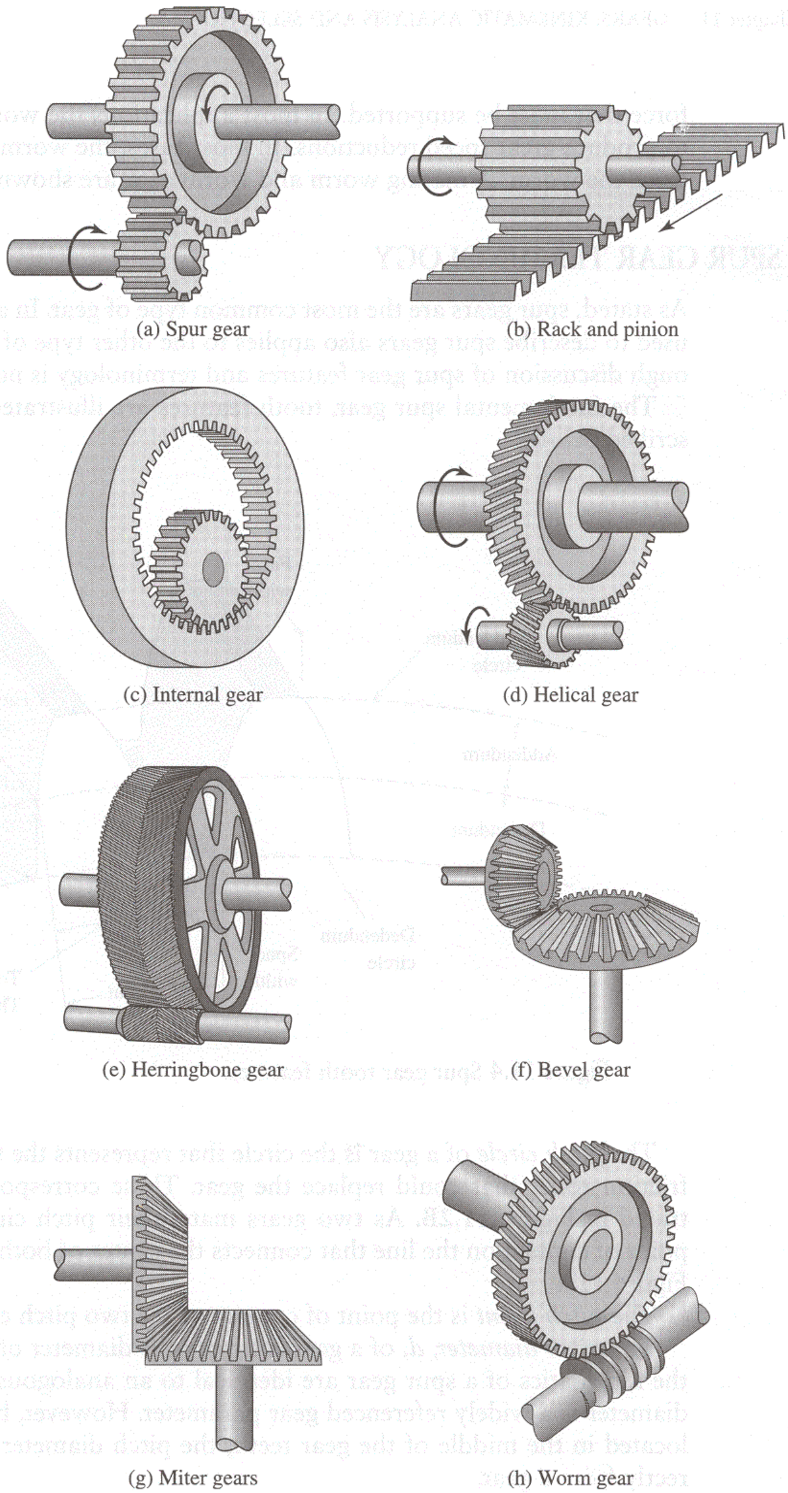


Figure 11.3 Gear types.

force that must be supported. In most applications the worm drives the worm gear to produce great speed reductions. In most cases, the worm drives. The gear can not drive the worm. A mating worm and worm gear are shown in Figure 11.3H.

11.3 SPUR GEAR TERMINOLOGY

As stated, spur gears are the most common type of gear. In addition, the terminology used to describe spur gears also applies to the other type of gears. Therefore, a thorough discussion of spur gear features and terminology is necessary.

The fundamental spur gear, tooth features are illustrated in Figure 11.4 and described below.

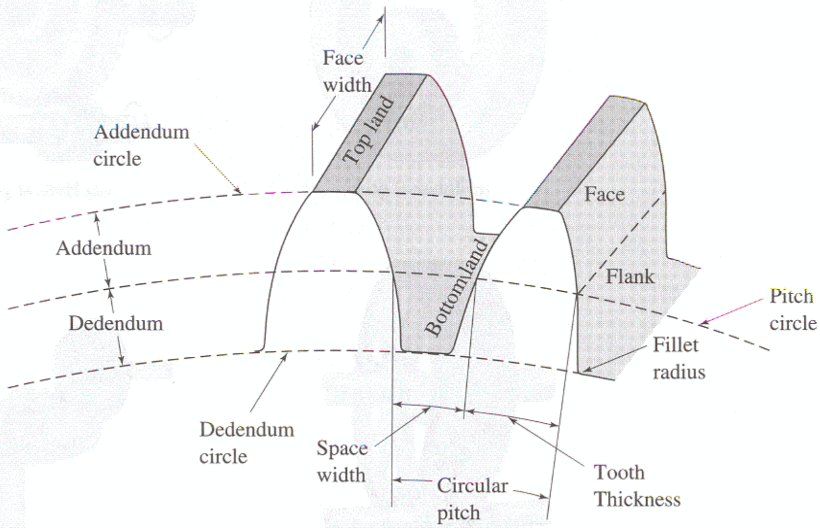


Figure 11.4 Spur gear tooth features.

The *pitch circle* of a gear is the circle that represents the size of the corresponding friction roller that could replace the gear. These corresponding rollers were illustrated in Figure 11.2B. As two gears mate, their pitch circles are tangent, with a point of contact on the line that connects the center of both circles. This is shown in Figure 11.4.

The *pitch point* is the point of contact of the two pitch circles.

The *pitch diameter, d,* of a gear is simply the diameter of the pitch circle. Because the kinematics of a spur gear are identical to an analogous friction roller, the pitch diameter is a widely referenced gear parameter. However, because the pitch circle is located in the middle of the gear teeth, the pitch diameter cannot be measured directly from a gear.

The *number of teeth, N,* is simply the total number of teeth on the gear. Obviously, this value must be an integer because fractional teeth cannot be used.

The *circular pitch, p,* is the distance measured along the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth of the gear. The circular pitch can be calculated from the number of teeth and the pitch diameter of a gear. The governing equation is as follows:

(11.1)
$$p = \frac{\pi d}{N}$$

The *base circle* of a gear is the circle from which the curved shape of the gear tooth is constructed. Details on the generation of the curved tooth profile are presented in the following section.

The *base diameter,* d_b, is simply the diameter of the base circle.

The *face width, F,* is the length of the gear tooth parallel with the shaft axis.

The *addendum, a,* is the radial distance from the pitch circle to the top of a gear tooth.

The *dedendum, b,* is the radial distance from the pitch circle to the bottom of a gear tooth.

The *whole depth,* h_T, is the height of a gear tooth and is the sum of the addendum and dedendum.

The *clearance, c,* is the amount that the dedendum exceeds the addendum. This is the room between the top of a gear tooth and the bottom of the mating gear tooth.

The *backlash, B,* is the amount that the width of a tooth space exceeds the thickness of a gear tooth, measured on the pitch circle.

The *diametral pitch,* P_d, or simply pitch, actually refers to the tooth size and has become a standard for tooth size specifications. Formally, the diametral pitch is the number of teeth per inch of pitch diameter:

(11.2)
$$P_d = \frac{N}{d}$$

The diametral pitch is a commonly referenced gear parameter in the United States Customary Units. Again, it is a relative measure of the size of gear tooth. The standard tooth sizes and their diametral pitches are shown in Figure 11.5. Although mating gears can have different diameters and number of teeth, mating gears must have the same diametral pitch. This fact should be obvious when recalling that the diametral pitch is a measure of tooth size.

The diametral pitch cannot be measured directly from a gear; yet, it is an extremely common referenced value. Tooling for commercially available spur gears are stocked in standardized diametral pitches as shown in Table 11.1. Although no physical significance exists, preference for standard diametral pitches is given to even integers. Sheet metal gauges which allow measurement of the standard diametral pitches are available.The units of diametral pitch are the reciprocal of inches; yet, it is not customary to specify units when expressing numerical values.

The *module, m,* is a commonly referenced gear parameter in the SI unit system:

(11.3)
$$m = \frac{d}{N}$$

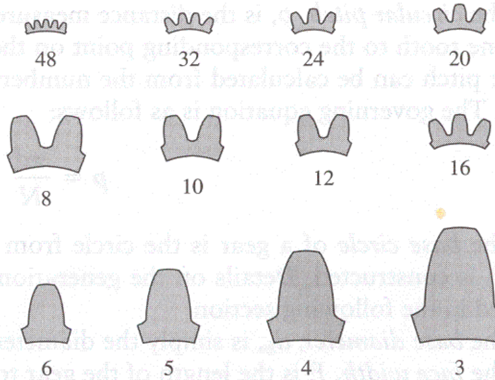


Figure 11.5 Standard tooth sizes (shown one-half actual size).

The module is also a relative measure of tooth size and is theoretically the reciprocal of the diametral pitch. However, because it is referenced in the SI system, the module has units of millimeters. Therefore, the module and diametral pitch are not numerically reciprocal. The relationship between diametral pitch and module accounting for units is as follows:

$$m = \frac{25.4}{P_d} \tag{11.4}$$

As with the diametral pitch, tooling for commercially available metric spur gears is stocked in standardized modules. Common values are shown in Table 11.2.

Substituting Equation 11.2 and 11.3 into 11.1, the circular pitch can also be computed as:

$$p = \frac{\pi d}{N} = \frac{\pi}{P_d} = \pi m \tag{11.5}$$

The *pressure angle*, ϕ, is the angle between a line tangent to the base circles, of mating gears, and a line perpendicular to the surfaces of the teeth at the contact point. The line tangent to the pitch circles is termed the pitch line. The line, perpen-

TABLE 11.1 Standard Diametral Pitches

Coarse	Pitch	Fine	Pitch
2	6	20	80
2.25	8	24	96
2.5	10	32	120
3	12	40	150
4	16	48	200
		64	

TABLE 11.2 Standard Metric Modules

1	4	16
1.25	5	20
1.5	6	25
2	8	32
2.5	10	40
3	12	50

dicular to the surfaces of the teeth at the contact point, is termed the pressure line. Therefore, the pressure angle is measured between the pitch line and the pressure line. The pressure angle is illustrated in Figure 11.6.

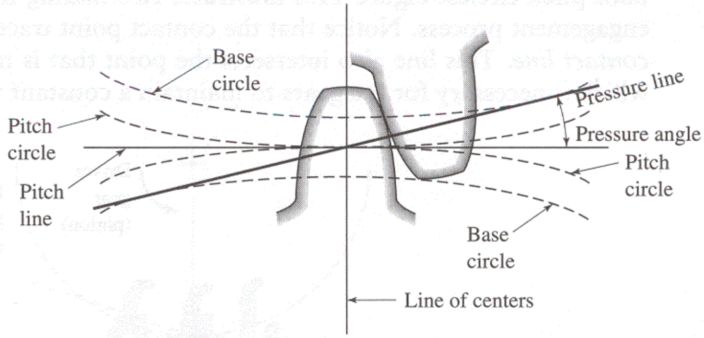


Figure 11.6 Pressure angle.

The pressure angle affects the relative shape of a gear tooth as shown in Figure 11.7. Although gears can be manufactured in a wide range of pressure angles, most gears are standardized at 20° and 25°. Gears with 14-1/2° pressure angles were widely used, but are now considered obsolete. They are still manufactured as replacements for older gear trains still in use. Because the pressure angle affects the shape of a tooth, two mating gears must also have the same pressure angle.

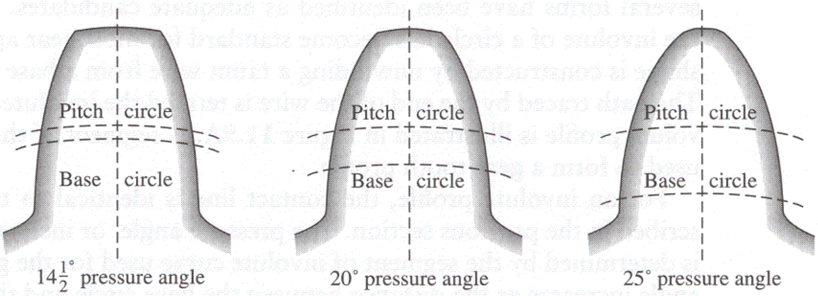


Figure 11.7 Pressure angle influence on tooth shapes.

Recall that forces are transmitted perpendicular to the surfaces in contact. Therefore, the force acting on a tooth is along the pressure line. As is discussed in the next section, gear teeth are shaped to maintain a constant pressure angle during engagement. Gears with smaller pressure angles efficiently transfer torque and apply less radial loads onto the shaft and supporting bearings. However, as the pressure angles are reduced, a greater tendency exists for gear teeth to interfere as they engage.

11.4 INVOLUTE TOOTH PROFILES

In order to achieve smooth motion, a gear tooth must have a shape that keeps the driven gear rotating at a constant velocity throughout the engagement and disengagement process. Stated more concisely, gears need to have a constant velocity ratio. Kinematically, this condition requires that the path of the gear tooth contact point is a straight line. That line must also intersect the point that is common to both pitch circles. Figure 11.8 illustrates two mating teeth at three intervals of the engagement process. Notice that the contact point traces a straight line, termed the *contact line*. This line also intersects the point that is tangent to both pitch circles, which is necessary for the gears to maintain a constant velocity ratio.

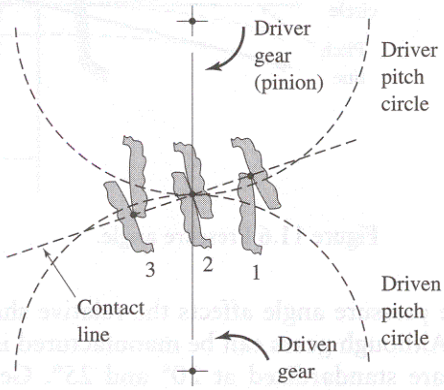


Figure 11.8 Gear mating process.

Discovering a tooth shape that fulfills the condition is not a trivial task; however, several forms have been identified as adequate candidates. Of the possible shapes, the involute of a circle has become standard for most gear applications. An involute shape is constructed by unwinding a taunt wire from a base circle with diameter d_b. The path traced by the end of the wire is termed the involute curve of a circle. An involute profile is illustrated in Figure 11.9A. A segment of this involute curve is then used to form a gear tooth profile.

For an involute profile, the contact line is identical to the pressure line, as described in the previous section. The pressure angle, or inclination of the contact line, is determined by the segment of involute curve used for the gear tooth. The pressure angle increases as the distance between the base circle and the pitch circle increases.

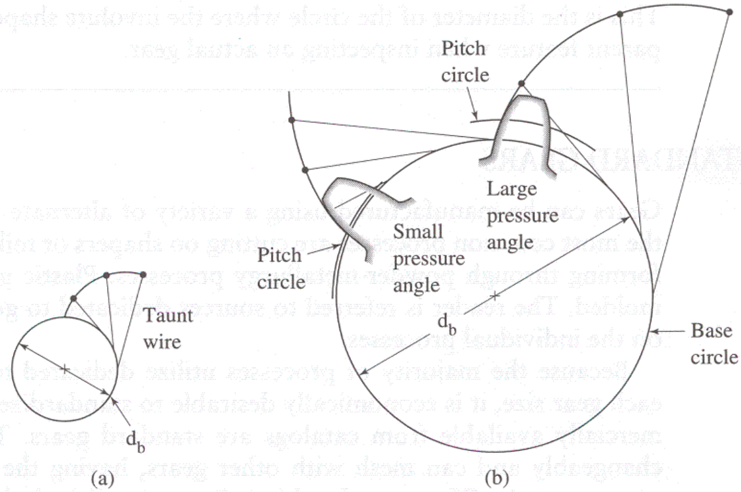


Figure 11.9 Involute gear tooth.

This is shown in Figure 11.9B. The relationship between the pressure angle, pitch diameter, and base circle diameter is expressed by:

$$d_b = d \cos \phi \tag{11.6}$$

Because the definition of an involute only extends from a base circle, any portion of tooth profile inside the base circle is not an involute. It is common to machine this portion as a radial line and a fillet to the dedendum circle. The portion of the tooth inside the base circle is not designed to be contacted by a mating gear tooth. Such contact would result in interference.

EXAMPLE PROBLEM 11.1

A 20° full-depth, involute spur gear with 35 teeth has a diametral pitch of 10. Determine the diameter of the pitch circle, the circular pitch, and the base circle.

Solution:

The pitch diameter can be computed by rearranging Equation 11.2:

$$d = \frac{N}{P_d} = \frac{35}{10} = 3.5 \text{ in}$$

The circular pitch can be computed from Equation 11.5:

$$p = \frac{\pi}{P_d} = \frac{\pi}{(10 \text{ in}^{-1})} = 0.314 \text{ in}$$

The base circle can be computed directly from Equation 11.6:

$$d_b = d \cos \phi = 3.5\cos(20°) = 3.289 \text{ in.}$$

This is the diameter of the circle where the involute shape originates. It is not an apparent feature when inspecting an actual gear.

11.5 STANDARD GEARS

Gears can be manufactured using a variety of alternate processes. For metal gears, the most common processes are cutting on shapers or milling machines, casting, and forming through powder-metallurgy processes. Plastic gears are typically injection molded. The reader is referred to sources dedicated to gear manufacture for details on the individual processes.

Because the majority of processes utilize dedicated tooling, which is unique to each gear size, it is economically desirable to standardize the gear sizes. Gears commercially available from catalogs are standard gears. These gears are sold interchangeably and can mesh with other gears, having the same diametral pitch and pressure angle. Of course, for this to be accomplished, the manufacturers' must follow a standard convention to form the details of the gear tooth profile.

As stated, any two involute gears with the same diametral pitch and pressure angle will mate. Therefore, gear teeth have been standardized based on the diametral pitch and pressure angle. As mentioned in Section 11.2, standard pressure angles are 14-1/2°, 20°, and 25°. The 14-1/2° pressure angle is becoming obsolete and used mainly as a replacement gear.

The diametral pitch is a measure of tooth size. In applications where the transmitted forces are high, larger teeth, having smaller values of diametral pitch, are required. Gears are used in a great range of applications from mechanical watches with low forces to large steel rolling mills with extremely large forces. Therefore, a wide range of diametral pitches must be available. The standardized values of diametral pitch were given in Table 11.1.

Most gear tooth features, as identified in Section 11.3 and Figure 11.3, are standardized relative to the diametral pitch and pressure angle. The governing relationships are given in Table 11.3. The American Gear Manufacturers Association (AGMA) is the primary technical organization that oversees this standardization scheme.

EXAMPLE PROBLEM 11.2

Consider the 20° full-depth, involute spur gear, with 35 teeth and a diametral pitch of 10 from Example Problem 11.1. Determine the diameter of the addendum circle, dedendum circle, and the clearance.

Solution:

The addendum circle is the outer diameter of the gear. The addendum is the distance from the pitch circle on a gear tooth to the top of the tooth. The standard distance for this gear can be computed from the equations in Table 11.3:

$$a = \frac{1}{P_d} = \frac{1}{10} = 0.100 \text{ in}$$

TABLE 11.3 Standard Spur Gear Tooth Proportions.

Tooth Feature	Coarse ($P_d < 20$)	Pitch		Fine Pitch ($P_d > 20$)
Pressure angle, ϕ	14-1/2°	20°	25°	20°
Addendum, a	$\frac{1}{P_d}$	$\frac{1}{P_d}$	$\frac{1}{P_d}$	$\frac{1}{P_d}$
Dedendum, b	$\frac{1.25}{P_d}$	$\frac{1.25}{P_d}$	$\frac{1.25}{P_d}$	$0.002 + \frac{1.2}{P_d}$
Working depth, h_k	$\frac{2}{P_d}$	$\frac{2}{P_d}$	$\frac{2}{P_d}$	$\frac{2}{P_d}$
Whole depth, h_t	$\frac{2.25}{P_d}$	$\frac{2.25}{P_d}$	$\frac{2.25}{P_d}$	$0.002 + \frac{2.2}{P_d}$
Circular tooth thickness, t	$\frac{\pi}{2P_d}$	$\frac{\pi}{2P_d}$	$\frac{\pi}{2P_d}$	$\frac{1.5708}{P_d}$
Fillet radius, r_f	$\frac{0.3}{P_d}$	$\frac{0.3}{P_d}$	$\frac{0.3}{P_d}$	—
Min. clearance, c	$\frac{0.25}{P_d}$	$\frac{0.25}{P_d}$	$\frac{0.25}{P_d}$	$0.002 + \frac{0.2}{P_d}$
Clearance (ground tooth), c	$\frac{0.35}{P_d}$	$\frac{0.35}{P_d}$	$\frac{0.35}{P_d}$	$0.002 + \frac{0.35}{P_d}$
Min top land width	$\frac{0.25}{P_d}$	$\frac{0.25}{P_d}$	$\frac{0.25}{P_d}$	—
AGMA standard	201.02A	201.02	201.02	207.04
Face width	$\frac{12}{P_d}$	$\frac{12}{P_d}$	$\frac{12}{P_d}$	$\frac{12}{P_d}$

Notice that this is the distance between the radii of the pitch circle and the addendum circle. Therefore, the diameter of the addendum circle is offset a distance, a, on both sides of the pitch circle. In Example Problem 11.1, the pitch diameter is 3.50 in. Therefore, the diameter of the addendum circle can be computed as:

$$d_a = d + 2a = 3.5 + 2(0.100) = 3.7 \text{ in}$$

In a similar fashion, the dedendum is the distance between the radii of the pitch circle and the dedendum circle. Therefore, the dedendum can be computed as:

$$b = \frac{1.250}{P_d} = \frac{1.250}{10} = 0.125 \text{ in}$$

and the dedendum circle diameter is:

$$d_b = d - 2b = 3.5 - 2(0.125) = 3.25 \text{ in}$$

Notice that the base circle diameter from this Example Problem is 3.289 in. Comparing this to the dedendum circle reveals that a short portion of the gear tooth profile is inside the base circle. On a radial basis, the length of this short portion of tooth profile is determined by:

$$\frac{(3.289)}{2} - \frac{(3.250)}{2} = 0.019$$

Recall that the definition of an involute only extends from a base circle. This short portion of tooth profile is not an involute and should not be contacted by the mating gear tooth.

Finally, the clearance is the amount that the dedendum exceeds the addendum. This is the room between the top of a gear tooth and the bottom of the mating gear tooth. The standard distance for this gear can be computed using the equations in Table 11.2:

$$c = \frac{0.25}{P_d} = \frac{0.25}{10} = 0.025 \text{ in}$$

Notice that the clearance is greater than the distance of the noninvolute portion of the gear tooth. Thus, contact between gear teeth on this portion is not expected.

11.6 RELATIONSHIPS OF GEARS IN MESH

Two gears in contact were shown in Figure 11.3. As two gears mesh, the smaller gear is commonly termed the *pinion* and the larger is referred to as the *bull gear* or simply the *gear.* Recall that in order for two gears to mate, they must have the same diametral pitch and pressure angle. Relationships of two mating gears are discussed in the following sections.

11.6.1 Center Distance

The *center distance, c,* is defined as the center-to-center distance between two mating gears. This is also the distance between the shafts that are carrying the gears. For the common configuration of external gears (Figure 11.3), the distance can be written as:

(11.7)
$$c_{\text{external gears}} = r_1 + r_2 = \frac{(d_1 + d_2)}{2}$$

because:

$$d = \frac{N}{P_d}$$

Equation 11.7 can be rewritten as:

(11.8)
$$c_{\text{external gears}} = \frac{(N_1 + N_2)}{2\ P_d}$$

For internal gears (Figure 11.3C), the center distance is the difference in the pitch radii and can be written as:

(11.9)
$$c_{\text{internal gears}} = r_2 - r_1 = \frac{(d_2 - d_1)}{2} = \frac{(N_2 - N_1)}{2\,P_d}$$

EXAMPLE PROBLEM 11.3

Two 5 pitch, 20° full-depth gears are used on a small construction site concrete mixer. The gears transmit power from a small engine to the mixing drum. This machine is shown in Figure 11.10. The pinion has 15 teeth and the gear has 30 teeth. Determine the center distance.

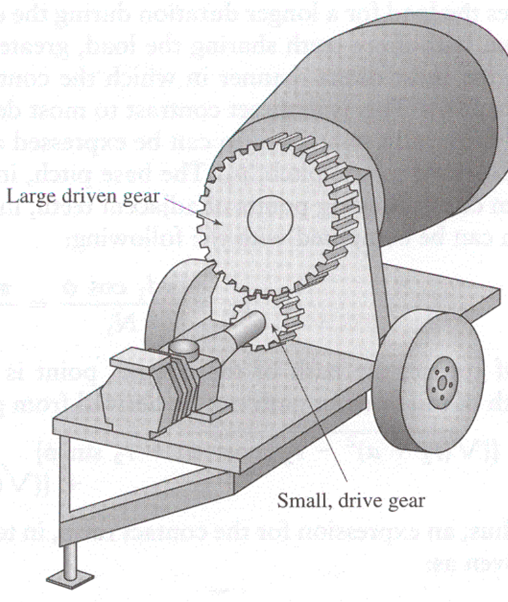


Figure 11.10 Concrete mixer for Example Problem 11.3.

Solution:

The pitch diameters of both gears can be determined from Equation 11.2:

$$d_1 = \frac{N_1}{P_d} = \frac{15}{5} = 3.0 \text{ in}$$

$$d_2 = \frac{N_2}{P_d} = \frac{30}{5} = 6.0 \text{ in}$$

Because these gears are external, the center distance can be found from Equation 11.7:

$$c = \frac{(d_1 + d_2)}{2} = \frac{(3.0 \text{ in} + 6.0 \text{ in})}{2} = 4.5 \text{ in}$$

11.6.2 Contact Ratio

The contact ratio is the average number of teeth that are in contact an any instant. Obviously, the contact ratio must exceed 1 because contact between gears must not be lost. In practice, contact ratios should be greater than 1.2. Robust designs have contact ratios of 1.4 or 1.5. To illustrate the principle, a contact ratio of 1.2 indicates that one pair of teeth is always in contact, and a second pair of teeth is in contact 20% of the time.

Greater contact ratio values result in smoother action because another gear tooth shares the load for a longer duration during the engaging/disengaging process. In addition, with more teeth sharing the load, greater power may be transmitted. However, the most direct manner in which the contact ratio can be increased is to use larger gears. This is in direct contrast to most design goals of compactness.

Numerically, contact ratio can be expressed as the length of the path of contact, divided by the base pitch, p_b. The base pitch, in turn, is defined as the distance between corresponding points of adjacent teeth, measured on the base circle. The base pitch can be computed with the following:

(11.10) $$p_b = \frac{\pi d_1 \cos \phi}{N_1} = \frac{\pi d_2 \cos \phi}{N_2}$$

Of course, the path of the contact point is a straight line (Section 11.4). The length of this contact path can be derived from geometry [1] by:

(11.11) $$L = \{(\sqrt{(r_2 + a)^2 - r_2^{\,2} \cos^2 \phi}) - r_2 \sin \phi\} + \{(\sqrt{(r_1 + a)^2 - r_1^{\,2} \cos^2 \phi}) - r_1 \sin \phi\}$$

Thus, an expression for the contact ratio, in terms of the gear tooth geometry, can be given as:

(11.12) $$\text{Contact ratio} = \frac{L}{p_b}$$

EXAMPLE PROBLEM 11.4

For the concrete mixer gears described in Example Problem 11.3, determine the contact ratio.

Solution:

The pitch radii of both gears can be determined from the pitch diameters:

$$r_1 = \frac{d_1}{2} = \frac{3.0 \text{ in}}{2} = 1.5 \text{ in}$$

$$r_2 = \frac{d_2}{2} = \frac{6.0 \text{ in}}{2} = 3.0 \text{ in}$$

From Table 11.3, the addendum for 20° full-depth teeth is:

$$a = \frac{1}{P_d} = \frac{1}{5} = 0.20 \text{ in}$$

The base pitch is calculated from Equation 11.10:

$$p_b = \frac{\pi d_1 \cos \phi}{N_1} = \frac{\pi(3.0 \text{ in})\cos(20°)}{15} = 0.6890 \text{ in}$$

The length of the contact line is calculated from Equation 11.11:

$$L = \{(\sqrt{(r_2 + a)^2 - r_2^2 \cos^2 \phi}) - r_2 \sin \phi\} + \{(\sqrt{(r_1 + a)^2 - r_1^2 \cos^2 \phi}) - r_1 \sin \phi\}$$

$$= \{(\sqrt{(3.0 + 0.2)^2 - (3.0)^2\cos^2(20°)}) - (3.0)\sin(20°)\} + \{(\sqrt{(1.5 + 0.2)^2 - (1.5)^2\cos^2(20°)}) - (1.5)\sin(20°)\} = 0.9255 \text{ in}$$

Then the contact ratio is determined from Equation 11.12:

$$\text{Contact ratio} = \frac{L}{P_b} = \frac{0.9255 \text{ in}}{0.6890 \text{ in}} = 1.3433$$

Although this ratio is acceptable, larger values (1.4–1.5) are desirable.

11.6.3 Interference

Gear teeth have involute profiles between the base circle and the addendum circle. When a gear with few teeth and small pressure angles is constructed, the dedendum circle is considerably smaller than the base circle of the involute. Therefore, the tooth between the base circle and the dedendum is not an involute. If the mating gear tooth were to contact this portion of the tooth, the fundamental condition for constant velocity ratio would be violated. This condition is termed *interference* and, as it occurs, the teeth can exhibit drastic noise, vibration, and wear.

Interference is induced as designers attempt to make gear assemblies compact by using too few teeth on the gears. Interference commonly occurs when a small gear mates with a much larger one. A relationship that can be used to determine the necessary number of teeth in the gear to avoid interference has been derived [5]. Equation 11.12 determines the largest number of teeth in the gear to ensure no interference. The relationship is given as a function of the number of teeth in the mating pinion, along with the pressure angle and addendum size:

$$N_2 < \frac{\{N_1^2 \sin^2 \phi - 4k^2\}}{4k - 2N_1 \sin^2 \phi} \quad (11.13)$$

where k is defined from the addendum relation:

$$a = \frac{k}{P_d}$$

Equation 11.10 can be used to tabulate suitable combinations of gears that avoid interference. These combinations are given in Table 11.4.

TABLE 11.4 Gear Teeth Combinations to Ensure No Interference

Number of Pinion Teeth	ϕ = 14-1/2° Maximum Number of Gear Teeth	Number of Pinion Teeth	ϕ = 20° Maximum Number of Gear Teeth	Number of Pinion Teeth	ϕ = 25° Maximum Number Gear Teeth
23	26	13	16	9	13
24	32	14	26	10	32
25	40	15	45	11	249
26	51	16	101	12	∞
27	67	17	1309		
28	92	18	∞		
29	133				
30	219				
31	496				
32	∞				

Note from Table 11.4 that 14-1/2° pinions with more than 32 teeth can mate with any size gear without interference; however, any 14-1/2° pinion with fewer than 23 teeth experiences interference, regardless of the size of the mating gear. Such limits can be gathered for other standard pressure angles.

It is apparent from Table 11.4 that involute gear teeth with a 25° pressure angle permit usage gears with fewer teeth, without interference. As a result, more compact gear assemblies can be produced. This is the primary reason for the popularity of 25° teeth and the obsolescence of 14-1/2° teeth.

For the extreme case where a pinion can mate with any other gear, $N_2 = \infty$, can be substituted into Equation 11.12. This provides the size of pinion that can mate with any gear. As already mentioned, a 14-1/2 pinion with 32 teeth exhibits such properties. Once $N_2 = \infty$ is substituted, the following relationship is derived:

$$N_1 > \frac{2k}{\sin^{-2} \phi} \tag{11.14}$$

It should be noted that a gear with $N_2 = \infty$ would also have an infinite pitch radius. This is the concept behind a rack, as shown in Figure 11.3D. Thus, Equation 11.11 must be met to ensure that a gear mates with a rack and avoids interference.

EXAMPLE PROBLEM 11.5

For the gears of the concrete mixer described in Example Problem 11.3, determine whether interference is a concern.

Solution:

From Table 11.4, it is demonstrated that a 20° full-depth tooth with 15 teeth cannot mate with a gear with more than 45 teeth without interference.

The same result can be obtained from Equation 11.13. From Table 11.2, the addendum needed is:

$$a = \frac{1}{P}$$

therefore:

$$k = 1$$

Equation 11.12 can be used to check for interference problems:

$$N_2 < \frac{\{N_1^{\,2} \sin^2 \phi - 4k^2\}}{4k - 2N_1 \sin^2 \phi}$$

$$N_2 < \frac{\{15^2 \sin^2(20°) - 4(1)^2\}}{4(1) - 2(15)\sin^2(20°)}$$

$$N_2 < 45.48$$

The number of teeth on the driven gear is 30, less than the limiting value of 45.48 computed above. Therefore, interference is not a foreseeable problem.

11.6.4 Undercutting

Interference can also be avoided by removing the material on the gear tooth between the base circle and dedendum circle. This is the portion of the gear tooth that is not an involute and would interfere with the mating tooth. An undercut gear tooth is shown in Figure 11.11.

Undercutting obviously reduces the strength of the gear, thus reducing the power that can be safely transmitted. In addition, it also reduces the length of contact, which reduces the contact ratio and results in rougher and noisier gear action. Therefore, undercutting should be avoided unless the application absolutely requires a compact gearset. In these cases, advanced kinematic and strength analyses and experiments are necessary to verify proper operation.

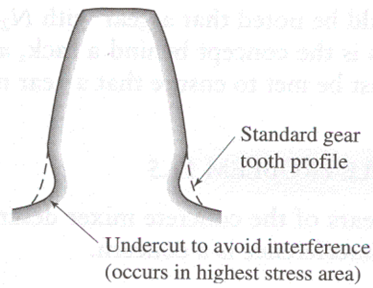


Figure 11.11 Undercut gear tooth.

11.6.5 Backlash

As stated in Section 11.3, backlash is the amount that the width of a tooth space exceeds the thickness of a gear tooth, measured on the pitch circle. In more practical terms, it is the amount that a gear can turn without its mating gear turning. Although backlash may seem undesirable, some backlash is necessary to provide for lubrication on the gear teeth. Gears that run continuously in one direction can actually have considerable backlash. Gears that frequently start/stop or reverse direction should have closely controlled backlash.

A nominal value of backlash is designed into a gear tooth profile. The amount of backlash determines the thickness of a gear tooth because backlash is a measure of the tooth thickness to the tooth space. Recommended values of backlash are specified by the American Gear Manufacturer's Association. Although these values are somewhat conservative, general power transmitting gears have recommended backlash values of:

$$\frac{0.05}{P_d} < B_{\text{recommended}} < \frac{0.1}{P_d}$$

For commercially available stock gears, backlash values are considerably higher to allow for greater flexibility in applications. The backlash values of these gears are typically:

$$\frac{0.3}{P_d} < B_{\text{stock gears}} < \frac{0.5}{P_d}$$

Therefore, great care must be taken when specifying stock gears for applications with reversing directions or frequent start/stop sequences.

Backlash values are strongly influenced by any variation in the center distance of the gears. Of course, in any production environment, the center distance of two gears varies. However, a deviation is the nominal center distance can be purposely specified by the designer to adjust the backlash to a desired range. The backlash variation that will be encountered with a variation in the center distance can be approximated by the following relationship:

$$\Delta B \approx 2\,(\Delta C)\tan\phi \tag{11.15}$$

Equation 11.14 can be used with Equation 11.7 or 11.8 to specify a center distance that produces backlash values to be maintained in the range given above. Reducing the center distance reduces the backlash, and vice-versa.

EXAMPLE PROBLEM 11.6

The gears for the concrete mixer described in Example Problem 11.3 are catalog items, with a designed backlash of $\frac{0.4}{P_d}$. Specify a center distance that reduces the backlash to an AGMA recommended value of $\frac{0.1}{P_d}$.

Solution:

For 5-pitch gears, the designed backlash is

$$B_{\text{designed}} = \frac{0.4}{P_d} = \frac{0.4}{5} = 0.08 \text{ in}$$

The AGMA recommendation is:

$$B_{\text{recommended}} = \frac{0.1}{P_d} = \frac{0.1}{5} = 0.02 \text{ in}$$

Rearranging Equation 11.15 gives:

$$\Delta C \approx \frac{\Delta B}{(2\tan\phi)}$$

$$= \frac{(0.02 - 0.08)}{2\tan(20°)} = 0.0824 \text{ in}$$

From Example Problem 11.3, the nominal center distance was determined as 4.5 in. Therefore, to adjust the backlash value, the center distance should be reduced to:

$$C_{\text{adjusted}} = 4.5 - 0.0824 = 4.4176 \text{ in}$$

11.6.6 Operating Pressure Angle

As mentioned in preceding sections, the pressure angle defines the line of action of the force onto the gear teeth. The designated pressure angle is cut or formed into the gear tooth and affects the actual shape of the tooth (Figure 11.7).

It should be mentioned that as the center distance of the mating gears deviates from the nominal value, the actual pressure angle during operation differs from the designated value. In other words, two 20° gears may actually have a greater pressure angle during operation by increasing the center distance from the nominal value. The relationship that can be used to determine the amount of variance is derived [1] as:

(11.16)
$$\cos \phi_{operating} = \left\{\frac{C_{nominal}}{C_{operating}}\right\}\cos \phi_{nominal}$$

Applications that require precise calculation of the actual force being transmitted should use this operating pressure angle. This reflects the actual performance of the gear forces.

EXAMPLE PROBLEM 11.7

For the gears of the concrete mixer described in Example Problem 11.3, determine the operating pressure angle when the center distance is measured at 4.4176 in as in Example Problem 11.6.

Solution:

From the numbers in Example Problem 11.6 and Equation 11.16, the following can be determined:

$$\cos \phi_{operating} = \left\{\frac{C_{nominal}}{C_{operating}}\right\}\cos \phi_{nominal}$$

$$= \left\{\frac{4.5}{4.4176}\right\}\cos 20° = 0.9572$$

and:

$$\phi_{operating} = 16.82°$$

11.7 SPUR GEAR KINEMATICS

A basic function of gears is to provide a constant velocity ratio between their respective shafts. A pair of gears that have a constant velocity ratio means that the driven gear maintains a uniform speed as long as the driver gear rotates at a constant speed. This condition led to the development of the involute tooth profile.

A pair of mating spur gears are shown in Figure 11.12.

Formally, the *velocity ratio, VR,* is defined as the angular speed of the driver gear (gear 1) divided by the angular speed of the driven gear (gear 2):

(11.17)
$$VR = \frac{\omega_{driver}}{\omega_{driven}} = \frac{\omega_1}{\omega_2}$$

where:

$$\omega = \text{angular speed}$$

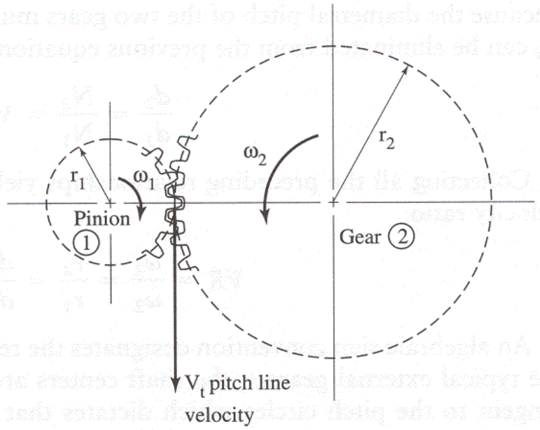


Figure 11.12 Kinematics of meshing gears.

Because a ratio is valid regardless of units, the velocity ratio can be defined in terms of revolutions per minute, radians per time, or any other convenient set of rotational velocity units. In practice, a velocity ratio of 3 would be indicated as 3:1, and pronounced "three to one." Likewise, a velocity ratio of 1/3 would be indicated as 1:3, and pronounced "one to three."

The *pitch line velocity*, v_t, is defined as the velocity of the pitch point of the two mating gears. This velocity is also illustrated in Figure 11.12. It should be apparent that the pitch line velocity of both gears is identical because one gear tooth pushes the mating tooth. Therefore, the pitch line velocity is a linear one and can be related to the rotational velocities of the gears and their pitch radii using Equation 5.5:

(11.18)
$$v_t = r_1\omega_1 = r_2\omega_2$$

Note that, as in Chapter 5, the angular velocity in this equation must be specified in radians per unit time.

Rearranging this relationship gives the following equation:

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = VR$$

introducing the pitch diameters:

$$\frac{d_2}{d_1} = \frac{(2r_2)}{(2r_1)} = \frac{r_2}{r_1} = VR$$

and introducing the diametral pitch and number of teeth:

$$\frac{d_2}{d_1} = \frac{\dfrac{N_2}{P_d}}{\dfrac{N_1}{P_d}} = VR$$

Because the diametral pitch of the two gears must be identical for the teeth to mate, P_d can be eliminated from the previous equation, yielding:

$$\frac{d_2}{d_1} = \frac{N_2}{N_1} = VR$$

Collecting all the preceding relationships yields a comprehensive definition of a velocity ratio:

(11.19) $$VR = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1} = \frac{N_2}{N_1}$$

An algebraic sign convention designates the relative direction of gear rotations. In the typical external gearset, the shaft centers are on opposite sides of the common tangent to the pitch circles which dictates that the gears rotate in opposite directions. To signify this fact, the velocity ratio is given a negative value.

For internal gears, as shown in Figure 11.3C, the shaft centers are on the same side of the common tangent to the pitch circles. This dictates that the gears rotate in the same direction. Thus, the velocity ratio is given a positive value.

As discussed in the introduction, many gears are used in applications where the speed from a power source must be reduced. Therefore, it is typical to have velocity ratios greater than one. As can be seen from Equation 11.17, this indicates that the drive gear rotates faster than the driven gear, which is the case in speed reductions.

EXAMPLE PROBLEM 11.8

A set of gears is used to reduce the speed from an electric motor to a shaft driving a grocery checkout conveyor (Figure 11.13). The gear on the motor shaft is a 10-pitch pinion, has 15 teeth, and drives at 1800 rpm, clockwise. Determine the speed of the mating gear, which has 45 teeth. Also calculate the pitch line velocity.

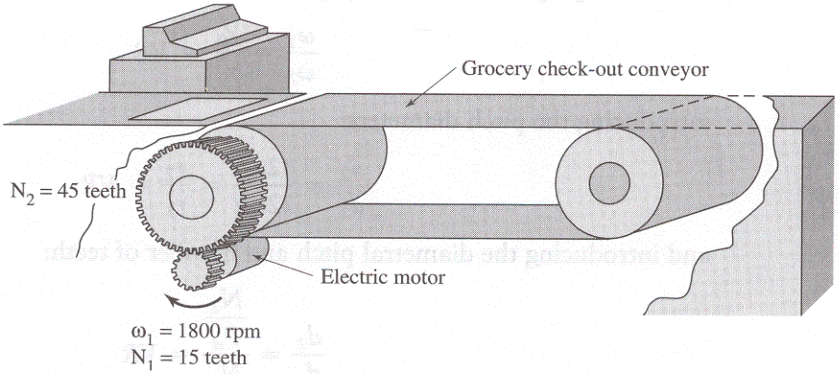


Figure 11.13 Checkout conveyor for Example Problem 11.8.

Solution:

The velocity ratio can be computed from Equation 11.18:

$$VR = \frac{N_2}{N_1} = \frac{45}{15} = -3$$

In practice, this value would be commonly expressed as a 3:1 gear ratio. Note that the negative value indicates that the gears rotate in opposite directions. This is consistent with external gears.

The angular velocity of the driven gear can be computed by rearranging Equation 11.17:

$$\omega_2 = \frac{\omega_1}{VR} = \frac{1800 \text{ rpm}}{(-3)}$$

$$= 600 \text{ rpm, counterclockwise}$$

The pitch diameters are computed from Equation 11.2:

$$d_1 = \frac{N_1}{P_d} = \frac{15}{10} = 1.5 \text{ in}$$

$$d_2 = \frac{N_2}{P_d} = \frac{45}{10} = 4.5 \text{ in}$$

The pitch line velocity can be computed from Equation 11.18:

$$v_t = r_1\omega_1$$

$$r_1 = \frac{1.5}{2} = 0.75 \text{ in}$$

$$\omega_1 = (1800 \text{ rpm})\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right]$$

$$= 11309.7\frac{\text{rad}}{\text{min}}$$

$$v_t = (0.75 \text{ in})\left(11309.7\frac{\text{rad}}{\text{min}}\right) = 8482.3\frac{\text{in}}{\text{min}}$$

converting units:

$$v_t = 8482.3\frac{\text{in}}{\text{min}}\left[\frac{1 \text{ ft}}{12 \text{ in}}\right] = 706.9\frac{\text{ft}}{\text{min}}$$

11.8 SPUR GEAR SELECTION

In a design situation, gears must be selected to accomplish a desired task. Often this task is to achieve a desired velocity ratio. Because the majority of gears in operation are AGMA standard, the designer only needs to determine the key parameters.

These parameters are the diametral pitch, pressure angle, and number of teeth on each gear. Most other gear features can be determined using the AGMA standard relationships presented in previous sections.

In the typical design situation, the first selection parameter is an appropriate diametral pitch. Because the diametral pitch is the relative size of a gear tooth, it stands to reason that the transmitted forces and the gear material properties influence this decision. Precise selection criteria involve calculation of gear tooth stresses and contact pressures. The calculation procedures are outlined in the AGMA specifications. This level of detail is beyond the scope of this text.

Conservative estimates of appropriate diametral pitches can be readily obtained from most commercial gear suppliers. The suppliers use the AGMA standards to determine the power-carrying capabilities of their stock gears. From this data, an estimate of suitable diametral pitch can be made with knowledge of the nominal power that is transmitted by the gear pair, the rotational speed of the pinion, and the gear material. As an example, such data is presented in Table 11.5. This table gives suitable diametral pitches of 20° mild steel gears with standard face width, based on pinion speed and the power transmitted. Similar tables exist for alternate pressure angles and materials. The use of these tables can be illustrated from an example.

TABLE 11.5 Suitable Diametral Pitches for 20°, Mild-Steel Gears with Standard Face Width

Power	Pinion rpm								
HP	50	100	300	600	900	1200	1800	2400	3600
0.05	20	20	24	32	32	32	32	32	32
0.10	16	20	20	24	24	24	32	32	32
0.25	12	16	20	20	24	24	24	24	24
0.33	10	12	16	20	20	24	24	24	24
0.50	10	12	16	20	20	20	20	24	24
0.75	8	10	12	16	16	20	20	20	20
1.0	6	10	12	16	16	16	20	20	20
1.5	6	8	12	12	16	16	16	16	20
2.0	6	6	10	12	12	12	16	16	16
3.0	5	6	8	10	12	12	12	12	16
5.0	4	5	6	8	10	10	12	12	12
7.5	4	5	6	8	8	8	10	10	10
10	3	4	6	6	6	8	8	8	10
15	2	4	5	6	6	6	6	6	8
20	2	3	4	5	6	6	6	6	—
25	—	3	4	5	5	5	6	5	—
30	—	2	4	4	5	5	5	—	—
40	—	2	3	4	4	—	—	—	—
50	—	—	3	4	4	—	—	—	—

EXAMPLE PROBLEM 11.9

A pair of mild steel gears is selected for the concrete mixer described in Example Problem 11.3. The mixer is driven by a 10-hp engine at a speed of 1200 rpm. Determine an appropriate diametral pitch.

Solution:

Mild steel gears are specified, which are capable of handling 10 hp at a pinion speed of 1800 rpm. From interference criteria, Table 11.3 shows that an 18-tooth pinion with a pressure angle of 20° can mate with any other gear. Using an 18 toth pinion, Table 11.4 suggests that a diametral pitch of 8 is used to transfer the power. Therefore, an 18-tooth, mild steel pinion with a diametral pitch of 8 should be suitable. For a more reliable selection, thorough strength analysis should be performed.

The second parameter that should be selected is a pressure angle. As mentioned,the standard values of pressure angles is 14-1/2°, 20°, and 25°. Recall that 14-1/2° is recommended for replacement on existing machinery. Gears with pressure angles of 20° are well suited for general applications. Gears with pressure angles of 25° can be smaller without a concern for interference, but have less efficient force transmission. Therefore, they are best suited for high speed and lower power applications.

Finally, the number of gear teeth should be determined. This decision is typically influenced by the desired velocity ratio. In general, smaller gears are preferred because they minimize size, weight, and cost. Of course, the minimum size is determined by interference criteria. The number of teeth on a gear also must be an integer. Although this statement seems obvious, it must be a constant consideration, as obtaining an integer solution can be difficult. In addition, gear manufacturers do not stock gears with tooth increments of one, and the catalog should be consulted when deciding on the number of teeth.

EXAMPLE PROBLEM 11.10

A gear reducer is used on a concept for a small trolling motor for fishing boats. The gears must transmit 5 hp from an electric motor at 900 rpm to the propeller at 320 rpm. Select a set of gears to accomplish this task.

Solution

Because this application involves general gearing, a pressure angle of 20° is used. Referring to Table 11.4, an estimate of a suitable diametral pitch is as follows:

$$P_d = 10$$

The required velocity ratio is:

$$VR = \frac{\omega_{\text{driver}}}{\omega_{\text{driven}}} = \frac{900 \text{ rpm}}{320 \text{ rpm}} = 2.8125$$

Rearranging Equation 11.18 yields:

$$N_{\text{driven}} = N_{\text{driver}} \left(\frac{\omega_{\text{driver}}}{\omega_{\text{driven}}} \right)$$

Because a smaller assembly is generally preferred, values of pinion (driver) teeth are substituted beginning with the smallest possible pinion. Note that an iterative procedure must be used because the number of teeth must be an integer (Table 11.6):

$$N_{\text{driven}} = N_{\text{driver}} \left(\frac{900}{320} \right)$$

TABLE 11.6 Iterations for Example Problem 11.10

No. of Pinion Teeth	No. of Gear Teeth
13	36.56
14	39.375
15	42.14
16	45

The smallest integer combination is 16 and 45 teeth. Also, from the preceding discussion, a suitable diametral pitch is 9. Finally, the corresponding pitch diameters and center distance are as follows:

$$d_1 = \frac{N_1}{P_d} = \frac{16}{10} = 1.6 \text{ in}$$

$$d_2 = \frac{N_2}{P_d} = \frac{45}{10} = 4.5 \text{ in}$$

$$c_{\text{external gears}} = \frac{(d_1 + d_2)}{2} = \frac{(1.6 + 4.5)}{2} = 3.05 \text{ in}$$

Often gears must be selected to alter the velocity ratio between shafts of an existing machine. Another problem occurs when the shafts must be spaced at a specific distance due to other constraints. Both of these situations place a limit on the center distance of the gears. In these situations, the number of teeth selected for each gear may not be the smallest possible, but are needed to fill the distance between the shafts. Also, a larger tooth than necessary can be used to help fill the distance between shafts. Finally, some deviation from the target ratio may be needed to specify standard gears. In general, the relationships explained throughout this chapter can be used to specify any gearset. The following examples illustrate some possible scenarios.

EXAMPLE PROBLEM 11.11

A pair of gears is powered by an electric motor and used to drive the spindle of a lathe at 200 rpm. This drive system is illustrated in Figure 11.14. The 1-hp motor will be replaced by a more efficient, but higher speed motor, rated at 600 rpm. To accomplish this alteration, a new set of gears must be selected that will maintain the spindle speed at 200 rpm. However, the gears are mounted in an elaborate housing, which cannot be modified. Therefore, the center distance between the gears must remain at 7.5 in. Specify a set of gears that can be used.

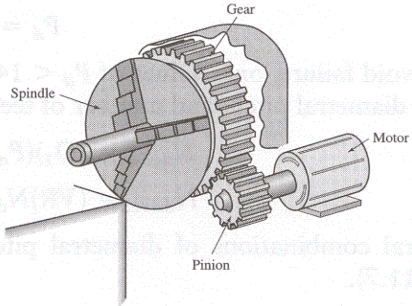


Figure 11.14 Lathe drive for Example Problem 11.11.

Solution

The main parameters in this problem are the velocity ratio and the center distance. The required velocity ratio is:

$$VR = \frac{\omega_{driver}}{\omega_{driven}} = \frac{600 \text{ rpm}}{200 \text{ rpm}} = 3.0$$

Therefore:

$$VR = \frac{d_2}{d_1} = 3.0$$

which can be rewritten as:

$$d_2 = 3\, d_1$$

In addition, the center distance is:

$$c_{\text{external gears}} = \frac{(d_1 + d_2)}{2} = 7.5$$

Using these relationships, appropriate pitch diameters can be algebraically determined by:

$$\frac{(d_1 + 3d_1)}{2} = 7.5$$

$$\frac{4d_1}{2} = 7.5$$

solving:

$$d_1 = 3.75 \text{ in}$$

and:

$$d_2 = 3(3.75) = 11.25 \text{ in}$$

The problem now reduces to finding a suitable diametral pitch and number of teeth that result in the required pitch diameters. Because this application involves general gearing, a pressure angle of 20° is used. Referring to Table 11.4, an estimate of a suitable diametral pitch is determined by:

$$P_d = 14$$

To avoid failure, only values of $P_d < 14$ are considered. By relating the pitch diameter, diametral pitch, and number of teeth, the following can be calculated:

$$N_{\text{drive}} = (D_1)(P_d) = 3.75\ P_d$$

$$N_{\text{driven}} = (VR)N_{\text{driver}} = 3N_{\text{driver}}$$

Several combinations of diametral pitch and number of teeth are considered (Table 11.7).

TABLE 11.7 Iterations for Example Problem 11.11

Diametral Pitch	No. of Pinion Teeth	No. of Gear Teeth
14	52.5	157.5
12	45	135
10	37.5	112.5
8	30	90
6	22.5	67.5
5	18.75	56.25
4	15	45

Recall that only an integer number of teeth is feasible. It appears that three of the above combinations are possible:

$$P_d = 12\ N_1 = 45\ N_2 = 135$$

$$P_d = 8\ N_1 = 30\ N_2 = 90$$

$$P_d = 4\ N_1 = 15\ N_2 = 45$$

The best alternative would depend on availability of standard gears, cost, and weight of the gearset. Notice that the output speed will be exactly 320 rpm. In many situations, the driven speed can not be exactly obtained. The next problem illustrates such a case.

EXAMPLE PROBLEM 11.12

A gear-driven exhaust fan and housing is shown in Figure 11.15. To improve the air flow, the speed of the fan needs to be increased to 620 rpm and must be as close to this speed as possible. The existing 3-hp motor will be used, which operates at 1750 rpm. The housing should not be altered, which has a bearing system with a center distance of 5.5 in. Select a set a gears for this application.

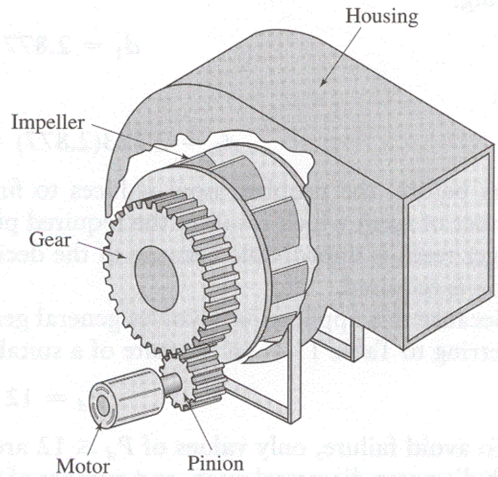


Figure 11.15 Exhaust fan for Example Problem 11.12.

Solution:

As in Example Problem 11.11, the main parameters in this problem are the velocity ratio and the center distance. The required velocity ratio is:

$$VR = \frac{\omega_{\text{driver}}}{\omega_{\text{driven}}} = \frac{1750 \text{ rpm}}{620 \text{ rpm}} = 2.823$$

therefore:

$$VR = \frac{d_2}{d_1} = 2.832$$

which can be rewritten as:

$$d_2 = 2.823 d_1$$

This problem is complicated by a nonfractional velocity ratio. It will be be impossible to obtain a driven speed at exactly 620 rpm. As before, the center distance is:

$$c_{\text{external gears}} = \frac{(d_1 + d_2)}{2} = 5.5 \text{ in}$$

Using these relationships, appropriate pitch diameters can be algebraically determined by:

$$\frac{(d_1 + 2.823d_1)}{2} = 5.5$$

$$\frac{3.823d_1}{2} = 5.5$$

solving:

$$d_1 = 2.877 \text{ in}$$

and:

$$d_2 = 2.823(2.877) = 8.123 \text{ in}$$

As before, the problem now reduces to finding a suitable diametral pitch and number of teeth which result in the required pitch diameters. As mentioned, finding integer teeth is improbable because of the decimal velocity ratio. An interactive solution is required.

Because this application involves general gearing, a pressure angle of 20° is used. Referring to Table 11.4, an estimate of a suitable diametral pitch is determined by:

$$P_d = 12$$

To avoid failure, only values of $P_d \leq 12$ are considered. By relating the velocity, pitch diameter, diametral pitch, and number of teeth, the following can be calculated:

$$N_{\text{driver}} = (D_1)(P_{\text{d}}) = 2.877\ P_d$$

$$N_{\text{driven}} = (VR)\ N_{\text{driver}} = 2.823\ N_{\text{driver}}$$

$$\omega_{\text{driven}} = \left(\frac{N_{\text{driver}}}{N_{\text{driven}}}\right)\omega_{\text{driver}} = \left(\frac{N_{\text{driver}}}{N_{\text{driven}}}\right)1750 \text{ rpm}$$

Several combinations of diametral pitch and number of teeth are considered (Table 11.8).

TABLE 11.8 Iterations for Example Problem 11.12

Diametral Pitch	No. of Pinion Teeth	No. of Gear Teeth	Output Velocity (rpm)
12	34.44 ≈ 35	98.81 ≈ 99	618.7
10	28.77 ≈ 29	82.15 ≈ 82	618.9
8	23.01 ≈ 23	64.97 ≈ 65	619.2
6	17.26 ≈ 17	47.99 ≈ 48	619.8
5	14.38 ≈ 14	39.52 ≈ 40	612.5
4	11.51 ≈ 11	31.05 ≈ 31	620.9

Because only integer numbers of teeth are feasible, all solutions were rounded. Rounding produces deviations from the target output velocity; thus, the resulting output rotational velocities were tabulated. Note that the last option induces interference when observing the limits in Table 11.4. Therefore, only the top 5 combinations are feasible candidates.

Again, the best alternative depends on availability of standard gears, cost, and weight of the gearset.

11.9 RACK AND PINION KINEMATICS

A gear rack was briefly discussed in Section 11.2 and is illustrated in Figure 11.3B. It is used to convert rotational motion of a pinion to translating motion of the rack. The most noteworthy application is the rack and pinion steering in automobiles. In this application, the rotational motion from the steering wheel pushes the rear of the front wheels, steering the car in a new direction; the motion, thus, transfers from rotational to linear. A rack and pinion can also be operated such that the linear motion of the rack rotates the pinion.

As briefly mentioned in Section 11.5.3, a rack is a special case of a spur gear. As the diameter of a gear becomes very large, the local profile of the teeth resemble a rack. In fact, mathematically, a rack can be treated as a spur gear with an infinite pitch diameter. Therefore, all geometric properties that were introduced for spur gears also apply to a rack. The only difference is that instead of referring to a pitch diameter, a rack has a pitch line.

From a kinematic standpoint, the rotational motion of the pinion and the linear motion of the rack can be related through concepts presented in Chapter 5, Equation 5.5 . The rack displacement equation can be given as:

(11.20)
$$\Delta s_{rack} = r(\Delta\theta) = \frac{(d_{pinion})(\Delta\theta_{pinion})}{2}$$

where—$\Delta\theta_{pinion}$—must be specified in radians. The linear velocity of the rack is given as:

(11.21)
$$v_{rack} = \omega r = \frac{(d_{pinion})(\omega_{pinion})}{2}$$

EXAMPLE PROBLEM 11.13

A rack and pinion is used on a drill press as shown in Figure 11.16. The 16-pitch pinion has 16 teeth. Determine the distance that the handle (and pinion) must be rotated in order to advance the drill 0.75 in.

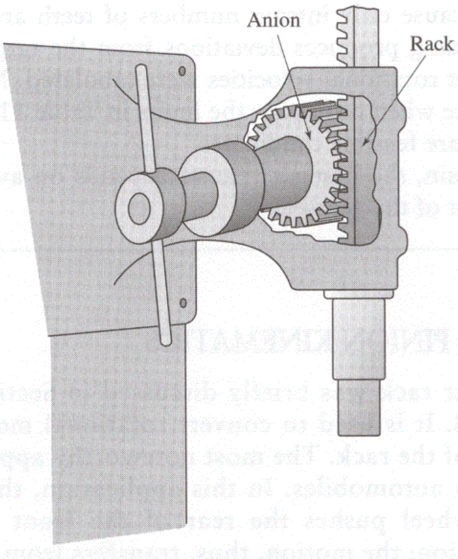


Figure 11.16 Rack and pinion drill press.

Solution:

From Equation 11.19, the rotation of the pinion is desired:

$$\Delta\theta_{\text{pinion}} = \frac{2\Delta s_{\text{rack}}}{d_{\text{pinion}}}$$

and:

$$d_{\text{pinion}} = \frac{N_{\text{pinion}}}{P_d} = \frac{16}{16} = 1.0 \text{ in}$$

$$\Delta\theta_{\text{pinion}} = 2\frac{(0.75 \text{ in})}{(1.0 \text{ in})} = 1.5 \text{ radians}$$

converting to degrees:

$$\Delta\theta_{\text{pinion}} = 1.5 \text{ radians}\left(\frac{180°}{\pi \text{ rad}}\right) = 85.94°$$

EXAMPLE PROBLEM 11.14

For the drill press described in Example Problem 11.13, determine the speed that the pinion must be rotated in order to advance the drill at a rate of 12 in/min.

Solution:

From Equation 11.20, the rotational speed of the pinion is determined by:

$$\omega_{pinion} = \frac{2v_{rack}}{d_{pinion}} = \frac{2(12 \text{ in/min})}{(10 \text{ in})} = 24\frac{\text{rad}}{\text{min}}$$

converting to revolutions per minute:

$$\omega_{pinion} = 24\frac{\text{rad}}{\text{min}}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 3.82 \text{ rpm}$$

11.10 HELICAL GEAR KINEMATICS

Helical gears were introduced in Section 11.2 and illustrated in Figure 11.3D. The development of helical gears actually resulted from machinists who discovered that stepped gears ran smoother and quieter than spur gears. A stepped gear consisted of a number of thin spur gears placed side by side, with each gear rotated a small angle relative to the adjacent gear. The resulting stacked gear did not exhibit the same large impact that two teeth usually have when they come into contact (e.g., ordinary spur gears).

Helical gears are the extreme case of stepped gears, where the teeth are not stepped, but inclined to the axis of the gear. When used on parallel shafts, helical gears provide overlapping tooth contact. That is, when the front edge of a tooth comes into contact and begins to take the transmitted load, the back edge of the previous tooth is also in contact. This results in the smoother and quieter operation, as a tooth loads gradually. For these reasons, helical gears are often preferred, even though they are more difficult to manufacture and, consequently, more expensive.

Helical gears are designated as either right-hand or left-hand, depending on the slope of the inclined teeth. A helical gear with teeth that slope down toward the left is designated as a left-hand helix. Conversely, a helical gear with teeth that slope down toward the right is designated as a right-hand helix. The upper helical gear illustrated in Figure 11.3D is a left-hand gear.

Helical gears can also be used on nonparallel shafts, without altering the inherent geometry. Such a configuration is termed *crossed helical gears*. However, with crossed configurations, the forces required to drive the gearset increase dramatically with the shaft angle. Therefore, such configurations are recommended for lower power transmitting applications.

The geometric and kinematic relationships for helical gears are very similar to spur gears. The major difference is the definition of a *helix angle*, φ, which is the angle of inclination of the teeth. This angle is illustrated with the right-hand helical gear shown in Figure 11.17.

A cross section view through a helical gear, perpendicular to the gear axis, appears identical to a spur gear. This view is generated by section A-A in Figure 11.16, and termed the *transverse section*. The tooth geometric properties defined for spur

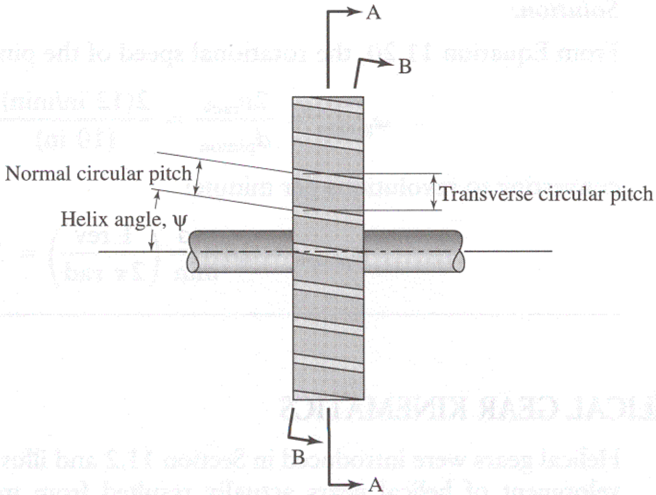


Figure 11.17 Helical gear geometry.

gears can be used for helical gears. To avoid confusion, these properties are designated as transverse properties. The transverse circular pitch, transverse pressure angle, and transverse diametral pitch are identical to the corresponding spur gear definitions. The transverse circular pitch is shown in Figure 11.17.

Some additional geometric properties are defined by viewing a cross section, normal to the gear teeth. This view would be generated by section B-B in Figure 11.17, and termed the *normal section.*

The *normal circular pitch, p^n,* is defined as the distance between corresponding points on a gear, measured on the pitch circle and normal to the gear tooth. The normal circular pitch is also shown in Figure 11.16. The normal circular pitch can be related to the transverse circular pitch through trigonometry:

(11.22) $$p^n = p \cos \varphi$$

The *normal diametral pitch, $P_d^{\,n}$,* is defined using the normal circular pitch in a similar fashion as Equation 11.5:

(11.23) $$P_d^{\,n} = \frac{\pi}{p^n}$$

A *normal module, m^n,* is similarly defined:

(11.24) $$m^n = \pi p^n$$

also from trigonometry:

(11.25) $$P_d = P_d^{\,n} \cos \varphi$$

(11.26) $$m = \frac{m^n}{\cos \varphi}$$

A *normal pressure angle*, ϕ^n, is also defined from the tooth form in this normal view. The normal pressure angle can also be related to the transverse pressure angle by the following:

$$\tan \phi^n = \tan \phi \cos \varphi \tag{11.27}$$

Helical gears are rarely used interchangeably and, therefore, no standard tooth systems exist, as those described for spur gears. The preferred dimensions are usually dependent on the manner in which the helical gear is formed. When the gear is cut through a hobbing operation, the normal diametral pitch should conform to the standards listed in Table 11.1. Conversely, when a gear is cut on a shaper, the transverse diametral pitch should conform to values listed in Table 11.1.

The helix angle for most gears varies between 15° and 45°. Because the teeth are at an angle to the shaft, a thrust load is produced with mating helical gears. The thrust force varies directly with the tangent of the helix angle and, therefore, larger helix angles require sufficient axial gear and shaft support.

For parallel shaft applications, the velocity ratio presented in Equation 11.19 is also applicable to helical gears. Two additional requirements, beyond those for spur gears, for proper meshing of helical gears include:

1. The gears must have equal helix angles.
2. The helix on the two mating gears must be opposite hand. That is, one gear must have a left-hand helix and the other a right-hand one.

The presence of the helix angle also aids in the avoidance of interference. An equation similar to Equation 11.13 has been derived for helical gears. Thus, the

TABLE 11.9 Minimum Helical Gear Teeth to Avoid Interference

	Normal	Pressure	Angle, ϕ^n
Helix Angle	14-1/2°	20°	25°
0 (Spur gear)	32	17	12
5°	32	17	12
10°	31	17	12
15°	29	16	11
20°	27	15	10
22.5°	25	14	10
25°	24	13	9
30°	21	12	8
35°	18	10	7
40°	15	8	6
45°	12	7	5

minimum number of pinion teeth that can be used, mating with any size gear, without interference concerns is written as follows:

$$N_1 > \frac{2k \cos \varphi}{\sin^2 \phi} \tag{11.28}$$

Values generated from this equation are condensed into Table 11.9.

EXAMPLE PROBLEM 11.15

In order to reduce the noise in a gear drive, two 12-pitch gears with 20 and 65 teeth are to be replaced with helical gears. The new set of gears must have the same velocity ratio. Because the same housing will be used, the center distance must also remain the same. Assume that the helical gears will be formed with a hob.

Solution

The original velocity ratio and center distance must be computed as follows:

$$VR = \frac{N_{\text{driven}}}{N_{\text{driver}}} = \frac{65}{20} = 3.25$$

$$c_{\text{external gears}} = \frac{(N_1 + N_2)}{2P_d} = \frac{(20 + 65)}{2(12)} = 3.4$$

Because the gears will be cut with a hob, the normal diametral pitch should conform to the standards listed in Table 11.1. The original gears had a diametral pitch of 12; thus, it is assumed that the teeth have sufficient strength. The helical gears then are selected with a normal diametral pitch of 12.

By substituting Equation 11.22 into Equation 11.7, the following calculations can be made:

$$c_{\text{external gears}} = \frac{(N_1 + N_2)}{2P_d^{\,n} \cos \varphi} = \frac{(N_1 + N_2)}{2(12 \cos \varphi)} = 3.4$$

also:

$$\frac{N_2}{N_1} = 3.25$$

or:

$$N_2 = 3.25N_1$$

therefore:

$$\frac{(N_1 + 3.25N_1)}{24 \cos \varphi} = 3.4$$

which reduces to:

$$\cos \varphi = \frac{N_1}{19.2}$$

This equation reveals that N_1 must be less than 19.2 for this application. By trial, the following combinations are considered in Table 11.10.

TABLE 11.10 Iterations for Example Problem 11.15.

Pinion Teeth N_1	Gear Teeth N_2	Normal Diametral Pitch $P_d{}^n$	Helix Angle φ	Diametral Pitch P_d
19	61.75	12	8.27	11.88
18	58.50	12	20.36	11.25
17	55.25	12	27.70	9.62
16	52	12	33.55	9.00

The first solution to generate integer numbers for both teeth will be used. A 16 tooth pinion and a 52 tooth gear having a helical angle of 33.55° is selected. Notice that from the interference criteria in Table 11.9, a normal pressure angle of either 20° or 25° can be used.

11.11 BEVEL GEAR KINEMATICS

Bevel gears were introduced in Section 11.2 and illustrated in Figure 11.3F. Bevel gears are used for transmitting motion between two shafts that intersect. One of the most important properties of a bevel gear arrangement is the *shaft angle, Σ*. The shaft angle is defined as the angle between the centerlines of the supporting shafts. Common bevel gear applications consist of shafts that intersect at right angles or have a shaft angle of 90°.

As discussed in Section 11.1 and illustrated in Figure 11.2, spur gears exhibit the same kinematics as two friction rollers. In a similar fashion, bevel gears can be replaced by two friction cones. With this conical geometry, the depth of the gear teeth tapers from the outside toward the middle. Most geometric tooth features used with spur gears, such as the pitch diameter and addendum, apply to bevel gears. This can be seen from the axial section of two mating bevel gears shown in Figure 11.18. Because the tooth tapers, tooth features are measured at the outside edge of the tooth.

The angular velocity ratio, as presented for spur gears in Equation 11.19, is also applicable to bevel gears. The diametral pitch and pressure angle also have the same definition as spur gears, and must be identical for bevel gears to mate. The diametral pitch for bevel gears typically follows the standard values as presented in Table 11.1. Most bevel gears are made with a pressure angle of 20°; however, the tooth form is

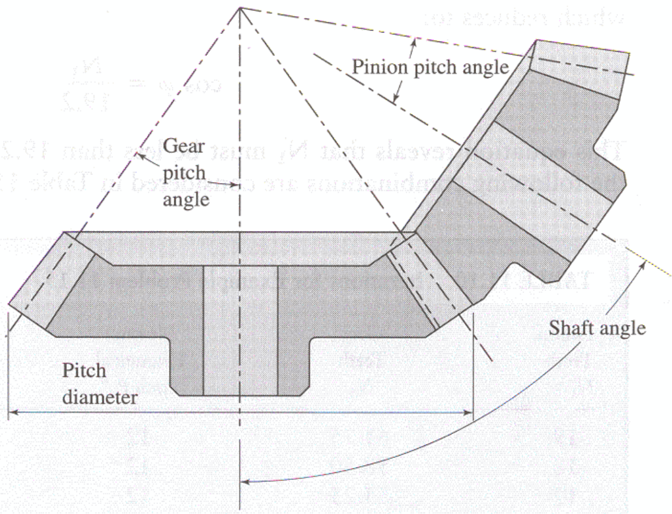


Figure 11.18 Mating bevel gears.

usually not an involute due to the difficulty in manufacturing. Alternate profiles have been developed, trademarked by vendors, and serve as competitive features.

In addition to diametral pitch and pressure angle, bevel gears are classified by their *pitch angle*, γ. The pitch angle is the generating angle of the cone upon which the gear is constructed. The pitch angles are labeled for the two mating gears shown in Figure 11.17. The pitch angle of each gear is a function of the velocity ratio and can be given as:

$$\tan \gamma_{\text{pinion}} = \frac{\sin\Sigma}{\left\{\cos \Sigma + \left(\dfrac{N_{\text{gear}}}{N_{\text{pinion}}}\right)\right\}} \tag{11.29}$$

$$\tan \gamma_{\text{gear}} = \frac{\sin \Sigma}{\left\{\cos \Sigma + \dfrac{N_{\text{pinion}}}{N_{\text{gear}}}\right\}} \tag{11.30}$$

Because the pitch cone is a function of the velocity ratio, a single bevel gear cannot be replaced to alter the ratio, as was the case for spur gears. Thus, bevel gears are sold as a set.

In Figure 11.17, it is apparent that the sum of the pitch angles for the two mating gears must equal the shaft angle. Thus:

$$\Sigma = \gamma_{\text{pinion}} + \gamma_{\text{gear}} \tag{11.31}$$

A miter gear, as shown in Figure 11.3G, is a special case of a bevel gear, with a shaft angle 90°, and a velocity ratio of one. Using Equation 11.21 and 11.22, the pitch angle for both miter gears equals 45°.

The mounting of bevel gears is critical. For ideal mating, the apex of the cones for both gears must be at the same location. Any deviation could cause excessive backlash or interference. Due to the inherent geometry of bevel gears, at least one gear must be attached to the end of a cantilevered shaft. This configuration lends itself to excessive deflections, which can also result in problems with backlash.

Axial thrust loads developed by mating bevel gears always tend to separate the gears. This can contribute to shaft deflection and must also be considered. Of course the shaft support bearings must also be configured to withstand this thrust force.

EXAMPLE PROBLEM 11.16

A pair of bevel gears have 18 and 27 teeth, and are used on shafts that intersect each other at an angle of 70°. Determine the velocity ratio and the pitch angles of both gears.

Solution

The velocity ratio can be computed from Equation 11.16:

$$VR = \frac{N_{gear}}{N_{pinion}} = \frac{27 \text{ teeth}}{18 \text{ teeth}} = 1.5$$

The pitch angles can be computed from Equations 11.21 and 11.22:

$$\tan \gamma_{pinion} = \frac{\sin \Sigma}{\left\{\cos \Sigma + \left(\frac{N_{gear}}{N_{pinion}}\right)\right\}}$$

$$= \frac{\sin (70°)}{\{(\cos 70°) + (1.5)\}} = 0.510$$

$$\gamma_{pinion} = 27.02°$$

$$\tan \gamma_{gear} = \frac{\sin \Sigma}{\left\{\cos \Sigma + \left(\frac{N_{pinion}}{N_{gear}}\right)\right\}}$$

$$= \frac{\sin (70°)}{\left\{(\cos 70°) + \left(\frac{18}{27}\right)\right\}} = 0.932$$

$$\gamma_{gear} = 42.98°$$

11.12 WORM GEAR KINEMATICS

A worm and worm gear is described in Section 11.2 and illustrated in Figure 11.3H. A worm and worm gear is used to transfer motion between nonparallel and nonintersecting shafts. With a worm gearset, large velocity ratios can be obtained in a rather limited space. The small gear is termed the *worm,* and the larger is termed the *worm gear, worm wheel,* or simply the *gear.*

The worm resembles a screw and often the teeth on the worm are referred to as threads (Figure 11.3H). Worms are commonly available with single, double, and quadruple threads. Thus, the *number of worm teeth (threads),* N_w, is an important property. The concept of multiple threads superimposed on a single worm is illustrated in Figure 11.19.

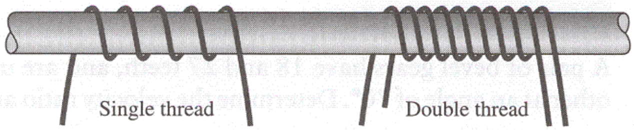


Figure 11.19 Multiple thread concept.

The tooth form of the worm gear is typically an involute. It is also common to cut the teeth concave across the face, so they better conform to the cylindrical worm. This technique is termed *enveloping worm gear teeth.* It is an attempt to provide a larger contact patch on which the forces are transferred. The worm may also be cut with a concave length, so it better conforms to the round worm gear. When both options are incorporated, the worm gearset is known as double-enveloping, thus providing a larger contact patch and greater power transmission. For such configurations, the worm and worm gear are not interchangeable and, thus, are sold as a set.

The worm gear is actually an extreme case of a helical gear with a large helix angle, which wraps the tooth around the gear. Therefore, the worm is described by all the geometric properties of a helical gear given in Section 11.6. The values of diametral pitch typically conform to the standards in Table 11.1. The pressure angles also conform to the 14-1/2°, 20°, and 25° standards used with helical gears. In practice, the pressure angle is also selected based on the lead angle of the worm, as will be discussed later.

The worm is described by the number of threads, as discussed above, the *worm pitch diameter,* d_w, the *pitch,* p_w, and the *lead angle,* λ. The worm pitch diameter is determined similar to that in spur gears, as the diameter of the circle that remains tangent to the pitch diameter of the worm gear. The worm pitch is also similar to the definition for spur gears, and is the distance between corresponding points on adjacent teeth (threads). These worm geometric properties are illustrated in Figure 11.20.

Also shown in Figure 11.20 is the lead angle, which is the angle of inclination of the teeth (threads). It can be computed from a trigonometric relationship to the other worm features:

$$\tan \lambda = \frac{N_w p_w}{\pi d_w} \qquad (11.32)$$

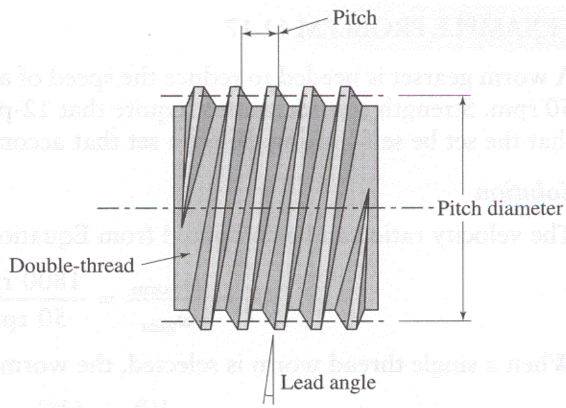


Figure 11.20 Worm geometry.

For a mating worm gearset, the pitch of the worm must be the same as the pitch of the worm gear. Thus from Equation 11.1:

$$p_w = p_{\text{gear}} = \frac{\pi}{P_d} \tag{11.33}$$

For shafts that are at 90°, which is the usual case, the lead angle of the worm must equal the helix angle of the worm gear.

The velocity ratio of a worm gearset is computed as the number of teeth on the worm gear divided by the number of threads of the worm:

$$VR = \frac{N_{\text{Gear}}}{N_{\text{w}}} \tag{11.34}$$

This is also identical to the spur gear application.

In most gearsets, the worm is the driver, thereby making the set a speed reducer. Most sets are irreversible, in that the worm cannot turn the gear because a substantial friction force develops between the teeth. Irreversible drives are also referred to as self-locking. Worms must have a lead angle greater than approximately 10°, to be able to drive the mating worm gear. This would result in a *reversible gearset,* but is highly uncommon.

Although irreversibility may sound like a pitfall, distinct advantages exist. For example, lifting equipment typically requires that the load be held in an upward position, even as the power source is removed, such as a motor being turned off. Because the worm cannot rotate the worm gear, the load is locked in an upright position. This braking action is used in several mechanical devices, such as hoists, jacks, and lifting platforms. For these cases, but the strength of the teeth, and the predictability of friction must be analyzed to ensure safety.

EXAMPLE PROBLEM 11.17

A worm gearset is needed to reduce the speed of an electric motor from 1800 rpm to 50 rpm. Strength considerations require that 12-pitch gears be used, and it is desired that the set be self-locking. Select a set that accomplishes this task.

Solution

The velocity ratio can be computed from Equation 11.17:

$$VR = \frac{\omega_{worm}}{\omega_{gear}} = \frac{1800 \text{ rpm}}{50 \text{ rpm}} = 36$$

When a single thread worm is selected, the worm gear must have:

$$N_{Gear} = \frac{VR}{N_w} = \frac{(36)}{(1)} = 36 \text{ teeth}$$

From Equation 11.33, and using a diametral pitch of 12, the pitch of the worm is determined by:

$$p_w = \frac{\pi}{P_d} = \frac{\pi}{12} = 0.2618 \text{ in}$$

Because self-locking is desired, a conservative lead angle of 5° is used. Equation 11.21 is used to determine the following:

$$\tan \lambda = \frac{N_w\, p_w}{\pi d_w}$$

$$\tan 5° = \frac{(1)(0.2618)}{\pi d_w}$$

solving:

$$d_w = 1.0499 \text{ in}$$

The pitch diameters of the worm gear are:

$$d_{gear} = \frac{N_{gear}}{P_d} = \frac{36 \text{ teeth}}{12} = 3.0 \text{ in}$$

Finally, the center distance is:

$$c = \frac{(d_{worm} + d_{gear})}{2} = \frac{(1.0499 + 3.0)}{2} = 2.0250 \text{ in}$$

11.13 GEAR TRAINS

A gear train is a series of mating gearsets. Gear trains are commonly used to achieve large speed reductions. Many mechanical power sources, such as engines, turbines, and electric motors, operate efficiently at high speeds (1800–10,000 rpm). Many uses for this power, such as garage door openers, automotive drive wheels, and ceiling fans, require low speeds (10–100 rpm) for operation. Therefore, a desire to achieve large-velocity reductions is common, and the use of gear trains is very common.

For example, it may be desired to reduce the speed of a shaft from 1800 rpm to 10 rpm. Thus, a velocity reduction of 180:1 is required. If this reduction were attempted with one gearset, Equation 11.19 would reveal that the driven gear would be 180 times larger than the drive gear. Obviously, the driven gear would be tremendously large, heavy, and expensive.

A second, more logical option is to reduce the speed in steps, through a series of gear pairs. This strategy cascades the rotational velocities towards the desired output velocity. This is exactly the logic behind gear trains.

When multiple gear pairs are used in a series, the overall velocity ratio is termed a *train value, TV.* The train value is defined as the input velocity to the gear train, divided by the output velocity from the train. This is consistent with the definition of a velocity ratio. A train value is the product of the velocity ratio of the individual mating gear pairs that comprise the train. In equation form, it is stated as follows:

(11.35) $$TV = \frac{\omega_{in}}{\omega_{out}} = (VR_1)(VR_2)(VR_3)\ .\ .\ .$$

The algebraic sign resulting from the multiplication of individual velocity ratios determines the relative rotational direction of input and output shafts. Positive values reveal that the input and output shafts rotate in the same direction, and negative values indicate opposite rotation.

EXAMPLE PROBLEM 11.18

A gear train is shown in Figure 11.21. The gears have the following properties:

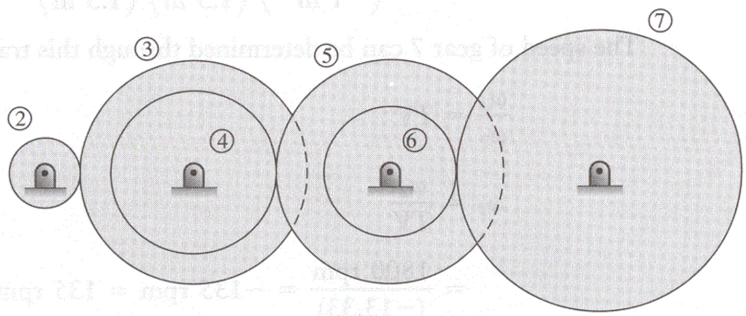


Figure 11.21 Gear train for Example Problem 11.13.

Gear 2: $N_2 = 12$ teeth and $P_d = 12$
Gear 3: $D_3 = 2.5$ in
Gear 4: $N_4 = 15$ teeth
Gear 5: $D_5 = 3.0$ in and $P_d = 10$
Gear 6: $D_6 = 1.5$ in and $P_d = 8$
Gear 7: $N_7 = 32$ teeth

Determine the rotational velocity of gear 7 as gear 2 drives at 1800 rpm, counterclockwise. Also determine the distance between the shafts that carry gears 2 and 7.

Solution:

In order to calculate the train value, consistent properties of the gears must be determined. For this problem, gear pitch diameters are used and must be computed:

$$D_2 = \frac{N_2}{P_d} = \frac{12}{12} = 1 \text{ in}$$

Gear 4 mates with gear 5 and must have an identical diametral pitch:

$$D_5 = \frac{N_5}{P_d} = \frac{15}{10} = 1.5 \text{ in}$$

Likewise, gear 7 mates with gear 6 and must have an identical diametral pitch:

$$D_7 = \frac{N_6}{P_d} = \frac{32}{8} = 4 \text{ in}$$

The train value can then be computed as:

$$TV = (VR_{2\text{-}3})\,(VR_{4\text{-}5})\,(VR_{6\text{-}7})$$

$$= \left(\frac{-D_3}{D_2}\right)\left(\frac{-D_5}{D_4}\right)\left(\frac{-D_7}{D_6}\right)$$

$$= \left(\frac{-2.5 \text{ in}}{1 \text{ in}}\right)\left(\frac{-3 \text{ in}}{1.5 \text{ in}}\right)\left(\frac{-4 \text{ in}}{1.5 \text{ in}}\right) = -13.33$$

The speed of gear 7 can be determined through this train value:

$$\frac{\omega_2}{\omega_7} = TV$$

$$\omega_7 = \frac{\omega_2}{TV}$$

$$= \frac{1800 \text{ rpm}}{(-13.33)} = -135 \text{ rpm} = 135 \text{ rpm (clockwise)}$$

The center distance between gears 2 and 7 can be determined by stacking the pitch radii from all gears between 2 and 7. This can be seen in Figure 11.21

$$C = r_2 + r_3 + r_4 + r_5 + r_6 + r_7$$

$$= \left(\frac{1 \text{ in}}{2}\right) + \left(\frac{2.5 \text{ in}}{2}\right) + \left(\frac{1.5 \text{ in}}{2}\right) + \left(\frac{3 \text{ in}}{2}\right) + \left(\frac{1.5 \text{ in}}{2}\right) + \left(\frac{4 \text{ in}}{2}\right) = 6.75 \text{ in}$$

EXAMPLE PROBLEM 11.19

Design a gear train that yields a train value of +300:1. From interference criteria, no gear should have fewer than 15 teeth and, due to size restrictions, no gear can have more than 75 teeth.

Solution:

With the restrictions placed on gear size used in this train, the maximum individual velocity ratio is determined by:

$$VR_{max} = \frac{N_2}{N_1} = \frac{75}{15} = 5$$

As with all design problems, more than one possible solution exists. Because a train value is the product of individual velocity ratios, one solution can be obtained by factoring the train value into values no greater than the maximum individual velocity ratios. For this problem, no factor can be greater than 5:

$$TV = 300 = (-5)(-60)$$
$$= (-5)(-5)(12)$$
$$= (-5)(-5)(-4)(-3)$$

Therefore, a gear train with gear pairs that have individual velocity ratios of −5, −5, −4, and −3 nets a train value of 300. A negative value is used for the individual velocity ratios because it is desirable to use the more common external gears:

$$VR_1 = -5, \text{ use external gears with } N_1 = 15 \text{ and } N_2 = 75$$
$$VR_1 = -5, \text{ use external gears with } N_3 = 15 \text{ and } N_4 = 75$$
$$VR_1 = -4, \text{ use external gears with } N_5 = 15 \text{ and } N_6 = 60$$
$$VR_1 = -3, \text{ use external gears with } N_7 = 15 \text{ and } N_8 = 45$$

In general, when using external gears that produce opposite rotations, an even number of gear pairs must be used to produce a positive train value. Because the solution for the above example has four gear pairs, the output rotation occurs in the same direction as the input.

11.14 IDLER GEARS

Consider the gear train shown in Figure 11.22. Notice that the middle gear mates with the small gear to form the first ratio. The middle gear also mates with the large gear to form a second ratio. As always, the train value can be computed as the product of the velocity ratios:

$$TV = (VR_1)(VR_2) = \left(\frac{-d_3}{d_2}\right)\left(\frac{-d_4}{d_3}\right)$$

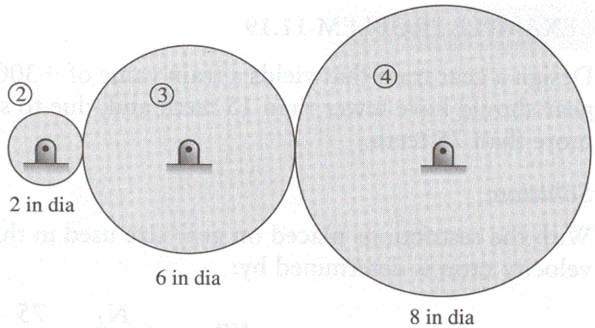


Figure 11.22 Gear train with an idler gear.

$$TV = \left(\frac{-6}{2}\right)\left(\frac{-8}{6}\right) = +\frac{8}{2} = +4$$

Notice that d_3 appears in both the numerator and the denominator. In this situation, the influence of the middle gear is negated. This gear arrangement creates a train value of:

$$TV = \left(\frac{-d_3}{d_2}\right)\left(\frac{-d_4}{d_3}\right) = \frac{+d_4}{d_2}$$

Therefore, the train value is only dependent on the size of the first and last gears. The diameter, or the number of teeth, of the center gear does not influence the train value. The center gear is termed an *idler gear.* Its function is to alter the direction of the output motion, yet not affect the magnitude of that motion. To illustrate this function, consider an arrangement where gear 2 mates directly with gear 4. The resulting train value would be:

$$TV = (-VR_1) = \frac{-d_4}{d_2}$$

Thus, the idler gear serves to reverse the direction of the output. As mentioned, the size of the idler gear does not influence the kinematics of the train. In practice, this idler gear can be sized to conveniently locate the centers of the input and output gears. Of course, because all three gears mesh, they must have identical diametral pitches and pressure angles.

11.15 PLANETARY GEAR TRAINS

The gear trains presented in preceding sections all had gear centers attached to fixed bodies. With planetary gear trains this restriction is removed. In these trains, a link which holds the center of the gears is allowed to move. A planetary gear train, which is also called an *epicyclic train,* is shown in Figure 11.23.

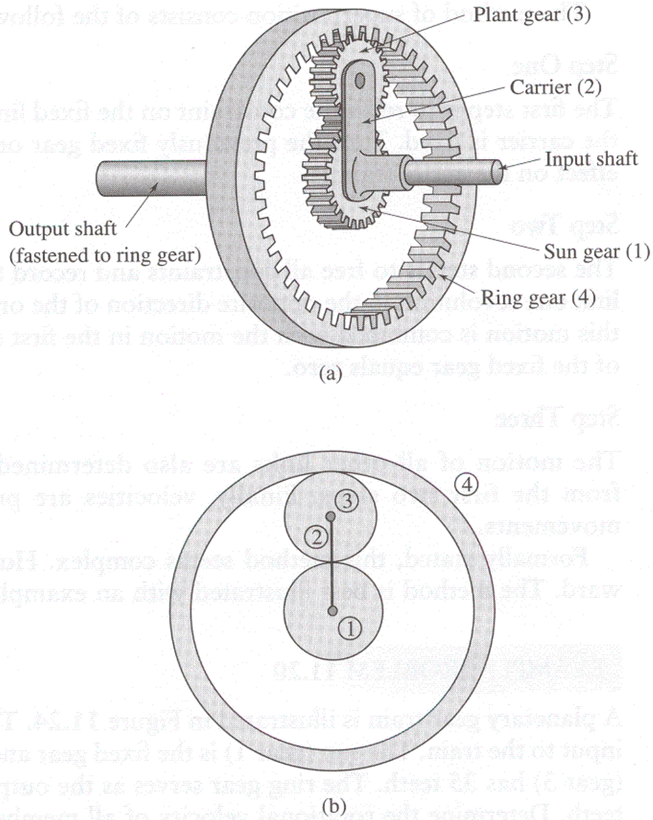


Figure 11.23 A planetary gear train.

Planetary trains can be used to achieve large-speed reductions in a more compact space than in a conventional gear train. However, a greater benefit is the ability to readily alter the train value. Because all links are capable of moving, one can alter the train value by holding different gears or carriers. In practice, switching the fixed link is accomplished with brake or clutch mechanisms, thus releasing one link and fixing another. For this reason, planetary gear trains are very common in automotive transmissions.

Because the motion can resemble the planets rotating about the sun in our solar system, the term *planetary gear train* was founded. Expanding on this comparison, the center gear is called the *sun*. Gears that revolve around the sun are called *planets*. Finally, the train is encased in the internal gear termed the *ring gear*. These gears are labeled in Figure 11.23.

The motion of a planetary gear train is not always as intuitive as fixed-center trains. As gears and carriers rotate, the motion can appear rather complex. To analyze the motion of a planetary gear train, the method of superposition can be used to "step through" the gear movements.

The method of superposition consists of the following:

Step One

The first step is to relax the constraint on the fixed link and temporarily assume that the carrier is fixed. Turn the previously fixed gear one revolution and calculate the effect on the entire train.

Step Two

The second step is to free all constraints and record the movement of rotating each link one revolution in the opposite direction of the original rotation in Step One. As this motion is combined with the motion in the first step, the superimposed motion of the fixed gear equals zero.

Step Three

The motion of all other links are also determined by combining the rotations from the first two steps. Finally, velocities are proportional to the rotational movements.

Formally stated, this method seems complex. However, it is rather straightforward. The method is best illustrated with an example problem.

EXAMPLE PROBLEM 11.20

A planetary gear train is illustrated in Figure 11.24. The carrier (link 2) serves as the input to the train. The sun (gear 1) is the fixed gear and has 30 teeth. The planet gear (gear 3) has 35 teeth. The ring gear serves as the output from the train and has 100 teeth. Determine the rotational velocity of all members of this gear train when the input shaft rotates at 1200 rpm clockwise.

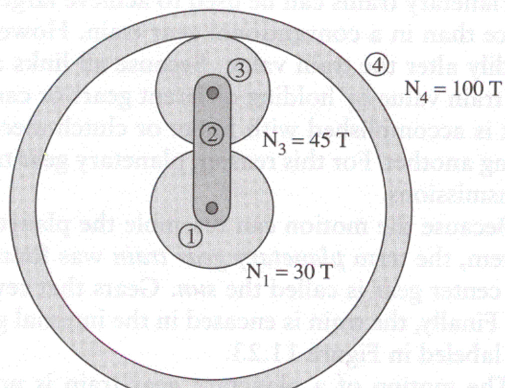


Figure 11.24 Planetary train for Example Problem 11.20.

Solution:

The first step is to temporarily fix the carrier, then compute the motions of all gears as the previously fixed gear rotates one revolution. Thus, the following can be determined:

Gear 1 rotates one revolution:

$$\Delta\theta_1 = +1 \text{ revolution.}$$

Gear 3 rotates ($VR_{2\text{-}3}$) as much as gear 2:

$$\Delta\theta_3 = (VR_{2\text{-}3})\ (\Delta\theta_1)$$
$$= -(30/45)(+1 \text{ rev}) = -0.667 \text{ revolutions}$$

Gear 4 rotates ($VR_{3\text{-}4}$) as much as gear 3:

$$\Delta\theta_4 = (VR_{3\text{-}4})\ (\Delta\theta_3)$$
$$= (VR_{2\text{-}3})\ (VR_{3\text{-}4})(\Delta\theta_1)$$
$$= \left(-30/35\right)\left(\frac{+35}{100}\right)(+1 \text{ rev})$$
$$= -0.3 \text{ revolutions}$$

The second step rotates all links -1 revolution. This returns the sun gear to its original position, yielding a net movement of zero.

The method of superposition involves combining these two motions, resulting in the actual planetary gear train motion. Thus, the rotations from both steps are algebraically added together. The two steps are summarized in Table 11.11.

TABLE 11.11 Tabulating Planetary Gear Analysis for Example Problem 11.20

Link	Sun	Planet	Ring	Carrier
Step 1:				
Rotate with fixed carrier	+1	−0.857	−0.3	0
Step 2:				
Rotate all links	−1	−1	−1	−1
Total rotations	0	−1.857	−1.3	−1

The velocities can be determined by using the ratios of rotations:

$$\omega_{sun} = \left(\frac{0}{-1}\right) = 0 \text{ rpm}$$

$$\omega_{planet} = \left(\frac{-1.857}{-1}\right)\omega_{carrier} = (-1.857)(1200 \text{ rpm}) = +2228 \text{ rpm}$$

$$= 2228 \text{ rpm (clockwise)}$$

$$\omega_{ring} = \left(\frac{-1.3}{-1}\right)\omega_{carrier} = (+1.3)(1200 \text{ rpm}) = +1560 \text{ rpm}$$

$$= 1560 \text{ rpm (clockwise)}$$

EXAMPLE PROBLEM 11.21

A planetary gear train is illustrated in Figure 11.25. The carrier (link 2) serves as the input to the train. The ring gear (gear 2) is the fixed gear and has 120 teeth. The planet gear (gear 2) has 40 teeth. The sun gear serves as the output from the train and has 30 teeth. Determine the rotational velocity of all members of this gear train when the input shaft rotates at 1200 rpm clockwise.

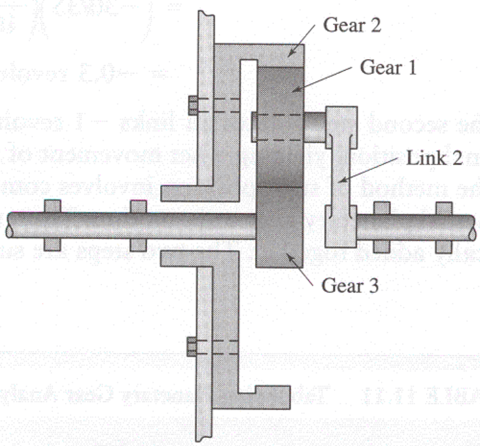


Figure 11.25 Planetary train for Example Problem 11.21.

Solution:

The first step is to temporarily fix the carrier. Then compute the motions of all gears as the previously fixed gear rotates one revolution.

The second step rotates all links -1 revolution. This returns the ring gear to its original position, yielding a net movement of zero.

The two steps are summarized in Table 11.12.

The velocities can be determined by using the ratios of rotations:

$$\omega_{ring} = \left(\frac{0}{-1}\right) = 0 \text{ rpm}$$

TABLE 11.12

Link	Sun	Planet	Ring	Carrier
Step 1: Rotate with fixed carrier	$\left(\frac{-120}{40}\right)\left(\frac{-40}{25}\right) = +4.0$	$\left(\frac{-100}{40}\right) = -2.5$	+1	0
Step 2: Rotate all links	−1	−1	−1	−1
Total rotations	+3.0	−4.0	0	−1

$$\omega_{planet} = \left(\frac{-3.5}{-1}\right)\omega_{carrier} = (4.0)(1200 \text{ rpm}) = +4800 \text{ rpm} = 4800 \text{ rpm (clockwise)}$$

$$\text{where } \omega_{earner} = 1200 \text{ rpm CW}$$

$$\omega_{ring} = \left(\frac{+3.0}{-1}\right)\omega_{carrier} = (-3.0)(1200 \text{ rpm}) = -3600 \text{ rpm} = 3600 \text{ rpm (counter-clockwise)}$$

PROBLEMS

Spur Gear Geometry Problems

For Problems 11–1 through 11–3, determine the following:

1. The pitch circle diameter.
2. The diameter of the base circle.
3. The diameter of the addendum circle.
4. The diameter of the dedendum circle.
5. The circular pitch.

11–1. A 20°, full-depth, involute spur gear with 18 teeth has a diametral pitch of 12.

11–2. A 20°, full-depth, involute spur gear with 48 teeth has a diametral pitch of 8.

11–3. A 14-1/2°, full-depth, involute spur gear with 40 teeth has a diametral pitch of 16. Determine the following:

11–4. A 25° spur gear with 21 teeth has a metric module of 4. Determine the pitch circle diameter.

For Problems 11–5 through 11–8, determine the following:

1. The center distance.
2. The contact ratio.
3. Whether interference is a problem.
4. A center distance that reduces backlash from a vendor value of $0.4/P_d$ to an AGMA recommended value of $0.1/P_d$.

11–5. Two 12-pitch, 20°, full-depth, involute spur gears are used on a industrial circular saw for cutting timber. The pinion has 18 teeth and the gear has 42.

11–6. Two 4-pitch, 20°, full-depth, involute spur gears are used on a tumbling machine for deburring steel-stamped parts. The pinion has 12 teeth and the gear has 28.

11–7. Two plastic, 48-pitch, 25°, full-depth, involute spur gears are used on an electric shaver. The pinion has 18 teeth and the gear has 42.

11–8. Two 16-pitch, 14-1/2°, full-depth, involute spur gears are used on a machine shop lathe. The pinion has 16 teeth and the gear has 72.

Problems 11–9 through 11–14, determine the following:

1. Their pitch diameters.
2. The center distance.

11–9. Two mating 12-pitch gears have 18 external and 48 internal teeth, respectively.

11–10. Two mating 20-pitch gears have 15 external and 60 internal teeth, respectively.

11–11. Two mating gears have 18 and 48 teeth, respectively and a center distance of 4.125.

11–12. Two mating gears have 20 and 45 teeth, respectively and a center distance of 3.25.

11–13. An 8-pitch, 18-tooth pinion mates with an internal gear of 64 teeth.

11–14. A 12-pitch, 24-tooth pinion mates with an internal gear of 108 teeth.

Problems With Gear Kinematics

For Problems 11–15 through 11–18, determine the following:

1. The speed of the gear.
2. The pitch line velocity.

11–15. An 8-pitch, 18-tooth pinion rotates clockwise at 1150 rpm and mates with a 64-tooth gear.

11–16. A 20-pitch, 15-tooth pinion rotates clockwise at 1725 rpm and mates with a 60-tooth gear.

11–17. A 6-pitch, 21-tooth pinion rotates clockwise at 850 rpm and mates with a 42-tooth gear.

11–18. A 24-pitch, 24-tooth pinion rotates clockwise at 1725 rpm and mates with a 144-tooth gear.

11–19. Two 10-pitch gears are to be mounted 12 inches apart and have a velocity ratio of 5:1. Find the pitch diameters and the number of teeth on both gears.

11–20. Two 16-pitch gears are to be mounted 3.75 inches apart and have a velocity ratio of 4:1. Find the pitch diameters and the number of teeth on both gears.

Gear Selection Problems

11–21. Two 32-pitch gears are to be mounted 2.25 inches apart and have a velocity ratio of 8:1. Find the pitch diameters and the number of teeth on both gears.

11–22. Two gears are to be mounted 5 inches apart and have a velocity ratio of 4:1. Find appropriate pitch diameters, diametral pitches, and the number of teeth on both gears that will be suitable.

11–23. Two gears are to be mounted 3.5 inches apart and have a velocity ratio of 6:1. Find appropriate pitch diameters, diametral pitches, and the number of teeth on both gears that will be suitable.

11–24. Two gears are to be mounted 10 inches apart and have a velocity ratio of 3:1. Find appropriate pitch diameters, diametral pitches, and the number of teeth on both gears that will be suitable.

11–25. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 5 hp. The pinion drives at 1800 rpm. Determine an appropriate diametral pitch for the gears.

11–26. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 2.5 hp. The pinion drives at 1500 rpm. Determine an appropriate diametral pitch for the gears.

11–27. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 8 hp. The pinion drives at 1500 rpm and the gear must rotate as close to 200 rpm as possible. Determine an appropriate set of gears for this application.

11–28. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 10 hp. The pinion drives at 800 rpm and the gear must rotate as close to 180 rpm as possible. Determine an appropriate set of gears for this application.

11–29. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 1 hp. The pinion drives at 1725 rpm and the gear must rotate as close to 560 rpm as possible. Determine an appropriate set of gears for this application.

11–30. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 10 hp. The pinion drives at 1175 rpm and the gear must rotate as close to 230 rpm as possible. Determine an appropriate set of gears for this application.

11–31. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 10 hp. The gears must fit into a housing where the center distance is 8 in. The pinion drives at 1175 rpm and the gear must rotate as close to 170 rpm as possible. Determine an appropriate set of gears for this application.

11–32. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 3 hp. The gears must fit into a housing, where the center distance is 14 in. The pinion will be driven at 1750 rpm and the gear must rotate as close to 290 rpm as possible. Determine an appropriate set of gears for this application.

11–33. A pair of 20°, mild steel gears are to be selected for an application where they need to transfer 20 hp. The gears must fit into a housing where the center distance is 20 in. The pinion drives at 825 rpm and the gear must rotate as close to 205 rpm as possible. Determine an appropriate set of gears for this application.

Rack and Pinion Problems

11–34. A rack and pinion will be used for a height adjustment on a camera stand. The 24-pitch pinion has 18 teeth. Determine the angle that the handle (and pinion) must rotate to raise the camera 5 in.

11–35. A rack and pinion will be used to lower a drill on a drill press. The 16-pitch pinion has 20 teeth. Determine the angle that the handle (and pinion) must rotate to raise the camera 3 in.

11–36. An 8-pitch, 18-tooth pinion is used to drive a rack. Determine the distance that the rack travels when the pinion rotates 3 revolutions.

11–37. A 12-pitch, 24-tooth pinion is used to drive a rack. Determine the distance that the rack travels when the pinion rotates 5 revolutions.

11–38. A rack and pinion will be used for a steering mechanism. The 12-pitch pinion has 18 teeth. Determine the required speed of the pinion if the rack must be driven at a rate of 50 in/min.

11–39. A rack and pinion will be used for a steering mechanism. The 10-pitch pinion has 20 teeth. Determine the required speed of the rack if the pinion rotates at a rate of 80 rpm.

Helical Gear Problems

For Problems 11–40 and 11–41, determine the following:

1. The pitch diameters.
2. The normal diametral pitch.
3. The normal circular pitch.
4.Whether interference is a problem.

11–40. A pair of helical gears has a 20° pressure angle, a 45° helix angle, and an 8 diametral pitch. The pinion has 16 teeth and the gear has 32 teeth.

11–41. A pair of helical gears has a 14-1/2° pressure angle, a 30° helix angle, and a 12 diametral pitch. The pinion has 16 teeth and the gear has 48 teeth.

11–42. In order to reduce the noise of a gear drive, two 8-pitch spur gears with 20 and 40 teeth are to be replaced with helical gears. The new set must have the same velocity ratio and center distance. Specify two helical gears, which will be formed on a hob, to accomplish the task.

11–43. In order to reduce the noise of a gear drive, two 12-pitch spur gears with 18 and 54 teeth are to be replaced with helical gears. The new set must have the same velocity ratio and center distance. Specify tow helical gears, which will be formed on a hob, to accomplish the task.

Bevel Gear Problems

11–44. A pair of bevel gears have 20 and 75 teeth, and are used on shafts that intersect each other at an angle of 90°. Determine the velocity ratio and the pitch angles of both gears.

11–45. A pair of bevel gears have 20 and 75 teeth, and are used on shafts that intersect each other at an angle of 60°. Determine the velocity ratio and the pitch angles of both gears.

11–46. A pair of bevel gears have 18 and 90 teeth, and are used on shafts that intersect each other at an angle of 75°. Determine the velocity ratio and the pitch angles of both gears.

Worm Gear Problems

11–47. A worm gearset is needed to reduce the speed of an electric motor from 3600 to 60 rpm. Strength considerations require that 16-pitch gears be used, and it is desired that the set be self-locking. Specify a set that accomplishes the task.

11–48. A worm gearset is needed to reduce the speed of an electric motor from 1800 to 18 rpm. Strength considerations require that 12-pitch gears be used, and it is desired that the set be self-locking. Specify a set that accomplishes the task.

11–49. A worm gearset is needed to reduce the speed of an electric motor from 3600 to 40 rpm. Strength considerations require that 20-pitch gears be used, and it is desired that the set be self-locking. Specify a set that accomplishes the task.

Gear Train Problems

11–50. A gear train is shown in Figure P11.50. The gears have the following properties: $N_2 = 18$ teeth; $N_3 = 72$ teeth and $P_d = 10$; $N_4 = 16$ teeth and $P_d = 8$; and $N_5 = 48$ teeth. Determine the velocity of gear 5, as gear 2 drives at 1200 rpm, clockwise. Also determine the center distance between gears 2 and 5.

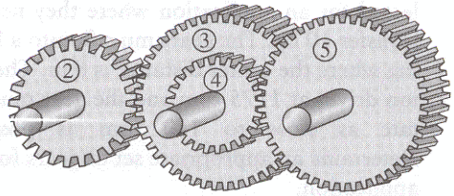


Figure P11.50 Problems 50 and 51.

11–51. A gear train is shown in Figure P11.50. The gears have the following properties: $N_2 = 20$ teeth and $P_d = 10$; $N_3 = 6$ in; $d_4 = 2$ in and $P_d = 8$; and $N_5 = 48$ teeth. Determine the velocity of gear 5, as gear 2 drives at 1800 rpm, counterclockwise. Also determine the center distance between gears 2 and 5.

11–52. A gear train is shown in Figure P11.52. The gears have the following properties: $N_2 = 15$ teeth; $N_3 = 90$ teeth and $P_d = 16$; $N_4 = 15$ teeth; $N_5 = 75$ teeth; $N_6 = 75$ teeth and $P_d = 12$; $N_7 = 15$ teeth; and $N_8 = 60$ teeth and $P_d = 8$. Determine the velocity of gear 8, as gear 2 drives at 3600 rpm, clockwise. Also determine the center distance between gears 2 and 8.

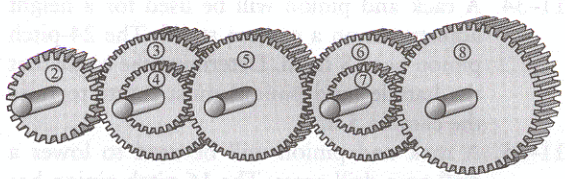


Figure P11.52 Problems 52 and 53.

11–53. A gear train is shown in Figure P11.52. The gears have the following properties: $N_2 = 16$ teeth and $P_d = 16$; $d_3 = 8$ in; $d_4 = 1.25$ in; $N_5 = 50$ teeth and $P_d = 10$; $d_6 = 5.5$ in; $N_7 = 1.5$ in and $P_d = 8$; and $N_8 = 56$ teeth. Determine the velocity of gear 8, as gear 2 drives at 1200 rpm, counterclockwise. Also determine the center distance between gears 2 and 8.

11–54. A gear train is shown in Figure P11.54. The gears have the following properties: $N_1 = 20$ teeth and $P_d = 16$; $d_2 = 8$ in; and $d_3 = 1.5$ in and $P_d = 10$. Determine the distance that the rack moves for each revolution of gear. Also determine the center distance between gears 1 and 3.

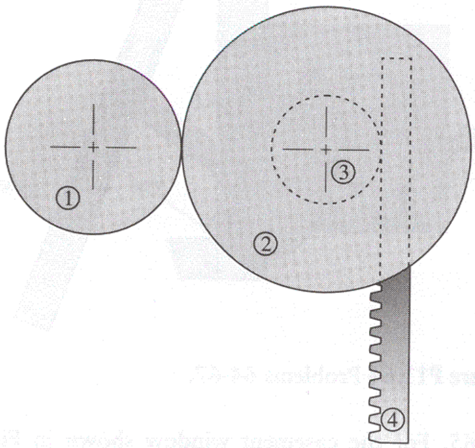


Figure P11.54 Problems 54-56.

11–55. A gear train is shown in Figure P11.54. The gears have the following properties: $N_1 = 18$ teeth and $P_d = 10$; $d_2 = 5.5$ in; and $d_3 = 1.25$ in and $P_d = 4$. Determine the required speed of gear 1 for the rack to move at a rate of 50 in/min.

11–56. For the gear train is shown in Figure P11.54. The gears have the following properties: $d_1 = 2.5$ in; $N_2 = 75$ teeth and $P_d = 10$; and $N_3 = 24$ teeth. Determine the required diametral pitch of the rack for the rack to move 0.5 in for each revolution of gear 1.

11–57. A gear train is shown in Figure P11.57. The gears have the following properties: $N_1 = 16$ teeth and $P_d = 16$; $d_2 = 8$ in; $N_3 = 20$ teeth; and $N_4 = 50$ teeth. Determine the velocity of gear 4, as gear 1 drives at 1800 rpm.

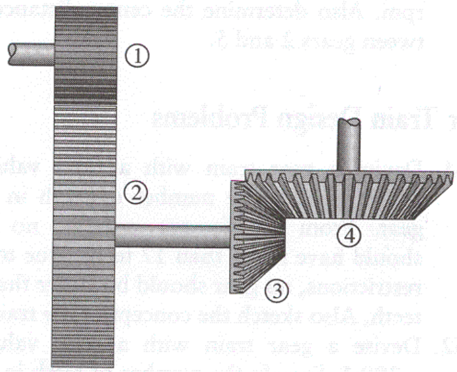


Figure P11.57 Problems 57 and 58.

11–58. For the gear train shown in Figure P11.57, the gears have the following properties: $N_1 = 17$ teeth and $P_d = 20$; $d_2 = 4$ in; $N_3 = 18$ teeth; and $N_4 = 36$ teeth.Determine the required velocity of gear 1 for gear 4 to drive at 380 rpm.

11–59. A gear train is shown in Figure P11.59. The gears have the following properties: $N_{worn} = 1$ threads; $N_2 = 45$ teeth; $N_3 = 18$ teeth and $P_d = 16$; $d_4 = 6$ in; and $N_5 = 80$ teeth. Determine the velocity of gear 5, as gear 1 drives at 1800 rpm. Also determine the center distance between gears 2 and 5.

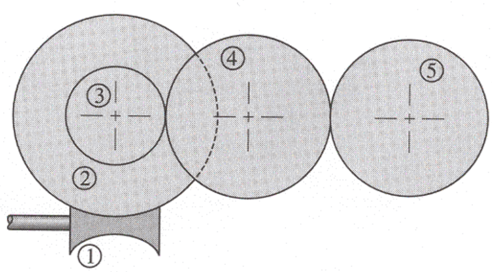


Figure P11.59 Problems 59 and 60.

11–60. For the gear train shown in Figure P11.59, the gears have the following properties: N_{worm} = 2 threads; N_2 = 60 teeth; N_3 = 18 teeth and P_d = 12; d_4= 6 in; and N_5 = 54 teeth. Determine the required velocity of gear 1 (the worm), to enable gear 5 to drive at 28 rpm. Also determine the center distance between gears 2 and 5.

Gear Train Design Problems

11–61. Devise a gear train with a train value of 400:1. Specify the number of teeth in each gear. From interference criteria, no gear should have fewer than 17 teeth. Due to size restrictions, no gear should be larger than 75 teeth. Also sketch the concept of the train.

11–62. Devise a gear train with a train value of −200:1. Specify the number of teeth in each gear. From interference criteria, no gear should have fewer than 17 teeth. Due to size restrictions, no gear should be larger than 75 teeth. Also sketch the concept of the train.

11–63. Devise a gear train with a train value of −900:1. Specify the number of teeth in each gear. From interference criteria, no gear should have fewer than 17 teeth. Due to size restrictions, no gear should be larger than 75 teeth. Also sketch the concept of the train.

Gear Driven Mechanism Problems

11–64. A casement window opening mechanism is shown in Figure P11.64. The gears have the following properties: d_1 = 1 in; N_2 = 30 teeth and P_d = 20; N_3 = 18 teeth and P_d = 18; and d_4= 4 in. Starting at the configuration shown, with β = 20°, graphically determine (using either manual drawing techniques or CAD) the angular rotation of the window when the crank rotates one revolution.

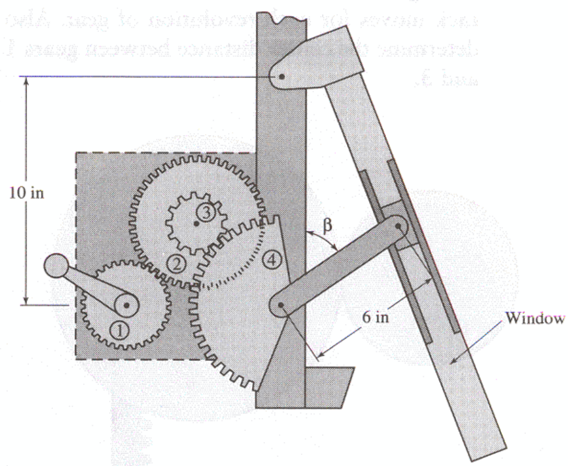


Figure P11.64 Problems 64-67.

11–65. For the casement window shown in Figure 11.64, analytically determine the angular rotation of the window when the crank rotates one revolution using the configuration shown (β = 20°).

11–66. For the casement window mechanism shown in Figure P11.64, the gears have the following properties: d_1 = .75 in; N_2 = 48 teeth and P_d = 32; N_3 = 16 teeth and P_d = 32; d_4= 4 in. Starting at the configuration shown (β = 20°), graphically determine (using either manual drawing techniques or CAD) the rotational speed with which the window opens when the crank rotates at a constant rate of 20 rpm.

11–67. For Problem 11–66, analytically determine the rotational speed with which the window opens from the configuration shown ($\beta = 20°$), when the crank rotates at a constant rate of 20 rpm.

Planetary Gear Train Problems

11–68. A planetary gear train is shown in Figure P11.68. The carrier (link 2) serves as the input to the train. The sun (gear 1) is fixed and has 16 teeth with a diametral pitch of 16. The planet gear (gear 3) has a 2-in pitch diameter. The ring serves as the output from the train and has a 5-in pitch diameter. Determine the rotational velocity of all members of this gear train when the input shaft rotates at 1800 rpm, clockwise.

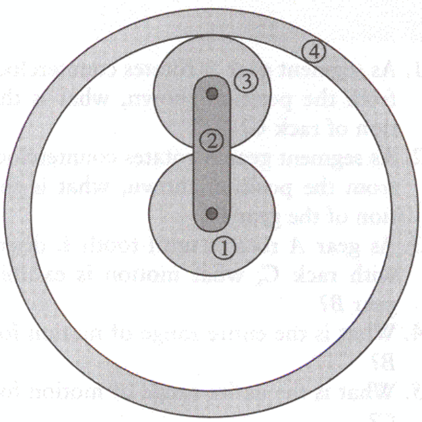


Figure P11.68 Problems 68 and 69.

11–69. For the planetary gear train is shown in Figure P11.68. The carrier (link 2) serves as the input to the train. The sun (gear 1) serves as the output gear and has 18 teeth with a diametral pitch of 12. The planet gear (gear 3) has a 2.5-in pitch diameter. The ring gear is fixed and has a 6.5-in pitch diameter. Determine the rotational velocity of all members of this gear train when the input shaft rotates at 800 rpm, counterclockwise.

11–70. A planetary gear train is shown in Figure P11.70. The carrier (link 2) serves as the input to the train. The sun (gear 1) is fixed and has a 1.25-in pitch diameter with a diametral pitch of 16. Gear 3 has 42 teeth and gear 4 has 21 teeth. Gear 5 has 32 teeth and is keyed to the same shaft as gear 4. Gear 5 mates with the ring gear (gear 6), which serves as the output from the train, and has 144 teeth. Determine the rotational velocity of all members of this gear train when the input shaft rotates at 680 rpm, clockwise.

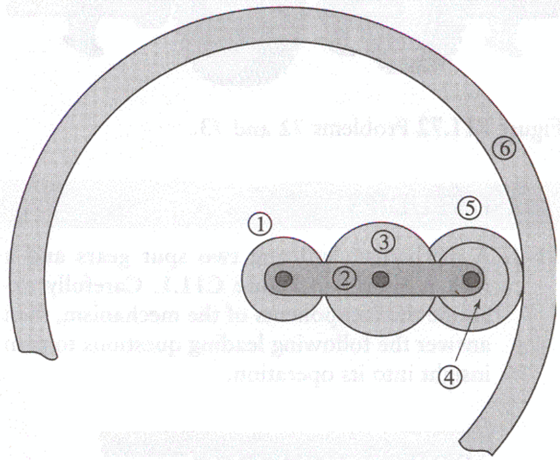


Figure P11.70 Problems 70 and 71.

11–71. A planetary gear train is shown in Figure P11.70. The carrier (link 2) serves as the input to the train. The sun (gear 1) serves as the output from the train and has a 1.0-in pitch diameter with a diametral pitch of 20. Gear 3 has 45 teeth and gear 4 has 20 teeth. Gear 5 has 30 teeth and is keyed to the same shaft as gear 4. Gear 5 mates with the ring gear (gear 6), which is fixed, and has 150 teeth. Determine the rotational velocity of all members of this gear train when the input shaft rotates at 1125 rpm, counterclockwise.

11–72. A planetary gear train is shown in Figure P11.72. The carrier (link 2) serves as the input to the train. Gear 2 is fixed and has a 48 teeth with a diametral pitch of 12. Gear 1 has 24 teeth, gear 3 has a pitch diameter of 2.5 in, and gear 4 has 35 teeth and a diametral pitch of 10. Determine the rotational velocity of all members of this gear train when the input shaft rotates at 900 rpm, clockwise.

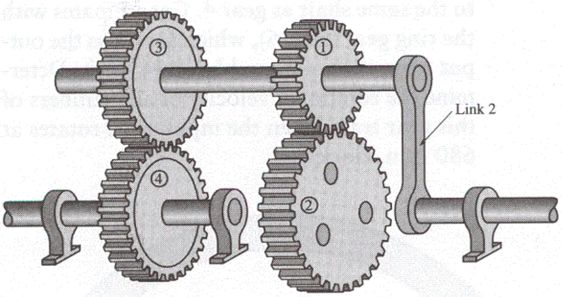


Figure P11.72 Problems 72 and 73.

11–73. A planetary gear train is shown in Figure P11.72. The carrier (link 2) serves as the input to the train. Gear 2 is fixed and has a 4.0-in pitch diameter with a diametral pitch of 10. Gear 1 has 25 teeth, gear 3 has a pitch diameter of 2.5 in, and gear 4 has 32 teeth and a diametral pitch of 8. Determine the rotational velocity of all members of this gear train when the output shaft rotates at 210 rpm, clockwise.

CASE STUDIES

11–1. A mechanism utilizing two spur gears and a rack is shown in Figure C11.1. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

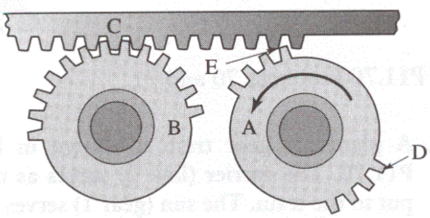


Figure C11.1 (Courtesy, Industrial Press)

1. As segment gear *A* rotates counterclockwise from the position shown, what is the motion of rack *C*?
2. As segment gear A rotates counterclockwise from the position shown, what is the motion of the gear *B*?
3. As gear *A* rotates until tooth *E* disengages with rack *C*, what motion is exhibited in gear *B*?
4. What is the entire range of motion for gear *B*?
5. What is the entire range of motion for rack *C*?
6. What is the purpose of this mechanism?
7. What are possible operating problems with this mechanism?

11–2. A device from a wire-forming machine is shown in Figure C11.2. Link *B* and spur gear *C* are keyed to the same shaft. Likewise, link *E* and spur gear *D* are keyed to the same shaft. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

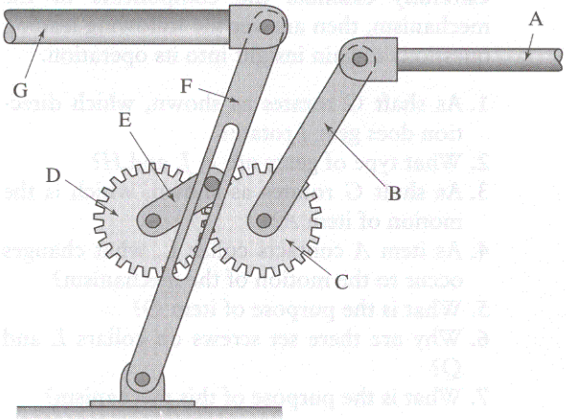


Figure C11.2 (Courtesy, Industrial Press)

1. As link *A* moves to the left, what is the motion of link *B*?
2. As link *A* moves to the left, what is the motion of gear *C*?
3. As link *A* moves to the left, what is the motion of gear *D*?
4. As link *A* moves to the left, what is the motion of link *E*?
5. As link *A* moves to the left, what is the motion of link *F*?
6. As link *A* moves to the left, what is the motion of link *G*?
7. Describe specifically the motion given to *G* as link *A* reciprocates back and forth.
8. How would the motion of link *G* be altered, if the mechanism were assembled, such that everything appeared identical except link *E* rotated clockwise 90°?

11–3. A device that controls the motion of a gear attached to gear *D* is shown in Figure C11.3. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

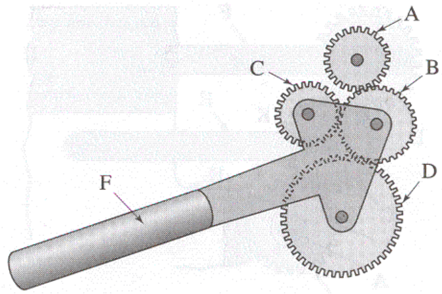


Figure C11.3 (Courtesy, Industrial Press)

1. As gear *A* rotates clockwise, what is the motion of gear *B*?
2. As gear *A* rotates clockwise, what is the motion of gear *C*?
3. As gear *A* rotates clockwise, what is the motion of gear *D*?
4. As the handle *F* is forced upward, what happens to the mating gears?
5. As gear *A* rotates clockwise, what are the motions of gears *B*, *C*, and *D*?
6. What is the purpose of this mechanism?
7. What problems may occur when operating this mechanism?

11–4. A device that drives a piston (*G*) is shown in Figure C11.4. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

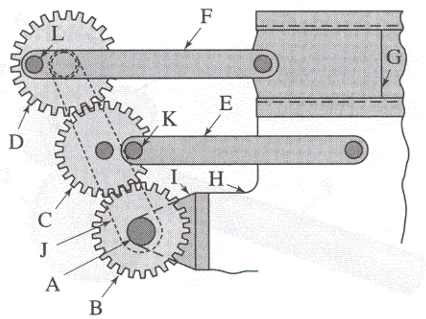


Figure C11.4 (Courtesy, Industrial Press)

1. As gear *B* rotates clockwise, what is the motion of gear *C*?
2. As gear *B* rotates clockwise, what is the motion of gear *D*?
3. If link *J* were hinged at *A*, but were not attached to gear *B*, what motion would link J exhibit and what would cause this motion?
4. What is the motion of the center of gear *C*?
5. What is the motion of piston *G*?
6. What is the purpose of this mechanism?

11–5. A device is shown in Figure C11.5. Shaft *C* is a free running fit through gears *H* and *J*, but item *K* is attached with a pin to the shaft. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

1. As shaft *G* rotates as shown, which direction does gear *J* rotate?
2. What type of gears are *F*, *J*, and *H*?
3. As shaft *G* rotates as shown, which is the motion of item *A*?
4. As item *A* contacts collar *L*, what changes occur to the motion of the mechanism?
5. What is the purpose of item *O*?
6. Why are there set screws on collars *L* and *Q*?
7. What is the purpose of this mechanism?

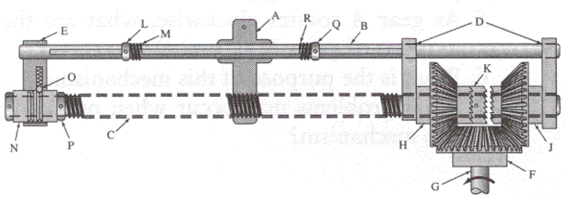


Figure C11.5 (Courtesy, Industrial Press)

12 Belt and Chain Drives

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Describe the advantages of a belt drive.
2. Identify several different types of belt designs.
3. Determine the geometric relationships of a belt drive.
4. Analyze the kinematic relationships of two shafts coupled by a belt drive.
5. Describe the advantages of a chain drive.
6. Identify several different types of chain designs.
7. Determine the geometric relationships of a chain drive.
8. Analyze the kinematic relationships of two shafts coupled by a chain drive.

12.1 INTRODUCTION

The primary function of a belt or chain drive is identical to a gear drive. All three of these mechanisms are used to transfer power between rotating shafts. However, the use of gears becomes impractical when the distance between the shafts is large. Both belt and chain drives offer the flexibility of efficient operation at large and small center distances.

Consider the chain on a bicycle. This mechanism is used to transfer the motion and forces of the rotating pedal assembly to the rear wheel. The distance between these two rotating components is considerable, and a gear drive would be unreasonable. Additionally, the velocity ratio of the chain drive can be readily altered by relocating the chain to an alternate set of sprockets. Thus, a slower pedal rotation, but greater forces are needed to maintain the identical rotation of the rear wheel. The velocity ratio of a belt drive can be similarly altered. Changing a velocity ratio on a gear drive is a much more complex process, as in an automotive transmission.

Belt and chain drives are commonly referred to as flexible connectors. These two types of mechanisms can be "lumped together" because the kinematics are identical. The determination of the kinematics and forces in belt and chain drives is the purpose of this chapter. Because the primary motion of the shafts is pure rotation, graphical solutions do not provide any insight. Therefore, only analytical techniques are practical, and are introduced in this chapter.

12.2 BELTS

The function of a belt drive is to transmit rotational motion and torque from one shaft to another, smoothly, quietly, and inexpensively. Belt drives provide the best overall combination of design flexibility, low cost, low maintenance, ease of assembly, and space savings.

Compared to other forms of power transmission, the advantages of belt drives include:

- They are less expensive than gear or chain drives.
- They have flexible shaft center distances, where gear drives are restricted.
- They operate smoothly and with less noise at high speeds.
- They can be designed to slip when an overload occurs in the machine.
- They require no lubrication, as do chains and gears.
- They can be used in more than one plane.
- They are easy to assemble, install, and have flexible tolerances.
- They require little maintenance.
- They do well in absorbing shock loading.

Belts are typically made of continuous construction of materials, such as rubberized fabric, rubberized cord, reinforced plastic, leather, and fabric (i.e., cotton or synthetic fabric). Many belt shapes are commercially available and are listed below.

1. A *flat belt* is shown in Figure 12.1A. This belt is the simplest type, but is typically limited to low-torque applications because the driving force is restricted to pure friction between the belt and the pulley.
2. A *V-belt* is shown in Figure 12.1B. This is the most widely used type of belt, particularly in automotive and industrial machines. The V shape causes the belt to wedge tightly into the pulley, increasing friction and allowing higher operating torque.
3. A *multi-V-belt* is shown in Figure 12.1C. This belt design is identical to several V-belts placed side-by-side, but is integrally connected. It is used to increase the amount of power transferred.
4. A *cog* belt is shown in Figure 12.1D. This belt design is similar to V-belts but has grooves formed on the inner surface. This feature increases belt flexibility, allowing the belt to turn smaller radii. Thus, they can be used on smaller pulleys, reducing the size of the drive.
5. A *timing-belt* is shown in Figure 12.1E. This belt design has gear-like teeth which engage with mating teeth on the pulleys. This arrangement combines the flexibility of a belt with the positive grip of a gear drive. This belt is widely used in applications where relative positioning of the respective shafts is desired.

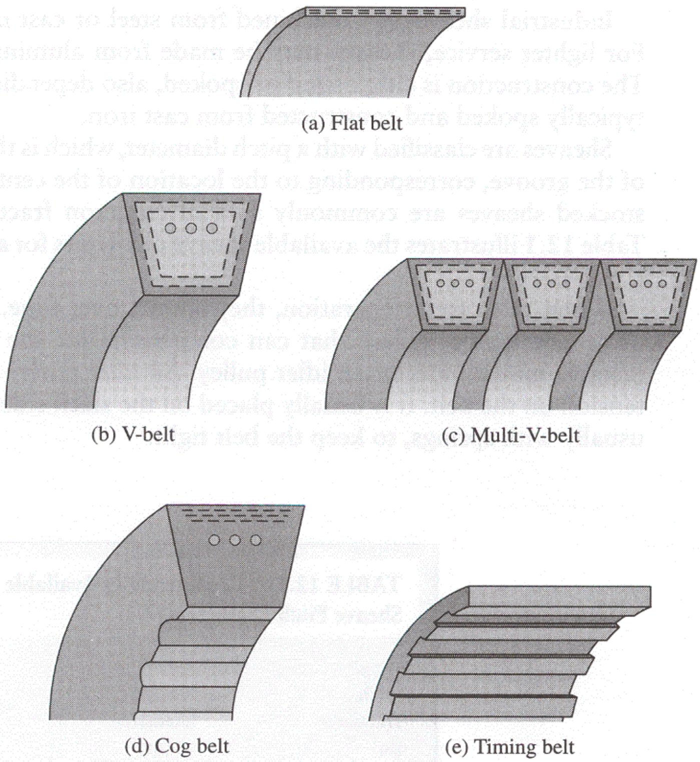


Figure 12.1 Types of belts.

Pulleys, also referred to as *sheaves*, are the wheels that are connected to the shafts. The pulleys have a groove around the outside, with a shape to match that of the belt. A V-belt sheave is shown in Figure 12.2.

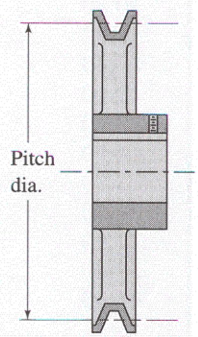


Figure 12.2 Single groove V-belt sheave.

Industrial sheaves are machined from steel or cast iron, depending on diameter. For lighter service, sheaves may be made from aluminum, plastic, or die-cast zinc. The construction is either solid or spoked, also depending on size. Large sheaves are typically spoked and constructed from cast iron.

Sheaves are classified with a pitch diameter, which is the diameter across the middle of the groove, corresponding to the location of the center of the belt. Commercially stocked sheaves are commonly sold in common fractional-inch inside diameters. Table 12.1 illustrates the available sheave diameters for an average sized V-belt design ("5V").

When belts are in operation, they stretch over time. Machines that utilize a belt drive need some feature that can compensate for the belt stretch, such as an adjustable motor base, or an idler pulley. An *idler pulley* is used to maintain constant tension on the belt. It is usually placed on the slack side of the belt and is preloaded, usually with springs, to keep the belt tight.

TABLE 12.1 Commercially Available "5V" Sheave Pitch Diameters

Pitch Diameter (in)	Outside Diameter (in)	Inside Groove Diameter (in)
7.00	7.10	5-3/8
7.40	7.50	5-3/4
7.90	8.00	6-1/4
8.40	8.50	6-3/4
8.90	9.00	7-1/4
9.15	9.25	7-3/8
9.65	9.75	7-7/8
10.20	10.30	8-7/16
10.80	10.90	9
11.70	11.80	10
12.40	12.50	10-3/4
13.10	13.20	11-7/16
13.90	14.00	12-1/4
14.90	15.00	13-1/4
15.90	16.00	14-1/4
21.10	21.20	19-3/8
27.90	28.00	26-1/4
37.40	37.50	35-3/4
49.90	50.00	48-1/4

Note that other belt size sheaves have different commercially available sheaves. The 5V series is a mid-range belt drive and is given as an illustration.

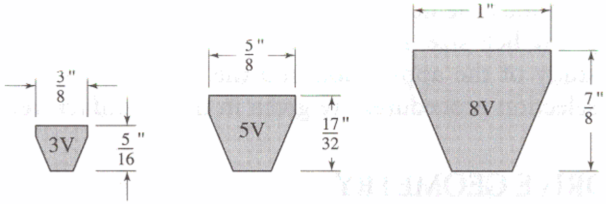


Figure 12.3 Industrial narrow section V-belts.

As stated, V-belts are the most widely used type of belt. Commercially available, industrial V-belts are made to one of the standard sizes shown in Figure 12.3. Of course, the larger cross sections are able to transmit greater power. Often several belts are used on multiple groove pulleys to increase the amount of power transmitted by the belt drive. A rough guide to V-belt selection is given in Table 12.2. The power values are listed "per belt." When the belt drive musttransfer 6 hp using a three-groove belt, each of the three belts must be capable of carrying 2 hp.

TABLE 12.2 Industrial V-belt Selection Chart

	Nominal Driver Speed (rpm)						
HP per belt	435	585	690	870	1160	1750	3500
1/2	3V	3V	3V	3V	3V	3V	3V
3/4	3V	3V	3V	3V	3V	3V	3V
1	3V	3V	3V	3V	3V	3V	3V
1-1/2	3V	3V	3V	3V	3V	3V	3V
2	3V	3V	3V	3V	3V	3V	3V
3	3V	3V	3V	3V	3V	3V	3V
5	3V	3V	3V	3V	3V	3V	3V
7-1/2	3V	3V	3V	3V	3V	3V	3V
10	3V	3V	3V	3V	3V	3V	3V
15	5V	3V	3V	3V	3V	3V	3V
20	5V	3V	3V	3V	3V	3V	3V
25	5V	5V	3V	3V	3V	3V	3V
30	5V	5V	5V	3V	3V	3V	3V
40	5V	5V	5V	5V	3V	3V	3V
50	5V	5V	5V	5V	5V	3V	3V
60	5V	5V	5V	5V	5V	3V	3V
75	5V	5V	5V	5V	5V	5V	3V
100	5V	5V	5V	5V	5V	5V	3V
125	8V	5V	5V	5V	5V	5V	3V
150	8V	8V	5V	5V	5V	5V	5V
200	8V	8V	8V	5V	5V	5V	5V
250	8V	8V	8V	8V	5V	5V	5V

It must be noted that Table 12.2 gives only a rough guide to selecting an appropriate belt size. It is important to select the most suitable belt drive with a detailed study of the application and the power transmission requirements. These detailed selection procedures are given in the manufacturers' catalogs.

12.3 BELT DRIVE GEOMETRY

A belt drive is intended to provide a constant velocity ratio between the respective shafts. A sketch of the basic geometry in a belt drive is shown in Figure 12.4.

As stated above, the *pitch diameter, D,* of the sheave is measured to the point in the groove where the center of the belt sits. This is slightly smaller than the outside diameter of the sheave. Note that the diameters shown for the sheaves in Figures 12.2 and 12.4 are the pitch diameters.

The *center distance, C,* is the distance between the center of the driver and driven sheaves. Of course, this is also the distance between the two shafts coupled by the belt drive. Small center distances can cause fatigue, with frequent maximum loading on the belt sections as it enters the small sheave. Large center distances, with the long unsupported span can cause belt whip and vibrations. Normal center distances for V-belts should be in the following range:

$$D_2 < C < 3(D_1 + D_2)$$

The *belt length, L,* is the total length of the belt. Specifically, the outside length is usually specified. This is the dimension obtained by wrapping a tape measure around the outside of the belt in the installed position. Belts are commercially available at specified lengths. Table 12.3 illustrates the available lengths for an average sized V-belt design ("5V").

The center distance and pitch length can be mathematically related[2]:

(12.1)
$$L = 2C + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C}$$

and:

(12.2)
$$C = \frac{B + \sqrt{B^2 - 32(D_2 - D_1)^2}}{16}$$

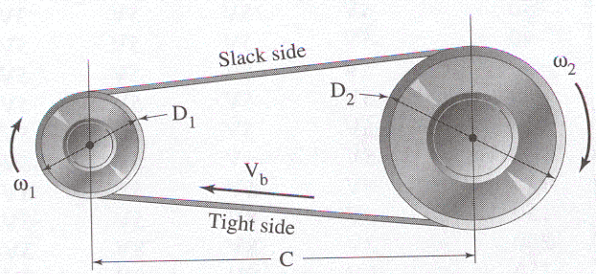


Figure 12.4 Belt drive geometry.

TABLE 12.3 Available "5V" Belt Lengths

50.0	90.0	160.0	280.0
53.0	95.0	170.0	300.0
56.0	100.0	180.0	315.0
60.0	106.0	190.0	335.0
63.0	112.0	200.0	355.0
67.0	118.0	212.0	
71.0	125.0	224.0	
75.0	132.0	236.0	
80.0	140.0	250.0	
85.0	150.0	265.0	

where:

$$B = 4L - 2\pi(D_2 + D_1) \tag{12.3}$$

The *angle of contact,* θ, of is a measure of the angular engagement of the belt on each sheave. It can be computed for each sheave as follows:

$$\theta_1 = 180° - 2\sin^{-1}\left\{\frac{D_2 - D_1}{2C}\right\} \tag{12.4}$$

$$\theta_2 = 180° + 2\sin^{-1}\left\{\frac{D_2 - D_1}{2C}\right\} \tag{12.5}$$

The power ratings for commercially available belts are for the same size drive and driven sheaves. Thus, the "rated" angle of contact is 180°. For smaller angles, the amount of friction that can be developed around the sheave is reduced and, therefore, the amount of power that a belt can transfer is reduced. Table 12.4 shows the percentage of actual rated power than can be transferred by a belt, riding over a sheave with a contact angle smaller than 180°. Belt manufacturers suggest keeping the contact angle greater than 120°, when possible.

TABLE 12.4 Reduced Power Capability with Contact Angle

Angle of Contact, θ	180°	160°	140°	120°	100°	80°
Actual Capability (% of Rated Power)	100%	95%	89%	82%	74%	63%

12.4 BELT DRIVE KINEMATICS

In an identical manner as gear drives, the *velocity ratio, VR,* is defined as the angular speed of the driver sheave (sheave 1) divided by the angular speed of the driven sheave (sheave 2):

(12.6)
$$VR = \frac{\omega_{driver}}{\omega_{driven}} = \frac{\omega_1}{\omega_2}$$

where:

ω = angular speed

Because a ratio is valid regardless of units, the velocity ratio can be defined in terms of revolutions per minute, radians per time, or any other convenient set of rotational velocity units. Using the same logic as the derivation of Equation 11.19 yields the following equation:

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = VR$$

Introducing the pitch diameters gives:

$$\frac{D_2}{D_1} = \frac{2r_2}{2r_1} = \frac{r_2}{r_1} = VR$$

Thus, a comprehensive definition of a velocity ratio is given as:

(12.7)
$$VR = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{D_2}{D_1}$$

Notice that for the typical arrangement, as shown in Figure 12.3, the sheaves rotate in the same direction. Crossed drives or serpentine drives, as shown in Figure 12.5, can be used to reverse the direction of sheave rotation.

Many industrial applications require belts to reduce the speed of a power source. Therefore, it is typical to have velocity ratios greater than one. As can be seen from Equation 12.6, this indicates that the drive sheave rotates faster than the driven sheave, which is the case in speed reductions.

The *belt speed,* v_b, is defined as the linear velocity of the belt. The magnitude of this velocity corresponds to the magnitude of the linear velocity of a point on the pitch diameter of each sheave. Therefore, the belt speed can be related to the rotational velocities of the sheaves and their pitch radii using Equation 4.5:

(12.8)
$$v_b = r_1\omega_2 = \frac{D_1}{2}\omega_1 = r_2\omega = \frac{D_2}{2}\omega_2$$

Note that, as in Chapter 5, the angular velocity in this equation must be specified in radians per unit time.

A belt transfers maximum power at speeds of 4000 to 5000 fpm (ft/min). Therefore, it is best to design a belt drive to operate in this range. Large sheaves for in-

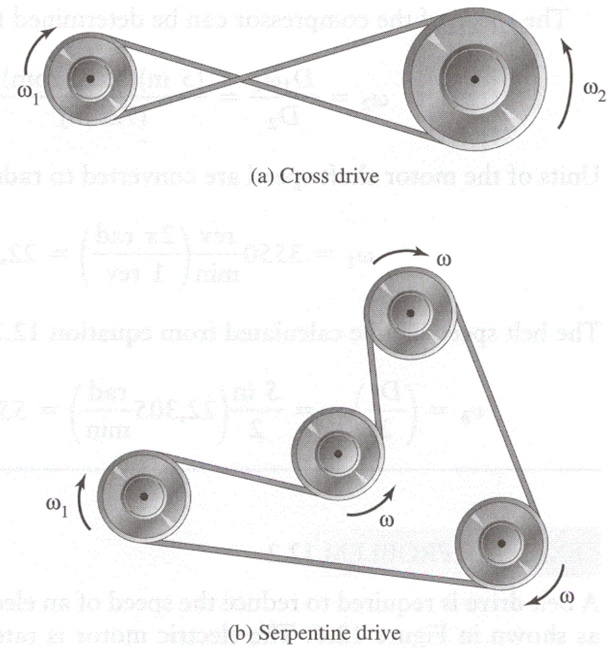


Figure 12.5 Alternate forms of belt drives.

dustrial use are cast iron and typically are limited to a maximum belt speed of 6500 fpm. This is because the inertial forces created by the normal acceleration become excessive. Special balance may be needed for speeds exceeding 5000 fpm, as vibration can be caused by the centrifugal acceleration. Finally, another type of drive, specifically chains, is typically more desirable for speeds under 1000 fpm.

EXAMPLE PROBLEM 12.1

A belt drive is used to transmit power from an electric motor to a compressor for a refrigerated truck. A design is required when the truck is parked and the engine is not running yet the compressor must still operate. The 10-hp electric motor is rated at 3550 rpm, and motor sheave diameter is 5 in. The compressor sheave is 7.5 in diameter. Determine the appropriate industrial belt size, operating speed of the compressor, and the belt speed.

Solution:

With a 10-hp motor driving at 3550 rpm, Table 12.2 suggests using a 3V belt.

From equation 12.8, the velocity ratio is determined by:

$$VR = \frac{D_2}{D_1} = \frac{7.5 \text{ in}}{5 \text{ in}} = 1.5$$

The speed of the compressor can be determined from rewriting equation 12.8:

$$\omega_2 = \frac{D_1\omega_1}{D_2} = \frac{(5\text{ in})(3550\text{ rpm})}{(7.5\text{ in})} = 2367\text{ rpm}$$

Units of the motor shaft speed are converted to radians per unit time:

$$\omega_1 = 3550\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 22{,}305\text{ rad/min}$$

The belt speed can be calculated from equation 12.7:

$$v_b = \left(\frac{D_1}{2}\right)\omega_1 = \frac{5\text{ in}}{2}\left(22{,}305\frac{\text{rad}}{\text{min}}\right) = 55762\frac{\text{in}}{\text{min}} = 4647\text{ fpm}$$

EXAMPLE PROBLEM 12.2

A belt drive is required to reduce the speed of an electric motor for a grinding wheel, as shown in Figure 12.6. The electric motor is rated at 1725 rpm, and a grinding wheel speed of approximately 600 rpm is desired. Find suitable sheave diameters of "5V" stock pulleys listed in Table 12.1. Also, select a suitable belt length and center distance.

Solution:

The respective shaft speeds are as follows:

$$\omega_1 = 1725\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 10{,}838\frac{\text{rad}}{\text{min}}$$

$$\omega_2 = 600\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 3770\frac{\text{rad}}{\text{min}}$$

A belt speed of 4000 to 5000 fpm is optimal. Rewriting equation 12.7 yields the following:

$$D_1 = 2\left(\frac{v_b}{\omega_1}\right) = 2\left(\frac{4500\frac{\text{ft}}{\text{min}}}{10{,}838\frac{\text{rad}}{\text{min}}}\right)$$
$$= 0.83\text{ ft} = 9.96\text{ in}$$

Selecting a driver sheave of 12.20 in, from Table 12.1, yields a belt speed of:

$$v_b = \frac{D_1}{2}\omega_1 = \frac{10\text{ in}}{2}10{,}838\frac{\text{rad}}{\text{min}}$$
$$= 54190\frac{\text{in}}{\text{min}} = 4515\text{ fpm}$$

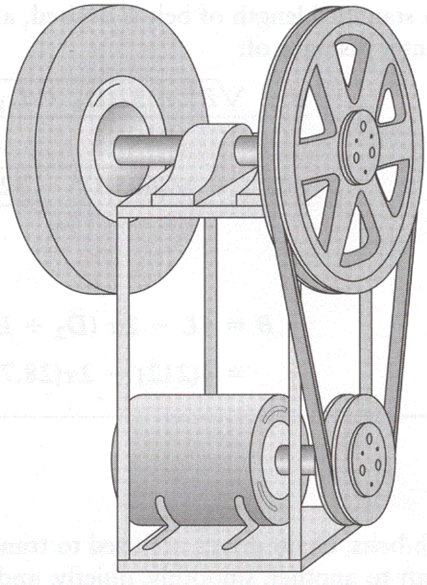


Figure 12.6 Grinding wheel for Example Problem 12.2.

From equation 12.8, the desired velocity ratio is determined by:

$$VR = \frac{\omega_1}{\omega_2} = \frac{10{,}838\frac{\text{rad}}{\text{min}}}{3770\frac{\text{rad}}{\text{min}}} = 2.87$$

And the resulting driven sheave diameter is calculated as follows:

$$D_2 = (VR)(D_1) = 2.87(10 \text{ in}) = 28.7 \text{ in}$$

The closest stock sheave of 29 in is selected. Rewriting equation 12.8, the actual grinding wheel speed is:

$$\omega_2 = \frac{\omega_1 D_1}{D_2} = \frac{(1725 \text{ rpm})(10 \text{ in})}{29 \text{ in}} = 595 \text{ rpm}$$

The suggested center distance for belt drives is:

$$D_2 < C < 3(D_1 + D_2)$$

Substituting values for this application yields:

$$28.7 \text{ in} < C < 116.0 \text{ in}$$

A mid value of 72 in is tentatively selected. Substituting into Equation 12.1 gives:

$$L = 2C + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C}$$

$$= 2(72 \text{ in}) + 1.57(28.7 + 9.96) + \frac{(28.7 - 9.96)^2}{4(72)} = 205.9 \text{ in}$$

Since a standard length of belt is desired, a 212 in belt will be used. This requires a true center distance of:

$$C = \frac{B + \sqrt{B^2 - 32(D_2 - D_1)^2}}{16}$$

$$= \frac{605.1 + \sqrt{(605.1)^2 - 32(28.7 - 9.96)^2}}{16} = 75.1 \text{ in}$$

where

$$B = 4L - 2\pi\,(D_2 + D_1)$$

$$= 4(212) - 2\pi(28.7 + 9.96) = 605.1$$

12.5 CHAINS

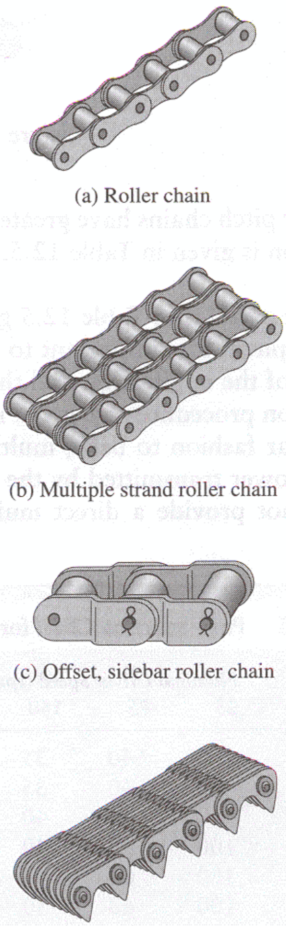
As with belts, chain drives are used to transmit rotational motion and torque from one shaft to another, smoothly, quietly, and inexpensively. Chain drives provide the flexibility of a belt drive with the positive engagement feature of a gear drive. Therefore, chain drives are well suited for applications with large distances between the respective shafts: slow speed and high torque.

Compared to other forms of power transmission, the advantages of chain drives include:

- They are less expensive than gear drives.
- They have no slippage, as with belts, and provide a more efficient power transmission.
- They have flexible shaft center distances, where gear drives are restricted.
- They are more effective at lower speeds than belts.
- They have lower loads on the shaft bearings because initial tension is not required as with belts.
- They have a longer service life and do not deteriorate with factors such as heat, oil, or age, as do belts.
- They require little adjustment, while belts require frequent adjustment.

Chains are made from a series of interconnected links. Many types of chain designs are commercially available and are listed below.

1. A *roller chain* is shown in Figure 12.7A. This is the most common type of chain used for power transmission. Large roller chains are rated to over 600 hp. The roller chain design provides quiet and efficient operation, but must be lubricated.
2. A *multiple-strand roller chain* is shown in Figure 12.7B. This design uses multiple standard roller chains built into parallel strands. This increases the power capacity of the chain drive.
3. An *offset sidebar roller chain* is shown in Figure 12.7C. This is less expensive than a roller chain, but has slightly less power capability. It also has an open



(a) Roller chain

(b) Multiple strand roller chain

(c) Offset, sidebar roller chain

(d) Silent chain

Figure 12.7 Types of chains.

construction that allows it to withstand dirt and contaminants, which can wear-out other chains. These chains are often used on construction equipment.

4. An *inverted tooth, silent chain* is shown in Figure 12.7D. This is the most expensive chain to manufacture. It is can be efficiently used in applications that require high-speed, smooth, and quiet power transmission. Lubrication is required to keep these chains in reliable operation. They are common in machine tools, pumps, and power drive units.

Chains are classified by a *pitch, p,* which is the distance between the link pins that connect the adjacent links. The pitch is illustrated in Figure 12.8. Roller chains have a size designation ranging from 25 to 240. This designation is the pitch of the chain in one-eightieth of an inch. Thus, a 120 chain has a pitch of 120/80 or 1-1/2 inches.

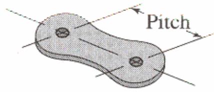


Figure 12.8 Chain pitch.

The larger pitch chains have greater power capacity. A rough guide to roller chain pitch selection is given in Table 12.5. These power values are listed as "horsepower per chain."

It must be noted that Table 12.5 gives only a rough guide to selecting an appropriate chain pitch. It is important to select the most suitable chain drive with a detailed study of the application and the power transmission requirements. These detailed selection procedures are given in the manufacturers' catalogs.

In a similar fashion to belts, multiple-strand chains can be used to increase the amount of power transmitted by the chain drive. However, using a multiple-strand chain does not provide a direct multiple of the single-strand capacity. When the

TABLE 12.5 Pitch Selection Chart for Roller Chains

	Nominal Driver Speed (rpm)								
HP per chain	25	75	150	300	600	900	1200	1800	2400
1/2	60	40	35	35	—	—	—	—	—
3/4	80	40	35	35	—	—	—	—	—
1	80	40	40	35	35	—	—	—	—
1-1/2	100	50	40	40	35	35	—	—	—
2	100	60	50	40	40	35	—	—	—
3	100	60	50	40	40	35	35	—	—
5	120	80	60	50	50	40	35	35	35
7-1/2	140	80	80	60	50	40	35	40	40
10	140	100	80	60	50	50	40	40	50
15	160	100	100	80	60	50	50	50	—
20	180	120	100	80	60	60	50	60	—
25	200	140	100	80	80	60	60	80	—
30	200	140	100	100	80	60	60	—	—
40	240	160	120	100	80	80	8	—	—
50	240	180	140	100	100	80	100	—	—
60	—	180	160	120	100	100	—	—	—
75	—	200	160	120	100	100	—	—	—
100	—	200	180	140	100	120	—	—	—
125	—	240	200	160	140	—	—	—	—
150	—	—	200	160	160	—	—	—	—
200	—	—	240	180	160	—	—	—	—
250	—	—	240	200	—	—	—	—	—

TABLE 12.6 Multi-Strand Factor

Number of roller chain strands	2	3	4	5	6	8	10
Multi-strand factor	1.7	2.5	3.3	3.9	4.6	6.2	7.5

chain drive requires multiple strands, equation 12.9 is used to calculate the power transmitted through each chain. A multi-strand factor has been experimentally determined and is tabulated in Table 12.6:

(12.9)
$$\text{Power per chain} = \frac{\text{total power transmitted}}{\text{multi-strand factor}}$$

Sprockets are the toothed wheels connected to the shafts. The sprocket teeth are designed with geometry that conforms to the chain link. This positive engagement prohibits slippage. Sprockets are classified with a pitch diameter, which is the diameter across the middle of the teeth that corresponds with the center of the chain. Stock sprockets are available with virtually any integer number of teeth. A sprocket, designed to mate with a roller chain, is shown in Figure 12.9.

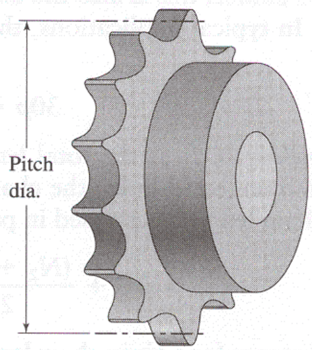


Figure 12.9 Roller chain sprocket.

12.6 CHAIN DRIVE GEOMETRY

The basic geometry in a chain drive is virtually identical to that of a belt drive, as shown in Figure 12.10.

The *number of teeth, N,* in the sprocket is a commonly referenced property. Sprockets should have at least 17 teeth, unless they operate at very low speeds, under 100 rpm. The larger sprocket should normally have no more than 120 teeth.

As before, the *pitch diameter, D,* of the sprocket is measured to the point on the teeth where the center of the chain rides. This is slightly smaller that the outside diameter of the sprocket. Note that the diameters shown for the sprockets in Figure

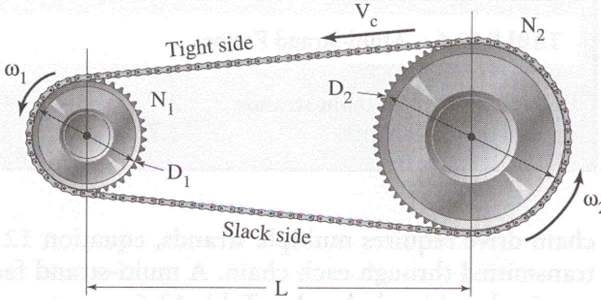


Figure 12.10 Chain drive geometry.

12.10 are the pitch diameters. The pitch diameter of a sprocket with N teeth for a chain with a pitch of p is determined by:

(12.10)
$$D = \frac{p}{\sin\left(\frac{180°}{N}\right)}$$

The *center distance,* C, is the distance between the center of the driver and driven sprockets. Of course, this is also the distance between the two shafts coupled by the chain drive. In typical applications, the center distance should be in the following range:

$$30p < C < 50p$$

The *chain length,* L, is the total length of the chain. Because the chain is comprised of interconnected links, the chain length must be an integral multiple of the pitch. The chain length, expressed in pitches, can be computed as:

(12.11)
$$L = 2C + \frac{(N_2 + N_1)}{2} + \left\{\frac{(N_2 - N_1)^2}{4\pi^2 C}\right\}$$

The center distance for a given chain length, expressed in pitches, can be computed as:

(12.12)
$$C = \frac{1}{4}\left[L - \frac{(N_2 + N_1)}{2} + \sqrt{\left\{\frac{L - (N_2 - N_1)}{2}\right\}^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}}\right]$$

The *angle of contact,* θ, is a measure of the angular engagement of the chain on each sprocket. It can be computed as:

(12.13)
$$\theta_1 = 180° - 2\sin^{-1}\left\{\frac{(N_2 - N_1)}{2C}\right\}$$

(12.14)
$$\theta_2 = 180° + 2\sin^{-1}\left\{\frac{(N_2 - N_1)}{2C}\right\}$$

Again, C is the center distance, expressed in pitches. Chain manufacturers suggest keeping the angle of contact greater than 120°, when possible.

Finally, when in operation, chains have a tight side and a slack side. In most applications, chain drives should be designed so that the slack side is on the bottom or lower side. Due to the direction of shaft rotation and the relative positions of the drive and driven shafts, the arrangement shown in Figure 12.10 has the slack side on the bottom.

12.7 CHAIN DRIVE KINEMATICS

Once again, the *velocity ratio, VR,* is defined as the angular speed of the driver sprocket (sprocket 1) divided by the angular speed of the driven sprocket (sprocket 2). Using the same derivations as for gear and belt drives, the velocity ratio consists of:

(12.15)
$$VR = \frac{\omega_{\text{driver}}}{\omega_{\text{driver}}} = \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} = \frac{N_2}{N_1}$$

Because a ratio is valid regardless of units, the velocity ratio can be defined in terms of revolutions per minute, radians per time, or any other convenient set of rotational velocity units.

Many industrial applications require chains to reduce the speed of a power source. Therefore, it is typical to have velocity ratios greater than one. As can be seen from equation 12.16, this indicates that the drive sprocket rotates faster than the driven sprocket, which is the case in speed reductions.

Similar to belts, the *chain speed,* v_c, is defined as the linear velocity of the chain. The magnitude of this velocity corresponds to the magnitude of the linear velocity of a point on the pitch diameter of each sprocket. Consistent with the belt speed, the chain speed can be computed as:

(12.16)
$$v_c = \frac{D_1}{2}\omega_1 = \frac{D_2}{2}\omega_2$$

In equation 12.16, the rotation velocities must be stated in radians per unit time.

Lubrication for the chain is important in maintaining long life for the drive. Recommended lubrication methods are primarily dictated by the speed of the chain. The recommended lubrication is as follows:

- Low speed ($v_c < 650$ fpm): manual lubrication, where the oil is periodically applied to the links of the chain.
- Moderate speed ($650 < v_c < 1500$ fpm): bath lubrication, where the lowest part of the chain dips into a bath of oil.
- High speed ($1500 \text{ fpm} < v_c$): oil stream lubrication, where a pump delivers a continuous stream onto the chain.

EXAMPLE PROBLEM 12.3

A single-strand roller chain drive connects a 10-hp engine to a lawn waste chipper/shredder. As the engine operates at 1200 rpm, the shredding teeth should rotate at 240 rpm. The drive sprocket has 18 teeth. Determine an appropriate pitch for

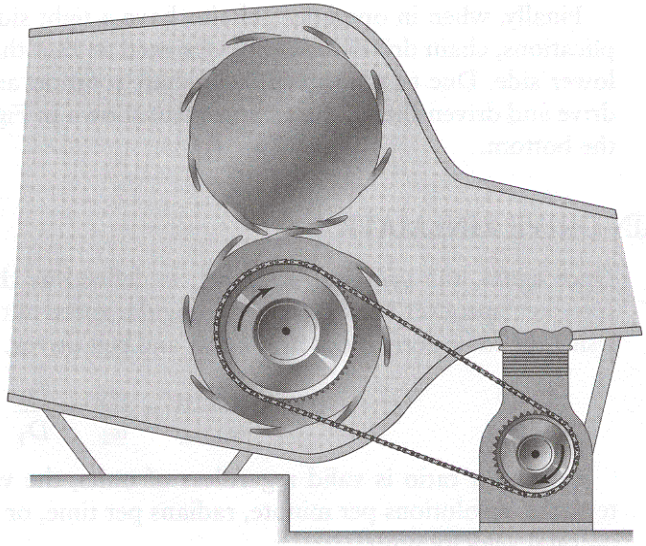


Figure 12.11 Chipper/shredder for Example Problem 12.3.

the chain, the number of teeth and, on the driven sprocket, the pitch diameters of both sprockets and the chain speed.

Solution:

With a 10-hp engine driving a sprocket at 1200 rpm, Table 12.5 specifies that a 40-pitch chain is appropriate.

By rewriting equation 12.14, the number of teeth needed on the driven sprocket can be determined:

$$N_2 = N_1\left(\frac{\omega_1}{\omega_2}\right) = 18\left\{\frac{1200\text{ rpm}}{(240\text{ rpm})}\right\} = 90\text{ teeth}$$

A No. 40 roller chain has a pitch of:

$$p = \frac{40}{80} = \frac{1}{2}\text{ in}$$

From equation 12.9, the pitch diameters of the sprockets are:

$$D_1 = \frac{p}{\sin\left(\frac{180°}{N_1}\right)} = \frac{0.5\text{ in}}{\sin\left(\frac{180°}{18\text{ teeth}}\right)} = 2.88\text{ in}$$

$$D_2 = \frac{p}{\sin\left(\frac{180°}{N_2}\right)} = \frac{0.5\text{ in}}{\sin\left(\frac{180°}{90\text{ teeth}}\right)} = 14.33\text{ in}$$

The chain speed can be calculated from equation 12.16:

$$\omega_1 = 1200\frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 10{,}838\frac{\text{rad}}{\text{min}}$$

$$v_c = \left(\frac{D_1}{2}\right)\omega_1 = \left(\frac{2.88 \text{ in}}{2}\right)10{,}838\frac{\text{rad}}{\text{min}} = 15603\frac{\text{in}}{\text{min}} = 1300 \text{ fpm}$$

With a chain speed of 1300 fpm, a bath lubrication system for the chain is desired.

PROBLEMS

Belt Drive Problems

Use the following information to solve Problems 12–1 through 12–4. A motor is used with a belt drive to rotate the crank of a mechanism for an industrial sewing machine. The 1-hp motor runs at 1725 rpm. The diameter of the motor sheave is 3.5 in and the sheave on the sewing machine crank is 8 in.

12–1. Determine an appropriate belt size and the speed of the sewing machine crank.

12–2. Calculate the belt speed and compare this to the ideal range for V-belts.

12–3. A center distance of 23 in is used. Compare this with the ideal range and calculate the associated belt length.

12–4. The belt has a maximum rating of 3 hp. When a center distance of 23 in is used, calculate the contact angle of the belt around the smaller sheave and determine the power that can be transmitted in this drive.

12–5. Two sheaves have diameters of 5 and 12 in, respectively. Determine the center distance of the drive utilizing a 72-in long belt. Compare this center distance to the ideal range.

12–6. Two sheaves have diameters of 8 and 12 in, respectively. Determine the center distance of the drive utilizing an 88-in long belt. Compare this center distance with the ideal range.

12–7. Two sheaves have diameters of 8 and 24 in, respectively. Determine the center distance of the drive utilizing a 104-in long belt. Compare this center distance with the ideal range.

Use the following information to solve Problems 12–8 through 12–11. A belt drive transfers power from an 7.5-hp electric motor to an exhaust fan. The diameters of the motor and fan sheaves are 5 in and 12 in, respectively. The motor is rated at 1150 rpm.

12–8. Determine an appropriate belt size and the speed of the fan.

12–9. Calculate the belt speed and compare this to the ideal range for V-belts.

12–10. The belt has a maximum rating of 10 hp. When a center distance of 39 in is used, calculate the contact angle of the belt around the smaller sheave and determine the power that can be transmitted in this drive.

12–11. A center distance of 39 in is used. Compare this with the ideal range and calculate the associated belt length.

Use the following information for Problems 12–12 through 12–15. A drill press drives with a 3-hp, 1725 rpm motor. The spindle of a drill press should operate at approximately 1100 rpm. The spindle sheave diameter is 12 in.

12–12. Determine the correct motor sheave diameter, which is available only in integer-inch sizes, and then calculate the resulting spindle speed.

12–13. Calculate the belt speed and compare this to the ideal range for V-belts.

12–14. The belt is rated at 5 hp. The center distance is 14 in; calculate the contact angle of the belt around the smaller sheave and determine the power that can be transmitted in this drive.

12–15. A center distance of 14 in is used. Compare this with the ideal range for belts, and calculate the associated belt length.

Use the following information to solve Problems 12–16 through 12– 18. A 1–1/2-hp, 1175-rpm motor operates a furnace blower at approximately 800 rpm.

12–16. Find suitable sheave diameters of stock sheave, which are available in integer-inch sizes.

12–17. The belt is rated at 2 hp. Sheave pitch diameters of 8 and 11 in are used with a center distance of 10 in. Calculate the contact angle of the belt around the smaller sheave and determine the power that can be transmitted in this drive.

12–18. Sheave pitch diameters of 8 and 11 in are used qith a center distance of 10 in. Compare the center distance with the ideal range for belt drives, and calculate the associated belt length.

Use the following information to solve Problems 12–19 through 12–21. A 1-hp, 1750-rpm motor operates a woodworking ban saw at approximately 600 rpm.

12–19. Find an appropriate belt size and determine suitable stock sheave diameters, which are available in integer-inch pitch diameters.

12–20. The belt is rated at 1-1/2 hp. Sheave diameters of 5 and 13 in are used with an 85-in belt. Calculate the required center distance, the contact angle of the belt around the smaller sheave, and determine the power that can be transmitted in this drive.

12–21. Sheave diameters of 5 and 11 in are used with a center distance of 20 in. Compare this center distance with the ideal range and calculate the associated belt length.

Chain Drive Problems

Use the following information to solve Problems 12–22 through 12–25. A 25-hp diesel engine drives a gear box which used a chain to drive to a concrete mixer. The gear box output shaft and, thus the driving sprocket, operates at 180 rpm. The pitch diameter of the driving sprocket is approximately 6 in, and the diameter of the sprocket on the concrete mixer is approximately 36 in.

12–22. Determine the speed of the concrete mixer.

12–23. Calculate the chain speed and determine the type of lubrication recommended.

12–24. An 80-pitch chain is used with a 40-in center distance. Calculate the number of teeth on the sprockets and the contact angle of chain around the smaller sheave.

12–25. A No. 80 chain is used with sprockets of 20 and 120 teeth. Specify a suitable center distance and calculate the associated number of links required to fabricate the chain.

12–26. Two sprockets have 20 and 48 teeth. Determine the center distance of the drive utilizing a No. 60 chain constructed with 90 links.

12–27. Two sprockets have 20 and 64 teeth. Determine the center distance of the drive utilizing a No. 100 chain constructed with 130 links.

12–28. Two sprockets have 18 and 52 teeth. Determine the center distance of the drive utilizing a No. 80 chain constructed with 80 links.

Use the following information to solve Problems 12–29 through 12–32. A 220-rpm shaft is used with a No. 40 chain to drive a liquid agitator. The agitator should operate at approximately 60 rpm. The agitator sprocket has 50 teeth.

12–29. Determine an appropriate shaft sprocket, and then calculate the sprocket pitch diameters and the resulting agitator speed.

12–30. Sprockets of 18 and 50 teeth have been selected. Calculate the chain speed and determine the type of lubrication recommended.

12–31. Sprockets of 18 and 50 teeth have been selected, along with a center distance of 18.3 in. Calculate the pitch diameters of the sprockets and the contact angle of chain around the smaller sprocket.

12–32. Sprockets of 18 and 50 teeth have been selected, along with a center distance of 18.3 in. Compare this center distance with the ideal range, and calculate the associated number of links required to fabricate the chain.

12–33. A shaft rotating at 120 rpm transfers 1-1/2 hp to operate a pump at 50 ± 1 rpm. Determine a suitable chain pitch and a pair of sprockets. Also determine an appropriate center distance and the number of links required to construct the chain.

12–34. A shaft rotating at 275 rpm transfers 10 hp to a mining conveyor at 65 ± 1 rpm. Determine a suitable chain pitch and a pair of sprockets. Also determine an appropriate center distance and the number of links required to construct the chain.

CASE STUDIES

12–1. The device shown in Figure C12.1 drives a chute that funnels individual beverage bottles into twelve-pack containers. Pin *C* is rigidly attached to one link of the chain. Yoke *D* is rigidly welded to rod *E*, which extends to the chute (not shown). Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

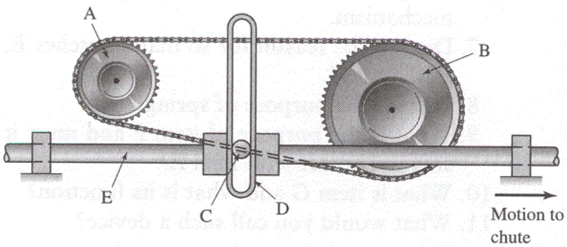


Figure C12.1 (Courtesy, Industrial Press)

1. As sprocket *A* drives clockwise, determine the motion of sprocket *B*.
2. As sprocket *A* drives clockwise, specify the instantaneous motion of pin *C*.
3. As sprocket *A* drives clockwise, specify the instantaneous motion of yoke *D*.
4. How far must sprocket *A* rotate to move pin *C* onto a sprocket?
5. What happens to the motion of rod *E* when the pin rides onto the sprocket?
6. What happens to rod *E* when pin *C* is on the top potion of the chain drive?
7. Discuss the overall motion characteristics of rod *E*.

12–2. The sheave shown in Figure C12.2 drives shaft *A*, which drives a log splitter (not shown). Notice that the sheave is split into two halves, labeled *B* and *C*. These two halves are threaded together at *D*. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

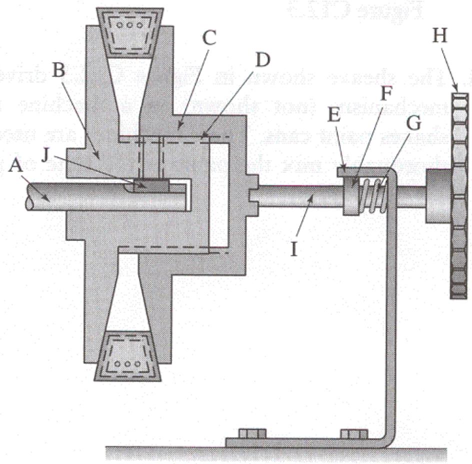


Figure C12.2 (Courtesy, Industrial Press)

1. As handle *H* rotates, what is the motion of shaft *I*?
2. As handle *H* rotates, what is the motion of the right half, *C*, of the sheave?
3. What is the resulting effect on the sheave by rotating handle *H*?
4. What is item *J* and what is its function?
5. What is the purpose of item *F* and must it stay in contact with item *E*?
6. What is item *G* and what is its function?
7. What would you call such a device?

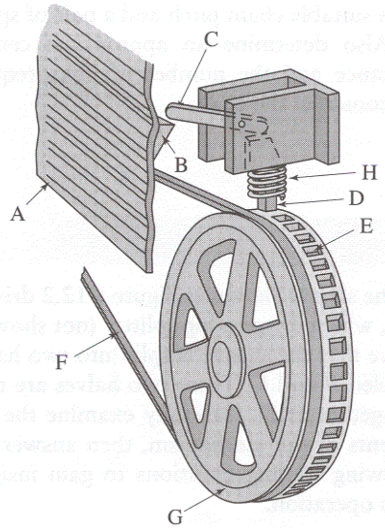


Figure C12.3

12–3. The sheave shown in Figure C12.3 drives a mechanism (not shown) in a machine that shakes paint cans. These machines are used to thoroughly mix the paint at the time of purchase and are common at most paint retail locations. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into its operation.

1. As tab *B* is forced upward into lever *C*, determine the motion of item *D*.
2. As tab *B* is forced upward into lever *C*, determine the resulting action of sheave *G*.
3. As tab *B* is forced upward into lever *C*, determine the resulting action of the paint shaking mechanism.
4. As tab *B* is forced upward into lever *C*, determine the resulting action of belt *F*.
5. As tab *B* is forced upward into lever *C*, determine the resulting action of the driving sheave.
6. Item *A* is the door to the paint shaking compartment; discuss the purpose of the mechanism.
7. Discuss the reason for so many notches *E*, in sheave *G*.
8. Discuss the purpose of spring *H*.
9. What is the purpose of item *F* and must it stay in contact with item *E*?
10. What is item *G* and what is its function?
11. What would you call such a device?

13 Screw Mechanisms

OBJECTIVES *Upon completion of this chapter, the student will be able to:*

1. Describe the operation of a screw mechanism.
2. Become familiar with thread features and standard thread forms.
3. Understand the function of a ball screw.
4. Determine whether a thread is self-locking.
5. Compute the torque required to rotate a screw and the efficiency of a screw joint.
6. Compute the kinematic relationships of a general screw mechanism.
7. Understand the operation of a differential screw and compute the kinematic relationships.
8. Understand the operation of an auger and compute the kinematic relationships.

13.1 INTRODUCTION

In general, screw mechanisms are designed to convert rotary motion to linear motion. Consider a package for a stick deodorant. As the knob turns, the deodorant stick either extends or retracts into the package. Inside the package, a screw turns, which pushes a nut and the deodorant stick along the thread. Thus, a "disposable" screw mechanism is used in the deodorant package. This same concept is commonly used in automotive jacks, some garage door openers, automotive seat adjustment mechanisms, and milling machines tables.

The determination of the kinematics and forces in a screw mechanism is the purpose of the chapter. Because the motion of a nut on a thread is strictly linear, graphical solutions do not provide any insight. Therefore, only analytical techniques are practical and are introduced in this chapter.

13.2 THREAD FEATURES

For a screw to function, there must be two mating parts, one with an internal thread and the other with an external thread. The external threads are turned on the surface of a shaft or stud, such as a bolt or screw. The internal threads can be tapped into a part, such as a cast housing or, more commonly, inside a nut. Whenever possible, the selection of a thread should be standard to improve interchangeability for maintenance or replacement. Threads, whether internal or external, are classified with the following features.

Two most common features of a thread are the pitch and pitch diameter. The *pitch, p,* is the distance measured parallel to the axis from a point on one thread to the corresponding point on the adjacent thread. The *pitch diameter, D,* is the diameter measured from a point halfway between the tip and root of the thread profile, through the axis and to the corresponding point on the opposite side. Figure 13.1 illustrates these properties.

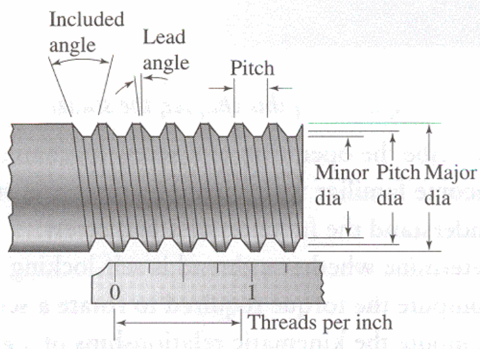


Figure 13.1 Thread profile.

Other pertinent features of a screw thread (Figure 13.1) include the *major diameter,* the *minor diameter,* the *lead angle,* and the *included angle.* In the United States Customary Unit System, the number of *threads per inch, n,* along the length of the screw is more common than the pitch. The threads-per-inch value is related to the pitch through the following equation:

(13.1) $$n = \frac{1}{p}$$

13.3 THREAD FORMS

Thread form defines the shape of the thread. The features illustrated in the previous section, the thread features were illustrated on a unified thread form. Regardless, these definitions are applicable to all thread forms. The most popular thread forms include the following:

1. *Unified threads* are the most common threads used on fasteners and small mechanisms. They are also commonly used for positioning mechanisms. Figure 13.2A illustrates the profile of a unified thread. It is described as a sharp, triangular tooth. The dimensions of a unified thread have been standardized and are given in Table 13.1. Unified threads are designated as either coarse pitch (UNC) or fine pitch (UNF).

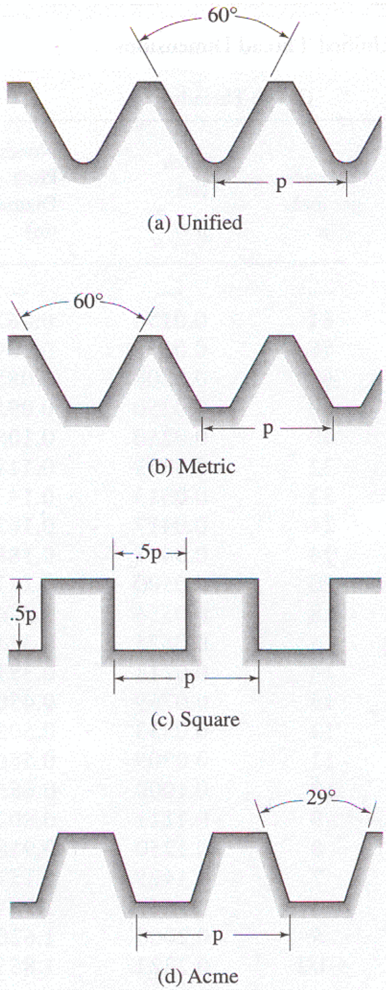


Figure 13.2 Thread forms.

A standard unified thread is specified by the size, threads per inch, and whether it a coarse or fine pitch. Standard thread designations would appear as:

10 - 32 UNF
1/2 - 13 UNC

2. *Metric thread* forms are also described as a sharp, triangular shape, but with a flat root. However, the standard dimensions ar emetric measure and coordinated through the International Organization for Standardization (ISO) . The tread shape is shown in Figure 13.2B. Standard metric thread dimensions are given in Table 13.2.

TABLE 13.1 Standard Unified Thread Dimensions

		Coarse Threads			Fine Threads		
Size	Nominal Major Diamter in	Threads per inch, n	Pitch (in) $p = \frac{1}{n}$	Nominal Pitch Diameter (in)	Threads per inch, n	Pitch (in) $p = \frac{1}{n}$	Nominal Pitch Diameter (in)
0	0.0600	—	—	—	80	0.0125	0.0519
1	0.0730	64	0.0156	0.0629	72	0.0139	0.0640
2	0.0860	56	0.0179	0.0744	64	0.0156	0.0759
3	0.0990	48	0.0208	0.0855	56	0.0179	0.0874
4	0.1120	40	0.0250	0.0958	48	0.0208	0.0985
5	0.1250	40	0.0250	0.1088	44	0.0227	0.1102
6	0.1380	32	0.0313	0.1177	40	0.0250	0.1218
8	0.1640	32	0.0313	0.1437	36	00278	0.1460
10	0.1900	24	0.0417	0.1629	32	0.0313	0.1697
12	0.2160	24	0.0417	0.1889	28	0.0357	0.1928
1/4	0.2500	20	0.0500	0.2175	28	0.0357	0.2268
5/16	0.3125	18	0.0556	0.2764	24	0.0417	0.2854
3/8	0.3750	16	0.0625	0.3344	24	0.0417	0.3479
7/16	0.4375	14	0.0714	0.3911	20	0.0500	0.4050
1/2	0.5000	13	0.0769	0.4500	20	0.0500	0.4675
9/16	0.5625	12	0.0833	0.5084	18	0.0556	0.5264
5/8	0.6250	11	0.0909	0.5660	18	0.0556	0.5889
3/4	0.7500	10	0.1000	0.6850	16	0.0625	0.7094
7/8	0.8750	9	0.1111	0.8028	14	0.0714	0.8286
1	1.0000	8	0.1250	0.9188	12	0.0833	0.9459
1-1/4	1.2500	7	0.1429	1.1572	12	0.0833	1.1959
1-1/2	1.5000	6	0.1667	1.3917	12	0.0833	1.4459
1-3/4	1.7500	5	0.2000	1.6201	—	—	—
2	2.0000	4-1/2	0.2222	1.8557	—	—	—

A standard metric thread is specified by an "M" metric designation, the nominal major diameter and pitch. A standard thread designation would appear as:

$$M10 \times 1.5$$

3. *Square threads,* as the name implies, are a square, flat top thread. They are strong and were originally designed to transfer power. A square thread form is shown in Figure 13.2C. Although they efficiently transfer large loads, these threads are difficult to machine with perpendicular sides. The square threads have been generally replaced by ACME threads.
4. *ACME threads* are similar to square threads, but with sloped sides. They are commonly used when rapid movement is required or large forces are transmitted. An ACME thread form is shown in Figure 13.2D. The standard ACME

TABLE 13.2 Standard Metric Thread Dimensions

Nominal Major Diamter (mm)	Coarse Threads: Pitch (mm) $p = \frac{1}{n}$	Coarse Threads: Nominal Pitch Diameter (mm)	Fine Threads: Pitch (mm) $p = \frac{1}{n}$	Fine Threads: Nominal Pitch Diameter (mm)
1	0.25	0.84	—	—
1.6	0.35	1.37	0.20	1.47
2	0.40	1.74	0.25	1.84
2.5	0.45	2.20	0.35	2.27
3	0.50	2.67	0.35	2.77
4	0.70	3.54	0.50	3.67
5	0.80	4.47	0.50	4.67
6	1.00	5.34	0.75	5.51
8	1.25	7.18	1.00	7.34
10	1.50	9.01	1.25	9.18
12	1.75	10.85	1.25	11.18
16	2.00	14.68	1.50	15.01
20	2.50	18.35	1.50	19.01
24	3.00	22.02	2.00	22.68
30	3.50	27.69	2.00	28.68
36	4.00	33.36	3.00	34.02
42	4.50	39.03	—	—
48	5.00	44.70	—	—

screw thread dimensions are given in Table 13.3. This thread is the most common form used in screw mechanisms for industrial machines. Advantages are low cost and ease of manufacture. Disadvantages include low efficiencies, as will be discussed later, and difficulty in predicting service life.

13.4 BALL SCREWS

Ball screws have also been designed to convert rotary motion of either the screw or nut into relatively slow linear motion of the mating member, along the screw axis. However, a ball screw has drastically less friction than a traditional screw configuration. The sliding contact between the screw and nut have been replaced with rolling contact of balls in grooves along the screw. Thus, a ball screw requires less power to drive a load. A ball screw is shown in Figure 13.3.

The operation of a ball screw is smooth because the rolling balls eliminate the "slip-stick" motion caused by the friction of a traditional screw and nut. However, because of the low friction of a ball screw, a brake must usually be used to hold the load in place.

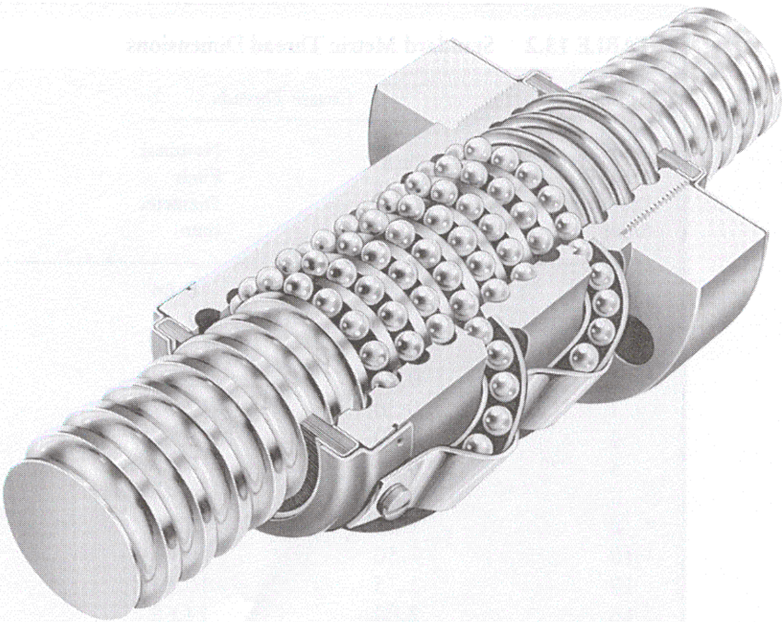


Figure 13.3 Ball screw.

The kinematics of a ball screw are identical to those of a traditional screw. Therefore, a distinction is not required when performing a kinematic analysis. The following concepts apply to both traditional and ball screws.

13.5 LEAD

In determining the motion of a screw mechanism, the lead of the screw is a critical parameter and must be understood. The *lead, L,* is the distance along the screw axis that a nut travels with one revolution of the screw. For most screws, the lead is identical to the pitch. However, screws are available with single or double threads. Thus, the *number of threads,* N_t, superimposed on a screw is an important property. The concept of multiple threads superimposed on a single screw is illustrated in Figure 13.4.

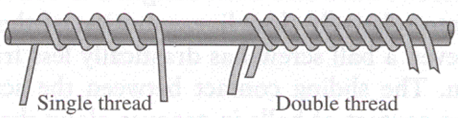


Figure 13.4 Multiple thread concept.

The lead can be computed as:

$$L = N_t p \tag{13.2}$$

TABLE 13.3 Standard ACME Thread Dimensions

Nominal Major Diamter (in)	Threads per inch, n	Pitch (in) $p = \frac{1}{n}$	Nominal Pitch Diameter (in)
1/4	16	0.0625	0.2043
5/16	14	0.0714	0.2614
3/8	12	0.0833	0.3161
7/16	12	0.0833	0.3783
1/2	10	0.1000	0.4306
5/8	8	0.125	0.5408
3/4	6	0.1667	0.6424
7/8	6	0.1667	0.7663
1	5	0.2000	0.8726
1-1/8	5	0.2000	0.9967
1-1/4	5	0.2000	1.1210
1-3/8	4	0.2500	1.2188
1-1/2	4	0.2500	1.3429
1-3/4	4	0.2500	1.5916
2	4	0.2500	1.8402
2-1/4	3	0.3333	2.0450
2-1/2	3	0.3333	2.2939
2-3/4	3	0.3333	2.5427
3	2	0.5000	2.7044
3-1/2	2	0.5000	3.2026
4	2	0.5000	3.7008
4-1/2	2	0.5000	4.1991
5	2	0.5000	4.6973

A *lead angle*, λ, can be defined as the angle of inclination of the threads. It can be computed from a trigonometric relationship to the other screw features:

(13.3)
$$\tan \lambda = \frac{N_t p}{\pi D} = \frac{L}{\pi D}$$

When a screw thread is very steep and has large lead angles, the torque required to push a load along a screw can become large. Typical screws have lead angles that range from approximately 2° to 6°.

Additionally, small lead angles prohibit a load to "slide down a screw" due to gravity. The friction force and shallow thread slope combine to lock the load in place. This is known as self-locking and is desirable for lifting devices. For example, a car jack requires that the load be held in an upward position, even as the power source is removed. When the thread is self-locking, the load is locked in an upright position. This braking action is used in several mechanical devices, but the strength of the thread, and the predictability of friction must be analyzed to ensure safety.

Mathematically, the condition that must be met for self-locking is as follows:

(13.4) $$\mu > \tan \lambda$$

In Equation 13.4, μ is the coefficient of friction of the nut/thread interface. For traditional threads, common values for the coefficient of friction are:

$\mu = 0.10$ for very smooth, well-lubricated surfaces
$\mu = 0.15$ for general machined screws with well-lubricated surfaces
$\mu = 0.20$ for general machined screws with ordinary surfaces

Special surface treatments and coatings can reduce these values by at least half. Ball screws, with inherent low friction, are virtually never self-locking.

13.6 SCREW KINEMATICS

From a kinematic viewpoint, the screw joint connects two bodies, and couples two degrees of freedom. Typically the joint is configured such that one body will translate with a rotational input from the other body. Depending on the constraints of the two bodies, the following relative motions are possible:

- *Translation of the nut as the screw rotates:* Occurs when the screw is unable to translate and nut is unable to rotate.
- *Translation of the screw as the nut rotates:* Occurs when the nut is unable to translate and screw is unable to rotate.
- *Translation of the screw as it rotates:* Occurs when the nut is fully constrained against any motion.
- *Translation of the nut as it rotates:* Occurs when the screw is fully constrained against any motion.

Regardless of the actual system configuration, the relative motion is the same. A given rotation produces a resulting translation. Therefore, equations are developed to describe the relative motion, and the absolute motion can be determined when examining the actual system configuration. A notation is made where:

- A is the part which is allowed to rotate.
- B is the other part joined by the screw joint.

As previously defined, the lead of a screw is the distance along the screw axis that a nut travels with one revolution of the screw. Therefore, the relative displacement of B relative to A is calculated as follows:

(13.5) $$s_{B/A} = L\theta_A$$

Differentiating yields equations for velocity and acceleration:

(13.6) $$v_{B/A} = L\omega_A$$

(13.7) $$a_{B/A} = L\alpha_A$$

Note that the lead, L, is specified as the relative displacement per revolution. Therefore, in this instance, *the angular motion must be specified in revolutions.*

Thus, ω_A should be specified in revolutions per minute (or second) and α_A should be specified in revolutions per squared minute (or squared second).

The direction of the relative motion depends on the hand designation of the thread. Screws and the mating nuts are classified as either right-hand or left-hand. A right-hand thread is most common. For this threaded joint, the screw advances into the nut when the screw rotates clockwise. A right-hand thread slopes downward to the left on an external thread when the axis is horizontal. The slope is opposite on an internal thread. The opposite, left-hand configuration produces the opposite motion.

The following examples illustrate the determination of screw kinematics.

EXAMPLE PROBLEM 13.1

An automated wine press is shown in Figure 13.5. A single thread, 3/4 × 6 ACME screw thread moves the ram. The screw is rotated at 80 rpm. Determine the advancing speed of the ram.

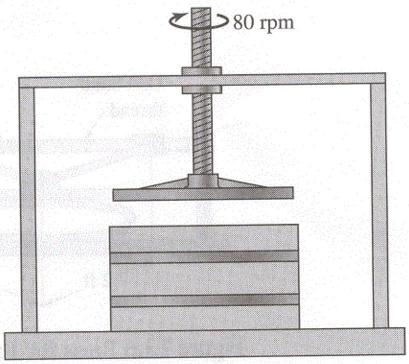


Figure 13.5 Wine press for Example Problem 13.1.

Solution:

In this configuration, the nut is fixed. The screw is allowed to turn and translate. Therefore, according to the developed notation:

Part A is the screw.
Part B is the nut.

A single thread, 3/4 × 6 ACME screw has the following properties:

Number of threads per inch: $n = 6$

Pitch: $p = \dfrac{1}{n} = \dfrac{1}{6} = 0.167$ in

Number of threads: $N_t = 1$

Lead: $L = N_t p = 0.167 \dfrac{\text{in}}{\text{rev}}$

As part A, the screw, rotates with a velocity of 80 rpm, the relative velocity of B (nut) with respect to A (screw) is as follows:

$$v_{B/A} = L\omega_A$$

$$= \left(0.167\frac{\text{in}}{\text{rev}}\right)\left(80\frac{\text{rev}}{\text{min}}\right) = 13.36\frac{\text{in}}{\text{min}}$$

Because the nut is fixed, the computed velocity is the absolute velocity of the advancing screw.

EXAMPLE PROBLEM 13.2

A screw-operated press is shown in Figure 13.6. The screw has a single 1/2 × 10 ACME thread, both in a right-hand and left-hand orientation as shown. The handle rotates counterclockwise at 45 rpm to drive the pressure plate downward. In the position shown, with $\beta = 25°$, determine the velocity of the pressure plate.

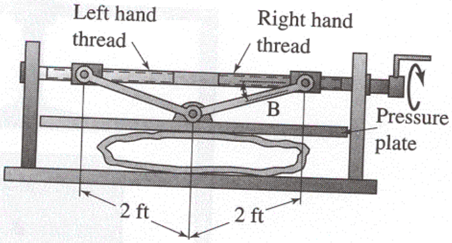


Figure 13.6 Press for Example Problem 13.2.

Solution:

A single thread, 1/2 × 10 ACME screw has the following properties:

Number of teeth per inch: $n = 10$

Pitch: $p = \frac{1}{n} = \frac{1}{10} = 0.10$ in.

Number of threads: $N_t = 1$

Lead: $L = N_t p = 0.10\frac{\text{in}}{\text{rev}}$

A kinematic diagram of this mechanism is shown in Figure 13.7. 13. By calculating the mobility of the mechanism, 5 links are identified. There are also 4 pin joints. Therefore:

$$n = 5 \quad j_p = 5 \text{ (3 pins \& 2 sliding joints)} \quad j_h = 0$$

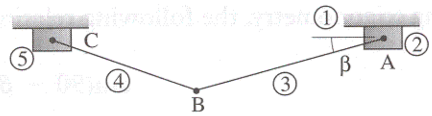


Figure 13.7 Kinematic diagram for Example Problem 13.2.

and:

$$F = 3(n-1) - 2j_p - j_h$$
$$= 3(5-1) - 2(5) - 0 = 12 - 10 = 2$$

With two degrees of freedom, both nuts must be driven. The screw configuration shown in Figure 13.6 does drive both nuts.

From Figure 13.6, the screw is free to rotate, but fixed against axial displacement. With the opposite hand threads, the two nuts also move in opposite directions. Therefore, the relative velocity of the nut with respect to the screw (Equation 13.6) is the absolute velocity of each advancing nut. As the screw rotates with a velocity of 45 rpm, the nut advances at a rate of:

$$v_{\text{nut/screw}} = L\omega_{\text{screw}}$$
$$= \left(0.10\frac{\text{in}}{\text{rev}}\right)\left(45\frac{\text{rev}}{\text{min}}\right) = 4.5\frac{\text{in}}{\text{min}}$$

Thus:

$$v_A = 4.5\frac{\text{in}}{\text{min}}\leftarrow \quad \text{and} \quad v_C = 4.5\frac{\text{in}}{\text{min}}\rightarrow$$

A velocity equation can be written as:

$$v_B = v_A +> v_{B/A} = v_C +> v_{C/A}$$

A velocity diagram is formed from both velocity equations. Notice that because of symmetry, the displacement and velocity of B is vertical (Figure 13.8).

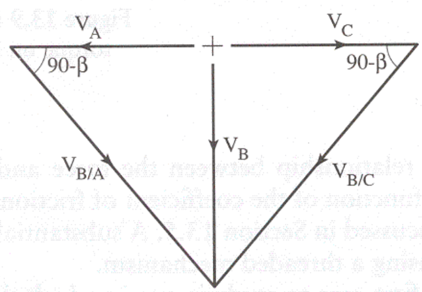


Figure 13.8 Velocity diagram for Example Problem 13.2.

Using trigonometry, the following relationship can be written:

$$\tan(90 - \beta) = \frac{v_B}{v_A}$$

and:

$$v_B = v_A \tan(90 - \beta)$$

for the case shown:

$$\beta = 25°$$

$$v_B = \left(4.5 \frac{\text{in}}{\text{min}}\right) \tan(90° - 20°) = 12.4 \frac{\text{in}}{\text{min}} \downarrow$$

13.7 SCREW FORCES AND TORQUES

The torque and force acting on a screw and nut assembly are shown in Figure 13.9.

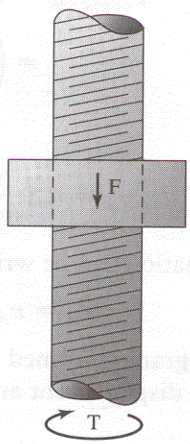


Figure 13.9 Force and torque on a screw.

The relationship between the force and torque have been derived[1] and are a strong function of the coefficient of friction, μ, between the thread and nut. Friction was discussed in Section 13.5. A substantial amount of energy can be lost to friction when using a threaded mechanism.

The first case to study is one in which the motion of the nut occurs in the opposite direction from the applied force acting on a nut. This is commonly referred to a case of lifting or pushing a load. The required torque to accomplish this motion is calculated as follows:

$$T = \left(\frac{FD}{2}\right)\left[\frac{(L + \pi\mu D)}{(\pi D - \mu L)}\right] \tag{13.8}$$

where:

F = applied force on nut
D = pitch diameter of threads
L = lead of threads
μ = coefficient of friction between nut and threads

The second case to be studied is one in which the motion of the nut is in the same direction as the force acting on the nut. in essence the load assists the motion of the nut. This is commonly referred to a case of *lowering a load*. The required torque to accomplish this motion is as follows:

$$T = \left(\frac{FD}{2}\right)\left[\frac{(\pi\mu D - L)}{(\pi D + \mu L)}\right] \tag{13.9}$$

An *efficiency, e,* can be defined as the percentage of power that is transferred through the threads to the nut. It is the ratio of torque required to raise the load in the absence of friction, to the torque required to raise a load with friction. Again, a closed form equation has been derived for efficiency [1]:

$$e = \left(\frac{L}{\pi D}\right)\left[\frac{(\pi D \cos \alpha - \mu L)}{(\pi\mu D + L \cos \alpha)}\right] \tag{13.10}$$

In addition to the previously defined quantities the *included thread angle, α,* is used. This angle was illustrated in Figure 13.2. Standard values include:

Unified thread: $\alpha = 30°$
Metric thread: $\alpha = 30°$
Square thread: $\alpha = 0°$
ACME thread: $\alpha = 14.5°$

Threaded screws typically have efficiencies between 20 and 50%. Thus, a substantial amount of energy is lost to friction. As opposed to threaded screws, ball screws have efficiencies in excess of 90%. For ball screws, the operational torque equations can be estimated as:

(13.11) To lift a load: $T = 0.177\ FL$

(13.12) To lower a load: $T = 0.143\ FL$

EXAMPLE PROBLEM 13.3

A screw jack mechanism is shown in Figure 13.10. A belt/sheave is used to rotate a nut, mating with a single thread, 1 × 5 ACME screw, to raise the jack. Notice that a pin is used in a groove on the screw to prevent the screw from rotating. The nut ro-

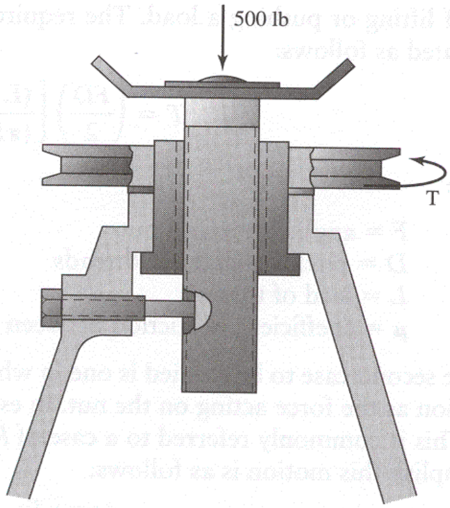


Figure 13.10 Jack for Example Problem 13.3.

tates at 300 rpm. Determine the lifting speed of the jack, the torque required, and the efficiency of the jack.

Solution:

A single thread, 1 × 5 ACME screw has the following properties:

Number of teeth per inch: $n = 5$

Pitch: $p = \frac{1}{n} = \frac{1}{5} = 0.20$ in.

Number of threads: $N_t = 1$

Lead: $L = N_t p = 0.20 \frac{\text{in}}{\text{rev}}$

In Figure 13.10, the nut is fixed from translation. Therefore, the velocity computed with Equation 13.6 is that of the advancing screw. As the nut rotates with a velocity of 300 rpm, the thread advances through the nut at a rate of:

$$v_{\text{screw}} = L\omega_{\text{nut}}$$

$$= \left(0.20\frac{\text{in}}{\text{rev}}\right)\left(300\frac{\text{rev}}{\text{min}}\right) = 60\frac{\text{in}}{\text{min}}\uparrow$$

The torque required to raise the load is dependent on the estimated coefficient of friction between the threads and nut. Because this jack configuration is used in standard industrial settings, a coefficient of friction of 0.2 is assumed. In Table 13.1, the nominal pitch diameter for 1 × 5 ACME threads is 0.8726 in. Also, for all ACME threads, the included angle is 29°. Therefore, the torque can be computed from Equation 13.8:

$$T = \left(\frac{FD}{2}\right)\left[\frac{(L + \pi\mu D)}{(\pi D - \mu L)}\right]$$

$$= \frac{(500 \text{ lb})(0.8726 \text{ in})}{2}\left\{\frac{[(0.20 + \pi(0.2)(0.8726)]}{[\pi(0.8726) - (0.2)(0.2)]}\right\} = 60.4 \text{ in}$$

Finally, efficiency can be computed from Equation 13.10:

$$e = \left(\frac{L}{\pi D}\right)\left[\frac{(\pi D \cos\alpha - \mu L)}{(\pi\mu D + L\cos\alpha)}\right]$$

$$= \frac{(0.2)}{\pi(0.8726)}\left\{\frac{[\pi(0.8726)\cos(29) - (0.2)(0.2)]}{[\pi(0.2)(0.8726) + (0.2)(\cos 29°)]}\right\}$$

$$= 0.24$$

An efficiency of 0.24 reveals that only 24% of the power transferred to the nut is ddelivered into lifting the weight. The remaining 76% is lost in friction. If these values were not acceptable, a ball screw could be substituted for the ACME thread. A ball screw has an efficiency of approximately 90%, but also a significantly higher cost. However, recall that a ball screw is not self locking, and does not maintain the load at an elevated level.

13.8 DIFFERENTIAL SCREWS

A differential screw is a mechanism designed to provide very fine motions. Although they can be made in several forms, one common form is shown in Figure 13.11. This particular differential screw configuration consists of two different threads on the same axis and one sliding joint.

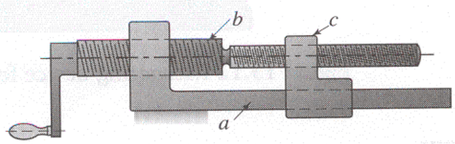


Figure 13.11 A differential screw.

In Figure 13.11, as the handle turns one revolution, thread A rotates one revolution and advances a distance equal to the lead of thread A. Of course, the motion of thread B is identical to thread A because it is machined onto the same shaft as A.

Thus, thread B also rotates one revolution and advances a distance equal to the lead of thread A. As thread B rotates one revolution, nut C is retracted because it is able to rotate. Specifically, as thread B rotates one revolution, nut C is retracted a distance equal to the lead of thread B. However, because thread B already advances one revolution, the net motion of nut C is the difference between the lead of threads A and B. Thus, this screw arrangement with different leads is called a the *differential screw*.

For differential screws, the kinematic relationship can be modified as follows:

(13.13) $$s_{nut} = (L_A - L_B)\,\theta_{screw}$$

(13.14) $$v_{nut} = (L_A - L_B)\omega_{screw}$$

(13.15) $$a_{nut} = (L_A - L_B)\alpha_{screw}$$

Again note that the lead, L, is specified as the nut displacement per revolution. Therefore, in this rare instance, *the angular motion should be specified in revolutions.*

When the lead of the two threads are close, small motions of the nut can be produced. This configuration is popular for fine adjustments of precision equipment at a relatively low cost.

EXAMPLE PROBLEM 13.4

A device that is intended to gage the length of parts is shown in Figure 13.12. The concept utilizes a differential screw, such that the rotation of knob A slides nut D until it is firmly pressed against part E. Nut D also has a pointer that can be used to determine the length of part E. The objective is to configure the system, such that one rotation of knob A causes a 1-mm traverse of nut D. Select threads B and C to accomplish this requirement.

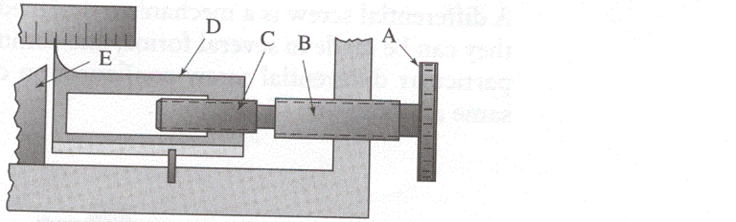


Figure 13.12 Measuring device for Example Problem 13.4.

Solution:

Use the following differential screw kinematics equation:

$$s_{nut} = (L_B - L_C)\theta_{screw}$$

$$1 \text{ mm} = (L_B - L_C)\ (1 \text{ rev})$$

$$(L_B - L_C) = 1\frac{\text{mm}}{\text{rev}}$$

Several arrangements are possible. The standard threads listed in Table 13.2 are utilized. These have a single thread, so that the lead and pitch are identical. Then two threads need to be selected that have a difference in pitch of 1.0 mm. Arbitrarily select course pitch threads:

For thread *B*: M16 × 2.0
For thread C: M6 × 1.0

13.9 AUGER SCREWS

Many centuries ago, Archimedes ingeniously applied a screw mechanism to lifting water, which is now known as the "Archimedes Screw" (Figure 13.13). As the screw rotates, each thread of the screw transports a certain amount of water.

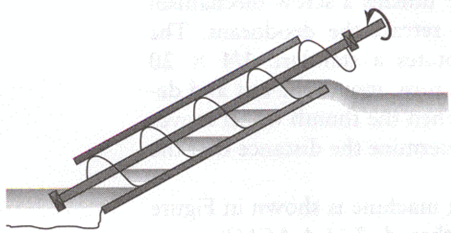


Figure 13.13 Archimedes screw.

With this screw mechanism, the mating nut is actually the fluid being transported. This form is still utilized today to transport many different types of material. Common applications include transporting molten plastic into molds, moving salt from dump trucks through spreaders for icy winter roads, digging fence post holes in soil, and moving cattle feed through long troughs. This screw mechanism is more commonly referred to as an auger.

The kinematic equations presented in Equations 13.11, 13.12, and 13.13 can be used to determine the motion of the material being transported, given the motion of the auger. Consistent with standard screws, a pitch or lead of an auger blade is defined. The volumetric transport rate is then a function of the clearance between auger blades, which traps the material being transported. This can be mathematically written as:

Volume through auger:

$$= (\text{volume trapped between auger blades})(L_{\text{auger}})\theta_{\text{screw}} \tag{13.16}$$

Volumetric flow through auger:

$$= (\text{volume trapped between auger blades})(L_{\text{auger}})\omega_{\text{screw}} \tag{13.17}$$

Volumetric acceleration through auger:

$$= (\text{volume trapped between auger blades})(L_{\text{auger}})\alpha_{\text{screw}} \tag{13.18}$$

PROBLEMS

Screw Thread Geometry Problems

For Problems 13–1 through 13–4, compute the lead angle. Also, determine whether it is self-locking when the thread is general machined quality. For a self-locking thread, what coefficient of friction would prevent self-locking?

13–1. 1/4 × 20 UNC thread.

13–2. 1/4 × 28 UNF thread.

13–3. M16 × 0.2 thread.

13–4. ACME 1-1/8 × 5 thread.

13–5. As described in the introduction, a stick deodorant package utilizes a screw mechanism to advance and retract the deodorant. The thumb wheel rotates a standard 1/4 × 20 thread which, in turn, moves the nut and deodorant stick. When the thumb wheel moves 3 revolutions, determine the distance that the stick advances.

13–6. A tension testing machine is shown in Figure P13.6. A single thread, 2 × 4 ACME screw thread, moves the nut. When the screw rotates at 40 rpm, determine the advancing speed of the ram.

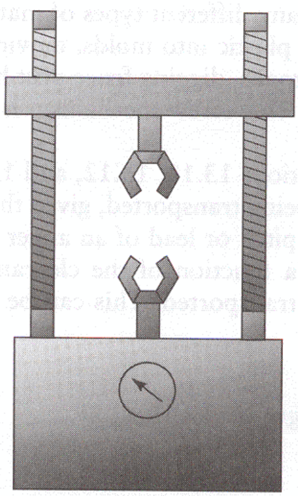


Figure P13.6 Problems 6 and 7.

13–7. When the screw rotates 10 revolutions in the tension machine described in Problem 13–6, determine the displacement of the ram.

13–8. An automated garage door opener is shown in Figure P13.8. A single thread, 1 × 5 ACME screw thread, moves the nut. The screw is rated at 1200 rpm; determine the speed of the door.

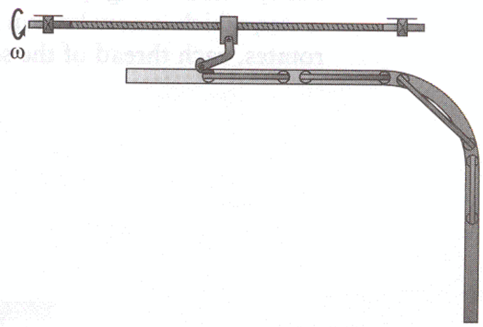


Figure P13.8 Problems 8 and 9.

13–9. For the garage door opener described in Problem 13–8, determine the amount that the door lowers for each 10 revolutions of the ACME screw.

13–10. Compute the lead angle of the ACME 1 × 5 thread used on the garage door in Problem 13–8. When the thread is well-lubricated, determine whether it is self-locking. For a self-locking thread, what coefficient of friction would prevent self-locking?

Screw Kinematics Problems

13–11. An adjustable work table mechanism is shown in Figure P13.11. The input shaft rotates at 600 rpm. Bevel gears, with a velocity ratio of 5:1, couple the motion from the input shaft to the rotating nut. A stud from the table has a unified 1/2 -13 thread that mates with the rotating nut. Determine the speed of the rising table.

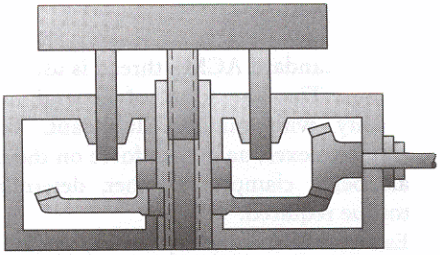


Figure P13.11 Problems 11 and 12.

13–12. For the work table described in Problem 13–11, determine the amount that the table lowers for each 10 revolutions of the input shaft.

13–13. The automated press described in Example Problem 13–2 is positioned such that β = 25°. Graphically determine the distance that the pressure plate lowers when the crank rotates 25 revolutions.

13–14. The automated press described in Example Problem 13–2 is positioned such that β = 30°. Analytically determine the distance that the pressure plate lowers when the crank rotates 10 revolutions.

13–15. For the automated press described in Example Problem 13–2, determine the velocity of the pressure plate when β = 45°.

13–16. The automated press described in Example Problem 13–2 is positioned such that β = 45°. Graphically determine the distance that the pressure plate lowers when the crank rotates 50 revolutions.

13–17. The automated press described in Example Problem 13–2, is positioned such that β = 45°. Analytically determine the distance that the pressure plate lowers, when the crank is rotated 75 revolutions.

13–18. For the automated press described in Example Problem 13–2, determine the velocity of the pressure plate when β = 65°.

13–19. The automated press described in Example Problem 13–2 is positioned such that β = 65°. Graphically determine the distance that the pressure plate lowers when the crank rotates 35 revolutions.

13–20. The automated press described in Example Problem 13–2, is positioned such that β = 65°. Analytically determine the distance that the pressure plate lowers, when the crank is rotated 15 revolutions.

13–21. A screw-operated press is shown in Figure P13.21. The screw has a single 1/2 × 10 ACME thread, both in a right-hand and left-hand orientation as shown. The handle rotates counter-clockwise at 45 rpm to drive the pressure plate downward. In the position shown, with β = 25°, determine the velocity of the pressure plate.

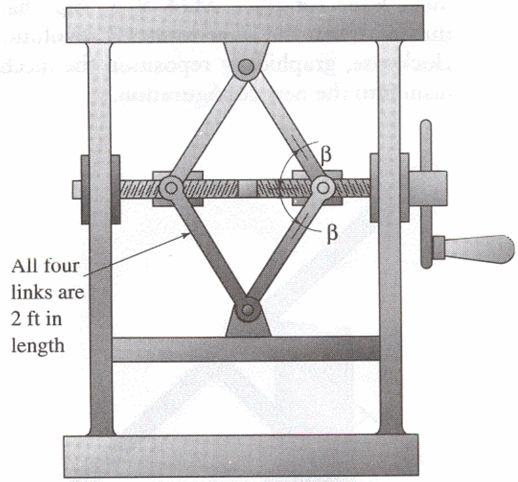


Figure P13.21 Problems 21-28.

13–22. The automated press described in Problem 13–21 is positioned such that β = 25°. Graphically determine the distance that the pressure plate lowers, when the crank rotates 20 revolutions.

13–23. The automated press described in Problem 13–21, is positioned such that β = 25°. Analytically determine the distance that the pressure plate raises, when the crank is rotated 10 revolutions.

13–24. For the automated press described in Problem 13–21, determine the velocity of the pressure plate when β = 45°.

13–25. The automated press described in Problem 13–2 is positioned such that β = 45°. Graphically determine the distance that the pressure plate lowers when the crank rotates 15 revolutions.

13–26. For the automated press described in Problem 13–21, determine the velocity of the pressure plate when β = 65°.

13–27. The automated press described in Problem 13–21 is positioned such that β = 65°.

Graphically determine the distance that the pressure plate lowers when the crank rotates 30 revolutions.

13–28. The automated press described in 13.21 is positioned such that $\beta = 65°$. Analytically determine the distance that the pressure plate raises, when the crank is rotated 30 revolutions.

13–29. The drawing table shown in Figure P13.29 has an adjustable height/angle. The adjustment lever rotates a M24 × 3, right-hand thread. When the lever rotates 3 revolutions clockwise, graphically reposition the mechanism into the new configuration.

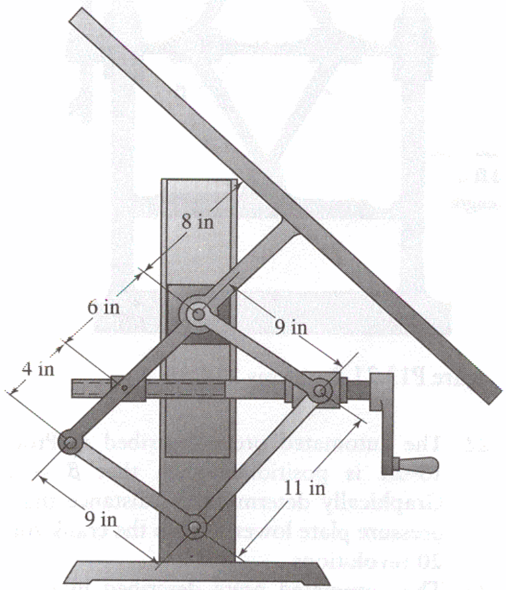


Figure P13.29 Problems 29-32.

13–30. For the drawing table described in Problem 13–29 graphically reposition the mechanism into the new configuration when the lever rotates 5 revolutions clockwise.

13–31. For the drawing table described in Problem 13–29 graphically reposition the mechanism into the new configuration when the lever rotates 7 revolutions clockwise.

13–32. For the drawing table described in Problem 13–29 determine the velocity of the roller A and angular velocity of the table when the lever rotates at 30 rpm, clockwise.

Screw Force Problems

13–33. A 1/2″ standard ACME thread is used on a C-clamp. This thread is of general machine quality with minimal lubricant. For the clamp to exert aa 500-lb force on the materials being clamped together, determine the torque required.

13–34. Estimate the efficiency of the C-clamp described in Problem 13–29.

13–35. A jack uses a double-thread ACME thread with a major diameter of 25 mm and a pitch of 5 mm. The jack is intended to lift 4000 N. Determine:

a. The lead angle.
b. Whether the jack is self-locking.
c. The torque to raise the load.
d. The torque to lower the load.
e. The efficiency of the jack.

13–36. A jack uses a double-thread 1 × 5 ACME thread. The jack is intended to lift 2000 lb. Determine:

a. The lead angle.
b. Whether the jack is self-locking.
c. The torque to raise the load.
d. The torque to lower the load
e. The efficiency of the jack.

13–37. For the table in Problem 13–11, the thread is of general machine quality with minimal lubricant. The table supports 75 lb; determine the torque that must be transferred to the nut.

13–38. Estimate the efficiency of the screw used in the table, described in Problem 13–33.

Differential Screw Problems

13–39. A differential screw is to be used in a measuring device similar to the one described in Example Problem 13–4. Select two standard threads such that one rotation of the knob creates a 0.5-mm traverse of the nut.

13–40. A differential screw is to be used in a measuring device similar to the one described in Example Problem 13–4. Select two standard threads such that one rotation of the knob creates a 0.25-inch traverse of the nut.

13–41. A differential screw is to be used in a measuring device similar to the one described in Example Problem 13–4. Select two standard threads such that one rotation of the knob creates a 0.05-inch traverse of the nut.

CASE STUDIES

C13–1 The device shown in Figure C13.1 utilizes a screw mechanism. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into the operation.

1. What is the hand designation for thread *B*?
2. What is the hand designation for thread *E*?
3. When handle *A* rotates counterclockwise, what is the motion of nut *C*?
4. When handle *A* rotates counterclockwise, what is the motion of slide *H*?

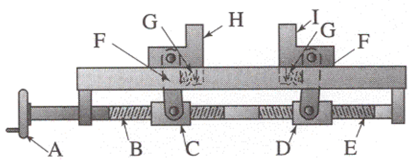


Figure C13.1 (Courtesy, Industrial Press)

5. When handle *A* rotates counterclockwise, what is the motion of nut *D*?
6. When handle *A* rotates counterclockwise, what is the motion of slide *I*?
7. What is the function of both links labeled *F*?
8. What is component *G* and what is its function(s)?
9. What is the function of this device, and what would you call it?

C13–2 The device shown in Figure C13.2 utilizes a screw mechanism. Carefully examine the components of the mechanism, then answer the following leading questions to gain insight into the operation.

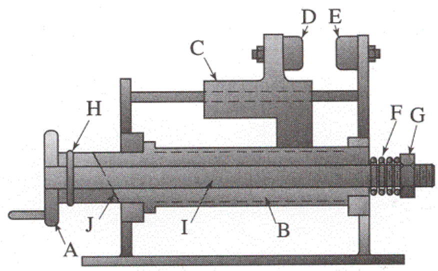


Figure C13.2 (Courtesy, Industrial Press)

1. As hand wheel *A* rotates counterclockwise, what is the motion of shaft *I*?
2. What is the function of pin *H*?
3. As hand wheel *A* rotates counterclockwise, what is the motion of threaded sleeve *B*?
4. What actually couples the motion of shaft *I* and threaded sleeve *B*?
5. Threaded sleeve *B* has right-hand threads; as hand wheel *A* rotates counterclockwise, what is the motion of nut *C*?
6. What happens to this device when pads *D* and *E* contact?
7. What component is *F* and what is its function?
8. What is the function of this device?
9. What would happen if nut *G* were tightened?
10. What would happen to this device if the interface *J* were designed with a more vertical slope?

References

1. Barton, Lyndon, *Mechanism Analysis: Simplified Graphical and Analytical Techniques,* 2nd ed., Marcel Dekker Inc., 1993.
2. Baumeister, Theodore III, & Avallone Eugene, *Mark's Standard Handbook for Mechanical Engineers,* 9th ed., McGraw-Hill Book Company, 1987.
3. Chaney, Ward, & Kincaid, David, *Numerical Mathematics and Computing,* Brooks Publishing Company, 1980.
4. Chironis, Nicholas, & Sclater, Neil, *Mechanisms and Mechanical Drives Sourcebook,* McGraw-Hill Book Company, 1996.
5. Erdman, Arthur, & Sandor, George, *Mechanism Design Vol 1: Analysis and Synthesis,* 3rd ed., Prentice Hall, 1997.
6. Hrones, J., & Nelson, G., Analysis of the Four-Bar Linkage, The Technology Press of the Massachusetts Institute of Technology, 1951.
7. Jensen Preben, *Cam Design and Manufacture,* The Industrial Press, 1965.
8. Jensen, Preben W., *Classical Modern Mechanisms for Engineers and Inventors,* Marcel Dekker, Inc., 1991.
9. Jones Franklin, et al., *Ingenious Mechanisms for Designers & Inventors, Vols I–IV,* Industrial Press Inc., 1983.
10. Shigley, Joseph, & Uicker, John Jr., *Theory of Machines and Mechanisms,* 2nd ed., McGraw-Hill Book Company, 1995.
11. *Working Model Demonstration Guide,* Knowledge Revolution Inc., 1995.

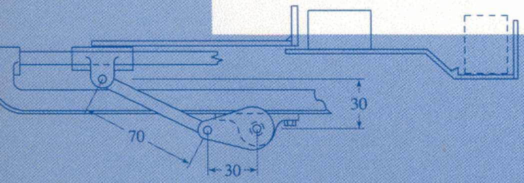
Index

For more information about this book or any of Prentice Hall's other new technology titles, visit our Web site at http://www.prenhall.com

Machines and Mechanisms: Applied Kinematic Analysis applies kinematic theories, both graphical and analytical, to real-world machines. It is intended to bridge the gap between a theoretical study of kinematics and the application to practical mechanisms. This approach of applying theoretical developments to practical problems is consistent with the philosophy of engineering technology programs.

Distinctive features of this book include:

- Focus on the application of kinematic theories to practical mechanisms. Machinery that contains mechanisms is pictured throughout the text.
- Use of both graphical techniques and analytical methods in the analysis of mechanisms
- Case studies at the end of every chapter that illustrate a mechanism used on industrial equipment
- Suggestions for implementing the graphical techniques on computer-aided design (CAD) systems
- Introduction of mechanism force analysis
- Example problems that illustrate the application of every major concept
- Real machines used in example problems
- Numerous end-of-chapter problems, most of which include the machine that relies on the mechanism being analyzed
- Suggestions for using programmable devices (calculators, spreadsheets, math software, etc.) for analytical solution procedures



ISBN 0-13-597915-3
90000>
9 780135 979150