APPLIED MECHANICAL DESIGN

by

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&

M. R. Harris

"The essence of knowledge is, having it, to apply it."

Confucius
(551 – 479 B.C.)

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PREFACE

About the Authors

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Alan Hosking M.S.E.(Mech.) Lon; M.I.D. (Aust.) has had over 30 years experience on major mechanical and structural design projects in private industry and with the Commonwealth Government. For the past 11 years he has been a part-time Teacher in Mechanical Design with the Royal Melbourne Institute of Technology. During the same period he has served as the Examiner in Mechanical Design for the Education Department of Victoria and has worked on a number of educational committees.

Malcolm Harris Dip. Mechanical Eng. (G.I.A.E.) T.Tr.I.C. (Ed. Dept. - Vic.) has 10 years experience on Mechanical Design in private industry and 13 years teaching experience in the training of Mechanical Engineering Assistants and Design Draftsmen. For a number of years he served as Moderator with Alan Hosking on the preparation of Mechanical Design examination papers for the Education Department of Victoria. At present Head - Mechanical Engineering - Box Hill College of Technical and Further Education, Victoria.

How the book came to be written

The changeover to the SI system of Engineering units left a serious void in the availability of suitable text books for teaching and practical design aids. The effect was probably felt more severely in the Mechanical field than any other.

A successful design book must cater for "local" conditions i.e. availability and specification of raw materials and correct use of legally accepted Engineering Codes. Numerous "case history" design examples and exercises are essential also, if an "Applied Design" text book is to be useful. This book attempts to cover all the above requirements.

Acknowledgments

Department of Mechanical and Production Engineering - Royal Melbourne Institute of Technology.
The authors wish to acknowledge the assistance given by the above department in permitting their notes to be used in the preparation of the following chapters:

Chapter 5	Belts
Chapter 6	Chain drives
Chapter 8	Couplings
Chapter 11	Keys

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Box Hill, Castlemaine, Caulfield, Dandenong, Collingwood, Footscray, Geelong (Gordon Inst.), Moorabbin, Preston, R.M.I.T., Swinburne, Wangaratta and Yallourn.
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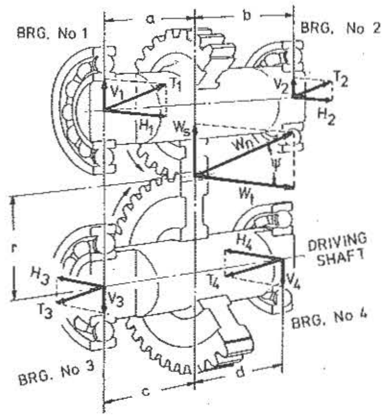
Engineering firms

Various engineering firms have contributed articles which have been included throughout this book. Acknowledgments are given in the appropriate chapters.



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General

Use of Codes

Extracts from various Australian and British engineering codes are used throughout this book.

These extracts are reproduced by permission of the

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and

British Standards Institution.

Because code material is reproduced in extract form, use out of context could result in misinterpretation. Readers are therefore advised to consult the standard code concerned when use in practice is contemplated.

Codes are subject to alteration.

Authors' Comments

The majority of answers to set exercises are calculated to a significant degree of accuracy for the benefit of students. This avoids any doubt as to correctness of method. In most practical applications there is no need for such precision.

It is the endeavour of the authors to keep this book abreast of continually changing technology. Comments and contributions will be welcomed and should be forwarded to the Publishers. Acknowledgment will be given as appropriate.

Despite all reasonable efforts by authors, publishers and printers, some errors may creep into the printing of any book; consequently any errors reported will be gratefully noted: subsequent printings will benefit greatly.

Alan K. Hosking.
Malcolm R. Harris.

CONTENTS

CHAPTER 8.

COUPLINGS.

Rigid, flexible and kinematic flexible. Selection criteria.

CHAPTER 9.

WORK, ENERGY AND TORQUE.

Design criteria. Worked examples and exercises.

CHAPTER 10.

STRESS CONCENTRATION.

Theory of static failure (maximum normal and shear stresses). Fatigue failure. Stress concentration. Design criteria. Dynamic loading. Impact effects. Worked examples and exercises.

CHAPTER 11.

KEYS.

Parallel, tapered, woodruff and round. Design criteria (strength, bearing and shear stress). Taper pins. Hardened set screw drives. Worked examples and exercises.

CHAPTER 12.

STRUCTURAL DESIGN TO AS 1250 - 1981.

Structural and machine design applications. Bolt and weld design. Tension members. Compression members (columns and struts). Beams (use of all B.H.P. Rolled Steel Sections, including square and rectangular bars and hollow sections). Encastre and continuous beams. Deflection of beams (area-moment method). Commercial, medium and high strength bolts. Worked examples and exercises.

CHAPTER 13.

SELECTION OF MATERIALS.

Factors affecting selection (cost, strength, rigidity, hardness, ductility, weight, wear and corrosion resistance, friction). Materials (structural steel, cast steel, cast iron, alloy cast iron, brass, bronze, aluminium, zinc, die casting). Surface hardening. Surface protection (galvanizing, zinc coatings, painting).

CHAPTER 14.

INERTIA.

Dynamics of a rigid body, translational, rotational and plane motion. Inertia-force method of analysis. Centre of percussion. Acceleration of a geared system. Choice of motor power. Worked examples and exercises.

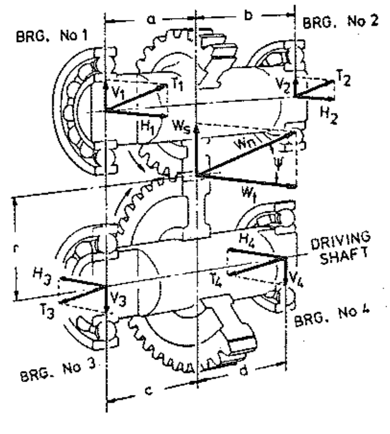
CHAPTER 15.

FRICTION EQUIPMENT.

Nature and laws of friction. Inclined plane. Power screws (square, acme, vee). Brakes (block, band, multi-block). Power ratings of friction materials. Linings. Energy dissipation. Clutches (plate, multi-plate, cone). Capstans. Design criteria and selection. Brake design to AS 1418 and AS 1735. Commercial selection of brakes. Worked examples and exercises.

CHAPTER 1

FUNDAMENTAL DESIGN PRINCIPLES and GENERAL DESIGN DATA



CHAPTER 1

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FUNDAMENTAL DESIGN PRINCIPLES AND GENERAL DESIGN DATA

1.1 SUMMARY OF FUNDAMENTAL TERMS

At the outset students should familiarize themselves with the following fundamental engineering terms.

Only the briefest summary will be given here. (Alphabetically listed)

Bearing Pressure (Also known as Bearing Stress)

Basically this equals load divided by projected area. For a bolt, rivet or pin,

$$\text{Bearing pressure} = \frac{\text{load}}{\text{diameter x length resisting load}}$$

Buckling, Elastic

This is where a member under load, recovers its original shape when unloaded.

Buckling, Plastic

This occurs when portions of a member are stressed beyond the elastic limit.

Creep

Some metals when subjected to a constant tensile load, at an elevated temperature, undergo a time-dependent increase in length, known as creep.

Concurrency, Point of

Where three non-parallel co-planar forces are in equilibrium the lines of action must intersect at a common point.

When two forces are in equilibrium they must be equal, opposite and co-linear.

Damping capacity

The amount of work dissipated into heat by a unit volume of material during a completely reversed cycle of unit stress. Refer to Chapter 13, page 13.5.

Dynamics

The study of forces which act on parts.
Dynamics may be subdivided into:

(i) statics - deals with bodies at rest or in equilibrium.
(ii) kinetics - deals with accelerating bodies, this involves mass and motion.

Efficiency (η)

$$\eta = \frac{\text{Useful work done}}{\text{Energy supplied}} \qquad \left(\eta = \frac{\text{M.A.}}{\text{V.R.}} \text{ where these terms are applicable}\right)$$

Elastic limit

The least stress that will cause permanent set.

Elasticity, Modulus of (E)

Also known as Young's Modulus.

$$E = \frac{\text{tensile stress}}{\text{tensile strain}}$$ (For steel this is determined within the elastic limit range.)

Endurance Limit

Also known as Fatigue Limit (see Fatigue Limit).

Equilibrant

This is defined as equal and opposite to the resultant. It is the force required to keep a system in equilibrium.

Equilibrium

A system of forces acting on a body produces equilibrium if, when acting together, the forces have no tendency to produce a change in the body's translatory (straight line) motion, or its rotatory motion.

Equilibrium, Condition for Static

The three laws of static equilibrium for a set of co-planar forces:-

(1) The sum of the forces in any chosen direction in the plane must be zero.

(2) The sum of the forces in any second chosen direction in the plane muat be zero.

(3) The sum of the moments about any point in the plane must be zero.

Equilibrium, Rotational

A body having a plane of symmetry is in rotational equilibrium if it is not rotating, or if it is rotating at constant angular speed about an axis of symmetry.

Equilibrium, Translational

A body is in translational equilibrium if it is at rest, or is moving at constant speed in a straight line.

Factor of Safety (F of S) or Factor, Safety

This is a number used to divide into the appropriate stress value of the material to obtain the allowable (sometimes called working) stress.

In design for strength, to avoid fracture or permanent deformation of parts subjected to static loads the appropriate property for brittle materials is the ultimate strength.

For ductile materials subjected to static loading it is the yield point. For cyclical loading, where infinite life is required, it is the endurance limit.

In Structural Engineering the F of S is 1.67. This is also referred to as the "load factor".

In Mechanical Engineering the general basic safety factor on endurance applications is 2.0.

Shock and service factors are applied to the load values and not to the allowable stresses.

Factor, Service (S.F.)

A number by which the actual power being transmitted through a driving element is multiplied to obtain the design power.
This number covers such considerations as type of prime mover, hours per day of use, etc.

Fatigue Limit

This is the maximum stress that a material subjected to a completely reversed load can withstand for an infinite number of load applications before failure will occur.

Free Body Diagram (F.B.D.)

Condition where an element, or part of an element, is taken from a mechanism or structure and the external forces acting on that element to keep it in equilibrium or condition of acceleration are shown.

Free body diagram analysis can also be done on complete assemblies, sub-assemblies or partial assemblies.

This method of analysis enables complex problems to be solved relatively simply.

Friction, Coefficient of (μ)

There are two values:-

(1) Coefficient of friction, static (μ_s).

This is the value commonly used. It is calculated for the conditions when the body is just on the point of sliding.

(2) Coefficient of friction, kinetic (μ_k).

This is calculated for the conditions when steady sliding has been attained. The value is always less than the static value.

Hooke's Law

For many materials stress is proportional to strain provided the stress is less than a value characteristic of the substance. This stress is termed the "limit of proportionality".

Inertia - Translational Case

A resultant force acting on a body moving with linear acceleration is balanced by the inertia "force" of the body. This inertia "force" acts in the opposite direction to the mathematical positive acceleration and through the mass centre.

The inertia - force method of analysis may be applied to a free body diagram. The rules for this procedure are given in the Design Technique Section (page 1.13).

Although in actual fact the inertia "force" does not exist, the advantage offered by the above method is that it reduces the problem to a study in statics i.e., it places the body in "equilibrium".

- Rotational Case (Bodies having a plane of symmetry and rotating about an axis of symmetry.)

A resultant torque acting on a body, such as defined above, moving with angular acceleration is balanced by the inertia "torque" of the body.

The Inertia "torque" acts in the opposite direction to the mathematical positive angular acceleration.

The Inertia "torque" is equal to I ϕ

where I = mass moment of inertia of body

ϕ = angular acceleration.

- Rotational Case. (Bodies having a plane of symmetry and rotating about an axis which is not an axis symmetry.)

Bodies rotating as above are referred to as compound pendulums. The theory for this case is more complex than for the previous one. A practical application is given in Chapter 14 page 14.5 (Centre of Percussion) and Worked Example No. 5 (page 14.11 and page 1.14).

Load Application

Depending upon the method of application of a load there are three classifications of stress.

(1) Static Stress
(2) Cyclic or Fatigue Stress
(3) Impact or Shock Stress

Mechanical Advantage (M.A.)

$$\text{M.A.} = \frac{\text{Load}}{\text{Effort}}$$

Modulus, Shear (G)

Also known as Modulus of Rigidity.

$$G = \frac{\text{shear stress}}{\text{shear strain}}$$ (For steel this is determined within the elastic limit range.)

Modulus, Young's (E)

Also known as Modulus of Elasticity.

$$E = \frac{\text{tensile stress}}{\text{tensile strain}}$$ (For steel this is determined within the elastic limit range.)

Proportionality, Limit of

The point at which a steel no longer obeys Hooke's law.

Reactions

Externally applied forces require externally applied reactions.

Internally applied forces are reacted out internally.

Resilience

This is defined as the quantity of stored energy in a member under load that is returned upon release of the load.

A material without resilience would be stressed infinitely if subjected to a shock load.

Resultant

- is the vector sum of a number of given forces.

Rigidity, Modulus of (G)

Also known as Shear Modulus.
(see Modulus, Shear)

Strength, Ultimate

Also known as Ultimate Stress. It is the stress that causes fracture. Ultimate-tension, compression or shear strength, may be defined as:

$$\frac{\text{maximum load}}{\text{original cross-sectional area}}$$

Stress, Allowable.

Also known as Safe Working Stress.

The portion of the Ultimate Strength, Yield Strength or Endurance Limit (as applicable) which may be safely used in design.

Stress, Axial

Acts parallel to a member's major axis.

Stress, Bearing

Also known as Bearing Pressure (see Bearing Pressure).

Stress, Cyclical or Fatigue

Stresses caused by loads that are not static but continuously vary in magnitude. The stresses may be totally tensile, totally compressive or there may be some degree of reversal.

In mechanical design we are concerned with any of the stress variations shown in Fig. 1 of Chapter 10 (page 10.1)

In structural design fatigue loading is uncommon.
Fluctuation in stress not involving tensile stress is not considered a fatigue situation. The exceptions to this rule are certain types of welding situations. Guidance on this point is given in the A.S. Structural Code.

Stress, Normal

(Due to bending in a beam.)
This is covered in detail in the structural section, Chapter 12, page12.5 et seq.
Acts normal (perpendicular) to the stressed surface.

Stress, Proof

The term pertains to acceptance tests of metals and specifies a tensile stress which must be sustained without deformation in excess of a specified amount, generally 0.2%.

Stress, Residual

These are those internal, inherent, trapped, locked up stresses that exist within a material as a result of conditions other than external loading.

Some causes of residual stress:-

(1) Cold working. Rolling or machining.

(2) Heating and cooling. Heat treatment, surface hardening, welding etc.

(3) Over stressing.

Stress, Shear

There are three fundamental types of shear stress.

(1) Direct shear stress.
(2) Torsional shear stress.
(3) Beam shear stress.

This is covered in detail in the structural section, Chapter 12, pages 12.60 to 12.63.

Stress, Temperature

Stress created when a body is heated or cooled and free expansion or contraction is prevented or restricted.

Stress, Ultimate

Also known as Ultimate Strength. (See Strength, Ultimate.)

Stress, Safe Working

Also known as Allowable Stress.

The portion of the Ultimate Strength, Yield Strength or Endurance Limit (as applicable) which may be safely used in design.

Stress, Yield

Only a few materials exhibit a true yield point. Yield stress is the lowest stress at which strain increases without increase in stress. For static applications of load using steel, the working stress is based on yield stress.

Velocity Ratio (V.R.)

$$\text{V.R.} = \frac{\text{Distance travelled by effort}}{\text{Distance travelled by load in same time}}$$

Yield Point

The point at which a material reaches the plastic state, i.e. permanent change of form as a result of stress. The property of "plasticity" may be regarded as the exact opposite to that of "elasticity". A sudden stretch is observed at yield point.

See also Stress, Yield.

1.2 INTRODUCTION TO DESIGN

1.2.1 DESIGN THEORY

Definition of Design

The process in which one uses the tools of Engineering - Mathematics, Graphics, English, Scientific Principles in order to produce a plan, which when carried to completion, will supply a practical need.

Design Cycle

A machine is designed by creatively blending theory and experience and applying it to known materials and existing fabricating processes. The "new" machine builds up experience and may lead to further knowledge of related theory, thus completing a cycle as indicated in Fig. 1.

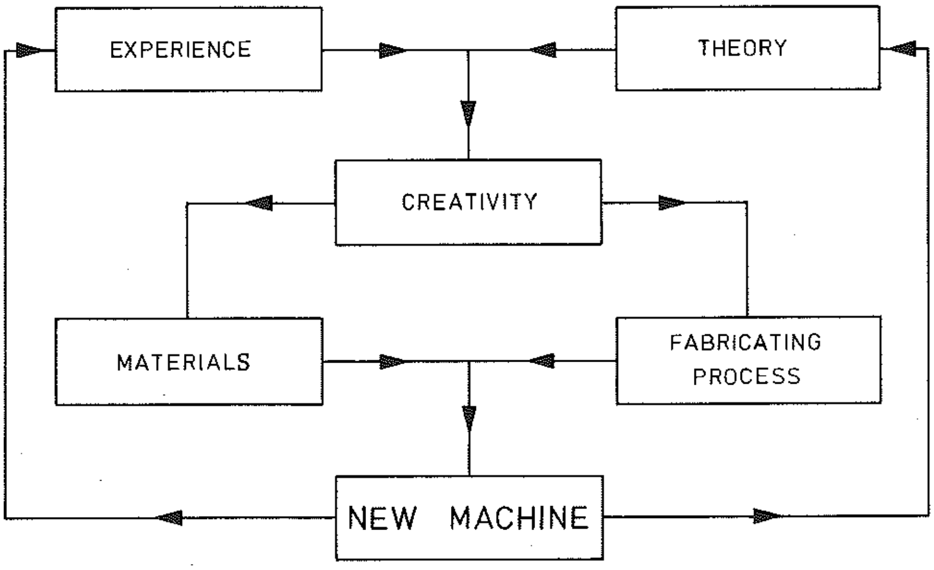


FIG. 1

Aspects of Design

1. Design problems have no unique answers. This is contrary to scientific problems. There are possibly many good solutions to design problems. Even a "poor" solution today may be a "good" solution tomorrow. The aim in this book is to study the method of obtaining various satisfactory solutions to machine design problems.

   This could include the use of such knowledge already acquired in the study of Mechanics, Thermodynamics, Heat Transfer, Material Manufacturing Processes, Combustion and effects of Vibration to name a few. Mechanical design covers a broad spectrum of activity: Research Projects; Nuclear and conventional Power Plants; Turbines, Turbojets, Rockets, Production Machinery, Automobiles, Washing Machines, etc.

Design Decisions

To design is to make decisions. Considering automobile manufacture: there are numerous types all successfully serving a need, but each type has its own identification due to different design decisions, i.e., valve arrangement, piston sizes, etc.

When considering the variety of products produced such as lawn mowers, bicycles, pumps, lathes, etc; it can be seen that each design has many solutions. A decision one makes influences other decisions in the design of a product, i.e., an automotive manufacturer makes a decision to use a compression ratio of 7.8 instead of 7.2. The higher compression ratio places more force on pistons, block, conrods etc., creating the need for stronger construction requirements, heavier materials etc.

It is therefore apparent that just one design decision difference can greatly change similar products.

Poor decisions will result in poor design. New materials, better mechanical arrangements, etc., will affect the design and bring about improvements in products.

Decisions are always compromises
i.e., the Engineer prefers the stronger material but it is expensive so he compromises. He prefers light weight, but not too much deflection so he compromises.

Thus design is a study of decisions, which requires a broad comprehensive knowledge, a wealth of experience and careful judgment; but most of all creative imagination.

The Nature of Creative Thinking

1. Preparation

   - exploration of all possible solutions to a problem.

2. Incubation

   - the sorting of the solution.

3. Illumination

   - the recognition of "the" suitable solution.
     This may happen instantly or over a period of time.

4. Verification

   - solution analysed, tested and found to be satisfactory.

Creative thinking in Engineering, in general, follows three forms:

a) The improvement of existing solutions

   (i) Methods which employ theory or laws

   - mathematical laws may be used, but this often involves somewhat arbitrary and not well founded assumptions, such as friction, air and wind resistance.

   This simplifies the problem but it also limits its scope.

(ii) Methods which employ experimental evidence

i.e., small scale models under controlled laboratory conditions.

(iii) Empirical Methods

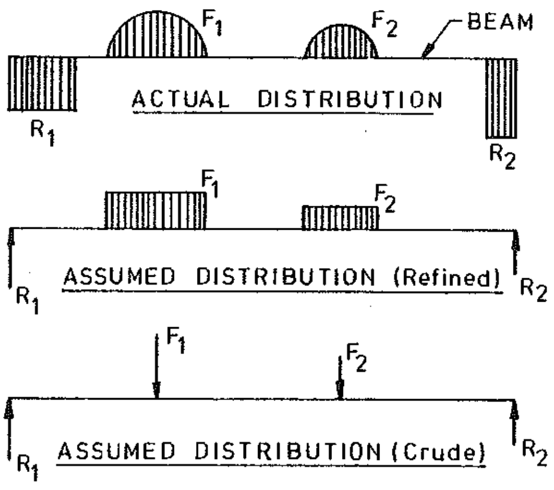
- the Design Engineer uses his experience and sense of "feel" to solve a problem. Empirical methods are used where a new field of work is investigated, i.e., the first internal combustion engine was built before the study of combustion had occurred.

The degree of empirical method used in design has been tremendously reduced during the past 20 to 30 years as the result of analytical and experimental techniques.

(b) The analysis of particular problems

Simplifying assumptions
- simplifying the problem so that it can be analysed by known methods.

See Fig. 2 for example.



The assumption (simplification) selected, depends on the degree of accuracy required.

FIG. 2

(c) The shaping and synthesizing of the parts

The bringing together of all design aspects during or before analysis.

Design Procedures

Students can build up experience by observation in laboratories, workshops, industrial visits, critical examination of machine units, and where possible, by disassembling machines and components.

Design does not consist of a fixed series of steps which must be followed in order to provide a solution. However a logical sequence of steps does exist. In some cases, one or more of the steps may be omitted.

1. Need - Design seeks to satisfy a need.

2. Defining the Problems - The machine is defined in terms of power input and output; use, weight, etc; noise level, environmental conditions; or other limiting factors. Specifications should be as accurate as possible.

3. Examination of Feasibility - Meeting the need within the limits of the specifications may not be feasible for economic or technical reasons.

4. Preliminary Design Alternatives - Generally various solutions exist. Elements can be arranged in various ways to convert the input into the required output.

5. Final Design Alternatives - Upon more detailed investigations, some of the possibilities in (4) cannot satisfy design requirements. Actual mechanical design is carried out here with a thorough analysis of strains and stresses. Strength, wear, rigidity, corrosion resistance, noise level etc., must all be checked.

6. Final Design Selection - The alternatives in (5) all satisfy the specification and represent suitable solutions. One of these must now be selected as the final design. In industry, this may involve management, sales, servicing, and other divisions in addition to the engineers. It will be necessary to produce some drawings (e.g. assembly drawings) of the various alternatives.

7. Plans & Drawings - A layout or assembly drawing often indicates assembly difficulties, helps in visualizing final shapes, aids in shaping frames etc. The drawings are an economical and simple means of communicating ideas and information, hence they must be

   CLEAR, CONCISE, COMPLETE.

   Enough views, sections and notes must be given to show all detail. The main view of a part should show it in the position it will occupy in the machine. There should be no occasion for questions, guessing or scaling.

   Revision of the drawings must be allowed for, due to faults picked up in the assembly stage; improvements in production or assembly methods, and so on. Hence all drawings require a clear numbering of reference system.

8. Manufacture or Production - As mentioned above, some revision of steps (6) and (7) may be necessary.

1.2.2 GENERAL PRINCIPLES

Design work in general involves 3 criteria (in reference to member sizing).

(1) Stress

(2) Deflection

(3) Proportion

These should be considered in each and every design and checked where applicable.

(1) Stress

This check is an obvious and primary requirement. Any machine or structural element will yield or fail (depending upon the type of material and type of load application) once a certain stress is reached. A load factor or safety factor will prevent this condition.

(2) Deflection

Large deflections in one member can cause dangerous strains and secondary stresses in other members of a structure. This is because the nature of the normally operating forces in the network can be altered. Elastic failure of the assembly then becomes a possibility.

For these reasons, deflections should be kept within the normally accepted limits for each particular application.

(3) Proportion

(a) Sometimes members can fail at stresses much lower than normal design methods would indicate. This is because elastic buckling has occurred due to the member being too slender i.e., incorrectly proportioned.

(b) Occasionally a member size has to be increased over design to give a pleasing appearance to the final assembly.

(c) Member size is sometimes governed by welding or jointing requirements etc.

Structures

If a structure as a whole is in equilibrium then any joint, sub-assembly, element or portion of an element isolated as a free body diagram must also be in equilibrium.

Machine Design

(1) It is a principle of machine design to design all members for the motive power (e.g. kW rating of the electric drive motor) - efficiencies and design factors considered if appropriate. It is incorrect to design the machine for the actual power requirement of the operation being performed; the motive power is invariably larger because:-

(a) Efficiencies have to be considered.

(b) The actual power requirement at the input rarely corresponds exactly to stock sizes of motors and engines.

The nearest size of motor or engine above requirements is generally chosen.

(2) In any machine design problem involving varying load conditions, the maximum condition is taken and assumed as constant.

(3) Any prime mover must supply sufficient starting torque to satisfactorily bring the machine to working speed. The starting torque must overcome working load and friction plus inertia of all components in the mechanism. (This principle is demonstrated in Chapter 2, pages 2.36 to 2.40 and Worked Example in AS 1403 Shaft Design Code.)

As a general guide, if the prime mover gives 150% of the required full load normal working speed torque, then this covers most applications.

IMPORTANT:

In using the various formulas of design ensure that the units are consistent.

1.3 DESIGN TECHNIQUE

1.3.1 GENERAL TECHNIQUES

Form of Computations

1. Each sheet should carry the designer's name, date, and consecutive numbers if more than one sheet is used.

2. One side only of the paper should be used and a 25 mm strip on the left edge of the sheet should be reserved for binding.

3. If possible, a sketch should be drawn for every problem. The sketch should be neat and correctly drawn and may be used to show the arrangement of the parts and such data as dimensions, speeds, materials, etc. For data that cannot be shown on the sketch, a list should be used.

4. Each step should be set off, labelled or lettered, so that divisions of work can be readily identified. This corresponds to using paragraphs in ordinary written matter and is helpful in making computations understandable.

5. All pertinent assumptions should be stated and references given.

6. Equations should be solved for the unknown quantity before numerical substitutions are made, wherever possible.

7. The numerical substitutions should follow the literal equations and be in order, term for term.

8. Units should not be placed in numerical equations. If a check on units is desirable, a separate dimensional equation should be used.

9. Units should be indicated for every separate value or result that has units whether it be data, intermediate calculated value, or final result.

10. Long equations should be avoided if possible; instead, intermediate values or particular physical quantities should be calculated separately before being inserted into the parent equation.

11. An unjustified number of significant figures should not be used.

12. Calculated dimensions of parts should be rounded out to nominal sizes and indicated, as, for example,

    $$d = 2.95; \text{ use } 3.$$

13. Values in an equation in formal computations should not be cancelled. Cancelling values would be confusing. Do any cancelling off to one side.

14. Where at a later stage a revised computation is necessary the original computation should be kept for possible future reference and the revised computation added as an errata sheet. This enables comparisons to be made.

15. All results should be considered from the standpoint of their reasonability.

Methods of Problem Solving

A. For the Mathematical Case use the following method.

(1) Draw a diagram. Add the relevant information.

(2) Rule three columns.

Column (a) Write all the formulas directly applicable to that situation.

Column (b) List all the known values.

Column (c) List the requirements.

(3) Proceed with the calculations. Form equations where necessary.

B. For the Inertia Case use one of the following methods.

Using the inertia-force method applied to a free body diagram.

(1) Draw a free body diagram of the element or one for each element in the case of a system.

(2) Place on the diagram the values of each force that is external to that element and acting directly on the element. Self weight is an external force.

Translational Motion - Rules

(a) The force causing acceleration acts in the direction of the mathematical positive acceleration. It is always an external force. (See Fig. 3.)

(b) The inertia "force" acts in the opposite direction to the mathematical positive acceleration and through the mass centre.

(c) Friction always opposes the direction of the motion.

(d) A force has no effect in a direction at right angles to itself.

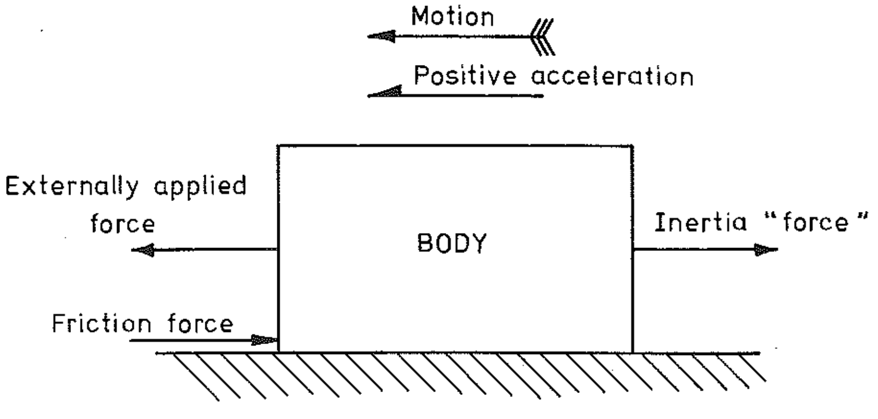


FIG. 3

Rotational Motion - Rules

(a) The torque causing angular acceleration acts in the direction of the mathematical positive angular acceleration. It is always an external torque.

(b) (i) The inertia "force" acts in the opposite direction to the mathematical positive angular acceleration and through the centre of percussion. This represents a compound pendulum arrangement. (See Fig. 4a)

(ii) If a body has a plane of symmetry and rotates about an axis of symmetry (in this case there is no centre of percussion) it is only necessary to apply a reversed couple (the inertia "torque") to hold the system in equilibrium. Apply this couple in the opposite direction to the mathematical positive angular acceleration. (See Fig. 4b)

(c) Friction always opposes the motion.

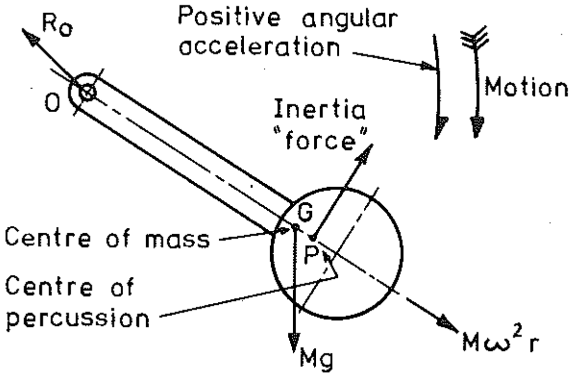


FIG. 4(a)

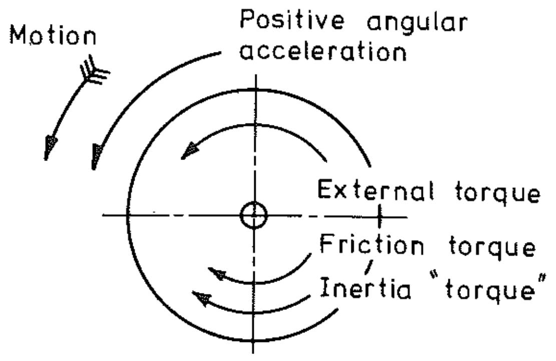


FIG. 4(b)

(3) Apply the rules of static equilibrium.

NOTE: The equations of rectilinear and/or rotational motion will have to be applied at the appropriate stage.

C. For cases involving Mechanisms, Structures, Problems in Statics use Free Body diagrams. (Refer page 1.3 and Chapter 16 Worked Example No. 2, page 16.7) The diagram can be applied to the complete assembly, a sub-assembly or each individual element.

D. Check Your Answers

In practical application always check your answer where possible.

This can be done two ways:-

(1) Do the problem again using a completely different method.

(2) Balance your forces e.g., with a framework loaded with an external load:

(a) calculate the external reactions.

(b) start again at the load application point and work through all members to each support and check whether the vector addition of all the member forces at the point equals the numerical value of the reaction.

1.3.2 SPECIFIC TECHNIQUES

Two-Force Member

Forces act at two points only, usually at its ends. When isolated as a free body, it is then in equilibrium under the action of two forces. These forces must be equal, opposite and co-linear. When a two force member is cut, the stress within it is known to act along its axis.

Three-Force Member

Forces act at three or more points. Forces act transversally to its axis, causing it to bend therefore the stresses are complex - a combination of axial, bending and shear.

A three-force member should never be cut during a force analysis. The reacting forces at any support of a three force member can never be parallel to the axis of the member.

Substituting a given Force by a Force and a Moment

Fig. 5. shows a body having a force P applied at A.
This system may be replaced, for the convenience of problem solving, by the arrangement shown in Fig. 6 with force P and Moment Pd applied at B.

Both systems are equivalent.

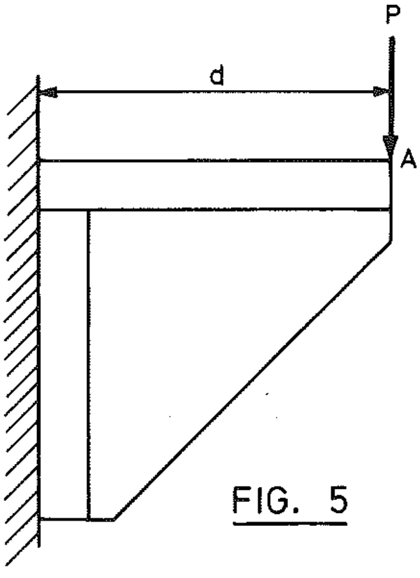


FIG. 5

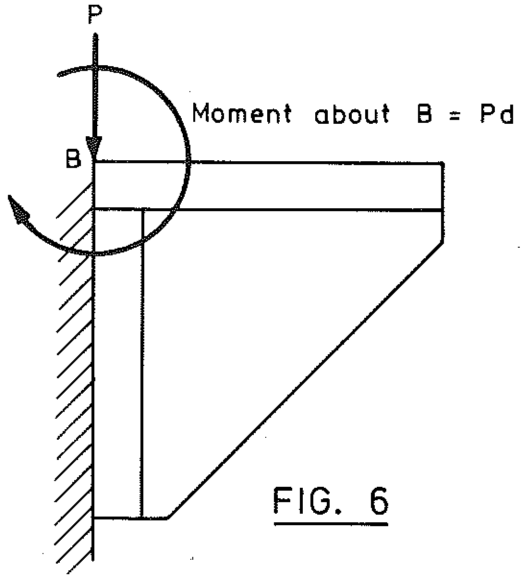


FIG. 6

Superposition

With certain exceptions, the effect (stress, strain, deflection) produced on an elastic system by any final state of loading is the same whether the forces that constitute that loading system are applied simultaneously or in any given sequence.

The final result can be obtained by algebraic or graphical addition of the effect of each single force.

This theorem does not apply if:-

(a) The applied forces cause deformations that produce other forces e.g., an arch.

(b) One force alters the character or effect of another force.

Method of Joints

Never cut through a member in which a bending moment acts.

Should a pin ended member be subjected to bending it is permissible to disconnect the member at the pins and analyse it as a free body diagram.

A member in which a bending moment acts places an "inclined" force on the joint (not a purely axial force). Therefore, if the joint is considered in isolation this "inclined" force must be used.

Tension and Compression Members in Structures and Mechanisms

The usual method of indicating whether a member is a tension or compression element is to place arrows on the member near the joints. Refer to Fig. 7.

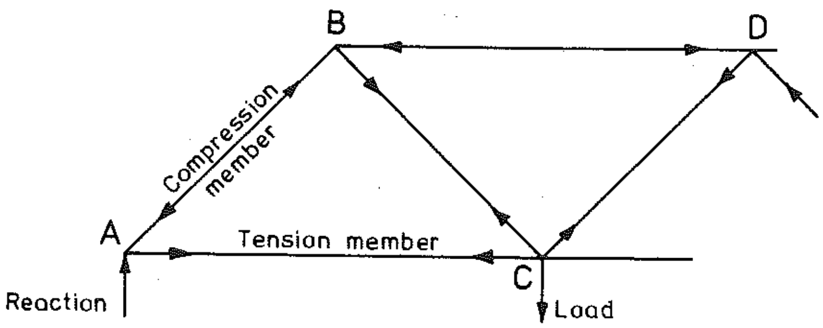


FIG. 7

An arrow head near a joint pointing towards the joint shows the member on which it lies is in compression; an arrow head away from the joint shows the member to be in tension.

On first sight it would appear that member AC is in compression and members AB and BD are in tension but the arrows indicate the direction in which the member is acting on the joint.

Frictionless surface

Where friction at a reaction is ignored then that reaction can only be normal to the surface
OR
a frictionless surface can only give a normal reaction.

Miscellaneous

A force may be replaced by its components without changing its total effect.

The sum of the potential and kinetic energy of a body acted on by gravity alone is constant.
The point of application of an external force acting on a body may be transmitted anywhere along its line of action without changing other external forces acting on the body.

1.4 COMMONLY USED FORMULAS AND VALUES

Students of mechanical design will find the following formulas and values handy for quick reference (no explanation of symbols is given).

These formulas and values are either not mentioned specifically in any of the following chapters or may be used in several different situations.

E steel = 207 000 MPa (AS 1250 uses E = 200 000 MPa)

G steel = 82 700 MPa

MPa = p.s.i. x 0.006 895

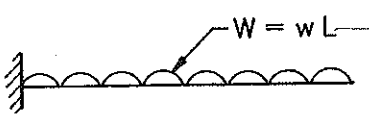
1 lb. f = 4.448 N

Pa = 1 N/m^2

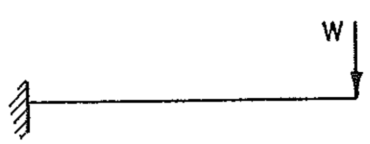
kPa = 10^3 N/m^2 = kN/m^2

MPa = 10^6 N/m^2 = MN/m^2 = 1 N/mm^2

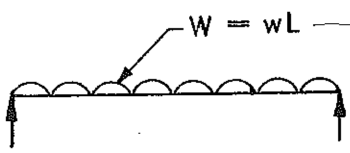
1 H.P. = 0.746 kW



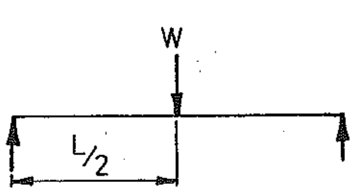
Max. B.M. = $\frac{WL}{2}$ or $\frac{wL^2}{2}$ Max. Deflection = $\frac{WL^3}{8\ E.I.}$



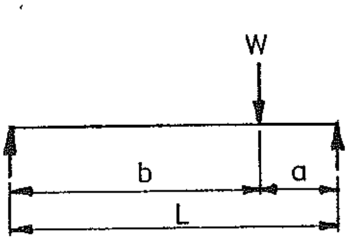
Max. B.M. = WL Max. Deflection = $\frac{WL^3}{3\ E.I.}$



Max. B.M. = $\frac{WL}{8}$ = $\frac{wL^2}{8}$ Max. Deflection = $\frac{5}{384}\ \frac{WL^3}{E.I.}$



Max. B.M. = $\frac{WL}{4}$ Max. Deflection = $\frac{WL^3}{48\ E.I.}$



Max. B.M. = $\frac{Wab}{L}$

Deflection at centre =

$$\frac{WL^3}{48\ E.I.}\ \left(\frac{3a}{L} - 4\left(\frac{a}{L}\right)^3\right) \text{-------}$$

-------This value is always within 2.5% of the maximum value.

FIG. 8

Other standard B.M. and deflection formulae can be found in various structural design books.

$$v = u + at$$
$$s = ut + \tfrac{1}{2} at^2$$
$$v^2 = u^2 + 2as$$
$$s = \frac{(u + v)\,t}{2}$$

$$\omega_2 = \omega_1 + \phi t$$
$$\theta = \omega_1 t + \tfrac{1}{2}\phi t^2$$
$$\omega_2^2 = \omega_1^2 + 2\phi\theta$$
$$\theta = \frac{(\omega_1 + \omega_2)\,t}{2}$$

$$F = Ma$$
$$B.M. = FZ$$
$$Z = \frac{I}{y}$$
$$Z = \frac{\pi d^3}{32}$$ For round shaft
$$Z_p = \frac{\pi d^3}{16} = \frac{2J}{d}$$
$$Z = \frac{bd^2}{6}$$
$$I_{zz} = I_{xx} + I_{yy}$$
$$I = Ak^2$$
$$arc = r\theta$$
$$a = r\phi$$
$$F = \mu R$$

$$T = I\phi$$
$$T = F_s Z_p$$
$$\theta = \frac{T\ell}{GJ}$$
$$I = \frac{\pi d^4}{64}$$
$$J = \frac{\pi d^4}{32}$$
$$I = \frac{bd^3}{12}$$
$$I_{qq} = I_{xx} + Ah^2$$
$$F = M\omega^2 r = \frac{Mv^2}{r}$$
$$v = r\omega$$
$$\pi^c = 180^\circ$$
$$1^c = 57.3^\circ$$

$$I = Mk_o^2$$
$$k_o = \frac{d}{\sqrt{8}}$$
$$k = \frac{d}{4}$$
$$I_h = \frac{\pi(d^4 - di^4)}{64}$$
$$Z_h = \frac{\pi(d^4 - di^4)}{32d}$$
$$k_h = \frac{\sqrt{d^2 + di^2}}{4}$$
$$J_h = \frac{\pi(d^4 - di^4)}{32}$$
$$Z_{ph} = \frac{\pi(d^4 - di^4)}{16d}$$
$$k_{oh} = \sqrt{\frac{d^2 + di^2}{8}}$$

$$P = \frac{T\omega}{1000} = \frac{2\pi rNF}{60 \times 1000} = \frac{2\pi NT}{60 \times 1000}$$
$$\omega = \frac{2\pi N}{60}$$
$$P = \frac{Fv}{1000} = \frac{Fs}{1000t}$$

$$K.E. = \tfrac{1}{2} Mv^2 \text{ and } \tfrac{1}{2} I\omega^2$$
Surface speed = $N \times \pi d$
$$P.E. = W \times h$$

Change in K.E. = Work done

Unit of work = N.m = Joule

1 Joule/sec = 1 Watt

1.5 PERMISSIBLE STRESSES AND SAFETY FACTORS FOR ALL COMMON MACHINE ELEMENTS

Table No. 1. has been compiled detailing references to safety factors and permissible stresses for all common machine elements.

For simplicity the aim is to keep one stress level for each particular application applying such shock and service factors to the basic load on each element as required. (Do not apply factor upon factor through a system.)

The basic load is calculated from the motor (or other motive) power (efficiencies considered where appropriate).

Shock Factors and Service Factors (Refer Table No. 1.)

In many design situations these factors are not required.

For the choice of proprietary items such as gear boxes, couplings etc., the Manufacturer's recommendation should be accepted.

A Designer's knowledge of the job should also be a consideration.

TABLE No. 1

(See also Section 1.7, page 1.27 & Appendices No. 3 & No. 4)

ELEMENT	SAFETY FACTOR	APPLICATION OF SHOCK FACTOR OR SERVICE FACTOR	PERMISSIBLE STRESS
BEARINGS			
1. Anti-friction	-	To basic bearing load.	Permissible loads from Manufacturer's catalogue.
2. Plain	-	To basic bearing load.	Permissible loads from Manufacturer's catalogue. For M.S. and gunmetal or Phos. bronze elements refer to Chap. 7, pages 7.22 and 7.23.
BELTS			
1. Flat	-	To power transmitted	Working tension N/mm of width/ply from Manufacturer's catalogue.
2. Vee	-	To power transmitted	Choice from Manufacturer's catalogue.
CHAIN DRIVES	Approx. 10. (see Note 1) On ultimate strength.	To input power see Table No. 5 Chapter 6, page 6.8	Breaking strength from Manufacturer's catalogue.
COUPLINGS	-	To power transmitted	Choice from Manufacturer's catalogue.

Table No. 1. (Cont.)

ELEMENT	SAFETY FACTOR	APPLICATION OF SHOCK FACTOR OR SERVICE FACTOR	PERMISSIBLE STRESS
ELECTRIC MOTORS	-	-	Can run continuously at about 105% full load torque. Momentary overloads say of 5 seconds duration at 250% full load torque twice per hour acceptable.
GEARS			
Spur and helical	Item 64 BS 436	To input power to pinion. Table No. 1 Chap. 3 page 3.3	Refer to BS 436 and Table No. 4 page 1.26
GEAR BOXES	-	To input power	Choice from Manufacturer's catalogue.
KEYS			
(a) Bearing on steel	*2.0	To power transmitted	(a) Bearing 0.5 F_y or 0.6 F_y (see Note 2)
(b) Bearing on C.I.	*6.0 on ultimate compressive stress.		(b) 138 - 166 MPa
(c) Shear on key	*2.0		(c) Shear 0.25 F_y
	(*see Note 2 page 1.22.	Also page 2.11.)	
SHAFTS			
1. Rotating solid and hollow	2.0 (see Note 3 page 1.23. Also page 2.11.)	K_m and K_t and stress concentration factors all applied to the basic torque and/or B.M. Table No. 1 Chapter 2, page 2.6 and Charts 6 to 16, in Chapter 10.	Maximum permissible shear stress $= \dfrac{\text{endurance limit, } F_e}{2 \times 2}$ for shafts subjected to combined loading or torque only. For shafts subjected to bending only, maximum permissible tensile stress = $\dfrac{\text{endurance limit, } F_e}{2}$ In all cases where a keyway exists at the section under consideration multiply the maximum permissible stress x 0.75

Table No. 1. (Cont.)

ELEMENT	SAFETY FACTOR	APPLICATION OF SHOCK FACTOR OR SERVICE FACTOR	PERMISSIBLE STRESS
2. STATIONARY			
(a) Solid * Loads must be essentially static.	1.67 (see Note 4 page 1.23)	K_m and K_t and stress concentration factors all applied to the basic torque and/or B.M. Table No. 1 Chapter 2, page 2.6 and Charts 6 to 16 from Chapter 10.	Maximum permissible shear stress = $0.3F_y$ for shafts subjected to combined loading or torque only. For shafts subjected to bending only, maximum permissible tensile stress = $0.6\ F_y$. In all cases where a keyway exists at the section under consideration multiply the maximum permissible stress x 0.75.
(b) Hollow * Loads must be essentially static.	1.92 (see Note 5 page 1.24)	K_m and K_t and stress concentration factors all applied to the basic torque and/or B.M. Table No. 1 Chapter 2, page 2.6 and Charts 6 to 16 from Chapter 10.	Maximum permissible shear stress = $0.26\ F_y$ for shafts subjected to combined loading or torque only. For shafts subjected to bending only, maximum permissible tensile stress = $0.52\ F_y$. Keyway allowance as above.

* Stationary shafts with reversing loads or severely fluctuating loads should be designed for endurance, see "Shafts 1. Rotating".

SPRINGS	1.5 to 2.0	Designer's decision on applied load. (Definitions of service given in Chapter 4 page 4.6.	For general purpose springs made from "Carbon steel spring wire Table 1 - AS 1472 - 1979" refer Table No. 2, page 1.22.
STRUCTURAL (Static)	Basic load factor implied by AS 1250 Structural Code = 1.67 (see Note 6, page 1.24)	To primary load	Refer to AS 1250 - 1981 Structural Code.
*STRUCTURAL (Endurance)	-	To primary load	Refer to AS 1250 - 1981 Structural Code Appendix B

*May also be useful in some mechanical situations

TABLE NO. 2

Spring design stresses. (Ref., Westinghouse Electric Corporation.)

Material: Carbon steel spring wire Table 1 AS 1472 - 1979

WIRE DIAMETER mm	SEVERE SERVICE MPa	AVERAGE SERVICE MPa	LIGHT SERVICE MPa
Up to 2.16	414	520	640
2.17 to 4.70	380	475	585
4.71 to 8.13	330	414	510
8.14 to 13.46	290	360	450
13.47 to 24.6	250	310	385
24.7 to 38	220	275	345

NOTE: 1

F of S = 10 by common usage. Covers wear effects, poor lubrication and dynamic effects. Applies mainly to chains in "open" drive situations.

Safety factors from Fenner Dodge roller chain catalogue are as follows:

up to approximately 12 on small chains (say 12.7 mm pitch)

down to approximately 7 on large chains (say 25.4 mm pitch).

NOTE: 2

The design of keys involves two checks:-

(i) Bearing stress on key or as is more often the case the bearing stress on the weaker element, the shaft or C.I. or steel sprocket etc.

Bearing stresses are simply compressive stresses which occur over short longitudinal lengths and in rather confined stress situations. Because of this confinement, transverse compressive stresses are also developed and so the effective yield stress of the steel is raised.

Thus the F of S of 2.0 (an apparent factor) would increase. Practical application has shown that it is quite satisfactory to have working bearing stresses up to 20 per cent above 0.5 F_y if necessary in high torque situations. (See footnote.)

Maximum permissible compressive stress on grey C.I. (say Meehanite G.E.) = 138 MPa (Crane Code C.B. 2 Rule 4.20).

Ultimate compressive stress of above C.I. = 825 MPa.

$\therefore$ F of S $= \dfrac{825}{138} = 6$ (apparent factor) on ultimate strength.

(Footnote. The 20% increase brings bearing stress level to that recommended by U.S. navy design practice.)

(ii) Shear stress on key

Key steel is generally an imported steel. One key steel is an American steel sold under the trade name "MAK-A-KEY".

Specifications:-

Ultimate tensile	517-586 MPa
Yield tensile	414-480 MPa
Brinell hardness	165-180

By using the maximum-shear-stress theory maximum permissible shear

$$\text{stress} = \frac{F_y}{2 \times 2} = 0.25\ F_y.$$

SUMMARY:-

The following stresses can be commonly used in key design.

(a)	Bearing on F_y 250 M.S. and C.S.	125 MPa-150 MPa	Assuming normal drive conditions.
(b)	Bearing on grey C.I.	138 MPa-166 MPa	
(c)	Shear stress on "MAK-A-KEY" steel	100 MPa	

NOTE: 3

The basic safety factor of 2.0 has been chosen arbitrarily to cover inertia of starting and miscellaneous unknowns.

The maximum-shear-stress theory has been used to determine the allowable shear stress for the torsion and combined torsion and bending cases. The factor here is 0.5 applied to the allowable stress in tension/compression (endurance).

The maximum-shear-stress theory states that yielding of a steel specimen subjected to a combined stress condition takes place when the maximum shear stress reaches a value equal to the shear stress at yielding in a simple tensile test.

In practice the maximum-shear-stress theory errs on the side of safety when there are principal stresses of unlike signs such as exist in a shaft under combined loading.

(In other engineering applications, where principal stresses have like signs the maximum shear stress depends on the maximum principal stress alone and the maximum-shear-stress theory is less conservative.)

More exact analysis by other theories, namely the "Mises criteria" and the "Distortion-energy theory" shows that the 0.5 factor referred to above may be increased to 0.58 and 0.577 respectively.

No account is taken in the approach to shaft design of the frequency of application of stress reversals.

NOTE: 4

Maximum permissible tensile stress in a round beam = $0.75\ F_y$ AS 1250 5.2 (1).

Using a factor of 1.25 to allow for machine design considerations give a tensile

$$\text{stress of} \quad \frac{0.75}{1.25} F_y = 0.6\ F_y.$$

Using the maximum-shear-stress theory (refer Chap. 2, page 2.2) gives a maximum permissible shear stress of 0.3 F_y.

$$F \text{ of } S = \frac{1}{0.6} = 1.67$$

NOTE: 5

Maximum permissible tensile stress in a round hollow beam

$= 0.66\ F_y$ AS 1250 5.2 (2).

Using a factor of 1.25 to allow for machine design considerations gives a tensile stress of $\frac{0.66}{1.25}\ F_y = 0.52\ F_y$.

Using the maximum-shear-stress theory (refer Chap. 2, page 2.2) gives a maximum permissible shear stress of 0.26 F_y.

$$F \text{ of } S = \frac{1}{0.52} = 1.92$$

This F of S is higher than for solid shafts because of the lower reserve "plastic" capacity possessed by the hollow shaft.

GENERAL COMMENTS OF NOTES: 3, 4 and 5.

The maximum-shear-stress theory (Guest's theory) has been chosen for simplicity. Where the maximum-shear-stress theory is not used to determine the allowable working shear stress for the torsion and the combined torsion and bending cases the maximum principal stress may determine the shaft size under certain combinations of loading.

All shafts should be checked for deflection.

Beam Shear — This is rarely a consideration in shaft design. If this check is required the true maximum beam shear stress should be limited to the shear stress figures quoted in Table No. 1. This procedure will err on the side of safety.

NOTE: 6

AS 1250 allows 0.6 F_y as the permissible stress in tension or compression.

$$0.6\ F_y = \frac{F_y}{F \text{ of } S}$$

$$\therefore\ F \text{ of } S = 1.67$$

Welds in endurance applications

Machine design

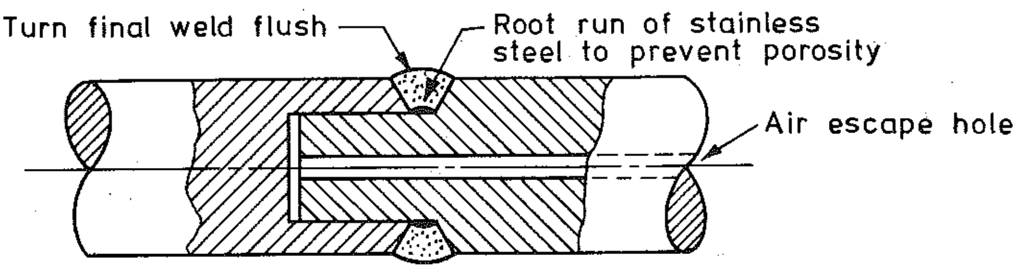
Transverse fillet welds should never be used for fatigue applications because of dangerous stress concentrations at the toe of the weld. Endurance applications (e.g. shafts) where the weld is in the combined (complex) stress condition should be avoided where possible. Maximum permissible stress permitted where welds are in fatigue situations:-

Max. stress $= \frac{1}{5}$ of endurance limit of weld metal or parent metal, whichever is least (Reference: Engineering Testing and Research Services Pty. Ltd.)

With M.S. this means (Refer also to pages 12.55 and 12.56.)

$$\frac{207}{5} = 40 \text{ MPa tension/compression}$$

$$= 20 \text{ MPa max. permissible shear stress}$$



TYPICAL JOIN IN A ROTATING MILD STEEL SHAFT

FIG. 9

Steel specifications

For static applications where the permissible stress is based on F_y, Appendix C of AS 1250 gives details. F_y is governed by material thickness and the maximum section thickness must be used.

Common structural mild steels are:

grade 250 AS 1204, sections and plates
grade 250 & 200 AS 1163, hollow sections.

Endurance limit

According to a paper titled "Shortcuts for Designing Shafts" by H.A. Borchardt, published in Machine Design, Vol. 45, No. 3, 8th February 1973:

The fatigue limit of polished steel is more closely related to ultimate tensile strength than any other characteristic. Unless experimental data is available, its value may be taken as 45% of the tensile strength for values up to 930 MPa. The ratio decreases beyond this range. A fine machined finish gives only slightly inferior results.

Strength properties are also size dependent; refer to "Size factor", Chapter 2, page 2.30.

The information given in Table No. 3, page 1.26, applies to bright ("turned") steels and finely shop machined hot-rolled (black) steels.
The values quoted are minimum and conservative.
Manufacturing processes which improve the properties of a material (cold rolling, cold drawing etc.) are not taken into account.

TABLE NO. 3

Steel	Ult. (F_{ult})	Yield (F_y)	End. Limit (F_e)
Hot rolled structural steel AS 1204 grade 250	410 MPa	250 MPa	207 MPa
Common shafting materials			
bright steel AS 1443-1983 black steel AS 1442-1983			
- CS 1020	400 MPa	200 MPa	180 MPa
- CS 1030	500 MPa	250 MPa	225 MPa
- CS 1040	540 MPa	270 MPa	243 MPa

All values are "average" values and are size dependent.

TABLE NO. 4

Gear design factors

Values for other materials can be found in Appendix A of BS 436.

Materials can be the same for a pinion and its mating gear.

Material	S_c	S_b
Cast iron (not heat treated) 185 MPa tensile	6.9	40
Mild steel, grade 250 AS 1204	7.0	96
CS 1020 AS 1442 AS 1443	7.0	96
CS 1030 AS 1442 AS 1443	8.3	114
CS 1040 AS 1442 AS 1443	9.7	130
En 8 (normalised)	9.7	130
Cast steel (not heat treated) 600 MPa tensile	9.7	130
Steel - normalised, forged 0.4 C	9.7	117
Steel - normalised, forged 0.55 C	13.8	150

1.6 EFFICIENCIES OF MACHINES AND MACHINE ELEMENTS

General guide only.

Bush (grease lubricated).	96%
Bushes (grease lubricated) on lineshafting	94%
Roller bearings	98% - 99%
Ball bearings	99% - 99.5%
Spur gear, including bearings	
(anti-friction) Cast teeth	93%
Cut teeth	96%
Helical gear, including bearings	
(anti-friction) Cut teeth	95%
Bevel gear, including bearings	
(anti-friction) Cast teeth	92%
Cut teeth	95%
Flat belting	96% - 98%
Vee belts	98%
Roller chain	96% - 98%
Hydraulic jack	80% - 90%
Hydraulic couplings, max.	98%

Proprietary gear boxes - refer to Manufacturer's catalogue.

1.7 APPLICATION OF SHOCK FACTORS, SERVICE FACTORS AND SAFETY FACTORS

In all engineering design work not covered by Code requirements, choice of the above factors is the Designer's decision. This would be governed by a "value judgment" based on his knowledge of the job and possible design conditions to be encountered.

Shock Factors

These can apply to the design of any element.

Service Factors and Safety Factors

One of the major points dictating the choice of these two factors is the starting torque conditions applied to the system. This is fully discussed on page 2.11. For example, the choice of a D.O.L. squirrel cage electric motor could typically govern selection of design factors as follows:

Service Factor required to be applied to the design of -
gears, belt drives, bearings, couplings, clutches.

The choice of a service factor is often a matter of experience. Table No 1 on page 3.3 gives an indication for gear design. These factors could also be used for the other elements listed.

Chain drives - see Table No 5, page 6.8.

Keys - apply a service factor equal to the K_t factor (see Table No 1, page 2.6) used for the shaft design.

Other points affecting the selection of service factors are type of machine, machine duty and hours per day operation.

Safety Factor to be increased on the design of - shafts, keys, couplings.

Worked Example No 1

A food mixing machine is driven by a D.O.L. squirrel cage electric motor. This motor gives a T_{max} of 2.6 times full load torque.

Working conditions result in minor shock. Rotation is always in the same direction i.e. torque is static.

Using traditional design methods, show how:

(1) the motor power is determined.

(2) shock factors, service factors and safety factors are applied to each element in the system at the design stage.

Answer

(1) Start at the load application point and determine the maximum load generally applied, during normal full speed working operation. Using this load, work back towards the motor and using all the element efficiencies, determine the motor power requirements.

Choose a motor. Because there will rarely be a stock motor to give the exact power requirements, a slightly larger motor will be used. The name plate power of this motor then becomes the basic power on which the system is designed. In high efficiency systems it is common practice then, to assume that this power is applied to all elements i.e. efficiency is 100% throughout. In low efficiency systems, the exact power in each element can be determined.

Let the basic power in each element be "P".

(2) Bearings

Determine load on each bearing caused by transmitting power P.

* Multiply each load by a S.F. of 1.2, Table No 1, page 3.3, to obtain the Design Load.

Anti-friction bearings.
Using the design loads and further factors from the manufacturer's catalogue, choose appropriate bearings.

Plain bearings.
Using the design loads, design the bearings according to the type of bearing.

* Note: Marginally lower values of S.F. may be used in bearing design because starting torque conditions can be disregarded. (S K F Aust. (Sales) P/L.)

Belts

Flat

Determine the Design Power by multiplying P by a S.F. of 1.4, Table No 1, page 3.3.

Vee

Design according to manufacturer's catalogue using power P.

Chain Drives

Determine the Design Power by multiplying P by a S.F. of 1.3, Table No 5, page 6.8.

Calculate the working chain pull from the design power and use a F of S of approximately 10, to choose a chain from the manufacturer's catalogue.

Clutches

Proprietary clutches

Choose according to instructions in manufacturer's catalogue using power P.

Designed clutches

It is extremely difficult here to lay down firm rules for choice of design factors. These are largely a matter of experience and investigation. Suitable factors for this application could be:

service factor 1.4
safety factor 1.5 - assuming clutch is disengaged when machine is started.

Design power = P x 1.4 x 1.5

Ample scope for torque adjustment should be provided with friction clutches e.g. plate and cone types.

Couplings

Proprietary couplings

Choose according to instructions in manufacturer's catalogue using power P. Sometimes special instructions apply where high starting torque is involved.

Designed couplings

Service factor 1.4, Table No 1, page 3.3.
Safety factor 3.0 minimum (as for rotating shafts) for design of parts.

Design power = P x 1.4

Slip coupling

Determine torque applied to coupling by power P. Choose the torque at which slipping is required and select or design coupling for this torque.

Shear pin couplings

Determine torque applied to coupling by power P. Choose the torque at which pin failure is required. Select or design a normal strength coupling except for the pin which is to shear at the chosen torque.

Gears

Multiply power P by a S.F. of 1.4, Table No 1, page 3.3, to obtain the design power.

Gear Boxes

Proprietary boxes can be chosen from the manufacturer's catalogue, using their recommended factors together with power P.

Keys

Design power = P x (1.0 to 1.5)

Design keys using a F of S of 3.0, as for rotating shafts.

Bearing stresses may be increased by at least 20%. (Refer also to Table No 1, page 1.20.)

Shafts

1. Rotating

Determine shaft loadings caused by power P.

Design shaft using the appropriate method from those listed on page 2.12. Use K_m = 1.5-2.0 and K_t = 1.0-1.5, Table No 1, page 2.6 together with a F of S of 3.0.

2. Stationary

Determine suitable design loadings.

Design shaft using the appropriate method from those listed on page 2.12. Use K_m and K_t = 1.5-2.0, Table No 1, page 2.6; F of S, see page 1.21.

Note:

(i) Alternatively, the Distortion-energy method may be used to design the above shafts. See page 2.30.

(ii) Always determine the actual mechanical properties of the chosen size of shaft material.
Strength details diminish with increasing size.

Springs

Design according to type of service, see page 4.6.

Structural

Standard design according to AS 1250-1981 for static or fatigue loading. The fatigue loading section (Appendix B) covers both reversal fatigue and tensile fatigue.

Design for Inertia

The question arises sometimes with Designers as to whether the motor they have chosen when designing a machine will actually start the mechanism. The concern is that the inertia is so large that the prime mover will be unable to accelerate the unit to its working speed.

The vast majority of machines are not inertia applications. There is generally ample torque available from the drive source at the instant of starting to satisfactorily accelerate all items to working speed, within a reasonable time. (A flywheel is an obvious example of an inertia application but there are few others.)

Two categories of prime movers are of concern:

(1) Electric motors, generally D.O.L. squirrel cage.

(2) Other types, I.C. engines, steam engines etc.

(1) Most machines and mechanisms should be designed as indicated on page 1.28, item (1). If the Designer is concerned about inertia effects, the machine can be checked to see that the motor chosen has sufficient average starting torque available, (the torque-speed curve is required,) to accelerate the elements to working speed within the time allowed by the motor manufacturer. The motor manufacturer should be consulted on the question of maximum permissible run-up time for a particular motor; also the number of starts per hour is a consideration. The inertia calculations are protracted, as the mass moment of inertia of all parts will have to be related to the motor shaft. (Compound pendulum elements, refer Chapt. 14.)

During the normal design procedure, page 1.28, correctly chosen factors of safety will take care of any larger forces that occur during the starting period.

When a machine is clearly an inertia application, the various elements would have to be designed for the forces that occurred during the starting period. Factors of safety to be appropriate to the application.

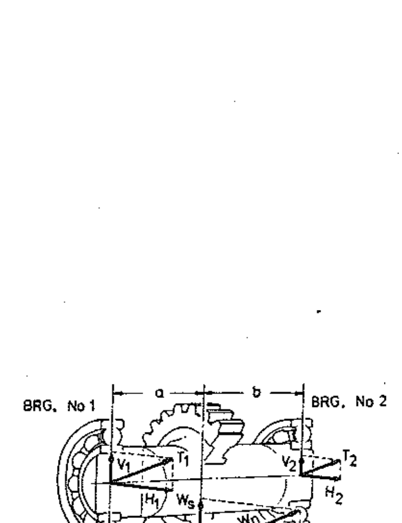
Designers should also be aware of the dangers to machine parts when fierce braking is involved.

(2) Again, the torque-speed curve will be required. Care should be taken to see that, at the operating speed, there is still sufficient torque available for starting considerations (generally a maximum of 150% working torque is sufficient). The engine manufacturer should be consulted at time of design.

* * * * * * * * * * * *

CHAPTER 2

SHAFTS



CHAPTER 2

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SHAFTS

The most basic point in all of mechanical and structural design is to have a thorough grasp of elementary stress analysis.
Mastery of the subject of shaft design for example is impossible without this understanding.

The theory has two fundamental foundations:

(1) The principles of equilibrium.

(2) The elastic properties of solids.

Strain causes stress and in studying strain there are two elementary motions of the planes of molecules.

(1) Normal movement (causes tensile or compressive stress).

(2) Tangential movement (causes shear stress).

These two can be combined to give inclined stress. The following is a brief summary of simple stress analysis, the proofs of which can be found in any standard book on mechanics.

Glossary of terms used in the theory leading to the derivation of the Equivalent Torque formula on page 2.5.

F = load, tensile, compressive or shear, N.

T = applied torque, N.mm.

M = applied bending moment, N.mm.

T_e = equivalent torque, N.mm.

S_t = a tensile stress, MPa.

S_T = a second tensile stress, MPa.

S_c = a compressive stress, MPa.

S_s = a shear stress, MPa.

f_t = principal stress, MPa.

f_s = maximum shear stress, MPa.

Z_p = polar section modulus, mm^3.

d = shaft diameter, mm.

(a) Simple Tension (or compression)

Fig. 1. This not only causes tensile (or compressive) stress but a maximum shear stress also, equal to half the tensile (or compressive) stress, acting at 45° to the axis of the load. This forms the basis of the maximum-shear-stress theory. The maximum-shear-stress theory states that yielding begins whenever the maximum shear stress equals the shear stress corresponding to the yield strength in a simple tension test (refer also Chap. 1, page 1.23).

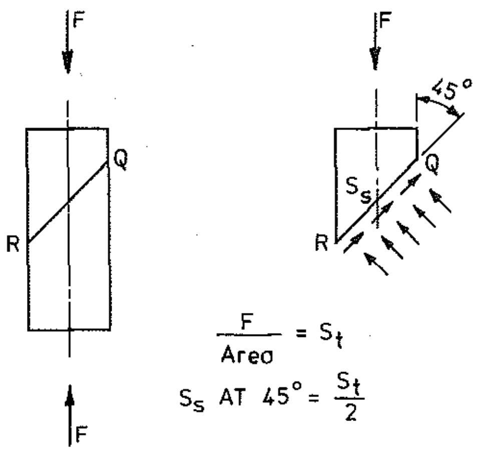


FIG. 1

(b) Pure Shear

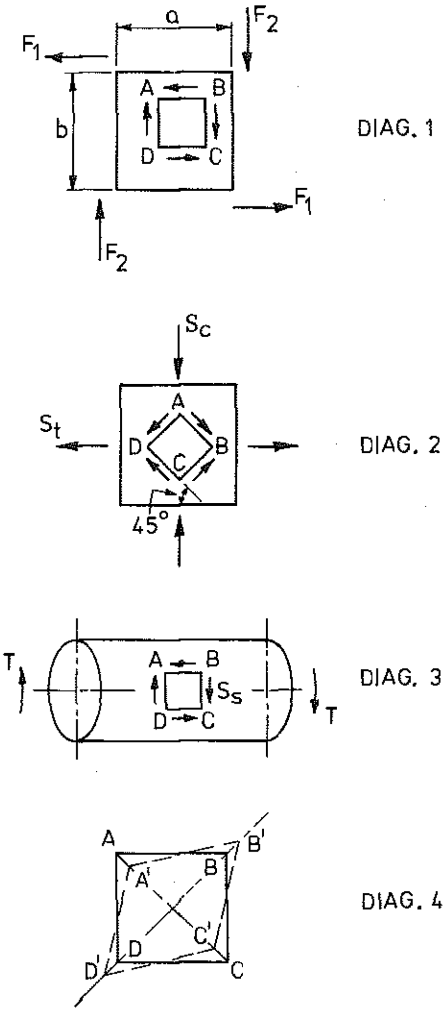


FIG. 2

This condition can be produced by any of the methods shown in Fig. 2. In the first diagram a rectangular block of unit thickness is loaded by forces F_1 and F_2 both uniformly distributed over the surfaces on which they are acting. The block is in equilibrium so that $F_1 \times b = F_2 \times a$. Equal shear stresses act on vertical and horizontal planes, therefore an elementary cube ABCD has on all four faces a shear stress

$$S_s = \frac{F_1}{a} = \frac{F_2}{b}.$$

In the second diagram a rectangular block is subjected to equal compressive and tensile stresses. Equal shear stresses act on all planes at 45° to the faces so that an elementary cube ABCD has on all four faces a shear stress $S_s = S_t = S_c$

In the third diagram a circular shaft is under a torque T. Any small cube ABCD has a shear stress acting on each of its four faces.

SUMMARY

The result of strain produced by pure shear is shown in Fig. 2 fourth diagram.

Any cube ABCD is deformed into a rhombohedron A'B'C'D'. From a study of the diagonals it appears that a shear strain does not cause only shear stresses, but causes tensile and compressive stresses of magnitude equal to the shear stresses on planes inclined at 45° with the planes of shear. These last are called "diagonal tension" and "diagonal compression".

Diagonal compression is one of the main considerations in the design of large plate girders. It will cause failure at shear stresses much lower than the girder will apparently theoretically take. Failure of this nature is termed "elastic shear buckling".

This leads to the principle of Complementary Shear.
This theory states that if at any point in a material under strain there exists a shear stress across one plane there must exist an equal shear stress across a second plane at right angles to the first, and there is a definite relation between the senses of the two.

(c) Direct Shear

This occurs when a member is subjected to a pair of equal, opposite and parallel forces which are so near to being co-linear that the material between is resisting a negligible bending moment. Rivets, keys and pins are examples of this condition and the shear stress is simply taken as uniformly distributed.
Sections (b) and (c) are important principles to understand in the design of concrete foundations where it is necessary to determine whether beam shear or punching shear is the criterion (concrete being three times as strong in punching shear as opposed to beam shear).

(d) Bulk Strain

Fig. 3. This produces only normal movement of all parallel pairs of planes and hence for a decrease in volume compressive stresses are set up and for increase in volume tensile stresses are produced. The compressive or tensile stresses are numerically equal to applied bulk stress.

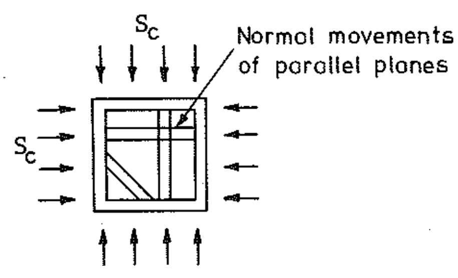


FIG. 3

(e) Theorem of Principal Stress

This theorem is of major importance. At any point in a material under strain there exists two planes at right angles to one another, across which the stresses are entirely normal (i.e. there are no shear stresses across these planes). The stress across one of these planes is the algebraical maximum stress at the point, and that across the other plane is the algebraical minimum stress at the point. The two planes are known as the "principal planes" at the point, and the stresses across them as the "principal stresses" at the point. Consider a small portion of material under the action of tensile stresses S_t and S_T and complimentary shear S_s, Fig. 4.

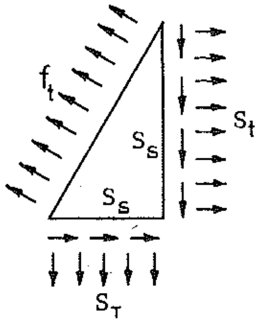


FIG. 4

$$f_t \text{(principal stress)} = \frac{S_t + S_T \pm \sqrt{(S_t - S_T)^2 + 4\,S_s^2}}{2}$$

Maximum Shear Stress

The maximum shear stress at a point acts across planes inclined at 45° to the principal planes and equals one half of the algebraic difference between the principal stresses.

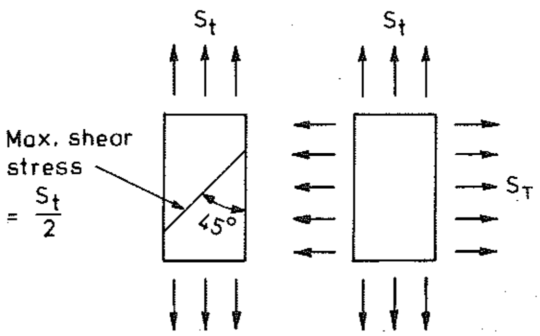


FIG. 5

In calculating the principal stresses and hence the maximum shear stress, compression is taken as negative tension. This now requires further examination. From a study of Fig. 5 it will become obvious that with like principal stresses (taking $S_t > S_T$) the greatest shear stress acting in a body will be determined by the greater principal stress alone. The value of the shear stress will be $\frac{S_t}{2}$ and will act across the plane shown. This particular condition is met in the longitudinal and circumferential stresses in a boiler shell.

Extension of Theory to Shaft Design

Refer to Figs. 6 and 7. Let "M" the bending moment and "T" the torque act at the same cross-section. It can be seen that S_T is absent.

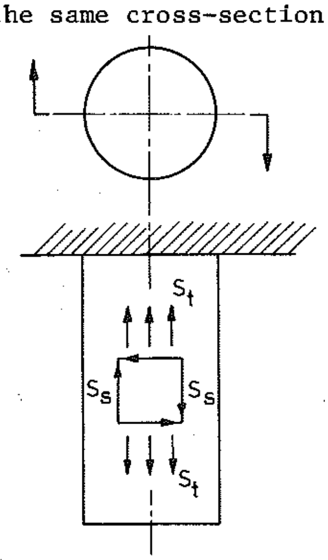


FIG. 6

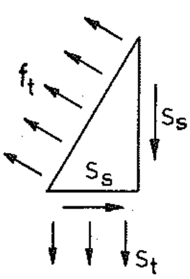


FIG. 7

$$f_t \text{ (principal stress)} = \frac{S_t \pm \sqrt{S_t^2 + 4\,S_s^2}}{2}$$

Now

$$S_t = \frac{32M}{\pi d^3} \quad \text{and} \quad S_s = \frac{16T}{\pi d^3}$$

The result indicates unlike principal stresses as the quantity under the square root sign is greater than S_t^2, therefore the maximum shear stress is determined by both principal stresses.

Ductile Shafts

Ductile material like mild steel fails by shear therefore the maximum-shear-stress theory as propounded by Guest is used. Guest's theory as tested by practical experiments is found to err on the side of safety, nonetheless it is the design procedure most widely used.

It can be easily shown that for a round shaft

$$T_e \text{ (equivalent torque)} = \sqrt{M^2 + T^2} = \frac{f_s \pi d^3}{16}$$

where f_s = maximum shear stress operating in shaft.

Derivation of Equivalent Torque Formula

Refer to Figs. 6 and 7, page 2.4.

Let "M" the bending moment and "T" the torque act at the same cross-section. It can be seen that S_T is absent.

$$f_t \text{ (principal stress)} = \frac{S_t \pm \sqrt{S_t^2 + 4\,S_s^2}}{2}$$

$$\therefore \quad f_{t1} = \frac{S_t}{2} + \sqrt{\tfrac{1}{4}\,S_t^2 + S_s^2} \text{ ------------------- } (1)$$

and $$f_{t2} = \frac{S_t}{2} - \sqrt{\tfrac{1}{4}\,S_t^2 + S_s^2} \text{ ------------------- } (2)$$

now $$S_t = \frac{32M}{\pi d^3} \text{ --------------------------------------- } (3)$$

and $$S_s = \frac{16T}{\pi d^3} \text{ --------------------------------------- } (4)$$

Subtract (2) from (1) and divide by 2, this equals the maximum shear stress f_s that is operating in the shaft.

$$f_s = \sqrt{\tfrac{1}{4}\,S_t^2 + S_s^2}$$

Inserting values from (3) and (4)

$$f_s = \sqrt{\tfrac{1}{4}\left(\frac{32M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\therefore \quad f_s = \frac{T_e}{Z_p}$$

Determination of Maximum Permissible Working Stress

Ductile materials only

Maximum permissible stresses for all combinations of shaft loadings are given in Table No. 1., Chapter 1, pages 1.19 - 1.21.

(Where the maximum-shear-stress theory is used to determine working stresses it is always T_e that governs shaft size in the case of combined loading. M_e, Equivalent Bending Moment, will always give a smaller diameter. The theory behind this method is not covered in this book.)

The maximum permissible working stress for all rotating (or even oscillating) shafts is based on endurance limit regardless of the operating speed.

The maximum permissible working stress for stationary shafts is based on yield point only if the load is essentially static. If the load is reversing or severely fluctuating the maximum permissible working stress should be as for rotating shafts. Explaining the problem from another viewpoint, the important question to answer is whether the stress being considered is static or reversing (or fluctuating).

Bending Moments in Two Planes

It is sometimes convenient to consider bending moments in two planes when calculating shaft designs. (See page 3.19, Q8.)

These bending moments can simply be added with vectors (or by Pythagoras' theorem if planes are at 90°) at critical sections.

Factors for Bending Moment and Torque

(i) Shock Factors

These are given in Table No. 1. for stationary and rotating shafts.

Note regarding rotating shafts:-
In the absence of shock, no factor on B.M. is theoretically necessary as the working stress is based on endurance limit and hence shaft failure will not occur.

However it is a common practice throughout engineering to apply a small factor to the B.M. This causes only a marginal increase in shaft size.

TABLE NO. 1

	K_m	K_t static	K_t reversal
Stationary Shafts:			Design procedure as for rotating shafts.
Load gradually applied	1.0	1.0	
Load suddenly applied	1.5-2.0	1.5-2.0	
Rotating Shafts:			
Load gradually applied	1.0-1.5	1.0	1.0-1.5
Load suddenly applied, minor shock	1.5-2.0	1.0-1.5	1.5-2.0
Load suddenly applied, heavy shock	2.0-3.0	1.5-3.0	2.0-3.0

(ii) Stress Concentration Factors

Allowance for stress concentration must be made where applicable in the various shaft design formulas.

Stress concentration takes place at a change of diameter (critical section). With a live shaft a cyclical reversal of bending stress necessitates a factor being applied to the calculated bending moment. Torque is generally static and therefore no factor is required in most cases. As a common rule in design a small amount of stress concentration is tolerable in a ductile material when not subject to stress reversal. If reversal of torque is present a factor is necessary as with the bending moment and these factors can be obtained from Charts 6 to 16 in Chapter 10.

Among other conditions which can be described as "stress raisers" are keyways, holes and screw threads.

GLOSSARY OF TERMS

M	=	Applied bending moment, N.mm
T	=	Applied torque, N.mm
T_e	=	Equivalent torque, N.mm
K_m	=	Shock factor in bending
K_t	=	Shock factor in torsion
K_C, K_T	=	Shock factor for shaft under axial load, compression or tension respectively (use as for K_m, Table 1 page 2.6).
K_{fm}	=	Stress concentration factor for shaft in bending only.
K_{fc}	=	Combined stress concentration factor for shaft in bending.
K_{ft}	=	Stress concentration factor for shaft in torsion.
K_{fT}	=	Stress concentration factor for shaft in tension.
K_{fTc}	=	Combined stress concentration factor for shaft in tension.

Values for stress concentration factors can be taken from Charts 6 to 16 in Chapter 10. These Charts apply essentially to reversed loading. Members stressed by static loading (most torque loading is static) may be stronger in practice than the theory indicates. Therefore the value of K_f becomes a value judgment in static applications and in some circumstances is taken as 1.0.

F_s, f_s	=	Permissible and actual shear stress, MPa. } Chap. 1, Table No. 1,
F_t, f_t	=	Permissible and actual tensile stress, MPa. } pages 1.20 and 1.21.
Z_p	=	Polar section modulus, mm³, $(\frac{\pi d^3}{16})$; $Z_{ph} = \frac{\pi(d^4 - di^4)}{16d}$
Z	=	'Beam' section modulus, mm³, $(\frac{\pi d^3}{32})$; $Z_h = \frac{\pi(d^4 - di^4)}{32d}$
d	=	External diameter of shaft, mm

Glossary of terms (Cont.)

A = $\frac{di}{d}$ = ratio of internal to external diameters.

F_y = yield point stress of the material, MPa (Table No. 2, page 2.9)

F_e = endurance limit of material, MPa (Table No. 2, page 2.9)

W = Axial tension or compression, N

γ = Column action factor

= unity for a tensile load

For a compressive load

$$\gamma = \frac{1}{1 - (0.0044\,\frac{nL}{k})} \quad \ldots \text{ for } \frac{nL}{k} < 150$$

$$= \frac{F_y}{\pi^2 E}\left(\frac{nL}{k}\right)^2 \quad \ldots \text{ for } \frac{nL}{k} > 150$$

See footnote

where n = 1.0 for hinged ends (self-aligning bearings)
= 0.79 for partly restrained ends (rigid bearings)
= 0.67 for fixed ends

k = radius of gyration (orthogonal), mm

L = actual span of shaft, mm

E = Young's modulus, MPa

Shafts With Axial Loading. Refer also to pages 2.12 and 2.13.

The following formula is based on the maximum-shear-stress theory.

$$d = \sqrt[3]{\frac{16\sqrt{\{K_m K_f M + \frac{1}{8}\gamma K_{C\text{ or }T}\, K_f\, W d\,(1 + A^2)\}^2 + (K_t K_{ft} T)^2}}{\pi F_s\,(1 - A^4)}}$$

Where

K_f = stress concentration factor (refer page 2.7)

The process of designing shafts having axial loading is iterative as "d" appears on both sides of the equation.

Footnote. Computer analysis has shown that the ASME shaft design formulas given for γ in previous printings of this text (prior to 9th reprint) had several inconsistencies. Consequently the required adjustments have been carried out; the resulting formulas are given in the glossary of terms and specifically apply to steels having F_y 250 to 300 MPa. For other steels a conservative approach would be to neglect the 150 slenderness ratio limit and use the larger of the two γ values. Even though the ASME code has been withdrawn by the U.S.A. Standards Institute the above formulas for γ and hence 'd' still have good practical use.

TABLE NO. 2

Engineering properties of shafting steels

Steel	Ult. (F_{ult})	Yield (F_y)	End. Limit (F_e)
Hot rolled structural steel AS 1204 grade 250	410 MPa	250 MPa	207 MPa
Common shafting materials			
bright steel AS 1443-1983 black steel AS 1442-1983			
- CS 1020	400 MPa	200 MPa	180 MPa
- CS 1030	500 MPa	250 MPa	225 MPa
- CS 1040	540 MPa	270 MPa	243 MPa

All values are "average" values and are size dependent.

Brittle Materials

Brittle materials fail in tension and so maximum principal stress "f" (tension) is the design criterion. Rankine developed the following formula from this theory,

$$T_e = M + \sqrt{M^2 + T^2} = \frac{f\pi d^3}{16}$$

where T_e is the imaginary torque which if applied alone would produce a shear stress equal to the maximum principal stress. An alternative form of the above equation is now quoted.

$$M_e = \frac{M + \sqrt{M^2 + T^2}}{2} = \frac{f\pi d^3}{32}$$

where M_e is termed the Equivalent Bending Moment and is the bending moment which if acting alone would produce the same maximum principal stress.

Stress concentrations at critical sections should be carefully checked as brittle materials have no yield point and hence plastic flow is absent.

Deflection Limits for Shafts

General

Torsional. 3° per metre.

Lateral.

Simply supported 0.8 mm per metre of span

Cantilever (overhung) 1.6 mm per metre of span

Cont. on page 2.10

Deflection Limits for Shafts. (Cont.)

Gears

Torsional   0.25° per metre between adjacent gears mounted on the same shaft.

Lateral

Lateral deflection of shafts carrying gears should be limited to contain "error in action" between gear teeth to the figures given in Table No. 3.

These figures apply to well cut commercial gears having pitch line velocities of up to 75 m/min. and are for the purpose of keeping noise and wear factors within reasonable bounds.

It must be realised that these limits are for ideal machine design applications e.g. gear boxes.

The limits cannot always be maintained e.g. designs having overhung shafts. In the latter case we rely on small tooth deflections and "wearing in" of gear teeth.

TABLE NO. 3

MODULE	ERROR IN ACTION
25	0.12 mm
12	0.10 mm
8	0.08 mm
6	0.07 mm
5	0.06 mm
4 and less	0.05 mm

Deflection Formulas

Torsional $\theta = \frac{T\ell}{GJ}$ radians

Lateral. Use normal beam formulas. (Refer Chapter 1, page 1.17.)

Inertia Loading On Shafts. (Refer also to pages 2.36 and 14.18.)

The basic safety factor of 2.0 on stress allows for inertia of starting in normal design application.

The average starting torque given by most squirrel cage electric motors is well in excess of 2.0 x normal maximum full load running torque, when starting Direct-on-Line (D.O.L.). This means that by standard calculation methods a shaft may be stressed beyond yield point at the instant of starting.

Nonetheless practical application proves the chosen safety factor of 2.0 is satisfactory for the following reason.

In general with steel, as the speed of loading is increased, the yield strength has a noticeable increase. "Run-up" time with normal motor use is well under one second, which corresponds with "high speed" loading. This is effective in raising the yield point of the steel sufficiently to make the design safe.

However, designers are advised to increase the basic safety factor to at least 3.0 where the prime mover places a starting torque of between 160% and 260% full load torque on the system. (With an electric motor the reference would be either the starting torque or the pull-out torque, whichever is greater.) This may avoid shaft damage, especially in the case of a jam load. Other elements in the system may need special consideration also. Where motor starting or pull-out torque of 300% to 350% full load torque is encountered, a basic safety factor of 4.0 may be advisable. The choice of a safety factor in non-code work is the Designer's decision and is a "value judgment" on the possible design conditions to be encountered. General guidance on torque-speed characteristics of squirrel cage electric motors is given on pages 17.4 and 17.5.

In the rarer cases, where d.c. motors or High Torque a.c. motors are used, extremely high starting torques, up to five times full load torque, can be encountered. Safety factors for shaft design should be correspondingly increased, unless the design procedure shown in the AS 1403 Shaft Design Code is used. Methods of reducing starting torque are discussed in Appendix C of AS 1403. Additionally, with excessively high starting torques, appropriately increased safety factors will be required on all elements in the system. Comment on the safety factor for chain drives is given on pages 6.8 and 6.9.

A fluid coupling presents a slightly different shaft loading condition. This is discussed on pages 8.7 and 8.8. It is worth noting that the output torque characteristics of a fluid coupling, during the starting period, can be modified by varying the amount of oil within the coupling.

True inertia loading where motor size is determined by torque of acceleration, rather than power of normal running, would necessitate special loading considerations in the shaft design. The prolonged "run-up" times (say 2 to 5 seconds or more) encountered, would suggest that the maximum torque loading imposed on the system, by the motor, during the "run-up" period, be used as a basis for the design. This torque would be greater than that resulting from the normal maximum full load torque as calculated from the motor name plate power. Basic safety factor should be 2.0 minimum. Alternatively, AS 1403 may be used. With most inertia applications, power requirements at normal running speed are negligible.

Beam Shear Stress

This is rarely if ever a consideration in shaft design. If this check is required, recommended stresses are given in Table No. 1, Chap. 1, pages 1.19-21.

TABLE NO. 4

Metric bar sizes in mm.

Always check with manufacturer for availability at time of design.

CS 1020 1.6 to 250; CS 1030 1.6 to 150; CS 1040 1.6 to 250.

Below are the standard sizes from 14 mm upwards.

14	17	20	23	27	40	52	65	80	95	120	150	250
15	18	21	24	30	45	56	70	85	100	130	160	
16	19	22	25	35	50	60	75	90	110	140	180	

Summary of Shaft Design

The following categories of shaft design should be considered. Ductile materials only.

Rotating Shafts

Bending only.	Section 1.
Torque only.	Section 2.
Combined bending and torque.	Section 3.
Any of above plus axial loading.	Section 4.

Stationary Shafts

Bending only.	Section 5.
Torque only.	Section 6.
Combined bending and torque.	Section 7.
Any of above plus axial loading.	Section 4.

Loads on stationary shafts must be essentially static if the maximum permissible working stress is to be based on yield point. For reversing or severely fluctuating loads maximum permissible working stress is to be determined as for rotating shafts i.e. as an endurance application.

Summary of Sections

Section 1

Rotating shafts.

Bending only. $K_m K_{fm} M = F_t Z \quad (\text{or } Z_h)$

Section 2

Rotating shafts.

Torque only. $K_t K_{ft} T = F_s Z_p \quad (\text{or } Z_{ph})$

Section 3

Rotating shafts.

Combined bending and torque.

$$T_e = \sqrt{(K_m K_{fc} M)^2 + (K_t K_{ft} T)^2} = F_s Z_p \quad (\text{or } Z_{ph})$$

Section 4

Rotating or stationary shafts. Refer also to page 2.8.

M and/or T plus axial loading.

$$d = \sqrt[3]{\frac{16\sqrt{\{K_m K_f M + \frac{1}{8}\gamma K_{C \text{ or } T} K_f W d (1+A^2)\}^2 + (K_t K_{ft} T)^2}}{\pi F_s (1 - A^4)}}$$

This is a formula covering all possibilities of loading.

Note: (i) If $T = 0$ still use F_s.

(ii) In the axial load component of the above formula neglect K_f if shaft is under compressive load only.

Section 5

Stationary shafts.

Bending only. $K_m K_{fm} M = F_t Z$ (or Z_h)

Section 6

Stationary shafts.

Torque only. $K_t K_{ft} T = F_s Z_p$ (or Z_{ph})

Section 7

Stationary shafts.

Combined bending and torque.

$$T_e = \sqrt{(K_m K_{fc} M)^2 + (K_t K_{ft} T)^2} = F_s Z_p \quad (\text{or } Z_{ph})$$

Deflection Formulas

Torsional. $\theta = \frac{T\ell}{GJ}$ radians

T = Torque applied over length "ℓ", N.mm

ℓ = Length of shaft under consideration, mm

G = Modulus of rigidity (shear modulus), MPa

J = Polar second moment of area (polar moment of inertia), mm^4

Lateral. Use normal beam formulas. (Refer Chapter 1, page 1.17.)

NOTE: Torsional and lateral deflections are calculated on basic loads; do not include shock or service factors.

Suggested deflection limits were given on pages 2.9 and 2.10.

Excessive deflections (lateral and/or torsional) should be avoided because they:

(i) can cause secondary stresses.

(ii) can result in unsatisfactory machine performance. A whipping shaft can cause shock loadings in transmission.

(iii) can in some circumstances cause bad gear meshing.

(iv) can look unsightly.

WORKED EXAMPLE NO. 1

Section 1 Type shafts

Design the shaft shown in Fig. 8.

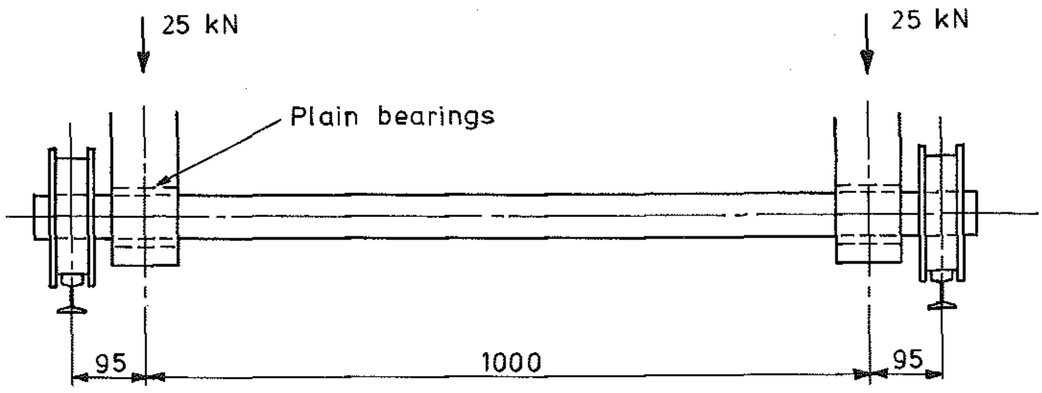


FIG. 8

Answer

Neglect any small amount of torsion in shaft due to bearing friction.

Use CS 1030, $F_e = 225$ MPa, Table No. 2, page 2.9.

$$\therefore F_t = \frac{225}{2} = 112.5 \text{ MPa}$$

$$M = F_t Z \quad \therefore Z = \frac{M}{F_t} = \frac{25\ 000 \times 95}{112.5} = 21\ 111 \text{ mm}^3$$

$$Z = \frac{\pi d^3}{32} = 21\ 111 \text{ from which } d = 59.91 \text{ mm}$$

Nearest standard size above = 60 mm, Table No. 4, page 2.11.

Check lateral deflection

(i) Assume shaft is restrained at the bearings i.e. the ends are true cantilevers; $\delta = \frac{WL^3}{3\ EI}$ (Standard deflection formula.)

$$I = \frac{\pi d^4}{64} = \frac{\pi 60^4}{64} = 636\ 172.5 \text{ mm}^4$$

$$\therefore \delta = \frac{25\ 000 \times 95^3}{3 \times 207\ 000 \times 636\ 172.5} = 0.054 \text{ mm}$$

Permissible δ = 1.6 mm per m of span, page 2.9.

= 1.6 x 0.095

= 0.15 mm > 0.054 mm

(ii) Assume the shaft pivots at each bearing as it would with self aligning bearings.

$$\delta = \frac{Wa\ (3\ a\ell - 4\ a^2)}{12\ E\ I} \qquad \text{(Standard deflection formula.)}$$

Where $W = 25\ 000 + 25\ 000 = 50\ 000$ N

$a = 95$ mm

$\ell = 1000 + 2 \times 95 = 1190$ mm

$$\therefore \delta = \frac{50\ 000 \times 95\ (3 \times 95 \times 1190 - 4 \times 95^2)}{12 \times 207\ 000 \times 636\ 172.5}$$

$$= 0.91 \text{ mm}$$

This represents a considerable increase in deflection when compared with case (i). The reason for this is that when the bearings do not restrain the shaft, the shaft bows between the bearings and this considerably accentuates the end deflections.

WORKED EXAMPLE NO. 2

Section 2 Type shafts

A solid shaft is to transmit 75 kW at 150 r.p.m.
Determine the shaft size using CS 1020 bright steel.

Answer

$$P = \frac{T\omega}{1000}$$

$$\therefore T = \frac{1000\ P}{\omega} = \frac{1000 \times 75 \times 60}{2\pi \quad 150} = 4774.65 \text{ N.m}$$

CS 1020, F_e = 180 MPa, using the maximum-shear-stress theory and a basic safety factor of 2.0.

$$F_s = \frac{180}{2 \times 2} = 45 \text{ MPa}$$

$$T = F_s Z_p \qquad \therefore Z_p = \frac{T}{F_s} = \frac{4774.65 \times 10^3}{45} = 106\ 103.33 \text{ mm}^3$$

$$\therefore Z_p = \frac{\pi d^3}{16} = 106\ 103.33 \text{ from which } d = 81.45 \text{ mm}$$

Nearest standard size above = 85 mm, Table No. 4, page 2.11.

Check torsional deflection

$$\theta = \frac{T\ell}{GJ}, \text{ assume } \ell = 1000 \text{ mm.}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi 85^4}{32} = 5.1248 \times 10^6 \text{ mm}^4$$

Worked Example No. 2. (Cont.)

$$\therefore \theta = \frac{4774.65 \times 10^3 \times 1000}{82\ 700 \times 5.1248 \times 10^6} = 0.0113^c = 0.65°$$

Permissible θ = $3°/m > 0.65°/m$, page 2.9.

WORKED EXAMPLE NO. 3

Section 3 Type shafts

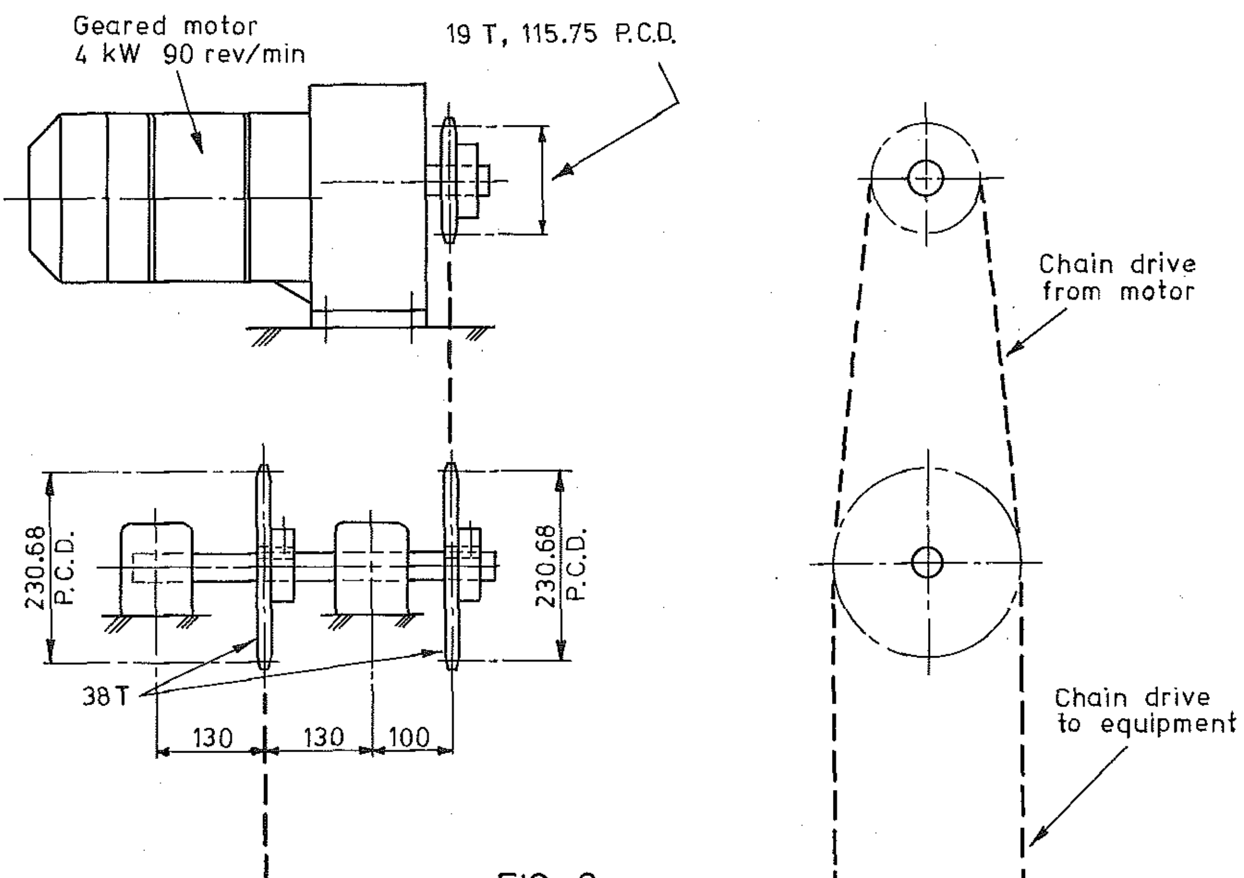


FIG. 9

Fig. 9. shows the drive for some mining equipment.

(i) Draw a B.M. diagram for the shaft.

(ii) Calculate the shaft size on the basis of strength only. Both sprockets are keyed to the shaft.

Answer

Shaft speed $= 90 \times \frac{19}{38} = 45$ r.p.m.

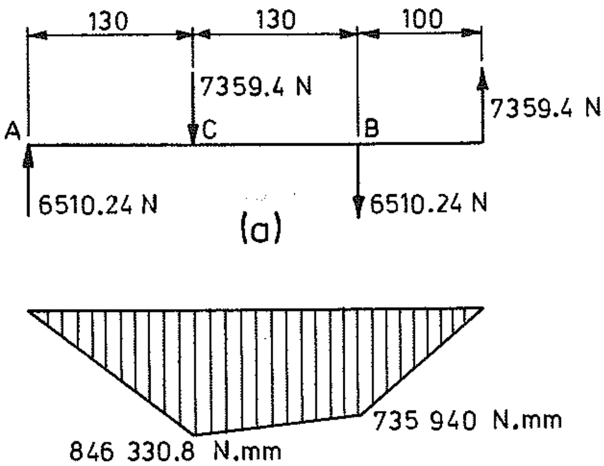
$$P = \frac{T\omega}{1000}$$

$$\therefore T = \frac{1000\ P}{\omega} = \frac{1000 \times 4 \times 60}{2\pi \quad 45} = 848.83 \text{ N.m}$$

Worked Example No. 3. (Cont.)

$$\text{Force in chains} = \frac{848.83 \times 10^3 \times 2}{230.68} = 7359.4 \text{ N}$$

Assume chain forces are vertical. Refer to Fig. 10(a) for an idealised load diagram.



(b)

FIG. 10

Moments about A

$$7359.4 \times 130 + 260\,B = 7359.4 \times 360$$

$$\therefore B = 6510.24 \text{ N}$$

Moments about B

$$7359.4 \times 100 + 7359.4 \times 130 = 260\,A$$

$$\therefore A = 6510.24 \text{ N}$$

Check. By inspection, upward forces = downward forces.

$$M_B = 7359.4 \times 100 = 735\ 940 \text{ N.mm}$$

$$M_C = 7359.4 \times 230 - 6510.24 \times 130$$

$$= 846\ 330.8 \text{ N.mm}$$

Refer to Fig. 10(b) for B.M. Diagram

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{846\ 330.8^2 + 848\ 830^2}$$

$$= 1\ 198\ 661 \text{ N.mm}$$

Use CS 1030, F_e = 225 MPa, keyway allowance to be made.

$$\therefore F_s = 0.75 \times \frac{225}{2 \times 2} = 42.2 \text{ MPa}$$

$$T_e = F_s Z_p$$

$$\therefore Z_p = \frac{T_e}{F_s} = \frac{1\ 198\ 661}{42.2} = 28\ 404.29 \text{ mm}^3$$

$$\therefore Z_p = \frac{\pi d^3}{16} = 28\ 404.29 \text{ from which } d = \underline{52.5 \text{ mm}}$$

Note: The motor used in this example is a squirrel cage type. If it is started D.O.L., then a basic F of S of 3.0 should be used because of the high starting torque. The high starting torque and/or high pull-out torque of such a drive places great loads on shafts and other components of the driven mechanism.

T_{max} can be as high as 2.6 x rated torque.

See also page 2.11. If a F of S of 3.0 is used, d = 60.1 mm.

WORKED EXAMPLE NO. 4

Section 4 Type shafts

(a) Rotating shaft

A certain solid shaft in a food processing plant has an axial compressive load of 8t. Consider the shaft is pin ended and the span is 400 mm. Working conditions are minor shock and there is a keyway at the critical section.

The applied moment = 4000 N.m

The applied torque = 2000 N.m

Determine if a 120 mm diameter shaft of CS 1030 is satisfactory.

Answer

$$k = \frac{120}{4} = 30$$

$$\frac{nL}{k} = \frac{1.0 \times 400}{30} = 13.33 < 150$$

$$\therefore \quad \gamma = \frac{1}{1 - (0.0044 \frac{nL}{k})}$$

$$= \frac{1}{1 - (0.0044 \times 13.33)} = 1.0623$$

CS 1030, F_e = 225 MPa, Table No. 2, page 2.9.

$$\therefore \quad F_s = \frac{225}{2x2} \times 0.75 = 42.19 \text{ MPa.}$$

$$d = \sqrt[3]{\frac{16\sqrt{\{K_m M + \frac{1}{8}\gamma K_C W d (1 + A^2)\}^2 + (K_t T)^2}}{\pi F_s (1 - A^4)}}$$

K_m and K_C = 1.75 (K factors Table No. 1, page 2.6)

K_t = 1.25; A = 0

$$\therefore \quad d = \sqrt[3]{\frac{16\sqrt{\{1.75x4000x10^3 + \frac{1}{8} 1.0623x1.75x8000gx120\}^2 + (1.25x2000x10^3)^2}}{\pi\ 42.19}}$$

$$= 104.75 < 120 \text{ mm}$$

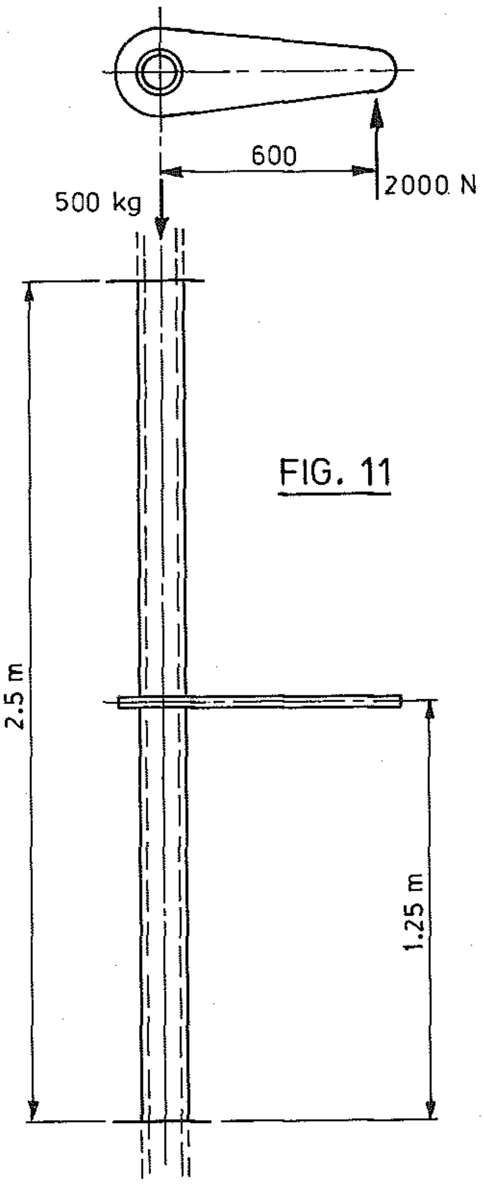


FIG. 11

(b) Stationary shaft

Fig. 11. shows a hollow stationary shaft which forms part of some mechanical equipment. The nominated size is 101.6 O.D. x 4.5 wall from F_y 250 MPa tube.

Check the suitability of the above tube.

Assumptions: (i) the shaft (tube) has pin ends.

(ii) the loads are always in the one direction.

(iii) the torque is reacted out at the lower support.

Engineering details of tube from Manufacturer's catalogue:

Area of cross section .... 1360 mm^2

I .... 1.61×10^6 mm^4

J .... 3.22×10^6 mm^4

k .... 34.4 mm

Z .... 31.7×10^3 mm^3

Z_p .... 63.4×10^3 mm^3

Mass .... 10.7 kg/m

Answer

$$M = \frac{WL}{4} = \frac{2000 \times 2500}{4} = 1\ 250\ 000 \text{ N.mm}$$

$$T = W \times \text{radius} = 2000 \times 600$$

$$= 1\ 200\ 000 \text{ N.mm}$$

$$\frac{nL}{k} = \frac{1.0 \times 2500}{34.4} = 72.67 < 150$$

$$\gamma = \frac{1}{1 - (0.0044 \frac{nL}{k})}$$

$$= \frac{1}{1 - (0.0044 \times 72.67)} = 1.47$$

$$F_y = 250 \text{ MPa}$$

$$\therefore F_s = 0.26 F_y = 0.26 \times 250 = 65 \text{ MPa}$$

$$A = \frac{di}{d} = \frac{101.6 - 2 \times 4.5}{101.6} = 0.9114$$

$$W = 500 + 2.5 \times 10.7 = 526.8 \text{ kg (conservatively)}$$

$$d = \sqrt[3]{\frac{16\sqrt{\{M + \frac{1}{8}\gamma W d (1 + A^2)\}^2 + T^2}}{\pi F_s (1 - A^4)}}$$

$$= \sqrt[3]{\frac{16\sqrt{\{1\ 250\ 000 + \frac{1}{8} \times 1.47 \times 526.8g \times 101.6(1+0.9114^2)\}^2 + 1\ 200\ 000^2}}{\pi \times 65 \ (1 - 0.9114^4)}}$$

$\therefore$ $d = 77.81 \text{ mm} < 101.6 \text{ mm}$

Check lateral deflection

$$\delta = \frac{WL^3}{48EI} = \frac{2000 \times 2500^3}{48 \times 207\ 000 \times 1.61 \times 10^6} = 1.95 \text{ mm}$$

δ = permissible = 0.8 mm/m span, page 2.9

= 0.8×2.5

= $2.0 \text{ mm} > 1.95 \text{ mm}$

Check torsional deflection

$$\theta = \frac{T\ell}{GJ} = \frac{1\ 200\ 000 \times 1250}{82\ 700 \times 3.22 \times 10^6} = 0.0056^c = 0.32°$$

Permissible $\theta = 3°/\text{m} = 3 \times 1.25 = 3.75° > 0.32°$

WORKED EXAMPLE NO. 5

Section 5 Type shafts

Refer to Fig. 8. page 2.14. Suppose now that the shaft is stationary and the wheels are fitted with bearings; determine the required size of shaft.

Answer

Use CS 1030, F_y = 250 MPa, Table No. 2, page 2.9

$\therefore$ $F_t = 0.6\ F_y = 0.6 \times 250 = 150 \text{ MPa}$

$M = F_t Z$

$\therefore$ $Z = \frac{M}{F_t} = \frac{25\ 000 \times 95}{150} = 15\ 833.33 \text{ mm}^3$

$Z = \frac{\pi d^3}{32} = 15\ 833.33$ from which $d = 54.43$ mm

Nearest standard size above = 56 mm, Table No. 4, page 2.11.

Check lateral deflection

$$\delta = \frac{WL^3}{3\ EI} \quad \text{(Standard deflection formula.)}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi 56^4}{64} = 482\ 749.7\ \text{mm}^3$$

$$\therefore \delta = \frac{25\ 000 \times 95^3}{3 \times 207\ 000 \times 482\ 749.7} = 0.072\ \text{mm}$$

Permissible δ = 1.6 mm per m of span = 1.6 x 0.095 = 0.15 mm > 0.072 mm

WORKED EXAMPLE NO. 6

Section 6 Type shafts

A solid round "shaft" is required to act as an anchor to resist a torque of 6000 N.m. The load is suddenly applied but always in the one direction.

Answer

Try CS 1040, F_y = 270 MPa, Table No. 2, page 2.9.

$$F_s = 0.3\ F_y = 0.3 \times 270 = 81\ \text{MPa.}$$

$$K_t T = F_s Z_p$$

$$\therefore Z_p = \frac{K_t T}{F_s}, \quad \text{select } K_t = 1.5 \text{ from Table No. 1, page 2.6.}$$

$$\therefore Z_p = \frac{1.5 \times 6000 \times 10^3}{81} = 111\ 111.11\ \text{mm}^3$$

$$Z_p = \frac{\pi d^3}{16} = 111\ 111.11 \text{ from which } d = 82.7\ \text{mm}$$

Nearest standard size above = 85 mm, Table No. 4, page 2.11.

Check torsional deflection

$$\theta = \frac{T\ell}{GJ}, \text{ assume } \ell = 1000\ \text{mm}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi 85^4}{32} = 5.1248 \times 10^6\ \text{mm}^4$$

$$\therefore \theta = \frac{6000 \times 10^3 \times 1000}{82\ 700 \times 5.1248 \times 10^6} = 0.0142^c = 0.81°$$

Permissible θ = 3°/m > 0.81°/m, page 2.9.

WORKED EXAMPLE NO. 7

Section 7 Type shafts

Determine a suitable size for the shaft shown in Fig. 12. Use CS 1030 material.

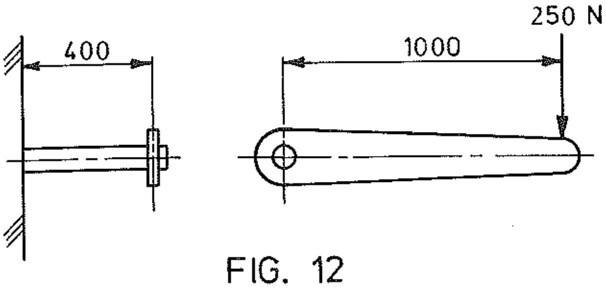


FIG. 12

Answer

First design for strength

$M = 250 \times 400 = 100\ 000$ N.mm

$T = 250 \times 1000 = 250\ 000$ N.mm

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{100\ 000^2 + 250\ 000^2}$$

$$= 269\ 258.24 \text{ N.mm}$$

CS 1030, $F_y = 250$ MPa, Table No. 2, page 2.9.

$F_s = 0.3\,F_y = 0.3 \times 250 = 75$ MPa

$$T_e = F_s\,Z_p$$

$$\therefore Z_p = \frac{T_e}{F_s} = \frac{269\ 258.24}{75} = 3590.1 \text{ mm}^3$$

$$Z_p = \frac{\pi d^3}{16} = 3590.1 \text{ from which } d = 26.35 \text{ mm}$$

nearest standard size above = 27 mm, Table No. 4, page 2.11.

Secondly check for lateral deflection

$$\delta = \frac{WL^3}{3EI} \;;\quad I = \frac{\pi d^4}{64} = \frac{\pi 27^4}{64} = 26\ 087 \text{ mm}^4$$

$$\therefore \delta = \frac{250 \times 400^3}{3 \times 207\ 000 \times 26\ 087} = 0.99 \text{ mm}$$

Permissible δ = 1.6 mm/m of span, page 2.9.

$= 1.6 \times 0.4 = 0.64$ mm $<$ 0.99 mm

Try 30 mm diameter shaft.

$$I = \frac{\pi d^4}{64} = \frac{\pi 30^4}{64} = 39\ 760.78 \text{ mm}^4$$

$$\therefore \delta = \frac{WL^3}{3EI} = \frac{250 \times 400^3}{3 \times 207\ 000 \times 39\ 760.78} = 0.65 \text{ mm which is only}$$

marginally in excess of the maximum permissible deflection of 0.64 mm.

Thirdly check for torsional deflection

$$\theta = \frac{T\ell}{GJ} ; \quad J = \frac{\pi d^4}{32} = \frac{\pi 30^4}{32} = 79\ 521.56\ \text{mm}^4$$

$$\therefore \quad \theta = \frac{250\ 000 \times 400}{82\ 700 \times 79\ 521.56} = 0.0152^c = 0.87°$$

Maximum permissible $\theta = 3°/\text{m} = 3 \times 0.4 = 1.2° > 0.87°$, page 2.9.

EXERCISES

1. Fig. 13. shows a stationary axle used in a machine trolley.

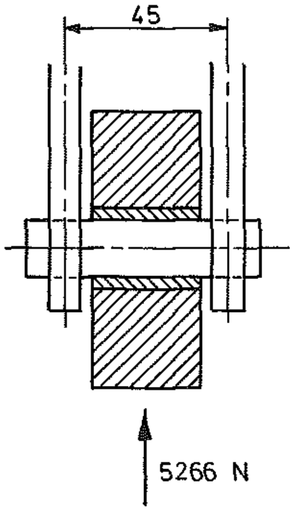


FIG. 13

(a) Determine the required size for the axle if CS 1030 steel is used.

Assume: (i) the load is suddenly applied, $K_m = 1.5$

(ii) the loads and reactions are concentrated.

(b) Calculate the actual axle deflection and the maximum permissible deflection if 20 mm diameter material is chosen.

Ans. (a) 18.2 mm

(b) 0.0061 mm
0.036 mm

2. Fig. 14. shows a shaft drive to a machine. Half the power is taken off at each pinion.

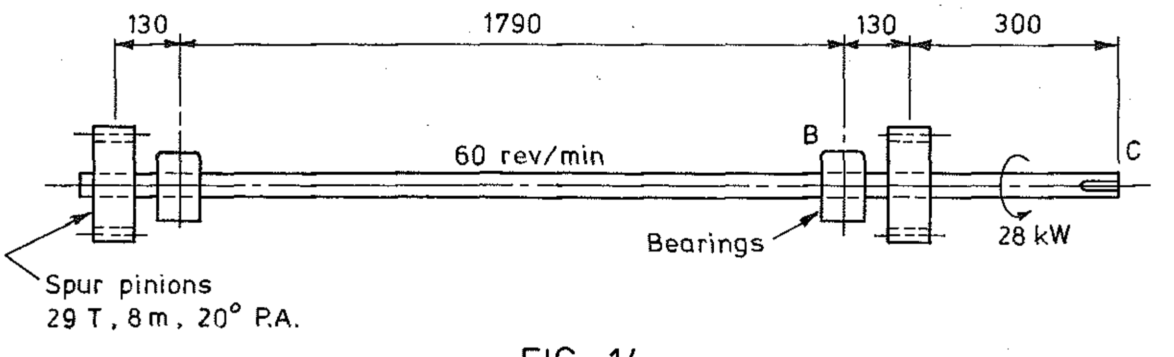


FIG. 14

(a) Determine the required size of shaft using CS 1020 material. There are two critical positions to be checked:

(i) at the power input, point C. There is a keyway at this position.

(ii) at point B.

(b) Suppose 90 mm stock is chosen for the job: calculate the lateral deflection at each pinion.

and (c) Determine the actual and permissible torsional deflections of the shaft:

(i) from C to the first pinion.

(ii) between the two pinions.

Ans.	(a)(i)	87.61 mm	(b)	0.486 mm (self aligning brgs.)	(c)(i)	0.144°; 0.9°
	(ii)	73.22 mm		0.0225 mm (plain brgs.)	(ii)	0.491°; 0.513°

3. A shaft in some mechanical equipment is subject to a bending moment of 400 N.m, while transmitting 5 kW at 870 r.p.m. If an axial tension load of 60 000 N is then applied to the shaft which is 50 mm diameter of CS 1030 determine f_s and F_s.

Ans. 31.66 MPa
56.25 MPa

4. Fig. 15. shows a stationary shaft which has to resist the loads indicated. If the loads are suddenly applied, K_t = 1.75, (always in the direction shown) determine:

(a) a suitable diameter for the shaft if CS 1030 material is used.

(b) the actual and maximum permissible torsional deflections.

Ans. (a) 28.75 mm (use diam. 30).

(b) 1.743°/m (for diam. 30).
3.0°/m

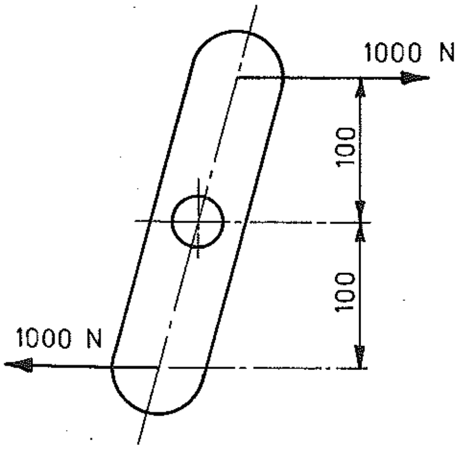


FIG. 15

5. Fig. 16. shows a hollow stationary shaft with an off centre load.
F_y shaft = 210 MPa.
Assume ends are pinned.

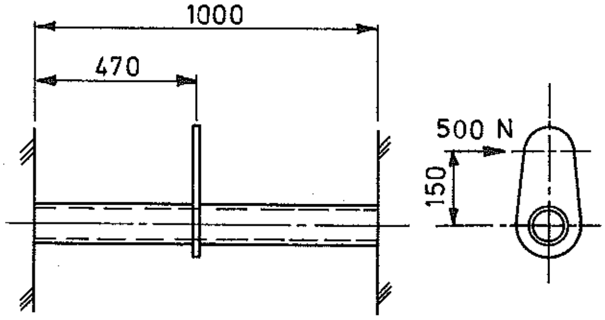


FIG. 16

(a) Draw B.M. and torque diagrams for the shaft.

(b) Determine the required section modulus.

(c) To keep deflections within permissible limits a tube 42.4 outside diameter x 4 wall is finally chosen. Determine:

(i) the lateral deflection in the centre and the maximum permissible deflection.

(ii) the maximum torsional deflection and the maximum permissible torsional deflection.

From the Manufacturer's catalogue, $I = 0.0942 \times 10^6\ \text{mm}^4$

$J = 0.1884 \times 10^6\ \text{mm}^4$

Answers:

(b) $Z_p = 2394.493\ \text{mm}^3$

(c) (i) 0.5314 mm
0.8 mm

(ii) 0.1462°/m
3°/m

6. A solid shaft of CS 1020 is to transmit 120 kW at 250 r.p.m. Service conditions are minor shock $K_t = 1.5$. There is a key in the shaft. Determine:

(a) the required shaft diameter.

(b) the actual torsional deflection and the maximum permissible torsional deflection if a 110 mm diameter shaft is used.

Ans. (a) 101.236 mm

(b) 0.221°/m
3°/m

7. A rotating shaft of CS 1040 carries a bending moment of 45 kN.m where torque is 57 kN.m. Loading conditions are minor shock $K_m = 1.5$ and $K_t = 1.25$.

There is a keyway at the position of maximum moment.

Design the above shaft on the basis of strength only.

Ans. 222.2 mm

8. The machine mechanism shown in Fig. 17. has a hollow shaft at the operating end. Welded to this shaft is a lever. When the lever is in its lowermost position a load "F" is attached. The load is then raised by operating the rope wheel. The lever has 60° of movement and there is a holding brake in the arrangement.
The overall efficiency is 95%.
Determine:

(a) the maximum value of F.

(b) the required section modulus for the hollow shaft. Use a steel equivalent to CS 1020.

(c) the actual and maximum permissible torsional deflection of the shaft if 33.7 outside diameter x 4.0 wall steel tube is used.

Ans. (a) 145.26 kg

(b) $Z_p = 4481.141\ mm^3$

(c) 0.46°; 0.975°

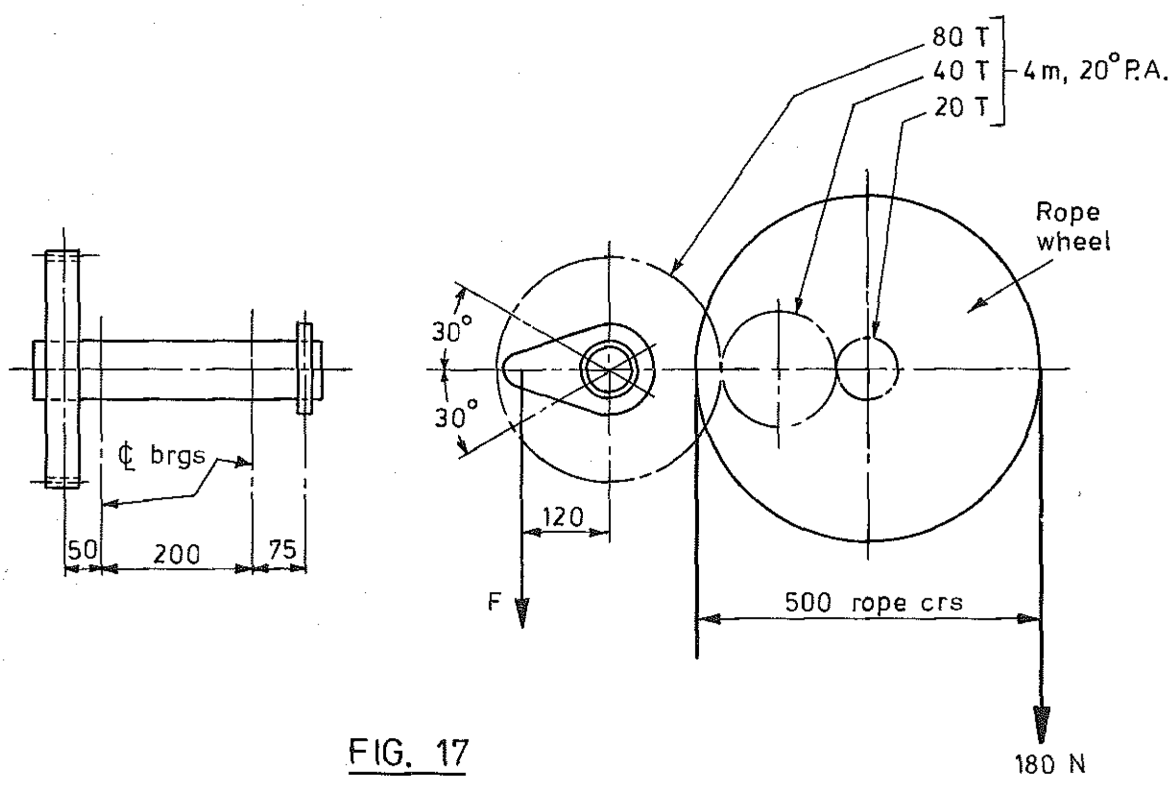


FIG. 17

9. The stationary shaft shown in Fig. 18. is the anchor point for some mechanical equipment. The nominated size of tube is 114.3 outside diameter x 4.5 wall in material equivalent to CS 1020.

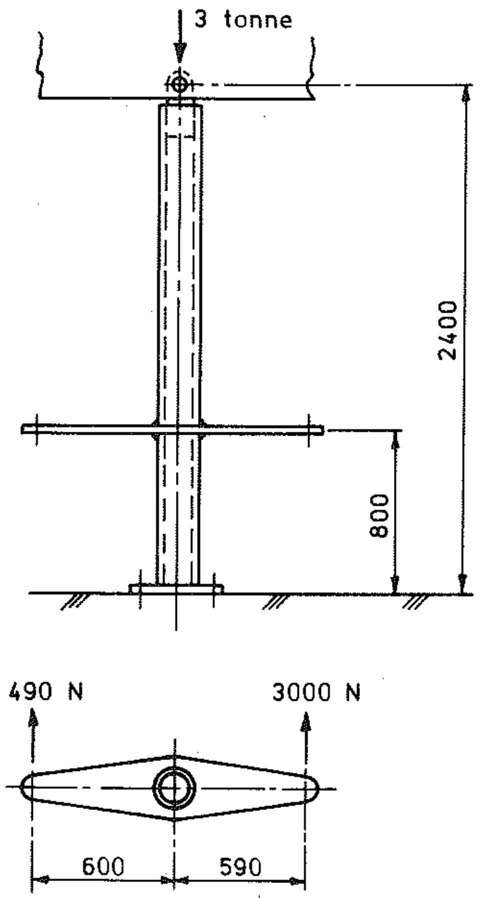


FIG. 18

The axial load is reacted out at the base but the tube is torsionally restrained at each end.

Engineering details of tube from Manufacturer's catalogue.

Area of cross section	1540 mm^2
I	2.33×10^6 mm^4
J	4.66×10^6 mm^4
k	38.9 mm
mass	12.1 kg/m

Determine:

(a) the suitability of the nominated tube.

(b) the approximate maximum lateral deflection and the maximum permissible lateral deflection.

(c) the actual and maximum permissible torsional deflections.

Ans.			
	(a)	102.781 mm < 114.3 mm	
	(b)	1.775 mm	1.92 mm
	(c)	0.146°/m	3°/m

10. Examination question, R.M.I.T., 1979.
Figs. 19 and 20, page 2.29 , show diagrammatic assemblies of a Rumbling Machine used to clean small castings.

In answering the following questions use minor shock factors as follows:

shafts 1.5 on M and 1.25 on T, all other machine elements 1.25.

(a) The drive connection is to be by roller chain. Determine the safety factor for the nominated size.

(b) Determine the bending moment (excluding shock factor) at:

(i) the bearing support, point A.

(ii) the bolted stub shaft connection, point B.

10. (c) Using CS 1030 material determine the required size for the drive side stub shaft.

(d) Fig. 20, page 2.29 shows the suggested design for the bolted drive side stub shaft connection.

Determine the value of:

(i) the interaction formula for the bolts. (Refer to pages 12.29 to 12.43.)

(ii) f_{pf} and F_{pf}. (Refer to pages 12.29 to 12.43.)

(e) Calculate the following values for the drive key in the C.I. chain wheel (assume a 60 mm diameter shaft is used).

(i) f_s and F_s for the key. (ii) f_c and F_c.

Ans. (a) 9.93 (b) (i) 948 613.62 N.mm (c) 59.386 mm
(ii) 559 443.88 N.mm

(d) (i) 0.432
(ii) 32.89 MPa 525 MPa

(e) (i) 44.44 MPa 103.5 MPa
(ii) 145.43 MPa 125-150 MPa

Authors' comments

(i) The above analysis assumes the motor is not started D.O.L. In the stub shaft connection design, it may be advisable to use a further factor of 1.5 on bolt loads to allow for endurance type loading.
Refer to page 12.44.

(ii) The student should now repeat the question assuming the motor is started D.O.L. Pull-out torque = 260% full load torque. Loading conditions impose minor shock.

Follow the instructions given in Worked Example No. 1, page 1.28.

The factors of 1.4 and 3.0 quoted for "Designed couplings" can be used for the design of the bolts in the stub shaft connection. These factors will also cover any endurance type loading that will occur during normal full speed running. Use 16 mm diameter bolts. Permissible stresses for the bolts can be:

$$F_{tf} = 144 \times \frac{1.67}{3} = 80 \text{ MPa}$$

$$F_{vf} = 80 \times \frac{1.67}{3} = 44 \text{ MPa}$$

$$F_{pf} = 525 \times \frac{1.67}{3} = 292 \text{ MPa}$$

Shaft factors can be as before:
1.5 on M and 1.25 on T.
The service factor on key design can be 1.25. Assume a 20 x 12 x 76 key in a 70 mm diameter shaft.

Ans. (a) 9.55 (b) (i) 948 613.62 N.mm (c) 67.98 mm
(ii) 559 443.88 N.mm

(d) (i) 0.558
(ii) 27.63 MPa 292 MPa

(e) (i) 34.28 MPa 69 MPa
(ii) 114.26 MPa 83-100 MPa.
Slightly overstressed but not seriously for crushing stress.

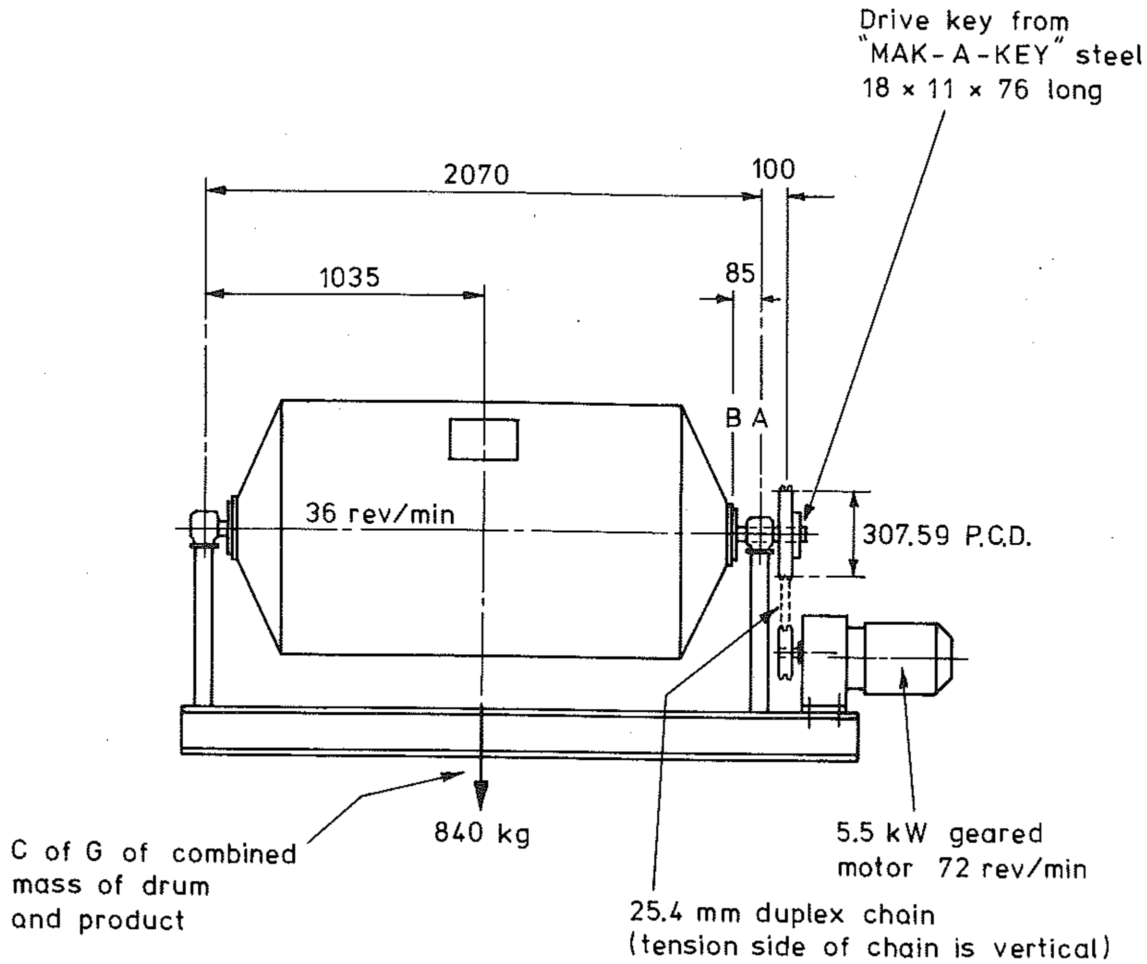

Drive key from
"MAK-A-KEY" steel
18 × 11 × 76 long
2070
100
1035
85
B A
36 rev/min
307.59 P.C.D.
840 kg
C of G of combined
mass of drum
and product
5.5 kW geared
motor 72 rev/min
25.4 mm duplex chain
(tension side of chain is vertical)

FIG. 19

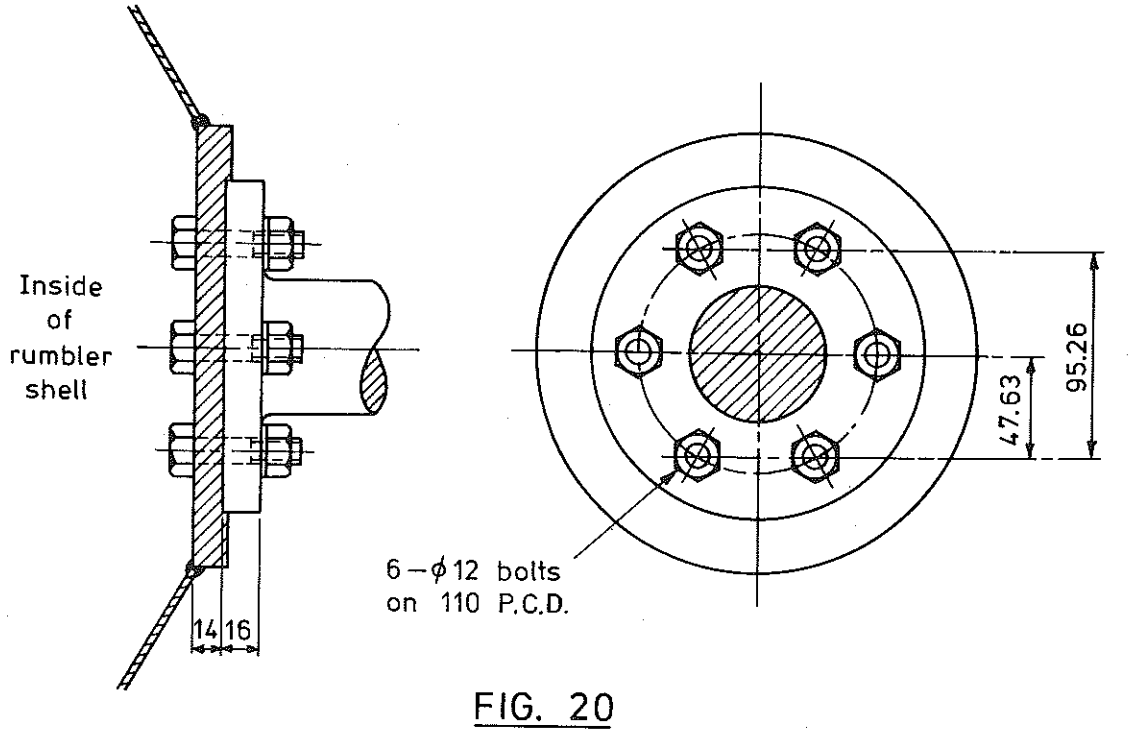

Inside
of
rumbler
shell
14
16
6 – φ 12 bolts
on 110 P.C.D.
95.26
47.63

FIG. 20

THE DISTORTION-ENERGY THEORY

Variously known as the shear-energy theory, Hencky-von Mises theory or octahedral shear stress theory.

This theory, like the maximum-shear-stress theory is used to define the beginning of yield in a ductile material. It does however predict yielding with a greater accuracy and forms the basis of AS 1403, 'DESIGN OF ROTATING STEEL SHAFTS', see page 2.36.

Derivation of theory

Glossary of terms

E	=	Young's modulus	S_{sr}	=	reversed shear stress
F_e	=	endurance stress	S_t	=	a tensile stress
F_{sy}	=	yield stress in shear	S_{tm}	=	mean tensile stress
F_y	=	yield stress in tension	S_{tr}	=	reversed tensile stress
FS	=	factor of safety	$s_{1,2,3}$	=	applied principal stresses
f_s	=	maximum shear stress	s_{av}	=	average principal stress
S_{eq}	=	equivalent state of stress	U	=	total strain energy
S_m	=	mean stress	$U_{1,2,3}$	=	strain energy in principal direction
S_r	=	reversed stress	U_d	=	strain energy for distortion
S_s	=	a shear stress	U_v	=	strain energy for volume change
S_{sm}	=	mean shear stress	$\varepsilon_{1,2,3}$	=	strain
			ν	=	Poisson's ratio

Ductile materials loaded hydrostatically yield at stresses much greater than the value given in a simple tension test. A similar phenomenon can be noticed with any element under a triaxial stress state where $s_1 > s_2 > s_3$. The latter element undergoes angular distortion. If the energy of this distortion is equated to the distortion energy of a simple tension test, a useful algebraic expression can be formed, which defines the beginning of yield for a triaxial state.

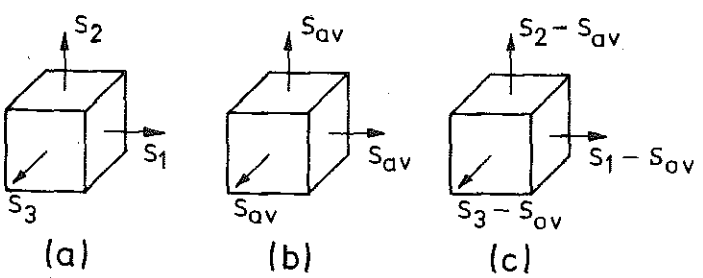


FIG. 21

Fig. 21(a) shows a unit cube with triaxial tension stresses $s_1 > s_2 > s_3$. The cube undergoes both angular distortion and volume change. Then,

$$U_1 = \frac{s_1\varepsilon_1}{2} \quad ; \quad U_2 = \frac{s_2\varepsilon_2}{2} \quad ; \quad U_3 = \frac{s_3\varepsilon_3}{2}$$

(Now typically $\varepsilon_1 = \frac{s_1}{E} - \frac{\nu s_2}{E} - \frac{\nu s_3}{E}$)

$$\therefore \quad U = U_1 + U_2 + U_3 = \frac{1}{2E}[s_1^2 + s_2^2 + s_3^2 - 2\nu(s_1 s_2 + s_2 s_3 + s_3 s_1)] \quad \ldots\ldots(5)$$

Let $s_{av} = \frac{s_1 + s_2 + s_3}{3}$ as shown in Fig. 21(b).

This cube will be under hydrostatic tension stress and will undergo a volume change only.

The remaining stresses $s_1 - s_{av}$, $s_2 - s_{av}$ and $s_3 - s_{av}$ can be applied to a cube as shown in Fig. 21(c), page 2.30. This cube has angular distortion but no volume change.

To obtain the strain energy to produce volume change only, substitute s_{av} for s_1, s_2 and s_3 in equation (5).

$$U_v = \frac{1}{2E}(3s_{av}^2 - 2\nu \times 3s_{av}^2) = \frac{3s_{av}^2}{2E}(1 - 2\nu) \quad \ldots\ldots (6)$$

Now substituting $s_{av} = \frac{s_1 + s_2 + s_3}{3}$ into equation (6) and simplifying

$$U_v = \frac{1 - 2\nu}{6E}(s_1^2 + s_2^2 + s_3^2 + 2s_1 s_2 + 2s_2 s_3 + 2s_3 s_1) \quad \ldots\ldots (7)$$

The distortion energy can then be obtained by subtracting equation (7) from equation (5),

thus

$$U_d = U - U_v = \frac{1 + \nu}{3E}\left[\frac{(s_1 - s_2)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2}{2}\right] \quad \ldots\ldots (8)$$

For a simple tension test, $s_1 = F_y$ and $s_2 = s_3 = 0$.

$$\therefore \quad U_d = \frac{1 + \nu}{3E} F_y^2 \quad \ldots\ldots (9)$$

Equate Eqs. (8) and (9)

$$2F_y^2 = (s_1 - s_2)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2 \quad \ldots\ldots (10)$$

Equation (10) defines the beginning of yield for a triaxial state. If $s_3 = 0$ the biaxial stress condition is stated.

$$F_y^2 = s_1^2 - s_1 s_2 + s_2^2 \quad \text{.......} \quad (11)$$

For a purely torsional condition, $s_2 = -s_1$ and $f_s = s_1$

$$\therefore \quad F_{sy} = 0.577\, F_y$$

This compares with $0.5\, F_y$ for the maximum-shear-stress theory.

Shaft Design Formulas, Distortion-energy Theory

Equation (11) can also be written

$$\left(\frac{F_y}{FS}\right)^2 = s_1^2 - s_1 s_2 + s_2^2 \quad \text{.......} \quad (12)$$

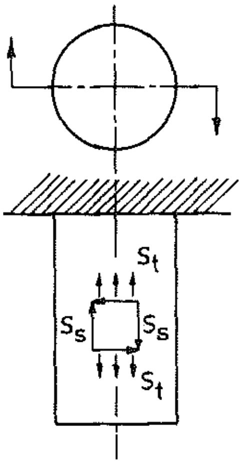


FIG. 22

A shaft is a special case of loading (see Fig. 22) where,

$$s_{1,2} = \frac{S_t}{2} \pm f_s \quad \text{.......} \quad (13)$$

and $$f_s = [(\tfrac{1}{2} S_t)^2 + S_s^2]^{\frac{1}{2}} \quad \text{.......} \quad (14)$$

Substituting equations (13) and (14) into (12)

$$\frac{F_y}{FS} = (S_t^2 + 3S_s^2)^{\frac{1}{2}} \quad \text{.......} \quad (15)$$

For a fluctuating stress condition Soderberg gives an equivalent state of stress

$$S_{eq} = S_m + \frac{F_y}{F_e} S_r \quad \text{.......} \quad (16)$$

Substituting equation (16) into (15)

$$\frac{F_y}{FS} = \left[(S_{tm} + \frac{F_y}{F_e} S_{tr})^2 + 3\,(S_{sm} + \frac{F_y}{F_e} S_{sr})^2 \right]^{\frac{1}{2}} \quad \text{.......} \quad (17)$$

Equation (17) may also be expressed in terms of shaft diameter, applied torque and applied bending moment. This is similar to the procedure used in the maximum-shear-stress theory of shaft design, see page 2.5. Repeating this, transposing for d and adding the appropriate design factors

$$d = \left(\frac{32\ FS}{F_y\ \pi} \left[K_m^2\,(M_a + K_f \frac{F_y}{F_e} M_r)^2 + 0.75\ K_t^2\,(T_a + K_f \frac{F_y}{F_e} T_r)^2 \right]^{\frac{1}{2}} \right)^{\frac{1}{3}}$$

The following is a formula for solid or hollow shafts with or without axial loading

$$d = \left(\frac{32\ FS}{F_y\ \pi\ (1 - A^4)} \left[K_m^2 \left(M_a + K_f \frac{F_y}{F_e} M_r + \frac{1}{8} \alpha K_f W d (1 + A^2) \right)^2 + 0.75\ K_t^2 \left(T_a + K_f \frac{F_y}{F_e} T_r\right)^2 \right]^{\frac{1}{2}} \right)^{\frac{1}{3}}$$

where

$A = \frac{d_i}{d}$ = ratio of internal to external diameters.

d = diameter of shaft or outside diameter of tube, mm.

F_e = endurance limit of material, MPa.

= $0.45\ F_{ult}$ for low carbon steel.

= $0.3\ F_{ult}$ to $0.6\ F_{ult}$ for high tensile and alloy steels.

F_y = yield point stress of material, MPa

= $0.5\ F_{ult}$ for low carbon steel

= $0.8\ F_{ult}$ for high tensile steel

FS = factor of safety, see pages 2.10-2.11.

Due to the more accurately defined procedure of the distortion-energy method, lower safety factors than those specified on pages 2.10 and 2.11 can be used. The designer however, must be satisfied that the shaft will withstand both the starting conditions and any possible load jam situation.

K_m, K_t = shock factor in bending and torsion.

= 1.0 for gradually applied or steady loads

= 1.0 to 1.5 for minor shocks

= 1.5 to 2.0 for heavy shocks

K_m and K_t values are also applicable to axial loads if present.

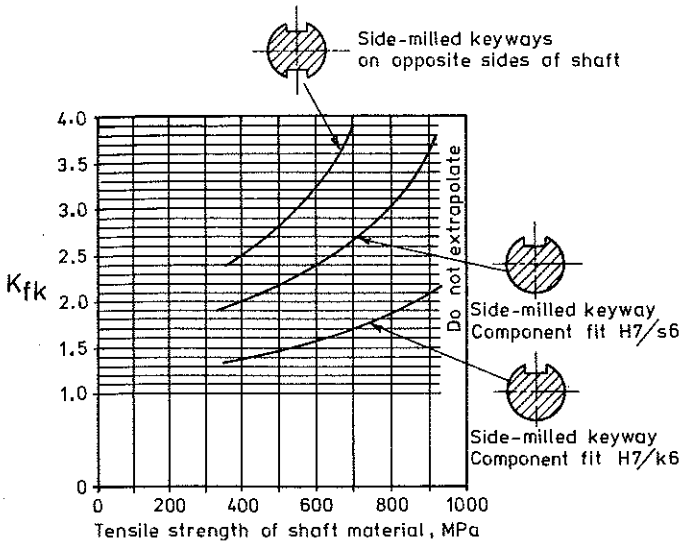
K_f = appropriate stress concentration factor (see page 2.7)

Additionally,

* K_{fk} = stress concentration factor for keyway, see Fig. 23, page 2.34

* K_{fb} = stress concentration factor for rolling contact bearing, see Fig. 24, page 2.34

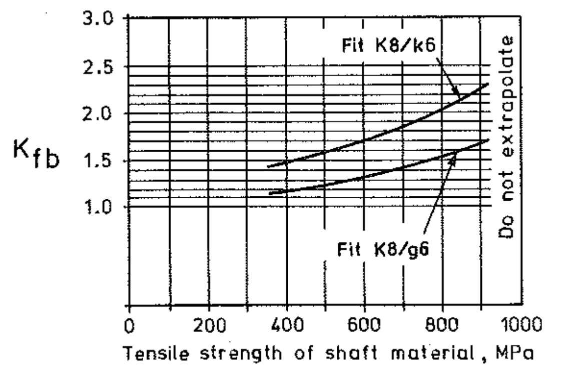
* Press fits create both stress concentration and fretting corrosion. (Figs. 23 and 24 have been reproduced from AS 1403,'DESIGN OF ROTATING STEEL SHAFTS', by permission of the Standards Association of Australia.



Notes:
1. For end-milled keyway with blind-end, multiply factor K_{fk} by 1.1
2. Values may be interpolated for fits between H7/s6 and H7/k6

STRESS-RAISING FACTOR K_{fk} FOR KEYED COMPONENT

FIG. 23



Note: Values may be interpolated for fits between K8/k6 and K8/g6 which are recommended by the bearing manufacturers.

STRESS-RAISING FACTOR K_{fb} FOR SHAFT FITTED WITH ROLLING-CONTACT BEARING

FIG. 24

Where two stress raisers are adjacent to each other they may superimpose. If they are greater than $\frac{d}{4}$ apart, use the larger value. For closer spacing, the larger value should be used and augmented by a percentage of the lower value.
The percentages according to Borchardt are:

between $\frac{d}{6}$ and $\frac{d}{4}$, 10%
less than $\frac{d}{6}$, 20%
— See also Rule 8.2, AS 1403

The application of a K_f factor to M_a and T_a is the designer's decision. Where a keyway exists at the section under consideration, the use of K_{fk} would be advisable. Other factors are optional.

M_a = average or mean bending moment being applied during normal full load shaft operation. (M_a generally = 0 for a revolving shaft.) N.mm

M_r = value of reversed bending moment being applied during normal full load shaft operation, N.mm

T_a = average or mean torque being applied during normal full load shaft operation, N.mm

T_r = value of reversed torque being applied during normal full load shaft operation. (For the majority of shafts T_r = 0.) N.mm

W = axial load, tension or compression, N.
If W is compressive only, neglect K_f in the term

$$\frac{1}{8}\gamma K_f W d (1 + A^2)$$

γ = column action factor, see page 2.8.

Useful references:

R.B. Waterhouse,
Fretting Corrosion,
Pergamon Press.

R.E. Peterson,
Stress Concentration Design Factors,
John Wiley and Sons, Inc., N.Y.

<u>Exercises</u>

Repeat exercises 1 to 10 using the distortion-energy method of design.

<u>Answers</u> (Factor of safety in parenthesis.)

<u>1.</u> 18.2 mm (1.67)

<u>2.</u>(a)(i) 81.95 mm (2.0) with $K_{fk} = 1.4$
(ii) 71.09 mm (2.0)

<u>3.</u> FS = 3.74

<u>4.</u> 27.4 mm (1.67)

<u>5.</u> Z_p = 2362.88 mm^3 (1.92)

<u>6.</u> 94.69 mm (2.0) with $K_{fk} = 1.4$

<u>7.</u> 222.34 mm (2.0)

<u>8.</u> Z_p = 3796.44 mm^3 (2.0)

<u>9.</u> FS = 2.68

<u>10.</u> 56.68 mm (2.0) ; 64.88 mm (3.0)

11. Fig. 25 shows a vertical shaft which transmits a reversing torque of 5300 N.m. The bending moment at the critical section is 4200 N.m. Loads can be considered as heavy shock. Shaft material has the following properties:

F_{ult} = 490 MPa ; F_y = 265 MPa ; F_e = 255 MPa

Determine the shaft diameter using FS = 1.4 and K_m, K_t = 1.75 and an assumed fit of H7/k6.

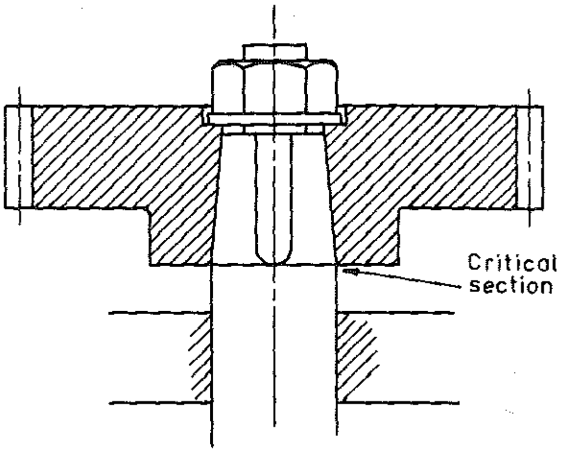


FIG. 25

Ans. 99.26 mm

Design of shafts to AS 1403
'DESIGN OF ROTATING STEEL SHAFTS'

The previous theory in this chapter represented the traditional approach to shaft design in machine design and general mechanical situations. The design method was suitable for any starting torque or pull-out torque an 'off the shelf' squirrel cage motor could give, say 260% full load torque. However, motors having much higher starting torque and pull-out torque are available and there is the possibility of shaft damage unless care is taken. Also severe braking conditions, if present, can present a danger.

AS 1403 offers refinements to cover inertia conditions; also procedures which enable shaft diameters to be kept as small as possible, where this is required.

The latest revision of AS 1403 has been extended to include shafts in a number of general mechanical situations. Specifically, the standard applies to the design of rotating steel shafts which are subjected to torsional, bending and axial-tensile loads either singly or in combination.

In general principle, AS 1403 uses the peak torques, moments and axial loads that occur during the acceleration (or deceleration) period. Thus, exact loads are used in lieu of nominal loads. (Nominal loads being those of normal full speed running conditions.) Because of this, lower safety factors (down to 1.2) are used. In cases where inertia is not significant AS 1403 allows a simplified method to be used for the load calculations. Thus, designs can be done for both 'inertia significant' and 'inertia not significant' cases. Table 1, AS 1403, gives formulas for the torque calculation for each case both for acceleration and deceleration.

The low safety factors are augmented by other considerations.

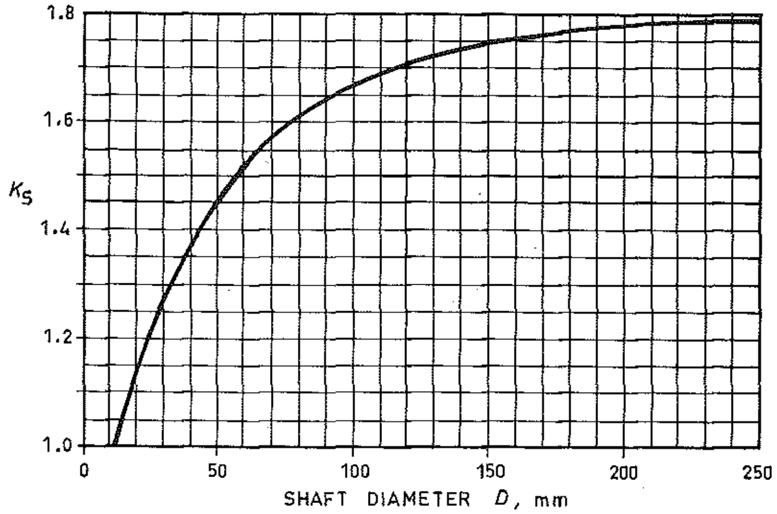
These include: (1) size factor.
(2) stress-raising factors.
(3) corrosion.

(1) Size factor.

The endurance limit for polished or finely machined steel is generally designated as 100%. This applies for diameters up to approximately 10 mm. The fatigue limit is smaller for larger diameters, being about 85% for a 25 mm diameter shaft and 55% for a 250 mm diameter.

Fig. 26 shows this graphically.
On the ordinate is the size factor K_s in lieu of percentage.

Fig. 26, which is Fig. 1 in AS 1403, 'Design of rotating steel shafts' is reproduced by permission of the Standards Association of Australia.



SIZE FACTOR K_s

FIG. 26

F_R values for shafting steels are: CS1020 216 MPa; CS1030 265 MPa; CS1040 292 MPa.

(2) Stress-raising factors.

These are due to fillets, fitted components, keyways, grooves, holes etc.

These are covered in Figs. 2 to 10 in AS 1403.

(3) Corrosion. See Clause 7.2, AS 1403.

Table 2, AS 1403, lists the four basic diameter formulas to be used, together with various choices of operating conditions. A trial diameter is necessary (except where the diameter is dictated by practical considerations) and this may be obtained by using Appendix A, AS 1403. The graph in the Appendix has two lines; the top line is for low strength steel while the lower line is for high strength steel.

Appendices D, E and F, AS 1403, give sample calculations of shaft designs for various cases.

Codes that require the use of AS 1403 are AS 1418 SAA Crane Code and AS 1755 SAA Conveyor Safety Code. The latter Code includes belt conveyors, bucket elevators, screw conveyors, screw elevators and various other types of equipment used for moving and/or lifting materials.

However, Statutory bodies governing the design of 'Code' equipment have a reasonable attitude towards the use of Codes. Their primary concern is in the safety of the various machine elements. Special circumstances exist sometimes where shaft diameters smaller than those indicated by the use of AS 1403 may be satisfactory. This can always be checked out with the particular Statutory Authority concerned, at the outset of a design.

An abbreviated Worked Example now follows, which illustrates the use of the graphs and equations in AS 1403. Notation is given in the code.

<u>Worked example No. 8.</u>

Check the theoretical diameter of the following CS1020 shaft.

Peak loadings are:
reversing torque 64 N.m
bending moment 105 N.m

The shaft has a 40 mm basic diameter stepped up to 45 mm diameter with a 1.25 mm shoulder radius. An end milled keyway finishes within 5 mm of the step.
Component fit, H7/k6.
Loading conditions require the use of Formula 3, Table 2, AS 1403.

<u>Answer</u>

From Fig. 3 AS 1403

$$\frac{D_1}{D} = \frac{45}{40} = 1.125$$

$$\therefore \Delta = 0.09$$

From Fig. 4 AS 1403

$$Z = \frac{R}{D} + \Delta$$

$$= \frac{1.25}{40} + 0.09 = 0.12$$

$\therefore$ K = 1.55 (for F_u = 400 MPa, see Table No. 2, page 2.9)

Keying of component, Fig. 7 AS 1403

$K = 1.4 \times 1.1 = 1.54$

Clause 8.2(d), AS 1403

Distance between stress raising characteristics = 5 mm

$0.16\ D = 0.16 \times 40 = 6.4\ \text{mm} > 5\ \text{mm}$

$\therefore\ K = 1.55 + 0.2 \times 1.54 = 1.858$

From Fig. 1 AS 1403

K_s for 40 mm diameter = 1.37

$F_R = 0.45\ F_u$ from Notation (Clause 4) in AS 1403

$= 0.45 \times 480 = 216$ MPa (refer to page 2.36)

Formula 3, Table 2, AS 1403

$$D^3 = \frac{10^4\ F_s}{F_R} K_s K \sqrt{\left(M_q + \frac{P_q D}{8000}\right)^2 + \frac{3}{4} T_q^2}$$

$$= \frac{10^4 \times 1.2}{216} \times 1.37 \times 1.858 \sqrt{105^2 + \frac{3}{4} \times 64^2}$$

$$= 16\ 790.248$$

$\therefore\ D = 25.6\ \text{mm} < 40\ \text{mm}$

$\therefore$ design is adequate.

Exercises

12. Redesign the shaft of Worked example No. 8 with the following variations:

Peak torque	=	150 N.m (non-reversing)
Peak moment	=	180 N.m
Axial tension load	=	10 kN.

Keyway is side milled (swept end).

A 10% corrosion allowance factor is required, i.e. a 1.1 factor on the R.H.S. of Formula 4, Table 2, AS 1403.

Ans. D = 33.59 mm

13.(a) Refer to Exercise 5, page 16.11.

Using AS 1403 design the diameter of the head shaft at the bearing on the drive side.

Use CS1020 material and K8/k6 bearing fit. Elevator operation dictates the use of Formula 4 in Table 2, AS 1403.

Determine:

(i) Mass M of I of elevator parts with respect to headshaft drive input position, for acceleration phase.

($23.0258\ \text{kg.m}^2$)

Cont. on page 2.39

(ii) Mass M of I of complete system with respect to motor shaft for acceleration phase.

(0.009956 kg.m^2)

(iii) Motor torque available for acceleration.

(6.7279 N.m)

(iv) M_q and T_q (443.1585 N.m; 511.139 N.m)

(v) Diameter of shaft. (39.79 mm)

(b) Using Guest's formula and a F of S of 3.0 determine the required shaft diameter.

(38.05 mm)

14.

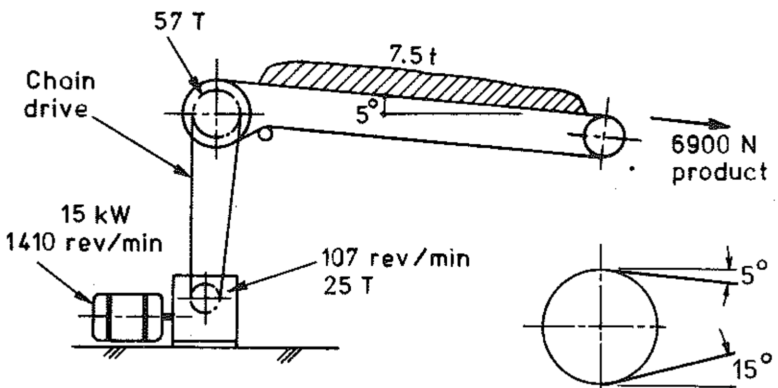
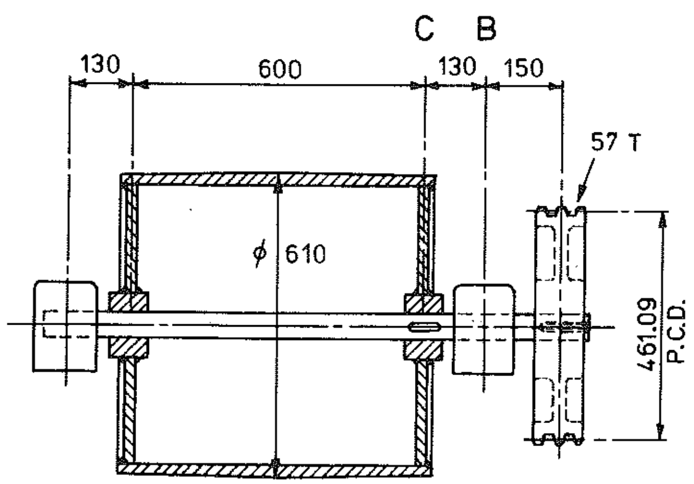


FIG. 27

Fig. 27 shows a schematic arrangement of a troughed belt conveyor for transporting coal.

Fig. 28 indicates the design layout of the headshaft.

Determine the required size of the shaft at points B and C using AS 1403 and compare with the sizes obtained when using Guest's formula incorporating a basic safety factor of 3.0. Material CS1020.



Mass of pulley = 250 kg
Mass of sprocket = 60 kg

FIG. 28

Engineering details:

Mass M of I of conveyor and product at headshaft input position for acceleration phase,
conveyor .... 418 kg.m^2
product .... 727 kg.m^2

Mass M of I of gear box and complete drive system at motor shaft for acceleration phase .... 0.1106 kg.m^2

Efficiencies:

Drive to headshaft input 94%
Headshaft 96%

Component of conveyor product opposing motion (conveyor frictional allowance included) .... 6900 N

Coefficient of friction, belt to head pulley 0.35

Angle of wrap at head pulley .... 200°

Average starting torque of motor .... 260% F.L. torque

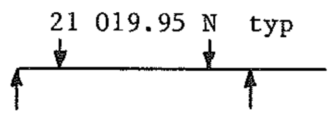
Procedural requirements:

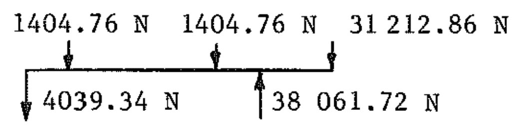
(a) Assume the conveyor has to be started in the fully loaded condition and belt slip does not occur.
Assume μ remains constant (conservative approach).

(i) Determine the motor torque available for acceleration.
(186.5085 N.m)

(ii) Calculate the torque applied to the headshaft, T_q.
(7060.2695 N.m)

(iii) From (ii) draw loading diagrams for the headshaft in the horizontal and vertical planes. For simplicity, assume 100% headshaft efficiency. The drive side of the chain is vertical.





(iv) State M_q at B and C.

(4681.929 N.m
4673.662 N.m)

(v) Determine the required diameters at B using a K8/k6 bearing fit and C using an H7/k6 fit and Formula 4 in Table 2, AS 1403.

(95.42 mm; 95.92 mm)

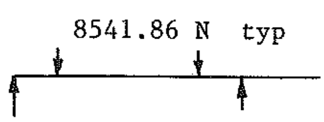
(b) Assume the conveyor has to be started in the empty condition and belt slip does not occur; determine the torque applied to the headshaft, T_q.
(6092 N.m)

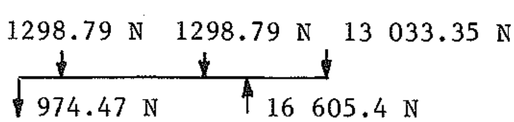
(c) Assume the conveyor is running at working speed and full motor power is being utilised.

(i) Determine the torque being applied to the headshaft.

(2869.07 N.m)

(ii) From (i) draw loading diagrams for the headshaft in the horizontal and vertical planes. For simplicity assume 100% headshaft efficiency. The drive side of the chain is vertical.





(iii) Using Guest's formula calculate the required diameters at B and C.
F of S = 3.0.

(83.84 mm; 91.81 mm)

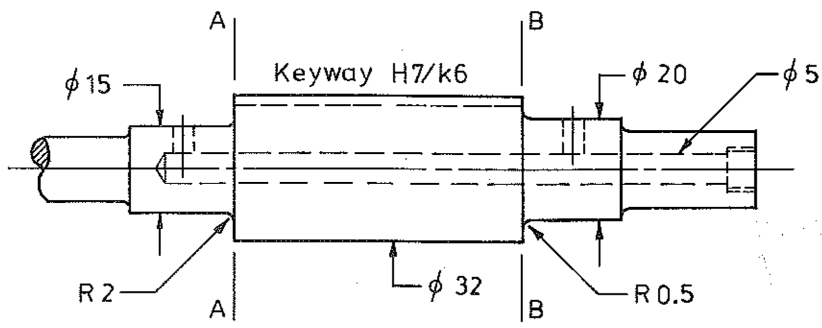


FIG. 29

15. Fig. 29 shows a suggested design for a revolving steel shaft subject to the following loads at normal operating speed:

bending moment at AA = 4 N.m
torque at AA = 2 N.m (reversing)

bending moment at BB = 12 N.m
torque at BB = 10 N.m (reversing)

Shock free

Starting torque = twice full load torque

Determine the minimum permissible diameters for the steps at AA and BB making due allowance for the central hole. Material of construction CS1020.

Three methods are to be used:

(a) AS 1403 'Inertia not significant'.

(b) Distortion energy theory, F of S = 3.0

(c) Maximum shear stress theory, F of S = 3.0

Ans. (a) 10.41 & 16.09 mm (b) 11.04 & 18.635 mm (c) 10.52 & 18.215 mm

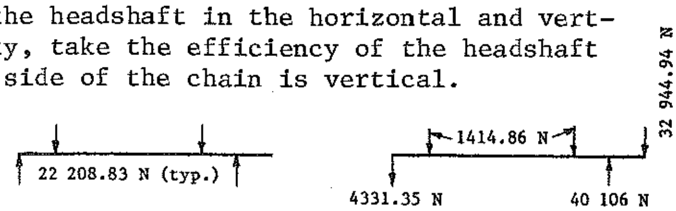
16. Repeat Exercise 13 using the 'Inertia not significant' method from AS 1403.

Ans. 40.819 mm

17. Repeat Exercise 14 using the 'Inertia not significant' method from AS 1403. (Refer particularly to Exercise 14(a), (i) to (v)).

(i) Draw loading diagrams for the headshaft in the horizontal and vertical planes. For simplicity, take the efficiency of the headshaft itself as 100%. The drive side of the chain is vertical.

Ans.



(ii) State M_q at B and C and T_q.

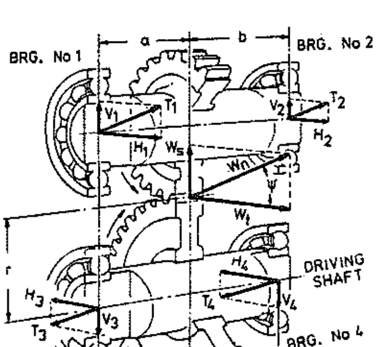
Ans. 4941.741 N.m, 4941.878 N.m, 7459.593 N.m

(iii) Determine the required diameters at B using a K8/k6 bearing fit and C using an H7/k6 fit and Formula 4 in Table 2, AS 1403.

Ans. 97.679 mm and 98.224 mm

CHAPTER 3

GEARS



CHAPTER 3

****

G E A R S

Toothed gearing is used to connect two shafts where the distance between them is not too large.

Gearing is one method of obtaining a positive drive; it gives a constant speed ratio between the shafts.

This chapter deals with two types of gears:-

(i) Spur Gears

If the teeth are parallel to the axis of the cylinder, the gear is called a straight spur or simply a spur gear.

(ii) Helical Gears

If the teeth are inclined to the axis of the cylinder, the gear is called a helical gear.
Only helical gears suitable for connecting parallel shafts are considered.

Design will be done to the B.S. 436 code (AS B61), Machine Cut Gears
A. Helical & straight spur.

Form of Teeth. 20° Involute system.

The essential definitions of gearing elements are shown in Fig. 1, page 3.2.

Tooth module is the height of the addendum in mm. Alternatively, tooth module may be defined as the number of mm in the pitch circle diameter for each tooth in the gear i.e. $m = D/T$. These definitions apply specifically to spur gears.

Standard normal modules according to BS 436 : Part 2 : 1970 are (non-preferred sizes in parentheses):

1, (1.125), 1.25, (1.375), 1.5, (1.75), 2, (2.25), 2.5, (2.75), 3, (3.5), 4, (4.5), 5, (5.5), 6, (7), 8, (9), 10, (11), 12, (14), 16, (18), 20, (22), 25, (28), 32, (36), 40, (45), 50.

Base Circle

This is the circle from which the involute tooth curve is developed.

Hunting Tooth

It is common practice to make one of a pair of meshing gears to have a "hunting tooth".

Then, each tooth of one gear will encounter each tooth of the other equally often, and wear will be equalized. Any pair of gears will have a hunting tooth if the highest common factor to the numbers of teeth is unity.

In other words the numbers must be prime to each other.

Examples:-

Prime to each other	Not prime to each other
21 and 41	21 and 39
30 and 61	30 and 62

SPUR GEAR NOMENCLATURE

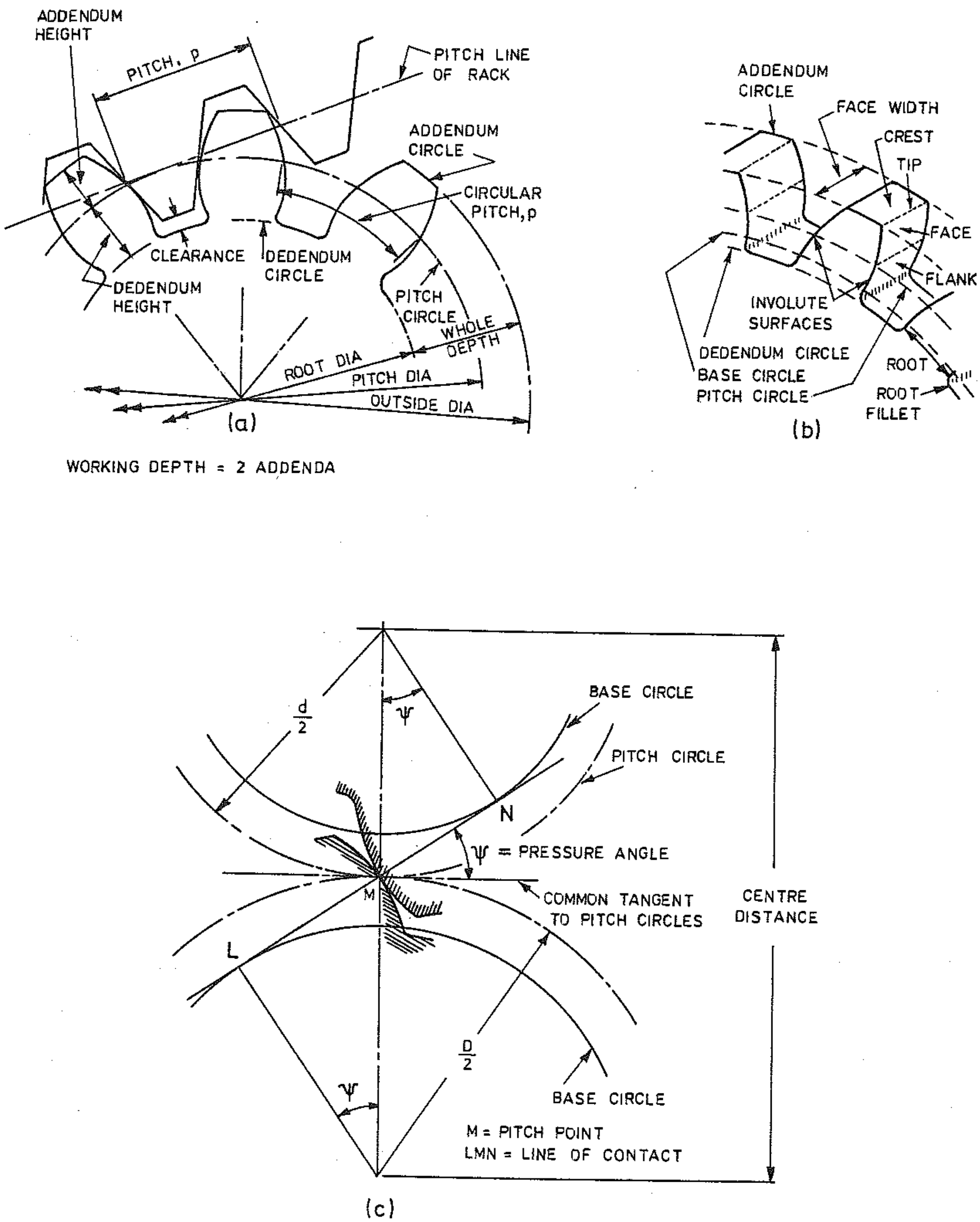


FIG. 1

Ratio: The maximum ratio of spur and helical gears is about 10.1 although it is generally kept at 3:1 maximum.

Service Factors (See also Clause 67, BS 436.)

These are applied to the input power to a pair of gears, refer to Table No.1. Factors for electric motor drives other than D.O.L., to be chosen to suit the individual application.

TABLE NO. 1

Based on AGMA Standard 420.03 - 1963

PRIME MOVER	UNIFORM	MODERATE SHOCK	HEAVY SHOCK
Electric motor - squirrel cage D.O.L.	1.20	1.40	1.60
Multicylinder internal-combustion engine	1.25	1.50	2.00
Single-cylinder internal-combustion engine	1.50	1.75	2.25

Module Selection Chart

Fig. 2. is a graph of R.P.M. against power which enables a tooth module to be selected.

The selection has to be for the pinion (ideally in the 16 to 20 teeth range otherwise some interpolation is necessary). This graph is based on the Lewis formula and specifically applies to materials in the mild steel range.

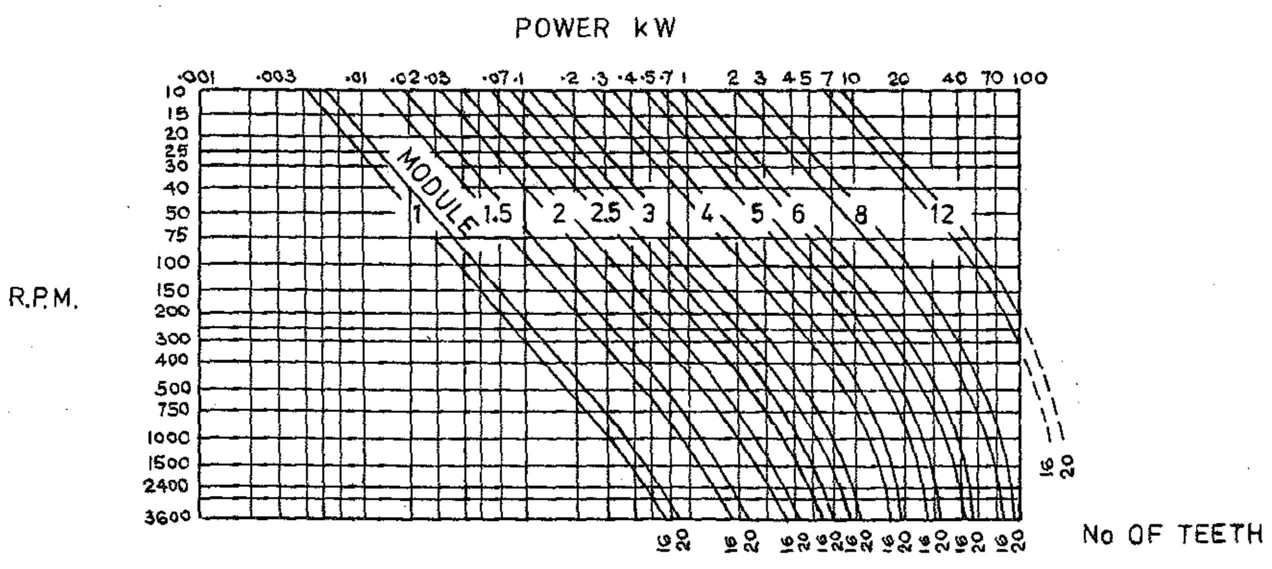


FIG. 2

MODULE SELECTION CHART

SPUR GEARS AND PARALLEL HELICAL GEARS

Materials of Gears

Some of the more common materials of manufacture and their design factors are given in Table No. 2, page 3.5.
Normally it is the "pinion for wear" that determines the face width of a pair of meshing gears. Consequently the pinion material is generally the higher grade. However, it is quite satisfactory to make the pinion and gear from the same material specification if this is convenient.
1000 MPa steel is about the highest tensile steel that can be gear cut. Higher tensile steel needs to be gear cut in the annealed condition and then heat treated to the required tensile strength.
No grinding of teeth or any other machining operation except grinding of the bore is required after heat treatment.
Heat treated gears are best avoided if possible.

GLOSSARY OF TERMS (Essentially from BS 436.A.)

Refer also to BS 2519: Part 1 and Part 2: 1976

A	=	wheel addendum, mm
a	=	pinion addendum, mm
B	=	wheel dedendum, 1.25 module (standard tooth)
b	=	pinion dedendum, 1.25 module (standard tooth)
C	=	centre distance, mm
D	=	wheel pitch diameter, mm
d	=	pinion pitch diameter, mm
I	=	wheel root diameter, mm
i	=	pinion root diameter, mm
J	=	wheel outside diameter, mm
j	=	pinion outside diameter, mm
k_p	=	pinion correction factor
k_w	=	wheel correction factor
m	=	module, mm
p	=	circular pitch in plane of rotation, mm
p_n	=	normal pitch, mm (refers to helical gears) (p_n is the pitch of the cutter used)
T	=	number of teeth in wheel
t	=	number of teeth in pinion
W_a	=	axial (thrust) force, N
W_n	=	normal force acting between gear teeth i.e. the force absolute, N
W_s	=	separating force, N
W_t	=	tangential force, N
Δ	=	extension of centre distance factor
σ	=	helix angle in degrees
ψ	=	pressure angle, 20°

The following power formulas apply:-

$$kW_{(wear)} = \frac{X_c \; S_c \; Z \; F \; N \; T \; m^{1.8}}{10^7}$$

where X_c is from Chart 11 in BS 436 (reproduced on page 3.16)

S_c is the surface stress factor, Table No. 2

Z (zone factor) is from Chart 8 in BS 436 for spur gears (reproduced on page 3.15)

<u>or</u> is from Chart 7 in BS 436 for helical gears having a helix angle of 30° (reproduced on page 3.15)

F	=	face width, mm	T	=	number of teeth
N	=	R.P.M.	m	=	module, mm

$$kW_{(strength)} = \frac{X_b \; Y \; S_b \; F \; N \; T \; m^2}{1.9 \times 10^7}$$

where: X_b is from Chart 10 in BS 436 (reproduced on page 3.16)

Y (strength factor) is from Chart 9 in BS 436 for spur gears and helical gears having a helix angle of 30° (reproduced on page 3.16)

S_b is the bending stress factor, Table No. 2

Other letters as previously stated.

Gear design factors — TABLE NO. 2

Values for other materials can be found in Appendix A of BS 436.
Materials can be the same for a pinion and its mating gear.

Material	S_c	S_b
Cast iron (not heat treated) 185 MPa tensile	6.9	40
Mild steel, grade 250 AS 1204	7.0	96
CS 1020 AS 1442 AS 1443	7.0	96
CS 1030 AS 1442 AS 1443	8.3	114
CS 1040 AS 1442 AS 1443	9.7	130
En 8 (normalized)	9.7	130
Cast steel (not heat treated) 600 MPa tensile	9.7	130
Steel - normalised, forged 0.4 C	9.7	117
Steel - normalised, forged 0.55 C	13.8	150

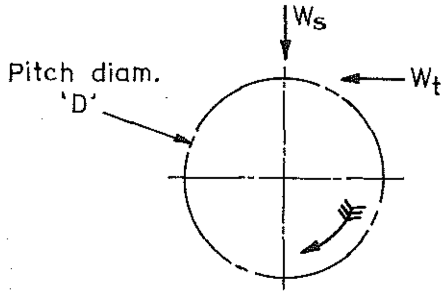
SPUR GEARS

D = T x m = Wheel pitch diameter
d = t x m = Pinion pitch diameter

J = (T + 2) m = Wheel outside diameter
j = (t + 2) m = Pinion outside diameter

The driving force W_n acting between the surfaces of the teeth is normal to the surfaces and acts along the line which is tangent to both base circles.

The normal force may be replaced by components W_t and W_s acting perpendicular and parallel to the centre line of the tooth, refer to Fig. 3.



Free body diagram
of driving gear

$$W_t = \frac{\text{Torque} \times 2}{D}$$

$$W_s = W_t \tan \psi$$

FIG. 3

The radial (separating force) W_s produces a compressive force on the tooth and may be used with W_t for shaft design and bearing calculations.
The tangential component W_t causes bending stresses on the tooth and is used as a basis for the design of the tooth.

Minimum Number of Teeth

To avoid undercutting of the root of the tooth the minimum number of teeth should be 17 with a pressure angle of 20°.

Undercutting reduces tooth strength and gives a poor action with mating teeth. This condition can be avoided somewhat by applying positive addendum modification to the pinion. This is discussed on pages 3.9 to 3.11.

Tooth Size and Face Width

For best proportions the face width of a spur gear should lie between 8 and 12 times the tooth module.

It is usual for the face width of the pinion to be slightly greater (say 4 to 8 mm depending upon size) than that of the gear. This is mainly for appearance and to give a wider tolerance for assembly. It is bad practice not to have full face contact for the gear.

Parallel Helical Gears

$$D = \frac{T \times m}{\cos \sigma} = \text{Wheel pitch diameter}$$

$$d = \frac{t \times m}{\cos \sigma} = \text{Pinion pitch diameter}$$

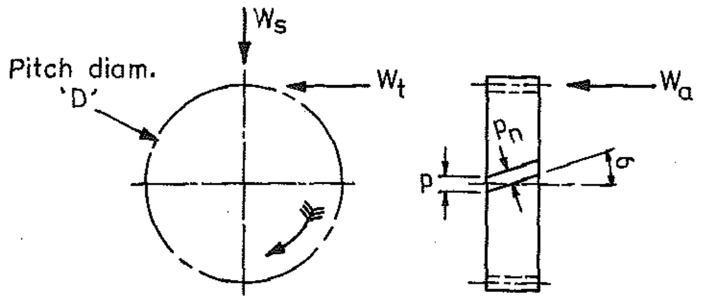
$$J = \frac{T \times m}{\cos \sigma} + 2\,m = \text{Wheel outside diameter}$$

$$j = \frac{t \times m}{\cos \sigma} + 2\,m = \text{Pinion outside diameter}$$

Because of gradual engagement of the teeth, helical gears run more smoothly and quietly than spur gears.

They may be operated at pitch line velocities of 300 m/min. or more.

Refer to Fig. 4.



Free body diagram of driving gear

FIG. 4

$$W_t = \frac{\text{Torque} \times 2}{D}$$

$$W_s = \frac{W_t \tan \psi}{\cos \sigma}$$

$$W_a = W_t \tan \sigma$$

Note:

For helical gears the 'normal' module and 'normal' pressure angle are used.

Three components of force act.

(1) Tangential force W_t

(2) Separating force W_s

(3) Thrust (axial) force W_a

Minimum Number of Teeth

Minimum number of teeth to avoid undercutting

$$T_{min.} = \cos^3\sigma \left(\frac{2}{\sin^2\Psi}\right)$$

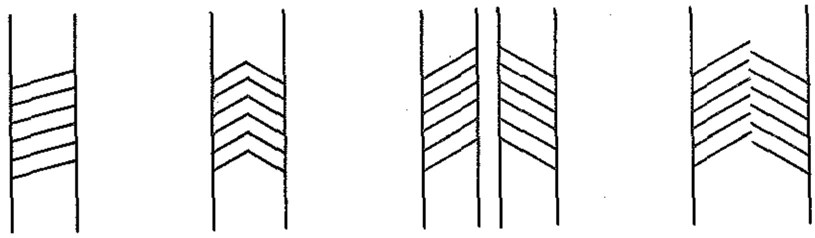
Refer to Table No. 3.

TABLE NO. 3

σ in degrees	T_{min} Ψ 20°
0	17
10	16
15	15
20	14
25	13
30	11
35	10

Helix angle for parallel shafts should preferably be less than 20° in order to prevent excessive thrust (single helical gears only).

The four basic types of helical teeth are shown in Fig. 5, page 3.8.



SINGLE | DOUBLE (HERRINGBONE) | * DOUBLE WITH GAP | DOUBLE STAGGERED (WUEST)

*Used for high speeds (say over 150 m/min.) where gears run in oil.
Gap allows oil to escape and so reduce noise etc.

FIG. 5

Minimum Face Width

To obtain smooth running gears, the helix angle should be such that one end of the tooth remains in contact until the opposite end of the following tooth has formed a bearing.

Minimum face width for single helical gears,

$$F = \frac{p}{\tan \sigma}$$

$$\text{Now } p_n = p \cos \sigma = \pi m$$

$$\therefore \quad F = \frac{p_n}{\tan \sigma \quad \cos \sigma} = \frac{\pi m}{\tan \sigma \quad \cos \sigma} = \frac{\pi m}{\sin \sigma}$$

For double helical gears,

$$F = \frac{2p}{\tan \sigma} = \frac{2\pi m}{\tan \sigma \cos \sigma} = \frac{2\pi m}{\sin \sigma}$$

Again, as for spur gears the face width of the pinion should be slightly larger than that of the gear.

Double helical gears give zero thrust force. Helix angles commonly used with this type of gear are 15°, 25°, 30°, and for high speeds (turbine drives) 45°.

Hand of Helix

A helical gear is designated "right hand" when a point traced along the helix in a clockwise direction moves away from the observer.

When a point is traced along the helix in an anti-clockwise direction and moves away from the observer the gear is designated "left hand".

This is shown diagrammatically in Fig. 6.

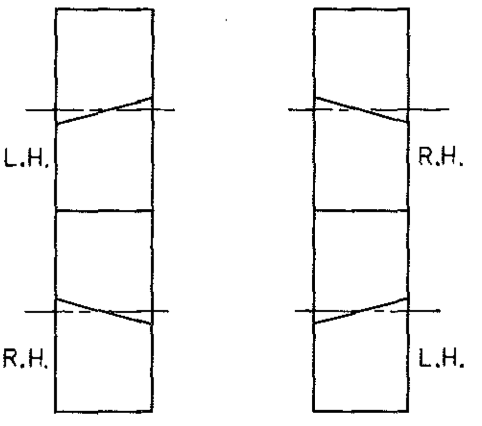


FIG. 6

Use of "Power Formulas".

In applying the power formulas as noted on page 3.5 for the design of parallel helical gears, note the following:

Zone factor, Z. The zone factor for 30° helix angle shall be in accordance with Chart 7, page 3.15. For other helix angles the zone factor obtained from Chart 7 shall be multiplied by $0{\cdot}75\ \sec^2\sigma$.

Strength factor, Y. The strength factor for helical gears with 30° helix angle and 20° normal pressure angle shall be in accordance with Chart 9, page 3.16, provided that the face width is sufficient to give overlap.

For other helix angles the strength factor obtained from Chart 9, page 3.16, shall be multiplied by $1.33\ \cos^2\sigma$.

Correction.

The shape of an involute of a circle varies with the size of the circle and thus in a series of gears all of the same module, pressure angle and tooth proportions, the shape of the teeth will vary with the number of teeth in the gear. This is indicated below and in Fig. 7.

12 t, 3 m, $\Psi = 20°$

$d = t\,m = 12 \times 3 = 36$ mm.

Base circle diameter $= 36 \cos 20°$

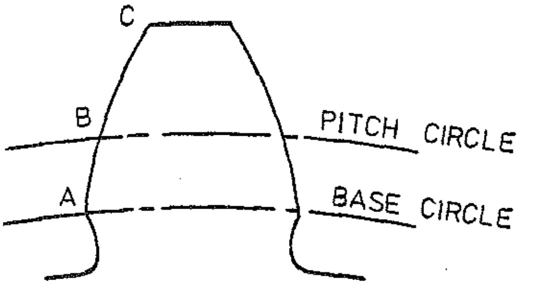
$= 33.829$ mm

Radial distance between base and pitch circle diameters

$$= \frac{36 - 33.829}{2} = 1.086 \text{ mm}$$

Dedendum $= 1.25\ m = 1.25 \times 3 = 3.75$ mm

$\therefore$ tooth space extends inside base circle.



DIAGRAMMATIC REPRESENTATION OF UNDERCUTTING

FIG. 7

Since an involute cannot extend inside its base circle it follows that portion of the profile below the base circle is not an involute. The shape of this portion is formed by the corner of the cutter that generates the gear. This portion of the profile plays no part in the tooth action when the gear is meshed with another.

The working profile of the tooth is ABC (Fig. 7.) of which AB is inside and BC outside the pitch circle.

If this gear meshes with a similar 12 T gear then the portion AB will engage with a portion similar to BC of the mating gear. As the length of these portions are unequal there will be considerable sliding between the tooth profiles.

The teeth are thinner below than at the pitch circle i.e. they are said to be undercut and are relatively weak. Undercutting also reduces the contact ratio (average number of pairs of teeth in contact, 1.2 being an optimum figure).
As the number of teeth in the pinion increases, the radial distance between the base and pitch circles increases, thus increasing the length of tooth profile in working contact.

The contact ratio increased, undercutting is decreased, the relative average radius of curvature is increased, resulting in less sliding between the faces in contact and generally in a stronger tooth.

All spur gears generated from the same basic rack will mesh correctly with each other. This however, is only true of the involute portions left by the generations. When the axis of symmetry of the basic rack coincides with the pitch line, it produces "standard" (i.e. unmodified) teeth and the flanks of small pinions may be undercut. This occurs when the number of teeth is less than 17 for spur gears having a pressure angle of 20°.

The result, as before, reduces the contact ratio and weakens the teeth. This can be avoided by applying positive addendum modification (or correction).

Correction for involute gear teeth means that the outside diameter and root diameter of the gear are changed slightly. A positive correction requires the outside diameter to be slightly larger than the standard. The depth of the teeth is the same as standard (except case (d) under "Modification of spur and helical gears to BS 436") and the standard cutter is still used to generate the teeth.

Generally speaking, it is desirable to apply more than the minimum amount of addendum modification necessary to avoid undercutting. The portion of the involute curve near the base circle has little capacity to resist surface loading since it has a comparatively small radius of curvature; the sliding velocity tends to be high and the rolling velocity low.

For this reason, many gears which do not need addendum modification merely to avoid undercutting, can, with advantage, be modified. This achieves better conditions of surface contact and relative motion and a better balance of conditions between wheel and pinion. Superior zone and strength factors result also and this is reflected by a discontinuity in some of the design curves that are given in BS 436.

Modification of spur and helical gears to BS 436

The recommended values for the addendum are given by:

$$a = m(1 + k_p) \qquad A = m(1 + k_w)$$

k_p and k_w are determined thus:

(a) If $(t + T)\sec^3\sigma$ be not less than 60

$$k_p = 0.4\left(1 - \frac{t}{T}\right) \quad \underline{\underline{\text{or}}} \quad 0.02\,(30 - t\sec^3\sigma),$$

whichever is the greater,

and $k_w = -k_p$

(b) If $(t + T) \sec^3 \sigma$ be less than 60 then the centre distance shall be extended by an amount Δm.

$$k_p = 0.02\ (30 - t \sec^3 \sigma)$$

$$k_w = 0.02\ (30 - T \sec^3 \sigma)$$

Values of Δ can be obtained from Table No. 4.

TABLE NO. 4.

$k_p + k_w$	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
Δ	0.10	0.15	0.20	0.24	0.29	0.33	0.38	0.42	0.46	0.50	0.54	0.57	0.61	0.63	0.66

(c) For internal gears:

$$k_p = 0.4 \qquad\qquad k_w = -k_p$$

irrespective of numbers of teeth.

(d) To maintain the tip thickness if the number of teeth in the pinion is such that $t \sec^3 \sigma$ is less than 17, the "corrected" outside diameter shall be reduced by an amount

$m \times 0.04\ (17 - t \sec^3 \sigma)$,

the pitch and root diameters remaining unaltered.

Design examples are given in BS 436, Appendix B.

Worked Example No. 1.

Determine the face width of the following spur gears:

power 1.5 kW

pinion 32 T 22 r.p.m., 0.4% normalised steel.

gear 44 T 16 r.p.m., cast steel.

4 hours per day operation.

Answer.

By interpolation using the graph Fig. 2, page 3.3, 6 module would seem satisfactory.

	Pinion	Gear	
S_c	9.7	9.7	Table No. 2, page 3.5.
S_b	117	130	Table No. 2, page 3.5.
X_c	0.775	0.825	Chart 11, page 3.16.
X_b	0.625	0.64	Chart 10, page 3.16.
Y	0.70	0.645	Chart 9, page 3.16.
Z	2.40	2.40	Chart 8, page 3.15.

Worked Example No. 1. (Cont.)

Pinion (wear). Transposing the formula on page 3.5

$$F = \frac{kW \times 10^7}{X_c \ S_c \ Z \ N \ T \ m^{1.8}}$$

$$= \frac{1.5 \times 10^7}{0.775 \times 9.7 \times 2.4 \times 22 \times 32 \times 6^{1.8}} = 46.94 \text{ mm}$$

Pinion (strength). Transposing the formula on page 3.5

$$F = \frac{kW \times 1.9 \times 10^7}{X_b \ Y \ S_b \ N \ T \ m^2}$$

$$= \frac{1.5 \times 1.9 \times 10^7}{0.625 \times 0.70 \times 117 \times 22 \times 32 \times 6^2} = 21.97 \text{ mm}$$

Similarly for the gear.

Gear (wear)

$$F = \frac{kW \times 10^7}{X_c \ S_c \ Z \ N \ T \ m^{1.8}}$$

$$= \frac{1.5 \times 10^7}{0.825 \times 9.7 \times 2.4 \times 16 \times 44 \times 6^{1.8}} = 44.1 \text{ mm}$$

Gear (strength)

$$F = \frac{kW \times 1.9 \times 10^7}{X_b \ Y \ S_b \ N \ T \ m^2}$$

$$= \frac{1.5 \times 1.9 \times 10^7}{0.64 \times 0.645 \times 130 \times 16 \times 44 \times 6^2} = 20.96 \text{ mm}$$

Summary: Face width determined by pinion for wear = 46.94 mm

Ideal face width to be between 8 m and 12 m, page 3.6

= 8 x 6 and 12 x 6

= 48 mm and 72 mm

∴ make pinion 52 mm face width

and gear 48 mm face width.

Worked Example No. 2.

Design a pair of single helical gears to transmit 46 kW for 8 hours per day. Helix angle to be 20°. Pinion speed = 750 r.p.m., 3:1 reduction approximately. Allow a hunting tooth.

Materials:

pinion, CS 1040

gear, cast steel

Worked Example No. 2. (Cont.)

Answer.

Using graph Fig. 2, page 3.3, 8 module is indicated

	Pinion 20 T 750 r.p.m.	Gear 61 T 245.9 r.p.m.	
S_c	9.7	9.7	Table No. 2, page 3.5
S_b	130	130	Table No. 2, page 3.5
X_c	0.32	0.40	Chart 11, page 3.16
X_b	0.31	0.38	" 10 " 3.16
Y	$0.70 \times 1.33 \cos^2 20°$ $= 0.822$	$0.59 \times 1.33 \cos^2 20°$ $= 0.693$	" 9 " 3.16
Z	$2.6 \times 0.75 \sec^2 20°$ $= 2.208$	$2.6 \times 0.75 \sec^2 20°$ $= 2.208$	" 7 " 3.15

Pinion (wear). Transposing of formula on page 3.5

$$F = \frac{kW \times 10^7}{X_c \ S_c \ Z \ N \ T \ m^{1.8}}$$

$$= \frac{46 \times 10^7}{0.32 \times 9.7 \times 2.208 \times 750 \times 20 \times 8^{1.8}} = 106 \text{ mm}$$

Pinion (strength). Transposing the formula on page 3.5

$$F = \frac{kW \times 1.9 \times 10^7}{X_b \ Y \ S_b \ N \ T \ m^2}$$

$$= \frac{46 \times 1.9 \times 10^7}{0.31 \times 0.822 \times 130 \times 750 \times 20 \times 8^2} = 27.48 \text{ mm}$$

Gear (wear)

$$F = \frac{kW \times 10^7}{X_c \ S_c \ Z \ N \ T \ m^{1.8}}$$

$$= \frac{46 \times 10^7}{0.40 \times 9.7 \times 2.208 \times 245.9 \times 61 \times 8^{1.8}} = 84.78 \text{ mm}$$

Gear (strength)

$$F = \frac{kW \times 1.9 \times 10^7}{X_b \ Y \ S_b \ N \ T \ m^2}$$

$$= \frac{46 \times 1.9 \times 10^7}{0.38 \times 0.693 \times 130 \times 245.9 \times 61 \times 8^2} = 26.59 \text{ mm}$$

Pinion for wear dictates a face width of 106 mm.

Worked Example No. 2. (Cont.)

$$F_{minimum} = \frac{\pi\ m}{\tan\sigma\ \cos\sigma}\ ,\quad \text{page 3.8}$$

$$= \frac{\pi\ 8}{\tan\ 20°\ \cos\ 20°} = 73.48\ \text{mm} < 106\ \text{mm}$$

An improved proportion of gear may be designed by recalculation using 9 module.

Worked Example No. 3.

Determine the three components of tooth loading for the gears designed in the previous example.

Answer.

$$\text{Pinion.} \quad d = \frac{t \times m}{\cos\ \sigma} \quad \text{(page 3.6)}$$

$$= \frac{20 \times 8}{\cos\ 20°} = 170.268\ \text{mm}$$

$$\text{Power} = \frac{\text{torque}.\ \omega}{1000}$$

$$\therefore\ \text{torque} = \frac{1000\ \text{Power}}{\omega} = \frac{1000 \times 46 \times 60}{2\pi \times 750} = 585.69\ \text{N.m}$$

Refer to Fig. 4, page 3.7.

$$W_t = \frac{\text{torque}}{\frac{d}{2}} = \frac{585.69 \times 10^3 \times 2}{170.268} = \underline{6879.63\ \text{N}}$$

$$W_s = \frac{W_t\ \tan\ \Psi}{\cos\ \sigma} = \frac{6879.63\ \tan\ 20°}{\cos\ 20°} = \underline{2664.68\ \text{N}}$$

$$W_a = W_t\ \tan\ \sigma = 6879.63\ \tan\ 20° = \underline{2504\ \text{N}}$$

* * * * * * * * * * *

For Exercises see page 3.17.

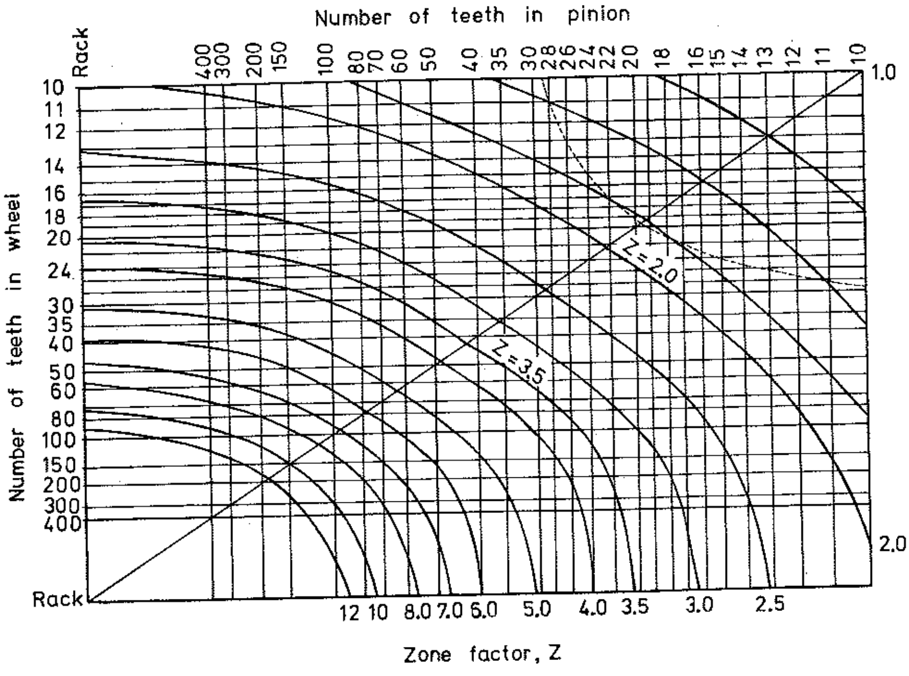


CHART 7
BS 436

HELICAL GEARS, 30° HELIX ANGLE
20° NORMAL PRESSURE ANGLE

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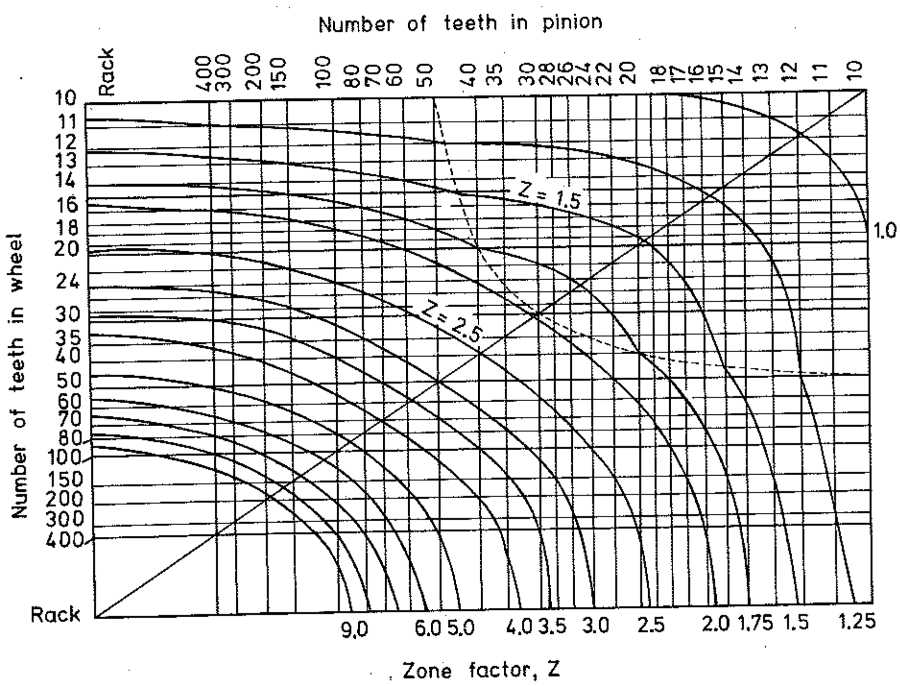


CHART 8
BS 436

SPUR GEARS, 20° PRESSURE ANGLE

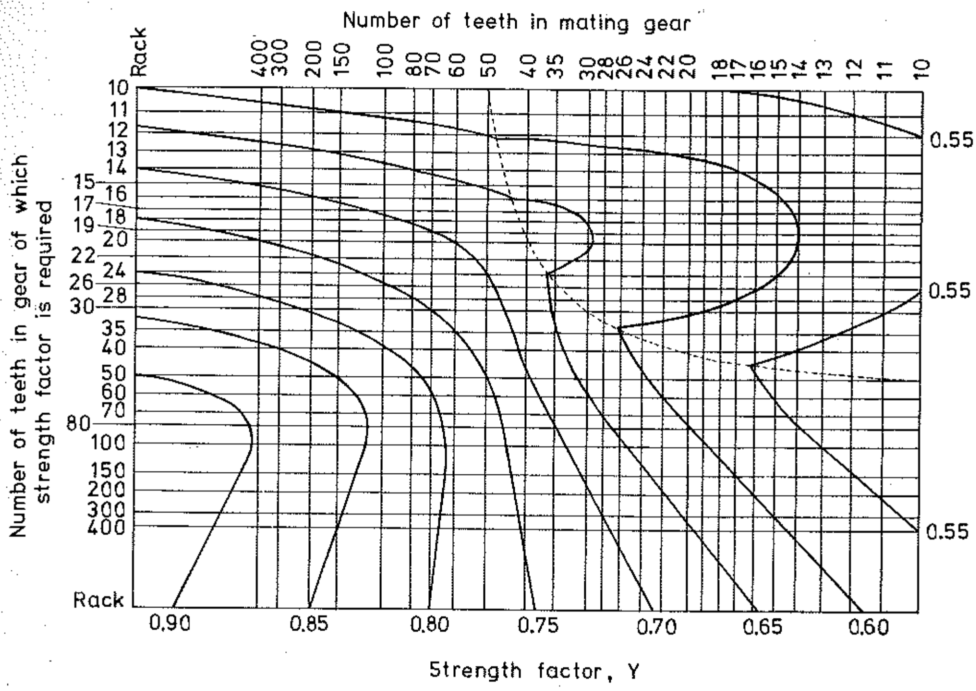


CHART 9
BS 436

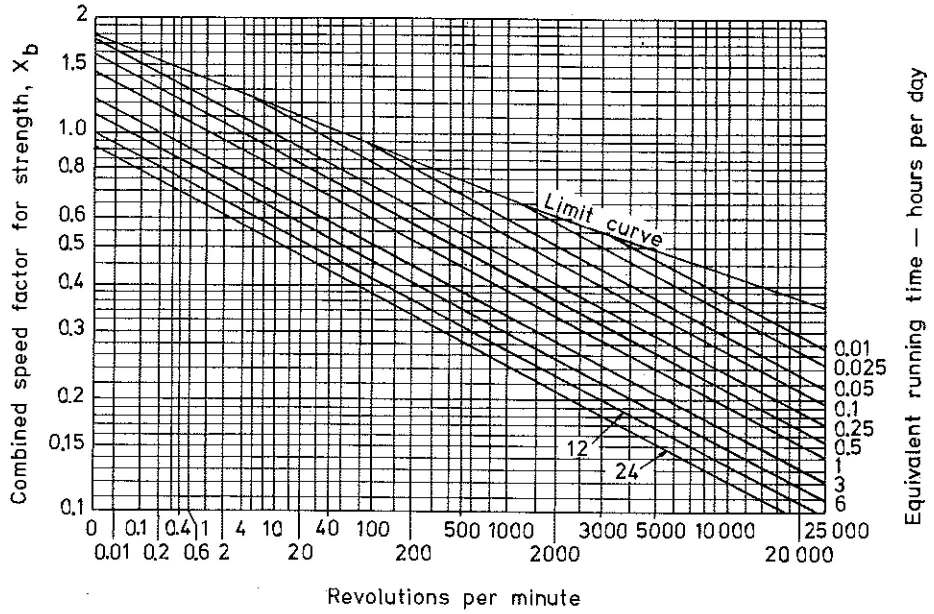


CHART 10
BS 436

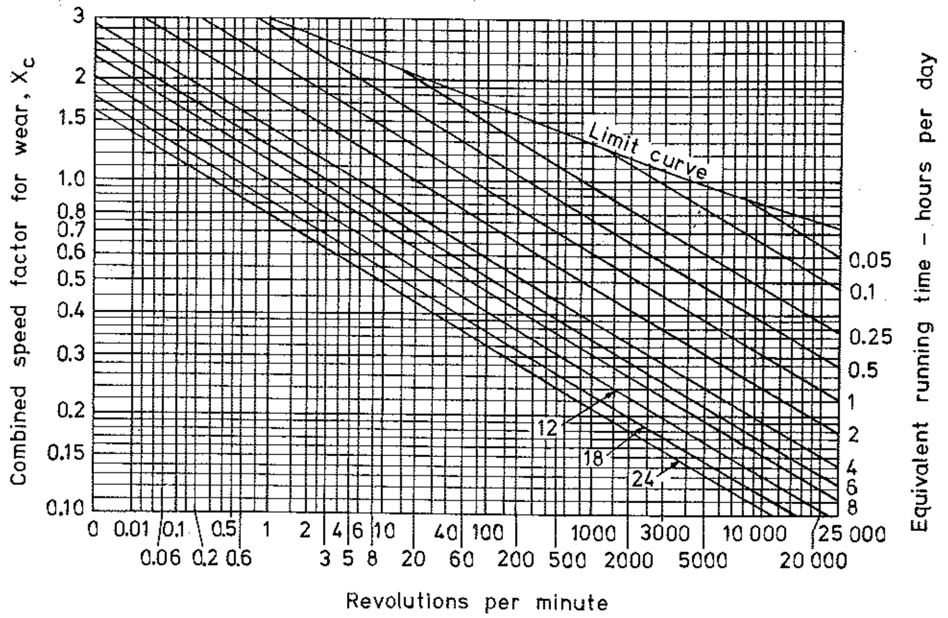


CHART 11
BS 436

Exercises

1. Fig. 8, below, shows a two stage spur gear box incorporating a flange mounted motor. The unit is mounted so that the two output shafts from the gear box point vertically downwards. The first output shaft is only for auxiliary operations; no power is taken. The second output shaft is the operational one.

   9 hours per day operation is required.

   Load conditions are moderate shock.

   Drive is from a 7.5 kW 1435 r.p.m. (full load speed) squirrel cage electric motor.

   Material of pinions .... CS 1040
   Material of gears .... cast steel

   (a) Determine the required face width of the gears.
   Neglect inefficiencies. Use an S.F. (Service Factor) of 1.25.

   (b) Calculate the maximum load that each bearing will have to take.

   The radial load on the lower end of the operational shaft is balanced.
   The total vertical load on this shaft is 130 kg.

   Ans. (a) 1st stage 40.28 mm   2nd stage 61.2 mm

   (b) First shaft

   Top bearing 599.17 N   Lower bearing 1601.49 N

   Operational shaft

   Top bearing 4893.46 N radial, 1275.3 N axial

   Lower bearing 2490.42 N radial

   These are the basic loads based on 7.5 kW and 100% η.

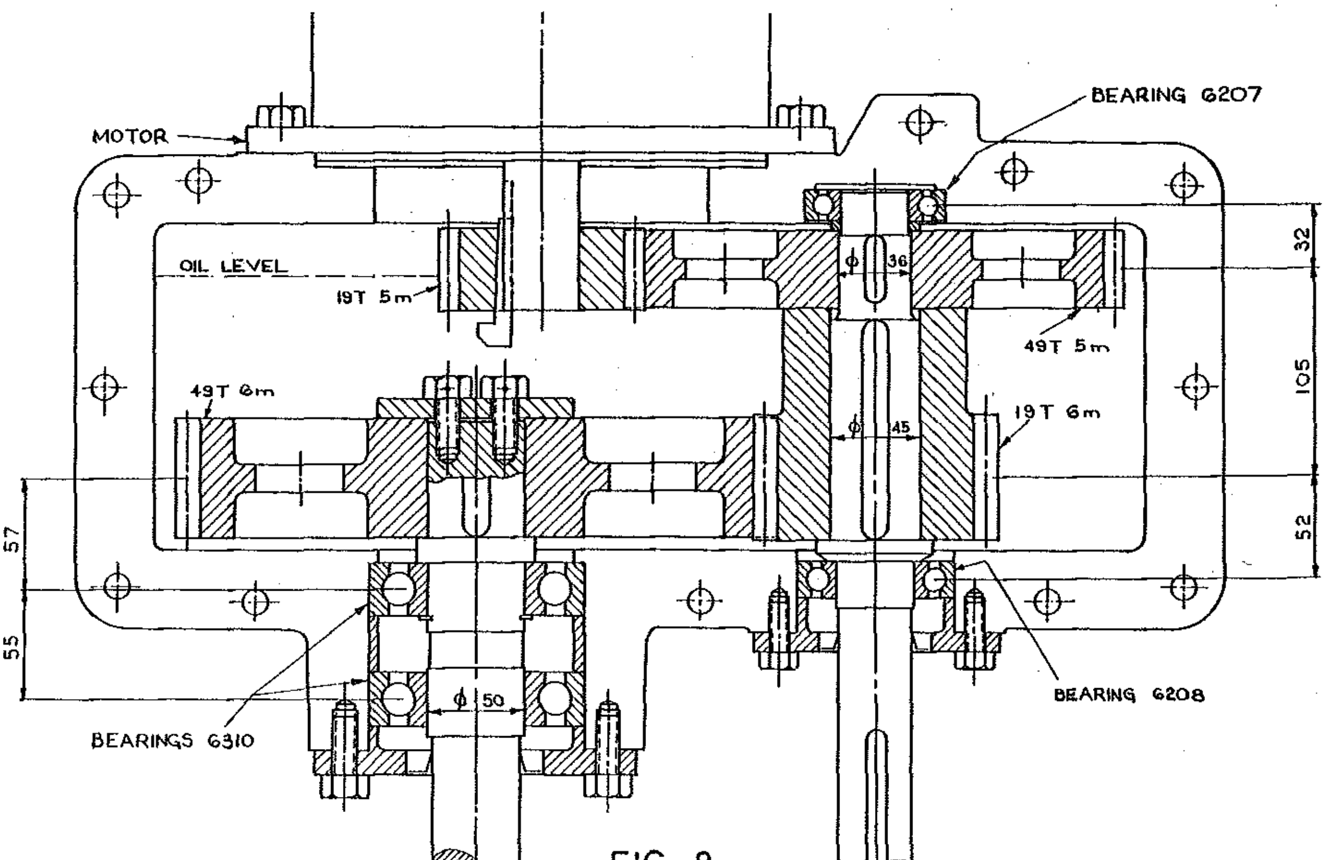


FIG. 8

2. Redesign the gears specified in Worked Example No. 2 using 50 kW and a 9 module tooth.

State: required face width,
minimum permissible face width, W_t, W_s and W_a

Ans. 93.18 mm, 82.67 mm, 6647.04 N, 2574.6 N, 2419.33 N

3. Fig. 9, below, shows a special purpose mechanism.

(a) What maximum torques could occur concurrently at gears A and B? Assume 100% efficiency.

(b) The mechanism is to operate 6 hours/day.
The 36T gear is to be made from 0.4% C normalised steel.
Determine the required face width of the gears on the basis of the 36T gear for wear and strength.
The rack is constrained to move horizontally.
The design is to be done in accordance with BS 436:

(i) design on maximum conditions, Clause 68.

(ii) X_b to be half the normal value for six hours/day operation as there is reverse bending on the teeth, Clause 70.

(iii) X_c is to be for 12 hours/day operation as the power goes in and out on the one gear.

Ans. (a) Gear A 272.84 N.m Gear B 204.63 N.m
(b) Wear 47.27 mm Strength 47.4 mm

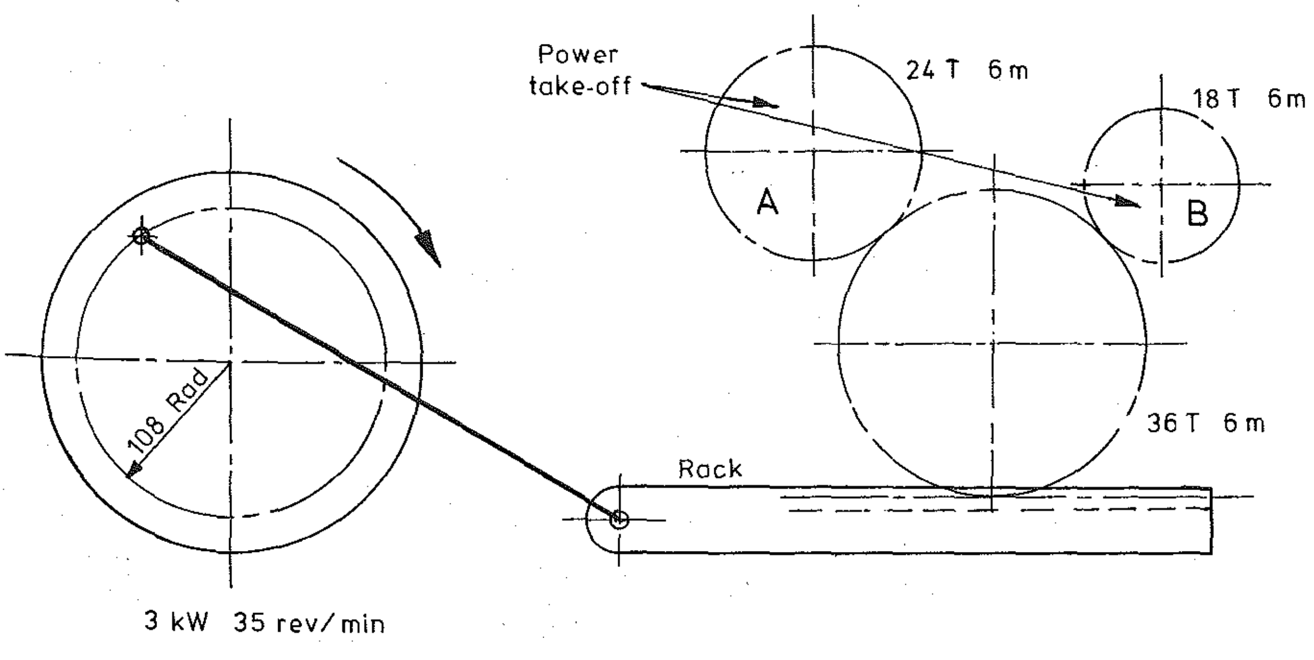


FIG. 9

4. Refer to Fig. 14, Chapter 2, page 2.23

Suppose the pinions are manufactured from 0.55 C normalised steel and the power input is increased to 40 kW for 8 hours/day operation. The meshing gears each have 70 teeth and are manufactured from normal grade cast steel,

determine:

(a) a suitable tooth module

(b) the required face width

(c) the recommended limits within which the face width should ideally lie.

Ans. (a) 10 mm module	(b)	Pinion (wear)	99.55 mm	(c) 80-120 mm
		" (strength)	41.14 mm	
		Gear (wear)	120.38 mm	
		" (strength)	46.63 mm	

5. A pair of meshing spur gears has $t = 20$, $T = 80$, $m = 2.5$.
If corrected teeth are used,

determine: A, a, B, b, J, j, D, d, I, i and C.

Ans. (mm) 1.75, 3.25, 3.875, 2.375, 203.5, 56.5, 200, 50, 192.25, 45.25, 125.

6. A pair of meshing spur gears has $t = 10$, $T = 50$, $m = 2$.
If corrected teeth are used,

determine: A, a, B, b, J, j, D, d, I, i and C.

Ans. (mm) 1.2, 2.8, 3.3, 1.7, 102.4, 25.6, 100, 20, 93.4, 16.6, 60.

7. A pair of meshing spur gears has $t = 20$, $T = 25$, $m = 2$.
If corrected teeth are used,

determine: A, a, B, b, J, j, D, d, I, i and C.

Ans. (mm) 2.2, 2.4, 2.3, 2.1, 54.4, 44.8, 50, 40, 45.4, 35.8, 45.58.

8. Refer to Fig. 10.

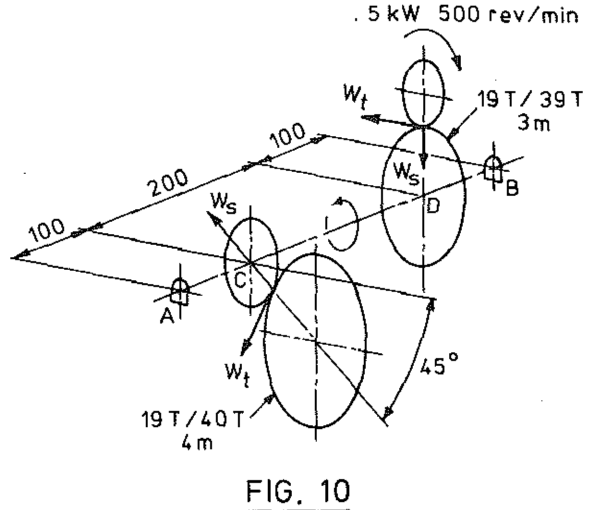


FIG. 10

The input of 5 kW results in forces being applied to the intermediate shaft AB as shown. A and B represent support bearings Assume 100% efficiency.

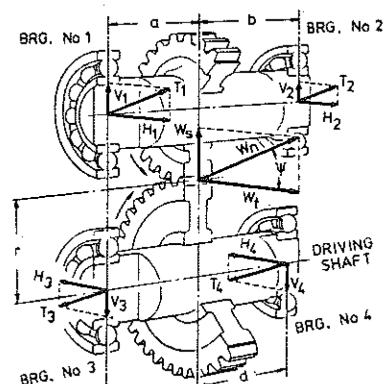
Draw B.M. diagrams for the above shaft for both the horizontal and vertical planes and hence determine the B.M. absolute at C and D.

Ans. Bending moments in N.m

C (vert)	204.48	C (horiz)	456.8
D (vert)	149.46	D (horiz)	375.6
C (abs)	500.56	D (abs)	404.3

CHAPTER 4

SPRINGS



CHAPTER 4
****

S P R I N G S

Springs are mechanical elements intended to take a large amount of strain. They have the following variety of applications in machinery.

(1) To control forces due to impact or shock loading and control vibration.

(2) To control motions e.g. valve springs, brake springs.

(3) To store energy e.g. clock springs.

(4) To measure forces e.g. scales.

The load on any spring should be kept within the elastic limit; hence the change in length of a spring (those covered in this book) will be proportional to the applied load (Hooke's Law).

The following theory applies to compression and tension helical springs.

GLOSSARY OF TERMS

P = axial load, N

d = diameter of wire, mm

δ = deflection of spring, mm

C = spring index $= \frac{D}{d}$

F_s, f_s = permissible and actual shear stress, MPa

K = Wahl's factor

h = chock length, mm

E = energy stored in a spring, N.mm

D = mean diameter of coils, mm

p = pitch of coils, mm

n = number of active coils

G = modulus of rigidity, MPa

$F_{t(ult)}$ = ultimate tensile strength of wire, MPa

H = free length, mm

S = spring stiffness or spring rate $= \frac{P}{\delta}$, N/mm

Free Length

The overall length of the unloaded spring.

Chock Length

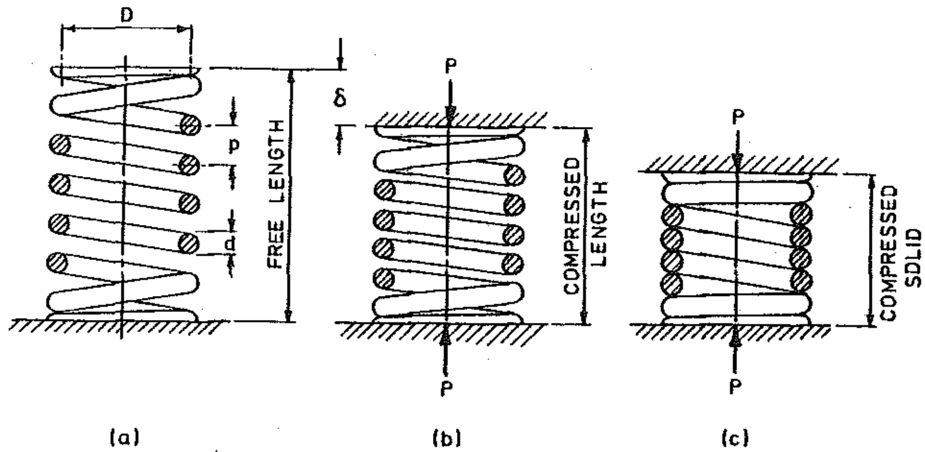
(Compressed or solid length.) The overall length of the spring when it is compressed solid.

Active Coil

A coil that contributes fully to the forces acting in a loaded spring.

Inactive Coil

Any end coil that is just for the purpose of giving support to the load. It contributes nothing to the forces acting in a loaded spring.



HELICAL COMPRESSION SPRING

FIG. 1

Spring Theory

Refer to Fig. 1.

(a) This shows a helical coil spring having end coils suitable for supporting a compression load.

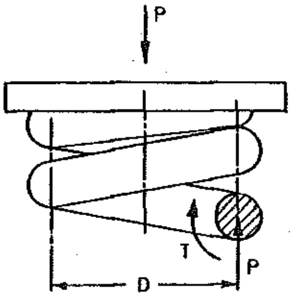
(b) A load P has compressed the spring, and amount δ. The end coils of the spring are inactive.

Fig. 2 shows the top active coil of the spring. This free body diagram is in equilibrium under the action of the force P and the resisting torque T.

$$T = \frac{PD}{2}$$

The shearing stress due to the torque T is

$$f_s = \frac{8PD}{\pi d^3} \quad \text{-----------------} \quad (1)$$



LOADING ON HELICAL SPRING

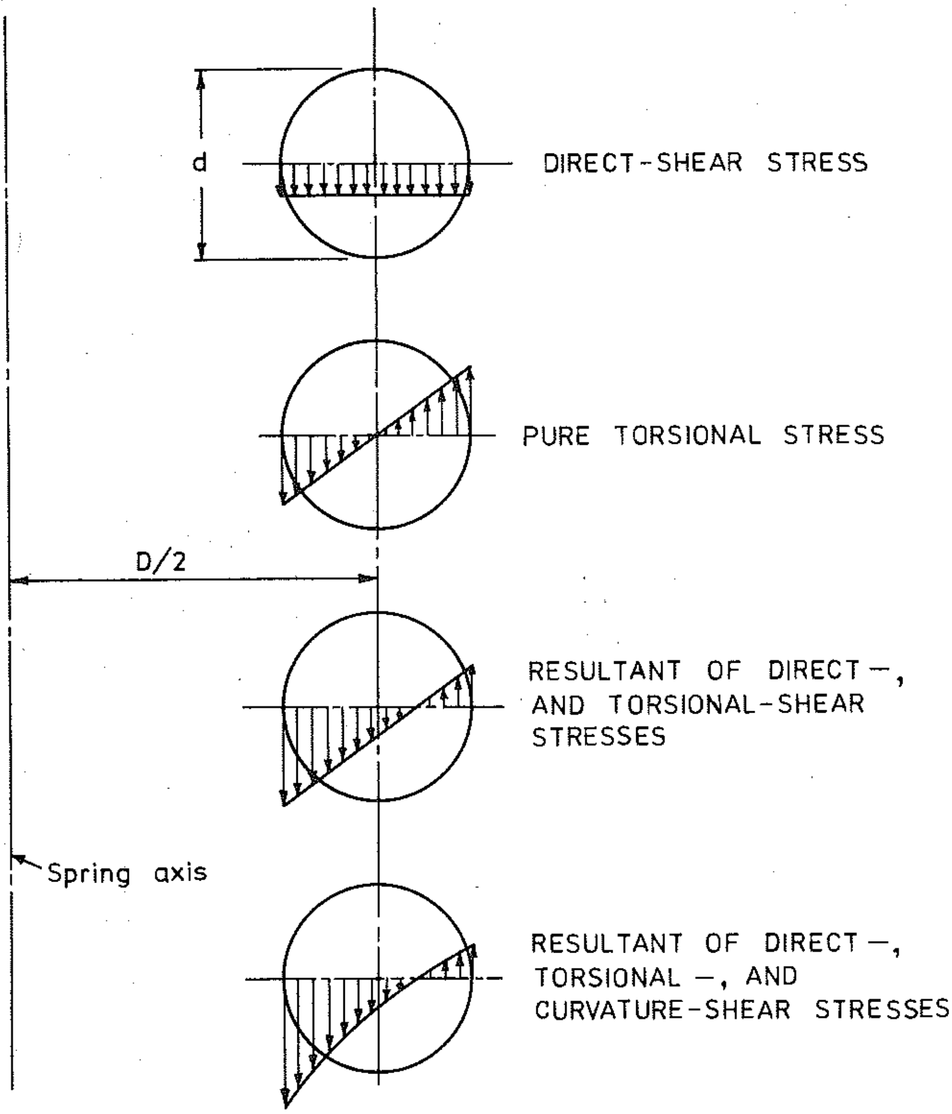
FIG. 2

Additionally there is the direct shear stress due to load P. Taking this stress as "average" across the section and adding it to the torsional shear stress the maximum shear stress is obtained.

Fig. 3 indicates the predominance of shear stress on the inside of the coil.

$$f_s = \frac{8PD}{\pi d^3} + \frac{4\,P}{\pi d^2} = \frac{8\,PD}{\pi d^3}\left(1 + \frac{1}{2c}\right)$$

(The torsional shear stress is always much larger than the direct shear stress.)



PREDOMINANCE OF SHEAR STRESS ON INSIDE OF COIL

FIG. 3

Wahl's Factor

A.M. Wahl developed a factor which may be used with equation (1) that allows for the effect of both direct shear and wire curvature.

$$f_s = K \frac{8\ PD}{\pi\ d^3} = K \frac{8\ PC}{\pi d^2}$$

Where $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C}$

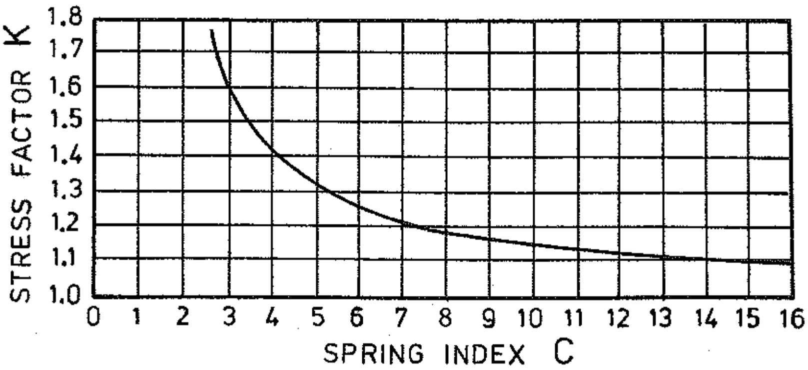


FIG. 4

Fig. 4 is a graph which gives the value of K for various values of C.

The ideal value of C is between 8 and 12 because:-

(1) Wahl's factor is small for this range.

(2) Good spring proportions result.

Deflection

By equating the work done in deflecting a spring to the strain energy in the twisted wire a useful design equation can be developed.

The equation would need to relate deflection to axial load, spring dimensions and material properties. The bending of the wire in close coiled springs is negligible therefore the strain energy of bending can be neglected.

Refer to Fig. 1.

$$P \propto \delta \quad \text{(Hooke's Law)}$$

$\therefore$ Work done = average force x deflection

$$= \frac{P\delta}{2}$$

Assume bar is twisted through an angle of θ radians by a torque T

$$\therefore \quad \text{Strain energy} = \frac{T\theta}{2}$$

$$\text{Total angle of twist} = \frac{T\ell}{GJ}$$

ℓ = length of twisted wire, mm

J = polar second moment of area (polar moment of inertia), mm^4

Let ϕ = helix angle of spring coil (this is generally about 5° or less).

The active length of wire $= \dfrac{\pi D n}{\cos \phi}$

$\simeq \pi D n$ when ϕ is small

Equating work done to strain energy

$$\frac{P\delta}{2} = \frac{T\theta}{2} \qquad \text{-------------- (2)}$$

By simple substitution of the previously listed values into equation (2) it can be easily shown that

$$\delta = \frac{8\ P\ D^3\ n}{G\ d^4} = \frac{8\ P\ C^3\ n}{G\ d}$$

Values for G:
Table No. 3, page 4.8.
Table B3, page 4.14.

Also from the above equation

$$\frac{P}{\delta} = \frac{G\ d}{8\ C^3\ n}$$

For a particular spring the right hand side of the above equation is constant; hence

$\dfrac{P}{\delta}$ (S) is constant.

$\dfrac{P}{\delta}$ is termed the spring rate and by definition it is the force required to cause unit deflection in a spring.

Slenderness Ratio

Compression springs may fail by sidewise buckling if the free length is more than four times the mean diameter. To prevent buckling, springs may be guided on a central rod or assembled in a tube. It should be noted however that springs increase in diameter when compressed because of changing slope of the coils and unwinding effect. Care must therefore be taken in a design assembly to prevent binding of the coils and subsequent wear.

Fig. 5 shows a curve which may be used to determine the likelihood of buckling of a compression spring. The curve is for springs having squared, ground ends mounted so that the ends move in the axial direction only. Buckling will occur at points above the curve.

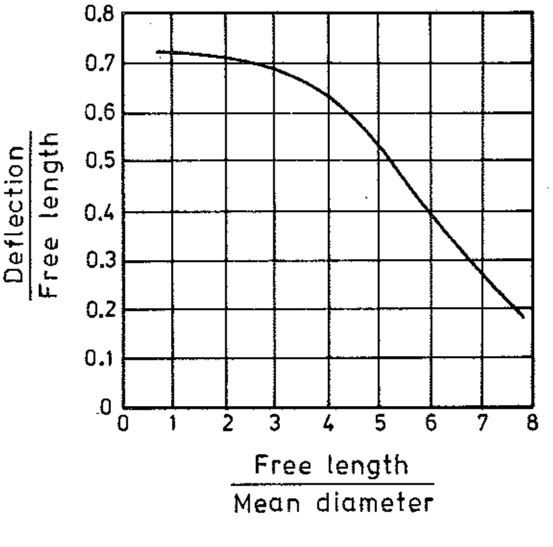


FIG. 5

Service

Before choosing a working stress for a spring the service conditions have to be categorized.

The three service conditions are defined as follows:-

1. Severe service

   Rapid continuous loading where the ratio of minimum to maximum stress is 0.5 or less, e.g. valve springs.

2. Average service

   As for severe service but with only intermittent operation, e.g. engine-governor springs.

3. Light service

   Loads that are static or very infrequently varied, e.g. safety valve springs.

Overstressing

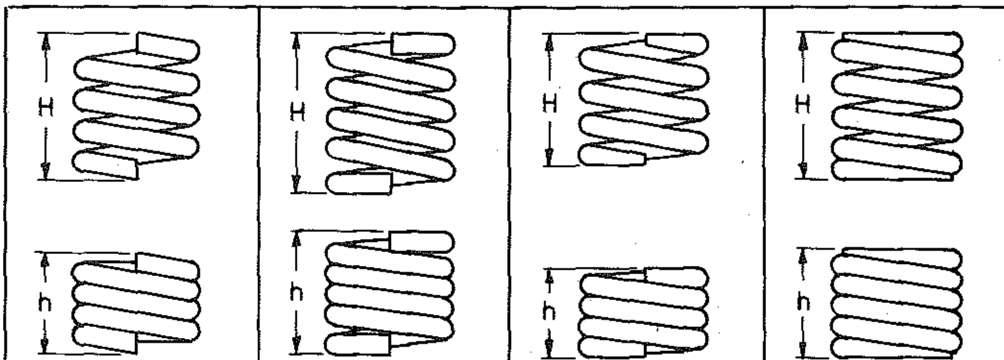
In spring design, the possibility of overstressing should always be considered.

This can happen with a compression spring if there is a likelihood of the spring being compressed beyond its design deflection to the chock length. If so, redesign should be considered.

End Coil Design

Fig. 6 shows some of the different end coil designs that are used for compression springs.

Overall lengths of helical springs having different forms of end coils



A	B	C	D
Plain ends not squared	Ends squared by bending	Ends squared by grinding	Closed end coils squared by grinding
H = np + d h = nd + d	H = np + 3d h = nd + 3d	H = np h = nd	H = np + 2d h = nd + 2d
H = h + δ (all cases)			

FIG. 6

Energy in Springs

When a helical spring is stretched or compressed the wire is twisted within its elastic limit during normal application. Upon release of the load, the spring returns to its natural shape, thus releasing its stored energy.

$$\text{Energy} = \text{average force} \times \text{deflection}$$

$$= \frac{P\delta}{2}$$

$$\text{Also} \quad P = S\delta$$

$$\therefore E = \frac{S\delta^2}{2}$$

Tension Springs

In designing tension springs the weakening effect of end connections should be considered.

Where an attachment hook is used, special care should be taken. A sharp radius should be avoided at the point where the hook meets the end coil. Failure is a very real possibility at this point because of stress concentration.

To allow for the above consideration a reduction in design stress of 20-25% is commonly made.

'n' should be taken as the actual number of coils, including partial coils (as all coils are 'active').

Table No. 1 — Metric spring wire sizes

Always check with manufacturer for availability at time of design.

0.56,	0.63,	0.71,	0.80,	0.90,	1.00,	1.12,	1.25,	1.40,
1.60,	1.80,	2.00,	2.24,	2.50,	2.80,	3.15,	3.55,	4.0,
4.5,	5.0,	5.6,	6.3,	7.1,	8.0,	9.0,	10.0,	11.2,
15,	16,	17,	18,	19,	20,	21.		

Larger sizes also available.

Spring Design Stresses (Refer Table No. 2)

Material: Carbon Steel Spring Wire, Table 1, Range 3 stresses, AS 1472.

Table No. 2 (Ref., Westinghouse Electric Corporation.)

WIRE DIAMETER mm	SEVERE SERVICE MPa	AVERAGE SERVICE MPa	LIGHT SERVICE MPa
Up to 2.16	414	520	640
2.17 to 4.70	380	475	585
4.71 to 8.13	330	414	510
8.14 to 13.46	290	360	450
13.47 to 24.6	250	310	385
24.7 to 38	220	275	345

Materials of construction and permissible stresses

For general purpose engineering applications:

Carbon steel spring wire, Table 1, Range 3 stresses, AS 1472

This is a hard-drawn wire containing

0.45% - 0.85% C, 0.10% - 0.35% Si, 0.40% - 1.00% Mn.

Permissible shear stresses for the above wire are given in Table No. 2, page 4.7.

Springs can be made from many other materials, e.g. brass, bronze, piano wire etc. In the absence of comprehensive details on engineering properties of chosen materials, F_s can be conservatively determined as follows:

Light service $F_s = 0.20F_{t(ult)}$; F of S = 1.25 to 2.15 (essentially static stress condition)

Average service $F_s = 0.16F_{t(ult)}$; F of S = 1.56 to 2.68 (essentially static stress condition)

Severe service $F_s = 0.125F_{t(ult)}$; F of S = 2.00 to 3.44 (fatigue loading, i.e. fluctuating stress)

Factors of safety are determined as follows:

(1) $F_y = 0.5F_{t(ult)}$ and using the maximum-shear-stress theory $F_{sy} = 0.5F_y$ then $F_{sy} = 0.5 \times 0.5F_{t(ult)} = 0.25F_{t(ult)}$

(2) $F_y = 0.75F_{t(ult)}$ and using the distortion-energy theory $F_{sy} = 0.577F_y$ then $F_{sy} = 0.577 \times 0.75F_{t(ult)} = 0.43F_{t(ult)}$

This means that, for example, for 'light service' the F of S will be in the range of 0.25/0.20 = 1.25 to 0.43/0.20 = 2.15.

Note: For 'severe service' the F of S may be governed by the stress fluctuation range and number of load applications. For extreme cases of fatigue loading a more sophisticated design analysis can be done; refer to Chapter 10 and Appendix No. 3.

The engineering properties of one common non-ferrous material, phosphor bronze AS 518 are given in Table No. 3.

TABLE No. 3

Round bar cold drawn spring hard temper. AS 518 grade phosphor bronze.
Sn 4% - 6%, P 0.10% - 0.35%, Al 0.01% max., Pb 0.02% max., (AS 1573)

Diam. mm	Tensile strength MPa (ult)	E MPa	G MPa
up to 0.6	990	116 000	43 500
0.7 to 1.6	930	116 000	43 500
1.7 to 3.0	900	116 000	43 500
3.1 to 6.0	870	116 000	43 500

Heat treatment

In general, coil springs (ferrous and non-ferrous) are wound in the cold drawn spring hard condition and then (generally) stress relieved. Springs from some grades of large diameter steel bar are heat treated to the required temper grade after manufacture.

Spring hard temper is the highest grade of hardness and tensile strength possible commensurate with reasonable 'non-brittleness'.

Non-ferrous materials are cold rolled or cold drawn to the following main temper grades:

soft, ¼ hard, ½ hard, hard, spring hard.

Square wire.

For springs made of square steel wire multiply values of:

P by 1.2

δ by 0.59

Students should note before studying the following examples that spring design is an iterative process. There is no way of applying previous formulas and obtaining some unique answer.

Space restrictions will often dictate either allowed length or diameter and sometimes both.

"C" should ideally be between 8 and 12 as previously explained.

Worked example No. 1

When a coil spring with a spring rate of 17.5 N/mm is compressed 32 mm the coils are closed.

Determine:

(a) the required wire diameter.
(b) the required coil diameter.
(c) the number of active coils necessary.

Summarise the answers in a form suitable for a purchasing officer to order the spring.

Note:

Include Wahl's factor.
Design for light service.
Ensure that the proportions of the spring are such that it will not buckle sideways.

Answer.

(a) Choose $C = 8$

$F_s = 510$ MPa (guessed value only at this stage; from Table No.2).

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

$$= \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184 \text{ (or use Fig. 4).}$$

$$f_s = \frac{8PC}{\pi d^2} \times K$$

$$\therefore \; d^2 = \frac{8PCK}{\pi f_s} = \frac{8 \times 17.5 \times 32 \times 8 \times 1.184}{\pi \times 510} = 26.485$$

$$\therefore \; d = 5.15 \text{ mm}$$

(a) (Cont.)
From Table No. 1

use d $\underline{5.6 \text{ mm}}$

(b) $D = Cd = 8 \times 5.6 = \underline{\underline{44.8 \text{ mm}}}$

Check actual stress with these proportions.

$$f_s = \frac{8\ P\ C}{\pi\ d^2} \times K$$

$$= \frac{8 \times 17.5 \times 32 \times 8 \times 1.184}{\pi \times 5.6^2}$$

$$= 430.7 \text{ MPa} < 510 \text{ MPa (see Table No. 2).}$$

(c) $$\delta = \frac{8\ P\ C^3\ n}{G\ d}$$

$$\therefore\ n = \frac{\delta\ G\ d}{8\ P\ C^3}$$

$$= \frac{32 \times 82\ 700 \times 5.6}{8 \times 17.5 \times 32 \times 8^3}$$

$$= 6.46 \quad \text{say } \underline{\underline{7}}$$

Check length of spring, refer to Fig. 6, case D,
$h = nd + 2d = 7 \times 5.6 + 2 \times 5.6 = 50.4$ mm

Free length of spring $= 50.4 + 32$
$= 82.4$ mm say 83 mm

<u>Sideways buckling</u>:

$$\frac{H}{D} = \frac{83}{44.8} = 1.85; \qquad \frac{\delta}{H} = \frac{32}{83} = 0.39$$

Plotting these values on the graph, Fig.5, indicates that the spring will not buckle.

<u>Summary</u>:

Helical compression spring, carbon steel spring wire, Table 1,
Range 3 stresses, AS 1472.
44.8 mm mean diameter
5.6 mm diameter wire
free length 83 mm
7 active coils + 1 extra coil each end, closed and squared by grinding.

Worked example No. 2

Fig. 7 shows part of a clutch mechanism. There are four such assemblies equally spaced on the complete clutch. The clutch is to be held together with an operating force of 1000 N when new.

After 1 mm of wear has occurred on each lining the operating force must not be less than 750 N.

(a) Design the spring. Space restricts the mean diameter to 25 mm maximum.

(b) Determine the maximum total force F required to hold the clutch in the disengaged position.

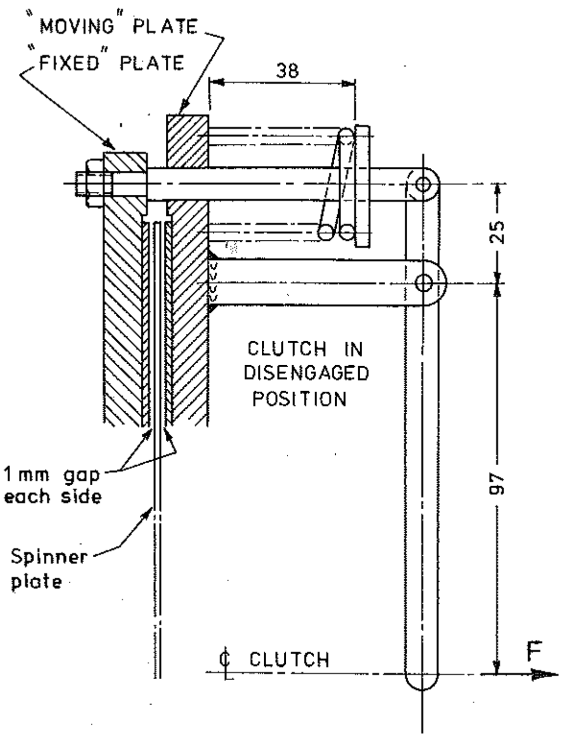


FIG. 7

ANSWER.

(a) First determine a suitable spring rate. Maximum permissible drop in operating force

$$= 1000 - 750 = 250 \text{ N}$$

$$= \frac{250}{4} = 62.5 \text{ N per spring.}$$

$$\therefore \quad \text{spring rate} = \frac{62.5}{2} = 31.25 \text{ N/mm}$$

For conservatism allow 25 N/mm

$\therefore$ for each spring $P = \dfrac{1000}{4} = 250$ N at 40 mm length which equals 300 N at 38 mm length.

$$\therefore \quad H = 40 + \frac{250}{25} = 50 \text{ mm}$$

$$\delta = \frac{8\,P\,D^3\,n}{Gd^4}$$

$$\therefore \quad n = \frac{\delta\,G\,d^4}{8\,P\,D^3}$$

Try $C = 6$

$$\frac{D}{d} = 6 \quad \therefore \quad d = \frac{D}{6} = \frac{25}{6} = 4.16 \text{ mm}$$

From Table No. 1 use 4 mm wire

$$\therefore \quad n = \frac{(50 - 38)\ 82\ 700 \times 4^4}{8 \times 300 \times 25^3} = 6.8 \quad \text{say 7 coils}$$

Check to see that the spring will fit into the 38 mm space

$$h = nd + 2d$$

$$= 7 \times 4 + 2 \times 4 = 36 \text{ mm} < 38 \text{ mm}$$

Check stress

$$f_s = K \frac{8\ P\ D}{\pi\ d^3}$$

$$C = \frac{25}{4} = 6.25, \text{ from Fig. 4} \quad K = 1.24$$

$$\therefore \quad f_s = 1.24 \ \frac{8 \times 300 \times 25}{\pi \quad 4^3} = 370 \text{ MPa}$$

$$F_s = 380 \text{ MPa "Severe service" from Table No. 2.}$$

Summary: Helical compression spring, carbon steel spring wire, Table 1, Range 3 stresses, AS 1472.
25 mm mean diameter
4 mm diameter wire
Free length = 50 mm
7 active coils + 1 extra coil each end, closed and squared by grinding.

(b) Students should attempt this lever exercise themselves.

The answer is,

$$F = \underline{\underline{245.9 \text{ N}}}$$

Exercises

1. Refer to Fig. 8.
Two helical compression springs are used to take a shock loading delivered when a balance weight from a machine falls. The weight touches the springs at a velocity of 5 m/s.

Calculate:
(a) the maximum deflection the springs will have to accommodate.
(b) The stored energy each spring will have at the point of maximum deflection.
(c) the deflection the springs will have when the system reaches its final equilibrium position.

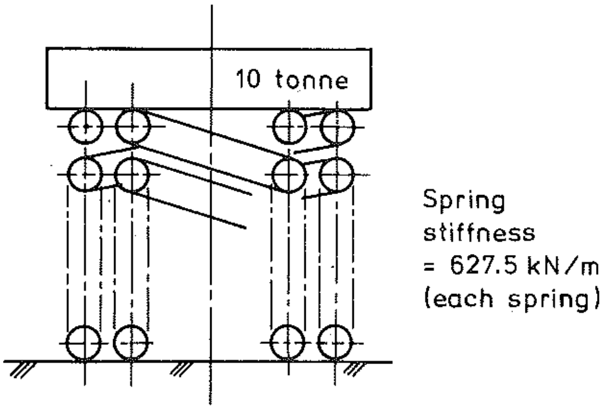


FIG. 8

Ans. (a) 0.53 m (b) 88 559.41 N.m. (c) 0.078 m

2. A spring for a can making machine is subjected to a load of 650 N maximum to 220 N minimum. The spring deflection during this range of loads is 20 mm. The spring is chock (nominally) at the 650 N load. The mean diameter of the spring is to be 72 mm. The spring is considered to be under severe service loading. Design the spring and specify all the necessary details for ordering purposes.

Ans. 9 active coils + 1 extra each end etc.
$H = 129.2$ mm
$d = 9$ mm

Carbon steel spring wire, Table 1, Range 3 stresses, AS 1472.

($f_s = 193$ MPa; $F_s = 290$ MPa)

3. Design a compression spring from carbon steel spring wire. The spring is to give 550 N force at 150 mm compression. However, during operation the compression can be 210 mm at which point the spring must not be overstressed. The pitch diameter is to be 75 mm, C = 8 approximately. Use Wahl's factor and light service.

State:-
- (a) The wire diameter.
- (b) The maximum shear stress that acts and the maximum permissible shear stress.
- (c) The number of working coils.

Ans. (a) 9 mm (b) 237.3 MPa, 450 MPa (c) 44

4. (a) For the return mechanism shown in Fig. 9 determine the force F which must be applied to the lever arm ABC to extend the spring 40 mm. (Assume severe service conditions.)

Spring specifications:

20 mm diameter wire (carbon steel)
8 working coils
130 mm inside diameter

(b) Determine the values of f_s and F_s.

Ans. (a) 612.59 N (b) 140.1 MPa, 187.5 - 200 MPa

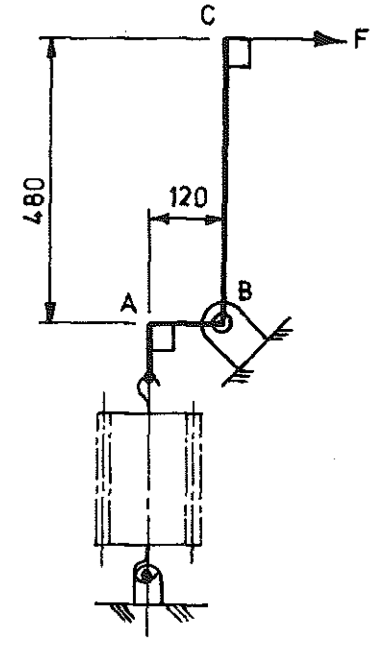


FIG. 9

5. A frictionless trolley of mass 500 kg rolls from rest 250 mm down a 45° incline. The trolley then strikes four buffer springs which provide a maximum deflection of 150 mm. (Assume each spring is chock at this point although in practice the spring should still have a few mm deflection in reserve. The designer should be aware that in the latter case it would be advisable to check the spring for f_s with the spring chock.)

(a) Design the springs using C = 8 and d = 20 mm. Use average service and end condition D, Fig. 6.

State:

f_s, F_s, n, H. (Use carbon steel spring wire.)

(b) Calculate the deflection the springs will have when the system reaches its final equilibrium position.

Ans. (a) 278.86 MPa; 310 MPa; 13; 450 mm.
(b) 28.125 mm

Design of springs to B.S. 1726 : Part 1 : 1964

"Helical Compression Springs"

The following rules from Appendix B of the above Code are applicable to the design of springs with circular wire or bar section.

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b. Formulae for springs with circular wire or bar section.

(i) Rate

$$S = \frac{P}{\delta} = \frac{d^4 G}{8 n D^3} = \frac{d G}{8 n C^3}$$

NOTE. Some authorities introduce rate correction factors depending on coil curvature. They are very small and can safely be ignored as their influence on the result is negligible compared with that of the uncertainty regarding the true number of working coils.

(ii) Shear stress.

$$f_s = \frac{8 P D K}{\pi d^3} \quad \text{where} \quad K = \frac{C + 0.2}{C - 1}$$

The following figures are given.

B3 Modulus of Rigidity

			MPa
All carbon, low alloy & martensitic steels	79 300	to	82 700
Austenitic stainless steel			69 000
Phosphor bronze	43 100	to	43 500
Hard drawn brass wire			36 200
Monel	62 000	to	65 500
Inconel	72 400	to	75 800
Nimonic '90'	79 300	to	86 200
Copper-beryllium			41 400
Titanium alloys	34 500	to	41 400

B4 Design Stresses

Recommended values of permissible shear stresses for the design of general purpose helical compression springs when compressed coil to coil are as follows:

B.S.	1429	En.42 B, C and D	695 MPa	
B.S.	1429	En.44 B and C	770 MPa	
B.S.	1429	En.45 and En.45A	850 MPa	
B.S.	1429	En.47 and En.50	850 MPa	
B.S.	2056	En.56	850 MPa	
B.S.	2056	En.58A	40%	of the tensile strength
B.S.	5216	En.49B, C and D	40%	of the tensile strength
B.S.	2873		35%	of the tensile strength
B.S.	2786		35%	of the tensile strength
B.S.	2803		40%	of the tensile strength

Details of the above B.S. numbers are as follows.

Cont. on page 4.15

BS 1429 : 1980 Annealed round steel wire for general engineering springs.

Annealed wire has tensile strength ranging from 730 - 800 MPa. Springs are cold formed and then hardened and tempered.

BS 2056 : 1983 Specification for stainless steel wire for mechanical springs.

Specifies requirements for two austenitic, one precipitation hardening and one martensitic type of stainless steel wire suitable for the manufacture of springs for general mechanical purposes.

BS 5216 : 1975 Patented cold drawn carbon steel wire for mechanical springs.

BS 2873 : 1969 Copper and copper alloys. Wire.

BS 2786 : 1963 Brass wire for springs. 2/1 brass.

Supplied in the extra hard condition.
Tensile strength approximately 770 MPa.

BS 2803 : 1980 Pre-hardened and tempered carbon and low alloy round steel wire for springs for general engineering purposes.

Gives details for plain carbon and low alloy round steel wire.

Tensile strengths of the various grades of wire are contained in the above codes.

The B.S. 1726 Code does offer further guidance on the design of helical compression springs in the following categories.:

Springs with square and rectangular bar section.

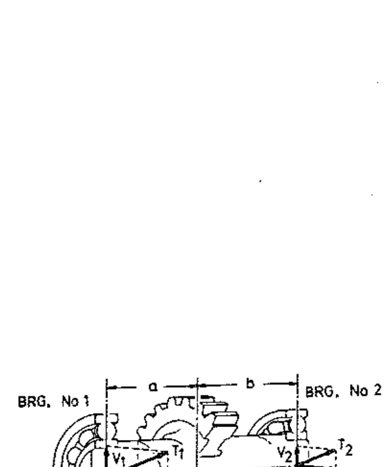
Transverse stability with different end support conditions.

Natural frequency of vibration.

Transverse loading.

CHAPTER 5

BELTS



B E L T S

The three main categories of belt design are:-

(1) Flat belt drives (2) Belt conveyors (3) Vee belt drives

The information that follows in this chapter should be applied together with the chosen belt manufacturer's engineering data.

Glossary of Terms (design formula terms only)

T_{T1} = total tension on tight side of belt, N

T_{T2} = total tension on slack side of belt, N

T_1 = tension on tight side of belt, N (neglecting centrifugal tension)

T_2 = tension on slack side of belt, N (neglecting centrifugal tension)

T_c = centrifugal tension, N

T_o = initial belt tension, N

T_E = effective belt tension, N

T = torque, N.m

P = power, kW

ω = speed of shaft in radians/sec.

N = speed of shaft in r.p.m.

m = mass of belt in kg/m.

W = weight of product on belt, N

v = velocity of belt in metres/sec.

D = diameter of pulley, m

r = radius of pulley in metres

θ = angle of wrap of belt in radians

C = centre distance of pulleys, m

μ = coefficient of friction, belt to pulley

μ_s = coefficient of friction, belt to support surface

General Theory on Design of Belt Drives

Belt tensions

Difference in tensions (Effective pull)

From an inspection of Fig. 1 it can be seen that the belt tension T_1 is pulling the pulley in an anti-clockwise direction while the belt tension T_2 is pulling the pulley in the clockwise direction, but as T_1 is larger than T_2 the pulley will rotate in an anti-clockwise direction.

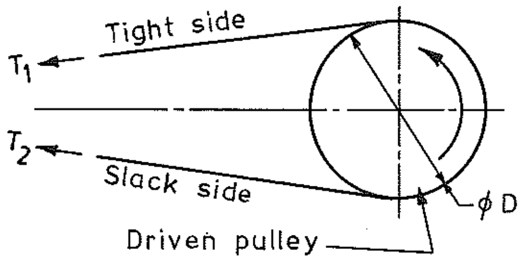


FIG. 1

The turning effect of the belt on the pulley is:

$$T_1 \times \frac{D}{2} - T_2 \times \frac{D}{2} = (T_1 - T_2)\,\frac{D}{2}$$

In designing a belt the aim is to make the turning effect and hence (T_1-T_2) as large as possible. The difference in belt tensions $(T_1 - T_2)$ is known as the <u>effective pull</u> "T_E" of the belt.

Sum of Tensions

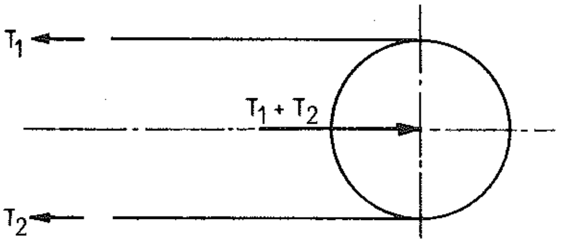


FIG. 2

The pulley is held in position by a shaft mounted in bearings. The load on the shaft will be the sum of the tensions $T_1 + T_2$ (Fig. 2) when the slack side and tight side of the belt are parallel (even when the sides are not parallel; this is roughly true). Hence it is desirable to keep the sum of the tensions $T_1 + T_2$ as small as possible.

Tension Ratio

Of course $T_1 + T_2$ is always larger than $T_1 - T_2$, but these two requirements ($T_1 - T_2$ large and $T_1 + T_2$ small) are fulfilled when the tension ratio T_1/T_2 is made as large as possible.

Consider two examples, both having the same effective pull: -

EXAMPLE 1.

$T_1/T_2 = 2$

Let $T_1 = 6 \quad \therefore \quad T_2 = 3$

$T_1 - T_2 = 3$

$T_1 + T_2 = 9$

EXAMPLE 2.

$T_1/T_2 = 4$

Let $T_1 = 4 \quad \therefore \quad T_2 = 1$

$T_1 - T_2 = 3$

$T_1 + T_2 = 5$

It can be seen that these two drives would transmit the same power at the same speed (since $T_1 - T_2$ is the same in both cases). But, in Ex. 1, the load on the shafts and bearings is nearly twice as great as in 2, also in 1 the maximum tension in the belt T_1 is 50% greater. Hence, when the tension ratio is lower (Ex. 1) the shafts, bearings, belts, and perhaps pulleys, would all need to be stronger and more expensive.

NOTE: $\frac{T_1}{T_2}$ should be kept large.

Optimum Tension Ratio

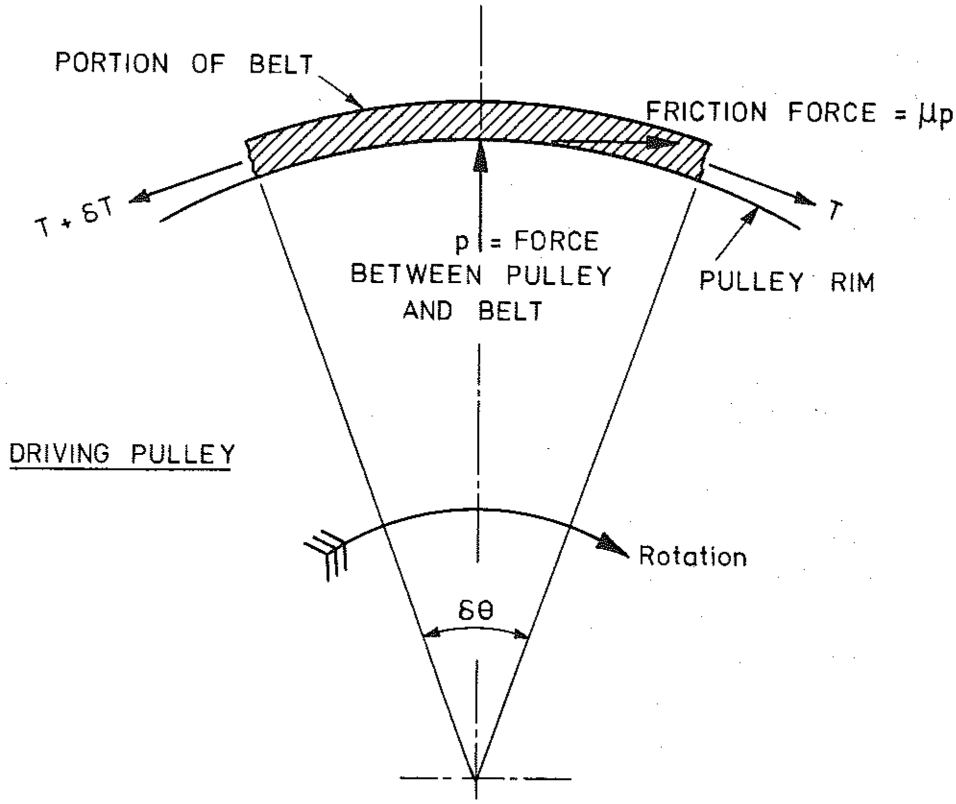


FIG. 3(a)

Consider portion of belt shown in Fig. 3a.

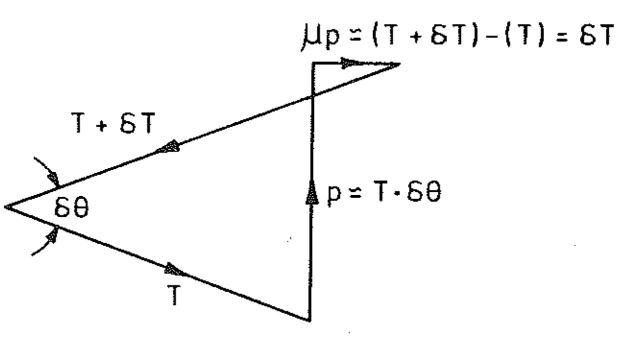


FIG. 3(b)

The force polygon for this portion of belt is shown in Fig. 3b.

When $\delta\theta$ is very small:-

$p \cong T.\delta\theta$ (where $\delta\theta$ is in radians)

and $\mu p \cong \delta T$

Where μ = coefficient of friction.

Hence $\mu(T.\delta\theta) = \delta T$

$$\therefore \quad \mu\delta\theta = \frac{\delta T}{T}$$

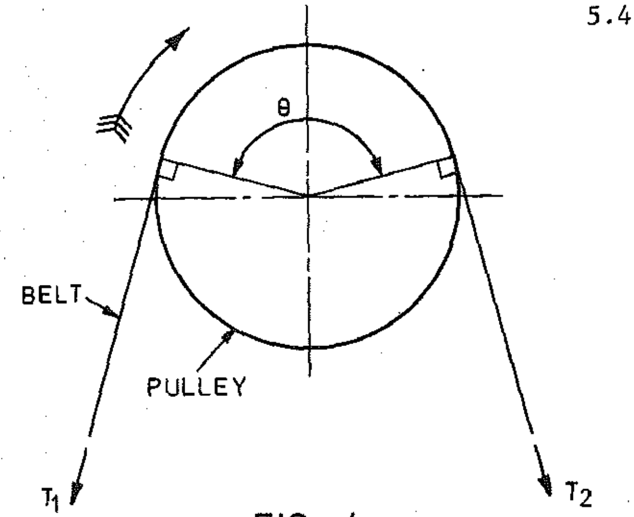


FIG. 4

Taking limits and integrating:-

$$\therefore\ \mu\int_0^\theta d\theta = \int_{T_2}^{T_1}\frac{dT}{T} \quad \text{(see Fig. 4 for limits)}$$

$$\therefore\ \mu\theta = \ell n\,\frac{T_1}{T_2}$$

$$\therefore\ \frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{------ (A)}$$

Hence the tension ratio when the belt is just on the point of slipping is $e^{\mu\theta}$

where e = 2.718 (base of natural logs)

μ = Coefficient of friction between belt and pulley

θ = Arc of contact in <u>radians</u> = $2\ \text{arcos}\left[\left(\frac{D_1}{2} - \frac{D_2}{2}\right)\frac{1}{C}\right]$

Thus for the tension ratio T_1/T_2 to be large, $e^{\mu\theta}$ must be large; this means that both μ and θ should be as large as possible.

μ is controlled by the materials of the belt and pulley, it can be slightly increased for leather belts by using a belt dressing.

θ is always smaller on the small pulley so the belt will slip there first.
The arc of contact (θ) can be improved by:-

(i) Small ratio
(ii) Long centre distance
(iii) Jockey pulley
(iv) Crossed belt.

(See Fig. 5.)

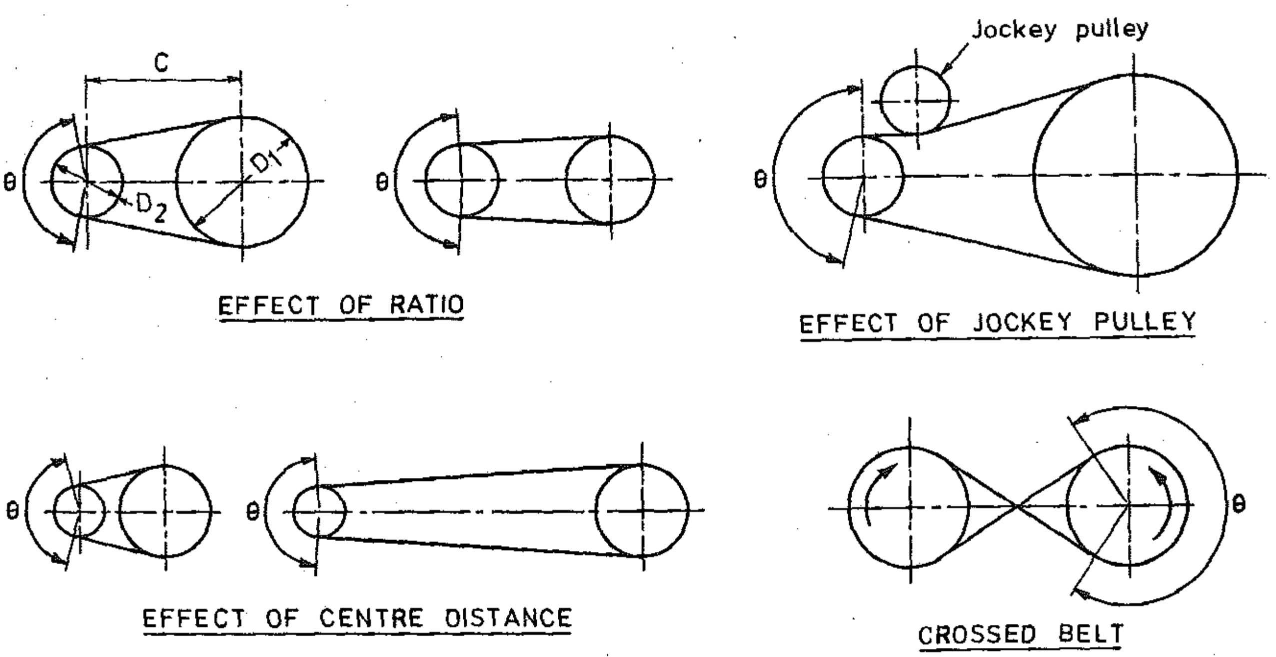


FIG. 5

This last method is not recommended, because of the increased wear on the belt. It is used, however, to give reversal of direction of rotation.

The ideal value for the tension ratio is $e^{\mu\theta}$; it cannot be higher because the belt would then slip, if it is lower the belt will drive satisfactorily but, as already indicated, this will result in additional stress in the belt and increased load on the shafts and bearings.

However, except with an automatic take-up, it is impossible to maintain the tension at an exact value.

Hence, in practice, the tension ratio is usually between $e^{\mu\theta}$ and $\frac{1}{2}e^{\mu\theta}$ when transmitting full load. At light loads the tension ratio will be very much lower of course, however the total tension of $T_1 + T_2$ remains roughly the same at all loads.

Centrifugal Effect.

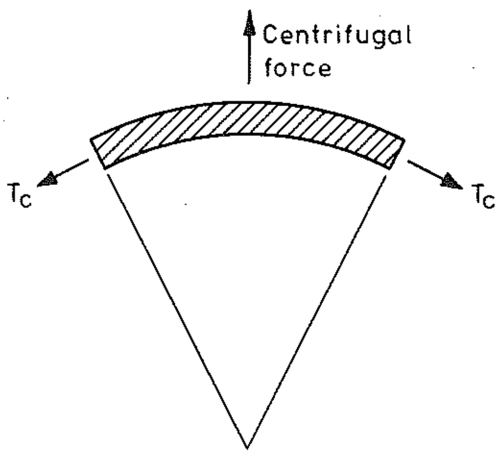


FIG. 6

If centrifugal force is present (as in all practical drives) the tension on each side will be increased by an amount T_c which is required to balance the centrifugal force (see Fig. 6).

Total tension on tight side (T_{T1})

$$T_{T1} = T_1 + T_c$$

$$\therefore \quad T_1 = T_{T1} - T_c$$

Total tension on slack side (T_{T2})

$$T_{T2} = T_2 + T_c$$

$$\therefore \quad T_2 = T_{T2} - T_c$$

Substituting these values in equation (A), we get:-

$$\frac{T_1}{T_2} = \frac{T_{T1} - T_c}{T_{T2} - T_c} = e^{\mu\theta} \quad \text{------------(B)}$$

An inspection of this equation will show that, for given values of T_{T1} and $e^{\mu\theta}$, the larger the centrifugal tension T_c the larger T_{T2} must be. But, if T_{T2} is increased without any increase in T_{T1} then the difference of tensions $T_{T1} - T_{T2}$ ($= T_1 - T_2$) & hence the torque will be reduced. Thus the effect of the centrifugal tension is to decrease the torque which can be transmitted by the belt.

It will be shown later that, for a given belt, the centrifugal tension depends on the square of the speed.

Hence as the belt speed increases, the torque decreases. The effect is greatest at higher speeds.

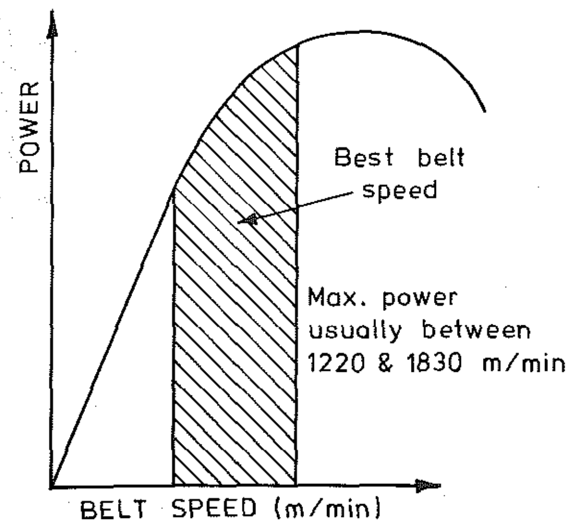


FIG. 7

Power

The power transmitted by a belt is proportional to the product of torque and speed (R.P.M.). Hence at low speeds, the faster the belt runs, the more power it will transmit but, at higher speeds, the effect of the centrifugal tension becomes so large that the power transmitted begins to decrease as the speed increases.

This is shown in Fig. 7.

Determination of T_c

Where the speed of belting is above 900 m/min allowance should be made for the effect of centrifugal tension.

Refer to Fig. 8.

The centrifugal force per metre length of arc will be given by

$$f = \frac{mv^2}{r} \text{ newtons}$$

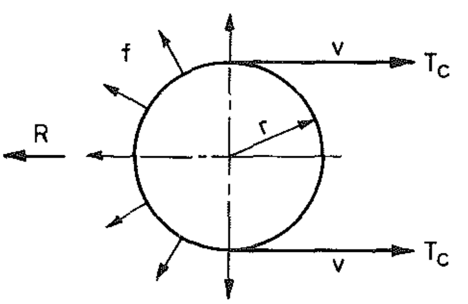


FIG. 8

These radial forces will have a resultant R directed towards the left and will be balanced by tensions T_c in the belt, which are in addition to those required for driving purposes. The case is analogous to a boiler shell subjected to internal pressure.

$$R = f \times 2r = mv^2/r \times 2r = 2\ mv^2 \text{ newtons}$$

$$\text{Also } 2T_c = R \qquad \therefore \quad T_c = mv^2$$

The power formula for a flat belt drive may be written

$$P = \left(T_{T1} - T_c\right)\left(1 - \frac{1}{e^{\mu\theta}}\right) v$$

This power is a maximum when

$$\frac{d}{dv}\left[\left(T_{T1} - T_c\right) v\right] = 0 \qquad \text{i.e. } \frac{d}{dv}\left(T_{T1}v - mv^3\right) = 0$$

$$\text{i.e. } mv^2 = \frac{1}{3} T_{T1} \qquad \text{or} \qquad T_c = \frac{1}{3} T_{T1}$$

V-Belt Drives

On account of the wedging action of a V-belt in the groove of the pulley (or sheave) the effective friction force is greater than in a flat belt drive. To ensure wedging action in groove, the belt should make contact with the sides of the groove, but not the bottom.

Effect of Wedging Action

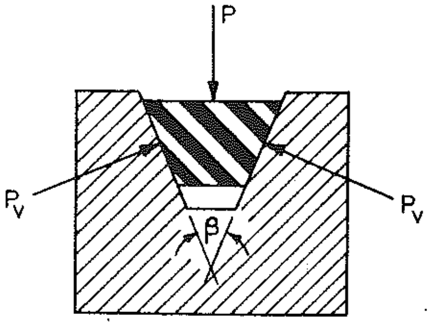


FIG. 9

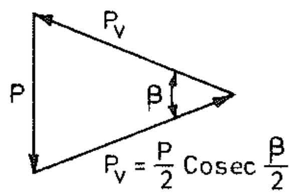


FIG. 10

Consider a section of a V-belt drive as shown in Fig. 9.

The force between the V-belt and the pulley:

$$P_v = \frac{P}{2} \operatorname{cosec} \frac{\beta}{2} \quad \text{(see Fig. 10)}$$

The frictional force on the V-belt =

$$2\mu P_v = \mu P \operatorname{cosec} \frac{\beta}{2}$$

Hence, for a V-belt, the frictional force is greater than for a flat belt by a factor V which is given by:

$$V = \operatorname{cosec} \frac{\beta}{2}$$

With a groove angle of 38°, V = 3.07

Thus it can be seen that, using a V-groove of 38°, has the same effect as increasing the coefficient of friction to 3 times its actual value. This means that, for a V-pulley to develop the same frictional force as a flat pulley, the tensions in the V-belt can be very much lower.

(1) Flat Belt Drives

Flat belting is still the logical selection to a great many driving problems. It still remains the major means of transmitting power in agricultural machinery. The familiar power take-off on most tractors is the starting point of many flat belt drives to welding plants, pumps, saw benches, grinding wheels and machine tools.

There are many variations of flat belt drives:-

Horizontal

Right-angled

Quarter turn

Crossed belt

Advantages of flat belt drives:-

May be used on very long centre distances.

Relatively inexpensive.

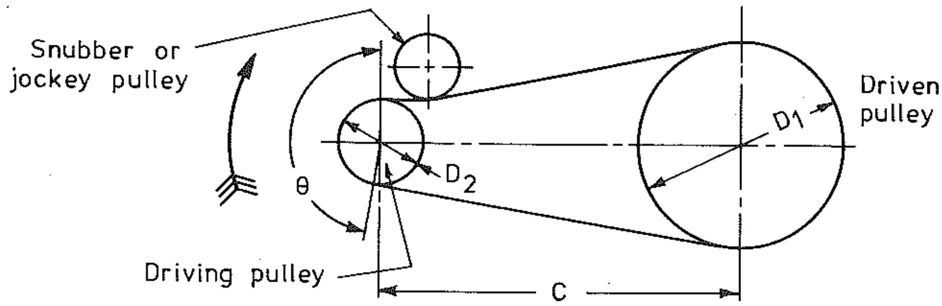
Service factors.

Use those shown in Table No. 1 on page 3.3.

Design power, P = motor power x S.F.

The following theory deals specifically with horizontal belt drives. Fig. 11 shows the components of a flat belt drive. Wherever possible the slack side of the belt should be kept to the top so that the force of gravity increases the wrap of the belt around the pulleys.

Because of belt slip and creep the driven pulley will rotate at a slightly slower speed than the pulley ratios would indicate. The velocity ratio does not remain quite constant. Allowance for the first point can be made by a small change in one of the pulley diameters.



$$\text{Length of belt} = \frac{\pi}{2}(D_1 + D_2) + 2C + \frac{(D_1 - D_2)^2}{4C} \text{ approx.}$$

FIG. 11

Coefficient of Friction

Common values.	Rubber belts on iron or steel pulleys	0.25
	Rubber belts on lagged pulleys	0.35

The smaller diameter pulley is the one on which the tendency to slip is the greater (the smaller pulley has the smaller arc of contact).

It is therefore essential for its diameter to be known in fixing the correct belt to be used for any drive, for this pulley limits the power that the belt will transmit.

Snubber Pulley

The standard method of increasing the arc of contact on the small pulley is to use a snubber (or jockey) pulley.

This idler pulley should be applied as closely as possible to the small pulley on the slack side of the belt.

Limits on Pulley sizes etc.

Each belt manufacturer has his recommended limits on minimum pulley diameters and number of plies etc. For practical application the manufacturer's catalogue should be consulted.

Tables No. 1 to No. 3 (page 5.9) are taken from various sources and give students some idea on values that could be expected.

[The authors express thanks to Dunlop Industrial for a number of contributions that have been used throughout this chapter.]

FLAT TRANSMISSION BELTING

Table No. 1.

Width of belt mm.	Min. No. Plies	Max. No. Plies
50	2	4
75	3	5
100	3	6
125	4	6
150	4	6
200	4	6
250	4	8

Table No. 2. Minimum pulley diameters mm.

	Speed metres/min.					
Plies	300	600	900	1200	1500	1800
3	75	75	100	100	125	125
4	100	125	125	150	175	200
5	200	200	200	225	250	300
6	250	300	360	360	410	460
7	360	410	460	510	560	610
8	460	560	610	660	710	860

Table No. 3.

	Mass of belt per mm. width per metre length per ply.	Maximum working tension.
Duck 28 oz	0.0014 kg	4.38 N/mm./ply
Duck 32 oz	0.0016 kg	5.25 N/mm./ply
Duck 36 oz	0.0018 kg	5.60 N/mm./ply
Duck 42 oz	0.0021 kg	7.00 N/mm./ply

= or kN/ply metre.

Conveyor belting

Table No. 4.

Width of belt mm.	Min. No. plies	Max. No. plies
300	3	4
350	3	4
400	4	5
460	4	5
510	4	6
610	5	7
760	5	8
920	6	8
1220	7	8

Table No. 5. Recommended pulley diameters mm.

Plies	Drive	Head	Tail	Snub
3	380	300	250	250
4	510	410	300	300
5	610	510	410	410
6	760	610	510	410
7	915	710	610	510
8	1020	860	760	510
9	1120	1020	915	610
10	1270	1170	1070	760

Crowning of Pulleys

Generally with transmission belting a long length of unsupported belt approaches the pulley. Under these conditions crowning helps the belt to track correctly. This effect is due to the tendency of the two sides of the belt section, viewed separately, to ride in towards the high point on the pulley, which is the crown of the camber. Fig. 12 shows the correct amount of camber. Where pulley camber is too great, the belt will in time acquire a glazed surface in the centre, as a result of carrying the whole load and slipping, because the edges of the belt are not in contact with the pulley.

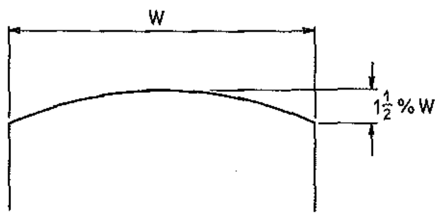


FIG. 12

Note: Excessive crowning is detrimental to the belt and results in uneven tension distribution. With some types of belting materials, such as nylon, flat faced pulleys (or pulleys with a very small amount of crowning) are preferred. Always check with the belt manufacturer.

Tension during acceleration period.

Care should be taken to keep maximum belt tension during the acceleration period within acceptable limits. The application of the correct service factor at the design stage helps in avoiding any problem. If the starting torque is severe, the belt manufacturer can provide maximum permissible acceleration tensions for more precise calculations.

Summary of design formulas for Flat Belt Drives.

$$v = \frac{\pi DN}{60} \quad ; \quad \frac{T_1}{T_2} = e^{\mu\theta} = \frac{T_{T1} - T_c}{T_{T2} - T_c} \quad ; \quad T_c = mv^2$$

$$P = \frac{T\omega}{1000} = \frac{T_E\, v}{1000} \quad ; \quad \omega = \frac{2\pi N}{60}$$

$$T = (T_1 - T_2)r = (T_{T1} - T_{T2})r = T_E\, r$$

$$T_E = T_1 - T_2 = T_{T1} - T_{T2}$$; Design power, P = motor power x S.F.

Belt width = T_{T1} ÷ (No. plies x max. permissible working tension)

$$\sqrt{T_{T1}} + \sqrt{T_{T2}} = 2\sqrt{T_o}$$ (C. Barth, Trans. ASME, vol. 31, p.29, 1909.)

Alternatively, $T_o \cong \dfrac{T_{T1} + T_{T2}}{2}$ (Dunlop Industrial)

With tension pulley type belt drives, the initial tension is from a practical viewpoint not affected by centrifugal action since its resulting belt stretch is taken up by the movement of either the tension pulley or motor weight.

(2) Belt Conveyors

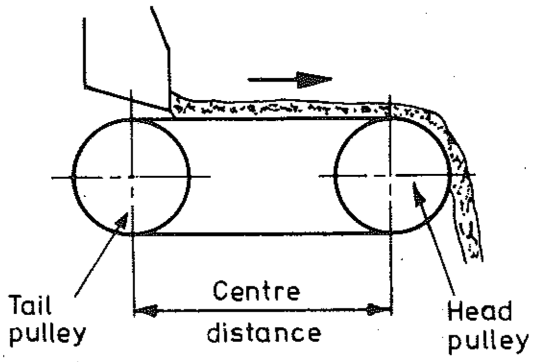
Belt conveyors are now one of the most common methods of materials handling. A wide variety of materials can be transported including coal, ore, grain and food stuffs.

As for flat belt drives the manufacturer's catalogue should be consulted for guidance on design details not covered in this section.

Fig. 13. shows elements of a simple belt conveyor.

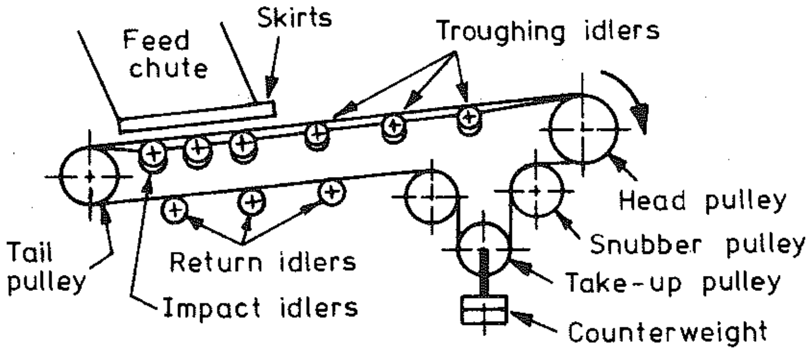
Fig. 14. shows the component parts of a more complex belt conveyor; there are many other variations of this arrangement.

Tables No. 3 to 5 (page 5.9) give a guide for students on design limits for pulleys and belt plies.



BASIC BELT CONVEYOR

FIG. 13



BELT CONVEYOR TERMINOLOGY

FIG. 14

Coefficient of Friction - Common values.

Rubber belts on iron or steel pulleys	0.25
Rubber belts on lagged pulleys	0.35

Crowning of Pulleys

Flat faced pulleys are favoured for belt conveyors these days. Most belts are fairly well guided along their length anyway.

There are some types of belts on the market for which it is not permissible to have a crowned pulley (e.g., rayon belts when subjected to high tension and steel cord belts).

If crowning is favoured, 5 mm on the radius for each metre width of pulley is recommended for pulleys up to 1 m wide. Over 1 m width, total crowning to be 5 mm.

NOTE: The crowning of pulleys tends to keep the belt controlled only when the belt as a whole does not slip.

A slipping belt will run off a crowned face quicker than off a flat face.

Take-up method

Take-up for tail pulleys can be accomplished by two basic methods:-

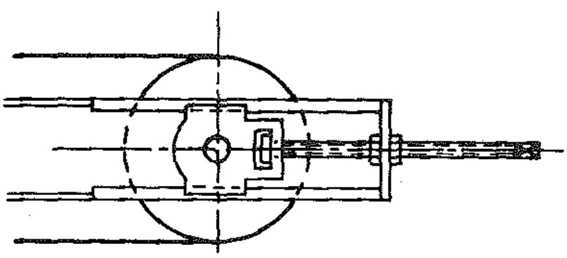
(a) Screw type (b) Gravity type

(a) Screw type.

Refer to Fig. 15. This design is suitable for conveyors of up to 30 m centres. Larger centres tend to require excessively long screws.

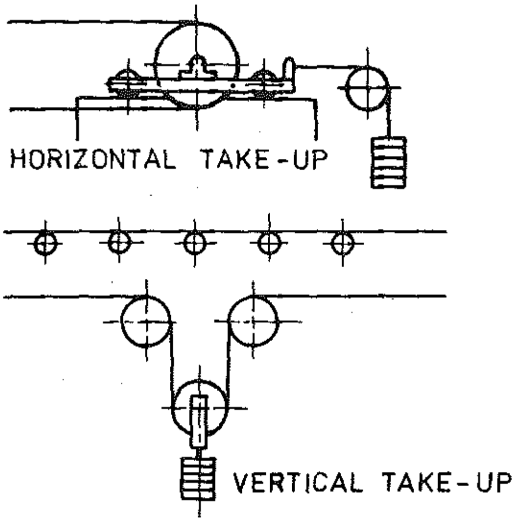
(b) Gravity type.

Refer to Fig. 16. These designs are generally used on conveyors that are too long for screw type take-ups.



SCREW TAKE-UP

FIG. 15



GRAVITY TAKE-UPS

FIG. 16

Take-up Distance

2% of centre distance plus 0.6 m for gravity take-up.
3% of centre distance for screw type take-up.

Also refer to manufacturer's catalogue.

The two basic types of belt conveyors which will be covered are:-

(a) Belt conveyors having the loaded side supported on idlers.

(b) Belt conveyors having the loaded side fully supported by sheet metal or hardboard.

(a) A troughed conveyor can be either horizontal or inclined.

Power requirements:-

(i) Power for horizontal component of product movement. (from manufacturer's catalogue)

(ii) Power to drive empty conveyor. (from manufacturer's catalogue)

(iii) For an inclined conveyor the power for lifting product =

$$\frac{\text{N/sec of product lifted x height in m}}{1000} \text{ kW}$$

Total power at headshaft = (i) + (ii) + (iii)

Tension during the acceleration and braking period.

Where excessive starting torque (say D.O.L.) or braking torque is applied to a conveyor, the maximum allowable operating tension for the belt should be reduced. This ensures the belt will not be overstressed during starting or stopping. Manufacturers will give recommended tensions for these conditions. They will also give figures for the maximum permissible belt tension during the actual acceleration or braking period, if more precise calculations are required.

However, in most cases, drives designed for running conditions will handle the added load due to acceleration.

The take-up method affects the calculation of acceleration tension.

Constant tension take-up.

This would include gravity, spring and hydraulic types. $T_2 = T_o$ under all conditions. Then,

$$T_1 \text{ (acceleration)} \cong \frac{\% \text{ F.L.T.}}{100} \times T_E + T_2$$

where T_E is the value for normal full load, full speed conveyor operation.

According to Dunlop Industrial, an increased coefficient of friction is developed for the starting period under these conditions. See Exercise No 4.

Screw type take-up.

In this case, T_1 and T_2 will vary according to the applied torque. Hence, if the coefficient of friction is assumed constant, conservatively high values for the acceleration tension will be determined. See Exercise No 4.

In actual fact, average tension tends to remain constant for all operating conditions, e.g. running, accelerating, braking, loaded, unloaded, etc. There is a consequential temporary increase in the value of μ.

(b) Sometimes a belt conveyor having a perfectly flat conveying surface is required. In this case the belt is fully supported for its entire length on hardboard or sheet metal on the conveying side. Usually this type of design is restricted to short conveyors.

The conveyor can be either horizontal or slightly inclined.

Power requirements:- The power can be determined from T_E

Horizontal conveyor.
Refer to Fig. 17.

$$T_E = \mu_s W$$

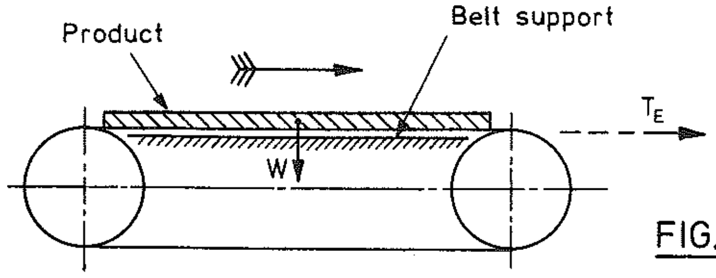


FIG. 17

Inclined conveyor. Refer to Fig. 18.

$$T_E = \mu_s W \cos\phi + W \sin\phi$$

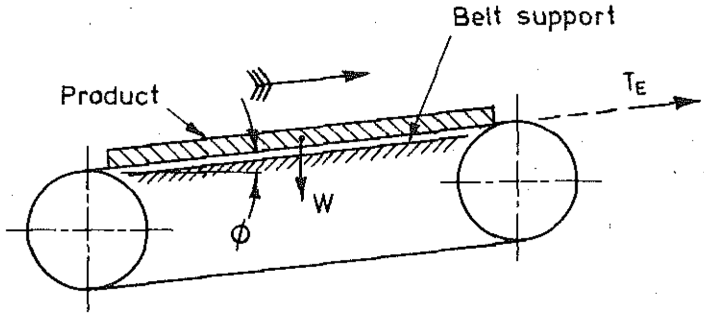


FIG. 18

Summary of Design Formulas for Type (a) and (b) Belt Conveyors.

$$v = \frac{\pi DN}{60} \quad ; \quad \frac{T_1}{T_2} = e^{\mu\theta} \quad ; \quad P = \frac{T\omega}{1000} = \frac{T_E v}{1000}$$

$$\omega = \frac{2\pi N}{60} \quad ; \quad T = (T_1 - T_2)r = T_E r \quad ; \quad T_E = T_1 - T_2$$

Belt width = $T_1 \div$ (N^{o} plies x max. permissible working tension)

For screw type take-up, $\sqrt{T_1} + \sqrt{T_2} = 2\sqrt{T_o}$

(C. Barth, Trans. ASME, vol. 31, p.29, 1909.)

Alternatively, $T_o \cong \dfrac{T_1 + T_2}{2}$ (Dunlop Industrial)

For constant tension take-up, $T_o = T_2$

(3) Vee Belt Drives

These drives are designed almost totally from the manufacturer's catalogue.

Advantages of V-belts; Compared with flat belts

Belt tension reduced hence the load on the shafts and bearings is reduced.

Close centre distances can be used so that the drive is compact, thus fitting nicely into a machine assembly.

Large ratios can be used even when the centre distance is comparatively short.

The slip between belt and pulley is negligible, resulting in longer life of the belt.

Compared with Gears, Chains and Couplings

V-belts cushion the motor and bearings against load fluctuations. (This is an important advantage of any belt drive as compared with a positive drive.)

They are quieter than gear or chain drives.

They do not need the extreme accuracy of alignment required for couplings and gears. For high speed shafts, the initial cost is lower than gears or chains.

The maintenance is usually much less than that for gears and chains, which require adequate lubrication.

The power which can be transmitted by a V-belt drive depends on:-

(i) Cross section of Belt
(ii) Speed of Belt
(iii) Diameter of Pulleys
(iv) Length of Belt
(v) Number of Belts
(vi) Arc of Contact
(vii) Service Conditions

Cross-section of Belt

(i) V-belts are manufactured in five standard sizes. These are designated by A,B,C,D and E. The included angle is 40° in each case.

(ii) Speed of Belt. As with flat belts, the power transmitted by a V-belt drive increases with increase in speed until a maximum value is reached (at about 1500 m/min); after this, any further increase in speed results in a decrease in power. (See Fig. 7, page 5.6.) The permissible effective pull (T_E) for a V-belt decreases with increasing speed.

(iii) Diameter of Pulleys. The smaller the pulley, the more severe will be the bending stresses as the belt wraps around it. This is allowed for by reducing the permissible effective pull (T_E) for the belt. The size of the larger pulley of a drive has a small effect on the life of the belt, but this is neglected in these notes.

(iv) Length of Belt. The severity of service of a V-belt drive depends on belt length because the shorter the belt the more often it flexes around the pulley. However, this effect is frequently neglected except in very short drives.

(v) Number of Belts. To keep the bending stress to a minimum, it would be best to use a large number of small belts. However, this requires a wider sheave which increases the load overhang (distance from the resultant belt force to the bearings) which, in turn, increases the shaft stresses and bearing loads, all of which raise the cost of the drive. Hence good design requires suitable compromise.

(vi) Arc of Contact. This is the angle for which the belt is in contact with the pulley. As the arc of contact decreases, there is a greater tendency for the belt to slip, hence the power which can be transmitted is, in general, reduced by a factor which depends on the arc of contact.

Table No. 6. gives a general guide on this point.

(vii) Service Conditions. These include:- Shock, overload, life, etc.

TABLE NO. 6 - ARC OF CONTACT FACTOR

$\frac{D_1 - D_2}{C}$	0	0.35	0.68	1.0	1.2	1.4
Arc of Contact	180°	160°	140°	120°	105°	90°
Factor V to V	1.05	1.0	0.94	0.87	0.80	0.72
V to flat	0.79	0.84	0.88	0.87	0.80	0.72

Life of Belts

In designing a V-belt drive, it should be remembered that additional V-belts usually result in longer belt life. On the other hand, it may be possible to use less V-belts than the calculated number, but this will greatly reduce the life of the belts.

To obtain long belt life, it is important that correct belt tension be maintained at all times. This is best done by providing for automatic take-up, either by a large diameter Jockey Pulley, or better still, by a Pivoted Motor, either of which may be operated by a spring or by gravity.

The tension ratio T_1/T_2 for V-belts transmitting full power, should be about 7. However, frequently the belts are tighter than they need to be. In these cases the tension ratio may be as low as 3. This will, of course, shorten the life of the belts.

V-Pulley to Flat-Pulley Drives

In some cases, a saving in initial cost can be made by using a flat pulley for the large diameter one. This is particularly true where the flat pulley is already available. The arc of contact on the flat pulley should be large enough to balance the wedging action of the V-belt in the small pulley. The balance point is 130° (see Table 6). If the arc of contact on the small pulley is less than this, it makes no difference to the driving if the large pulley is grooved or not. However, if there is excessive slipping, due to shock or overload, the V-belts will slide off the flat pulley.

The flat pulley should be about 25 mm wider than the V pulley and preferably not crowned, although slight crowning would be satisfactory. To obtain the pitch diameter of the flat pulley, add the belt thickness to the outside diameter. As a general rule, the centre distance should be between 0.75 and 1.5 times the large pulley diameter and the speed ratio 3:1 or higher.

Worked example No. 1.

Design a flat belt drive to transmit 15 kW from a 1432 r.p.m. full load speed motor using a 2:1 reduction at 1.5 m centres. Use a C.I. pulley.

Answer

Assume a 4 ply belt (36 oz) x 150 mm wide.

Mass of belt from Table No. 3. (page 5.9) $= 0.0018 \times 150 \times 4 = 1.08$ kg/m

From Table No. 2, page 5.9, assume a 200 mm diameter pulley.

$$v = \frac{\pi DN}{60} = \frac{\pi \times 0.2 \times 1432}{60} = 14.996 \text{ m/sec.}$$

$$T_c = mv^2 = 1.08 \times 14.996^2 = 242.87 \text{ N}$$

By a scale set out, θ on the small pulley = 170° (approx.) = 2.97^c

$$\frac{T_{T1} - T_c}{T_{T2} - T_c} = e^{\mu\theta}$$

$$\frac{T_{T1} - 242.87}{T_{T2} - 242.87} = e^{0.25 \times 2.97}$$

From which

$$T_{T1} = 2.1\, T_{T2} - 267.45 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$P = \frac{T\omega}{1000}$$

$$\therefore \quad T = \frac{1000\, P}{\omega} = \frac{1000 \times 15 \times 60}{2\ \pi\ 1432} = 100 \text{ N.m}$$

$$(T_{T1} - T_{T2})r = T$$

$$(T_{T1} - T_{T2})0.1 = 100$$

From which

$$T_{T1} = T_{T2} + 1000 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving equations (1) and (2) gives

$$T_{T1} = 2152 \text{ N}$$

$$T_{T2} = 1152 \text{ N}$$

From Table No. 3, page 5.9

36 oz duck belt = 5.6 N/mm/ply

If a 4 ply belt is used then belt width $= \dfrac{2152}{4 \times 5.6}$

$= 96$ mm < 150 mm

The foregoing represents the initial attempt at the drive design. A further trial could now be made with a lighter narrower belt.

Worked Example No. 2.

If the initial design made in the previous example was accepted determine the required value of the initial tension.

Answer

$$\sqrt{T_{T1}} + \sqrt{T_{T2}} = 2\sqrt{T_o} \qquad \text{Alternatively, } T_o = \frac{T_{T1} + T_{T2}}{2}$$

$$\sqrt{2152} + \sqrt{1152} = 2\sqrt{T_o} \qquad = \frac{2152 + 1152}{2}$$

$$\therefore\ T_o = 1613\ \text{N} \qquad \therefore\ T_o = 1652\ \text{N}$$

Worked Example No. 3.

Design the salient features of a conveyor for transporting coal.
The width of the belt is to be 600 mm, speed is to be 90 m/min.

The power as worked out from instructions in the manufacturer's catalogue is 15 kW.

The head (drive) pulley is to be lagged and is to have an angle of wrap of 200°.

Answer

$$200° = 3.49^c$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore T_1 = e^{0.35 \times 3.49}\ T_2$$

$$T_1 = 3.39\ T_2 \ \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$T_E = T_1 - T_2$$

$$\text{and } T_E = \frac{1000\ P}{v} = \frac{1000 \times 15 \times 60}{90}$$

$$= 10\ 000\ \text{N.}$$

$$\therefore 10\ 000 = T_1 - T_2 \ \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Solving equations (1) and (2) yields

$$T_1 = 14\ 184 \text{ N and } T_2 = 4184 \text{ N}$$

$$\text{Tension per mm of width} = \frac{14\ 184}{600} = 23.64 \text{ N}$$

Try 32 oz duck (Table No. 3, page 5.9.)

$$\text{No. of plies} = \frac{23.64}{5.25} = 4.5 \quad \text{say } 5.$$

Fig. 19 shows suggested pulley sizes as taken from Table No. 5, page 5.9.

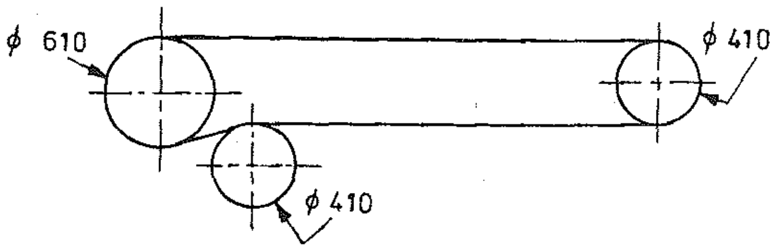


FIG. 19

Worked Example No. 4..

A 10 metre long x 450 mm wide Storage Conveyor, Fig. 20, page 5.20, is required to carry letters and small parcels at a mail exchange.
The maximum mass on the belt at any one time is 1.5 tonne.

The belt is to run at 25 m/min. and will be fully supported on masonite, $\mu = 0.15$.

The head (drive) pulley will be lagged and have an angle of wrap of 225°.

Determine:

(a) the motor power requirements.

(b) the number and specification of plies in the belt.

(c) the recommended diameters of head (drive), tail and snubber pulleys.

(d) the initial belt tension necessary. Screw type take-up.

Answer

(a) $T_1 - T_2 = T_E = \mu_s W$

$$= 0.15 \times 1500\ g = 2207 \text{ N}$$

$$P = \frac{T_E\ v}{1000} = \frac{2207 \times 25}{60 \times 10^3} = 0.92 \text{ kW.}$$

To allow for inefficiencies, drag on side walls of trough and other unknowns, use a 1.5 kW motor.

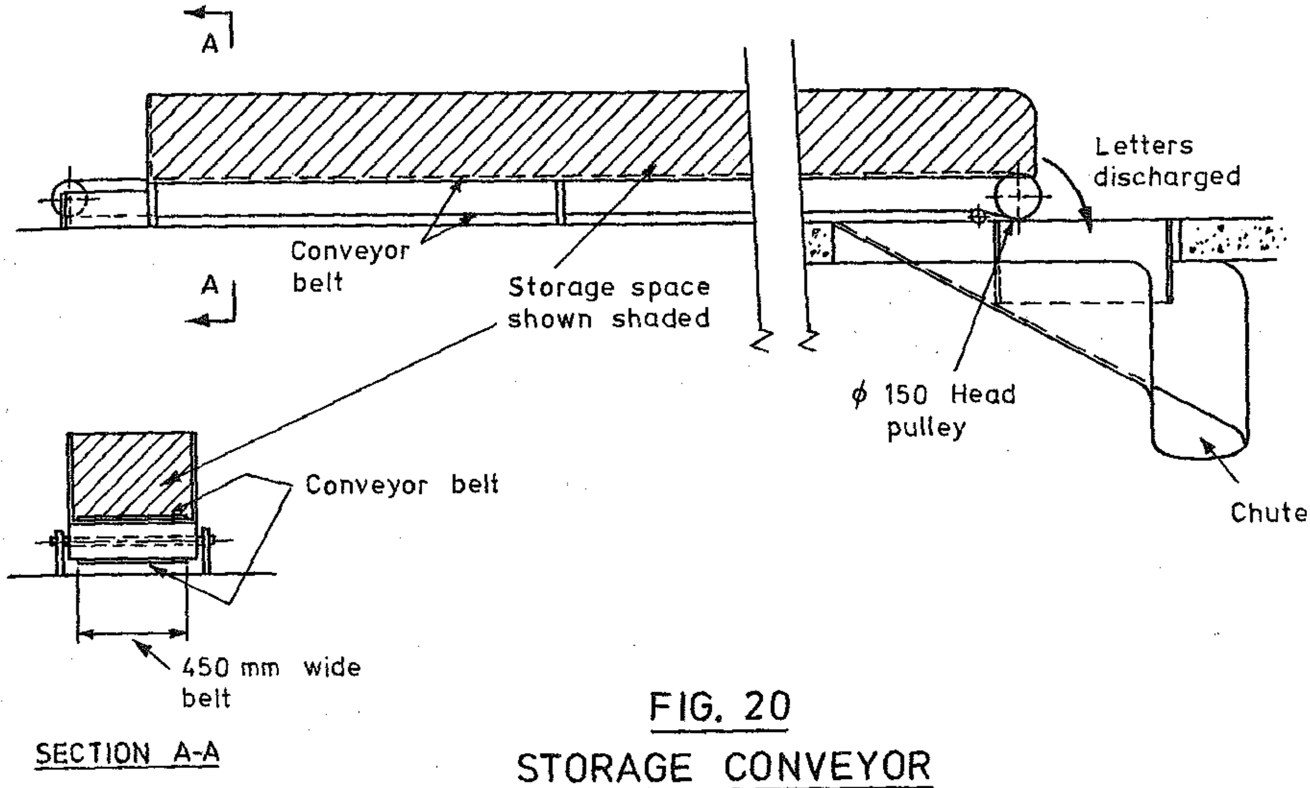


FIG. 20
STORAGE CONVEYOR

(b) $\theta = 225° = 3.93^c$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \quad T_1 = e^{0.35 \times 3.93}\, T_2 = 3.96T_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$T_E = T_1 - T_2 = \frac{1000\ P}{v} = \frac{1000 \times 1.5 \times 60}{25}$$

$$\therefore \quad T_1 - T_2 = 3600 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Solving equations (1) and (2) gives $T_1 = 4816$ N

$T_2 = 1216$ N

Tension/mm width $= \dfrac{4816}{450} = 10.7$ N

Try 28 oz duck, Table No. 3, page 5.9

Number of plies $= \dfrac{10.7}{4.38} = 2.4$ say 3

(c) From Table No. 5, page 5.9,

drive pulley	380 diameter
tail pulley	250 diameter
snubber pulley	250 diameter

(d) $\sqrt{T_1} + \sqrt{T_2} = 2\sqrt{T_o}$

$\sqrt{4816} + \sqrt{1216} = 2\sqrt{T_o}$

$\therefore\ T_o = 2718$ N

Alternatively, $T_o = \dfrac{T_1 + T_2}{2}$

$= \dfrac{4816 + 1216}{2}$

$\therefore\ T_o = 3016$ N

EXERCISES

1. Design a flat belt drive to transmit 26 kW from a 1432 r.p.m. full load speed motor using a 2.3 : 1 reduction at 1.7 m centres.
Use a C.I. pulley. Iterative design suggests the following figures:

    5 ply belt (32 oz) x 150 wide
    small pulley diameter = 250 with an angle of wrap of 169°.

    (a) Using a similar method to that shown in Worked example No. 1. determine the actual belt width required.

    (b) If the initial design was accepted calculate the required initial belt tension.

    (a) 117.36 mm
    (b) 2335.74 N

2. A belt connects two flat pulleys. The smaller pulley is 300 mm in diameter and runs at 200 r.p.m.

    The angle of wrap on this pulley is 160° and the coefficient of friction = 0.25.

    The belt is on the point of slipping when 2.6 kW is being transmitted.

    Determine the power that will be transmitted:

    (a) When the initial tension is increased by 10%

    (b) When the coefficient of friction is increased by 10% while keeping the initial tension the same.

    (a) 2.86 kW
    (b) 2.85 kW

3. Refer to Fig. 21.

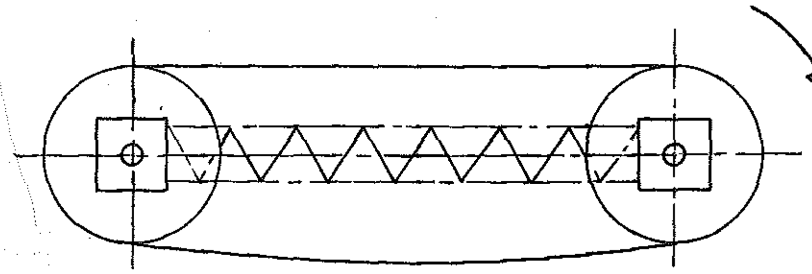


FIG. 21

Two pulleys are held apart by a spring with a compressive force S. The diameter of each pulley is D.

Coefficient of friction between pulley and belt = μ. Neglect centrifugal tension.

Determine the maximum torque which may be transmitted.

$$\text{Ans. } \frac{SD}{2} \times \frac{e^{\mu\pi} - 1}{e^{\mu\pi} + 1}$$

4. An inclined troughed belt conveyor is to transport 10 tonne per hour of product.

Height of lift = 20 m.
Speed of belt = 35 m/min.
Head pulley diameter = 400 mm lagged.
Power requirements from manufacturer's catalogue:
  Horizontal component of product movement = 0.85 kW.
  Empty conveyor = 0.25 kW.
Angle of wrap at head (drive) pulley = 220°.

(a) Determine the total power requirement at the head shaft.

(b) If a 2.2 kW motor is chosen for the job calculate the belt tensions assuming full motor power is being used at the head shaft.

(c) If a 28 oz duck belt is used for the conveyor, determine the number of plies required. Belt width = 450 mm.

(d) Initial tension is acquired by the use of a counterweight. Determine a suggested only weight necessary.

(e) If a screw take-up is used, state the initial belt tension required.

(f) If the motor is started D.O.L. and gives 160% F.L.T., state the approximate acceleration tension for (d) and (e)

(a) 1.645 kW	(b) T_1 = 5102.23 N T_2 = 1330.8 N	(c) 2.59 say 3
(d) 271.32 kg	(e) 2911.17 N	(f) 7365 N 8163.6 N*

*If average tension is assumed to be constant, then this figure would be 6233.7 N. This then requires a μ of 0.9 during the acceleration period. As this is unattainable the answer would be to increase T_o say to 4000 N;

μ then becomes 0.51 during the acceleration period. The magnitude of initial belt tension should be the minimum that will result in acceptable conveyor operation. The summary is shown in Fig. 22, page 5.23.

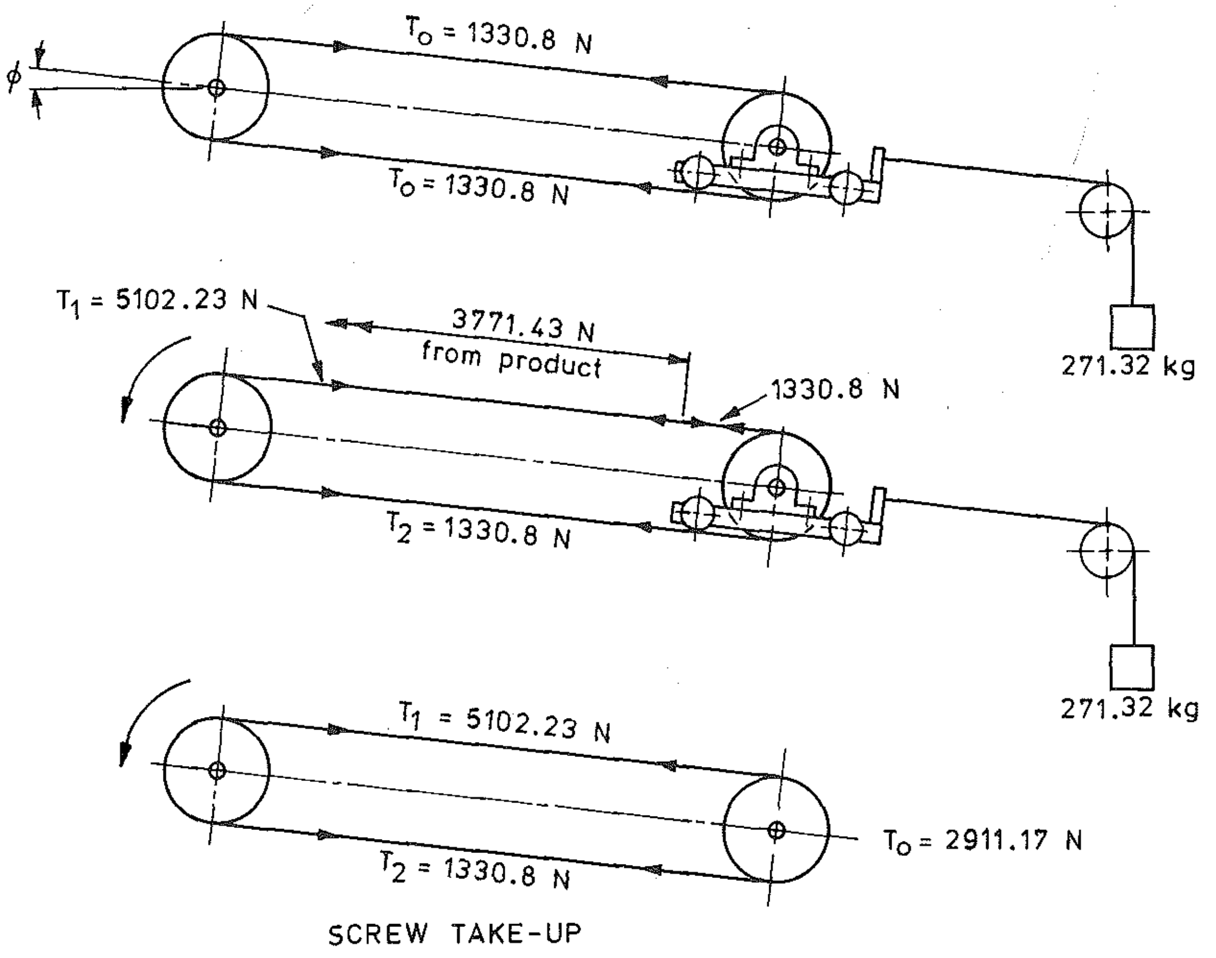


FIG. 22

5. Refer to Worked example No. 4. If this conveyor has to lift the mail (conveyor angle 5°) determine:

   (a) the motor power requirements if the drive efficiency is 92%.

   (b) the belt tensions if a 2.2 kW motor is chosen for the job. Assume the full motor power is used at the head (drive) pulley.

   (c) the number of plies necessary if a 32 oz duck belt is chosen for the conveyor.

(a) 1.58 kW

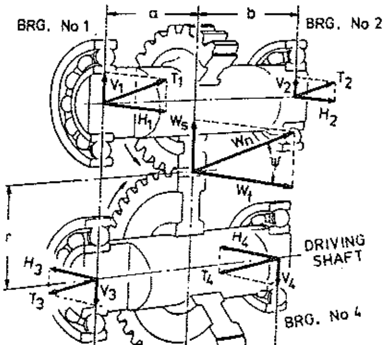
(b) T_1 = 7063.8 N, T_2 = 1783.8 N

(c) 2.99 say 3

Comment: Space limitations caused the 150 mm drive pulley to be chosen for the conveyor shown in Fig. 20. Ideally, in Exercise 5 a 28 oz 4 ply belt and a larger diameter pulley would be more in accordance with general practice.

CHAPTER 6

CHAIN DRIVES



CHAPTER 6
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C H A I N   D R I V E S

Thanks are expressed to Renold Australia Pty. Ltd. for their help in compiling this chapter.

Transmission Chains

Transmission chains are used for the transmission of power between two or more shafts and the arrangement is described as a chain drive. Wheels with specially shaped teeth on them are attached to the shafts and the chain engages with the teeth to transmit the power.

Precision Roller Chains

The universally accepted type of chain used for the transmission of power is the Precision Roller Chain. It is called a Precision Roller Chain because its components are made to fine tolerances, but it is generally referred to simply as a Roller Chain.

Roller chain is a highly efficient and versatile means of transmitting mechanical power. Other types of chain have been almost completely replaced now by roller chain.

The roller chain (Fig. 1) consists essentially of alternately assembled inner and outer links. An inner link consists of two steel side plates held rigidly together with hardened steel bushes. A hardened steel roller mounted on each bush between the plates is free to rotate on the bush. The outer link consists of two steel side plates held rigidly together by two hardened steel pins.

On assembly the pins are riveted to one side plate and then passed through one bush on each of two adjacent inner links. The other side plate is pressed over the ends of the pins and the ends are riveted over.

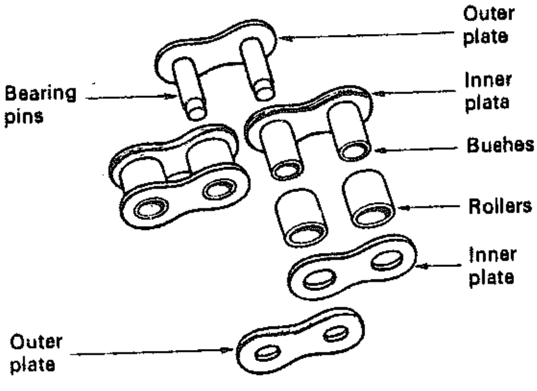


FIG. 1

Size of Chain

In roller chains the size means the pitch of the chain (i.e., the centre distance of adjacent pins).

Types of Precision Roller Chain

(Refer also to the manufacturer's catalogue)

(a) British Standard Series Chains.

(i) Short pitch chains are manufactured in single, double, and triple widths (usually described as simple, duplex and triplex chains) to British standard specifications.

Breaking loads of the more generally used chains are shown in Table No. 1.

These chains are used for the majority of chain drives where a high standard of performance and low wear with consequent long life are the important requirements.

(ii) Long Pitch chains are single width chains manufactured to British standard specifications and are used for drives with large centre distances and light loads.

TABLE NO. 1.

CHAIN PITCH IN mm	CHAIN TYPE	AVERAGE ULTIMATE STRENGTH NEWTONS
9.5	single	9810
	duplex	18 640
	triplex	26 490
12.7	single	18 640
	duplex	35 320
	triplex	51 990
15.9	single	24 525
	duplex	49 050
	triplex	73 575
19.0	single	29 430
	duplex	58 860
	triplex	88 290
25.4	single	58 860
	duplex	117 720
	triplex	176 580

(b) American Standard Series Chains.

These chains are manufactured in widths varying from simple to octuplex. Compared with British standard chains, American standard chains have a higher breaking load and so are less likely to fail under shock loads or occasional overloads but they have a smaller bearing area and so will have a much shorter life.

Other Types of Transmission Chain

(a) Bush chain is a precision chain similar to roller chain, but the rollers have been omitted. It is made in two types (i) short pitch (ii) long pitch and these have the same breaking load respectively as short pitch and long pitch roller chains.

Bush chain is sometimes used for slow moving applications and also where gumming up of the rollers could occur.

(b) Cranked Link chain is a heavy duty roller chain made only in large sizes and is used for oil well and excavator drives and similar heavy rough machinery. Wheels for the chain usually have cast teeth.

(c) Coventry Mark 5 chain is a light duty roller chain which was developed originally to replace the malleable iron chain and is particularly suitable for use under rough and exposed conditions. The component parts of the chain are treated to make them moderately corrosive resistant.
Wheels for this chain usually have cast teeth.

Standard attachments are available to enable the chain to be used as a conveyor.

(d) Inverted Tooth chain as shown in Fig. 2. is a plate link chain with teeth on the inside of the chain. It usually has a central guide link in the chain to prevent it from running off the wheel while in motion.

It is comparatively quiet and very efficient. The most common application of this chain is for timing internal combustion engines but it has many other applications.

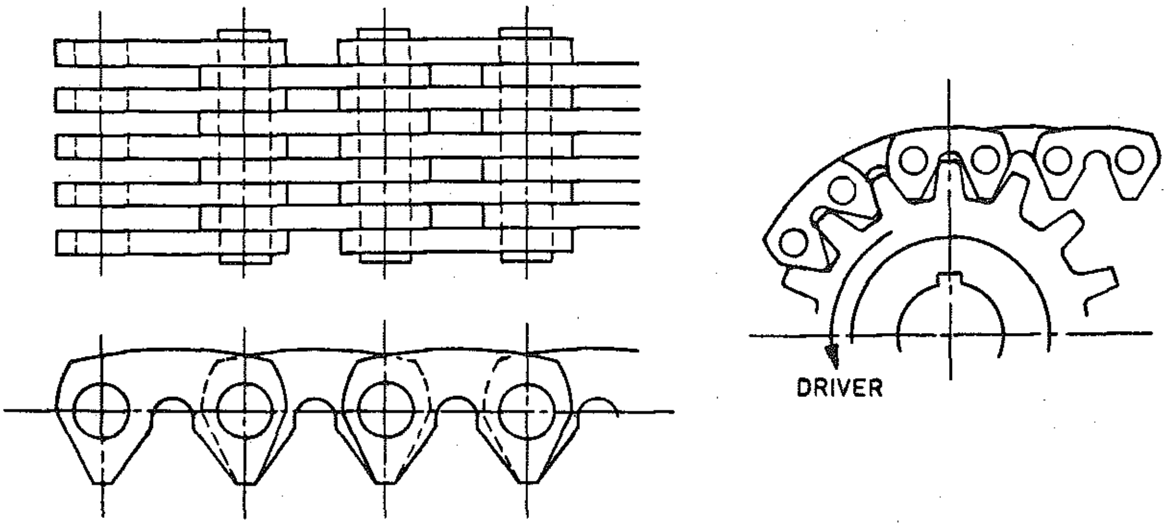


FIG. 2

Wheels for Chain Drives

Chain wheels are frequently called sprockets or sprocket wheels.

Wheels for use with precision roller chains have machine cut teeth and the standard numbers of teeth are 17, 19, 21, 23, 25, 38, 57, 76, 95 and 114. However, wheels with any number of teeth can be made for special cases. Wheels having up to and including 29 teeth are called pinions while those having 30 or more teeth are called simply wheels.

The pitch of the wheel implies the chordal pitch and is, of course, identical with the chain pitch.

Types of Wheels

Several types of wheels are manufactured and full details of the wheels available from stock can be obtained from manufacturer's catalogues.

Drive Applicational Technique

The notes given below are general recommendations, and should be followed in the selection and installation of a chain drive, in order that satisfactory performance and drive life may be ensured.

Chain Pitch

The Selection Charts in manufacturer's catalogues give alternative sizes of chains which may be used to transmit the load. The smallest pitch of simple chain should be used, as this normally results in the most economical drive. If the simple chain does not satisfy the requirements dictated by space limitations, high speed, quietness, or smoothness of running, use a smaller pitch of duplex or triplex chain. When the power requirement at a given speed is beyond the capacity of a single strand of chain, two or more strands may be used.

Refer to "Chain Choice" at end of chapter.

Maximum Operating Speeds

For normal drives, experience has established a maximum pinion speed for each pitch of chain. These speeds, which relate to pinions having 17 to 25 teeth inclusive, are given in Table No. 2. They are applicable only if the method of lubrication provided is in line with the recommendations given in Table No. 4 (page 6.8).

TABLE NO. 2.

Chain Pitch	Normal Maximum pinion speed
mm.	RPM
9.5	5000
12.7	3750
15.9	2750
19.0	2000
25.4	1500
31.8	1200
38.1	900
44.5	700
50.8	550
63.5	450
76.2	300

Number of Teeth in Wheels

Minimum number of teeth

Four important advantages of a chain drive are dependent directly upon the minimum number of teeth in the pinion. The advantages are smooth uniform flow of power, quietness of operation, high efficiency and long life; the reason for their dependence being that the chain forms a polygon on the wheel.

Thus, when the wheel speed is constant, the chain speed (due to the many-sided shape of its path around the wheel teeth) is subject to a regular cyclic variation. This cyclic variation becomes less marked as the path of the chain approaches more nearly that of a true circle, and in fact becomes insignificant for most applications as the number of teeth in the wheel exceeds 19.

The effect of this cyclic variation can best be shown in the extreme case of a pinion with the absolute minimum number of teeth, i.e. three. In this instance, for each revolution of the pinion, the chain is subjected to a three-phase cycle; each phase being associated with the engagement of a single tooth. As the tooth comes into engagement, for a sixth of a revolution the effective distance, or driving radius, from the wheel centre to the chain is gradually doubled; for the remaining sixth of a revolution, it falls back to its original position.

Thus, as the linear speed of the chain is directly related to the effective driving radius of the pinion, the chain speed fluctuates by 50% six times during each revolution of the pinion.
Refer to Fig. 3.

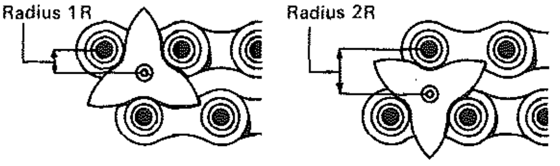


FIG. 3

As the graph (Fig. 4) shows, the percentage of cyclic speed variation decreases rapidly as more teeth are added. With a pinion of 19 teeth, therefore, this cyclic speed variation is negligible; hence we recommend that pinions used in normal application drives running at medium to maximum speeds, should have not less than 19 teeth.

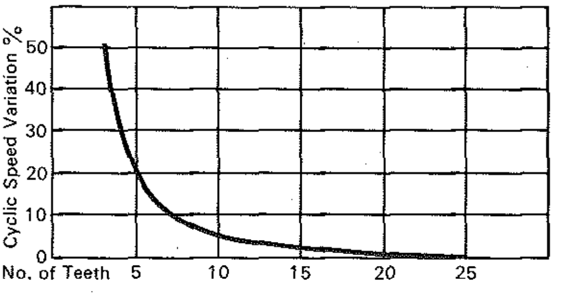


FIG. 4

Additional factors, besides cyclic variation, also affect drives where the number of teeth in the pinion is small. As the number of teeth decreases, so the gearing becomes rougher and energy is dissipated by impact and friction; self-excited chain strand vibration is induced, and chain life is drastically reduced with the increased angle of joint articulation.

There are, however, applications where space saving is a vital design requirement and the speed/power conditions are so conservative that the disadvantages of small numbers of teeth (i.e. below 17) remain unobtrusive so that a compact, satisfactory drive is possible, e.g. office machinery, hand-operated drives, mechanisms, etc. The limiting conditions with steady loading for using small numbers of teeth are shown in Table No. 3.

TABLE NO. 3

TEETH	PERCENTAGE OF MAXIMUM RATED RPM	PERCENTAGE OF MAXIMUM RATED POWER
11	20	30
13	30	40
15	50	60
17	80	90

Even number of teeth

Most drives have an even number of pitches in the chain and by using a pinion with an odd number of teeth, uniform wear distribution over both chain and pinion teeth is ensured. The need to use a pinion with an even number of teeth is exceptional and, therefore, such a pinion should not be used unless specific need due to ratio or space makes its use imperative.

Sometimes because of space limitations a cranked link has to be used to join a length of chain. In this case the chain has an odd number of pitches. However, it is only on rare occasions that this has to be done.

Maximum number of teeth

The maximum number of teeth in any wheel should not exceed 150. This limitation is due to the fact that for a given elongation of chain due to wear the working pitch diameter of the chain on the wheel increases in proportion to the nominal pitch diameter. Even though the tooth height reduces slightly as the number of teeth increases, a wheel of 150 teeth represents the maximum which will permit the accommodation of maximum chain wear.

Minimum total number of teeth in wheels

It is good practice to have the sum of the teeth in both wheels operated by the same chain not less than 50, e.g. on a 1:1 ratio drive both pinions should have not less than 25 teeth.

Centre Distance

For optimum wear life, centre distance between two wheels should normally be within the range 30 to 80 times the chain pitch. Drive proposals with centre distances below 30 pitches or greater than 2m should receive special consideration.

The minimum centre distance is sometimes governed by the amount of chain lap on the pinion, the normal recommendation for this being not less than 7 teeth in engagement with the chain. See Fig. 5.

On two-point drives this lap is obtained when the centre distance is equal to or greater than the difference between the pitch diameter of the wheel and the pitch diameter of the pinion.

The centre distance is also governed by the desirability of using a chain having an even number of pitches. This is due to the fact that the use of cranked links, i.e. of a chain having an odd number of pitches, should wherever possible, be avoided.

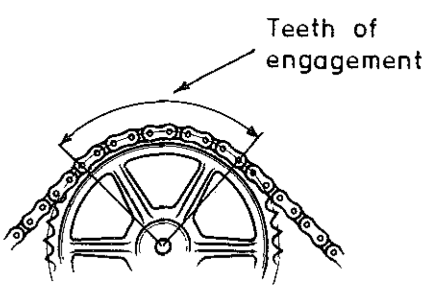


FIG. 5

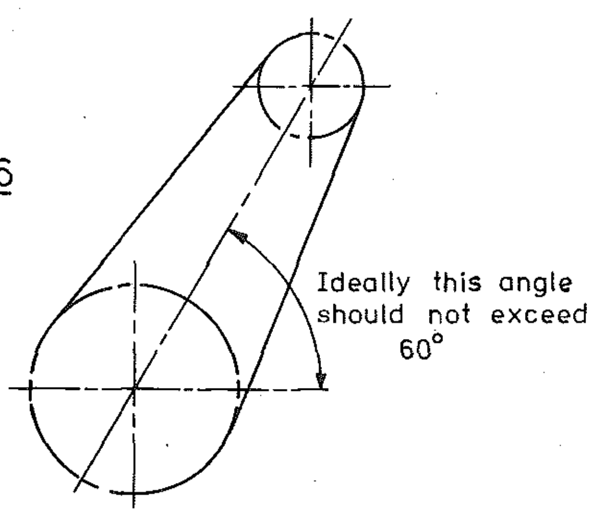
For a drive in the horizontal plane the shortest centre distance possible should be used consonant with recommended chain lap on the pinion.

Lie of Drive (Fig. 6.)

Drives may be arranged to run horizontally, inclined or vertically. In general, the loaded strand of the chain may be uppermost or lowermost as desired. Where the lie of the drive is vertical, or nearly so, it is preferable for the pinion to be above the wheel; however, even with a drive of vertical lie it is quite feasible for the pinion to be lowermost provided care is taken that correct adjustment is maintained.

The general recommendation is not to exceed an angle to the horizontal of 60°.

FIG. 6



Arc of Contact

To ensure proper load distribution on the teeth the arc of contact between the chain and wheel should be 120°. For wheels of 30 or more teeth and for driven wheels it may, in certain circumstances, be reduced to 90°.

Horizontal Drives

A drive in a horizontal plane (i.e., vertical shafts) is not recommended as the weight of the chain produces excessive wear on the side of the teeth.

Also, if the chain is at all slack it tends to run off the wheels. However, in certain cases, where the speed is low, horizontal drives may be used provided that the centre distance is small or the chain is supported between the wheels.

Adjustment

There are two basic forms of adjustment:

(i) by movement of one of the shafts - Fig. 7.

(ii) by the use of a jockey - Fig. 8.

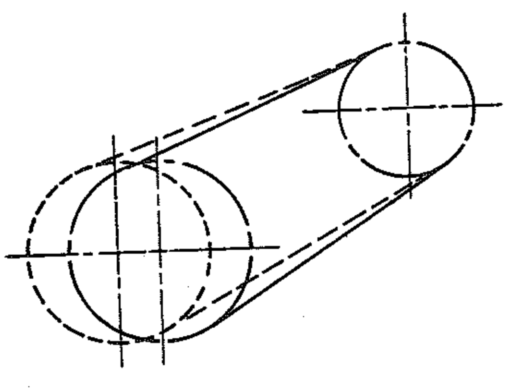


FIG. 7

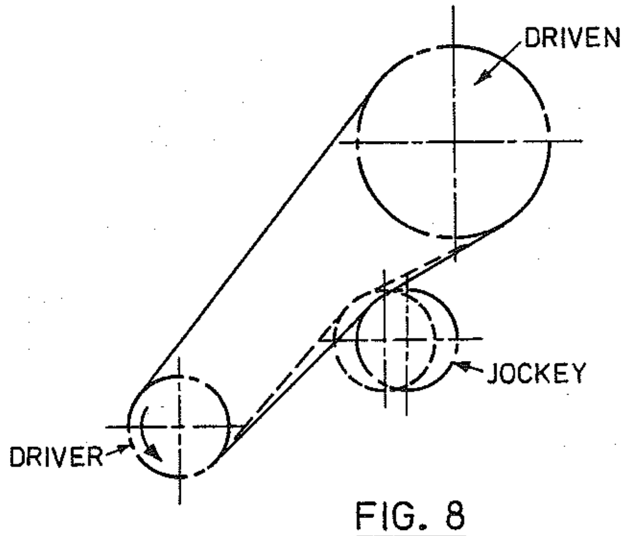


FIG. 8

The amount of adjustment by either method should be sufficient to take up chain wear amounting to 2 pitches or 2% elongation above the nominal chain length, whichever is the greater.

The jockey may be adjusted manually or automatically. It should normally be on the unloaded side of the chain and will be most effective if it is on the outside of the chain near to the larger wheel. In the initial position of any jockey there must be an arc of contact of at least 3 teeth. In any position there should be a free length of at least 4 pitches between the jockey and the nearest wheel.

The general aim should be to use not less than 19 teeth for any jockey and to ensure that the number of teeth is such that the R.P.M. of the jockey does not exceed the maximum R.P.M. recommended. Where space limitations are a problem jockey sprockets down to 15 teeth may be used.

Lubrication

It is essential that a roller chain should be properly lubricated particularly when it is travelling at high speeds. The suggested methods of lubrication are shown in Table No. 4.

TABLE NO. 4.

Power Transmitted	Chain Speed	Method of Lubrication
Low Power	Low Speed	Occasional greasing
Up to 38 kW	Up to 365 m/min	Drip Feed
Up to 38 kW	Up to 610 m/min	Oil Bath
Any power	Any speed	Oil Pump and Jets

For slow-running drives which are necessarily exposed and where drip-feed lubrication or brushing on of heavy oil is not acceptable chains should be removed periodically, thoroughly cleaned by scrubbing in kerosene and allowed to dry. They should then be immersed in a bath of pure mineral grease of medium weight, the grease being heated sufficiently to assist penetration into the chain joints. The grease should be allowed to cool before the chain is removed and surplus grease wiped away. Wheel teeth should be cleaned before refitting the chain. It must be emphasised that application of grease to the outside of a chain is of negligible value as a means of lubricating the chain joints, although it may afford some protection against corrosion.

Chain Choice

Where chain life is not important (e.g., a drive which is used only occasionally) the drive may be designed by dividing the breaking load by a suitable factor of safety, generally about 10, see Table No. 1, Chapter No. 1. (page 1.19).

Service Factors

Table No. 5. gives service factors for various types of industrial conditions.

TABLE NO. 5.

TYPE OF DRIVEN LOAD	INTERNAL COMBUSTION ENGINE MECHANICAL DRIVE	INTERNAL COMBUSTION ENGINE HYDRAULIC DRIVE	ELECTRIC MOTOR
Smooth	1.2	1.0	1.0
Med. Shock	1.4	1.2	1.3
Heavy Shock	1.7	1.4	1.5

Shock loading is a different type of loading to momentary overloads or high starting torque.

Because of the large safety factors associated with chain selection momentary overloading and high starting torque do not normally require the use of a service factor.

Worked example

The drive for a bucket elevator is shown in Fig. 9.
Check the safety factor on the chain and state whether the chain selected is satisfactory.

Answer

The following design method is the one generally used for "open" chain drives, grease lubricated, occasionally maintained. Chain choice is an iterative process. The details given in Fig. 9. have been arrived at after a couple of trials.

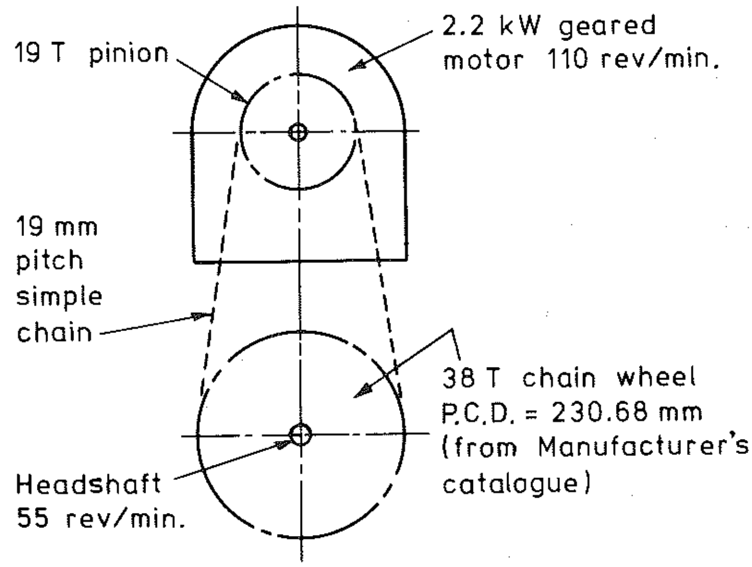


FIG. 9

$$\text{Speed of headshaft} = 55 \text{ r.p.m.}$$

$$= \frac{2\pi \times 55}{60} = 5.76 \text{ radians/sec.}$$

$$P = \frac{T\omega}{1000}$$

$$\therefore \quad T = \frac{1000\ P}{\omega}$$

$$= \frac{1000 \times 2.2}{5.76}$$

$$= 381.94 \text{ N.m (for headshaft)}$$

$$\therefore \quad \text{Force in chain} = 381.94 \times \frac{2}{0.230\ 68} = 3\ 311.43 \text{ N.}$$

Refer to Table No. 1 (page 6.2).

Breaking load of 19.0 mm pitch simple roller chain = 29 430 N.

$$\text{Safety Factor} = \frac{29\ 430}{3311.43} = 8.89$$

This safety factor is satisfactory as it is very close to 10.0.

EXERCISES

1. Fig. 10 shows a chain drive from a squirrel cage electric motor to a mixing machine.

   (a) Choose a chain from Table No. 1, (page 6.2) which will give a safety factor of approximately 10.

   (b) State the exact value of the resultant safety factor.

(a) 25.4 mm simple.

(b) 6.89 (13.78 if duplex chain is used).

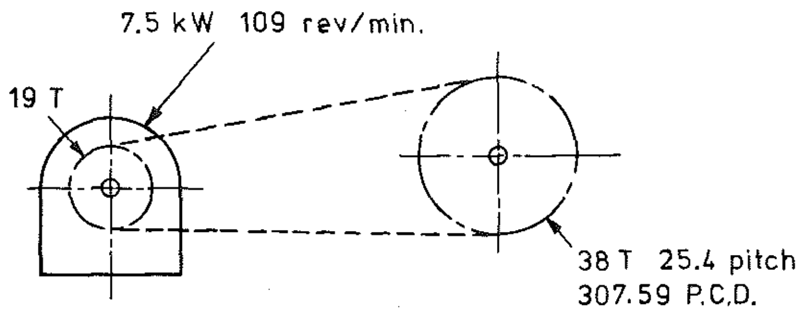


FIG. 10

2. Fig. 11 shows the plan view of the drive to the main shaft of a machine.

   Determine the roller chain size and state the safety factor.

25.4 duplex; 9.48

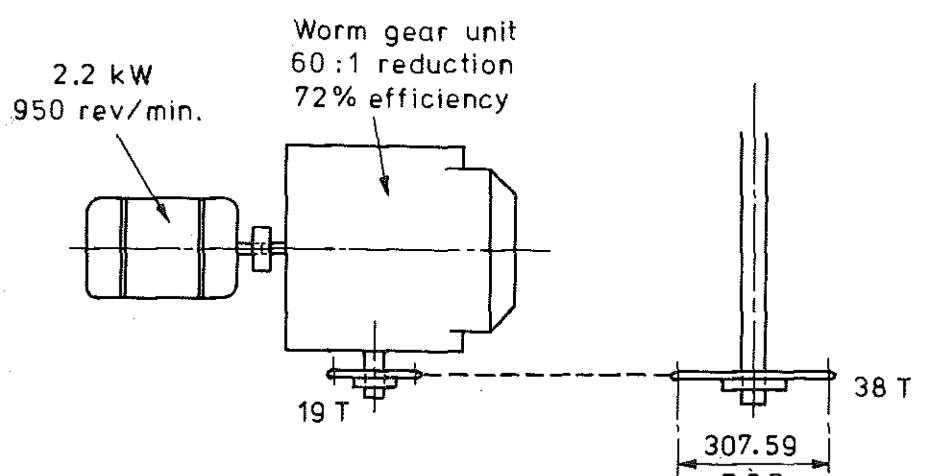


FIG. 11

CONVEYOR CHAINS

These are used for the mechanical transfer of materials or objects between various locations. They are made with attachments to which can be connected whatever other fittings are necessary for the particular application.

Types of Conveyor Chains

Conveyor Roller Chain:

This is the most important type of conveyor chain. It is similar in construction to precision roller chain, but usually has a longer pitch and larger diameter rollers. There are two types:-

(a) Hollow bearing pin chain which is particularly suitable for fixing attachments to it by bolting through the hollow bearing pins.
Hollow bearing pin chain is also used for heating ovens and similar applications because it cools more quickly than solid bearing pin type chain.

(b) Solid bearing pin chain which has exactly the same gearing dimensions as the equivalent hollow bearing pin chain but it is more robust.

Attachments for Conveyor Roller Chain

Standard attachments can be fitted to the chain and users can easily bolt slats, trays, buckets, wire mesh, staybars and spigot pins to them. Some of the more common types of attachments are shown in Fig. 12. In some cases the attachments are integral with the chain link plate, but in others they are either bolted through the hollow bearing pins or welded or bolted to the link plates.

Other types of Conveyor Roller Chain

Standard transmission chain can be adapted to suit light conveyors. The types are:-

(a) Precision roller chain, incorporating standard attachments which can be built in on one or both sides of the chain as required. When used on one side only, the whole load is carried by this side of the chain and this means the chain will break at half the usual load.

(b) Coventry Mark 5 roller chain (see under transmission chain).

(c) Bush chain (see under transmission chain).

Brass blocks can be built into the inner links of either short or long pitch bush chains and held in position by the link plates.
Various attachments can be fixed to the blocks.

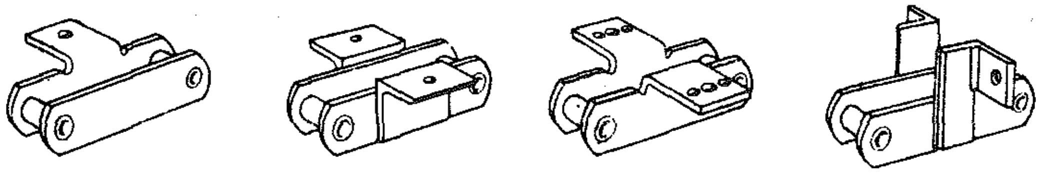


FIG. 12

Conveyor Roller Chain Selection.

A. Total Pull in Chain

For the general case of an inclined conveyor, the total pull in the chain P(N) is derived from three sources:

1. The pull (P_1) required to lift the load (W) on the conveyor plus the weight of the working side of the conveyor chain including slats etc., $\frac{W_1}{2}$, where W_1 = total weight of chain plus attachments.

$$P_1 = (W + \frac{W_1}{2}) \sin \phi \quad \text{where } \phi \text{ is the angle of inclination.}$$

<u>Note</u>: (i) P_1 is zero for a horizontal conveyor.

(ii) If the size of the chain to be used is not known, assume a weight for the chain. If the weight finally selected is heavier or much lighter than the assumed weight, it may be necessary to re-check.

2. The pull (P_2) required to overcome the frictional resistance of the load and the working side of the chain, slats etc.

$$P_2 = \mu(W + \frac{W_1}{2}) \cos \phi \quad \text{where } \mu \text{ is the coefficient of friction.}$$

<u>Note</u>: The frictional resistance is present whether the conveyor is inclined or horizontal.

Approximate values for the coefficient of friction (μ) are:

Steel chains or slats sliding on steel tracks	=	0.25
Wooden slats sliding on steel tracks	=	0.40

Where the chain is supported on rollers, which is the usual case, the coefficient of friction will be reduced in the ratio of the roller diameter (D) to the bush diameter (d)

i.e. $\mu_1 = \mu \times \frac{d}{D}$

3. The pull (P_3) required to overcome friction of the return chain, slats etc.

$$P_3 = \frac{W_1}{2} (\mu \cos \phi - \sin \phi)$$

For horizontal conveyors, this reduces to

$$P_3 = \frac{W_1}{2} \mu$$

If the angle of inclination is greater that the friction angle, P_3 will be negative and is then neglected.

B. Working Loads on Conveyor Chains

When the chain is operating in clean conditions and is regularly lubricated the working load (i.e. the total load calculated as set out above) may be $\frac{1}{10}$ to $\frac{1}{8}$ of the breaking strength.

If these conditions do not exist, the maximum permissible working load must be further reduced by the following factors:

Poor lubrication	- moderately clean conditions,	1.5
No lubrication	- dirty conditions,	2.0
No lubrication	- abrasive conditions ,	2.5

C. Power Required to Drive the Conveyor.

$$\text{Power, kW} = \frac{\text{pull in chain, } P(N) \times \text{chain speed (m/s)}}{1000}$$

This determines the power required for the fully loaded conveyor during operation. When selecting the motor or prime mover, allowance must be made for starting effort (about 150% of that when running) and losses in the conveyor drive.

Note: The preceding theory on conveyor chain assumes a single strand conveyor. Where two strands of chain are used in a conveyor the theory must be appropriately modified.

Slat Band Chains:

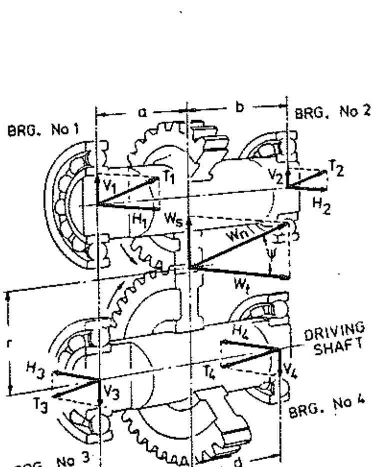
A single plane chain provides a level moving platform for the steady conveyance of bottles or similar objects between operations.

To provide a slat conveyor which is not limited to movement in one straight line, biplanar or multiplanar chains are used.

Details on the above chains can be obtained from the manufacturer.

CHAPTER 7

BEARINGS



CHAPTER 7
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B E A R I N G S

Introduction

When one mechanical member rests on another and there is relative motion between them, they constitute a bearing. However, bearings are generally regarded as separate elements, which are inserted between the two mechanical members, and which permit controlled relative motion between them.

A bearing has ONE main function:-
To lessen the friction between the two members.

All rotating parts of machinery are supported by a type of bearing. Generally speaking, bearings may be broadly classified into TWO main groups.:

(A) Plain bearings.

(B) Rolling contact bearings (anti-friction bearings).

(A) PLAIN BEARINGS (Also known as JOURNAL bearings, SLEEVE bearings or BUSHES).

In this type of bearing there is a sliding contact between the shaft and the bearing. Plain bearings offer the simplest and most economical means of supporting shafts. They have no moving parts and are usually nothing more than one piece of metal enclosing a shaft.

NOTE: A plain bearing can be split. e.g., the "big end" bearing of a con-rod.

(a) Journal Bearings (Bushes)
Refer to Fig. 1.

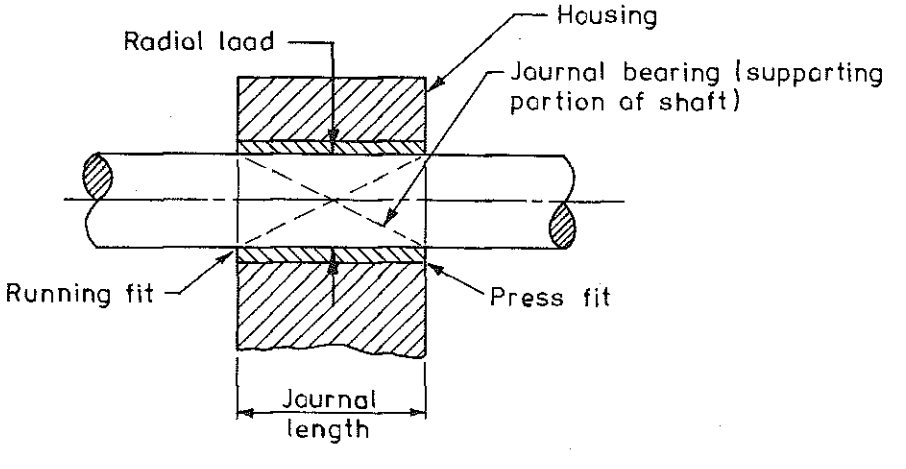


FIG. 1

This type of bearing is intended to resist radial loading, i.e., loading which is perpendicular to the axis of the shaft. The word 'journal' refers to the supporting portion of the shaft.

In the simplest form of journal bearing there is no bush. The journal revolves in a hole in the main member. This is acceptable in certain cases e.g. a shaft with only occasional use, hand operated shafts etc. Alternatively the shaft may be the stationary member.

(b) Thrust Bearings
Refer to Fig. 2.

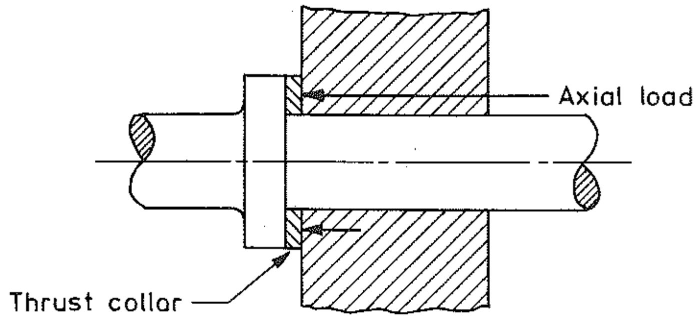


FIG. 2

This type of bearing is used to resist thrust or axial loading, i.e., loading which is along the axis of the shaft.

A plain thrust bearing may be in the form of a collar, washer or a series of washers, and usually bears against a collar which is an integral part of the shaft.

(c) Flanged Bearings
Refer to Fig. 3.

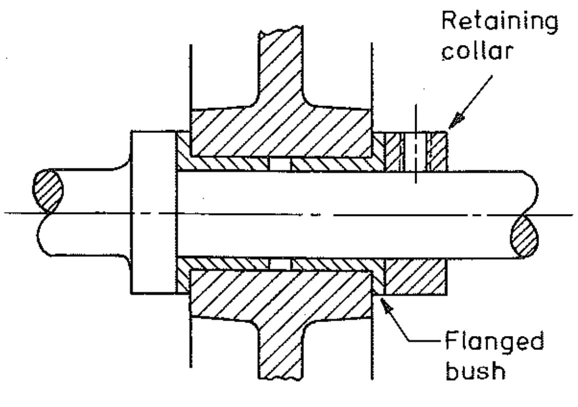


FIG. 3

This type of bearing is used when there is a combination of both radial and thrust loading. It is therefore a combination of both the journal and thrust bearings and as its name suggests, resembles a bush with a flange attached.

Plain bearings are generally made from a metal that has good wearing qualities. Metals most commonly used for this purpose are phosphor-bronze, brass, gunmetal and cast iron.

NOTE: Plain bearings are also manufactured from 'teflon', which as well as having good wearing qualities, has the distinct advantage that it can be operated at much higher speeds than the metallic bushes - in the region of 6000 R.P.M.

Other non-metallic bearing materials are nylon and ferobestos.

The shaft should have a smooth finish and should be harder than the bearing material. The smoother and harder the shaft, the better the operating performance.

For practical reasons, the length of the bearing should be between one and two times the shaft diameter, and the outside diameter approximately 25% larger than the shaft diameter.
Refer to Fig. 4.

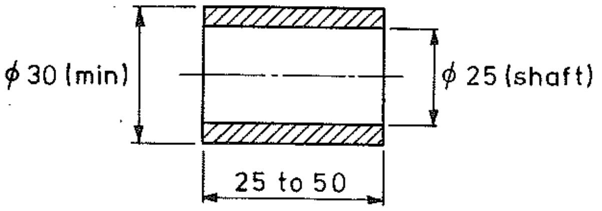


FIG. 4

Plain bearings are a commercial item and are readily obtainable in a large combination of diameters and lengths.

NOTE: Always refer to manufacturer's catalogues when selecting a plain bearing and choose a stock item if possible.

Lubrication of Plain Bearings

(1) Self-lubrication.

A majority of the plain bearings used today are the self-lubricating type, i.e., the bearing is made of porous material and is impregnated with oil or some other suitable lubricant, which automatically lubricates the bearing surfaces during operation. This type of bearing is a great asset where regular lubrication is difficult or sometimes impossible to supply.

(2) External lubrication

An external supply of lubricant is fed to the bearing by various means.
It is used:-

(a) for bearings which are not self-lubricating e.g., phosphor bronze, brass or gunmetal. Oil or grease can be used.

(b) as a supplementary lubrication for self-lubricating bearings, in order to give longer life and better performance.

Oil only should be used.

Lubrication Methods

Refer to Fig. 5, page 7.5, for examples of lubrication methods applicable to rotating shafts and rotating housings.

When lubrication is supplied to a bush it is desirable to have either:

(1) A flat on the shaft (in the case of stationary shafts).
The flat (say 1.5 mm deep) should be slightly shorter in length than the bush and on the non-pressure side.

or (2) A network of suitable grooves in the bush. Longitudinal grooves generally stop short of the ends of the bush.

In the case of reciprocating motion there should be sufficient oil or grease grooves to ensure lubrication to all parts of the journal.

Advantages of Plain Bearings

(1) Initial cost is lower in most cases.

(2) Less radial space required than for rolling contact bearings.

(3) Better suited to overload and shock conditions.

(4) Quieter operation than rolling contact bearings, especially after wear has taken place.

(5) Less difficulty with fatigue.

(6) Less easily injured by foreign matter.

Disadvantages of Plain Bearings

(1) Limited to relatively low speeds (metallic types).

(2) Require constant supervision for lubrication.
(Not self-lubricating types.)

(3) Relatively high rate of wear.

(B) ROLLING CONTACT BEARINGS. (Ball, Roller and Needle bearings)

These are sometimes called antifriction bearings because with this type of bearing, friction has been reduced to a minimum. Rolling contact bearings may be classified into three main groups:-

(1) those capable of taking radial loads only

(2) those capable of taking thrust loads only

(3) those capable of taking a combination of both radial and thrust loads.

(1) Radial loads only
Cylindrical roller bearings
Needle bearings

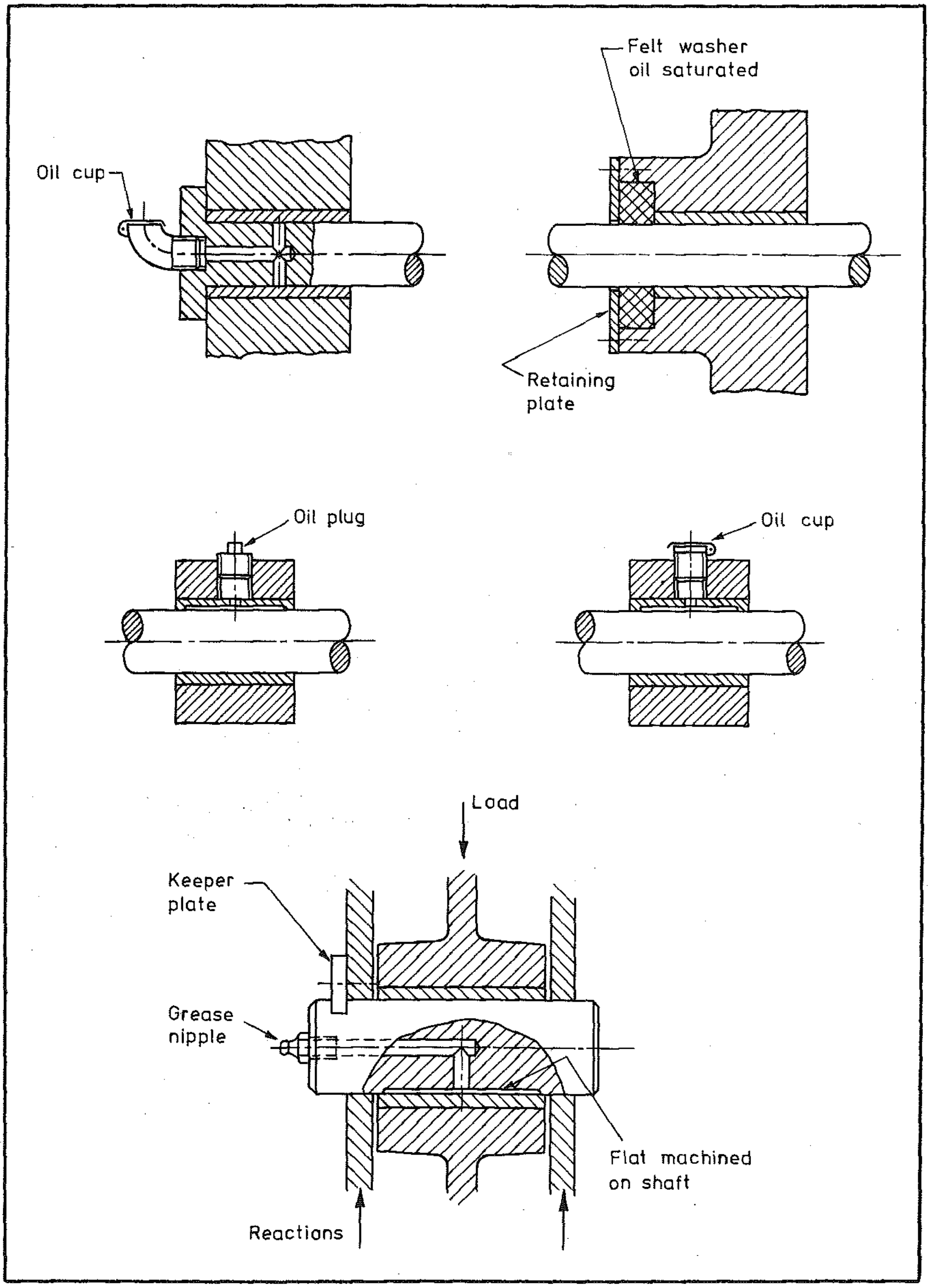

Felt washer
oil saturated
Oil cup
Retaining
plate
Oil plug
Oil cup
Load
Keeper
plate
Grease
nipple
Flat machined
on shaft
Reactions

FIG. 5

(2) Thrust loads only
    Thrust ball bearings
    Angular contact thrust ball bearings

(3) Radial and thrust loads
    Deep groove ball bearings
    Self aligning ball and roller bearings
    Angular contact ball bearings
    Spherical roller bearings, double row
    Spherical roller bearings, single row
    Tapered roller bearings
    Spherical roller thrust bearings

Most rolling contact bearings consist of the following components:-

(a) an inner ring (inner race)
(b) an outer ring (outer race)
(c) a set of rolling elements
(d) a 'cage' (separator)

The cage separates the rolling elements and spaces them evenly around the periphery of the inner race.
Refer to Fig. 6.

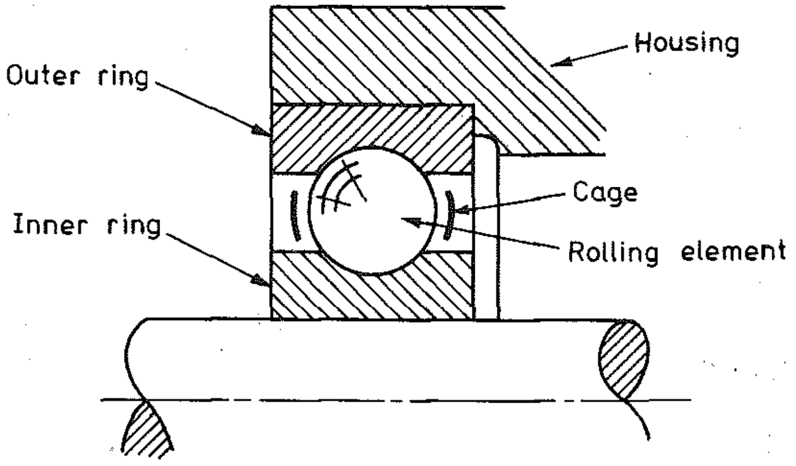


FIG. 6

Discussion will be confined to the following main types of antifriction bearings.

(1) Single row deep groove ball bearings.

Refer to Fig. 7(a).

Owing to the groove depth, ball size and high degree of conformity between balls and grooves, this bearing can deal with considerable axial load in addition to radial load.

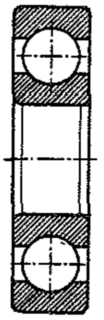


FIG. 7(a)

(2) <u>Self-aligning ball bearings.</u>

Refer to Fig. 7(b).

The self-aligning ball bearing has two rows of balls with a common sphered track in the outer ring. This form of track gives the bearing its self-aligning property and permits automatic adjustment to minor angular displacements to the shaft due to mounting errors. It also prevents the bearing from exerting even the slightest bending influence on the shaft. This bearing will take small axial loads.

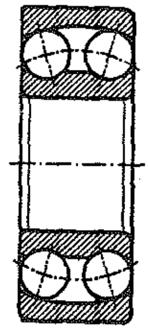


FIG. 7(b)

Used where bending is a significant feature of the shaft design or accurate mounting is difficult. Examples: Shafts with overhung loads, line shafting.

(3) <u>Self-aligning roller bearings.</u>

Refer to Fig. 7(c).

As for self-aligning ball bearings except that spherical rollers replace the balls. Will take heavy axial loads.

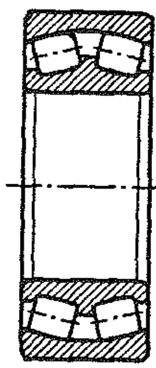


FIG. 7(c)

(4) <u>Single thrust ball bearings.</u>

Refer to Fig. 7(d).

The single thrust ball bearing has one row of balls between two washers. Deals exclusively with axial loads acting in one direction.

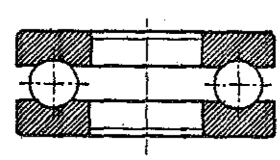
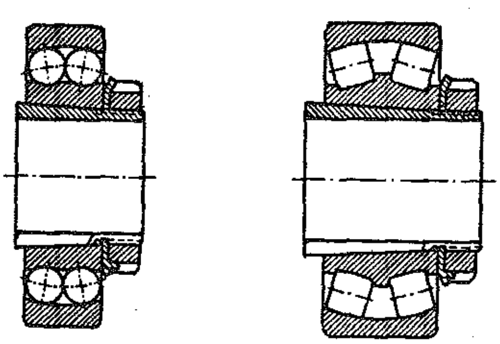


FIG. 7(d)

(5) <u>Adapter bearings.</u>

Refer to Figs. 7(e) and (f).

These are normal self-aligning ball or roller bearings fitted with a tapered split sleeve (adapter sleeve), lock washer and nut. After the assembly is placed in position on a shaft the lock washer is tightened. This forces the bearing up the adapter sleeve taper and clamps the bearing assembly in position.



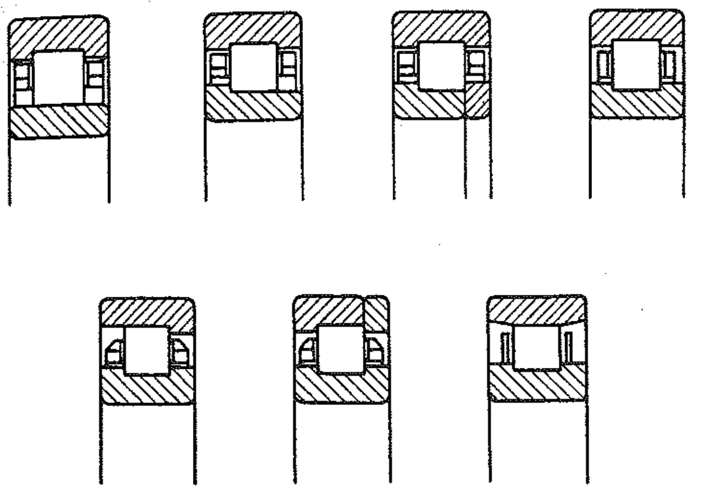
(e) (f)

FIG. 7

Shaft tolerance does not have to be close. This type of bearing is ideal for bright rolled mild steel shafting.

(6) Cylindrical roller bearings.

Refer to Fig. 7(g) for a few of the many different variations of this type of bearing.



The rollers of the cylindrical roller bearing are guided by flanges on one of the bearing rings, the other ring usually having no flanges. This design has the merit of permitting relative axial displacement of shaft and bearing housing within certain limits. Bearings with a flange on the other ring as well can locate the shaft axially, provided that the axial load is not great. Dismantling is easy, even when both bearing rings have a tight fit.
This bearing is suitable for comparatively heavy radial loads and for use at high speeds.

FIG. 7(g)

(7) Single row angular contact ball bearings.

Refer to Fig. 7(h).

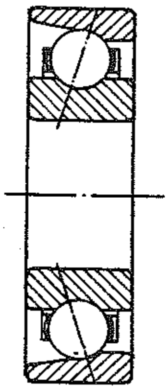


FIG. 7(h)

In the single row angular contact ball bearing the ball tracks are so disposed that a line through the ball contact points forms an acute angle with the bearing axis.
This feature makes the bearing particularly suitable for heavy axial loads. A bearing of this type must always be adjusted towards another bearing capable of dealing with axial forces in the opposite direction. It cannot be separated into its component parts except by the use of force.

This is a more specialised bearing.
The assemblies can be "normal" or in certain cases in machine design "preloaded".

(8) Tapered roller bearings.

Refer to Fig. 7(i).

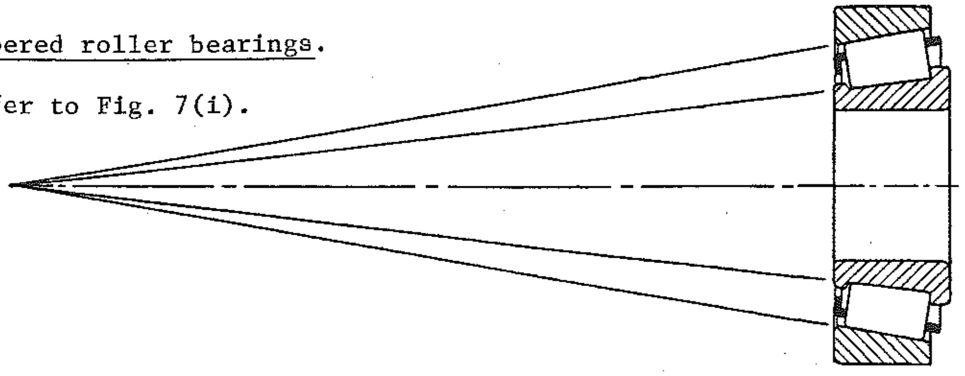
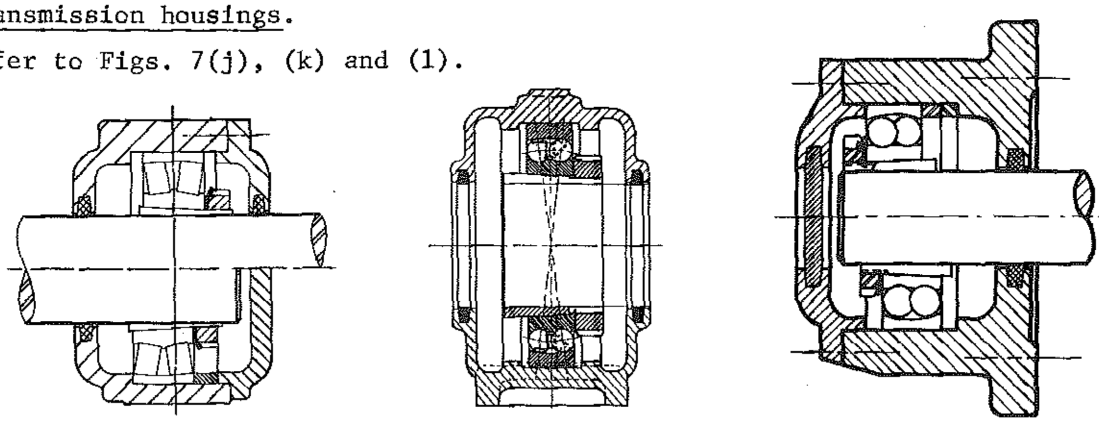


FIG. 7(i)

Since the axes of its rollers and tracks form an angle with the shaft axis, the tapered roller bearing is especially suitable for carrying radial and axial forces acting simultaneously. In cases where the axial forces are very considerable, a series of bearings with a steep taper angle is available. Whichever design is used, the bearing must always be adjusted towards another bearing capable of dealing with axial forces acting in the opposite direction. The taper roller bearing is a separable type; its inner ring with rollers and its outer ring are mounted separately.

(9) Transmission housings.

Refer to Figs. 7(j), (k) and (l).



(j) (k) (l)

FIG. 7

These are a few of the very wide range of split and solid housings (generally of C.I.) available fitted with adapter bearings.

Bearing Assemblies (for bearings shown in Fig. 7(a), (b), and (c), pages 7.6 and 7.7).

When designing bearing assemblies ensure that the bearings cannot be pre-loaded i.e., loaded with forces that are not design loads. Pre-loads can be caused by differential thermal expansion of the shaft and housings or by imperfections in the manufacture of these parts. (Deliberately pre-loaded bearing assemblies are a highly specialised form of design which is only discussed briefly in this book in the section on single row angular contact ball bearings, pages 7.8 and 7.14.)

Two assembly conditions arise for consideration:
(a) rotating shaft. (b) rotating housing.

There are two basic methods of designing a bearing assembly to avoid pre-loading:
(1) The floating bearing method.
(2) The fixed bearings method.

Firstly it must be understood that whichever ring of the bearing rotates, has the tighter fit; the inner ring for a rotating shaft or the outer ring for a rotating housing. This is of importance where method (1) is used. In this method one bearing is fixed on the shaft and in the housing; the second bearing, the non-locating bearing, has only one ring, the one having the tighter fit, axially located, the other ring must be free to move axially in relation to the shaft or housing.

Where the bearings are arranged so that axial location of the shaft is given by each bearing in one direction only, method (2), it is sufficient for the rings to be located at one side only. This arrangement is mainly used for short shafts, with either rotating shaft or rotating housing.

(1) Refer to Fig. 8, page 7.10.

This shows the floating bearing method which is generally used for a rotating

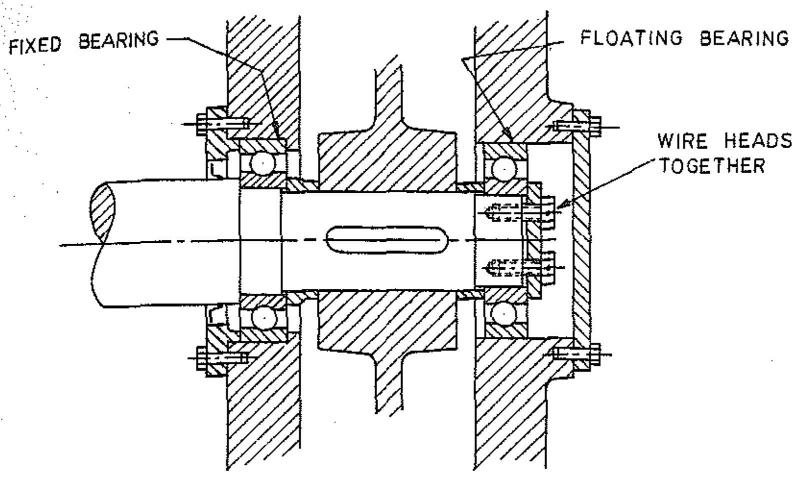


FIG. 8

shaft. The bearing at one end is fixed in position on the shaft and in the housing. At the opposite end the bearing is fixed only on the shaft, the outer race having the ability to "float" axially, thus avoiding any likelihood of pre-loading the bearing assembly.

(2) Refer to Fig. 9, page 7.11.

This shows an alternative bearing assembly which requires a little more care with machining if pre-loading is to be avoided. Typically in this method the parts are made and assembled with the exception that one spacer or the spigot on one cover is left unmachined in length. This part is omitted from the preliminary assembly.

The required length is measured from the partial assembly and then the appropriate part is completed. In actual fact the complete assembly is given approximately 0.05 mm axial float.

Bearing assemblies for bearings shown in Fig. 7(g), (h) and (i), page 7.8.

(1) Cylindrical roller bearings.

Typical bearing mounting No. 1, refer to Fig. 10.

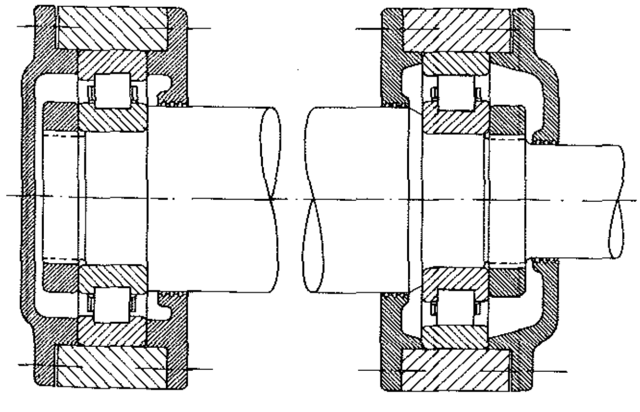


FIG. 10

The shaft shown in this example is the rotating member so the inner rings of the bearings are an interference fit. The outer rings are normally made a transition fit, but a tighter fit may be used if necessary. The shaft is located axially in both directions by the four ribbed roller bearing (these ribs are known alternatively as lips or flanges). It is suitable for heavy radial loading and can also accommodate light, intermittent or reversing, axial loads provided the speed is not high. Limited axial expansion can be accommodated as there is no danger of the two bearings being nipped together axially. The ribs of the locating bearing are backed-up by deep abutments to minimise shear stresses being set up in the ribs. It is important that these deep abutments are flat and square with the axis of rotation, otherwise the ribs could become distorted resulting in premature failure of the bearing. (Cont. on page 7.12.)

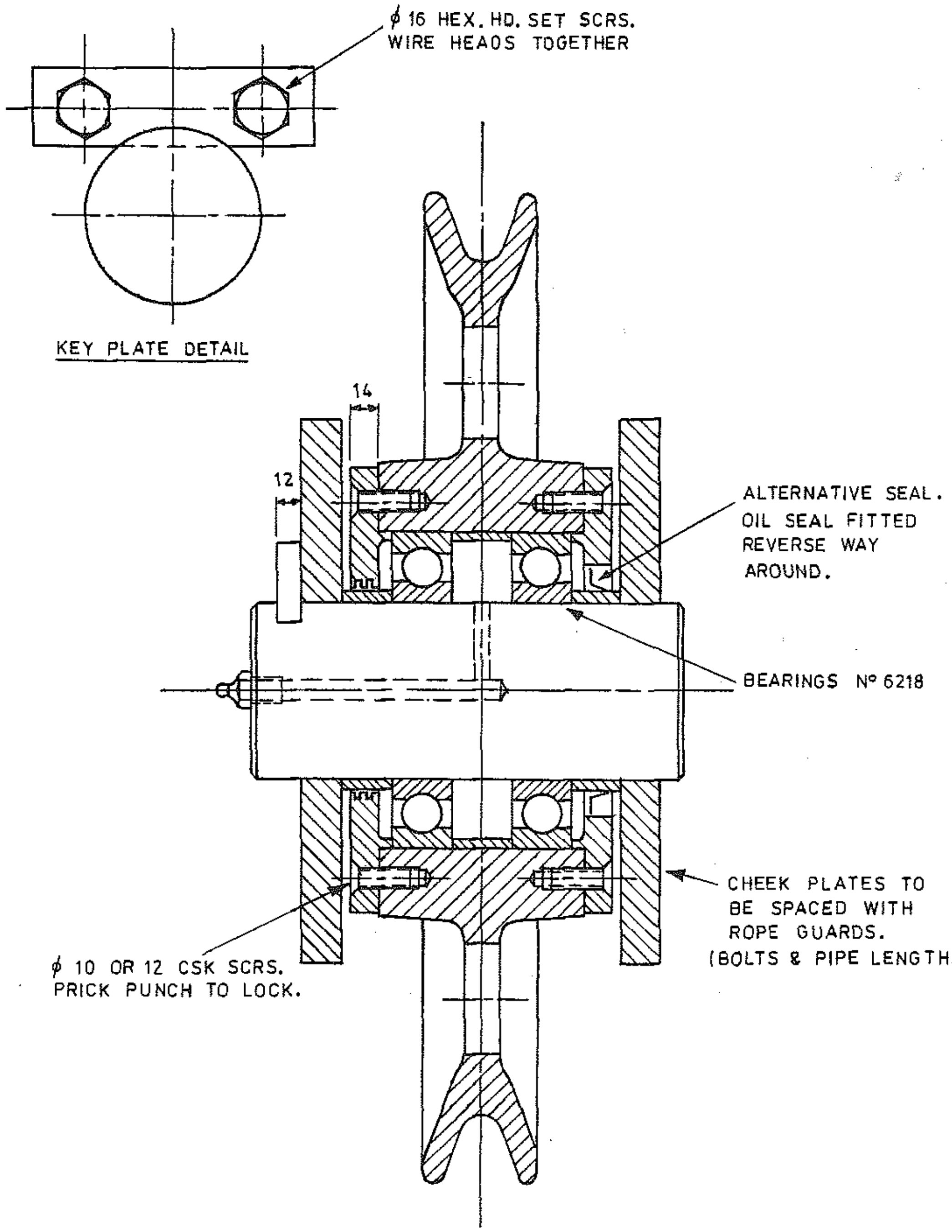


FIG. 9

Fig. 9 shows a typical rope sheave assembly. As it is sometimes customary to arrange several sheaves on a common shaft, the sheaves and bearings should be as compact as possible to give the least pulley block width. Also, in these situations one grease fill lasts for several years.

Typical bearing mounting No. 2, refer to Fig. 11.

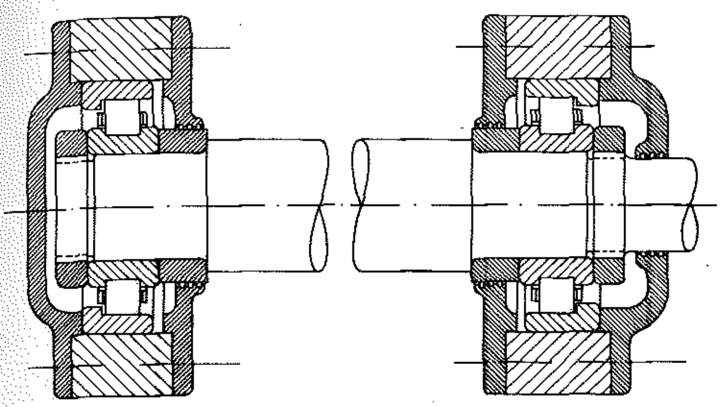


FIG. 11

The above arrangement is one where an abutment can be provided only on one side of a roller bearing outer ring and the roller bearing with double ribbed inner ring, single ribbed outer ring is particularly advantageous. If an outer ring should attempt to move axially then positive restraint is provided in one direction by the abutment and in the other by the rollers. The outer rings are mounted offset to prevent the rollers binding on the ring ribs. Again, the ribs are backed-up by deep abutments to prevent shear stresses being set up in the ribs. It is important that these deep abutments are flat and square with the axis of rotation, otherwise the ribs could become distorted.
As the inner rings rotate they must be made interference fits on the shaft. Normally, the outer rings are interference fits when not clamped endwise as is the case in this arrangement.

(2) Single row angular contact ball bearings.

Typical bearing mounting No. 1, refer to Fig. 12.

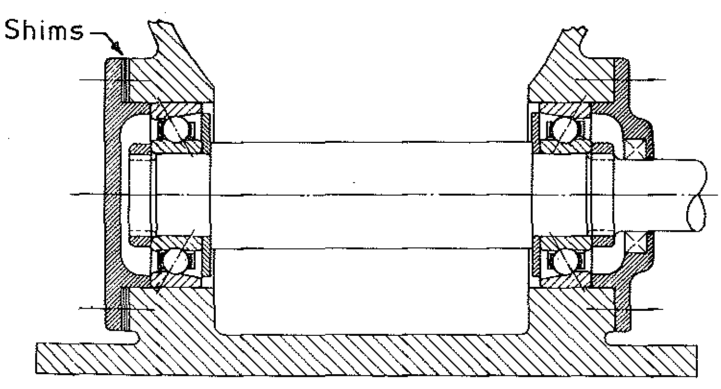


FIG. 12

This is the common method of arranging angular contact bearings when the shaft is the rotating component. They will take axial loads in either direction in addition to carrying radial loads. The open sides of the outer rings face one another and, consequently, the lines joining the points of contact between the balls and the raceways of each bearing converge towards one another as they approach the shaft.

To remove unwanted play from the two bearings and to ensure that the balls are maintained in the correct running position, adjustment is carried out through the outer ring of one bearing by means of shims. The outer rings must be sliding fits in the housings and the inner rings are interference fits on the shaft. A rotating shaft usually runs at a higher temperature than its stationary housing and, as this causes expansion of the shaft relative to the housing, a small end movement should remain in the arrangement after adjustment. If this point is ignored the two bearings would become nipped together as a result of relative expansion. Because of the expansion problem this arrangement is only used when the distance between the bearings is short.

Typical bearing mounting No. 2, refer to Fig. 13, page 7.13.

The bearings in this arrangement face the opposite way to those in Mounting (1); the lines of contact between the balls and their raceways diverge as they

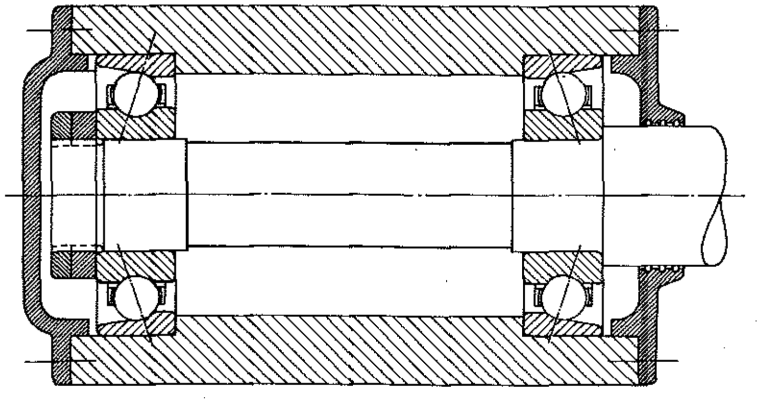


FIG. 13

approach the shaft and give a more rigid mounting than the previous application for moment loading.

Adjustment is carried out through one of the inner rings which must, therefore, be a sliding fit on the shaft. Consequently, this arrangement should only be used with a stationary shaft.

A small end movement should normally be left after adjustment to allow for any possible thermal expansion. The bearing centres are very short and rigidity is the primary requirement. The bearings can be axially pre-loaded providing due care is taken to avoid any overload condition. (Pre-loaded bearings are discussed on page 7.14).

To facilitate assembly of the inner ring of the inner bearing, the length of shaft between the bearings should be reduced marginally in diameter.

Typical bearing mounting No. 3, refer to Fig. 14.

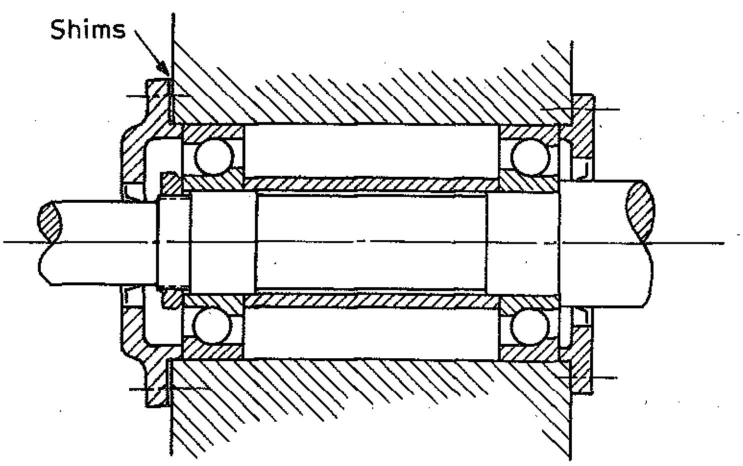


FIG. 14

This is a variation of the arrangement shown in Fig. 12. It is also a favoured method for use with a rotating shaft.

The outer rings would be sliding fits in the housing and the inner rings interference fits on the shaft. The shaft should be reduced marginally in diameter for almost the full length of the spacer. This facilitates assembly of the inner ring of the inner bearing. (A small length of seating should be left for each end of the spacer.) Relief should be machined in the housing also, when this would facilitate bearing assembly.

Preload. This is an initial predetermined internal thrust (axial) load imposed on the bearings to provide both radial and axial rigidity. This is accomplished in various ways depending on the type of application. There are several methods of obtaining the necessary amount of preload. The most important are:

- shims.
- spacers.
- springs.
- manual adjustment.

Normally, the preload should not exceed from one-fifth to one-third of the rated capacity of the bearing, and preferably only slightly above the minimum for satisfactory work.

Preloading is a specialised type of assembly and it is preferable to consult the Bearing Company in individual cases.

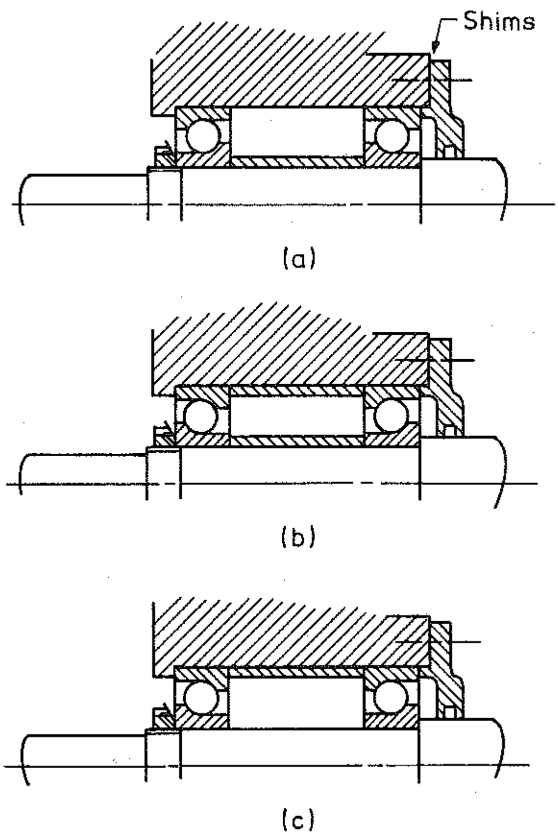


Fig. 15(a)

Preload may be adjusted by the cover shims or spacer length.

Fig. 15(b)

Preload may be adjusted by varying either of the spacer lengths.

Fig. 15(c)

Preload can be varied by nut adjustment.

FIG. 15

(3) Tapered roller bearings

On account of the tapered construction of a Tapered Roller Bearing, an applied radial load sets up a thrust reaction which must be resisted by another bearing. Hence it is not generally practicable to mount Tapered Roller Bearings singly, they must be used in pairs.

There are two fundamental methods of using such a pair of bearings.:

The indirect mounting, in which the small ends of the rollers point inwards, Fig. 16(a).

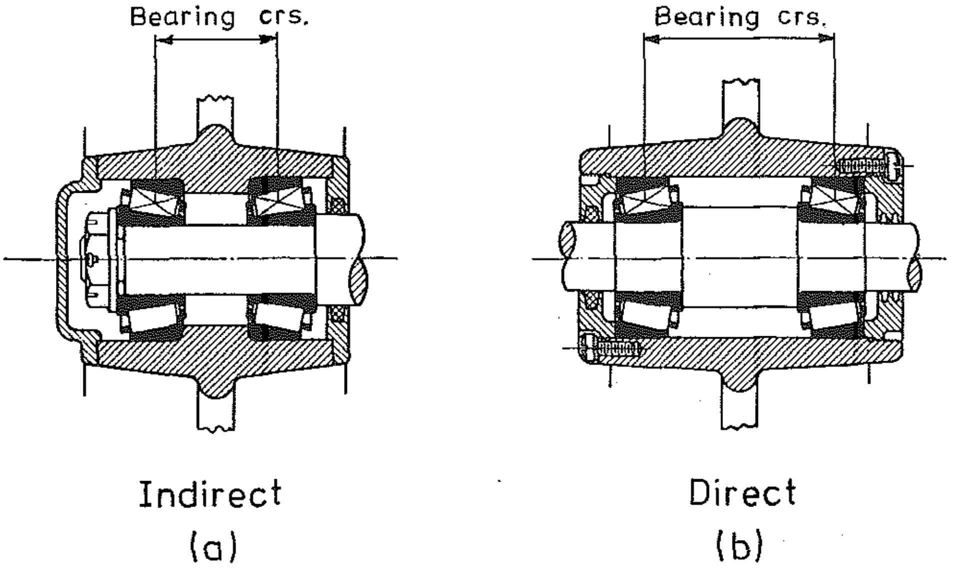


FIG. 16

The direct mounting, in which the small ends of the rollers point outwards, Fig. 16(b).

Indirect mounting

Indirect mounting of Tapered Roller Bearings is usual when space is limited, and it is essential to have the greatest stability of mounting possible, e.g., wheels, etc., mounted on stationary shafts. The spacing of bearing centres should be at least 10 per cent, and preferably 15 per cent to 20 per cent of the diameter of the wheel they carry.

Direct mounting

Direct mounting is used for applications where ample bearing spread can be obtained and where it is desirable to make adjustment by the cups, such as in a high speed application, where the cone must be a press fit upon the shaft.

The increased stability of the indirect over the direct mounting may be better understood from Figs. 17(a) and (b), where, in the direct mounting, the lines of resultant force normal to the cone raceways cut the common axis of the bearings a distance B1 apart. In the indirect mounting the corresponding distance B2 is seen to be much greater, although the distance between the bearings A is the same in each case.

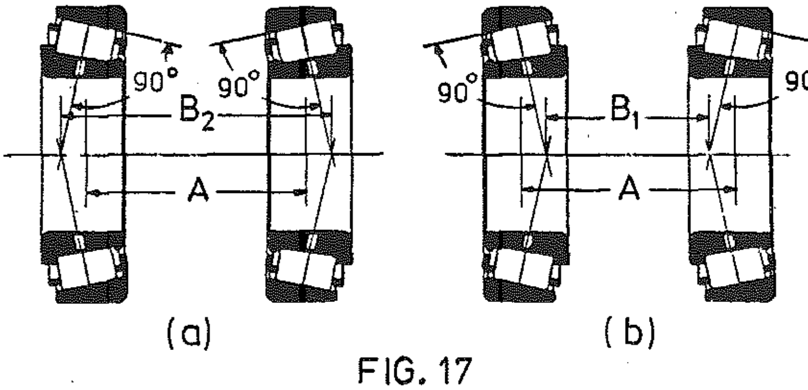


FIG. 17

A typical Tapered Roller Bearing mounting is shown in Fig. 18.

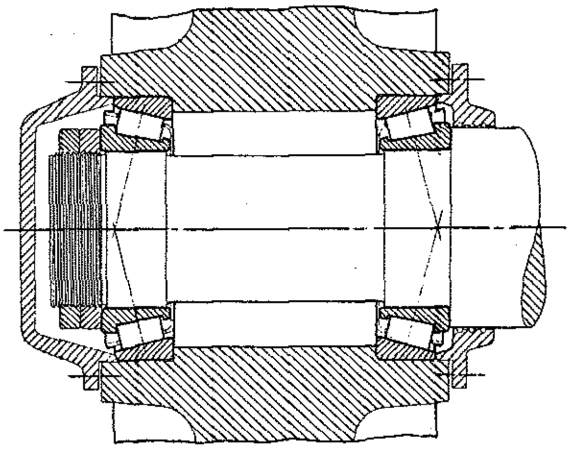


FIG. 18

There are many factors affecting the choice and mounting of Tapered Roller bearings. They are a specialised type of bearing and it is advisable in individual problems to seek the help of the manufacturer. Complete books are devoted to instruction on the use of Tapered Roller Bearings.

Footstep bearing

Refer to Fig. 19.

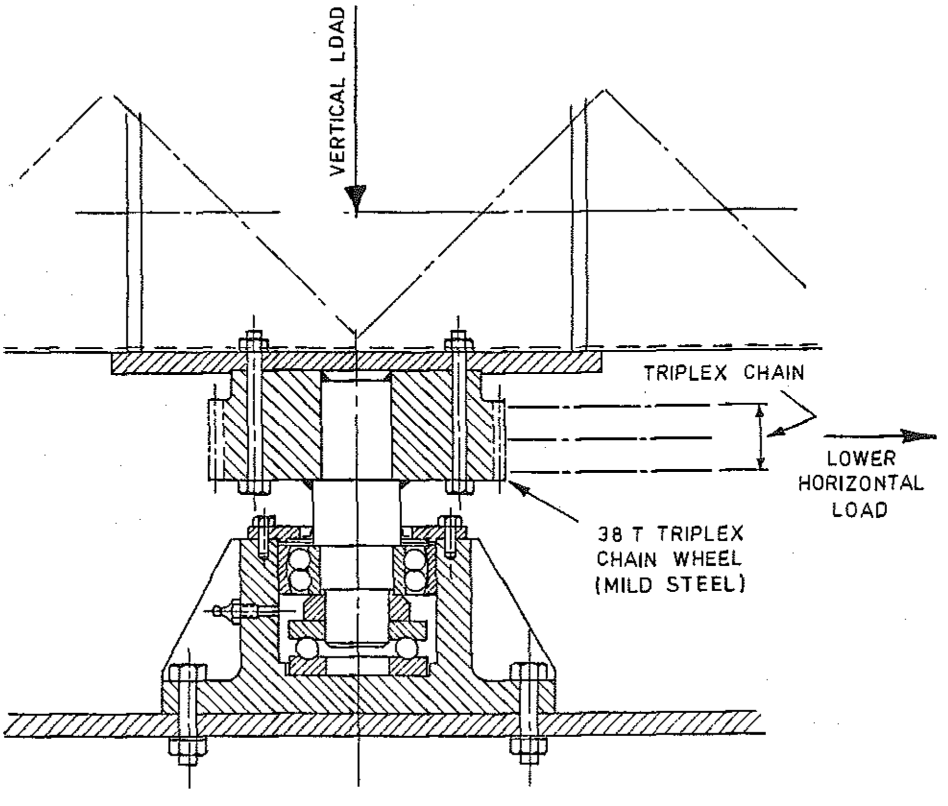


FIG. 19

This is a bearing arrangement which takes the vertical and horizontal loads at the base of a vertical shaft.

Selecting a rolling contact bearing

Machine designers have a large variety of bearing types and sizes at their disposal from which to make a choice. Each of these types have characteristics which make it best for a certain application. Although selection can become a complicated problem the following list could serve as a general guide for most applications:-

(1) Generally, ball bearings are less expensive in the smaller sizes where lighter loads are involved, while roller bearings are less expensive in the larger sizes where heavier loads are involved.

(2) Roller bearings are more satisfactory under shock or impact loading than ball bearings.

(3) To accommodate any misalignment between shaft and housing, a 'self-aligning' type of bearing should be used. Self-aligning bearings are necessary where bending deflection is a significant feature of the shaft design.

(4) Single row thrust ball bearings should be subjected to pure thrust loads only and the shaft speed should be moderately low. To accommodate thrust loading at high shaft speeds, it is better to use a bearing that will take a combination of both types of loads.

(5) Self-aligning ball bearings and cylindrical roller bearings have the lowest friction.

(6) Some bearings (e.g. single row deep groove ball bearings) are available with built-in seals so that the bearings can be pre-lubricated and thus operate for much longer periods of time without requiring attention. They can be purchased as single or double sealed or shielded. High speed should be avoided to prevent generation of excessive heat and subsequent melting of the grease.

Lubrication of rolling contact bearings

Lubricants are used in bearings for two main reasons:-

(1) to reduce friction between the surfaces in contact

(2) to act as coolants and hence help to dissipate any heat which may be generated in the bearing

Lubricants also help to prevent corrosion of the bearing and they act as a barrier against the entry of foreign matter into the bearing.

The bearings may be:-

(a) lubricated during usage by providing lubricant continuously or intermittently

(b) pre-lubricated, so that no additional lubricant has to be provided during a set usage of the bearing

Conventional types of lubricants can be generally classified into three groups:-

(1) Oils.
(2) Greases.
(3) Solid-film luhricants.

Oils

The most important aspect to consider when selecting an oil to lubricate a bearing is the viscosity of the oil, i.e. the ability of the oil to flow and withstand loading over a range of temperatures.

The following types of oil lubrication can be employed according to the bearing set-up and requirements:-

(1) Intermittent oiling - usually carried out by hand, the oil reaching the bearing through an access hole.

(2) Drop feed - oil is released from a reservoir at a predetermined rate through some type of variable valve.

(3) Oil bath - this is a satisfactory method for low and medium speeds. The static level of the oil should not be higher than the centre of the lowest rolling element of the bearing being lubricated (if possible), otherwise overheating of the bearing may result.

(4) Oil splash - the system predominantly used in gearboxes, when the same lubricant is suitable for both gears and bearings. Care should be taken that the "splash" is sufficient to adequately lubricate but not heavy enough to flood the bearing.

(5) Circulation - the system used to lubricate heavy duty bearings. A pump is used to circulate the oil and so ensure a positive supply. Care should be taken to ensure that the oil flows freely and does not become "captive" in any part of the system.

(6) Oil mist - used for bearings which are operating continuously at high speeds. The oil is metered, atomized by compressed air, mixed with filtered air and supplied to the bearings under pressure. It is most important to "wet" the bearings when using this type of lubrication, i.e., commence lubrication of the bearings before the machinery is put into operation.

Advantages of oil lubrication

(1) Easier to drain and refill, which is important if lubricating intervals are close together.

(2) Oil used to lubricate bearings might also be usable in other parts of the machine.

(3) More effective than grease in dissipating heat from the bearing and its housing.

(4) Can be used over a greater range of speeds than grease.

(5) Readily feeds into all parts of the bearing.

(6) Helps to carry away foreign matter which may have entered the bearing.

GREASES

Greases are mostly mineral oils which have been thickened by the addition of some kind of metallic soap. Grease is usually applied in either of the following ways:

(1) Intermittent greasing - grease is pumped into the bearing at regular intervals using a conveniently placed nipple and grease gun. Care should be taken not to "over lubricate".

(2) Prepacking - the bearings are prepacked in grease so that they will last for a predetermined time without needing maintenance.

Some of the advantages of "prepacking" are:-

(i) grease might be harmful to other parts of the machinery.

(ii) space limitations might eliminate the use of a grease filled housing.

(iii) some housings cannot be kept free of contaminating material.

(iv) lubrication could be dangerous or even impossible to perform.

(v) lubrication could be overlooked.

NOTE: In either of the grease lubrication methods mentioned above there is no danger with very slow speed bearings in packing them fully with grease. It is with medium and high speed applications where this should be avoided because of the danger of overheating. In these cases the available space should be no more than one third filled with grease.

Advantages of Grease Lubrication

(1) Does not flow as readily as oil so it is easily contained in a housing.

(2) Leak-proof systems are not necessary.

(3) Less maintenance is required.

(4) Has better sealing qualities than oil and therefore helps to keep foreign matter out of the bearing.

Solid-film Lubricants

These are a relatively modern means of lubricating bearings. They are usually applied as a dry powder and have the advantages that they offer greater resistance to penetration by "rough" surfaces and they shear more easily than the conventional types of lubricants. (An advantage when starting machinery.)

Stringent requirements involving cost, load capacity, temperature resistance, operating environment, corrosiveness, abrasiveness etc. limit the selection of solid-film lubricants to a few types.

The types most commonly used are:-

graphite, molybdenum disulphide ("molybond"),tungsten disulphide, nylon (low loads only)

Advantages of rolling contact bearings

(1) Starting friction is low, (good for intermittent use and low starting temperatures).

(2) Loads can be inclined at almost any angle.

(3) Thrust components can be carried.

(4) Maintenance costs are relatively low.

(5) Bearings are easily replaced when worn.

(6) Less axial space required than for plain bearings.

Disadvantages of rolling contact bearings

(1) More expensive than plain bearings in most cases.

(2) Failure of one rolling element means failure of the complete bearing.

(3) More critical tolerancing required for both shaft and housing.

Sealing of rolling contact bearing assemblies

There are two main reasons for sealing bearing assemblies.:

(1) To keep out dirt, grit, water, chemicals and any other foreign matter.

(2) To keep in the lubricant or alternatively to let excess lubricant (generally grease) escape.
Allowing the excess grease to escape prevents the generation of pressure build-up and excessive heat.

Types of seals.

(1) Rubbing seals.

(2) Non-rubbing seals.

(3) Combination seals.

(1) Rubbing seals.

The two common types would be:

(a) felt
and (b) leather and synthetic rubber.

a). Felt seals must used in combination either with sheet metal pressings, Fig. 20(a) or a deep machined taper sided groove, which the felt must fit snugly and completely fill. These grooves are indicated in their typical form in the housings shown in Fig. 7(j), (k) and (l), page 7.9. Felt seals are only used for grease lubrication.

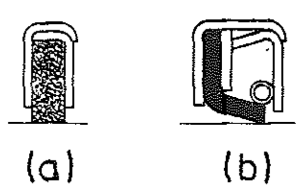


FIG. 20

b). Most leather and synthetic seals are of a proprietary make. Although commonly known as "oil seals" they can be used for either oil or grease applications. They are available as complete assemblies in a metal pressing, with the seal backed up by a spring. Fig. 20(b) shows a typical oil seal. The spring maintains adequate contact force; the seal can follow the non-uniform rotational movement of the shaft to some degree. Fig. 9, page 7.11 shows a simple practical application. The seals can be fitted either way depending on the type of duty required.

Reversed assembly is sometimes used to allow excess grease to escape where there is the likelihood of over-filling of the housing with grease.

A considerable variety of oil seal designs are available. These can be seen in the various manufacturer's catalogues.

Sometimes two seals are used, one facing in each direction. The space between the seals in this case should be filled with grease.

(2) Non-rubbing seals.

Annular grooves would be the most common type.

Other types are less commonly used, such as labyrinths and flingers.

Annular grooves are shown as an alternative application in Fig. 9, page 7.11. Three grooves are an ideal number to use where space is available. When these grooves are used with grease, the grooves pack hard with grease and make a close seal.

Groove size is generally about 1.5 to 3 mm square. Some designers prefer grooves that taper in towards the top. Shaft clearance where shaft bending is not a major problem should be about 0.8 mm on diameter for shafts up to 50 mm diameter. For larger shafts, diametral clearances of 1.3 mm have been successfully used.

(3) Combination seals.

These become rather specialised.
They are used sometimes in extremely dirty conditions. Various combinations are adopted by designers and the utilisation of the rubbing and non-rubbing principles in conjunction proves very effective.

The authors wish to acknowledge the assistance given in the preparation of the foregoing section to the following bearing companies:

Bearing Service Pty. Ltd.
R H P Bearings Australia Ltd.
S K F Australia (Sales) Pty. Ltd.

DESIGN OF BEARINGS

Plain Bearings

Glossary of Terms

N = speed of rotation in r.p.m.

p = bearing pressure on projected area, MPa

W = load supported by bearing, N

Q = reaction opposing W, N

L = bearing length, mm

d = diameter of journal, mm

r = radius of journal, mm

V = rubbing speed of journal, m/min.

T = torque required to overcome bearing friction, N.mm

ϕ = angle of sliding friction, degrees

μ = coefficient of bearing friction = $\tan\phi$

C_1 = a constant, see Table No. 3, page 7.25.

C_2 = a constant, see Table No. 4, page 7.25.

D_1 = outside diameter of a thrust bearing, mm

D_2 = inside diameter of a thrust bearing, mm

The design of plain journal bearings depends on two main conditions:-

(1) Rubbing speed

(2) Pressure on projected area. The formulas applicable are:-

$$V = \frac{\pi d N}{1000}$$

$$p = \frac{W}{L \times d}$$

Permissible pressures (stresses) (adapted from the CB.2 Crane and Hoist Code)

Table No. 2, page 7.23 gives permissible pressures between mild steel shafts and phosphor bronze or gunmetal bushes. The pressures assume a thin film of oil or grease exists at all times.

Temperature of operation is to be tolerably cool.

Firstly reference must be made to Table No. 1. to determine a mechanism class before establishing a permissible pressure from Table No. 2.

Conditions where bearing failure does not cause immediate danger to personnel allow twice the pressures listed in Table No. 2,(page 7.23).

For slow oscillatory movements permissible pressures are three times those listed in Table. No. 2, (page 7.23).
Table No. 2 may also be used for mild steel shafting on grey cast iron.

TABLE NO. 1. (Tables No. 1 and No. 2 and Rule 5.5.4 (above) are reproduced from the SAA Crane and Hoist Code CB.2 - 1960 (now AS 1418) by permission of the Standards Association of Australia.)

MECHANISM CLASS	RUNNING TIME PER DAY - HOURS	LIFE - HOURS
1	½	2500
2	3/4	3750
3	1½	7500
4	not less than 3	not less than 15 000

TABLE NO. 2.

SPEED OF RUBBING	BEARING PRESSURE IN MPa											
	CLASS 1				CLASS 2 and 3				CLASS 4			
	Bearing diam. mm				Bearing diam. mm				Bearing diam. mm			
m/min	25	50	100	150	25	50	100	150	25	50	100	150
0 - 8	4.6	6.2	9.3	12.4	3.1	4.1	6.2	8.3	2.3	3.1	4.6	6.2
8.1 - 24	4.3	5.6	7.7	9.9	3.0	3.7	5.1	6.6	2.1	2.8	3.8	5.0
24.1 - 60	3.5	4.0	4.8	5.4	2.3	2.7	3.2	3.6	1.8	2.0	2.4	2.7
60.1 - 120	2.5	2.7	2.9	3.2	1.7	1.8	1.9	2.1	1.2	1.3	1.4	1.6
Over 120	1.5	1.7	1.9	2.1	1.0	1.1	1.2	1.4	0.76	0.83	0.97	1.0

Phosphor bronze and gunmetal bushes are used where very high loads are encountered.

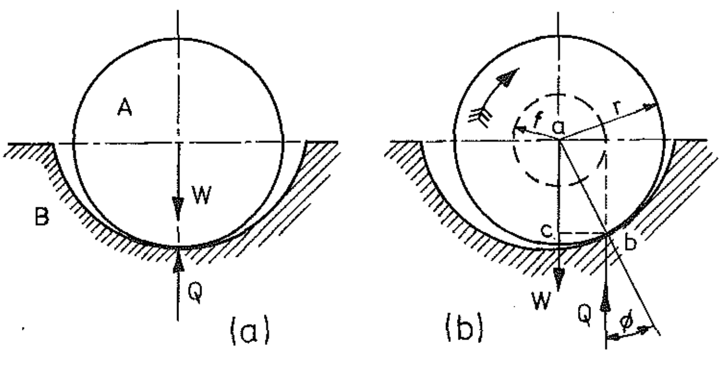
In less critical situations (including hand power operation) unbushed grey cast iron on mild steel shafting is satisfactory. The free graphite in grey cast iron acts as a lubricant and makes it wear resistant.

White "chilled" cast iron bushes are used on case hardened steel shafts in cases where lubrication is either undesirable or impossible. Their use is generally limited to slow speed hanger or steady bearings.

Permissible bearing pressures on non-metallic bushes (e.g., teflon, nylon and ferobestos etc, etc.) may be ascertained from the manufacturer's catalogue.

Friction circle for a journal

Refer to Fig. 21(a). This shows a journal A resting within a loosely fitting bearing B. When the journal is at rest, the journal load W is balanced by reaction Q. Supposing a couple of moment T is then applied to the journal and steady rotation occurs as shown in Fig. 21(b). The applied torque will cause the journal to roll up the bearing to point b after which steady slipping will occur.



Friction of a loose bearing

FIG. 21

The position of b is determined by the fact that force Q at b must make an angle ϕ with the normal ab. ϕ is the angle of sliding friction.

As W and Q are equal, a couple W x bc occurs, which is balanced by the torque T.

Hence,

$$T = W \times bc$$

$$\text{Also, } \frac{bc}{ac} = \tan\phi \cong \frac{bc}{ab}$$

$$\therefore \; bc = ab \tan\phi = r \tan\phi$$

$$\therefore \; T = Wr \tan\phi$$

$$\text{or } T = Wr\,\mu$$

T will be in N.mm if r is in mm and W is in N. It can be seen that Q is tangential to a circle of radius f, drawn around centre a.
This circle, whose radius is equal to bc, is known as the friction circle.

Hence,

$$\text{Radius of friction circle} = f = r \tan\phi \text{ very nearly}$$
$$= \mu\, r \text{ mm},$$

where $\mu = \tan\phi$, is the coefficient of friction

r = radius of journal in mm.

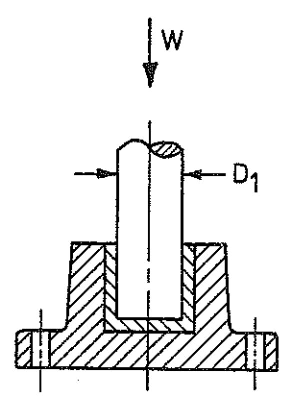


FIG. 22

Flat Pivot Bearing (Also called a footstep or step bearing.)

Refer to Fig. 22

$$p = \frac{4\,W}{\pi\,D_1^2}$$

It is probable that the value of the torque of friction T of the thrust surface lies between

$1/3\ \mu W\ D_1$ (assumption of uniform bearing pressure)

and $1/4\ \mu W\ D_1$ (assumption of uniform wear)

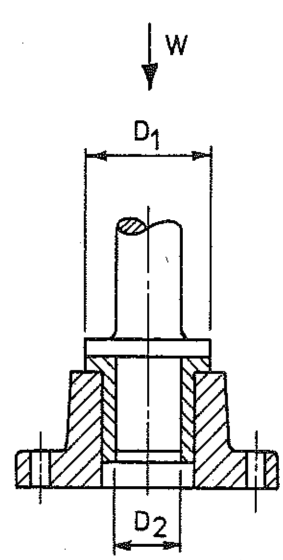


FIG. 23

Collar Bearing

Refer to Fig. 23.

$$p = \frac{4\,W}{\pi(D_1^2 - D_2^2)}$$

In the case of a collar, no great error will be made when calculating the torque of friction by assuming that the resultant force acts at the mean radius:

$$T = 1/4\ \mu W\ (D_1 + D_2)$$

Estimation of coefficient of friction

The majority of plain bearings come within the classification of "imperfect lubrication", i.e. there is some breakdown of the film of lubrication resulting in partial bearing surface contact.

The following formula enables an estimate of μ to be made under this condition:

$$\mu = \frac{C_1 \; C_2}{97} \sqrt[4]{\frac{P}{V}}$$

Louis Illmer,
High Pressure Lubrication,
Trans. ASME, 1924, p.833.

The coefficient of friction for a reasonably lubricated journal is conservatively 0.08 to 0.14 while if the journal is perfectly dry the coefficient may range from 0.25 to 0.40.

The value of μ is for all practical purposes independent of the materials in contact for lubricated bearings. Some material combinations however have better bearing characteristics than others.

Experiments by Tower and Molesworth indicate average values for μ of 0.04 to 0.06 for well lubricated journal and thrust bearings.

TABLE NO. 3

TYPE OF LUBRICATION	WORKMANSHIP	ATTENDANCE	LOCATION	C_1
Bath or oil ring	Excellent	Good	Clean and Protected	1
Oil by drip feed	Good	Reasonable	Normal	2
Occasionally —— oil cup or grease nipple	Fair	Poor	Unfavourable conditions	4-6

TABLE NO. 4

EXAMPLE OF BEARING	C_2
Rigid rotating and oscillating journals	1
Journals lacking rigidity	2
Rotating flat surfaces, collar and footstep bearings	2
Sliding flat surfaces, slipper bearings	2-3
Worm type gears	3-4
Power screws	4-6

Rolling contact bearings

These are chosen by following the instructions given in the manufacturers' catalogues.

The following design data is required to enable a selection to be made:-

Design load on bearing. Refer to Worked Example No. 1, page 1.28.

Speed of shaft or housing in r.p.m.

Life required of bearing in hours

In most cases two load values are applicable to a bearing selection:-

(1) Dynamic load

(2) Static load

The exception is where the speed of rotation is less than about 20 r.p.m., then the bearing is chosen on static load only.
In other words all applied loads are taken as static loads. Bearing manufacturers recommend that the loads should be multiplied by a duty factor (generally about 1.35) before a bearing selection is made (in the case of selection on static load only). For spherical roller thrust bearings use duty factor $\geqq 2.0$ or 1.25 if the rings are fully supported.

Worked Example

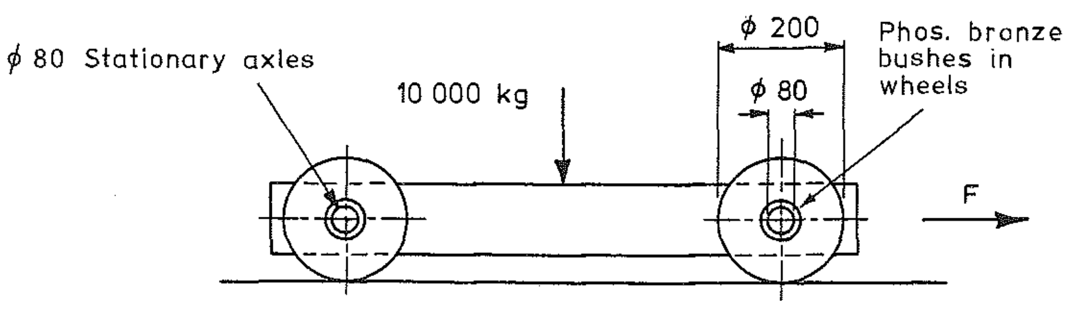


FIG. 24

Fig. 24 shows a trolley used for transporting steel bars. Determine the force "F" required to haul the trolley. The stationary axles are made of mild steel.
The wheels are fitted with grease lubricated phosphor bronze bushes 120 mm long.
Hauling speed = 15 m/min.

Answer

$$\text{Weight on each wheel} = \frac{10\ 000\ g}{4} = 24\ 525\ \text{N}$$

$$p = \frac{W}{L \times d} = \frac{24\ 525}{120 \times 80} = 2.56\ \text{MPa}$$

$$\text{R.P.M. of wheel} = \frac{15}{\pi \times 0.2} = 23.87$$

$$V = \pi d N = \pi \times 0.08 \times 23.87 = 6\ \text{m/min.}$$

$$\mu = \frac{C_1\ C_2}{97} \sqrt[4]{\frac{p}{V}} = \frac{6 \times 1}{97} \sqrt[4]{\frac{2.56}{6}} = 0.05$$

In practice μ is commonly taken as 0.08 to 0.14, allow 0.1.

Torque offered by each bearing, $T = W\,r\,\mu$

$= 24\ 525 \times 40 \times 0.1 = 98\ 100$ N.mm

For 4 wheels bearing torque $= 98\ 100 \times 4 = 392\ 400$ N.mm

Torque to overcome bearing friction

$= F \times$ radius of wheel

$392\ 400 = F \times 100$

$F = 3924$ N (Refer Fig. 25.)

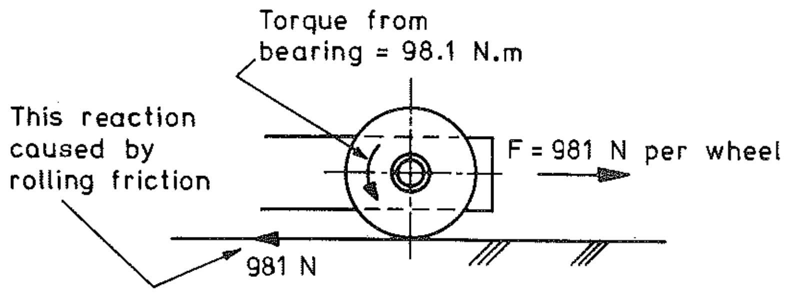


FIG. 25

Exercises on Chapter 7.

(1) Refer to Fig. 24, page 7.26.

If the load is increased to 11 000 kg and the force F is applied 300 mm above the wheel centre line find:-

(a) The value of F if μ is taken as 0.10.

(b) The vertical reaction acting on each wheel. Wheel centres = 1400 mm.

(c) The theoretical value of μ for the most heavily loaded bearing.

(a) 4316.4 N (b) Front 27 440 N (c) 0.0514
Rear 26 515 N

(2) The trolley in Exercise (1) operates for 1½ hours/day and a 6000 hours life is expected. Bearing failure will not endanger personnel. Determine the maximum permissible bearing pressure and compare this with the actual bearing pressure.

Approx. 10.72 MPa by interpolation.
2.86 MPa

(3) In a machine for raising a load of 230 kg the load is suspended from a rope wound round a drum A, 200 mm diameter, to the rope centres (Fig. 26, page 7.28). The axle on which the drum is fixed has journals 38 mm in diameter, and is rotated by a toothed wheel B, 460 mm diameter, to which a force P is applied. $\mu = 0.10$.

Determine:

(a) the mechanical advantage of the machine.

(b) the machine efficiency.

Ans. (a) 2.239
(b) 97.325%

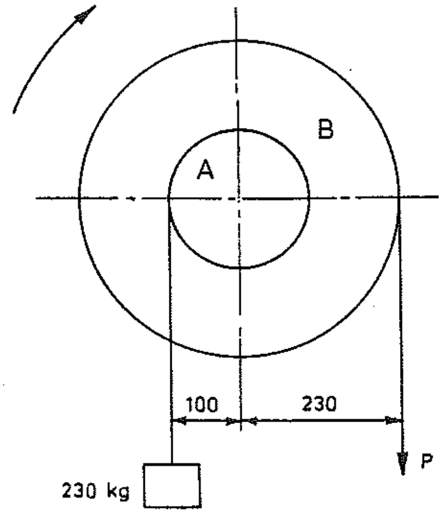


FIG. 26

(4) A vertical shaft is supported on a flat pivot bearing 50 mm in diameter and carries a load of 70 kg.

The shaft revolves at 300 r.p.m.

Take $\mu = 0.04$.

Calculate the frictional torque and power absorbed by the bearing:

(a) assuming uniform bearing pressure.

(b) assuming uniform wear.

Ans. (a) 457.8 N.mm, 0.0144 kW (b) 343.35 N.mm, 0.0108 kW

(5) Refer to the trolley shown in Fig. 27.

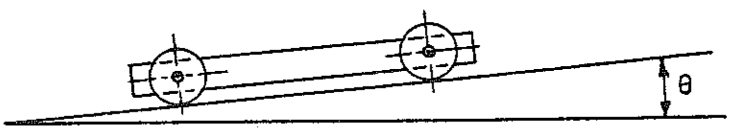


FIG. 27

The wheels and axles are 250 mm and 70 mm in diameter respectively. The axles revolve in plain bearings attached to the trolley frame. $\mu = 0.14$.

The total mass of the trolley and load = 12 tonne.

Determine the minimum angle θ which will allow the loaded trolley to roll down the plane unassisted.

Ans. $\theta = 2.24°$

(6) Refer to Exercise No. 5.

(a) What force acting parallel to the plane is necessary to haul the loaded trolley up a 5° incline?

(b) If two falls of rope are used in the winching system calculate the rope tension.

(c) If the hauling speed parallel to the plane is 25 m/min. determine the size of winch motor required.

Drive system efficiency is 89%.

Ans. (a) 14 857 N (b) 7428.5 N (c) 6.96 kW

(7) A 60 mm diameter shaft runs in a phos. bronze bush 65 mm long.

Speed of rotation = 22 rev/min.

Applied load = 20 000 N.

Running time = $1\frac{1}{4}$ hours/day and life expectancy = 7000 hours.

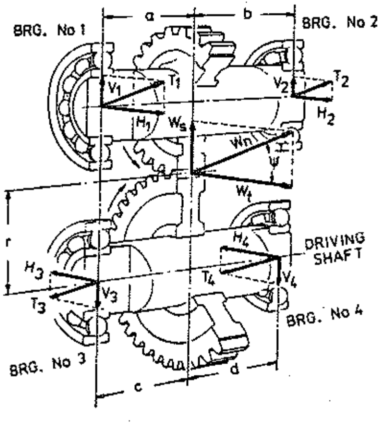
Bearing failure represents danger to personnel.

Determine if the design is satisfactory.

Ans. $p = 5.13$ MPa
p (permiss.) $= 4.52$ MPa
(by interpolation)

CHAPTER 8

COUPLINGS



CHAPTER 8

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COUPLINGS

Couplings are used to join two co-axial shafts in such a manner, that both shafts revolve together permanently. If exact alignment of both shafts is essential, rigid couplings are used; however, if this is not necessary it is more convenient to use flexible couplings which allow for a certain amount of eccentricity, and sometimes for slight angular misalignment or a combination of both. See Fig. 1.

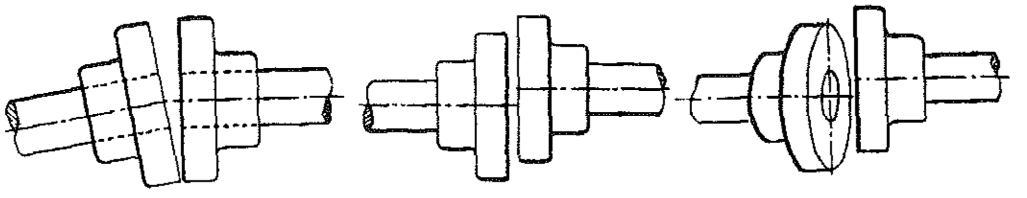


FIG. 1

Shaft couplings serve several purposes in machinery. These are:-

(1) To provide a disconnection point to enable repairs and alterations to be done.

(2) To allow for shaft misalignment.

(3) To reduce or eliminate transference of shock from one unit to another.

(4) To give protection against overloads.

Not all couplings have each of these characteristics.
Couplings have to be chosen on the type of duty required.

Some of the couplings commonly used in industry are described in the following articles.

(a) Rigid Couplings

This type of coupling has no flexibility. The shafts must be very accurately aligned to prevent unnecessary loads on the coupling, shafts and bearings.

Rigid couplings are sometimes used to make up a long transmission shaft consisting of two or more parts. This is necessary sometimes because shafts are only manufactured in certain lengths. (6 m to 10 m length).

The rigid couplings mostly used are:

the flanged coupling, Fig. 2.
the shell coupling (or muff coupling), Fig. 3, and the Sellers or compression coupling, Fig. 4.

The Flanged Coupling is very simple and inexpensive; however, the bolts holding the two halves together must be fitted to ensure accurate alignment of the bores in both halves. This can often be more easily achieved by turning a recess in one half, concentric with the bore to take a spigot on the second half, which is a push fit in the recess and also concentric with the bore. If this is done, the bolt holes can be more easily lined up. In order to ensure safety while in rotation, the nuts and bolt heads are not allowed to project.

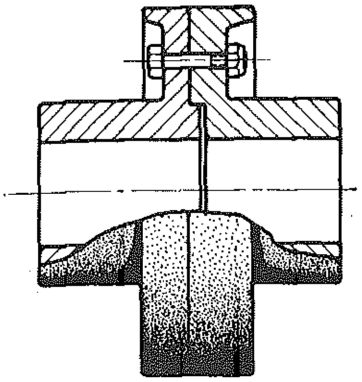
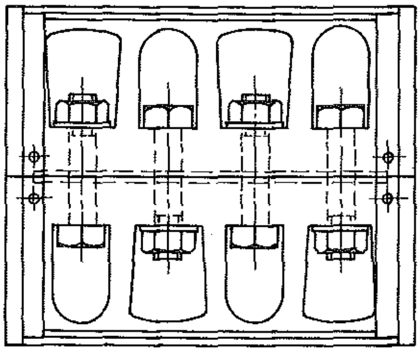


FIG. 2



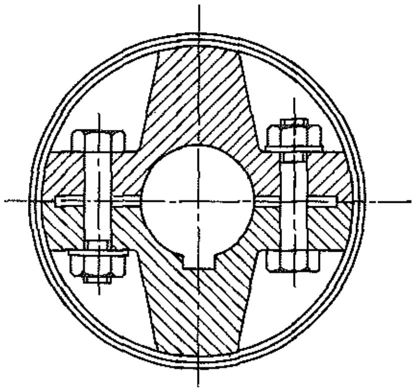


FIG. 3

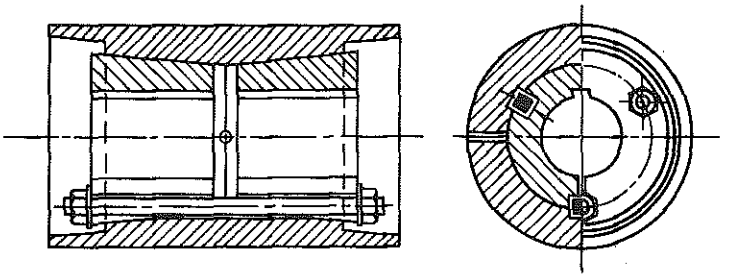


FIG. 4

The Shell Coupling is made in two halves, the plane of splitting being parallel to the axis and the two halves are bolted together as shown in Fig. 3. It is smaller in diameter than a corresponding flanged coupling. A parallel keyway and key are used to transmit the torque. Here the bolt heads and nuts are also in a cavity to avoid any accident which may occur during rotation due to projecting parts. The shell coupling, as well as the Sellers coupling, is used when the large flange of the flanged coupling cannot be accommodated.

The Sellers or Compression coupling consists of an outer shell which has two opposite conical bores into which two conical shells fit. Each conical shell is made flexible by splitting and the bores which accommodate the shafts on these shells are concentric with the outer conical surfaces. Three square bolts are used to draw the shells in the outer sleeve thus pressing them on the shaft ends. Sometimes a hole is drilled in the centre of the outer sleeve. This hole is used to observe the shaft ends which must be in the centre of the coupling when tightened. As can be seen, the nuts at both ends of the bolts are in recesses provided in the outer shell for protection and safety. (Fig. 4.)

(b) Flexible Couplings

These couplings depend upon some type of resilient element being interposed between the coupling halves to give them their flexibility.

This flexibility allows for lateral and angular misalignment of the shafts and reduces transmission of shock and vibration.

Some specific types of flexible couplings are now discussed in the following articles.

Rubber Bush Coupling

Refer to Fig. 5. This type of coupling consists of two halves, one half being fitted with rigid pins. These pins fit into steel bushed rubber bushes or sometimes a series of rubber rings in the other half.

Thus, small amounts of shaft misalignment can be accommodated.

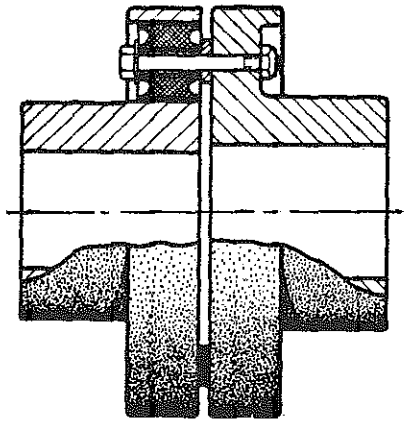


FIG. 5

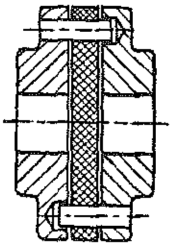


FIG. 6

Flexible disc coupling

Refer to Fig. 6. Pins are fitted to each half of the coupling. These pins engage a rubber impregnated fabric, flexible disc in between the coupling halves.

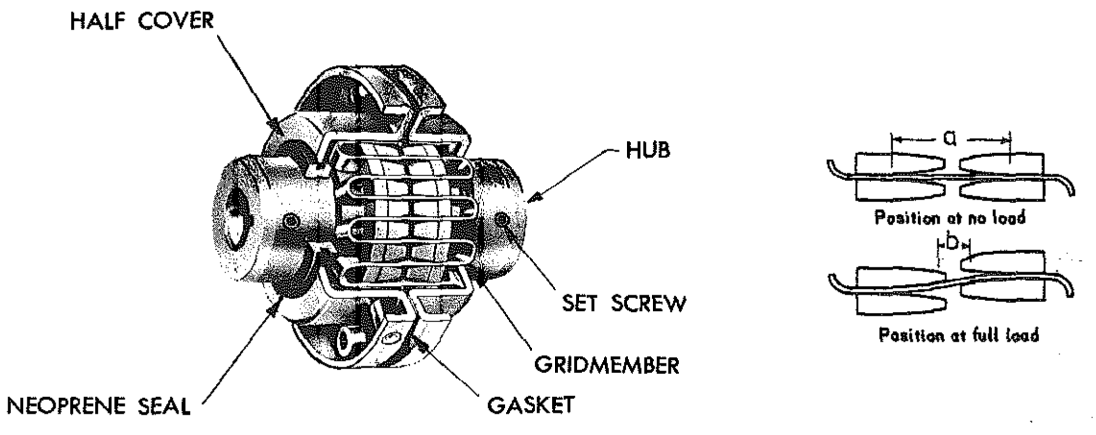


FIG. 7

Falk coupling

Refer to Fig. 7. Each half of this coupling has a number of slots. A continuous spring lies in these slots and so connects the coupling halves and gives them their flexible characteristic.

The sides of the slots are curved as shown in the detail. This causes the effective length of each driving element to change from "a" at no load to "b" at full load. This means that as the torque increases the coupling becomes torsionally stiffer.

Bonded couplings

Refer to Fig. 8. This type of coupling consists of two steel halves bonded together with rubber of 40 to 65 Shore hardness.

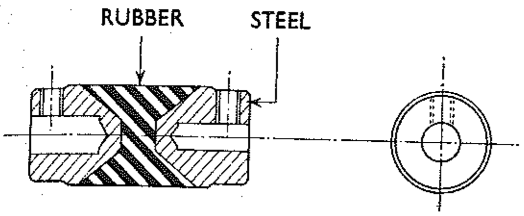


FIG. 8

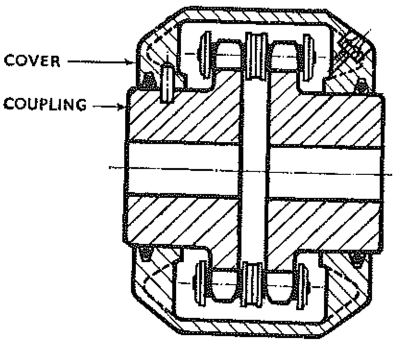


FIG. 9

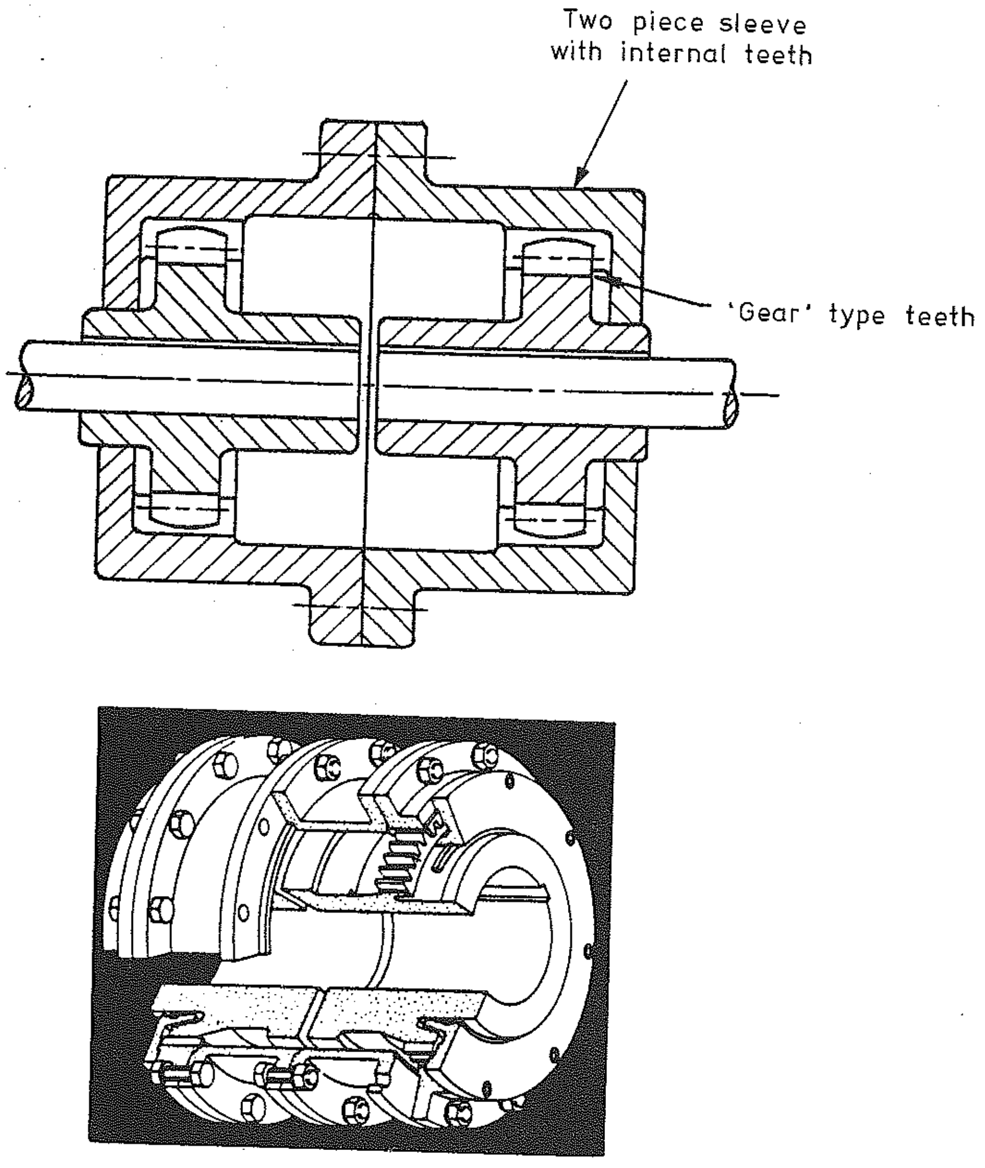


FIG. 10

Chain couplings

Refer to Fig. 9. This type of coupling is constructed of two sprockets mounted face to face connected with a length of roller chain wrapped around the teeth as shown. The enclosing case is grease filled.

Gearflex couplings

Refer to Fig. 10. This coupling is of the double engagement type and of all steel construction. A splined hub is fitted to each shaft end and a sleeve cut with internal splines, engages the splines of the hubs.

The engaging teeth are near the perimeter and widely spaced so that the error between the splines is only a small fraction of the corresponding error of alignment between the shafts. The coupling is suitably filled with oil and under the centrifugal action of its rotation forces the lubricant outwards to provide a film which acts as a cushion between the engaging surfaces of the teeth. In slow speed or reversing work a heavy lubricant is used and when the direction of rotation is changed the pockets of oil between the teeth act as a dashpot, cushioning the action at the moment of reversal.

The outer sleeve is held in position by bearing rings making a seal on the shaft hubs, thus retaining the oil, while, by the correct placing of the sealing point, permitting the necessary slight rocking action without the need for non-metallic sealing rings.

Application

Gearflex couplings are suitable for the heaviest of duties at low or high speeds and especially where a high factor of safety against shock loads is required as in steel mill practice. Moreover, as it is all metal it is suitable for high operating temperatures as met with in refineries and chemical works. It is not, however, suitable where torsional flexibility is a major requirement.

(c) Couplings with kinematic flexibility

Kinematics is the study of relative motion between parts of a machine. Two types of coupling which allow this action are now discussed.

Universal coupling (Hooke's coupling)

Refer to Fig 11.

This coupling is used to join two intersecting shafts. Angles of up to 30° or more may be accommodated but efficiency drops off noticeably, refer to the graph Fig. 12.

The angular velocity ratio for two shafts connected by one universal coupling is not constant. This condition can be corrected by using two couplings correctly mounted. The driving and driven shafts should be at the same angle with the intermediate shaft and the fork members on each end of the latter should be in the same plane.

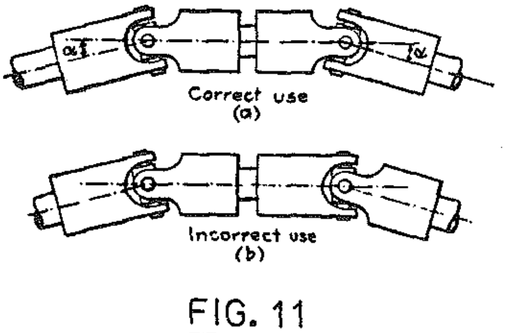


FIG. 11

EFFICIENCY

The efficiency of a needle bearing universal joint is shown in the graph below and compares joint angle and efficiency. (Compliments of Repco Universal Joints Co.)

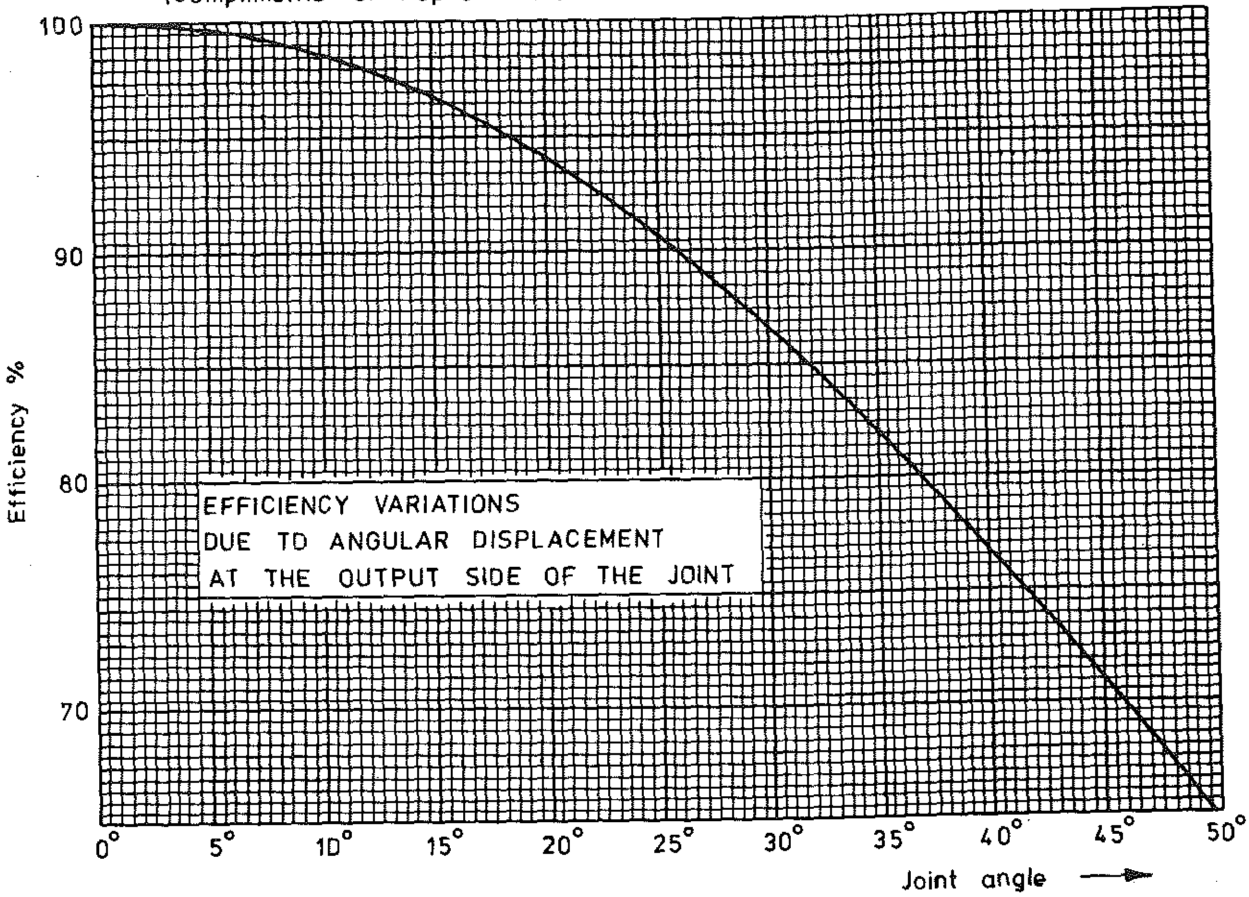


FIG. 12

Oldham coupling

Refer to Fig. 13.

Lateral misalignment of two shafts may be accommodated by the use of this coupling. The coupling consists of three parts, two hubs and a central floating piece. The central piece has two tongues at 90°, one on each face. These match slots in the hub pieces. As drawn, tongue "x" allows relative lateral movement while tongue "y" allows relative vertical movement of the parts. The net result of these components of motion allows lateral misalignment of the shafts as they rotate.

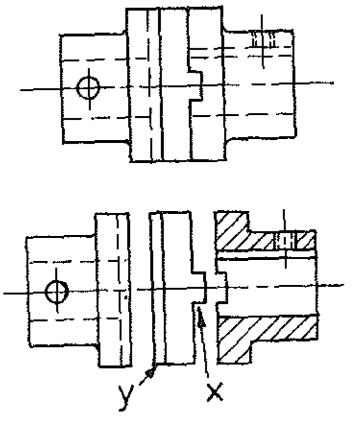


FIG. 13

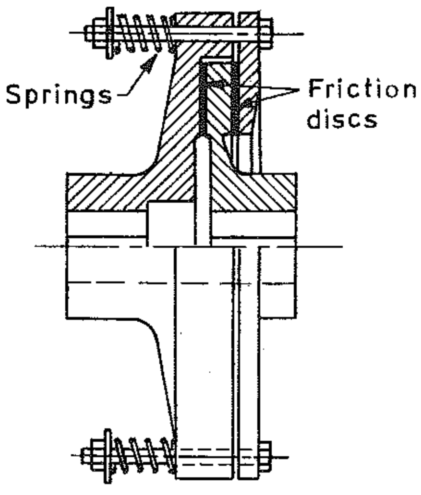


FIG. 14

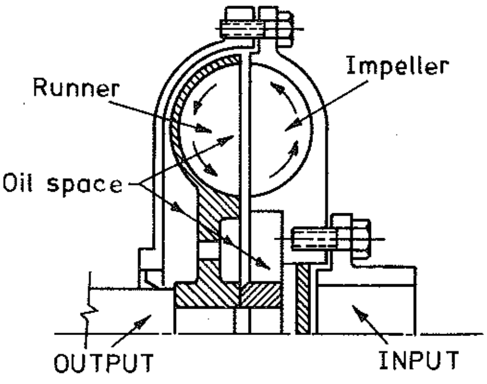


FIG. 15

(d) Slip couplings

Several forms of this type are available. Essentially they permit relative rotation i.e., slip between the coupling halves at a predetermined torque. This prevents overloading of machinery parts and the likelihood of subsequent damage.

An elementary type of slip coupling is shown in Fig. 14.

By adjusting the spring tensions the pressure on the friction discs is modified, thus adjusting the "limit torque" of the coupling.

(e) Shear pin couplings

These are basically one of the rigid or flexible couplings modified and fitted with a carbon steel pin which will shear at a pre-determined torque.

(f) Fluid coupling

Refer to Fig. 15.

A fluid coupling is a device for connecting a driving shaft to a driven shaft; the torque on each shaft is the same, but the speed of the driven shaft is slower by the amount of slippage.

The fluid coupling (Fig. 15.) consists of an impeller, which is the driving member, and a runner, which is the driven member. They are enclosed by a housing, which may be a part of the impeller, and are filled with a hydraulic fluid. A series of radial blades, usually straight, is mounted in both the impeller and runner. When the impeller is rotated, centrifugal force directs the fluid outward. At the same time, it is carried across the space and its momentum delivered to the blades of the runner, causing the runner to rotate.

The output torque of these couplings is always equal to full input torque. This is their great advantage. Every internal combustion engine and electric motor has some speed at which it develops more torque than at any other.

With fluid couplings, the prime mover can turn at that most effective speed, regardless of the speed of the output shaft, delivering maximum torque even if the load is at a complete standstill. With these couplings, an engine cannot be stalled by application of load, however great, although the coupling output shaft itself may stall. An electric motor, hydraulically coupled to a stalled output shaft will drop back to maximum torque speed, pulling sufficient excess current to enable the overload safely to cut out before the coupling gets too hot. The coupling

(f) Fluid coupling (cont.)

delivers the maximum torque of which the prime mover is capable, to overcome a momentary demand which would otherwise stop the power source completely.

The cushion effect of the fluid coupling protects the prime mover and driven mechanism because the flexibility of the coupling prevents transmission of torsional vibration and shock loads through gears, belts or chains. When a hydraulic coupling is incorporated in any drive it breaks the solid connection between the power source and the driven mechanism and provides a point at which power is transmitted entirely as "kinetic energy", which is the mass and velocity of moving oil. Incoming power rotates the impeller, the multiple blades of which throw a continuous stream of oil against the blades of the runner causing the latter to revolve and thus deliver power to the output shaft.

The coupling is so made as to be entirely reversible, transmitting torque in either direction of rotation, both horizontal and vertical, with the same full efficiency. In normal operation, at rated kW, slip is two per cent to three per cent, making power efficiency 97 per cent or better, torque transmission being constant at 100 per cent.

A further refinement in the use of a fluid coupling is that of limiting the torque applied to a transmission system (to say twice the maximum rated torque requirement of the driven machine or, if this is not known then to twice the full load torque of the motor). This is accomplished by filling the fluid coupling with oil to the appropriate level.

Fig. 16. shows graphically the operation of a fluid coupling.

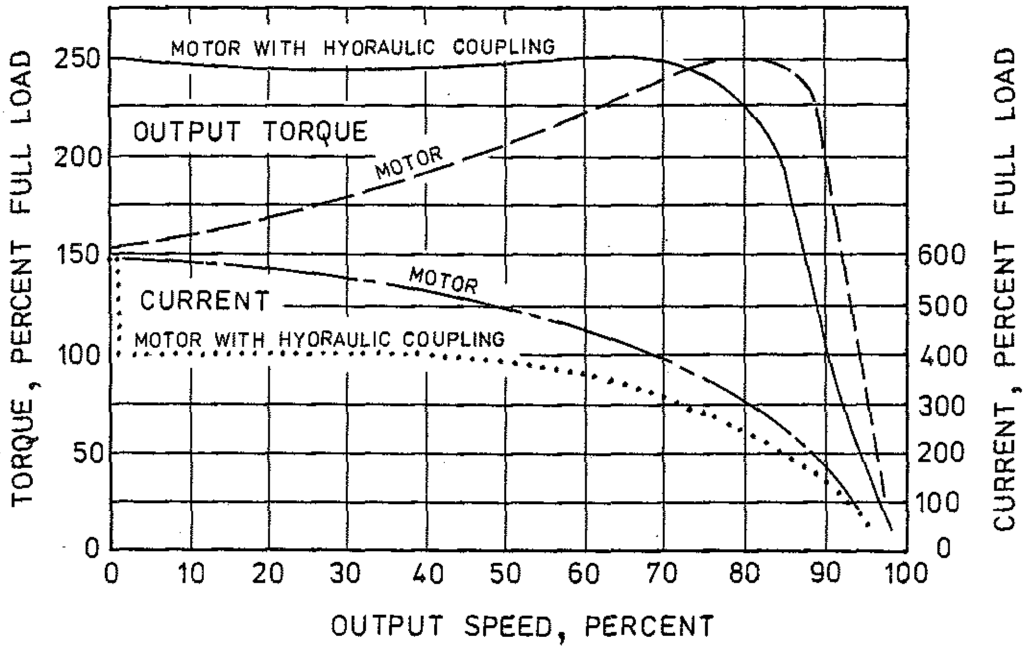


FIG. 16

Acknowledgments.

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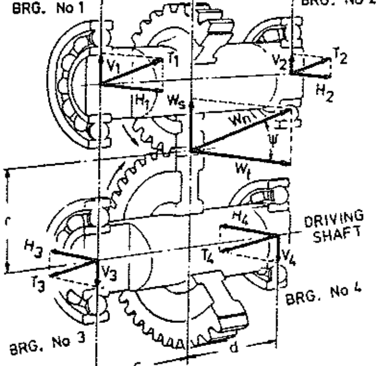
CHAPTER 9

WORK, ENERGY and TORQUE



CHAPTER 9

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WORK, ENERGY AND TORQUE

GLOSSARY OF TERMS

F = force, N

W = work, N.m

P = power, kW

T = torque, N.m

M = mass, kg

N = speed in r.p.m.

ω = speed in radians/sec

v = linear velocity, m/s

s = linear distance, m

h = height, m

t = time, seconds

θ = angular distance in radians

I = mass moment of inertia, $kg.m^2$

r = radius of path, m

α = angle of inclination of force, degrees

Work - Linear motion

Whenever the point of application of a force is moved any distance along its line of action, work is done.

(If there is no movement along the line of action of a force then there is no work done by that force regardless of its magnitude.)

$$W = Fs$$

Unit of work = N.m = joule

Refer to Fig. 1.

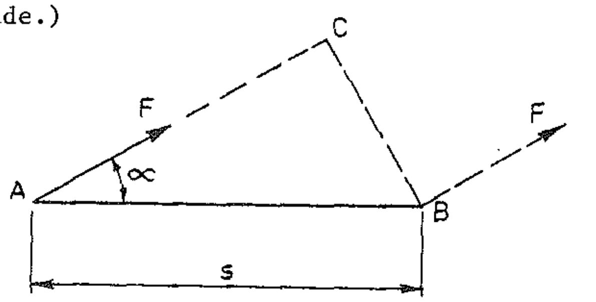


FIG. 1

If the point of application of F moves a distance, s, from A to B the resolved part of the displacement in the direction of the force is AC.

$$W = F \times AC = F \times AB \cos \alpha = F\, s \cos \alpha$$

Work is a scalar quantity.

Work - Rotational motion.

If a couple T acts on a body so as to produce an angular displacement θ about an axis perpendicular to the plane of the couple, the work done $W = T\,\theta$

Work is a scalar quantity.

Power

Power is the rate of doing work. It is the work done in unit time.
Units of power N.m/sec.
= joule/sec.

1 joule/sec = 1 watt

Energy

Energy is the capacity for doing work.
The mechanical forms of energy are:-
potential, strain and kinetic.

Units of energy are the N.m

Potential energy:-

If a body of mass M is raised a height h then the body possesses an amount of energy Mgh with respect to the original datum level.

Strain energy:-

If a spring of stiffness S is extended or compressed a distance δ then the spring has strain energy of amount $\frac{S\delta^2}{2}$. It will do an amount of work $\frac{S\delta^2}{2}$ in returning to the unstrained condition. (Refer to chapter 4, page 4.7.)

Kinetic energy:-

Linear motion.

If a body of mass M is moving with a velocity v it has kinetic energy.

$$KE = \tfrac{1}{2} M v^2$$

It will do this amount of work in being brought to rest.

Rotational motion.

If a body has a mass moment of inertia I about an axis of rotation and an angular speed ω about that axis then

$$KE = \tfrac{1}{2} I \omega^2$$

It will do this amount of work in being brought to rest.

Summary of Design Formulas

$W = F \times s \qquad W = T \times \theta \qquad K.E. = \tfrac{1}{2} M v^2 \qquad K.E. = \tfrac{1}{2} I\omega^2$

$P.E. = Mgh \qquad P = \frac{T \times \theta}{1000\ t} = \frac{T \times \omega}{1000}$

$P = \frac{F \times s}{1000\ t} = \frac{F \times v}{1000}$

Summary of Design Formulas (Cont.)

$$v = \frac{2 \pi r N}{60} \qquad \omega = \frac{2 \pi N}{60}$$

Relation between linear and angular motion

$$v = \omega r \qquad s = \theta r$$

Worked Example No. 1.

A bucket elevator has to lift 7.5 Tonne per hour of product a height of 40 m. Allow an overall efficiency of 50% (allows for general friction plus "digging" effect of buckets).

Find (a) The work done in 1 hour by the drive motor.

(b) The power requirements of the drive motor.

Answer.

(a) $W = \frac{7500\ g \times 40}{0.5} = 5.886 \times 10^6$ N.m (joule)

(b) $P = \frac{5.886 \times 10^6}{60 \times 60 \times 1000} = 1.64$ kW

Worked Example No. 2.

A pile driver of mass 100 kg strikes the top of the pile after having fallen 7 m. It drives the pile into the ground a distance of 150 mm. Determine the average force of the blow.

Answer.

Velocity of pile driver at instant of impact =

$v = \sqrt{2gh} = \sqrt{2g7} = 11.72$ m/sec

$KE = \frac{1}{2} M v^2 = \frac{1}{2} \times 100 \times 11.72^2 = 6867.92$ N.m

This should equal the potential energy

$PE = Mgh = 100\ g \times 7 = 6867$ N.m

The pile stops in 150 mm, then the average force =

$\frac{6867}{0.15} = 45\ 780$ N.

To this must be added the dead weight of the pile driver

Total $F = 45\ 780 + 100\ g$

$= 46\ 761$ N

Worked Example No. 3.

A rope brake is fitted to a flywheel 1.1 m in diameter (rope centres). Speed = 220 r.p.m. It is desired to absorb 7 kW.
What should be the difference in the pulls at the two ends of the rope?

Answer.

$$P = \frac{T\omega}{1000} = \frac{F \times r \times \omega}{1000}$$

$$\therefore \quad 7 = F \times \frac{1.1}{2} \times \frac{2\pi \times 220}{60 \times 1000}$$

$$\therefore \quad F = 552.4 \text{ N}$$

Worked Example No. 4.

Fig. 2 shows a rotary pellet mixing machine. For the position shown the C of G of the product is at its furthermost distance from the vertical centre line.

Speed of rotation = 5 r.p.m.

Mass of product = 7920 kg

Determine the power requirements if the machine overall efficiency = 91%

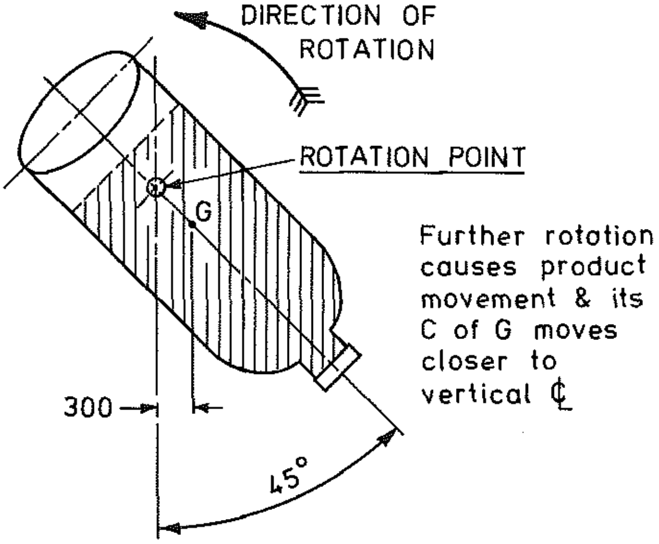


FIG. 2

Answer.

The loading conditions on the drive vary. In accordance with one of the principles given in the section "Introduction to Design" in Chapter No. 1, we will take the maximum load condition and assume as constant.

$$P = \frac{T\omega}{1000} = \frac{7920\,g \times 0.3 \times 2\pi 5}{1000 \times 60 \times 0.91} = 13.41 \text{ kW}$$

Worked Example No. 5.

Refer to Fig. 3.
A barge is hauled along by a rope. If the tension in the rope is 1000 N determine the work done in hauling the barge 2000 m.

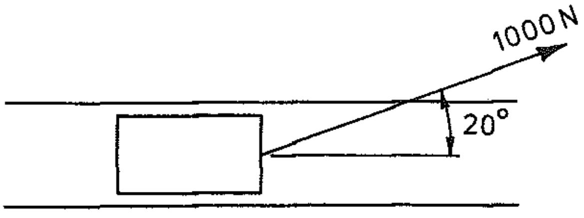


FIG. 3

$$W = Fs \cos \alpha$$

$$= 1000 \times 2000 \cos 20°$$

$$= 1.88 \times 10^6 \text{ N.m}$$

Worked Example No. 6.

Fig. 4. shows a hoisting drum with R.H. and L.H. grooving. There are four falls of rope supporting the 2 tonne load. Rope centres on drum = 1 m.

Speed of lift = 8 m/min

If the overall efficiency of the drive = 90% calculate the power requirements of the motor.

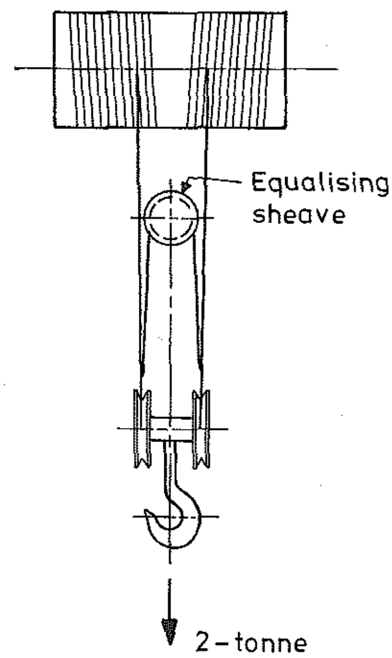


FIG. 4

Answer

$$\text{Load in each rope} = \frac{2000\ g}{4} = 4905\ \text{N}$$

$$\text{Linear speed of rope} = 2 \times 8 = 16\ \text{m/min}$$

$$\text{Speed of drum} = \frac{v}{r} = \frac{16}{60} \times \frac{1}{0.5} = 0.53\ \text{rad/s.}$$

$$\text{Torque on drum} = 2 \times 4905 \times 0.5 = 4905\ \text{N.m}$$

$$P = \frac{T\omega}{1000} = \frac{4905 \times 0.53}{1000 \times 0.9} = 2.89\ \text{kW}$$

Power and torque through geared systems.

Power could be defined as the rate of applying torque.

As the speed (r.p.m.) reduces through a gear system so the torque increases. The power is reduced progressively according to the inefficiencies.

Worked Example No. 7.

Fig. 5. shows a triple reduction gear train. The efficiency of each reduction is 98% and the input power is 10 kW at 960 r.p.m.

Determine the power and torque at shafts A, B, C, and D.

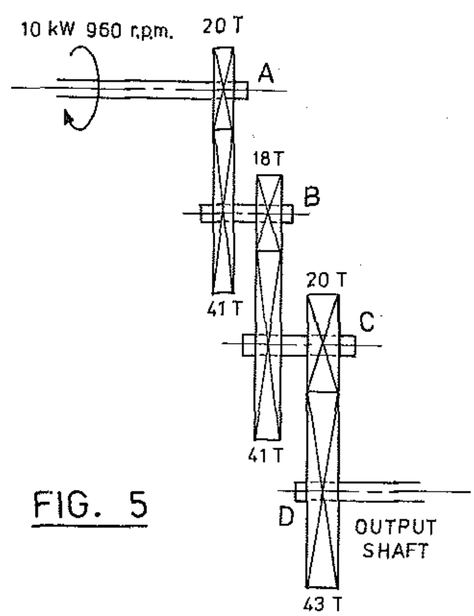


FIG. 5

Answer

$$\text{Power at shaft A} = \underline{10\ \text{kW}}$$

$$P = \frac{T\ \omega}{1000}$$

$$\therefore T = \frac{1000\ P}{\omega}$$

$$= \frac{1000 \times 10 \times 60}{2\pi\ \ 960} = \underline{99.47\ \text{N.m}}$$

$$\text{Power at shaft B} = 0.98 \times 10 = 9.8 \text{ kW}$$

$$\text{Speed of shaft B} = 960 \times \frac{20}{41} = 468.\ 29 \text{ r.p.m.}$$

$$\text{Torque at shaft B} = \frac{9.8 \times 1000 \times 60}{2\pi \quad 468.29} = 199.84 \text{ N.m}$$

$$\text{Power at shaft C} = 0.98 \times 9.8 = 9.6 \text{ kW}$$

$$\text{Speed of shaft C} = 468.29 \times \frac{18}{41} = 205.59 \text{ r.p.m.}$$

$$\text{Torque at shaft C} = \frac{9.6 \times 1000 \times 60}{2\pi \quad 205.59} = 445.9 \text{ N.m}$$

$$\text{Power at shaft D} = 0.98 \times 9.6 = 9.41 \text{ kW}$$

$$\text{Speed of shaft D} = 205.59 \times \frac{20}{43} = 95.62 \text{ r.p.m.}$$

$$\text{Torque at shaft D} = \frac{9.41 \times 1000 \times 60}{2\pi \quad 95.62} = 939.75 \text{ N.m}$$

Reversibility (overhauling)

A machine in which the frictional resistances are small may reverse if the effort is released. To investigate this point, consider the machine, Fig. 6, when the load Mg is being raised.

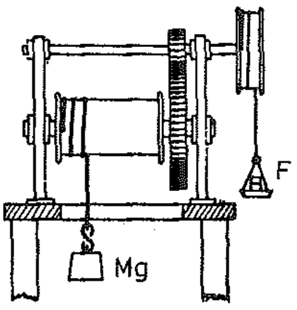


FIG. 6

Let the applied effort F act through at distance H in raising the load a distance h

Energy supplied = FH

Work done = Mgh

Energy wasted = FH − Mgh .... (1)

Now let F be removed and let the conditions be such that the load is just able to reverse the machine. Let the load descend through a height h.

Then, energy supplied and wasted in machine = Mgh ....(2)

Assuming that this waste has the same value as when the load is being raised, we have from (1) and (2)

$$FH - Mgh = Mgh$$

$$FH = 2\ Mgh \ ....(3)$$

$$\text{Now efficiency} = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}$$

$$\therefore \text{ from (3) efficiency} = \frac{Mgh}{FH} = \tfrac{1}{2}$$

Hence, when the load is being raised, the efficiency will be 50% for reversal to be possible if F is removed. Any value of the efficiency exceeding 50% would be accompanied by the same effect.

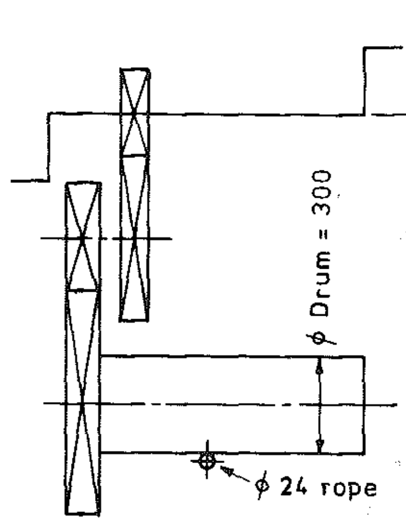
The foregoing theory does not apply to a power screw; refer to Chapter 15, page 15.5.

EXERCISES.

1. Fig. 7. gives a diagrammatic layout of a winch.

   Find: (a) mechanical advantage
   (b) velocity ratio
   (c) efficiency

   Answers. (a) 14.824
   (b) 21.111
   (c) 70.22%



Both reductions 3:1
Load = 68 kg
Total effort = 45 N

FIG. 7

2. Refer to Worked example No. 7, page 9.5. Supposing the power required at shaft D is 15 kW, determine the power and torque requirements at shafts C, B, and A.

   Ans. Shaft C 15.306 kW
   710.94 N.m

   Shaft B 15.62 kW
   318.49 N.m

   Shaft A 15.94 kW
   158.53 N.m

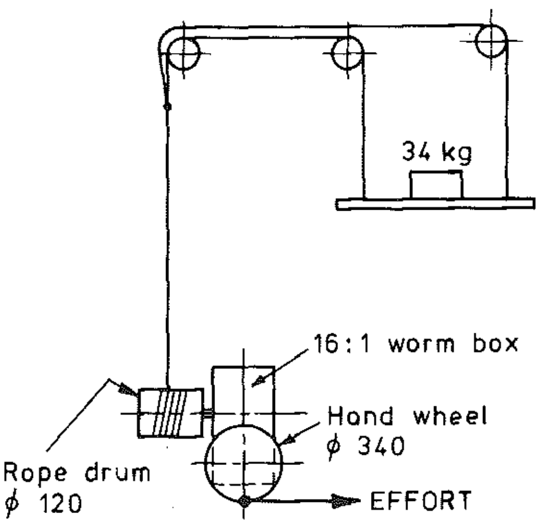


FIG. 8

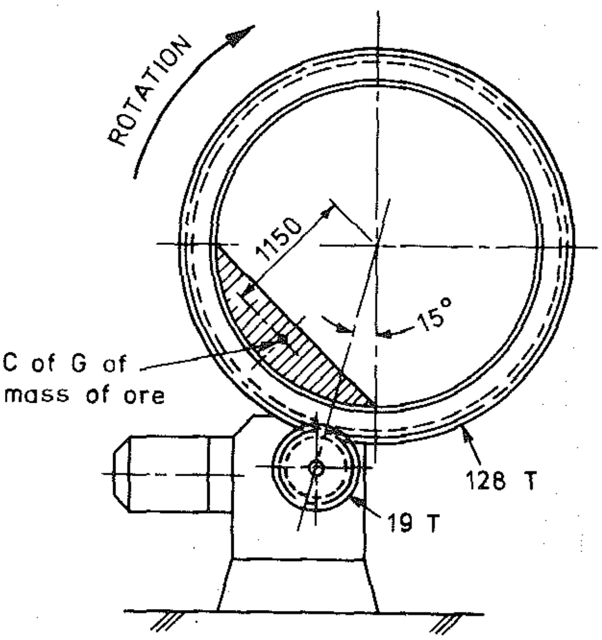


FIG. 9

3. Fig. 8 shows a portable hoist used for the maintenance of electrical equipment.

   (a) If the effort required to raise the load is 60 N and that for lowering is 18 N determine the raising and lowering efficiencies of the gear box.

(b) Determine the work done by the effort in lifting the load 1 m.

(c) Determine the work done by the effort in lowering the load 1 m.

Ans. (a) 12.26%
40.88% (b) 2720.56 N.m (c) 815.9 N.m

4. A solid cylindrical flywheel is 1.2 m in diameter and has a mass of 910 kg. If the axle is 150 mm in diameter and the coefficient of journal friction is 0.15, find the time required for the flywheel to coast to rest from a speed of 500 r.p.m.

Ans. 85.4 seconds

5. Fig. 9 shows a cross-section through a rotary kiln. During operation the free surface of the product is constant at 45°.

Determine the power requirements of the geared motor drive.

Mass of product .... 30 tonne.

Efficiencies:

Kiln support system 93%
Drive system 89.5%
Geared motor output speed .... 18 r.p.m.

Ans. 80.45 kW

6. Fig. 10 shows a 300 kg load being hauled up a cable-way by a hauling rope.

The speed of the hauling rope is 18 m/min.

Determine:

(a) the angle θ the load will assume when being hauled.

(b) the power requirements of the hauling motor if the overall system efficiency is 91%.

Ans. (a) 71.25° approx.
(b) 0.795 kW

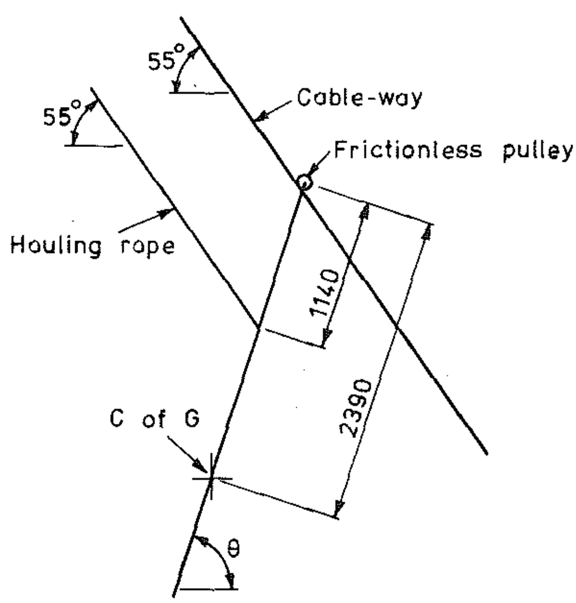


FIG. 10

7. A block and tackle having three sheaves in each block is used to raise a load of 230 kg.
The lower block has a mass of 4.5 kg and 15% of the work input is used in overcoming friction.

Determine:

(a) the effort

(b) the efficiency of the machine at this load.

Ans. (a) 451.07 N
(b) 83.37%

8. Fig. 11 shows a tilting table for use in an abattoirs. The load remains fixed with reference to the table during the tilting operation. What is the maximum tangential force required on the handle to perform the operation?

The machine efficiency is 90%.

Ans. 226.55 N

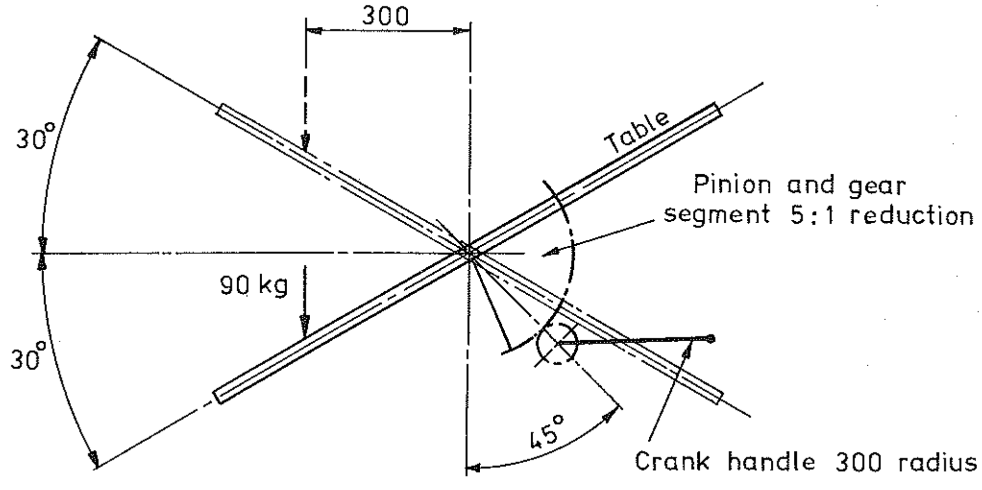
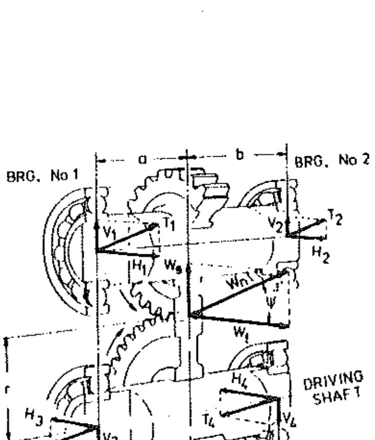


FIG. 11

CHAPTER 10

STRESS CONCENTRATION



CHAPTER 10
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STRESS CONCENTRATION

(REFER ALSO TO APPENDIX No. 3)

Basic Theory of Static Failure

For an in-depth study of the various theories of static failure students are referred to the numerous books on strength of materials.

Of the various theories of failure of machine parts only two will be mentioned here in brief summary.

(1) Maximum Normal Stress Theory (Rankine's Theory)

This theory assumes that failure will occur when the maximum normal stress (principal stress) reaches the yield stress as determined by a simple tensile test.

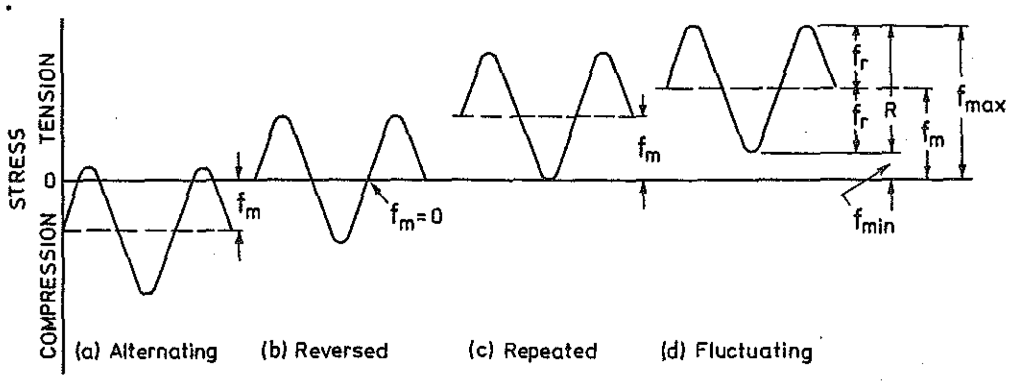
Observation of tension tests indicates that brittle materials (as C.I.) break by direct stress i.e. on a section perpendicular to the line of loading and Rankine's Theory is generally accepted for such materials.

(2) Maximum Shear Stress Theory (Guest's Theory)

Ductile materials usually break on surfaces inclined at 45° to the line of loading i.e., along the surfaces of maximum shear stress. In tension tests the maximum shear stress is half of the direct stress (since the other principal stress is zero) hence Guest's Theory is commonly accepted for ductile materials.
Guest's theory errs on the side of safety as actual tests show that the average shear strength of steel is approximately 60% of the tensile strength.

Fatigue Failure

In order to design a member for fatigue loading, the stress range with regard to time must be known. Fig. 1 shows the classically accepted stress-time variations.



FATIGUE-STRESS VARIATIONS

FIG. 1

The following relationships apply:-

$$f_m = \frac{f_{max} + f_{min}}{2} \; ; \qquad f_r = \frac{f_{max} - f_{min}}{2} \; ; \qquad R = 2f_r$$

where: f_m = mean stress f_{max} = maximum stress

f_{min} = minimum stress R = Range of stress

It is a well proven fact that a metal subjected to a continuously varying stress will fail at a lower stress than the same metal subjected to a single application of a load. Fatigue failure is a progressive fracture starting at a point of stress concentration. The stress concentration point may be very minute and difficult to detect.

There is a noticeable difference in the type of fracture produced in a ductile material when failure occurs under fatigue loading as compared with static loading.

With a ductile material, failure under static loading is always preceded by plastic flow hence the ruptured surface has a fibrous structure.

In the case of a fatigue fracture a fatigue crack spreads progressively from a point of stress concentration until the area of solid metal is so small it cannot sustain the load. Fracture then occurs suddenly. The fractured surface shows two clear regions. The first is the rubbing action as the crack propogates and the second is where sudden fracture has taken place. The second zone is similar to the fractured surface of a brittle material that has failed under static tensile loading.

There are three significant points controlling fatigue failures:-

(1) Magnitude of the maximum tensile stress.
This is because fatigue cracks only spread in regions of tensile stress.

(2) Magnitude of the stress fluctuation.

(3) Number of cycles of stress fluctuation.

The endurance limit for a metal is generally determined by doing a series of rotating beam tests on specimens of the material.
The mean stress is zero in these tests, as in Fig. 1. (b), page 10.1.
The results are plotted on an S-N graph, Fig. 2 (typical only).

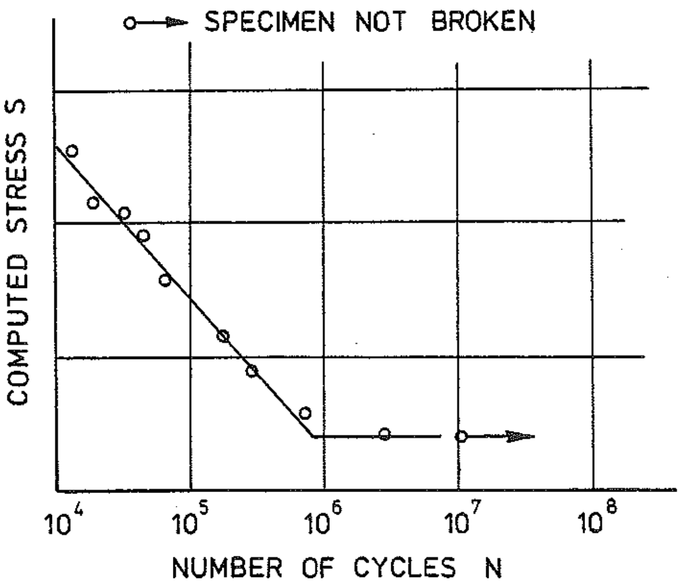


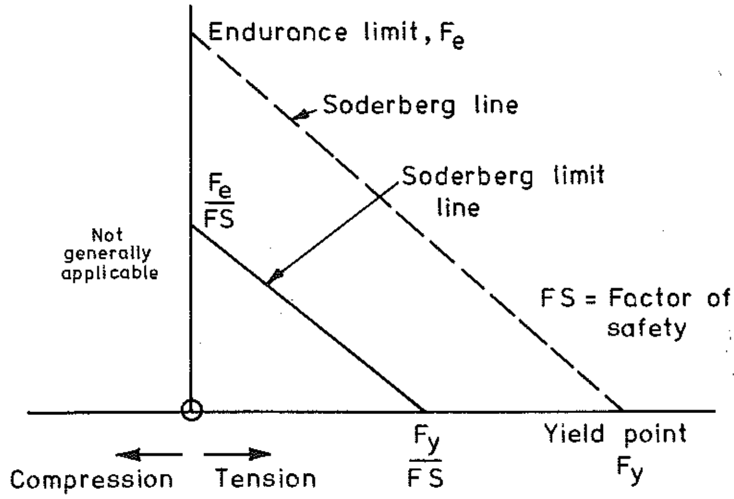
FIG. 2

The number of cycles of stress that a metal can resist before failure occurs increases with a decrease in stress.

Ferrous metals have a fatigue or endurance limit. This is the stress which causes failure at an infinite number of cycles.
The fatigue limit can be represented by a horizontal line on the S-N graph (Fig. 2.) to which S-N curve is asymptotic.

A true fatigue limit is not observed for non-ferrous metals.
It is usual here to specify a fracture stress at a certain number of cycles.
This is referred to as "finite life".

The case when the mean stress on a member is not zero can be represented on a Soderberg diagram, refer to Fig. 3.



SODERBERG DIAGRAM

FIG. 3

If the point for the actual alternating stress and mean stress falls below the Soderberg limit line then the design is safe.
Refer to the numerical example at the end of this chapter.

Stress Concentration

Many machine members have local points at which the stress is much higher than elementary analysis would indicate. Such stress concentrations may eventually cause a crack which will lead to failure of the member.

Some "stress raisers" are:-

(1) Sharp corners (2) Internal cracks

(3) Cavities in welds (4) Holes and notches

(5) Flaws and scratches on the external surface

(6) Abrupt changes in section

(7) Points or small areas supporting concentrated loads.

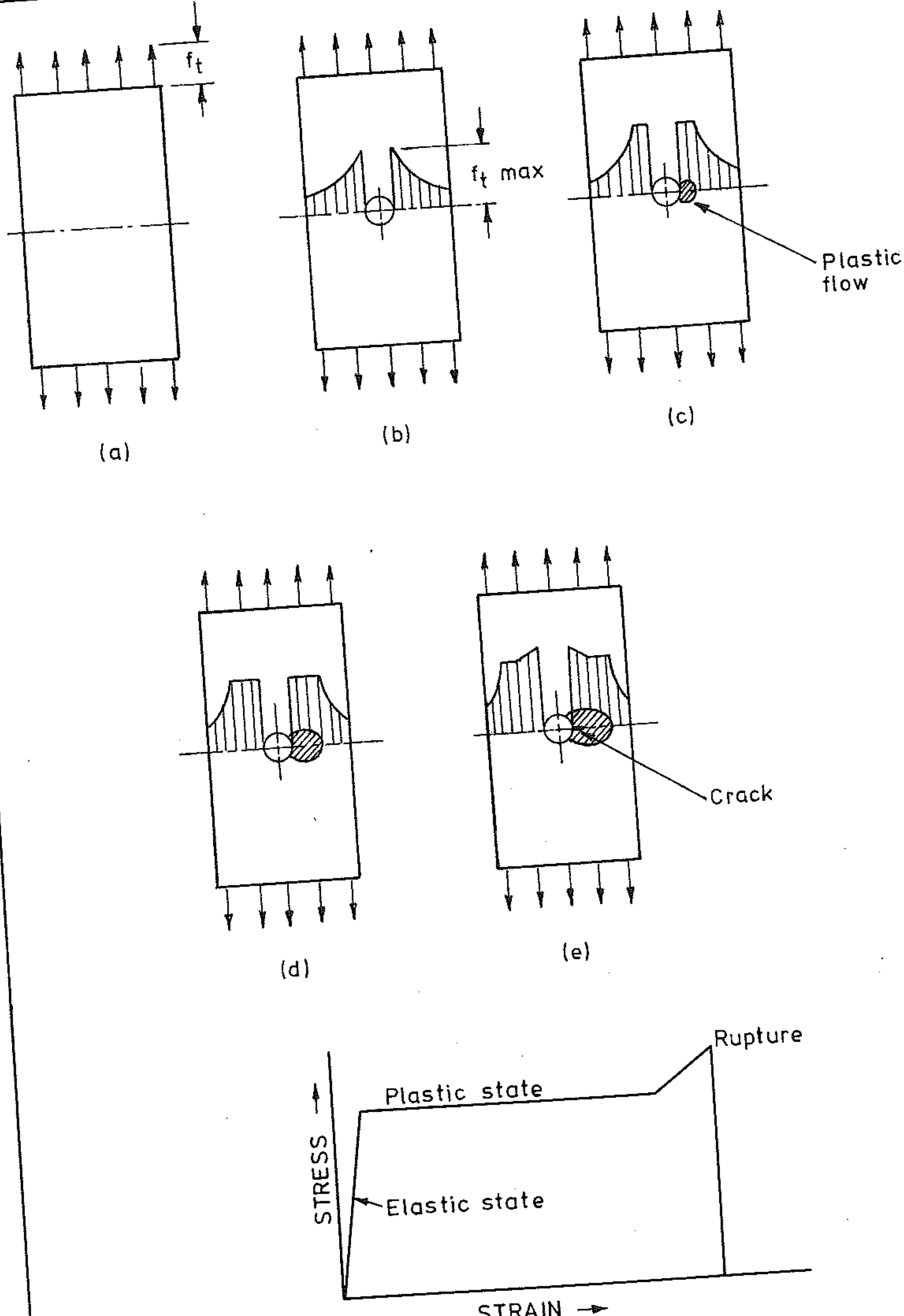
The seriousness of stress concentration depends on:-

(1) Properties of the material

(2) Nature of load i.e., static or cyclic.

Brittle materials such as cast iron exhibit no yield point therefore failure is sudden. Stress concentrations are serious in brittle materials regardless of the nature of the load.

Stress concentrations are moderately serious in ductile materials subjected to static loading. Refer to Fig. 4, page 10.4. This shows a rectangular bar under the action of a tensile load.



Idealised stress-strain diagram for ductile material

FIG. 4

At (a) the stress is uniformly distributed throughout the length of the bar. However, if a hole is drilled in the bar the stress distribution at that section will alter as shown at (b). A maximum stress will be induced at the edge of the hole. As the load is increased the yield point of the material will eventually be reached as shown at (c). Beyond the yield point there is a region of plastic flow where considerable strain takes place without increase in stress. Therefore, as the load is still further increased, the stress approaches uniform distribution as shown at (d). Finally when the load is too great, a crack will form as shown at (e) and the member will fail.

It is evident then that plastic flow in ductile materials lessens the seriousness of stress concentrations when the loading is static (refer also to the idealised stress-strain diagram on Fig. 4).

However, with cyclic loading, stress concentrations are serious in a ductile material because the ductility of the member is ineffective in relieving the concentrations of stress. If the stress at any point is above the endurance limit a crack will in all probability develop which will lead to failure.

Refer to Fig. 5, page 10.6.

At (x) $f_t \max = f\left(1 + \frac{2a}{b}\right)$ and at (y) $f_t \max = f\left(1 + \frac{2b}{a}\right)$

These solutions are exact and not mere approximations and are applicable to all proportions of ellipses. If the two axes of the ellipse are equal, it degenerates into a circle, and the maximum stress is three times the mean. If, on the other hand, the minor axis is $\frac{1}{1000}$ of the major axis, the ellipse approximates to a mere crack in the plate; and if the major axis be perpendicular to the line of the pull, the maximum stress becomes 2001 f; whilst if the ratio of the axis be 10 000 to 1, the maximum stress will be no less than 20 001 f.

In the above discussion the hole is assumed to be so small that it does not appreciably affect the cross-sectional area of the bar.
For bars having circular holes of diameter greater than one-sixth of the bar width use Chart No. 5, page 10.15.

It should be pointed out that the formulas applicable to Fig. 5 (x) and (y), page 10.6, and the information given in Charts No. 1. to 16, pages 10.13 to 10.20. are essentially applicable to reversed loading. Members stressed by static loading may be stronger in practice than the theory indicates.

Stress - Concentration Design Factors

The main loadings we are concerned about in machine members are:-

(1) Tension (2) Bending

(3) Torsional and direct shear (4) Combined loading.

A stress concentration factor is required to allow for the increase in stress where the geometry of the member causes local stress increase. These "K" factors are given in Charts No. 1 to No. 16 (pages 10.15 to 10.20) and were determined either mathematically or by the photo-elastic method.

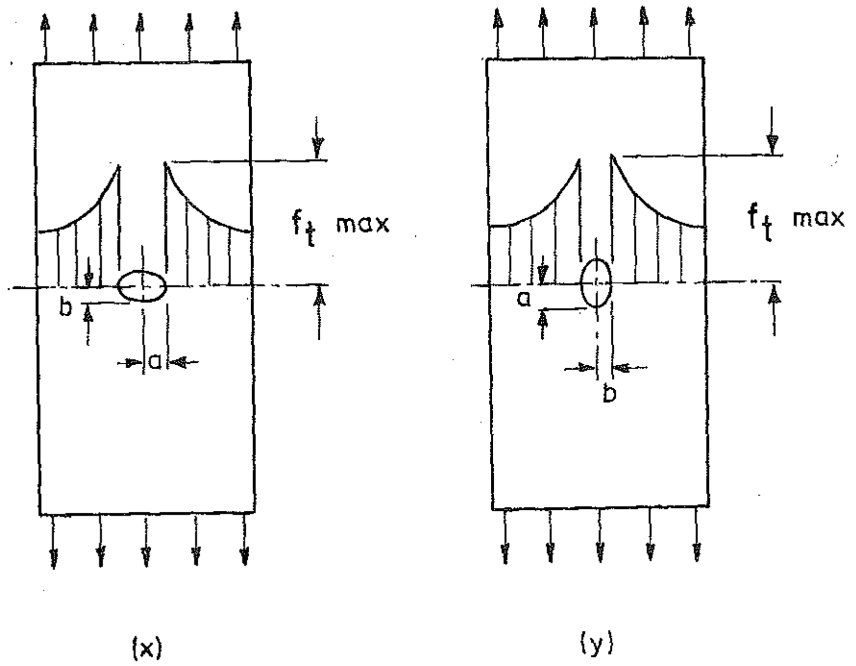


FIG. 5

Fig. 6, (page 10.7) illustrates some simple loading cases with the appropriate solutions. The nominal stress is always calculated on the minimum cross-section.

In shaft design, Charts No. 6. to No. 16 (pages 10.15 to 10.20) apply. However, we are only concerned commonly with Charts No. 13. to No. 15.

Chart 13 K_{fm} Stepped shaft ------- bending only. (Page 10.19)

Chart 14 K_{fc} Stepped shaft ------- combined bending and torque. (Page 10.19.) (See footnote)

Chart 15 K_{ft} Stepped shaft ------- torsion only. (Page 10.20.)

Minimising stress concentration.

(1) Avoid sharp corners. Use generous radii.

(2) Avoid abrupt changes in cross-section.

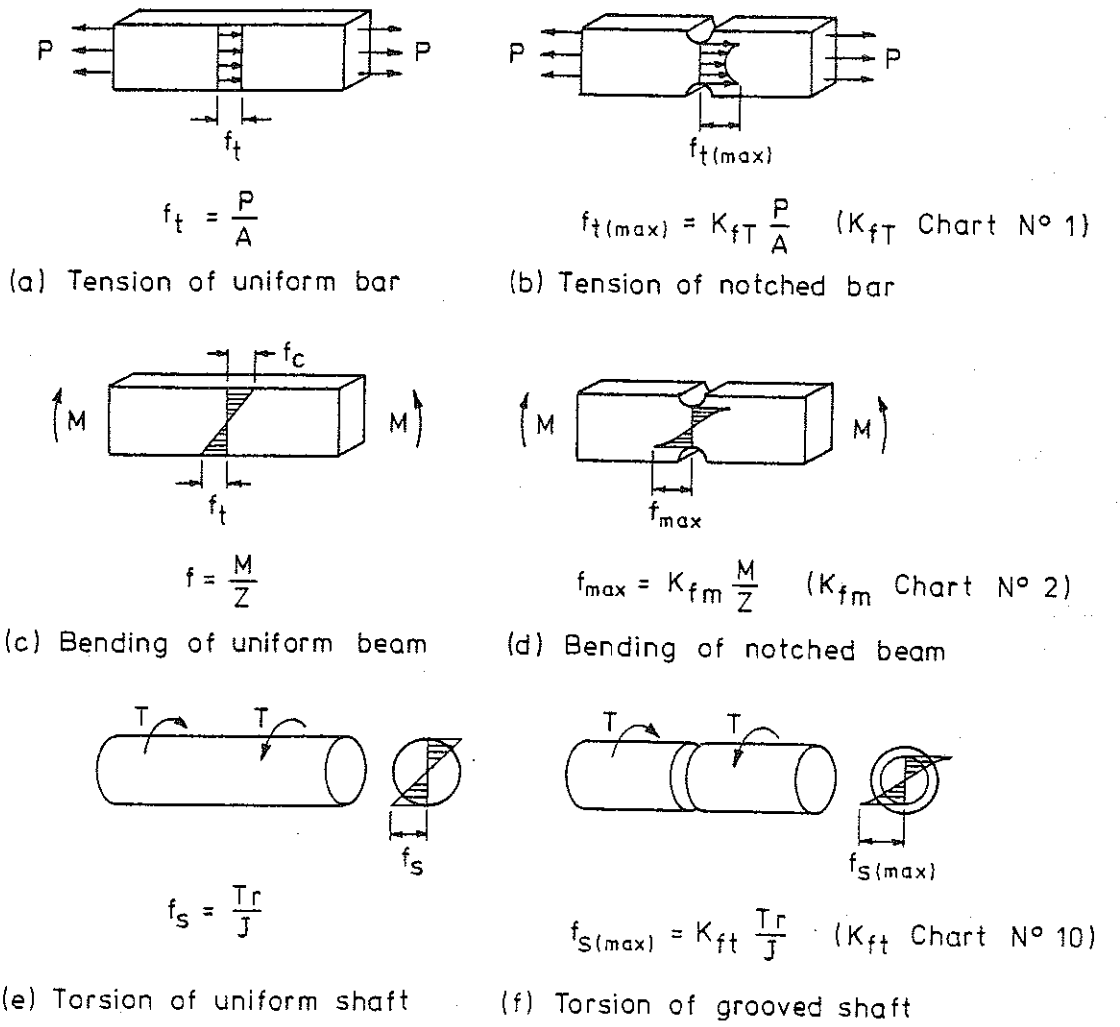
(3) Make machined surfaces as smooth as possible.

(4) Ensure good quality welding.

Notch Sensitivity.

This is the degree to which the theoretical effect of stress concentration is actually reached. It is determined experimentally. Theoretical factors are used without considering notch sensitivity when experimental factors are unavailable.

[Footnote: The term "combined" means that the type of loading stated (tension or bending) is accompanied by torsion (i.e. a shear stress).]



Simple cases of loading without and with stress concentration.

FIG. 6

Dynamic Loading.

This implies any load that is suddenly applied. Two cases arise.:

(i) Shock loading. This occurs when the load is suddenly applied but without velocity of approach.

(ii) Impact loading. This occurs when the load has a velocity of approach relative to a member.

Dynamic loading also refers to a force whose point of application is constantly moving (e.g. a vehicle). The effect is a complex phenomenon.

Impact effect.

The impact effect of a load is due to vibrations set up in the members of the system. These vibrations result in momentary strains greater than occur with the load steady, and thus give stresses greater than with the load steady.

Vibration under load may result from:-

(1) Rapidity of application of load.

(2) Resonance between a member and a periodically varying load.

(3) Blows or shocks.

Rapidity of application of load.

In static loading the load is applied very slowly, so that the strain increases gradually with the load, and there exists at all times equilibrium between the stresses across any section and the applied forces (e.g. loading a specimen in a testing machine). The system acquires no appreciable velocity, so that if loading is discontinued at any stage, the members are immediately at rest in a state of equilibrium under the applied forces and the internal stresses; there is no tendency to "overshoot" the existing deflection due to the load and members having acquired a momentum. On the other hand, if the whole load is suddenly applied there exists at first no stresses to balance it, and the load and members acquire momentum. When the deflection corresponding to the same load gradually applied is reached, the acquired momentum causes motion to continue until the kinetic energy possessed by the load and members is absorbed as energy of elastic strain. Under this increased deflection the resultant of the stresses is greater than the load, and the whole system starts to return again. When it again reaches its static deflection, it has momentum, and "overshoots" its equilibrium position in the opposite direction. It then continues to vibrate either side of the static equilibrium position until the energy of the vibration is dissipated, and the system settles to a condition of equilibrium at the static deflection.

Although the yield strength of steel rises somewhat with speed of loading, static permissible design stresses are always used with shock loading conditions. Recommended shock load factors are given in the various design sections of this book.

Where severe shock loading conditions exist, the steel selected should have a high yield strength and be reasonably ductile to allow for stress relieving at regions of high stress concentration.

Impact load factor

Refer to Fig. 7.

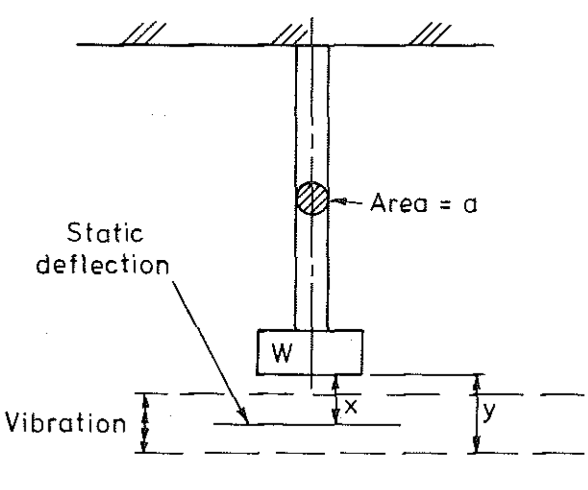


FIG. 7

Let W = Applied load without velocity of approach.

a = cross-sectional area of member

x = static deflection

y = maximum deflection under vibration

f = static stress

f_1 = stress with deflection "y"

Read again the previous section "Rapidity of application of load".

Then, $f\,a = W$ ------------(1)

Assuming stress proportional to strain (elastic limit not exceeded during vibration)

$$\frac{f_1}{f} = \frac{y}{x}; \qquad \therefore \ f_1 = f. \frac{y}{x} \text{ ---------- } (2)$$

Now, with deflection "y" the energy of elastic strain of the member must equal the potential energy lost by the load.

$$\therefore \ \tfrac{1}{2} . f_1 . a. y = w.y \text{ -------------- } (3)$$

Substituting from (2) into (3):-

$$\tfrac{1}{2} . f. a. \frac{y^2}{x} = W.y$$

and substituting from (1):-

$$\tfrac{1}{2} . W . \frac{y^2}{x} = W.y. \qquad \therefore \ y = 2x$$

Thus sudden application of the whole load causes twice the strain, and consequently twice the stress, as the same load gradually applied. This assumption is on the safe side and represents the upper limit. Further academic study shows that sudden application of a load normally causes between 1.63 and something less than twice the static stress.

Worked Example No. 1.

A solid stationary shaft of CS 1030 material is subjected to a fluctuating B.M. which results in tensile stresses ranging from 35 MPa to 139 MPa. Using a Soderberg diagram, determine whether the design is safe.

Answer.

$$f_m = \frac{f_{max} + f_{min}}{2} = \frac{139 + 35}{2} = 87 \text{ MPa}$$

$$f_r = \frac{f_{max} - f_{min}}{2} = \frac{139 - 35}{2} = 52 \text{ MPa}$$

From Table No. 3. in Chapter 1, page 1.26.

CS 1030 Yield stress = 250 MPa

Endurance limit = 225 MPa

From Table No. 1 in Chapter 1, page 1.20.
Safety Factor for endurance stress (e.g. a rotating shaft) = 2.0
This factor is applied to the endurance limit stress on the Soderberg diagram.

$$\frac{225}{2} = 112.5 \text{ MPa}$$

Safety factor for a solid stationary shaft = 1.67. (Refer page 1.21.)
This factor is applied to the yield point stress on the Soderberg diagram.

$$\frac{250}{1.67} = 150 \text{ MPa}$$

Refer to Fig. 8.

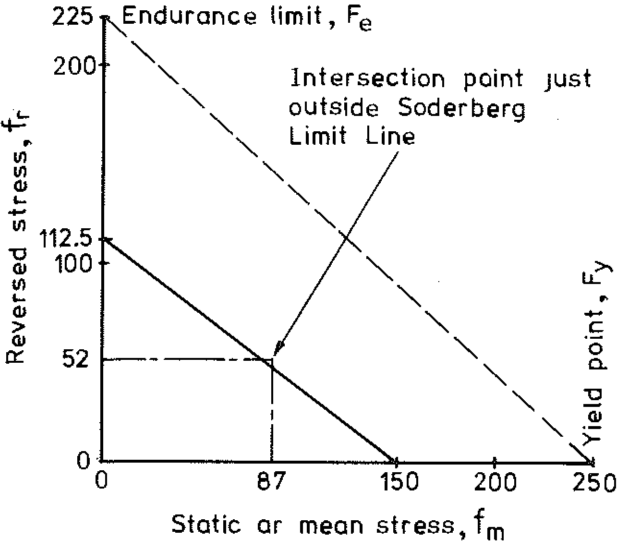


FIG. 8

Plotting the stress point on a Soderberg diagram shows the design to be marginally unsafe.

General equation of Soderberg Limit Line

$$F_r = \frac{F_e}{F \text{ of } S} - \frac{F_e}{F_y} \times f_m$$

Specific equation

$$F_r = 112.5 - \frac{112.5}{150} f_m$$

$$= 112.5 - \frac{112.5}{150} \times 87$$

$$\therefore \quad F_r = 47.25 \text{ MPa} < f_r \text{ of } 52 \text{ MPa}$$

$\therefore$ design not acceptable.

(F_r represents maximum permissible reversed stress.)

Worked Example No. 2.

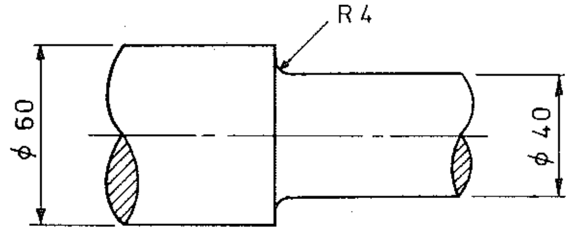


FIG. 9

Fig. 9 shows a shaft of CS 1020 which is subjected to repeated torque reversals of 310 N.m.

Determine f_s and F_s.

Answer.

$$\frac{D}{d} = \frac{60}{40} = 1.5$$

$$\frac{r}{d} = \frac{4}{40} = 0.1$$

From Chart No. 15, page 10.20 $K_{ft} = 1.43$

$$Z_p = \frac{\pi d^3}{16} = \frac{\pi\, 40^3}{16} = 12\,566.37 \text{ mm}^3$$

$$K_{ft}\, T = f_s\, Z_p \qquad \text{(Refer to pages 2.12 \& 2.13)}$$

$$\therefore \quad f_s = \frac{K_{ft}\, T}{Z_p} = \frac{1.43 \times 310 \times 10^3}{12\,566.37} = \underline{35.28 \text{ MPa}}$$

CS 1020 F_e = 180 MPa (Refer Table No. 3, Chapter 1, page 1.26.)

$\therefore$ $F_s = \frac{180}{2 \times 2} = \underline{45 \text{ MPa}} > 35.28$ MPa (Refer to pages 1.20 & 1.21)

Worked Example No. 3.

Refer to Fig. 10.

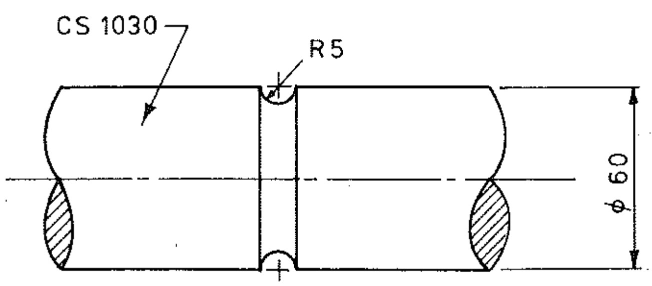


FIG. 10

(a) State the maximum bending moment that can be applied to the shaft (no torque):

(i) when the bending moment is static.

(ii) when the bending moment is continually reversed.

(b) State the maximum torque that the shaft can transmit (no bending moment).

(i) when the torque is static.

(ii) when the torque is continually reversed.

Answer

(a) (i) $\frac{r}{d} = \frac{5}{50} = 0.1$ $\qquad \frac{D}{d} = \frac{60}{50} = 1.2$

From Chart No. 8. $K_{fm} = 1.88$ (page 10.16)

$K_{fm}M = F_tZ \qquad \therefore \quad M = \frac{F_t \; Z}{K_{fm}}$ (Refer to page 2.13)

$F_t = 0.6\ F_y$, Table No. 1, Chapter 1, page 1.21.

F_y CS 1030 = 250 MPa Table No. 3, Chapter 1, page 1.26.

$\therefore \quad F_t = 0.6 \times 250 = 150$ MPa

$\therefore \quad M = 150 \times \frac{\pi\ 50^3}{32} \times \frac{1}{1.88} \times \frac{1}{10^3} = \underline{979.14 \text{ N.m}}$

(ii) F_e CS 1030 = 225 MPa Table No. 3, Chapter 1, page 1.26.

$$\therefore \quad F_t = \frac{225}{2} = 112.5 \text{ MPa}$$ (Refer to pages 1.20 & 1.21)

$$\therefore \quad M = \frac{F_t Z}{K_{fm}}$$ (pages 2.12 & 2.13)

$$= 112.5 \times \frac{\pi\, 50^3}{32} \times \frac{1}{1.88} \times \frac{1}{10^3}$$

$$= \underline{734.35 \text{ N.m}}$$

(b) (i) From Chart No. 10, page 10.17, $K_{ft} = 1.46$

$$K_{ft}\, T = F_s\, Z_p \qquad \therefore \quad T = \frac{F_s\, Z_p}{K_{ft}}$$ (Refer to pages 2.12 & 2.13)

$F_s = 0.3\ F_y$, Table No. 1, Chapter 1, page 1.21.

F_y CS 1030 = 250 MPa Table No. 3, Chapter 1, page 1.26

$$\therefore \quad F_s = 0.3 \times 250 = 75 \text{ MPa}$$

$$\therefore \quad T = 75 \times \frac{\pi\, 50^3}{16} \times \frac{1}{1.46} \times \frac{1}{10^3} = \underline{1260.81 \text{ N.m}}$$

(ii) F_e CS 1030 = 225 MPa Table No. 3, Chapter 1, page 1.26

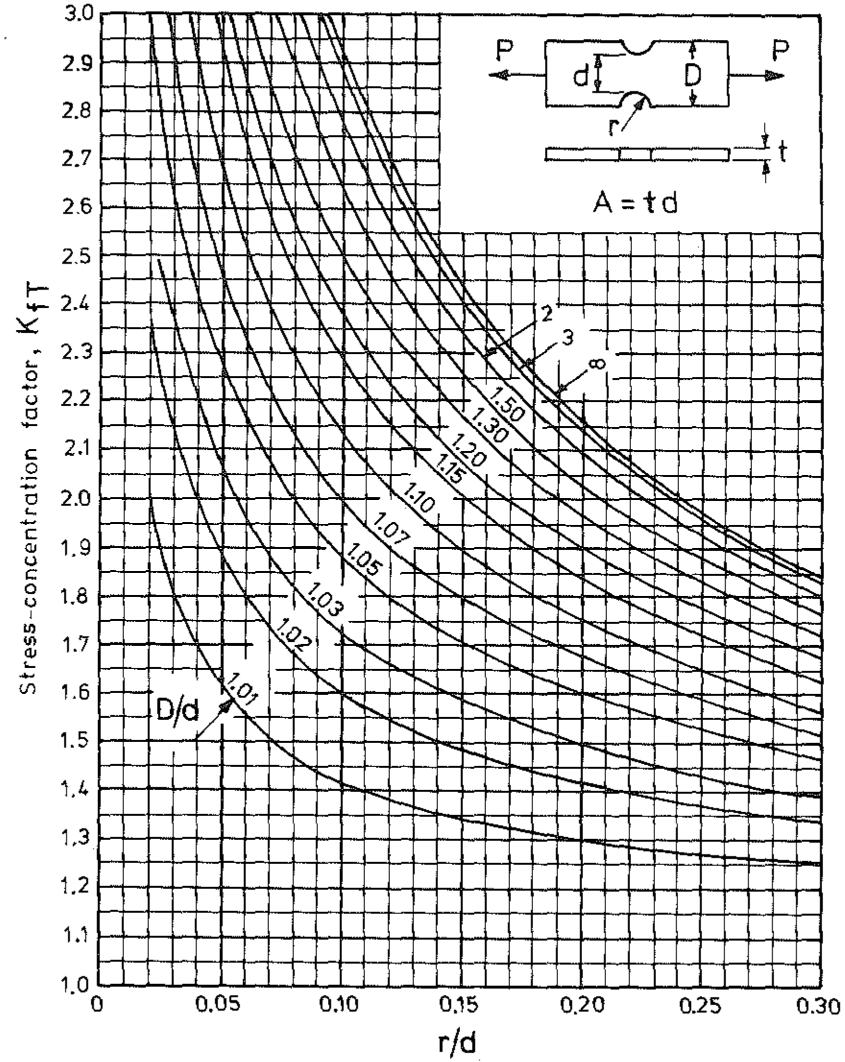
$$\therefore \quad F_s = \frac{225}{2 \times 2} = 56.25 \text{ MPa}$$ (Refer to pages 1.20 & 1.21)

$$\therefore \quad T = 56.25 \times \frac{\pi\, 50^3}{16} \times \frac{1}{1.46} \times \frac{1}{10^3}$$ (formula as in (b)(i))

$$= \underline{945.61 \text{ N.m}}$$

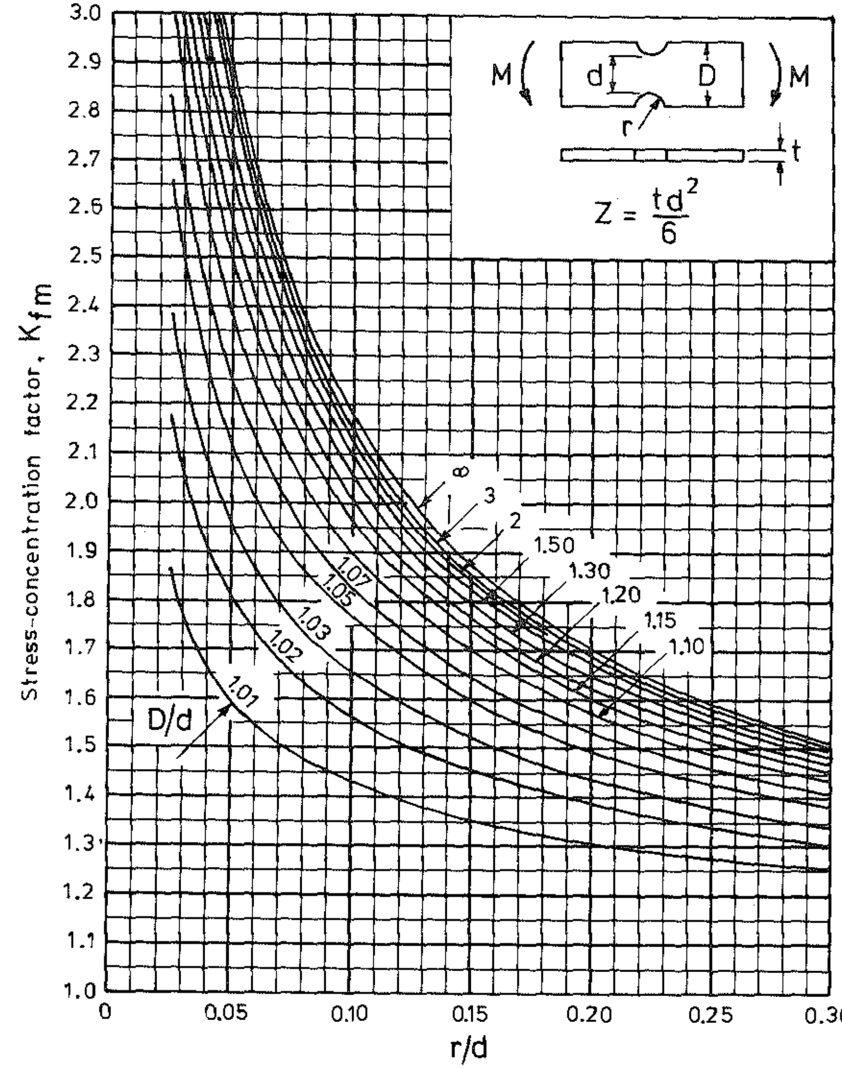
* * * * * * * * * * *

For Exercises see page 10.21



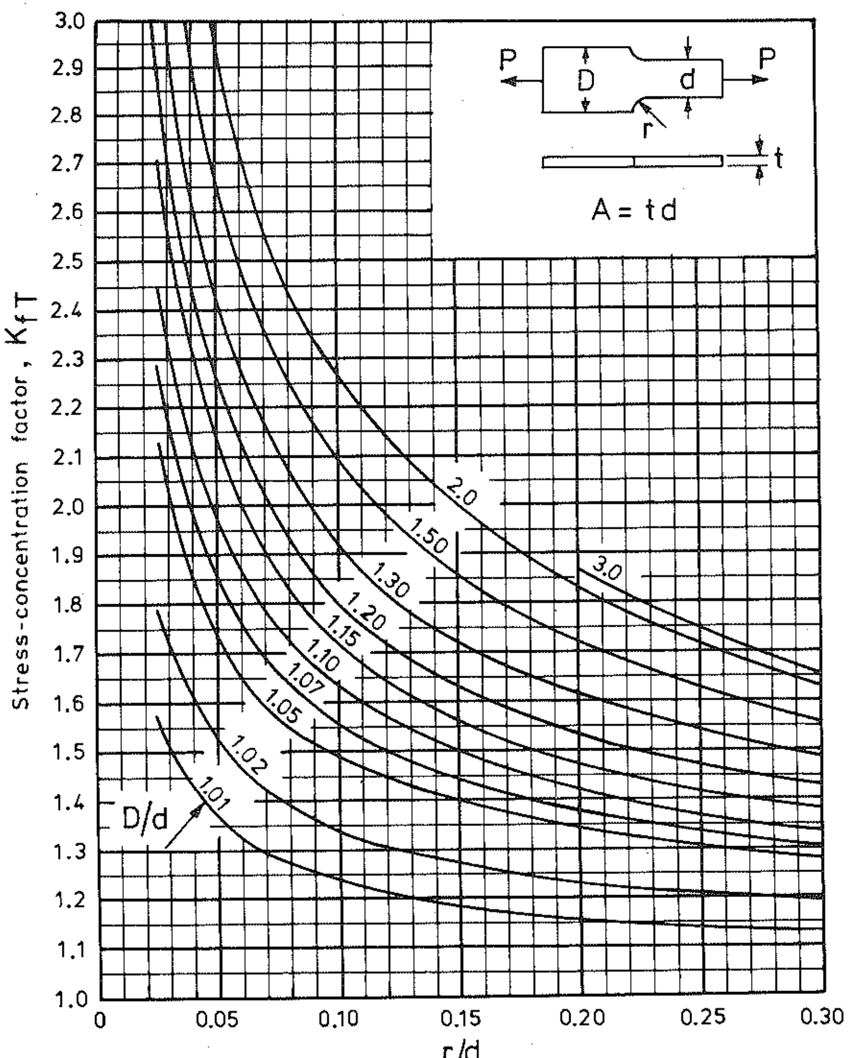
Stress-concentration factor
for notched flat bar in tension, K_{fT}

CHART No 1



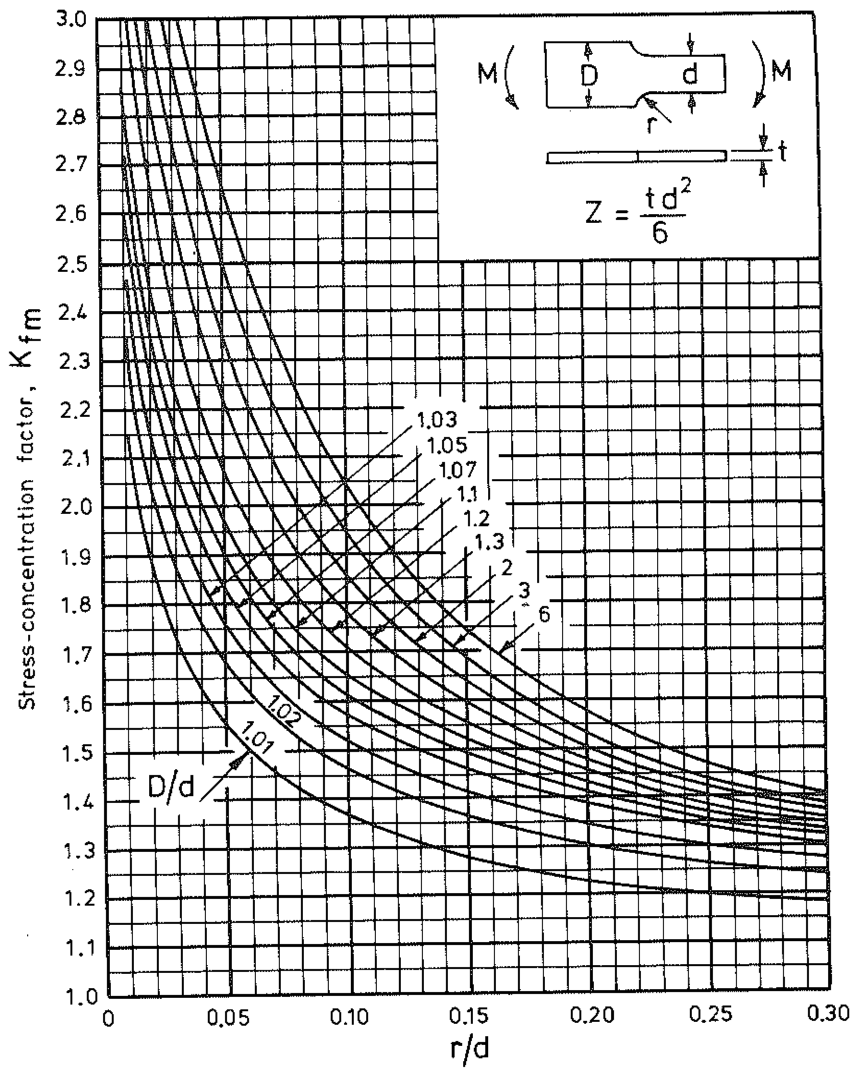
Stress-concentration factor
for notched flat bar in bending, K_{fm}

CHART No 2



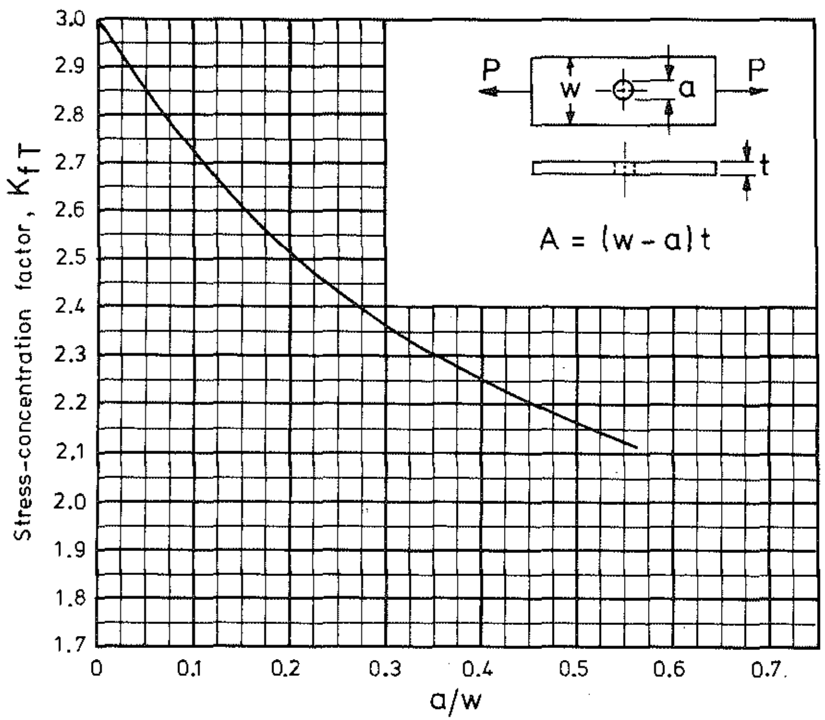
Stress-concentration factor
for filleted flat bar in tension, K_{fT}

CHART No 3



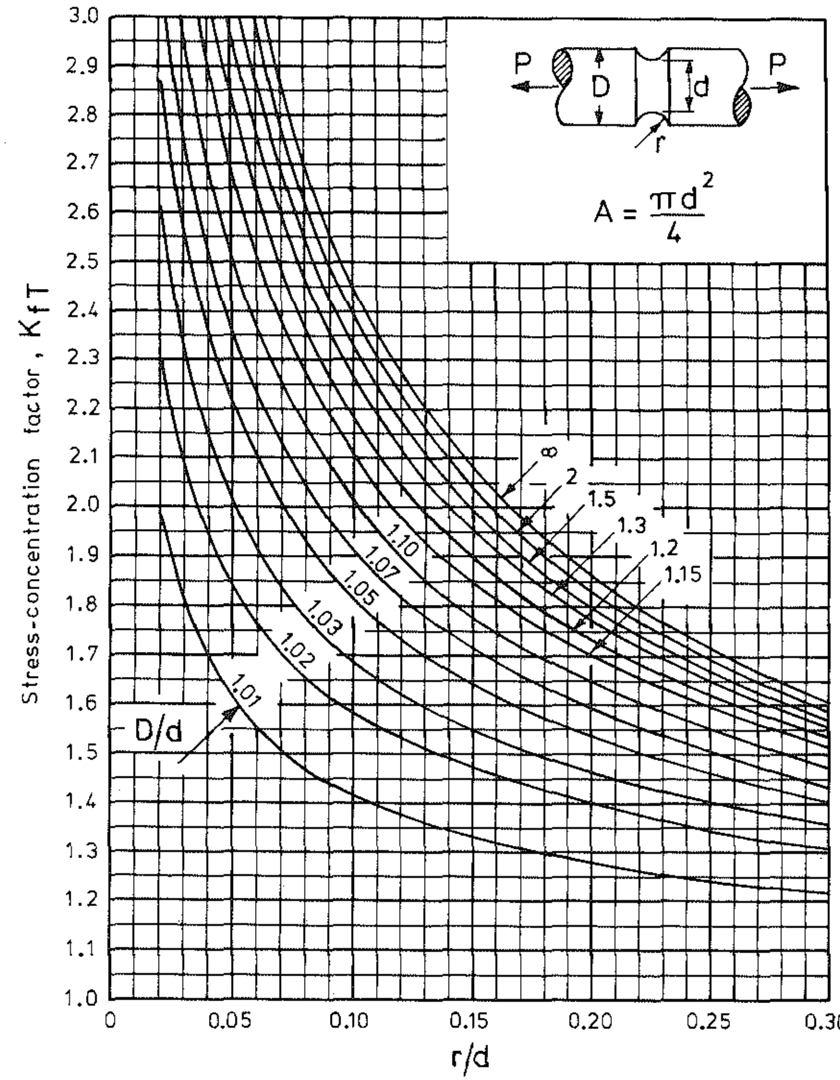
Stress-concentration factor
for filleted flat bar in bending, K_{fm}

CHART No 4



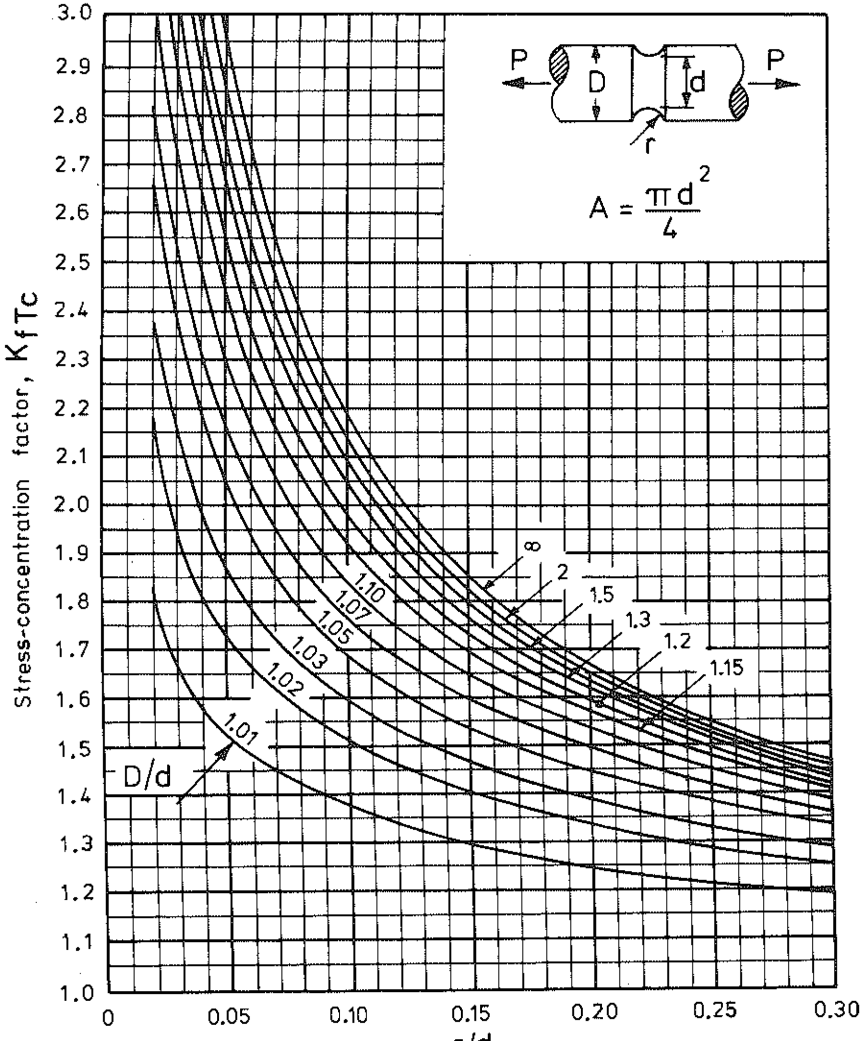
Stress-concentration factor
for plate with hole, in tension, K_{fT}

CHART No 5



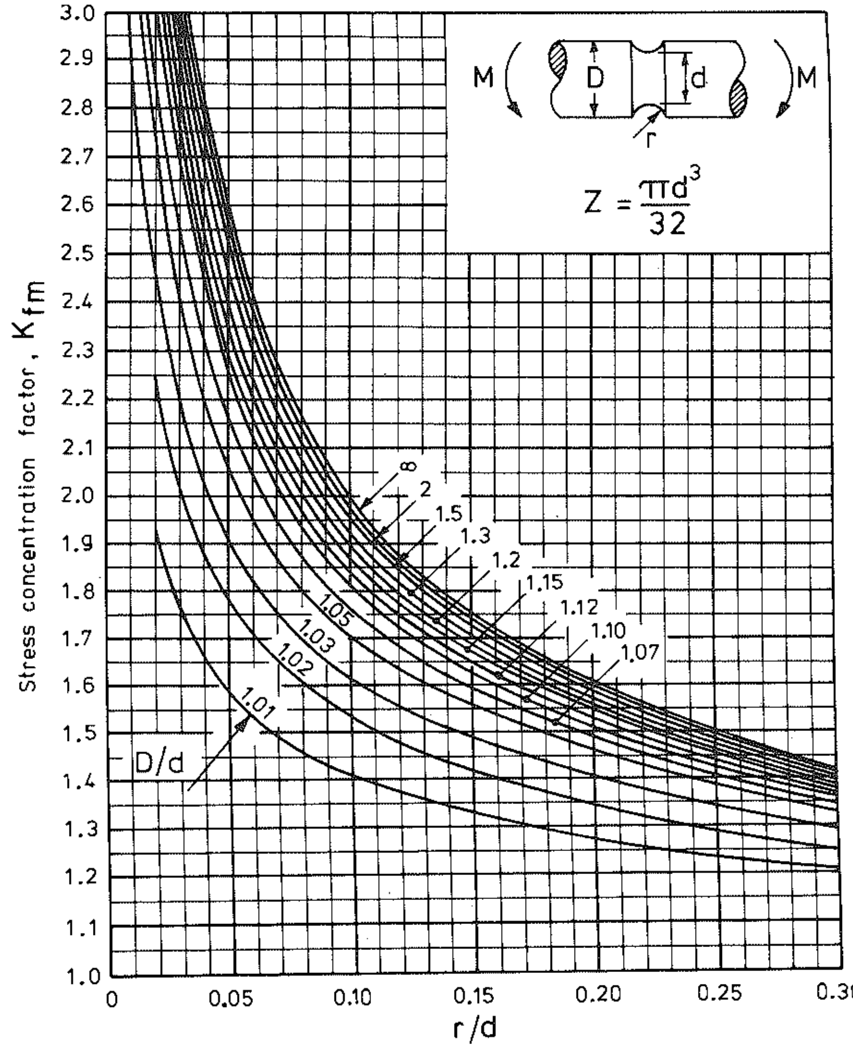
Stress-concentration factor
for grooved shaft in tension, K_{fT}

CHART No 6



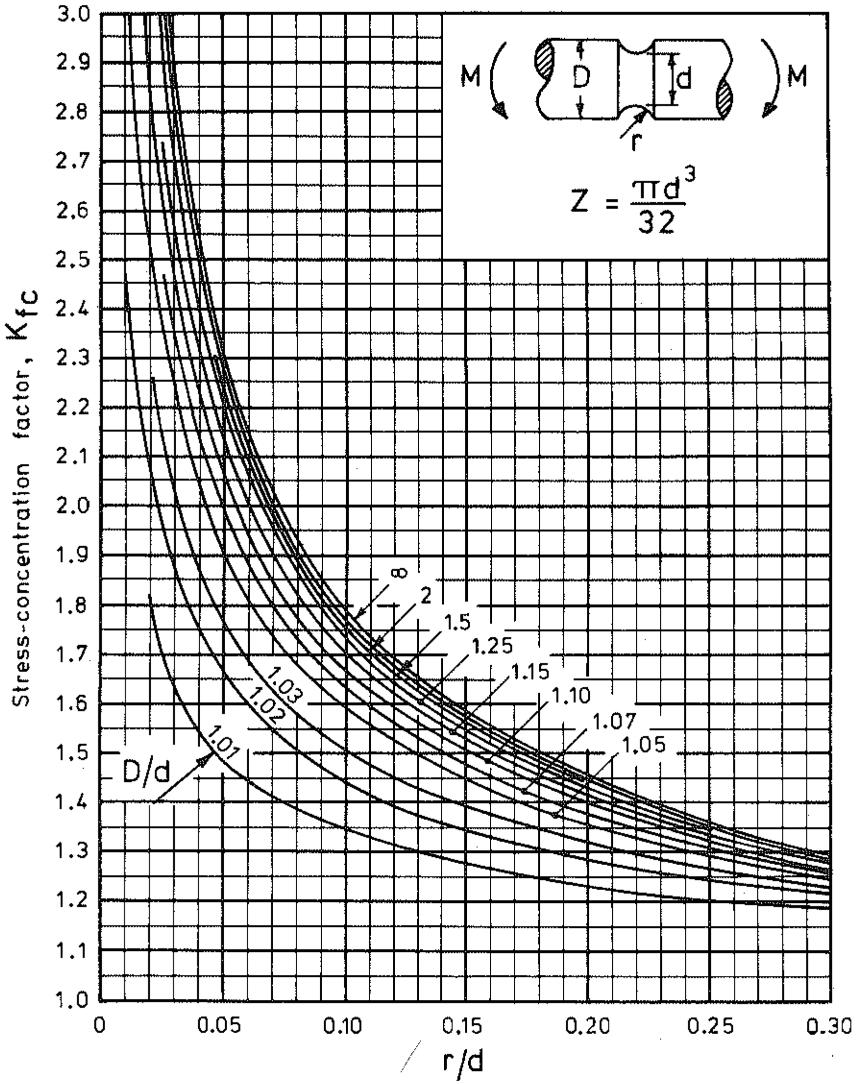
Combined stress-concentration factor
for grooved shaft in tension, K_{fTc}

CHART No 7



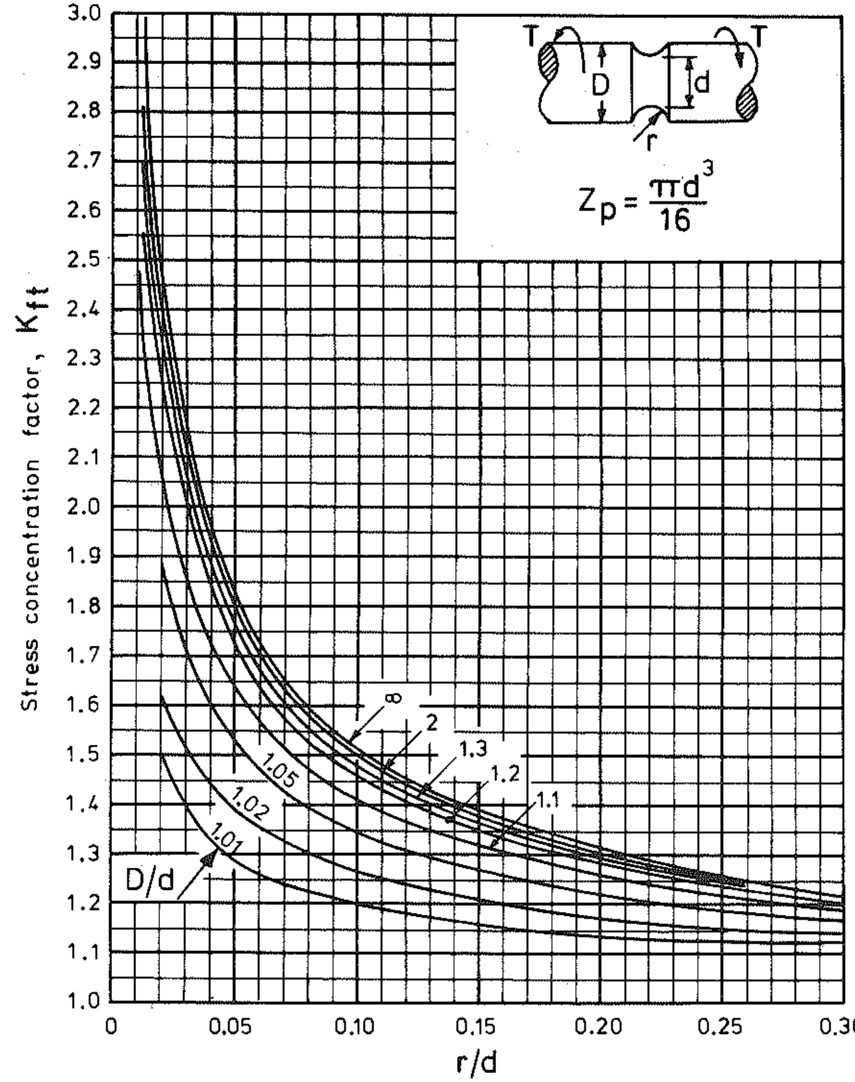
Stress-concentration factor
for grooved shaft in bending, K_{fm}

CHART No 8



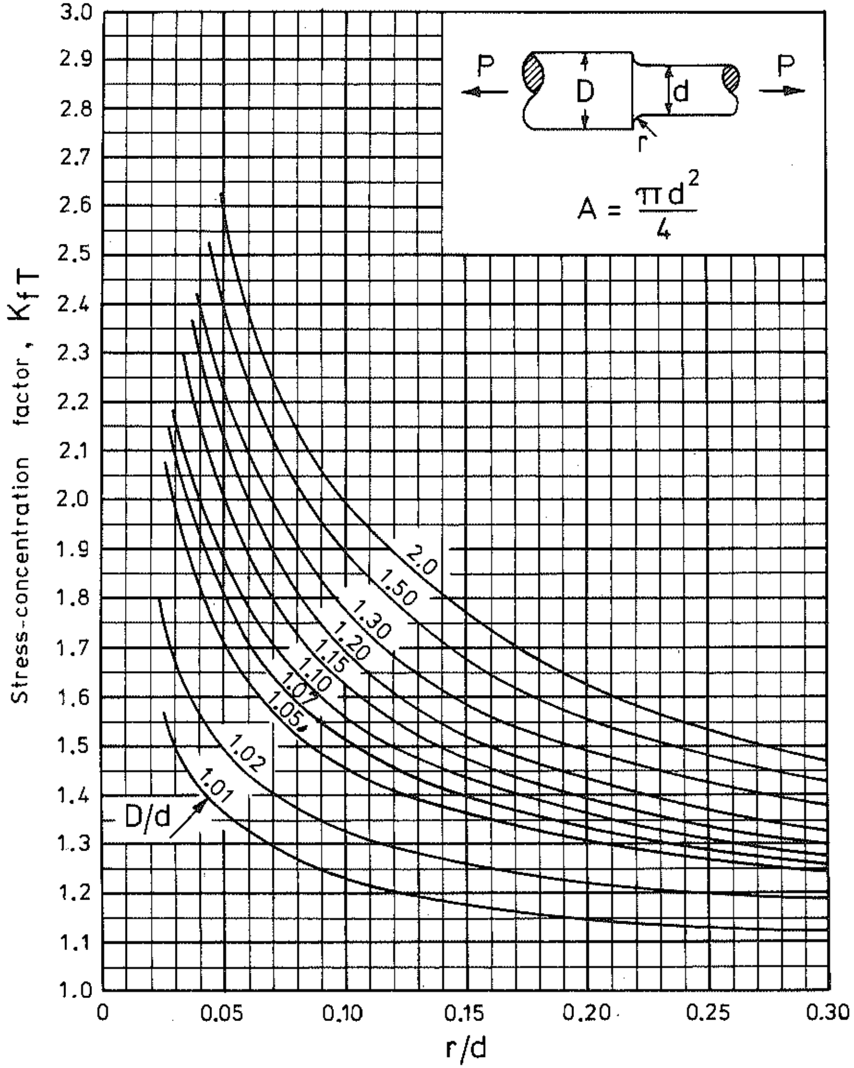
Combined stress-concentration factor for grooved shaft in bending, K_{fc}

CHART No 9



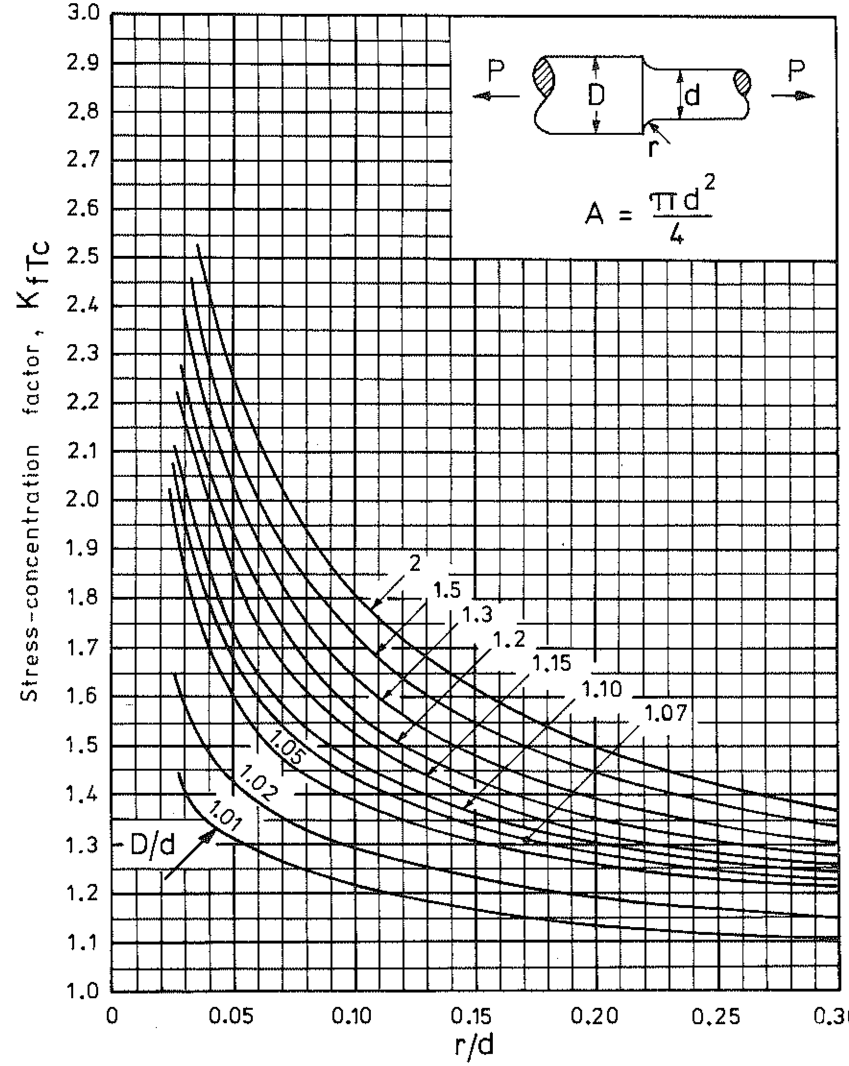
Stress-concentration factor for grooved shaft in torsion, K_{ft}

CHART No 10



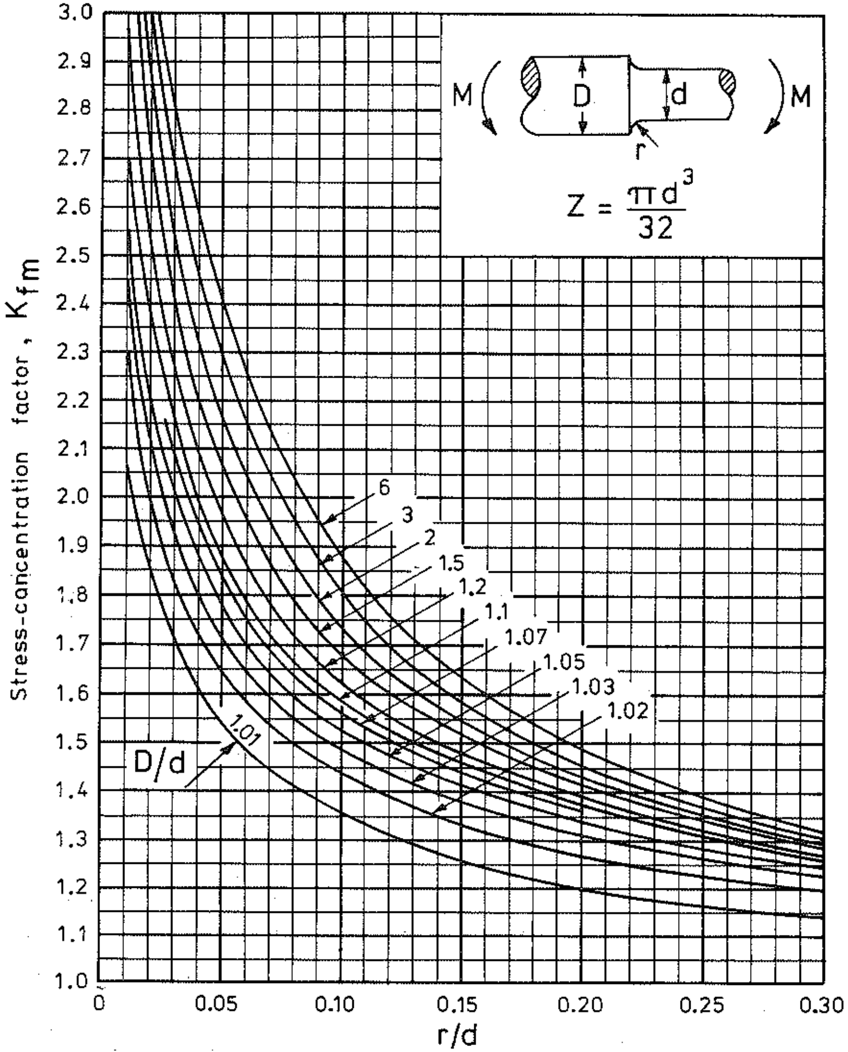
Stress-concentration factor
for stepped shaft in tension, K_{fT}

CHART No 11



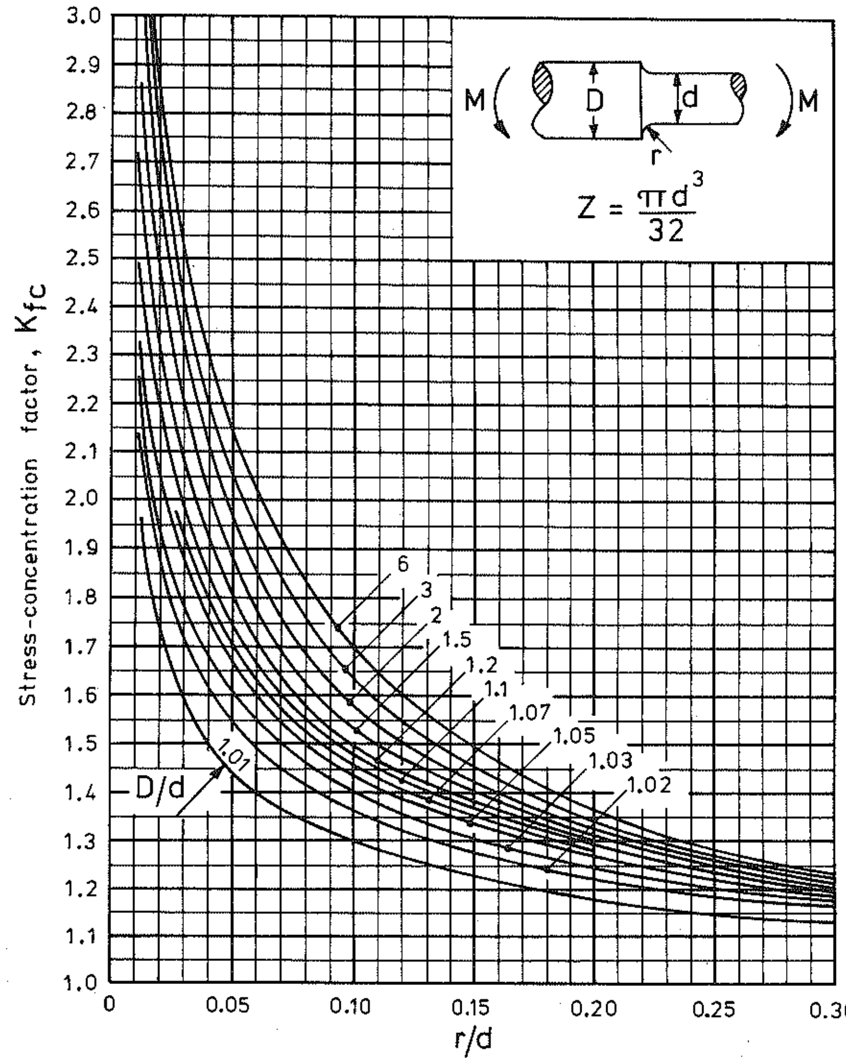
Combined stress-concentration factor
for stepped shaft in tension, K_{fTc}

CHART No 12



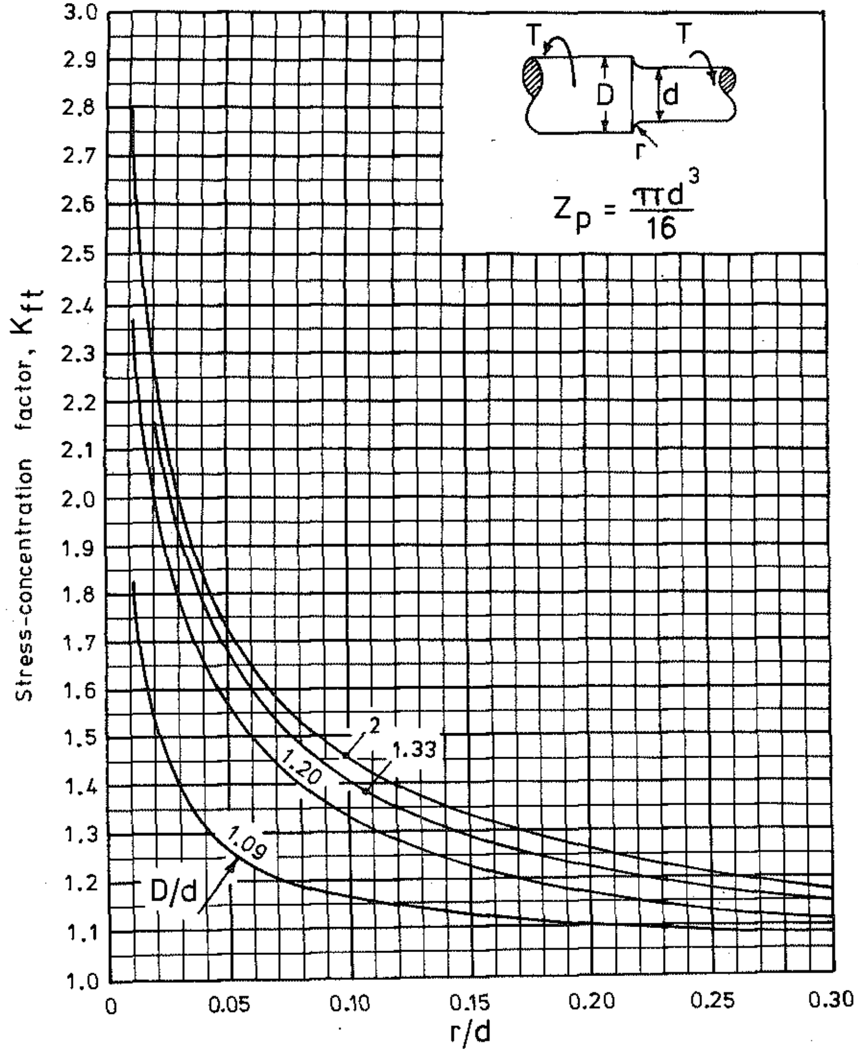
Stress-concentration factor for stepped shaft in bending, K_{fm}

CHART No 13



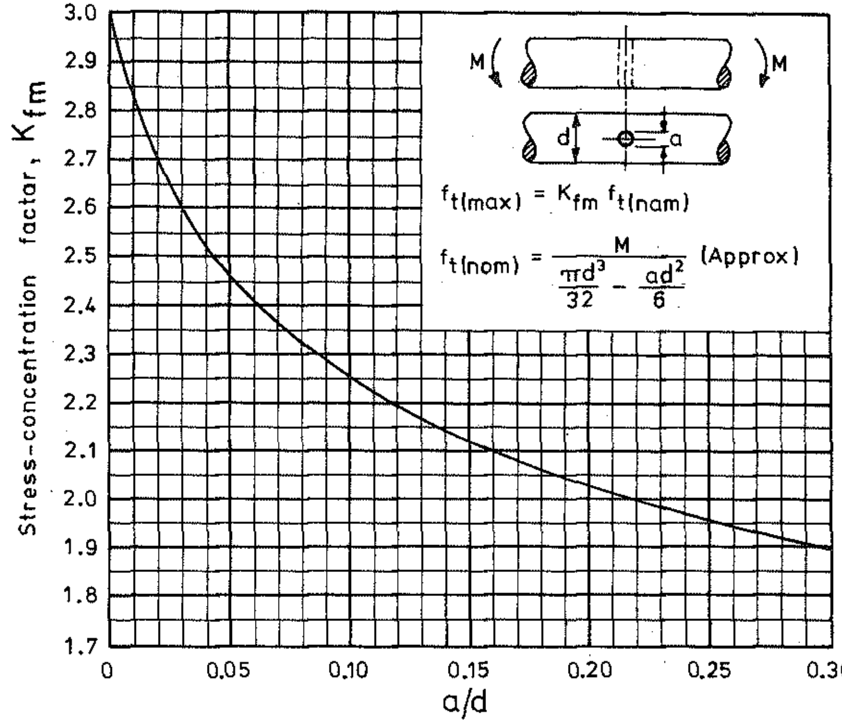
Combined stress-concentration factor for stepped shaft in bending, K_{fc}

CHART No 14



Stress-concentration factor
for stepped shaft in torsion, K_{ft}

CHART No 15



Stress-concentration factor
for a shaft with transverse hole, K_{fm}

CHART No 16

Exercises.

In the following exercises:

(i) use the AS 1250 Structural Code for static applications

(ii) use a basic F of S = 2.0 with F_e for endurance applications.

1. Determine the maximum tensile stress that occurs in the machine member shown in Fig. 11.

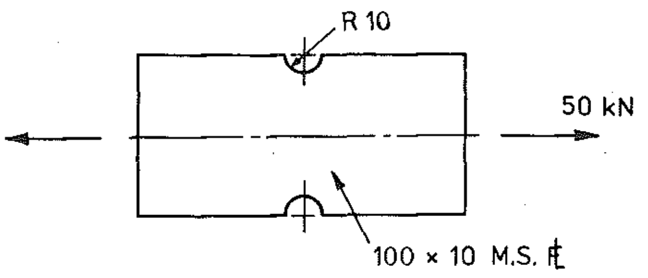


FIG. 11

Ans. 140.63 MPa

2. Refer to Fig. 11.

If the tensile force is replaced by a bending moment in the plane of the paper, assuming buckling is prevented, determine:

(a) the maximum permissible moment value if it is static.

(b) the maximum permissible moment value if it is continually reversed.

Ans. (a) 973.62 N.m
(b) 587.23 N.m

3. Determine the maximum tensile stress acting in the member shown in Fig. 12.

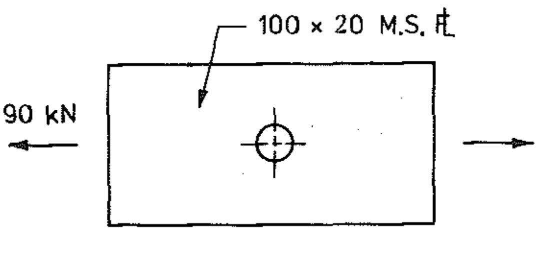


FIG. 12

(a) (i) if the hole is 6 mm in diameter.
(ii) if the hole is 30 mm in diameter.

(b) State F_t
(i) for static loading
(ii) for reversed loading

Ans. (a) (i) 143.62 MPa
(ii) 152.36 MPa

(b) (i) 150 MPa
(ii) 103.5 MPa

4. Refer to Fig. 13.

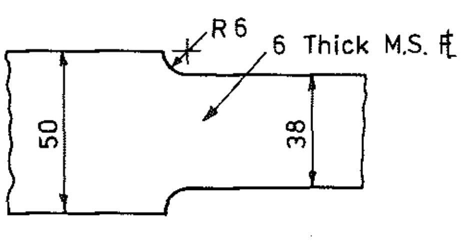


FIG. 13

(a) What is the maximum permissible static tensile loading that the member will take?

(b) What is the maximum permissible static bending moment in the plane of the paper that the member will take? Assume buckling is prevented.

Ans. (a) 20.56 kN
(b) 157.33 N.m

5. A stationary cantilever shaft of CS 1040 is shown in Fig. 14. It is subjected to cyclical loading of 52 N in one direction, to 156 N in the opposite direction.
Analyse the change of cross section with a Soderberg diagram.

Experimental notch sensitivity, $q = 0.9$.

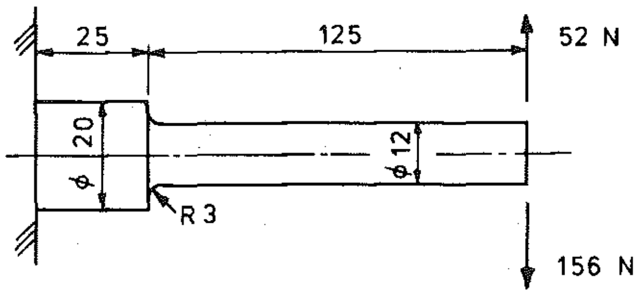


FIG. 14

[By mathematical definition

$$q = \frac{K_F - 1}{K_f - 1} \quad \text{or} \quad K_F = 1 + q \; (K_f - 1)$$

where q = an experimental notch sensitivity value due to stress concentration. Values range from 0 to 1.0.

K_f = theoretical value of stress concentration factor.

K_F = the actual value of the stress concentration factor.]

Safety factors to be as for normal shaft design:

2.0 on F_e, 1.67 on F_y.

Ans. f_m = 50.39 MPa
f_r = 100.77 MPa
(F_r = 83.64 MPa)
Intersection point outside Soderberg limit line.

6. A certain valve spring is made from steel with the following shear stress properties:

F_{se} = 310 MPa, F_{sy} = 550 MPa

The maximum force on spring = 300 N.
The minimum force on spring = 160 N.
Mean diameter = 55 mm
Wire diameter = 6 mm

Construct a Soderberg diagram to check whether the spring is safe. Use Wahl's factor and a safety factor of 2.0.

f_m = 172.8 MPa
f_r = 52.6 MPa
F_r = 57.6 MPa
} Shear stresses

Intersection point under Soderberg Limit Line.

Note:
The above method represents a conservative approach to spring design analysis. A more accurate method is offered by a "modified" Soderberg diagram. Refer to:

Hall, Holowenko, Laughlin,
Machine Design, SI Units,
McGraw - Hill Book Company, 1980.

7. Refer to Fig. 15.

The stationary axle shown is subjected to a fluctuating load which varies from 4.5 kN to 21.5 kN.

By doing a Soderberg analysis for the change of cross-section, determine the suitability of the design. Use CS1030 material. A shock factor of 1.5 is required in the calculations. Safety factors 2.0 and 1.67 as appropriate.

Ans. $f_m = 73.47$ MPa $f_r = 48.04$ MPa $F_r = 57.29$ MPa

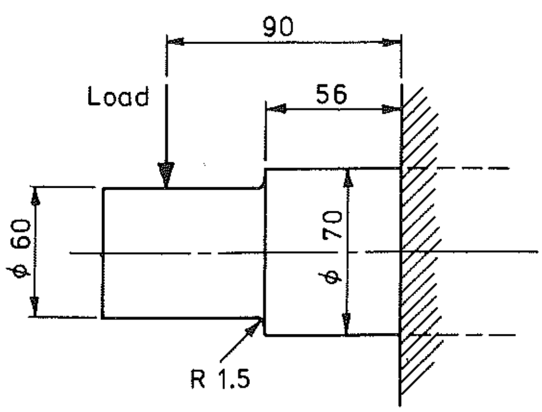


FIG. 15

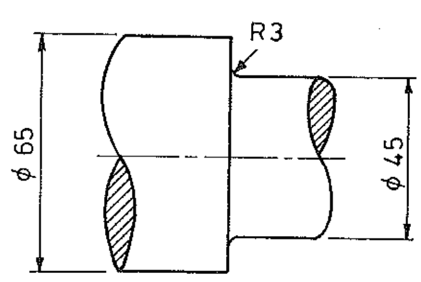


FIG. 16

8. Refer to Fig. 16.

The above shaft is subjected to repeated reversals of bending moment (550 N.m) and torque (310 N.m).

Shock factors on bending and torque = 1.5

Material of construction is CS1040.

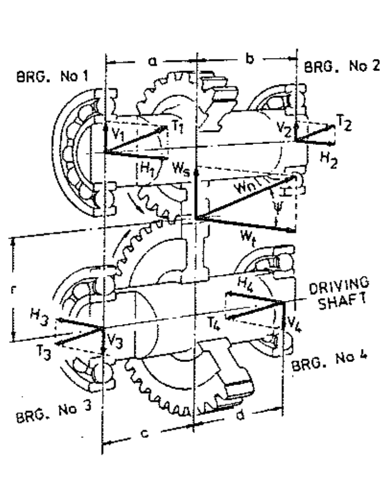
(i) Determine f_s and F_s and state whether the design is satisfactory using the maximum shear stress theory. F of S = 2.0.

(ii) Calculate the required size of the smaller diameter, d, using the 'Distortion energy theory'. F of S = 2.0.

Ans. (i) $f_s = 88.5$ MPa; $F_s = 60.75$ MPa (ii) 50.54 mm
Design unsatisfactory

CHAPTER 11

KEYS



CHAPTER 11

K E Y S   A N D   P I N S

A key is a removable fastener used to prevent relative rotational movement and generally axial movement also between two machine parts such as a pulley and shaft.

A key is made of key steel and is most commonly fitted partly into a shallow slot in the shaft and partly into a shallow slot in the hub of the pulley or gear. See Fig. 1.

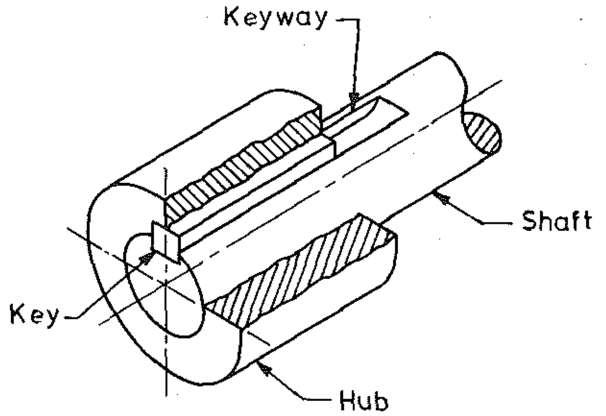


FIG. 1

For very light power requirements a set screw in the hub is satisfactory. The set screw is tightened against the shaft (or preferably against a flat spot on the shaft to enable the hub to be dismantled easily when required). For most applications however a more positive and stronger drive connection is required.

The following types of "keys" will be discussed:-

(1) Parallel keys
(2) Taper keys
(3) Woodruff keys
(4) Round keys
(5) Taper pins
(6) Hardened set screw drives

(1) Parallel Key. (Also called a feather key)

This type of key is a neat fit in the sides of the keyways in both the hub and the shaft (three classes of fit are available - free, normal and close). When it is assembled, there is a small amount of clearance between the top of the key and the bottom of the keyway in the hub. To prevent any axial movement of the hub, a set screw (often a square head, cup point screw) can be provided in the hub directly over the key. There are other more elaborate means also such as spacing collars together with an end plate on the shaft for locating a hub (see Fig. 8. Chapter 7). Parallel keys may be square or rectangular, refer to Fig. 2.

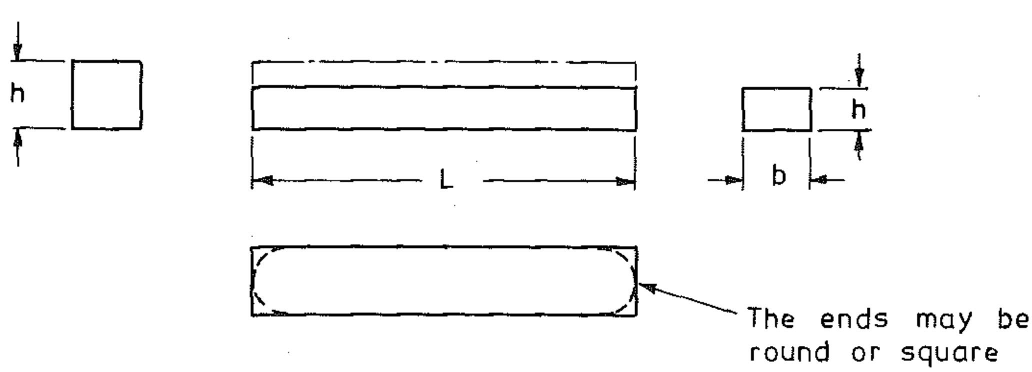


FIG. 2

There are two methods of machining keyways in shafts for parallel keys.

(a) Using a side and face cutter (Sled or Sledge runner keyway, figure 3a);

(b) Using an end mill (End milled keyway, figure 3b).

Generally, using a side and face cutter is the cheaper and easier way of machining a keyway particularly if it runs to the end of the shaft. But this type of keyway (figure 3a) requires a long "run-out" at the end of the keyway and this is not always desirable; also some provision must be made for preventing axial movement of the key.

With an end milled keyway (see figure 3b) the end (or ends) of the keyway can be used to prevent axial movement of the key in one (or both) directions.

See Table No. 1 for standard key sizes.

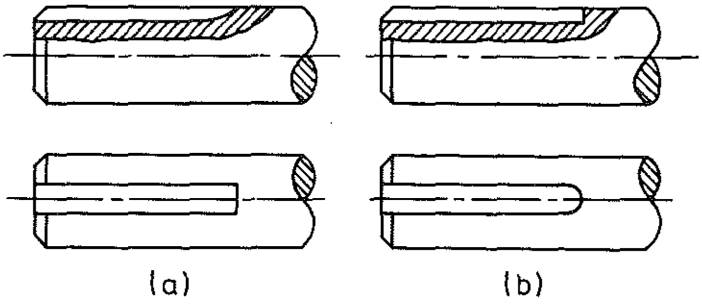


FIG. 3

*Table No. 1 (Reference BS 4235 : Part 1 : 1972 - Parallel and taper keys.)

Diameter Over	to	Section b x h	Diameter Over	to	Section b x h	Diameter Over	to	Section b x h
6	8	2 x 2	50	58	16 x 10	170	200	45 x 25
8	10	3 x 3	58	65	18 x 11	200	230	50 x 28
10	12	4 x 4	65	75	20 x 12	230	260	56 x 32
12	17	5 x 5	75	85	22 x 14	260	290	63 x 32
17	22	6 x 6	85	95	25 x 14	290	330	70 x 36
22	30	8 x 7	95	110	28 x 16	330	380	80 x 40
30	38	10 x 8	110	130	32 x 18	380	440	90 x 45
38	44	12 x 8	130	150	36 x 20	440	500	100 x 50
44	50	14 x 9	150	170	40 x 22			

(2) Taper Keys

The taper key is tapered on the top of the key only; the depth of the keyway in the hub is also tapered to match. The key is tapered so that it can be driven into place to form a self locking wedge between the hub and the shaft. It wedges the hub and the shaft solidly together thereby producing a large frictional force between them.

When assembled there are three classes of fit available for the sides of the key - free, normal and close.

Gib head keys are good for very high torque drives especially in cases where the drive is reversible.

Types of taper keys

(a) Gib-head key. Fig. 4.

This is the most common type of taper key. The purpose of the gib-head is to provide a means of readily withdrawing the key from its assembled position. The key is usually of rectangular cross-section but may be square.

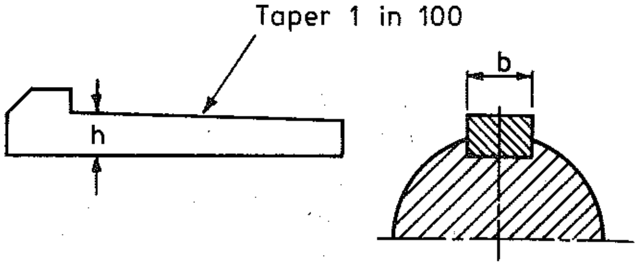


FIG. 4

(b) Plain Taper Key. Fig. 5.

The thicker end is sometimes extended in length so that a hole can be drilled through this portion of the key. This hole serves the same purpose as the gib-head.

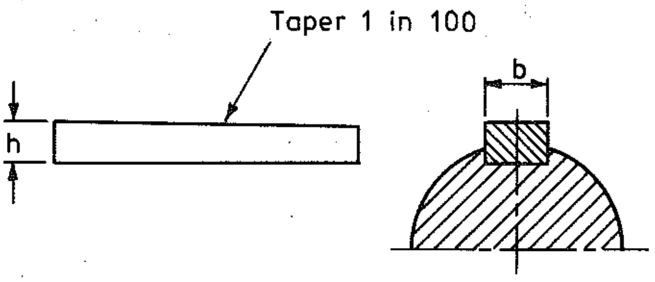


FIG. 5

There are two methods of machining keyways in shafts for taper keys. They are the same as for parallel keys.
See Table No. 1 for standard key sizes.

(3) Woodruff Keys

The woodruff key has the shape of a segment of a circle and is manufactured from a round bar. The width of the key b is usually about 1/6, or slightly more, of the diameter of the shaft. Refer to Fig. 6.

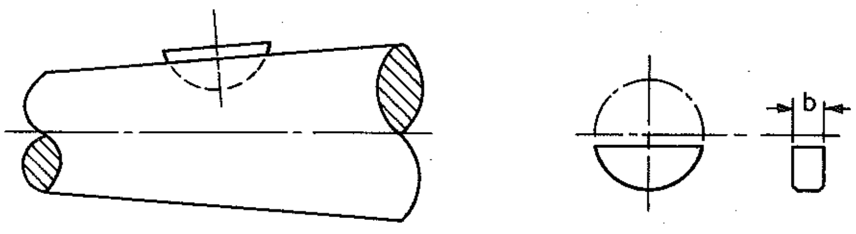


FIG. 6

The keyway in the shaft, cut with a special key-seat cutter, is deep compared with the keyways for other types of keys and the tendency of the key to roll over when transmitting the load is considerably reduced.

A normal keyway is cut in the hub. The key is a transition fit in the keyway in the shaft and a clearance fit in the keyway in the hub.

The Woodruff key is generally used for light drives and is specially suitable for use with tapered shafts because it can align itself with the hub.

Key and Keyway sizes are given in BS 4235 : Part 2 : 1977 (ISO 3912 - 1977).

(4) Round Key. Fig. 7.

The keyways for a round key are not cut in the usual manner. Instead, a hole is drilled parallel to the axis of the shaft, half in the shaft and half in the hub. The drilled hole is then reamed either parallel or taper and the appropriate pin is inserted.

Sometimes the hole is tapped and a screw is used as the key (which is then called a "Scotch Key"). This has the advantage that the one item prevents both rotational and axial movement.

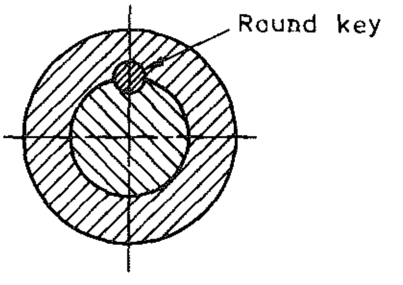


FIG. 7

The round key is suitable for heavy loads since the absence of sharp corners reduces the high stress concentrations common to rectangular keyways. However, the round key is only suitable for use when the end of the shaft and the hub are flush.

Design of Keys

Transmission of loads by parallel and taper keys

(a) Parallel Keys. Fig. 8.

When the key is transmitting the load from the hub to the shaft, the hub presses on the key with a force F1; the key in turn presses on the shaft which reacts with a force F2 on the key. These two forces are not in line hence they tend to roll the key. This rolling is prevented by the reactions F3 and F4 from the hub and the shaft.

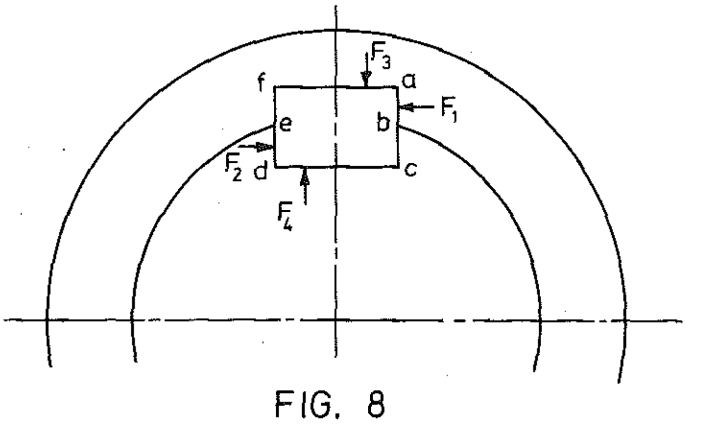


FIG. 8

The forces F1 and F2 cause a crushing stress which for simplicity is assumed to be uniform on the faces ab and ed. These forces (F1 and F2) also cause a shear stress in the key. Again, for simplicity, this stress is assumed to be uniformly distributed over the area resisting the shear. Thus a key must be designed to resist crushing and shearing forces.

(b) Taper Keys. Fig. 9.

When the taper key is driven into position, the hub and the shaft are drawn solidly together on the side opposite the key. When the load is first applied, a frictional force is developed between the shaft and hub. When the load becomes too great for the frictional force to transmit it, slight slippage occurs and the load is then carried (or assumed to be carried) by crushing and shearing forces on the key.

In designing a taper key the friction force is always ignored and the key is treated the same way as a parallel key.

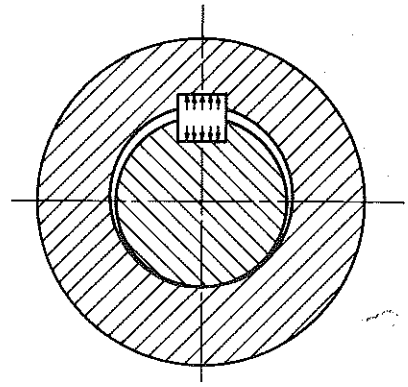


FIG. 9

Assumptions.

For design purposes the following assumptions are made:

(Note: None of these assumptions are true in actual practice but they are simple and lead to a satisfactory design.)

(a) The force F acts on the key at the surface of the shaft (i.e. a distance $\frac{D}{2}$ from the centre of the shaft) see Fig. 10.

(b) The force F is evenly distributed along the length of the key.

(c) The crushing stress is uniform over the portion of the side of the key in the keyway.

(d) The shearing stress is uniform over the area resisting shear.

(e) Friction does not assist in carrying the load.

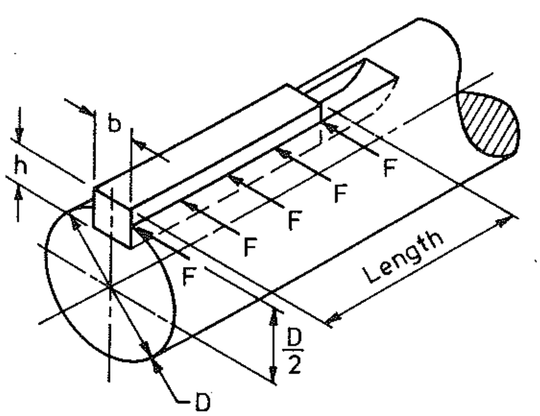


FIG. 10

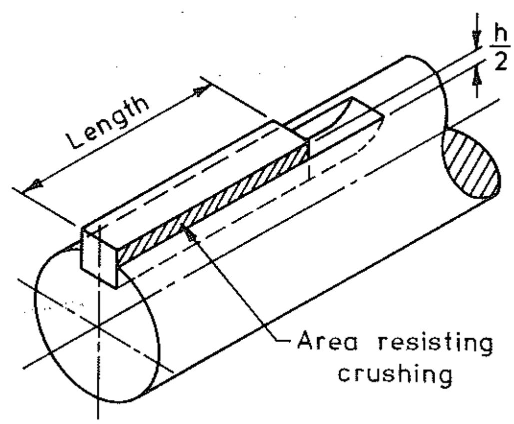


FIG. 11

Strength of a Key

When designing a key two stress checks are necessary:-

(a) Crushing (or bearing) stress. Fig. 11.
The weaker element in the connection is the governing factor in this check. This is generally either the shaft or the hub.
Refer to Table No. 1 in Chapter No. 1, page 1.20.

Area resisting crushing (bearing) = A_c

$$A_c = \text{length of key} \times \frac{\text{thickness of key}}{2}$$

$$A_c = L \times \frac{h}{2}$$

If F_c is the allowable average crushing (bearing) stress, the force which can be transmitted by the key is equal to

$$\underline{\underline{F_c \times A_c}}$$

(b) Shearing stress of key. Fig. 12.

Area resisting shear = A_s

$$A_s = \text{length of key} \times \text{width of key}$$
$$= L \times b$$

If F_s is the allowable shearing stress for the key, the force which can be transmitted by the key is equal to

$$\underline{\underline{F_s \times A_s}}$$

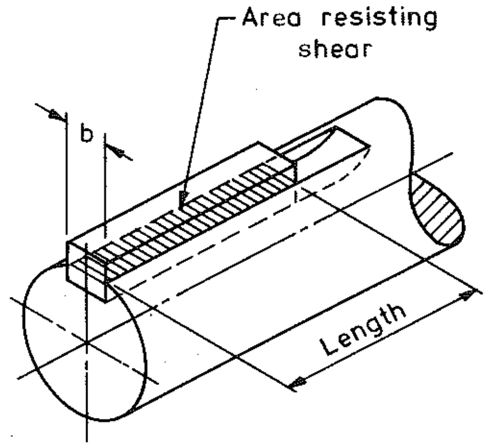


FIG. 12

"Two - Key" Designs

Sometimes in exceptional circumstances two keys at 90° or 120° are required to transmit a drive. This design should be avoided where possible due to the accurate machining required to line up the keyways in the hub and shaft.

Length of a Key

An ideal proportion for the length of a key is 1.5 times the diameter of the shaft but this is by no means mandatory.

Worked Example No. 1.

Glossary of additional terms

P =	power transmitted in kW	ω =	shaft speed in radians/sec.
T =	torque, N.m	f_c =	actual crushing (bearing) stress, MPa.
N =	shaft speed in r.p.m.	f_s =	actual shearing stress on key, MPa.

Check the design of the following key for driving a large gear.

Power 30 kW at 20.9 r.p.m.
Key size = 32 x 18 x 175 long
Shaft diameter = 120 mm.
Material of key F_y = 414 MPa

Material of shaft and gear F_y = 250 MPa.

Answer.

Refer to Table No. 1 in Chapter No. 1. (Page 1.20.)

$F_c = 0.5\ F_y$

$= 0.5 \times 250 = 125$ MPa (This stress may be increased to 150 MPa in accordance with Note 2 (i) on page 1.22 and Summary page 1.23.)

$F_s = 0.25\ F_y$

$= 0.25 \times 414 = 103.5$ MPa 62.5

$$T = \frac{1000\ P}{\omega} \quad \left(\text{and } \omega = \frac{2\ \pi\ N}{60}\right) = \frac{1000 \times 30 \times 60}{2\ \pi\ 20.9}$$

$\frac{500 \times 60}{2\pi\ 3} = 1592$ Nm

$\therefore\ T = 13\ 707$ N.m

$$\text{Tangential force "F" on key} = \frac{13\ 707}{0.06} = 228\ 450\ \text{N}$$

265 26 N

$$f_c = \frac{228\ 450}{9 \times 175} = 145\ \text{MPa}\ (> 125\ \text{MPa} < 150\ \text{MPa})$$

61.

$$f_s = \frac{228\ 450}{32 \times 175} = 40.8\ \text{MPa}\ (< 103.5\ \text{MPa})$$

Result: - Key design safe.

Woodruff keys

The strength of a Woodruff key is calculated in the same way as that of a parallel or taper key.

Round Key

The key transmits the load by crushing and shearing forces. Its strength is determined in the same way as that of a parallel or taper key. In this case, the area resisting crushing = projected area of half the key

$= L \times \frac{d}{2}$ where d = dia. of key

Area resisting shear = $L \times d$

(5) Taper Pins

Taper pins are used as fasteners for light drives. They are usually fitted through the shaft and the hub on a diameter as shown in Fig. 13(a), but may be fitted with the centre line of the pin tangential to the shaft and hub as shown in Fig. 13(b).

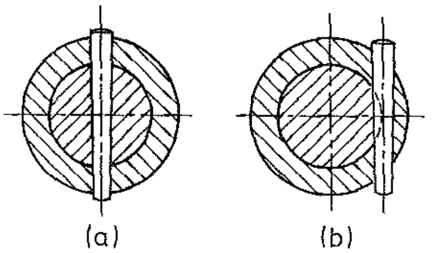


FIG. 13

For highly stressed shafts, the position shown in Fig. 13(b), is preferable as it does not cause such a large stress concentration as the position shown in Fig. 13(a),but is more difficult to produce hence it is usually not done.

The hole for the taper pin should be drilled and reamed with a taper reamer after the parts have been assembled.

Since taper pins are suitable only for relatively light loads, when inserted as shown in Fig. 13a, they can serve as safety shear pins designed to shear off when a certain predetermined overload is reached. The pin will fail without causing much damage to the hub or shaft. In use, taper pins are subjected to double shear.

For simplicity in calculations, the mean diameter is assumed to be the diameter of the large end of the pin. This, of course, is not correct, but is sufficiently accurate considering that, in practice, neither the load nor the strength of the material is known with any certainty.

The taper of the pin is 1 in 48 on the diameter.
The diameter of a taper pin used on a particular job is usually about one quarter of the shaft diameter.

Determination of Pin Size

Glossary of terms

F = tangential driving force on one side, N

F_s = allowable shear stress in pin, MPa

T = torque being transmitted, N.mm

r = radius of shaft, mm

D = diameter of shaft, mm

d = diameter of taper pin, mm

$$\frac{\pi d^2}{4} F_s = F$$

Also $$F = \frac{T}{r} \times \frac{1}{2} = \frac{T}{D}$$

$$\frac{\pi d^2}{4} F_s = \frac{T}{D}$$

From which $$d = 1.13 \sqrt{\frac{T}{DF_s}}$$

Worked Example No. 2

Refer to Fig. 14

A hollow M.S. shaft (F_y = 250 MPa) from a special gear box has to have a universal coupling attached. The torque to be transmitted is 152 000 N.mm.

Determine the size of mild steel taper pin.

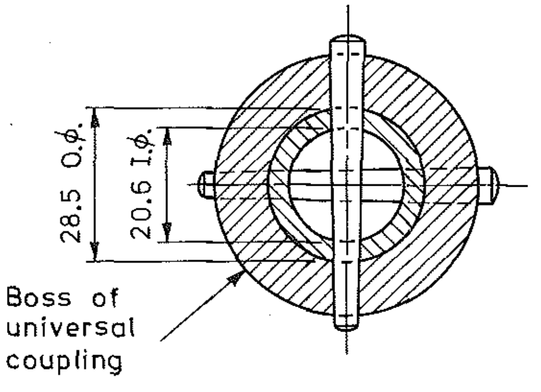


FIG. 14

Answer

As for a parallel or taper key use (in this case all materials are M.S.)

$$F_s = 0.25\ F_y = 0.25 \times 250 = 62.5 \text{ MPa (Refer page 1.20)}$$

$$F_c = 0.5\ F_y = 0.5 \times 250 = 125 \text{ MPa (Refer page 1.20)}$$

Preliminary design has indicated that two taper pins at 90° will be necessary.

$$d = 1.13 \sqrt{\frac{T}{DF_s}} = 1.13 \times \sqrt{\frac{76\ 000}{28.5 \times 62.5}} = 7.38 \text{ mm}$$

Nearest standard size is 8 mm. As the shaft is hollow the crushing (bearing) area is small therefore a check will have to be made.

$$F = \frac{76\ 000}{14.25 \times 2} = 2666.7 \text{ N}$$

$$f_c = \frac{2666.7}{8 \times (14.25-10.3)} = 84.4 \text{ MPa} < 125 \text{ MPa}$$

Result:- Taper pin design satisfactory.

(6) Hardened Set Screw Drives

Refer to Fig. 15.

Hard set screws are frequently used to fix the hub to the shaft when the load to be transmitted is comparatively small.

There is no standard relationship between the shaft size and the size of the set screw. However, the following formula is a good guide.

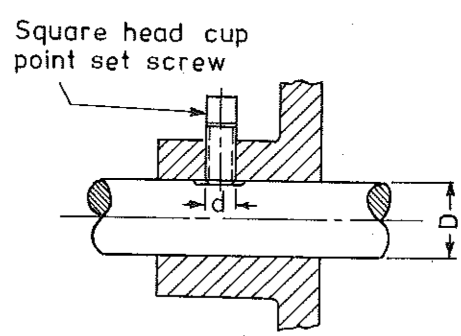


FIG. 15

$$\text{Dia. of set screw} = \frac{D}{4} + 3 \quad \text{where } D = \text{shaft dia. in mm.}$$

Experiments have been made in an attempt to determine the safe holding force of a set screw. The results of such experiments must be reduced by a fairly large factor to allow for the uncertainty of the tightening of the screw and of the uncertainty of the frictional forces involved. By doing this, the following formula was obtained.

Safe Holding Force $F = 11\ d^2$ N where d is the diameter of the set screw in mm.

It is advisable to file or machine a flat on the shaft to act as a seat for the end of the screw. This will facilitate disassembly as the hub & shaft will be prevented from binding together because of shaft scoring.

Worked Example No. 3

A pulley is to be fixed to a 16 mm diameter shaft which revolves at 200 r.p.m. Select a suitable set screw and determine the maximum power which can be transmitted by it.

Answer

Dia. of set screw $d = \frac{D}{4} + 3$ where d = dia. of set screw.
D = dia. of shaft.

$$= \frac{16}{4} + 3 = 7 \text{ mm}$$

As this is not a standard size, use 8 mm diameter.

Safe holding force $= 11\, d^2\, N = 11 \times 8^2 = 704\ N$

$$P = \frac{T\omega}{1000} = \frac{704 \times 0.008 \times 2\pi \times 200}{1000 \times 60} = 0.118 \text{ kW}$$

Exercises.

1. A 75 mm diameter shaft revolves at 330 r.p.m. A vee pulley transmitting 50 kW is keyed to the shaft with a 22 x 14 x 128 long parallel key.

   Materials of construction:

   key.............. F_y = 414 MPa

   shaft............ CS1040

   vee pulley....... grey cast iron

   Check the safety of the key design.

   State: (a) f_s and F_s

   (b) f_c and F_c

   (a) 13.7 MPa
   103.5 MPa

   (b) 43.1 MPa
   135 - 162 MPa

2. A 30 mm diameter shaft is to transmit 2 kw at 100 r.p.m. A sprocket is to be attached to the shaft with a M.S. (F_y = 250 MPa) taper pin.

   Determine the minimum size of the pin.

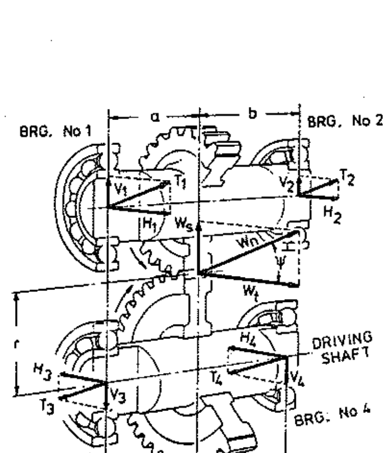
   11.4 mm

3. Repeat Exercises 1 and 2 for when the drive motors are started D.O.L. with 220% F.L.T.

   Ex. 1. (a) 13.7 MPa (b) 43.1 MPa Ex. 2. 14 mm
   69 MPa 90-108 MPa

CHAPTER 12

STRUCTURAL DESIGN
to
AS 1250-1981



CHAPTER 12
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STRUCTURAL DESIGN TO AS 1250 - 1981

(Includes Machine Design Applications
- Bolt Design (pages 12.44)
- Weld Design (pages 12.55)

Structural design work is governed by the AS 1250 - 1981 SAA Steel Structures Code. The stress tables and other information given in this Code greatly simplifies otherwise difficult problems.

Glossary of terms:-

All terms are specified in the notation listed in the Code.

This Chapter is intended primarily for mechanical designers who are involved in minor structural design e.g., small support systems, bolted, pinned and welded joints etc.

Specifically the following topics will be covered:-

NOTE: The values of F_y for various thicknesses of rolled steel sections, flat and plate, are given in Appendix C of AS 1250.

The "simple design method" will be used throughout this chapter.
This method assumes that members are joined by connections having relatively low moment capacity. These are coloquially known as pin jointed structures.
Refer to Rule 3.2.2 (a) and (c) AS 1250.

(1) TENSION MEMBERS

In an axially loaded tension member, the average tensile stress calculated on the effective sectional area shall not exceed

$$F_{at} = 0.60\ F_y \qquad \text{Rule 7.1}$$

The general equation for axial stress in tension members is

$$f_{at} = \frac{P}{A_s}$$

A_s is the gross cross-sectional area in cases where the stress distribution is clearly uniform.

It is more often the case in practice however, that end connections produce a non-uniform stress distribution within the member. This means that an effective cross-sectional area has to be determined.

No allowance for stress concentration around holes need be made in tension members such as are discussed in this section. (When designing pinned joints, Section 6, stress concentration allowance is required.)

Connections in structural tension members shall be designed for at least the forces stated in Rules 7.5.1, 7.5.2 and 9.1 AS 1250.

Connections in tension members for general mechanical equipment have no particular governing clauses.

Effective cross-sectional areas

A tension member which is bolted at one or both ends clearly has a reduced effective cross-sectional area caused by the bolt holes.

The usual practice is for a bolt hole to be 2 mm larger in diameter than the bolt. The minimum size bolt commonly used in structural work is 16 mm diameter, except for "non-strength" members, such as handrails etc., where it is permissible to use 12 mm diameter bolts.

Worked Example No. 1.

Determine the maximum value of P applicable in Fig. 1 (a) and (b). Base calculations on plate value only.

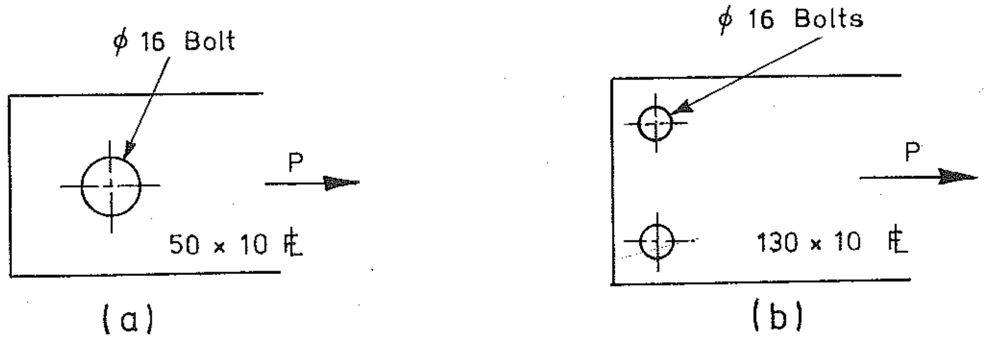


FIG. 1

Answer

Fig. 1 (a)

$A_s = (50-18)\ 10 = 320\ \text{mm}^2$

$F_y = 260$ MPa (App. C AS 1250)

$P = 0.6 \times 260 \times 320$

$= \underline{49\ 920\ \text{N}}$

Fig. 1 (b)

$A_s = (130-2 \times 18)\ 10 = 940\ \text{mm}^2$

$F_y = 260$ MPa (App. C AS 1250)

$P = 0.6 \times 260 \times 940$

$= \underline{146\ 640\ \text{N}}$

Staggering of fasteners

Sometimes fasteners are staggered in order to keep the width of the joint in a tension member to a minimum. In such cases it is possible for a critical section to be along a zig-zag line through the holes as shown in Fig. 2, page 12.3.

The procedure for calculating the effective cross-sectional area is given in Rule 4.4.2 in AS 1250. This rule is known as "Cochranes $\frac{s^2}{4g}$ rule". × mean thickness

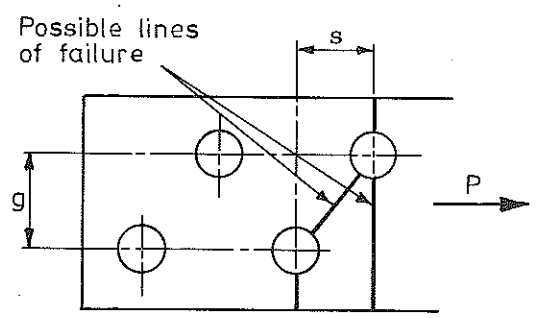


FIG. 2

Worked Example No. 2.

Determine the critical section of the plate shown in Fig. 3.

Answer

Firstly consider section A-A

$$A_s = 90 \times 12 - 12\ (16 + 2)$$

$$= 864\ \text{mm}^2$$

Secondly consider section A-B

$$A_s = 90 \times 12 - \{2T\ (\phi + 2) - \frac{s^2\ T}{4g}\}$$

$$= 1080 - \{2 \times 12 \times (16 + 2) - \frac{40^2 \times 12}{4 \times 40}\}$$

$$= 768\ \text{mm}^2$$

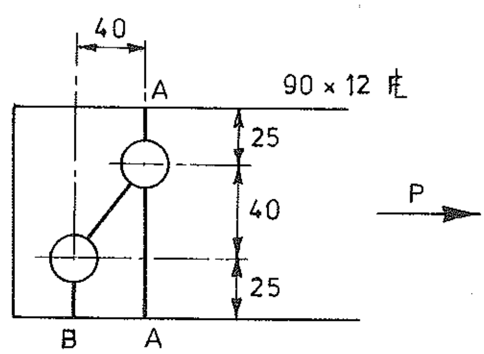


FIG. 3

Hence the critical section is along A-B.

The above method can be applied to handle cases of three or more staggered holes.
Refer to Fig. 4.

The sections to be checked are:-

XACZ (Use previous formula)

XABCZ In this case the formula becomes:-

Area to be deducted from gross area wT

= Σ sum of sectional areas of holes A, B, and C

$$- \left(\frac{s_1{}^2\ T}{4g_1} + \frac{s_2{}^2\ T}{4g_2} \right)$$

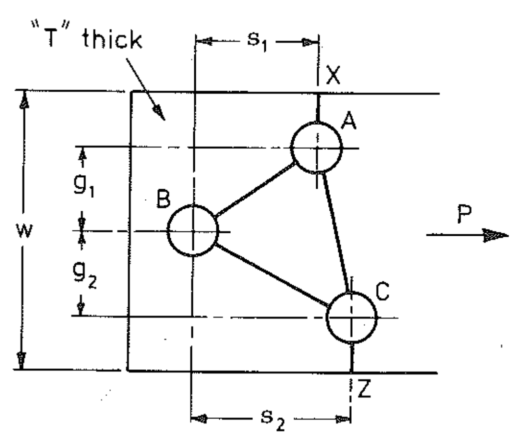


FIG. 4

In the case of non-planar sections such as angles, the gauge is measured along the centre line of the thickness, see Fig. 5. The formula for A_s is then applied in the same manner as before.

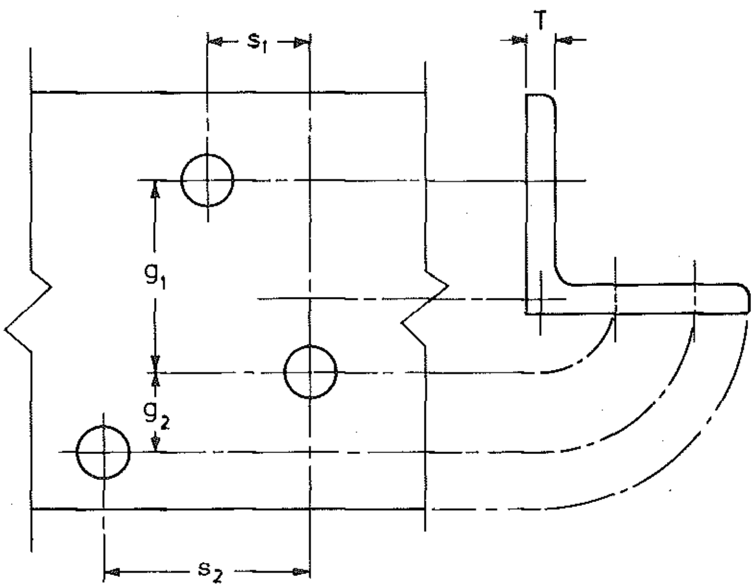


FIG. 5

The formula for critical section determination when fasteners are staggered can be rationalised to give a value of minimum stagger for cases where a deduction of one hole only is required.

This formula becomes,

Minimum stagger, mm, $s_{min.} = 2\sqrt{g\,\phi}$

where g = gauge, mm

ϕ = hole diameter, mm

Minimum and maximum edge and pitch distances for bolts are given in Section 9 of the AS 1250 Code.

End connections providing non-uniform stress distribution

If the end connections of a tension member do not provide uniform force distribution in the member then a "reduced" or "effective" area must be used to compute the average tensile stress f_{at}.

The effective cross-sectional areas for a number of rolled steel sections under various loading conditions can be determined from Rule 7.3.2 in AS 1250.

Maximum slenderness ratio of tension members

Slenderness ratio $= \frac{\ell}{r}$

where ℓ = effective or unsupported length of member.

r = least radius of gyration of the net section of the member between connections.

Table 4.6 AS 1250 limits the slenderness ratio of tension members to 300 maximum. The purposes of this restriction are:-

(a) to reduce noise caused by braces vibrating against other components

(b) to avoid the difficulties experienced in applying sufficient force to straighten members to a visually acceptable degree.

Excessively slender tension members may indeed be ineffective due to the fact that too great a movement of end connection points is required to enable a tension force to be taken.

Axial tension and bending.

A member subjected to both axial tension and bending shall be proportioned in accordance with Rules 8.2 and 8.3.2 AS 1250.

Exercises

1. Determine the maximum permissible value of P in Fig. 6. Base calculation on plate value only.

(158 400 N)

2. Find the maximum permissible value of P in Fig. 7. Base calculation on angle value only.

(180 911.5 N)

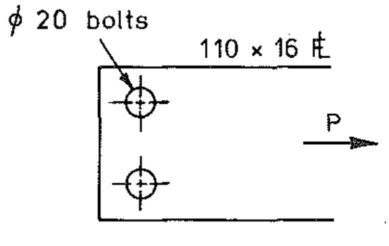


FIG. 6

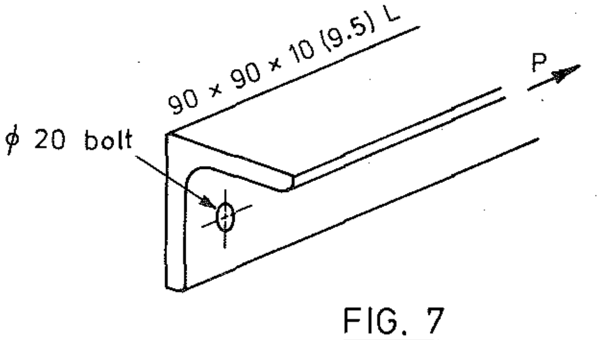


FIG. 7

(2) BEAMS

Discussion will be confined to cantilevers and simply supported beams. Only rolled steel sections will be considered, as follows:-

(i) Universal beams and columns, taper flange beams and channels.

(ii) Angles.

(iii) Solid round and square bars.

(iv) Solid rectangular bars.

(v) Hollow sections.

The design of plate girders and compound beams is a more involved process which is outside the scope of this book.

(a) Beams bent about the axis of maximum strength

(i) Universal beams and columns, taper flange beams and channels

In general the following factors should be considered when designing a beam:-

(aa) Bending stresses.

(bb) Shear stress.

(cc) Web crushing due to compression.

(dd) Web buckling due to compression.

(ee) Deflection (check to see that it is not unacceptably large).

(ff) Web buckling due to bending load (not applicable to the rolled steel members listed in this section).

(gg) Web buckling due to shear (not applicable to the rolled steel members listed in this section).

(aa) Bending stresses

Most of the rolled steel sections listed above (in 2 (a) (i)) are what is termed "compact" i.e., the thicknesses of the flanges and webs are large enough to prevent both:

local buckling of the section

lateral buckling of the section (not to be confused with "lateral buckling" of the beam as a whole).

The Sections will therefore reach their full plastic moment unless the beam as a whole is slender, $\ell/r > 70$ approx., in which case it may buckle laterally "as a whole".

The exceptions are a few of the universal columns in the remote instance when they are used in beam action.

Absolute maximum permissible stress (compression or tension)

$F_b = 0.66\ F_y$ Rule 5.2(2) AS 1250

Maximum permissible compressive stress

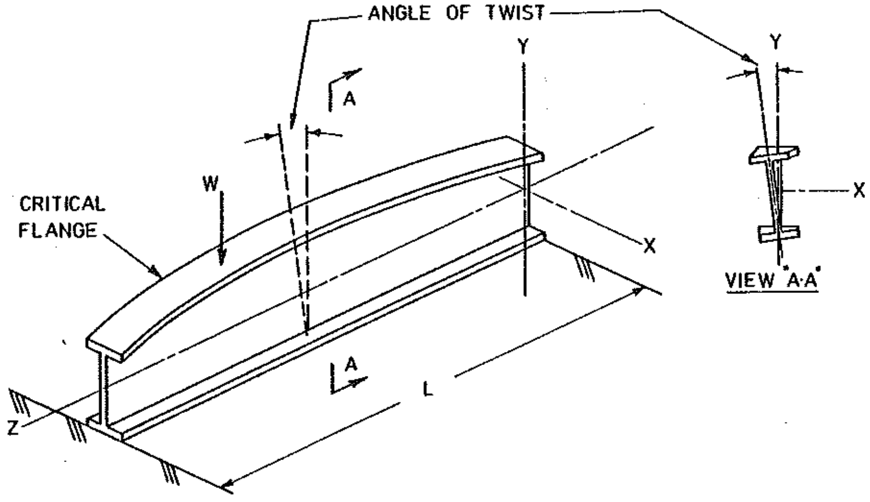
Apply Rule 5.3 AS 1250.

Mechanical designers would not normally be concerned with this rule. It mainly applies to I sections with very thin flanges. The only rolled steel sections mentioned in 2(a)(i) to be affected by the above rule are a few of the universal columns when used as beams.

Lateral stability

Torsional flexural buckling of a beam

The strength of a beam will be reduced if it is not restrained from lateral (sideways) movement. Torsional flexural buckling (colloquially termed lateral buckling) occurs when a beam as a whole "twists" and moves sideways, see Fig. 8, page 12.7. Table 5.4.1 (3) AS 1250 gives permissible stresses (tension or compression) for the rolled steel sections mentioned under 2(a)(i), page 12.6.



TORSIONAL FLEXURAL BUCKLING OF A SIMPLY SUPPORTED BEAM WITHOUT INTERMEDIATE LATERAL RESTRAINTS

FIG. 8

Two values are required before a permissible stress can be determined from the above table,

$$\frac{\ell}{r_y} \quad \text{and} \quad \frac{D}{T}$$

Effective length of a beam "ℓ"

Simply supported beam

This is the effective length between lateral restraints to the critical flange (see "Critical flange", page 12.8)

Alternatively, in the absence of such restraints, it is, for the simplified approach used in this book, the span of the beam. However, there are certain conditions which necessitate the effective length being multiplied by factors. These conditions are discussed in some of the following sections, see page 12.8.

Effective length is the length used in the slenderness ratio $\frac{\ell}{r_y}$ when determining the maximum permissible bending stress from Table 5.4.1 (3) AS 1250.

Cantilever

See page 12.9 (second paragraph).

Effective span of a beam "L"

Effective span is used for calculating in-plane bending moments and deflections.

For a simply supported beam it is generally the distance between the centres of the "stiff bearing" (see Fig. 9, page 12.8) portions of the supports.

For a cantilever it is the distance from the support to the outer end of the beam.

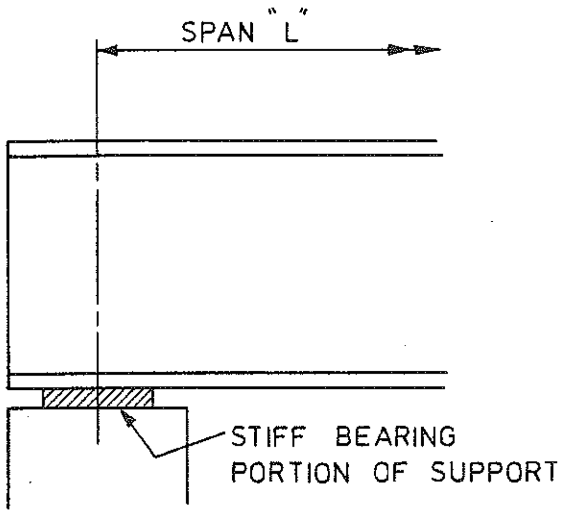


FIG. 9

Critical flange

This is best explained with two examples; a simply supported beam and a cantilever both with gravity loads on either upper or lower flange.

The critical flange of the simply supported beam is the upper (compression) flange.

The critical flange for the cantilever is the upper (tension) flange. However, once the outer end of a cantilever is braced the critical flange for any intermediate bracing is the compression flange.

Lateral restraints should be capable of taking 2½% of the force in the critical flange. Rule 3.3.4.3 AS 1250.

Beams (including cantilevers) with critical flange loading unrestrained laterally

Rule 5.9.6 AS 1250.

If a load is applied to the critical flange and both the flange and load are free to move laterally the effective length of the beam "ℓ" for the purposes of determining the maximum permissible bending stress shall be multiplied by 1.20.

Crane wheels are an example of an unrestrained load. However, it is worth noting that most loads bring with them their own constraint e.g., an incoming beam.

Beams on seats

Rule 5.9.3.2 AS 1250.

This rule concerns simply supported beams which are supported on their bottom flanges only, see Fig. 10, page 12.9. There is clearly very little torsional end restraint to prevent the beam at that point from twisting.

The rule covers this by requiring a 20% increase in effective length "ℓ" when determining the maximum permissible bending stress i.e. a 1.20 factor. This is additional to the 1.20 factor described in the previous section if applicable.

An alternative approach would be to use load bearing stiffeners to keep ϕ zero at the ends. The design of these stiffeners would have to be in accordance with Rule 5.13.2.4 AS 1250 (refer to section (c) in Worked Example No. 3, page 12.15).

A cantilever without lateral restraints (a basic cantilever) must have full torsional restraint at the support if "ℓ" is to be taken as equal to "L". Effective lengths of cantilevers with other degrees of torsional restraint and/or lateral supports are given in Rules 5.9.4 and 5.9.5 AS 1250.

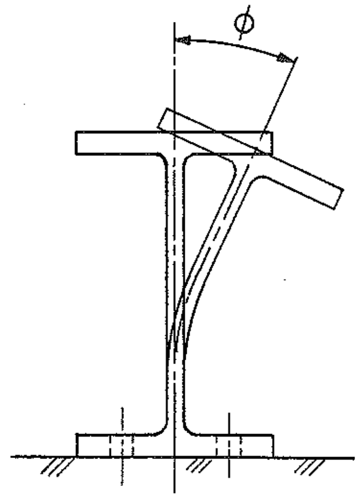
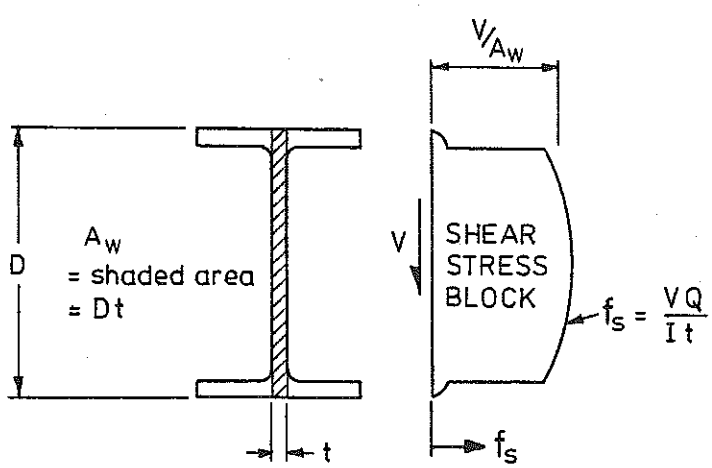


FIG. 10

(bb) Shear stress

Transverse shear stress is the only shear stress of interest in the selection of a rolled steel section. This is the vertical shear across the beam section (perpendicular to the neutral axis). This shear stress varies over the beam section, from zero at the extreme fibres, to a maximum at the neutral axis. Because of this, the flanges of an I beam or channel contribute very little to the beam's resistance to transverse shear (see Fig. 11).



SHEAR IN AN I SECTION

FIG. 11

AS 1250 gives two alternative methods to be used regarding the shear stress problem. Both methods give the same net result.

The absolute maximum permissible shear stress in a beam, F_{vm} = 0.45 F_y Rule 5.10.1 AS 1250.

However, if "average" shear stress is used, the maximum permissible shear stress, F_v = 0.37 F_y Rule 5.10.2 (1) AS 1250.

Additionally $F_v = \dfrac{592\ 000}{(d_1/t)^2}$, Rule 5.10.2 (2) AS 1250

However, this rule only controls the design of the web when $\frac{d_1}{t} \geq 80$ (for $F_y = 250$ MPa).

Rule 5.10.2 (2) AS 1250 has no influence on any of the rolled steel sections mentioned in 2(a)(i), page 12.6.

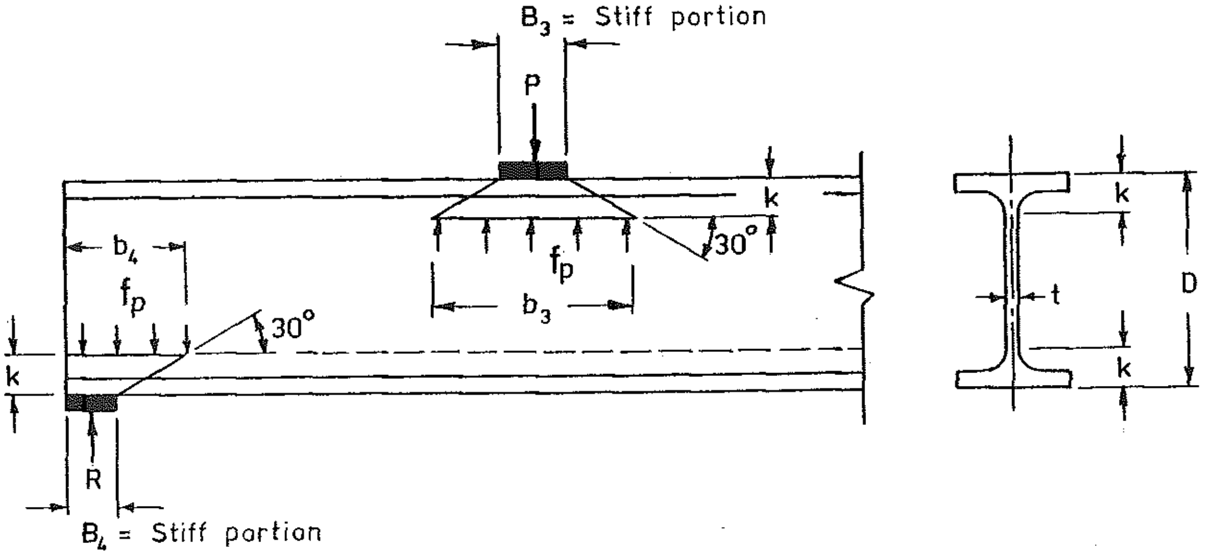
The effective sectional area of an I beam or channel

A_W = D.t, Rule 5.10.4 (a) AS 1250.

$$f_v = \frac{\text{Maximum shear force "V"}}{A_W} \leq 0.37\ F_y$$

(cc) Web crushing or crimping due to compression.

At points in a beam where relatively heavy loads are concentrated, the roof of the web is subjected to a maximum bearing stress and this may crush the web, see Fig. 12.



CRUSHING OF THE WEB AT THE JUNCTION WITH THE FLANGE FILLET BY CONCENTRATED LOADS AND REACTIONS ACTING ON THE FLANGES.

FIG. 12

At the support a similar situation arises. There is a vertical reaction at the bottom of the beam but there is no compression on the top flange. Clearly the vertical compression diminishes with height and so the maximum bearing stress occurs at the narrowest section; this is where the web fillet meets the straight of the web.

AS 1250 specifies an angle of dispersion for the loads of 30°.

The maximum permissible bearing stress is $F_p = 0.75\ F_y$, Rule 5.11 AS 1250. If the permissible bearing stress is exceeded a stress reduction can be achieved as follows:-

Increasing the length of stiff bearing.

Choosing a beam with a greater web thickness.

Providing load bearing stiffeners. These would be designed according to Rule 5.13.2.1 AS 1250.

It is worth mentioning however that it is uncommon for a rolled steel section to require load bearing stiffeners. The case cited in Worked Example No. 4, page 12.17 was an extreme one.

(dd) Web buckling due to compression

This is another effect brought about by point loads and reactions. These cause high local compression forces in the web of a beam and so the web acts as a column; it will tend to buckle at the neutral axis in its weak direction.

The rolled steel sections being considered each have a web thickness of sufficient size to ensure that web crushing occurs first.

Only "built up" beams with relatively thin webs will buckle before crushing takes place.

(ee) Deflection

Beam deflections are always calculated using basic (unfactored) loads.

Large deflections in a beam do not necessarily represent a structural failure. However, it is usual to limit the deflection of structural members for the following reasons:-

Asthetics

Excessive deflection gives a poor and "unsafe looking" appearance. The total deflection at a point is not necessarily given by the beam deflection but may be the cumulative effect of a number of members in the structure (supports etc).

Springiness and poor vibration characteristics

Platforms for example should be limited to say 10 mm deflection under live load otherwise they feel unsafe to personnel.

Possible damage to attached materials

This would include such items as sheeting and plaster.

AS 1250 Appendix A gives suggested only limits on beam deflection. These are:-

main beams, $\frac{\text{span}}{360}$ or $\frac{\text{cantilever length}}{180}$

The ratio of $^{L}/_{D}$ of a beam can give a "rule of thumb" indication of how important deflection will be in the design. As a general guide, if the $^{L}/_{D}$ ratio is greater than about 25 then deflection will be the governing factor and not stress.

Determination of beam deflection.

Simple cases

Deflection formulas for elementary loadings are given in Chapter No. 1, page 1.17 under the heading "Commonly used formulas and values".

Complex cases

Most structural design text books contain useful tables giving deflection formulas for many unusual loading conditions.

Sometimes however, a fundamental analysis must be done. Two cases arise:

(aaa) Multiple loadings with constant M of I.

(bbb) Multiple loadings with varying M of I.

(aaa) - One straight forward method is to use the principle of superposition. Refer to Worked Example No. 6, page 12.20.

(bbb) - The solution to this type of problem is best done by the moment of area method i.e. graphical integration. Refer to "Deflection of beams - the area-moment method", page 12.73.

(ff) and (gg) - These points are not applicable to the design of rolled steel sections.

(ii) Angles

The loads for these are based on an elastic torsional analysis and a series of associated experiments. The use of the AISC (Australia) "Safe Load Tables for Structural Steel" is recommended. Rule 5.4.2. AS 1250.

(iii) Solid round and square bars

The maximum permissible bending stress $F_b = 0.75\ F_y$ Rule 5.2(1) AS 1250.

(iv) Solid rectangular bars

These present a severe buckling problem. Formulas offering solution can be taken from "Formulas for Stress and Strain" by Roark and Young.

(v) Hollow sections

Hollow sections are not usually affected by any type of buckling unless the wall thickness is unusually thin.

The effective cross-sectional areas for the purpose of shear stress calculations are given in Rule 5.10.4(b) and (c) AS 1250.

Manufacturers recommend that the maximum permissible bending stress, $F_b = 0.60\ F_y$.

(b) Beams bent about the axis of minimum strength.

As buckling is a search for a minimum energy position and a beam bent about its weak axis is already in its most stable, lowest energy state, it cannot buckle laterally, i.e. fail in the torsional-flexural buckling mode.

For a solid rectangular bar

$$F_b = 0.75\ F_y \quad \text{Rule 5.2 (1) AS 1250}$$

For hollow sections

$$F_b = 0.60\ F_y \text{ (conservatively) manufacturer's recommendation}$$

For other sections

$$F_b = 0.66\ F_y \quad \text{Rule 5.2 (2) AS 1250.}$$

Maximum permissible compressive stress

See Rule 5.3 AS 1250.

This rule mainly applies to I beams which have very thin flanges. Some universal columns are governed by the above rule when used in beam action.

(c) Beams with biaxial bending.

Apply Rule 8.3.3 AS 1250.

GENERAL SUMMARY

The design details (as appropriate) given in Section 2(a)(i) apply in principle to all beams. This section has been covered quite extensively.

Shear stress was not mentioned in all sections because the provision is the same for all beams. i.e.,

$$F_{vm} = 0.45\ F_y \quad \text{Rule 5.10.1 AS 1250,}$$

unless some "effective area" is used in an "average" shear stress calculation in which case

$$F_v = 0.37\ F_y \quad \text{Rule 5.10.2 (1) AS 1250.}$$

$$F_v = 592\ 000/\left(\frac{d_1}{t}\right)^2 \quad \text{Rule 5.10.2 (2) AS 1250.}$$

Allowable bearing stress is the same for all parts of a beam design,

$$F_p = 0.75\ F_y \quad \text{Rule 5.11 AS 1250.}$$

Maximum permissible slenderness ratio of a beam,

$\frac{\ell}{r_{min}}$, is to be 300 according to Table 4.6 in AS 1250.

Worked Example No. 3.

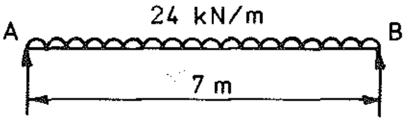


FIG. 13(a)

Refer to Fig. 13(a)
Neither the 610 UB 101 nor the load have lateral supports between A and B. The beam is to have torsional end restraints.
The length of the stiff bearing at each end support = 100 mm

Fully design the beam.

Answer

(a) Firstly check the bending stress. The load and critical flange are free to move laterally therefore the factor on effective length = 1.20, Rule 5.9.6 AS 1250.

The beam has torsional end restraints therefore the factor on effective length = 1.0.

$$\ell = 7000 \times 1.2 \times 1.0 = 8400 \text{ mm}$$

$$r_y = 47.5 \text{ mm (B.H.P. Handbook)}$$

$$\frac{\ell}{r_y} = \frac{8400}{47.5} = 177 < 300$$

$$\frac{D}{T} = 40.7 \quad \text{(B.H.P. Handbook)}$$

Table 5.4.1(3) AS 1250 $F_{bcx} = \underline{\underline{71 \text{ MPa}}}$

$$W = 24\,000 \times 7 + 101\, g \times 7$$

$$= 174.9 \times 10^3 \text{ N}$$

$$\text{Maximum bending moment} = \frac{WL}{8}$$

$$= \frac{174.9 \times 10^3 \times 7000}{8}$$

$$= 153 \times 10^6 \text{ N.mm.}$$

$$f_{bcx} = \frac{M}{Z_x} = \frac{153 \times 10^6}{2510 \times 10^3} = \underline{\underline{61 \text{ MPa}}} < 71 \text{ MPa}$$

Beam is safe in bending.

(b) Shear stress check

$$\text{Maximum shear force, } V = \frac{W}{2} = 87.45 \times 10^3 \text{ N}$$

$$f_v = \frac{V}{A_w}$$

$$= \frac{87.45 \times 10^3}{602.2 \times 10.6} = \underline{\underline{13.7 \text{ MPa}}}$$

Next determine maximum permissible shear stress.
Greatest section thickness = T = 14.8 mm from B.H.P. Handbook.

$$F_v = 0.37\, F_y, \quad \text{Rule 5.10.2(1) AS 1250}$$

$F_y = 250$ MPa, Appendix C AS 1250

$\therefore \quad F_v = 0.37 \times 250$

$= \underline{\underline{92.5 \text{ MPa}}} > 13.7$ MPa

Beam is safe in shear.

(c) Check web crushing.
Refer to Fig. 12, page 12.10.

$b_4 = B_4 + k \cot. 30°$

$= 100 + 27.5 \cot. 30° = 147.6$ mm

$$f_p = \frac{W}{2} \times \frac{1}{b_4 t} = \frac{174.9 \times 10^3}{2 \times 147.6 \times 10.6}$$

$$= 55.9 \text{ MPa}$$

$F_p = 0.75 F_y = 0.75 \times 250 = 187.5 \text{ MPa} > 55.9$ MPa

Stiffeners will not be required for the load but will be required to provide torsional end restraint as specified in the question. These will have to be designed in accordance with Rule 5.13.2.4 AS 1250.

$$I_s = \frac{D^3T}{125} \times \frac{R}{W}$$

$$= \frac{602.2^3 \times 14.8}{125} \times \frac{1}{2}$$

$$= 12.93 \times 10^6 \text{ mm}^4$$

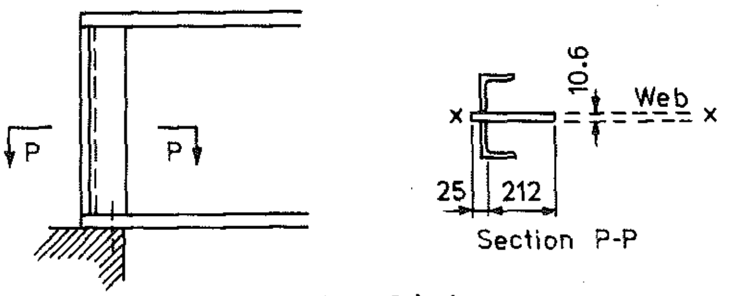


FIG. 13(b)

Refer to Fig. 13(b).

Try 2 ———— 102 x 102 x 8 angles.

Length of web contributing to stiffener section

= 20t, Rule 4.3.1(b) AS 1250

= 20 x 10.6 = 212 mm

Total area of stiffener

$$A = 2 \times 1540 + 237 \times 10.6 = 5592 \text{ mm}^2$$

I_{xx} of stiffener

$$= 2(1.5 \times 10^6 + 1540 \times 79.4^2) + \frac{237 \times 10.6^3}{12}$$

$$= 22.44 \times 10^6 \text{ mm}^4 > 12.93 \times 10^6 \text{ mm}^4$$

Additionally each support system has to withstand a lateral force of 2½% of the maximum flange force, acting at the centroid of the critical flange, Rule 5.9.2.1 AS 1250.

In this case (refer to Fig. 13(c))

F (conservatively) = Area of flange x maximum fibre stress

$= 61 \times 227.6 \times 14.8$

$= 205.48 \times 10^3$ N

2½% F = 5137 N

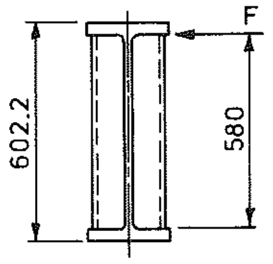


FIG 13(c)

$$f_{bcx} = \frac{M}{Z} = \frac{5137 \times 580}{22.44 \times 10^6} \times 107.3$$

$$= 14.25 \text{ MPa} < 0.66 F_y$$

(d) Check deflection.

$$\delta = \frac{5}{384} \frac{WL^3}{E I}$$

$$= \frac{5}{384} \frac{174.9 \times 10^3 \times 7000^3}{207\,000 \times 757 \times 10^6}$$

$$= \underline{\underline{5 \text{ mm}}}$$

Maximum permissible deflection

$$= \frac{L}{360}, \text{ Appendix A AS 1250}$$

$$= \frac{7000}{360} = \underline{\underline{19.4 \text{ mm}}} > 5\text{mm}$$

orked Example No. 4.

Refer to Fig. 14(a)

Design load bearing stiffeners for the load application point of a cantilever beam.

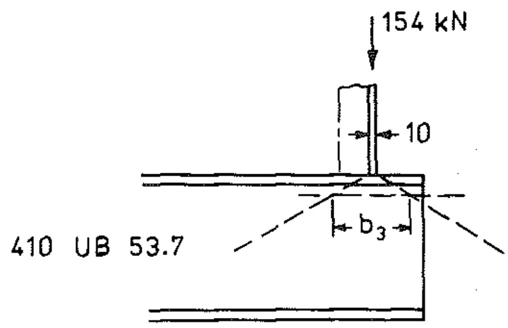


FIG. 14(a)

nswer

Firstly we will prove that load bearing stiffeners are required.

Refer to Fig. 12, page 12.10.

$b_3 \;=\; B_3 + 2\ k \cot 30°$

$\;\;\;\;\;= \;10 + 2 \times (10.9 + 10.2) \cot 30°$

$\;\;\;\;\;= \;83$ mm

Largest section thickness of beam

$= \;10.9$ mm $\;\therefore\; F_y \;=\; 260$ MPa, Appendix C AS 1250.

$F_p \;=\; 0.75\ F_y$, Rule 5.11 AS 1250

$\;\;\;\;\;=\; 0.75 \times 260 \;=\; 195$ MPa

$$f_p = \frac{\text{Reaction}}{b_3\ t}$$

$$= \frac{154 \times 10^3}{83 \times 7.6} = 244 \text{ MPa} > 195 \text{ MPa}$$

Therefore load bearing stiffeners are required in accordance with Rules 5.13.2.2(b) and 5.13.2.3 AS 1250.

Try 2 ———— 76 x 76 x 8 angles, Fig. 14(b).

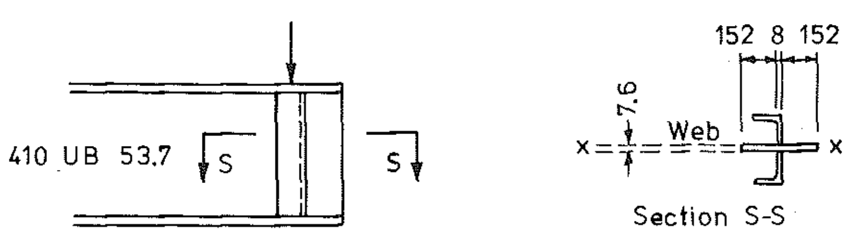


FIG. 14(b)

Length of web contributing to stiffener section:

20t (each side) = 20 x 7.6 = 152 mm.

Total area of stiffener

$$A = 312 \times 7.6 + 2 \times 1140$$

$$= 4651 \text{ mm}^2.$$

I_{xx} of stiffener

$$= 2\,(0.609 \times 10^6 + 1140 \times 58.2^2) + 312 \times \frac{7.6^3}{12}$$

$$= 8.95 \times 10^6 \text{ mm}^4$$

$$r_x = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{8.95 \times 10^6}{4651}} = 43.87 \text{ mm}$$

$$\frac{\ell}{r_x} = \frac{0.7 \times 380.8}{43.87} = 6 \;;\; F_{ac} = 156 \text{ MPa, Table 6.1.1 AS 1250.}$$

(Fy for angles = 260 MPa)

$$f_{ac} = \frac{154\,000}{4651} = 33 \text{ MPa} < 156 \text{ MPa}$$

$\therefore$ Stiffeners adequate.

Worked Example No. 5.

Determine the required diameter of a structural mild steel bar which is to support 1 - tonne.

Refer to Fig. 15 for further details.

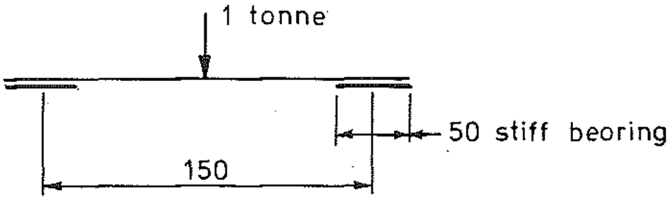


FIG. 15

Answer

(a) Bending stress.

$$B.M. = \frac{WL}{4} = \frac{1000\text{ g} \times 150}{4} = 367.875 \times 10^3 \text{ N.mm}$$

$$B.M. = F_b\, Z$$

$$\therefore \text{ required } Z = \frac{BM}{F_b} \text{ ------------------- (1)}$$

$F_b = 0.75\, F_y$ Rule 5.2(1) AS 1250

F_y = 250 MPa (assumed)

$$\therefore F_b = 0.75 \times 250 = 187.5 \text{ MPa}$$

Substituting into equation (1)

$$\therefore \quad Z = \frac{367.875 \times 10^3}{187.5} = 1962 \text{ mm}^3 = \frac{\pi d^3}{32}$$

from which $d = 27.14$ mm

nearest standard size = $\underline{\underline{30 \text{ mm}}}$

(b) Shear stress

Maximum shear force, $V = \frac{1000\ g}{2} = 4905$ N

(self weight of bar negligible)

$$f_{vm} = \frac{4}{3} \times \frac{V}{A} \quad \text{(Refer to page 12.63)}$$

$$= \frac{4905}{\frac{\pi}{4} \times 30^2} \times \frac{4}{3} = 9.3 \text{ MPa}$$

$F_{vm} = 0.45\ F_y$, Rule 5.10.1 AS 1250

$F_y = 250$ MPa, Appendix C AS 1250

$$\therefore \quad F_{vm} = 0.45 \times 250 = 112.5 \text{ MPa} > 9.3 \text{ MPa}$$

(c) Bearing stress at support

$$f_p = \frac{R}{\text{projected area}}$$

$$= \frac{4905}{30 \times 50} = 3.3 \text{ MPa}$$

$F_p = 0.75\ F_y$ Rule 5.11.1 AS 1250

$= 0.75 \times 250$

$= 187.5$ MPa > 3.3 MPa

(d) Deflection

$$\delta = \frac{WL^3}{48\ EI}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi 30^4}{64} = 39.76 \times 10^3 \text{ mm}^4$$

$$\therefore \quad \delta = \frac{4905 \times 150^3}{48 \times 207\ 000 \times 39.76 \times 10^3} = 0.04 \text{ mm}$$

$$\delta_{max} = \frac{L}{360} \qquad \text{Rule A2.1 AS 1250}$$

$$= \frac{150}{360} = 0.42 \text{ mm} > 0.04 \text{ mm}$$

Worked Example No. 6.

Determine the deflection at end A of the beam shown in Fig. 16(a).

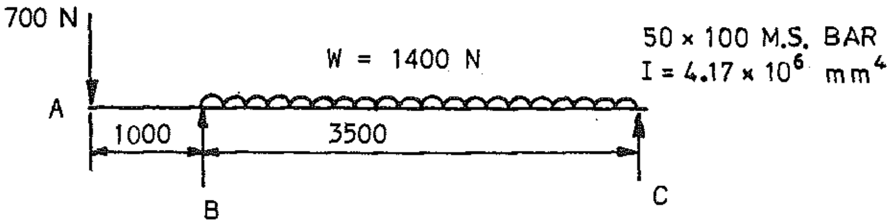


FIG. 16(a)

Answer

The deflection of A is the sum of δ_1 produced by the U.D. load and δ_2 produced by the concentrated load each computed independently.

Firstly the U.D. load, refer to Fig. 16(b)

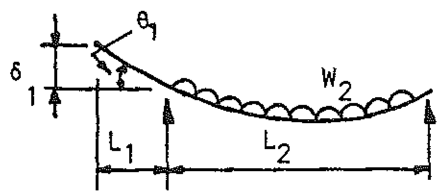


FIG. 16(b)

$$\theta_1 = \frac{W_2 \, L_2^2}{24 \, E \, I}$$

and $\delta_1 = L_1 \, \theta_1$

$$= 1000 \times \frac{1400 \times 3500^2}{24 \times 207\,000 \times 4.17 \times 10^6}$$

$$= 0.83 \text{ mm up}$$

δ_2 is the sum of the deflection the 700 N load would produce if the beam were fixed at the left support and the deflection due to the fact that it actually slopes there, refer to Fig. 16(c).

$$\delta_2 = \frac{W_1 \, L_1^3}{3 \, E \, I}$$

$$= \frac{700 \times 1000^3}{3 \times 207\,000 \times 4.17 \times 10^6}$$

$$= 0.27 \text{ mm down}$$

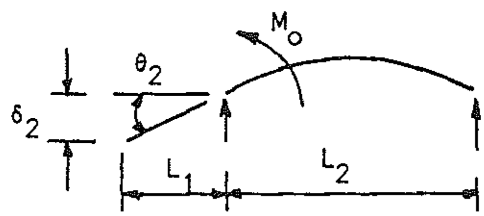


FIG. 16(c)

Deflection due to slope

$$\theta_2 = \frac{M_o \, L_2}{3 \, E \, I} \qquad \text{and} \quad \delta_2 = \theta_2 \, L_1$$

$$\therefore \delta_2 = \frac{700 \times 1000 \times 3500}{3 \times 207\,000 \times 4.17 \times 10^6} \times 1000$$

$$= 0.95 \text{ mm down}$$

Resultant downward deflection

$$= 0.95 + 0.27 - 0.83 = \underline{\underline{0.39 \text{ mm}}}$$

EXERCISES

3. A 200 x 75 x 22.9 parallel flange, channel beam has a span of 4000 mm. Torsional end restraints are provided. A central laterally restrained load of 2.75 tonne is applied. This load is bolted to the support beam.

Determine:

(a) f_{bcx} and F_{bcx} (143.6 MPa; 161.76 MPa)

(b) f_v and F_v (11.62 MPa; 96.2 MPa)

(c) The actual and maximum permissible deflections.

(9.6 mm; 11.1 mm)

4. Determine the minimum size of square structural mild steel bar to support a concentrated load of 0.75 tonne over a span of 135 mm. Neglect self weight of bar. Assume Fy = 250 MPa. For your chosen size of bar, state:

(a) f_b and F_b (20 x 20 bar, 186.24 MPa, 187.5 MPa)

(b) f_{vm} and F_{vm} (13.8 MPa, 112.5 MPa)

(c) The actual and maximum permissible deflections. (0.137 mm, 0.375 mm)

5. A 610 UB 101 supports a U.D. load of 58 kN/m over a span of 7 m. It has lateral supports to the critical flange every 1 m.
Torsional end restraints are provided.

Determine:

(a) f_{bcx} and F_{bcx} (144 MPa, 165 MPa)

(b) f_v and F_v (32.3 MPa, 92.5 MPa)

(c) The actual and maximum permissible deflections. (11.8 mm, 19.4 mm).

6. A 250 UB 31.4 is loaded as shown in Fig. 17.

Determine the deflection at the centre of the span.

Ans. (2.88 mm inc. self wt.)

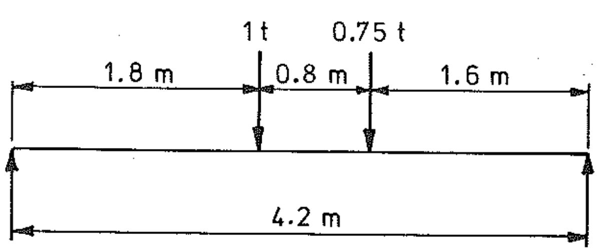


FIG. 17

(3) COMPRESSION MEMBERS

The following types of compression members will be considered:

(i) struts and columns

(ii) eccentrically loaded columns and beam columns.

(i) Struts and columns

Typical compression members are columns in building frameworks and struts in triangulated frameworks; the axial force is assumed to act along the gravity axis of the member.

The two main factors to be considered are:

(a) slenderness ratio, $\frac{\ell}{r_{min}}$ typically. (Limits, refer Table 4.6, AS 1250.)

(b) end conditions.

End connection conditions govern the effective length of a strut or column. Fig. 18, page 12.23 gives details.

Table 6.1.1 AS 1250 lists permissible stresses for compression members according to slenderness ratio and F_y. The Table is based on formula 6.1.1. Formula 6.1.1 reduces to $F_{ac} = 0.60\ F_y$ at low slenderness ratios (say $\frac{\ell}{r_{min}} < 10$).

No deduction in cross-sectional area need be made because of bolt holes in a strut or column. Specifically this would apply at the end connections and/or splices. Refer to Rule 4.1(b)(ii) AS 1250.

Design of angles as struts

Refer also to the companion book 'Applied Structural Design' for a comprehensive design discussion.

Most applications of angle struts occur in trusses, transmission towers and tower type supports. In the majority of these cases, the angle is connected through one leg only and so the member is subject to both axial load and bending.

(a) Single angle, single bolted struts will generally deflect about the vv axis. Slenderness ratio $= L/r_{vv}$ and provided that this is greater than approximately 100, the eccentric loading may be ignored and one half of the appropriate stress given in Table 6.1.1, AS 1250, used. This is a conservative approach.

(b) Single angle, multiple bolted or welded struts. Refer to the companion book.

Worked Example No. 7

Find the greatest load a 250 UC 89 could take in each of the three cases shown in Fig. 19, page 12.24.

Self weight of column $= 89.5 \times 5g \div 10^3 = 4.39$ kN

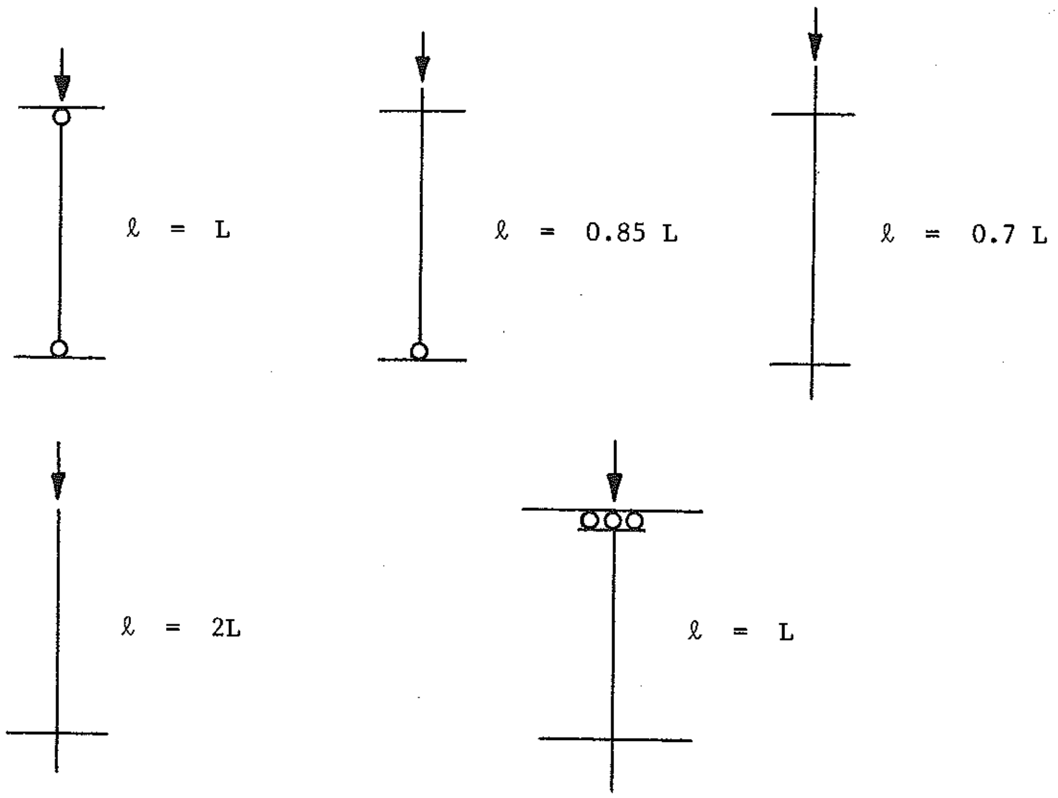
Answer

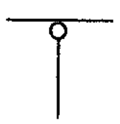
Case (1) $\frac{\ell}{r_y} = \frac{5000}{65.2} = 76.7$

Cont. on page 12.24

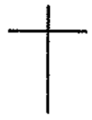
Design of Columns to AS 1250 - 1981

Refer to Rules 6.3.1 and 6.3.2 (a) and (b).



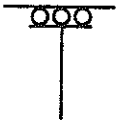


Free to rotate but not deflect.



Not free to do anything, i.e. fixed.

Free to do anything.



Free to deflect, but not to rotate.

These rules cover intermediate and long columns $\frac{\ell}{r_{min}} > 10$

For short columns $\frac{\ell}{r_{min}} < 10$ the maximum permissible compressive stress is 0.6 Fy.

FIG. 18

Maximum section thickness of 250 UC 89

= 17.3 mm $\therefore$ F_y = 250 MPa, Appendix C AS 1250

From Table 6.1.1 AS 1250 F_{ac} = 108 MPa

$\therefore$ Maximum permissible load = F_{ac} x area of cross-section - self weight.

$$= \frac{108 \times 11\ 400}{1000} - 4.39 = 1226.81 \text{ kN}$$

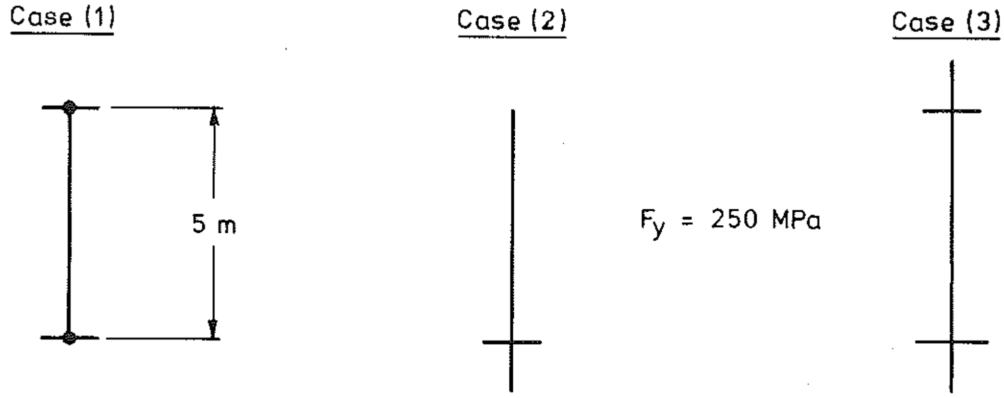


FIG. 19

Case (2)

$$\frac{\ell}{r_y} = \frac{2 \times 5000}{65.2} = 153.4$$

From Table 6.1.1 AS 1250 F_{ac} = 38 MPa

$\therefore$ Maximum permissible load $= \frac{38 \times 11\ 400}{1000} - 4.39 = 428.81$ kN

Case (3)

$$\frac{\ell}{r_y} = \frac{5000 \times 0.7}{65.2} = 53.7$$

From Table 6.1.1 AS 1250 F_{ac} = 133 MPa

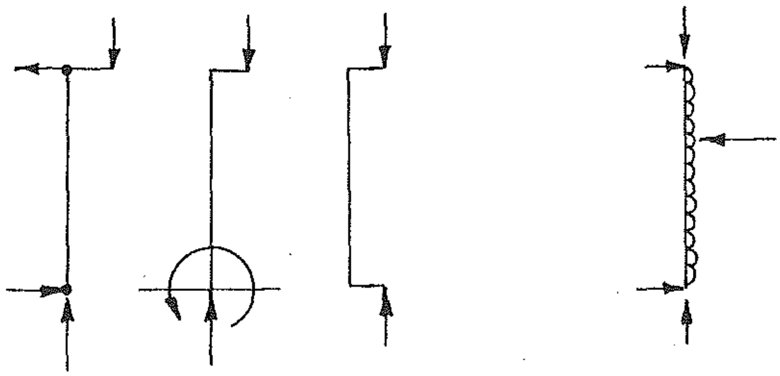
$\therefore$ Maximum permissible load $= \frac{133 \times 11\ 400}{1000} - 4.39 = 1511.81$ kN

(ii) Eccentrically loaded columns and beam columns

This section deals with two different loading conditions encountered with columns:

(a) Eccentrically loaded columns

With these columns the axially applied load does not coincide with the gravity line of the column; hence a bending moment and an axial stress occur simultaneously. This is in fact the most common type of column. See Fig. 20(a).



(a) (b)

FIG. 20

(b) Beam columns

These columns have an axially applied load occurring simultaneously with a bending moment caused by transverse loading. See Fig. 20(b).

Both cases detailed above are covered by Rules 8.2 and 8.3.1(a) and (b) AS 1250. Rules 8.3.1(a) and (b) allow for the complexities caused by the bending stresses.

Note that when $\frac{f_{ac}}{F_{ac}} < 0.15$ the simplified version of the interaction (or "unity") formula is used, Rule 8.3.1(a)(2) AS 1250.

Comment on notation for terms used in the first interaction equation of Rule 8.3.1(a), equation 8.3.1(a)(1) AS 1250.

C_m is a complex factor involving the end moments of the column.
As a blanket rule for straight forward design this factor can be taken as 1.0. This is a conservative approach.

F_{oc} is the Euler elastic failure stress of a column and is equal to $\frac{\pi^2 E}{(\frac{\ell}{r})^2}$.

(AS 1250 uses E = 200 000 MPa.)

Values of F_{oc} are listed in the right hand side columns of Table 6.1.1 AS 1250.

Eccentricities brought about by different types of connections

Rule 6.4.1 AS 1250. The summation of this rule is shown in Fig. 21(a), (b) and (c). page 12.26.

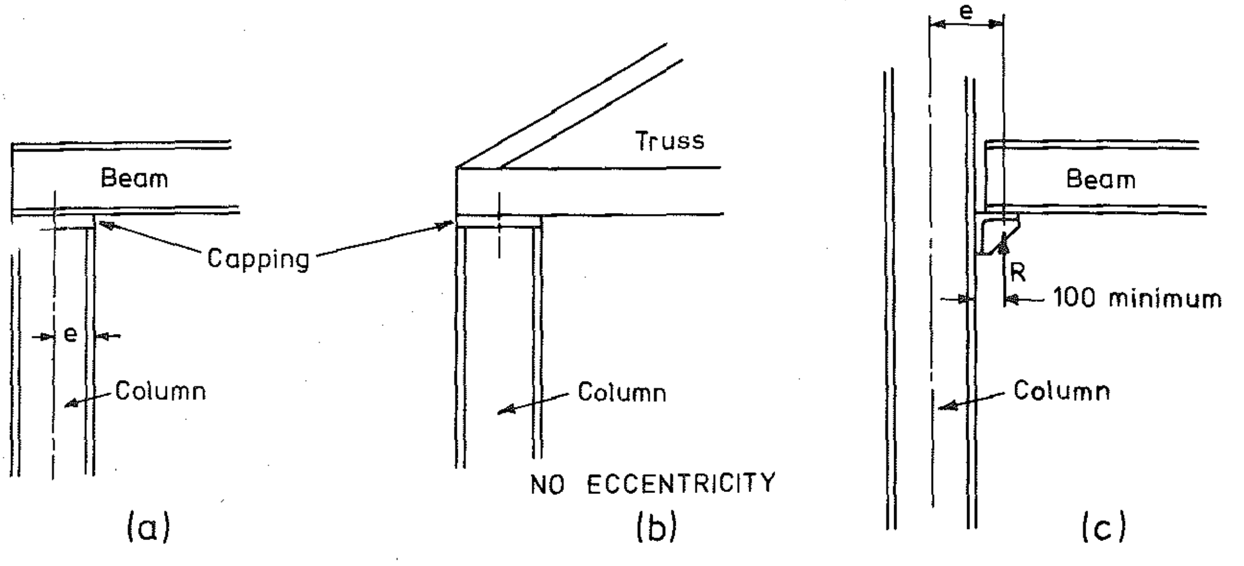


FIG. 21

Splices and end connections for structural compression members shall be designed for at least the forces stated in Rules 6.5.4 and 9.1 AS 1250.

Splices and end connections in compression members for general mechanical equipment have no particular governing clauses.

Worked Example No. 8.

Refer to Worked Example No. 3, page 12.13.

Suppose a 50 kN axial load is applied additionally to the lateral load.

Check the safety of the resultant "beam-column".

Answer

$$f_{ac} = \frac{50\ 000}{12\ 900} = 3.88 \text{ MPa.}$$

$$\frac{\ell}{r_y} = \frac{7000}{47.5} = 147$$

From Table 6.1.1 AS 1250 for $F_y = 250$ MPa

$$F_{ac} = 41 \text{ MPa}$$

$$\frac{f_{ac}}{F_{ac}} = \frac{3.88}{41} = 0.09 \quad \therefore \text{ low axial loading. (Because} < 0.15)$$

$$\frac{f_{bcx}}{F_{bcx}} = \frac{61}{71} = 0.86 \text{ (from Worked Example No. 3, page 12.13).}$$

$\therefore$ applying the simplified version of the interaction formula from Rule 8.3.1(a) AS 1250.

$$\frac{f_{ac}}{F_{ac}} + \frac{f_{bcx}}{F_{bcx}} = 0.09 + 0.86 = 0.95 < 1.0$$

$\therefore$ beam - column is safe.

Worked Example No. 9.

Refer to Fig. 22.

Check the column shown, assuming $\ell = L$ for column action i.e. pin ends.

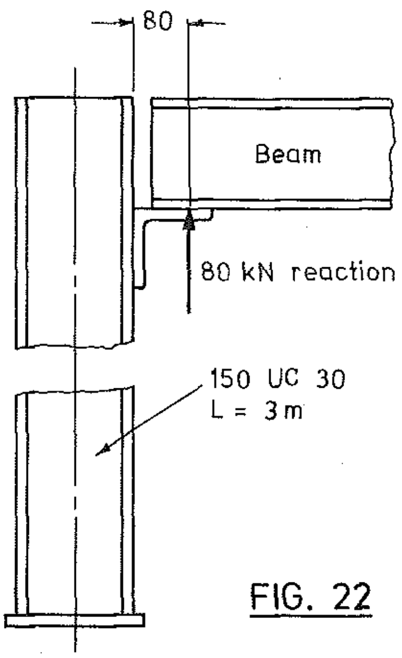


FIG. 22

Answer

Maximum section thickness = T = 9.37 mm

$\therefore$ F_y = 260 MPa, Appendix C AS 1250

$$\frac{\ell}{r_y} = \frac{3000}{38.2} = 79$$

From Table 6.1.1 AS 1250 F_{ac} = 108 MPa

Total column load = P + self weight
= 80 000 + 3 x 30 g = 80 883 N

$$\therefore \quad f_{ac} = \frac{80\ 883}{3820} = 21.2 \text{ MPa}$$

$\dfrac{f_{ac}}{F_{ac}} = \dfrac{21.2}{108} = 0.20 > 0.15$ $\therefore$ high axial loading. Rule 8.3.1(a) AS 1250

B.M. = Pe

Although Fig. 22 shows a distance from the column face to the reaction of 80 mm, we must assume a distance of 100 mm minimum, Rule 6.4.1 AS 1250.

$$\therefore \quad e = \frac{157.5}{2} + 100 = 178.75 \text{ mm}$$

$$\therefore \quad \text{B.M.} = 80\ 000 \times 178.75 = 14.3 \times 10^6 \text{ N.mm.}$$

$$\therefore \quad f_{bcx} = \frac{\text{B.M.}}{Z_{xx}} = \frac{14.3 \times 10^6}{221 \times 10^3} = 64.7 \text{ MPa}$$

$\dfrac{\ell}{r_y}$ = 79 (as before) $\dfrac{D}{T}$ = 16.8

$\therefore$ F_{bcx} = 164 MPa, Table 5.4.1(3) AS 1250.

$$\frac{\ell}{r_x} = \frac{3000}{67.5} = 45$$

$\therefore$ F_{ocx} = 975 MPa Table 6.1.1 AS 1250.

Applying Rule 8.3.1(a) AS 1250 and neglecting the "y" term as there is only bending about the x-x axis.

$$\frac{f_{ac}}{F_{ac}} + \frac{C_m \, f_{bcx}}{(1 - \frac{f_{ac}}{0.6\ F_{ocx}})\ F_{bcx}}$$

$$= 0.20 + \frac{1.0 \times 64.7}{(1 - \frac{21.2}{0.6 \times 975})\ 164} = 0.62 < 1.0$$

$\therefore$ Column is safe.

NOTE: A full check should be done for the beam design alone as shown in Section 2, Rule 8.2 AS 1250. Column is clearly safe for column action alone, Rule 8.2 AS 1250.

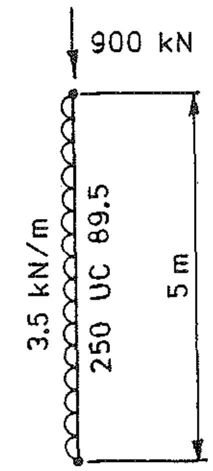


FIG. 23

EXERCISES

7. Determine the value of the interaction (or "unity") formula for the beam-column shown in Fig. 23. The column has torsional restraints at the ends but the beam load and critical flange are not restrained laterally. Assume the beam-column is pin ended. (The beam is being bent about its axis of maximum strength.)

Ans. (0.802)

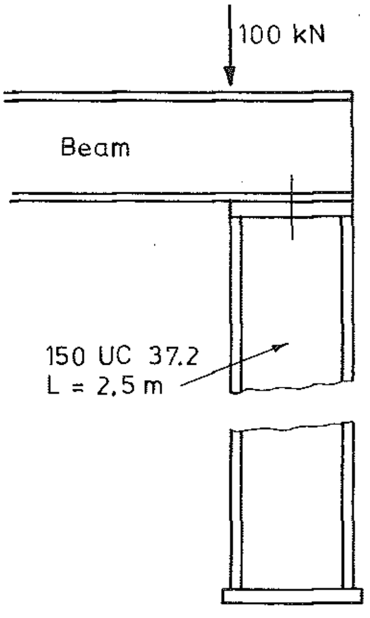


FIG. 24

8. Determine the value of the interaction formula for the eccentrically loaded column shown in Fig. 24.

Assume ℓ = L for column action i.e. pin ends. 100 kN is the resultant beam load on the column.

Ans. (0.3511)

(4) BOLTED JOINTS.

This section deals with various types of joints bolted together with "ISO Metric commercial bolts" to AS 1111; the material of these bolts being mild steel, F_y = 240 MPa.

They are referred to as Strength Grade 4.6 (see also page 12.79)

All bolted structural joints shall be designed for a force of at least 25 kN, *Rule 9.1 AS 1250. The requirement of this clause is additional to those requirements of Rules 6.5.4, 7.5.1 and 7.5.2 AS 1250 listed below.

Code restrictions on connections for compression members.

Refer to Rule 6.5 AS 1250.

Rule 6.5.4 AS 1250 specifies that splices and end connections of compression members must be designed for the greater of:

(a) 0.50 times the maximum permissible compressive force in the member; and

(b) 1.05 times the actual compressive force in the member.

Code restrictions on bolted (and welded) connections of tension members.

Rule 7.5.1 AS 1250 specifies that connections at the ends of tension members must be designed for at least the following forces:

(i) 0.50 x the maximum permissible force in the member.

(ii) 1.05 x the actual tensile force in the member.

The maximum of (i) and (ii) must be used.

Splices in tension members.

Rule 7.5.2 AS 1250 is similar to Rule 7.5.1 and additionally restates the 25 kN provision of Rule 9.1 AS 1250.

Bolted joints in general mechanical equipment have no particular governing clauses. Some of the examples given in this section are from mechanical equipment.

In any assemblage (tension or compression) where rotation of a connection is possible the minimum number of fasteners is two, except that a single pin may be used.

*According to Rule 9.1 of AS 1250 the minimum force for which a connection should be designed is 25 kN. However, the intent of the Code applies to large scale structural work; the design of smaller structures, especially those that represent no danger to personnel, is at the Designer's discretion. This is providing that the design can be backed up by sound calculations.

Modes of Failure of Bolted Joints

Primary Loading

(a) Bolt shear (single or double shear)

(b) Bolt in tension

(c) Bearing on connected member

(d) Net section yielding

(e) Gross section yielding

(f) Shearing along two longitudinal planes

(g) Tearing in one plane (splitting)

Eccentric loading (primary and secondary loads)

(h) Combined stresses, tension and shear.

(i) Primary and secondary shear.

(j) Tension only.

Bolted components should where possible be designed so that the line of action of the resultant force on the component passes through the centre of areas of the cross-sections of the bolts resisting the load.

If this loading condition is achieved the loads on the individual bolts can be taken as equal provided that elastic deflection is not a critical feature.

When the resultant force does not pass through the centre of area of the bolts the component is said to be loaded eccentrically (moment loading or secondary loading alternative terms).

The bolts have to resist indirect loading, resulting from the tendency of the component to rotate in addition to the direct loading.

These loads may be tensile, shear or a combination of both.

Permissible stresses. Reference AS 1250 1981 and (a) to (j) above.

(a) Table 9.5.2 80 MPa.

(b) Table 9.5.2 144 MPa.

(c) Table 9.5.2 2.1 F_y and $\dfrac{F_y\ e_d}{1.4\ d_f}$

(d) Rule 7.1 0.6 F_y

(e) Rule 7.1 0.6 F_y

(f) Governed by end distance, Table 9.6.2 and Rule 9.6.2.2.

(g) Governed by end distance. See Table 9.6.2 and Rule 9.6.2.2.

(h) Rule 9.5.3 and Table 9.5.2.

(i) Table 9.5.2 80 MPa.

(j) Table 9.5.2 144 MPa.

TABLE NO. 1(a) AREAS IN SQUARE mm

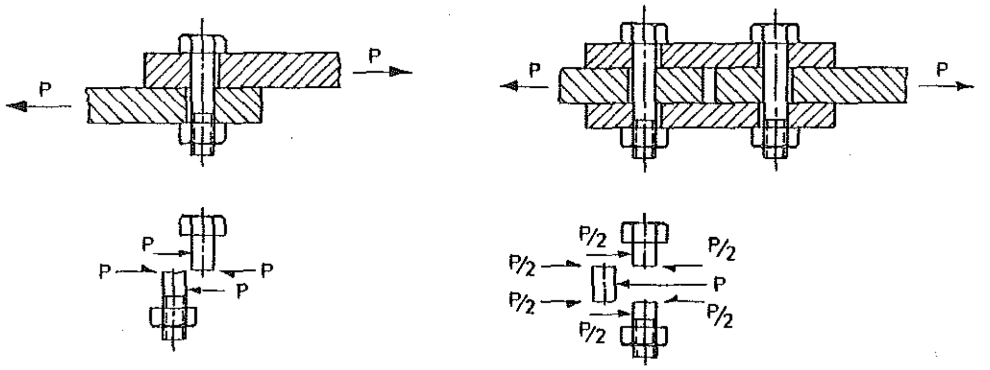
BOLT		M5	M6	M8	M10	M12	M16	M20	M24	M30	M36
SHANK AREA	A_D	20	28	50	79	113	201	314	452	706	1016
STRESS AREA	A_s	14.2	20.1	36.6	58	84.3	157	245	353	561	817
CORE AREA	A_c	12.7	17.9	32.8	52.3	76.2	144	225	324	519	759

(a) Bolt Shear

This type of failure is indicated in Fig. 25, page 12.31.

The act of tightening the bolt forces the plates together thus creating a frictional force. This however is difficult to calculate, and so in practice the strength of the bolt only, is allowed in the design of a joint.

(The design of joints using friction grip bolts is covered at the end of this Chapter, page 12.80 et seq.)



(a) BOLTS IN SINGLE SHEAR

(b) BOLTS IN DOUBLE SHEAR

FIG. 25

The ideal assembly for a bolt is to have about one thread projecting into the connected member after tightening. The full shear value of the bolt is thus attained, as the shank area, A_D, resists the shear force.

If the threads project into the shear plane, then the core area only, A_C, resists the shear force.

Due to the long length of thread on Commercial Bolts, most practical applications of these bolts will result in the thread being in the shear plane.

Shear stress is taken as "average" across the area of the bolt. Peak stresses are ignored because if any zone is overstressed yielding occurs. This has the effect of reducing the difference between peak stress value and average stress value. The shear stress permitted by Code allows for possible local overstress.

Tests on bolted joints have indicated that the level of any initial clamping force has no significant effect on the ultimate shear strength.

(b) Bolt tension

Tensile stresses in bolts.

The act of initially tightening a bolt applies an axial load and hence axial stress.

What then is the effect of applying an external load in addition?

The resultant axial load in a bolt depends upon:

(i) initial tension caused by tightening.
(ii) the external load.
(iii) the relative elastic yielding or "springiness" of the bolt and connected members.

If a very soft gasket is used as shown in Fig. 26(a), page 12.32, then the resultant bolt load will be approximately equal to the sum of the initial tension plus the external load.

If however, a solid metal to metal (no gasket) connection is used as shown in Fig. 26(b), page 12.32, the resultant bolt load will be either the initial tension or the external load, whichever is greater.

Structural applications are represented by the conditions of Fig. 26(b), page 12.32.

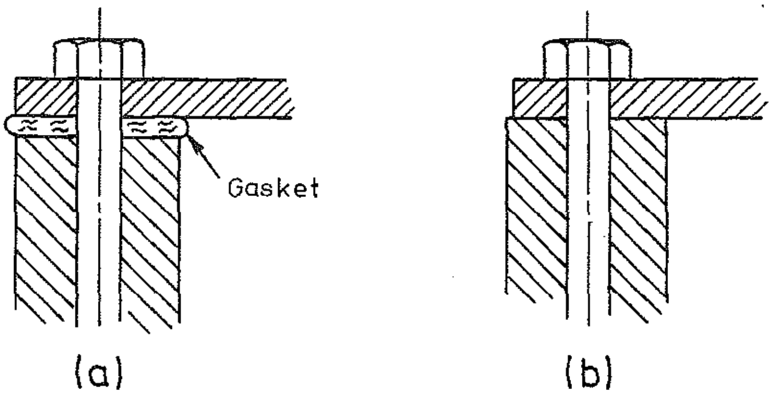


FIG. 26

Tensile stress, $f_{tf} = \frac{\text{tensile load}}{\text{stress area}}$

Stress areas as defined in Rule 4.5.1 AS 1250 are listed in Table No. 1(a) for various bolt diameters. (See page 12.30.)

The method of designing bolts for use with soft gaskets is given later in this Chapter under the heading "Bolt design - machine design applications", page 12.44.

Sometimes "prying" action increases the tensile force in a bolt. This action is indicated in Fig. 27. Tests, however, show that prying force on a bolt is rarely greater that 20% of the simple tension load irrespective of the various formulas giving prying force. Refer also to Gorenc, see References, page 12.90.

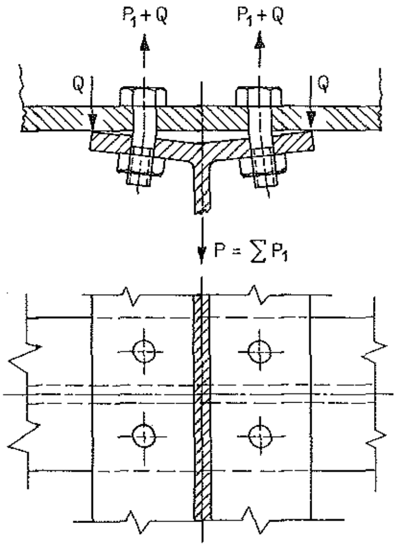


FIG. 27

According to tests, the torsional stresses induced by torquing or tightening a bolt have a negligible effect on its tensile strength.

(c) Bearing stress

The general formula for bearing stress is:

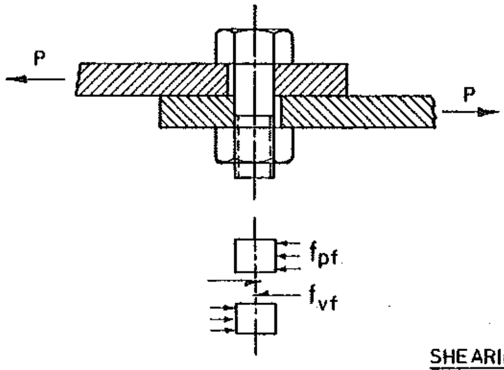
$$f_{pf} = \frac{\text{load}}{\text{bolt diam. x length resisting load}}$$

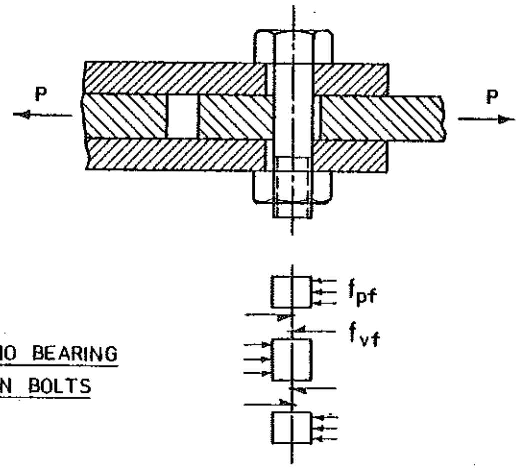
(regardless of whether the threads are in the shear plane or not).

Refer to Fig. 28, page 12.33.

In practical application it has been found that the critical bearing stress is on the connected member and not on the bolt. The bolt seats itself into and elongates the bolt hole.

Due to the fact that the bolts clamp the plates together when tightened there is a frictional force aiding the joint. However, this force is neglected in design work, as previously mentioned, and we "assume" the joint slips and the full load is active in producing bearing stress.





SHEARING AND BEARING STRESSES IN BOLTS

(a) SINGLE SHEAR ACTION

(b) DOUBLE SHEAR ACTION

FIG. 28

Bearing stresses are simply compressive stresses set up over short distances. When they occur in very confined situations such as a bolted joint, transverse compressive stresses are also developed. This effectively raises the yield point stress of the steel. This is the reason why such high design bearing stresses are allowed.

$$F_{pf} = 2.10\ F_y\text{, Table 9.5.2, AS 1250}$$

$$\text{also}\quad F_{pf} \leq \frac{F_y\ e_d}{1.4\ d_f}$$

(d) Net section yielding

Refer to Fig. 29(a).

This failure mode is governed by edge distance.

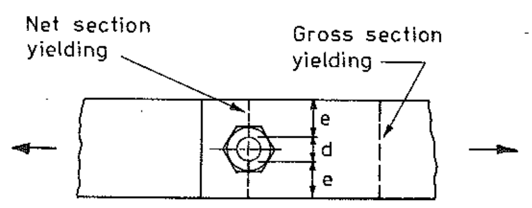


FIG. 29(a)

(e) Gross section yielding

The stress in the "body" of a connection (refer to Fig. 29(a) must be equal to or below the permissible stress.

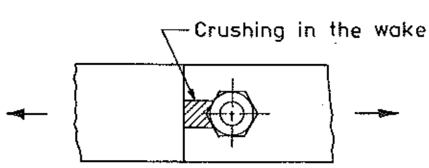


FIG. 29(b)

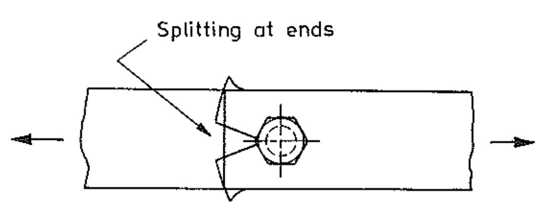


FIG. 29(c)

(f) Shearing along two longitudinal planes

Refer to Fig. 29(b).

(g) Tearing in one plane (splitting)

Refer to Fig. 29(c).

In normal circumstances failure modes (f) and (g) will not occur if the edge (or end) distances specified in Rule 9.6.2. AS 1250 are used.

Code restrictions on bolt pitches, edge and end distances.

To avoid joint failure by plate tearing, shear or buckling (in compressive joints) the Code specifies:

Minimum pitch, Rule 9.6.1. AS 1250 (also Rule 9.6.2.2. AS 1250 may affect minimum pitch).

Minimum edge distance Table 9.6.2. AS 1250 and Rule 9.6.2.2 AS 1250

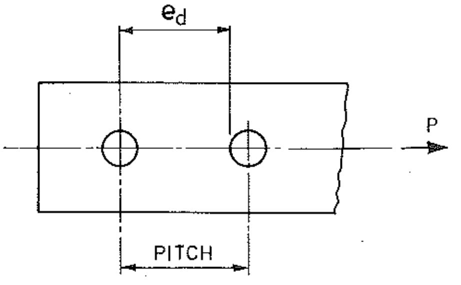
Maximum pitch, Rule 9.6.3. AS 1250

Maximum edge distance, Rule 9.6.4 AS 1250

The maximum number of bolts in line in direction of force is recommended as being five, Table 9.5.2. Footnote 1 AS 1250.

Minimum pitch Rule 9.6.1 AS 1250

The edge of a plate or section when calculating permissible bearing stress (Table 9.5.2. AS 1250) or end distance requirements, Rule 9.6.2.2. AS 1250, must include the edge of an adjacent fastener hole.
The minimum bolt pitch should exceed the minimum edge distance, refer to Fig. 30.



$e_d \geqslant \frac{1400\,P}{F_y\,t}$ Rule 9.6.2.2

FIG. 30

Minimum edge distance, Rule 9.6.2. AS 1250

Some explanation of the application of this Rule may be useful.

Table No. 1(b) gives details of minimum edge distances for various types of plates and sections.

TABLE NO. 1(b)

Bolt diam. mm.	Sheared or hand flame cut edge mm.	Rolled plate: Machine flame cut, sawn or planed edge mm.	Rolled edge of a rolled section mm.	$e_d = \frac{1400\,P}{F_y\,t}$ mm.
	1.75 d_f	1.50 d_f	1.25 d_f	
10	17.5	15	12.5	6.8
12	21	18	15	9.7
16	28	24	20	17.3
20	35	30	25	27.1
24	42	36	30	38.9
30	52.5	45	37.5	60.8
36	63	54	45	87.5

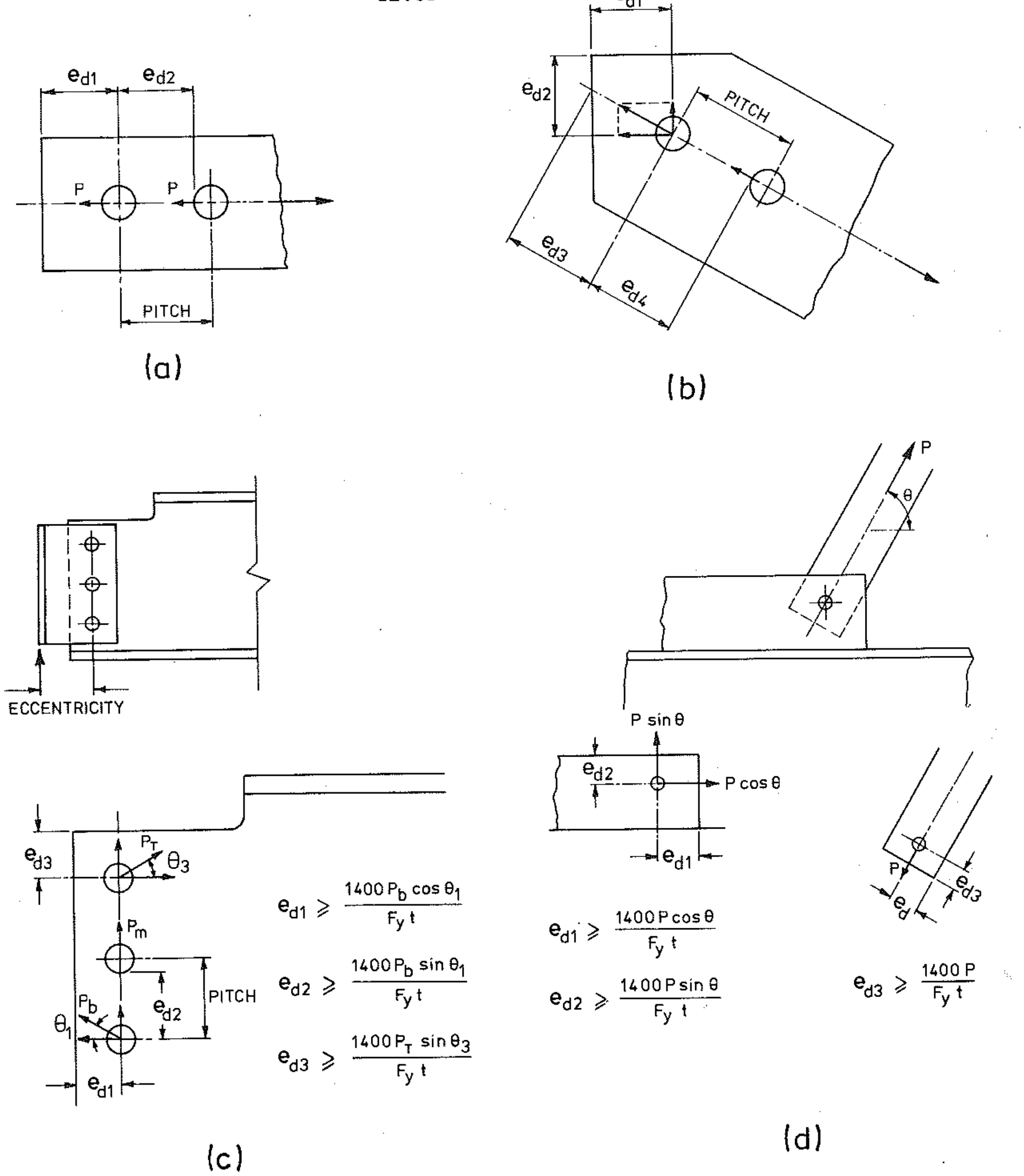


FIG. 31

Additionally, for interest, in the R.H.S. column are edge distances which are in accordance with Rule 9.6.2.2. AS 1250. These have been calculated using 5 mm M.S. bar, F_y = 260 MPa (about the thinnest plate commonly used in structural work) and bolts in the operating mode shown in Fig. 28(a), page 12.33. The bolt shanks are stressed to 80 MPa shear stress.

The formula given in Rule 9.6.2.2 AS 1250 was derived as follows:

$$f_{pf} = \frac{P}{d_f t} \leq \frac{F_y e_d}{1.4 d_f} \quad \text{for P in N}$$

from which $e_d \geq \dfrac{1.4 P}{F_y t}$ ...for P in N <u>or</u> $\dfrac{1400 P}{F_y t}$ ...for P in kN.

t = THICKNESS

$P = b_v$

Minimum edge distance in direction of component of force. Rule 9.6.2.2 AS 1250.

This provision is intended to apply to simple situations (Figs. 31(a) and (b)), page 12.35, as well as more complicated ones (Figs. 31(c) and (d)), page 12.35.

Eccentric loading

(h) Combined stresses, tension and shear.

See Worked Example No. 13, page 12.40.

(i) Primary and secondary shear.

See Worked Example No. 14, page 12.41.

(j) Tension only.

See Worked Example No. 15, page 12.42.

Worked Example No. 10.

Refer to Exercise 2, page 12.5.

The angle shown in Fig. 7, page 12.5, is a tension member in a truss and has to sustain a working load of 90 kN. The chords to which the angle connects are 12 mm thick.

Determine the number of ϕ 20 bolts required. Each bolt is in single shear and the shank is in the shear plane.

Answer

Using Rule 7.5.1 AS 1250

(i) $0.50 \times 180\,911.5 = 90\,455.75$ N

(ii) $1.05 \times 90\,000 = 94\,500$ N

Using Rule 9.1 AS 1250

Minimum force = 25 kN

$\therefore$ Bolt connection has to withstand 94 500 N.
ϕ 20 bolt single shear value $= A_D \times F_{vf}$

Values of A_D are given in Table No. 1(a), page 12.30.

$\therefore$ shear value $= 314 \times 80 = 25\,120$ N.

$$\therefore\ n = \frac{94\,500}{25\,120} = 3.8 \text{ say } \underline{4 - \text{bolts}}$$

Now check bearing stress.
The maximum bearing stress occurs on the tension angle (the thinner member).

Use a standard end distance of 35 mm (from Table No. 1(b), page 12.34).

F_y of angle = 260 MPa App. C AS 1250

$$P \text{ per bolt} = \frac{94\,500}{4} = 23\,625 \text{ N}$$

$$\therefore \quad f_{pf} = \frac{23\ 625}{20 \times 9.5} = 124.3 \text{ MPa}$$

$$F_{pf} = 2.1\ F_y \quad \text{or} \quad \frac{F_y\ e_d}{1.4\ d_f}$$ ... Table 9.5.2, AS 1250 (use least value)

$$= 2.1 \times 260 \quad \text{or} \quad \frac{260 \times 35}{1.4 \times 20}$$

$$= 546 \text{ MPa} \quad \text{or} \quad 325 \text{ MPa}$$

$$f_{pf} = 124.3 \text{ MPa} < 325 \text{ MPa} \quad \therefore \text{ design safe}$$

Worked Example No. 11.

Supposing the truss mentioned in the previous example has a compression member 1500 mm long which is required to carry a load of 93 kN.

Check the suitability of a 90 x 90 x 10 angle and determine the number of ϕ 20 bolts required.

Answer

Assume the brace has two or more bolts in line at each end.

From Graph No. 2 in 'Applied Structural Design' or by calculation, the above angle will take a maximum permissible compressive load of 129 205 N.

Applying Rule 6.5.4, AS 1250

(a) $0.50 \times 129\ 205 = 64\ 602.5$ N

(b) $1.05 \times 93\ 000 = 97\ 650$ N

Applying Rule 9.1 AS 1250

Minimum force = 25 kN

$\therefore$ bolt connection has to withstand 97 650 N

From previous example, bolt value = 25 120 N

$$\therefore \quad n = \frac{97\ 650}{25\ 120} = 3.9 \quad \text{say } \underline{4 - \text{bolts}}.$$

Now check bearing stress.
The maximum bearing stress occurs on the compression angle (the thinner member).

Assume centre distance of holes $=$ 60 mm
($>$ 2.5 d_f, Rule 9.6.1 AS 1250).

$$F_y \text{ of angle} = 260 \text{ MPa} \quad \text{App. C AS 1250}$$

$$P \text{ per bolt} = \frac{97\ 650}{4} = 24\ 412.5 \text{ N}$$

$$e_d = 60 - \left(\frac{20 + 2}{2} \right) = 49 \text{ mm}$$

$$f_{pf} = \frac{24\ 412.5}{20 \times 9.5} = 128.5 \text{ MPa}$$

$$F_{pf} = 2.1\ F_y \quad \text{or} \quad \frac{F_y\ e_d}{1.4\ d_f} \quad \text{... Table 9.5.2, AS 1250 (use least value)}$$

$$= 2.1 \times 260 \quad \text{or} \quad \frac{260 \times 49}{1.4 \times 20}$$

$$= 546 \text{ MPa} \quad \text{or} \quad 455 \text{ MPa}$$

$$f_{pf} = 128.5 \text{ MPa} < 455 \text{ MPa} \quad \therefore \text{ design safe}$$

orked Example No. 12.

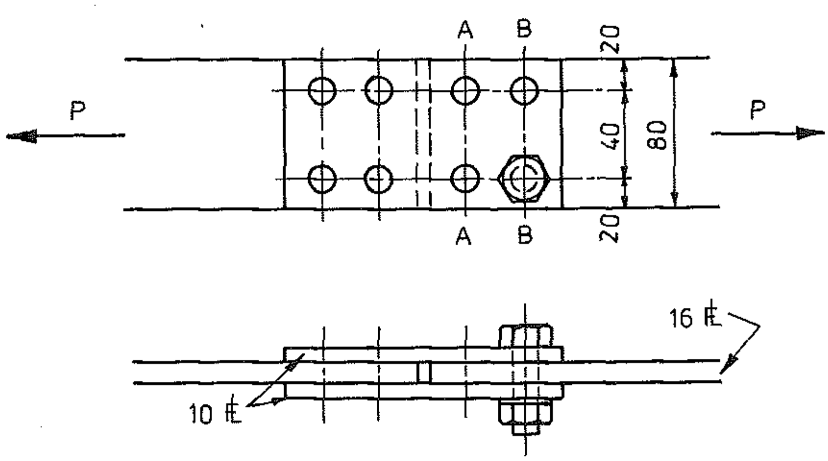


FIG. 32

heck the splice in a tie, as shown in Fig. 32.
he load is 100 kN. Bolt shanks are in the shear plane.

nswer

Assume 4 ϕ 16 bolts and space according to AS 1250.

Critical section for plate is "B-B"

$A_s = 16 \ \{80 - 2\ (16 + 2)\}$

$= 704 \text{ mm}^2$

F_y plate = 250 MPa, App. C AS 1250

$\therefore$ Allowable load on plate

$= 704 \times 0.6 \times 250 = 105\ 600$ N

The critical tensile stress is in the tie plate as the combined areas of the cover plates are far in excess of the tie plate area.

Applying Rule 7.5.2 AS 1250

(i) $0.50 \times 105\ 600 = 52\ 800$ N
(ii) $1.05 \times 100\ 000 = 105\ 000$ N
(iii) 25 000 N

Applying Rule 9.1 AS 1250

Minimum force = 25 kN

$\therefore$ Design load for splice = 105 000 N

Check fasteners according to Table 9.5.2 AS 1250

Bolt value double shear $= 2 \times A_D \times F_{vf}$

$= 2 \times 201 \times 80 = 32\ 160$ N

$$\therefore \quad n = \frac{105\ 000}{32\ 160} = 3.3 \text{ say 4 bolts.}$$

Bolt bearing value. The critical bearing value will occur on the plate as $16 < 10 + 10$.

End distance = 28 mm Table No. 1(b), page 12.34.

F_y of plate = 250 MPa App. C AS 1250

$$P \text{ per bolt} = \frac{105\ 000}{4} = 26\ 250 \text{ N}$$

$$f_{pf} = \frac{26\ 250}{16 \times 16} = 102.5 \text{ MPa}$$

$$F_{pf} = 2.1\ F_y \text{ or } \frac{F_y\ e_d}{1.4\ d_f}$$ ... Table 9.5.2, AS 1250 (use least value)

$$= 2.1 \times 250 \text{ or } \frac{250 \times 28}{1.4 \times 16}$$

$= 525$ MPa or 312.5 MPa

$f_{pf} = 102.5 \text{ MPa} < 312.5 \text{ MPa}$ $\therefore$ design safe

The bearing check on bolt pitch is left to the student.

Worked Example No. 13.

Determine the value of the interaction equation for the bolts and check the bearing value of the bracket shown in Fig. 33. Assume the bolt shanks are in the shear plane.

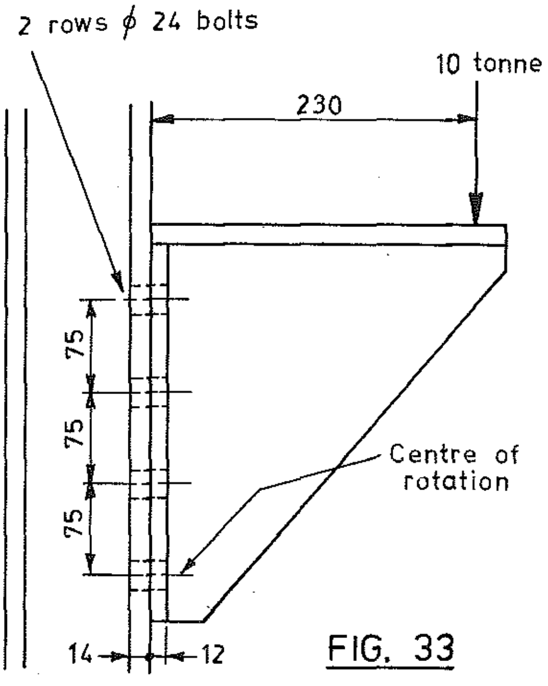


FIG. 33

Answer

The primary stress is shear.

$$\text{Primary shear force} = \frac{10\ 000\ g}{8}$$

$$= 12\ 262.5 \text{ N}$$

$$\therefore\ f_{vf} = \frac{\text{Shear load}}{A_D} = \frac{12\ 262.5}{452}$$

$$= 27.1 \text{ MPa.}$$

The secondary stress is tensile.

Bolt modulus

$$= \frac{2 \times 75^2 + 2 \times 150^2 + 2 \times 225^2}{225}$$

$$= 700 \text{ mm units.}$$

$$\text{Applied moment} = 98\ 100 \times 230$$

$$= 22\ 563\ 000 \text{ N.mm}$$

Tension in top two bolts

$$= \frac{22\ 563\ 000}{700} = 32\ 233 \text{ N}$$

$$\therefore\ f_{tf} = \frac{\text{Tensile load}}{A_s} = \frac{32\ 233}{353} = 91.3 \text{ MPa}$$

Applying Rule 9.5.3 AS 1250

with F_{vf} = 80 MPa and F_{tf} = 144 MPa from Table 9.5.2 AS 1250

$$\left(\frac{f_{vf}}{F_{vf}}\right)^2 + \left(\frac{f_{tf}}{F_{tf}}\right)^2$$

$$= \left(\frac{27.1}{80}\right)^2 + \left(\frac{91.3}{144}\right)^2 = 0.52 < 1.0$$

The design should now be checked for bearing stress as shown in principle in Worked Examples Nos. 10, 11 and 12, pages 12.36 - 12.38.

Worked Example No. 14.

Check the connection shown in Fig. 34. Bolt shanks are in the shear plane.

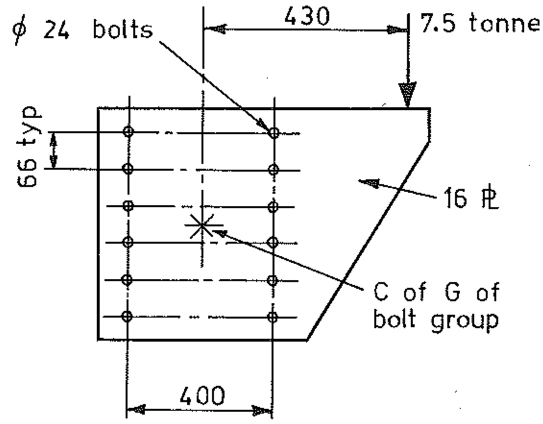


FIG. 34

Answer

The loads on the bolts consist of primary and secondary shear. Primary shear on each bolt

$$= \frac{7500\ g}{12} = 6131\ \text{N}$$

Next step is to find the secondary shear force

$$I_p = I_{xx} + I_{yy}$$

$$= 4 \times 33^2 + 4 \times 99^2 + 4 \times 165^2 + 12 \times 200^2$$

$$= 632\ 460\ \text{mm units}$$

$$\therefore \text{ Bolt modulus (polar)} = \frac{632\ 460}{\sqrt{200^2 + 165^2}}$$

$$= 2439.3\ \text{mm units.}$$

Secondary shear force on each corner bolt

$$= \frac{7500\ g \times 430}{2439.3} = 12\ 970\ \text{N}$$

Fig. 35 shows how the primary and secondary shear forces are added graphically. The shear force absolute = 18 100 N approximately.

This maximum shear force occurs on the upper and lower bolt on the right hand side of the bolt group. The upper and lower bolts on the left hand side of the bolt group resist a smaller absolute shear force; the student can check these points.

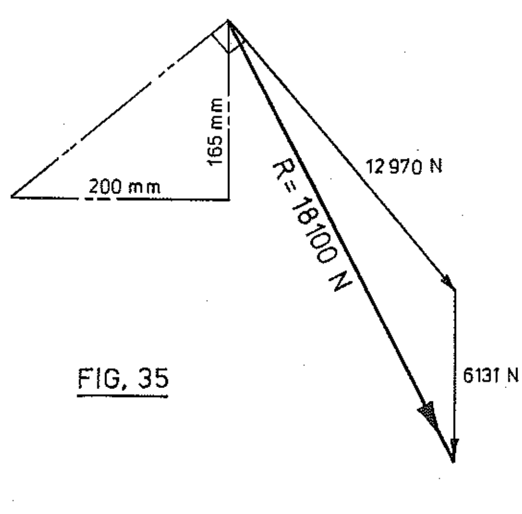


FIG. 35

$$\therefore f_{vf} = \frac{\text{Shear load}}{A_D} = \frac{18\ 100}{452} = 40\ \text{MPa}$$

From Table 9.5.2 AS 1250.

$$F_{vf} = 80\ \text{MPa} > 40\ \text{MPa}$$

Maximum bearing stress on connected member

$$f_{pf} = \frac{\text{load}}{d_f t} = \frac{18\ 100}{24 \times 16} = 47.1 \text{ MPa.}$$

From Table 9.5.2 AS 1250

$$F_{pf} = 2.1\ F_y$$

F_y of 16 plate = 250 MPa App. C AS 1250

$$\therefore\ F_{pf} = 2.1 \times 250 = 525 \text{ MPa} > 47.1 \text{ MPa}$$

Also the edge distances and bolt pitches must be checked according to Rule 9.6.2.2 AS 1250. The principles are shown in Figs. 30 and 31, pages 12.34 and 12.35.

Worked Example No. 15.

Determine the size of bolt required for the bracket shown in Fig. 36.

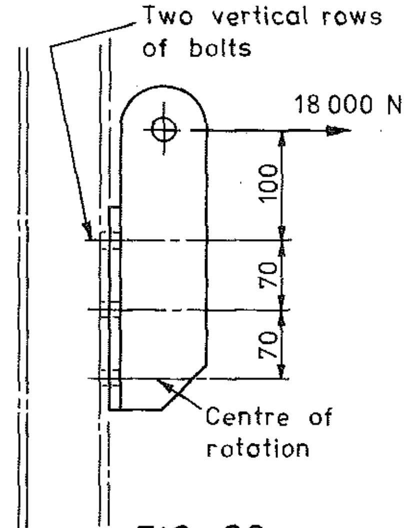


FIG. 36

Answer

Applied moment = 18 000 x 240

= 4 320 000 N.mm

$$\text{Bolt modulus} = \frac{2 \times 70^2 + 2 \times 140^2}{140}$$

= 350 mm units.

$$\text{Force in each outer bolt} = \frac{4\ 320\ 000}{350} = 12\ 343 \text{ N}$$

From Table 9.5.2 AS 1250 F_{tf} = 144 MPa.

$\therefore$ Stress area required, A_s = Tensile load/F_{tf}

$$= \frac{12\ 343}{144} = 85.7 \text{ mm}^2$$

From Table No. 1(a), page 12.30, use M16 bolts.

EXERCISES

9. Refer to Fig. 37.

In this structural end connection the applied load P = 50 kN. The bolt shanks are in the shear plane.

(a) Determine the maximum permissible force for the member. Three checks should be done:

(i) Section AA for the 12 flat.
(ii) Section BB for the 12 flat.

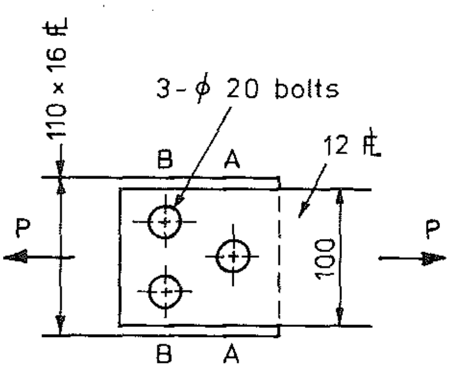


FIG. 37

(iii) Section BB for the 16 flat (F_y differs for each flat).

(b) Determine the strength of the connection according to the bolts for:

(i) single shear action; and
(ii) bearing, e_d = 35 mm.

(c) Check that the connection design complies with Rule 7.5.1, AS 1250.

Ans. (a) (i) 146 kN (ii) 129.9 kN (iii) 158.4 kN
(b) (i) 75.36 kN (ii) 234 kN
(c) 0.5 x 129.9 = 64.95 kN < 75.36 kN
1.05 x 50 = 52.50 kN < 75.36 kN

10. Find the safe load P the joint shown in Fig. 38 can carry. Assume the bolt threads are in the shear plane. This connection forms part of some mechanical equipment.

Ans. P = 23.04 kN
(P = 23.04/1.05 = 21.94 kN if joint were in a structural connection)

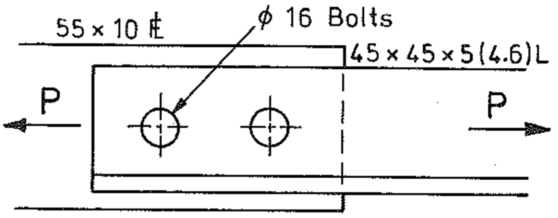


FIG. 38

11. Determine the size of bolts required to bolt a steel eye piece to a container as shown in Fig. 39.

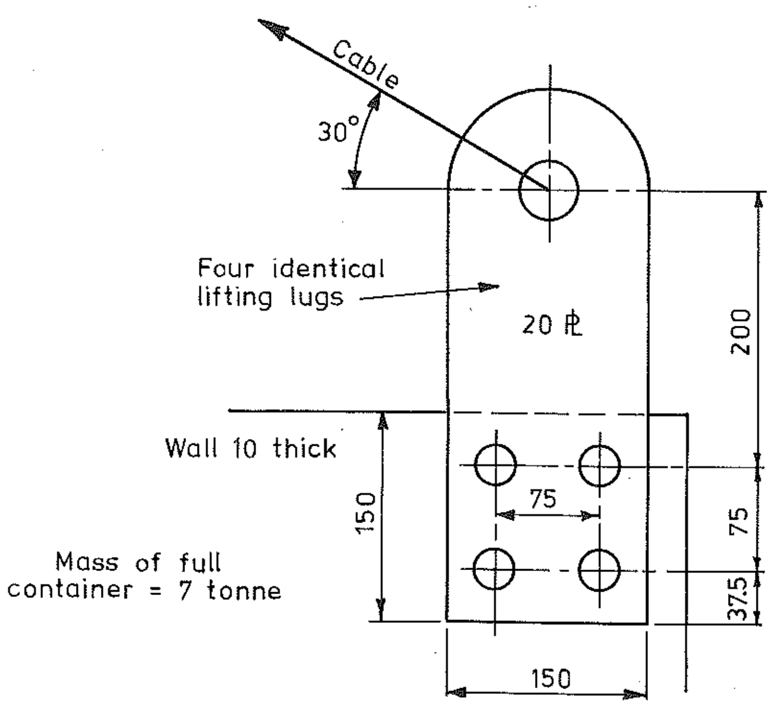


FIG. 39

Assume:
(i) the bolt shanks are in the shear plane.
(ii) the cable is in the plane of the paper.

Ans. Shank area = 525 mm^2. Use ϕ30 bolts. No factor was applied to the load.

Bolt design - machine design applications. (Refer acknowledgment page 12.90)

Tensile fatigue or shear stress reversal fatigue

Any moderate application of this type can be designed as follows:

design load = 1.5 x maximum applied load

Use the stresses from Table 9.5.2 AS 1250 with this increased load.

AS 1250 (Appendix B5) is the main Australian Code dealing with design for fatigue in bolted connections. The permissible stress in the base metal of various bolted connections subject to tensile or stress reversal fatigue conditions is given in Appendix B of AS 1250. There is no recommendation given for bolts subject to combined shear and tension under fatigue conditions.

Bolted joints using gaskets.

The tightening of a bolt by turning the nut will induce a very complex state of stress in the bolt including torsion, direct tension, crushing of flank surface and shear across the root of the thread. None of these can be accurately calculated, even if the torque is known.

As a result of experience however, simplifying assumptions can be made which enable satisfactory designs to be accomplished.

If a joint is required to be pressure tight a gasket of some kind must be used. In such joints, the tightened bolt will compress the gasket and the load will be F_i on both the bolt and packing i.e. F_i is the counter action between flange and packing.

If e = elongation of bolt in mm per N load.

c = compression of gasket in mm per N load.

The bolt will elongate $F_i\,e$ mm and the packing will compress $F_i\,c$ mm.

If additional load is applied (e.g. in the form of internal pressure etc.) this will cause further elongation of the bolt and thus slackening (diminishing) the compression of the packing. This, however, will mean a change in the interaction of flange and packing. If we denote the applied load per bolt by F_a, the new interacting force by C, the total load on the bolt will be -

$$F = C + F_a \quad \text{i.e.} \quad C = F - F_a$$

The additional elongation of the bolt will now = $(F - F_i)\,e$

and the compression of the packing will diminish by the same amount i.e.,

$(F_i - C)\,c = (F - F_i)\,e$ but $C = F - F_a$ and thus

$$(F_i - F + F_a)\,c = (F - F_i)\,e$$

$$F\,(e + c) = F_i\,(e + c) + F_a\,.c$$

$$F = F_i + \frac{c\,F_a}{e + c} \quad \text{or} \quad F = F_i + K\,F_a$$

The factor $K = \dfrac{c}{e + c}$ is called gasket factor and determines the proportion in which the applied load will be transferred to the bolt.

For steel bolts e is a constant, and the value of K will only depend on the material used for packing.

For very soft packings c is large compared with e,

hence $K = \frac{c}{c + e} \cong 1.$

For metal to metal joint (no gasket) $c = 0$ and $K = 0$. Values of K for different materials are given in Table No. 2.

TABLE NO. 2.

GASKET	K
Soft packing with studs	1.00
Soft packing with bolts	0.75
Asbestos	0.60
Soft Copper	0.50
Hard Copper	0.25
Metal to metal (no packing)	0.0

Note: a service factor may be applied to F_a. Two cases arise for consideration:

(a) Metal to metal joints, $K = 0$. To minimise possibility of fatigue failure of bolts under highly fluctuating stresses, $F_i > F_a$

$$\text{i.e. } F_i = S.F. \times F_a$$

(b) Flexible gasket joints, $K > 0$. The stress in bolts in this application varies with load and under rapidly fluctuating loads, bolts can be subject to fatigue. A service factor will minimise this condition.

If a pressure tight joint is required, the value $C = F - F_a$ must be large enough to keep the joint tight.

Bolts in gasketed joints do not normally have to be tightened to the same tension as "structural type" connections. Reasonably low tensions (say up to 50% of the yield point tension as a maximum) should keep the joint pressure tight.

Torque spanners enable a good estimate of stress to be made in a design. With such precautions the initial tension can be controlled. The smallest size of bolt generally used in these circumstances is 16 mm diameter. Alloy steels can be used in cases where very high stresses are encountered.

The following formula relates tightening torque with bolt tension:

$$T = \frac{0.2\, d_s\, F_i}{10^3} \quad \text{(Do not use for stainless steel bolts.)}$$

Where T = tightening torque, N.m

d_s = shank diameter of bolt, mm

F_i = induced bolt tension, N.

The formula applies to coarse threads that are lightly oiled. Fine threads require approximately 10% extra torque.

For bolts with special finishes or heavily greased, the torque induced tension relationship is altered. Table No. 3, page 12.46, lists factors by which the torque should be multiplied to correct for various surface conditions.

TABLE NO. 3.

SURFACE CONDITION		FACTOR
Galvanized	- Degreased	2.1
	- Lightly oiled	1.1
Zinc plated	- Degreased	1.9
	- Lightly oiled	0.9
Cadmium plated	- Degreased	1.0
	- Lightly oiled	0.7
Phosphated and oiled		0.7
Standard finish plus heavy grease		0.7

For stainless steel bolts with coarse threads lightly oiled:

$$T = \frac{0.28\ d_s\ F_i}{10^3}$$

Fine threads require approximately 10% extra torque.

With all the above information a sample design can be attempted.

Worked Example No. 16.

The cover of a 250 mm inside diameter hydraulic cylinder is bolted down with 12 bolts.

The internal pressure is 1.8 MPa.

Using a hard copper gasket determine the size of bolt required. Use metric hexagon commercial bolts to AS 1111, $F_y = 240$ MPa.

Initial tightening should be to 50% of the yield tension of the bolt on lightly oiled threads. Working stress should be less than 65% yield stress.

State the torque requirement for tightening the bolts.

Answer

The design process is iterative.
Try ϕ 20 bolts.
Initial tensile force in bolt,

$$F_i = 0.5 \times 240\ A_s = 0.5 \times 240 \times 245 = 29\ 400\ \text{N}$$

$$\text{Total applied load} = \frac{\pi\ D^2}{4} \times 1.8$$

$$= \frac{\pi\ 250^2}{4} \times 1.8 = 88\ 357.3\ \text{N}$$

$$F_a \text{ per bolt} = \frac{88\ 357.3}{12} = 7363\ \text{N}$$

$$F = F_i + K F_a = 29\,400 + 0.25 \times 7363 = 31\,240.75 \text{ N}$$

$$f_{tf} = \frac{F}{A_s} = \frac{31\,240.75}{245} = 127.5 \text{ MPa.}$$

$$65\% \text{ of } 240 = 156 \text{ MPa} > 127.5 \text{ MPa.}$$

$$T = \frac{0.2\, d_s\, F_i}{10^3} = \frac{0.2 \times 20 \times 29\,400}{10^3} = 117.6 \text{ N.m.}$$

EXERCISES

12. An engine cylinder is fastened to the crank case by eight M12 alloy steel bolts, F_y = 620 MPa. The maximum gas pressure in the cylinder is 3 MPa and the inside diameter is 125 mm. Assume the gasket is "soft" (K = 0.75). Initial bolt tension is to be 50% of yield tension.

Determine:

(i) the working stress in each bolt.

(ii) the torque requirement for initially tightening the bolts. The threads are unplated and lightly oiled.

Ans. (i) 350.9 MPa.
(ii) 62.72 N.m

(5) WELDED JOINTS.

All welded structural connections shall be designed for a force of at least 25 kN, Rule 9.1 AS 1250.

This is additional to:
Rule 6.5.4 AS 1250 for compression members.
Rules 7.5.1 and 7.5.2 AS 1250 for tension members.

Welded connections in general mechanical equipment have no particular governing clauses. Some of the examples in this section are from mechanical equipment.

Fillet weld design.

The failure mode for fillet welds is in the minimum area (middle plane) which is termed the maximum effective throat thickness,

$$t = \frac{W}{\sqrt{2}}, \text{ refer to Fig. 40.}$$

Strictly, all forces should be resolved with respect to the failure plane defined above, refer to Fig. 41, page 12.48.

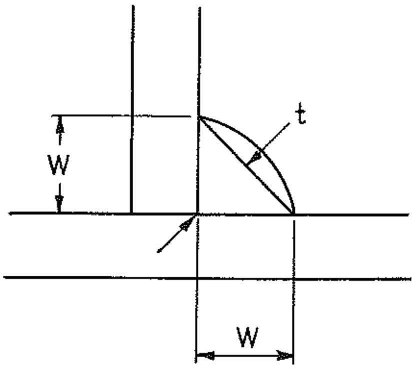


FIG. 40

Any longitudinal direct tensile forces, F_L, do not act on the failure plane and can normally be ignored. These "direct" forces are an uncommon form of loading.

All other forces can be resolved into the three orthogonal components F_1, F_2 and F_3. These have to be then resolved further on to the "failure" plane. F_1 and F_2 each have components normal (direct stress) to and along (shear stress) the "failure" plane; F_3 is already a shear on the plane.

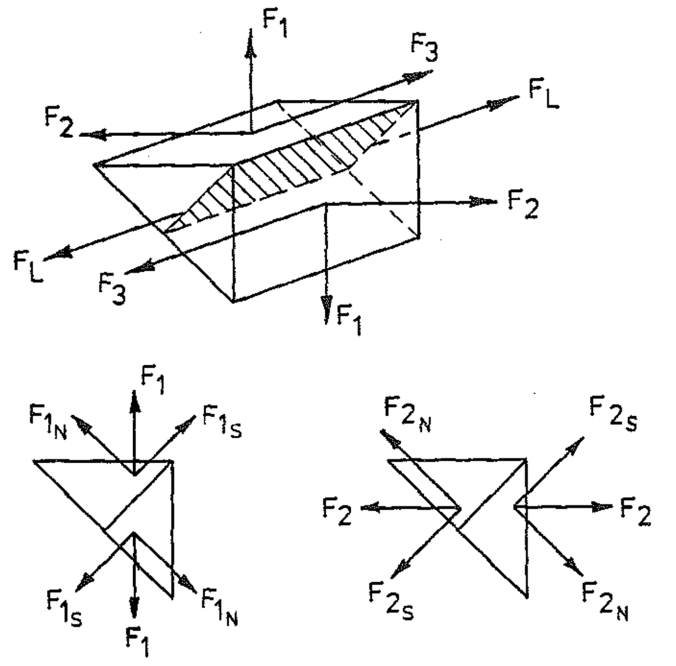


FIG. 41

Design problems involving only F_3 are straight forward. The shear stress on the weld is f_v;

$$f_v = \frac{F_3}{\text{failure area}}$$

The maximum permissible shear stress

$F_v = 0.33\ F_{uw}$ Rule 9.8.2 (b) AS 1250.

If forces F_1 and F_2 are present, the direct and normal stress components can be combined using the Mises criterion.

$$\sqrt{f_n^2 + 3(f_v^2 \text{ transverse} + f_v^2 \text{ long.})} \leq 0.57\ F_{uw}$$

Rule 9.8.2 (b) AS 1250.

However, it can be shown mathematically from the above formula that a fillet weld loaded transversally is only 13 per cent stronger (a little more according to practical experiments) than one loaded longitudinally. Therefore, design problems can be greatly simplified by considering all loads to be longitudinal shear stresses on the weld throat. This assumption will be conservative.

This leads into what is termed the <u>line method of weld design</u>.

Stress in a fillet weld shall be considered as shear stress on the throat <u>for any direction of applied load</u>.

Stress distribution in fillet welds is very complex and design can only be carried out if some simplifications are introduced, providing that there is experimental evidence confirming that such assumptions err on the conservative side.

The permissible stress on fillet welds is based on the nominal strength of the electrode, F_{uw}, since it is the weld metal that is critically stressed and not the parent metal.

The two electrodes in common use are:

E41XX $F_{uw} = 410$ MPa

E48XX $F_{uw} = 480$ MPa

A complete coverage of the use of electrodes is given in AS 1554, Part 1.

The maximum permissible stress on the effective area of the fillet is:-

$$F_v = 0.33\ F_{uw}$$

For an electrode where F_{uw} = 410 MPa (E41XX)

$$F_v = 0.33 \times 410 = 135 \text{ MPa}$$

The allowable parallel load per mm length of weld in a statically loaded fillet weld is

$$P_{allow} = F_v \times A = 135 \times 0.707\ W = 95.5\ W \text{ N/mm}$$

A = throat area of 1 mm of weld at 45° which is 0.707 W

W = leg size in mm.

P = allowable load in N.

Efficiency of joint

$$= \frac{\text{strength of weld}}{\text{strength of member O/S the joint}} \times 100$$

When bending and torsion are encountered the weld is treated as a line with no cross-sectional area.

<u>Summary of formulas used in line method of weld design.</u>

<u>Bending</u>

$$f = \frac{M}{Z}$$

f = stress in N/mm^2

M = Bending moment N.mm

Z = Section modulus mm^3

If the section modulus of weld, treated as a line, is determined the units are mm^2. The "stress" will then be in N/mm.

Thus $$f_w = \frac{M}{Z_w}$$

f_w = "stress" in N/mm

M = Bending moment N.mm

Z_w = Line modulus for bending, mm^2.

A similar analysis can be done for torsion.

Where both primary and secondary forces are present, the "stresses" are added by means of vectors and the resultant equated to the allowable "stress".

For torsion

$$f_w = \frac{T \times c}{J_W}$$

f_w = "stress" in N/mm

T = applied torque N.mm

J_W = Line moment of inertia (2nd moment) - polar mm^3.

c = distance of C of G of weld design to furthermost point.

Tension or Compression

$$f_W = \frac{\text{Load}}{L_W}$$

L_W = length of weld mm.

Direct shear

$$f_w = \frac{\text{Load}}{L_W}$$

Table No. 4, page 12.53, gives properties of various weld groups, treated as lines.

Butt weld design.

Complete penetration.

In a full penetration butt weld there is very good mixing of the electrode and parent metal hence F_y can be taken as the minimum value of the components being joined.

Nominally there is a "full strength" connection. Rule 9.8.2 (a) AS 1250.

Incomplete penetration.

For incomplete penetration butt welds F_y shall be taken as 0.50 F_{uw}, Rule 9.8.2 (a) AS 1250.

Further instructions covering special cases of butt and fillet welds are given in Rules 9.8.3 and 9.8.4 AS 1250.

Structural design - reversal of stress applications - endurance.

This type of problem is not common in structural work.
Where reversal of stress is involved, instructions are given in Appendix B of AS 1250. In summary, permissible shear stress on welds in structural endurance situations is limited to 27.5 MPa where complete reversal occurs; Table B3 AS 1250, case F, refers to shear stress.

Worked Example No. 17.

Determine the size of fillet weld required to join the two plates shown in Fig. 42, page 12.51. The welded joint must be at least as strong as the 250 x 12 plate.

Use E41XX rods (F_{uw} = 410 MPa).

Answer

F_y plate = 260 MPa, App. C AS 1250.

Maximum permissible load plate will carry

$$P = 0.6 \times 260 \times 250 \times 12 = 468\ 000\ \text{N}$$

$$f_W = \frac{P}{L_W} = \frac{468\ 000}{500} = 936\ \text{N/mm}$$

$$P_{allow} = 95.5\ W \quad \text{N/mm}$$

$$\therefore\ 95.5\ W = 936$$

$$\therefore\ W = \underline{9.8 \text{ say } 10 \text{ mm}}$$

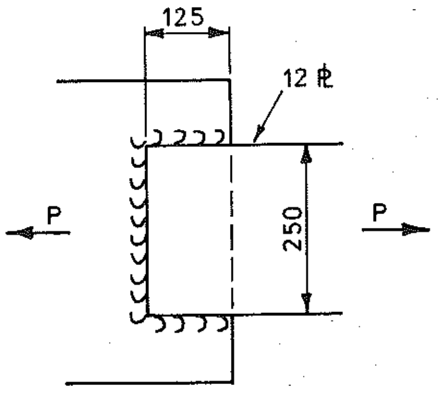


FIG. 42

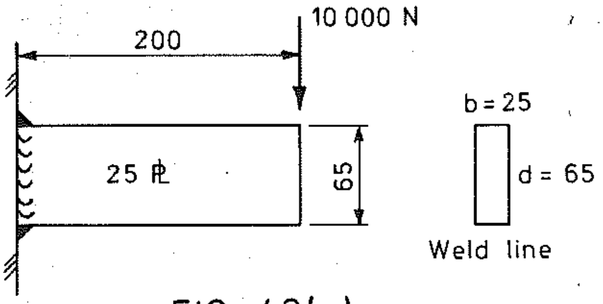


FIG. 43(a)

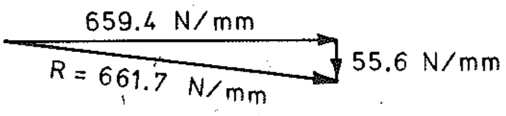


FIG. 43(b)

Worked Example 18(a).

Refer to Fig. 43(a).
Determine the size of fillet weld required. Use E41XX rods.

Answer

From Table No. 4, page 12.53.

$$Z_W = bd + \frac{d^2}{3} = 25 \times 65 + \frac{65^2}{3} = 3033 \text{ units.}$$

$$\text{Primary "stress"} = \frac{\text{Load}}{L_W} = \frac{10\ 000}{2(65 + 25)} = 55.6\ \text{N/mm}$$

Secondary "stress" $= \dfrac{M}{Z_W} = \dfrac{10\ 000 \times 200}{3033} = 659.4$ N/mm

From Fig. 43(b), page 12.51, the resultant "stress" = 661.7 N/mm

$\therefore\ 95.5\ W = 661.7$

from which $W = 6.93$ mm say use 8 mm fillet weld.

Worked Example No. 18(b).

Refer to Fig. 44(a).

Determine the size of fillet weld required. Use E41XX rods.

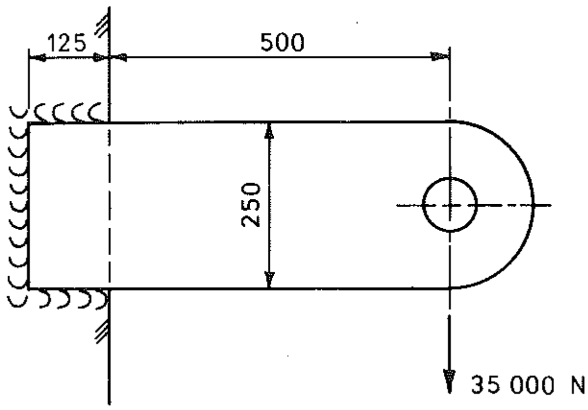


FIG. 44(a)

Answer

From Table No. 4, page 12.53.

$$J_W = \frac{(2b + d)^3}{12} - \frac{b^2\ (b + d)^2}{2b + d}$$

$b = 125$ and $d = 250$

by substitution $J_W = 6\ 022\ 135$ mm units

From Fig. 44(b)

$$N_y = \frac{b^2}{2b + d}$$

$b = 125$ and $d = 250$

by substitution $N_y = 31.25$ mm

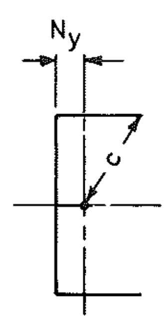


FIG. 44(b)

$$\therefore\ c = \sqrt{\left(\frac{250}{2}\right)^2 + (125 - 31.25)^2}$$

$= 156.25$ mm

Torsion being applied to weld group

$= 35\ 000 \times \{500 + (125 - 31.25)\} = 20\ 781\ 250$ N.mm

Primary "stress" $= f_W = \dfrac{\text{Load}}{L_W} = \dfrac{35\ 000}{500} = 70$ N/mm

Secondary "stress" $= f_W = \dfrac{Tc}{J_W} = \dfrac{20\ 781\ 250 \times 156.25}{6\ 022\ 135}$

$= 539$ N/mm

TABLE No 4

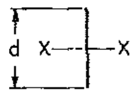
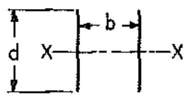
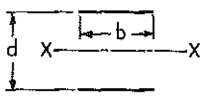
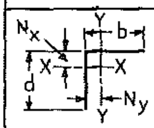
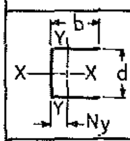
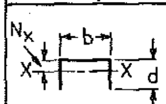
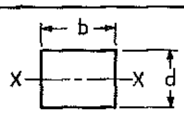
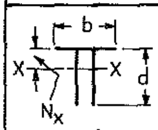
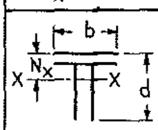
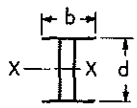
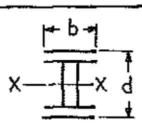
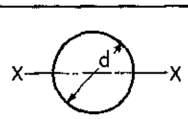
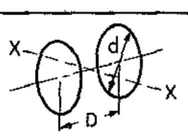
Properties of weld treated as a line		
Outline of welded joint b = width d = depth	Bending about horizontal axis X–X	Twisting
	$Z_w = \dfrac{d^2}{6}$	$J_w = \dfrac{d^3}{12}$
	$Z_w = \dfrac{d^2}{3}$	$J_w = \dfrac{d(3b^2+d^2)}{6}$
	$Z_w = bd$	$J_w = \dfrac{b^3+3bd^2}{6}$
 $N_x = \dfrac{d^2}{2(b+d)}$ $N_y = \dfrac{b^2}{2(b+d)}$	$Z_w = \underset{\text{top}}{\dfrac{4bd+d^2}{6}} = \underset{\text{bottom}}{\dfrac{d^2(4b+d)}{6(2b+d)}}$	$J_w = \dfrac{(b+d)^4-6b^2d^2}{12(b+d)}$
 $N_y = \dfrac{b^2}{2b+d}$	$Z_w = bd + \dfrac{d^2}{6}$	$J_w = \dfrac{(2b+d)^3}{12} - \dfrac{b^2(b+d)^2}{(2b+d)}$
 $N_x = \dfrac{d^2}{b+2d}$	$Z_w = \underset{\text{top}}{\dfrac{2bd+d^2}{3}} = \underset{\text{bottom}}{\dfrac{d^2(2b+d)}{3(b+d)}}$	$J_w = \dfrac{(b+2d)^3}{12} - \dfrac{d^2(b+d)^2}{(b+2d)}$
	$Z_w = bd + \dfrac{d^2}{3}$	$J_w = \dfrac{(b+d)^3}{6}$
 $N_x = \dfrac{d^2}{b+2d}$	$Z_w = \underset{\text{top}}{\dfrac{2bd+d^2}{3}} = \underset{\text{bottom}}{\dfrac{d^2(2b+d)}{3(b+d)}}$	$J_w = \dfrac{(b+2d)^3}{12} - \dfrac{d^2(b+d)^2}{(b+2d)}$
 $N_x = \dfrac{d^2}{2(b+d)}$	$Z_w = \underset{\text{top}}{\dfrac{4bd+d^2}{3}} = \underset{\text{bottom}}{\dfrac{4bd^2+d^3}{6b+3d}}$	$J_w = \dfrac{d^3(4b+d)}{6(b+d)} + \dfrac{b^3}{6}$
	$Z_w = bd + \dfrac{d^2}{3}$	$J_w = \dfrac{b^3+3bd^2+d^3}{6}$
	$Z_w = 2bd + \dfrac{d^2}{3}$	$J_w = \dfrac{2b^3+6bd^2+d^3}{6}$
	$Z_w = \dfrac{\pi d^2}{4}$	$J_w = \dfrac{\pi d^3}{4}$
	$Z_w = \dfrac{\pi d^2}{2} + \pi D^2$	

Fig. 44(c) shows the critical points of the weld group and the method of determining the absolute "stress".

$$95.5\ W = 585$$

$$\therefore\ W = 6.13 \quad \text{say 8mm}$$

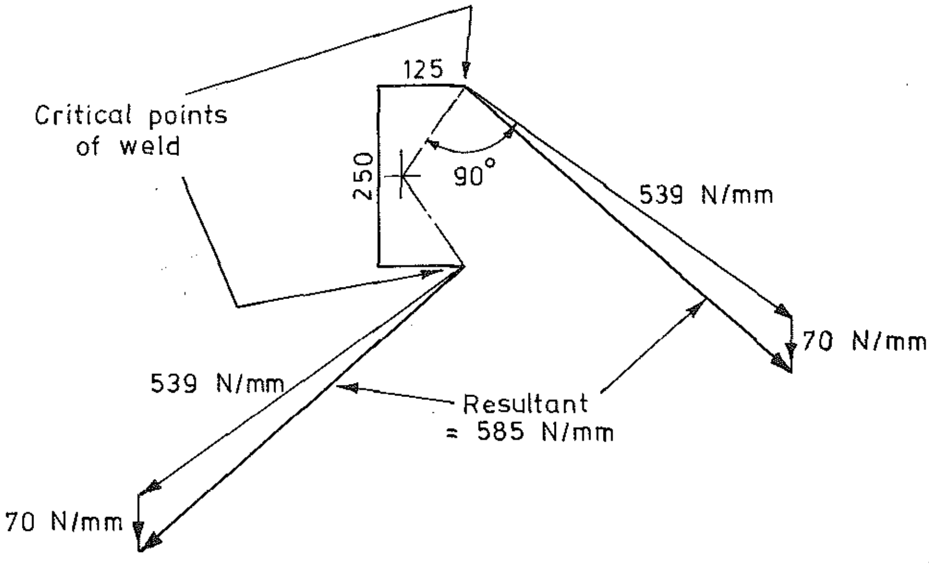


FIG. 44(c)

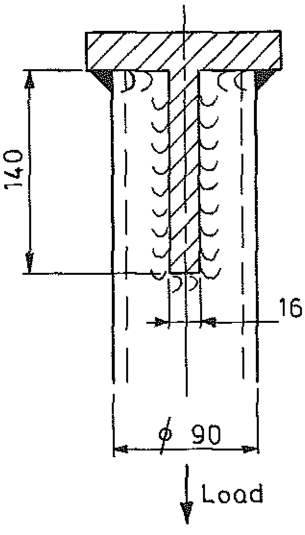


FIG. 45

EXERCISES

13. Refer to Fig. 45.

    A pipe is slotted and welded to the upper chord of a truss with a continuous 6 mm fillet weld. If E41XX rods are used calculate the maximum permissible load which can be applied to the pipe.

    Ans. $\dfrac{482.9}{1.05} = 459.9$ kN.

14. Refer to Fig. 39, page 12.43.

    If the eye piece were to be welded instead of bolted, determine the size of fillet weld required.
    Weld the eye piece to the container along all edges i.e. 600 mm total length of weld. Use E41XX rods.

    Ans. (2.3 mm, use a 5 mm fillet)

15. Calculate the size of fillet weld required to attach the bracket to the wall as illustrated in Fig. 46, page 12.55.
    Materials are mild steel. Use E41XX rods.

    Ans. (4.78 mm, use a 6 mm fillet)

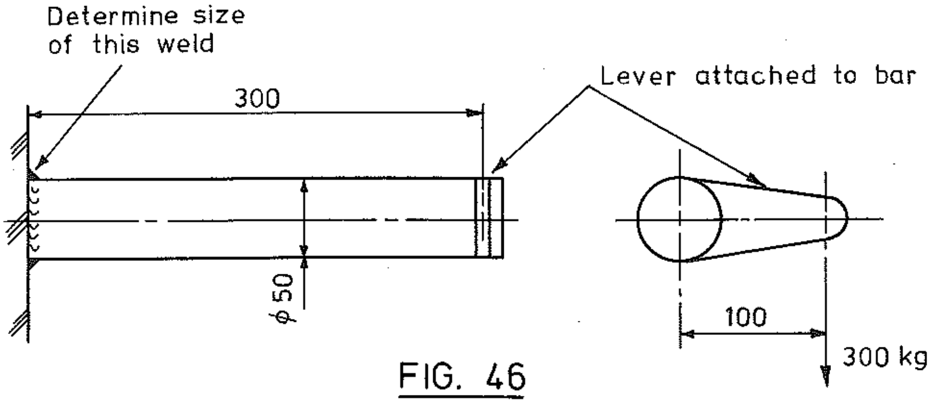


FIG. 46

Weld design - machine design applications

Reversal of stress - endurance

In general, for any endurance application involving welding, the maximum permissible shear stress in the weld metal = $\frac{1}{5}$ of the endurance limit in shear of the weld metal or parent (base) metal, whichever is least (page 1.25).

Further guidance on the maximum permissible stresses for both the weld metal and base metal, for particular applications, is given in Appendix B of AS 1250. These stresses are separated into four categories according to the number of expected loading cycles.

The figures in AS 1250 are for the stress range, F_{sr}. (R < + 1, fluctuating load, to R = -1, complete reversal. R is the ratio of minimum stress to the maximum stress; tension taken as positive, compression negative.) This is in accordance with modern thinking, that the stress range is the dominating factor in fatigue design. (No increase in F_{sr} is allowed for high strength structural steel.)

Worked Example No. 20

Refer to Fig. 47. This machine member is made from 40 x 10 M.S. flat. A reversing load of 220 N is applied at the end.

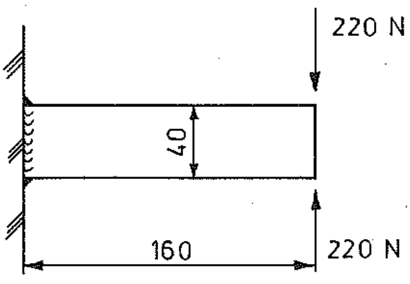


FIG. 47

Determine:

(i) whether the stated size of flat is satisfactory.

(ii) the required size of fillet weld to attach the arm to the M.S. support.

Answer

(i) $M = f_b Z$

$$\therefore f_b = \frac{M}{Z} = \frac{220 \times 160 \times 6}{10 \times 40 \times 40} = 13.2 \text{ MPa}$$

The particular case of reversed bending on a fillet weld is not specifically mentioned in App. B of AS 1250. The nearest choice for the base metal check would be example 20, Fig. B1, AS 1250.

Tables B2 and B3 of AS 1250 then specify F_{sr} = 30 MPa (tension, or reversal) for this case.

This means that for complete reversal F_b (max.) = 15 MPa > 13.2 MPa

(ii) Primary weld "stress" (shear)

$$f_w = \frac{\text{load}}{L_w} = \frac{220}{100} = 2.2 \text{ N/mm}$$

Secondary "stress" (bending)

$$Z_w = bd + \frac{d^2}{3} \quad \text{(Table No. 4, page 12.53)}$$

$$= 10 \times 40 + \frac{40^2}{3} = 933.33 \text{ mm units}$$

$$M = f_w Z_w$$

$$\therefore f_w = \frac{M}{Z_w} = \frac{220 \times 160}{933.33} = 37.7 \text{ N/mm}$$

$$\therefore f_w \text{ (absolute)} = \sqrt{2.2^2 + 37.7^2} = 37.76 \text{ N/mm}$$

M.S., F_e = 207 MPa; E41XX rods, $F_e = 0.45 \times 410 = 184.5$ MPa (page 1.25)

The least value = 184.5 MPa

$$\therefore F_w = \frac{184.5}{5 \times 2} = 18.45 \text{ MPa}$$

The allowable load per mm of weld

$$P_{allow} = F_w \times A = 18.45 \times 0.707W = 13W \text{ N/mm}$$

$$\therefore 13W = 37.7$$

$$\therefore W = 2.9 \text{ mm}$$

EXERCISES

16.

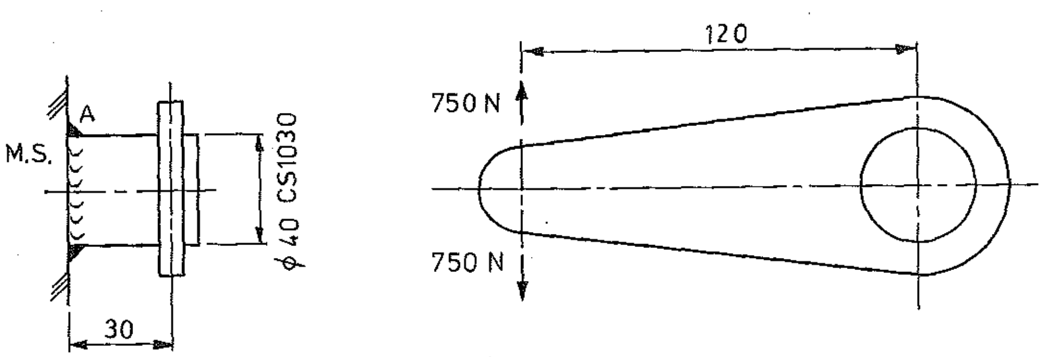


FIG. 48

Refer to Fig. 48. This shows a lever subjected to a constant reversal of load.

(i) Calculate the maximum shear stress, f_s, in the base metal.

(A suitable value for F_s would be $0.6F_b = 0.6 \times 15 = 9$ MPa. F_b from Worked Example No. 20.)

(ii) Determine the size of the fillet weld required at A. Use E41XX rods.

Ans. (i) 7.38 MPa (ii) 3.2 mm

(6) PINS

All pins for structural connections shall be designed for a force of at least 25 kN, Rule 9.1 AS 1250.

Pins for mechanical equipment have no particular governing clauses. Some of the examples in this section are from mechanical equipment.

Modes of failure of pinned joints

(i) Pin in bending.
(ii) Pin in shear.
(iii) Bearing on link.
(iv) Gross section yielding.
(v) Net section yielding.
(vi) Shearing along two planes.
(vii) Tearing in one plane (splitting).
(viii) Dishing.

(i) Bending moment $= \frac{WL}{4}$, Rule 9.7.2 AS 1250.

$F_b = 0.75\ F_y$, Rule 5.2(1) AS 1250.

(ii) $F_v = 0.33\ F_y$, Table 9.5.2, AS 1250,
based on an 'average' shear stress calculation.

(iii) $F_p = 0.75\ F_y$, Rules 6.9 and 7.6 AS 1250.

The stress situation is relatively unconfined and so the lower 0.75 F_y value applies.
If pin seizure is likely to occur in situations where there is slight pin rotation the bearing stress should be limited to 80 to 100 MPa.

Points (iv) to (viii) are covered by the following rules from the AS 1250 Code.

Refer to Fig. 49, page 12.58.

(iv)	Rule 7.1	$A_S \geq P/\ 0.6\ F_y$
(v)	Rule 7.5.3 (a)	$(A_{aa} + A_{cc}) > P/\ 0.45\ F_y$
(vi)	Rule 7.5.3 (b)	$A_{bb} > A_s$
(vii)	Rule 7.5.3 (c)	$A_S < 12\ T_1^2$
(viii)	Rule 7.5.3 (d)	$(A_{aa} + A_{cc}) > 1.33\ A_S$

Rule (d) effectively covers Rule (a).

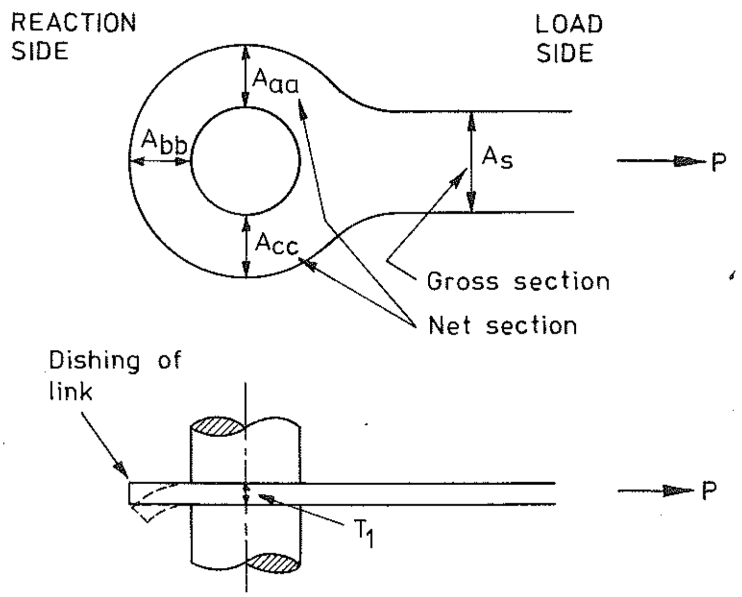


FIG. 49

Worked Example No. 21.

Design the pin in the connection shown in Fig. 50 to withstand a load of 60 kN.

Answer

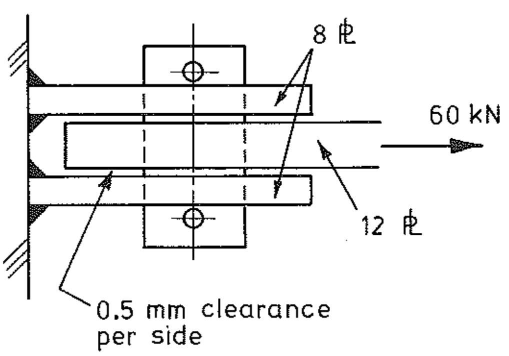


FIG. 50

Bending moment on pin $= \frac{WL}{4}$

$$= \frac{60\ 000 \times 21}{4} = 315\ 000 \text{ N.mm}$$

$F_b = 0.75\ F_y$.

Use CS 1030 bright steel, $F_y = 250$ MPa

$\therefore\ F_b = 0.75 \times 250 = 187.5$ MPa.

B.M. $= F_b\ Z$

$$\therefore\ Z = \frac{\text{B.M.}}{F_b} = \frac{315\ 000}{187.5} = 1680 \text{ mm}^3$$

$$Z = \frac{\pi d^3}{32} = 1680$$

from which $d = 25.77$ mm
use ϕ 27 material.

Check now for shear stress.

$$f_v = \frac{\text{load}}{\text{area}} = \frac{30\ 000 \times 4}{\pi\ 27^2} = 52.4 \text{ MPa}$$

$F_v = 0.33\ F_y = 0.33 \times 250 = 82.5$ MPa > 52.4 MPa.

Bearing check.

$$f_p = \frac{60\ 000}{27 \times 12} = 185.2 \text{ MPa}$$

$$F_p = 0.75\ F_y$$

F_y of 12 plate $=$ 260 MPa App. C. AS 1250.

$\therefore\ F_p = 0.75 \times 260 = 195$ MPa > 185.2 MPa

EXERCISES (General)

17. Fig. 51 shows the anchor point for a guy on some mechanical equipment.

(a) For the pin determine:-

(i) f_b and F_b

(ii) f_v and F_v

(iii) f_p and F_p

(b) f_{tf} and F_{tf} for the thread.

(c) the size of fillet weld required using E41XX rods. Each plate is welded along the outside and inside edge.

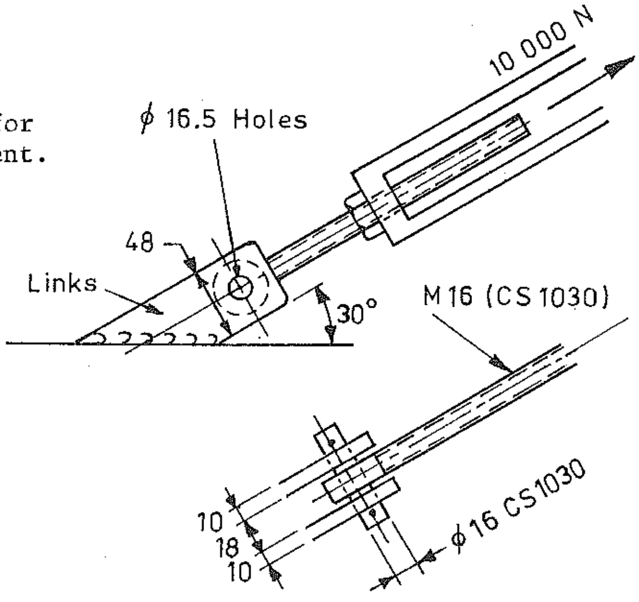


FIG. 51

(d) the average and permissible tensile stresses at the net section of the links.

(a)	(i)	174.1 MPa.	187.5 MPa.
	(ii)	24.9 MPa.	82.5 MPa.
	(iii)	39.06 MPa.	187.5 MPa.
(b)	63.7 MPa.	150 MPa.	
(c)	0.27 mm	use 4mm	
(d)	15.9 MPa.	117 MPa.	

18. For the stationary bracket shown in Fig. 52, page 12.60, determine:

(a) (i) The required diameter of shaft. Design as a stationary shaft using CS 1030.

(ii) The actual and permissible lateral deflections.

(iii) The actual and permissible torsional deflections.

(b) The size of fillet weld required to join the shaft to the mounting plate. Use E41XX rods.

(c) (i) The value of the interaction equation for the bolts. Bolt shanks are in the shear plane.

(ii) f_{pf} and F_{pf} for the most highly stressed bolt.

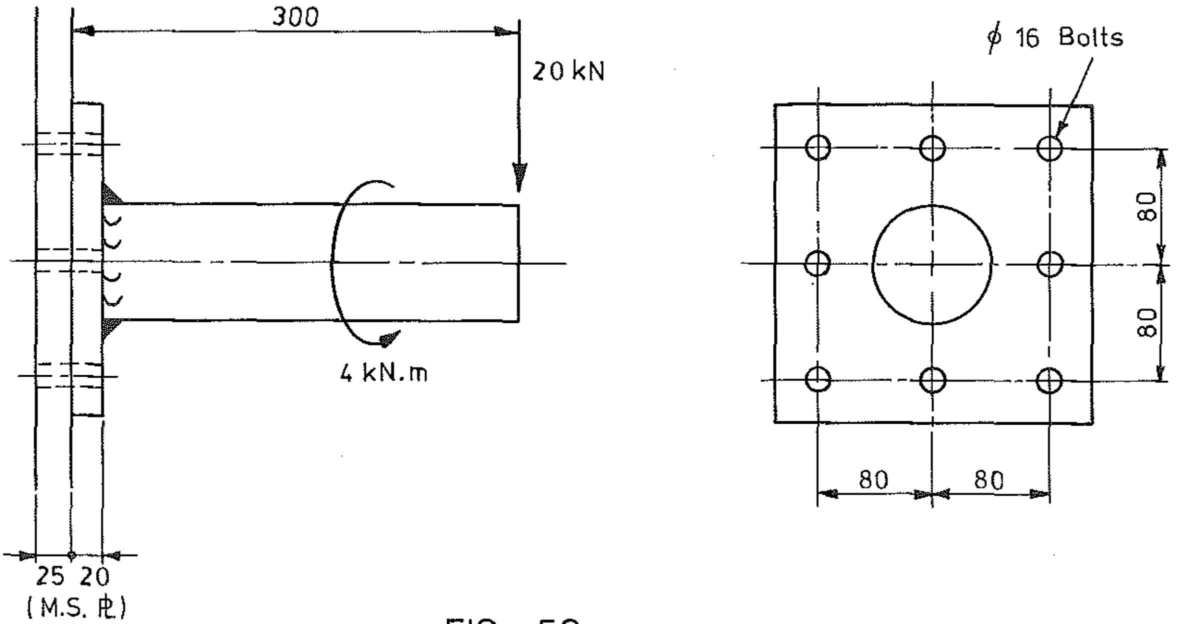


FIG. 52

Ans.	(a)	(i)	77.6	use 80 mm
		(ii)	0.35 mm	0.448 mm
		(iii)	0.193°	0.84°
	(b)		12.42 say 15 mm	
	(c)	(i)	0.463	
		(ii)	24.5 MPa	525 MPa

SHEAR STRESS

There are three basic types of applied shear stress.

(a) Direct shear stress

An example of this type of shear stress is shown in Fig. 25, page 12.31.

The shear forces being applied to each bolt are so near to being co-linear that no significant bending moment is being applied to the bolt shank.

The shear stress is considered as being constant over the entire area of the bolt.

i.e. $f_{vf} = \dfrac{\text{Load}}{A_s}$

(assuming the shank is in the shear plane).

This is not strictly correct as a photo-elastic analysis shows but is acceptable for practical purposes.

(b) Torsional shear stress

An example of this type of shear stress is a shaft transmitting a torque.

The shear stress is a maximum on the outer fibres diminishing steadily (in direct proportion to the radius) to zero at the neutral axis, refer to Fig. 53.

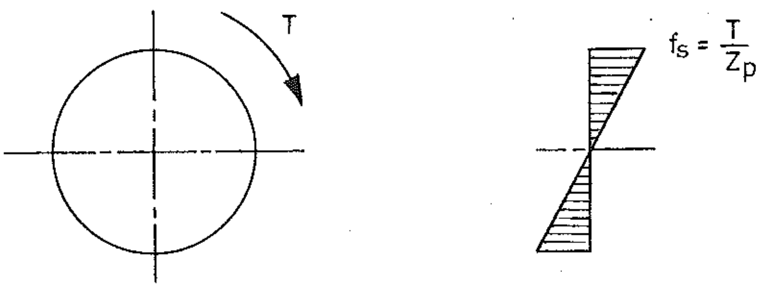


FIG. 53

(c) Beam shear stress

This type of shear stress occurs in any loaded beam.

The shear stress is accompanied by a bending moment and is more complex than the first two mentioned shear stresses.

Shear stresses act in the vertical direction at all points on the section, and at any point the theorem of complementary shear states that there must exist a horizontal shear stress numerically equal to the vertical shear stress, see Fig. 54.

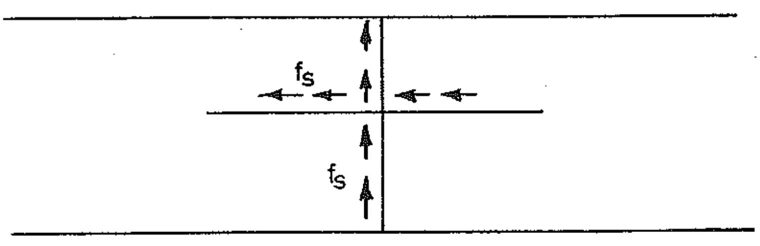


FIG. 54

The existence of the horizontal shear stresses may be demonstrated by comparing the behaviour of a solid beam, with one of the same depth made up of separate units laid one above the other, as in Fig. 55.

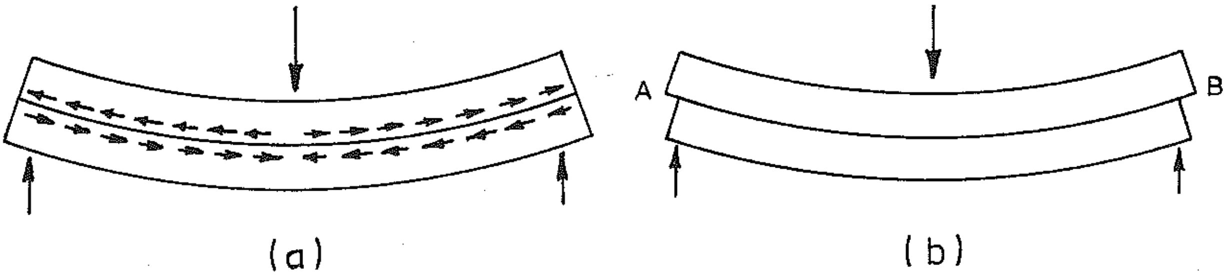


FIG. 55

The slide along AB in Fig. 55(b), page 12.61, is prevented in Fig. 55(a), page 12.61, by horizontal shear stresses being developed.

The value of the shear stress varies from point to point over the section, and it is necessary to derive an expression by which its value at any desired point may be computed. Since at every point the horizontal shear stress and vertical shear stress are equal, either one may be computed, and in practice it is simpler to compute the horizontal stress.

Let the shear stress be f_s at point "A" a distance "z" from the neutral axis on section 1 - 1, where the B.M. = M and the shear force = V, Fig. 56(a).

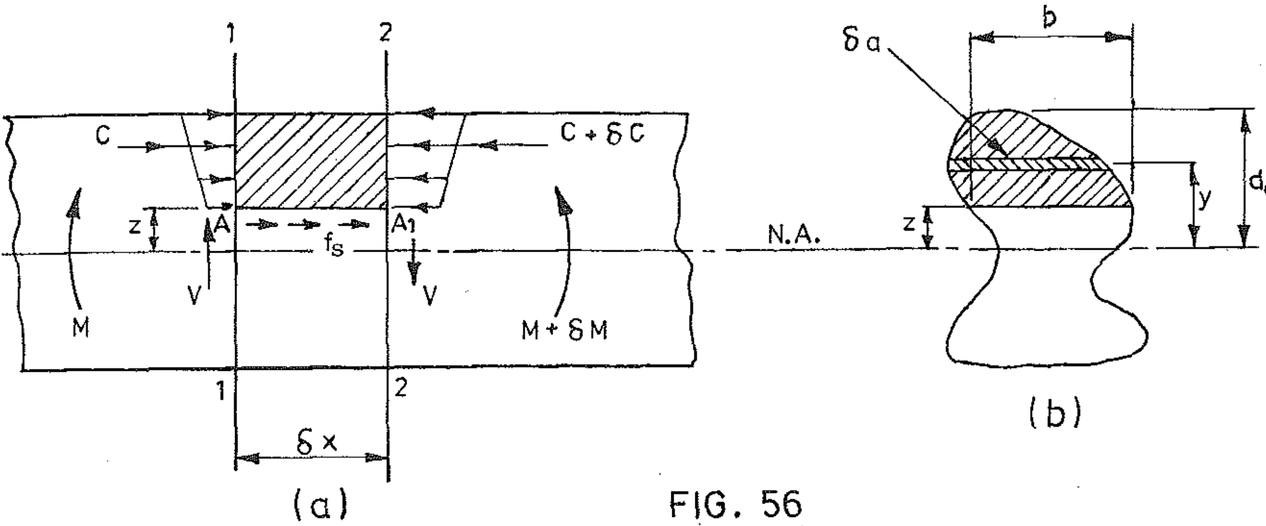


FIG. 56

At a closely adjacent section 2 - 2, a distance "δx" from 1 - 1, let the B.M. = $M + \delta M$ and the S.F. = V.
Draw a horizontal plane through A to intersect section 2 - 2 at A_1, and let the width of beam at this place = "b", Fig. 56(b).

Consider the equilibrium of the portion of the beam contained between sections 1 - 1, 2 - 2, the plane A - A, and the outer surface of the beam, as shown cross hatched in Fig. 56(a) and (b).

Acting on the face A - 1 there are the normal compressive stresses due to the B.M. "M", which will have a resultant "C", and on face A_1 - 2 there are somewhat greater stresses due to the slightly greater moment "$M + \delta M$" these having a resultant "$C + \delta C$". In order to provide the balance of horizontal forces the horizontal shear stress f_s across plane A-A, must act from left to right, as shown in Fig. 56(a); and resolving horizontally for equilibrium:

$$C + f_s\, b\, \delta x = C + \delta C$$

$$\text{or } f_s = \frac{1}{b}\frac{\delta C}{\delta x} \quad \text{------------------------ (i)}$$

$$\text{but } C = \sum_{z}^{d_c} \frac{My}{I}\,\delta a = \frac{M}{I}\sum_{z}^{d_c} \delta a \times y$$

$$= \frac{M}{I}\,Q, \text{ where } Q = \text{1st moment about the N.A. of the area cross hatched in Fig. 56(b).}$$

Then $C + \delta C = \frac{M + \delta M}{I} \times Q$

$\therefore \delta C = \frac{Q}{I} \times \delta M$ ---------------------- (ii)

substituting in (i)

$f_s = \frac{Q}{b\,I} \times \frac{\delta M}{\delta x}$ ---------------------- (iii)

But considering the balance of moments for the equilibrium of the whole portion of the beam between sections 1 - 1 and 2 - 2.

$M + V\delta x = \delta M + M$

or $\frac{\delta M}{\delta x} = V$ ---------------------------- (iv)

(i.e. the rate of change of B.M. = S.F.)

Substituting from (iv) into (iii) the final expression for the shear stress f_s is obtained in the form:

$f_s = \frac{VQ}{bI}$ ------------------------------ (v)

Equation (v) gives the value of the shear stress at any point of any section.

Maximum shear stress for various sections.

"I" beams. This was covered in Section No. 2, page 12.9.
Refer also to Fig. 11, page 12.9.

Round sections.

$f_{vm} = \frac{4}{3} \frac{V}{A}$ (By applying equation (v))

Square and rectangular sections.

$f_{vm} = \frac{3}{2} \frac{V}{A}$ (By applying equation (v))

Round hollow sections.

For thin walled circular tubes.

$f_{vm} \cong 2 \frac{V}{A}$

THE "BEAM" FORMULA.

Bending stresses in beams.

The external forces acting upon the portion of a beam to either side of a given section cause two effects at the section, (1) a B.M. and (2) a S.F. The stresses exerted upon this portion by the other portion must balance both of these effects, and thus two sets of stresses must exist, (1) stresses due to B.M., and (2) stresses due to S.F. Since the stresses (1) are normal to the section and stresses (2) are tangential (or shear) they do not affect one another, and each may be considered separately. (In the special case of pure or circular bending, the S.F. equals zero.)

Analysis of strain at a section.

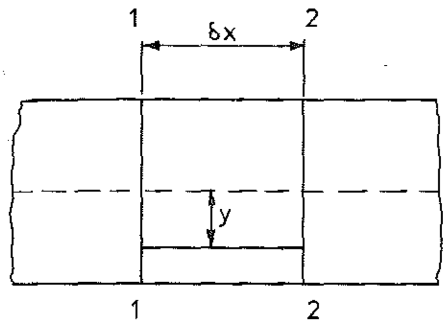


FIG. 56(c)

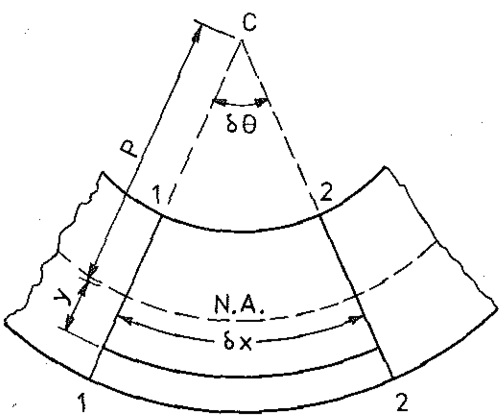


FIG. 56(d)

Consider strain due to B.M. at a section 1 - 1 taken at right angles to the length of the beam, Fig. 56(c). If a second section 2 - 2 be taken at an infinitesimal distance "δx" from 1 - 1, the bending of the beam results in these sections originally parallel, intersecting at the centre of curvature C of the bent beam, Fig. 56(d). No matter what shape of curve the beam bends into, an infinitesimal length in the neighbourhood of any point, coincides with the arc of the circle of curvature at that point: thus, provided 1 - 1 and 2 - 2 are taken indefinitely close together, the portion of the beam between may be treated as bent into a circular arc; and thus, after bending, the sections 1 - 1 and 2 - 2 will be radii, and will intersect at the centre of curvature C.

Material near the bottom of the beam is stretched, and that near the top is compressed; somewhere in between must exist a layer neither stretched nor compressed, constituting the Neutral Plane. The intersection of the neutral plane with the plane of symmetry gives the longitudinal Neutral Axis (N.A.). Tensile and compressive strains are proportional to distance from the N.A.

Consider strain of a line distant "y" from the N.A., Fig. 56(c). After strain this becomes bent into an arc about the centre of curvature C, Fig. 56(d).

Original length of line $= \delta x$

Length after bending $= (P + y).\delta\theta - P.\delta\theta = y.\delta\theta$

Now the N.A. is unchanged in length; therefore its length after strain remains equal to δx.

Thus $\delta x = P.\delta\theta$

∴ Change in length of line $= (P + y).\delta\theta = y.\delta\theta$

and strain of line $= \dfrac{y.\delta\theta}{P.\delta\theta} = \dfrac{y}{P}$

This applies to a line at any distance "y", either below or above the N.A.; provided that "y" be accounted + below the N.A. and - above the N.A., and that tension be considered + and compression -.

Thus the strain at section 1 - 1 due to B.M. consists of normal tension one side of the N.A. and normal compression the other side, each being proportional to the distance from the N.A.

Calculation of stress at the section.

Fig. 56(e) shows the stresses exerted by the portion to the right of section 1 - 1 on the portion to the left of that section. These vary linearly from a maximum compression at the top, through zero at the N.A., to a maximum tension at the bottom; the compressive stresses above the N.A. have a resultant C, and the tensile stresses below the N.A. have a resultant T, and since these are the only forces acting on the left hand portion in the direction of the span it is seen, by resolving in that direction for equilibrium of the L.H. portion, that C = T.

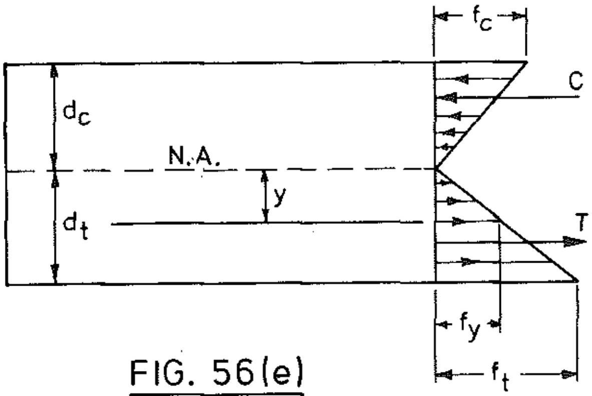


FIG. 56(e)

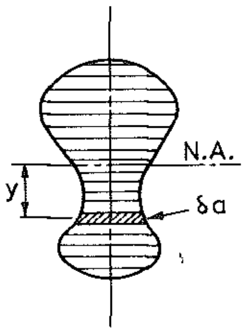


FIG. 56(f)

In order to calculate the values of "C" and "T" the section is divided up into an indefinitely large number of infinitesimal areas by lines parallel to the N.A., Fig. 56(f). Consider the area "δa" at distance "y" from the N.A., the stress at that area being "fy". Then at any point $fy = E.\frac{y}{P}$.

$$\text{Specifically, } T = \sum_{o}^{d_t} E.\frac{y}{P}.\delta a \quad \text{and} \quad C = \sum_{o}^{d_c} E.\frac{y}{P}.\delta a_1$$

Since "C" and "T" are equal and opposite parallel forces they constitute a couple whose moment is termed the "Moment of Resistance" (M of R) of the section. The M of R is calculated by taking the sum of the moments about the N.A. of all the forces across the elementary areas, noting that both tensile and compressive forces have moments about the N.A. in the same sense. Thus:-

$$\text{M of R} = \sum_{o}^{d_t} E.\frac{y^2}{P}.\delta a + \sum_{o}^{d_c} E.\frac{y^2}{P}.\delta a_1$$

$$= \frac{EI}{P}, \text{ where } I = \text{Moment of Inertia} \text{ ——}$$

(Second Moment) of the area about the N.A.

If the B.M. at the section is "M", it is clear that for equilibrium, the M of R must equal the B.M.

$$\therefore \; M = \frac{EI}{P} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

To determine the maxima stresses at the section due to B.M., note that:

$$f_t = \frac{E.d_t}{P} \quad \text{and} \quad f_c = \frac{E.d_c}{P}$$

$$\therefore \quad \frac{E}{P} = \frac{f_t}{d_t} = \frac{f_c}{d_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Substituting (2) into (1)

$$f_t = \frac{M}{I/d_t} = \frac{M}{Z_t} \quad \text{where} \quad Z_t = \text{Section Modulus in tension}$$

and $$f_c = \frac{M}{I/d_c} = \frac{M}{Z_c} \quad \text{where} \quad Z_c = \text{Section Modulus in compression.}$$

If the N.A. is an axis of symmetry,

$$d_t = d_c \quad \text{and} \quad \therefore \quad Z_t = Z_c \quad (= Z, \text{ say}).$$

In that case

$$f_t = f_c = \frac{M}{Z}$$

or $$M = fZ$$

The foregoing "beam" formula theory was based on the following assumptions:

(1) Sections plane and at right angles to the length before bending remain plane and at right angles to the length after bending.

(2) The elastic limit is nowhere exceeded, so that stress is everywhere proportional to strain.

(3) The modulus of elasticity connecting strain and stress is Young's Modulus. This has the same value for both tension and compression.

(4) The beam has a plane of symmetry parallel to its length, and all loads or the resultants of all loads are applied in this plane.

S U P P L E M E N T A R Y   S E C T I O N

This section contains theory on more advanced structural work to assist the mechanical designer who may be occasionally faced with such problems. The methods given are basic and conservative and are primarily intended as a "service" to mechanical plant equipment.

Fixed, built-in or encastre beams.

Up to this point only simply supported beams have been considered. However, on occasions the ends of a beam are fixed so as to prevent rotation under the action of superimposed loads.

The B.M. for such a beam can be considered in two parts:

(i) the free or positive B.M. diagram which would have occurred if the beam had been simply supported i.e. the ends were free to rotate.

and (ii) the negative or fixing moments resulting from restraints imposed on the ends of the beam by the supports.

Fig. 57 shows a general example of a fixed, built-in or encastre beam. The shaded areas show the final bending moment diagram.

Fig. 59, page 12.66 shows the formulas for a number of simple cases of fixed beam loading.

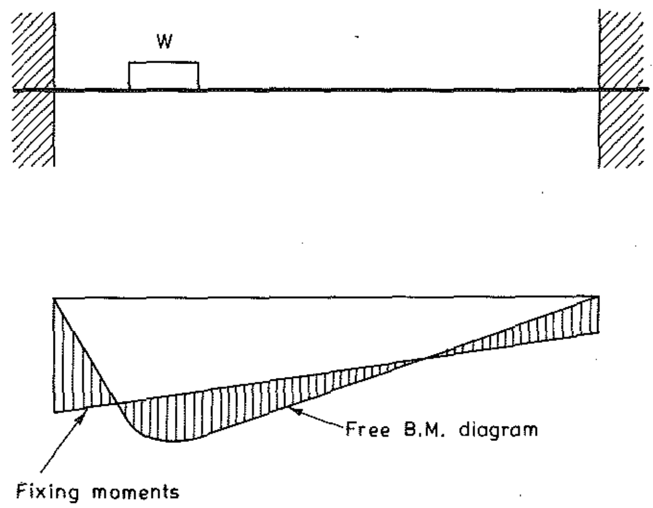


FIG. 57

Propped cantilevers

These are beams which are "fixed" at one end and simply supported at the other.

Fig. 60, page 12.66 gives formulas for some basic loading cases of a propped cantilever.

Effective length of a beam "ℓ"

Refer to page 12.7.

(i) Fixed, built-in or encastre beams.

Refer to Rule 5.9.3.1(c) AS 1250

$\ell = 0.7\ L$

(ii) Propped cantilevers.

Conservatively ℓ = L although some benefit is obtained from the fact that the beam is "fixed" at one end.

EXERCISES.

19. Refer to Fig. 58, (For the purpose of the exercise neglect self weight of beam.)

(a) Construct B.M. and S.F. diagrams and determine the reactions and beam deflection.

(b) Repeat question (a) considering the beam as a propped cantilever with end A fixed.

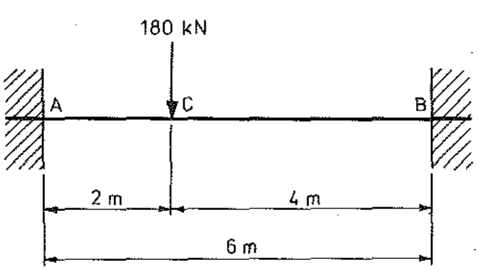


FIG. 58

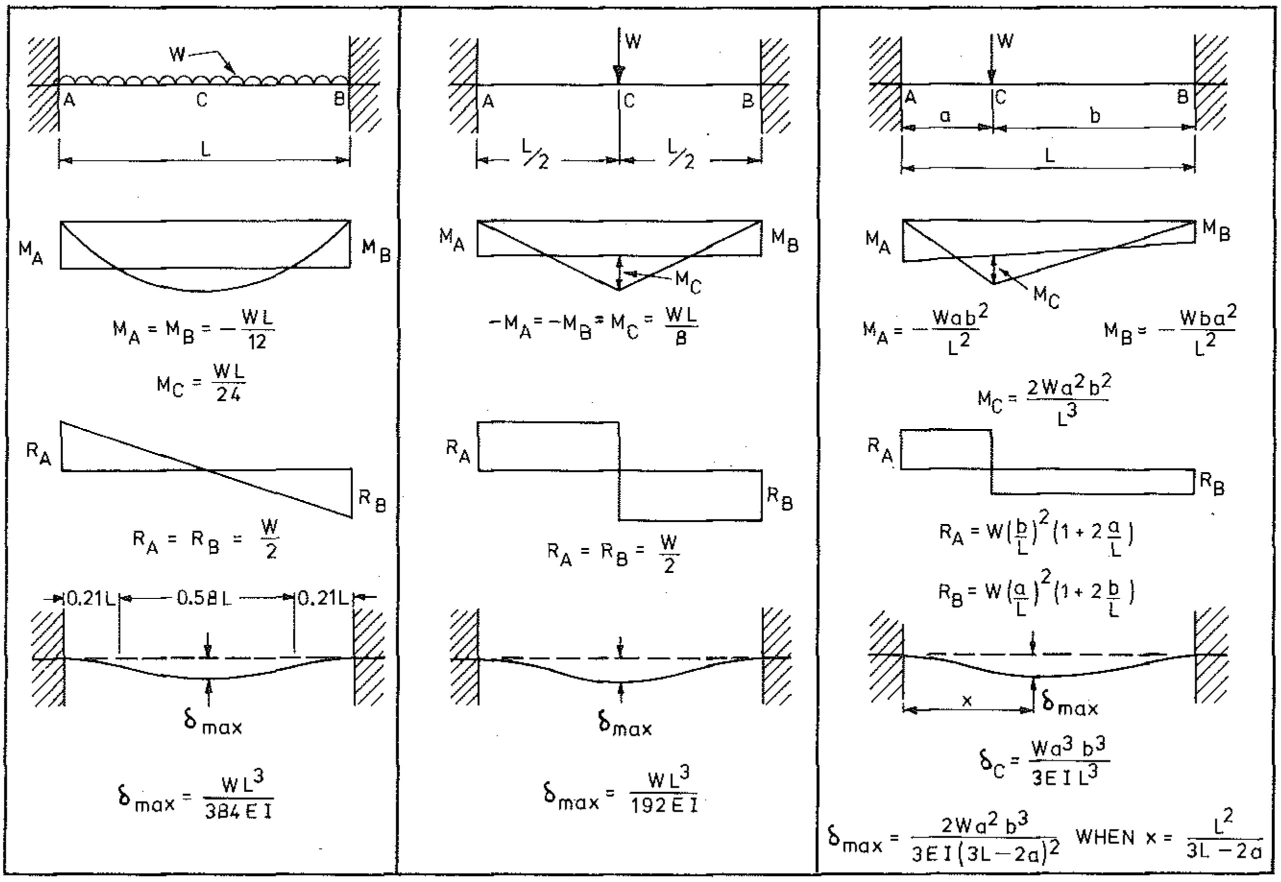

W
A
C
B
L
M_A
M_B
$M_A = M_B = -\frac{WL}{12}$
$M_C = \frac{WL}{24}$
R_A
R_B
$R_A = R_B = \frac{W}{2}$
0.21 L
0.58 L
0.21 L
δ_{max}
$\delta_{max} = \frac{WL^3}{384EI}$
W
A
C
B
L/2
L/2
M_A
M_B
M_C
$-M_A = -M_B = M_C = \frac{WL}{8}$
R_A
R_B
$R_A = R_B = \frac{W}{2}$
δ_{max}
$\delta_{max} = \frac{WL^3}{192EI}$
W
A
C
B
a
b
L
M_A
M_B
M_C
$M_A = -\frac{Wab^2}{L^2}$
$M_B = -\frac{Wba^2}{L^2}$
$M_C = \frac{2Wa^2b^2}{L^3}$
R_A
R_B
$R_A = W\left(\frac{b}{L}\right)^2\left(1+2\frac{a}{L}\right)$
$R_B = W\left(\frac{a}{L}\right)^2\left(1+2\frac{b}{L}\right)$
x
δ_{max}
$\delta_C = \frac{Wa^3b^3}{3EIL^3}$
$\delta_{max} = \frac{2Wa^2b^3}{3EI(3L-2a)^2}$ WHEN $x = \frac{L^2}{3L-2a}$

FIG. 59

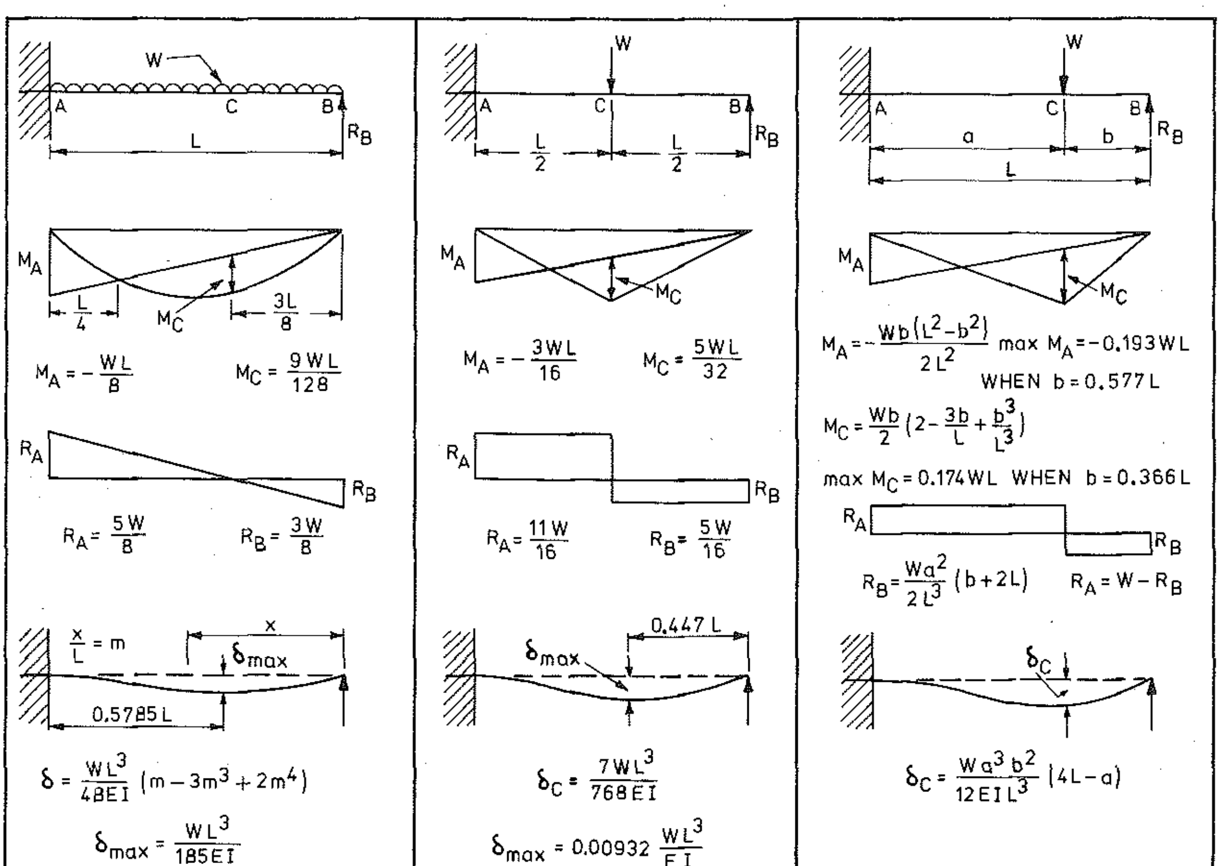

W
A
C
B
R_B
L
M_A
M_C
$\frac{L}{4}$
$\frac{3L}{8}$
$M_A = -\frac{WL}{8}$
$M_C = \frac{9WL}{128}$
R_A
R_B
$R_A = \frac{5W}{8}$
$R_B = \frac{3W}{8}$
$\frac{x}{L} = m$
x
δ_{max}
0.5785 L
$\delta = \frac{WL^3}{48EI}(m-3m^3+2m^4)$
$\delta_{max} = \frac{WL^3}{185EI}$
W
A
C
B
R_B
$\frac{L}{2}$
$\frac{L}{2}$
M_A
M_C
$M_A = -\frac{3WL}{16}$
$M_C = \frac{5WL}{32}$
R_A
R_B
$R_A = \frac{11W}{16}$
$R_B = \frac{5W}{16}$
0.447 L
δ_{max}
$\delta_C = \frac{7WL^3}{768EI}$
$\delta_{max} = 0.00932\,\frac{WL^3}{EI}$
W
A
C
B
R_B
a
b
L
M_A
M_C
$M_A = -\frac{Wb(L^2-b^2)}{2L^2}$ max $M_A = -0.193WL$
WHEN $b = 0.577L$
$M_C = \frac{Wb}{2}\left(2-\frac{3b}{L}+\frac{b^3}{L^3}\right)$
max $M_C = 0.174WL$ WHEN $b = 0.366L$
R_A
R_B
$R_B = \frac{Wa^2}{2L^3}(b+2L)$
$R_A = W - R_B$
δ_C
$\delta_C = \frac{Wa^3b^2}{12EIL^3}(4L-a)$

FIG. 60

(c) Repeat question (a) considering the beam as simply supported.

(d) Compare the respective values of (a), (b) and (c).

(a) $M_A = -160$ kN.m, $M_B = -80$ kN.m, $M_C = 106.67$ kN.m

$R_A = 133.3$ kN, $R_B = 46.7$ kN

$\delta_{max} = \dfrac{156.74}{E\ I}$ units

(b) $M_A = -200$ kN.m, $M_B = 0$, $M_C = 106.67$ kN.m

$R_A = 153.33$ kN, $R_B = 26.67$ kN

$\delta_C = \dfrac{195.56}{E\ I}$ units

(c) $M_A = M_B = 0$, $M_C = 240$ kN.m

$R_A = 120$ kN, $R_B = 60$ kN

$\delta_{centre} = \dfrac{690}{E\ I}$ units

20. Refer to Fig. 61.
Determine F_{bcx} for the following cases:

(a) simply supported beam
(b) fixed beam
(c) propped cantilever

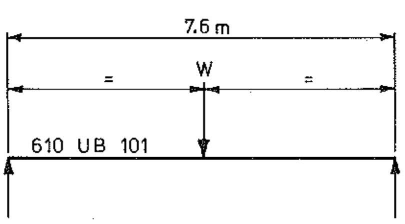


FIG. 61

For the purpose of the exercise assume:

(i) torsional end restraint is present at the supports.

(ii) the applied load is laterally supported but not rigidly connected to the beam.

(a) 80.6 MPa (b) 123 MPa (c) 80.6 MPa (conservatively)

21. Repeat Ex. 20 for the case where the load is rigidly connected to the critical flange of the beam.

(a) 152.9 MPa (b) 165 MPa (c) 152.9 MPa (conservatively)

22. Repeat Ex. 20 for the case where the load and critical flange are free to move laterally.

(a) 62.8 MPa (b) 101.3 MPa (c) 62.8 MPa (conservatively)

Continuous beams

A beam resting on more than two supports and covering more than one span is called a continuous beam

Fig. 62 shows the general loading case for a continuous beam.

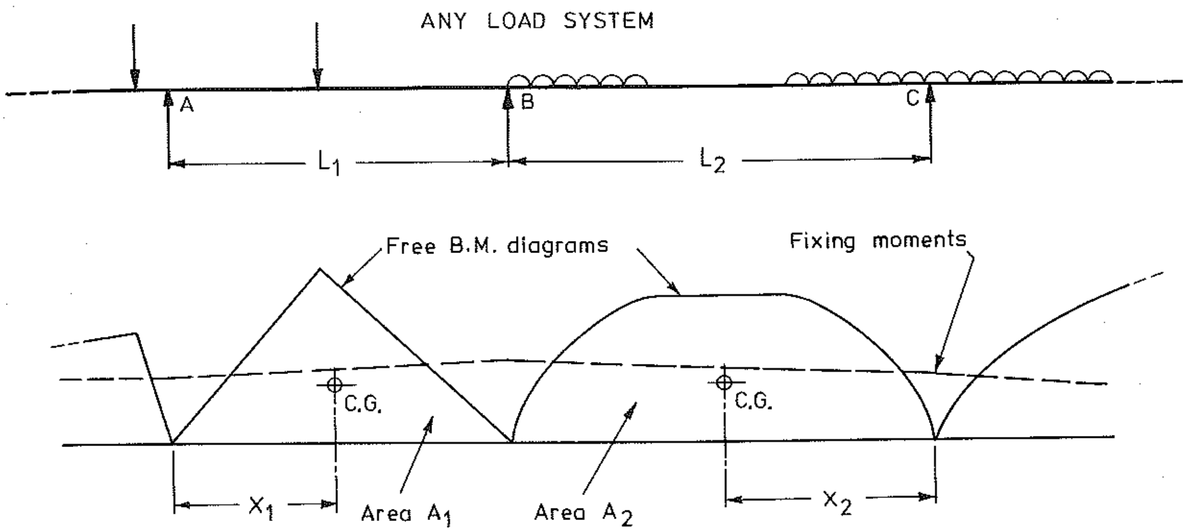


FIG. 62

The solution to this type of beam problem consists of evaluating the fixing, negative or "hogging" moment at each support. This can be done using Clapeyron's Theorem of Three Moments. The theorem is applied to any two adjacent spans and its general form is as follows:

$$M_A \frac{L_1}{I_1} + 2\, M_B \left(\frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + M_C \frac{L_2}{I_2} = -6 \left(\frac{A_1\, x_1}{L_1\, I_1} + \frac{A_2\, x_2}{L_2\, I_2} \right)$$

Where the beam is of constant cross-section the theorem simplifies to the following expression:

$$M_A\, L_1 + 2\, M_B \left(L_1 + L_2 \right) + M_C\, L_2 = -6 \left(\frac{A_1\, x_1}{L_1} + \frac{A_2\, x_2}{L_2} \right)$$

The general equation for the reaction at any support "n" of a continuous beam is:

$$R_n = R_n^{1} + R_n^{11} + \frac{M_{n-1} - M_n}{\ell_n} + \frac{M_{n+1} - M_n}{\ell_{n+1}}$$

where $R_n^{1} + R_n^{11}$ is the magnitude of the reaction at the support "n" that would be obtained if the continuous beam were cut at all intermediate supports and thus transformed into a system of simply supported beams.

Three general cases arise:

(1) Simple supports at each end.
(2) Cantilever end support.

(3) End supports fixed.

One example of each type will be given. For the purpose of the examples beam self-weight has been neglected.

Worked Example No. 22.

A continuous beam ABC, Fig. 63(a) is loaded as shown, draw bending moment and shear force diagrams.

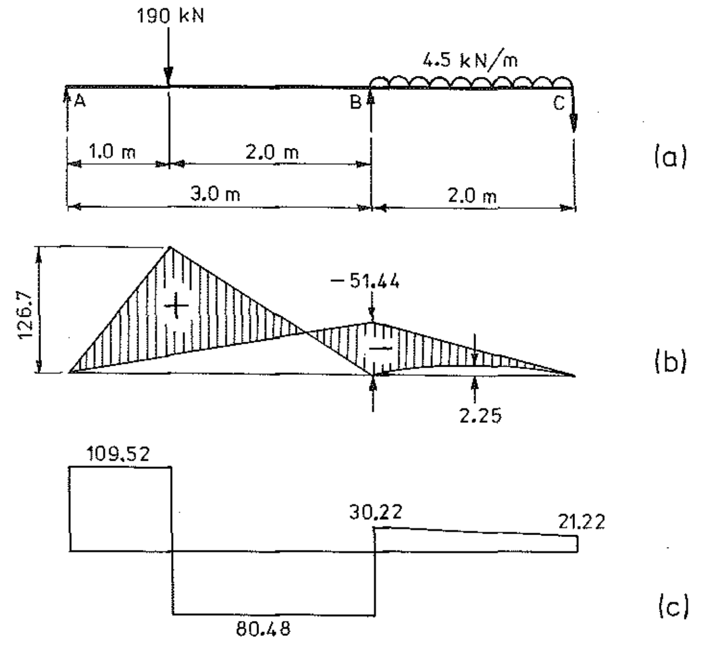


FIG. 63

Applying Clapeyron's theorem,

$$L_1 = 3.0 \text{ m} \quad \text{and} \quad L_2 = 2.0 \text{ m}$$

$$A_1 = \frac{190 \times 1 \times 2}{3} \times \frac{3}{2} = 190 \text{ kN.m}^2$$

$$A_2 = \frac{9 \times 2}{8} \times \frac{2 \times 2}{3} = 3 \text{ kN.m}^2$$

$$x_1 = 1.33 \text{ m} \quad \text{and} \quad x_2 = 1.0 \text{ m}$$

$$\therefore \quad M_A \times 3 + 2M_B\,(3 + 2) + M_C \times 2 = -6\left(\frac{190 \times 1.33}{3} + \frac{3 \times 1.0}{2}\right)$$

Since A and C are simple supports

$$M_A = M_C = 0$$

$$\therefore \quad M_B = -51.44 \text{ kN.m}$$

Applying the general equation for support reactions quoted previously:

$$R_A = \frac{190 \times 2}{3} + \frac{-51.44 - 0}{3} = 109.52 \text{ kN}$$

$$R_B = \frac{190 \times 1}{3} + \frac{4.5 \times 2}{2} + \frac{0 + 51.44}{3} + \frac{0 + 51.44}{2} = 110.7 \text{ kN}$$

$$R_C = \frac{4.5 \times 2}{2} + \frac{-51.44 - 0}{2} = -21.22 \text{ kN}$$

Check: upward forces total 220.22 kN
downward forces total 220.22 kN

By simple addition and subtraction the shear force diagram can be drawn, Fig. 63(c), page 12.69.

Worked Example No. 23.

A continuous beam ABCDE, Fig. 64(a) is loaded as shown, draw bending moment and shear force diagrams.

The second moments of area of the various sections are as follows:

$$I_{AC} = 53 \times 10^6 \text{ mm}^4$$

$$I_{CD} = 66 \times 10^6 \text{ mm}^4$$

$$I_{DE} = 86 \times 10^6 \text{ mm}^4$$

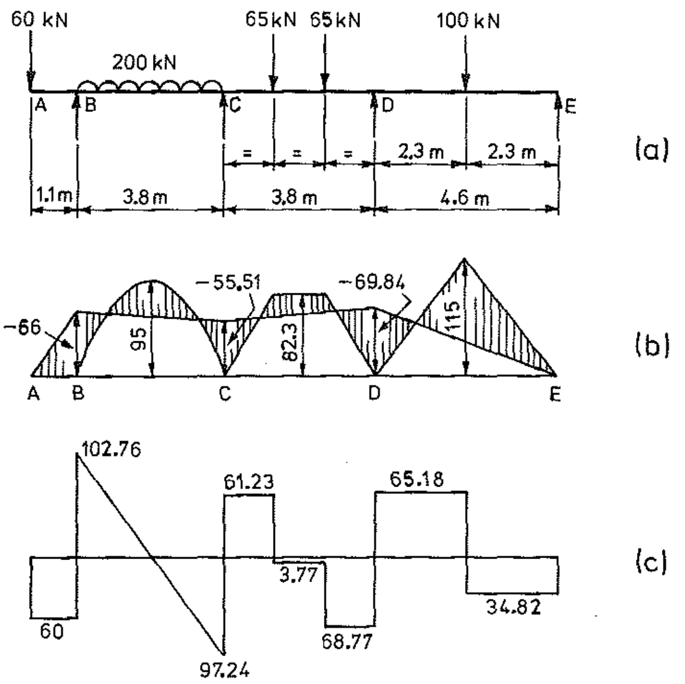


FIG. 64

Applying Clapeyron's theorem,

$$A_1 = \frac{200 \times 3.8}{8} \times \frac{3.8 \times 2}{3} = 240.67 \text{ kN.m}^2$$

$$A_2 = \frac{65 \times 3.8}{3} \times \frac{3.8 \times 2}{3} = 208.58 \text{ kN.m}^2$$

$$A_3 = \frac{100 \times 4.6}{4} \times \frac{4.6}{2} = 264.5 \text{ kN.m}^2$$

$$M_B = -60 \times 1.1 = -66 \text{ kN.m}; \quad M_E = 0$$

Consider spans BC and CD

$$\frac{-66 \times 3.8}{53 \times 10^6} + 2\, M_C \left(\frac{3.8}{53 \times 10^6} + \frac{3.8}{66 \times 10^6} \right) + M_D \frac{3.8}{66 \times 10^6}$$

.....cont. page 12.71

$$= -6\left(\frac{240.67 \times 1.9}{3.8 \times 53 \times 10^6} + \frac{208.58 \times 1.9}{3.8 \times 66 \times 10^6}\right)$$

$$\therefore\ 0.2585\ M_C + 0.0576\ M_D = -18.3717 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Next consider spans CD and DE

$$\frac{M_C \times 3.8}{66 \times 10^6} + 2M_D\left(\frac{3.8}{66 \times 10^6} + \frac{4.6}{86 \times 10^6}\right)$$

$$= -6\left(\frac{208.58 \times 1.9}{3.8 \times 66 \times 10^6} + \frac{264.5 \times 2.3}{4.6 \times 86 \times 10^6}\right)$$

$$\therefore\ 0.0576\ M_C + 0.2221\ M_D = -18.7077 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

From equations (1) and (2)

$$M_C = -55.51 \text{ kN.m}$$

$$M_D = -69.84 \text{ kN.m}$$

$$R_B = 60 + 100 + \frac{-55.51 + 66}{3.8} = 162.76 \text{ kN}$$

$$R_C = 100 + 65 + \frac{-66 + 55.51}{3.8} + \frac{-69.84 + 55.51}{3.8} = 158.47 \text{ kN}$$

$$R_D = 65 + 50 + \frac{-55.51 + 69.84}{3.8} + \frac{0 + 69.84}{4.6} = 133.95 \text{ kN}$$

$$R_E = 50 + \frac{-69.84 - 0}{4.6} = 34.82 \text{ kN}$$

Check: upward forces total 490 kN
downward forces total 490 kN

By simple addition and subtraction the shear force diagram can be drawn, Fig. 64(c), page 12.70.

Worked Example No. 24.

A continuous beam ABC, Fig. 65(a), page 12.72, is built-in at A and C and loaded as shown. Draw bending moment and shear force diagrams.

Built-in beams are best solved by imagining a mirror image of the span at the fixed support being added to the existing beam as shown in Fig. 65(b), page 12.72.

This means that the 'negative moment' at points X and Y is equal to the negative moment at B.

Applying Clapeyron's theorem,

$$A_1 = \frac{130 \times 3.6}{8} \times \frac{3.6 \times 2}{3} = 140.4 \text{ kN.m}^2$$

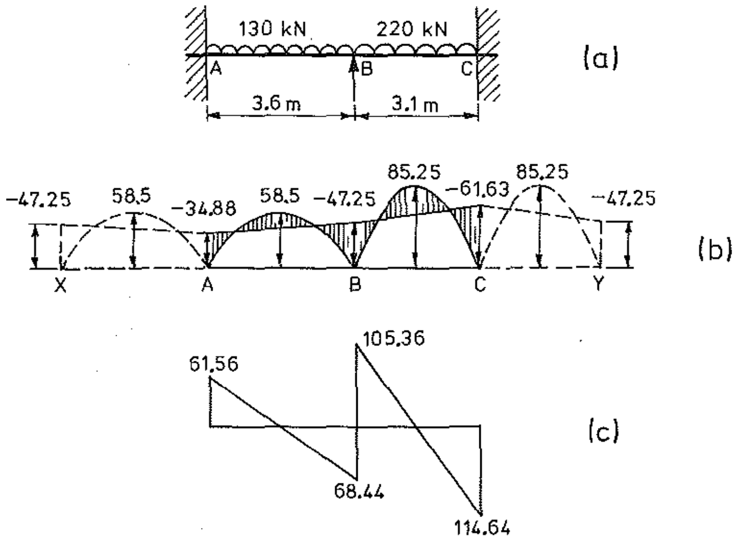


FIG. 65

$$A_2 = 140.4 \text{ kN.m}^2$$

$$A_3 = \frac{220 \times 3.1}{8} \times \frac{3.1 \times 2}{3} = 176.18 \text{ kN.m}^2$$

$$A_4 = 176.18 \text{ kN.m}^2$$

Firstly considering spans AX and AB

$$M_X \times 3.6 + 2\, M_A\, (3.6 + 3.6) + M_B \times 3.6$$

$$= -6 \left(\frac{140.4 \times 1.8}{3.6} + \frac{140.4 \times 1.8}{3.6} \right)$$

Now $M_X = M_B \quad \therefore \quad 2\, M_A + M_B = -117$ .... (1)

Secondly considering spans AB and BC

$$M_A \times 3.6 + 2\, M_B\, (3.6 + 3.1) + M_C \times 3.1$$

$$= -6 \left(\frac{140.4 \times 1.8}{3.6} + \frac{176.18 \times 1.55}{3.1} \right)$$

$3.6\, M_A + 13.4\, M_B + 3.1\, M_C = -949.74$ .... (2)

Thirdly considering spans CB and CY

$$M_B \times 3.1 + 2\, M_C\, (3.1 + 3.1) + M_Y \times 3.1$$

$$= -6 \left(\frac{176.18 \times 1.55}{3.1} + \frac{176.18 \times 1.55}{3.1} \right)$$

Now $M_Y = M_B \quad \therefore \quad M_B + 2\, M_C = -170.5$ .... (3)

From equations (1), (2) and (3)

$$M_A = -34.88 \text{ kN.m}$$

$$M_B = -47.25 \text{ kN.m}$$

$$M_C = -61.63 \text{ kN.m}$$

$$R_A = 65 + \frac{-47.25 + 34.88}{3.6} = 61.56 \text{ kN}$$

$$R_B = 65 + 110 + \frac{-34.88 + 47.25}{3.6} + \frac{-61.63 + 47.25}{3.1} = 173.8 \text{ kN}$$

$$R_C = 110 + \frac{-47.25 + 61.63}{3.1} = 114.64 \text{ kN}$$

Check: upward forces total 350 kN
downward forces total 350 kN

By simple addition and subtraction the shear force diagram can be drawn, Fig. 65(c), page 12.72.

Deflection of beams - the area-moment method

Refer to Fig. 66.

The cantilever shown is built in at X and carries a point load at Z. Under the action of this load the cantilever will no longer be horizontal, except at X. The slope and deflection will increase from X to Z.

At Y a distance ds from X the slope will be $d\theta$ and the deflection $d\delta$.

It is well known that

$$\frac{M}{EI} = \frac{1}{R} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where R = the radius of curvature of the member.

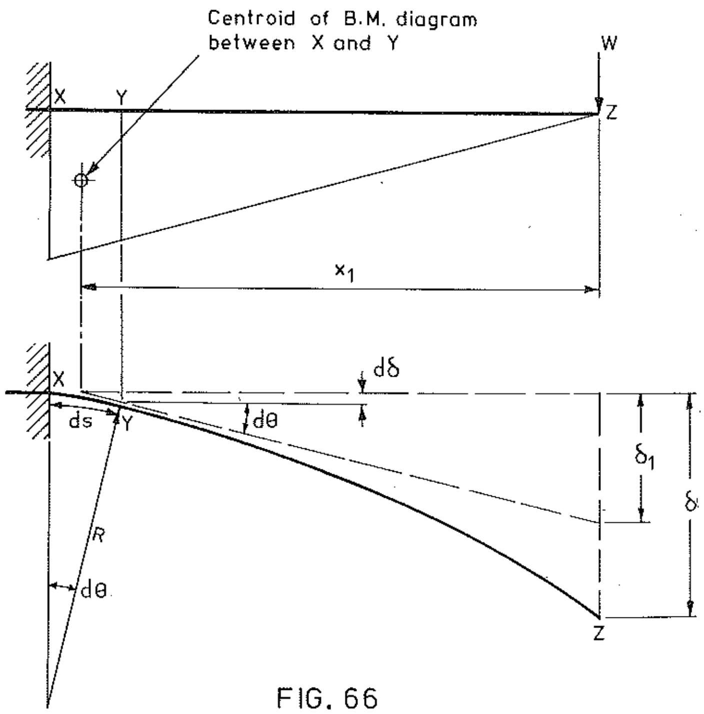


FIG. 66

Returning to Fig. 66, consider the short length ds between X and Y, it can be seen that the deflection δ_1 at Z due to the bending of ds alone can be determined as follows:

$$\delta_1 = d\theta \cdot x_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Now $ds = R \cdot d\theta$

$$\text{or } d\theta = \frac{ds}{R}$$ and by substituting from equation (1)

$$d\theta = \frac{M \cdot ds}{EI} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

substituting (3) into (2)

$$\delta_1 = \frac{M \cdot x_1 \cdot ds}{EI}$$

$$\text{hence} \quad \delta = \int_Z^X \frac{M \cdot x \cdot ds}{EI} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

In equation (4) M . ds is the area of the B.M. diagram and x is the lever arm between the centroid and the point of deflection under consideration,

$$\therefore \delta = \frac{\Sigma Ax}{EI}$$

where A is the area of the B.M. diagram.

Worked Example No. 25.

Fig. 67 shows a 250 U.B. 31.4 beam loaded as shown. $I_{XX} = 44.4 \times 10^6 \text{ mm}^4$.

Determine:

(a) the deflection under the load.
(b) the maximum deflection.
(c) the deflection at the centre of the span.

NOTE: For the purpose of the example beam self-weight has been neglected.

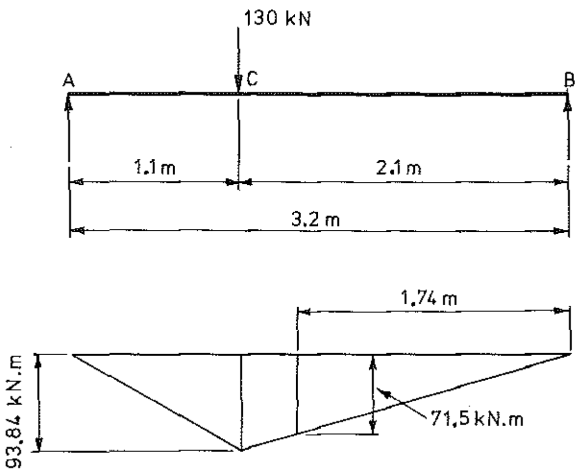


FIG. 67

Answer

$$\text{(a)} \quad M_C = \frac{Wab}{L} = \frac{130 \times 1.1 \times 2.1}{3.2} = 93.84 \text{ kN.m}$$

Consider the B.M. as a load and take moments about A,

$$3.2\, R_B = \frac{93.84 \times 1.1}{2} \times \frac{1.1 \times 2}{3} + \frac{93.84 \times 2.1}{2} \times \left(1.1 + \frac{2.1}{3}\right)$$

$$\therefore R_B = 67.25 \text{ kN.m}^2$$

Deflection at centre of span

$$\text{B.M. at centre of span} = 93.84 \times \frac{1.6}{2.1} = 71.5 \text{ kN.m}$$

$$\text{Secondary B.M.} = 67.25 \times 1.6 - \frac{71.5 \times 1.6}{2} \times \frac{1.6}{3}$$

$$= 77.09 \text{ kN.m}^3$$

$$\therefore \quad \delta \quad = \quad \frac{77.09 \times 10^{12}}{207\ 000 \times 44.4 \times 10^{6}} \quad = \quad 8.39 \text{ mm}$$

Deflection under load

$$\text{Secondary B.M.} \quad = \quad 67.25 \times 2.1 - \frac{93.84 \times 2.1}{2} \times \frac{2.1}{3} \quad = \quad 72.25 \text{ kN.m}^3$$

$$\therefore \quad \delta \quad = \quad \frac{72.25 \times 10^{12}}{207\ 000 \times 44.4 \times 10^{6}} \quad = \quad 7.86 \text{ mm}$$

Maximum deflection

Firstly it is necessary to find the position of the maximum deflection.

Because $\delta = \frac{\text{Secondary B.M.}}{EI}$ it is clear that the maximum deflection occurs where the secondary B.M. is a maximum. As with other cases of loading the maximum B.M. corresponds with the position of zero shear.

The point of zero shear will always be situated between the load and the span centre.

Let x = the distance of point X of zero secondary shear from B.

$$\text{B.M. at X} \quad = \quad \frac{x}{2.1} \cdot 93.84 \quad = \quad 44.69x \text{ kN.m}$$

The area of the B.M. diagram on length x = the secondary reaction.

$$\therefore \quad 44.69x \cdot \frac{x}{2} \quad = \quad 67.25$$

from which $x = 1.74$ m

$$\text{Secondary B.M.} \quad = \quad 67.25 \times 1.74 - 67.25 \times \frac{1.74}{3} \quad = \quad 78 \text{ kN.m}^3$$

$$\therefore \quad \delta \quad = \quad \frac{78 \times 10^{12}}{207\ 000 \times 44.4 \times 10^{6}} \quad = \quad 8.5 \text{ mm}$$

Worked Example No. 26.

Fig. 68 shows details of a loaded continuous beam.

The deflections in any span can readily be determined by using the area-moment method; each span being treated in turn.

Span AB $\quad I_{XX} = 85 \times 10^{6}$ mm^4

There are two B.M. diagrams in this span; one positive and one negative. Taking these as loads, the secondary reaction at A

$$= \frac{166.75 \times 4.6}{2} \times \frac{1}{2} - \frac{101.7 \times 4.6}{2} \times \frac{1}{3}$$

$$= 113.8 \text{ kN.m}^2$$

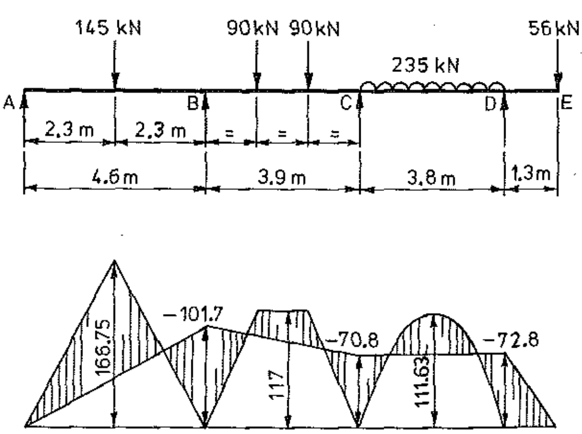


FIG. 68

The maximum deflection occurs at the point of maximum secondary B.M. Let this point be X, a distance x from A where the net area of the B.M. diagram on length x equals the secondary reaction.

$$\therefore \quad 113.8 \quad = \quad [\left(\frac{166.75}{2.3} \cdot \frac{x^2}{2} \right) - \left(\frac{101.7}{4.6} \cdot \frac{x^2}{2} \right)]$$

from which $x = 2.13$ m

The maximum deflection can now be determined by the principles previously discussed.

$$\delta = \frac{[\,113.8 \times 2.13 - (113.8 \times \frac{2.13}{3})] \times 10^{12}}{207\,000 \times 85 \times 10^6} = 9.18 \text{ mm}$$

Span BC $\quad I_{XX} = 62 \times 10^6 \text{ mm}^4$

Again, taking the B.M. diagrams as loads the secondary reaction at B

$$= \frac{117 \times 2.6}{2} - \frac{70.8 \times 3.9}{2} - \frac{30.9 \times 3.9}{2} \times \frac{2}{3} = -26.13 \text{ kN.m}^2$$

and the secondary reaction at C

$$= \frac{117 \times 2.6}{2} - \frac{70.8 \times 3.9}{2} - \frac{30.9 \times 3.9}{2} \times \frac{1}{3} = -6.05 \text{ kN.m}^2$$

As both secondary reactions are negative the beam must be hogging at each support. This means there will be

(a) either one point of maximum upward deflection

or (b) three points of maximum deflection:
two upwards towards the ends of the span and one downwards near the centre.

Because of the loading conditions case (b) will apply.

The point of maximum deflection nearest to B will occur at point X, a distance x from B. At this point the sum of the reaction and the area of the positive B.M. diagram on length x is equal to the area of the negative B.M. diagram on length x.

$$\text{i.e.} \quad 26.13 + \frac{117}{1.3} \cdot \frac{x^2}{2} = x \left(\frac{101.7 + 70.8 + \left(\frac{3.9 - x}{3.9} \right) \cdot 30.9}{2} \right)$$

from which $x = 0.3$ m

The deflection will be

$$\frac{[\left(-26.13 \times 0.3 \right) - \left(\frac{117}{1.3} \times \frac{0.3^2}{6} \right) + \left(99.3 \times \frac{0.3^2}{2} \right) + \left(2.4 \times \frac{0.3^2}{2} \times \frac{2}{3} \right)] \times 10^{12}}{207\,000 \times 62 \times 10^6}$$

$= -0.36$ mm (the negative sign indicates an upward deflection).

The maximum deflection nearest to C will occur at point X_1, a distance x_1 from

C such that

$$6.05 + \left(\frac{117}{1.3} \cdot \frac{x_1^{\,2}}{2}\right) = x_1\left(\frac{70.8 + 70.8 + \frac{30.9\ x_1}{3.9}}{2}\right)$$

from which $x_1 = 0.09$ m

The deflection will be

$$\frac{\left[(-6.05 \times 0.09) - \left(\frac{117}{1.3} \times \frac{0.09^2}{6}\right) + \left(70.8 \times \frac{0.09^2}{2}\right) + \left(0.713 \times \frac{0.09^2}{2} \times \frac{1}{3}\right)\right] \times 10^{12}}{207\ 000 \times 62 \times 10^6}$$

= −0.03 mm (the negative sign indicates an upward deflection).

At the centre, the point of maximum deflection will occur at point X_2, a distance x_2 from C such that

$$6.05 + \frac{117 \times 1.3}{2} + 117\ (x_2 - 1.3) = x_2\left(\frac{70.8 + 70.8 + \frac{30.9\ x_2}{3.9}}{2}\right)$$

from which $x_2 = 1.79$ m

The downward deflection will be

$$\frac{\left[(-6.05 \times 1.79) - \left(\frac{117 \times 1.3}{2} \times 0.92\right) - \left(\frac{117 \times 0.49^2}{2}\right) + \left(\frac{70.8 \times 1.79^2}{2}\right) + \left(\frac{14.18 \times 1.79^2}{2} \times \frac{1}{3}\right)\right] \times 10^{12}}{207\ 000 \times 62 \times 10^6}$$

= 2.04 mm

<u>Span CD</u> $I_{XX} = 58 \times 10^6\ \text{mm}^4$

The end moments acting on this span are approximately equal. Therefore it is close enough for practical purposes to assume a constant negative moment of 71.8 kN.m is acting throughout the length of the beam. The upward deflection at the centre for this type of loading is given by the formula $\frac{ML^2}{8EI}$.

The resultant downward deflection in the centre of the span will be

$$\frac{5}{384}\ \frac{WL^3}{EI} - \frac{ML^2}{8EI}$$

$$= \left(\frac{5}{384} \times \frac{235 \times 3.8^3}{207\ 000 \times 58 \times 10^6} - \frac{71.8 \times 3.8^2}{8 \times 207\ 000 \times 58 \times 10^6}\right) \times 10^{12}$$

= 3.19 mm

Span DE $\quad I_{XX} = 58 \times 10^6 \text{ mm}^4$

The deflection at E can be determined by following the procedure set out in Worked Example No. 6, page 12.20.

One additional influence has to be included; that of the negative B.M. at support C.

Following the steps of calculation in Worked Example No. 6, page 12.20, the following values can be found:

$$\delta_1 = -15.3 \text{ mm}$$

$$\delta_2 = 13.4 \text{ mm}$$

Let δ_3 = the downward deflection at E due to the negative moment at C.

The slope at D will be:

$$\theta_3 = \frac{M_C L_2}{6EI} \quad \text{(see Fig. 69)}$$

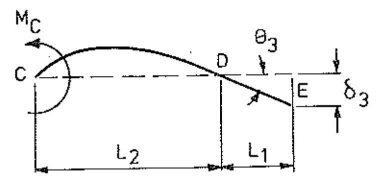


FIG. 69

$$\therefore \delta_3 = L_1 \theta_3$$

$$= 1.3 \times \frac{70.8 \times 3.8}{6 \times 207\ 000 \times 58 \times 10^6} \times 10^{12} = 4.9 \text{ mm}$$

Resultant deflection

$$= -15.3 + 13.4 + 4.9$$

$$= \underline{3.0 \text{ mm (downward)}}$$

EXERCISES

23. Fig. 70 shows the critical loading diagram for the rudder post of a yacht.

 (a) Determine the bending moment absolute at the following positions:

  (i) At bearing support B.
  (ii) At bearing support C.
  (iii) In the centre of span AB.
  (iv) In the centre of span BC.

 (b) Determine the bearing reactions absolute at A, B and C.

 (c) Using shock factors of 2.0 on B.M. and 1.5 on torque, calculate the required diameter of rudder post if CS1040 material is used.

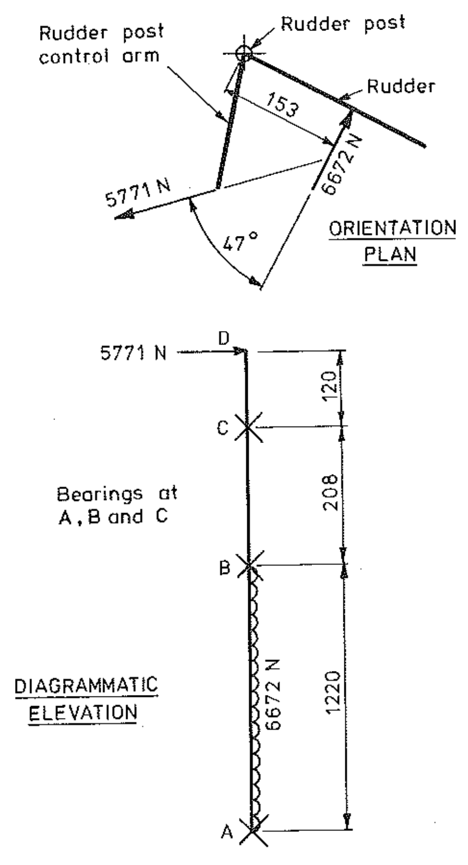


FIG. 70

Ans. (a)(i) 903 964 N.mm (ii) 692 520 N.mm
 (iii) 566 187.5 N.mm (iv) 318 721.9 N.mm

 (b) 2595.9 N, 11 011 N, 12 568.2 N

 (c) 58.35 mm

STRUCTURAL BOLTS AND BOLTING TECHNIQUES

Four main types of structural steel bolts are available:

1. Commercial bolts, strength grade 4.6, plain or coated.
2. Medium strength or tower bolts, strength grade similar to 5.6, hot dip galvanized only.
3. High strength bolts, strength grade 8.8, plain or coated.
4. High strength interference body or bearing bolts, plain or coated.

Strength designations, metric bolts

The strength of metric structural bolts is normally specified in terms of the tensile strength of the threaded fastener and defined according to the ISO strength grade system, which consists of two numbers separated by a 'full point'. The first number of the designation represents one hundredth of the nominal tensile strength (MPa) and the number following the point represents one tenth of the ratio between nominal yield stress and nominal tensile strength expressed as a percentage.

For example a grade 4.6 bolt has:

Tensile strength of 4 x 100 = 400 MPa
Yield stress of 0.6 x 400 = 240 MPa

1. Commercial bolts (low carbon steel bolts).

   The design of this type of bolting has already been covered. See pages 12.29 to 12.43.

2. Medium strength bolts.

   Medium strength galvanized bolts stronger than the regular low carbon steel bolts were developed originally for use on galvanized transmission towers and are known as transmission tower bolts. They have found numerous applications in galvanized structures of other types with resulting advantages and economies.

   As maximum shear strength values are required the thread is kept out of the shear plane. Transmission towers are often erected in high snow country and it is also necessary to have a bolt with good low temperature notch toughness.

   Galvanized transmission tower bolts are made with short thread lengths and specified notch ductility to meet these requirements.

3. High strength bolts.

   The introduction of high strength bolting using structural bolts to AS 1252 'High strength steel bolts with associated nuts and washers' has brought improved economy and efficiency to the fabrication of galvanized structures by permitting:

   1. Smaller bolts of higher strength
   2. Fewer bolts and bolt holes, resulting in:

      (a) lower fabrication cost for members.
      (b) faster erection and reduced erection cost.

   In structural applications high strength structural bolts are commonly used in M20 and M24 metric diameter in both flexible and rigid connections.

Larger sizes (M30, M36) of the high strength structural bolt should be used with caution, especially when full tensioning to the requirements of AS 1511, High-strength Structural Bolting Code, is required. This is because on-site tensioning can be difficult and requires special equipment to achieve the minimum bolt tensions specified in AS 1511.

Applications of high strength bolts

It is important to note that the high strength structural bolt may be used in three ways, applying different design values in each case:

Procedure 8.8/S - Snug tight
Procedure 8.8/TF - Friction type joint (fully tight)
Procedure 8.8/TB - Bearing type joint (fully tight)

The designer must therefore indicate the level of bolt tightening required in both drawings and specifications, and ensure that this information is conveyed to all those involved in installation, tightening and inspection.

Procedure 8.8/S - High strength structural bolts used snug tight.

Snug tight is defined in Australian Standard 1511 'SAA high strength structural bolting code' as the full effort of a man on a standard podger spanner or the point at which there is a change in note or speed of rotation when a pneumatic impact wrench starts impacting solidly.

High strength structural bolts in the snug tight condition may be used in flexible joints where their extra capacity can make them more economic than commercial bolts. The level of tightening achieved is adequate for joint designs where developed bolt tension is not significant. Behaviour of the bolt under applied loads is well known and accepted.

Procedure 8.8/TF - Friction type joints.

In a friction type joint shear loads are transmitted from one member to another by means of the friction developed between mating surfaces. The friction force is controlled by the tension or pre-load developed in the bolt, and high-bolt tension in a correctly designed joint produces great friction forces across mating surfaces. The slip factor or coefficient of friction of the mating surfaces must be taken into account in design.

AS 1511 specifies that friction type joints must be used where no slip is acceptable. They should also be used in applications where joints are subject to severe stress reversals or fluctuations as in dynamically loaded structures such as bridges, except in special circumstances as determined by the engineer. Where the choice is optional, bearing type joints are more economic than friction type.

Fig. 71, page 12.81 shows the principle of the high strength friction-type bolted connection.

Procedure 8.8/TB - Bearing type joints.

The principle of the high strength bearing type joint is the same as in conventional low and medium strength bolted joints where loads between members are carried by bolts in shear and bearing. The advantage of high strength structural bolts used in bearing type applications is that considerably higher loads can be carried by fewer bolts.

Because loads are not transmitted by friction between mating surfaces the slip factors of surface coatings are not considered in design as is the case in procedure 8.8/TF.

General

When a high strength bolt is tightened in accordance with AS 1511 it is loaded to approximately its minimum proof load, specified in AS 1252.

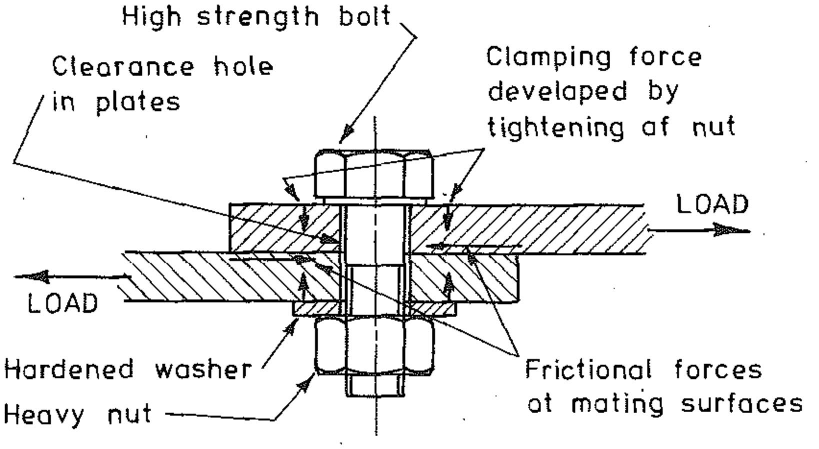


FIG. 71

4. High strength bearing bolts.

The interference body high strength bolt or Bearing Bolt is made to the mechanical property requirement of Australian Standard B157, the imperial precursor of AS 1252. Nuts and washers also comply with AS B157.

The Bearing Bolt features a specially knurled shank larger in diameter than the nominal thread size to provide a tight driving fit in 1.6 mm clearance holes. Knurls have a rounded profile and grooves are both circumferential and helical to facilitate driving. During installation, the knurls indent grooves in the hole, developing an interference fit and preventing significant slip in service. Hole preparation involves reaming of holes to ensure a tight fit on installation.

The Bearing Bolt may be used in both bearing type and friction type joints but is particularly applicable to joints between coated members where the slip factor may be lower than for uncoated steel. It develops the same high clamping force and frictional resistance as a high strength hexagon head bolt and the interference fit prevents or minimises amplitude of slip between members.

High strength bearing bolts can be useful where there is a need for non-slip joints but where poor interface conditions make friction-type joints (procedure 8.8/TF) impracticable. These bolts are not presently covered by an Australian Standard, and are only available (in imperial sizes) to special order.

The name should not be confused with a high strength structural bolt to AS 1252 used in a bearing type joint (bolting procedure 8.8/TB).

Fig. 72, page 12.82 illustrates a high strength bearing bolt.

Table No. 5, page 12.82 gives details of the various tightening procedures and engineering details of structural bolts.

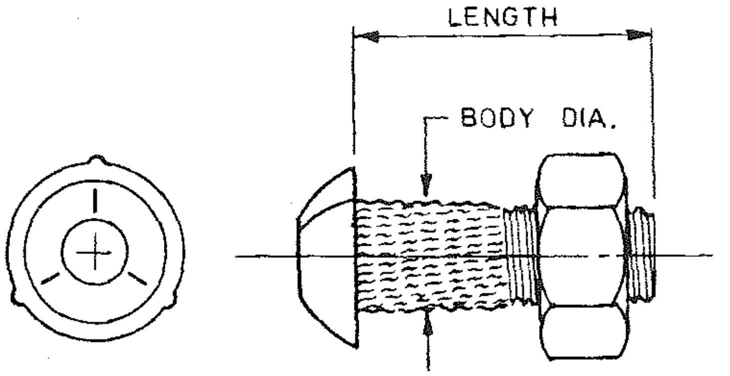


FIG. 72

TABLE NO. 5.

Bolting Procedure		Strength Grade	Tensile Strength MPa	Yield Strength MPa	Name	Australian Standard	Remarks
		Details of bolt used					
4.6/S		4.6	400	240	Commercial	AS 1111	Least costly and most commonly available 4.6 Grade bolt. Use Snug tightened.
8.8/S		8.8	800	640	High Strength Structural	AS 1252	Bolts are used Snug tightened. The high strength structural bolt has large bolt head and nut because it is designed to withstand full tensioning (see 8.8/T procedures below.) However, it can also be used in a snug tight condition.
8.8/T	8.8/TF	8.8	800	640	High Strength Structural Bolt - Friction type joint	AS 1252	In both applications, bolts are fully Tensioned to the requirements of AS 1511. Cost of tensioning is an important consideration in the use of these bolting procedures.
	8.8/TB	8.8	800	640	High Strength Structural Bolt - Bearing type joint		

The system shown in Table No. 5, page 12.82 identifies the bolt being used by stating its strength grade designation (4.6 or 8.8) followed by the installation procedure letter.

S .... snug
T .... full tensioning to AS 1511.

For the 8.8/T tightening procedures the joint type is specified by an additional letter.

F .... friction joint.
B .... bearing joint.

Procedure 4.6/S refers to commercial bolts of Strength Grade 4.6 conforming to AS 1111, tightened using a standard wrench to a "snug-tight" condition.

Procedure 8.8/S refers to any bolt of Strength Grade 8.8, tightened using a standard wrench to a "snug-tight" condition in the same way as for procedure 4.6/S. Essentially, these bolts are used as higher grade commercial bolts in order to increase the capacity to certain connection types. In practice, they will normally be high strength structural bolts of Grade 8.8 to AS 1252.

Procedures 8.8/TF and 8.8/TB (or 8.8/T when referring generally to both types) refer specifically to high strength structural bolts of Strength Grade 8.8 conforming to AS 1252, <u>fully tensioned in a controlled manner to the requirements of AS 1511</u>. (See reference.)

[Reference: Firkins, A. and Hogan T.J., "Bolting of Steel Structures", AISC, 1984]

<u>Summary</u>.

In the design of bolts in bolted connections, two design codes are relevant, according to the bolting procedure:

4.6/S, 8.8/S procedures .... AS 1250

8.8/TF, 8.8/TB procedures ...... AS 1511

Tower bolts to AS 1559 may have working stresses as specified in AS 1250.

Other types of bolts e.g. Metric Hexagon Precision Bolts to AS 1110 etc. (refer Rule 2.2.1 AS 1250) and not tightened in accordance with the methods specified in AS 1511 may have working stresses as specified in AS 1250.

Allowable bolt loads for the 8.8/S bolting procedure are given in Table No. 6, page 12.84. Edge distance stipulations and permissible bearing stress are specified in AS 1250. See pages 12.34 to 12.36.

Note: Sometimes two extra symbols are added to the bolting procedure designations 4.6/S, 8.8/S and 8.8/TB.

N: bolt in shear with threads iNcluded in the shear plane (e.g. 8.8N/S).
X: bolt in shear with threads eXcluded from the shear plane (e.g. 8.8X/S).

<u>Bolts in line</u>

For long lines of bolts, the conventional assumption of a uniform distribution of load on each bolt in the line due to an applied shear force is no longer justified. Some allowance is required to be made for the non-uniformity of shear forces which results in higher loads on the extreme bolts in the line.

AS 1511 gives reduction provisions for 8.8/TB bolts while for 8.8/TF bolts no reduction is necessary since shear transfer is by interface friction.

TABLE NO. 6. 8.8/S bolting procedure

Nominal diam. of bolt	Shear (single shear) F_{vf} = 208 MPa		Axial tension F_{tf} = 374 MPa
	Threads included in shear plane B_{VN}	Threads excluded from shear plane B_{VX}	B_t
	kN	kN	kN
M16	29.9	41.7	58.6
M20	46.7	65.2	91.5
M24	67.2	93.8	132
M30	108	147	210
M36	158	211	305
	8.8 N/S	8.8 X/S	

Design of High Strength Bolting to AS 1511

The maximum permissible loads for these bolts are:-

(i) Tensile force - restricted to 60% of minimum induced bolt tension.
Refer to Table No. 7, page 12.85.
In fatigue conditions - restricted to 50% of minimum induced bolt tension.
Refer to Table No. 7, page 12.85.

(ii) Shear force, friction type joint - for bolts subject to pure shear, allowable load per interface is given by:

$$\frac{\text{slip factor}}{\text{load factor}} \times \text{minimum bolt tension}$$

Slip factor refer to Table No. 8, page 12.86.
Load factor refer to Table No. 9, page 12.86.

For joints subject to combined shear and tension, the following interaction formula applies:-

* FRICTION JOINT ONLY

$$\frac{b_t}{B_t} + \frac{b_v}{B_V} \leq 1.0$$

where b_t = tensile force in the bolt;

B_t = maximum permissible tensile force;

b_v = shear force in the bolt;

B_V = maximum permissible shear force;

(iii) Shear force, bearing type joint (Procedure 8.8/TB) - for bolts subject to pure shear, allowable bolt loads are given in Table No. 10, page 12.86.

Higher values than for friction type joints result, providing joint slippage is of no design consequence. Bearing type connections resist the applied load by a combination of interface friction and shear resistance of the bolt shank. However, the friction resistance is ignored in design computations since it is hard to evaluate after slip has occurred.

The bearing stress and edge distance limitations should be identical to those specified in AS 1250. See pages 12.34 to 12.36.

Joints subject to combined shear and tensile forces are governed by the interaction formula:-

* BEARING JOINT

$$\left(\frac{b_t}{B_t}\right)^2 + \left(\frac{b_v}{B_V}\right)^2 \leq 1.0$$

AS 1511 imposes the following limitations on the use of bearing-type joints:

(a) where slip is not acceptable in shear or moment connections, friction-type joints must be used in lieu;

(b) bearing-type joints are precluded for joints subject to severe stress reversal or fluctuation, unless special circumstances exist.

TABLE NO. 7. * FRICTION JOINT 8.8/TF

Nominal diam. of bolt	Minimum bolt tension kN	60% tension kN	50% tension kN
M16	95	57	48
M20	145	87	73
M24	210	126	105
M30	335	201	168
M36	490	294	245

TABLE NO. 8 (Gorenc, see "References", page 12.90).

Surface Preparation	Mean Slip Factor	Minimum Slip Factor
Bare steel as rolled	0.35	
Flame cleaned plain steel	0.48	0.20
Shot blasted plain steel	0.57	0.40
Red lead primed surface	0.07	0
Lacquer varnish	0.24	0.10
Hot-dip galvanized surface	0.19	0.15
Galvanized and grit blasted or wire brushed	0.30 to 0.40	0.20
Inorganic zinc rich paint over shot blasted surface	0.51	0.40
Shot blasted and zinc sprayed	0.65	0.50

TABLE NO. 9

Load factor	Application
1.4	AS 1511 now requires the load factor of 1.4 to be applied to all load combinations specified in AS 1250, Rule 3.3.1.

TABLE NO. 10. 8.8/TB bolting procedure

Nominal diam. of bolt	Permissible shear loads, kN	
	Threads included in shear plane, N	Threads excluded from shear plane, X
M16	30	42
M20	47	65
M24	67	94
M30	108	147
M36	158	211

Methods of full tensioning.

The High Strength Bolting Code AS 1511 allows two methods of tensioning.

1. Part turn method of tightening.

2. Direct tension indicator method.

Worked Example No. 27.

Two bare M.S. flats are joined using a lap joint bolted together with two M20 bolts using procedure 8.8/TF.

Determine the maximum permissible tensile working load that could be applied to the flats.

Answer

Allowable load per bolt

$$= \frac{\text{slip factor}}{\text{load factor}} \times \text{minimum bolt tension}$$

slip factor = 0.35, Table No. 8, page 12.86.

load factor = 1.4, Table No. 9, page 12.86.

Minimum bolt tension = 145 kN, Table No. 7, page 12.85.

$$\therefore \text{ Allowable bolt load} = \frac{0.35}{1.4} \times 145 = 36.25 \text{ kN}$$

$$\therefore \text{ Allowable joint load} = 36.25 \times 2 = 72.5 \text{ kN}$$

Worked Example No. 28.

Suppose in the previous example the bolting procedure was changed to 8.8X/TB.

Determine the maximum permissible static working load.

Answer

From Table No. 10 , page 12.86.

Permissible bolt load = 65 kN

$\therefore$ Permissible load on joint = 65 x 2 = 130 kN

Worked Example No. 29.

A statically loaded M24 bolt tightened to procedure 8.8/TF has a tensile load of 59 kN concurrently with a shear load of 10 kN. The steel is hot dip galvanized.

Check the value of the interaction formula.

Answer

$$\frac{b_t}{B_t} + \frac{b_v}{B_V} \leq 1.0$$

$$B_V = \frac{\text{slip factor}}{\text{load factor}} \times \text{minimum bolt tension}$$

Slip factor = 0.19, Table No. 8, page 12.86.

Load factor = 1.4, Table No. 9, page 12.86.

Minimum bolt tension = 210 kN, Table No. 7, page 12.85.

$$\therefore \quad B_V = \frac{0.19}{1.4} \times 210 = 28.5 \text{ kN}$$

$$b_v = 10 \text{ kN}$$

$$b_t = 59 \text{ kN}$$

B_t = 126 kN, Table No. 7, page 12.85.

$$\therefore \quad \text{Interaction formula} = \frac{59}{126} + \frac{10}{28.5} = \underline{0.82}$$

Worked Example No. 30.

A statically loaded M20 bolt tightened to procedure 8.8X/TB has a tensile load of 30 kN concurrently with a shear load of 12 kN.

Check the value of the interaction equation.

Answer

$$\left(\frac{b_t}{B_t}\right)^2 + \left(\frac{b_v}{B_V}\right)^2 \leq 1.0$$

$$b_t = 30 \text{ kN}$$

B_t = 87 kN, Table No. 7, page 12.85.

$$b_v = 12 \text{ kN}$$

B_V = 65 kN, Table No. 10, page 12.86.

$\therefore$ Interaction equation

$$= \left(\frac{30}{87}\right)^2 + \left(\frac{12}{65}\right)^2 = \underline{0.15}$$

EXERCISES

24. Two flame cleaned M.S. flats are joined using a lap joint. Two M24 bolts are used tightened with procedure 8.8/TF.

Determine the maximum permissible tensile working load that could be applied along the flats.

Ans. 144 kN

25. A statically loaded M20 bolt is tightened to procedure 8.8/TF. If a tensile load of 30 kN and a shear load of 4.9 kN are applied, check the value of the interaction equation.

The steel surfaces are plain and flame cleaned.

Ans. 0.44

26. A statically loaded M24 bolt is tightened to procedure 8.8N/TB.

A tensile load of 77 kN and a shear load of 39.5 kN are applied.

Check the value of the interaction equation.

Ans. 0.72

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The authors express thanks for permission to use the following publications in the preparation of the foregoing section "Structural bolts and bolting techniques".

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Acknowledgment

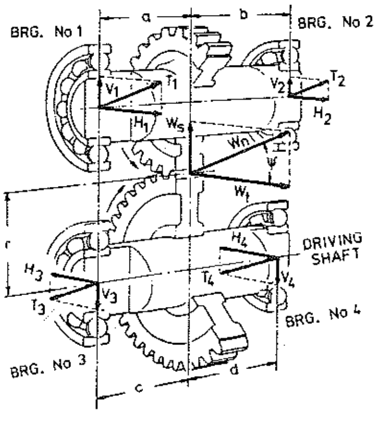
The authors express thanks for the assistance given by Ajax Nettlefolds Pty. Ltd. in preparing the section 'Bolt design - machine design applications' in this chapter.

A further useful reference to the design of gasketted joints can be found in,

Standards Association of Australia (SAA),
Unfired Pressure Vessels,
AS 1210.

CHAPTER 13

SELECTION of MATERIALS



CHAPTER 13
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SELECTION OF MATERIALS

Factors Affecting the Selection of Materials (REFER ALSO TO APPENDICES No. 1 & 2)

Cost In most cases this is one of the primary considerations.

There are no unique answers to engineering problems. Using higher grades of materials may increase costs in one area and reduce them in other areas (e.g., may enable smaller bearings to be used etc., etc.)

Machining and manufacturing costs may vary significantly from one proposed design to another. The question of fabrication versus casting may arise.

The type of duty to which a machine will be applied is a consideration. Expensive materials could not be justified on a machine required only for occasional use.

All of the above questions can only be answered by a complete cost analysis in the case of acceptable alternative designs.

Strength and Rigidity

The lowest strength grade of material commensurate with the application should normally be used. High strength steels in general have lower ductility which is undesirable in members subjected to stress concentration (read also section on "Hardness and ductility".)

Rigidity should be such that working deflections are within recommended limits. It is interesting to note that all steels have approximately the same elastic and torsion modulii therefore rigidity will not be improved by changing from a low grade to a high grade material. Rigidity can only be improved by increasing the second moment of area - orthogonal (I) or - polar (J) of the member as applicable.

Hardness and ductility

Hardness is a desirable property in bearing surfaces which have relative motion.

Hardness and ductility generally have reciprocal relationship in steel. Ductility is advantageous to relieve stress concentrations in static loading but with cyclic loading its value is minimal.

Weight

In some instances considerable weight is necessary e.g., foundations on which there is a resultant uplift.

Other cases require extreme lightness e.g. aircraft parts. In these instances aluminium and magnesium alloys can be used.

Resistance to Wear

This is important in bearing applications where the surfaces have relative motion. Also with members used in abrasive applications like earth moving and digging equipment.

esistance to Corrosion

The effect of corrosion is especially important where there is stress concentration in cyclic loading.

Some members are exposed to corrosive environments. Examples of this are pumps, some chemical handling equipment and outdoor applications.

riction

Low friction is desirable in plain (bush) bearings. This depends upon:-

(1) The materials in contact. Some combinations are superior to others.

(2) Surface finish.

(3) Lubrication.

High friction is necessary in other instances e.g., brake linings, clutch linings and power belt drives.

ATERIALS

teel

Steel in one of its various grades is by far the most common building material in mechanical and structural engineering.

Steel is an alloy of iron and carbon. Among the other elements found in steel are silicon, phosphorus, sulphur, manganese and chromium.

Steels may be roughly classified as straight carbon steels and alloy steels. A straight carbon steel is a steel that owes its properties chiefly to various percentages of carbon without substantial amounts of other alloying elements. When steel contains about 0.30% or less it may be called a low carbon steel, from about 0.30% to about 0.70% a medium carbon steel, and from about 0.70% to 1.3% a high carbon steel. An alloy steel is a steel to which some element in addition to carbon has been added to improve or change the physical properties.

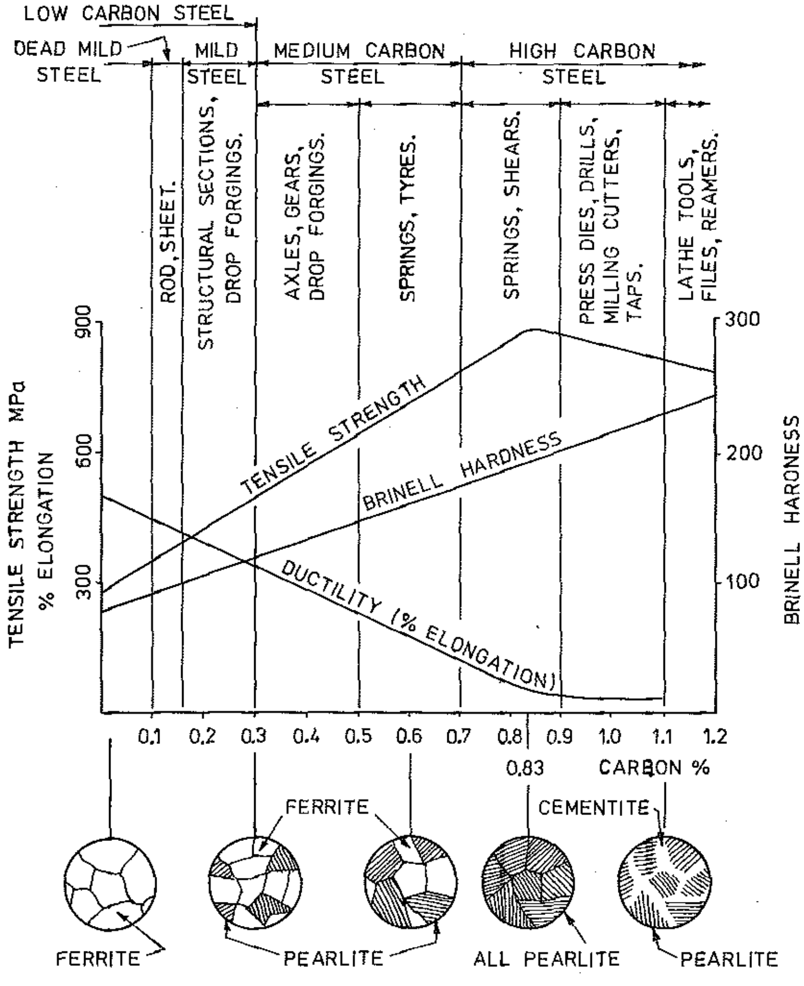
A low carbon steel does not contain enough carbon to cause it to harden to any great extent when heated to a certain temperature and quenched in oil, water or brine. It may be heat treated to increase its strength marginally or case hardened to increase its resistance to wear.

Medium and high carbon steels respond well to heat treatment. The higher the carbon content the better the response. Tool steel is an example of high carbon steel.

Refer to Fig. 1. (page 13.3) for a graphical representation of plain carbon steels.

Alloy steels are those steels which contain some alloying elements, such as: chromium, vanadium, nickel, molybdenum, tungsten, etc., which give them some peculiar characteristic not possessed by ordinary steel. Alloys are put into steels for the following reasons: to secure greater hardness, to secure greater toughness or strength, to enable the steel to hold its size and shape during hardening, or to enable the steel to retain its hardness at high temperatures.

High speed steel is an alloy steel used to make tool bits, forged cutting tools, milling machine cutters, reamers and broaches. It contains a high percentage of tungsten, chromium, vanadium and carbon. These elements give the steel the ability to retain sufficient hardness at high temperatures to cut metal.



MICROSTRUCTURE, MECHANICAL PROPERTIES, AND USES OF PLAIN CARBON STEELS.

FIG. 1

Stainless steel holds an important place in industry. The main alloying elements are chromium (about 18%) and nickel (about 8%). Stainless steel comes in many specifications and owes its stainless and corrosion resistant properties to the formation of a very thin, usually invisible, oxide film which forms on the surface. Main uses for stainless steel are in the food and chemical industries.

Shafts

The three grades of commercial mild steel shafting are

CS 1020, CS 1030, and CS 1040.

These grades can be obtained as either black or bright surface finish. Structural grade round mild steel bar is not commonly used for the manufacture of shafts.

Structural Steel

All structural steel has a black surface finish.

The least costly and most commonly used steel for general constructional purposes is mild steel, alternatively known as carbon steel or normal strength steel. Structural mild steel contains about 0.2% carbon and has a yield point strength of 250 MPa nominally. The yield point strength varies slightly with the maximum section thickness and the Structural Code gives exact values for all sections.

Rolled structural sections are also made in high tensile steel, yield point 350 MPa nominally. The range of sections however is limited and they are not always readily obtainable.

Cast Steel

Cast steel either plain or alloyed is used where castings of improved properties over the grades of cast iron are required.

Table No. 1 gives a guide on some of the cast steels available and their applications. Some types of cast steel will accept heat treatment.

The foundry should be consulted before a final decision is made. Most foundries have very helpful literature available detailing their range of steels.

TABLE NO. 1

NO.	C %	Mn %	Si %	Cr %	S %	P %	U.T.S. MPa	Y.P. MPa	TYPE OF STEEL
1	0.16-.22	0.6-.8	0.2-.4				430	250	Plain Carbon
2	0.3-.35	0.6-.8	0.3-.4				540	430	Plain Carbon
3	0.4-.45	0.6-.8	0.3-.4				660	360	Plain Carbon
4	0.45-.55	0.6-.7	0.3-.4				725	400	Plain Carbon
5	0.35-.4	0.6-.8	0.3-.4	0.4-.6			695	460	Chrome
6	0.7-.8	0.6-.8	0.3-.4	0.7-.9			925		Chrome
7	1-1.35	11-14	1.0		0.06	0.12	770		Austenitic Manganese

NO.	PROPERTIES	APPLICATIONS
1.	As for Mild Steel	Bed plates, brackets, levers, bearing housings and general machinery castings
2.	Strength and wear resistance a little better than M.S.	Gears
3.	High Strength, good wear resistance, reduced toughness and machinability	Gears

Cont. on page 13.5

NO.	PROPERTIES	APPLICATIONS
4.	Designed for flame hardening. Tough core, wear resistant face.	Gears, clutches.
5.	Hardness and wear resistance good.	Truck wheels, Gears.
6.	Withstands abrasion. Reasonable shock resistance.	Stamper shoes and dies, roller shells.
7.	Work hardening steel used for wear resistance. Peening action must be present with wearing action.	Teeth, liner plates, crusher jaws, ball mill liners, grader cutting edges, grader teeth.

Cast Iron

Cast iron is an iron carbon alloy containing 2.5% to 3.5% carbon & 0.25% to 3% silicon.

Cast iron is the cheapest of the cast metals and can be cast as plain or alloyed to almost any desired shape and/or size. The surface finish is good, being superior to that of cast steel.

The damping capacity of cast iron is high. Damping capacity is the energy dissipated as heat per unit volume during a completely reversed cycle of stress. It is related to the internal friction in the material.

High damping capacity prevents the accumulation of serious resonant stresses and decreases vibrations.
Cast iron is therefore very suitable for the manufacture of crankshafts, camshafts and bases and frames for machinery.

Basic forms of cast iron:

(1) Grey iron
(2) White iron

(1) Grey iron.

This is by far the most common grade of iron. It is used for the manufacture of pulleys, sprockets, cylinder blocks, gears and various types of housings.

Grey cast iron has an important place in engineering for the following reasons:

(a) It is a relatively cheap metal.

(b) It is easy to melt and cast. Compared to steel it has the following advantages.:

  (i) Does not penetrate the sand to the same extent, therefore the casting finish is superior.

  (ii) Has a lower melting point; CI, 1100°C; steel, 1600°C.

(iii) Has a smaller contraction upon solidification;
CI, 8 mm/m; steel, 21 mm/m.

(iv) The final product is more corrosion resistant.

(c) It has free graphite flakes. This means:

(i) It is easy to machine because the graphite lubricates the cutting tool.

(ii) It makes a good "natural" bearing, because the graphite acts as a lubricant. This makes grey cast iron wear resistant.

(iii) It is good for damping vibrations as the graphite causes discontinuities in the metal structure.

(d) It has high compressive strength (but poor tensile strength). A general grade of grey cast iron would have the following strength specifications:

Ultimate tensile strength	210 MPa
Ultimate compressive strength	825 MPa

Composition:
C 3.1%, Si 1.75%, Mn 0.5%, S 0.1%, P 0.35%

(2) White Iron

Although white iron is brittle, its extreme hardness, toughness and wear resistance makes it suitable for use in stone and ore crushers, chilled rolls and plough shares. It cannot be machined.

White iron is made by casting the molten metal into a "chill" mould of by embedding chills into the sand. This gives a casting which is grey except at the chill areas. Here the carbon is present predominantly as iron carbide. This gives white iron its engineering properties.

Composition:
C 3%, Si 0.8%, S 0.15%, P 0.4%, Mn 0.45%

Mottled iron is the name given to a structure which is half grey and half white. It has no engineering use.

Alloy Cast Irons

As with steel, alloying elements can be added to cast iron to give it superior engineering properties. Amongst the elements which can be added are:

silicon, nickel, chromium, copper, molybdenum, magnesium and cerium.

Silicon tends to produce grey iron.

Nickel tends to produce grey iron and also refines the grain structure to produce a stronger casting.
Austenitic (non-magnetic) cast iron contains about 15% nickel. Austenitic iron is heat and corrosion resistant and has good application in the chemical industry.

Chromium tends to give a white iron structure. This is useful for the production of piston rings and cylinder liners.

Copper has similar effects to nickel.

Molybdenum is generally used together with other alloying elements to strengthen cast iron.
0.5% Mo together with 2% Ni produces acicular iron which is a hard tough material with a tensile strength of about 540 MPa.

Magnesium (in the form of a magnesium-nickel alloy) and/or cerium are added to molten cast iron to form spheroidal graphite cast iron or "nodular iron". This is a high strength alloy in which the graphite is present in the form of spheres or nodules. These break up the general matrix and so the iron becomes ductile instead of brittle. Castings come from the mould in this condition therefore no heat treatment is necessary as with malleable iron. Spheroidal graphite iron has about the strength and shock resistance of steel but the corrosion resistance and cheapness of cast iron.

Malleable cast iron

These castings are first produced in the white iron form and then heat treated by one of two methods to make them malleable.

(1) The Whiteheart process.

(2) The Blackheart process.

(1) Whiteheart process.

This method produces a material with a white fracture.

The castings are packed in boxes of iron oxide ore (alternatively a gaseous mixture is used) and placed in a furnace for a number of days at a temperature of about 950° C. Under these conditions most of the carbon is oxidised from the iron. Upon cooling, the remaining carbon forms pearlite; the cooled structure consists of pearlite and ferrite and is very malleable.

The iron has a tensile strength of about 370 MPa.

(2) Blackheart process.

This method produces a material with a grey fracture.

The castings are packed in sand and held in a furnace for a number of days at about 850° C. All the carbide, including that of the pearlite, is decomposed to give "temper carbon" (free carbon) in ferrite.

Uses of malleable cast iron

Malleable iron is the cheapest shock resisting material.
Uses include agricultural machinery,pipe fittings and parts for automobiles.

Brass and Bronze

Brass (a copper-zinc alloy), bronze (a copper-tin alloy) and all copper based alloys have good wear and corrosion resistant properties.

α Brass.
This contains a maximum of 30% zinc. It is a homogeneous solid solution at room temperature and is therefore ductile and readily cold worked.

Its main applications are for deep-drawing and cold rolling to thin sheet.

$\alpha\beta$ Brass.
This alloy contains 40% zinc or more and can only be cold worked with frequent intermediate annealing.

Its main applications are for castings and hot worked forgings.

Manganese bronze.

This is the misleading name given to a brass containing 40% zinc. Small amounts of other strengthening elements have been added; iron, tin, manganese and aluminium.

Its main uses are for castings (e.g. marine propellers) and rolled and extruded sections.

Bronze (plain tin bronze and phosphor bronze).

Plain tin bronze is a copper-tin alloy containing up to 12% tin.

When lead and phosphorous are added the alloy becomes known as phosphor bronze.

Bronzes make excellent plain bearings.

Gunmetal

Gunmetal, although so-called, is no longer used for making guns.

It contains 88% copper, 10% tin and 2% zinc.

It is used for castings of many types where strength and corrosion resistance are required. It makes excellent plain bearings.

Monel

Monel is 32% copper, 68% nickel.
It is used where a combination of high strength, toughness and resistance to chemical attack is required. It can also hold its strength at high temperatures.

Aluminium

Aluminium is a relatively modern metal, it being first produced in 1825.

Pure aluminium has a tensile strength of only 90 MPa. Because it is soft it tends to tear if machined therefore it has little use in its pure condition. However, once alloyed with other elements its properties are greatly enhanced and it becomes an important engineering material.

The greatest advantages offered by aluminium alloys are their extreme lightness (density one third of that of steel) and corrosion resistance.

Uses of aluminium alloys:
Aircraft construction, lorry bodies, bicycle parts, motor car parts, portable equipment, pistons, tubes for food, foil wrapping etc., etc.

Aluminium casting alloys.

One very good aluminium alloy contains about 8% copper and 1% iron.
In the cast condition its ultimate tensile strength is 140 MPa. This alloy can be used for transmission housings and crankcases.

Many alloys are obtainable. Among the best known are duralumin and duralium, each containing amongst other elements, 4% copper. The ultimate strength of these materials is 400 MPa tensile, equal to mild steel.

Aluminium alloys are readily extruded into a variety of ornamental or engineering sections (angles, channels etc.).

Aluminium sections are weldable but the design technique involved is specialised and beyond the scope of this book.

The corrosion resistance offered by aluminium is due to the formation of an oxide coating 0.0002 mm thick on the surface. This takes place immediately the material is exposed to the air.

Aluminium alloys containing magnesium have a "self healing" coating. This coating immediately reoxides if the surface of the material is scratched.

Anodising is a process which artificially increases the thickness of the protective coating to about 0.02 mm. Coloured dyes can be added to the anodising process to give highly decorative metallic finishes.

Zinc

The chief uses of zinc in industry are as follows:

(1) Die casting. Zinc base alloys are eminently suited to this process. They are easy to handle and give a good quality product.

(2) Used with copper to form brass.

(3) Used in various ways for corrosion protection. See "Surface Protection", page 13.10.

(4) Used as the negative electrode in a dry cell battery.

Note: points (3) and (4) utilise the electronegative nature of zinc while point (1), the low melting point.

Die-casting

There are two main groups of die-casting.:

Gravity die-casting in which the molten alloy is poured from a ladle into the die.

Pressure die-casting in which the metal is injected into the die at high pressure.

The three main metal groups into which die-casting may be divided are:

zinc based alloys.
aluminium based alloys.
brass.

The dies themselves are generally made from alloy steel.

Die-castings have extensive use in domestic appliances e.g. vacuum cleaner impellers, door and drawer handles etc. In the automotive field many complex carburettor bodies are die castings.

Die casting makes it possible to secure high accuracy and uniformity together with a good surface finish. Subsequent machining can be virtually eliminated, flash trimming being the main requirement. Complex shapes can be relatively easily handled. For large numbers of castings the process is rapid and cheap.

Surface Hardening

Carburizing and nitriding.

These processes may be applied to certain grades of steel for the purpose of producing a hard wearing surface combined with a tough shock resisting core.

Carburizing of low carbon steels is the process of forcing carbon into steel by packing it in a carburizing material and heating it to a temperature of 930° C.

The length of time the furnace is held at this temperature is governed by the depth of the case required. After the steel is removed from the furnace it is normalized and heat treated. The resulting surface is as hard as tool steel.

Nitriding is a process of using nitrogen to produce a hard case. Ammonia is used, which dissociates in contact with steel at 500° C. Fe_4N is taken up.
Special steels containing aluminium, chromium, or molybdenum are more easily treated and form a thicker case.

Flame and induction hardening.

Medium carbon and many low alloy steels are suitable for hardening by these methods.

The methods consist of heating the surface of the steel to above hardening temperature and then quenching. The surface layer upon cooling experiences a phase change to martensite and so there is a volume increase. This volume increase produces a residual compressive stress which gives a dense, hard, wear resistant surface. Endurance is also improved because fatigue failure cannot start in regions of compressive stress.

Flame hardening is a mechanically operated process, the apparatus consisting essentially of an oxy-acetylene blow-pipe flame with a water jet for the rapid quenching of the surface of the steel when it has been heated.

Induction hardening is a method where the steel is heated by a high-frequency electric current and quenched by flushing with water supplied through holes in the inductor blocks. The process takes a few seconds only and almost any depth of hardening can be obtained.

Both the above methods are suitable for pins, shafts, and journals of crank shafts.

Surface Protection

There are numerous approaches to the prevention of corrosion of steel but none so effective, practical and economic as metallic zinc coatings applied by the hot dip galvanizing process. Zinc is strongly resistant to the corrosive action of normal environments and therefore provides long term protection.

Hot dip galvanizing prevents corrosion of steel by:

(1) Providing a tough, durable barrier coating of metallic zinc which completely seals the steel from corrosive environments.

(2) Cathodic or sacrificial protection. The zinc coating corrodes very slowly and preferentially to the steel base, preventing corrosion of small areas of steel which may be exposed through accidental damage to the coating or at cut edges or drilled holes.

The life expectancy of a galvanized coating depends on the thickness of the zinc and the environment to which it is exposed. Even under adverse conditions, commercial hot dip galvanizing will be effective for many years.

Other metallic zinc coatings for steel

Zinc plating is an economic, versatile and effective method of applying a protective coating to small steel components, and it is the most widely used method of applying metallic zinc coatings to small fasteners.

There is in general an economic upper limit to the zinc coating mass which can be applied by plating however, and zinc plating is not recommended for outdoor exposure without supplementary coatings, although heavier plated zinc coatings are applied to certain roofing fasteners.

Sheradizing is used for zinc coating small, complex steel parts such as fasteners, springs and chain links. Articles are coated by tumbling with zinc dust at about 370° C, just below the melting point of zinc, until a zinc-iron alloy coating of the desired thickness is formed by diffusion. The dark grey sheradized coating is hard, abrasion resistant and uniform in thickness over the whole surface of the article.

The uniformity of the coating allows suitably dimensioned threaded parts to be sheradized after manufacture without the need to re-tap threads.

Mechanical plating or peen plating is an electroless plating method used to deposit uniform coatings of ductile metals onto a metal substrate, using mechanical energy. It is ideally suited to plating zinc onto steel parts, particularly threaded components and close tolerance items. Zinc coatings are deposited in the range 5 to 150 μm thickness.

After precleaning, parts are tumbled together with zinc powder, glass beads, impact media, water, and a suitable 'promoter'. The glass beads peen and cold weld the zinc powder to the steel surface. Coating thickness is controlled by time and the quantity of zinc powder added.

Zinc spraying or zinc metallising allows coating of fabricated items which cannot be galvanized because of their size or because coating must be performed on site. Zinc spraying has the advantage that zinc coatings up to 250 μm thick, equivalent to 1500 g/m^2 can be applied by either manual or mechanised methods. The steel surface must be prepared by grit blasting. The resulting zinc coating provides cathodic protection for the underlying steel in the same way as a galvanized coating.

Zinc rich coatings consist of zinc dust in organic or inorganic vehicle/binders. Surface preparation by abrasive blast cleaning is necessary, and coatings may be applied by brush or spray. Zinc rich coatings are barrier coatings which also provide cathodic protection to small exposed areas of steel, provided the steel surface is properly prepared, and the paint conforms to relevant Australian Standards 2105 and 2204.

Suitable zinc rich paint coatings provide a useful repair coating for damaged galvanized coatings.

Preconstruction primers are relatively thin weldable zinc rich coatings used widely for ship building, storage tanks, and similar steel plate constructions, intended for subsequent top coating.

Information on these zinc coating processes and their applications is available from Australian Zinc Development Association.

Painting

Painting galvanized steel.

'Duplex' coatings of galvanizing-plus-paint are often the most economic solution to the problem of protecting steel in highly corrosive environments. Such systems provide a synergistic effect in which life of the combined coatings exceeds the total life of the two coatings if they were used alone.

In general, correctly chosen galvanized coatings used alone provide the most economic corrosion protection for steel. When coatings are painted it is usually for aesthetic reasons, for identification or warning, for camouflage, or for added corrosion resistance under severe service or exposure conditions.

In many applications duplex systems of galvanizing-plus-paint are an ideal combination. The galvanized coating provides a stable base which greatly increases paint life, while the paint film protects the galvanized coating to give a synergistic effect in which the combination lasts considerably longer than the total life of each coating alone.

The synergistic effect of duplex galvanizing-plus-paint systems

The longer life of correctly chosen and applied paint coatings on galvanized steel surfaces results from the stable zinc substrate which prevents initiation of corrosion at pores and scratches, and resulting creep corrosion beneath the paint film.

Weathering of exposed galvanized surfaces until all bright zinc has changed to a dull surface layer may aid adhesion of some paints, provided any loosely adherent surface particles have been brushed from the surface.

Painting of plain steel.

Painting still remains a very popular method of surface protection. With the present level of technology, there are paints suitable for all surfaces under most environmental conditions.

There is ample literature available from paint manufacturers to describe their products.

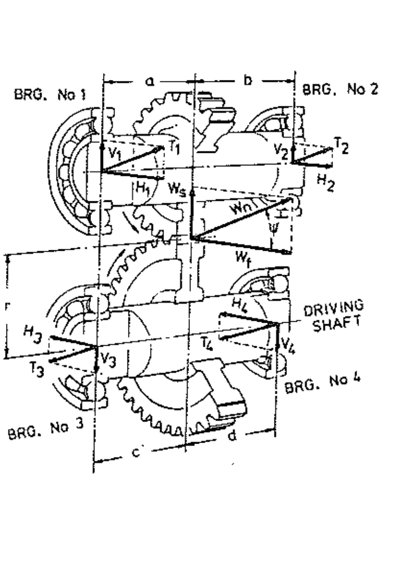
Reference

Galvanizers Association of Australia,
Galvanizing,
Australian Zinc Development Association.

The authors express thanks for permission to use the above publication in the preparation of the section on "Surface Protection" in this chapter.

CHAPTER 14

INERTIA



CHAPTER 14

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I N E R T I A

Inertia is tbe inability of a body to change its state of rest or of uniform motion in a straight line. The only way the state of rest or motion can be changed is by the application of a force that is external to the system.

The above is a re-statement of Newton's first law of motion.

Glossary of main terms

I = moment of inertia, kg.m^2

k_o = radius of gyration, m

T = torque, N.m

ϕ = angular acceleration, rad/s^2

M = mass, kg

W = weight = Mg, N

ω = angular velocity, rad/s

R = radius, m

r = distance from centre of rotation to centre of mass, m

q = distance from centre of rotation to centre of percussion, P, m

h = distance between parallel axes, m

a = linear acceleration, m/s^2

G = gear ratio or centre of gravity as applicable.

P = centre of percussion

θ = angle travelled, rad.

η = efficiency

$T_{1,2,3,4}$ = rope tension, N

μ = coefficient of friction

N = speed, r.p.m.

F = force, N

Q = centripetal force, N

Dynamics of a rigid body in translation

Any particle of a rigid body in translation has the same acceleration as any other particle.

The effective force on a body is the product of its mass and acceleration.

$$F = Ma$$

"Inertia - force" method of analysis

The inertia-force method of solution may be applied provided the effective forces for the body are reversed and added to the external force system. This has the effect of holding the system in equilibrium for the instant for study purposes only. The reversed effective (inertia) forces do not actually exist.
The equations of static equilibrium may then be used.

The rules for this method of analysis were given in the section "Design technique" in Chapter No. 1.

Worked Example No. 1.

A body of mass 5.5 kg is released from the top end of a 20° incline. The coefficient of friction = 0.3. Find the force required to stop the body in 25 mm after it has travelled 915 mm.

Answer

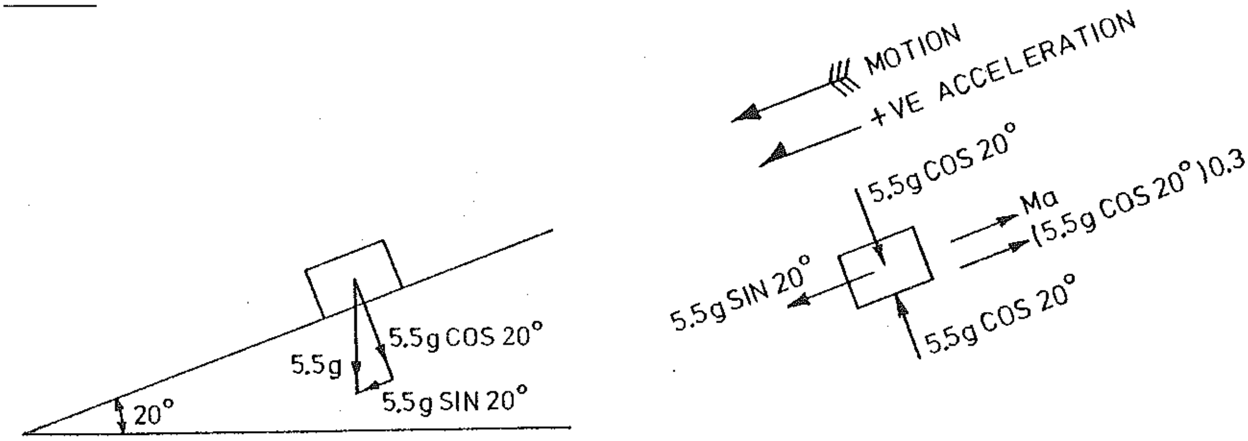


FIG. 1

Refer to Fig. 1. A free body diagram of the body has been drawn and all the external forces shown. Resolving parallel to the plane:

$$5.5\ g \sin 20° - (5.5\ g \cos 20°)\ 0.3 - 5.5a = 0$$

$$\therefore \quad a = 0.59\ \text{m/s}^2$$

Now to find the velocity after 915 mm apply the appropriate law of rectilinear motion.

$$v^2 = u^2 + 2\ as$$

$$v^2 = 2 \times 0.59 \times 0.915$$

$$\therefore v = 1.04\ \text{m/s}$$

The body is now stopped in 25 mm, find the deceleration.

$$v^2 = u^2 + 2as$$

$$0 = 1.04^2 + 2a \times 0.025$$

$$\therefore a = -21.63 \text{ m/s}^2$$

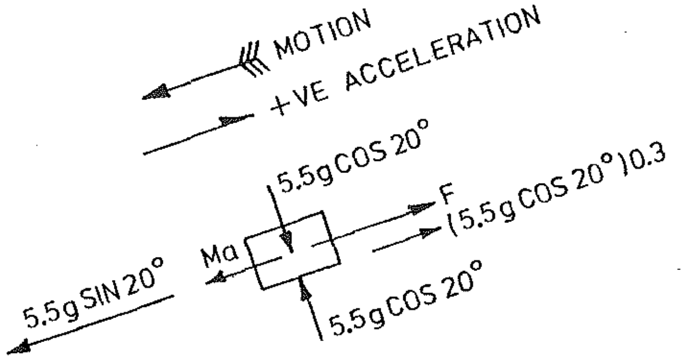


FIG. 2

Refer to Fig. 2.

Resolving parallel to plane:

$$F + (5.5g \cos 20°)\, 0.3 - 5.5g \sin 20° - 5.5 \times 21.63 = 0$$

From which $\underline{F = 122.21 \text{ N}}$

Dynamics of a rigid body in rotation.

Inertia force in rotation. Refer to Fig. 3.

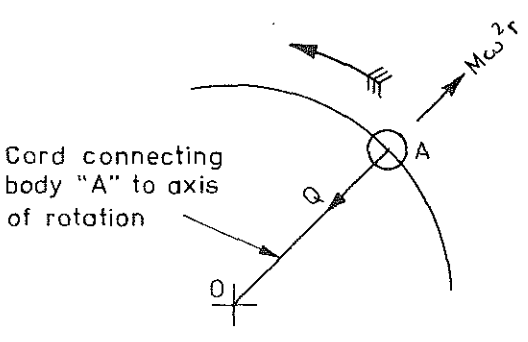


FIG. 3

For balance of forces at the body A the centripetal force Q may be considered as being in equilibrium with an equal and opposite inertia "force" of magnitude $M\omega^2r$. This inertia "force" is a reactive force, since of itself it cannot cause motion. For example, if the cord is cut the active force Q disappears and the body A moves in a straight line tangential to the circular path. It does not fly radially outwards.

The inertia "force", $M\omega^2r$, is termed centrifugal force.

The proof and theory behind the foregoing can be found in lessons on applied mechanics.

Axial mass moment of inertia, I

The axial moment of inertia of an element of mass is the product of the mass of the element and the square of the distance of the element from the axis.

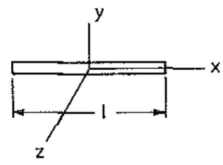
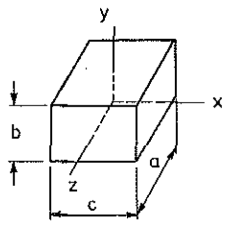
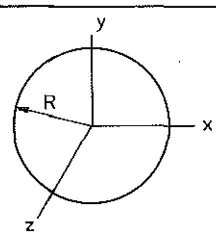
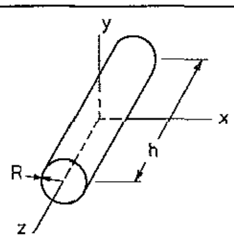
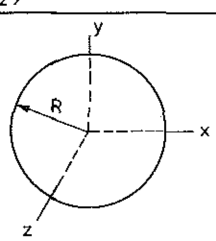
The centroidal mass moments of inertia for some common shapes are given in Table No. 1, page 14.4.

For a thin laminar essentially in the xy plane

$$I_{zz} = I_{xx} + I_{yy}$$

In this case I_{zz} is called the polar moment of inertia.

TABLE Nº 1

FIGURE	NAME	I_x	I_y	I_z
	SLENDER BAR	—	$\frac{1}{12}Ml^2$	$\frac{1}{12}Ml^2$
	RECTANGULAR PARALLELEPIPED	$\frac{1}{12}M(a^2+b^2)$	$\frac{1}{12}M(a^2+c^2)$	$\frac{1}{12}M(b^2+c^2)$
	THIN CIRCULAR DISC	$\frac{1}{4}MR^2$	$\frac{1}{4}MR^2$	$\frac{1}{2}MR^2$
	RIGHT CIRCULAR CYLINDER	$\frac{1}{12}M(3R^2+h^2)$	$\frac{1}{12}M(3R^2+h^2)$	$\frac{1}{2}MR^2$
	SPHERE	$\frac{2}{5}MR^2$	$\frac{2}{5}MR^2$	$\frac{2}{5}MR^2$

Parallel axis theorem.

The parallel axis theorem states that the moment of inertia of a body about an axis is equal to the moment of inertia I about a parallel axis through the centre of gravity of the body plus the product of the mass of the body and the square of the distance between the two parallel axes.

$$I_{pp} = I_{xx} + M h^2$$

Radius of gyration, k_o.

The radius of gyration of a body with respect to an axis is defined mathematically as the square root of the quotient of its moment of inertia divided by the mass.

$$k_o = \sqrt{\frac{I}{M}}$$

Units

The units of mass moment of inertia are kg.m^2.
(since I = mass x radius of gyration^2)

Equations of motion

If a body rotates about an axis of symmetry the following inertia equation holds:

$$T = I\phi$$

If a body has a plane of symmetry and rotates about a fixed axis perpendicular to this plane then the forces acting are as shown in Fig. 4.

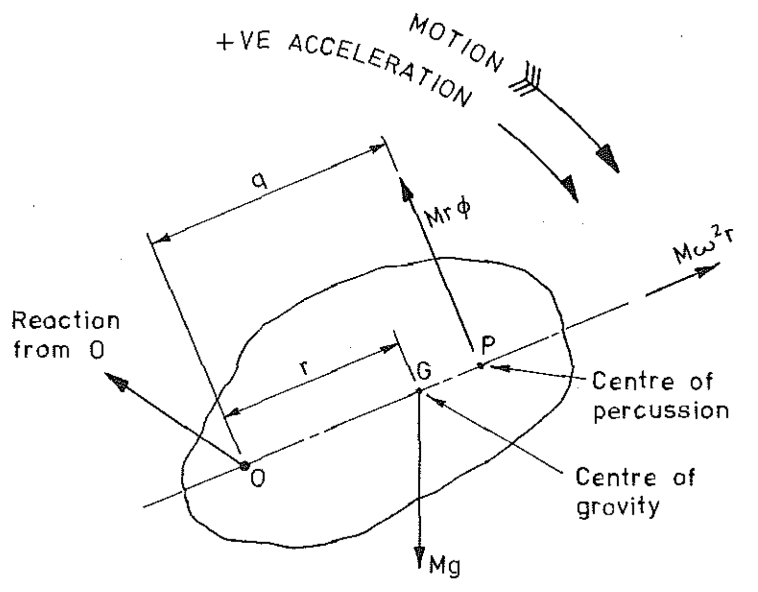


FIG. 4

The reversed "force" $M\omega^2 r$ must act away from the centre of rotation and the reversed "force" $Mr\phi$ must oppose the direction of the mathematical positive acceleration. "Force" $Mr\phi$ acts at a distance q from O such that

$$q = \frac{k_o^2}{r} \quad \text{(centre of percussion).}$$

Centre of Percussion

The centre of percussion is the point at which, if a "normal" impulse force is applied there is no impulse felt at the axis of rotation.

In the context of this chapter the centre of percussion is the point through which the resultant of the inertia "forces" acts.

The theory on centre of percussion is derived in lessons on applied mechanics.

"Inertia - force"method of analysis.

As with linear motion this method of solution may also be used for rotational analysis.

The rules were given in the section "Design technique" in Chapter No. 1.

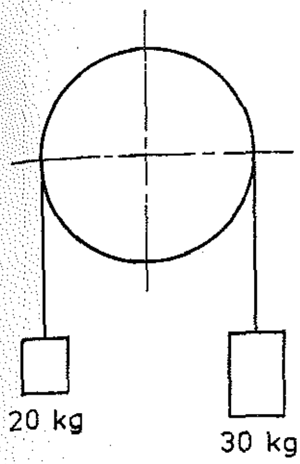


FIG. 5(a)

Worked Example No. 2.

Refer to Fig 5(a)

The system shown accelerates from rest for 3 seconds before a brake is applied which brings the system to rest in 2.5 seconds. The constant frictional torque = 3 N.m. The pulley is a thin disc 200 mm diameter of 10 kg mass. The effect of the shaft can be neglected.

Find: (a) The acceleration of the masses during the first part of the motion.

(b) The value of the brake torque necessary to bring the system to rest.

Answer

(a) I of wheel $= \frac{1}{2} MR^2$ (Table No. 1, page 14.4)

$= \frac{1}{2}\, 10 \times 0.1^2 = 0.05$ kg.m²

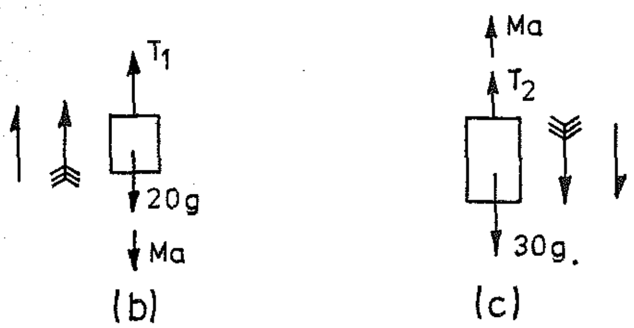


FIG. 5

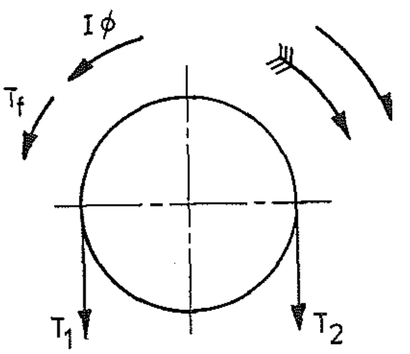


FIG. 5(d)

Refer to Figs. 5 (b) and (c) for free body diagrams of the masses. The following equations can be formed:-

$$T_1 = 20g + 20a$$

$$T_2 = 30g - 30a$$

Subtract T_1 from T_2

$$T_2 - T_1 = 10g - 50a \text{ ---------------(1)}$$

Fig 5 (d) shows a free body diagram of the wheel. The following equation can be formed.

$$(T_2 - T_1)\, 0.1 = T_f + I\phi$$

Now $a = R\,\phi$

$$\therefore \quad 0.1\, T_2 - 0.1\, T_1 = 3 + \frac{0.05a}{0.1}$$

$$\therefore \quad T_2 - T_1 = 30 - 5\,a \text{ -------------------(2)}$$

$$T_2 - T_1 = 10g - 50\,a \text{ -----------------(1)}$$

Subtract (1) from (2) and solve for "a"

$a = \underline{1.24\ m/s^2}$

(b) Speed of masses after 3 seconds

$= 1.24 \times 3 = 3.72\ m/s$

Deceleration of weights in 2.5 seconds

$= \frac{3.72}{2.5} = 1.49\ m/s^2$

Refer to Figs. 5 (e) and (f) for free body diagrams of the masses.

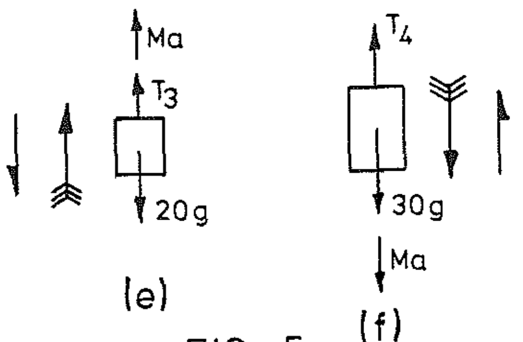


FIG. 5

$T_3 = 20g - 20a$

$T_4 = 30g + 30a$

Subtract T_3 from T_4

$T_4 - T_3 = 10g + 50a$ -------------(1)

Fig. 5 (g) shows a free body diagram of the wheel.

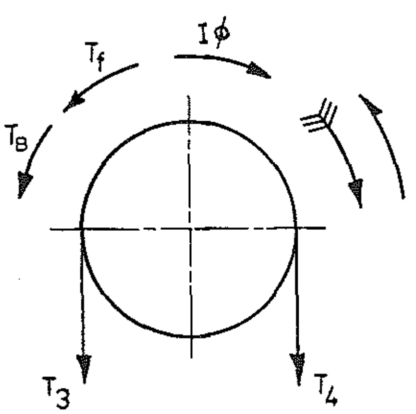


FIG. 5(g)

$(T_4 - T_3)\ 0.1 + I\phi = T_f + T_B$

from which $T_4 - T_3 = 22.55 + 10\ T_B$ ----------(2)

$T_4 - T_3 = 10g + 50\ a$ ------------(1)

subtract (1) from (2) and solve for T_B,

$T_B = \underline{15\ N.m}$

Worked Example No. 3.

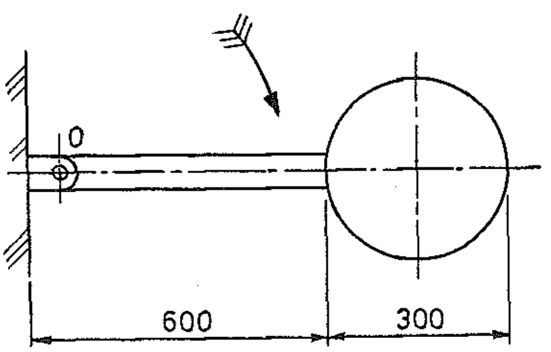


FIG. 6(a)

A homogeneous sphere of mass 110 kg is attached to a slender rod of mass 24 kg. In the horizontal position shown in Fig. 6 (a) the angular speed of the system is 8 rad/s. Determine the magnitude of the angular acceleration of the system and the reaction at 0 on the rod.

Answer

Locate the centroid of the system with reference to the pin O.

$$r = \frac{24 \times 300 + 110 \times 750}{134 \times 1000} = 0.6694 \text{ m}$$

The total moment of inertia, I, equals the moment of inertia of the rod about its end ($\frac{1}{3} M\ell^2$) plus the transferred moment of inertia of the sphere ($\frac{2}{5} MR^2 + Mh^2$) where

$$h = 0.75 \text{ m}$$

$$I = \frac{1}{3}\ 24 \times 0.6^2 + \frac{2}{5}\ 110 \times 0.15^2 + 110 \times 0.75^2$$

$$= 65.745 \text{ kg.m}^2$$

$$k_o^2 = \frac{I}{M} = \frac{65.745}{134} = 0.4906 \text{ m}^2$$

Point P is $\frac{k_o^2}{r} = \frac{0.4906}{0.6694} = 0.733$ m to the right of O.

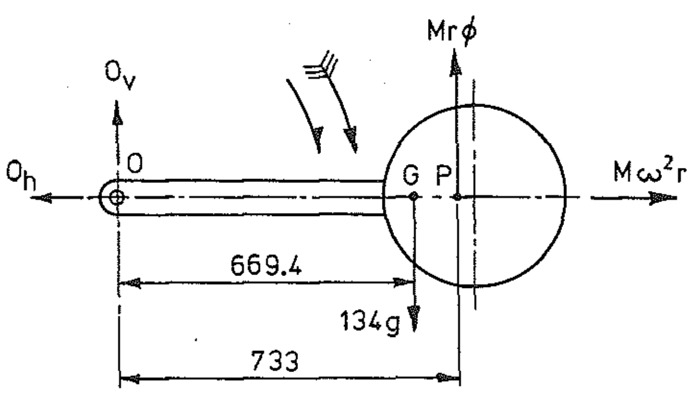


FIG. 6(b)

A free body diagram of the system is shown in Fig. 6 (b)

Taking moments about O,

$$134 \times 0.6694\ \phi \times 0.733 - 134g \times 0.6694 = 0$$

$$\therefore\ \phi = \underline{13.4 \text{ rad/s}^2}$$

Sum forces horizontally to obtain

$$O_h = M\ \omega^2\ r = 134 \times 8^2 \times 0.6694 = 5740.8 \text{ N}$$

Sum forces vertically to obtain

$$O_v = 134\ g - 134 \times 0.6694 \times 13.4 = 112.6 \text{ N}$$

The absolute reaction at O

$$= \sqrt{O_h^2 + O_v^2} = \underline{5741.9 \text{ N}}$$

Acceleration of a geared system

In Fig. 7 two shafts A and B are geared together, so that B rotates at G times the speed of A, i.e. $G = N_b/N_a$.

The total mass moment of inertia of the masses attached to A is I_a and of those attached to B is I_b.

If the angular acceleration of shaft A is ϕ, what torque must be applied to the shaft A? Since the shaft B turns at G times the speed of shaft A, the rate of change of the speed of shaft B with respect to time must necessarily be G times the rate of change of the speed of the shaft A with respect to time, or

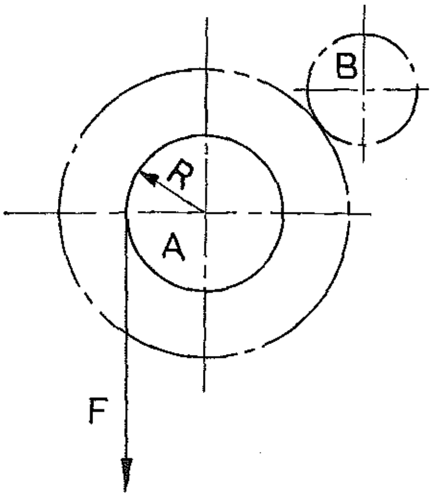


FIG. 7

$$\phi_b = G\,\phi_a$$

It follows that the torque required for the angular acceleration of B is given by

$$T_b = I_b\,\phi_b = G\,I_b\,\phi_a$$

But to provide a torque T_b on shaft B, the torque applied to shaft A must be $G\,T_b$, i.e. a torque of $G^2\,I_b\,\phi_a$ must be applied to shaft A in order to accelerate shaft B.

In addition the torque required to accelerate shaft A by itself is equal to $I_a\,\phi_a$.

Hence the total torque which must be applied to shaft A in order to accelerate the geared system is given by:

$$T = I_a\,\phi_a + G^2\,I_b\,\phi_a = \phi_a(I_a + G^2\,I_b) = \phi_a\,I$$

where, $I = I_a + G^2\,I_b$

"I" may be regarded as the equivalent mass moment of inertia of the system referred to shaft A.

If the efficiency of the gearing between the two shafts A and B is η, then the torque which must be applied to A in order to accelerate B

$$= G\,T_b/\eta = G^2\,I_b\,\phi_a/\eta,$$

the total torque applied to shaft A in order to accelerate the geared system

$$= T = I_a\,\phi_a + G^2\,I_b\,\phi_a/\eta$$

and the equivalent mass moment of inertia of the geared system referred to shaft A

$$= I = I_a + G^2\,I_b/\eta$$

For a system in which a number of shafts are geared together in series, the equivalent inertia referred to shaft A is evidently given by:

$$I = I_a + \Sigma\ (G_x^2\ I_x/\eta_x)$$

where I_x is the mass moment of inertia of shaft X, G_x is the ratio of the speed of shaft X to the speed of shaft A and η_x is the overall efficiency of the gearing from shaft A to shaft X.

If each pair of gear wheels is assumed to have the same efficiency η, the overall efficiency from shaft A to shaft x is given by:

$$\eta_x = \eta^i$$

where i is the number of gear pairs through which the power is transmitted from A to X.

Let us suppose that the torque required to accelerate the system shown in Fig. 7, page 14.9, is applied by means of a force F which acts tangentially to a drum or pulley of radius R.

Then $F\ R = \phi_a\ I = \phi_a\ (I_a + G^2\ I_b)$

But the tangential acceleration of the drum "a" = ϕ_a R, or $\phi_a = a/R$ so that

$$F\ R = (a/R)\ (I_a + G^2 . I_b)$$

$$\therefore\ F = a\ (I/R^2)\ (I_a + G^2\ I_b) = M_e a$$

where $M_e = (I/R^2)\ (I_a + G^2\ I_b)$

This may be regarded as the equivalent mass of the system referred to the line of action of the accelerating force F.

Worked Example No. 4.

The moment of inertia of A, Fig. 7,(page 14.9) is 8.5 kg.m^2 and that of B is 0.6 kg.m^2. The shaft B runs at five times the speed of A. A mass of 70 kg is hung from a rope wrapped around a drum of effective radius 200 mm which is keyed to shaft A.

(a) If the mass is allowed to fall freely, find its acceleration.

(b) What would be the acceleration, if the efficiency of the gearing were 90%?

Answer

(a) $M_e = (\frac{1}{R^2})\ (I_a + G^2\ I_b) = \frac{1}{0.2^2}\ (8.5 + 5^2 \times 0.6)$

$= 587.5$ kg

The total equivalent mass to be accelerated

$= 587.5 + 70 = 657.5$ kg

But the accelerating force is provided by the pull of gravity on the mass suspended from the rope

$$\therefore \quad a = \frac{70g}{657.5} = \underline{1.04 \text{ m/s}^2}$$

(b) If the efficiency of the gearing is 90%, the equivalent mass of the geared system referred to the circumference of the drum

$$= \left(\frac{1}{R^2}\right) \left(I_a + \frac{G^2 I_b}{\eta}\right)$$

$$= \left(\frac{1}{0.2^2}\right) \left(8.5 + \frac{5^2 \times 0.6}{0.9}\right) = 629.2 \text{ kg}$$

and the total equivalent mass to be accelerated

$$= 629.2 + 70 = 699.2 \text{ kg}$$

$$\therefore \quad a = \frac{70g}{699.2} = 0.98 \text{ m/s}^2$$

Worked Example No. 5.

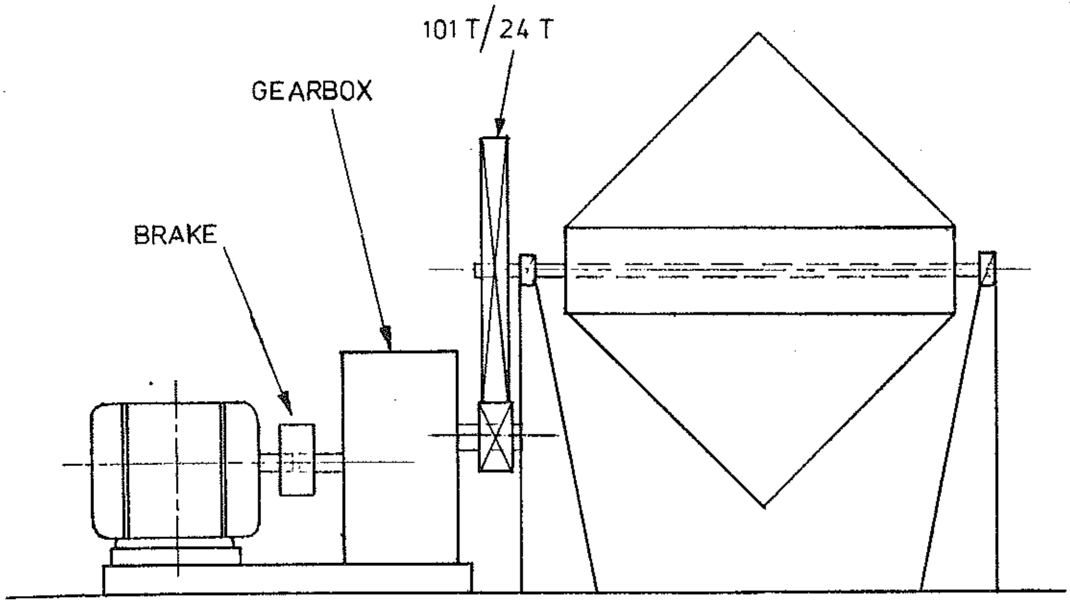


FIG. 8(a)

Double Cone Blending Machine

A double Cone Blending Machine is a unit used for the mixing and/or blending of powders or pellets. The machine is generally about 60 per cent full with the product being treated, this degree of loading giving the best mixing condition.

Fig. 8 (a) shows a Double Cone Blender body mounted on two support pedestals. The body construction consists of two cone frustrums fixed to a centre cylindrical section. Material of construction is M.S. plate reinforced. The centre cylindrical section is fixed to a shaft which turns in bearings attached to the pedestals. The complete body assembly is weighted where necessary so as to be reasonably well balanced dynamically when empty.

Speed of rotation is 5. R.P.M. approximately.

A brake has to be chosen for the machine. The most severe position for brake operation is the 315° position as shown in Fig. 8 (b). The brake has to stop the machine in 13° of body movement i.e., between 308½° to 321½°. The assumed position of the C of G of the product is shown. This is the furthermost position that it is possible for the product C of G to be from the vertical centre line of rotation.

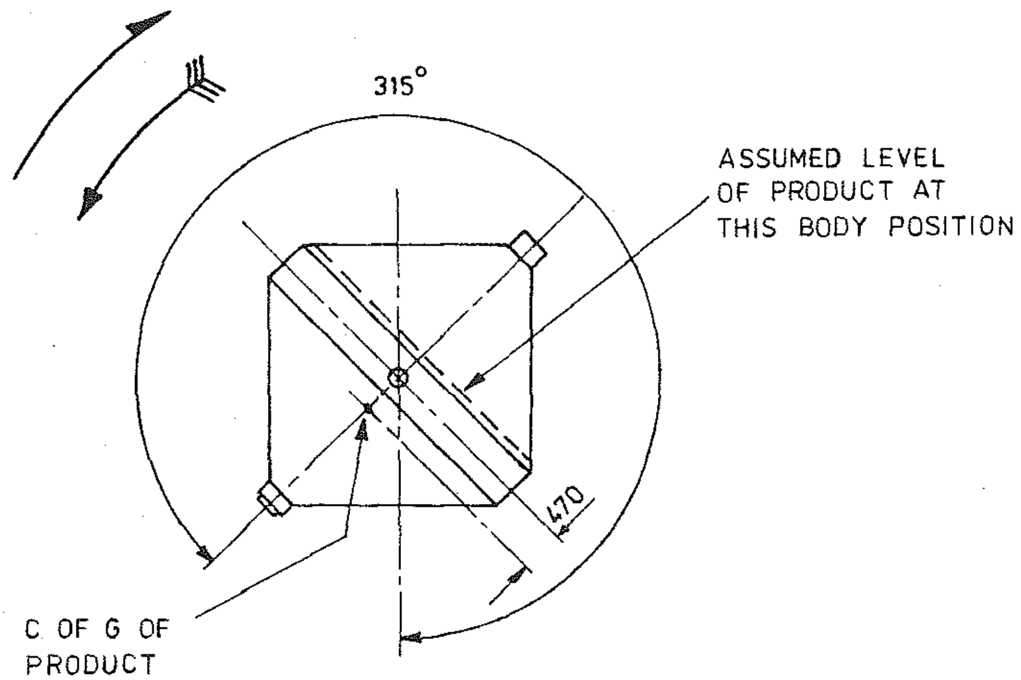


FIG. 8(b)

Engineering details:-

Mass M of I of blender body and attachments (plus product) about its axis of rotation .... 32 000 kg.m^2

Mass M of I of all other parts of system referred to the motor shaft for the deceleration phase .... 0.6725 kg.m^2

Mass of product .... 10 960 kg

Mass of empty blender body .... 5350 kg

Gear box reduction .... 71.3 : 1

Spur gear reduction .... 101 T to 24 T

Motor speed (over-run speed) .... 1540 r.p.m.

Efficiencies:-

Spur gear reduction 97%

Gear box 95%

Determine the torque requirements of the brake:-

(a) by the inertia - force method

(b) by two different energy concepts

Answer

(a) System energy to be dissipated by brake (neglecting loaded blender body)

$$= \tfrac{1}{2} I\omega^2 = \tfrac{1}{2} \times 0.6725 \times \left(\frac{2\pi \;\; 1540}{60}\right)^2 = 8745.02 \text{ N.m}$$

13° of blender body movement

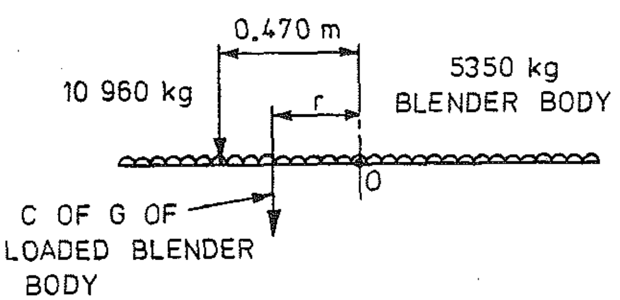
$$= 13 \times 71.3 \times \frac{101}{24} \times \frac{2\pi}{360} = 68.08 \text{ rad. movement of brake drum}$$

$$E = T\,\theta \quad \text{from which} \quad T = E/\theta$$

$$\therefore \quad T \text{ (brake drum torque)} = \frac{8745.02}{68.08} = 128.45 \text{ N.m}$$

Inertia - force method for blender body forces.

Refer to Fig. 9 (a) and (b).



SIMPLIFIED DIAGRAM OF LOADED BLENDER BODY

FIG. 9(a)

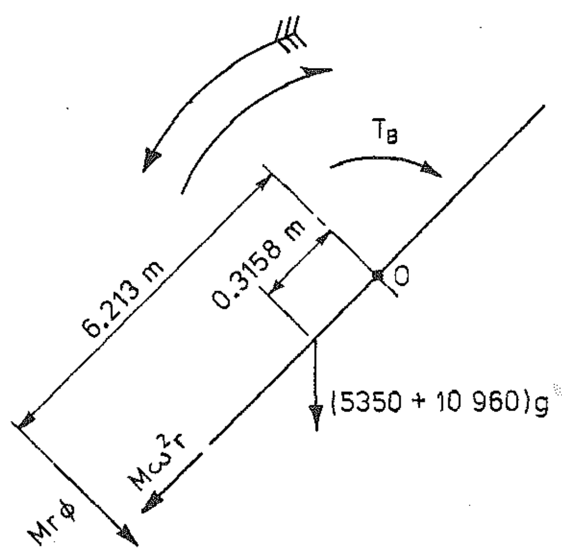


FIG. 9(b)

M of I of loaded body = 32 000 kg.m²

$$I = M k_o^2$$

$$32\,000 = (5350 + 10\,960)\, k_o^2$$

$$\therefore \quad k_o^2 = 1.962 \text{ m}^2$$

Determine the C of G of loaded blender body.

Take moments about 0(Fig. 9 (a) page 14.13)

$10\ 960 \times 0.47 = (5350 + 10\ 960)r$

$\therefore \quad r = 0.3158$ m

Distance from 0 to centre of percussion

$$q = \frac{k_o^2}{r} = \frac{1.962}{0.3158} = 6.213 \text{ m}$$

$$\text{Overall reduction} = 71.3 \times \frac{101}{24} = 300.054{:}1$$

$$\therefore \text{ Blender speed} = \frac{1540}{300.054} \times \frac{2\pi}{60} = 0.538 \text{ rad/s}$$

Blender to stop in $13° = 0.227$ rad

$$\omega_2^2 = \omega_1^2 + 2\phi\theta$$

$$0 = 0.538^2 + 2\ \phi\ 0.227$$

$$\therefore \phi \quad 0.637 \text{ rad/s}^2$$

$$Mr\phi = (5350 + 10\ 960) \times 0.3158 \times 0.637 = 3281 \text{ N}$$

Applying the calculated information to a free body diagram (Fig. 9 (b), page 14.13) and taking moments about 0.

$$T_B = 3281 \times 6.213 + (5350 + 10\ 960)g \times 0.3158 \sin 45°$$

$$= 56\ 113.79 \text{ N.m}$$

$$\therefore \text{ Torque at brake} = \frac{56\ 113.79}{300.054} \times 0.97 \times 0.95 = 172.33 \text{ N.m}$$

$$\text{Total torque at brake} = 172.33 + 128.45 = 300.8 \text{ N.m}$$

(b) (i) Summarising the equivalent mass moment of inertia formulas applicable to this type of example:-

Equivalent mass M. of I. at motor shaft for the deceleration phase

$$I = I_A + \frac{I_B}{G^2}\eta \text{ ---------------- (1)}$$

For the acceleration phase

$$I = I_A + \frac{I_B}{G^2\eta} \text{ ------------------ (2)}$$

Formula (1) is applicable in this case.

$G = 300.054 : 1$ from part (a) of example

$$I = 0.6725 + \frac{32\ 000}{300.054^2} \times 0.95 \times 0.97 = 1.0 \text{ kg.m}^2$$

Motor speed $=$ 1540 r.p.m. $=$ 161.268 rad/s

Total kinetic energy in system

$$= \tfrac{1}{2} I \omega^2 = \tfrac{1}{2} \times 1.0 \times 161.268^2 = 13\,003.7 \text{ N.m}$$

Refer to Fig. 9. (c) (A and B represent C of G of product.)

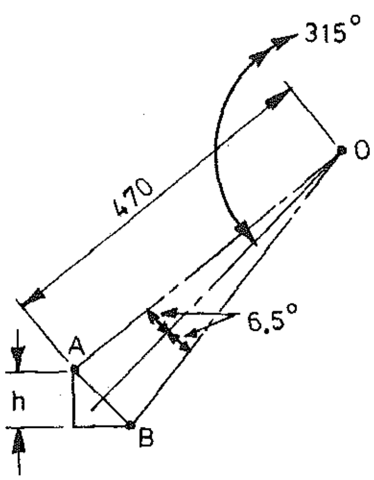


FIG 9(c)

$$AB = 2 \times 470 \sin 6\tfrac{1}{2}° = 106.411 \text{ mm}$$

$$\therefore h = \frac{106.411}{1000\sqrt{2}} = 0.075\,244 \text{ m}$$

$$\therefore \text{P.E.} = Mgh = 10\,960g \times 0.075\,244$$

$$= 8090.054 \text{ N.m}$$

$\therefore$ Total energy to be dissipated by brake

$$= 13\,003.7 + 8090.054 \times 0.95 \times 0.97$$

$$= 20\,458.69 \text{ N.m}$$

From part (a) brake drum is to stop in 68.08 rad.

$$\therefore \text{Brake torque, } T = \frac{E}{\theta} = \frac{20\,458.69}{68.08}$$

$$= 300.5 \text{ N.m}$$

(ii) The kinetic energy the brake has to dissipate is 13 003.7 N.m (from (b) (i)),

$$\therefore \text{Brake torque, } T = \frac{13\,003.7}{68.08} = 191 \text{ N.m}$$

Alternatively $T = I\,\phi$

$I = 1.0$ kg.m^2 from part b (i)

$\phi = 0.637$ rad/s^2 for Blender body from part (a)

thus $\phi = 0.637 \times 300.054$ for Motor Shaft.

$= 191$ rad/s^2

$$\therefore T = 1.0 \times 191 = 191 \text{ N.m}$$

Additionally there is the static torque from the product, refer to Fig. 9 (d)

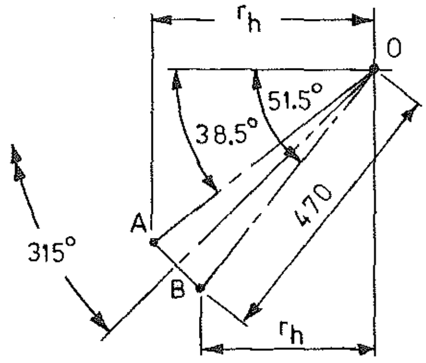


FIG. 9(d)

Distance r_h when brake is initially applied

$$= \frac{470 \quad \cos \quad 38.5°}{1000} = 0.3678 \text{ m}$$

Distance r_h when blender has stopped

$$= \frac{470 \quad \cos \quad 51.5°}{1000} = 0.2926 \text{ m}$$

from which average r_h = 0.3302 m

Average torque on blender from product (again A and B represent C of G of product)

$$= 10\ 960g \times 0.3302 = 35\ 502.31 \text{ N.m}$$

$\therefore$ Torque required at brake,

$$T = 191 + \frac{35\ 502.31}{300.054} \times 0.95 \times 0.97 = \underline{300.03 \text{ N.m}}$$

Dynamics of a rigid body in plane motion

This section of applied mechanics deals with bodies having a combination of translational and rotational motion while moving in a plane.

Worked Example No. 6.

A 300 kg mass wheel with a diameter of 800 mm rolls without slipping down a plane inclined at an angle of 25° with the horizontal (Fig. 10)

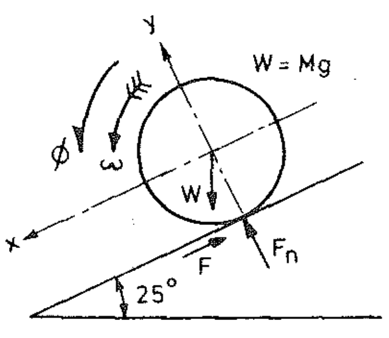


FIG. 10

F and the acceleration of the mass centre.

Answer.

Fig. 10 shows the force system acting on the wheel.

Considerable difficulty is usually experienced in indicating the direction of the friction force F. (F may have any value between - μN and μN.) In this case the friction must act up the plane; otherwise the wheel would slip down the plane. Also, friction is the only force which has a moment about the mass centre and therefore is the force causing the angular acceleration.

$$\Sigma T = I \phi$$

Refer to Fig. 10.

$$I = \tfrac{1}{2} MR^2 \text{ (Table No. 1, page 14.4)}$$

$$= \tfrac{1}{2} \ 300 \times 0.4^2$$

$$= 24 \text{ kg.m}^2$$

The equations of motion are:-

$$300g \sin 25° - F = 300\,a \quad \text{----------} \quad (1)$$

$$F_n - 300g \cos 25° = 0 \quad \text{----------} \quad (2)$$

$$F \times 0.4 = 24\phi \quad \text{----------} \quad (3)$$

$$a = R\phi$$

$$\therefore \quad \phi = \frac{a}{0.4} \quad \text{----------} \quad (4)$$

Substitute (4) into (3) to obtain $F = 150\,a$

Substitute $F = 150a$ into (1) to obtain

$$a = 2.764 \text{ m/s}^2$$

then $F = \underline{414.6 \text{ N}}$

Worked Example No. 7.

Refer to Fig. 11 (a)

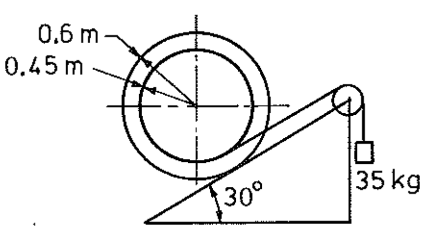


FIG. 11a

Assuming the pulley to be weightless and frictionless, determine the smallest coefficient of static friction to cause the cylinder to roll.

For the cylinder, mass = 70 kg and $k_o = 0.38$ m.

Answer

Assume the cylinder is rolling down the plane and the weight is ascending. Since pure rolling is specified. (Refer to Figs. 11 (b) and (c).)

$$a = R\,\phi = 0.6\,\phi$$

a_1 has the same magnitude as the component of the absolute acceleration of point A parallel to the plane.

By kinematic considerations (study of relative motion between parts)

$$a_A = a_{A/O} + a$$

Dealing only with components parallel to the plane,
the component $a_{A/O} = OA \times \phi = 0.45\,\phi$ up the plane since ϕ is anticlockwise.

The value of $a = 0.6\,\phi$ down the plane.

$$I = M\,k_o^2 = 70 \times 0.38^2 = 10.108 \text{ kg.m}^2$$

Refer to Fig. 11 (b) and (c)

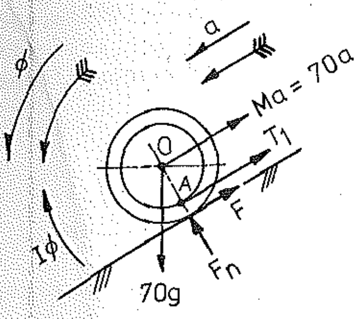


FIG. 11b

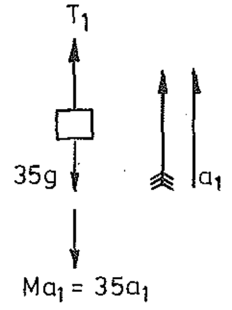


FIG. 11c

$T_1 \;=\; 35g \;+\; 35\,a_1$ ------------------(1)

$T_1 \;+\; F \;+\; 70a \;=\; 70g\,\sin\,30°$ ------(2)

$F \;\times\, 0.6 \;+\; T_1 \,\times\, 0.45 \;=\; 10.108\;\phi$ ---(3)

$a \;=\; 0.6\;\phi$ ------------------(4)

$a_1 = 0.6\;\phi \,-\, 0.45\;\phi \;=\; 0.15\;\phi$ --------(5)

Solution of the above equations yields

$F = -\,202.255$ N i.e. F is acting down the plane (in the opposite direction to that assumed).

By inspection $F_n \;=\; 70g\,\cos\,30° \;=\; 594.7$ N

Thus the required value of

$$\mu \;=\; \frac{202.2548}{594.7} \;=\; \underline{\underline{0.34}}$$

Choice of motor power.

The selection of an electric motor for the great majority of engineering applications is based on the power required for normal full speed operation.

However, occasionally an "extreme inertia" application is encountered where a check must also be done on the "torque for acceleration" requirements to see whether the motor selection is governed by this aspect.

Basically this method involves determining the average torque required during the acceleration period using the maximum permissible "run-up" time of the motor. The motor manufacturer will give the information regarding "run-up" time which is based on the heat generated and disippated in the motor windings. The greater the number of starts per hour the shorter the permissible "run-up" time.

The "torque for acceleration" is then matched against the torque-speed graphs of a number of motors within the required range to enable a suitable selection to be made.

The following analysis is for a squirrel-cage motor started D.O.L. without the use of any special control equipment - thermals etc. Several starts per hour would be allowed. For other types of electric motors and special starting equipment, the appropriate torque-speed curves would be required. Drives with fluid couplings in the system need special consideration as do systems with other types of prime movers, diesel engines, steam engines etc.

General Guide

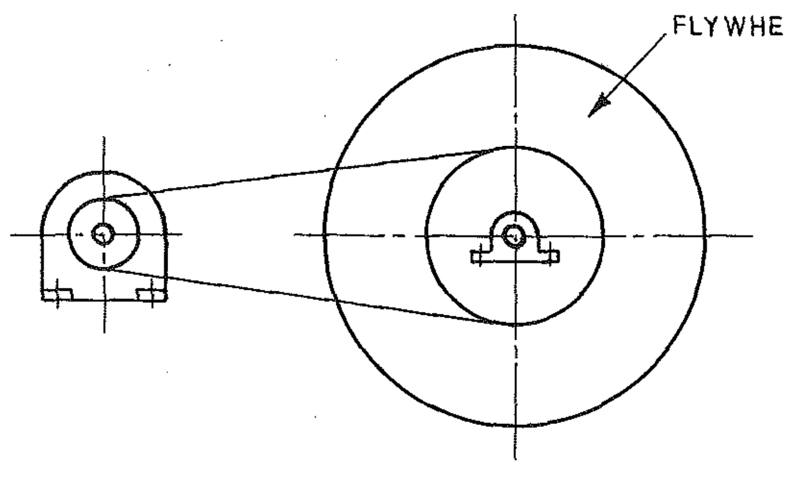
A general guide to motor selection is that if the load torque at normal full speed running is greater than half the torque required for initial acceleration to full speed, the motor power can be safely based on full speed normal running conditions. Torques should be related to the motor shaft.

Five seconds should generally be the maximum "run-up" time.

Most squirrel cage electric motors give two to three times their own full load torque during the acceleration period (when started D.O.L).

Worked Example No. 8.

Determine the motor size required to perform the duty specified in Fig. 12.



Total M. of I. of flywheel shaft assy.	24.3 $kg.m^2$
M. of I. of motor plus drive sprocket.	0.018 $kg.m^2$
Constant frictional torque from bearings on flywheel shaft.	0.34 N.m
Motor speed (synchronous)	1000 rev/min.
Drive reduction and efficiency.	3 : 1 98%
Run-up time for motor.	5 seconds.

FIG. 12

Answer

Power of normal operation.

Motor will be overpowered as far as normal operation is concerned therefore take operating speed as the synchronous speed i.e. 1000 R.P.M.

$$kW = T\omega/1000 = 0.34 \times \frac{1}{3} \times \frac{1}{0.98} \times \frac{2\pi 1000}{60 \times 1000}$$

$$\therefore kW = 0.0121$$

Torque of acceleration.

1000 R.P.M. = 104.7 rad. per sec.

Angular acceleration of motor shaft $= \frac{104.7}{5} = 20.94$ rad. per sec^2.

Total M of I of complete system with respect to motor (for acceleration).

$= \frac{24.3}{3^2 \times 0.98} + 0.018 = 2.77$ kg.m^2 (Refer page 14.14 (b)(i) case 2.)

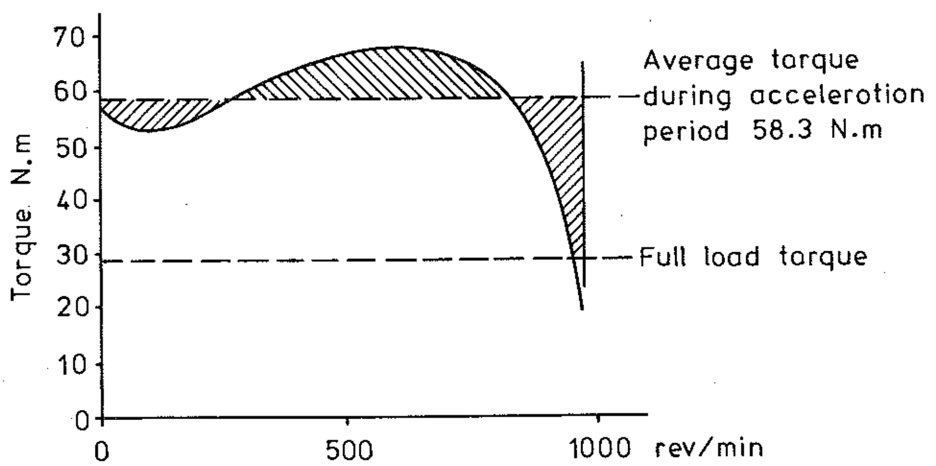
Average torque for acceleration

$= I\,\phi$ + constant frictional torque

$= 2.77 \times 20.94 + 0.34 \times \frac{1}{3} \times \frac{1}{0.98} = 58.1$ N.m

A motor must be chosen which will give at least this average torque during the acceleration period.

Fig. 13 shows how the torque required is matched against the torque-speed curve of a suitable motor.



SPEED AND TORQUE CHARACTERISTICS OF A 3 kW 1000 rev/min SQUIRREL CAGE MOTOR ON D.O.L. START.

FIG. 13

Thus it can be seen that the motor power is determined by the torque requirements during the acceleration period and not by power requirements during normal full speed operation.

EXERCISES

No. 1.

Fig. 14 represents a tilt hammer hinged at A and raised 30° ready to strike an object B. If the total mass of the hammer is 5 kg, the distance of its centre of gravity G from A is 600 mm and its radius of gyration about the axis of the hinge is 670 mm, calculate:-

(a) The force of the blow on B which may be assumed to take place in 0.004 sec.

(b) The maximum reaction offered by the hinge.

(a) 2712.4 N

(b) Reaction Vert. = 52.4 N up.
Reaction Horiz. = 39.3 N to left.

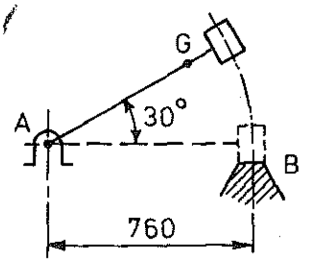


FIG. 14

No. 2.

Refer to worked example No. 4.

Determine the torque required on shaft B to retard the load in 75 mm of travel when it is descending at a speed of 2 m/s. Allow 90% efficiency for the gearing and bearings.

(706.14 N.m)

No. 3.

(a) A counterweighted furnace door (Fig. 15) is being lowered at a speed of 7.62 m/min. A brake on the winding drum is then applied which stops the door in 38 mm of (door) movement.

Determine the rope tensions:-
(i) During the steady speed period.
(ii) During the retardation period.

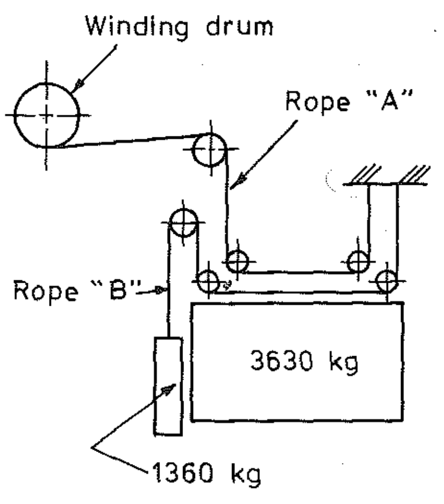


FIG. 15

(b) The winding drum is then reversed and the door takes 3.4 seconds to attain a steady speed of 7.62 m/min.

Determine the rope tensions:-
(i) During the steady speed period of upward movement of the door.
(ii) During the acceleration period.

(c) Determine the power requirements of the motor during steady speed operation.

The system efficiency = 93%

(a) (i) Rope A 4463.6 N (ii) Rope A 5425 N
Rope B 13 341.6 N Rope B 12 765 N

No. 3 Cont.

(b) (i) Rope A 4463.6 N
Rope B 13 341.6 N

(ii) Rope A 4633 N
Rope B 13 240 N

(c) 1.22 kW

No. 4.

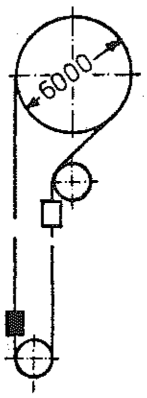


FIG. 16

Material is lifted from a deep mine by the balanced hoist shown in Fig. 16. The mass of the unloaded cage is 6800 kg and of the loaded cage 11 000 kg. The mass of the rope is 20 000 kg. The head pulley which is 6 m in diameter has a mass moment of inertia of 2000 $kg.m^2$. During hoisting operations the pulley is uniformly accelerated, acquiring a speed of 48 r.p.m. in 20 seconds then revolves at constant speed and finally is retarded and brought to rest in 20 seconds. The constant torque of friction is 2500 N.m.

(a) Calculate the torque required during the acceleration period.

(b) Determine the power requirements during the period of constant speed.

(c) What torque is required at the head pulley during the deceleration period?

(a) 211 656 N.m (b) 630.5 kW (c) 40 556 N.m in direction of rotation.

No. 5.

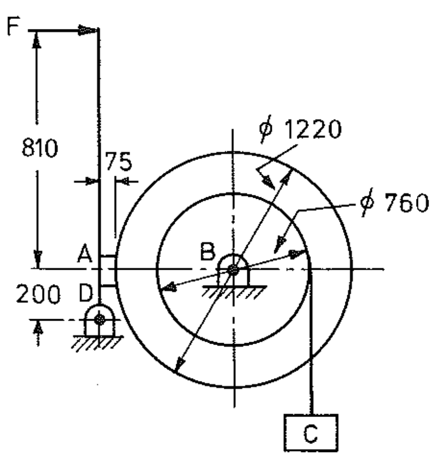


FIG. 17

In Fig. 17 the mass C of 12 kg is moving down with a velocity of 5 m/sec. The M of I of drum is 16 $kg.m^2$ and it rotates in frictionless bearings. The coefficient of friction between the brake A and the drum is 0.40.

(a) What force F is necessary to stop the system in two seconds?

(b) What is the reaction at D on the rod?

(a) 150.63 N

(b) Reaction vert. down 264.58 N
Reaction horiz. to right 510.83 N

No. 6.

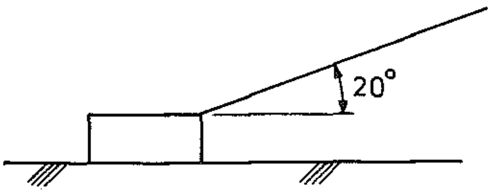


FIG. 18

Fig. 18 shows a 180 kg mass being hauled along a flat surface ($\mu = 0.3$) by a rope. The rope tension is 1300 N during the three seconds acceleration time required to bring the load to working speed.

Find:

(a) The working speed of the load.

(b) The work done by the power source in bringing the load to working speed.

(c) The power being supplied by the power source the instant before working speed is reached.

(d) The power being supplied by the power source after working speed has been attained.

(e) The number of joules of work required to accelerate the load if friction were absent. Compare this number of joules with the kinetic energy of the load.

(a) 13.7541 m/s	(b) 25 203 J	(c) 16.8 kW
(d) 6.57 kW	(e) 17 025.8 J	

No. 7.

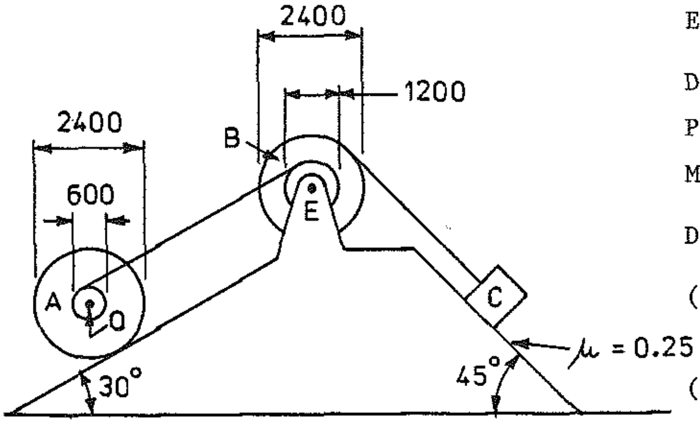


FIG. 19

Assume that the disc A in Fig. 19 rolls without slipping.

Engineering details:-

Disc A, M = 73 kg., k_o = 0.9 m

Pulley B, M = 88 kg., k_o = 0.75 m

Mass of C = 136 kg.

Determine:-

(a) The acceleration of the mass centre of disc A.

(b) The rope tensions.

(a) 1.1965 m/s^2

(b) $T_1 = 395.6$ N

$T_2 = 300.6$ N

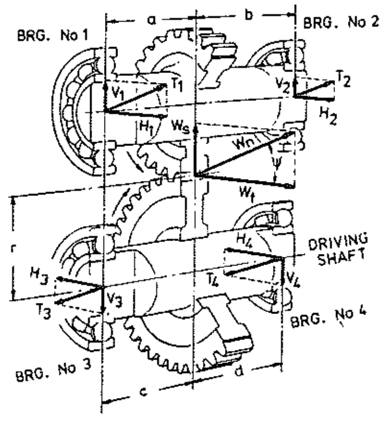
No. 8.

Two spheres A and B are held at the top of a 10° inclined plane.
The sloping length of the plane = 1 m.
Sphere A is solid, 800 mm diameter, mass = 1000 kg and M of I = 64 kg.m^2.
Sphere B is hollow, 800 mm outside diameter, mass = 1000 kg and M of I = 70.85 kg.m^2.

If the spheres are released and assuming pure rolling occurs determine how much sooner sphere A will reach the bottom of the incline. Ans. 0.019 sec.

CHAPTER 15

FRICTION EQUIPMENT



CHAPTER 15
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FRICTION EQUIPMENT

Nature of Friction

When two bodies are pressed together it will be found that there is a resistance offered to sliding of one upon the other. This resistance is called the force of friction.

The force of friction always opposes the direction of motion, or, if the body is at rest, the force tends to prevent motion.

Let a body "A" rest on a plane "B", see Fig. 1. The mutual force perpendicular to the surfaces is "R". Let a force "P" parallel to the surfaces in contact, be applied. If "P" is not large enough to produce sliding, or, if sliding with steady speed takes place, "B" will apply to "A" a frictional force "F" equal and opposite to "P". See Fig. 2.

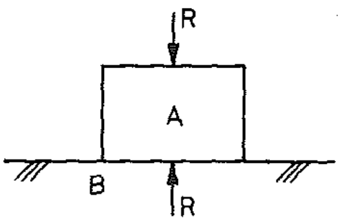


FIG. 1

The force "F" may have any value lower than a certain maximum, which depends on the magnitude of "R" and on the nature and condition of the surfaces in contact. If "P" is less than the maximum value of "F", sliding will not occur; sliding will be on the point of occurring when "P" is equal to the maximum possible value of "F" ("F" is then called limiting friction).

It has been found that the frictional resistance offered after steady sliding has been attained is less than that offered when the body is on the point of sliding.

Refer again to Fig. 2.

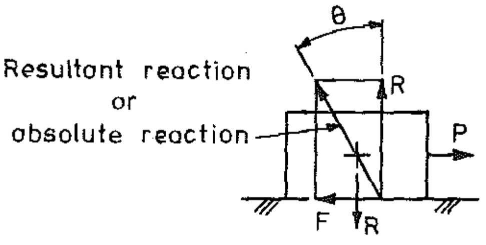


FIG. 2

The angle θ is a maximum when friction has its limiting value. θ is termed the angle of friction or the limiting angle of resistance; the direction of the absolute reaction between the two surfaces cannot be inclined to the normal at an angle greater than the angle of friction.

When steady sliding is occurring θ has a smaller value and is termed the angle of sliding friction.

Let F_s = frictional resistance in N when the body is on the point of sliding.

F_k = frictional resistance in N when steady sliding has been attained.

R = perpendicular force in N between the surfaces in contact.

$$\text{Then } \mu_s = \frac{F_s}{R} \quad \text{and} \quad \mu_k = \frac{F_k}{R}$$

are called respectively the static and kinetic coefficients of friction. It is important to understand that the coefficient of friction reduces at elevated temperatures. This is a significant point in brake design where continuous operation is involved and brake fade is to be minimised.

Angle of repose is the angle to which a plane may be raised before an object resting on it will slide.

Laws of Friction

For dry clean surfaces the laws of Coulomb state:-

(1) The frictional resistance is approximately proportional to the load on the rubbing surfaces.

(2) The frictional resistance is slightly greater for large areas and small pressures than for small areas and large pressures.

(3) The frictional resistance, except for low speeds, decreases as the velocity increases.

For very well lubricated surfaces the following rules approximately apply:-

(1) The frictional resistance is independent of the load.

(2) The frictional resistance is proportional to the speed of rubbing.

(3) For all practical purposes the frictional resistance is independent of the respective materials. Some materials however have better bearing characteristics than others, characteristics such as:-

lubrication adsorbing properties,
compressive strength,
corrosion resistance,
general compatability with mating surface.

For surfaces that are imperfectly lubricated the laws of friction are intermediate between those for dry surfaces and those for very well lubricated surfaces.

Friction on Inclined Planes

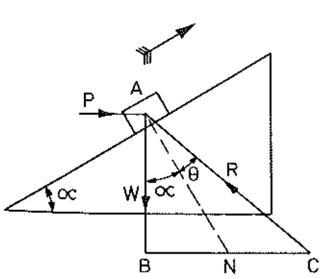


FIG. 3

In Fig. 3. is shown a block of weight "W" sliding steadily up a plane of inclination α to the horizontal, under the action of a horizontal force "P". Draw AN perpendicular to the plane; then the angle between "W" and AN is equal to α.
Draw AC at an angle θ (the angle of sliding friction) to AN; the resultant reaction "R" from the plane will act along line CA.

It can be shown from the triangle of forces that

$$P = W \left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} \right)$$

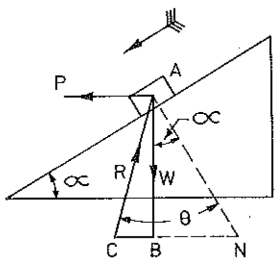


FIG. 4

Fig. 4. shows the case of the block sliding down the plane. "R" acts at an angle θ to AN but this time on the opposite side.

$$P = W \left(\frac{\mu - \tan \alpha}{1 + \mu \tan \alpha} \right)$$

In the last case it will be noted that, if θ is less than α, the block will slide down the plane unassisted. Rest is possible, unaided, if α is equal to the limiting angle of resistance.

When the force "P" is parallel to the plane the forces are as shown in Figs. 5 and 6.

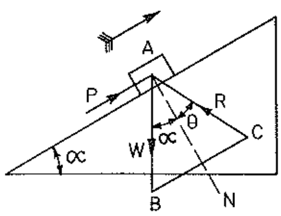


FIG. 5

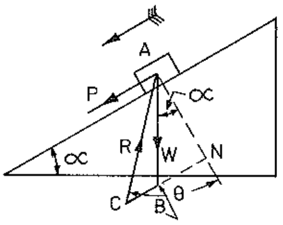


FIG. 6

Fig. 5 for sliding up the plane $\quad P = W (\sin \alpha + \mu \cos \alpha)$

Fig. 6 for sliding down the plane $\quad P = W (\mu \cos \alpha - \sin \alpha)$

Power Screws

Power screws are used to move machine parts against resisting forces. Good examples of this are lifting jacks and lead screws of lathes.

Three design situations occur.:

(1) The nut has axial motion against an applied load while the screw rotates in its bearings. This is the most common design.

(2) The screw rotates and moves axially against the applied load while the nut remains stationary.

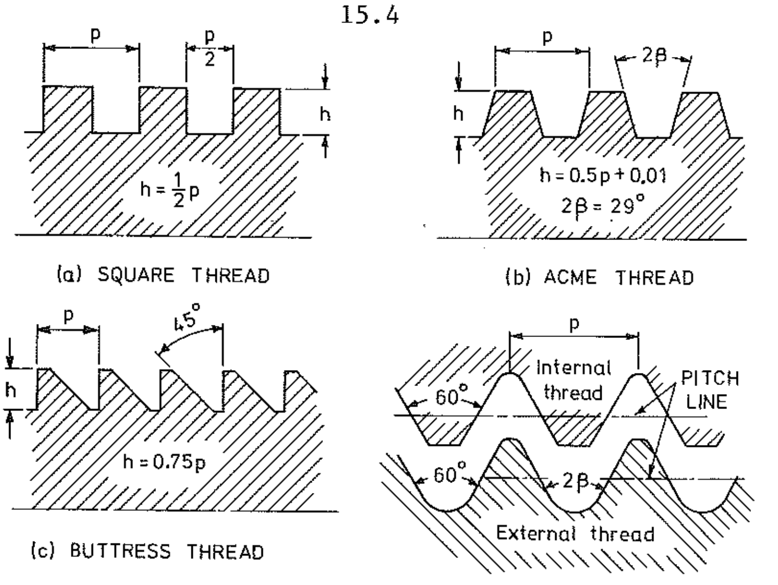
(3) The nut rotates and the screw moves axially with no rotation.

Forms of threads.

Common thread forms are shown in Fig. 7, page 15.4.

The most common thread used in power screws is the square thread.

In the following articles discussion will centre on the square thread although design formulas will also be given for the V thread.



(a) SQUARE THREAD (b) ACME THREAD

(c) BUTTRESS THREAD (d) METRIC THREAD

FIG. 7

Friction of a Screw

A square threaded screw is analogous to a block on an inclined plane and acted on by a horizontal force. The above screw can be regarded as an inclined plane wrapped around a cylinder. A simplification of the analogy is shown in Fig. 8, which is a development of one turn of thread. The force "P" may be assumed as acting at the mean radius of the screw, see Fig. 9.

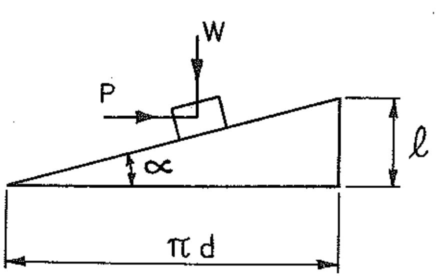


FIG. 8

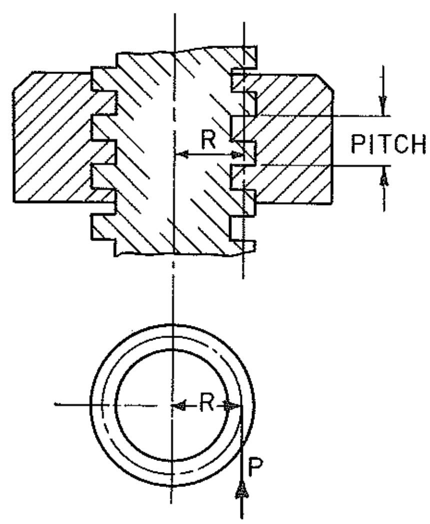


FIG. 9

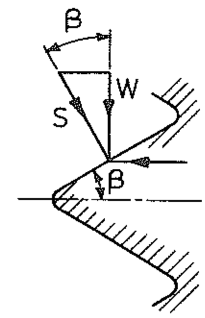


FIG. 10

Glossary of Terms

W = axial load, N

d = diameter of mean helix, mm

α = lead angle or helix angle, degrees

θ = friction angle, degrees

2β = included thread angle, degrees (see Fig. 10.)

μ = coefficient of thread friction = $\tan\theta$

ℓ = lead of thread, mm

T = torque required to overcome thread friction and to move load, N.mm.

(By derivation $T = \frac{Pd}{2}$, refer to Figs. 8 and 9).

Glossary of Terms (Cont.)

T_o = torque required to move load, neglecting friction, N.mm.

η = efficiency of screw.

From Fig. 8, page 15.4, $\tan \alpha = \frac{\ell}{\pi d}$

We have already shown that for raising "W"

$$P = W \left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} \right)$$

By substituting $\frac{\ell}{\pi d}$ for $\tan \alpha$ we get

$$P = W \left(\frac{\pi \mu d + \ell}{\pi d - \mu \ell} \right)$$

$$\therefore T = \frac{Wd}{2} \left(\frac{\pi \mu d + \ell}{\pi d - \mu \ell} \right)$$

Alternatively $T = \frac{Wd}{2} \left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} \right) = \frac{Wd}{2} \tan (\alpha + \theta)$

similarly for lowering "W"

$$T = \frac{Wd}{2} \left(\frac{\pi \mu d - \ell}{\pi d + \mu \ell} \right)$$

OR $T = \frac{Wd}{2} \left(\frac{\mu - \tan \alpha}{1 + \mu \tan \alpha} \right) = - \frac{Wd}{2} \tan (\alpha - \theta)$

A screw becomes non-reversing (self-locking) when the angle of friction is greater than the helix angle.
{The efficiency is not the governing factor e.g. a screw may have an efficiency of less than 50% and be reversible.}

For a non-reversing screw, "T" is positive, indicating that a torque must be applied to the screw to lower the load. For a reversing screw, T is negative, indicating that a torque must be applied to the screw to prevent the load from descending.

The efficiency of a screw

$$\eta = \frac{T_o}{T} = \frac{Wd}{2} \tan \alpha \div T$$

$$= \frac{Wd}{2} \times \frac{\ell}{\pi d} \times \frac{1}{T}$$

$$\therefore \eta = \frac{W \ell}{2 \pi T}$$

An alternative expression for efficiency,

$$\eta = \frac{\ell}{\pi d} \left(\frac{\pi d - \mu \ell}{\ell + \pi \mu d} \right)$$

Angular, or V, Thread Refer Fig. 10, page 15.4.

The normal load on the thread of an angular, or V, type is oblique to the axial load. This means the normal load will be greater than for a square thread.
Hence, the frictional force will be proportionally greater so the friction terms in the previous equations should be divided by cos β. The net result of this adjustment gives the following equations:-

Raising load,

$$T = \frac{Wd}{2}\left(\frac{\pi\mu d \sec\beta + \ell}{\pi d - \mu\ell\sec\beta}\right) \quad \underline{\text{or}} \quad \frac{Wd}{2}\left(\frac{\cos\beta\tan\alpha + \mu}{\cos\beta - \mu\tan\alpha}\right)$$

$$\text{and} \quad \eta = \frac{\tan\alpha\,\{1 - (\mu\sec\beta\tan\alpha)\}}{\tan\alpha + (\mu\sec\beta)} \quad \underline{\text{or}} \quad \tan\alpha\,\frac{(\cos\beta - \mu\tan\alpha)}{(\cos\beta\tan\alpha + \mu)}$$

Lowering load,

$$T = \frac{Wd}{2}\left(\frac{\pi\mu d\sec\beta - \ell}{\pi d + \mu\ell\sec\beta}\right) \quad \underline{\text{or}} \quad \frac{Wd}{2}\left(\frac{\mu - \cos\beta\tan\alpha}{\cos\beta + \mu\tan\alpha}\right)$$

Collar Friction

The load acting on a nut produces an axial force on a screw. This axial force has to be resisted by a collar of some type. This loading action is shown in Fig. 11. The load is assumed to be concentrated at the mean collar diameter d_c.

Let T_c = torque of collar friction

μ = coefficient of collar friction

W = axial load on screw.

$$\text{then } T_c = \frac{\mu W d_c}{2}$$

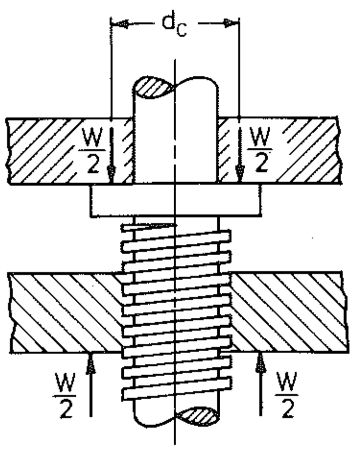


FIG. 11

The overall efficiency of a screw mechanism may be obtained by dividing the torque required to move the load neglecting all friction, by the torque required to move the load, including thread and collar friction.

Coefficient of Friction

Some guidance was given on this point in Chapter No. 7, page 7.25 under the heading "Estimation of coefficient of friction".
However, many practical investigations have been carried out and the results of these are listed in Table No. 1, page 15.7. The value of the coefficient of friction was (in a practical sense) independent of combinations of materials, load and rubbing speed, except under starting conditions.

(Table No 1, refer Ham and Ryan, An Experimental Investigation of Thread Friction of Screw Threads, University of Illinois Bull. 247, 1932.)

TABLE NO. 1.

DESCRIPTION	VALUE OF μ
Threads of top quality workmanship, good materials well run in and lubricated.	0.10
Threads of average quality workmanship and materials. Maintenance reasonable.	0.125
Threads of poor quality workmanship and materials. Newly machined surfaces poorly lubricated. Slow surface speeds.	0.15
Collar friction	As for thread friction.
Starting conditions	1.33 times value for running conditions.

Screw Stresses

Glossary of terms not already covered. Refer to Fig. 12.

d_o = major (outside) diameter of thread, mm.

d_r = minor (root) diameter of thread, mm.

t = width of thread, mm.

n = number of threads in engagement.

S_s, s_s = permissible and actual average shear stress in threads of nut and screw, MPa.

F_b, f_b = permissible and actual average bearing stress between threads, MPa.

L = length of screw between load points, mm.

k = orthogonal radius of gyration of minor diameter of thread = $\frac{d_r}{4}$ mm.

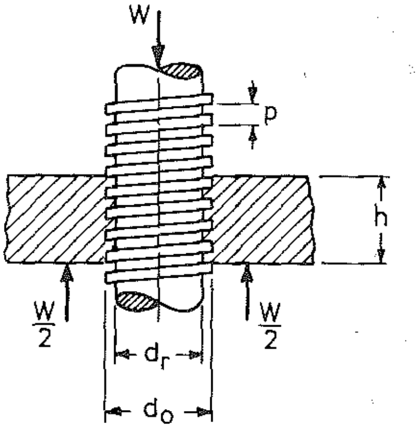


FIG. 12

F_s = maximum permissible shear stress in body of screw when considered as a shaft, MPa.

N = speed of screw in r.p.m.

ω = speed of screw in radians/sec.

p = pitch, mm. (ℓ = p for single start thread only, see Worked example No. 1, page 15.8.)

The following are the possible modes of failure in a screw and nut design.

(1) Combination of the following items as appropriate:-

(a) Direct tensile or compressive stress in body of screw due to load "W".

(b) Column action.

(c) Torsional stress in body of screw from torque required to rotate screw.

(d) Bending stress from load.

All these conditions are covered by adapting the formula for shafts with axial loading from Section 4 of Chapter 2, page 2.12.

$$d_r = \sqrt[3]{\frac{16\sqrt{\{M + \frac{1}{8}\gamma W d_r (1 + A^2)\}^2 + T^2}}{\pi F_s (1 - A^4)}}$$

The process of using this formula is an iterative one.
The slenderness ratio of a screw should be kept reasonably low, 100 being a conservative value where column action is encountered.

(2) Shear of threads in screw.

$$s_s \text{ (screw)} = \frac{W}{n \pi d_r t}$$

(3) Shear of threads on nut.

$$s_s \text{ (nut)} = \frac{W}{n \pi d_o t}$$

(4) Bearing stress between threads.

$$f_b = \frac{4 W}{n \pi (d_o^2 - d_r^2)}$$

Worked Example No. 1.

A square thread and nut are to be used as a tilting mechanism on a food processing machine. Duration of tilting operation = 3/4 hour per day. The distance between the load points on the screw = 1400 mm maximum. Screw speed = 250 r.p.m. The load points can be considered as pin connections.
The load = 2000 kg. The drive end of the screw is in a gear box therefore neglect collar friction.

(a) Choose a screw from Table No. 2, page 15.9, and design the screw and nut mechanism.

(b) Determine the efficiency of the screw.

(c) State whether the screw is reversible.

(d) Determine the power requirements.

(e) Calculate the screw torque required to lower the load.

TABLE NO. 2. Single start square threads.

OUTSIDE DIAMETER	PITCH "p"	ROOT DIAMETER
48	10	38
50	12	38
56	12	44
65	14	51
75	15	60
90	16	74
100	18	82

The term "square thread" refers to the fact that the side of the thread is square with the root. The width of a square thread does not necessarily have to equal the depth.

Answer.

(a) First check the screw body. (Item 1 under "Screw Stresses", page 15.8.)

$$\frac{L}{k} = 100$$

$$\frac{1400}{k} = 100 \qquad \therefore \quad k = 14 \text{ mm}$$

Root diameter = 14 x 4
= 56 mm.

> Note:-
> The method of drive may have an effect on some aspects of the design.
>
> Starting torque, design torque and safety factor.
>
> Refer to page 2.11.

Choose a screw 75 mm outside diam.
60 mm root diam.

$\therefore$ Mean diam. = (75 + 60) ÷ 2 = 67.5 mm

For newly machined surfaces

μ = 0.15 (Table No. 1, page 15.7.)

$$\tan\alpha = \frac{\ell}{\pi d} = \frac{15}{\pi 67.5} = 0.0707$$

Note.
For a single start screw ℓ = p
For a double start screw ℓ = 2p
For a triple start screw ℓ = 3p
Etc. etc.

For raising the load

$$T = \frac{Wd}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu \tan\alpha}\right)$$

$$\therefore T = \frac{2000\ g \quad 67.5}{2} \left(\frac{0.0707 + 0.15}{1 - 0.0707 \times 0.15}\right)$$

$$= 147\ 708.5 \text{ N.mm}$$

From Chapter 2, Section 4, page 2.12, also page 2.8.

$$\frac{L}{k} = \frac{1400 \times 4}{60} = 93.3$$

$$\gamma = \frac{1}{1 - (0.0044\ \frac{L}{k})}$$

$$= \frac{1}{1 - (0.0044 \times 93.3)} = 1.7$$

Use CS 1030 material, endurance limit = 225 MPa. (Refer page 2.9)

$$\therefore F_s = \frac{225}{2 \times 2} = 56.25 \text{ MPa}$$ (Stress is essentially reversing because torque is reversing.)

$$d_r = \sqrt[3]{\frac{16\sqrt{(M + \frac{1}{8}\gamma W d_r (1 + A^2)^2 + T^2}}{\pi\ F_s\ (1 - A^4)}}$$

Substituting the appropriate values and omitting the terms not relevant

$$d_r = \sqrt[3]{\frac{16\sqrt{(\frac{1}{8} \times 1.7 \times 2000g \times 60)^2 + 147\ 708.5^2}}{\pi\ 56.25}}$$

$$= 29.7 \text{ mm} < 60 \text{ mm}$$

Therefore the screw body design is safe.

Now check shear stress in screw thread.
(Item 2 under "Screw Stresses", page 15.8.)
Material of screw to be CS 1030 F_y = 250 MPa. (Refer page 2.9)
Use a safety factor of 2.0 on yield and apply the maximum shear stress theory:-

$$S_s = \frac{250}{2 \times 2} = 62.5 \text{ MPa}$$ (Stress is essentially static.)

Make nut 75 mm long
Pitch of screw = 15 mm (From Table No. 2, page 15.9.)

$$\therefore n = \frac{75}{15} = 5 \quad \text{and} \quad t = \frac{15}{2} = 7.5 \text{ mm}$$

$$s_s = \frac{W}{n\pi d_r\ t} = \frac{2000\ g}{5\pi 60 \times 7.5} = 2.8 \text{ MPa} < 62.5 \text{ MPa}$$

Now check shear stress in threads of nut. (Item 3 under "Screw Stresses", page 15.8.) Use phosphor bronze. Three grades would be suitable:-

AS 1565	C90250	230 MPa Ult. 130 MPa Yield	C90710	220 MPa Ult. 130 MPa Yield	C90810	220 MPa Ult. 130 MPa Yield

Try AS 1565 C90810. Use a safety factor of 2.0 on yield and apply the maximum shear stress theory:

$$S_s = \frac{130}{2 \times 2} = 32 \text{ MPa} \quad \text{(Stress is essentially static.)}$$

$$s_s = \frac{W}{n \pi d_o t} = \frac{2000 \; g}{5 \; \pi \; 75 \quad x \quad 7.5} = 2.2 \text{ MPa} < 32 \text{ MPa}$$

(Alternative approach. As the percentage elongation of the above cast phosphor bronzes is only 2% to 9%, (reference AS 1565 - 1985,) as compared with 20% for mild steel, some authorities would recommend design for brittle mode failure.

This means that the ultimate tensile strength, together with a safety factor of between 4 and 6, would have to be used as a basis for the determination of working stresses.)

Now check bearing stress between threads. (Item 4 under "Screw Stresses", page 15.8.)

$$f_b = \frac{4W}{n \; \pi \; (d_o^2 - d_r^2)} = \frac{4 \times 2000 \; g}{5 \; \pi \; (75^2 - 60^2)} = 2.5 \text{ MPa}$$

By reference to Tables No. 1 & 2 in Chapter 7, pages 7.22-23, the allowable pressure for a class 2 mechanism (3/4 hour/day operation) can be estimated.

$$\begin{aligned} \text{Rubbing speed} = V &= \pi \; d \; N \\ &= \pi \times 0.0675 \quad x \quad 250 \\ &= 53 \text{ m/min.} \end{aligned}$$

The allowable pressure F_b is between 2.7 and 3.2 MPa. (Refer page 7.23)

The actual pressure f_b is only 2.5 MPa.

(b) $$\eta = \frac{W\ell}{2 \; \pi \; T} = \frac{2000 \; g \quad 15}{2 \; \pi \; 147 \; 708.5}$$

$$= 0.3171 = 31.71\%$$

(c) $\text{Tan } \propto \; = \; 0.0707 < \tan \theta \; = \; 0.15$

As friction angle is greater than the helix angle the screw is non-reversible.

(d) $$P = \frac{T \; \omega}{1000} = \frac{147 \; 708.5 \quad x \quad 2 \; \pi \; 250}{1000 \quad x \quad 1000 \quad x \quad 60} = 3.87 \text{ kW}$$

(e) $$T = \frac{Wd}{2} \left(\frac{\mu - \tan \propto}{1 + \mu \tan \propto} \right)$$

(e) (Cont.)

$$= \frac{2000\ g\ \ 67.5}{2} \quad \left(\frac{0.15 - 0.0707}{1 + 0.0707 \times 0.15}\right)$$

$$= 51\ 959.5 \text{ N.mm}$$

i.e. a torque must be applied to lower the load.

BRAKES

General

Brakes are units for arresting motion. The action involves rubbing friction and the resultant loss of system energy is dissipated in the brake mainly as heat.

The location of the pins and levers in a brake assembly gives the brake its particular characteristics leading to the following terminology:-

Self energising (servo positive)
Servo negative
Self-locking
Simple, two way and differential band brake.

In all types of pivoting arm brakes the pin reactions can be readily attained from an analysis of a free body diagram of the arm.

Discussion will be limited to a few of the more common types of brakes encountered in industry.

Single block brake Refer to Figs. 13 and 14 and Table No. 3, page 15.13.

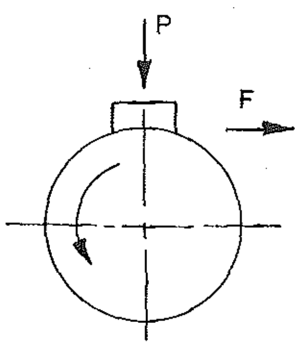


FIG. 13

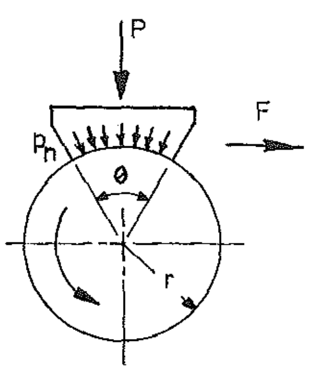


FIG. 14

Glossary of terms

P = operating force on block in radial direction, N

T = torque being applied, N.m

F = tangential frictional force, N.

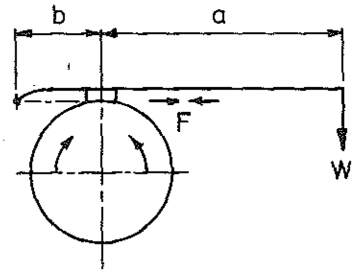
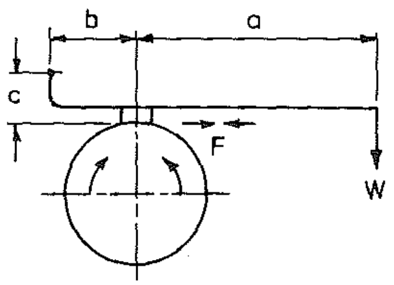
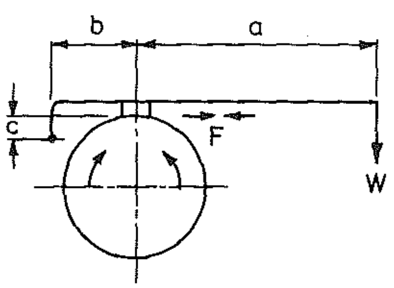
r = radius of drum, mm.

θ = angle of contact in radians.

H = heat generated in one brake application, kJ/s

μ = coefficient of friction for materials of block and drum.

TABLE N° 3

BLOCK BRAKES	CLOCKWISE ROTATION	ANTI-CLOCKWISE ROTATION	REMARKS
	$W = \frac{Fb}{a+b}\left(\frac{1}{\mu}\right)$	$W = \frac{Fb}{a+b}\left(\frac{1}{\mu}\right)$	Condition is the same for rotation in either direction.
	$W = \frac{Fb}{a+b}\left(\frac{1}{\mu} + \frac{c}{b}\right)$ Unfavourable. Effect of "c" is to increase "W" or "a"	$W = \frac{Fb}{a+b}\left(\frac{1}{\mu} - \frac{c}{b}\right)$ Favourable. Effect of "c" is to reduce "W" or "a"	Brake is self-locking in one direction when $\mu c \geqslant b$. In the other direction of rotation the effect of "c" is for brake to lift off drum.
	$W = \frac{Fb}{a+b}\left(\frac{1}{\mu} - \frac{c}{b}\right)$ Favourable. Effect of "c" is to reduce "W" or "a"	$W = \frac{Fb}{a+b}\left(\frac{1}{\mu} + \frac{c}{b}\right)$ Unfavourable. Effect of "c" is to increase "W" or "a"	

Glossary of terms (Cont.)

μ' = equivalent coefficient of friction

ω = drum speed at start of brake application, rad/s

w = width of braking face, mm.

V = peripheral velocity of drum, m/s, at start of brake application.

p_n = normal pressure between brake drum and block, MPa

W = force on end of operating lever, N.

A single block brake consists of a block which is forced against a rotating drum by a lever. The contact pressure distribution may be regarded as simple for θ less than 60° to complex for θ = 60° or greater. Table No. 3. lists formulas for the simple condition.

Where θ is less than 60° it may be assumed that pressure is uniformly distributed over the contact faces.

$$F = \mu p_n w r \theta = \mu P; \quad H = \frac{\mu PV}{1000} = \frac{T\omega}{1000}$$

Where θ is = 60° or greater the force analysis is a complex exercise in applied mechanics. The net result is

$$F = \frac{4 \mu P \sin \frac{\theta}{2}}{\theta + \sin \theta}$$

In the above equation, the quantity

$\mu \left(\frac{4 \sin \frac{\theta}{2}}{\theta + \sin \theta} \right)$ may be termed the "equivalent coefficient of friction" and

denoted μ', hence

$$F = \mu' P; \quad H = \frac{\mu' PV}{1000} = \frac{T\omega}{1000}$$

Generally unit pressure is less at the ends than the centre with a complex single block brake. Table No. 3, page 15.13, applies also to the complex brake as long as μ is replaced by μ'.
An approximate method for determining pressure distribution on a rigidly mounted shoe is given in Worked Example No. 4, page 15.40.

Double block brake

Glossary of additional terms

D = brake drum diameter, mm.

T = torque, N.m

S = spring force, N.

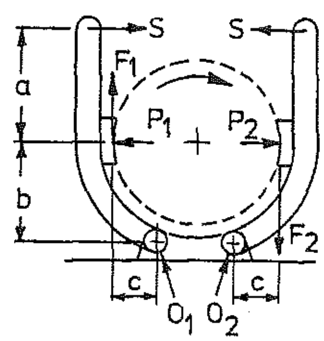


FIG. 15

Fig. 15 shows one form of a two shoe (double-block) spring-set brake. A spring suitably arranged provides the force "S" which applies the brake. A solenoid, thrustor or other suitable unit is required for releasing the brake and holding it in the "off" position.

$$H = \frac{\mu \text{ (or } \mu') (P_1 + P_2) V}{1000} = \frac{T\omega}{1000}$$

Sometimes the blocks (brake shoes) are pivoted on the operating yokes to provide uniform contact between the lining and the wheel. The tangential frictional force on the shoe applies a moment to the shoe which in turn gives a different contact pressure distribution to that of a solid block. However, if the pivot is placed as close as possible to the brake lining only a small moment is introduced and the pressure distribution difference is small enabling the previous formulas for "F" to be used satisfactorily.

The following formulas apply to Fig. 15, page 15.14:-

$$\Sigma M_{01} = S(a + b) + F_1 c - P_1 b = 0$$

$$\Sigma M_{02} = S(a + b) - F_2 c - P_2 b = 0$$

$$\frac{F_1}{P_1} = \frac{F_2}{P_2} = \mu \text{ (or } \mu' \text{)}$$

$$T = \frac{F_1 + F_2}{1000} \quad \frac{D}{2}$$

If the brake always operates in the one direction lining wear will be uneven because the values of F_1 and F_2 are different. If pivot points 0_1 and 0_2 are located on the lines of action of F_1 and F_2 lining wear will be equal. The reason for having the pivot points as shown is to enable a smaller spring force to be used.

Drum size

In designing a block brake it is usual to have a drum diameter in proportion to the associated equipment. Should the brake drum be attached to an electric motor then the centre line height of the brake should be about the same as that of the motor. A suitable drum diameter will automatically result in most cases.

After the forces P_1 and P_2 (see Fig. 15, page 15.14) have been calculated, the lining width may then be determined from the "pV" value (see "Power rating of friction materials", page 15.22).

Drum ratios ($\frac{W}{D}$) should lie between ¼ and ½ otherwise a new diameter should be chosen.

If the ratio is less than ¼ the yoke or shoe may lack lateral stability because of being too narrow. If the ratio is greater than ½ it may be difficult to obtain a reasonably uniform contact pressure between linings and drum. This will lead to uneven lining wear; also bad cooling will result.

Brake shoe or yoke section

A brake shoe (or yoke in the case of a pivoted shoe) is for practical design purposes a curved beam. If the radius of curvature at the neutral axis is more than 10 times the depth of the "beam" (shoe or yoke) no great error is involved in using the straight beam formulas. However, where curvature is sharp the error is significant. In this case formulas for curved beams can be taken from books like "Formulas for Stress and Strain" by Roark.

Band Brakes

Glossary of Terms

T_1 = band tight side tension, N

T_2 = band slack side tension, N

Glossary of Terms (Cont.)

F = tangential frictional force, N, = $(T_1 - T_2)$

T = drum torque, N.m

D = drum diameter, m

r = drum radius, m

V = peripheral velocity of drum, m/sec, at start of brake application.

ω = drum speed, radians/sec $= \frac{2 \pi N}{60}$

N = drum speed, r.p.m. (at start of brake application)

θ = angle of wrap, radians

μ = coefficient of friction, lining to drum.

P = power, kW

w = width of brake band, mm.

p = normal contact pressure between lining and drum in MPa (either average or maximum).

A_c = area of contact of lining, mm^2.

H = heat generated in one brake application, kJ/sec.

n = number of blocks in a multi-block band brake.

θ_1 = semi-angle of wrap, degrees.

W = force on end of operating lever, N

Band brakes, or devices employing friction on the same principle, are as old as engineering. The self-wrapping or energising property of these brakes is similar to that of a rope round a rotating capstan or a lashing round a spar. In each case, the brake band or rope has a magnifying effect on the grip exerted on drum, capstan or spar, provided that the torque which is to be resisted acts in the same direction as the operating pull on the band or rope. Conversely, if the torque opposes the operating force, the band brake will be correspondingly ineffective. It is usual practice to arrange a band brake so that self-energisation augments brake torque in a unidirectional brake. This magnification or self-energising property is evident as soon as torque is present with or without relative motion in the mechanism; i.e., a band brake is just as effective under static as kinetic conditions.

The band brake consists of a flexible steel band lined with friction material, the latter usually having an arc of embrace of the order of 270°. Tightening or slackening of the band on to or away from the rotating member may be carried out by many different mechanical arrangements, of which space does not permit detailed descriptions or appraisals.

Band brakes are, in general, positive in operation and can be prone to such troubles as grabbing and judder. Many of these conditions arise from mechanical causes which can be avoided at the design stage. Heavy loads can be applied to band brake anchorages and any weakness at these points may lead to flexing and distortion. Such features react unfavourably on the brake performance if they cause locking of the anchor pins or band distortion.

Friction linings tend to reduce band flexibility to a degree which depends upon the type of lining used. A cotton-base type, or a semi-flexible asbestos-base material, may be mounted in continuous strips, but the more rigid moulded types should be mounted in short segments with a small gap between each, which will help to preserve the flexibility of the steel band.

When the band is tightened on to the rotating member, the friction between them provides the tangential braking force F. Owing to the direction of rotation of the rotating member, the tension in the band varies between the ends and these tight and slack tensions are denoted by T_1 and T_2 respectively.

It can be shown that the limiting ratio between these tensions is given by the formula $\frac{T_1}{T_2} = e^{\mu\theta}$ and that by observation of Figs. 16 and 17, $T_1 = T_2 + F$.

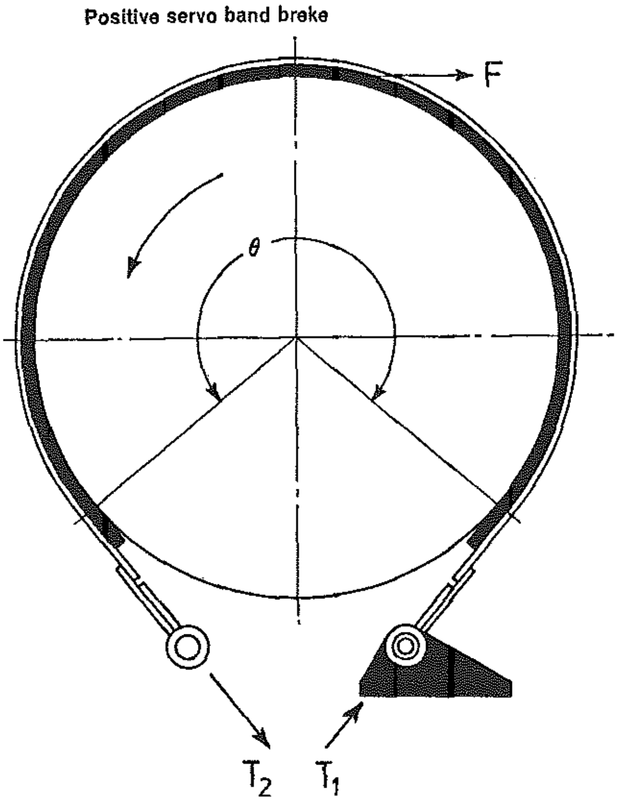


FIG. 16

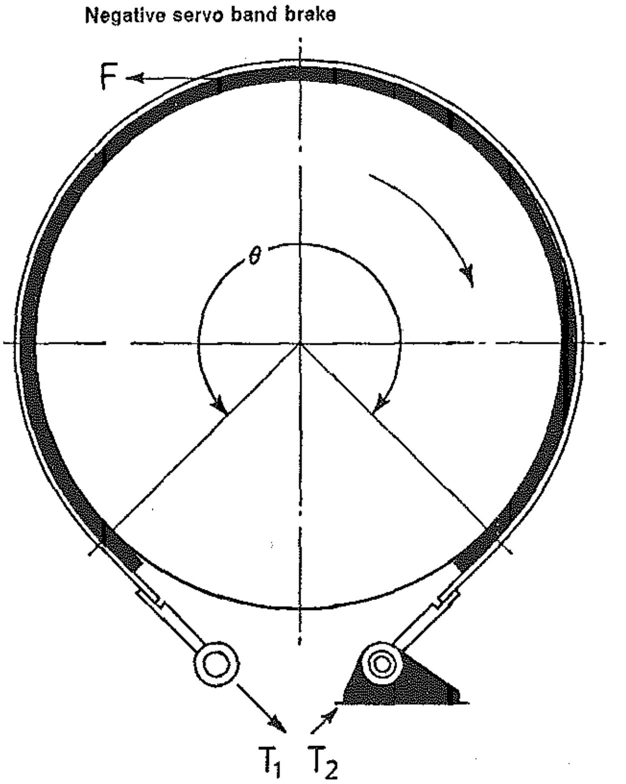


FIG. 17

A combination of these expressions gives a further two expressions which are of more value in the design of band brakes, namely

$$T_1 = \frac{F \times e^{\mu\theta}}{e^{\mu\theta} - 1} \quad \text{and} \quad T_2 = \frac{F}{e^{\mu\theta} - 1}$$

A study of the above formulae will show that it is advantageous wherever possible to make the tight side (T_1) the fixed end, and the slack side (T_2) the operating end, where the force required to actuate the brake is at a minimum. When the above conditions are applied the brake is termed a positive-servo one, but when the tight side (T_1) is the operating end the brake is termed negative-servo.

TABLE NO. 4.

VALUES OF $e^{\mu\theta}$

θ degrees	μ = 0.5	μ = 0.4	μ = 0.3	μ = 0.2	μ = 0.1
30	1.30	1.23	1.17	1.11	1.05
60	1.69	1.52	1.37	1.23	1.11
90	2.19	1.88	1.60	1.37	1.17
120	2.85	2.31	1.88	1.52	1.23
150	3.70	2.85	2.19	1.69	1.30
180	4.81	3.51	2.57	1.88	1.37
210	6.25	4.33	3.00	2.08	1.44
240	8.13	5.34	3.51	2.31	1.52
270	10.55	6.59	4.11	2.57	1.60
300	13.72	8.12	4.81	2.85	1.69
330	17.82	10.00	5.63	3.16	1.78
360	23.16	12.34	6.59	3.51	1.87

Table No. 4. gives values of $e^{\mu\theta}$ for various angles and various coefficients of friction in order to facilitate calculation.

Table No. 5, page 15.19, shows details of the various types of band brakes and gives formulas for operating force. The alternative procedure in band brake analysis (preferred by many designers) is the fundamental approach. This is done by taking moments about the pivot point on a free body diagram of the operating arm; see Worked Example No. 5, page 15.42.

Table No. 5, page 15.19, however gives a good pictorial representation of band brake geometry.

Case 1 & 2.

Simple band brake. Cannot be designed as self-locking.

Case 3.

Differential band brake. This brake can be made self-locking for one direction of rotation only i.e., it can be designed to allow motion in one direction only.

A self-locking band brake requires a negative force on the operating arm to release it i.e., a force in the opposite direction to normal. After the brake has locked, the band tensions T_1 and T_2 will increase and as the band is no longer on the point of slipping the formula

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{cannot be used.}$$

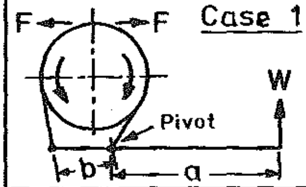
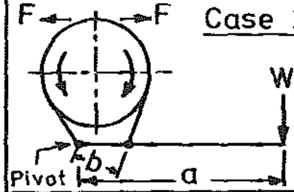
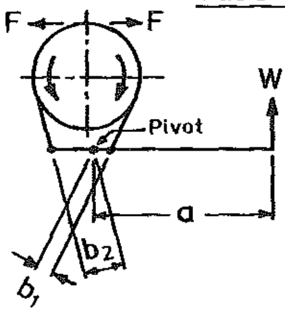
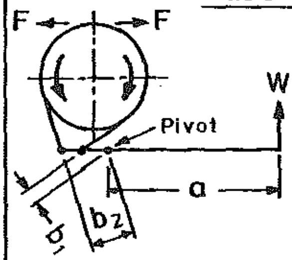
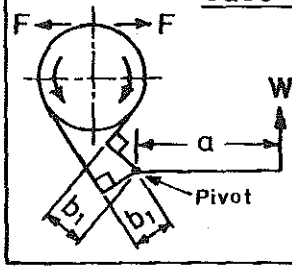
In problems involving "case 3" type differential band brakes, first check to see whether or not the brake is self-locking.

If it is then do not use $\frac{T_1}{T_2} = e^{\mu\theta}$ for that direction of rotation.

Case 4.

Differential band brake.

TABLE N° 5

APPLICATION	CLOCKWISE ROTATION	ANTI-CLOCK ROTATN	REMARKS
Case 1 	$W = \frac{Fb}{a}\left(\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right)$ Unfavourable	$W = \frac{Fb}{a}\left(\frac{1}{e^{\mu\theta} - 1}\right)$ Favourable	
Case 2 	$W = \frac{Fb}{a}\left(\frac{1}{e^{\mu\theta} - 1}\right)$ Favourable	$W = \frac{Fb}{a}\left(\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right)$ Unfavourable	
Case 3 	$W = \frac{F}{a}\left(\frac{b_2 e^{\mu\theta} - b_1}{e^{\mu\theta} - 1}\right)$ Unfavourable	$W = \frac{F}{a}\left(\frac{b_2 - b_1 e^{\mu\theta}}{e^{\mu\theta} - 1}\right)$ Favourable, but care must be taken to prevent self-locking.	If $b_2 \leqslant b_1 e^{\mu\theta}$ W is 0 or negative & brake is either self-locking or requires a negative force at W to prevent self-locking.
Case 4 	$W = \frac{F}{a}\left(\frac{b_2 e^{\mu\theta} + b_1}{e^{\mu\theta} - 1}\right)$ Unfavourable	$W = \frac{F}{a}\left(\frac{b_1 e^{\mu\theta} + b_2}{e^{\mu\theta} - 1}\right)$ Favourable	
Case 5 	If $b_2 = b_1$ the same force W is required for rotation in either direction and $W = \frac{Fb_1}{a}\left(\frac{e^{\mu\theta} + 1}{e^{\mu\theta} - 1}\right)$ Case 5		

Case 5.

When the band brake of "case 4" has $b_2 = b_1$ it becomes a two-way band brake. This type of band brake functions equally well for either direction of rotation because the moment arms of the tight and slack tensions about the pivot point are equal.

General notes on band brake design.

The maximum angle of wrap from a practical standpoint is 270°.

The best material for the band is spring steel. For example, a holding band brake 760 mm diameter could have a 75 mm x 3 mm spring steel band.

The width of a band should be about one-seventh of the diameter maximum.

Further formulas applicable to band brake design are:-

$$T = (T_1 - T_2)\, r$$

For a "V" band brake $\dfrac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec}\beta}$ where

$$\beta = \text{semi-angle of the "V"}$$

$$P = \frac{(T_1 - T_2)\, V}{1000} = \frac{T\omega}{1000}$$

Maximum normal pressure developed between contact surfaces

$$p_{max} = \frac{T_1}{1000\ wr}$$

Average normal pressure developed between contact surfaces

$$p_{av} = \frac{T_1}{1000\ wr\ \mu\ \theta} \left(\frac{e^{\mu\theta} - 1}{e^{\mu\theta}} \right)$$

This pressure is used in the "heat generated" calculation.

Suitable drum diameters usually fall between the following limits although many satisfactory designs are in operation which are outside this range.

$$\sqrt[3]{\frac{T}{34\ 500}} < D < \sqrt[3]{\frac{T}{27\ 600}}$$

Another relationship offered in deciding a drum diameter is

$$\left(\frac{fkW}{760} \right) < D < \left(\frac{fkW}{570} \right)$$

Where fkW is the maximum power to be dissipated in any 15 minute period.

The heat generated during the application of a brake is given by

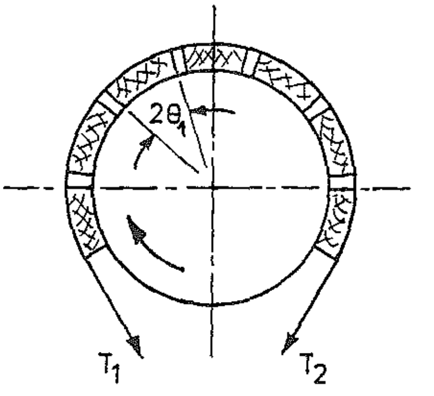
$$H = \frac{p_{av}\ A_c\ V\ \mu}{1000}$$

Multi-block band brake.

Fig. 18, page 15.21, shows a slight variation to the normal band brake. This design consists of a number of blocks attached to a band.

The following formula is applicable to this condition:-

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan \theta_1}{1 - \mu \tan \theta_1}\right)^n$$



MULTI BLOCK BAND BRAKE

FIG. 18

Power rating of friction materials

Table No. 6 gives a guide on "μ" values for various facing materials and their allowable unit pressures.

TABLE NO. 6. Design values for brake linings (traditional values).

FACING MATERIAL		DESIGN VALUES OF μ	PERMISSIBLE AVERAGE UNIT PRESSURE "p_{av}" IN MPa.	
			V = 1 m/sec	V = 10 m/sec
C.I. on C.I.	Dry	0.20	0.34	
	Oily	0.07	0.34	
Wood on C.I.	Dry	0.25-0.30	0.55-0.69	0.14-0.17
	Oily	0.08-0.12	0.55-0.69	0.14-0.17
Leather on C.I.	Dry	0.40-0.50	0.06-0.10	
	Oily	0.15	0.06-0.10	
Asbestos fabric on metal	Dry	0.35-0.40	0.62-0.69	0.17-0.21
	Oily	0.25	0.62-0.69	0.17-0.21
Moulded asbestos on metal.	Dry	0.30-0.35	1.03-1.21	0.21-0.28

Heating of brake facing. p_{av} V ratings (traditional values).

Recommended limits are:-

pV = 1.0 for continuous service with short rest periods and poor radiation.

pV = 2.0 for intermittent operation with long rest periods and poor radiation values, e.g. wood blocks.

pV = 3.0 for continuous operation and with good radiation as with an oil bath.

Limiting values for applied effort

Hand operated brakes - not greater than 155 N

Foot operated brakes - not greater than 310 N

The following information on friction materials has been supplied from the Ferodo Ltd. publication "Friction Materials for Engineers" and is intended as a guide in general design work. Designers are referred to the above text for more advanced work.

Three types of organic brake and clutch linings are in general use.

(1) Woven cotton

Such linings are essentially non-metallic, and are used fairly extensively in the engineering field, where advantage is taken of their high friction. In many instances they are employed to replace wood and leather on very old brakes and clutches still in use.

Further, they offer a good compromise where mating surfaces are not sufficiently obdurate for asbestos-based linings. They can only be used in applications where comparatively low temperatures are encountered as their temperature range is limited.

(2) Solid-woven asbestos linings

Linings containing asbestos still remain possibly the best type of material for high duty brakes and clutches.

(3) Moulded linings

These can be made of asbestos based material, rigid, flexible or semi-flexible.

Power ratings of friction materials (Ferodo types)

Refer to Table No. 7, page 15.23.

The power rating used is a mean value calculated from the mean speed during the application. At the beginning of a brake application (if the vehicle or machinery is brought to a standstill) the instantaneous power rating will be double the quoted mean value, falling to zero at the end of the application. Similarly, the pressure utilised is the mean effective pressure on the friction material and takes no account of the facts that the pressure will vary substantially from the tip to the heel of a shoe and that the mean pressure between leading and trailing shoes will be substantially different because of their different degrees of self-energisation. The power rating utilised, therefore, is an overall average of all these factors.

Table No. 7, page 15.23 indicates approximate ratings for different applications; it must be emphasised that these are based on general experience. However, in the case where a brake or clutch is operating satisfactorily, this may prove a better guide for design for similar duty than the general figures given in the table.

TABLE NO. 7.

THE POWER RATING OF ORGANIC FRICTION MATERIALS FOR INDUSTRIAL APPLICATIONS.

Type of duty	Cooling Conditions	Typical applications	Mean rating kW/m² Drum Brakes	Plate clutches and plate type disc brakes	Cone clutches
Intermittent duty or infrequent full duty applications	Time between applications permits the assembly to cool to the ambient temperature before each application	Emergency brakes, safety brakes, torque limiting clutches, safety clutches	1734	578	770
Normal intermittent duty	Time between applications permits some cooling to take place between applications but the residual bulk temperature builds up to a moderate level with successive applications	All general duty applications - winding engines, cranes, winches, lifts	578	230	385
Heavy duty where life is critical and applications frequent	Frequency of applications too high to permit any appreciable cooling between applications	Excavators, presses, drop stamps, haulage gear	290 or less according to application	115	230

Allowable lining pressures, self-servo factor and stability.

Refer to Table No. 8, page 15.24.

The torque output of a brake or clutch obviously depends on the operating force providing the load on the linings. In addition it depends upon the geometrical construction of the brake or clutch and in particular the shoe arrangement. A brake which gives a very high torque capacity for a low operating force is said to have a high self-servo factor, as the extra torque is obtained by virtue of some self-wrapping action under the frictional loads.

The self-servo factor may be considered as unity for two flat planes rubbing together where frictional drag is directly proportional to the load applied, and any variation in the friction coefficient has a directly proportionate effect on the drag.

Where a brake has a high self-wrapping action, any frictional change or mechanical distortion has an exaggerated effect upon the torque output, so that a low energising force can be adopted only at the expense of the stability of the brake.

Conversely, if large operating forces are available the brake can be designed to minimise frictional changes and give exceptional stability. The designer must choose, in accordance with the requirements of the particular application, a type of brake which will give him the required compromise between self-servo action and stability. Table No. 8 gives an approximate rating of different types of brake from this viewpoint, together with the angle of wrap of the lining around the drum which is generally accepted as giving a satisfactory design.

TABLE NO. 8.

SELF-SERVO FACTOR AND STABILITY

Type of brake or clutch	Self-servo factor	Stability	Angle of wrap	Mean lining pressure range MPa
Band brakes operating with the rotation of the drum	Very high	Very low	270°	0.07 - 0.7
Drum brakes with two leading shoes	High	Low	90-110° per shoe (2 shoes)	0.07 - 0.7
Drum brakes with one leading and one trailing shoe	Moderate	Moderate	90-110° per shoe (2 shoes)	0.07 - 0.7
Suspended shoe drum brakes	Unity	Unity	90-110° per shoe (2 shoes) for drum brakes	0.07 - 0.7
Plate clutches and plate-type brakes	Unity	Unity	Not applicable	0.07 - 0.35
Spot-type disc brakes	Unity	Unity	Not applicable	0.35 - 1.7
Drum brakes with two trailing shoes	Low	High	90-110° per shoe (2 shoes)	0.07 - 0.7
Band brakes operating against the rotation of the drum	Very low	Very high	270°	0.07 - 0.7
Cone clutches	Not applicable	Not applicable	360°	0.07 - 0.35

Rivet holding capacity

Riveting is still the most widely used method for the attachment of friction linings. Semi-tubular rivets are most commonly employed, with flat or countersunk heads (150°), in both brass and copper.

Aluminium rivets are used only to a small degree, these having a solid shank. They are easy to clench but have undesirable scoring propensities when in contact with ferrous mating surfaces.

Copper rivets are more easily clenched than brass, and while copper has the lower physical strength it is considered adequate, bearing in mind that its strength is several times greater than that of any asbestos-based friction material.

The number and distribution of rivets required to ensure that the lining is kept in intimate contact with the shoe is usually appreciably above that required for adequate rivet holding strength. If the layout is designed to ensure that the lining is held firmly to the shoe it is therefore usually more than adequate from a strength point of view. There will always be exceptions to the rule, which must each be considered individually.

The following proportions are current practice in the automotive and industrial fields.

TABLE NO. 9.

LINING AREAS PER RIVET

	Lining thickness mm	Diameter of rivet mm	Lining area per rivet mm^2		
Industrial	6	5 and 6	1775	to	2580
	10	6 " 8	2910	"	3875
	12	6 " 8	3230	"	4200
	19	10	5160	"	9040

Table No. 9 shows that as the lining thickness increases, the area of lining per rivet also increases. The reason is that a larger diameter of rivet is employed pro rata to the lining thickness, with a correspondingly increased amount of lining material under the rivet head.

Coefficient of friction

μ values for the cotton and asbestos materials mentioned at the outset of this section range from 0.35 to 0.5 in the majority of cases. This is for dry ambient conditions.

Under wet conditions the value may drop to about 0.15.

Refer to the Manufacturer for the exact value applicable to the chosen material.

Energy dissipation capacity of brakes.

The main functions of a dynamic brake (as opposed to a holding brake) are to arrest motion and convert the mechanical energy into heat and dissipate this heat without overheating of the brake linings.

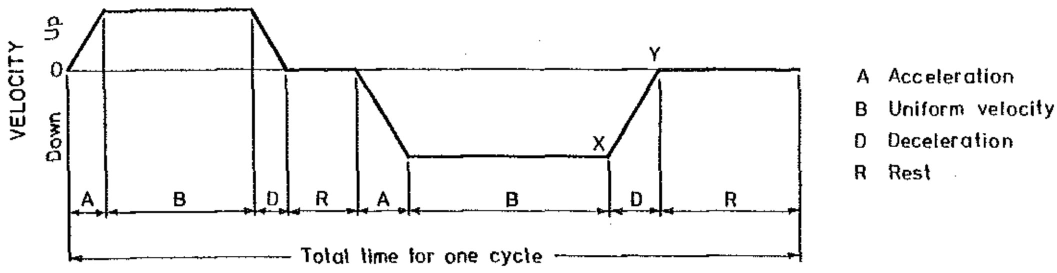
To correctly design a brake we need to determine:-

(1) The energy capacity requirement.

(2) The torque requirement.

The total energy that a brake must absorb equals the total energy of the moving parts of the mechanical system minus frictional losses in the elements of the system.

Consider the simple operation of a hoist raising and lowering a load.



HOISTING AND LOWERING CYCLE

FIG. 19

Fig. 19 gives a graphical presentation of the operation.
The most severe requirement of the brake occurs at XY as the weight is being lowered and brought to rest.

GLOSSARY OF TERMS

M = mass being lowered, kg.

v = initial linear velocity of load, m/s

E_t = total energy to be absorbed by brake, N.m.

T = torque on brake drum, N.m.

t = time of application of brake, seconds.

N = initial speed of brake drum, r.p.m.

θ = angle through which brake drum turns during time t, radians.

I = polar mass moment of inertia of each rotating part, kg.m^2

ω = angular velocity of each rotating part, radians/sec.

The initial kinetic energy of the load is

$$E_k = \frac{Mv^2}{2}$$

The change in potential energy of the load is

$$E_p = \tfrac{1}{2} M\, gvt$$

The initial kinetic energy of rotation of all the rotating parts such as gears, shafts, drum, rotor of motor etc, etc, is

$$E_r = \tfrac{1}{2} I\, \omega^2$$

The sum of the three above equations gives the total energy to be absorbed by the brake.

$$E_t = E_k + E_p + E_r$$ (Efficiencies should be taken into account when using this formula; Refer to Exercise 11, page 15.50.)

This energy will equal the work done by the brake during the time of its application.

$$E_t = T\theta$$

$$\therefore\quad T = \frac{E_t}{\theta} \qquad \text{now} \qquad \theta = \frac{2\pi Nt}{2 \times 60}$$

$$\therefore\quad T = \frac{60\, E_t}{\pi\, Nt}$$

For further techniques in brake selection refer to page 15.53 and Chapter 14.

CLUTCHES

Clutches, like couplings,are used to connect co-axial shafts, the difference being that while couplings establish a permanent connection, clutches make it possible to disconnect or connect the driven shaft while the driving shaft is continuously rotating.

They are used mostly where it would be inconvenient to stop and start the driving shaft whenever the driven shaft must be stopped. (Examples: Motor car engine to driving axle connection or a lineshaft driven single pulley gear box.)

A clutch normally consists of two halves, one being fixed to the driving or driven shaft while the second half is free to slide axially on the other shaft on a parallel feather key, refer to Fig. 20. The fixed half carries a bearing in which the opposite shaft end rotates thus ensuring the axial alignment of the two halves.

Clutches are classified as:

1. Positive action clutches.

2. Friction clutches.

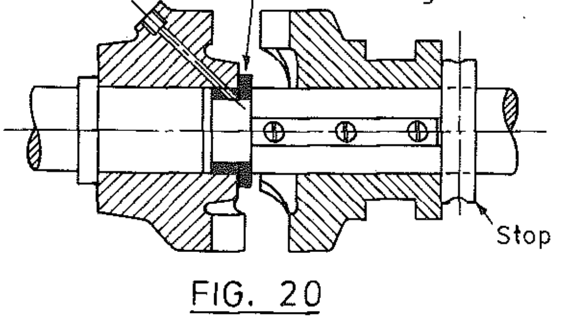


FIG. 20

A positive action clutch can be used only for slow speeds, since due to its action the stationary shaft is suddenly brought to full speed which results in considerable shock. (Alternatively, the clutch can be engaged with both shafts stationary.)

For medium or high speeds, friction clutches must be used to ensure a gradual acceleration from zero to full speed, and eliminate shock, which could cause failure of some of the parts.

An example of a positive action clutch is the jaw, claw or dog clutch, see Fig. 21.

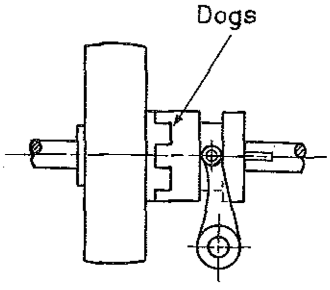


FIG. 21

A large variety of friction clutches have been designed for various purposes. They are mechanically, electrically or hydraulically (fluid or air pressure) operated. Only mechanically operated types will be considered here.

Perhaps the oldest type of friction clutch is the cone clutch with cast iron friction surfaces or with lined surfaces. The lining is a material used in brakes to increase friction and thus increase the torque capacity while minimising wear due to sliding.

Fig. 22 shows the construction of a simple cone clutch, the force P_a applied axially to the sliding half will result in a force R normal to the cone surfaces and the frictional force thus created will cause the two halves to rotate together and transmit the torque.

The plate clutch (Fig. 23) was the forerunner of the multiple disc clutch (Fig. 24, page 15.31) used in automotive practice.

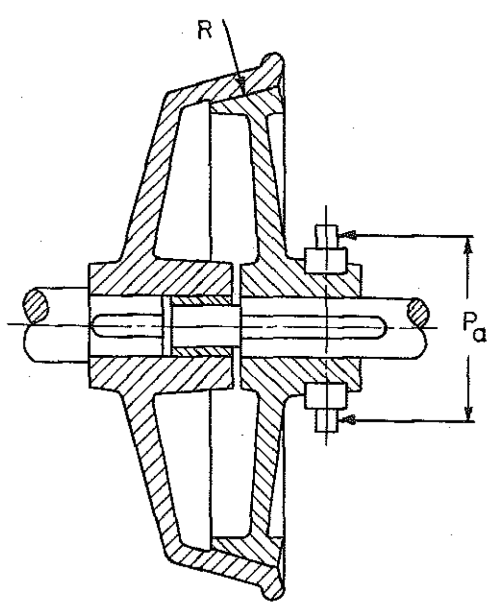
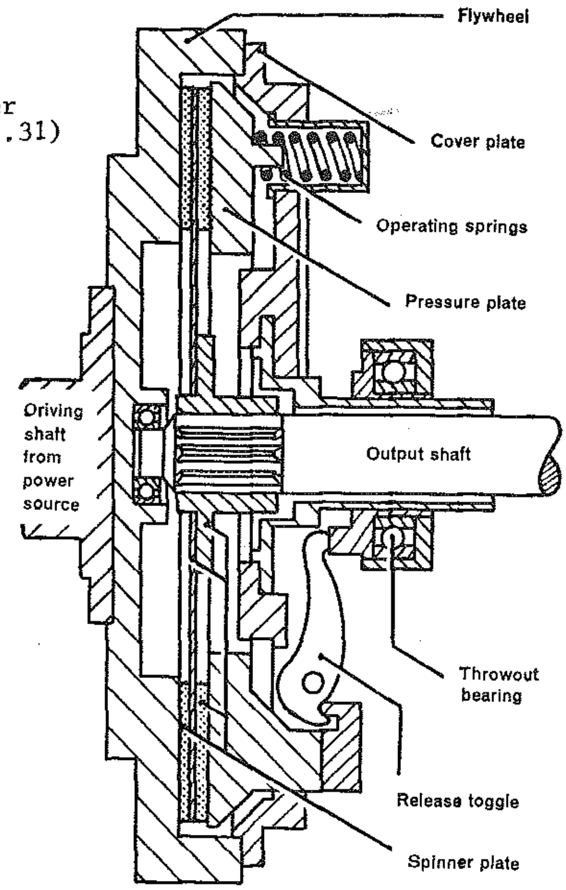


FIG. 22



Typical single plate clutch

FIG. 23

Theory of single plate clutch Refer to Fig. 23, page 15.28.

The whole of the clutch mechanism is rotating with the input member. The driven shaft passes through the centre of the clutch shell and is splined at the end to take the spinner plate which is situated between the driving and pressure plates. The spinner plate is lined on both sides with friction facings.

When the clutch is operative the pressure plate clamps the spinner plate against the driving plate by means of the springs, and the drive is then transmitted through the pressure and driving plate to the spinner and so to the transmission shaft.

To disengage the clutch, the pressure plate must be withdrawn to release the clutch plate. This is effected by depressing the throw-out bearing which, being coupled to the pressure plate by a series of toggles, pulls this plate off against the operating springs.

Plate clutch formulas.

Glossary of terms

P = power being transmitted, kW

T = torque being transmitted, N.m

P_a = clamping or axial load, N.

N = operating speed of clutch, rev/min.

ω = operating speed of clutch, rad/sec.

μ = coefficient of friction of contact surfaces.

A = area of contact surfaces, mm^2.

n = number of effective friction surfaces (i.e. pairs of contacting surfaces).

H = heat generated in one clutch application, kJ/s

D = outside diameter of clutch facing, mm.

d = inside diameter of clutch facing, mm.

r = mean radius, mm, $= \dfrac{D + d}{4}$

p_{av} = average normal pressure between contact surfaces, MPa.

p_{min} = minimum normal pressure between contact surfaces, MPa.

p_{max} = maximum normal pressure between contact surfaces, MPa.

$$T = \frac{P_a \mu n r}{1000} \qquad H = \frac{p_{av} A\mu\omega r}{1000^2} = \frac{T\omega}{1000}$$

$$P_a = \frac{P\ 1000^2}{\omega \mu n r} \qquad p_{max} = \frac{P_a}{2\pi\left(\frac{D}{2} - \frac{d}{2}\right)\frac{d}{2}}$$

$$P = \frac{P_a \, \omega \, \mu \, n \, r}{1000^2} \qquad p_{min} = \frac{P_a}{2\pi \left(\frac{D}{2} - \frac{d}{2}\right) \frac{D}{2}}$$

$$p_{av} = \frac{P_a}{\frac{\pi}{4}\left(D^2 - d^2\right)}$$

The maximum pressure occurs at the smaller radius. (After initial wear has taken place and the discs have worn down to the point where uniform wear becomes possible, the greatest pressure must occur at the smallest radius in order for the wear to be uniform
i.e. p x velocity = Const.).

All the above formulas are based on the theory of uniform wear (as opposed to the theory of uniform pressure). When using these formulas always incorporate a suitable service factor and/or shock factor. Each factor is usually between 1.25 and 2.0. A suitable safety factor is also required - up to 2.0.

Design power = nominal power x service factor x shock factor x safety factor.

Power rating for friction materials.

Basically, the same information applies to clutches as applied to brakes. Refer to the sections "Power rating of friction materials" (page 15.22) and "Coefficient of friction" (page 15.25). Table No. 6 does not apply to clutches; Table No. 10 gives some general values which are applicable to clutches.

Note regarding plate clutches:-

Experience has shown that the applied load is not transmitted as efficiently to the friction surfaces in a plate clutch as in a drum brake. Further, the full effective torque radius may not always be developed. For all friction materials it is therefore wise to adopt a somewhat lower coefficient of friction for design purposes than would be used on a drum brake application.

TABLE NO. 10 (traditional values).

FACING MATERIAL	COEFFICIENT OF FRICTION	UNIT PRESSURE MPa
C.I. on C.I. Dry	0.20	0.28 - 0.41
C.I. on C.I. Oily	0.07	0.28 - 0.41
C.I. on steel Dry	0.30	0.28 - 0.41
C.I. on steel Oily	0.10	0.28 - 0.41
Leather on C.I. Dry	0.50	0.07 - 0.08
Leather on C.I. Oily	0.15	0.07 - 0.08
Cork on C.I. Dry	0.35	0.014 - 0.034
Cork on C.I. Oily	0.30	0.014 - 0.034
Asbestos fabric on metal Dry	0.40	0.21 - 0.41
Asbestos fabric on metal Oily	0.25	0.21 - 0.41
Moulded asbestos on metal Dry	0.30	0.21 - 0.41
Wood on C.I. Dry	0.30	0.17 - 0.34
Wood on C.I. Oily	0.12	0.17 - 0.34

Multi-plate clutch. (Fig. 24.)

Multi-plate clutches have friction linings on both sides of alternate plates. Where large torques are to be transmitted, a multi-plate clutch can be used.

A multi-plate clutch has the advantage of limiting the clamping or axial load, P_a.

The torque transmitted by a single plate clutch is determined from the equation on page 15.29. The term "n" (pairs of contacting surfaces) is unity for a single plate clutch, but is 8 for a multi-plate clutch, as shown in Fig. 24.

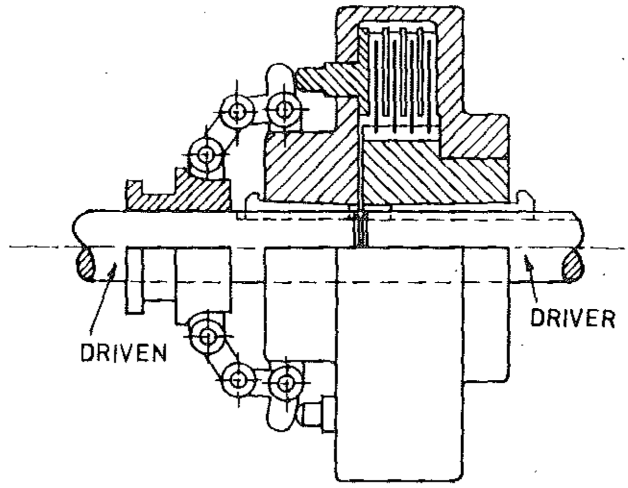


FIG. 24

Theory of Cone clutch design. (Refer to Fig. 25.)

Again reference has been made to the Ferodo Ltd. publication "Friction Materials for Engineers" in compiling the following information. Designers are referred to this book for more detailed design information.

The cone clutch, embodying the mechanical advantage of the wedge, reduces the axial force necessary to transmit a given torque, without recourse to levers between friction faces and throw-out bearing.

It has greater facilities for heat dissipation than a plate clutch of similar dimensions; hence it may be somewhat more heavily rated. However, it is not suitable for extreme speeds as the pressure normal to the friction surfaces acts in conjunction with centrifugal force loading.

A cone clutch uses the mechanical advantage of the simple wedge to multiply the axial clamping pressure to the value required normal to the friction face. The mechanical advantage of a wedge varies as the reciprocal of the sine of the wedge angle, and as the coned friction face of the clutch is equivalent to two similar wedges back to back, the similar ratio in a cone clutch will be

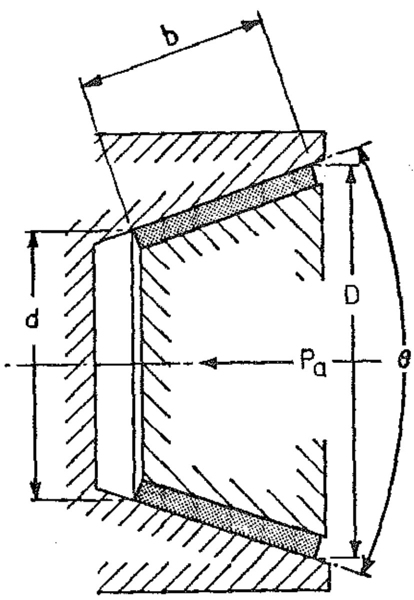


FIG.25

$$\frac{1}{\sin \frac{1}{2}\theta} \quad \text{where } \theta = \text{included or total angle.}$$

To obtain the best mechanical advantage in a cone clutch it is advisable to keep the included angle between 24° and 36°. If this angle is too large the effect of the wedge action is lost with a subsequent increase in the operating pressure. On the other hand, if the angle is small enough the clutch will become self-sustaining. A cone clutch fitted with a friction lining having a coefficient of 0.3 will be self-sustaining up to an included angle of 10° to 12° and similarly for a coefficient of friction of 0.5 it will be self-sustaining up to an angle of 16°. The disadvantages of a self-sustaining clutch are that a large withdrawal force is required and it is very fierce in take-up when the clutch is engaged.

As already mentioned, the female member of a cone clutch usually forms the steel or cast-iron surface with which the friction-fabric cone lining is brought into contact by axial displacement of the lined male member. This is dictated by good design as the rim of the female portion must withstand both centrifugal force loads and the bursting pressure of the application force normal to the friction surfaces. Rivet or screw holes in the rim create discontinuities and so reduce strength. Again, in such a unit the female member is preferably driving so that the throw-out bearing, connected to the male member, is stationary when loaded.

The mating metal surface of a cone clutch must be given the highest polish commercially practicable. This perfection is not only reflected in a low rate of lining wear, but makes for smooth engagement and easy disengagement. A cone clutch surface with a gramophone-record scroll finish resulting from the lathe feed gear will have a screw-in action for the same rotation. Such a clutch will be uncontrollably fierce in take-up and require considerable force to free it.

As it is general practice to attach the cone lining to the male member of the clutch, most of the heat units released during each engagement period are absorbed by the metal female member. The latter has most of the favourable heat dissipation facilities inherent in a brake drum housing of an internal expanding brake, and, therefore, higher unit loading of cone friction facings can be tolerated than would be the case in a plate clutch.

Summary of cone clutch features and uses

The cone clutch, embodying the mechanical advantage of the wedge, reduces the axial force necessary to transmit a given torque, without recourse to levers between friction faces and throw-out bearing.

It has greater facilities for heat dissipation than a plate clutch of similar dimensions and may be somewhat more heavily rated.

It is simple in construction and manufacture.

It is unsuitable for extreme speeds as the pressure normal to the friction surfaces acts in conjunction with centrifugal force loading.

It is inherently fierce in take-up characteristics if reasonable advantage is taken of its wedge action.

(These last two features were responsible for the disappearance of the cone clutch from automotive engineering.)

The cone clutch in general engineering is now confined to small and medium power transmission in machinery of the more rugged and abused typed.

Typical instances are contractors' plant, such as road rollers, concrete mixers and jib cranes.

Design of cone clutch linings (if linings are used).

In the design of cone clutches, it is essential to observe the following precaution.

Where the friction lining is fixed to the male member, the metal female cone surface must coincide with or overlap the large diameter of the new lining to prevent formation of a ridge or shoulder in the latter.

Similarly, where the friction cone is attached to the female member, the small diameter of the metal male cone must coincide with or overlap the small bore of the lining for the same reason. Fig. 26 illustrates the essential nature of this requirement.

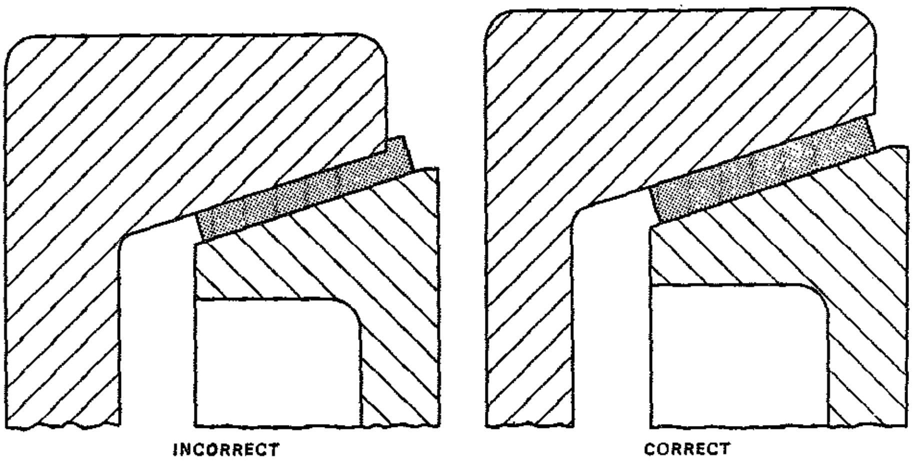


FIG. 26

Glossary of terms Refer to Fig. 25, page 15.31.

P = power being transmitted, kW.

P_a = axially applied force to engage clutch, N.

P_d = axially applied force to disengage clutch, N.

T = torque being transmitted, N.m.

N = operating speed of clutch, r.p.m.

ω = operating speed of clutch, rad/sec.

θ = total or included angle of cone, degrees.

μ = coefficient of friction of contact surfaces.

p_{av} = average normal pressure between contact surfaces, MPa.

p_{max} = maximum normal pressure between contact surfaces, MPa.

A = area of contact surfaces, mm^2.

b = lining width, mm.

H = heat generated in one clutch application, kJ/s.

r = mean radius, mm, $\frac{D + d}{4}$

NOTE: In this case the difference between friction radius and mean radius of contact surfaces is very small, therefore mean radius may be used.

Cone clutch formulas.

$$T = \frac{P_a \mu r}{\sin\frac{\theta}{2} \times 1000} \qquad p_{av} = \frac{T \times 1000}{r \mu A}$$

$$P = \frac{P_a \mu r}{\sin\frac{\theta}{2}} \times \frac{\omega}{1000^2} \qquad H = \frac{p_{av} A\mu\omega r}{1000^2} = \frac{T\omega}{1000}$$

$$\omega = \frac{2\pi N}{60} \qquad p_{max} = \frac{P_a}{2\pi\left(\frac{D-d}{2}\right)\frac{d}{2}}$$

If no relative rotary motion occurs during clutch engagement i.e., a stationary engagement is effected,

$$Pa = \frac{1000\ T}{\mu r}\left(\sin\frac{\theta}{2} + \mu \cos\frac{\theta}{2}\right)$$

The maximum pressure occurs at the smallest radius.

All the above formulas are based on the theory of uniform wear (as opposed to the theory of uniform pressure).

Ordinarily, with the cone angles commonly used, no force is necessary to disengage the parts, although it is possible that if $\mu \cos\frac{\theta}{2} > \sin\frac{\theta}{2}$, an axial force P_d will be necessary to disengage the parts:-

$$P_d = \frac{P_a}{\sin\frac{\theta}{2}}\left(\mu \cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)$$

When using the formulas for cone clutch design incorporate suitable service and/or shock factors plus a safety factor. See page 15.30.

CAPSTANS.

A capstan is simply a spool shaped cylinder. In its common form it is attached to an extension on the drum shaft of a winch as shown in Fig. 27, page 15.35.
A hemp rope is wound two or three times around the capstan; one end of the rope is attached to a load while the other end is controlled by a workman. By means of a small force from the workman a large weight may be moved, raised or lowered. A capstan has great application in jobbing and erection work.

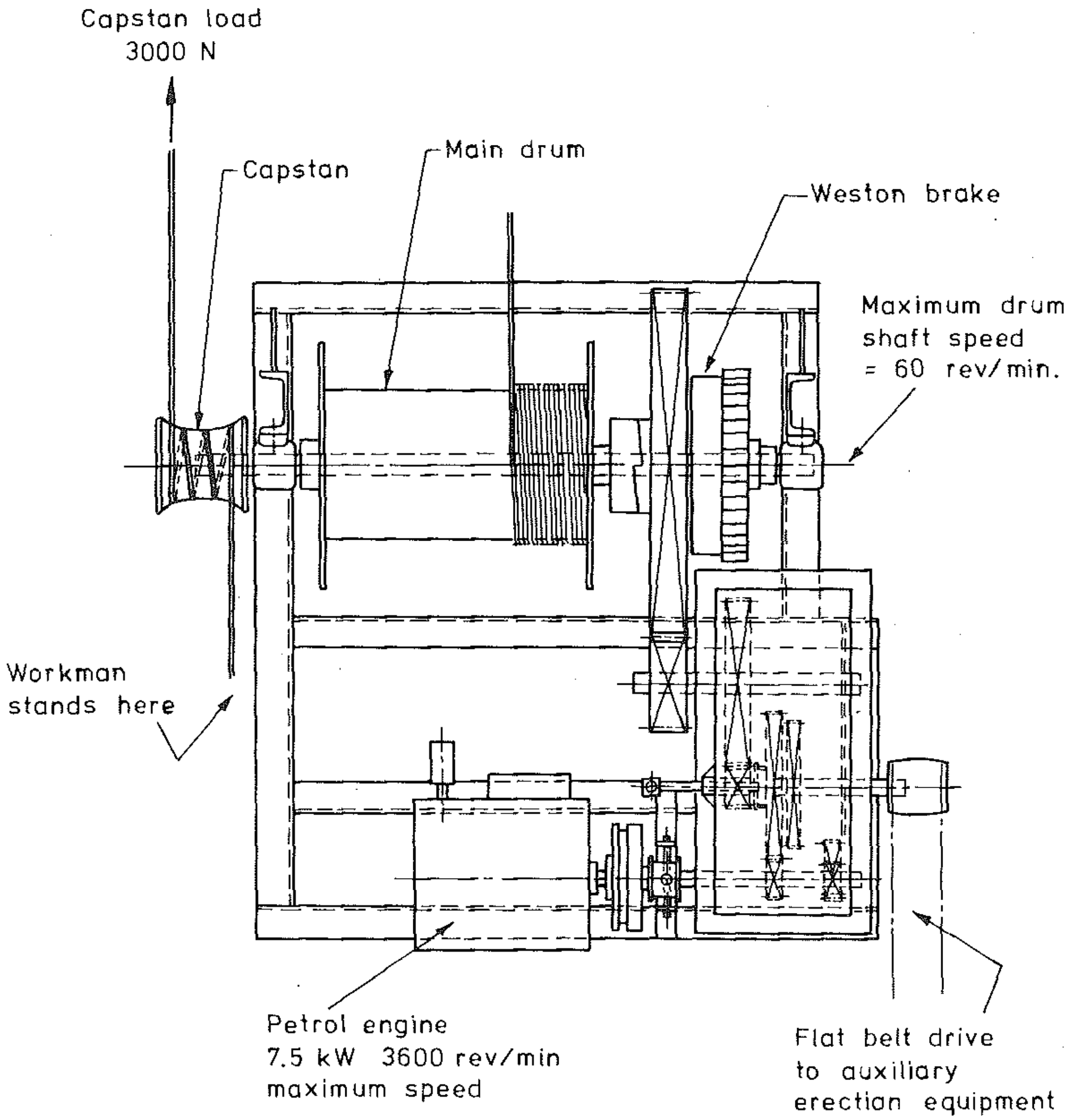
Design of a capstan.

Glossary of terms

T_1 = tension in rope from load, N.

T_2 = tension in rope from operator, N.
Generally allow 110 N to 140 N for one man.
A man can exert 0.15 kW of power for a working day or 4.5 kW in a single action.

θ = angle of wrap of rope, radians.



GENERAL ASSEMBLY OF ERECTION WINCH

FIG. 27

Glossary of terms (Cont.)

μ = coefficient of friction rope to capstan.

= 0.25 for hemp rope on metal, dry.

= 0.15 for hemp rope on metal, greasy.

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Worked Example No. 2.

Refer to Fig. 27, page 15.35.

The capstan is 140 mm diameter and a 12 mm diameter rope is used.

If the speed of rotation is 60 r.p.m. determine:

(a) The number of turns of rope required around the capstan (assume dry conditions).

(b) The power supplied to the capstan from the motor.

(c) The power absorbed by the load.

(d) The power supplied by the workman.

Answer.

(a)
$$T_1 = 3000 \text{ N}$$
$$T_2 = 125 \text{ N (say)}$$
$$\mu = 0.25$$
$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$\frac{3000}{125} = e^{0.25\theta}$$

From which θ = 12.71 radians = 2.02 turns.

(b) Use 2 - turns and calculate the value of T_2.

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{and} \quad \frac{3000}{T_2} = e^{0.25 \times 4\pi}$$

$$\therefore \quad T_2 = 129.64 \text{ N}$$

$$\text{Rope speed} = \frac{60\,\pi\,(0.140 + 0.012)}{60}$$

$$= 0.478 \text{ m/sec.}$$

Power supplied to capstan from motor

$$= \frac{(3000 - 129.64) \times 0.478}{1000} = 1.372 \text{ kW}$$

(c) Power absorbed by load

$$= \frac{3000 \times 0.478}{1000} = 1.434 \text{ kW}$$

(d) Power supplied by workman

$$= 1.434 - 1.372 = 0.062 \text{ kW.}$$

Worked Example No. 3.

A 700 mm diameter brake drum contacts a single shoe as shown in Fig. 28. The brake is to operate against 225 N.m torque at 500 r.p.m.

$\mu = 0.3$

Determine:

(a) the normal force P on the shoe

(b) the required force W to apply the brake for clockwise rotation

(c) the required force W to apply the brake for anti-clockwise rotation

(d) the dimension that c would need to be to make the brake self-locking. (Refer to Table No. 3, page 15.13.)

(e) the rate of heat generated, H.

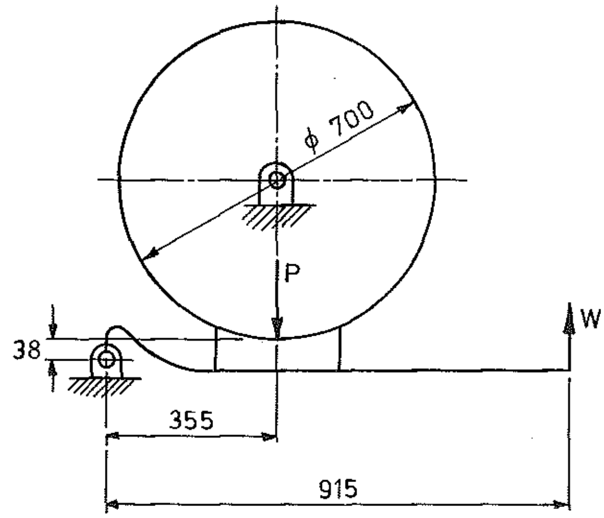


FIG. 28

Answer.

(Refer to page 15.13 for formulas.)

(a) $T = \mu Pr$

$$\therefore \quad P = \frac{T \times 1000}{\mu r} = \frac{225 \times 1000}{0.3 \times 350} = 2142.9 \text{ N}$$

(b) $$W = \frac{Fb}{a + b}\left(\frac{1}{\mu} - \frac{c}{b}\right)$$

$$= \frac{2142.9 \times 0.3 \times 355}{560 + 355}\left(\frac{1}{0.3} - \frac{38}{355}\right)$$

$$= 804.7 \text{ N}$$

(c) $$W = \frac{Fb}{a + b}\left(\frac{1}{\mu} + \frac{c}{b}\right)$$

$$= \frac{2142.9 \times 0.3 \times 355}{560 + 355}\left(\frac{1}{0.3} + \frac{38}{355}\right)$$

$$= 858.1 \text{ N}$$

(d) For self-locking, which can only occur for clockwise rotation of the drum,

$$\mu c \geq b \qquad \text{or} \qquad c \geq \frac{b}{\mu} = \frac{355}{0.3} = 1183.33 \text{ mm}$$

(e) $$H = \frac{\mu P V}{1000} = \frac{0.3 \times 2142.9 \times \pi \times 0.7 \times 500}{1000 \times 60}$$

$$= 11.78 \text{ kJ/s}$$

Worked Example No. 4.

Fig. 29(a) shows a double block brake which is to have a rating of 240 N.m at 600 r.p.m.

The drum diameter is to be 200 mm; each lining has an angle of contact of 120°. $\mu = 0.3$.
Service conditions require a pV = 2.0 maximum value.

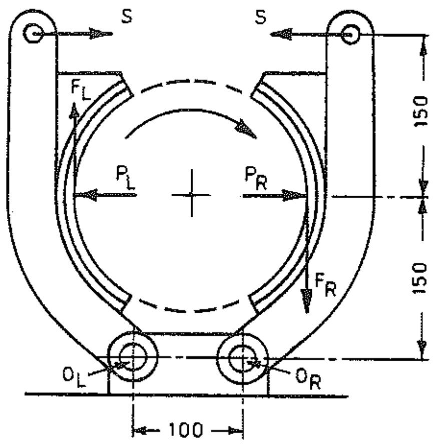


FIG. 29(a)

Determine:

(a) the spring force required

(b) the shoe width necessary

(c) the average normal pressure between lining and drum

(d) the maximum normal pressure between lining and drum

(e) the heat rate being generated

(f) the total amount of heat generated if the brake has to be applied for 6 seconds at full capacity to stop the load

(g) the maximum permissible value of the mean rating in kW/m² if solid woven, asbestos based friction material is used for the linings
Compare with the actual figure.

Answer. (Refer to page 15.14 for formulas.)

(a) $$\mu' = \mu \left(\frac{4 \sin \frac{\theta}{2}}{\theta + \sin \theta} \right) = 0.3 \left(\frac{4 \sin 60°}{2.094 + \sin 120°} \right)$$

$$= 0.3511$$

$$\frac{F}{P} = 0.3511 \quad \therefore \quad P = 2.85 \text{ F}$$

Take moments about O_L for L.H.S. shoe

$$300 \text{ S} + 50 \text{ F}_L - 150 \text{ P}_L = 0$$

$$300 \text{ S} + 50 \text{ F}_L - 150 \times 2.85 \text{ F}_L = 0$$

$$\therefore \text{ F}_L = 0.795 \text{ S}$$

Take moments about O_R for R.H.S. shoe

$$300\ S - 50\ F_R - 150\ P_R = 0$$

$$300\ S - 50\ F_R - 150 \times 2.85\ F_R = 0$$

$$\therefore\ F_R = 0.628\ S$$

From the torque requirements

$$F_L + F_R = \frac{T}{r}$$

$$0.795\ S + 0.628\ S = \frac{240 \times 1000}{100}$$

$$\therefore\ \underline{S = 1686.6\ N}$$

(b) $pV = 2.0$

In designing block brakes p in the above formula is usually based on the projected area.

Projected bearing area for one shoe

$$= w\ 200 \sin 60° = 173.2\ w\ mm^2.$$

The <u>average</u> force P acting on a shoe

$$= \frac{P_L + P_R}{2}$$

$$= \frac{2.85 \times 1686.6\ (0.795 + 0.628)}{2}$$

$$= 3420\ N$$

$$V = \frac{2\ \pi\ rN}{60 \times 1000} = \frac{2\ \pi\ 100 \times 600}{60 \times 1000} = 6.28\ m/s, \quad pV = \frac{3420 \times 6.28}{173.2\ w} = 2.0$$

from which $w = \underline{62\ mm \text{ say } 70\ mm}$

$\frac{w}{D} = \frac{70}{200} = 0.35$ which is between ¼ and ½ and this is satisfactory.

(c) Consider the L.H.S. shoe

$$\mu' p_n\ wr\theta = F = 0.795\ S$$

$$\therefore\ p_n = \frac{0.795\ S}{\mu'\ wr\ \theta}$$

$$p_n = \frac{0.795 \times 1686.6 \times 57.3}{0.3511 \times 70 \times 100 \times 120}$$

$= \underline{0.26\ \text{MPa}}$ (is approximate for such a large angle of contact).

(d) Intensity of pressure and pressure distribution on the lining.

The position of maximum pressure and pressure distribution on the lining can also be shown graphically as illustrated on the left-hand shoe of Fig. 29(a), page 15.38. The method of construction is as follows (refer to Fig. 29(b)):

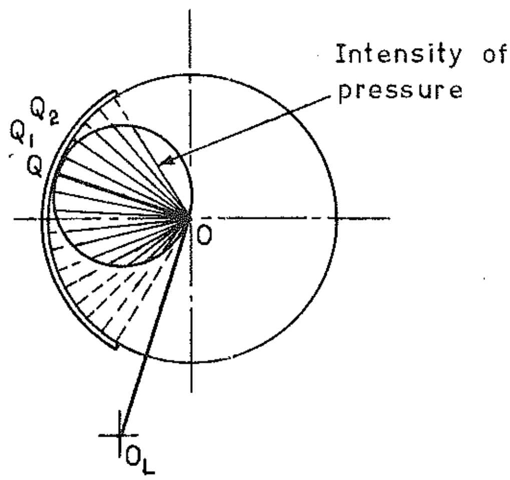


FIG. 29(b)

1. Join O_L (the centre point of the shoe pivot) and O (the centre of the drum).
2. From O draw OQ at right angles to OO_L.
   The point at which OQ intersects the lining is the point at which the maximum pressure will be located.
   Let Q then be the point of intersection.
3. Adopting OQ as a diameter, construct a circle with centre on OQ.
4. From point O construct lines at regular intervals intersecting the circumference of circle diameter OQ.

   The length of these chords OQ_1, OQ_2, etc., will be proportional to the pressure on the lining along these chords.

By measurement and simple arithmetic the average length of chords OQ_1, OQ_2, etc. can be determined as 71 units; this represents the average normal pressure p_n.
The maximum chord length = 100 units and this represents the maximum normal pressure.

$71 : p_n$ (average) : : $100 : P_n$ (maximum)

$\therefore\ 71 : 0.26 : : 100 : p_n$ (maximum)

from which p_n (maximum) $= \underline{0.37\ \text{MPa}}$

(e) $H = \dfrac{\mu' PV}{1000} = \dfrac{0.3511 \times 2.85 \times 1686.6\ (0.795 + 0.628)\ \pi\ 0.2 \times 600}{1000 \times 60}$

$= \underline{15.1\ \text{kJ/s}}$

(f) Total heat $= \dfrac{15.1 \times 6}{2} = \underline{45.3\ \text{kJ}}$

(g) From Table No. 7, page 15.23, permissible mean rating for intermittent duty $= 578\ \text{kW/m}^2$.

Now find actual figure.

(g) $$P = \frac{T\omega}{1000} = \frac{240 \times 2\pi\, 600}{1000 \times 60} = 15.1 \text{ kW}$$

$$\text{Lining area} = \frac{240}{360} \times 0.07 \times 2\pi\, 0.1$$

$$= 0.0293 \text{ m}^2$$

$$\text{Mean rating} = \frac{1}{2} \times \frac{15.1}{0.0293} = 258 \text{ kW/m}^2$$

Worked Example No. 5.

A differential band brake has a force of 220 N applied at the end of the operating lever as shown in Fig. 30.

μ = 0.4. Drum speed = 400 r.p.m. Drum width = 20 mm.

(a) (i) If a clockwise torque of 45 N.m is applied, determine T_1 and T_2.

(ii) Calculate the maximum unit pressure between the drum and lining.

(b) (i) What is the maximum torque that the brake may sustain for anti-clockwise drum rotation?

(ii) Calculate the maximum unit pressure between the drum and lining.

(iii) Determine the heat rate being generated.

(iv) State the total amount of heat generated if the brake has to be applied for 3 seconds at full capacity to stop the load.

(v) State the maximum permissible value of the mean rating in kW/m² if solid woven, asbestos based friction material is used for the lining. Normal intermittent duty is required.
Compare with the actual figure.

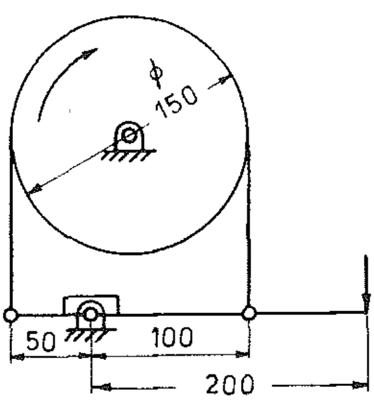


FIG. 30

Answer (Refer to page 15.20 for formulas)

(a) (i) From Table No. 5, page 15.19.

$$b_2 = 100$$

$$b_1 e^{\mu\theta} = 50e^{0.4\pi} = 175.7$$

As $b_2 < b_1 e^{\mu\theta}$ brake is self-locking.

Take moments about pivot

$$50\ T_1 + 220 \times 200 - 100\ T_2 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

From the torque relationship

$$0.075\ (T_1 - T_2) = 45 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Solving equations (1) and (2) gives

$$\underline{T_1 = 2080\text{ N}} \qquad \text{and} \qquad \underline{T_2 = 1480\text{ N}}$$

(ii) $$p_{max} = \frac{T_1}{1000\ w\ r} = \frac{2080}{100 \times 20 \times 0.075}$$

$$= \underline{1.39\text{ MPa}}$$

(b) (i) $$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.4\pi} = 3.51$$

$$\therefore\ T_1 = 3.51\ T_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Take moments about pivot

$$100\ T_1 = 220 \times 200 + 50\ T_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Solving equations (3) and (4) gives

$$T_1 = 513.09\text{ N}$$

$$T_2 = 146.18\text{ N}$$

$$T = (T_1 - T_2)r = (513.09 - 146.18)\ 0.075$$

$$= \underline{27.52\text{ N.m}}$$

(ii) $$p_{max} = \frac{T_1}{1000\ w\ r} = \frac{513.09}{1000 \times 20 \times 0.075}$$

$$= \underline{0.34\text{ MPa}}$$

(iii) $$p_{av} = \frac{T_1}{1000\ wr\mu\ \theta}\ \left(\frac{e^{\mu\theta} - 1}{e^{\mu\theta}}\right)$$

$$= \frac{513.09}{1000 \times 20 \times 0.075 \times 0.4 \times \pi}\ \left(\frac{3.51 - 1}{3.51}\right)$$

$$= 0.195\text{ MPa}$$

$$V = \frac{\pi DN}{60} = \frac{\pi \times 0.15 \times 400}{60}$$

$$= 3.1416\text{ m/s}$$

$$A_c = \frac{\pi \times 150 \times 20}{2} = 4712.4 \ \text{mm}^2$$

$$H = \frac{p_{av} A_c \mu V}{1000}$$

$$H = \frac{0.195 \times 4712.4 \times 0.4 \times 3.1416}{1000}$$

$$= 1.15 \ \text{kJ/s}$$

(iv) Heat $= \dfrac{1.15 \times 3}{2} = \underline{1.73 \ \text{kJ}}$

(v) From Table No. 7, page 15.23, permissible mean rating $= \underline{578 \ \text{kW/m}^2}$

$$P = \frac{T\omega}{1000} = \frac{27.52 \times 2\pi\ 400}{1000 \times 60}$$

$$= 1.15 \ \text{kW}$$

$A_c = 4712.4 \ \text{mm}^2$ (from (iii)) $= 0.0\ 047\ 124 \ \text{m}^2$

Actual mean rating $= \dfrac{1}{2} \times \dfrac{1.15}{0.0\ 047\ 124}$

$= \underline{122 \ \text{kW/m}^2}$

Worked Example No. 6.

Refer to Fig. 31. A cone clutch (leather on C.I., $\mu = 0.25$) has to be designed to transmit 15 kW at 900 r.p.m.; the mean diameter of the clutch = 320 mm.

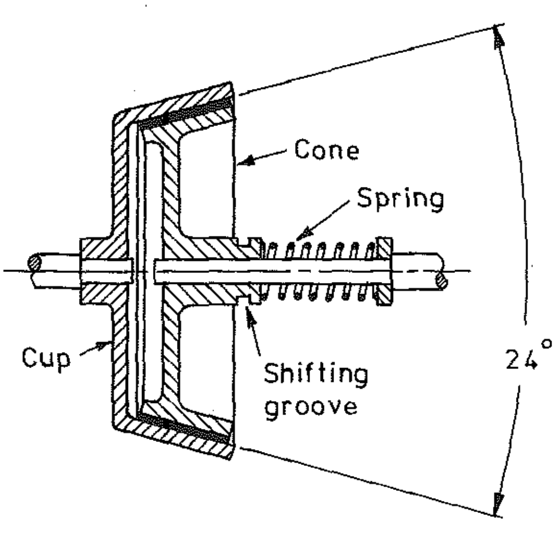


FIG. 31

(a) What should be the strength of the spring?

(b) Determine the required outside and inside diameter for the clutch; $p_{av} = 0.07$ MPa, $p_{max} = 0.08$ MPa.

(c) (i) Determine if a force is required to disengage the clutch.

(ii) Calculate the magnitude of the disengaging force if one is necessary.

Answer (Refer to page 15.34 for formulas)

(a) $$P = \frac{P_a \mu r}{\sin \frac{\theta}{2}} \times \frac{\omega}{1000^2}$$

$$P_a = \frac{1000^2 \sin\frac{\theta}{2}\ P}{\mu\ r\ \omega} = \frac{1000^2 \sin 12° \times 15 \times 60}{0.25 \times 160 \times 2\pi\ 900}$$

$$= 827.3 \text{ N}$$

(b) $$p_{av} = \frac{T \times 1000}{r\ \mu\ A} \qquad \therefore\ A = \frac{T \times 1000}{p_{av}\ r\ \mu}$$

$$T = \frac{1000\ P}{\omega} = \frac{1000 \times 15 \times 60}{2\pi\ 900} = 159.16 \text{ N.m}$$

$$\therefore\ A = \frac{159.16 \times 1000}{0.07 \times 160 \times 0.25}$$

$$= 56\ 842.9 \text{ mm}^2$$

$$A = 2\pi r b$$

$$\therefore\ b = \frac{A}{2\ \pi\ r} = \frac{56\ 842.9}{2\pi\ \ 160} = 56.5 \text{ mm say } 60 \text{ mm}$$

$$\therefore\ D = 2r + b\sin\frac{\theta}{2} = 2 \times 160 + 60 \sin 12° = 332.5 \text{ mm}$$

$$d = 2r - b\sin\frac{\theta}{2} = 2 \times 160 - 60 \sin 12° = 307.5 \text{ mm}$$

$$p_{max} = \frac{P_a}{2\ \pi\ (\frac{D - d}{2})\ \frac{d}{2}}$$

$$= \frac{827.3}{2\ \pi\ (\frac{332.5 - 307.5}{2})\ \frac{307.5}{2}}$$

$$= 0.0685 \text{ MPa} < 0.08 \text{ MPa}$$

$$p_{av} = \frac{T \times 1000}{r\ \mu\ A}; \qquad A = 60\ 318.6 \text{ mm}^2$$

$$= \frac{159.16 \times 1000}{160 \times 0.25 \times 60\ 318\ .6}$$

$$= 0.066 \text{ MPa} < 0.07 \text{ MPa}$$

(c) (i) $$\mu \cos\frac{\theta}{2} = 0.25 \times \cos 12° = 0.2445$$

$$\sin\frac{\theta}{2} = \sin 12° = 0.2079$$

$\mu \cos \frac{\theta}{2} \quad > \quad \sin \frac{\theta}{2}$ $\quad \therefore$ a force will be necessary to disengage the clutch.

(ii) $$P_d = \frac{P_a}{\sin \frac{\theta}{2}} \left(\mu \cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)$$

$$= \frac{827.3}{\sin 12°} \left(0.25 \cos 12° - \sin 12° \right)$$

$$= \underline{145.6 \text{ N}}$$

Worked Example No. 7.

Refer to the plate clutch shown in Fig. 32.

$N = 400$ r.p.m.; $\mu = 0.25$

Determine:

(a) The power the clutch will transmit.

(b) p_{av}, p_{max} and p_{min} when the clutch is in operation.

(c) The heat rate being generated during engagement.

(d) The amount of heat generated if the slipping time is 0.38 sec.

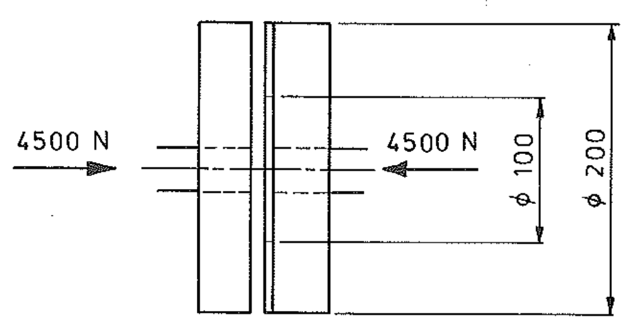


FIG. 32

Answer (Refer to pages 15.29 & 15.30 for formulas.)

(a) $$P = \frac{P_a \omega \mu n r}{1000^2}$$

$$= \frac{4500 \times 2\pi\, 400 \times 0.25 \times 1 \times 75}{1000^2 \times 60}$$

$$= \underline{3.53 \text{ kW}}$$

(b) $$p_{av} = \frac{P_a}{\frac{\pi}{4}(D^2 - d^2)} = \frac{4500}{\frac{\pi}{4}(200^2 - 100^2)} = \underline{0.191 \text{ MPa}}$$

$$p_{max} = \frac{P_a}{2\pi \left(\frac{D}{2} - \frac{d}{2}\right) \frac{d}{2}} = \frac{4500}{2\pi \left(\frac{200}{2} - \frac{100}{2}\right) \frac{100}{2}} = \underline{0.287 \text{ MPa}}$$

$$p_{min} = \frac{P_a}{2\pi \left(\frac{D}{2} - \frac{d}{2}\right) \frac{D}{2}} = \frac{4500}{2\pi \left(\frac{200}{2} - \frac{100}{2}\right) \frac{200}{2}} = \underline{0.143 \text{ MPa}}$$

(c) $H = 3.53$ kJ/s (from (a))

(d) Heat $= \dfrac{3.53 \times 0.38}{2} = \underline{0.67 \text{ kJ}}$

EXERCISES

(1) A right-hand and a left-hand square-threaded screw is used as a strainer.

If the details are as follows:

outside diameter = 28 mm
root diameter = 22 mm
pitch = 6 mm, single start, μ = 0.15

find the couple required to tighten against a pull of 4500 N.

Ans. 25 764.6 N.mm

(2) A screw of CS 1040 steel drives a phosphor bronze nut of AS 1565-906D specification. The axial load developed is 200 kN.

Details of the screw are:

outside diameter 100 m
root diameter 84 mm
pitch 16 mm single start
speed 150 r.p.m. at 1.5 hours/day operation.

(a) From Tables Nos. 1 and 2 of Chapter 7, pages 7.22 & 7.23. determine a suggested value for F_b. Bearing failure presents no danger.

(b) Determine the required nut length on the basis of:

(i) shear stress in screw threads
(ii) shear stress in nut threads
(iii) bearing stress.

Ans. (a) 6.4 MPa
(b) (i) 22.46 mm
(ii) 39.18 mm
(iii) 216.24 mm

(3) A sluice gate has a mass of 700 kg and the friction force, due to water pressure, resisting the opening operation is 2300 N.

The valve stem is non-rotating and is raised by rotating a wheel having internal threads acting as a nut on the valve stem.

The wheel is supported by a collar 42 mm inside diameter and 75 mm outside diameter.

Details of thread of valve stem:

outside diameter 42 mm
root diameter 34 mm
pitch 8 mm single start
μ collar = 0.25;
μ thread = 0.10

Determine:

(a) the torque which must be applied to the wheel to raise the gate.
(b) the overall efficiency of the mechanism.
(c) the maximum shear stress in the body of the screw.

Ans. (a) 96 319 N.mm
(b) 12.12%
(c) 6.32 MPa

(4) An M64 screw passes through a nut which rests on a ball-thrust washer. The angle of the thread vee is 60°, the pitch of the screw is 6 mm (single start) and the mean diameter = 60 mm.
If the axial load is 5 tonne, $\mu = 0.15$ and collar friction can be neglected, find:

(a) the torque required on the nut to raise the load.
(b) the screw efficiency.
(c) the torque required on the nut to lower the load.

Ans. (a) 303 383.2 N.mm
(b) 15.44%
(c) 206 891.3 N.mm

(5) Fig. 33 shows a single block brake.

(a) Determine the value of the torque which may be resisted by this brake.

$\mu = 0.3$

The angle subtended at the centre of the drum by the shoe = 65°.

(b) Calculate the value of the average normal pressure between the shoe and drum.

The shoe width = 80 mm.

Ans. (a) 51 500 N.mm
(b) 0.115 MPa

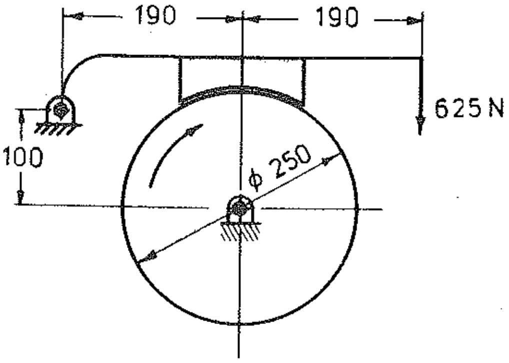


FIG. 33

(6) Fig. 34 shows a double block brake actuated by a system of levers. The drum speed is 80 r.p.m. and $\mu = 0.3$.

Determine:

(a) the braking torque.
(b) the heat rate being generated.
(c) the total amount of heat generated if the brake has to be applied for 5 seconds at full capacity to stop the load.
(d) the average normal pressure between drum and lining for L.H.S. shoe.

$w = 170$ mm.

Ans. (a) 1577.94 N.m
(b) 13.22 kJ/s
(c) 33 kJ
(d) 0.3 MPa

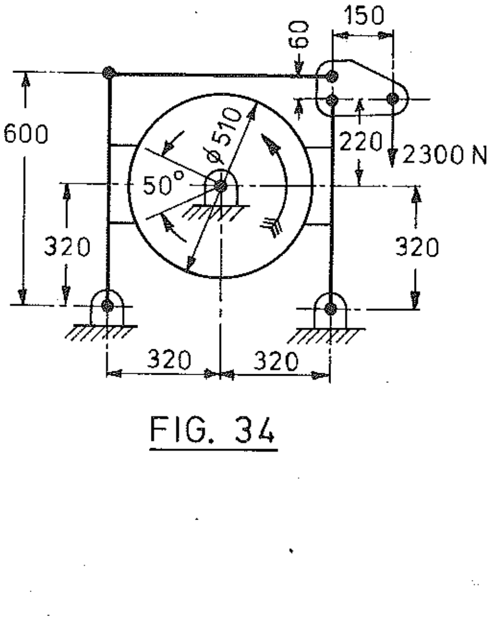


FIG. 34

(7) The double block brake shown in Fig. 35 has to absorb 26 kW at 300 r.p.m.

$\mu = 0.3$

Heavy duty classification is required, $pV = 1.0$.

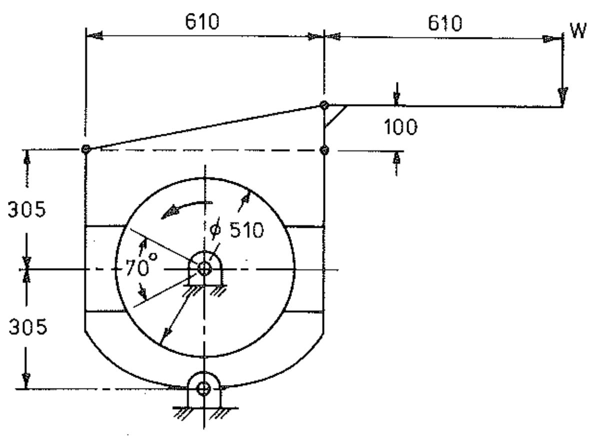


FIG. 35

Determine:

(a) the value of W.

(b) the minimum shoe width necessary.

(c) the average normal pressure between lining and drum if the shoe width chosen = 150 mm. Use the most heavily loaded shoe.

(d) the maximum normal pressure between lining and drum if the shoe width chosen is 150 mm. Use the most heavily loaded shoe.

(e) the heat rate being generated.

(f) the total amount of heat generated if the brake has to be applied for 4 seconds at full capacity to stop the load.

(g) the maximum permissible value of the mean rating in kW/m² if solid woven, asbestos based friction material is used for the linings.

Compare with the actual figure.

Ans. (a) 419.03 N (b) 139.58 mm (c) 0.392 MPa (d) 0.42 MPa

(e) 26 kJ/s (f) 52 kJ (g) 290 kW/m² 139 kW/m²

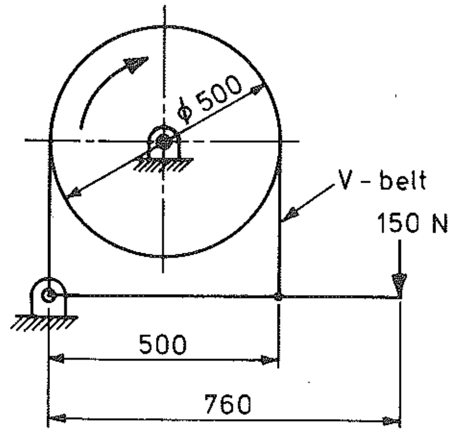


FIG. 36

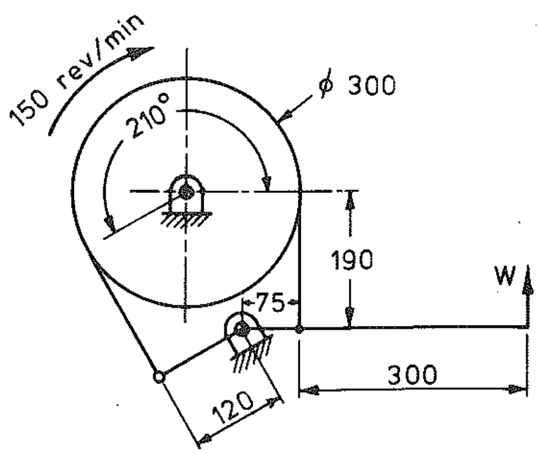


FIG. 37

(8) Fig. 36 shows a band brake which uses 38° V-belts.
Coefficient of friction = 0.24.
Determine the power rating at 420 r.p.m.

Ans. 22.9 kW

(9) Refer to the band brake shown in Fig. 37, page 15.48.
The coefficient of friction = 0.17 and the bands are fastened normal to the operating lever.
The power absorbed = 5.5 kW at normal intermittent duty.

(a) (i) Determine the value of W.

(ii) State the maximum recommended width for the band.

(iii) Calculate the maximum unit pressure between drum and lining if a 42 mm wide band is chosen.

(iv) Determine the heat rate being generated.

(v) Find the total amount of heat generated if the brake has to be applied for 3.5 seconds at full capacity to stop the load.

(vi) State the maximum permissible value of the mean rating in kW/m^2 if solid woven, asbestos based friction material is used for the linings.

Compare with the actual figure.

(b) (i) Show that the brake is self-locking for anti-clockwise rotation.

(ii) If an anti-clockwise torque of 250 N.m is applied determine the band tensions.

(iii) What force W will be required to release the brake with the above torque applied?

Ans. (a) (i) 1070.9 N (ii) 42.86 mm (iii) 0.799 MPa
(iv) 5.5 kJ/s (v) 9.625 kJ (vi) 578 kW/m^2
119.1 kW/m^2

(b) (ii) T_1 = 4444.4 N; T_2 = 2777.8 N
(iii) 102 N down.

(10) Fig. 38 shows a holding brake which must resist a torque of 1350 N.m in an anti-clockwise direction. A spring force of 550 N is applied to the end of the operating lever to ensure the band is initially in firm contact with the drum. A motor and worm gear box arrangement releases the brake.

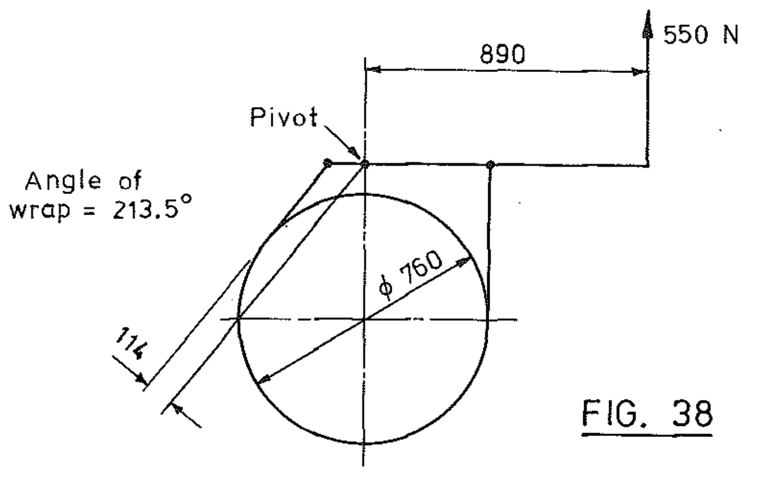


FIG. 38

The cotton based friction brake lining gives a coefficient of friction of 0.5.

(a) Determine the band tensions.

(b) Calculate the maximum unit pressure between the lining and drum if the lining is 90 mm wide.

(c) What is the maximum torque the brake will sustain for clockwise rotation?

Ans. (a) $T_1 = 6915.4$ N $T_2 = 3362.8$ N

(b) 0.2 MPa (c) 433.73 N.m

(11) Fig. 39 shows a hoisting system for raising and lowering a 3 tonne load. A brake is fitted to the drive motor shaft.

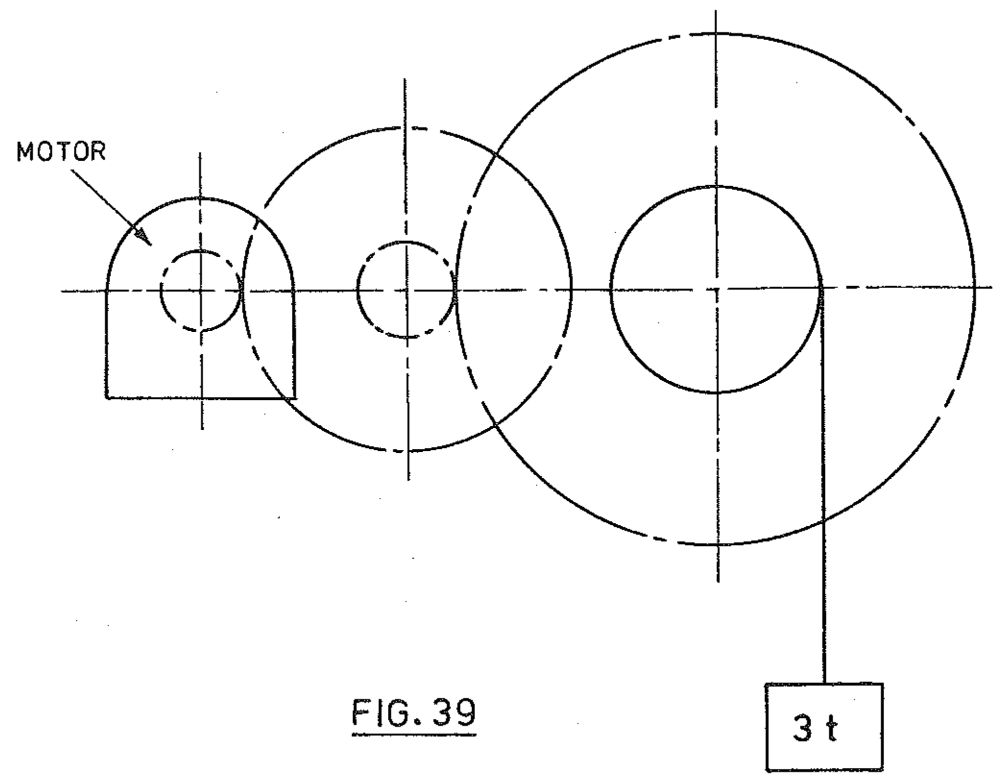


FIG. 39

Engineering details:

Electric motor .... 700 r.p.m. full load speed.
Gear reductions .... 5.1 each
Drum diameter .... 1.0 m rope centres.

Mass M. of I. of each shaft about its own axis of rotation:

motor shaft .... 0.76 $kg.m^2$
intermediate shaft .... 160 $kg.m^2$
drum shaft .... 1400 $kg.m^2$

Determine:

(a) the power requirements of the motor if each gear reduction assembly is 95% efficient.

(b) the torque requirement of the brake if the system has to be stopped in 1.1 seconds when the load is descending:

(i) if the gearing system were 100% efficient.
(ii) if each reduction efficiency is taken into account.

(c) the distance in which the load will stop when it is ascending and the brake is applied. (Use a brake torque of 1300 N.m.)

(i) if the gearing system were 100% efficient.
(ii) if each reduction efficiency is taken into account.

Ans.	(a) 47.81 kW	(b) (i) 1295 N.m	(c) (i) 0.3016 m
		(ii) 1193.93 N.m	(ii) 0.2918 m

(12) A cone clutch is to have a design rating of 200 N.m at 1250 r.p.m. The included angle = 13°, b = 64 mm, D = 350 mm and μ = 0.25.

(a) Find the axial force P_a required to transmit the torque.

(b) Find the axial force P_a required to engage the clutch if both halves are stationary.

(c) Determine p_{av} and p_{max} when the clutch is in operation.

(d) (i) Show that a force is required to disengage the clutch.
(ii) Determine the magnitude of the disengaging force.

(e) Calculate the heat rate being generated during engagement.

(f) If the slipping time is 0.27 seconds determine the amount of heat generated.

(g) Calculate the mean rating required for the linings and compare with the recommended value (Table No. 7, page 15.23.)

Use solid woven, asbestos based friction material.

Duty is very severe.

Ans.	(a) 528.4 N	(b) 1687.95 N	(c) 0.0677 MPa 0.0692 MPa
	(d)(ii) 631.07 N	(e) 26.18 kJ/s	(f) 3.53 kJ
	(g) 189.9 kW/m^2 230 kW/m^2		

(13) Fig. 40 shows a multiple disc clutch. The plates shown at x are of steel and are set on splines on shaft B to permit axial motion (except the last disc).

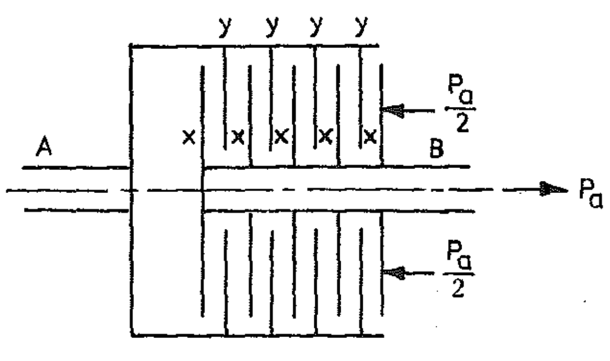


FIG. 40

The plates shown at y are bronze and are set in splines in the housing of shaft A.

The number of pairs of surfaces transmitting the torque is one less than the total number of discs. The number of surface pairs should be even if the design is such that no thrust bearings are needed.

Design a clutch similar to the above to transmit 5.2 kW at 900 r.p.m. The clutch is to run in oil for better heat dissipation and smoother engagement.

Engineering details:

D = 140 mm; d = 75 mm; μ = 0.12
p_{av} = 0.35 MPa maximum.

(13) (a) Determine the number of discs required. Use an even number of contact pairs.

(b) What axial force will be required for operation?

(c) Calculate p_{av} and p_{max}.

Ans. (a) 3 steel and 2 bronze (b) 2138.5 N
(c) 0.195 MPa; 0.279 MPa

(14) A single plate clutch with a lining on each side of the spinner plate has to transmit 24 N.m torque.

Use a service factor of 1.5.

μ = 0.25 for the asbestos based friction linings.
p_{av} = 0.30 MPa maximum value.

(a) Determine the inside and outside diameters of the clutch plate linings.

The ratio of the mean radius of the linings to lining width = 4.5.

(b) Calculate the axial force required to operate the clutch.

Ans. (a) D = 123.56 mm; d = 98.84 mm
(b) 1294.97 N

(15) Refer to Fig. 41.

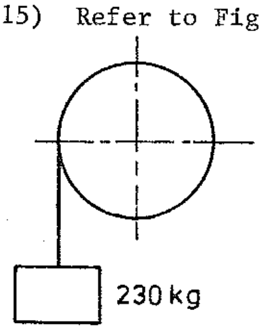


FIG. 41

A brake drum shaft allows the load to be lowered at a constant speed of 180 m/minute. The drum has a mass of 140 kg and a radius of gyration of 185 mm. The rope centres on the drum = 410 mm.

(a) Calculate the energy in the system.

(b) If the load is to be stopped in 0.5 seconds how much additional braking torque must be applied?

Ans. (a) 1548.07 N.m
(b) 423.14 N.m

(16) (a) If the load in Exercise 15 is at rest what torque must a clutch have in order to give the load an upward velocity of 180 m/min. in a slipping time of 0.3 seconds?

(b) Determine the heat rate being generated in the clutch.

Ans. (a) 1167.77 N.m
(b) 6.77 kJ/s

(17) A motorised capstan is shown in Fig. 42. A workman takes a 25 mm greasy hemp rope four times around the cast iron drum and applies a 140 N load at one end; the opposite end is attached to a load. The drum is 250 mm in diameter and rotates at 26 r.p.m.

(a) What load can be raised?
(b) What size motor is necessary allowing 94% overall efficiency?

Ans. (a) 619 kg (b) 2.363 kW

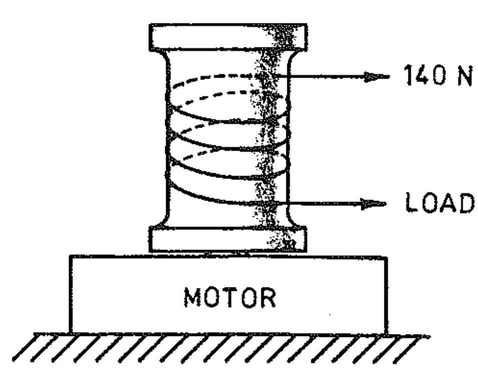


FIG. 42

Code requirements on brake design

AS 1418, Part 1 - 1986 SAA Crane Code
AS 1735, Part 2 - 1986 SAA Lift Code

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SAA Crane Code

Clause 7.6.2 Size and characteristics

Each brake shall be of torque rating, braking characteristics and heat-dissipation characteristics appropriate to its application on the crane, and shall have effective range of torque adjustment and adjustment to compensate for wear to obviate undue loss of braking efficiency during periods of time between normal servicing. At the end of such adjustment range, the brake shall comply with this Clause.

Brakes in hoisting motions and luffing motions shall be capable of exerting a restraining torque of 1.4 times the maximum static torque applied to the brake when the crane is suspending the safe working load.

Brakes in motions other than hoisting and luffing shall be capable of bringing the fully loaded crane to rest without shock in the shortest time consistent with safe working, and shall arrest the crane safely under all in-service conditions.

Clause 7.6.5.4 Springs. (Refer also to page 4.14 of this text.)

Springs shall be of the compression type and shall be manufactured from an appropriate grade of spring steel. Helical compression springs shall comply with BS 1726, Part 1 so that —

(a) the pitch of the spring coils shall not allow a broken spring to intercoil when the spring is in the minimum working load condition;

(b) when the spring is closed solid, the stress is not greater than the permissible design stress specified in BS 1726, Part 1, Appendix B, Paragraph B4; and

(c) where the spring is used on cranes of Classes C6, C7 and C8 and/or only one spring is used to apply the brake, the stress at maximum operating deflection does not exceed 75 percent of the permissible design stress specified in BS 1726, Part 1, Appendix B, Paragraph B4.

Springs may be designed in accordance with other approved standards.

Design stresses for brake components. Refer to Appendices Nos 3 and 4.

SAA Lift Code

Clause 7.2 Assumed loadings. Lift machine members in bending, shear, tension, or compression shall be capable of sustaining twice the actual computed static load with rated load in the lift car.

Members subject to torsion shall be capable of sustaining twice the actual static out-of-balance load with rated load in the lift car.

Clause 7.3 Factors of safety. The factors of safety for the design of driving machines, based on the loadings specified in Clause 7.2, shall be not less than —

(a) for steel, based on yield strength with an elongation not less than 14 percent in a gauge length of 50 mm 2;
(b) for steel, based on yield strength with an elongation less than 14 percent in a gauge length of 50 mm 2.5;
(c) for ductile metals other than steel (i.e. those with an elongation not less than 14 percent in a gauge length of 50 mm) based on yield strength .. 2.5;
(d) for grey cast iron in compression, based on tensile strength . . . 5;
(e) for grey cast iron in tension or bending, based on tensile strength ... 6.

Materials of gear teeth shall comply with the strength requirements specified in Clause 7.8.2.

NOTE: The above factors of safety provide for the abnormal and infrequent stresses resulting from safety gear and buffer engagement, which are included in the loadings specified in Clause 7.2. Components designed with these factors of safety are normally considered to have adequate reserve strength to prevent failure due to fatigue.

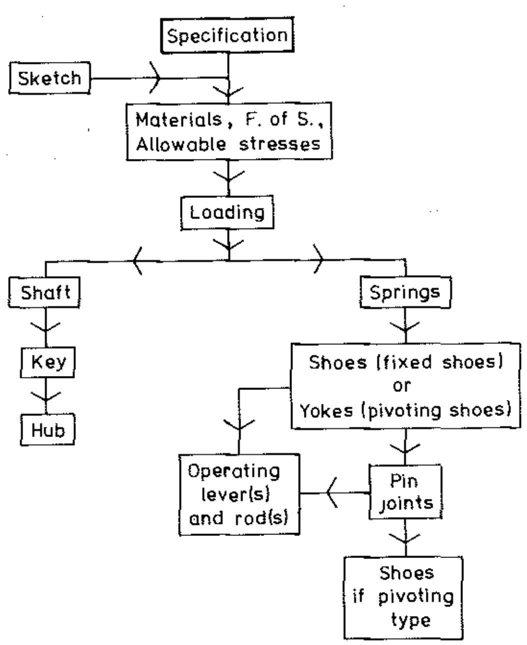
Clause 7.10 Brakes (partial extract). Every lift machine shall be provided with a brake complying with the following requirements:

(a) The brake shall be mechanically applied and electrically held off.
(b) During normal operation, the brake shall not be released unless power is applied to the lift motors.
(d) Toggles shall not be used for the normal operation of the brake.
(e) Where springs are used to apply brake shoes, the springs shall be in compression and adequately guided and supported. In the event of failure of any one spring, the brake shall continue to be operable to a significant extent.
(h) The brake shall be capable of stopping and holding the lift car with 125 percent of its rated load, from a test speed of 110 percent of the rated speed.
(k) Brakes shall have not less than two brake shoes. (Not service lifts.)

Design of components for shoe brake.

Outline design sequence.

Design sequence to be applied according to whether brake has fixed or pivoting shoes.



Notes on calculations, etc.

1. Sketch.
   Sketch an assembly of the brake on a "design by proportion" basis. Refer to pages 15.14 and 15.15.

2. Materials, F of S and allowable stresses.
   Suggested materials:
   M.S. - shoes, levers, yokes.
   CS1020, 1030 or 1040 (bright) - shaft, pins, rods.
   C.I. (or alternatively M.S.) - drum, base.
   Key steel - key.
   Spring steel - springs.
   F of S and allowable stresses to be determined by traditional methods or Code requirements as necessary.

3. Loading.
   Determine design torque from inertia or Code requirements as necessary. Obtain corresponding pin loads and spring operating force.

   Assume suitable lever layout for initial calculations such as that shown in Fig. 43.

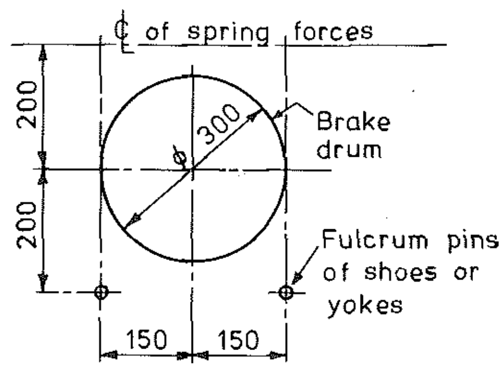


FIG. 43

4. Shaft, key and hub.
   Shaft and key to be designed by traditional or Code methods as required. Key length chosen should be between 1.25 (Deutschman) and 2.25 (Siegel) times the shaft diameter.

   Make O.D. of hub ≅ 1.75 x shaft diameter to allow for keyway. Hub length to suit key.

5. Springs.
   Design to traditional or Code requirements to suit application.

   In the absence of better information assume spring force is increased by about 15% during disengagement with new linings, and allow for maximum wear in each lining of about 2 mm.

6. Shoes i.e. fixed shoes.

   Determine minimum bearing area required and corresponding width of shoe.

   Calculate maximum B.M. acting on shoe and hence suitable cross-sectional dimensions. Use a simplified loading and geometry, together with either straight or curved beam formulas, see page 15.15 and Fig. 44 (page 15.56).

   Determine suitable diameter for

6. Yokes i.e. pivoting shoes.

   Determine maximum B.M. acting on yoke and hence suitable cross-sectional dimensions (ignoring shoe pivot pin hole for initial calculations).

   Calculate suitable diameter for each pin.

   Recheck yoke cross-sectional dimensions allowing for shoe pivot pin hole.

   Design operating lever(s) and rod(s).

6. Shoes i.e. fixed shoes (cont.).

each pin after the general method shown on pages 12.57 to 12.59. (Note. Account for total load on each pin, including component due to friction, where applicable.)

Design operating lever(s) and rod(s). Cranked levers to be designed as curved beams.

All stresses for shoes, pins, rods and levers to be determined by traditional or Code requirements as applicable.

6. Yokes i.e. pivoting shoes (cont.).

For further appropriate comments read "6. Shoes i.e. fixed shoes."

7. Shoes.

Determine minimum bearing area required and corresponding width of shoe.

Calculate minimum required thickness of shoe based on simplified loading and geometry as shown in Fig. 45.

Stresses to be determined by traditional or Code requirements as applicable.

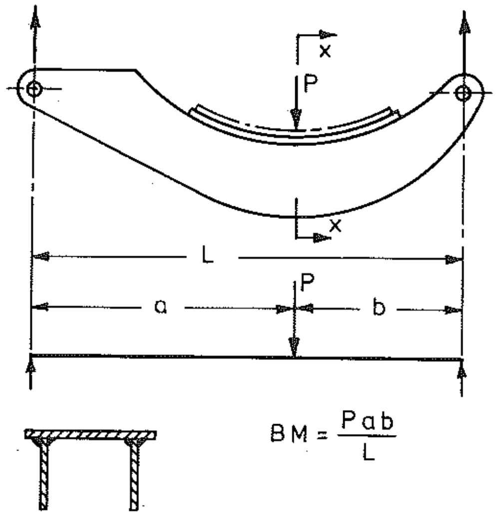


FIG. 44

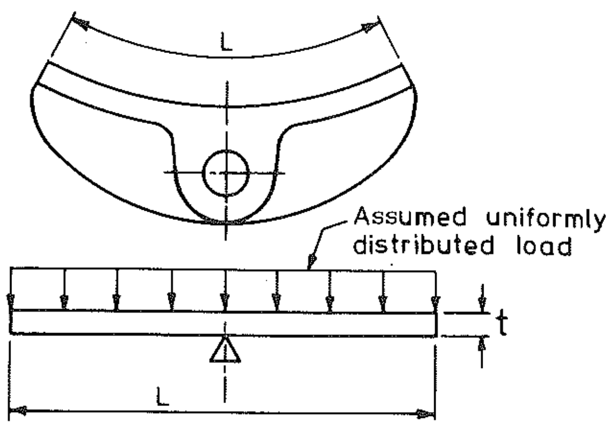


FIG. 45

References.

Ferodo Ltd.,
Friction Materials for Engineers,
Ferodo Ltd., 1961.

The authors wish to express thanks to Ferodo Ltd. for permission to use the above publication in the preparation of the sections on brakes and clutches in this chapter.

Newcastle Technical College, N.S.W., School of Mechanical Engineering.

The authors wish to acknowledge the assistance given by Mr James Baker, of the above College, in permitting their notes to be used in the preparation of "Design of components for shoe brake", pages 15.54 to 15.56.

Notes on electromagnetic brake selection.

In many engineering design situations a proprietary brake can be chosen from a manufacturer's catalogue. However this must not be done haphazardly. Full operating requirements on each particular application should be carefully ascertained.

The authors are indebted to GEC Industrial Products, a Division of GEC Australia Limited, for the following instructions on brake selection.

Magnetic brakes, both a.c. and d.c., have become well established for use on cranes, hoists, haulages, rolling mill auxiliaries, machine tools, and wherever positive stopping is important. It is found, however, that the majority are selected on insufficient data. Usually the brake manufacturer is only given the motor power and speed (the brake duty, if added, is seldom fully described). For some applications this may give satisfactory results, but there are many cases where trouble follows in the form of heavy maintenance or unsatisfactory operation.

A common criticism of magnetic brakes is that they are 'too fierce', stopping the load too quickly and causing excessive stress in the mechanical parts. A clapper type brake will necessarily act more suddenly than a solenoid-operated type with dashpots, but the real cause of the trouble is generally that the brake used gives a larger torque than is necessary. In most cases this is caused by the lack of sufficient data when the brake size was calculated. Consider a brake selected only from the motor power and speed. Assuming the brake drum is on the motor shaft (the first motion shaft) and the braking torque is made equal to the full load motor torque, then,

$$\text{brake torque } T = \frac{1000\ P}{\omega} \quad \text{N.m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where P is in kW and $\omega = \frac{2\pi N}{60}$.

Since the motor rating always includes its own safety factor or service factor the brake selected will probably apply more torque than that necessary to deal with the load. The stresses set up in the working parts of the system will thus be increased, perhaps unnecessarily.

The effect of friction torque may be appreciable in some applications but the high efficiency of modern gears and bearings is reducing its importance. Its precise value is difficult to estimate, but in any case it assists the stopping of the load and is generally ignored.

Further troubles arising from haphazard brake selection are rapid wear of brake linings and, in a.c. clapper brakes, an undue amount of maintenance on the clapper laminations. These are caused by the frequency of operation being beyond the designed capacity of the brake. All brakes absorb mechanical energy by converting it to heat, which is mainly dissipated by the brake drum. If the brake is operated too often it will absorb energy beyond the capacity of the drum, and the temperature of the brake lining will be raised above its working limit, causing rapid wear. A further consequence (with a.c. clapper brakes) is increased 'burring' of the laminations, necessitating adjustment after a very short time.

Thus it can be seen that the advantages of a well designed brake may be lost by choosing an unsuitable size or type for the particular application.

Braking torque required.

The brake may be required to do one of three things:

(a) bring a load to rest without consideration of time or distance.
(b) stop a load within a given time, distance, or number of revolutions, or
(c) hold a stationary load.

(a) If it is desired to bring a free running load to rest irrespective of braking time, the brake is usually designed to give from half- to full-load torque, calculated from (1). With a load of very high inertia, however, it may be necessary to check the amount of energy to be dissipated by the brake drum.

(b) For stopping a load under some limiting condition, such as a specified braking time, distance or number of revolutions, the problem is not quite so simple.

If it is again assumed that the brake drum is on the motor shaft, (the first motion shaft) then the total system inertia must be referred to this shaft. Some methods of brake torque calculation have already been demonstrated, see pages 14.11 to 14.16 and 15.26 to 15.27.

If there is linear motion involved in the system then this must be related to the brake drum shaft.

Let M = Total mass of components having linear speed, kg.

v = linear speed of components, m/s

N = speed of brake drum shaft, rev/min.

ω = speed of brake drum shaft, rad/s

I_{EL} = equivalent mass moment of inertia of components with linear motion, $kg.m^2$

I_{ER} = equivalent mass moment of inertia of components with rotational motion, $kg.m^2$

I_E = equivalent rotational mass moment of inertia related to rotational full-load speed of the braking medium, $kg.m^2$

$= I_{EL} + I_{ER}$

ϕ = angular deceleration of brake drum shaft, rad/s^2

then $$\tfrac{1}{2} I_{EL}\omega^2 = \tfrac{1}{2} Mv^2$$

$$\therefore \quad I_{EL} = M\left(\frac{v}{\omega}\right)^2 = M\left(\frac{60v}{2\pi N}\right)^2$$

and $$T = I_E\phi$$

Occasionally there are special cases where particularly accurate stopping is required. Two other factors become involved : the time required for the electrical operation of any controlling devices and the time required for the application of the brake. Designs of this type are best referred to the brake manufacturers.

(c) To calculate the braking torque required to hold a load (when already stationary) is not difficult. In the case of cranes, the brake has to be designed with a load factor of 1.4, AS 1418 Rule 8.3.5, see page 15.53. (If the crane is required to operate under excessively severe conditions then a heat dissipation calculation would also be necessary.)

Frequency of operation

Any of the foregoing operations may have to be performed frequently. The brake drum must then be able to dissipate energy fast enough to avoid overheating. Frequency of operation does not affect the braking torque required since the energy to be dissipated in stopping a given load, and therefore the heating of the drum is always the same.

Choosing the correct brake.

When determining the best size of brake for a given application, it has been usual to consider only the required braking torque. Experience has shown that for heavy service, the energy absorbed by the brake in bringing the moving parts to rest must be considered in order to prevent overheating and consequent rapid wearing of the lining. Unsatisfactory operation may arise from failure to allow for this factor.

In Table No. 11 are values for the maximum rate at which a drum can dissipate the absorbed energy without overheating. The figures quoted are for asbestos linings and are based on a brake drum temperature of 120°C using a well-proportioned cast iron brake drum with free air flow for cooling round the sides. Ambient temperature 30°C.

TABLE NO. 11.

Brake size mm	Braking torque (N.m). Typical magnetic brake	Energy dissipation of drum per hour		Max safe speed of drum rev/min
		Stationary kJ	Additional when rotating kJ per rev/min	
102	24	170	0.53	5750
152	80	350	1.09	3800
203	162	600	1.84	2850
254	340	915	2.80	2300
305	540	1290	3.93	1900

Worked Example No 8

An engineering application has the following requirements:
power of motor 13.5 kW, energy of moving parts to be dissipated = 4434 N.m, number of stops per hour = 240, speed = 725 rev/min., braking torque = 150% full load torque of motor, ratio of moving time to total time, 1:8.

Choose a brake.

(a) Braking torque $= \dfrac{1000\ P}{\omega} \times 1.5$

$$= \frac{1000 \times 13.5 \times 60}{2\pi\ 725} \times 1.5 = 266.72 \text{ N.m}$$

Table No 11 shows that on the basis of braking torque a 254 mm brake is required.

(b) The energy to be absorbed per hour
= energy of moving parts x number of stops per hour

$$= \frac{4434 \times 240}{1000} = 1064.16 \text{ kJ}$$

The energy that can be dissipated by a 254 mm brake is 915 kJ/hour while the drum is stationary with an additional 2.80 kJ per rev/min for the time during which the drum rotates.

Thus,

Total energy dissipated per hour =

(cont. on page 15.60)

$$\text{Energy dissipated by drum when stationary} + \left(\frac{\text{Rotating time}}{\text{Total time}} \times \begin{array}{c}\text{Energy}\\ \text{dissipated}\\ \text{per rev/min}\end{array} \times \text{rev/min}\right)$$

$$= 915 + \left(\frac{1}{8} \times 2.80 \times 725\right)$$

$$= 1168.75 \text{ kJ}$$

Thus the 254 mm brake is capable of absorbing the required 1064.16 kJ.

Referring to Table No 11 again it is seen that on the basis of both braking torque and drum heating a 254 mm brake is still required.

(c) Maximum speed of drum.
The speed of the drum as stated does not exceed 725 rev/min so the 254 mm brake would be satisfactory.

EXERCISES.

(18) It is required to stop a load of 136 kg having a radius of gyration of 0.15 m in four revolutions of the brake drum, the full load speed being 480 rev/min.

Determine the brake torque.

(153.8 N.m)

(19) A load of 9000 kg is moving in a straight line at a speed of 9 m/min. The drive motor has a full load speed of 730 rev/min and inertia of 0.72 $kg.m^2$. A second motion shaft is running at 240 rev/min and carries parts whose inertia is 2.11 $kg.m^2$.

Determine: (a) I_E if the system is 100% efficient.

(0.9828 $kg.m^2$)

(b) I_E for the deceleration phase if the shaft drive connection is 95% efficient.

(0.9714 $kg.m^2$)

(20) (a) A load of 13.5 tonne is being lowered from a 2.4 m diameter drum. The drum has an inertia of 840 $kg.m^2$.

If the load has to be brought to rest with a retardation of 5 m/sec^2, determine the braking torque required on the drum.

(243.464 kN.m)

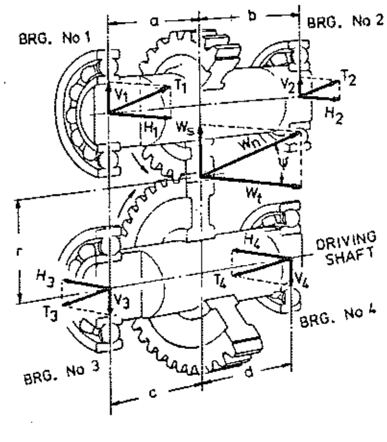
(b) The above drum is driven by an electric motor through a geared system. A brake is provided on the motor shaft. The I_E of the system (excluding part (a)) is 0.49 $kg.m^2$ and the drive system efficiency is 86%. The overall reduction = 500 : 1.

Determine the torque requirements of the brake.

(1439.6 N.m)

CHAPTER 16

TECHNIQUE
of
APPLIED MECHANICS



CHAPTER 16
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TECHNIQUE OF APPLIED MECHANICS

Applied mechanics forms the basic foundation of design work.
Applied mechanics is best mastered by doing numerical problems, the solid theory being only a small portion of the subject.

Generally the best approach in teaching is to give numerous solved problems of the types commonly encountered in design work, followed by set exercises.

The modern technique of problem solution is the free body diagram method. The rules for this method were given in the Section "Design Technique" in Chapter 1, page 1.13. It can be said with confidence that the great majority of all mechanics problems can be solved using free body diagrams. The basic method has already been demonstrated in problems in Chapter 14. Now more advanced problems can be attempted. An automatic brake has been chosen and presented in "project" form followed by a partially completed support framework analysis.

Worked Example No. 1.

Automatic brake for manually operated lift.

General description. Refer Fig. 1.

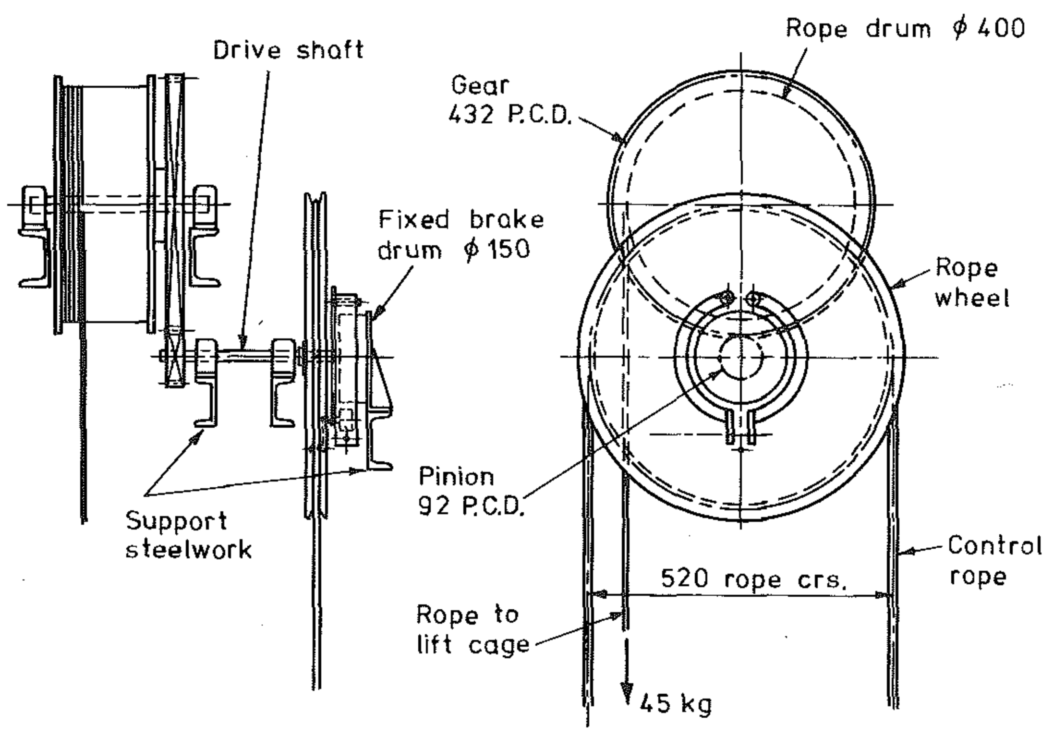


FIG. 1

The drive shaft has a pinion at one end which meshes with a gear on the rope drum shaft. The opposite end of the drive shaft has a brake mounting plate fixed to it on which the brake shoes, spring, actuating lever and pins are assembled. The brake drum is fixed to the lift framework. The rope wheel for controlling lift operations runs "free" on the drive shaft.

Operation of brake. Refer to Fig. 2.

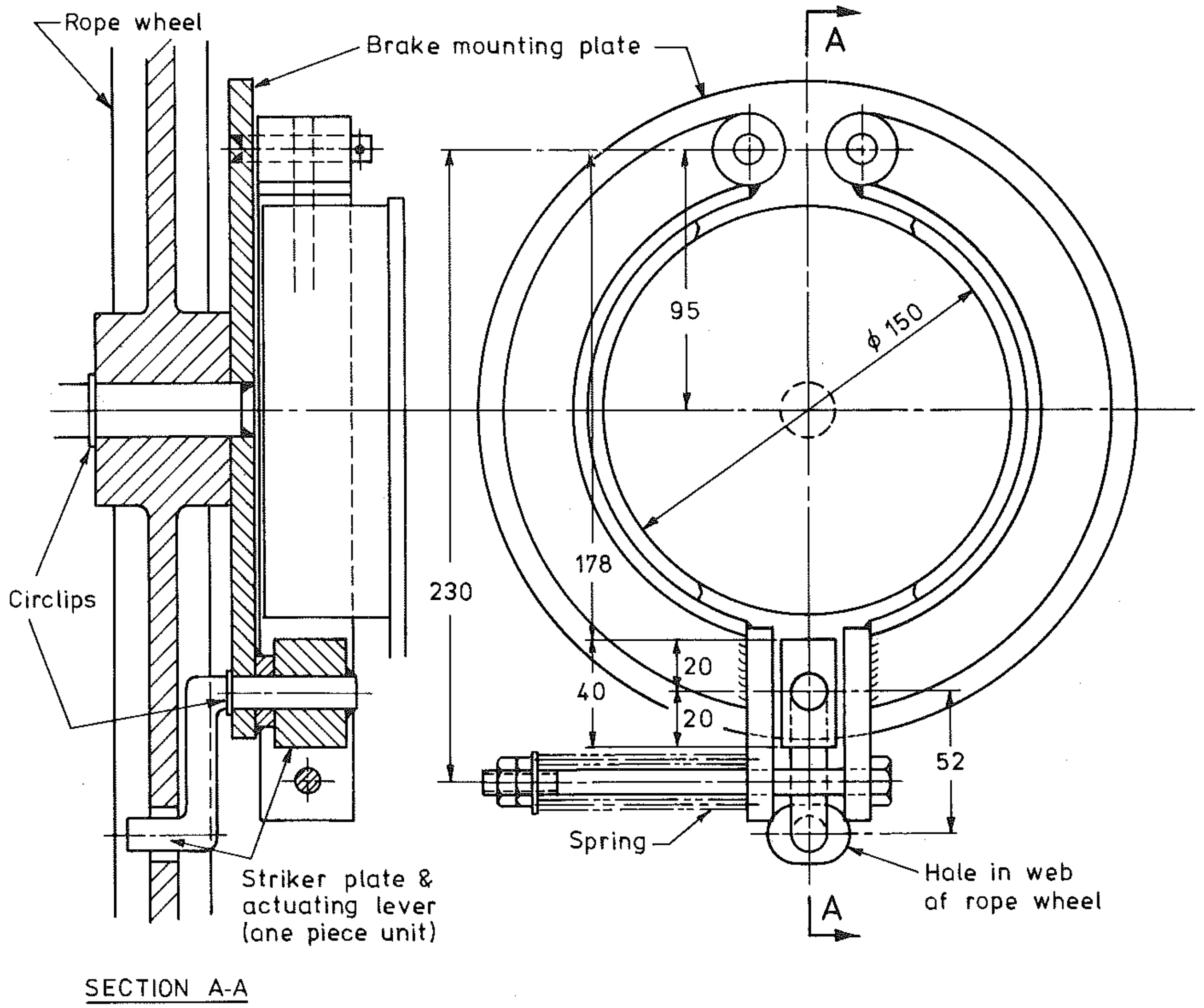


FIG. 2

When the control rope is pulled for the lifting or lowering operation, the rope wheel rotates "free" on the drive shaft until the hole in the web of the wheel strikes the end of the actuating lever.

Further rotation of the rope wheel then causes the actuating lever to turn slightly on its own pivot. This results in the striker plate rotating, forcing the brake shoes apart so releasing their grip on the fixed brake drum.

From this point on, the rope wheel, mounting plate with the brake shoes and drive shaft rotate as a unit and the load is raised or lowered as appropriate. When the pull on the control rope is released the brake applies automatically.

The test load to be lifted on the single fall of rope is 45 kg.

Use this load as your design figure.

(a) Design the operating spring.

(b) Determine the rope pull force required to release the brake.
Compare this force with the force required on the control rope if there were no brake. Comment on any difference.

(c) Do a "follow through" analysis of the brake i.e. draw free body diagrams of all elements in the brake system in turn showing that each one is in equilibrium.

<u>NOTE</u>: (i) coefficient of friction, brake lining to shoe, $\mu = 0.25$

(ii) angle of contact for each shoe = 120°

<u>ANSWER</u>

(a) Tangential force on brake drum

$$= 45\ g \times \frac{400}{432} \times \frac{92}{150} = 250.7\ \text{N}$$

Refer to Fig. 3(a), page 16.5.

$$\frac{F_1}{P_1} = \frac{F_2}{P_2} = \mu\left(\frac{4\ \sin\frac{\theta}{2}}{\theta + \sin\theta}\right) = \mu'$$

$$= 0.25\left(\frac{4\ \sin 60°}{2.0944 + \sin 120°}\right) = 0.293$$

from which $P_1 = \dfrac{F_1}{0.293}$ and $P_2 = \dfrac{F_2}{0.293}$

$$\Sigma M_{o_1} = S\ (95 + 135) - F_1 55 - P_1 95 = 0$$

from which $F_1 = 0.606\ S$

$$\Sigma M_{o_2} = S\ (95 + 135) + F_2 55 - P_2 95 = 0$$

from which $F_2 = 0.854\ S$

$$F_1 + F_2 = 250.7$$

$$0.606\ S + 0.854\ S = 250.7$$

$$\therefore\ S = 171.71\ \text{N}$$

Spring design: Refer to Chapter 4, page 4.1 for terminology.

171.71 N at say 25 mm compression.

Try D = 16 mm and d = 2.80 mm

$$\therefore \quad C = \frac{16}{2.8} = 5.71$$

$$f_s = K \frac{8PD}{\pi d^3}$$

$$= 1.27 \frac{8 \times 171.71 \times 16}{\pi \, 2.8^3} = 404.75 \text{ MPa}$$

Maximum permissible stress = 475 MPa
(Table No. 2, Chapter 4, page 4.7)

$$\delta = \frac{8 \, PD^3 n}{G \, d^4}$$

$$\therefore \quad n = \frac{\delta \, Gd^4}{8 \, PD^3} = \frac{25 \times 82\,700 \times 2.8^4}{8 \times 171.71 \times 16^3} = 22.6 \text{ say } 23$$

Chock length = (23 + 2) 2.8 = 70 mm

Free length = 70 + 25 = 95 say 98 mm

$$\text{Spring rate} = \frac{171.71}{25} = 6.87 \text{ N/mm}$$

∴ spring force P at chock

$$= (98 - 70)\, 6.87 = 192.36 \text{ N}$$

$$f_s = 1.27 \frac{8 \times 192.36 \times 16}{\pi \, 2.8^3} = 453.42 \text{ MPa} < 475 \text{ MPa}$$

Spring Specification:

13.2 mm inside diameter 2.8 mm diameter carbon steel spring wire
Table 1 AS 1472 - 1973.

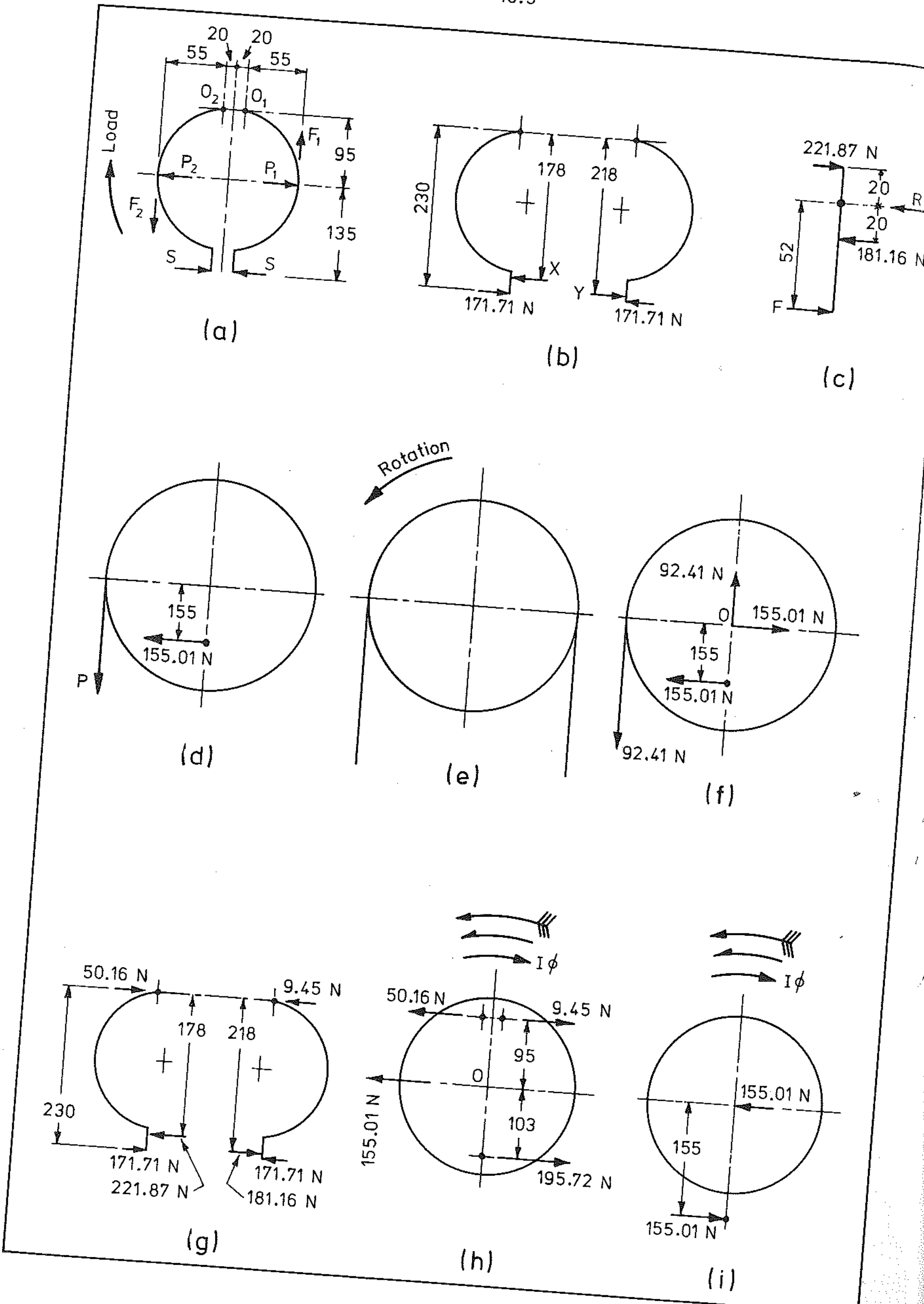
98 mm free length.

23 working coils plus one extra each end closed and ground square.

NOTE: The type of spring required in the brake is a slow rate spring. This is to minimise the spring force difference when wear has taken place on the linings (the spring force will become progressively less).

(b) Refer to Fig. 3(b), page 16.5.
This gives free body diagrams of the brake shoes when the striker plate is on the point of just opening the shoes.

{The control rope force depends on the brake spring force only and is independent of the load in the lift cage or whether the load is being raised or lowered.}



20
20
55
55
O2
O1
Load
F1
95
P2
P1
F2
135
S
S
(a)
230
178
218
X
Y
171.71 N
171.71 N
(b)
221.87 N
20
20
R
52
181.16 N
F
(c)
Rotation
155
155.01 N
P
(d)
(e)
92.41 N
O
155.01 N
155
155.01 N
92.41 N
(f)
50.16 N
9.45 N
178
218
230
171.71 N
221.87 N
171.71 N
181.16 N
(g)
Iϕ
50.16 N
9.45 N
95
O
155.01 N
103
195.72 N
(h)
Iϕ
155.01 N
155
155.01 N
(i)

FIG. 3

The striker plate contacts the shoes at X and Y (assuming the load is being raised).

$$\text{Force at X} = 171.71 \times \frac{230}{178} = 221.87 \text{ N}$$

$$\text{Force at Y} = 171.71 \times \frac{230}{218} = 181.16 \text{ N}$$

Refer to Fig. 3(c), page 16.5, for a free body diagram of the actuating lever.

Force on end of actuating lever

$$= \frac{221.87 \times 20 + 181.16 \times 20}{52} = 155.01 \text{ N}$$

Fig. 3(d), page 16.5, indicates a free body diagram of the rope wheel. Taking moments about centre

$$\text{Rope pull} = \frac{155.01 \times 155}{260} = 92.41 \text{ N}$$

Force on control rope if there were no brake in system.

$$= 45 \text{ g } 200 \times \frac{92}{432} \times \frac{1}{260} = 72.32 \text{ N}$$

The excessive rope pull force of 92.41 - 72.32 = 20.09 N will accelerate the system.

(c) Consider the lifting operation as shown in Fig. 3(e), page 16.5.

Free body diagram of rope wheel, Fig. 3(f), page 16.5.

Moments about 0.

$$155.01 \times 155 = 92.41 \times 260$$

$$24\,026.6 = 24\,026.6 \text{ (checks)}$$

Lateral forces balance, vertical forces balance.

Free body diagram of actuating lever, Fig 3(c), page 16.5.

Moments about F.

$$181.16 \times 32 + R\,52 = 221.87 \times 72$$

$$181.16 \times 32 + (155.01 + 221.87 - 181.16)52 = 221.87 \times 72$$

$$15\,974.6 = 15\,974.6 \text{ (checks)}$$

Free body diagram of shoes, refer to Fig. 3(g), page 16.5.

Left hand shoe, moments about spring force:

$$221.87\,(230 - 178) = 50.16 \times 230$$

$$11\,537.24 \cong 11\,536.8 \quad \text{(checks)}$$

Right hand shoe, moments about spring force:

$$181.16\ (230 - 218) = 9.45 \times 230$$
$$2173.92 \cong 2173.5 \quad \text{(checks)}$$

Free body diagram of backing plate, refer to Fig 3(h), page 16.5.

It has already been shown that the load will be accelerated.

Moments about 0:

$$I\phi = 195.72 \times 103 + 50.16 \times 95 - 9.45 \times 95$$
$$= 24\ 026.6 \text{ N.mm}$$

($I\phi$ is the inertia "torque".)

Free body diagram of complete assembly less rope wheel, Fig, 3(i), page 16.5.

$$I\phi = 155.01 \times 155$$
$$= 24\ 026.6 \text{ N.mm (checks with previous calculations).}$$

Static torque applied to brake drum from load

$$= 250.7\ \{\text{from part (a)}\} \times 75$$
$$= 18\ 802.5 \text{ N.mm}$$

Torque to accelerate system

$$= 24\ 026.6 - 18\ 802.5$$
$$= 5224.1 \text{ N.mm}$$

Rope force to accelerate system

$$= \frac{5224.1}{260} = 20.09 \text{ N (checks with result from Section (b)).}$$

<u>Worked Example No. 2</u>.

The free body diagram method can be used on support structures.

Refer to Fig. 4, page 16.8.

Two identical frames are connected laterally and 1600 N load is applied as shown (this equals 800 N per frame).
The complete assembly runs on frictionless wheels.

All connections are pinned except insofar as members ABC, DEF and GCE are continuous in themselves.

Calculate the B.M. and axial force acting in each member and show these on a force diagram.

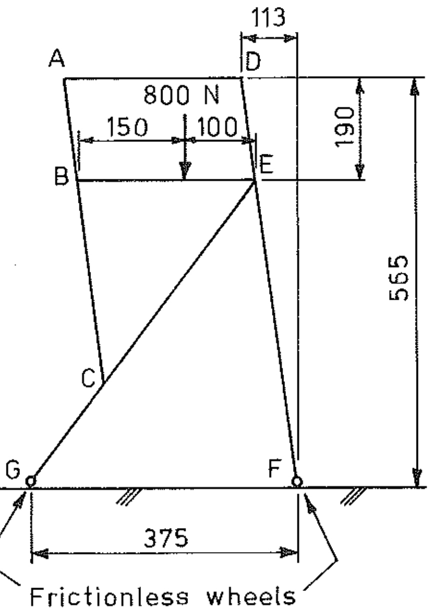


FIG. 4

ANSWER

First determine the reactions.

Moments about G.

375 F = 800 x 200 (By scaling)

$\therefore$ F = 426.67 N

Moments about F.

375 G = 800 x 175 (By scaling)

$\therefore$ G = 373.33 N

check 373.33 + 426.67 = 800 N

Load = 800 N

Fig. 5(a), page 16.9, shows a free body diagram of member DEF.
Member AD must be in compression.

Take moments about E.

426.67 x 75 = 190 D

$\therefore$ D = 168.42 N and $\therefore$ AD = 168.42 N compression.

Resolve at points A and D, Fig 5(b), page 16.9, to give the following results:

Portion DE of member DEF has 34 N compression.

Portion AB of member ABC has 34 N tension.

Resolve at points G and F, Fig. 5(c), page 16.9, to give the following results:

Portion FE of member DEF has 420 N compression.

Fig. 5(d), page 16.9, shows a free body diagram of member BE

$$\text{B.M.} = \frac{Wab}{L} = \frac{800 \times 0.15 \times 0.1}{0.25} = 48 \text{ N.m.}$$

Taking moments gives reactions of:

E = 480 N and B = 320 N

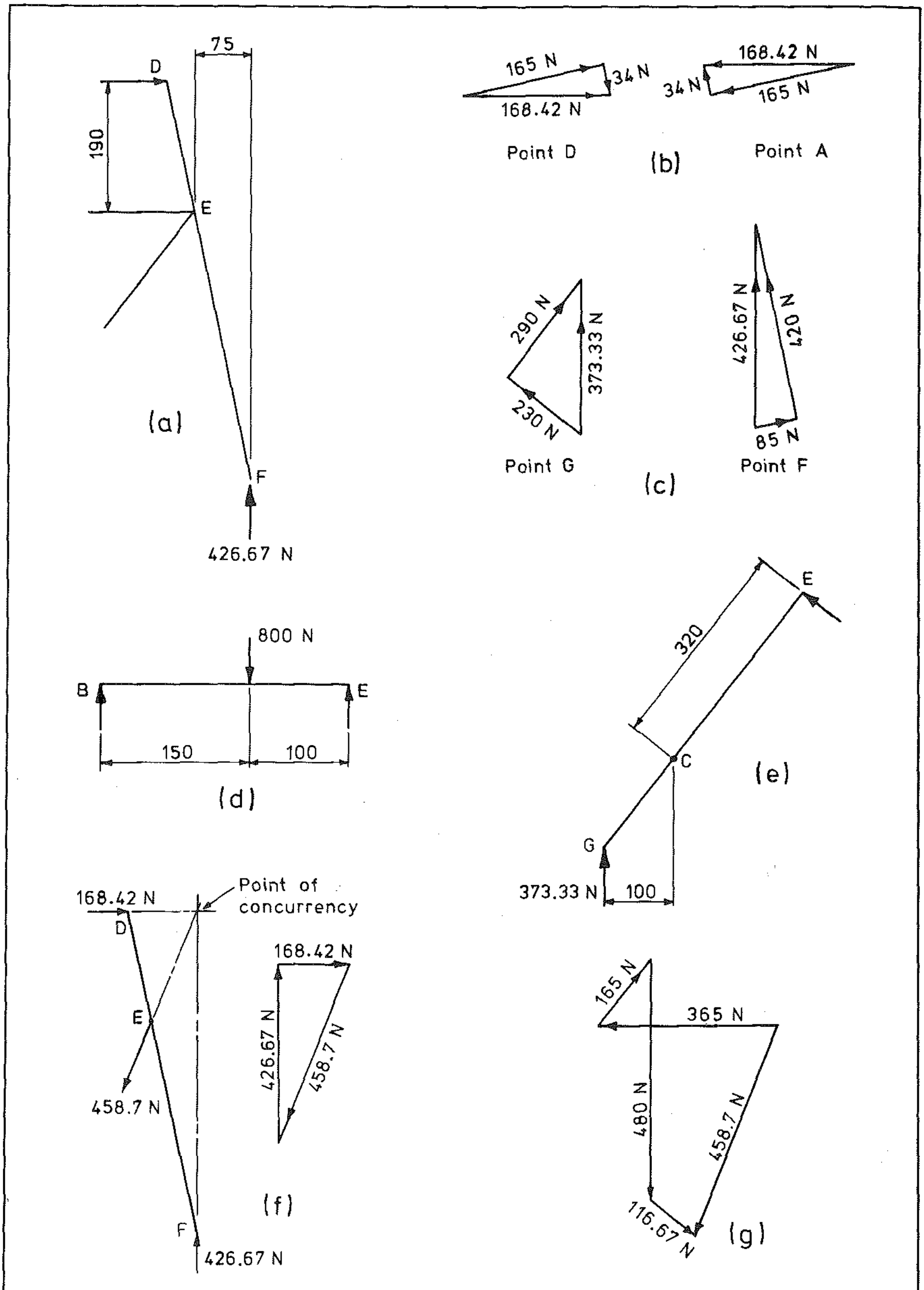
These reactions represent a component only of the absolute force acting at each point.

Fig 5(e), page 16.9, gives a free body diagram of member GCE.

Moments about C:

373.33 x 100 = E x 320

$\therefore$ E = 116.67 N.



75
D
190
E
(a)
F
426.67 N
165 N
34 N
168.42 N
Point D
(b)
168.42 N
34 N
165 N
Point A
290 N
373.33 N
230 N
Point G
426.67 N
420 N
85 N
Point F
(c)
800 N
B
E
150
100
(d)
E
320
C
(e)
G
373.33 N
100
168.42 N
Point of concurrency
D
E
458.7 N
F
426.67 N
168.42 N
426.67 N
458.7 N
(f)
165 N
365 N
480 N
458.7 N
116.67 N
(g)

FIG. 5

This represents another component of the force absolute acting at point E. (the component causing the B.M. in GCE).

A free body diagram of member DEF is shown in Fig. 5(f), page 16.9. The 458.7 N force is the absolute load acting on the member at point E. By drawing a polygon of forces with this force and the "normal" components of the forces from members EB and ECG the axial forces in the latter two members can be established; refer to Fig. 5(g), page 16.9.

The student is now left to complete the problem.

The answer is given in Fig. 6.

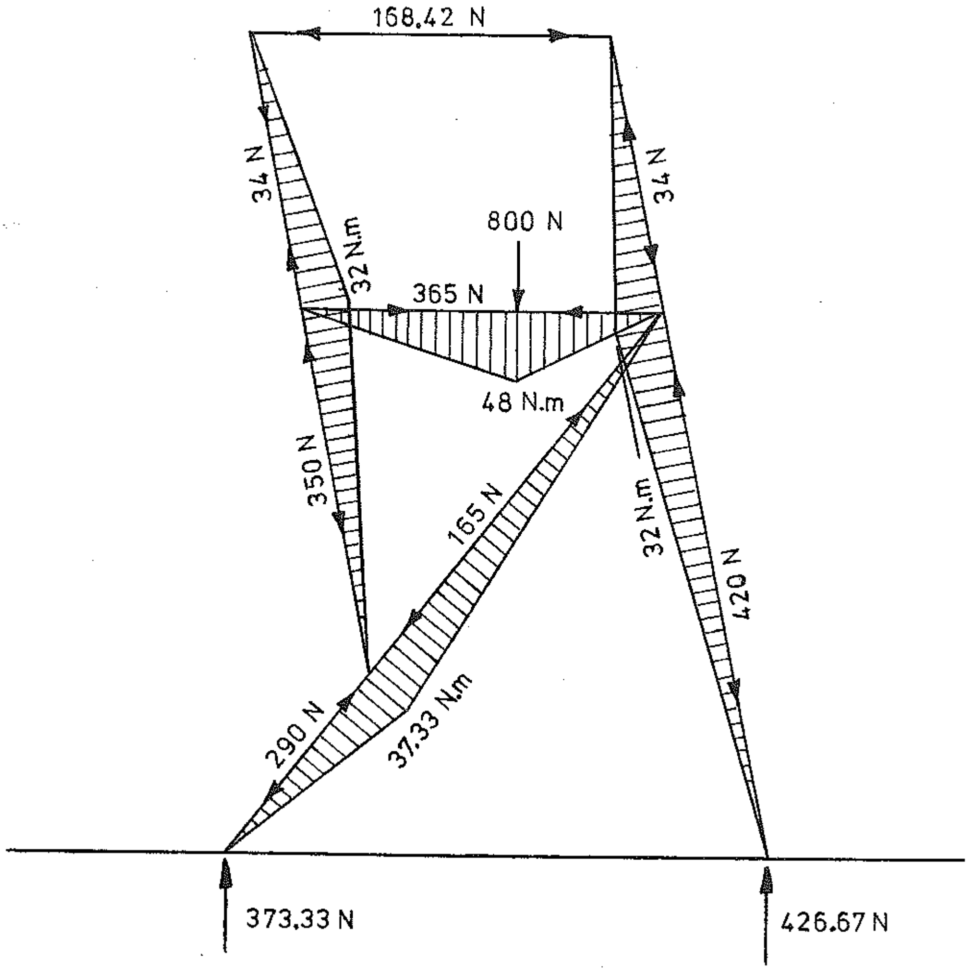


FIG. 6

EXERCISES.

1. A disc of M of I 20 kg.m^2 is rotating at 300 r.p.m. A second disc of M of I 50 kg.m^2 is brought into firm contact with the first. The second disc is initially not rotating but two seconds after contact the composite assembly is rotating at uniform speed.

   Neglect any frictional losses.

   (a) What is the final speed of the composite assembly?

   (b) (i) What is the total initial energy in the systems?
   (ii) What is the final energy in the composite system?

(iii) Explain why there is an energy difference.

(a) 85.714 r.p.m.

(b) (i) 9869.588 N.m
(ii) 2819.900 N.m

2. The link AB, Fig. 7, in a machine mechanism has a mass of 4.5 kg. The linear acceleration of the centre of gravity, G, at the instant shown is 60 m/s^2 and the angular acceleration of the link around G is 120 rad/s^2 clockwise.

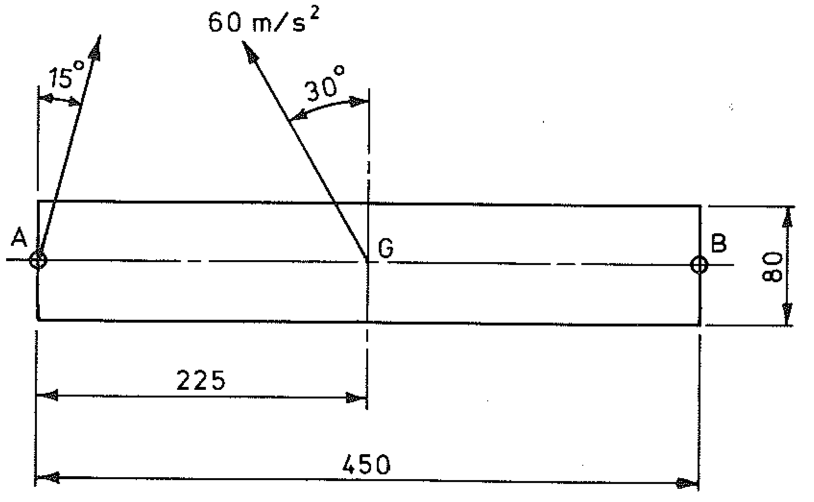


FIG. 7

The link is revolving in the horizontal plane.

Find the forces which must be applied through pins A and B to give the stated motion (the direction only of the force at A is given in Fig. 7).

F_A = 143 N; F_B = 197 N at 29°.

3. Repeat Exercise 2 for rotation in the vertical plane.

F_A = 165.85 N; F_B = 213 N at 33.5°

4. Repeat Exercise 3 for the case where the angular acceleration is about a point 100 mm to the right of A.

The directions of the force at A and the linear acceleration of G are unaltered.

F_A = 252.85 N; F_B = 202.5 N at 10°.

5. Fig. 8, page 16.12, gives the diagrammatic representation of a bucket elevator.

Engineering details:

Elevator bead and boot shaft assemblies, M of I = 1.3 $kg.m^2$ each
Mass of elevator chain = 76 kg each side
Mass of buckets = 30 kg each side
Mass of product = 50 kg total on lifting side.

Overall efficiency allowing for digging effect of buckets = 50%

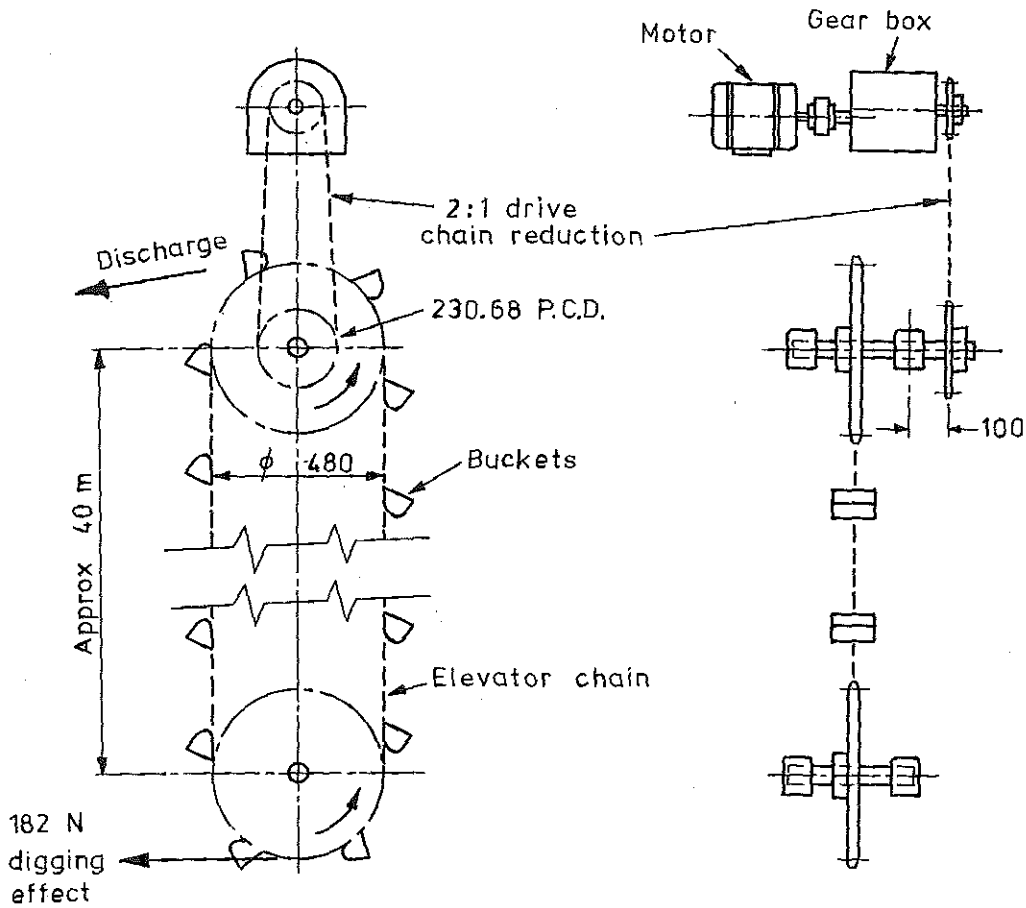


FIG. 8

Individual efficiencies:

motor to elevator headshaft	=	95%
elevator headshaft assembly	=	95%
elevator chain	=	80%
elevator bootshaft assembly	=	95%
digging effect of buckets	=	182 N

M of I of motor, gear box and chain drive reduction about motor shaft for acceleration phase = 0.0012 $kg.m^2$

Gear box output speed (full load) = 111 r.p.m.

Motor shaft speed (full load) = 2920 r.p.m.

(a) At what theoretical speed should the headshaft run to ensure discharge of product from buckets?

(b) Determine the motor power required for normal full speed running using the information nominated in the "Engineering details" and Fig. 8.

(c) Calculate the pull in the drive chain from the gear box for normal full speed running. Assume a 1.5 kW motor is chosen for the job.

(d) Determine the pull in the drive chain from the gear box if the motor has to be started under full load conditions.
The motor gives torque at starting (T_{max}) of 225% full load torque.

(a) 61.05 r.p.m. (b) 1.368 kW (c) 2125.76 N (d) 4431.59 N

6. Five smooth steel pipes each of mass 500 kg are placed between container walls as shown in Fig. 9.

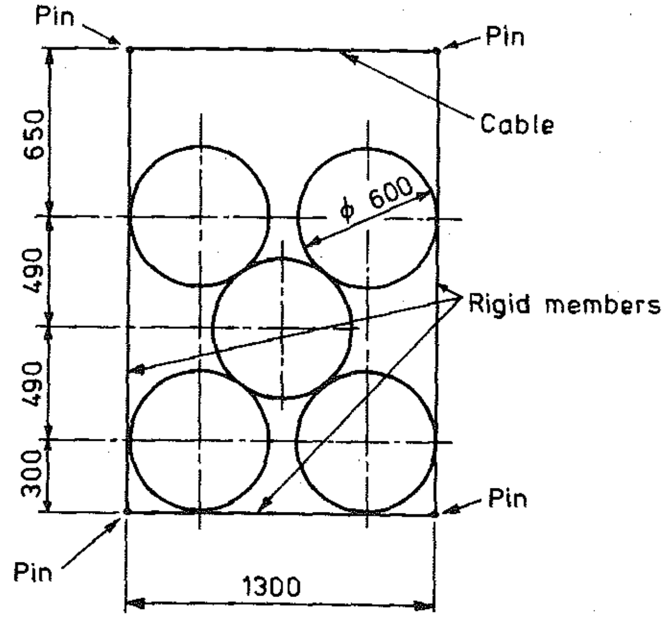


FIG. 9

Determine the force in the cable.

Ans. 3132 N.

7. Refer to the Dump Truck shown in Fig. 10.

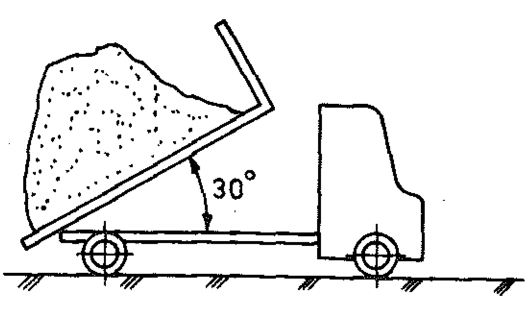


FIG. 10

Calculate the minimum required acceleration of the truck, to the right, which will cause the sand mass to slide down the box. $\mu = 0.68$

Ans. $0.723\ m/s^2$

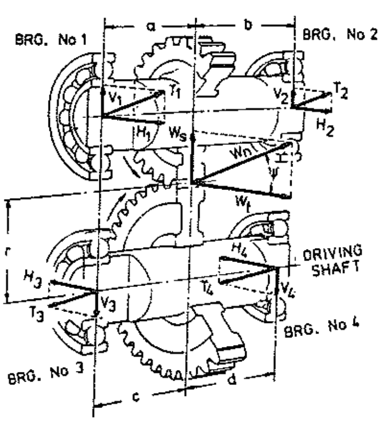
8. A pair of freight car wheels with axle has a mass of 400 kg. Wheel diameter = 0.8 m, $I = 60\ kg.m^2$. The assembly is moved along horizontal rails by a horizontal force of 900 N applied at the C of G of the system.

Determine the linear acceleration.

Ans. $1.16\ m/s^2$

CHAPTER 17

CHOICE
of
ELECTRICAL EQUIPMENT



CHAPTER 17
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CHOICE OF ELECTRICAL EQUIPMENT

The authors are indebted to Paul Niehoff, Electrical Engineer with the Victorian Railways Department for his assistance in compiling this Chapter.

The intention is:

(i) to provide essential "back-ground" information in electrical knowledge for the Mechanical Designer engaged in general design work in industry.

(ii) to give detailed information on some of the electrical equipment commonly used.

Some prior elementary knowledge of electrical terminology and theory is required. This chapter is not intended as a teaching document from first principles.

Electric Power

An adult male working at his maximum effort can develop about 0.3 kilowatt of power for approximately one hour.

Power x time = Energy or Work

Electrical energy is measured in kilowatt hour, then

kilowatt x time in hours = kilowatt hour.

Three men working at maximum effort for one hour will do the equivalent work of approximately 1 kWh.
On today's costs, for the same price we can do electrically what it would take 150 men to do.
Again, if a particular electrical generating station produces one million kilowatts it would take 3.3 million men working at maximum capacity to produce the same power.

Sources of electrical energy.

1. Fossil fuels are burnt to provide heat energy which is converted to mechanical energy, then converted to electrical energy. Total efficiency is about 30 per cent maximum.

2. The mechanical potential energy of water at a height is converted to kinetic mechanical energy and this is converted to electrical energy. Total efficiency is about 70 per cent.

Types of current.

(a) Three phase A.C.
(b) Single phase A.C.
(c) Direct current D.C.

(a) Three phase current is transmitted using three active lines, a neutral and an earth. Its basic use is industrial.
Reasons for use of three phase electricity:
In single phase circuits the power is pulsating from zero to full, twice per cycle. When the power factor is poor, the power is negative for parts of each cycle.

With three phase circuits, although the power in each phase is pulsating, the power supplied to a balanced load is constant. Because of this, the operating characteristics of three phase apparatus are usually superior to those of single phase apparatus e.g. constant torque instead of pulsating torque.

Three phase machinery is smaller, lighter and more efficient than single phase equipment. Also three phase circuits require only 0.866 times as much copper to transmit the same power as single phase circuits.

(b) Single phase current is transmitted using one active line, a neutral and an earth. Its basic use is domestic.

(c) Only a minute proportion of all electricity generated is direct current. It is required occasionally for special purposes such as certain types of control equipment etc. (The notable exceptions to the above are trains and trams which favour the use of D.C.)

Electric motors.

The following types of electric motors will now be discussed:

Single phase

Three phase
(1) Squirrel cage
(2) High torque
(3) Slip ring
(4) Synchronous

Direct current

Geared

Vari-speed

Single phase motors.

This type of motor has a relatively small application in industry.

The complexities in the starting equipment places them at a disadvantage when compared with the three phase squirrel cage induction motor. Also the maximum power of single phase motors is very limited; commonly about 4 kW is the largest unit obtainable.

The main uses of single phase motors are in the domestic field and in other locations where only single phase power is available.

The following methods of starting are available:

(a) split phase
(b) capacitor
(c) repulsion - induction
(d) universal

(a) Split phase starting gives low starting torque and is generally used only on motors of 0.25 kW or less.

(b) Capacitor starting gives high starting torque. A centrifugal switch is involved in the circuit and this cuts out the capacitor at 70% to 80% of full speed.

(c) Repulsion - induction starting is usually found only in older equipment since capacitor start motors do the same job and require less maintenance.

(d) Universal motors (modified D.C. series motors) are used for electric drills, vacuum cleaners, sewing machines etc. They can be run on D.C. or A.C. power.

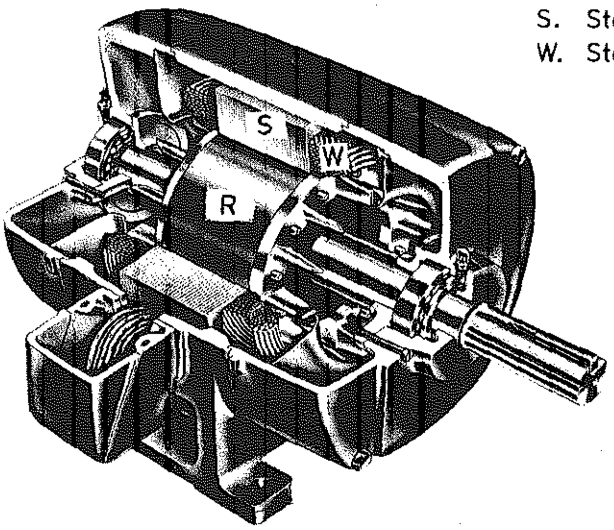
Three phase motors.

(1) Squirrel cage motor

The most common type of motor for industrial use is the three phase squirrel-cage induction motor. This motor is simple, rugged and foolproof, and the current is induced in the rotor bars by induction, so that slip rings or commutator is not required.

Fig. 1 shows a cut-away view of such a motor.

POLYPHASE INDUCTION MOTOR



R. Rotor assembly
S. Stator assembly
W. Stator windings

A cutaway view of a squirrel-cage induction motor (General Electric Company)

FIG. 1

Overload capacity.

The maximum momentary overload to which induction motors may be subjected is limited by the pull-out torque.

Motors having continuous maximum rating will withstand the following momentary overloads in torque for 15 seconds:

up to and including 40 kW	100%
over 40 kW up to 380 kW	75%
over 380 kW	60%

Speed and direction of rotation.

The full load speed of induction motors is less than the no-load speed, the difference varying from approximately four per cent for small motors to one per cent for large 50 cycle motors. In the rare case of full load over-run the motor speed will be correspondingly greater than the no load (synchronous) speed. Table No. 1 gives an indication of the speed characteristics of squirrel cage motors.

The direction of rotation of any motor can be reversed by interchanging any two of the stator supply connections.

Table No. 1

No. of poles	No load (synchronous) speed r.p.m.	Approximate full load speed r.p.m.
2	3000	2900
4	1500	1410
6	1000	950
8	750	710

Starting characteristics

When selecting a squirrel-cage motor, care should be exercised to ensure that the torque developed at starting is greater than the requirements of the load. The starting torque of the motor depends on the voltage applied to the stator terminals, so a choice must be made from three types of starter:

direct on line, D.O.L.

star delta

auto-transformer.

When star delta starters are used, additional connections from the stator windings are necessary and must be specified when ordering the motor.

Fig. 2 shows a typical torque-speed curve for a squirrel-cage motor when started D.O.L.

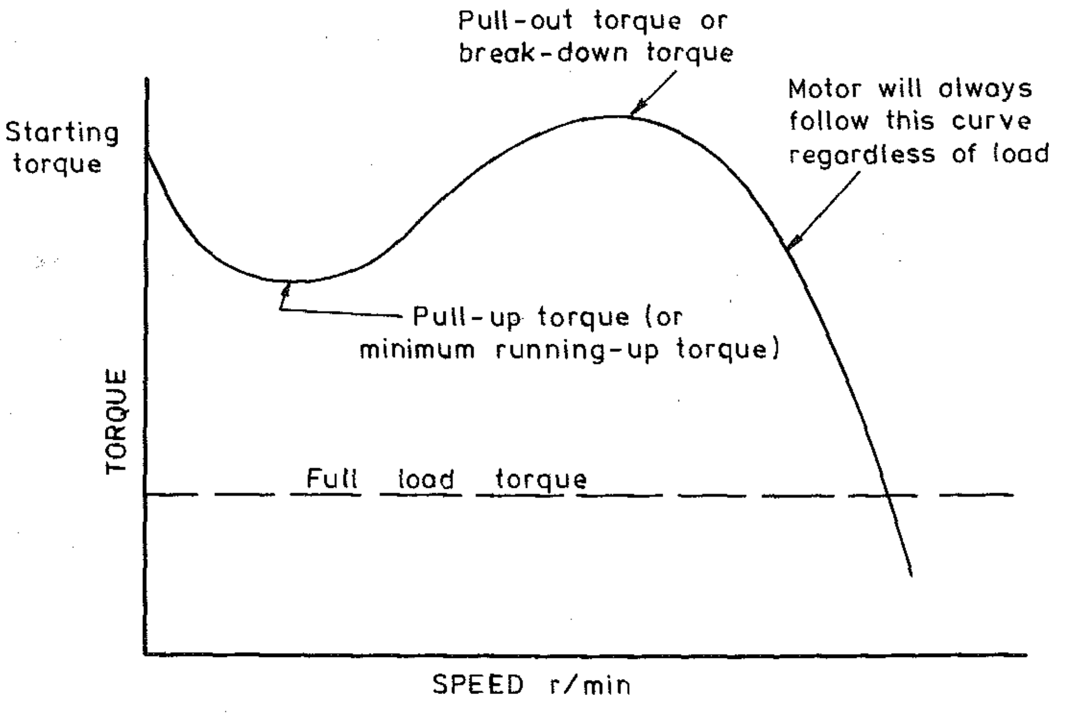


FIG. 2

Starting characteristics for different starters are given in Table No. 2. These figures represent average values for 50 cycle motors; they are not necessarily suitable for every installation and lower values of starting current and/or higher values of starting torque can be provided when necessary for a particular application.

Table No. 2

Starting characteristics for different starters controlling squirrel cage motors.

Type of starter	Direct on line	Star-Delta	Auto-Transformer 50% tap	Auto-Transformer 65% tap	Auto-Transformer 80% tap
Percentage of line voltage applied to motor terminals	100	58	50	65	80
Percentage of full-load torque at start (approx)	150 to 200	90	25	42	64
Percentage of full-load current at start (approx)	600 to 700	200	150	250	380

Refer to the section in Chapter 14 entitled "Choice of motor power" page 14.18, for the applied mechanics involved in motor selection when inertia (i.e. "torque for acceleration") is the controlling factor. This shows the practical use of a motor "torque-speed" curve.

Notwithstanding the above, the great majority of industrial applications have no unusual requirement and are quite satisfactorily handled by "off the shelf" motors. Motors obtainable today are of balanced electrical design and provide ample starting torque for ordinary purposes.

(2) High torque induction motors.

High torque, high slip squirrel cage induction motors are used in flywheel type applications e.g. a punch press. In this operation the flywheel slows as it transfers some of its kinetic energy to the load. This means that a motor is required that is capable of speed variation to transfer enough energy to the system to carry the load through the peak in the duty cycle.

The motor must also be capable of quickly accelerating the system back to its original speed thus replacing the lost kinetic energy to the flywheel in preparation for the next cycle.

The high torque induction motor meets all the above requirements.

(3) Slip ring motors.

The windings of "wound rotor" induction motors are connected to slip rings by means of which variable external resistance can be connected in the rotor circuit for the purpose of keeping down starting currents and/or controlling speed.

Slip ring motors are often used for more severe service conditions e.g. mine hoisting equipment.

These motors are capable of developing starting torques up to their maximum torque, but when starting against a load exceeding full load the corresponding starting current will be slightly greater than the normal current for that load.

For example, with full-load torque at starting, the current taken will be the normal full-load current, but when starting against 1.75 times full-load the current will be approximately 2¼ times the normal full-load current.

The main disadvantage of the slip ring motor is that its efficiency is reduced due to heat losses in the external resistances. Also a certain minimum amount of maintenance is required on the slip rings.

(4) Synchronous motors.

These are motors in which the rotor is an electro-magnet which rotates synchronously with the rotating field produced by the stator. This means there is no speed variation from no load to full load.

The synchronous motor is not self starting. If it drives a D.C. generator, the generator can be used as a starting motor. More usually however the synchronous motor is started as a three phase induction motor and when at 95% synchronous speed the rotor is connected to D.C. and pulls into step.

Synchronous motors are often used on large reciprocating compressors, 15 kW to above 75 kW.

The big advantage of the synchronous motor is that with excess D.C. rotor current, it can be used to correct power factor for a number of motors.

If a synchronous motor goes out of synchronous speed it must be re-started.

D.C. Motors.

In general, direct current motors are used where special operating characteristics, such as good speed control, are required or where direct current is the only kind available.

Three types of D.C. motor are available:

(a) Shunt (b) Series (c) Compound

(a) The shunt motor is the most common D.C. motor. The armature and field windings are in parallel across the supply. It is a constant speed motor although speed does decrease slightly with load. (However for any setting of the controller, motor speed is relatively constant.) It is easy to vary speed above normal.

For starting, torque is proportional to current.

(b) With the series motor the armature and field windings are in series. Speed varies greatly with load and the motor must not be unloaded otherwise the speed becomes so great that the motor will be destroyed. Series motors are suitable for starting very heavy loads e.g. winches and cranes or for electric traction purposes.

For starting, torque is proportional to current squared.

(c) The compound motor is a hybrid between shunt and series. It has a fixed no load speed, greater torque at starting than a shunt motor but poorer speed regulation.

Geared motors

These are a very popular choice of unit where low speeds are required. Fig. 3 shows the construction of a geared motor. The most common type of motor used is a flange mounted T.E.F.C. squirrel-cage motor. (T.E.F.C. - totally enclosed fan cooled.)

The output speed from the gear box is fixed. A wide choice of speeds are available from 450 r.p.m. down to 10 r.p.m.
(Ultra slow speed units are available for the lower power range of motors. Output speeds can be as low as a fraction of one r.p.m.)

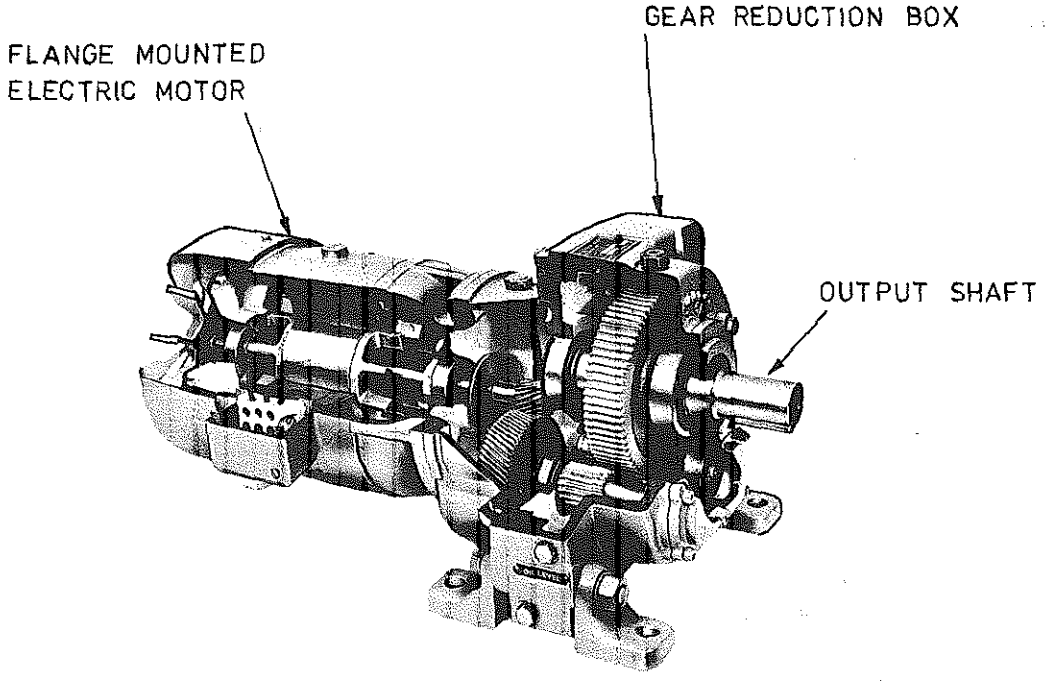


FIG. 3

Mounting of geared motors.

Solid foundations will assure maximum efficiency and minimum noise level. Avoid mounting on light fabricated steel frames or brackets which may amplify gear noise. If necessary, shims should be provided at each fixing bolt to prevent possible distortion of gear box when bolting to foundation. Output shaft and driven shaft must be correctly aligned to prevent overloads on the bearings and undue strains on the gear housing.

Connection of driven load.

The diameter of pulleys, sprockets or pinions should be selected to keep the overhung load on the output shaft within the figure recommended for the geared motor to be used.

The centre line of the load pull should be applied as close as possible to the bearing housing. It should not be applied beyond the mid-point of the shaft extension. For pinion drives it is essential that the mating gear should be concentric about the axis of rotation.

When couplings are used, care should be exercised to line up both parts correctly for angularity and concentricity. Couplings, sprockets and pinions, etc., should be fitted to the output shaft with care. They must never be driven on by hammering. This may damage the bearings and the circlip location. Ease of assembly may be achieved by heating the drive part to about 120° - 150° C prior to the fitting. The drive part should be a 'snug' fit on output shaft.

Efficient lubrication

Standard geared motors are supplied for normal horizontal mounting with the correct quantity of oil for efficient lubrication.

They may be mounted in positions with the output shaft inclined at angles not exceeding 15° above or below the horizontal. For inclined mounting in excess of 15° the oil should be drained off and the gearbox charged completely with grease.

Vari-speed motors (mechanical type)

These units consist of an electric motor coupled to a "variable speed reduction box". The reduction box consists of a friction wheel arrangement or adjustable belt drive etc. The output shaft of the reduction box generally has a speed variation range of 4:1 to 6:1.

Motor enclosures

Various types of motor housings are available depending upon service requirements.

The principal types are listed below.

(a) Totally enclosed, T.E.

Totally enclosed fan cooled, T.E.F.C.

The T.E.F.C. represents the most common type of enclosure.

The above types of motors have been developed for service where they operate in dusty, damp and exposed conditions. All foreign matter is positively excluded from the inside of the motor.

(b) Weather proof motors.

This type of motor is necessary for permanent "out of doors" operation.

Totally enclosed and T.E.F.C. motors can easily be made weatherproof.

(c) Explosion proof motors

These motors are totally enclosed and are of heavy construction with specially designed spigots and flanges.

The use of these motors is necessary in locations where there exists hazardous dust or explosive vapours.

(d) Screen protected, drip proof and splash proof.

All the above represent enclosures which offer a lesser degree of protection for the inside of a motor.

Rating.

All motors are designed on the basis of maximum permissible temperature rise. For T.E.F.C. squirrel cage motors this is 65° C in a maximum ambient of 40° C for continuous maximum rating (C.M.R.)

Motors may have other than continuous rating e.g. 1 hour, ½ hour, 15 minutes etc. This kind of rating is common for motors having intermittent loads, such as crane motors, elevators and some types of machine tools, so that they may be operated for a short time at relatively high loads followed by a cooling period. The rating and corresponding temperature rise are stamped on the motor name plate.

Protective devices

These must be incorporated into a circuit to protect motor windings and circuit wiring. The re-winding of a burnt out motor is an expensive operation. A few of the methods of protection are listed below:

Fuses

Thermal relays

Circuit breakers

Electromagnetic trips

Fuses for motor protection need to be delay action type since the starting current can be much greater than the running current. The fuse must not blow during starting but must blow if excessive current continues for some time.

Motor starters usually have an overcurrent circuit breaker and an undervoltage circuit breaker.

The overcurrent circuit breaker is to prevent overheating of the motor or wiring during a fault or overload. It is usually delayed in operation so that the starting current, which is greater than the full load current, will not cause it to be tripped.

The undervoltage circuit breaker is used to allow for power failures. It prevents automatic restarting where this could be unsafe and also ensures that the motor will have to go through the normal starting sequence after any stoppage.

Fig. 4 shows a typical circuit for a D.O.L. motor starter controlling a three phase motor. Pushing start button energises contactor coil which puts power on to motor. Contactor coil maintains its energisation until the stop button is pushed or the power is disconnected or the thermal overload operates.

Power factor.

This is a complex electrical phenomonen encountered when using A.C. By definition it is the cosine of the angle (ϕ) between the voltage and current.

Power = volts x amps x cos ϕ

The greater the power factor the greater is the power for the same current. Normally the power factor is about 0.8.

When motors are run on part load only there is a reduction in power factor. This is undesirable because it increases power loss in the lines and transformers. Also the output from the power-supply generator is reduced.

In a plant where a large number of small motors run on part load, power factor loss can become a severe problem.

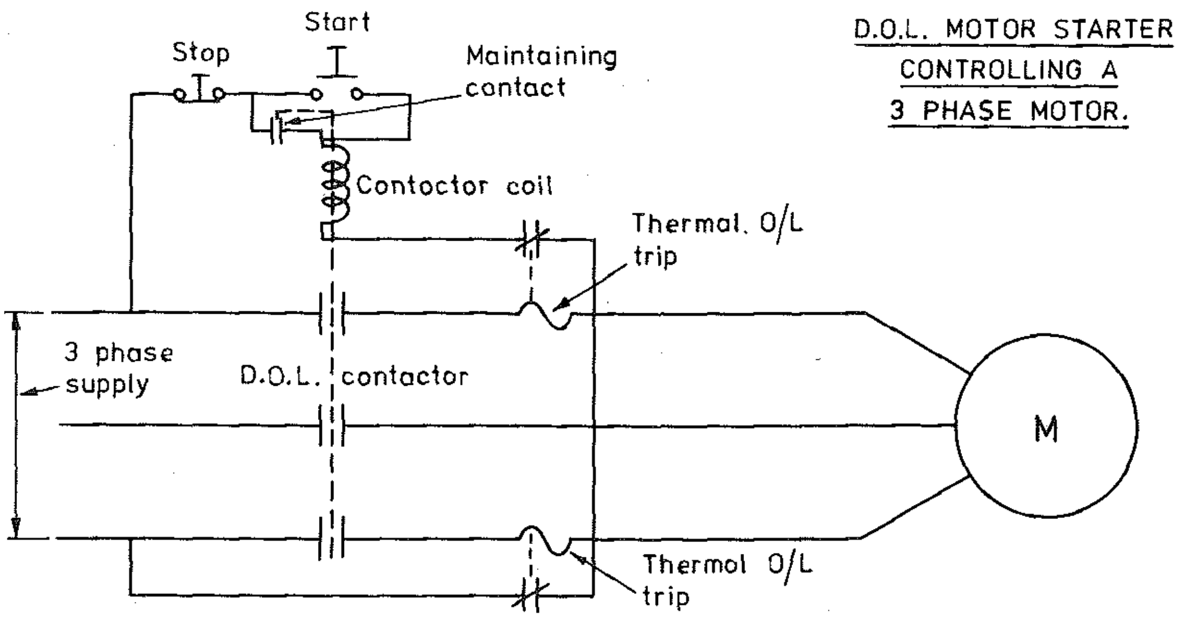


FIG. 4

The Supply Authority may require the consumer to maintain a certain power factor and so power factor correction equipment or synchronous motors may be used as required.

Limit switches

Limit switches in one or other of their different and varied forms are the basis of numerous electrical control circuits for either A.C. or D.C. current.

The primary function of a limit switch is to prevent over-travel of machinery - usually by opening the control circuit of the driving motor at a predetermined point in the travel of the machine or mechanism. In addition, limit switches serve many other purposes, either for normal operation or as emergency devices. They are often used to provide slowdown at a point ahead of the stopping point, so that stop may be more accurate. They may also act to vary the speed of a machine in different parts of its operating cycle, or to reverse its rotation, or to interlock the operation of two or more machines, and have many applications for the control of indicating circuits.

Speed control of A.C. motors

(Compliments of GEC Machines, a Division of GEC Australia Limited.)

Variable frequency inverters are now available which enable the speed of A.C. motors, particularly squirrel cage motors, to be varied.

The control equipment converts the fixed frequency and voltage of the supply mains to a controllable frequency and voltage. In this way, the speed of the motor can be varied infinitely.

The following control modes are available:

local/remote, auto/manual, forward/reverse, adjustable acceleration and deceleration (1 - 25 seconds) and speed control potentiometer.

Controlled starting and acceleration reduce mechanical stresses in the motor and driven equipment, extending the life of the motor and improving overall reliability. In many installations this smooth controlled start mode can facilitate a reduction in the mechanical service factor, in turn reducing the overall cost of the installation.

Starting torque can be maximised while high inrush currents are eliminated; refer to the graph, Fig. 5.

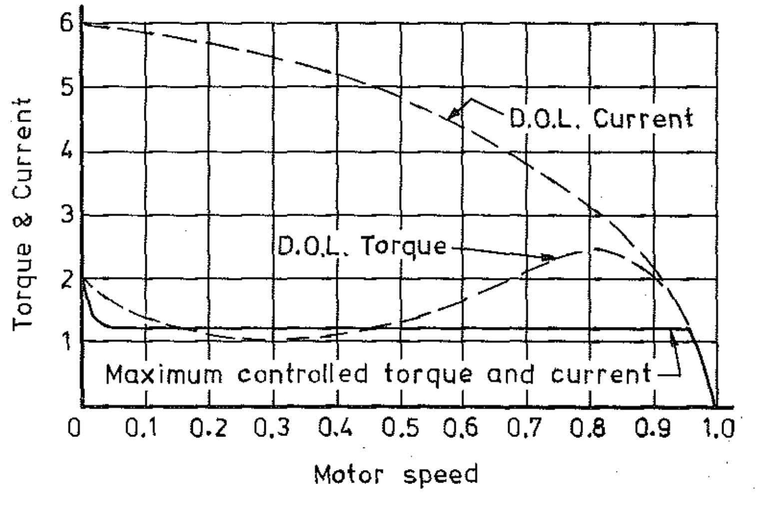


FIG. 5

Speed control range is 20:1 which means that a motor with a synchronous speed of 1000 rev/min can be operated to as low as 50 rev/min. Motors can also run at higher than normal maximum speed, further increasing the speed control range.

Torque is nominally constant although a constant power operation over a 1.5:1 speed range is programmable. At continuous running on any set speed, full load torque is available.

Inverters are ideal for fan and pump installations replacing throttles and dampers.

In other applications, inverters result in a drive system which provides smooth versatile control at high efficiency where other drive systems such as eddy-current and fluid couplings are frequently more expensive to install and significantly less efficient.

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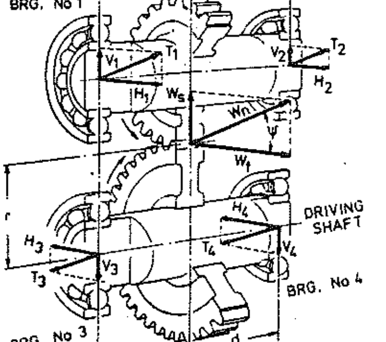
APPENDICES



APPENDICES

Appendix No. 1

MATERIAL PROPERTIES (Supplementary to Chapter 13)

Strength

This can refer to ultimate value in tension, compression or shear.

Tensile strength, however, is generally the one given for most materials as it seems, intuitively, the easiest to appreciate. Simply defined, it is the stress that causes fracture.

Stiffness

Stiffness is not related to strength. Young's modulus is the measure of the stiffness (or flexibility or springiness) of a material. Young's modulus is the stress which if applied to a material and the material did not break, would cause it to double in length. Concrete is stiff but weak (in tension). Steel is strong and stiff while rubber is flexible (low E value).

Strength and stiffness are the best two properties whereby a material can be described.

Hardness

Hardness is a measure of the resistance offered by a metal to plastic deformation by indentation or penetration.

Hardness scales are: Brinell, Vickers and Rockwell.

Toughness

Toughness is the property of a material which enables it to resist the propogation of cracks. It requires a good combination of ductility and reasonably high tensile strength.

Brittleness

This is the property of a material which causes it to fracture without visible plastic deformation. It is a lack of ductility and ability to resist the propogation of cracks.

Ductility

In its original meaning, ductility was defined as the property of a material which allowed it to be drawn out by tension to a smaller section e.g. a wire being made by drawing out the metal through a hole.

Present day usage of the term 'ductility' denotes general 'workability' i.e. plastic deformation under any type of applied stress. A ductile material will always deform plastically before fracture.

Malleability

This is the property of a material which allows it to be beaten, rolled or pressed into plates; malleability is similar to ductility.

Malleability allows plastic deformation in compression, without rupture.

Work hardening or cold working

This takes place when a metal is plastically deformed; it results in an increase in hardness and/or mechanical strength. The type and extent of cold working may be varied. With cold rolling, drawing and upsetting the effect will extend completely through the metal. However, too much cold working will result in a brittle product. Cold working may be restricted to the surface layers (i.e. skin hardening) of a metal by peening, frictional wear or machining operations.

Plastic deformation

This condition involves the permanent change in form of material as a result of applied stress. It is the opposite property to that of 'elasticity'.

Rimmed steel, semi-killed and killed steel

(1) Rimmed steel

This is a low carbon steel which has been incompletely deoxidized. This is done in such a way that the resulting ingot will have a rim (bottom and sides) which is relatively free of carbon and impurities. The centre will have some blowholes plus impurities and carbon.

(2) Semi-killed steel

A steel that is incompletely deoxidized so that sufficient carbon monoxide is evolved during the manufacturing process to offset solidification shrinkage.

By far the greatest range of common steels (shafting steels - CS1020, CS1030 and CS1040 and structural Grade 250) are semi-killed. Used where severe mechanical properties are not required.

(3) Killed steel

A steel which is fully deoxidized with aluminium or silicon. Used where severe mechanical properties and enhanced notch ductility are required; Grades 250L0 and 250L15.

Steel plates of even higher notch ductility are available to Grade AS 1548 for boilers and unfired pressure vessels.

Strain ageing embrittlement

This phenomonen occurs with susceptible steels; semi-killed steels come into this category. Strain ageing occurs with the passage of time following cold working and results in changed mechanical properties. Severe embrittlement is the most serious consequence.

Strain ageing occurs at atmospheric temperatures, however, the process can be accelerated greatly at high temperatures such as those experienced during galvanizing.

When working semi-killed steels, preventative action should be taken. If parts are cold worked then they should afterwards be stress relieved (600° C) or if they are going to be highly stressed in service then normalized. Galvanizing, if required, follows the stress relieving or normalizing.

Alternatively, hot working can be done at 900°C without further heat treatment.

Damping capacity and resilience

(1) Damping capacity

The property of a metal which allows it to absorb vibratory strains. This is a very desirable ability where a machine element is subjected to vibration e.g. a machine bed plate. With cast iron, the inclusion of free graphite gives a structural discontinuity which results in high damping capacity. Damping capacity results in vibrational energy being converted to heat.

(2) Resilience

The property of a material to store energy (within the elastic limit) when strained and release it when the load is removed. It is the opposite to damping capacity where elastic energy is converted to heat and so is irrecoverable.

Appendix No. 2

HEAT TREATMENT TERMS (Supplementary to Chapter 13)

The following terms briefly explain the operations used in heat treatment. Heat treatment procedures apply predominantly to ferrous metals (steels).

Transformation range (critical range or critical temperatures)

These are the temperatures at which some definite change takes place in the physical properties of a metal (particularly applied to the iron-carbon system).

Hardening and tempering.

(1) Hardening

This term implies greatly increasing the hardness of an alloy by heat treatment.

With steel, the process involves heating it to a temperature above or within the transformation range and then quenching it in a suitable medium such as oil, brine or water.

(2) Tempering

After the hardening operation a steel is in a stressed condition and is too brittle for most practical uses. It is therefore re-heated to a temperature below the transformation range and then cooled at a suitable rate.

The tempering temperature is governed by the required function of the component. A normal cutting blade needs to be hard but not tough as it is not subjected to shock loading. With a punch however, the predominant property requirement is toughness as the punch is subjected to shock loading.

After the operations of hardening and tempering, a steel should be in its optimum condition of strength, bardness, ductility and toughness for the particular purpose.

To harden a piece of steel satisfactorily it should be in the softened or annealed condition initially.

Annealing, normalizing and stress relieving

(1) Annealing

This process is to put steel into its softest possible condition in preparation for further processing. All internal stresses are relieved. The steel is heated to above its normal hardening temperature followed by slow cooling; generally within the furnace.

Non-ferrous metals are often annealed after cold working to soften the component. Thus, copper tubes must be annealed after cold bending to shape or they will be hard and brittle.

(2) Normalizing

This involves heating a steel to a temperature above the normal hardening temperature followed by cooling freely in air. This refines the grain size and puts steel back into its 'normal' condition. It restores stability (toughness) to a material (for example after cold working or prolonged high temperature).

It can be said that both annealing and normalizing relieve stress in steel. Broadly speaking however, annealing would be used where the parts are to be worked further and normalizing where the components are ready for service.

(3) Stress relieving

Heating a steel to a temperature below the recrystallization range. After soaking for the required length of time the component is allowed to cool uniformly. The only purpose of the procedure is for the removal of internal stresses.

The term applies also to non-ferrous metals.

Surface hardening methods

Steels with carbon content below approximately 0.3% cannot be hardened to any noticeable degree by heating to a certain temperature and quenching. However a number of surface hardening techniques may be used on such steels.

Many engineering components require a combination of wear and shock resistance. This can be attained by providing a hard outer case and a tough core such as surface hardening will give. Gudgeon pins and gears are examples of this category.

(1) Case hardening

This term is generally restricted to carburizing. It consists of heating a component to the austenitic state while in contact with a source of carbon. The depth of carbon penetration will depend on the length of heating time. A depth of 1 mm may be achieved in about 8 hours. This may be followed by appropriate heat treatment to refine the core and case.

(2) Cyaniding

Components are immersed in high temperature molten cyanides for a number of hours. This produces a carbon-rich skin. The above procedure is followed by quenching or heat treatment.

(3) Nitriding

Special alloy steel is required (containing aluminium) which is heated in contact with partially dissociated ammonia. The nitrogen reacts with the steel to form a hard nitride case. The process is performed at a temperature below the transformation range. Correct core conditions are attained by heat treatment before nitriding and no subsequent heat treatment is necessary.

(4) Carbo-nitriding

A process where a component is heated in an atmosphere of carburizing gas and ammonia. This is followed by quenching or heat treatment.

(5) Further types of surface hardening

Flame hardening and induction hardening

Both these processes involve heating and quenching; this means the steel used must respond to hardening by this method i.e. have a suitably high carbon content.

Appendix No. 3

DESIGNING FOR FATIGUE (Supplementary to Chapter 10)

Modern rules of fatigue design dictate that stress range is the dominant factor to be considered together with the number of load applications (see pages A8 & A9). The severity of stress concentrations is the third point to be considered.

According to testing laboratories fatigue failures constitute the most common source of failure in machinery and structures. Unfortunately fatigue failures rarely give warning of impending failure. Fatigue failures have become more prevalent in recent years because of:

- the increased utilization of aircraft, motor vehicles and other forms of machinery,
- the continuing trend towards higher strength/weight and power/weight ratios,
- more refined static design techniques and the use of higher working stresses,
- increased speeds of operation, with the resultant accumulation of large numbers of cycles in a small period of time,
- the use of materials of ultra-high static strength.

Fatigue cracks generally start on the outer surface of a component as surface crystals offer less resistance to deformation than interior ones. Further, failure always commences in places of tensile stress; fluctuation of stress which does not involve tensile stress is now known not to propogate cracks.

Table No. 1 gives details of Ultimate Tensile Strength and Fatigue Strength for a number of materials, see page A7.

Surface finish in general has only a minor effect on fatigue strength when compared with the influences of the various 'design' stress concentrators, refer to Table No. 2. The hot rolled surfaces of mild steel only marginally decrease its endurance limit.

General features associated with welding are listed in Table No. 3. These contribute to major reductions in the fatigue strengths of metals, sometimes down to 20% of that of the base metal at the joint.

TABLE No. 2

Fatigue Strength Reduction Resulting from
Different Machining Processes

Finish	Steel $F_{t\ ult}$ = 480 MPa		Steel $F_{t\ ult}$ = 1170 MPa		Aluminium Alloy	
	%	K_f	%	K_f	%	K_f
'Basic' high polish	0	1.0	0	1.0	0	1.0
Coarse turning	15	1.2	30	1.4	15	1.2
Fine turning	10	1.1	25	1.3	0	1.0
Coarse file	15	1.2	25	1.3		
Smooth file	10	1.1	10	1.1		
Fine emery	0	1.0	10	1.1		
Coarse grinding			25	1.3	15	1.2
Fine grinding	0	1.0	10	1.1	10	1.1

Tables No. 1, No. 2 and No. 3 are taken from

Mann, J.Y.,
Fatigue of Materials,
Melbourne University Press.

(Pages 50, 79 and 93 resp.)

TABLE No. 1

Unnotched Fatigue Strength (at room temperature) of Different Materials in Rotating Bending
(R = -1)

Material	$F_{t\ ult}$ MPa	F_e MPa	Ratio: $\frac{F_e}{F_{t\ ult}}$
Iron, commercially pure (annealed)	315	180	0.57
Cast iron, flake	375	150	0.40
Cast iron, nodular	745	250	0.34
Mild steel (0.15%C) (annealed)	410	230	0.56
Carbon steel (0.36%C) (annealed)	580	260	0.45
Carbon steel (0.36%C) (Q. & T.)	860	400	0.47
Carbon steel (0.75%C) (annealed)	785	295	0.38
Carbon steel (0.75%C) (Q. & T.)	1030	420	0.41
2½% Ni steel (Q. & T.)	970	575	0.59
Cr-Mo steel (Q. & T.)	960	470	0.49
Ni-Cr-Mo steel (Q. & T.)	1240	500	0.40
Ni-Cr-Mo steel (Q. & T.)	1930	655	0.34
18Cr-8Ni stainless steel (cold drawn)	885	510	0.58
12% Mn steel (quenched)	1105	450	0.41
Copper, pure (annealed)	215	60	0.28
Brass 60 Cu-40 Zn (annealed)	375	140	0.37
Brass 70 Cu-30 Zn (annealed)	310	95	0.31
Brass 90 Cu-10 Zn (hard drawn)	495	145	0.29
Phosphor bronze (annealed)	450	140	0.31
Aluminium bronze (9.5% Al) (annealed)	565	200	0.35
Monel (Cu-Ni) (annealed)	540	235	0.44
Monel (Cu-Ni) (cold drawn)	725	305	0.42
Nickel alloy (Nimonic 80)	1005	310	0.31
Aluminium, pure (annealed)	70	20	0.29
Aluminium, pure (cold worked)	130	45	0.35
Al-7% Mg alloy (annealed)	240	115	0.48
Al-7% Mg alloy (cold worked)	330	145	0.44
Duralumin (2024) (annealed)	185	90	0.49
Duralumin (2024) (solut. treat. and aged)	470	140	0.30
Al-Zn-Mg alloy (7075) (solut. treat. and aged)	575	150	0.26
Magnesium, pure (extruded)	225	70	0.31
Mg-Al-Zn (heat treated)	330	130	0.39
Mg-Zn-Zr (heat treated)	380	140	0.37
Titanium, commercially pure (rolled)	605	360	0.60
Ti-4Al-2.5Sn (heat treated)	915	525	0.57
Ti-6Al-4V (heat treated)	1060	565	0.53
Ti-4Mn-4Al (heat treated)	1070	620	0.58
Lead, pure (annealed)	14	2	0.14
Polymethylmethacrylate plastic	58	20	0.34

F_e is at 10^7 cycles for steels; 5×10^7 to 10×10^7 for non-ferrous metals.

TABLE No. 3

Features associated with welding

(a) a cast structure, frequently containing slag inclusions, porosity, shrinkage cracks, etc., compared with a wrought structure in the adjoining bar or plate;

(b) the metallurgical notch effect of the change in metal structure at the plate/weld junction;

(c) the normalized condition of the weld and the adjacent plate material;

(d) residual internal stresses resulting from the uneven cooling of the weldment or metallurgical changes during cooling;

(e) the geometric stress concentration effect of the change in section at the plate/weld junction;

(f) undercutting along a weld bead causing additional stress concentration;

(g) the rough surface ripples or craters of the weld with a multitude of small geometric stress concentrators;

(h) stress concentration resulting from lack of penetration or lack of fusion at the root of welds.

In general, these refer more particularly to the welding of wrought materials. In the welding of castings, some of the features may be of much less significance.

Structural design

As many common design loads have only about a 5% chance of occurrence very few serious stress fluctuations will be encountered, hence fatigue will rarely be a problem.

Nonetheless, Appendix B of AS 1250 gives full details to be used in cases where fatigue is encountered. Such information may be of use in some cases of mechanical design.

Table B1, AS 1250, gives four loading conditions:

No. 1	20 000 to 100 000 loading cycles
No. 2	100 000 to 500 000 loading cycles
No. 3	500 000 to 2 000 000 loading cycles
No. 4	over 2 000 000 loading cycles

It can be seen from the above table that no fatigue is predicted for less than 20 000 loading cycles.

Table B3, AS 1250, lists a number of different categories of structural situations, (connections etc.) A to F, together with the permissible stress ranges, F_{sr}, applicable to each loading condition. Categories B to F deal with the permissible stress ranges in welds and base metals (this is an alternative method of fatigue analysis based on the 'permissible stress range concept').

Category A concerns plain mild steel members in either tension or bending, refer to Fig. 1.

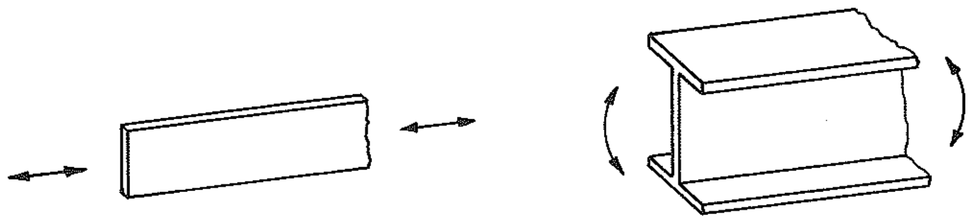


FIG. 1

The permissible stress ranges for this basic condition are given in Table No.4.

TABLE No. 4

Loading condition	Permissible stress range
1	F_{sr1} = 410 MPa
2	F_{sr2} = 245 MPa
3	F_{sr3} = 165 MPa
4	F_{sr4} = 165 MPa

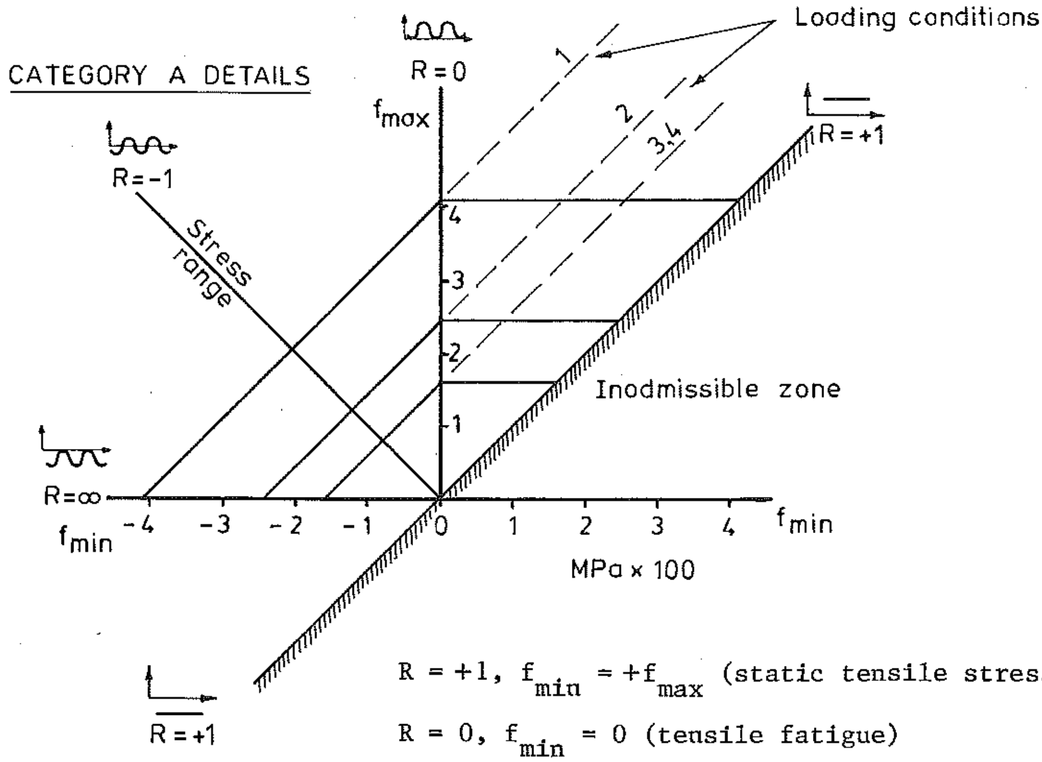
The stress range figures given in AS 1250 apply for all structural steel grades and are based on extensive tests carried out in U.S.A. on full size specimens.

Appendix B in AS 1250 applies predominantly to weld type details. It has been shown that all grades of structural steel exhibit about the same fatigue strength for identical weld details.

It should be realised also that when using Appendix B, AS 1250, that maximum stresses should not exceed those given elsewhere in the code.

Refer to Fig. 2; this is a 'Ros (AWS) diagram'. These diagrams are often used to illustrate fatigue results. R is the ratio of minimum stress to maximum stress. The graph shown demonstrates the simplified approach adopted by the AS 1250 Code, but which agrees with the extensive practical tests previously mentioned. The figures of 410 MPa, 245 MPa and 165 MPa on the ordinate correspond with the ultimate tensile strength of mild steel, the yield point stress and the $0.66F_y$ stress limit respectively. It can be seen that for R > 0 stress fluctuations are ignored.

Appendix B, AS 1250, should be studied to see the various types of situations (connections etc.) which can be encountered.



$R = +1$, $f_{min} = +f_{max}$ (static tensile stress)

$R = 0$, $f_{min} = 0$ (tensile fatigue)

$R = -1$, $f_{min} = -f_{max}$ (reversal of stress)

$R = \infty$, $f_{max} = 0$ (compression fatigue)

$R = +1$, $f_{min} = -f_{max}$ (static compressive stress)

FIG. 2

Mechanical design

A greater variety of fatigue situations can be encountered in mechanical design and because of the more severe applications greater refinements in design procedure are required.

The following types of fatigue loads have to be considered:

(1) Complete reversal of stress (tension/compression)

Infinite life
Finite life

(2) Fluctuating tensile stress

Infinite life
Finite life

(3) Complete reversal of shear stress

Infinite life
Finite life

(4) Fluctuating shear stress

Infinite life
Finite life

The following relationships for steels are useful when definite properties are not known.

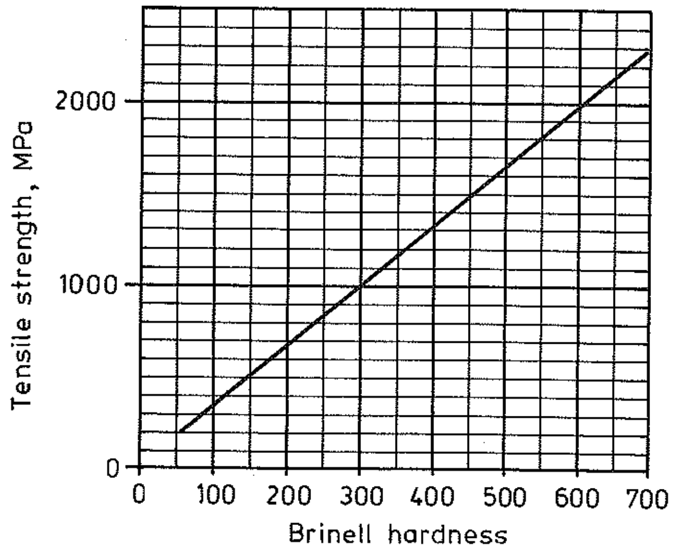
$$F_y = 0.5F_{t\ ult} \quad \text{(low strength steels)}$$
$$= 0.75F_{t\ ult} \quad \text{(high strength steels)}$$

$F_e \cong 0.45F_{t\ ult}$ ... see also Table No. 1

$$F_{sy} = 0.5F_y \text{ to } 0.577F_y$$

$$F_{s\ ult} = 0.6F_{t\ ult}$$

$$F_{se} = 0.55F_e$$



Relationship between Brinell hardness
and ultimate tensile strength of steel

FIG. 3

The following general definitions will apply:

Fatigue strength

This is the value of the stress at which the fatigue life is N cycles. The stress cycle should be stated, i.e. mean stress and either the alternating stress or maximum stress of the cycle.

Fatigue limit

This is the stress below which a metal may endure an infinite number of stress cycles without fracture. Again, the type of stress cycle should be specified as for 'Fatigue strength'.

(1) Complete reversal of stress (tension/compression)

For the design of components in this category an S-N diagram is required with the alternating stress amplitude on the ordinate and the log of the number of cycles on the abscissa. (Alternatively log-log scales may be used.) Both the fatigue strength and fatigue limit may be obtained from this graph, i.e. infinite life and finite life. In the absence of experimental data a probable S-N graph may be constructed (see Worked Example No. 1).

Worked Example No. 1

Tests on a certain sample of 0.36%C steel indicate a tensile strength ($F_{t\ ult}$) of 860 MPa.

Construct an S-N diagram and state the fatigue strength at 100 000 cycles and the fatigue limit.

Answer

No fatigue is predicted for less than 10^3 cycles and 10^6 is taken as the limiting number.

From Table No. 1, F_e = 400 MPa ($\cong$ 0.47 x 860)

For the graph $F_{t\ ult}$ is generally taken as 0.9 x actual value (0.9 x 860 = 774 MPa)

Fig. 4 shows the resulting graph.

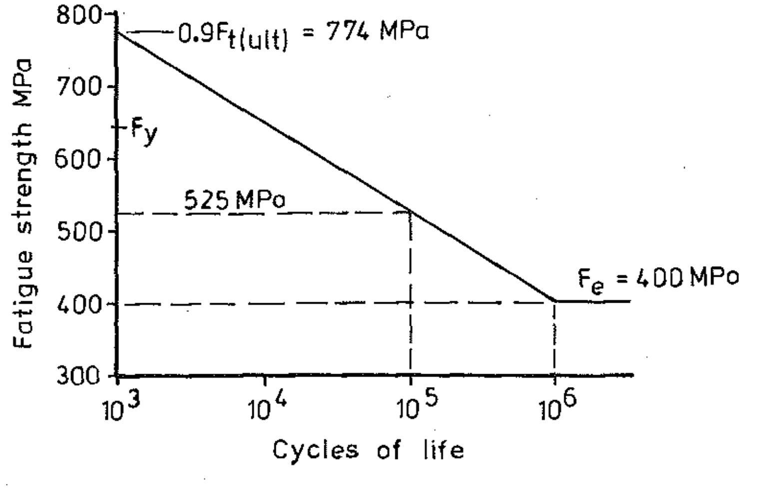


FIG. 4

From the graph:

Fatigue strength at 100 000 cycles = 525 MPa
Fatigue limit = 400 MPa

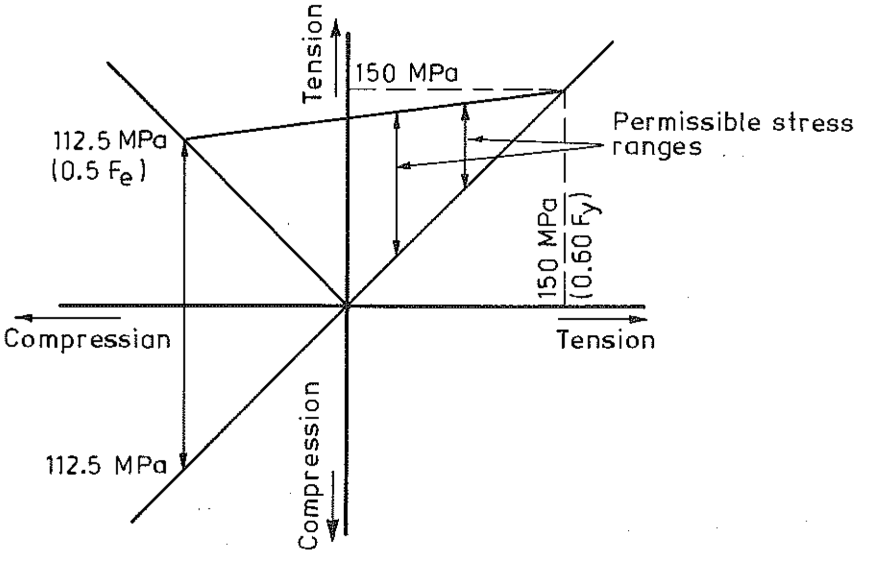
These figures can be used in design with the appropriate factor of safety applied.

(2) Fluctuating tensile stress

Infinite life

These stresses can be represented on a 'Ros' (AWS) diagram. Fig. 5, which is self explanatory, illustrates how, as the maximum tensile stress rises then the permissible range for the fluctuating tensile stress diminishes. This is one of the traditionally accepted methods for machine design. The diagram shown is for infinite life but they may also be constructed for any degree of finite life; this condition is shown dotted.

'Ros' diagrams are usually modified in practice to incorporate permissible stress values. Refer to Fig. 6.



Sample only, of permissible working stresses, CS1030

FIG. 6

Finite life

Where finite life for fluctuating tensile stress is required it is necessary to conduct a series of fatigue tests and plot an S-N graph. This is rarely warranted in normal practice, therefore it is preferable to design (conservatively) as for 'infinite life'.

(3) Complete reversal of shear stress

As for Category (1) type design, an S-N diagram is required. In the absence

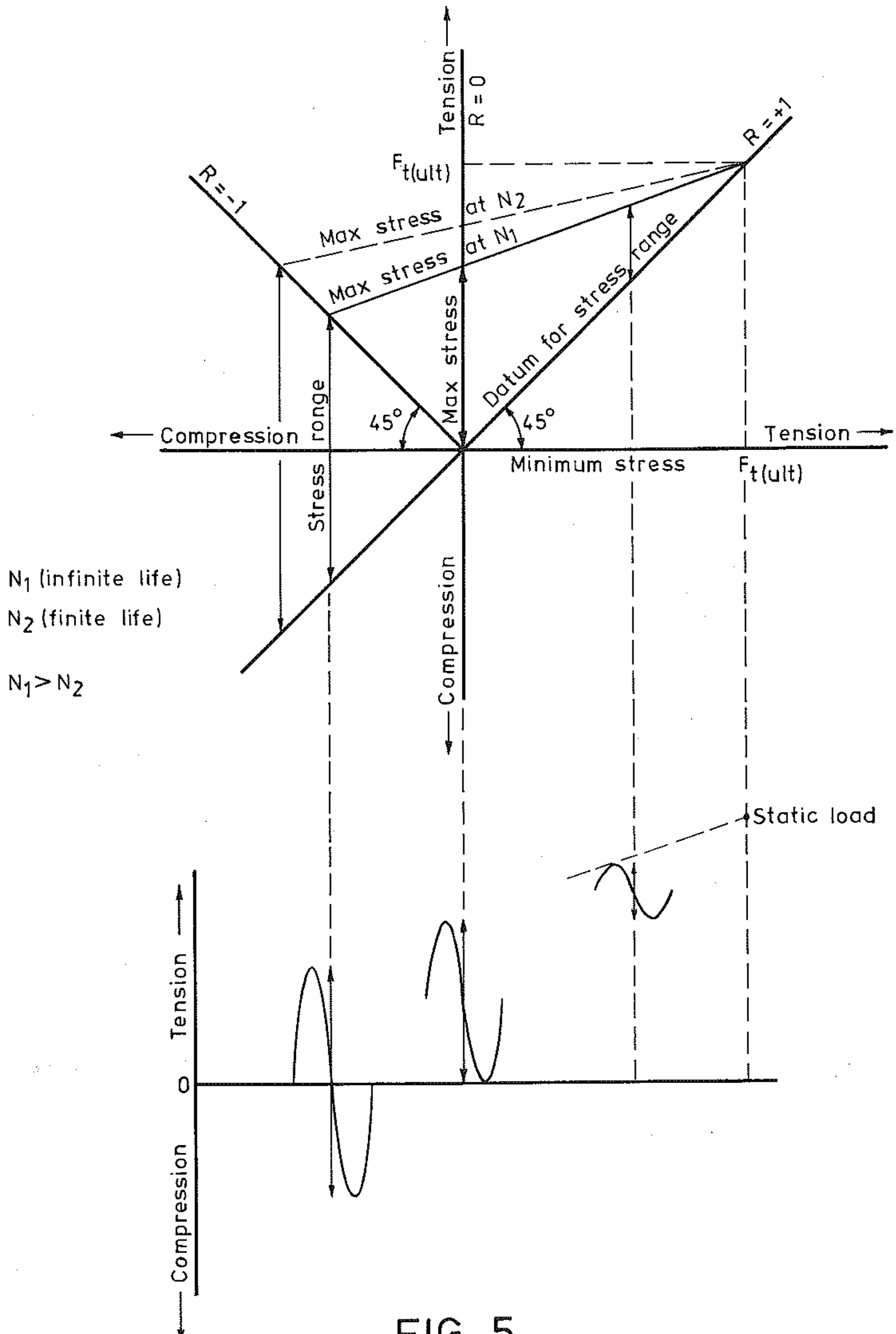

Tension
R = 0
R = +1
$F_{t(ult)}$
R = -1
Max stress at N_2
Max stress at N_1
Max stress
Datum for stress range
Stress range
45°
45°
Compression
Tension
Minimum stress
$F_{t(ult)}$
Compression
N_1 (infinite life)
N_2 (finite life)
$N_1 > N_2$
Static load
Tension
0
Compression

FIG. 5

of experimental data a probable graph can be drawn (see Worked Example No. 2).

Worked Example No. 2

Draw an S-N diagram for reversal of shear stress for the steel specified in Worked Example No. 1.

Specify the fatigue strength at 100 000 cycles and the fatigue limit.

Answer

$F_{sy} = 0.5F_y$

$F_y = 0.75\ F_{t\ ult}$

$\therefore F_{sy} = 0.5 \times 0.75 \times 860$

$\cong 320$ MPa

$F_e = 400$ MPa

$F_{se} = 0.55F_e$

$= 0.55 \times 400$

$= 220$ MPa

$F_{s\ ult} = 0.6F_{t\ ult}$

$= 0.6 \times 860$

$= 516$ MPa

Value of $F_{s\ ult}$ for graph $= 0.9 \times 516 = 464$ MPa

Fig. 7 shows the resulting graph.

Fatigue strength at 100 000 cycles = 300 MPa
Fatigue limit = 220 MPa

These figures can be used in design with the appropriate factor of safety applied.

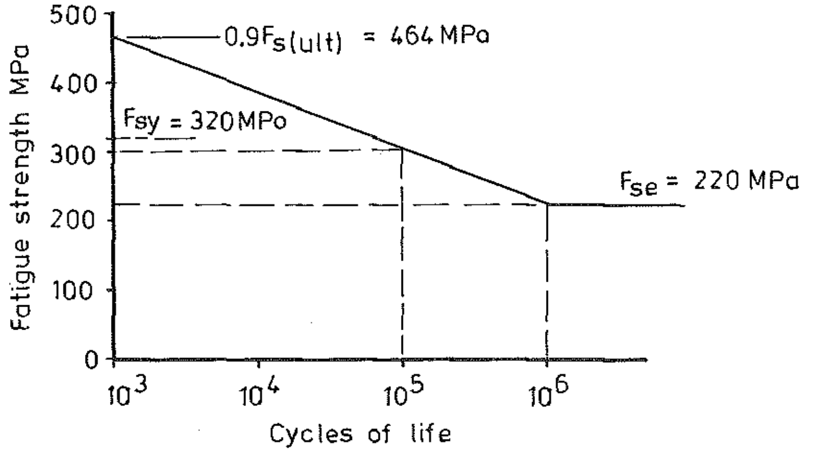


FIG. 7

(4) Fluctuating shear stress

Infinite life

When experimental observations are plotted the resulting graph is as shown in

Fig. 8. The theoretical plot (modified Goodman line) is also shown on the graph.

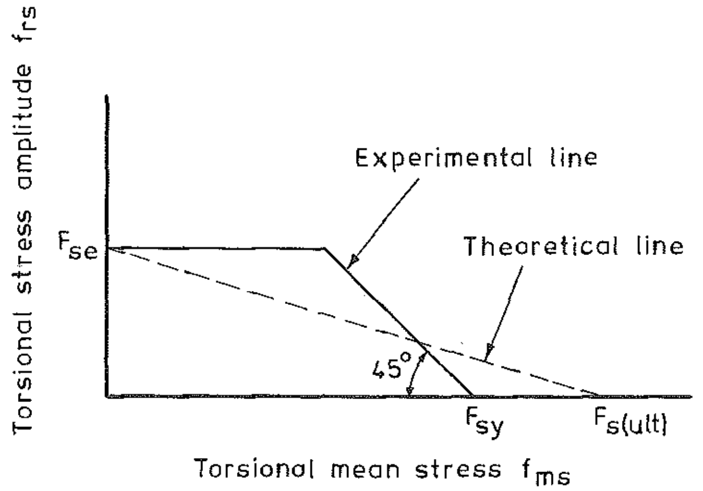


FIG. 8

The interesting result is that, up to a certain point the mean stress has no influence on the endurance limit.

Hence, the following relationships are established:

fatigue failure will occur if $f_{rs} = F_{se}$

and

static failure will occur if $f_{ms\ max} = f_{rs} + f_{ms} = F_{sy}$

A factor of safety can be applied to these relationships to form a design basis.

Finite life

Again, as with previous categories an experimental S-N curve would have to be drawn. If this is not feasible, then design would have to be to infinite life.

Worked Example No. 3

A certain spring is subjected to repeated maximum shear stress fluctuations as follows:

f_{ms} = 173 MPa

f_{rs} = 53 MPa

If the spring is made from a wire of $F_{t\ ult}$ of 1250 MPa, determine the factor of safety against failure.

Answer

From previously stated relationships:

Cont. on page A17

$$F_y = 0.75F_{t\ ult} = 0.75 \times 1250 = 938 \text{ MPa}$$

$$F_{sy} = 0.577F_y = 0.577 \times 938 = 540 \text{ MPa}$$

$$F_e = 0.45F_{t\ ult} = 0.45 \times 1250 = 562 \text{ MPa}$$

$$F_{se} = 0.55F_e = 0.55 \times 562 = 310 \text{ MPa}$$

<u>Fatigue failure</u>

$$f_{rs} = 53 \text{ MPa}, \ F_{se} = 310 \text{ MPa}$$

$$F \text{ of } S = 310/53 = \underline{5.85}$$

<u>Static failure</u>

$$\begin{aligned} f_{ms\ max} &= f_{rs} + f_{ms} \\ &= 53 + 173 \\ &= 226 \text{ MPa} \end{aligned}$$

$$F_{sy} = 540 \text{ MPa}$$

$$F \text{ of } S = 540/226 = \underline{2.4}$$

<u>The effective factor of safety = 2.4</u>

General considerations

The endurance limit figures for metals are invariably given for unnotched specimens (i.e. polished and without stress raisers). The size of specimens generally ranges up to about 12 mm for round test pieces and 40 mm width for flat bars.

However, in actual design situations a number of modifying factors have to be considered as discussed below.

(1) Size factor

The unnotched fatigue strength of a specimen in axial loading appears to be nearly independent of the cross-sectional size. However, when there is a stress gradient across the section, e.g. a shaft under bending and/or torsional loading or when the member is under the influence of a non-uniform stress distribution then the fatigue strength diminishes with increased size. This effect is demonstrated by the graph from the AS 1403 shaft design code (see Fig. 26, chapter 2, of this text).

(2) Stress concentration factor

Design calculations should be based on the actual maximum stress at a particular section (including the effects of any stress concentrations if present).

If more than one stress raiser occurs at a section then the effect of the second raiser should be included as noted on page 2.34.

The stress concentration factors given elsewhere in this text are based on mathematical and/or photo-elastic analysis. Under various fatigue type loadings, these factors are sometimes realised to a lesser extent; this effect is termed 'notch sensitivity' (denoted 'q'). Where experimental values of 'q' are not known then its value can be conservatively taken as unity, i.e. the full theoretical value of K can be used. In fact, for zero mean stress conditions this procedure is quite realistic.

(3) Surface finish factor

This has been previously mentioned and K_f factors are given in Table No. 2.

These factors indicate the reduction in the fatigue strength of three materials. As can be seen, a fine machine finish results in only a small reduction in fatigue strength with lower strength steels.

(4) Fretting

Fretting is a phenomonen which occurs when two closely fitting surfaces which are normally fixed in relation to eachother are subject to relative vibrational movement. The movement can be microscopic in magnitude, nonetheless, pits form on the surfaces of the components with the resultant generation of powdered materials (generally oxides of the metals). Fluctuating external load particularly favours the propogation of fretting. Fretting can, in some cases seriously reduce the fatigue strength of a component; typical examples which fall into this category are gears and like members which are clamped, keyed, press-fitted or shrunk on to shafts, bolted and pinned assemblies.

One of the major factors influencing 'fretting fatigue' is the clamping force between the mating parts. Fatigue strength for steel reduces progressively from zero clamping force until at a clamping force of 120 MPa, the endurance limit is only about 55% of that of the unclamped material.

Other moderating factors are the number of fretting cycles and the magnitude of the relative movement. The AS 1403 shaft design code gives a design factor which allows for fretting influence.

Acknowledgment

The authors express thanks for permission to use several tables and general comments from the following text:

Mann, J.Y.,
Fatigue of Materials,
Melbourne University Press.

*   *   *   *   *   *   *

Appendix No. 4

DESIGNING FOR STRENGTH

It is expedient at this time, with the advent of the latest SAA Crane Code, AS 1418.1-1986, to summarise the main requirements for the design of mechanisms, particularly allowable stresses, strength basis only. These allowable stress details may be useful in other areas of design and can also be used as a comparison with other methods.

Design of mechanism ... Clause 7.3.1

The design of the crane mechanism shall be on the following basis:

(a) Manually operated — strength basis only.

(b) Power-operated —

  (i) strength basis; and

  (ii) life basis —

    A. wear; and
    B. fatigue (finite or infinite).

Summary of Design for strength ... Clause 7.3.2

For manually operated mechanisms the design shall be based on static loading with a duty factor of 1.1 applied to the load.

For power operated mechanisms the duty factor can be ascertained from the Crane Code. The factor will contain an allowance for load inertia and can be from 1.1 to 2.2 depending upon the class and duty of the crane. Additionally, a dead load factor of 1.0 to 1.2 could be required.

The following formulas are a summary of those given in Clause 7.3.8, AS 1418.1.

(a) Tension — $f_t = \frac{P}{A}$ $\left(\sigma_z = \frac{P_z}{A_z}\right)$

(b) Compression — $f_c = \frac{P}{A}$ $\left(\sigma_d = \frac{P_d}{A_d}\right)$

(Slenderness ratio also has to be considered if applicable; in such cases refer to Table 6.1.1, AS 1250.)

(c) Bending — $f_b = \frac{M}{Z}$ $\left(\sigma_b = \frac{M_b}{Z_b}\right)$

(d) Beam shear stress — $f_s = \frac{VQ}{bI}$ $\left(\tau_q = \frac{QS}{IT}\right)$

This is the true maximum shear stress, refer to page 12.63.

(e) Torsional shear — $f_s = \frac{T}{Z_p}$ $\left(\tau_e = \frac{M_T}{Z_{ps}}\right)$
(for solid members only)

For hollow members use Z_{ph}

(f) Rolling pressure — Refer to AS 1418.1

(g) For multi-axial stresses and/or normal and shear stresses acting simultaneously, the most unfavourable reference stress (principal stress) shall be calculated from the following equation —

$$f_t \text{ (principal stress)} = (f_x^{\,2} + f_y^{\,2} - f_x.f_y + 3f_s^{\,2})^{\frac{1}{2}} \quad \ldots \text{ Mises}$$

$$\sigma_v \text{ (principal stress)} = (\sigma_x^{\,2} + \sigma_y^{\,2} - \sigma_x.\sigma_y + 3\tau^2)^{\frac{1}{2}}$$

For a statically loaded member under the action of combined bending and torsion, the formula simplifies to —

$$f_t \text{ (principal stress)} = (f_x^{\,2} + 3f_s^{\,2})^{\frac{1}{2}}$$

Permissible stresses

The Crane Code considers the yield point or the 0.2 percent limit of the material in a component as the strength under static stress.

For materials (predominantly steels) in which the yield stress is not greater than 0.7 times the ultimate tensile strength, then Clause 7.3.9, AS 1418.1, gives the following limits for permissible stresses for strength design —

$$F_c,\ F_t,\ \text{and } F_b = 0.67F_y$$

$$\text{and } F_s = 0.577F_t = 0.387F_y$$

When the yield stress is greater than 0.7 times the ultimate tensile strength (some higher grades of steel come into this category), then Clause 7.3.7, AS 1418.1, specifies that a 'fictitious yield point' shall be used in place of F_y in the above formula for F_c, F_t and F_b. The following equation should be applied to determine this allowable yield stress —

$$F_{yf} = \frac{F_y + 0.7F_{t(ult)}}{2} \qquad \left(\sigma_{EF} = \frac{\sigma_E + 0.7\sigma_B}{2}\right)$$

This ensures that when the component is in operation the true yield stress will not be exceeded.

The aforementioned Clauses from the AS 1418.1 Crane Code are reproduced by permission of the Standards Association of Australia.

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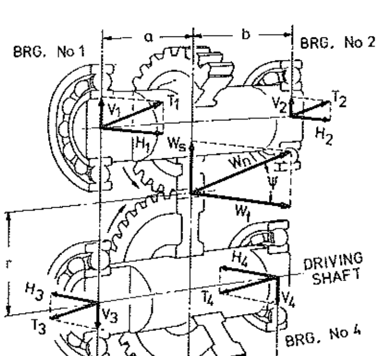
INDEX


BRG. No 1
a
b
BRG. No 2
V_1
T_1
V_2
T_2
H_1
W_s
H_2
W_n
ψ
W_t
r
H_4
DRIVING SHAFT
H_3
T_4
V_4
T_3
V_3
BRG. No 4
BRG. No 3
c
d

INDEX

SUPPLEMENTARY INDEX

SUPPLEMENTARY INDEX