Monetary Theory and Policy

Second Edition

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The MIT Press
Cambridge, Massachusetts
London, England

This book was set in Times New Roman on 3B2 by Asco Typesetters, Hong Kong and was printed and bound in the United States of America.

Library of Congress Cataloging-in-Publication Data

Walsh, Carl E.
Monetary theory and policy / Carl E. Walsh. — 2nd ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-262-23231-6
1. Monetary policy. 2. Money. I. Title.
HG230.3.W35 2003
332.4′6—dc21 2002044919

10 9 8 7 6 5 4 3 2 1

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Preface

Monetary economics has continued to be a rapidly expanding research area since the first edition of *Monetary Theory and Policy* was published. In updating and revising the book for a second edition, my objective has been to incorporate the new insights gained from this recent research. Virtually every chapter has been revised, with an eye to improving the exposition and incorporating recent research. At the time of the first edition, the use of models based on dynamic optimization and nominal rigidities in consistent general equilibrium frameworks was still relatively new. Since then, however, these models have been used to explore a wide variety of interesting theoretical and policy questions and have motivated new empirical work that has contributed to our knowledge of macroeconomic and monetary phenomena. Issues such as optimal policy, the role of policy instrument rules, policy and macroeconomic stability, and discretion versus commitment are just some of the topics on which interesting new insights have been gained. In addition to the work on what are generally referred to as *new Keynesian models*, this new edition includes discussion of such topics as the fiscal theory of the price level, inflation targeting, Taylor rules, and liquidity traps.

I have received many comments from users of the first edition, and these comments have served to guide me in revising the material. I would like particularly to thank Henning Bohn, Betty Daniel, Jordi Galí, Eric Leeper, Tim Fuerst, Ed Nelson, Federico Ravenna, and Kevin Salyer for very helpful comments on early draft versions of some of the chapters of the second edition. The comments and suggestions of Julia Chiriaeva, Nancy Jianakoplos, Stephen Miller, Jim Nason, Claudio Shikida, and students and participants in courses I have taught based on the first edition at the IMF Institute, the Bank of Spain, the Bank of Portugal, the Bank of England, the University of Olso, and the Swiss National Bank Studeniencentrum Gerzensee all contributed to improving the material. Wei Chen, Ethel Wang, and Jamus Lim, graduate students at the University of California, Santa Cruz (UCSC), also offered helpful comments and assistance in preparing this new edition.

The first edition was immensely improved by the thoughtful comments of many individuals who took the time to read parts of earlier drafts. Luigi Buttiglione, Marco Hoeberichts, Michael Hutchison, Francesco Lippi, Jaewoo Lee, Doug Pearce, Gustavo Piga, Glenn Rudebusch, Willem Verhagen, and Chris Waller provided me with many insightful and useful comments, all of which led to significant improvements. Students at Stanford and UCSC gave me important feedback on draft material; Peter Kriz, Jerry McIntyre, Fabiano Schivardi, Alina Carare, and especially Jules Leichter deserve special mention. A very special note of thanks is due Lars Svensson and Berthold Herrendorf. Each made extensive comments on complete drafts of the first edition. Attempting to address the issues they raised greatly improved the final product; it would have been even better if I had had the time and energy to follow all their suggestions. I would also like to thank Terry Vaughan and Elizabeth Murry, my editors at MIT Press, and Nancy Lombardi, the production editor for both the first and second editions, for their excellent assistance on the manuscript. Needless to say, all the remaining weaknesses and errors are my own responsibility.

Monetary Theory and Policy

Introduction

Monetary economics investigates the relationship between real economic variables at the aggregate level—such as real output, real rates of interest, employment, and real exchange rates—and nominal variables—such as the inflation rate, nominal interest rates, nominal exchange rates, and the supply of money. So defined, monetary economics has considerable overlap with macroeconomics more generally, and these two fields have, to a large degree, shared a common history over most of the past 50 years. This statement was particularly true during the 1970s after the monetarist/Keynesian debates led to a reintegration of monetary economics with macroeconomics. The seminal work of Robert Lucas (1972) provided theoretical foundations for models of economic fluctuations in which money was the fundamental driving factor behind movements in real output. The rise of real-business-cycle models during the 1980s and early 1990s, building on the contribution of Kydland and Prescott (1982) and focusing explicitly on nonmonetary factors as the driving forces behind business cycles, tended to separate monetary economics from macroeconomics. More recently, the real-business-cycle approach to aggregate modeling has been used to incorporate monetary factors into dynamic general equilibrium models. Today, both macroeconomics and monetary economics share the common tools associated with dynamic, stochastic approaches to modeling the aggregate economy.

Despite these close connections, a book on monetary economics is not a book on macroeconomics. The focus in monetary economics is distinct, emphasizing price-level determination, inflation, and the role of monetary policy. This book provides coverage of the most important topics in monetary economics and of some of the models that economists have employed as they attempt to understand the interactions between real and monetary factors. It deals with topics in both monetary theory and monetary policy and is designed for second-year graduate students specializing in monetary economics, for researchers in monetary economics wishing to have a systematic summary of recent developments, and for economists working in policy institutions such as central banks. It can also be used as a supplement for first-year graduate courses in macroeconomics, as it provides a more in-depth treatment of inflation and monetary policy topics than is customary in graduate macro textbooks. The chapters on monetary policy may be useful for advanced undergraduate courses.

Besides incorporating a discussion of the basic empirical approaches to estimating the impact of monetary policy, the major theoretical models are linked to the calibration and simulation techniques commonly used in macroeconomics and to the class of models commonly used for policy analysis. The use of dynamic simulations to evaluate the quantitative significance of the channels through which monetary policy and inflation affect the economy is one innovation of this book. Modern approaches to monetary policy stress the need to understand the incentives facing central banks and to model the strategic interactions between the central bank and the private sector. This approach receives extensive coverage, and this represents a second innovation of the book. A third innovation is the focus on interest rates in the discussion of monetary policy. Few central banks conduct monetary policy oriented toward the quantity of money, yet until the mid-1990s, most treatments of monetary policy emphasized money-supply control and money demand.

When one is writing a book such as this, several organizational approaches present themselves. Monetary economics is a large field, and one must decide whether to provide broad coverage, giving students a brief introduction to many topics, or to focus more narrowly and in more depth. I have chosen to focus on particular models, models that monetary economists have employed to address topics in theory and policy. I have tried to stress the major topics within monetary economics in order to provide sufficiently broad coverage of the field, but the focus within each topic is often on a small number of papers or models that I have found useful for gaining insight into a particular issue. As an aid to students, derivations of basic results are often quite detailed, but deeper technical issues of existence, multiple equilibria, and stability receive somewhat less attention. This choice was not made because the latter are unimportant; instead, the relative emphasis reflects an assessment that to do these topics justice, while still providing enough emphasis on the core insights offered by monetary economics, would have required a much longer book. By reducing the dimensionality of problems and by not treating them in full generality, I hoped to achieve the right balance of insight, accessibility, and rigor. The many references will serve to guide students to the extensive treatments in the literature of all the topics touched upon in this book.

Monetary economics today is dominated by three alternative modeling strategies. The first two, representative-agent models and overlapping-generations models, share a common methodological approach in building equilibrium relationships explicitly on the foundations of optimizing behavior by individual agents. The third approach is based on sets of equilibrium relationships that are often not derived directly from any decision problem. Instead, they are described as *ad hoc* by critics and as *convenient approximations* by proponents. The latter characterization is generally more appropriate, and these models have demonstrated great value in helping economists understand issues in monetary economics. In this book, we will deal with models in the representative-agent class and with ad hoc models of the type more common in policy analysis.

There are several reasons for ignoring the overlapping-generations, or OLG, approach. First, systematic expositions of monetary economics from the perspective

of overlapping generations are already available. For example, Sargent (1987) and Champ and Freeman (1994) cover many topics in monetary economics using OLG models. Second, many of the issues one studies in monetary economics require understanding the time-series behavior of macroeconomic variables such as inflation or the relationship between money and business cycles. It is helpful if the theoretical framework one uses can be mapped directly into implications for behavior that can be compared with actual data. This mapping is more easily done with infinite-horizon representative-agent models than with OLG models. This advantage, in fact, is one reason for the popularity of real-business-cycle models that employ the representative-agent approach, and so a third reason for limiting the coverage to representative-agent models is that they provide a close link between monetary economics and other popular frameworks for studying business-cycle phenomena. Fourth, monetary policy issues are generally related to the dynamic behavior of the economy over time periods associated with business-cycle frequencies, and here again the OLG framework seems less directly applicable. Finally, OLG models emphasize the store-of-value role of money at the expense of the medium-of-exchange role that money plays in facilitating transactions. McCallum (1983b) has argued that some of the implications of OLG models that contrast most sharply with the implications of other approaches (the tenuousness of monetary equilibria, for example) are directly related to the lack of a medium-of-exchange role for money.

A book on monetary theory and policy would be seriously incomplete if it were limited to representative-agent models alone. A variety of ad hoc models have played, and continue to play, important roles in influencing the way economists, and perhaps more importantly policy makers, think about the role of monetary policy. These models can be very helpful in highlighting key issues affecting the linkages between monetary and real economic phenomena. No monetary economist's tool kit is complete without them. But it is important to begin with more fully specified models so that one has some sense of what is missing in the simpler models. In this way, one is better able to judge whether the ad hoc models are likely to provide insight into particular questions.

The book divides naturally into chapters on models with flexible prices (chapters 2–4), chapters on money in the short run when nominal rigidities are important (chapters 5–7), and chapters on policy topics (chapters 8–11). In covering topics of monetary theory, major emphasis is placed on the money-in-the-utility-function (chapter 2) and cash-in-advance approaches (chapter 3) to integrating money into general equilibrium frameworks and on the role of inflation as a tax instrument (chapter 4). Stochastic versions for the basic models are calibrated, and simulations are used to illustrate how monetary factors affect the behavior of the economy. Such simulations aid in assessing the ability of the models to capture correlations observed in actual data.

The link between the dynamic general equilibrium models of chapters 2–4 and the models employed for monetary policy analysis in chapters 8–11 is developed in chapters 5–7. I have tried to emphasize that the standard models employed in policy analysis can be viewed as linear approximations to general equilibrium models based on firm micro foundations. An extension to the open economy is provided (chapter 6), and credit channels for monetary policy are discussed (chapter 7).

A survey of the recent literature on game-theoretic approaches to the study of monetary policy provides the starting point for the main coverage of policy topics (chapter 8). The details of policy implementation and operating procedures are important, particularly for empirical work that attempts to measure the impact of policy actions (chapter 9). Since most central banks use interest-rate-oriented procedures to implement policy, the book provides a discussion of interest rates and monetary policy, concluding with a discussion of recent models that are explicit in dropping the quantity of money from monetary policy analysis (chapter 10). Chapter 11, which is new to the second edition, explores the policy implications of the recent generation of new Keynesian models.

There is one traditionally important topic missing from this book—money demand. Academic models in monetary economics have, by long tradition, treated the quantity of money as central to the field. Such a focus leads naturally to an emphasis on the links between the direct instruments of monetary policy (open market operation, discount or other interest-rate-setting policy, reserve requirements) and the money supply and on the determinants of money demand. The interaction of money demand and money supply then serves to determine the quantity of money and the economy's price level. A large branch of the literature in monetary economics has concentrated on understanding the determinants of the demand for money and on developing and estimating empirical models of money demand. While attention will be paid to the demand for money at a theoretical level, the analysis of money demand is of less relevance now than it has been in the past. This change has occurred because, to a large extent, central banks operate today by employing a short-term interest rate as their policy operating target, with a deemphasis on the quantity of money. Such a focus reduces the importance of money-demand and money-supply analysis, and this reduced relevance is reflected in the coverage of this book.

This book is about monetary theory and the theory of monetary policy. Occasional references to empirical results are made, but no attempt has been made to provide a systematic survey of the vast body of empirical research in monetary economics. Most of the debates in monetary economics, however, have at their root issues of fact that can only be resolved by empirical evidence. Empirical evidence is needed to choose between theoretical approaches, but theory is also needed to interpret empirical evidence. How one links the quantities in the theoretical model to measurable data is critical, for example, in developing measures of monetary policy actions that can be used to estimate the impact of policy on the economy. Because empirical evidence aids in discriminating between alternative theories, it is helpful to begin with a brief overview of some basic facts. Chapter 1 does so, providing a discussion that focuses primarily on the estimated impact of monetary policy actions on real output. Here, as in the chapters that deal with some of the institutional details of monetary policy, the evidence comes primarily from research on the United States. However, an attempt has been made to cite cross-country studies and to focus on empirical regularities that seem to characterize most industrialized economies.

Chapters 2–4 emphasize the role of inflation as a tax, using models that provide the basic micro foundations of monetary economics. These chapters cover topics of fundamental importance for understanding how monetary phenomena affect the general equilibrium behavior of the economy and how nominal prices, inflation, money, and interest rates are linked. Because the models studied in these chapters assume that prices are perfectly flexible, they are most useful for understanding longer-run correlations between inflation, money, and output and cross-country differences in average inflation. However, they do have implications for short-run dynamics as real and nominal variables adjust in response to aggregate productivity disturbances and random shocks to money growth. These dynamics are examined by employing simulations based on linear approximations around the steady-state equilibrium.

Chapters 2 and 3 employ a neoclassical growth framework to study monetary phenomena. The neoclassical model is one in which growth is exogenous, and money either has no effect on the real economy's long-run steady state or has effects that are likely to be small empirically. However, because these models allow one to calculate the welfare implications of exogenous changes in the economic environment, they provide a natural framework for examining the welfare costs of alternative steady-state rates of inflation. They also, as the real-business-cycle literature has shown, can be simulated to generate artificial time series to study their implications for short-run cyclical fluctuations. Since policy can be expressed in terms of both exogenous shocks and endogenous feedbacks from real shocks, the models can be used to study how economic fluctuations depend on monetary policy.

In chapter 4, the focus turns to public finance issues associated with money, inflation, and monetary policy. The ability to create money provides governments with a means of generating revenue. As a source of revenue, money creation, along with the inflation that results, can be analyzed from the perspective of public finance as one among many tax tools available to governments.

Beginning with chapter 5, issues related to the short-run impact of monetary policy and to a number of topics that are relevant for understanding the conduct of monetary policy take center stage. The critical difference between the models of chapters 2–4 and those of chapters 5–11 lies in the price-adjustment mechanism. Understanding the impacts that monetary disturbances have on the real economy over time intervals measured in months or quarters requires one to drop the assumption used in chapters 2–4 that the aggregate price level immediately adjusts to ensure that equilibrium in all markets is continuously maintained.

Chapter 5 begins by reviewing some attempts to replicate the empirical evidence on the short-run effects of monetary policy shocks while still maintaining the assumption of flexible prices. Lucas's misperceptions model provides an important example of one such attempt. These efforts provide some insights into money-output links, but they are unable to mimic the persistence of the estimated impacts of monetary shocks on output. To do so requires that prices, wages, or both adjust sluggishly in response to economic disturbances. Chapter 5 discusses some important models of price and inflation adjustment, and discusses the foundations of new Keynesian models built on the joint foundations of optimizing behavior by economic agents and nominal rigidities.

Chapter 6 extends the analysis to the open economy by focusing on two questions. First, what additional channels from monetary policy actions to the real economy are present in the open economy that were absent in the closed-economy analysis? Second, how does monetary policy affect the behavior of nominal and real exchange rates? New channels through which monetary policy actions are transmitted to the real economy are present in open economies and involve exchange-rate movements and interest-rate linkages.

While the channels of monetary policy emphasized in traditional models operate primarily through interest rates and exchange rates, an alternative view is that credit markets play an independent role in affecting the transmission of monetary policy actions to the real economy. The nature of credit markets, and their role in the transmission process, are affected by market imperfections arising from imperfect information. Chapter 7 examines theories that stress the role of credit and credit-market imperfections in the presence of moral hazard, adverse selection, and costly monitoring.

Chapters 8–11 focus more directly on policy topics. These topics include strategic models of monetary policy, operating procedures and policy implementation, and the

role of interest rates and interest-rate-oriented policies. Also discussed are some simple frameworks that have proven useful in discussions of short-run monetary policy.

Chapter 8 discusses monetary policy objectives and then turns to consider the ability of policy authorities to achieve these objectives. Understanding monetary policy requires an understanding of how policy actions affect macro variables (the topic of chapters 2–7), but it also requires models of policy behavior to understand why particular policies are undertaken. A large body of research over the past two decades has used game-theoretic concepts to model the monetary policy maker as a strategic agent. These models have provided new insights into the rules-versus-discretion debate, provided positive theories of inflation, and provided justification for many of the actual reforms of central banking legislation that have been implemented in recent years.

In chapter 9, the focus turns to monetary policy implementation. Here, the discussion deals with the monetary-instrument-choice problem and monetary-policy operating procedures. A long tradition in monetary economics has debated the usefulness of monetary aggregates versus interest rates in the design and implementation of monetary policy, and chapter 9 reviews the approach economists have used to address this issue. A simple model of the market for bank reserves is used to stress how the observed responses of short-term interest rates and reserve aggregates will depend on the operating procedures used in the conduct of policy. A basic understanding of policy implementation is important for empirical studies that attempt to measure changes in monetary policy.

Traditionally, economists have employed simple models in which the money stock or even inflation is assumed to be the direct instrument of policy. In fact, most central banks have employed interest rates as their operational policy instrument, so chapter 10 emphasizes explicitly the role of a short-term interest rate as the instrument of monetary policy. Issues such as price-level determinacy under interest-rate-policy rules, the term structure of interest rates, and simple simulation models for policy analysis are discussed. Chapter 11 discusses monetary analysis using new Keynesian models. These models have provided new insights into the roles of discretion and commitment and the design of targeting rules and instrument rules.

1 Empirical Evidence on Money, Prices, and Output

1.1 Introduction

In this chapter, some of the basic empirical evidence on money, inflation, and output is reviewed. This review serves two purposes. First, these basic "facts" about both the long-run and the short-run relationships can serve as benchmarks for judging theoretical models. Second, reviewing the empirical evidence provides an opportunity to discuss the approaches monetary economists have taken to estimate the effects of money, and monetary policy, on real economic activity. The discussion will focus heavily on evidence from vector autoregressions (VARs), since these have served as a primary tool for uncovering the impact of monetary phenomena on the real economy. The findings obtained from VARs have been criticized, and these criticisms, as well as other methods that have been used to investigate the money-output relationship, are also discussed.

1.2 Some Basic Correlations

What are the basic empirical regularities that monetary economics must explain? Monetary economics focuses on the behavior of prices, monetary aggregates, nominal and real interest rates, and output, so a useful starting point is to summarize briefly what macroeconomic data tell us about the relationships among these variables.

1.2.1 Long-Run Relationships

A nice summary of long-run monetary relationships is provided by McCandless and Weber (1995). They examine data covering a 30-year period from 110 countries using several definitions of money. By examining average rates of inflation, output growth, and the growth rates of various measures of money over a long period of time and for many different countries, McCandless and Weber provide evidence on relationships that are unlikely to be dependent on unique, country-specific events (such as the particular means employed to implement monetary policy) that might influence the actual evolution of money, prices, and output in a particular country. Based on their analysis, two primary conclusions emerge.

The first is that the correlation between inflation and the growth rate of the money supply is almost 1, varying between 0.92 and 0.96, depending on the definition of the money supply used. This strong positive relationship between inflation and money growth is consistent with many other studies based on smaller samples of countries and different time periods.[1] This correlation is normally taken to support one of the basic tenets of the quantity theory of money: a change in the growth rate of money induces "an equal change in the rate of price inflation" (Lucas 1980b, p. 1005). This high correlation does not, however, have any implications for causality. If the countries in the sample had followed policies under which money-supply growth rates were exogenously determined, then the correlation could be taken as evidence that money growth causes inflation, with an almost one-to-one relationship between the two. An alternative possibility, equally consistent with the high correlation, is that other factors generate inflation, and central banks allow the growth rate of money to adjust. Any theoretical model not consistent with a roughly one-for-one long-run relationship between money growth and inflation, though, would need to be questioned.[2]

The appropriate interpretation of money-inflation correlations, both in terms of causality and in terms of tests of long-run relationships, also depends on the statistical properties of the underlying series. As Fischer and Seater (1993) note, one cannot ask how a permanent change in the growth rate of money affects inflation unless actual money growth has exhibited permanent shifts. They show how the order of integration of money and prices influences the testing of hypotheses about the long-run relationship between money growth and inflation. In a similar vein, McCallum (1984b) demonstrates that regression-based tests of long-run relationships in monetary economies may be misleading when expectational relationships are involved.

McCandless and Weber's second general conclusion is that there is no correlation between either inflation or money growth and the growth rate of real output. Thus, there are countries with low output growth and low money growth and inflation, and countries with low output growth and high money growth and inflation—and countries with every other combination as well. This conclusion is not as robust as the money-growth–inflation one; McCandless and Weber report a positive correlation between real growth and money growth, but not inflation, for a subsample of OECD countries. Kormendi and Meguire (1984) for a sample of almost 50 countries and Geweke (1986) for the United States argue that the data reveal no long-run effect of money growth on real output growth. Barro (1995, 1996) reports a negative correlation between inflation and growth in a cross-country sample. Bullard and Keating (1995) examine the post–World War II data from 58 countries, concluding for the sample as a whole that the evidence that permanent shifts in inflation produce permanent effects on the level of output is weak, with some evidence of positive effects of inflation on output among low-inflation countries and zero or negative effects for higher-inflation countries.[3] Similarly, Boschen and Mills (1995b) conclude that permanent monetary shocks in the United States made no contribution to permanent shifts in GDP. Thus, there is somewhat greater uncertainty as to the relationship between inflation and real growth, and other measures of real economic activity such as unemployment, in the long run, but the general consensus is well summarized by the proposition, "about which there is now little disagreement,... that there is no long-run trade-off between the rate of inflation and the rate of unemployment" (Taylor 1996, p. 186).

Monetary economics is also concerned with the relationship between interest rates, inflation, and money. According to the Fisher equation, the nominal interest rate equals the real return plus the expected rate of inflation. If real returns are independent of inflation, then nominal interest rates should be positively related to expected inflation. This relationship is an implication of the theoretical models discussed throughout this book. In terms of long-run correlations, it suggests that the level of nominal interest rates should be positively correlated with average rates of inflation. Because average rates of inflation are positively correlated with average money growth rates, nominal interest rates and money growth rates should also be positively correlated. Monnet and Weber (2001) examine annual average interest rates and money growth rates over the period 1961 to 1998 for a sample of 31 countries. They find a correlation of 0.87 between money growth and long-term interest rates. For the developed countries, the correlation is somewhat smaller (0.70); for the developing countries, it is 0.84, although this falls to 0.66 when Venezuela is excluded.[4] This evidence is consistent with the Fisher equation.

Mishkin (1992) examines the interest rate–inflation correlation directly by testing for the presence of a long-run relationship between the two. Using U.S. data, Mishkin finds the evidence to be consistent with the Fisher relationship.

1. Examples include Lucas (1980b), Geweke (1986), and Rolnick and Weber (1994), among others. A nice graph of the close relationship between money growth and inflation for high-inflation countries is provided by Abel and Bernanke (1995, p. 242). Hall and Taylor provide a similar graph for the G-7 countries (Hall and Taylor 1997, p. 115). As will be noted, however, the interpretation of correlations between inflation and money growth can be problematic.

2. Haldane (1997) finds, however, that the money growth rate–inflation correlation is much less than 1 among low-inflation countries.

3. Kormendi and Meguire (1985) report a statistically significant positive coefficient on average money growth in a cross-country regression for average real growth. This effect, however, is due to a single observation (Brazil), and the authors report that money growth becomes insignificant in their growth equation when Brazil is dropped from the sample. They do find a significant negative effect of monetary volatility on growth.

4. Venezuela's money growth rate averaged over 28%, the highest among the countries in Monnet and Weber's sample.

1.2.2 Short-Run Relationships

The long-run empirical regularities of monetary economies are important for gauging how well the steady-state properties of a theoretical model match the data. Much of our interest in monetary economics, however, arises because of a need to understand how monetary phenomena in general, and monetary policy in particular, affect the behavior of the macroeconomy over time periods of months or quarters. Short-run dynamic relationships between money, inflation, and output reflect both the way in which private agents respond to economic disturbances and the way in which the monetary policy authority responds to those same disturbances. For this reason, short-run correlations are likely to vary both across countries, as different central banks implement policy in different ways, and across time in a single country, as the sources of economic disturbances vary.

Some evidence on short-run correlations for the United States are provided in figure 1.1. The figure shows correlations between the detrended log of real GDP and three different monetary aggregates, each in detrended log form as

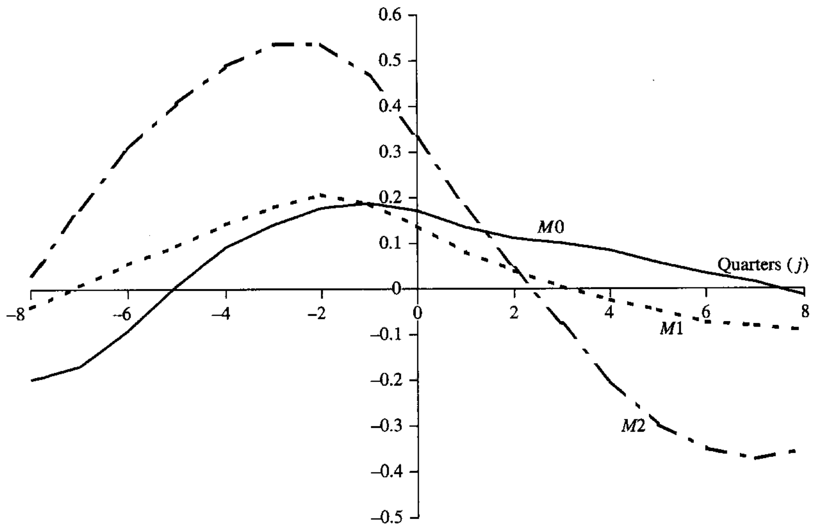

Figure 1.1
Dynamic Correlations, GDP_t and M_{t+j}: 1967:1–2000:4

well.[5] Data are quarterly from 1967:1 to 2000:4, and the figure plots the correlation between real GDP_t and M_{t+j} against j, where M represents a monetary aggregate. The three aggregates are the monetary base (sometimes denoted as $M0$), $M1$ and $M2$. $M0$ is a narrow definition of the money supply, consisting of total reserves held by the banking system plus currency in the hands of the public. $M1$ consists of currency held by the nonbank public, travelers checks, demand deposits, and other checkable deposits. $M2$ consists of $M1$ plus savings accounts and small-denomination time deposits plus balances in retail money market mutual funds.

As figure 1.1 shows, the correlations with real output change substantially as one moves from $M0$ to $M2$. The narrow measure $M0$ is positively correlated with real GDP at both leads and lags. In contrast, $M2$ is positively correlated at lags but negatively correlated at leads. In other words, high GDP (relative to trend) tends to be proceeded by high values of $M2$ but followed by low values. The positive correlation between GDP_t and M_{t+j} for $j < 0$ indicates that movements in money lead movements in output. This timing pattern played an important role in Friedman and Schwartz's classic and highly influential *Monetary History of the United States* (Friedman and Schwartz 1963b). The larger correlations between GDP and $M2$ arise in part from the endogenous nature of an aggregate such as $M2$, depending as it does on banking sector behavior as well as that of the nonbank private sector (see King and Plosser 1984, Coleman 1996).

Figure 1.2 shows the cross-correlations between detrended real GDP and several interest rates and between detrended real GDP and the detrended GDP deflator. The interest rates range from the federal funds rate, an overnight interbank rate that is the rate used by the Federal Reserve to implement monetary policy, to the 1-year and 10-year rates on government bonds. The three interest rate series display similar correlations with real output, although the correlations become smaller for the longer-term rates. Low interest rates tend to lead output, while a rise in output tends to be followed by higher interest rates.

In contrast, the GDP deflator tends to be below trend when output is above trends, but increases in real output tend to be followed by increases in prices. Kydland and Prescott (1990) have argued that the negative contemporaneous correlation between the output and price series suggests that supply shocks, and not demand shocks, must be responsible for business cycle fluctuations. Aggregate supply shocks would cause prices to be countercyclical, while demand shocks would be expected to make prices procyclical. However, if prices are sticky, a demand shock would initially raise

5. Trends are estimated using a Hodrick-Prescott filter.

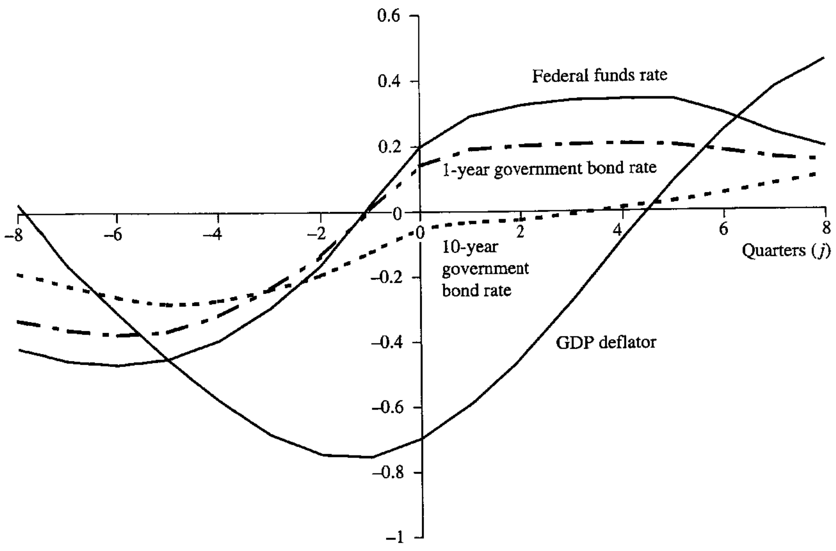

Figure 1.2
Dynamic Correlations, Output, Prices, and Interest Rates: 1967:1–2000:4

output above trend, while prices respond very little. If prices do eventually rise while output eventually returns to trend, prices could be rising as output is falling, producing a negative unconditional correlation between the two even though it is demand shocks that generate the fluctuations (Ball and Mankiw 1994; Judd and Trehan 1995). Den Haan (2000) examines forecast errors from a vector autoregression (see section 1.3.4) and finds that price and output correlations are positive for short forecast horizons and negative for long forecast horizons. This pattern seems consistent with demand shocks playing an important role in accounting for short-run fluctuations, while supply shocks play a more important role in the long-run behavior of output and prices.

While figures 1.1 and 1.2 give one summary of the joint behavior of money, prices, interest rates, and output, at least for the United States, one of the challenges of monetary economics is to determine the degree to which these data reveal causal relationships, relationships that should be expected to appear in data from other countries and during other time periods, or relationships that depend on the particular characteristics of the policy regime under which monetary policy is conducted.

1.3 Estimating the Effect of Money on Output

Almost all economists accept that the long-run effects of money fall entirely, or almost entirely, on prices, with little impact on real variables, but most economists also believe that monetary disturbances can have important effects on real variables such as output in the short run.[6] As Lucas puts it in his Nobel lecture: "This tension between two incompatible ideas—that changes in money are neutral unit changes and that they induce movements in employment and production in the same direction—has been at the center of monetary theory at least since Hume wrote" (Lucas 1996, p. 664).[7] The time-series correlations presented in the previous subsection are suggestive of the short-run relationships between money and income, but the evidence for the effects of money on real output is based on more than these simple correlations.

The tools that have been employed to estimate the impact of monetary policy have evolved over time as the result of developments in time-series econometrics and changes in the specific questions posed by theoretical models. In this section, we review some of the empirical evidence on the relationship between monetary policy and U.S. macro behavior. One objective of this literature has been to determine whether monetary policy disturbances actually have played an important role in U.S. economic fluctuations. Equally important, the empirical evidence is useful in judging whether the predictions of different theories about the effects of monetary policy are consistent with the evidence. Among the excellent recent discussions of these issues are Leeper, Sims, and Zha (1996) and Christiano, Eichenbaum, and Evans (1999), where the focus is on the role of identified VARs in estimating the effects of monetary policy, and King and Watson (1996), where the focus is on using empirical evidence to distinguish among competing business-cycle models.

1.3.1 The Evidence of Friedman and Schwartz

Friedman and Schwartz's classic study of the relationship between money and business cycles (M. Friedman and Schwartz 1963b) probably still represents the most influential empirical evidence that money does matter for business-cycle fluctuations. Their evidence, based on almost 100 years of data from the United States, relied heavily on patterns of timing; systematic evidence that money growth rate changes lead changes in real economic activity is taken to support a causal interpretation in

6. For an exposition of the view that monetary factors have not played an important role in U.S. business cycles, see Kydland and Prescott (1990).

7. The reference is to David Hume's 1752 essays *Of Money* and *Of Interest*.

which money causes output fluctuations. This timing pattern shows up most clearly in figure 1.1 with $M2$.

Friedman and Schwartz concluded that the data "decisively support treating the rate of change series [of the money supply] as conforming to the reference cycle positively with a long lead" (M. Friedman and Schwartz 1963a, p. 36). That is, faster money growth tends to be followed by increases in output above trend, and slowdowns in money growth tend to be followed by declines in output. The inference Friedman and Schwartz drew was that variations in money growth rates cause, with a long (and variable) lead, variations in real economic activity.

The nature of this evidence for the United States is apparent in figure 1.3, which shows two money supply measures and detrended real GDP. The monetary aggregates in the figure, $M1$ and $M2$, are quarterly observations on the deviations of the actual series from trend. The sample period is 1967:1–2001:4, so this figure starts after the Friedman and Schwartz study ends. The figure reveals slowdowns in money leading most business cycle downturns through the early 1980s. However, the pattern is not so apparent after 1982. B. Friedman and Kuttner (1992) have documented the seeming breakdown in the relationship between monetary aggregates and real output; this changing relationship between money and output has affected the manner in which monetary policy has been conducted, at least in the United States (see chapter 9).

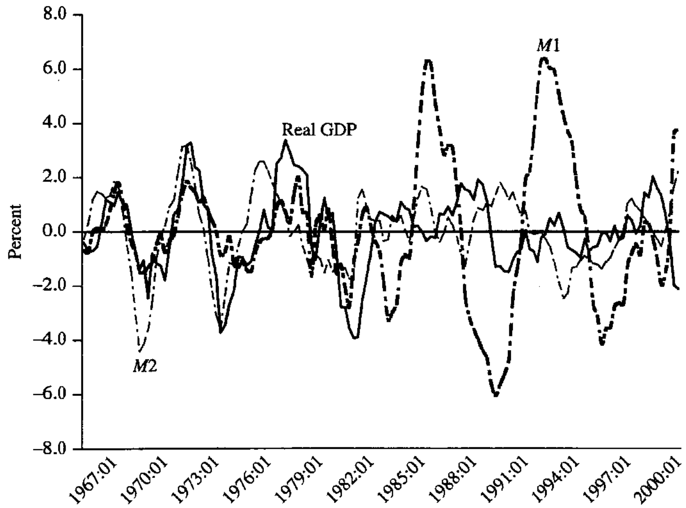

Figure 1.3
Detrended Money and Real GDP

While suggestive, evidence based on timing patterns and simple correlations may not indicate the true causal role of money. Since the Federal Reserve and the banking sector respond to economic developments, movements in the monetary aggregates are not exogenous, and the correlation patterns need not reflect any causal effect of monetary policy on economic activity. If, for example, the central bank is implementing monetary policy by controlling the value of some short-term market interest rate, the nominal stock of money will be affected both by policy actions that change interest rates and by developments in the economy that are not related to policy actions. An economic expansion may lead banks to expand lending in ways that produce an increase in the stock of money, even if the central bank has not changed its policy. If the money stock is used to measure monetary policy, the relationship observed in the data between money and output may reflect the impact of output on money and not the impact of money and monetary policy on output.

Tobin (1970) was the first to model formally the idea that the positive correlation between money and output, the correlation that Friedman and Schwartz interpreted as providing evidence that money caused output movements, could, in fact, reflect just the opposite—output might be causing money. A more modern treatment of what is known as the *reverse causation argument* is provided by King and Plosser (1984). They show that inside money, the component of a monetary aggregate such as $M1$ that represents the liabilities of the banking sector, is more highly correlated with output movements in the United States than is outside money, the liabilities of the Federal Reserve. King and Plosser interpret this finding as evidence that much of the correlation between broad aggregates such as $M1$ or $M2$ and output arises from the endogenous response of the banking sector to economic disturbances that are not the result of monetary policy actions. More recently, Coleman (1996), in an estimated equilibrium model with endogenous money, finds that the implied behavior of money in the model cannot match the lead-lag relationship in the data. Specifically, a money-supply measure such as $M2$ leads output, whereas Coleman finds that his model implies that money should be more highly correlated with lagged output than with future output.[8]

The endogeneity problem is likely to be particularly severe if the monetary authority has employed a short-term interest rate as its main policy instrument, and

8. Lacker (1988) shows how the correlations between inside money and future output could also arise if movements in inside money reflect new information about future monetary policy.

this has generally been the case in the United States. Changes in the money stock will then be endogenous and cannot be interpreted as representing policy actions. Figure 1.4 shows the behavior of two short-term nominal interest rates, the three-month Treasury-bill rate (3MTB) and the federal funds rate, together with detrended real GDP. Like figure 1.3, figure 1.4 provides some support for the notion that monetary policy actions have contributed to U.S. business cycles. Interest rates have typically increased prior to economic downturns. But whether this is evidence that monetary policy has caused or contributed to cyclical fluctuations cannot be inferred from the figure; the movements in interest rates may simply reflect the Fed's response to the state of the economy.

Simple plots and correlations are suggestive, but they cannot be decisive. Other factors may be the cause of the joint movements of output, monetary aggregates, and interest rates. The comparison with business-cycle reference points also ignores much of the information about the time-series behavior of money, output, and interest rates that could be used to determine what impact, if any, monetary policy has on output. And the appropriate variable to use as a measure of monetary policy will depend on how policy has been implemented.

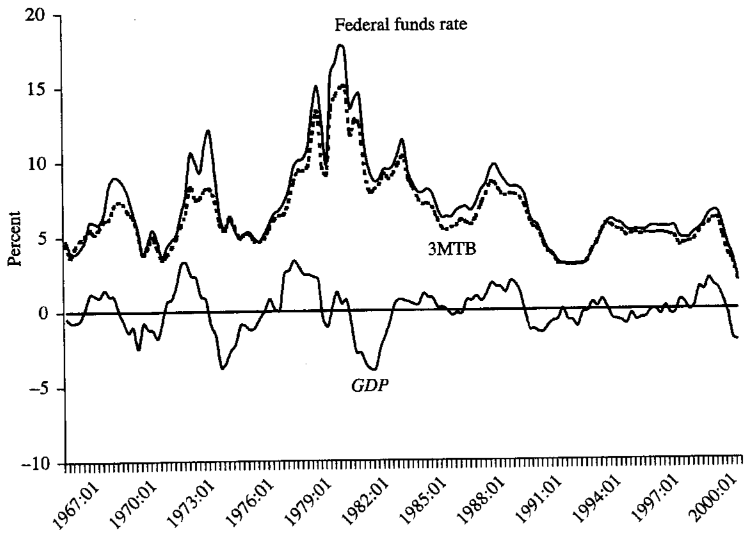

Figure 1.4
Interest Rates and Detrended Real GDP

One of the earliest time-series econometric attempts to estimate the impact of money was due to M. Friedman and Meiselman (1963). Their objective was to test whether monetary or fiscal policy was more important for the determination of *nominal* income. To address this issue, they estimated the following equation:[9]

$$y_t^n \equiv y_t + p_t = y_0^n + \sum_{i=0} a_i A_{t-i} + \sum_{i=0} b_i m_{t-i} + \sum_{i=0} h_i z_{t-i} + u_t, \qquad (1.1)$$

where y^n denotes the log of nominal income, equal to the sum of the logs of output and the price level, A is a measure of autonomous expenditures, and m is a monetary aggregate; z can be thought of as a vector of other variables relevant for explaining nominal income fluctuations. Friedman and Meiselman reported finding a much more stable and statistically significant relationship between output and money than between output and their measure of autonomous expenditures. In general, they could not reject the hypothesis that the a_i coefficients were zero, while the b_i coefficients were always statistically significant.

The use of equations such as (1.1) for policy analysis was promoted by a number of economists at the Federal Reserve Bank of St. Louis, so regressions of nominal income on money are often called *St. Louis equations* (see Andersen and Jordon 1968, B. Friedman 1977a, Carlson 1978). Because the dependent variable is nominal income, the St. Louis approach does not address directly the question of how a money-induced change in nominal spending is split between a change in real output and a change in the price level. The impact of money on nominal income was estimated to be quite strong, and Andersen and Jordon (1968, p. 22) concluded that "Finding of a strong empirical relationship between economic activity and ... monetary actions points to the conclusion that monetary actions can and should play a more prominent role in economic stabilization than they have up to now."[10]

The original Friedman-Meiselman result generated responses by Modigliani and Ando (1976) and De Prano and Mayer (1965), among others. This debate emphasized that an equation such as (1.1) is misspecified if m is endogenous. To illustrate the point with an extreme example, suppose that the central bank is able to

9. This is not exactly correct; because Friedman and Meiselman included "autonomous" expenditures as an explanatory variable, they also used consumption as the dependent variable (basically, output minus autonomous expenditures). They also reported results for real variables as well as nominal ones. Following modern practice, (1.1) is expressed in terms of logs; Friedman and Meiselman estimated their equation in levels.

10. B. Friedman (1977a) argued that updated estimates of the St. Louis equation did yield a role for fiscal policy, although the statistical reliability of this finding was questioned by Carlson (1978). Carlson also provides a bibliography listing many of the papers on the St. Louis equation (see his footnote 2, p. 13).

manipulate the money supply to offset almost perfectly shocks that would otherwise generate fluctuations in nominal income. In this case, y^n would simply reflect the random control errors the central bank had failed to offset. As a result, m and y^n might be completely uncorrelated, and a regression of y^n on m would not reveal that money actually played an important role in affecting nominal income. If policy is able to respond to the factors generating the error term u_t, m_t and u_t will be correlated, ordinary least squares estimates of (1.1) will be inconsistent, and the resulting estimates will depend on the manner in which policy has induced a correlation between u and m. Changes in policy that altered this correlation will also alter the least squares regression estimates one would obtain in estimating (1.1).

1.3.2 Granger Causality

The St. Louis equation related nominal output to the past behavior of money. Similar regressions employing *real* output have also been used to investigate the connection between real economic activity and money. In an important contribution, Sims (1972) introduced the notion of *Granger causality* into the debate over the real effects of money. A variable X is said to Granger cause Y if and only if lagged values of X have marginal predictive content in a forecasting equation for Y. In practice, testing whether money Granger causes output involves testing whether the a_i coefficients equal zero in a regression of the form

$$y_t = y_0 + \sum_{i=1} a_i m_{t-i} + \sum_{i=1} b_i y_{t-i} + \sum_{i=1} c_i z_{t-i} + e_t, \tag{1.2}$$

where key issues involve the treatment of trends in output and money, the choice of lag lengths, and the set of other variables (represented by z) that are included in the equation.

Sims's original work used log levels of U.S. nominal GNP and money (both $M1$ and the monetary base). He found evidence that money Granger caused GNP. That is, the past behavior of money helped to predict future GNP. However, using the index of industrial production to measure real output, Sims (1980) found that the fraction of output variation explained by money was greatly reduced when a nominal interest rate was added to the equation (so that z consisted of the log price level and an interest rate). Thus, the conclusion seemed sensitive to the specification of z. Eichenbaum and Singleton (1986) found that money appeared to be less important if the regressions were specified in log first difference form rather than in log levels with a time trend. Stock and Watson (1989) provided a systematic treatment of the trend specification in testing whether money Granger causes real output. They concluded

that money does help to predict future output (they actually use industrial production) even when prices and an interest rate are included.

A large literature has examined the value of monetary indicators in forecasting output. One interpretation of Sims's finding was that including an interest rate reduced the apparent role of money because, at least in the United States, a short-term interest rate, rather than the money supply, provided a better measure of monetary policy actions (see chapter 9). B. Friedman and Kuttner (1992) and Bernanke and Blinder (1992), among others, have looked at the role of alternative interest rate measures in forecasting real output. Friedman and Kuttner examined the effects of alternative definitions of money and different sample periods, concluding that the relationship in the United States is unstable and deteriorated in the 1990s. Bernanke and Blinder find that the federal funds rate "dominates both money and the bill and bond rates in forecasting real variables."

Regressions of real output on money were also popularized by Barro (1977, 1978, 1979b) as a way of testing whether only unanticipated money mattered for real output. By dividing money into anticipated and unanticipated components, Barro obtained results suggesting that only the unanticipated part affected real variables (see also Barro and Rush 1980 and the critical comment by Small 1979). Subsequent work by Mishkin (1982) found a role for anticipated money as well. Cover (1992) employs a similar approach and finds differences in the impacts of positive and negative monetary shocks. Negative shocks are estimated to have significant effects on output, while the effect of positive shocks is usually small and statistically insignificant.

1.3.3 Policy Uses

Before reviewing other evidence on the effects of money on output, it is useful to ask whether equations such as (1.2) can be used for policy purposes. That is, can a regression of this form be used to design a policy rule for setting the central bank's policy instrument? If it can, then the discussions of theoretical models that form the bulk of this book would be unnecessary, at least from the perspective of conducting monetary policy.

Suppose that the estimated relationship between output and money takes the form

$$y_t = y_0 + a_0 m_t + a_1 m_{t-1} + c_1 z_t + c_2 z_{t-1} + u_t. \tag{1.3}$$

According to (1.3), systematic variations in the money supply affect output. Consider the problem of adjusting the money supply to reduce fluctuations in real output. If this objective is interpreted to mean that the money supply should be manipulated to minimize the variance of y_t around y_0, then m_t should be set equal to

$$m_t = -\frac{a_1}{a_0} m_{t-1} - \frac{c_2}{a_0} z_{t-1} + v_t$$
$$= \pi_1 m_{t-1} + \pi_2 z_{t-1} + v_t, \tag{1.4}$$

where for simplicity we have assumed that the monetary authority's forecast of z_t is equal to zero. The term v_t represents the control error experienced by the monetary authority in setting the money supply. Equation (1.4) represents a feedback rule for the money supply whose parameters are themselves determined by the estimated coefficients in the equation for y. A key assumption is that the coefficients in (1.3) are independent of the choice of the policy rule for m. Substituting (1.4) into (1.3), output under the policy rule given in (1.4) would be equal to $y_t = y_0 + c_1 z_t + u_t + a_0 v_t$.

Notice that a policy rule has been derived using only knowledge of the policy objective (minimizing the expected variance of output) and knowledge of the estimated coefficients in (1.3). No theory of how monetary policy actually affects the economy was required. Sargent (1976) showed, however, that the use of (1.3) to derive a policy feedback rule may be inappropriate. To see why, suppose that real output actually depends only on unpredicted movements in the money supply; only surprises matter, with predicted changes in money simply being reflected in price-level movements with no impact on output.[11] From (1.4), the unpredicted movement in m_t is just v_t, so let the true model for output determination be

$$y_t = y_0 + d_0 v_t + d_1 z_t + d_2 z_{t-1} + u_t. \tag{1.5}$$

Now from (1.4), $v_t = m_t - (\pi_1 m_{t-1} + \pi_2 z_{t-1})$, so output can be expressed equivalently as

$$y_t = y_0 + d_0[m_t - (\pi_1 m_{t-1} + \pi_2 z_{t-1})] + d_1 z_t + d_2 z_{t-1} + u_t$$
$$= y_0 + d_0 m_t - d_0 \pi_1 m_{t-1} + d_1 z_t + (d_2 - d_0 \pi_2) z_{t-1} + u_t, \tag{1.6}$$

which has exactly the same form as (1.3). Equation (1.3), which was initially interpreted as consistent with a situation in which systematic feedback rules for monetary policy could affect output, is *observational equivalent* to (1.6), which was derived under the assumption that systematic policy had no effect and only money surprises mattered. The two are observationally equivalent since the error term in both (1.3) and (1.6) is just u_t; both equations fit the data equally well.

11. The influential model of Lucas (1972) has this implication. See chapter 5.

A comparison of (1.3) and (1.6) reveals another important conclusion. The coefficients of (1.6) are functions of the parameters in the policy rule (1.4). Thus, changes in the conduct of policy, interpreted to mean changes in the feedback-rule parameters, will change the parameters estimated in an equation such as (1.6) (or in a St. Louis–type regression). This is an example of the Lucas critique (Lucas 1976): empirical relationships are unlikely to be invariant to changes in policy regimes.

Of course, as Sargent stressed, it may be that (1.3) is the true structure that remains invariant as policy changes. In this case, (1.5) will not be invariant to changes in policy. To demonstrate this point, note that (1.4) implies

$$m_t = (1 - \pi_1 \mathrm{L})^{-1}(\pi_2 z_{t-1} + v_t),$$

where L is the lag operator.[12] Hence, we can write (1.3) as

$$y_t = y_0 + a_0 m_t + a_1 m_{t-1} + c_1 z_t + c_2 z_{t-1} + u_t$$
$$= y_0 + a_0(1 - \pi_1 \mathrm{L})^{-1}(\pi_2 z_{t-1} + v_t)$$
$$+ a_1(1 - \pi_1 \mathrm{L})^{-1}(\pi_2 z_{t-2} + v_{t-1}) + c_1 z_t + c_2 z_{t-1} + u_t$$
$$= (1 - \pi_1) y_0 + \pi_1 y_{t-1} + a_0 v_t + a_1 v_{t-1} + c_1 z_t$$
$$+ (c_2 + a_0 \pi_2 - c_1 \pi_1) z_{t-1} + (a_1 \pi_2 - c_2 \pi_1) z_{t-2} + u_t - \pi_1 u_{t-1}, \tag{1.7}$$

where we have now expressed output as a function of lagged output, the z variable, and money surprises (the v realizations). If this were interpreted as a policy invariant expression, one would conclude that output was independent of any predictable or systematic feedback rule for monetary policy; only unpredicted money appears to matter. Yet, under the hypothesis that (1.3) is the true invariant structure, changes in the policy rule (the π_i coefficients) will cause the coefficients in (1.7) to change.

Note that when we started with (1.5) and (1.4), we derived an expression for output that was observationally equivalent to (1.3). When we started with (1.3) and (1.4), however, we ended up with an expression for output that was not equivalent to (1.5); (1.7) contains lagged values of output, v, and u and two lags of z, while (1.5) contains only the contemporaneous values of v and u and one lag of z. These differences would allow one to distinguish between the two, but they arise only because this example placed a priori restrictions on the lag lengths in (1.3) and (1.5). In general, we would not have the type of a priori information that would allow us to do this.

12. That is, $\mathrm{L}^i x_t = x_{t-i}$.

The lesson from this simple example is that we cannot design policy without a theory of how money affects the economy. Theory should identify whether the coefficients in a specification of the form (1.3) or in a specification such as (1.5) will remain invariant as policy changes. While output equations estimated over a single policy regime may not allow us to identify the true structure, information from several policy regimes might succeed in doing so. If a policy regime change means that the coefficients in the policy rule (1.4) have changed, this would serve to identify whether an expression of the form (1.3) or one of the form (1.5) was policy invariant.

1.3.4 The VAR Approach

Most recent empirical studies of monetary policy and real economic activity have adopted a vector autoregression (VAR) framework. The use of VARs to estimate the impact of money on the economy was pioneered by Sims (1972, 1980). The development of the approach as it has moved from bivariate (Sims 1972) to trivariate (Sims 1980) to larger and larger systems, and the empirical findings the literature has produced, are summarized by Leeper, Sims, and Zha (1996). Christiano, Eichenbaum, and Evans (1999) provide a thorough discussion of the use of VARs to estimate the impact of money, and they provide an extensive list of references to work in this area.[13]

Suppose that we consider a bivariate system in which y_t is the natural log of real output at time t and x_t is a candidate measure of monetary policy such as a measure of the money stock or a short-term market rate of interest.[14] The VAR system can be written as

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = A(\mathrm{L})\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix}, \tag{1.8}$$

where $A(\mathrm{L})$ is a 2×2 matrix polynomial in the lag operator L and u_{it} is a time t serially independent innovation to the ith variable. These innovations can be thought of as linear combinations of independently distributed shocks to output (e_{yt}) and to policy (e_{xt}):

$$\begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} = \begin{bmatrix} e_{yt} + \theta e_{xt} \\ \phi e_{yt} + e_{xt} \end{bmatrix} = \begin{bmatrix} 1 & \theta \\ \phi & 1 \end{bmatrix}\begin{bmatrix} e_{yt} \\ e_{xt} \end{bmatrix} = B\begin{bmatrix} e_{yt} \\ e_{xt} \end{bmatrix}. \tag{1.9}$$

13. Two references on the econometrics of VARs are Hamilton (1994) and Maddala (1992).

14. How one measures monetary policy is a critical issue in the empirical literature and is a topic of open and ongoing debate (see, for example, Romer and Romer 1989, Bernanke and Blinder 1992, Gordon and Leeper 1994, Christiano, Eichenbaum, and Evans 1996a, Bernanke and Mihov 1998, Rudebusch 1997, Leeper, Sims, and Zha 1996, Leeper 1997, Christiano, Eichenbaum, and Evans 1999). Zha (1997) provides a useful discussion of the general identification issues that arise in attempting to measure the impact of monetary policy. We will return to this issue in chapter 9.

The one-period ahead error made in forecasting the policy variable x_t is equal to u_{xt}, and, since from (1.9), $u_{xt} = \phi e_{yt} + e_{xt}$, these errors are caused by the exogenous output and policy disturbances e_{yt} and e_{xt}. Letting $\sum_u$ denote the 2×2 variance-covariance matrix of the u_{it}'s, $\sum_u = B\sum_e B'$ where $\sum_e$ is the (diagonal) variance matrix of the e_{it}'s.

The random variable e_{xt} represents the exogenous shock to policy. If we wish to determine the role of policy in *causing* movements in output or other macro variables, it is the effect of e_x on these variables that we need to estimate. As long as $\phi \neq 0$, the innovation to the observed policy variable x_t will depend both on the shock to policy e_{xt} and on the nonpolicy shock e_{yt}; obtaining an estimate of u_{xt} does not provide a measure of the policy shock unless $\phi = 0$.

To make the example even more explicit, suppose the VAR system is

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix}, \tag{1.10}$$

with $0 < a_1 < 1$. Then $x_t = u_{xt}$ and $y_t = a_1 y_{t-1} + u_{yt} + a_2 u_{xt-1}$, and we can write y_t in moving average form as

$$y_t = \sum_{i=0}^{\infty} a_1^i u_{yt-i} + \sum_{i=0}^{\infty} a_1^i a_2 u_{xt-i-1}.$$

Estimating (1.8) yields estimates of $A(\mathrm{L})$ and $\sum_u$, and from these we can calculate the effects of u_{xt} on $\{y_t, y_{t+1}, \ldots\}$. If one interpreted u_x as an exogenous policy disturbance, then the implied response of $y_t, y_{t+1}, \ldots$ to a policy shock would be[15]

$$0, \quad a_2, \quad a_1 a_2, \quad a_1^2 a_2, \ldots$$

To estimate the impact of a policy shock on output, however, we need to calculate the effect on $\{y_t, y_{t+1}, \ldots\}$ of a realization of the policy shock e_{xt}. In terms of the true underlying structural disturbances e_y and e_x, (1.9) implies

$$\begin{aligned} y_t &= \sum_{i=0}^{\infty} a_1^i (e_{yt-i} + \theta e_{xt-i}) + \sum_{i=0}^{\infty} a_1^i a_2 (e_{xt-i-1} + \phi e_{yt-i-1}) \\ &= e_{yt} + \sum_{i=0}^{\infty} a_1^i (a_1 + a_2\phi) e_{yt-i-1} + \theta e_{xt} + \sum_{i=0}^{\infty} a_1^i (a_1\theta + a_2) e_{xt-i-1}, \end{aligned} \tag{1.11}$$

15. This represents the response to an nonorthogonalized innovation. The basic point, however, is that if θ and ϕ are nonzero, the underlying shocks are not identified, so the estimated response to u_x or to the component of u_x that is orthogonal to u_y will not identify the response to the policy shock e_x.

so that the impulse response function giving the true response of y to the exogenous policy shock e_x is

$$\theta, \quad a_1\theta + a_2, \quad a_1(a_1\theta + a_2), \quad a_1^2(a_1\theta + a_2), \ldots$$

This response involves the elements of $A(L)$ *and* the elements of B. And while $A(L)$ can be estimated from (1.8), B and $\sum_e$ are not identified without further restrictions.[16]

Two basic approaches to solving this identification problem have been followed. The first imposes additional restrictions on the matrix B that links the observable VAR residuals to the underlying structural disturbances (see 1.9). This approach has been used by Sims (1972), Bernanke (1986), Walsh (1987), Sims (1988), Bernanke and Blinder (1992), Gordon and Leeper (1994), and Bernanke and Mihov (1998), among many others. If policy shocks affect output with a lag, for example, the restriction that $\theta = 0$ would allow the other parameters of the model to be identified. The second approach achieves identification by imposing restrictions on the long-run effects of the disturbances on observed variables. For example, the assumption of long-run neutrality of money would imply that a monetary policy shock (e_x) has no long-run permanent effect on output. In terms of the example that led to (1.11), long-run neutrality of the policy shock would imply that $\theta + (a_1\theta + a_2)\sum a_1^i = 0$ or $\theta = -a_2$. Examples of this approach include Blanchard and Watson (1986), Blanchard (1989), Blanchard and Quah (1989), Judd and Trehan (1989), Hutchison and Walsh (1992), and Galí (1992). The use of long-run restrictions is criticized by Faust and Leeper (1997).

In Sims (1972), the nominal money supply ($M1$) was treated as the measure of monetary policy (the x variable), and policy shocks were identified by assuming that $\phi = 0$. This approach corresponds to the assumption that the money supply is predetermined and that policy innovations are exogenous with respect to the nonpolicy innovations (see 1.9). In this case, $u_{xt} = e_{xt}$, so from the fact that $u_{yt} = \theta e_{xt} + e_{yt} = \theta u_{xt} + e_{yt}$, θ can be estimated from the regression of the VAR residuals u_{yt} on the VAR residuals u_{xt}.[17] This corresponds to a situation in which the policy variable x does not respond contemporaneously to output shocks, perhaps because of information lags in formulating policy. However, if x depends contemporaneously on nonpolicy disturbances as well as policy shocks (i.e., $\phi \neq 0$), using u_{xt} as an estimate of e_{xt} will compound the effects of e_{yt} on u_{xt} with the effects of policy actions.

16. In this example, the three elements of $\sum_u$, the two variances and the covariance term, are functions of the four unknown parameters: ϕ, θ, and the variances of e_y and e_x.

17. This represents a Choleski decomposition of the VAR residuals with the policy variable ordered first.

An alternative approach seeks a policy measure for which $\theta = 0$ is a plausible assumption; this corresponds to the assumption that policy shocks have no contemporaneous impact on output.[18] This type of restriction is imposed by Bernanke and Blinder (1992) and Bernanke and Mihov (1998). How reasonable such an assumption might be clearly depends on the unit of observation. In annual data, the assumption of no contemporaneous effect would be implausible; with monthly data, it might be much more plausible.

This discussion has, for simplicity, treated both y and x as scalars. In fact, neither assumption is appropriate. We are usually interested in the effects of policy on several dimension of an economy's macroeconomic performance, and policy is likely to respond to unemployment and inflation, as well as other variables, so y would normally be a vector of nonpolicy variables. Then the restrictions that correspond to either $\phi = 0$ or $\theta = 0$ may be less easily justified. While one might argue that policy does not respond contemporaneously to unemployment when the analysis involves monthly data, this is not likely to be the case with respect to market interest rates. And, using the same example, one might be comfortable assuming that the current month's unemployment rate is unaffected by current policy actions, but this would not be true of interest rates, since financial markets will respond immediately to policy actions.

In addition, there generally is no clear scalar choice for the policy variable x. If policy were framed in terms of strict targets for the money supply, for a specific measure of banking sector reserves, or for a particular short-term interest rate, then the definition of x might be straightforward. In general, however, several candidate measures of monetary policy will be available, all depending in various degrees on both policy actions and nonpolicy disturbances. What constitutes an appropriate candidate for x, and how x depends on nonpolicy disturbances, will depend on the operating procedures the monetary authority is following as it implements policy.

Money and Output Sims (1992) provides a useful summary of the VAR evidence on money and output from France, Germany, Japan, the United Kingdom, and the United States. He estimates separate VARs for each country, using a common specification that includes industrial production, consumer prices, a short-term interest rate as the measure of monetary policy, a measure of the money supply, an exchange rate index, and an index of commodity prices. Sims orders the interest rate variable first. This corresponds to the assumption that $\phi = 0$; innovations to the interest rate variable potentially affect the other variables contemporaneously (Sims uses monthly

18. This represents a Choleski decomposition with output ordered before the policy variable.

data), while the interest rate is not affected contemporaneously by innovations in any of the other variables.[19]

The response of real output to an interest rate innovation is similar for all five of the countries Sims examines. In all cases, monetary shocks lead to an output response that is usually described as following a hump-shaped pattern. The negative output effects of a contractionary shock, for example, build to a peak after several months and then gradually die out.

Eichenbaum (1992) presents a comparison of the estimated effects of monetary policy in the United States using alternative measures of policy shocks, discussing how different choices can produce puzzling results, or at least puzzling relative to certain theoretical expectations. He based his discussion on the results obtained from a VAR containing four variables: the price level and output (these correspond to the elements of y in 1.8), $M1$ as a measure of the money supply, and the federal funds rate as a measure of short-term interest rates (these correspond to the elements of x). He considers interpreting shocks to $M1$ as policy shocks versus the alternative of interpreting funds-rate shocks as policy shocks. He finds that a positive innovation to $M1$ is followed by an increase in the federal funds rate and a decline in output. This result is puzzling if $M1$ shocks are interpreted as measuring the impact of monetary policy. An expansionary monetary policy shock would be expected to lead to increases in both $M1$ and output. The interest rate was also found to rise after a positive $M1$ shock, also a potentially puzzling result; a standard model in which money demand varies inversely with the nominal interest rate would suggest that an increase in the money supply would require a decline in the nominal rate to restore money-market equilibrium. D. Gordon and Leeper (1994) show that a similar puzzle emerges when total reserves are used to measure monetary policy shocks; positive reserve innovations are found to be associated with increases in short-term interest rates and unemployment increases. The suggestion that a rise in reserves or the money supply might raise, not lower, market interest rates generated a large literature that attempted to search for a liquidity effect of changes in the money supply (e.g., Reichenstein 1987; Christiano and Eichenbaum 1992a; Leeper and Gordon 1992; Strongin 1995, Hamilton 1996).

When Eichenbaum used innovations in the short-term interest rate as a measure of monetary policy actions, a positive shock to the funds rate represented a contractionary policy shock. No output puzzle was found in this case; a positive interest-rate shock was followed by a decline in the output measure. Instead, what has been called

19. Sims notes that the correlations among the VAR residuals, the u_{it}'s, are small so that the ordering has little impact on his results (i.e., sample estimates of ϕ and θ are small).

the *price puzzle* emerges; a contractionary policy shock is followed by a rise in the price level. The effect is small and temporary (and barely statistically significant) but still puzzling. The most commonly accepted explanation for the price puzzle is that it reflects the fact that the variables included in the VAR do not span the full information set available to the Fed. Suppose the Fed tends to raise the funds rate whenever it forecasts that inflation might rise in the future. To the extent that the Fed is unable to offset the factors that led it to forecast higher inflation, or to the extent that the Fed acts too late to prevent inflation from rising, the increase in the funds rate will be followed by a rise in prices. This interpretation would be consistent with the price puzzle. One solution is to include commodity prices or other asset prices in the VAR. Since these prices tend to be sensitive to changing forecasts of future inflation, they serve as a proxy for some of the Fed's additional information (Sims 1992; Chari, Christiano, and Eichenbaum 1995; Bernanke and Mihov 1998). Sims (1992) shows that the price puzzle is not confined to U.S. studies. He reports VAR estimates of monetary policy effects for France, Germany, Japan, and the United Kingdom, as well as for the United States, and in all cases, a positive shock to the interest rate leads to a positive price response. These price responses tend to become smaller, but do not in all cases disappear, when a commodity price index and a nominal exchange rate are included in the VAR.

An alternative interpretation of the price puzzle is provided by Barth and Ramey (2001). They argue that contractionary monetary policy operates on aggregate supply as well as aggregate demand. For example, an increase in interest rates raises the cost of holding inventories and, as a consequence, acts as a positive cost shock. This negative supply effect raises prices and lowers output. Such an effect is called the *cost channel* of monetary policy. In this interpretation, the price puzzle is simply evidence of the cost channel rather than evidence that the VAR is misspecified. Barth and Ramey combine industry-level data with aggregate data in a VAR and report evidence supportive of the cost channel interpretation of the price puzzle.

The Funds Rate as a Measure of U.S. Monetary Policy One difficulty in measuring the impact of monetary policy shocks arises when operating procedures change over time. The best measure of policy during one period may no longer accurately reflect policy in another period if the implementation of policy has changed. Many authors have argued that over most of the past 35 years, the federal funds rate has been the key policy instrument in the United States, suggesting that unforecasted changes in this interest rate may provide good estimates of policy shocks. This view has been argued, for example, by Bernanke and Blinder (1992) and Bernanke and Mihov (1998). While the Fed's operating procedures have varied over time, the funds rate is likely to be the best indicator of policy in the United States during the pre-1979 and post-1982 periods.[20] Policy during the period 1979–1982 is less adequately characterized by the funds rate.[21] Since our objective in this section is only to give a general sense of the empirical evidence on the impact of policy shocks, we will use the funds rate as an indicator of policy.

While researchers have disagreed on the best means of identifying policy shocks, there has been a surprising consensus on the general nature of the economic responses to monetary policy shocks. A variety of VARs estimated for a number of countries all indicate that, in response to a policy shock, output follows a hump-shaped pattern in which the peak impact occurs several quarters after the initial shock. Monetary policy actions appear to be taken in anticipation of inflation, so that a price puzzle emerges if forward-looking variables such as commodity prices are not included in the VAR.

Figure 1.5 shows impulse responses of output and the price level obtained when the funds rate is taken as the measure of monetary policy. The point estimates of the responses are shown as solid lines; dashed lines display a two standard deviation confidence interval. The figure is based on a VAR estimate using monthly data for the period 1965:01 to 2001:12. The variables in the VAR are the log of the CPI, a measure of output, the log of $M1$, and the federal funds rate. Because GDP is not available at a monthly frequency, the standard practice is to use a variable such as the Federal Reserve's Index of Industrial Production as the monthly measure of real economic activity.[22] This provides a much narrower index of activity than does GDP, so the estimates reported here use instead the log of the Department of Commerce's Index of Coincident Indicators (ICI) as the proxy for real activity.[23] The log of $M1$ is included as a measure of the money supply, and the funds rate (FF) is included as a measure of short-term market interest rates. By ordering the funds rate last, the identifying restriction is that the other variables do not respond contemporaneously to a shock to the funds rate ($\theta = 0$ in terms of 1.9). Given that the results are based on monthly data, this is probably a reasonable restriction.

20. Chapter 9 provides a brief history of Fed operating procedures.

21. During this period, nonborrowed reserves were set to achieve a level of interest rates consistent with the desired monetary growth targets. In this case, the funds rate may still provide a satisfactory policy indicator. Cook (1989) finds that most changes in the funds rate during the 1979–1982 period reflected policy actions. See chapter 9 for a discussion of operating procedures and the reserve market.

22. Bernanke and Mihov (1998) report VAR estimates using a monthly GDP series that they have constructed.

23. See Walsh and Wilcox (1995). This index is no longer produced by the Department of Commerce, but it is maintained by the Conference Board.

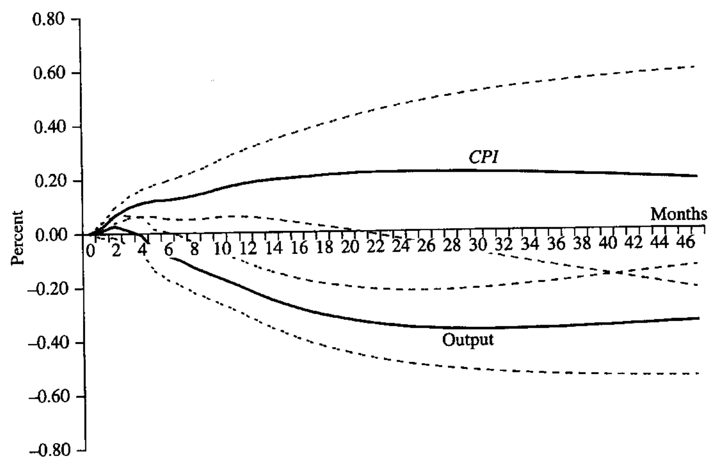

Figure 1.5
Output and Price Response to a Funds-Rate Shock (1965–2001)

Exogenous shocks to the funds rate have transitory output effects that die out over time but leave the funds rate above its baseline for more than a year. Output (as measured by ICI) follows a hump-shaped pattern in response to a contractionary policy shock. Although the response is statistically insignificant for most of the first year after a shock, output then declines below the baseline, with the peak effect occurring more than two years after the shock. This hump-shaped pattern is a very common finding (see, for example, Leeper, Sims, and Zha 1996 for U.S. evidence and Sims 1992 for international evidence). The price puzzle is also illustrated in figure 1.5; contractionary monetary policy, as measured by a positive interest rate innovation, is estimated to lead to a rise in the price level. The impulse response functions indicate that the major impact of policy shocks only occurs with quite a long lag.

The impacts of monetary policy shocks have not remained constant over time. Figure 1.6 illustrates the estimated impulse responses to a funds rate shock when the sample is restricted to end in 1979. Output responds more strongly, the peak effect occurs earlier, and output returns to trend more quickly, as does the price level.

If monetary policy shocks cause output movements, how important have these shocks been in accounting for actual business-cycle fluctuations? Leeper, Sims, and Zha (1996) conclude that monetary policy shocks have been relatively unimportant.

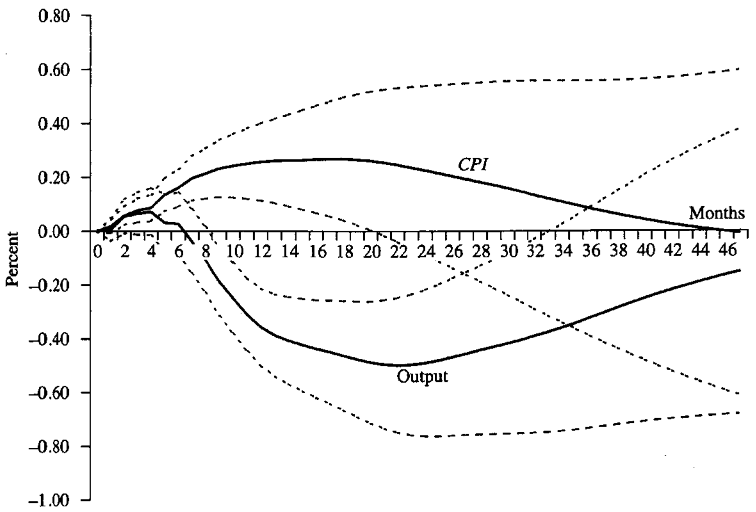

Figure 1.6
Output and Price Response to a Funds-Rate Shock (1965–1979)

However, their assessment is based on monthly data for the period from the beginning of 1960 until early 1996. This sample contains several distinct periods characterized by differences in the procedures used by the Fed to implement monetary policy, and the contribution of monetary shocks may have differed over various subperiods. Christiano, Eichenbaum, and Evans (1999) conclude that estimates of the importance of monetary policy shocks for output fluctuations are sensitive to the way monetary policy is measured. When they use a funds-rate measure of monetary policy, policy shocks account for 21% of the 4-quarter ahead forecast error variance for quarterly real GDP. This figure rises to 38% of the 12-quarter ahead forecast error variance. Smaller effects are found using policy measures based on monetary aggregates. Christiano, Eichenbaum, and Evans find that very little of the forecast error variance for the price level can be attributed to monetary policy shocks.

Criticisms of the VAR Approach Measures of monetary policy based on the estimation of VARs have been criticized on several grounds.[24] First, some of the

24. These criticism are detailed in Rudebusch (1998).

impulse responses do not accord with most economists' priors. In particular, the price puzzle, the finding that a contractionary policy shock, as measured by a funds-rate shock, tends to be followed by a rise in the price level is troublesome. As noted earlier, the price puzzle can be solved by including oil prices or commodity prices in the VAR system, and the generally accepted interpretation is that, lacking these inflation-sensitive prices, a standard VAR misses important information that is available to policy makers. A related but more general point is that many of the VAR models used to assess monetary policy fail to incorporate forward-looking variables. Central banks look at a lot of information in setting policy. Because policy is likely to respond to forecasts of future economic conditions, VARs may attribute the subsequent movements in output and inflation to the policy action. However, the argument that puzzling results indicate a misspecification implicitly imposes a prior belief about what the correct effects of monetary shocks should look like. Eichenbaum (1992), in fact, argues that short-term interest-rate innovations have been used to represent policy shocks in VARs because they produce the types of impulse response functions for output that economists expect.

In addition, the residuals from the VAR regressions that are used to represent exogenous policy shocks often bear little resemblance to standard interpretations of the historical record of past policy actions and periods of contractionary and expansionary policy (Sheffrin 1995; Rudebusch 1998). They also differ considerably depending on the particular specification of the VAR. Rudebusch reports low correlations between the residual policy shocks he obtains based on funds rate futures and those obtained from a VAR by Bernanke and Mihov. How important this finding is depends on the question of interest. If the objective is to determine whether a particular recession was caused by a policy shock, then it is important to know if and when the policy shock occurred. If alternative specifications provide differing and possibly inconsistent estimates of when policy shocks occurred, then their usefulness as a tool of economic history would be limited. If, however, the question of interest is how the economy responds when a policy shock occurs, then the discrepancies among the VAR residual estimates may be of less importance. Sims (1998a) argues that in a simple supply-demand model, different authors using different supply-curve shifters may obtain quite similar estimates of the demand-curve slope (since they all obtain consistent estimators of the true slope). At the same time, they may obtain quite different residuals for the estimated supply curve. If our true interest is in the parameters of the demand curve, the variations in the estimates of the supply shocks may not be of importance. Thus, the type of historical analysis based on a VAR, as in Walsh (1993b), is likely to be more problematic than the use of a VAR to determine the way the economy responds to exogenous policy shocks.

While VARs focus on residuals that are interpreted as policy shocks, the systematic part of the estimated VAR equation for a variable such as the funds rate can be interpreted as a policy reaction function; it provides a description of how the policy instrument has been adjusted in response to lagged values of the other variables included in the VAR system. Rudebusch (1998) has argued that the implied policy reaction functions look quite different than results obtained from more direct attempts to estimate reaction functions or to model actual policy behavior.[25] A related point is that VARs are typically estimated using final, revised data and will therefore not capture accurately the historical behavior of the monetary policy maker who is reacting to preliminary and incomplete data. Woolley (1995) shows how the perception of the stance of monetary policy in the United States in 1972, and President Richard Nixon's attempts to pressure Fed Chairman Arthur F. Burns into adopting a more expansionary policy, were based on initial data on the money supply that were subsequently very significantly revised.

At best the VAR approach identifies only the effects of monetary policy shocks, shifts in policy unrelated to the endogenous response of policy to developments in the economy. Yet most, if not all, of what one thinks of in terms of policy and policy design represents the endogenous response of policy to the economy, and "most variation in monetary policy instruments is accounted for by responses of policy to the state of the economy, not by random disturbances to policy" (Sims 1998a, p. 933). So it is unfortunate that a primary empirical tool—VAR analysis—used to assess the impact of monetary policy is uninformative about the role played by policy rules. If policy is completely characterized as a feedback rule on the economy, so that there are no exogenous policy shocks, then the VAR methodology would conclude that monetary policy doesn't matter. Yet while monetary policy is not causing output movements in this example, it does not follow that policy is unimportant; the response of the economy to nonpolicy shocks may depend importantly on the way monetary policy endogenously adjusts.

Cochrane (1998b) makes a similar point that is related to the issues discussed in section 1.3.3. In that section, it was noted that one must know whether it is anticipated money that has real effects (as in 1.3) or whether it is unanticipated money that matters (as in 1.5). Cochrane argues that, while most of the VAR literature has focused on issues of lag length, detrending, ordering, and variable selection, there is another fundamental identification issue that has been largely ignored—is it anticipated or unanticipated monetary policy that matters? If only unanticipated policy matters, then the subsequent systematic behavior of money after a policy shock is irrelevant. This means that the long hump-shaped response of real variables to a policy shock must be due to inherent lags of adjustment and the propagation mechanisms that characterize the structure of the economy. If anticipated policy matters, then subsequent systematic behavior of money after a policy shock is relevant. This means that the long hump-shaped response of real variables to a policy shock may only be present because policy shocks are followed by persistent, systematic policy actions. If this is the case, the direct impact of a policy shock, if it were not followed by persistent policy moves, would be small.

25. For example, Taylor (1993a) has employed a simple interest-rate rule that closely matches the actual behavior of the federal funds rate in recent years. As Khoury (1990) notes in her survey of many studies of the Fed's reaction function, few systematic conclusions have emerged from this empirical literature.

Attempts have been made to use VARs frameworks to assess the systematic effects of monetary policy. Sims (1998b), for example, estimates a VAR for the interwar years and uses it to simulate the behavior of the economy if policy had been determined according to the feedback rule obtained from a VAR estimate using postwar data.

1.3.5 Structural Econometric Models

The empirical assessment of the effects of alternative feedback rules for monetary policy has traditionally been carried out using structural macroeconometric models. During the 1960s and early 1970s, the specification, estimation, use, and evaluation of large-scale econometric models for forecasting and policy analysis represented a major research agenda in macroeconomics. Important contributions to our understanding of investment, consumption, the term structure, and other aspects of the macroeconomy grew out of the need to develop structural equations for various sectors of the economy. An equation describing the behavior of a policy instrument such as the federal funds rate was incorporated into these structural models, allowing model simulations of alternative policy rules to be conducted. These simulations would provide an estimate of the impact on the economy's dynamic behavior of changes in the way policy was conducted. For example, a policy under which the funds rate was adjusted rapidly in response to unemployment movements could be contrasted with one in which the response was more muted.

A key maintained hypothesis, one necessary to justify this type of analysis, was that the estimated parameters of the model would be invariant to the specification of the policy rule. If this were not the case, then one could no longer treat the model's parameters as unchanged when altering the monetary policy rule (as the example in section 1.3.3 shows). In a devastating critique of this assumption, Lucas (1976) argued that economic theory predicts that the decision rules for investment, consumption, and expectations formation will not be invariant to shifts in the systematic behavior of policy. The Lucas critique emphasized the problems inherent in the assumption, common in the structural econometric models of the time, that expectations adjust adaptively to past outcomes.

While large-scale econometric models of aggregate economies continued to play an important role in discussions of monetary policy, they fell out of favor among academic economists during the 1970s, in large part as a result of Lucas's critique, the increasing emphasis on the role of expectations in theoretical models, and the dissatisfaction with the empirical treatment of expectations in existing large-scale models. The academic literature witnessed a continued interest in small-scale rational-expectations models, both single and multicountry versions (for example, the work of Taylor 1993b), as well as the development of larger-scale models (Fair 1984), all of which incorporated rational expectations into some or all aspects of the model's behavioral relationships. Recent examples of small models based on rational expectations and forward-looking behavior include Fuhrer (1994a, 1997c), Fuhrer and Moore (1995a, 1995b), Ireland (1997), and Rotemberg and Woodford (1997). The structure of these models will be discussed in chapters 5, 10, and 11.

Larger-scale econometric models have proven useful to central banks in providing answers to questions related to the design and implementation of monetary policy, and within the past few years, a new generation of large-scale econometric policy models have come into use. Brayton and Tinsley (1996) and Brayton, Mauskopf, Reifschneider, Tinsley, and Williams (1997) provide a description of the Federal Reserve Board's FRB/US model.[26] Levin, Rogers, and Tryon (1997) discuss the Federal Reserve's international, multicountry FRB/Global model. These econometric models are designed to address specific questions of relevance for the actual design of monetary policy. The FRB/US model is structured to allow simulations to be conducted under alternative assumptions about expectations formation.

Work reported in R. Bryant, Hooper, and Mann (1992) evaluates the implications for policy experiments of a variety of empirical econometric models. Other countries have also actively developed econometric models for policy work combining both estimated and calibrated relationships. For example, Poloz, Rose, and Tetlow (1994) describe the Bank of Canada's econometric model, while Black et al. (1997) discuss the Core Model employed at the Reserve Bank of New Zealand.

The previous section presented evidence derived from a VAR on the output effects of a funds-rate shock. The VAR approach has been criticized for associating policy shocks with VAR residuals that do not closely correspond to other measures of

26. Brayton and Mauskopf (1985) provide a discussion of an earlier-generation FRB model, while Brayton, Levin, Tryon, and Williams (1997) describe the evolution of the Board's macro model.

monetary policy actions. The results from a VAR analysis, such as the impulse response functions shown in figures 1.5 and 1.6, can be compared to the findings from a large-scale structural model such as the FRB/US. Brayton and Tinsley (1996) report the output effects of an increase in the funds rate. The funds rate is increased for one quarter, with its subsequent behavior determined by the estimated policy reaction function embedded in the model. Brayton and Tinsley find an output response that looks very similar to figures 1.5 and 1.6, the response function from a simple VAR. Output follows a hump-shaped pattern, although the peak effect occurs after slightly more than one year in the FRB/US model simulation, while figures 1.5 and 1.6 suggests that it takes two years for output to reach its low point. When the FRB/US model is simulated using a small VAR to generate expectations, the output decline is larger and the peak decline occurs somewhat later. As a qualitative description of the output effects of monetary policy, however, the simple VARs give answers that are quite similar to that of the large FRB/US structural model. The simulation of a change in the inflation target reported by Black et al. (1997) using the New Zealand model also yields a qualitatively similar estimate of the output effects of monetary policy.

1.3.6 Alternative Approaches

Although the VAR approach has been the most commonly used empirical methodology in recent years, and although the results that have emerged provide a fairly consistent view of the impact of monetary policy shocks, other approaches have also influenced views on the role policy has played. Two such approaches, one based on deriving policy directly from a reading of policy statements, the other based on case studies of disinflations, have influenced academic discussions of monetary policy.

Narrative Measures of Monetary Policy An alternative to the VAR statistical approach is to develop a measure of the stance of monetary policy from a direct examination of the policy record. In recent years, this approach has been taken by Romer and Romer (1989) and by Boschen and Mills (1991), among others.[27]

Boschen and Mills develop an index of policy stance that takes on integer values from -2 (strong emphasis on inflation reduction) to $+2$ (strong emphasis on "promoting real growth"). Their monthly index is based on a reading of the Fed's Federal Open Market Committee (FOMC) policy directives and the records of the FOMC meetings. Boschen and Mills show that innovations in their index

27. Boschen and Mills (1991) provide a discussion and comparison of some other indices of policy. For a critical view of Romer and Romer's approach, see Leeper (1993).

corresponding to expansionary policy shifts are followed by subsequent increases in monetary aggregates and declines in the federal funds rate. They also conclude that all the narrative indices they examine yield relatively similar conclusions about the impact of policy on monetary aggregates and the funds rates. And, in support of the approach used in section 1.3.4, Boschen and Mills conclude that the funds rate is a good indicator of monetary policy. These findings are extended in Boschen and Mills (1995a), which compares several narrative-based measures of monetary policy, finding them to be associated with permanent changes in the level of $M2$ and the monetary base and temporary changes in the funds rate.

Romer and Romer (1989) used the Fed's "Record of Policy Actions" and, prior to 1976 when they were discontinued, minutes of FOMC meetings to identify episodes in which policy shifts occurred that were designed to reduce inflation. They find six different months during the postwar period that saw such contractionary shifts in Fed policy: October 1947, September 1955, December 1968, April 1974, August 1978, and October 1979. Leeper (1993) argues that the Romer-Romer index is equivalent to a dummy variable that picks up large interest-rate innovations. Hoover and Perez (1994) provide a critical assessment of the Romers' narrative approach, noting that the Romer dates are associated with oil price shocks, while Leeper (1997) finds that the exogenous component of the Romers' policy variable does not produce dynamic effects on output and prices that accord with general beliefs about the effects of monetary policy.

The narrative indices of Boschen and Mills and the dating system employed by Romer and Romer to isolate episodes of contractionary policy provide a useful and informative alternative to the VAR approach that associates policy shocks with serially uncorrelated innovations. The VAR approach attempts to identify exogenous shifts in policy; the estimated effects of these exogenous shifts are the conceptual parallels to the comparative static exercises for which theoretical models make predictions. To determine whether the data are consistent with a model's predictions about the effects of an exogenous policy action, we need to isolate empirically such exogenous shifts. Doing so, however, does not yield a measure of whether policy is, on net, expansionary or contractionary.[28] The narrative indices can provide a better measure of the net stance of policy, but they capture both exogenous shifts in policy and the endogenous response of monetary policy to economic developments. It is presumably the latter that accounts for most of the changes we observe in policy variables such as the funds rate as policy responds to current and future expected

28. Although Bernanke and Mihov (1998) use their VAR estimates in an attempt to develop such a measure.

economic conditions. In fact, a major conclusion of Leeper, Sims, and Zha (1996), and one they view as not surprising, is that most movements in monetary policy instruments represent responses to the state of the economy, not exogenous policy shifts.

Case Studies of Disinflations Case studies of specific episodes of disinflation provide, in principle, an alternative means of assessing the real impact of monetary policy. Romer and Romer's approach to dating periods of contractionary monetary policy is one form of case study. However, the most influential example of this approach is that of Sargent (1986), who examined the ends of several hyperinflations. As we will discuss more fully in chapter 5, the distinction between anticipated and unanticipated changes in monetary policy has played an important role during the past 30 years in academic discussions of monetary policy, and a key hypothesis is that anticipated changes should affect prices and inflation, with little or no effect on real economic activity. This implies that a credible policy to reduce inflation should succeed in actually reducing inflation without causing a recession. This implication contrasts sharply with the view that any policy designed to reduce inflation would succeed only by inducing an economic slowdown and temporarily higher unemployment.

Sargent tested these competing hypotheses by examining the ends of the post–World War I hyperinflations in Austria, Germany, Hungary, and Poland. In each case, Sargent found that the hyperinflations ended abruptly. In Austria, for example, prices rose by over a factor of 20 from December 1921 to August 1922, an annual inflation rate of over 8800%. Prices then stopped rising in September 1922, actually declining by more than 10% during the remainder of 1922. While unemployment did rise during the price stabilizations, Sargent concluded that the output cost "was minor compared with the $220 billion GNP that some current analysts estimate would be lost in the United States per one percentage point inflation reduction" (Sargent 1986, p. 55). Sargent's interpretation of the experiences in Germany, Poland, and Hungary is similar. In each case, the hyperinflation was ended by a regime shift that involved a credible change in monetary and fiscal policy designed to reduce government reliance on inflationary finance. Because the end of inflation reduced the opportunity cost of holding money, money demand grew and the actual stock of money continued to grow rapidly after prices had stabilized.

Sargent's conclusion that the output costs of these disinflations were small has been questioned, as have the lessons he drew for the moderate inflations experienced by the industrialized economies in the 1970s and early 1980s. As Sargent noted, the end of the hyperinflations "were not isolated restrictive actions within a given set of rules of the game" but represented changes in the rules of the game, most importantly in the ability of the fiscal authority to finance expenditures by creating money. In contrast, the empirical evidence from VARs of the type discussed earlier in this chapter reflects the impact of policy changes within a given set of rules.

Schelde-Andersen (1992) and Ball (1993) provide more recent examples of the case-study approach. In both cases, the authors examine disinflationary episodes in order to estimate the real output costs associated with reducing inflation.[29] Their cases, all involving OECD countries, represent evidence on the costs of ending moderate inflations. Ball calculates the deviation of output from trend during a period of disinflation and expresses this as a ratio to the change in trend inflation over the same period. The 65 disinflation periods he identifies in annual data yield an average sacrifice ratio of 0.77%; each percentage point reduction in inflation was associated with a 0.77% loss of output relative to trend. The estimate for the United States was among the largest, averaging 2.3% based on annual data. The sacrifice ratios are negatively related to nominal wage flexibility; countries with greater wage flexibility tend to have smaller sacrifice ratios. The costs of a disinflation also appear to be larger when inflation is brought down more gradually over a longer period of time.[30]

The case-study approach can provide interesting evidence on the real effects of monetary policy. Unfortunately, as with the VAR and other approaches, the issue of identification needs to be addressed. To what extent have disinflations been exogenous, so that any resulting output or unemployment movements can be attributed to the decision to reduce inflation? If policy actions depend on whether they are anticipated or not, then estimates of the cost of disinflating obtained by averaging over episodes, episodes that are likely to have differed considerably in terms of whether the policy actions were expected or, if announced, credible, may yield little information about the costs of ending any specific inflation.

1.4 Summary

The consensus from the empirical literature on the long-run relationship between money, prices, and output is clear. Money growth and inflation essentially display a

29. See also R. Gordon (1982) and R. Gordon and King (1982).

30. Brayton and Tinsley (1996) show how the costs of disinflation can be estimated under alternative assumptions about expectations and credibility using the FRB/US structural model. Their estimates of the sacrifice ratio, expressed in terms of the cumulative annual unemployment rate increase per percentage point decrease in the inflation rate, range from 2.6 under imperfect credibility and VAR expectations to 1.3 under perfect credibility and VAR expectations. Under full-model expectations, the sacrifice ratio is 2.3 with imperfect credibility and 1.7 with full credibility.

correlation of 1; the correlation between money growth or inflation and real output growth is probably close to 0, although it may be slightly positive at low inflation rates and negative at high rates.

The consensus from the empirical literature on the short-run effects of money is that exogenous monetary policy shocks produce hump-shaped movements in real economic activity. The peak effects occur after a lag of several quarters (as much as two or three years in some of the estimates) and then die out. The exact manner in which policy is measured makes a difference, and using an incorrect measure of monetary policy can significantly affect the empirical estimates one obtains.

There is less consensus, however, on the effects not of policy shocks but of the role played by the systematic feedback responses of monetary policy. Structural econometric models have the potential to fill this gap, and they are widely used in policy-making settings. Disagreements over the "true" structure and the potential dependence of estimated relationships on the policy regime have, however, posed problems for the structural modeling approach. A major theme of the next 10 chapters is that the endogenous response of monetary policy to economic developments can have important implications for the empirical relationships observed among macroeconomic variables.

2 Money-in-the-Utility Function

2.1 Introduction

The neoclassical growth model, due to Ramsey (1928) and Solow (1956), provides the basic framework for much of modern macroeconomics. Solow's growth model has just three key ingredients: a production function allowing for smooth substitutability between labor and capital in the production of output, a capital accumulation process in which a fixed fraction of output is devoted to investment each period, and a labor supply process in which the quantity of labor input grows at an exogenously given rate. Solow showed that such an economy would converge to a steady-state growth path along which output, the capital stock, and the effective supply of labor all grew at the same rate.

When the assumption of a fixed savings rate is replaced by a model of forward-looking households choosing savings and labor supply to maximize lifetime utility, the Solow model becomes the foundation for dynamic stochastic models of the business cycle. Productivity shocks or other real disturbances affect output and savings behavior, with the resultant effect on capital accumulation propagating the effects of the original shock over time in ways that can mimic some features of actual business cycles (see Cooley 1995).

The neoclassical growth model is a model of a nonmonetary economy, and while goods are exchanged and transactions must be taking place, there is no medium of exchange—that is, no "money"—that is used to facilitate these transactions. Nor is there an asset, like money, that has a zero nominal rate of return and is therefore dominated in rate of return by other interest-bearing assets. To employ the neoclassical framework to analyze monetary issues, a role for money must be specified so that the agents will wish to hold positive quantities of money. A positive demand for money is necessary if, in equilibrium, money is to have positive value.[1]

A fundamental question in monetary economics is the following: How should we model the demand for money? How do real economies differ from Arrow-Debreu economies in ways that give rise to a positive value for money? Three general approaches to incorporating money into general equilibrium models have been followed: (1) assume that money yields direct utility by incorporating money balances directly into the utility functions of the agents of the model (Sidrauski 1967); (2) impose transactions costs of some form that give rise to a demand for money, either

1. This is just another way of saying that we would like the money price of goods to be bounded. If the price of goods in terms of money is denoted by P, then 1 unit of money will purchase $1/P$ units of goods. If money has positive value, $1/P > 0$ and P is bounded ($0 < P < \infty$). Bewley (1983) refers to the issue of why money has positive value as the *Hahn problem* (Hahn 1965).

by making asset exchanges costly (Baumol 1952; Tobin 1956), requiring that money be used for certain types of transactions (Clower 1967), assuming that time and money can be combined to produce transaction services that are necessary for obtaining consumption goods, or assuming that direct barter of commodities is costly (Kiyotaki and Wright 1989); or (3) treat money like any other asset used to transfer resources intertemporally (Samuelson 1958). All involve shortcuts in one form or another; some aspects of the economic environment are simply specified exogenously in order to introduce a role for money. This can be a useful device, allowing one to focus attention on questions of primary interest without being unduly distracted by secondary issues. But our confidence in the ability of a model to answer the questions we bring to it is reduced if those aspects that are simply specified exogenously appear to be critical to the issue of focus. An important consideration in evaluating different approaches will be to determine whether conclusions generalize beyond the specific model or are dependent on the exact manner in which a role for money has been introduced. We will see examples of results that are robust, such as the connection between money growth and inflation, and others that are sensitive to the specification of money's role, such as the impact of inflation on the steady-state capital stock.

In this chapter, we develop the first of these three approaches by incorporating into the basic neoclassical model agents whose utility depends directly on their consumption of goods *and* their holdings of money.[2] Given suitable restrictions on the utility function, such an approach can guarantee that, in equilibrium, agents choose to hold positive amounts of money so that money will be positively valued. The money-in-the-utility function, or MIU, model we begin with is originally due to Sidrauski (1967), and it has been used widely to study a variety of issues in monetary economics.[3] The model can be used to examine the critical issues in monetary economics—the relationship between money and prices, the effects of inflation on equilibrium, and the optimal rate of inflation. To better understand the role of money in such a framework, a log-linear approximation for which analytic solutions can be derived is also studied. This allows us to calculate the macro time series behavior that the model implies. We can then determine whether the model is capable of generating the type of time series behavior we actually observe in macro-

2. The second approach, focusing on the transactions role of money, will be discussed in chapter 3. The third approach has been developed primarily within the context of overlapping-generation models; see Sargent (1987) or Champ and Freeman (1994).

3. Patinkin (1965, chapter 4) provides an earlier discussion of an MIU model, although he does not integrate capital accumulation into his model. However, the first order condition for optimal money holdings that he presents (see his equation 1, page 89) is equivalent to the one we will derive in the next section.

economic data, as well as assess the quantitative effects of inflation on the real economy.

2.2 The Basic MIU Model

To develop the basic MIU approach, we will ignore uncertainty and any labor-leisure choice, focusing instead on the implications of the model for money demand, the value of money, and the costs of inflation. Suppose that the utility function of the representative household takes the form

$$U_t = u(c_t, z_t),$$

where z_t is the flow of services yielded by money holdings and c_t is time t per capita consumption. Utility is assumed to be increasing in both arguments, strictly concave and continuously differentiable. The demand for monetary services will always be positive if we assume that $\lim_{z\to 0} u_z(c, z) = \infty$ for all c, where $u_z = \partial u(c, z)/\partial z$.

What constitutes z_t? If we wish to maintain the assumption of rational economic agents, then presumably what enters the utility function cannot just be the number of dollars (or yen or marks) that the individual holds. What should matter is the command over goods that are represented by those dollar holdings, or some measure of the transaction services, expressed in terms of goods, that money yields. In other words, z should be related to something like the number of dollars, M, times their price $(1/P)$ in terms of goods: $M(1/P) = M/P$. If the service flow is proportional to the real value of the stock of money, then we can set z equal to real per capita money holdings:

$$z_t = \frac{M_t}{P_t N_t} \equiv m_t.$$

To ensure that a monetary equilibrium exists, it is often assumed that, for all c, there exists a finite $\bar{m} > 0$ such that $u_m(c, m) \leq 0$ for all $m > \bar{m}$. This means that the marginal utility of money eventually becomes negative for sufficiently high money balances. The role of this assumption will be made clear when the existence of a steady state is discussed later. It is, however, not necessary for the existence of equilibrium, and some common functional forms often employed for the utility function (and that will be used later in this chapter) do not satisfy this condition.[4]

4. For example, $u(c, m) = \log c + b \log m$ does not exhibit this property since $u_m = b/m > 0$ for all finite m.

Assuming that money enters the utility function is often criticized on the grounds that money itself is intrinsically useless (as with a paper currency), and that it is only through its use in facilitating transactions that it yields valued services. Approaches that emphasize the transactions role of money will be discussed in chapter 3, but as will be shown there, models in which money helps to reduce the time needed to purchase consumption goods can also be represented by the money in the utility function approach adopted in this chapter.[5]

The representative household is viewed as choosing time paths for consumption and real money balances subject to budget constraints to be specified below, with total utility given by

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \tag{2.1}$$

where $0 < \beta < 1$ is a subjective rate of discount.

Equation (2.1) implies a much stronger notion of the utility provided by holding money than simply that the household would prefer having more money than less money. If the marginal utility of money is positive, then (2.1) implies that, *holding constant the path of real consumption for all* t, the individual's utility is increased by an increase in money holdings. That is, even though the money holdings are never used to purchase consumption, they yield utility. This should seem strange; we usually think the demand for money is instrumental in that we hold money to engage in transactions leading to the purchase of the goods and services that actually yield utility. All this is just a reminder that putting money in the utility function may be a useful shortcut for ensuring that there is a demand for money, but it is just a shortcut.[6]

To complete the specification of the model, assume that households can hold money, that bonds pay a nominal interest rate i_t, and physical capital. Physical capital produces output according to a standard neoclassical production function. Given its current income, its assets, and any net transfers received from the government (τ_t), the household allocates its resources between consumption, gross investment in physical capital, and gross accumulation of real money balances and bonds.

5. Brock (1974), for example, develops two simple transactions stories that can be represented by putting money directly in the utility function. See also Feenstra (1986).

6. In some environments, money might yield utility, even if never actually spent, if it is held for insurance purchases. For example, Imrohoroḡlu (1992) studies a model in which agents can insure against income fluctuations only by holding "money."

If the rate of depreciation of physical capital is δ, the aggregate economy-wide budget constraint of the household sector takes the form

$$Y_t + \tau_t N_t + (1-\delta)K_{t-1} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \tag{2.2}$$

where Y_t is aggregate output, K_{t-1} is the aggregate stock of capital at the start of period t, and $\tau_t N_t$ is the aggregate real value of any lump-sum transfers (taxes if negative).

The timing implicit in this specification of the MIU model assumes that it is the household's real money holdings at the end of the period, M_t/P_t, *after* having purchased consumption goods, that yield utility. Carlstrom and Fuerst (2001) have criticized this timing assumption, arguing that the appropriate way to model the utility from money is to assume that money balances available *before* going to purchase consumption goods yield utility. As they demonstrate, alternative timing assumptions can affect the correct definition of the opportunity cost of holding money and whether multiple real equilibria can be ruled out. Because it is standard in the MIU approach to assume that it is end-of-period money holdings that yield utility, we will continue to maintain that assumption in our development of the MIU model.[7]

The aggregate production function relates output Y_t to the available capital stock K_{t-1} and employment N_t: $Y_t = F(K_{t-1}, N_t)$. Assuming that this production function is linear homogeneous with constant returns to scale, output per capita at time t will be a function of the per capita capital stock:[8]

$$y_t = f\left(\frac{k_{t-1}}{1+n}\right), \tag{2.3}$$

where n is the population growth rate (assumed to be constant). Note the assumption that output is produced in period t using capital carried over from period $t-1$. The production function is assumed to be continuously differentiable and to satisfy the usual Inada conditions ($f_k \geq 0, f_{kk} \leq 0, \lim_{k\to 0} f_k(k) = \infty, \lim_{k\to\infty} f_k(k) = 0$).

7. Problems 1 and 2 at the end of the chapter ask you to derive the first order conditions for money holdings under an alternative timing assumption.

8. That is, if $Y_t = F(K_{t-1}, N_t)$, where Y is output, K is the capital stock, and N_t is labor input, and $F(\lambda K, \lambda N) = \lambda F(K, N) = \lambda Y$, we can write $Y_t/N_t \equiv y_t = F(K_{t-1}, N_t)/N_t = F(K_{t-1}/N_t, 1) \equiv f(k_{t-1}/(1+n))$, where $n = (N_t - N_{t-1})/N_{t-1}$ is the constant labor-force growth rate. In general, a lowercase letter will denote the per capita value of the corresponding uppercase variable.

Dividing both sides of the budget constraint (2.2) by the population N_t, the per capita version becomes

$$\omega_t \equiv f\left(\frac{k_{t-1}}{1+n}\right) + \tau_t + \left(\frac{1-\delta}{1+n}\right)k_{t-1} + \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{(1+\pi_t)(1+n)}$$

$$= c_t + k_t + m_t + b_t, \tag{2.4}$$

where τ_t is the rate of inflation, $b_t = B_t/D_tN_t$, and $m_t = M_t/P_tN_t$.

The household's problem is to choose paths for c_t, k_t, b_t, and m_t to maximize (2.1) subject to (2.4). This is a problem in dynamic optimization, and it is convenient to formulate the problem in terms of the value function. The value function gives the maximized value of utility the household can achieve by behaving optimally, given its current state.[9] The state variable for the problem is the household's initial resources ω_t. The value function, defined as the present discounted value of utility if the household optimally chooses consumption, capital holdings, bond holdings, and money balances, is given by

$$V(\omega_t) = \max\{u(c_t, m_t) + \beta V(\omega_{t+1})\}, \tag{2.5}$$

where the maximization is subject to the budget constraint (2.4) and

$$\omega_{t+1} = \frac{f(k_t)}{1+n} + \tau_{t+1} + \left(\frac{1-\delta}{1+n}\right)k_t + \frac{(1+i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)}.$$

Using (2.4) to express k_t as $\omega_t - c_t - m_t - b_t$ and making use of the definition of ω_{t+1}, (2.5) can be written as

$$V(\omega_t) = \max\Bigg\{u(c_t, m_t) + \beta V\left(\frac{f(\omega_t - c_t - m_t - b_t)}{1+n} + \tau_{t+1} + \left(\frac{1-\delta}{1+n}\right)(\omega_t - c_t - m_t - b_t) + \frac{(1+i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)}\right)\Bigg\}$$

with the maximization problem now an unconstrained one over c_t, b_t, and m_t. The first order necessary conditions for this problem are

$$u_c(c_t, m_t) - \frac{\beta}{1+n}[f_k(k_t) + 1 - \delta]V_\omega(\omega_{t+1}) = 0 \tag{2.6}$$

9. For introductions to dynamic optimization designed for economists see Sargent (1987), Lucas and Stokey (1989), Dixit (1990), Chiang (1992), Obstfeld and Rogoff (1996), or Ljungquist and Sargent (2000).

$$\frac{1+i_t}{(1+\pi_{t+1})(1+n)} - \left[\frac{f_k(k_t) + 1 - \delta}{1+n}\right] = 0 \tag{2.7}$$

$$u_m(c_t, m_t) - \beta\left[\frac{f_k(k_t) + 1 - \delta}{1+n}\right]V_\omega(\omega_{t+1}) + \frac{\beta V_\omega(\omega_{t+1})}{(1+\pi_{t+1})(1+n)} = 0 \tag{2.8}$$

together with the transversality conditions

$$\lim_{t\to\infty} \beta^t \lambda_t x_t = 0, \quad \text{for } x = k, b, m, \tag{2.9}$$

where λ_t is the marginal utility of period t consumption. The envelope theorem implies

$$\lambda_t = u_c(c_t, m_t) = V_\omega(\omega_t). \tag{2.10}$$

The first order conditions have straightforward interpretations. Since initial resources ω_t must be divided between consumption, capital, bonds, and money balances, each use must yield the same marginal benefit at an optimum allocation. Using (2.6) and (2.10), (2.8) can be written as

$$u_m(c_t, m_t) + \frac{\beta u_c(c_{t+1}, m_{t+1})}{(1+\pi_{t+1})(1+n)} = u_c(c_t, m_t), \tag{2.11}$$

which states that the marginal benefit of adding to money holdings at time t must equal the marginal utility of consumption at time t. The marginal benefit of additional money holdings has two components. First, money directly yields utility u_m. Second, real money balances at time t add $1/(1+\pi_{t+1})(1+n)$ to real per capita resources at time $t+1$; this addition to ω_{t+1} is worth $V_\omega(\omega_{t+1})$ at $t+1$, or $\beta V_\omega(\omega_{t+1})$ at time t. Thus, the total marginal benefit of money at time t is $u_m(c_t, m_t) + \beta V_\omega(\omega_{t+1})/(1+\pi_{t+1})(1+n)$.

From (2.6), (2.7), and (2.11),

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = 1 - \left[\frac{1}{(1+\pi_{t+1})(1+n)}\right]\frac{\beta u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)}$$

$$= 1 - \frac{1}{(1+r_t)(1+\pi_{t+1})}$$

$$= \frac{i_t}{1+i_t} \equiv \Upsilon_t, \tag{2.12}$$

where $1 + r_t \equiv f_k(k_t) + 1 - \delta$ is the real return on capital. To interpret (2.12), consider a very simple choice problem in which the agent must pick x and z to maximize

$u(x, z)$ subject to a budget constraint of the form $x + pz = y$, where p is the relative price of z. The first order conditions imply $u_z/u_x = p$. Comparing this to (2.12) shows that Υ has the interpretation of the relative price of real money balances in terms of the consumption good. The marginal rate of substitution between money and consumption is set equal to the price, or opportunity cost, of holding money. The opportunity cost of holding money is directly related to the nominal rate of interest. The household could hold one unit less of money, purchasing instead a bond yielding a nominal return of i; the real value of this payment is $i/(1+\pi)$, and since it is received in period $t+1$, its present value is $i/(1+r)(1+\pi) = i/(1+i)$.[10] Because money is assumed to pay no rate of interest, the opportunity cost of holding money is affected both by the real return on capital and by the rate of inflation. If the price level is constant (so $\pi = 0$), then the forgone earnings from holding money rather than capital are determined by the real return to capital. If the price level is rising ($\pi > 0$), this causes the real value of money to decline, adding to the opportunity cost of holding money.

Equation (2.6) for capital holdings has a similar interpretation; the net marginal return from holding additional capital, $\beta[f_k(k_t) + (1-\delta)]V_\omega(\omega_{t+1})/(1+n)$, must equal the marginal utility of consumption.

Equation (2.7) links the nominal return on bonds, inflation, and the real return on capital. It can be written as

$$1 + i_t = [1 + f_k(k_t) - \delta](1 + \pi_{t+1}) = (1 + r_t)(1 + \pi_{t+1}). \tag{2.13}$$

This relationship between real and nominal rates of interest is called the *Fisher relationship* after Irving Fisher (1896). It expresses the gross nominal rate of interest as equal to the gross real return on capital times 1 plus the expected rate of inflation. If we note that $(1+x)(1+y) \approx 1 + x + y$ when x and y are small, (2.13) is often written as

$$i_t = r_t + \pi_{t+1}.$$

Equations (2.6) to (2.8), together with the budget constraint (2.4), characterize the household's choice of consumption, money, bond, and capital holdings at each point in time. Equilibrium also requires that the nominal demand for money equals the nominal supply of money (assumed to be exogenous). In addition, since we have

10. Suppose households gain utility from the real money balances they have at the start of period t rather than the balances they hold at the end of the period, as we have been assuming. Then the marginal rate of substitution between money and consumption will be set equal to i_t (see Lucas 1982; Carlstrom and Fuerst 2001). See also problem 1 at the end of this chapter.

assumed that all households are identical, the stock of bonds must equal zero in equilibrium.

In deriving the first order conditions for the household's problem, we could have, equivalently, assumed that the household rented its capital to firms, receiving a rental rate of r_k, and sold its labor services at a wage rate of w. Household income would then be $r_k k + w$ (expressed on a per-capita basis). With competitive firms hiring capital and labor in perfectly competitive factor markets under constant returns to scale, $r_k = f'(k)$ and $w = f(k) - kf'(k)$, so household income would be $r_k k + w = f_k(k)k + [f(k) - kf_k(k)] = f(k)$, as in (2.4).[11]

While we could use this system to study analytically the dynamic behavior of the economy (for example, see Sidrauski 1967, Fischer 1979b, Blanchard and Fischer 1989), we will instead focus first on the properties of the steady-state equilibrium. And, since our main focus here is not on the exogenous growth generated by population growth, it will provide some slight simplification to set $n = 0$ in the following. After we have examined the steady state, we will study the dynamic properties by examining the time-series behavior of macroeconomic variables implied by a stochastic version of the model, a version that also includes uncertainty, a labor-leisure choice, and variable employment.

2.2.1 Steady-State Equilibrium

Consider the properties of this economy when it is in a steady-state equilibrium with $n = 0$ and the nominal supply of money is growing at the rate θ. Let the superscript ss denote values evaluated at the steady state. The steady-state values of consumption, the capital stock, real money balances, inflation, and the nominal interest rate must satisfy the first order necessary conditions for the household's decision problem given by (2.6)–(2.8), the economy-wide budget constraint, and the specification of the exogenous growth rate of M. These conditions can be written as

$$u_c(c^{ss}, m^{ss}) - \beta[f_k(k^{ss}) + 1 - \delta]V_\omega(\omega^{ss}) = 0 \tag{2.14}$$

$$\frac{1 + i^{ss}}{1 + \theta} - [f_k(k^{ss}) + 1 - \delta] = 0 \tag{2.15}$$

$$u_m(c^{ss}, m) - \beta[f_k(k^{ss}) + 1 - \delta]V_\omega(\omega^{ss}) + \frac{\beta V_\omega(\omega^{ss})}{1 + \theta} = 0 \tag{2.16}$$

11. This follows from Euler's theorem: If the aggregate constant-returns-to-scale production function is $F(N, K)$, then $F(N, K) = F_N N + F_K K$. In per capita terms, this becomes $f(k) = F_N + F_K k = w + rk$ if labor and capital are paid their marginal products.

$$f(k^{ss}) + \tau^{ss} + (1-\delta)k^{ss} + \frac{m^{ss}}{1+\theta} = c^{ss} + k^{ss} + m^{ss}, \tag{2.17}$$

where $\omega^{ss} = f(k^{ss}) + \tau^{ss} + (1-\delta)k^{ss} + m^{ss}/(1+\pi)$. In (2.14)–(2.17), use has been made of the fact that, in the equilibrium of this representative agent model, $b = 0$. We have also used the result that the rate of inflation is determined by the growth rate of the nominal quantity of money. This is simply an implication of the steady-state property that real, per capita money holdings are constant in the steady state, and a constant value of real money balances requires prices to change at the same rate as the nominal stock of money, so $\pi^{ss} = \theta$.[12]

Notice that in (2.14)–(2.17), money appears only in the form of m, real money balances. Thus, any change in the nominal quantity of money that is matched by a proportional change in the price level, leaving m unchanged, has no effect on the economy's real equilibrium. This is described by saying that the model exhibits the *neutrality of money*. If prices do not adjust immediately in response to a change in M, then a model might display nonneutrality with respect to changes in M in the short run but still exhibit monetary neutrality in the long run once all prices have adjusted. In fact, this will be the case with the models used in chapters 5–11 to examine issues related to short-run monetary policy.

Using (2.10), (2.14) implies that $1 = \beta[f_k(k^{ss}) + 1 - \delta]$, or

$$f_k(k^{ss}) = \frac{1}{\beta} - 1 + \delta. \tag{2.18}$$

This equation defines the steady-state capital-labor ratio k^{ss}. If the production function is Cobb-Douglas, say $f(k) = k^{\alpha}$, then $f_k(k) = \alpha k^{\alpha-1}$ and we have

$$k^{ss} = \left(\frac{\alpha\beta}{1+\beta(\delta-1)}\right)^{\frac{1}{1-\alpha}}. \tag{2.19}$$

What is particularly relevant for our purposes is the implication from (2.18) that the steady-state capital-labor ratio is independent of 1) all parameters of the utility function other than the subjective discount rate β and 2) the steady-state rate of inflation π^{ss}. In fact, k^{ss} depends only on the production function, the depreciation rate, and the discount rate. It is independent of the rate of inflation.

Because changes in the nominal quantity of money are engineered in this model by making lump-sum transfers to the public, the real value of these transfers is equal

12. If the population is growing at the rate n, then $1 + \pi^{ss} = (1 + \theta^{ss})/(1 + n)$.

to $(M_t - M_{t-1})/P_t$. Hence, steady-state transfers are given by $\tau^{ss} = m^{ss} - m^{ss}/(1+\pi^{ss}) = \theta m^{ss}/(1+\theta)$, so the budget constraint (2.17) reduces to

$$c^{ss} = f(k^{ss}) - \delta k^{ss}. \tag{2.20}$$

The steady-state level of consumption per capita is completely determined once we know the level of steady-state capital. If we again assume that $f(k) = k^{\alpha}$, k^{ss} is given by (2.19) and

$$c^{ss} = \left(\frac{\alpha\beta}{1+\beta(\delta-1)}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\alpha\beta}{1+\beta(\delta-1)}\right)^{\frac{1}{1-\alpha}}.$$

Steady-state consumption per capita depends on the parameters of the production function (α), the rate of depreciation (δ), and the subjective rate of time discount (β).

This model exhibits a property called the *superneutrality of money*; the steady-state values of the capital-labor ratio, consumption, and output are all independent of the rate of inflation. That is, not only is money neutral, so that proportional changes in the *level* of nominal money balances and prices have no real effects, but changes in the *rate of growth* of nominal money also have no effect either on the steady-state capital stock or, therefore, on output or per capita consumption. Since the real rate of interest is equal to the marginal product of capital, it also is invariant across steady states that differ only in their rates of inflation.

An important distinction is that between changes in the *level* of the nominal supply of money and changes in the *rate of growth* of the nominal money supply. In all the models we will examine, the nominal money stock enters in the form M/P. Thus, proportional changes in the level of M and P, changes that leave M/P unaffected, have no real effects. A model displays the property of *superneutrality* if the real equilibrium is independent of the rate of growth of the nominal money supply. Thus, the Sidrauski MIU model possesses the properties of both neutrality and superneutrality.

To understand why superneutrality holds, note that from (2.10), $u_c = V_{\omega}(\omega_t)$, so using (2.6),

$$u_c(c_t, m_t) = \beta[f_k(k_t) + 1 - \delta]u_c(c_{t+1}, m_{t+1}),$$

or

$$\frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} = \frac{1/\beta}{f_k(k_t) + 1 - \delta}. \tag{2.21}$$

Recall from (2.18) that the right side of this expression is equal to 1 in the steady state. If $k < k^{ss}$ so that $f_k(k) > f_k(k^{ss})$, then the right side is smaller than 1, and the marginal utility of consumption will be declining over time. It will be optimal to postpone consumption to accumulate capital and have consumption grow over time (so u_c declines over time). As long as $f_k + 1 - \delta > 1/\beta$, this process continues, but as the capital stock grows, the marginal product of capital declines until eventually $f_k(k) + 1 - \delta = 1/\beta$. The converse holds if $k > k^{ss}$. Consumption remains constant only when $f_k + 1 - \delta = 1/\beta$. If an increase in the rate of inflation were to induce households to accumulate more capital, this would lower the marginal product of capital, leading to a situation in which $f_k + 1 - \delta < 1/\beta$. Households would then want their consumption path to decline over time, so they would immediately attempt to increase current consumption and reduce their holdings of capital. The value of k consistent with a steady state is independent of the rate of inflation.

What is affected by the rate of inflation? One thing we should expect is that the interest rate on any asset that pays off in units of money at some future date will be affected; the real value of those future units of money will be affected by inflation, and this will be reflected in the interest rate required to induce individuals to hold the asset, as shown by (2.13). To understand this equation, consider the nominal interest rate that an asset must yield if it is to give a real return of r_t in terms of the consumption good. That is, consider an asset that costs 1 unit of consumption in period t and yields $(1 + r_t)$ units of consumption at $t + 1$. In units of money, this asset costs P_t units of money at time t. Since the cost of each unit of consumption at $t + 1$ is P_{t+1} in terms of money, the asset must pay an amount equal to $(1 + r)P_{t+1}$. Thus, the nominal return is $[(1 + r_t)P_{t+1} - P_t]/P_t = (1 + r_t)(1 + \pi_{t+1}) - 1 \equiv i_t$. In the steady state, $1 + r^{ss} = 1/\beta$ and $\pi^{ss} = \theta$, so the steady-state nominal rate of interest is given by $[(1 + \theta)/\beta] - 1$ and varies (approximately) one for one with inflation.[13]

Existence of the Steady State To ensure that a steady-state monetary equilibrium exists, there must exist a positive but finite level of real money balances m^{ss} that satisfies (2.12), evaluated at the steady-state level of consumption. If utility is separable in consumption and money balances, say $u(c, m) = v(c) + \phi(m)$, this condition can be written as $\phi_m(m^{ss}) = \Upsilon^{ss} v_c(c^{ss})$. The right side of this expression is a positive constant; the left side approaches ∞ as $m \to 0$. If $\phi_m(m) \leq 0$ for all m greater than some finite level, a steady-state equilibrium with positive real money balances is guaranteed to exist. This was the role of the earlier assumption that the marginal

13. Outside of the steady state, the nominal rate can still be written as the sum of the expected real rate plus the expected rate of inflation, but there is no longer any presumption that short-run variations in inflation will leave the real rate unaffected.

utility of money eventually becomes negative. Note that this assumption is not necessary; $\phi(m) = \log m$ yields a positive solution to (2.12) as long as $\Upsilon^{ss} v_c(c^{ss}) > 0$. When utility is not separable, we can still write (2.12) as $u_m(c^{ss}, m^{ss}) = \Upsilon^{ss} u_c(c^{ss}, m^{ss})$. If $u_{cm} < 0$ so that the marginal utility of consumption decreases with increased holdings of money, both u_m and u_c decrease with m and the solution to (2.12) may not be unique; multiple steady-state equilibria may exist.[14]

When $u(c, m) = v(c) + \phi(m)$, the dynamics of real balances around the steady state can be described easily by multiplying both sides of (2.11) by M_t and noting that $M_{t+1} = (1 + \theta)M_t$:

$$B(m_{t+1}) \equiv \frac{\beta}{1 + \theta} v_c(c_{ss}) m_{t+1} = [v_c(c^{ss}) - \phi_m(m_t)] m_t \equiv A(m_t), \tag{2.22}$$

which gives a difference equation in m. The properties of this equation have been examined by Brock (1974) and Obstfeld and Rogoff (1983, 1986). A steady-state value for m satisfies $B(m^{ss}) = A(m^{ss})$. The functions $B(m)$ and $A(m)$ are illustrated in figure 2.1. For the case drawn, $\lim_{m \to 0} \phi_m m = 0$ so there are two steady-state solutions to (2.22), one at m' and one at 0. Only one of these involves positive real money balances (and a positive value for money). If $\lim_{m \to 0} \phi_m m = \bar{m} > 0$, then $\lim_{m \to 0} A(m) < 0$ and there is only one solution. Paths for m_t originating to the right of m' involve $m_{t+s} \to \infty$ as $s \to \infty$. When $\theta \geq 0$ (nonnegative money growth), such explosive paths for m, involving a price level going to zero, violate the transversality condition that the discounted value of asset holdings must go to zero (see Obstfeld and Rogoff 1983, 1986).[15] When $\lim_{m \to 0} A(m) < 0$, paths originating to the left of m' converge to $m < 0$; but this is clearly not possible, since real balances cannot be negative. For the case drawn in figure 2.1, however, some paths originating to the left of m' converge to 0 without ever involving negative real balances. For example, a path that reaches m'' at which $A(m'') = 0$ then jumps to $m = 0$. Along such an equilibrium path, the price level is growing faster than the nominal money supply (so that m declines). Even if $\theta = 0$, so that the nominal money supply is constant, the equilibrium path would involve a speculative hyperinflation with the price level going to infinity.[16] Unfortunately, Obstfeld and Rogoff show that the conditions needed to

14. For more on the conditions necessary for the existence of monetary equilibria, see Brock (1974, 1975) and Bewley (1983).

15. Obstfeld and Rogoff (1986) show that any such equilibrium path with an implosive price level violates the transversality condition unless $\lim_{m \to \infty} \phi(m) = \infty$. This condition is implausible, as it would require that the utility yielded by money be unbounded.

16. The hyperinflation is labeled speculative since it is not driven by fundamentals, such as the growth rate of the nominal supply of money.

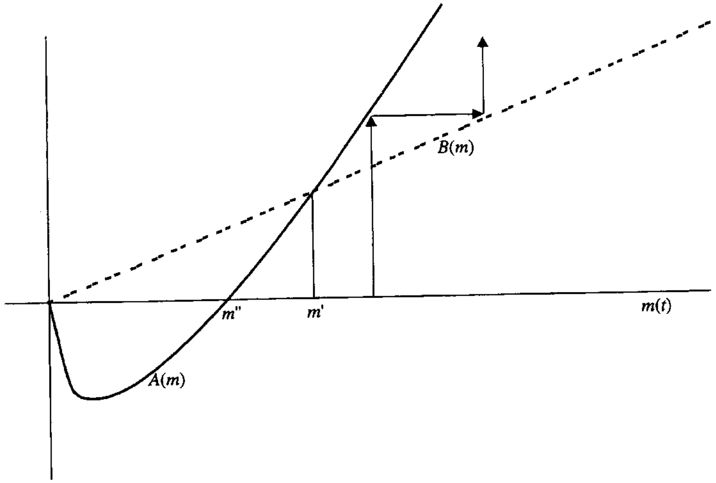

Figure 2.1
Steady-State Real Balances (Separable Utility)

ensure $\lim_{m \to 0} \phi_m m = \tilde{m} > 0$ so that speculative hyperinflations can be ruled out are restrictive. They show that $\lim_{m \to 0} \phi_m m > 0$ implies $\lim_{m \to 0} \phi(m) = -\infty$; essentially, money must be so necessary that the utility of the representative agent goes to minus infinity if her real balances fall to zero.[17]

When paths originating to the left of m' cannot be ruled out, the model exhibits multiple equilibria. For example, suppose that the nominal stock of money is held constant, with $M_t = M_0$ for all $t > 0$. Then there is a rational expectations equilibrium path for the price level and real money balances starting at any price level P_0 as long as $M_0/P_0 < m'$. Chapter 4 examines an approach called the *fiscal theory of the price level*, which argues that the initial price level may be determined by fiscal policy.

2.2.2 The Interest Elasticity of Money Demand

Returning to (2.12), this characterizes the demand for real money balances as a function of the nominal rate of interest and real consumption. For example, suppose

17. Speculative hyperinflations are shown by Obstfeld and Rogoff to be ruled out if the government holds real resources to back a fraction of the outstanding currency. This ensures a positive value below which the real value of money cannot fall.

that the utility function in consumption and real balances is of the constant elasticity of substitution (CES) form:

$$u(c_t, m_t) = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}}, \tag{2.23}$$

with $0 < a < 1$ and $b > 0$, $b \neq 1$. Then

$$\frac{u_m}{u_c} = \left(\frac{1-a}{a}\right)\left(\frac{c_t}{m_t}\right)^b$$

and (2.12) can be written as[18]

$$m_t = \left(\frac{1-a}{a}\right)^{\frac{1}{b}}\left(\frac{i}{1+i}\right)^{-\frac{1}{b}} c_t. \tag{2.24}$$

In terms of the more common specification in log form used to model empirical money demand equations (Goldfeld and Sichel 1990),

$$\log \frac{M_t}{P_t N_t} = \frac{1}{b} \log\left(\frac{1-a}{a}\right) + \log c - \frac{1}{b} \log \frac{i}{1+i}, \tag{2.25}$$

which gives the real demand for money as a negative function of the nominal rate of interest and a positive function of consumption.[19] The consumption (income) elasticity of money demand is equal to 1 in this specification. The elasticity of money demand with respect to the opportunity cost variable $\Upsilon_t = i_t/(1+i_t)$ is $1/b$. For simplicity, this will often be referred to as the *interest elasticity of money demand.*[20]

For $b = 1$, the CES specification becomes $u(c_t, m_t) = c_t^a m_t^{1-a}$. Note from (2.25) that in this case, the consumption (income) elasticity of money demand and the elasticity with respect to the opportunity cost measure Υ_t are both equal to 1.

18. In the limit, as $b \to \infty$, (2.24) implies that $m = c$. This is then equivalent to the cash-in-advance models we will examine in chapter 3.

19. The standard specification of money demand would use income in place of consumption, although see Mankiw and Summers (1986).

20. The elasticity of money demand with respect to the nominal interest rate is

$$-\frac{\partial m_t}{\partial i_t}\frac{i_t}{m_t} = \frac{1}{b}\frac{1}{1+i_t}.$$

Empirical work often estimates money-demand equations in which the log of real money balances is a function of log income and the *level* of the nominal interest rate. The coefficient on the nominal interest rate is then equal to the semielasticity of money demand with respect to the nominal interest rate $(m^{-1}\partial m/\partial i)$, which for (2.25) is $1/bi(1+i)$.

While the parameter b governs the interest elasticity of demand, the steady-state level of money holdings depends on the value of a. From (2.24), the ratio of real money balances to consumption in the steady state will be[21]

$$\frac{m^{ss}}{c^{ss}} = \left(\frac{1-a}{a}\right)^{\frac{1}{b}}\left(\frac{\Pi^{ss}-\beta}{\Pi^{ss}}\right)^{-\frac{1}{b}},$$

where $\Pi^{ss} = 1 + \pi^{ss}$ is the gross rate of inflation. The ratio of m^{ss} to c^{ss} is decreasing in a; an increase in a reduces the weight given to real money balances in the utility function and results in smaller holdings of money (relative to consumption) in the steady state. Increases in inflation also reduce the ratio of money holdings to consumption by increasing the opportunity cost of holding money.

Empirical Evidence on the Interest Elasticity of Money Demand The empirical literature on money demand is vast. See, for example, the references in Judd and Scadding (1982), Laidler (1985), or Goldfeld and Sichel (1990) for earlier surveys. More relevant for our discussion is Holman (1998), who directly estimates the parameters of the utility function under various alternative specifications of its functional form, including (2.23), using annual U.S. data from 1889 to 1991.[22] She obtains estimates of b of around 0.1 and a of around 0.95. This value of b implies an elasticity of money demand equal to 10. However, in shorter samples, the data fail to reject $b = 1$, the case of Cobb-Douglas preferences, indicating that the interest elasticity of money demand is estimated very imprecisely.

Chari, Kehoe, and McGrattan (2000) estimate (2.25) using quarterly U.S. data and the $M1$ definition of money. They obtain an estimate for a of around 0.94 and an estimate of the interest elasticity of money demand of 0.39, implying a value of b on the order of $1/.39 \approx 2.6$. Hoffman, Rasche, and Tieslau (1995) conduct a cross-country study of money demand and find a value of around 0.5 for the United States and Canada money demand interest elasticity, with somewhat higher values for the United Kingdom and lower values for Japan and Germany. An elasticity of 0.5 implies a value of 2 for b. Ireland (2001a) estimates the interest elasticity as part of a general equilibrium model and obtains a value of 0.19 for the pre-1979 period and 0.12 for the post-1979 period. These translate into values for b of 5.26 and 8.33, respectively.

Most empirical estimates of the interest elasticity of money demand employ aggregate time-series data. At the household level, many U.S. households hold

21. This makes use of the fact that $1 + i = R\Pi = \Pi/\beta$ in the steady state.

22. Holman (1998) considers a variety of specifications for the utility function, including Cobb-Douglas ($b = 1$) and nested CES functions of the form we will use later in section 2.5.

no interest-earning assets, so the normal substitution between money and interest-earning assets as the nominal interest rate changes is absent. As nominal interest rates rise, more households find it worthwhile to hold interest-earning assets. Changes in the nominal interest rate then affect both the extensive margin (the decision whether to hold interest-earning assets) and the intensive margin (the decision on how to allocate wealth between money and interest-earning assets). Mulligan and Sala-i-Martin (2000) focus on these two margins and use cross-sectional evidence on household holdings of financial assets to estimate money demand interest elasticity. They find that the elasticity increases with the level of nominal interest rates and is low at low nominal rates of interest.

2.2.3 Limitations

Before moving on to use this framework to analyze the welfare cost of inflation, we need to consider the limitations of the money in the utility approach. In the MIU model, there is a clearly defined reason for individuals to hold money—it provides direct utility. However, this essentially "solves" the problem of generating a positive demand for money by assumption; it doesn't address the reasons why money, particularly money in the form of unbacked pieces of paper, might yield utility. The money in the utility function approach has to be thought of as a shortcut for a fully specified model of the transactions technology faced by households that gives rise to a positive demand for a medium of exchange.[23]

Shortcuts are often extremely useful. But one problem with such a shortcut is that it does not provide any real understanding of, or possible restrictions on, such quantities as u_m or u_{cm} that play a role in determining equilibrium and the outcome of comparative static exercises. One possible story that can generate money in the utility function is a shopping-time story, and we will return to this idea in chapter 3.

2.3 The Welfare Cost of Inflation

Because money holdings yield direct utility and higher inflation reduces real money balances, inflation generates a welfare loss. This raises two questions: 1) how large is the welfare cost of inflation? and 2) is there an optimal rate of inflation that maximizes the steady-state welfare of the representative household? While chapter 4 (and chapter 11) will provide a more detailed discussion of the optimal rate of inflation,

23. For a general equilibrium analysis of asset prices in an MIU framework, see LeRoy (1984a, 1984b).

we can illustrate here some important results on both of these questions, taking them in reverse order.

The second question—the optimal rate of inflation—was originally addressed by Bailey (1956) and M. Friedman (1969). Their basic intuition was the following. The private opportunity cost of holding money depends on the nominal rate of interest (see 2.12). The social marginal cost of producing money—that is, running the printing presses—is essentially zero. The wedge that arises between the private marginal cost and the social marginal cost when the nominal rate of interest is positive generates an inefficiency. This inefficiency would be eliminated if the private opportunity cost were also equal to zero, and this will be the case if the nominal rate of interest equals zero. But $i = 0$ requires that $\pi = -r/(1+r) \approx -r$. So the optimal rate of inflation is a rate of *deflation* approximately equal to the real return on capital.[24]

In the steady state, real money balances are directly related to the inflation rate, so the optimal rate of inflation is also frequently discussed under the heading of the *optimal quantity of money*. With utility depending directly on m, one can think of the government choosing its policy instrument θ (and therefore π) to achieve the steady-state optimal value of m. Steady-state utility will be maximized when $u(c^{ss}, m^{ss})$ is maximized subject to the constraint that $c^{ss} = f(k^{ss}) - \delta k^{ss}$. But since c^{ss} is independent of θ, the first order condition for the optimal θ is just $u_m(\partial m/\partial \theta) = 0$, or $u_m = 0$, and from (2.12), this occurs when $i = 0$.[25]

The major criticism of this result is that of Phelps (1973), who pointed out that money growth generates revenue for the government—the inflation tax. The implicit assumption so far has been that variations in money growth are engineered via lump-sum transfers. Any effects on government revenue can be offset by a suitable adjustment in these lump-sum transfers (taxes). But if governments only have distortionary taxes available for financing expenditures, then reducing inflation tax revenues to achieve the Friedman rule for optimal inflation requires that the lost revenue be replaced through increases in other distortionary taxes. To minimize the total distortions associated with raising a given amount of revenue, it may be optimal to rely to some degree on the inflation tax. Reducing the nominal rate of interest to zero would increase the inefficiencies generated by the higher level of other taxes needed to replace lost inflation tax revenues. Recent work has reexamined these results (see

24. Since $(1+i) = (1+r)(1+\pi)$, $i = 0$ implies $\pi = -r/(1+r) \approx -r$.

25. Note that the earlier assumption that the marginal utility of money goes to zero at some finite level of real balances ensures that $u_m = 0$ has a solution with $m < \infty$. While we focus here on the steady state, a more appropriate perspective for addressing the optimal inflation question would not restrict attention solely to the steady state. The more general case is considered in chapter 4.

Chari, Christiano, and Kehoe 1991, 1996; Correia and Teles 1996, 1999; Mulligan and Sala-i-Martin 1997). The revenue implications of inflation and optimal inflation are major themes of chapter 4.

Now let's return to the first question posed previously—what is the welfare cost of inflation? Beginning with Bailey (1956), this welfare cost has been calculated from the area under the money demand curve (showing money demand as a function of the nominal rate of interest), since this provides a measure of the consumer surplus lost as a result of having a positive nominal rate of interest. Figure 2.2 is based on the money demand function given by (2.24) with $a = 0.9$ and $b = 2.56$. At a nominal interest rate of i^*, the deadweight loss is measured by the shaded area under the money demand curve.

Because nominal interest rates reflect *expected* inflation, calculating the area under the money demand curve will provide a measure of the costs of anticipated inflation and is therefore appropriate for evaluating the costs of alternative constant rates of inflation. There are other costs of inflation associated with tax distortions and with variability in the rate of inflation; these are discussed in the survey on the costs of inflation by Driffill, Mizon, and Ulph (1990).

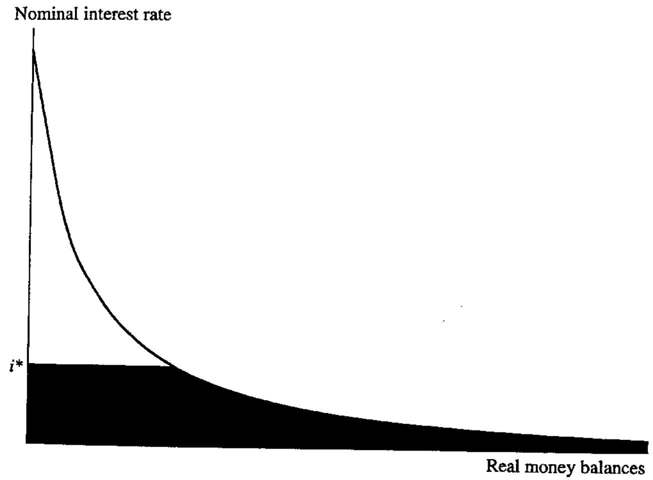

Figure 2.2
The Welfare Costs of Inflation

Lucas (1994) provides estimates of the welfare costs of inflation by starting with the following specification of the instantaneous utility function:

$$u(c,m) = \frac{1}{1-\sigma}\left\{\left[c\varphi\left(\frac{m}{c}\right)\right]^{1-\sigma} - 1\right\}. \tag{2.26}$$

With this utility function, (2.12) becomes

$$\frac{u_m}{u_c} = \frac{\varphi'(x)}{\varphi(x) - x\varphi'(x)} = \frac{i}{1+i} = \Upsilon, \tag{2.27}$$

where $x \equiv m/c$.[26] Normalizing so that steady-state consumption equals 1, $u(1,m)$ will be maximized when $\Upsilon = 0$, implying that the optimal x is defined by $\varphi'(m^*) = 0$. Lucas proposes to measure the costs of inflation by the percentage increase in steady-state consumption necessary to make the household indifferent between a nominal interest rate of i and a nominal rate of 0. If this cost is denoted $w(\Upsilon)$, it is defined by

$$u(1 + w(\Upsilon), m(\Upsilon)) \equiv u(1, m^*), \tag{2.28}$$

where $m(\Upsilon)$ denotes the solution of (2.27) for real money balances evaluated at steady-state consumption $c = 1$.

Suppose, following Lucas, that $\varphi(m) = [1 + Bm^{-1}]^{-1}$ where B is a positive constant. Solving (2.27), one obtains $m(i) = B^{.5}\Upsilon^{-.5}$.[27] Note that $\varphi' = 0$ requires that $m^* = \infty$. But $\varphi(\infty) = 1$ and $u(1,\infty) = 0$, so $w(\Upsilon)$ is the solution to $u(1 + w(\Upsilon), B^{.5}\Upsilon^{-.5}) = u(1,\infty) = 0$. Using the definition of the utility function, we obtain $1 + w(\Upsilon) = 1 + \sqrt{B\Upsilon}$, or

$$w(\Upsilon) = \sqrt{B\Upsilon}. \tag{2.29}$$

Based on U.S. annual data from 1900 to 1985, Lucas reports an estimate of .0018 for B. Hence, the welfare loss arising from a nominal interest rate of 10% would be $\sqrt{(.0018)(.1/1.1)} = .013$, or just over 1% of aggregate consumption.

Because U.S. government bond yields were around 10% in 1979 and 1980, we can use 1980 aggregate personal consumption expenditures of \$2447.1 trillion to get a rough estimate of the dollar welfare loss (although consumption expenditures includes purchases of durables). In this example, 1.3% of \$2447.1 trillion is about \$32 billion. Since this is the annual cost in terms of steady-state consumption, we need the present discounted value of \$32 billion. Using a real rate of return of 2%, this amounts to \$32(1.02)/.02 = \$1.632 trillion; at 4%, the cost would be \$832 billion.

26. In the framework Lucas employs, the relevant expression is $u_m/u_c = i$; problem 1 at the end of the chapter provides an example of the timing assumptions Lucas employed.

27. Lucas starts with the assumption that money demand is equal to $m = Ai^{-.5}$ for A equal to a constant. He then derives $\varphi(m)$ as the utility function necessary to generate such a demand function, where $B = A^2$.

An annual welfare cost of \$32 billion seems a small number, especially when compared to the estimated costs of reducing inflation. For example, Ball (1993) reports a "sacrifice ratio" of 2.4% of output per 1% of inflation reduction for the United States. Because inflation was reduced from about 10% to about 3% in the early 1980s, Ball's estimate would put the cost of this disinflation at approximately 17% of GDP (2.4% times an inflation reduction of 7%). Based on 1980 GDP of \$3776.3 trillion (1987 prices), this would be \$642 billion. This looks large when compared to the \$32 billion annual welfare cost, but the trade-off starts looking more worthwhile if the costs of reducing inflation are compared to the present discounted value of the annual welfare cost. (See also Feldstein 1979.)

Gillman (1995) provides a useful survey of different estimates of the welfare cost of inflation. The estimates differ widely. One important reason for these differences arises from the choice of the base inflation rate. Some estimates compare the area under the money demand curve between an inflation rate of zero and, say, 10%. This is incorrect in that a zero rate of inflation still results in a positive nominal rate (equal to the real rate of return) and therefore a positive opportunity cost associated with holding money. Gillman concludes, based on the empirical estimates he surveys, that a reasonable value of the welfare cost of inflation for the United States is in the range of 0.85% to 3% of real GNP per percentage rise in the nominal interest rate above zero, a loss in 2002 dollars of \$88 to \$310 billion per year.[28]

The area under the demand curve is a partial equilibrium measure of the welfare costs of inflation. Gomme (1993) and Dotsey and Ireland (1996) examine the effects of inflation in general equilibrium frameworks that allow for the supply of labor and the average rate of economic growth to be affected. Gomme finds that, even though inflation reduces the supply of labor and economic growth, the welfare costs are small because of the increased consumption of leisure that households enjoy. Dostey and Ireland find much larger welfare costs of inflation in a model that generates an interest elasticity of money demand that matches estimates for the United States. (See also De Gregorio 1993 and Imrohoroğlu and Prescott 1991.)

The Sidrauski model provides a convenient framework for calculating the steady-state welfare costs of inflation, both because the lower level of real money holdings that result at higher rates of inflation has a direct effect on welfare when money enters the utility function and because the superneutrality property of the model means that the other argument in the utility function, real consumption, is invariant across different rates of inflation. This latter property simplifies the calculation since

28. These estimates apply to the United States, which has experienced relatively low rates of inflation. They may not be relevant for high-inflation countries.

it is not necessary to account for both variations in money holdings and variations in consumption when making the welfare cost calculation. Of critical importance, however, is the ability of the MIU function approach to allow the costs of inflation to be calculated based on a model of money demand that is consistent with optimizing behavior on the part of economic agents.

2.4 Extensions

2.4.1 Interest on Money

If the welfare costs of inflation are related to the positive private opportunity costs of holding money, an alternative to deflation as a means of eliminating these costs would be the payment of explicit interest on money. There are obvious technical difficulties in paying interest on cash, but ignoring these, assume that the government pays a nominal interest rate of i^m on money balances. Assume further that these interest payments are financed by lump-sum taxes s. The household's budget constraint, (2.4), now becomes (setting $n = 0$)

$$f(k_{t-1}) - s_t + \tau_t + (1-\delta)k_{t-1} + (1+r_{t-1})b_{t-1} + \frac{1+i_t^m}{1+\pi_t}m_{t-1} = c_t + k_t + m_t + b_t \tag{2.30}$$

and the first order condition (2.8) becomes

$$-u_c(c_t, m_t) + u_m(c_t, m_t) + \frac{\beta(1+i_t^m)V_\omega(\omega_{t+1})}{(1+\pi_{t+1})} = 0, \tag{2.31}$$

while (2.12) is now

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t - i_t^m}{1+i_t}.$$

The opportunity cost of money is related to the interest rate gap $i - i_m$, which represents the difference between the nominal return on bonds and the nominal return on money. If $\theta = 0$ so that the rate of inflation in the steady state is also zero, the optimal quantity of money, the quantity such that $u_m = 0$, can be achieved if $i_m = r$.

The assumption that the interest payments are financed by the revenue from lump-sum taxes is critical for this result. One of the problems at the end of this chapter considers what happens if the government simply finances the interest payments on money by printing more money.

2.4.2 Nonsuperneutrality

Calculations of the steady-state welfare costs of inflation in the Sidrauski model are greatly simplified by the fact that the model exhibits superneutrality. But how robust is the result that money is superneutral? The empirical evidence of Barro (1995) suggests that inflation has a negative effect on growth, a finding inconsistent with superneutrality.[29] One channel through which inflation can have real effects in the steady state is introduced if households have a labor supply choice. That is, suppose utility depends on consumption, real money holdings, and leisure:

$$u = u(c, m, l). \tag{2.32}$$

The economy's production function becomes

$$y = f(k, 1-l), \tag{2.33}$$

where the total supply of time is normalized to equal 1 so that labor supply is just $1 - l$. The additional first order condition implied by the optimal choice of leisure is

$$\frac{u_l(c, m, l)}{u_c(c, m, l)} = f_n(k, 1-l). \tag{2.34}$$

Now, both steady-state labor supply and consumption may be affected by variations in the rate of inflation. Specifically, an increase in the rate of inflation reduces holdings of real money balances. If this affects the marginal utility of leisure, then (2.34) implies that the supply of labor will be affected, leading to a change in the steady-state per capita stock of capital, output, and consumption. But why would changes in money holdings affect the marginal utility of leisure? Because money has simply been assumed to yield utility, with no explanation for the reason, it is difficult to answer this question. In chapter 3 we will examine a model in which money helps to reduce the time spent in carrying out the transactions necessary to purchase consumption goods; in this case, a rise in inflation would lead to more time spent engaged in transactions, and this would raise the marginal utility of leisure. But one might expect that this channel is unlikely to be important empirically, so superneutrality may remain a reasonable first approximation to the effects of inflation on steady-state real magnitudes.

29. Of course, the empirical relationship may not be causal; both growth and inflation may be reacting to common factors. As noted in chapter 1, McCandless and Weber (1995) find no relationship between inflation and average real growth.

Equation (2.34) suggests that if u_l/u_c were independent of m, then superneutrality would hold. This is the case since the steady-state values of k, c, and l could then be found from

$$\frac{u_l}{u_c} = f_n(k^{ss}, 1 - l^{ss}),$$

$$f_k(k^{ss}, 1 - l^{ss}) = \frac{1}{\beta} - 1 + \delta,$$

and

$$c^{ss} = f(k^{ss}, 1 - l^{ss}) + \delta k^{ss}.$$

If u_l/u_c does not depend on m, these three equations determine the steady-state values of consumption, capital, and labor independently of inflation. So superneutrality reemerges when the utility function takes the general form $u(c, m, l) = v(c, l)g(m)$. While variations in inflation will affect the agent's holdings of money, the consumption-leisure choice will not be directly affected. As McCallum (1990a) notes, Cobb-Douglas specifications, which are quite commonly used, satisfy this condition. So with a Cobb-Douglas utility function, the ratio of the marginal utility of leisure to the marginal utility of consumption will be independent of the level of real money balances, and superneutrality will hold.

Another channel through which inflation can affect the steady-state stock of capital occurs if money is entered directly into the production function (Fischer 1974). Since steady states with different rates of inflation will have different equilibrium levels of real money balances, they will also then have different marginal products of capital if the capital-labor ratios are the same. With the steady-state marginal product of capital determined by $1/\beta - 1 + \delta$ (see 2.18), the two steady states can have the same marginal product of capital only if their capital-labor ratios differ. If $\partial MPK/\partial m > 0$ (so that money and capital are complements), higher inflation, by leading to lower real money balances, also leads to a lower steady-state capital stock.[30] This is the opposite of the *Tobin effect*; Tobin (1965) argued that higher inflation would induce a portfolio substitution toward capital that would increase the steady-state capital-labor ratio (see also Stein 1969, Fischer 1972). For higher inflation to be associated with a higher steady-state capital-labor ratio requires that

30. That is, in the steady state, $f_k(k^{ss}, m^{ss}) = \beta^{-1} - 1 + \delta$, where $f(k, m)$ is the production function and f_i denotes the partial with respect to the ith argument. It follows that $dk^{ss}/dm^{ss} = -f_{km}/f_{kk}$, so with $f_{kk} \leq 0$, $\text{sign}(dk^{ss}/dm^{ss}) = \text{sign}(f_{km})$.

$\partial MPK/\partial m < 0$ (that is, higher money balances reduce the marginal product of capital; money and capital are substitutes in production).

This discussion actually has, by ignoring taxes, excluded what is probably the most important reason that superneutrality may fail in actual economies. Taxes generally are not indexed to inflation and are levied on nominal capital gains instead of real capital gains. Effective tax rates will depend on the inflation rate, generating real effects on capital accumulation and consumption as inflation varies. (See, for example, Feldstein 1978, Summers 1981, and Feldstein 1998). We will return to this issue in chapter 4.

2.5 Dynamics in an MIU Model

The analysis of the MIU function approach has, up to this point, focused on steady-state properties. We are also interested in understanding the implications of the model for the dynamic process the economy follows as it adjusts in response to exogenous disturbances. Even the basic Sidrauski model can exhibit nonsuperneutralities during the transition to the steady state. For example, Fischer (1979a) has shown that, for the constant relative risk aversion class of utility functions, the rate of capital accumulation is positively related to the rate of money growth except for the case of log separable utility.[31]

In addition, theoretical and empirical work in macroeconomics and monetary economics are closely tied, and it is important to reflect on how the theoretical models can help us understand actual observations on inflationary experiences. One way to do this is to use a theoretical model to generate artificial data by simulating the model economy; comparing the simulated data with actual data generated by real economies provides a means of validating the model. This approach has been popularized by the real-business-cycle literature (see Cooley 1995). Since we can vary the parameters of our theoretical models in ways that we cannot vary the characteristics of real economies, simulation methods allow us to answer a variety of "what if" questions. For example, how does the dynamic response to a temporary change in the growth rate of the money supply depend on the degree of intertemporal substitution characterizing individual preferences?

31. Superneutrality holds during the transition if $u(c, m) = \ln(c) + b\ln(m)$. The general class of utility functions Fischer considers is of the form $u(c, m) = \frac{1}{1-\Phi}(c^a m^b)^{1-\Phi}$; log utility obtains when $\Phi = 1$. See also Asako (1983), who shows that faster money growth can lead to slower capital accumulation under certain conditions if c and m are perfect complements. These effects of inflation on capital accumulation apply during the transition from one steady-state equilibrium to another; they differ therefore from the Tobin effect of inflation on the steady-state capital-labor ratio.

It can also be helpful to have an analytic solution to a model; often explicit solutions help to indicate whether simulation results are likely to be sensitive to parameter values and to highlight directly the mechanisms through which changes in the processes followed by the exogenous variables lead to effects on the endogenous variables and to alterations in the equilibrium decision rules of the agents in the model. Campbell (1994) has proposed using log-linear approximation for studying a basic nonmonetary real-business-cycle model, and this approach has been further extended by Uhlig (1999). In addition, easily adaptable programs for solving log-linear dynamic stochastic models are now freely available.[32]

In this subsection, we apply these methods to the Sidrauski model. But rather than assuming that utility depends just on consumption and money holdings, we also allow utility to depend on the representative agent's consumption of leisure. This introduces a labor supply decision into the analysis, an important and necessary extension for studying business-cycle fluctuations since employment variation is an important characteristic of cycles. By specifying functional forms for the utility function (as we did, for example, in 2.26) and for the production function, we can develop approximations around the steady state that can be solved numerically. We also need to add a source, or sources, of exogenous shocks that disturb the system from its steady-state equilibrium. The two types of shocks we will consider will be productivity shocks, the driving force in real-business-cycle models, and shocks to the growth rate of the nominal stock of money.

We follow the standard specification in dynamic general equilibrium models by assuming that output is produced using capital and labor according to a Cobb-Douglas, constant returns to scale production function. Consistent with the real-business-cycle literature, we incorporate a stochastic disturbance to total factor productivity, so that

$$Y_t = e^{z_t} K_{t-1}^{\alpha} N_t^{1-\alpha} \tag{2.35}$$

with $0 < \alpha < 1$ and

$$z_t = \rho z_{t-1} + e_t \tag{2.36}$$

is the process followed by the productivity shock. We assume that e_t is a serially uncorrelated mean zero process and $|\rho| < 1$. Note the timing convention in (2.35): the capital carried over from period $t-1$, K_{t-1}, is available for use in producing output during period t.

32. For example, Matlab programs provided by Harald Uhlig can be obtained from http://www.wiwi.hu-berlin.de/wpol/html/toolkit.htm and Paul Söderlind's Gauss and Matlab programs are available at http://www.hh.se/personal/psoderlind/.

We also need to specify the process followed by the nominal stock of money. In previous sections, we let θ denote the growth rate of the nominal money supply. Assume then that the average growth rate is θ^{ss} and let $u_t \equiv \theta_t - \theta^{ss}$ be the deviation in period t of the growth rate from its unconditional average value. This deviation will be treated as a stochastic process given by

$$u_t = \gamma u_{t-1} + \phi z_{t-1} + \varphi_t, \quad 0 \le \gamma < 1, \tag{2.37}$$

where φ_t is a white noise process. This formulation allows the growth rate to display persistence (if $\gamma > 0$), to respond to the real productivity shock z, and to be subject to random disturbances through the realizations of φ.

For the utility function, assume a nested CES specification given by

$$u(c_t, m_t, l_t) = \frac{[ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi \frac{l_t^{1-\eta}}{1-\eta}, \tag{2.38}$$

with $0 < a < 1$, $b, \eta, \Phi, \Psi > 0$, $b, \eta, \Phi \neq 1$.[33] As discussed by King, Plosser, and Rebelo (1988) and Cooley and Prescott (1995), preferences of the form given in (2.38) are consistent with steady-state growth. In the limiting case with $\Phi = b = 1$, preferences over consumption and money holdings are log linear ($u = a \ln c_t + (1-a) \ln m_t + \Psi l_t^{1-\eta}/(1-\eta)$). Fischer (1979b) showed that the transition paths are independent of the money supply in this case since the marginal rate of substitution between leisure and consumption is independent of real money balances. With the specification in (2.38), utility can also be written as

$$\frac{C_t^{1-\Phi}}{1-\Phi} + \Psi \frac{l_t^{1-\eta}}{1-\eta},$$

where $C_t = [ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}}$ is a composite consumption good that depends on the level of goods consumption (c) and real money balances (m).

Normalizing the total available time for work and leisure to one, labor supply is given by

$$n_t = 1 - l_t.$$

In what follows, we will make use of the notational convention that uppercase letters denote economy-wide variables, lowercase letters denote random disturbances

33. This specification differs from that used in the first edition. The earlier specification forced the income and interest rate elasticities of money demand to be equal.

and variables expressed in per capita terms, and the superscript ss indicates the steady-state value of a variable. Since the focus will be on the dynamic behavior around the steady state, a hat (ˆ) will indicate the percentage deviation of a variable from its steady-state level. There will be two important exceptions: m, m^{ss}, and $\hat{m}$ will refer to *real* money balances per capita while M will represent the aggregate *nominal* stock of money, and $\hat{r}$ and $\hat{\imath}$ will be expressed as percentage rates rather than as percentage deviations around the steady state.

Given these functional forms for the production function and the utility function, we need to set up the household's decision problem, find the first order conditions, and then solve for the paths of the endogenous variables that are consistent with these first order conditions and with the equilibrium conditions of the model. To obtain solutions, however, we will linearize the model around the steady state. The details of the derivations involved in each of these steps can be found in the chapter appendix (section 2.7).

2.5.1 The Steady State

The appendix shows how the steady-state values can be expressed in terms of the basic parameters of the model. Letting $\Theta \equiv 1 + \theta^{ss}$ be 1 plus the average growth rate of the nominal supply of money, the steady-state values for the endogenous variables of the model are given in table 2.1.

Table 2.1 reveals some of the key properties of this money in the utility function model and how variations in the steady-state rate of inflation will affect output, the capital stock, and consumption. The first thing to note is that, with the exception of real money balances relative to the capital stock, the other ratios are all independent of the steady-state growth rate of the nominal supply of money. However, the steady-state *levels* of the capital stock, output, and consumption will depend on the money growth rate through the effects of inflation on labor supply, with inflation-induced changes in n^{ss} affecting y^{ss}, c^{ss}, and k^{ss} equiproportionally. To determine how n^{ss} may depend on the rate of inflation, we need to use the first order condition from the labor-leisure choice (2.34).

Table 2.1
Steady-State Values

R^{ss}	$\frac{y^{ss}}{k^{ss}}$	$\frac{c^{ss}}{k^{ss}}$	$\frac{m^{ss}}{k^{ss}}$	$\frac{n^{ss}}{k^{ss}}$
$\frac{1}{\beta}$	$\frac{1}{\alpha}(R^{ss} - 1 + \delta)$	$\frac{y^{ss}}{k^{ss}} - \delta$	$\left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta - \beta}{\Theta}\right)^{-\frac{1}{b}}\left(\frac{c^{ss}}{k^{ss}}\right)$	$\left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{1}{1-\alpha}}$

Using the function forms for the production function and the utility function, the appendix shows that the steady-state level of n is determined as the solution to

$$\frac{(n^{ss})^{\Phi}}{(1-n^{ss})^{\eta}} = H\left[1 + \left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta - \beta}{\Theta}\right)^{\frac{b-1}{b}}\right]^{\frac{b-\Phi}{1-b}}, \tag{2.39}$$

where

$$H \equiv \left(\frac{1-\alpha}{\Psi}\right)\left(\frac{y^{ss}}{k^{ss}}\right)^{-\frac{\alpha}{1-\alpha}} a^{\frac{1-\Phi}{1-b}}\left(\frac{c^{ss}}{k^{ss}}\right)^{-\Phi}\left(\frac{k^{ss}}{n^{ss}}\right)^{-\Phi}$$

is independent of Θ. The left side of (2.39) is increasing in n^{ss}. The effect of faster money growth, Θ, on the right side depends on the sign of $\xi \equiv (b - \Phi)$. If this is positive (negative), faster money growth, which leads to a higher steady-state rate of inflation and a higher nominal interest rate, decreases (increases) the right side of (2.39). This produces a fall (rise) in steady-state employment (and hence in output, consumption, and capital stock). When $\xi > 0$,

$$u_{cm} = a(1-a)\xi[a(c)^{1-b} + (1-a)(m)^{1-b}]^{\frac{b-\Phi}{1-b}-1}(mc)^{-b} > 0$$

so that consumption and money are Edgeworth complements. Higher inflation that reduces real money balances decreases the marginal utility of consumption. This causes households to substitute toward leisure and to decrease labor supply. Hence, output falls. These effects go in the opposite direction if consumption and money are Edgeworth substitutes.

In standard calibration exercises, preferences are often taken to be log separable ($\Phi = 1$), while our earlier discussion of empirical evidence on money demand suggested that b was on the order of 2.5. For these parameter values, $\xi > 0$ and $u_{cm} > 0$; consumption and money are Edgeworth complements. In this case, faster money growth reduces m and decreases the marginal utility of consumption. Households substitute away from labor and toward leisure. Steady-state employment, output, and consumption fall. If $\Phi = b$, neither the marginal utility of consumption nor the marginal utility of leisure depends on the level of real money balances. In this case, n^{ss} is independent of money growth and superneutrality is obtained. In this case, only m^{ss}/k^{ss} will depend on the growth rate of money.

The terms involving money growth also drop out of the right side of (2.39) when $b = 1$; n^{ss} and all the other real variables $(y^{ss}, k^{ss}, c^{ss}, R^{ss})$ are then independent of the inflation rate. The steady-state level of real money balances, however, is still a function of the rate of inflation.

2.5.2 The Linear Approximation

The steps involved in obtaining the linear approximation around the steady state are contained in the appendix to this chapter and follow the approach of Campbell (1994) and Uhlig (1999). The resulting linearized system consists of the exogenous processes for the productivity shock and the money growth rate plus the following eight equations that can be solved for the capital stock, money holdings, output, consumption, employment, the real rate of interest, the nominal interest rate, and the inflation rate:

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + z_t \tag{2.40}$$

$$\left(\frac{y^{ss}}{k^{ss}}\right)\hat{y}_t = \left(\frac{c^{ss}}{k^{ss}}\right)\hat{c}_t + \hat{k}_t - (1-\delta)\hat{k}_{t-1} \tag{2.41}$$

$$\hat{r}_t = \alpha\left(\frac{y^{ss}}{k^{ss}}\right)(\mathrm{E}_t\hat{y}_{t+1} - \hat{k}_t) \tag{2.42}$$

$$\mathrm{E}_t[\Omega_1(\hat{c}_{t+1} - \hat{c}_t) - \Omega_2(\hat{m}_{t+1} - \hat{m}_t)] - \hat{r}_t = 0 \tag{2.43}$$

$$[\hat{y}_t - \Omega_1\hat{c}_t + \Omega_2\hat{m}_t] = \left(1 + \eta\frac{n^{ss}}{1-n^{ss}}\right)\hat{n}_t \tag{2.44}$$

$$\hat{\imath}_t = \hat{r}_t + \mathrm{E}_t\hat{\pi}_{t+1} \tag{2.45}$$

$$\hat{m}_t = \hat{M}_t - \hat{p}_t = \hat{c}_t - \left(\frac{1}{b}\right)\hat{\imath}_t \tag{2.46}$$

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + u_t, \tag{2.47}$$

where $\Omega_1 = \gamma\Phi + (1-\gamma)b$, $\Omega_2 = (b-\Phi)(1-\gamma)$, and $\gamma = [1 + a^{-1}(1-a) \times (m^{ss}/c^{ss})^{1-b}]^{-1}$.

Equation (2.40) is the economy's production function in which output deviations from the steady state are a linear function of the percentage deviations of the capital stock and labor supply from the steady state plus the productivity shock. Equation (2.41) is the resource constraint derived from the condition that output equals consumption plus investment. Deviations of the marginal product of capital are tied to deviations of the real return by (2.42). Equations (2.43)–(2.46) are derived from the representative household's first-order conditions for consumption, leisure, and money holdings. Finally, (2.47) relates the change in the deviation from steady state of real money balances to the inflation rate and the growth of the nominal money stock. To complete the specification, the exogenous disturbances for productivity and nominal money growth were given earlier by (2.36) and (2.37).

One conclusion follows immediately from inspecting this system. If $\Phi = b$, (2.43) becomes

$$\Omega_1(\mathrm{E}_t\hat{c}_{t+1} - \hat{c}_t) - \hat{r}_t = 0$$

so that, together with (2.40)–(2.42) and (2.44), we can solve for $\hat{y}, \hat{c}, \hat{r}, \hat{k}$, and $\hat{n}$ independently of the money supply process and inflation. This implies that superneutrality will characterize dynamics around the steady state as well as the steady state itself. Thus, the system will exhibit superneutrality along its dynamic adjustment path.[34]

While separability allows the real equilibrium to be solved for, independent of money and inflation, it has more commonly been used in monetary economics to allow the study of inflation and money growth to be conducted independent of the real equilibrium. When $\Phi = b$, (2.46) and (2.47) constitute a two-equation system in inflation and real money balances, with u representing an exogenous random disturbance and $\hat{c}$ and $\hat{r}$ determined by (2.40)–(2.44) and exogenous to the determination of inflation and real money balances. Equation (2.46) can then be written as

$$\mathrm{E}_t\pi_{t+1} \equiv \mathrm{E}_t\hat{p}_{t+1} - \hat{p}_t = -b(\hat{M}_t - \hat{p}_t) + x_t.$$

This is an expectational difference equation that can be solved for the equilibrium path of $\hat{p}$ for a given process for the nominal money supply and the exogenous variable $x = b\hat{c}_{t+1} - \hat{r}_t$. Models of this type have been widely employed in monetary economics, and we will return to study them in chapter 4.

A second conclusion revealed by the dynamic system is that when money does matter (i.e., when $\Phi \neq b$), it is only *anticipated* changes in money growth that matter. To see this, suppose $\gamma = \phi = 0$, so that $u_t = \varphi_t$ is a purely unanticipated change in the growth rate of money that has no effect on anticipated future values of money growth. Now consider a positive realization of φ_t (nominal money growth is faster than average). This increases the nominal stock of money. If $\gamma = \phi = 0$, future money growth rates are unaffected by the value of φ_t. This means that future expected inflation, $\mathrm{E}_t\pi_{t+1}$, is also unaffected. Therefore, a permanent jump in the price level that is proportional to the unexpected rise in the nominal money stock leaving m_t unaffected also leaves (2.40)–(2.46) unaffected. From (2.47), for φ_t to have

34. This result, for the preferences given by (2.38), generalizes the findings of Brock (1974) and Fischer (1979a).

no effect on $\hat{m}_t$ requires that $\pi_t = \varphi_t$. So an unanticipated money growth rate disturbance has no real effects and simply leads to a one-period change in the inflation rate (and a permanent change in the price level). Unanticipated money doesn't matter.[35]

Now consider what happens when we continue to assume that $\phi = 0$ but allow γ to differ from zero. In the United States, money growth displays positive serial correlation, so assume that $\gamma > 0$. A positive shock to money growth ($\varphi_t > 0$) now has implications for the future growth rate of money. With $\gamma > 0$, future money growth will be above average, so expectations of future inflation will rise. From (2.46), however, we can see that for real consumption and the expected real interest rate to remain unchanged in response to a rise in expected future inflation, current real money balances must fall. This means that $\hat{p}_t$ would need to rise more than in proportion to the rise in the nominal money stock. But when $\Omega_2 \neq 0$, the decline in $\hat{m}_t$ affects the first order conditions given by (2.44) and (2.46), so the real equilibrium will not remain unchanged. Monetary disturbances have real effects by affecting the expected rate of inflation.

To actually determine how the equilibrium responds to money growth rate shocks and how the response depends quantitatively on γ and ϕ, we will employ numerical methods. That means we need to assign specific values to the parameters of the model, a task to which we now turn.

2.5.3 Calibration

Thirteen parameters appear in the equations that characterize behavior around the steady state: $\alpha, \delta, \rho, \sigma_e^2, \beta, a, b, \eta, \Phi, \Theta, \gamma, \phi, \sigma_\varphi^2$. Some of these parameters are common to standard real-business-cycle models; for example, Cooley and Prescott (1995, p. 22) report values of, in our notation, α (the share of capital income in total income), δ (the rate of depreciation of physical capital), ρ (the autoregressive coefficient in the productivity process), σ_e (the standard deviation of productivity innovations), and β (the subjective rate of time discount in the utility function). These values are based on a time period equal to three months (one quarter). We adopt Cooley and Prescott's values except for the depreciation rate δ; Cooley and Prescott calibrate $\delta = .012$ based on a model that explicitly incorporates growth. For our purposes, we use the somewhat higher value of .019 given in Cooley and Hansen (1995, p. 201). For the average growth rate of the nominal money stock, an annual rate of 5% would imply a quarterly value of 1.0125 for Θ. Cooley and Hansen (1989) report a value of .0089 for σ_φ, the standard deviation of innovations to the money growth

35. During the 1970s, macroeconomics was heavly influenced by a model developed by Lucas (1972) in which only unanticipated changes in the money supply had real effects. See chapter 5.

rate. We will consider various values for the autoregression coefficient for money growth, γ, and the coefficient on the productivity shock, ϕ, to see how the implications of the model are affected by the manner in which money growth evolves.

The remaining parameters are those in the utility function. The value of Ψ can be chosen so that the steady-state value of n^{ss} is equal to one-third, as in Cooley and Prescott. For a and b, values were chosen to be consistent with the estimated values reported in Chari, Kehoe, and McGrattan (2000). The appendix shows that the steady-state value of real money balances relative to consumption is equal to $(a\Upsilon^{ss}/(1-a))^{-1/b}$, $\Upsilon^{ss} = (\Theta - \beta)/\Theta$. The benchmark parameter values of $a = 0.95$, $b = 2.56$, and $\Theta = 1.0125$ imply a value of 1.38 for m^{ss}/c^{ss}. This value is somewhat high if m is interpreted as corresponding to an aggregate such as $M1$ (expressed at quarterly rates as in the model, $M1/c$ is about 0.7 for the United States), while it is too low if m is viewed as corresponding to $M3$ (the ratio of $M3$ to consumption is over 4 in the United States).

The inverse of the intertemporal elasticity of substitution, Φ, is set equal to 2 in the benchmark simulations. With $b = 2.56$, this means $b - \Phi > 0$ and faster expected money growth will decrease employment and output. Finally, η is set equal to 1. With $n^{ss} = 1/3$, a value of $\eta = 1$ yields a labor supply elasticity of $[\eta n^{ss}/(1-n^{ss})]^{-1} = 2$.

These parameter values are summarized in table 2.2. Using the information in this table, the steady-state values for the variables reported in table 2.1 can be evaluated. These are given in table 2.3. The effect of money growth on the steady-state level of employment can be derived using (2.39). The elasticity of the steady-state labor supply with respect to the growth rate of the nominal money supply depends on the sign of u_{cm}; this, in turn, depends on the sign of $b - \Phi$. For the benchmark parameter values, this is positive. With Φ less than b, the marginal utility of consumption is

Table 2.2
Baseline Parameter Values

α	δ	β	Φ	η	a	b	Θ	ρ	σ_e	σ_φ
0.36	0.019	0.989	2	1	0.95	2.56	1.0125	0.95	0.007	0.0089

Table 2.3
Steady-State Values at Baseline Parameter Values

R^{ss}	$\frac{y^{ss}}{k^{ss}}$	$\frac{c^{ss}}{k^{ss}}$	$\frac{m^{ss}}{k^{ss}}$	$\frac{n^{ss}}{k^{ss}}$
1.011	0.084	0.065	0.089	0.021

Table 2.4
Implied Contemporaneous Correlations

	$\Phi = 2$			$\Phi = 3$		
	s.d.	*s.d.* rel. to y	Corr. with y	*s.d.*	*s.d.* rel. to y	Corr. with y
y	1.174	1.00	1.00	1.088	1.00	1.00
n	0.432	0.368	0.95	0.315	0.268	0.89
c	0.295	0.252	0.94	0.234	0.200	0.95
x	4.235	3.608	1.00	4.039	3.713	1.00
r	0.032	0.028	0.97	0.030	0.028	0.97
m	0.291	0.248	0.94	0.228	0.194	0.95
i	0.016	0.013	0.97	0.019	0.018	0.96
π	0.881	0.751	−0.11	0.872	0.744	−0.09

increasing in real money balances. Hence, higher inflation decreases the marginal utility of consumption, increases the demand for leisure, and decreases the supply of labor (see 2.34). If $b - \Phi$ is negative, higher inflation leads to a rise in labor supply and output. The dependence of the elasticity of labor with respect to inflation on the partial derivatives of the utility function in a general MIU model is discussed more fully by Wang and Yip (1992).

2.5.4 Simulation Results

Table 2.4 shows the summary statistics implied by the model for the major variables for the baseline parameter values. Deviations of nominal money growth from its average growth rate of 1.25% per quarter ($\Theta = 1.0125$) are white noise (i.e., $\gamma = \phi = 0$ in 2.37). The table also shows the implied contemporaneous correlation with output for two different values of Φ, one less than b and one greater than b.[36] Statistics for investment are also included.[37] Recall that the transitional dynamics exhibit superneutrality when $\Phi = b$. When $\Phi = 2$ ($\Phi = 3$), the marginal utility of consumption is increasing (decreasing) in real money balances (see 2.38).[38]

We now want to see how the properties of the model vary as the time-series properties of the growth rate of the money stock vary. Cooley and Hansen (1989) report

36. The reported correlations and standard deviation are for data first detrended using a Hodrick-Prescott filter.

37. If $\hat{x}_t$ is the percent deviation of investment around the steady state, then $\delta\hat{x}_t = \hat{k}_t - (1-\delta)\hat{k}_{t-1}$.

38. Brock (1974) shows that multiple steady-state equilibria may exist if $u_{cm} < 0$, as is the case when $\Phi > b$. For the utility function used here, however, the steady state is unique. As discussed below, $\Phi > b$ is required if faster money growth is to induce a rise in output if $\gamma > 0$.

Table 2.5
Effects of the Money Process: $\gamma = 0.5$

	$\phi = 0$			$\phi = 0.15$			$\phi = -0.15$		
	s.d.	*s.d.* rel. to y	Corr. with y	*s.d.*	*s.d.* rel. to y	Corr. with y	*s.d.*	*s.d.* rel. to y	Corr. with y
y	1.174	1.00	1.00	1.173	1.00	1.00	1.174	1.00	1.00
n	0.432	0.368	0.95	0.432	0.368	0.95	0.433	0.368	0.95
c	0.295	0.252	0.94	0.295	0.251	0.94	0.296	0.252	0.94
x	4.235	3.608	1.00	4.235	3.609	1.00	4.235	3.607	1.00
r	0.032	0.027	0.97	0.032	0.028	0.97	0.032	0.028	0.97
m	0.319	0.271	0.86	0.239	0.203	0.79	0.394	0.336	0.91
i	0.333	0.284	0.04	0.383	0.327	0.60	0.367	0.313	−0.54
π	0.908	0.773	−0.11	0.853	0.727	0.08	0.845	0.720	−0.32

estimates of approximately .5 for γ, the autoregressive coefficient for money growth. Table 2.5 illustrates the effects of setting $\gamma = 0.5$ combined with various values for ϕ, beginning with $\phi = 0$.[39] The table is based on $\Phi = 2$. The major effect of ϕ is on the behavior of inflation and the nominal rate of interest. When money growth does not respond to a productivity shock or when it decreases in response (i.e., when $\phi \leq 0$), output and inflation are negatively correlated, as the positive shock to productivity increases output and reduces prices. When $\phi > 0$, however, the output–inflation correlation becomes positive.

As table 2.5 shows, the manner in which the growth rate of the nominal money supply responds to the real productivity shock has only very small effects on the variances of real output, consumption, and investment. The channels through which money affects the real economy in this MIU model are very weak. That is, while monetary policy matters in this equilibrium model, it doesn't matter very much. When $\phi = -0.15$, a positive technology shock leads to lower expected money growth and inflation. Lower expected inflation raises real money balances, raises the marginal utility of consumption, and increases the labor supply when, as in the case here, $b > \Phi$. Hence, employment and output are slightly higher after a technology shock when $\phi = -0.15$. Conversely, when $\phi = 0.15$, a positive technology shock leads to

39. Cooley and Hansen report a value of .0089 for the standard deviation of u_t when $M_t = \gamma M_{t-1} + (1-\gamma)\Theta + u_t$. When ϕz_{t-1} is added to this specification of the money supply process, the variance of u is adjusted to keep the standard deviation of $M_t - \gamma M_{t-1} - (1-\gamma)\Theta$ equal to .0089.

higher expected inflation, and employment and output respond less than in the base case. Simulations reveal these differences to be quite small.

Consistent with the earlier discussion, the monetary shock φ_t affects the labor-leisure choice only when the nominal money growth rate process exhibits serial correlation ($\gamma \neq 0$) or responds to the technology shock ($\phi \neq 0$). But for the base value of .5 for γ, the effect of a money growth shock on n_t is very small. As (2.34) showed, variations in money holdings can affect the agent's labor-leisure choice by affecting the ratio of the marginal utility of leisure to the marginal utility of consumption. A positive realization of φ_t implies a rise in expected inflation when money growth is positively serially correlated ($\gamma > 0$); this reduces holdings of real money balances (m), and with $\Phi = 2 < b$, lowers the marginal utility of consumption and causes the agent to substitute toward leisure. As a consequence, labor supply and output fall. If $\Phi > b$, higher expected inflation (and therefore lower real money balances) would raise the marginal utility of consumption and lead to a decrease in leisure demand; labor supply and output would rise in this case.

Figure 2.3 shows that the magnitude of the effect of monetary shocks on output and labor is small, but the effects clearly depend on the degree of persistence in the

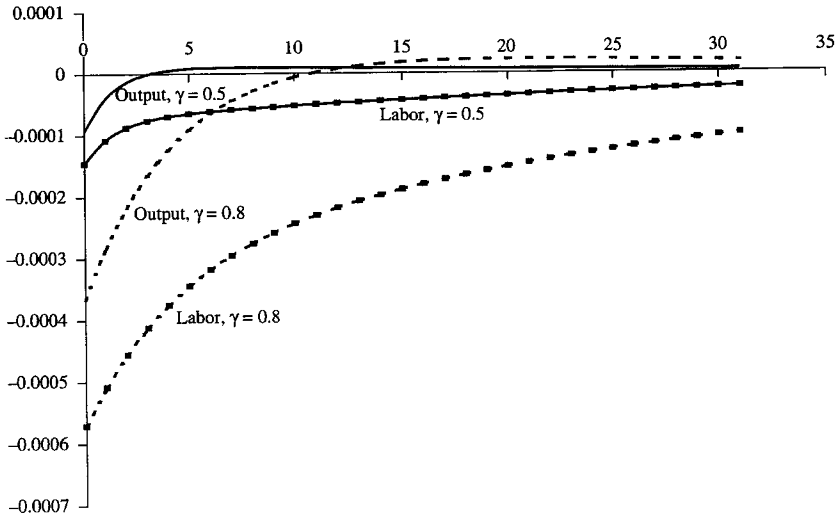

Figure 2.3
Output and Labor Responses to a Money Growth Shock

money growth process. Higher values of γ generate larger effects on labor input and output.

Finally, figure 2.4 shows how the nominal interest rate and the inflation rate response depend on γ. Notice that a positive monetary shock *increases* the nominal rate of interest. Monetary policy actions that increase the growth rate of money are usually thought to reduce nominal interest rates, at least initially. The negative effect of money on nominal interest rates is usually called the *liquidity effect*, and it arises if an increase in the nominal quantity of money also increases the real quantity of money because nominal interest rates would need to fall to ensure that real money demand also increased. However, in the MIU model, prices have been assumed to be perfectly flexible; the main effect of money growth rate shocks when $\gamma > 0$ is to increase expected inflation and raise the nominal interest rate. Because prices are perfectly flexible, the monetary shock generates a jump in the price level immediately. The real quantity of money actually falls, consistent with the decline in real money demand that occurs as a result of the increase in the nominal interest rate.

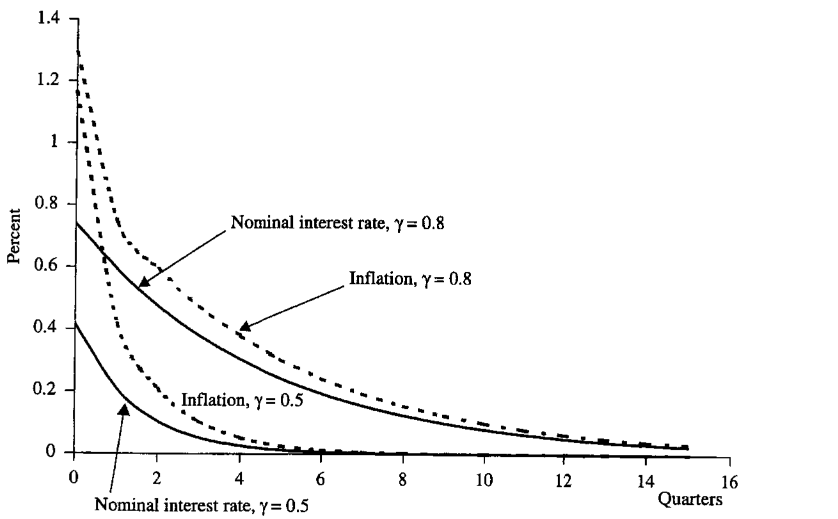

Figure 2.4
Nominal Interest Rate and Inflation Response to a Money Growth Shock (solid lines, nominal interest rate response; dashed lines, inflation response)

2.6 Summary

Assuming that holdings of real money balances yield direct utility is a means of ensuring a positive demand for money so that, in equilibrium, money is held and has value. This assumption is clearly a shortcut; it does not address the issue of why money yields utility, nor does it address the issue of why certain pieces of paper that we call money yield utility but other pieces of paper presumably do not.

The Sidrauski model, because it assumes that agents act systematically to maximize utility, allows welfare comparisons to be made. The model can be used to assess the welfare costs of inflation and to determine the optimal rate of inflation. Friedman's conclusion that the optimal inflation rate is the rate that produces a zero nominal rate of interest is one that we will see is quite robust.

Finally, by developing a linear approximation to the basic money in the utility function model (augmented to include a labor supply choice), we were able to show how the economy responded to shocks to the growth rate of the money supply and how the short-run dynamic adjustment to real productivity shocks was affected by the feedback rule followed by money growth. For the benchmark values of the model's parameters, however, the effects of variation in the money growth process on output are quantitatively quite small in the flexible-price model examined in this chapter.

2.7 Appendix: Solving for the Dynamics in the MIU Model

The linear approximation to the MIU model studied in section 2.5 is derived in this appendix. After setting up the household's decision problem and deriving the first order necessary conditions, the properties of the steady state are determined. Then, the first order conditions, together with the economy's resource constraint and production function, are linearized around the steady state.

2.7.1 The Decision Problem

The household's problem is conveniently expressed using the value function. In studying a similar problem without a labor-leisure choice (see section 2.2), the state could be summarized by $\omega_t = y_t + \tau_t + (1-\delta)k_{t-1} + [(1+i_{t-1})/(1+\pi_t)]b_{t-1} + [1/(1+\pi_t)]m_{t-1}$, where y_t is per capita income, k_{t-1} is the per capita (or household) stock of capital at the start of the period, b_{t-1} and m_{t-1} are real bond and money balances at the end of the previous period, and τ_t is the real transfer payment received at the start of period t. Now, however, output y_t will depend on the household's current period choice of labor supply, so it cannot be part of the state vector

for period t. Instead, let $a_t = \tau_t + [(1+i_{t-1})/(1+\pi_t)]b_{t-1} + [1/(1+\pi_t)]m_{t-1}$ be the household's real financial wealth plus transfer at the start of period t, and define the value function $V(a_t, k_{t-1})$ as the maximum present value of utility the household can achieve if the current state is (a_t, k_{t-1}). If n_t denotes the fraction of time the household devotes to market employment, output per household y_t is given by

$$y_t = f(k_{t-1}, n_t, z_t).$$

The value function for the household's decision problem is defined by

$$V(a_t, k_{t-1}) = \max\{u(c_t, m_t, 1-n_t) + \beta \mathrm{E}_t V(a_{t+1}, k_t)\}, \tag{2.48}$$

where the maximization is over $(c_t, m_t, b_t, k_t, n_t)$ and is subject to

$$f(k_{t-1}, n_t, z_t) + (1-\delta)k_{t-1} + a_t \geq c_t + k_t + b_t + m_t \tag{2.49}$$

$$a_{t+1} = \tau_{t+1} + \left(\frac{1+i_t}{1+\pi_{t+1}}\right)b_t + \frac{m_t}{1+\pi_{t+1}}. \tag{2.50}$$

Note that the presence of uncertainty arising from the stochastic productivity and money growth rate shocks means that it is the expected value of $V(a_{t+1}, k_t)$ that appears in the value function (2.48). The treatment of a_t as a state variable assumes that the money growth rate is known at the time the household decides on c_t, k_t, b_t, and m_t since φ_t determines the current value of the transfer τ_t. We will also assume that the productivity disturbance z_t is known at the start of period t.

Using (2.49) to eliminate k_t and (2.50) to substitute for a_{t+1}, the value function can be rewritten as

$$\begin{aligned} V(a_t, k_{t-1}) = \max_{c_t, m_t, n_t} \Bigg\{ & u(c_t, m_t, 1-n_t) \\ & + \beta \mathrm{E}_t V\Bigg(\tau_{t+1} + \left(\frac{1+i_t}{1+\pi_{t+1}}\right)b_t + \frac{m_t}{1+\pi_{t+1}}, f(k_{t-1}, n_t, z_t) \\ & + (1-\delta)k_{t-1} + a_t - c_t - b_t - m_t\Bigg)\Bigg\}, \end{aligned}$$

where this is now an unconstrained maximization problem. The first order necessary conditions are

$$u_c(c_t, m_t, 1-n_t) - \beta \mathrm{E}_t V_k(a_{t+1}, k_t) = 0 \tag{2.51}$$

$$\beta \mathrm{E}_t\left(\frac{1+i_t}{1+\pi_{t+1}}\right) V_a(a_{t+1}, k_t) - \beta \mathrm{E}_t V_k(a_{t+1}, k_t) = 0 \tag{2.52}$$

$$u_m(c_t, m_t, 1-n_t) + \beta \mathrm{E}_t\left[\frac{V_a(a_{t+1}, k_t)}{1+\pi_{t+1}}\right] - \beta \mathrm{E}_t V_k(a_{t+1}, k_t) = 0 \tag{2.53}$$

$$-u_l(c_t, m_t, 1-n_t) + \beta \mathrm{E}_t V_k(a_{t+1}, k_t) f_n(k_{t-1}, n_t, z_t) = 0 \tag{2.54}$$

$$V_a(a_t, k_{t-1}) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t) \tag{2.55}$$

$$V_k(a_t, k_{t-1}) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t)[1 - \delta + f_k(k_{t-1}, n_t, z_t)]. \tag{2.56}$$

Updating (2.56) one period and using (2.55), one obtains

$$V_k(a_{t+1}, k_t) = \mathrm{E}_t\{[1 - \delta + f_k(k_t, n_{t+1}, z_{t+1})] V_a(a_{t+1}, k_t)\}.$$

Now using this to substitute for $V_k(a_{t+1}, k_t)$ in (2.51) yields

$$u_c(c_t, m_t, 1-n_t) - \beta \mathrm{E}_t\{[1 - \delta + f_k(k_t, n_{t+1}, z_{t+1})] V_a(a_{t+1}, k_t)\} = 0. \tag{2.57}$$

When it is recognized that $u_c(c_t, m_t, 1-n_t) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t)$ from (2.51), (2.53), (2.57), and (2.55) take the same form as (2.8), (2.6), and (2.10), the first order conditions for the basic Sidrauski model that did not include a labor-leisure choice. Equation (2.54) can be written, using (2.51), as

$$\frac{u_l(c_t, m_t, 1-n_t)}{u_c(c_t, m_t, 1-n_t)} = f_n(k_{t-1}, n_t, z_t),$$

which states that at an optimum, the marginal rate of substitution between consumption and leisure must equal the marginal product of labor.

The equilibrium values of consumption, capital, money holdings, and labor supply must satisfy the conditions given in (2.51)–(2.55). These conditions can be simplified, however. Using (2.51), (2.53), (2.54), and (2.57) can be rewritten as

$$u_c(c_t, m_t, 1-n_t) = u_m(c_t, m_t, 1-n_t) + \beta \mathrm{E}_t\left[\frac{u_c(c_{t+1}, m_{t+1}, 1-n_{t+1})}{1+\pi_{t+1}}\right] \tag{2.58}$$

$$u_l(c_t, m_t, 1-n_t) = u_c(c_t, m_t, 1-n_t) f_n(k_{t-1}, n_t, z_t) \tag{2.59}$$

$$u_c(c_t, m_t, 1-n_t) = \beta \mathrm{E}_t R_t u_c(c_{t+1}, m_{t+1}, 1-n_{t+1}), \tag{2.60}$$

where in (2.60)

$$R_t = 1 - \delta + f_k(k_t, n_{t+1}, z_{t+1}) \tag{2.61}$$

is 1 plus the marginal product of capital net of depreciation. In addition, the economy's aggregate resource constraint, expressed in per capita terms, requires that

$$k_t = (1-\delta)k_{t-1} + y_t - c_t, \tag{2.62}$$

while the production function implies

$$y_t = f(k_{t-1}, n_t, z_t). \tag{2.63}$$

Finally, real money balances evolve according to

$$m_t = \left(\frac{1+\theta_t}{1+\pi_t}\right) m_{t-1}, \tag{2.64}$$

where θ_t is the growth rate of the nominal stock of money.

To complete the model, it remains to specify the process governing the productivity disturbance z_t and the growth rate of money θ_t. Following the real business cycle literature, let

$$z_t = \rho z_{t-1} + e_t, \tag{2.65}$$

where e_t is a serially uncorrelated mean zero process and $|\rho| < 1$. For money growth, define $u_t = \theta_t - \theta^{ss}$ as the deviation of money growth around its average value θ^{ss}. This deviation will be treated as a stochastic process given by

$$u_t = \gamma u_{t-1} + \phi z_{t-1} + \varphi_t, \quad 0 \le \gamma < 1, \tag{2.66}$$

where φ_t is a serially uncorrelated mean zero innovation.

Equations (2.58)–(2.66) are a nonlinear system of equations to determine the equilibrium values of the model's nine endogenous variables: $y_t, c_t, k_t, m_t, n_t, R_t, \pi_t, z_t$, and u_t.

2.7.2 Functional Forms

In order to study the properties of this nonlinear system of equations, we will evaluate them using specific function forms for the utility function and the production function. For the production function, we follow common practice in assuming a constant returns to scale, Cobb-Douglas production function expressed in per capita terms:[40]

$$y_t = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

For the utility function, we assume a nested CES functional form:

$$u(c_t, m_t, 1-n_t) = \frac{[a c_t^{1-b} + (1-a) m_t^{1-b}]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta}$$

40. Recalling that n is the fraction of time the representative household spends in employment, aggregate output is equal to $Y = e^z (kN)^{\alpha} (nN)^{1-\alpha} = e^z (K)^{\alpha} (nN)^{1-\alpha}$ if there are N households in the economy.

with $0 < a < 1$ and $b, \eta, \Phi, \Psi > 0$. The parameter Φ is the coefficient of relative risk adversion (the inverse of the elasticity of intertemporal substitution), and b is the inverse of the interest elasticity of real money demand.

Using the assumed functional forms, and letting $X_t \equiv ac_t^{1-b} + (1-a)m_t^{1-b}$, (2.58)–(2.61) become

$$\frac{u_m}{u_c} = \left(\frac{1-a}{a}\right)\left(\frac{m_t}{c_t}\right)^{-b} = \frac{i_t}{1+i_t} \tag{2.67}$$

$$\frac{u_l}{u_c} = \frac{\Psi(1-n_t)^{-\eta}}{aX_t^{\frac{b-\Phi}{1-b}}c_t^{-b}} = (1-\alpha)\frac{y_t}{n_t} \tag{2.68}$$

$$X_t^{\frac{b-\Phi}{1-b}}ac_t^{-b} = \beta \mathrm{E}_t(R_t X_{t+1}^{\frac{b-\Phi}{1-b}}ac_{t+1}^{-b}) \tag{2.69}$$

$$R_t = \alpha\frac{\mathrm{E}_t y_{t+1}}{k_t} + 1 - \delta. \tag{2.70}$$

2.7.3 The Steady State

In the steady state, $R^{ss} = \beta^{-1}$. This condition together with (2.70) implies that the steady-state output-capital ratio is equal to

$$\left(\frac{y^{ss}}{k^{ss}}\right) = \frac{1}{\alpha}\left(\frac{1}{\beta} - 1 + \delta\right).$$

From the production function, $(y^{ss}/k^{ss}) = (n^{ss}/k^{ss})^{1-\alpha}$, or

$$\left(\frac{n^{ss}}{k^{ss}}\right) = \left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{1}{1-\alpha}} = \left[\frac{1}{\alpha}\left(\frac{1}{\beta} - 1 + \delta\right)\right]^{\frac{1}{1-\alpha}}.$$

It follows from the aggregate resource constraint that

$$\left(\frac{c^{ss}}{k^{ss}}\right) = \left(\frac{y^{ss}}{k^{ss}}\right) - \delta = \frac{1}{\alpha}\left(\frac{1-\beta}{\beta}\right) + \left(\frac{1-\alpha}{\alpha}\right)\delta.$$

Since $(1+i^{ss}) = R^{ss}(1+\pi^{ss})$ and $1+\pi^{ss} = \Theta$,

$$\frac{i^{ss}}{1+i^{ss}} = \frac{\Theta - \beta}{\Theta}.$$

Therefore, using the utility function to evaluate (2.67) in the steady state yields

$$\left(\frac{m^{ss}}{c^{ss}}\right) = \left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta-\beta}{\Theta}\right)^{-\frac{1}{b}}$$

and

$$\frac{m^{ss}}{k^{ss}} = \left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta-\beta}{\Theta}\right)^{-\frac{1}{b}}\left(\frac{c^{ss}}{k^{ss}}\right). \tag{2.71}$$

These results for the steady-state ratios are collected in table 2.1.

To go from the steady-state ratios of table 2.1 to the steady-state levels, it is necessary to determine the steady-state value of n since it is through its effect on n^{ss} that inflation may affect the steady state. For this, we need to use the first order condition given by (2.68). In the steady state, this can be written as

$$\Psi(1-n^{ss})^{-\eta} = (1-\alpha)\left(\frac{y^{ss}}{n^{ss}}\right)(X^{ss})^{\frac{b-\Phi}{1-b}}a(c^{ss})^{-b}.$$

The steady-state value of X is $a(c^{ss})^{1-b}\left[1 + \left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta-\beta}{\Theta}\right)^{\frac{b-1}{b}}\right]$, while $\left(\frac{y^{ss}}{n^{ss}}\right) = \left(\frac{y^{ss}}{k^{ss}}\right)\left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{-1}{1-\alpha}} = \left(\frac{y^{ss}}{k^{ss}}\right)^{-\frac{\alpha}{1-\alpha}}$. Making use of these steady-state values,

$$\Psi(1-n^{ss})^{-\eta} = (1-\alpha)\left(\frac{y^{ss}}{k^{ss}}\right)^{-\frac{\alpha}{1-\alpha}}\left(a(c^{ss})^{1-b}\left[1 + \left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta-\beta}{\Theta}\right)^{\frac{b-1}{b}}\right]\right)^{\frac{b-\Phi}{1-b}}a(c^{ss})^{-b}$$

$$= (1-\alpha)\left(\frac{y^{ss}}{k^{ss}}\right)^{-\frac{\alpha}{1-\alpha}}\left[1 + \left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta-\beta}{\Theta}\right)^{\frac{b-1}{b}}\right]^{\frac{b-\Phi}{1-b}}a^{\frac{1-\Phi}{1-b}}(c^{ss})^{-\Phi}.$$

Finally, replacing $(c^{ss})^{-\Phi}$ with $(c^{ss}/k^{ss})^{-\Phi}(k^{ss}/n^{ss})^{-\Phi}(n^{ss})^{-\Phi}$, we obtain

$$\frac{(n^{ss})^{\Phi}}{(1-n^{ss})^{\eta}} = H\left[1 + \left(\frac{a}{1-a}\right)^{-\frac{1}{b}}\left(\frac{\Theta-\beta}{\Theta}\right)^{\frac{b-1}{b}}\right]^{\frac{b-\Phi}{1-b}},$$

where

$$H \equiv \left(\frac{1-\alpha}{\Psi}\right)\left(\frac{y^{ss}}{k^{ss}}\right)^{-\frac{\alpha}{1-\alpha}}a^{\frac{1-\Phi}{1-b}}\left(\frac{c^{ss}}{k^{ss}}\right)^{-\Phi}\left(\frac{k^{ss}}{n^{ss}}\right)^{-\Phi}$$

is independent of inflation and the rate of money growth. This is (2.39) of the text.

2.7.4 The Linear Approximation

Next, we need to linearize the model around the steady state. With the exception of interest rates and inflation, variables will be expressed as percentage deviations around the steady state. Percentage deviations of a variable u around its steady-state value will be denoted by $\hat{u}$, where $u_t \equiv u^{ss}(1 + \hat{u}_t)$.

Three basic rules will be followed in deriving approximations (see Uhlig 1999). First, for two variables u and w,

$$uw = u^{ss}(1+\hat{u})w^{ss}(1+\hat{w}) = u^{ss}w^{ss}(1+\hat{u}+\hat{w}+\hat{u}\hat{w}) \approx u^{ss}w^{ss}(1+\hat{u}+\hat{w}).$$

That is, we will assume that product terms like $\hat{u}\hat{w}$ are approximately equal to zero. Second,

$$u^a = (u^{ss})^a(1+\hat{u})^a \approx (u^{ss})^a(1+a\hat{u}),$$

which can be obtained as a repeated application of the first rule. Finally,

$$\ln u = \ln u^{ss}(1+\hat{u}) = \ln u^{ss} + \ln(1+\hat{u}) \approx \ln u^{ss} + \hat{u}.$$

By applying these rules, we can obtain a system of linear equations that characterize the dynamic behavior of the MIU model for small deviations around its steady state.

We begin with (2.64), (2.62), and (2.63) since these equations are the most straightforward.

Equation (2.64) can be written as $m^{ss}(1+\hat{m}_t) = \Theta(1+u_t)m^{ss}(1+\hat{m}_{t-1})/\Theta(1+\hat{\pi}_t)$, or approximately,

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + u_t. \tag{2.72}$$

The resource constraint (2.62) can be written in terms of percentage deviations from the steady state as

$$k^{ss}(1+\hat{k}_t) = k^{ss}(1-\delta)(1+\hat{k}_{t-1}) + y^{ss}(1+\hat{y}_t) - c^{ss}(1+\hat{c}_t).$$

Subtracting $y^{ss} = c^{ss} + \delta k^{ss}$ and dividing by k^{ss}, we obtain

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \left(\frac{y^{ss}}{k^{ss}}\right)\hat{y}_t - \left(\frac{c^{ss}}{k^{ss}}\right)\hat{c}_t. \tag{2.73}$$

The aggregate production function can be expressed in terms of percentage deviations around the steady state as

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + z_t. \tag{2.74}$$

Equations (2.68)–(2.69) are more complicated, but we can derive approximations for them using the same methods used to obtain the other approximations. It will simplify matters to introduce a new variable, λ_t, equal to the marginal utility of consumption:

$$\lambda_t = aX_t^{\frac{b-\Phi}{1-b}}c_t^{-b}.$$

Since we have introduced the new variable λ_t, we need to derive the approximation for it. This is

$$\hat{\lambda}_t = -[\gamma\Phi + (1-\gamma)b]\hat{c}_t + (b-\Phi)(1-\gamma)\hat{m}_t, \tag{2.75}$$

where $\gamma = a(c^{ss})^{1-b}/[a(c^{ss})^{1-b} + (1-a)(m^{ss})^{1-b}]$.[41] Equations (2.68)–(2.69) now become

$$\frac{\Psi(1-n_t)^{-\eta}}{\lambda_t} = (1-\alpha)\frac{y_t}{n_t}$$

$$\lambda_t = \beta E_t(R_t\lambda_{t+1}).$$

The first of these can be approximated by first noting that it can be rewritten as $\Psi(1-n_t)^{-\eta} = (1-\alpha)y_t n_t^{-1}\lambda_t$. Now $(1-n_t)^{-\eta} = l_t^{-\eta}$, and this can be approximated by recognizing that $l^{ss}(1+\hat{l}_t) = 1 - n^{ss}(1+\hat{n}_t)$ so that $\hat{l}_t = -\left(\frac{n^{ss}}{l^{ss}}\right)\hat{n}_t$. Hence, $(1-n_t)^{-\eta} = (l^{ss})^{-\eta}(1-\eta\hat{l}_t) \approx (l^{ss})^{-\eta}\left(1+\eta\left(\frac{n^{ss}}{l^{ss}}\right)\hat{n}_t\right)$. So (2.68) is then approximated by

$$\left[1+\eta\left(\frac{n^{ss}}{l^{ss}}\right)\right]\hat{n}_t = \hat{y}_t + \hat{\lambda}_t. \tag{2.76}$$

Equation (2.67) can be approximated as

$$\hat{m}_t = \hat{c}_t - \left(\frac{1}{b}\right)\hat{\imath}_t. \tag{2.77}$$

41. Note that

$$X^{ss}(1+\hat{x}_t) = [a(c^{ss})^{1-b}(1+(1-b)\hat{c}_t) + (1-a)(m^{ss})^{1-b}(1+(1-b)\hat{m}_t)].$$

It follows that

$$\hat{x}_t = (1-b)[\gamma\hat{c}_t + (1-\gamma)\hat{m}_t].$$

Thus the marginal utility of consumption (the left side of the Euler condition) can be approximated by

$$\begin{aligned} ax_t^{\frac{b-\Phi}{1-b}}c^{-b} &\approx a(x^{ss})^{\frac{b-\Phi}{1-b}}(c^{ss})^{-b}\left[1+\left(\frac{b-\Phi}{1-b}\right)\hat{x}_t - b\hat{c}_t\right] \\ &= a(x^{ss})^{\frac{b-\Phi}{1-b}}(c^{ss})^{-b}\{1+(b-\Phi)[\gamma\hat{c}_t+(1-\gamma)\hat{m}_t] - b\hat{c}_t\} \\ &= a(x^{ss})^{\frac{b-\Phi}{1-b}}(c^{ss})^{-b}\{1-[\gamma\Phi+(1-\gamma)b]\hat{c}_t + (b-\Phi)(1-\gamma)\hat{m}_t\}. \end{aligned}$$

Therefore,

$$\hat{\lambda}_t = -[\gamma\Phi + (1-\gamma)b]\hat{c}_t + (b-\Phi)(1-\gamma)\hat{m}_t.$$

We next turn to (2.70), the condition linking the real rate of interest and the expected marginal product of capital. This can be written as

$$R_t = \alpha\left(\frac{y^{ss}}{k^{ss}}\right)\mathrm{E}_t\left(\frac{1+\hat{y}_{t+1}}{1+\hat{k}_t}\right) + 1 - \delta$$

$$\approx \alpha\left(\frac{y^{ss}}{k^{ss}}\right)(1 + \mathrm{E}_t\hat{y}_{t+1} - \hat{k}_t) + 1 - \delta.$$

With $R^{ss} = \alpha\left(\frac{y^{ss}}{k^{ss}}\right) + 1 - \delta$, our approximation around the steady state is given by

$$\hat{r}_t \equiv R_t - R^{ss} = \alpha\left(\frac{y^{ss}}{k^{ss}}\right)(\mathrm{E}_t\hat{y}_{t+1} - \hat{k}_t). \tag{2.78}$$

Notice that $\hat{r}$ is measured as a percentage rate, not as a percentage deviation around the steady state.

The Fisher condition linking the real and nominal interest rates to expected inflation takes the form

$$\hat{\imath}_t = \hat{r}_t + \mathrm{E}_t\hat{\pi}_{t+1}. \tag{2.79}$$

Now consider (2.69). Since $\beta R^{ss} = 1$, this Euler condition becomes

$$\hat{\lambda}_t = \hat{r}_t + \mathrm{E}_t\hat{\lambda}_{t+1}. \tag{2.80}$$

The equations describing the evolution of the exogenous processes z and φ are straightforward, given by (2.65) and (2.66), but they are repeated here for completeness:

$$z_t = \rho z_{t-1} + e_t \tag{2.81}$$

$$u_t = \gamma u_{t-1} + \phi z_{t-1} + \varphi_t. \tag{2.82}$$

Equations (2.72)–(2.82) constitute a linearized version of Sidrauski's MIU model. These equations represent a linear system of difference equations involving expectational variables. The endogenous state variables are k and m; the endogenous jump variables are y, c, n, π, i, r, and λ; the exogenous variables are z and u.

2.7.5 Solving Linear Rational Expectations Models with Forward-Looking Variables

This sections provides a brief overview of the approach used to solve linear rational expectations models. This discussion follows Uhlig (1999), to which the reader is referred for more details. General discussions can be found in Farmer (1993, chapter

3) or the user's guide in Hoover, Hartley, and Salyer (1998). See also Turnovsky (1995).

Let $x_{1t} = (k_t, m_t)'$ be the vector of endogenous state variables, and let $x_{2t} = (y_t, c_t, n, \pi_t, i, r_t, \lambda_t)'$ be the vector of other endogenous variables. The equilibrium conditions of the MIU model can be written in the form

$$Ax_{1t} + Bx_{1t-1} + Cx_{2t} + D\psi_t = 0$$

$$F\mathrm{E}_t x_{1t+1} + Gx_{1t} + Hx_{1t-1} + J\mathrm{E}_t x_{2t+1} + Kx_{2t} + M\psi_t = 0$$

$$\psi_{t+1} = N\psi_t + \varepsilon_{t+1},$$

where $\psi_t = (z_t, u_t)'$. It is assumed that C is of full column rank and that the eigenvalues of N are all within the unit circle.

Then if an equilibrium solution to this system of equations exists, it takes the form of stable laws of motion

$$x_{1t} = Px_{1t-1} + Q\psi_t$$

$$x_{2t} = Rx_{1t-1} + S\psi_t$$

for x_{1t} and x_{2t}. When C is a square invertible matrix, Uhlig proves that P satisfies the quadratic matrix equation

$$(F - JC^{-1}A)P^2 - (JC^{-1}B - G + KC^{-1}A)P - KC^{-1}B + H = 0$$

and the equilibrium is stable if and only if all the eigenvalues of P are less than unity in absolute value. The matrix R is given by

$$R = -C^{-1}(AP + B),$$

while Q and S are given by

$$(N' \otimes (F - JC^{-1}A) + I_k \otimes (JR + FP + G\_KC^{-1}A))\, vec(Q)$$
$$= vec((JC^{-1}D - L)N + KC^{-1}D - M)$$

and

$$S = -C^{-1}(AQ + D).$$

Uhlig provides a fuller discussion and treats the case in which C is $l \times n$ with $l > n$.

For the MIU model given by (2.72)–(2.82), there are 6 equations that do not involve expectations of future variables and 7 elements of x_{2t}, so C is 6×7 and is of less than full rank. We need to eliminate expectations from one additional equation.

One way to do this is to eliminate $E_t\hat{y}_{t+1}$ from (2.78). Use the production function (2.74) and the labor market equilibrium condition (2.76) to eliminate n_t between them and obtain an expression for y_t of the form

$$\hat{y}_t = \frac{\left(1+\eta\frac{N^{ss}}{L^{ss}}\right)(\alpha\hat{k}_{t-1}+z_t)+(1-\alpha)\hat{\lambda}_t}{\alpha+\eta\frac{N^{ss}}{L^{ss}}}.$$

Advancing this one period, taking expectations, and using (2.80) and (2.81),

$$E_t\hat{y}_{t+1} = \frac{\left(1+\eta\frac{N^{ss}}{L^{ss}}\right)(\alpha\hat{k}_t+\rho z_t)+(1-\alpha)(\hat{\lambda}_t-\hat{r}_t)}{\alpha+\eta\frac{N^{ss}}{L^{ss}}}.$$

Equation (2.78) now becomes

$$\hat{r}_t = \alpha\left(\frac{Y^{ss}}{K^{ss}}\right)\left[\frac{\left(1+\eta\frac{N^{ss}}{L^{ss}}\right)(\alpha\hat{k}_t+\rho z_t)+(1-\alpha)(\hat{\lambda}_t-\hat{r}_t)}{\alpha+\eta\frac{N^{ss}}{L^{ss}}}-\hat{k}_t\right]$$

$$= \alpha\left(\frac{Y^{ss}}{K^{ss}}\right)\left[\frac{(\alpha-1)\eta\left(\frac{N^{ss}}{L^{ss}}\right)\hat{k}_t+\left(1+\eta\frac{N^{ss}}{L^{ss}}\right)\rho z_t+(1-\alpha)\hat{\lambda}_t}{\left(\alpha+\eta\frac{N^{ss}}{L^{ss}}\right)+\alpha(1-\alpha)\left(\frac{Y^{ss}}{K^{ss}}\right)}\right]. \qquad (2.83)$$

Since this produces an equation for the real return in which no expectational terms appear, replacing (2.78) with (3.72) yields a system that can be solved easily using Uhlig's programs. The Matlab program used for the simulations in this chapter (and other chapters of this book) can be found at http://econ.ucsc.edu/~walshc/2ed/programs/.

2.8 Problems

1. The MIU model of section 2.2 implied that the marginal rate of substitution between money and consumption was set equal to $i_t/(1+i_t)$ (see 2.12). That model assumed that agents entered period t with resources ω_t and used those to purchase capital, consumption, nominal bonds, and money. The real value of these money

holdings yielded utility in period t. Assume instead that money holdings chosen in period t do not yield utility until period $t+1$. Utility is $\sum\beta^i U(c_{t+i}, M_{t+i}/P_{t+i})$ as before, but the budget constraint takes the form

$$\omega_t = c_t + \frac{M_{t+1}}{P_t} + b_t + k_t,$$

and the household chooses c_t, k_t, b_t, and M_{t+1} in period t. The household's real wealth, ω_t, is given by

$$\omega_t = f(k_{t-1}) + (1-\delta)k_{t-1} + (1+r_{t-1})b_{t-1} + m_t.$$

Derive the first-order condition for the household's choice of M_{t+1} and show that

$$\frac{U_m(c_{t+1}, m_{t+1})}{U_c(c_{t+1}, m_{t+1})} = i_t.$$

(Suggested by Kevin Salyer.)

2. (Carlstrom and Fuerst 2001): Assume that the representative household's utility depends on consumption and the level of real money balances available for spending on consumption. Let A_t/P_t be the real stock of money that enters the utility function. If capital is ignored, the household's objective is to maximize $\sum\beta^i U(c_{t+i}, A_{t+i}/P_{t+i})$ subject to the budget constraint

$$Y_t + \frac{M_{t-1}}{P_t} + \tau_t + \frac{(1+i_{t-1})B_{t-1}}{P_t} = C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t},$$

where income Y_t is treated as an exogenous process. Assume that the stock of money that yields utility is the real value of money holdings after bonds have been purchased but before income has been received or consumption goods have been purchased:

$$\frac{A_t}{P_t} = \frac{M_{t-1}}{P_t} + \tau_t + \frac{(1+i_{t-1})B_{t-1}}{P_t} - \frac{B_t}{P_t}.$$

a. Derive the first order conditions for B_t and for A_t.

b. How do these conditions differ from those obtained in the text?

3. (Calvo and Leiderman 1992): A commonly used specification of the demand for money, originally due to Cagen (1956), assumes that $m = Ae^{-\alpha i_t}$, where A and α are parameters and i is the nominal rate of interest. In the Sidrauski (1967) model,

assume that utility is separable in consumption and real money balances: $u(c_t, m_t) = w(c_t) + v(m_t)$, and further assume that $v(m_t) = m_t(B - D \ln m_t)$, where B and D are positive parameters. Show that the demand for money is given by $m_t = Ae^{-\alpha_t i_t}$, where $A = e^{(\frac{B}{D}-1)}$ and $\alpha_t = w'(c_t)/D$.

4. Assume that $m_t = Ae^{-\alpha i_t}$, where A and α are constants. Calculate the welfare cost of inflation in terms of A and α, expressed as a percentage of steady-state consumption (normalized to equal 1). Does the cost increase or decrease with α? Explain why.

5. Suppose $W = \sum \beta^i(\ln c_t + m_t e^{-\gamma m_t})$, $\gamma > 0$, and $\beta = 0.95$. Assume that the production function is $f(k_t) = k_t^{.5}$ and $\delta = 0.02$. What rate of inflation maximizes steady-state welfare? How do real money balances at the welfare-maximizing rate of inflation depend on γ?

6. Suppose that the utility function (2.38) is replaced by

$$u(c_t, m_t, l_t) = \left(\frac{1}{1-\Phi}\right)\{[ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1}{1-b}} l_t^{1-\eta}\}^{1-\Phi}.$$

a. Derive the first order conditions for the household's optimal money holdings.

b. Show how (2.43) and (2.44) are altered with this specification of the utility function.

7. Suppose the utility function (2.38) is replaced by

$$u(c_t, m_t, l_t) = \left(\frac{1}{1-\Phi}\right)[a_c c_t^{1-b} + a_m m_t^{1-\gamma} + a_l l^{1-\eta}]^{1-\Phi}.$$

a. Derive the first order conditions for the household's optimal money holdings.

b. Show how (2.43) and (2.44) are altered with this specification of the utility function.

8. Suppose a nominal interest rate of i_m is paid on money balances. These payments are financed by a combination of lump-sum taxes and printing money. Let a be the fraction financed by lump-sum taxes. The government's budget identity is $\tau_t + v_t = i_m m_t$, with $\tau_t = a i_m m_t$ and $v = \theta m_t$. Using Sidrauski's model, do the following:

a. Show that the ratio of the marginal utility of money to the marginal utility of consumption will equal $r + \pi - i_m = i - i_m$. Explain why.

b. Show how $i - i_m$ is affected by the method used to finance the interest payments on money. Explain the economics behind your result.

9. Suppose money is a productive input into production so that the aggregate production function becomes $y = f(k, m)$. Incorporate this modification into the model of section 2.2. Is money still superneutral? Explain.

10. In Sidrauski's MIU model augmented to include a variable labor supply, money is superneutral if the representative agent's preferences are given by

$$\sum \beta^i u(c_{t+i}, m_{t+i}, l_{t+i}) = \sum \beta^i (c_{t+i} m_{t+i})^b l_{t+i}^d$$

but not if they are given by

$$\sum \beta^i u(c_{t+i}, m_{t+i}, l_{t+i}) = \sum \beta^i (c_{t+i} + km_{t+i})^b l_{t+i}^d.$$

Discuss. (Assume output depends on capital and labor and the aggregate production function is Cobb-Douglas.)

11. Suppose the representative agent does not treat τ_t as a lump-sum transfer, but instead assumes her transfer will be proportional to her own holdings of money (since in equilibrium, $\tau = \theta m$). Solve for the agent's demand for money. What is the welfare cost of inflation?

12. By simulating the model of section 2.5.2, investigate the impact of a money growth shock on employment as η varies from 1 to 10. Provide an intuitive explanation for your findings. Assume $u_t = \gamma u_{t-1} + \phi_t$, where ϕ_t is white noise and $\gamma = .5$. How do your results change if $\gamma = .8$?

13. By simulating the model of section 2.5.2, investigate the impact of a money growth shock on employment as b varies from .1 to 20. Assume $u_t = \gamma u_{t-1} + \phi_t$, where ϕ_t is white noise and $\gamma = .5$.

3 Money and Transactions

3.1 Introduction

The previous chapter introduced a role for money by assuming that individuals derive direct utility from holding real money balances. Models that generate a positive demand for money in this way have been criticized for solving the problem of creating positive value for money by simply assuming the problem away; postulating that money yields direct utility guarantees that money will be valued (as long as the utility function is suitably defined). Yet we usually think of money as yielding utility through use—we value money because it is useful in facilitating transactions. As described by Clower (1967), goods buy money and money buys goods, but goods don't buy goods. And because goods don't buy goods, a medium of exchange that serves to aid the process of transacting has value.

A medium of exchange that facilitates transactions yields utility indirectly by allowing certain purchases to be made or by reducing the costs associated with transactions. The demand for money is then determined by the nature of the economy's transactions technology. The first formal models of money demand that emphasized the role of transactions costs were due to Baumol (1952) and Tobin (1956).[1] Niehans (1978) developed a systematic treatment of the theory of money in which transactions costs play a critical role. These models were partial equilibrium models, focusing on the demand for money as a function of the nominal interest rate and income. In keeping with the approach used in examining money-in-the-utility (MIU) models, our focus in this chapter will be on *general* equilibrium models in which the demand for money arises from its use in carrying out transactions.

The first model we examine in this chapter is one in which time and money are used to produce transaction services that are required to purchase consumption goods. The consumer must balance the opportunity cost of holding money against the value of leisure in deciding how to combine time and money to purchase consumption goods. The production technology used to produce transaction services determines how much time must be spent "shopping" for given levels of consumption and money holdings. Higher levels of money holdings reduce the time needed for shopping, thereby increasing the individual agent's leisure. When leisure enters the utility function of the representative agent, such shopping time models provide a link between the MIU approach of the previous chapter and models of money that focus more explicitly on transactions services and money as a medium of exchange.

1. Jovanovic (1982) and Romer (1986) embed the Baumol-Tobin model in general equilibrium frameworks.

Most of this chapter, however, will be devoted to the study of models which impose a rigid restriction on the nature of transactions. Rather than allowing substitutability between time and money in carrying out transactions, *cash-in-advance* (CIA) models simply require that money balances be held to finance certain types of purchases; without money, these purchases cannot be made. CIA models, like the MIU models of the previous chapter, assume that money is special; unlike other financial assets, it either yields direct utility, and therefore belongs in the utility function, or it has unique properties that allow it to be used to facilitate transactions. This chapter concludes with a look at some recent attempts based on search theory to explain how the nature of transactions gives rise to money.

Just as in chapter 2, the assumption that prices and wages are perfectly flexible will be mantained throughout this chapter. Thus, the focus is on flexible price models that emphasize the transactions role of money. The approaches adopted in these models can also be used to incorporate money into models in which prices and/or wages are sticky. The implications of introducing nominal rigidities into general equilibrium models of monetary economies are discussed in chapters 5, 6, 8, 10, and 11.

3.2 Shopping Time Models

A direct approach to modeling the role of money in facilitating transactions is to assume that the purchase of goods requires the input of transaction services, and these services are, in turn, produced by money and time. Larger holdings of money allow the household to reduce the time it must devote to producing transaction services.[2]

Suppose that purchasing consumption requires transactions services ψ, with units chosen so that consumption of c requires transaction services $\psi = c$. These transaction services are produced with inputs of real cash balances $m \equiv M/P$ and shopping time n^s:

$$\psi = \psi(m, n^s) = c, \tag{3.1}$$

where $\psi_m \geq 0$, $\psi_{n^s} \geq 0$, and $\psi_{mm} \leq 0$, $\psi_{n^s n^s} \leq 0$. This specification assumes that it is the agent's holdings of *real* money balances that produce transaction services; a change in the price level requires a proportional change in nominal money holdings to generate the same level of real consumption purchases, holding shopping time n^s

2. See Brock (1974) for an earlier use of a shopping time model to motivate an MIU approach. The use of a shopping time approach to the study of the demand for money is presented in McCallum and Goodfriend (1987) and Croushore (1993).

constant. Rewriting (3.1) in terms of the shopping time required for given levels of consumption and money holdings,

$$n^s = g(c, m); \quad g_c > 0, \quad g_m \leq 0.$$

Household utility is assumed to depend on consumption and leisure: $v(c, l)$. Leisure is equal to $l = 1 - n - n^s$, where n is time spent in market employment and n^s is time spent shopping. Total time available is normalized to equal 1. With shopping time n^s an increasing function of consumption and a decreasing function of real money holdings, time available for leisure is $1 - n - g(c, m)$. We can now define a function

$$u(c, m, n) \equiv v[c, 1 - n - g(c, m)]$$

that gives utility as a function of consumption, labor supply, and money holdings. Thus, a simple shopping time model can motivate the appearance of an MIU function and, more importantly, can help determine the properties of the partial derivatives of the function u with respect to m. By placing restrictions on the partial derivatives of the shopping time production function $g(c, m)$, we potentially can determine what restrictions might be placed on the utility function $u(c, m, n)$. For example, if the marginal productivity of money goes to zero for some finite level of real money balances $\bar{m}$, that is, $\lim_{m \to \bar{m}} g_m = 0$, then this property will carry over to u_m.

In the MIU function model of chapter 2, higher expected inflation lowers money holdings, but the effect on consumption depends on the sign of u_{cm}.[3] The shopping time model implies that $u_m = -v_l g_m \geq 0$, so

$$u_{cm} = (v_{ll} g_c - v_{cl}) g_m - v_l g_{cm}. \tag{3.2}$$

The sign of u_{cm} will depend on such factors as the effect of variations in leisure time on the marginal utility of consumption (v_{cl}) and the effect of variations in consumption on the marginal productivity of money in reducing shopping time (g_{cm}). In the benchmark MIU model of the previous chapter, u_{cm} was taken to be positive.[4] Relating u_{cm} to the partials of the underlying utility function v and the transactions production function g can suggest whether this assumption was reasonable. From (3.2), the assumption of diminishing marginal utility of leisure ($v_{ll} \leq 0$) and $g_m \leq 0$ implies that $v_{ll} g_c g_m \geq 0$. If greater consumption raises the marginal productivity of

3. This is a statement about the partial equilibrium effect of inflation on the representative agent's decision. In general equilibrium, consumption is independent of inflation in models that display superneutrality.

4. This corresponded to $b > \Phi$ in the benchmark utility function used in chapter 2.

money in reducing shopping time ($g_{cm} \leq 0$), then $-v_l g_{cm} \geq 0$ as well. Wang and Yip (1992) characterize the situation in which these two dominate so that $u_{cm} \geq 0$ as the transaction services version of the MIU model. In this case, the MIU model implies that a rise in expected inflation would lower m and u_c, and this would lower consumption, labor supply, and output (see section 2.3.2). If consumption and leisure are strong substitutes so that $v_{cl} \leq 0$, then u_{cm} could be negative, a situation Wang and Yip describe as corresponding to an asset substitution model. With $u_{cm} < 0$, a monetary injection that raises expected inflation will increase consumption, labor supply, and output.

The household's intertemporal problem analyzed in the appendix to chapter 2 for the MIU model can be easily modified to incorporate a shopping time role for money. The household's objective is to maximize

$$\sum \beta^i v[c_{t+i}, 1 - n_{t+i} - g(c_{t+i}, m_{t+i})]; \quad 0 < \beta < 1$$

subject to

$$f(k_{t-1}, n_t) + \tau_t + (1-\delta)k_{t-1} + \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{1+\pi_t} = c_t + k_t + b_t + m_t, \tag{3.3}$$

where f is a standard neoclassical production function, k is the capital stock, δ is the depreciation rate, b and m are real bond and money holdings, and τ is a real lump-sum transfer from the government.[5] Defining $a_t = \tau_t + [(1+i_{t-1})b_{t-1} + m_{t-1}]/(1+\pi_t)$, the household's decision problem can be written in terms of the value function $V(a_t, k_{t-1})$:

$$V(a_t, k_{t-1}) = \max\{v[c_t, 1 - n_t - g(c_t, m_t)] + \beta V(a_{t+1}, k_t)\},$$

where the maximization is subject to the constraints $f(k_{t-1}, n_t) + (1-\delta)k_{t-1} + a_t = c_t + k_t + b_t + m_t$ and $a_{t+1} = \tau_{t+1} + [(1+i_t)b_t + m_t]/(1+\pi_{t+1})$. Proceeding as in chapter 2 by using these two constraints to eliminate k_t and a_{t+1} from the expression for the value function, the necessary first order conditions for real money holdings, capital holdings, and labor supply are

$$v_c - v_l g_c - \beta V_k(a_{t+1}, k_t) = 0 \tag{3.4}$$

$$-v_l g_m + \beta \frac{V_a(a_{t+1}, k_t)}{1+\pi_{t+1}} - \beta V_k(a_{t+1}, k_t) = 0 \tag{3.5}$$

5. Note that we have assumed that transaction services are needed only for the purchase of consumption goods and not for the purchase of capital goods. In the next section, we will see that alternative treatments of investment and the transactions technology have implications for the steady state.

$$-v_l + \beta V_k(a_{t+1}, k_t) f_n(k_{t-1}, n_t) = 0 \tag{3.6}$$

$$V_a(a_t, k_{t-1}) = \beta V_k(a_{t+1}, k_t) \tag{3.7}$$

$$V_k(a_t, k_{t-1}) = \beta V_k(a_{t+1}, k_t)[f_k(k_{t-1}, n_t) + 1 - \delta]. \tag{3.8}$$

Letting w_t denote the marginal product of labor (i.e., $w_t = f_n(k_{t-1}, n_t)$), (3.6) and (3.7) yield $v_l = w_t V_a(a_t, k_{t-1})$. This implies that (3.4) can be written as

$$u_c(c_t, l_t) = V_a(a_t, k_{t-1})[1 + w_t g_c(c_t, m_t)]. \tag{3.9}$$

The marginal utility of consumption is set equal to the marginal utility of wealth, $V_a(a_t, k_{t-1})$, plus the cost, in utility units, of the marginal time needed to purchase consumption. The increased shopping time involved in additional consumption is g_c, its value in terms of goods is obtained by multiplying this by the real wage w, and its value in terms of utility is $V_a(a; k)wg_c$.

With $g_m \leq 0$, $v_l g_m = V_a w g_m$ is the value in utility terms of the shopping time savings that results from additional holdings of real money balances. Equations (3.5) and (3.7) imply that money will be held to the point where the marginal net benefit, equal to the value of shopping time savings plus the discounted value of money's wealth value in the next period, or $-v_l g_m + \beta V_a(a_{t+1}, k_t)/(1+\pi_{t+1})$, just equals the net marginal utility of wealth. Letting $R_t = f_k(k_t, n_{t+1}) + 1 - \delta$ denote 1 plus the real rate of return net of depreciation, the first order condition for optimal money holdings, together with (3.7) and (3.8), implies

$$\begin{aligned}
-v_l g_m &= \beta V_k(a_{t+1}, k_t) - \beta \frac{V_a(a_{t+1}, k_t)}{1+\pi_{t+1}} \\
&= V_a(a_t, k_{t-1})\left[1 - \beta \frac{V_a(a_{t+1}, k_t)/V_a(a_t, k_{t-1})}{1+\pi_{t+1}}\right] \\
&= V_a(a_t, k_{t-1})\left\{1 - \left[\frac{1}{R_t(1+\pi_{t+1})}\right]\right\} \\
&= V_a(a_t, k_{t-1})\left(\frac{i_t}{1+i_t}\right),
\end{aligned} \tag{3.10}$$

where $i_t = R_t(1+\pi_{t+1}) - 1$ is the nominal rate of interest and, using (3.7) and (3.8), $\beta(V_a(a_{t+1}, k_t)/V_a(a_t, k_{t-1})) = R_t^{-1}$.

Further insight can be gained by using (3.6) and (3.7) to note that (3.10) can also be written as

$$-w_t g_m = \frac{i_t}{1+i_t}. \tag{3.11}$$

The left side of this equation is the value of the transactions time saved by holding additional real money balances. At the optimal level of money holdings, this is just equal to the opportunity cost of holding money, $i/(1+i)$.

Since no social cost of producing money has been introduced, optimality would require that the private marginal product of money, g_m, be driven to zero. Equation (3.5) implies that $g_m = 0$ if and only if $i = 0$; we thus obtain the standard result for the optimal rate of inflation as seen earlier in the MIU model.

The chief advantage of the shopping time approach as a means of motivating the presence of money in the utility function is its use in tying the partials of the utility function with respect to money to the specification of the production function relating money, shopping time, and consumption. But this representation of the medium-of-exchange role of money is also clearly a shortcut. The transaction services production function $\psi(m, n^s)$ is simply postulated; this approach does not help to determine what constitutes money. Why, for example, do certain types of green paper facilitate transactions (at least in the United States), while yellow pieces of paper don't? Section 3.5 will review models based on search theory that attempt to derive money demand from a more primitive specification of the transactions process.

3.3 CIA Models

The shopping time model allows time and money to serve as substitutes in carrying out transactions. A somewhat more direct approach, proposed by Clower (1967) and developed formally by Grandmont and Younes (1972) and Lucas (1980a), captures the role of money as a medium of exchange by requiring explicitly that money be used to purchase consumption goods. Such a requirement can also be viewed as replacing the substitution possibilities between time and money highlighted in the shopping time model with a transactions technology in which shopping time is zero if $M/P \geq c$ and infinite otherwise (McCallum 1990a). This specification can be represented by assuming the individual faces, in addition to a standard budget constraint, a CIA constraint.[6]

6. Boianovsky (2002) discusses the early use in the 1960s of a CIA constraint by the Brazilian economist Mario Simonsen.

The exact form of the CIA constraint depends on which transactions or purchases are subject to the CIA requirements. For example, both consumption goods and investment goods might be subject to the requirement. Or only consumption might be subject to the constraint. Or only a subset of all consumption goods may require cash for their purchase. The constraint will also depend on what constitutes cash. Can bank deposits that earn interest, for example, also be used to carry out transactions? As we will see, the exact specification of the transactions subject to the CIA constraint can be important.

Timing assumptions also are important in CIA models. In Lucas (1982), agents are able to allocate their portfolio between cash and other assets at the start of each period, after observing any current shocks but prior to purchasing goods. This timing is often described by saying that the asset market opens first and then the goods market opens. If there is a positive opportunity cost of holding money and the asset market opens first, agents will only hold an amount of money that is just sufficient to finance their desired level of consumption. In Svensson (1985), the goods market opens first. This implies that agents have available for spending only the cash carried over from the previous period, and so cash balances must be chosen before agents know how much spending they will wish to undertake. For example, if uncertainty is resolved *after* money balances are chosen, an agent may find that she is holding cash balances that are too low to finance her desired spending level. Or she may be left with more cash than she needs, thereby forgoing interest income.

To understand the structure of CIA models, the next subsection reviews a simplified version of a model due to Svensson (1985). The simplification involves eliminating uncertainty. Once the basic framework has been reviewed, however, we consider a stochastic CIA model that was developed by Cooley and Hansen (1989) as a means of studying the role of money in a stochastic dynamic general equilibrium model in which business cycles are generated by both real productivity shocks and shocks to the growth rate of money. Developing a linearized version of the Cooley-Hansen model will serve to illustrate how the CIA approach differs from the MIU approach discussed in chapter 2.

3.3.1 The Certainty Case

This section develops a simple CIA model. Issues arising in the presence of uncertainty are postponed until section 3.3.2. The timing of transactions and markets follows Svensson (1985), although the alternative timing used by Lucas (1982) is also discussed. After the model and the agent's decision problem are set out in the next subsection, the steady state is examined and the welfare costs of inflation in a CIA model are discussed.

The Model Consider the following representative agent model. The agent's objective is to choose a path for consumption and asset holdings to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{3.12}$$

for $0 < \beta < 1$, where $u(.)$ is bounded, continuously differentiable, strictly increasing, and strictly concave, and the maximization is subject to a sequence of CIA and budget constraints. The agent enters the period with money holdings M_{t-1} and receives a lump-sum transfer T_t (in nominal terms). If goods markets open first, the CIA constraint takes the form

$$P_t c_t \leq M_{t-1} + T_t,$$

where c is real consumption, P is the aggregate price level, and T is the nominal lump-sum transfer. In real terms,

$$c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} = \frac{m_{t-1}}{\Pi_t} + \tau_t, \tag{3.13}$$

where $m_{t-1} = M_{t-1}/P_{t-1}$, $\Pi_t = P_t/P_{t-1}$ is 1 plus the inflation rate, and $\tau_t = T_t/P_t$. Note the timing: M_{t-1} refers to nominal money balances chosen by the agent in period $t-1$ and carried into period t. The real value of these balances is determined by the period t price level P_t. Since we have assumed away any uncertainty, the agent knows P_t at the time M_{t-1} is chosen. This specification of the CIA constraint assumes that income from production during period t will not be available for consumption purchases during period t.

The budget constraint, in nominal terms, is

$$\begin{aligned} P_t\omega_t \equiv P_t f(k_{t-1}) + (1-\delta)P_t k_{t-1} + M_{t-1} + T_t + I_{t-1}B_{t-1} \\ \geq P_t c_t + P_t k_t + M_t + B_t, \end{aligned} \tag{3.14}$$

where ω_t is the agent's time t real resources, consisting of income generated during period t $f(k_{t-1})$, the undepreciated capital stock $(1-\delta)k_{t-1}$, money holdings, the transfer from the government, and gross nominal interest earnings on the agent's $t-1$ holdings of nominal one-period bonds, B_{t-1}. $I_{t-1} = 1 + i_{t-1}$ is the gross nominal return from period $t-1$ to period t. Physical capital depreciates at the rate δ. These resources are used to purchase consumption, capital, bonds, and nominal money holdings that are then carried into period $t+1$. Dividing through by the time t price level, the budget constraint can be rewritten in real terms as

$$\omega_t \equiv f(k_{t-1}) + (1-\delta)k_{t-1} + \tau_t + \frac{m_{t-1} + I_{t-1}b_{t-1}}{\Pi_t} \geq c_t + m_t + b_t + k_t, \tag{3.15}$$

where m and b are real cash and bond holdings. Note that real resources available to the representative agent in period $t+1$ are given by

$$\omega_{t+1} = f(k_t) + (1-\delta)k_t + \tau_{t+1} + \frac{m_t + I_t b_t}{\Pi_{t+1}}. \tag{3.16}$$

The period t nominal interest factor I_t divided by Π_{t+1} is the gross real rate of return from period t to $t+1$ and can be denoted by $R_t \equiv I_t/\Pi_{t+1}$. With this notation, (3.16) can be written as

$$\omega_{t+1} = f(k_t) + (1-\delta)k_t + \tau_{t+1} + R_t a_t - (i_t/\Pi_{t+1})m_t,$$

where $a_t \equiv m_t + b_t$ is the agent's holding of nominal financial assets (money and bonds). This form highlights that there is a cost to holding money when the nominal interest rate is positive. This cost is i_t/Π_{t+1}; since this is the cost in terms of period $t+1$ real resources, the discounted cost at time t of holding an additional unit of money is $i_t/R_t\Pi_{t+1} = i_t/(1+i_t)$. This is the same expression for the opportunity cost of money obtained in chapter 2 in an MIU model.

Equation (3.13) is based on the timing convention that goods markets open before asset markets. The model of Lucas (1982) assumes the reverse, and individuals can engage in asset transactions at the start of each period before the goods market has opened. In the present model, this would mean that the agent enters period t with financial wealth, which can be used to purchase nominal bonds B_t or carried as cash into the goods market to purchase consumption goods. The CIA constraint would then take the form

$$c_t \leq \frac{m_{t-1}}{\Pi_t} + \tau_t - b_t. \tag{3.17}$$

In this case, the household is able to adjust its portfolio between money and bonds before entering the goods market to purchase consumption goods. To understand the implications of this alternative timing, suppose there is a positive opportunity cost of holding money. Then, if the asset market opens first, the agent will only hold an amount of money that is just sufficient to finance the desired level of consumption. Since the opportunity cost of holding m is positive whenever the nominal interest rate is greater than zero, (3.17) will always hold with equality as long as the nominal rate of interest is positive. When uncertainty is introduced, the CIA constraint may not bind when (3.13) is used and the goods market opens before the asset market. For example, if period t's income is uncertain and is realized after M_{t-1} has been chosen, a bad income realization may cause the agent to reduce consumption to a point where the CIA constraint is no longer binding. Or a disturbance that causes an unexpected price decline might, by increasing the real value of the agent's money holdings, result in a nonbinding constraint.[7] Since we are dealing with a non-stochastic environment in this section, the CIA constraint will bind under either timing assumption if the opportunity cost of holding money is positive. For a complete discussion and comparison of alternative assumptions about the timing of the asset and goods markets, see Salyer (1991). In the remainder of this chapter, we shall follow Svensson (1985) in using (3.13) and assume that consumption in period t is limited by the cash carried over from period $t-1$ plus any net transfer.

The choice variables at time t are c_t, m_t, b_t, and k_t. An individual agent's state at time t can be characterized by her resources ω_t and her real cash holdings m_{t-1}; both are relevant since consumption choice is constrained by the agent's resources and by cash holdings. To analyze the agent's decision problem, we can define the value function

$$V(\omega_t, m_{t-1}) = \max\{u(c_t) + \beta V(\omega_{t+1}, m_t)\}, \tag{3.18}$$

where the maximization is subject to the budget constraint (from 3.15)

$$\omega_t \geq c_t + m_t + b_t + k_t, \tag{3.19}$$

the CIA constraint (3.13), and the definition of ω_{t+1} given by (3.16). Using this expression for ω_{t+1} in (3.18) and letting λ_t (μ_t) denote the Lagrangian multiplier associated with the budget constraint (the CIA constraint), the first order necessary conditions for the agent's choice of consumption, capital, bond, and money holdings take the form[8]

$$u_c(c_t) - \lambda_t - \mu_t = 0 \tag{3.20}$$

$$\beta[f_k(k_t) + 1 - \delta]V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0 \tag{3.21}$$

$$\beta R_t V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0 \tag{3.22}$$

$$\beta\left[R_t - \frac{i_t}{\Pi_{t+1}}\right]V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) - \lambda_t = 0. \tag{3.23}$$

7. While uncertainty may cause the CIA constraint to not bind, it does not follow that the nominal interest rate will be zero. If money is held, the constraint must be binding in some states of nature. The nominal interest rate will equal the discounted expected value of money; see problem 4.

8. The first order necessary conditions also include the transversality conditions.

From the envelope theorem,

$$V_\omega(\omega_t, m_{t-1}) = \lambda_t \tag{3.24}$$

$$V_m(\omega_t, m_{t-1}) = \left(\frac{1}{\Pi_t}\right)\mu_t. \tag{3.25}$$

From (3.24), λ_t is equal to the marginal utility of wealth. According to (3.20), the marginal utility of consumption exceeds the marginal utility of wealth by the value of liquidity services, μ_t. The individual must hold money in order to purchase consumption, so the "cost," to which the marginal utility of consumption is set equal, is the marginal utility of wealth plus the cost of the liquidity services needed to finance the transaction.

In terms of λ, (3.22) becomes

$$\lambda_t = \beta R_t \lambda_{t+1}, \tag{3.26}$$

which is a standard asset pricing equation and is a familiar condition from problems involving intertemporal optimization. Along the optimal path, the marginal cost (in terms of today's utility) from reducing wealth slightly, λ_t, must equal the utility value of carrying that wealth forward one period, earning a gross real return R_t, where tomorrow's utility is discounted back to today at the rate β; that is, $\lambda_t = \beta R_t \lambda_{t+1}$ along the optimal path.

Using (3.24) and (3.25), the first order condition (3.23) can be expressed as

$$\lambda_t = \beta\left(\frac{\lambda_{t+1} + \mu_{t+1}}{\Pi_{t+1}}\right). \tag{3.27}$$

Equation (3.27) can also be interpreted as an asset pricing equation for money. The price of a unit of money in terms of goods is just $1/P_t$ at time t; its value in utility terms is λ_t/P_t. Now by dividing (3.27) through by P_t, it can be rewritten as $\lambda_t/P_t = \beta(\lambda_{t+1}/P_{t+1} + \mu_{t+1}/P_{t+1})$. Solving this equation forward[9] implies that

$$\frac{\lambda_t}{P_t} = \sum_{i=1}^{\infty} \beta^i \left(\frac{\mu_{t+i}}{P_{t+i}}\right). \tag{3.28}$$

From (3.25), μ_{t+i}/P_{t+i} is equal to $V_m(\omega_{t+i}, m_{t+i-1})/P_{t+i-1}$. This last expression, though, is just the partial of the value function with respect to time $t+i-1$ *nominal*

9. For references on solving difference equations forward in the context of rational expectations models, see Blanchard and Kahn (1980) or McCallum (1989).

money balances:

$$\frac{\partial V(\omega_{t+i}, m_{t+i-1})}{\partial M_{t+i-1}} = V_m(\omega_{t+i}, m_{t+i-1})\left(\frac{\partial m_{t+i-1}}{\partial M_{t+i-1}}\right)$$
$$= \frac{V_m(\omega_{t+i}, m_{t+i-1})}{P_{t+i-1}}$$
$$= \left(\frac{\mu_{t+i}}{P_{t+i}}\right).$$

This means we can rewrite (3.28) as

$$\frac{\lambda_t}{P_t} = \sum_{i=1}^{\infty} \beta^i \frac{\partial V(\omega_{t+i}, m_{t+i-1})}{\partial M_{t+i-1}}.$$

In other words, the current value of money in terms of utility is equal to the present value of the marginal utility of money in all future periods. Equation (3.28) is an interesting result; it says that money is just like any other asset in the sense that we can think of its value (i.e., its price today) as equal to the present discounted value of the stream of returns generated by the asset. In the case of money, these returns take the form of liquidity services. If the CIA constraint were not binding, these liquidity services would not have value ($\mu = V_m = 0$) and neither would money. But if the constraint is binding, then money has value because it yields valued liquidity services.[10]

The result that the value of money, λ/P, satisfies an asset pricing relationship is not unique to the CIA approach. For example, a similar relationship is implied by the MIU approach. The model employed in our analysis of the MIU approach (see the appendix to chapter 2), implied that

$$\frac{\lambda_t}{P_t} = \beta\left(\frac{\lambda_{t+1}}{P_{t+1}}\right) + \frac{u_m(c_t, m_t)}{P_t},$$

which can be solved forward to yield

$$\frac{\lambda_t}{P_t} = \sum_{i=0}^{\infty} \beta^i \left[\frac{u_m(c_{t+i}, m_{t+i})}{P_{t+i}}\right].$$

10. Bohn (1991b) analyzes the asset pricing implications of a CIA model. See also Salyer (1991).

Here, the marginal utility of money u_m plays a role exactly analogous to that played by the Lagrangian on the CIA constraint μ. The one difference is that in the MIU approach, m_t yields utility at time t, while in the CIA approach, the value of money accumulated at time t is measured by μ_{t+1} since the cash cannot be used to purchase consumption goods until period $t+1$.[11]

An expression for the nominal rate of interest can be obtained by using (3.26) and (3.27) to obtain $\lambda_t = \beta R_t \lambda_{t+1} = \beta(\lambda_{t+1} + \mu_{t+1})/\Pi_{t+1}$, or $R_t \Pi_{t+1} \lambda_{t+1} = (\lambda_{t+1} + \mu_{t+1})$. Since $I_t = 1 + i_t = R_t \Pi_{t+1}$, the nominal interest rate is given by

$$i_t = \left(\frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_{t+1}}\right) - 1 = \frac{\mu_{t+1}}{\lambda_{t+1}}. \tag{3.29}$$

Thus, the nominal rate of interest is positive if and only if money yields liquidity services ($\mu_{t+1} > 0$). In particular, if the nominal interest rate is positive, the CIA constraint is binding ($\mu > 0$).

We can use the relationship between the nominal rate of interest and the Lagrangian multipliers to rewrite the expression for the marginal utility of consumption, given in (3.20), as $u_c = \lambda(1 + \mu/\lambda) = \lambda(1 + i) \geq \lambda$. Since λ represents the marginal value of income, the marginal utility of consumption exceeds that of income whenever the nominal interest rate is positive. Even though the economy's technology allows output to be directly transformed into consumption, the "price" of consumption is not equal to 1; it is $1 + i$ since the household must hold money to finance consumption. Thus, in this CIA model, a positive nominal interest rate acts as a tax on consumption; it raises the price of consumption above its production cost.[12]

The CIA constraint holds with equality when the nominal rate of interest is positive, so $c_t = M_{t-1}/P_t + \tau_t$. Since the lump-sum monetary transfer τ_t is equal to $(M_t - M_{t-1})/P_t$, this implies that $c_t = M_t/P_t = m_t$. Consequently, the consumption velocity of money is identically equal to 1 (velocity $= P_t c_t / M_t = 1$). Since actual velocity varies over time, CIA models have been modified in ways that break this tight link between c and m. One way to avoid this is to introduce uncertainty (see

11. Carlstrom and Fuerst (2001) argue that utility at time t should depend on money balances available for spending during period t, or M_{t-1}/P_t. This would make the timing more consistent with CIA models. With this timing, m_t is chosen at time t but yields utility at $t+1$. In this case, $\lambda_t/P_t = \sum_{i=1}^{\infty} \beta^i [u_m(c_{t+i}, m_{t+i})/P_{t+i}]$, and the timing is the same as in the CIA model.

12. In the shopping time model, the tax on consumption was related to the time cost of shopping. Recalling that money reduced shopping time ($g_m \leq 0$); (3.9) and (3.11) imply that

$$u_c = V_a\left[1 - \left(\frac{i_t}{1+i_t}\right)\left(\frac{g_c}{g_m}\right)\right] \geq V_a.$$

Svensson 1985). If money balances have to be chosen prior to the resolution of uncertainty, it may turn out that the desired level of consumption is less than the amount of real money balances being held. In this case, some money balances will be unspent, and velocity can be less than 1. Velocity may also vary if the CIA constraint only applies to a subset of consumption goods. Then variations in the rate of inflation can lead to substitution between goods whose purchase requires cash and those whose purchase does not.

The Steady State If we restrict ourselves to considering the steady state, (3.26) implies that $R = 1/\beta$, and $i = \Pi/\beta - 1 \approx 1/\beta - 1 + \pi$. Thus, the Fisher relationship, in which the nominal interest rate moves one for one with variations in the rate of inflation, holds. In addition, (3.21) gives the steady-state capital stock as the solution to

$$f_k(k^{ss}) = R - 1 + \delta = \frac{1}{\beta} - 1 + \delta.$$

So this CIA model, like the Sidrauski MIU model, exhibits superneutrality. The steady-state capital stock depends only on the time preference parameter β, the rate of depreciation δ, and the production function. It is independent of the rate of inflation. Since steady-state consumption is equal to $f(k^{ss}) - \delta k^{ss}$, it too is independent of the rate of inflation.[13]

We have seen already that the marginal utility of consumption could be written as the marginal utility of wealth (λ) times 1 plus the nominal rate of interest, reflecting the opportunity cost of holding the money required to purchase goods for consumption. Using (3.29), the ratio of the liquidity value of money, measured by the Lagrangian multiplier μ, to the marginal utility of consumption is

$$\frac{\mu}{u_c} = \frac{\mu}{\lambda(1+i)} = \frac{i}{1+i}.$$

This expression is exactly parallel to our result in the MIU framework, where the ratio of the marginal utility of money to the marginal utility of consumption was equal to the nominal interest rate divided by 1 plus the nominal rate, that is, the relative price of money in terms of consumption.

13. The expression for steady-state consumption can be obtained from (3.15) by noting that $m_t = \tau_t + m_{t-1}/\Pi_t$ and, with all households identical, $b = 0$ in equilibrium. Then (3.15) reduces to

$$c^{ss} + k^{ss} = f(k^{ss}) + (1-\delta)k^{ss}$$

or $c^{ss} = f(k^{ss}) - \delta k^{ss}$.

With the CIA constraint binding, real consumption is equal to real money balances. In the steady state, constant consumption implies that the stock of nominal money balances and the price level must be changing at the same rate. Define θ as the growth rate of the nominal quantity of money (so that $T_t = \theta M_{t-1}$); then

$$\pi^{ss} = \theta^{ss}.$$

The steady-state inflation rate is, as usual, determined by the rate of growth of the nominal money stock.

The Welfare Costs of Inflation The CIA model, because it is based explicitly on behavioral relationships consistent with utility maximization, can be used to assess the welfare costs of inflation and to determine the optimal rate of inflation. The MIU approach of chapter 2 had very strong implications for the optimal inflation rate. Steady-state utility of the representative household was maximized when the nominal rate of interest equaled zero. We have already suggested that this conclusion continues to hold when money produces transaction services.

In the basic CIA model, however, there is no optimal rate of inflation that maximizes the steady-state welfare of the representative household. The reason follows directly from the specification of utility as a function only of consumption and the result that consumption is independent of the rate of inflation (superneutrality). Steady-state welfare is equal to

$$\sum_{t=0}^{\infty} \beta^t u(c^{ss}) = \frac{u(c^{ss})}{1-\beta}$$

and is invariant to the inflation rate. Comparing across steady states, any inflation rate is as good as any other.[14]

This finding is not robust to modifications in the basic CIA model. In particular, once we extend the model to incorporate a labor-leisure choice, consumption will no longer be independent of the inflation rate and there will be a well-defined optimal rate of inflation. Because leisure can be "purchased" without the use of money (i.e., leisure is not subject to the CIA constraint), variations in the rate of inflation will affect the marginal rate of substitution between consumption and leisure (see section 3.3.2). With different inflation rates leading to different levels of steady-state consumption and leisure, steady-state utility will be a function of inflation. This type of

14. By contrast, the optimal rate of inflation was well defined even in the basic Sidrauski model that exhibited superneutrality, since real money balances vary with inflation and directly affect utility in an MIU model.

substitution plays an important role in the model of Cooley and Hansen (1989), which will be discussed in the next section; in their model, inflation leads to an increased demand for leisure and a reduction in labor supply. But before including a labor-leisure choice, we will briefly review some other modifications of the basic CIA model, modifications that will, in general, generate a unique optimal rate of inflation.

CASH AND CREDIT GOODS Lucas and Stokey (1983, 1987) introduced the idea that the CIA constraint may only apply to a subset of consumption goods. They modeled this by assuming that the representative agent's utility function is defined over consumption of two types of goods: "cash" goods and "credit" goods. In this case, paralleling (3.20), the marginal utility of cash goods will be equated to $\lambda + \mu \geq \lambda$, while the marginal utility of credit goods will be equated to λ. Hence, the CIA requirement for cash goods drives a wedge between the marginal utilities of the two types of goods. It is exactly as if the consumer faces a tax of $\mu/\lambda = i$ on purchases of the cash good. Higher inflation, by reducing holdings of real cash balances, serves to raise the tax on cash goods and generates a substitution away from the cash good and toward the credit good. (See also Hartley 1988.)

The obvious difficulty with this approach is that the classifications of goods into cash and credit goods is exogenous. And it is common to assume a one-good technology so that the goods are not differentiated by any technological considerations. The advantage of these models is that they can produce time variation in velocity. Recall that in the basic CIA model, any equilibrium with a positive nominal rate of interest is characterized by a binding CIA constraint, and this means that $c = m$. With both cash and credit goods, m will equal the consumption of cash goods, allowing the ratio of total consumption to money holdings to vary with expected inflation.[15]

CIA AND INVESTMENT GOODS A second modification to the basic model involves extending the CIA constraint to cover investment goods. In this case, the inflation tax applies to both consumption and investment goods. Higher rates of inflation will tend to discourage capital accumulation, and Stockman (1981) showed that higher inflation would lower the steady-state capital-labor ratio (see problem 5 at the end of the chapter).[16]

15. Woodford (1998a) studies a model with a continuum of goods indexed by $i \in [0, 1]$. A fraction s, $0 \leq s \leq 1$, are cash goods. He then approximates a cashless economy by letting $s \to 0$.

16. Abel (1985) studies the dynamics of adjustment in a model in which the CIA constraint applies to both consumption and investment.

IMPLICATIONS FOR OPTIMAL INFLATION In CIA models, inflation acts as a tax on goods or activities whose purchase requires cash. This tax then introduces a distortion by creating a wedge between the marginal rates of transformation implied by the economy's technology and the marginal rates of substitution faced by consumers. Since the CIA model, like the MIU model of the previous chapter, offers no reason for such a distortion to be introduced (there is no inefficiency that calls for Pigovian taxes or subsidies on particular activities, and the government's revenue needs can be met through lump-sum taxation), optimality calls for setting the inflation tax equal to zero. The inflation tax is directly related to the nominal rate of interest; a zero inflation tax is achieved when the nominal rate of interest is equal to zero.

3.3.2 A Stochastic CIA Model

While the models of Lucas (1982), Svensson (1985), and Lucas and Stokey (1987) provide theoretical frameworks for assessing the role of inflation on asset prices and interest rates, they do not provide any guide to the empirical magnitude of inflation effects or to the welfare costs of inflation. What one would like is a dynamic equilibrium model that one could simulate under alternative monetary policies—for example, for alternative steady-state rates of inflation—in order to assess quantitatively the effects of inflation. Such an exercise is conducted by Cooley and Hansen (1989, 1991).

Cooley and Hansen follow the basic framework of Lucas and Stokey (1987). However, important aspects of their specification include 1) the introduction of capital and, consequently, an investment decision; 2) the introduction of a labor-leisure choice; and 3) the identification of consumption as the cash good and investment and leisure as credit goods.

Inflation represents a tax on the purchases of the cash good, and therefore higher rates of inflation shift household demand away from the cash good and toward the credit good. In Cooley and Hansen's formulation, this implies that higher inflation increases the demand for leisure. One effect of inflation, then, is to reduce the supply of labor. This then reduces output, consumption, investment, and the steady-state capital stock.

Cooley and Hansen express welfare losses across steady states in terms of the consumption increase (as a percentage of output) required to yield the same utility as would arise if the CIA constraint were nonbinding.[17] For a 10% inflation rate, they

17. Refer to Cooley and Hansen (1989, section II) or Hansen and Prescott (chapter 2 in Cooley 1995) for discussions of the computational aspects of this exercise.

report a welfare cost of inflation of 0.387% of output if the CIA constraint is assumed to apply at a quarterly time interval. Not surprisingly, if the constraint binds only at a monthly time interval, the cost falls to 0.112% of output. These costs are small. For much higher rates of inflation, they start to look significant. For example, with a monthly time period for the CIA constraint, a 400% annual rate of inflation generates a welfare loss equal to 2.137% of output. The welfare costs of inflation are discussed further in chapter 4.

The Basic Model To model the behavior of the representative agent faced with uncertainty and a CIA constraint, assume that the agent's objective is to maximize

$$\mathrm{E}_0 \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - n_{t+i}) = \mathrm{E}_0 \sum_{i=0}^{\infty} \beta^i \left[\frac{c_{t+i}^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_{t+i})^{1-\eta}}{1-\eta} \right], \tag{3.30}$$

with $0 < \beta < 1$. Here c_t is real consumption, while n_t is labor supplied to market activities, expressed as a fraction of the total time available, so that $1 - n_t$ is equal to leisure time.[18] The parameters Φ, Ψ, and η are restricted to be positive.

Households supply labor and rent capital to firms which produce goods. The household enters each period with nominal money balances M_{t-1} and receives a nominal lump-sum transfer equal to T_t.[19] In the aggregate, this transfer is related to the growth rate of the nominal supply of money. Letting the stochastic variable θ_t denote the rate of money growth ($M_t = (1 + \theta_t)M_{t-1}$), the per capita transfer will equal $\theta_t M_{t-1}$. At the start of period t, θ_t is known to all households.

The CIA constraint is taken to apply only to the purchase of consumption goods;

$$P_t c_t \leq M_{t-1} + T_t,$$

where P_t is the time t price level. Note that time t transfers are available to be spent in period t. In real terms, the CIA constraint becomes

$$c_t \leq \frac{m_{t-1}}{\Pi_t} + \tau_t. \tag{3.31}$$

18. In order to allow for comparison between the MIU model developed earlier and a CIA model, I have modified the preference function used earlier, (2.38) of chapter 2, by setting $a = 1$ and $b = 0$ so that real balances do not yield direct utility. The resulting utility function given in (3.30) differs from Cooley and Hansen's specification; they assume that the preferences of the identical (ex ante) households are log separable in consumption and leisure, a case obtained when $\Phi = \eta = 1$.

19. Cooley and Hansen distinguish between individual specific decision variables and the corresponding aggregate per capita quantity, although in equilibrium the two will be equal. In order to simplify the notation, we will only call attention to this distinction when necessary to avoid confusion.

Here $\Pi_t = P_t/P_{t-1} = 1 + \pi_t$ is equal to 1 plus the rate of inflation. Following Cooley and Hansen, we will only consider equilibria in which (3.31) holds with equality.[20]

In addition to the CIA constraint, the household faces a flow budget constraint of the form

$$y_t + (1-\delta)k_{t-1} + \left(\frac{1+i_{t-1}}{1+\pi_t}\right)b_{t-1} + \frac{m_{t-1}}{\Pi_t} + \tau_t \geq c_t + k_t + b_t + m_t, \tag{3.32}$$

where $0 \leq \delta \leq 1$ is the depreciation rate.

The individual's decision problem is characterized by the value function

$$V(k_{t-1}, b_{t-1}, m_{t-1}) = \max \left[\frac{c_t^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta} + \beta \mathrm{E}_t V(k_t, b_t, m_t) \right],$$

where the maximization is with respect to $\{c_t, n_t, k_t, b_t$, and $m_t\}$ and is subject to the constraints (3.32) and (3.31).

The economy's technology is given by a Cobb-Douglas constant returns to scale production function, expressed in per capita terms as

$$y_t = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}, \tag{3.33}$$

where $0 \leq \alpha \leq 1$. The exogenous productivity shock z_t follows

$$z_t = \rho z_{t-1} + e_t$$

with $0 \leq \rho \leq 1$. The innovation e_t has mean zero and variance σ_e^2.

The final aspect of the model is a specification for the behavior of the growth rate of the nominal money supply θ_t. Adopting the process used in chapter 2, let u_t be equal to the deviation of money growth around the steady state: $u_t = \theta_t - \theta^{ss}$. Assume

$$u_t = \gamma u_{t-1} + \phi z_{t-1} + \varphi_t,$$

where φ_t is a white noise innovation with variance σ_φ^2.

The first order conditions for the representative agent's decision problem are presented in the chapter appendix (see section 3.6).

20. If agents choose their money holdings after observing current shocks but before going to the goods market to purchase consumption goods, as in Lucas (1982), the CIA constraint will always be binding if the nominal interest rate is positive. In problem 9, you are asked to find the equilibrium conditions for a CIA model with this alternative timing and to determine if the equilibrium conditions are identical to those in the model based on Cooley and Hansen's specification.

The Steady State The steady-state values of the ratios that were reported in table 2.1 of chapter 2 for the MIU model are also the steady-state values for the CIA model, the one exception being that for real money balances. In the steady state, the CIA constraint is binding as long as the nominal rate of interest is positive. Hence, $c^{ss} = m^{ss}/\Pi^{ss} + \tau^{ss} = m^{ss}$, so $m^{ss}/k^{ss} = c^{ss}/k^{ss}$. The other values reported in table 2.1 hold for the CIA model; even though the method used to generate a demand for money has changed, the steady-state values of the output-capital, capital-labor, and consumption-labor ratios are unchanged. Note that none of these steady-state ratios depend on the growth rate of the nominal money supply.

What will depend on the money growth rate, and therefore on the rate of inflation, will be the steady-state value of labor supply. The appendix shows that n^{ss} satisfies

$$(1-n^{ss})^{-\eta}(n^{ss})^{\Phi} = \frac{(1-\alpha)}{\Psi}\left(\frac{\beta}{\Theta}\right)\left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{\Phi-\alpha}{1-\alpha}}\left(\frac{c^{ss}}{k^{ss}}\right)^{-\Phi}, \tag{3.34}$$

where $\Theta = 1+\theta$. Since the left side of this expression is increasing in n^{ss}, a rise in Θ, which implies a rise in the inflation rate, lowers the steady-state labor supply. This is the source of the welfare cost of inflation in this CIA model. The elasticity of labor supply with respect to the growth rate of money is negative.

It is useful to note the similarity between the expression for steady-state labor supply in the CIA model and the corresponding expression given in (2.39) in chapter 2 that was obtained in the MIU model. With the MIU specification, faster money growth had an ambiguous effect on the supply of labor. With the calibrated values of the parameters of the utility function used in chapter 2, money and consumption were complements, so higher inflation, by reducing real money holdings, lowered the marginal utility of consumption and also reduced the supply of labor.

Calibration In order to assess the effects of money in this CIA model, we need to assign values to the specific parameters; that is, we need to calibrate the model. The steady state depends on the values of $\alpha, \beta, \delta, \eta, \Psi$, and Φ. The baseline value reported in table 2.2 for the MIU model will be employed for the CIA model as well. This implies that $\alpha = .36$, $\beta = .989$, and $\delta = .019$. Assuming $\eta = 1$ implies that the utility is log linear in leisure. The value of Ψ is then determined so that the steady-state value of N is .31. For the baseline parameters, this yields $\Psi = 1.34$. To maintain comparability with the MIU model, the utility function parameter Φ will be set equal to 2 for the baseline solutions.

Dynamics The dynamic implications of the CIA model can be explored by obtaining a log-linear approximation around the steady state. The derivation of the approximation is contained in the appendix. It is convenient to use the fact that $c_t = m_t$ in equilibrium to eliminate consumption from the system and to include directly the marginal utility of wealth, denoted by $\hat{\lambda}_t$, instead (see the appendix). As in chapter 2, a variable $\hat{x}$ denotes the percentage deviation of x around the steady state.[21] The CIA model can then be approximated around the steady state by the following eight equations:

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + z_t \tag{3.35}$$

$$\left(\frac{y^{ss}}{k^{ss}}\right)\hat{y}_t = \left(\frac{c^{ss}}{k^{ss}}\right)\hat{m}_t + \hat{k}_t - (1-\delta)\hat{k}_{t-1} \tag{3.36}$$

$$\hat{r}_t = \alpha\left(\frac{y^{ss}}{k^{ss}}\right)(\mathrm{E}_t\hat{y}_{t+1} - \hat{k}_t) \tag{3.37}$$

$$\hat{\lambda}_t = \mathrm{E}_t\hat{\lambda}_{t+1} + \hat{r}_t \tag{3.38}$$

$$\hat{y}_t + \hat{\lambda}_t = \left(1 + \eta\frac{n^{ss}}{1-n^{ss}}\right)\hat{n}_t \tag{3.39}$$

$$\hat{\imath}_t = \hat{r}_t + \mathrm{E}_t\hat{\pi}_{t+1} \tag{3.40}$$

$$\hat{\lambda}_t = -\mathrm{E}_t[\Phi\hat{m}_{t+1} + \hat{\pi}_{t+1}] \tag{3.41}$$

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + u_t. \tag{3.42}$$

Note that the first six equations (the production function, the resource constraint, the marginal product of capital equation, the Euler condition [once it is noted that in the MIU model the marginal utility of wealth was equal to $-\Omega_1\hat{c}_t + \Omega_2\hat{m}_t$], the labor-leisure condition, and the Fisher equation) are identical to those found with the MIU approach except that the CIA constraint allows $\hat{m}$ to replace $\hat{c}$ in the resource constraint (3.36). The critical differences between the two approaches appear in a comparison of (3.41) with (2.46) of chapter 2. In the MIU model, utility depended directly on m, so (2.46) is the money demand condition derived from the first order condition for the household's holdings of real money balances. In the CIA model, (3.41) reflects the presence of the nominal interest rate as a tax on consumption in the CIA model. Equation (3.60) of the appendix shows that $\lambda_t = \beta\mathrm{E}_t(\lambda_{t+1} + \mu_{t+1})/\Pi_{t+1}$.[22] Since the marginal utility of consumption is equated to $\lambda + \mu$, this becomes $\lambda_t = \beta\mathrm{E}_t c_{t+1}^{-\Phi}/\Pi_{t+1} = \beta\mathrm{E}_t m_{t+1}^{-\Phi}/\Pi_{t+1}$. Linearizing this equation produces (3.41).

21. The exceptions again being that $\hat{r}$ and $\hat{\imath}$ are expressed in percentage terms (e.g., $\hat{r}_t = R_t - R^{ss}$).

22. Equation (3.27) is the corresponding equation for the nonstochastic CIA model of section 3.3.1.

Table 3.1
Baseline Parameters

	s.d.	s.d. rel. to y	Corr. with y
y	1.033	1.000	1.00
n	0.221	0.213	0.86
c	0.385	0.372	0.97
x	3.286	3.182	1.00
r	0.014	0.014	0.85
m	0.385	0.372	0.97
i	0.006	0.006	0.83
π	0.902	0.873	−0.14

Recall that the MIU displayed short-run dynamics in which the real variables such as output, consumption, the capital stock, and employment were independent of the nominal money supply process when utility was log linear in consumption and money balances.[23] Although $\hat{m}$ does not directly enter the utility function in the CIA model, note that in the case of log utility in consumption (that is, when $\Phi = 1$), the short-run real dynamics in the CIA model are not independent of the process followed by $\hat{m}$, as they were in the MIU model. We can see this by noting that (3.39), (3.41), and (3.42) imply, when $\Phi = 1$, that

$$\hat{\lambda}_t = -\mathrm{E}_t(\hat{m}_{t+1} + \hat{\pi}_{t+1}) = -(\hat{m}_t + \mathrm{E}_t u_{t+1}) = \left(1 + \eta \frac{n^{ss}}{1 - n^{ss}}\right)\hat{n}_t - \hat{y}_t.$$

Thus, variations in the expected future growth rate of money, $\mathrm{E}_t u_{t+1}$, force adjustment to either $\hat{y}$, $\hat{c}$ ($\hat{m}$), or $\hat{n}$ (or all three). In particular, for given output and consumption, higher expected money growth (and therefore higher expected inflation) produces a fall in $\hat{n}_t$. This is the effect discussed above by which higher inflation reduces labor supply and output.

The current growth rate of the nominal money stock, u_t, and the current rate of inflation, π_t, only appear in the form $u_t - \pi_t$ (see 3.42). Hence, as we saw in the MIU model, *unanticipated* monetary shocks affect only current inflation and have no real effects unless they affect expectations of future money growth (i.e., unless $\mathrm{E}_t u_{t+1}$ is affected).

Table 3.1, which should be compared with table 2.4, shows the contemporaneous correlations implied by the CIA model for the benchmark parameter values. The

23. This was the case in which $\Phi = b = 1$.

Table 3.2
Effects of the Money Process ($\gamma = 0.5$)

	$\phi = 0$			$\phi = 0.15$			$\phi = -0.15$		
	s.d.	s.d. rel. to y	Corr. with y	s.d.	s.d. rel. to y	Corr. with y	s.d.	s.d. rel. to y	Corr. with y
y	1.033	1.000	1.00	0.982	1.000	1.00	1.084	1.000	1.00
n	0.223	0.215	0.86	0.166	0.169	0.69	0.289	0.266	0.92
c	0.461	0.447	0.82	0.395	0.403	0.81	0.508	0.469	0.86
x	3.388	3.271	0.96	3.338	3.399	0.97	3.394	3.131	0.97
r	0.014	0.014	0.85	0.013	0.014	0.81	0.015	0.014	0.88
m	0.461	0.447	0.82	0.395	0.403	0.81	0.508	0.469	0.86
i	0.265	0.257	0.00	0.337	0.343	0.67	0.330	0.304	−0.68
π	1.018	0.986	−0.14	0.971	0.989	0.02	0.933	0.861	−0.32

effects of altering the money growth rate process are illustrated in table 3.2. Comparing tables 2.5 and 3.2 reveals that the money growth rate process has much larger real effects in the CIA framework. In particular, the variance of output declines by 4% when ϕ is increased from 0 to 0.15. When money growth is positively serially correlated ($\gamma = .5$), a positive money growth rate disturbance ($u_t > 0$) reduces employment, and the effect of the u disturbance is significantly larger than was found with the MIU model.

Table 3.2 shows how variability depends on the response of money growth to productivity shocks. The economy's response to a productivity shock is decreasing in ϕ. For example, when ϕ is negative, a positive productivity shock implies that money growth will decline in the future. Consequently, expected inflation also declines. The resulting reduction in the nominal interest rate lowers the effective inflation tax on consumption and increases labor supply. In contrast, when ϕ is positive, a positive productivity shock increases expected inflation and reduces labor supply. This tends to partially offset the effect of the productivity shock on output. Thus, output variability is less when ϕ is positive than when it is zero or negative.

The behavior of output and labor supply in response to a positive shock to the growth rate of the nominal money supply is shown in figure 3.1 for $\gamma = 0.5$ (the baseline value) and $\gamma = 0.8$. The response of the nominal interest rate is shown in figure 3.2. Greater persistence of the money growth rate process leads to larger movements in expected inflation in response to a monetary shock. This, in turn, produces larger adjustments of labor supply and output. As illustrated in figure 3.1, a positive shock, by raising the expected rate of inflation and thereby increasing the inflation tax on consumption, induces a substitution toward leisure that lowers labor

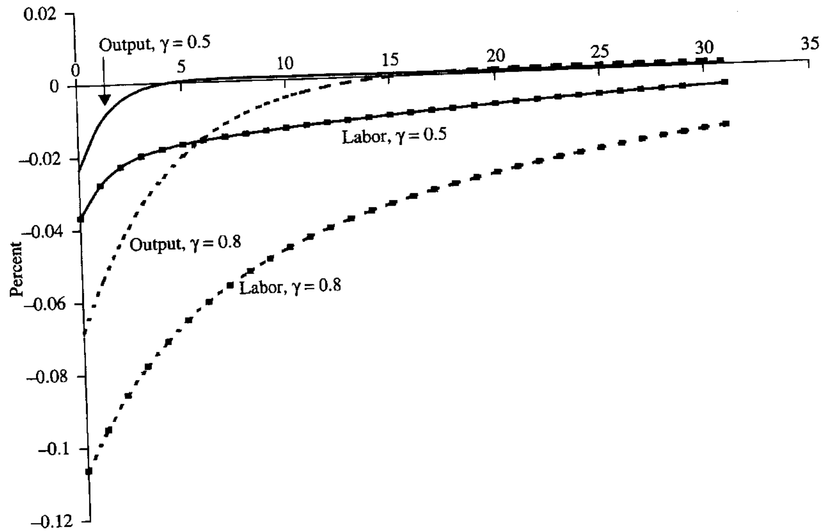

Figure 3.1
Output and Labor Response to a Money Growth Shock

supply. Comparing this figure with figure 2.2 reveals that a money growth shock has a much larger real impact in the CIA model than in the MIU model of chapter 2. As was also the case with the MIU model, a positive money growth shock, by raising expected inflation when $\gamma > 0$, raises the nominal rate of interest.

3.4 Other Approaches

3.4.1 Real Resource Costs

An alternative approach to the CIA or shopping time models is to assume that transaction costs take the form of real resources that are used up in the process of exchange (Brock 1974, 1990). An increase in the volume of goods exchanged leads to a rise in transaction costs, while higher average real money balances, for a given volume of transactions, lower costs. In a shopping time model, these costs are time costs and so enter the utility function indirectly by affecting the time available for leisure.

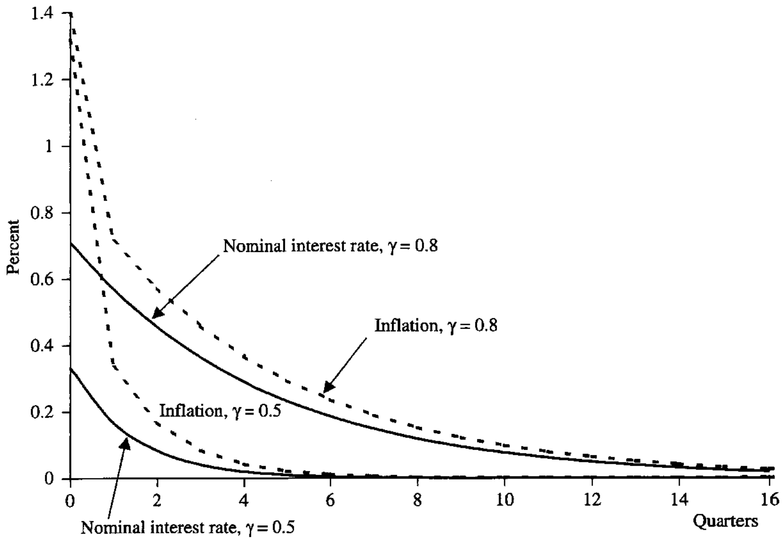

Figure 3.2
Nominal Interest Rate and Inflation Response to a Money Growth Shock (solid lines, nominal interest rate response; dashed lines, inflation response)

If goods must be used up in transacting, the household's budget constraint must be modified, for example by adding a transaction costs term $\Upsilon(c, m)$ that depends on the volume of transactions (represented by c) and the level of money holdings. The budget constraint (3.15) then becomes

$$f(k_{t-1}) + (1-\delta)k_{t-1} + \tau_t + R_{t-1}b_{t-1} + \frac{m_{t-1}}{\Pi_t} \geq c_t + m_t + b_t + k_t + \Upsilon(c_t, m_t).$$

Feenstra (1986) considers a variety of transaction costs formulations to show that they all lead to the presence of a function involving c and m appearing on the right side of the budget constraint. He also shows that transaction costs satisfy the following condition for all $c, m \geq 0$: Υ is twice continuously differentiable and $\Upsilon \geq 0$; $\Upsilon(0, m) = 0$; $\Upsilon_c \geq 0$; $\Upsilon_m \leq 0$; $\Upsilon_{cc}, \Upsilon_{mm} \geq 0$; $\Upsilon_{cm} \leq 0$; and $c + \Upsilon(c, m)$ is quasi-convex, with expansion paths having a nonnegative slope. These conditions all have intuitive meaning: $\Upsilon(0, m) = 0$ means that the consumer bears no transaction costs if consumption is zero. The sign restrictions on the partial derivatives reflect the assumptions that transaction costs rise at an increasing rate as consumption increases and that money has positive but diminishing marginal productivity in reducing transaction costs. The assumption that $\Upsilon_{cm} \leq 0$ means that the marginal transaction costs of additional consumption do not increase with money holdings. Expansion paths with nonnegative slopes imply that $c + \Upsilon$ increases with income. Positive money holdings can be ensured by the additional assumption that $\lim_{m \to 0} \Upsilon_m(c,m) = -\infty$; that is, money is essential.

Now consider how the MIU approach compares to a transactions-cost approach. Suppose we define a function $W(x,m)$ with the following properties: for all $x, m \geq 0$, W is twice continuously differentiable and satisfies $W \geq 0$; $W(0,m) = 0$; $W(x,m) \to \infty$ as $x \to \infty$ for fixed m; $W_m \geq 0$; $0 \leq W_x \leq 1$; $W_{xx} \leq 0$; $W_{mm} \leq 0$; $W_{xm} \geq 0$; W is quasi-concave with Engel curves with a nonnegative slope.

Now simplify by dropping capital and consider the following two static problems representing simple transactions-cost and MIU approaches:

$$\max U(c) \text{ subject to } c + \Upsilon(c,m) + b + m = y \tag{3.43}$$

and

$$\max V(x,m) \text{ subject to } x + b + m = y, \tag{3.44}$$

where $V(x,m) = U[W(x,m)]$. These two problems are equivalent if (c^*, b^*, m^*) solves (3.43) if and only if (x^*, b^*, m^*) solves (3.44) with $x^* = c^* + \Upsilon(c^*, m^*)$. Feenstra shows that equivalence holds if the functions $\Upsilon(c,m)$ and $W(x,m)$ satisfy the stated conditions.

This "functional equivalence" (Wang and Yip 1992) between the transactions cost and MIU approaches suggests that conclusions derived within one framework will also hold under the alternative approach. However, this equivalence is obtained by redefining variables. So, for example, the "consumption" variable x in the utility function is equal to consumption *inclusive* of transactions costs (i.e., $x = c + \Upsilon(c,m)$) and is therefore not independent of money holdings. At the very least, the appropriate definition of the consumption variable needs to be considered if one attempts to use either framework to draw implications for actual macro time series.[24]

3.4.2 Search

Both the MIU and CIA approaches are useful alternatives for introducing money into a general equilibrium framework. However, neither approach is very specific

24. When distortionary taxes are introduced, Mulligan and Sala-i-Martin (1997) show that the functional equivalence between the two approaches can depend on whether money is required to pay taxes.

about the exact role played by money. MIU models assume the direct utility yielded by money proxies for the services money produces in facilitating transactions. However, the nature of these transactions and, more importantly, the resource costs they might involve and how these costs might be reduced by holding money are not specified. Use of the CIA model is motivated by appealing to the idea that some form of nominal asset is required to facilitate transactions. Yet the constraint used is extreme, implying that there are no alternative means of carrying out certain transactions. The CIA constraint is meant to capture the role of money as a medium of exchange, but in this case we might wish to start from a specification of the transactions technology to understand why some commodities and assets serve as money and others do not.

A number of papers have employed search theory to motivate the development of media of exchange (examples include Jones 1976; Diamond 1983; Kiyotaki and Wright 1989, 1993; Oh 1989; Trejos and Wright 1993, 1995; Ritter 1995; Rupert, Schindler, and Wright 2001; and Shi 1995). In these models, individual agents must exchange the goods they produce (or with which they are endowed) for the goods they can consume. During each period, individuals randomly meet other agents; exchange takes place if it is mutually beneficial. In a barter economy, exchange is possible only if an agent holding good i and wishing to consume good j (call this an ij agent) meets an individual holding good j who wishes to consume good i (a ji agent). This requirement is known as the *double coincident of wants* and limits the feasibility of direct barter exchange when production is highly specialized. Trade could occur if agent ij meets a ki agent for $k \neq j$ as long as exchange of goods is costless and the probability of meeting a jk agent is the same as meeting a ji agent. In this case, agent ij would be willing to exchange i for k (thereby becoming a kj agent).

In the basic Kiyotaki-Wright model, direct exchange of commodities is assumed to be costly, but there exists a fiat money that can be traded costlessly for commodities. The assumption that there exists money with certain exchange properties (costless trade with commodities) serves a role similar to that of putting money directly into the utility function in the MIU approach or specifying that money must be used in certain types of transactions in the CIA approach.[25] Whether an agent will accept money in exchange for goods will depend on the probability the agent places on being able later to exchange money for a consumption good.

25. In an early analysis, Alchian (1977) attempted to explain why there might exist a commodity with the types of exchange properties assumed in the new search literature. He stressed the role of information and the costs of assessing quality. Any commodity whose quality can be assessed at low cost can facilitate the acquisition of information about other goods by serving as a medium of exchange.

Suppose agents are endowed with a new good according to a Poisson process with arrival rate a.[26] Trading opportunities arrive at rate b. A successful trade can occur if there is a double coincidence of wants. If x is the probability that another agent chosen at random is willing to accept the trader's commodity, the probability of a double coincidence of wants is x^2. A successful trade can also take place if there is a single coincidence of wants (i.e., one of the agents has a good the other wants), if one agent has money and the other agent is willing to accept it. That is, a trade can take place when an ij agent meets a jk agent if the ij agent has money and the jk agent is willing to accept it.

In this simple framework, agents can be in one of three states; an agent can be waiting for a new endowment to arrive (state 0), can have a good to trade and be waiting to find a trading partner (state 1), or can be holding money and be waiting for a trading opportunity (state m). Let N_0, N_1, and N_m denote the fraction of the population in each state, and define V_i as the value function of an agent currently in state i. Let μ be the fraction of traders with money, and let s be the probability that a random trader accepts money. If β is the rate of time preference, the following conditions must hold for a representative agent j:

$$\beta V_0 = a(V_1 - V_0),$$

$$\beta V_1 = b(1-\mu)x^2(U - \varepsilon + V_0 - V_1) + b\mu x \max_{s_j} s_j(V_m - V_1),$$

and

$$\beta V_m = b(1-\mu)sx(U - \varepsilon + V_0 - V_m).$$

The first of these equations states that the return from waiting for a new endowment is equal to the arrival rate of goods times the value from switching from state 0 to state 1 (having a good to trade). The return to agent j from having a good, βV_1, is equal to the sum of two terms. The first is the probability $b(1-\mu)x^2$ that agent j meets another agent with a commodity to trade (rather than money) and the double coincidence of wants is satisfied, times the consumption value of the trade U minus the costs of commodity trading ε, plus the value from switching from state V_1 to state V_0 (waiting for another endowment). The second is the probability that agent j meets a trader with money who wants j's good (a single coincidence of wants) times the probability that agent j will accept money s_j times the value of switching from state 1 to state m. Note that s_j is chosen optimally to maximize agent j's gain. Finally, the

26. In Kiyotaki and Wright (1993), this is interpreted as a production technology.

last of the three conditions requires that the return from holding money equal the arrival rate of trading opportunities (i.e., meeting someone with no money and a good that agent j wants) times the net gain from the resulting consumption and switch to state V_0.

In a steady state, $s_j = s$ for all j and the proportions in each of the three states must remain constant. This requires that

$$aN_0 = b(1-\mu)x^2 N_1 + b(1-\mu)sxN_m \tag{3.45}$$

and

$$b\mu sxN_1 = b(1-\mu)sxN_m \Rightarrow N_1 = \frac{1-\mu}{\mu}N_m. \tag{3.46}$$

The left side of (3.45) is the flow out of state 0 (waiting for a new commodity endowment). The right side of (3.45) is the flow into state 0. The first term represents the flow into state 0 arising from successful commodity trades. With probability $b(1-\mu)x^2$, an agent meets another agent with a commodity to trade (rather than money), and the double coincidence of wants is satisfied. The last term arises from agents meeting other agents for whom a single coincidence is satisfied and the trader is willing to accept money.

The second steady-state condition, (3.46), equates the flow of commodity traders who successfully trade goods for money (i.e., the flow into state m) to the flow of money holders who successfully trade for a consumption good (i.e., the flow out of state m).

Because $N_0 + N_1 + N_m = 1$, (3.45) can be rewritten as

$$a(1 - N_1 - N_m) = b(1-\mu)x^2 N_1 + b(1-\mu)sxN_m.$$

Now using (3.46), this becomes

$$N_m = \frac{a\mu}{\varphi(\mu, s)},$$

where $\varphi(\mu, s) = \{a + b(1-\mu)[\mu xs + (1-\mu)x^2]\}$.

In this environment, three equilibria are possible. Suppose $s < x$; then the probability of making a trade holding money is less than the probability of making a trade holding a commodity. In this case, individuals will prefer to hold on to their good when they meet another trader (absent a double coincidence) rather than trade for money. With no one willing to trade for money, money will be valueless in equilibrium. A second equilibrium arises when $s > x$. In this case, holding money makes a successful trade more likely than continuing to hold a commodity. So every agent will be willing to hold money, and in equilibrium, all agents will be willing to accept money in exchange for goods. If $s = x$, then what Kiyotaki and Wright characterize as a mixed-monetary equilibrium exists; agents accept money with probability x as long as they believe other agents will accept it with probability x.

The Kiyotaki-Wright model emphasizes the exchange process and the possibility for an intrinsically valueless money to be accepted in trade. It does so, however, by assuming a fixed rate of exchange—one unit of money is exchanged for one unit of goods whenever a trade takes place. The value of money in terms of goods is either 0 (in a nonmonetary equilibrium) or 1. In Trejos and Wright (1995), however, the goods price of money is determined endogenously as part of the equilibrium. This price is the outcome of a bargaining process between buyers and sellers who meet through a process similar to that in Kiyotaki and Wright.

Following Trejos and Wright (1993), assume that the consumption of q units yields utility $u(q)$ and can be produced at a cost (in units of utility) of $c(q)$. If agents could consume their own production, q would be determined by the condition that marginal utility equals marginal cost: $u_q(q) = c_q(q)$. Suppose, however, that agents cannot consume their own output. Instead, they must trade. To keep the example simple, assume that barter is prohibitively expensive, so consumption requires meeting someone willing to accept money for goods. Once two traders meet, they bargain over how much of the consumption good will be traded for one unit of money. Thus, money is assumed to be indivisible, while goods are infinitely divisible (i.e., all trades involve \$1, but the quantity of goods exchanged for that dollar may vary).[27]

Assume that while bargaining, the buyer and seller cannot search for alternative trading opportunities.[28] Trejos and Wright analyze a situation in which the seller and buyer participate in bargaining rounds, each of which consists of one party making a "take it or leave it" offer. As the time between rounds goes to zero, the bargaining solution is a Nash equilibrium that can be found as the value of q that maximizes $[u(q) + V_s][V_b - c(q)]$ where V_s and V_b are the value functions for a seller and a buyer. For any quantity transacted Q, these must satisfy

$$\beta V_s = b\mu x[V_b - V_s - c(Q)] \tag{3.47}$$

and

27. Shi (1997) extends the Kiyotaki-Wright search model to include divisible goods *and* divisible money. Shi (1999) analyzes inflation and its effects on growth in a search model.

28. Trejos and Wright also consider the situation in which agents can look for alternative bargaining opportunities between rounds and cases in which barter is feasible. See Trejos and Wright (1993).

$$\beta V_b = b(1-\mu)x[u(Q) + V_s - V_b]. \tag{3.48}$$

Solving these two equations,

$$V_s = \frac{b^2x^2\mu(1-\mu)u(Q) - b\mu x[\beta + b(1-\mu)x]c(Q)}{(\beta + b\mu x)[\beta + b(1-\mu)x] - b^2x^2\mu(1-\mu)}$$

$$V_b = \frac{b(1-\mu)x(\beta + b\mu x)u(Q) - b^2x^2\mu(1-\mu)c(Q)}{(\beta + b\mu x)[\beta + b(1-\mu)x] - b^2x^2\mu(1-\mu)}.$$

In addition, for a buyer to accept a bargain, it must be the case that $u(q) + V_s - V_b \geq 0$; the utility from consuming q plus the gain from changing from a buyer to a seller must be nonnegative. For a seller to accept a bargain, it must be that $-c(q) + V_b - V_s \geq 0$; the gain from switching from being a seller to a buyer must exceed the cost of producing q.

In an equilibrium, q must maximize $[u(q) + V_s][V_b - c(q)]$, V_s V_b are functions of Q given by the solutions to (3.47) and (3.48), and $q = Q$. Assuming an interior solution, the first order condition for q is

$$T(q, Q) = u_q(q)[V_b(Q) - c(q)] - c_q(q)[u(q) + V_s(Q)] = 0,$$

where V_s and V_b have been written explicitly as functions of Q. Trejos and Wright (1995) prove that $T(q, q) = 0$ has a unique solution with $q > 0$.[29] By substituting the expressions for V_s and V_b into $T(q, q) = 0$, one can study the effect of changes in the primitive parameters of the model such as the fraction of the population holding units of money, μ, on the value of money as measured by the quantity of goods one unit of money can command, q (see Trejos and Wright 1993). However, as Marshall (1993) stresses, the search-theoretic models still lack any connection between standard monetary policy instruments (even as basic as the quantity of money) and the variables in the theory. For example, changes in μ are changes in the cross-sectional distribution of money, not changes in the quantity of money.

The search-theoretic approach to monetary economics provides a natural framework for addressing a number of issues. Ritter (1995) has used it to examine the conditions necessary for fiat money to arise, linking it to the credibility of the issuer. Governments lacking credibility would be expected to overissue the currency to gain seigniorage. In this case, agents would be unwilling to hold the fiat money. Soller and Waller (2000) use a search-theoretic approach to study the coexistence of legal and

29. Since $-c(q) + V_b - V_s \geq 0$ and $u(q) + V_s - V_b \geq 0$ in this equilibrium, Trejos and Wright label it unconstrained. If barter is possible, there can exist a constrained monetary equilibrium in which $c(q) = V_b - V_s$. See Shi (1995).

illegal currencies. By stressing the role of money in facilitating exchange, the search-theoretic approach emphasizes the role of money as a medium of exchange. The approach also emphasizes the social aspect of valued money; agents are willing to accept fiat money only in environments in which they expect others similarly to accept money.[30]

3.5 Summary

The models we have examined in this and the previous chapter are variants of Walrasian economies in which prices are perfectly flexible and adjust to ensure that market equilibrium is continuously maintained. The MIU, CIA, and shopping time models as well as the other approaches discussed all represent means of introducing valued money into the Walrasian equilibrium. Each approach captures some aspects of the role that money plays in facilitating transactions.

Despite the different approaches, several conclusions are common to all. First, because the price level is completely flexible, the value of money, equal to 1 over the price of goods, behaves like an asset price. The return money yields, however, differs in the various approaches. In the MIU approach, the marginal utility of money is the direct return, while in the CIA model, this return is measured by the Lagrangian multiplier on the CIA constraint. In the shopping time model, the return arises from the time savings provided by money in carrying out transactions, and the value of this time savings depends on the real wage.

All these models have similar implications for the optimal rate of inflation. An efficient equilibrium will be characterized by equality between social and private costs. Because the social cost of producing money is zero, the private opportunity cost of holding money must be zero in order to achieve optimality. The private opportunity cost is measured by the nominal interest rate, so the optimal rate of inflation in the steady state is the rate that achieves a zero nominal rate of interest. While this result is quite general, two important considerations—the effects of inflation on government revenues and the interaction of inflation with other taxes in a nonindexed tax system—have been ignored. These will be among the topics of chapter 4.

Finally, the class of models studied in this chapter are among the basic frameworks monetary economists have found useful for understanding the steady-state implications of inflation and the steady-state welfare implications of alternative rates of inflation. However, the dynamics implied by these flexible-price models fail to capture the short-run behavior that appears to characterize modern economies. That is perhaps not surprising; most economists believe that sluggish wage and price adjustment, absent from the models of this chapter, play critical roles in determining the short-run real effects of monetary disturbances and monetary policy. Although systematic monetary policy can have real effects with flexible prices, simulations suggest that these effects are small, at least at moderate inflation rates. To understand how the observed short-run behavior of money, interest rates, the price level, and output might be generated in a monetary economy, we need to introduce nominal rigidities, a topic discussed in chapter 5.

30. Samuelson (1958) is one of the earliest modern treatments of money as a social construct.

3.6 Appendix: The CIA Approximation

The method used to obtain a linear approximation around the steady state for the CIA model is discussed here. Since the approach is similar to the one followed for the MIU model of chapter 2 (and discussed in the appendix to that chapter), some details are skipped.

3.6.1 The Basic Decision Problem

The utility function of the representative agent is given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta} \right] \tag{3.49}$$

$0 < \beta < 1$; c_t is real consumption, while n_t is labor supplied to market activities expressed as a fraction of the total time available; $1 - n_t$ is equal to leisure time. The parameters Φ, Ψ, and η are restricted to be positive.

The CIA constraint is assumed to apply only to the purchase of consumption goods, and in real terms, it takes the form

$$c_t \leq \frac{m_{t-1}}{\Pi_t} + \tau_t \equiv a_t, \tag{3.50}$$

where $\Pi_t = P_t/P_{t-1} = 1 + \pi_t$ is equal to 1 plus the rate of inflation. From the definition of the transfer payment, $a_t = m_t$, but this is an equilibrium condition; the individual takes a_t as given while deciding on a level of real money balances to hold. In equilibrium, the demand for money by the representative agent must equal the per capita stock of money.

The household also faces a flow budget constraint of the form

$$e^{z_t}k_{t-1}^{\alpha}n_t^{1-\alpha} + (1-\delta)k_{t-1} + \left(\frac{1+i_{t-1}}{1+\pi_t}\right)b_{t-1} + a_t \geq c_t + k_t + b_t + m_t. \quad (3.51)$$

Notice that we have assumed that the production technology is a Cobb-Douglas with constant returns to scale, $0 \leq \alpha \leq 1$. The exogenous productivity shock z_t follows

$$z_t = \rho z_{t-1} + e_t,$$

with $0 \leq \rho \leq 1$. The innovation e_t has mean zero and variance σ_e^2.

The individual's decision problem is characterized by the value function

$$V(k_{t-1}, b_{t-1}, a_t) = \max\left\{\frac{c_t^{1-\Phi}}{1-\Phi} + \Psi\frac{(1-n_t)^{1-\eta}}{1-\eta} + \beta \mathrm{E}_t V(k_t, b_t, a_{t+1})\right\},$$

where the maximization is with respect to $\{c_t, n_t, k_t, b_t, \text{ and } m_t\}$ and is subject to the constraints (3.51) and (3.50).

Let λ_t be the Lagrangian associated with the budget constraint, and let μ_t be the Lagrangian associated with the CIA constraint. The first order necessary conditions for c_t, m_t, b_t, and n_t, along with the budget constraint and the CIA constraint, are

$$c_t^{-\Phi} = \lambda_t + \mu_t \quad (3.52)$$

$$\lambda_t = \beta \mathrm{E}_t\left[\frac{V_a(k_t, b_t, a_{t+1})}{\Pi_{t+1}}\right] \quad (3.53)$$

$$\lambda_t = \beta \mathrm{E}_t V_b(k_t, b_t, a_{t+1}) \quad (3.54)$$

$$-\Psi(1-n)^{-\eta} + (1-\alpha)e^{z_t}k_{t-1}^{\alpha}n_t^{-\alpha}\lambda_t = 0 \quad (3.55)$$

$$V_a(k_{t-1}, b_{t-1}, a_t) = \lambda_t + \mu_t \quad (3.56)$$

$$V_k(k_{t-1}, b_{t-1}, a_t) = [\alpha e^{z_t}k_{t-1}^{\alpha-1}n_t^{1-\alpha} + 1 - \delta]\lambda_t \quad (3.57)$$

$$V_b(k_{t-1}, b_{t-1}, a_t) = \beta\left(\frac{1+i_{t-1}}{1+\pi_t}\right)\lambda_t. \quad (3.58)$$

Let $y_t = e^{z_t}k_{t-1}^{\alpha}n_t^{1-\alpha}$. The first order conditions (3.52)–(3.58) can be rewritten compactly as

$$c_t^{-\Phi} = \lambda_t + \mu_t \quad (3.59)$$

$$\lambda_t = \beta \mathrm{E}_t\left[\frac{\lambda_{t+1} + \mu_{t+1}}{\Pi_{t+1}}\right] \quad (3.60)$$

$$\Psi(1-n)^{-\eta} = (1-\alpha)\left(\frac{y_t}{n_t}\right)\lambda_t \quad (3.61)$$

$$\lambda_t = \beta \mathrm{E}_t R_t \lambda_{t+1} \quad (3.62)$$

$$\lambda_t = \beta \mathrm{E}_t\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\lambda_{t+1}, \quad (3.63)$$

where $R_t = [\alpha y_{t+1}/k_t + 1 - \delta]$.

3.6.2 The Steady State

The steady-state values of the ratios that were reported in table 2.1 for the MIU model also characterize the steady state for the CIA model. In a steady state, (3.62) yields the standard result that

$$\beta R^{ss} = \beta\left[\alpha\left(\frac{y^{ss}}{k^{ss}}\right) + 1 - \delta\right] = 1$$

or $R^{ss} = 1/\beta$ and

$$\left(\frac{y^{ss}}{k^{ss}}\right) = \frac{1}{\alpha}[R^{ss} - 1 + \delta].$$

From the economy's resource constraint, $y^{ss} = c^{ss} + \delta k^{ss}$, so

$$\left(\frac{c^{ss}}{k^{ss}}\right) = \left(\frac{y^{ss}}{k^{ss}}\right) - \delta.$$

With a binding CIA constraint, $c^{ss} = \tau^{ss} + m^{ss}/(1+\pi^{ss})$, but in a steady state with m constant, $\tau^{ss} + m^{ss}/(1+\pi^{ss}) = m^{ss}$. Thus, $c^{ss} = m^{ss}$.

The production function implies that $y^{ss}/k^{ss} = (n^{ss}/k^{ss})^{1-\alpha}$, or

$$\left(\frac{n^{ss}}{k^{ss}}\right) = \left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{1}{1-\alpha}}.$$

This leaves n^{ss} to be determined. From the first order condition for the household's choice of n,

$$\Psi(1-n^{ss})^{-\eta} = (1-\alpha)\left(\frac{y^{ss}}{n^{ss}}\right)\lambda^{ss} = (1-\alpha)\left(\frac{y^{ss}}{k^{ss}}\right)\left(\frac{n^{ss}}{k_{ss}}\right)^{-1}\lambda^{ss}, \quad (3.64)$$

so the only remaining value to find is λ^{ss}. From (3.59), $(c^{ss})^{-\Phi} = \lambda^{ss} + \mu^{ss}$, and (3.60) then yields $\lambda^{ss} = \frac{\beta}{\Theta}(c^{ss})^{-\Phi}$, where $\Theta = 1 + \theta^{ss} = \Pi^{ss}$. Combining this with (3.64) and recalling that $\left(\frac{n^{ss}}{k^{ss}}\right) = \left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{1}{1-\alpha}}$ results in

$$(1-n^{ss})^{-\eta}(n^{ss})^{\Phi} = \frac{(1-\alpha)}{\Psi}\left(\frac{\beta}{\Theta}\right)\left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{\Phi-\alpha}{1-\alpha}}\left(\frac{c^{ss}}{k^{ss}}\right)^{-\Phi}.$$

3.6.3 The Linear Approximation

Expressions linear in the percentage deviations around the steady state can be obtained for the economy's resource constraint, the production function, the definition of the marginal product of capital, and the first order conditions for consumption, money holdings, and labor supply, just as was done for the MIU model of chapter 2. The economy's resource constraint, the production function, and the definition of the expected marginal product of capital are identical to those of the MIU model,[31] so they are simply stated here:

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \left(\frac{y^{ss}}{k^{ss}}\right)\hat{y}_t - \left(\frac{c^{ss}}{k^{ss}}\right)\hat{c}_t \tag{3.65}$$

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + z_t \tag{3.66}$$

$$\hat{r}_t = \alpha\left(\frac{y^{ss}}{k^{ss}}\right)(\mathrm{E}_t\hat{y}_{t+1} - \hat{k}_t). \tag{3.67}$$

From the CIA constraint, $c_t = m_t$ in an equilibrium with a positive nominal rate of interest. Eliminating consumption and noting that $\lambda_{t+1} + \mu_{t+1} = c_{t+1}^{-\Phi} = m_{t+1}^{-\Phi}$ yields

$$\lambda_t = \beta\mathrm{E}_t\left[\frac{m_{t+1}^{-\Phi}}{\Pi_{t+1}}\right].$$

Linearizing this equation, together with (3.61)–(3.63) around the steady state, one obtains

$$\hat{\lambda}_t = -\beta\mathrm{E}_t[\Phi\hat{m}_{t+1} + \hat{\pi}_{t+1}] = -\Phi\beta(\hat{m}_t + \gamma u_t + \phi z_t) - \beta(1-\Phi)\mathrm{E}_t\hat{\pi}_{t+1} \tag{3.68}$$

$$\left(1 + \eta\frac{n^{ss}}{1-n^{ss}}\right)\hat{n}_t = \hat{y}_t + \hat{\lambda}_t \tag{3.69}$$

$$\hat{\lambda}_t = \mathrm{E}_t\hat{\lambda}_{t+1} + \hat{r}_t \tag{3.70}$$

31. See the appendix to chapter 2.

and

$$\hat{\lambda}_t = \hat{\imath}_t - \mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\hat{\lambda}_{t+1}. \tag{3.71}$$

Notice that in (3.68), (3.42) has been used to replace $\mathrm{E}_t\hat{m}_{t+1}$ by $\hat{m}_t - \mathrm{E}_t\hat{\pi}_{t+1} + \mathrm{E}_t u_{t+1} = \hat{m}_t - \mathrm{E}_t\hat{\pi}_{t+1} + \gamma u_t + \phi z_t$. Equations (3.70) and (3.71) imply that

$$\hat{\imath}_t = \hat{r}_t + \mathrm{E}_t\pi_{t+1}.$$

Proceeding as was done in chapter 2, the production function and the labor market equilibrium condition can be used to eliminate expected future output from (3.67), yielding

$$\hat{r}_t = \alpha\left(\frac{Y^{ss}}{K^{ss}}\right)\left[\frac{(\alpha-1)\eta\left(\frac{N^{ss}}{L^{ss}}\right)\hat{k}_t + \left(1+\eta\frac{N^{ss}}{L^{ss}}\right)\rho z_t + (1-\alpha)\hat{\lambda}_t}{\left(\alpha+\eta\frac{N^{ss}}{L^{ss}}\right) + \alpha(1-\alpha)\left(\frac{Y^{ss}}{K^{ss}}\right)}\right]. \tag{3.72}$$

The Matlab program used to simulate this CIA model can be found at http://econ.ucsc.edu/~walshc/2ed/programs/.

3.7 Problems

1. Suppose the production function for shopping takes the form $\psi = c = e^x(n^s)^a m^b$, where a and b are both positive but less than 1 and x is a productivity factor. The agent's utility is given by $v(c,l) = c^{1-\Phi}/(1-\Phi) + l^{1-\eta}/(1-\eta)$, where $l = 1 - n - n^s$ and n is time spent in market employment.

a. Derive the transaction time function $g(c,m) = n^s$.

b. Derive the money in the utility function specification implied by the shopping production function. How does the marginal utility of money depend on the parameters a and b? How does it depend on x?

c. Is the marginal utility of consumption increasing or decreasing in m?

2. Define superneutrality. Carefully explain whether the Cooley-Hansen CIA model exhibits superneutrality. What role does the CIA constraint play in determining whether superneutrality holds?

3. Is the steady-state equilibrium in the Cooley-Hansen CIA model affected by any of the following modifications? Explain.

a. Labor is supplied inelasticly (normalize so that $n = 1$, where n is the supply of labor).

b. Purchases of capital are also subject to the CIA constraint (i.e., one needs money to purchase both consumption and investment goods).

c. The growth rate of money follows the process $u_t = \gamma u_{t-1} + \varphi_t$, where $0 < \gamma < 1$ and φ is a mean zero, independently and identically distributed process.

4. Use (3.60), (3.62), and (3.63) to show that the nominal interest rate is positive as long as the CIA constraint is expected to bind in the future.

5. MIU and CIA models are alternative approaches to constructing models in which money has positive value in equilibrium.

a. What strengths and weaknesses do you see in each of these approaches?

b. Suppose you wanted to study the effects of the growth of credit cards on money demand. Which approach would you adopt? Why?

6. Consider the model of section 3.1. Suppose that money is required to purchase both consumption and investment goods. The CIA constraint then becomes $c_t + x_t \leq m_{t-1}/\Pi_t + \tau_t$, where x is investment. Assume that the aggregate production function takes the form $y_t = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}$. Show that the steady-state capital-labor ratio is affected by the rate of inflation. Does a rise in inflation raise or lower the steady-state capital-labor ratio? Explain.

7. Consider the following model:

$$\text{Preferences: } E_t \sum_{i=0}^{\infty} \beta^i [\ln c_{t+i} + \theta \ln d_{t+i}]$$

$$\text{Budget constraint: } c_t + d_t + m_t + k_t = Ak_{t-1}^{a} + \tau_t + \frac{m_{t-1}}{1+\pi_t} \tag{3.73}$$

$$\text{CIA constraint: } c_t \leq \tau_t + \frac{m_{t-1}}{1+\pi_t}, \tag{3.74}$$

where m denotes real money balances and π_t is the inflation rate from period $t-1$ to period t. The two consumption goods, c and d, represent cash (c) and credit (d) goods. The net transfer τ is viewed as a lump-sum payment (or tax) by the household.

a. Does this model exhibit superneutrality? Explain.

b. What is the rate of inflation that maximizes steady-state utility?

8. Consider the following model:

$$\text{Preferences: } E_t \sum_{i=0}^{\infty} \beta^i [\ln c_{t+i} + \ln d_{t+i}]$$

$$\text{Budget constraint: } c_t + d_t + m_t + k_t = Ak_{t-1}^{a} + \tau_t + \frac{m_{t-1}}{1+\pi_t} + (1-\delta)k_{t-1},$$

where m denotes real money balances and π_t is the inflation rate from period $t-1$ to period t. Utility depends on the consumption of two types of good; c must be purchased with cash, while d can be purchased using either cash or credit. The net transfer τ is viewed as a lump-sum payment (or tax) by the household. If a fraction θ of d is purchased using cash, then the household also faces a CIA constraint that takes the form

$$c_t + \theta d_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t.$$

What is the relationship between the nominal rate of interest and whether the CIA constraint is binding? Explain. Will the household ever use cash to purchase d (i.e., will the optimal θ ever be greater than zero)?

9. Suppose the representative household enters period t with nominal money balances M_{t-1} and receives a lump-sum transfer T_t. During period t, the bond market opens first, and the household receives interest payments and purchases nominal bonds in the amount B_t. With its remaining money $(M_{t-1} + T_t + I_{t-1}B_{t-1} - B_t)$, the household enters the goods market and purchases consumption goods subject to

$$P_t c_t \leq M_{t-1} + T_t + I_{t-1}B_{t-1} - B_t.$$

The household receives income at the end of the period, and ends period t with nominal money holdings M_t given by

$$M_t = P_t[e^{z_t} K_{t-1}^{\alpha} N_t^{1-\alpha} + (1-\delta)K_{t-1} - K_t - c_t] + M_{t-1} + T_t + I_{t-1}B_{t-1} - B_t.$$

If the household's objective is to maximize

$$\mathrm{E}_0 \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - N_{t+i}) = \mathrm{E}_0 \sum_{i=0}^{\infty} \beta^i \left[\frac{c_{t+i}^{1-\Phi}}{1-\Phi} + \Psi \frac{(1 - N_{t+i})^{1-\eta}}{1-\eta} \right],$$

do the equilibrium conditions differ from (3.59)–(3.63)?

10. Trejos and Wright (1993) find that if no search is allowed while bargaining takes place, output tends to be too low (the marginal utility of output exceeds the marginal production costs). Show that output is also too low in a basic CIA model. (For simplicity, assume that only labor is needed to produce output according to the production function $y = n$.) Does the same hold true in an MIU model?

4 Money and Public Finance

4.1 Introduction

Inflation is a tax. And as a tax, it both generates revenue for the government and distorts private sector behavior. The previous two chapters focused on these distortions. In the Sidrauski model, inflation distorts the demand for money, thereby generating welfare effects because real money holdings directly yield utility. In the Cooley-Hansen cash-in-advance (CIA) model, inflation serves as an implicit tax on consumption, so a higher inflation rate generates a substitution toward leisure, leading to lower labor supply, output, and consumption.

In our analysis of these distortions, we ignored the revenue side of the inflation tax except to note that the Friedman rule for the optimal rate of inflation may need to be modified if the government does not have lump-sum sources of revenue available. Any change in inflation that affects the revenue from the inflation tax will have budgetary implications for the government. If higher inflation allows other forms of distortionary taxation to be reduced, this fact must be incorporated into any assessment of the costs of the inflation tax. In this chapter, we introduce the government sector's budget constraint and examine the revenue implications of inflation. Doing so allows us to focus more explicitly on the role of inflation in a theory of public finance and to draw on the literature on optimal taxation in analyzing the effects of inflation.

A public-finance approach yields several insights. Among the most important is the recognition that fiscal and monetary policies are linked through the government sector's budget constraint. Variations in the inflation rate can have implications for the fiscal authority's decisions about expenditures and taxes, and, conversely, decisions by the fiscal authority can have implications for money growth and inflation. When inflation is viewed as a distortionary revenue-generating tax, the degree to which it should be relied upon depends on the set of alternative taxes available to the government and on the reasons individuals hold money. Whether the most appropriate strategy is to think of money as entering the utility function as a final good or as serving as an intermediate input into the production of transaction services can have implications for whether money should be taxed. The optimal-tax perspective also has empirical implications for inflation.

In the next section, the consolidated government's budget identity is set out, and some of the revenue implications of inflation are examined. Section 4.3 introduces various assumptions that can be made about the relationship between monetary and fiscal policies. Section 4.3.1 discusses situations of fiscal dominance in which a fixed amount of revenue must be raised from the inflation tax. It then discusses the equilibrium relationship between money and the price level. Section 4.3.2 turns to recent theories that emphasize what has come to be called the *fiscal theory of the price level*.

In section 4.4, inflation revenue (seigniorage) and other taxes are brought together to analyze the joint determination of the government's tax instruments. This theme is developed first in a partial equilibrium model, and then Friedman's rule for the optimal inflation rate is revisited. The implications of optimal Ramsey taxation for inflation are discussed. Finally, section 4.5 contains a brief discussion of some additional effects that arise when the tax system is not fully indexed.

4.2 Budget Accounting

To obtain goods and services, governments in market economies need to generate revenue. And one way that they can obtain goods and services is to print money that is then used to purchase resources from the private sector. However, to understand the revenue implications of inflation (and the inflation implications of the government's revenue needs), we must start with the government's budget constraint.[1]

Consider the following identity for the fiscal branch of a government:

$$G_t + i_{t-1}B_{t-1}^T = T_t + (B_t^T - B_{t-1}^T) + RCB_t, \tag{4.1}$$

where all variables are in nominal terms. The left side consists of government expenditures on goods, services, and transfers G_t, plus interest payments on the outstanding debt $i_{t-1}B_{t-1}^T$ (the superscript T denoting total debt, assumed to be one period in maturity, where debt issued in period $t-1$ earns the nominal interest rate i_{t-1}), and the right side consists of tax revenue T_t, plus new issues of interest-bearing debt $B_t^T - B_{t-1}^T$, plus any direct receipts from the central bank RCB_t. As an example of RCB, the U.S. Federal Reserve turns over to the Treasury almost all the interest earnings on its portfolio of government debt.[2] We will refer to (4.1) as the *Treasury's budget constraint.*

The monetary authority, or central bank, also has a budget identity that links changes in its assets and liabilities. This takes the form

$$(B_t^M - B_{t-1}^M) + RCB_t = i_{t-1}B_{t-1}^M + (H_t - H_{t-1}), \tag{4.2}$$

where $B_t^M - B_{t-1}^M$ is equal to the central bank's purchases of government debt, $i_{t-1}B_{t-1}^M$ is the central bank's receipt of interest payments from the Treasury, and $H_t - H_{t-1}$ is the change in the central bank's own liabilities. These liabilities are

1. Bohn (1992) provides a general discussion of government deficits and accounting.

2. In 2001, the Federal Reserve banks turned over \$27 billion to the Treasury (*88nd Annual Report of the Federal Reserve System* 2001, p. 383). Klein and Neumann (1990) show how the revenue generated by seigniorage and the revenue received by the fiscal branch may differ.

called *high-powered money* or sometimes the *monetary base* since they form the stock of currency held by the nonbank public plus bank reserves, and they represent the reserves private banks can use to back deposits under a fractional reserve system. Changes in the stock of high-powered money lead to changes in broader measures of the money supply, measures that normally include various types of bank deposits as well as currency held by the public (see chapter 9).

By letting $B = B^T - B^M$ be the stock of government interest-bearing debt held by the public, the budget identities of the Treasury and the central bank can be combined to produce the consolidated government-sector budget identity:

$$G_t + i_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}). \tag{4.3}$$

From the perspective of the consolidated government sector, only debt held by the public (i.e., outside the government sector) represents an interest-bearing liability.

According to (4.3), the dollar value of government purchases G_t, plus its payment of interest on outstanding privately held debt $i_{t-1}B_{t-1}$, must be funded by revenue that can be obtained from one of three alternative sources. First, T_t represents revenues generated by taxes (other than inflation). Second, the government can obtain funds by borrowing from the private sector. This borrowing is equal to the change in the debt held by the private sector, $B_t - B_{t-1}$. Finally, the government can print currency to pay for its expenditures, and this is represented by the change in the outstanding stock of noninterest-bearing debt, $H_t - H_{t-1}$.

We can divide (4.3) by P_tY_t, where P_t is the price level and Y_t is real output, to obtain

$$\frac{G_t}{P_tY_t} + i_{t-1}\left(\frac{B_{t-1}}{P_tY_t}\right) = \frac{T_t}{P_tY_t} + \frac{B_t - B_{t-1}}{P_tY_t} + \frac{H_t - H_{t-1}}{P_tY_t}.$$

Note that terms like B_{t-1}/P_tY_t can be multiplied and divided by $P_{t-1}Y_{t-1}$, yielding

$$\frac{B_{t-1}}{P_tY_t} = \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\right)\left(\frac{P_{t-1}Y_{t-1}}{P_tY_t}\right)$$
$$= b_{t-1}\left[\frac{1}{(1+\pi_t)(1+\mu_t)}\right],$$

where $b_{t-1} = B_{t-1}/P_{t-1}Y_{t-1}$ represents real debt relative to income, π_t is the inflation rate, and μ_t is the growth rate of real output.[3] Employing the convention that low-

3. If n is the rate of population growth and λ is the growth rate of real per capita output, then $1+\mu = (1+n)(1+\lambda)$.

ercase letters denote variables deflated by the price level and by real output, the government's budget identity is

$$g_t + \bar{r}_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + h_t - \frac{h_{t-1}}{(1+\pi_t)(1+\mu_t)}, \tag{4.4}$$

where $\bar{r}_{t-1} = (1+i_{t-1})/[(1+\pi_t)(1+\mu_t)] - 1$ is the *ex post* real return from $t-1$ to t. For simplicity, in the following we will abstract from real income growth by setting $\mu_t = 0$.

To highlight the respective roles of anticipated and unanticipated inflation, let r_t be the *ex ante* real rate of return and let π_t^e be the expected rate of inflation; then $1 + i_{t-1} = (1 + r_{t-1})(1 + \pi_t^e)$. Adding $(r_{t-1} - \bar{r}_{t-1})b_{t-1} = (\pi_t - \pi_t^e)(1 + r_{t-1})b_{t-1}/(1+\pi_t)$ to both sides of (4.4) and rearranging, the budget constraint becomes

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + \left(\frac{\pi_t - \pi_t^e}{1+\pi_t}\right)(1 + r_{t-1})b_{t-1} + \left[h_t - \left(\frac{1}{1+\pi_t}\right)h_{t-1}\right]. \tag{4.5}$$

The third term on the right side of this expression, involving $(\pi_t - \pi_t^e)b_{t-1}$, represents the revenue generated when unanticipated inflation reduces the real value of the government's outstanding interest-bearing nominal debt. To the extent that inflation is anticipated, it will be reflected in higher nominal interest rates that the government must pay. Inflation by itself does not reduce the burden of the government's interest-bearing debt; only unexpected inflation has such an effect.

The last bracketed term in (4.5) represents seigniorage, the revenue from money creation. Seigniorage can be written as

$$s_t \equiv \frac{H_t - H_{t-1}}{P_t Y_t} = (h_t - h_{t-1}) + \left(\frac{\pi_t}{1+\pi_t}\right)h_{t-1}. \tag{4.6}$$

Seigniorage arises from two sources. First, $h_t - h_{t-1}$ is equal to the change in real high-powered money holdings relative to income. Since the government is the monopoly issuer of high-powered money, an increase in the amount of high-powered money that the private sector is willing to hold allows the government to obtain real resources in return. In a steady-state equilibrium, h is constant, so this source of seigniorage then equals zero. The second term in (4.6) is normally the focus of analyses of seigniorage because it can be nonzero even in the steady state. To maintain a constant level of real money holdings relative to income, the private sector needs to increase its nominal holdings of money at the rate π (approximately) to offset the effects of inflation on real holdings. By supplying money to meet this demand, the government is able to obtain goods and services or reduce other taxes.[4]

If we denote the growth rate of the nominal monetary base H by θ, the growth rate of h will equal $(\theta - \pi)/(1+\pi) \approx \theta - \pi$. In a steady state, h will be constant, implying that $\pi = \theta$.[5] In this case, (4.6) shows that seigniorage will equal

$$\left(\frac{\pi}{1+\pi}\right)h = \left(\frac{\theta}{1+\theta}\right)h. \tag{4.7}$$

For small values of the rate of inflation, $\pi/(1+\pi)$ is approximately equal to π, so s can be thought of as the product of a tax rate of π, the rate of inflation, and a tax base of h, the real stock of base money. Since base money does not pay interest, its real value is depreciated by inflation whether inflation is anticipated or not.

The definition of s would appear to imply that the government receives no revenue if inflation is zero. But this inference neglects the real interest savings to the government of issuing h, which is noninterest-bearing debt, as opposed to b, which is interest-bearing debt. That is, for a given level of the government's total real liabilities $d = b + h$, interest costs will be a decreasing function of the fraction of this total that consists of h. A shift from interest-bearing to noninterest-bearing debt would allow the government to reduce total tax revenues or increase transfers or purchases.

This observation suggests that one should consider the government's budget constraint expressed in terms of the total liabilities of the government. Using (4.5) and (4.6), we can rewrite the budget constraint as[6]

$$g_t + r_{t-1}d_{t-1} = t_t + (d_t - d_{t-1}) + \left(\frac{\pi_t - \pi_t^e}{1+\pi_t}\right)(1 + r_{t-1})d_{t-1} + \left(\frac{i_{t-1}}{1+\pi_t}\right)h_{t-1}. \tag{4.8}$$

4. With population and real income growth, (4.6) becomes

$$s_t = (h_t - h_{t-1}) + \left[\frac{(1+\pi_t)(1+n_t)(1+\lambda_t) - 1}{(1+\pi_t)(1+n_t)(1+\lambda_t)}\right]h_{t-1},$$

where n is the rate of population growth and λ is the rate of per capita income growth. Private sector nominal money holdings increase to offset inflation and population growth. In addition, if the elasticity of real money demand with respect to income is equal to 1, real per capita demand for money will rise at the rate λ. Thus, the demand for nominal balances rises approximately at the rate $\pi + n + \lambda$ when h is constant.

5. With population and income growth, the growth rate of h is approximately equal to $\theta - \pi - n - \lambda$. In the steady state, this equals zero, or $\pi = \theta - n - \lambda$.

6. To obtain this, add $r_{t-1}h_{t-1}$ to both sides of (4.5).

Seigniorage, defined as the last term in (4.8), becomes

$$\bar{s} = \left(\frac{i}{1+\pi}\right)h. \tag{4.9}$$

This shows that the relevant tax rate on high-powered money depends directly on the nominal rate of interest. Thus, under the Friedman rule for the optimal rate of inflation, which calls for setting the nominal rate of interest equal to zero (see chapters 2 and 3), the government collects no revenue from seigniorage. The budget constraint also illustrates that any change in seigniorage requires an offsetting adjustment in the other components of (4.8). Reducing the nominal interest rate to zero implies that the lost revenue must be replaced by an increase in other taxes, real borrowing that increases the government's net indebtedness, or reductions in expenditures.

The various forms of the government's budget identity suggest at least three alternative measures of the revenue governments generate through money creation. First, the measure that might be viewed as appropriate from the perspective of the Treasury is simple *RCB*, total transfers from the central bank to the Treasury (see 4.1). For the United States, King and Plosser (1985) report that the real value of these transfers amounted to 0.02% of real GNP during the 1929–1952 period and 0.15% of real GNP in the 1952–1982 period. Under this definition, shifts in the ownership of government debt between the private sector and the central bank affect the measure of seigniorage even if high-powered money remains constant. That is, from (4.2), if the central bank used interest receipts to purchase debt, B^M would rise, *RCB* would fall, and the Treasury would, from (4.1), need to raise other taxes, reduce expenditures, or issue more debt. But this last option means that the Treasury could simply issue debt equal to the increase in the central bank's debt holdings, leaving private debt holdings, government expenditures, and other taxes unaffected. Thus, changes in *RCB* do not represent real changes in the Treasury's finances and are therefore not the appropriate measure of seigniorage.

A second possible measure of seigniorage is given by (4.6), the real value of the change in high-powered money. King and Plosser report that s equaled 1.37% of real GNP during 1929–1952 but only 0.3% during 1952–1982. This measure of seigniorage equals the revenue from money creation for a given path of interest-bearing government debt. That is, s equals the total expenditures that could be funded, holding constant other tax revenues and the total private sector holdings of interest-bearing government debt. While s, expressed as a fraction of GNP, was quite small during the postwar period in the United States, King and Plosser report much higher values for other countries. For example, it was more than 6% of GNP in Argentina and over 2% in Italy.

Finally, (4.9) provides a third definition of seigniorage as the nominal interest savings from issuing noninterest-bearing as opposed to interest-bearing debt.[7] Using the four- to six-month commercial paper rate as a measure of the nominal interest rate, King and Plosser report that this measure of seigniorage equaled 0.2% of U.S. GNP during 1929–1952 and 0.47% during 1952–1982. This third definition equals the revenue from money creation for a given path of total (interest- and noninterest-bearing) government debt; it equals the total expenditures that could be funded, holding constant other tax revenues and the total private sector holdings of real government liabilities.

The difference between s and $\bar{s}$ arises from alternative definitions of fiscal policy. To understand the effects of monetary policy, we normally want to consider changes in monetary policy while holding fiscal policy (and perhaps other things also) constant. Suppose tax revenues t are simply treated as lump-sum taxes. Then one definition of fiscal policy would be in terms of a time series for government purchases and interest-bearing debt: $\{g_{t+i}, b_{t+i}\}_{i=0}^{\infty}$. Changes in s, together with the changes in t necessary to maintain $\{g_{t+i}, b_{t+i}\}_{i=0}^{\infty}$ unchanged, would constitute monetary policy. Under this definition, monetary policy would change the total liabilities of the government (i.e., $b+h$). An open market purchase by the central bank would, ceteris paribus, lower the stock of interest-bearing debt held by the public. The Treasury would then need to issue additional interest-bearing debt to keep the b_{t+i} sequence unchanged. Total government liabilities would rise. Under the definition $\bar{s}$, fiscal policy sets the path $\{g_{t+i}, d_{t+i}\}_{i=0}^{\infty}$ and monetary policy determines the division of d between interest- and noninterest-bearing debt but not its total.

4.2.1 Intertemporal Budget Balance

The budget relationships derived in the previous section link the government's choices concerning expenditures, taxes, debt, and seigniorage at each point in time. However, unless there are restrictions on the government's ability to borrow or to raise revenue from seigniorage, (4.8) places no real constraint on expenditure or tax choices. If governments, like individuals, are constrained in their ability to borrow, then this constraint limits the government's choices. To see exactly how it does so requires that we focus on the intertemporal budget constraint of the government.

7. And these are not the only three possible definitions. See King and Plosser (1985) for an additional three.

Ignoring the effect of surprise inflation, the single-period budget identity of the government given by (4.5) can be written as

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + s_t.$$

Assuming the interest factor r is a constant (and is positive),[8] this equation can be solved forward to obtain

$$(1+r)b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{t_{t+i}}{(1+r)^i} + \sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} + \lim_{i\to\infty} \frac{b_{t+i}}{(1+r)^i}. \quad (4.10)$$

The government's expenditure and tax plans are said to satisfy the requirement of intertemporal budget balance (the *no Ponzi condition*) if the last term in (4.10) equals zero:

$$\lim_{i\to\infty} \frac{b_{t+i}}{(1+r)^i} = 0. \quad (4.11)$$

In this case, the right side of (4.10) becomes the present discounted value of all current and future tax and seigniorage revenues, and this is equal to the left side, which is the present discounted value of all current and future expenditures plus current outstanding debt (principal plus interest). In other words, the government must plan to raise sufficient revenue, in present value terms, to repay its existing debt and finance its planned expenditures. Defining the primary deficit as $\Delta = g - t - s$, intertemporal budget balance implies, from (4.10), that

$$(1+r)b_{t-1} = -\sum_{i=0}^{\infty} \frac{\Delta_{t+i}}{(1+r)^i}. \quad (4.12)$$

Thus, if the government has outstanding debt ($b_{t-1} > 0$), the present value of future primary deficits must be negative (i.e., the government must run a primary surplus in present value). This surplus can be generated through adjustments in expenditures, taxes, or seigniorage.

Is (4.12) a constraint on the government? Must the government (the combined monetary and fiscal authorities) pick expenditures, taxes, and seigniorage to ensure that (4.12) holds for all possible values of the initial price level and interest rates? Or is it an equilibrium condition that need only hold at the equilibrium price level and interest rate? Buiter (2002) argues strongly that the intertemporal budget balance condition represents a constraint on government behavior. This is the perspective that will be taken in section 4.3.1. Sims (1994), Woodford (1995, 2001), and Cochrane (1998a) argue that (4.12) is an equilibrium condition. This is the perspective that will be taken in section 4.3.2.

8. With population growth and trend income growth, the relevant discount factor is $r - n - \lambda$.

If the requirement of intertemporal budget balance is a constraint, does it impose any testable restrictions on the behavior of government deficits? After all, a long sequence of primary deficits could be consistent with intertemporal budget balance as long as the government is expected to run sufficiently large primary surpluses sometime in the future. A closely related issue is whether a long sequence of primary deficits has any implications for future seigniorage. Often, concern is expressed that deficits might imply higher future inflation as seigniorage is used to generate the necessary future surpluses. Most of the empirical work in this area has focused on a narrower test: is the time-series behavior of expenditures, tax revenues, and debt consistent with intertemporal budget balance? A finding that it isn't might imply that the historical patterns are expected to change. (If they weren't expected to change, then the public would believe the government is borrowing and running a Ponzi scheme. If such is the case, why are they buying the government's debt? That is, why are they lending to the government?)

To determine the empirical restrictions that intertemporal budget balance might impose on the time-series behavior of expenditures, taxes, and debt, suppose that the first difference of the primary deficit is stationary (this assumption is stronger than necessary; see Trehan and Walsh 1991). This supposition allows Δ_t to be integrated of order 1, denoted $I(1)$. Then intertemporal budget balance holds if Δ_t and b_{t-1} are cointegrated. In other words, if there exists a linear combination of the primary deficit and the stock of debt that is stationary, then intertemporal balance holds.

A number of authors have applied tests for intertemporal budget balance to determine if deficit processes are sustainable. Often the results have been mixed, depending on the country examined and the time period used. Trehan and Walsh (1988) failed to reject intertemporal budget balance for the United States based on data covering 1890–1986. Other authors have rejected budget balance using post-1960 U.S. data (see Hamilton and Flavin 1986, Wilcox 1989, and Hakkio and Rush 1991).

This analysis has assumed that r remains constant and that $r > 0$. With income growth, this condition requires that $r > \mu > 0$, a condition associated with dynamic efficiency (see Abel, Mankiw, Summers, and Zeckhauser 1989). But, in fact, the safe real rate of return in the United States is less than the economy's average growth rate, a situation that can arise in a stochastic environment. Bohn (1991d, 1995) analyzes sustainability in a stochastic environment and argues that, in the absence of lump-sum taxes, even seemingly prudent fiscal policies such as running a balanced budget may be unsustainable. The problem arises if the growth rate of real income is a unit root process that can take on negative values. In this case, there is a positive probability of large income declines that can make the debt-to-income ratio become large enough to threaten sustainability. At a minimum, Bohn's results are a reminder that the simple implications of debt sustainability derived in the case of certainty may not carry over to more realistic settings.

4.3 Money and Fiscal Policy Frameworks

Most analyses of monetary phenomena and monetary policy assume, usually without statement, that variations in the stock of money matter but that how that variation occurs does not. The nominal money supply could change due to a shift from tax-financed government expenditures to seigniorage-financed expenditures. Or it could change as the result of an open market operation in which the central bank purchases interest-bearing debt, financing the purchase by an increase in noninterest-bearing debt, holding other taxes constant (see 4.2). Because these two means of increasing the money stock have differing implications for taxes and the stock of interest-bearing government debt, they may lead to different effects on prices and/or interest rates.

The government sector's budget constraint links monetary and fiscal policies in ways that can matter for determining how a change in the money stock affects the equilibrium price level.[9] The budget link also means that one needs to be precise about defining monetary policy as distinct from fiscal policy. An open market purchase increases the stock of money, but by reducing the interest-bearing government debt held by the public, it has implications for the future stream of taxes needed to finance the interest cost of the government's debt. So an open market operation potentially has a fiscal side to it, and this fact can lead to ambiguity in defining what one means by a change in monetary policy, *holding fiscal policy constant.*

The literature in monetary economics has analyzed several alternative assumptions about the relationship between monetary and fiscal policies. In most traditional analyses, fiscal policy is assumed to adjust to ensure that the government's intertemporal budget is always in balance, while monetary policy is free to set the nominal money stock or the nominal rate of interest. This situation is described as a *Ricardian regime* (Sargent 1982), one of monetary dominance, or one in which fiscal policy is passive and monetary policy is active (Leeper 1991). The models of chapters 2 and 3 implicitly fall into this category in that fiscal policy was ignored and monetary policy determined the price level. Traditional quantity theory relationships were obtained—one-time proportional changes in the nominal quantity of money led to equal proportional changes in the price level.

9. See, for example, Sargent and Wallace (1981) and Wallace (1981). The importance of the budget constraint for the analysis of monetary topics is clearly illustrated in Sargent (1987).

If fiscal policy affects the real rate of interest,[10] then the price level is not independent of fiscal policy, even under regimes of monetary dominance. A balanced budget increase in expenditures that raises the real interest rate raises the nominal interest rate and lowers the real demand for money. Given an exogenous path for the nominal money supply, the price level must jump up to reduce the real supply of money.

A second policy regime is one in which the fiscal authority sets its expenditure and taxes without regard to any requirement of intertemporal budget balance. If the present discounted value of these taxes is not sufficient to finance expenditures (in present value terms), seigniorage must adjust to ensure that the government's intertemporal budget constraint is satisfied. This regime is one of fiscal dominance (or active fiscal policy) and passive monetary policy, as monetary policy must adjust to deliver the level of seigniorage required to balance the government's budget. Prices and inflation are affected by changes in fiscal policy because these fiscal changes, if they require a change in seigniorage, alter the current and/or future money supply. Aiyagari and Gertler (1985), following Sargent (1982), describe this regime as *non-Ricardian*, although more recent usage describes any regime in which taxes and/or seigniorage always adjust to ensure that the government's intertemporal budget constraint is satisfied as Ricardian. Regimes of fiscal dominance are analyzed in section 4.3.1.

Finally, a third regime that has attracted recent attention leads to what has become known as the fiscal theory of the price level (Sims 1994; Woodford 1995; 2001; Cochrane 1998a). In this regime, the government's intertemporal budget constraint may not be satisfied for arbitrary price levels. Following Woodford (1995), these regimes are described as non-Ricardian. The intertemporal budget constraint is satisfied only at the equilibrium price level, and the government's nominal debt plays a critical role in determining the price level. The fiscal theory of the price level is analyzed in section 4.3.2.

4.3.1 Fiscal Dominance, Deficits, and Inflation

The intertemporal budget constraint implies that any government with a current outstanding debt must run, in present value terms, future surpluses. One way to

10. That is, if Ricardian equivalence does not hold.

generate a surplus is to increase revenues from seigniorage, and for that reason, economists have been interested in the implications of budget deficits for future money growth. Two questions have formed the focus of studies of deficits and inflation: First, do fiscal deficits necessarily imply that inflation will eventually occur? Second, if inflation is not a necessary consequence of deficits, is it in fact a historical consequence?

The literature on the first question has focused on the implications for inflation if the monetary authority must act to ensure that the government's intertemporal budget is balanced. This interpretation views fiscal policy as set independently, so that the monetary authority is forced to generate enough seigniorage to satisfy the intertemporal budget balance condition. Leeper (1991) describes such a situation as one in which there is an active fiscal policy and a passive monetary policy. It is also described as a situation of *fiscal dominance.*

From (4.12), the government's intertemporal budget constraint takes the form

$$b_{t-1} = -R^{-1}\sum_{i=0}^{\infty} R^{-i}(g_{t+i} - t_{t+i} - s_{t+i}),$$

where $R = 1 + r$ is the gross real interest rate, $g_t - t_t - s_t$ is the primary deficit, and s_t is real seigniorage revenue. Let $s_t^f \equiv t_t - g_t$ be the primary *fiscal* surplus (i.e., tax revenues minus expenditures but excluding interest payments and seigniorage revenue). Then the government's budget constraint can be written as

$$b_{t-1} = R^{-1}\sum_{i=0}^{\infty} R^{-i}s_{t+i}^f + R^{-1}\sum_{i=0}^{\infty} R^{-i}s_{t+i}. \tag{4.13}$$

The current real liabilities of the government must be financed by, in present value terms, either a fiscal primary surplus $R^{-1}\sum_{i=0}^{\infty} R^{-i}s_{t+i}^f$ or seigniorage.

Given the real value of the government's liabilities b_{t-1}, (4.13) illustrates what Sargent and Wallace (1981) described as "unpleasant monetarist arithmetic" in a regime of fiscal dominance. If the present value of the fiscal primary surplus is reduced, the present value of seigniorage must rise to maintain (4.13). Or, for a given present value of s^f, an attempt by the monetary authority to reduce inflation and seigniorage today must lead to higher inflation and seigniorage in the future, because the present discounted value of seigniorage cannot be altered. The mechanism is straightforward; if current inflation tax revenues are lowered, the deficit grows and the stock of debt rises. This implies an increase in the present discounted value of future tax revenues, including revenues from seigniorage. If the fiscal authority does not adjust, the monetary authority will be forced eventually to produce higher inflation.[11]

The literature on the second question—has inflation been a consequence of deficits historically?—has focused on estimating empirically the effects of deficits on money growth. Joines (1985) finds money growth in the United States to be positively related to major war spending but not to nonwar deficits. Grier and Neiman (1987) summarize a number of earlier studies of the relationship between deficits and money growth (and other measures of monetary policy) in the United States. That the results are generally inconclusive is perhaps not surprising, as the studies they review were all based on postwar but pre-1980 data. Thus, the samples covered periods in which there was relatively little deficit variation and in which much of the existing variation arose from the endogenous response of deficits to the business cycle as tax revenues varied procyclically.[12] Grier and Neiman do find that the structural (high-employment) deficit is a determinant of money growth. This finding is consistent with that of King and Plosser (1985), who report that the fiscal deficit does help to predict future seigniorage for the United States. They interpret this as mixed evidence for fiscal dominance.

Demopoulos, Katsimbris, and Miller (1987) provide evidence on debt accommodation for eight OECD countries. These authors estimate a variety of central bank reaction functions (regression equations with alternative policy instruments on the left-hand side) in which the government deficit is included as an explanatory variable. For the post–Bretton Woods period, they find a range of outcomes, from no accommodation by the Federal Reserve and the Bundesbank to significant accommodation by the Bank of Italy and the Nederlandse Bank.

One objection to this empirical literature is that simple regressions of money growth on deficits, or unrestricted VAR used to assess Granger causality (i.e., whether deficits contain any predictive information about future money growth), ignore information about the long-run behavior of taxes, debt, and seigniorage that is implied by intertemporal budget balance. Intertemporal budget balance implies a cointegrating relationship between the primary deficit and the stock of debt. This link between the components of the deficit and the stock of debt restricts the time-series behavior of expenditures, taxes, and seigniorage, and this fact in turn implies that

11. In a regime of monetary dominance, the monetary authority can determine inflation and seigniorage, in which case the fiscal authority must adjust either taxes or spending to ensure that (4.13) is satisfied.

12. For that reason, some of the studies cited by Grier and Neiman employed a measure of the high-employment surplus (i.e., the surplus estimated to occur if the economy had been at full employment). Grier and Neiman conclude, "The high employment deficit (surplus) seems to have a better 'batting average.' . . ." (p. 204).

empirical modeling of their behavior should be carried out within the framework of a vector error correction model (VECM).[13]

Suppose $X_t = (g_t \quad T_t \quad b_{t-1})$, where $T = t + s$ is defined as total government receipts from taxes and seigniorage. If the elements of X are nonstationary, intertemporal budget balance implies that the deficit inclusive of interest, or $(1 \quad -1 \quad r)X_t = \beta' X_t = g_t - T_t + rb_{t-1}$, is stationary. Hence, $\beta' = (1 \quad -1 \quad r)$ is a cointegrating vector for X. The appropriate specification of the time-series process is then a VECM of the form

$$C(L)\Delta X_t = -\alpha\beta' X_t + e_t. \tag{4.14}$$

The presence of the deficit inclusive of interest, $\beta' X_t$, ensures that the elements of X cannot drift too far apart; doing so would violate intertemporal budget balance.

Bohn (1991a) has estimated a model of the form (4.14) using U.S. data from 1800 to 1988. Unfortunately for our purposes, Bohn does not treat seigniorage separately, and thus his results are not directly relevant for determining the effects of spending or tax shocks on the adjustment of seigniorage. He does find, however, that one-half to two-thirds of a deficit initiated by a tax revenue shock are eventually eliminated by spending adjustments, while about one-third of spending shocks are essentially permanent and result in tax changes.

Ricardian and (Traditional) Non-Ricardian Fiscal Policies Sargent and Wallace's (1981) unpleasant monetarist arithmetic reminds us that fiscal policy and monetary policy are linked. This also means that changes in the nominal quantity of money engineered through lump-sum taxes and transfers (as in chapters 2 and 3) may have different effects than changes introduced through open market operations in which noninterest-bearing government debt is exchanged for interest-bearing debt. In an early contribution, Metzler (1951) argued that an open market purchase, that is, an increase in the nominal quantity of money held by the public and an offsetting reduction in the nominal stock of interest-bearing debt held by the public, would raise the price level less than proportionally to the increase in M. An open market operation would, therefore, affect the real stock of money and lead to a change in the equilibrium rate of interest. Metzler assumed that households' desired portfolio holdings of bonds and money depended on the expected return on bonds. An open market operation, by altering the ratio of bonds to money, requires a change in the rate of interest to induce private agents to hold the new portfolio composition of bonds and money. A price-level change proportional to the change in the nominal

13. See Engle and Granger (1987).

money supply would not restore equilibrium, because it would not restore the original ratio of nominal bonds to nominal money.

An important limitation of Metzler's analysis was its dependence on portfolio behavior that was not derived directly from the decision problem facing the agents of the model. The analysis was also limited in that it ignored the consequence for future taxes of shifts in the composition of the government's debt, a point made by Patinkin (1965). We have seen that the government's intertemporal budget constraint requires the government to run surpluses in present value terms equal to its current outstanding interest-bearing debt. An open market purchase by the monetary authority reduces the stock of interest-bearing debt held by the public. This reduction will have consequences for future expected taxes in ways that critically affect the outcome of policies that affect the stock of interest-bearing debt.

Sargent and Wallace (1981) have shown that the "backing" for government debt, whether it is ultimately paid for by taxes or by printing money, is important in determining the effects of debt issuance and open market operations. This finding can be illustrated following the analysis of Aiyagari and Gertler (1985). They use a two-period overlapping-generations model that allows debt policy to affect the real intergenerational distribution of wealth. This effect is absent from the representative-agent models we have been using, but the representative-agent framework can still be used to show how the specification of fiscal policy will have important implications for conclusions about the link between the money supply and the price level.[14]

In order to focus on debt, taxes, and seigniorage, set government purchases equal to zero and ignore population and real income growth, in which case the government's budget constraint takes the simplified form

$$(1 + r_{t-1})b_{t-1} = t_t + b_t + s_t, \tag{4.15}$$

with s_t denoting seigniorage.

In addition to the government's budget constraint, we need to specify the budget constraint of the representative agent. Assume that this agent receives an exogenous endowment y in each period and pays (lump-sum) taxes t_t in period t. She also receives interest payments on any government debt held at the start of the period; these payments, in real terms, are given by $(1 + i_{t-1})B_{t-1}/P_t$, where i_{t-1} is the nominal interest rate in period $t-1$, B_{t-1} is the number of bonds held at the start of period t, and P_t is the period t price level. We can write this equivalently as $(1 + r_{t-1})b_{t-1}$, where $r_{t-1} = (1 + i_{t-1})/(1 + \pi_t) - 1$ is the ex post real rate of interest.

14. See also Woodford (1995, 2001) and section 4.3.2.

Finally, the agent has real money balances equal to $M_{t-1}/P_t = (1+\pi_t)^{-1} m_{t-1}$ that are carried into period t from period $t-1$. The agent allocates these resources to consumption, real money holdings, and real bond purchases:

$$c_t + m_t + b_t = y + (1 + r_{t-1})b_{t-1} + \frac{m_{t-1}}{1+\pi_t} - t_t. \tag{4.16}$$

Aiyagari and Gertler (1985) ask whether the price level will depend only on the stock of money or whether debt policy and the behavior of the stock of debt might also be relevant for price level determination. They assume that the government sets taxes to back a fraction ψ of its interest-bearing debt liabilities, with $0 \le \psi \le 1$. If $\psi = 1$, government interest-bearing debt is completely backed by taxes in the sense that the government commits to maintaining the present discounted value of current and future tax receipts equal to its outstanding debt liabilities. Such a fiscal policy is called Ricardian by Sargent (1982).[15] If $\psi < 1$, Aiyagari and Gertler characterize fiscal policy as non-Ricardian. To avoid confusion with the more recent interpretations of non-Ricardian regimes (see section 4.3.2), let regimes where $\psi < 1$ be referred to as *traditional non-Ricardian* regimes. In such regimes, seigniorage must adjust to maintain the present value of taxes plus seigniorage equal to the government's outstanding debt.

Let T_t now denote the present discounted value of taxes. Under the assumed debt policy, the government ensures that $T_t = \psi(1 + r_{t-1})b_{t-1}$ since $(1 + r_{t-1})b_{t-1}$ is the net liability of the government (including its current interest payment). Because T_t is a present value, we can also write

$$T_t = t_t + \mathrm{E}_t\left(\frac{T_{t+1}}{1+r_t}\right) = t_t + \mathrm{E}_t\left[\frac{\psi(1+r_t)b_t}{(1+r_t)}\right]$$

or $T_t = t_t + \psi b_t$. Now because $T_t = \psi(1 + r_{t-1})b_{t-1}$, it follows that

$$t_t = \psi(R_{t-1}b_{t-1} - b_t), \tag{4.17}$$

where $R = 1 + r$. Similarly, $s_t = (1 - \psi)(R_{t-1}b_{t-1} - b_t)$. With taxes adjusting to ensure that the fraction ψ of the government's debt liabilities is backed by taxes, the remaining fraction, $1 - \psi$, represents the portion backed by seigniorage.

15. It is more common for Ricardo's name to be linked with debt in the form of the Ricardian Equivalence Theorem, under which shifts between debt and tax financing of a given expenditure stream have no real effects. See Barro (1974) or Romer (2001). Ricardian Equivalence holds in the representative-agent framework we are using; the issue is whether debt policy, as characterized by ψ, matters for price-level determination.

Given (4.17), the household's budget constraint (4.16) becomes

$$y + (1-\psi)R_{t-1}b_{t-1} + \frac{m_{t-1}}{1+\pi_t} = c_t + m_t + (1-\psi)b_t.$$

In the Ricardian case ($\psi = 1$), all terms involving the government's debt drop out; only the stock of money matters. If $\psi < 1$, however, debt does not drop out. We can then rewrite the budget constraint as $y + R_{t-1}w_{t-1} = c_t + w_t + i_{t-1}m_{t-1}/(1+\pi_t)$, where $w = m + (1-\psi)b$, showing that the relevant measure of household income is $y + R_{t-1}w_{t-1}$ and this is then used to purchase consumption, financial assets, or money balances (where the opportunity cost of money is $i/(1+\pi)$). With asset demand depending on ψ through w_{t-1}, the equilibrium price level and nominal rate of interest will generally depend on ψ.[16]

While we have derived the representative agent's budget constraint and shown how it is affected by the means the government uses to back its debt, to actually determine the effects on the equilibrium price level and nominal interest rate, we must determine the agent's demand for money and bonds and then equate these demands to the (exogenous) supplies. To illustrate the role of debt policy, assume log separable utility, $\ln c_t + \delta \ln m_t$, and consider a perfect foresight equilibrium. From chapter 2, we know that the marginal rate of substitution between money and consumption will be set equal to $i_t/(1+i_t)$. With log utility, this implies $m_t = \delta c_t(1+i_t)/i_t$. The Euler condition for the optimal consumption path yields $c_{t+1} = \beta(1+r_t)c_t$. Using these in the agent's budget constraint,

$$\begin{aligned} y + R_{t-1}w_{t-1} &= c_t + w_t + \left(\frac{i_{t-1}}{1+\pi_t}\right)\delta\left(\frac{1+i_{t-1}}{i_{t-1}}\right)\frac{c_t}{\beta(1+r_{t-1})} \\ &= \left(1 + \frac{\delta}{\beta}\right)c_t + w_t. \end{aligned}$$

In equilibrium, $c_t = y$, so this becomes $R_{t-1}w_{t-1} = (\delta/\beta)y + w_t$. If we consider the steady state, $w_t = w_{t-1} = w^{ss} = \delta y/\beta(R-1)$. But $w = [M + (1-\psi)B]/P$, so the equilibrium steady-state price level is equal to

$$P^{ss} = \left(\frac{\beta r^{ss}}{\delta y}\right)[M + (1-\psi)B]. \tag{4.18}$$

If government debt is entirely backed by taxes ($\psi = 1$), we get the standard result; the

16. In this example, $c = y$ in equilibrium since there is no capital good that would allow the endowment to be transferred over time.

price level is proportional to the nominal stock of money. The stock of debt has no effect on the price level. With $0 < \psi < 1$, however, both the nominal money supply and the nominal stock of debt play a role in price level determination. Proportional changes in M *and* B produce proportional changes in the price level; increases in the government's total nominal debt, $M + B$, raise P^{ss} proportionately.

In a steady state, all nominal quantities and the price level must change at the same rate since real values are constant. Thus, if M grows, then B must also grow at the same rate. The real issue is whether the composition of the government's liabilities matters for the price level. To focus more clearly on that issue, let $\lambda = M/(M+B)$ be the fraction of government liabilities that consists of noninterest-bearing debt. Since open market operations affect the relative proportions of money and bonds in government liabilities, open market operations determine λ. Equation (4.18) can then be written as

$$P^{ss} = \left(\frac{\beta r^{ss}}{\delta y}\right)[1 - \psi(1-\lambda)](M+B).$$

Open market purchases (an increase in λ) that substitute money for bonds but leave $M + B$ unchanged raise P^{ss} when $\psi > 0$. The rise in P^{ss} is not proportional to the increase in M. Shifting the composition of its liabilities away from interest-bearing debt reduces the present discounted value of the private sector's tax liabilities by less than the fall in debt holdings; a rise in the price level proportional to the rise in M would leave households' real wealth lower (their bond holdings are reduced in real value, but the decline in the real value of their tax liabilities is only $\psi < 1$ times as large).

Leeper (1991) argues that even if $\psi = 1$ on average (that is, all debt is backed by taxes), the means used to finance shocks to the government's budget have important implications. He distinguishes between active and passive policies; in an active monetary policy and a passive fiscal policy, monetary policy acts to target nominal interest rates and does not respond to the government's debt, while fiscal policy must then adjust taxes to ensure intertemporal budget balance (i.e., a Ricardian fiscal policy). Conversely, in an active fiscal policy and a passive monetary policy, the monetary authority must adjust seigniorage revenues to ensure intertemporal budget balance, while fiscal policy does not respond to shocks to debt. Leeper shows that the inflation and debt processes are unstable if both policy authorities follow active policies, while there is price level indeterminacy if both follow passive policies.

The Government Budget Constraint and the Nominal Rate of Interest Earlier, we examined Sargent and Wallace's unpleasant monetarist arithmetic using (4.13).

Given the government's real liabilities, the monetary authority would be forced to finance any difference between these real liabilities and the present discounted value of the government's fiscal surpluses. Fiscal considerations determine the money supply, but the traditional quantity theory holds and the price level is proportional to the nominal quantity of money. Suppose, however, that the initial nominal stock of money is set exogenously by the monetary authority. Does this mean that the price level is determined solely by monetary policy, with no effect of fiscal policy? In fact, the following example illustrates how fiscal policy can affect the initial equilibrium price level, even when the initial nominal quantity of money is given and the government's intertemporal budget constraint must be satisfied at all price levels.

Consider a perfect foresight equilibrium. In such an equilibrium, the government's budget constraint must be satisfied and the real demand for money must equal the real supply of money. The money-in-the-utility function (MIU) model of chapter 2 can be used, for example, to derive the real demand for money. That model implied that agents would equate the marginal rate of substitution between money and consumption to the cost of holding money, where this cost depended on the nominal rate of interest:

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1+i_t}.$$

Using the utility function employed in chapter 2,[17] this condition implies that

$$m_t = \frac{M_t}{P_t} = \left[\left(\frac{i_t}{1+i_t}\right)\left(\frac{a}{1-a}\right)\right]^{-\frac{1}{b}} c_t.$$

Evaluated at the economy's steady state, this can be written as

$$\frac{M_t}{P_t} = f(R_{m,t}), \tag{4.19}$$

where $R_m = 1 + i$ is the gross nominal rate of interest and $f(R_m) = [a(R_m - 1)/R_m(1-a)]^{-\frac{1}{b}}c$. Given the nominal interest rate, (4.19) implies a proportional relationship between the nominal quantity of money and the equilibrium price level. If the initial money stock is M_0, then the initial price level is $P_0 = M_0/f(R_m)$.

17. In chapter 2 we assumed that

$$u(c_t, m_t) = \frac{[ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1-\Phi}{1-b}}}{1-\Phi}.$$

The government's budget constraint must also be satisfied. In a perfect-foresight equilibrium, there are no inflation surprises, so the government's budget constraint given by (4.5) can be written as

$$g_t + rb_{t-1} = t_t + (b_t - b_{t-1}) + m_t - \left(\frac{1}{1+\pi_t}\right)m_{t-1}. \tag{4.20}$$

Now consider a stationary equilibrium in which government expenditures and taxes are constant, as are the real stocks of government interest-bearing debt and money. In such a stationary equilibrium, the budget constraint becomes

$$g + \left(\frac{1}{\beta} - 1\right)b = t + \left(\frac{\pi_t}{1+\pi_t}\right)m = t + \left(\frac{\beta R_m - 1}{\beta R_m}\right)f(R_m), \tag{4.21}$$

where we have used the steady-state results that the gross real interest rate is $1/\beta$, $R_m \equiv (1+\pi_t)/\beta$, and real money balances must be consistent with the demand given by (4.19).

Suppose the fiscal authority sets g, t, and b. Then (4.21) determines the nominal interest rate R_m. With g, t, and b given, the government needs to raise $g + (1/\beta - 1)b - t$ in seigniorage. The nominal interest rate is determined by the requirement that this level of seigniorage be raised.[18] Because the nominal interest rate is equal to $(1+\pi_t)/\beta$, we can alternatively say that fiscal policy determines the inflation rate. Once the nominal interest rate is determined, the initial price level is given by (4.19) as $P_0 = M_0/f(R_m)$, where M_0 is the initial stock of money. In subsequent periods, the price level is equal to $P_t = P_0(\beta R_m)^t$, where $\beta R_m = (1+\pi_t)$ is the gross inflation rate. The nominal stock of money in each future period is endogenously determined by $M_t = P_t f(R_m)$. In this case, even though the monetary authority has set M_0 exogenously, the initial price level is determined by the need for fiscal solvency since the fiscal authority's budget requirement (4.21) determines R_m and therefore the real demand for money. The initial price level is proportional to the initial money stock, but the factor of proportionality $(1/f(R_m))$ is determined by fiscal policy, and both the rate of inflation and the path of the future nominal money supply are determined by the fiscal requirement that seigniorage equal $g + (1/\beta - 1)b - t$.

If the fiscal authority raises expenditures, holding b and t constant, then seigniorage must rise. The equilibrium nominal interest rate rises to generate this additional

18. The nominal interest rate that raises seigniorage equal to $g + (1/\beta - 1)b - t$ may not be unique. A rise in R_m increases the tax rate on money, but it also erodes the tax base by reducing the real demand for money. A given amount of seigniorage may be raised with a low tax rate and a high base or a high tax rate and a low base.

seigniorage.[19] With a higher R_m, the real demand for money falls, and this increases the equilibrium value of the initial price level P_0, even though the initial nominal quantity of money is unchanged.

Equilibrium Seigniorage Under a regime of fiscal dominance, seigniorage is determined by the requirement that (4.13) hold. Suppose that, given its expenditures and other tax sources, the government has a fiscal deficit of Δ^f that must be financed by money creation. When will it be feasible to raise Δ^f in a steady-state equilibrium? And what will be the equilibrium rate of inflation?

The answers to these questions would be straightforward if there were a one-to-one relationship between the revenue generated by the inflation tax and the inflation rate. If this were the case, the inflation rate would be uniquely determined by the amount of revenue that must be raised. But the inflation rate affects the base against which the tax is levied. For a given base, a higher inflation rate raises seigniorage, but a higher inflation rate raises the opportunity cost of holding money and reduces the demand for money, thereby lowering the base against which the tax is levied. This raises the possibility that a given amount of revenue can be raised by more than one rate of inflation. For example, the nominal rate of interest R_m that satisfies (4.21) may not be unique.

It will be helpful to impose additional structure so that we can say more about the demand for money.[20] The standard approach used in most analyses of seigniorage is to specify directly a functional form for the demand for money as a function of the nominal rate of interest. An early example of this approach, and one of the most influential, is that of Cagan (1956); a more recent example is Bruno and Fischer (1990). We will return to this approach, but we start by following Calvo and Leiderman (1992) in using a variant of the Sidrauski model of chapter 2 to motivate a demand for money. That is, suppose the economy consists of identical individuals, and the utility of the representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \tag{4.22}$$

where $0 < \beta < 1$, c is per capita consumption, m is per capita real money holdings, and the function $u(.)$ is strictly concave and twice continuously differentiable. The

19. This assumes that the economy is on the positively sloped portion of the Laffer curve so that raising the tax rate increases revenue.

20. The base for the inflation tax is the stock of high-powered money. Earlier, this measure of money was denoted by H. For simplicity, we will follow the literature in referring to this as the *quantity of money* and denote its real value by m.

representative agent chooses consumption, money balances, and holdings of interest-earning bonds to maximize the expected value of (4.22), subject the following budget constraint:

$$c_t + b_t + m_t = y_t - \tau_t + (1+r)b_{t-1} + \frac{m_{t-1}}{\Pi_t},$$

where b is the agent's holdings of bonds, y is real income, τ is equal to the net taxes of the agent, r is the real rate of interest, assumed constant for simplicity, and $\Pi_t \equiv P_t/P_{t-1} = 1 + \pi_t$, where π_t is the inflation rate. Thus, the last term in the budget constraint, m_{t-1}/Π_t, is equal to the period t real value of money balances carried into period t, that is, M_{t-1}/P_t, where M represents nominal money holdings. We will restrict attention to perfect-foresight equilibria.

If we define w_t as the agent's real wealth in period t, $w_t = b_t + m_t$, and let $R_t = 1 + r_t$, then the budget constraint can be rewritten as

$$c_t + w_t = y_t - \tau_t + R_{t-1}w_{t-1} - \left(\frac{R_{t-1}\Pi_t - 1}{\Pi_t}\right)m_{t-1}$$

$$= y_t - \tau_t + R_{t-1}w_{t-1} - \left(\frac{i_{t-1}}{\Pi_t}\right)m_{t-1}$$

by using the fact that $R\Pi = 1 + i$, where i is the nominal rate of interest. Writing the budget constraint in this way, we can see that the cost of holding wealth in the form of money, as opposed to interest-earning bonds, is i/Π.[21] The first order condition for optimal money holdings sets the marginal utility of money equal to the cost of holding money times the marginal utility of wealth. Since the interest forgone by holding money in period t is a cost that is incurred in period $t+1$, this cost must be discounted back to period t using the discount factor β to compare with the marginal utility of money in period t. Thus, $u_m(c_t, m_t) = \beta(i_t/\Pi_{t+1})\mathrm{E}_t u_c(c_{t+1}, m_{t+1})$. But the standard Euler condition for optimal consumption implies that $u_c(c_t, m_t) = \beta R_t \mathrm{E}_t u_c(c_{t+1}, m_{t+1})$. Combining these first order conditions yields

$$u_m(c_t, m_t) = \left(\frac{i_t}{R_t\Pi_{t+1}}\right)u_c(c_t, m_t) = \left(\frac{i_t}{1+i_t}\right)u_c(c_t, m_t). \qquad (4.23)$$

Now suppose the utility function takes the form $u(c_t, m_t) = \ln c_t + m_t(B - D\ln m_t)$.

21. Recall from the derivation of (4.8) that the term for the government's revenue from seigniorage was $(i_{t-1}/\Pi_t)h_{t-1}$. Comparing this to the household's budget constraint (with $h_{t-1} = m_{t-1}$) shows that the cost of holding money is exactly equal to the revenue obtained by the government.

If we use this functional form in (4.23), we obtain

$$m_t = Ae^{-\omega_t/Dc_t}, \qquad (4.24)$$

where $A = e^{(\frac{B}{D}-1)}$ and $\omega = i/(1+i)$. Equation (4.24) provides a convenient functional representation for the demand for money.

Since the time of Cagan's seminal contribution to the study of seigniorage and hyperinflations (Cagan 1956; see pages 158–161), many economists have followed him in specifying a money demand function of the form $m = Ke^{-\alpha\pi^e}$; (4.24) shows how something similar can be derived from an underlying utility function. As Calvo and Leiderman (1992) point out, the advantage is that one sees how the parameters K and α depend on more primitive parameters of the representative agent's preferences and how they may actually be time dependent. For example, α depends on c_t and, as a consequence, will be time dependent unless K varies appropriately or c itself is constant.

The reason for deriving the demand for money as a function of the rate of inflation is that, having done so, we can now express seigniorage as a function of the rate of inflation. Recall from (4.9) that seigniorage was equal to $im/(1+\pi) = (1+r)im/(1+i)$. Using our expression for the demand for money, steady-state seigniorage is equal to

$$\bar{s} = (1+r)\left(\frac{i}{1+i}\right)A\exp\left[-\frac{i}{Dc(1+i)}\right].$$

If we assume that superneutrality characterizes the model, then c will be constant in the steady state and independent of the rate of inflation. The same will be true of the real rate of interest.

To determine how seigniorage varies with the rate of inflation, think of choosing $\omega = i/(1+i)$ through the choice of π. Then $\bar{s} = (1+r)\omega Ae^{-\omega/Dc}$ and

$$\frac{\partial\bar{s}}{\partial\omega} = (1+r)Ae^{-\omega/Dc}\left[1 - \frac{\omega}{Dc}\right] = \frac{\bar{s}}{\omega}\left[1 - \frac{\omega}{Dc}\right].$$

Since $\partial\bar{s}/\partial\pi = (\partial\bar{s}/\partial\omega)(\partial\omega/\partial i)(\partial i/\partial\pi) = (\partial\bar{s}/\partial\omega)(1+r)/(1+i)^2$, the sign of $\partial\bar{s}/\partial\pi$ will be determined by the sign of $(\partial\bar{s}/\partial\omega)$, and that, in turn, depends on the sign of $1 - (\omega/Dc)$. As illustrated in figure 4.1, seigniorage increases with inflation initially but eventually begins to decline with further increases in π as the demand for real balances shrinks.[22]

22. Whether a Laffer curve exists for seigniorage depends on the specification of utility. For example, in chapter 2 we saw that with a CES utility function, the demand for money was given by $m_t = A[i/(1+i)]^{-\frac{1}{b}}c_t$, where A is a constant. Hence, seigniorage is $A[i/(1+i)]^{1-\frac{1}{b}}c_t$, which is monotonic in i.

Figure 4.1
Seigniorage as a Function of Inflation

To determine the inflation rate that maximizes seigniorage, note that $\partial \bar{s}/\partial \pi = 0$ if and only if

$$\omega = \frac{i}{1+i} = Dc \quad \text{or} \quad \pi^{\max} = \left(\frac{1}{1+r}\right)\left(\frac{1}{1-Dc}\right) - 1.$$

For inflation rates less than $\pi^{\max}$, the government's revenue is increasing in the inflation rate. The effect of an increase in the tax rate dominates the effect of higher inflation in reducing the real demand for money. As inflation increases above $\pi^{\max}$, the tax base shrinks sufficiently that revenues from seigniorage decline. Consequently, governments face a seigniorage Laffer curve; raising inflation beyond a certain point results in lower real tax revenue.

Cagan's Model Since 1970, the consumer price index for the United States has risen just over 4.5-fold; that is inflation.[23] In Hungary, the index of wholesale prices was 38,500 in January 1923 and 1,026,000 in January 1924, one year later, a 27-fold increase; that is hyperinflation (Sargent 1986, p. 64).

23. The CPI was equal to 38.8 in 1970 and had reached 179.5 in May 2002.

One of the earliest studies of the dynamics of money and prices during hyperinflation was done by Cagan (1956). We will follow Cagan in using continuous time. Suppose the real per capita fiscal deficit that needs to be financed is exogenously given and is equal to Δ^f. Using (4.7), this means that

$$\Delta^f = \frac{\dot{H}}{H}\frac{H}{PY} = \theta h.$$

The demand for real balances will depend on the nominal interest rate and therefore the expected rate of inflation. Treating real variables such as the real rate of interest and real output as constant (which is appropriate in a steady state characterized by superneutrality and is usually taken as reasonable during hyperinflations since all the action involves money and prices), write the demand for the real monetary base as $h = \exp(-\alpha\pi^e)$. Then the government's revenue requirement implies that

$$\Delta^f = \theta e^{-\alpha\pi^e}. \tag{4.25}$$

We also know that for h to be constant in equilibrium requires that $\pi = \theta - \mu$, where μ is the growth rate of real income. And in a steady-state equilibrium, $\pi^e = \pi$, so (4.25) becomes

$$\Delta^f = \theta e^{-\alpha(\theta-\mu)}, \tag{4.26}$$

the solution(s) of which give the rates of money growth that are consistent with raising the amount Δ^f through seigniorage. The right side of (4.26) equals zero when money growth is equal to zero, rises to a maximum at $\theta = (1/\alpha)$, and then declines.[24] That is, for rates of money growth above $(1/\alpha)$ (and therefore inflation rates above $(1/\alpha) - \mu$), higher inflation actually leads to lower revenues because the tax base falls sufficiently to offset the rise in inflation. Thus, any deficit less than $\Delta^* = (1/\alpha)\exp(\alpha\mu - 1)$ can be financed by either a low rate of inflation or a high rate of inflation.

Figure 4.2, based on Bruno and Fischer (1990), illustrates the two inflation rates consistent with seigniorage revenues of Δ^f. The curve SR is derived from (4.25) and shows, for each rate of money growth, the expected rate of inflation needed to generate the required seigniorage revenues.[25] The 45° line gives the steady-state inflation

24. More generally, with h a function of the nominal interest rate and r a constant, seigniorage can be written as $s = \theta h(\theta)$. This is maximized at the point where the elasticity of real money demand with respect to θ is equal to -1: $\theta h'(\theta)/h = -1$.

25. That is, SR plots $\pi^e = (\ln\theta - \ln\Delta^f)/\alpha$. A reduction in θ continues to yield Δ^f only if money holdings rise, and this would require a fall in expected inflation.

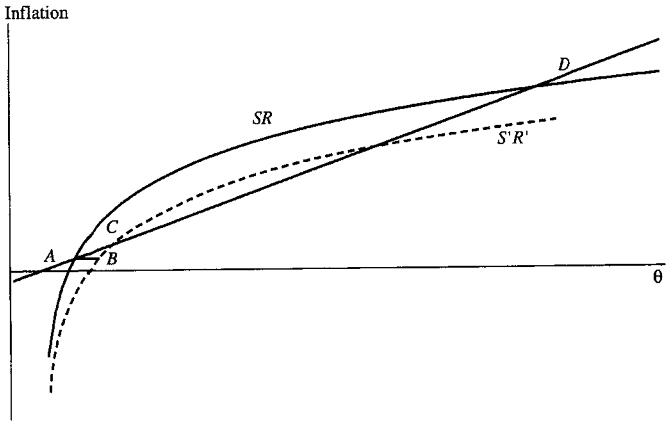

Figure 4.2
Money Growth and Seigniorage Revenue

rate as a function of the money growth rate: $\pi^e = \pi = \theta - \mu$. The two points of intersection labeled A and D are the two solutions to (4.26).

What determines whether, for a given deficit, the economy ends up at the high inflation equilibrium or the low inflation equilibrium? Which equilibrium is picked out depends on the stability properties of the economy. Determining this, in turn, requires a more complete specification of the dynamics of the model. Recall that the demand for money depends on expected inflation through the nominal rate of interest, while the inflation tax rate depends on actual inflation. In considering the effects of variations in the inflation rate, we need to determine how expectations will adjust. Cagan (1956) addressed this by assuming that expectations adjust adaptively to actual inflation:

$$\frac{\partial \pi^e}{\partial t} = \dot{\pi}^e = \eta(\pi - \pi^e), \tag{4.27}$$

where η captures the "speed of adjustment" of expectations. A low η implies that expectations respond slowly to inflation forecast errors. Since $h = \exp(-\alpha\pi^e)$, differentiate this expression with respect to time, obtaining

$$\frac{\dot{h}}{h} = \theta - \mu - \pi = -\alpha\dot{\pi}^e.$$

Solving for π using (4.27) yields $\pi = \theta - \mu + \alpha\dot{\pi}^e = \theta - \mu + \alpha\eta(\pi - \pi^e)$ or $\pi = (\theta - \mu - \alpha\eta\pi^e)/(1 - \alpha\eta)$. Substituting this back into the expectations adjustment equation gives

$$\dot{\pi}^e = \frac{\eta(\theta - \mu - \pi^e)}{1 - \alpha\eta}, \tag{4.28}$$

which implies that the low inflation equilibrium will be stable as long as $\alpha\eta < 1$. This requires that expectations adjust sufficiently slowly ($\eta < 1/\alpha$).

If we assume that expectations adjust adaptively and sufficiently slowly, consider what happens when the deficit is increased. Since the demand for real money balances depends on expected inflation, and because the adjustment process does not allow the expected inflation rate to jump immediately, the higher deficit can be financed by an increase in the rate of inflation (assuming the new deficit is still below the maximum that can be financed, Δ^*). Since actual inflation now exceeds expected inflation, $\dot{\pi}^e > 0$ and π^e begins to rise. The economy converges into a new equilibrium at a higher rate of inflation.

In terms of figure 4.2, an increase in the deficit shifts the SR line to the right to $S'R'$ (for a given expected rate of inflation, money growth must rise in order to generate more revenue). Assume that initially the economy is at point A, the low inflation equilibrium. Budget balance requires that the economy be on the $S'R'$ line, so θ jumps to the rate associated with point B. But now, at point B, inflation has risen and $\pi^e < \pi = \theta - \mu$. Expected inflation rises (as long as $\alpha\eta < 1$; see 4.28), and the economy converges to C. The high inflation equilibrium, in contrast, is unstable.

Adaptive expectations of the sort Cagan assumed disappeared from the literature under the onslaught of the rational-expectations revolution begun by Lucas and Sargent in the early 1970s. If agents are systematically attempting to forecast inflation, then their forecast will depend on the actual process governing the evolution of inflation; rarely will this imply an adjustment process such as (4.27). Stability in the Cagan model also requires that expectations not adjust too quickly ($\eta < 1/\alpha$), and this requirement conflicts with the rational-expectations notion that expectations adjust quickly in response to new information. Bruno and Fischer (1990) show that, to some degree, assuming agents adjust their holdings of real money balances slowly plays a role under rational expectations similar to the role played by the slow adjustment of expectations in Cagan's model in ensuring stability under adaptive expectations.

Rational Hyperinflation Why do countries find themselves in situations of hyperinflation? Most explanations of hyperinflation point to fiscal sources as the chief culprit. Governments that are forced to print money to finance real government expenditures often end up generating hyperinflations. In that sense, rapid money growth does lead to hyperinflation, consistent with the relationship between money growth and inflation implied by the models we have examined, but money growth is no longer exogenous. Instead, it is endogenously determined by the need to finance a fiscal deficit.

Two explanations for the development of hyperinflation suggest themselves. In the Cagan model with adaptive expectations, suppose that $\alpha\eta < 1$ so that the low inflation equilibrium is stable. Now suppose that a shock pushes the inflation rate above the high inflation equilibrium (above point D in figure 4.2). If that equilibrium is unstable, the economy continues to diverge—moving to higher and higher rates of inflation. So one explanation for hyperinflations is that they represent situations in which exogenous shocks push the economy into an unstable region.

Alternatively, suppose the deficit that needs to be financed with seigniorage grows. If it rises above Δ^*, the maximum that can be financed by money creation, the government finds itself unable to obtain enough revenue, so it runs the printing presses faster, further reducing the real revenue it obtains and forcing it to print money even faster. Most hyperinflations have occurred after wars (and on the losing side). Such countries face an economy devastated by war and a tax system that no longer functions effectively. At the same time, there are enormous demands on the government for expenditures to provide the basics of food and shelter and to rebuild the economy. Revenue needs outpace the government's ability to raise tax revenues. The ends of such hyperinflations usually involve a fiscal reform that allows the government to reduce its reliance on seigniorage (see Sargent 1986).

When expected inflation falls in response to the reforms, the opportunity cost of holding money is reduced and the demand for real money balances rises. Thus, the growth rate of the nominal money supply normally continues temporarily at a very high rate after a hyperinflation has ended. A similar, if smaller-scale, phenomenon, occurred in the United States in the mid-1980s. The money supply, as measured by $M1$, grew very rapidly. At the time, there were concerns that this growth would lead to a return of higher rates of inflation. Instead, it seemed to reflect the increased demand for money resulting from the decline in inflation from its peak levels in 1979–1980. The need for real money balances to grow as inflation is reduced often causes problems for establishing and maintaining the credibility of policies designed to reduce inflation. If a disinflation is credible, so that expected inflation falls, it may be necessary to increase the rate of growth of the nominal money supply temporarily. But when inflation and rapid money growth are so closely related, letting money growth rise may be misinterpreted as a signal that the central bank has given up on its disinflation policy.

Fiscal theories of seigniorage, inflation, and hyperinflations are based on fundamentals—there really is a deficit that needs to be financed, and that is what leads to money creation. An alternative view of hyperinflations is that they are simply *bubbles*, similar to bubbles in financial markets. Such phenomena are based on the possibility of multiple equilibria in which expectations can be self-fulfilling.

To illustrate this possibility, suppose the real demand for money is given by, in log terms,

$$m_t - p_t = -\alpha(\mathrm{E}_t p_{t+1} - p_t),$$

where $\mathrm{E}_t p_{t+1}$ denotes the expectation formed at time t of time $t+1$ prices and $\alpha > 0$. This money demand function is the log version of Cagan's demand function. We can rearrange this equation to express the current price level as

$$p_t = \left(\frac{1}{1+\alpha}\right)m_t + \left(\frac{\alpha}{1+\alpha}\right)\mathrm{E}_t p_{t+1}. \tag{4.29}$$

Suppose that the growth rate of the nominal money supply process is given by $m_t = \theta_0 + (1-\gamma)\theta_1 t + \gamma m_{t-1}$. Since m is the log money supply, the growth rate of the money supply is $m_t - m_{t-1} = (1-\gamma)\theta_1 + \gamma(m_{t-1} - m_{t-2})$, and the trend (average) growth rate is θ_1. Given this process, and the assumption that agents make use of it and the equilibrium condition (4.29) in forming their expectations, one solution for the price level is given by

$$\begin{aligned} p_t &= \frac{\alpha[\theta_0 + (1-\gamma)\theta_1(1+\alpha)]}{1+\alpha(1-\gamma)} + \left[\frac{\alpha(1-\gamma)\theta_1}{1+\alpha(1-\gamma)}\right]t + \left[\frac{1}{1+\alpha(1-\gamma)}\right]m_t \\ &= A_0 + A_1 t + A_2 m_t. \end{aligned}$$

That this is a solution can be verified by noting that it implies that $\mathrm{E}_t p_{t+1} = A_0 + A_1(t+1) + A_2\mathrm{E}_t m_{t+1} = A_0 + A_1(t+1) + A_2[\theta_0 + (1-\gamma)\theta_1(t+1) + \gamma m_t]$; substituting this into (4.29) yields the proposed solution. Under this solution, the inflation rate $p_t - p_{t-1}$ converges to θ_1, the average growth rate of the nominal supply of money.[26]

Consider, now, an alternative solution:

$$p_t = A_0 + A_1 t + A_2 m_t + B_t, \tag{4.30}$$

where B_t is time varying. We are interested in determining whether there exists a B_t process consistent with (4.29). Substituting the new proposed solution into the

26. This follows since $p_t - p_{t-1} = A_1 + A_2(m_t - m_{t-1})$ converges to $A_1 + A_2\theta_1 = \theta_1$.

equilibrium condition for the price level yields

$$A_0 + A_1 t + A_2 m_t + B_t = \frac{m_t}{1+\alpha} + \frac{\alpha[A_0 + A_1(t+1) + A_2\gamma m_t + \mathrm{E}_t B_{t+1}]}{1+\alpha},$$

which, to hold for all realizations of the nominal money supply, requires that, as before, $A_0 = \alpha[\theta_0 + (1-\gamma)\theta_1(1+\alpha)]/[1+\alpha(1-\gamma)]$, $A_1 = \alpha(1-\gamma)\theta_1/[1+\alpha(1-\gamma)]$, and $A_2 = 1/[1+\alpha(1-\gamma)]$. This then implies that the B_t process must satisfy

$$B_t = \left(\frac{\alpha}{1+\alpha}\right)\mathrm{E}_t B_{t+1},$$

which holds if B follows the explosive process

$$B_{t+1} = kB_t \tag{4.31}$$

for $k = (1+\alpha)/\alpha > 1$. In other words, (4.31) is an equilibrium solution for *any* process satisfying (4.31). Since B grows at the rate $k - 1 = 1/\alpha$, and since α, the elasticity of money demand with respect to expected inflation, is normally thought to be small, its inverse would be large. The actual inflation rate along a bubble solution path could greatly exceed the rate of money growth.

Obstfeld and Rogoff (1983, 1986) have considered whether speculative hyperinflations are consistent with equilibrium when agents are utility maximizing. As discussed in section 2.2.1, they show that speculative hyperinflation in unbacked fiat money systems cannot generally be ruled out. Equilibrium paths may exist along which real money balances eventually converge to zero as the price level goes to $+\infty$. (See also section 4.3.2.)

The methods developed to test for bubbles are similar to those that have been employed to test for intertemporal budget balance. For example, if the nominal money stock is nonstationary, then the absence of bubbles implies that the price level will be nonstationary *but cointegrated with the money supply*. This is a testable implication of the no-bubble assumption. Equation (4.31) gives the simplest example of a bubble process. Evans (1991) shows how the cointegration tests can fail to detect bubbles that follow periodically collapsing processes. For more on asset prices and bubbles, see Shiller (1981), Mattey and Meese (1986), West (1987, 1988), Diba and Grossman (1988a, 1988b), and Evans (1991).

4.3.2 The Fiscal Theory of the Price Level

Recently, a number of researchers have examined models in which fiscal factors replace the money supply as the key determinant of the price level (see Leeper 1991; Sims 1994; Woodford 1995, 1998b, 2001; Bohn 1998b; Cochrane 1998a; Kocherlakota and Phelen 1999; Daniel 2001, the excellent discussions by Carlstrom and Fuerst 1999b and by Christiano and Fitzgerald 2000, and references they list, and the criticisms of the approach by McCallum 2001a and Buiter 2002). The *fiscal theory of the price level* raises some important issues for both monetary theory and monetary policy.

There are two ways fiscal policy might matter for the price level. First, equilibrium requires that the real quantity of money equal the real demand for money. If fiscal variables affect the real demand for money, the equilibrium price level will also depend on fiscal factors (see section 4.3.1). This, however, is not the channel emphasized in fiscal theories of the price level. Instead, these theories focus on a second aspect of monetary models—there may be multiple price levels consistent with a given nominal quantity of money and equality between money supply and money demand. Fiscal policy may then determine which of these is the equilibrium price level. And in some cases, the equilibrium price level picked out by fiscal factors may be independent of the nominal supply of money.

In contrast to the standard monetary theories of the price level, the fiscal theory assumes that the government's intertemporal budget equation represents an equilibrium condition rather than a constraint that must hold for all price levels. At some price levels, the intertemporal budget constraint would be violated. Such price levels are not consistent with equilibrium. Given the stock of nominal debt, the equilibrium price level must ensure that the government's intertemporal budget is balanced.

The next subsection illustrates why the requirement that the real demand for money equal the real supply of money may not be sufficient to uniquely determine the equilibrium price level, even for a fixed nominal money supply. The subsequent subsection shows how fiscal considerations may serve to pin down the equilibrium price level.

Multiple Equilibria The traditional quantity theory of money highlights the role the nominal stock of money plays in determining the equilibrium price level. Using the demand for money given by (4.19), we obtained a proportional relationship between the nominal quantity of money and the equilibrium price level that depended on the nominal rate of interest. However, the nominal interest rate is also an endogenous variable, so (4.19) by itself may not be sufficient to determine the equilibrium price level. Because the nominal interest rate depends on the rate of inflation, (4.19) can be written as

$$\frac{M_t}{P_t} = f\left(R_t \frac{P_{t+1}}{P_t}\right),$$

where R is the gross real rate of interest. As we saw in section 4.3.1, this forward difference equation in the price level may be insufficient to determine a unique equilibrium path for the price level.

Consider a perfect-foresight equilibrium with a constant nominal supply of money, M_0. Suppose the real rate of return is equal to its steady-state value of $1/\beta$, and the demand for real money balances is given by (4.19). We can then write the equilibrium between the real supply of money and the real demand for money as

$$\frac{M_0}{P_t} = g\left(\frac{P_{t+1}}{P_t}\right), \quad g' < 0.$$

Under suitable regularity conditions on $g(\ )$, this condition can be rewritten as

$$P_{t+1} = P_t g^{-1}\left(\frac{M_0}{P_t}\right) \equiv \phi(P_t). \tag{4.32}$$

Equation (4.32) defines a difference equation in the price level. One solution is $P_{t+i} = P^*$ for all $i \geq 0$, where $P^* = M_0 g(1)$. In this equilibrium, the quantity theory holds, and the price level is proportional to the money supply.

This constant price level equilibrium is not, however, the only possible equilibrium. As we saw in section 4.3.1 (and in chapter 2), there may be equilibrium price paths starting from $P_0 \neq P^*$ that are fully consistent with the equilibrium condition (4.32). For example, in figure 4.3, the convex curve shows $\phi(P_t)$ as an increasing function of P_t. Also shown in the figure is the 45° line. Using the fact that $g^{-1}(M_0/P^*) = 1$, the slope of $\phi(P_t)$, evaluated at P^*, is

$$\begin{aligned}\phi'(P^*) &= g^{-1}(M_0/P^*) - [\partial g^{-1}(M_0/P^*)/\partial(M_0/P^*)](M_0/P^*)\\ &= 1 - [\partial g^{-1}(M_0/P^*)/\partial(M_0/P^*)](M_0/P^*) > 1.\end{aligned}$$

Thus, ϕ cuts the 45° line from below at P^*. Any price path starting at $P_0 = P' > P^*$ is consistent with (4.32) and involves a positive rate of inflation. As the figure illustrates, $P \to \infty$, but the equilibrium condition (4.32) is satisfied along this path. As the price level explodes, real money balances go to zero. But this is consistent with private agents' demand for money because inflation, and therefore nominal interest rates, are rising, lowering the real demand for money. Any price level to the right of P^* is a valid equilibrium. These equilibria all involve speculative hyperinflations. (Equilibria originating to the left of P^* eventually violate a transversality condition since M/P is exploding as $P \to 0$.) By itself, (4.32) is not sufficient to uniquely determine the equilibrium value of P_0, even though the nominal quantity of money is fixed.

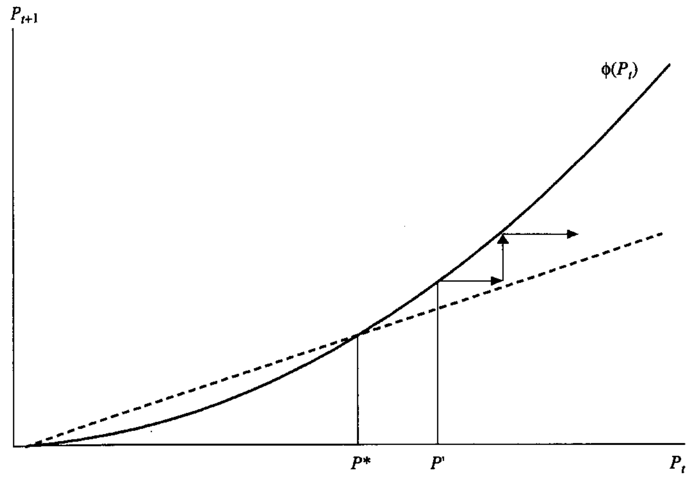

Figure 4.3
Equilibrium with a Fixed Nominal Money Supply

The Fiscal Theory Standard models in which equilibrium depends on forward-looking expectations of the price level, a property of the models discussed in chapters 2 and 3, generally have multiple equilibria. Thus, an additional equilibrium condition may be needed to uniquely determine the price level. The fiscal theory of the price level focuses on situations in which the government's intertemporal budget constraint may supply the additional equilibrium condition.

THE BASIC IDEA The fiscal theory can be illustrated in the context of a model with a representative household and a government, but with no capital. The implications of the fiscal theory will be easiest to see if attention is restricted to perfect-foresight equilibria.

The representative household chooses its consumption and asset holdings optimally, subject to an intertemporal budget constraint. Suppose the period t budget constraint of the representative household takes the form

$$D_t + P_t y_t - T_t \geq P_t c_t + M_t^d + B_t^d = P_t c_t + \left(\frac{i_t}{1+i_t}\right) M_t^d + \left(\frac{1}{1+i_t}\right) D_{t+1}^d,$$

where D_t is the household's beginning-of-period financial wealth and $D_{t+1}^d = (1+i_t)B_t^d + M_t^d$. The superscripts denote that M^d and B^d are the household's demand for money and interest-bearing debt. In real terms, this budget constraint becomes

$$d_t + y_t - \tau_t \geq c_t + m_t^d + b_t^d = c_t + \left(\frac{i_t}{1+i_t}\right)m_t^d + \left(\frac{1}{1+r_t}\right)d_{t+1}^d,$$

where $\tau_t = T_t/P_t$, $m_t^d = M_t^d/P_t$, $1 + r_t = (1+i_t)(1+\pi_{t+1})$, and $d_t = D_t/P_t$. Let

$$\lambda_{t,t+i} = \prod_{j=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)$$

be the discount factor, with $\lambda_{t,t} = 1$. Under standard assumptions, the household intertemporal budget constraint takes the form

$$d_t + \sum_{i=0}^{\infty}\lambda_{t,t+i}(y_{t+i} - \tau_{t+i}) = \sum_{i=0}^{\infty}\lambda_{t,t+i}\left[c_{t+i} + \left(\frac{i_{t+i}}{1+i_{t+i}}\right)m_{t+i}^d\right]. \tag{4.33}$$

Household choices must satisfy this intertemporal budget constraint. The left side is the present discounted value of the household's initial real financial wealth and after-tax income. The right side is the present discounted value of consumption spending plus the real cost of holding money. This condition holds with equality because any path of consumption and money holdings for which the left side exceeded the right side would not be optimal; the household could increase its consumption at time t without reducing consumption or money holdings at any other date. As long as the household is unable to accumulate debts that exceed the present value of its resources, the right side cannot exceed the left side.

The budget constraint for the government sector, in nominal terms, takes the form

$$P_t g_t + (1+i_{t-1})B_{t-1} = T_t + M_t - M_{t-1} + B_t. \tag{4.34}$$

Dividing by P_t, this can be written as

$$g_t + d_t = \tau_t + \left(\frac{i_t}{1+i_t}\right)m_t + \left(\frac{1}{1+r_t}\right)d_{t+1}.$$

Recursively substituting for future values of d_{t+i}, this budget constraint implies that

$$d_t + \sum_{i=0}^{\infty}\lambda_{t,t+i}[g_{t+i} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T\to\infty}\lambda_{t,t+T}d_T, \tag{4.35}$$

where $\bar{s}_t = i_t m_t/(1+i_t)$ is the government's real seigniorage revenue. In previous

sections, we assumed that the expenditures, taxes, and seigniorage choices of the consolidated government (the combined monetary and fiscal authorities) were constrained by the requirement that $\lim_{T\to\infty}\lambda_{t,t+T}d_T = 0$ for *all* price levels P_t. Policy paths for $(g_{t+i}, \tau_{t+i}, s_{t+i}, d_{t+i})_{i\geq 0}$ such that

$$d_t + \sum_{i=0}^{\infty}\lambda_{t,t+i}[g_{t+i} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T\to\infty}\lambda_{t,t+T}d_T = 0$$

for all price paths p_{t+i}, $i \geq 0$ are called *Ricardian* policies. Policy paths for $(g_{t+i}, \tau_{t+i}, \bar{s}_{t+i}, d_{t+i})_{i\geq 0}$ for which $\lim_{T\to\infty}\lambda_{t,t+T}d_T$ may not equal zero for all price paths are called *non-Ricardian*.[27]

Now consider a perfect-foresight equilibrium. Regardless of whether the government follows a Ricardian or a non-Ricardian policy, equilibrium in the goods market in this simple economy with no capital requires that $y_t = c_t + g_t$. The demand for money must also equal the supply of money: $m_t^d = m_t$. Substituting $y_t - g_t$ for c_t and m_t for m_t^d in (4.33) and rearranging yields

$$d_t + \sum_{i=0}^{\infty}\lambda_{t,t+i}\left[g_{t+i} - \tau_{t+i} - \left(\frac{i_{t+i}}{1+i_{t+i}}\right)m_{t+i}\right] = 0. \tag{4.36}$$

Thus, an implication of the representative household's optimization problem and market equilibrium is that (4.36) must hold in equilibrium. Under Ricardian policies, (4.36) does not impose any additional restrictions on equilibrium since the policy variables are always adjusted to ensure that this condition holds. Under a non-Ricardian policy, however, it does impose an additional condition that must be satisfied in equilibrium. To see what this condition involves, we can use the definition of d_t and seigniorage to write (4.36) as

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty}\lambda_{t,t+i}[\tau_{t+i} + \bar{s}_{t+i} - g_{t+i}]. \tag{4.37}$$

At time t, the government's outstanding nominal liabilities D_t are predetermined by past policies. Given the present discounted value of the government's future surpluses

27. Notice that this usage differes somewhat from the way Sargent (1982) and Aiyagari and Gertler (1985) employed the terms. In these earlier papers, a Ricardian policy was one in which the fiscal authority fully adjusted taxes to ensure intertemporal budget balance for all price paths. A non-Ricardian policy was a policy in which the monetary authority was required to adjust seigniorage to ensure intertemporal budget balance for all price paths. Both of these policies would be labeled Ricardian under the current usage of the term.

(the right side 4.37), the only endogenous variable is the current price level P_t. The price level must adjust to ensure that (4.37) is satisfied.

Equation (4.37) is an equilibrium condition under non-Ricardian policies, but it is not the only equilibrium condition. It is still the case that real money demand and real money supply must be equal. Suppose the real demand for money is given by (4.19), rewritten here as

$$\frac{M_t}{P_t} = f(1 + i_t). \tag{4.38}$$

Equations (4.37) and (4.38) must both be satisfied in equilibrium. However, which two variables are determined jointly by these two equations depends on the assumptions that are made about fiscal and monetary policies. For example, suppose the fiscal authority determines g_{t+i} and τ_{t+i} for all $i \geq 0$, and the monetary authority pegs the nominal rate of interest $i_{t+i} = \bar{\imath}$ for all $i \geq 0$. Seigniorage is equal to $\bar{\imath} f(1 + \bar{\imath})/(1 + \bar{\imath})$ and so is fixed by monetary policy. With this specification of monetary and fiscal policies, the right side of (4.37) is given. Since D_t is predetermined at date t, (4.37) can be solved for the equilibrium price level P_t^* given by

$$P_t^* = \frac{D_t}{\sum_{i=0}^{\infty} \lambda_{t,t+i}[\tau_{t+i} + \bar{s}_{t+i} - g_{t+i}]}. \tag{4.39}$$

The current nominal money supply is then determined by (4.38):

$$M_t = P_t^* f(1 + \bar{\imath}).$$

One property of this equilibrium is that changes in fiscal policy (g or τ) directly alter the equilibrium price level, even though seigniorage as measured by $\sum_{i=0}^{\infty} \lambda_{t,t+i}\bar{s}_{t+i}$ is unaffected.[28] The finding that the price level is uniquely determined by (4.39) contrasts with a standard conclusion that the price level is indeterminate under a nominal interest rate peg. This conclusion is obtained from (4.38): with i pegged, the right side of (4.38) is fixed, but this only determines the real supply of money. Any price level is consistent with equilibrium, as M then adjusts to ensure that (4.38) holds.

Critical to the fiscal theory is the assumption that (4.37), the government's intertemporal budget constraint, is an *equilibrium* condition that holds at the equilibrium price level and not a condition that must hold at all price levels. This means that at price levels not equal to P_t^*, the government is planning to run surpluses (including

28. A change in g or τ causes the price level to jump, and this transfers resources between the private sector and the government. This transfer can also be viewed as a form of seigniorage.

seigniorage) whose real value, in present discounted terms, is not equal to the government's outstanding real liabilities. The government does not need to ensure that (4.37) holds for all price levels. Similarly, it means that the government could cut current taxes, leaving current and future government expenditures and seigniorage unchanged, and not simultaneously plan to raise future taxes. When (4.37) is interpreted as a budget *constraint* that must be satisfied for all price levels, then any decision to cut taxes today (and so lower the right side of 4.37) must be accompanied by planned future tax increases to leave the right side unchanged.

In standard infinite-horizon, representative-agent models, a tax cut (current and future government expenditures unchanged) has no effect on equilibrium (i.e., Ricardian equivalence holds) because the tax reduction does not have a real wealth effect on private agents. They recognize that in a Ricardian regime, future taxes have risen in present value terms by an amount exactly equal to the reduction in current taxes. Alternatively expressed, the government cannot engineer a permanent tax cut unless government expenditures are also cut (in present value terms). Because the fiscal theory of the price level assumes that (4.37) holds only when evaluated at the equilibrium price level, the government can plan a permanent tax cut. If it does, the price level must rise to ensure that the new, lower value of discounted surpluses is again equal to the real value of government debt.

An interest rate peg is just one possible policy specification. As an alternative, suppose as before that the fiscal authority sets the paths for g_{t+i} and τ_{t+i}, but now suppose that the government adjusts tax revenues to offset any variations in seigniorage. In this case, $\tau_{t+i} + \bar{s}_{t+i}$ becomes an exogenous process. Then (4.37) can be solved for the equilibrium price level, independent of the nominal money stock. Equation (4.38) must still hold in equilibrium. If the monetary authority sets M_t, this equation determines the nominal interest rate that ensures that the real demand for money is equal to the real supply. If the monetary authority sets the nominal rate of interest, (4.38) determines the nominal money supply. The extreme implication of the fiscal theory (relative to traditional quantity theory results) is perhaps most stark when the monetary authority fixes the nominal supply of money: $M_{t+i} = \bar{M}$ for all $i \geq 0$. Then, under a fiscal policy that makes $\tau_{t+i} + \bar{s}_{t+i}$ an exogenous process, the price level is proportional to D_t and, for a given level of D_t, is independent of $\bar{M}$.

EMPIRICAL EVIDENCE ON THE FISCAL THEORY Under the fiscal theory of the price level, (4.37) holds at the equilibrium value of the price level. Under traditional theories of the price level, (4.37) holds for all values of the price level. If we only observe equilibrium outcomes, it will be impossible empirically to distinguish between the two theories. As Sims (1994, p. 381) puts it, "Determinacy of the price level under any policy depends on the public's beliefs about what the policy authority would do under conditions that are never observed in equilibrium."

Canzoneri, Cumby, and Diba (2001) examine VAR evidence on the response of U.S. liabilities to a positive innovation to the primary surplus. Under a non-Ricardian policy, a positive innovation to $\tau_t + \bar{s}_t - g_t$ should increase D_t/P_t (see 4.37) unless it also signals future reductions in the surplus, that is, unless $\tau_t + \bar{s}_t - g_t$ is negatively serially correlated. The authors argue that in a Ricardian regime, a positive innovation to the current primary surplus will reduce real liabilities. This can be seen by writing the budget constraint (4.34) in real terms as

$$d_{t+1} = R[d_t - (\tau_t + s_t - g_t)]. \tag{4.40}$$

Examining U.S. data, they find that the responses are inconsistent with a Ricardian regime. Increases in the surplus are associated with declines in current and future real liabilities, and the surplus does not display negative serial correlation.

Cochrane points out the fundamental problem with this test: both (4.40) and (4.37) must hold in equilibrium, so it can be difficult to develop testable restrictions that can distinguish between the two regimes. The two regimes have different implications only if we can observe nonequilibrium values of the price level.

Bohn (1998a) has examined the U.S. deficit and debt processes and concludes that the primary surplus responds positively to the debt to GDP ratio. In other words, a rise in the debt to GDP ratio leads to an increase in the primary surplus. Thus, the surplus does adjust, and Bohn finds that it responds enough to ensure that the intertemporal budget constraint is satisfied. This is evidence that the fiscal authority seems to act in a Ricardian fashion.

Finally, there is an older literature that attempted to estimate whether fiscal deficits tended to lead to faster money growth. Such evidence might be interpreted to imply a Ricardian regime of fiscal dominance. Some of this literature was reviewed in section 4.3.1.

4.4 Optimal Taxation and Seigniorage

If the government can raise revenue by printing money, how much should it raise from this source? If only distortionary revenue sources are available, it will generally be desirable to raise some revenue from all available sources in order to minimize the overall distortions from raising a given amount of revenue. As first noted by Phelps (1973), this suggests that an optimal tax package should include some seigniorage.

If the objective of the government is to raise a given amount of revenue while causing the minimum deadweight loss from tax-induced distortions, then the government should generally set its tax instruments so that the marginal distortionary cost per dollar of revenue raised is equalized across all taxes. This prescription links the optimal inflation tax to a more general problem of determining the optimal levels of all tax instruments. If governments are actually attempting to minimize the distortionary costs of raising revenue, then the optimal tax literature provides a positive theory of inflation.

This basic idea is developed in the next subsection and was originally used by Mankiw (1987) to explain nominal interest rate setting by the Federal Reserve. However, the implications of this approach are rejected for the industrialized economies (Poterba and Rotemberg 1990, Trehan and Walsh 1990), although this may not be too surprising because seigniorage plays a fairly small role as a revenue source for these countries. Calvo and Leiderman (1992) have used the optimal tax approach to examine the experiences of some Latin American economies, with more promising results. An excellent survey of optimal seigniorage that links the topic with the issues of time inconsistency treated in chapter 8 can be found in Herrendorf (1997). Subsection 4.4.2 considers the role inflation might play as an optimal response to the need to finance temporary expenditure shocks. Section 4.4.3 revisits Friedman's rule for the optimal rate of inflation.

4.4.1 A Partial Equilibrium Model

In this section, we assume the government has available to it two revenue sources. The government can also borrow. It needs to finance a constant, exogenous level of real expenditures g, plus interest on any borrowing. To simplify the analysis, the real rate of interest is assumed to be constant, and we specify ad hoc descriptions of both money demand and the distortions associated with the two tax instruments.

With these assumptions, the basic real budget identity of the government can be obtained by dividing (4.3) by the time t price level to obtain

$$b_t = Rb_{t-1} + g - \tau_t - s_t, \tag{4.41}$$

where R is the gross interest factor (i.e., 1 plus the rate of interest), τ is non-seigniorage tax revenue, and s is seigniorage revenue. Seigniorage is given by

$$s_t = \frac{M_t - M_{t-1}}{P_t} = m_t - \frac{m_{t-1}}{1 + \pi_t}. \tag{4.42}$$

Taking expectations of (4.41) conditional on time-t information and recursively solving forward yields the intertemporal budget constraint of the government:

$$\mathrm{E}_t \sum_{i=0}^{\infty} R^{-i}(\tau_{t+i} + s_{t+i}) = Rb_{t-1} + \left(\frac{R}{R-1}\right)g. \tag{4.43}$$

Note that, given b_{t-1}, (4.43) imposes a constraint on the government, since $\mathrm{E}_t \lim_{i\to\infty} R^{-i}b_{t+i}$ has been set equal to zero. Absent this constraint, the problem of choosing the optimal time path for taxes and seigniorage becomes trivial. Just set both equal to zero and borrow continually to finance expenditures plus interest, because debt never needs to be repaid.

The government is assumed to set τ_t and the inflation rate π_t, as well as planned paths for their future values to minimize the present discounted value of the distortions generated by these taxes, taking as given the inherited real debt b_{t-1}, the path of expenditures, and the financing constraint given by (4.43). The assumption that the government can commit to a planned path for future taxes and inflation is an important one. Much of chapter 8 deals with outcomes when governments cannot precommit to future policies.

In order to illustrate the key implications of the joint determination of inflation and taxes, assume that the distortions arising from income taxes are quadratic in the tax rate: $(\tau_t + \phi_t)^2/2$, where ϕ is a stochastic term that allows the marginal costs of taxes to vary randomly.[29] Similarly, costs associated with seigniorage are taken to equal $(s_t + \varepsilon_t)^2/2$, where ε is a stochastic shift in the cost function. Thus, the present discounted value of tax distortions is given by

$$\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} R^{-i}[(\tau_{t+i} + \phi_{t+i})^2 + (s_{t+i} + \varepsilon_{t+i})^2]. \tag{4.44}$$

The government's objective is to choose paths for the tax rate and inflation to minimize (4.44) subject to (4.43).

Letting λ represent the Lagrangian multiplier associated with the intertemporal budget constraint, the necessary first order conditions for the government's setting of τ and s take the form

$$\mathrm{E}_t(\tau_{t+i} + \phi_{t+i}) = \lambda, \quad i \ge 0$$

$$\mathrm{E}_t(s_{t+i} + \varepsilon_{t+i}) = \lambda, \quad i \ge 0.$$

29. This approach follows that of Poterba and Rotemberg (1990), who specify tax costs directly, as we are doing here, although they assume a more general function form for which the quadratic specification is a special case. See also Trehan and Walsh (1988).

These conditions simply state that the government will arrange its tax collections to equalize the marginal distortionary costs across tax instruments, that is, $\mathrm{E}_t(\tau_{t+i} + \phi_{t+i}) = \lambda = \mathrm{E}_t(s_{t+i} + \varepsilon_{t+i})$ for each $i \ge 0$, and across time, that is, $\mathrm{E}_t(\tau_{t+i} + \phi_{t+i}) = \mathrm{E}_t(\tau_{t+j} + \phi_{t+j})$ and $\mathrm{E}_t(s_{t+i} + \varepsilon_{t+i}) = \mathrm{E}_t(s_{t+j} + \varepsilon_{t+j})$ for all i and j.

The first of these conditions implies that $\tau_t + \phi_t = \lambda = s_t + \varepsilon_t$ and represents an *intratemporal* optimality condition. It implies that changes in the government's revenue needs lead the tax rate and the inflation rate to move in the same direction. The value of λ will depend on the total revenue needs of the government; increases in $Rg/(R-1) + Rb_{t-1}$ will cause the government to increase the revenue raised from both tax sources. Thus, we would expect to observe τ_t and s_t moving in similar directions (given ϕ_t and ε_t).

Intertemporal optimality requires that marginal costs be equated across time periods for each tax instrument:

$$\mathrm{E}_t\tau_{t+1} = \tau_t - \mathrm{E}_t\phi_{t+1} + \phi_t \tag{4.45}$$

and

$$\mathrm{E}_t s_{t+1} = s_t - \mathrm{E}_t\varepsilon_{t+1} + \varepsilon_t. \tag{4.46}$$

These intertemporal conditions lead to standard *tax-smoothing* conclusions; for each tax instrument, the government will equate the expected marginal distortionary costs in different time periods. If the random shocks to tax distortions follow $I(1)$ processes such that $\mathrm{E}_t\phi_{t+1} - \phi_t = \mathrm{E}_t\varepsilon_{t+1} - \varepsilon_t = 0$, these intertemporal optimality conditions imply that both τ and s follow Martingale processes, an implication of the tax-smoothing model originally developed by Barro (1979a). If $\mathrm{E}_t\varepsilon_{t+1} - \varepsilon_t = 0$, (4.46) implies that changes in seigniorage revenues should be unpredictable based on information available at time t.

Changes in revenue sources might be predictable, and still be consistent with this model of optimal taxation, if the expected $t+1$ values of ϕ and/or ε, conditional on period-t information, are nonzero. For example, if $\mathrm{E}_t\varepsilon_{t+1} - \varepsilon_t > 0$, that is, if the distortionary cost of seigniorage revenue were expected to rise, it would be optimal to plan to reduce future seigniorage.

Using a form of (4.46), Mankiw (1987) argued that the near random walk behavior of inflation (actually nominal interest rates) is consistent with U.S. monetary policy having been conducted in a manner consistent with optimal finance considerations. Poterba and Rotemberg (1990) provide some cross-country evidence on the joint movements of inflation and other tax revenues. In general, this evidence is not favorable to the hypothesis that inflation (or seigniorage) has been set on the basis of optimal finance considerations. While Poterba and Rotemberg find the predicted positive relationship between tax rates and inflation for the United States and Japan, there is a negative relationship for France, Germany, and the United Kingdom.

The implications of the optimal finance view of seigniorage are, however, much stronger than simply that seigniorage and other tax revenues should be positively correlated. Since the unit root behavior of both s and τ arises from the same source (their dependence on $Rg/(R-1) + Rb_{t-1}$ through λ), the optimizing model of tax setting has the joint implication that both tax rates and inflation should contain unit roots (they respond to permanent shifts in government revenue needs) and that they should be cointegrated.[30] Trehan and Walsh (1990) show that this implication is rejected for U.S. data.

The optimal finance view of seigniorage fails for the United States because seigniorage appears to behave more like the stock of debt than like general tax revenues. Under a tax-smoothing model, temporary variations in government expenditures should be met through debt financing. Variations in seigniorage should reflect changes in expected permanent government expenditures or, from (4.46), stochastic shifts in the distortions associated with raising seigniorage (due to the ε realizations). In contrast, debt should rise in response to a temporary revenue need (such as a war) and then gradually decline over time. However, as figure 4.4 shows, the behavior of seigniorage, particularly during the World War II period, mimics that of the deficit much more than it does that of other tax revenues.

One drawback of this analysis is that the specification of the government's objective function is ad hoc; the assumed form of the tax distortions was not related in any way to the underlying sources of the distortions in terms of the allocative effects of taxes or the welfare costs of inflation. These costs depend on the demand for money; therefore, the specification of the distortions should be consistent with the particular approach used to motivate the demand for money.

Calvo and Leiderman (1992) provide an analysis of optimal intertemporal inflation taxation using a money demand specification that is consistent with utility maximization. They show that the government's optimality condition requires that the nominal rate of interest vary with the expected growth of the marginal utility of consumption. Optimal tax considerations call for high taxes when the marginal utility of consumption is low and low taxes when the marginal utility of consumption is high. Thus, models of inflation in an optimal finance setting will generally imply restrictions on the joint behavior of inflation and the marginal utility of consumption, not just on inflation alone. Calvo and Leiderman estimate their model using

30. That is, if ϕ and ε are $I(0)$ processes, then τ and s are $I(1)$ but $\tau - s = \varepsilon - \phi$ is $I(0)$.

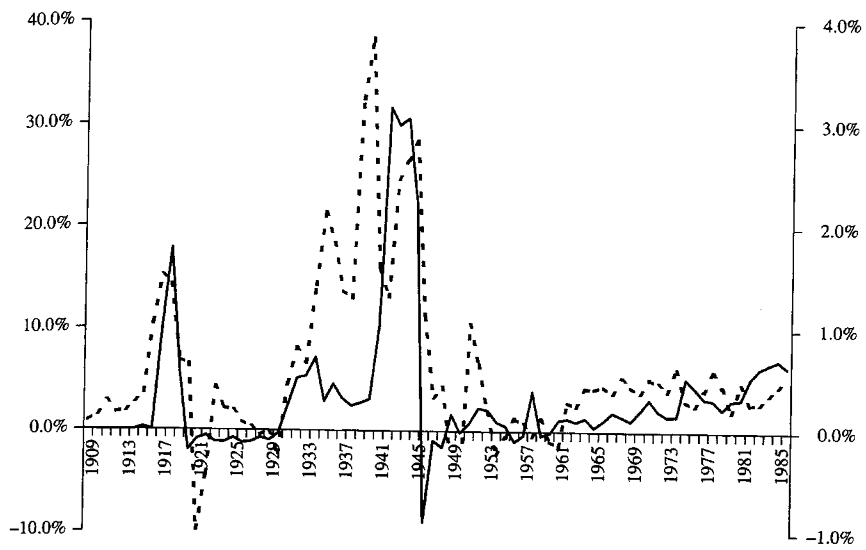

Figure 4.4
U.S. Deficits and Seigniorage 1909–1986 (as a percentage of GDP) (solid line, deficit (left scale); dashed line, seigniorage (right scale))

data from three countries that have experienced periods of high inflation: Argentina, Brazil, and Israel. While the overidentifying restrictions implied by their model are not rejected for the first two countries, they are for Israel.

4.4.2 Optimal Seigniorage and Temporary Shocks

The prescription to smooth marginal distortionary costs over time implies that tax levels are set on the basis of some estimate of permanent expenditure needs. In Barro's original formulation, temporary fluctuations in government expenditures did not lead to tax adjustments; instead, temporary increases in expenditures were deficit financed; periods of temporarily low expenditures were used to generate the surpluses needed to retire the previously issued debt. To allow tax rates to fluctuate in response to temporary and unanticipated fluctuations in expenditures would result in a higher total efficiency loss in present value terms because of the distortions induced by non-lump-sum taxes. As extended to seigniorage by Mankiw (1987), the same argument implies that seigniorage should be set on the basis of permanent expenditure needs and not adjusted in response to unanticipated temporary events.

The allocative distortions induced by the inflation tax, however, were shown in chapters 2 and 3 to be based on anticipated inflation. Consumption, labor-supply, and money-holding decisions are made by households on the basis of expected inflation, and for this reason, variations in expected inflation generate distortions. In contrast, unanticipated inflation has wealth effects but no substitution effects. It serves, therefore, as a form of lump-sum tax. Given real money holdings, which are based on the public's expectations about inflation, a government interested in minimizing distortionary tax costs should engineer a surprise inflation. If sufficient revenue could be generated in this way, socially costly distortionary taxes could be avoided.[31]

Unfortunately, private agents are likely to anticipate that the government will have an incentive to attempt a surprise inflation, and the outcome in such a situation will be the major focus of chapter 8. But suppose the government can commit itself to, on average, only inflating at a rate consistent with its revenue needs based on average expenditures. That is, average inflation (and other taxes as well) is set according to permanent expenditures, as implied by the tax-smoothing model. But if there are unanticipated fluctuations in expenditures, these should be met through socially costless unanticipated inflation.

Calvo and Guidotti (1993) make this argument rigorous. They show that when the government can commit to a path for anticipated inflation, it is optimal for unanticipated inflation to respond flexibly to unexpected disturbances. Recall from figure 4.3 that seigniorage in the United States followed a pattern that appeared to be more similar to that of the federal government deficit than to a measure of the average tax rate. During war periods, when most of the rise in expenditures could be viewed as temporary, taxes were not raised sufficiently to fund the war effort. Instead, the U.S. government borrowed heavily, just as the Barro tax-smoothing model implies. But the United States did raise the inflation tax; seigniorage revenues rose during the war, falling back to lower levels at the war's conclusion. This behavior is much closer to that implied by Calvo and Guidotti's theory than to the basic implications of Mankiw's.

4.4.3 Friedman's Rule Revisited

The preceding analysis has gone partway toward integrating the choice of inflation with the general public finance choice of tax rates, and the discussion was motivated by Phelps's conclusion that some revenue should be raised from the inflation tax

31. Auernheimer (1974) provides a guide to seigniorage for an "honest" government, one that does not generate revenue by allowing the price level to jump unexpectedly, even though this would represent an efficient lump-sum tax.

if only distortionary tax sources are available. However, this conclusion has been questioned by Kimbrough (1986a, 1986b), Faig (1988), Chari, Christiano, and Kehoe (1991, 1996), and Correia and Teles (1996, 1999).[32] They show that there are conditions under which Friedman's rule for the optimal inflation rate—a zero nominal rate of interest—continues to be optimal even in the absence of lump-sum taxes. Mulligan and Sala-i-Martin (1997) provide a general discussion of the conditions necessary for taxing (or not taxing) money.

This recent literature integrates the question of the optimal inflation tax into the general problem of optimal taxation. By doing so, the analysis can build on findings in the optimal tax literature that identify situations in which the structure of optimal indirect taxes calls for different final goods to be taxed at the same rate or for the tax rate on goods that serve as intermediate inputs to be zero (see Diamond and Mirrlees 1971, Atkinson and Stiglitz 1972). An MIU approach, for example, treats money as a final good; in contrast, a shopping time model, or a more general model in which money serves to produce transaction services, treats money as an intermediate input. Thus, it is important to examine what implications these alternative assumptions about the role of money might have for the optimal tax approach to inflation determination, and how optimal inflation tax results might depend on particular restrictions on preferences or on the technology for producing transaction services.

The Basic Ramsey Problem The problem of determining the optimal structure of taxes to finance a given level of expenditures is called the *Ramsey problem*, after the classic treatment of Frank Ramsey (1928). In the representative-agent models we have been using, the Ramsey problem involves setting taxes to maximize the utility of the representative agent, subject to the government's revenue requirement.

The following static Ramsey problem, based on Mulligan and Sala-i-Martin (1997), can be used to highlight the key issues. The utility of the representative agent depends on consumption, real money balances, and leisure:

$$u = u(c, m, l).$$

Agents maximize utility subject to the following budget constraint:

$$f(n) \geq (1 + \tau)c + \tau_m m, \tag{4.47}$$

where $f(n)$ is a standard production function, $n = 1 - l$ is the supply of labor, c is consumption, τ is the consumption tax, $\tau_m = i/(1 + i)$ is the tax on money, and m

32. An early example of the use of optimal tax models to study the optimal inflation rate issue is Drazen (1979). See also Walsh (1984). A recent survey is Chari and Kehoe (1999).

is the household's holdings of real money balances. The representative agent picks consumption, money holdings, and leisure to maximize utility, taking the tax rates as given. Letting λ be the Lagrangian multiplier on the budget constraint, the first order conditions from the agent's maximization problem are

$$u_c = \lambda(1+\tau) \tag{4.48}$$

$$u_m = \lambda\tau_m \tag{4.49}$$

$$u_l = \lambda f'. \tag{4.50}$$

From these first order conditions and the budget constraint, the choices of c, m, and l can be expressed as functions of the two tax rates: $c(\tau,\tau_m)$, $m(\tau,\tau_m)$, and $l(\tau,\tau_m)$.

The government's problem is to set τ and τ_m to maximize the representative agent's utility, subject to three types of constraints. First, the government must satisfy its budget constraint; tax revenues must be sufficient to finance expenditures. This constraint takes the form

$$\tau c + \tau_m m \geq g, \tag{4.51}$$

where g is real government expenditures. These expenditures are taken to be exogenous. Second, the government is constrained by the fact that consumption, labor supply, and real money must be consistent with the choices of private agents. That means that (4.48)–(4.50) represent constraints on the government's choices. Finally, the government is constrained by the economy's resource constraint:

$$f(1-l) \geq c + g. \tag{4.52}$$

The government's problem is to pick τ and τ_m to maximize $u(c,m,l)$ subject to (4.48)–(4.52).

There are two approaches to solving this problem. The first approach, often called the *dual approach*, employs the indirect utility function to express utility as a function of taxes. These tax rates are treated as the government's control variables, and the optimal values of the tax rates are found by solving the first order conditions from the government's optimization problem. The second approach, called the *primal approach*, treats quantities as the government's controls. The tax rates are found from the representative agent's first order conditions to ensure that private agents choose the quantities that solve the government's maximization problem. We start with the dual approach. The primal approach will be employed later in this section.

The government's problem can be written as

$$\max_{\tau,\tau_m}\{v(\tau,\tau_m) + \mu[\tau_m m(\tau,\tau_m) + \tau c(\tau,\tau_m) - g] + \theta[f(1-l(\tau,\tau_m)) - c(\tau,\tau_m) - g]\},$$

where $v(\tau,\tau_m) = u[c(\tau,\tau_m), m(\tau,\tau_m), l(\tau,\tau_m)]$ is the indirect utility function, and μ and θ are Lagrangian multipliers on the budget and resource constraints. Notice that we have incorporated the constraints represented by (4.48)–(4.50) by writing consumption, money balances, and leisure as functions of the tax rates. The first order conditions for the two taxes are

$$v_\tau + \mu(\tau_m m_\tau + c + \tau c_\tau) - \theta(f'l_\tau + c_\tau) \leq 0$$

$$v_{\tau_m} + \mu(m + \tau_m m_{\tau_m} + \tau c_{\tau_m}) - \theta(f'l_{\tau_m} + c_{\tau_m}) \leq 0,$$

where $v_\tau = u_c c_\tau + u_m m_\tau + u_l l_\tau$ and $v_{\tau_m} = u_c c_{\tau_m} + u_m m_{\tau_m} + u_l l_{\tau_m}$. These conditions will hold with equality if the solution is an interior one with positive taxes on both consumption and money. If the left side of the second first order condition is negative when evaluated at a zero tax on money, then a zero tax on money ($\tau_m = 0$) will be optimal. From the resource constraint (4.52), $-f'l_x - c_x = 0$ for $x = \tau, \tau_m$ since g is fixed, so the two first order conditions can be simplified to yield

$$u_c c_\tau + u_m m_\tau + u_l l_\tau + \mu(\tau_m m_\tau + c + \tau c_\tau) \leq 0 \tag{4.53}$$

$$u_c c_{\tau_m} + u_m m_{\tau_m} + u_l l_{\tau_m} + \mu(m + \tau_m m_{\tau_m} + \tau c_{\tau_m}) \leq 0. \tag{4.54}$$

The first three terms in the first of these equations can be written using the agent's first order conditions and the budget constraint (4.47) as

$$u_c c_\tau + u_m m_\tau + u_l l_\tau = u_l\left(\frac{u_c}{u_l}c_\tau + \frac{u_m}{u_l}m_\tau + l_\tau\right) = \left(\frac{u_l}{f'}\right)[(1+\tau)c_\tau + \tau_m m_\tau + f'l_\tau].$$

However, differentiating the budget constraint (4.47) by τ yields

$$c + (1+\tau)c_\tau + \tau_m m_\tau + f'l_\tau = 0,$$

so $u_c c_\tau + u_m m_\tau + u_l l_\tau = -(u_l/f')c$. Thus, (4.53) becomes

$$\left(\frac{u_l}{f'}\right)c \geq \mu(\tau_m m_\tau + c + \tau c_\tau),$$

while following similar steps implies that (4.54) becomes

$$\left(\frac{u_l}{f'}\right)m \geq \mu(m + \tau_m m_{\tau_m} + \tau c_{\tau_m}).$$

Hence, if the solution is an interior one with positive taxes on consumption and money holdings,

$$\frac{m}{c} = \frac{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}}{\tau_m m_\tau + c + \tau c_\tau}. \tag{4.55}$$

To interpret this condition, note that $v_{\tau_m} = u_c c_{\tau_m} + u_m m_{\tau_m} + u_l l_{\tau_m} = -u_l m/f'$ is the effect of the tax on money on utility, while $v_\tau = u_c c_\tau + u_m m_\tau + u_l l_\tau = -u_l c/f'$ is the effect of the consumption tax on utility. Thus, their ratio, m/c, is the marginal rate of substitution between the two tax rates, holding constant the utility of the representative agent.[33] The right side of (4.55) is the marginal rate of transformation, holding the government's revenue constant. At an optimum, the government equates the marginal rates of substitution and transformation.

Our interest is in determining when the Friedman rule, $\tau_m = 0$, is optimal. Assume, following Friedman, that at a zero nominal interest rate, the demand for money is finite. Since the tax on consumption must be positive if the tax on money is zero (since the government does need to raise revenue), (4.53) will hold with equality. Then

$$\frac{m}{c} \geq \frac{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}}{\tau_m m_\tau + c + \tau c_\tau} \tag{4.56}$$

or

$$\frac{m}{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}} \geq \frac{c}{\tau_m m_\tau + c + \tau c_\tau}.$$

The left side is proportional to the marginal impact of the inflation tax on utility per dollar of revenue raised. The right side is proportional to the marginal impact of the consumption tax on utility per dollar of revenue raised. If the inequality is strict at

33. That is, if $v(\tau, \tau_m)$ is the utility of the representative agent as a function of the two tax rates, then $v_\tau d\tau + v_{\tau_m} d\tau_m = 0$ yields

$$\frac{d\tau}{d\tau_m} = -\frac{v_{\tau_m}}{v_\tau} = -\frac{m}{c}.$$

$\tau_m = 0$, then the distortion caused by using the inflation tax (per dollar of revenue raised) exceeds the cost of raising that same revenue with the consumption tax. Thus, it is optimal to set the tax on money equal to zero if

$$\frac{m}{c} \geq \frac{m + \tau c_{\tau_m}}{c + \tau c_\tau} \tag{4.57}$$

or (since $c_\tau \leq 0$)

$$\frac{m}{c} \leq \frac{c_{\tau_m}}{c_\tau}, \tag{4.58}$$

where these expressions are evaluated at $\tau_m = 0$.[34]

Mulligan and Sala-i-Martin (1997) consider (4.56) for a variety of special cases that have appeared in the literature. For example, if utility is separable in consumption and money holdings, then $c_{\tau_m} = 0$; in this case, the right side of (4.58) is equal to zero, while the left side is positive. Hence, (4.58) cannot hold and it is optimal to tax money.

A second case that leads to clear results occurs if $c_{\tau_m} > 0$. In this case, the right side of (4.58) is negative (since $c_\tau < 0$, an increase in the consumption tax reduces consumption). Because the left side is nonnegative, $m/c > c_{\tau_m}/c_\tau$ and money should always be taxed. This corresponds to a case in which money and consumption are substitutes so that an increase in the tax on money (which reduces money holdings) leads to an increase in consumption. Finally, if money and consumption are complements, $c_{\tau_m} < 0$. The ratio c_{τ_m}/c_τ is then positive, and whether money is taxed will depend on a comparison of m/c and c_{τ_m}/c_τ. Recall that the calibration exercises in chapter 2 used parameter values that implied that m and c were complements.

Chari, Christiano, and Kehoe (1996) examine the optimality of the Friedman rule in an MIU model with taxes on consumption, labor supply, and money. They show that if preferences are homothetic in consumption and money balances and separable in leisure, the optimal tax on money is zero. When preferences satisfy these assumptions, we can write

$$u(c, m, l) = \bar{u}[s(c, m), l],$$

where $s(c, m)$ is homothetic.[35] Mulligan and Sala-i-Martin (1997) show that in

34. This is Proposition 2 in Mulligan and Sala-i-Martin (1997, p. 692).

35. Homothetic preferences imply that $s(c, m)$ is homogeneous of degree 1 and that s_i is homogeneous of degree 0. With homothetic preferences, indifference curves are parallel to each other, with constant slope along any ray; $u_2(c, m)/u_1(c, m) = f(m/c)$.

this case,

$$\frac{m}{c} = \frac{c_{\tau_m}}{c_\tau},$$

so (4.58) implies that the optimal tax structure yields $\tau_m = 0$.

Chari, Christiano, and Kehoe relate their results to the optimal taxation literature in public finance. Atkinson and Stiglitz (1972) show that if two goods are produced under conditions of constant returns to scale, a sufficient condition for uniform tax rates is that the utility function is homothetic. With equal tax rates, the ratio of marginal utilities equals the ratio of producer prices. To see how this applies in the present case, suppose the budget constraint for the representative household takes the form

$$(1 + \tau_t^c)Q_t c_t + M_t + B_t = (1 - \tau_t^h)Q_t(1 - l_t) + (1 + i_{t-1})B_{t-1} + M_{t-1},$$

where M and B are the nominal money and bond holdings, i is the nominal rate of interest, Q is the producer price of output, and τ^c and τ^h are the tax rates on consumption (c) and hours of work ($1 - l$). In addition, we have assumed that the production function exhibits constant returns to scale and that labor hours, $1 - l$, are transformed into output according to $y = 1 - l$. Define $P \equiv (1 + \tau^c)Q$. Household real wealth is $w_t = (M_t + B_t)/P_t = m_t + b_t$, and the budget constraint can be written as

$$\begin{aligned} c_t + w_t &= \left(\frac{1 - \tau_t^h}{1 + \tau_t^c}\right)(1 - l_t) + (1 + r_{t-1})b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} \\ &= (1 - \tau_t)(1 - l_t) + (1 + r_{t-1})w_{t-1} - \left(\frac{i_{t-1}}{1 + \pi_t}\right)m_{t-1}, \end{aligned} \tag{4.59}$$

where $1 - \tau_t \equiv (1 - \tau_t^h)/(1 + \tau_t^c)$ and $(1 + r_{t-1}) = (1 + i_{t-1})/(1 + \pi_t)$, and $\pi_t = P_t/P_{t-1} - 1$. Thus, the consumption and labor taxes only matter through the composite tax τ, so without loss of generality, set the consumption tax equal to zero. If the representative household's utility during period t is given by $\bar{u}[s(c, m), l]$ and the household's objective is to maximize $E_t \sum_{i=0}^{\infty} \beta^i \bar{u}[s(c_{t+i}, m_{t+i}), l_{t+i}]$ subject to the budget constraint given by (4.59), then the first order conditions for the household's decision problem imply that consumption, money balances, and leisure will be chosen such that

$$\frac{\bar{u}_m(c_t, m_t, l_t)}{\bar{u}_c(c_t, m_t, l_t)} = \frac{s_m(c_t, m_t)}{s_c(c_t, m_t)} = \frac{i_t}{1 + i_t} \equiv \tau_{m,t}.$$

With the production costs of money assumed to be zero, the ratio of marginal utilities differs from the ratio of production costs unless $\tau_{m,t} = 0$. Hence, with preferences that are homothetic in c and m, the Atkinson-Stiglitz result implies that it will be optimal to set the nominal rate of interest equal to zero.

Correia and Teles (1999) consider other cases in which (4.57) holds so that the optimal tax on money equals zero. They follow M. Friedman (1969) in assuming a satiation level of money holdings m^* such that the marginal utility of money is positive for $m < m^*$ and nonpositive for $m \geq m^*$. This satiation level can depend on c and l. Correia and Teles show that the optimal tax on money is zero if $m^* = \bar{k}c$ for a positive constant $\bar{k}$. They also show that the optimal tax on money is zero if $m^* = \infty$. Intuitively, at an optimum, the marginal benefit of additional money holdings must balance the cost of the marginal effect on government revenues. This contrasts with the case of normal goods, where the marginal benefit must balance the costs of the marginal impact on the government's revenue *and* the marginal resource cost of producing the goods. Money, in contrast, is assumed to be costless to produce. At the satiation point, the marginal benefit of money is zero. The conditions studied by Correia and Teles ensure that the marginal revenue effect is also zero.

We can recover Friedman's rule for the optimal rate of inflation even in the absence of lump-sum taxes. But it is important to recognize that the restrictions on preferences necessary to restore Friedman's rule are very strong and, as discussed by Braun (1991), different assumptions about preferences will lead to different conclusions. The assumption that the ratio of the marginal utilities of consumption and money is independent of leisure can certainly be questioned. However, it is very common in the literature to assume separability between leisure, consumption, and money holdings. The standard log utility specification, for example, displays this property and so would imply that a zero nominal interest rate is optimal.

A CIA Model The examples so far have involved MIU specifications. Suppose instead that the consumer faces a CIA constraint on a subset of its purchases. Specifically, assume that c_1 represents cash goods, while c_2 represents credit goods. Let l denote leisure. The household's objective is to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i U(c_{1,t+i}, c_{2,t+i}, l_{t+i})$$

subject to the budget constraint

$$(1 + \tau_t^c)Q_t(c_{1,t} + c_{2,t}) + M_t + B_t = (1 - \tau_t^h)Q_t(1 - l_t) + (1 + i_{t-1})B_{t-1} + M_{t-1},$$

where variables are as defined above. In addition, the CIA constraint requires that

$$c_{1,t} \leq \frac{m_{t-1}}{1 + \pi_t}.$$

Before considering when the optimal inflation tax might be positive, suppose we ignore the credit good c_2 for the moment so that the model is similar to the basic CIA model studied in chapter 3. Recall that inflation served as a tax on labor supply in that model. But according to the budget constraint, the government already has, in τ^h, a tax on labor supply. Thus, the inflation tax is redundant.[36] Because it is redundant, the government can achieve an optimal allocation without using the inflation tax.

In a cash-and-credit-good economy, the inflation tax is no longer redundant if the government cannot set different commodity taxes on the two types of goods. So if we return to the model with both cash and credit goods, the first order conditions for the household's decision problem imply that consumption and leisure will be chosen such that

$$\frac{U_1(c_{1,t}, c_{2,t}, l_t)}{U_2(c_{1,t}, c_{2,t}, l_t)} = 1 + i_t.$$

The analysis of Atkinson and Stiglitz (1972) implies that if preferences are homothetic in $c_{1,t}$ and $c_{2,t}$, the ratio of the marginal utility of cash and credit goods should equal 1, the ratio of their production prices. This occurs only if $i = 0$; hence, homothetic preferences imply that the nominal rate of interest should be set equal to zero. But this is just the Friedman rule for the optimal rate of inflation.

Thus, the optimal inflation tax should be zero if for all $\lambda > 0$,

$$\frac{U_1(\lambda c_{1,t}, \lambda c_{2,t}, l_t)}{U_2(\lambda c_{1,t}, \lambda c_{2,t}, l_t)} = \frac{U_1(c_{1,t}, c_{2,t}, l_t)}{U_2(c_{1,t}, c_{2,t}, l_t)}, \tag{4.60}$$

in which case the utility function has the form

$$U(c_1, c_2, l) = V(\phi(c_1, c_2), l),$$

where ϕ is homogeneous of degree 1. If this holds, the government should avoid using the inflation tax even though it must rely on distortionary taxes. Positive nominal rates of interest impose an efficiency cost by distorting the consumer's choice between cash and credit goods.[37]

36. See Chari, Christiano, and Kehoe (1996).

37. As Chari, Christiano, and Kehoe (1996) note, the preference restrictions are sufficient for the Friedman rule to be optimal but not necessary. For example, in the cash/credit model, suppose preferences are not homothetic and the optimal tax structure calls for taxing credit goods more heavily. A positive nominal interest rate taxes cash goods, and negative nominal rates are not feasible. Thus, a corner solution can arise in which the optimal nominal interest rate is zero. Note that this assumes that the government cannot impose separate goods taxes on cash and credit goods.

How reasonable is this condition? Recall that we have offered no explanation for why one good is a cash good and the other is a credit good. This distinction has simply been assumed, and therefore it is difficult to argue intuitively why the preferences for cash and credit goods should (or should not) satisfy condition (4.60). In aggregate analysis, it is common to combine all goods into one composite good; this is standard in writing utility as $u(c, l)$, with c representing an aggregation over all consumption goods. Interpreting c as $\phi(c_1, c_2)$, that is, interpreting ϕ as an aggregator function, implies that preferences would satisfy the properties necessary for the optimal inflation tax to be zero. However, this is not an innocuous restriction. It requires, for example, that the ratio of the marginal utility of coffee at the local coffee cart (a cash good) to that of books at the bookstore (a credit good) remain constant if coffee and book consumption double.

Money as an Intermediate Input The approach in the previous subsections motivated a demand for money by imposing a CIA constraint that applied to a subset of goods or by including real money balances as an element of the representative agent's utility function. If the role of money arises because of the services it provides in facilitating transactions, then it might be more naturally viewed as an intermediate good, a good used as an input in the production of the final goods that directly enter the utility function. The distinction between final goods and intermediate goods is important for determining the optimal structure of taxation; Diamond and Mirrlees (1971), for example, showed that under certain conditions it may be optimal to tax only final goods. In particular, when the government can levy taxes on each final good, intermediate goods should not be taxed.

The importance of money's role as an intermediate input was first stressed by Kimbrough (1986a, 1986b) and Faig (1988).[38] Their work suggested that the Friedman rule might apply even in the absence of lump-sum taxes, and conclusions to the contrary arose from the treatment of money as a final good that enters the utility function directly. Under conditions of constant returns to scale, the Diamond-Mirrlees result called for efficiency in production, implying that money and labor inputs into producing transactions should not be taxed. Since the MIU approach is usually used as a shortcut for modeling situations in which money serves as a medium of exchange by facilitating transactions, the work of Kimbrough and Faig indicates that such shortcuts can have important implications. However, the requirement that taxes be available for every final good is not satisfied in practice, and the properties of the transactions technology of the economy are such that, until

38. See also Guidotti and Végh (1993).

these are better understood, there is no clear case for assuming constant returns to scale.

Correia and Teles (1996) have provided further results on the applicability of the Friedman rule. They show that Friedman's result holds for any shopping time model in which shopping time is a homogeneous function of consumption and real money balances. To investigate this result, and to illustrate the primal approach to the Ramsey problem, consider a generalized shopping time model in which money and time are inputs into producing transaction services. Specifically, assume that the representative agent has a total time allocation normalized to 1 which can be allocated to leisure (l), market activity (n), or shopping (n^s):

$$l_t + n_t + n_t^s = 1. \tag{4.61}$$

Shopping time depends on the agent's choice of consumption and money holdings, with n_t^s increasing in c_t and decreasing in m_t according to the shopping production function

$$n_t^s = G(c_t, m_t).$$

Assume that G is homogeneous of degree η so that we can write $G(\lambda_t c_t, \lambda_t m_t) = \lambda_t^\eta G(c_t, m_t)$. Letting $\lambda_t = 1/c_t$,

$$n_t^s = c_t^\eta G\left(1, \frac{m_t}{c_t}\right) \equiv c_t^\eta g\left(\frac{m_t}{c_t}\right).$$

In addition, assume that g is a convex function, $g' \leq 0, g'' \geq 0$, which implies that shopping time is nonincreasing in m_t/c_t but real money balances exhibit diminishing marginal productivity. Constant returns to scale corresponds to $\eta = 1$. Assume that there exists a level of real balances relative to consumption $\bar{\mu}$ such that $g'(x) = 0$ for $x \geq \bar{\mu}$, corresponding to a satiation level of real balances.

The representative agent chooses paths for consumption, labor supply, money holdings, and capital holdings to maximize

$$\sum_{i=0}^{\infty} \beta^i u\left[c_{t+i}, 1 - n_{t+i} - c_{t+i}^\eta g\left(\frac{m_{t+i}}{c_{t+i}}\right)\right] \tag{4.62}$$

subject to the following budget constraint:

$$w_t \equiv \left(\frac{1 + i_{t-1}}{1 + \pi_t}\right) d_{t-1} - \left(\frac{i_{t-1}}{1 + \pi_t}\right) m_{t-1} \geq c_t + d_t - (1 - \tau_t) f(n_t), \tag{4.63}$$

where $f(n_t)$ is a standard, constant returns to scale, neoclassical production function, τ_t is the tax rate on income, $d_t = m_t + b_t$ is total real assets holdings, equal to government interest-bearing debt holdings (b_t) plus real money holdings, i_{t-1} is the nominal interest rate from $t-1$ to t, and π_t is the inflation rate from $t-1$ to t. Notice that we have ignored capital accumulation in this analysis. Further assume that initial conditions include $M_{t-1} = B_{t-1} = 0$, where these are the nominal levels of money and bond stocks. A final, important assumption in Correia and Teles's analysis is that $f(n) = 1 - l - n^s$.[39]

The government's optimal tax problem is to pick time paths for τ_{t+i} and i_{t+i} to maximize (4.62) subject to the economy resource constraint $c_t + g_t \leq 1 - l_t - n_t^s$ and to the requirement that consumption and labor supply be consistent with the choices of private agents. Following Lucas and Stokey (1983), this problem can be recast by using the first order conditions from the individual agent's decision problem to express, in terms of the government's tax instruments, the equilibrium prices that will support the paths of consumption and labor supply that solve the government's problem. This leads to an additional constraint on the government's choices and can be summarized in terms of an *implementability condition.*

To derive this implementability condition, we start with the first order conditions for the representative agent's problem. Define the value function

$$v(w_t) = \max\left\{u\left[c_t, 1 - n_t - c_t^\eta g\left(\frac{m_t}{c_t}\right)\right] + \beta v(w_{t+1})\right\},$$

where the maximization is subject to the budget constraint (4.63). Letting λ_t denote the Lagrangian multiplier associated with the time t budget constraint, the first order conditions imply

$$u_c - u_l\left(\eta g - \frac{m}{c} g'\right) c_t^{\eta-1} = \lambda_t \tag{4.64}$$

$$u_l = \lambda_t (1 - \tau_t) \tag{4.65}$$

$$-u_l g' c_t^{\eta-1} = \lambda_t I_t \tag{4.66}$$

$$\lambda_t = \beta R_t \lambda_{t+1}, \tag{4.67}$$

where $I_t = i_t/(1 + i_t)$ and the real interest rate is $R_t = (1 + i_t)/(1 + \pi_{t+1})$.

39. Notice that the utility function in (4.62) can be written as $v(c_{t+i}, m_{t+i}, n_{t+i})$ and so can be used to justify an MIU function (see also section 3.1 of chapter 3). When the shopping time function takes the form assumed here, Correia and Teles (1999) show that $m^* = \bar{k}c$ for a positive constant $\bar{k}$, where m^* is the satiation level of money balances such that $g'(m^*/c) = 0$. As noted earlier, the optimal tax on money is zero when $m^* = \bar{k}c$.

The next step is to recast the budget constraint (4.63). This constraint can be written as

$$R_{t-1}d_{t-1} = \sum_{i=0}^{\infty} D_i[c_{t+i} - (1-\tau_{t+i})(1-l_{t+i}-n^s_{t+i}) + R_{t-1+i}I_{t-1+i}m_{t-1+i}], \quad (4.68)$$

where we have imposed a no-Ponzi condition and the discount factor D_i is defined as $D_i = 1$ for $i = 0$ and $D_i = \prod_{j=1}^{i} R^{-1}_{t+j-1}$ for $i \geq 1$. Since we have assumed that the initial stocks of money and bonds equal zero, $d_{t-1} = 0$, so the right side of (4.68) must also equal zero.[40] The implementability condition is obtained by replacing the prices in this budget constraint using the first order conditions of the agent's problem to express the prices in terms of quantities.[41]

Recalling that $c^\eta g = n^s$, first multiply and divide the intertemporal budget constraint by λ_{t+i}; then use the result from the first order conditions (4.67) that $D_i = \beta^i D_0 \lambda_{t+i}/\lambda_t$ to write (4.68) as[42]

$$\sum_{i=0}^{\infty} \beta^i[\lambda_{t+i}c_{t+i} - \lambda_{t+i}(1-\tau_{t+i})(1-l_{t+i}-n^s_{t+i}) + \lambda_{t+i}I_{t+i}m_{t+i}] = 0.$$

Now use the first order conditions (4.64)–(4.66) to obtain

$$\sum_{i=0}^{\infty} \beta^i\left\{\left[u_c - u_l\left(\eta g - u_l\frac{m}{c}g'\right)c^{\eta-1}_{t+i}\right]c_{t+i} - u_l(1-l_{t+i}-n^s_{t+i}) - u_l\frac{m_{t+i}}{c_{t+i}}g'c^\eta_{t+i}\right\} = 0.$$

40. If the government's initial nominal liabilities were positive, it would be optimal to immediately inflate away their value, as this would represent a nondistortionary source of revenue. It is to avoid this outcome that the initial stocks are assumed to be zero.

41. The price of consumption is 1, the price of leisure is $1-\tau$, and the price of real balances is I.

42. This uses the fact that $m_{t-1} = 0$, so we can write

$$\sum_{i=0} D_i R_{t-1+i} I_{t-1+i} m_{t-1+i} = \sum_{i=1} D_i R_{t-1+i} I_{t-1+i} m_{t-1+i} = \sum_{i=1} D_{i-1} I_{t-1+i} m_{t-1+i}$$
$$= \sum_{i=1} \frac{D_{i-1}}{\lambda_{t-1+i}} \lambda_{t-1+i} I_{t-1+i} m_{t-1+i} = \sum_{i=0} \frac{D_i}{\lambda_{t+i}} \lambda_{t+i} I_{t+i+i} m_{t+i}$$
$$= \frac{1}{\lambda_t}\sum_{i=0} \beta^i \lambda_{t+i} I_{t+i+i} m_{t+i}.$$

Since the term $u_l \frac{m}{c} g' c^\eta_{t+i}$ appears twice, with opposite signs, these cancel, and this condition becomes

$$\sum_{i=0}^{\infty} \beta^i[u_c c_{t+i} - u_l(1-l_{t+i}) + u_l(1-\eta)n^s_{t+i})] = 0. \quad (4.69)$$

Equation (4.69) is the implementability condition. The government's problem now is to choose c_{t+i}, m_{t+i}, and l_{t+i} to maximize the utility of the representative agent, subject to the economy's resource constraint, the production function for shopping time, and (4.69), that is, $\max \sum \beta^i u(c_{t+i}, l_{t+i})$ subject to (4.69) and $c_t + g_t \leq (1 - l_t - n^s_t)$ where $n^s_t = g(m_t/c_t)c^\eta_t$. This formulation of the Ramsey problem illustrates the primal approach; the first order conditions for the representative agent are used to eliminate prices from the agent's intertemporal budget constraint.

Since m appears in this problem only in the production function for shopping time, the first order condition for the optimal choice of m_t is

$$[\beta^i \psi u_l(1-\eta) - \mu_{t+i}]g' = 0, \quad (4.70)$$

where $\psi \geq 0$ is the multiplier on the implementability constraint (4.69) and $\mu \geq 0$ is the multiplier on the resource constraint. Correia and Teles show that $\beta^i \psi u_l(1-\eta) - \mu_{t+i} = 0$ cannot characterize the optimum, so for (4.70) to be satisfied requires that $g' = 0$. From the first order conditions in the representative agent's problem, $-u_l g' c^{\eta-1}_t = \lambda_t I_t$; this implies that $g' = 0$ requires $I = 0$. That is, the nominal rate of interest should equal zero and the optimal tax on money should be zero.

The critical property of money, according to Correia and Teles, is its status as a free primary good. *Free* in this context means that it can be produced at zero variable cost. The costless production assumption is standard in monetary economics, and it provided the intuition for Friedman's original result. With a zero social cost of production, optimality requires that the private cost also be zero. This occurs only if the nominal rate of interest is zero.

We have now seen that there are general cases in which Phelps's conclusion does not hold. Even in the absence of lump-sum taxation, optimal tax policy should not distort the relative price of cash and credit goods or distort money holdings. But, as discussed by Braun (1991) and Mulligan and Sala-i-Martin (1997), different assumptions about preferences or technology can lead to different conclusions. Correia and Teles (1999) attempt to quantify the deviations from the Friedman rule when the preference and technology restrictions required for a zero nominal interest rate to be optimal do not hold. They find that the optimal nominal rate of interest is still close to zero.

4.5 Nonindexed Tax Systems

Up to this point, our discussion has assumed that the tax system is indexed so that taxes are levied on real income; a one-time change in all nominal quantities and the price level would leave the real equilibrium unchanged. This assumption requires that a pure price change have no effect on the government's real tax revenues or the tax rates faced by individuals and firms in the private sector. Most actual tax systems, however, are not completely indexed to ensure that pure price-level changes leave real tax rates and real tax revenue unchanged. Inflation-induced distortions generated by the interaction of inflation and the tax system have the potential to be much larger than the revenue-related effects on which most of the seigniorage and optimal inflation literature has focused. Feldstein (1998) provides an analysis of the net benefits of reducing inflation from 2% to zero,[43] and he concludes that for his preferred parameter values, the effects due to reducing distortions related to the tax system are roughly twice those associated with the change in government revenue.

One important distortion arises when nominal interest income, and not real interest income, is taxed. After-tax real rates of return will be relevant for individual agents in making savings and portfolio decisions, and if nominal income is subject to a tax rate of τ, the real after-tax return will be

$$\begin{aligned} r_a &= (1-\tau)i - \pi \\ &= (1-\tau)r - \tau\pi, \end{aligned}$$

where $i = r + \pi$ is the nominal return and r is the before-tax real return. Thus, for a given pretax real return r, the after-tax real return is decreasing in the rate of inflation.

To see how this distortion affects the steady-state capital-labor ratio, consider the basic MIU model of chapter 2 with an income tax of τ on total nominal income. Nominal income is assumed to include any nominal capital gain on capital holdings:

$$Y_t \equiv P_t f(k_{t-1}) + i_{t-1}B_{t-1} + P_tT_t + (P_t - P_{t-1})(1-\delta)k_{t-1}.$$

The representative agent's budget constraint becomes

$$(1-\tau)Y_t = P_tc_t + P_tk_t - P_t(1-\delta)k_{t-1} + (B_t - B_{t-1}) + (M_t - M_{t-1}),$$

where M is the agent's nominal money holdings, B is his bond holdings, and P_tT_t is a

43. Feldstein allows for an upward bias in the inflation rate, as measured by the consumer price index, so that his estimates apply to reducing consumer price inflation from 4% to 2%.

nominal transfer payment.[44] In real terms, the budget constraint becomes[45]

$$\begin{aligned} &(1-\tau)\left[f(k_{t-1}) + \frac{i_{t-1}b_{t-1}}{1+\pi_t} + T_t\right] - \tau\left(\frac{\pi_t}{1+\pi_t}\right)(1-\delta)k_{t-1} \\ &\quad = c_t + k_t - (1-\delta)k_{t-1} + \left(b_t - \frac{b_{t-1}}{1+\pi_t}\right) + \left(m_t - \frac{m_{t-1}}{1+\pi_t}\right). \end{aligned}$$

Assuming the agent's objective is to maximize the present discounted value of expected utility, which depends on consumption and money holdings, the first order conditions for capital and bonds imply, in the steady state,

$$(1-\tau)f_k(k) + \left[\frac{1+(1-\tau)\pi}{1+\pi}\right](1-\delta) = \frac{1}{\beta} \tag{4.71}$$

and

$$(1-\tau)\left(\frac{1+i}{1+\pi}\right) + \frac{\tau}{1+\pi} = \frac{1}{\beta}. \tag{4.72}$$

The steady-state capital-labor ratio is determined by

$$f_k(k^{ss}) = \left(\frac{1}{1-\tau}\right)\left\{\frac{1}{\beta} - \left[\frac{1+(1-\tau)\pi}{1+\pi}\right](1-\delta)\right\}.$$

Because $[1+(1-\tau)\pi]/(1+\pi)$ is decreasing in π, k^{ss} is decreasing in the inflation rate. Higher inflation leads to larger nominal capital gains on existing holdings of capital, and since these are taxed, inflation increases the effective tax rate on capital.

Equation (4.72) can be solved for the steady-state nominal rate of interest to yield

$$1 + i^{ss} = \frac{1}{\beta}\left(\frac{1+\pi}{1-\tau}\right) - \frac{\tau}{1-\tau}.$$

Thus, the pretax real return on bonds, $(1+i)/(1+\pi)$, increases with the rate of inflation, implying that nominal rates rise more than proportionately with an increase in inflation.

44. For simplicity, assume that T is adjusted in a lump-sum fashion to ensure that variations in inflation and the tax rate on income leave the government's budget balanced. Obviously, if lump-sum taxes actually were available, the optimal policy would involve setting $\tau = 0$ and following Friedman's rule for the optimal rate of inflation. The purpose here is to examine the effects of a nonindexed tax system on the steady-state capital stock in the easiest possible manner.

45. This formulation assumes that real economic depreciation is tax deductible. If depreciation allowances are based on historical nominal cost, a further inflation-induced distortion would be introduced.

It is important to recognize that we have examined only one aspect of the effects of inflation and the tax system.[46] Because of the taxation of nominal returns, higher inflation distorts the individual's decisions, but it also generates revenue for the government that, with a constant level of expenditures (in present value terms), would allow other taxes to be reduced. Thus, the distortions associated with the higher inflation are potentially offset by the reduction in the distortions caused by other tax sources. As noted earlier, however, Feldstein (1998) argues that the offset is only partial, leaving a large net annual cost of positive rates of inflation. Feldstein identifies the increased effective tax rate on capital that occurs because of the treatment of depreciation and the increased subsidy on housing associated with the deductibility of nominal mortgage interest in the United States as important distortions generated by higher inflation interacting with a nonindexed tax system. Including these effects with an analysis of the implications for government revenues and, consequently, possible adjustments in other distortionary taxes, Feldstein estimates that a 2% reduction in inflation (from 2% to zero) increases net welfare by 0.63% to 1.01% of GDP annually. These figures assume an elasticity of savings with respect to the after-tax real return of 0.4 and a deadweight loss of taxes of between 40 cents for every dollar of revenue (leading to the 0.63% figure) and $1.50 per dollar of revenue (leading to the 1.01% figure). Since these are annual gains, the present discounted value of permanently reducing inflation to zero would be quite large.

4.6 Summary

Monetary and fiscal actions are linked through the government's budget constraint. Under Ricardian regimes, changes in the money stock or its growth rate will require some other variable in the budget constraint—taxes, expenditures, or borrowing—to adjust. With fiscal dominance, changes in government taxes or expenditures can require changes in inflation. Under non-Ricardian regimes, changes in government debt affect prices even if monetary policy is exogenous. A complete analysis of price level determinacy requires a specification of the relationship between fiscal and monetary policies.

Despite this and despite the emphasis budget relationships have received in the work of Sargent and Wallace and the work on the fiscal theory of the price level initiated by Sims and Woodford, much of monetary economics ignores the implications

46. Feldstein, Green, and Sheshinski (1978) used a version of Tobin's money and growth model (Tobin 1965) to explore the implications of a nonindexed tax system when firms use both debt and equity to finance capital.

of the budget constraint. This is valid in the presence of lump-sum taxes; any effects on the government's budget can simply be offset by an appropriate variation in lump-sum taxes. Traditional analyses that focus only on the stock of high-powered money are also valid when governments follow a Ricardian policy of fully backing interest-bearing debt with tax revenues, either now or in the future. In general, though, we should be concerned with the fiscal implications of any analysis of monetary policy, since changes in the quantity of money that alter the interest payments of the government have implications for future tax liabilities.

4.7 Problems

1. Suppose real income grows at the rate $\mu_t > 0$. How are (4.6) and (4.7) affected? Does seigniorage depend on μ? Explain.

2. Consider the version of the Sidrauski (1967) model studied in problem 3 of chapter 2. Utility was given by $u(c_t, m_t) = w(c_t) + v(m_t)$, with $w(c_t) = \ln c_t$ and $v(m_t) = m_t(B - D \ln m_t)$, where B and D are positive parameters. Approximate steady-state revenues from seigniorage are given by θm, where θ is the growth rate of the money supply.

a. Is there a "Laffer curve" for seigniorage (i.e., are revenues increasing in θ for all $\theta \leq \theta^*$ and decreasing in θ for all $\theta > \theta^*$ for some θ^*?

b. What rate of money growth maximizes steady-state revenues from seigniorage?

c. Assume now that the economy's rate of population growth is λ and reinterpret m as real money balances per capita. What *rate of inflation* maximizes seigniorage? How does it depend on λ?

3. Suppose the government faces the following budget identity:

$$b_t = Rb_{t-1} + g_t - \tau_t y_t - s_t,$$

where the terms are one-period debt, gross interest payments, government purchases, income tax receipts, and seigniorage. Assume seigniorage is given by $f(\pi_t)$, where π is the rate of inflation. The interest factor R is constant, and the expenditure process $\{g_{t+i}\}_{i=0}^{\infty}$ is exogenous. The government sets time paths for the income tax rate and for inflation to minimize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i [h(\tau_{t+i}) + k(\pi_{t+i})],$$

where the functions h and k represent the distortionary costs of the two tax sources. Assume that the functions h and k imply positive and increasing marginal costs of both revenue sources.

a. What is the *intratemporal* optimality condition linking the choices of τ and π at each point in time?

b. What is the *intertemporal* optimality condition linking the choice π at different points in time?

c. Suppose $y = 1$, $f(\pi) = a\pi$, $h(\tau) = b\tau^2$, and $k(\pi) = c\pi^2$. Evaluate the inter- and intratemporal conditions. Find the optimal settings for τ_t and π_t in terms of b_{t-1} and $\sum R^{-i} g_{t+i}$.

d. Using your results from part c, when will optimal financing imply constant planned tax rates and inflation over time?

4. Suppose utility is given by $U = c^{1-\sigma}/(1-\sigma) + m^{1-\theta}/(1-\theta)$. Find the function $\phi(P)$ defined in (4.32) and verify that it has the shape shown in figure 4.3. Solve for the stationary equilibrium price level P^* such that $P^* = \phi(P^*)$.

5. Consider (4.37) implied by the fiscal theory of the price level. Seigniorage $\bar{s}_t$ was defined as $i_t m_t/(1+i_t)$. Assume that the utility function of the representative agent takes the form $u(c,m) = \ln c + b \ln m$. Show that $\bar{s}_t = bc_t$ and that the price level is independent of the nominal supply of money as long as $\tau_t - g_t + bc_t$ is independent of M_t.

6. Mankiw (1987) suggested that the nominal interest rate should evolve as a random walk under an optimal tax policy. Suppose that the real rate of interest is constant and that the equilibrium price level is given by (4.29). Suppose that the nominal money supply is given by $m_t = m_t^p + v_t$, where m_t^p is the central bank's planned money supply and v_t is a white noise control error. Let θ be the optimal rate of inflation. There are different processes for m^p that lead to the same average inflation rate but different time-series behavior of the nominal interest rate. For each of the processes for m_t^p given below, demonstrate that average inflation is θ. Is the nominal interest rate a random walk?

a. $m_t^p = \theta(1-\gamma)t + \gamma m_{t-1}$;

b. $m_t^p = m_{t-1} + \theta$.

7. Consider the optimal tax problem of section 4.4.3. The government wishes to maximize $u(c,m,l) = v(c,m) + \phi(l)$ subject to the economy's resource constraint: $f(1-l) \geq c + g$.

a. Derive the implementability constraint by using the first order conditions (4.48)–(4.50) to eliminate the tax rates from the representative agent's budget constraint (4.47).

b. Set up the government's optimization problem and derive the first order conditions.

c. Show that the first order condition for m is satisfied if $v_m = v_{mc} = v_{mm} = 0$. Argue that these conditions are met if the satiation level m^* is equal to ∞.

8. Suppose the Correia-Teles model of section 4.4.3 is modified so that output is equal to $f(n)$, where f is a standard neoclassical production function exhibiting positive but diminishing marginal productivity of n. Show that if $f(n) = n^a$ for $a > 0$, the optimality condition given by (4.70) continues to hold.

5 Money, Output, and Inflation in the Short Run

5.1 Introduction

Chapter 1 provided evidence that monetary policy actions have effects on real output that persist for appreciable periods of time. The empirical evidence from the United States is consistent with the notion that positive monetary shocks lead to a hump-shaped positive response of output, and Sims (1992) finds similar patterns for other OECD economies. We have not yet discussed why such a response is produced. Certainly the models of chapters 2–4 did not seem capable of producing such an effect. So why does money matter?[1] Is it only through the tax effects that arise from inflation? Or are there other channels through which monetary actions have real effects? This question is critical for any normative analysis of monetary policy, since designing good policy requires understanding how monetary policy affects the real economy and how changes in the way policy is conducted might affect economic behavior.

In the models examined in earlier chapters, monetary disturbances did cause output movements, but these movements arose from substitution effects induced by expected inflation. The simulation exercises suggested that these effects were too small to account for the empirical evidence on the output responses to monetary shocks. In addition, the evidence in many countries is that inflation responds only slowly to monetary shocks.[2] If actual inflation responds gradually, so should expectations. Thus, the evidence does not appear supportive of theories that require monetary shocks to affect labor-supply decisions and output by causing shifts in expected inflation.

In this chapter, the focus shifts away from the role of inflation as a tax and toward the effects of policy-induced changes in real interest rates that affect aggregate spending decisions. In making this shift, we move from the general equilibrium models built on the joint foundations of individual optimization and flexible prices to the class of general equilibrium models built on optimizing behavior and nominal rigidities that are employed in most discussions of monetary policy issues.

We begin this chapter with a discussion of models that maintain the assumption that nominal wages and prices are flexible, exploring channels for money to have real effects that were missing in chapters 2–4. These channels involve informational or distributional effects. As we will see, these attempts to account for the empirical evidence on the short-run effects of monetary policy have so far had mixed success. We

1. For a survey on this topic, see Blanchard (1990). See also Romer (1996, chapters 5 and 6).

2. For example, see Nelson (1998) or Christiano, Eichenbaum, and Evans (1999) for evidence on the United States. Sims (1992) and Taylor (1993b) provide evidence for other countries.

then turn to models in which monetary policy and monetary disturbances have important short-run effects on real economic activity because of nominal wage and price rigidities.

It is easy to see why nominal price stickiness is important. As we have seen in the previous chapters, the nominal quantity of money affects equilibrium in two ways. First, its rate of change affects the rate of inflation. Changes in expected inflation affect the opportunity cost of holding money, leading to real effects on labor-leisure choices and the choice between cash and credit goods. However, these substitution effects seem small empirically. Second, money appears in household budget constraints, cash-in-advance (CIA) constraints, and utility functions in the form of real money balances. If prices are perfectly flexible, changes in the nominal quantity of money via monetary policy actions will not necessarily affect the real supply of money. When prices are sticky, however, changing the nominal stock of money does initially alter the real stock of money. These changes then affect the economy's real equilibrium. Short-run price and wage stickiness implies a much more important role for monetary disturbances and monetary policy.

5.2 Flexible Prices

In chapters 2–4 the emphasis was on the importance of anticipated money growth that affected expectations of inflation, and it was through substitution effects induced by expected inflation that money mattered. To account for the empirical evidence on the short-run impact of money, models that maintain the assumption of price flexibility need to introduce new channels through which money can affect the real equilibrium. In this section we review two attempts to resolve the tension between the long-run neutrality of money and the short-run real effects of money while maintaining the assumption that wages and prices are flexible. The first approach focuses on misperceptions about aggregate economic conditions; the second focuses on trading restrictions in financial markets.

5.2.1 Imperfect Information

During the 1960s the need to reconcile the long-run neutrality of money with the apparent short-run nonneutrality of money was not considered a major research issue in macroeconomics. Models used for policy analysis incorporated a Phillips curve relationship between wage (or price) inflation and unemployment that allowed for a long-run trade-off between the two. In 1968, Milton Friedman and Edmund Phelps (M. Friedman 1968; Phelps 1968) independently argued on theoretical

grounds that the inflation-unemployment trade-off was only a short-run trade-off at best; attempts to exploit the trade-off by engineering higher inflation to generate lower unemployment would ultimately result only in higher inflation.

Milton Friedman (1968, 1977) reconciled the apparent short-run trade-off with the neutrality of money by distinguishing between actual real wages and perceived real wages.[3] The former were relevant for firms making hiring decisions; the latter were relevant for workers making labor-supply choices. In a long-run equilibrium, the two would coincide; the real wage would adjust to clear the labor market. Since economic decisions depend on real wages, the same labor-market equilibrium would be consistent with any level of nominal wages and prices or any rate of change of wages and prices that left the real wage equal to its equilibrium level.

An unexpected increase in inflation would disturb this real equilibrium. As nominal wages and prices rose more rapidly than previously expected, workers would see their nominal wages rising but would initially not realize that the prices of all the goods and services they consumed were also rising more rapidly. They would misinterpret the nominal wage increase as a rise in their real wage. Labor supply would increase, shifting the labor-market equilibrium to a point of higher employment and lower actual real wages. As workers then engaged in shopping activities, they would discover that not only the nominal price of their labor services had risen unexpectedly, but all prices had risen. Real wages had actually fallen, not risen. The labor supply curve would shift back, and the initial equilibrium would be restored eventually.

The critical insight is that changes in wages and prices that are unanticipated generate misperceptions about relative prices (the real wage in Friedman's version). Economic agents, faced with what they perceive to be changes in relative prices, alter their real economic decisions, and the economy's real equilibrium is affected. Once expectations adjust, however, the economy's natural equilibrium is reestablished. Expectations, and the information on which they are based, become central to understanding the effects of money.

The Lucas Model Friedman's insight was given an explicit theoretical foundation by Lucas (1972). Lucas showed how unanticipated changes in the money supply could generate short-run transitory movements in real economic activity. He did so by analyzing the impact of monetary fluctuations in an overlapping-generations environment with physically separate markets. The demand for money in each location was made random by assuming that the allocation of the population to each

3. A nice exposition of Friedman's model is provided by Rasche (1973).

location was stochastic.[4] The key features of this environment can be illustrated by employing the analogy of an economy consisting of a large number of individual islands. Agents are randomly reallocated among islands after each period, so individuals care about prices on the island they currently are on and prices on other islands to which they may be reassigned. Individuals on each island are assumed to have imperfect information about aggregate economic variables such as the nominal money supply and price level. Thus, when individuals observe changes in the prices on their island, they must decide whether they reflect purely nominal changes in aggregate variables or island-specific relative price changes.

To illustrate how variations in the nominal quantity of money can have real affects when information is imperfect, assume a basic money-in-the-utility (MIU) function model, such as the one developed in chapter 2, but simplify it in three ways. First, ignore capital. This choice implies that only labor is used to produce output and, with no investment, equilibrium requires that output equal consumption. Second, assume that money is the only available asset. Third, assume that the monetary transfers associated with changes in the nominal quantity of money are viewed by agents as being proportional to their own holdings of cash. This change has substantive implications and is not done just to simplify the model. It implies that the transfers will appear to money holders as interest payments on their cash holdings. This approach eliminates inflation-tax effects so that we can concentrate on the role of imperfect information.[5]

Suppose the aggregate economy consists of several islands, indexed by i; thus, x^i denotes the value of variable x on island i, while x denotes its economy-wide average value. Using the model from the appendix to chapter 2, we can express equilibrium deviations from the steady state on each island by the following three conditions:[6]

$$y_t^i = (1-\alpha)n_t^i \tag{5.1}$$

$$\left[1+\eta\left(\frac{n^{ss}}{l^{ss}}\right)\right]n_t^i = y_t^i + \lambda_t^i \tag{5.2}$$

4. In Lucas's formulation, agents had two-period lives; young agents were distributed randomly to each location.

5. Recall that in chapter 2, transfers were viewed as lump-sum. With higher inflation, the transfers rose (as the seigniorage revenues were returned to private agents), but each individual viewed these transfers as unrelated to his or her own money holdings. If the transfers are viewed as interest payments, higher inflation does not raise the opportunity cost of holding money since the interest payment on cash also rises. In this case, money is superneutral.

6. All variables are expressed as natural log deviations around steady-state values. Since all values will be in terms of deviations, the "hat" notation of chapters 2–4 will be dropped for convenience. For an early exposition of a linearized version of Lucas's model, see McCallum (1984a).

$$m_t^i - p_t^i = y_t^i + \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[\mathrm{E}^i\tau_{t+1} - (\mathrm{E}^i p_{t+1} - p_t^i) + \mathrm{E}^i(\lambda_{t+1} - \lambda_t^i)], \tag{5.3}$$

where $\lambda_t^i = -\Omega_1 y_t^i + \Omega_2(m_t^i - p_t^i)$ is the marginal utility of consumption.[7] Note that the goods market equilibrium condition, $y_t = c_t$, has been used and, in contrast to chapters 2–4, m^i now denotes the *nominal* supply of money on island i. Equation (5.1) is the production function linking labor input (n_t^i) to output.[8] Equation (5.2) comes from the first order condition linking the marginal utility of leisure, the marginal utility of consumption, and the real wage.[9] Equation (5.3) is derived from the first order condition for the individual agent's holdings of real money balances. This first order condition requires that reducing consumption at time t slightly, thereby carrying higher money balances into period $t+1$ and then consuming them, must, at the margin, have no effect on total utility over the two periods. In the present context, the cost of reducing consumption in period t is the marginal utility of consumption; the additional money balances yield the marginal utility of money in period t and a gross return of T_{t+1}/Π_{t+1} in period $t+1$, where T_{t+1} is the gross nominal transfer per dollar on money holdings and Π_{t+1} is 1 plus the inflation rate from t to $t+1$. This return can be consumed at $t+1$, yielding, in terms of period t utility, $\beta(T_{t+1}/\Pi_{t+1})$ times the marginal utility of consumption, where β is the representative household's discount factor. Linearizing the result around the steady state leads to (5.3); details can be found in the appendix.

If agents are reallocated randomly across islands in each period, then the relevant period $t+1$ variables in (5.3) are aggregate per capita real money balances, $m_{t+1} - p_{t+1}$, consumption, c_{t+1}, and the nominal transfer τ_{t+1}. However, information, and therefore expectations, will differ across islands, so the expectations operator has the superscript i.

7. The model assumes that the representative agent's utility function is

$$u(C_t, M_t/P_t, 1-N_t) = \frac{[aC_t^{1-b} + (1-a)(M_t/P_t)^{1-b}]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi\frac{[(1-N_t)^{1-\eta}]}{1-\eta}.$$

The parameters in (5.1)–(5.3) are $\Omega_1 = [\gamma\Phi + (1-\gamma)b]$, $\Omega_2 = (b-\Phi)(1-\gamma)$ and $\gamma = a/[a+(1-a)\cdot[(M/CP)^{ss}]^{1-b}]$.

8. Note that any productivity disturbance has been eliminated since the focus will be on monetary disturbances.

9. Equation (5.2) arises from the requirement that the marginal utility of leisure ($[\eta n^{ss}/(1+n^{ss})]n_t$ in percentage deviation around the steady state) equal the real wage times the marginal utility of consumption. The marginal product of labor (the real wage) is equal to $(1-\alpha)Y/N$, or $y-n$ in terms of percentage deviations. The marginal utility of consumption, in derivation form, is $-\Omega_1 y_t^i + \Omega_2(m_t^i - p_t^i)$.

The final component of the model is the specification of the nominal money supply process. Assume the aggregate average nominal money supply evolves as[10]

$$m_t = \gamma m_{t-1} + v_t + u_t. \tag{5.4}$$

The aggregate supply is assumed to depend on two shocks, v and u, assumed to have zero means and variances σ_v^2 and σ_u^2. The difference between the two is that we assume v is public information, while u is not. Including both will help to illustrate how imperfect information (in this case about u) will influence the real effects of money shocks. The nominal money stock on island i is given by

$$m_t^i = \gamma m_{t-1} + v_t + u_t + u_t^i,$$

where u^i is an island-specific money shock that averages to zero across all islands and has variance σ_i^2. If the aggregate money stock at time $t-1$, as well as v, is public information, then observing the island-specific nominal money stock m_t^i allows individuals on island i to infer $u_t + u_t^i$ but not u and u^i separately. This is important because only u affects the aggregate money stock (see 5.4) and, as long as $\gamma \neq 0$, knowledge about u would be useful in forecasting m_{t+1}.

Since $m_{t+1} = \gamma m_t + v_{t+1} + u_{t+1} = \gamma(\gamma m_{t-1} + v_t + u_t) + v_{t+1} + u_{t+1}$, the expectation of the time $t+1$ money supply, conditional on the information available on island i, will be $\mathrm{E}^i m_{t+1} = \gamma^2 m_{t-1} + \gamma v_t + \gamma \mathrm{E}^i u_t$. But what will $\mathrm{E}^i u_t$ equal? If expectations are equated with linear least squares projections,

$$\mathrm{E}^i u_t = \kappa(u_t + u_t^i),$$

where $\kappa = \sigma_u^2/(\sigma_u^2 + \sigma_i^2)$, $0 \le \kappa \le 1$. If aggregate money shocks are large relative to island-specific shocks (i.e., σ_u is large relative to σ_i), κ will be close to 1, as movements in $u + u^i$ are interpreted as predominantly reflecting movements in the aggregate shock u. In contrast, if the variance of the island-specific shocks is large, κ will be close to 0 as movements in $u + u^i$ are interpreted as predominantly reflecting island-specific shocks.

Using (5.1)–(5.3), section 5.7.1 of the appendix to this chapter shows that the equilibrium solutions for the price level and employment are given by

$$p_t = \gamma m_{t-1} + v_t + \left(\frac{\kappa + K}{1 + K}\right) u_t \tag{5.5}$$

10. With money supply changes engineered via transfers,

$$\tau_t = m_t - m_{t-1} = (\gamma - 1)m_{t-1} + v_t + u_t.$$

and

$$n_t = A(m_t - p_t) = A\left(\frac{1-\kappa}{1+K}\right)u_t, \tag{5.6}$$

where A and K depend on the underlying parameters of the model and are given in the appendix.

Equation (5.6) reveals Lucas's basic result; aggregate monetary shocks, represented by u, have real effects on employment (and therefore output) if and only if there is imperfect information ($\kappa < 1$). Publicly announced changes in the money supply, represented by the v shocks, have no real effects on output (v does not appear in 5.6) but simply move the price level one for one (v has a coefficient equal to 1 in 5.5). But the u shocks will affect employment and output if private agents are unable to determine whether the money stock movements they observe on island i (m^i) reflect aggregate or island-specific movements. Predictable movements in money (captured here by γm_{t-1}) or announced changes (captured by v) have no real effects. Unanticipated changes in the money supply, represented by the errors agents make in inferring u (given by $(1-\kappa)u$), will have real effects.

Equation (5.6) can be rewritten in a form that emphasizes the role of "money surprises" in producing employment and output effects. From (5.4), we can write $u_t = m_t - \mathrm{E}(m_t \mid \Gamma_{t-1}, v_t)$ where $\mathrm{E}(m_t \mid \Gamma_{t-1}, v_t)$ denotes the expectation of m_t conditional on aggregate information on variables dated $t-1$ or earlier, summarized by the information set Γ_{t-1} and the announced money injection v_t. Thus,

$$n_t = A\left(\frac{1-\kappa}{1+K}\right)[m_t - \mathrm{E}(m_t \mid \Gamma_{t-1}, v_t)].$$

Equations of this form provided the basis for the empirical work of Barro (1977, 1978) and others in testing whether unanticipated or anticipated changes in money matter for real output.

In writing employment as a function of money surprises, it is critically important to specify correctly the information set on which agents base their expectations. In empirical work, this information set is often assumed to consist simply of lagged values of the relevant variables. But in our example, $\mathrm{E}(m_t \mid \Gamma_{t-1}) = \gamma m_{t-1}$ and $m_t - \mathrm{E}(m_t \mid \Gamma_{t-1}) = u_t + v_t \neq u_t$. Misspecifying the information set can create difficulties in testing models that imply only surprises matter.

Because we derived (5.6) directly from a model consistent with optimizing behavior, we are able to relate the effects of an unanticipated money supply shock on employment to the basic underlying parameters of the production and utility functions.[11] Using the basic parameter values given in table 2.2 of chapter 2, $A/[1+K] = .002$. This implies that even if κ is close to zero, the elasticity of employment with respect to a money surprise is tiny; a 10% surprise increase in the money supply would raise employment by .02% and output by less than $(1-\alpha) * .02\% = .64 * .02\% = .01\%$.[12]

The impact of money surprises in this example works through labor supply decisions. An increase in real money balances raises the marginal utility of consumption and induces agents to increase consumption and labor supply (since $\Omega_2 > 0$). This effect is larger the more willing agents are to substitute consumption over time. Thus, the impact of a money surprise is larger when the degree of intertemporal substitution is larger.[13] The effect of a money surprise on output is increasing in the wage elasticity of labor supply.[14]

The basic idea behind Lucas's island model is that unpredicted variations in money generate price movements that agents may misinterpret as relative price movements. If a general price rise is falsely interpreted to be a rise in the relative price of what the individual or firm sells, the price rise will induce an increase in employment and output. Once individuals and firms correctly perceive that the price rise was part of an increase in all prices, output returns to its former equilibrium level.

Implications Lucas's model makes clear the important distinction between expected and unexpected variations in money. Economic agents face a signal-extraction problem because they have imperfect information about the current money supply. If all changes in the nominal supply of money were perfectly predictable, money would have no real effects. Short-run fluctuations in the money supply are likely to be at least partially unpredictable, so they will cause output and employment movements. In this way, Lucas was able to reconcile the neutrality of money in the long run with important real effects in the short run. Sargent and Wallace (1975) and Barro (1976) provide important early contributions that employed the general approach pioneered by Lucas to examine its implications for monetary policy issues.

11. McCallum (1984a) presents a linearized approximation to Lucas's model within an overlapping-generations framework. See also Romer (1996). However, both simply postulate some of the basic behavioral relationships of the model.

12. This uses the baseline value of .36 for α.

13. See Barro and King (1984).

14. In the Federal Reserve's FRB/US structural econometric model, the wage elasticity of labor supply is assumed to be zero (Brayton and Tinsley 1996), so that this channel for misperceptions of money disturbances to have real effects is absent.

Lucas's model has several important testable implications, and these were the focus of a great deal of empirical work in the late 1970s and early 1980s. A first implication is that the distinction between anticipated and unanticipated money matters. Barro (1977, 1978, 1979b) was the first to directly examine whether output was related to anticipated or unanticipated money. He concluded that the evidence supported Lucas's model, but subsequent empirical work by Mishkin (1982) and others showed that both anticipated and unanticipated money appear to influence real economic activity. A survey of the general approach motivated by Lucas's work and of the empirical literature can be found in chapter 2 of Barro (1981).

A second implication is that the short-run relationship between output and inflation will depend on the relative variance of real and nominal disturbances. The parameter κ in (5.6) depends on the predictability of aggregate changes in the money supply, and this can vary across time and across countries. Lucas (1973) examined the slopes of short-run Phillips curves in a cross-country study and showed that, as predicted by his model, there was a positive correlation between the slope of the Phillips curve and the relative variance of nominal aggregate volatility. A rise in aggregate volatility (an increase in σ_u^2 in the version of Lucas's model developed in the previous section) implies that an observed increase in prices is more likely to be interpreted as resulting from an aggregate price increase. A smaller real response occurs as a result, and aggregate money surprises have smaller real effects.

A third influential implication of Lucas's model was demonstrated by Sargent and Wallace (1975) and became known as the *policy irrelevance hypothesis*. If changes in money have real effects only when they are unanticipated, then any policy that generates systematic, predictable variations in the money supply will have no real effect. For example, (5.6) shows that employment, and therefore output, are independent of the degree of serial correlation in m as measured by γ. Because the effects of lagged money on the current aggregate money stock are completely predictable, no informational confusion is created and the aggregate price level simply adjusts, leaving real money balances unaffected (see 5.5). A similar conclusion would hold if policy responded to lagged values of u (or to lagged values of anything else), as long as private agents knew the rule being followed by the policy maker.

The empirical evidence that both anticipated and unanticipated money affect output implies, however, that the policy irrelevance hypothesis does not hold. Systematic responses to lagged variables seem to matter, and therefore the choice of policy rule is not irrelevant for the behavior of real economic activity.

The misperceptions model in the original form developed by Lucas, and popularized by Sargent and Wallace (1975) and Barro (1976), who employed tractable log-linear models based on Lucas's theory, is no longer viewed as an adequate explanation for the short-run real effects of monetary policy. It has had, and continues to have, however, an enormous influence on modern monetary economics. For example, the finding that announced changes in money (the v term in our example) have no real effects implies that inflation could be reduced at no output cost simply by announcing a reduction in money growth. But such announcements must be credible so that expectations are actually reduced as money growth falls; disinflations will be costly if announcements are not credible. This point has produced a large literature on the role of credibility, a literature discussed in chapter 8.

In most work in monetary economics, imperfect information no longer plays a major role as the source of monetary nonneutrality. Instead, the assumption of flexible prices is dropped and prices and/or wages are assumed to be sticky (see section 5.3). However, Mankiw and Reis (2002) have argued that *sticky information*—the slow dispersal of information about macroeconomic conditions—can help account for the sluggish adjustment of prices that appears to characterize the data.

5.2.2 Limited Participation and Liquidity Effects

The impact of a monetary disturbance on market interest rates can be decomposed into its effect on the expected real rate of return and its effect on the expected inflation rate. If money growth is positively serially correlated, an increase in money growth will be associated with higher future inflation and, therefore, higher *expected* inflation. As we saw in chapters 2 and 3, the flexible price MIU and CIA models implied that faster money growth would immediately increase nominal interest rates.

Most economists, and certainly monetary policy makers, believe that central banks can reduce short-term nominal interest rates and can do so by employing policies that lead to faster growth in the money supply. This belief is often interpreted to mean that faster money growth will initially cause nominal interest rates to fall, an impact called the *liquidity effect*. This effect is usually viewed as an important channel through which a monetary expansion affects real consumption, investment, and output.[15]

A number of authors have explored flexible-price models in which monetary injections reduce nominal interest rates (Lucas 1990; Christiano 1991; Christiano and Eichenbaum 1992a, 1995; Fuerst 1992; Dotsey and Ireland 1995; King and Watson 1996; Cooley and Quadrini 1999). These models generate effects of monetary shocks on real interest rates by imposing restrictions on the ability of agents to engage in

15. A thorough discussion of possible explanations of liquidity effects is provided by Ohanian and Stockman (1995) and Hoover (1995).

certain types of financial transactions.[16] For example, Lucas modifies a basic CIA framework in a way that allows him to study effects that arise when monetary injections are not distributed equally across a population of otherwise representative agents. If a monetary injection affects agents differentially, a price-level increase proportional to the aggregate change in the money stock will not restore the initial real equilibrium. Some agents will be left with higher real money holdings, others with lower real balances.

Fuerst (1992) and Christiano and Eichenbaum (1992a) introduce a liquidity effect by modifying a basic CIA model to distinguish between households, firms, and financial intermediaries. Households can allocate resources between bank deposits and money balances that are then used to finance consumption. Intermediaries lend out their deposits to firms that borrow to finance purchases of labor services from households. After households have made their choice between money and bank deposits, financial intermediaries receive lump-sum monetary injections. Only firms and intermediaries interact in financial markets after the monetary injection.[17]

In a standard representative-agent CIA model, monetary injections are distributed proportionately to all agents. Thus, a proportional rise in the price level leaves all agents with the same level of real money balances as previously. In contrast, if the injections initially affect only the balance sheets of the financial intermediaries, a new channel is introduced by which employment and output will be affected. As long as the nominal interest rate is positive, intermediaries will wish to increase their lending in response to a positive monetary injection. To induce firms to borrow the additional funds, the interest rate on loans must fall. Hence, a liquidity effect is generated; interest rates decline in response to a positive monetary injection.[18] The restrictions on trading mean that cash injections create a wedge between the value of cash in the hands of household members shopping in the goods market and the value of cash in

16. The first limited-participation models were due to Grossman and Weiss (1983) and Rotemberg (1984). Models that restrict financial transactions can be viewed as variants of the original Baumol-Tobin models with infinite costs for certain types of transactions, rather than the finite costs of exchanging money and interest earning assets assumed by Baumol (1952) and Tobin (1956).

17. Allowing for heterogeneity greatly complicates the analysis, but these limited-participation models overcome this problem by following the modeling strategy introduced by Lucas (1980), in which each representative "family" consists of a household supplying labor and purchasing goods, a firm hiring labor, producing goods, and borrowing from the intermediary, and an intermediary. At the end of each period, the various units of the family are reunited and pool resources. As a result, there can be heterogeneity within periods as the new injections of money affect only firms and intermediaries, but between periods all families are identical, so the advantages of the representative-agent formulation are preserved.

18. Expected inflation effects will also be at work, so the net impact on nominal interest rates will depend on, among other things, the degree of positive serial correlation in the growth rate of the money supply.

the financial market.[19] Because Fuerst and Christiano and Eichenbaum assume that firms must borrow to fund their wage bill, the appropriate marginal cost of labor to firms is the real wage times the gross rate of interest on loans. The interest-rate decline generated by the liquidity effect lowers the marginal cost of labor; at each real wage, labor demand increases. As a result, equilibrium employment and output rise.

Assessment Models that generate real effects of money by restricting financial transactions can account for nominal (and real) interest-rate declines in response to monetary policy shocks. But as Dotsey and Ireland (1995) show, this class of models does not account for interest-rate effects of the magnitude actually observed in the data. Similarly, King and Watson (1996) find that monetary shocks do not produce significant business-cycle fluctuations in their version of a limited-participation model (which they call a *liquidity-effect model*). Christiano, Eichenbaum, and Evans (1997) show that their limited-participation model is able to match evidence on the effects of monetary shocks on prices, output, real wages, and profits only if the labor-supply wage elasticity is assumed to be very high. They argue that this outcome is due, in part, to the absence of labor-market frictions in the current generation of limited-participation models.

Because limited-participation models were developed to account for the observation that monetary injections lower market interest rates, the real test of whether they have isolated an important channel through which monetary policy operates must come from evaluating their other implications. Christiano, Eichenbaum, and Evans (1997) examine real wage and profit movements to test their models. They argue that limited-participation models are able to account for the increase in profits that follows a monetary expansion. A further implication of such models relates to the manner in which the impact of monetary injections will change over time as financial sectors evolve and the cost of transactions falls. Financial markets today are very different than they were 25 years ago, and these differences should show up in the way money affects interest rates now as compared to 25 years ago.[20] While financial market frictions are likely to be important in understanding the impact effects of monetary policy actions on short-term market interest rates, the relevance of the

19. In Fuerst (1992), this wedge is measured by the difference between the Lagrangian multiplier on the household's CIA constraint and that on the firm's CIA constraint. A cash injection lowers the value of cash in the financial market and lowers the nominal rate of interest.

20. Cole and Ohanian (2002) argue that the impact of money shocks in the United States has declined with the ratio of $M1$ to nominal GDP, a finding consistent with the implications of limited-participation models.

channels emphasized in limited-participation models for understanding the broader effects of monetary policy on the aggregate economy remains an open debate.

5.3 Nominal Rigidities

Whereas flexible-price, imperfect-information models enjoyed popularity during the 1970s and flexible-price, limited-participation models have attracted attention only more recently, most macroeconomic models attribute the short-run real effects of monetary disturbances to the presence of nominal wage and/or price rigidities. These rigidities mean that nominal wages and prices fail to adjust immediately and completely to changes in the nominal quantity of money. In the 1980s, it was common to impose the assumption that prices (or wages) were fixed for one period (see chapter 8). This modification increases the impact monetary disturbances have on real output but cannot account for persistent real effects of monetary policy. The model of staggered-overlapping and multi-period nominal wage contracts due to Taylor (1979, 1980) can generate the persistent output responses observed in the data, but Taylor's model was not based on an explicit model of optimizing behavior by workers or firms. The literature in recent years has turned to models of monopolistic competition and price stickiness in which the decision problem faced by firms in setting prices can be made explicit.

The next subsection briefly examines a model that adds one-period nominal wage rigidity to the MIU model of chapter 2. While this approach is not based on optimizing behavior by wage setters, it leads to a reduced-form model that has been widely used in monetary economics. This model plays an important role in chapter 8. We then turn to a model in which monopolisticly competitive firms optimally adjust prices every two periods. This leads to a dynamic adjustment of prices that is similar to that proposed by Taylor (1979, 1980, 1993b).[21] We then develop a model of price adjustment based on Calvo (1983), one that has formed the foundation for many recent models used in monetary policy analysis. Section 5.4 shows how a model of price stickiness can be embedded in a general equilibrium framework to yield a simple model useful for addressing a number of monetary policy issues.

5.3.1 Wage Rigidity in an MIU Model

One way to introduce nominal price stickiness is to modify a flexible-price model, such as the MIU model of chapter 2, by simply assuming that prices and/or wages

21. Section 5.3.3 below contains a discussion of some alternative price adjustment specifications. Coverage of a variety of price-adjustment models can be found in Romer (2001, chapter 6).

are set at the start of each period and are unresponsive to developments within the period. In chapter 2 a linear approximation was used to examine the time-series implications of an MIU model. Wages and prices were assumed to adjust to ensure market equilibrium, and, as a consequence, the behavior of the money supply mattered only to the extent that anticipated inflation was affected. A positive disturbance to the growth rate of money would, assuming that the growth rate of money was positively serially correlated, raise the expected rate of inflation, leading to a rise in the nominal rate of interest that affects labor supply and output. These last effects depended on the form of the utility function; if utility was separable in money, changes in expected inflation had no effect on labor supply or real output. Introducing wage stickiness into an MIU model will serve to illustrate the effect such a modification has on the impact of monetary disturbances.

Suppose we use the linear approximation to the Sidrauski MIU model developed in chapter 2. To simplify the model, we assume utility is separable in consumption and money holdings ($b = \Phi$, or $\Omega_2 = 0$ in terms of the parameters of the model used in chapter 2). This implies that money and monetary shocks have no effect on output when prices are perfectly flexible.[22] In addition, the capital stock is treated as fixed, and investment is zero. This follows McCallum and Nelson (1999), who argue that, for most monetary policy and business-cycle analyses, fluctuations in the stock of capital do not play a major role. The equations characterizing equilibrium in the resulting MIU model are

$$y_t = (1 - \alpha)n_t + e_t \tag{5.7}$$

$$y_t = c_t \tag{5.8}$$

$$y_t - n_t = w_t - p_t \tag{5.9}$$

$$\Phi \mathrm{E}_t(c_{t+1} - c_t) - r_t = 0 \tag{5.10}$$

$$\eta\left(\frac{n^{ss}}{1 - n^{ss}}\right)n_t + \Phi c_t = w_t - p_t \tag{5.11}$$

$$m_t - p_t = c_t - \left(\frac{1}{b}\right)i_t \tag{5.12}$$

$$i_t = r_t + \mathrm{E}_t p_{t+1} - p_t \tag{5.13}$$

$$m_t = \gamma m_{t-1} + s_t. \tag{5.14}$$

22. From (5.6), money surprises also have no effect on employment and output when $\Omega_2 = 0$ in Lucas's imperfect-information model.

The system is written in terms of the price level p rather than the inflation rate, and, in contrast to the notation of chapter 2, m represents the nominal stock of money. To briefly review these equations, (5.7) is the economy's production function in which output deviations from the steady state are a linear function of the deviations of labor supply from the steady state and a productivity shock. Equation (5.8) is the resource constraint derived from the condition that, in the absence of investment or government purchases, output equals consumption. Labor demand is derived from the condition that labor is employed up to the point where the marginal product of labor equals the real wage. With the Cobb-Douglas production function underlying (5.7), this condition, expressed in terms of percentage deviations from the steady state, can be written as (5.9).[23] Equations (5.10) and (5.12) are derived from the representative household's first order conditions for consumption, leisure, and money holdings. Equation (5.13) is the Fisher equation linking the nominal and real rates of interest. Finally, (5.14) gives the exogenous process for the nominal money supply.[24]

When prices are flexible, (5.7)–(5.11) form a system of equations that can be solved for the equilibrium time paths of output, labor, consumption, the real wage, and the real rate of interest. Equations (5.12)–(5.14) then determine the evolution of real money balances, the nominal interest rate, and the price level. Thus, realizations of the monetary disturbance s_t have no effect on output when prices are flexible. This version of the MIU model displays the *classical dichotomy* (Modigilani 1963; Patinkin 1965); real variables such as output, consumption, investment, and the real interest rate are determined independently of both the money supply process and money demand factors.[25]

Now suppose the nominal wage rate is set prior to the start of the period, and that it is set equal to the level *expected* to produce the *real* wage that equates labor supply and labor demand. Since workers and firms are assumed to have a real wage target in mind, the nominal wage will adjust fully to reflect expectations of price-level

23. If $Y = \bar{K}^{\alpha}N^{1-\alpha}$, then the marginal product of labor is $(1-\alpha)Y/N$, where $\bar{K}$ is the fixed stock of capital. In log terms, the real wage is then equal to $\ln W - \ln P = \ln(1-\alpha) + \ln Y - \ln N$, or, in terms of deviations from steady state, $w - p = y - n$.

24. Alternatively, the nominal interest rate i_t could be taken as the instrument of monetary policy, with (5.12) then determining m_t.

25. This is stronger than the property of monetary superneutrality, in which the real variables are independent of the money-supply process. For example, Lucas's model does not display the classical dichotomy as long as $\Omega_2 \neq 0$ because the production function, the resource constraint, and the labor-supply condition cannot be solved for output, consumption, and employment without knowing the real demand for money, since real balances enter (5.2).

changes held at the time the nominal wage is set. This means that the information available at the time the wage is set, and on which expectations will be based, will be important. If unanticipated changes in prices occur, the actual real wage will differ from its expected value. In the standard formulation, firms are assumed to determine employment on the basis of the actual, realized real wage. If prices are unexpectedly low, the actual real wage will exceed the level expected to clear the labor market, and firms will reduce employment.[26]

The equilibrium level of employment and the real wage with flexible prices can be obtained by equating labor supply and labor demand (from 5.11 and 5.9) and then by using the production function (5.7) and the resource constraint (5.8) to obtain

$$n_t^* = \left[\frac{1-\Phi}{1+\bar{\eta}+(1-\alpha)(\Phi-1)}\right]e_t = b_0 e_t$$

and

$$\omega_t^* = \left[\frac{\bar{\eta}+\Phi}{1+\bar{\eta}+(1-\alpha)(\Phi-1)}\right]e_t = b_1 e_t,$$

where n^* is the flex-price equilibrium employment, ω^* is the flex-price equilibrium real wage, and $\bar{\eta} \equiv \eta n^{ss}/(1-n^{ss})$.

The contract nominal wage w^c will satisfy

$$w_t^c = \mathrm{E}_{t-1}\omega_t^* + \mathrm{E}_{t-1}p_t. \tag{5.15}$$

With firms equating the marginal product of labor to the actual real wage, actual employment will equal $n_t = y_t - (w_t^c - p_t) = y_t - \mathrm{E}_{t-1}\omega_t^* + (p_t - \mathrm{E}_{t-1}p_t)$, or, using the production function and noting that $\mathrm{E}_{t-1}\omega_t^* = -\alpha\mathrm{E}_{t-1}n_t^* + \mathrm{E}_{t-1}e_t$,

$$n_t = \mathrm{E}_{t-1}n_t^* + \left(\frac{1}{\alpha}\right)(p_t - \mathrm{E}_{t-1}p_t) + \left(\frac{1}{\alpha}\right)\varepsilon_t, \tag{5.16}$$

where $\varepsilon_t = (e_t - \mathrm{E}_{t-1}e_t)$. Equation (5.16) shows that employment deviates from the expected flexible price equilibrium level in the face of unexpected movements in prices. An unanticipated increase in prices reduces the real value of the contract wage and leads firms to expand employment. An unexpected productivity shock ε_t raises the marginal product of labor and leads to an employment increase.

26. This implies that the real wage falls in response to a positive money shock. Using a VAR approach based on U.S. data, Christiano, Eichenbaum, and Evans (1997) find that an expansionary monetary policy shock actually leads to a slight increase in real wages.

By substituting (5.16) into the production function, one obtains

$$y_t = (1-\alpha)\left[\mathrm{E}_{t-1}n_t^* + \left(\frac{1}{\alpha}\right)(p_t - \mathrm{E}_{t-1}p_t) + \left(\frac{1}{\alpha}\right)\varepsilon_t\right] + e_t,$$

which implies that

$$y_t - \mathrm{E}_{t-1}y_t^* = a(p_t - \mathrm{E}_{t-1}p_t) + (1+a)\varepsilon_t, \tag{5.17}$$

where $\mathrm{E}_{t-1}y^* = (1-\alpha)\mathrm{E}_{t-1}n_t^* + \mathrm{E}_{t-1}e_t$ is expected equilibrium output under flexible prices and $a = (1-\alpha)/\alpha$. Innovations to output are positively related to price innovations. Thus, monetary shocks that produce unanticipated price movements directly affect real output.

The linear approximation to the MIU model, augmented with one-period nominal wage contracts, produces one of the basic frameworks often used to address policy issues. This framework generally assumes serially uncorrelated disturbances, so the aggregate supply equation (5.17) becomes

$$y_t = a(p_t - \mathrm{E}_{t-1}p_t) + (1+a)\varepsilon_t, \tag{5.18}$$

and the demand side often consists of a simple quantity equation of the form

$$m_t - p_t = y_t. \tag{5.19}$$

This model can be obtained from the model of the appendix by letting $b \to \infty$; this implies that the interest elasticity of money demand goes to zero. According to (5.18), a 1% deviation of p from its expected value will cause a $(1-\alpha)/\alpha \approx 1.8\%$ deviation of output if the benchmark value of 0.36 is used for α. To solve the model for equilibrium output and the price level, given the nominal quantity of money, note that (5.19) and (5.14) imply

$$p_t - \mathrm{E}_{t-1}p_t = m_t - \mathrm{E}_{t-1}m_t - (y_t - \mathrm{E}_{t-1}y_t) = s_t - y_t.$$

Substituting this result into (5.18), one obtains

$$y_t = \left(\frac{a}{1+a}\right)s_t + \left(\frac{1+a}{1+a}\right)\varepsilon_t = (1-\alpha)s_t + \varepsilon_t. \tag{5.20}$$

A 1% money surprise increases output by $1-\alpha \approx 0.64\%$. Notice that in (5.18), the coefficient a on price surprises depends on parameters of the production function. This is in contrast to Lucas's misperceptions model, in which the impact on output of a price surprise depends on the variances of shocks (see 5.6). The model consisting of (5.18) and (5.19) will play an important role in the analysis of monetary policy in chapter 8.

Benassy (1995) shows how one-period wage contracts affect the time-series behavior of output in a model similar to the one used here but in which capital is not ignored. However, the dynamics associated with consumption smoothing and capital accumulation are inadequate on their own to produce anything like the output persistence that is revealed by the data.[27] That is why real business-cycle models assumed that the productivity disturbance itself is highly serially correlated. Because we have assumed that nominal wages are fixed for only one period, the estimated effects of a monetary shock on output die out almost completely after one period.[28] This would continue to be the case even if the money shock were serially correlated. While serial correlation in the s_t shock would affect the behavior of the price level, this will be incorporated into expectations, and the nominal wage set at the start of $t+1$ will adjust fully to make the expected real wage (and therefore employment and output) independent of the predictable movement in the price level. Just adding one-period sticky nominal wages will not capture the persistent effects of monetary shocks, but it will significantly influence the impact effect of a money shock on the economy.

5.3.2 Imperfect Competition and Price Stickiness

The previous section illustrated how wage inflexibility has quite dramatic effects on the impact of monetary disturbances. Except for the assumption that the nominal wage was set one period in advance, the model was identical to an equilibrium model of an economy characterized by perfect competition. Several authors have argued that nominal rigidities arise because of small *menu costs,* essentially fixed costs, associated with changing wages or prices. As economic conditions change, a firm's *optimal* price will also change, but if there are fixed costs of changing prices, it may not be optimal for the firm to adjust its price continuously to economic changes. Only if the firm's actual price diverges sufficiently from the equilibrium price will it be worthwhile to bear the fixed cost and adjust prices. The macroeconomic implications of menu cost models were first explored by Akerlof and Yellen (1985) and Mankiw (1985) and are surveyed by Romer (2001, chapter 6). Ball and Romer (1991) show how small menu costs can interact with imperfect competition in either goods or labor markets to amplify the impact of monetary disturbances, create strategic complementaries, and lead potentially to multiple equilibria. While menu costs

27. Cogley and Nason (1995) demonstrate this for standard real business-cycle models.

28. In Benassy's model with parameters $\alpha = 0.40$ and rate of depreciation $\delta = 0.019$, equilibrium output (expressed as a deviation from trend) is given by $y_t \approx 0.6 * (1 + .006L - .002L^2 \ldots)(m_t - m_t^e)$, so that the effects of a money surprise die out almost immediately (Benassy, 1995, equation 51, p. 313).

rationalize sluggish price-setting behavior, such costs may seem implausible as the reason monetary disturbances have significant real effects. After all, adjusting production is also costly, and it is difficult to see why shutting down an assembly line is less costly than reprinting price catalogs. And computers have lowered the cost of changing prices for most retail establishments, though it seems unlikely that this has had an important effect on the ability of monetary authorities to have short-run real effects on the economy. Money seems to matter in important ways because of nominal rigidities, but we do not have a satisfactory integration of microeconomic models of nominal adjustment with monetary models of macroeconomic equilibrium.

A problem with simply introducing nominal wage rigidity into an otherwise competitive model is that any sort of nominal rigidity naturally raises the question of who is setting wages and prices, a question the perfectly competitive model begs. Once we need to address the issue of price setting, we must examine models that incorporate some aspect of imperfect competition, such as monopolistic competition.

A Basic Model of Monopolistic Competition To explore the implications of nominal rigidities, a basic model that incorporates monopolistic competition among intermediate goods producers is developed in this subsection. Examples of similar models include Blanchard and Kiyotaki (1987), Ball and Romer (1991), Beaudry and Devereux (1995), and King and Watson (1996). Imperfect competition can lead to aggregate demand externalities (Blanchard and Kiyotaki 1987), equilibria in which output is inefficiently low, and multiple equilibria (Ball and Romer 1991, Rotemberg and Woodford 1995), but imperfect competition alone does not lead to monetary nonneutrality. If prices are free to adjust, one-time, permanent changes in the level of the money supply induce proportional changes in all prices, leaving the real equilibrium unaffected. Price stickiness remains critical to generating significant real effects of money. Our example follows Chari, Kehoe, and McGrattan (2000), and in the following subsection, we add price stickiness by assuming that intermediate goods producers engage in multi-period, staggered price setting.

Let Y_t be the output of the final good; it is produced using inputs of the intermediate goods according to

$$Y_t = \left[\int Y_t(i)^q \, di\right]^{\frac{1}{q}}, \quad 0 < q \leq 1, \tag{5.21}$$

where $Y_t(i)$ is the input of intermediate good i. Firms producing final goods operate in competitive output markets and maximize profits given by $P_t Y_t - \int P_t(i) Y_t(i) \, di$, where P is the price of final output and $P(i)$ is the price of input i. The first order conditions for profit maximization by final goods producers yield the following input demand functions for intermediate good i:

$$Y_t^d(i) = \left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-q}} Y_t. \tag{5.22}$$

Final goods firms earn zero profit as long as

$$P_t = \left[\int P_t(i)^{\frac{q}{q-1}} di\right]^{\frac{q-1}{q}}.$$

Each intermediate good is produced according to a constant returns to scale, Cobb-Douglas production function:

$$Y_t(i) = K_t(i)^a L_t(i)^{1-a}, \tag{5.23}$$

where K and L denote capital and labor inputs purchased in competitive factor markets at prices r and W. The producer of good $Y(i)$ chooses $P(i)$, $K(i)$, and $L(i)$ to maximize profits subject to the demand function (5.22) and the production function (5.23). Intermediate profits are equal to

$$\begin{aligned}\pi_t(i) &= P_t(i)Y_t(i) - r_tK_t(i) - W_tL_t(i)\\ &= [P_t(i) - P_tV_t]\left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-q}} Y_t,\end{aligned} \tag{5.24}$$

where V_t is equal to minimized unit costs of production (so P_tV_t is nominal unit cost). The first order condition for the value of $P_t(i)$ that maximizes profits for the intermediate goods–producing firm is

$$\left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-q}} Y_t - \frac{1}{1-q}[P_t(i) - P_tV_t]\left[\frac{P_t}{P_t(i)}\right]^{\frac{2-q}{1-q}}\left(\frac{1}{P_t}\right) Y_t = 0.$$

After some rearranging, this yields

$$P_t(i) = \frac{P_tV_t}{q}. \tag{5.25}$$

Thus, the price of intermediate good i is set as a constant markup $1/q$ over unit nominal costs PV.

For the intermediate goods producers, labor demand involves setting

$$\frac{W_t}{P_t(i)} = q\left[\frac{(1-\alpha)Y_t(i)}{L_t(i)}\right], \tag{5.26}$$

where W_t is the nominal wage rate and $(1-\alpha)Y_t(i)/L_t(i)$ is the marginal product of labor. In a symmetric equilibrium, all intermediate firms charge the same relative price, employ the same labor and capital inputs, and produce at the same level, so $P_t(i) = P_t(j) = P_t$, and (5.26) implies

$$L_t = \frac{q(1-\alpha)Y_t}{W_t/P_t}. \tag{5.27}$$

Firms will be concerned with their relative price, not the absolute price level, so money remains neutral. As (5.26) and (5.27) show, proportional changes in all nominal prices (i.e., $P(i)$, P, and W) leave firm i's optimal relative price and aggregate labor demand unaffected. If we do not alter the household's decision problem from our earlier analysis, neither consumption, labor supply, nor investment decisions would be altered by proportional changes in all nominal prices and the nominal stock of money.[29]

To complete the specification of the model, the aggregate demand for labor given by (5.27) must be equated to the aggregate labor supply derived from the outcome of household choices. In the flexible-price models examined so far, labor market equilibrium with competitive factor markets required that the marginal rate of substitution between leisure and consumption be equal to the real wage, which, in turn, was equal to the marginal product of labor. With imperfect competition, (5.27) shows that q drives a wedge between the real wage and the marginal product of capital.[30] Thus, labor market equilibrium requires that

$$\frac{U_l}{U_c} = \frac{W}{P} = qMPL \leq MPL. \tag{5.28}$$

If we now linearize the model around the steady state, q drops out of the labor-market equilibrium condition because of the way in which it enters multiplicatively.[31]

Adding Price Stickiness Chari, Kehoe, and McGrattan (2000) introduce price stickiness into their model by following Taylor (1979, 1980), who argued that the

29. The household's budget constraint is altered since real profits of the intermediate goods producers must be paid out to households. However, as (5.24) shows, nominal profits are homogeneous of degree 1 in prices, so their real value will be homogeneous of degree 0. Thus, proportional changes in the nominal money stock and all prices leave the household's budget constraint unaffected.

30. In their calibrations, Chari, Kehoe, and McGrattan use a value of 0.9 for q.

31. The ex-ante marginal product of capital is $q\alpha \mathrm{E}_t Y_{t+1}/K_t$. In the steady state, this is equal to $(1/\beta) + \delta - 1$, so the percentage deviation of the marginal product around its steady state value is $r_t = q\alpha(Y/K)(\mathrm{E}_t\hat{y}_{t+1} - \hat{k}_t) = [(1/\beta) + \delta - 1](\mathrm{E}_t\hat{y}_{t+1} - \hat{k}_t)$, which is also independent of q.

presence of multi-period nominal contracts, with only a fraction of wages or prices negotiated each period, could generate the type of real output persistence in response to monetary shocks observed in the data. When setting a price during period t that will remain in effect for several periods, a firm will base its decisions on its expectations of conditions in future periods. But the aggregate price level will also depend on those prices set in earlier periods that are still in effect. This imparts both forward-looking and backward-looking aspects to the aggregate price level and, as Taylor showed, provides a framework capable of replicating aggregate dynamics.

To develop a simple example, suppose that each intermediate goods–producing firm sets its price $P(i)$ for two periods, with half of all firms adjusting in each period.[32] Thus, if $i \in [0, 0.5)$, assume that $P(i)$ is set in period t, $t+2$, $t+4$, and so on. If $i \in [0.5, 1]$, the firm sets prices in periods $t+1$, $t+3$, and so on. Since we will only consider symmetric equilibria in which all firms setting prices at time t pick the same price, we can drop the index i and let $\bar{P}_{t+j}$ denote the intermediate goods price set in period $t+j$ for periods $t+j$ and $t+j+1$.

Consider a firm i setting its price in period t. This price will be in effect for periods t and $t+1$. Thus, if R_t is the gross interest rate, $\bar{P}_t$ will be chosen to maximize

$$\mathrm{E}_t\left[(\bar{P}_t - P_t V_t)\left(\frac{P_t}{\bar{P}_t}\right)^{\frac{1}{1-q}} Y_t + R_t^{-1}(\bar{P}_t - P_{t+1}V_{t+1})\left(\frac{P_{t+1}}{\bar{P}_t}\right)^{\frac{1}{1-q}} Y_{t+1}\right],$$

which represents the expected discounted profits over periods t and $t+1$.[33] After some manipulation of the first order condition, one obtains

$$\bar{P}_t = \frac{\mathrm{E}_t(P_t^\theta V_t Y_t + R_t^{-1} P_{t+1}^\theta V_{t+1} Y_{t+1})}{q\mathrm{E}_t(P_t^{\frac{1}{1-q}} Y_t + R_t^{-1} P_{t+1}^{\frac{1}{1-q}} Y_{t+1})}, \tag{5.29}$$

where $\theta = (2-q)/(1-q)$. If prices are set for only one period, the terms involving $t+1$ drop out and one obtains the earlier pricing equation (5.25).

What does (5.29) imply about aggregate price adjustment? Let $\bar{p}$, p, and v denote percentage deviations of $\bar{P}$, P, and V around a zero-inflation steady state. If we ignore discounting for simplicity, (5.29) can be approximated in terms of percentage

32. This is a form of non-state-contingent pricing; prices are set for a fixed length of time, regardless of economic conditions.

33. Chari, Kehoe, and McGrattan (2000) consider situations in which a fraction $1/N$ of all firms set prices each period for N periods. They can then vary N to examine its role in affecting aggregate dynamics. They alter the interpretation of the time period so that N always corresponds to one year; thus, varying N alters the degree of staggering. They conclude that N has little effect.

deviations around the steady state as

$$\bar{p}_t = \tfrac{1}{2}(p_t + \mathrm{E}_t p_{t+1}) + \tfrac{1}{2}(v_t + \mathrm{E}_t v_{t+1}). \tag{5.30}$$

The average price of the final good, expressed in terms of deviations from the steady state, is $p_t = \frac{1}{2}(\bar{p}_{t-1} + \bar{p}_t)$, where $\bar{p}_{t-1}$ is the price of intermediate goods set at time $t-1$ and $\bar{p}_t$ is the price set in period t. Similarly, $\mathrm{E}_t p_{t+1} = \frac{1}{2}(\bar{p}_t + \bar{p}_{t+1})$. Substituting these expressions into the equation for $\bar{p}_t$ yields

$$\bar{p}_t = \tfrac{1}{2}\bar{p}_{t-1} + \tfrac{1}{2}\mathrm{E}_t\bar{p}_{t+1} + (v_t + \mathrm{E}_t v_{t+1}).$$

This reveals the backward-looking (via the presence of $\bar{p}_{t-1}$) and forward-looking (via the presence of $\mathrm{E}_t\bar{p}_{t+1}$ and $\mathrm{E}_t v_{t+1}$) nature of price adjustment.

The variable v_t is the deviation of minimized unit costs from its steady state. Suppose this is proportional to output: $v_t = \gamma y_t$.[34] If we further assume a simple money demand equation of the form $m_t - p_t = y_t$, we obtain

$$\begin{aligned}\bar{p}_t &= \frac{1}{2}\bar{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\bar{p}_{t+1} + \gamma(y_t + \mathrm{E}_t y_{t+1})\\ &= \frac{1}{2}\bar{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\bar{p}_{t+1} + \gamma(m_t - p_t + \mathrm{E}_t m_{t+1} - \mathrm{E}_t p_{t+1})\\ &= \frac{1}{2}\left(\frac{1-\gamma}{1+\gamma}\right)(\bar{p}_{t-1} + \mathrm{E}_t\bar{p}_{t+1}) + \left(\frac{\gamma}{1+\gamma}\right)(m_t + \mathrm{E}_t m_{t+1}).\end{aligned} \tag{5.31}$$

This is a difference equation in $\bar{p}$. It implies that the behavior of prices set during period t will depend on prices set during the previous period, on prices expected to be set during the next period, and on the path of the nominal money supply over the two periods during which $\bar{p}_t$ will be in effect. For the case in which m_t follows a random walk (so that $\mathrm{E}_t m_{t+1} = m_t$), the solution for $\bar{p}_t$ is

$$\bar{p}_t = a\bar{p}_{t-1} + (1-a)m_t, \tag{5.32}$$

where $a = (1-\sqrt{\gamma})(1+\sqrt{\gamma})$ is the root less than 1 of $a^2 - 2(1+\gamma)a/(1-\gamma) + 1 = 0$.[35] Since the aggregate price level is an average of prices set at t and $t-1$,

$$p_t = a p_{t-1} + \tfrac{1}{2}(1-a)(m_t + m_{t-1}). \tag{5.33}$$

34. The coefficient γ will depend on the elasticity of labor supply with respect to the real wage. See Chari, Kehoe, and McGrattan (2000) and section 5.4.

35. See problem 2 at the end of the chapter.

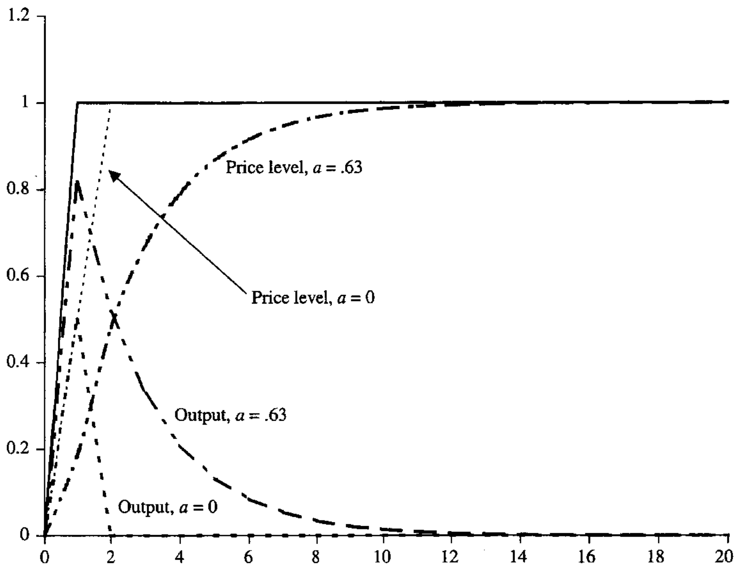

Figure 5.1
The Effects of a Money Shock with Staggered Price Adjustment

Taylor (1979, 1980) demonstrated that a price adjustment equation of the form given by (5.33) is capable of mimicking the dynamic response of U.S. prices.[36] The response, however, depends critically on the value of a (which, in turn, depends on γ). Figure 5.1 shows the response of the price level and output for $\gamma = 1$ ($a = 0$) and $\gamma = 0.05$ ($a = 0.63$). This latter value is the one Taylor finds matches U.S. data, and, as the figure shows, an unexpected, permanent increase in the nominal money supply produces a rise in output with a slow adjustment back to the baseline, mirrored by a gradual rise in the price level. Though the model assumes that prices are set for only two periods, the money shock leads to a persistent, long-lasting effect on output with this value of γ.

Chari, Kehoe, and McGrattan assume that employment must be consistent with household labor supply choices, and they show that γ is a function of the parameters

36. Taylor's actual model was based on nominal wage adjustment rather than price adjustment as presented here.

of the representative agent's utility function. They argue that a very high labor-supply elasticity is required to obtain a value of γ on the order of 0.05. With a low labor-supply elasticity, as seems more plausible, γ will be greater than or equal to 1. If this is the case, the Taylor model is not capable of capturing realistic adjustment to monetary shocks. Ascari (2000) reaches similar conclusions in a model that is similar to the framework in Chari, Kehoe, and McGrattan (2000) but that follows Taylor's original work in making wages sticky rather than prices. However, rather than drawing the implication that staggered price (or wage) adjustment is unimportant for price dynamics, the assumption that observed employment is consistent with the labor-supply behavior implied by the model of the household can be questioned. Models that interpret observed employment as tracing out a labor-supply function typically have difficulty matching other aspects of labor market behavior (Christiano and Eichenbaum 1992b).

5.3.3 Inflation Persistence

The assumption of two-period nominal price stickiness employed in the previous section demonstrates how a nominal rigidity could be introduced into a model based on optimizing firm behavior. Nelson (1998) has argued that models such as the one developed by Chari, Kehoe, and McGrattan cannot account for the sluggish response of *inflation* that seems to characterize the data. In this section, we follow Roberts (1995) and discuss models of price-level adjustment that attempt to capture the dynamic aspects of inflation adjustment. Models due to Taylor (1979, 1980), Calvo (1983), and Fuhrer and Moore (1995a) are developed. An important distinction turns out to be whether it is the price level or the inflation rate that is sticky.

Before proceeding, it is important to distinguish between two sources of persistence in observed inflation. Inflation can display persistence if money growth rates display persistence. If this were the only sense in which inflation exhibits persistence, it could easily be explained within the context of flexible-price models. The behavior of inflation would simply reflect the behavior of the money growth rate; if the way policy is conducted introduces a high degree of serial correlation into the money growth process, then this will be reflected in the behavior of the inflation rate.[37] It is a second source of inflation persistence that is the focus here. In response to serially uncorrelated monetary policy shocks (measured either by money growth rates or by interest rate movements), the response of inflation appears to follow a highly serially correlated pattern. This is the sense of persistence with which we are concerned here.

37. Dittman, Gavin, and Kydland (2002) provide an example of a flexible-price model in which the central bank's policy causes real output persistence to generate inflation persistence.

Taylor's Model Taylor (1979, 1980) originally developed his model in terms of nominal wage-setting behavior, so we follow that approach here. With prices assumed to be a constant markup over wage costs, the adjustment of wages translates directly into an adjustment equation for prices.

Assume that wages are set for two periods with one half of all contracts negotiated each period. Let x_t equal the log contract wage set at time t. The average wage faced by the firm is equal to $w_t = (x_t + x_{t-1})/2$ since in period t, contracts set in the previous period (x_{t-1}) are still in effect. Assuming a constant markup, the log price level is given by

$$p_t = w_t + \mu,$$

where μ is the log markup. For convenience, normalize so that $\mu = 0$.

For workers covered by the contract set in period t, the average expected real wage over the life of the contract is $\frac{1}{2}[(x_t - p_t) + (x_t - \mathrm{E}_t p_{t+1})] = x_t - \frac{1}{2}(p_t + \mathrm{E}_t p_{t+1})$.[38] In Taylor (1980), the expected average real contract wage is assumed to be increasing in the level of economic activity, represented by log output:

$$x_t = \tfrac{1}{2}(p_t + \mathrm{E}_t p_{t+1}) + k y_t. \tag{5.34}$$

With $p_t = .5(x_t + x_{t-1})$,

$$\begin{aligned} p_t &= \frac{1}{2}\left[\frac{1}{2}(p_t + \mathrm{E}_t p_{t+1}) + k y_t + \frac{1}{2}(p_{t-1} + \mathrm{E}_{t-1} p_t) + k y_{t-1}\right] \\ &= \frac{1}{4}[2p_t + \mathrm{E}_t p_{t+1} + p_{t-1} + \eta_t] + \frac{k}{2}(y_t + y_{t-1}), \end{aligned}$$

where $\eta_t \equiv \mathrm{E}_{t-1} p_t - p_t$ is an expectational error term. Rearranging,

$$p_t = \tfrac{1}{2} p_{t-1} + \tfrac{1}{2}\mathrm{E}_t p_{t+1} + k(y_t + y_{t-1}) + \tfrac{1}{2}\eta_t. \tag{5.35}$$

The basic Taylor specification leads to inertia in the aggregate price level. The value of p_t is influenced both by expectations of future prices and by the price level in the previous period.

Expressed in terms of the rate of inflation $\pi_t = p_t - p_{t-1}$, (5.35) implies

$$\pi_t = \mathrm{E}_t \pi_{t+1} + 2k(y_t + y_{t-1}) + \eta_t. \tag{5.36}$$

38. It would be more appropriate to assume that workers care about the present discounted value of the real wage over the life of the contract. This would lead to a specification of the form $.5(1+\beta)x_t - .5(p_t + \beta \mathrm{E}_t p_{t+1})$ for $0 < \beta < 1$, where β is a discount factor.

The key implication of (5.36) is that although prices display inertia, the inflation rate need not exhibit inertia. This is important, as can be seen by considering the implications of Taylor's model for a policy of disinflation. Suppose that the economy is in an initial, perfect-foresight equilibrium with a constant inflation rate π_1. Now suppose that in period $t-1$, a policy maker announces a policy that will lower the inflation rate to π_2 in period t and then maintain inflation at this new lower rate. Using (5.35) and the definition of η_t, it can be shown that this disinflation has no impact on total output. As a consequence, inflation can be costlessly reduced. The price level is sticky in Taylor's specification, but the rate at which it changes—the rate of inflation—is not. The backward-looking aspect of price behavior causes unanticipated reductions in the *level* of the money supply to cause real output declines. Prices set previously are too high relative to the new path for the money supply; only as contracts expire can their real value be reduced to levels consistent with the new, lower money supply. However, as Ball (1994a) has shown, price rigidities based on such backward-looking behavior need not imply that policies to reduce inflation by reducing the *growth rate* of money will cause a recession. Since m continues to grow, just at a slower rate, the real value of preset prices continues to be eroded, unlike the case of a level reduction in m.[39]

Calvo's Model An alternative model of staggered price adjustment is due to Calvo (1983). He assumed that firms adjust their prices infrequently and that opportunities to adjust arrived as an exogenous Poisson process. Each period, there is a constant probability $1-\omega$ that the firm can adjust its price; the expected time between price adjustments is $1/(1-\omega)$. Because these adjustment opportunities occur randomly, the interval between price changes for an individual firm is a random variable.

Following Rotemberg (1987), suppose the representative firm i sets its price to minimize a quadratic loss function that depends on the difference between the firm's actual price in period t, p_{it}, and its optimal price, p_t^*.[40] This latter price might denote the profit-maximizing price for firm i in the absence of any restrictions or costs associated with price adjustment. If the firm can adjust at time t, it will set its price to minimize

$$\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^i (p_{it+j} - p_{t+j}^*)^2, \tag{5.37}$$

39. For example, when the policy to reduce inflation from π_1 to π_2 is announced in period $t-1$, $\mathrm{E}_{t-1}\pi_t$ falls. For a given level of output, this decline would reduce π_{t-1}. If the policy maker acts to keep inflation unchanged at the time of the announcement (i.e., π_{t-1}), output must increase.

40. Since all firms are assumed to be identical (except for the timing of their price adjustments), the subscript i on p^* is dropped.

subject to the assumed process for determining when the firm will next be able to adjust. If only the terms in (5.37) involving the price set at time t are written out, they are

$$(p_{it} - p_t^*)^2 + \omega\beta \mathrm{E}_t(p_{it} - p_{t+1}^*)^2 + \omega^2\beta^2 \mathrm{E}_t(p_{it} - p_{t+2}^*)^2 + \cdots$$

or

$$\sum_{j=0}^{\infty} \omega^i \beta^i \mathrm{E}_t(p_{it} - p_{t+i}^*)^2,$$

since ω^i is the probability that the firm has not adjusted after i periods so that the price set at t still holds in $t+i$. Thus, the first order condition for the optimal choice of p_{it} requires that

$$p_{it}\sum_{j=0}^{\infty} \omega^i \beta^i - \sum_{j=0}^{\infty} \omega^i \beta^i \mathrm{E}_t p_{t+i}^* = 0.$$

Rearranging, and letting x_t denote the optimal price set at t by all firms adjusting their price,

$$x_t = (1 - \omega\beta)\sum_{j=0}^{\infty} \omega^i \beta^i \mathrm{E}_t p_{t+i}^*. \tag{5.38}$$

The price set by the firm at time t is a weighted average of current and expected future values of the target price p^*. If ω is small, the expected time until the firm can next adjust its price is short. In this case, less weight is placed on future p^*'s.

Equation (5.38) can be rewritten as

$$x_t = (1 - \omega\beta)p_t^* + \omega\beta \mathrm{E}_t x_{t+1}.$$

If the price target p^* depends on the aggregate price level and output, we can replace p_t^* with $p_t + \gamma y_t + \varepsilon_t$, where ε is a random disturbance to capture other determinants of p^*. In section 5.4.1, the firm's optimal price will be shown to be a function of its marginal cost, which, in turn, can be related to a measure of output.

With a large number of firms, a fraction $1 - \omega$ will actually adjust their price each period, and the aggregate price level can be expressed as $p_t = (1 - \omega)x_t + \omega p_{t-1}$. We then have the following two equations to describe the evolution of x_t and p_t:

$$x_t = (1 - \omega\beta)(p_t + \gamma y_t + \varepsilon_t) + \omega\beta \mathrm{E}_t x_{t+1} \tag{5.39}$$

$$p_t = (1 - \omega)x_t + \omega p_{t-1}. \tag{5.40}$$

To obtain an expression for aggregate inflation, update (5.40) by one period and take expectations to obtain $\mathrm{E}_t p_{t+1} = (1 - \omega)\mathrm{E}_t x_{t+1} + \omega p_t$. This can be rewritten as $(1 - \omega)\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} + (1 - \omega)p_t$. Use this to eliminate $\mathrm{E}_t x_{t+1}$ from (5.39), and then use the resulting expression to eliminate x_t from (5.40), yielding

$$p_t = (1 - \omega)(1 - \omega\beta)(p_t + \gamma y_t + \varepsilon_t) + \omega\beta[\mathrm{E}_t \pi_{t+1} + (1 - \omega)p_t] + \omega p_{t-1}.$$

Collecting terms, and noting that $\pi_t = p_t - p_{t-1}$,

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \left[\frac{(1 - \omega)(1 - \omega\beta)}{\omega}\right](\gamma y_t + \varepsilon_t) \\ &= \beta \mathrm{E}_t \pi_{t+1} + \gamma' y_t + \varepsilon_t'. \end{aligned} \tag{5.41}$$

Comparing this to the inflation equation from Taylor's model, (5.36), shows them to be quite similar. Current inflation depends on expectations of future inflation and on current output. One difference is that in deriving an inflation equation based on Calvo's specification, expected future inflation has a coefficient equal to the discount factor $\beta < 1$. In deriving an expression for inflation using Taylor's specification, however, we ignored discounting in (5.34), the equation giving the value of the contract wage. A further difference between the Taylor model and the Calvo model has been highlighted by Kiley (2002). He shows that Taylor-type staggered adjustment models display less persistence than the Calvo-type partial adjustment model when both are calibrated to produce the same average frequency of price changes. Under the Taylor model, for example, suppose contracts are negotiated every two periods. The average frequency of wage changes is one-half—half of all wages adjust each period—and no wage remains fixed for more than two periods. In contrast, suppose $\omega = 1/2$ in the Calvo model. The expected time between price changes is two periods, so on average, prices are adjusted every two periods. However, many prices will remain fixed for more than two periods. For instance, $\omega^3 = .125$ of all prices remain fixed for at least three periods. In general, the Calvo model implies that there is a tail of the distribution of prices that consists of prices that have remained fixed for many periods, while the Taylor model implies that no wages remain fixed for longer than the duration of the longest contract.

One attractive aspect of Calvo's model is that it shows how the coefficient on output in the inflation equation depends on the frequency with which prices are adjusted. A rise in ω, which means that the average time between price changes for an individual firm increases, causes γ' in (5.41) to decrease. Output movements have a smaller impact on current inflation, holding expected future inflation constant. Because opportunities to adjust prices occur less often, current demand conditions become less important.

Fuhrer and Moore's Specification Disinflations are costly; they have normally been accompanied by below-trend growth and higher than average unemployment.[41] This suggests that it may be the inflation rate and not just the price level that exhibits some degree of stickiness. This view has been adopted by Fuhrer and Moore (1995a) in modeling the inflation process. Fuhrer and Moore assume that wage negotiations are conducted in terms of the wage relative to an average of real contract wages in effect over the life of a contract.[42] Specifically, define the real value of contracts negotiated at time t as $x_t - p_t \equiv \psi_t$. Define the index of average real contract wages in contracts still in effect at time t as

$$v_t = \tfrac{1}{2}(\psi_t + \psi_{t-1}). \tag{5.42}$$

Fuhrer and Moore assume that in setting ψ_t, agents take two factors into account. First, they attempt to achieve a current real contract wage equal to the expected average of the real contract index over the two-period life of the contract, $\frac{1}{2}(v_t + E_t v_{t+1})$. Second, the real contract wage can deviate from this average expected index to reflect the current state of the business cycle, ky_t.[43] Combining these assumptions with (5.42) yields

$$\begin{aligned}\psi_t &= \tfrac{1}{2}(v_t + E_t v_{t+1}) + ky_t \\ &= \tfrac{1}{4}(\psi_{t-1} + 2\psi_t + E_t\psi_{t+1}) + ky_t \\ &= \tfrac{1}{2}(\psi_{t-1} + E_t\psi_{t+1}) + 2ky_t.\end{aligned}$$

Recalling that $\psi_t = x_t - p_t$, this can be rewritten as

$$x_t - p_t = \tfrac{1}{2}[x_{t-1} - p_{t-1} + E_t(x_{t+1} - p_{t+1})] + 2ky_t, \tag{5.43}$$

which highlights the difference between the Taylor specification in (5.34) and the

41. Ball (1993, 1994b) provides some evidence on the costs of disinflations. As noted in chapter 1, Sargent (1986) has argued that hyperinflations have often ended with relatively little output cost.

42. Buiter and Jewitt (1981) provide the first analysis of this type of contracting model.

43. Fuhrer and Moore also allow the real contract wage to respond to the expected state of the business cycle by including $E_t y_{t+1}$ in the equation for ψ_t. We follow Roberts (1997) in excluding this factor in order to focus on the main differences between the Taylor specification and that of Fuhrer and Moore.

Fuhrer-Moore specification. After some rearranging, (5.43) can be written in terms of the rate of change in the contract wage as

$$\Delta x_t \equiv x_t - x_{t-1} = \tfrac{1}{2}(\pi_t + E_t\pi_{t+1}) + 2ky_t. \tag{5.44}$$

With the price level equal to $\frac{1}{2}(x_t + x_{t-1})$, inflation is given by $\pi_t = \frac{1}{2}(\Delta x_t + \Delta x_{t-1})$, implying

$$\pi_t = \tfrac{1}{2}(\pi_{t-1} + E_t\pi_{t+1}) + 2k(y_t + y_{t-1}) + \eta_t, \tag{5.45}$$

where $\eta_t = -(\pi_t - E_{t-1}\pi_t)$.[44] Contrasting this with the inflation equation (5.36) shows that the Fuhrer-Moore specification imparts a sluggishness to inflation adjustment; new information about current or future monetary policy that becomes available at the start of period t can be reflected in $E_t\pi_{t+1}$ but not, by definition, in π_{t-1}. Therefore, the flexibility of current inflation to jump in response to new information is limited. In the Fuhrer-Moore specification, the backward-looking nature of the *inflation* process implies that reductions in the growth rate of money will be costly in terms of output.

Whether price stickiness or inflation stickiness best characterizes the actual inflation processes is an open empirical issue. Fuhrer and Moore (1995a) argue that their specification fits U.S. data better than the Taylor model does. Roberts (1997) provides some evidence favoring the sticky-price version. However, he also shows that (5.45) could arise in a sticky-price model if expectations are not rational. Recall that η_t in (5.36) was equal to $-(\pi_t - E_{t-1}\pi_t)$. Under the assumption of rational expectations, this expectational error will be uncorrelated with information available at time $t-1$, the date at which the expectation is formed. Suppose instead that expectations are actually better proxied as an average of rational expectation and a simple extrapolation of current inflation. In this case, the expectation of future inflation in (5.36) would be replaced by $\frac{1}{2}(E_t\pi_{t+1} + \pi_t)$, while the previous period's expectation of π_t would equal $\frac{1}{2}(E_{t-1}\pi_t + \pi_{t-1})$. With these substitutions, the Taylor model can be written as

$$\pi_t = \tfrac{1}{2}(\pi_{t-1} + E_t\pi_{t+1}) + 2k(y_t + y_{t-1}) + \eta_t',$$

which has exactly the same form as (5.45). Based on an analysis of survey measures of inflation expectations for the United States, Roberts concludes that the evidence

44. Using (5.44), $\pi_t = \frac{1}{2}(\Delta x_t + \Delta x_{t-1})$, so

$$\begin{aligned}\pi_t &= \tfrac{1}{4}(\pi_t + E_t\pi_{t+1}) + ky_t + \tfrac{1}{4}(\pi_{t-1} + E_{t-1}\pi_t) + ky_{t-1} \\ &= \tfrac{1}{4}(\pi_t + E_t\pi_{t+1}) + ky_t + \tfrac{1}{4}(\pi_{t-1} + \pi_t) + ky_{t-1} + \tfrac{1}{4}\eta_t.\end{aligned}$$

Rearranging yields (5.45).

supports the view that inflation-rate stickiness actually arises from the presence of less than perfectly rational expectations.[45] Ireland (2001a) has estimated a price adjustment model that allows price stickiness and inflation stickiness to be distinguished. He finds that the U.S. data reject inflation stickiness in favor of price stickiness.

The distinction between price stickiness and inflation stickiness is important for understanding the costs of policies to lower inflation and the role of credibility. Disinflations must inevitably create recessions if inflation is sticky; they need to do so under price-level stickiness only if the policy lacks full credibility. In evaluating the eventual behavior of inflation after the adjustment to a change in policy, the assumption of rational expectations is likely to be appropriate. Otherwise, one is left with the unsatisfactory presumption that the public never fully learns about the policy. During transitional periods, as the way in which policy is conducted changes, the assumption of fully rational expectations may be inappropriate; backward-looking expectational behavior may play an important role in expectations formation. Predicting the effects of a change in policy may require that both types of behavior be recognized.[46]

5.4 A New Keynesian Model for Monetary Analysis

In the 1970s, 1980s, and early 1990s, the standard models used for most monetary policy analysis combined the assumption of nominal rigidity with a simple structure linking the quantity of money to aggregate spending. This linkage was usually directly through a quantity theory equation in which nominal demand was equal to the nominal money supply, often with a random disturbance included, or through a traditional textbook *IS-LM* model. While the theoretical foundations of these models were weak, the approach proved remarkably useful in addressing a wide range of monetary policy topics. More recently, attention has been placed on ensuring that the model structure is consistent with the underlying behavior of optimizing economic agents. The standard approach today builds on a dynamic, stochastic, general equilibrium framework based on optimizing behavior, combined with some form of nominal wage and/or price rigidity. Early examples of models with these properties

45. The dependence of expected inflation on lagged actual inflation need not reflect less than fully rational expectations. For example, suppose the public is uncertain as to the central bank's target rate of inflation. The public may base its beliefs about this target value on observed inflation under Bayesian updating. Erceg and Levin (2002) provide a recent development of this idea.

46. Taylor (1975) provided an early analysis of monetary policy during the transition to rational expectations. In his model, the monetary authority could influence the rate at which the public learned about policy by following a randomized rule for inflation.

include those of Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1995, 1997), and McCallum and Nelson (1999).

This section shows how a basic MIU model, combined with the assumption of monopolistically competitive goods markets and price stickiness, can form the basis for a simple linear macroeconomic model that is useful for policy analysis. The model is a consistent general equilibrium model in which all agents face well-defined decision problems and behave optimally. Three key modifications of the MIU model of chapter 2 will be made. First, endogenous variations in the capital stock are ignored. This follows McCallum and Nelson (1999), who argue that little is lost for the purposes of short-run business-cycle analysis by assuming an exogenous process for the capital stock. They show that, at least for the United States, there is little relationship between the capital stock and output at business-cycle frequencies. Endogenous capital stock dynamics play a key role in equilibrium business-cycle models in the real business-cycle tradition, but as Cogley and Nason (1995) show, the response of investment and the capital stock to productivity shocks actually contributes little to the dynamics implied by such models. For simplicity, then, the capital stock will be ignored.[47]

The second key modification is to incorporate differentiated goods whose individual prices are set by monopolistically competitive firms facing Calvo-type price stickiness. In the basic model, nominal wages will be allowed to fluctuate freely, although section 5.5 will explore the implications of assuming that both prices and wages are sticky.

Third, monetary policy is represented by a rule for setting the nominal rate of interest. The nominal quantity of money is then endogenously determined to achieve the desired nominal interest rate. Most central banks today use a short-term nominal interest rate as their instrument for implementing monetary policy. In the United States, for example, the Federal Reserve establishes a target for the federal funds interest rate. There are important issues involved in choosing between money supply policy procedures and interest rate procedures, and some of these will be discussed in chapter 9.

These three modifications yield a framework, often referred to as *new Keynesian*, that is consistent with optimizing behavior by private agents and incorporates nominal rigidities, yet is simple enough for use in exploring a number of policy issues. The resulting version of the MIU model can be linked directly to the more traditional aggregate supply-demand (AS-IS-LM) model that long served as one of the work-

47. However, Dotsey and King (2001) and Christiano, Eichenbaum, and Evans (2001) have emphasized the importance of variable capital utilization for understanding the behavior of inflation.

horses for monetary policy analysis. McCallum and Nelson (1999) also provide a version of an AS-IS-LM model derived as an approximation to a dynamic, stochastic, general equilibrium model, and Galí (2002) discusses the derivation of the model's equilibrium conditions.[48] The focus of this section is on the foundations of the new Keynesian model. This model is then used in chapters 10 and 11 to explore a variety of monetary policy issues.

5.4.1 The Basic Model

The model consists of households that supply labor, purchase goods for consumption, and hold money and bonds and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977). Each firm sets the price of the good it produces, but not all firms reset their price in each period. Households and firms behave optimally; households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. The central bank, in contrast to households and firms, is not assumed to behave optimally; chapters 8 and 11 examine the implications of optimizing behavior by the central bank.

Households The preferences of the representative household are defined over a composite consumption good C_t, real money balances M_t/P_t, and leisure $1 - N_t$, where N_t is the time devoted to market employment. Households maximize the expected present discounted value of utility:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \tag{5.46}$$

The composite consumption good consists of differentiated products produced by monopolistically competitive final goods producers (firms). There is a continuum of such firms of measure 1, and firm j produces good c_j. The composite consumption good that enters the household's utility function is defined as

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \theta > 1. \tag{5.47}$$

The parameter θ governs the price elasticity of demand for the individual goods.

48. See also Jeanne (1998).

The household's decision problem can be dealt with in two stages. First, regardless of the level of C_t the household decides on, it will always be optimal to purchase the combination of individual goods that minimizes the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of C_t, the household chooses C_t, N_t, and M_t optimally.

Dealing first with the problem of minimizing the cost of buying C_t, the household's decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} \, dj$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t, \tag{5.48}$$

where p_{jt} is the price of good j. Letting ψ_t be the Lagrangian multiplier on the constraint, the first order condition for good j is

$$p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0.$$

Rearranging, $c_{jt} = (p_{jt}/\psi_t)^{-\theta} C_t$. From the definition of the composite level of consumption (5.47), this implies

$$C_t = \left[\int_0^1 \left[\left(\frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \left(\frac{1}{\psi_t} \right)^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t.$$

Solving for ψ_t,

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t. \tag{5.49}$$

The Lagrangian multiplier is the appropriately aggregated price index for consumption. The demand for good j can then be written as

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t. \tag{5.50}$$

The price elasticity of demand for good j is equal to θ. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and, as a consequence, individual firms have less market power.

Given the definition of the aggregate price index in (5.49), the budget constraint of the household is, in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right)N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1})\left(\frac{B_{t-1}}{P_t}\right) + \Pi_t, \tag{5.51}$$

where M_t (B_t) is the household's nominal holdings of money (one-period bonds). Bonds pay a nominal rate of interest i_t. Real profits received from firms are equal to Π_t.

In the second stage of the household's decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (5.46) subject to (5.51). This leads to the following conditions, which, in addition to the budget constraint, must hold in equilibrium:[49]

$$C_t^{-\sigma} = \beta(1+i_t)\mathrm{E}_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}; \tag{5.52}$$

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1+i_t}; \tag{5.53}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{5.54}$$

These conditions represent the Euler condition for the optimal intertemporal allocation of consumption, the intratemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money, and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage.

Firms Firms maximize profits, subject to three constraints. The first is the production function summarizing the available technology. For simplicity, we have ignored capital, so output is a function solely of labor input N_{jt} and an aggregate productivity disturbance Z_t:

$$c_{jt} = Z_t N_{jt}, \quad \mathrm{E}(Z_t) = 1,$$

where constant returns to scale has been assumed. The second constraint on the firm is the demand curve each firm faces. This is given by (5.50). The third constraint is that each period some firms are not able to adjust their price. The specific model of

49. See chapter 2 for further discussion of these first order conditions in an MIU model.

price stickiness we will use is due to Calvo (1983).[50] Each period, the firms that adjust their price are randomly selected, and a fraction $1-\omega$ of all firms adjust while the remaining ω fraction do not adjust. The parameter ω is a measure of the degree of nominal rigidity; a larger ω implies that fewer firms adjust each period and that the expected time between price changes is longer. Those firms that do adjust their price at time t do so to maximize the expected discounted value of current and future profits. Profits at some future date $t+s$ are affected by the choice of price at time t only if the firm has not received another opportunity to adjust between t and $t+s$. The probability of this is ω^s.[51]

Before analyzing the firm's pricing decision, consider its cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written, in real terms, as

$$\min_{N_t}\left(\frac{W_t}{P_t}\right)N_t + \varphi_t(c_{jt} - Z_t N_{jt}).$$

where φ_t is equal to the firm's real marginal cost. The first order condition implies

$$\varphi_t = \frac{W_t/P_t}{Z_t}. \tag{5.55}$$

The firm's pricing decision problem then involves picking p_{jt} to maximize

$$\mathrm{E}_t\sum_{i=0}^{\infty}\omega^i\Delta_{i,t+i}\left[\left(\frac{p_{jt}}{P_{t+i}}\right)c_{jt+i} - \varphi_{t+i}c_{jt+i}\right],$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i(C_{t+i}/C_t)^{-\sigma}$. Using the demand curve (5.50) to eliminate c_{jt}, this objective function can be written as

$$\mathrm{E}_t\sum_{i=0}^{\infty}\omega^i\Delta_{i,t+i}\left[\left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\theta} - \varphi_{t+i}\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta}\right]C_{t+i}.$$

While individual firms produce differentiated products, they all have the same production technology and face demand curves with constant and equal demand elas-

50. See section 5.3.3.

51. In this formulation, the degree of nominal rigidity, as measured by ω, is constant, and the probability that a firm has adjusted its price is a function of time but not of the current state. State-dependent pricing models have been developed by Dotsey, King, and Wolman (1999) based on fixed costs of adjustment and by Kiley (2000) based on information costs. See also Wolman (1999). Haubrich and King (1991) developed a model with endogenous price stickiness in which nominal contracts provide insurance in the presence of random monetary injections that are distributed unequally across agents.

ticities. In other words, they are essentially identical, except that they may have set their current price at different dates in the past. However, all firms adjusting in period t face the same problem, so all adjusting firms will set the same price. Let p_t^* be the optimal price chosen by all firms adjusting at time t. The first order condition for the optimal choice of p_t^* is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta)\left(\frac{p_t^*}{P_{t+i}}\right) + \theta \varphi_{t+i} \right] \left(\frac{1}{p_t^*}\right) \left(\frac{p_t^*}{P_{t+i}}\right)^{-\theta} C_{t+i} = 0. \tag{5.56}$$

Using the definition of $\Delta_{i,t+i}$, (5.56) can be rearranged to yield

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}. \tag{5.57}$$

Consider the case in which all firms are able to adjust their prices every period ($\omega = 0$). When $\omega = 0$, (5.57) reduces to

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) \varphi_t = \mu \varphi_t. \tag{5.58}$$

Each firm sets its price p_t^* equal to a markup $\mu > 1$ over its nominal marginal cost $P_t \varphi_t$. This is the standard result in a model of monopolistic competition. Because price exceeds marginal cost, output will be inefficiently low. When prices are flexible, all firms charge the same price. In this case, $p_t^* = P_t$ and $\varphi_t = 1/\mu$. Using the definition of real marginal cost, this means

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu}$$

in a flexible-price equilibrium. However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization. This condition implies, from (5.54), that

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu} = \frac{\chi N_t^{\eta}}{C_t^{-\sigma}}. \tag{5.59}$$

Let $\hat{x}_t$ denote the percentage deviation of a variable X_t around its steady state and let the superscript f denote the flexible-price equilibrium. Then, approximating

(5.59) around the steady state yields $\eta \hat{n}_t^f + \sigma \hat{c}_t^f = \hat{z}_t$. From the production function, $\hat{y}_t^f = \hat{n}_t^f + \hat{z}_t$, and because output is equal to consumption in equilibrium, $\hat{y}_t^f = \hat{c}_t^f$. Combining these conditions, the flexible-price equilibrium output $\hat{y}_t^f$ can be expressed as

$$\hat{y}_t^f = \left(\frac{1+\eta}{\sigma+\eta}\right) \hat{z}_t. \tag{5.60}$$

When prices are sticky ($\omega > 0$), output can differ from the flexible-price equilibrium level. Because it will not adjust its price every period, the firm must take into account expected future marginal cost as well as current marginal cost whenever it has an opportunity to adjust its price. Equation (5.57) shows how adjusting firms set their price, conditional on the current aggregate price level P_t. This aggregate price index is an average of the price charged by the fraction $1-\omega$ of firms setting their price in period t and the average of the remaining fraction ω of all firms setting their price in earlier periods. However, because the adjusting firms were selected randomly from among all firms, the average price of the nonadjusters is just the average price of all firms that prevailed in period $t-1$. Thus, from (5.49), the average price in period t satisfies

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \tag{5.61}$$

Equations (5.57) and (5.61) can be approximated around a zero average inflation, steady-state equilibrium to obtain an expression for aggregate inflation (see the appendix, section 5.7, for details) of the form

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \tilde{\kappa} \hat{\varphi}_t, \tag{5.62}$$

where

$$\tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

is an increasing function of the fraction of firms able to adjust each period and $\hat{\varphi}_t$ is real marginal cost, expressed as a percentage deviation around its steady-state value.

Equation (5.62) is often referred to as the *new Keynesian Phillips curve*. Unlike more traditional Phillips curve equations, the new Keynesian Phillips curve implies that real marginal cost is the correct driving variable for the inflation process. It also implies that the inflation process is forward-looking, with current inflation a function of expected future inflation. When a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods. Solving (5.62) forward,

$$\pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \hat{\varphi}_{t+i},$$

which shows that inflation is a function of the present discounted value of current and future real marginal costs.

The new Keynesian Phillips curve also differs from traditional Phillips curves in having been derived explicitly from a model of optimizing behavior on the part of price setters, conditional on the assumed economic environment (monopolistic competition, constant elasticity demand curves, and randomly arriving opportunities to adjust prices).[52] One advantage such a derivation provides is that it reveals how $\tilde{\kappa}$, the impact of real marginal cost on inflation, depends on the structural parameters β and ω. An increase in β means that the firm gives more weight to future expected profits. As a consequence, $\tilde{\kappa}$ declines; inflation is less sensitive to current marginal costs. Increased price rigidity (a rise in ω) reduces $\tilde{\kappa}$; with opportunities to adjust arriving less frequently, the firm places less weight on current marginal cost (and more on expected future marginal costs) when it does adjust its price.

Equation (5.62) implies that inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves. However, real marginal costs can be related to an output gap measure. The firm's real marginal cost is equal to the real wage it faces divided by the marginal product of labor (see 5.55). In a flexible price equilibrium, all firms set the same price, so (5.58) implies that the real marginal cost will equal its steady-state value of $1/\mu$. Because nominal wages have been assumed to be completely flexible, the real wage must, according to (5.54), equal the marginal rate of substitution between leisure and consumption. Expressed in terms of percentage deviations around the steady state, (5.54) implies that $\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \sigma \hat{y}_t$. Recalling that $\hat{c}_t = \hat{y}_t$ and $\hat{y}_t = \hat{n}_t + \hat{z}_t$, the percentage deviation of real marginal cost around its steady-state value is

$$\begin{aligned}\hat{\varphi}_t &= (\hat{w}_t - \hat{p}_t) - (\hat{y}_t - \hat{n}_t) \\ &= (\sigma + \eta)\left[\hat{y}_t - \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\right].\end{aligned}$$

52. Note the similarity between (5.62) and (5.36) and (5.41), which were not based on models of optimizing behavior.

But from (5.60), this can be written as

$$\hat{\varphi}_t = \gamma(\hat{y}_t - \hat{y}_t^f), \tag{5.63}$$

where $\gamma = \sigma + \eta$. Using this result, the inflation adjustment equation (5.62) becomes

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \tag{5.64}$$

where $\kappa = \gamma\tilde{\kappa} = \gamma(1-\omega)(1-\beta\omega)/\omega$ and $x_t \equiv \hat{y}_t - \hat{y}_t^f$ is the gap between actual output and flexible-price equilibrium output.

The preceding assumes that firms face constant returns to scale. If, instead, each firm's production function is $c_{jt} = Z_t N_{jt}^a$, where $0 < a \le 1$, then the results must be modified slightly. When $a < 1$, firms with different production levels face different marginal costs, and real marginal cost for firm j will equal

$$\varphi_{jt} = \frac{W_t/P_t}{aZ_t N_{jt}^{a-1}} = \frac{W_t/P_t}{ac_{jt}/N_{jt}}.$$

Marginal cost for an individual firm can be related to average marginal cost, $\varphi_t = (W_t/P_t)/(aC_t/N_t)$, by using the production function and the demand relationship (5.50) to write

$$\varphi_{jt} = \varphi_t\left(\frac{C_t/N_t}{c_{jt}/N_{jt}}\right) = \varphi_t\left(\frac{C_t}{c_{jt}}\right)^{\frac{a-1}{a}} = \varphi_t\left(\frac{p_{jt}}{P_t}\right)^{\frac{\theta(a-1)}{a}}.$$

Hence, in terms of deviations around the steady state,

$$\hat{\varphi}_{jt} = \hat{\varphi}_t - \left[\frac{\theta(1-a)}{a}\right](\hat{p}_{jt} - \hat{p}_t).$$

Firms with relatively high prices (and therefore low output) have relatively low real marginal costs. In the case of constant returns to scale ($a = 1$), all firms face the same marginal cost. Sbordone (2002) and Galí, Gertler, and Lopez-Salido (2001) show that the new Keynesian inflation adjustment equation becomes[53]

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \tilde{\kappa}\left[\frac{a}{a+\theta(1-a)}\right]\hat{\varphi}_t.$$

In addition, the labor market equilibrium condition under flexible prices becomes

$$\frac{W_t}{P_t} = \frac{aZ_t N_t^{a-1}}{\mu} = \frac{\chi N_t^{\eta}}{C_t^{-\sigma}},$$

53. See the appendix, section 5.7.3, for further details on the derivation.

which implies that flexible-price output is

$$\hat{y}_t^f = \left[\frac{1+\eta}{1+\eta+a(\sigma-1)}\right]\hat{z}_t.$$

When $a = 1$, this reduces to (5.60).

Equation (5.64) relates output, in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity, to inflation. It forms one of the two key components of an optimizing model that can be used for monetary policy analysis. The other component is a linearized version of the household's Euler condition, (5.52). Before we use this model, however, the empirical evidence on the new Keynesian Phillip curve is discussed.

5.4.2 Evaluating the New Keynesian Inflation Adjustment Equation

The empirical evidence reviewed in chapter 1 indicated that inflation responds sluggishly to economic shocks. While (5.64) is based on a theory of sluggish *price* adjustment, it implies that inflation, as a purely forward-looking variable, can jump immediately in response to changes in output or expected inflation. The underlying model of sluggish price adjustment does not impart any inherent dynamics to the inflation process; any dynamics exhibited by inflation simply reflect the dynamic process that characterizes the output gap. For example, suppose x_t follows an exogenous $AR(1)$ process: $x_t = \rho x_{t-1} + e_t$. To solve for the equilibrium process for inflation, assume that $\pi_t = Ax_t$, where A is an unknown parameter. Then $\mathrm{E}_t\pi_{t+1} = A\mathrm{E}_t x_{t+1} = A\rho x_t$, and

$$\pi_t = Ax_t = \beta A\rho x_t + \kappa x_t \Rightarrow A = \frac{\kappa}{1-\beta\rho}.$$

If we multiply π_t by $(1-\rho\mathrm{L})$, where L is the lag operator, $(1-\rho\mathrm{L})\pi_t = (1-\rho\mathrm{L})Ax_t = Ae_t$, so $\pi_t = \rho\pi_{t-1} + Ae_t$. The dynamics characterizing inflation depend solely on the serial correlation in x_t in the form of the parameter ρ. The fact that prices are sticky makes no additional contribution to the resulting dynamic behavior of inflation. In addition, e_t, the innovation to x_t, has its maximum impact on inflation immediately, with inflation then reverting to its steady-state value at a rate governed by ρ.

Nelson (1998) has evaluated the ability of optimizing models of price adjustment to match U.S. inflation data. He concludes that inflation displays much more persistence than is consistent with these models. This point is also made by Estrella and Fuhrer (2002) who, in criticizing the dynamics implied by inflation adjustment

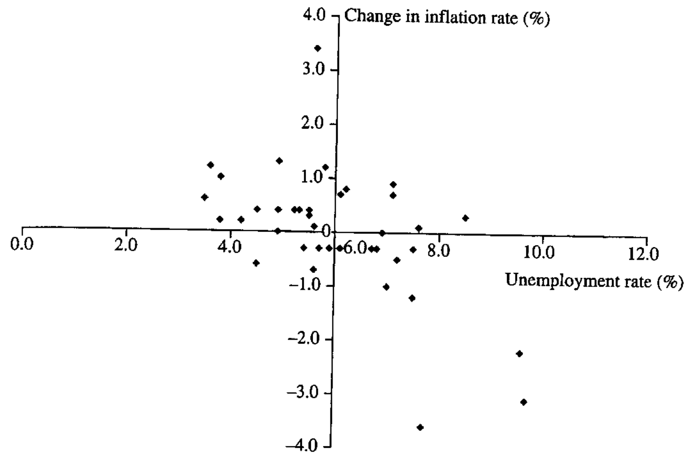

Figure 5.2
Change in Inflation Versus Unemployment (1960–1998)

equations of the form given by (5.64), point out their counterfactual implications. For example, (5.64) implies that $\beta\mathrm{E}_t\pi_{t+1} - \pi_t = -\kappa x_t$. If we let u_{t+1} denote the error in forecasting future inflation, this can be written as $\beta\pi_{t+1} - \pi_t = -\kappa x_t + \beta(\pi_{t+1} - \mathrm{E}_t\pi_{t+1}) = -\kappa x_t + \beta u_{t+1}$. Since $\beta \approx 1$ in quarterly data,

$$\pi_{t+1} - \pi_t \approx -\kappa x_t + u_{t+1}.$$

An increase in the output gap should lead to a *fall* in future inflation. Under the assumption that the unemployment rate can proxy for the output gap (with an increase in unemployment associated with a fall in the gap), a rise in unemployment should be associated with a rise in inflation. Figure 5.2 suggests that this implication is not supported by the U.S. data.

Attempts to estimate (5.64) have not been successful. In fact, when x_t is proxied by detrended real GDP, the estimated coefficient on the gap measure in quarterly data is actually negative (Galí and Gertler 1999, Sbordone 2001), although Roberts (1995) found a small positive coefficient using annual data.[54] One interpretation is that standard output gap measures, typically derived by detrending GDP, are poor

54. According to the derivation followed to obtain (5.64), there is no error term in the equation, so, in theory, the model should fit the data exactly. Empirical specifications add a stochastic error term. Chapter 11 discusses the implications of this disturbance term for monetary policy.

proxies for the theoretically correct measure of the deviation from the flexible price equilibrium level of output. Neiss and Nelson (2002) report better results using a measure of the output gap that more closely corresponds to the theoretically correct gap measure.

In order to capture the inflation persistence found in the data, it is common to augment the basic forward-looking inflation adjustment equation with the addition of lagged inflation, yielding an equation of the form

$$\pi_t = (1-\phi)\beta \mathrm{E}_t\pi_{t+1} + \kappa x_t + \phi\pi_{t-1}. \tag{5.65}$$

In this formulation, the parameter ϕ is often described as a measure of the degree of backward-looking behavior in price setting. Fuhrer (1997b) finds little role for future inflation once lagged inflation is added to the inflation adjustment equation. Rudebusch (2002a) estimates (5.65) using U.S. data and argues that ϕ is on the order of 0.7, suggesting that inflation is predominantly backward-looking.

Both Rudebusch and Fuhrer employ a statistically based measure of the output gap—detrended real GDP. Galí and Gertler (1999) argue that the model of inflation adjustment should be tested by estimating (5.62) directly using real marginal cost rather than using an output gap variable to proxy for marginal cost. They conclude that lagged inflation is much less important than suggested by Rudebusch and Fuhrer if real marginal cost is used in place of an output gap measure.[55]

Real marginal cost is equal to the ratio of real wages to the marginal product of labor. If the aggregate production function is given by $Y_t = A_t N_t^a K_t^{1-a}$, real marginal cost, φ_t, is equal to

$$\varphi_t = \frac{(W_t/P_t)}{a(Y_t/N_t)} = \left(\frac{1}{a}\right)\left(\frac{W_t N_t}{P_t Y_t}\right).$$

Thus, real marginal cost is proportional to labor's share of total output. Galí and Gertler then assume that a fraction λ of the firms that are allowed to adjust each period simply set $p_{jt} = \bar{\pi} p_{t-1}^*$, where $\bar{\pi}$ is the average inflation rate and p_{t-1}^* is the price chosen by optimizing firms in the previous period. They show that the inflation adjustment equation then becomes

$$\pi_t = \left(\frac{1}{\delta}\right)[\beta\omega \mathrm{E}_t\pi_{t+1} + (1-\lambda)\bar{\kappa}\hat{\varphi}_t + \lambda\pi_{t-1}] + \varepsilon_t, \tag{5.66}$$

55. Galí, Gertler, and López-Salido (2001) estimate (5.62) using European data and find support for the basic specification. Roberts (2001) finds a role for more lags of inflation than the theory implies.

where $\bar{\kappa} = (1-\omega)(1-\omega\beta)$ and $\delta = \omega + \lambda[1-\omega(1-\beta)]$. Based on U.S. data, their estimate of the coefficient on π_{t-1} is in the range 0.25 to 0.4, suggesting that the higher weight on lagged inflation obtained when the output gap is used reflects the fact that the gap may be a poor proxy for real marginal cost.[56] Sbordone (2002) also reports evidence in favor of the implied dependence of inflation on expected future inflation and real marginal cost. These results suggest that it is the link between marginal cost and output that is the problem and not the link between marginal cost and inflation. This is perhaps not surprising. To go from marginal cost to an output gap measure, real wages were replaced by the marginal rate of substitution between leisure and consumption (using 5.54). This procedure assumed that, while prices were sticky, nominal wages were perfectly flexible so that the real wage could adjust to maintain workers on their labor supply curve. If nominal wages are also sticky, a gap can open between the real wage and the marginal rate of substitution between leisure and consumption. The implications of nominal wage stickiness are discussed in section 5.5.

Note that as the fraction of backward-looking firms approaches 1, the inflation rate given by (5.66) converges to

$$\pi_t = \left(\frac{\omega\beta}{1+\omega\beta}\right)\mathrm{E}_t\pi_{t+1} + \left(\frac{1}{1+\omega\beta}\right)\pi_{t-1},$$

which has as its solution $\pi_t = \pi_{t-1} = 0$; inflation is independent of marginal cost and output. This conclusion is a result of the specific form of nonoptimizing behavior Galí and Gertler assume. Since backward-looking firms base their price on what the optimizing firms did in the previous period, the model is not well defined when there are no optimizing firms at all (i.e., in the limit as $\lambda \to 1$).

Christiano, Eichenbaum, and Evans (2001) make a distinction between firms that reoptimize in setting their price and those that do not. In their formulation, each period a fraction $1-\omega$ of all firms optimally set their price. The remaining firms either adjust their price based on the average rate of inflation, so that $p_{jt} = \bar{\pi} p_{jt-1}$ where $\bar{\pi}$ is the average inflation rate, or they adjust based on the most recently observed rate of inflation, so that $p_{jt} = \pi_{t-1} p_{jt-1}$. The first specification leads to (5.62) when the steady-state inflation rate is zero. The second specification results in an inflation adjustment equation of the form

$$\pi_t = \left(\frac{\beta}{1+\beta}\right)\mathrm{E}_t\pi_{t+1} + \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \tilde{\kappa}\hat{\varphi}_t.$$

56. See also Sbordone (2001).

The presence of lagged inflation in this equation introduces inertia into the inflation process.

5.4.3 General Equilibrium

We now have all the components of a simple general equilibrium model that is consistent with optimizing behavior on the part of households and firms. Because consumption is equal to output in this model (there is no government or investment since capital has been ignored), (5.52), (5.57), and (5.61) provide the equilibrium conditions that determine output, the price set by firms adjusting their price, and the aggregate price level once the behavior of the nominal rate of interest is specified. With the nominal interest rate treated as the monetary policy instrument, (5.53) simply determines the nominal quantity of money in equilibrium.

Equation (5.52) can be approximated around the zero-inflation steady state as

$$\hat{y}_t = \mathrm{E}_t\hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t\pi_{t+1}). \tag{5.67}$$

Expressing this in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$,

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t\pi_{t+1}) + u_t, \tag{5.68}$$

where $u_t \equiv \mathrm{E}_t\hat{y}_{t+1}^f - \hat{y}_t^f$ depends only on the exogenous productivity disturbance (see 5.60). Combining (5.68) with (5.64) gives a simple two-equation, forward-looking, rational-expectations model for inflation and the output gap measure x_t. For convenience, (5.64) is repeated here:

$$\pi_t = \beta \mathrm{E}_t\pi_{t+1} + \kappa x_t. \tag{5.69}$$

This two-equation model represents the equilibrium conditions for a well-specified general equilibrium model. The equations appear broadly similar, however, to the types of aggregate demand and aggregate supply equations commonly found in intermediate-level macroeconomics textbooks. Equation (5.68) represents the demand side of the economy (an expectational, forward-looking IS curve), while the new Keynesian Phillips curve (5.69) corresponds to the supply side. In fact, both equations are derived from well-specified optimization problems, with (5.68) based on the Euler condition for the representative agent's decision problem and (5.69) derived from the pricing decisions of individual firms.

There is a long tradition of using two equation, aggregate demand-aggregate supply (AD-AS) models in intermediate-level macroeconomic and monetary policy analysis. The derivations leading to (5.68) and (5.69) reveal how these models are

related to the theoretical models employed in earlier chapters. Models in the AD-AS tradition are often criticized as "starting from curves" rather than starting from the primitive tastes and technology from which behavioral relationships can be derived, given maximizing behavior and a market structure (Sargent 1982). This criticism does not apply to (5.68) and (5.69). The parameters appearing in these two equations are explicit functions of the underlying structural parameters of the production and utility functions, and the assumed process for price adjustment.[57] And (5.68) and (5.69) contain expectations of future variables; the absence of this type of forward-looking behavior is a critical shortcoming of older AD-AS frameworks. The importance of incorporating a role for future income has been emphasized by Kerr and King (1996).

Uniqueness of the Equilibrium Equations (5.68) and (5.69) contain three variables: the output gap, inflation, and the nominal interest rate. The model can be closed by assuming that the central bank implements monetary policy through control of the nominal interest rate. The linearized version of (5.53) can then be used to find the equilibrium nominal money supply.[58] Alternatively, if the central bank implements monetary policy by setting a path for the nominal supply of money, (5.68) and (5.69), together with the linearized version of (5.53), determine x_t, π_t, and $\hat{\imath}_t$.[59]

If a policy rule for the nominal interest rate is added to the model, this must be done with care to ensure that the policy rule does not render the system unstable or introduce multiple equilibria. For example, suppose monetary policy is represented by the following rule for $\hat{\imath}_t$:

$$\hat{\imath}_t = \rho_r\hat{\imath}_{t-1} + v_t. \tag{5.70}$$

This specification makes the nominal interest rate an exogenous $AR(1)$ process with innovation v_t. Combining (5.70) with (5.68) and (5.69), the resulting system of equations can be written as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sigma^{-1} \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \hat{\imath}_t \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_r & 0 & 0 \\ \sigma^{-1} & 1 & 0 \\ 0 & -\kappa & 1 \end{bmatrix} \begin{bmatrix} \hat{\imath}_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} v_t \\ -u_t \\ 0 \end{bmatrix}.$$

57. The process for price adjustment, however, has not been derived from the underlying structure of the economic environment.

58. Important issues of price-level determinancy arise under interest-rate-setting policies, and these will be discussed in chapter 10.

59. An alternative approach, discussed in chapters 8 and 11, specifies an objective function for the monetary authority and then derives the policy maker's decision rule for setting the nominal interest rate.

Premultiplying both sides by the inverse of the matrix on the left produces

$$\begin{bmatrix} \hat{\imath}_t \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = M \begin{bmatrix} \hat{\imath}_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} v_t \\ -u_t \\ 0 \end{bmatrix}, \tag{5.71}$$

where

$$M = \begin{bmatrix} \rho_r & 0 & 0 \\ \frac{1}{\sigma} & 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ 0 & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Equation (5.71) has a unique, stationary solution for the output gap, inflation, and the nominal interest rate if and only if the number of eigenvalues of M outside the unit circle is equal to the number of forward-looking variables, in this case, two (see Blanchard and Kahn 1980). One eigenvalue is ρ_r, which is inside the unit circle. Thus, stability and uniqueness require that both eigenvalues of

$$\begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}$$

be outside the unit circle. However, only the largest eigenvalue of this matrix is outside the unit circle, implying that multiple bounded equilibria exist and that the equilibrium is locally indeterminate. Stationary sunspot equilibria are possible.

This example illustrates that an exogenous policy rule—one that does not respond to the endogenous variables x and π—introduces the possibility of multiple equilibria. To understand why, consider what would happen if expected inflation were to rise. Because (5.70) does not allow for any endogenous feedback from this rise in expected inflation to the nominal interest rate, the real interest rate must fall. This decline in the real interest rate is expansionary, and the output gap increases. The rise in output increases actual inflation, according to (5.69). Thus, a change in expected inflation, even if due to factors unrelated to the fundamentals of inflation, can set off a self-fulfilling change in actual inflation.

This discussion suggests that a policy which raised the nominal interest rate when inflation rose, and raised $\hat{\imath}_t$ enough to increase the real interest rate so that the output gap fell, would be sufficient to ensure a unique equilibrium. For example, suppose the nominal interest rate responds to inflation according to the rule

$$\hat{\imath}_t = \delta\pi_t + v_t. \tag{5.72}$$

Combining (5.72) with (5.68) and (5.69), $\hat{\imath}_t$ can be eliminated and the resulting system written as

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = N \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix},$$

where

$$N = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & \frac{(\beta\delta - 1)}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Bullard and Mitra (2002) show that a unique stationary equilibrium exists as long as $\delta > 1$.[60] Setting $\delta > 1$ is referred to as the *Taylor principle* because John Taylor was the first to stress the importance of interest-rate rules that called for responding more than one for one to changes in inflation.

Suppose that, instead of reacting solely to inflation, as in (5.72), the central bank responds to both inflation and the output gap according to

$$\hat{\imath}_t = \delta_\pi \pi_t + \delta_x x_t + v_t.$$

This type of policy rule is called a *Taylor rule* (Taylor 1993a), and variants of it have been shown to provide a reasonable empirical description of the policy behavior of many central banks (Clarida, Galí, and Gertler 2000). With this policy rule, the condition necessary to ensure that the economy has a unique stationary equilibrium becomes[61]

$$\kappa(\delta_\pi - 1) + (1 - \beta)\delta_x > 0. \tag{5.73}$$

Stability now depends on both the policy parameters δ_π and δ_x.

The Monetary Transmission Mechanism The model consisting of (5.68) and (5.69) assumes that the impact of monetary policy on output and inflation operates through

60. If the nominal interest rate is adjusted in response to expected future inflation (rather than current inflation), multiple solutions again become possible if $\hat{\imath}_t$ responds too strongly to $\mathrm{E}_t\hat{\pi}_{t+1}$. See Clarida, Galí, and Gertler (2000).

61. See Bullard and Mitra (2002).

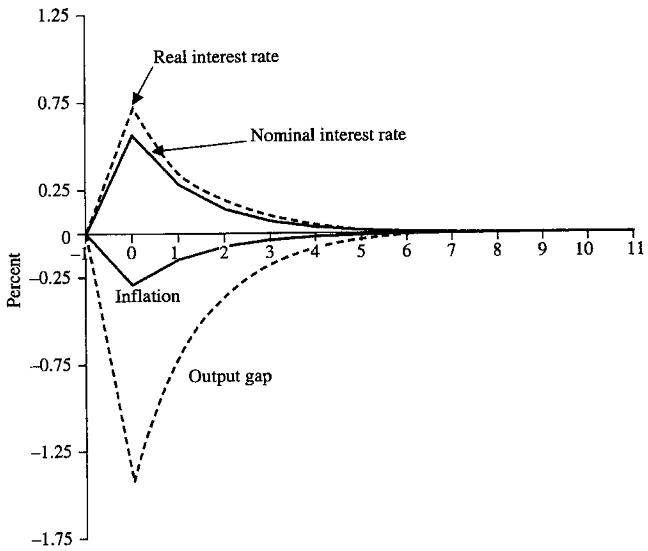

Figure 5.3
Output, Inflation, and Real Interest Rate Responses to a Policy Shock in the New Keynesian Model

the real rate of interest. As long as the central bank is able to affect the real interest rate through its control of the nominal interest rate, monetary policy can affect real output. Changes in the real interest rate alter the optimal time path of consumption. An increase in the real rate of interest, for instance, leads households to attempt to postpone consumption. Current consumption falls relative to future consumption.[62]

Figure 5.3 illustrates the impact of a monetary policy shock (an increase in the nominal interest rate) in the model consisting of (5.68), (5.69), and the policy rule (5.72). The parameter values used in constructing the figure are $\beta = 0.99$, $\sigma = \eta = 1$, $\delta = 1.5$, and $\omega = 0.8$. In addition, the policy shock v_t in the policy rule is assumed to follow an $AR(1)$ process given by $v_t = \rho_v v_{t-1} + \varepsilon_t$, with $\rho_v = 0.5$. The rise in the nominal rate causes inflation and the output gap to fall immediately. This reflects

62. Just as was the case with the forward-looking inflation-adjustment equation, the forward-looking Euler equation implies counterfactual dynamics (Estrella and Fuhrer 2002). Equation (5.68) implies that $E_t\hat{c}_{t+1} - \hat{c}_t = \sigma^{-1}(\hat{\imath}_t - E_t\pi_{t+1})$, so that a rise in the real interest rate means that consumption must increase from t to $t+1$.

the forward-looking nature of both variables. In fact, all the persistence displayed by the responses arises solely from the serial correlation introduced into the process for the monetary shock v_t. If $\rho_v = 0$, all variables return to their steady-state values in the period after the shock.[63]

To emphasize the interest rate as the primary channel through which monetary influences affect output, it can be convenient to express the output gap as a function of an *interest rate gap*, the gap between the current interest rate and the interest rate consistent with the flexible-price equilibrium. For example, let $\hat{r}_t \equiv \hat{\imath}_t - E_t\pi_{t+1}$ be the real interest rate and write (5.68) as

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{r}_t - \tilde{r}_t),$$

where $\tilde{r}_t \equiv -\sigma u_t$. Woodford (2000b) has labeled $\tilde{r}_t$ the *Wicksellian real interest rate.* It is the interest rate consistent with output equaling the flexible-price equilibrium level. If $\hat{r}_t = \tilde{r}_t$ for all t, then $x_t = 0$ and output is kept equal to the level that would arise in the absence of nominal rigidities. The interest rate gap $\hat{r}_t - \tilde{r}_t$ then summarizes the effects on the actual equilibrium that are due to nominal rigidities.[64]

The basic interest rate transmission mechanism for monetary policy could be extended to include effects on investment spending if capital were reintroduced into the model (Christiano, Eichenbaum, and Evans 2001; Dotsey and King 2001). Increases in the real interest rate would reduce the demand for capital and lead to a fall in investment spending. In the case of both investment and consumption, monetary policy effects are transmitted through interest rates.

In addition to these interest rate channels, monetary policy is often thought to affect the economy either indirectly through credit channels or directly through the quantity of money. Since most measures of the money supply consist largely of bank liabilities, measures of money and measures of bank credit tend to move together. They represent different sides of the banking sector's balance sheet. Chapter 7 discusses the role of credit channels in the monetary transmission process.

It is sometimes argued that changes in the money supply have direct effects on aggregate demand that are independent of the interest rate channels that operate on consumption and investment. Real money holdings represent part of household wealth; an increase in real balances should induce an increase in consumption

63. See Galí (2002) for a discussion of the monetary transmission mechanism incorporated in the basic new Keynesian model.

64. Neiss and Nelson (2001) use a structural model to estimate the real interest-rate gap $\hat{r}_t - \tilde{r}_t$ and find that it has value as a predictor of inflation.

spending through a wealth effect. This channel is often called the *Pigou effect* and was viewed as generating a channel through which price-level declines during a depression would eventually increase real balances and household wealth sufficiently to restore consumption spending. During the Keynesian/monetarist debates of the 1960s and early 1970s, some monetarists argued for a direct wealth effect that linked changes in the money supply directly to aggregate demand (Patinkin 1965). The effect of money on aggregate demand operating through interest rate effects was viewed as a Keynesian interpretation of the transmission mechanism, although most monetarists argued that changes in monetary policy lead to substitution effects over a broader range of assets than Keynesians normally considered. Because wealth effects are likely to be small at business-cycle frequencies, most simple models used for policy analysis ignore them.[65]

Direct effects of the quantity of money are not present in the model we have been using; the quantity of money does not appear in either (5.68) or (5.69). The underlying model was derived from an MIU model, and the absence of money in (5.68) and (5.69) results from the assumption that the utility function is separable. If utility is not separable, then changes in the real quantity of money alter the marginal utility of consumption. This would affect the model specification in two ways. First, the real money stock would appear in the household's Euler condition and therefore in (5.68). Second, to replace real marginal cost with a measure of the output gap in (5.69), the real wage was equated to the marginal rate of substitution between leisure and consumption, and this would also involve real money balances if utility is nonseparable (see problem 10). Thus, the absence of money constitutes a special case. However, McCallum and Nelson (1999) and Woodford (2001a) have both argued that the effects arising with nonseparable utility are quite small, so that little is lost by assuming separability. Ireland (2001c) finds little evidence for nonseparable preferences in a model estimated on U.S. data.

The quantity of money is not totally absent from the underlying model, since (5.53) also must hold in equilibrium. Linearizing this equation around the steady state yields

$$\hat{m}_t - \hat{p}_t = \left(\frac{1}{b}\right)(\sigma\hat{y}_t - \hat{\imath}_t). \tag{5.74}$$

Given the nominal interest rate chosen by the monetary policy authority, this equation determines the nominal quantity of money. Alternatively, if the policy maker

65. Empirical models used for policy analysis by agencies such as the Federal Reserve usually incorporate wealth effects, although these include much broader definitions of wealth than simply real money balances. Wealth effects then arise from the impact of interest rates on asset prices. For a recent analysis of the real balance effect, see Ireland (2001b).

sets the nominal quantity of money, then (5.68), (5.69), and (5.74) must all be used to solve jointly for x_t, π_t, and $\hat{\imath}_t$.

Adding Economic Disturbances As the model consisting of (5.68) and (5.69) stands, there are no underlying nonpolicy disturbances that might generate movements in either output gap or inflation other than the productivity disturbance. It is common to see stochastic disturbances arising from other sources also included in these equations.

Suppose the representative household's utility from consumption is subject to random shocks that alter the marginal utility of consumption. Specifically, let the utility function in (5.46) be modified to include a taste shock ψ:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(\psi_{t+1} C_{t+i})^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-b} - \chi\frac{N_{t+i}^{1+\eta}}{1+\eta}\right]. \tag{5.75}$$

The Euler condition (5.52) becomes

$$\psi_t^{1-\sigma} C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)(\psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}),$$

which, when linearized around the zero-inflation steady state yields

$$\hat{c}_t = E_t\hat{c}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - E_t\pi_{t+1}) + \left(\frac{\sigma-1}{\sigma}\right)(E_t\psi_{t+1} - \psi_t). \tag{5.76}$$

If, in addition to consumption by households, the government purchases final output G_t, the goods market equilibrium condition becomes $Y_t = C_t + G_t$. When this is expressed in terms of percentage deviations around the steady state, one obtains

$$\hat{y}_t = \left(\frac{C}{Y}\right)^{ss}\hat{c}_t + \left(\frac{G}{Y}\right)^{ss}\hat{g}_t.$$

Using this equation to eliminate $\hat{c}_t$ from (5.76) and then replacing $\hat{y}_t$ with $x_t + \hat{y}_t^f$ yields an expression for the output gap $(\hat{y}_t - \hat{y}_t^f)$,

$$x_t = E_t x_{t+1} - \tilde{\sigma}^{-1}(\hat{\imath}_t - E_t\pi_{t+1}) + \xi_t, \tag{5.77}$$

where $\tilde{\sigma}^{-1} = \sigma^{-1}(C/Y)^{ss}$ and

$$\xi_t \equiv \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{C}{Y}\right)^{ss}(E_t\psi_{t+1} - \psi_t) - \left(\frac{G}{Y}\right)^{ss}(E_t\hat{g}_{t+1} - \hat{g}_t) + (E_t\hat{y}_{t+1}^f - \hat{y}_t^f).$$

Equation (5.77) is a typical forward-looking aggregate demand specification. It represents the Euler condition consistent with the representative household's intertemporal optimality condition linking consumption levels over time. It is also consistent with the resource constraint $Y = C + G$. The disturbance term arises from taste shocks that alter the marginal utility of consumption, shifts in government purchases, and shifts in the flexible-price equilibrium output. In each case, it is expected changes in ψ, g, and $\hat{y}^f$ that matter. For example, for a given value of $E_t x_{t+1}$, if government purchases are expected to rise so that $E_t \hat{g}_{t+1} - \hat{g}_t > 0$, current output falls. Given expected future output, an expected rise in government purchases implies that future consumption must fall. This reduces current consumption.

The source of a disturbance term in the inflation adjustment equation is both more critical for policy analysis and more controversial. While policy analysis will be taken up in chapters 8–11, it is easy to see why exogenous shifts in (5.69) can have important implications for policy. Two commonly assumed objectives of monetary policy are to maintain a low and stable average rate of inflation and to stabilize output around full employment. These two objectives are often viewed as presenting central banks with a trade-off. A supply shock, such as an increase in oil prices, increases inflation and reduces output. To keep inflation from rising calls for contractionary policies that would exacerbate the decline in output; stabilizing output calls for expansionary policies that would worsen inflation. However, if the output objective is interpreted as meaning that output should be stabilized around its flexible-price equilibrium level, then (5.69) implies that the central bank can always achieve a zero output gap (i.e., keep output at its flexible-price equilibrium level) and keep inflation equal to zero. Solving (5.69) forward yields

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}.$$

By keeping current and expected future output equal to the flexible-price equilibrium level, $E_t \hat{x}_{t+i} = 0$ for all i and inflation remains equal to zero. This will not be the case if an error term is added to the inflation adjustment equation. If

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \tag{5.78}$$

then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i}.$$

As long as $\sum_{i=0}^{\infty} \beta^i E_t e_{t+i} \neq 0$, maintaining $\sum_{i=0}^{\infty} \beta^i E_t x_{t+i} = 0$ is not sufficient to ensure that inflation always remains equal to zero. A trade-off between stabilizing output and stabilizing inflation can arise. Disturbance terms in the inflation adjustment equation are often called *cost shocks* or *inflation shocks*. Since e shocks, unless they are permanent, ultimately affect only the price level, they are also called *price shocks*.

Clarida, Galí, and Gertler (2001) suggest one means of including a stochastic shock in the inflation adjustment equation. They add a stochastic *wage markup* to represent deviations between the marginal rate of substitution between leisure and consumption and the real wage. Thus, the labor-supply condition (5.54) becomes

$$\left(\frac{\chi N_t^{\eta}}{C_t^{-\sigma}}\right) e^{\mu_t^w} = \frac{W_t}{P_t},$$

where μ_t^w is a random disturbance.[66] This could arise from shifts in tastes that affect the marginal utility of leisure. Or, if labor markets are imperfectly competitive, it could arise from stochastic shifts in the markup of wages over the marginal rate of substitution (Clarida, Galí, and Gertler 2002). When linearized around the steady state, one obtains

$$\eta \hat{n}_t + \sigma \hat{c}_t + \mu_t^w = \hat{w}_t - \hat{p}_t. \tag{5.79}$$

The real marginal cost variable becomes

$$\varphi_t = (\eta \hat{n}_t + \sigma \hat{c}_t) - (\hat{y}_t - \hat{n}_t) + \mu_t^w,$$

and this suggests that the inflation adjustment equation becomes

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \tilde{\kappa} x_t + \tilde{\kappa} \mu_t^w. \tag{5.80}$$

In this formulation, μ_t^w is the source of inflation shocks.

While this approach appears to provide an explanation for a disturbance term to appear in the inflation adjustment equation, if μ_t^w reflects taste shocks that alter the marginal rate of substitution between leisure and consumption, then μ_t^w also affects the flexible-price equilibrium level of output. The same would be true if μ_t^w is a markup due to imperfect competition in the labor market. Thus, if the output gap variable in the inflation adjustment equation is correctly measured as the deviation of

66. With the utility function given in (5.75), this becomes

$$\left(\frac{\chi N_t^{\eta}}{C_t^{-\sigma}}\right)\left(\frac{e^{\mu_t^w}}{\psi_t^{1-\sigma}}\right) = \frac{W_t}{P_t},$$

showing that μ_t^w affects the labor-market condition in a manner similar to a taste shock.

output from the flexible-price equilibrium level, μ_t^w no longer has a separate, independent impact on π_t.

5.5 Sticky Wages and Prices

Erceg, Henderson, and Levin (2000) have employed the Calvo specification to incorporate sticky wages *and* sticky prices into an optimizing framework.[67] The goods market side of their model is identical in structure to the one developed in section 5.4. In the labor market, however, they assume that individual households supply differentiated labor services; firms combine these labor services to produce output. Output is given by a standard production function, $F(N_t, K_t)$, but the labor aggregate is a composite function of the individual types of labor services:

$$N_t = \left[\int_0^1 n_{jt}^{\frac{\gamma-1}{\gamma}} dj\right]^{\frac{\gamma}{\gamma-1}}, \quad \gamma > 1,$$

where n_{jt} is the labor from household j that the firm employs. With this specification, households face a demand for their labor services that depends on the wage they set relative to the aggregate wage rate. Erceg, Henderson, and Levin assume that a randomly drawn fraction of households optimally set their wage each period, just as the models of price stickiness assume that only a fraction of firms adjust their price each period (see also Christiano, Eichenbaum, and Evans 2001 and Sbordone 2001).

The model of inflation adjustment based on the Calvo specification implies that inflation depends on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equals the gap between the real wage and the marginal product of labor (*mpl*). Thus, letting ω_t denote the real wage,

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa(\omega_t - mpl_t). \tag{5.81}$$

Similarly, wage inflation responds to a gap variable, but this time the appropriate gap depends on a comparison between the real wage and the household's marginal rate of substitution between leisure and consumption. With flexible wages, as in the earlier sections where only prices were assumed to be sticky, workers are always on their labor supply curves; despite price stickiness, nominal wages can adjust to ensure that the real wage equals the marginal rate of substitution between leisure and consumption (*mrs*). When nominal wages are also sticky, however, ω_t and mrs_t can

67. Other models incorporating both wage and price stickiness include those of Guerrieri (2000), Ravenna (2000), Christiano, Eichenbaum, and Evans (2001), and Sbordone (2001, 2002).

differ. If $\omega_t < mrs_t$, workers will want to raise their nominal wage when the opportunity to adjust arises. Letting π_t^w denote the rate of nominal wage inflation, Erceg, Henderson, and Levin show that

$$\pi_t^w = \beta \mathrm{E}_t \pi_{t+1}^w + \kappa^w(mrs_t - \omega_t). \tag{5.82}$$

From the definition of the real wage,

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t. \tag{5.83}$$

Equations (5.81)–(5.83) constitute the inflation adjustment block of an optimizing model with both wage and price rigidities.

Christiano, Eichenbaum, and Evans (2001) have estimated such a model using U.S. data.[68] They report that wage rigidity, not price rigidity, seems to be the key in accounting for the observed dynamics of inflation and output. However, a model with sticky wages and flexible prices implies that real wages should move countercyclically; a monetary policy expansion raises the price level, and the resulting decline in real wages induces firms to increase employment and output. The empirical evidence for the United States suggests that real wages are procyclical, although not strongly so. This is not necessarily inconsistent with the basic monetary transmission mechanism since cycles are presumable caused by nonmonetary forces (that may cause wages to move procyclically) as well as monetary disturbances, so the observed correlations will reflect the net balance of several types of disturbances. Huang and Liu (2002) argue that wage stickiness is more important than price stickiness for generating output persistence. In contrast, Goodfriend and King (2001), while accepting that nominal wages are sticky, argue that the long-term nature of employment relationships means that nominal wage rigidity has little implication for real resource allocation.

5.6 Summary

Three major issues in monetary economics have been addressed in this chapter. First, we have examined how the models of chapters 2–4, models that were useful for examining issues such as the welfare cost of inflation and the optimal inflation tax, need to be modified to account for the short-run effects of monetary factors on the economy. While informational and portfolio channels through which money has

68. Their model also includes a much richer real side, including habit persistence in consumption and variable capital utilization.

short-run real effects even with flexible prices were discussed, most monetary models designed to address short-run monetary issues assume that wages and/or prices do not adjust instantaneously in response to changes in economic conditions. Second, we have examined some standard models of price adjustment. Finally, we have seen how new Keynesian models recently employed to study policy issues can be viewed as linear approximations to fully specified general equilibrium models based on optimizing behavior combined with nominal rigidities.

The framework of section 5.4 can be used to study a variety of monetary policy issues. However, an important omission must first be addressed; the analysis has so far dealt only with a closed economy. Monetary policy can affect the economy through additional channels once the linkages between economies are recognized. Domestic output and prices will depend on exchange rates, which, in turn, may depend on monetary policy. These open economy issues are discussed in the next chapter.

5.7 Appendix

5.7.1 An Imperfect-Information Model

Lucas's imperfect-information model generated real output effects of monetary surprises because individual agents faced a signal extraction problem. If prices in local markets rose, agents needed to estimate to what extent this represented an economy-wide general rise in the price level versus a rise in local prices relative to the economy-wide average.

The equilibrium in local market i, or island i, could be represented by the following three equations, where the goods equilibrium condition $y^i = c^i$ has been used:

$$y_t^i = (1-\alpha)n_t^i \tag{5.84}$$

$$\left[1+\eta\left(\frac{n^{ss}}{l^{ss}}\right)\right]n_t^i = (1-\Omega_1)y_t^i + \Omega_2(m_t^i - p_t^i) \tag{5.85}$$

and

$$\begin{aligned} m_t^i - p_t^i = y_t^i + \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[\mathrm{E}^i\tau_{t+1} - (\mathrm{E}^i p_{t+1} - p_t^i) \\ - \Omega_1\mathrm{E}^i(y_{t+1} - y_t^i) + \Omega_2\mathrm{E}^i(m_{t+1} - p_{t+1} - m_t^i + p_t^i)], \end{aligned} \tag{5.86}$$

where $\Omega_1 = [\gamma\Phi + (1-\gamma)b]$, $\Omega_2 = (b-\Phi)(1-\gamma)$, and

$$\gamma = \frac{a}{a + (1-a)\left[\left(\frac{M}{CP}\right)^{ss}\right]^{1-b}}.$$

The parameters β, Φ, b, and η are from the utility function of the representative agent:

$$u\left(C_t, \frac{M_t}{P_t}, 1-N_t\right) = \frac{\left[aC_t^{1-b} + (1-a)\left(\frac{M_t}{P_t}\right)^{1-b}\right]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi\frac{[(1-N_t)^{1-\eta}]}{1-\eta}.$$

The appendix to chapter 2 contains a more complete derivation of the basic MIU model. Equation (5.85) is derived from the condition that the marginal utility of leisure divided by the marginal utility of consumption must equal the marginal product of labor. Equation (5.86) is derived from the first order condition that, for an agent on island i,

$$u_c^i(t) = u_m^i(t) + \beta\mathrm{E}^i\left(\frac{T_{t+1}}{\Pi_{t+1}}\right)u_c(t+1), \tag{5.87}$$

where the left side is the utility cost of reducing consumption marginally in order to hold more money, and the right side is the return from higher money holdings. This return consists of the direct utility yield $u_m^i(t)$, plus the utility from using the real balances to increase consumption in period $t+1$. With transfers viewed as proportional to money holdings, the individual treats money as if it yielded a real return of T_{t+1}/Π_{t+1}. Given the assumed utility function, both sides of (5.87) can be divided by $u_c^i(t)$ and written as

$$1 = \left(\frac{1-a}{a}\right)\left(\frac{M_t^i/P_t^i}{C_t^i}\right)^{-b} + \beta\mathrm{E}^i\left(\frac{T_{t+1}}{\Pi_{t+1}}\right)\left(\frac{X_{t+1}^{\frac{b-\Phi}{1-b}}C_{t+1}^{-b}}{X_t^{\frac{b-\Phi}{1-b}}C_t^{-b}}\right),$$

where

$$X_t = aC_t^{1-b} + (1-a)\left(\frac{M_t}{P_t}\right)^{1-b}.$$

Expressed in terms of percentage deviations around the steady state (denoted by lowercase letters), the two terms on the right side become

$$\left(\frac{1-a}{a}\right)\left(\frac{M_t^i/P_t^i}{C_t^i}\right)^{-b} \approx \left(\frac{1-a}{a}\right)\left[\left(\frac{M/P}{C}\right)^{ss}\right]^{-b}[1 + bc_t^i - b(m_t^i - p_t^i)]$$

and

$$\beta \mathrm{E}^i\left(\frac{T}{\Pi}\right)\left(\frac{X_{t+1}^{\frac{b-\Phi}{1-b}}C_{t+1}^{-b}}{X_t^{\frac{b-\Phi}{1-b}}C_t^{-b}}\right) \approx \beta \mathrm{E}^i\left(\frac{T^{ss}}{\Pi^{ss}}\right)(1+\tau_{t+1}-\pi_{t+1}^i)$$
$$\times\,[1-\Omega_1\Delta c_{t+1}+\Omega_2(\Delta m_{t+1}^i-\Delta p_{t+1})]$$
$$\approx \beta \mathrm{E}^i[1+\tau_{t+1}-\pi_{t+1}^i-\Omega_1\Delta c_{t+1}+\Omega_2(\Delta m_{t+1}^i-\Delta p_{t+1})],$$

where Δ is the first difference operator ($\Delta c_{t+1} = c_{t+1} - c_t^i$) and we have used the fact that in the steady state, $T^{ss} = \Pi^{ss}$. This condition also implies

$$\left(\frac{1-a}{a}\right)\left[\left(\frac{M/P}{C}\right)^{ss}\right]^{-b} + \beta\left(\frac{T^{ss}}{\Pi^{ss}}\right) = 1, \quad \text{or} \quad \left(\frac{1-a}{a}\right)\left[\left(\frac{M/P}{C}\right)^{ss}\right]^{-b} = 1-\beta, \tag{5.88}$$

so the first order condition becomes

$$0 = (1-\beta)[bc_t^i - b(m_t^i + p_t^i)] + \beta \mathrm{E}^i[\tau_{t+1} - \pi_{t+1}^i - \Omega_1\Delta c_{t+1} + \Omega_2(\Delta m_{t+1}^i - \Delta p_{t+1})],$$

which can be rearranged to yield (5.3).

The nominal money supply on island i is assumed to evolve according to

$$m_t^i = \gamma m_{t-1} + v_t + u_t + u_t^i,$$

with u^i equal to an island-specific money shock that averages to zero across all islands and has variance σ_i^2. Both v and u are aggregate disturbances (common across all islands), each assumed to have zero mean and variances σ_v^2 and σ_u^2. The value of v is announced (or observed) at the start of period t. The aggregate average nominal money supply evolves as

$$m_t = \gamma m_{t-1} + v_t + u_t$$

and the aggregate transfer τ_t is given by

$$\tau_t = m_t - m_{t-1} = (\gamma - 1)m_{t-1} + v_t + u_t.$$

Individuals on island i observe the island-specific nominal money stock m_t^i. This allows them to infer $u_t + u_t^i$ but not u and u^i separately. The expectation of the time $t+1$ money supply, conditional on the information available on island i, will be $\mathrm{E}^i m_{t+1} = \gamma^2 m_{t-1} + \gamma v_t + \gamma \mathrm{E}^i u_t$. Equating expectations with linear least squares projections, $\mathrm{E}^i u_t = \kappa(u_t + u_t^i)$, where $\kappa = \sigma_u^2/(\sigma_u^2 + \sigma_i^2)$. Hence,

$$\mathrm{E}_t\tau_{t+1} = \mathrm{E}_t m_{t+1} - \mathrm{E}^i m_t = (\gamma-1)\mathrm{E}^i m_t + \mathrm{E}^i[v_{t+1} + u_{t+1}]$$
$$= (\gamma - 1)[\gamma m_{t-1} + v_t + \kappa(u_t + u_t^i)].$$

Eliminating output and the expected transfer from (5.84)–(5.86), these equations yield the following two equations for employment and prices:

$$n_t^i = \left[\frac{\Omega_2}{1+\eta\left(\frac{n^{ss}}{l^{ss}}\right) - (1-\Omega_1)(1-\alpha)}\right](m_t^i - p_t^i) = A(m_t^i - p_t^i) \tag{5.89}$$

$$m_t^i - p_t^i = (1-\alpha)n_t^i + \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)\Omega_2 \mathrm{E}^i[m_{t+1} - p_{t+1} - m_t + p_t^i]$$
$$-\left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)(1-\alpha)\Omega_1 \mathrm{E}^i\Delta n_{t+1}$$
$$+\left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[(\gamma-1)\mathrm{E}^i m_t - \mathrm{E}^i p_{t+1} + p_t^i]. \tag{5.90}$$

By substituting (5.89) into (5.90), we can obtain a single equation that involves the price process and the exogenous nominal money supply process:

$$(m_t^i - p_t^i) = (1-\alpha)A(m_t^i - p_t^i)$$
$$+\left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[\Omega_2 - (1-\alpha)\Omega_1 A]\mathrm{E}^i[m_{t+1} - p_{t+1} - m_t + p_t^i]$$
$$+\left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[(\gamma-1)\mathrm{E}^i m_t - \mathrm{E}^i p_{t+1} + p_t^i]. \tag{5.91}$$

Equation (5.91) can be solved using the method of undetermined coefficients (see McCallum 1989, Attfield, Demery, and Duck 1991). This method involves guessing a solution for p_t^i and then verifying that the solution is consistent with (5.91). Since m_t depends on m_{t-1}, v_t, u_t, and u_t^i, our guess for the minimum state variable solution (McCallum 1983a) for the equilibrium price level takes the following form:

$$p_t^i = a_1 m_{t-1} + a_2 v_t + a_3 u_t + a_4 u_t^i, \tag{5.92}$$

where the a_j's are yet-to-be-determined parameters. Equation (5.92) implies that the aggregate price level is $p_t = a_1 m_{t-1} + a_2 v_t + a_3 u_t$, so

$$\mathrm{E}^i p_{t+1} = a_1 \mathrm{E}^i m_t = a_1(\gamma m_{t-1} + v_t + \mathrm{E}^i u_t)$$
$$= a_1[\gamma m_{t-1} + v_t + \kappa(u_t + u_t^i)].$$

We are now in a position to evaluate all the terms in (5.91). The left-hand side of (5.91) is equal to

$$(m_t^i - p_t^i) = (\gamma - a_1)m_{t-1} + (1 - a_2)v_t + (1 - a_3)u_t + (1 - a_4)u_t^i,$$

while the three terms on the right-hand side equal

$$(1-\alpha)A[(\gamma - a_1)m_{t-1} + (1 - a_2)v_t + (1 - a_3)u_t + (1 - a_4)u_t^i],$$

$$B(\gamma - 1)[\gamma m_{t-1} + v_t + \kappa(u_t + u_t^i)]$$

$$- B[a_1(\gamma m_{t-1} + v_t + \kappa(u_t + u_t^i)) - (a_1 m_{t-1} + a_2 v_t + a_3 u_t + a_4 u_t^i)],$$

where $B = (\beta/b(1-\beta))[\Omega_2 - (1-\alpha)\Omega_1 A]$, and

$$\left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)(\gamma - 1)[\gamma m_{t-1} + v_t + \kappa(u_t + u_t^i)]$$

$$- \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[a_1(\gamma m_{t-1} + v_t + \kappa(u_t + u_t^i)) - (a_1 m_{t-1} + a_2 v_t + a_3 u_t + a_4 u_t^i)].$$

For the two sides of (5.91) to be equal for all possible realizations of m_{t-1}, v_t, u_t, and u_t^i requires that the following hold: the coefficient on m_{t-1} on the right side must be equal to the coefficient on the left side, or

$$(\gamma - a_1) = (1-\alpha)A(\gamma - a_1) + B(\gamma - 1)(\gamma - a_1) - Ba_1(\gamma - a_1)$$
$$+ \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)(\gamma - 1)(\gamma - a_1),$$

which holds if $a_1 = \gamma$; the coefficient of v_t on the right side must be equal to the coefficient of v_t on the left side, or

$$(1 - a_2) = (1-\alpha)A(1 - a_2) + B(\gamma - 1) - B(a_1 - a_2)$$
$$+ \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)(\gamma - 1)(1 - a_1 - a_2)$$

or $a_2 = 1$ (since $a_1 = \gamma$); the coefficient of u_t on the right side must be equal to the coefficient of u_t on the left side, or

$$(1 - a_3) = (1-\alpha)A(1 - a_3) + B(\gamma - 1)\kappa - B(a_1\kappa - a_3)$$
$$+ \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[(\gamma - 1)\kappa - (a_1\kappa - a_3)],$$

which implies

$$(1 - a_3) = (1-\alpha)A(1 - a_3) + B(a_3 - \kappa) + \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)(a_3 - \kappa),$$

or

$$a_3 = \frac{\kappa + K}{1 + K} < 1,$$

where

$$K = \frac{1 - (1-\alpha)A}{\left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)}.$$

Finally, the coefficient on u_t^i on the right side must be equal to its coefficient on the left side, or

$$(1 - a_4) = (1-\alpha)A(1 - a_4) + B(\gamma - 1)\kappa - B(a_1\kappa - a_4)$$
$$+ \left(\frac{1}{b}\right)\left(\frac{\beta}{1-\beta}\right)[(\gamma - 1)\kappa - (a_1\kappa - a_4)]$$

or $a_4 = a_3$.

Combining these results, we have the expressions for the equilibrium economy-wide price level and employment given by (5.5) and (5.6).

5.7.2 A Sticky-Wage MIU Model

In section 5.3.1, an MIU model was modified to include one-period nominal wage contracts. The equations characterizing equilibrium in the flexible-price MIU model were given by (5.7)–(5.14). Output was shown to equal

$$y_t - \mathrm{E}_{t-1}y_t^* = a(p_t - \mathrm{E}_{t-1}p_t) + (1 + a)\varepsilon_t, \tag{5.93}$$

where $\mathrm{E}_{t-1}y^* = (1-\alpha)\mathrm{E}_{t-1}n_t^* + \mathrm{E}_{t-1}e_t$ is the expected equilibrium output under flexible prices, $a = (1-\alpha)/\alpha$, and

$$y_t^* = \left[\frac{1 + \bar{\eta}}{1 + \bar{\eta} + (1-\alpha)(\Phi - 1)}\right]e_t = b_2 e_t.$$

The aggregate demand side of this economy consists of (5.10) and (5.12)–(5.14). Making use of the economy's resource constraint, (5.10) can be written as

$$y_t = \mathrm{E}_t y_{t+1} - \left(\frac{1}{\Phi}\right)r_t. \tag{5.94}$$

Using the Fisher equation, (5.13), and (5.94), the money demand condition becomes

$$m_t - p_t = y_t - \left(\frac{1}{b}\right)[r_t + \mathrm{E}_t p_{t+1} - p_t]$$
$$= y_t - \frac{\Phi}{b}[\mathrm{E}_t y_{t+1} - y_t] - \left(\frac{1}{b}\right)(\mathrm{E}_t p_{t+1} - p_t).$$

Notice that expected future income affects the demand for money. Higher expected income raises the expected real interest rate for a given level of current output, and this implies lower money demand.

We can now collect the equations of the model:

$$\text{Aggregate supply:} \quad y_t = b_2 \mathrm{E}_{t-1} e_t + a(p_t - \mathrm{E}_{t-1} p_t) + (1+a)\varepsilon_t$$

$$\text{Aggregate demand:} \quad y_t = \mathrm{E}_t y_{t+1} - \left(\frac{1}{\Phi}\right) r_t$$

$$\text{Money demand:} \quad m_t - p_t = y_t - \frac{\Phi}{b}[\mathrm{E}_t y_{t+1} - y_t] - \left(\frac{1}{b}\right)(\mathrm{E}_t p_{t+1} - p_t).$$

$$\text{Fisher equation:} \quad i_t = r_t + \mathrm{E}_t p_{t+1} - p_t.$$

To complete the solution to the model, assume that the productivity shock e_t and the money supply shock s_t are both serially and mutually uncorrelated. Then $\mathrm{E}_{t-1} e_t = \mathrm{E}_{t-1} y_t^* = 0$. The model reduces to

$$y_t = a(p_t - \mathrm{E}_{t-1} p_t) + (1+a)\varepsilon_t$$
$$m_t - p_t = \left(1 - \frac{\Phi}{b}\right) y_t - \left(\frac{1}{b}\right)(\mathrm{E}_t p_{t+1} - p_t)$$

and

$$m_t = m_{t-1} + s_t.$$

Combining the first and second of these equations,

$$[1 + b + a(b - \Phi)]p_t = bm_t + a(b - \Phi)\mathrm{E}_{t-1} p_t - (1+a)(b-\Phi)\varepsilon_t + \mathrm{E}_t p_{t+1}. \quad (5.95)$$

Guess a solution of the form $p_t = \gamma_1 m_{t-1} + \gamma_2 s_t + \gamma_3 \varepsilon_t$. Then $\mathrm{E}_{t-1} p_t = \gamma_1 m_{t-1}$ and $\mathrm{E}_t p_{t+1} = \gamma_1 m_t = \gamma_1 m_{t-1} + \gamma_1 s_t$. Substituting these expressions into (5.95),

$$[1 + b + a(b - \Phi)](\gamma_1 m_{t-1} + \gamma_2 s_t + \gamma_3 \varepsilon_t)$$
$$= b(m_{t-1} + s_t) + a(b - \Phi)\gamma_1 m_{t-1} - (1+a)(b-\Phi)\varepsilon_t + \gamma_1 m_{t-1} + \gamma_1 s_t.$$

Equating the coefficients on either side, γ_1, γ_2, and γ_3 must satisfy

$$[1 + b + a(b - \Phi)]\gamma_1 = b + a(b - \Phi)\gamma_1 + \gamma_1 \Rightarrow \gamma_1 = 1$$

$$[1 + b + a(b - \Phi)]\gamma_2 = b + \gamma_1 \Rightarrow \gamma_2 = \frac{1+b}{1 + b + a(b - \Phi)}$$

$$[1 + b + a(b - \Phi)]\gamma_3 = (1+a)(b - \Phi) \Rightarrow \gamma_3 = -\left[\frac{(1+a)(b-\Phi)}{1 + b + a(b - \Phi)}\right].$$

To determine the impact of a money shock s_t on output, note that $p_t - \mathrm{E}_{t-1} p_t = \gamma_2 s_t + \gamma_3 \varepsilon_t$, so

$$y_t = a(p_t - \mathrm{E}_{t-1} p_t) + (1+a)\varepsilon_t$$
$$= a\gamma_2 s_t + (a\gamma_3 + 1 + a)\varepsilon_t.$$

From the definition of γ_2,

$$y_t = \left[\frac{a(1+b)}{1 + b + a(b - \Phi)}\right] s_t + (1+a)\left[\frac{1+b}{1 + b + a(b - \Phi)}\right]\varepsilon_t.$$

Using the parameter values from table 2.2, the coefficient on s_t is equal to 1.39. Letting $b \to \infty$ yields (5.20).

5.7.3 The New Keynesian Phillips Curve

In this appendix, (5.57) and (5.61) are used to obtain an expression for the deviations of the inflation rate around its steady-state level. We will assume that the steady state involves a zero rate of inflation. Let $Q_t = p_t^*/P_t$ be the relative price chosen by all firms that adjust their price in period t. The steady-state value of Q_t is $Q = 1$; this is also the value Q_t equals when all firms are able to adjust every period. Dividing (5.61) by P_t, one obtains $1 = (1-\omega)Q_t^{1-\theta} + \omega(P_{t-1}/P_t)^{1-\theta}$. Expressed in terms of percentage deviations around the zero-inflation steady state, this becomes

$$0 = (1-\omega)\hat{q}_t - \omega\pi_t \Rightarrow \hat{q}_t = \left(\frac{\omega}{1-\omega}\right)\pi_t. \quad (5.96)$$

To obtain an approximation to (5.57), note that it can be written as

$$\left[\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}\right] Q_t = \mu \left[\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}\right]. \quad (5.97)$$

In the flexible-price equilibrium with zero inflation, $Q_t = \mu\varphi_t = 1$. The left side of (5.97) is approximated by

$$\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right) + \left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\hat{q}_t + C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i[(1-\sigma)\mathrm{E}_t\hat{c}_{t+i} + (\theta-1)(\mathrm{E}_t\hat{p}_{t+i} - \hat{p}_t)].$$

The right side is approximated by

$$\mu\left\{\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\varphi + \varphi C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i[\mathrm{E}_t\hat{\varphi}_{t+i} + (1-\sigma)\mathrm{E}_t\hat{c}_{t+i} + \theta(\mathrm{E}_t\hat{p}_{t+i} - \hat{p}_t)]\right\}.$$

Setting these two expressions equal and noting that $\mu\varphi = 1$ yields

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t + \sum_{i=0}^{\infty}\omega^i\beta^i[(1-\sigma)\mathrm{E}_t\hat{c}_{t+i} + (\theta-1)(\mathrm{E}_t\hat{p}_{t+i} - \hat{p}_t)]$$

$$= \sum_{i=0}^{\infty}\omega^i\beta^i[\hat{\varphi}_{t+i} + (1-\sigma)\mathrm{E}_t\hat{c}_{t+i} + \theta(\mathrm{E}_t\hat{p}_{t+i} - \hat{p}_t)].$$

Canceling the terms that appear on both sides of this equation leaves

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t = \sum_{i=0}^{\infty}\omega^i\beta^i(\mathrm{E}_t\hat{\varphi}_{t+i} + \mathrm{E}_t\hat{p}_{t+i} - \hat{p}_t),$$

or

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t = \sum_{i=0}^{\infty}\omega^i\beta^i(\mathrm{E}_t\hat{\varphi}_{t+i} + \mathrm{E}_t\hat{p}_{t+i}) - \left(\frac{1}{1-\omega\beta}\right)\hat{p}_t.$$

Multiplying by $1-\omega\beta$ and adding $\hat{p}_t$ to both sides yields

$$\hat{q}_t + \hat{p}_t = (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i(\mathrm{E}_t\hat{\varphi}_{t+i} + \mathrm{E}_t\hat{p}_{t+i}).$$

The left side is the optimal nominal price $\hat{p}_t^* = \hat{q}_t + \hat{p}_t$, and this is set equal to the expected discounted value of future nominal marginal costs. This equation can be rewritten as $\hat{q}_t + \hat{p}_t = (1-\omega\beta)(\hat{\varphi}_t + \hat{p}_t) + \omega\beta(\mathrm{E}_t\hat{q}_{t+1} + \mathrm{E}_t\hat{p}_{t+1})$. Rearranging this expression yields

$$\hat{q}_t = (1-\omega\beta)\hat{\varphi}_t + \omega\beta(\mathrm{E}_t\hat{q}_{t+1} + \mathrm{E}_t\hat{p}_{t+1} - \hat{p}_t)$$

$$= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(\mathrm{E}_t\hat{q}_{t+1} + \mathrm{E}_t\pi_{t+1}).$$

Now using (5.96) to eliminate $\hat{q}_t$, one obtains

$$\left(\frac{\omega}{1-\omega}\right)\pi_t = (1-\omega\beta)\hat{\varphi}_t + \omega\beta\left[\left(\frac{\omega}{1-\omega}\right)\mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\pi_{t+1}\right]$$

$$= (1-\omega\beta)\hat{\varphi}_t + \omega\beta\left(\frac{1}{1-\omega}\right)\mathrm{E}_t\pi_{t+1}.$$

Multiplying both sides by $(1-\omega)/\omega$ produces the forward-looking new Keynesian Phillips curve:

$$\pi_t = \kappa\hat{\varphi}_t + \beta\mathrm{E}_t\pi_{t+1},$$

where

$$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}.$$

When production is subject to diminishing returns to scale, firm-specific marginal costs may differ from the average marginal cost. Let $A = \theta(1-a)/a$. All firms adjusting at time t set their relative price such that

$$\hat{q}_t + \hat{p}_t = (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i(\mathrm{E}_t\hat{\varphi}_{jt+i} + \mathrm{E}_t\hat{p}_{t+i})$$

$$= (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i[\mathrm{E}_t\hat{\varphi}_{t+i} - A(\hat{q}_t + \hat{p}_t - \mathrm{E}_t\hat{p}_{t+i}) + \mathrm{E}_t\hat{p}_{t+i}].$$

This equation can be rewritten as

$$\hat{q}_t + \hat{p}_t = (1-\omega\beta)(\hat{\varphi}_t - A\hat{q}_t + \hat{p}_t)$$

$$+ \omega\beta(1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i[\mathrm{E}_t\hat{\varphi}_{t+1+i} - A(\hat{q}_t + \hat{p}_t - \mathrm{E}_t\hat{p}_{t+1+i}) + \mathrm{E}_t\hat{p}_{t+1+i}].$$

By rearranging this equation and recalling that $q_t = \omega\pi_t/(1-\omega)$, one obtains

$$\left(\frac{\omega}{1-\omega}\right)(1+A)\pi_t = (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A)\left[\left(\frac{\omega}{1-\omega}\right)\mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\pi_{t+1}\right]$$

$$= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A)\left(\frac{1}{1-\omega}\right)\mathrm{E}_t\pi_{t+1}.$$

Multiplying both sides by $(1-\omega)/\omega(1+A)$ produces

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \left(\frac{\tilde{\kappa}}{1+A}\right)\hat{\varphi}_t.$$

5.8 Problems

1. Assume that nominal wages are set for one period but that they can be indexed to the price level:

$$w_t^c = w_t^0 + b(p_t - \mathrm{E}_{t-1}p_t),$$

where w^0 is a base wage and b is the indexation parameter $(0 \le b \le 1)$.

a. How does this change modify the aggregate supply equation (5.17)?

b. Suppose the demand side of the economy is represented by a simple quantity equation, $m_t - p_t = y_t$ and assume $m_t = v_t$, where v_t is a mean zero shock. Assume the indexation parameter is set to minimize $\mathrm{E}_{t-1}(n_t - \mathrm{E}_{t-1}n_t^*)^2$ and show that the optimal degree of wage indexation is increasing in the variance of v and decreasing in the variance of e (Gray 1978).

2. The Chari, Kehoe, and McGrattan (2000) model of price adjustment led to (5.32). Using (5.31), show that the parameter a in (5.32) equals $(1-\sqrt{\gamma})/(1+\sqrt{\gamma})$.

3. Equation (5.30) was obtained from (5.29) by assuming that $R = 1$. Show that, in general, if R is constant but $R^{ss} > 1$,

$$\bar{p}_t = \left(\frac{R^{ss}}{1+R^{ss}}\right)\left[p_t + \frac{1}{R^{ss}}\mathrm{E}_t p_{t+1}\right] + \left(\frac{R^{ss}}{1+R^{ss}}\right)\left[v_t + \frac{1}{R^{ss}}\mathrm{E}_t v_{t+1}\right].$$

4. The basic Taylor model of price-level adjustment was derived under the assumption that the nominal wage set in period t remained unchanged for periods t and $t+1$. Suppose instead that each period t contract specifies a nominal wage x_t^1 for period t and x_t^2 for period $t+1$. Assume these are given by $x_t^1 = p_t + \kappa y_t$ and $x_t^2 = \mathrm{E}_t p_{t+1} + \kappa \mathrm{E}_t y_{t+1}$. The aggregate price level at time t is equal to $p_t = \frac{1}{2}(x_t^1 + x_{t-1}^2)$. If aggregate demand is given by $y_t = m_t - p_t$ and $m_t = m_0 + \omega_t$, what is the effect of a money shock ω_t on p_t and y_t? Explain why output shows no persistence after a money shock.

5. Suppose that the nominal money supply evolves according to $m_t = \mu + \gamma m_{t-1} + s_t$ for $0 < \gamma < 1$ and s_t, a white noise control error. If the rest of the economy is characterized by (5.7)–(5.13), solve for the equilibrium expressions for the price level, output, and the nominal rate of interest. What is the effect of a positive money shock $(s_t > 0)$ on the nominal rate? How does this result compare to the $\gamma = 1$ case discussed in the appendix? Explain.

6. An increase in average inflation lowers the real demand for money. Demonstrate this by using the steady-state version of the model given by (5.7)–(5.13), assuming that the nominal money supply grows at a constant trend rate μ so that $m_t = \mu t$, to show that real money balances $m_t - p_t$ are decreasing in μ.

7. Consider a simple forward-looking model of the form

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}(i_t - \mathrm{E}_t \pi_{t+1}) + u_t,$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t.$$

Suppose policy reacts to the output gap:

$$i_t = \delta x_t.$$

Write this system in the form given by (5.71). Are there values of δ that ensure a unique stationary equilibrium? Are there values that do not?

8. Consider the model given by

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}(i_t - \mathrm{E}_t \pi_{t+1})$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

Suppose policy sets the nominal interest rate according to a policy rule of the form

$$i_t = \phi_1 \mathrm{E}_t \pi_{t+1}$$

for the nominal rate of interest.

a. Write this system in the form $\mathrm{E}_t z_{t+1} = M z_t + \eta_t$, where $z_t = [x_t, \pi_t]'$.

b. For $\beta = 0.99$, $\kappa = 0.05$, and $\sigma = 1.5$, plot the absolute values of the eigenvalues of M as a function of $\phi_1 > 0$.

c. Are there values of ϕ_1 for which the economy does not have a unique stationary equilibrium?

9. Suppose the economy is described by the basic new Keynesian model consisting of

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}(i_t - \mathrm{E}_t \pi_{t+1})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t.$$

a. If $\phi_x = 0$, explain intuitively why $\phi_\pi > 1$ is needed to ensure that the equilibrium will be unique.

b. If both ϕ_π and ϕ_x are nonnegative, the condition given by (5.73) implies that the economy can still have unique, stable equilibrium even when

$$1 - \frac{(1-\beta)\phi_x}{\kappa} < \phi_\pi < 1.$$

Explain intuitively why some values of $\phi_\pi < 1$ are still consistent with uniqueness when $\phi_x > 0$.

10. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}\left(\frac{M_t}{P_t}\right)^{1-b}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t^a$.

a. Show that the household's first order condition for labor supply takes the form

$$\eta \hat{n}_t + \sigma \hat{c}_t - \mu_t^w = \hat{w}_t - \hat{p}_t,$$

where $\mu_t^w = (1-b)(\hat{m}_t - \hat{p}_t)$.

b. Derive an expression for the flexible-price equilibrium output $\hat{y}_t^f$ and the output gap $x_t = \hat{y}_t - \hat{y}_t^f$.

c. Does money affect the flexible-price equilibrium? Does the nominal interest rate? Explain.

6 Money and the Open Economy

6.1 Introduction

The analysis in chapters 2–5 was conducted within the context of a closed economy. Many useful insights into monetary phenomena can be obtained while still abstracting from the linkages that tie different economies together, but clearly many issues do require an open-economy framework if they are to be adequately addressed. New channels through which monetary factors can influence the economy arise in open economies. Exchange-rate movements, for example, play an important role in the transmission process that links monetary disturbances to output and inflation movements.[1] Open economies face the possibility of economic disturbances that originate in other countries, and this raises questions of monetary policy design that are absent in a closed-economy environment: Should policy respond to exchange-rate movements? Should monetary policy be used to stabilize exchange rates? Should national monetary policies be coordinated?

In this chapter, we begin section 6.2 by studying a two-country model based on Obstfeld and Rogoff (1995, 1996). The two-country model has the advantage of capturing some of the important linkages between economies while still maintaining a degree of simplicity and tractability. It can be used to examine monetary policy interactions that are absent from the closed-economy models utilized in previous chapters. Because an open economy is linked to other economies, policy actions in one economy have the potential to affect equilibrium in other economies. Spillovers can occur. Policy actions in one country will depend on the response of monetary policy in the other. Often, because of these spillovers, countries attempt to coordinate their policy actions. The role of policy coordination is examined in section 6.3.

Section 6.4 considers the case of a small open economy. In the open-economy literature, a small open economy denotes an economy that is too small to affect world prices, interest rates, or economic activity. Since many countries really are small relative to the world economy, the small-open-economy model provides a framework that is relevant for studying many policy issues.

The analyses of policy coordination and the small open economy are conducted using models that are similar to those used in section 5.3.1. In these models, behavioral relationships are specified directly rather than derived from underlying assumptions about the behavior of individuals and firms. As a result, the frameworks are of limited use for conducting normative analysis since they are not able to make predictions about the welfare of the agents in the model. This is one reason for beginning the discussion of the open economy with the Obstfeld-Rogoff model; it is

1. For empirical evidence on international business cycles, see Backus and Kehoe (1992).

based explicitly on the assumption of optimizing agents and therefore offers a natural metric—in the form of the utility of the representative agent—for addressing normative policy questions. This chapter ends by returning, in section 6.4, to a class of models based on optimizing agents and nominal rigidities. These models are the open-economy counterparts of the new Keynesian models of section 5.4.

6.2 The Obstfeld-Rogoff Two-Country Model

Obstfeld and Rogoff (1995, 1996) examine the linkages between two economies within a framework that combines three fundamental building blocks we have already seen. The first is an emphasis on intertemporal decisions by individual agents; foreign trade and asset exchange open up avenues for transferring resources over time that are not available in a closed economy. A temporary positive productivity shock that raises current output relative to future output induces individuals to increase consumption both now and in the future as they try to smooth the path of consumption. Since domestic consumption rises less than domestic output, the economy increases its net exports, thereby accumulating claims against future foreign output. These claims can be used to maintain higher consumption in the future after the temporary productivity increase has ended. The trade balance therefore plays an important role in facilitating the intertemporal transfer of resources.

Monopolistic competition in the goods market is the second building block of the Obstfeld-Rogoff model. As we saw in chapter 5, this by itself has no implications for the effects of monetary disturbances, but it does set the stage for the third aspect of their model—sticky prices. Since we have already discussed these basic building blocks, we will focus on the new aspects introduced by open-economy considerations. Detailed derivations of the various components of the model are provided in the appendix to this chapter. It will simplify the exposition to deal with a nonstochastic model in order to highlight the new considerations that arise in the open-economy context.

Each of the two countries is populated by a continuum of agents, indexed by $z \in [0,1]$, who are monopolistic producers of differentiated goods. Agents $z \in [0,n]$ reside in the home country, while agents $z \in (n,1]$ reside in the foreign country. Thus, n provides an index of the relative sizes of the two countries. If the countries are of equal size, $n = \frac{1}{2}$. Foreign variables are denoted by a superscript asterisk (*).

The present discounted value of lifetime utility of a domestic resident j is

$$U^j = \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j + b \log \frac{M_t^j}{P_t} - \frac{k}{2} y_t(j)^2 \right], \tag{6.1}$$

where C_t^j is agent j's period-t consumption of the composite consumption good, defined by

$$C_t^j = \left[\int_0^1 c_t^j(z)^q \right]^{\frac{1}{q}}, \quad 0 < q < 1, \tag{6.2}$$

and consumption by agent j of good z is $c^j(z)$, $z \in [0,1]$. The aggregate domestic price deflator P is defined as

$$P_t = \left[\int_0^1 p_t(z)^{\frac{q}{q-1}} \right]^{\frac{q-1}{q}}. \tag{6.3}$$

This price index P depends on the prices of all goods consumed by domestic residents (the limits of integration run from 0 to 1). It incorporates prices of both domestically produced goods $\{p(z) \text{ for } z \in [0,n]\}$ and foreign-produced goods $\{p(z) \text{ for } z \in (n,1]\}$. Thus, P corresponds to a consumer price index concept of the price level, not a GDP price deflator that would include only the prices of domestically produced goods.

Utility also depends on the agent's holdings of real money balances. Agents are assumed to hold only their domestic currency, so M_t^j/P_t appears in the utility function (6.1). Since agent j is the producer of good j, the effort of producing output $y_t(j)$ generates disutility. A similar utility function is assumed for residents of the foreign country:

$$U^{*j} = \sum_{t=0}^{\infty} \beta^t \left[\log C_t^{*j} + b \log \frac{M_t^{*j}}{P_t^*} - \frac{k}{2} y_t^*(j)^2 \right],$$

where C^{*j} and P^* are defined analogously to C^j and P.

Agent j will pick consumption, money holdings, holdings of internationally traded bonds, and output of good j to maximize utility subject to the budget constraint

$$P_t C_t^j + M_t^j + P_t T_t + P_t B_t^j \le p_t(j) y_t(j) + R_{t-1} P_t B_{t-1}^j + M_{t-1}^j.$$

The gross real rate of interest is denoted R, and T represents real taxes minus transfers. Bonds purchased at time $t-1$, B_{t-1}^j, yield a gross real return R_{t-1}. As in our analysis of chapter 2, the role of T will be to allow for variations in the nominal supply of money, with $P_t T_t = (M_t - M_{t-1})$. Dividing the budget constraint by P_t, one obtains

$$C_t^j + \frac{M_t^j}{P_t} + T_t + B_t^j \le \left[\frac{p_t(j)}{P_t} \right] y_t(j) + R_{t-1} B_{t-1}^j + \left(\frac{1}{1+\pi_t} \right) \frac{M_{t-1}^j}{P_{t-1}}, \tag{6.4}$$

where π_t is the inflation rate from $t-1$ to t. To complete the description of the agent's decision problem, we need to specify the demand for the good the agent produces. This specification is provided in the appendix (section 6.7.1). The appendix shows that the following necessary first order conditions can be derived from the individual consumer/producer's decision problem:

$$C^j_{t+1} = \beta R_t C^j_t \tag{6.5}$$

$$ky^j_t = q\left(\frac{1}{C^j_t}\right)\left(\frac{y^j_t}{C^w_t}\right)^{q-1} \tag{6.6}$$

$$\frac{M^j_t}{P_t} = bC^j_t\left(\frac{1+i_t}{i_t}\right), \tag{6.7}$$

together with the budget constraint (6.4) and the transversality condition

$$\lim_{i\to\infty}\prod_{s=0}^{i} R^{-1}_{t+s-1}\left(B^j_{t+i} + \frac{M^j_{t+i}}{P_{t+i}}\right) = 0.$$

In these expressions, i_t is the nominal rate of interest, defined as $R_t(1+\pi_{t+1}) - 1$.

In (6.6), $C^w_t \equiv nC_t + (1-n)C^*_t$ is world consumption, where $C_t = \int_0^n C^j_t\, dj$ and $C^*_t = \int_n^1 C^j_t\, dj$ equal total home and foreign consumption. Equation (6.5) is a standard Euler condition for the optimal consumption path. Equation (6.6) states that the ratio of the marginal disutility of work to the marginal utility of consumption must equal the marginal product of labor.[2] Equation (6.7) is the familiar condition for the demand for real balances of the domestic currency, requiring that the ratio of the marginal utility of money to the marginal utility of consumption equal $i_t/(1+i_t)$. Similar expressions hold for the foreign consumer/producer.

We have yet to introduce the exchange rate and the link between prices for similar goods in the two countries. Let S_t denote the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. A rise in S_t means that the price of foreign currency has risen in terms of domestic currency; consequently, a unit of domestic currency buys fewer units of foreign currency. So a rise in S_t corresponds to a fall in the value of the domestic currency.

While S_t is the exchange rate between the two currencies, the exchange rate between goods produced domestically and goods produced in the foreign economy

2. See (6.100) in the appendix.

will play an important role. The law of one price requires that good z sell for the same price in both the home and foreign countries when expressed in a common currency.[3] This requires

$$p(z) = Sp^*(z).$$

It follows from the definitions of the home and foreign price levels that

$$P_t = S_t P^*_t. \tag{6.8}$$

Any equilibrium must satisfy the first order conditions for the agent's decision problem, the law-of-one-price condition, and the following additional market-clearing conditions:

$$\text{Goods market clearing:}\quad C^w_t = n\left[\frac{p_t(h)}{P_t}\right]y_t(h) + (1-n)\left[\frac{p^*_t(f)}{P^*_t}\right]y^*_t(f) \equiv Y^w_t,$$

where $p(h)$ and $y(h)$ are the price and output of the representative home good (and similarly for $p^*(f)$ and $y^*(f)$), and

$$\text{Bond market clearing:}\quad nB_t + (1-n)B^*_t = 0.$$

From the structure of the model, it should be clear that one-time proportional changes in the nominal home money supply, all domestic prices, and the nominal exchange rate leave the equilibrium for all real variables unaffected—the model displays monetary neutrality. An increase in M accompanied by a proportional decline in the value of home money in terms of goods (i.e., a proportional rise in all $p(j)$) and a decline in the value of M in terms of M^* (i.e., a proportional rise in S) leaves equilibrium consumption and output in both countries, together with prices in the foreign country, unchanged.

If we consider the model's steady state, the budget constraint (6.4) becomes

$$C = \frac{p(h)}{P}y(h) + (R-1)B, \tag{6.9}$$

3. While the law of one price is intuitively appealing and provides a convenient means of linking the prices $p(j)$ and $p^*(j)$ to the nominal exchange rate, it may be a poor empirical approximation. In a study of prices in different U.S. cities, Parsley and Wei (1996) find rates of price convergence to be faster than in cross-country comparisons, and they conclude that tradable-goods prices converge quickly. Even so, the half-life of a price difference among U.S. cities for tradables is estimated to be on the order of 12–15 months.

where B is the steady-state real stock of bonds held by the home country. For the foreign country,

$$C^* = \frac{p(f)}{P} y(f) - (R-1)\left(\frac{n}{1-n}\right) B. \tag{6.10}$$

These two equations imply that real consumption equals real income (the real value of output plus income from net asset holdings) in the steady state.

6.2.1 The Linear Approximation

It will be helpful to develop a linear approximation to the basic Obstfeld-Rogoff model in terms of percentage deviations around the steady state. This serves to make the linkages between the two economies clear and provides a base of comparison when, in the following section, we employ a more traditional open-economy model that is not directly derived from the assumption of optimizing agents. Using lower-case letters to denote percentage deviations around the steady state, the equilibrium conditions can be expressed as[4]

$$p_t = np_t(h) + (1-n)[s_t + p_t^*(f)] \tag{6.11}$$

$$p_t^* = n[p_t(h) - s_t] + (1-n)p_t^*(f) \tag{6.12}$$

$$y_t = \frac{1}{1-q}[p_t - p_t(h)] + c_t^w \tag{6.13}$$

$$y_t^* = \frac{1}{1-q}[p_t^* - p_t^*(f)] + c_t^w \tag{6.14}$$

$$nc_t + (1-n)c_t^* = c_t^w \tag{6.15}$$

$$c_{t+1} = c_t + r_t \tag{6.16}$$

$$c_{t+1}^* = c_t^* + r_t \tag{6.17}$$

$$(2-q)y_t = (1-q)c_t^w - c_t \tag{6.18}$$

$$(2-q)y_t^* = (1-q)c_t^w - c_t^* \tag{6.19}$$

$$m_t - p_t = c_t - \delta(r_t + \pi_{t+1}) \tag{6.20}$$

$$m_t^* - p_t^* = c_t^* - \delta(r_t + \pi_{t+1}^*), \tag{6.21}$$

4. In chapters 2–4, percentage deviations around the steady state were denoted by $\hat{x}$.

where $\delta = \beta/(\bar{\Pi} - \beta)$ and $\bar{\Pi}$ is 1 plus the steady-state rate of inflation (assumed to be equal in both economies). Equations (6.11) and (6.12) express the domestic and foreign price levels as weighted averages of the prices of home- and foreign-produced goods expressed in a common currency. The weights depend on the relative sizes of the two countries as measured by n. Equations (6.13) and (6.14) are derived from (6.92) of the appendix and give the demand for each country's output as a function of world consumption and relative price. Increases in world consumption (c^w) increase the demand for the output of both countries, while demand also depends on a relative price variable. Home country demand, for example, falls as the price of home production $p(h)$ rises relative to the home price level. Equation (6.15) defines world consumption as the weighted average of consumption in the two countries.

Equations (6.16)–(6.21) are from the individual agent's first order conditions (6.5), (6.6), and (6.7). The first two of these equations are simply the Euler condition for the optimal intertemporal allocation of consumption; the change in consumption is equal to the real rate of return. Equations (6.18) and (6.19) are implied by optimal production decisions. Finally, (6.20) and (6.21) give the real demand for home and foreign money as functions of consumption and nominal interest rates. While both countries face the same real interest rate r_t, nominal interest rates may differ if inflation rates differ between the two countries.

The equilibrium path of home and foreign production (y_t, y_t^*), home, domestic, and world consumption (c_t, c_t^*, c_t^w), prices and the nominal exchange rate $(p_t(h), p_t, p_t^*(f), p_t^*, s_t)$, and the real interest rate (r_t) must be consistent with these equilibrium conditions.[5] Note that subtracting (6.12) from (6.11) implies

$$s_t = p_t - p_t^*, \tag{6.22}$$

while the addition of n times (6.13) and $(1-n)$ times (6.14) yields the goods market-clearing relationship equating world production to world consumption: $ny_t + (1-n)y_t^* = c_t^w$.

6.2.2 Equilibrium with Flexible Prices

The linear version of the two-country model serves to highlight the channels that link open economies. Using this framework, we first discuss the role of money when

5. Equations (6.20)–(6.21) differ somewhat from Obstfeld and Rogoff's specification due to differences in the methods used to obtain linear approximations. See Obstfeld and Rogoff (1996, chapter 10).

6. Recall from the discussion in chapter 2 that the dynamic adjustment outside the steady state is independent of money when utility is log separable, as assumed in (6.1). This result would also characterize this open-economy model if it were modified to incorporate stochastic uncertainty due to productivity and money growth rate disturbances.

prices are perfectly flexible. As in the closed-economy case, the real equilibrium is independent of monetary phenomena when prices can move flexibly to offset changes in the nominal supply of money.[6] Prices and the nominal exchange rate will depend on the behavior of the money supplies in the two countries, and the adjustment of the nominal exchange rate becomes part of the equilibrating mechanism that insulates real output and consumption from monetary effects.

The assumption of a common capital market, implying that consumers in both countries face the same real interest rate, means from the Euler conditions (6.16) and (6.17) that $c_{t+1} - c^*_{t+1} = c_t - c^*_t$; any difference in relative consumption is permanent. And world consumption c^w is the relevant scale variable for demand facing both home and domestic producers.

Real-Monetary Dichotomy With prices and the nominal exchange rate free to adjust immediately in the face of changes in either the home or foreign money supply, the model displays the classical dichotomy discussed in section 5.3.1 under which the equilibrium values of all real variables can be determined independently of the money supply and money demand factors. To see this, define the two relative price variables $\chi_t \equiv p_t(h) - p_t$ and $\chi^*_t \equiv p^*_t(f) - p^*_t$. Equations (6.11) and (6.12) imply that

$$n\chi_t + (1-n)\chi^*_t = 0,$$

while (6.13) and (6.14) can be rewritten as

$$y_t = -\frac{\chi_t}{1-q} + c^w_t$$

$$y^*_t = -\frac{\chi^*_t}{1-q} + c^w_t.$$

These three equations, together with (6.15)–(6.17), suffice to determine the real equilibrium. The money demand equations (6.20) and (6.21) determine the price paths, while (6.22) determines the equilibrium nominal exchange rate given these price paths. Thus, an important implication of this model is that monetary policy (defined as changes in nominal money supplies) has no short-run effects on the real interest rate, output, or consumption in either country. Rather, only nominal interest rates, prices, and the nominal exchange rate are affected by variations in the nominal money stock. One-time changes in m produce proportional changes in p, $p(h)$, and s.

Changes in the growth rate of m affect inflation and nominal interest rates. Equation (6.20) shows that inflation affects the real demand for money, so different rates of inflation are associated with different levels of real money balances. Changes in nominal money growth rates produce changes in the inflation rate and nominal interest rates, thereby affecting the opportunity cost of holding money and, in equilibrium, the real stock of money. The price level and the nominal exchange rate jump to ensure that the real supply of money is equal to the new real demand for money.

Equations (6.21) can be subtracted from (6.20), yielding

$$m_t - m^*_t - (p_t - p^*_t) = (c_t - c^*_t) - \delta(\pi_{t+1} - \pi^*_{t+1}),$$

which, using (6.22), implies[7]

$$m_t - m^*_t - s_t = (c_t - c^*_t) - \delta(s_{t+1} - s_t). \tag{6.23}$$

Solving this forward for the nominal exchange rate, the no-bubbles solution is

$$s_t = \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i[(m_{t+i} - m^*_{t+i}) - (c_{t+i} - c^*_{t+i})]. \tag{6.24}$$

Since (6.16) and (6.17) imply that $c_{t+i} - c^*_{t+i} = c_t - c^*_t$, the expression for the nominal exchange rate can be rewritten as

$$s_t = -(c_t - c^*_t) + \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i(m_{t+i} - m^*_{t+i}).$$

The current nominal exchange rate depends on the current and future paths of the nominal money supplies in the two countries and on consumption differentials. The exchange rate measures the price of one money in terms of the other, and, as (6.24) shows, this depends on the relative supplies of the two monies. An increase in one country's money supply relative to the other's depreciates that country's exchange rate. From the standard steady-state condition that $\beta R^{ss} = 1$ and the definition of δ as $\beta/(\bar{\Pi} - \beta)$, the discount factor in (6.24), $\delta/(1+\delta)$, is equal to $\beta/\bar{\Pi} = 1/R^{ss}\bar{\Pi} = 1/(1+i^{ss})$. Future nominal money supply differentials are discounted by the steady-state nominal rate of interest. Because agents are forward-looking in their decision making, it is only the present discounted value of the relative money supplies that matters. In other words, the nominal exchange rate depends on a measure of the permanent money supply differential. Letting $x_{t+i} \equiv (m_{t+i} - m^*_{t+i}) - (c_{t+i} - c^*_{t+i})$, the equilibrium condition for the nominal exchange rate can be written as

$$s_t = \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i x_{t+i} = \frac{1}{1+\delta}x_t + \frac{\delta}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i x_{t+1+i}$$

$$= \frac{1}{1+\delta}x_t + \frac{\delta}{1+\delta}s_{t+1}.$$

7. This uses the fact that $\pi_{t+1} - \pi^*_{t+1} = (p_{t+1} - p^*_{t+1}) - (p_t - p^*_t) = s_{t+1} - s_t$.

Rearranging and using (6.24) yields

$$s_{t+1} - s_t = -\frac{1}{\delta}(x_t - s_t)$$

$$= -\frac{1}{\delta}\left[(m_t - m_t^*) - \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i (m_{t+1+i} - m_{t+1+i}^*)\right].$$

Analogously to Friedman's permanent income concept, the term

$$\frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i (m_{t+1+i} - m_{t+1+i}^*)$$

can be interpreted as the permanent money supply differential. If the current value of $m - m^*$ is high relative to the permanent value of this differential, the nominal exchange rate will fall (the home currency will appreciate). If s_t reflects the permanent money supply differential at time t, and m_t is temporarily high relative to m_t^*, then the permanent differential will be lower beginning in period $t+1$. As a result, the home currency appreciates.

An explicit solution for the nominal exchange rate can be obtained if specific processes for the nominal money supplies are assumed. To take a very simple case, suppose m and m^* each follow constant, deterministic growth paths given by

$$m_t = m_0 + \mu t$$

and

$$m_t^* = m_0^* + \mu^* t.$$

Strictly speaking, (6.24) applies only to deviations around the steady state and not to money-supply processes that include deterministic trends. However, it is very common to specify (6.20) and (6.21), which were used to derive (6.24), in terms of the log levels of the variables, perhaps adding a constant to represent steady-state levels. The advantage of interpreting (6.24) as holding for the log levels of the variables is that we can then use it to analyze shifts in the trend growth paths of the nominal money supplies, rather than just deviations around the trend. It is important to keep in mind, however, that the underlying representative-agent model implies that the interest rate coefficients in the money-demand equations are functions of the steady-state rate of inflation. We assume this is the same in both countries, implying that the

δ parameter is the same as well. The assumption of common coefficients in two-country models is common, and we will maintain it in the following examples. The limitations of doing so should be kept in mind.

Then (6.24) implies

$$s_t = -(c_t - c_t^*) + \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i [m_0 - m_0^* + (\mu - \mu^*)(t+i)]$$

$$= s_0 + (\mu - \mu^*)t - (c_t - c_t^*),$$

where $s_0 = m_0 - m_0^* + \delta(\mu - \mu^*)$.[8] In this case, the nominal exchange rate has a deterministic trend equal to the difference in the trend of money growth rates in the two economies (also equal to the inflation rate differentials since $\pi = \mu$ and $\pi^* = \mu^*$). If domestic money growth exceeds foreign money growth ($\mu > \mu^*$), s will rise over time to reflect the falling value of the home currency relative to the foreign currency.

Uncovered Interest Parity Real rates of return in the two countries have been assumed to be equal, so the Euler conditions for the optimal consumption paths (6.16 and 6.17) imply the same expected consumption growth in each economy. It follows from the equality of real returns that nominal interest rates must satisfy $i_t - \pi_{t+1} = r_t = i_t^* - \pi_{t+1}^*$, and this means, using (6.22), that

$$i_t - i_t^* = \pi_{t+1} - \pi_{t+1}^* = s_{t+1} - s_t.$$

The nominal interest rate differential is equal to the actual change in the exchange rate in a perfect-foresight equilibrium. This equality would not hold in the presence of uncertainty, since the variables dated $t+1$ would need to be replaced with their expected values, conditional on the information available at time t. In this case,

$$E_t s_{t+1} - s_t = i_t - i_t^*, \tag{6.25}$$

and nominal interest rate differentials would reflect expected exchange-rate changes. If the home country has a higher nominal interest rate in equilibrium, its currency must be expected to depreciate (s must be expected to rise) to equalize real returns across the two countries.

This condition, known as *uncovered nominal interest parity*, links interest rates and exchange rate expectations in different economies if their financial markets are integrated. Under rational expectations, we can write the actual exchange rate at $t+1$ as

8. This uses the fact that $\sum_{i=0}^{\infty} ib^i = b/(1-b)^2$ for $|b| < 1$.

equal to the expectation of the future exchange rate plus a forecast error φ_t uncorrelated with $E_t s_{t+1}$: $s_{t+1} = E_t s_{t+1} + \varphi_t$. Uncovered interest parity then implies

$$s_{t+1} - s_t = i_t - i_t^* + \varphi_{t+1}.$$

The ex post observed change in the exchange rate between times t and $t+1$ is equal to the interest-rate differential at time t plus a random, mean zero forecast error. Since this forecast error will, under rational expectations, be uncorrelated with information, such as i_t and i_t^*, that is known at time t, we can recast uncovered interest parity in the form of a regression equation:

$$s_{t+1} - s_t = a + b(i_t - i_t^*) + \varphi_{t+1}, \tag{6.26}$$

with the null hypothesis of uncovered interest parity implying that $a = 0$ and $b = 1$. Unfortunately, the evidence rejects this hypothesis.[9] In fact, estimated values of b are often negative.

One interpretation of these rejections is that the error term in an equation such as (6.26) is not simply due to forecast errors. Suppose, more realistically, that (6.25) does not hold exactly:

$$E_t s_{t+1} - s_t = i_t - i_t^* + v_t,$$

where v_t captures factors such as risk premia that would lead to divergences between real returns in the two countries. In this case, the error term in the regression of $s_{t+1} - s_t$ on $i_t - i_t^*$ becomes $v_t + \varphi_{t+1}$. If v_t and $i_t - i_t^*$ are correlated, ordinary least squares estimates of the parameter b in (6.26) will be biased and inconsistent.

Correlation between v and $i - i^*$ might arise if monetary policies are implemented in a manner that leads the nominal interest-rate differential to respond to the current exchange rate. For example, suppose that the monetary authority in each country tends to tighten policy whenever its currency depreciates. This could occur if the monetary authorities are concerned with inflation; depreciation raises the domestic currency price of foreign goods and raises the domestic price level. To keep the example simple for illustrative purposes, suppose that as a result of such a policy, the nominal interest rate differential is given by

$$i_t - i_t^* = \mu s_t + u_t, \quad \mu > 0,$$

9. For a summary of the evidence, see Froot and Thaler (1990). See also McCallum (1994a), Eichenbaum and Evans (1995), and Schlagenhauf and Wrase (1995).

where u_t captures any other factors affecting the interest rate differential.[10] Assume u is an exogenous, white noise process. Substituting this into the uncovered interest parity condition yields

$$E_t s_{t+1} = (1+\mu)s_t + u_t + v_t, \tag{6.27}$$

the solution to which is[11]

$$s_t = -\left(\frac{1}{1+\mu}\right)(u_t + v_t).$$

Since this solution implies that $E_t s_{t+1} = -E_t(u_{t+1} + v_{t+1})/(1+\mu) = 0$, the interest parity condition is then given by

$$E_t s_{t+1} - s_t = -\left(\frac{1}{1+\mu}\right)(u_t + v_t) = i_t - i_t^* + v_t$$

or $i_t - i_t^* = (u_t - \mu v_t)/(1+\mu)$.

What does this imply for tests of uncovered interest parity? From the solution for s_t, $s_{t+1} - s_t = -(u_{t+1} - u_t + v_{t+1} - v_t)/(1+\mu)$. The probability limit of the interest-rate coefficient in the regression of $s_{t+1} - s_t$ on $i_t - i_t^*$ is equal to

$$\frac{cov(s_{t+1} - s_t, i_t - i_t^*)}{var(i_t - i_t^*)} = \frac{\dfrac{1}{(1+\mu)^2}(\sigma_u^2 - \mu\sigma_v^2)}{\dfrac{1}{(1+\mu)^2}(\sigma_u^2 + \mu^2\sigma_v^2)} = \frac{\sigma_u^2 - \mu\sigma_v^2}{\sigma_u^2 + \mu^2\sigma_v^2},$$

which will not generally equal 1, the standard null in tests of interest parity. If $u \equiv 0$, the probability limit of the regression coefficient is $-1/\mu$. That is, the regression estimate uncovers the policy parameter μ. Not only would a regression of the change in the exchange rate on the interest differential not yield the value of 1 predicted by the uncovered interest parity condition, but the estimate would be negative.

McCallum (1994a) develops more fully the argument that rejections of uncovered interest parity may arise because standard tests compound the parity condition with

10. The rationale for such a policy is clearly not motivated within the context of a model with perfectly flexible prices in which monetary policy has no real effects. And notice that we are treating nominal interest rates, rather than the nominal money supply, as the policy instrument, an issue that will be addressed in chapters 9 and 10. The general point is to illustrate how empirical relationships such as (6.26) can depend on the conduct of policy.

11. From (6.27), the equilibrium exchange-rate process must satisfy $E_t s_{t+1} = (1+\mu)s_t + u_t + v_t$ so that the state variables are u_t and v_t. Following McCallum (1983a), the minimal state solution takes the form $s_t = b_0 + b_1(u_t + v_t)$. This implies that $E_t s_{t+1} = b_0$. So the interest parity condition becomes $b_0 = (1+\mu)(b_0 + b_1(u_t + v_t)) + u_t + v_t$, which will hold for all realizations of u and v if $b_0 = 0$ and $b_1 = -1/(1+\mu)$.

the manner in which monetary policy is conducted. Although uncovered interest parity is implied by the model independently of the manner in which policy is conducted, the outcomes of statistical tests may in fact be dependent on the behavior of monetary policy, since policy may influence the time-series properties of the nominal-interest-rate differential.

As noted earlier, tests of interest parity often report negative regression coefficients on the interest rate differential in (6.26). This finding is also consistent with the empirical evidence reported by Eichenbaum and Evans (1995). They estimate the impact of monetary shocks on nominal and real exchange rates and interest-rate differentials between the United States and France, Germany, Italy, Japan, and the United Kingdom. A contractionary U.S. monetary-policy shock leads to a persistent nominal and real appreciation of the dollar and a fall in $i_t^* - i_t + s_{t+1} - s_t$, where i is the U.S. interest rate and i^* is the foreign rate. Uncovered interest parity implies that this expression should have expected value equal to zero, yet it remains predictably low for several months. Rather than leading to an expected depreciation that offsets the rise in i, excess returns on U.S. dollar securities remain high for several months following a contractionary U.S. monetary-policy shock.[12]

6.2.3 Sticky Prices

Just as was the case with the closed economy, flexible-price models of the open economy appear unable to replicate the size and persistence of monetary shocks on real variables. And just as with closed-economy models, this can be remedied by the introduction of nominal rigidities. Obstfeld and Rogoff (1996, chapter 10) provide an analysis of their basic two-country model under the assumption that prices are set one period in advance.[13] The presence of nominal rigidities leads to real effects of monetary disturbances through the channels discussed in chapter 5, but new channels through which monetary disturbances have real effects are now also present.

Suppose $p(h)$, the domestic currency price of domestically produced goods, is set one period in advance and fixed for one period. A similar assumption is made for the foreign currency price of foreign-produced goods, $p^*(f)$. Although $p(h)$ and $p^*(f)$ are preset, the aggregate price indices in each country will fluctuate with the nominal

12. Eichenbaum and Evans measure monetary policy shocks in a variety of ways (VAR innovations to nonborrowed reserves relative to total reserves, VAR innovations to the federal funds rate, and Romer and Romer's 1989 measures of policy shifts). However, the identification scheme used in their VARs assumes that policy does not respond contemporaneously to the real exchange rate. This means that the specific illustrative policy response to the exchange rate that led to (6.27) is ruled out by their framework.

13. They also consider the case in which nominal wages are preset.

exchange rate according to (6.11) and (6.12). Nominal depreciation, for example, raises the domestic price index p by increasing the domestic currency price of foreign-produced goods. This introduces a new channel, one absent in a closed economy, through which monetary disturbances can have an immediate impact on the price level. Recall that in the closed economy, there is no distinction between the price of domestic output and the general price level. Nominal price rigidities implied that the price level cannot adjust immediately to monetary disturbances. Exchange-rate movements alter the domestic currency price of foreign goods, allowing the consumer price index to move in response to such disturbances even in the presence of nominal rigidities.

Now suppose that in period t the home country's money supply rises unexpectedly relative to that of the foreign country.[14] Under Obstfeld and Rogoff's simplifying assumption that prices adjust completely after one period, both economies return to their steady state one period after the change in m. But during the one period in which product prices are preset, real output and consumptions levels will be affected.[15] And these real effects mean that the home country may run a current account surplus or deficit in response to the change in m. This effect on the current account alters the net asset positions of the two economies and can affect the new steady-state equilibrium.

Interpreting the model consisting of (6.11)–(6.21) as applying to deviations around the initial steady state, the Euler conditions (6.16) and (6.17), imply that $c_{t+1} - c_{t+1}^* = c_t - c_t^*$. Since the economies are in the new steady state after one period (i.e., in $t+1$), $c_{t+1} - c_{t+1}^* \equiv \mathscr{C}$ is the steady-state consumption differential between the two countries. But since we also have that $c_t - c_t^* = c_{t+1} - c_{t+1}^* = \mathscr{C}$, this relationship implies that relative consumption in the two economies immediately jumps in period t to the new steady-state value. Equation (6.23), which expresses relative money demands in the two economies, can then be written $m_t - m_t^* - s_t = \mathscr{C} - \delta(s_{t+1} - s_t)$. Solving this equation forward for the nominal exchange rate (assuming no bubbles),

$$s_t = -\mathscr{C} + \frac{\delta}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{1}{1+\delta}\right)^i (m_{t+i} - m_{t+i}^*).$$

14. An unexpected change is inconsistent with the assumption of perfect foresight implicit in the nonstochastic version of the model derived earlier. However, the linear approximation will continue to hold under uncertainty if future variables are replaced with their mathematical expectation.

15. If we consider a situation in which the economies are initially in a steady state, the preset values for $p(h)$ and $p^*(f)$ will equal zero.

If the change in $m_t - m_t^*$ is a permanent one-time change, we can let $\Omega \equiv m - m^*$ without time subscripts denote this permanent change. The equilibrium exchange rate is then equal to

$$s_t = -\mathscr{C} + \frac{\delta}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{1}{1+\delta}\right)^i \Omega = \Omega - \mathscr{C}. \tag{6.28}$$

Since $\Omega - \mathscr{C}$ is a constant, (6.28) implies that the exchange rate jumps immediately to its new steady state following a permanent change in relative nominal money supplies. If relative consumption levels do not adjust (i.e., if $\mathscr{C} = 0$), then the permanent change in s is just equal to the relative change in nominal money supplies Ω. An increase in m relative to m^* (i.e., $\Omega > 0$) produces a depreciation of the home country currency. If $\mathscr{C} \neq 0$, then changes in relative consumption affect the relative demand for money from (6.20) and (6.21). For example, if $\mathscr{C} > 0$, consumption, as well as money demand, in the home country is higher than initially. Equilibrium between home money supply and home money demand can be restored with a smaller increase in the home price level. Since $p(h)$ and $p^*(f)$ are fixed for one period, the increase in p necessary to maintain real money demand and real money supply equal is generated by a depreciation (a rise in s). The larger the rise in home consumption, the larger the rise in real money demand and the smaller the necessary rise in s. This is just what (6.28) says.

While we have determined the impact of a change in $m - m^*$ on the exchange rate, given $\mathscr{C}$, the real consumption differential is itself endogenous. To determine $\mathscr{C}$ requires that we work through several steps. First, the linear approximation to the current account relates the home country's accumulation of net assets to the excess of its real income over its consumption: $b = y_t + [p_t(h) - p_t] - c_t = y_t - (1-n)s_t - c_t$, where $p_t(h) - p_t = -(1-n)s_t$ follows from (6.11) and the fact that $p_t(h)$ is fixed (and equal to zero) during period t. Similarly, for the foreign economy, $-nb/(1-n) = y_t^* + ns_t - c_t^*$. Together, these imply

$$\frac{b}{1-n} = (y_t - y_t^*) - (c_t - c_t^*) - s_t. \tag{6.29}$$

From (6.13) and (6.14), $y_t - y_t^* = s_t/(1-q)$, so (6.29) becomes

$$\frac{b}{1-n} = \left(\frac{q}{1-q}\right)s_t - (c_t - c_t^*) = \left(\frac{q}{1-q}\right)s_t - \mathscr{C}, \tag{6.30}$$

where we have made use of the definition of $\mathscr{C}$ as the consumption differential.

The last step is to use the steady-state relationship between consumption, income, and asset holdings given by (6.9) and (6.10) to eliminate b in (6.30) by expressing it in terms of the exchange-rate and consumption differences. In the steady state, b is constant and current accounts are zero, so consumption equals real income inclusive of asset income. In terms of the linear approximation, (6.9) and (6.10) become

$$c = rb + y + [p(h) - p] = rb + y - (1-n)[s + p^*(f) - p(h)] \tag{6.31}$$

and

$$c^* = -\left(\frac{n}{1-n}\right)rb + y^* + n[s + p^*(f) - p(h)]. \tag{6.32}$$

From the steady-state labor-leisure choice linking output and consumption given in (6.18) and (6.19), $(2-q)(y-y^*) = -(c-c^*)$, and from the link between relative prices and demand from (6.13) and (6.14), $y - y^* = [s + p^*(f) - p(h)]/(1-q)$. Using these relationships, we can now subtract (6.32) from (6.31), yielding

$$\begin{aligned}\mathscr{C} &= \left(\frac{1}{1-n}\right)rb + (y - y^*) - [s + p^*(f) - p(h)] \\ &= \left(\frac{1}{1-n}\right)rb + q(y - y^*) \\ &= \left(\frac{1}{1-n}\right)rb + \left(\frac{q}{q-2}\right)\mathscr{C}.\end{aligned}$$

Finally, this yields

$$b = \left(\frac{1-n}{r}\right)\left(\frac{2}{2-q}\right)\mathscr{C}. \tag{6.33}$$

Substituting (6.33) into (6.30), $[2/(2-q)]\mathscr{C}/r = qs_t/(1-q) - \mathscr{C}$. Solving for s in terms of $\mathscr{C}$,

$$s_t = \psi\mathscr{C}, \tag{6.34}$$

where $\psi = (1-q)[1 + 2/r(2-q)]/q > 0$. But from (6.28) we have already seen that $s_t = \Omega - \mathscr{C}$, so $\psi\mathscr{C} = \Omega - \mathscr{C}$. It follows that the consumption differential is $\mathscr{C} = \Omega/(1+\psi)$. The equilibrium nominal exchange-rate adjustment to a permanent change in the home country's nominal money supply is then given by

$$s_t = \left(\frac{\psi}{1+\psi}\right)\Omega < \Omega.$$

With $\psi > 0$, the domestic monetary expansion leads to a depreciation that is less than proportional to the increase in m. This induces an expansion in domestic production and consumption. Consumption rises by less than income, so the home country runs a current account surplus and accumulates assets that represent claims against the future income of the foreign country. This allows the home country to maintain higher consumption forever. As we have seen, consumption levels jump immediately to their new steady state with $\mathscr{C} = \Omega/(1+\psi) > 0$. To provide a rough magnitude for ψ, if $q = .9$ and $r = .05$, $\psi \approx 4$, so $s_t = .8\Omega$ while $\mathscr{C} = .2\Omega$.

The two-country model employed in this section has the advantage of being based on the clearly specified decision problems faced by agents in the model. As a consequence, the responses of consumption, output, interest rates, and the exchange rate are consistent with optimizing behavior. Unanticipated monetary disturbances can have a permanent impact on real consumption levels and welfare when prices are preset. These effects arise because the output effects of a monetary surprise alter each country's current account, thereby altering their relative asset positions. A monetary expansion in the home country, for instance, produces a currency depreciation and a rise in the domestic price level p. This, in turn, induces a temporary expansion in output in the home country (see 6.13). With consumption determined on the basis of permanent income, consumption rises less than output, leading the home country to run a trade surplus as the excess of output over domestic consumption is exported. As payment for these exports, the home country receives claims against the future output of the foreign country. Home consumption does rise, even though the increase in output lasts only one period, as the home country's permanent income has risen by the annuity value of its claim on future foreign output.

A domestic monetary expansion leads to permanently higher real consumption for domestic residents; welfare is increased. This observation suggests that each country has an incentive to engage in a monetary expansion. However, a joint proportionate expansion of each country's money supply leaves $m - m^*$ unchanged. There are then no exchange-rate effects, and relative consumption levels do not change. After one period, when prices fully adjust, a proportional change in $p(h)$ and $p^*(f)$ returns both economies to the initial equilibrium. But because output is inefficiently low because monopolistic competition is present, the one-period rise in output does increase welfare in both countries. Both countries have an incentive to expand their money supplies, either individually or in a coordinated fashion. But the analysis we have carried out has involved changes in money supplies that were unexpected. If they had been anticipated, the level at which price setters would set individual goods prices would have incorporated expectations of money-supply changes. As we have seen in chapter 5, fully anticipated changes in the nominal money supply will not have the real effects that unexpected changes do when there is some degree of nominal wage or price rigidity. As we will see in chapter 8, the incentive to create surprise

expansions can, in equilibrium, lead to steady inflation without the welfare gains an unanticipated expansion would bring.

Unanticipated, permanent changes in the money supply can have permanent effects on the international distribution of wealth in the Obstfeld-Rogoff model when there are nominal rigidities. Corsetti and Presenti (2001) have developed a two-country model with similar micro foundations as the Obstfeld-Rogoff model. In Corsetti and Presenti's model, preferences are specified so that changes in national money supplies do not cause wealth redistributions. Corsetti and Presenti assume the elasticity of output supply with respect to relative prices and the elasticity of substitution between the home-produced and foreign-produced goods are both equal to 1. Thus, an increase in the relative price of the foreign good (i.e., a decline in the terms of trade) lowers the purchasing power of domestic agents, but it also leads to an increase in the demand for domestic goods that increases nominal incomes. These two effects cancel each other out, leaving the current account and international lending and borrowing unaffected. By eliminating current account effects, the Corsetti–Presenti model allows for a tractable closed-form solution with one-period nominal wage rigidity, and it allows them to determine the impact of policy changes on welfare.

A large and growing literature has studied open-economy models that are explicitly based on optimizing behavior by firms and households but which also incorporate nominal rigidities. Besides the work of Obstfeld and Rogoff (1995, 1996) and Corsetti and Presenti (2001), examples include Betts and Devereux (2000), Obstfeld and Rogoff (2000), Benigno and Benigno (2001), Corsetti and Presenti (2002), and Kollmann (2001). Lane (2001) and Engle (2002) provide surveys of the "new open economy macroeconomics."

6.3 Policy Coordination

An important issue facing economies linked by trade and capital flows is the role to be played by policy coordination. Monetary policy actions by one country will affect other countries, leading to spillover effects that open the possibility of gains from policy coordination. As demonstrated in the previous section, the real effects of an unanticipated change in the nominal money supply in the two-country model depend on how $m - m^*$ is affected. A rise in m, holding m^* unchanged, will produce a home country depreciation, shifting world demand toward the home country's output. With preset prices and output demand determined, the exchange-rate movement represents an important channel through which a monetary expansion affects domestic output. If both monetary authorities attempt to generate output expansions by increasing their money supplies, this exchange-rate channel will not operate, since the exchange rate depends on the relative money supplies. Thus, the impact of an unanticipated change in m depends critically on the behavior of m^*.

This dependence raises the issue of whether there are gains from coordinating monetary policy. Hamada (1976) is closely identified with the basic approach that has been used to analyze policy coordination, and in this section, we develop a version of his framework. Canzoneri and Henderson (1989) provide an extensive discussion of monetary policy coordination issues; a survey is provided by Currie and Levine (1991).

Suppose we consider a model with two economies. We will assume that each economy's policy authority can choose its inflation rate and, because of nominal rigidities, monetary policy can have real effects in the short run. In this context, a complete specification of policy behavior is more complicated than in a closed-economy setting; we need to specify how each national policy authority interacts strategically with the other policy authority. We will examine two possibilities. First, we consider coordinated policy, meaning that inflation rates in the two economies are chosen jointly to maximize a weighted sum of the objective functions of the two policy authorities. Second, we consider noncoordinated policy, with the policy authorities interacting in a Nash equilibrium. In this setting, each policy authority sets its own inflation rate to maximize its objective function, taking as given the inflation rate in the other economy. These clearly are not the only possibilities. One economy may act as a Stackelberg leader, recognizing the impact its choice has on the inflation rate set by the other economy. Reputational considerations along the lines we will study in chapter 8 can also be incorporated into the analysis (see Canzoneri and Henderson 1989).

6.3.1 The Basic Model

The two-country model is specified as a linear system in log deviations around a steady state and represents an extension to the open-economy environment of the sticky-wage, AS-IS model of chapter 5. The LM relationship is dispensed with by assuming that the monetary policy authorities in the two countries set the inflation rate directly. An asterisk will denote the foreign economy, and ρ will be the real exchange rate, defined as the relative price of home and foreign output, expressed in terms of the home currency; a rise in ρ represents a real depreciation for the home economy. If s is the nominal exchange rate and $p(h)$ and $p^*(f)$ are the prices of home and foreign output, then $\rho = s + p^*(f) - p(h)$. The model should be viewed as an approximation that is appropriate when nominal wages are set in advance so that unanticipated movements in inflation affect real output. In addition to aggregate

supply and demand relationships for each economy, an interest parity condition links the real interest-rate differential to anticipated changes in the real exchange rate:

$$y_t = -b_1\rho_t + b_2(\pi_t - \mathrm{E}_{t-1}\pi_t) + e_t \tag{6.35}$$

$$y_t^* = b_1\rho_t + b_2(\pi_t^* - \mathrm{E}_{t-1}\pi_t^*) + e_t^* \tag{6.36}$$

$$y_t = a_1\rho_t - a_2r_t + a_3y_t^* + u_t \tag{6.37}$$

$$y_t^* = -a_1\rho_t - a_2r_t^* + a_3y_t + u_t^* \tag{6.38}$$

$$\rho_t = r_t^* - r_t + \mathrm{E}_t\rho_{t+1}. \tag{6.39}$$

Equations (6.35) and (6.36) relate output to inflation surprises and the real exchange rate. A real exchange-rate depreciation reduces home aggregate supply by raising the price of imported materials and by raising consumer prices relative to producer prices. This latter effect increases the real wage in terms of producer prices. Equations (6.37) and (6.38) make demand in each country an increasing function of output in the other to reflect spillover effects that arise as an increase in output in one country raises demand for the goods produced by the other. A rise in ρ_t (a real domestic depreciation) makes domestically produced goods less expensive relative to foreign goods and shifts demand away from foreign output and toward home output.

To simplify the analysis, the inflation rate is treated as the choice variable of the policy maker. An alternative approach to treating inflation as the policy variable would be to specify money demand relationships for each country and then take the nominal money supply as the policy instrument. This would complicate the analysis without offering any new insights.

A third approach would be to replace r_t with $i_t - \mathrm{E}_t\pi_{t+1}$, where i_t is the nominal interest rate, and treat i_t as the policy instrument. An advantage of this approach is that it more closely reflects the way most central banks actually implement policy. Because a number of new issues arise under nominal interest rate policies (see section 6.5 and chapters 9 and 10), we will simply interpret policy as choosing the rate of inflation in order to focus, in this section, on the role of policy coordination. Finally, a further simplification is reflected in the assumption that the parameters (the a_i's and b_i's) are the same in the two countries.

Demand (u_t, u_t^*) and supply (e_t, e_t^*) shocks are included to introduce a role for stabilization policy. These disturbances are assumed to be mean zero, serially uncorrelated processes, but we allow them to be correlated so that it will be possible to distinguish between common shocks that affect both economies and asymmetric shocks that originate in a single economy.

Equation (6.39) is an uncovered interest rate parity condition. Rewritten in the form $r_t = r_t^* + E_t\rho_{t+1} - \rho_t$, it implies that the home country real interest rate will exceed the foreign real rate if the home country is expected to experience a real depreciation.

Evaluating outcomes under coordinated and noncoordinated policies requires some assumption about the objective functions of the policy makers. In models built more explicitly on the behavior of optimizing agents, alternative policies could be ranked according to their implications for the utility of the agents in the economies. This approach will be pursued in chapter 11, but here we simply follow a common approach in which polices are evaluated on the basis of loss functions that depend on output variability and inflation variability:

$$V_t = \sum_{i=0}^{\infty} \beta^i (\lambda y_{t+i}^2 + \pi_{t+i}^2) \tag{6.40}$$

$$V_t^* = \sum_{i=0}^{\infty} \beta^i [\lambda (y_{t+i}^*)^2 + (\pi_{t+i}^*)^2]. \tag{6.41}$$

The parameter β is a discount factor between 0 and 1. The weight attached to output fluctuations relative to inflation fluctuations is λ. While these objective functions are ad hoc, they capture the idea that policy makers prefer to minimize output fluctuations around the steady state and fluctuations of inflation.[16] Objective functions of this basic form have played a major role in the analysis of policy, and we will make extensive use of them in chapter 8. Equations (6.40) and (6.41) reflect the assumption that steady-state output will be independent of monetary policy, so policy should focus on minimizing fluctuations around the steady state, not on the level of output.

The model can be solved to yield expressions for equilibrium output in each economy and for the real exchange rate. To obtain the real exchange rate, first subtract foreign aggregate demand (6.38) from domestic aggregate demand (6.37), using the interest parity condition (6.39) to eliminate $r_t - r_t^*$. This process yields an expression for $y_t - y_t^*$. Next, subtract foreign aggregate supply (6.36) from domestic aggregate

16. The steady-state values of y and y^* are zero by definition. The assumption that the policy loss functions depend on the variance of output around its steady-state level, and not on some higher output target, is critical for the determination of average inflation. Chapter 8 deals extensively with the time-inconsistency issues that arise when policy makers target a level of output that exceeds the economy's equilibrium level.

supply (6.35) to yield a second expression for $y_t - y_t^*$. Equating these two expressions and solving for the equilibrium real exchange rate leads to the following:

$$\rho_t = \frac{1}{B}\{b_2(1+a_3)[(\pi_t - E_{t-1}\pi_t) - (\pi_t^* - E_{t-1}\pi_t^*)]$$
$$+ (1+a_3)(e_t - e_t^*) - (u_t - u_t^*) + a_2 E_t\rho_{t+1}\}, \tag{6.42}$$

where $B \equiv 2a_1 + a_2 + 2b_1(1+a_3) > 0$. An unanticipated rise in domestic inflation relative to unanticipated foreign inflation or in e_t relative to e_t^* will increase domestic output supply relative to foreign output. Equilibrium requires a decline in the relative price of domestic output; the real exchange rate rises (depreciates), shifting demand toward domestic output. If the domestic aggregate demand shock exceeds the foreign shock, $u_t - u_t^* > 0$, the relative price of domestic output must rise (ρ must fall) to shift demand toward foreign output. A rise in the expected future exchange rate also leads to a rise in the current equilibrium ρ. If ρ were to increase by the same amount as the rise in $E_t\rho_{t+1}$, the interest differential $r_t - r_t^*$ would be left unchanged, but the higher ρ would, from (6.35) and (6.36), lower domestic supply relative to foreign supply. So ρ rises by less than the increase in $E_t\rho_{t+1}$ to maintain goods market equilibrium.[17]

Notice that (6.42) can be written as $\rho_t = AE_t\rho_{t+1} + v_t$, where $0 < A < 1$ and v_t is white noise, since the disturbances are assumed to be serially uncorrelated and the same will be true of the inflation forecast errors under rational expectations. It follows that $E_t\rho_{t+1} = 0$ in any no-bubbles solution. The expected future real exchange rate would be nonzero if either the aggregate demand or aggregate supply shocks were serially correlated.

Now that we have obtained an expression for the equilibrium real exchange rate, this can be substituted into the aggregate supply relationships (6.35) and (6.36) to yield

$$y_t = b_2A_1(\pi_t - E_{t-1}\pi_t) + b_2A_2(\pi_t^* - E_{t-1}\pi_t^*)$$
$$- a_2A_3E_t\rho_{t+1} + A_1e_t + A_2e_t^* + A_3(u_t - u_t^*) \tag{6.43}$$

$$y_t^* = b_2A_2(\pi_t - E_{t-1}\pi_t) + b_2A_1(\pi_t^* - E_{t-1}\pi_t^*)$$
$$+ a_2A_3E_t\rho_{t+1} + A_2e_t + A_1e_t^* - A_3(u_t - u_t^*). \tag{6.44}$$

17. The coefficient on $E_t\rho_{t+1}$, a_2/B is less than 1 in absolute value.

The A_i parameters are given by

$$A_1 = \frac{2a_1 + a_2 + b_1(1 + a_3)}{B} > 0$$

$$A_2 = \frac{b_1(1 + a_3)}{B} > 0$$

$$A_3 = \frac{b_1}{B} > 0.$$

Equations (6.43) and (6.44) reveal the spillover effects through which the inflation choice of one economy affects the other economy when $b_2 A_2 \neq 0$. An increase in inflation in the home economy (assuming it is unanticipated) leads to a real depreciation. This occurs since unanticipated inflation leads to a home output expansion (see 6.35). Equilibrium requires a rise in demand for home country production. In the closed economy, this occurs through a fall in the real interest rate. In the open economy, an additional channel of adjustment arises from the role of the real exchange rate. Given that $E_t \rho_{t+1} = 0$, the interest parity condition (6.39) becomes $\rho_t = r_t^* - r_t$, so, for given r_t^*, the fall in r_t requires a rise in ρ_t (a real depreciation), which also serves to raise home demand.

The rise in ρ_t represents a real appreciation for the foreign economy, and this raises consumer-price wages relative to producer-price wages and increases aggregate output in the foreign economy (see 6.36). As a result, an expansion in the home country produces an economic expansion in the foreign country. But as (6.42) shows, a surprise inflation by both countries leaves the real exchange rate unaffected. It is this link that opens the possibility that outcomes will depend on the extent to which the two countries coordinate their policies.

6.3.2 Equilibrium with Coordination

To focus on the implications of policy coordination, we will restrict attention to the case of a common aggregate supply shock, common in the sense that it affects both countries. That is, suppose $e_t = e_t^* \equiv \varepsilon_t$, where ε_t is the common disturbance. For the rest of this section, we assume $u \equiv u^* \equiv 0$, so that ε represents the only disturbance.

In solving for equilibrium outcomes under alternative policy interactions, the objective functions (6.40) and (6.41) simplify to a sequence of one-period problems (the problem is a static one with no link between periods). Assuming that the policy authority is able to set the inflation rate after observing the supply shock ε_t, the decision problem under a coordinated policy is

$$\min_{\pi, \pi^*} \left\{ \tfrac{1}{2}[\lambda y_t^2 + \pi_t^2] + \tfrac{1}{2}[\lambda (y_t^*)^2 + (\pi_t^*)^2] \right\}$$

subject to (6.43) and (6.44).[18] The first order conditions are

$$\begin{aligned} 0 &= \lambda b_2 A_1 y_t + \pi_t + \lambda b_2 A_2 y_t^* \\ &= (1 + \lambda b_2^2 A_1^2 + \lambda b_2^2 A_2^2)\pi_t + 2\lambda b_2^2 A_1 A_2 \pi_t^* + \lambda b_2 \varepsilon_t \\ 0 &= \lambda b_2 A_2 y_t + \lambda b_2 A_1 y_t^* + \pi_t^* \\ &= (1 + \lambda b_2^2 A_1^2 + \lambda b_2^2 A_2^2)\pi_t^* + 2\lambda b_2^2 A_1 A_2 \pi_t + \lambda b_2 \varepsilon_t, \end{aligned}$$

where we have used the fact that $A_1 + A_2 = 1$ and the result that the first order conditions imply $E_{t-1}\pi_t = E_{t-1}\pi_t^* = 0$.[19] Solving these two equations yields the equilibrium inflation rates under coordination:

$$\pi_{c,t} = \pi_{c,t}^* = -\left(\frac{\lambda b_2}{1 + \lambda b_2^2}\right)\varepsilon_t \equiv -\theta_c \varepsilon_t. \tag{6.45}$$

Both countries maintain equal inflation rates. In response to an adverse supply shock ($\varepsilon < 0$), inflation in both countries rises to offset partially the decline in output. Substituting (6.45) into the expressions for output and the equilibrium real exchange rate,

$$y_{c,t} = y_{c,t}^* = \left(\frac{1}{1 + \lambda b_2^2}\right)\varepsilon_t < \varepsilon_t$$

and

$$\rho_t = 0.$$

The policy response acts to offset partially the output effects of the supply shock. The larger the weight placed on output in the loss function (λ), the larger the inflation response and the more output is stabilized. Because both economies respond symmetrically, the real exchange rate is left unaffected.[20]

6.3.3 Equilibrium without Coordination

When policy is not coordinated, some assumption must be made about the nature of the strategic interaction between the two separate policy authorities. One natural

18. In defining the objective function under coordinated policy, we have assumed that each country's utility receives equal weight.

19. Writing out the first order condition for π_t in full, $0 = \pi_t + \lambda b_2^2(A_1^2 + A_2^2)(\pi_t - E_{t-1}\pi_t) + 2\lambda b_2^2 A_1 A_2(\pi_t^* - E_{t-1}\pi_t^*) + 2\lambda b_2 \varepsilon_t$. Taking expectations conditional on time $t-1$ information (i.e., prior to the realization of ε_t), we obtain $E_{t-1}\pi_t = 0$.

20. This would not be the case in response to an asymmetric supply shock. See problem 4.

case to consider corresponds to a Nash equilibrium; the policy authorities choose inflation to minimize loss, taking as given the inflation rate in the other economy. An alternative case arises when one country behaves as a Stackelberg leader, taking into account how the other policy authority will respond to the leader's choice of inflation. We will analyze the Nash case, leaving the Stackelberg case to be studied as a problem at the end of the chapter.

The home policy authority picks inflation to minimize $\lambda y_t^2 + \pi_t^2$, taking π_t^* as given. The first order condition is

$$0 = \lambda b_2 A_1 y_t + \pi_t$$
$$= (1 + \lambda b_2^2 A_1^2)\pi_t + \lambda b_2^2 A_1 A_2 \pi_t^* + \lambda b_2 A_1 \varepsilon_t$$

so that the home country's reaction function is

$$\pi_t = -\left(\frac{\lambda b_2^2 A_1 A_2}{1 + \lambda b_2^2 A_1^2}\right)\pi_t^* - \left(\frac{\lambda b_2 A_1}{1 + b_2^2 A_1^2}\right)\varepsilon_t. \tag{6.46}$$

A rise in the foreign country's inflation rate is expansionary for the domestic economy (see 6.43). The domestic policy authority lowers domestic inflation to partially stabilize domestic output. A parallel treatment of the foreign country policy authority's decision problem leads to the reaction function

$$\pi_t^* = -\left(\frac{\lambda b_2^2 A_1 A_2}{1 + \lambda b_2^2 A_1^2}\right)\pi_t - \left(\frac{\lambda b_2 A_1}{1 + \lambda b_2^2 A_1^2}\right)\varepsilon_t. \tag{6.47}$$

Jointly solving these two reaction functions for the Nash equilibrium inflation rates yields

$$\pi_{N,t} = \pi_{N,t}^* = -\left(\frac{\lambda b_2 A_1}{1 + \lambda b_2^2 A_1}\right)\varepsilon_t \equiv -\theta_N \varepsilon_t. \tag{6.48}$$

How does stabilization policy with noncoordinated policies compare with the coordinated policy response given in (6.45)? Since $A_1 < 1$,

$$|\theta_N| < |\theta_c|.$$

Policy responds less to the aggregate supply shock in the absence of coordination, and as a result, output fluctuates more:

$$y_{N,t} = y_{N,t}^* = \left(\frac{1}{1 + \lambda b_2^2 A_1}\right)\varepsilon_t > \left(\frac{1}{1 + \lambda b_2^2}\right)\varepsilon_t.$$

Because output and inflation responses are symmetric in the Nash equilibrium, the real exchange rate does not respond to ε_t.

Why does policy respond less in the absence of coordination? For each individual policy maker, the perceived marginal output gain from more inflation when there is an adverse realization of ε reflects the two channels through which inflation affects output. First, surprise inflation directly increases real output because of the assumption of nominal rigidities. This direct effect is given by the term $b_2(\pi_t - \mathrm{E}_{t-1}\pi_t)$ in (6.35). Second, for given foreign inflation, a rise in home inflation leads to a real depreciation (see 6.42) and, again from (6.35), the rise in ρ_t acts to lower output, reducing the net impact of inflation on output. With π^* treated as given, the exchange-rate channel implies that a larger inflation increase is necessary to offset the output effects of an adverse supply shock. Since inflation is costly, the optimal policy response involves a smaller inflation response and less output stabilization. With a coordinated policy, the decision problem faced by the policy authority recognizes that a symmetric increase in inflation in both countries leaves the real exchange rate unaffected. With inflation perceived to have a larger marginal impact on output, the optimal response is to stabilize more.

The loss functions of the two countries can be evaluated under the alternative policy regimes (coordination and noncoordination). Because the two countries have been specified symmetrically, the value of the loss function will be the same for each. For the domestic economy, the expected loss when policies are coordinated is equal to

$$L^c = \frac{1}{2}\left(\frac{1}{1 + \lambda b_2^2}\right)\lambda\sigma_\varepsilon^2.$$

When policies are determined in a Nash noncooperative equilibrium,

$$L^N = \frac{1}{2}\left[\frac{1 + \lambda b_2^2 A_1^2}{(1 + \lambda b_2^2 A_1)^2}\right]\lambda\sigma_\varepsilon^2.$$

Because $0 < A_1 < 1$, it follows that $L^c < L^N$; coordination achieves a better outcome than occurs in the Nash equilibrium.

This example appears to imply that coordination will always dominate noncoordination. It is important to recall that we considered the case in which the only source of disturbance was a common aggregate-supply shock. The case of asymmetric shocks is addressed in problem 4. But even when there are only common shocks, coordination need not always be superior. Rogoff (1985a) provides a counterexample. His argument is based on a model in which optimal policy is time inconsistent, a topic we will cover in chapter 8, but we can briefly describe the intuition behind Rogoff's results. A coordinated monetary expansion leads to a larger short-run real-output expansion because it avoids changes in the real exchange rate. But this fact increases the incentive to engineer a surprise monetary expansion if the policy makers believe the natural rate of output is too low. Wage and price setters will anticipate this tactic, together with the associated higher inflation. Equilibrium involves higher inflation, but because it has been anticipated, output (which depends on inflation surprises) does not increase. Consequently, coordination leads to better stabilization but higher average inflation. If the costs of the latter are high enough, noncoordination can dominate coordination.

The discussion of policy coordination serves to illustrate several important aspects of open-economy monetary economics. First, the simple framework introduced in this section provides a two-country model that closely corresponds to the type of closed-economy aggregate-supply, aggregate-demand model discussed in section 5.3.1. This framework has been used to analyze many policy topics. Second, the model incorporated several channels through which monetary policy can affect the real economy that are absent when the analysis is limited to a closed economy. The real exchange rate is the relative price of output in the two countries, so it plays an important role in equilibrating relative demand and supply in the two countries. Third, foreign shocks matter for the domestic economy; both aggregate-supply and aggregate-demand shocks originating in the foreign economy affect output in the domestic economy. As (6.43) and (6.44) show, however, the model implies that common demand shocks that leave $u - u^*$ unaffected have no effect on output levels or the real exchange rate. Because these shocks do affect demand in each country, a common demand shock raises real interest rates in each country. Fourth, policy coordination can matter.

While the two-country model of this section is useful, it has several omissions that may limit the insights that can be gained from its use. First, the aggregate-demand and aggregate-supply relationships are not derived explicitly within an optimizing framework. As we saw in chapter 5 and in the Obstfeld-Rogoff model, expectations of future income will play a role when consumption is determined by forward-looking, rational economic agents. Second, there is no role for current-account imbalances to affect equilibrium through their effects on foreign asset holdings. Third, no distinction has been drawn between the price of domestic output and the price index relevant for domestic residents. The loss function for the policy maker may depend on consumer price inflation. Fourth, the inflation rate was treated as the instrument of policy, directly controllable by the central bank. This is an obvious simplification, one that abstracts from the linkages (and slippages) between the

actual instruments of policy and the realized rate of inflation. Although such simplifications are useful for addressing many policy issues, chapters 9–11 will examine these linkages in more detail. Finally, the model, like the Obstfeld-Rogoff example, assumed one-period nominal contracts. Such a formulation fails to capture the persistence that generally characterizes actual inflation and the lags between changes in policy and the resulting changes in output and inflation.

6.4 The Small Open Economy

A two-country model provides a useful framework for examining policy interactions in an environment in which developments in one economy affect the other. For many economies, however, domestic developments have little or no impact on other economies. Decisions about policy can, in this case, treat foreign interest rates, output levels, and inflation as exogenous because the domestic economy is small relative to the rest of the world. The small open economy is a useful construct for analyzing issues when developments in the country of interest are unlikely to influence other economies.

In the small-open-economy case, the model of the previous section simplifies to become

$$y_t = -b_1\rho_t + b_2(p_t - \mathrm{E}_{t-1}p_t) + e_t \tag{6.49}$$

$$y_t = a_1\rho_t - a_2 r_t + u_t \tag{6.50}$$

$$\rho_t = r_t^* - r_t + \mathrm{E}_t\rho_{t+1}. \tag{6.51}$$

The real exchange rate ρ is equal to $s + p^* - p$, where s is the nominal exchange rate and p^* and p are the prices of foreign and domestic output, all expressed in log terms. The aggregate supply relationship has been written in terms of the unanticipated price level rather than unanticipated inflation.[21] The dependence of output on price surprises arises from the presence of nominal wage and price rigidities. With foreign income and consumption exogenous, the impact of world consumption on the domestic economy can be viewed as one of the factors giving rise to the disturbance term u_t.

Consumer prices in the domestic economy are defined as

$$q_t = hp_t + (1 - h)(s_t + p_t^*), \tag{6.52}$$

21. Since $p_t - \mathrm{E}_{t-1}p_t = p_t - p_{t-1} - (\mathrm{E}_{t-1}p_t - p_{t-1}) = \pi_t - \mathrm{E}_{t-1}\pi_t$, the two formulations are equivalent.

where h is the share of domestic output in the consumer price index, while the Fisher relationship links the real rate of interest appearing in (6.50) and (6.51) with the nominal interest rate,

$$r_t = i_t - \mathrm{E}_t p_{t+1} + p_t. \tag{6.53}$$

Uncovered interest parity links nominal interest rates. Since i^* will be exogenous from the perspective of the small open economy, we can write (6.51) as

$$i_t = \mathrm{E}_t s_{t+1} - s_t + i_t^*, \tag{6.54}$$

where $i^* = r^* + \mathrm{E}_t p_{t+1}^* - p_t^*$. Finally, real money demand is assumed to be given by

$$m_t - q_t = y_t - ci_t + v_t. \tag{6.55}$$

Notice that the basic structure of the model, like the closed-economy models of chapter 5 based on wage and/or price rigidity, displays the classical dichotomy between the real and monetary sectors if wages are flexible. That is, if wages adjust completely to equate labor demand and labor supply, the price-surprise term in (6.49) disappears.[22] In this case, (6.49)–(6.51) constitute a three-equation system for real output, the real interest rate, and the real exchange rate. Using the interest parity condition to eliminate r_t from the aggregate demand relationship, and setting the resulting expression for output equal to aggregate supply, yields the following equation for the equilibrium real exchange rate in the absence of nominal rigidities:

$$(a_1 + a_2 + b_1)\rho_t = a_2(r_t^* + \mathrm{E}_t\rho_{t+1}) + e_t - u_t.$$

This can be solved forward for ρ_t:

$$\rho_t = \sum_{i=0}^{\infty} d^i \mathrm{E}_t\left(\frac{a_2 r_{t+i}^* + e_{t+i} - u_{t+i}}{a_1 + a_2 + b_1}\right)$$

$$= d\sum_{i=0}^{\infty} d^i \mathrm{E}_t r_{t+i}^* + \frac{e_t - u_t}{a_1 + a_2 + b_1},$$

where $d \equiv a_2/(a_1 + a_2 + b_1) < 1$ and the second equals sign follows from the assumption that e and u are serially uncorrelated processes. The real exchange rate

22. Recall from chapter 5 that the assumption behind an aggregate-supply function such as (6.49) is that nominal wages are set in advance on the basis of expectations of the price level, while actual employment is determined by firms on the basis of realized real wages (and therefore on the actual price level).

responds to excess supply for domestic output; if $e_t - u_t > 0$, a real depreciation increases aggregate demand and lowers aggregate supply to restore goods market equilibrium.

The monetary sector consists of (6.52)–(6.55), plus the definition of the nominal exchange rate as $s_t = \rho_t - p_t^* + p_t$. When wages and prices are flexible, these determine the two price levels p (the price of domestic output) and q (the consumer price index). From the Fisher equation, the money demand equation, and the definition of q_t,

$$m_t - p_t = y_t + (1-h)\rho_t - c(r_t + \mathrm{E}_t p_{t+1} - p_t) + v_t.$$

Because the real values are exogenous with respect to the monetary sector when there are no nominal rigidities, this equation can be solved for the equilibrium value of p_t:

$$p_t = \left(\frac{1}{1+c}\right)\sum_{i=0}^{\infty}\left(\frac{c}{1+c}\right)^i \mathrm{E}_t(m_{t+i} - z_{t+i} - v_{t+i}),$$

where $z_{t+i} \equiv y_{t+i} + (1-h)\rho_{t+i} - cr_{t+i}$. The equilibrium p_t depends not just on the current money supply, but also on the expected future path of m. Since (6.52) implies $p_t = q_t - (1-h)\rho_t$, the equilibrium behavior of the domestic consumer price index q_t follows from the solutions for p_t and ρ_t.

When nominal wages are set in advance, the classical dichotomy no longer holds. With $p_t - \mathrm{E}_{t-1}p_t$ affecting the real wage, employment, and output, any disturbance in the monetary sector that was unanticipated will affect output, the real interest rate, and the real exchange rate. Since the model does not incorporate any mechanism to generate real persistence, these effects last only for one period.

With nominal wage rigidity, monetary policy affects real aggregate demand through both interest-rate and exchange-rate channels. As can be seen from (6.50), these two variables appear in the combination $a_1\rho_t - a_2 r_t$. For this reason, the interest rate and exchange rate are often combined to create a *monetary conditions index*; in the context of the present model, this index would be equal to $r_t - a_1\rho_t/a_2$. Variations in the real interest rate and real exchange rate that leave this linear combination unchanged would be neutral in their impact on aggregate demand, since the reduction in domestic aggregate demand caused by a higher real interest rate would be offset by a depreciation in the real exchange rate.

6.4.1 Flexible Exchange Rates

Suppose that nominal wages are set in advance, but the nominal exchange rate is free to adjust flexibly in the face of economic disturbances. In addition, assume that monetary policy is implemented through control of the nominal money supply. In this case, the model consisting of (6.49)–(6.54) can be reduced to two equations involving the price level, the nominal exchange rate, and the nominal money supply (see the appendix for details). Equilibrium will depend on expectations of the period $t+1$ exchange rate, and the response of the economy to current policy actions may depend on how these expectations are affected.

To determine how the exchange rate and the price level respond to monetary shocks, we will assume a specific process for the nominal money supply. To allow for a distinction between transitory and permanent monetary shocks, assume

$$m_t = \mu + m_{t-1} + \varphi_t - \gamma\varphi_{t-1}, \quad 0 \le \gamma \le 1, \tag{6.56}$$

where φ is a serially uncorrelated white noise process. If $\gamma = 0$, m_t follows a random walk with drift μ; innovations φ have a permanent impact on the level of m. If $\gamma = 1$, the money supply is white noise around a deterministic trend. If $0 < \gamma < 1$, a fraction $(1-\gamma)$ of the innovation has a permanent effect on the level of the money supply.

To analyze the impact of foreign price shocks on the home country, let

$$p_t^* = \pi^* + p_{t-1}^* + \phi_t, \tag{6.57}$$

where ϕ is a random white noise disturbance. This allows for a constant average foreign inflation rate of π^* with permanent shifts in the price path due to the realizations of ϕ.

Using the method of undetermined coefficients, the appendix shows that the following solutions for p_t and s_t are consistent with (6.49)–(6.54), and with rational expectations:

$$p_t = k_0 + m_{t-1} + \frac{B_2[1 + c(1-\gamma)]}{K}\varphi_t - \gamma\varphi_{t-1} + \frac{[(A_2 - B_2)u_t - A_2 e_t - B_2 v_t]}{K} \tag{6.58}$$

$$s_t = d_0 + m_{t-1} - p_{t-1}^* - \phi_t - \frac{B_1[1 + c(1-\gamma)]}{K}\varphi_t - \gamma\varphi_{t-1} + \frac{[(B_1 - A_1)u_t + A_1 e_t + B_1 v_t]}{K}, \tag{6.59}$$

where $A_1 = h - a_1 - a_2$, $A_2 = 1 + c - A_1 > 0$, $B_1 = -(a_1 + a_2 + b_1 + b_2) < 0$, $B_2 = a_1 + a_2 + b_1 > 0$, and $K = -[(1+c)B_1 + b_2 A_1]$. The constant k_0 is given by

$$k_0 = (1+c)\mu + \left[c - \frac{a_2(1 - h - b_1)}{a_1 + b_1}\right] r^*,$$

and $d_0 = k_0 - \pi^*$.

Of particular note is the way a flexible exchange rate insulates the domestic economy from the foreign price shock ϕ. Neither p_{t-1}^* nor ϕ_t affects the domestic price level under a flexible exchange-rate system (see 6.58). Instead, (6.59) shows how they move the nominal exchange rate to maintain the domestic currency price of foreign goods, $s + p^*$, unchanged. This insulates the real exchange rate and domestic output from fluctuations in the foreign price level.

With $B_2[1 + c(1-\gamma)]/K > 0$ and $-B_1[1 + c(1-\gamma)]/K > 0$, a positive monetary shock increases the equilibrium price level and the nominal exchange rate. That is, the domestic currency depreciates in response to a positive money shock. The effect is offset partially in the following period if $\gamma > 0$. The shape of the exchange rate response to a monetary shock is shown in figure 6.1 for different values of γ.

The $\gamma < 1$ cases in figure 6.1 illustrate Dornbusch's overshooting result (Dornbusch 1976). To the extent that the rise in m is permanent (i.e., $\gamma < 1$), the price level

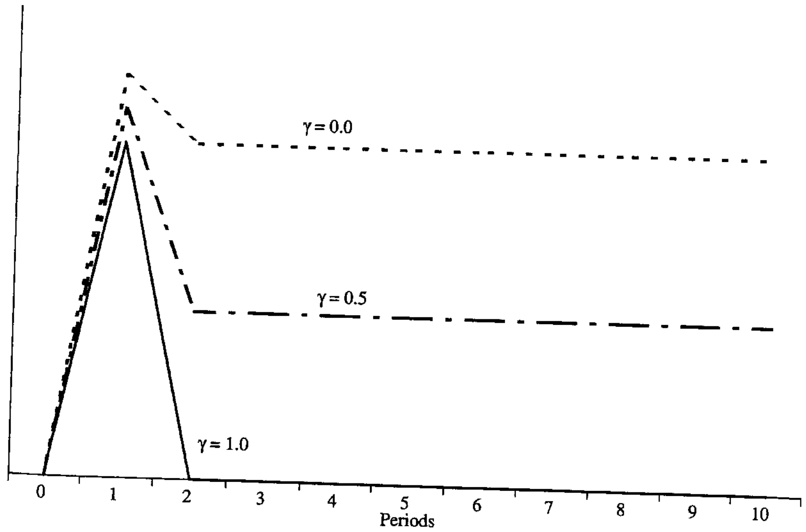

Figure 6.1
Nominal Exchange Rate Response to a Monetary Shock

and the nominal exchange rate eventually rise proportionately. With one-period nominal rigidities, this occurs in period 2. A rise in the nominal money supply that increases the real supply of money reduces the nominal interest rate to restore money market equilibrium. From the interest parity condition, the domestic nominal rate can fall only if the exchange rate is expected to fall. Yet the exchange rate will be higher than its initial value in period 2, so to generate an expectation of a fall, s must rise more than proportionately to the permanent rise in m. It is then expected to fall from period 1 to period 2; the nominal rate overshoots its new long-run value.

The Dornbusch overshooting result stands in contrast to Obstfeld and Rogoff's conclusion, derived in section 6.2.3, that a permanent change in the nominal money supply does not lead to overshooting. Instead, the nominal exchange rate jumps immediately to its new long-run level. This difference results from the ad hoc nature of aggregate demand in the model of this section. In the Obstfeld-Rogoff model, consumption is derived from the decision problem of the representative agent. As we saw in section 6.2, the Euler condition for consumption linked consumption choices over time. The desire to smooth consumption implies that consumption immediately jumps to its new equilibrium level. As a result, exchange-rate overshooting is eliminated in the basic Obstfeld-Rogoff model.

One implication of the overshooting hypothesis is that exchange-rate movements should follow a predictable or forecastable pattern in response to monetary shocks. A positive monetary shock leads to an immediate depreciation followed by an appreciation. The path of adjustment will depend on the extent of nominal rigidities in the economy, since these influence the speed with which the economy adjusts in response to shocks. Such a predictable pattern is not clearly evident in the data. In fact, nominal exchange rates display close to random-walk behavior over short time periods (Meese and Rogoff 1983). In a VAR-based study of exchange-rate responses to U.S. monetary shocks, Eichenbaum and Evans (1995) do not find evidence of overshooting, but they do find sustained and predictable exchange-rate movements following monetary-policy shocks. A monetary contraction produces a small initial appreciation, with the effect growing so that the dollar appreciates for some time. However, in a study based on more direct measurement of policy changes, Bonser-Neal, Roley, and Sellon (1998) find general support for the overshooting hypothesis. They measure policy changes by using data on changes in the Federal Reserve's target for the federal funds rate, rather than the actual funds rate, and restrict attention to time periods during which the funds rate was the Fed's policy instrument.

6.4.2 Fixed Exchange Rates

Under a system of fixed exchange rates, the monetary authority is committed to using its policy instrument to maintain a constant nominal exchange rate. This commitment requires that the monetary authority stand ready to buy or sell domestic currency for foreign exchange to maintain the fixed exchange rate. When it is necessary to sell foreign exchange, such a policy will be unsustainable if the domestic central bank's reserves of foreign exchange are expected to go to zero. Such expectations can produce speculative attacks on the currency.[23] In our analysis here, we will deal only with the case of a sustainable fixed rate. And to draw the sharpest contrast with the flexible-exchange-rate regime, we will assume that the exchange rate is pegged. In practice, most fixed-exchange-rate regimes allow rates to fluctuate within narrow bands.[24]

Normalizing the fixed rate at $s_t = 0$ for all t, the real exchange rate equals $p_t^* - p_t$. Assuming the foreign price level follows (6.57), the basic model becomes

$$y_t = -b_1(p_t^* - p_t) + b_2(p_t - \mathrm{E}_{t-1}p_t) + e_t$$

$$y_t = a_1(p_t^* - p_t) - a_2 r_t + u_t$$

$$r_t = r^* + \pi^* - (\mathrm{E}_t p_{t+1} - p_t).$$

The nominal interest rate has been eliminated, since the interest-parity condition and the fixed-exchange-rate assumption imply that $i = r^* + \pi^*$. These three equations can be solved for the price level, output, and the domestic real interest rate. The money demand condition plays no role because m must endogenously adjust to maintain the fixed exchange rate.

Solving for p_t,

$$p_t = \frac{(a_1 + b_1)p_t^* - a_2(r^* + \pi^*) + a_2\mathrm{E}_t p_{t+1} + b_2\mathrm{E}_{t-1}p_t + u_t - e_t}{a_1 + a_2 + b_1 + b_2}.$$

Using the method of undetermined coefficients, one obtains

$$p_t = p_t^* - \frac{a_2 r^*}{a_1 + b_1} + \frac{u_t - e_t - b_2\phi_t}{a_1 + a_2 + b_1 + b_2}. \tag{6.60}$$

Comparing (6.60) to (6.58) reveals some of the major differences between the fixed and flexible exchange-rate systems. Under fixed exchange rates, the average

23. See Krugman (1979) and Garber and Svensson (1995).

24. Exchange-rate behavior under a target zone system was first analyzed by Krugman (1991).

domestic rate of inflation must equal the foreign inflation rate: $E(p_{t+1} - p_t) = E(p_{t+1}^* - p_t^*) = \pi^*$. The foreign price level and foreign price shocks (ϕ) affect domestic prices and output under the fixed-rate system. But domestic disturbances to money demand or supply (φ and v) have no price level or output effects. This situation is in contrast to the case under flexible exchange rates and is one reason why high-inflation economies often attempt to fix their exchange rates with low inflation countries. But when world inflation is high, a country can maintain lower domestic inflation only by allowing its nominal exchange rate to adjust.

The effects on real output of aggregate demand and supply disturbances also depend on the nature of the exchange-rate system. Under flexible exchange rates, a positive aggregate-demand shock increases prices and real output. Goods-market equilibrium requires a rise in the real interest rate and a real appreciation. By serving to equilibrate the goods market and partially offset the rise in aggregate demand following a positive realization of u, the exchange-rate movement helps stabilize aggregate output. As a result, the effect of u on y is smaller under flexible exchange rates than under fixed exchange rates.[25]

The choice of exchange-rate regime influences the manner in which economic disturbances affect the small open economy. While the model examined here does not provide an internal welfare criterion (such as the utility of the representative agent in the economy), such models have often been supplemented with loss functions depending on output or inflation volatility (such as we used in section 6.3.2 to study policy coordination), which are then used to rank alternative exchange-rate regimes. Based on such measures, the choice of an exchange-rate regime should depend on the relative importance of various disturbances. If volatility of foreign prices is of major concern, a flexible exchange rate will serve to insulate the domestic economy from real exchange-rate fluctuations that would otherwise affect domestic output and prices. If domestic monetary instability is a source of economic fluctuations, a fixed-exchange-rate system provides an automatic monetary response to offset such disturbances.

The role of economic disturbances in the choice of a policy regime is an important topic of study in monetary policy analysis. It figures most prominently in discussion of the choice between using an interest rate or a monetary aggregate as the instrument of monetary policy. This topic forms the major focus of chapter 9.

25. This is consistent with the estimates for Japan reported in Hutchison and Walsh (1992). Obstfeld (1985) discusses the insulation properties of exchange rate systems.

6.5 Open-Economy Models with Optimizing Agents and Nominal Rigidities

Chapter 5 developed a model based on optimizing households and firms but in which prices were sticky. This model could be summarized in terms of an expectational IS curve, relating aggregate output to expected future output and the real interest rate, and an inflation adjustment relationship, in which current inflation was a function of expected future inflation and real marginal cost. Real marginal cost was a function of output relative to the flexible-price equilibrium level of output. The model was closed by assuming that the monetary authority determined the nominal rate of interest. In recent years, a number of authors have extended this basic model framework to the open-economy context. Examples include McCallum and Nelson (2000b); Clarida, Galí, and Gertler (2001, 2002); Gertler, Gilchrist, and Natalucci (2001); and Monacelli (2002). These models differ from the models discussed in section 6.2 in that an inflation-adjustment model based on Calvo (1983) replaces the assumption of one-period nominal rigidity. In this section, the focus is on models that are directly in the new Keynesian tradition of the closed-economy models studied in chapter 5. The basic structure of the model is based on Clarida, Galí, and Gertler (2001, 2002).

6.5.1 A Basic Open-Economy Model

Suppose there are two countries. The home country is denoted by the superscript h, and the superscript f denotes the foreign country. The countries share the same preferences and technologies. Both produce traded consumption goods which are imperfect substitutes in utility.

Households Households consume a CES composite of home and foreign goods, defined as

$$C_t = \left[(1-\gamma)^{\frac{1}{a}}(C_t^h)^{\frac{a-1}{a}} + \gamma^{\frac{1}{a}}(C_t^f)^{\frac{a-1}{a}}\right]^{\frac{a}{a-1}} \tag{6.61}$$

for $a > 1$. The domestic household's relative demand for C^h and C^f will depend on their relative prices. Given the CES specification for preferences,[26]

$$\frac{C_t^h}{C_t^f} = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{P_t^h}{P_t^f}\right)^{-a}. \tag{6.62}$$

26. This is obtained by minimizing the cost $P_t^h C_t^h + P_t^f C_t^f$ of achieving a given level of C_t.

Household utility depends on its consumption of the composite good and on its labor supply. Assume that

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}. \tag{6.63}$$

Intertemporal optimization implies the standard Euler condition,

$$C_t^{-\sigma} = \beta \mathrm{E}_t R_t \left(\frac{P_t^c}{P_{t+1}^c}\right) C_{t+1}^{-\sigma}, \tag{6.64}$$

where R_t is the (gross) nominal rate of interest and P_t^c is an aggregate consumer price index, defined as

$$P_t^c = [(1-\gamma)(P_t^h)^{1-a} + \gamma(P_t^f)^{1-a}]^{\frac{1}{1-a}}, \tag{6.65}$$

where P_t^h (P_t^f) is the average price of domestically (foreign) produced consumption goods.

Optimal labor-leisure choice requires that the marginal rate of substitution between leisure and consumption equal the real wage. This condition takes the form

$$\frac{N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t^c}, \tag{6.66}$$

where W_t is the nominal wage.

Assume that the law of one price holds. This implies that

$$P_t^f = S_t P_t^*, \tag{6.67}$$

where P_t^* is the foreign currency price of foreign-produced goods and S_t is the nominal exchange rate (price of foreign currency in terms of domestic currency). For simplicity, assume that all foreign goods sell for the price P_t^f. This specification assumes complete exchange rate pass-through. It will be useful to define the terms of trade, the relative price of foreign and domestic goods:

$$\Delta_t \equiv \frac{P_t^f}{P_t^h} = \frac{S_t P_t^*}{P_t^h}.$$

As was the case with the new Keynesian model of chapter 5, the focus will be on percentage deviations around the steady state. Let lowercase letters denote percentage deviation around the steady state of the corresponding uppercase letter. Then, using the definition of the terms of trade,

$$p_t^c = (1-\gamma)p_t^h + \gamma p_t^f = p_t^h + \gamma\delta_t, \tag{6.68}$$

and (6.61)–(6.66) can be written as

$$c_t = (1-\gamma)c_t^h + \gamma c_t^f \tag{6.69}$$

$$c_t^f = -a\delta_t + c_t^h \tag{6.70}$$

$$c_t = \mathrm{E}_t c_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1}^c) \tag{6.71}$$

and

$$\eta n_t + \sigma c_t = w_t - p_t^h - \gamma\delta_t, \tag{6.72}$$

where $\pi_t^c = p_t^c - p_{t-1}^c$. Combining (6.69) and (6.70), $c_t = c_t^h - \gamma a \delta_t$. Defining π_t^h as $p_t^h - p_{t-1}^h$,

$$\pi_t^c = p_t^c - p_{t-1}^c = \pi_t^h + \gamma(\delta_t - \delta_{t-1}). \tag{6.73}$$

Note that the real interest rate is defined in terms of consumer price inflation π_t^c. In turn, this measure of inflation depends on the rate of inflation of domestically produced goods π_t^h and the rate of change in the terms of trade.

Clarida, Galí, and Gertler (2001) add a stochastic *wage markup* μ_t^w to (6.72) to represent deviations from the marginal rate of substitution between leisure and consumption:

$$\eta n_t + \sigma c_t + \mu_t^w = w_t - p_t^h - \gamma\delta_t. \tag{6.74}$$

They motivate this markup as arising from the monopoly power of labor suppliers who set wages as a markup over the marginal rate of substitution. The markup is then assumed to be subject to exogenous stochastic variation.

Domestic Firms The analysis of domestic firms parallels the approach followed in section 5.4, and further details can be found there.

In each period, there is a fixed probability $1-\omega$ that the firm can adjust its price. When it can adjust, it does so to maximize the expected discounted value of profits. Each domestic firm faces the identical production function,

$$Y_t^h = e^{\varepsilon_t} N_t,$$

and a constant elasticity demand curve for its output. The firm's real marginal cost is equal to

$$MC_t = \frac{W_t / P_t^h}{e^{\varepsilon_t}},$$

where W_t/P_t^h is the real product wage and e^{ε_t} is the marginal product of labor. In terms of percentage deviations around the steady state, this expression becomes

$$mc_t = w_t - p_t^h - \varepsilon_t. \tag{6.75}$$

Following the derivation of chapter 5, the inflation rate for the price index of domestically produced goods is

$$\pi_t^h = \beta \mathrm{E}_t \pi_{t+1}^h + \kappa mc_t, \tag{6.76}$$

where $\kappa = (1-\omega)(1-\beta\omega)/\omega$. Combining (6.76) with (6.73),

$$\pi_t^c = \beta \mathrm{E}_t \pi_{t+1}^c + \kappa mc_t - \beta\gamma(\mathrm{E}_t\delta_{t+1} - \delta_t) + \gamma(\delta_t - \delta_{t-1}). \tag{6.77}$$

Inflation, as measured by the consumer price index, depends on expected future inflation and real marginal cost, the same two factors that appeared in the closed-economy inflation adjustment equation. In the open economy, the current and expected future changes in the terms of trade also affect inflation. A decline in the terms of trade ($\delta_t - \delta_{t-1} < 0$) implies a decrease in the relative price of foreign goods. Because foreign goods prices are included in the consumer price index (see 6.68), a fall in their relative prices reduces inflation. An expected future appreciation also reduces current consumer price inflation since, for a given $E_t\pi_{t+1}^c$, this rise in the terms of trade must imply a fall in expected future domestic goods inflation and therefore a fall in current domestic goods inflation.

The Foreign Country To keep the analysis simple, assume that the foreign country is large relative to the home country. This is taken to mean that it is unnecessary to distinguish between consumer price inflation and domestic inflation in the foreign country, and that domestic output and consumption are equal. Goods produced in the home country are sold to domestic residents and to foreigners. Let c_t^{h*} be the foreign country's consumption of the domestically produced good (as a percentage deviation from the steady state). The foreign country's demand for the home country's output depends on the terms of trade. Assuming that foreign households have the same preferences as those of the home country (so the demand elasticity is the same),

$$c_t^{h*} = a\delta_t + y_t^f, \tag{6.78}$$

where y_t^* is foreign income. The Euler condition for foreign country households implies

$$y_t^f = \mathrm{E}_t y_{t+1}^f - \left(\frac{1}{\sigma}\right)(i_t^f - \mathrm{E}_t\pi_{t+1}^f),$$

or

$$\rho_t^f \equiv i_t^f - E_t\pi_{t+1}^f = \sigma(\mathrm{E}_t y_{t+1}^f - y_t^f). \tag{6.79}$$

Equilibrium Conditions Equilibrium requires that production equal consumption. For domestically produced output, this requires that

$$y_t = (1-\gamma)c_t^h + \gamma c_t^{h*}. \tag{6.80}$$

In addition, uncovered interest parity implies

$$i_t = i_t^f + \mathrm{E}_t s_{t+1} - s_t,$$

or in real terms,

$$i_t - \mathrm{E}_t\pi_{t+1}^h = \rho_t^f + (\mathrm{E}_t\delta_{t+1} - \delta_t).$$

Using (6.78) and the earlier result that $c_t = c_t^h - \gamma a\delta_t$, (6.80) can be rewritten as

$$y_t = (1-\gamma)c_t + (2-\gamma)\gamma a\delta_t + \gamma y_t^f.$$

This relationship between domestic output and consumption can be used to eliminate c_t from the Euler condition (6.71), yielding

$$y_t = \mathrm{E}_t y_{t+1} - (2-\gamma)\gamma a(\mathrm{E}_t\delta_{t+1} - \delta_t) - \gamma\mathrm{E}_t(y_{t+1}^f - y_t^f) - \left(\frac{1-\gamma}{\sigma}\right)(i_t - \mathrm{E}_t\pi_{t+1}^c).$$

Making use of (6.73) and the uncovered interest parity condition, one obtains, after some manipulation,

$$y_t = \mathrm{E}_t y_{t+1} - \left(\frac{1+w}{\sigma}\right)\left[i_t - \mathrm{E}_t\pi_{t+1}^h - \left(\frac{w}{1+w}\right)\rho_t^f\right], \tag{6.81}$$

where $w = \gamma(\sigma a - 1)(2-\gamma)$.

6.5.2 The Flexible-Price Equilibrium

Assume that the productivity and wage markup disturbances are mean zero, white noise disturbances. Let z_t^o denote the flexible-price equilibrium value of a variable z_t (still expressed as percentage deviations around the steady state). Then the flexible-price equilibrium satisfies

$$y_t^o = c_t^o \tag{6.82}$$

$$\varepsilon_t = (\sigma+\eta)y_t^o - \eta\varepsilon_t + \gamma\delta_t^o + \mu_t^w \tag{6.83}$$

$$\rho_t^o = r_t - \mathrm{E}_t\pi_{t+1}^h = \rho_t^f + \mathrm{E}_t\delta_{t+1}^o - \delta_t^o \tag{6.84}$$

$$\rho_t^o = \left(\frac{\sigma}{1+w}\right)(\mathrm{E}_t y_{t+1}^o - y_t^o) + \left(\frac{w}{1+w}\right)\rho_t^f. \tag{6.85}$$

Equation (6.83) is the labor market equilibrium condition, equating, in percentage deviations terms, the marginal product of labor to the marginal rate of substitution between leisure and consumption. It follows from (6.82), the aggregate production relationship $y_t = n_t + \varepsilon_t$, and (6.72). Equation (6.85) for the domestic real interest rate in the flexible-price equilibrium is obtained from the Euler condition (6.81).

Because of the earlier assumption of white noise disturbances, set all expected future values equal to zero.[27] From (6.84), $\delta_t^o = \rho_t^f - \rho_t^o$; the terms of trade are equal to the real interest-rate differential. Combining the last three equilibrium conditions and noting that all future expected values are zero,

$$\begin{aligned}\delta_t^o &= \left(\frac{\sigma}{1+w}\right)[(\mathrm{E}_t y_{t+1}^f - y_t^f) - (\mathrm{E}_t y_{t+1}^o - y_t^o)] \\ &= \left(\frac{\sigma}{1+w}\right)(y_t^o - y_t^f).\end{aligned}$$

Using this expression for δ_t^o to eliminate it from the labor-market equilibrium condition equation (6.83), the flexible-price equilibrium output is equal to

$$y_t^o = \frac{(1+\eta)\varepsilon_t - \mu_t^w + \left(\frac{\gamma\sigma}{1+w}\right)y_t^f}{\sigma + \eta + \left(\frac{\gamma\sigma}{1+w}\right)}.$$

6.5.3 Deviations from the Flexible-Price Equilibrium

When prices are sticky, output and the terms of trade can differ from their flexible-price equilibrium values. In the closed-economy model of chapter 5, the model of households and firms could be reduced to a two-equation system (the expectational IS equation and the inflation-adjustment equation) expressed in terms of inflation and the gap between output and the flexible-price equilibrium output. As we will see, the open-economy model can also be expressed in a similar form.

27. We have not yet specified the behavior of the monetary authority in setting the nominal interest rate; our assumption here is that no dynamics that would make expected future values nonzero are introduced by policy.

Define the output gap x_t as

$$x_t \equiv y_t - y_t^o.$$

Real marginal cost, given by (6.75), is equal to the gap between the real product wage and the marginal product of labor. When prices are sticky, the real wage can deviate from the marginal product of labor, but with flexible wages, the real consumption wage is still equal to the marginal rate of substitution between leisure and consumption. Thus, using (6.74),

$$mc_t = [(\sigma+\eta)y_t - \eta\varepsilon_t + \gamma\delta_t + \mu_t^w] - \varepsilon_t.$$

In the flexible-price equilibrium, the marginal product of labor, ε_t, is equal to $(\eta+\sigma)y_t^o - \eta\varepsilon_t + \gamma\delta_t^o + \mu_t^w$ (see 6.83). Hence,

$$\begin{aligned}mc_t &= [(\sigma+\eta)y_t - \eta\varepsilon_t + \gamma\delta_t + \mu_t^w] - [(\sigma+\eta)y_t^o - \eta\varepsilon_t + \gamma\delta_t^o + \mu_t^w] \\ &= (\sigma+\eta)x_t + \gamma(\delta_t - \delta^o).\end{aligned}$$

The terms of trade deviation $\delta_t - \delta_t^o$ can be related to the output gap using the Euler condition and the interest parity condition:

$$\delta_t - \delta_t^o = \left(\frac{\sigma}{1+w}\right)x_t. \tag{6.86}$$

Substituting this expression for $\delta_t - \delta_t^o$ into the previous expression for marginal cost and using (6.76) implies that domestic inflation is[28]

$$\pi_t^h = \beta\mathrm{E}_t\pi_{t+1}^h + \kappa\left[\sigma + \eta + \left(\frac{\gamma\sigma}{1+w}\right)\right]x_t. \tag{6.87}$$

From (6.81),

$$\begin{aligned}x_t &= \mathrm{E}_t x_{t+1} - \left(\frac{1+w}{\sigma}\right)\left[i_t - \mathrm{E}_t\pi_{t+1}^h - \left(\frac{w}{1+w}\right)\rho_t^f\right] + \mathrm{E}_t y_{t+1}^o - y_t^o \\ &= \mathrm{E}_t x_{t+1} - \left(\frac{1+w}{\sigma}\right)[i_t - \mathrm{E}_t\pi_{t+1}^h - \rho_t^o], \qquad (6.88)\end{aligned}$$

where ρ_t^o is the equilibrium real interest rate under flexible prices, given by (6.85).

28. Clarida, Galí, and Gertler (2002) assume that the stochastic wage markup μ_t^w does not affect the flexible-price equilibrium, so their expression for inflation includes a term involving μ_t^w.

Equations (6.86)–(6.88), when combined with a policy rule for determining the nominal rate of interest, provide a simple model of a small open economy with a flexible exchange rate and sticky domestic goods prices. Interestingly, disturbances originating from fluctuations in foreign income or interest rates only affect demand in the domestic economy via the real interest rate term ρ_t^o. In fact, when ρ_t^o is viewed as an exogenous demand disturbance, (6.87) and (6.88) are essentially identical to the closed-economy, new Keynesian model of chapter 5.

There are only two differences from the closed-economy model. First, the presence of the parameter w implies a larger real interest-rate impact on demand in the open economy. This effect is due to the impact of the interest rate on the terms of trade. From the interest parity condition, a rise in the domestic real interest rate causes an appreciation (a fall in the terms of trade). This acts to shift demand away from domestically produced goods and toward foreign goods. This shift strengthens the impact of the rise in the real interest rate on domestic demand. Second, the relationship between marginal cost and the output gap variable depends on γ and w. A rise in the gap increases δ_t (see 6.86). This leads to an increase in consumer prices relative to domestic producer prices and increases the real wage faced by domestic producers. This channel then increases the impact of x_t on current inflation.

6.5.4 Extensions

The model of the previous section assumed that foreign-produced goods were consumption goods. These goods entered directly into the household's utility function. The presence of imported consumption goods meant that it was necessary to distinguish between the consumer price index and the producer price index. Changes in the terms of trade created a wedge between the behavior of these two price indexes. An alternative approach, developed by McCallum and Nelson (2000b), makes the assumption that foreign imported goods serve as producer inputs. All consumer goods are domestically produced with labor and imported inputs. Treating imports as inputs simplifies the model somewhat, as there is now only a single price index; consumer and producer prices are the same. Again, however, the model reduces to a structure that differs from a closed-economy model only in the values of some of the coefficients.

Adolfson (2001), Corsetti and Presenti (2002), and Monacelli (2002) provide examples of models that allow for incomplete pass-through. When pass-through is incomplete, the law of one price no longer holds. The law of one price allows the domestic currency price of foreign goods, p_t^f, to be expressed as $s_t + p_t^*$, where s_t is the nominal exchange rate and p_t^* is the foreign currency price of foreign goods (all expressed as percentage deviations from their steady-state values). The terms of trade are then equal to $s_t + p_t^* - p_t^h$ under the law of one price. With incomplete pass-though, however, p_t^f and $s_t + p_t^*$ can differ. Following Monacelli (2002), the real exchange rate q_t can be written as[29]

$$q_t = s_t + p_t^* - p_t^c = \psi_t + (1-\gamma)\delta_t,$$

where $\psi_t = s_t + p_t^* - p_t^f$ measures the deviation from the law of one price. In the model of the previous section, the law of one price held, so ψ_t was identically equal to zero. Suppose pass-through is incomplete because of nominal rigidity in the price of imports, with only a fraction of importers adjusting their price each period, as in a standard Calvo-type model of price adjustment. Then the variable ψ_t represents the marginal cost of importers, and Monacelli shows that the rate of inflation in the average domestic currency price of foreign imports takes the form

$$\pi_t^f = \beta \mathrm{E}_t \pi_{t+1}^f + \kappa^f \psi_t,$$

where $\pi_t^f = p_t^f - p_{t-1}^f$ and the parameter κ^f depends on the fraction of import prices that adjust each period.

Following the earlier notation by letting z_t^o denote the flex-price equilibrium, Monacelli (2002) shows that the output gap can be written as

$$x_t = \left(\frac{1+w}{\sigma}\right)(\delta_t - \delta_t^o) + \left[\frac{1+\gamma(\sigma a - 1)}{\sigma}\right](\psi_t - \psi_t^o),$$

where $w = \gamma(\sigma a - 1)(2-\gamma)$ as before. The real marginal cost of domestic firms can then be expressed as

$$mc_t - mc_t^o = \left(\frac{\sigma}{1+w} + \eta\right)x_t + \left(1 - \frac{1+\gamma(\sigma a - 1)}{1+w}\right)(\psi_t - \psi_t^o).$$

Inflation in domestic producer prices is

$$\pi_t^h = \beta \mathrm{E}_t \pi_{t+1}^h + \kappa(mc_t - mc_t^o).$$

Thus, deviations from the law of one price, as measured by $\psi_t - \psi_t^o$, directly affect inflation through their impact on marginal cost.

Clarida, Galí, and Gertler (2002) relax the assumption that the foreign country is large and examine the role of policy coordination. To carry out this examination,

29. This uses (6.68), which defines the consumer price index as $(1-\gamma)p_t^h + \gamma p_t^f$ and the definition of the terms of trade, $\delta_t = p_t^f - p_t^h$.

assume that both countries are subject to nominal price rigidities. In this case, (6.86) is modified to become

$$\tilde{\delta}_t = \left(\frac{\sigma}{1+w}\right)(\tilde{x}_t - \tilde{x}_t^f),$$

where $\tilde{\delta}_t$, $\tilde{x}_t$, and $\tilde{x}_t^f$ are defined as gaps relative to the outcome when prices are flexible in *both* countries. Using this equation to derive real marginal cost, Clarida, Galí, and Gertler find that

$$\pi_t^h = \beta \mathrm{E}_t \pi_{t+1}^h + \kappa \tilde{x}_t + \kappa \tilde{x}_t^f$$

for domestic goods inflation in the home country, with inflation in the foreign country satisfying a similar equation. The spillover effects of the output gap on inflation in the other country give rise, in general, to gains from policy coordination.

6.6 Summary

This chapter has reviewed various models that are useful for studying aspects of open-economy monetary economics. A two-country model whose equilibrium conditions were consistent with optimizing agents was presented. This model, based on the work of Obstfeld and Rogoff, preserved the classical dichotomy between real and monetary factors when prices and wages were assumed to be perfectly flexible. In this case, the price level and the nominal exchange rate could be expressed simply in terms of the current and expected future paths of the nominal money supplies in the two countries.

Two primary lessons are that new channels by which monetary factors affect the economy are present in an open economy, and the choice of exchange-rate regime has important implications for the role of monetary policy. With sticky nominal wages, monetary factors have important short-run effects on the real exchange rate. Exchange-rate movements alter the relative price of domestic and foreign goods, leading to impacts on aggregate demand and supply. In addition, consumer prices, because they are indices of domestic currency prices of domestically produced and foreign-produced goods, will respond to exchange-rate movements.[30] While the models of this chapter have not addressed the issue of inflation persistence, the impact of exchange-rate movements on consumer price inflation suggests that monetary policy may have a quicker effect on inflation in more open economies.

30. Duguay (1994) provides evidence on these channels in the case of Canada.

Unlike the analysis of the closed economy, the Obstfeld-Rogoff model implied that monetary-induced output movements would have persistent real effects by altering the distribution of wealth between economies. In general, standard open-economy frameworks used for policy analysis assume that the real effects of monetary policy arise only in the short run and are due to nominal rigidities. Over time, as wages and prices adjust, real output, real interest rates, and the real exchange rate return to equilibrium levels that are independent of monetary policy. This long-run neutrality means that these models, like their closed-economy counterparts, imply that the long-run effects of monetary policy fall on prices, inflation, nominal interest rates, and the nominal exchange rate. One implication is that the rate of inflation is an appropriate long-run objective for monetary policy, while the growth rate of real output and the level of the real exchange rate are not. In the short run, however, monetary policy can have important effects on the manner in which real output and the real exchange rate fluctuate around their longer-run equilibrium values.

The final section of the chapter reviewed some extensions of new Keynesian models to open-economy settings. In some cases, these models resulted in equilibrium reduced-form equations for the output gap and inflation that were identical in form to the closed-economy equivalents.

6.7 Appendix

6.7.1 The Obstfeld-Rogoff Model

This appendix provides a derivation of some of the components of the Obstfeld-Rogoff (1995, 1996) model.

Individual Product Demand The demand functions faced by individual producers are obtained as the solution to the following problem:

$$\max \left[\int_0^1 c(z)^q \, dz\right]^{\frac{1}{q}} \quad \text{subject to} \quad \int_0^1 p(z)c(z)\, dz = Z$$

for a given total expenditure Z. Letting θ denote the Lagrangian multiplier associated with the budget constraint, the first order conditions imply, for all z,

$$c(z)^{q-1}\left[\int_0^1 c(z)^q \, dz\right]^{\frac{1}{q}-1} = \theta p(z).$$

For any two goods z and z', therefore, $[c(z)/c(z')]^{q-1} = p(z)/p(z')$, or

$$c(z) = c(z')\left[\frac{p(z')}{p(z)}\right]^{\frac{1}{1-q}}.$$

If this expression is substituted into the budget constraint, we obtain

$$\int_0^1 p(z)c(z')\left[\frac{p(z')}{p(z)}\right]^{\frac{1}{1-q}} dz = c(z')p(z')^{\frac{1}{1-q}}\left[\int_0^1 p(z)^{\frac{q}{q-1}} dz\right] = Z. \tag{6.89}$$

Using the definition of P given in (6.3), both sides of (6.89) can be divided by P to yield

$$\frac{c(z')p(z')^{\frac{1}{1-q}}\left[\int_0^1 p(z)^{\frac{q}{q-1}} dz\right]}{\left[\int_0^1 p(z)^{\frac{q}{q-1}} dz\right]^{\frac{q-1}{q}}} = \frac{Z}{P}.$$

This can be simplified to

$$c(z')\left[\frac{p(z')}{P}\right]^{\frac{1}{1-q}} = \frac{Z}{P} \quad \text{or} \quad c(z') = \left[\frac{p(z')}{P}\right]^{\frac{1}{q-1}} C, \tag{6.90}$$

where $C = \frac{Z}{P}$ is total real consumption of the composite good. Equation (6.90) implies that the demand for good z by agent j is equal to $c^j(z) = [p(z)/P]^{1/(q-1)}C^j$, so the world demand for product z will be equal to

$$\begin{aligned} y_t^d(z) &\equiv n\left[\frac{p_t(z)}{P_t}\right]^{\frac{1}{q-1}} C_t + (1-n)\left[\frac{p_t^*(z)}{P_t^*}\right]^{\frac{1}{q-1}} C_t^* \\ &= \left[\frac{p_t(z)}{P_t}\right]^{\frac{1}{q-1}} C_t^w, \end{aligned} \tag{6.91}$$

where $C^w = nC + (1-n)C^*$ is world real consumption. Notice that we have used the law of one price here, since it implies that the relative price for good z is the same for home and foreign consumers: $p(z)/P = Ep^*(z)/EP^* = p^*(z)/P^*$. Finally, note that (6.91) implies

$$p_t(z) = P_t\left[\frac{y_t^d(z)}{C_t^w}\right]^{q-1}. \tag{6.92}$$

The Individual's Decision Problem Each individual begins period t with existing asset holdings B_{t-1}^j and M_{t-1}^j and chooses how much of good j to produce (subject to the world demand function for good j), how much to consume, and what levels of real bonds and money to hold. These choices are made to maximize utility given by (6.1) and subject to the following budget constraint:

$$C_t^j + B_t^j + \frac{M_t^j}{P_t} \le \frac{p_t(j)y_t(j)}{P_t} + R_{t-1}B_{t-1}^j + \frac{M_{t-1}^j}{P_t} + \tau_t,$$

where τ_t is the real net transfer from the government and R_t is the real gross rate of return. From (6.92), agent j's real income from producing $y(j)$ will be equal to $y(j)^q(C_t^w)^{1-q}$, so the budget constraint can be written as

$$C_t^j + B_t^j + \frac{M_t^j}{P_t} = y_t(j)^q(C_t^w)^{1-q} + R_{t-1}B_{t-1}^j + \frac{M_{t-1}^j}{P_t} + \tau_t. \tag{6.93}$$

The value function for the individual's decision problem is

$$V(B_{t-1}^j, M_{t-1}^j) = \max\left\{\log C_t^j + b\log\frac{M_t^j}{P_t} - \frac{k}{2}y_t(j)^2 + \beta V(B_t^j, M_t^j)\right\},$$

where the maximization is subject to (6.93). Letting λ denote the Lagrangian multiplier associated with the budget constraint, first order conditions are

$$\frac{1}{C_t^j} - \lambda_t = 0 \tag{6.94}$$

$$\frac{b}{M_t^j} + \beta V_2(B_t^j, M_t^j) - \frac{\lambda_t}{P_t} = 0 \tag{6.95}$$

$$-ky_t(j) + \lambda_t q y_t(j)^{q-1}(C_t^w)^{1-q} = 0 \tag{6.96}$$

$$\beta V_1(B_t^j, M_t^j) - \lambda_t = 0 \tag{6.97}$$

$$V_1(B_{t-1}^j, M_{t-1}^j) = \lambda_t R_{t-1} \tag{6.98}$$

$$V_2(B_{t-1}^j, M_{t-1}^j) = \frac{\lambda_t}{P_t}. \tag{6.99}$$

We also have the transversality condition $\lim_{i\to\infty}\prod_{k=0}^{i} R_{t+s-1}(B_{t+i}^j + M_{t+i}^j/P_{t+i}) = 0$.

These first order conditions lead to the standard Euler condition for consumption with log utility:

$$C_{t+1}^j = \beta R_t C_t^j,$$

which is obtained using (6.94), (6.97), and (6.98). Equations (6.96) and (6.94) imply that the optimal production level the individual chooses satisfies

$$y_t(j)^{2-q} = \frac{q}{k}\frac{(C_t^w)^{1-q}}{C_t^j}. \tag{6.100}$$

Equation (6.95) yields an expression for the real demand for money,

$$\frac{M_t^j}{P_t} = bC_t^j\left(\frac{1+i_t}{i_t}\right),$$

where $(1+i_t) = R_{t+1}P_{t+1}/P_t$ is the gross nominal rate of interest from period t to $t+1$. This expression should look familiar from chapter 2.

6.7.2 The Small Open-Economy Model

This appendix employs the method of undetermined coefficients to obtain the equilibrium exchange rate and price level processes consistent with (6.49)–(6.57). The equations of the model are repeated here, where the real exchange rate ρ_t has been replaced by $s_t + p_t^* - p_t$, r_t by $r^* - (s_t + p_t^* - p_t) + \mathrm{E}_t(s_{t+1} + p_{t+1}^* - p_{t+1})$, i_t by $i_t^* + \mathrm{E}_t s_{t+1} - s_t$, and q_t by $p_t + (1-h)(s_t + p_t^* - p_t)$:

$$y_t = -b_1(s_t + p_t^* - p_t) + b_2(p_t - \mathrm{E}_{t-1}p_t) + e_t \tag{6.101}$$

$$y_t = a_1(s_t + p_t^* - p_t) - a_2[r^* - (s_t + p_t^* - p_t) + \mathrm{E}_t(s_{t+1} + p_{t+1}^* - p_{t+1})] + u_t \tag{6.102}$$

$$m_t - [p_t + (1-h)(s_t + p_t^* - p_t)] = y_t - c(i_t^* + \mathrm{E}_t s_{t+1} - s_t) + v_t \tag{6.103}$$

$$m_t = \mu + m_{t-1} + \varphi_t - \gamma\varphi_{t-1}, \quad 0 \le \gamma \le 1 \tag{6.104}$$

$$p_t^* = \pi^* + p_{t-1}^* + \phi_t. \tag{6.105}$$

Substituting the aggregate demand relationship (6.102), the money supply process (6.104), and the foreign price process (6.105) into the money demand equation (6.103) yields, after some rearrangement,

$$\begin{aligned} A_1 p_t + A_2 s_t = {} & C_0 + \mu + m_{t-1} + \varphi_t - \gamma\varphi_{t-1} - (1 - h + a_1)(p_{t-1}^* + \phi_t) \\ & - a_2 \mathrm{E}_t p_{t+1} + (a_2 + c)\mathrm{E}_t s_{t+1} - u_t - v_t, \end{aligned} \tag{6.106}$$

where $A_1 = h - a_1 - a_2$, $A_2 = 1 - h + a_1 + a_2 + c > 0$ and $C_0 = (c + a_2)r^* - (1 - h + a_1 - a_2 - c)\pi^*$. In deriving (6.106), two additional results have been used: from (6.105), $\mathrm{E}_t p_{t+1}^* = 2\pi^* + p_{t-1}^* + \phi_t$ and $i_t^* = r^* + \mathrm{E}_t p_{t+1}^* - p_t^* = r^* + \pi^*$.

Using the aggregate supply and demand relationships (6.101) and (6.102),

$$\begin{aligned} B_1 p_t + B_2 s_t = {} & -b_2 \mathrm{E}_{t-1} p_t + a_2(\mathrm{E}_t s_{t+1} - \mathrm{E}_t p_{t+1}) - (a_1 + b_1)\pi^* \\ & + a_2(r^* + \pi^*) + e_t - u_t - (a_1 + b_1)(p_{t-1}^* + \phi_t), \end{aligned} \tag{6.107}$$

where $B_1 = -(a_1 + a_2 + b_1 + b_2) < 0$ and $B_2 = a_1 + a_2 + b_1 > 0$.

The state variables at time t are m_{t-1}, p_t^* and the various random disturbances. To rule out possible bubble solutions, we follow McCallum (1983a) and hypothesize minimum state variable solutions of the form[31]

$$p_t = k_0 + m_{t-1} + k_1 p_{t-1}^* + k_2\varphi_t + k_3\varphi_{t-1} + k_4 u_t + k_5 e_t + k_6 v_t + k_7\phi_t$$

$$s_t = d_0 + m_{t-1} + d_1 p_{t-1}^* + d_2\varphi_t + d_3\varphi_{t-1} + d_4 u_t + d_5 e_t + d_6 v_t + d_7\phi_t.$$

These imply

$$\mathrm{E}_{t-1}p_t = k_0 + m_{t-1} + k_1 p_{t-1}^* + k_3\varphi_{t-1}$$

$$\begin{aligned} \mathrm{E}_t p_{t+1} &= k_0 + m_t + k_1 p_t^* + k_3\varphi_t \\ &= k_0 + \mu + m_{t-1} + (1 + k_3)\varphi_t - \gamma\varphi_{t-1} + k_1(\pi^* + p_{t-1}^* + \phi_t) \end{aligned}$$

and

$$\mathrm{E}_t s_{t+1} = d_0 + \mu + m_{t-1} + (1 + d_3)\varphi_t - \gamma\varphi_{t-1} + d_1(\pi^* + p_{t-1}^* + \phi_t).$$

These expressions for p_t and s_t, together with those for the various expectations of p and s, can be substituted into (6.106) and (6.107). These then yield a pair of equations that must be satisfied by each pair (k_i, d_i). For example, the coefficients on p_{t-1}^* in (6.106) and (6.107) must satisfy

$$A_1 k_1 + A_2 d_1 = -(1 - h + a_1) - a_2 k_1 + (a_2 + c)d_1$$

and

$$B_1 k_1 + B_2 d_1 = a_2(d_1 - k_1) - b_2 k_1 - (a_1 + b_1).$$

Using the definitions of A_i and B_i to cancel terms, the second equation implies that $d_1 = k_1 - 1$. Substituting this back into the first equation yields $k_1 = 0$. Therefore, the solution pair is $(k_1, d_1) = (0, 1)$. Repeating this process yields the values for (k_i, d_i) reported in (6.58) and (6.59).

6.8 Problems

1. Suppose $m_t = m_0 + \gamma m_{t-1}$ and $m_t^* = m_0^* + \gamma^* m_{t-1}^*$. Use (6.24) to show how the behavior of the nominal exchange rate under flexible prices depends on the degree of serial correlation exhibited by the home and foreign money supplies.

31. We have set the coefficient on m_{t-1} equal to 1 in these trial solutions. It is easy to verify that this assumption is in fact correct.

2. In the model of section 6.3 used to study policy coordination, aggregate demand shocks were set equal to zero in order to focus on a common aggregate supply shock. Suppose instead that the aggregate supply shocks are zero, and the demand shocks are given by $u \equiv x + \phi$ and $u^* \equiv x + \phi^*$, so that x represents a common demand shock and ϕ and ϕ^* are uncorrelated country-specific demand shocks. Derive policy outcomes under coordinated and (Nash) noncoordinated policy setting. Is there a role for policy coordination in the face of demand shocks? Explain.

3. Continuing with the same model as in the previous question, how are real interest rates affected by a common aggregate demand shock?

4. Policy coordination with asymmetric supply shocks: Continuing with the same model as in the previous two questions, assume that there are no demand shocks but that the supply shocks e and e^* are uncorrelated. Derive policy outcomes under coordinated and uncoordinated policy setting. Does coordination or noncoordination lead to a greater inflation response to supply shocks? Explain.

5. Assume that the home country policy maker acts as a Stackelberg leader and recognizes that foreign inflation will be given by (6.47). How does this change in the nature of the strategic interaction affect the home country's response to disturbances?

6. In a small open economy with perfectly flexible nominal wages, the text showed that the real exchange rate and domestic price level were given by

$$\rho_t = \sum_{i=0}^{\infty} d^i \mathrm{E}_t \left(\frac{a_2 r^*_{t+i} + e_{t+i} - u_{t+i}}{a_1 + a_2 + b_1} \right)$$

and

$$p_t = \left(\frac{1}{1+c}\right) \sum_{i=0}^{\infty} \left(\frac{c}{1+c}\right)^i \mathrm{E}_t(m_{t+i} - z_{t+i} - v_{t+i}),$$

where $z_{t+i} \equiv y_{t+i} + (1-h)\rho_{t+i} - cr_{t+i}$. Assume that $r^* = 0$ for all t and that e, u, and $z + v$ all follow first order autoregressive processes (e.g., $e_t = \rho_e e_{t-1} + x_{e,t}$ for x_e white noise). Let the nominal money supply be given by

$$m_t = g_1 e_{t-1} + g_2 u_{t-1} + g_3(z_{t-1} + v_{t-1}).$$

Find equilibrium expressions for the real exchange rate, the nominal exchange rate, and the consumer price index. What values of the parameters g_1, g_2, and g_3 minimize fluctuations in s_t? In q_t? In ρ_t? Are there any conflicts between stabilizing the exchange rate (real or nominal) and stabilizing the consumer price index?

7. Equation (6.42) for the equilibrium real exchange rate in the two-country model of section 6.3.1 takes the form $\rho_t = A\mathrm{E}_t\rho_{t+1} + v_t$. Suppose $v_t = \gamma v_{t-1} + \psi_t$, where ψ_t is a mean-zero, white-noise process. Suppose the solution for ρ_t is of the form $\rho_t = bv_t$. Find the value of b. How does it depend on γ?

8. Section 6.5.1 demonstrated how a simple open-economy model with nominal price stickiness could be expressed in a form that paralleled the closed-economy new Keynesian model of chapter 5. Would this same conclusion result in a model of sticky wages with flexible prices? What if both wages and prices are sticky?

7 The Credit Channel of Monetary Policy

7.1 Introduction

The previous chapters have illustrated several channels through which monetary factors and monetary policy may affect the real economy. When prices are flexible, anticipated inflation affects the opportunity cost of holding money. This effect taxes money holdings, reducing utility directly if real balances enter agents' utility functions or indirectly in cash-in-advance (CIA) models by raising the total cost of goods whose purchase requires cash. When prices or nominal wages are sticky, changes in the current and expected future paths of the money supply affect real aggregate demand and output. These effects operated, in the closed economy, through interest rates and the impact interest rates have on real aggregate demand.

In the open economy, exchange-rate channels operate. The exchange rate affects the domestic price level directly by influencing the domestic currency price of imports. A depreciation raises the domestic consumer price index. Because exchange rates respond quickly to interest-rate changes, this exchange-rate-to-inflation channel speeds up the impact of monetary policy on domestic inflation. Exchange-rate movements also alter relative prices when nominal wages or prices are sticky, and these real-exchange-rate effects induce substitution effects between domestic and foreign goods, thereby influencing aggregate demand and supply.

Many economists, however, have argued that monetary policy has direct effects on aggregate spending that do not operate through traditional interest-rate or exchange-rate channels, and a large literature in recent years has focused on credit markets as playing a critical role in the transmission of monetary policy actions to the real economy. Money has traditionally played a special role in macroeconomics and monetary theory because of the relationship between the nominal stock of money and the aggregate price level. The importance of money for understanding the determination of the general level of prices and average inflation rates, however, does not necessarily imply that the stock of money is the key variable that links the real and financial sectors or the most appropriate indicator of the short-run influence of financial factors on the economy.

The *credit view* stresses the distinct role played by financial assets and liabilities. Rather than aggregate all nonmoney financial assets into a single category called *bonds*, the credit view argues that macroeconomic models need to distinguish between different nonmonetary assets, either along the dimension of bank versus nonbank sources of funds or along the more general dimension of internal versus external financing. The credit view also highlights heterogeneity among borrowers, stressing that some borrowers may be more vulnerable to changes in credit conditions than others. Finally, investment may be sensitive to variables such as net worth or cash flow if agency costs associated with imperfect information or costly monitoring create a wedge between the cost of internal and external finance. A rise in interest rates may have a much stronger contractionary impact on the economy if balance sheets are already weak, introducing the possibility that nonlinearities in the impact of monetary policy may be important.

Discussions of the credit channel often distinguish between a *bank lending channel* and a broader financial-accelerator mechanism.[1] The bank lending channel emphasizes the special nature of bank credit and the role of banks in the economy's financial structure. In the bank lending view, banks play a particularly critical role in the transmission of monetary-policy actions to the real economy. Policy actions that affect the reserve positions of banks will generate adjustments in interest rates and in the components of the banking sector's balance sheet. Traditional models of the monetary transmission mechanism focus on the impact of these interest-rate changes on money demand and on consumption and investment decisions by households and firms. The ultimate effects on bank deposits and the supply of money are reflected in adjustments to the liability side of the banking sector's balance sheet.

The effects on banking-sector reserves and interest rates also influence the supply of bank credit, the asset side of the balance sheet. If banks cannot offset a decline in reserves by adjusting securities holdings or raising funds through issuing nonreservable liabilities (such as CDs in the United States), bank lending must contract. If banking lending is *special* in the sense that bank borrowers do not have close substitutes for obtaining funds, variation in the availability of bank lending may have an independent impact on aggregate spending. Key then to the bank lending channel is the lack of close substitutes for deposit liabilities on the liability side of the banking sector's balance sheet and the lack of close substitutes for bank credit on the part of borrowers.

Imperfect information plays an important role in credit markets, and bank credit may be special, that is, have no close substitutes, because of information advantages banks have in providing both transactions services and credit to businesses. Small firms in particular may have difficulty obtaining funding from nonbank sources, so a contraction in bank lending will force these firms to contract their activities.

The broad credit channel is not restricted to the bank lending channel. Credit-market imperfections may characterize all credit markets, influencing the nature of financial contracts, raising the possibility of equilibria with rationing, and creating a

1. A variety of excellent surveys and overviews of the credit channel are available. These include Gertler (1988), Bernanke (1993), Gertler and Gilchrist (1993), Ramey (1993), Kashyap and Stein (1994), Bernanke and Gertler (1995), Cecchetti (1995), Hubbard (1995), and Bernanke, Gertler, and Gilchrist (1999).

wedge between the costs of internal and external financing. This wedge arises because of agency costs associated with information asymmetries and the inability of lenders to monitor borrowers costlessly. As a result, cash flow and net worth become important in affecting the cost and availability of finance and the level of investment spending. A recession that weakens a firm's sources of internal finance can generate a *financial-accelerator* effect; the firm is forced to rely more on higher-cost external funds just at the time the decline in internal finance drives up the relative cost of external funds. Contractionary monetary policy that produces an economic slowdown will reduce firm cash flow and profits. If this policy increases the external finance premium, there will be further contractionary effects on spending. In this way, the credit channel can serve to propagate and amplify an initial monetary contraction.

Financial-accelerator effects can arise from the adjustment of asset prices to contractionary monetary policy. Borrowers may be limited in the amount they can borrow by the value of their assets that can serve as collateral. A rise in interest rates that lowers asset prices reduces the market value of borrowers' collateral. This reduction in value may then force some firms to reduce investment spending as their ability to borrow declines.

The credit channel also operates when shifts in monetary policy alter either the efficiency of financial markets in matching borrowers and lenders or the extent to which borrowers face rationing in credit markets so that aggregate spending is influenced by liquidity constraints. There are several definitions of nonprice credit rationing. Jaffee and Russell (1976) define credit rationing as existing when, at the quoted interest rate, the lender supplies a smaller loan than the borrower demands. Jaffee and Stiglitz (1990), however, point out that this practice represents standard price rationing; larger loans will normally be accompanied by a higher default rate and therefore carry a higher interest rate. Instead, Jaffee and Stiglitz characterize "pure credit rationing" as occurring when, among a group of agents (firms or individuals) who appear to be identical, some receive loans and others do not. Stiglitz and Weiss (1981) define equilibrium credit rationing as being present whenever "either (a) among loan applicants who appear to be identical some receive a loan and others do not, and the rejected applicants would not receive a loan even if they offered to pay a higher interest rate; or (b) there are identifiable groups of individuals in the population who, with a given supply of credit, are unable to obtain loans at any interest rate, even though with a larger supply of credit, they would" (pp. 394–395). The critical aspect of this definition is that at the market equilibrium interest rate, there is an unsatisfied demand for loans that cannot be eliminated through higher interest rates. Rejected loan applicants cannot succeed in getting a loan by offering to pay a higher interest rate.

It is important to recognize that credit rationing is sufficient but not necessary for a credit channel to exist. A theme of Gertler (1988), Bernanke and Gertler (1989), and Bernanke (1993) is that agency costs in credit markets will vary countercyclically; a monetary tightening that raises interest rates and generates a real economic slowdown will cause firm balance sheets to deteriorate, raising agency costs and lowering the efficiency of credit allocation. Changes in credit conditions are not reflected solely in interest-rate levels. Thus, the general issue is to understand how credit-market imperfections affect the macroeconomic equilibrium and the channels through which monetary policy actions are transmitted to the real economy.

Critical to the presence of a distinct credit channel is the presence of imperfections in financial markets. The first task, then, is to review theories of credit-market imperfections based on adverse selection, moral hazard, and monitoring costs; this is done in section 7.2. These theories help to explain many of the distinctive features of financial markets, from collateral to debt contracts to the possibility of credit rationing. This material provides the microfoundations for the macroeconomic analysis of credit channels in section 7.3. Section 7.4 reviews the empirical evidence on the role played by credit channels in the transmission of monetary-policy actions. The main focus of the chapter will be on credit markets for firms undertaking investment projects. This approach is chosen primarily for convenience; the theoretical models may also be applied to the consumer loan market, and there is evidence that a significant fraction of households behave as if they face liquidity constraints that link consumption spending more closely to current income than would be predicted by forward-looking models of consumption.[2]

7.2 Imperfect Information in Credit Markets

The role of credit effects in the transmission of monetary policy arises as a result of imperfect information between parties in credit relationships. The information that each party to a credit transaction brings to the exchange will have important implications for the nature of credit contracts, the ability of credit markets to match borrowers and lenders efficiently, and the role played by the rate of interest in allocating credit among borrowers. The nature of credit markets can lead to distinct roles for

2. Empirical evidence on consumption and liquidity constraints can be found in Campbell and Mankiw (1989, 1991), who provide estimates of the fraction of liquidity-constrained households for a number of OECD countries.

different types of lenders (e.g., bank versus nonbank) and different types of borrowers (e.g., small firms versus large firms).

Whereas some of the early, large-scale econometric models did incorporate credit-rationing channels, the modern analysis of credit markets from the perspective of imperfect-information theory dates from Jaffee and Russell (1976), Keeton (1979), and Stiglitz and Weiss (1981). These models of credit rationing rely on imperfect information, although the exact mechanisms emphasized by different authors—adverse selection, moral hazard, or monitoring costs—have varied. These models generally imply that in some circumstances the lender's expected profits will decline with an increase in the interest rate charged to borrowers. Lenders will not raise interest rates on loans past the point at which expected profits start to decline even if there are borrowers who would be willing to borrow at higher interest rates. Equilibrium may then be characterized by an excess demand for loans and rationing.

Microeconomic models of credit market imperfection emphasize the effects of imperfect information on the relationship between borrowers and lenders. Adverse selection, moral hazard, and costly monitoring affect the nature of credit contracts and may lead to the presence of equilibrium rationing. These models highlight the importance of information in the market for credit.

7.2.1 Adverse Selection

Jaffee and Russell (1976) analyze a credit-market model in which there are two types of borrowers, "honest" ones who always repay and "dishonest" ones who repay only if it is in their interest to do so. Ex ante, the two types appear identical to lenders. Default is assumed to impose a cost on the defaulter, and dishonest borrowers default whenever the loan repayment amount exceeds the cost of default. By assuming a distribution of default costs across the population of borrowers, Jaffee and Russell show that the fraction of borrowers who default is increasing in the loan amount.[3] In a pooling equilibrium, lenders offer the same loan contract (interest rate and amount) to all borrowers, since they are unable to distinguish between the two types.[4] If lenders operate with constant returns to scale, if there is free entry, and if funds are available to lenders at an exogenously given opportunity cost, then the equilibrium loan rate must satisfy a zero-profit condition for lenders. Since the expected return on a loan is less than or equal to the interest rate charged, the actual

3. See Smith (1983) for a general equilibrium version of Jaffee and Russell's model using an overlapping-generations framework.

4. This ignores the possibility of separating equilibrium in which the lender offers two contracts and the borrowers (truthfully) signal their type by the contract they choose.

interest rate on loans must equal or exceed the opportunity cost of funds to the lenders.[5]

The effects of borrower heterogeneity and imperfect information on credit-market equilibria can be illustrated following Stiglitz and Weiss (1981). The lender's expected return on a loan is a function of the interest rate charged and the probability that the loan will be repaid, but individual borrowers differ in their probabilities of repayment. Suppose borrowers come in two types. Type G repays with probability q_g; type B repays with probability q_b. If lenders can observe the borrower's type, each type will be charged a different interest rate to reflect the differing repayment probabilities. If the supply of credit is perfectly elastic at the opportunity cost of r, and lenders are risk neutral and able to lend to a large number of borrowers so that the law of large numbers holds, then all type Gs can borrow at an interest rate of r/q_g, while type Bs borrow at $r/q_b > r/q_g$. At these interest rates, the lender's expected return from lending to either type of borrower is equal to her opportunity cost of r. No credit rationing occurs; riskier borrowers are simply charged higher interest rates.

Now suppose the lender cannot observe the borrower's type. It may be the case that changes in the terms of a loan (interest rate, collateral, amount) affect the mix of borrower types the lender attracts. If increases in the loan interest rate shift the mix of borrowers, raising the fraction of type Bs, the expected return to the lender might actually decline with higher loan rates because of adverse selection. In this case, further increases in the loan rate would lower the lender's expected profits, even if an excess demand for loans remains. The intuition is similar to that of Akerlof's market for lemons (Akerlof 1970). Assume that a fraction g of all borrowers are of type G. Suppose the lender charges an interest rate of r_l such that $gq_gr_l + (1-g)q_br_l = r$, or $r_l = r/[gq_g + (1-g)q_b]$. At this loan rate, the lender earns her required return of r if borrowers are drawn randomly from the population. But at this rate, the pool of borrowers is no longer the same as in the population at large. Since $r/q_g < r_l < r/q_b$, the lender is more likely to attract type B borrowers, and the lender's expected return would be less than r.

Loans are, however, characterized by more than just their interest rate. For example, suppose a loan is characterized by its interest rate r_l, the loan amount L, and the collateral the lender requires C. The probability that the loan will be repaid

5. If the probability of default was zero, the constant-returns-to-scale assumption with free entry would ensure that lenders charge an interest rate on loans equal to the opportunity cost of funds. If default rates are positive, then the expected return on a loan is less than the actual interest rate charged, and the loan interest rate must be greater than the opportunity cost of funds.

depends on the (risky) return yielded by the borrower's project. If the project return is R, then the lender is repaid if

$$L(1+r_l) < R + C.$$

If $L(1+r_l) > R + C$, the borrower defaults and the lender receives $R + C$.

Suppose the return R is $R' + x$ with probability $\frac{1}{2}$ and $R' - x$ with probability $\frac{1}{2}$. The expected return is R', while the variance is x^2. An increase in x represents a mean preserving spread in the return disturbance and corresponds to an increase in the project's risk. Assume that $R' - x < (1+r_l)L - C$ so that the borrower must default when the bad outcome occurs. If the project pays off $R' + x$, the borrower receives $R' + x - (1+r_l)L$; if the bad outcome occurs, the borrower receives $-C$, that is, any collateral is lost. The expected profit to the borrower is

$$\mathrm{E}\pi^B = \tfrac{1}{2}[R' + x - (1+r_l)L] - \tfrac{1}{2}C.$$

Define

$$x^*(r_l, L, C) \equiv (1+r_l)L + C - R'. \tag{7.1}$$

Expected profits for the borrower are positive for all $x > x^*$. This critical cutoff value of x is increasing in r_l. Recall that increases in x imply an increase in the project's risk, as measured by the variance of returns. An increase in the loan rate r_l increases x^*, and this implies that some borrowers with less risky projects will find it unprofitable to borrow if the loan rate rises, while borrowers with riskier projects will still find it worthwhile to borrow. Because the borrower can lose no more than her collateral in the bad state, expected profits are a convex function of the project's return and therefore increase with an increase in risk (for a constant mean return).

While the expected return to the firm is increasing in risk, as measured by x, the lender's return is decreasing in x. To see this point, note that the lender's expected profit is

$$\mathrm{E}\pi^L = \tfrac{1}{2}[(1+r_l)L] + \tfrac{1}{2}[C + R' - x] - (1+r)L,$$

where r is the opportunity cost of funds to the lender. The lender's expected profit decreases with x. Because the lender receives a fixed amount in the good state, the lender's expected return is a concave function of the project's return and therefore decreases with an increase in risk.

Now suppose there are two groups of borrowers, those with $x = x_g$ and those with $x = x_b$, with $x_g < x_b$. Type x_g's have lower-risk projects. From (7.1), if the loan rate

r_l is low enough such that $x_b > x_g \geq x^*(r_l, L, C)$, then both types will find it profitable to borrow. If each type is equally likely, the lender's expected return is

$$\begin{aligned} \mathrm{E}\pi^L &= \tfrac{1}{4}[(1+r_l)L + C + R' - x_g] + \tfrac{1}{4}[(1+r_l)L + C + R' - x_b] - (1+r)L \\ &= \tfrac{1}{2}[(1+r_l)L + C + R'] - \tfrac{1}{4}(x_g + x_b) - (1+r)L, \quad x^*(r_l, L, C) \leq x_g, \end{aligned}$$

which is increasing in r_l. But as soon as r_l increases to the point where $x^*(r_l, L, C) = x_g$, any further increase causes all x_g types to stop borrowing. Only x_b types will still find it profitable to borrow, and the lender's expected profit falls to

$$\mathrm{E}\pi^L = \tfrac{1}{2}[(1+r_l)L + C + R'] - \tfrac{1}{2}x_b - (1+r)L, \quad x_g \leq x^*(r_l, L, C) \leq x_b.$$

As a result, the lender's expected profit as a function of the loan rate is increasing for $x^*(r_l, L, C) \leq x_g$ and then falls discretely at $1 + r_l = [x_g - C + R']/L$ as all low-risk types exit the market. This is illustrated in figure 7.1, where r^* denotes the loan rate that tips the composition of the pool of borrowers. For loan rates between r_1 and r^*, both types borrow and the lender's expected profit is positive. Expected profits are again positive for loan rates above r_2, but in this region only x_b types borrow.

The existence of a local maximum in the lender's profit function at r^* introduces the possibility that credit rationing will occur in equilibrium. Suppose at r^* there remains an excess demand for loans. Type x_gs would not be willing to borrow at a rate above r^*, but type x_bs would. If the lender responds to the excess demand by raising the loan rate, expected profits fall. Equilibrium may involve a loan rate of r^*, with some potential borrowers being rationed.[6] Thus, adverse selection provides one rationale for a lender's profit function that is not monotonic in the loan rate. Equilibrium credit rationing may exist because lenders find it unprofitable to raise the interest rate on loans even in the face of an excess demand for loans.

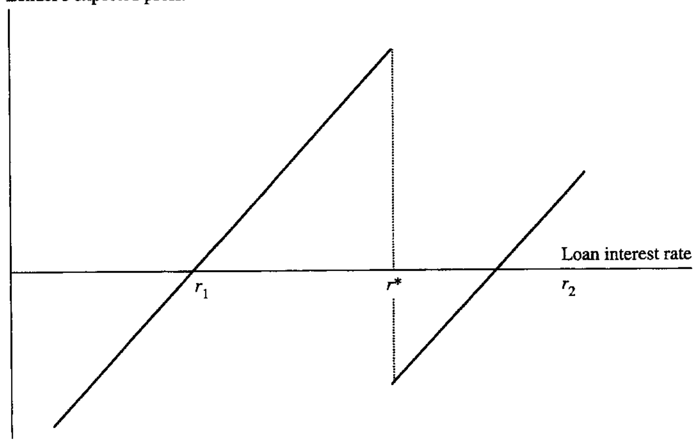

Figure 7.1
Expected Loan Profit with Adverse Selection

7.2.2 Moral Hazard

Moral hazard can arise in credit markets when the borrower's behavior is influenced by the terms of the loan contract. In the model of the previous section, the borrower decided whether to borrow, but the project's return was exogenous. Borrowers differed in terms of the underlying riskiness of their projects, and adverse selection occurred as loan rate changes affected the pool of borrowers. Suppose instead that each borrower can choose between several projects of differing risk. If the lender cannot monitor this choice, a moral-hazard problem arises. The lender's expected return may not be monotonic in the interest rate charged on the loans. Higher loan rates lead the borrower to invest in riskier projects, lowering the expected return to the lender.

To illustrate this situation, again following Stiglitz and Weiss (1981), suppose the borrower can invest either in project A, which pays off R^a in the good state and 0 in the bad state, or in project B, which pays off $R^b > R^a$ in the good state and 0 in the bad state. Suppose the probability of success for project A is p^a and p^b for project B, with $p^a > p^b$. Project B is the riskier project. Further, assume the expected payoff from A is higher: $p^aR^a > p^bR^b$. By investing in A, the borrower's expected return is

$$\mathrm{E}\pi^A = p^a[R^a - (1+r_l)L] - (1-p^a)C,$$

where the borrower loses collateral C if the project fails. The expected return from project B is

$$\mathrm{E}\pi^B = p^b[R^b - (1+r_l)L] - (1-p^b)C.$$

The expected returns on the two projects depend on the interest rate on the loan r_l. It is straightforward to show that

$$\mathrm{E}\pi^A > \mathrm{E}\pi^B$$

if and only if

$$\frac{p^aR^a - p^bR^b}{p^a - p^b} > (1+r_l)L - C.$$

6. As figure 7.1 suggests, if the demand for loans is strong enough, the lender may be able to raise the loan rate sufficiently so that expected profits do rise.

The left side of this condition is independent of the loan rate, but the right side is increasing in r_l. Define r_l^* as the loan rate at which the expected returns to the borrower from the two projects are equal. This occurs when

$$(1+r_l^*)L - C = \frac{p^a R^a - p^b R^b}{p^a - p^b}.$$

For loan rates less than r_l^*, the borrower will prefer to invest in project A; for loan rates above r_l^*, the riskier project B is preferred. The expected payment to the lender, therefore, will be $p^a(1+r_l)L + (1-p^a)C$ if $r_l < r_l^*$, and $p^b(1+r_l)L + (1-p^b)C$ for $r_l > r_l^*$. Since

$$p^a(1+r_l^*)L + (1-p^a)C > p^b(1+r_l^*)L + (1-p^b)C, \qquad (7.2)$$

the lender's profits fall as the loan rate rises above r^*; the lender's profits are not monotonic in the loan rate.[7] Just as in the example of the previous subsection, this leads to the possibility that credit rationing may characterize the loan market's equilibrium.

7.2.3 Monitoring Costs

The previous analysis illustrated how debt contracts in the presence of adverse selection or moral hazard could lead to credit rationing as an equilibrium phenomenon. One limitation of the discussion, however, was the treatment of the nature of the loan contract—repayment equal to a fixed interest rate times the loan amount in some states of nature, zero or a predetermined collateral amount in others—as exogenous. Williamson (1986, 1987a, 1987b) has illustrated how debt contracts and credit rationing can arise, even in the absence of adverse-selection or moral-hazard problems, if lenders must incur costs to monitor borrowers.[8] The intuition behind his result is straightforward. Suppose the lender can observe the borrower's project outcome only at some positive cost. Any repayment schedule that ties the borrower's payment to the project outcome would require that the monitoring cost be incurred; otherwise, the borrower always has an incentive to underreport the success of the project. Expected monitoring costs can be reduced if the borrower is monitored only in some states of nature. If the borrower reports a low project outcome and defaults

7. To see this, note that using the definition of r_l^* implies that the left side of (7.2) is equal to $p^a[(1+r_l^*)L - C] = p^a\left(\frac{p^aR^a - p^bR^b}{p^a-p^b}\right)$, while the right side is equal to $p^b[(1+r_l^*)L - C] = p^b\left(\frac{p^aR^a-p^bR^b}{p^a-p^b}\right)$. The direction of the inequality follows since $p^a > p^b$.

8. Townsend (1979) provided the first analysis of optimal contracts when it is costly to verify the state.

on the loan, the lender incurs the monitoring cost to verify the truth of the report. If the borrower reports a good project outcome and repays the loan, the lender does not need to incur the monitoring cost.

Following Williamson (1987a), assume there are two types of agents, borrowers and lenders. Lenders are risk neutral and have access to funds at an opportunity cost of r. Each lender takes r as given and offers contracts to borrowers that yield, to the lender, an expected return of r. Assume there are two periods. In period 1, lenders offer contracts to borrowers who have access to a risky investment project that yields a payoff in period 2 of $x \in [0, \bar{x}]$. The return x is a random variable, drawn from a distribution known to both borrowers and lenders. The actual realization is observed costlessly by the borrower; the lender can observe it by first paying a cost of c. This assumption captures the idea that borrowers are likely to have better information about their own projects than do lenders. Lenders can obtain this information by monitoring the project, but such monitoring is costly.

In period 2, after observing x, the borrower reports the project outcome to the lender. Let this report be x^s. While x^s must be in $[0, \bar{x}]$, it need not equal the true x, since the borrower will have an incentive to misreport if doing so is in the borrower's own interest. By choice of normalization, projects require an initial resource investment of 1 unit. Although borrowers have access to an investment project, we assume they have no resources of their own, so to invest they must obtain resources from lenders.

Suppose that monitoring occurs whenever $x^s \in S \subset [0, \bar{x}]$. Otherwise, the lender does not monitor. Denote by $R(x)$ the payment from the borrower to the lender if $x^s \in S$ and monitoring takes place. Because the lender monitors and therefore observes x, the repayment can be made a function of the actual x. The return to the lender net of monitoring costs is $R(x) - c$. If the reported value $x^s \notin S$, then no monitoring occurs and the borrower pays $K(x^s)$ to the lender. This payment can only depend on the signal, not the true realization of x, since the lender cannot verify the latter. In this case, the return to the lender is simply $K(x^s)$. Whatever the actual value of $x^s \notin S$, the borrower will report the value that results in the smallest payment to the lender; hence, if monitoring does not occur, the payment to the lender must be equal to a constant, $\bar{K}$.[9] Since all loans are for 1 unit, $\bar{K} - 1$ is the interest rate on the loan when $x^s \notin S$.

If the reported signal is in S, then monitoring occurs so that the lender can learn the true value of x. The borrower will report a x^s in S only if it is in her best

9. That is, suppose x_1 and x_2 are project-return realizations such that the borrower would report x_1^s and $x_2^s \notin S$. If reporting x_1^s results in a larger payment to the lender, the borrower would always report x_2^s.

interest—that is, reporting $x^s \in S$ must be incentive compatible. For this to be the case, the net return to the borrower when $x^s \in S$, equal to $x - R(x)$, must exceed the return from reporting a signal not in S, $x - \bar{K}$. That is, incentive compatibility requires that

$$x - R(x) > x - \bar{K} \quad \text{or} \quad \bar{K} > R(x) \quad \text{for all } x^s \in S.$$

The borrower will report a signal that leads to monitoring only if $R(x) < \bar{K}$ and will report a signal not in S (so that no monitoring occurs) if $R(x) \geq \bar{K}$.

The optimal contract is a payment schedule $R(x)$ and a value $\bar{K}$ that maximizes the borrower's expected return, subject to the constraint that the lender's expected return be at least equal to her opportunity cost r. Letting $\Pr[x < y]$ denote the probability that x is less than y, the expected return to the borrower can be written as the expected return conditional on monitoring occurring, $\mathrm{E}[x - R(x) \mid R(x) < \bar{K}]$ times the probability that $R(x) < \bar{K}$, plus the expected return conditional on $R(x) \geq \bar{K}$ so that no monitoring occurs, times the probability that $R(x) \geq \bar{K}$:

$$\mathrm{E}[R^b] \equiv \mathrm{E}[x - R(x) \mid R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \mathrm{E}[x - \bar{K} \mid R(x) \geq \bar{K}] \Pr[R(x) \geq \bar{K}]. \tag{7.3}$$

The optimal loan contract maximizes this expected return subject to the constraint that the lender's expected return be at least r:

$$\mathrm{E}[R(x) - c \mid R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \bar{K} \Pr[R(x) \geq \bar{K}] \geq r. \tag{7.4}$$

The solution to this problem, and therefore the optimal loan contract, has $R(x) = x$. In other words, if the borrower reports a signal that leads the lender to monitor, then the lender takes the entire actual project return. This result corresponds to a loan default in which the lender takes over the project, incurs the monitoring cost c (which in this case we can think of as a liquidation cost), and ends up with $x - c$. If the project earns a sufficient return—that is, $R(x) = x \geq \bar{K}$—then the borrower pays the lender the fixed amount $\bar{K}$. Since $\bar{K}$ is independent of the realization of x, no monitoring is necessary. The presence of monitoring costs and imperfect information leads to the endogenous determination of the optimal loan contract.

The proof that $R(x) = x$ whenever monitoring takes place is straightforward. In equilibrium, the constraint given by (7.4) will be satisfied with equality. Otherwise, the payment to the leader could be reduced in some states, which would increase the expected return to the borrower. Hence,

$$\mathrm{E}[R(x) - c \mid R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \bar{K} \Pr[R(x) \geq \bar{K}] = r.$$

Any contract that called for $R(x) < x$ for some realizations of x could be replaced by another contract that increases repayment slightly when monitoring occurs but lowers $\bar{K}$ to decrease the range of x for which monitoring actually takes place. This can be done such that the lender's expected profit is unchanged.[10] Using the constraint for the lender's expected return, the expected return to the borrower can be written as

$$\begin{aligned}
\mathrm{E}[R^b] &= \mathrm{E}[x - R(x) \mid R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \{\mathrm{E}[x \mid R(x) \geq \bar{K}] - \bar{K}\} \Pr[R(x) \geq \bar{K}] \\
&= \mathrm{E}[x - R(x) \mid R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \mathrm{E}[x \mid R(x) \geq \bar{K}] \Pr[R(x) \geq \bar{K}] \\
&\quad - \{r - \mathrm{E}[R(x) - c \mid R(x) < \bar{K}] \Pr[R(x) \geq \bar{K}]\} \\
&= \mathrm{E}[x - c \mid R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \mathrm{E}[x \mid R(x) \geq \bar{K}] \Pr[R(x) \geq \bar{K}] - r \\
&= \mathrm{E}[x] - c \Pr[R(x) < \bar{K}] - r,
\end{aligned} \tag{7.5}$$

where $\Pr[R(x) < \bar{K}]$ is the probability that monitoring occurs. Equation (7.5) shows that the expected return to the borrower is decreasing in $\bar{K}$. Any contract that lowers $\bar{K}$ and reduces the probability of monitoring while leaving the lender with an expected return of r will be strictly preferred by the borrower. Such a contract can be constructed if $R(x) < x$.[11]

To make the example more specific, suppose x is uniformly distributed on $[0, \bar{x}]$. The expected return to the lender is equal to

$$\int_0^{\bar{K}} (x - c)\frac{1}{\bar{x}}\,dx + \int_{\bar{K}}^{\bar{x}} \bar{K}\frac{1}{\bar{x}}\,dx.$$

The first term is the expected return to the lender if the borrower defaults, an outcome that occurs whenever $x < \bar{K}$; the probability of this outcome is $\bar{K}/\bar{x}$. The second term is the fixed payment received by the lender whenever $x \geq \bar{K}$, an outcome

10. $R(x) > x$ is ruled out by the assumption that the borrower has no other resources. If $R(x) < x$ for some x for which monitoring occurs, then the new contract, which increases $R(x)$ in those states, increases $R(x) - c$ when monitoring does occur. For a given $\bar{K}$, this increases $\mathrm{E}[R(x) - c \mid R(x) < \bar{K}]$, making the lender's expected profit greater than r. Since $\bar{K}$ is then lowered, monitoring occurs in fewer states, thereby reducing the lender's expected profit so that it again equals r.

11. One implication of (7.5) is that the borrower bears the cost of monitoring; the expected return to the borrower is equal to the total expected project return net of the opportunity cost of funds (r) and expected monitoring costs ($c \Pr[R(x) < \bar{K}]$).

that occurs with probability $[\bar{x} - \bar{K}]/\bar{x}$. Evaluating the expected return and equating it to r yields the following condition to determine $\bar{K}$:

$$\left[\frac{1}{2}\left(\frac{\bar{K}^2}{\bar{x}}\right) - c\left(\frac{\bar{K}}{\bar{x}}\right)\right] + \bar{K}\left[1 - \left(\frac{\bar{K}}{\bar{x}}\right)\right] = r.$$

If $(\bar{x} - c)^2 > 2\bar{x}r$, this quadratic has two real solutions, one less than $\bar{x} - c$ and one greater than $\bar{x} - c$.[12] However, the effect of $\bar{K}$ on the lender's expected return is

$$\frac{\bar{K}}{\bar{x}} - \frac{c}{\bar{x}} + \left(1 - \frac{2\bar{K}}{\bar{x}}\right) = 1 - \frac{c + \bar{K}}{\bar{x}}$$

which becomes negative for $\bar{K} > \bar{x} - c$. This means that when the loan repayment amount is large, further increases in the contracted repayment would actually lower the lender's expected return; loan contracts with less monitoring (a lower $\bar{K}$) would be preferred by both borrower and lender; $\bar{K} > \bar{x} - c$ cannot be an equilibrium.

When the lender's expected profits are no longer monotonic in the loan interest rate but can actually decrease at higher interest rates, the possibility exists of an equilibrium in which some borrowers face credit rationing. In a nonrationing equilibrium, all borrowers receive loans.[13] The expected rate of return r is determined by the condition that loan demand equal loan supply, and the gross interest rate on loans, $\bar{K}$, is less than $\bar{x} - c$. In a credit rationing equilibrium, $\bar{K} = \bar{x} - c$, and not all potential borrowers receive loans. Even though there are unsatisfied potential borrowers, the interest rate on loans will not rise because the lenders' expected profits are decreasing in the loan rate when $\bar{K} > \bar{x} - c$. Even though all potential borrowers were assumed to be identical ex ante, some receive loans while others do not. The ones that do not get loans would be willing to borrow at an interest rate above the market rate, yet no lenders are willing to lend.

Williamson's model illustrates that neither adverse selection nor moral hazard is necessary for rationing to characterize credit markets. The presence of monitoring costs can account for both the general form of loan contracts in which monitoring occurs only when the borrower defaults—in which case the lender takes over the entire project's return—and for rationing to arise in some equilibria.

12. These are given by

$$\bar{x} - c \pm \sqrt{(\bar{x} - c)^2 - 2\bar{x}r}.$$

13. A complete specification of the model requires assumptions on the number of (potential) borrowers and lenders that ensures an upward-sloping supply curve of funds. See Williamson (1987a) for details on one such specification.

7.2.4 Agency Costs

Adverse selection, moral hazard, and monitoring costs all arise as important factors in any relationship in which a principal delegates decision-making authority to an agent. In credit markets, the lender delegates to a borrower control over resources. The inability to monitor the borrower's actions or to share the borrower's information gives rise to agency costs. Bernanke and Gertler (1989) and Gertler (1988) have emphasized the role of agency costs that made external funding sources more expensive for firms than internal sources. As a consequence, a firm's balance sheet plays a role in affecting the cost of finance. In recessions, internal sources of funds decline, forcing firms to turn to external sources. But the deterioration of the firm's balance sheet worsens the agency problems and increases the cost of external funds, thereby further contracting investment spending and contributing to the recession. Thus, credit conditions can play a role in amplifying the impact of other shocks to the economy and affecting their propagation throughout the economy and through time.

In the model of Bernanke and Gertler (1989), firms are assumed to be able to observe the outcome of their own investment projects costlessly; others must incur a monitoring cost to observe project outcomes. Firms and lenders are assumed to be risk neutral. Firms are indexed by efficiency type ω, distributed uniformly on $[0, 1]$. More efficient types (ones with low ωs) need to invest fewer inputs in a given project. Projects themselves require inputs of $x(\omega)$, yielding gross payoff κ_1 with probability π_1 and $\kappa_2 > \kappa_1$ with probability $\pi_2 = 1 - \pi_1$. The function $x(.)$ is increasing in ω. The expected project return, $\pi_1\kappa_1 + \pi_2\kappa_2$, will be denoted κ. The realized outcome of a particular project can be observed costlessly by the firm undertaking the project and at cost c by others. Firms are assumed to have internal sources of financing equal to S; S is assumed to be less that $x(0)$, so that even the most efficient firm must borrow to undertake a project. Finally, let r denote the opportunity cost of funds to lenders; firms that do not undertake a project also receive this rate on their funds.[14]

If lenders could observe project outcomes costlessly, equilibrium would involve lenders financing all projects whose expected payoff exceeds their opportunity cost of rx. Thus, all firms whose ω is less than a critical value ω^* defined by

$$\kappa - rx(\omega^*) = 0$$

would receive loans. Firms with $\omega < \omega^*$ borrow $B \equiv x(\omega) - S$.

14. Bernanke and Gertler develop a general equilibrium model; we describe a partial equilibrium version to focus here on the role played by credit market imperfections in investment decisions.

With imperfect information, the firm clearly has an incentive to always announce that the bad outcome, κ_1, occurred. It will never pay for the lender to incur the monitoring cost if the firm announces κ_2. Let p be the probability that the firm is audited (i.e., the lender pays the monitoring cost to observe the true outcome) when the firm announces κ_1. Let P_1^a be the payment to the firm when κ_1 is announced and auditing takes place, P_1 the payment when κ_1 is announced and no auditing occurs, and P_2 the payment if κ_2 is announced. The optimal lending contract must maximize the expected payoff to the firm, subject to several constraints. First, the lender's expected return must be at least as great as her opportunity cost rB. Second, the firm must have no incentive to report the bad state when in fact the good state occurred. Third, even in the bad state, limited liability requires that P_1^a and P_1 be nonnegative. The optimal contract is characterized by the values of $\{p, P_1^a, P_1, P_2\}$ that solve

$$\max \pi_1[pP_1^a + (1-p)P_1] + \pi_2 P_2$$

subject to

$$\pi_1[\kappa_1 - p(P_1^a + c) - (1-p)P_1] + \pi_2[\kappa_2 - P_2] \geq rB \tag{7.6}$$

$$P_2 \geq (1-p)(\kappa_2 - \kappa_1 + P_1) \tag{7.7}$$

$$P_1^a \geq 0 \tag{7.8}$$

$$P_1 \geq 0 \tag{7.9}$$

and $0 \leq p \leq 1$.

Only the constraint given by (7.7) may require comment. The left side is the firm's payment in the good state. The right side gives the firm's payment if the good state occurs but the firm reports the bad state. After reporting the bad state, the firm is audited with probability p. So with probability $1-p$ the firm is not audited, turns over κ_1 to the lender, and receives P_1. But the firm now gets to keep the amount $\kappa_2 - \kappa_1$ since, by assumption, the good state had actually occurred. If (7.7) is satisfied, the firm has no incentive to conceal the truth in announcing the project outcome.

Assuming an interior solution, the first order necessary conditions for this problem are

$$\pi_1[(P_1^a - P_1) + \mu_1(P_1 - P_1^a - c)] + \mu_2(\kappa_2 - \kappa_1 + P_1) = 0 \tag{7.10}$$

$$\pi_1 p(1-\mu_1) + \mu_3 = 0 \tag{7.11}$$

$$\pi_1(1-p)(1-\mu_1) - \mu_2(1-p) + \mu_4 = 0 \tag{7.12}$$

$$\pi_2(1-\mu_1) + \mu_2 = 0, \tag{7.13}$$

where the μ_is are the (nonnegative) Lagrangian multipliers associated with the constraints (7.6)–(7.9).

Since $\mu_3 \geq 0$, (7.11) implies that $\mu_1 \geq 1$. This means the constraint on the lender's return (7.6) holds with equality. With $\pi_1[\kappa_1 - p(P_1^a + c) - (1-p)P_1] + \pi_2[\kappa_2 - P_2] - r(x - S) = 0$, this can be added to the objective function, yielding an equivalent problem that the optimal contract solves given by $\max[\pi_1(\kappa_1 - pc) + \pi_2\kappa_2]$, subject to (7.7) and the nonnegative constraints on P_1^a and P_1. However, $\pi_1(\kappa_1 - pc) + \pi_2\kappa_2 = \kappa - \pi_1 pc$, and with κ an exogenous parameter, this new problem is equivalent to minimizing expected auditing costs $\pi_1 pc$.

If the return to the lender, rB, is less than the project return even in the bad state κ_1, then no auditing is ever necessary and $p = 0$. Agency costs are zero, therefore, whenever $\kappa_1 \geq rB$. Recall that the amount borrowed, B, was equal to $x(\omega) - S$, where S represented the firm's internal funds invested in the project, so the no-agency-cost condition can be written

$$S \geq x(\omega) - \frac{\kappa_1}{r} \equiv S^*(\omega).$$

Any type ω with internal funds greater than or equal to $S^*(\omega)$ can always repay the lender, so no auditing on the project is required. When $S < S^*(\omega)$, a situation Bernanke and Gertler label as one of *incomplete collateralization*, constraints (7.6)–(7.9) all hold with equality. Since auditing is costly, the optimal auditing probability is just high enough to ensure that the firm truthfully reports the good state when it occurs. From the incentive constraint (7.7), $P_2 = (1-p)(\kappa_2 - \kappa_1)$ since $P_1 = P_1^a = 0$ (the firm keeps nothing in the bad state). Substituting this into the lender's required-return condition (7.6),

$$p = \frac{r[x(\omega) - S] - \kappa_1}{\pi_2(\kappa_2 - \kappa_1) - \pi_1 c}.$$

The auditing probability is decreasing in the return in the bad state (κ_1) and the firm's own contribution S. If the firm invests little in the project and borrows more, then the firm receives less of the project's return in the good state, increasing its incentive to falsely claim that the bad state occurred. To remove this incentive, the probability of auditing must rise.

Bernanke and Gertler characterize the expected costs of project auditing, $\pi_1 pc$, as the *agency costs due to asymmetric information*. As they show, some firms with intermediate values of ω (i.e., neither the most nor the least efficient) will find that the investment project is not worth undertaking if they have only low levels of internal funds to invest. The probability of auditing that lenders would require makes agency costs too high to justify the investment. If the firm had a higher level of internal funds, it would undertake the project. Even though the opportunity costs of funds r and the project inputs x and returns (κ_1 and κ_2) have not changed, variations in S can alter the number of projects undertaken. This illustrates how investment levels may depend on the firm's internal sources of financing. Agency costs drive a wedge between the costs of internal and external funds, so investment decisions will depend on variables such as cash flow that would not play a role if information were perfect. Since a recession will worsen firms' balance sheets, reducing the availability of internal funds, the resulting rise in agency costs and the reduction in investment may serve to amplify the initial cause of a recession.

7.3 Macroeconomic Implications

The presence of credit market imperfections can play a role in determining how the economy responds to economic disturbances and how these disturbances are propagated throughout the economy and over time. Various partial equilibrium models have provided insights into how imperfect information and costly state verification affect the nature of credit market equilibria. The next step is to embed these partial equilibrium models of the credit market within a general equilibrium macro model so that the qualitative and quantitative importance of credit channels can be assessed. As Bernanke, Gertler, and Gilchrist (1996) discuss, there are difficulties in taking this step. For one, distributional issues are critical. Private sector borrowing and lending do not occur in a representative-agent world, so agents must differ in ways that give rise to borrowers and lenders. And both the source of credit and the characteristics of the borrower matter, so not all borrowers and not all lenders are alike. Changes in the distribution of wealth or the distribution of cash flow can affect the ability of agents to obtain credit.

7.3.1 A Simple Model with Bank Loans

Bernanke and Blinder (1988) provide a modified version of a traditional IS-LM framework in which they incorporate a bank lending channel. The standard IS-LM model distinguishes between money and bonds as the only two financial assets. Money is assumed to pay a zero nominal interest rate, so the nominal rate determined in the IS-LM analysis is the return on bonds. Bernanke and Blinder modify this framework by distinguishing between money, bonds, and bank loans. With three

financial assets, the model will determine the interest rates on bonds and loans and the level of output consistent, for a given price level, with equilibrium in the money market, the market for bank loans, and the equality of output and aggregate demand. Since the focus here is on how monetary policy affects aggregate demand, we can ignore the supply side of the model and simply treat the price level as given.

The bank lending channel can be illustrated by adding a stylized banking sector to an otherwise standard IS-LM model. Banks are assumed to hold reserves (R), bonds (B), and loans (L) as assets; their liabilities are deposits (D). The representative bank's balance sheet is

$$B + L + R = D.$$

Assume that excess reserves are zero so that reserves are held only to meet the reserve requirement:

$$R^d = \sigma D,$$

where σ is the required reserve ratio on deposits. Loans and bond holdings must then sum to $(1-\sigma)D$. Bernanke and Blinder specify directly the banking sector's portfolio demands for bonds and loans as functions of total available assets after meeting reserve requirements and the returns on bonds I_b and loans I_l:

$$\frac{B}{(1-\sigma)D} = b(I_b, I_l): \quad b_b \geq 0, b_l \leq 0,$$

where b_j is the partial derivative with respect to I_j. The fraction of the bank's net of required reserves assets held in loans is assumed to be decreasing in i_b and increasing in i_l:

$$\frac{L}{(1-\sigma)D} = 1 - b(I_b, I_l) \equiv l^s(I_b, I_l); \quad l_b^s \leq 0, l_l^s \geq 0.$$

This equation gives the supply of loans.

In equilibrium, bank reserve demand must equal the reserve supply determined by the central bank, and the implied level of deposits supported by the supply of reserves must equal the demand for deposits by the nonbank public. Let R^s denote reserve supply, and let deposit demand depend on output (positively) and the interest rate on bonds (negatively). Then, equating reserve supply and demand, we can approximate the deviations around the steady state as

$$r^s = y_t - ci_b + v, \tag{7.14}$$

where lowercase letters denote percentage deviations around the steady state, so r^s and y are the percentage deviations of R^s and output around their steady-state values. To allow for money demand shocks (actually, deposit demand shocks), the random error v is included in (7.14).

Loan demand is assumed to depend on the interest rate on loans and the level of economic activity:

$$L^d = l^d(I_l, Y): \quad l_l^d \leq 0, l_y^d \geq 0.$$

Assuming no credit rationing, equilibrium in the market for loans requires

$$l^d(I_l, Y) = l^s(I_b, I_l)(1-\sigma)D = l^s(I_b, I_l)\left(\frac{1-\sigma}{\sigma}\right)R,$$

which can be approximated around the steady state as

$$l_l^d i_l + l_y^d y = l_b^s i_b + l_l^s i_l + r^s + \omega'$$

or

$$i_l = h_1 i_b + h_2 y - h_3 r^s + \omega, \tag{7.15}$$

where $h_1 = -l_b^s/(l_l^d - l_l^d)$, $h_2 = -l_y^s/(l_l^d - l_l^s)$, and $h_3 = -1/(l_l^d - l_l^s)$ are all positive, and $\omega = \omega'/(l_l^d - l_l^s)$ is a random disturbance that could incorporate both credit supply and credit demand shocks. An adverse credit supply shock would correspond to a positive realization of ω that increases the loan interest rate for given levels of reserve, output, and the bond rate. A positive credit demand shock would also correspond to a positive realization of ω.

Equation (7.15) can be combined with the deposit demand equation (7.14) and an IS relationship that links output demand to the interest rates on loans and bonds and a random disturbance:

$$y = -\phi_1 i_l - \phi_2 i_b + u. \tag{7.16}$$

Equations (7.14), (7.15), and (7.16) constitute a three-equation system to determine aggregate demand and the two interest rates as a function of reserve supply. Substituting the loan market condition (7.15) into the IS function (7.16), the loan rate can be eliminated, yielding

$$y = \frac{\phi_1 h_3 r^s - (\phi_2 + \phi_1 h_1) i_b + u - \phi_1 \omega}{1 + \phi_1 h_2}. \tag{7.17}$$

This modified IS curve reveals the key difference between a model that distinguishes

between bonds and loans and the standard IS-LM model: the quantity of reserves appears in the IS curve as long as $\phi_1 h_3 \neq 0$.

The framework suggested by Bernanke and Blinder attempts to capture in a simple way the additional linkages that arise through the bank lending version of the credit view. The approach, based on direct specification of behavioral relationships, is more in keeping with an older tradition in monetary economics, as exemplified by the work of Tobin and Brainard (1963), Tobin (1969), or Brunner and Meltzer (1972, 1988).[15] It does, however, provide a simple means of highlighting the critical factors in the bank lending channel. First, bank loans must be essential for spending ($\phi_1 \neq 0$). Suppose firms could substitute away from bank loans to other forms of intermediated credit with little cost. In this case, a rise in i_l might have a large impact on the quantity of bank loans outstanding, but ϕ_1 would be small and there would be little impact on aggregate spending. Second, the bank lending channel requires that $h_3 = -1/(l_l^d - l_l^s) \neq 0$. If loan supply or demand is perfectly elastic, then $h_3 = 0$ and the bank lending channel would not operate. Loan demand will be very elastic if alternative credit sources are available that serve as close substitutes for bank lending.

To investigate the conditions under which loan supply might be perfectly elastic, it is useful to be more explicit about the representative bank's portfolio decisions. Assume, in contrast to the above, that deposit liabilities come in two forms: demand deposits D_d that for simplicity will be assumed to pay a zero rate of interest but are subject to a reserve requirement ratio of σ, and D_{od}, other nonreservable deposits on which the bank pays an interest rate of i_d. The representative bank maximizes profits given by

$$i_b B + i_l L - i_d D_{od} - C(L, D_d, D_{od}) \tag{7.18}$$

subject to the balance sheet constraint

$$B + L = (1-\sigma)D_d + D_{od}, \tag{7.19}$$

where $C(l, D_d, D_{od})$ is the bank's cost function.[16] Assume that the return on loans is a decreasing function of the quantity of loans: $i_l(L)$, $i_l' \leq 0$. Assume also that the bank faces an upward-sloping supply of deposits function $D_{od}(i_d)$, $D_{od}'(i_d) \geq 0$. This

15. The Tobin-Brainard approach, with its emphasis on asset substitutability and the role of money substitutes, was labeled the *New View*. This was done to differentiate it from an older approach that built on money-multiplier analysis. In principle, there is no conflict between these approaches, but in practice, the money-multiplier view was characterized as ignoring the behavioral determinants of the ratios in the multiplier expressions. See Walsh (1992).

16. It is assumed that there are no significant costs associated with holding bonds.

latter function might arise from the types of credit market imperfections discussed previously since D_{od} represents a liability of the bank. The first order conditions imply that i_l and i_d satisfy

$$i_l = i_b + C_1 - i_l' L,$$

which implies that the marginal loan return $(i_l + i_l' L - C_1)$ is equated with the opportunity cost of funds i_b, and

$$i_d = i_b - \frac{C_3 D_{od}'}{1 + D_{od}'}, \tag{7.20}$$

which implies that the marginal cost of funds, $i_d + C_3 D_{od}'/(1 + D_{od}')$, is also equated with the opportunity cost of funds i_b. If the cost function is separable in demand deposits, then these two equations determine the interest rate on nondemand deposit liabilities and loan supply, $L^s = \theta(i_l - i_b - C_1)$, where $\theta = -1/i_l' > 0$, independent of reserves and demand deposits.

Equating loan supply and loan demand,

$$i_l = i_b + C_1 - i_l' l^d(i_l, y). \tag{7.21}$$

Equations (7.20) and (7.21) determine i_l and i_d as functions of i_b.[17] Bond holdings are residually determined by the bank's balance sheet and are equal to

$$B = \left(\frac{1-\sigma}{\sigma}\right) R + D_{od}(i_d) - L.$$

A change in reserves that changes demand deposits is entirely reflected in security holdings. In the aggregate, an increase in reserves will lower i_b and increase loan supply, but the effects operate through traditional interest-rate channels. With banks holding securities and having access to managed liabilities, such as CDs, the marginal return on loans will be set equal to the marginal opportunity cost of funds. Variations in reserve supply will affect the quantity of bank loans, but they will do so through the banking sector's response to changes in market interest rates (see Romer and Romer 1990).

If the cost function is not separable in demand deposits, then D_d affects C_1 and C_3, the marginal costs of servicing loans and other deposits. This might be the case if the provision of transactions accounts lowers the costs to banks of monitoring bor-

17. The loan demand function and the other deposit supply functions must be used to eliminate L and D from the cost function.

rowers. If loans and demand deposits are complements in the bank's cost function, then a change in reserves that lowers deposits may directly raise the cost of loans, leading to a shift in the loan supply function. This shift would then represent a distinct bank lending channel leading to a drop in loans in addition to the reduction that results from the impact of a rise in i_b on the loan interest rate.

Bank lending channels were likely to have been more important during periods in which financial markets were more heavily regulated. In the United States, Regulation Q imposed limits on the deposit interest rates banks and other financial institutions could offer and limited the ability of banks to offset declines in banking sector reserves. In an environment in which interest rates are free to adjust, the bank lending channel of monetary policy is likely to be of lesser importance. Such a conclusion is still consistent with the notion that banks cater to a particular clientele. For that reason, most discussions of the credit channel focus on the characteristics of borrowers and not on those of lenders. This "broad" credit channel (Oliner and Rudebusch 1995) does not give a prominent role to banks but instead stresses the general implications of credit market imperfections for different types of borrowers.

7.3.2 General Equilibrium Models

The micro literature on imperfect information provides insights into the structure of credit markets. Embedding these insights in a macroeconomic framework to determine how credit markets affect the nature of the equilibrium and the manner in which the economy responds to macro disturbances is much more difficult. In representative-agent models, no lending actually takes place. And with all agents identical, the distinctive features of credit markets that have been emphasized in the literature on credit channels are absent. Incorporating heterogeneity among agents in a tractable general equilibrium model is difficult, particularly when the nature of debt and financial contracts in the model economy should be derived from the characteristics of the basic technology and informational assumptions of the model environment.

Two early examples of general equilibrium models designed to highlight the role of credit factors were due to Williamson (1987b) and Bernanke and Gertler (1989). In these models, credit markets play an important role in determining how the economy responds to a real productivity shock. Williamson embeds his model of financial intermediation with costly monitoring (discussed in section 2.3) in a dynamic general equilibrium model. In response to shocks to the riskiness of investment, credit rationing increases, loans from intermediaries fall, and investment declines. The decline in investment reduces future output and contributes to the propagation of the initial shock. Bernanke and Gertler (1989) incorporate the model of costly state verification reviewed in section 2.4 into a general equilibrium framework in which shocks to productivity drive the business-cycle dynamics. A positive productivity shock increases the income of the owners of the production technology; this rise in their net worth lowers agency costs associated with external financing of investment projects, allowing for increased investment. This serves to propagate the shock through time.

Kiyotaki and Moore (1997) have developed a model that illustrates the role of net worth and credit constraints on equilibrium output. In their model economy, there are two types of agents. One group, called *farmers*, can combine their own labor with land to produce output. They can borrow to purchase additional land but face credit constraints in so doing. These constraints arise because a farmer's labor input is assumed to be critical to production—once a farmer starts producing, no one else can replace her—and the farmer is assumed to be unable to precommit to work. Thus, if any creditor attempts to extract too much from a farmer, the farmer can simply walk away from the land, leaving the creditor with only the value of the land; all current production is lost. The inability to precommit to work plays a role similar to the assumption of cost state verification; in this case, the creditor is unable to monitor the farmer to ensure that she continues to work. As a result, the farmer's ability to borrow will be limited by the collateral value of her land.

Letting k_t denote the quantity of land cultivated by farmers, output by farmers is produced according to a linear technology:

$$y_{t+1}^f = (a+c)k_t,$$

where ck_t is nonmarketable output ("bruised fruit" in the farmer analogy) that can be consumed by the farmer.

The creditors in Kiyotaki and Moore's model are called *gatherers*. They too can use land to produce output, using a technology characterized by decreasing returns to scale. The output of gatherers is

$$y_{t+1}^g = G(\bar{k} - k_t); \quad G' \geq 0; G'' \leq 0,$$

where $\bar{k}$ is the total fixed stock of land, so $\bar{k} - k_t$ is the land cultivated by gatherers.

Utility of both farmers and gatherers is assumed to be linear in consumption, although gatherers are assumed to discount the future more. Because of the linear utility, and the assumption that labor generates no disutility, the socially efficient allocation of the fixed stock of land between the two types of agents would ensure

that the marginal product of land is equalized between the two production technologies, or

$$G'(\bar{k} - k^*) = a + c, \tag{7.22}$$

where k^* is the efficient amount of land allocated to farmers.

We can now consider the market equilibrium. Taking the gatherers first, given that they are not credit constrained and have linear utility, the real rate of interest will simply equal the inverse of their subjective rate of time preference: $R = 1/\beta$.[18] Again exploiting the unconstrained nature of the gatherers's decision, the value of a unit of land, q_t, must satisfy

$$q_t = \beta[G'(\bar{k} - k_t) + q_{t+1}].$$

The present value of a unit of land is just equal to the discounted marginal return G' plus its resale value at time $t+1$. Since $\beta = R^{-1}$, this condition can be rewritten as

$$\frac{1}{R}G'(\bar{k} - k_t) = q_t - \frac{q_{t+1}}{R} \equiv u_t. \tag{7.23}$$

The variable u_t will play an important role in the farmers' decision problem. To interpret it, q_{t+1}/R is the present value of land in period $t+1$. This represents the collateralized value of a unit of land; a creditor who lends q_{t+1}/R or less against a piece of land is sure of being repaid. The price of a unit of land at time t is q_t, so u_t is the difference between the cost of the land and the amount that can be borrowed against the land. It thus represents the down payment a farmer will need to make in order to purchase more land.

Kiyotaki and Moore construct the basic parameters of their model to ensure that farmers will wish to consume only their nonmarketable output (ck_{t-1}). Farmers then use the proceeds of their marketable output plus new loans minus repayment of old loans (inclusive of interest) to purchase more land. However, the maximum a farmer can borrow will be the collateralized value of the land, equal to $q_{t+1}k_t/R$. Hence, if b_t is the farmer's debt,

$$b_t \leq \frac{q_{t+1}k_t}{R}. \tag{7.24}$$

18. The standard Euler condition for optimal consumption requires that $u_c(t) = \beta R u_c(t+1)$, where $u_c(s)$ is the marginal utility of consumption at date s. With linear utility, $u_c(t) = u_c(t+1) = h$ for some constant h. Hence, $h = \beta R h$ or $R = 1/\beta$.

This can be shown to be a binding constraint in equilibrium, and the change in the farmer's land holdings will be

$$q_t(k_t - k_{t-1}) = ak_{t-1} + \frac{q_{t+1}k_t}{R} - Rb_{t-1},$$

where b_{t-1} is debt incurred in the previous period. Rearranging,

$$k_t = \frac{(a + q_t)k_{t-1} - Rb_{t-1}}{u_t}. \tag{7.25}$$

The numerator of this expression represents the farmer's net worth—current output plus land holdings minus existing debt. With u_t equal to the required down payment per unit of land, the farmer invests her entire net worth in purchasing new land.

To verify that the borrowing constraint is binding, it is necessary to show that the farmer always finds it optimal to use all marketable output to purchase additional land (after repaying outstanding loans). Suppose instead that the farmer consumes a unit of output over and above ck_{t-1}. This yields marginal utility u_c (a constant by the assumption of linear utility), but by reducing the farmer's land in period t by $1/u_t$, this additional consumption costs

$$u_c\left[\beta_f \frac{c}{u_t} + \beta_f^2\left(\frac{a}{u_t}\left(\frac{c}{u_{t+1}} + \beta_f\left(\frac{a}{u_{t+1}}\left(\frac{c}{u_{t+2}} + \cdots\right)\cdots\right)\cdots\right)\cdots\right)\right]$$

since the $1/u_t$ units of land purchased at time t would have yielded additional consumption c/u_t plus marketable output a/u_t that could have been used to purchase more land that would have yielded c/u_{t+1} in consumption, and so on. Each of these future consumption additions must be discounted back to time t using the farmer's discount rate β_f. As will be demonstrated below, the steady-state value of u will be a. Making this substitution, the farmer will always prefer to use marketable output to purchase land if

$$1 < \left[\beta_f \frac{c}{a} + \beta_f^2\left(\frac{a}{a}\left(\frac{c}{a} + \beta_f\left(\frac{a}{a}\left(\frac{c}{a} + \cdots\right)\cdots\right)\cdots\right)\cdots\right)\right] = \frac{\beta_f}{1 - \beta_f}\frac{c}{a}$$

or

$$\frac{a + c}{a} > \frac{1}{\beta_f} > R. \tag{7.26}$$

Kiyotaki and Moore assume that c is large enough to ensure that this condition holds. This means farmers would always like to postpone consumption and will borrow as much as possible to purchase land. Hence, the borrowing constraint will bind.

Equation (7.25) can be written as $u_t k_t = (a + q_t)k_{t-1} - Rb_{t-1}$. But $Rb_{t-1} = q_t k_{t-1}$ from (7.24), so $u_t k_t = ak_{t-1}$. Now using (7.23) to eliminate u_t, the capital stock held by farmers satisfies the following difference equation:

$$\frac{1}{R}G'(\bar{k} - k_t)k_t = ak_{t-1}. \tag{7.27}$$

Assuming standard restrictions on the gatherers's production function, (7.27) defines a convergent path for the land held by farmers.[19] The steady-state value of k is then given as the solution k^{ss} to

$$\frac{1}{R}G'(\bar{k} - k^{ss}) = a. \tag{7.28}$$

Multiplying through by R, $G'(\bar{k} - k^{ss}) = Ra$. From (7.23) this implies

$$u^{ss} = a.$$

Equation (7.28) can be compared with (7.22), which gives the condition for an efficient allocation of land between farmers and gatherers. The efficient allocation of land to farmers, k^*, was such that $G'(\bar{k} - k^*) = a + c > Ra = G'(\bar{k} - k^{ss})$, where the inequality sign is implied by (7.26). Since the marginal product of gatherers's output is positive but declines with the amount of land held by gatherers, it follows that $k^{ss} < k^*$. The market equilibrium is characterized by too little land in the hands of farmers. As a consequence, aggregate output is too low.

Using the definition of u, the steady-state price of land is equal to $q^{ss} = Ra/(R - 1)$, and steady-state debt is equal to $b^{ss} = q^{ss}k^{ss}/R = ak^{ss}/(R - 1)$. The farmer's debt repayments each period are then equal to $Rb^{ss} = [R/(R - 1)]ak^{ss} > ak^{ss}$.

Kiyotaki and Moore extend this basic model to allow for reproducible capital and are able to study the dynamics of the more general model. The simple version, though, allows the key channels through which credit affects the economy's equilibrium to be highlighted. First, output is inefficiently low due to borrowing restrictions; even though farmers have access to a technology that, at the steady state, is more productive than that of gatherers, they cannot obtain the credit necessary to purchase additional land. Second, the ability of farmers to obtain credit is limited by their net worth. Equation (7.25) shows how the borrowing constraint makes land holdings at time t dependent on net worth (marketable output plus the value of existing land holdings minus debt). Third, land purchases by farmers will depend on asset prices.

19. As long as $G'(\bar{k} - k)$ is monotonically increasing in k, $G'(\bar{k}) < a$, and $G'(0) > a$, there will be a single stable equilibrium.

A fall in the value of land that is expected to persist (so q_t and q_{t+1} both fall) reduces the farmers' net worth and demand for land. This follows from (7.25), which can be written as $k_t = (q_t k_{t-1}/u_t) + (ak_{t-1} - Rb_{t-1})/u_t$. A proportional fall in q_t and q_{t+1} leaves the first term, $q_t k_{t-1}/u_t$, unchanged. The second term increases in absolute value, but at the steady state, $Rb > ak$, so this term is negative. Thus, farmers' net worth declines with a fall in land prices.

These mechanisms capture the financial accelerator effects, as can be seen by considering the effects of an unexpected but transitory productivity shock. Suppose the output of both farmers and gatherers increases unexpectedly at time t. If the economy was initially at the steady state, then if Δ is the productivity increase for farmers, (7.25) implies

$$u(k_t)k_t = (a + \Delta a + q_t - q^{ss})k^{ss}, \tag{7.29}$$

since $q^{ss}k^{ss} = Rb^{ss}$ from the borrowing constraint and we have written the required down payment u as a function of k.[20] Two factors are at work in determining the impact of the productivity shock on the farmers' demand for land. First, because marketable output rises by Δak^{ss}, this directly increases farmers' demand for land. Second, the term $(q_t - q^{ss})k^{ss}$ represents a capital gain on existing holdings of land. Both factors act to increase farmers' net worth and their demand for land.

One way to highlight the dynamics is to examine a linear approximation to (7.29) around the steady state. Letting e denote the elasticity of the user cost of land $u(k)$ with respect to k, the left side of (7.29) can be approximated by

$$ak^{ss}[1 + (1 + e)\hat{k}],$$

where we have used the fact that $u(k^{ss}) = a$ and have let $\hat{x}$ denote the percentage deviation of a variable x around the steady state.[21] The right side is approximated by

$$(a + \Delta a + q^{ss}\hat{q}_t)k^{ss}.$$

Equating these two and using the steady-state result that $q^{ss} = Ra/(R-1)$ yields

$$(1 + e)\hat{k} = \Delta + \frac{R}{R-1}\hat{q}_t. \tag{7.30}$$

The capital gain effect on farmers' land purchases is, as Kiyotaki and Moore emphasize, scaled up by $R/(R-1) > 1$ because farmers are able to leverage their net worth. This factor can be quite large; if $R = 1.05$, the coefficient on $\hat{q}_t$ is 21.

20. Recall that $u_t = G'(\bar{k} - k_t)/R$ from (7.23).

21. The elasticity e is equal to $[u'(k^{ss})k^{ss}]/u(k^{ss}) = u'(k^{ss})k^{ss}/a$, where u' denotes the derivative of u with respect to k. Since u is increasing in $\bar{k} - k$, $u' < 0$.

Consistent with the notion of the financial accelerator, the asset price effects of the temporary productivity shock reinforce the original disturbance. These effects also generate a channel for persistence. When more land is purchased in period t, the initial rise in aggregate output persists.[22]

7.3.3 Agency Costs and General Equilibrium

Carlstrom and Fuerst (1997) embed a model of agency costs based on Bernanke and Gertler (1989) in a general equilibrium framework that can then be used to investigate the model's qualitative and quantitative implications. In particular, they are able to study the way agency costs arising from costly state verification affect the impact that shocks to net worth have on the economy.[23]

In their model, entrepreneurs borrow external funds in an intraperiod loan market to invest in a project that is subject to idiosyncratic productivity shocks. Suppose entrepreneur j has a net worth of n_j and borrows $i_j - n_j$. The project return is $\omega_j i_j$, where ω_j is the idiosyncratic productivity shock. Entrepreneurs have private information about this shock, while lenders can observe it only by incurring a cost. If the interest rate on the loan to entrepreneur j is r_j^k, then the borrower defaults if

$$\omega_j < \frac{(1 + r_j^k)(i_j - n_j)}{i_j} \equiv \bar{\omega}_j.$$

If the realization of ω_j is less than $\bar{\omega}_j$, the entrepreneur's resources, $\omega_j i_j$, are less than the amount needed to repay the loan, $(1 + r_j^k)(i_j - n_j)$. If default occurs, the lender monitors the project at a cost μi_j.

Carlstrom and Fuerst derive the optimal loan contract between entrepreneurs and lenders and show that it is characterized by i_j and $\bar{\omega}_j$. Given these two parameters, the loan interest rate is

$$1 + r_j^k = \frac{\bar{\omega}_j i_j}{i_j - n_j}.$$

Suppose the distribution function of ω_j is $\Phi(\omega_j)$. The probability of default is $\Phi(\bar{\omega}_j)$. Let q denote the end-of-period price of capital. If the entrepreneur does not default, she receives $q\omega_j i_j - (1 + r_j^k)(i_j - n_j)$. If the borrower defaults, she receives

22. Recall that at the margin, farmers are more productivity than gatherers; a shift of land from gatherers to farmers raises total output.

23. See also Kocherlakota (2000).

nothing. If $f(\bar{\omega})$ is defined as the fraction of expected net capital output received by the entrepreneur, then

$$qi_jf(\bar{\omega}_j) \equiv q\left\{\int_{\bar{\omega}}^{\infty} \omega i_j\Phi(d\omega) - [1-\Phi(\bar{\omega}_j)](1+r_j^k)(i_j-n_j)\right\}$$
$$= qi_j\left\{\int_{\bar{\omega}}^{\infty} \omega\Phi(d\omega) - [1-\Phi(\bar{\omega}_j)]\bar{\omega}_j\right\}. \quad (7.31)$$

The expected income of the lender is

$$q\left\{\int_0^{\bar{\omega}} \omega i_j\Phi(d\omega) - \mu i_j\Phi(\bar{\omega}_j) + [1-\Phi(\bar{\omega}_j)](1+r_j^k)(i_j-n_j)\right\}.$$

If $g(\bar{\omega}_j)$ is defined as the fraction of expected net capital output received by the lender, then

$$qi_jg(\bar{\omega}_j) \equiv qi_j\left\{\int_0^{\bar{\omega}} \omega\Phi(d\omega) - \mu\Phi(\bar{\omega}_j) + [1-\Phi(\bar{\omega}_j)]\bar{\omega}_j\right\}. \quad (7.32)$$

By adding together (7.31) and (7.32), one finds that

$$f(\bar{\omega}_j) + g(\bar{\omega}_j) = 1 - \mu\Phi(\bar{\omega}_j) < 1. \quad (7.33)$$

Hence, the total expected income to the entrepreneur and the lender is less than the total expected project return (the fractions sum to less than 1) because of the expected monitoring costs.

The optimal lending contract maximizes $qif(\bar{\omega})$ subject to

$$qig(\bar{\omega}) \geq i - n \quad (7.34)$$

and

$$qif(\bar{\omega}) \geq n,$$

where, for convenience, the j notation has been dropped. The first constraint reflects the assumption these are intraperiod loans, so the lender just needs to be indifferent between lending and retaining funds. The second constraint must hold if the entrepreneur is to participate; it ensures that the expected payout to the entrepreneur is greater than the net worth the entrepreneur invests in the project. Carlstrom and Fuerst show that this second constraint always holds, so it will be ignored in the following. Using (7.33), the optimal loan contract solves

$$\max_{i,\bar{\omega}}\{qif(\bar{\omega}) + \lambda[qi(1-\mu\Phi - f(\bar{\omega})) - i + n]\}.$$

The first order conditions for i and $\bar{\omega}$ are

$$qf(\bar{\omega}) + \lambda[q(1-\mu\Phi - f(\bar{\omega})) - 1] = 0 \quad (7.35)$$

and

$$qif'(\bar{\omega}) - \lambda qi(\mu\phi + f'(\bar{\omega})) = 0,$$

where $\phi = \Phi'$ is the density function for ω. Solving this second equation for λ,

$$\lambda\left[1 + \frac{\mu\phi(\bar{\omega})}{f'(\bar{\omega})}\right] = 1. \quad (7.36)$$

Now multiplying both sides of (7.35) by $[1 + \mu\phi(\bar{\omega})/f'(\bar{\omega})]$ and using (7.36) yields, after some rearrangement,

$$q\left[1 - \mu\Phi + \mu\phi(\bar{\omega})\frac{f(\bar{\omega})}{f'(\bar{\omega})}\right] = 1. \quad (7.37)$$

Finally, from the constraint (7.34),

$$qig(\bar{\omega}) = i - n. \quad (7.38)$$

Equation (7.37) determines $\bar{\omega}$ as a function of the price of capital q, the distribution of the shocks, and the cost of monitoring. All three of these factors are the same for all entrepreneurs, so all borrowers face the same $\bar{\omega}$, justifying us in dropping the j subscript. Writing $\bar{\omega} = \bar{\omega}(q)$, investment i can be expressed using (7.38) as a function of q and n:

$$i(q,n) = \left[\frac{1}{1 - qg(\bar{\omega}(q))}\right]n. \quad (7.39)$$

Expected capital output is

$$I^s(q,n) = i(q,n)[1 - \mu\Phi(\bar{\omega})]. \quad (7.40)$$

The optimal contract has been derived while taking the price of capital, q, as given. In a general equilibrium analysis, this price must also be determined. To complete the model specification, assume that firms produce output using a standard neoclassical production function employing labor and capital:

$$Y_t = \theta_t F(K_t, H_t)$$

where θ_t is an aggregate productivity shock. Factor markets are competitive. Households supply labor and rent capital to firms. If households wish to accumulate more capital, they can purchase investment goods at the price q_t from a mutual fund that lends to entrepreneurs. These entrepreneurs then create capital goods using the project technology described above and end the period by making their consumption decision.[24] This last choice then determines the net worth entrepreneurs carry into the following period.

If net worth is constant, Carlstrom and Fuerst show that their general equilibrium model can be mapped into a standard real-business-cycle model with capital adjustment costs. They argue, therefore, that agency costs provide a means of endogenizing adjustment costs. Because net worth is not constant in their model, however, variations in entrepreneur net worth can serve to propagate shocks over time. For example, a positive productivity shock increases the demand for capital, and this pushes up the price of capital. By increasing entrepreneurs' net worth, the rise in the price of capital increases the production of capital (see 7.40). By boosting the return on internal funds, the rise in the price of capital also induces entrepreneurs to reduce their own consumption to build up additional net worth. The endogenous response of net worth causes investment to display a hump-shaped response to an aggregate productivity shock. This type of response is more consistent with empirical evidence than is the response predicted by a standard real-business-cycle model in which the maximum impact of a productivity shock on investment occurs in the initial period.

7.3.4 Agency Costs and Sticky Prices

In chapter 5, it was emphasized that nominal rigidities play an important role in transmitting monetary policy disturbances to the real economy. Bernanke, Gertler, and Gilchrist (1999) have combined nominal rigidities with an agency cost model to explore the interactions between credit market factors and price stickiness. They developed a tractable model with the complications introduced by both credit factors and sticky prices by employing a model with three types of agents—households, entrepreneurs, and retailers. Entrepreneurs borrow to purchase capital. Costly state verification in the Bernanke, Gertler, and Gilchrist model implies that investment will depend positively on entrepreneurs' net worth, just as it did in the Carlstrom and Fuerst (1997) model (see 7.39). Entrepreneurs use capital and labor to produce

24. Carlstrom and Fuerst assume that entrepreneurs discount the future more heavily than households and that their utility is linear. The Euler condition for entrepreneurs is

$$q_t = \beta\gamma \mathrm{E}_t[q_{t+1}(1-\delta) + F_K(t+1)]\left[\frac{q_{t+1}f(\bar{\omega}_{t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1})}\right] \quad 0 < \gamma < 1,$$

where the first term on the right side is the return to capital and the second term is the additional return on internal funds.

wholesale goods. These wholesale goods are sold in a competitive goods market to retailers. Retailers use wholesale goods to produce differentiated consumer goods that are sold to households. Wholesale prices are flexible, but retail prices are sticky. This model exhibits a financial accelerator (Bernanke, Gertler, and Gilchrist 1996): movements in asset prices affect net worth and amplify the impact of an initial shock to the economy.

Sticky price adjustment in the retail sector is modeled following Calvo. The probability that an individual retail firm can adjust its price is $1-\omega$ each period. When a firm does adjust, it sets its price optimally. As a result, the rate of inflation of retail prices is given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa mc_t,$$

where mc_t is the percentage deviation of real marginal cost in the retail sector from its steady-state value.[25] Since retail firms simply purchase wholesale goods at the competitive wholesale price P_t^w and resell these goods to households, real marginal cost for retailers is just the ratio of wholesale to retail prices. The inflation equation can therefore be written as

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \kappa z_t,$$

where z_t is the percentage deviation of the retail price markup over its steady-state value.

The retail price markup affects equilibrium in the labor market. With competitive factor markets, the real product wage is equal to the marginal product of labor and the real wage in terms of retail prices is equal to the marginal rate of substitution between leisure and consumption. Thus, with a Cobb-Douglas, constant returns to scale, production function and the utility function given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta},$$

the labor market equilibrium condition can be expressed in terms of percentage deviations around the steady state as

$$y_t - n_t - z_t = \eta n_t + \sigma c_t.$$

Bernanke, Gertler, and Gilchrist (1999) calibrate a log-linearized version of their model to study the role the financial accelerator plays in propagating the impact of a

25. For details on the derivation of the inflation-adjustment equation in a Calvo-type model of price stickiness, see chapter 5.

monetary policy shock. They find that it increases the real impact of a policy shock. A positive nominal interest-rate shock reduces the demand for capital, and this lowers the price of capital. The decline in the value of capital lowers entrepreneurs' net worth. As a consequence, the finance premium demanded by lenders rises, and this further reduces investment demand. Thus, a multiplier effect operates to amplify the initial impact of the interest-rate rise. The contraction in the wholesale sector lowers wholesale prices relative to sticky retail prices. The retail price markup increases, reducing retail price inflation.

7.4 Does Credit Matter?

The theoretical work incorporating imperfect information in models of credit provides important insights into the nature of credit transactions, the characteristics of financial contracts, and the properties of credit market equilibria. A separate set of issues revolves around whether these aspects of credit markets are important for determining the impact of monetary policy on the economy and/or play an independent role as a source of economic disturbances. If credit channels are important for the monetary transmission process, then evolution in financial markets, whether caused by changes in regulations or to financial innovations, may change the manner in which monetary policy affects the real economy. It also implies that the level of real interest rates may not provide a sufficient indicator of the stance of monetary policy. And credit shocks may have played an independent role in creating economic fluctuations. In this section, the empirical evidence on the credit channel is reviewed. The coverage will be selective, as a number of recent surveys exist that discuss (and extend) the empirical work in the area (Gertler 1988, Gertler and Gilchrist 1993, Ramey 1993, Kashyap and Stein 1994, Hubbard 1995, Bernanke, Gertler, and Gilchrist 1996).

In an influential article, Bernanke (1983) provided evidence consistent with an important role for nonmonetary financial factors in accounting for the severity of the Great Depression in the United States. After controlling for unexpected money growth, he found that proxies for the financial crises of the early 1930s contributed significantly to explaining the growth rate of industrial production in his regression analysis.[26] If pure monetary causes were responsible for the decline in output during the Depression, the other measures of financial disruptions should not add explanatory power to the regression.

26. Bernanke employed the real change in the deposits at failing banks and the real change in the liabilities of failing businesses as his measures of the financial crises.

As Bernanke notes, his evidence is "not inconsistent" with the proposition that the financial crisis in the United States represented a distinct nonmonetary channel through which real output was affected during the Depression. The evidence is not conclusive, however, because an alternative hypothesis is simply that the Depression itself was the result of nonmonetary factors (or at least factors not captured by unanticipated money growth) and that these factors caused output to decline, businesses to fail, and banks to close. By controlling only for unanticipated money growth, Bernanke's measures of financial crisis may only be picking up the effects of the underlying nonmonetary causes of the Depression. Still, Bernanke's results offered support for the notion that the massive bank failures of the 1930s in the United States were not simply a sideshow but were at least partially responsible for the output declines.

Attempts to isolate a special role for credit in more normal business-cycle periods have been plagued by what are essentially similar identification problems. Are movements in credit aggregates a reflection of shifts in demand resulting from effects operating through the traditional money channel or do they reflect supply factors that constitute a distinct credit channel? Most macroeconomic variables behave similarly under either a money view or a credit view, so distinguishing between the two views based on time-series evidence is difficult. For example, under the traditional money channel view, a contractionary shift in monetary policy raises interest rates and reduces investment spending. The decline in investment is associated with a decline in credit demand, so quantity measures of both bank and nonbank financing should fall. The competing theories are not sufficiently powerful to allow us to draw sharp predictions about the timing of interest-rate, money, credit, and output movements that would allow the alternative views to be tested. As a consequence, much of the empirical work has focused on compositional effects, seeking to determine whether there are differential impacts of interest-rate and credit movements that might distinguish between the alternative views. After some of the evidence on the bank lending channel is considered, the evidence on the broad credit channel is discussed.

7.4.1 The Bank Lending Channel

Banks play an important role in discussions of the monetary transmission mechanism, but the traditional approach stresses the role of bank liabilities as part of the money supply. Part of the reason for the continued focus on the liabilities side is the lack of convincing empirical evidence that bank lending plays a distinct role in the transmission process through which monetary policy affects the real economy. As Romer and Romer (1990) state:

> A large body of recent theoretical work argues that the Federal Reserve's leverage over the economy may stem as much from the distinctive properties of the loans that banks make as from the unique characteristics of the transaction deposits that they receive.... Examining the behavior of financial variables and real output in a series of episodes of restrictive monetary policy, we are unable to find any support for this view. (pp. 196–197)

One of the first attempts to test for a distinct bank lending channel was that of S. King (1986). He found that monetary aggregates were better predictors of future output than were bank loans. More recently, Romer and Romer (1990) and Ramey (1993) have reached similar conclusions. Unfortunately, our theories are usually not rich enough to provide sharp predictions about timing patterns that are critical for drawing conclusions from evidence on the predictive content of macro variables. This is particularly true when behavior depends on forward-looking expectations. Anticipations of future output movements can lead to portfolio and financing readjustments that will affect the lead-lag relationship between credit measures and output. Because a decline in output may be associated with inventory buildups, the demand for short-term credit can initially rise, and the existence of loan commitments will limit the ability of banks to alter their loan portfolios quickly. These factors make money-credit-output timing patterns difficult to interpret.

In part, Romer and Romer's negative assessment quoted above reflects the difficult identification problem mentioned earlier. A policy-induced contraction of bank reserves will lead to a fall in both bank liabilities (deposits) and bank assets (loans and securities). With both sides of the banking sector's balance sheet shrinking, it is clearly difficult to know whether to attribute a subsequent decline in output to the money channel, the credit channel, or both.[27] Kashyap, Stein, and Wilcox (1993) address this problem by examining the composition of credit between bank and nonbank sources. Under the money view, a contractionary policy raises interest rates, lowering aggregate demand and the total demand for credit. Consequently, all measures of outstanding credit should decline. Under the bank lending view, the contractionary policy has a distinct effect in reducing the supply of bank credit. With bank credit less available, borrowers will attempt to substitute other sources of credit, and the relative demand for nonbank credit should rise. Thus, the composition of credit should change if the bank lending view is valid, with bank credit falling more in response to contractionary monetary policy than other forms of credit.

27. The identification problems are not quite so severe in attempting to estimate the role of credit-supply versus credit-demand shocks on the economy. A contractionary bank credit-supply shock would generally lower loan quantity and raise loan interest rates; a contraction in loan quantity caused by a demand shock would lower loan interest rates.

Kashyap, Stein, and Wilcox do find evidence for the bank lending channel when they examine aggregate U.S. data on bank versus nonbank sources of finance, the latter measured by the stock of outstanding commercial paper. Using the Romer and Romer (1989) dates to identify contractionary shifts in monetary policy,[28] Kashyap, Stein, and Wilcox find that the financing mix shifts away from bank loans following a monetary contraction. However, this occurs primarily because of a rise in commercial paper issuance, not a contraction in bank lending.

Evidence based on aggregate credit measures can be problematic, however, if borrowers are heterogeneous in their sensitivity to the business cycle and in the types of credit they use. For example, the sales of small firms fluctuate more over the business cycle than those of large firms, and small firms are more reliant on bank credit than large firms that have greater access to the commercial paper market. Contractionary monetary policy that causes both small and large firms to reduce their demand for credit will cause aggregate bank lending to fall relative to nonbank financing as small firms contract more than large firms. This could account for the behavior of the debt mix even in the absence of any bank lending channel. Oliner and Rudebusch (1995, 1996b) argue that this is exactly what happens. They use disaggregate data on large and small firms and show that, in response to a monetary contraction, there is no significant effect on the mix of bank/nonbank credit used by either small or large firms. Instead, the movement in the aggregate debt mix arises because of a general shift of short-term debt away from small firms and toward large firms. They conclude that the evidence does not support the bank lending channel as an important part of the transmission process of monetary policy. Similar conclusions are reached by Gertler and Gilchrist (1994) in an analysis also based on disaggregated data.

While the bank lending channel as part of the monetary policy transmission process may not be operative, it might still be the case that shifts in bank loan supply are a cause of economic fluctuations. In the United States, the 1989–1992 period generated a renewed interest in credit channels and monetary policy.[29] An unusually large decline in bank lending and stories, particularly from New England, of firms facing difficulty borrowing, led many to seek evidence that credit markets played an independent role in contributing to the 1990–1991 recession. One difficulty in attempting

28. Romer and Romer (1989) base their dating of monetary policy shifts on a reading of FOMC documents. See chapter 1.

29. See, for example, Bernanke and Lown (1992), the papers collected in Federal Reserve Bank of New York (1994), and Peek and Rosengren (1995).

to isolate the impact of credit supply disturbances is the need to separate movements caused by a shift in credit supply from movements due to changes in credit demand.

Walsh and Wilcox (1995) estimate a monthly VAR in which bank loan supply shocks are identified with innovations in the prime lending rate. They show that their estimated loan supply innovations are related to changes in bank capital ratios, changes in required reserves, and the imposition of credit controls. This provides some evidence that the innovations are actually picking up factors that affect the supply of bank loans. While prime rate shock are estimated to lower loan quantity and output, they were not found to play a major causal role in U.S. business cycles, although their role was somewhat atypically large during the 1990–1991 recession.

7.4.2 The Broad Credit Channel

The broad credit channel for the transmission of monetary policy is based on the view that credit market imperfections are not limited to the market for bank loans but instead are important for understanding all credit markets. With agency costs creating a wedge between internal and external finance, measures of cash flow, net worth, and the value of collateral should affect investment spending in ways not captured by traditional interest-rate channels. The evidence in support of a broad credit channel has recently been surveyed by Bernanke, Gertler, and Gilchrist (1996), who conclude: "we now have fairly strong evidence—at least for the case of firms—that downturns differentially affect both the access to credit and the real economic activity of high-agency-cost borrowers" (p. 14).

Hubbard (1995) and Bernanke, Gertler, and Gilchrist (1996) list three empirical implications of the broad credit channel. First, external finance is more expensive for borrowers than internal finance. This should apply particularly to uncollateralized external finance. Second, because the cost differential between internal and external finance arises from agency costs, the gap should depend inversely on the borrower's net worth. A fall in net worth raises the cost of external finance. Third, adverse shocks to net worth should reduce borrowers' access to finance, thereby reducing their investment, employment, and production levels.

If, as emphasized under the broad credit channel, agency costs increase during recessions and in response to contractionary monetary policy, then the share of credit going to low-agency-cost borrowers should rise. Bernanke, Gertler, and Gilchrist characterize this as the *flight to quality*. Aggregate data are likely to be of limited usefulness in testing such a hypothesis, since most data on credit stocks and flows are not constructed based on the characteristics of the borrowers. Because small firms are presumably subject to higher agency costs than large firms, much of the evidence

for a broad credit channel has been sought by looking for differences in the behavior of large and small firms in the face of monetary contractions.

Gertler and Gilchrist (1994) document that small firms do behave differently than large firms over the business cycle, being much more sensitive to cyclical fluctuations. Kashyap, Lamont, and Stein (1994) find that inventory investment by firms without access to public bond markets appears to be affected by liquidity constraints.[30] Oliner and Rudebusch (1996a) assess the role of financial factors by examining the behavior of small and large firms in response to changes in monetary policy. Interest-rate increases in response to a monetary contraction lower asset values and the value of collateral, increasing the cost of external funds relative to internal funds. Since agency problems are likely to be more severe for small firms than for large firms, the linkage between internal sources of funds and investment spending should be particularly strong for small firms after a monetary contraction. Oliner and Rudebusch do find that the impact of cash flow on investment increases for small firms, but not for large firms, when monetary policy tightens.

7.5 Summary

The economics of imperfect information has provided numerous insights into the structure of credit markets. Adverse selection and moral hazard account for many of the distinctive features of credit contracts when monitoring is costly. Credit-market imperfections commonly lead to situations in which the lender's expected profits are not monotonic in the interest rate charged on a loan; expected profits initially rise with the loan rate but can then reach a maximum before declining. This produces the possibility that equilibrium may be characterized by credit rationing; excess demand fails to induce lenders to raise the loan rate, as doing so lowers their expected profits. Perhaps more important, balance sheets matter. Variations in borrower's net worth affect their ability to gain credit. A recession that lowers cash flows or a decline in asset prices that lowers net worth will reduce credit availability and increase the wedge between the costs of external and internal finance. The resulting impact on aggregate demand can generate a financial accelerator effect.

Kashyap and Stein (1994) summarize the general state of the credit view among monetary economists:

> Still, the failure of the lending view to be widely embraced cannot be completely ascribed to theoretical discomfort—it has also suffered until recently from a lack of clear-cut, direct empirical support. (p. 7)

30. They focus on the 1981–1982 recession in the United States, a recession typically attributed to tight monetary policy.

Despite Kashyap and Stein's conclusion that such direct empirical support now exists, skepticism still surrounds the existence, and importance, of the credit channel. Certainly the evidence on the empirical importance of a distinct bank lending channel for monetary policy is mixed. While periods of monetary contraction are followed by a fall in bank credit relative to open market credit, this may reflect simple composition effects and not a bank lending channel. The access to managed liabilities also suggests that variations in banking sector reserves caused by changes in monetary policy will affect bank lending mainly through traditional interest-rate channels.

The evidence for a broad credit channel or for financial accelerator effects is more favorable. Recessions are associated with a flight to quality. Small firms, a group likely to face large agency costs in obtaining external financing, are affected more severely during recessions. Net worth and cash flow do seem to affect investment, inventory, and production decisions.

For the analysis of monetary policy, it is important to know whether the general level of interest rates adequately captures the effects operating through credit channels. If so, then the traditional approach, which focuses on interest rates as the key linkage between monetary policy and the real economy, may prove sufficient for the analysis of many issues. The distinction between the credit and money views of the transmission process becomes more important if the nonlinearities suggested by the financial accelerator are quantitatively important. If they are, then the impact of monetary policy will depend on the initial condition of firm and household balance sheets.

8 Discretionary Policy and Time Inconsistency

8.1 Introduction

Macroeconomic equilibrium depends on both the current and expected future behavior of monetary policy. Illustrations of this dependence were seen most clearly in the equilibrium expressions for the price level in the money-in-the-utility function (MIU) model, the cash-in-advance (CIA) model, the models of hyperinflation, and the equilibrium expression for the nominal exchange rate. If policy behaves according to a systematic rule, the rule can be used to determine rational expectations of future policy actions under the assumption that the central bank continues to behave according to the policy rule. In principle, one could derive an "optimal" policy rule by specifying an objective function for the central bank and then determining the values of the parameters in the policy rule that maximized the expected value of the objective function.

But what ensures that the central bank will find it desirable to behave according to such a policy rule? Absent enforcement, it may be optimal to deviate from the rule once private agents have made commitments based on the expectation that the rule will be followed. Firms and workers may agree to set nominal wages or prices based on the expectation that monetary policy will be conducted in a particular manner, yet once these wage and price decisions have been made, the central bank may have an incentive to deviate from actions called for under the rule. The rule may not be incentive compatible. If deviations from a strict rule are possible—that is, if the policy makers can exercise discretion—then agents will need to consider the policy makers' incentive to deviate; they can no longer simply base their expectations on the policy rule that the policy makers say they will follow.

Much of the modern analysis of monetary policy has focused on the incentives central banks face when actually setting their policy instrument. Following the seminal contribution of Kydland and Prescott (1977), attention has been directed to issues of central bank credibility and the ability to precommit to policies. Absent some means of committing in advance to take specific policy actions, central banks may find that they face incentives to act in ways that are inconsistent with their earlier plans and announcements.

A policy is *time consistent* if an action planned at time t for time $t+i$ remains optimal to implement when time $t+i$ actually arrives. The policy can be state contingent; that is, it can depend on the realization of events that are unknown at time t when the policy is originally planned. But a time-consistent policy is one in which the planned response to new information remains the optimal response once the new information arrives. A policy is *time inconsistent* if at time $t+i$ it will not be optimal to respond as originally planned.

The analysis of time inconsistency in monetary policy is important for two reasons. First, it forces one to examine the incentives faced by central banks. The impact of current policy is often dependent on the public's expectations, either about current policy or about future policy actions. To predict how policy affects the economy, we need to understand how expectations will respond, and this understanding can only be achieved if policy behaves in a systematic manner. Just as with a study of private-sector behavior, an understanding of systematic behavior by the central bank requires an examination of the incentives the policy maker faces. And by focusing on the incentives faced by central banks, models of time inconsistency have had an important influence as *positive* theories of observed rates of inflation. These models provide the natural starting point for attempts to explain the actual behavior of central banks and actual policy outcomes.

Second, if time inconsistency is important, then models that help us to understand the incentives faced by policy makers and the nature of the decision problems they face are important for the normative task of designing policy-making institutions. Recent years have seen the reform and redesign of the central banks of many nations. In order to influence these reform efforts, monetary economists need models that provide help in understanding how institutional structures affect policy outcomes.

In the next section, we develop a framework, originally due to Barro and Gordon (1983a), that, despite its simplicity, has proven useful for studying problems of time inconsistency in monetary policy. The discretionary conduct of policy, meaning that the central bank is free at any time to alter its instrument setting, is shown to produce an average inflationary bias; equilibrium inflation exceeds the socially desired rate. This bias arises from a desire for economic expansions above the economy's equilibrium output level (or for unemployment rates below the economy's natural rate) and the inability of the central bank to commit credibly to a low rate of inflation. Section 8.3 examines some of the solutions that have been proposed for overcoming this inflationary bias. Central banks very often seem to be concerned with their reputations, and subsection 8.3.1 examines how such a concern might reduce or even eliminate the inflation bias. Subsection 8.3.2 considers the possibility that society, or the government, might wish to delegate responsibility for monetary policy to a central banker with preferences between employment and inflation fluctuations that differ from those of society as a whole. Since the inflation bias can be viewed as arising because the central bank faces the wrong incentives, a third approach to solving the inflation bias problem is to design mechanisms for creating the right incentives. This approach is discussed in subsection 8.3.3. Subsection 8.3.4 considers the role of institutional structures in solving the inflation bias problem arising from discretion. Finally, the role of explicit targeting rules is studied in subsection 8.3.5.

Though the models of sections 8.2 and 8.3, with their focus on the inflationary bias that can arise under discretion, have played a major role in the academic literature on inflation, their success as positive theories of inflation—that is, as explanations for the actual historical variations of inflation both over time and across countries—is open to debate. Section 8.4 discusses the empirical importance of the inflation bias in accounting for episodes of inflation.

Differences in institutional structures, differences that presumably affect the incentives faced by central bankers in different countries, seem to influence macroeconomic outcomes. Specifically, the degree of independence from political influence enjoyed by central banks is, at least for the developed economies, negatively correlated with average rates of inflation; increased central bank independence is associated with lower average inflation. Section 8.5 examines this empirical evidence and discusses its implications for central bank design and the role of institutions. Lessons we can draw from the issues covered in this chapter are summarized in section 8.6.[1]

8.2 Inflation Under Discretionary Policy

If inflation is costly (even a little), and if there is no real benefit to having 5% inflation on average as opposed to 1% inflation or 0% inflation, why do we observe average rates of inflation that are consistently positive? In recent years, most explanations of positive average rates of inflation have built on the time-inconsistency analysis of Kydland and Prescott (1977) and Calvo (1978).[2] The basic insight is that while it may be optimal to achieve a low average inflation rate, such a policy is not time consistent. If the public were to expect low inflation, the central bank would face an incentive to inflate at a higher rate. Understanding this incentive, and believing the policy maker will succumb to it, the public correctly anticipates a higher inflation rate.

8.2.1 Policy Objectives

To determine the central bank's policy choice, we need to specify the preferences of the central bank. It is standard to assume that the central bank's objective function

1. In chapter 11, recent work reexamining the issues of commitment and discretion in forward-looking optimizing models with nominal rigidities of the type introduced in chapter 5 will be discussed.

2. For a survey dealing with time-inconsistency problems in the design of both monetary and fiscal policies, see Persson and Tabellini (1990). Cukierman (1992) also provides an extensive discussion of the theoretical issues related to the analysis of inflation in models in which time inconsistency plays a critical role. Persson and Tabellini's survey of political economy covers many of the issues discussed in this chapter (Persson and Tabellini 1999). See also Driffill (1988).

involves output (or employment) and inflation, although the exact manner in which output has been assumed to enter the objective function has taken two different forms. In the formulation of Barro and Gordon (1983b), the central bank's objective is to maximize the expected value of

$$U = \lambda(y - y_n) - \tfrac{1}{2}\pi^2, \tag{8.1}$$

where y is output, y_n is the economy's natural rate of output, and π is the inflation rate. More output is preferred to less output with constant marginal utility, so output enters linearly, while inflation is assumed to generate increasing marginal disutility and so enters quadratically. The parameter λ governs the relative weight that the central bank places on output expansions relative to inflation stabilization. Often the desire for greater output is motivated by an appeal to political pressure on monetary policy that is due to the effects of economic expansions on the reelection prospects of incumbent politicians.[3] Alternatively, distortions due to taxes, monopoly unions, or monopolistic competition may lead y_n to be inefficiently low. The exact motivation for the output term in (8.1) is not particularly important. What is critical is that the central bank would like to expand output, but, because of the standard specification of the aggregate-supply function (see, for example, 5.18), it can do so only by generating inflation surprises. For discussions of alternative motivations for this type of loss function, see Cukierman (1992).

The other standard specification for preferences assumes that the central bank desires to minimize the expected value of a loss function that depends on output and inflation fluctuations. Thus, the *loss* function is quadratic in both output and inflation and takes the form

$$V = \tfrac{1}{2}\lambda(y - y_n - k)^2 + \tfrac{1}{2}\pi^2. \tag{8.2}$$

The key aspect of this loss function is the parameter k. The assumption is that the central bank desires to stabilize both output and inflation, inflation around zero but output around $y_n + k$, which exceeds the economy's equilibrium output of y_n by the constant k.[4] Because the expected value of V involves the variance of output, the loss function (8.2) will generate a role for stabilization policy that is absent when the central bank cares only about the level of output, as in (8.1).

3. The influence of reelections on the central bank's policy choices is studied by Fratianni, von Hagen, and Waller (1997) and Herrendorf and Neumann (forthcoming).

4. See (8.3). Note that the inflation term in (8.1) and (8.2) could be replaced by $\frac{1}{2}(\pi - \pi^*)^2$ if the monetary authority has a target inflation rate π^* that differs from zero.

There are several common interpretations for the assumption that $k > 0$, and these parallel the arguments for the output term in the linear preference function (8.1). Most often, some appeal is made to the presence of labor-market distortions (a wage tax, for example) that lead the economy's equilibrium rate of output to be inefficiently low. A rationale can also be provided by the presence of monopolistic competitive sectors, which also causes equilibrium output to be inefficiently low. Attempting to use monetary policy to stabilize output around $y_n + k$ then represents a second-best solution (the first best would involve eliminating the original distortion). An alternative interpretation is that k arises from political pressure on the central bank. Here the notion is that elected officials have a bias for economic expansions since expansions tend to increase their probability of reelection. Since, as we will see, the presence of k leads to a third-best outcome, this second interpretation motivates institutional reforms designed to minimize political pressures on the central bank.

The two alternative objective functions (8.1) and (8.2) are clearly closely related. Expanding the term involving output in the quadratic loss function, (8.2) can be written as

$$V = -\lambda k(y - y_n) + \tfrac{1}{2}\pi^2 + \tfrac{1}{2}\lambda(y - y_n)^2 + \tfrac{1}{2}k^2.$$

The first two terms are the same as the linear utility function (with signs reversed because V is a loss function), showing that the assumption of a positive k is equivalent to the presence of a utility gain from output expansions above y_n. In addition, V includes a loss arising from deviations of output around y_n (the $\lambda(y - y_n)^2$ term). This introduces a role for stabilization policies that is absent when the policy maker's preferences are assumed to be strictly linear in output. The final term involving k^2 is simply a constant and so has no effect on the central bank's decisions.[5]

The alternative formulations reflected in (8.1) and (8.2) produce many of the same insights. Following Barro and Gordon (1983b), we will work initially with the function (8.1) that is linear in output. The equilibrium concept in the basic Barro-Gordon model is noncooperative Nash. Given the public's expectations, the central bank's policy choice maximizes its objective function (or, equivalently, minimizes its loss function), given the public's expectations. The assumption of rational expectations

5. See Cukierman (1992) for more detailed discussions of alternative motivations that might lead to objective functions of the forms given by either (8.1) or (8.2). In an open-economy framework, Bohn (1991c) shows how the incentives for inflation will depend on foreign-held debt denominated in the domestic currency. In chapter 11, the objective function for the central bank will be derived as an approximation to the utility of the represented agent. Under certain conditions, such an approximation yields an objective function similar to (8.2).

implicitly defines the loss function for private agents as $L^P = \mathrm{E}(\pi - \pi^e)^2$; given the public's understanding of the central bank's decision problem, their choice of π^e is optimal.

Before turning to the determination of equilibrium output and inflation, it is worth noting that the assumption in both the linear and quadratic versions of the central bank's objective function is that inflation stabilization, and not price-level stabilization, is the appropriate objective of monetary policy. Even in countries with legislative price-stability objectives such as New Zealand, the operational conduct of policy has focused on achieving a desired inflation target. Under a price-level objective, fluctuations that cause the price level to deviate from target must then be offset; a price rise then requires a deflation to reduce the price level. Under a policy that cares only about the inflation rate, such a price rise is permanent; policy ensures that the inflation rate returns to target, but no attempt is made to restore the initial price level. The general presumption is that the costs of a policy of maintaining a stable price level would be higher inflation and output variability; the gain would be in lower long-term price-level uncertainty. For discussions of this issue, see Fischer (1994), Goodhart and Viñals (1994), and the papers in Bank of Canada (1993). Svensson (1999b) and Vestin (2001) show that this general presumption can be overturned (an issue discussed further in chapter 11).

A final point to note is that the tax distortions of inflation analyzed in chapter 4 were a function of anticipated inflation. Fluctuations in unanticipated inflation caused neutral price-level movements, while expected inflation altered nominal interest rates and the opportunity cost of money, leading to tax effects on money holdings, the consumption of cash goods, and the supply of labor. As we will see, equilibrium inflation will depend on the central bank's evaluation of the marginal costs and benefits of inflation. If the costs of inflation arise purely from expected inflation, while surprise inflation generates economic expansions, than a central bank would perceive only benefits from attempting to produce unexpected inflation. Altering the specification of the central bank's objective function in (8.2) or (8.1) to depend only on output and expected inflation would then imply that the equilibrium inflation rate could be infinite (see Auernheimer 1974, Calvo 1978, and problem 5 at the end of this chapter).

8.2.2 The Economy

The specification of the economy is quite simple and follows the analysis of Barro and Gordon (1983a, 1983b). Aggregate output is given by a Lucas-type aggregate supply function of the form

$$y = y_n + a(\pi - \pi^e) + e. \tag{8.3}$$

This can be motivated as arising from the presence of one-period nominal wage contracts set at the beginning of each period based on the public's expectation of the rate of inflation. If actual inflation exceeds the expected rate, real wages will be eroded and firms will expand employment. If actual inflation is less than the rate expected, realized real wages will exceed the level expected and employment will be reduced. One can derive (8.3) from the assumption that output is produced according to a Cobb-Douglas production function in which output is a function of labor input, the nominal wage is set at the start of the period at a level consistent with the labor-market equilibrium (given expectations of inflation), and firms base actual employment levels on the realized real wage. A critical discussion of this basic aggregate-supply relationship can be found in Cukierman (1992, chapter 3).[6]

The rest of the model is a simple link between inflation and the policy authority's actual policy instrument,

$$\pi = \Delta m + v, \tag{8.4}$$

where Δm is the growth rate of the money supply (the first difference of the log nominal money supply), assumed to be the central bank's policy instrument, and v is a velocity disturbance. The private sector's expectations are assumed to be determined prior to the central bank's choice of a growth rate for the nominal money supply. Thus, in setting Δm, the central bank will take π^e as given. We will also assume that the central bank can observe e (but not v) prior to setting Δm; this assumption generates a role for stabilization policy. Finally, assume that e and v are uncorrelated.[7]

The sequence of events is important. First, the private sector sets nominal wages based on its expectations of inflation. Thus, in the first stage, π^e is set. Then the supply shock e is realized. Because expectations have already been determined, they do not respond to the realization of e. Policy can respond, however, and the policy instrument Δm is set after the central bank has observed e. The velocity shock v is then realized, and actual inflation and output are determined.

Several important assumptions have been made here. First, as with most models involving expectations, the exact specification of the information structure is impor-

6. If the aggregate-supply equation is substituted into the central bank's preference function, both (8.1) and (8.2) can be written in the form $\bar{U}(\pi - \pi^e, \pi, e)$. Thus, the general framework is one in which the central bank's objective function depends on both surprise inflation and actual inflation. In addition to the employment motives mentioned above, one could emphasize the desire for seigniorage as leading to a similar objective function since surprise inflation, by depreciating the real value of both interest-bearing and noninterest-bearing liabilities of the government, produces larger revenue gains for the government than does anticipated inflation (which only erodes noninterest-bearing liabilities).

7. This basic framework can be viewed as a special case of the one employed in section 5.3.1 of chapter 5.

tant. Most critically, we have assumed that private agents must commit to nominal wage contracts before the central bank has to set the rate of growth of the nominal money supply. This means that the central bank has the opportunity to surprise the private sector by acting in a manner that differs from what private agents had expected when they locked themselves into nominal contracts. We have also assumed that the money growth rate, rather than an interest rate, is the policy instrument. In fact, central banks have more often employed a short-term interest rate as the instrument of policy (see chapters 9 and 10). If the main objective is to explain the determinants of average inflation rates, the distinction between money and interest rates as the policy instrument is probably not critical. However, the models we will examine are often used to account for stabilization issues as well, and here the appropriate modeling of the choice of the policy instrument is more important. In keeping with the literature based on Barro and Gordon (1983a), we will assume that the central bank sets money growth as its policy instrument. Given the focus on inflation, it will also be convenient at times simply to treat the inflation rate as the policy instrument. The basic model also assumes that the central bank can react to the realization of the supply shock e while the public commits to wage contracts prior to observing this shock. This informational advantage on the part of the central bank introduces a role for stabilization policy and is meant to capture the fact that policy decisions can be made more frequently than are most wage and price decisions. This means the central bank can respond to economic disturbances before private agents have had the chance to revise all nominal contracts.

Finally, the assumption that v is observed after Δm is set is not critical. It is easy to show that the central bank will always adjust Δm to offset any observed or forecastable component of the velocity shock, and this is why the rate of inflation itself is often treated as the policy instrument. Output and inflation will only be affected by the component of the velocity disturbance that was unpredictable at the time policy was set. This is modeled by assuming that v is realized after the central bank chooses its policy.

8.2.3 Equilibrium Inflation

Since we are assuming that the central bank acts before observing the disturbance v, its objective will be to maximize the expected value of U, where the central bank's expectation is defined over the distribution of v. Substituting (8.3) and (8.4) into the central bank's objective function yields

$$U = \lambda[a(\Delta m + v - \pi^e) + e] - \tfrac{1}{2}(\Delta m + v)^2.$$

The first order condition for the optimal choice of Δm, conditional on e and taking π^e as given, is

$$a\lambda - \Delta m = 0$$

or

$$\Delta m = a\lambda > 0. \tag{8.5}$$

Given this policy, actual inflation will equal $a\lambda + v$. Because private agents are assumed to understand the incentives facing the central bank—that is, are rational—they use (8.5) in forming their expectations about inflation. With private agents forming expectations prior to observing the velocity shock v, (8.4) and (8.5) imply

$$\pi^e = \mathrm{E}[\Delta m] = a\lambda.$$

Thus, average inflation is fully anticipated.

The equilibrium when the central bank acts with discretion in setting Δm produces a positive average rate of inflation equal to $a\lambda$. This has no effect on output, since the private sector completely anticipates inflation at this rate ($\pi^e = a\lambda$). The economy suffers from positive average inflation to no benefit. The size of the bias is increasing in the effect of a money surprise on output, a, since this parameter governs the marginal benefit in the form of extra output that can be obtained from an inflation surprise. The larger is a, the greater is the central bank's incentive to inflate. Recognizing this fact, private agents anticipate a higher rate of inflation. The inflation bias is also increasing in the weight the central bank places on its output objective, λ. A small λ implies that the gains from economic expansion are low relative to achieving inflation objectives, so the central bank has less of an incentive to generate inflation.

Why does the economy end up with positive average inflation even though it confers no benefits and the central bank dislikes inflation? The central bank is acting systematically to maximize the expected value of its objective function, so it weighs the costs and benefits of inflation in setting its policy. At a zero rate of inflation, the marginal benefit of generating a little inflation is positive since, with wages set, the effect of an incremental rise in inflation on output is equal to $a > 0$. The value of this output gain is $a\lambda$. This is illustrated in figure 8.1 by the horizontal line at a height equal to $a\lambda$. The marginal cost of inflation is equal to π. At a planned inflation rate of zero, this marginal cost is zero, so the marginal benefit of inflation exceeds the marginal cost. But the marginal cost rises (linearly) with inflation, as illustrated in figure 8.1. At an expected inflation rate of $a\lambda$, the marginal cost equals the marginal benefit.

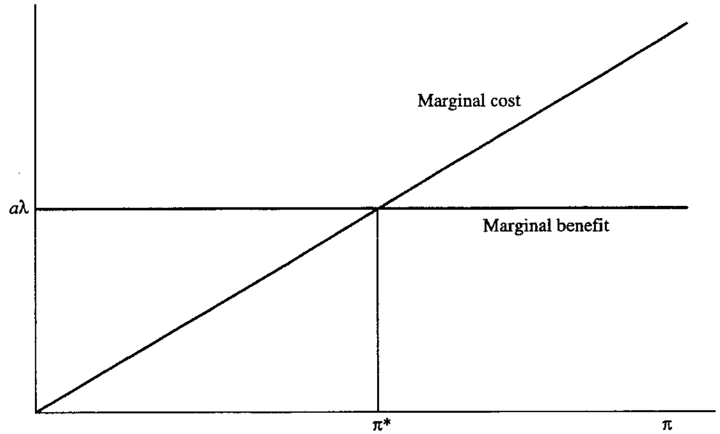

Figure 8.1
Equilibrium Inflation Under Discretion (Linear Objective Function)

Under this discretionary policy outcome, expected utility of the central bank is equal to

$$\begin{aligned} \mathrm{E}[U^d] &= \mathrm{E}\big[\lambda(av+e) - \tfrac{1}{2}(a\lambda+v)^2\big] \\ &= -\tfrac{1}{2}[a^2\lambda^2 + \sigma_v^2], \end{aligned}$$

where $\mathrm{E}[v] = \mathrm{E}[e] = 0$ and σ_v^2 is the variance of the random inflation control error v. Expected utility is decreasing in the variance of the random control error v and decreasing in the weight placed on output relative to inflation objectives (λ) since a larger λ increases the average rate of inflation. While the control error is unavoidable, the loss due to the inflation bias arises from the monetary authority's fruitless attempt to stimulate output.

The outcome under discretion can be contrasted with the situation in which, for some as yet unexplained reason, the monetary authority is able to commit to setting money growth always equal to zero: $\Delta m = 0$. In this case, $\pi = v$ and expected utility would equal

$$\mathrm{E}[U^c] = \mathrm{E}\big[\lambda(av+e) - \tfrac{1}{2}v^2\big] = -\tfrac{1}{2}\sigma_v^2 > \mathrm{E}[U^d].$$

The central bank (and society if the central bank's utility is interpreted as a social welfare function) would be better off if it were possible to commit to a zero-money-growth-rate policy. Discretion, in this case, generates a cost.

As noted earlier, an alternative specification of the central bank's objectives focuses on the loss associated with output and inflation fluctuations around desired levels. This alternative formulation, given by the loss function (8.2), leads to the same basic conclusions. Discretion will lead to an average inflation bias and lower expected utility. In addition, though, specifying the loss function so that the central bank cares about output fluctuations means that there will be a potential role for policy to reduce output fluctuations caused by the supply shock e.

Substituting (8.3) and (8.4) into the quadratic loss function (8.2) yields

$$V = \tfrac{1}{2}\lambda[a(\Delta m + v - \pi^e) + e - k]^2 + \tfrac{1}{2}(\Delta m + v)^2.$$

If Δm is chosen after observing the supply shock e, but before observing the velocity shock v, to minimize the expected value of the loss function, the first order condition for the optimal choice of Δm, conditional on e and taking π^e as given, is

$$a\lambda[a(\Delta m - \pi^e) + e - k] + \Delta m = 0$$

or

$$\Delta m = \frac{a^2\lambda\pi^e + a\lambda(k-e)}{1+a^2\lambda}. \tag{8.6}$$

There are two important differences to note in comparing (8.5), the optimal setting for money growth from the model with a linear objective function, to (8.6). First, the aggregate-supply shock appears in (8.6); because the central bank wants to minimize the variance of output around its target level, it will make policy conditional on the realization of the supply shock. Thus, an explicit role for stabilization policies arises that will involve trading off some inflation volatility for reduced output volatility. Second, the optimal policy depends on private-sector expectations about inflation.

Private agents are assumed to understand the incentives facing the central bank, so they use (8.6) in forming their expectations about inflation. However, private agents are atomistic; they do not take into account the effect their choice of expected inflation might have on the central bank's decision.[8] With expectations formed prior to

8. This assumption is natural in the context of individual firms and workers determining wages and prices. If nominal wages are set in a national bargaining framework, for example, by a monopoly union and employer representatives, then it may be more appropriate to assume that wages are set strategically, taking into account the impact of the wage decision on the incentives faced by the central bank. The case of a monopoly union has been analyzed by Tabellini (1988) and Cubitt (1992). See also Cukierman and Lippi (2001).

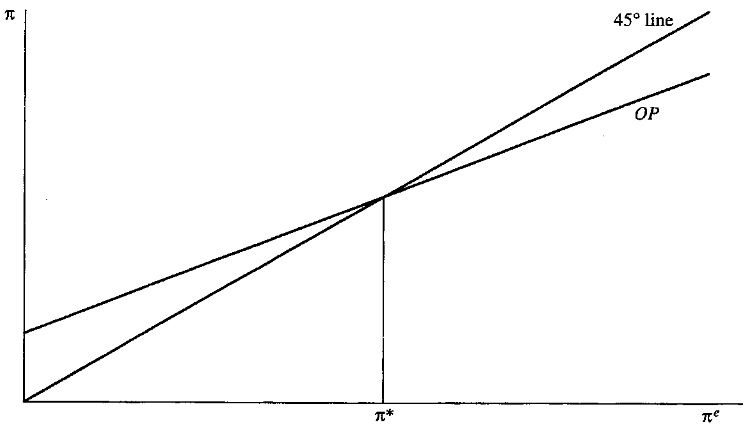

Figure 8.2
Equilibrium Inflation Under Discretion (Quadratic Loss Function)

observing the aggregate-supply shock e, (8.4) and (8.6) imply

$$\pi^e = \mathrm{E}[\Delta m] = \frac{a^2\lambda\pi^e + a\lambda k}{1+a^2\lambda}.$$

Solving for π^e yields $\pi^e = a\lambda k > 0$. Substituting this back into (8.6) and using (8.4) gives an expression for the equilibrium rate of inflation:

$$\pi^d = \Delta m + v = a\lambda k - \left(\frac{a\lambda}{1+a^2\lambda}\right)e + v, \tag{8.7}$$

where the superscript d stands for discretion. Note that the equilibrium when the central bank acts with discretion implies a positive average rate of inflation equal to $a\lambda k$. This has no effect on output, since the private sector completely anticipates this rate ($\pi^e = a\lambda k$). The size of the inflation bias is increasing in the distortion (k), the effect of a money surprise on output (a), and the weight the central bank places on its output objective (λ).[9]

If, for the moment, we ignore the random disturbances e and v, the equilibrium with the quadratic loss function can be illustrated using figure 8.2. Equation (8.6) is

9. In a model with monetary and fiscal policy authorities, Dixit and Lambertini (2002) show that if fiscal policy is optimally designed to eliminate the distortions behind k, the central bank's objective function can be reduced to $\frac{1}{2}\lambda(y-y_n)^2 + \frac{1}{2}\pi^2$. This would eliminate the average inflation bias.

shown, for $e = 0$, as the straight-line OP (for optimal policy), giving the central bank's reaction function for its optimal inflation rate as a function of the public's expected rate of inflation. The slope of this line is $a^2\lambda/(1+a^2\lambda) < 1$, with intercept $a\lambda k/(1+a^2\lambda) > 0$. An increase in the expected rate of inflation requires that the central bank increase actual inflation by the same amount in order to achieve the same output effect, but because this action raises the cost associated with inflation, the central bank finds it optimal to raise π by less than the increase in π^e. Hence the slope is less than 1. The positive intercept reflects the fact that, if $\pi^e = 0$, the central bank's optimal policy is to set a positive rate of inflation. In equilibrium, expectations of private agents must be consistent with the behavior of the central bank. In the absence of any random disturbances, this requires that $\pi^e = \pi$. Thus, equilibrium must lie along the 45° line in figure 8.2.

An increase in k, the measure of the output distortion, shifts the OP line upward and leads to a higher rate of inflation in equilibrium. An increase in a, the impact of an inflation surprise on real output, has two effects. First, it increases the slope of the OP line; by increasing the output effects of an inflation surprise, it raises the marginal benefit to the central bank of more inflation. By increasing the impact of an inflation surprise on output, however, a rise in a reduces the inflation surprise needed to move output to $y_n + k$, and, if λ is large, the intercept of OP could actually fall. The net effect of a rise in a, however, is to raise the equilibrium inflation rate (see 8.7, which shows that the equilibrium inflation rate when $e = 0$ is $a\lambda k$, which is increasing in a).

The coefficient on e in (8.7) is negative; a positive supply shock leads to a reduction in money growth and inflation. This response acts to reduce the impact of e on output (the coefficient on e in the output equation becomes $1/(1+a^2\lambda)$, which is less than 1). The larger the weight on output objectives (λ), the smaller the impact of e on output. In contrast, a central bank that places a larger relative weight on inflation objectives (a small λ) will engage in less output stabilization.

Using (8.7), the loss function under discretion is

$$V^d = \frac{1}{2}\lambda\left[\left(\frac{1}{1+a^2\lambda}\right)e + av - k\right]^2 + \frac{1}{2}\left[a\lambda k - \left(\frac{a\lambda}{1+a^2\lambda}\right)e + v\right]^2. \tag{8.8}$$

The unconditional expectation of this loss is

$$\mathrm{E}[V^d] = \frac{1}{2}\lambda(1+a^2\lambda)k^2 + \frac{1}{2}\left[\left(\frac{\lambda}{1+a^2\lambda}\right)\sigma_e^2 + (1+a^2\lambda)\sigma_v^2\right], \tag{8.9}$$

where σ_x^2 denotes the variance of x.

Now suppose that the central bank had been able to precommit to a policy rule prior to the formation of private expectations. Because there is a role for stabilization policy in the present case (i.e., the monetary authority would like to respond to the supply shock e), the policy rule will not simply be a fixed growth rate for Δm, as it was in the previous case when the central bank's objective function was a linear function of output. Instead, suppose the central bank is able to commit to a policy rule of the form

$$\Delta m^c = b_0 + b_1 e.$$

In the present linear-quadratic framework, a linear rule such as this will be optimal. Given this rule, $\pi^e = b_0$. Now substituting this into the loss function gives

$$V^c = \tfrac{1}{2}\lambda[a(b_1 e + v) + e - k]^2 + \tfrac{1}{2}[b_0 + b_1 e + v]^2. \tag{8.10}$$

Under a commitment policy, the central bank commits itself to particular values of the parameters b_0 and b_1 *prior* to the public's forming its expectations of inflation and *prior* to observing the particular realization of the shock e. Thus, b_0 and b_1 are chosen to minimize the unconditional expectation of the loss function. Solving the minimization problem, the optimal policy under precommitment is

$$\Delta m^c = -\frac{a\lambda}{1 + a^2\lambda} e. \tag{8.11}$$

Note that average inflation under precommitment will be zero ($b_0 = 0$), but the response to the aggregate-supply shock is the same as under discretion (see 8.7). The unconditional expectation of the loss function under precommitment is

$$\mathrm{E}[V^c] = \frac{1}{2}\lambda k^2 + \frac{1}{2}\left[\left(\frac{\lambda}{1 + a^2\lambda}\right)\sigma_e^2 + (1 + a^2\lambda)\sigma_v^2\right], \tag{8.12}$$

which is strictly less than the loss under discretion. Comparing (8.9) and (8.12), the "cost" of discretion is equal to $(a\lambda k)^2/2$, which is simply the loss attributable to the nonzero average rate of inflation.

The inflation bias that arises under discretion occurs for two reasons. First, the central bank has an incentive to inflate once private sector expectations are set. Second, the central bank is unable to precommit to a zero average inflation rate. To see why it cannot commit, suppose the central bank announces that it will deliver zero inflation. If the public believes the announced policy, and therefore $\pi^e = 0$, it is clear from (8.5) or (8.6) that the optimal policy for the central bank to follow would involve setting a positive average money growth rate, and the average inflation rate would be positive. So the central bank's announcement would not be believed in the first place. The central bank cannot believably commit to a zero inflation policy

because under such a policy (i.e., if $\pi = \pi^e = 0$) the marginal cost of a little inflation is $\partial\frac{1}{2}\pi^2/\partial\pi = \pi = 0$, while the marginal benefit is $a\lambda > 0$ under the linear objective function formulation, or $-a^2\lambda(\pi - \pi^e) + a\lambda k = a\lambda k > 0$ under the quadratic formulation. Because the marginal benefit exceeds the marginal cost, the central bank has an incentive to break its commitment.

Society is clearly worse off under the discretionary policy outcome because it experiences positive average inflation with no systematic improvement in output performance. This result fundamentally alters the long-running debate in economics over rules versus discretion in the conduct of policy. Prior to Kydland and Prescott's analysis of time inconsistency, economists had debated whether monetary policy should be conducted according to a simple rule, such as Milton Friedman's k% growth rate rule for the nominal supply of money, or whether central banks should have the flexibility to respond with discretion. With the question posed in this form, the answer is clearly that discretion is better. After all, if following a simple rule is optimal, under discretion one could always choose to follow such a rule. Thus, one could do no worse under discretion, and one might do better. But as the Barro-Gordon model illustrates, one might actually do worse under discretion. Restricting the flexibility of monetary policy may result in a superior outcome. To see this, suppose the central bank is forced (somehow) to set $\Delta m = 0$. This avoids any average inflation bias, but it also prevents the central bank from engaging in any stabilization policy. With the loss function given by (8.2), the unconditional expected loss under such a policy rule is $\frac{1}{2}\lambda(\sigma_e^2 + k^2) + \frac{1}{2}(1 + a^2\lambda)\sigma_v^2$. If this is compared to the unconditional expected loss under discretion, $\mathrm{E}[V^d]$, given in (8.8), the zero money growth rule will be preferred if

$$\left(\frac{a^2\lambda^2}{1 + a^2\lambda}\right)\sigma_e^2 < (a\lambda k)^2.$$

The left side measures the gains from stabilization policy under discretion; the right side measures the cost of the inflation bias that arises under discretion. If the latter is greater, expected loss is lower if the central bank is forced to follow a fixed money growth rule. Whether following a simple rule, thereby limiting the central bank's ability to respond to new circumstances, or allowing discretion, thereby generating an average inflation bias, will result in better policy outcomes becomes an open question.

By focusing on the strategic interaction of the central bank's actions and the public's formation of expectations, the Barro-Gordon model provides a simple but rich game-theoretic framework for studying monetary policy outcomes. The approach emphasizes the importance of understanding the incentives faced by the central bank in order to understand policy outcomes. It also helps to highlight the role of credibility, illustrating why central bank promises to reduce inflation may not be believed. The viewpoint provided by models of time inconsistency contrasts sharply with the traditional analysis of policy outcomes as either exogenous or as determined by a rule that implicitly assumes an ability to precommit.

8.3 Solutions to the Inflation Bias

Following Barro and Gordon (1983a), a large literature developed to examine alternative solutions to the inflationary bias that arises under discretion.[10] Because the central bank is assumed to set the inflation rate so that the marginal cost of inflation (given expectations) is equal to the marginal benefit, most solutions alter the basic model to raise the marginal cost of inflation as perceived by the central bank. For example, the first class of solutions we will examine incorporates notions of reputation into a repeated-game version of the basic framework. Succumbing to the temptation to inflate today worsens the central bank's reputation for delivering low inflation; as a consequence, the public expects more inflation in the future, and this response lowers the expected value of the central bank's objective function. By "punishing" the central bank, the loss of reputation raises the marginal cost of inflation.

The second class of solutions we will examine can also be interpreted in terms of the marginal cost of inflation. Rather than viewing inflation as imposing a reputational cost on the central bank, one could allow the central bank to have preferences that differ from those of society at large so that the marginal cost of inflation as perceived by the central bank is higher. One way to do this is simply to select as the policy maker an individual who places a larger than normal weight on achieving low inflation and then give that individual the independence to conduct policy. Another way involves thinking of the policy maker as an executive whose compensation package is structured so as to raise the marginal cost of inflation. Or, if the inflation bias arises from political pressures on the central bank, institutions might be designed to reduce the effect of the current government on the conduct of monetary policy.

Finally, a third class of solutions involves imposing limitations on the central bank's flexibility. The most common such restriction is a targeting rule that requires the central bank to achieve a preset rate of inflation or imposes a cost related to deviations from this target. An analysis of inflation targeting is important because

10. See Persson and Tabellini (1990) for an in-depth discussion of much of this literature. Many of the most important papers are collected in Persson and Tabellini (1994a).

many central banks have adopted inflation targeting as a framework for the conduct of policy.[11]

Before considering these solutions, however, it is important to note that the tradition in the monetary policy literature has been to assume that the underlying cause of the bias, the desire for economic expansions captured either by the presence of output in the case of the linear objective function (8.1) or by the parameter k in the quadratic loss function (8.2), is given. Clearly, policies that might eliminate the factors that create a wedge between the economy's equilibrium output and the central bank's desired level would lead to the first-best outcome in the Barro-Gordon model.

8.3.1 Reputation

One potential solution to an inflationary bias is to force the central bank to bear some cost if it deviates from its announced policy of low inflation, thereby raising the marginal cost of inflation as perceived by the central bank. One form such a cost might take is a lost reputation. The central bank might, perhaps through its past behavior, demonstrate that it will deliver zero inflation despite the apparent incentive to inflate. If the central bank then deviates from the low-inflation solution, its credibility is lost and the public expects high inflation in the future. That is, the public employs a trigger strategy. The folk theorem for infinite-horizon repeated games (Fudenberg and Maskin 1986) suggests that equilibria exist in which inflation remains below the discretionary equilibrium level as long as the central bank's discount rate is not too high. Hence, as long as the central bank cares enough about the future, a low-inflation equilibrium can be supported. This idea is developed in the next subsection.

An alternative approach, considered in subsection 8.3.1, is to consider situations in which the public may be uncertain about the true preferences of the central bank. In the resulting imperfect-information game, the public's expectations concerning inflation must be based on its beliefs about the central bank's preferences or *type*. Based on observed outcomes, these beliefs evolve over time, and central banks may have incentives to affect these beliefs through their actions. A central bank willing to accept some inflation in return for an economic expansion may still find it optimal initially to build a reputation as an anti-inflation central bank.

A Repeated Game The basic Barro-Gordon model is a one-shot game; even if the central bank's objective is to maximize $E_t \sum_{i=0} \beta^i U_{t+i}$, where U_t is defined by (8.1)

11. Mishkin and Schmidt-Hebbel (2001) identify 19 countries whose central banks have adopting policies to target inflation. See also Carare and Stone (2002).

and β is a discount factor $(0 < \beta < 1)$, nothing links time-t decisions with future periods.[12] Thus, the inflation rate in each period $t+s$ is chosen to maximize the expected value of U_{t+s}, and the discretionary equilibrium of the one-shot game is a noncooperative Nash equilibrium of the repeated game. Barro and Gordon (1983b) evaluate the role of reputation by considering a repeated game in which the choice of inflation at time t can affect expectations about future inflation. The public bases its expectations on the most recent rate of inflation, and Barro and Gordon examine whether inflation rates below the one-shot discretionary equilibrium rate can be sustained in a trigger strategy equilibrium.

To illustrate their approach, suppose that the central bank's objective is to maximize the expected present discounted value of (8.1) and that the public behaves in the following manner: If in period $t-1$ the central bank delivered an inflation rate equal to what the public had expected (i.e., the central bank did not fool them in the previous period), the public expects an inflation rate in period t of $\bar{\pi} < a\lambda$. But if the central bank did fool them, the public expects the inflation rate that would arise under pure discretion, $a\lambda$. The hypothesized behavior of the public is summarized by

$$\pi_t^e = \bar{\pi} < a\lambda \quad \text{if } \pi_{t-1} = \pi_{t-1}^e$$
$$\pi_t^e = a\lambda \quad \text{otherwise.}$$

It is important to note that this trigger strategy involves a one-period punishment. If, after deviating and inflating at a rate that differs from $\bar{\pi}$, the central bank can deliver an inflation rate of $a\lambda$ for one period, the public again expects the lower rate $\bar{\pi}$.[13]

The central bank's objective is to maximize the expected present value of its period-by-period loss function:

$$\sum_{i=0}^{\infty} \beta^i \mathrm{E}_t(U_{t+i}),$$

where U_t is given by (8.1). Previously, the central bank's actions at time t had no effects in any other period. Consequently, the problem simplified to a sequence of one-period problems, a situation that is no longer true in this repeated game with reputation. Inflation at time t affects expectations at time $t+1$ and therefore the

12. The same clearly applies to the case of a quadratic objective function of the form (8.2).

13. This type of one-period punishment strategy has little to commend itself in terms of plausibility. It does, however, provide a useful starting point for analyzing a situation in which the central bank might refrain from inflating at the discretionary rate because it recognizes that the public will subsequently expect higher inflation.

expected value of U_{t+1}. The question is whether equilibria exist for inflation rates $\bar{\pi}$ that are less than the outcome under pure discretion.

Suppose that the central bank has set $\pi_s = \bar{\pi}$ for all $s < t$. Under the hypothesis about the public's expectations, $\pi_t^e = \bar{\pi}$. What can the central bank gain by deviating from the $\bar{\pi}$ equilibrium? If we ignore any aggregate supply shocks (i.e., $e \equiv 0$) and assume that the central bank controls inflation directly, then setting inflation a little above $\bar{\pi}$, say at $\pi_t = \varepsilon > \bar{\pi}$, increases the time-$t$ value of the central bank's objective function by

$$\left[a\lambda(\varepsilon - \bar{\pi}) - \tfrac{1}{2}\varepsilon^2\right] - \left[-\tfrac{1}{2}\bar{\pi}^2\right] = a\lambda(\varepsilon - \bar{\pi}) - \tfrac{1}{2}(\varepsilon^2 - \bar{\pi}^2).$$

This is maximized for $\varepsilon = a\lambda$, the inflation rate under discretion. So if the central bank deviates, it will set inflation equal to $a\lambda$ and gain

$$G(\bar{\pi}) \equiv a\lambda(a\lambda - \bar{\pi}) - \tfrac{1}{2}[(a\lambda)^2 - \bar{\pi}^2] = \tfrac{1}{2}(a\lambda - \bar{\pi})^2 \geq 0.$$

Barro and Gordon refer to this as the *temptation* to cheat. The function $G(\bar{\pi})$ is shown as the dashed line in figure 8.3. It is positive for all $\bar{\pi}$ and reaches a minimum at $\bar{\pi} = a\lambda$.

Cheating carries a cost because, in the period following a deviation, the public will punish the central bank by expecting an inflation rate of $a\lambda$. Since $a\lambda$ maximizes the central bank's one-period objective function for any expected rate of inflation, the

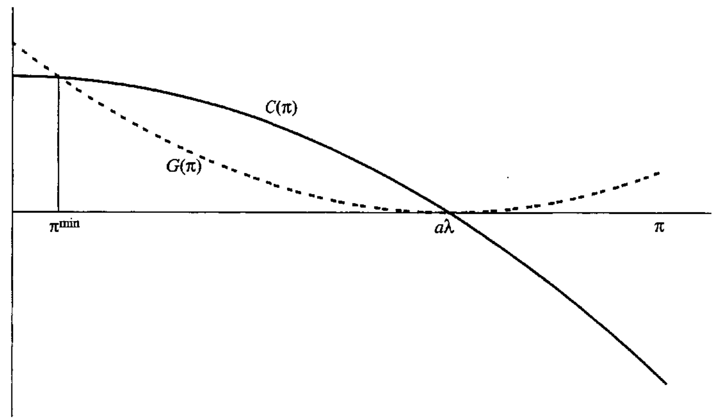

Figure 8.3
Temptation and Enforcement

central bank sets $\pi_{t+1} = a\lambda$. The subsequent loss, relative to the $\bar{\pi}$ inflation path, is given by

$$C(\bar{\pi}) \equiv \beta\left(-\frac{1}{2}\bar{\pi}^2\right) - \beta\left(-\frac{1}{2}a^2\lambda^2\right) = \frac{\beta}{2}[(a\lambda)^2 - \bar{\pi}^2]. \tag{8.13}$$

Since the loss occurs in period $t+1$, we multiply it by the central bank's discount factor β. Barro and Gordon refer to this as the *enforcement*. The function $C(\bar{\pi})$ is decreasing for $\bar{\pi} > 0$ and is shown as the solid line in figure 8.3.

The central bank will deviate from the proposed equilibrium if the gain (the temptation) exceeds the loss (the enforcement). Any $\bar{\pi}$ such that $C(\bar{\pi}) \geq G(\bar{\pi})$ can be supported as an equilibrium; with the loss exceeding the gain, the central bank has no incentive to deviate. As shown in figure 8.3, $C(\pi) < G(\pi)$ for inflation rates less than $\pi^{\min} \equiv (1-\beta)a\lambda/(1+\beta) < a\lambda$. Because $\pi^{\min} > 0$, the trigger strategy cannot support the socially optimal, zero-inflation outcome. However, any inflation rate in the interval $[\pi^{\min}, a\lambda]$ is sustainable. The minimum sustainable inflation rate $\pi^{\min}$ is decreasing in β; the greater the weight the central bank places on the future, the greater the enforcement mechanism provided by the public's expectations and the lower the inflation rate that can be sustained.[14]

This example is a simple illustration of a trigger strategy. The public expects one rate of inflation ($\bar{\pi}$ in this example) as long as the central bank "behaves," and it expects a different, higher rate of inflation if the central bank misbehaves. But how does the public coordinate on this trigger strategy? If the public is atomistic, each member would take the expectations of others as given in forming her own expectations, and the notion of public coordination makes little sense. This problem is even more severe when multiperiod punishment periods are considered, in which the public expects high inflation for some fixed number of period greater than one. Again, how is this expectation determined?

One way to solve the coordination problem is to assume that the central bank plays a game against a monopoly union.[15] With only one agent in the private sector (the union), the issue of atomistic agents coordinating on a trigger strategy no longer arises. Of course, the coordination problem has, in some sense, been solved by simply

14. With the central bank's objective given by (8.2), a zero inflation rate can be supported with a one-period punishment trigger strategy of the type considered as long as the central bank places sufficient weight on the future. In particular, zero inflation is an equilibrium if $\beta/(\alpha^2+\beta) > 1$. See problem 6 at the end of this chapter.

15. Tabellini (1988) studies the case of a monopoly union in the Barro-Gordon framework, although he focuses on imperfect information about the central bank's type, a topic discussed below. See also Cubitt (1992).

assuming it away, but it is also the case that many countries have labor markets that are dominated by national unions and business organizations that negotiate over wages.[16]

The general point, though, is that the reputational solution works because the loss of reputation represents a cost to the central bank. Raising the marginal cost of inflation lowers the equilibrium rate of inflation. If $C(\bar{\pi}) > G(\bar{\pi})$, the central bank will not have an incentive to cheat, and inflation at the rate $\bar{\pi}$ can be supported. But can it really? Suppose the central bank does cheat. Will it be in the interests of a private sector that has somehow coordinated on a trigger strategy to actually punish the central bank? If by punishing the central bank the private sector also punishes itself, the threat to punish may not be credible. If punishment is not credible, the central bank is not deterred from cheating in the first place.

The credibility of trigger strategies in the context of the Barro-Gordon model (with the utility function 8.1) have been examined by al-Nowaihi and Levine (1994). They consider the case of a single monopoly union and show that if one requires that the punishment hurt the central bank but not the private sector (i.e., consider only equilibria that are renegotiation-proof), then the only equilibrium is the high-inflation discretionary equilibrium. Thus, it would appear that trigger strategies will not support a low-inflation equilibrium.

Requiring that the punishment hurt only the central bank imposes strong restrictions on the possible equilibria. Adopting a weaker notion of renegotiation, al-Nowaihi and Levine introduce the concept of *chisel-proof credibility*[17] by asking, if the central bank cheats just a little, will the public be better off simply acquiescing, or will it be better off punishing? They show that the lowest inflation rate that can be supported in a chisel-proof equilibrium is positive but less than the discretionary rate.

This discussion of trigger strategy equilibria assumed that the trigger was pulled whenever inflation deviated from its optimal value. If inflation differed from $\bar{\pi}$, this outcome revealed to the public that the central bank had cheated. But for such strategies to work, the public must be able to determine whether the central bank cheated. If inflation depends not just on the central bank's policy but also on the outcome of a random disturbance, as in (8.4), then the trigger strategy must be based directly on the central bank's policy instrument rather than on the realized rate of inflation. Simply observing the actual rate of inflation may only reveal the net effects

16. Al-Nowaihi and Levine (1994) provide an interpretation in terms of a game involving successive governments rather than a monopoly union. See also Herrendorf and Lockwood (1997).

17. See also Herrendorf (1995a).

of both the central bank's policy actions and the realizations of a variety of random effects that influence the inflation rate.

This consequence raises a difficulty, one first analyzed by Canzoneri (1985). Suppose that inflation is given by $\pi = \Delta m + v$. In addition, suppose that the central bank has a private, unverifiable forecast of v—call it v^f—and that Δm can be set conditional on v^f. Reputational equilibria will now be harder to sustain. Recall that the trigger strategy equilibrium required that the public punish the central bank whenever the central bank deviated from the low-inflation policy. In the absence of private information, the public can always determine whether the central bank deviated by simply looking at the value of Δm. When the central bank has private information on the velocity shock, it should adjust Δm to offset v^f. So if the central bank forecasts a negative v, it should raise Δm. Simply observing, ex post, a high value of Δm, therefore, will not allow the public to determine if the central bank cheated; the central bank can always claim that v^f was negative and that it had not cheated.[18]

Canzoneri shows that a trigger-strategy equilibrium can be constructed in which the public assumes that the central bank cheated whenever the implicit forecast error of the central bank is too large. That is, a policy designed to achieve a zero rate of inflation would call for setting $\Delta m = -v^f$, and this might involve a positive rate of money growth. Whenever money growth is too high, that is, whenever $\Delta m > -\bar{v}$ for some $\bar{v}$, the public assumes that the central bank has cheated. The public then expects high inflation in the subsequent period; high expected inflation punishes the central bank. The constant $\bar{v}$ is chosen to ensure that the central bank has no incentive to deviate from the zero-inflation policy. This equilibrium leads to a situation in which there are occasional periods of inflation; whenever the central bank's forecast for the random variable v takes on a value such that $\Delta m = -v^f > -\bar{v}$, expected inflation (and actual inflation) rises. One solution to this problem may involve making policy more transparent by establishing targets that allow deviations to be clearly observed by the public. Herrendorf (1995b), for example, argues that a fixed-exchange-rate policy may contribute to credibility since any deviation is immediately apparent. This solves the Canzoneri problem; the public does not need to verify the central bank's private information about velocity. If the central bank has private information on the economy that would call, under the optimal commitment policy, for a change in the exchange rate, a fixed-exchange-rate regime will limit the flexibility of

18. Herrendorf (1995b) considers situations in which v has a bounded support $[\underline{v}, \bar{v}]$. If the optimal commitment policy is $\Delta m = 0$, then as long as $\underline{v} \leq \pi \leq \bar{v}$ the public cannot tell whether the central bank cheated. However, if $\pi > \bar{v}$ the public knows the central bank cheated. Thus, the probability of detection is $\text{Prob}(v > \bar{v} - m)$.

the central bank to act on this information. Changing the exchange rate would signal to the public that the central bank was attempting to cheat. As a result, a trade-off between credibility and flexibility in conducting stabilization policy can arise.

Central Bank Types In Canzoneri (1985), the central bank has private information about the economy in the form of an unverifiable forecast of an economic disturbance. The public doesn't know what the central bank knows about the economy, and, more importantly, the public cannot, ex post, verify the central bank's information. An alternative aspect of asymmetric information involves situations in which the public is uncertain about the central bank's true preferences. Backus and Driffill (1985), Barro (1986), Cukierman and Meltzer (1986), Vickers (1986), Tabellini (1988), Andersen (1989), Mino and Tsutsui (1990), Cukierman and Liviatan (1991), Cukierman (1992), Drazen and Masson (1994), Garcia de Paso (1993), Ball (1995), Herrendorf (1995b), al-Nowaihi and Levine (1996), Briault, Haldane, and King (1996), Nolan and Schaling (1996), and Walsh (2000), among others, study models in which the public is uncertain about the central bank's *type*, usually identified either as its preference between output and inflation stabilization or as its ability to commit. In these models, the public must attempt to infer the central bank's type from its policy actions, and equilibria in which central banks may deviate from one-shot optimal policies in order to develop reputations have been studied (for a survey, see Rogoff 1989). In choosing its actions, a central bank must take into account the uncertainty faced by the public, and it may be advantageous for one type of bank to mimic the other type to conceal (possibly only temporarily) its true type from the public.

In one sense, the discussion of models with uncertainty over the central bank type is out of place here, since our focus is on solutions to the inflationary bias that arises under policy discretion. These models are more appropriately viewed as offering positive theories of inflation, and we will return to them again in discussing the ability of the time-inconsistency problem to explain actual inflation experiences. But it is useful to discuss these models here as well, because they correspond to game-theoretic notions of reputation, and reputational effects are widely viewed as relevant for explaining how low-inflation equilibria might be sustained. That is, they do explain how the inflation bias that can arise under discretion might be solved by the central bank's decision to preserve its reputation.

In one of the earliest reputational models of monetary policy, Backus and Driffill (1985) assumed that governments (or central banks) come in two types: optimizers who always act to maximize the expected present discounted value of a utility function of the form (8.1) and single-minded inflation fighters who always pursue a policy of zero inflation. Alternatively, the inflation-fighter types can be described as having access to a precommitment technology. The government in office knows which type it is, but this information is unverifiable by the public. Simply announcing it is a zero-inflation government would not be credible, since the public realizes that an optimizing central bank would also announce that it is a strict inflation fighter to induce the public to expect low inflation.[19]

Initially, the public is assumed to have prior beliefs about the current government's type (where these beliefs come from is unspecified, and therefore there will be multiple equilibria, one for each set of initial beliefs). If the government is actually an optimizer and ever chooses to inflate, its identity is revealed, and from then on the public expects the equilibrium inflation rate under discretion. To avoid this outcome, the optimizing government may have an incentive to conceal its true identity by mimicking the zero-inflation type, at least for a while. Equilibrium may involve pooling in which both types behave the same way. In a finite-period game, the optimizer will always inflate in the last period because there is no future gain from further attempts at concealment.

Backus and Driffill solve for the equilibrium in their model by employing the concept of a *sequential equilibrium* (Kreps and Wilson 1982) for a finitely repeated game. Let π_t^d equal the inflation rate for period t set by a zero-inflation (a "dry") government, and let π_t^w be the rate set by an optimizing (a "wet") government. We start in the final period T. The zero-inflation type always sets $\pi_T^d = 0$, while the optimizing type will always inflate in the last period at the discretionary rate $\pi_T^w = a\lambda$. With no further value in investing in a reputation, a wet government just chooses the optimal inflation rate derived from the one-period Barro-Gordon model analyzed earlier.

In periods prior to T, however, the government's policy choice affects its future reputation, and it may therefore benefit a wet government to choose a zero rate of inflation in order to build a reputation as a dry. Thus, equilibrium may consist of an initial series of periods in which the wet government mimics the dry government, and inflation is zero. For suitable values of the parameters, the sequential equilibrium concept that Backus and Driffill employ also leads to mixed strategies in which the wet government inflates with some probability. So the wet government randomizes; if the outcome calls for it to inflate, the government is revealed as wet, and from then on, inflation is equal to $a\lambda$. If it doesn't inflate, the public updates its beliefs about the government's type using Bayes's rule.

19. Vickers (1986) assumes that the types differ with respect to the weight placed on inflation in the loss function. In Tabellini (1988) the "tough" type has $\lambda = 0$ (i.e., no weight on output), while the "weak" type is characterized by a $\lambda > 0$. Cukierman and Liviatan (1991) assume that the types differ in their ability to commit.

Ball (1995) develops a model of inflation persistence based on the same notion of central bank types used by Backus and Driffill (1985) and Barro (1986). That is, one type, type D, always sets inflation equal to zero, while type W acts opportunistically to minimize the expected discounted value of a quadratic loss function of the form

$$L^W = \sum_{i=0}^{\infty} \beta^i [\lambda(y_{t+i} - y_n - k)^2 + \pi_{t+i}^2], \tag{8.14}$$

where $0 < \beta < 1$. To account for shifts in policy, Ball assumes that the central bank type follows a Markov process. If the central bank is of type D in period t, then the probability that the central bank is still type D in period $t+1$ is d; the probability that the bank switches to type W in $t+1$ is $1-d$. Similarly, if the period-t central bank is type W, then the $t+1$ central bank is type W with probability w and type D with probability $1-w$.

The specification of the economy is standard, with output a function of inflation surprises and an aggregate supply shock:

$$y_t = y_n + a(\pi_t - \pi_t^e) + e_t. \tag{8.15}$$

To capture the idea that economies are subject to occasional discrete supply shocks, Ball assumes that e takes on only two possible values: 0 with probability $1-q$ and $\bar{e} < 0$ with probability q. If shifts in policy and supply shocks are infrequent, then $1-d$, $1-w$, and q are all small.

The timing in this game has the public forming expectations of inflation; then the supply shock and the central bank type are determined. It is assumed that the realization of e but not of the central bank type is observable. Finally, the central bank sets π. In this game, there are many possible equilibria, depending on how the public is assumed to form its expectations about the central bank type. Ball considers a perfect Nash equilibrium concept in which actions depend only on variables that directly affect current payoffs. Such equilibria are Markov perfect equilibria (Maskin and Tirole 1988) and rule out the types of trigger-strategy equilibria considered, for example, by Barro (1986).[20] Ball then shows that such an equilibrium exists and involves the W type setting $\pi = 0$ as long as $e = 0$; if $e = \bar{e}$, the W type inflates at the discretionary rate. Since this reveals the identify of the central bank (i.e., as a type W), inflation remains at the discretionary rate $a\lambda k$ until such time as a type D central

20. In the trigger-strategy equilibria, current actions depend on π_{t-1} even though payoffs do not depend directly on lagged inflation.

bank takes over. At this point, inflation drops to zero, remaining there until a bad supply shock is again realized.[21]

This outcome predicts periodic and persistent bouts of inflation in response to adverse economic disturbances. This prediction for inflation appears to provide a good representation of actual inflation experiences, at least in the developed economies over the past 40 years.

One undesirable aspect of the Backus-Driffill framework is its assumption that one government, the dry government, is simply an automaton, always playing zero inflation. While serving a useful purpose in allowing one to characterize how beliefs about type might affect the reputation and the behavior of a government that would otherwise like to inflate, the myopic behavior of the dry government is unsatisfactory; such a government might also wish to signal its type to the public or otherwise attempt to differentiate itself from a wet type.

One way a dry government might distinguish itself would be to announce a planned or target rate of inflation and then build credibility by actually delivering on its promises. In the Backus-Driffill model, the dry government could be thought of as always announcing a zero target for inflation, but as Cukierman and Liviatan (1991) note, even central banks that seem committed to low inflation often set positive inflation targets, and they do so in part because low inflation is not perfectly credible. That is, if the public expects a positive rate of inflation because the central bank's true intentions are unknown, then even a dry central bank may feel the need to partially accommodate these expectations. Doing otherwise would produce a recession.

To model this type of situation, Cukierman and Liviatan assume that there are two potential government or central bank types, D and W, that differ in their ability to commit. Type D commits to its announced policy; type W cannot precommit. In contrast to Backus and Driffill, Cukierman and Liviatan allow their central banks to make announcements, and the D type is not simply constrained always to maintain a zero rate of inflation. If the public assigns some prior probability to the central bank being type W, type D's announcement will not be fully credible. As a result, a type D central bank may find it optimal to announce a positive rate of inflation.

To show the effect on inflation of the public's uncertainty about the type of central bank in office, the basic points can be illustrated within the context of a two-period

21. For this to be an equilibrium, the discount factor must be large but not too large. As in standard reputational models, the type W central bank must place enough weight on the future to be willing to mimic the type D in order to develop a reputation for low inflation. However, if the future receives too much weight, the type W will be unwilling to separate, that is, inflate, when the bad shock occurs. See Ball (1995).

model. To determine the equilibrium behavior of inflation, we need to solve the model backward by first considering the equilibrium during the last period.

Assume that both central bank types share a utility function that is linear in output and quadratic in inflation, as given by (8.1). With utility linear in output, stabilization will not play a role, so, let output be given by (8.3) with $e \equiv 0$. In the second period, reputation has no further value, so the type W central bank will simply set inflation at the optimal discretionary rate $a\lambda$. To determine D's strategy, however, we need to consider whether the equilibrium will be a separating, pooling, or mixed-strategy equilibrium. In a separating equilibrium, the behavior of the central bank during the first period reveals its identity; in a pooling equilibrium, both types behave the same way during the first period, so the public will remain uncertain as to the true identity of the bank. A mixed-strategy equilibrium would involve type W mimicking type D, with a positive probability less than 1.

Since a separating equilibrium is a bit simpler to construct, let us start with that case. With first-period behavior revealing its type, the public, in period 2, now knows the identity of the central bank. Since type D is able to commit, its optimal policy is to announce a zero rate of inflation for period 2. The public, knowing that type D is truthful, expects a zero inflation rate, and in equilibrium $\pi_2^D = 0$.

In the first period of a separating equilibrium, the public is uncertain about the type of central bank actually in power. Suppose the public assigns an initial probability q to the central bank being of type D. Since we are, by hypothesis, considering a separating equilibrium in which the W type reveals itself by inflating at a rate that differs from the announced rate, the type W will choose to inflate at the rate $a\lambda$ because this value maximizes its utility function.[22] So if the type D announces π^a, then the public will expect an inflation rate of $\pi_1^e = q\pi^a + (1-q)a\lambda$.[23] Our last step to fully characterize the separating equilibrium is to determine the optimal announcement (since the D type actually inflates at the announced rate and the W type inflates at the rate $a\lambda$).

If future utility is discounted at the rate β, the utility of the type D central bank is given by

$$\begin{aligned}U_{sep}^D &= \lambda(y_1 - y_n) - \tfrac{1}{2}\pi_1^2 + \beta\left[\lambda(y_2 - y_n) - \tfrac{1}{2}\pi_2^2\right]\\ &= a\lambda(\pi_1 - \pi_1^e) - \tfrac{1}{2}\pi_1^2,\end{aligned}$$

22. Recall that with the utility function (8.1), the central bank's optimal period-1 inflation rate is independent of the expected rate of inflation.

23. The W type will also announce the same inflation rate as the type D, since doing otherwise would immediately raise the public's expectations about first-period inflation and lower type W's utility.

since, in period 2, $y_2 = y_n$ and $\pi_2^D = 0$. The type D picks first-period inflation subject to $\pi_1 = \pi^a$ and $\pi_1^e = q\pi^a + (1-q)a\lambda$. This yields

$$\pi_1^D = (1-q)a\lambda \le a\lambda.$$

The role of credibility is clearly illustrated in this result. If the central bank were known to be of type D, that is, if $q = 1$, it could announce and deliver a zero rate of inflation. The possibility that the central bank might be of type W, however, forces the D type to actually announce, and deliver, a positive rate of inflation. The public's uncertainty leads it to expect a positive rate of inflation; the type D central bank could announce and deliver a zero rate of inflation, but doing so would create a recession whose cost outweighs the gain from a lower inflation rate.

To summarize, in a separating equilibrium, the type W inflates at the rate $a\lambda$ in each period, while the type D inflates at the rate $(1-q)a\lambda$ during the first period and 0 during the second period. Since expected inflation in the first period is $q(1-q)a\lambda + (1-q)a\lambda = (1-q^2)a\lambda$, which is less than $a\lambda$ but greater than $(1-q)a\lambda$, output is above y_n if the central banker is actually type W and below y_n if the banker is type D.

What happens in a pooling equilibrium? A pooling equilibrium requires that the W type not only make the same first-period announcement as the D type, but also that it pick the same actual inflation rate in period 1 (otherwise, it would reveal itself). In this case, the D type faces period-2 expectations $\pi_2^e = q\pi_2^a + (1-q)a\lambda$.[24] Since this is just like the problem we analyzed above for the first period of the separating equilibrium, $\pi_2^D = \pi_2^a = (1-q)a\lambda > 0$. The type D inflates at a positive rate in period 2, since her announcement lacks complete credibility. In the first period of a pooling equilibrium, however, things are different. In a pooling equilibrium, the D type knows that the W type will mimic whatever the D type does. And the public knows this also, so both types will inflate at the announced rate of inflation and $\pi_1^e = \pi_1^a$. In this case, with the announcement fully credible, the D type will announce and deliver $\pi_1 = 0$.

To summarize, in the pooling equilibrium inflation will equal 0 in period 1 and either $(1-q)a\lambda$ or $a\lambda$ in period 2, depending on which type is actually in office. In

24. In the pooling equilibrium, first-period outcomes do not reveal any information about the identity of the central bank type, so the public continues to assess the probability of a type D as equal to q. This would not be the case if the equilibrium involved the W type following a mixed strategy in which it inflates in period 1 with probability $p < 1$. In a sequential Bayesian equilibrium, the public updates the probability of a D type on the basis of the period-1 outcomes using Bayes's rule.

the separating equilibrium, inflation will equal $(1-q)a\lambda$ in period 1 and 0 in period 2 if the central bank is of type D, and $a\lambda$ in both periods if the central bank is of type W.

Which equilibrium will occur? If the type W separates by inflating at the rate $a\lambda$ during period 1, its utility will be $a\lambda[a\lambda - (1-q^2)a\lambda] - \frac{1}{2}(a\lambda)^2 - \beta\frac{1}{2}(a\lambda)^2$, or

$$U_{sep}^W = (a\lambda)^2\left[q^2 - \tfrac{1}{2}(1+\beta)\right].$$

If type W deviates from the separating equilibrium and mimics type D instead by only inflating at the rate $(1-q)a\lambda$ during period 1, it will achieve a utility payoff of $a\lambda[(1-q)a\lambda - (1-q^2)a\lambda] - \frac{1}{2}[(1-q)a\lambda]^2 + \beta a\lambda(a\lambda - 0) - \beta\frac{1}{2}(a\lambda)^2$, or

$$U_m^W = \tfrac{1}{2}(a\lambda)^2(q^2 - 1 + \beta),$$

since mimicking fools the public into expecting zero inflation in period 2. Type W will separate if and only if $U_{sep}^W > U_m^W$, which occurs when

$$\beta < q^2/2 \equiv \underline{\beta}. \tag{8.16}$$

Thus, the separating equilibrium occurs if the public places a high initial probability on the central bank's being type D (q is large). In this case, type D is able to set a low first-period rate of inflation, and the W type does not find it worthwhile to mimic. Only if type W places a large weight on being able to engineer a surprise inflation in period 2 (i.e., β is large), would deviating from the separating equilibrium be profitable.[25]

Suppose $\beta \ge \underline{\beta}$; will pooling emerge? Not necessarily. If the type W pools, her utility payoff will be

$$a\lambda[0] - \tfrac{1}{2}[0]^2 + \beta a\lambda[a\lambda - \pi_2^e] - \beta\tfrac{1}{2}(a\lambda)^2$$

or, since $\pi_2^e = q\pi_2^a + (1-q)a\lambda = (1-q^2)a\lambda$,

$$U_p^W = \beta(a\lambda)^2\left(q^2 - \tfrac{1}{2}\right).$$

If the type W deviates from the pooling equilibrium, she will generate an output expansion in period 1, but by revealing her identity, period 2 inflation is fully antici-

25. Walsh (2000) shows that a separating equilibrium is less likely if inflation is determined by the type of forward-looking new Keynesian Phillips curves discussed in chapter 5. When current inflation depends on expected future inflation, a type W whose identity is revealed in the the first period suffers an immediate rise in inflation as expected future inflation rises.

pated and output equals y_n. Thus, deviating gives type W a payoff of $a\lambda[a\lambda] - \frac{1}{2}[a\lambda]^2 + \beta a\lambda[0] - \beta\frac{1}{2}[a\lambda]^2$ or

$$U^W_{dev} = \tfrac{1}{2}(a\lambda)^2(1-\beta).$$

By comparing the incentive for W to deviate from a pooling equilibrium, the pooling outcome is an equilibrium whenever

$$\beta > \frac{1}{2q^2} \equiv \bar{\beta} \tag{8.17}$$

since in this case $U^W_p > U^W_{dev}$. If β is large enough, meaning $\beta > 1/(2q^2)$, type W places enough weight on the future that she is willing to forgo the temptation to inflate immediately, and zero inflation is the equilibrium in period 1. Of course, in period 2, there is no further value in maintaining a reputation, so type W inflates at the rate $a\lambda$. Equation (8.17) shows that the critical cutoff value for β depends on q, the prior probability the public assigns to a type D setting policy. A larger q makes pooling an equilibrium for more values of β, so that even less patient type W's will find it advantageous to not deviate from the pooling equilibrium. If q is large, then the public thinks it likely that the central bank is a type D. This leads them to expect low inflation in period 2, so the output gains of inflating at the rate $a\lambda$ will be large. By pooling during period 1, a type W can then benefit from causing a large expansion in period 2. If the type W deviates and reveals her type during period 1, the first-period output gain is independent of q.[26] So a rise in q leaves the period 1 advantage of deviating unchanged while increasing the gain from waiting until period 2 to inflate.

Comparing (8.16) and (8.17) shows that $\underline{\beta} < \bar{\beta}$, so there will be a range of values for the discount factor for which neither the separating nor the pooling outcomes will be an equilibrium. For β in this range, there will be mixed-strategy equilibria (see Cukierman and Liviatan 1991 for details).

This model reveals how public uncertainty about the intentions of the central bank affects the equilibrium inflation rate. In both the separating equilibrium and the mixed-strategy equilibrium, the type D central bank inflates in the first period even though it is (by assumption) capable of commitment and always delivers on its announcements.

The formulation of Cukierman and Liviatan also provides a nice illustration of the role that announcements can play in influencing the conduct of policy. It also illustrates why central banks might be required to make announcements about their inflation plans. The type D central bank is clearly better off making announcements; as long as $q > 0$, making an announcement allows the type D to influence expectations and reduce the first-period inflation rate (this occurs in the separating and pooling equilibria, as we have shown, but it also occurs in mixed-strategy equilibria). Even when there may be incentives to manipulate announcements, they can serve to constrain the subsequent conduct of policy. They may also convey information about the economy if the central bank has private and unverifiable information such as its own internal forecast of economic conditions.[27]

26. This is because expected inflation equals zero during the first period of a pooling equilibrium. Consequently, the output expansion of inflating at the rate $a\lambda$ is $a(a\lambda - 0) = a^2\lambda$, which is independent of q.

8.3.2 Preferences

An alternative approach to solving the inflationary bias of discretion focuses directly on the preferences of the central bank. This branch of the literature has closer connections with the extensive empirical work that has found, at least for the industrialized economies, that average inflation rates across countries are negatively correlated with measures of the degree to which a central bank is independent of the political authorities.[28] If the central bank is independent, then one can begin to think of the preferences of the central bank as differing from those of the elected government. And if they can differ, then one can ask how they might differ and how the government, through its appointment process, might influence the preferences of the central bank(er).

Rogoff (1985b) was the first to analyze explicitly the issue of the optimal preferences of the central banker.[29] He did so in terms of the relative weight the central banker places on his inflation objective. In the objective function (8.2), λ measures the weight on output relative to a weight normalized to 1 on inflation objectives. Rogoff concluded that the government should appoint as central banker someone who places greater relative weight on the inflation objective than does society (the government) as a whole. That is, the central banker should have preferences that are of the form given by (8.2) but with a weight on inflation of $1 + \delta > 1$. Rogoff characterized such a central banker as more *conservative* than society as a whole. This is usefully described as *weight conservatism* (Svensson 1997b), since there are other

27. See Persson and Tabellini (1993), Muscatelli (1999), and Walsh (1999).

28. This empirical work will be discussed in section 8.5.

29. Interestingly, Barro and Gordon recognized that outcomes could be improved under discretion by distorting the central banker's preferences so that "there is a divergence in preferences between the principal (society) and its agent (the policymaker)" (Barro and Gordon 1983a, p. 607, footnote 19). This insight is also relevant for the contracting approach to be discussed in section 8.3.3.

interpretations of conservatism; for example, the central bank might have a target inflation rate that is lower than that of the government. In most of the literature, however, conservative is interpreted in terms of the weight placed on inflation objectives relative to output objectives.

The intuition behind Rogoff's result is easily understood by referring back to (8.7), which showed the inflation rate under discretion for the quadratic loss function (8.2). If the central banker conducting monetary policy has a loss function that differs from (8.2) only by placing weight $1+\delta$ on inflation rather than 1, then inflation under discretion will equal

$$\pi^d(\delta) = \Delta m + v = \frac{a\lambda k}{1+\delta} - \left(\frac{a\lambda}{1+\delta+a^2\lambda}\right)e + v. \tag{8.18}$$

The equilibrium inflation rate is a function δ. Two effects are at work. First, the inflation bias is reduced, since $1+\delta > 1$. This tends to reduce the social loss function (the loss function with weight 1 on inflation and λ on output). But the coefficient on the aggregate-supply shock is also reduced; stabilization policy is distorted, and the central bank responds too little to e. As a consequence, output fluctuates more than is socially optimal in response to supply shocks. The first effect (lower average inflation) makes it optimal to appoint a central banker who places more weight on inflation than does society; this is usually interpreted to mean that society should appoint a conservative to head the central bank. But the second effect (less output stabilization) limits how conservative the central banker should be.

Using (8.18), we can evaluate the government's loss function V as a function of δ. By then minimizing the government's expected loss function with respect to δ, we can find the *optimal preferences* for a central banker. The expected value of the government's objective function is

$$\begin{aligned}
\mathrm{E}[V] &= \frac{1}{2}\mathrm{E}(\lambda\{a[\pi^d(\delta) - \pi^e] + e - k\}^2 + [\pi^d(\delta)]^2) \\
&= \frac{1}{2}\left[\lambda k^2 + \lambda\left(\frac{1+\delta}{1+\delta+a^2\lambda}\right)^2\sigma_e^2 + a^2\lambda\sigma_v^2\right] \\
&\quad + \frac{1}{2}\left[\left(\frac{a\lambda k}{1+\delta}\right)^2 + \left(\frac{a\lambda}{1+\delta+a^2\lambda}\right)^2\sigma_e^2 + \sigma_v^2\right],
\end{aligned}$$

where we have used (8.18) to replace π^e with $a\lambda k/(1+\delta)$ under the assumption that the public knows δ when forming its expectations. Minimizing this expression with

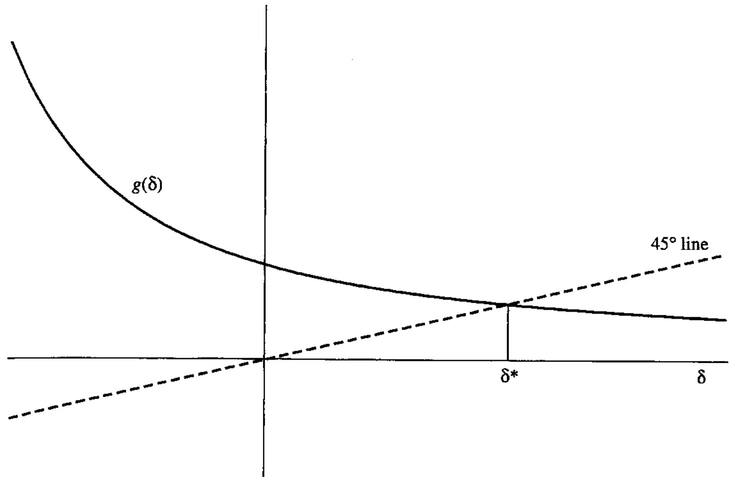

Figure 8.4
The Optimal Degree of Conservatism

respect to δ yields, after some manipulation, the following condition that must be satisfied by the optimal value of δ:

$$\delta = \left(\frac{k^2}{\sigma_e^2}\right)\left(\frac{1+\delta+a^2\lambda}{1+\delta}\right)^3 \equiv g(\delta). \tag{8.19}$$

The function $g(\delta)$ is shown in figure 8.4.[30] Equation (8.19) is satisfied where $g(\delta)$ crosses the 45° line. Since $\lim_{\delta\to\infty} g(\delta) = k^2/\sigma_e^2 > 0$, the intersection always occurs in the range $\delta \in (0, \infty]$; given the trade-off between distorting the response of policy to aggregate-supply shocks and reducing the average inflation bias, it is always optimal to appoint a central banker who places more weight ($\delta > 0$) on inflation objectives than the government itself does.

Rogoff's solution is often characterized as involving the appointment of a conservative to head an independent central bank. The concept of *independence* means that, once appointed, the central banker is able to set policy without interference or restriction, and she will do so to minimize her own assessment of social costs. Thus,

30. See Eijffinger, Hoeberichts, and Schaling (1995) for a discussion of this graphical representation of the determinants of the optimal degree of conservatism. Eijffinger and Schaling (1995) extend the framework to an open-economy context.

the inflation bias problem is solved partially through delegation; the government delegates responsibility for monetary policy to an independent central bank. The benefit of this independence is lower average inflation; the cost depends on the realization of the aggregate-supply shock. If shocks are small, the gain in terms of low inflation clearly dominates the distortion in stabilization policy; if shocks are large, the costs associated with the stabilization distortion can dominate the gain from low inflation.[31]

Lohmann (1992) shows that the government can do even better if it appoints a weight-conservative central banker but limits the central bank's independence. If the aggregate-supply shock turns out to be too large, the government overrides the central banker, where the critical size determining what is too large is determined endogenously as a function of the costs of overriding. The knowledge that she can be overridden also affects the way the central banker responds to shocks that are less than the threshold level that triggers an override. By responding more actively to large shocks, the central banker is able to extend the range of shocks over which she maintains independence.

This type of outcome—a relatively independent and conservative central bank whose independence may be overruled in the face of large economic disturbances—provides an appealing model of the structure of monetary policy in a country such as Germany. There the Bundesbank was perceived to place a large weight on achieving low inflation and to enjoy a high degree of political independence. Yet, at the time of the unification of East and West Germany, the Bundesbank recommended against exchanging East for West German marks at a one-for-one exchange rate but was overruled by the government of Helmut Kohl. While exchange-rate policy in Germany, as in other countries, is generally under the government's control and not the central bank's, this example illustrates how even independent central banks do not operate in a political vacuum.

Rogoff's solution highlights a trade-off: one can reduce the bias but only at the cost of distorting stabilization policy. One implication is that countries with central banks which place a high weight on inflation objectives should have, on average, lower inflation, but they should also experience greater output variance. The variance of output is equal to

$$\left(\frac{1+\delta}{1+\delta+a^2\lambda}\right)^2\sigma_e^2 + a^2\sigma_v^2,$$

31. Since society is better off appointing a conservative, the expected gain from low inflation exceeds the expected stabilization cost, however.

and this is increasing in δ. Highly independent central banks are presumed to place more weight on achieving low inflation, and a large literature has investigated the finding that measures of central bank independence are negatively correlated with average inflation, at least for the industrialized economies (see Cukierman 1992; Eijffinger and de Haan 1996). Alesina and Summers (1993) have shown, however, that such measures do not appear to be correlated with the variance of real output. This runs counter to the implications of the Rogoff model.

Solving the inflationary bias of discretionary policy through the appointment of a conservative raises several issues. First, how does the government identify the preference parameter δ? Second, how does it commit to a δ? Once expectations are set, the government has an incentive to fire the conservative central banker and replace her with one sharing the government's preferences. Third, what prevents further delegation? If a government with preference parameter 1 finds it optimal to delegate monetary policy to a central banker with preference parameter $1+\delta > 1$, the same argument implies that it will be optimal for the conservative central bank to delegate policy to an even more conservative staff member with preference parameter $1+\delta' > 1+\delta > 1$. Finally, the focus on preferences, as opposed to incentives, clouds the model's implications for institutional structure and design. Should institutions be designed to generate appropriate incentives for policy makers? Or does good policy simply require putting the right people in charge?

8.3.3 Contracts

The problems that occur under discretion arise because central banks respond optimally to the incentive structure they face, but the incentives are wrong. This conclusion suggests that rather than relying on the central banks having the right preferences, one might try to affect the incentives the central banks face. But this requires first determining what incentives central banks *should* face.

The appropriate perspective for addressing such issues is provided by the principal-agent literature.[32] A key insight that motivated the large literature expanding on the analysis of the time inconsistency of optimal plans was the recognition that central banks respond to the incentives they face. These incentives may be shaped by the institutional structure within which policy is conducted. For example, as we have already seen, Lohmann shows how policy is affected when the central banker knows she will be overridden by the government if the economy is subject to a disturbance that is "too" big. Rogoff (1985b, p. 1180) argues that targeting rules might be enforced by making the monetary authority's budget depend on adherence to the rule. In a similar vein, Garfinkel and Oh (1993) suggest that a targeting rule might be

32. This section draws heavily on Walsh (1995a). See also Persson and Tabellini (1993) and Waller (1995).

enforced by legislation punishing the monetary authority if it fails to achieve the target. Such institutional aspects of the central bank's structure and its relationship with the government can be thought of as representing a *contract* between the government and the monetary authority. The conduct of monetary policy is then affected by the contract the government offers to the central bank.

The government's (or perhaps society's) problem can be viewed as that of designing an optimal incentive structure for the central bank. Following Walsh (1995a), the most convenient way to determine an optimal incentive structure is to assume that the government can offer the head of the central bank a state-contingent wage contract. Such a contract allows one to derive explicitly the manner in which the bank's incentives should be dependent on the state of the economy. While there are numerous reasons to question the effectiveness and implementability of such employment contracts in the context of monetary-policy determination, a (possibly) state-contingent wage contract for the central banker represents a useful fiction for deriving the optimal incentive structure with which the central bank should be faced and provides a convenient starting point for the analysis of optimal central bank incentives.[33]

The basic structure of the model is identical to that used earlier, consisting of an aggregate-supply relationship given by (8.3), a link between money growth and inflation given by (8.4), and an objective function that depends on output fluctuations and inflation variability, as in (8.2). The private sector's expectations are assumed to be determined prior to the central bank's choice of a growth rate for the nominal money supply. Thus, in setting Δm, the central bank will take π^e as given. We will also assume that the central bank can observe the supply shock e prior to setting Δm, since this will generate a role for stabilization policy. The disturbance v in the link between money growth and inflation is realized after the central bank sets Δm. Finally, assume that e and v are uncorrelated.

Monetary policy is conducted by an independent central bank, one which shares the government's preferences, V, but which also receives a monetary transfer payment from the government. This payment can be thought of either as the direct income of the central banker or as the budget of the central bank. Or the transfer payment can be viewed more broadly as reflecting legislated performance objectives for the central bank. Let t represent the transfer to the central bank, and assume that the central bank's utility is given by

$$\mathrm{U} = t - V.$$

33. Walsh (2002a) demonstrates that a dismissal rule can, in some circumstances, substitute for a state contingent wage contract in affecting the central bank's incentives.

That is, the central bank cares about both the transfer it receives and the social loss generated by inflation and output fluctuations. The central bank sets Δm to maximize the expected value of $\mathbf{U}$, conditional on the realization of e. The problem faced by the government (the principal) is to design a transfer function t that induces the central bank to choose $\Delta m = \Delta m^c(e)$, where Δm^c is the socially optimal commitment policy. As we have already seen, the optimal commitment policy in this framework is $\Delta m^c(e) = -a\lambda e/(1 + a^2\lambda)$ (see 8.11).

If the government can verify e ex post, there are clearly many contracts that would achieve the desired result. For example, any contract that imposes a large penalty on the central bank if Δm deviates from Δm^c will ensure that Δm^c is chosen. However, the difficulty of determining both the possible states of nature ex ante and the actual realization of shocks ex post makes such contracts infeasible. This task is particularly difficult if the central bank must respond to a forecast of e, since its internal forecast might be difficult to verify ex post, leading to the problems of private information highlighted by Canzoneri's (1985) analysis. Therefore, consider a transfer function $t(\pi)$ that makes the government's payment to the central bank contingent on the observed rate of inflation. The transfer function implements the optimal policy $\pi^c(e) = \Delta m^c(e) + v$ if π^c maximizes $\mathrm{E}^{cb}[t(\pi(e)) - V]$ for all e, where $\mathrm{E}^{cb}[\ ]$ denotes the central bank's expectation conditional on e.

The first order condition for the central banker's problem can be solved for $\Delta m^{cb}(e)$, the optimal discretionary policy:

$$\Delta m^{cb}(e) = \frac{a\lambda k}{1 + a^2\lambda} + \left(\frac{a^2\lambda}{1 + a^2\lambda}\right)\pi^e + \frac{\mathrm{E}^{cb}(t')}{1 + a^2\lambda} - \left(\frac{a\lambda}{1 + a^2\lambda}\right)e, \tag{8.20}$$

where $t' = \partial t(\pi)/\partial\pi$. The last term in (8.20) shows that the optimal discretionary policy response to the supply shock is equal to the response under the optimal commitment policy Δm^c. This is important because it implies that the government's objective will be to design a contract that eliminates the inflationary bias while leaving the central bank free to respond with discretion to e. Taking expectations of (8.20) and letting $\mathrm{E}[\ ]$ denote the public's expectation, we obtain

$$\mathrm{E}[\Delta m^{cb}(e)] = \pi^e = a\lambda k + \mathrm{E}[t'(\pi)].$$

When this is substituted back into (8.20), we obtain

$$\Delta m^{cb}(e) = a\lambda k + \mathrm{E}[t'(\pi(e))] - \frac{\mathrm{E}[t'] - \mathrm{E}^{cb}[t']}{1 + a^2\lambda} - \frac{a\lambda}{1 + a^2\lambda}e.$$

Setting $\Delta m^{cb}(e)$ equal to the optimal commitment policy $\Delta m^c(e)$ for all e requires that the first three terms vanish. They will vanish if $t(\pi)$ satisfies

$$t' = \frac{\partial t}{\partial \pi} = -a\lambda k.$$

The optimal commitment policy can be implemented, therefore, by the linear transfer function

$$t = t_0 - a\lambda k\pi.$$

The constant t_0 is set to ensure that the expected return to the central banker is sufficient to ensure her participation.[34] Presenting the central banker with this incentive contract achieves the dual objectives of eliminating the inflationary bias while still ensuring optimal stabilization policy in response to the central bank's private information about the aggregate-supply shock.

Why does the transfer function take such a simple linear form? Recall that the time-consistent policy under discretion resulted in an inflationary bias of $a\lambda k$. The key insight is that this is constant; it does not vary with the realization of the aggregate-supply shock. Therefore, the incentive structure for the central bank just needs to raise the marginal cost of inflation (from the perspective of the central banker) by a constant amount; that is what the linear transfer function does. Because the bias is independent of the realization of the underlying state of nature, it is not necessary for the government to actually verify the state, and so the presence of private information about the state of the economy on the part of the central bank does not affect the ability of the linear contract to support the optimal policy. This case contrasts sharply with the one in which reputation is relied on to achieve low average inflation (Canzoneri 1985).

One interpretation of the linear-inflation-contract result is that it simply points out that the Barro-Gordon framework is too simple to adequately capture important aspects of monetary-policy design. In this view, there really is a trade-off between credibility and flexibility, and the fact that this trade-off can be made to disappear so easily represents a methodological criticism of the Barro-Gordon model.[35] Several authors have explored modifications to the Barro-Gordon model that allow this trade-off to be reintroduced. They do so by making the inflation bias state dependent. In this way, the linear contract, which raises the marginal cost of inflation by a

34. This is known as the *individual rationality constraint*. Since $\partial\pi/\partial m = 1$, a contract of the form $t_0 - akm$ based on the observed rate of money growth would also work. Chortareas and Miller (forthcoming) analyze the case in which the government also cares about the cost of the contract.

35. This argument is made by Canzoneri, Nolan, and Yates (1997).

constant amount for all state realizations, cannot achieve the socially optimal commitment policy. If the penalty cannot be made state contingent, then average inflation can be eliminated, but inflation will remain too volatile. For example, Walsh (1995c), Canzoneri, Nolan, and Yates (1997), and Herrendorf and Lockwood (1997) introduce a state-contingent bias by modifying the basic model structure. Walsh assumes that there exists a flexible wage sector in addition to a nominal-wage-contract sector. Herrendorf and Lockwood assume that labor-market participants can observe a signal that reveals information about aggregate-supply shocks prior to forming nominal wage contracts. Canzoneri, Nolan, and Yates assume that the central bank has an interest-rate-smoothing objective. Herrendorf and Lockwood (1997) and Muscatelli (1998) show that when the inflation bias is state-contingent rather than constant, as in the Baro-Gordon model, there can be a role for a linear inflation contract, as in Walsh (1995a), *and* a conservative central banker, as in Rogoff (1985b). Schellekens (2002) considers delegation to a central bank with preferences that are generalized from the standard quadratic form. He examines the connection between optimal conservatism and cautionary policy arising from model uncertainty.

The contracting approach has been developed further in Persson and Tabellini (1993). Walsh (1995c, 2002a) shows how the properties of a linear inflation contract can be mimicked by a dismissal rule under which the central banker is fired if inflation ever rises above a critical level. Lockwood (1997), Jonsson (1995, 1997), and Svensson (1997b) have shown how linear inflation contracts are affected when the inflation bias is time dependent because of persistence in the unemployment process. *Persistence* means that a surprise expansion in period t reduces unemployment (increases output) in period t, but it also leads to lower expected unemployment in periods $t+1$, $t+2$, and so on. Thus, the benefits of a surprise inflation are larger, leading to a higher average inflation rate under discretionary policy. The bias at time t, though, will depend on the unemployment rate at $t-1$, since, with persistence, unemployment at $t-1$ affects the average unemployment rate expected for period t. Therefore, the inflation bias will be time varying. The simple linear contract with a fixed weight on inflation will no longer be optimal if the inflation bias is state dependent. However, a state-contingent contract can support the optimal commitment policy.

Like the Rogoff conservative-central-banker solution, the contracting solution *relocates* the commitment problem that gives rise to the inflation bias in the first place.[36] Jensen (1997) shows how the ability of an incentive contract for the central

36. McCallum (1995, 1997b) has emphasized the relocation issue with respect to the contracting approach. A similar criticism applies to the conservative central banker solution as well.

banker to solve an inflation bias is weakened when the government can undo the contract ex post. In the case of the conservative central banker, the proposed solution assumes that the government cannot commit to a specific inflation policy but can commit to the appointment of an agent with specific preferences. In the contracting case, the government is assumed to be able to commit to a specific contract. Both of these assumptions are plausible; relocating the commitment problem is often a means of solving the problem. Confirmation processes, together with long terms of office, can reveal the appointee's preferences and ensure that policy is actually conducted by the appointed agent. Incentives called for in the contracting approach can similarly be thought of as aspects of the institutional structure and may therefore be more difficult to change than actual policy instrument settings.

As al-Nowaihi and Levine (1996) argue, relocation can allow the government to commit credibly to a contract or to a particular appointee if the process is public. If contract renegotiations or the firing of the central banker are publicly observable, then it may be in the interest of the government to forgo any short-term incentive to renegotiate in order to develop a reputation as a government that can commit. Thus, the transparency of any renegotiation serves to support a low-inflation equilibrium; relocating the time-inconsistency problem can solve it.[37]

The type of policy transparency emphasized by al-Nowaihi and Levine characterizes the policy process established under the 1989 central banking reform in New Zealand. There the government and the Reserve Bank establish short-run inflation targets under a Policy Targets Agreement (PTA). The PTA can be renegotiated, and once current economic disturbances have been observed, both the government and the Reserve Bank have incentives to renegotiate the target (Walsh 1995c). Because this renegotiation must be public, however, reputational considerations may sustain an equilibrium in which the targets are not renegotiated. When we turn to an analysis of inflation-targeting rules, we will see that inflation targeting may also replicate the optimal incentives called for under the linear inflation contract (Svensson 1997b).

Dixit and Jensen (2001) have extended the contracting approach to the case of a monetary union in which member governments offer the common central bank incentive contracts designed to influence monetary policy. They show that if the central bank cares about the incentives it receives and about the union-wide inflation rate, the central bank implements a policy that leads to average inflation that is too low and stable. The central bank implements a weighted average of each member

37. See also Herrendorf (1995a, 1998), who develops a similar point using inflation targeting, and Walsh (2002a), who shows that the government will find it advantageous to carry out a dismissal rule policy under which the central banker is fired if inflation exceeds a critical level.

country's desired policy only if the central bank cares only about the contract incentives. Hence, mandating that the central bank achieve price stability would result in a deflationary bias under discretion.

8.3.4 Institutions

One interpretation of the contracting approach is that the incentive structures might be embedded in the institutional structure of the central bank. If institutions are costly to change, then institutional reforms designed to raise the costs of inflation can serve as commitment devices. Incorporating a price-stability objective directly in the central bank's charter legislation, for example, might raise the implicit penalty (in terms of institutional embarrassment) the central bank would suffer if it failed to control inflation. Most discussions of the role of institutional structure and inflation have, however, focused on the effects alternative structures have on the extent to which political pressures affect the conduct of monetary policy.

A starting point for such a focus is Alesina's model of policy in a two-party system.[38] Suppose there is uncertainty about the outcome of an approaching election, and suppose the parties differ in their economic policies, so that inflation in the postelection period will depend on which party wins the election. Let the parties be denoted A and B. The inflation rate expected if party A wins the election is π^A; inflation under party B will be π^B. Assume $\pi^A > \pi^B$. If the probability that party A wins the election is q, then expected inflation prior to the election will be $\pi^e = q\pi^A + (1-q)\pi^B$. Since q is between 0 and 1, expected inflation falls in the interval $[\pi^B, \pi^A]$. If postelection output is equal to $y = a(\pi - \pi^e)$, where π is actual inflation, then the election of party A will generate an economic expansion (since $\pi^A - \pi^e = (1-q)(\pi^A - \pi^B) > 0$), while the election of party B will lead to an economic contraction ($\pi^B - \pi^e = q(\pi^B - \pi^A) < 0$).

This very simple framework provides an explanation for a political business cycle that arises because of policy differences between parties and electoral uncertainty. Because parties are assumed to exploit monetary policy to get their desired inflation rate, and because election outcomes cannot be predicted with certainty, inflation surprises will occur after an election. Alesina and Sachs (1988) provide evidence for this theory based on U.S. data, while Alesina and Roubini (1992) examine OECD countries. Faust and Irons (1996), however, conclude that there is little evidence from the United States to support the hypothesis that political effects generate monetary-policy surprises.

38. For a discussion of this model, see Alesina (1987), Alesina and Sachs (1988), Alesina and Roubini (1992, 1997), and Drazen (2000).

Waller (1989, 1992) shows how the process used to appoint members of the central bank's policy board can influence the degree to which partisan political factors are translated into monetary-policy outcomes. If policy is set by a board whose members serve overlapping but noncoincident terms, the effect of policy shifts resulting from changes in government is reduced. In a two-party system in which nominees forwarded by the party in power are subject to confirmation by the out-of-power party, the party in power will nominate increasingly moderate candidates as elections near. Increasing the length of terms of office for central bank board members also reduces the role of partisanship in monetary policy making.[39] Waller and Walsh (1996) consider a partisan model of monetary policy. They focus on the implications for output of the degree of partisanship in the appointment process and the term length of the central banker. Similarly, Alesina and Gatti (1995) show that electorally induced business cycles can be reduced if political parties jointly appoint the central banker.

While most work has focused on the appointment of political nominees to the policy board, the Federal Reserve's policy board (the FOMC) includes both political appointees (the governors) and nonappointed members (the regional bank presidents).[40] Faust (1996) provides an explanation for this structure by developing an overlapping-generations model in which inflation has distributional effects. If monetary policy is set by majority vote, excessive inflation results as the (larger) young generation attempts to transfer wealth from the old generation. If policy is delegated to a board consisting of one representative from the young generation and one from the old, the inflationary bias is eliminated. Faust argues that the structure of the FOMC takes the shape it does because of the advantages of delegating to a board in which the relative balance of different political constituencies differs from that of the voting public as a whole.

Who makes policy and who appoints the policy makers can affect policy outcomes, but institutional design also includes mechanisms for accountability, and these can affect policy as well. Minford (1995), in fact, argues that democratic elections can enforce low-inflation outcomes if voters punish governments that succumb to the temptation to inflate, while Lippi (1997) develops a model in which rational voters choose a weight-conservative central banker. O'Flaherty (1990) shows how finite term lengths can ensure accountability, while Walsh (1995c) shows that the

39. See also Havrilesky and Gildea (1992) and Garcia de Paso (1994). For some recent empirical evidence in support of these models, see Mixon and Gibson (2002).

40. And Havrilesky and Gildea (1991, 1995) argue that the voting behavior of regional bank presidents and board governors differs, with regional bank presidents tending to be tougher on inflation; this conclusion is disputed by Tootell (1991).

type of dismissal rule incorporated into New Zealand's Reserve Bank Act of 1989 can partially mimic an optimal contract.

The launch of the European Central Bank in 2000 helped to focus attention on the role institutions and their formal structure play in affecting policy outcomes. Because the individual member countries in a monetary union may face different economic conditions, disagreements about the common central bank's policies may arise. Dixit (2000) has used a principal-agent approach to study policy determination in a monetary union. With a single central bank determining monetary policy for a union of countries, the central bank is the agent of many principals. Each principal may try to influence policy outcomes, and the central bank may need to appease its principals to avoid noncooperative outcomes.

Dixit shows that the central bank's decision problem must take into account the individual incentive compatibility constraints that require all principals to accept a continuation of the policy the central bank chooses. For example, if one country has an large adverse shock, the central bank may have to raise inflation above the optimal commitment level to ensure the continued participation in the union of the affected country. When the incentive constraint binds, policy will diverge from the full-commitment case in order to secure the continued participation of the union members. Dixit shows that when countries are hit by different shocks, it is the incentive constraint of the worst-hit country that is binding—policy must shade toward what that country would want. If the costs of overturning the central bank's policy (and thereby reverting to the discretionary equilibrium) are high enough, there will be some range of asymmetric shocks within which it is possible to sustain the full-commitment policy.

8.3.5 Targeting Rules

The contracting approach focuses on the incentive structure faced by the central bank; once the incentives are correct, complete flexibility in the actual conduct of policy is allowed. This allows the central bank to respond to new and possibly unverifiable information. An alternative approach acts to reduce the problems arising from discretion by *restricting* policy flexibility. The gold standard or a fixed-exchange-rate regime provide examples of situations in which policy flexibility is deliberately limited; Milton Friedman's proposal that the Fed be required to maintain a constant growth rate of the money supply is another famous example. A wide variety of rules designed to restrict the flexibility of the central bank have been proposed and analyzed. The cost of reduced flexibility depends on the nature of the economic disturbances affecting the economy and the original scope for stabilization policies in the first place, while the gain from reducing flexibility takes the form of a lower average inflation rate.

In this section, we examine targeting rules, that is, rules under which the central bank is judged in part on its ability to achieve a prespecified value for some macro variable. Inflation targeting is currently the most commonly discussed form of targeting, and some form of inflation targeting has been adopted in Canada, Israel, Mexico, Sweden, the United Kingdom, and New Zealand. The mandate of the European Central Bank to pursue price stability as its sole objective can also be viewed as representing a form of inflation targeting.[41] Fixed or target-zone exchange-rate systems also can be interpreted as targeting regimes. The central bank's ability to respond to economic disturbances, or to succumb to the temptation to inflate, is limited by the need to maintain an exchange-rate target. When the lack of credibility is a problem for the central bank, committing to maintaining a fixed nominal exchange rate against a low-inflation country can serve to import credibility. Giavazzi and Pagano (1988) provide an analysis of the advantages of "tying one's hands" by committing to a fixed exchange rate.

Flexible Targeting Rules Suppose the central bank cares about output and inflation stabilization but is, in addition, penalized for deviations of actual inflation from a target level.[42] In other word, the central bank's objective is to minimize

$$V^{cb} = \tfrac{1}{2}\lambda \mathrm{E}_t(y_t - y_n - k)^2 + \tfrac{1}{2}\mathrm{E}_t(\pi_t - \pi^*)^2 + \tfrac{1}{2}h\mathrm{E}_t(\pi_t - \pi^T)^2, \tag{8.21}$$

where this differs from (8.2) in that π^* now denotes the socially optimal inflation rate (which may differ from zero), and the last term represents the penalty related to deviations from the target inflation rate π^T. The parameter h measures the weight placed on deviations from the target inflation rate. We will refer to targeting rules of this form as *flexible targeting rules*. They do not require that the central bank hit its target exactly; instead, one can view the last term as representing a penalty suffered by the central bank based on how large the deviation from the target turns out to be. This type of targeting rule allows the central bank to trade off achieving its inflation target for achieving more desired values of its other goals.

The rest of the model consists of an aggregate supply function and a link between the policy instrument, the growth rate of money, and inflation:

41. For recent discussions of inflation targeting experiences, see Ammer and Freeman (1995), Haldane (1995), McCallum (1997a), Mishkin and Posen (1997), Bernanke, Lauback, Mishkin, and Posen (1998), and the papers in Leiderman and Svensson (1995) and Lowe (1997). See also section 11.5.

42. The central bank might be required to report on its success or failure in achieving the target, with target misses punished by public censoring and embarrassment or by some more formal dismissal procedure.

$$y_t = y_n + a(\pi_t - \pi^e) + e_t$$

and

$$\pi_t = \Delta m_t + v_t,$$

where v is a velocity disturbance. It will be assumed that the public's expectations are formed prior to observing either e or v, but the central bank can observe e (but not v) before setting Δm.

Before deriving the policy followed by the central banker, note that the socially optimal commitment policy is given by[43]

$$\Delta m_t^S = \pi^* - \left(\frac{a\lambda}{1 + a^2\lambda}\right)e_t. \tag{8.22}$$

Now consider policy under discretion. Using the aggregate-supply function and the link between inflation and money growth, the loss function (8.21) can be written as

$$V^{cb} = \tfrac{1}{2}\lambda \mathrm{E}[a(\Delta m + v - \pi^e) + e - k]^2 + \tfrac{1}{2}\mathrm{E}(\Delta m + v - \pi^*)^2 + \tfrac{1}{2}h\mathrm{E}(\Delta m + v - \pi^T)^2.$$

The first order condition for the optimal choice of Δm, taking expectations as given, is

$$a^2\lambda(\Delta m - \pi^e) + a\lambda(e - k) + (\Delta m - \pi^*) + h(\Delta m - \pi^T) = 0.$$

Solving yields

$$\Delta m = \frac{a^2\lambda\pi^e - a\lambda e + a\lambda k + \pi^* + h\pi^T}{1 + h + a^2\lambda}. \tag{8.23}$$

Assuming rational expectations, $\pi^e = \Delta m^e = (a\lambda k + \pi^* + h\pi^T)/(1 + h)$, since the public forms expectations prior to knowing e. Substituting this result into (8.23) yields the time-consistent money growth rate:

$$\begin{aligned}\Delta m^T &= \frac{a\lambda k + \pi^* + h\pi^T}{1 + h} - \left(\frac{a\lambda}{1 + h + a^2\lambda}\right)e \\ &= \pi^* + \frac{a\lambda k}{1 + h} + \frac{h(\pi^T - \pi^*)}{1 + h} - \left(\frac{a\lambda}{1 + h + a^2\lambda}\right)e. \end{aligned} \tag{8.24}$$

43. This is obtained by substituting the commitment policy $\Delta m = b_0 + b_1 e$ into the social objective function

$$\tfrac{1}{2}[\lambda \mathrm{E}(y - y_n - k)^2 + \mathrm{E}(\pi - \pi^*)^2]$$

and minimizing the unconditional expectation with respect to b_0 and b_1.

If the target inflation rate is equal to the socially optimal inflation rate ($\pi^T = \pi^*$), (8.24) reduces to

$$\Delta m^T = \pi^* + \frac{a\lambda k}{1+h} - \left(\frac{a\lambda}{1+h+a^2\lambda}\right)e. \tag{8.25}$$

Setting $h = 0$ yields the time-consistent discretionary solution *without* targeting:

$$\Delta m^{NT} = \pi^* + a\lambda k - \left(\frac{a\lambda}{1+a^2\lambda}\right)e, \tag{8.26}$$

with the inflation bias equal to $a\lambda k$.

Comparing (8.22), (8.25), and (8.26) reveals that the targeting penalty reduces the inflation bias from $a\lambda k$ to $a\lambda k/(1+h)$. The targeting requirement imposes an additional cost on the central bank if it allows inflation to deviate too much from π^T; this raises the marginal cost of inflation and reduces the time-consistent inflation rate. The cost of this reduction in the average inflation bias is the distortion that targeting introduces into the central bank's response to the aggregate supply shock e. Under pure discretion, the central bank responds optimally to e (note that the coefficient on the supply shock is the same in (8.26) as in (8.22)), but the presence of a targeting rule distorts the response to e. Comparing (8.25) with (8.22) shows that the central bank will respond too little to the supply shock (the coefficient falls from $a\lambda/(1+a^2\lambda)$ to $a\lambda/(1+h+a^2\lambda)$).

This trade-off between bias reduction and stabilization response is one we have seen earlier in discussing Rogoff's model.[44] Note that if $\pi^T = \pi^*$, the central bank's objective function can be written as

$$V^{cb} = \tfrac{1}{2}\lambda \mathrm{E}(y_t - y_n - k)^2 + \tfrac{1}{2}(1+h)\mathrm{E}(\pi - \pi^*)^2. \tag{8.27}$$

It is apparent from (8.27) that the parameter h plays exactly the same role that Rogoff's degree of conservatism played. From the analysis of Rogoff's model, we know that the optimal value of h will be positive, so that the total weight placed on the inflation objective exceeds society's weight, which is equal to 1. A flexible inflation target, interpreted here as a value for h that is positive, leads to an outcome that dominates pure discretion.[45]

44. Canzoneri (1985), Garfinkel and Oh (1993), and Garcia de Paso (1993, 1994) consider multiperiod targeting rules as solutions to this trade-off between stabilization and inflation bias. Defining money growth or inflation targets as averages over several periods restricts average inflation while allowing the central bank more flexibility in each period to respond to shocks.

45. That is, of course, unless h is too large.

While we have just highlighted the connection between an inflation-targeting rule and Rogoff's conservative central banker approach, Svensson (1997b) has shown that a similar connection exists between inflation targeting and the linear inflation contract. Svensson demonstrates that the optimal linear inflation contract can be implemented if the central bank is required to target an inflation rate π^T that is actually less than the socially optimal rate of inflation. To see how this result is obtained, let $H = 1 + h$, replace π^* with π^T in (8.27), and expand the resulting second term so that the expression becomes

$$\begin{aligned} V^{cb} &= \tfrac{1}{2}\lambda \mathrm{E}(y_t - y_n - k)^2 + \tfrac{1}{2}H\mathrm{E}(\pi - \pi^* + \pi^* - \pi^T)^2 \\ &= \tfrac{1}{2}\lambda \mathrm{E}(y_t - y_n - k)^2 + \tfrac{1}{2}H\mathrm{E}(\pi - \pi^*)^2 + D\mathrm{E}(\pi - \pi^*) + C, \end{aligned}$$

where $D = H(\pi^* - \pi^T)$ and $C = \frac{1}{2}H(\pi^* - \pi^T)^2$. Since C is a constant, it does not affect the central bank's behavior. Notice that V^{cb} is equal to $V + \frac{1}{2}h\mathrm{E}(\pi - \pi^*)^2 + D\mathrm{E}(\pi - \pi^*) + C$. This is exactly equivalent to the incentive structure established under the optimal linear inflation contract if and only if $h = 0$ and $D = -a\lambda k$. The condition $h = 0$ is achieved if the central banker is not a weight conservative but instead shares society's preferences (so $H = 1$); the condition $D = -a\lambda k$ is then achieved if

$$\pi^T = \pi^* - a\lambda k < \pi^*.$$

Thus, the optimal linear contract can be implemented by assigning to the central bank an inflation target that is actually below the rate that is socially preferred. But at the same time, policy should be assigned to an agent who has the same preferences between inflation and output stabilization as society in general.

Strict Targeting Rules The preceding analysis considered a flexible targeting rule. The central bank was penalized for deviations of π around a targeted level but was not required to achieve the target precisely. This flexibility allowed the central bank to trade off the objective of meeting the target against achieving its other objectives. Often, however, targeting is analyzed in terms of strict targets; the central bank is required to achieve a specific target outcome, regardless of the implications for its other objectives. For an early analysis of strict targeting regimes, see Aizenman and Frankel (1986).

As an example, consider a strict money growth rate target under which the central bank is required to set the growth rate of the money supply equal to some constant:[46]

46. Alternatively, the targeting rule could require the central bank to minimize $\mathrm{E}(\Delta m - \Delta m^T)^2$. However, this occurs if the central bank sets policy such that $\mathrm{E}(\Delta m) = \Delta m^T$. If Δm is controlled exactly, this is equivalent to $\Delta m = \Delta m^T$.

$$\Delta m = \Delta m^T.$$

Since the desired rate of inflation is π^*, it makes sense to set $\Delta m^T = \pi^*$, and the public will set $\pi^e = \pi^*$. With this rule in place, we can now evaluate the social loss function. If social loss is given by

$$V = \tfrac{1}{2}\lambda \mathrm{E}_t(y_t - y_n - k)^2 + \tfrac{1}{2}\mathrm{E}_t(\pi_t - \pi^*)^2,$$

then under a strict money growth rate target it takes the value

$$V(\Delta m^T) = \tfrac{1}{2}[\lambda k^2 + \lambda \sigma_e^2 + (1 + a^2\lambda)\sigma_v^2].$$

Recall that under pure discretion, the expected value of the loss function was, from (8.9),

$$V^d = \frac{1}{2}\lambda(1 + a^2\lambda)k^2 + \frac{1}{2}\left[\left(\frac{\lambda}{1 + a^2\lambda}\right)\sigma_e^2 + (1 + a^2\lambda)\sigma_v^2\right].$$

Comparing these two, we have

$$V(\Delta m^T) - V^d = -\frac{1}{2}(a\lambda k)^2 + \frac{1}{2}\left(\frac{a^2\lambda^2}{1 + a^2\lambda}\right)\sigma_e^2.$$

Notice that this can be either positive or negative. It is more likely to be negative (implying that the strict money growth rate target is superior to discretion) if the underlying inflationary bias under discretion, $a\lambda k$, is large. Since the strict targeting rule ensures that average inflation is π^*, it eliminates any inflationary bias, so the gain is larger, the larger the bias that arises under discretion. However, discretion is more likely to be preferred to the strict rule when σ_e^2 is large. The strict targeting rule eliminates any stabilization role for monetary policy. The cost of doing so will depend on the variance of supply shocks. Eliminating the central bank's flexibility to respond to economic disturbances increases welfare if

$$k > \sigma_e \sqrt{\frac{1}{1 + a^2\lambda}}.$$

If σ_e^2 is large, pure discretion, even with its inflationary bias, may still be the preferred policy (Flood and Isard 1988).

Another alternative targeting rule that has often been proposed focuses on nominal income (see, for example, Hall and Mankiw 1994). If we interpret $y - y_n$ as the percentage output deviation from trend, we can approximate a nominal income rule as requiring that

$$y - y_n + \pi = g^*,$$

where g^* is the target growth rate for nominal income. Because the equilibrium growth rate of $y - y_n$ is zero (because it is a deviation from trend) and the desired rate of inflation is π^*, we should set $g^* = 0 + \pi^* = \pi^*$. Under this rule, expected inflation is $\pi^e = g^* - \mathrm{E}(y - y_n) = g^* - 0 = g^* = \pi^*$. Aggregate output is given by

$$y = y_n + a(\pi - \pi^e) + e = y_n + a(y_n - y) + e \Rightarrow y - y_n = \left(\frac{1}{1 + a}\right)e,$$

because $\pi = g^* - (y - y_n) = \pi^e - (y - y_n)$ under the proposed rule. A positive supply shock that causes output to increase will induce a contraction designed to reduce the inflation rate to maintain a constant rate of nominal income growth. The decline in inflation (which is unanticipated because it was induced by the shock e), acts to reduce output and partially offset the initial rise. With the specification used here, exactly $a/(1 + a)$ of the effect of e is offset. Substituting this result back into the policy rule implies that $\pi = \pi^* - e/(1 + a)$.

Using these results, the expected value of the social loss function is

$$V(g^*) = \frac{1}{2}\lambda k^2 + \frac{1}{2}\left[\frac{1 + \lambda}{(1 + a)^2}\right]\sigma_e^2.$$

In the present model, nominal income targeting stabilizes real output more than pure discretion (and the optimal commitment policy) if $a\lambda < 1$. In this example, we assumed that the central bank could control nominal income growth exactly. If, as is more realistic, this is not the case, a term caused by control errors will also appear in the expected value of the loss function.

Nominal income targeting imposes a particular trade-off between real income growth and inflation in response to aggregate-supply disturbances. The social loss function does not weigh output fluctuations and inflation fluctuations equally (unless $\lambda = 1$), but nominal income targeting does. Nevertheless, nominal income targeting is often proposed as a "reasonably good rule for the conduct of monetary policy" (Hall and Mankiw 1994). For analyses of nominal income targeting, see Bean (1983), Frankel and Chinn (1995), McCallum (1988), Taylor (1985), and West (1984). Targeting rules in new Keynesian models are discussed in section 11.5.

The analysis of targeting rules has much in common with the analysis of monetary-policy operating procedures, the topic of chapter 9. Targeting rules limit the flexibility of the central bank to respond as economic conditions change. Thus, the manner in which disturbances will affect real output and inflation will be affected by the choice of targeting rule. For example, a strict inflation or price-level rule forces real output to absorb all the effects of an aggregate productivity disturbance. Under a nominal income rule, such disturbances are allowed to affect both real output and the price level. As with operating procedures, the relative desirability of alternative rules will depend both on the objective function and on the relative variances of different types of disturbances.

8.4 Is the Inflation Bias Important?

During the past twenty years, a large literature has focused on issues related to the inflationary bias that might arise when monetary policy is conducted with discretion. Despite this academic interest, some have questioned whether this whole approach has anything to do with explaining actual episodes of inflation. Do these models provide useful frameworks for positive theories of inflation?

Surprisingly, there has been little attempt to test directly for the inflation bias. Since monetary models imply that the behavior of real output should be the same whether average inflation is 0% or 10%, the very fact that most economies have consistently experienced average inflation rates well above zero for extended periods of time might be taken as evidence for the existence of an inflation bias. However, in earlier chapters we examined theories of inflation based on optimal tax considerations that might imply nonzero average rates of inflation, although few argue that tax considerations alone could account for the level of inflation observed during the 1970s in most industrialized economies (or for the observed variations in inflation).

One relevant piece of evidence is provided by Romer (1993). He argues that the average inflation bias should depend on the degree to which an economy is open. A monetary expansion produces a real depreciation, raising the price of foreign imports. This increases inflation as measured by the consumer price index, raising the inflationary cost of the monetary expansion.[47] As a result, a given output expansion caused by an unanticipated rise in the domestic price level brings with it a larger inflation cost in terms of an increase in consumer price inflation. In addition, the output gain from such an expansion will be reduced if domestic firms use imported

47. That is, output depends on domestic price inflation π_d and is given by

$$y = y_n + \alpha(\pi_d - \pi_d^e),$$

while consumer price inflation is equal to

$$\pi_{cpi} = \theta\pi_d + (1 - \theta)s,$$

where s is the rate of change of the nominal exchange rate and θ is the share of domestic output in the consumer price index.

intermediate goods or if nominal wages are indexed. In terms of our basic model, this could be interpreted either as a lowering of the benefits of expansion relative to the costs of inflation or as a reduction in the output effects of unanticipated inflation. Consequently, the weight on output, λ, should be smaller (or the weight on inflation larger) in more open economies, and the coefficient of the supply curve, a, should be smaller. Since the inflation bias is increasing in $a\lambda$ (see, for example, 8.7), the average inflation rate should be lower in more open economies.

Romer tests these implications using data on 114 countries for the post-1973 period. Using the import share as a measure of openness, he finds the predicted negative association between openness and average inflation. The empirical results, however, do not hold for the OECD economies. For the highly industrialized, high-income countries, openness is unrelated to average inflation.[48]

Temple (2002) examines the link between inflation and the slope of the Phillips curve linking inflation and output (represented by the value of the parameter a in 8.3) and finds little evidence that a is smaller in more open economies. To account for Romer's finding that openness is associated with lower inflation, he suggests that inflation may be more costly in open economies because it is associated with greater real exchange-rate variability. In this case, the parameter λ would be smaller in a more open economy, as the central bank places relatively more weight on inflation objectives. As a result, average inflation would be lower in open economies, as Romer found.

Romer's test focuses on one of the factors (openness) that might affect the incentive to inflate. If central banks respond systematically to the costs and benefits of inflation, variations in the incentive to inflate across countries should be reflected, ceteris paribus, in variations in actual inflation rates. In the next section, we will examine some empirical evidence that focuses more directly on the role of institutional structure and policy outcomes. In testing for the role of the biases arising under time inconsistency on actual inflation, however, it is important to keep in mind that observed inflation is an equilibrium outcome, and a low observed inflation rate need not imply the absence of time-inconsistency problems. As we saw in our examination of reputational models, for example, equilibrium may involve pooling in which even an opportunistic central bank delivers low inflation, at least for a while. There are, in fact, several reasons for questioning the empirical relevance of time

48. Terra (1998) argues that Romer's results are driven by the countries in his sample that are severely indebted. However, Romer (1998) notes in reply that the relationship between indebtedness and the openness–inflation correlation disappears when one controls for central bank independence. This suggests that both indebtedness and inflation are more severe in countries that have not solved the policy commitment problem.

inconsistency as a factor in monetary policy. Some economists have argued that time inconsistency just isn't a problem. For example, Taylor (1983) points out that society finds solutions to these sorts of problems in many other areas (patent law, for example) and that there is no reason to suppose that the problem is particularly severe in the monetary-policy arena.[49] Institutional solutions, such as separating responsibility for monetary policy from the direct control of elected political officials, may reduce or even eliminate the underlying bias toward expansions that leads to excessively high average inflation under discretion.[50]

Ireland (1998) argues that the behavior of inflation in the United States is consistent with the Barro-Gordon model if one allows for time variation in the natural rate of unemployment. Ireland assumes that the central bank's objective is to minimize

$$V = \tfrac{1}{2}\lambda(u - ku_n)^2 + \tfrac{1}{2}\pi^2,$$

where u is the unemployment rate and u_n is the natural rate of unemployment. It is assumed that $k < 1$ so that the central bank attempts to target an unemployment rate below the economy's natural rate. Ireland assumes that u_n is unobservable but varies over time and is subject to permanent stochastic shifts. As a result, the average inflation rate varies with these shifts in u_n; when u_n rises, average inflation also rises (see problem 7). Ireland finds support for a long-run (cointegrating) relationship between unemployment and inflation in the United States. However, this is driven by the rise in inflation in the 1970s that coincided with the rise in the natural rate of unemployment as the baby boom generation entered the labor force. Whether the latter was the cause of the former is more difficult to determine, and Europe in the 1990s certainly experienced a rise in average unemployment rates with a fall rather than an increase in average inflation.

One of the hallmarks of the time-inconsistency literature is its attempt to think seriously about the incentives facing policy makers. As such, it contrasts sharply with the older tradition in monetary policy in which the policy maker was simply assumed to follow an arbitrary (or perhaps optimal) rule. The newer view stresses that policy makers may face incentives to deviate from such rules. McCallum (1995, 1997b), however, has argued that central banks can be trusted not to succumb to the incentive to inflate, since they know that succumbing leads to a bad equilibrium. But such

49. As Taylor puts it, "In the Barro-Gordon inflation-unemployment model, the superiority of the zero inflation policy is just as obvious to people as the well-recognized patent problem is in the real world. It is therefore difficult to see why the zero inflation policy would not be adopted in such a world" (1983, p. 125).

50. In the following section, some empirical evidence on the role of central bank independence that supports this hypothesis is reviewed.

a view ignores the basic problem; even central banks that want to do the right thing may face the choice of either inflating or causing a recession. In such circumstances, the best policy may not be to cause a recession. For example, consider Cukierman and Liviatan's type D policy maker. Such a policy maker is capable of committing to and delivering on a zero-inflation policy, but if the public assigns some probability to the possibility that a type W might be in office, even the type D ends up inflating. If central banks were to define their objectives in terms of stabilizing output around the economy's natural rate (i.e., $k \equiv 0$), then there would be no inflationary bias; central banks would deliver the socially optimal policy. However, this corresponds to a situation in which there is no bias, not to one in which an incentive to inflate exists but the central bank resists it.

An alternative criticism of the time-inconsistency literature questions the underlying assumption that the central bank cannot commit. Blinder (1995), for example, argues that the inherent lags between a policy action and its effect on inflation and output serve as a commitment technology. Inflation in period t is determined by policy actions taken in earlier periods, so if the public knows past policy actions, the central bank can never produce a surprise inflation.

The presence of lags does serve as a commitment device. If outcomes today are entirely determined by actions taken earlier, the central banker is clearly committed; nothing she can do will affect today's outcome. And few would disagree that monetary policy acts with a (long) lag. But appealing to lags solves the time-inconsistency problem by eliminating *any* real effects of monetary policy. That is, there is no incentive to inflate because expansionary monetary policy does not affect real output or unemployment. If this were the case, central banks could costlessly disinflate; seeing a shift in policy, private agents could all revise nominal wages and prices before any real effects occurred. If monetary policy does have real effects, even if these occur with a lag, the inflationary bias under discretion will reappear.

To see this, suppose we follow Fischer (1977) in assuming two-period overlapping nominal-wage contracts, giving rise to an aggregate supply function of the form

$$y_t = y_n + \tfrac{1}{2}a(\pi_t - \mathrm{E}_{t-1}\pi_t) + \tfrac{1}{2}a(\pi_t - \mathrm{E}_{t-2}\pi_t), \tag{8.28}$$

where $\mathrm{E}_s\pi_t$, $s \le t$ denotes expectations of inflation during period t formed at the end of period s. If money growth affects inflation with a one-period lag so that $\pi_t = \Delta m_{t-1}$, (8.28) becomes

$$y_t = y_n + \tfrac{1}{2}a(\Delta m_{t-1} - \Delta m_{t-1}) + \tfrac{1}{2}a(\Delta m_{t-1} - \mathrm{E}_{t-2}\Delta m_{t-1}) = \tfrac{1}{2}a(\Delta m_{t-1} - \mathrm{E}_{t-2}\Delta m_{t-1}),$$

where $\mathrm{E}_{t-2}\Delta m_{t-1}$ is the public's expectation formed at the end of $t-2$ of money growth for period $t-1$. Monetary-policy actions taken in period $t-1$ affect real output in period t because nominal-wage contracts negotiated at the start of period $t-1$ (i.e., at the end of period $t-2$ and based on $E_{t-2}\Delta m_{t-1}$) are still in effect. In setting Δm_t, the central bank's objective is to minimize the expected value of the loss function for period $t+1$, taking $E_{t-1}\Delta m_t$ as given. The loss function becomes

$$V_{t+1} = \frac{1}{2}[\lambda(y_{t+1} - y_n - k)^2 + \pi_{t+1}^2]$$
$$= \frac{1}{2}\lambda\left[\frac{1}{2}a(\Delta m_t - E_{t-1}\Delta m_t) - k\right]^2 + \frac{1}{2}\Delta m_t^2.$$

The first order condition implies $\Delta m_t = \left(1+\frac{1}{4}a^2\lambda\right)^{-1}\left[\frac{1}{4}a^2\lambda E_{t-1}\Delta m_t + \frac{1}{2}a\lambda k\right]$. Expected money growth is then equal to $\frac{1}{2}a\lambda k$, and the time-consistent equilibrium under discretion leads to an inflation rate of $\frac{1}{2}a\lambda k > 0$. The presence of a lag between money growth and inflation reduces the inflation bias from $a\lambda k$ to $\frac{1}{2}a\lambda k$, but the bias is still positive. Thus, if monetary policy has real effects, lags between the setting of the policy instruments and the effects on real output do not serve to eliminate the inflation bias completely.

In the models that have been used in the time-inconsistency literature, monetary policy affects real output through its effect on inflation—more specifically, by creating inflation surprises. The empirical evidence from most countries, however, indicates that policy actions affect output before inflation is affected.[51] Policy actions can be observed long before the effects on inflation occur. But for this to represent a commitment technology that can overcome the time-inconsistency problem requires that the observability of policy eliminate its ability to affect real output. It is the ability of monetary policy to generate real output effects that leads to the inflationary bias under discretion, and the incentive toward expansionary policies exists as long as monetary policy can influence real output. The fact that the costs of an expansion in terms of higher inflation only occur later actually increases the incentive for expansion if the central bank discounts the future.

A serious criticism of explanations of actual inflation episodes based on the Barro-Gordon approach relates to the assumption that the central bank and the public understand that there is no long-run trade-off between inflation and unemployment. The standard aggregate-supply curve, relating output movements to inflation surprises, implies that the behavior of real output (and unemployment) will be independent of the average rate of inflation. However, many central banks in the 1960s and into the 1970s did not accept this as an accurate description of the economy. Phillips curves were viewed as offering a menu of inflation-unemployment combinations from which policy makers could choose. Actual inflation may have reflected policy

51. Kiley (1996) presents evidence for the United States, Canada, Great Britain, France, and Germany.

makers' misconceptions about the economy rather than their attempts to engineer surprise inflations that would not be anticipated by the public.

These criticisms, while suggesting that the simple models of time inconsistency may not account for all observed inflation, do not mean that time-inconsistency issues are unimportant. Explaining actual inflationary experiences will certainly involve consideration of the incentives faced by policy makers and the interaction of the factors such as uncertainty over policy preferences, responses to shocks, and a bias toward expansions that play a key role in models of discretionary policy.[52] So the issues that are central to the time-inconsistency literature do seem relevant for understanding the conduct of monetary policy. At the same time, important considerations faced by central banks are absent from the basic models generally used in the literature. For example, the models have implications for average inflation rates but usually do not explain variations in average inflation over longer time periods.[53] Yet one of the most important characteristics of inflation during the past 40 years in the developed economies is that it has varied; it was low in the 1950s and early 1960s, much higher in the 1970s, and lower again in the mid-1980s and 1990s. Thus, average inflation changes, but it also displays a high degree of persistence.

This persistence does not arise in the models we have examined so far. Reputational models can display a type of inflation persistence; inflation may remain low in a pooling equilibrium; then, once the high-inflation central bank reveals itself, the inflation rate jumps and remains at a higher level. But this description does not seem to capture the manner in which a high degree of persistence is displayed in the response of actual inflation to economic shocks that, in principle, should cause only one-time price-level effects. For example, consider a negative supply shock. When the central bank is concerned with stabilizing real output, such a shock leads to a rise in the inflation rate. This reaction seems consistent with the early 1970s, when the worldwide oil price shock is generally viewed as being responsible for the rise in inflation. In the models of the previous sections, the rise in inflation lasts only one period. The shock may have a permanent effect on the price level, but it cannot account for persistence in the inflation rate. Ball (1991, 1995) has argued, however,

52. And in reputational solutions, observed inflation may remain low for extended periods of time even though the factors highlighted in the time-inconsistency literature play an important role in determining the equilibrium.

53. Potential sources of shifts in the discretionary average rate of inflation would be changes in labor-market structure that affect the output effects of inflation (the a parameter in the basic model), shifts in the relative importance of output expansions or output stabilization in the policy maker's objective functions (the λ parameter), or changes in the percentage gap between the economy's natural rate of output (unemployment) and the socially desired level (the parameter k).

that inflation results from an adverse shock and that once inflation increases, it remains high for some time. Eventually, policy shifts do bring inflation back down. Models of unemployment persistence based on labor-market hysteresis, such as those developed by Lockwood and Philippopoulos (1994), Jonsson (1995), Lockwood (1997), and Svensson (1997b), also imply some inflation persistence. A shock that raises unemployment now also raises expected unemployment in the future. This increases the incentive to generate an expansion and so leads to a rise in inflation both now and in the future. But these models imply that inflation gradually returns to its long-run average and so cannot account for the shifts in policy that often seem to characterize disinflations.

One model that does display such shifts was discussed earlier. Ball (1995) accounts for shifts in policy by assuming that the central bank type can change between a zero-inflation type and an optimizing type according to a Markov process. With imperfect information, the public must attempt to infer the current central bank's type from inflation outcomes. The wet type mimics the zero-inflation type until an adverse disturbance occurs. If such a shock occurs and the central bank is a wet type, inflation rises. This increase reveals the central bank's type, so the public expects positive inflation, and, in equilibrium, inflation remains high until a dry type takes over. As a result, the model predicts the type of periodic and persistent bouts of inflation that seem to have characterized inflation in many developed economies.

A number of authors have suggested that central banks now understand the dangers of having an overly ambitious output target and, as a consequence, these central bank now target the output gap $y_t - y_n$; in other words, $k = 0$. With the standard quadratic loss function, the inflation bias under discretion is zero when $k = 0$. Cukierman (2002), however, has shown that an inflation bias reemerges if central bank preferences are asymmetric. He argues that central banks are not indifferent between $y_t - y_n > 0$ and $y_t - y_n < 0$ even if the deviations are of equal magnitude. A central bank that views a 1% fall in output below y_n as more costly than a 1% rise above y_n will tend to err in the direction of an overly expansionary policy. As a result, an average inflation bias reemerges even though $k = 0$. Ruge-Mucia (forthcoming), in contrast, considers the case of a central bank with an inflation target and asymmetric preferences over target misses. He shows that if the central bank prefers undershooting its target rather than overshooting it, average inflation will tend to be less than the target inflation rate.

Finally, in an important contribution, Sargent (1999) studies the case of a central banker in a Barro-Gordon world who must learn about the structure of the economy. Initially, if the central bank believes it faces a Phillips curve trade-off between output and inflation, it will attempt to expand the economy. The equilibrium involves the standard inflation bias. As new data reveal to the central bank that the Phillips curve is vertical and that it has not gained an output expansion despite the inflation, the equilibrium can switch to a zero-inflation path. However, the apparent conquest of inflation is temporary, and the equilibrium can alternate between periods of high inflation and periods of low inflation.

8.5 Do Central Banking Institutions Matter?

Both the academic literature on discretionary policy and the policy discussions surrounding the design of new policy-making institutions for the European Union and the emerging nations of Eastern Europe have generated increased interest in the role institutional structures play in affecting both policy and macroeconomic outcomes. While Olson (1996) argues strongly that policies and institutions are critical in accounting for cross-country differences in real economic growth, the focus in the monetary literature has been predominantly on the implications of alternative institutional structures for the conduct of monetary policy and for inflation.

By far the greatest attention has been focused on the relationship between the political independence of central banks and the resulting average inflation rates in different countries. If political pressures lie behind the bias toward economic expansion that the Barro-Gordon model shows leads to an inflationary bias, then central banks that are less subject to political influences should be able to deliver consistently lower inflation.

This proposition has been intensively investigated. Beginning with Bade and Parkin (1984), various authors have constructed measures of central bank independence and have examined the relationship across countries between independence and average inflation or measures of real economic performance.[54] Much of this literature is surveyed by Cukierman (1992) and Eijffinger and de Haan (1996). The general conclusion is that central bank independence among the industrial economies is negatively correlated with average inflation; greater independence is associated with lower average inflation. Using a measure of independence that was constructed by Cukierman, Webb, and Neyapti (1992) based on the legal charters of central banks, and which they call LVAU, figure 8.5 shows the relationship between independence and average inflation during 1973–1979 and 1980–1993 for a sample of 20 OECD

54. See Alesina (1988), Grilli, Masciandaro, and Tabellini (1991), and Eijffinger and Schaling (1993).

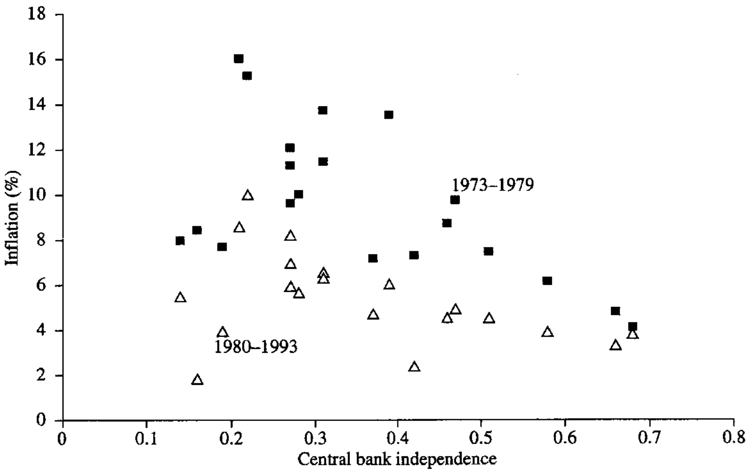

Figure 8.5
Average Inflation Versus Central Bank Independence

countries.[55] This same information is summarized in table 8.1 showing simple regressions of average inflation on central bank independence.

In addition to being statistically significant, the estimated effect of central bank independence on inflation is of an economically significant magnitude as well. For example, the 1973–1979 regression predicts that changing the degree of independence from that of the Bank of England (with a value of $LVAU$ equal to 0.31, as measured prior to recent changes in the relationship between the Bank of England and the Treasury) to that of the Bundesbank (an $LVAU$ of 0.66) would be associated with a reduction of 4 percentage points in average annual inflation.

One difficulty in interpreting this empirical literature in terms of the time-inconsistency literature arises because the link between central bank independence, as measured by the various indices, and the inflation bias under discretion is ambiguous. Delegating monetary policy to an independent central bank that has the same

55. The countries included in the sample are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Luxembourg, Norway, the Netherlands, New Zealand, Spain, Sweden, Switzerland, the United Kingdom, and the United States. The index of central bank independence employed is that due to Cukierman, Webb, and Neyapti (1992) and is reported in table 19.4 of Cukierman (1992).

Table 8.1
Central Bank Independence and Inflation

	Const.	$LVAU$	R^2
1973–1979	0.140[a]	−0.122[a]	0.35
1980–1993	0.073[a]	−0.054[b]	0.18

a. Significant at the 1% level.
b. Significant at the 5% level.

objective function as the government would not reduce the time-consistent inflation rate.[56] However, independence is often interpreted in terms of the weight placed on inflation objectives; thus, independence may imply greater conservativeness in the sense used by Rogoff (1985b). Such an interpretation is consistent with greater independence being negatively correlated with average inflation (see 8.18), but under this interpretation, greater independence should also be associated with less activist stabilization policies and, as a result, higher output variance. The relationship between the volatility of real output and other measures of real economic activity has been examined by Alesina and Summers (1993), Cukierman, Kalaitzidakis, Summers, and Webb (1993), Eijffinger and Schaling (1993), and Pollard (1993). These authors find no relationship between central bank independence and real economic volatility. This finding is inconsistent with the notion that independence is associated with a greater weight on inflation objectives. However, the finding has been interpreted to mean that granting greater central bank independence is a free lunch; average inflation is reduced, with no cost in terms of greater output instability. Note that this interpretation requires that the empirical correlations be given a causal interpretation, but the empirical findings are also consistent with the implication from the contracting approach that there need be no inherent trade-off between reducing the inflation bias and achieving optimal stabilization.

Debelle and Fischer (1994), Froyen and Waud (1995), Walsh (1995d), and A. M. Fischer (1996) report that greater central bank independence is associated with greater short-run costs of disinflations. If causality runs from central bank independence to greater nominal rigidities that raise the sacrifice ratio associated with disinflations, then moves by high-inflation countries to grant their central banks more independence may not lower the costs of reducing inflation. Causality could, however, run the other way. Disinflations will tend to be costly when the effects of

56. Herrendorf and Neumann (forthcoming) argue that a politically independent central bank is less likely to care about the government's reelection chances and is therefore less likely to use monetary policy in an attempt to create surprise inflation.

surprise inflation on output are large. This occurs when the parameter a in our basic model is large. But a large value for a also implies that the inflation bias under discretion will be large, so countries with large as (and high sacrifice ratios) may be more likely to have established independent central banks to avoid the inflation bias.

In support of the hypothesis that central bank independence can affect the slope of the short-run Phillips curve (i.e., the value of a), Walsh (1995d) shows that an increased focus on inflation objectives by the central bank can raise the degree of nominal-wage rigidity by leading to less nominal-wage indexation. Hutchison and Walsh (1998) do find that the short-run Phillips curve in New Zealand has become flatter (i.e., a has increased) since the 1989 reform of the Reserve Bank of New Zealand.

The appropriate interpretation of the empirical evidence on independence and inflation will also depend on the meaning of independence and the benefits independence confers. Suppose independence enhances the central bank's reputation in a way that reduces the inflation bias.[57] This reduction in the inflation bias lowers the optimal degree of conservativeness, since the stabilization costs of a conservative now become relatively larger. So independence may shift the relationship between average inflation and output variability, influencing the cross-country evidence on central bank independence and inflation. Or, if independence and conservatism are interpreted to be associated with a lower target for inflation, rather than a greater weight on inflation objectives, increased independence will lower average inflation but will not lead to an increase in the variability of output (Svensson 1997b).

An alternative interpretation is that the inflation bias arises from political pressures, perhaps related to electoral considerations, and that central bank independence reduces this bias by reducing direct political pressures on the central bank. That is, it is the greater autonomy associated with independence, and not a change in preferences, that accounts for the lower average inflation. By limiting the underlying source of the bias toward expansion, independence would act to reduce average inflation but would still allow the central bank to respond to economic disturbances. Independence need not imply a rise in output volatility. Waller and Walsh (1996) parameterize independence in terms of the political partisanship in the appointment process for the central banker and in the length of the term of office. They show that greater independence—a smaller role for partisan politics in the appointment process or a longer term of office—can reduce output volatility as well as average inflation.[58]

57. Herrendorf and Lockwood (forthcoming).

58. Alesina and Gatti (1995) make a similar point by considering a two-party model in which the central banker is chosen jointly by the two political parties prior to an election. They show that electorally induced political business cycles are reduced.

The type of independence might also matter. Debelle and Fischer (1994) draw a distinction between *instrument independence* and *goal independence*. The former refers to the ability of a central bank to sets its policy instruments without interference in the pursuit of its policy goals. The latter refers to the ability of the central bank to set policy goals. The Federal Reserve, for example, has both instrument and goal independence, while the Reserve Bank of New Zealand has instrument independence, but the goals of policy are set by the Reserve Bank Act of 1989, which specifies price stability as the sole objective of monetary policy. Prior to May 6, 1997, the Bank of England had neither instrument nor goal independence.[59] Debelle and Fischer report some (weak) empirical evidence that the presence of a formal goal of price stability (that is, a lack of goal independence) and instrument independence are related to low-inflation outcomes. Other aspects of the political linkages between the central bank and the government, such as appointment procedures, do not seem to matter.

The critical issue is the extent to which these correlations between inflation and measures of independence represent causal relationships. Will increasing the political insulation of a country's central bank result in lower average inflation without producing any detrimental effects on real economic performance? Posen (1993, 1995) has argued strongly that average inflation and the degree of central bank independence are jointly determined by the strength of political constituencies opposed to inflation; in the absence of these constituencies, simply increasing a central bank's independence will not cause inflation to be lower. Cargill (1995a, 1995b) has also questioned the causal significance of the statistical correlations between measures of central bank independence based on descriptions of the legal structure of the bank. In particular, Cargill emphasizes the case of Japan, where low inflation coexisted with a very dependent central bank. McKinnon and Ohno (1997) argue that U.S. pressure on Japan over trade disputes accounts for deflationary pressures in Japan and explains Japan's low inflation despite a politically dependent central bank. In this case, the political pressures on a dependent central bank supported a low-inflation policy, in contrast to the general presumption that political pressures are always in favor of inflationary policies.

One problem with regressions of the form reported in table 8.1 is that they fail to correct for country-specific factors that may affect inflation and may also be correlated with the measure of central bank independence. If countries with independent central banks differ systematically from countries with dependent central banks in

59. The Bank of England was granted instrument independence by the Labour government elected on May 3, 1997. Some other recent central bank reforms are discussed in Walsh (1995b).

ways that are associated with lower inflation, the regressions in table 8.1 will attribute the low inflation to central bank independence. One approach to correcting for potential bias is to include other determinants of inflation in the empirical analysis. Campillo and Miron (1997) have shown that central bank independence has no explanatory power for cross-country variation in average inflation once other potential determinants of inflation are included in the analysis. They argue instead that the degree of openness is an important factor in explaining inflation, a result consistent with Romer (1993) and Jonsson (1995). Campillo and Miron also find a significant effect of the debt-to-GDP ratio in accounting for cross-country inflation variation, higher values of this ratio being associated with higher average inflation. Using a different approach, Johnson and Siklos (1994) find little relationship between the measured responses of short-term interest rates (the policy instrument) to political influences and measures of independence based on central banking laws.[60]

While the more recent empirical work has cast doubt on the causal role played by central bank independence in determining inflation, the earlier findings have played an important role in shaping central banking reforms around the world as countries attempt to design institutional structures that will support desirable policy outcomes. However, a complete understanding of the relationship between average inflation and central bank independence, even if the correlation is not causal, will require a better understanding of the factors that have historically led to variations in central bank independence across countries. What determines central bank independence? If independence has been employed as a means of reducing the inflation bias that might arise from political pressures on the central bank, then those countries facing the greatest bias would have had the most incentive to establish independent central banks. De Haan and van't Hag (1995) develop some implications of the hypothesis that the delegation of monetary policy to an independent central bank has been used as a commitment device and test these using data from OECD economies. Eijffinger and de Haan (1996) show how central bank independence, interpreted in terms of the degree of conservativeness, is related to factors such as the natural rate of unemployment (assumed to be related to the incentive for expansions) that are suggested by the basic Rogoff model.

60. The discussion here applies only to developed economies. There is no statistical relationship between measures of central bank independence and average inflation among the developing economies. Cukierman (1992) has argued for the use of a measure of central bank governor turnover as a proxy for independence. Among the developing economies, high turnover is associated with high inflation.

8.6 Summary

Many countries experience, for long periods of time, average inflation rates that clearly exceed what would seem to be reasonable estimates of the socially desired inflation rate. The time-inconsistency literature originated as a positive attempt to explain this observation. In the process, the approach has made important methodological contributions to monetary-policy analysis by emphasizing the need to treat central banks as responding to the incentives they face.

As positive theories of actual inflation, the evidence for this approach is promising but mixed. In assessing the ability of this approach to account for actual inflation, one can distinguish between attempts to explain the time-series experience of a single country and the cross-sectional evidence on inflation differences among countries. The factors emphasized in this literature—central bank preferences, the short-run real effects of surprise inflation, the rate at which the central bank discounts the future, the effects of political influences on the central bank—are quite different from the factors that receive prominence in the optimal taxation models of inflation of chapter 4.[61] Although a large number of empirical studies of the industrialized economies have found that indices of central bank political independence are negatively related to average inflation, evidence also suggests the importance of financing considerations.

Perhaps the most important contribution of the literature on time inconsistency, however, has been to provide theoretical frameworks for thinking formally about credibility issues, on the one hand, and the role of institutions and political factors, on the other, in influencing policy choices. By emphasizing the interactions of the incentives faced by the policy makers, the structure of information about the economy and about the central bank's preferences, and the public's beliefs, the models examined in this chapter provide a critical set of insights that have influenced the recent debates over rules, discretion, and the design of monetary-policy institutions.

8.7 Problems

1. Assume that firms maximize profits in competitive factor markets, with labor the only variable factor of production. Output is produced according to the production function $Y = AL^{\alpha}$, $0 < \alpha < 1$. Labor is supplied inelastically. Nominal wages are

61. These two literatures are linked, however, by Herrendorf (1997) in his survey on theories of seigniorage.

set at the start of the period at a level consistent with market clearing, given expectations of the price level. Actual employment is determined by firms once the actual price level is observed. Show that, in log terms, output is given by $y = \alpha l^* + \left(\frac{\alpha}{1-\alpha}\right)(p - p^e) + \ln A$, where l^* is the log labor supply.

2. Suppose an economy is characterized by the following three equations:

$$\pi = \pi^e + ay + e$$

$$y = -br + u$$

$$\Delta m - \pi = -di + y + v,$$

where the first equation is an aggregate-supply function written in the form of an expectations-augmented Phillips curve, the second is an IS or aggregate-demand relationship, and the third is a money-demand equation, where Δm denotes the growth rate of the nominal money supply. The real interest rate is denoted by r and the nominal rate by i, with $i = r + \pi^e$. Let the central bank implement policy by setting i to minimize the expected value of $\frac{1}{2}[\lambda(y-k)^2 + \pi^2]$, where $k > 0$. Assume that the policy authority has forecasts e^f, u^f, and v^f of the shocks but that the public forms its expectations prior to the setting of i and without any information on the shocks.

a. Assume that the central bank can commit to a policy of the form $i = c_0 + c_1 e^f + c_2 u^f + c_3 v^f$ prior to knowing any of the realizations of the shocks. Derive the optimal commitment policy (i.e., the optimal values of c_0, c_1, c_2, and c_3).

b. Derive the time-consistent equilibrium under discretion. How does the nominal interest rate compare to the case under commitment? What is the average inflation rate?

3. Verify that the optimal commitment rule that minimizes the unconditional expected value of the loss function given by (8.10) is $\Delta m^c = -a\lambda e/(1 + a^2\lambda)$.

4. Suppose the central bank acts under discretion to minimize the expected value of (8.2). The central bank can observe e prior to setting Δm, but v is observed only after policy is set. Assume, however, that e and v are correlated and that the expected value of v, conditional on e, is $\mathrm{E}[v|e] = qe$, where $q = \sigma_{v,e}/\sigma_e^2$ and $\sigma_{v,e}$ is the covariance between e and v.

a. Find the optimal policy under discretion. Explain how policy depends on q.

b. What is the equilibrium rate of inflation? Does it depend on q?

5. Since the tax distortions of inflation are related to expected inflation, suppose the loss function (8.2) is replaced by

$$L = \lambda(y - y_n - k)^2 + (\pi^e)^2,$$

where $y = y_n + a(\pi - \pi^e)$. How is figure 8.2 modified by this change in the central bank's loss function? Is there an equilibrium inflation rate? Explain.

6. Based on Jonsson (1995) and Svensson (1997b): Suppose (8.3) is modified to incorporate persistence in the output process:

$$y_t = (1-\theta)y_n + \theta y_{t-1} + a(\pi_t - \pi_t^e) + e; \quad 0 < \theta < 1.$$

Suppose that the policy maker has a two-period horizon with objective function given by

$$L = \min \mathrm{E}[L_t + \beta L_{t+1}],$$

where $L_i = \frac{1}{2}[\lambda(y_i - y_n - k)^2 + \pi_i^2]$.

a. Derive the optimal commitment policy.

b. Derive the optimal policy under discretion without commitment.

c. How does the presence of persistence ($\theta > 0$) affect the inflation bias?

7. Suppose the central bank's objective is to minimize

$$V = \tfrac{1}{2}\lambda(u - ku_n)^2 + \tfrac{1}{2}\pi^2,$$

where u is the unemployment rate and u_n is the natural rate of unemployment, with $k < 1$. If the economy is described by

$$u = u_n - a(\pi - \pi^e),$$

what is the equilibrium rate of inflation under discretion? How does a fall in u_n affect the equilibrium rate of inflation?

8. Suppose that the private sector forms expectations according to

$$\pi_t^e = \pi^* \quad \text{if } \pi_{t-1} = \pi_{t-1}^e$$

$$\pi_t^e = a\lambda k \quad \text{otherwise.}$$

If the central bank's objective function is the discounted present value of the single period loss function given by (8.2) and its discount rate is β, what is the minimum value of π^* that can be sustained in equilibrium?

9. Based on Cukierman and Liviatan (1991): Assume that there are two central bank types with common preferences given by (8.1), but type D always delivers what it announces, while type W acts opportunistically. Assume that output is given by (8.3), with $e \equiv 0$. Using a two-period framework, show how output behaves under each type in (a) a pooling equilibrium and (b) a separating equilibrium. Are there any values of β such that welfare is higher if a type W central bank is setting policy?

10. Assume that nominal wages are set at the start of each period but that wages are partially indexed against inflation. If w^c is the contract base nominal wage, the actual nominal wage is $w = w^c + \kappa(p_t - p_{t-1})$, where κ is the indexation parameter. Show how indexation affects the equilibrium rate of inflation under pure discretion. What is the effect on average inflation of an increase in κ? Explain why.

11. Beetsma and Jensen (1998): Suppose the social loss function is equal to

$$V^s = \tfrac{1}{2}\mathrm{E}[\lambda(y - y_n - k)^2 + \pi^2)]$$

and the central bank's loss function is given by

$$V^{cb} = \tfrac{1}{2}\mathrm{E}[(\lambda - \theta)(y - y_n - k)^2 + (1 + \theta)(\pi - \pi^T)^2)] + t\pi,$$

where θ is a mean zero stochastic shock to the central bank's preferences, π^T is an inflation target assigned by the government, and $t\pi$ is a linear inflation contract with t a parameter chosen by the government. Assume that the private sector forms expectations before observing θ. Let $y = y_n + (\pi - \pi^e) + e$ and $\pi = \Delta m + v$. Finally, assume that θ and the supply shock e are uncorrelated.

a. Suppose the government only assigns an inflation target (so $t = 0$). What is the optimal value for π^T?

b. Now suppose the government only assigns a linear inflation contract (so $\pi^T = 0$). What is the optimal value for t?

c. Is the expected social loss lower under the inflation target arrangement or the inflation contract arrangement?

9 Monetary-Policy Operating Procedures

9.1 Introduction

Previous chapters treated the nominal money supply or even inflation as the variable directly controlled by the monetary policy maker. While this approach does allow for an analysis of many important issues, it ignores the actual problems surrounding policy implementation. Central banks do not directly control the nominal money supply, inflation, or long-term interest rates likely to be most relevant for aggregate spending. Instead, narrow reserve aggregates, such as the monetary base, or very short-term interest rates, such as the federal funds rate in the United States, are the variables over which the central bank can exercise close control. Chapter 1 claimed that a short-term interest rate typically provides a better measure of monetary-policy actions than measures of the money supply do, but no explanation was given for why this might be the case. We have also not yet discussed the specific relationship between short-term interest rates, other reserve aggregates such as nonborrowed reserves or the monetary base, and the broader monetary aggregates such as $M1$ or $M2$. And there has been no discussion of the factors that might explain why many central banks choose to use a short-term interest rate rather than a monetary aggregate as their instrument for implementing monetary policy. These issues will be addressed in this chapter.

The actual implementation of monetary policy involves a variety of rules, traditions, and practices, and these collectively are called *operating procedures*. Operating procedures differ according to the actual instrument the central bank uses in its daily conduct of policy, the operating target whose control is achieved over short horizons (a short-term interest rate versus a reserve aggregate, for example), the conditions under which the instruments and operating targets are automatically adjusted in light of economic developments, the information about policy and the types of announcements the monetary authority might make, its choice of variables for which it establishes targets (e.g., for money-supply growth or the inflation rate), and whether these targets are formal or informal.

The objective in examining monetary-policy operating procedures is to understand what instruments are actually under the control of the monetary authority, the factors that determine the optimal instrument choice, and how the choice of instrument affects the manner in which short-term interest rates, reserve aggregates, or the money stock might reflect policy actions and nonpolicy disturbances. We first examine the factors that determine the optimal choice of an operating procedure. Then we focus on the relationship between the choice of operating procedure, the response of the market for bank reserves to various economic disturbances, and the way policy should be measured.

9.2 From Instruments to Goals

There are many interesting insights that can be obtained by ignoring the linkages between the actual tools controlled by the central bank and instead simply assuming that money growth, or even the inflation rate, can be controlled directly. But central banks do not directly control inflation, nor do their policy actions have effects on real economic activity that are always easy to determine. At best, policy actions taken today affect these variables several months in the future. The lags in the effects of policy, together with the infrequency with which observations on aggregate price indices and output become available, greatly complicate the implementation of monetary policy, even when there is agreement on the objectives. How should the central bank adjust its actual policy tools in response to economic developments? Should it focus only on its ultimate goals such as the inflation rate? Should it attempt to maintain some measure of the money supply on a target path? Should it keep a short-term nominal interest rate constant?

Discussions of monetary-policy implementation focus on *instruments*, *operating targets*, *intermediate targets*, and *policy goals*. Instruments are the variables that are directly controlled by the central bank. These typically include an interest rate charged on reserves borrowed from the central bank, the reserve requirement ratios that determine the level of reserves banks must hold against their deposit liabilities, and the composition of the central bank's own balance sheet (its holdings of government securities, for example). The instruments of policy are usually manipulated to achieve a prespecified value of an operating target, typically some measure of bank reserves (total reserves, borrowed reserves, or nonborrowed reserves—the difference between total and borrowed reserves), a very short-term rate of interest, usually an overnight interbank rate (the federal funds rate in the case of the United States), or a monetary-conditions index that combines an interest rate and the exchange rate.

Goals, such as inflation or deviations of unemployment from the natural rate, are the ultimate variables of interest to policy makers; instruments are the actual variables under their direct control. Intermediate target variables fall between operating targets and goals in the sequence of links that run from policy instruments to real economic activity and inflation. Because observations on some or all of the goal variables are usually obtained less frequently than are data on interest rates, exchange rates, or monetary aggregates, the behavior of these latter variables can often provide the central bank with information about economic developments that will affect the goal variables. For example, faster than expected money growth may signal that real output is expanding more rapidly than was previously thought. The central bank might change its operating target (in this case, raise the interbank rate or contract reserves) to keep the money growth rate on a path believed to be consistent with achieving its policy goals. In this case, money growth is serving as an intermediate target variable.

Instruments, operating targets, intermediate targets, and goals have been described in a sequence running from the instruments directly controlled by the central bank to goals, the ultimate objectives of policy. Actually, policy design operates in the reverse fashion: from the goals of policy, to the values of the intermediate targets consistent with the goals, to the values of the operating targets needed to achieve the intermediate targets, and finally to the instrument settings that yield the desired values of the operating targets. In earlier chapters, inflation and the money supply were treated as policy instruments, ignoring the linkages from reserve markets to interest rates to banking-sector behavior to aggregate demand. Similarly, it is often useful to ignore reserve-market behavior and treat an operating target variable, such as the overnight interbank interest rate or a reserve aggregate, as the policy instrument. Since these variables can be controlled closely over short time horizons, they are often also described as *policy instruments.*

A related point to note is that the definition of a policy instrument is often model dependent. Many of the models discussed in chapter 8, for example, treated the inflation rate as the direct instrument of monetary policy. The assumption was not that the central bank directly set prices in the economy; rather, it was that the central bank, through its actual policy tools, could exercise sufficiently close control over the inflation rate that errors in the control process could be ignored for the purpose of the particular model. Similarly, in chapter 10, a short-term rate of interest will be treated as the instrument of monetary policy. Again, this should be interpreted to mean that the central bank, by engaging in open-market operations (its actual instrument), can control the interest rate, so that for many purposes, we can simply ignore the reserve market and treat the short-term interest rate as if it were set directly by the central bank.

9.3 The Instrument Choice Problem

If the monetary policy authority can choose between employing an interest rate or a monetary aggregate as its policy tool, which should it choose? The classic analysis of this question is due to Poole (1970). He showed how the stochastic structure of the economy—the nature and relative importance of different types of disturbances—would determine the optimal instrument. The analysis of the instrument choice problem also provides a framework for discussing the role of policy targets, intermediate targets, and information in the conduct of policy.

9.3.1 Poole's Analysis

Suppose the central bank must set policy before observing the current disturbances to the goods and money markets, and assume that information on interest rates, but not output, is immediately available. This informational assumption reflects a situation in which the central bank can observe market interest rates essentially continuously, but data on inflation and output might be available only monthly or quarterly. In such an environment, the central bank will be unable to determine from a movement in market interest rates the exact source of any economic disturbances that might be affecting the economy. To make a simple parallel with a model of supply and demand, observing a rise in price does not indicate whether there has been a positive shock to the demand curve or a negative shock to the supply curve. Only by observing both price and quantity can these two alternatives be distinguished, since a demand shift would be associated with a price and quantity rise, while a supply shift would be associated with a rise in price and a decline in quantity. At the macro level, an increase in the interest rate could be due to expanding aggregate demand (which might call for contractionary monetary policy to stabilize output) or an exogenous shift in money demand (which might call for letting the money supply expand). With imperfect information about economic developments, it will be impossible to determine the source of shocks that have caused interest rates to move.

Poole asked whether, in this environment, the central bank should try to hold market interest rates constant or should hold a monetary quantity constant while allowing interest rates to move. And he assumed that the objective of policy was to stabilize real output, so that he answered his question by comparing the variance of output implied by the two alternative policies.

Poole treated the price level as fixed, and to highlight his basic results, we will do so as well. Since the instrument-choice problem primarily relates to the decision to hold either a market rate or a monetary quantity constant over a fairly short period of time (say, the time between policy board meetings), ignoring price-level effects is not unreasonable as a starting point for the analysis. A simple variant of the basic IS-LM model in log terms that can be used to derive Poole's results is given by

$$y_t = -\alpha i_t + u_t \tag{9.1}$$

$$m_t = y_t - c i_t + v_t. \tag{9.2}$$

Equation (9.1) represents an aggregate-demand relationship in which output is a decreasing function of the interest rate; demand also depends on an exogenous disturbance u_t with variance σ_u^2. Equation (9.2) gives the demand for money as a decreasing function of the interest rate and an increasing function of output. Money demand is subject to a random shock v_t with variance σ_v^2. Equilibrium requires that the demand for money equal the supply of money m_t. For simplicity, u and v will be treated as mean zero, serially and mutually uncorrelated processes. These two equations represent a simple IS-LM model of output determination, given a fixed price level.[1]

The final aspect of the model is a specification of the policy maker's objective, assumed to be the minimization of the variance of output deviations:

$$\mathrm{E}[y_t]^2, \tag{9.3}$$

where all variables have been normalized so that the economy's equilibrium level of output in the absence of shocks is $y = 0$. Because the central bank's loss function is quadratic in output around the true steady-state value of zero, the problem of time inconsistency that was the focus of chapter 8 will not arise.

The timing is as follows: the central bank sets either i_t or m_t at the start of the period; the stochastic shocks u_t and v_t occur, determining the values of the endogenous variables (either y_t and i_t if m_t is the policy instrument or y_t and m_t if i_t is the policy instrument).

When the money stock is the policy instrument, (9.1) and (9.2) can be solved jointly for equilibrium output:

$$y_t = \frac{\alpha m_t + c u_t - \alpha v_t}{\alpha + c}.$$

Then, setting m_t such that $\mathrm{E}[y_t] = 0$,[2] we obtain $y_t = (c u_t - \alpha v_t)/(\alpha + c)$. Hence, the value of the objective function under a money-supply procedure is

$$\mathrm{E}_m[y_t]^2 = \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}, \tag{9.4}$$

where the assumption that u and v are uncorrelated has been used.

Under the alternative policy, i_t is the policy instrument, and (9.1) can be solved directly for output. That is, the money market condition is no longer needed, although it will determine the level of m_t necessary to ensure money market equilibrium. By fixing the rate of interest, the central bank lets the money stock adjust

1. Note that the price level has been normalized to equal 1 so that the log of the price level is zero; $p = 0$. The income elasticity of money demand has also been set equal to 1.

2. This just requires $m = 0$ because of our normalizations.

endogenously to equal the level of money demand given by the interest rate and the level of income. Setting i_t such that $E[y_t] = 0$, output will equal u_t and

$$E_i[y_t]^2 = \sigma_u^2. \tag{9.5}$$

The two alternative policy choices can be evaluated by comparing the variance of output implied by each. The interest-rate operating procedure is preferred to the money-supply operating procedure if and only if

$$E_i[y_t]^2 < E_m[y_t]^2$$

and, from (9.4) and (9.5), this condition is satisfied if and only if

$$\sigma_v^2 > \left(1 + \frac{2c}{\alpha}\right)\sigma_u^2. \tag{9.6}$$

Thus, an interest-rate procedure is more likely to be preferred when the variance of money demand disturbances is larger, the LM curve is steeper (the slope of the LM curve is $1/c$), and the IS curve is flatter (the slope of the IS curve is $-1/\alpha$). Conversely, the money-supply procedure will be preferred if the variance of aggregate-demand shocks (σ_u^2) is large, the LM curve is flat, or the IS curve is steep.[3]

If only aggregate-demand shocks are present (i.e., $\sigma_v^2 = 0$), a money rule leads to a smaller variance for output. Under a money rule, a positive IS shock leads to an increase in the interest rate. This acts to reduce aggregate spending, thereby partially offsetting the original shock. Since the adjustment of i acts to automatically stabilize output, preventing this interest-rate adjustment by fixing i leads to larger output fluctuations. If only money-demand shocks are present (i.e., $\sigma_u^2 = 0$), output can be stabilized perfectly under an interest-rate rule. Under a money rule, money-demand shocks cause the interest rate to move to maintain money market equilibrium; these interest-rate movements then lead to output fluctuations. With both types of shocks occurring, the comparison of the two policy rules depends on the relative variances of u and v, as well as on the slopes of the IS and LM curves, as shown by (9.6).

This framework is quite simple and ignores many important factors. To take just one example, no central bank has direct control over the money supply. Instead, control can be exercised over a narrow monetary aggregate such as the monetary base, and variations in this aggregate are then associated with variations in broader

3. In the context of an open economy in which the IS relationship is $y_t = -\alpha_1 i_t + \alpha_2 s_t + u_t$, where s_t is the exchange rate, Poole's conclusions go through without modification if the central bank's choice is expressed not in terms of i_t but in terms of the monetary conditions index $i_t - (\alpha_2/\alpha_1)s_t$.

measures of the money supply. To see how the basic framework can be modified to distinguish between the base as a policy instrument and the money supply, suppose the two are linked by

$$m_t = b_t + hi_t + \omega_t, \tag{9.7}$$

where b is the (log) monetary base, and the money multiplier ($m_t - b_t$ in log terms) is assumed to be an increasing function of the rate of interest (i.e., $h > 0$). In addition, ω_t is a random money-multiplier disturbance. Equation (9.7) could arise under a fractional reserve system in which excess reserves are a decreasing function of the rate of interest.[4] Under an interest-rate procedure, (9.7) is irrelevant for output determination, so $E_i(y_t)^2 = \sigma_u^2$, as before. But now, under a monetary-base operating procedure,

$$y_t = \frac{(c+h)u_t - \alpha v_t + \alpha\omega_t}{\alpha + c + h}$$

and

$$E_b(y_t)^2 = \left(\frac{1}{\alpha + c + h}\right)^2 [(c+h)^2\sigma_u^2 + \alpha^2(\sigma_v^2 + \sigma_\omega^2)].$$

The interest-rate procedure is preferred over the monetary-base procedure if and only if

$$\sigma_v^2 + \sigma_\omega^2 > \left[1 + \frac{2(c+h)}{\alpha}\right]\sigma_u^2.$$

Because ω shocks do not affect output under an interest-rate procedure, the presence of money-multiplier disturbances makes a base rule less attractive and makes it more likely that an interest-rate procedure will lead to a smaller output variance. This simple extension reinforces the basic message of Poole's analysis; increased financial sector volatility (money-demand or money-multiplier shocks in the model used here) increases the desirability of an interest-rate-oriented policy procedure over a monetary-aggregate procedure. If money demand is viewed as highly unstable and difficult to predict over short time horizons, greater output stability can be achieved by stabilizing interest rates, letting monetary aggregates fluctuate. If, however, the main source of short-run instability arises from aggregate spending, a policy that stabilizes a monetary aggregate will lead to greater output stability.

4. See, for example, Modigliani, Rasche, and Cooper (1970) or McCallum and Hoehn (1983).

This analysis is based on the realistic assumption that policy is unable to identify and respond to underlying disturbances. Instead, policy is implemented by fixing, at least over some short time interval, the value of an operating target or policy instrument. As additional information about the economy is obtained, the appropriate level at which to fix the policy instrument changes. So the critical issue is not so much which variable is used as a policy instrument, but how that instrument should be adjusted in light of new but imperfect information about economic developments.

Poole's basic model ignores such factors as inflation, expectations, and aggregate-supply disturbances. These factors, and many others, have been incorporated into models examining the choice between interest-rate- and monetary-aggregate-oriented operating procedures. B. Friedman (1990) contains a useful and comprehensive survey. In addition, as Friedman stresses, the appropriate definition of the policy maker's objective function is unlikely to be simply the variance of output once inflation is included in the model. Poole's approach provides a means of determining the optimal instrument choice, given a particular objective function. In the case examined here, the objective was the minimization of the variance of output. While it is often useful to analyze policy under the assumption that policy is implemented using a specific instrument, it is also important to recognize that the choice of instrument is an endogenous decision of the policy maker and is therefore dependent on the objectives of monetary policy.

9.3.2 Policy Rules and Information

The alternative policies considered in the previous subsection can be viewed as special cases of the following policy rule:[5]

$$b_t = \mu i_t. \tag{9.8}$$

According to (9.8), the monetary authority adjusts the base, its actual instrument, in response to interest-rate movements. The parameter μ, both its sign and its magnitude, determine how the base is varied by the central bank as interest rates vary. If $\mu = 0$, then $b_t = 0$, and we have the case of a monetary-base operating procedure in which b is fixed (at zero by our normalization) and is not adjusted in response to interest-rate movements. If $\mu = -h$, then (9.7) implies that $m_t = \omega_t$, and we have the case of a money-supply operating procedure in which the base is automatically

5. Recall that we have normalized constants in equations such as (9.8) to be zero. More generally, we might have a rule of the form $b_t = b_0 + \mu(i_t - \mathrm{E}i_t)$, where b_0 is a constant and $\mathrm{E}i_t$ is the expected value of i_t. As we will see in chapter 10, issues of price-level indeterminacy can arise if the average value of b_t is not tied down (as it is in this case by b_0).

adjusted to keep m_t equal to zero on average; the actual value of m_t varies as a result of the control error ω_t. In this case, b_t is the policy instrument and m_t is the operating target. As we will see, an interest-rate operating target involves letting $\mu \to \infty$. Equation (9.8) is called a *policy rule* in that it provides a description of how the policy instrument, in this case, the monetary base b_t, is set.

By combining (9.8) with (9.1) and (9.2), we obtain

$$i_t = \frac{v_t - \omega_t + u_t}{\alpha + c + \mu + h}, \tag{9.9}$$

so that large values of μ reduce the variance of the interest rate. As $\mu \to \infty$, we approximate the interest-rate operating procedure in which i_t is set equal to a fixed value (zero due to the normalization). By representing policy in terms of the policy rule and then characterizing policy in terms of the choice of a value for μ, we can consider intermediate cases to the extreme alternatives considered in section 9.3.1.

Substituting (9.9) into (9.1), output is given by

$$y_t = \frac{(c + \mu + h)u_t - \alpha(v_t - \omega_t)}{\alpha + c + \mu + h}.$$

From this expression, we can calculate the variance of output:

$$\sigma_y^2 = \frac{(c + \mu + h)^2 \sigma_u^2 + \alpha^2(\sigma_v^2 + \sigma_\omega^2)}{(\alpha + c + \mu + h)^2}.$$

If we minimize this with respect to μ, the optimal policy rule (in the sense of minimizing the variance of output) is given by

$$\mu^* = -(c + h) + \frac{\alpha(\sigma_v^2 + \sigma_\omega^2)}{\sigma_u^2}. \tag{9.10}$$

In general, neither the interest-rate ($\mu \to \infty$), nor the base ($\mu = 0$), nor the money-supply ($\mu = -h$) operating procedures will be optimal. Instead, Poole (1970) demonstrated that the way policy (in the form of the setting for b_t) should respond to interest-rate movements will depend on the relative variances of the three underlying economic disturbances.

To understand the role these variances play, suppose first that $v \equiv \omega \equiv 0$ so that $\sigma_v^2 = \sigma_\omega^2 = 0$; there are no shifts in either money demand or money supply, given the base. In this environment, the basic Poole analysis concludes that a base rule dominates an interest-rate rule. Equation (9.10) shows that one can do even better than fixing the monetary base if b_t is actually reduced when the interest rate rises (i.e.,

$b_t = -(c+h)i_t$). With interest-rate movements signaling aggregate-demand shifts (since u_t is the only source of disturbance), a rise in the interest rate indicates that $u_t > 0$. A policy designed to stabilize output should reduce m_t; this decline in m_t can be achieved by reducing the base. Rather than "leaning against the wind" to offset the interest-rate rise, the central bank should engage in a contractionary policy that pushes i_t up even further.

When σ_v^2 and σ_ω^2 are positive, interest-rate increases may now be the result of an increase in money demand or a decrease in money supply. Since the appropriate response to a positive money-demand shock or a negative money-supply shock is to increase the monetary base and offset the interest-rate rise (i.e., it *is* appropriate to lean against the wind), $\mu^* > -(c+h)$; it will become optimal to actually increase the base as $\sigma_v^2 + \sigma_\omega^2$ becomes sufficiently large.

The value for the policy rule parameter in (9.10) can also be interpreted in terms of a signal extraction problem faced by the policy authority. Recall that the basic assumption in the Poole analysis was that the policy maker could observe, and react to, the interest rate, but perhaps because of information lags, the current values of output and the underlying disturbances could not be observed. Suppose instead that the shocks u, v, and e are observed, and the central bank can respond to them. That is, suppose the policy rule could take the form $b_t = \mu_u u_t + \mu_v v_t + \mu_\omega \omega_t$ for some parameters μ_u, μ_v, and μ_ω. If we substitute this policy rule into (9.1) and (9.2), we obtain

$$y_t = \frac{(c+h+\alpha\mu_u)u_t - \alpha(1-\mu_v)v_t + \alpha(1+\mu_\omega)\omega_t}{\alpha+c+h}.$$

In this case, which corresponds to a situation of perfect information about the basic shocks, it is clear that the variance of output can be minimized if $\mu_u = -(c+h)/\alpha$, $\mu_v = 1$, and $\mu_\omega = -1$.

If the policy maker cannot observe the underlying shocks, then policy will need to be set on the basis of forecasts of these disturbances. Given the linear structure of the model, the optimal policy can be written $b_t = \mu_u \hat{u}_t + \mu_v \hat{v}_t + \mu_\omega \hat{\omega}_t = -[(c+h)/\alpha]\hat{u}_t + \hat{v}_t - \hat{\omega}_t$, where $\hat{u}_t, \hat{v}_t$, and $\hat{\omega}_t$ are the forecasts of the shocks. In the Poole framework, the central bank observes the interest rate and can set policy conditional on i_t. Thus, the forecasts of shocks will depend on i_t and will take the form $\hat{u}_t = \delta_u i_t$, $\hat{v}_t = \delta_v i_t$, and $\hat{\omega}_t = \delta_\omega i_t$. The policy rule can then be written as

$$b_t = -\left(\frac{c+h}{\alpha}\right)\hat{u}_t + \hat{v}_t - \hat{\omega}_t = \left(-\frac{c+h}{\alpha}\delta_u + \delta_v - \delta_\omega\right)i_t. \tag{9.11}$$

Using this policy rule to solve for the equilibrium interest rate, determining the δ_is from the assumption that forecasts are equal to the projections of the shocks on i_t, it is straightforward to verify that the coefficient on i_t in the policy rule (9.11) is equal to the value μ^* given in (9.10).[6] Thus, the optimal policy response to interest-rate movements represents an optimal response to the central bank's forecasts of the underlying economic disturbances, where these forecasts are based on the observed interest rates.

9.3.3 Intermediate Targets

The previous subsection showed how the optimal response coefficients in the policy rule could be related to the central bank's forecast of the underlying disturbances. This interpretation of the policy-rule parameter is important, since it captures a very general way of thinking about policy. When the central bank faces imperfect information about the shocks to the economy, it should respond based on its best forecasts of these shocks.[7] In our example, the only information variable available was the interest rate, so forecasts of the underlying shocks were based on i. In more general settings, information on other variables may be available on a frequent basis, and this should also be used in forecasting the sources of economic disturbances. Examples of such information variables include—besides market interest rates—exchange rates, commodity prices, and asset prices.[8]

Policy design needs to recognize that the central bank must respond to partial and incomplete information about the true state of the economy. Given such circumstances, monetary policy is often formulated in practice in terms of intermediate targets. Intermediate targets are variables whose behavior provides information useful in forecasting the goal variables.[9] Deviations in the intermediate targets from their expected paths indicate a likely deviation of a goal variable from its target and signal the need for a policy adjustment. For example, if money growth, which is observed weekly, is closely related to subsequent inflation, which is observed only monthly,

6. See problem 2.

7. Brainard (1967) showed that this statement is no longer true when there is uncertainty about the model parameters in additional to the additive uncertainty considered here. Parameter uncertainty makes it optimal to adjust less than completely. See section 11.3.6.

8. As discussed in chapter 1, commodity prices eliminate the price puzzle in VAR estimates of monetary policy effects because of the informational role they appear to play.

9. See Kareken, Muench, and Wallace (1973) and B. Friedman (1975, 1977b, 1990) for early treatments of the informational role of intermediate targets. More recently, Svensson (1997a, 1999c) has stressed the role of inflation forecasts as an intermediate target. Bernanke and Woodford (1997) show, however, how multiple equilibria may arise if policy is based on private sector forecasts, which are, in turn, based on expectations of future policy.

then faster than expected money growth signals the need to tighten policy. When action is taken to keep the intermediate target variable equal to its target, the hope is that policy will be adjusted automatically to keep the goal variables close to their targets as well.[10]

To see the role of intermediate targets in a very simple framework, consider the following aggregate-supply, aggregate-demand, and money-demand system, expressed in terms of the rate of inflation:

$$y_t = a(\pi_t - \mathrm{E}_{t-1}\pi_t) + z_t \tag{9.12}$$

$$y_t = -\alpha(i_t - \mathrm{E}_t\pi_{t+1}) + u_t \tag{9.13}$$

$$m_t - p_t = m_t - \pi_t - p_{t-1} = y_t - ci_t + v_t. \tag{9.14}$$

Equation (9.12) is a standard Lucas supply curve, (9.13) gives aggregate demand as a decreasing function of the expected real interest rate, and (9.14) is a simple money-demand relationship. Assume that each of the three disturbances z, u, and v follows a first order autoregressive process:

$$z_t = \rho_z z_{t-1} + e_t$$

$$u_t = \rho_u u_{t-1} + \varphi_t$$

$$v_t = \rho_v v_{t-1} + \psi_t,$$

where $0 < \rho_i < 1$ for $i = z, u, v$. The innovations e, φ, and ψ are assumed to be mean zero and serially and mutually uncorrelated processes. The interest rate i is taken to be the policy instrument of the monetary authority.

Suppose that the monetary authority's objective is to minimize the expected squared deviations of the inflation rate around a target level π^*. Hence, i_t is chosen to minimize[11]

$$V = \mathrm{E}(\pi_t - \pi^*)^2. \tag{9.15}$$

To complete the model, we need to specify the information structure. Suppose that i_t must be set before observing e_t, φ_t, or ψ_t but that y_{t-1}, π_{t-1}, and m_{t-1} (and therefore $p_{t-1}, z_{t-1}, u_{t-1}$, and v_{t-1}) are known when i_t is set. The optimal setting for the policy

10. B. Friedman (1990) and McCallum (1990b) provide discussions of the intermediate target problem.

11. Note that for this example we have replaced the loss function in output deviations with one involving only inflation stabilization objectives. As is clear from (9.12), stabilizing inflation to minimize unexpected movments in π is consistent with minimizing output variability if there are no supply disturbances ($z \equiv 0$). If the loss function depends on output and inflation variability and there are supply shocks, the optimal policy will depend on the relative weight placed on these two objectives.

instrument can be found by solving for the equilibrium price level in terms of the policy instrument and then evaluating the loss function given by (9.15).

Solving the model is simplified by recognizing that i_t will always be set to ensure that the expected value of inflation equals the target value π^*. Actual inflation will differ from π^* because policy cannot respond to offset the effects of the shocks to aggregate supply, aggregate demand, or money demand, but policy will offset any expected effects of lagged disturbances to ensure that $\mathrm{E}_{t-1}\pi_t = \mathrm{E}_t\pi_{t+1} = \pi^*$. Using this result, (9.12) can be used to eliminate y_t from (9.13) to yield

$$\pi_t = \frac{(a+\alpha)\pi^* - \alpha i_t + u_t - z_t}{a}. \tag{9.16}$$

If the policy maker had full information on u_t and z_t, the optimal policy would be to set the interest rate equal to $i_t^* = \pi^* + (1/\alpha)(u_t - z_t)$ since this would yield $\pi_t = \pi^*$. If policy must be set prior to observing the realization of the shocks at time t, the optimal policy can be obtained by taking expectations of (9.16), conditional on time $t-1$ information, yielding the optimal setting for i_t:

$$\hat{\imath} = \pi^* + \left(\frac{1}{\alpha}\right)(\rho_u u_{t-1} - \rho_z z_{t-1}). \tag{9.17}$$

Substituting (9.17) into (9.16) shows that the actual inflation rate under this policy is equal to[12]

$$\pi_t(\hat{\imath}) = \pi^* + \frac{\varphi_t - e_t}{a} \tag{9.18}$$

and the value of the loss function is equal to

$$V(\hat{\imath}) = \left(\frac{1}{a}\right)^2(\sigma_\varphi^2 + \sigma_e^2),$$

where σ_x^2 denotes the variance of a random variable x.

An alternative approach to setting policy in this example would be to derive the money supply consistent with achieving the target inflation rate π^* and then set the interest rate to achieve this level of m_t. Using (9.14) to eliminate i_t from (9.13),

$$y_t = \left(\frac{\alpha}{\alpha+c}\right)(m_t - \pi_t - p_{t-1} - v_t) + \left(\frac{c}{\alpha+c}\right)(u_t + \alpha\pi^*).$$

12. Note that under this policy, $\mathrm{E}_{t-1}\pi_t = \pi^*$, as we assumed.

Using the aggregate-supply relationship (9.12), the equilibrium inflation rate is

$$\pi_t = \pi^* + \frac{1}{a}\left[\left(\frac{\alpha}{\alpha + c}\right)(m_t - \pi_t - p_{t-1} - v_t) + \left(\frac{c}{\alpha + c}\right)(u_t + \alpha\pi^*) - z_t\right]$$

$$= \frac{[a(\alpha + c) + \alpha c]\pi^* + \alpha(m_t - p_{t-1} - v_t) + cu_t - (\alpha + c)z_t}{a(\alpha + c) + \alpha}.$$

The value of m_t consistent with $\pi_t = \pi^*$ is therefore

$$m_t^* = (1 - c)\pi^* + p_{t-1} - \left(\frac{c}{\alpha}\right)u_t + \left(1 + \frac{c}{\alpha}\right)z_t + v_t.$$

If the money supply must be set before observing the time-t shocks, the optimal target for m is

$$\hat{m}_t = (1 - c)\pi^* + p_{t-1} - \left(\frac{c}{\alpha}\right)\rho_u u_{t-1} + \left(1 + \frac{c}{\alpha}\right)\rho_z z_{t-1} + \rho_v v_{t-1}. \tag{9.19}$$

As can be easily verified, the interest rate consistent with achieving the targeted money supply $\hat{m}_t$ is just $\hat{\imath}_t$, given by (9.17). Thus, an equivalent procedure for deriving the policy that minimizes the loss function is to first calculate the value of the money supply consistent with the target for π and then set i equal to the value that achieves the targeted money supply.

Now suppose the policy maker can observe m_t and respond to it. Under the policy that sets i_t equal to $\hat{\imath}$, (9.14) implies that the actual money supply will equal $m_t = \pi_t(\hat{\imath}) + p_{t-1} + y_t(\hat{\imath}) - c\hat{\imath}_t + v_t$, which can be written as[13]

$$m_t(\hat{\imath}) = \hat{m}_t - \left(\frac{1}{a}\right)e_t + \left(1 + \frac{1}{a}\right)\varphi_t + \psi_t. \tag{9.20}$$

Observing how m_t deviates from $\hat{m}_t$ reveals information about the shocks, and this information can be used to adjust the interest rate to keep inflation closer to target. For example, suppose aggregate-demand shocks (φ) are the only source of uncertainty (i.e., $e \equiv \psi \equiv 0$). A positive aggregate-demand shock ($\varphi > 0$) will, for a given nominal interest rate, increase output and inflation, both of which contribute to an increase in nominal money demand. Under a policy of keeping i fixed, the policy maker automatically allows reserves to increase, letting m rise in response to the increased demand for money. Thus, an increase in m_t above $\hat{m}_t$ would signal that the nominal interest rate should be increased to offset the demand shock. Responding to the money supply to keep m_t equal to the targeted value $\hat{m}_t$ would achieve the ultimate goal of keeping the inflation rate equal to π^*. This is an example of an intermediate targeting policy; the nominal money supply serves as an intermediate target, and by adjusting policy to achieve the intermediate target, policy is also better able to achieve the target for the goal variable π_t.

13. Substitute the solution (9.18) into the aggregate-supply function to yield $y(\hat{\imath}_t) = \varphi_t - e_t + z_t = \varphi_t + \rho_z z_{t-1}$. Using this result in (9.17) and (9.18) in (9.14),

$$m(\hat{\imath}_t) = \pi^* + \frac{\varphi_t - e_t}{a} + p_{t-1} + \varphi_t + \rho_z z_{t-1} - c\hat{\imath}_t + v_t$$

$$= \pi^* + \left(\frac{\varphi_t - e_t}{a}\right) + p_{t-1} + \varphi_t + \rho_z z_{t-1} - c\left[\pi^* + \frac{\rho_u u_{t-1} - \rho_z z_{t-1}}{\alpha}\right] + v_t.$$

Collecting terms and using (9.18) yields (9.20).

Problems arise, however, when there are several potential sources of economic disturbances. Then it can be the case that the impact on the goal variable of a disturbance would be exacerbated by attempts to keep the intermediate target variable on target. For example, a positive realization of the money-demand shock ψ_t does not require a change in i_t to maintain inflation on target.[14] But (9.20) shows that a positive money-demand shock causes m_t to rise above the target value $m_t(\hat{\imath})$. Under a policy of adjusting i to keep m close to its target, the nominal interest rate would be raised, causing π to deviate from π^*. Responding to keep m on target will not produce the appropriate policy for keeping π on target.

Automatically adjusting the nominal interest rate to ensure that m_t always equals its target $\hat{m}_t$ requires that the nominal interest rate equal[15]

$$i_t^T = \hat{\imath}_t + \frac{(1 + a)\varphi_t - e_t + a\psi_t}{ac + \alpha(1 + a)}. \tag{9.21}$$

In this case, inflation is equal to

$$\pi_t(i_t^T) = \pi^* + \left(\frac{1}{a}\right)[-\alpha(i_t^T - \hat{\imath}) + \varphi_t - e_t]$$

$$= \pi^* + \frac{c\varphi_t - (\alpha + c)e_t - \alpha\psi_t}{ac + \alpha(1 + a)}.$$

Comparing this expression for inflation to $\pi_t(\hat{\imath}_t)$ from (9.18), the value obtained when

14. Equation (9.18) shows that inflation is independent of v_t.

15. Note that this discussion does not assume that the realizations of the individual disturbances can be observed by the policy maker; as long as m_t is observed, i_t can be adjusted to ensure that $m_t = \hat{m}_t$, and this results in i being given by (9.21). Equation (9.21) is obtained by solving (9.12)–(9.14) for m_t as a function of i_t and the various disturbances. Setting this expression equal to $\hat{m}_t$ yields the required value of i_t^T.

information on the money supply is not used, we can see that the impact of an aggregate-demand shock, φ, on the price level is reduced $(c/[ac+\alpha(1+a)] < 1/a)$; because a positive φ shock tends to raise money demand, the interest rate must be increased to offset the effects on the money supply to keep m on target. This interest-rate increase acts to offset partially the impact of a demand shock on inflation. The impact of an aggregate-supply shock (e) under an intermediate money targeting policy is also decreased. However, money demand shocks, ψ_t, now affect inflation, something they did not do under a policy of keeping i equal to $\hat{\imath}$; a positive ψ_t tends to increase m above target. If i is increased to offset this shock, inflation will fall below target.

The value of the loss function under the money targeting procedure is

$$V(i_t^T) = \left(\frac{1}{ac+\alpha(1+a)}\right)^2 [c^2\sigma_\varphi^2 + (\alpha+c)^2\sigma_e^2 + \alpha^2\sigma_\psi^2].$$

Comparing this to $V(\hat{\imath})$, the improvement from employing an intermediate targeting procedure in which the policy instrument is adjusted to keep the money supply on target will be decreasing in the variance of money-demand shocks, σ_ψ^2. As long as this variance is not too large, the intermediate targeting procedure will do better than a policy of simply keeping the nominal rate equal to $\hat{\imath}$. If this variance is too large, the intermediate targeting procedure will do worse.

An intermediate targeting procedure represents a rule for adjusting the policy instrument to a specific linear combination of the new information contained in movements of the intermediate target. Using (9.20) and (9.21), the policy adjustment can be written as

$$\begin{aligned} i_t^T - \hat{\imath} &= \left[\frac{a}{ac+\alpha(1+a)}\right](m_t(\hat{\imath}) - \hat{m}) \\ &= \mu^T[m_t(\hat{\imath}) - \hat{m}]. \end{aligned}$$

In other words, if the money supply realized under the initial policy setting $(m_t(\hat{\imath}))$ deviates from its expected level $(\hat{m})$, the policy instrument is adjusted. Because the money supply will deviate from target due to φ and e shocks, which do call for a policy adjustment, as well as ψ shocks, which do not call for any change in policy, an *optimal* adjustment to the new information in money-supply movements would depend on the relative likelihood that movements in m are caused by the various possible shocks. An intermediate target rule, by adjusting to deviations of money from target in a manner that does not take into account whether fluctuations in m are more likely to be due to φ or e or ψ shocks, represents an inefficient use of the information in m.

To derive the optimal policy response to fluctuations in the nominal money supply, let

$$\begin{aligned} i_t - \hat{\imath} &= \mu(m_t - \hat{m}) \\ &= \mu x_t, \end{aligned} \tag{9.22}$$

where $x_t = (1+a^{-1})\varphi_t - a^{-1}e_t + \psi_t$ is the new information obtained from observing m_t.[16] Under an intermediate targeting rule, the monetary authority would adjust its policy instrument to minimize deviations of the intermediate target from the value consistent with achieving the ultimate policy target, in this case an inflation rate of π^*. But under a policy that optimally uses the information in the intermediate target variable, μ will be chosen to minimize $\mathrm{E}(\pi_t - \pi^*)$, not $\mathrm{E}(m_t - \hat{m})$. Using (9.22) in (9.16), one finds that the value of μ that minimizes the loss function is

$$\mu^* = \frac{1}{\alpha}\left[\frac{a(1+a)\sigma_\varphi^2 + a\sigma_e^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2}\right].$$

This is a messy expression, but some intuition for it can be gained by recognizing that if the policy maker could observe the underlying shocks, (9.16) implies that the optimal policy involves setting the nominal interest rate i equal to $\hat{\imath} - \frac{1}{\alpha}(\varphi_t - e_t)$. The policy maker cannot observe φ_t or e_t, but information that can be used to estimate them is available from observing the deviation of money from its target. As already shown, observing m_t provides information on the linear combination of the underlying shocks given by x_t. Letting $\mathrm{E}^x[\ ]$ denote expectations conditional on x, the policy instrument should be adjusted according to

$$i(x_t) = \hat{\imath} + \frac{1}{\alpha}(\mathrm{E}^x\varphi_t - \mathrm{E}^x e_t). \tag{9.23}$$

16. The expression for x_t is obtained by solving (9.12)–(9.14) for m_t as a function of the interest rate, yielding

$$m_t = \pi_t + p_{t-1} + y_t - ci_t + v_t = \left[\pi^* + \frac{y_t - z_t}{a}\right] + p_{t-1} + y_t - ci_t + v_t$$

or

$$\begin{aligned} m_t &= \left[\pi^* + \frac{-\alpha i_t + \alpha\pi^* + u_t - z_t}{a}\right] + p_{t-1} + [-\alpha i_t + \alpha\pi^* + u_t] - ci_t + v_t \\ &= (1+\alpha(1+a^{-1}))\pi^* + p_{t-1} - (c+\alpha(1+a^{-1}))i_t - a^{-1}z_t + (1+a^{-1})u_t + v_t \end{aligned}$$

so that, conditional on i_t,

$$m_t - \mathrm{E}_{t-1}m_t = -a^{-1}e_t + (1+a^{-1})\varphi_t + \psi_t \equiv x_t.$$

Evaluating these expectations gives

$$\mathrm{E}^x\varphi_t = \left[\frac{a(1+a)\sigma_\varphi^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2}\right]x_t,$$

and

$$\mathrm{E}^x e_t = \left[\frac{-a\sigma_e^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2}\right]x_t.$$

Substituting these expressions into (9.23) yields

$$i(z_t) = \hat{\imath} + \left(\frac{1}{\alpha}\right)\left[\frac{a(1+a)\sigma_\varphi^2 + a\sigma_e^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2}\right]x_t$$
$$= \hat{\imath} + \mu^* x_t.$$

Under this policy, the information in the intermediate target is used optimally. As a result, the loss function is reduced relative to a policy that adjusts i to keep the money supply always equal to its target,

$$V^* \leq V(i^T),$$

where V^* is the loss function under the policy that adjusts i according to $\mu^* z_t$.

As long as money-demand shocks are not too large, an intermediate targeting procedure does better than following a policy rule that fails to respond at all to new information. The intermediate targeting rule does worse, however, than a rule that optimally responds to the new information. This point was first made by Kareken, Muench, and Wallace (1973) and B. Friedman (1975).

Despite the general inefficiency of intermediate targeting procedures, central banks often implement policy as if they were following an intermediate targeting procedure, adjusting their policy instrument in order to keep some intermediate target on track. During the 1970s, there was strong support in the United States for using money growth as an intermediate target. Support faded in the 1980s, when money demand became significantly more difficult to predict.[17] The Bundesbank (prior to being superseded by the European Central Bank) and the Swiss National Bank continued to formulate policy in terms of money growth rates that can be interpreted as intermediate targets.[18] Other central banks seem to use the nominal exchange rate as an intermediate target. Recently, many central banks have shifted to using inflation itself as an intermediate target.

17. B. Friedman and Kuttner (1996) examine the behavior of the Fed during the era of monetary targeting.

Intermediate targets do provide a simple framework for responding automatically to economic disturbances. The model of this section can be used to evaluate desirable properties that characterize good intermediate targets. The critical condition in our example is that σ_ψ^2 be small. Since ψ_t represents the innovation or shock to the money-demand equation, intermediate monetary targeting will work best if money demand is relatively predictable. Often this has not been the case. The unpredictability of money demand is an important reason that most central banks moved away from using monetary targeting during the 1980s. The shock ψ can also be interpreted as arising from control errors. For example, if we had assumed that the monetary base was the policy instrument, unpredictable fluctuations in the link between the base and the monetary aggregate being targeted (corresponding to the ω disturbance in 9.7) would reduce the value of an intermediate targeting procedure. Controllability is therefore a desirable property of an intermediate target.

Lags in the relationship between the policy instrument, the intermediate target, and the final goal variable represent an additional important consideration that we have not dealt with explicitly. The presence of lags introduces no new fundamental issues; as our simple framework shows, targeting an intermediate variable allows policy to respond to new information, either because the intermediate target variable is observed contemporaneously (as in the example) or because it helps to forecast future values of the goal variable. In either case, adjusting policy to achieve the intermediate target forces policy to respond to new information is a manner that is generally suboptimal. But this inefficiency will be smaller if the intermediate target is relatively easily controllable (i.e., σ_ψ^2 is small), yet is highly correlated with the variable of ultimate interest (i.e., σ_φ^2 and σ_e^2 are large), so that a deviation of the intermediate variable from its target provides a clear signal that the goal variable has deviated from its target.

9.3.4 Real Effects of Operating Procedures

The traditional analysis of operating procedures focuses on volatility; the operating procedure adopted by the central bank affects the way disturbances influence the variability of output, prices, real interest rates, and monetary aggregates. The average

18. Laubach and Posen (1997) argue that the targets were used to signal policy intentions rather than serving as strict intermediate targets.

values of these variables, however, is treated as independent of the choice of operating procedure. Canzoneri and Dellas (1998) show that the choice of procedure can have a sizable effect on the average level of the real rate of interest by affecting the variability of aggregate consumption.

The standard Euler condition relates the current marginal utility of consumption to the expected real return and the future marginal utility of consumption:

$$u_c(c_t) = \beta R_{ft} \mathrm{E}_t u_c(c_{t+1}),$$

where β is the discount factor, R_{ft} is the gross risk-free real rate of return, and $u_c(c_t)$ is the marginal utility of consumption at time t. The right side of this expression can be written as

$$\beta R_{ft} \mathrm{E}_t u_c(c_{t+1}) \approx \beta R_{ft} u_c(\mathrm{E}_t c_{t+1}) + \tfrac{1}{2} \beta R_{ft} u_{ccc}(c_{t+1}) \operatorname{Var}_t(c_{t+1}),$$

where u_{ccc} is the third derivative of the utility function and $\operatorname{Var}_t(c_{t+1})$ is the conditional variance of c_{t+1}. If the variance of consumption differs under alternative monetary policy operating procedures, then either the marginal utility of consumption must adjust (i.e., consumption will change) or the risk-free real return must change. Because the expected real interest rate can be expressed as the sum of the risk-free rate and a risk premium, average real interest rates will be affected if the central bank's operating procedure affects R_{ft} or the risk premium.

Canzoneri and Dellas develop a general equilibrium model with nominal wage rigidity and simulate the model under alternative operating procedures (interest rate targeting, money targeting, and nominal income targeting). They find that real interest rates, on average, are highest under a nominal interest rate targeting procedure. To understand why, suppose the economy is subject to money-demand shocks. Under a procedure that fixes the nominal money supply, such shocks induce a positive correlation between consumption (output) and inflation. This generates a negative risk premium (when consumption is lower than expected, the ex post real return is high because inflation is lower than expected). A nominal interest-rate procedure accommodates money-demand shocks and so results in a higher average risk premium. By calibrating their model and conducting simulations, Canzoneri and Dellas conclude that the choice of operating procedure can have a significant effect on average real interest rates.

9.4 Operating Procedures and Policy Measures

Understanding a central bank's operating procedures for implementing policy is important for two reasons. First, it is important in empirical work to distinguish

between endogenous responses to developments in the economy and exogenous shifts in policy. But whether movements in a monetary aggregate or a short-term interest rate are predominantly endogenous responses to disturbances unrelated to policy shifts or are exogenous shifts in policy will depend on the nature of the procedures used to implement policy. Thus, some understanding of operating procedures is required for empirical investigations of the impact of monetary policy. We have already seen in chapter 1 that estimates of the effects of monetary policy can be sensitive to the way policy is measured.

Second, operating procedures, by affecting the automatic adjustment of interest rates and monetary aggregates to economic disturbances, can have implications for the macro equilibrium. For example, operating procedures that lead the monetary authority to smooth interest-rate movements can introduce a unit root into the price level,[19] and in the models examined in chapters 2 and 3, the economy's response to productivity shocks was shown to depend on how the money supply was adjusted (although the effects were small).

Analyses of operating procedures are based on the market for bank reserves. In the United States, this is the federal funds market. While the focus of the discussion is on the United States and the behavior of the Federal Reserve, similar issues arise in the analysis of monetary policy in other countries, although institutional details can vary considerably. Discussions of operating procedures in major OECD countries can be found in Batten, Blackwell, Kim, Nocera, and Ozeki (1990); Bernanke and Mishkin (1992); Kasman (1993); Morton and Wood (1993); and Borio (1997).

9.4.1 Money Multipliers

Theoretical models of monetary economies often provide little guidance to how the quantity of money appearing in the theory should be related to empirical measures of the money supply. If m is viewed as the quantity of the means of payment used in the conduct of exchange, then cash, demand deposits, and other checkable deposits should be included in the empirical correspondence.[20] If m is viewed as a variable set by the policy authority, then an aggregate such as the monetary base, which represents the liabilities of the central bank and so can be directly controlled, would be more appropriate. The *monetary base* is equal to the sum of the reserve holdings of

19. See Goodfriend (1987) and Van Hoose (1989).

20. Whether these difference components of money should simply be added together, as they are in monetary aggregates such as $M1$ and $M2$, or whether the components should be weighted to reflect their differing degree of liquidity is a separate issue. Barnett (1980) has argued for the use of divisia indices of monetary aggregates. See also Spindt (1985).

the banking sector and the currency held by the nonbank public.[21] These are liabilities of the central bank and can be affected by open market operations. Most policy discussions, however, focus on broader monetary aggregates, but these are not the direct instruments of monetary policy. A traditional approach to understanding the linkages between a potential instrument such as the monetary base and the various measures of the money supply is to express broader measures of money as the product of the monetary base and a money multiplier. Changes in the money supply can then be decomposed into those resulting from changes in the base and those resulting from changes in the multiplier. The multiplier is developed using definitional relationships, combined with some simple behavioral assumptions.

A central bank can control the monetary base through open market operations. By purchasing securities, the central bank can increase the supply of bank reserves and the base. Securities sales reduce the base.[22] Denoting total reserves by TR and currency by C, the monetary base MB is given by

$$MB = TR + C.$$

In the United States, currency represents close to 90% of the base. Aggregates such as the monetary base and total reserves are of interest because of their close connection to the actual instruments central banks can control and because of their relationship to broader measures of the money supply.

In the United States, the monetary aggregate $M1$ is equal to currency in the hands of the public plus demand deposits and other checkable deposits. If the deposit component is denoted D and there is a reserve requirement ratio of rr against all such deposits, we can write

$$MB = RR + ER + C = (rr + ex + c)D,$$

where total reserves have been divided into required reserves (RR) and excess reserves (ER), and where $ex = ER/D$ is the ratio of excess reserves to deposits that banks choose to hold and $c = C/D$ is the currency-to-deposit ratio. This relationship allows us to write

$$M1 = D + C = (1 + c)D = \left(\frac{1 + c}{rr + ex + c}\right)MB. \tag{9.24}$$

21. There are two commonly used data series on the U.S. monetary base—one produced by the Board of Governors of the Federal Reserve System and one by the Federal Reserve Bank of St. Louis. The two series treat vault cash and the adjustment for changes in reserve requirements differently.

22. In the United States, daily Fed interventions are chiefly designed to smooth temporary fluctuations and are conducted mainly through repurchase and sale-purchase agreements rather than outright purchases or sales.

Equation (9.24) is a very simple example of money-multiplier analysis; a broad monetary aggregate such as $M1$ is expressed as a multiplier, in this case $(1 + c)/(rr + ex + c)$, times the monetary base. Changes in the monetary base translate into changes in broader measures of the money supply, given the ratios rr, ex, and c. Of course, the ratios rr, ex, and c need not remain constant as MB changes. The ratio ex is determined by bank decisions and the Fed's policies on discount lending, while c is determined by the decisions of the public concerning the level of cash they wish to hold relative to deposits. The usefulness of this money-multiplier framework was illustrated by M. Friedman and Schwartz (1963b), who employed it to organize their study of the causes of changes in the money supply.

In terms of an analysis of the market for bank reserves and operating procedures, the most important of the ratios appearing in (9.24) is ex, the excess reserve ratio. Since reserves earn no interest,[23] banks face an opportunity cost in holding excess reserves. As market interest rates rise, banks will tend to hold a lower average level of excess reserves. This drop in ex will work to increase $M1$. This implies that, holding the base constant, fluctuations in market interest rates will induce movements in the money supply.

9.4.2 The Reserve Market

In the United States, the Federal Reserve engages in open-market operations that affect the supply of reserves in the banking system and the federal funds rate, the interest rate banks in need of reserves pay to borrow reserves from banks with surplus reserves. Variations in the total quantity of bank reserves are associated with movements in broader monetary aggregates such as measures of the money supply ($M1$, $M2$, etc.). Similarly, movements in the funds rate influence other market interest rates. It is by intervening in the reserve market that the Fed attempts to affect the money supply, market interest rates, and, ultimately, economic activity and inflation.[24] The way reserve market variables (various reserve aggregates and the funds rate) respond to disturbances depends on the operating procedure being followed by the Fed. One objective in developing a model of the reserve market is to disentangle movements in reserves and the funds rate that are due to nonpolicy sources from those caused by exogenous policy actions.

Models of the reserve market generally have a very simple structure; reserve demand and reserve supply interact to determine the funds rate. Reserve demand

23. This statement is not true of all countries. For example, in New Zealand, reserves earn an interest rate set 300 basis points below the seven-day market rate.

24. In the United States, the development of the modern reserves market dates from the mid-1960s. See Meulendyke (1998).

arises primarily from the requirement that banks hold reserves equal to a specified fraction of their deposit liabilities; consequently, variations in the public's demand for bank liabilities will alter the banking sector's demand for reserves. The focus of these models is on the way reserves and interest rates react to shocks under alternative operating procedures. Hamilton (1996) provides a model that emphasizes the microstructure of the reserve market, while Bartolini, Bertola, and Prati (2002) develop a model designed to capture the day-to-day operations of the reserves market when the central bank targets the funds rate.

In the United States, banks are required to maintain an average reserve level equal to a fraction of their deposit liabilities; the fraction is set by the Federal Reserve.[25] These required reserves represent the bulk of reserve holdings, but the banking system does hold, on average, a level of reserves slightly greater than its level of required reserves. These excess reserves holdings are needed to meet the daily unpredictable net inflow or outflow of funds that each bank faces.[26] Excess reserves, when added to required reserves, yield total reserve holdings: $TR = RR + ER$. To give some sense of the magnitudes involved, in June 2002, seasonally adjusted total reserves of U.S. depository institutions averaged \$39.3 billion, of which \$38.0 billion were required reserves. By contrast, $M1$ averaged \$1.2 trillion in June 2002, and $M2$ averaged \$5.6 trillion. An economic expansion that increases the demand for money on the part of the public will lead to an increase in the banking sector's demand for reserves as required reserves rise with the growth of deposits.

The demand for reserves will also depend on the costs of reserves and on any factors that influence money demand—aggregate income, for example. In order to focus on the very short-run determination of reserve aggregates and the funds rate, factors such as aggregate income and prices are simply treated as part of the error term in the total reserve demand relationship, allowing us to write:

$$TR^d = -ai^f + v^d, \tag{9.25}$$

where TR^d represents total reserve demand, i^f is the funds rate (the rate at which a bank can borrow reserves in the private market), and v^d is a demand disturbance. This disturbance will reflect variations in income or other factors that produce fluc-

25. The actual precedure in the United States involves maintaining an average reserve level over a two-week maintenance period based on the average level of deposit balances two weeks earlier. For a discussion of these points, see Hamilton (1996).

26. Models of excess reserve holdings are generally based on inventory-theoretic models. Banks hold an inventory of reserves to balance stochastic payment flows. There is a cost asssociated with holding excess reserve balances that are too large (reserves don't pay interest) and with holding balances that are too small (the cost of borrowing reserves to offset a deficit position).

tuations in deposit demand. One interpretation of (9.25) is that it represents a relationship between the innovations in total reserve demand and the funds rate after the lagged effects of all other factors have been removed. For example, Bernanke and Mihov (1998) attempt to identify policy shocks by focusing on the relationships among the innovations to reserve demand, reserve supply, and the funds rate obtained as the residuals from a vector autoregression (VAR) model of reserve market variables. They characterize alternative operating procedures in terms of the parameters linking these innovations.[27]

The total supply of reserves held by the banking system can be expressed as the sum of the reserves that banks have borrowed from the Federal Reserve System plus nonborrowed reserves:

$$TR_t^s = BR_t + NBR_t.$$

The Federal Reserve can control the stock of nonborrowed reserves through open market operations; by buying or selling government securities, the Fed affects the stock of nonborrowed reserves. For example, a purchase of government debt by the Fed raises the stock of nonborrowed reserves when the Fed pays for its purchase by crediting the reserve account of the seller's bank with the amount of the purchase. Open market sales of government debt by the Fed reduce the stock of nonborrowed reserves. So the Fed can, even over relatively short time horizons, exercise close control over the stock of nonborrowed reserves.

The stock of borrowed reserves depends on the behavior of private banks and on their decisions about borrowing from the Fed (borrowing from the discount window). Bank demand for borrowed reserves will depend on the opportunity cost of borrowing from the Fed (the discount rate) and the cost of borrowing reserves in the federal funds market (the federal funds rate). An increase in the funds rate relative to the discount rate makes borrowing from the Fed more attractive and leads to an increase in bank borrowing. The elasticity of borrowing with respect to the spread between the funds rate and the discount rate will depend on the Fed's management of the discount window. Traditionally, the Fed has maintained the discount rate below the federal funds rate. This creates an incentive for banks to borrow reserves at the discount rate and then lend these reserves at the higher market interest rates. To prevent banks from exploiting this arbitrage opportunity, the Fed used nonprice methods to ration bank borrowing. This nonprice rationing affects the degree to

27. Kasa and Popper (1995) employ a similar approach to study monetary policy in Japan. Leeper, Sims, and Zha (1996) develop a more general formulation of the links between reserve market variables in an identified VAR framework.

which banks turn to the discount window to borrow as the incentive to do so, the spread between the funds rate and the discount rate, widens. Banks must weight the benefits of borrowing reserves in a particular week against the possible cost in terms of reduced future access to the discount window. Banks reduce their current borrowing if they expect the funds rate to be higher in the future because they prefer to preserve their future access to the discount window, timing their borrowing for periods when the funds rate is high. Therefore, borrowing decisions depend on the expected future funds rate as well as the current funds rate:

$$BR_t = b_1(i_t^f - i_t^d) - b_2 \mathrm{E}_t(i_{t+1}^f - i_{t+1}^d) + v_t^b, \tag{9.26}$$

where i^d is the discount rate (a policy variable) and v^b is a borrowing disturbance.

In 2002, the Fed proposed changing the way it administers the discount window. Under the new proposals, the discount rate would be set above the federal funds rate. Banks that qualify for *primary credit* could borrow at a rate 1% above the funds rate; *secondary credit* would be available at a rate 1.5% above the funds rate. By converting the discount rate into a penalty rate, the arbitrage opportunity created when the discount rate is below the funds rate will be eliminated. With a penalty rate, the need for nonprice rationing at the discount window is reduced. Because empirical work on the U.S. reserve market relies on data from periods when the discount rate was kept below the funds rate, our model of the reserve market will assume that $i^f > i^d$.

The simplest versions of a reserve market model often postulate a borrowing function of the form

$$BR_t = b(i_t^f - i_t^d) + v_t^b. \tag{9.27}$$

The manner in which an innovation in the funds rate affects borrowings, given by the coefficient b in (9.27), will vary, depending on how such a funds rate innovation affects expectations of future funds rate levels. Suppose, for example, that borrowings are actually given by (9.26) and that policy results in the funds rate following the process $i_t^f = \rho i_{t-1}^f + \xi_t$. Then $\mathrm{E}_t i_{t+1}^f = \rho i_t^f$ and, from (9.26), $BR_t = b i_t^f$, where $b = b_1 - \rho b_2$.[28] A change in operating procedures that leads the funds rate to be more highly serially correlated (increases ρ) will reduce the response of borrowings to the funds rate–discount rate spread.[29] While relationships such as (9.27) can help

28. For simplicity, this ignores the discount rate i^d for the moment.

29. Goodfriend (1983) provides a formal model of borrowed reserves; see also Waller (1990). For a discussion of how alternative operating procedures affect the the relationship between the funds rate and reserve aggregates, see Walsh (1982). Attempts to estimate the borrowings function can be found in Peristiani (1991) and Pearce (1993).

us to understand the linkages that affect the correlations among reserve market variables for a given operating procedure, we should not expect the parameter values to remain constant across operating procedures.

To complete the reserve market model, we need to specify the Fed's behavior in setting nonborrowed reserves. To consider a variety of different operating procedures, assume that the Fed can respond contemporaneously to the various disturbances to the reserve market, so that nonborrowed reserves are given by

$$NBR_t = \phi^d v_t^d + \phi^b v_t^b + v_t^s, \tag{9.28}$$

where v^s is a *monetary policy shock*. Different operating procedures will be characterized by alternative values of the parameters ϕ^d and ϕ^b.[30]

Equilibrium in the reserve market requires that total reserve demand equal total reserve supply. This condition is stated as

$$TR_t^d = BR_t + NBR_t. \tag{9.29}$$

If a month is the unit of observation, reserve market disturbances are likely to have no contemporaneous effect on real output or the aggregate price level.[31] Using this identifying restriction, Bernanke and Mihov (1998) obtain estimates of the innovations to TR, BR, i^f, and NBR from a VAR system that also includes GDP, the GDP deflator, and an index of commodity prices but in which the reserve market variables are ordered last.[32] Whether any of these VAR residuals can be interpreted directly as a measure of the policy shock v^s will depend on the particular operating procedure being used. For example, if $\phi^d = \phi^b = 0$, (9.28) implies that $NBR = v^s$; this corresponds to a situation in which the Fed does not allow nonborrowed reserves to be affected by disturbances to total reserve demand or to borrowed reserves, so the innovation to nonborrowed reserves can be interpreted directly as a policy shock. Under such an operating procedure, using nonborrowed reserve innovations (i.e., NBR) as the measure of monetary policy, as Christiano and Eichenbaum (1992a) do, is correct. However, if either ϕ^d or ϕ^b differs from zero, NBR will reflect nonpolicy shocks as well as policy shocks.

30. Note that ϕ^d and ϕ^b correspond to ϕ in (1.9) of chapter 1 since they reflect the impact of nonpolicy originating disturbances on the policy variable NBR.

31. Referring back to the discussion in section 1.3.4, this assumption corresponds to the use of the assumption that $\theta = 0$ to identify VAR innovations.

32. The commodity price index is included to eliminate the price puzzle discussed in chapter 1. This creates a potential problem for Bernanke and Mihov's identification scheme, since forward-looking variables such as asset prices, interest rates, and commodity prices may respond immediately to policy shocks. See the discussion of this issue in Leeper, Sims, and Zha (1996), who distinguish between policy, banking sector, production, and information variables.

Substituting (9.25), (9.27), and (9.28) into the equilibrium condition (9.29) and solving for the innovation in the funds rate yields

$$i_t^f = \left(\frac{b}{a+b}\right)i_t^d - \left(\frac{1}{a+b}\right)[v_t^s + (1+\phi^b)v_t^b - (1-\phi^d)v_t^d]. \tag{9.30}$$

The reduced-form expressions for the innovations to borrowed and total reserves are then found to be

$$BR_t = -\left(\frac{ab}{a+b}\right)i_t^d - \left(\frac{1}{a+b}\right)[bv_t^s - (a-b\phi^b)v_t^b - b(1-\phi^d)v_t^d] \tag{9.31}$$

$$TR_t = -\left(\frac{ab}{a+b}\right)i_t^d + \left(\frac{1}{a+b}\right)[av_t^s + a(1+\phi^b)v_t^b + (b+a\phi^d)v_t^d]. \tag{9.32}$$

How does the Fed's operating procedure affect the interpretation of movements in nonborrowed reserves, borrowed reserves, and the federal funds rate as measures of monetary policy shocks? Under a federal funds rate operating procedure, the Fed offsets total reserve demand and borrowing demand disturbances so that they do not affect the funds rate. According to (9.30), this policy requires that $\phi^b = -1$ and $\phi^d = 1$. In other words, a shock to borrowed reserves leads to an equal but opposite movement in nonborrowed reserves to keep the funds rate (and total reserves) unchanged (see 9.28), while a shock to total reserve demand leads to an equal change in reserve supply through the adjustment of nonborrowed reserves. The innovation in nonborrowed reserves is equal to $v^s - v^b + v^d$ and so does not reflect solely exogenous policy shocks.

Under a nonborrowed reserve procedure, $\phi^b = 0$ and $\phi^d = 0$ as innovations to nonborrowed reserves reflect policy shocks. In this case, (9.30) becomes

$$i_t^f = \left(\frac{b}{a+b}\right)i_t^d - \left(\frac{1}{a+b}\right)(v_t^s + v_t^b - v_t^d), \tag{9.33}$$

so innovations in the funds rate reflect both policy changes and disturbances to reserve demand and the demand for borrowed reserves. In fact, if v^d arises from shocks to money demand that lead to increases in measured monetary aggregates, innovations to the funds rate can be positively correlated with innovations to broader monetary aggregates. Positive innovations in an aggregate such as $M1$ would then appear to increase the funds rate, a phenomenon found in the VAR evidence reported in chapter 1.

From (9.31), a borrowed reserves policy corresponds to $\phi^d = 1$ and $\phi^b = a/b$, since adjusting nonborrowed reserves in this manner insulates borrowed reserves from nonpolicy shocks. That is, nonborrowed reserves are fully adjusted to accommodate fluctuations in total reserve demand. Under a borrowed reserves procedure, innovations to the funds rate are, from (9.30),

$$i_t^f = \left(\frac{b}{a+b}\right)i_t^d - \left(\frac{1}{a+b}\right)\left[v_t^s + \left(1+\frac{a}{b}\right)v_t^b\right],$$

so the funds rate reflects both policy and borrowing disturbances.

Table 9.1 summarizes the values of ϕ^d and ϕ^b that correspond to different operating procedures.

In general, the innovations in the observed variables can be written (ignoring discount rate innovations) as

$$\begin{bmatrix} i_t^f \\ BR_t \\ NBR_t \end{bmatrix} \equiv u_t = \begin{bmatrix} -\dfrac{1}{a+b} & -\dfrac{1+\phi^b}{a+b} & \dfrac{1-\phi^d}{a+b} \\ -\dfrac{b}{a+b} & \dfrac{a-b\phi^b}{a+b} & \dfrac{b(1-\phi^d)}{a+b} \\ 1 & \phi^b & \phi^d \end{bmatrix} \begin{bmatrix} v_t^s \\ v_t^b \\ v_t^d \end{bmatrix} \equiv Av_t. \tag{9.34}$$

By inverting the matrix A, we can solve for the underlying shocks, the vector v, in terms of the observed innovations u: $v = A^{-1}u$. This operation produces

$$\begin{bmatrix} v_t^s \\ v_t^b \\ v_t^d \end{bmatrix} = \begin{bmatrix} b\phi^b - a\phi^d & -(\phi^d+\phi^b) & 1-\phi^d \\ -b & 1 & 0 \\ a & 1 & 1 \end{bmatrix} \begin{bmatrix} i_t^f \\ BR_t \\ NBR_t \end{bmatrix}.$$

Hence,

$$v_t^s = (b\phi^b - a\phi^d)i_t^f - (\phi^d + \phi^b)BR_t + (1-\phi^d)NBR_t, \tag{9.35}$$

so that the policy shock can be recovered as a specific linear combination of the innovations to the funds rate, borrowed reserves, and nonborrowed reserves. From

Table 9.1
Parameters Under Alternative Operating Procedures

	Operating Procedure			
	Funds Rate	Nonborrowed	Borrowed	Total
ϕ^d	1	0	1	$-\frac{b}{a}$
ϕ^b	−1	0	$\frac{a}{b}$	−1

the parameter values in table 9.1, we have the following relationship between the policy shock and the VAR residuals:

$$\text{Funds Rate Procedure:} \quad v_t^s = -(b+a)i_t^f$$

$$\text{Nonborrowed Procedure:} \quad v_t^s = NBR_t$$

$$\text{Borrowed Reserves Procedure:} \quad v_t^s = -\left(1+\frac{a}{b}\right)BR_t$$

$$\text{Total Reserves Procedure:} \quad v_t^s = \left(1+\frac{a}{b}\right)TR_t$$

Policy shock cannot generally be identified with innovations in any one of the reserve market variables. Only for specific values of the parameters ϕ^d and ϕ^b, that is, for specific operating procedures, might the policy shock be recoverable from the innovation to just one of the reserve market variables.

9.4.3 Reserve Market Responses

This section will use the basic reserve market model to discuss how various disturbances affect reserve quantities and the funds rate under alternative operating procedures. Figure 9.1 illustrates reserve market equilibrium between total reserve demand and supply. For values of the funds rate less than the discount rate, reserve supply is vertical and equal to nonborrowed reserves. With the discount rate serving as a penalty rate, borrowed reserves fall to zero in this range, so that total reserve supply is just NBR. As the funds rate increases above the discount rate, borrowings become positive (see 9.27) and the total supply of reserves increases. Total reserve demand is decreasing in the funds rate according to (9.25).

Consider first a positive realization of the policy shock v^s. The effects on i^f, BR, and NBR can be found from the first column of the matrix A in (9.34). The policy shock increases nonborrowed reserves (we could think of it as initiating an open market purchase that increases banking sector reserve assets). In figure 9.1, the reserve supply curve shifts to the right horizontally by the amount of the increase in NBR. Given the borrowed reserves and total reserve demand functions, this increase in reserve supply causes the funds rate to fall. Bank borrowing from the Fed decreases because the relative cost of borrowed reserves $(i^d - i^f)$ has risen, partially offsetting some of the increase in total reserve supply.[33] A policy shock is associated

33. This analysis assumes that the discount rate has not changed; the Fed could, for example, change the discount rate to keep $i^f - i^d$ constant and keep borrowed reserves unchanged. Since the total supply of reserves has increased, the funds rate must fall, so this would require a cut in the discount rate.

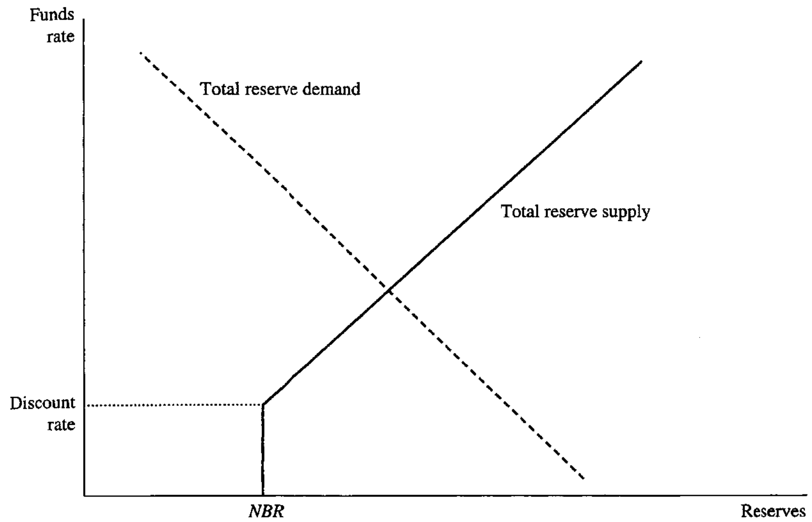

Figure 9.1
The Reserves Market

with an increase in total reserves, a fall in the funds rate, and a fall in borrowed reserves.

It is the response to nonpolicy disturbances that will differ, depending on the operating procedures (see the second and third columns of A; the elements of these columns depend on the ϕ^j parameters). Suppose there is a positive disturbance to total reserve demand, $v^d > 0$. This shifts total reserve demand to the right from RD to $R'D'$, as shown in figure 9.2. In the absence of any policy response (i.e., if $\phi^d = 0$), the funds rate increases (shown as the move from point A to point B in figure 9.2). This increase reduces total reserve demand (if $a > 0$), offsetting to some degree the initial increase in reserve demand. The rise in the funds rate induces an increase in reserve supply as banks increase their borrowing from the Fed. Under a funds rate operating procedure, however, $\phi^d = 1$; the Fed lets nonborrowed reserves rise by the full amount of the rise in reserve demand to prevent the funds rate from rising. Both reserve demand and reserve supply shift to the right by the amount of the disturbance to reserve demand, and the new equilibrium is at point C with an unchanged funds rate. Thus, total reserve demand shocks are completely accommodated under a funds rate procedure. If the positive reserve demand shock originated from an increase in

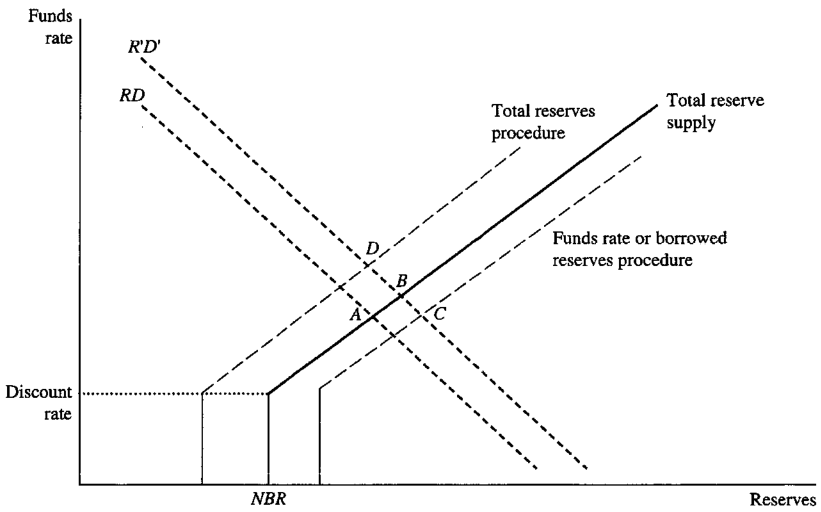

Figure 9.2
The Reserves Market Response to a Reserve Demand Increase

the demand for bank deposits as a result of an economic expansion, a funds rate procedure automatically accommodates the increase in money demand and has the potential to produce procyclical movements of money and output.[34]

In contrast, under a total reserves operating procedure, the Fed would adjust nonborrowed reserves to prevent v^d from affecting total reserves. From (9.32), this requires that $\phi^d = -b/a$; nonborrowed reserves must be reduced in response to a positive realization of v^d. It is not sufficient to just hold nonborrowed reserves constant; the rise in the funds rate caused by the rise in total reserve demand will induce an endogenous rise in reserve supply as banks increase their borrowing from the Fed. To offset this, nonborrowed reserves are reduced. Equilibrium under a total reserves procedure is at point D in figure 9.2. Thus, while a funds rate procedure offsets none of the impact of a reserve demand shock on total reserves, a total reserves procedure offsets all of it.

34. Since we have defined operating procedures in terms of the innovations to reserves and the funds rate, we have not said anything about the extent to which the funds rate might be adjusted in subsequent periods to offset movements in reserve demand induced by output or inflation.

Under a nonborrowed reserve procedure, $\phi^d = 0$; hence, a positive shock to reserve demand raises the funds rate and borrowed reserves. Total reserves rise by $-ai^f + v^d = [b/(a+b)]v^d < v^d$. So reserves do rise (in contrast to the case under a total reserves procedure) but by less than under a funds rate procedure.

Finally, under a borrowed reserves procedure, a positive shock to total reserve demand will, by increasing the funds rate, also tend to increase bank borrowing. To hold borrowed reserves constant, the Fed must prevent the funds rate from rising (i.e., it must keep $i^f = 0$; see 9.27). This objective requires letting nonborrowed reserves rise. So in the face of shocks to total reserve demand, a funds rate operating procedure and a borrowed reserves procedure lead to the same response. In terms of figure 9.2, both a funds rate procedure and a borrowed reserves procedure result in a new equilibrium at point C. As (9.33) shows, however, a borrowed reserves operating procedure is an inefficient procedure for controlling the funds rate in that it allows disturbances to the borrowings function (i.e., v^b shocks) to affect the finds rate. These results are summarized in table 9.2.

Now suppose there is a positive shock to bank borrowing; $v^b > 0$. The increase in borrowed reserves, by increasing total reserves, will lower the funds rate. Under a funds rate procedure, the Fed prevents this outcome by reducing nonborrowed reserves ($\phi^b = -1$) to fully neutralize the effect of v^b on the total reserve supply. The same response would occur under a total reserves operating procedure. In contrast, under a nonborrowed reserves procedure, $\phi^b = 0$, so the increase in borrowed reserves also increases total reserve supply, and the funds rate must decline to clear the reserve market. These results are summarized in table 9.3.

While the focus has been on the reserve market, it is important to keep in mind that the purpose of reserve-market intervention by the Fed is not to affect the funds rate or reserve measures themselves. The Fed's objective is to influence its policy-goal variables such as the rate of inflation. The simple money-multiplier framework that was discussed earlier provides a link between the reserve market and other factors

Table 9.2
Response to a Positive Reserve Demand Shock

	Operating Procedure			
	FF	BR	NBR	TR
Funds rate	0	0	+	+
Total reserves	+	+	+	0
Nonborrowed reserves	+	+	0	−
Borrowed reserves	0	0	+	+

Table 9.3
Response to a Positive Shock to Borrowed Reserves

	Operating Procedure			
	FF	BR	NBR	TR
Funds rate	0	−	−	0
Total reserves	0	+	+	0
Nonborrowed reserves	−	+	0	−
Borrowed reserves	+	0	+	+

affecting the supply of money. The observed quantities of the broader monetary aggregates then reflect the interaction of the supply of and demand for money. Movements in the funds rate are linked to longer-term interest rates through the term structure, a topic discussed in chapter 10.

9.4.4 A Brief History of Fed Operating Procedures

The model of the reserve market provides a very simple framework for analyzing how observable variables such as the funds rate and reserve aggregates respond to disturbances under alternative operating procedures. In the United States, the operating procedure employed by the Fed has changed over time. This fact, in turn, implies that the manner in which the reserve market has responded to disturbances has varied and that the appropriate measure of policy shocks has also changed.

Fed operating procedures have been discussed by various authors,[35] and major studies of operating procedures have been undertaken by the Federal Reserve (Federal Reserve 1981; Goodfriend and Small 1993). Over the past 30 years in the United States, most monetary economists have identified four different regimes, each defined according to the basic operating procedure the Fed followed. Chronologically, these correspond to periods of funds-rate, nonborrowed-reserves, borrowed-reserves, and funds-rate operating procedures, although in no case did the Fed's behavior reflect pure examples of any one type.[36]

1972–1979 The first period dates from the end of the Bretton Woods exchange-rate system in the early 1970s to October 6, 1979. The Fed is usually described as having followed a federal-funds-rate operating procedure during this period. Under such a

35. Examples include Walsh (1990), Goodfriend (1991, 1993), Strongin (1995), Meulendye (1998), and the references they cite.

36. From 1975 to 1993, the Fed announced targets for various monetary aggregates, and these played a role as intermediate targets during some periods; see B. Friedman and Kuttner (1996).

policy, the Fed allowed nonborrowed reserves to adjust automatically to stabilize the funds rate within a narrow band around its target level. Thus, a shock to total reserve demand that, in the absence of a policy response, would have led to an increase in both the funds rate and borrowed reserves was offset by open market purchases that expanded nonborrowed reserves sufficiently to prevent the funds rate from rising (i.e., $\phi^d = 1$). As a result, expansions in reserve demand were fully accommodated by increases in reserve supply.[37]

A funds-rate operating procedure only implies that shocks to the funds rate are offset initially; the targeted funds rate could, in principle, respond strongly beginning in period $t+1$. However, the funds-rate operating procedure came under intense criticism during the 1970s because of the Fed's tendency to stabilize interest rates for longer periods of time. Such interest-rate-smoothing behavior can have important implications for price-level behavior (Goodfriend 1987). Because a rise in the price level will increase the nominal demand for bank deposits as private agents attempt to maintain their real money holdings, periods of inflation will lead to increases in the nominal demand for bank reserves. If the central bank holds nonborrowed reserves fixed, the rising demand for reserves pushes up interest rates, thereby moderating the rise in money demand and real economic activity. If the central bank instead attempts to prevent interest rates from rising, it must allow the reserve supply to expand to accommodate the rising demand for reserves. Thus, interest-rate-stabilizing policies can automatically accommodate increases in the price level, contributing to ongoing inflation. Under some circumstances, an interest-rate policy can even render the price level indeterminate; an arbitrary change in the price level produces a proportionate change in nominal money demand, which the central bank automatically accommodates to keep interest rates from changing.[38] Since market interest rates incorporate a premium for expected inflation, an increase in expected inflation would, under a policy of stabilizing market interest rates, also be automatically accommodated.

Recall from our reserve-market model that under a funds-rate procedure, nonborrowed reserves are automatically adjusted to offset the impact on the funds rate of shocks to total reserve demand and to borrowed reserves. In terms of the model

37. While the discussion here focuses on reserve market adjustments, changes in the funds rate target then lead to changes in market interest rates. For evidence, see Cook and Hahn (1989), Rudebusch (1995), or Roley and Sellon (1996). International evidence on the response of market interest rates to changes in the short-run interest rate used to implement policy can be found in Buttiglione, Del Giovane, and Tristani (1996).

38. The compatibility of interest-rate rules for monetary policy and price-level determinacy will be discussed in detail in chapter 10.

parameters, this adjustment requires that $\phi^d = 1$ and $\phi^b = -1$. Bernanke and Mihov (1998), using both monthly and biweekly data, report that these restrictions are not rejected for the period 1972:11 to 1979:09. Thus, innovations in the funds rate provide an appropriate measure of monetary policy during this period.

1979–1982 In October 1979, as part of a policy shift to lower inflation, the Fed moved to a nonborrowed-reserves operating procedure. An operating procedure that focused on a reserve quantity was viewed as more consistent with reducing money growth rates to bring down inflation.

The Fed had, in fact, begun announcing target growth rates for several monetary aggregates in 1975. Under the Humphrey-Hawkins Act, the Fed was required to establish monetary targets and report these to Congress.[39] Because growth-rate target ranges were set for several measures of the money supply (there were targets for $M1$, $M2$, $M3$, and debt), the extent to which these targets actually influenced policy was never clear. The move to a nonborrowed-reserves operating procedure was thought by many economists to provide a closer link between the policy instrument (nonborrowed reserves) and the intermediate target of policy (the monetary growth targets). B. Friedman and Kuttner (1996) provide an evaluation of the actual effects of these targets on the conduct of policy.

Under a nonborrowed-reserves procedure, an increase in expected inflation would no longer automatically lead to an accommodative increase in bank reserves. Instead, interest rates would be allowed to rise, reducing nominal asset demand and restraining money growth. Similarly, if money growth rose above the Fed's target growth rate, reserve demand would rise, pushing up the funds rate. The resulting rise in the funds rate would tend to reduce money demand automatically.

Whether the Fed actually followed a nonborrowed-reserves procedure after October 1979 has often been questioned. Figure 9.3 plots the federal funds rate from 1965 to 2001, demarcating October 1979 by the first vertical line. The funds rate was clearly both higher and more volatile after the switch in policy procedures than before.[40] Many commentators felt that the policy shift in late 1979 was designed to allow the Fed to increase interest rates substantially while reducing the political

39. The targets for $M1$ for the period 1975–1986 and for $M2$ and $M3$ for the period 1975–1991 are reported in Bernanke and Mishkin (1992, table 1, pp. 190–191). Preliminary targets for the following calendar year were set each July and confirmed in January. Discussions of the targets can be found in the various issues of the Federal Reserve's "Monetary Report to Congress." The Fed stopped setting growth rate targets for $M1$ after 1986 because of the apparent breakdown in the relationship between $M1$ and nominal income.

40. Much of the increased volatility in early 1980 was caused by the imposition and then removal of credit controls.

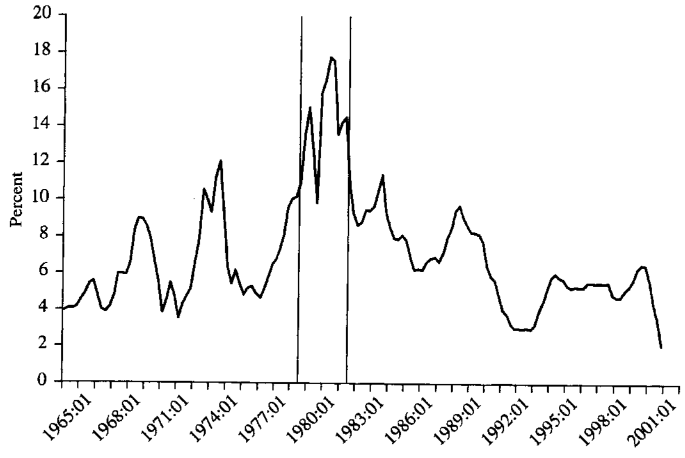

Figure 9.3
Federal Funds Rate, 1965–2001

pressures on the Fed to prevent rates from rising. Under the former funds-rate procedure, changes in short-term interest rates were (correctly) perceived as reflecting Fed decisions. By adopting a nonborrowed-reserves operating procedure and focusing more on achieving its targeted growth rates for the money supply, the Fed could argue that the high interest rates were due to market forces and not Fed policy. Cook (1989) estimates, however, that fully two-thirds of all funds-rate changes during this period were the result of "judgmental" Fed actions; only one-third represented automatic responses to nonpolicy disturbances.

The 1979–1982 period was characterized by increased attention by the Fed to its monetary targets. In principle, nonborrowed reserves were adjusted to achieve a targeted growth rate for the money stock. If the money stock was growing faster than desired, the nonborrowed reserve target would be adjusted downward to place upward pressure on the funds rate. This, in turn, would reduce money demand and tend to bring the money stock back on target. As a result, market interest rates responded sharply to each week's new information on the money supply. If the money supply exceeded the market's expectation, market interest rates rose in anticipation of future policy tightening (Roley and Walsh 1985 and the references listed there).

The actual practice under the nonborrowed-reserves procedure was complicated by several factors. First, the Fed established and announced targets for several different definitions of the money stock.[41] This policy reduced the transparency of the procedure, since often one monetary aggregate might be above its target while another would be below, making the appropriate adjustment to the nonborrowed-reserves path unclear. Second, because of the system of lagged reserve accounting then in effect, required reserves were predetermined each week. The level of reserves a bank was required to hold during week t was based on its average deposit liabilities during week $t-2$. With reserve demand essentially predetermined each week, variations in the funds rate had little contemporaneous effect on reserve demand. Changes in reserve supply would require large swings in the funds rate to equilibrate the reserve market. A rise in interest rates had no immediate effect on the banking sector's reserve demand, leading to a delay in the impact of a policy tightening on money growth. This system was criticized as reducing the ability of the Fed to control the growth rate of the monetary aggregates. See McCallum and Hoehn (1983).[42]

Referring back to our earlier reserve market model, with $\phi^d = \phi^b = 0$ under a nonborrowed-reserves operating procedure, (9.30) implies that

$$i^f_{NBR} = \left(\frac{b}{a+b}\right) i^d - \left(\frac{1}{a+b}\right)[v^s + v^b - v^d], \tag{9.36}$$

so that, ignoring discount rate changes, the variance of funds-rate innovations rises from $\sigma_s^2/(a+b)^2$ under a pure funds-rate operating procedure to $[\sigma_s^2 + \sigma_d^2 + \sigma_b^2]/(a+b)^2$ under a pure nonborrowed-reserves operating procedure, where σ_i^2 is the variance of v^i for $i = s, d, b$. The variance of funds-rate innovations is decreasing in a, and with lagged reserve accounting, $a = 0$, further increasing the variance of the funds rate. Changes in reserve supply would require large swings in the funds rate to equilibrate the reserve market.

In practice, it was argued that the Fed actually set its nonborrowed-reserves target so as to achieve the level of the funds rate it desired. That is, the Fed started with a desired path for the money stock; since equilibrium required that money demand equal money supply, it used an estimated money-demand function to determine the level of the funds rate consistent with the targeted level of money demand. Then, based on total reserve demand (predetermined under lagged reserve accounting) and

41. The Fed established target cones for each aggregate. For example, the target cone for $M1$ set in January 1980 was 4.0% to 6.5% from a base of the actual level of $M1$ in the fourth quarter of 1979. The use of actual levels as the base for new target cones resulted in base drift; past target misses were automatically incorporated into the new base. See Broaddus and Goodfriend (1984). For a discussion of the optimal degree of base drift, see Walsh (1986).

42. Lagged reserve accounting was replaced by the current system of contemporaneous reserve accounting in 1984. See Hamilton (1996) for a detailed discussion of the reserve accounting system.

an estimated borrowed-reserve function, it determined the level of nonborrowed reserves required to achieve the desired funds rate. A nonborrowed-reserves operating procedure designed to achieve a desired funds rate is simply an inefficient funds-rate procedure. However, by shifting the focus of policy away from a concern for stabilizing interest rates, the 1979 policy shift did reflect a substantive policy shift consistent with reducing the rate of inflation.

Using biweekly data for the period October 1979 to October 1982, Bernanke and Mihov (1998) report estimates of ϕ^d and ϕ^b; neither estimate is statistically significantly different from zero. These estimates are consistent, then, with the actual use of a nonborrowed-reserves operating procedure during this period.

Key to a nonborrowed-reserves operating procedure is the need to predict the relationship between changes in nonborrowed reserves and the resulting impact on broader monetary aggregates, inflation, and real economic activity. During the late 1970s and early 1980s, there seemed to be a fairly stable relationship between monetary aggregates such as $M1$ and nominal income. This relationship could be used to work backward from a desired path of nominal income growth to a growth path for $M1$ to a growth path for nonborrowed reserves. Unfortunately, this relationship appeared to break down in the early and mid-1980s (see, e.g., B. Friedman and Kuttner 1996). In the absence of a reliable link between reserve measures and nominal income, the Fed eventually moved away from a nonborrowed-reserves operating procedure.

1982–1988 After 1982, the Fed generally followed a borrowed-reserves operating procedure. As noted earlier, such a procedure is, in practice, similar to a funds-rate operating procedure, at least in the face of reserve demand shocks (see table 9.2). The basic Poole analysis implied that an interest-rate-oriented operating procedure will tend to dominate a monetary-aggregates-oriented one as the variance of money-demand shocks rises relative to aggregate-demand shocks. B. Friedman and Kuttner (1996) provide a plot of the ratio of the variance of money-demand shocks to the variance of aggregate-demand shocks based on an estimated VAR. The plot shows this ratio reaching a minimum during 1981 and then steadily increasing. The shift back to an interest-rate operating procedure after 1982 is consistent with the normative recommendations of Poole's model. The second vertical line in figure 9.3 marks the shift away from the previous nonborrowed-reserves procedure.

From our earlier discussion, a borrowed-reserves operating procedure implies values of 1 and a/b for ϕ^d and ϕ^b. Bernanke and Mihov (1998) obtain point estimates for ϕ^d and ϕ^b for February 1984 to October 1988 that are more consistent with a funds-rate procedure ($\phi^d = 1$; $\phi^b = -1$) than with a borrowed-reserves procedure.

However, for biweekly data during the post-1988 period, Bernanke and Mihov find estimates consistent with a borrowed-reserves procedure with $a = 0$. This last parameter restriction is in agreement with the characterization of policy provided by Strongin (1995).

Cosimano and Sheehan (1994) estimate a biweekly reserve-market model using data from 1984 to 1990. Their results are consistent with a borrowed-reserves procedure over this period and not with a funds-rate procedure, although they note that actual policy under this procedure was similar to what would occur under a funds-rate procedure. The evidence also suggests that after the October 1987 stock market crash, the Fed moved toward a more direct funds-rate procedure.

1988–the Present Since the late 1980s, the Fed has targeted the funds rate directly ($\phi^d = 1; \phi^b = -1$). Open market operations are conducted once each day, so the actual funds rate can fluctuate slightly around the target rate on a daily basis. One of the biggest changes in operating procedures since 1990 is the increase in the transparency with which the Fed conducts monetary policy. Since 1994, the FOMC has announced its policy decisions at the time they are made. These announced changes in the target rate receive prominent coverage in the press, and the FOMC's press releases serve to convey its assessment of the economy to the public and to give some signal of possible future changes in policy. For example, during 2000 the FOMC increased its funds-rate target out of concern that historically low unemployment and rapid economic growth would trigger an increase in inflation, and its press release after its May 16, 2000, meeting stated that

> The Federal Open Market Committee voted today to raise its target for the federal funds rate by 50 basis points to 6-1/2 percent.... Increases in demand have remained in excess of even the rapid pace of productivity-driven gains in potential supply....

Beginning in January 2001, the FOMC's concerns shifted as signs appeared that the economy was entering a recession.[43] After cutting the funds rate on January 3, the FOMC reduced its target again at its January 30–31 meeting, and its press release included the following statement:

> Against the background of its long-run goals of price stability and sustainable economic growth and of the information currently available, the Committee believes that the risks are weighed mainly towards conditions that may generate economic weakness in the foreseeable future.

43. The NBER Business Cycle Dating Committee subsequently identified March 2001 as the start of a recession.

While these statements contribute to policy transparency, the Fed, unlike many other central banks, has never formally translated its "long-run goals of price stability" into an explicit target for the rate of inflation.[44]

While we have followed Bernanke and Mihov in using ϕ^d and ϕ^b to characterize different operating procedures, the parameters a and b in the total-reserves demand (9.25) and the borrowed-reserves (9.27) relationships may also vary under difference operating procedures. For example, models of bank borrowing from the discount window (e.g., Goodfriend 1983) imply that the slope of the borrowings function should depend on the operating procedure being employed.[45] Evidence supporting this hypothesis is reported by Pearce (1993). As noted in section 9.4.2, the coefficient b should depend on the time-series process that characterizes the funds rate. If changes in the funds rate are very persistent, b will tend to be smaller than under a procedure that leads to more transitory changes in the funds rate.

9.4.5 Other Countries

The preceding discussion focused on the United States. If measuring monetary policy requires an understanding of operating procedures, then the appropriate measure of policy in the United States will not necessarily be appropriate for other countries. Operating procedures generally depend on the specific institutional structure of a country's financial sector, and the means used to implement monetary policy have varied over time in most countries as financial markets have evolved as the result of either deregulation or financial innovations. Borio (1997) provides a survey of policy implementation in the industrial economies. Detailed discussions of the operating procedures in France, Germany, Japan, the United Kingdom, and the United States can be found in Batten, Blackwell, Kim, Nocera, and Ozeki (1990). Bernanke and Mishkin (1992) provide case studies of monetary policy strategies in the United States, the United Kingdom, Canada, Germany, Switzerland, and Japan. These countries, plus France, are discussed in Kasman (1993) and Morton and Wood (1993). The behavior of the Bundesbank is examined by Clarida and Gertler (1997). Cargill, Hutchison, and Ito (1997) provide a discussion of Japan. Goodhart and Viñals (1994) discuss policy behavior in a number of European and Antipodian countries.

The experiences with monetary-policy operating procedures in all these countries have been broadly similar over the past 20 years. Beginning in the mid-1970s, many

44. Inflation targeting is discussed in chapters 8 and 11.

45. See (9.26) and the discussion after (9.27).

countries publicly established monetary targets. Since 1975, the U.S. Federal Reserve has been required by Congress to set targets for monetary aggregates. Germany, Canada, and Switzerland began announcing money targets in 1975, the United Kingdom in 1976, and France in 1977. The weight placed on these targets has, however, varied greatly over time. In general, the financial innovations that occurred in the 1980s, together with significant deregulation of financial markets that took place after 1985, reduced reliance on monetary targets. This finding is consistent with the implications of Poole's model, which suggested that increased financial-market instability that makes money demand less predictable would lessen the advantages of any monetary-aggregates-oriented operating procedure. The past 10 years have seen a shift toward interest-rate operating procedures in all these countries except Switzerland.

Morton and Wood (1993) argue that a common theme among the six industrial countries they examine has been the move to more flexible interest-rate policies. Rather than rely on officially established interest rates, often combined with direct credit controls, central banks have moved toward more market-oriented interest-rate policies.[46] These involve control over a reserve aggregate (such as nonborrowed reserves in the United States) through which the central bank influences liquidity in the money market. This provides the central bank with control over a short-term money-market rate that balances reserve supply and demand. Typically, central banks do not intervene in the market continuously; instead they usually estimate reserve demand and then add or subtract bank reserves to achieve the targeted interbank interest rate. Because these operations are based on reserve projections and because actual reserve demand may differ from projections, the actual value of the interest rate can differ from the central bank's target. However, by intervening daily, the central bank can keep target deviations quite small.[47]

Finally, it is worth emphasizing that the choice of operating procedure is, in principle, distinct from the choice of ultimate goals and objectives of monetary policy. For example, a policy under which price stability is the sole objective of monetary policy could be implemented through either an interest-rate procedure or a reserve-aggregate procedure. A policy that incorporates output-stabilization or exchange-rate considerations can similarly be implemented through different procedures. The choice of operating procedure is significant, however, for interpreting the short-term

46. Similarly, Kasman (1993) notes that innovation and liberalization in financial markets have made the institutional setting in which policy is conducted increasingly similar among the industrial countries.

47. In an interesting variation, the Reserve Bank of New Zealand appears to implement policy by simply making announcements—so-called open-mouth operations (Guthrie and Wright 2000).

response of financial markets to economic disturbances. And inefficient procedures can introduce unnecessary volatility into financial markets.

9.5 Problems

1. Suppose the basic Poole model (9.1 and 9.2) is modified by allowing the disturbances to be serially correlated. Specifically, assume that the disturbance in (9.1) is given by $u_t = \rho_u u_{t-1} + \varphi_t$, while the disturbance in (9.2) is given by $v_t = \rho_v v_{t-1} + \psi_t$, where φ and ψ are white noise processes (assume that all shocks can be observed with a one-period lag). Assume that the central bank's loss function is $\mathrm{E}(y_t)^2$.

a. Under a money-supply operating procedure, derive the value of m_t that minimizes $\mathrm{E}(y_t)^2$.

b. Under an interest-rate operating procedure, derive the value of i_t that minimizes $\mathrm{E}(y_t)^2$.

c. Explain why your answers in (a) and (b) depend on ρ_u and ρ_v.

d. Does the choice between a money-supply procedure and an interest-rate procedure depend on the ρ_is? Explain.

e. Suppose the central bank sets its instrument for two periods (for example, $m_t = m_{t+1} = m^*$) to minimize $\mathrm{E}(y_t)^2 + \beta\mathrm{E}(y_{t+1})^2$, where $0 < \beta < 1$. How is the instrument choice problem affected by the ρ_is?

2. Solve for the δ_is appearing in (9.11) and show that the optimal rule for the monetary base is the same as that implied by the value of μ^* given in (9.10).

3. Suppose the money demand relationship is given by $m = -c_1 i + c_2 y + v$. Show how the choice of an interest-rate versus a money-supply operating procedure depends on c_2. Explain why the choice depends on c_2.

4. Prices and aggregate-supply shocks can be added to Poole's analysis by using the following model:

$$y_t = y_n + a(\pi_t - \mathrm{E}_{t-1}\pi_t) + e_t$$

$$y_t = y_n - \alpha(i_t - \mathrm{E}_t\pi_{t+1}) + u_t$$

$$m_t - p_t = c_0 - ci_t + y_t + v_t.$$

Assume that the central bank's objective is to minimize $\mathrm{E}[\lambda(y - y_n)^2 + \pi^2]$, and that disturbances are mean-zero, white noise processes. Both the private sector in setting

$E_{t-1}\pi_t$ and the monetary authority in setting its policy instrument must act prior to observing the current values of the disturbances.

a. Calculate the expected loss function if i_t is used as the policy instrument. (Hint: Given the objective function, the instrument will always be set to ensure that expected inflation is equal to zero.)

b. Calculate the expected loss function if m_t is used as the policy instrument.

c. How does the instrument choice comparison depend on

i. the relative variances of the aggregate-supply, demand, and money-demand disturbances?

ii. the weight on stabilizing output fluctuations λ?

5. Using the intermediate target model of section 9.3.3 and the loss function (9.15), rank the policies that set i_t equal to $\hat{\imath}_t$, i_t^T, and $\hat{\imath}_t + \mu^* x_t$.

6. Show that if the nominal interest rate is set according to (9.17), the expected value of the nominal money supply is equal to $\hat{m}$ given in (9.19).

7. Suppose the central bank is concerned with minimizing the expected value of a loss function of the form

$$L = E(TR)^2 + \chi E(i^f)^2,$$

which depends on the variances of innovations to total reserves and the funds rate (χ is a positive parameter). Using the reserve-market model of this chapter, find the values of ϕ^d and ϕ^b that minimize this loss function. Are there conditions under which a pure nonborrowed-reserves or a pure borrowed-reserves operating procedures would be optimal?

10 Interest Rates and Monetary Policy

10.1 Introduction

Most central banks in the major industrialized economies implement policy by intervening in the money market to achieve a target level for a short-term interest rate.[1] In the United States, the Federal Reserve sets a target level for the federal funds rate and then influences the supply of bank reserves to maintain the funds rate at the targeted value. The funds rate serves as the operational target for policy, and because it can be closely controlled, it can for most purposes be treated as the instrument of policy. Nevertheless, the theoretical models examined in chapters 2–4 generally treated monetary policy as if it were implemented through the use of a money-supply operating procedure or by some sort of policy rule oriented toward the control of a monetary aggregate. Treating the nominal money supply as the instrument of monetary policy is the approach taken in most undergraduate textbooks in which financial-market equilibrium is summarized by an LM curve.[2] This perspective emphasizes the role of money-demand disturbances in affecting the link between the money supply, interest rates, and real economic activity.

If the central bank can directly control short-term interest rates, the predictability (or unpredictability) of money demand becomes less relevant. Instead, the linkages between the very short-term interest rate the central bank controls and the broad range of market interest rates that affect investment and consumption spending, as well as the link between interest rates and exchange rates, become of critical importance.

In this chapter, we examine the implications of using a nominal interest rate as the operational instrument of monetary policy. The actual policy instrument of central banks is usually the supply of reserve assets, but these can be adjusted to maintain close control of a short-term rate such as the interbank overnight rate. Under the assumption that components of aggregate spending are more closely linked to movements in long-term interest rates, monetary policy actions affecting short-term interest rates are linked to the aggregate economy through the term structure of interest rates. The term structure and the relationship between long-term rates and expected inflation become important in policy design. By treating an interest rate as the variable under the control of the central bank, one obtains a framework that more closely matches the way in which most policy makers view policy.

1. This is certainly true in the United States. As discussed in Bernanke and Mishkin (1992), Kasman (1993), and Morton and Wood (1993), the cental banks in the major OCED countries also use short-term market interest rates as their instrument of policy.

2. See, for example, Abel and Bernanke (1995), Hall and Taylor (1997), and Mankiw (1997).

10.2 Interest-Rate Rules and the Price Level

In this section, we explore the implications for the price level of policies that focus on the interest rate. As the models of chapters 2 and 3 showed, the steady-state real rate of interest is determined by the marginal product of capital, and so, in the long run, monetary policy has no effect on real rates of return.[3] Monetary policy can affect nominal rates, both in the short run and in the long run, but the Fisher relationship links the real rate, expected inflation, and the nominal rate of interest. Targets for nominal interest rates and inflation cannot be independently chosen, and controlling the nominal interest rate has important implications for the behavior of the aggregate price level.

10.2.1 Price-Level Determinacy

Section 5.3.1 made use of the following model, expressed in terms of the price level:

$$y_t = y^c + a(p_t - \mathrm{E}_{t-1}p_t) + e_t \tag{10.1}$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t \tag{10.2}$$

$$m_t - p_t = y_t - ci_t + v_t \tag{10.3}$$

$$i_t = r_t + (\mathrm{E}_t p_{t+1} - p_t), \tag{10.4}$$

where y, m, and p are the natural logs of output, the money stock, and the price level, and r and i are the real and nominal rates of interest. In chapter 5, the specification of the model was completed by treating the nominal money supply as the policy instrument; the only exception occurred in section 5.4. We want to change this treatment, returning to the approach of section 5.4, by assuming that the nominal interest rate is the policy instrument. Given real output, the price level, and the nominal interest rate, the nominal money supply will be determined endogenously by the money-demand equation (10.3). Although central banks may closely control the nominal rate i, it is the expected real rate of interest r that influences consumption and investment decisions and therefore aggregate demand.[4] This distinction has important implications for the feasibility of an interest targeting rule.

3. If money is superneutral, this will certainly be the case; in models in which money is not superneutral, the simulations in chapters 2 and 3 indicated that the effect of inflation on equilibrium real returns was small.

4. Term structure considerations are postponed until section 10.3.

Suppose that the central bank conducts policy by pegging the nominal interest rate at some targeted value:

$$i_t = i^T. \tag{10.5}$$

Under an interest rate peg, the basic aggregate demand and supply system given by (10.1), (10.2), and (10.4) become

$$y_t = y^c + a(p_t - \mathrm{E}_{t-1}p_t) + e_t \tag{10.6}$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t \tag{10.7}$$

$$i^T = r_t + (\mathrm{E}_t p_{t+1} - p_t). \tag{10.8}$$

The money demand equation (10.3) is no longer relevant because the central bank must allow the nominal money stock to adjust to the level of money demand at the targeted interest rate and the equilibrium level of output.

Note that the price level only appears in the form of an expectation error (i.e., as $p_t - \mathrm{E}_{t-1}p_t$ in the aggregate-supply equation) or as an expected rate of change (i.e., as $\mathrm{E}_t p_{t+1} - p_t$ in the Fisher equation). This structure implies that the price *level* is indeterminate. That is, if the sequence $\{p^*_{t+i}\}_{i=0}^{\infty}$ is an equilibrium, so is any sequence $\{\hat{p}_{t+i}\}_{i=0}^{\infty}$ where $\hat{p}$ differs from p^* by any constant κ: $\hat{p}_t = p^*_t + \kappa$ for all t. Since κ is an arbitrary constant, $p^*_t - \mathrm{E}_{t-1}p^*_t = \hat{p}_t - \mathrm{E}_{t-1}\hat{p}_t$; hence, y_t is the same under either price sequence. From (10.7), the equilibrium real interest rate is equal to $(\alpha_0 - y_t + u_t)/\alpha_1$, so it too is the same. With expected inflation the same under either price sequence, the only restriction on the price path is that the expected rate of inflation be such that $i^T = (\alpha_0 - y_t + u_t)/\alpha_1 + \mathrm{E}_t p^*_{t+1} - p^*_t$.

The indeterminacy of the price level is perhaps even more apparent if (10.6)–(10.8) are rewritten explicitly in terms of the rate of inflation. By adding ap_{t-1} to and subtracting it from the supply function, the equilibrium conditions become

$$y_t = y^c + a(\pi_t - \mathrm{E}_{t-1}\pi_t) + e_t$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t$$

$$i^T = r_t + \mathrm{E}_t \pi_{t+1}.$$

These three equations can be solved for output, the real rate of interest, and the rate of inflation. Since the price level does not appear, it is formally indeterminate.[5]

5. Employing McCallum's minimum state solution method (McCallum 1983a), the equilibrium inflation rate is $\pi_t = i^T + (y^c - \alpha_0)/\alpha_1 + (u_t - e_t)/a$ when u and e are serially uncorrelated and the target nominal interest rate is expected to remain constant. In this case, $\mathrm{E}_t\pi_{t+1} = i^T + (y^c - \alpha_0)/\alpha_1$, so permanent changes in the target rate i^T do not affect the real interest rate: $r_t = i^T - \mathrm{E}_t\pi_{t+1} = -(y^c - \alpha_0)/\alpha_1$.

As stressed by McCallum (1986), the issue of indeterminacy differs from the problem of multiple equilibria. The latter involves situations in which multiple equilibrium price paths are consistent with a given path for the nominal supply of money. We saw one example of such a multiplicity of equilibria when studying models of hyperinflation in chapter 4. With indeterminacy, neither the price level nor the nominal supply of money is determined by the equilibrium conditions of the model. If the demand for real money balances is given by (10.3), then the price sequence p^* is associated with the sequence $m_t^* = p_t^* + y_t - ci_t^T + v_t$, while $\hat{p}$ is associated with $\hat{m}_t = \hat{p}_t + y_t - ci_t^T + v_t = m^* + \kappa$. The price sequences p^* and $\hat{p}$ will be associated with different paths for the nominal money stock.

Intuitively, if all agents expect the price level to be 10% higher permanently, such an expectation is completely self-fulfilling. To peg the nominal rate of interest, the central bank simply lets the nominal money supply jump by 10%. This stands in contrast to the case in which the central bank controls the nominal quantity of money; a jump of 10% in the price level would reduce the real quantity of money, thereby disturbing the initial equilibrium. Under a rule such as (10.5) that has the policy maker pegging the nominal interest rate, the central bank lets the nominal quantity of money adjust as the price level does, leaving the real quantity unchanged.[6]

Price-level indeterminacy is often noted as a potential problem with pure interest-rate pegs; if private agents don't care about the absolute price level—and under pure interest-rate control, neither does the central bank—nothing pins down the price level. Simply pegging the nominal interest rate does not provide a *nominal anchor* to pin down the price level. However, this problem will not arise if the central bank's behavior does depend on a nominal quantity such as the nominal money supply.

For example, suppose the nominal money supply (or a narrow reserve aggregate) is the actual instrument used to affect control of the interest rate, and assume it is adjusted in response to interest-rate movements (Canzoneri, Henderson, and Rogoff 1983; McCallum 1986):

$$m_t = \mu_0 + m_{t-1} + \mu(i_t - i^T). \tag{10.9}$$

Under this policy rule, the monetary authority adjusts the nominal money-supply growth rate, $m_t - m_{t-1}$, in response to deviations of the nominal interest rate from its target value. If i_t fluctuates randomly around the target i^T, then the average rate of money growth will be μ_0. As $\mu \to \infty$, the variance of the nominal rate around the targeted value i^T will shrink to zero, but the price level can remain determinate.

6. See Patinkin (1965) for an early discussion of price-level determinacy and Schmidt-Grohe and Uribe (2000a) for a more recent discussion.

To verify these claims, we need to solve for the equilibrium price level (verifying that a determinate solution exists) and then show that the variance of the nominal interest rate around i^T can be made arbitrarily small by increasing μ. Employing the method of undetermined coefficients (see Sheffrin 1983, McCallum 1989, or Attfield, Demery, and Duck 1991), the model can be solved by first postulating candidate solutions for the price level and the nominal interest rate of the form

$$p_t = b_{10} + b_{11}m_{t-1} + b_{12}e_t + b_{13}u_t + b_{14}v_t \tag{10.10}$$

$$i_t = b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t, \tag{10.11}$$

where the b_{ij}'s are as yet unknown coefficients whose values we need to determine. Equations (10.10) and (10.11) express the equilibrium values of p_t and i_t as functions of the contemporaneous disturbances e_t, u_t, and v_t and the state variable m_{t-1}.

Using (10.1) and (10.2) to solve for r_t and substituting the result into (10.4) yields

$$i_t = \left(\frac{\alpha_0 - y^c}{\alpha_1}\right) - \frac{a}{\alpha_1}(p_t - \mathrm{E}_{t-1}p_t) + \frac{1}{\alpha_1}(u_t - e_t) + (\mathrm{E}_t p_{t+1} - p_t), \tag{10.12}$$

while substituting (10.6) into (10.3) and using the policy rule (10.9) yields

$$p_t = \mu_0 + m_{t-1} + (\mu + c)i_t - \mu i^T - y^c - a(p_t - \mathrm{E}_{t-1}p_t) - e_t - v_t. \tag{10.13}$$

This procedure gives two equations to solve for the equilibrium price level and the nominal interest rate, but they both involve expectational variables. These can be evaluated using the assumed solutions (10.10) and (10.11):

$$\mathrm{E}_{t-1}p_t = b_{10} + b_{11}m_{t-1}$$

and

$$\begin{aligned}\mathrm{E}_t p_{t+1} &= b_{10} + b_{11}m_t = b_{10} + b_{11}[\mu_0 + m_{t-1} + \mu(i_t - i^T)] \\ &= b_{10} + b_{11}\mu_0 + b_{11}m_{t-1} - b_{11}\mu i^T \\ &\quad + b_{11}\mu(b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t).\end{aligned}$$

These expressions for expectations can be substituted into (10.12) and (10.13) to yield

$$\begin{aligned}i_t = {} & \frac{\alpha_0 - y^c}{\alpha_1} - \frac{a}{\alpha_1}(b_{12}e_t + b_{13}u_t + b_{14}v_t) + \frac{1}{\alpha_1}(u_t - e_t) + b_{11}\mu_0 - b_{11}\mu i^T \\ & + b_{11}\mu(b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t) - b_{12}e_t - b_{13}u_t - b_{14}v_t\end{aligned}$$

and

$$p_t = \mu_0 + m_{t-1} - \mu i^T - y^c + (\mu + c)(b_{20} + b_{21}m_{t-1} + b_{22}e_t + b_{23}u_t + b_{24}v_t)$$
$$- a(b_{12}e_t + b_{13}u_t + b_{14}v_t) - e_t - v_t.$$

Equating these to the trial solutions (10.11) and (10.10) implies the following values for the unknown b_{ij}s:[7]

$$b_{10} = \mu_0 - \mu i^T - y^c + (\mu + c)b_{20}$$
$$b_{11} = 1$$
$$b_{12} = -\frac{\alpha_1(1-\mu) + (\mu + c)}{\varphi}$$
$$b_{13} = \frac{\mu + c}{\varphi}$$
$$b_{14} = -\frac{\alpha_1(1-\mu)}{\varphi}$$
$$b_{20} = \frac{\alpha_0 - y^c}{\alpha_1(1-\mu)} + \frac{\mu_0 - \mu i^T}{1-\mu}$$
$$b_{21} = 0$$
$$b_{22} = \frac{\alpha_1 - 1}{\varphi}$$
$$b_{23} = \frac{1+a}{\varphi}$$
$$b_{24} = \frac{\alpha_1 + a}{\varphi},$$

where $\varphi = \alpha_1(1-\mu)(1+a) + (\mu + c)(\alpha_1 + a)$.

Collecting these results, the equilibrium nominal interest rate is given by

$$i_t = b_{20} + \left(\frac{1}{\varphi}\right)[(\alpha_1 - 1)e_t + (1+a)u_t + (\alpha_1 + a)v_t]. \tag{10.14}$$

7. There is a second solution with $b_{11} = 1/\mu$ and $b_{21} = (1-\mu)/[\mu(\mu + c)]$. McCallum (1986) shows that the solution in the text is the minimal state-variable solution. See also McCallum (1983a).

The nominal rate will fluctuate around i^T if $b_{20} = i^T$, which occurs when $i^T = r + \mu_0$, where $r = (\alpha_0 - y^c)/\alpha_1$ is the average real rate of interest. As a result, the target nominal rate i^T and the average money-growth rate μ_0 cannot be chosen independently. Since $\lim_{\mu\to\infty}|\varphi| = \infty$ and $\lim_{\mu\to\infty} b_{20} = i^T$, (10.14) implies that the variance of the nominal rate around i^T goes to zero as μ becomes arbitrarily large. At the same time,

$$\lim_{\mu\to\infty} b_{10} = \mu_0 - y^c + \lim_{\mu\to\infty}[\mu(b_{20} - i^T) + cb_{20}]$$
$$= \mu_0 - y^c - \left(\frac{\alpha_0 - y^c}{\alpha_1} + \mu_0 - i^T\right) + ci^T$$
$$= \mu_0 + ci^T - y^c < \infty.$$

It follows that

$$\lim_{\mu\to\infty} p_t = \mu_0 - y^c + ci^T + m_{t-1} - \left(\frac{1}{a}\right)\left[e_t - \left(\frac{1}{1-\alpha_1}\right)u_t - \left(\frac{\alpha_1}{1-\alpha_1}\right)v_t\right],$$

which remains well defined.

One property of (10.9) is that the nominal money stock is $I(1)$. That is, m_t is nonstationary and integrated of order 1. This property of m causes the price level to be nonstationary also.[8] One implication is that the error variance of price-level forecasts increases with the forecast horizon.

As McCallum (1986) demonstrates, a different equilibrium describing the stochastic behavior of the nominal interest rate and the price level is obtained if the money supply process takes the trend stationary form

$$m_t = \mu' + \mu_0 t + \mu(i_t - i^T) \tag{10.15}$$

even though (10.15) and (10.9) both imply that the average growth rate of money will equal μ_0 (see problem 2). With the money supply process (10.15), the equilibrium price level is trend stationary, and the forecast error variance does not increase without limit as the forecast horizon increases.

It is not surprising that (10.9) and (10.15) lead to different solutions for the price level. Under (10.9), the nominal money supply is a nontrend stationary process; random target misses have permanent effects on the future level of the money supply and therefore on the future price level. In contrast, (10.15) implies that the nominal

8. In contrast, the nominal interest rate is stationary since both the real rate of interest and the inflation rate (and therefore expected inflation) are stationary.

money supply is trend stationary. Deviations of money from the deterministic growth path $\mu' + \mu_0 t$ are temporary, so the price level is also trend stationary.

This discussion leads to two conclusions. First, monetary policy can be implemented to reduce fluctuations in the nominal interest rate without leading to price-level indeterminacy. The Canzoneri, Henderson, and Rogoff (1983) and McCallum (1986) papers showed that by adjusting the money supply aggressively in response to interest-rate movements, a central bank can reduce the variance of the nominal rate around its target level while leaving the price level determinate. However, the level at which the nominal rate can be set is determined by the growth rate of the nominal money supply. The choice of i^T determines μ_0 (or, equivalently, the choice of μ_0 determines the feasible value of i^T). Targets for the nominal interest rate and rate of inflation cannot be independently determined.

Second, the underlying behavior of the nominal money supply is not uniquely determined by the assumption that the nominal rate is to be fixed at i^T; this target can be achieved with different money supply processes. And the different processes for m will lead to different behavior of the price level. A complete description of policy, even under a nominal interest-rate targeting policy, requires a specification of the underlying money supply process.

Models of interest rate targeting are relevant for understanding actual policy. Barro (1989) analyzes interest-rate targeting under the assumption that the target for the nominal rate follows a random walk.[9] Based on U.S. data, Barro finds that the model's predictions for the time-series behavior of nominal rates, the money supply, and inflation are consistent with his interest-rate targeting specification. Rudebusch (1995) provides an empirical model of changes in the federal funds rate target set by the Federal Reserve's FOMC and demonstrates how this target-setting behavior helps account for the behavior of longer-term interest rates. Taylor (1993a) has proposed a simple rule for the nominal interest rate that mimics actual Federal Reserve behavior (see section 5.4), and Clarida, Galí, and Gertler (2000) have estimated variants of the Taylor rule for several major central banks.

10.2.2 Interest-Rate Policies in General Equilibrium

The analysis in the previous section employed a model that was not derived directly from the assumption of optimizing behavior on the part of the agents in the economy. One disadvantage of such models is that there is no natural welfare measure

9. Barro takes the actual policy instrument to be the nominal money supply, with the feedback rule for m determined by the desire to minimize a loss function that depends on the variance of the nominal rate around its target and the variance of one-step-ahead price-level forecast errors.

that can be used to evaluate alternative policies. Assuming that the central bank is concerned with output and inflation variability is probably reasonable, but to derive conclusions about optimal policies, one would like to be able to evaluate the welfare of the representative agent under alternative policies.

Among the more recent papers employing general equilibrium, representative-agent models to study interest-rate policies are those of Carlstrom and Fuerst (1995, 1997) and Woodford (1999b). Carlstrom and Fuerst address welfare issues associated with interest-rate policies. They employ a cash-in-advance (CIA) framework in which consumption must be financed from nominal money balances. As we saw in chapter 3, a positive nominal interest rate represents a distorting tax on consumption, affecting the household's choice between cash goods (i.e., consumption) and credit goods (i.e., investment and leisure). Introducing one-period price stickiness into their model, Carlstrom and Fuerst (1997) conclude that a constant nominal interest rate eliminates the distortion on capital accumulation, an interest-rate peg Pareto dominates a fixed money rule, and for any interest-rate peg, there exists a money growth process that replicates the real equilibrium in the flexible-price version of their model. That is, an appropriate movement in the nominal money growth rate can undo the effects of the one-period price stickiness.

To illustrate the basic issues in a very simple manner, consider the following five equilibrium conditions for a basic CIA economy with a positive nominal interest rate:

$$\frac{u_{c,t}}{1+i_t} = \beta \mathrm{E}_t R_t \left(\frac{u_{c,t+1}}{1+i_{t+1}}\right)$$

$$\frac{u_{l,t}}{u_{c,t}} = \frac{MPL_t}{1+i_t}$$

$$R_t = 1 + \mathrm{E}_t(MPK_{t+1})$$

$$m_t = \frac{M_t}{P_t} = c_t$$

$$1 + i_{t+1} = \mathrm{E}_t\left(\frac{R_t P_{t+1}}{P_t}\right),$$

where $u_{c,t}$ is the marginal utility of consumption at time t, β is the subjective rate of time preference, R_t is 1 plus the real rate of return, $u_{l,t}$ is the marginal utility of leisure at time t, i_t is the nominal interest rate, MPL_t (MPK_t) is the marginal product of labor (capital), P_t is the price level, and m_t is the level of *real* money balances. The first of these five equations can be derived from a basic CIA model by recalling that $u_{c,t} = (1 + i_t)\lambda_t$, where λ_t is the time-t marginal value of wealth. (This assumes that assets markets open before goods markets; see chapter 3.) Since $\lambda_t = \beta E_t R_t \lambda_{t+1}$ (see 3.26), it follows that $u_{c,t}/(1+i_t) = \lambda_t = \beta E_t R_t u_{c,t+1}/(1+i_{t+1})$. The second equation equates the marginal rate of substitution between leisure and wealth to the marginal product of labor, again using the result that $\lambda_t = u_{c,t}/(1+i_t)$. The third equation is the definition of the real return on capital. The fourth equation is the binding CIA constraint that determines the demand for money as a function of the level of consumption. The final equation is simply the Fisher relationship linking nominal and real returns. The fourth and fifth equations of this system, as Woodford (1999c) emphasizes, are traditionally interpreted as determining the price level and the nominal interest rate for an exogenous nominal money supply process. The model could be completed by adding the production function and the economy-wide resource constraint.

Rebelo and Xie (1999) argue that this CIA economy will replicate the behavior of a nonmonetary real economy under any nominal interest-rate peg. To demonstrate the conditions under which their result holds, assume that the nominal interest rate is pegged at a value $\bar{\imath}$ for all t. Under an interest-rate peg, the first two equations of the basic CIA model become

$$\frac{u_{c,t}}{1+\bar{\imath}} = \beta E_t R_t \left(\frac{u_{c,t}}{1+\bar{\imath}}\right) \Rightarrow u_{c,t} = \beta E_t R_t u_{c,t}$$

and

$$\frac{u_{l,t}}{u_{c,t}} = \frac{MPL_t}{1+\bar{\imath}}.$$

The Euler condition is now identical to the form obtained in a real, nonmonetary economy, an economy not facing a CIA constraint.[10] The level at which the nominal interest rate is pegged only appears in the labor-market equilibrium condition. Thus, Rebelo and Xie conclude that if labor supply is inelastic, the equilibrium with an interest-rate peg is the same as the equilibrium in the corresponding nonmonetary real economy. Any equilibrium of the purely real economy can be achieved by a CIA model with a nominal interest-rate peg if labor supply is inelastic. If labor supply is elastic, however, the choice of $\bar{\imath}$ does have effects on the real equilibrium.

10. If output follows an exogenous process and all output is perishable, equilibrium requires that c_t equal output; the Euler condition then determines the real rate of return.

Under an interest-rate peg, the price-level process must satisfy

$$E_t\left(\frac{R_t P_t}{P_{t+1}}\right) = 1 + \bar{\imath},$$

while the nominal money supply must satisfy

$$M_t = P_t c_t.$$

These requirements do not, however, uniquely determine the nominal money-supply process. For example, suppose the utility of consumption is $\ln c_t$. Then $u_{c,t} = 1/c_t$, and the Euler condition under an interest-rate peg can be written as

$$\frac{1}{c_t} = \frac{P_t}{M_t} = \beta E_t R_t \left(\frac{P_{t+1}}{M_{t+1}}\right).$$

Rearranging this equation yields

$$1 = \beta E_t R_t \left(\frac{P_{t+1}}{P_t} \frac{M_t}{M_{t+1}}\right).$$

If this equation is linearized around the steady state, one obtains

$$r_t + E_t \pi_{t+1} - E_t \mu_{t+1} = \bar{\imath} - E_t \mu_{t+1} = 0,$$

where $E_t \mu_{t+1}$ is the expected growth rate of money. In this formulation, while real money balances are determined ($m_t = c_t$), there are many nominal money supply processes consistent with equilibrium, as long as they all generate the same expected rate of nominal money growth.

As we saw in section 10.2.1, the price level is indeterminate under such an interest-rate pegging policy. However, assuming that P_t is predetermined due to price-level stickiness still allows the money-demand equation and the Fisher equation to determine P_{t+1} and m_t (and so the implied nominal supply of money) without affecting the real equilibrium determined by the Euler condition. In that sense, Carlstrom and Fuerst (1997) conclude that there exists a path for the nominal money supply in the face of price stickiness that leads to the same real equilibrium under an interest-rate peg as would occur with a flexible price level.

Carlstrom and Fuerst (1995) provide some simulation evidence to suggest that nominal interest-rate pegs dominate constant money growth rate policies. While this suggests that a constant nominal interest-rate peg is desirable within the context of their model, Carlstrom and Fuerst do not explicitly derive the optimal policy.

Instead, their argument is based on quite different grounds than the traditional Poole (1970) argument for an interest-rate-oriented policy. In Poole's analysis, stabilizing the interest rate served to insulate the real economy from purely financial disturbances. In contrast, Carlstrom and Fuerst appeal to standard tax-smoothing arguments to speculate, based on intertemporal tax considerations, that an interest-rate peg might be optimal.

The tax-smoothing argument for an interest-rate peg is suggestive, but it is unlikely to be robust in the face of financial market disturbances. For example, in an analysis of optimal policy defined as money growth rate control, Ireland (1996) introduces a stochastic velocity shock by assuming that the CIA constraint applies to only a time-varying fraction v_t of all consumption. In this case, the CIA constraint takes the form $P_t v_t c_t \leq Q_t$, where Q_t is the nominal quantity out of which cash goods must be purchased. It is straightforward to show that the Euler condition must be modified in this case to become

$$\frac{u_c(c_t)}{1+v_t i_t} = \beta \mathrm{E}_t R_t \left(\frac{u_c(c_{t+1})}{1+v_{t+1} i_{t+1}}\right).$$

If $v_t \equiv 1$, the case considered by Carlstrom and Fuerst is obtained. If v_t is random, eliminating the intertemporal distortion requires that $v_t i_t$ be pegged and that the nominal interest rate vary over time to offset the stochastic fluctuations in v_t. The introduction of a stochastic velocity disturbance suggests that an interest-rate peg would not be optimal.

10.2.3 Liquidity Traps

The Euler condition and the Fisher equation from a standard money-in-the-utility function (MIU) model can be combined and written as $u_{c,t}/(1+i_t) = \beta \mathrm{E}_t P_t u_{c,t+1}/P_{t+1}$ or, in the absence of uncertainty,

$$\frac{P_t(1+i_t)}{P_{t+1}} = \frac{u_{c,t}}{\beta u_{c,t+1}} \equiv z_t. \tag{10.16}$$

Following Woodford (2000), an interest-rate policy can be written as

$$1 + i_t = \phi(P_t, z_t).$$

The function $\phi(P_t, z_t)$ specifies the setting for the policy instrument (the nominal rate i_t) as a function of the current price level and the variable z_t, which captures the real factors that determine the marginal utility of consumption. Woodford labels policies of this form *Wicksellian* policies. Under such a policy, equilibrium, if it exists, is a

sequence for the price level that satisfies (10.16). Using the policy rule in (10.16), this equilibrium condition becomes

$$P_t \phi(P_t, z_t) = P_{t+1} z_t. \tag{10.17}$$

Equilibrium conditions such as (10.17) were used in chapters 2 and 4 to illustrate how a monetary economy may have multiple equilibria. In those earlier chapters, the nominal quantity of money was assumed to be fixed, and then it was demonstrated that there existed multiple price paths consistent with equilibrium. Suppose P^* is the stationary solution to (10.17): $P^* \phi(P^*, z) = P^* z$, where, for simplicity, z_t is treated as constant. Initial price levels greater than P^* are consistent with a perfect-foresight equilibrium. Along price paths originating above P^*, hyperinflations occurred, with real money balances shrinking toward zero. These equilibrium paths involve rising inflation and an increasing nominal interest rate. Because the opportunity cost of holding money rises, the demand for real money balances falls, so that equilibrium between money demand and supply is maintained.

When the price path originates at a value less than P^*, the argument was made that such price paths, involving explosive deflations, could be ruled out as perfect-foresight equilibria. With real money balances going to infinity, the transversality condition for the representative agent's optimization problem would eventually be violated (see section 2.2.1).

Recently, Benhabib, Schmit-Grohé, and Uribe (2001a, 2001b, 2002) and Schmit-Grohé and Uribe (2001b) have argued that deflationary paths originating from initial price levels less than P^* cannot be ruled out. Their argument is based on the observation that the nominal rate of interest cannot fall below zero. Explosive deflations would eventually force the nominal interest rate to zero, but the nominal rate is then prevented from falling further. They argue that simple and seemingly reasonable monetary policy rules that follow the Taylor principle in changing the nominal interest rate more than one for one in response to changes in inflation may actually lead to macroeconomic instability that would force the economy into a liquidity trap—a situation of zero nominal interest rates.

To illustrate this possibility, rewrite the equilibrium condition (10.16) as

$$1 + i_t = \left(\frac{P_{t+1}}{P_t}\right) z,$$

where the real factors summarized by z that determine the real interest rate are taken to be constant. Taking logs of both sides, this equilibrium relationship can be approximated by

$$i_t = \pi_{t+1} + \log(z).$$

Now suppose the central bank follows an interest-rate rule in which it responds to inflation:

$$i_t = r^* + \pi^* + \delta(\pi_t - \pi^*), \tag{10.18}$$

where π^* is the central bank's target inflation rate and $r^* \equiv \log(z)$ is the equilibrium rate. This policy rule can be viewed as a simple form of the Taylor rule introduced in chapter 5. The Taylor principle calls for ensuring that the nominal interest rate responds more than one for one to changes in inflation: $\delta > 1$.

If these two equations are combined, the equilibrium process for the inflation rate becomes

$$\pi_{t+1} = \pi^* + \delta(\pi_t - \pi^*),$$

which is unstable for $\delta > 1$, that is, for policy rules following the Taylor principle. The dynamics of the model are illustrated in figure 10.1. A stationary equilibrium exists with inflation equal to π^*. However, for inflation rates that start out below the target rate π^*, π declines. If the rate of deflation is bounded below by the zero bound on nominal interest rates, however, the economy converges to a zero nominal rate liquidity trap. The resulting equilibrium at π^{**} is stable.

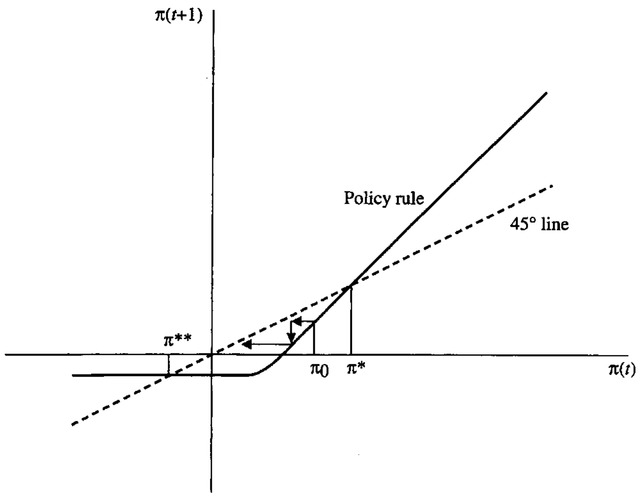

Figure 10.1
The Dynamics of a Liquidity Trap

This simple example has expected future inflation depend on current inflation through the assumed policy rule. Standard stability arguments in the presence of forward-looking jump variables rely on notions of saddle-path stability in which the inflation rate would jump to put the economy on a stable path converging to the unique, stationary steady state. In the present context, this would involve current inflation jumping immediately to equal π^*, the unique value consistent with a stationary equilibrium. That is, the only perfect-foresight, stationary equilibrium in a neighborhood of π^* is that associated with inflation equal to the target rate π^*. In contrast, in a neighborhood around the deflationary equilibrium π^{**}, there are many equilibrium paths consistent with a perfect-foresight equilibrium. Given this non-uniqueness or indeterminacy, sunspot equilibria are possible. If inflation starts out just to the left of π^*, the central bank cuts the nominal rate in an attempt to lower the real rate and stimulate the economy. But instead, this policy reaction simply generates expectations of lower inflation, causing actual inflation to decline further. Expressed in terms of the quantity of money, the lower nominal rate increases the demand for real money balances, forcing a fall in the price level and pushing the economy into a deflationary equilibrium.

Solutions How can the economy get out of a liquidity trap? First, it is worth noting that, in general, optimal monetary policy in the absence of nominal rigidities requires that the nominal interest rate equal zero. Rather than being a bad outcome, converging to a zero nominal interest rate is optimal, as it eliminates the wedge between the private and social opportunity costs of money. If, however, for reasons not specified in the simple model used here, the liquidity trap equilibrium is a bad outcome, then one must question the assumption that the policy maker follows an ad hoc, nonoptimal decision rule such as (10.18).

Suggestions for getting out of a liquidity trap have involved both fiscal and monetary policies. Suppose fiscal policy is non-Ricardian.[11] The government could promise to run huge deficits whenever the inflation rate gets too low (Benhabib, Schmit-Grohé, and Uribe 2002). According to the fiscal theory of the price level, this action, by increasing the government's total stock of nominal debt, would increase the equilibrium price level. This policy would rule out the low-inflation equilibrium by producing expectations of higher inflation whenever inflation becomes too low.

Ireland (2001b) departs from the standard representative-agent framework to show that a traditional *real balance effect* can eliminate liquidity traps.[12] In his model,

11. Under a non-Ricardian fiscal policy, the government's intertemporal budget constraint holds only at the equilibrium price level. See chapter 4.

12. See also McCallum (2000).

there are two overlapping generations. In the liquidity trap, nominal interest rates are zero, and the demand for real money balances is indeterminate. As a consequence, variations in the nominal stock of money may not affect the price level—there is a real indeterminacy (of real money balances). However in a steady state with a zero nominal interest rate, prices are falling, so the nominal stock of money must also decline to keep real balances constant. This requires taxing the young to reduce the money supply. With population growth, we have a world in which Ricardian equivalence does not hold. The future taxes necessary to reduce M will be paid, in part, by future generations, so the present discounted value of these taxes to the current generation is less than the value of their money holdings. In this environment, money is wealth, and aggregate demand depends on the real stock of money. This uniquely determines the level of real balances in equilibrium. But if M/P is uniquely determined, then varying M must always affect P, even in the liquidity trap.

If the central bank can conduct open market operations in an asset that is an imperfect substitute for money, monetary policy can still affect inflation, even in a liquidity trap. McCallum (2000) and Svensson (2001), for example, argue that a central bank can generate inflation by depreciating its currency, while Goodfriend (2000) considers the effects of open market operations in long-term bonds. By increasing the equilibrium price level, and thereby causing private agents to expect a positive rate of inflation, such policies can prevent nominal interest rates from falling to zero.

10.3 The Term Structure of Interest Rates

The distinction between real and nominal rates of interest is critical for understanding monetary policy issues, but another important distinction is that between short-term and long-term interest rates. Because aggregate spending decisions are generally viewed as more closely related to long-term interest rates, while the opportunity cost of holding money is best represented by short-term interest rates, the appropriate interest rates in the aggregate-demand relationship and the money-demand relationship are not the same. Changes in the short-term interest rate that serves as the operational target for implementing monetary policy will affect aggregate spending decisions only if longer-term rates of interest are affected. While the use of an interest-rate-oriented policy reduces the importance of money demand in the transmission of policy actions to the real economy, it raises to prominence the role played by the term structure of interest rates.

The exposition here builds on the expectations theory of the term structure. For a systematic discussion of the theory of the term structure, see Cox, Ingersoll, and

Ross (1985), Shiller (1990), or Campbell and Shiller (1991). Our objective is to illustrate how the term structure depends on the conduct of monetary policy. Under the expectations hypothesis of the term structure, long-term nominal interest rates depend on expectations of future nominal short-term interest rates. These future short-term rates will be functions of monetary policy, so expectations about future policy play an important role in determining the shape of the term structure.

10.3.1 The Expectations Theory of the Term Structure

Under the expectations theory of the term structure, the n-period interest rate equals an average of the current short-term rate and the future short-term rates expected to hold over the n-period horizon. For example, if $i_{n,t}$ is the nominal yield to maturity at time t on an n-period discount bond, while i_t is the one-period rate, the pure expectations hypothesis in the absence of uncertainty would imply that[13]

$$(1 + i_{n,t})^n = \prod_{i=0}^{n-1}(1 + i_{t+i}).$$

This condition ensures that the holding period yield on the n-period bond is equal to the yield from holding a sequence of one-period bonds. Taking logs of both sides and recalling that $\ln(1 + x) \approx x$ for small x yields a common approximation:

$$i_{n,t} = \frac{1}{n}\sum_{i=0}^{n-1} i_{t+i}.$$

Since an n-period bond becomes an $n - 1$ period bond after one period, these two relationships can also be written as

$$(1 + i_{n,t})^n = (1 + i_t)(1 + i_{n-1,t+1})^{n-1}$$

and

$$i_{n,t} = \left(\frac{1}{n}\right) i_t + \left(\frac{n-1}{n}\right) i_{n-1,t+1}.$$

These conditions will not hold exactly under conditions of uncertainty for two reasons. First, if risk-neutral investors equate expected one-period returns, then the one-period rate $1 + i_t$ will equal $E_t(1 + i_{n,t})^n/(1 + i_{n-1,t+1})^{n-1}$, which, from Jensen's

13. A constant risk premium could easily be incorporated. A time-varying risk premium will be added to the analysis below.

inequality, is not the same as $(1+i_{n,t})^n = (1+i_t)\mathrm{E}_t(1+i_{n-1,t+1})^{n-1}$.[14] Second, Jensen's inequality implies that $\ln \mathrm{E}_t(1+i_{n-1,t+1})$ is not the same as $\mathrm{E}_t \ln(1+i_{n-1,t+1})$. For illustrating the basic issues involving the term structure of interest rates and the role of monetary policy, however, we will ignore these issues, and it will be sufficient to simplify by dealing only with one- and two-period interest rates. Letting $I_t \equiv i_{2,t}$ be the two-period rate (the long-term interest rate), the term structure equation becomes

$$(1+I_t)^2 = (1+i_t)(1+\mathrm{E}_t i_{t+1}), \tag{10.19}$$

and this will be approximated as

$$I_t = \tfrac{1}{2}(i_t + \mathrm{E}_t i_{t+1}). \tag{10.20}$$

The critical implication of this relationship for monetary policy is that the current structure of interest rates will depend on current short rates *and* on market expectations of future short-term rates. Since the short rate is affected by monetary policy, I_t will depend on expectations about future policy.

The one-period ahead forward rate is defined as

$$f_t^1 = \frac{(1+I_t)^2}{1+i_t} - 1.$$

If the pure expectations hypothesis of the term structure holds, (10.19) implies that f_t^1 is equal to the market's expectation of the future one-period rate. Hence, forward rates derived from the term structure are often used to gain information on expectations of future interest rates (see Dahlquist and Svensson 1996; Rudebusch 2002b).

Equation (10.20) has a direct and testable empirical implication. Subtracting i_t from both sides, the equation can be rewritten as

$$I_t - i_t = \tfrac{1}{2}(\mathrm{E}_t i_{t+1} - i_t).$$

If the current two-period rate is greater than the one-period rate (i.e., $I_t - i_t > 0$), then agents must expect the one-period rate to rise ($\mathrm{E}_t i_{t+1} > i_t$). Because we can always write $i_{t+1} = \mathrm{E}_t i_{t+1} + (i_{t+1} - \mathrm{E}_t i_{t+1})$, it follows that

$$\begin{aligned}\tfrac{1}{2}(i_{t+1} - i_t) &= I_t - i_t + \tfrac{1}{2}(i_{t+1} - \mathrm{E}_t i_{t+1}) \\ &= a + b(I_t - i_t) + \theta_{t+1},\end{aligned} \tag{10.21}$$

14. Suppose $P_{n,t}$ is the time-t price of an n-period discount bond. Then $P_{n,t}^{-1} = (1+i_{n,t})^n$. Since at time t this becomes an $n-1$ period bond, the one-period gross return is

$$\mathrm{E}_t P_{n-1,t+1}/P_{n,t} = \mathrm{E}_t(1+i_{n,t})^n/(1+i_{n-1,t})^{n-1}.$$

where $a = 0$, $b = 1$, and $\theta_{t+1} = \frac{1}{2}(i_{t+1} - \mathrm{E}_t i_{t+1})$ is the error the private sector makes in forecasting the future short-term interest rate. Under the assumption of rational expectations, θ_{t+1} will be uncorrelated with information available at time t. In this case, (10.21) forms a regression equation that can be estimated consistently by least squares. Unfortunately, estimates of such equations usually reject the joint hypothesis that $a = 0$ and $b = 1$, generally obtaining point estimates of b significantly less than 1. Some of this empirical evidence is summarized in Rudebusch (1995, table 1, p. 249) and McCallum (1994b, table 1). As we will see, the observed relationship between long and short rates, as well as the way in which interest rates react to monetary policy, can depend on the manner in which policy is conducted.

10.3.2 Policy and the Term Structure

In this section, we use a simple model to illustrate how the behavior of nominal interest rates will depend on the money supply process. Consider the following model:

$$R_t = q_t \tag{10.22}$$

$$R_t = \tfrac{1}{2}[i_t - \mathrm{E}_t\pi_{t+1} + \mathrm{E}_t(i_{t+1} - \pi_{t+2})] \tag{10.23}$$

$$m_t - p_t = -a i_t + v_t \tag{10.24}$$

$$m_t = \gamma m_{t-1} + \varphi_t, \quad 0 < \gamma < 1, \tag{10.25}$$

where $\pi_t = p_t - p_{t-1}$. This simple model incorporates the assumption that output and the long-term (in this case, two-period) real interest rate are exogenous, with the gross long-term real rate R_t equal to a stochastic, mean-zero random variable q_t. R_t is equal to the average of the current real short-term rate $i_t - \mathrm{E}_t\pi_{t+1}$ and the expected future real short rate $\mathrm{E}_t(i_{t+1} - \pi_{t+2})$. The real demand for money is decreasing in the nominal short-term interest rate and is subject to a random shock v_t. Finally, the nominal money supply is assumed to follow a first order autoregressive process, subject to a control error φ_t. Note that this process implies that $\mathrm{E}_t m_{t+1} = \gamma m_t$.

By using (10.24) to eliminate i_t and $\mathrm{E}_t i_{t+1}$ from (10.23), this system of four equations implies that the equilibrium process for the price level must satisfy the following expectations difference equation:

$$2aq_t = (1+a)p_t + \mathrm{E}_t p_{t+1} - a\mathrm{E}_t p_{t+2} - (1+\gamma)m_t + v_t. \tag{10.26}$$

To find the solution for the short-term interest rate, we can employ the method of undetermined coefficients. Since the relevant state variables in (10.26) are m_t, q_t, and v_t, suppose we guess a solution of the form

$$p_t = b_1 m_t + b_2 q_t + b_3 v_t.$$

This implies that $E_t p_{t+1} = b_1 \gamma m_t$ and $E_t p_{t+2} = b_1 E_t m_{t+2} = b_1 \gamma^2 m_t$. Using these in (10.26), we find that the equilibrium solution for the price level is

$$p_t = \left[\frac{1}{1+a(1-\gamma)}\right] m_t + \left(\frac{1}{1+a}\right)(2aq_t - v_t). \tag{10.27}$$

From the money demand equation, $i_t = (v_t + p_t - m_t)/a$, so, given the equilibrium process for p_t,

$$i_t = -\left[\frac{1-\gamma}{1+a(1-\gamma)}\right] m_t + \left(\frac{1}{1+a}\right)(2q_t + v_t) \tag{10.28}$$

and the two-period nominal rate is

$$\begin{aligned} I_t &= \frac{1}{2}(i_t + E_t i_{t+1}) \\ &= \frac{1}{2}\left\{-\left[\frac{1-\gamma^2}{1+a(1-\gamma)}\right] m_t + \left(\frac{1}{1+a}\right)(2q_t + v_t)\right\}. \end{aligned} \tag{10.29}$$

Equation (10.29) illustrates how the long-term rate will depend on the money-supply process, where this process is characterized in this example by the parameter γ.

From (10.29), the impact of an innovation to m_t (i.e., a φ shock) on the current long rate is equal to

$$-\frac{1}{2}\left[\frac{1-\gamma^2}{1+a(1-\gamma)}\right] < 0,$$

which depends on the parameter γ. The greater the degree of serial correlation in the money-supply process (the larger is γ), the smaller the effect on i_t of a change in m_t. The effect of a money innovation on the slope of the term structure, $I_t - i_t$, is equal to $\frac{1}{2}(1-\gamma)^2/[1+a(1-\gamma)]$, and this also depends on γ. High persistence in the money-supply process produces a flatter term structure. To take the extreme case, suppose $\gamma = 1$; the nominal money supply follows a random walk with innovation φ_t. An innovation implies a permanent change in the *level* of the money supply. This causes a proportionate change in the price level (the coefficient on m_t in (10.27) is equal to 1 if $\gamma = 1$), but there is no impact on the expected rate of inflation. With the real rate exogenous, the nominal interest rate adjusts only in response to changes in expected inflation, so with $\gamma = 1$, changes in m have no effect on the nominal interest rate. If $\gamma < 1$, an unexpected increase in m causes the expectation of a subsequent decline in m and p. It is this expectation of a deflation that lowers the nominal rate of interest.

Similarly, the impact of real interest rate disturbances on nominal rates will depend on the money-supply process if policy responds to real disturbances. If, for example, the money-supply process is modified to become $m_t = m_{t-1} + \psi q_{t-1} + \varphi_t$ so that the growth rate of m_t depends on the real rate shock, it can be shown that the equilibrium short-term rate is

$$i_t = \left(\frac{2+\psi}{1+a}\right) q_t + \left(\frac{1}{1+a}\right) v_t.$$

If $\psi > 0$, an increase in the real rate ($q > 0$) induces an increase in the nominal money supply the following period. This increase implies that the money supply is expected to grow ($E_t m_{t+1} - m_t = \psi q_t > 0$), so expected inflation rises. This increases the positive impact of q_t on the short-term nominal rate.

These results are illustrative, showing how interest-rate responses depend on expectations of the future money supply and, consequently, on the systematic behavior of m. The exact mechanism highlighted in these examples requires that a monetary innovation (i.e., $\varphi > 0$) generate an expected deflation in order for nominal rates to decline, since the real rate has been treated as exogenous.

The dependence of interest rates and the term structure on monetary policy implies that the results of empirical studies of the term structure should depend on the operating procedures followed by the central bank. McCallum (1994b), Rudebusch (1995), Fuhrer (1996), and Balduzzi, Bertola, Foresi, and Klapper (1998) have examined the connection between the Fed's tendency to target interest rates, the dynamics of short-term interest rates, and empirical tests of the expectations model of the term structure.

This dependence can be seen most easily by employing a setup similar to that used by McCallum. Consider the following two-period model of nominal interest rates in which, as before, I is the two-period rate and i is the one-period rate:

$$I_t = \tfrac{1}{2}(i_t + E_t i_{t+1}) + \xi_t, \tag{10.30}$$

where ξ is a random variable that represents a time-varying term premium. Equation (10.20) implied that the pure expectations model of the term structure holds exactly, without error; the term premium ξ introduced in (10.30) allows for a stochastic deviation from the exact form of the expectations hypothesis. Variation in risk factors might account for the presence of ξ. Suppose further that the term premium is serially correlated:

$$\xi_t = \rho\xi_{t-1} + \eta_t, \tag{10.31}$$

where η_t is a white noise process.

If we let $\varepsilon_{t+1} = i_{t+1} - \mathrm{E}_t i_{t+1}$ be the expectational error in forecasting the future one-period rate, (10.30) implies that

$$\tfrac{1}{2}(i_{t+1} - i_t) = I_t - i_t - \xi_t + \tfrac{1}{2}\varepsilon_{t+1}, \tag{10.32}$$

which is usually interpreted to mean that the slope coefficient in a regression of one-half the change in the short rate on the spread between the long rate and the short rate should equal 1. We have previously noted that actual estimates of this slope coefficient have generally been much less than 1 and have even been negative.

The final aspect of the model is a description of the behavior of the central bank. Since many central banks use the short-term interest rate as their operational policy instrument, and since they often engage in interest-rate smoothing, McCallum assumes that $i_t = i_{t-1} + \mu(I_t - i_t) + \zeta_t$.[15] However, problems of multiple equilibria may arise when policy responds to forward-looking variables such as I_t (see Bernanke and Woodford 1997 and problem 6). To avoid this possibility, assume that policy adjusts the short-term rate according to

$$i_t = i_{t-1} - \mu\xi_t + \zeta_t, \tag{10.33}$$

where ζ is a white noise process and $|\mu| < 1$. According to (10.33), a rise in the risk premium in the long-term rate induces a policy response that lowers the short rate. Exogenous changes in risk that alter the term structure might also affect consumption or investment spending, leading the central bank to lower short-term interest rates to counter the contractionary effects of a positive realization of ξ_t. Because we have not introduced any real explanation for either ξ or why policy might respond to it, it is important to keep in mind that this is only an illustrative example that will serve to suggest how policy behavior might affect the term structure.

Equations (10.30)–(10.33) form a simple model that can be used to study how policy responses to the term structure risk premium (i.e., μ) affect the observed relationship between short-term and long-term interest rates. From (10.33), $\mathrm{E}_t i_{t+1} = i_t - \mu\rho\xi_t$, so

$$I_t = \frac{1}{2}(i_t + \mathrm{E}_t i_{t+1}) + \xi_t = i_t + \left(1 - \frac{\mu\rho}{2}\right)\xi_t.$$

15. McCallum actually allows the coefficient on i_{t-1} in (10.33) to differ from 1.

This implies that

$$\left(1 - \frac{\mu\rho}{2}\right)^{-1}(I_t - i_t) = \xi_t.$$

Using this result, (10.32) can be written as

$$\frac{1}{2}(i_{t+1} - i_t) = I_t - i_t - \left(1 - \frac{\mu\rho}{2}\right)^{-1}(I_t - i_t) + \frac{1}{2}\varepsilon_{t+1}$$

or

$$\frac{1}{2}(i_{t+1} - i_t) = -\left(\frac{\mu\rho}{2 - \mu\rho}\right)(I_t - i_t) + \frac{1}{2}\varepsilon_{t+1}, \tag{10.34}$$

so that we would expect the regression coefficient on $I_t - i_t$ to be $-\mu\rho/(2 - \mu\rho)$ and not 1. In other words, the estimated slope of the term structure, even when the expectations model is correct, will depend on the serial correlation properties of the term premium (ρ) and on the policy response to the spread between long and short rates (μ). The problem arises even though (10.34) implies that $\frac{1}{2}(i_{t+1} - i_t) = a + b(I_t - i_t) + x_{t+1}$, with $a = 0$ and $b = 1$, because the error term x_{t+1} is equal to $-\xi_t + \frac{1}{2}\varepsilon_{t+1}$; since this is correlated with $I_t - i_t$, ordinary least squares is an inconsistent estimator of b.

Rather than employing an equation such as (10.33) to represent policy behavior, Rudebusch (1995) uses data from periods of funds-rate targeting (1974–1979 and 1984–1992) to estimate a model of the Federal Reserve's target for the funds rate. He is then able to simulate the implied behavior of the term structure, using the expectations hypothesis to link funds-rate behavior to the behavior of longer-term interest rates. He finds that the manner in which the Fed has adjusted its target can account for the failure of the spread between long and short rates to have much predictive content for changes in long rates, at least at horizons of 3 to 12 months (that is, for the failure to obtain a coefficient of 1, or even a significant coefficient, in a regression of $\frac{1}{2}(i_{t+1} - i_t)$ on $(I_t - i_t)$). Thus, if the three-month rate exceeds the funds rate, (10.21) would appear to predict a rise in the funds rate. As Rudebusch demonstrates, the Fed tends to set its target for the funds rate at a level it expects to maintain. In this case, any spread between the funds rate and other rates has no implications for future changes in the funds rate (in terms of 10.33, $\mu \approx 0$). Only as new information becomes available might the target funds rate change.

Fuhrer (1996) provides further evidence on the relationship between the Fed's policy rule and the behavior of long-term interest rates. He estimates time-varying parameters of a policy reaction rule for the funds rate consistent with observed long-term rates. Agents are assumed to use the current parameter values of the policy rule to forecast future short rates.[16] Fuhrer argues that the parameters he obtains are consistent with general views on the evolution of the Fed's reaction function. Balduzzi, Bertola, Foresi, and Klapper (1998) find that during the 1989–1996 period of federal funds rate targeting in the United States, the term structure was consistent with a regime in which changes in the target for the funds rate occurred infrequently but were partially predictable. In a related literature, Mankiw and Miron (1986) and Mankiw, Miron, and Weil (1987) have studied how the founding of the Federal Reserve affected the seasonal behavior of interest rates (see also Fishe and Wohar 1990; Angelini 1994a, 1994b; and Mankiw, Miron, and Weil 1994).

10.3.3 Expected Inflation and the Term Structure

The term structure plays an important role as an indicator of inflationary expectations. Since market interest rates are the sum of an expected real return and an expected inflation premium, the nominal interest rate on an n-period bond can be expressed as

$$i_t^n = \frac{1}{n}\sum_{i=0}^{n} \mathrm{E}_t r_{t+i} + \frac{1}{n}\mathrm{E}_t\bar{\pi}_{t+n},$$

where $\mathrm{E}_t r_{t+i}$ is the one-period real rate expected at time t to prevail at $t+i$ and $\mathrm{E}_t\bar{\pi}_{t+n} \equiv \mathrm{E}_t p_{t+n} - p_t$ is the expected change in log price from t to $t+n$. If real rates are stationary around a constant value $\bar{r}$, then $\frac{1}{n}\sum_{i=0}^{n} \mathrm{E}_t r_{t+i} \approx \bar{r}$ and

$$i_t^n \approx \bar{r} + \frac{1}{n}\mathrm{E}_t\bar{\pi}_{t+n}.$$

In this case, fluctuations in the long rate will be caused mainly by variations in expected inflation. Based on a study of interest rates on nominal and indexed government bonds in the United Kingdom, Barr and Campbell (1997) conclude that "almost 80% of the movement in long-term nominal rates appears to be due to changes in expected long-term inflation." For this reason, increases in long-term nominal rates of interest are often interpreted as signaling an increase in expected inflation.

Figure 10.2 shows U.S. quarterly data on nominal interest rates of various maturity. The three rates shown are the federal funds rate (an overnight rate on bank

16. As Fuhrer notes, this behavior is not fully rational since agents presumably learn that the policy rule changes over time. However, the time-varying parameters approximately follow a random walk process, so using the current values to forecast future policy does not introduce large, systematic errors.

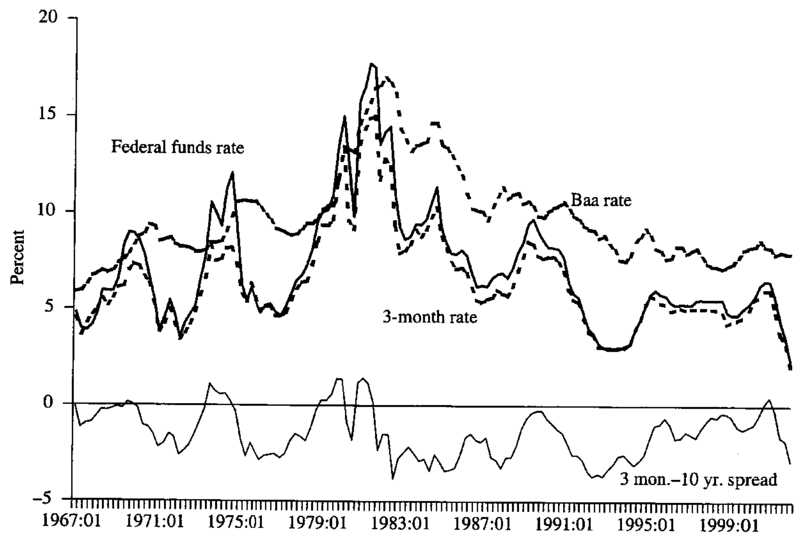

Figure 10.2
Short- and Long-Term Interest Rates: 1967–2001

reserves), the rate on three-month Treasury bills, and the interest rate on Baa-rated corporate bonds. The patterns exhibited by the rates are quite similar; that is why most theoretical models focus on a single representative interest rate. All nominal interest rates rose during the 1960s and 1970s as inflation increased, reflecting the relationship between expected inflation and nominal interest rates. All rates trended downward during the 1980s as inflation and expected inflation declined. Rates of different maturity do display different behavior over shorter time intervals. The early 1990s and 2001 provide examples of periods in which short rates declined much more than long rates.

Also shown in figure 10.2 is the spread between the 3-month Treasury bill rate and the 10-year government bond rate. This spread is normally negative, but its peaks have occurred at times typically associated with contractionary monetary policy; 1967, 1975, 1979–1980, 1990, and 2001 are periods of economic slowdowns or recessions. Several authors have found that interest-rate spreads have predictive value for forecasting future real output (Stock and Watson 1989; Bernanke and Blinder 1992; B. Friedman and Kuttner 1992). Friedman and Kuttner, for example, report that the spread between the commercial paper rate and the Treasury bill rate has predictive content for real output in the United States.

Short-run changes in long rates, particularly those following changes in monetary policy, are often interpreted as signaling the market's assessment of future inflation. Thus, a policy-induced rise in short rates that is accompanied by a decline in long rates would be interpreted as meaning that the contractionary policy (the rise in short rates) is expected to lower future inflation, thereby lowering nominal long-term interest rates and future short-term rates. Conversely, a cut in the short-run policy rate that is accompanied by a rise in long rates would provide evidence that the central bank was following an inflationary policy. Goodfriend (1993) provides an interpretation of U.S. monetary policy in the period 1979–1992 based on the notion that long-term interest rates provide important information on market inflation expectations.

Buttiglione, Del Giovane, and Tristani (1996) have examined the impact of policy rate changes on forward rates in OECD countries. Under the hypothesis that changes in monetary policy do not affect the expected real interest rate far in the future, changes in the forward rates implied by the term structure should reflect the impact of the policy change on expected future inflation. The forward interest rate on a one-period discount bond n periods in the future can be derived from the rates on n and $n+1$ period bonds and is equal to

$$f_t^n = \frac{(1+i_{n+1,t})^{n+1}}{(1+i_{n,t})^n} - 1 \approx (n+1)i_{n+1,t} - ni_{n,t}.$$

Thus, if long-term expected real rates are constant, then for large n, $f_t^n \approx \bar{r} + \mathrm{E}_t\bar{\pi}_{t+n+1} - \mathrm{E}_t\bar{\pi}_{t+n} = \bar{r} + \mathrm{E}_t(p_{t+n+1} - p_{t+n})$ or $f_t^n \approx \bar{r} + \mathrm{E}_t\pi_{t+n+1}$. The forward rate then provides a direct estimate of future expected rates of inflation.[17] Interestingly, Buttiglione, Del Giovane, and Tristani find that a contractionary shift in policy (a rise in the short-term policy interest rate) lowered forward rates for some countries and raised them for others. The response of forward rates was closely related to a country's average inflation rate; for low-inflation countries, a policy action that increased short-term rates was estimated to lower forward rates. This response is consistent with the hypothesis that the increases in the short rate represented a credible policy expected to reduce inflation. In countries with high-inflation experiences, increases in short rates were not associated with decreases in forward rates.

17. Söderlind and Svensson (1997) provide a survey of techniques for estimating market expectations from the term structure.

A key maintained hypothesis in the view that movements in interest rates reveal information about inflation expectations is that the Fisher hypothesis, the hypothesis that nominal interest rates will incorporate a premium for expected inflation, holds. Suppose that the real rate is stationary around an average value of $\bar{r}$. Then, since $i_t = r_t + \pi_{t+1}^e = r_t + \pi_{t+1} + e_{t+1}$, where e_{t+1} is the inflation forecast error (which is stationary under rational expectations), the ex post real rate $i_t - \pi_{t+1}$ is stationary. Thus, if the nominal interest rate and the inflation rate are nonstationary, they must be cointegrated under the Fisher hypothesis. This is the sense in which long-term movements in inflation should be reflected in the nominal interest rate. Mishkin (1992) has adopted this cointegrating interpretation of the Fisher relationship to test for the presence of a long-term relationship between inflation and nominal interest rates in the United States. If over a particular time period neither i nor π is integrated of order 1 but instead are both stationary, there is no real meaning to the statement that permanent shifts in the level of inflation should cause similar movements in nominal rates since such permanent shifts have not occurred. If either i or π is $I(1)$, they should both be $I(1)$, and they should be cointegrated. Mishkin finds the evidence to be consistent with the Fisher relationship.

10.4 Simple Models for Policy Analysis

While the basic aggregate-supply, aggregate-demand model typified by (10.1) to (10.4) has been widely used for policy analysis, it is too highly stylized and too heavily focused on the money supply as a policy tool to provide an adequate or completely satisfactory framework for policy analysis. Chapter 5 demonstrated how this simple framework could be derived as an approximation to a fully specified dynamic equilibrium model, but the impulse-response functions implied by that approximation failed to match the empirical evidence on the lags with which monetary-policy actions affect the real economy and inflation. And, by including only a single interest rate, the framework could not distinguish between the type of short-term rate commonly employed by central banks as a policy instrument and the longer-term rates relevant for consumption and investment decisions.

In recent years, there has been a revival of interest in empirically estimated models that can be used for monetary-policy analysis. Examples of both large- and small-scale models include Brayton, Levin, Tryon, and Williams (1997), Fuhrer (1997c), Ireland (1997), Reifschneider, Stockton, and Wilcox (1997), Rotemberg and Woodford (1997), and Rudebusch and Svensson (1999). In this section, we review a model based on Fuhrer and Moore (1995a, 1995b) and Fuhrer (1994a, 1997a) that incorporates the Fuhrer-Moore inflation specification (see section 5.3.3), a term structure relationship, and a policy rule specifying the behavior of the short-term nominal interest rate. This model structure helps to bridge the gap between the simple theoretical models used for gaining insights into policy questions and the large-scale empirical models commonly employed by central banks. This particular model is illustrative of others in the class; Taylor (1993b) has, for example, built upon similar foundations to develop open-economy and multicountry models for policy analysis. Other recent papers developing small models that could be used for policy analysis are Yun (1996); King and Wolman (1996); Fuhrer (1997c); McCallum and Nelson (1999); and Christiano, Eichenbaum, and Evans (2001). These latter models build more closely on the framework of the new Keynesian models discussed in section 5.4. Their use in policy analysis is discussed in chapter 11.

10.4.1 A Closed-Economy Model

The basic model consists of four equations. The first equation relates aggregate spending to lagged values of output and the long-term real interest rate; it corresponds to the aggregate-demand equation (10.2). The second equation is a term structure equation that links the long-term rate to current and expected future values of the short rate. The third equation is an inflation-adjustment equation in which current inflation depends on lagged and expected future inflation and output; this equation is based on the Fuhrer-Moore specification discussed in section 5.3.3. The final equation is a policy-reaction function that describes the evolution of the short-term interest rate, the latter assumed to be the instrument of monetary policy.

The two critical components are the inflation-adjustment equation and the policy-reaction function, so we can deal very briefly with the other two equations first. The aggregate spending relationship, corresponding to a traditional IS function, takes the form

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} - a_3 r_{t-1} + u_t, \qquad (10.35)$$

where y is the deviation of log output from its steady-state level, r is the real interest rate on long-term bonds, and u is an aggregate demand shock, taken to follow a first order autoregressive process:

$$u_t = \rho u_{t-1} + \varepsilon_t.$$

The parameters a_1 and a_2 will be important in governing the dynamic response of output to shocks. Monetary policy affects demand via r, so the assumption that time-t spending depends on r_{t-1} will (partially) account for a lagged response of output to policy changes.

If (10.35) is compared to the aggregate spending equation (5.67) that was derived in section 5.4.3 from a model of optimizing agents, three important differences are apparent. First, (10.35) incorporates lagged values of output and the real long-term interest rate. The presence of lagged variables will help the equation capture the dynamics in the data, but their presence is not motivated by any theoretical argument. Second, models of optimal consumption choice imply that current demand should depend on expected future income. Changes in current income that are expected to be permanent will have larger effects on spending than temporary changes will. Unless the lagged output terms are proxying for expected future output, this channel is absent from (10.35). Kerr and King (1996) discuss how the evaluation of interest-rate policy rules can be affected by the role of expected future output, while McCallum and Nelson (1999) conduct an empirical evaluation of alternative policy rules using a small model in which current aggregate demand depends on expectations of future output. Finally, (10.35) is based on the assumption that the long-term bond rate is the interest rate that is important for aggregate consumption and investment decisions. The models developed in chapter 5 contained only a single interest rate.

The long-term real rate is related, via the term structure, to the current short rate and expectations of future short rates. If expected real holding period yields are equalized on the long-term bond and the nominal federal funds rate i_t^f, then the two interest rates will be related by

$$r_t - D[\mathrm{E}_t r_{t+1} - r_t] = i_t^f - \mathrm{E}_t \pi_{t+1}, \qquad (10.36)$$

where D is Macaulay's duration.[18] If $r_t > i_t^f - \mathrm{E}_t\pi_{t+1}$, then expected real returns can be equal only if a capital loss is anticipated on the long-term bond. This implies that $\mathrm{E}_t r_{t+1} > r_t$; the long rate is expected to rise and the price of the long-term bond to fall. Conversely, expectations of a decline in the long rate, and a corresponding capital gain on long-term bonds, are consistent with the current long rate being less than the current expected real funds rate. Equation (10.36) can be solved forward to show that the current value of r_t depends on the current and expected future values of the real funds rate.

18. See Shiller (1990) for a discussion of duration. A long-term coupon bond involving payments at different dates in the future can be viewed as a sequence of discount bonds, one associated with each coupon payment of the original bond. Payments further into the future have smaller value today because of discounting. Macaulay's duration provides a measure of the effective term of the original bond by taking a weighted average of the terms of these discount bonds, with each term weighted by a discount factor. For discount bonds, duration is equal to the term of the bond. For coupon bonds, the duration is less than the term.

The most critical aspects of the model are those related to inflation adjustment and the setting of the policy instrument. Price and wage adjustment is based on the Fuhrer-Moore (1995a) model of multi-period, overlapping nominal contracts that lead inflation to depend on both past inflation and expected future inflation (see section 5.3.3). For our example, assume that contract prices x_t are set for two periods, with the aggregate price level p_t equal to a weighted average of contract prices set at time $t-1$ and t. Assuming that half of all contracts are negotiated each period,

$$p_t = \tfrac{1}{2}(x_t + x_{t-1}).$$

Define the real value of contracts negotiated at time t as $x_t - p_t \equiv z_t$. Define the index of average real contract prices negotiated in contracts still in effect at time t as

$$v_t = \tfrac{1}{2}[z_t + z_{t-1}]. \tag{10.37}$$

Fuhrer and Moore assume that in setting x_t, agents take two factors into account. First, they attempt to achieve a current real contract price equal to the expected average of the real contract index over the two-period life of the contract, $\frac{1}{2}(v_t + \mathrm{E}_t v_{t+1})$. Second, the contracted real price can deviate from this average expected index to reflect the current and expected state of the business cycle. This latter effect is taken to equal $(\gamma/2)(y_t + \mathrm{E}_t y_{t+1})$. Combining these assumptions with (10.37) yields

$$\begin{aligned} z_t &= \frac{1}{2}(v_t + \mathrm{E}_t v_{t+1}) + \frac{\gamma}{2}(y_t + \mathrm{E}_t y_{t+1}) \\ &= \frac{1}{4}(z_{t-1} + 2z_t + \mathrm{E}_t z_{t+1}) + \frac{\gamma}{2}(y_t + \mathrm{E}_t y_{t+1}) \\ &= \frac{1}{2}(z_{t-1} + \mathrm{E}_t z_{t+1}) + \gamma(y_t + \mathrm{E}_t y_{t+1}). \end{aligned} \tag{10.38}$$

Since $p_t = \frac{1}{2}(x_t + x_{t-1})$ and $x_t = z_t + p_t$, we can write the inflation rate as

$$\pi_t = p_t - p_{t-1} = z_t + z_{t-1}. \tag{10.39}$$

The inflation rate will depend on the real contract prices set at time t and $t-1$. The presence of z_{t-1} imparts a sluggishness to inflation adjustment; new information that becomes available at the start of period t about current or future monetary policy can be reflected in z_t but not, by definition, in z_{t-1}. This fact limits the flexibility of

current inflation to jump in response to new information. As shown in section 5.3.3, inflation can also be expressed as

$$\pi_t = \tfrac{1}{2}(\pi_{t-1} + \mathrm{E}_t \pi_{t+1}) + \gamma q_t + \eta_t, \tag{10.40}$$

where $\eta_t = -(\pi_t - \mathrm{E}_{t-1}\pi_t)$ and q_t depends on lagged, current, and expected future output.

The Fuhrer-Moore specification is closely related to, but distinct from, Taylor's original work on staggered, multi-period overlapping contracts and aggregate-price adjustment. In the specific example studied in section 5.3.1, Taylor's model of price-level adjustment led to a reduced-form expression for the price level in which p_t depends on p_{t-1} and $\mathrm{E}_t p_{t+1}$. The backward-looking aspect of price behavior causes unanticipated reductions in the money supply to cause real output declines. Only as contracts expire can their real value be reduced to levels consistent with the new, lower money supply. However, the inflation rate depends on $\mathrm{E}_t \pi_{t+1}$, not π_{t-1}, so the inflation process does not display stickiness. As Ball (1994a) has shown, price rigidities based on backward-looking behavior in the price-level process need not imply that policies to reduce inflation by reducing the growth rate of money will cause a recession. In the Fuhrer-Moore specification, the backward-looking nature of the *inflation* process implies that reductions in the growth rate of money will be costly in terms of output.

The final component of the Fuhrer-Moore model is a description of policy. The funds rate is taken to be the instrument of monetary policy, and the policy maker is assumed to respond to deviations of inflation from target, the output gap, and changes in output:

$$i_t^f = b_1 i_{t-1}^f + b_2(\pi_t - \pi^T) + b_3 y_t + b_4(y_t - y_{t-1}) + \varphi_t. \tag{10.41}$$

The parameters of the policy rule are all taken to be positive, with $b_1 \leq 1$. If inflation is above its target π^T, the funds rate is increased. Similarly, output above trend ($y_t > 0$) or an increase in output (even if the level is below trend) triggers funds-rate increases. In the terminology of Phillips (1957), the term $b_4(y_t - y_{t-1})$ represents a derivative response. The term φ_t is a mean-zero, serially uncorrelated random variable that represents a stochastic policy shock.[19]

Policy Shocks Recall from chapter 1 that monetary policy shocks, identified for the United States as shocks to the federal funds rate, produced a hump-shaped response in output that was spread over several quarters. The response of inflation was much

19. Fuhrer (1996) reports estimates of a policy rule for the Fed similar to (10.41). He finds that the parameters have evolved over time.

Table 10.1
Parameter Values (Baseline Case)

Coefficient	Value
a_1	1.53
a_2	−0.55
a_3	0.35
b_1	0.84
b_2	0.27
b_3	0.11
b_4	0.42
γ	0.002

more delayed. We can solve our modified Fuhrer-Moore policy model to see if it replicates these basic facts. Doing so requires that specific values be assigned to the parameters of the model. Fuhrer (1994a) reports full-information, maximum likelihood (FIML) estimates for a version of the model that incorporates a more complicated contracting specification than that of (10.38). Based on his estimates, table 10.1 shows the baseline parameter values to be used for the model.

Figure 10.3 shows the response of output, the long-term real rate, and inflation to a shock to the policy instrument. The output response is much more consistent with empirical estimates of the impact of policy shocks than those found in the equilibrium models of chapters 2 and 3 or in the simple sticky wage modification to an MIU model of chapter 5. The output response displays the hump-shaped pattern seen in estimated VARs for the United States and other industrial economies (Sims 1992, Taylor 1993b). The policy shock results in a rise in the short-term rate, which, because $b_1 = .84$, displays a great deal of persistence and returns only gradually to the baseline. The policy shock leaves the funds rate (not shown) above the baseline for about eight quarters, and the figure shows that the long-term rate rises but only about one-tenth as much as the funds rate. The long rate increase, with a one-period lag, causes real output to decline. Output reaches its trough almost two years after the contractionary policy shock. Inflation declines temporarily, but it does not respond as sharply as output.

The Role of the Policy Rule The simple Fuhrer-Moore model captures some of the basic stylized facts of a monetary shock for the United States. Of more interest than the impact of exogenous policy shocks, however, is the role the parameters of the policy rule play in affecting the behavior of the economy. Central banks can choose how to respond to inflation and output movements, and in the Fuhrer-Moore model

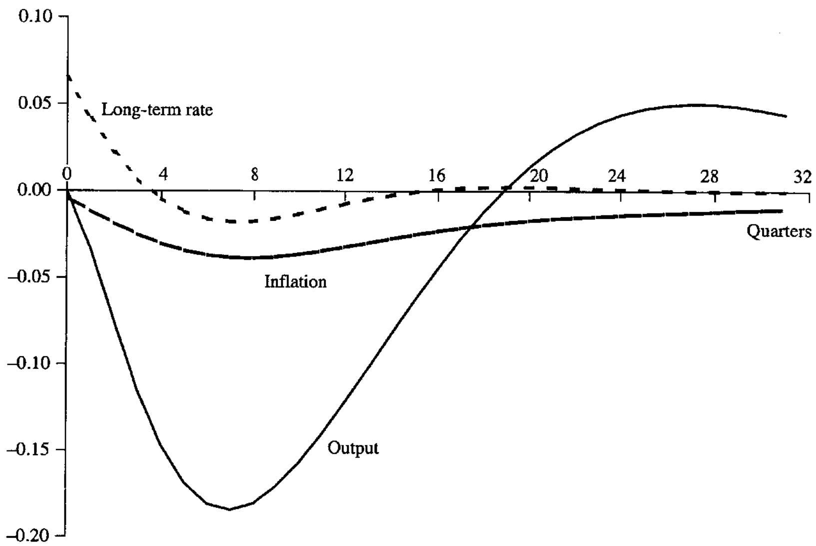

Figure 10.3
Response to a Policy Shock (Baseline Parameters)

these choices are captured by the parameters b_1 to b_4. The model can be used to examine how the response of long-term rates, output, and inflation to nonpolicy originating disturbances might vary with alternative values of these parameters.

Figures 10.4 and 10.5 show the responses of output and inflation to a demand shock (ε_t) for different values of the policy parameters b_2 and b_3. The outcomes for three alternatives are shown. As revealed by figure 10.3, the impact on output of an expansionary aggregate-demand shock depends importantly on the parameters of the monetary policy rule for the funds rate. The solid line is based on the parameter values in table 10.1. The alternatives represent stronger policy responses to inflation, output, or both. Increasing the reaction to deviations of output from trend from 0.11 to 1.0 significantly dampens the impact of a demand shock on output. When the funds rate is moved more aggressively, output is stabilized more effectively, and the standard deviation of output falls from 4.369 with the baseline parameters to 2.744.[20]

20. This is consistent with the argument of Ball (1999) that an increased response to output would provide better macroeconomic stabilization. See section 10.4.2.

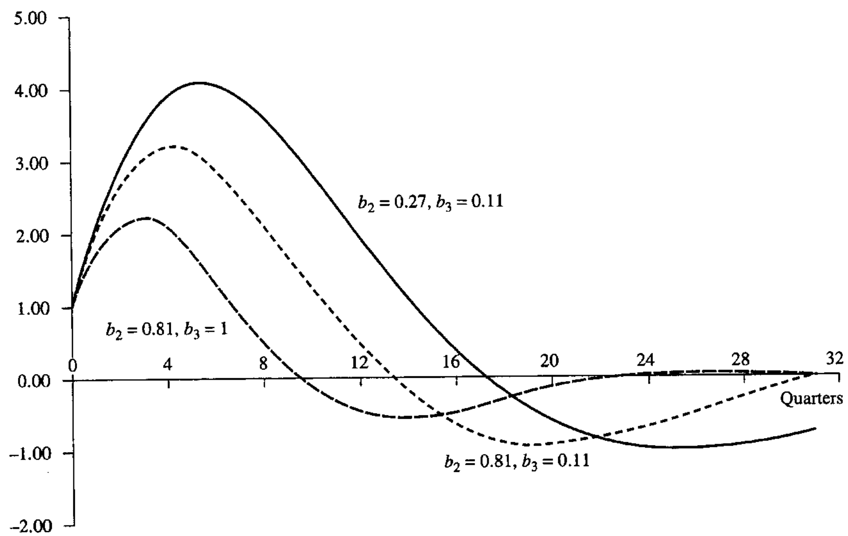

Figure 10.4
Output Response to a Demand Shock Under Alternative Policy Rules

For a given b_3, increases in b_2, the response to deviations of inflation from target, also act to stabilize output fluctuations, although the effects are small. Increasing b_2 by a factor of 3, from 0.27, the value estimated from U.S. data by Fuhrer (1994a), to 0.81 lowers the variance of output when $b_3 = 0.11$ but increases it (slightly) when $b_3 = 1$.

Figure 10.5 illustrates the effects of the policy-rule parameters on inflation. Aggregate-demand shocks lead to the greatest inflation responses for the baseline values. As with the output responses, increasing b_3 so that the funds rate is adjusted more strongly to output movements leads to significantly smaller inflation fluctuations. Notice that the persistence of the inflation response depends importantly on the policy rule.

The term structure behavior also is affected by the values of b_2 and b_3. A much more aggressive policy response to both inflation and output leads to greatly increased movements in the funds rate. The impact on long rates is much more muted because the stronger response of the funds rate when $b_2 = 0.81$ and $b_3 = 1$ is offset by the faster return to zero. Under the baseline values, movements in the funds rate are much more persistent.

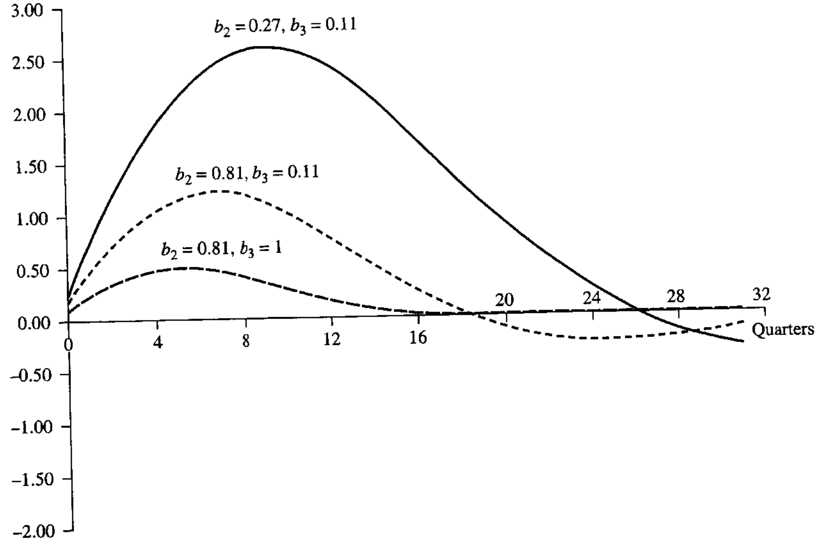

Figure 10.5
Inflation Response to a Demand Shock Under Alternative Policy Rules

These experiments illustrate how a small model can be used to analyze the impact of alternative monetary policy rules. The advantage of the model is the small number of parameters involved. These can be varied to gain insight into how changes in the systematic response of policy to economic conditions will alter the dynamic properties of output, inflation, and interest rates. As such, the model can serve as a laboratory for conducting policy experiments. Further examples of the use of small econometric models, as well as estimated VARs, for evaluating alternative policy rules include, in addition to the papers cited earlier, McCallum (1988) and Judd and Motley (1991, 1992). These last two papers simulate the impacts of alternative policy rules, focusing on the use of various interest-rate and monetary-base feedback rules.

The disadvantage of this approach is the absence of a direct link from the decisions of agents in the economy to the relationships characterizing aggregate economic behavior. As Lucas (1976) long ago emphasized, changes in the policy regime have the potential to alter the decision rules agents follow, leading to changes in the empirical regularities on which models such as the one studied in this section are based.

10.4.2 Optimal Policy

While the previous subsection demonstrated the role of alternative policy parameter values through simulations, the linear structure of the model means that one can obtain analytic solutions when the objective function is quadratic. To illustrate, this subsection further simplifies the model to obtain a particularly tractable version. The simplifications involve dropping the distinction between the short-term and long-term interest rates and ignoring the role of expected future inflation in the inflation-adjustment equations.[21] With these modifications, the model consists of the following two equations:

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} - a_3(i_{t-1} - \mathrm{E}_{t-1}\pi_t) + u_t \tag{10.42}$$

$$\pi_t = \pi_{t-1} + \gamma y_t + \eta_t. \tag{10.43}$$

The disturbances u_t and η_t are taken to be serially uncorrelated, with means equal to zero. Rudebusch and Svensson (1999) estimate a model similar to (10.42) and (10.43) and derive the optimal policy rule for their estimated model. They also use the model to evaluate a large number of instrument rules for setting the nominal rate of interest.

Policy actions (the setting of i_t) affect output and inflation with one-period lags in this formulation.[22] At time t, the choice of i_t affects y_{t+1} and π_{t+1}, but y_t and π_t are predetermined. Using the inflation equation, $\mathrm{E}_t\pi_{t+1} = \pi_t + \gamma \mathrm{E}_t y_{t+1}$, so period $t+1$ output will equal

$$\begin{aligned} y_{t+1} &= a_1 y_t + a_2 y_{t-1} - a_3(i_t - \pi_t - \gamma \mathrm{E}_t y_{t+1}) + u_{t+1} \\ &= \frac{a_1 y_t + a_2 y_{t-1} - a_3(i_t - \pi_t)}{1 - a_3\gamma} + u_{t+1}. \end{aligned}$$

With y_t, y_{t-1}, and π_t all predetermined when i_t is chosen, it will prove convenient to define

$$\theta_t \equiv \frac{a_1 y_t + a_2 y_{t-1} - a_3(i_t - \pi_t)}{1 - a_3\gamma} \tag{10.44}$$

21. Chapter 11 focuses directly on the policy implications arising from the forward-looking nature of inflation adustment, although Fuhrer (1997b) has argued, based on U.S. data, that forward-looking expectations may be unimportant empirically.

22. Svensson (1997b) has used a variant of this model in which the output variable in the inflation equation is lagged one period. This implies that policy affects output with a one-period lag and inflation with a two-period lag. The longer lag in the response of inflation is consistent with empirical evidence. McCallum (1997c) has shown, however, that the use of (10.43), in which lagged actual inflation appears with a coefficient equal to 1, can lead to problems of instability that would not arise if expectations of current or future inflation were included.

and treat this as the choice variable of the central bank. In terms of θ_t, then, the aggregate-demand and inflation equations become simply

$$y_{t+1} = \theta_t + u_{t+1} \tag{10.45}$$

and

$$\pi_{t+1} = \pi_t + \gamma\theta_t + v_{t+1}, \tag{10.46}$$

where $v_{t+1} = \gamma u_{t+1} + \eta_{t+1}$.

Suppose the objective of the policy maker is to pick θ_t at each point in time to minimize the expected loss given by[23]

$$L = \frac{1}{2}\mathrm{E}_t \sum_{i=1}^{\infty} \beta^i(\lambda y_{t+i}^2 + \pi_{t+i}^2). \tag{10.47}$$

The problem of minimizing expected loss subject to (10.45) and (10.46) is a simple problem in dynamic optimization. To solve it, and obtain the optimal decision rule of the central bank for setting θ_t (and therefore i_t), note that the only state variable at time t is π_t. We can therefore define the value function $V(\pi_t)$ as the expected presented value of the policy maker's loss function if θ_{t+i} is set optimally. The value function satisfies

$$V(\pi_t) = \min_{\theta_t} \mathrm{E}_t\left[\tfrac{1}{2}(\lambda y_{t+1}^2 + \pi_{t+1}^2) + \beta V(\pi_{t+1})\right],$$

where the minimization is subject to (10.45) and (10.46). Substituting these two constraints into the value function yields

$$V(\pi_t) = \min_{\theta_t} \mathrm{E}_t\left[\tfrac{1}{2}\lambda(\theta_t + u_{t+1})^2 + \tfrac{1}{2}(\pi_t + \gamma\theta_t + v_{t+1})^2 + \beta V(\pi_t + \gamma\theta_t + v_{t+1})\right].$$

The first order conditions are

$$(\lambda + \gamma^2)\theta_t + \gamma\pi_t + \gamma\beta \mathrm{E}_t V_\pi(\pi_{t+1}) = 0 \tag{10.48}$$

and, from the envelope theorem,

$$V_\pi(\pi_t) = \pi_t + \gamma\theta_t + \beta \mathrm{E}_t V_\pi(\pi_{t+1}).$$

Multiplying the second of these equations by γ and adding it to the first implies that

23. With y_t and π_t predetermined, the first term in the loss function is dated $t+1$.

$\gamma V_\pi(\pi_t) = -\lambda\theta_t$. Updating this one period and taking expectations, $\gamma\beta E_t V_\pi(\pi_{t+1})$ can be eliminated from (10.48), yielding

$$(\lambda + \gamma^2)\theta_t + \gamma\pi_t - \beta\lambda E_t\theta_{t+1} = 0.$$

Rearranging,

$$\theta_t = -\left(\frac{\gamma}{\lambda + \gamma^2}\right)\pi_t + \beta\left(\frac{\lambda}{\lambda + \gamma^2}\right)E_t\theta_{t+1}. \tag{10.49}$$

Given the linear-quadratic structure of this problem, the optimal decision rule will be of the form $\theta_t = B\pi_t$. This, in turn, implies that $E_t\theta_{t+1} = BE_t\pi_{t+1} = B(\pi_t + \gamma\theta_t)$. Substituting these expressions into (10.49), one obtains the following quadratic equation, whose solution yields the desired value of B:

$$\beta\lambda\gamma B^2 + (\beta\lambda - \lambda - \gamma^2)B - \gamma = 0.$$

Because $\pi_{t+1} = \pi_t + \gamma\theta_t + v_{t+1} = (1 + \gamma B)\pi_t + v_{t+1}$, stability of the inflation process requires $|1 + \gamma B| < 1$, so it is the negative solution for B that is relevant.

Recall that θ_t was defined in (10.44) as $[a_1 y_t + a_2 y_{t-1} - a_3(i_t - \pi_t)]/(1 - a_3\gamma)$, so the optimal rule for the actual policy instrument i_t is

$$i_t = \left(1 - \frac{B(1 - a_3\gamma)}{a_3}\right)\pi_t + \frac{a_1}{a_3}y_t + \frac{a_2}{a_3}y_{t-1},$$

which calls for the nominal interest rate to be adjusted on the basis of inflation and output. For the parameter values given in table 10.1 and $\beta = .989$, figure 10.6 shows the coefficient on inflation for this optimal policy rule as a function of λ, the relative weight on output fluctuations in the loss function. For the case of equal weight on output and inflation ($\lambda = 1$), the policy rule becomes

$$i_t = 1.50\pi_t + 4.37y_t - 1.57y_{t-1}.$$

Using the parameters proposed by Ball (1997) for a similar exercise, one obtains[24]

$$i_t = 1.48\pi_t + 0.8y_t.$$

24. Ball's model has only one lag of output in the aggregate spending equation. He views his model as appropriate for annual data, so the numbers reported are based on $\beta = .96$. His other parameter values are $a_1 = .8$, $a_3 = 1.0$, and $\gamma = 0.4$. Note that this implies a much stronger response of inflation to output (γ) and of spending to the interest rate (a_3). These changes affect mainly the coefficient on output. Ball also assumes that output enters with a lag in the inflation equation, so it should actually be y_{t-1} in the policy rule.

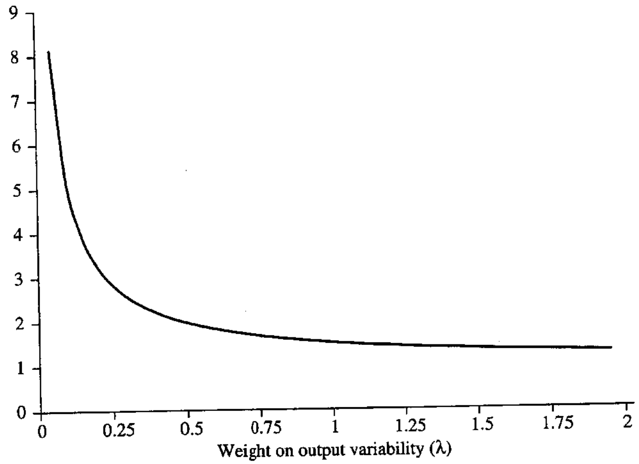

Figure 10.6
Inflation Coefficient in the Interest Rate Rule

Both of these rules for adjusting the nominal interest rate are similar to Taylor rules. Taylor (1993a) has shown that a rule of the form $i_t = 1.5\pi_t + 0.5y_t$ provides a good fit to the behavior of the federal funds rate in the United States. According to the Taylor rule, the nominal rate is increased more than one for one with any increase in inflation. This policy ensures a real rate response that will act to lower inflation. For a given inflation rate, the real rate is also increased in response to output expansions.

In the model of this section, policy can affect the volatility of output and inflation. An efficient policy rule is one that minimizes the variance of inflation for each variance of output around the natural rate. This then provides an efficiency frontier that shows the feasible set of output and inflation variances that can be achieved. Along this frontier, acting to stabilize inflation more requires that output become more volatile. The relative weight placed on output and inflation volatility, the parameter λ in the loss function equation (10.47), then determines where on this frontier the central bank should locate. Examples of recent works that try to map out the efficiency frontier and evaluate alternative policy rules include Fuhrer (1994a, 1997a) and Levin, Wieland, and Williams (1999) for the United States, de Brouwer and O'Regan (1997) for Australia, Amano, Coletti, and Macklem (1998) for Canada, and Batini and Haldane (1999) for the United Kingdom. Judd and Rudebusch (1997) show that the estimated parameters in a Taylor rule have varied with changes in the Fed chairmanship. See also section 11.5.3.

While the examples derived on the basis of the table 10.1 values from Fuhrer and Moore's and Ball's values have the basic form of a Taylor rule, they both imply relative more weight on output than characterizes the best fit to actual U.S. policy, a point emphasized by Ball (1999). Rudebusch (2002a) investigates a number of explanations including model and data uncertainty that might account for more "timid" responses than the optimal rules derived in simple models seem to suggest.

10.4.3 An Open-Economy Model

The basic Fuhrer-Moore model is a closed-economy model. Among the recent analyses of interest-rate rules using calibrated and estimated structural open-economy models are Batini and Haldane (1999) and Svensson (2000). Taylor (1989, 1993b) has worked extensively with multicountry versions of similar small-scale, rational expectations models designed to address policy issues.

To modify the Fuhrer-Moore model to incorporate open-economy considerations, the aggregate spending equation (10.35) needs to incorporate the effect of the real exchange rate on foreign and domestic demand. For a two-country model, the IS specifications are assumed to take the form

$$y_t = a_y y_{t-1} - a_r r_{t-1} + a_p \rho_{t-1} + a_y y_t^* + u_t \qquad (10.50)$$

$$y_t^* = a_y y_{t-1}^* - a_r r_{t-1}^* - a_p \rho_{t-1} + a_y y_t + u_t^*, \qquad (10.51)$$

where * denotes the foreign country, $\rho = s + p^* - p$ is the real exchange rate (where s is the nominal exchange rate), and we have simplified the dynamic structure by dropping the second lag of output. A rise in ρ representing a depreciation of the home currency shifts aggregate demand toward the home country ($a_p > 0$). Increases in income in either country increase aggregate demand in the other ($a_y > 0$).

Consumer prices in the home country are equal to $q_t = bp_t + (1-b)(s_t + p_t^*)$. Rewriting this as $q_t = p_t + (1-b)\rho_t$ and first differencing produces

$$\begin{aligned}\pi_t &= p_t - p_{t-1} + (1-b)(\rho_t - \rho_{t-1})\\ &= z_t + z_{t-1} + (1-b)(\rho_t - \rho_{t-1}),\end{aligned}$$

where domestic output price inflation is based on (10.39) and z_t is given by (10.38). A similar equation holds for foreign consumer price inflation:

$$\pi_t^* = z_t^* + z_{t-1}^* - (1-b)(\rho_t - \rho_{t-1}).$$

For each country, there is a term structure equation of the form given by (10.36), while policy, in the form of a rule for the short-term nominal interest rate, is represented by an equation of the form (10.41) for each country.

This framework is similar to that used for multi-country policy analysis by Taylor (1993b). The major differences (besides dealing only with two countries) are the term structure equation linking long-term and short-term interest rates and the use of the Fuhrer-Moore inflation-adjustment model rather than Taylor's original price-level adjustment model. To simulate the model, we use the same basic parameters used for the closed-economy exercise, with the exception that $a_y = 0.8$ and the new parameters introduced in the open-economy version are set at $a_r = .04$, $a_p = 0.1$, and $h = 0.8$.

Figure 10.7 illustrates the impact on domestic output and the real exchange rate of a domestic monetary-policy shock under the assumption that both home and foreign monetary policy is determined according to the policy rule (10.41). The rise in the short-term interest rate (the policy instrument) is followed by a decline in real output, with output following a typical hump-shaped pattern. The output decline is, from (10.50), induced by the rise in long-term rates and the real appreciation induced by the rise in interest rates. The real exchange rate does not overshoot; it falls below the

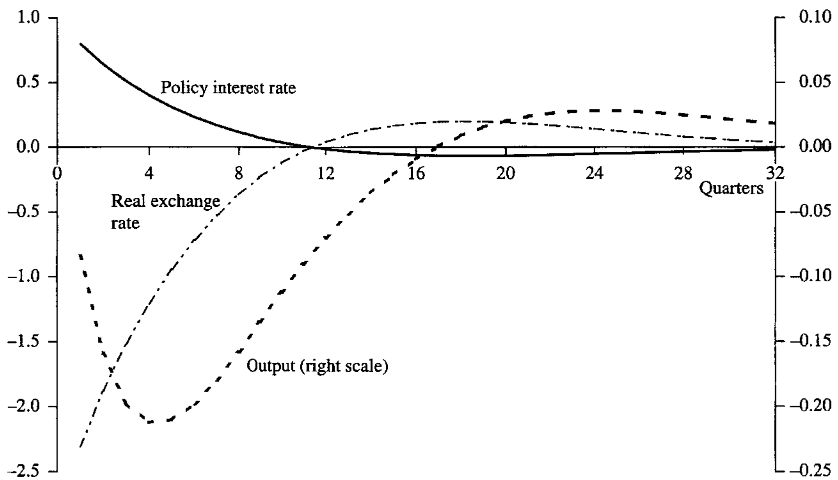

Figure 10.7
Response to a Home Policy Shock

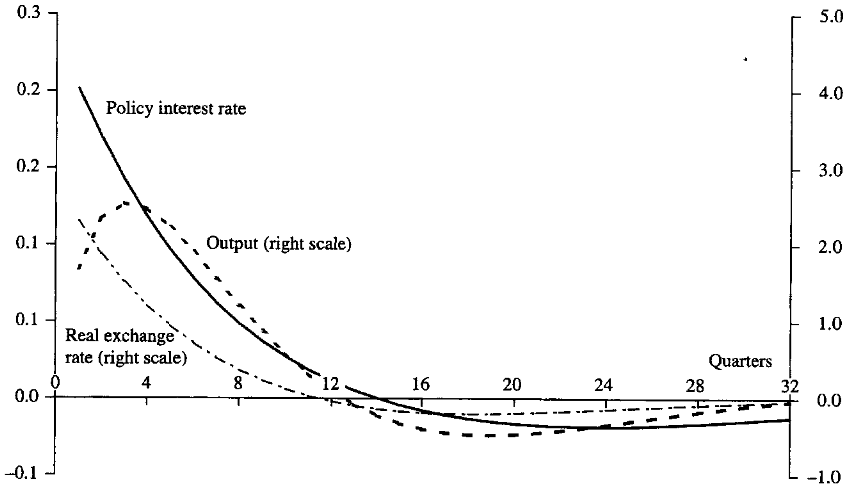

Figure 10.8
Response to a Foreign Policy Shock

baseline and then returns smoothly to its initial level. This behavior is consistent with the empirical evidence of Eichenbaum and Evans (1995) discussed in chapter 6.

Figure 10.8 illustrates the path followed by domestic real output, the real exchange rate, and the domestic monetary-policy instrument in response to a foreign monetary-policy shock. A contractionary foreign monetary policy induces a domestic output expansion. This occurs as a result of a domestic real depreciation that increased the demand for domestic output. The rise in the foreign short-term interest rate induces a rise in the domestic policy rate as well, and this dampens the domestic output expansion.

10.5 Summary

This chapter has examined a number of issues related to monetary policy. The general theme has been to focus on a short-term interest rate as the instrument of monetary policy and to move away from the traditional focus on monetary aggregates. With a short-term interest rate treated as the policy instrument, it is important to incorporate term-structure considerations into a policy model to provide a link

between the policy instrument and longer-term market interest rates that affect private-sector behavior.

The chapter concluded with simple examples of closed- and open-economy models that integrate simple aggregate supply and demand frameworks with dynamics that capture the types of persistent adjustments to monetary shocks that characterize actual U.S. behavior. Despite some obvious weaknesses, models of this type provide one convenient means of simulating the effects of alternative policy rules. They are useful in giving insight into the possible effects of alternative policy rules, providing a useful compromise between highly stylized theoretical models and large-scale econometric models. To the extent that the Lucas (1976) critique is empirically relevant, the parameters of these models may be altered by changes in policy behavior. Unlike the models built directly on foundations derived from the decision problems of the agents in the economy, the models of this section are ad hoc. But unlike those earlier models, they provide useful approximations to the actual data-generating processes. Monetary policy models with more clearly developed micro foundations are studied in chapter 11.

10.6 Problems

1. Suppose (10.1) is replaced by a Taylor sticky-price adjustment model of the type studied in chapter 5. Is the price level still indeterminate under the policy rule (10.5)? What if prices adjust according to the Fuhrer-Moore sticky inflation model?

2. Derive the values of the unknown coefficients in (10.10) and (10.11) if the money supply process is given by (10.15).

3. Suppose the money supply process in section 10.3.2 is replaced with

$$m_t = \rho m_{t-1} + \gamma q_{t-1} + \xi_t$$

so that the policy maker is assumed to respond with a lag to the real rate shock, with the parameter γ viewed as a policy choice. Thus, policy involves a choice of ρ and γ, with the parameter γ capturing the systematic response of policy to real interest-rate shocks. Show how the effect of q_t on the one- and two-period nominal interest rates depends on γ. Explain why the absolute value of the impact of q_t on the spread between the long and short rates increases with γ.

4. Suppose the money supply process in section 10.3.2 is replaced with

$$m_t = m_{t-1} + \xi_t - \gamma\xi_{t-1}.$$

Does i_t depend on γ? Does I_t? Explain.

5. Show that (10.36) implies that $r_t = \frac{1}{1+D}\sum_{i=0}^{\infty}\left(\frac{1}{1+D}\right)^i \mathrm{E}_t(i^f_{t+i} - \pi_{t+1+i})$.

6. (McCallum 1994a): Suppose the central bank adjusts the short-term rate i_t in respond to the slope of the term structure: $i_t = i_{t-1} + \lambda(I_t - i_t) + \zeta_t$, where ζ is a white noise process and $|\lambda| < 1$ and I_t is the two-period rate.

a. If the long-term rate is given by (10.30) and $\xi_t = \rho\xi_{t-1} + u_t$, show that the short-term rate must satisfy $(1+\lambda)i_t = i_{t-1} + \frac{\lambda}{2}(i_t + \mathrm{E}_t i_{t+1}) + \lambda\xi_t + \zeta_t$.

b. Now suppose the solution for i_t is of the form

$$i_t = \phi_0 + \phi_1 i_{t-1} + \phi_2\xi_t + \phi_3\zeta_t.$$

Assuming that private agents can observe the contemporaneous values of the two shocks ξ_t and ζ_t, show that

$$i_t = i_{t-1} + \frac{\lambda}{1 - \frac{\lambda\rho}{2}}\xi_t + \zeta_t$$

and

$$i_t = \frac{2}{\lambda}i_{t-1} + \frac{2}{1-\rho}\xi_t + \frac{2}{\lambda}\zeta_t$$

are both consistent with equilibrium but that the second of these solutions implies explosive behavior of the short rate.

7. Ball (1999) uses the following two-equation model:

$$y_{t+1} = a_1 y_t - a_2 r_t + u_{t+1}$$

$$\pi_{t+1} = \pi_t + \gamma y_t + \eta_{t+1}.$$

The disturbances u_t and η_t are taken to be serially uncorrelated. At time t, the policy maker chooses r_t, and the state variable at time t is $\pi_t + \gamma y_t \equiv \kappa_t$. Assume that the policy maker's loss function is given by (10.47). The optimal policy rule takes the form $x_t = A\kappa_t$, where $x_t \equiv a_1 y_t - a_2 r_t$. Derive the optimal value of A.

11 Policy Analysis in New Keynesian Models

11.1 The Basic New Keynesian Model

In recent years, the new Keynesian framework discussed in section 5.4 has become increasingly used for monetary policy analysis. Clarida, Galí, and Gertler (1999), Woodford (1999a, 2001a), McCallum and Nelson (1999), and Svensson and Woodford (1999, 2000), among others, have popularized this simple model for use in monetary policy analysis. Galí (2002) discusses some of the model's implications for monetary policy.

The new Keynesian model, in its basic form, consists of three components. First, the demand side is represented by a linear approximation to the representative household's Euler condition for optimal consumption (an *expectational IS curve*). Second, inflation adjustment is derived under the assumption of monopolistic competition, with individual firms adjusting prices in a staggered, overlapping fashion. Third, monetary policy is represented by a rule for setting the nominal rate of interest. This policy rule is either specified exogenously or derived from a specification of the central bank's objective function.

The first two components of the basic new Keynesian model take the form

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t\pi_{t+1}) + u_t \tag{11.1}$$

and

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \kappa x_t + e_t, \tag{11.2}$$

where x is the output gap, defined as output relative to the equilibrium level of output under flexible prices, i is the nominal rate of interest, and π is the inflation rate. All variables are expressed as percentage deviations around their steady-state values. The demand disturbance u can arise from taste shocks to the preferences of the representative household, fluctuations in the flexible-price equilibrium output level, or shocks to government purchases of goods and services. The e shock is variously called an *inflation shock*, a *cost shock*, or a *price shock*. We will refer to it as a *cost shock*. See section 5.4 for a discussion of how (11.1) and (11.2) can be derived as approximations to the equilibrium conditions of a well-specified general equilibrium model.

Both (11.1) and (11.2) contain expectational variables, and in both cases, these expectations are of future variables. The presence of expected future output in (11.1) implies that the future path of the one-period real interest rate matters for current demand. To see that this is the case, let $r_t \equiv i_t - \mathrm{E}_t\pi_{t+1}$ be the one-period real interest rate and then recursively solve (11.1) forward to yield

$$x_t = -\left(\frac{1}{\sigma}\right)\sum_{i=0}^{\infty}\mathrm{E}_t r_{t+i} + \sum_{i=0}^{\infty}\mathrm{E}_t u_{t+i}.$$

Changes in the one-period rate that are persistent influence expectations of future interest rates. Persistent changes therefore have stronger effects on x_t than more temporary changes in real interest rates. For example, suppose r_t follows an exogenous stochastic process given by $r_t = \gamma r_{t-1} + \xi_t$, where ξ_t is white noise. Then $\mathrm{E}_t r_{t+i} = \gamma^i r_t$, and the output gap is equal to

$$x_t = -\left[\frac{1}{\sigma(1-\gamma)}\right]r_t + \sum_{i=0}^{\infty}\mathrm{E}_t u_{t+i}.$$

If innovations to the real rate are very persistent (γ close to 1), $1/[\sigma(1-\gamma)]$ will be large and the innovations ξ_t have a large impact on x_t. If real rate innovations have only transitory effects on the real rate (γ close to 0), the realizations of ξ_t have a much smaller impact on output.

The inflation-adjustment equation also involves the expectation of a future variable, in this case future inflation. The Calvo model, combined with optimizing firms in an environment of monopolistic competition, implies that inflation depends on real marginal cost. When opportunities to adjust prices arrive infrequently, a firm must take into consideration both current real marginal cost and expected future real marginal cost when setting its price. It was shown in chapter 5 that real marginal cost could be expressed in terms of the output gap variable x_t, as is done in (11.2). Solving this equation forward,

$$\pi_t = \kappa\sum_{i=0}^{\infty}\beta^i\mathrm{E}_t x_{t+i} + \sum_{i=0}^{\infty}\beta^i\mathrm{E}_t e_{t+i}.$$

Inflation depends on the discounted value of current and expected future output gaps and cost shocks.

11.2 Policy Objectives

Given the specification of the economic environment that leads to (11.1) and (11.2), what are the appropriate objectives of the central bank? In chapter 8, policy analysis was conducted under the assumption that the central bank was concerned with minimizing a quadratic loss function that depended on output and inflation. Such an assumption is plausible but ultimately ad hoc. In the new Keynesian model, the description of the economy is based on an approximation to a fully specified general equilibrium model. Can we similarly develop a policy objective function that can be interpreted as an approximation to the utility of the representative household? Put differently, can we draw insights from the general equilibrium foundations of (11.1) and (11.2) to determine the basic objectives central banks should pursue? Woodford (2001a) has provided the most detailed analysis of the link between a welfare criterion derived as a log-linear approximation to the utility of the representative agent and the types of quadratic loss functions common in the literature.[1]

Woodford assumes that there is a continuum of differentiated goods $y_t(i)$ defined on the interval $[0,1]$ and that the representative household derives utility from consuming a composite of these individual goods. The composite consumption good is defined as

$$Y_t = C_t = \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}}. \tag{11.3}$$

In addition, each household produces one of these individual goods and experiences disutility from production. Suppose labor effort is proportional to output. The period utility of the representative agent is then assumed to be

$$V_t = U(Y_t, z_t) - \int_0^1 v(y_t(i), z_t)\,di, \tag{11.4}$$

where $v(y_t(i), z_t)$ is the disutility of producing good $y_t(i)$ and z_t is a vector of exogenous shocks. Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i V_{t+i} \approx -\Omega\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i[\pi_{t+i}^2 + \lambda(x_{t+i} - x^*)^2], \tag{11.5}$$

where the detailed derivation of (11.5) and the values of Ω and λ are given in the appendix to this chapter. In (11.5), x_t is the gap between output and the output level that would arise under flexible prices, and x^* is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the actual steady-state level of output.

1. See also Rotemberg and Woodford (1997).

Equation (11.5) looks a lot like the standard quadratic loss function used in chapter 8. There are, however, two critical differences. First, the output gap is measured relative to equilibrium output under flexible prices. In the traditional Barro-Gordon literature surveyed in chapter 8, the output variable was more commonly interpreted as output relative to trend or output relative to the natural rate of output. The natural rate of output varies with productivity shocks, but it is not the same as the flexible-price equilibrium level of output used to define the gap variable x_t. For example, in section 5.3.1, a Lucas-type aggregate-supply equation of the form $y_t = [a/(1-a)](p_t - \mathrm{E}_{t-1}p_t) + [1/(1-a)]\varepsilon_t$ was derived for the case of serially uncorrelated disturbances, when the production function (in log terms) was $y_t = an_t + \varepsilon_t$.[2] The standard interpretation of the natural rate is the output level in the absence of price surprises, or $y_t^n \equiv [1/(1-a)]\varepsilon_t$. This depends only on the productivity disturbance and the parameter α characterizing the production function. In contrast, the flexible-price equilibrium output, y_t^f, depends on the utility function of the households supplying labor. If the utility of the representative household is $C_t^{1-\sigma}/(1-\sigma) - N_t^{1+\eta}/(1+\eta)$, then the flexible-price output level satisfies (in log deviation terms) the labor-market equilibrium condition equating the marginal rate of substitution between leisure and consumption and the marginal product of labor, $\eta n_t + \sigma c_t = y_t - n_t$, the standard resource constraint, $c_t = y_t$, and the production function. Solving for y_t^f,

$$y_t^f = \left[\frac{1+\eta}{1+\eta+a(\sigma-1)}\right]\varepsilon_t \neq y_t^n.$$

The flexible-price output level reflects the change in equilibrium employment in the face of a productivity shock when both nominal wages and prices adjust. The natural rate is typically based on a model with nominal wage rigidity. In the face of a productivity shock, the equilibrium without price surprises is one in which neither nominal wages nor prices adjust. Employment increases in the face of a positive productivity shock and constant real wages. Output rises by $[1/(1-a)]\varepsilon_t > \varepsilon_t$ because of the rise in employment and the direct impact of the productivity shock. Worker preferences do not play a role in the definition of y_t^n, but they do play a role in the definition of y_t^f.

A second difference between (11.5) and the quadratic loss function of chapter 8 is the reason inflation variability enters the loss function. When prices are sticky,

2. In order to be consistent with the notation used later in this chapter, these expressions differ slightly from those appearing in chapter 5. The connection between the two sets of notations is that a here corresponds to $1-\alpha$ in chapter 5.

inflation results in an inefficient dispersion of output among the individual producers. The representative household's utility depends on its consumption of a composite good; faced with a dispersion of prices for the differentiated goods produced in the economy, the household buys more of the relatively cheaper goods and less of the relatively more expensive goods. Because of diminishing marginal utility, the increase in utility derived from consuming more of some goods is less than the loss in utility due to consuming less of the more expensive goods. Hence, price dispersion reduces utility. Similarly, dispersion on the production side is costly. The increased cost of producing more of some goods is greater than the cost saving from reducing production of other goods. For these reasons, price dispersion reduces utility, and, when each firm does not adjust its price every period, price dispersion is caused by inflation.

The efficiency distortion that leads to x^* was used in chapter 8 to motivate the presence of an overly ambitious output target in the central bank's objective function.[3] As a consequence, the presence of $x^* > 0$ implies that a central bank acting under discretion to maximize (11.5) would produce an average inflation bias. However, with average rates of inflation in the major industrialized economies remaining low during the 1990s, many authors now simply assume that $x^* = 0$. In this case, the central bank is concerned with stabilizing the output gap x_t, and no average inflation bias arises. If tax subsidies can be used to offset the distortions associated with monopolistic competition, then one could assign fiscal policy the task of ensuring that $x^* = 0$. In this case, the central bank has no incentive to create inflationary expansions, and average inflation will be zero under discretion. Dixit and Lambertini (2002) show that when both the monetary and fiscal authorities are acting optimally, the fiscal authority will use its tax instruments to set $x^* = 0$ and the central bank then ensures that inflation remains equal to zero. The source of the Barro-Gordon inflation bias is eliminated by fiscal policy.

The basic new Keynesian inflation-adjustment equation, derived in chapter 5, took the form

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

That is, no additional disturbance term, such as the e_t that was added to (11.2), appears. The absence of e implies that there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero. If $x_{t+i} = 0$ for all $i \geq 0$, then $\pi_{t+i} = 0$. In this case, if $x^* = 0$, perhaps because of fiscal subsidies designed to offset the distortion from monopolistic competition, a central bank that wants to maximize the expected utility of the representative

3. That is, $k > 0$ in the notation of chapter 8.

household will ensure that output is kept equal to the flexible-price equilibrium level of output. This also guarantees that inflation is equal to zero. Because firms adjust prices in a staggered manner, inflation generates a costly dispersion of prices; the central bank can eliminate this source of distortion by ensuring price stability. When firms do not need to adjust their prices, the fact that prices are sticky is no longer relevant. Thus, a key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.[4]

The optimality of zero inflation conflicts with the Friedman rule for optimal inflation. M. Friedman (1969) concluded that the optimal inflation rate must be negative to make the nominal rate of interest zero (see chapter 4). The reason we reach a different conclusion now is due to the absence of any explicit role for money when the utility approximation given by (11.5) is derived. In general, zero inflation still generates a monetary distortion. With zero inflation, the nominal rate of interest will be positive and the private opportunity cost of holding money will exceed the social cost of producing it. Khan, King, and Wolman (2000) and Adao, Corriea, and Teles (2001) consider models that integrate nominal rigidities and the Friedman distortion. Khan, King, and Wolman introduce money into a sticky-price model by assuming the presence of cash and credit goods, with money required to purchase cash goods. If prices are flexible, it is optimal to have a rate of deflation such that the nominal interest rate is zero. If prices are sticky, price stability would be optimal in the absence of the cash-in-advance (CIA) constraint. With both sticky prices and the monetary inefficiency associated with positive nominal interest rates, the optimal rate of inflation is less than zero but greater than the rate that yields a zero nominal interest rate. Khan, King, and Wolman conduct simulations in a calibrated version of their model and find that the relative price distortion dominates the Friedman monetary inefficiency. Thus, the optimal policy is close to the policy that maintains price stability.

In section 5.5, sticky wages as well as sticky prices were introduced. Inflation continued to depend on expected future inflation and real marginal cost, but with sticky wages, real marginal cost can no longer be measured by the gap between the household's marginal rate of substitution between leisure and consumption and the marginal product of labor; when wages are no longer flexible, the real wage can differ from the marginal rate of substitution. Erceg, Henderson, and Levin (2000) derive an

4. Notice that the conclusion that price stability is optimal is independent of the degree of nominal rigidity ω (see Adao, Correia, and Teles 1999). Steinsson (2000) shows that in the Galí and Gertler (1999) hybrid inflation model, in which lagged inflation appears in the inflation-adjustment equation, the loss function also includes a term in the squared change in inflation.

approximation to the utility of the representative household when both prices and wages are sticky. They show that wage stability is desirable because it eliminates dispersion of hours worked across households. Price stability is desirable because it eliminates price dispersion across goods. If real productivity shocks alter the real wage associated with the flexible-price equilibrium, monetary policy cannot simultaneously maintain $x_t = 0$ (output equal to the flexible-price level), price stability, and wage stability. When prices are sticky but wages are flexible, the nominal wage can adjust to ensure that labor-market equilibrium is maintained in the face of productivity shocks. Optimal policy should then aim to keep the price level stable. With sticky wages but flexible prices, any necessary adjustment of real wages can be achieved through changes in prices. Optimal policy should aim to keep nominal wages stable. With both sticky prices and sticky wages, trade-offs must be faced.

When wages are sticky, they adjust to the gap between the real wage and the marginal rate of substitution between leisure and consumption. When prices are sticky, they adjust to the gap between the marginal product of labor and the real wage. Galí, Gertler, and López-Salido (2002) define the *inefficiency gap* as the sum of these two gaps, the gap between the household's marginal rate of substitution between leisure and consumption (mrs_t) and the marginal product of labor (mpl_t). This inefficiency gap can be divided into its two parts, the wedge between the real wage and the marginal rate of substitution, which they label the *wage markup*, and the wedge between the real wage and the marginal product of labor, labeled the *price markup*. Based on U.S. data, they conclude that the wage markup accounts for most of the time-series variation in the inefficiency gap. This is consistent with the evidence of Christiano, Eichenbaum, and Evans (2001) and Sbordone (2001), suggesting that nominal wage rigidity is more important empirically than price rigidity.[5]

11.3 Optimal Commitment and Discretion

Suppose the central bank attempts to minimize a quadratic loss function such as (11.5), defined in terms of inflation and output relative to the flexible-price equilibrium.[6] Assume also that the steady-state gap between output and its efficient value is zero (i.e., $x^* = 0$). In this case, the central bank's loss function takes the form

5. Goodfriend and King (2001) argue that the long-term nature of employment relationships reduces the effects of nominal wage rigidity on real resource allocations.

6. Svensson (1999b, 1999c) argues that there is widespread agreement among policy makers and academics that inflation stability and output gap stability are the appropriate objectives of monetary policy.

$$L_t = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2). \tag{11.6}$$

This loss function is similar to the ones employed in chapter 8, with the exception that the output component does not include an overly ambitious output target of the sort that produces an average inflation bias in the Barro and Gordon (1983a) model. In the models used in chapter 8, the absence of an overly ambitious output target implied that commitment and discretionary policies would lead to the same outcomes. When forward-looking expectations play a role, as in (11.2), however, discretion leads to what is known as a *stabilization bias.* To illustrate this bias, we first derive the optimal policy under commitment.

11.3.1 Commitment

A central bank able to precommit chooses a path for current and future inflation and the output gap to minimize the loss function (11.6) subject to the expectational IS curve (11.1) and the inflation-adjustment equation (11.2). Let θ_{t+i} and ψ_{t+i} denote the Lagrangian multipliers associated with the period $t+i$ IS curve and the inflation-adjustment equation. The central bank's objective is to pick i_{t+i}, π_{t+i} and x_{t+i} to minimize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \{(\pi_{t+i}^2 + \lambda x_{t+i}^2) + \theta_{t+i}[x_{t+i} - x_{t+i+1} + \sigma^{-1}(i_{t+i} - \pi_{t+i+1}) - u_{t+i}]$$

$$+ \psi_{t+i}(\pi_{t+i} - \beta\pi_{t+i+1} - \kappa x_{t+i} - e_{t+i})\}.$$

The first order conditions for i_{t+i} take the form

$$\sigma^{-1}\mathrm{E}_t(\theta_{t+i}) = 0 \quad i \geq 0.$$

Hence, $\mathrm{E}_t\theta_{t+i} = 0$ for all $i \geq 0$. This finding reflects the fact that (11.1) imposes no real constraint on the central bank as long as there are no restrictions on, or costs associated with, varying the nominal interest rate. Given the central bank's optimal choices for the output gap and inflation, (11.1) simply determines the setting for i_t necessary to achieve the desired value of x_t. For that reason, it is often more convenient to treat x_t as if it were the central bank's policy instrument.

Setting $\mathrm{E}_t\theta_{t+i} = 0$, the remaining first order conditions can be written as

$$\pi_t + \psi_t = 0 \tag{11.7}$$

$$\mathrm{E}_t(\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0 \quad i \geq 1 \tag{11.8}$$

$$\mathrm{E}_t(\lambda x_{t+i} - \kappa\psi_{t+i}) = 0 \quad i \geq 0. \tag{11.9}$$

Equations (11.7) and (11.8) reveal the dynamic inconsistency that characterizes the optimal precommitment policy. At time t, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = -(\psi_{t+1} - \psi_t)$. But when period $t+1$ arrives, a central bank that reoptimizes will again obtain $\pi_{t+1} = -\psi_{t+1}$ as its optimal setting for inflation. That is, the first order condition (11.7) updated to $t+1$ will reappear.

An alternative definition of an optimal precommitment policy requires that the central bank implement conditions (11.8) and (11.9) for all periods, including the current period. Woodford (1999a) has labeled this the *timeless perspective* approach to precommitment. One can think of such a policy as having been chosen in the distant past, and the current values of the inflation rate and output gap are the values chosen from that earlier perspective to satisfy the two conditions (11.8) and (11.9). McCallum and Nelson (2000a) provide further discussion of the timeless perspective and argue that this approach agrees with the one commonly used in many studies of precommitment policies.

Combining (11.8) and (11.9), under the timeless perspective optimal commitment policy, inflation and the output gap satisfy

$$\pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1}) \tag{11.10}$$

for all $i \geq 0$. Using this equation to eliminate inflation from (11.2) and rearranging, one obtains

$$\left(1 + \beta + \frac{\kappa^2}{\lambda}\right)x_t = \beta\mathrm{E}_t x_{t+1} + x_{t-1} - \frac{\kappa}{\lambda}e_t. \tag{11.11}$$

The solution to this expectational difference equation for x_t will be of the form $x_t = a_x x_{t-1} + b_x e_t$. To determine the coefficients a_x and b_x, note that if $e_t = \rho e_{t-1} + \varepsilon_t$, the proposed solution implies $\mathrm{E}_t x_{t+1} = a_x x_t + b_x \rho e_t = a_x^2 x_{t-1} + (a_x + \rho) b_x e_t$. Substituting this into (11.11) and equating coefficients, the parameter a_x is the solution less than 1 of the quadratic equation

$$\beta a_x^2 - \left(1 + \beta + \frac{\kappa^2}{\lambda}\right)a_x + 1 = 0$$

and b_x is given by

$$b_x = -\left\{\frac{\kappa}{\lambda[1 + \beta(1 - \rho - a_x)] + \kappa^2}\right\}.$$

From (11.10), equilibrium inflation under the timeless perspective policy is

$$\pi_t = \left(\frac{\lambda}{\kappa}\right)(1 - a_x)x_{t-1} + \left[\frac{\lambda}{\lambda[1+\beta(1-\rho-a_x)]+\kappa^2}\right]e_t. \tag{11.12}$$

Woodford (1999a) has stressed that, even if $\rho = 0$, so that there is no natural source of persistence in the model itself, $a_x > 0$ and the precommitment policy introduces inertia into the output gap and inflation processes. Because the central bank responds to the lagged output gap (see 11.10), past movements in the gap continue to affect current inflation. This commitment to inertia implies that the central bank's actions at date t allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap. Equation (11.2) implies that the inflation impact of a positive cost shock, for example, can be stabilized at a lower output cost if the central bank can induce a fall in expected future inflation. Such a fall in expected inflation is achieved when the central bank follows (11.10).

Dennis (2001a) argues that the timeless precommitment policy is not unique. He notes that along the timeless perspective path, inflation satisfies $\pi_{t+i} = -(\psi_{t+i} - \psi_{t+i-1})$, implying that for time t, $\pi_t = -(\psi_t - \psi_{t-1})$. The Lagrangian ψ_{t-1} reflects the value the central bank places on past commitments. The fully optimal commitment policy ignores past commitments in the first period, setting $\psi_{t-1} = 0$ (see 11.7). Alternative values for ψ_{t-1} lead to alternative policy choices, all of which would be consistent with the timeless perspective. Because the timeless perspective commitment policy is not the solution to the policy problem under optimal commitment, the policy rule given by (11.10) may be dominated by other policy rules.

11.3.2 Discretion

When the central bank operates with discretion, it acts each period to minimize the loss function (11.6) subject to the inflation-adjustment equation (11.2). Because the decisions of the central bank at date t do not bind it at any future dates, the central bank is unable to affect the private sector's expectations about future inflation. Thus, the decision problem of the central bank becomes the single-period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to the inflation-adjustment equation (11.2).

The first order condition for this problem is

$$\kappa\pi_t + \lambda x_t = 0. \tag{11.13}$$

Notice that by combining (11.7) with (11.9) evaluated at time t, one obtains (11.13); thus, the central bank's first order condition relating inflation and the output gap at time t is the same under discretion or under the fully optimal precommitment policy (but not under the timeless perspective policy). The differences appear in subsequent periods. For $t+1$, under discretion $\kappa\pi_{t+1} + \lambda x_{t+1} = 0$, while under precommitment (from 11.8 and 11.9), $\kappa\pi_{t+1} + \lambda(x_{t+1} - x_t) = 0$.

The equilibrium expressions for inflation and the output gap under discretion can be obtained by using (11.13) to eliminate inflation from the inflation-adjustment equation. This yields

$$\left(1 + \frac{\kappa^2}{\lambda}\right)x_t = \beta \mathrm{E}_t x_{t+1} - \left(\frac{\kappa}{\lambda}\right)e_t. \tag{11.14}$$

Guessing a solution of the form $x_t = \delta e_t$, so that $\mathrm{E}_t x_{t+1} = \delta\rho e_t$, one obtains

$$\delta = -\left[\frac{\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right].$$

Equation (11.13) implies that equilibrium inflation under optimal discretion is

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)x_t = \left[\frac{\lambda}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t. \tag{11.15}$$

According to (11.15) the unconditional expected value of inflation is zero; there is no average inflation bias under discretion. However, there is a stabilization bias in that the response of inflation to a cost shock under discretion differs from the response under commitment. This can be seen by comparing (11.15) to (11.12).

11.3.3 Discretion Versus Commitment

The impact of a cost shock on inflation and the output gap under the timeless perspective optimal precommitment policy can be obtained by calibrating (11.2) and (11.10) and solving them numerically. Three unknown parameters appear in the model: β, κ, and λ. The discount factor, β, is set equal to 0.99, appropriate for interpreting the time interval as one quarter. A weight on output fluctuations of $\lambda = 0.25$ is used. This value is also used by Jensen (2002) and McCallum and Nelson (2000a). The parameter κ captures both the impact of a change in real marginal cost on inflation and the comovement of real marginal cost and the output gap. McCallum and Nelson (2000a) report that the empirical evidence is consistent with a value of κ in the range [0.01, 0.05]. Roberts (1995) reports higher values; his estimate of the coefficient on the output gap is about 0.3 when inflation is measured at an annual rate, so his estimate translates into a value for κ of 0.075. Whereas Jensen (2002) uses a baseline value of $\kappa = 0.1$, Walsh (2002b) uses 0.05. For the simulations, I use $\kappa = 0.05$. Finally, to focus on the inertia introduced by policy alone, ρ is set equal to zero.

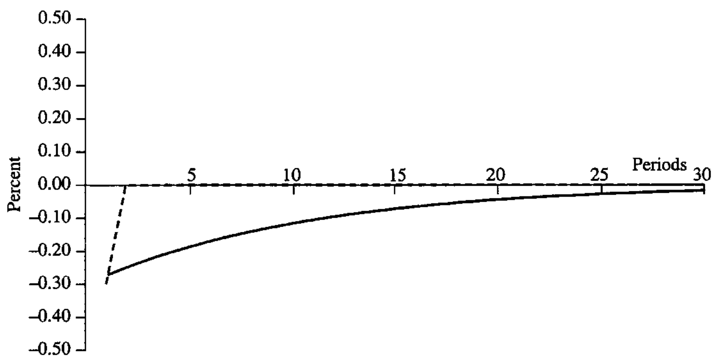

Figure 11.1
Output Gap Response to a Cost Shock: Timeless Precommitment and Pure Discretion (precommitment, solid line; discretion, dashed line)

The solid line in figure 11.1 (and figure 11.2) shows the response of the output gap (inflation) to a transitory, one standard deviation cost push shock under the optimal precommitment policy. Despite the fact that the shock itself has no persistence, the output gap displays strong positive serial correlation. By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in $E_t\pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.

Outcomes under optimal discretion are shown by the dashed lines in figures 11.1 and 11.2. There is no inertia under discretion; both the output gap and inflation return to their steady-state values in the period after the shock occurs. The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion.

The analysis so far has focused on the goal variables, inflation, and the output gap. Using (11.1), the associated setting for the interest rate can be derived. For example, under optimal discretion, the output gap is given by

$$x_t = -\left[\frac{\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t,$$

while inflation is given by (11.15). Using these to evaluate $E_t x_{t+1}$ and $E_t\pi_{t+1}$ and then solving for i_t from (11.1) yields

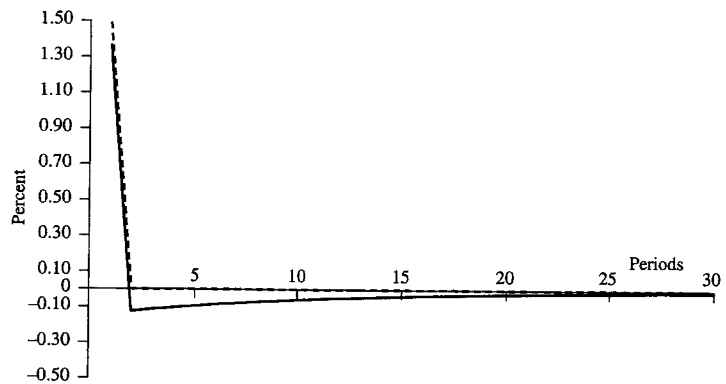

Figure 11.2
Response of Inflation to a Cost Shock: Timeless Precommitment and Pure Discretion (precommitment, solid line; discretion, dashed line)

$$\begin{aligned} i_t &= E_t\pi_{t+1} + \sigma(E_t x_{t+1} - x_t + u_t) \\ &= \left[\frac{\lambda\rho + (1-\rho)\sigma\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t + \sigma u_t. \end{aligned} \tag{11.16}$$

Equation (11.16) is the reduced-form solution for the nominal rate of interest. The nominal interest rate is adjusted to offset completely the impact of the demand disturbance u_t on the output gap. As a result, it affects neither inflation nor the output gap.

Chapter 5 illustrated how a policy that commits to a rule that calls for responding to the exogenous shocks renders the new Keynesian model's equilibrium indeterminate. Thus, it is important to recognize that (11.16) describes the *equilibrium* behavior of the nominal interest rate under optimal discretion; (11.16) is not an instrument rule (see Svensson and Woodford 1999).

11.3.4 Commitment to a Rule

In the basic Barro-Gordon model examined in chapter 8, the optimal commitment policy was a (linear) function of the state variables. In the present model consisting of (11.1) and (11.2), the only state variable is the current realization of the cost shock e_t. Suppose that the central bank can commit to a rule of the form[7]

7. This commitment does not raise the same uniqueness of equilibrium problem that would arise under a commitment to an instrument rule of the form $i_t = b_i e_t$. See problem 2.

$$x_t = b_x e_t. \tag{11.17}$$

What is the optimal value of b_x? With x_t given by (11.17), inflation satisfies

$$\pi_t = \beta E_t \pi_{t+1} + \kappa b_x e_t + e_t,$$

and the solution to this expectational difference equation is[8]

$$\pi_t = b_\pi e_t, \quad b_\pi = \frac{1 + \kappa b_x}{1 - \beta\rho}. \tag{11.18}$$

Using (11.17) and (11.18), the loss function can now be written as

$$E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) = \sum_{i=0}^{\infty} \beta^i \left[\left(\frac{1 + \kappa b_x}{1 - \beta\rho} \right)^2 + \lambda b_x^2 \right] e_t^2.$$

This is minimized when

$$b_x = -\left[\frac{\kappa}{\lambda(1 - \beta\rho)^2 + \kappa^2} \right].$$

Using this solution for b_x in (11.18), equilibrium inflation is given by

$$\pi_t = \left(\frac{1 + \kappa b_x}{1 - \beta\rho} \right) e_t = \left[\frac{\lambda(1 - \beta\rho)}{\lambda(1 - \beta\rho)^2 + \kappa^2} \right] e_t. \tag{11.19}$$

Comparing the solution for inflation under optimal discretion, given by (11.15), and the solution under commitment to a simple rule, given by (11.19), note that they are identical if the cost shock is serial uncorrelated ($\rho = 0$). If $0 < \rho < 1$, there is a stabilization bias under discretion relative to the case of committing to a simple rule.

Clarida, Galí, and Gertler (1999) have argued that this stabilization bias provides a rationale for appointing a Rogoff-conservative central banker when $\rho > 0$, even though in the present context there is no average inflation bias.[9] A Rogoff-conservative central banker places a weight $\hat{\lambda} < \lambda$ on gap fluctuations (see section 8.3.2). In a discretionary environment with such a central banker, (11.15) implies that inflation will equal

$$\pi_t = \left[\frac{\hat{\lambda}}{\hat{\lambda}(1 - \beta\rho) + \kappa^2} \right] e_t.$$

Comparing this with (11.19) reveals that if a central banker is appointed for whom $\hat{\lambda} = \lambda(1 - \beta\rho) < \lambda$, the discretionary solution will coincide with the outcome under commitment to the optimal simple rule. Such a central banker stabilizes inflation more under discretion than would be the case if the relative weight placed on output gap and inflation stability were equal to the weight in the social loss function, λ. Because the public knows inflation will respond less to a cost shock, future expected inflation rises less in the face of a positive e_t shock. As a consequence, current inflation can be stabilized with a smaller fall in the output gap. The inflation–output trade-off is improved.

Recall, however, that the notion of commitment used here is actually suboptimal. As we saw earlier, fully optimal commitment leads to inertial behavior in that future inflation depends not on the output gap but on the change in the gap.

11.3.5 Policy Trade-offs Under Discretion

When the cost shock is serially uncorrelated, neither inflation nor output displays any serial correlation under the optimal discretionary policy. Actual inflation and the output gap are given by the solution to (11.2) and (11.13), with expected future inflation equal to zero. A simple graphical presentation of the basic model aids in illustrating how the central bank's preferences affect the equilibrium variability of output and inflation. In figure 11.3, (11.2) is shown as a positive relationship between the output gap and inflation; the inflation-adjustment equation is drawn for a zero cost shock (the solid line) and a positive realization of the cost shock (the dashed line). The central bank's first order condition under discretion, (11.13), is negatively sloped, with slope equal to $-(\hat{\lambda}/\kappa)$, where $\hat{\lambda}$ is the weight on output variability in the central bank's loss function. Policy relationships are shown for two alternative values of $\hat{\lambda}$. The equilibrium output gap and the inflation rate must satisfy both relationships and so are given by the intersection of the two curves.

A positive cost shock leads to a rise in inflation and a fall in the output gap. The extent to which the shock affects output versus inflation depends on the weight $\hat{\lambda}$. A central bank that places a large weight on output variability (the solid policy line) allows inflation to increase more and output to fall less in response to the cost shock than would a central bank with a smaller $\hat{\lambda}$ (the dashed policy line).

8. To verify this is the solution, note that

$$\pi_t = \beta E_t \pi_{t+1} + \kappa b_x e_t + e_t = \beta b_\pi \rho e_t + \kappa b_x e_t + e_t$$
$$= [\beta b_\pi \rho + \kappa b_x + 1] e_t,$$

so that $b_\pi = \beta b_\pi \rho + \kappa b_x + 1 = (\kappa b_x + 1)/(1 - \beta\rho)$.

9. There is no average inflation bias because we have assumed that $x^* = 0$, ensuring that the central bank's loss function depends on output only through the gap between actual output and flexible-price equilibrium output.

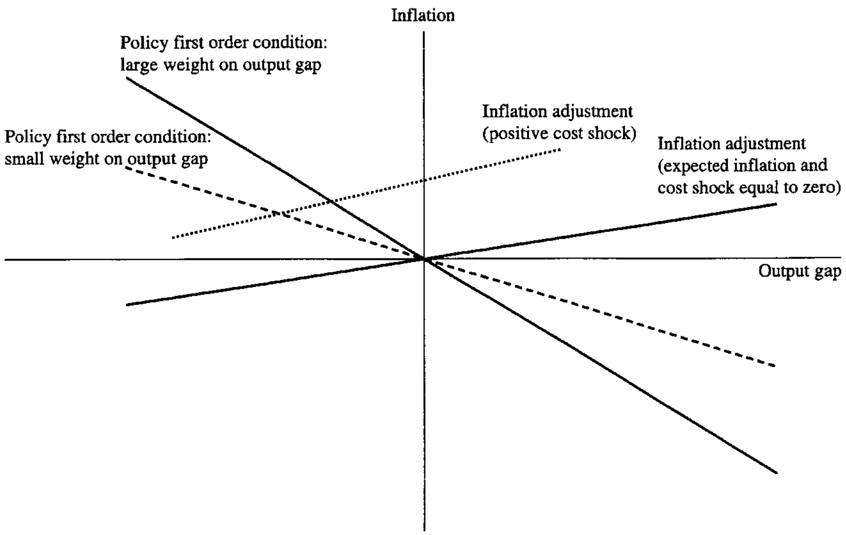

Figure 11.3
Equilibrium Output Gap and Inflation Under Discretion

In an economy subject to a sequence of mean-zero, serially uncorrelated cost shocks, the variances of inflation and the output gap will be functions of the policy parameter $\hat{\lambda}$. These variances are often used to evaluate alternative policies, and to see why, suppose the loss function (11.6) is multiplied by the constant $(1-\beta)$. This does not alter the central bank's first order condition and so leaves the choice of policy unaffected. The central bank minimizes

$$L(\hat{\lambda}) = (1-\beta)\mathrm{E}\sum_{i=0}^{\infty}\beta^i(\pi_{t+i}^2 + \hat{\lambda}x_{t+i}^2),$$

which, when $\hat{\lambda} = \lambda$, is just the unconditional expectation of the social loss function (11.6) multiplied by $(1-\beta)$. As $\beta \to 1$, this loss function approaches $\sigma_\pi^2 + \hat{\lambda}\sigma_x^2$. Thus, the asymptotic variances of the output gap and inflation can be used to compare alternative policy rules. As $\hat{\lambda}$ is varied, both σ_π^2 and σ_x^2 are affected. Plotting the variance combinations produces an efficiency frontier, showing the trade-off between output gap and inflation variability. Taylor (1993a) emphasizes that policy makers do not face a trade-off between the level of the output gap and the level of inflation.

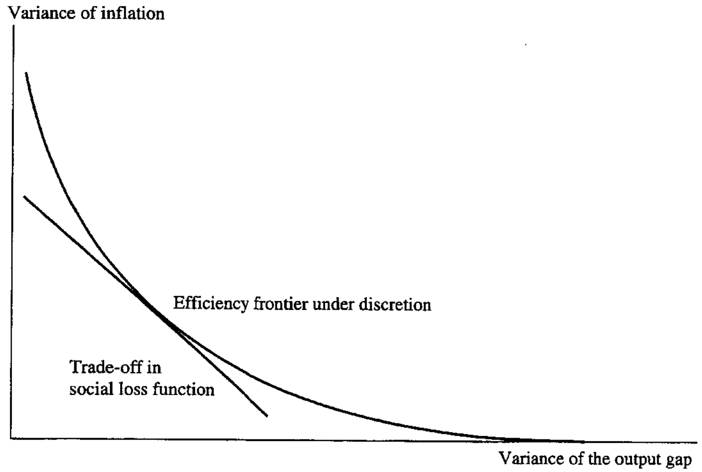

Figure 11.4
The Efficiency Frontier Under Discretion and the Social Marginal Rate of Substitution Between σ_π^2 and σ_x^2

Instead, they face a variance trade-off; greater inflation stability comes at the cost of greater output gap variability.

Figure 11.4 illustrates an efficiency frontier. Each point represents the asymptotic variances of x and π for a particular value of $\hat{\lambda}$. The social marginal rate of substitution between output gap and inflation variability is λ, and an *indifference curve* with slope λ is also shown. The optimal central banker has a $\hat{\lambda}$ that achieves the point of tangency. As the results of Clarida, Galí, and Gertler (1999) suggest, if the shock is serially uncorrelated, it is optimal to appoint a central banker who shares society's preferences ($\hat{\lambda} = \lambda$). If the cost shock is serially correlated, however, $\hat{\lambda} < \lambda$. Efficiency frontiers have also been used to evaluate simple rules for setting the central bank's policy instrument. Section 10.4.2 provided references to some of the literature examining these efficiency frontiers.

11.3.6 Model Uncertainty

Up to this point, the analysis has assumed that the central bank knows the true model of the economy with certainty. Fluctuations in output and inflation arose only from disturbances that took the form of additive errors. In this case, the linear-quadratic framework results in certainty equivalence holding; the central bank's actions depend on its expectations of future variables but not on the uncertainty associated with those expectations. When error terms enter multiplicatively, as occurs, for example, when the model's parameters are not known with certainty, equivalence will not hold. Brainard (1967) provided the classic analysis of multiplicative uncertainty. He showed that when there is uncertainty about the impact a policy instrument has on the economy, it will be optimal to respond more cautiously than would be the case in the absence of uncertainty.

Brainard's basic conclusion can be illustrated with a simple example. Suppose the inflation-adjustment equation given by (11.2) is modified to take the following form:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa_t x_t + e_t, \tag{11.20}$$

where $\kappa_t = \bar{\kappa} + v_t$ and v_t is a white noise stochastic process. In this formulation, the central bank is uncertain about the true impact of the gap x_t on inflation. For example, the central bank may have an estimate of the coefficient on x_t in the inflation equation, but there is some uncertainty associated with this estimate. The central bank's best guess of this coefficient is $\bar{\kappa}$, while its actual realization is κ_t. The central bank must choose its policy before observing the realization of v_t.

To analyze the impact uncertainty about the coefficient has on optimal policy, assume that the central bank's loss function is

$$L = \tfrac{1}{2}\mathrm{E}_t(\pi_t^2 + \lambda x_t^2)$$

and assume that policy is conducted with discretion. In addition, assume that the cost shock e_t is serially uncorrelated.

Under discretion, the central bank takes $\mathrm{E}_t\pi_{t+1}$ as given, and the first order condition for the optimal choice of x_t is

$$\mathrm{E}_t(\pi_t \kappa_t + \lambda x_t) = 0.$$

This equation can be rewritten as

$$\mathrm{E}_t[(\beta \mathrm{E}_t \pi_{t+1} + \kappa_t x_t + e_t)\kappa_t + \lambda x_t] = \beta\bar{\kappa}\mathrm{E}_t\pi_{t+1} + (\bar{\kappa}^2 + \sigma_v^2)x_t + \bar{\kappa}e_t + \lambda x_t = 0.$$

Solving for x_t, one obtains

$$x_t = -\left(\frac{\beta\bar{\kappa}\mathrm{E}_t\pi_{t+1} + \bar{\kappa}e_t}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right).$$

Since all stochastic disturbances have been assumed to be serially uncorrelated, expected inflation will be zero. Hence,

$$x_t = -\left(\frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t. \tag{11.21}$$

Equation (11.21) can be compared to the optimal discretionary response of the output gap to the cost shock when there is no parameter uncertainty. In this case, $\sigma_v^2 = 0$ and

$$x_t = -\left(\frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2}\right)e_t.$$

The presence of multiplicative parameter uncertainty ($\sigma_v^2 > 0$) reduces the impact of e_t on x_t. As uncertainty increases, it becomes optimal to respond less to e_t, that is, to behave more cautiously in setting policy.

Using (11.21) in the inflation-adjustment equation (11.20),

$$\pi_t = \kappa_t x_t + e_t = \left(\frac{\lambda + \sigma_v^2 - \bar{\kappa}(\kappa_t - \bar{\kappa})}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t = \left(\frac{\lambda + \sigma_v^2 - \bar{\kappa}v_t}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t.$$

Assuming that the two disturbances v_t and e_t are uncorrelated, the unconditional variance of inflation is equal to

$$\left[\frac{(\lambda + \sigma_v^2)^2 + \bar{\kappa}^2\sigma_v^2}{(\lambda + \bar{\kappa}^2 + \sigma_v^2)^2}\right]\sigma_e^2,$$

which is increasing in σ_v^2. In the presence of multiplicative uncertainty of the type modeled here, equilibrium output is stabilized more and inflation less in the face of cost shocks. The reason for this result is straightforward. With a quadratic loss function, the additional inflation variability induced by the variance in κ_t is proportional to x_t. Reducing the variability of x_t helps to offset the impact of v_t on the variance of inflation. It is optimal to respond more cautiously, thereby reducing the variance of x_t.

Brainard's basic result—multiplicative uncertainty leads to caution—is intuitively appealing, but it is not a general result. For example, Söderström (2002) examines a model in which there are lagged variables whose coefficients are subject to random shocks. He shows that in this case, optimal policy reacts more aggressively. For example, suppose current inflation depends on lagged inflation, but the impact of π_{t-1} on π_t is uncertain. The effect this coefficient uncertainty has on the variance of π_t depends on the variability of π_{t-1}. If the central bank fails to stabilize current inflation, it increases the variance of inflation in the following period. It can be optimal to respond more aggressively to stabilize inflation, thereby reducing the impact the coefficient uncertainty has on the unconditional variance of inflation.

A recent literature has combined the notion of parameter uncertainty with models of learning to examine the implications for monetary policy. Sargent (1999) incorporates least-squares learning in a Barro-Gordon model (see section 8.4). Wieland (2000a, 2000b) examines the trade-off between control and estimation that can arise under model uncertainty. A central bank may find it optimal to experiment, changing policy to generate observations that can help it learn about the true structure of the economy.

Another aspect of model uncertainty is measurement error or the inability to observe some relevant variables. For example, the flexible-price equilibrium level of output is needed to measure the gap variable x_t, but it is not directly observable. Svensson and Woodford (forthcoming) provide a general treatment of optimal policy when the central bank's problem involves both an estimation problem (determining the true state of the economy such as the value of the output gap) and a control policy (setting the nominal interest rate to affect the output gap and inflation). In a linear-quadratic framework in which private agents and the central bank have the same information, these two problems can be dealt with separately. Orphanides (2000) has emphasized the role the productivity slowdown played during the 1970s in causing the Fed to overestimate potential output.[10]

11.3.7 Endogenous Persistence

Empirical research on inflation has generally found that when lagged inflation is added to (11.2), its coefficient is statistically and economically significant. If lagged inflation affects current inflation, then even under discretion the central bank faces a dynamic optimization problem; decisions that affect current inflation also affect future inflation, and this intertemporal link must be taken into account by the central bank when setting current policy. Svensson (1999b) and Vestin (2001) illustrate how the linear-quadratic structure of the problem allows one to solve for the optimal discretionary policy.

To illustrate the effects introduced when inflation depends on both expected future inflation and lagged inflation, suppose (11.2) is replaced by

$$\pi_t = (1-\phi)\beta \mathrm{E}_t\pi_{t+1} + \phi\pi_{t-1} + \kappa x_t + e_t. \tag{11.22}$$

The coefficient ϕ measures the degree of backward-looking behavior exhibited by inflation.[11] If the central bank's objective is to minimize the loss function given by

10. See also Levin, Wieland, and Williams (1999), Ehrmann and Smets (2001), and Orphanides and Williams (2002).

11. Galí and Gertler (1999) and Christiano, Eichenbaum, and Evans (2001) develop inflation-adjustment equations in which lagged inflation appears by assuming that some fraction of firms do not reset their prices optimally (see section 5.4.2).

(11.6), the policy problem under discretion can be written in terms of the value function defined by

$$V(\pi_{t-1}, e_t) = \min_{\pi_t, x_t}\{[(\tfrac{1}{2})(\pi_t^2 + \lambda x_t^2) + \beta \mathrm{E}_t V(\pi_t, e_{t+1})]$$
$$+ \theta_t[\pi_t - (1-\phi)\beta\mathrm{E}_t\pi_{t+1} - \phi\pi_{t-1} - \kappa x_t - e_t]\}. \tag{11.23}$$

The value function depends on lagged inflation because it is an endogenous state variable.

Because the objective function is quadratic and the constraints are linear, the value function will be quadratic, and we can hypothesize that it takes the form

$$V(\pi_{t-1}, e_t) = a_0 + a_1 e_t + \tfrac{1}{2}a_2 e_t^2 + a_3 e_t \pi_{t-1} + a_4\pi_{t-1} + \tfrac{1}{2}a_5\pi_{t-1}^2. \tag{11.24}$$

As Vestin demonstrates, this guess is only needed to evaluate $\mathrm{E}_t V_\pi(\pi_t, e_{t+1})$, and $\mathrm{E}_t V_\pi(\pi_t, e_{t+1}) = a_3\mathrm{E}_t e_{t+1} + a_4 + a_5\pi_t$. If we assume that the cost shock is serially uncorrelated, $\mathrm{E}_t e_{t+1} = 0$ and, as a consequence, the only unknown coefficients in (11.24) that will play a role are a_4 and a_5.

The solution for inflation will take the form

$$\pi_t = b_1 e_t + b_2\pi_{t-1}. \tag{11.25}$$

Using this proposed solution, one obtains $\mathrm{E}_t\pi_{t+1} = b_2\pi_t$. This expression for expected future inflation can be substituted into (11.22) to yield

$$\pi_t = \frac{\kappa x_t + \phi\pi_{t-1} + e_t}{1-(1-\phi)\beta b_2}, \tag{11.26}$$

which implies $\partial\pi_t/\partial x_t = \kappa/[1-(1-\phi)\beta b_2]$.

Collecting these results, the first order condition for the optimal choice of x_t by a central bank whose decision problem is given by (11.23) is

$$\left[\frac{\kappa}{1-(1-\phi)\beta b_2}\right][\pi_t + \beta\mathrm{E}_t V_\pi(\pi_t, e_{t+1})] + \lambda x_t = 0. \tag{11.27}$$

By using (11.26) to eliminate x_t from (11.27) and recalling that $\mathrm{E}_t V_\pi(\pi_t, e_{t+1}) = a_4 + a_5\pi_t$, we obtain

$$\pi_t = \left[\frac{\Psi}{\kappa^2(1+\beta a_5) + \lambda\Psi^2}\right]\left[\lambda\phi\pi_{t-1} + \lambda e_t - \left(\frac{\beta\kappa^2}{\Psi}\right)a_4\right], \tag{11.28}$$

where $\Psi \equiv 1-(1-\phi)\beta b_2$.

From the envelope theorem and (11.27),

$$V_\pi(\pi_{t-1}, e_t) = a_3 e_t + a_4 + a_5 \pi_{t-1}$$
$$= \left[\frac{\phi}{1-(1-\phi)\beta b_2}\right][\pi_t + \mathrm{E}_t V_\pi(\pi_t, e_{t+1})] = -\left(\frac{\lambda\phi}{\kappa}\right)x_t.$$

Again using (11.26) to eliminate x_t,

$$V_\pi(\pi_{t-1}, e_t) = -\left(\frac{\lambda\phi}{\kappa}\right)\left[\frac{\Psi\pi_t - \phi\pi_{t-1} - e_t}{\kappa}\right]$$
$$= -\left(\frac{\lambda\phi}{\kappa}\right)\left[\frac{(\Psi b_2 - \phi)\pi_{t-1} + (\Psi b_1 - 1)e_t}{\kappa}\right]. \tag{11.29}$$

However, (11.24) implies that

$$V_\pi(\pi_{t-1}, e_t) = a_3 e_t + a_4 + a_5 \pi_t.$$

Comparing this with (11.29) reveals that $a_4 = 0$,

$$a_3 = \lambda\phi\left(\frac{1 - \Psi b_1}{\kappa^2}\right),$$

and

$$a_5 = \lambda\phi\left(\frac{\phi - \Psi b_2}{\kappa^2}\right).$$

Finally, substitute these results into (11.28) to obtain

$$\pi_t = \left[\frac{\Psi}{\kappa^2 + \beta\lambda\phi(\phi - \Psi b_2) + \lambda\Psi^2}\right][\lambda\phi\pi_{t-1} + \lambda e_t].$$

Equating coefficients with (11.25),

$$b_1 = \left[\frac{\lambda\Psi}{\kappa^2 + \beta\lambda\phi(\phi - \Psi b_2) + \lambda\Psi^2}\right]$$

and

$$b_2 = \left[\frac{\lambda\Psi\phi}{\kappa^2 + \beta\lambda\phi(\phi - \Psi b_2) + \lambda\Psi^2}\right]. \tag{11.30}$$

Because Ψ also depends on the unknown parameter b_2, (11.30) does not yield a convenient analytic solution. To gain insights into the effects of backward-looking

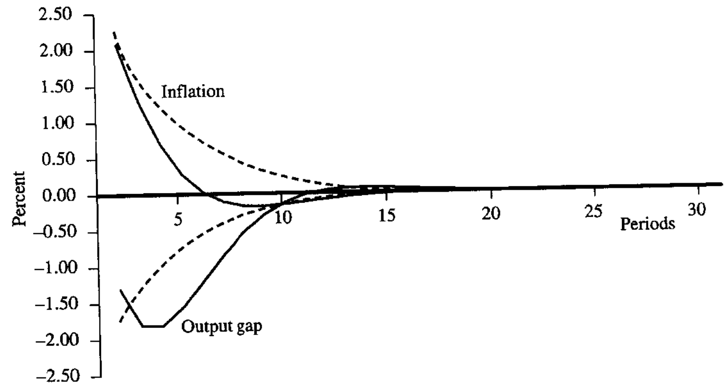

Figure 11.5
Responses to a Cost Shock with Endogenous Persistence ($\phi = 0.5$) (precommitment, solid line; discretion, dashed line)

aspects of inflation, it is useful to employ numerical techniques.[12] This is done to generate figure 11.5, which shows the response of the output gap and inflation under optimal discretion when $\phi = 0.5$. Also shown for comparison are the responses under the optimal commitment policy. While both the gap and inflation display more persistence than when $\phi = 0$ (see figures 11.1 and 11.2), inflation returns to zero more slowly under discretion.

11.4 Extensions to the Open Economy

Section 6.5 examined new open economy macroeconomic models build on the foundations of optimizing agents and sticky prices. Some versions of these open economy models could be reduced to a form that was isomorphic to the closed economy new Keynesian models we have been using. For example, the model of Clarida, Galí, and Gertler (2002) could be written in terms of an inflation-adjustment equation for the prices of domestically produced goods, given by

$$\pi_t^h = \beta \mathrm{E}_t \pi_{t+1}^h + \delta\left[\sigma + \eta + \left(\frac{\gamma\sigma}{1+w}\right)\right]x_t + \mu_t^w \tag{11.31}$$

12. The programs of Söderlind (1999) are used to find the optimal policy.

and a Euler condition of the form

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1+w}{\sigma}\right)[i_t - \mathrm{E}_t \pi_{t+1}^h - r_t^o]. \tag{11.32}$$

Equations (11.31) and (11.32) look just like a closed economy model, and that means that no new policy issues arise in the open economy case—that is, as long as policy objectives are defined in terms of x_t and π_t^h. Shifts in r_t^o, the flexible-price equilibrium real rate of interest, will affect the optimal nominal interest-rate setting (the instrument rule) but will not affect x or π^h. The stochastic mark-up term μ_t^w corresponds to the cost shock that generates policy trade-offs. See section 6.5 for the derivation of this model.

Clarida, Galí, and Gertler (2002) show that the appropriate loss function for the domestic central bank in a noncooperative equilibrium (i.e., in which the domestic central bank takes the other country's policy as given) is proportional to $(\pi_t^h)^2 + \lambda x_t^2$. The reasoning is parallel to the argument made in section 11.2 for the case of the closed economy. When prices are sticky and adjust in a staggered fashion, inflation produces a dispersion of relative prices that is inefficient. This production inefficiency is generated by the dispersion of prices among domestic firms, so it is the domestic price index rather than the consumer price index that the central bank should try to stabilize. With the presence of an inflation shock in the form of μ_t^w, the central bank still faces policy trade-offs.

If the law of one price does not hold, and traded-goods prices in the domestic currency are sticky, then the central bank's objective function will also involve inflation as measured by the consumer price index. When consumer price inflation matters, trade-offs are more complicated. An appreciation reduces firms' marginal cost and reduces domestically produced goods' inflation. As a consequence, consider what happens when the economy experiences a positive shock to aggregate spending. To stabilize the output gap, the central bank must raise the nominal interest rate. But this leads to an appreciation of the exchange rate and a decline in consumer price inflation. The important point is that the central bank cannot simultaneously stabilize the output gap and consumer price inflation. Demand shocks now lead to policy trade-offs, just as cost shocks do in the standard closed economy model.

11.5 Targeting Regimes and Instrument Rules

The analysis of optimal policy contained in section 11.3 specified an objective function for the central bank. The central bank was then assumed to behave optimally,

given its objective function and the constraints imposed on its choices by the structure of the economy. The specific objection function employed, given by (11.6), was motivated as an approximation to the utility of the representative household. One lesson from the Barro-Gordon literature on the time inconsistency of optimal policy was that policy outcomes could sometimes be improved by assigning the central bank an objective that differed from that of the public (Walsh 1995a). Rogoff (1985b) showed that when discretion led to an average inflation bias, this bias could be reduced by appointing as central banker a policy maker who placed more weight on inflation goals relative to output goals than society did. Section 11.3.4 showed that this result carries over to the forward-looking new Keynesian model even in the absence of an average inflation bias; a Rogoff-conservative central bank reduces the stabilization bias of pure discretion if the cost shock is serially correlated. A recent literature has developed to analyze the impact of alternative central bank objective functions on economic outcomes. These alternative objectives for the central bank usually involve a quadratic loss in the deviation of a variable or set of variables from prespecified target levels. As a consequence, the alternatives are often described as targeting regimes. Targeting regimes are discussed in the following subsection. An alternative approach is to assume that the central bank can commit to a simple feedback rule for its policy instrument. Instrument rules are considered in subsection 11.5.3.

11.5.1 Inflation Targeting

A policy regime in which the central bank is assigned an objective is commonly described as a *targeting regime*. A targeting regime is defined by 1) the variables in the central bank's loss function (the objectives) and 2) the weights assigned to these objectives, with policy implemented under discretion to minimize the expected discounted value of the loss function.[13] Perhaps the most widely discussed targeting regime is inflation targeting (Bernanke and Mishkin 1997; Svensson 1997a, 1997b, 1997d, 1999b, 1999c, 1999d; Svensson and Woodford 1999). Experiences with inflation targeting are analyzed by Ammer and Freeman (1995), Bernanke, Laubach, Mishkin, and Posen (1998), Mishkin and Schmidt-Hebbel (2001), Amato and

13. This definition of a targeting regime is consistent with that of Svensson (1999c), who states, "By a targeting rule, I mean, at the most general level, the assignment of a particular loss function to be minimized" (p. 617). An alternative interpretation of a targeting regime is that it is a rule for adjusting the policy instrument in the face of deviations between the current (or expected) value of the targeted variable and its target level (see, for example, McCallum 1990a and the references he cites). Jensen (2002) and Rudebusch (2002a) illustrate these two alternative interpretations of targeting.

Gerlach (2002), and the papers in Leiderman and Svensson (1995). Mishkin and Schmidt-Hebbel identify 19 countries as inflation targeters as of 2001, with New Zealand, in 1990, being the first country to have adopted formal targets for inflation.

Inflation targeting has been characterized in a variety of ways in the academic literature, and it has been implemented in different ways in the countries that have adopted inflation targeting as a framework for monetary policy. In general, the announcement of a formal target for inflation is a key component, and this is often accompanied by publication of the central bank's inflation forecasts. An inflation targeting regime can be viewed as the assignment to the central bank of an objective function of the form

$$L_t^{IT} = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i [(\pi_{t+i} - \pi^T)^2 + \lambda_{IT} x_{t+i}^2], \tag{11.33}$$

where π^T is the target inflation rate and λ_{IT} is the weight assigned to achieving the output gap objective relative to the inflation objective. λ_{IT} may differ from the weight placed on output gap stabilization in the social loss function (11.6). As long as $\lambda_{IT} > 0$, specifying inflation targeting in terms of the loss function (11.33) assumes that the central bank is concerned with output stabilization as well as inflation stabilization.[14]

In the policy problems we have been analyzing, the central bank's choice of its instrument i_t allows it to affect both output and inflation immediately. This absence of any lag between the time a policy action is taken and the time it affects output and inflation is unrealistic. If policy decisions taken in period t only affect future output and inflation, then the central bank must rely on forecasts of future output and inflation when making its policy choices. In analyzing the case of such policy lags, Svensson (1997a) and Svensson and Woodford (1999) emphasize the role of inflation-forecast targeting. To illustrate the role of forecasts in the policy process in a very simple manner, suppose the central bank must set i_t prior to observing any time-t information. This assumption implies that the central bank cannot respond to time-t shocks contemporaneously; information about shocks occurring in period t will affect the central bank's choice of i_{t+1} and, as a consequence, x_{t+1} and π_{t+1} can be affected. The model is otherwise given by (11.1) and (11.2), as before, with the additional assumption that the cost shock follows an $AR(1)$ process: $e_t = \rho e_{t-1} + \varepsilon_t$. Assume that the demand shock in (11.1) is serially uncorrelated. The central bank's

14. In the terminology of section 8.3.5, inflation targeting with the loss function (11.33) corresponds to a flexible targeting regime.

objective is to choose i_t to minimize

$$\mathrm{E}_{t-1} \sum_{i=0}^{\infty} \beta^i [(\pi_{t+i} - \pi^T)^2 + \lambda_{IT} x_{t+i}^2],$$

where the subscript on the expectations operator is now $t-1$ to reflect the information available to the central bank when it sets policy. The choice of i_t is subject to the constraints represented by (11.1) and (11.2). Taking expectations based on the central bank's information, these two equations can be written as

$$\mathrm{E}_{t-1} x_t = \mathrm{E}_{t-1} x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_{t-1}\pi_{t+1}) \tag{11.34}$$

and

$$\mathrm{E}_{t-1}\pi_t = \beta \mathrm{E}_{t-1}\pi_{t+1} + \kappa \mathrm{E}_{t-1} x_t + \rho e_{t-1}. \tag{11.35}$$

Under discretion, the first order condition for the central bank's choice of i_t implies that

$$\mathrm{E}_{t-1}[\kappa(\pi_t - \pi^T) + \lambda x_t] = 0. \tag{11.36}$$

Rearranging this first order condition yields

$$\mathrm{E}_{t-1} x_t = -\left(\frac{\kappa}{\lambda}\right)\mathrm{E}_{t-1}(\pi_t - \pi^T).$$

Thus, if the central bank forecasts that period-t inflation will exceed its target rate of inflation, it should adjust policy to ensure that the forecast of the output gap is negative.

Svensson and Woodford (1999) provide a detailed discussion of inflation-forecast targeting, and they focus on the implications for the determinacy of equilibrium under different specifications of the policy decision process. The possibility of multiple equilibria becomes particularly relevant if the central bank bases its own forecasts on private sector forecasts which are, in turn, based on expectations about the central bank's actions. Bernanke and Woodford (1997) provide a simple example of the problems that may arise if policy is based on private forecasts. Suppose inflation can be written as

$$\pi_{t+1} = s_t + u_t + \varepsilon_{t+1},$$

where s_t is a state variable, u_t is the central bank's policy instrument, and ε_{t+1} is an unforecastable error. Assume that s and ε are serially and mutually uncorrelated random variables with means equal to zero. Assume that the objective of the central bank is to set inflation equal to zero. If the central bank can observe s_t, optimal policy clearly involves setting $u_t = -s_t$.

Suppose private agents observe s_t but the central bank doesn't. Can the central bank use private sector inflation forecasts as a basis for setting policy? If the central bank could infer s_t from the private sector's forecasts, then inflation would equal $\pi_{t+1} = s_t - s_t + \varepsilon_{t+1} = \varepsilon_{t+1}$, but in this case, the private sector forecasts a zero inflation rate. If the public expects a zero rate of inflation, the central bank cannot infer s_t from the public's expectations, contrary to our supposition.

Bernanke and Woodford demonstrate this formally as follows. Let $\pi_t^f = \mathrm{E}[\pi_{t+1}|s_t]$ be the private agents' expectation of π_{t+1} conditional on observing s_t. Assume that the central bank can observe π_t^f; for example, the spread between index and non-index bonds provides a measure of the public's expectations about future inflation, and the central bank might consider using this information in setting its policy. Let the central bank set u_t on the basis of π_t^f:

$$u_t = \phi \pi_t^f. \tag{11.37}$$

Then

$$\pi_{t+1} = s_t + \phi \pi_t^f + \varepsilon_{t+1} = s_t + \phi \mathrm{E}[\pi_{t+1}|s_t] + \varepsilon_{t+1}$$

or

$$\pi_{t+1} = \left(\frac{1}{1-\phi}\right)(s_t + \varepsilon_{t+1})$$

as long as $\phi \neq 1$. Under this solution, $\pi_t^f = [1/(1-\phi)]s_t$, so (11.37) becomes

$$u_t = \left(\frac{\phi}{1-\phi}\right)s_t.$$

But then to achieve its zero expected inflation target, the central bank wants to set $u_t = -s_t$, which requires

$$\left(\frac{\phi}{1-\phi}\right) = -1,$$

which has no solution for ϕ. As this simple example demonstrates, policies that respond to private expectations, expectations which themselves depend on the central bank's policy, may not be consistent with a rational expectations equilibrium.

11.5.2 Other Targeting Regimes

Inflation targeting is just one example of a policy targeting regime.[15] A number of alternative targeting regimes have been analyzed in the literature. These include price level targeting (Dittmar, Gavin, and Kydland 1999; Svensson 1999d; Vestin 2001), nominal income growth targeting (Jensen 2002), hybrid price level–inflation targeting (Batini and Yates 2001), average inflation targeting (Nessén and Vestin 2000), and regimes based on the change in the output gap or its quasi-difference (Jensen and McCallum 2002; Walsh 2002b). In each case, it is assumed that, given the assigned loss function, the central bank chooses policy under discretion. The optimal values for the parameters in the assigned loss function, for example, the value of λ_{IT} in (11.33), are chosen to minimize the unconditional expectation of the social loss function (11.6).

The importance of forward-looking expectations in affecting policy choice is well illustrated by work on price level targeting. The traditional view argued that attempts to stabilize the price level, as opposed to the inflation rate, would generate undesirable levels of output variability. A positive cost shock that raised the price level would require a deflation to bring the price level back on target, and this deflation would be costly. However, as figure 11.2 shows, an optimal commitment policy that focuses on output and inflation stability also induces a deflation after a positive cost shock. By reducing $\mathrm{E}_t\pi_{t+1}$, such a policy achieves a better trade-off between inflation variability and output variability. The deflation generated under a discretionary policy concerned with output and price-level stability might actually come closer to the commitment policy outcomes than discretionary inflation targeting would. Using a basic new Keynesian model, Vestin (2001) shows that this intuition is correct. In fact, when inflation is given by (11.2) and the cost shock is serially uncorrelated, price-level targeting can replicate the timeless precommitment solution exactly if the central bank is assigned the loss function $p_t^2 + \lambda_{PL} x_t^2$, where λ_{PL} differs from the weight λ in the social loss function.

Jensen (2002) shows that a nominal income growth targeting regime can also dominate inflation targeting. Walsh (2002b) adds lagged inflation to the inflation-adjustment equation and shows that the advantages of price-level targeting over inflation targeting decline as the weight on lagged inflation increases. Walsh analyzes discretionary outcomes when the central bank targets inflation and the change in the output gap (a *speed limit* policy). Introducing the change in the gap induces inertial

15. See section 8.3.5 for a further analysis of targeting rules.

behavior similar to that obtained under precommitment. For empirically relevant values of the weight on lagged inflation (ϕ in the range 0.3 to 0.7), speed limit policies dominate price-level targeting, inflation targeting, and nominal income growth targeting. For ϕ below 0.3, price-level targeting does best. Svensson and Woodford (1999) have considered interest-rate-smoothing objectives as a means of introducing into discretionary policy the inertia that is optimal under commitment.

In the literature discussed here, numerical methods are used to evaluate outcomes under alternative objective functions. Dennis (2001b) attempts to estimate the Fed's objective function from output and inflation outcomes. He finds that for both the pre-Volker period (1996:1–1979:3) and the Volker-Greenspan era (1980:2–2000:2), the estimated weight on the output gap is zero. In contrast to loss functions such as that given by (11.33), Dennis finds a significant role for interest-rate smoothing and a lower inflation target in the Volker-Greenspan period than in the pre-Volker period.

11.5.3 Instrument Rules

The approach to policy analysis adopted in the preceding sections starts with a specification of the central bank's objective function and then derives the optimal setting for the policy instrument. An alternative approach specifies an instrument rule directly. The most famous of such instrument rules is the Taylor rule (Taylor 1993a). Taylor showed that the behavior of the federal funds interest rate in the United States from the mid-1980s through 1992 (when Taylor was writing) could be fairly well matched by a simple rule of the form

$$i_t = \pi_t + 0.5x_t + 0.5(\pi_t - \pi^T) + r^*,$$

where π^T was the target level of average inflation (Taylor assumed it to be 2%) and r^* was the equilibrium real rate of interest (Taylor assumed that this too was 2%). The Taylor rule for general coefficients is often written

$$i_t = r^* + \pi^T + \alpha_x x_t + \alpha_\pi(\pi_t - \pi^T). \tag{11.38}$$

The nominal interest rate deviates from the level consistent with the economy's equilibrium real rate and the target inflation rate if the output gap is nonzero or if inflation deviates from target. A positive output gap leads to a rise in the nominal rate, as does a deviation of actual inflation above target. With Taylor's original coefficients, $\alpha_\pi = 1.5$, so that the nominal rate is changed more than one for one with deviations of inflation from target. Thus, the rule satisfies the Taylor principle (see chapter 5); a greater than one-for-one reaction of i_t ensures that the economy has a

unique, stationary, rational expectations equilibrium. Lansing and Trehan (2001) explore conditions under which the Taylor rule emerges as the fully optimal instrument rule under discretionary policy.

A large literature has now developed that has estimated Taylor rules, or similar simple rules, for a variety of countries and time periods. For example, Clarida, Galí, and Gertler (2000) do so for the Federal Reserve, the Bundesbank, and the Bank of Japan. In their specification, however, actual inflation is replaced by expected future inflation so that the central bank is assumed to be forward-looking in setting policy. Estimates for the United States under different Federal Reserve chairmen are reported by Judd and Rudebusch (1997). In general, the basic Taylor rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate. However, Orphanides (2000) finds that when estimated using the data on the output gap and inflation actually available at the time policy actions were taken (i.e., using real-time data), the Taylor rule does much more poorly in matching the U.S. funds rate. Clarida, Galí, and Gertler (2000) find that the Fed moved the funds rate less than one for one during the period 1960–1979, thereby violating the Taylor principle. In a further example of the importance of using real-time data, however, Perez (2001) finds that when the Fed's reaction function is reestimated for this earlier period using real-time data, the coefficient on inflation is greater than 1.

When a policy interest rate such as the federal funds rate in the United States is regressed on inflation and output gap variables, the lagged value of the interest rate normally enters with a statistically significant and large coefficient. The interpretation of this coefficient on the lagged interest rate has been the subject of debate. One interpretation is that it reflects inertial behavior of the sort we saw in section 11.3.1 that would arise under an optimal precommitment policy. It has also been interpreted to mean that central banks adjust gradually toward a desired interest-rate level. For example, suppose that i_t^* is the central bank's desired value for its policy instrument. Suppose, however, that it wants to avoid large changes in interest rates. Such an interest-smoothing objective might arise from a desire for financial market stability. If the central bank adjusts i_t gradually toward i_t^*, then the behavior of i_t may be captured by a partial adjustment model of the form

$$i_t = i_{t-1} + \theta(i_t^* - i_{t-1}) = (1 - \theta)i_{t-1} + \theta i_t^*. \tag{11.39}$$

The estimated coefficient on i_{t-1} provides an estimate of $1 - \theta$. Values close to 1 imply that θ is small; each period the central bank closes only a small fraction of the gap between its policy rate and its desired value.

The view that central banks adjust slowly has been criticized. Sack (2000) and Rudebusch (2002b) argue that the presence of a lagged interest rate in estimated instrument rules is not evidence that the Fed acts gradually. Sack attributes the Fed's behavior to parameter uncertainty that leads the Fed to adjust the funds rate less aggressively than would be optimal in the absence of parameter uncertainty. Rudebusch argues that imperfect information about the degree of persistence in economic disturbances induces behavior by the Fed that appears to reflect gradual adjustment. He notes that if the Fed followed a rule such as (11.39), future changes in the funds rate would be predictable, but evidence from forward interest rates does not support the presence of predictable changes. Similarly, Lansing (2002) shows that the appearance of interest-rate smoothing can arise if the Fed uses real-time data to update its estimate of trend output each period. When final data are used to estimate a policy instrument rule, the serial correlation present in the Fed's real-time errors in measuring trend output will be correlated with lagged interest rates, creating the illusion of interest-rate-smoothing behavior by the Fed.

Analyzing Basic Instrument Rules While Taylor picked the coefficients in the original Taylor rule to provide a reasonable and simple representation of policy, subsequent research has focused on deriving the optimal values of these coefficients, under the assumption that the central bank can commit to the rule. That is, suppose the objective is to minimize the loss function given by (11.6), but rather than assume that the central bank optimizes directly, given the policy environment (discretion or commitment), assume that the central bank can commit to a simple rule of the form given by (11.38). What are the optimal values of α_x and α_π?[16] It is important to note that the "optimal" values of α_x and α_π are only optimal conditional on the central bank following an instrument rule of the form given by (11.38). If the central bank can commit, then the fully optimal policy will be the one studied in section 11.3.

To determine the optimal values of α_x and α_π, assume that the model of the economy is given by (11.1) and (11.2), repeated here for convenience:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1}) + u_t,$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t. \tag{11.40}$$

Using the assumed policy rule to eliminate i_t, the expectational IS relationship becomes

16. The optimal values for a backward-looking model were derived in section 10.4.2.

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\alpha_x x_t + \alpha_\pi \pi_t - \mathrm{E}_t \pi_{t+1}) + u_t. \tag{11.41}$$

Combining (11.41) and (11.40), the system can be written as

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \sigma^{-1}\alpha_x + \kappa(\sigma\beta)^{-1} & -(\sigma\beta)^{-1}(1 - \beta\alpha_\pi) \\ -\kappa\beta^{-1} & \beta^{-1} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} (\sigma\beta)^{-1} e_t - u_t \\ -\beta^{-1} e_t \end{bmatrix}.$$

Note that because this equation is expressed in terms of deviations from the flexible price level of output, r^* has been set equal to zero and, for simplicity, the target inflation rate has also been set equal to zero.

Unlike the cases analyzed in section 11.3, equilibrium does depend on the expectational IS relationship. When policy objectives were expressed in terms of the output gap and inflation, and policy was optimal, the nominal interest rate was always adjusted to achieve the desired trade-off between output and inflation. Realizations of the demand disturbance u_t were neutralized completely so that they affected neither the output gap nor inflation. This is no longer the case under an instrument rule such as (11.38). Demand disturbances cause the gap and inflation to fluctuate inefficiently. The argument for simple rules, then, relies not on their optimality but on their simplicity; they may serve as a useful benchmark for policy or aid in promoting policy transparency.

Numerous papers have, in recent years, analyzed the implications of different simple rules. For example, the Taylor (1999) volume contains a number of examples using various models to assess simple rules. McCallum (1999a) provides an extensive discussion of policy rules. Instrument rules in which the policy instrument is adjusted in response to nominal income or nominal income growth have also been analyzed (Rudebusch 2002a). These rules generally take the form

$$i_t = i_0 + a(z_t - z^*) \tag{11.42}$$

for some target variable z_t. For example, rather than viewing inflation targeting as the assignment of an objective function involving inflation target deviations, one might interpret inflation targeting as the use of an instrument rule of the form $i_t = i_0 + a(\pi_t - \pi^T)$.

McCallum (1999a) has argued for a research approach that analyzes the behavior of simple instrument rules in a variety of models. Given our uncertainty about the correct model of the economy, one should be concerned if a rule performs well in one class of models but very poorly in another class. A good instrument rule would be one that does well in a wide range of economic models even though it might not be optimal within any one class of models.

11.6 Appendix

11.6.1 Approximating Utility

In this appendix, details on the derivation of (11.5) are provided. The analysis is based on Woodford (2001a). To derive an approximation to the representative agent's utility, it is necessary to first introduce some additional notation. For any variable X_t, let $\bar{X}$ be its steady-state value, let X_t^* be its efficient level (if relevant), and let $\tilde{X}_t = X_t - \bar{X}$ be the deviation of X_t around the steady state. Let $\hat{X}_t = \log(X_t/\bar{X})$ be the log deviation of X_t around its steady-state value. Using a second order Taylor approximation, the variables $\tilde{X}_t$ and $\hat{X}_t$ can be related as

$$\tilde{X}_t = X_t - \bar{X} = \bar{X}\left(\frac{X_t}{\bar{X}} - 1\right) \approx \bar{X}\left(\hat{X}_t + \frac{1}{2}\hat{X}_t^2\right). \tag{11.43}$$

Employing this notation, we can develop a second order approximation to the utility of the representative household.

The first term on the right of (11.4) is the utility from consumption. This can be approximated around the steady state as

$$U(Y_t, z_t) \approx U(\bar{Y}, 0) + U_c\tilde{Y}_t + U_z z_t + \tfrac{1}{2}U_{cc}\tilde{Y}_t^2 + U_{c,z}z_t\tilde{Y}_t + \tfrac{1}{2}z_t'U_{z,z}z_t. \tag{11.44}$$

Using (11.43), and ignoring terms involving $\hat{Y}_t^i$ for $i > 2$ and terms such as $z_t\hat{Y}_t^2$, (11.44) becomes

$$U(Y_t, z_t) \approx U(\bar{Y}, 0) + U_c\bar{Y}\left(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right) + U_z z_t + \frac{1}{2}U_{cc}\bar{Y}^2\hat{Y}_t^2$$

$$+ U_{c,z}z_t\bar{Y}_t\hat{Y}_t + \frac{1}{2}z_t'U_{z,z}z_t$$

$$= \bar{Y}U_c\left\{\hat{Y}_t + \frac{1}{2}\left[1 + \frac{U_{cc}}{U_c}\bar{Y}\right]\hat{Y}_t^2 + \frac{U_{c,z}}{U_c}z_t\hat{Y}_t\right\} + t.i.p.,$$

where *t.i.p.* are terms independent of policy. The choice of terms to include in *t.i.p.* is based on the implication of the new Keynesian model that the steady state is independent of monetary policy. To simplify the approximation, define

$$\sigma = -\frac{\bar{Y}U_{cc}}{U_c}$$

as the coefficient of relative risk aversion, and let

$$\phi_t = -\frac{U_{c,z}}{\bar{Y}U_{cc}}z_t.$$

Then the approximation for $U(Y, z)$ becomes

$$U(Y_t, z_t) \approx \bar{Y}U_c\left\{\hat{Y}_t + \tfrac{1}{2}(1-\sigma)\hat{Y}_t^2 + \sigma\phi_t\hat{Y}_t\right\} + t.i.p.$$

We next need to analyze the second term on the right in (11.4), the term arising from the disutility of work. Expanding this around the steady state yields

$$v(y_t(i), z_t) = v(\bar{y}, 0) + v_y\tilde{y}(i) + v_z z_t + \tfrac{1}{2}v_{yy}\tilde{y}_t(i)^2 + v_{y,z}z_t\tilde{y}_t(i) + \tfrac{1}{2}z_t'v_{z,z}z_t.$$

By approximating $\tilde{y}_t(i)$ with $\bar{y}(\hat{y}_t(i) + \frac{1}{2}\hat{y}_t(i)^2)$, one obtains

$$v(y_t(i), z_t) \approx v_y\bar{y}\left\{\hat{y}_t(i) + \frac{1}{2}\hat{y}_t(i)^2 + \frac{1}{2}\frac{v_{yy}\bar{y}}{v_y}\hat{y}_t(i)^2 + \frac{v_{y,z}}{v_y}z_t\hat{y}_t(i)\right\} + t.i.p.$$

This last equation can be written as

$$v(y_t(i), z_t) \approx v_y\bar{y}\left\{\hat{y}_t(i) + \frac{1}{2}\left(1 + \frac{v_{yy}\bar{y}}{v_y}\right)\hat{y}_t(i)^2 + \frac{v_{y,z}}{v_y}z_t\hat{y}_t(i)\right\} + t.i.p.$$

$$= v_y\bar{y}\left\{\hat{y}_t(i) + \frac{1}{2}(1+\eta)\hat{y}_t(i)^2 - \eta q_t\hat{y}_t(i)\right\} + t.i.p.,$$

where

$$\eta = \frac{v_{yy}\bar{y}}{v_y}$$

and[17]

$$q_t = -\frac{v_{y,z}z_t}{v_{yy}\bar{y}}.$$

To proceed further, we need to recall the model of monopolistic competition underlying the new Keynesian framework. In a model of perfect competition, the household producer of good i would equate the marginal rate of substitution between leisure and consumption to the real wage, or $v_y/U_c = 1$, since the implicit production function is $y_t(i) = n_t(i)$. In the presence of monopolistic competition, $v_y/U_c = (\theta - 1)/\theta$ is 1 over the markup. Define $\Phi = 1/\theta$. Then $v_y/U_c = 1 - \Phi$. If the

17. If $v(y_t(i))$ is of the isoelastic form $y_t(i)^{1+\eta}/(1+\eta)$, then $v_{y,y}\bar{y}/v_y = \eta$.

distortion created by monopolistic competition is small, terms such as $\Phi\hat{y}^2$ and $\Phi q_t \hat{y}$ will be of second order, and we obtain

$$v(y_t(i), z_t) \approx U_c \overline{Y}_t \left[(1-\Phi)\hat{y}_t(i) + \tfrac{1}{2}(1+\eta)\hat{y}_t(i)^2 - \eta q_t \hat{y}_t(i)\right] + t.i.p.$$

Integrating over all goods, and using the relationship $\hat{Y} \approx \mathrm{E}_i \hat{y}(i) + \frac{1}{2}(1-\theta^{-1})\ var_i\ \hat{y}(i)$,

$$\int_0^1 v(y_t(i), z_t)\, di$$

$$\approx U_c \overline{Y}_t \left\{ (1-\Phi)\mathrm{E}\hat{y}_t(i) + \frac{1}{2}(1+\eta)[(\mathrm{E}\hat{y}_t(i))^2 + var_i\ \hat{y}_t(i)] - \eta q_t \mathrm{E}\hat{y}_t(i) \right\} + t.i.p.$$

$$= U_c \overline{Y}_t \left\{ (1-\Phi-\eta q_t)\hat{Y}_t + \frac{1}{2}(1+\eta)\hat{Y}_t^2 + \frac{1}{2}(\theta^{-1}+\eta)\ var_i\ \hat{y}_t(i) \right\} + t.i.p.,$$

where terms such as $var_i\ \hat{y}(i)^4$ and $\hat{y}(i)\ var_i\ \hat{y}(i)$ are set equal to zero.

Bringing together the results for the utility of consumption and the disutility of work,

$$V \approx \overline{Y} U_c \{ \hat{Y}_t + \tfrac{1}{2}(1-\sigma)\hat{Y}_t^2 + \sigma\phi_t \hat{Y}_t \} - U_c \overline{Y}_t \{ (1-\Phi-\eta q_t)\hat{Y}_t$$
$$+ \tfrac{1}{2}(1+\eta)\hat{Y}_t^2 + \tfrac{1}{2}(\theta^{-1}+\eta)\ var_i\ \hat{y}_t(i) \} + t.i.p.$$
$$= \overline{Y} U_c \{ [\Phi + \sigma\phi_t + \eta q_t]\hat{Y}_t - \tfrac{1}{2}(\sigma+\eta)\hat{Y}_t^2 - \tfrac{1}{2}(\theta^{-1}+\eta)\ var_i\ \hat{y}_t(i) \} + t.i.p.$$

To gain insight into this expression for utility, it will be useful to derive the equilibrium output level under flexible prices. In a flexible-price equilibrium, the marginal product of labor equals the markup arising from monopolistic competition times the marginal rate of substitution between leisure and consumption. Given the specification of the composite consumption good in (11.3), the markup equals $\theta/(\theta-1)$. Thus, in the flexible-price equilibrium,

$$\left(\frac{\theta}{\theta-1}\right)\frac{v_y}{U_c} = 1.$$

Multiply both sides of this expression by U_c and then use the approximations derived earlier to conclude that the flexible-price output level $\hat{Y}_t^f$ satisfies

$$\left(\frac{\theta}{\theta-1}\right)[v_y(\bar{y},0) + v_{yy}\overline{Y}\hat{Y}_t^f + v_{y,z}\hat{z}_t] = U_c(\overline{Y},0) + U_{cc}(\overline{Y},0)\overline{Y}\hat{Y}_t^f + U_{c,z}\hat{z}_t.$$

Dividing both sides by $U_c(\overline{Y},0) = \theta v_y(\bar{y},0)/(\theta-1)$,

$$\frac{v_{yy}\overline{Y}\hat{Y}_t^f + v_{y,z}\hat{z}_t}{v_y(\bar{y},0)} = \frac{U_{cc}(\overline{Y},0)\overline{Y}\hat{Y}_t^f + U_{c,z}\hat{z}_t}{U_c(\overline{Y},0)}$$

or

$$\eta\hat{Y}_t^f - \eta q_t = -\sigma\hat{Y}_t^f + \sigma\phi_t.$$

Solving for $\hat{Y}_t^f$,

$$\hat{Y}_t^f = \left(\frac{\sigma\phi_t + \eta q_t}{\sigma+\eta}\right).$$

The utility approximation can now be written as

$$V \approx -\left(\frac{1}{2}\right)(\sigma+\eta)\overline{Y}U_c\left\{ \hat{Y}_t^2 - 2\left[\frac{\Phi+\sigma\phi_t+\eta q_t}{\sigma+\eta}\right]\hat{Y}_t + \left(\frac{\theta^{-1}+\eta}{\sigma+\eta}\right) var_i\ \hat{y}_t(i) \right\} + t.i.p.$$

$$= -\left(\frac{1}{2}\right)(\sigma+\eta)\overline{Y}U_c\left\{ (x_t - x^*)^2 + \left(\frac{\theta^{-1}+\eta}{\sigma+\eta}\right) var_i\ \hat{y}_t(i) \right\} + t.i.p.,$$

where

$$x_t \equiv \hat{Y}_t - \hat{Y}_t^f$$

is the gap between output and the flexible-price equilibrium output, and

$$x^* \equiv \frac{\Phi}{\sigma+\eta}.$$

Letting $\overline{Y}^*$ be the steady-state, efficient level of output, x^* is equal to $\log(\overline{Y}^*/\overline{Y})$ and is a measure of the distortion created by the presence of monopolistic competition.

The next step in obtaining an approximation to the utility of the representative agent involves expressing the variance of $y_t(i)$ in terms of the dispersion of prices across firms.

With the assumed utility function, the demand for good i satisfies $y_t(i) = [p_t(i)/P_t]^{-\theta} Y_t$. Taking logs,

$$\log y_t(i) = \log Y_t - \theta(\log p_t(i) - \log P_t),$$

so

$$var_i \log y_t(i) = \theta^2\ var_i \log p_t(i).$$

Hence, we can evaluate alternative policies using as our welfare criterion

$$-\tfrac{1}{2}\bar{Y}U_c[(\sigma+\eta)(x_t-x^*)^2+(\theta^{-1}+\eta)\theta^2\, var_i \log p_t(i)]. \tag{11.45}$$

The last step in the approximation process is to relate $var_i \log p_t(i)$ to the average inflation rate across all firms. To do so, recall that the price-adjustment mechanism involves a randomly chosen fraction $1-\omega$ of all firms' optimally adjusting price each period (see chapter 5 for details). Define $\bar{P}_t \equiv \mathrm{E}_t \log p_t(i)$ and $\Delta_t \equiv var_i \log p_t(i)$. Then, since $var_i\, \bar{P}_{t-1}=0$, we can write

$$\begin{aligned}\Delta_t &= var_i[\log p_t(i)-\bar{P}_{t-1}]\\ &= \mathrm{E}_t[\log p_t(i)-\bar{P}_{t-1}]^2-[\mathrm{E}_t \log p_t(i)-\bar{P}_{t-1}]^2\\ &= \omega\mathrm{E}_t[\log p_{t-1}(i)-\bar{P}_{t-1}]^2+(1-\omega)(\log p_t^*-\bar{P}_{t-1})^2-(\bar{P}_t-\bar{P}_{t-1})^2,\end{aligned}$$

where p_t^* is the price set at time t by the fraction $1-\omega$ of firms that reset their price. Given that $\bar{P}_t=(1-\omega)\log p_t^*+\omega\bar{P}_{t-1}$,

$$\log p_t^*-\bar{P}_{t-1}=\left(\frac{1}{1-\omega}\right)(\bar{P}_t-\bar{P}_{t-1}).$$

Using this result,

$$\begin{aligned}\Delta_t &= \omega\Delta_{t-1}+\left(\frac{\omega}{1-\omega}\right)(\bar{P}_t-\bar{P}_{t-1})^2\\ &\approx \omega\Delta_{t-1}+\left(\frac{\omega}{1-\omega}\right)\pi_t^2.\end{aligned}$$

This implies

$$\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\Delta_{t+i}=\left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right]\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\pi_{t+i}^2+t.i.p.,$$

where the terms independent of policy also include the initial degree of price dispersion.

Combining this with (11.45), the present discounted value of the utility of the representative household can be approximated by

$$\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i V_{t+i}\approx -\Omega\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i[\pi_{t+i}^2+\lambda(x_{t+i}-x^*)^2],$$

where

$$\Omega=\frac{1}{2}\bar{Y}U_c\left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right](\theta^{-1}+\eta)\theta^2$$

and

$$\lambda=\left[\frac{(1-\omega)(1-\omega\beta)}{\omega}\right]\frac{(\sigma+\eta)}{(1+\eta\theta)\theta}.$$

11.6.2 Solving for Optimal Policy

The simulations used to obtain figures 11.1, 11.2, and 11.5 were conducted using the program of Söderlind (1999). These programs are available at http://www.hhs.se/personal/Psoderlind/.

Define $X_{1t}=[e_t u_t]'$ with innovation $[\varepsilon_t,\xi_t]'$, $X_{2t}=[x_t,\pi_t]'$, $\chi_{t+1}=[\varepsilon_{t+1},\xi_{t+1},0,0]'$, and let $Z_t=[X_{1t},X_{2t}]'$. Then the basic new Keynesian model can be written compactly as

$$\mathrm{E}_t Z_{t+1}=AZ_t+Bi_t+\chi_{t+1}. \tag{11.46}$$

The policy instrument i_t is set to minimize an objective function expressed as

$$L_t=(1-\beta)\mathrm{E}_t\sum\beta^i Z_{t+i}'QZ_{t+i}, \tag{11.47}$$

where Q depends on the specification of the single-period loss function. Under pure discretion, the period loss function is simply $\pi_t^2+\lambda x_t^2$, so

$$Q=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&\lambda&0\\0&0&0&1\end{bmatrix}.$$

Under optimal discretionary policy, the solution to the problem of minimizing (11.47) subject to (11.46) takes the form

$$i_t=-FX_{1t}$$

$$X_{1t}=M_1X_{1t-1}+M_2\chi_t$$

and

$$X_{2t}=M_3X_{1t}.$$

Details of the solution procedures are provided in Söderlind (1999).

11.7 Problems

1. Suppose the economy is characterized by (11.1) and (11.2), and let the cost shock be given by $e_t = \rho e_{t-1} + \varepsilon_t$. The central bank's loss function is (11.6). Assume that the central bank can commit to a policy rule of the form $\pi_t = \gamma e_t$.

a. What is the optimal value of γ?

b. Find the expression for equilibrium output gap under this policy.

2. In section 11.3.4, the case of commitment to a rule of the form $x_t = b_x e_t$ was analyzed. Does a unique, stationary, rational expectations equilibrium exist under such a commitment? Suppose instead that the central bank commits to the rule $i_t = b_i e_t$ for some constant b_i. Does a unique, stationary, rational expectations equilibrium exist under such a commitment? Explain why the two cases differ.

3. Suppose the economy is described by the following log-linearized system:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1}) + E_t(z_{t+1} - z_t) + u_t$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t,$$

where u_t is a demand shock, z_t is a productivity shock, and e_t is a cost shock. Assume that

$$u_t = \rho_u u_{t-1} + \xi_t$$

$$z_t = \rho_z z_{t-1} + \psi_t$$

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where ξ, ψ, and ε are white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\frac{1}{2}\mathrm{E}_t\left[\sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2)\right].$$

a. Derive the optimal time-consistent policy for the discretionary central banker. Write down the first order conditions and the reduced-form solutions for x_t and π_t.

b. Derive the interest-rate feedback rule implied by the optimal discretionary policy.

c. Show that under the optimal policy, nominal interest rates are increased enough to raise the real interest rate in response to a rise in expected inflation.

d. How will x_t and π_t move in response to a demand shock? To a productivity shock?

4. Consider the following model, where all variables are expressed in log deviations from the steady state:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right) r_t \tag{11.48}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa mc_t \tag{11.49}$$

$$y_t = a_t + n_t \tag{11.50}$$

$$mpl_t = a_t \tag{11.51}$$

$$mc_t = \omega_t - mpl_t \tag{11.52}$$

$$r_t = i_t - \mathrm{E}_t \pi_{t+1} \tag{11.53}$$

$$w_t = \beta \mathrm{E}_t w_{t+1} + \phi(mrs_t - \omega_t) \tag{11.54}$$

$$mrs_t = \eta n_t + \sigma y_t \tag{11.55}$$

$$\omega_t = \omega_{t-1} + w_t - \pi_t \tag{11.56}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t},$$

where ω is the real wage, w is the growth rate of nominal wages, mc is real marginal costs, mrs is the household's marginal rate of substitution between leisure and consumption, mpl is the marginal product of labor, $\varepsilon_{a,t}$ is a white noise i.i.d. productivity shock, $x_t = y_t - y_t^f$ is the output gap, and the policy rule is omitted.

a. Solve for the flexible-price/flexible-wage output y_t^f and real wage ω_t^f equilibrium levels.

b. Solve for equilibrium output if the central bank commits to a policy of price stability ($\pi_{t+i} = 0$ for all $i > 0$).

c. Solve for equilibrium output if the central bank commits to a policy of wage stability ($w_{t+i} = 0$ for all $i > 0$).

d. Briefly comment on how output responds to a productivity shock under the policy in part b. How does this response compare to the case under the policy in part c?

5. Suppose inflation adjustment is given by (11.22). The central bank's objective is to minimize

$$\left(\frac{1}{2}\right)\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i(\pi_{t+i}^2 + \lambda x_{t+i}^2)$$

subject to (11.22).

a. Using the programs available from Paul Söderlind's web site (http://www.hhs.se/personal/Psoderlind/Software/Software.htm), calculate the response of the output gap and inflation to a serially uncorrelated, positive cost shock for $\phi = 0$, 0.25, 0.5, 0.75, and 1 under the optimal discretionary policy.

b. Now do the same for the optimal commitment policy.

c. Discuss how the differences between commitment and discretion depend on ϕ, the weight on lagged inflation in the inflation-adjustment equation.

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