Fundamentals of Drilling Engineering

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Fundamentals of Drilling Engineering

Multiple Choice Questions and Workout Examples for Beginners and Engineers

M. Enamul Hossain

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To my grandfathers, the late Al-Haj Rahim Box Talukder (paternal) and the late Maheruzzaman Talukdar (maternal)

Contents

Preface

Petroleum energy continues to fuel the modern-day economy. In petroleum operations, drilling plays the most prominent role. Financially, drilling costs typically account for 50 to 80% of exploration finding costs, and 30 to 80% of field development costs. Drilling budget remains somewhat unaffected by the oil price. However, historically, whenever oil prices dwindle, the number of drilling rigs decreases. As we enter the Information Age, oil price declines correlate with decreasing drilling rig numbers but the oil production per new well continues to increase. This puts an extraordinary constraint on the efficiency of drilling technology and shows that today's drilling engineers must work with unparalleled efficiency. Anything that is done in petroleum drilling operations becomes a template for other drilling applications, such as exploration of other natural resources, environmental monitoring and remediation underground excavation and infrastructure development and general subsurface exploration. Yet, there is no textbook that has step-by-step procedures with explicit citation of worked out examples. The current book is impeccable in its timing, scope and content.

There is an ancient Chinese proverb: "I read and I forget; I see and I remember; I do and I understand." One of the most important reasons that our current education system is becoming so outdated is that our textbooks fail to capture the imagination beyond what could be learned from the Internet. It is not an exaggeration to say that the Information Age will soon see conventional textbooks become redundant as a result of freely available information. It is refreshing to see that this book has taken the approach of "doing" in order to elucidate complex scientific and engineering phenomena. Numerous worked out examples are given from all aspects of drilling engineering as well as cost analysis and well completion. Examples from horizontal and vertical drilling cases are given liberally. This dexterity makes the book suitable for both North America/Europe and the Middle East, where horizontal and vertical wells are prevalent, respectively. Questions are set in such a way that there is room for creative design and learning with maximum efficacy. Examples are such that the book is useful for both students and practicing engineers.

This book is useful for every student, academic and practicing engineer interested in knowing drilling operations and learning design techniques hands-on. Even though the book pertains to petroleum drilling, it is equally useful for drilling in other disciplines. The book will be remembered as a pioneering work both in content and form and as a template for future textbooks that should be written in a format different from old-style textbooks.

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Summary

Drilling engineering is one of the oldest technologies on earth. The technological advancement in this area is well recognized by scientists, engineers, researchers and petroleum industries. The technological advancement and good understanding in drilling engineering is the key to success in this information era. Unfortunately, the petroleum industry is one of the most hazardous and expensive branches of the industrial hub. The risk and uncertainty are due to the knowledge gap in drilling technology and the science behind it. In addition, there is a clear shortcoming in fundamental understanding of mathematical formulas derived from scientific theories due to the lack of enough practice on mathematical relations in the form of workout examples, multiple choice questions (MCQs) in the area of drilling engineering. To date there are several textbooks that explain and cover drilling operations with fundamentals of drilling engineering. Unfortunately, there is no book so far where enough workout examples, exercises, and MCQs exist as an independent book. As the first and only complete guide for petroleum engineering students, trainee engineers on basic drilling engineering and a milestone book for basic illustrations and understanding, this book is the best choice for the drilling engineering community. It will be a unique production in this discipline.

The book also covers the fundamental issues for beginners who are interested in learning drilling engineering through enough workout examples in each chapter, exercise, exercises for self-practice, enough MCQs to have a deep understanding on each topic and some self-practice MCQs. The book explains the concepts of the basic subject matter clearly and presents the existing knowledge ranges from the history of drilling technology to well completion. The book presents the engineering terminologies in a clear manner so that the beginner in drilling engineering would be able to understand the drilling engineering related formulas, mathematical models, correlations etc., with minimum effort. In addition, each chapter contains enough workout examples and exercises for a comprehensive understanding of the subject. This will make the reader interested in reading the book. The book explains the scientific concepts in the form of MCQs, mathematical formulations in a readable fashion very clearly. It includes all the basic aspects of drilling engineering including an introduction to drilling engineering, rig operations, drilling fluids, drilling hydraulics, well control and monitoring system, pore and fracture pressure estimation, basics of drillstring design, cementing jobs design, drillbit and casing design, horizontal and directional drilling, drilling cost analysis and finally, well completions. However, we believe that each chapter deserves to be a short book and we tried to focus on the most important concepts and main topics of the subject matter. The book is a foundation and resource guide, and an excellent resource for petroleum engineering students and drilling engineers who want to learn how to design and calculate different drilling engineering calculations in the classroom and field.

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1

Introduction

1.1 Introduction

This book is designed to help in solving the exercises and workout examples that are related to drilling engineering. This chapter introduces the fundamental aspects of the drilling engineering problems in general. Sets of multiple choice questions (MCQs) are included which are related to basic definitions about drilling engineering, importance and the procedure for drilling operations. The MCQs also cover the applications and history of drilling, the systematic approach and the aspects of sustainable drilling operations. The MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

1.2 Introduction to Drilling Engineering

Some scholars consider petroleum hydrocarbons to be the lifeblood of modern civilization. The complete cycle of petroleum operations includes seismic survey, exploration, field development, hydrocarbon production, refining, storage, transportation/distribution, marketing, and final utilization to the end user. The drilling technology has been developed through the efforts of many individuals, professionals, companies and organizations. This technology is a necessary step for petroleum exploration and production. Drilling is one of the oldest technologies in the world. *Drilling engineering is a branch of knowledge where the design, analysis and implementation procedure are completed to drill a well as sustainable as possible*. It is the technology used to utilize crude oil and natural gas reserves. The responsibilities of a drilling engineer are to facilitate the efficient penetration of the earth by wellbore and cementing operations from the surface to an optimum target depth that prevents any situation that may jeopardize the environment.

1.3 Importance of Drilling Engineering

It is well known that the petroleum industry drives the energy sector, which in turn drives modern civilization. Every day human beings are benefiting from the petroleum industry. Modern civilization is based on energy and hydrocarbon resources. The growth of human civilization and the necessities of livelihood over time inspired human beings to bore a hole for different reasons (such as drinking water, agriculture, hydrocarbon extraction for lighting, power generation, to assemble different mechanical parts, etc.). There is no surface hydrocarbon resource; rather, all resources are underground in this planet. To keep serving the whole civilization, drilling engineering has a significant role in this issue. Moreover, the world energy sector is dependent on drilling engineering. Without drilling a hole, how are we going to extract the hydrocarbon from underground and bring it to the surface of the earth? To the best of our knowledge, right now, there is no alternative technology available to extract hydrocarbon without drilling a hole. If the petroleum industry falls, the whole civilization will probably collapse. Therefore, for the survival of our existence, we need to know and keep updating our knowledge especially about the technology, drilling engineering. Based on this motivation, the human necessity of drilling a hole by excavation on earth has motivated researchers to develop different sophisticated technologies for drilling engineering. Drilling engineering has a vital role in our daily life, economy, society, and even in national and international politics.

1.4 Application of Drilling Engineering

In the development of human civilization over time, human beings have needed to make a hole in different objects for different purposes. This ranges from just a child playing a game with a toy to the drilling of a hole for the purpose of any scientific and technological usage. Humans have been using this technology for underground water withdrawal since ancient times. Drilling technology is a widely used expertise in the applied sciences and engineering such as manufacturing industries, pharmaceutical industries, aerospace, military defense, research laboratories, and any small-scale laboratory to a heavy industry like petroleum. Modern cities and urban areas use drilling technology to get underground water for drinking and household use. The underground water extraction by boring a hole is also used for agricultural irrigation purposes. Therefore, there is no specific field of application of this technology. It has been used in a wide range of fields based on its necessity. This book focuses on drilling a hole with the hope of hydrocarbon discovery; therefore, here the drilling engineering application means a shaftlike tool (i.e., drilling rig) with two or more cutting edges (i.e., drill bit) for making holes toward the underground hydrocarbon formation through the earth layers, especially by rotation. Hence the major application of drilling engineering is to discover and produce redundant hydrocarbon from a potential oil field.

1.5 Multiple Choice Questions

1. Technology necessary for extracting oil and gas reserves is
   a) Drilling
   b) Coiled tubing
   c) Hydraulic fracturing
   d) None of the above
2. Technology which is used to utilize crude oil and natural gas reserves is
   a) Reservoir engineering
   b) Stimulation technology
   c) Drilling technology
   d) All of the above
3. All the hydrocarbon resources present on the globe are found
   a) At surface
   b) Underground
   c) At rivers
   d) None of the above
4. The only method available to extract hydrocarbon reserves to date is
   a) Process engineering
   b) Production engineering
   c) Drilling technology
   d) All of the above
5. In early days, drilling was done to extract
   a) Underground water
   b) Coal
   c) Oil and gas reserves
   d) None of the above
6. Large deposits of untapped crude oil was mostly hidden below the surface until the middle of which century?
   a) 1700
   b) 1900
   c) 1800
   d) None of the above
7. In early days, oil seeps were used for
   a) Medicinal purpose
   b) Caulk boats
   c) In buildings
   d) Lubricating machinery
   e) All of the above
8. The first oil was drilled
   a) In Iraq and in 450 BC
   b) In China and in 347 BC
   c) In Macedonia and in 325 BC
   d) In Canada and in 1857

9. The petroleum industry in the Middle East was established by the
   a) Ninth century
   b) Eighth century
   c) Seventh century
   d) None of the above
10. In early days, the Chinese were using ______________ as modern drill pipe to extract oil.
    a) Bamboo
    b) Cable tool rig
    c) Rotary system
    d) None of the above
11. Mohammad ibn Zakariya Razi produced ______________ from petroleum using the distillation process in the ninth century.
    a) Diesel
    b) Kerosene
    c) Gas
    d) All of the above
12. Distillation process became available in Western Europe through Islamic Spain by
    a) Eleventh century
    b) Twelfth century
    c) Tenth century
    d) None of the above
13. In the West, ______________ was the first place of commercial production.
    a) United States
    b) Canada
    c) Brazil
    d) France
14. The first breakthrough in the oil industry's drilling history was the year ______________.
    a) 1839
    b) 1849
    c) 1859
    d) 1921
15. The first oil well drilled in the United States was ______________ ft deep.
    a) 59 feet
    b) 69 feet
    c) 79 feet
    d) 89 feet
16. The principal party in the oil industry is called
    a) Service company
    b) Operator company
    c) Contractor
    d) Consultant

17. The first task an operator has to do is the engagement of a
    a) Consultant
    b) Geologist
    c) Landman
    d) Surveyor
18. A ______________ is hired by the operator to acquire drilling rights.
    a) Landman
    b) Surveyor
    c) Geologists
    d) None of the above
19. A contractor who owns the drilling rig and employs the crew to drill the well is called
    a) Drilling contractor
    b) Service company
    c) Operator
    d) None of the above
20. Operator hires ______________ to conduct other rig jobs.
    a) Specialist consultants
    b) Geologists
    c) Landman
    d) Surveyor
21. Petroleum and mineral resources are usually owned by the ______________ of the host country.
    a) Gangsters
    b) Government
    c) Private Sector
    d) Bureaucrats
22. ______________ licenses allow licensees to drill for, develop and produce hydrocarbons from whatever depth is necessary.
    a) Exploration
    b) Production
    c) Drilling
    d) Seismic
23. A well that helps to determine the presence of hydrocarbons is called ______________ well.
    a) Wildcat
    b) Development
    c) Exploration
    d) None of the above

24. A well that is drilled to establish the extent (size) of reservoir is called ______________ well.
    a) Wildcat
    b) Development
    c) Appraisal
    d) None of the above

25. A well that is drilled in a proved production field or area to extract natural gas or crude oil is called ______________ well.
    a) Wildcat
    b) Development
    c) Appraisal
    d) None of the above

26. A well that is sealed and closed is called an
    a) Wildcat
    b) Development
    c) Appraisal
    d) Abandonment

27. ______________ licenses do not allow a company to drill any deeper than certain depth.
    a) Exploration
    b) Production
    c) Drilling
    d) Seismic

28. On average, only one in eight exploration wells are successful in ______________.
    a) Red Sea
    b) North Sea
    c) Atlantic Ocean
    d) Pacific Ocean

29. The role of drilling engineer during drilling operations is
    a) Planning
    b) Design
    c) Supervision
    d) All of the above

30. Which one is not a type of drilling well?
    a) Exploration well
    b) Appraisal well
    c) Development well
    d) Wild dog Well

31. It is believed that the revolution of modern civilization benefited much from the revolution of:
    a) Drilling technology
    b) Seismic technology
    c) Oil industry
    d) EOR technology
32. Considering the lifecycle of the well, drilling operations are required at
    a) The middle stage of the lifecycle
    b) The initial stage of the lifecycle
    c) The last stage of the lifecycle
    d) All of them
33. The main objective of drilling engineering is to
    a) Have sustainable drilling operations
    b) Explore deep reservoirs
    c) Increase productivity
    d) Avoid breaking the formations
34. A sustainable drilling operation can be achieved by
    a) Efficient designing
    b) Analyzing the drilling data
    c) Implementing the right procedures
    d) All of the above
35. The responsibilities of a drilling engineer are to
    a) Decide the location of the well to be drilled
    b) Prepare the well pad
    c) Determine the depth of the well
    d) None of the above
36. Which of the following does the drilling engineer has some flexibility to decide on?
    a) Well location
    b) Well trajectory
    c) Well depth
    d) Well cost
37. Which of the following does the drilling engineer not have flexibility to decide on?
    a) Well location
    b) Casing depths
    c) Holes diameter
    d) Rig maintenance
38. For which of the following operations does the drilling engineer need assistance and recommendations?
    a) Drilling fluids design
    b) Cementing design
    c) Drill bits design
    d) All of the above

39. Which of the following is not one of the technologies needed to prove the existence of petroleum accumulations?
    a) Seismic technology
    b) Reservoir simulation
    c) Formation evaluation
    d) Drilling technology
40. Petroleum accumulations can only be proved after
    a) Performing seismic operations
    b) Performing reservoir simulation
    c) Drilling a well
    d) Well stimulation
41. Commercial hydrocarbon accumulations can only be proved after performing
    a) Well testing
    b) Well logging
    c) Drilling a well
    d) None of the above
42. Human civilizations used drilling technology for
    a) Drilling water wells
    b) Mega constructions
    c) Drilling oil wells
    d) All of the above
43. The first oil discovery was in ______________, whereas the first nation to drill deep wells was ______________
    a) China and Babylon
    b) Kirkuk and Macedonia
    c) Iraq and China
    d) Kirkuk and Canada
44. Breakthrough of oil production all over the world was in
    a) The eighteenth century
    b) The nineteenth century
    c) The twentieth century
    d) The twenty-first century
45. The earliest known oil well was drilled in
    a) Canada
    b) United States
    c) Macedonia
    d) China
46. The first country to drill a commercial oil well was
    a) China
    b) Canada
    c) United States
    d) Iraq

47. A small exploratory oil well drilled in land not known to be an oil field to get the geological information is known as
    a) Wildcat well
    b) Appraisal well
    c) Pilot well
    d) Abandonment well
48. A well drilled in a land that has dry wells drilled earlier is a/an
    a) Appraisal well
    b) Wildcat well
    c) Development well
    d) Observation well
49. A well drilled in a land which has one discovery well drilled is a/an
    a) Exploration well
    b) Development well
    c) Wildcat well
    d) Appraisal well
50. A well drilled in a land known to have proven commercial oil is a/an
    a) Wildcat well
    b) Appraisal well
    c) Injection well
    d) None of the above

Answers: 1a, 2c, 3b, 4c, 5a, 6c, 7e, 8a, 9b, 10a, 11b, 12b, 13b, 14b, 15b, 16b, 17a, 18a, 19a, 20a, 21b, 22b, 23c, 24c, 25b, 26d, 27a, 28b, 29b, 30b, 31c, 32b, 33a, 34d, 35d, 36b, 37a, 38d, 39b, 40c, 41a, 42d, 43c, 44c, 45d, 46b, 47a, 48b, 49d, 50c.

1.6 Summary

This chapter developed the MCQs on some of the core issues related to drilling engineering. Even before starting drilling operations, many activities need to be completed to fulfill the different parties' requirements, which are well covered here. Moreover, this chapter added some more MCQs for the student's self-practice and the answers are given in Appendix B. The MCQs covered almost all the materials covered in this chapter.

1.7 MCQs (Self-Practices)

The solutions are in Appendix B.

1. A well which is drilled with no traces of oil accumulations is known as
    a) Abandonment well
    b) Appraisal well
    c) Dry well
    d) Wildcat well

2. All of the following well types can be converted into a producing well except
   a) Injection well
   b) Exploration well
   c) Observation well
   d) Dry well
3. A producing well should be ______________ in a sustainable fashion if it is not producing commercially any more
   a) Abandoned
   b) Shut in
   c) Stimulated
   d) None of the above
4. The first well that is drilled in a discovered field for gathering more information is called
   a) Exploration well
   b) Wildcat well
   c) Appraisal well
   d) All of the above
5. After drilling a wildcat well, which decision should be made right after completing the testing operation?
   a) Put the well on production
   b) Kill the well
   c) Abandon the well
   d) All of the above
6. The exploration well is drilled due to the following reason(s)—which one is the correct answer?
   a) Extent of the reservoir
   b) Oil existence
   c) Oil productivity estimations
   d) Production of oil
7. The appraisal well is drilled to find out
   a) Oil existence
   b) Rock and hydrocarbon properties
   c) Reservoir extent
   d) Reserve
8. The development well is drilled to
   a) Fix the designed rate
   b) Know the reservoir size
   c) Know the oil properties
   d) Extract hydrocarbons

9. Drilling operations can be categorized into three major steps—which one is not one of these categories?
   a) Drilling a hole
   b) Putting well on production
   c) Casing
   d) Completion of the well
10. Selection of the number of casing strings in a well is dependent on
    a) Well depth
    b) Complexity of the drilled formations
    c) Type of the well
    d) a and b
11. Casing sizes of the well is dependent on
    a) Well depth
    b) Well type
    c) Producing fluid type
    d) None of the above
12. Which of the following is the main purpose of conductor casing?
    a) Installation of BOP
    b) Prevent surface formations collapse
    c) Guide the drill bit vertically
    d) All of the above
13. All of the following are the main purposes of the surface casing except
    a) Prevent breaking the soft formations
    b) Isolate freshwater formations
    c) Decreasing well cost
    d) Install rig's BOP
14. If a well has three casing strings starting from its surface, these casings are called
    a) Conductor, surface and production
    b) Surface, intermediate and production
    c) Conductor, surface and intermediate
    d) Conductor, intermediate and production
15. The company that bears overall responsibilities of drilling operations is
    a) The drill bit company
    b) The rig contractor
    c) The drilling fluids company
    d) Drilling contractor
16. The company that may be responsible for slowing down the ROP is
    a) The rig contractor
    b) The drill bit company
    c) The service company
    d) All of them

17. Who is responsible for estimating the well drilling budget?
    a) The rig contractor
    b) Government
    c) Service companies
    d) Drilling team

18. Who is responsible for approving the well budget?
    a) Oil operator
    b) Government
    c) Service company
    d) a and b

19. Who is responsible for securing the well budget?
    a) Government
    b) Oil operator
    c) Service companies
    d) a and b

20. If a field is found which has no commercial hydrocarbon accumulations, who is going to pay the exploration expenses?
    a) Government
    b) Oil operator
    c) Land owner
    d) a and b

21. If a field is found which has commercial hydrocarbon accumulations, who is going to pay the exploration expenses?
    a) Service companies
    b) Government
    c) Oil operator
    d) b and c

22. If a well encounters problems during drilling and due to this problem the approved budget fails, which one of the following decisions should be made?
    a) Stop drilling the well and move to the next well
    b) Continue drilling at the expenses of the oil operator
    c) Request for supplementary budget
    d) None of the above

23. A well stopped during drilling due to bad weather for several days; who is going to pay the standby expenses?
    a) Oil operator
    b) Government
    c) a and b
    d) Service companies

24. Who will be the first responsible person if one of the drilling operations fails due to improper design implementation?
    a) The specified service company representative
    b) The oil operator representative
    c) The drilling engineer
    d) The government representative
25. In a well drilled with total cost which is less than the planned cost by 40%, this cost saving is considered as
    a) Saving oil operator's money
    b) Outstanding performance
    c) Bad cost estimates
    d) None of the above
26. Which one of the following wells is easier in estimating its drilling cost?
    a) Wildcat well
    b) Appraisal well
    c) Injection well
    d) Production well
27. Well cost estimation is difficult for explorations wells due to
    a) Difficulties in estimating the drilling rate
    b) Difficulties in estimating number of drill bits
    c) Difficulties in estimating other services cost
    d) All of the above
28. For a production section drilled vertically from 15,000 to 16,000 ft, what type of casing would need to be used for safety?
    a) Open hole without casing
    b) Casing string from surface to bottom
    c) Liner casing from last casing shoe to bottom
    d) None of the above
29. A well is planned to be drilled to a depth of 5,000 ft—what will be the suitable hole sections?
    a) One section
    b) Two sections
    c) Three sections
    d) Five sections
30. A well is planned to be drilled to a depth of 3,000 ft in two section—what will be the suitable section sizes?
    a) 36″ and 30″
    b) 30″ and 20″
    c) 20″ and 13 3/8″
    d) 13 3/8 and 9 5/8″

31. Technology necessary for extracting oil and gas reserves is
    a) Drilling
    b) Coiled Tubing
    c) Hydraulic Fracturing
    d) None of the above
32. Technology which is used to utilize the crude oil and natural gas reserves is
    a) Reservoir Engineering
    b) Stimulation Technology
    c) Drilling Technology
    d) All of the above

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2

Drilling Methods

2.1 Introduction

The rotary drilling rig and its components are the vehicle of modern drilling activities. In this method a downward force has to be applied on the tool that breaks the rock, and therefore it is an important parameter for an effective drilling operation. The conventional practice in the oil industry is to use heavy drillstring assembly, for which large capital expenses are required. However, while drilling, numerous challenges have to be faced throughout the drilling operations. Sets of multiple choice question (MCQs) are included which are related to different drilling systems including the rig and its components. The chapter covers almost all the mathematical calculations with enough workout examples. The solved examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

2.2 Different Mathematical Formulas and Examples

2.2.1 Power System

A typical arrangement of an engine with flywheel and pulley system is shown in Figure 2.1. The shaft power developed by an engine can be obtained by the following equation:

$$P_s = \omega T \tag{2.1}$$

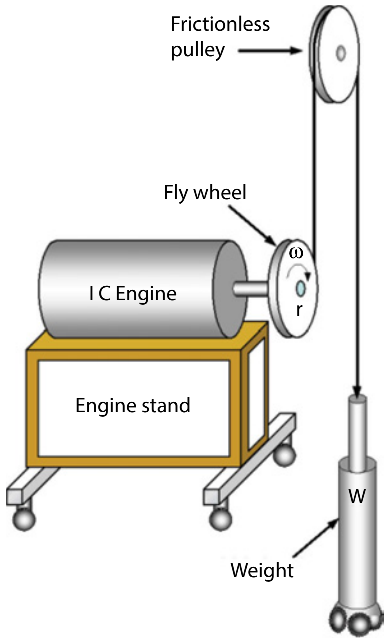

Figure 2.1 A typical IC engine power output.

where
P_s = Shaft power developed by an IC engine, *hp*
T = Output torque, $ft\text{-}lb_f$
ω = Angular velocity of the shaft, *rad/min*

In terms of revolution per minute, Eq. (2.1) can be written as:

$$P_s = \omega T = (2\pi N) \times (W r_{FW}) = 2\pi r_{FW} N W \tag{2.2}$$

In terms of velocity vector and for frictionless pulley system, Eq. (2.1) can be written as:

$$\bar{v} = 2\pi r_{FW} N \tag{2.3}$$

Power of shaft can also be written as:

$$P_s = \frac{Wd}{t} = W \cdot \bar{v} \tag{2.4}$$

where
d = distance travel by the weight on pulley, *ft*
N = revolution per minute, *rpm*
t = time required to travel the distance, *d*, *min*
W = Weight on pulley, lb_f
$\bar{v}$ = velocity vector, *ft/min*
r_{FW} = Radius of flywheel, *ft*

The overall engine power efficiency is determined as the power output by power input. Mathematically, it can be written as:

$$\eta_{ps} = \frac{\text{Power output}}{\text{Power input}} = \frac{P_s}{Q_i} = \frac{P_s}{w_f H_f} \tag{2.5}$$

where

η_{ps} = Overall engine efficiency of the power system
Q_i = power input to the IC engine, *hp*
w_f = the rate of fuel consumption by the engine, lb_m/min
H_f = heating value of fuel used in the engine, Btu/lb_m

Example 2.1: An internal combustion engine is run by diesel fuel in a rig side to generate power for the system. It gives an output torque of 1,600 $ft\text{-}lb_f$ at an engine speed of 1,150 *rpm*. The engine consumes fuel at a rate of 30 *gal/hr*. Calculate the wheel angular velocity, power output, overall efficiency of the IC engine.

Solution:

Given data:

T = Output torque = 1,600 $ft\text{-}lb_f$
N = revolution per minute = 1,150 *rpm*
w_f = the rate of fuel consumption by the engine = 30 *gal/hr*

Required data:

ω = Angular velocity of the shaft i.e., wheel angular velocity, *rad/min*
P_s = Shaft power developed by an IC engine i.e., power output, *hp*
η_{ps} = Overall engine efficiency of the power system i.e., IC engine, %

The angular velocity can be calculated by using the given equation:

$$\omega = 2\pi N = 2\pi \times 1{,}150 = \mathbf{7{,}225.68}\ \boldsymbol{rad/min}$$

The power output can be calculated using Eq. (2.1) as:

$$P_s = \omega T = \frac{(7{,}225.68\ rad/min) \times (1{,}600\ ft\text{-}lb_f)}{(33{,}000\ ft\text{-}lb_f/min) \big/ hp} = \mathbf{350.34}\ \boldsymbol{hp}$$

(***Note***: 1 $hp = 33{,}000\ ft\text{-}lb_f$ /min)

Since the engine is run by diesel fuel, therefore from Table 2.1 (Hossain and Al-Majed, 2014), the density ρ is 7.2 lb_m/gal and the heating value H_f is 19,000 Btu/lb_m. Therefore the fuel consumption rate w_f can be obtained by unit conversion as:

$$w_f = (30\ gal/hr) \times (7.2\ lb_m/gal) \times \left(\frac{1\ hr}{60\ minute}\right) = 3.6\ lb_m/min$$

Table 2.1 Derrick leg load distribution.

		Load on each derrick leg			
Load source	**Total load**	**Leg A**	**Leg B**	**Leg C**	**Leg D**
Hook load	W_{hl}	$W_{hl}/4$	$W_{hl}/4$	$W_{hl}/4$	$W_{hl}/4$
Fast line	T_f	$T_f/2$	–	$T_f/2$	–
Dead line	T_d	–	–	–	T_d
Total load on each derrick leg		$(W_{hl}/4) + (T_f/2)$	$W_{hl}/4$	$(W_{hl}/4) + (T_f/2)$	$W_{hl}/4+T_d$

Therefore, the total heat energy consumed by the IC engine i.e., input power can be calculated by using input power part of Eq. (2.5) as:

$$Q_i = w_f H_f = \frac{(3.6\ lb_m/min)\times(19{,}000\ Btu/lb_m)\times(779\ ft\text{-}lb_f/Btu)}{(33{,}000\ ft\text{-}lb_f/min)\big/hp}$$

$$= 1{,}614.65\ hp \qquad (\textbf{Note}:\ 1\ Btu = 779\ ft\text{-}lb_f)$$

Thus, the overall efficiency of the IC engine is obtained by using the Eq. (2.5) as:

$$\eta_{ps} = \frac{\text{Power output}}{\text{Power input}} = \frac{P_s}{Q_i} = \frac{350.34}{1{,}614.65} = \mathbf{0.2168\ or\ 21.68\%}$$

Example 2.2: An internal combustion engine is producing 251 *hp* power output at 6130 *rad/min* wheel angular velocity. The engine is consuming about 23 *gal/hr* of kerosene fuel that has a density of 0.79 *gm/cc*. Calculate the engine running speed, torque developed by the engine and efficiency of the engine.

Solution:

Given data:

P_s = Engine power = 251 *hp*
ω = Wheel angular speed = 6130 *rad/min*
ρ_{ker} = Kerosene density = 0.79 *gm/cc*
w_f = Engine fuel consumption = 23 *gal/hr*

Required data:

N = Engine running speed in *rpm*
T = Engine torque in $ft\text{-}lb_f$
η_{ps} = Efficiency of the engine

The engine running speed can be calculated using the following equation:

$$N = \frac{\omega}{2\pi} = \frac{6130(rad/min)}{2\pi} = \mathbf{976\ rpm}$$

The engine torque can be calculated after modifying Eq. (2.1) as below:

$$T = \frac{P_s}{\omega} = \frac{251(hp) \times 33{,}000\left(\frac{ft\text{-}lb_f}{min} \Big/ hp\right)}{6130(rad/min)} = \mathbf{1351\ lb_f\text{-}ft}$$

To calculate the efficiency of the engine, first we need to calculate the total heat energy consumed by the engine.

$$Q_i = w_f h_f$$

Heating value of kerosene (H_f) is 20,000 $\frac{Btu}{lb_m}$

$$w_f = \frac{23\left(\frac{gal}{hr}\right) \times 0.79\left(\frac{gm}{cc}\right) \times 8.34\left(ppg \Big/ \left(\frac{gm}{cc}\right)\right)}{60(min/hr)} = 2.53\frac{lb_m}{min}$$

So the total heat energy is equal to:

$$Q_i = \frac{2.53\left(\frac{lb_m}{min}\right) \times 20{,}000\left(\frac{Btu}{lb_m}\right) \times 779\left(ft\text{-}\frac{lb_f}{Btu}\right)}{33{,}000\frac{ft\text{-}lb_f}{min}/hp} = 1{,}195\ hp$$

Engine efficiency can now be calculated from Eq. (2.5):

$$\eta_{ps} = \frac{\text{Power output}}{\text{Power input}} = \frac{P_s}{Q_i} = \frac{251}{1{,}195} = 21\%$$

Example 2.3: A diesel engine was running at a speed of 1250 *rpm* at the drilling operations side. The driller noticed that the engine shaft output is 360 *hp*. He was trying to pull a drillstring of 600,000 lb_f. The engine was running for one hour. Calculate the wheel angular velocity, torque developed by the engine, the drillstring velocity, distance travelled by the drillstring.

Solution:

Given data:

N	= revolution per minute	=	1,250 *rpm*
W	= Weight on pulley	=	600,000 lb_f
P_s	= Shaft power developed by an IC engine i.e. power output	=	360 *hp*
t	= time required to travel the distance, d	=	1 week

Required data:

ω = Angular velocity of the shaft i.e. wheel angular velocity, *rad/min*
T = Output torque, *ft-lb$_f$*
$\bar{v}$ = the drillstring velocity i.e., velocity vector, *ft/min*
d = distance travel by the weight on pulley, *ft*

The angular velocity can be calculated by using the given equation:

$$\omega = 2\pi N = 2\pi \times 1{,}250 = \mathbf{7{,}854}\ \boldsymbol{rad/min}$$

The torque output is obtained using Eq. (2.1) as:

$$T = \frac{P_s}{\omega} = \frac{(360\ hp)\times\{(33{,}000\,ft\text{-}lb_f/min)/hp\}}{(7{,}854\ rad/min)} = \mathbf{1{,}512.61}\ \boldsymbol{ft\text{-}lb_f}$$

(***Note***: we know that 1 $hp = 33{,}000\,ft\text{-}lb_f/min$)

The drillstring velocity can be calculated using the Eq. (2.4) as:

$$\bar{v} = \frac{P_s}{W} = \frac{(360\ hp)\times\{(33{,}000\ ft\text{-}lb_f/min)/hp\}}{600{,}000\ lb_f} = \mathbf{19.8}\ \boldsymbol{ft/min}$$

As the engine was running for one hour, so the total distance traveled by the drill-string within one hour is obtained by using Eq. (2.4) as:

$$\frac{Wd}{t} = W\cdot\bar{v};\ \Rightarrow \frac{d}{t} = \bar{v}$$

So,

$$d = \bar{v}t = (19.8\,ft/min)\times(1\ hr\times 60\ min) = \mathbf{1188\ ft}$$

Example 2.4: A diesel engine gives an output torque of 1,740 *ft-lb$_f$* at an engine speed of 1,200 *rpm*. If the fuel consumption rate was 31.5 *gal/hr*, what is the output power and overall efficiency of the engine?

Solution:

Given data:

T = Output torque = 1740 *ft-lb$_f$*
N = revolution per minute = 1200 *rpm*
w_f = the rate of fuel consumption by the engine = 31.5 *gal/hr*

Required data:

P_s = Output power, *hp*
η_{ps} = Overall engine efficiency of the power system, %

The angular velocity can be calculated by using the given equation:

$$\omega = 2\pi N = 2\pi \times 1200 = 7539.822\ rad/min$$

The power output can be calculated using Eq. (2.1) as:

$$P_s = \omega T = \frac{(7539.822\ rad/min)\times(1740\,ft\text{ - }lb_f)}{(33{,}000\,ft\text{ - }lb_f/\mathrm{min})/_{hp}} = \mathbf{397.5}\ \boldsymbol{hp}$$

$$(\textbf{\textit{Note}}\text{: } 1\ hp = 33{,}000\,ft\text{ - }lb_f/\mathrm{min})$$

Since the engine is run by diesel fuel, therefore from Table 2.1, the density ρ is 7.2 *lbm/gal* and the heating value H_f is 19,000 Btu/lb_m. Therefore the fuel consumption rate w_f can be obtained by unit conversion as:

$$w_f = \left(31.5\frac{gal}{hr}\right)\times\left(\frac{7.2\ lb_m}{gal}\right)\times\left(\frac{1\ hr}{60\ minutes}\right) = 3.78\ lb_m/min$$

Therefore, the total heat energy consumed by the diesel engine i.e., input power can be calculated by using input power part of Eq. (2.5) as:

$$Q_i = w_f H_f = \frac{(3.78\ lb_m/min)\times(19{,}000\ Btu/lb_m)\times(779\ ft\text{-}lb_f/Btu)}{(33{,}000\ ft\text{-}lb_f/\mathrm{min})/_{hp}}$$

$$Q_i = 1695.3872\ hp \qquad (Note:\ 1\ Btu = 779\,ft\text{-}lb_f)$$

Thus, the overall efficiency of the diesel engine is obtained by using the Eq. (2.5) as:

$$\eta_{ps} = \frac{\text{Power output}}{\text{Power input}} = \frac{P_s}{Q_i} = \frac{397.5}{1695.3872} = \mathbf{0.2344}\ \boldsymbol{or}\ \mathbf{23.44\%}$$

Example 2.5: A diesel engine was running at an angular velocity of 8000 *rad/min* at the drilling operations side. The torque output of the engine is 1,612.61 $ft\text{-}lb_f$. The driller was trying to pull a drillstring 600,000 lb_f. The engine was running for one hour. Calculate the speed of the engine in *rpm*, engine shaft output in *hp*, the drillstring velocity, distance traveled by the drillstring.

Solution:

Given data:

ω = wheel angular velocity, *rad/min* = 8000 *rad/min*
T = Output torque = 1612.61 $ft\text{-}lb_f$
W = Weight on pulley = 600,000 lb_f

Required data:

N = revolution per minute, *rpm*
P_s = engine shaft output, *hp*
$\overline{v}$ = the drillstring velocity i.e., velocity vector, *ft/min*
d = distance traveled by the weight on pulley, *ft*

The speed of the engine can be calculated by using the given equation:

$$\omega = 2\pi N$$

$$2\pi N = 8000$$

$$N = 8000/2\pi = \mathbf{1273.24}\ \boldsymbol{rpm}$$

The engine shaft output is obtained using Eq. (2.1) as:

$$P_s = \omega T = \frac{(8000\ rad/min)\times(1612.61 ft\text{-}lb_f)}{(33{,}000 ft\text{-}lb_f/\text{min})/_{hp}} = \mathbf{390.94}\ \boldsymbol{hp}$$

$$(\textbf{\textit{Note}}\text{: 1 hp=33,000 } ft\text{-}lb_f/\text{min})$$

The drillstring velocity can be calculated using the Eq. (2.4) as:

$$\bar{v} = \frac{P_s}{W} = \frac{(390.94\ hp)\times\{(33000 ft\text{-}lb_f/\text{min})/hp\}}{600000\ lb_f} = \mathbf{21.5017}\ \boldsymbol{ft/min}$$

As the engine was running for one hour, so the total distance traveled by the drillstring within one hour is obtained by using Eq. (2.4) as:

$$\frac{Wd}{t} = W\cdot\bar{v}$$

$$\frac{d}{t} = \bar{v}$$

$$\text{so, } d = \bar{v}t = \left(21.5017\frac{ft}{min}\right)\times(1\ hr\times 60\ min) = \mathbf{1290.102}\ \boldsymbol{ft}$$

Example 2.6: A drilling rig diesel engine is consuming about 45 *gal/hr* of diesel fuel at running speed of 1,750 *rpm*. If the engine efficiency is estimated to be 29%, calculate the engine output power and the developed torque. The density of diesel is 7.2 *gal/min* and heating value is 19,000 Btu/lb_m.

Solution:

Given data:

w_f = Engine fuel consumption = 45 *gal/hr*

N = Engine running speed in rpm = 1,750 *rpm*

η_{ps} = Efficiency of the engine = 29%

Required data:

P_s = Engine output power in *hp*

T = Engine torque in $ft\text{-}lb_f$

To calculate the engine output power, we need first to calculate the total heat energy consumed by the engine.

$$w_f = \frac{45\left(\frac{gal}{hr}\right)\times 7.2(ppg)}{60(min/hr)} = 5.4\frac{lb_m}{min}$$

The total heat energy consumed by the engine is equal to:

$$Q_i = \frac{5.4\left(\frac{lb_m}{min}\right)\times 19{,}000\left(\frac{Btu}{lb_m}\right)\times 779\left(ft\text{-}\frac{lb_f}{Btu}\right)}{33{,}000\frac{ft\text{-}lb_f}{min}\Big/hp} = \mathbf{2{,}422}\ \boldsymbol{hp}$$

So, the engine power output can be calculated by modifying Eq. (2.5):

$$\text{Power output} = \text{Power input}\times\eta_{ps} = Q_i\times\eta_{ps}$$
$$= 2{,}422\times 0.29 = 702.4\ hp$$

Engine angular velocity is equal to:

$$\omega = 2\pi N = 2\pi\times 1750 = 10{,}996\ rad/min$$

So, the engine torque can now be calculated from Eq. (2.1):

$$T = \frac{P_s}{\omega} = \frac{702.4(hp)\times 33{,}000\left(\frac{ft\text{-}lb_f}{min}\Big/hp\right)}{10{,}996(rad/min)} = \mathbf{2{,}108}\ \boldsymbol{lb_f\text{-}ft}$$

Example 2.7: Shaft of an engine having a diameter of 4 *ft* was running at a velocity of 16,000 *ft/min* at the drilling operations side. The torque output of the engine is 1612.61 $ft\text{-}b_f$. The driller was trying to pull a drillstring 600,000 lb_f. The engine was running for one hour. Calculate the speed of the engine in *rpm*, engine shaft output in *hp*, the drillstring velocity, distance travelled by the drillstring.

Solution:

Given data:

V = wheel velocity = 16000 *ft/min*
d = diameter of the wheel = 4 *ft*
T = output torque = 1612.61 $ft\text{-}lb_f$
W = weight on pulley = 600,000 lb_f

Required data:

N = revolution per minute, *rpm*
P_s = engine shaft output, *hp*
$\bar{v}$ = the drillstring velocity i.e., velocity vector, *ft/min*
d = distance travelled by the weight on pulley, *ft*

We simply use $\omega = v/r$, but we must make sure that V and r have matching length units.

$$\omega = v\times\frac{1}{r} = \left(16000\,ft/min\times\frac{1}{2\ ft}\right) = 8{,}000\ rad/min$$

Note that the units actually come out as $1/min$; however, radians are suppressed unit with regards to angular velocity. So we write *(rad)/hr.* The speed of the engine can be calculated by using the given equation:

$$\omega = 2\pi N$$

$$8000 = 2\pi N$$

$$N = 8000/2\pi = \mathbf{1273.24\ \mathit{rpm}}$$

The engine shaft output is obtained using Eq. (2.1) as:

$$P_s = \omega T = \frac{\left(8000\dfrac{rad}{min}\right)\times(1612.61\,ft\text{-}lb_f)}{(33{,}000\,ft\text{-}lb_f/\text{min})/_{hp}} = \mathbf{390.94\ \mathit{hp}}$$

$$(\textbf{\textit{Note}}\text{: } 1\ hp = 33{,}000\,ft\text{-}lb_f/\text{min})$$

The drillstring velocity can be calculated using the Eq. (2.4) as:

$$\bar{v} = \frac{P_s}{W} = \frac{(390.94\ hp)\times\{(33000\,ft\text{-}lb_f/\text{min})/hp\}}{600000\ lb_f} = \mathbf{21.5017\ \mathit{ft/min}}$$

As the engine was running for one hour, the total distance traveled by the drill string within one hour is obtained by using Eq. (2.4) as:

$$\frac{Wd}{t} = W\cdot\bar{v}$$

$$\frac{d}{t} = \bar{v}$$

$$\text{So, } d = \bar{v}t = \left(21.5017\frac{ft}{min}\right)\times(1\ hr\times 60\ min) = \mathbf{1{,}290.102\ \mathit{ft}}$$

2.2.2 Hoisting System

The assembly and components of a hoisting system are shown in Figure 2.2. The compressive load of a derrick is calculated as the sum of the strengths of the four legs. Each leg is considered as a separate column and its strength is calculated at the weakest section. The wind load is specified in two ways, namely with or without pipe setback based on API derricks.

The wind load can be calculated as:

$$W_w = 0.004\ V^2 \tag{2.6}$$

where

W_w = wind load, lb_f/ft^2
V = wind velocity, *mph*

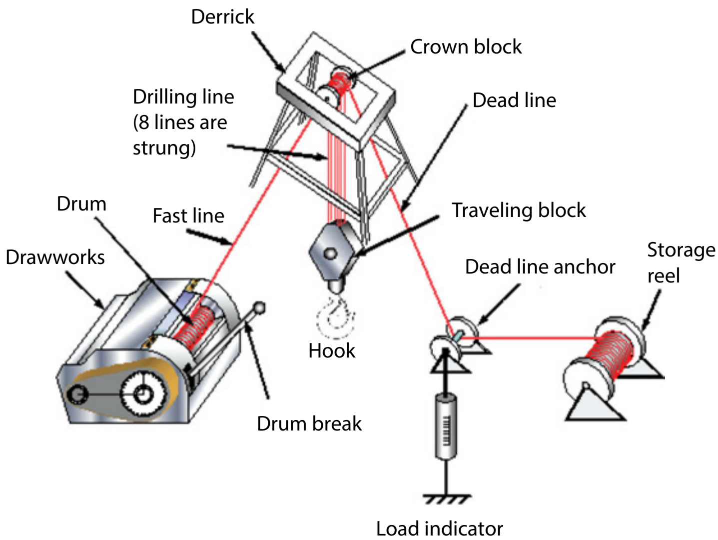

Figure 2.2 Different components of hoisting system.

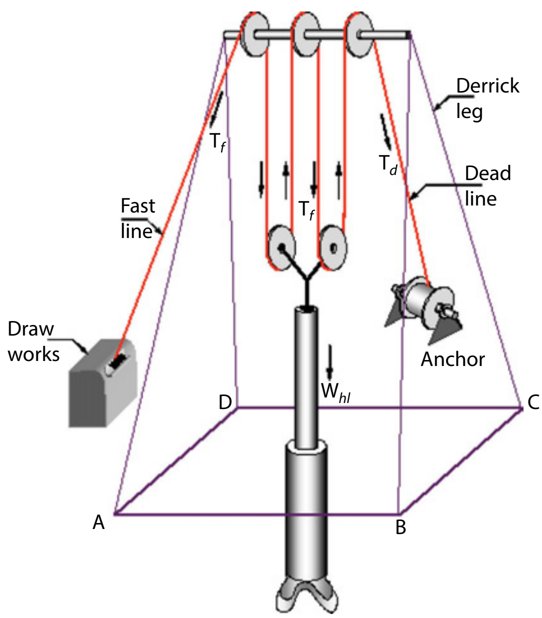

Figure 2.3 Derrick/mast of the hoisting system.

The total compressive load on derrick can be calculated using the block and tackle arrangement as shown in Figure 2.3. If the system has frictionless pulley, the following relationship is evident:

$$W_D = \frac{n+2}{n} W_{hl} \tag{2.7}$$

where
W_D = total compressive load on the derrick, lb_f
n = number of drilling lines through the traveling block
W_{hl} = hook load, lb_f

Example 2.8: During a drilling rig structure fatigue test, the operator measured the wind load of 0.5 *psi*. The rig has ten lines which are strung through the traveling block. A hook load of 250,000 lb_f is being hoisted. According to the API standard, calculate the wind velocity, and the total compressive load.

Solution:

Given data:
W_w = wind load = 0.5 *psi*
W_{hl} = hook load = 250,000 lb_f
n = number of drilling lines through the traveling block = 10

Required data:
V = wind velocity, *mph*
T = total compressive load on the derrick, lb_f

The wind velocity can be obtained using Eq. (2.6) as:

$$V^2 = \frac{W_w}{0.004} = \frac{(0.5\ lb_f/in^2)\times\{(144\ in^2)/1\ ft^2\}}{0.004}$$

$$(\textbf{\textit{Note}}\text{: we know that } 1\ ft_2 = 144\ in^2)$$

Therefore,

$$V = \mathbf{134}\ \boldsymbol{mph}$$

The total compressive load on the derrick is obtained using the Eq. (2.4) is:

$$W_D = \frac{n+2}{n}\ W_{hl} = \frac{(10+2)\times 250{,}000}{10} = \mathbf{300{,}000}\ \boldsymbol{lb_f}$$

***Example 2.9*:** During a rig structure fatigue test, the operator measured the wind load of 0.6 *psi*. The rig has sixteen lines which are strung through the traveling block. A hook load of 550,000 lb_f is being hoisted. According to the API standard, calculate the wind velocity, and the total compressive load.

Solution:

Given data:
W_w = wind load = 0.6 *psi*
W_{hl} = hook load = 550,000 lb_f
n = number of drilling lines through the traveling block = 16

Required data:
V = wind velocity, *mph*
T = total compressive load on the derrick, lb_f

The wind velocity can be obtained using Eq. (2.6) as:

$$V^2 = \frac{W_w}{0.004} = \frac{\left(0.6\dfrac{lbf}{in^2}\right)\times\{(144\ in^2)/1\,ft^2\}}{0.004}$$

$$(\textbf{\textit{Note}}\text{: we know that } 1\,ft_2 = 144\ in^2)$$

Therefore,

$$V = \mathbf{134}\,\boldsymbol{mph}$$

The total compressive load on the derrick is obtained using the Eq. (2.4) is:

$$W_D = \frac{n+2}{n}\ W_{hl} = \frac{(16+2)\times 550{,}000}{16} = \mathbf{618{,}750}\,\boldsymbol{lb_f}$$

Example 2.10: A rig is designed to withstand maximum wind speed of 250 *km/hr*. What will be the wind load that will be developed at that speed?

Solution:

Given data:

V = Maximum wind speed = 250 *km/hr*

Required data:

W_w = Wind load in lb_f/ft^2

Wind load can be estimated using Eq. (2.6):

$$W_w = 0.004\ V^2$$

Units of wind speed must first change to *mile/hr*:

$$V = \frac{250\dfrac{km}{hr}}{1.6\dfrac{km}{mile}} = \mathbf{156.25}\,\boldsymbol{mile/hr}$$

Now, wind load is equal to:

$$W_w = 0.004\ V^2 = 0.004 * 156.25^2 = \mathbf{97.9}\,\boldsymbol{lb_f/ft^2}$$

Using the fast line and dead line tension, the derrick load can also be calculated by:

Derrick load = hook load + Fast line load + dead line load

$$W_D = W_{hl} + T_f + T_d \tag{2.8}$$

where

T_f = tension (i.e., load) in the fast line, lb_f

T_d = tension (i.e., load) in the dead line, lb_f

In practical situation, the total derrick load is not distributed equally over all four derrick legs due to the placement of drawworks. Table 2.1 shows the load distribution for each leg where it is assumed that the four legs of the derrick are in equal distance.

The maximum equivalent derrick load can be written as:

$$W_{D_{max}} = \{W_{hl}/4 + T_d\} \times 4 \tag{2.9}$$

Derrick efficiency is written as:

$$\eta_D = \frac{W_D}{W_{D_{max}}} = \frac{W_D}{\{W_{hl}/4 + T_d\} \times 4} \tag{2.10}$$

The main function of block and tackle is to provide the *mechanical advantage* which can be written as:

$$M_{adv} = \frac{W_{hl}}{T_f} \tag{2.11}$$

where

M_{adv} = mechanical advantage of block and tackle

The tension in the tension (i.e., load) in the fast line can be written as:

$$T_f = \frac{W_{hl}}{n} \tag{2.12}$$

where

n = number of lines strung through the traveling block

The *ideal mechanical advantage* is

$$M_{iadv} = \frac{W_{hl}}{W_{hl}/n} = n \tag{2.13}$$

The power of the block and tackle can be defined mathematically:

$$P_{i_{bt}} = T_f v_f \tag{2.14}$$

where

$P_{i_{bt}}$ = input power of the block and tackle, *hp*

v_f = velocity of the fast line, *ft/min*

The output power or hook power is

$$P_{out_{bt}} = W_{hl} v_{bt} \tag{2.15}$$

where

$P_{out_{bt}}$ = output power of the block and tackle, *hp*

v_{bt} = velocity of the traveling block, *ft/min*

The traveling block velocity can be calculated using the following relation:

$$v_{bt} = \frac{v_f}{n} \tag{2.16}$$

The efficiency of the frictionless block and tackle can be obtained as the ratio of output power to input power which is obtained as:

$$\eta_{bt} = \frac{P_{out_{bt}}}{P_{i_{bt}}} = \frac{W_{hl} v_{bt}}{T_f v_f} = \frac{(T_f n) \times \left(\frac{v_f}{n}\right)}{T_f v_f} = 1 \tag{2.17}$$

If there is a friction, $\eta_{bt} = e^n = 0.98^n$. The actual block and tackle efficiency can be obtained as:

$$\eta_{bt} = \frac{P_{out_{bt}}}{P_{i_{bt}}} = \frac{W_{hl} v_{bt}}{T_f v_f} = \frac{W_{hl} \times \left(\frac{v_f}{n}\right)}{T_f v_f} = \frac{W_{hl}}{T_f n} \tag{2.18}$$

The fast line tensions in terms of hook load and block and tackle efficiency as:

$$T_f = \frac{W_{hl}}{\eta_{bt} n} \tag{2.19}$$

The tension in the deadline can be obtained as $T_d = W_{hl}/n$. The deadline derrick load is:

$$W_D = W_{hl} + \frac{W_{hl}}{\eta_{bt} n} + \frac{W_{hl}}{n} = \left\{\frac{(1 + \eta_{bt} + \eta_{bt} n)}{\eta_{bt} n}\right\} \times W_{hl} \tag{2.20}$$

The maximum derrick load is:

$$W_{D_{max}} = \left\{\frac{W_{hl}}{4} + \frac{W_{hl}}{n}\right\} \times 4 = \left(\frac{n+4}{n}\right) W_{hl} \tag{2.21}$$

The derrick efficiency is:

$$\eta_D = \frac{W_D}{W_{D_{max}}} = \frac{\left\{\frac{(1 + \eta_{bt} + \eta_{bt} n)}{\eta_{bt} n}\right\} \times W_{hl}}{\left(\frac{n+4}{n}\right) W_{hl}} = \frac{\eta_{bt}(n+1) + 1}{\eta_{bt}(n+4)} \tag{2.22}$$

Example 2.11: A 9 5/8-inch, 53.5 *lb/ft* casing string is set at a depth of 13,150 *ft*. Since this will be the heaviest casing string run, the maximum mast load must be calculated. Assuming that 10 lines run between the crown and the traveling blocks and neglecting buoyancy effects, calculate the maximum load.

Solution:

Given data:

W_{hl} = hook load = {(53.5 *lb/ft*) × (13150 *ft*)} = 703,525 lb_f
L_c = length of casing = 13,150 *ft*
OD_c = outer diameter of casing = 9 5/8 *in*
n = number of drilling lines through the traveling block = 10

Required data:
$W_{D_{max}}$ = Maximum derrick load, lb_f

If a frictionless pulley and block and tackle system is used, the fast line tension can be calculated using Eq. (2.12) as:

$$T_f = \frac{703{,}525\ lb_f}{10} = 70352.5\ lb_f$$

If we consider that the deadline has also the same tension, the maximum derrick load can be obtained using the Eq. (2.9) or Eq. (2.21) as:

$$W_{D_{max}} = \{W_{hl}/4 + T_d\} \times 4 = \left(\frac{703{,}525}{4} + 70352.5\right) \times 4 = \mathbf{984{,}935}\ \boldsymbol{lb_f}$$

$$W_{D_{max}} = \left(\frac{n+4}{n}\right) W_{hl} = \left(\frac{10+4}{10}\right) \times 703{,}525\ lb_f = \mathbf{984{,}935}\ \boldsymbol{lb_f}$$

Example 2.12: The total weight of 9,000 *ft* of 9 5/8-*inch* casing for a deep well is determined to be 400,000 *lbs*. Since this will be the heaviest casing string run, the maximum mast load must be calculated. Assuming that 10 lines run between the crown and the traveling blocks and neglecting buoyancy effects, calculate the maximum load.

Solution:
Given data:
Whl = hook load = 400,000 *lbf*
Lc = length of casing = 9,000 *ft*
ODc = outer diameter of casing = 9 5/8 *in*
n = number of drilling lines through the traveling block = 10

Required data:
$W_{D_{max}}$ = Maximum derrick load, lb_f

If frictionless pulley and block and tackle system is used, the fast line tension can be calculated using Eq. (2.12) as:

$$T_f = \frac{400{,}000\ lb_f}{10} = 40{,}000\ lb_f$$

If we consider that the deadline has also the same tension, the maximum derrick load can be obtained using the Eq. (2.9) or Eq. (2.21) as:

$$W_{D_{max}} = \{W_{hl}/4 + T_d\} \times 4 = \left(\frac{400{,}000}{4} + 40{,}000\right) \times 4 = \mathbf{560{,}000}\ \boldsymbol{lb_f}$$

$$W_{D_{max}} = \left(\frac{n+4}{n}\right) W_{hl} = \left(\frac{10+4}{10}\right) \times 400{,}000\ lb_f = \mathbf{560{,}000}\ \boldsymbol{lb_f}$$

This example demonstrates two additional points – the marginal decrease in mast load decreases with additional lines, and the total mast load is always greater than the load being lifted.

Example 2.13: The hoisting system of a rig derrick has a load of 350,000 lb_f. The input power of the drawworks for the rig can be a maximum of 530 *hp*. Eight drilling lines are strung between the crown block and traveling block. Assume that the rig floor is arranged as shown in Figure 2.3. Consider there is some loss of power due to friction within the hoisting system. Compute (1) the static tension in the fast line when upward motion is impending, (2) the mechanical advantage of the block and tackle, (3) the maximum hook horsepower available, (4) the maximum hoisting speed, (5) if a 90 *ft* stand is required to be pulled, what should be the required time, (6) the actual derrick load, (7) the maximum equivalent derrick load, and (8) the derrick efficiency factor.

Solution:

Given data:

W_{hl} = hook load = 350,000 lb_f
$P_{i_{bt}}$ = input power of the block and tackle = 530 *hp*
n = number of drilling lines through the traveling block = 8
L_s = length of stand = 90 *ft*

Required data:

T_f = tension (i.e., load) in the fast line, lb_f
M_{adv} = mechanical advantage
$P_{out_{bt}}$ = output power of the block and tackle, *hp*
v_{bt} = velocity of the traveling block, *ft/min*
t = time, *min*
W_D = actual load on the derrick, lb_f
$W_{D_{max}}$ = Maximum derrick load, lb_f
η_D = derrick efficiency, %

1. As the system is not frictionless, first we need to calculate the hoisting efficiency for eight numbers of drilling lines. The approximate block and tackle efficiency can be characterized by the following relationship:

$$\eta_{bt} = e^n = 0.98^n = 0.98^8 = 0.851$$

Therefore, the static tension in the fast line can be obtained using Eq. (2.19):

$$T_f = \frac{W_{hl}}{\eta_{bt} n} = \frac{350{,}000\ lb_f}{0.851 \times 8} = \mathbf{51{,}410\ lb_f}$$

2. The mechanical advantage of the block and tackle is given by Eq. (2.11) as:

$$M_{adv} = \frac{W_{hl}}{T_f} = \frac{350{,}000\ lb_f}{51{,}410\ lb_f} = \mathbf{6.81}$$

3. The maximum hook horsepower available can be obtained applying Eq. (2.17) as:

$$P_{out_{bt}} = \eta_{bt} \times P_{i_{bt}} = 0.851 \times 530 = \mathbf{451.03\ hp}$$

4. The maximum hoisting speed is the maximum velocity of the block and tackle that can be attained by the available hook power. Therefore the maximum velocity can be obtained by using Eq. (2.15) as:

$$v_{bt} = \frac{P_{out_{bt}}}{W_{hl}} = \frac{(451.03\ hp)\times\left(\dfrac{33{,}000\ ft\text{-}lb_f/min}{hp}\right)}{350{,}000\ lb_f}$$
$$= \mathbf{42.53\ ft/min}$$

5. To pull a 90 *ft* long stand, the time required can be estimated using the definition of speed as:

$$t = \frac{L_s}{v_{bt}} = \frac{90\,ft}{42.53\,ft/min} = \mathbf{2.12\,min}$$

6. The actual derrick load is given by Eq. (2.20):

$$W_D = \left\{\frac{(1+\eta_{bt}+\eta_{bt}n)}{\eta_{bt}n}\right\}\times W_{hl} = \left\{\frac{(1+0.851+0.851\times 8)}{0.851\times 8}\right\}$$
$$\times(350{,}000\ lb_f) = \mathbf{445{,}160.1\ lb_f}$$

7. The maximum derrick load can be obtained by using Eq. (2.21) as:

$$W_{D_{max}} = \left(\frac{n+4}{n}\right)W_{hl} = \frac{8+4}{8}\times(350{,}000\ lb_f) = \mathbf{525{,}000\ lb_f}$$

8. The derrick efficiency is given by Eq. (2.22):

$$\eta_D = \frac{W_D}{W_{D_{max}}} = \frac{445{,}160.1\ lb_f}{525{,}000\ lb_f} = 0.8479 = \mathbf{84.8\%}$$

Example 2.14: The hoisting system of a rig derrick has a load of 450,000 lb_f. The input power of the drawworks for the rig can be a maximum of 630 *hp*. Ten drilling lines are strung between the crown block and traveling block. Assume that the rig floor is arranged as shown in Figure 2.3. Consider there is some loss of power due to friction within the hoisting system. Compute (1) the static tension in the fast line when upward motion is impending, (2) the mechanical advantage of the block and tackle, (3) the maximum hook horsepower available, (4) the maximum hoisting speed, (5) if a 90 *ft* stand is required to be pulled, what should be the required time, (6) the actual derrick load, (7) the maximum equivalent derrick load, and (8) the derrick efficiency factor.

Solution:

Given data:

W_{hl} = hook load = 450,000 lb_f
$P_{i_{bt}}$ = input power of the block and tackle = 630 *hp*
n = number of drilling lines through the traveling block = 10
L_s = length of stand = 90 *ft*

Required data:

T_f = tension (i.e., load) in the fast line, lb_f
M_{adv} = mechanical advantage
$P_{out_{bt}}$ = output power of the block and tackle, *hp*
v_{bt} = velocity of the traveling block, *ft/min*
t = time, *min*
W_D = actual load on the derrick, lb_f
$W_{D_{max}}$ = Maximum derrick load, lb_f
η_D = derrick efficiency, %

1. As the system is not frictionless, then first we need to calculate the hoisting efficiency for ten numbers of drilling lines. The approximate block and tackle efficiency can be characterized by the following relationship:

$$\eta_{bt} = e^n = 0.98^n = 0.98^{10} = 0.817$$

Therefore, the static tension in the fast line can be obtained using Eq. (2.19):

$$T_f = \frac{W_{hl}}{\eta_{bt} n} = \frac{450{,}000\ lb_f}{0.817 \times 10} = 55079.559\ lb_f$$

2. The mechanical advantage of the block and tackle is given by Eq. (2.11) as:

$$M_{adv} = \frac{W_{hl}}{T_f} = \frac{450{,}000\ lb_f}{55079.559\ lb_f} = 8.170$$

3. The maximum hook horsepower available can be obtained by applying Eq. (2.17)

$$P_{out_{bt}} = \eta_{bt} \times P_{i_{bt}} = 0.817 \times 630 = \mathbf{514.71}\ \boldsymbol{hp}$$

4. The maximum hoisting speed is the maximum velocity of the block and tackle that can be attained by the available hook power. Therefore the maximum velocity can be obtained by using Eq. (2.15) as:

$$v_{bt} = \frac{P_{out_{bt}}}{W_{hl}} = \frac{(514.71\ hp) \times \left(\dfrac{33{,}000\ ft\text{-}lb_f/min}{hp} \right)}{450{,}000\ lb_f}$$
$$= \mathbf{37.7454}\ \boldsymbol{ft/min}$$

5. To pull a 90 ft long stand, the time required can be estimated using the definition of speed as:

$$t = \frac{L_s}{v_{bt}} = \frac{90\ ft}{37.7454\ ft/min} = \mathbf{2.384}\ \boldsymbol{min}$$

6. The actual derrick load is given by Eq. (2.20):

$$W_D = \left\{\frac{(1+\eta_{bt}+\eta_{bt}n)}{\eta_{bt}n}\right\} \times W_{hl}$$

$$= \left\{\frac{(1+0.817+0.817\times 10)}{0.817\times 10}\right\} \times (450{,}000\ lb_f)$$

$$W_D = \mathbf{550{,}079.559}\ \boldsymbol{lb_f}$$

7. The maximum derrick load can be obtained by using Eq. (2.21) as:

$$W_{D_{max}} = \left(\frac{n+4}{n}\right) \times W_{hl} = \frac{10+4}{10} \times (450{,}000\ lb_f)$$

$$= \mathbf{630{,}000}\ \boldsymbol{lb_f}$$

8. The derrick efficiency is given by Eq. (2.22):

$$\eta_D = \frac{W_D}{W_{D_{max}}} = \frac{550{,}079.559\ lb_f}{630{,}000\ lb_f} = .8731 = \mathbf{87.31\%}$$

Example 2.15: A diesel engine is run to generate power for the rig system. It gives an output torque rating of 1,500 *ft-lb$_f$* at an engine speed of 1,170 *rpm*. Consider that there is a friction loss in the pulley and block and tackle system. The hook load of the rig is 580,000 lb_f and there are ten drilling lines strung on the system. Find the output power of the engine, velocity of the fast line, tension of the fast line, velocity of the traveling block, power output of the block and tackle, efficiency of block and tackle.

Solution:

Given data:

T = Output torque = 1,500 *ft-lb$_f$*
N = revolution per minute = 1,170 *rpm*
W_{hl} = hook load = 580,000 lb_f
n = number of drilling lines through the traveling block = 10

Required data:

P_s = Shaft power developed by an IC engine i.e., power output, *hp*
T_f = tension (i.e., load) in the fast line, lb_f
v_f = velocity of the fast line, *ft/min*
v_{bt} = velocity of the traveling block, *ft/min*
$P_{out_{bt}}$ = output power of the block and tackle, *hp*
η_{bt} = efficiency of the block and tackle, %

The angular velocity can be calculated by using the given equation:

$$\omega = 2\pi N \times = 2\pi \times 1{,}170 = \mathbf{7{,}351.34}\ \boldsymbol{rad/min}$$

The power output can be calculated using Eq. (2.1) as:

$$P_s = \omega T = \frac{(7{,}351.34\ rad/min)\times(1{,}500\,ft\text{-}lb_f)}{33{,}000\,ft\text{-}lb_f/min\,/\,hp} = \mathbf{334.15}\ \boldsymbol{hp}$$

(***Note***: 1 $hp = 33{,}000\,ft\text{-}lb_f/\text{min}$)

If we consider that this engine power output will be only engaged by the hoisting system and there is no friction loss on the pulley, this engine power output would be considered as the power input for the block and tackle $(P_{i_{bt}})$. So, $P_{i_{bt}}$ = 334.15 *hp*. Tension in the fast line can be obtained using Eq. (2.12) as:

$$T_f = \frac{W_{hl}}{n} = \frac{580{,}000\ lb_f}{10} = \mathbf{58{,}000}\ \boldsymbol{lb_f}$$

Using Eq. (2.14), the velocity of the fast line can be obtained as:

$$v_f = \frac{P_{i_{bt}}}{T_f} = \frac{(334.15\ hp)\times(33{,}000\,ft - lb_f/min/hp)}{58{,}000\ \boldsymbol{lb_f}} = 190.12\,\boldsymbol{ft/min}$$

Equation (2.16) is used to calculate the velocity of traveling block as;

$$v_{bt} = \frac{v_f}{n} = \frac{190.12\,ft/min}{10} = \mathbf{19.0}\ \boldsymbol{ft/min}$$

The output power or hook power can be measured as the traveling block load (i.e., hook load) times the velocity of the traveling block i.e., using Eq. (2.15) as:

$$P_{out_{bt}} = W_{hl}v_{bt} = \frac{(580{,}000\ lb_f)\times(19.0\,ft/min)}{(33{,}000\,ft\text{-}lb_f/min)\,/\,hp} = \mathbf{333.94}\ \boldsymbol{hp}$$

The efficiency of the block and tackle can be given by Eq. (2.18) as:

$$\eta_{bt} = \frac{P_{out_{bt}}}{P_{i_{bt}}} = \frac{333.94\ hp}{334.15\ hp} = 0.99 = \mathbf{99\%}$$

Just to cross check, if we use the relationship $\eta_{bt} = e^n = 0.98^n$ for the efficiency, it becomes as $\eta_{bt} = e^n = 0.98^n = 0.98^{10} = 0.817 = 81.7\%$

Example 2.16: A diesel engine is run to generate power for the rig system. It gives an output torque rating of 1,000 $ft\text{-}lb_f$ at an engine speed of 1,070 *rpm*. Consider that there is a friction loss in the pulley and block and tackle system. The hook load of the rig is 500,000 lb_f and there are twelve drilling lines strung on the system. Find the output power of the engine, velocity of the fast line, tension of the fast line, velocity of the traveling block, power output of the block and tackle, efficiency of block and tackle.

Solution:

Given data:

T = Output torque = 1,000 *ft-lb*$_f$
N = revolution per minute = 1,070 *rpm*
n = number of drilling lines through the traveling block = 12
W_{hl} = hook load = 500,000 *lb*$_f$

Required data:

T_f = tension (i.e., load) in the fast line, *lb*$_f$
P_s = Shaft power developed by an IC engine i.e., power output, *hp*
$P_{out_{bt}}$ = output power of the block and tackle, *hp*
v_f = velocity of the fast line, *ft/min*
v_{bt} = velocity of the traveling block, *ft/min*
h_{bt} = efficiency of the block and tackle, %

The angular velocity can be calculated by using the given equation:

$$\omega = 2\pi N = 2\pi \times 1{,}070 = \mathbf{6723.008}\ \boldsymbol{rad/min}$$

The power output can be calculated using Eq. (2.1) as:

$$P_s = \omega T = \frac{(6723.008\ rad/min)\times(1000\,ft\text{-}lb_f)}{(33{,}000\,ft\text{-}lb_f/min)\big/hp} = \mathbf{203.727}\ \boldsymbol{hp}$$

(***Note***: 1 $hp = 33{,}000\,ft\text{-}lb_f/\text{min}$)

If we consider that this engine power output will be only engaged by the hoisting system and there is no friction loss on the pulley, this engine power output would be considered as the power input for the block and tackle ($P_{i_{bt}}$). So, $P_{i_{bt}} = 203.727\ hp$. Tension in the fast line can be obtained using Eq. (2.12) as:

$$T_f = \frac{W_{hl}}{n} = \frac{500{,}000\ lb_f}{12} = \mathbf{41666.67}\ \boldsymbol{lb_f}$$

Using Eq. (2.14), the velocity of the fast line can be obtained as:

$$v_f = \frac{P_{i_{bt}}}{T_f} = \frac{(203.727\ hp)\times(33{,}000\,ft\text{-}lb_f/min)\big/hp}{41666.67\ lb_f}$$
$$= \mathbf{161.351}\ \boldsymbol{ft/min}$$

Equation (2.16) is used to calculate the velocity of traveling block as:

$$v_{bt} = \frac{v_f}{n} = \frac{161.351\,ft/min}{12} = \mathbf{13.445}\ \boldsymbol{ft/min}$$

The output power or hook power can be measured as the traveling block load (i.e., hook load) times the velocity of the traveling block, i.e., using Eq. (2.15) as:

$$P_{out_{bt}} = W_{hl} v_{bt} = \frac{(500{,}000\ lb_f)\times(13.445\ ft/min)}{(33{,}000\ ft\text{-}lb_f/min)\big/hp} = \mathbf{203.71\ hp}$$

The efficiency of the block and tackle can be given by Eq. (2.18) as:

$$\eta_{bt} = \frac{P_{out_{bt}}}{P_{i_{bt}}} = \frac{203.71\ hp}{203.727\ hp} = 0.99 = 99\%$$

Just to cross check, if we use the relationship $\eta_{bt} = e^n = 0.98^n$ for the efficiency, it becomes as,

$$\eta_{bt} = e^n = 0.98^n = 0.98^{12} = 0.784 = 78.4\%$$

Example 2.17: A 9 5/8-inch, 50.5 *lb/ft* casing string is set at a depth of 10,150. Since this will be the heaviest casing string run, the maximum mast load must be calculated. Assuming that 10 lines run between the crown and the traveling blocks and neglecting buoyancy effects, calculate the maximum load.

<u>Solution</u>:

Given data:

W_{hl} = hook load = {(50.5 *lb/ft*) × (10150 *ft*)} = 512,575 lb_f
L_c = length of casing = 10,150 *ft*
OD_c = outer diameter of casing = 9 5/8 *in*
n = number of drilling lines through the traveling block = 10

Required data:

$W_{D_{max}}$ = Maximum derrick load, lb_f

If frictionless pulley and block and tackle system is used, the fast line tension can be calculated using Eq. (2.12) as:

$$T_f = \frac{512{,}575\ lb_f}{10} = 51257.5\ lb_f$$

If we consider that the deadline has also the same tension, the maximum derrick load can be obtained using the Eq. (2.9) or Eq. (2.21) as:

$$\textbf{Eq. (2.9):}\ W_{D_{max}} = \{W_{hl}/4 + T_d\}\times 4 = \left(\frac{512{,}575}{4} + 51257.5\right)\times 4$$
$$= \mathbf{717{,}605\ lb_f}$$

$$\textbf{Eq. (2.21):}\ W_{D_{max}} = \left(\frac{n+4}{n}\right)W_{hl} = \left(\frac{10+4}{10}\right)\times 512{,}575\ lb_f$$
$$= \mathbf{717{,}605\ lb_f}$$

Example 2.18: A 9 5/8-*inch*, 45.5 *lb/ft* casing string is set at a depth of 9,150. Since this will be the heaviest casing string run, the maximum mast load must be calculated. Assuming that 10 lines run between the crown and the traveling blocks and neglecting buoyancy effects, calculate the maximum load.

Solution:

Given data:

W_{hl} = hook load = {(45.5 *lb/ft*) × (9,150 *ft*)} = 416,325 lb_f

L_c = length of casing = 9,150 *ft*

OD_c = outer diameter of casing = 9 5/8 *in*

n = number of drilling lines through the traveling block = 10

Required data:

$W_{D_{max}}$ = Maximum derrick load, lb_f

If frictionless pulley and block and tackle system is used, the fast line tension can be calculated using Eq. (2.12) as:

$$T_f = \frac{416{,}325\ lb_f}{10} = 41632.5\ lb_f$$

If we consider that the deadline has also the same tension, the maximum derrick load can be obtained using the Eq. (2.9) or Eq. (2.21) as:

$$\textbf{Eq. (2.9)}:\ W_{D_{max}} = \{W_{hl}/4 + T_d\} \times 4 = \left(\frac{416{,}325}{4} + 41632.5\right) \times 4$$

$$= \mathbf{582{,}855}\ \boldsymbol{lb_f}$$

$$\textbf{Eq. (2.21)}:W_{D_{max}} = \left(\frac{n+4}{n}\right) W_{hl} = \left(\frac{10+4}{10}\right) \times 416325\ lb_f$$

$$= \mathbf{582{,}855}\ \boldsymbol{lb_f}$$

Example 2.19: A rig is hoisting a load of 450,000 lb_f. If 8 lines are strung through the traveling block, calculate the total compressive loads on the rig.

Solution:

Given data:

W_{hl} = Hook load = 450,000 lb_f

n = Number of lines = 8 lines

Required data:

W_D = Total compressive load, lb_f

The total compressive load can be calculated from Eq. (2.7)

$$W_D = \frac{n+2}{n} W_{hl} = \frac{8+2}{8} * 450{,}000 = 562{,}500\ lb_f$$

Example 2.20: A block and tackle system has 10 lines with efficiency of 0.81. The system can lift a maximum load of 600,000 lb_f at a speed of 100 *ft/min*. The diameter of draw-works drum is 30 *inches*. Calculate the fast line and dead line tensions, fast line and drum speeds, and the torque in the drum when no drilling line is reeled in the drum.

Solution:

Given data:

W_{hl} = Hook load = 600,000 lb_f
n = Number of lines = 10 *lines*
h_{bt} = Block and tackle efficiency = 0.81
v_{bt} = Block and tackle speed = 100 *ft*/min
d_{drum} = Drum diameter = 30 *inches*

Required data:

T_f = Fast line tension in lb_f
T_d = Dead line tension in lb_f
v_f = Fast line speed in *ft/min*
v_{drum} = Drum speed in *rpm*
T_{drum} = Drum torque in lb_f-*ft*

Fast line tension can be estimated using Eq. (2.19):

$$T_f = \frac{W_{hl}}{\eta_{bt} n} = \frac{600{,}000}{0.81 \times 10} = \mathbf{74{,}074}\ \boldsymbol{lb_f}$$

Dead line tension can be calculated using equation:

$$T_d = \frac{W_{hl}}{n} = \frac{600{,}000}{10} = \mathbf{60{,}000}\, \boldsymbol{lb_f}$$

Fast line speed can be determined from Eq. (2.16):

$$v_f = v_{bt}\ n = 100 * 10 = \mathbf{1{,}000}\ \boldsymbol{ft/min}$$

Drum speed can be calculated from the equation:

$$v_{drum} = \frac{v_f}{drum\ perimeter} = \frac{1{,}000}{\pi \times \left(\frac{30}{12}\right)} = \mathbf{127.3}\, \boldsymbol{rpm}$$

Drum torque where there is no line in the drum is equal to:

$$T_{drum} = T_f \times r_{drum} = 74{,}074 \times \frac{30}{2 * 12} = \mathbf{92{,}592.5}\, \boldsymbol{lb_f\text{-}ft}$$

2.2.3 Circulation System

Figure 2.4 depicts the valve and cylinder arrangement of a double acting (Figure 2.4a) and a single acting (Figure 2.4b) pumps.

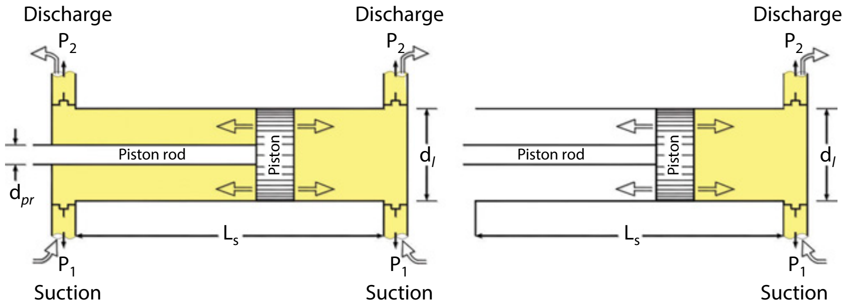

Figure 2.4 Valve and liner arrangement of mud circulating pumps.

The theoretical displacement from a double-acting pump is the volume of liquid displaced (Figure 2.4a):

$$V_{F1} = \frac{\pi}{4} d_l^2 L_s \tag{2.23}$$

where,

V_{F1} = volumetric displacement of liquid for a forward stroke with one piston, *in*³/*stroke*

d_l = liner diameter, *inch*

L_s = stroke length, *inch*

Similarly, for the backward stroke of each piston and liner system (Figure 2.4a), the volume of liquid displaced is

$$V_{B1} = \frac{\pi}{4} d_l^2 L_s - \frac{\pi}{4} d_{pr}^2 L_s = \frac{\pi}{4}(d_l^2 - d_{pr}^2) L_s \tag{2.24}$$

where,

V_{B1} = volumetric displacement of liquid for a backward stroke with one piston, *in*³/*stroke*

d_{pr} = piston diameter, *inch*

Therefore, for a double acting (duplex) pump having two cylinders, the total volumetric displacement of liquid per complete pump cycle is given as:

$$q_D = V_{F1} + V_{B1} = 2 \times \left\{ \frac{\pi}{4} d_l^2 L_s + \frac{\pi}{4}(d_l^2 - d_{pr}^2) L_s \right\} = \frac{\pi L_s}{2} \times \{2d_l^2 - d_{pr}^2\} \tag{2.25}$$

If the volumetric efficiency of pump (η_p) is considered, the total volumetric displacement per cycle or pump factor can be written as:

$$q_D = \frac{\pi L_s}{2} \times \{2d_l^2 - d_{pr}^2\} \times \eta_p \tag{2.26}$$

For N number of pump cycle:

$$q_{DN} = \frac{\pi L_s \eta_p}{2} \times \{2d_l^2 - d_{pr}^2\} \times N \tag{2.27}$$

where, N = number of pump cycle i.e., revolutions per minute of crank

$$= \frac{piston\ strokes/min}{4}$$

Since both pistons make a stroke in each direction for each revolution of the crank, there are four individual piston strokes per crank revolution. Therefore, for N revolution, Eq. (2.27) can be obtained as:

$$q_{DN} = \frac{\pi L_s \eta_p}{2} \times \{2d_l^2 - d_{pr}^2\} \times 4N \tag{2.28}$$

Now, pumps are commonly rated by hydraulic horsepower. If we assume that suction pressure is atmospheric, then work done per piston stroke can be calculated as:

$$W_P = P_d \times \left[\frac{\pi}{4}(d_l^2 - d_{pr}^2)\frac{L_s}{12}\right] \tag{2.29}$$

where,
W_P = work done per piston stroke, lb_f-ft
P_d = discharge pressure, *psig*

Since both pistons make a stroke in each direction for each revolution of the crank, there are four individual piston strokes per crank revolution. Therefore, for N revolution, Eq. (2.29) can be obtained as:

$$W_{PN} = P_d \times \left[\frac{\pi}{4}(d_l^2 - d_{pr}^2)\frac{L_s}{12}\right] \times 4N \tag{2.30}$$

where,
W_{PN} = work done per complete stroke, lb_f-ft

The power output for pump then can be obtained as:

$$P_{out_p} = \frac{P_d \times \left[\frac{\pi}{4}(d_l^2 - d_{pr}^2)\frac{L_s}{12}\right] \times 4N}{(33{,}000\,ft\text{-}lb_f/min)\big/hp \times \eta_{mp}} = \frac{P_d(d_l^2 - d_{pr}^2)L_s N}{126050.4 \times \eta_{mp}} \tag{2.31}$$

where,
W_{PN} = work done per complete stroke, lb_f-ft
P_{out_p} = output power for the duplex pump, *hp*
h_{mp} = mechanical efficiency of the duplex pump, %

For a mechanical efficiency of 85%, Eq. (2.31) can be reduced to the following equation:

$$P_{out_p} = \frac{P_d(d_l^2 - d_{pr}^2)L_s N}{107143} \tag{2.32}$$

In general, pumps are rated for hydraulic horsepower, maximum pressure, and maximum flow rate. The following equation is used to calculate the pump horsepower.

$$P_{hp} = \frac{\Delta pq}{1714} \tag{2.33}$$

where,
P_{hp} = pump horsepower, *hp*
Δp = increase in pressure, *psi,* which cannot be more than 3,500 *psi.*
q = flow rate, *gal/min*

As shown in Figure (2.4b) for a single acting pump (triplex), there is only one suction and delivery valve. which means there is no backward displacement. Therefore, the volumetric displacement by each piston stroke during one complete cycle is given by

$$q_S = \frac{\pi}{4} d_l^2 L_s \tag{2.34}$$

Thus the volumetric displacement per cycle for a single-acting pump having three cylinders with volumetric efficiency becomes as:

$$q_{ST} = \frac{3\pi}{3} d_l^2 L_s \times \eta_p \tag{2.35}$$

For N number of pump cycle, Eq. (2.35) can be written as:

$$q_{STN} = \frac{3\pi}{3} d_l^2 L_s \eta_p \times N \tag{2.36}$$

Example 2.21: Calculate the liner size required for a double-acting duplex pump where rod diameter is 2.5 *in*, stroke length is 22 *in* stroke, pump speed is 70 *strokes/min*. In addition, the maximum available pump hydraulic horsepower is 1200 *hp*. For optimum hydraulics, the pump recommended delivery pressure is 3,000 *psi*. Assume the volumetric efficiency of pump is 98%.

Solution:

Given data:

d_{pr} = piston rod diameter, *inch* = 2.5 *in*
L_s = stroke length, *inch* = 22 *in*
$4N$ = revolutions per minute of crank
= *piston strokes/min* = 70 *strokes/min*
P_{out_p} = output power for the duplex pump, *hp* = 1200 *hp*
P_d = discharge pressure, *psig* = 3,000 *psi* (here it is Δp for the pump)
η_p = volumetric efficiency of pump = 0.98

Required data:

d_l = liner diameter, *inch*

Using Eq. (2.26), the pump displacement for a duplex pump is given by

$$q_D = \frac{\pi L_s}{2} \times \{2d_l^2 - d_{pr}^2\} \times \eta_p = \frac{\pi \times (22\ in)}{2} \times \{2d_l^2 - (2.5\ in)^2\} \times 0.98,\ in^3/_{stroke}$$

$$= \left[33.85 \times \{2d_l^2 - (2.5\ in)^2\} in^3/_{stroke}\right] \times \frac{1\ gal}{231\ in^3} = \frac{\{2d_l^2 - (2.5\ in)^2\}}{6.82},\ \frac{gal}{stroke}$$

(***Note***: 1 gal = 231 in^3)

Using Eq. (2.28), the pump displacement for a duplex pump operating at 60 strokes/min is given by

$$q_{DN} = \frac{\pi L_s \eta_p}{2} \times \{2d_l^2 - d_{pr}^2\} \times 4N = \frac{\{2d_l^2 - (2.5\ in)^2\}}{6.82},\ \frac{gal}{stroke} \times 60 \frac{stroke}{min}$$

$$= \{17.6\ d_l^2 - 55\}\ gpm$$

Equation (2.33) gives the pump displacement as

$$P_{hp} = \frac{\Delta pq}{1714} \Rightarrow 1{,}200\ hp = \frac{(3{,}000\ psi)q}{1714} \Rightarrow q = 685.6\ gpm$$

Theoretically, these two pump displacement is equal therefore,

$$q_{DN} = q \Rightarrow \{17.6\ d_l^2 - 55\} = 685.6 \Rightarrow \boldsymbol{d_l} = 6.49\ \boldsymbol{in}$$

Example 2.22: Calculate the liner size required for a double-acting duplex pump where rod diameter is 3.0 *in*, stroke length is 25 *in* stroke; pump speed is 75 *strokes/min*. In addition the maximum available pump hydraulic horsepower is 1500 *hp*. For optimum hydraulics, the pump recommended delivery pressure is 3,500 *psi*. Assume the volumetric efficiency of pump is 98%.

Solution:

Given data:

P_d = discharge pressure, *psig* = 3000 *psi* (here it is Δp for the pump)
L_s = stoke length, inch = 25 *in*
$4N$ = revolution per minute of crank
= *piston strokes*/min = 75 *rpm*
P_{out_p} = output power for the duplex pump, *hp* = 1500 *hp*
d_{pr} = piston rod diameter, *inch* = 3.0 *in*
η_p = volumetric efficiency of pump = 0.98

Required data:

d_l = liner diameter, *inch*

Using Eq. (2.26), the pump displacement for a duplex pump is given by,

$$q_D = \frac{\pi L_s}{2} \times \{2d_l^2 - d_{pr}^2\} \times \eta_p$$

$$= \frac{\pi \times (25\ in)}{2} \times \{2d_l^2 - (2.5\ in)^2\} \times 0.98\ in^3/_{stroke}$$

$$q_D = \left[39.27 \times \{2d_l^2 - (2.5\ in)^2\} \times 0.98\ {in^3}/{stroke} \right] \times \frac{1\ gal}{231\ in^2}$$
$$= \frac{\{2d_l^2 - (2.5\ in)^2\}}{6.82}, \frac{gal}{stroke}$$
(***Note***: 1 gal = 231 in^3)

Using Eq. (2.28), the pump displacement for a duplex pump operating at 60 strokes/min is given by,

$$q_{DN} = \frac{\pi L_s \eta_p}{2} \times \{2d_l^2 - d_{pr}^2\} \times 4N = \frac{\{2d_l^2 - (2.5\ in)^2\}}{6.82}, \frac{gal}{stroke} \times 60 \frac{stroke}{min}$$
$$= \{17.6 d_l^2 - 55\} gpm$$

Equation (2.33) gives the pump displacement as,

$$P_{out_p} = \frac{\Delta p q}{1714}$$

$$1500\ hp = \frac{(3500\ psi) q}{1714}$$

$$q = 734.57\ gpm$$

Theoretically, these two pump displacement is equal therefore,

$$q_{DN} = q = \{17.6 d_l^2 - 55\} = 734.57$$

$$d_l = \mathbf{6.698\ in}$$

Example 2.23: A double-acting duplex pump has a rod of 2.25 *inches* diameter, 7 *inches* size and 25 *inches* liner length. The pump has the maximum hydraulic horsepower of 1,400 *hp*. If the required pump pressure is 2,400 *psi*. Calculate the pump speed in *strokes/min*. Assume displacement efficiency of 91%.

Solution:

Given data:

d_{pr} = Rod diameter = 2.25″
d_l = Liner diameter = 7.0″
L_s = Liner length = 25″
P_{out} = Pump hydraulic output power = 1,400 *hp*
P = Pump pressure = 2,400 *psi*
η_P = Pump displacement eff. = 0.91

Required data:

N = Pump speed in *strokes/min*

The displacement of this pump can be calculated using Eq. (2.26):

$$q_D = \frac{\pi L_S}{2\times 231\ (in^3/gal)}\times(2d_l^2 - d_{pr}^2)\eta_P = \frac{\pi\times 25}{2\times 231}\times(2\times 7^2 - 2.25^2)\times 0.91$$
$$= 14.4\ gals/stroke$$

The pump rate at the required pressure of 2,400 *psi* can be calculated from Eq. (2.33):

$$q = \frac{P_{out}\times 1714}{P} = \frac{1{,}400\times 1714}{2{,}400} = 1{,}000\ gals/min$$

Thus, the required strokes/min to get the same pump rate is equal to:

$$Pump\ speed = \frac{q}{q_D} = \frac{1{,}000}{14.4} = \mathbf{69.4\ OR\ 70\ strokes/min}$$

***Example 2.24*:** A single-acting triplex pump has been used in a drilling rig to provide a total pump rate of 750 *gals/min* at pressure of 1800 *psi* and pump speed of 100 *strokes/min*. If the liner diameter is 6 inches, determine the liner length and the pump output power. Assume a displacement efficiency of 95%.

Solution:

Given data:

d_l	=	Liner diameter	=	6.0″
q	=	Required pump rate	=	750 *gals/min*
P	=	Pump pressure	=	1,800 *psi*
η_P	=	Pump displacement eff.	=	95%
N	=	Pump speed in strokes/min	=	100

Required data:

L_s = Liner length

P_{out} = Pump hydraulic output power

Because the pump is single acting, only the forward stroke movement will be considered in the calculations. Thus, the volume displacement for a triplex pump can be calculated from Eq. (2.23):

$$q_D = \frac{3\times\frac{\pi}{4}\times d_l^2\times l\times\eta_P}{231\ in^3/gal} = \frac{3\times\frac{\pi}{4}\times 6^2\times l\times 0.95}{231\ in^3/gal} = 0.349\ l$$

The pump displacement can be calculated from the pump rate and pump speed as follows:

$$q_D = \frac{q}{N} = \frac{750}{100} = 7.5\ gals/stroke$$

Thus, the liner length is equal to:

$$l = \frac{q_D}{0.349} = \frac{7.5}{0.349} = 21.45 \; inches$$

The hydraulic output power can be calculated using Eq. (2.33):

$$P_{out} = \frac{Pq}{1{,}714} = \frac{2{,}400 \times 750}{1{,}714} = 1{,}050.2 \; \boldsymbol{hp}$$

Example 2.25: A double-acting duplex pump has a volumetric displacement of 16.5 *gals/stroke*. If the pump requires a 1,300 *hp* as an input power to be run at a 65 *strokes/min* that gives a discharge pressure of 1,750 *psi*, calculate the pump mechanical efficiency.

Solution:

Given data:

q_D	=	Volumetric displacement	=	16.5 *gals/stroke*
P_{input}	=	Power input	=	1,300 *hp*
P	=	Pump pressure	=	1,750 *psi*
N	=	Pump speed	=	65 *strokes/min*

Required data:

η_{mp} = Pump mechanical efficiency

To calculate the mechanical efficiency of the pump, we need to calculate the pump output power. From the given data, the pump flow rate is equal to:

$$q = q_D \times N = 16.5 \times 65 = 1{,}072.5 \; gals/min$$

The pump output power can now be calculated from Eq. (2.33):

$$P_{out} = \frac{PQ}{1714} = \frac{1{,}750 \times 1{,}072.5}{1{,}714} = 1{,}095 \; hp$$

Thus, the pump mechanical efficiency is equal to:

$$\eta_{mp} = \frac{P_{output}}{P_{input}} = \frac{1{,}095}{1{,}300} = \mathbf{84.2\%}$$

Example 2.26: A single-acting triplex pump of an output power of 1,250 *hp* is used to deliver the required rate at running speed of 110 *strokes/min*. The liner size and length are 5.5 and 30 *inches*, respectively. If the displacement efficiency is 90%, calculate the discharge pressure of the pump at that speed.

Solution:

Given data:

P_{out}	=	Pump output power	=	1,250 *hp*
N	=	Pump speed in strokes/min	=	110 *strokes/min*
d_l	=	Liner diameter	=	5.5″
L_s	=	Liner length	=	30″
η_P	=	Pump displacement eff.	=	90%

Required data:

P = Pump discharge pressure

To calculate the pump discharge pressure, we should first calculate the pump rate. Pump volume displacement can be calculated from Eq. (2.23):

$$q_D = \frac{3\times\frac{\pi}{4}\times d_l^2\times l\times\eta_P}{231\ in^3/gal} = \frac{3\times\frac{\pi}{4}\times 5.5^2\times 30\times 0.90}{231\ in^3/gal} = 8.33\ gals/stroke$$

Pump rate can now be calculated as follows:

$$q = q_D\times N = 8.33\times 110 = 916.4\ gal/min$$

Thus, pump discharge pressure can be calculated from Eq. (2.33):

$$P = \frac{P_{out}\times 1{,}714}{q} = \frac{1{,}250\times 1{,}714}{916.4} = \mathbf{2{,}338\ psi}$$

***Example 2.27*:** A double-acting triplex pump has a rod of 2.125 *inches* diameter, 5.5 *inches* size and 21 *inches* liner length. If it is required to pump a certain drilling mud at pumping rate of 750 *gpm*, what will be the pump speed in spm assuming displacement efficiency of 0.89%? What will be the pump speed if the efficiency was assumed to be 100%?

Solution:

Given data:

d_{pr}	=	Rod diameter	=	2.125″
d_l	=	Liner diameter	=	5.5″
L_s	=	Liner length	=	21″
q	=	Pumping rate	=	750 *gpm*
η_P	=	Pump displacement eff.	=	0.89

Required data:

N = Pump speed in strokes/min

Eq. (2.25) can be modified to be used for the double-acting triplex pump to determine its displacement as follows:

$$q_D = \frac{3\times\pi L_S}{4\times 231(in^3/gal)}\times(2d_l^2 - d_{pr}^2)\eta_P$$

$$= \frac{3\times\pi\times 25}{4\times 231}\times(2\times 5.5^2 - 2.125^2)\times 0.89 = 10.67\frac{gals}{stroke}$$

Thus, the pump should run at the following speed in *strokes/min* to get 750 *gpm*:

$$Pump\ speed = \frac{q}{q_D} = \frac{750}{10.67} = 70.3\ OR\ 71\ strokes/min$$

If the efficiency was assumed to be 100%, then the pump displacement will be equal to:

$$q_D = \frac{10.67\times 1.0}{0.89} = 11.99\ gals/stroke$$

The pump speed if we assume 100% efficiency will be equal to:

$$Pump\ speed = \frac{q}{q_D} = \frac{750}{11.99} = \mathbf{62.6\ \mathit{OR}\ 63\ \mathit{strokes/min}}$$

2.3 Multiple Choice Questions

1. Which one is not the basic rig classification?
   a) Onshore rig
   b) Cable tool rig
   c) Rotary rig
   d) Conventional rig
2. Earlier it was believed that hydrocarbons were present in only
   a) Soft formations
   b) Hard formations
   c) Carbonate formations
   d) All of the above
3. Which one is not a component of the cable tool rig?
   a) Derrick
   b) Crown block
   c) Rotary table
   d) Drilling cable
4. In cable tool drilling, drill cuttings are removed by
   a) Drilling mud
   b) Bailer
   c) Water
   d) Acidizing
5. Which one is not a basic component of the rotary rig?
   a) Kelly
   b) Annulus
   c) Rotary table
   d) Drill pipe
6. The first rotary drilling rig was developed in
   a) Sweden
   b) France
   c) Saudi Arabia
   d) Norway
7. The first rotary drilling rig was successfully used in Corsicana, Texas, in
   a) 1890
   b) 1880
   c) 1900
   d) 1920

8. The first oil well in the United States was drilled in 1859 with
   a) Cable tool rig
   b) Rotary rig
   c) Onshore rig
   d) None of the above

9. The first rotary drilling rigs were introduced in 1890 to cut
   a) Hard formations
   b) Soft formations
   c) Brittle formations
   d) Very hard formations

10. In rotary drilling, drill cuttings are removed by
    a) Drilling mud
    b) Bailer
    c) Water
    d) Acidizing

11. Which one is responsible for transmitting power to other rig systems?
    a) Hoisting system
    b) Power system
    c) Circulating system
    d) Rotary system

12. Which one of the following is responsible for lowering or lifting the drillstring, casing string in and out of the hole?
    a) Hoisting system
    b) Power system
    c) Circulating system
    d) Rotary system

13. Which one is a part of rotary drilling system?
    a) Hoisting system
    b) Power system
    c) Circulating system
    d) All of the above

14. Which one is not a component of power system of drilling rig?
    a) Drawworks
    b) Mud pumps
    c) Rotary table
    d) Drilling line

15. The steel structure part of rig which provides vertical height required to raise pipe sections is
    a) Derrick
    b) Draw works
    c) Crown block
    d) Traveling block

16. The total derrick load is not distributed equally over all four derrick legs due to the placement of
    a) Drilling line
    b) Draw works
    c) Crown block
    d) Traveling block
17. A parameter used to evaluate various drilling line arrangements is
    a) Derrick load
    b) Derrick efficiency
    c) Hook load
    d) Wind load
18. Crown block, traveling block and drilling line are components of
    a) Hook and load
    b) Block and tackle
    c) Block and load
    d) Fastline and deadline
19. The program carried out to maintain good condition of drilling line is
    a) Hook and load
    b) Slip and cut
    c) Slip and wear
    d) Wear and cut
20. Calculation made to assess the amount of wear on drilling line is
    a) Ton-miles
    b) Ton-meters
    c) Ton-foot
    d) Ton-km
21. The hook load is completely carried over by
    a) The traveling block
    b) Crown block
    c) Block and tackle
    d) Drilling line
22. The load imposed on the drawworks is equal to the ______________ in the fast line.
    a) Tension
    b) Compression
    c) Friction
    d) Shear
23. The ideal mechanical advantage assumes ______________ in the block and tackle.
    a) No friction
    b) 100% friction
    c) 50% friction
    d) 25% friction

24. Work done per unit time is the definition of ____________ of the block and tackle.
    a) Energy
    b) Power
    c) Tension
    d) Shear
25. The main function of drilling fluid is
    a) Remove cuttings
    b) Formation of mud cake
    c) Cools the bit
    d) All of above
26. Following is a type of drilling fluid
    a) Oil-based mud
    b) Water-based mud
    c) Invert oil emulsion
    d) All of above
27. Which one is <u>not</u> used to remove finer materials from the mud?
    a) Degasser
    b) Shale shaker
    c) Desander
    d) Centrifuge
28. Which one is a part of mud-cleaning process?
    a) Desander
    b) Desilter
    c) Degasser
    d) All of the above
29. Mud-circulating pumps are used to circulate drilling fluid at the desired
    a) Depth and temperature
    b) Pressure and temperature
    c) Temperature and volume
    d) Pressure and volume
30. The rotary system includes all of the equipment, which is used to attain
    a) Weight on bit
    b) Bit rotation
    c) Drill string load
    d) Derrick load
31. Which one is **not** the basic component of the power system?
    a) Rotary table
    b) Block and tackle
    c) Draw works
    d) Mud pump

32. Which one is **not** the function of the circulating system?
    a) To clean the hole of cuttings made by the bit
    b) To exert a hydrostatic pressure sufficient to prevent formation fluids entering the borehole
    c) To maintain the stability of the hole by depositing a thin mud-cake on the sides of the hole
    d) To give rotation to the rotary table

33. A mechanical device that suspends the weight of the drill pipe, provides for the rotation of the drill pipe beneath it while keeping the upper portion stationary, and permits the flow of drilling mud from the standpipe without leaking is named as
    a) Kelly
    b) Rotary table
    c) Swivel
    d) Drill Pipe

34. There are four major historical drilling techniques—which **one** is not the correct answer?
    a) Spring-pole drilling
    b) Cable tool drilling
    c) Conventional rotary drilling
    d) Drillstring assembly drilling

35. Which one is **not** the basic component of the hoisting system?
    a) Derrick and substructure
    b) Block and tackle
    c) Draw works
    d) Mud Pump

36. Which one is **not** associated with the rotary system?
    a) Kelly
    b) Annulus
    c) Rotary Table
    d) Drill Pipe

37. The main method of drilling wells before developing combustion engines was
    a) Human and animal's muscles
    b) Human brains
    c) Steam engines
    d) All of the above

38. The oldest drilling method used in drilling wells is
    a) Rotary drilling
    b) Cable-tool drilling
    c) Laser drilling
    d) None of the above

39. The major disadvantage of cable-tool drilling is
    a) Difficulties in drilling soft formations
    b) Drilling must be stopped to remove cuttings
    c) Blowout problems are common
    d) All of the above

40. The major difference between rotary drilling and cable-tool drilling is
    a) Method of drilling the rock
    b) How to remove the drilled cuttings
    c) The design of drillstring
    d) All of the above

41. Most of the rig power system is consumed in the
    a) Well control system
    b) Lighting system
    c) Circulating system
    d) All of the above

42. The generated rig power must be converted to ______________ in order to run and pull the drillstring
    a) Electrical power
    b) Mechanical power
    c) Hydraulic power
    d) None of the above

43. Which of the following component connects most of the hoisting system components together?
    a) Crown block
    b) Traveling block
    c) Drilling line
    d) Elevator

44. Maximum hook load occurred while
    a) Pulling drillstring up
    b) Running drillstring down
    c) Suspending the string
    d) None of the above

45. Which one of the following components is not the part of block and tackle?
    a) Crown block
    b) Traveling block
    c) Drilling line
    d) Drawworks

46. The most severe wear of the drilling line occurs when it passes through the
    a) Crown block sheaves
    b) Traveling block sheaves
    c) Draw works
    d) Dead line

47. To maintain the drilling line, which one of the following actions should be done frequently?
    a) Perform a specified cut and slip program
    b) Change all the drilling line
    c) Inspect all the drilling line after each well drilled
    d) All of the above

48. The name circulating system implies that the circulated mud
    a) Will be dumped at the mud pits
    b) Will be injected into the well
    c) Will be circulated again to the well after specific treatments
    d) All of the above

49. The first equipment to remove drill cuttings from the drilling mud is the
    a) Desander
    b) Desilter
    c) Degasser
    d) None of the above

50. Shale shakers are used to remove
    a) Gases from the drilling mud
    b) large cutting fragments
    c) Fine clays
    d) Small size solids and silts

51. Positive displacement pumps are used in drilling because it can
    a) Give high flow rates
    b) Give high discharge pressures
    c) Pump heavy drilling fluids
    d) All of the above

Answers: 1d, 2b, 3c, 4b, 5b, 6b, 7b, 8a, 9b, 10a, 11b, 12a, 13d, 14d, 15a, 16b, 17b, 18b, 19b, 20a, 21a, 22a, 23a, 24b, 25d, 26d, 27b, 28d, 29d, 30b, 31d, 32d, 33c, 34d, 35d, 36b, 37a, 38b, 39d, 40d, 41c, 42b, 43c, 44a, 45d, 46a, 47a, 48c, 49d, 50b, 51d.

2.4 Summary

This chapter developed the MCQs and examples based on all aspects related to a drilling rig and its components. The different formulations and mathematical models for drilling rig systems are explained with workout examples. The pump rating, capacity and design of pumps are also stated here. Moreover, this chapter added some more MCQs for the student's self-practice and the answers are given in Appendix B. The MCQs covered almost all the materials covered in this chapter.

2.5 Exercise and MCQs for Practice

2.5.1 Exercises (Solutions are in Appendix A)

Exercise 2.1: An internal combustion engine in a drilling rig is running at a speed of 1250 *rpm*. The engine is developing a torque of 1600 lb_f-ft and consuming about 27 *gal/hr* of diesel fuel that has a density of 7.2 ppg. Calculate the output power and the overall efficiency of the engine. ***Answers: 381 hp, 26.2%***

Exercise 2.2: A drilling rig is pulling a drill string at velocity of 30 *ft*/min. If the weight of the drill string is 450,000 lbf, calculate the output power and the developed torque if the rig engines are running at speed of 1350 *rpm*. ***Answers: 409.1 hp, 1592*** $\boldsymbol{lb_f}$***-ft***

Exercise 2.3: An internal combustion engine in a drilling rig has a flywheel diameter of 2 feet is running at a speed of 1200 *rpm*. The engine is developing a torque of 1,550 lb_f-ft. Calculate the flywheel velocity in *ft*/min, engine horsepower, and the maximum drillstring weight that the rig can be pulled if the required pulling velocity is 25ft/min. ***Answers: 7,540 ft/min, 354.2 hp, 467,544*** $\boldsymbol{lb_f}$

Exercise 2.4: A drilling rig engine that produces an output power of 450 *hp* at a torque of 2,500 lb_ff-ft is used to pull a drilling string that has a weight of 350,000 lb_f. Calculate the engine running speed in *rpm* and the maximum pulling velocity of the drilling string. ***Answers: 945.4 rpm, 42.4 ft/min***

Exercise 2.5: A 15,000 *ft* steel sand line cable weighing 1.1 lb_f/ft was suspended inside a well filled with 8.7 *ppg* brine. If an engine managed to reel the cable in 3.5 hours, what is the engine output power? ***Answer: 15.44 hp***

Exercise 2.6: A drilling rig is designed to have 8 lines to be strung between crown and traveling blocks. Approximate length of each line is 175 *ft* plus extra length of 300 *ft*. The draw works drum diameter is 36″ and drilling line diameter is 1.25″. If the plan is to have maximum of 4 laps in the drum, estimate the approximate length of the drum required to reel that drilling line. ***Answer: 4.27 ft***

Exercise 2.7: A hoisting system in a drilling rig has 2 *ft* diameter draw works drum that can rotate at a maximum speed of 159 *rpm* and output power of 450 *hp*. Ten lines are strung between crown and traveling blocks. The rig is drilling a vertical well using 9.7 *ppg* drilling fluid. Drilling string consists of 6 stands drill collars and 150 stands of drill pipes. Weight of one stand of drill collar in air is 8,650 lb_f and for the drill pipe is 2,380 lb_f, and all stands have same length, that is, 93 *ft*. Drilling engineers planned to pull out the drill string to change the bit. If the drilling foreman planned to pull every 30 stands at the same pulling speed, calculate the following:

a. Maximum speed available for the traveling block system to pull the drill string in *ft*/min?
b. How many stands need to be pulled before the hoisting system can be able to use his maximum pulling speed if the weight of the traveling block is 35,000 lb_f?

c. The time required to pull all the drill string out of the hole if disconnecting each stand takes approximately 0.75 minute. ***Answers: 100ft/min, 116 stands, 5.72 hrs***

Exercise 2.8: A hoisting system in a rig that is able to pull a maximum weight of 300,000 lb_f at a hook speed of 45 *ft*/min and drum speed of 51 *rpm*. Eight lines are strung between crown and traveling blocks. Calculate the minimum torque developed against the draw work drum due to the tension in the fast line. Assume block and tackle efficiency of 0.84. ***Answer: 51,154 lb_f-ft***

Exercise 2.9: A hoisting system has an output power of 550 *hp* being used in a drilling rig that has 10 lines strung between crown and traveling blocks. When the rig was pulling the drillstring at speed of 41.5 *ft*/min, the fast line tension read 54,130 lb_f. If the rig is using all the available output power of the draw work to pull the drillstring, calculate the block and tackle efficiency and the pulling speed when the fast line tension reads 44,045 lb_f. ***Answers: 0.81, 51 ft/min***

Exercise 2.10: A draw work has a drum diameter of 2.5 *ft* and is being used to pull a drillstring using 8 lines strung between crown and traveling blocks. When the hook was at the nearest point to the rig floor, the torque developed at the draw work drum was 66,813 lb_f-*ft* and only 8 reels of drilling line was in the drum. And when the hook was at the farthest point from the rig floor, the torque developed at the draw work drum was 93,300 lb_f-*ft* and 5 laps of drilling line was in the drum. Calculate the diameter of the drilling line used in this rig and the maximum pulling speed if the output power of the draw work is 500 *hp*. Assume block and tackle efficiency is 0.84. ***Answers: 1.5 inches, 45.9 ft/min***

Exercise 2.11: A double-acting duplex pump has a liner size and length of 7.5 and 25 inches, respectively. If the pump requires 1,400 *hp* output power to be run at 75 *strokes/min* and gives a pressure of 2,000 *psi*, calculate the diameter of the piston rod that satisfies the above required conditions. Assume displacement efficiency of 0.88 ***Answer: 2.37″***

Exercise 2.12: A new double-acting duplex pump is being installed in a drilling rig. The pump has 6.75″ liner size, 25″ liner length and 1.75″ piston rod diameter. The pump was tested for 5 minutes at the maximum allowable pump speed of 100 strokes/min using fresh water. The actual volume the pump displaced during the test duration was estimated to be 7,050 gallons. What is the displacement efficiency of this pump? ***Answer: 0.94***

Exercise 2.13: A single-acting triplex pump is required to deliver a pumping rate of 1,250 *gpm* at pressure of 1,750 *psi*. The size of each pump liner is 7.75″, whereas the length is 24″. If the displacement efficiency is 0.89, at what speed should the pump be run to deliver that rate? Also, what is the power output required to run the pump to achieve that pumping rate? ***Answers: 96stroke/min, 1,276 hp***

Exercise 2.14: A rig pump is planned to pump a drilling fluid at a pump speed of 84 *strokes/min* to give a pumping rate of 1,005 *gpm* at a stand pipe pressure of 1,750 *psi*. The pump displacement efficiency was previously estimated to be 0.92. When the pump was run at the planned speed, the driller noticed that the stand pipe pressure read 1,694 *psi*. Later, they realized that the pump displacement efficiency was not correct. Calculate the correct pump displacement efficiency, and the new pumping speed that would give the planned pumping rate. ***Answers: 0.95, 81 strokes/min***

Exercise 2.15: A rig company is planning to install new rig pumps that will be used in drilling a well. Hydraulics studies showed that it is required to have a total pumping rate of 1,850 *gpm*. A single-acting triplex pump of 6.25″ liner size and 14″ liner length is planned to be used. The maximum tested pump speed was 114 *strokes/min*, but the maximum allowable speed should only be 90% of the maximum tested speed. The pump displacement efficiency was estimated to be 0.93 How many pumps should they install to achieve the required pumping rate, and what will be the pumping speed if all the pumps are going to be run at the same speed? ***Answers: 4, 89 strokes/min***

Exercise 2.16: For a drilling rig having a maximum hook load of 250,000 lb_f, and 8 lines. Calculate i) fast line load (F_f), ii) dead-line load (F_s), and iii) static derrick load (F_d). ***Answers:*** 31,250 lb_f, 31,250 lb_f, 312,500 lb_f

Exercise 2.17: For problem 2.16, if hoisting system efficiency is 0.84, recalculate the derrick and lines loads? ***Answers:*** 37,202.4 lb_f, 31,250 lb_f, 318,452.4 lb_f

Exercise 2.18: If a rig has the following data: rig hoist load (W) = 200,000 lb_f, maximum draw works input power (P_i) = 800 *hp*, number of lines (n) = 10. Calculate i) static fast line tension (F_f), ii) maximum hook horsepower (P_h), iii) derrick load (F_d), iv) distribute the derrick load on its legs (A, B, C &D), v) maximum equivalent derrick load (F_e), vi) derrick efficiency factor (E_d). ***Answers:*** 24,691 lb_f, 648 ***hp***, 244,691 lb_f, **Leg *B*** = 50,000 lb_f, **Leg *A*** = 70,000 lb_f, **Leg *C*** = 62,346 lb_f, **Leg *D*** = 62,346 lb_f

2.5.2 Exercises (Self-Practices)

E2.1: A diesel engine gives an output torque of 1,740 *ft-lb*$_f$ at an engine speed of 1,200 *rpm*. If the fuel consumption rate was 31.5 *gal/hr*, what is the output power and overall efficiency of the engine? **Ans.** 397.5 hp; 23.4%

E2.2: An internal combustion engine is run by diesel fuel which gives an output torque of 1,300 *ft-lb*$_f$ at an engine speed of 1,000 *rpm*. The engine consumes fuel at a rate of 25 *gal/hr*. Calculate the wheel angular velocity, power output, overall efficiency of the IC engine. **Ans.**

E2.3: A diesel engine runs at a speed of 1,100 *rpm* and its engine power output 300 *hp*. If the system uses frictionless pulley, the draw works can handle to lower a drillstring of 500,000 *lb*$_f$. The drilling operations engine was continuing for three days. Calculate the wheel angular velocity, torque developed by the engine, the drillstring velocity, distance traveled by the drillstring, power input, and overall efficiency of the engine. **Ans.**

E2.4: A drilling rig has a hook load of 300,000 lb_f, which has eight drilling lines. A wind velocity of 100 *mph* is felt by the derrick. The rig has 10 lines which are strung through the traveling block. A hook load is being hoisted. According to the API standard, calculate the wind load and total compressive load. Assume that the block and tackle has the frictionless pulley. **Ans.**

E2.5: For a series of engine operations, the following data were obtained. The fuel used for running the engine was gasoline. Compute power output or break horsepower, overall engine efficiency for each engine speed, fuel consumptions in gal/day for 1,000 *rpm* and 850 *rpm* considering 8 hrs a day. **Ans.**

Engine speed (rpm)	Torque (ft-lb_f)	Fuel consumption (gal/hr)
1,350	1,500	27.0
1,150	1,650	20.0
1,000	1,700	18.0
850	1,750	15.5
700	1,800	13.0

E2.6: The total weight of 8,000 *ft* of 9 5/8-inch casing for a deep well is determined to be 344,000 lbs. Since this will be the heaviest casing string run, the maximum derrick load must be calculated. Assuming that 12 lines run between the block and tackle and neglecting buoyancy effects and friction, calculate the maximum derrick load. Also calculate each derrick leg load. **Ans.**

E2.7: The total casing weight is determined 440,000 lb_f for a 11,000 *ft* of 8 5/8-inch casing during a deep well casing operation. Assume that 14 lines are run with the hoisting system. As this casing string operation is the heaviest run, the maximum derrick load is needed to be calculated. Assume that there is a friction loss with the hoisting system and neglecting buoyancy effect, calculate the maximum derrick load. Also calculate each derrick leg load. **Ans.**

E2.8: The hoisting system of a rig derrick has a load of 440,000 lb_f. The input power of the drawworks for the rig can be a maximum of 560 *hp*. Fourteen drilling lines are strung between the crown block and traveling block. Assume that the rig floor is arranged as shown in Fig. 2.9. Consider there is some loss of power due to friction within the hoisting system. Compute (1) the static tension in the fast line when upward motion is impending, (2) the mechanical advantage of the block and tackle, (3) the maximum hook horsepower available, (4) the maximum hoisting speed, (5) if a 60 *ft* stand would require to pull, what should be required time, (6) the actual derrick load, (7) the maximum equivalent derrick load, and (8) the derrick efficiency factor.

E2.9: Calculate the liner size required for a triplex pump where rod diameter is 2.0 in, stroke length is 20 in stroke, pump speed is 80 strokes/min. In addition the maximum available pump hydraulic horsepower is 1000 *hp* and the delivery pressure is 3,000 *psi*. Assume the volumetric efficiency of pump is 98%.

Example 2.10: Calculate the liner size required for a double-acting duplex pump where rod diameter is 2.2 in, stroke length is 24 in stroke, pump speed is 75 strokes/min. In addition the maximum available pump hydraulic horsepower is 1300 *hp*. For optimum hydraulics, the pump recommended delivery pressure is 2,500 *psi*. Use the formula for calculating the volumetric efficiency of pump.

2.5.3 MCQs (Self-Practices)

1. Which pump can give higher flow rate than the others?
   a) Centrifugal pump
   b) Vacuum pump
   c) Helical pump
   d) Positive displacement pump
2. A duplex and double acting pump can deliver flow rate ______________ the triplex pump single acting if they have the same liner size and length
   a) Less than
   b) Equal to
   c) Greater than
   d) None of the above
3. For double-acting pumps, how many suction inlets and discharge outlets exist?
   a) Two suctions and two discharges
   b) Two suctions and one discharge
   c) One suction and two discharges
   d) One suction and one discharge
4. The component that provides the necessary rotation to turn the bit is the
   a) Kelly
   b) Rotary table
   c) Drill pipes
   d) Drill collars
5. The component that transmits the rotating action to the bit is
   a) Kelly
   b) Rotary table
   c) Drill pipes
   d) Drill collars
6. To add new drill pipe, the tool that is used to suspend the drillstring from the hole is called
   a) Elevator
   b) Slip
   c) Kelly
   d) All of the above

7. While making a new drill pipe, the drillstring will be suspended on the
   a) Elevator
   b) Rotary table
   c) Kelly
   d) All of the above
8. The off-shore rig that stands on the sea floor is called
   a) Submersible rig
   b) Semi-submersible rig
   c) Drill ship
   d) Jack-up rig
9. The immobile off-shore rig which is used to drill development wells is called
   a) Jack-up rig
   b) Submersible rig
   c) Semi-submersible rig
   d) Production platform
10. The main classification criterion for off-shore rigs is the
    a) Depth of sea bed
    b) Depth of the well
    c) The expected formation fluid
    d) All of the above
11. The equipment that is used to store and pressurize the hydraulic fluid for well control situations is the
    a) Accumulator
    b) Annular preventer
    c) Control panel
    d) BOP
12. ________ are effective at maintaining a seal around the drillpipe even as it rotates during drilling
    a) Pipe rams
    b) Annular blowout preventers
    c) Blind rams
    d) Shear rams
13. ______________ close around a drill pipe, restricting flow in the annulus between the outside of the drill pipe and the wellbore, but do not obstruct flow within the drill pipe
    a) Annular rams
    b) Blind rams
    c) Pipe rams
    d) Shear rams

14. ______________ cut through the drillstring or casing with hardened steel shears
    a) Annular rams
    b) Pipe rams
    c) Blind rams
    d) Shear rams

15. The rams which is used to control the well when there are no pipes in the hole is the
    a) Blind rams
    b) Shear rams
    c) Annular rams
    d) Pipe rams

16. If there is uncontrollable flow coming from the annulus, which of the following rams should be used to control the well?
    a) Shear rams
    b) Blind rams
    c) Annular rams
    d) Pipe rams

17. If there is uncontrollable flow coming from inside the drill pipe, which of the following rams should be used to control the well?
    a) Blind rams
    b) Pipe rams
    c) Shear rams
    d) None of the above

18. The part of the well control equipment which is used to circulate the mud during blow out situations is called
    a) Blow out preventer
    b) Choke manifold
    c) Poor boy degasser
    d) All of the above

19. The part of drillstring that keep drill pipes in tension during drilling is the
    a) Drill collars
    b) Kelly
    c) Hook
    d) None of the above

20. The part of the drilling equipment that can allow circulation and rotation while pulling the drilling string off bottom and running it back to bottom is
    a) Kelly
    b) Rotary table
    c) Circulating hoses
    d) Top drive system

21. The part of the drillstring that secures the required weight on the bit is the
    a) Heavy weight drill pipes
    b) Drill collars
    c) Drill pipes
    d) All of the above
22. The part of circulating system that allows drillstring rotation and at the same time allows mud circulation is the
    a) Kelly
    b) Rotary table
    c) Swivel
    d) All of the above
23. The sheaves of the crown block are
    a) Always greater than the sheaves of traveling block
    b) Always less than the sheaves of traveling block
    c) Equal to the sheaves of traveling block
    d) Less or greater than the sheaves of traveling block
24. The difference between the number of sheaves, and between crown block and traveling block is
    a) No difference
    b) Three sheaves
    c) Two sheaves
    d) One sheave
25. The part of the drillstring which is responsible for breaking up the formation rocks into fragments is the
    a) Drill bit
    b) Drill collar
    c) Drill pipe
    d) Drillstring
26. The following equipment are parts of circulating system—which one is not the correct answer?
    a) Drawworks
    b) Desander
    c) Shale shakers
    d) Degasser
27. The part of circulating system that is used to observe any loss or gain is the
    a) Settling tank
    b) Mixing tank
    c) Trip tank
    d) Regulator
28. The extra sheave in the crown block is required for the
    a) Fast line
    b) Dead line
    c) Sand line
    d) All of the above

29. The main objective of diverter system is to be used
    a) In shallow drilling before installing BOP
    b) In off-shore drilling before installing BOP
    c) In deep drilling after installing BOP
    d) None of the above
30. Diverter system is used
    a) As a blowout control equipment at shallow depths
    b) To install the BOP on top of it
    c) To divert the mud to the circulating system
    d) To control blowout
31. Which of the following is part of the circulating and rotating systems at the same time?
    a) Rotary table
    b) Drill pipes
    c) Stand pipe
    d) Generator
32. If you have a single-acting triplex pump, how many suction inlets and discharge outlets should you have?
    a) Three suctions and one discharge
    b) One suction and three discharges
    c) Three suctions and three discharges
    d) None of the above
33. Which of the following equipment is not a part of the well control system?
    a) Choke manifold
    b) Drill collar
    c) Accumulators
    d) Annular rams
34. All of the following equipment are not part of the hoisting system except
    a) Rotary table
    b) Mud pumps
    c) Drawworks
    d) Accumulator
35. Which of the following equipment is not a part of circulating system?
    a) Mud pumps
    b) Accumulator
    c) Shale shaker
    d) Drill string

2.6 Nomenclature

d	=	distance travel by the weight on pulley, *ft*
d_l	=	liner diameter, *inch*
d_{pr}	=	piston diameter, *inch*
H_f	=	heating value of fuel used in the engine, Btu/lb_m
L_S	=	stroke length, *inch*
M_{adv}	=	mechanical advantage
n	=	number of lines strung through the traveling block
N	=	number of pump cycle i.e. revolutions per minute of crank, $rpm = \frac{piston\ strokes/min}{4}$
n	=	number of drilling lines through the traveling block
P_d	=	discharge pressure, *psig*
P_{hp}	=	pump horsepower, *hp*
$P_{i_{bt}}$	=	input power of the block and tackle, *hp*
$P_{out_{bt}}$	=	output power of the block and tackle, *hp*
P_s	=	Shaft power developed by an IC engine, *hp*
P_{out_p}	=	output power for the duplex pump, *hp*
q	=	flow rate, *gal/min*
Q_i	=	power input to the IC engine, *hp*
r_{FW}	=	Radius of fly wheel, *ft*
T	=	Output torque, $ft\text{-}lb_f$
t	=	time required to travel the distance, *d*, *min*
T_f	=	tension (i.e., load) in the fast line, lb_f
T_d	=	tension (i.e., load) in the dead line, lb_f
V	=	wind velocity, *mph*
V_{B1}	=	volumetric displacement of liquid for a backward stroke with one piston, $in^3/stroke$
v_{bt}	=	velocity of the traveling block, *ft/min*
V_{F1}	=	volumetric displacement of liquid for a forward stroke with one piston, $in^3/stroke$
$\bar{v}$	=	velocity vector, *ft/min*
v_f	=	velocity of the fast line, *ft/min*
W	=	Weight on pulley, lb_f
w_f	=	the rate of fuel consumption by the engine, lb_m/min
W_w	=	wind load, lb_f/ft_2
W_D	=	total compressive load on the derrick, lb_f
W_{hl}	=	hook load, lb_f
W_P	=	work done per piston stroke, $lb_f\text{-}ft$
W_{PN}	=	work done per complete stroke, $lb_f\text{-}ft$
Δp	=	increase in pressure, *psi,* which cannot be more than 3,500 *psi*.
ω	=	Angular velocity of the shaft, *rad/min*
η_{mp}	=	mechanical efficiency of the duplex pump, %
η_{ps}	=	Overall engine efficiency of the power system

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3

Drilling Fluids

3.1 Introduction

The drilling-fluid system is one of the well-construction processes that remains in contact with the wellbore throughout the entire drilling operation. Sets of multiple choice question (MCQs) are presented in this chapter which are related to the drilling fluid technology. Workout examples related to mud engineering are extensively covered. The chapter covers almost all the mathematical calculations with enough workout examples. Workout examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

3.2 Different Mathematical Formulas and Examples

3.2.1 Solid Control

The following equation (Eq. 3.1) can be used to estimate the volume of solids entering to the mud system while drilling.

$$V_s = \frac{\pi(1-\phi_A)d_B^2}{4}R_{ROP} \tag{3.1}$$

where

d_B = bit diameter
V_s = solid volume of rock fragments entering the mud, i.e., volume of cuttings
R_{ROP} = rate of penetration of the bit
ϕ_A = average formation porosity

In field unit, Eq. (3.1) can be written as

$$V_s = \frac{\pi(1-\phi_A)d_B^2}{1029} R_{ROP} \tag{3.2a}$$

where

d_B = bit diameter, *in*
V_s = solid volume of rock fragments entering the mud, i.e., volume of cuttings, *bbl/hr*
R_{ROP} = rate of penetration of the bit, *ft/hr*
ϕ_A = average formation porosity, vol. fraction

If, V_s is in *tons/hr*, d_B is in *inch* and R_{ROP} is in *ft/hr*, Eq. (3.1) can be obtained as:

$$V_s = \frac{(1-\phi_A)d_B^2}{2262} R_{ROP} \tag{3.2b}$$

Example 3.1: A 15-in bit is used to drill a hole at a rate of 80 *ft/hr* where the porosity of the formation is 20%. Calculate the solid volume generated in this drilling operation. If the density of the solid is 910 lb_m/bbl, calculate the solid generation in tons/hr also.

Solution:

Given data:

d_B = bit diameter = 15 *in*
R_{ROP} = rate of penetration = 80 *ft/hr*
π_A = average porosity of the formation = 0.20

Required data:

V_S = solid volume generated by the bit in *bbl/hr*

The solid volume generated by the bit in *bbl/hr* can be calculated by using the Eq. (3.1) as:

$$\begin{aligned} V_s &= \frac{\pi(1-\phi_A)d_B^2}{4} R_{ROP} \\ &= \frac{\pi(1-0.20)(15^2\ in^2)}{4\times(231\ in^3/gal)\times(42\ gal/bbl)}\left(\frac{80\,ft}{hr}\times 12\ in/hr\right) \\ &= \mathbf{13.986\ bbl/hr} \end{aligned}$$

Same result can be obtained if we use Eq. (3.2a).

$$\text{In } tons/hr\text{: } 910\frac{lb_m}{bbl}\times 13.986\frac{bbl}{hr}\times\frac{1\ ton}{2000\ lb_m} = \mathbf{6.36\ tons/hr}$$

Same result can be obtained if we use Eq. (3.2b).

The solid volume generated by the bit in tons/hr can be calculated by using the Eq. (3.2b) as:

$$V_s = \frac{\pi(1-\phi_A)d_B^2}{2262}R_{ROP} = \frac{\pi(1-0.20)(15^2\ in^2)}{2262}\left(80\frac{ft}{hr}\right)$$
$$= \mathbf{20.0\ \mathit{tons/hr}}$$

***Example 3.2*:** A hole has been drilled by using a 17.5 *inches* bit at a drilling rate of 55 *ft/hr*. If the formation specific gravity and porosity were estimated to be 2.25, and 0.3 respectively. Calculate the solid volume generated after 1.75 *hrs*. If the produced cuttings at this time was estimated to be 8.0 *tons*, calculate the actual hole size.

Solution:

Given data:

d_B	=	Bit diameter	=	17.5 *inches*
R_{ROP}	=	rate of penetration	=	55 *ft/hr*
$\gamma_{cuttings}$	=	Cutting's specific gravity	=	2.25
ϕ_A	=	average porosity of the formation	=	0.3
t	=	drilling time	=	1.75 *hrs*
$M_{cuttings}$	=	Actual cutting mass	=	8.0 *tons*

Required data:

$V_{cuttings}$ = Volume of produced cuttings
d_{hole} = Hole diameter after enlargement

To calculate the volume of cuttings, first we will calculate the length that is drilled in 1.75 *hrs*:

$$L = R_{ROP} \times t = 55.5 \times 1.75 = 96.25\ ft$$

Now the volume of cuttings is equal to:

$$V_{cuttings} = \frac{\pi}{4}d_{bit}L \times (1-\phi_A) = \frac{\pi}{4} \times \frac{17.5^2}{144} \times 96.25 \times (1-0.3)$$
$$= \mathbf{112.54\ \mathit{ft}^3}$$

To calculate the hole diameter, first we change the mass of cutting to volume as below:

$$V = \frac{\text{mass}}{\text{density}} = \frac{8.0\ tons \times 2{,}200\left(\frac{lbm}{tons}\right)}{2.25 \times 62.4\frac{lbm}{cu\ ft}} = \frac{8.0 \times 2{,}200}{2.25 \times 62.4} = 125.36\ ft^3$$

Now, by knowing the length drilled we can calculate the actual hole diameter as follows:

$$V = 125.36 = \frac{\pi}{4} \times d_{hole}^2 L(1-\phi_A) = \frac{\pi}{4} \times 96.25 \times (1-0.3) \times d_{hole}^2$$
$$d_{hole} = \mathbf{1.54\ \mathit{ft}\ \mathit{OR}\ 18.47\ \mathit{inches}}$$

Example 3.3: For a typical North Sea well, it is given that the diameter of the bit is 26 in., rate of penetration is 62 *ft/hr*, and the average porosity is 25%. Find out the volume of cuttings.

Solution:

Given data:

d_B = bit diameter = 26 *in*

R_{ROP} = rate of penetration of the bit = 62 *ft/hr*

ϕ_A = average formation porosity = 0.25

Required data:

V_s = volume of cuttings

The volume of cuttings can be calculated by using the Eq. (3.1) as:

$$V_s = \frac{\pi(1-\phi_A)d_B^2}{4} R_{ROP} = \frac{\pi(1-0.25)\left(\frac{26}{12}ft\right)^2}{4}(62\text{ ft/hr})$$

$$= \mathbf{171.45\frac{ft^3}{hr}}$$

If we consider the specific gravity of drilled solids is approximately 2.6, so the density of the solid is $171.45\frac{ft^3}{hr} \times 62.4\frac{lb_m}{ft^3} = 10{,}698.5\frac{lb_m}{hr}$. So, the cuttings in $\frac{tons}{hr}$ will be $10{,}698.5\frac{lb_m}{hr} \times \frac{1\ ton}{2000\ lb_m} = \mathbf{5.35\frac{tons}{hr}}$.

Example 3.4: An 18-*in* bit is used to drill a hole at a rate of 72 *ft/hr* where the porosity of the formation is 25%. Calculate the solid volume generated this drilling operation. If the density of the solid is 960 *lbm/bbl*, calculate the solid generation in *tons/hr* also.

Solution:

Given data:

d_B = bit diameter = 18 *in*

R_{ROP} = rate of penetration = 72 *ft/hr*

ϕ_A = average porosity of the formation = 0.25

Required data:

V_s = solid volume generated by the bit in *bbl/hr*

The solid volume generated by the bit in *bbl/hr* can be calculated by using the Eq. (3.1) as:

$$V_s = \frac{\pi(1-\phi_A)d_B^{\,2}}{4} R_{ROP} = \frac{\pi(1-0.25)(18^2 in^2)}{4\times\left(231\frac{in^3}{gal}\right)\times\left(42\frac{gal}{bbl}\right)}\left(\frac{72\,ft}{hr}\times 12\frac{in}{hr}\right)$$

$$= \mathbf{16.996\ bbl/hr}$$

$$\text{In } tons\,/\,hr\text{: } 910\frac{lb_m}{bbl}\times 16.996\frac{bbl}{hr}\times\frac{1\ ton}{2000\ lb_m}=\mathbf{7.733\,tons/hr}$$

Example 3.5: A 22-*in* bit is used to drill a hole at a rate of 75 *ft/hr* where the volume of cuttings generated in this drilling operation is 149 ft^3/hr . Calculate the porosity of the formation.

<u>Solution</u>:

Given data:

d_B = bit diameter = 22 *in*
V_s = volume of cuttings = 149 ft^3/hr
R_{ROP} = rate of penetration = 75 *ft/hr*

Required data:

ϕ_A = average porosity of the formation = 0.25

The volume of cuttings can be calculated by using the Eq. (3.1) as:

$$V_s=\frac{\pi(1-\phi_A)d_B{}^2}{4}R_{ROP}=\frac{\pi(1-\phi_A)\left(\frac{22}{12}ft\right)^2}{4}(75\,ft/hr)$$

$$=148.49\frac{ft^3}{hr}$$

$$\phi_A=\mathbf{24.75\%}$$

Example 3.6: An intermediate hole was drilled using a 12.25 *inches* bit. The interval was described to be a consolidated rock with an estimated gravity and porosity of 2.9 and 0.1; respectively. The interval took 10.5 *hrs* to be drilled and produced about 16.5 *tons*. If there was no hole enlargement and no swelling of cuttings, calculate the rate of drilling at this section.

<u>Solution</u>:

Given data:

d_B = bit diameter = 12.25 *in*
$\gamma_{cuttings}$ = Cutting's specific gravity = 2.9
ϕ_A = average porosity of the formation = 0.10
t = drilling time = 10.5 *hrs*
$M_{cuttings}$ = Actual cutting mass = 16.5 *tons*

Required data:

R_{ROP} = rate of penetration in *ft/hr*

If there was no enlargement and no swelling occurred due to the water in the mud, the hole diameter will be exactly similar to the bit diameter. So by knowing the volume of the produced cuttings, we can calculate the length drilled, hence the ROP can be calculated. The volume of cuttings is equal to:

$$V = \frac{mass}{density} = \frac{16.5 \times 2{,}200 \dfrac{lbm}{tons}}{2.9 \times 62.4 \dfrac{lbm}{cu\ ft} \times 5.615 \dfrac{cu\ ft}{bbl}} = 35.73\ bbls$$

Now, we can use Eq. (3.2a) to get the length of the section drilled:

$$V_s = \frac{(1-\phi_A)d_B^2 L}{1029} = \frac{(1-0.1)\times 12.25^2 \times L}{1029} = 35.73$$
$$L = 272.36\ ft$$

The rate of penetration can now be calculated as below:

$$ROP = \frac{L}{t} = \frac{272.36}{10.5} = \mathbf{25.94\ ft/hr}$$

3.2.2 Mud Density

The conversion factors for mud density can be written as

$$\text{Specific Gravity } (SG) = gm/cm^3 = \frac{lb_m/gal}{8.33} = \frac{lb_m/ft^3}{62.3} \tag{3.3}$$

$$\text{Mud gradient } (MG_{FPS}) \text{ in } psi/f = \frac{lb_m/gal}{19.24} = \frac{lb_m/ft^3}{144} \tag{3.4}$$
$$= SG \times 0.433$$

$$\text{Mud gradient } (MG_{MKS}) \text{ in } kg/cm^2/m = SG \times 0.1 \tag{3.5}$$

Example 3.7: A mud engineer measured the density of the drilling fluid as 10 *ppg* in the rig side area. Calculate the specific gravity in gm/cm^3, mud gradient in *psi/ft* and $kg/cm^2/m$.

<u>Solution</u>:

Given data:

ρ_m = mud density = 10 *ppg*

Required data:

SG = specific gravity, gm/cm^3
MG_{FPS} = mud gradient, *psi/ft*
MG_{MKS} = mud gradient, $kg/cm^2/m$

The specific gravity of the mud in gm/cm^3 can be calculated by using the Eq. (3.3) as:

$$SG = \frac{lb_m/gal}{8.33} = \frac{10}{8.33} = \mathbf{1.2\ gm/cm^3}$$

The mud gradient (MG_{FPS}) in *psi/ft* can be calculated using Eq. (3.4) as:

$$MG_{FPS} = \frac{lb_m / gal}{19.24} = \frac{10}{19.24} = \mathbf{0.5197}\ \boldsymbol{psi/ft}$$

OR

$$MG_{FPS} = SG \times 0.433 = 1.2 \times 0.433 = \mathbf{0.5196}\ \boldsymbol{psi/ft}$$

The mud gradient (MG_{MKS}) in *kg/cm²/m* can be calculated using Eq. (3.5) as:

$$MG_{MKS} = SG \times 0.1 = 1.2 \times 0.1 = \mathbf{0.12}\ \boldsymbol{kg/cm^2/m}$$

Example 3.8: A mud engineer measured the density of the drilling fluid as 15.2 *ppg* in the rig side area. Calculate the specific gravity in gm/cm^3, mud gradient in *psi/ft* and $kg/cm^2/m$.

Solution:

Given data:

ρ_m = mud density = 15.2 *ppg*

Required data:

SG = specific gravity, gm/cm^3
MG_{FPS} = mud gradient, *psi/ft*
MG_{MKS} = mud gradient, $kg/cm^2/m$

The specific gravity of the mud in gm/cm^3 can be calculated by using the Eq. (3.3) as:

$$SG = \frac{lb_m / gal}{8.33} = \frac{15.2}{8.33} = \mathbf{1.824}\ \boldsymbol{gm/cm^3}$$

The mud gradient (MG_{FPS}) in *psi/ft* can be calculated using Eq. (3.4) as:

$$MG_{FPS} = SG \times 0.433 = 1.824 \times 0.433 = \mathbf{0.7901}\ \boldsymbol{psi/ft}$$

The mud gradient (MG_{MKS}) in $kg/cm^2/m$ can be calculated using Eq. (3.5) as:

$$MG_{MKS} = SG \times 0.1 = 1.824 \times 0.1 = \mathbf{0.1824}\ \boldsymbol{kg/cm^2/m}$$

3.2.3 Mud Viscosity

Mathematically viscosity can be expressed as

$$\mu = \frac{\text{Shearing stress}}{\text{rate of shearing strain}} = \frac{\tau_s}{\gamma} = \frac{F/A}{dv/dl} \tag{3.6}$$

Here,

A = cross-sectional area
l = layer thickness
F = force
v = velocity
μ = dynamic viscosity of the fluid between the plate
τ_s = shear stress
γ = shear rate
$\frac{dv}{dl}$ = velocity gradient along l-direction

Example 3.9: A moving plate with a velocity of 20 *cm/s* having a cross-sectional area of 10 cm^2 is placed 2 *cm* above a fixed plate. A force of 250 *dynes* is required to move the upper plate. Calculate the viscosity of the fluid.

Solution:

Given data:

v = velocity = 20 *cm/s*
A = cross-sectional area = 10 cm^2
l = layer thickness = 2 *cm*
F = force = 250 *dynes*

Required data:

μ = dynamic viscosity of the fluid between the plate, *cp*

The viscosity of the fluid can be calculated using Eq. (3.6) as:

$$\mu = \frac{F/A}{dv/dl} = \frac{(250\ dyne)/(10\ cm^2)}{(20\ cm/s)/(2\ cm)} = 2.5\frac{dynes}{cm^2} = \mathbf{25\ cp}$$

Example 3.10: Calculate the area of upper plate which is spaced 1 *cm* above a stationary plate. The viscosity of the fluid is 50 *cp* if a force of 1000 *dynes* is required to move the upper plate at a constant velocity of 10 *cm/s*.

Solution:

Given data:

v = velocity = 10 *cm/s*
l = layer thickness= 1 *cm*
F = force = 1000 *dynes*
μ = dynamic viscosity of the fluid = 50 *cp* = 5 *poise*

Required data:

A = Cross-sectional area

The viscosity of the fluid can be calculated using Eq. (3.6) as,

$$\mu = \frac{F/A}{dv/dl} = \frac{(1000\ dynes)/A}{(10\ cm/s)/(1\ cm)} = 5\ poise$$

$$\boldsymbol{A = 20\,cm^2}$$

Hossain *et al.* (2007) developed a new stress-strain relationship which can be written as:

$$\tau_{sT} = \left(\frac{\partial\sigma}{\partial T}\frac{\Delta T}{\alpha_D M_a}\right) \times \left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial\xi\partial x}\right)d\xi}{\Gamma(1-\alpha)}\right]^{0.5} \times \left(\frac{6K\mu_0\eta}{\frac{\partial p}{\partial x}}\right)^{0.5} \times e^{\left(\frac{E}{RT}\right)}\frac{du_x}{dy} \tag{3.7}$$

Here

E	=	activation energy for viscous flow, *KJ/mol*
K	=	operational parameter
M_a	=	Marangoni number
p	=	pressure of the system, N/m^2
R	=	universal gas constant, *kJ/mole – k*
T	=	temperature, °K
t	=	time, *s*
ξ	=	a dummy variable for time, i.e., real part in the plane of the integral
u_x	=	fluid velocity in porous media in the direction of *x* axis, *m/s*
y	=	distance from the boundary plan, *m*
σ	=	surface tension, *N/m*
α	=	fractional order of differentiation, dimensionless
α_D	=	thermal diffusivity, m^2/s
ΔT	=	$T_T - T_o$ = temperature difference between a temperature and a reference temperature, °K
μ	=	fluid dynamic viscosity, *Pa-s*
μ_o	=	fluid dynamic viscosity at reference temperature T_0, *Pa-s*
τ_{sT}	=	shear stress at temperature *T*, *Pa*
η	=	ratio of the pseudopermeability of the medium with memory to fluid viscosity, $\frac{m^3 s^{1+\alpha}}{kg}$
$\frac{du_x}{dy}$	=	velocity gradient along *y*-direction, $1/s$
$\frac{\partial\sigma}{\partial T}$	=	the derivative of surface tension σ with temperature and can be positive or negative depending on the substance, *N/m-K*

3.2.3.1 Measurement of Mud Viscosity

i) Marsh Funnel: Pitt (2000) introduced a new formula to measure the effective viscosity by Marsh funnel. For field use, the following equation is obtained as:

$$\mu_e = \rho_M (t_M - 25) \tag{3.8}$$

Here

m_e = the effective viscosity, *cp*
t_M = the Marsh funnel (quart) time, *s*
r_M = density of mud, *g/cm*^{3}

Example 3.11: A Marsh funnel is used to measure the density of the drilling fluid which is 1.2 *g/cm*^{3} in 50 *seconds*. Calculate the effective viscosity using Marsh funnel equation.

Solution:

Given data:

ρ_M = mud density = 1.2 g/*cm*^{3}
t_M = time = 50 *sec*

Required data:

μ_e = effective viscosity, *cp*

The effective viscosity of the mud in *cp*. can be calculated by using the Eq. (3.8) as:

$$\mu_e = \rho_M (t_M - 25) = 1.2(50 - 25) = \mathbf{30\ cp}$$

Example 3.12: The effective viscosity of the drilling fluid using Marsh funnel equation was found to be 30 *cp* in 50 *seconds.* Calculate the density of the drilling fluid obtained from Marsh Funnel experiment.

Solution:

Given data:

μ_e = effective viscosity = 30 *cp*
t_m = time = 50 *sec*

Required data:

ρ_m = mud density, *g/cm*^{3}

The density of the drilling fluid can calculated using the Eq. (3.8) as:

$$\rho_m = \frac{\mu_e}{(t_m - 25)} = \frac{30}{(50 - 25)} = \mathbf{1.2\ g/cm^3}$$

Example 3.13: Two drilling fluids have the same effective viscosity when measured using march funnel. Fluid A has a density of 10.1 *ppg* and read 49 *seconds* using the March funnel. If fluid B read 44 *seconds* in the March funnel, estimate the fluid B density.

Solution:

Given data:

ρ_A = Density of fluid A = 10.1 *ppg*
t_{MA} = Reading of fluid A = 49 *seconds*
t_{MB} = Reading of fluid B = 44 *seconds*

Required data:

ρ_B = Density of fluid B

Because two fluids have the same viscosity, we can calculate the viscosity of fluid A in order to calculate the density of fluid B. Fluid A effective viscosity can be calculated using Eq. (3.8):

$$\mu_{eA} = \rho_{MA}(t_{MA} - 25) = \frac{10.1}{8.34}(49 - 25) = 29.1\ cp$$

Using Eq. (3.8), we can calculate the density of fluid A to have the same viscosity:

$$\mu_{eB} = \rho_{MB}(t_{MB} - 25) = 29.1 = (44 - 25)\rho_{MB}$$

$$\rho_{MB} = 1.53\frac{gm}{cc}\ OR\ \mathbf{12.77\ \textit{ppg}}$$

***ii) Rotational Viscometer*:** During the measurement of apparent viscosity in centipoise, the dimensions of the bob and rotor are chosen so that the dial reading is equal to the apparent viscosity at a rotor speed of 300 *rpm*. Now, the apparent viscosity at any other speeds, N is given by:

$$\mu_{app} = \frac{300\ \pi_N}{N} \tag{3.9}$$

Here

Φ_N = torque reading from the dial at a speed N, *rpm*
N = rotor speed, *rpm*
μ_{app} = apparent viscosity at a speed N *rpm*, *cp*

Example 3.11: A mud sample in a rotational viscometer gives a dial reading of 45° at 600 *rpm* and a dial reading of 26° at 300 *rpm*. Compute the apparent viscosity of the mud at each rotor speed.

Solution:

Given data:

Φ_N = torque reading from the dial= 45° for 600 *rpm* = 26° for 300 *rpm*
N = rotor speed = 600 and 300 *rpm*

Required data:

μ_{app} = apparent viscosity at 600 *rpm* and 300 *rpm*, *cp*

The apparent viscosity of the mud in *cp* can be calculated by using the Eq. (3.9) as:

$$\mu_{app} = \frac{300\ \Phi_N}{N}$$

$$\mu_{app\_600} = \frac{300 \times 45}{600} = \mathbf{22.5\ cp}$$

$$\mu_{app\_300} = \frac{300 \times 27}{300} = \mathbf{27\ cp}$$

Example 3.12: The apparent viscosity of a mud sample at 300 *rpm* was found to be 32 *cp* and at 600 *rpm* was found to be 25 *cp*, from a rotational viscometer. Calculate the dial reading at 300 *rpm* and 600 *rpm*.

Solution:

Given data:

μ_{app_300} = apparent viscosity @ 300 *rpm* = 32 *cp*
μ_{app_600} = apparent viscosity @ 600 *rpm* = 25 *cp*
N = rotor speed = 600 *rpm* and 300 *rpm*

Required data:

Φ_N = torque reading from the dial for 600 *rpm* and 300 *rpm*

The torque reading from the dial can be calculated from the following equation Eq. (3.9) as:

$$\Phi_N = \frac{\mu_{app} \times N}{300}$$

$$\Phi_N = \frac{\mu_{app\_300} \times N}{300} = \frac{32 \times 300}{300} = 32^\circ$$

$$\Phi_N = \frac{\mu_{app\_600} \times N}{300} = \frac{25 \times 600}{300} = 50^\circ$$

The plastic viscosity is normally computed using the below relationship:

$$\mu_p = \Phi_{600} - \Phi_{300} \tag{3.10}$$

Here

Φ_{300} = torque reading from the dial at a speed of 300 *rpm*, *rpm*
Φ_{600} = torque reading from the dial at a speed of 600 *rpm*, *rpm*
μ_p = plastic viscosity, *cp*

The yield point can be computed using the following formula as:

$$\tau_B = f_{300} - \mu_p \tag{3.11}$$

Here

τ_B = the Bingham yield point, $\frac{lb_f}{100\ ft^2}$

From Eq. (3.10) and Eq. (3.11), yield point can be calculated as:

$$\tau_B = \Phi_{600} - 2\mu_p \tag{3.12}$$

In Eq. (3.9), using the dial deflection for 600 *rpm*, the apparent viscosity becomes

$$\mu_{app} = \frac{300\ \Phi_{600}}{600} = \frac{1}{2}\Phi_{600} \tag{3.13}$$

The use of Eq. (3.13) forms the Eq. (3.12) as:

$$\tau_B = 2(\mu_{app} - \mu_p) \tag{3.14}$$

True yield point can be defined using Figure 3.1 for plastic or Bingham fluids as:

$$\tau_{TB} = \frac{3}{4}\tau_B \tag{3.15}$$

Here

τ_{TB} = true Bingham yield point, $\frac{lb_f}{100\ ft^2}$

Example 3.13: Using the data of Example 3.11, compute the plastic viscosity yield point and true yield point of the mud sample.

Solution:

Given data:

Φ_N = torque reading from the dial = 45° for 600 *rpm* = 26° for 300 *rpm*

N = rotor speed = 600 and 300 *rpm*

Required data:

μ_p = plastic viscosity, *cp*

τ_B = Bingham yield point, $lb_f/100\ ft^2$

τ_T = true yield point, $lb_f/100\ ft^2$

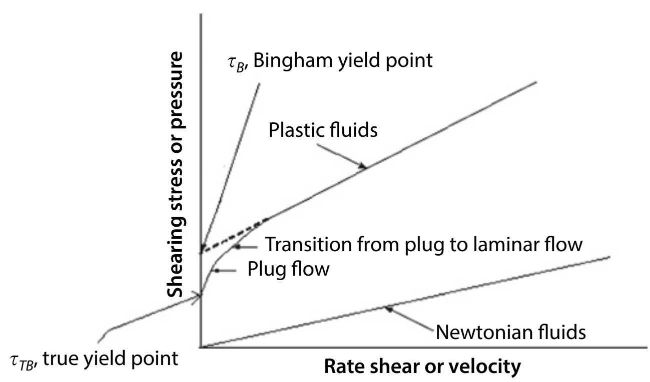

Figure 3.1 Flow behavior of fluids.

The plastic viscosity of the mud in *cp* can be calculated by using the Eq. (3.10) as:

$$\mu_p = \Phi_{600} - \Phi_{300} = 45 - 26 = \mathbf{19\ cp}$$

The yield point of the mud can be calculated by using the Eq. (3.11) as:

$$\tau_B = \Phi_{300} - \mu_p = 26 - 19 = \mathbf{7\ lb_f/100\ ft^2}$$

The true yield point of the mud can be calculated by using the Eq. (3.15) as:

$$\tau_T = \frac{3}{4}\tau_B = \frac{3}{4}\times 7 = \mathbf{5.25\ lb_f/100\ ft^2}$$

Example 3.14: Using the below data, compute the plastic viscosity, yield point and true yield point of the mud sample.

Solution:

Given data:

Φ_N = torque reading from the dial = 50° for 600 *rpm* = 32° for 300 *rpm*
μ_{app_300} = apparent viscosity @ 300 *rpm*=32 *cp*
μ_{app_600} = apparent viscosity @ 600 *rpm*=25 *cp*
N = rotar speed =600 *rpm* and 300 *rpm*

Required data:

μ_p = plastic viscosity, *cp*
τ_B = Bingham yield point, $lb_f/100\ ft^2$
τ_T = True yield point, $lb_f/100\ ft^2$

The plastic viscosity of the mud in *cp* can be calculated by using the Eq. (3.10) as:

$$\mu_p = \Phi_{600} - \Phi_{300} = 50 - 32 = \mathbf{18\ cp}$$

The yield point of the mud can be calculated by using the Eq. (3.11) as:

$$\tau_B = \Phi_{300} - \mu_p = 32 - 18 = \mathbf{14\ lb_f/100\ ft^2}$$

The true yield point of the mud can be calculated by using the Eq. (3.15) as:

$$\tau_T = \frac{3}{4}\tau_B = \frac{3}{4}\times 14 = \mathbf{10.5\ lb_f/100\ ft^2}$$

Example 3.15**:** A standard fluid that has a plastic viscosity of 24 *cp* and true yield pint of 5.9 $lb_f/100\ ft^2$. This fluid was put on the rotational viscometer that has an average reading error of 3.5%. What should be the reading of the instrument at 300 and 600 *rpm*?

Solution:

Given data:

μ_p	= Plastic viscosity	=	24 *cp*
τ_T	= True yield point	=	5.9 $lb_f/100\ ft^2$
e	= Viscometer's average error	=	3.5%

Required data:

ϕ_{300} = Reading at speed of 300 *rpm*

ϕ_{600} = Reading at speed of 600 *rpm*

By knowing the true yield point, we can calculate the yield point of the fluid using Eq. (3.15):

$$\tau_B = \frac{4}{3}\tau_T = \frac{4}{3}\times 5.9 = 7.87\ lb_f/100\ ft^2$$

Now we can calculate what will be the viscometer reading if it has no errors using equations (3.10) and (3.11):

$$\Phi_{300} = \tau_B + \mu_p = 7.87 + 24 = 31.87$$

$$\Phi_{600} = \mu_p + \Phi_{300} = 24 + 31.87 = 55.87$$

Because the instrument has an error of 3.5%, the actual reading that the viscometer should give is:

$$\Phi_{300} = 31.87\times(1-0.035) = \mathbf{30.75}$$

$$\Phi_{600} = 55.87\times(1-0.035) = \mathbf{53.91}$$

3.2.4 *pH* Determination

The *pH* of a solution is the logarithm of the reciprocal of the $[H^+]$ concentration in grams moles per liter, which can be mathematically expressed as

$$pH = \log\left[\frac{1}{(H^+)}\right] = -\log(H^+) \tag{3.16}$$

$$pOH = \log\left[\frac{1}{(OH^-)}\right] = -\log(OH^-) \tag{3.17}$$

where (H^+) and (OH^-) are the hydrogen and hydroxyl ion concentration in moles/liter.

Example 3.16: Find out the *pH* of an aqueous solution where both $[H^+]$ and $[OH^-]$ ions are same and equal to 1×10^{-7}. Find out the *pOH* also.

<u>Solution</u>:

Given data:

$[H^+]$ = hydrogen ion concentration = 1×10^{-7} *m/l*

$[OH^-]$ = hydroxyl ion concentration = 1×10^{-7} *m/l*

Required data:

pH = pH of drilling mud, *m/l*

pOH = pOH of drilling mud, *m/l*

The pH of the mud in *m/l* can be calculated by using the Eq. (3.16) and Eq. (3.17) as:

$$pH = \log\left[\frac{1}{(H^+)}\right] = \log\left[\frac{1}{1\times10^{-7}}\right] = -\log[1\times10^{-7}]$$
$$= -(-7) = \mathbf{7.00}$$

and

$$pOH = \log\left[\frac{1}{(OH^-)}\right] = \log\left[\frac{1}{1\times10^{-7}}\right] = -\log[1\times10^{-7}]$$
$$= -(-7) = \mathbf{7.00}$$

3.2.5 Determination of Liquid and Solids Content

The solids volume fraction of mud can be mathematically obtained as

$$f_s = 1 - f_w C_f - f_o \tag{3.18}$$

Here

C_f = volume increase factor due to the loss of dissolved salt during retorting

f_o = volume fraction of oil phase in the mud system, *vol/vol*

f_s = volume fraction of solids in the drilling mud, *vol/vol*

f_w = volume fraction of water phase in the mud system, *vol/vol*

Example 3.17: A 11 lb_m/gal saltwater mud is retorted and found to contain 8% oil and 72% water. If the chloride test shows the mud to have a chloride content of 79,000 *mg* Cl^-/L, find out the solid fraction of the mud. Assume that the mud is a sodium chloride mud. The solution has 12% salinity and NaCl has a volume increase factor of 1.045.

Solution:

Given data:

f_w = water volume fraction of drilling mud = 0.76

f_o = oil volume fraction of drilling mud = 0.08

C_f = volume increase factor due to the loss of dissolved salt during retorting = 1.045

Required data:

f_s = solids volume fraction of drilling mud, *vol/vol*

The solid volume fraction can be calculated by using the Eq. (3.18) as:

$$f_s = 1 - f_w C_f - f_o = 1 - (0.76)\times(1.045) - 0.08 = \mathbf{0.1258}$$

Example 3.18: A 13 *lbm/gal* saltwater mud is retorted and found to contain 9.5% oil and 70.5% water. If the chloride test shows the mud to have a chloride content of 79,000 mg Cl^-/L, find out the solid fraction of the mud. Assume that the mud is a sodium chloride mud. The solution has 14% salinity and NaCl has a volume increase factor of 1.045.

Solution:

Given data:

f_w = water volume fraction of drilling mud = 0.705
f_o = oil volume fraction of drilling mud = 0.095
C_f = volume increase factor due to the loss of dissolved salt during retorting = 1.045

Required data:

f_s = solids volume fraction of drilling mud, *vol/vol*

The solids volume fraction can be calculated by using the Eq. (3.18) as:

$$f_s = 1 - f_w C_f - f_o = 1 - (0.705) \times (1.045) - 0.095 = \mathbf{0.1682}$$

3.2.6 New Drilling Mud Calculations

Solid content of the mud can be calculated using the following equations.

$$V_{sc} + V_{m1} = V_{m2} \tag{3.19}$$

Here

V_{m1} = volume of initial mud (or any liquid) in mud calculation, *bbl*, *cc*
V_{m2} = volume of new mixture in mud calculation, *bbl*, *cc*
V_{sc} = volume of solids in mud calculation, *bbl*, *cc*

$$\rho_{sc} V_{sc} + \rho_{m1} V_{m1} = \rho_{m2} V_{m2} \tag{3.20}$$

Here

ρ_{sc} = density of solids, *gm/cc*
ρ_{m1} = density of initial mud, *gm/cc*
ρ_{m2} = density of new/final mud (i.e., fresh water and clay), *gm/cc*

Equation (3.19) and (3.20) can be solved for solid volume and product of solid volume and density as:

$$V_{sc} = \frac{V_{m2}(\rho_{m2} - \rho_{m1})}{\rho_{sc} - \rho_{m2}} \tag{3.21}$$

Equation (3.21) is not very useful because the net volume of a powdered solid is not readily measureable. However, the corresponding weight to add is:

$$\rho_{sc} V_{sc} = \frac{\rho_{sc} V_{m2}(\rho_{m2} - \rho_{m1})}{\rho_{sc} - \rho_{m2}} \tag{3.22}$$

In terms of volume percentage, Eq. (3.21) can be written as:

$$\frac{V_{sc}}{V_{m_2}} = \frac{\rho_{m_2} - \rho_{m_1}}{\rho_{sc} - \rho_{m_1}} \times 100 \tag{3.23}$$

The weight percentage can now be calculated as:

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} \tag{3.24}$$

Example 3.19: A 10.0 lb_m/gal mud contains clay of specific gravity of 2.5 and fresh water. Compute volume percentage and weight percentage of clay in this mud.

<u>Solution</u>:

Given data:

ρ_{sc} = density of solids (i.e., clay) = 2.5×8.33 = 20.825 *gm/cc*
ρ_{m1} = density of initial mud (i.e., fresh water) = 8.33 *gm/cc*
ρ_{m2} = density of final mud (i.e., fresh water and clay) = 10.0 *gm/cc*

Required data:

$\frac{V_{sc}}{V_{m2}}$ = volume in percentage

$\frac{\rho_{sc} V_{sc}}{\rho_{sc} V_{m2}}$ = weight in percentage

The solid volume in percentage can be calculated by using the Eq. (3.23) as:

$$\frac{V_{sc}}{V_{m_2}} = \frac{\rho_{m_2} - \rho_{m_1}}{\rho_{sc} - \rho_{m_1}} \times 100 = \frac{10.0 - 8.33}{20.825 - 8.33} \times 100 = \mathbf{13.37\%}$$

The solid weight in percentage can be calculated as:

$$\frac{\rho_{sc} V_{sc}}{\rho_{m2} V_{m2}} = \frac{\rho_{sc}}{\rho_{m2}} \frac{V_{sc}}{V_{m2}} = \frac{\rho_{sc}}{\rho_{m2}} \frac{(\rho_{m2} - \rho_{m1})}{\rho_{sc} - \rho_{m2}} \times 100$$
$$= \frac{20.825}{10} \frac{10.0 - 8.33}{20.825 - 8.33} \times 100 = \mathbf{27.83\%}$$

Example 3.20: A 12.0 *lbm/gal* mud contains clay of specific gravity of 3.0 and fresh water. Compute volume percentage and weight percentage of clay in this mud.

<u>Solution</u>:

Given data:

ρ_{sc} = density of solids (i.e., clay) = 3.0×8.33 = 24.99 *gm/cc*
ρ_{m1} = density of initial mud (i.e., fresh water) = 8.33 *gm/cc*
ρ_{m2} = density of final mud (i.e., fresh water and clay) = 12.0 *gm/cc*

Required data:

$\frac{V_{sc}}{V_{m2}}$ = volume in percentage

$\frac{\rho_{sc}V_{sc}}{\rho_{sc}V_{m2}}$ = weight in percentage

The solid volume in percentage can be calculated by using the Eq. (3.23) as:

$$\frac{V_{sc}}{V_{m_2}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{sc} - \rho_{m1}} \times 100 = \frac{12.0 - 8.33}{24.99 - 8.33} \times 100 = \mathbf{22.03\%}$$

The solid weight in percentage can be calculated as:

$$\frac{\rho_{sc}V_{sc}}{\rho_{m2}V_{m2}} = \frac{\rho_{sc}}{\rho_{m2}}\frac{V_{sc}}{V_{m2}} = \frac{\rho_{sc}}{\rho_{m2}}\frac{(\rho_{m2} - \rho_{m1})}{\rho_{sc} - \rho_{m2}} \times 100$$
$$= \frac{24.99}{12.0}\frac{(12.0 - 8.33)}{24.99 - 8.33} \times 100 = \mathbf{45.875\%}$$

3.2.7 Design of Mud Weight

In general drilling mud is composed of four major components – i) water or brine phase, ii) an oil phase, iii) low density solids, and iv) high density solids. These four components are immiscible, i.e., no component dissolves in any other component to any significant degree. This means that the four components form an ideal mixture. Mathematically, the sum of the four components volumes equals the total volume of the final mixture.

$$V_{mix} = V_w + V_o + V_{ls} + V_{hs} \tag{3.25}$$

Here
V_{mix} = volume of final mixture, i.e., mud
V_w = volume of the water phase in the mud system
V_o = volume of oil phase in the mud system
V_{ls} = volume of the low density solid in the mud system
V_{hs} = volume of the high density solid in the mud system

Employing Eq. (3.25), the total weight of the fluid mixture is simply the sum of the weights of the components. Conservation of mass ensures that total weight calculation is always correct.

$$\rho_{mix}V_{mix} = m_{mix} = \rho_w V_w + \rho_o V_o + \rho_{ls}V_{ls} + \rho_{hs}V_{hs} \tag{3.26}$$

Here
m_{mix} = mass of final mixture, i.e., mud
ρ_{mix} = overall density of fluid mixture, i.e., mud
ρ_w = density of water phase in the mud system
ρ_o = density of oil phase in the mud system
ρ_{ls} = density of the low density solid in the mud system
ρ_{hs} = density of the high density solid in the mud system

From Eq. (3.26) the overall density of the fluid mixture (i.e., mud) can be written as:

$$\rho_{mix} = \rho_w \frac{V_w}{V_{mix}} + \rho_o \frac{V_o}{V_{mix}} + \rho_{ls} \frac{V_{ls}}{V_{mix}} + \rho_{hs} \frac{V_{hs}}{V_{mix}} \tag{3.27}$$

In terms of volume fraction, Eq. (3.27) can be written as:

$$\rho_{mix} = \rho_w f_w + \rho_o f_o + \rho_{ls} f_{ls} + \rho_{hs} f_{hs} \tag{3.28}$$

where,

$$f_w + f_o + f_{ls} + f_{hs} = 1 \tag{3.29}$$

Here

f_w = volume fraction of water phase in the mud system
f_o = volume fraction of oil phase in the mud system
f_{ls} = volume fraction of low density solid in the mud system
f_{hs} = volume fraction of high density solid in the mud system

Example 3.21: A well is drilled to a depth of 8,000 *ft*. The top of the oil formation is at 7,600 *ft* and the bottom is at 8,000 *ft*. The pore pressure at 7,600 *ft* is 3600 *psi*. Calculate the following:

a. Calculate the mud weight in pcf to balance the pore pressure at 7600 *ft*.
b. What mud weight should be used to over balance the pore pressure by 300 *psi*?
c. What is the over balance pressure if 10.6 *ppg* mud is used?
d. If the fracture gradient of the formation at 7,600 *ft* is 0.75 *psi/ft*, what is the bottom hole pressure that will fracture the formation?
e. What is the surface pressure that will fracture the formation at 7,600 *ft* if the hole is full of water?

Solution:

Given data:

h_{TVD} = total vertical depth = 8,000 *ft*
h_{ot} = top of oil formation at a depth = 7,600 *ft*
h_{ob} = bottom of oil formation at a depth = 8,000 *ft*
P_{TVD} = pressure at a depth 7,600 *ft* = 3,600 *psi*

Required data:

a. $$\rho = \frac{P \times 144}{L} = \frac{3600 \times 144}{7600} = 68.21 \ pcf$$

b. $$\rho = \frac{P \times 144}{L} = \frac{3900 \times 144}{7600} = 73.89 \ pcf$$

c. $$P_h = \frac{\rho L}{144} = \frac{(10.6 \times 7.48)(7600)}{144} = 4184\, psi$$
Over balance $= 4184 - 3600 = 584\ psi$

d. Fracturing pressure = 7600 × 0.75 = 5700 *psi* @ 7600

e. $$P_s + P_h = P_b$$
$$P_s = P_b - P_h$$
$$= 5700 - \frac{62.4 \times 7600}{144}$$
$$= 5700 - 3293 = 2406\ psi$$

Example 3.22: A well is drilled to a depth of 10,500 *ft*. The top of the oil formation is at 8600 *ft* and the bottom is at 9,500 *ft*. The pore pressure at 8,600 *ft* is 4,600 *psi*. Calculate the following:

a. Calculate the mud weight in *pcf* to balance the pore pressure at 8,600 *ft*.
b. What mud weight should be used to over balance the pore pressure by 500 *psi*?
c. What is the over balance pressure if 13.6 *ppg* mud is used?
d. If the fracture gradient of the formation at 8,600 *ft* is 0.85 *psi/ft*, what is the bottom hole pressure that will fracture the formation?
e. What is the surface pressure that will fracture the formation at 8,600 *ft* if the hole is full of water?

Solution:

Given data:

h_{TVD}	=	total vertical depth	=	10,500 *ft*
h_{ot}	=	top of oil formation at a depth	=	8,600 *ft*
h_{ob}	=	bottom of oil formation at a depth	=	9,500 *ft*
P_{TVD}	=	pressure at a depth 8,600 *ft*	=	4,600 *psi*

Required data:

a. $$\rho = \frac{P \times 144}{L} = \frac{4600 \times 144}{8600} = 77.023\ pcf$$

b. $$\rho = \frac{P \times 144}{L} = \frac{5100 \times 144}{8600} = 85.395\ pcf$$

c. $$P_h = \frac{\rho L}{144} = \frac{(13.6 \times 7.48)(8600)}{144} = 6075.422\ psi$$
Over balance $= 6075.422 - 4600 = 1475.422\ psi$

d. Fracturing Pressure = 8600 × 0.85 = 7310 *psi* @8600 *ft*

e.
$$P_s + P_h = P_b$$
$$P_s = P_b - P_h$$
$$= 7310 - \frac{62.4 \times 8600}{144} = 7310 - 3726.67 = \mathbf{3583.33\ \textit{psi}}$$

***Example 3.23*:** A drilling engineer wants to prepare a drilling mud of volume of 4,450 cubic feet using water and Bentonite of 2.43 *gm/cc* density. The required final weight was calculated to be 9.4 *ppg*. Calculate the amount of Bentonite to be mixed in tons and the volume of water to be used in barrels.

Solution:

Given data:

V_{m2} = required volume of the mud = 4,450 cubic feet
ρ_{SC} = density of Bentonite = 2.47*gm/cc*
ρ_{m2} = required mud weight = 9.4 *ppg*

Required data:

m_{SC} = amount of Bentonite to be mixed in tons
V_{water} = volume of water in barrels

From the given data, we can calculate the volume percentage of solids using Eq. (3.23):

$$\frac{V_{SC}}{V_{m2}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{SC} - \rho_{m1}} \times 100 = (9.4 - 8.34)/(2.47 \times 8.34 - 8.34) = 8.65\%$$

Weight percentage can be calculated using Eq. (3.24):

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} = 8.65\% \times \frac{2.47 \times 8.34}{9.4} = 18.95\%$$

To calculate the m = amount of Bentonite in tons, first we need to calculate the mass of the final fluid:

$$\text{mass of mud} = \text{density of mud} \times \text{volume of mud}$$
$$= \frac{9.4}{8.34} \times 62.4 \times 4{,}450 = 312{,}973\ lbm$$

Now, the amount of Bentonite required can be calculated from the weight percentage and the mass of the mud as follows:

$$\text{mass}_{\text{bent}} = \text{Weight\%} \times \text{mass of mud} = 0.1895 \times 312{,}793$$
$$= 59{,}308\ lbm$$

By dividing the above value by 2,200, we can convert the amount to tons as follows:

$$mass_{bent} = \frac{59,308}{2,200} = 26.96\ tons$$

To calculate the required volume of water to make the above mud, we first calculate the amount of water in pounds using the difference between the mass of the final fluid and the mass of Bentonite as below:

$$mass_{water} = mass_{mud} - mass_{bent} = 312,793 - 59,308 = 253,485\ lbm$$

Now by using the density of water of 62.4 lb_m/ft^3, we can calculate the volume of water as below:

$$V_w = \frac{mass_w}{\rho_w} = \frac{253,485}{62.4} = 4,062.3\ ft^3\ \ OR\ 723.5\ bbls$$

***Example 3.24*:** A final volume of 1,750 *bbls* of drilling mud was planned to be prepared. An existing mud having a mud weight of 10.2 *ppg* will be used. It is needed to increase its density to 10.5 *ppg* by adding clay of 2.52 *gm/cc* density. Calculate the volume of the old mud to be taken and the amount of clay required in tons to get the desired mud weight.

Solution:

Given data:

V_{m2}	=	Final volume of the mud	=	1,750 *barrels*
ρ_{clay}	=	density of clay	=	2.52 *gm/cc*
$MW_{init.}$	=	Initial mud weight	=	10.2 *ppg*
MW_{Final}	=	Final mud weight	=	10.5 *ppg*

Required data:

m_{Bent}	=	amount of clay to be mixed in tons
V_{old}	=	Volume of old mud to be taken in barrels.

First we should calculate the volume percentage using Eq. (3.23):

$$\frac{V_{SC}}{V_{m2}} = (\rho_{m2} - \rho_{m1})/(\rho_{SC} - \rho_{m1}) \times 100$$
$$= (10.5 - 10.2)/(2.52 \times 8.34 - 10.2) = 2.77\%$$

The weight percentage can now be calculated using Eq. (3.24):

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} = 2.77\% \times \frac{2.52 \times 8.34}{10.5} = 5.55\%$$

The volume of final mud in cubic feet is equal to:

$$V_{m2} = 1,750\ bbls \times 5.615\ cu.ft/bbl = 9,826.3\ cu.ft$$

The amount of mud in pounds is equal to:

$$mass_{m2} = density \times volume = \frac{10.5}{8.34} \times 62.4 \times 9{,}8263 = 771{,}962\ lbm$$

The amount of clay required is equal to:

$$mass_{clay} = 0.0555 \times 771{,}962 = 42{,}844\ lbm$$

The amount of clay in tons is equal to:

$$mass_{clay} = \frac{42{,}844}{2{,}200} = \mathbf{19.47}\ \boldsymbol{tons}$$

To calculate the volume of the old mud to be taken to prepare 1,750 *bbls* of 10.5 *ppg* mud, first we should calculate the amount of the old mud in pounds:

$$mass_{m1} = mass_{m2} - mass_{clay} = 771{,}962 - 42{,}844 = 729{,}118\ lbm$$

The volume of the old mud to be taken is equal to:

$$V_{m1} = \frac{mass_{m1}}{\rho_{m1}} = \frac{729{,}118}{\left(\frac{10.2}{8.34} \times 62.4\right)} = 9{,}953.9\ ft^3$$

The volume of the old mud in barrels is equal to:

$$V_{m1} = \frac{9{,}953.9}{5.615} = \mathbf{1{,}701.5}\ \boldsymbol{bbls}$$

Example 3.25**:** An oil-based mud is prepared using 450 *bbls* of diesel which has a density of 54.5 lb_m/ft^3, 1,250 *bbls* of water and 75 *tons* of Bentonite. The mud has a density of 162.24 lb_m/ft^3. Calculate the mud weight and volume fractions of each component of the mud.

Solution:

Given data:

V_o = Diesel volume = 450 *barrels*
V_w = Water volume = 1,250 *barrels*
ρ_{Diesel} = Diesel density = 54.5 lb_m/ft^3
ρ_{Bent} = Density of Bentonite = 162.24 lb_m/ft^3
$mass_{Bent}$ = Mass of Bentonite = 75 *tons*

Required data:

MW_{mud} = Mud weight in *ppg*
f_o = Diesel volume fraction.
f_w = Water volume fraction.
f_S = Bentonite volume fraction.

To calculate the mud weight, we should calculate the total weight in total volume of the mixture. The mass of water is equal to:

$$mass_w = \rho_w \times V_w = 62.4 \times (1{,}250 \times 5.615) = 437{,}973.1\ lbm$$

The mass of diesel is equal to:

$$mass_o = \rho_o \times V_o = 54.5 \times (450 \times 5.615) = 137{,}707.9\ lbm$$

The total mass is equal to:

$$\begin{aligned} mass_{mix} &= mass_w + mass_o + mass_s \\ &= 137{,}707.9 + 437{,}973.1 + 75 \times 2{,}200 = 740{,}678\ lbm \end{aligned}$$

Volume of Bentonite can be calculated by knowing the porosity of powder to get the actual volume of the Bentonite as below:

$$V_s = \frac{mass_s}{\rho_s} = \frac{165{,}000}{162.24} = 1{,}017.1\ ft^3$$

The total volume of the mixture is equal to:

$$V_{mix} = V_o + V_w + V_s = 5.615(450 + 1{,}250) + 1{,}017.1 = 10{,}563\ ft^3$$

Mud weight can be calculated as:

$$MW_{mud} = \frac{mass_{mix}}{V_{mix}} = \frac{740{,}678}{10{,}563} \times \frac{8.34}{62.4} = \mathbf{9.37\ ppg}$$

Volume fraction of the mud components are:

$$f_o = \frac{V_o}{V_{mix}} = \frac{2{,}526.8}{10{,}563} = \mathbf{0.24}$$

$$f_w = \frac{V_w}{V_{mix}} = \frac{7{,}018.8}{10{,}563} = \mathbf{0.66}$$

$$f_s = \frac{V_s}{V_{mix}} = \frac{1{,}017.1}{10{,}563} = \mathbf{0.10}$$

***Example 3.26*:** A total volume of mud is 1,000 *bbls* that has a mud weight of 9.1 *ppg*. Calculate the volume fractions of water, Bentonite, and the weight of Bentonite used. Density of powder Bentonite is 156 lb_m/ft^3.

Solution:

Given data:

V_{mix} = Volume of the mud = 1,000 *barrels*

ρ_{mud} = Mud weight = 9.1 *ppg*

ρ_{Bent} = Density of Bentonite = 156 lb_m/ft^3

Required data:

f_w = Water volume fraction, fraction

f_{sc} = Bentonite volume fraction, fraction

W_{Bent} = Weight of Bentonite, tons

To get water volume used in preparing the mud, first we should calculate the weight and volume percentages using Eq. (3.23) and (3.24):

$$\frac{V_{SC}}{V_{m2}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{SC} - \rho_{m1}} \times 100 = \frac{9.1 - 8.34}{2.50 \times 8.34 - 8.34} \times 100 = 6.08\%$$

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} = 6.08\% \times \frac{2.50 \times 8.34}{9.1} = 13.92\%$$

From the value of volume fraction we can get the volume of Bentonite and volume of water as follows:

$$V_{SC} = 0.0608 \times V_{m2} = 0.0608 \times (1{,}000 \times 5.615) = 341.8\, ft^3$$

$$V_w = V_{m2} - V_{SC} = 5{,}615 - 341.8 = 5{,}273.2\, ft^3$$

Volume fractions can be calculated as below:

$$f_w = \frac{V_w}{V_{mix}} = \frac{5{,}273.2}{5{,}615} = \mathbf{0.94}$$

$$f_{sc} = \frac{V_{sc}}{V_{mix}} = \frac{341.8}{5{,}615} = \mathbf{0.06}$$

The weight of Bentonite can be calculated as below:

$$W_{Bent} = 0.1392 \times m_{m2} = 0.1392 \times \left(5{,}615 \times \frac{9.1}{8.34} \times 62.4\right)$$

$$= \mathbf{53{,}217\ lbm\ Or\ 24.2\ tons}$$

Example 3.27: A surface section of 2,000 *ft* and 17.5″ diameter has to be drilled using 9.2 *ppg* mud. The mud will be prepared using fresh water and attapulgite that has a density of 2.9 *gm/cc*. If the plan is to have mud volume greater than the section volume by 1.3, calculate the volume of water in barrels and the weight of attapulgite in tons to prepare the above mud.

Solution:

Given data:

L = Length of the section = 2,000 *ft*

d_h = Diameter of the hole = 17.5″ OR 1.458 *ft*

ρ_{mud} = Required mud weight = 9.2 *ppg*

ρ_{Attap} = Density of Attapulgite = 2.9 *gm/cc* OR 181 lb_m/ft^3

Required data:
V_w = Water volume in barrels
W_{Attp} = Weight of Bentonite in tons

First, we will calculate the required mud volume as below:

$$V_{m2} = \frac{\pi}{4} d_h^2 L \times 1.3 = \frac{\pi}{4} \times 1.458^2 \times 2{,}000 \times 1.3 = 4{,}343\ ft^3$$

Weight of the mud can be calculated as below:

$$mass_{m2} = \rho_{m2} \times V_{m2} = \frac{9.2}{8.34} \times 62.4 \times 4{,}343 = 298{,}940\ lbm$$

Volume and weight percentages can now be calculated using Eq. (3.23) and (3.24):

$$\frac{V_{SC}}{V_{m2}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{SC} - \rho_{m1}} \times 100 = \frac{9.2 - 8.34}{2.9 \times 8.34 - 8.34} \times 100 = 5.43\%$$

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} = 5.43\% \times \frac{2.9 \times 8.34}{9.2} = 14.27\%$$

The weight of the attapulgite can be calculated using the weight percentage as below:

$$\begin{aligned} W_{Attp} &= 0.1427 \times m_{m2} = 0.1427 \times 298{,}940 \\ &= \mathbf{42{,}660}\ \boldsymbol{lbm}\ \mathbf{OR\ 19.4}\ \boldsymbol{tons} \end{aligned}$$

To calculate the required volume of water, first we should calculate the weight of water as below:

$$\begin{aligned} mass_w &= mass_{m2} - mass_{sc} = 298{,}940 - 42{,}660 \\ &= 256{,}280\ lbm \end{aligned}$$

The water volume is equal to:

$$V_w = \frac{mass_w}{\rho_w} = \frac{256{,}280}{62.4} = \mathbf{4{,}107}\ \boldsymbol{ft^3}\ \boldsymbol{OR}\ \mathbf{731.4}\ \boldsymbol{bbls}$$

***Example 3.28*:** How many tons of Bentonite should be added to water to prepare 750 *bbls* of drilling mud? The mud weight is recorded as 9.5, and the density of Bentonite is 2.6 *gm/cc*.

Solution:

Given data:
V_m = Mud volume = 750 *bbls* = 4,211.3 ft^3
ρ_{mud} = Required mud weight = 9.5 *ppg*
ρ_{Bent} = Density of Bentonite = 2.6 *gm/cc* = 162.24 lb_m/ft^3

Required data:
W_{Bent} = Weight of Bentonite.

Using the available data, we can calculate the volume and weight percentages using Eq. (3.23) and (3.24):

$$\frac{V_{SC}}{V_{m2}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{SC} - \rho_{m1}} \times 100 = \frac{9.5 - 8.34}{2.6 \times 8.34 - 8.34} \times 100 = 8.69\%$$

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} = 8.69\% \times \frac{2.6 \times 8.34}{9.5} = 19.84\%$$

The mass of the mud is equal to:

$$mass_{mud} = \rho_{mud} V_{mud} = \left(\frac{9.5}{8.34} \times 62.4 \right) \times 4{,}211.3 = 299{,}332\ lbm$$

The mass of Bentonite can now be calculated using weight percent ratio:

$$W_{Bent} = 0.1984 \times m_m = 0.1984 \times 299{,}332 = 59{,}387.5\ lbm = \mathbf{27\ tons}$$

3.3 Multiple Choice Questions

1. Which one is an essential part of the rotary drilling system
   a) Drilling fluid
   b) Drilling engineer
   c) Landman
   d) Derrick man

2. Objectives of any mud program are
   a) Allow the target depth to be reached
   b) Minimize well costs
   c) Maximize production from the pay zone
   d) All of the above

3. In mud program, factors needing to be considered are
   a) Location of wells
   b) Expected lithology
   c) Mud properties
   d) All of the above

4. No one could successfully drill wells with the rotary method without using
   a) Drilling mud
   b) Bailer
   c) Water
   d) Acid

5. Which one is not a function of drilling mud?
   a) Remove cuttings
   b) Formation of mud cake
   c) Cooling of bit
   d) Drill pipe lubrication
6. The first-ever drilling fluid used was
   a) Oil
   b) Water
   c) Emulsion
   d) None of the above
7. Water was used as first drilling fluid in
   a) 1840
   b) 1845
   c) 1850
   d) 1855
8. Water was used as the first drilling fluid in 1845 *in*
   a) France
   b) Norway
   c) Saudi Arabia
   d) None of the above
9. Drilling fluids are classified as
   a) Liquid
   b) Gases
   c) Liquid-gas mixtures
   d) All of the above
10. In rotary drilling, drill cuttings are removed by
    a) Drilling mud
    b) Bailer
    c) Water
    d) Acidizing
11. Which one is responsible for circulating mud to the formation?
    a) Hoisting system
    b) Power system
    c) Circulating system
    d) Rotary system
12. A fresh water based drilling fluid with additives is commonly called
    a) Drilling mud
    b) Acid
    c) Chemical agents
    d) None of the above

13. ______________ must often be added to these fluids to overcome specific down-hole problems.
    a) Chelating agents
    b) Acids
    c) Additives
    d) None of the above

14. Drilling mud in which the continuous phase is water is called
    a) Oil-based muds
    b) Water-based muds
    c) Emulsion-based muds
    d) All of the above

15. Salt water drilling fluids are prepared from
    a) Brine water
    b) Sea water
    c) Dry sodium chloride
    d) All of the above

16. Salt water drilling fluids have a chloride content of
    a) 6,000 *mg/lt* to less 189,000 *mg/lt*
    b) 5,500 *mg/lt* to less 190,000 *mg/lt*
    c) 4,500 *mg/lt* to less 200,000 *mg/lt*
    d) None of the above

17. A mud with salt or calcium to reduce active clays hydration is called
    a) Oil-based mud
    b) Inhibited mud
    c) Dehydrated mud
    d) None of the above

18. Distinction between fresh-water and inhibited muds is based on
    a) Salt concentration
    b) Oil concentration
    c) Additive concentration
    d) None of the above

19. Sodium ion concentration in fresh-water mud is
    a) Less than 5000 *ppm*
    b) Less than 3000 *ppm*
    c) Greater than 3000 ppm and less than 5000 *ppm*
    d) None of the above

20. Low solid muds are those where solid contents are
    a) Less than 5%
    b) Between 5–10 %
    c) Less than 3%
    d) None of the above

21. The drilling mud with oil as solvent carrier for the solids content is known as
    a) Oil-based mud
    b) Water-based mud
    c) Emulsion-based mud
    d) None of the above
22. Which of the following is used as base fluids in OBM?
    a) Diesel
    b) Kerosene
    c) Fuel oils
    d) All of the above
23. Which of the following is used as drilling fluids in directional and horizontal wells?
    a) Oil-based mud
    b) Water-based mud
    c) Emulsion-based mud
    d) None of the above
24. Which of the following is the type of OBM?
    a) Invert emulsion oil-based mud
    b) Pseudo oil-based mud
    c) Full oil mud
    d) All of the above
25. Water content in OBM is less than
    a) 10%
    b) 3%
    c) 7%
    d) 5%
26. Which of the following is the continuous phase in invert emulsion oil-based?
    a) Mineral oil
    b) Brine
    c) $CaCl_2$ Brine
    d) None of the above
27. Air with additives is referred to as ______________.
    a) Pollution
    b) Foam
    c) Vapors
    d) None of the above
28. Biodegradable mixtures of surfactants are generally known as ______________
    a) Chelating agents
    b) Additives
    c) Foaming agents
    d) None of the above

29. Bentonite mixed with water produces slurry with ______________ greater than water.
    a) Viscosity
    b) Density
    c) Salt concentration
    d) All of the above
30. Use of additives are characterized mainly under,
    a) Chemical additives
    b) Additives for WBM
    c) Additives for OBM
    d) All of the above
31. The correct selection of drilling fluid directly affects
    a) The rate of penetration
    b) Drilling fluids cost
    c) Overall drilling cost
    d) All of the above
32. The following objectives are set for mud engineering. Which one is not the correct answer?
    a) Reach the target depth
    b) Minimize well cost
    c) Maximize the rate of penetration
    d) Enhance the oil production
33. One of the following mud properties is responsible for suspending the drill cuttings. Which one is the correct answer?
    a) Gel strength
    b) Mud density
    c) Yield strength
    d) Mud viscosity
34. The main purpose of preparing drilling mud during well control situations is to
    a) Remove the cuttings out of the well
    b) Provide sufficient hydrostatic pressure to control formation fluids
    c) Cool and lubricate the drill bit
    d) Form a low permeable mud cake
35. The mud property that is responsible for providing the necessary hydrostatic pressure is
    a) Mud viscosity
    b) Gel strength
    c) Mud weight
    d) All of the above

36. The mud properties that are responsible for removing the cutting out of the well are
    a) Gel strength and viscosity
    b) Gel strength and density
    c) Viscosity and density
    d) None of the above

37. What is the importance of removing drill cutting off the well?
    a) Reducing the horsepower required to run the circulating system
    b) Knowing some information about the drilled formation
    c) Reducing the needed circulating rate
    d) All of the above

38. Which of the following mud types is not commonly used in drilling?
    a) Liquid-based muds
    b) Oil-based muds
    c) Gas-based muds
    d) None of the above

39. What is the common drilling fluid type which is widely used in drilling?
    a) Compressed air
    b) Oil-based mud
    c) Foam-based mud
    d) Water-based mud

40. The main governing factor for selecting the type of drilling fluid is the
    a) Type of expected formation fluid
    b) Type of mud pumps
    c) Type of the formation
    d) All of the above

Answers: 1a, 2d, 3e, 4a, 5d, 6b, 7b, 8a, 9d, 10a, 11c, 12a, 13d, 14d, 15d, 16a, 17b, 18a, 19b, 20a, 21a, 22d, 23a, 24d, 25d, 26a, 27b, 28c, 29a, 30d, 31d, 32d, 33a, 34b, 35c, 36a, 37b, 38c, 39d, 40c.

3.4 Summary

The chapter presents almost all the formulas related to the drilling fluid. The workout examples and the MCQs are presented in a chronological manner. The exercise solutions are given in Appendix A. Moreover, this chapter added some more MCQs for the student's self-practice and the answers are given in Appendix B. The MCQs covered almost all the materials covered in this chapter.

3.5 Exercise and MCQs for Practice

3.5.1 Exercises (Solutions are in Appendix A)

Exercise 3.1: A production hole has been drilled in an off-shore area using an 8.5 inches bit at an average drilling rate of 17 *ft/hr*. The formation cuttings are stored in containers of 45 cubic feet volume to be dumped later in a safe manner. If the formation porosity was estimated to be 0.14, calculate the time required to fill one mud container. Assume the formation is consolidated and no hole enlargement. ***Answer: 7.8 hrs***

Exercise 3.2: An intermediate hole has been drilled using a 12.25 *inches* bit. The interval was described to be a consolidated rock with an estimated gravity and porosity of 2.9 and 0.1; respectively. The interval took 10.5 *hrs* to be drilled and produced about 16.5 *tons*. If there was no hole enlargement and no swelling of cuttings, calculate the rate of drilling at this section. ***Answer: 25.9 ft/hr***

Exercise 3.3: Two wells, A and B, have been drilled in the same area. Both wells drilled the same formation in the surface section. Surface section for well A was drilled using a 17.5 *inches* bit while well B drilled using a 20 *inches* bit. Both wells produced same cuttings volume at the same drilling time. If the hole size was assumed the same as the bit size, what is the ratio of the average drilling rate of the two bits at the specified drilling time? ***Answer: 1.31***

Exercise 3.4: Two fluids have densities of 9.6 *pgg* and 8.9 *ppg*, respectively. If both fluids read exactly the same reading in the March funnel at the same conditions, find out the relation between the viscosities of both fluids. And if the first fluid read 61 seconds, calculate the viscosity of both fluids. ***Answers: 1.079, 41.44 cp, 38.4 cp***

Exercise 3.5: A drilling fluid has a density of 9.3 *ppg* read 66 *seconds* in the March funnel. A viscosifying additive was added to the fluid that did not make any changes to its density. If the viscosity of the new fluid was increased by 1.12 of the old viscosity, what should be the March funnel reading of the new fluid? ***Answer: 70.9 seconds***

Exercise 3.6: A standard fluid that has a plastic viscosity of 28.5 *cp* and yield point of 8.9 $lb_f/100\ ft^2$ was used to calibrate a certain rotational viscometer. When the instrument ran at 600 rpm speed, it read 65.5 degrees. Calculate the instrument average error. ***Answer: 4.5%***

Exercise 3.7: A drilling fluid has a plastic viscosity of 22 *cp* and yield point of 6.25 $lb_f/100\ ft^2$ using a rotational viscometer instrument. If another fluid reads 0.85 of the reading of the first fluid at speed of 300 *rpm* and 42° at speed of 600 *rpm*, calculate the viscosity and yield point of the second fluid. ***Answers: 18 cp, 6*** $\boldsymbol{lb_f/100\ ft^2}$

Exercise 3.8: 54 tons of Bentonite 2.51 *gm/cc* was used to prepare a drilling mud by mixing it with water only. If the final density of the mud is 9.7 *ppg*, calculate the volume of water to be used and the volume of the drilling fluid that can be prepared using the above information. ***Answers: 1,250.6 bbls, 1,115.5 bbls***

Exercise 3.9: 29.55 *tons* 2.49 *gm/cc* of Bentonite was mixed with an old mud used in the previous section of a well. A 1,500 *bbls* of a new mud having mud density of 11.2 *ppg* was prepared. What were the density and the volume of the old mud? ***Answers: 10.7 ppg, 1,425.5 bbls***

Exercise 3.10: A drilling fluid is planned to be prepared using water and Bentonite that has a density of 2.5 *gm/cc*. If it is planned to prepare 800 *bbls* of drilling fluid having a mud weight of 8.8 *ppg*, calculate the amount of Bentonite to be used in tons and the volume of water required to prepare this mud. ***Answers: 11.71 tons, 770.6 bbls***

Exercise 3.11: If the maximum mud weight that can be achieved by mixing sodium chloride with water is 12.0 *ppg*, calculate the sodium chloride volume fraction that can be used to prepare the above mud weight. Sodium chloride density is 2.15 *gm/cc*. ***Answer: 0.38***

Exercise 3.12: An oil-based mud is prepared using a diesel that has a density of 54.5 lb_m/ft^3, 1,500 bbls of water and 115.5 *tons* of Barite that has a density of 262.08 lb_m/ft^3. If the mud weight of the prepared mud is 10.5 *ppg*, calculate the volume of diesel that used in preparing the above mud. ***Answer: 309.6 bbls***

Exercise 3.13: 60 *tons* of Barite was used together with water to develop 1,050 *bbls* of a drilling fluid. If the density of Barite is 4.2 *gm/cc*, calculate the volume of water used and the density of the mud developed. ***Answers: 960 bbls, 11.0 ppg***

Exercise 3.14: A drilling fluid was developed using water and Hematite that has a density of 5.05 *gm/cc*. Mud engineer took 1,350 bbls of water to prepare 1,425 *bbls* of the mud. Calculate the amount of Hematite used to prepare the above mud and the mud weight. ***Answers: 60.3 tons, 10.1 ppg***

Exercise 3.15: A 1,125 *bbls* of drilling mud that has mud weight of 10.4 *ppg* was diluted using 250 *bbls* of water. What will be the new mud weight after dilution? ***Answer: 10.0 ppg***

3.5.2 Exercises (Self-Practices)

E3.1: For a typical US Gulf Coast area, it is given that the diameter of the bit is 15 *in.*, rate of penetration is 100 *ft/hr*, and the average porosity is 25%. Find out the volume of cuttings in *bbl/hr* and *tons/hr*.

E3.2: The density of the drilling mud is measured as 80 lb_m/ft^3 in the rig side area. Calculate the specific gravity in gm/cm^3, mud gradient in *psi/ft* and $kg/cm^2/m$ **Ans.**

E3.3: A mud engineer measured the density of the drilling fluid as 75 lb_m/ft^3 in the rig side area. Calculate the specific gravity in gm/cm^3, mud gradient in *psi/ft* and $kg/cm^2/m$ **Ans.**

E3.4: During the drilling of hazardous formation, the mud engineer was trying to keep up the quality of drilling fluid. At a certain time, he measured the density of the drilling fluid as 12 *ppg* in the rig side area. Calculate the specific gravity in gm/cm^3, mud gradient in *psi/ft* and $kg/cm^2/m$. **Ans.**

E3.5: A Marsh funnel is used to measure the density of the drilling fluid which is 1.1 g/cm^3 in 40 *seconds*. Calculate the effective viscosity using Marsh funnel equation. **Ans.**

E3.6: A Marsh funnel is used for 45 *seconds* to to measure the density of the drilling fluid which is 1.3 g/cm^3. Calculate the effective viscosity using Marsh funnel equation. **Ans.**

E3.7: A mud sample in a rotational viscometer gives a dial reading of 46° at 600 *rpm* and a dial reading of 28° at 300 *rpm*. Compute the apparent viscosity of the mud at each rotor speed.

E3.8: Using the data of Exercise 3.6, compute the plastic viscosity and yield point of the mud sample. **Ans.**

E3.9: To calculate the different viscosity of a mud sample, a Fann V-G meter is used and the data measurements are $\Phi_{600} = 25°$ and $\Phi_{300} = 18°$. Calculate i) plastic viscosity, ii) apparent viscosity, iii) Bingham yield point, and true yield point. **Ans.**

E3.10: Find out the pH of a aqueous solution where both $[H^+]$ and $[OH^-]$ ions are 1×10^{-2} and 1×10^{-12} respectively. **Ans.**

E3.11: Find out the pH of both $[H^+]$ and $[OH^-]$ ions if an aqueous solution has the concentration of 1×10^{-9} for $[H^+]$ and 1×10^{-5} for $[OH^-]$ ions respectively. **Ans.**

E3.12: A 13 lb_m/gal drilling mud is retorted and found to contain 9% oil and 75% water. If the chloride test shows the mud to have a chloride content of 150,000 $mg\ Cl^-/L$, Find out the solid fraction of the mud. Assume that the mud is a calcium chloride mud. The solution has 14% salinity and $CaCl_2$ has a volume increase factor of 1.037. **Ans.**

E3.13: A 10 lb_m/gal drilling mud is retorted and found to contain 12% oil and 70% water. If the chloride test shows the mud need to have chloride content, find out the solid fraction of the mud. Assume that the mud is a sodium chloride mud. The solution has 18% salinity and NaCl has a volume increase factor of 1.075. **Ans.**

E3.14: A 20-in bit is used to drill a hole at a rate of 70 *ft/hr* where the porosity of the formation is 25%. Calculate the solid volume generated in this drilling operation. If the density of the solid is 910 lb_m/bbl, calculate the solid generation in tons/hr also. **Ans.**

E3.15: For a typical North Sea well, a 26-in bit is used to drill a hole at a rate of 60 *ft/hr* where the porosity of the formation is 25%. Calculate the solid volume generated this drilling operation. If the density of the solid is 910 lb_m/bbl, calculate the solid generation in tons/hr also. **Ans.**

3.5.3 MCQs (Self-Practices)

1. The name “water-based mud” is designated because
   a) The drilling mud is only fresh water
   b) The continuous phase in the mud is the fresh water
   c) It can only drill water zones
   d) All of the above
2. Fresh-water muds are the muds that
   a) Have less than 3,000 ppm Na^+ ions
   b) Do not have any free ions
   c) Have only positive ions
   d) None of the above
3. Oil-based mud is the drilling fluid that
   a) Does not have any amounts of water
   b) Has less amount of water
   c) Used to drill only the oil zones
   d) All of the above
4. What is the difference between the full-oil mud and inverted emulsion oil-based mud?
   a) Full-oil mud has very low water content as compared to the inverted mud
   b) Full-oil mud can give lower mud weights than the inverted mud
   c) Full-oil mud has more oil content as compared to inverted mud
   d) All of the above
5. Gas-based muds are very effective in drilling
   a) Salt formations
   b) Gas formations
   c) Consolidated formations
   d) Unconsolidated formations
6. One of the following additional equipment in circulating system is needed for drilling by gas-based mud. Which is the correct answer?
   a) Degasser
   b) Special top drive system
   c) Compressors
   d) None of the above
7. Which of the following mud type should be used in wildcat wells?
   a) Gas-based muds
   b) Foam muds
   c) Oil-based muds
   d) Water-based muds

8. Inhibited muds are used when?
   a) Swelled materials are present in the drilled formations
   b) The drilled formations have high H_2S contents
   c) The mud have high tendency for contaminations
   d) All of the above

9. The additive which is used to remove the calcium from the hard water is
   a) Caustic soda
   b) Soda ash
   c) Sodium borate
   d) Sodium carbonate

10. The additive which is used to increase the mud alkalinity is
    a) Sodium hydroxide
    b) Soda ash
    c) Sodium chloride
    d) All of the above

11. The additive which is used as a density control additive is
    a) Barite
    b) Hematite
    c) Siderite
    d) All of the above

12. The following materials are used as viscosifiers except
    a) Bentonite
    b) Guar gum
    c) Shredded wood
    d) Synthetic polymers

13. The following materials are not used as the controller of lost circulation materials except
    a) Soda ash
    b) Limestone
    c) Barite
    d) Bentonite

14. At surface, ______________ is added to the drilling fluid for removing the oxygen that is entrained into mud while depositing in the mud pit
    a) Sodium sulphite
    b) Sodium sulphite
    c) Sodium chloride
    d) Sodium hydroxide

15. Which of the following materials are used as filtration control?
    a) Polymers
    b) Asphalt
    c) Manganese oxide
    d) All of the above

16. First step in solid separation is to pass the drilling fluid through
    a) Shale shakers
    b) Settling tanks
    c) Desilter
    d) All of the above
17. Larger parts of drilled cuttings are removed from the mud using
    a) Settling tanks
    b) Waste pits
    c) Shale shakers
    d) None of the above
18. Smaller solids are removed from the mud using
    a) Desander
    b) Desilter
    c) Mud cleaner
    d) All of the above
19. Two holes, A and B, with the same depth are filled with different fluids. If the pressure at bottom of the well A is higher than the well B, the fluid inside A has
    a) Lower density than the other well
    b) Higher density than the other hole
    c) Same density in both holes
    d) None of the above
20. If mud A needs less pressure to be pumped compared with mud B, then mud A has ______________ as compared with mud B
    a) Low viscosity
    b) High viscosity
    c) Same viscosity
    d) None of the above
21. The definition of viscosity is
    a) The resistance of mud to create mud cake
    b) The resistance of mud to behave as a Newtonian fluid
    c) The resistance of mud to flow
    d) All of the above
22. The mud viscosity is measured using
    a) Marsh funnel
    b) Rotational viscometer
    c) Rotational flow meter
    d) a and b
23. The viscosity measuring device which is used to give only a comparative reading is
    a) Marsh funnel
    b) Rotational viscometer
    c) a and b
    d) None of the above

24. Gel strength of mud is defined as
    a) The force required to circulate the mud
    b) The force required to initiate a flow when flow stopped
    c) The force required to stop the flow of mud
    d) All of the above

25. Gel strength of mud is
    a) Directly related to shear stress
    b) Related to the viscosity of the mud
    c) A function of the inter particle forces
    d) All of the above

26. In general, pH test is used to measure of acidity or alkalinity through
    a) The concentration of hydrogen ions in any aqueous solution
    b) The reaction of hydrogen ions with any aqueous solution
    c) The ability of hydrogen ions to dissolve in any aqueous solution
    d) All of the above

27. Filtration properties of the drilling mud affects
    a) The borehole stability
    b) The formation damage
    c) The creation of filter cake
    d) All of the above

28. The excessive sand content in the drilling mud may result in
    a) The deposition of thick filter cake
    b) Settlement in the bottom of the hole
    c) Abrasion of pump parts
    d) All of the above

29. The term "water hardness" is mainly due to the presence of
    a) Calcium and sodium ions
    b) Magnesium and sodium ions
    c) Calcium and magnesium ions
    d) None of the above

30. Water hardness can be estimated using
    a) Titration methods
    b) Filtration methods
    c) Trial and error methods
    d) All of the above

31. Water is said be soft in terms of hardness if hard ions are in the range of
    a) Less than 17.1 *ppm*
    b) Less than 60 *ppm*
    c) Less than 120 *ppm*
    d) Less than 180 *ppm*

32. The water is said to be very hard if hard ions are in the range of
    a) More than 60 *ppm*
    b) More than 180 *ppm*
    c) Less than 180 *ppm*
    d) Less than 60 *ppm*
33. Chemical analysis of water which is used in preparing the drilling fluid is required for
    a) Compatibility studies
    b) Quality control
    c) Evaluation of pollution hazards
    d) All of the above
34. If chloride concentration of the mud is increased during drilling, it is an indication that the drilled formation
    a) Is a salt formation
    b) Is carbonate rock formation
    c) Has saline or salt formation water
    d) a and c
35. The cation exchange reaction which takes place between the mud and the formation will tend to
    a) Harden the water of the mud
    b) Neutralize the ions in the water of the mud
    c) Soften the water of the mud
    d) None of the above
36. Resistivity of mud decreases when
    a) Temperature decreases
    b) Temperature increases
    c) Temperature has no effect on resistivity
    d) None of the above
37. Which of the following mud types has the highest resistivity value?
    a) Fresh water-based mud
    b) Salt water-based mud
    c) Oil-based mud
    d) KCl brine mud
38. While pumping a mud down to a deep well, the mud density at the bottom of the well will be
    a) Lower than the mud weight at surface
    b) Higher than the mud weight at surface
    c) Same as the mud weight at surface
    d) Can be higher or lower

39. If a mud of a certain volume mixed with the same volume of fresh water, what will be the density of the mixture?
    a) Greater than the original density of the mud
    b) The average between mud density and water density
    c) Mud density will not be affected
    d) None of the above

40. The current trend of the development of drilling fluids is mainly focused on
    a) Developing an environmentally friendly drilling fluid
    b) Decreasing the mud cost by using cheaper materials and chemicals
    c) Increasing rate of penetration on harsh environment
    d) All of the above

41. The pH measurement is used as well to indicate the presence of ____________
    a) Contaminants such as cement or anhydrite
    b) Ion concentration
    c) Positive or negative charges
    d) Alkalinity or acidity of the mud

3.6 Nomenclature

A = cross-sectional area
d_B = bit diameter
C_f = volume increase factor due to the loss of dissolved salt during retorting
E = activation energy for viscous flow, *KJ/mol*
K = operational parameter
F = force
f_{ls} = volume fraction of low density solid in the mud system, *vol/vol*
f_{hs} = volume fraction of high density solid in the mud system, *vol/vol*
f_o = oil volume fraction of drilling mud, *vol/vol*
f_s = solids volume fraction of drilling mud, *vol/vol*
f_w = water volume fraction of drilling mud, *vol/vol*
l = layer thickness
p = pressure of the system, N/m^2
M_a = Marangoni number
m_{mix} = mass of final mixture, i.e., mud
N = rotor speed, *rpm*
R = universal gas constant, $kJ/mole - k$
R_{ROP} = rate of penetration of the bit, *ft/hr*
T = temperature, °K
t = time, *s*
t_M = the Marsh funnel (quart) time, s
u_x = fluid velocity in porous media in the direction of *x* axis, *m/s*
v = velocity
V_{ls} = volume of the low density solid in the mud system, *bbl, cc*
V_{hs} = volume of the high density solid in the mud system, *bbl, cc*

V_{mix} = volume of final mixture, i.e., mud, *bbl, cc*
V_{m1} = volume of initial mud (or any liquid) in mud calculation, *bbl, cc*
V_{m2} = volume of mixture in mud calculation, *bbl, cc*
V_s = solid volume of rock fragments entering the mud i.e. volume of cuttings, *bbl/hr*
V_{sc} = volume of solids in mud calculation, *bbl, cc*
V_w = volume of the water phase in the mud system, *bbl, cc*
V_o = volume of oil phase in mud the mud system, *bbl, cc*
y = distance from the boundary plan, *m*
α = fractional order of differentiation, dimensionless
α_D = thermal diffusivity, m^2/s
ρ_{ls} = density of the low density solid in the mud system, g/cm^3
ρ_{hs} = density of the high density solid in the mud system, g/cm^3
ρ_M = density of mud, g/cm^3
ρ_{m1} = density of initial mud, *gm/cc*
ρ_{m2} = density of final mud (i.e., fresh water and clay), *gm/cc*
ρ_{mix} = overall density of fluid mixture, i.e., mud, g/cm^3
ρ_w = density of water phase in the mud system, g/cm^3
ρ_o = density of oil phase in the mud system, g/cm^3
ρ_{sc} = density of solids, *gm/cc*
ϕ_A = average formation porosity
Φ_N = torque reading from the dial at a speed N, *rpm*
Φ_{300} = torque reading from the dial at a speed of 300 *rpm*, *rpm*
Φ_{600} = torque reading from the dial at a speed of 600 *rpm*, *rpm*
μ = viscosity of the fluid between the plate
τ_s = shear stress
γ = shear rate
ξ = a dummy variable for time, i.e., real part in the plane of the integral
σ = surface tension, *N/m*
ΔT = $T_T - T_o$ =temperature difference between a temperature and a reference temperature, °K
μ = fluid dynamic viscosity, *Pa-s*
μ_{app} = apparent viscosity at a speed *N rpm*, *cp*
μ_e = the effective viscosity, *cp*
μ_o = fluid dynamic viscosity at reference temperature T_0, *Pa-s*
μ_p = plastic viscosity, *cp*
τ_B = the Bingham yield point, $\dfrac{lb_f}{100\,ft^2}$
τ_{TB} = the true Bingham yield point, $\dfrac{lb_f}{100\,ft^2}$
τ_{sT} = shear stress at temperature *T*, *Pa*
η = ratio of the pseudo-permeability of the medium with memory to fluid viscosity, $\dfrac{m^3 s^{1+\alpha}}{kg}$

$\dfrac{du_x}{dy}$ = velocity gradient along y-direction, $1/s$

$\dfrac{dv}{dl}$ = velocity gradient along l-direction

$\dfrac{\partial \sigma}{\partial T}$ = the derivative of surface tension s with temperature and can be positive or negative depending on the substance, $N/m\text{-}K$

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4

Drilling Hydraulics

4.1 Introduction

Hydraulics can be defined as the study of the physical science and technology of the static and dynamic behavior of fluids under the influence of mechanical forces and/or pressure, and uses of that knowledge in the designing and controlling of machines. In drilling engineering, drilling hydraulics is an essential part of drilling operations where computation of pressure profiles along the wellbore and particularly in the annulus contributing to well safety and well integrity are done to improve the API recommended practice for drilling fluid rheology and drilling hydraulics estimation. Drilling hydraulics play a vital role while drilling activities continue to operate, which is also referred to as rig hydraulics. This chapter deals with sets of multiple choice questions (MCQs) which are related to the drilling hydraulics. Workout examples and exercises are presented here. Workout examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

4.2 Different Mathematical Formulas and Examples

4.2.1 Newtonian Fluid

The shear stress and the rate of deformation are normally expressed by Newton's law of viscosity, which can be written mathematically as:

$$\tau = \mu_d \frac{du_x}{dy} = \mu(\gamma) \tag{4.1}$$

where:

τ = shear stress, *Pa*

μ_d = dynamic viscosity, *Pa-s*

$\frac{du_x}{dy}$ or γ = the velocity gradient perpendicular to the direction of shear, or equivalently the strain rate, s^{-1}

Example 4.1: Calculate the shear stress of a drilling fluid which has a viscosity of 55 *cp* and a shear rate of 15 s^{-1}.

Solution:

Given data:

μ_d = Mud viscosity = 55 *cp* = 0.55 Poise = 0.55 *dyne.s/cm*^{2} = 0.55 *Pa-s*

$\frac{du_x}{dy}$ = γ = Shear rate = 15 s^{-1}

Required data:

τ = Shear stress, *Pa*

The shear stress of the fluid can be calculated using the Eq. (4.1) as:

$$\tau = \mu_d \frac{du_x}{dy} = 0.55\ Pa.s \times 15\ s^{-1} = \mathbf{8.25\,Pa}$$

Example 4.2: Calculate the viscosity of a fluid which has a shear stress of 4.5 *pa* and a shear rate of 10 s^{-1}.

Solution:

Given data:

$\frac{du_x}{dy}$ = γ = Shear rate = 10 s^{-1}

τ = Shear stress = 4.5 *pa*

Required data:

μ_d = fluid dynamic viscosity = 45 *cp*

The shear stress of the fluid can be calculated from the following equation Eq. (4.1) as:

$$\tau = \mu_d \frac{du_x}{dy} \text{ or } 4.5\ pa = \mu_d (10\ s^{-1})$$

$$\mu_d = 0.45\ pa\ s = 0.45\ poise = \mathbf{45\,cp}$$

4.2.2 Non-Newtonian Fluid

***i) Bingham Plastic Models*:**
It is defined in terms of the shear stress and shear rate, which are given by the following mathematical models:

$$\tau = \mu_p \gamma + \tau_y;\ \text{if } \tau > \tau_y \tag{4.2a}$$

$$\tau = \mu_p \gamma - \tau_y;\ \text{if } \tau < \tau_y \tag{4.2b}$$

where:
τ_y = a minimum shear stress that needs to initiate fluid flow, *Pa*
μ_p = Bingham plastic viscosity, *Pa-s*

Example 4.3: A moving plate is set 2 *cm* above a stationary plate which has a cross-sectional area of 25 cm^2. If a force of 250 dynes is required to just initiate the upper plate and a force of 550 dynes is needed to move the plate with a uniform velocity of 8 *cm/s*, calculate the yield point and plastic viscosity of the fluid.

<u>Solution</u>:
Given data:
L = Gap between the plates = 2 *cm*
A = Cross-sectional area of the upper plate = 25 cm^2
F_y = Force needed to initiate the plate movement (i.e., $\gamma = 0$)
= 250 *dynes*
V = Fluid velocity = 8 *cm/s*
F = Force needed to move the plate = 550 *dynes*

Required data:
τ_y = Yield point, *Pa*
μ_p = Plastic viscosity, *Pa-s*

The yield point (τ_y) needed to initialize the fluid can be calculated using Eq. (4.2a) as:

$$\tau = \mu_p \gamma + \tau_y = \mu_p \times 0 + \tau_y = \tau_y$$

Now, the definition of shear stress can be written using Eq. (3.6) as:

$$\tau = F/A = \frac{250\ dynes}{25\ cm^2} = 10\ dyne/cm^2$$

Therefore, $\tau_y = 10\ \boldsymbol{dynes/cm^2}$

The plastic viscosity can be calculated using the force needed to move the plate and with the fluid velocity and by Eq. (4.2a) as:

$$\mu_p = \frac{\tau - \tau_y}{\gamma} = \frac{F/A - \tau_y}{V/L} = \frac{550/25 - 10}{8/2} = 3.0\ dyne.s/cm^2$$

$$= 3\ Poise = \mathbf{300\ \mathit{cp}}$$

Example 4.4: A moving plate is set 2.5 *cm* above a stationary plate which has a cross-sectional area of 30 cm^2. If a force of 450 *dynes* is required to just initiate the upper plate and a force of 650 *dynes* is needed to move the plate with a uniform velocity of 8 *cm/s*, calculate the yield point and plastic viscosity of the fluid.

<u>Solution</u>:

Given data:

L = Gap between the plates = 2.5 *cm*

A = Cross-sectional area of the upper plate = 30 cm^2

F_y = Force needed to initiate the plate movement (i.e. $\gamma = 0$)
= 450 *dynes*

V = Fluid Velocity = 8 *cm*/s

F = Force needed to move the plate = 650 *dynes*

Required data:

μ_p = plastic viscosity, *cp*

τ_y = Yield point, *Pa-s*

The yield point (τ_y) needed to initialize the fluid can be calculated using Equation (4.2a) as:

$$\tau_y = \mu_p \gamma + \tau_y = \mu_p \times 0 + \tau_y = \tau_y$$

Now, the definition of shear stress can be written using the Eq. (3.6) as:

$$\tau = \frac{F}{A} = \frac{450\ dynes}{30\ cm^2} = 15\ dyne/cm^2$$

Therefore, $\tau_y = 15\ \boldsymbol{dynes/cm^2}$

The plastic viscosity can be calculated using the force needed to move the plate and with the fluid velocity and by Eq. (4.2a) as:

$$\mu_p = \frac{\tau - \tau_y}{\gamma} = \frac{F/A - \tau_y}{V/L} = \frac{650/30 - 15}{8/2.5} = 2.083\ dyne.\frac{s}{cm^2}$$
$$= 2.083\ Poise = \mathbf{208.3}\ \boldsymbol{cp}$$

***Example 4.5*:** A moving plate of a cross-sectional area of 34 cm^2 is positioned at 3 *cm* above a stationary plate. A force of 763 *dynes* is required to move the plate at a uniform velocity of 11 *cm/s*, and a force of 841 *dynes* is needed to move the plate at a uniform speed of 14 *cm/s*. Calculate the Calculate the force required to initiate the initial movement of the plate and the plastic viscosity.

<u>Solution</u>:

Given data:

L = Distance between the plates = 3.0 *cm*

A = Cross-sectional area of the upper plate = 34 cm^2

$F1 =$ Force required for first speed $= 763\ dynes$
$v1 =$ First speed of the plate $= 11\ cm/s$
$F2 =$ Force required for second speed $= 841\ dynes$
$v2 =$ Second speed of the plate $= 14\ cm/s$

Required data:
$F_y =$ Force to initiate the movement
$\mu_p =$ Plastic viscosity.

Eq. (4.2a) can be used to develop a relation between plastic viscosity and yield point using the given information:

$$\tau_1 = \frac{F_1}{A} = \frac{763}{34} = 22.4\ dynes/cm^2$$

$$\tau_2 = \frac{F_2}{A} = \frac{841}{34} = 24.7\ dynes/cm^2$$

$$\gamma_1 = \frac{v_1}{L} = \frac{11}{3} = 3.67\ s^{-1}$$

$$\gamma_2 = \frac{v_2}{L} = \frac{14}{3} = 4.67\ s^{-1}$$

Now two equations can be developed as follows:

$$\tau = \mu_p \gamma + \tau_y$$

$$22.4 = \mu_p 3.67 + \tau_y$$

$$24.7 = \mu_p 4.67 + \tau_y$$

Solving the above two equations together will give:

$$\mu_p = \mathbf{2.3}\ \boldsymbol{poise}\ \mathbf{OR}\ \mathbf{230}\ \boldsymbol{cp}$$

$$\tau_y = 14.0\ dynes/cm^2$$

The force required to initiate movement is equal to:

$$F_y = \tau_y A = 14.0 \times 34 = \mathbf{476}\ \boldsymbol{dynes}$$

For Bingham fluid, plastic viscosity, yield point, and zero-sec-gel can be calculated using FannV-G meter (See Chapter 3) reading with the following relationships.

$$\mu_p = \theta_{600} - \theta_{300} \tag{4.3a}$$

$$\tau_y = 2\theta_{300} - \theta_{600} \tag{4.3b}$$

$$\tau_0 = \theta_3 \tag{4.3c}$$

where:

θ_{600} = the Fann dial reading at 600 *rpm*
θ_{300} = the Fann dial reading at 300 *rpm*
θ_3 = the Fann dial reading at 3 *rpm*
τ_0 = yield point stress at dial reading at 3 *rpm*

Alternatively, the dial readings can be reverse calculated by using plastic viscosity, yield point, and zero-gel by rearranging the Eqs. (4.3a–c). It is noted that these reading give the corresponding plastic viscosity, and yield point in field unit.

$$\theta_{300} = \mu_p + \tau_y \tag{4.4a}$$

$$\theta_{600} = 2\mu_p + \tau_y \tag{4.4b}$$

$$\theta_3 = \tau_0 \tag{4.4c}$$

If the Fann V-G meter RPMs are any other readings except 300 and 600 *rpm*, the following equations can be used:

$$\mu_p = \frac{300}{N_2 - N_1}(\theta_{N_2} - \theta_{N_1}) \tag{4.5}$$

$$\tau_y = \theta_{N_1} - \mu_p \frac{N_1}{300} \tag{4.6}$$

where:

N_1, N_2 = The Fann rpm reading
θ_{N_1},θ_{N_2} = The Fann dial reading at N_1 and N_2 *rpm*

Example 4.5: In the drilling fluid laboratory, a student was conducting an experiment for the Bingham fluid where he was using the Fann V-G meter to measure the viscosity of the fluid and he found the following Fann data: θ_{300} = 30; θ_{600} = 55, and θ_{200} = 27; θ_{400} = 49. Calculate the plastic viscosity, and yield point of the fluid using Bingham plastic model.

Solution:

Given data:

θ_{300} = Dial reading at 300 *rpm* = 30
θ_{600} = Dial reading at 600 *rpm* = 55
θ_{200} = Dial reading at 200 *rpm* = 27
θ_{400} = Dial reading at 400 *rpm* = 49

Required data:

μ_p = Plastic viscosity, *Pa.s*
τ_y = Yield point, *Pa*

For 300 and 600 *rpm* reading, the plastic viscosity and yield point can be calculated using the Eqs. (4.3a) and (4.3b) as:

$$\mu_p = \theta_{600} - \theta_{300} = 55 - 30 = 25cp = 0.25\ Poise = 0.25\ dyne.s/cm^2$$
$$= \mathbf{0.25}\ \boldsymbol{Pa.s}$$

$$\tau_y = 2\theta_{300} - \theta_{600} = 2\times 30 - 55 = 5lb_f/100ft^2$$

$$= (5lb_f/100ft^2)\times\frac{dyne/cm^2}{4.79lb_f/100ft^2};\ (1\ Pa = 4.79lb_f/100ft^2)$$

$$= 1.04\ dyne/cm^2 = \mathbf{1.04\ Pa}$$

For the second set of reading, set of readings, we should use the Eqs. (4.5) and (4.6) as:

$$\mu_p = \frac{300}{N_2 - N_1}(\theta_{N_2} - \theta_{N_1}) = \frac{300}{400-200}(49-27) = 33\ cp = \mathbf{0.33\ Pa.s}$$

$$\tau_y = \theta_{N_1} - \mu_p\frac{N_1}{300} = 27 - 33\times\frac{200}{300} = 5\ lb_f/100ft^2$$

$$= \mathbf{1.04\ dyne/cm^2} = \mathbf{1.04\ Pa}$$

Example 4.6: In the drilling fluid laboratory, a student was conducting an experiment for the Bingham fluid where he was using the Fann V-G meter to measure the viscosity of the fluid and he found the following Fann data: θ_{300} = 35; θ_{600} = 60, and θ_{200} = 32; θ_{400} = 54. Calculate the plastic viscosity, and yield point of the fluid using Bingham plastic model.

Solution:

Given data:

θ_{300}= Dial reading at 300 *rpm* = 35
θ_{600}= Dial reading at 600 *rpm* = 60
θ_{200}= Dial reading at 200 *rpm* = 32
θ_{400}= Dial reading at 400 *rpm* = 54

Required data:

μ_p = Plastic viscosity, *Pa.s*
τ_y = Yield point, *Pa*

For 300 and 600 *rpm* reading, the plastic viscosity and yield point can be calculated using the Eqs. (4.3a) and (4.3b) as:

$$\mu_p = \theta_{600} - \theta_{300} = 60 - 35 = 25\ cp = 0.25\ Poise$$

$$= 0.25\ dyne.s/cm^2 = \mathbf{0.25\ Pa.s}$$

$$\tau_y = 2\theta_{300} - \theta_{600} = 2\times 35 - 60 = 10\ lb_f/100ft^2$$

$$\tau_y = (10\ lb_f/100ft^2)\times\frac{dyne/cm^2}{4.79\ lb_f/100ft^2};(1\ Pa = 4.79\ lb_f/100ft^2)$$

$$\tau_y = 2.087\ dynes/cm^2 = \mathbf{2.087\,Pa}$$

For the second set of reading set of readings, we should use the Eqs. (4.5) and (4.6) as:

$$\mu_p = \frac{300}{N_2 - N_1}(\theta_{N_2} - \theta_{N_1}) = \frac{300}{400-200}(54-32) = 33\ cp = \mathbf{0.33\ \textit{Pa.s}}$$

$$\tau_y = \theta_{N_1} - \mu_p \frac{N_1}{300} = 32 - 33 \times \frac{200}{300} = 10\ lb_f/100ft^2$$
$$= 2.087\ dynes/cm^2 = \mathbf{2.087\textit{Pa}}$$

***ii) Power-Law Models*:**
The rheological equation for the power law model can be given as:

$$\tau = K\gamma^{n_p} \tag{4.7}$$

where:

K = flow consistency index, $Pa.s^N$

$\gamma = \frac{du_x}{dy}$ = shear rate or velocity gradient perpendicular the plan of shear, s^{-1}

n_p = power-law exponent or flow behavior index, dimensionless

The apparent viscosity as a function of shear rate can be written as:

$$\tau_{app} = K\gamma^{n_p-1} \tag{4.8}$$

Example 4.7: A moving plate is set 2 *cm* above a stationary plate which has a cross-sectional area of 25 cm^2. Calculate the consistency index and flow-behavior index if a force of 250 *dyne* is required to move the upper plate at a constant velocity of 8 *cm/s* and a force of 300 dyne is needed to move the plate with a uniform velocity of 10 *cm/s*.

Solution:

Given data:

L	=	Gap between the plates	=	2 *cm*
A	=	Cross-sectional area of the upper plate	=	25 cm^2
F_1	=	Force needed to move the plate by 8 *cm/s*	=	250 *dynes*
V_1	=	First fluid velocity	=	8 *cm/s*
F_2	=	Force needed to move the plate by 10 *cm/s*	=	300 *dynes*
V_2	=	Second fluid velocity	=	10 *cm/s*

Required data:

K = Flow consistency index, $Pa.s^N$

n_p = Power-law exponent or flow behavior index, dimensionless

To calculate K and n_p, Eq. (4.7) is used at the two rates of shear observed and Eq. (3.4) is used for shear stress as:

$$\tau = F/A = K\gamma^{n_p}$$

$$\frac{250}{25} = K\left(\frac{8}{2}\right)^{n_p} \text{ and } \frac{300}{25} = K\left(\frac{10}{2}\right)^{n_p}$$

Now, dividing the second equation by first equation as:

$$\frac{300}{250} = \left(\frac{10}{8}\right)^{n_p}$$

Taking the *ln* of both sides and solving for n_p as:

$$n_p = \frac{ln(300/250)}{ln(10/8)} = \mathbf{0.817}$$

Substituting the value of *n* in the first equation above gives as:

$$\frac{250}{25} = K\left(\frac{8}{2}\right)^{n_p} \Rightarrow K = \frac{10}{(4)^{0.817}} = 3.222\ Pa.s^{0.817} = \mathbf{322.2\ \mathit{eq.cp}}$$

Example 4.8: A moving plate is set 2.5 *cm* above a stationary plate which has a cross-sectional area of 30 cm^2. Calculate the consistency index and flow-behavior index if a force of 300 *dyne* is required to move the upper plate at a constant velocity of 10 *cm/s* and a force of 350 *dyne* is needed to move the plate with a uniform velocity of 12 *cm/s*.

Solution:

Given data:

L	=	Gap between the plates	=	2.5 *cm*
A	=	Cross-sectional area of the upper plate	=	30 cm^2
F_1	=	Force needed to move the plate by 10 *cm/s*	=	300 *dynes*
V_1	=	First fluid Velocity	=	10 *cm/s*
F_2	=	Force needed to move the plate by 10 *cm/s*	=	350 *dynes*
V_2	=	Second fluid Velocity	=	12 *cm/s*

Required data:

K	=	Flow consistency index, $Pa.s^N$
n_p	=	Power-law exponent or flow behavior index, dimensionless

To calculate *K* and n_p, Eq. (4.7) is used at the two rates of shear observed and Eq. (3.4) is used for shear stress as:

$$\tau = F/A = K\gamma^{n_p}$$

$$\frac{300}{30} = K\left(\frac{10}{2.5}\right)^{n_p} \text{ and } \frac{350}{30} = K\left(\frac{12}{2.5}\right)^{n_p}$$

Now, dividing the second equation by first equation as:

$$\frac{350}{300} = \left(\frac{12}{10}\right)^{n_p}$$

Taking the *ln* of both sides and solving for n_p as:

$$n_p = \frac{\ln(350/300)}{\ln(12/10)} = \mathbf{0.845}$$

Substituting the value of *n* in the first equation above gives as:

$$\frac{300}{30} = K\left(\frac{10}{2.5}\right)^{n_p} \Rightarrow K = \frac{10}{(4)^{0.845}} = 3.099\ Pa.s^{0.845} = \boldsymbol{309.9\ eq.cp}$$

For power-law fluid, the flow behavior index can be calculated using Fann V-G meter which is written as:

$$n_p = \frac{\log_{10}\dfrac{\theta_{600}}{\theta_{300}}}{0.301} = 3.322\log\frac{\theta_{600}}{\theta_{300}} \tag{4.9}$$

In Eq. (4.9), if we use the modified power-law, it can be written using Eqs. (4.4a) and (4.4b) as:

$$n_p = 3.322\log\frac{2\mu_p + \tau_y}{\mu_p + \tau_y} \tag{4.10}$$

The consistency index, *K* can be written as:

$$K = \frac{5.11\ \theta_{300}}{511^{n_p}} \tag{4.11}$$

In Eq. (4.11), *K* is in eq. poise. It can be expressed in terms of $(lb_f \times sec^{-n})/100\,ft^2$ using the conversion factor of $1Pa = 4.79lb_f/100\,ft^2$.

K can also be calculated using the modified power-law which gives as:

$$K = \frac{2\mu_p + \tau_y}{1022^n} \tag{4.12}$$

If the Fann V-G meter RPMs are any other readings except 300 and 600 *rpm*, the following equations can be used:

$$n_p = \frac{\log\left(\dfrac{\theta_{N_2}}{\theta_{N_1}}\right)}{\log\left(\dfrac{N_2}{N_1}\right)} \tag{4.13}$$

$$K = \frac{5.11\theta_N}{(1.703 \times N)^{n_p}} \tag{4.14}$$

Example 4.9: In the drilling fluid laboratory, a student was conducting an experiment for the Bingham fluid where he was using the Fann V-G meter to measure the viscosity of the fluid and he found the following Fann data: $\theta_{300} = 30$; $\theta_{500} = 55$, and $\theta_{200} = 27$; $\theta_{400} = 49$. Calculate the consistency index and flow-behavior index for the power-law model.

Solution:

Given data:

θ_{300} = Dial reading at 300 rpm = 30
θ_{600} = Dial reading at 600 rpm = 55
θ_{200} = Dial reading at 200 rpm = 27
θ_{400} = Dial reading at 400 rpm = 49

Required data:

K = Flow consistency index, $Pa.s^{n_p}$
n_p = Power-law exponent or flow behavior index, dimensionless

For 300 and 600 rpm reading, K and n can be calculated using the Eqs. (4.9) and (4.11) as:

$$n_p = 3.322 \log \frac{\theta_{600}}{\theta_{300}} = 3.322 \log \frac{55}{30} = \mathbf{0.874}$$

and

$$K = \frac{5.11\ \theta_{300}}{511^{n_p}} = \frac{5.11 \times 30}{511^{0.874}} = \mathbf{0.658\,eq.\,poise}$$

In field unit:

$$K = 0.658 \times 4.79 = \boldsymbol{3.152\ lb_f/100\,ft^2}.\ [\text{Note: } 1\ Pa = 4.79 lb_f/100\,ft^2]$$

For the second set of readings, we should use the Eqs. (4.13) and (4.14) as:

$$n_p = \frac{\log\left(\dfrac{\theta_{N_2}}{\theta_{N_1}}\right)}{\log\left(\dfrac{N_2}{N_1}\right)} = \frac{\log\left(\dfrac{49}{27}\right)}{\log\left(\dfrac{400}{200}\right)} = \mathbf{0.8598}$$

$$K = \frac{5.11\ \theta_N}{(1.703 \times N)^{n_p}} = \frac{5.11 \times 27}{(1.703 \times 200)^{0.8598}} = \boldsymbol{0.9174\ eq.cp}$$

In field unit:

$$K = 0.9174 \times 4.79 = \boldsymbol{4.394\ lb_f/100\ ft^2}$$

Example 4.10: In the drilling fluid laboratory, a student was conducting an experiment for the Bingham fluid where he was using the Fann V-G meter to measure the viscosity of the fluid and he found the following Fann data: $\theta_{300} = 35$; $\theta_{600} = 60$, and $\theta_{200} = 32$; $\theta_{400} = 54$. Calculate the consistency index and flow-behavior index for the power-law model.

Solution:

Given data:

θ_{300} = Dial reading at 300 *rpm* = 35
θ_{600} = Dial reading at 600 *rpm* = 60
θ_{200} = Dial reading at 200 *rpm* = 32
θ_{400} = Dial reading at 400 *rpm* = 54

Required data:

K = Flow consistency index, $Pa.s^N$
n_p = Power-law exponent or flow behavior index, dimensionless

For 300 and 600 *rpm* reading, K and n can be calculated using the Eqs. (4.9) and (4.11) as:

$$n_p = 3.322 \log \frac{\theta_{600}}{\theta_{300}} = 3.322 \log \frac{60}{35} = \mathbf{0.777}$$

and

$$K = \frac{5.11\theta_{300}}{511^{n_p}} = \frac{5.11 \times 35}{511^{0.777}} = \mathbf{1.406}\ \boldsymbol{eq.poise}$$

In field unit:

$$K = 1.406 \times 4.79 = 6.735\ lb_f/100ft^2$$

$$[\text{Note: } 1\ Pa = 4.79\ lb_f/100ft^2]$$

For the second set of readings, we should use the Eqs. (4.13) and (4.14) as:

$$n_p = \frac{\log\left(\frac{\theta_{N_2}}{\theta_{N_1}}\right)}{\log\left(\frac{N_2}{N_1}\right)} = \frac{\log\left(\frac{54}{32}\right)}{\log\left(\frac{400}{200}\right)} = \mathbf{0.7548}$$

$$K = \frac{5.11\theta_N}{(1.703 \times N)^{n_p}} = \frac{5.11 \times 32}{(1.703 \times 200)^{0.7548}} = \mathbf{2.0055}\ \boldsymbol{eq.cp}$$

In field unit:

$$K = 2.0055 \times 4.79 = \mathbf{9.60}\ \boldsymbol{lb_f/100\ ft^2}$$

iii) Shear-thinning fluid models:

For the porous media, Chauveteru's form of the definition of porous media wall shear strain rate or in-situ shear rate is

$$\dot{\gamma}_{pm} = \frac{\alpha_{SF} u_x}{\sqrt{k\phi}} \tag{4.15}$$

where:

u_x = Fluid velocity in porous media in the direction of x axis, m/s
α_{SF} = Shape factor which is medium-dependent
$\dot{\gamma}_{pm}$ = Apparent shear rate within the porous medium, s^{-1}
k = Reservoir permeability, m^2
ϕ = porosity of fluid media, m^3/m^3

To model the bulk rheology of the non-Newtonian fluid, Carreau-Yasuda model may be written as:

$$\mu_{eff} = \mu_\infty + \frac{(\mu_0 - \mu_\infty)}{[1 + (\lambda \dot{\gamma}_{pm})^a]^{\frac{n_c}{a}}} \tag{4.16}$$

where:

a = Parameter in Carreau–Yasuda model, dimensionless
n_c = Power-law exponent for Carreau–Yasuda model, dimensionless
μ_{eff} = Fluid effective viscosity, *Pa-s*
μ_0 = Fluid dynamic viscosity at zero shear rate, *Pa-s*
μ_∞ = Fluid dynamic viscosity at infinite shear rate, *Pa-s*
λ = time constant in Carreau–Yasuda model, *s*

For apparent shear rate, Hossain *et al.* (2009) model can be written as:

$$\dot{\gamma}_{pm} = \frac{\alpha_{SF}}{\sqrt{k\phi}} \frac{\eta}{\Gamma(1-\alpha)} \int_0^t (t-\xi)^{-\alpha} \frac{\partial^2 p}{\partial \xi \partial x} d\xi \tag{4.17}$$

where:

p = pressure of the system, N/m^2
t = time, *s*
α = fractional order of differentiation, dimensionless
η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3 s^{1+\alpha}/kg$
ξ = a dummy variable for time, i.e., real part in the plane of the integral, *s*

To analyze the memory effect in the shear-thinning fluid viscosity, Hossain *et al.* (2009) proposed the following model for effective viscosity.

$$\mu_{eff} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{\left[1 + \left(\frac{\lambda \eta \alpha_{SF}}{\sqrt{k\phi}} \frac{\int_0^t (t-\xi)^{-\alpha} \frac{\partial^2 p}{\partial \xi \partial x} d\xi}{\Gamma(1-\alpha)}\right)^a\right]^{\frac{n_c}{a}}} \tag{4.18}$$

***iv) Herschel-Bulkley model*:**
Mathematically, Herschel-Bulkley model is defined as:

$$\tau = K\gamma^n + \tau_y \quad (4.19)$$

In Eq. (4.19), if $\tau_y = 0$, the Herschel-Bulkley model is reduced to the power-law model. On the other hand if $n = 1$, the model reduces to the Bingham plastic model.

4.2.3 Turbulent Flow

Reynolds number: Reynolds observed that when circulating Newtonian fluids through pipes the onset of turbulence was dependent on variables such as i) pipe diameter (d), ii) density of fluid®, iii) viscosity of fluid (μ), iv) average flow velocity (v). He also found that the onset of turbulence occurred when the above combination of these variables exceeded a value of 2100. Reynold's observation was very significant because it means that the onset of turbulence can be predicted for pipes of any size, and fluids of any density or viscosity, flowing at any rate through the pipe. This grouping of variables is generally termed a dimensionless group, which is known as the Reynolds number. Therefore, the onset of turbulence in pipe flow is characterized by the dimensionless group as:

$$N_{R_e} = \frac{\rho \bar{v} d_i}{\mu} \quad (4.20)$$

Here:

ρ = Fluid density, *gm/cc*
$\bar{v}$ = Avg. fluid velocity, *cm/s*
d_i = Pipe inner diameter, *cm.*
μ_d = Dynamic viscosity of fluid, *cp*
q = Circulating volume, *cc/s*

In field units, Reynolds number can be written as:

$$N_{R_e} = \frac{928\ \rho \bar{v} d_i}{\mu} \quad (4.21)$$

Here:

ρ = Fluid density, lb_m/gal
$\bar{v}$ = Avg. fluid velocity, *ft/s* = $\dfrac{q}{2.448\ d_i^2}$
d_i = Pipe inner diameter, *in.*
μ_d = Dynamic viscosity of fluid, *cp*
q = Circulating volume, *gal/min*

Example 4.11: While drilling, a 10.0 lb_m/gal of mud having a viscosity of 1.2 *cp* was being circulated through drillstring at a rate of 650 *gal/min*. If the internal diameter of the drillpipe is 5.0 *in*, determine the type of flow in the drillpipe of the circulating system.

Solution:

Given data:

ρ_m = Density of the mud = 10.0 lb_m/gal
μ_m = Viscosity of the mud = 1.2 cp
q = Circulating volume or volume flow rate = 650 gal/min
d_i = Inner diameter of drillpipe = 5.0 in

Required data:

Type of flow

The average fluid velocity can be calculated as:

$$\bar{v} = \frac{q}{2.448\ d_i^2} = \frac{650\ gal/min}{2.448 \times (5\ in)^2} = 10.62\ ft/s$$

Equation (4.21) is used to determine whether the fluid is laminar or turbulent

$$N_{R_e} = \frac{928\ \rho \bar{v} d_i}{\mu} = \frac{928 \times (10\ lb_m/gal) \times (10.62\ ft/s) \times (5\ in)}{(1.2\ cp)} = \mathbf{410{,}640}$$

Since the Reynolds number is considerably very high comparing with 2,100, the fluid of the drillpipe is in **turbulent flow**.

Example 4.12: While drilling, an 11.0 lb_m/gal of mud having a viscosity of 1.4 cp was being circulated through drillstring at a rate of 700 gal/min. If the internal diameter of the drillpipe is 5.5 in, determine the type of flow in the drillpipe of the circulating system.

Solution:

Given data:

ρ_m = Density of the mud = 11.0 lb_m/gal
μ_m = Viscosity of the mud = 1.4 cp
q = Circulating volume or volume flow rate = 700 gal/min
d_i = Inner diameter of drillpipe = 5.5 in

Required data:

Type of flow

The average fluid velocity can be calculated as:

$$\bar{v} = \frac{q}{2.448\ d_i^2} = \frac{700}{2.448 \times (5.5\ in)^2} = 9.453\ ft/s$$

Equation (4.21) is used to determine whether the fluid is laminar or turbulent

$$N_{R_e} = \frac{928 \rho \bar{v} d_i}{\mu} = \frac{928 \times (11\ lbm/gal) \times (9.453\ ft/s) \times (5.5\ in)}{(1.4\ cp)}$$
$$= \mathbf{379{,}092.308}$$

Since the Reynolds number is very high compared with 2,100, the fluid of the drillpipe is in **turbulent flow**.

4.2.4 Transitional Flow

The critical velocity can be determined for the Bingham plastic model as:

$$V_{cB} = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 12.34\ \rho_m d_i^2 \tau_y}}{\rho_m d_i} \tag{4.22}$$

where:

V_{cB} = Critical velocity for the Bingham plastic model, *ft/s*
ρ_m = Mud density, *ppg*
d_i = Pipe inner diameter, *in*

The critical flow rate can be determined for the Bingham plastic model as:

$$Q_{cB} = 2.448 \times V_{cB} \times d_i^2 \tag{4.23}$$

Here:

Q_{cB} = Critical flow rate for the Bingham plastic model, *gpm*

Example 4.13: While drilling, a 13.0 lb_m/gal of mud is used where Fann data was observed as $\theta_{300} = 25$; $\theta_{600} = 47$. The target depth was set at 10,000 *ft* (TVD). If the internal diameter of the drillpipe is 3.75 *in*, calculate the critical velocity inside the pipe and the critical flow rate.

Solution:

Given data:

ρ_m = Density of the mud = 13.0 lb_m/gal
θ_{300} = Dial reading at 300 *rpm* = 25
θ_{600} = Dial reading at 600 *rpm* = 47
TVD = Total vertical depth = 10,000 *ft*
d_i = Inner diameter of drillpipe = 3.75 *in*

Required data:

V_{cB} = Critical velocity inside the pipe, *ft/s*
Q_{cB} = The critical flow rate, *gpm*

For 300 and 600 *rpm* reading, the plastic viscosity and yield point can be calculated using the Eqs. (4.3a) and (4.3b) as:

$$\mu_p = \theta_{600} - \theta_{300} = 47 - 25 = 22\,cp$$

$$\tau_y = 2\theta_{300} - \theta_{600} = 2 \times 25 - 47 = 3\ lb_f/100\,ft^2$$

The critical velocity can be determined for the Bingham plastic model by using Eq. (4.22) as:

$$V_{cB} = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 12.34\ \rho_m d_i^2 \tau_y}}{\rho_m d_i}$$

$$= \frac{1.08 \times 22 + 1.08\sqrt{22^2 + 12.34 \times 13 \times 3.75^2 \times 3}}{13 \times 3.75} = \mathbf{2.37\ \mathit{ft/s}}$$

The critical flow rate can be determined for the Bingham plastic model by Eq. (4.23) as:

$$Q_{cB} = 2.448 \times V_{cB} \times d_i^2 = 2.448 \times 2.37 \times 3.75^2 = \mathbf{81.59\ \mathit{gpm}}$$

Example 4.14: While drilling, a 12.0 lb_m/gal of mud is used where Fann data was observed as $\theta_{300} = 20$; $\theta_{600} = 42$. The target depth was set at 8,000 *ft* (TVD). If the internal diameter of the drillpipe is 3.25 *in*, calculate the critical velocity inside the pipe and the critical flow rate.

Solution:

Given data:

ρ_m	=	mud density	=	12.0 lb_m/gal
θ_{300}	=	Dial reading at 300 *rpm*	=	20
θ_{600}	=	Dial reading at 600 *rpm*	=	42
TVD	=	Total vertical depth	=	8,000 *ft*
d_i	=	Inner diameter of drillpipe	=	3.25 *in*

Required data:

V_{cB}	=	Critical velocity inside the pipe, *ft/s*
Q_{cB}	=	The critical flow rate, *gpm*

For 300 and 600 *rpm* reading, the plastic viscosity and yield point can be calculated using the Eqs. (4.3a) and (4.3b) as:

$$\mu_p = \theta_{600} - \theta_{300} = 42 - 20 = 22\ cp$$

$$\tau_y = 2\theta_{300} - \theta_{600} = 2 \times 20 - 42 = 2 lb_f/100 ft^2$$

The critical velocity can be determined for the Bingham plastic model by using Eq. (4.22) as:

$$V_{cB} = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^{\ 2} + 12.34\rho_m d_i^{\ 2}\tau_y}}{\rho_m d_i}$$

$$= \frac{1.08 \times 22 + 1.08\sqrt{22^2 + 12.34 \times 12 \times 3.25^2 \times 2}}{12 \times 3.25}$$

$$V_{cB} = \mathbf{2.273\ \mathit{ft/s}}$$

$$Q_{cB} = 2.448 \times V_{cB} \times d_i^{\ 2} = 2.448 \times 2.273 \times 3.25^2 = \mathbf{58.773\ \mathit{gpm}}$$

4.2.5 Hydrostatic Pressure Calculation

The hydrostatic pressure of the drilling fluid is divided into three subsections for i) liquid column, ii) gas column, and iii) complex fluid column.

i) Liquid Columns

Figure 4.1 shows the free-body diagram from which variation of pressure with depth can be derived mathematically. The downward force acting on the fluid element can be calculated as:

$$F_{down} = pA \tag{4.24}$$

where:

F_{down} = Downward force on the fluid element applied by the fluid column above, lb_f
p = Pressure on the fluid element, *psig*
A = Inner cross-sectional area of the fluid column, in^2

The upward force acting on the fluid element can be calculated as:

$$F_{up} = \left(p + \frac{dp}{dL_{tvd}} \Delta L_{tvd} \right) A \tag{4.25}$$

where:

F_{up} = Upward force on the fluid element applied by the below fluid column, lb_f
L_{tvd} = Total vertical depth, *ft*
ΔL_{tvd} = Differential total vertical depth, *ft*
$\frac{dp}{dL_{tvd}}$ = Pressure gradient with respect to total vertical depth, *psig/ft*

Finally the weight of the fluid element itself is exerting a downward force which can be calculated as:

$$F_{self} = W_{sp} A \Delta L_{tvd} \tag{4.26}$$

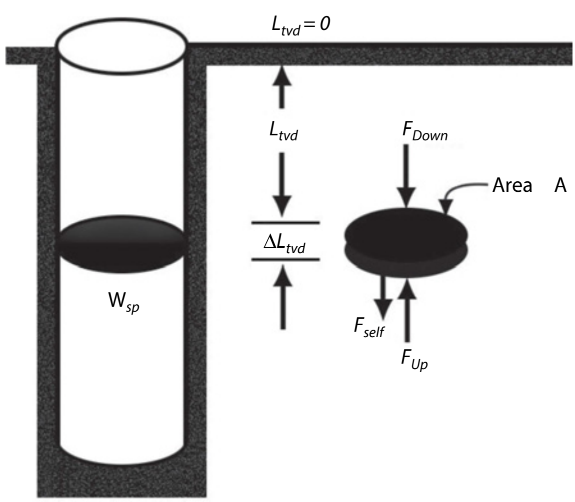

Figure 4.1 Fluid column showing distribution of forces acting on the fluid element.

where:
F_{self} = Fluid element's self-weight acting as a downward force, lb_f
W_{sp} = Specific weight of fluid, $lb_f/in^2\text{-}ft$

If we consider that fluid is at rest, there will not be any shear forces acting on the fluid element and thus all forces acting on the fluid element must be in equilibrium, i.e.:

$$F_{down} + (-F_{up}) + W_{sp} A \Delta L_{tvd} = 0 \tag{4.27}$$

$$pA - \left(p + \frac{dp}{dL_{tvd}} \Delta L_{tvd} \right) A + W_{sp} A \Delta L_{tvd} = 0 \tag{4.28}$$

$$pA - pA - \frac{dp}{dL_{tvd}} \mathrm{A} \Delta L_{tvd} + W_{sp} A \Delta L_{tvd} = 0 \tag{4.29}$$

$$dp = W_{sp} dL_{tvd} \tag{4.30}$$

Eq. (4.30) can be integrated for an incompressible fluid which gives the following final form:

$$p = W_{sp} L_{tvd} + p_o \tag{4.31}$$

where:
p_o = Surface pressure at L_{tvd} = 0 which is also the constant of the integral.

In general, the static surface pressure, p_o is zero unless the blowout preventer of the well is closed and the well is trying to flow. The specific weight of the liquid in field unit can be written as:

$$W_{sp} = 0.052\ \rho_m \tag{4.32}$$

where:
W_{sp} = Specific weight of fluid, $lb_f/in^2\text{-}ft$

Therefore, Eq. (4.31) can be written in field unit as:

$$p = 0.052\ \rho_m L_{tvd} + p_o \tag{4.33}$$

Since mud weights and well depths are often measured with different units, the constant of Eq. (4.32) will vary. Common forms of the hydrostatic pressure equation are as follows:

$$p = 0.052 \times (\text{mud weight, } lb_m/gal) \times (\text{depth, } ft),\ \text{where } p \text{ is in } psia \tag{4.34a}$$

$$p = 0.00695 \times (\text{mud weight, } lb_m/ft^3) \times (\text{depth, } ft),\ \text{where } p \text{ is in } psia \tag{4.34b}$$

$$p = 9.81 \times (\text{mud weight, } g/cm^3) \times (\text{depth, } m),\ \text{where } p \text{ is in } kPa \tag{4.34c}$$

If a column of fluid contains several mud weights, the total hydrostatic pressure is the sum of the individual fluid column or section:

$$p_t = \sum C\rho_i L_i \tag{4.35}$$

where:
p_t = Total hydrostatic pressure
C = Conversion constant
ρ_i = Mud weight for the section of interest
L_i = Length for the section of interest (which is part of L_{tvd})

Equivalent Mud Weight (EMW) in lb_m/gal (i.e., ppg) can be calculated as:

$$EMW = \frac{p_t}{0.052\ L_{tvd}} \tag{4.36a}$$

If the well is deviated α deg from the vertical, the EMW is given by:

$$EMW = \frac{p_t}{0.052\ D_m \cos\alpha} \tag{4.36b}$$

where:
D_m = Measured depth, ft

Equivalent circulating density (ECD) in ppg can be determined as:

$$ECD = \rho_{om} + \frac{\Delta p_{an}}{0.052\ L_{tvd}} \tag{4.37a}$$

where:
ρ_{om} = Original mud density, ppg
Δp_{an} = Annular pressure loss, psi

In deviated wells, vertical depth is used. In such case, Eq. (4.37a) can be modified for multiple sections as:

$$ECD = \rho_{om} + \frac{\sum_{i=1}^{n_w} \Delta p_{ani}}{0.052 \sum_{i=1}^{n_w} L_{tvdi}} \tag{4.37b}$$

where:
n_w = number of wellbore sections

Example 4.15: An intermediate casing string was cemented using the following muds: first section 7,000 *ft* was filled by 12.5 lb_m/gal mud, second section of 1500 *ft* was filled by 15.3 lb_m/gal mud and the last section was filled by 16 lb_m/gal mud. Calculate the total hydrostatic pressure at 11,000 *ft*. Convert the pressure at 11,000 *ft* to an equivalent mud weight and determine if it will exceed the fracture gradient of 14.2 lb_m/gal. Also calculate the ECD for an annular pressure loss gradient of 0.04 *psi/ft* and an original mud weight of 13.5 *ppg*.

Solution:

Given data:

ρ_i = Mud weight for the section of interest
= 12.5, 15.3, and 16 lb_m/gal
L_{tvd} = Total vertical depth = 11,000 *ft*

L_i = Length for the section of L_{tvd} = 7000, 1500 and 2500 respectively
Fracture gradient = 14.2 lb_m/gal
Annular pressure loss = 0.04 *psi/ft*

Required data:
p_t = Total hydrostatic pressure, *psia*
EMW = Equivalent mud weight, lb_m/gal
ECD = Equivalent circulating Density, lb_m/gal *(i.e ppg)*

Total hydrostatic pressure can be determined using Eq. (4.35) where *C* is equal to 0.052.

Therefore,

$$p_t = 0.052 \times (12.5\ lb_m/gal) \times (7000\ ft) + 0.052 \times (15.3\ lb_m/gal) \\ \times (1500\ ft) + 0.052 \times (16\ lb_m/gal) \times (2500\ ft)$$

$$p_t = \mathbf{7823.4}\ \boldsymbol{psia}$$

Equivalent mud weight (EMW) can be calculated using Eq. (4.36)

$$EMW = \frac{p_t}{0.052 L_{tvd}} = \frac{7823.4}{0.052 \times 11000} = \mathbf{13.677}\ \boldsymbol{lb_m/gal(or\ ppg)}$$

Therefore, the static hydrostatic pressure with a 13.677 lb_m/gal EMW will not exceed the fracture gradient of 14.2 lb_m/gal.

The ECD is calculated for the total depth of 11000 ft so Eq. (4.37a) is used for calculating ECD as:

$$ECD = 13.5 + \frac{0.04\ psi/ft \times 11000\,ft}{0.052\ psi/ft \times 11000\ ft} = \mathbf{14.269}\ \boldsymbol{ppg}$$

Example 4.16: An intermediate casing string was cemented using the following muds: first section 7,000 *ft* was filled by 11.5 lb_m/gal mud, second section of 1,500 *ft* was filled by 14.3 lb_m/gal mud and the last section was filled by 15 lb_m/gal mud. Calculate the total hydrostatic pressure at 11,000 *ft*. Convert the pressure at 11,000 *ft* to an equivalent mud weight and determine if it will exceed the fracture gradient of 13.2 lb_m/gal. Also calculate the ECD for an annular pressure loss gradient of 0.04 *psi/ft* and an original mud weight of 12.5 *ppg*.

Solution:

Given data:
ρ_i = Mud weight for the section of interest
= 11.5, 14.3, and 15 lb_m/gal
L_{tvd} = Total vertical depth = 11,000 *ft*
L_i = Length for the section of L_{tvd}
= 7,000', 1,500' and 2,500' respectively
Fracture gradient = 13.2 lb_m/gal
Annular pressure loss = 0.04 *psi/ft*

Required data:

p_t = Total hydrostatic pressure, *psia*
EMW = Equivalent mud weight, lb_m/gal
ECD = Equivalent circulating Density, lb_m/gal (i.e. *ppg*)

Total hydrostatic pressure can be determined using Eq. (4.35) where C is equal to 0.052. Therefore,

$$\begin{aligned} p_t &= 0.052 \times (11.5,\ lb_m/gal) \times (7000\,ft) + 0.052 \times (14.3,\ lb_m/gal) \\ &\quad \times (1500\,ft) + \ 0.052 \times (15,\ lb_m/gal) \times 2500\,ft \\ &\quad \{i.e.\ 11000 - (7000 + 1500) = \mathbf{7{,}251.4\ psia} \end{aligned}$$

Equivalent mud weight (EMW) can be calculated using Eq. (4.36)

$$EMW = \frac{p_t}{0.052\ L_{tvd}} = \frac{7{,}251.4\ psia}{0.052 \times 11{,}000\,ft} = \mathbf{12.67}\ \boldsymbol{lb_m/gal}(\mathbf{or}\ \boldsymbol{ppg})$$

Therefore, the static hydrostatic pressure with a 12.67 lb_m/gal EMW will not exceed the fracture gradient of 13.2 lb_m/gal.

The ECD is calculated for the total depth of 11,000 ft so Eq. (4.37a) is used for calculating ECD as:

$$ECD = 12.5 + \frac{0.04\frac{psi}{ft} \times 11{,}000\,ft}{0.052 \times 11{,}000\,ft} = 12.5 + 0.769 = \mathbf{13.27}\ \boldsymbol{ppg}$$

ii) Gas Columns

The gas behavior can be described using real gas equation defined by

$$p_a V = ZnRT \tag{4.38}$$

where:

p_a = Absolute pressure
V = Gas volume
Z = Universal gas constant
n = mole of gas = $\frac{m}{M}$
m = mass of gas
M = Gas molecular weight
R = Universal gas constant
T = Absolute temperature

Gas density can be expressed as a function of pressure using Eq. (4.38) which can be written as

$$\rho = \frac{m}{V} = \frac{Mp_a}{ZRT} \tag{4.39}$$

Equation (4.39) can be expressed in field unit for mud as

$$\rho_m = \frac{Mp_a}{80.3\ ZT} \tag{4.40}$$

where:

p_a = Absolute pressure, psia
M = Gas molecular weight, fraction
T = Absolute temperature, °R

For any long gas column, variation of gas density with depth can be written in terms of pressure gradient using Eq. (4.40) in Eq. (4.32) and then applying the product to Eq. (4.30) yields

$$dp = 0.052\frac{Mp_a}{80.3\ ZT}dL_{tvd} \tag{4.41}$$

If the gas deviation factor, Z is constant, Eq. (4.41) can be rearranged as integrate both sides as

$$\int_{p_0}^{p_a}\frac{1}{p}dp = \frac{M}{1,544\ ZT}\int_{L_0}^{L_{tvd}}dL_{tvd} \tag{4.42}$$

The final form of Eq. (4.42) gives

$$p_a = p_0 e^{\frac{M(L_{tvd}-L_0)}{1,544\ ZT}} \tag{4.43}$$

Example 4.17: The tubing of a well filled with methane (CH_4) gas to a vertical depth of 12,000 *ft*. The annular space is filled with a 10.5 lb_m/gal mud. Assume that the gas follows the ideal gas behavior. Calculate the amount by which the exterior pressure on the tubing exceeds the interior tubing pressure at 12,000 *ft* if the surface tubing pressure is 1,200 *psia* and the mean temperature is 150 °F. If the collapse resistance of the tubing is 8,500 *psi*, will the tubing collapse due to the high external pressure?

Solution:

Given data:

M = Gas molecular weight = $12 + 1 \times 4 = 16$
L_{tvd} = Total vertical depth = 12,000 *ft*
ρ_m = mud weight = 10.5 lb_m/gal
p_o = Surface pressure = 14.7 *psia*
p_{st} = Surface tubing pressure = 1,200 *psia*
T = Absolute temperature in °R = (460 + 150) = 610 °R
$p_{collapse}$ = Collapse tubing pressure = 8,500 *psi*

Required data:

Δp = Pressure difference between exterior and interior tubing, *psia*

The pressure in the annulus of the well can be calculated using Eq. (4.33) at a depth of 12,000 *ft* as:

$$p_{12,000_A} = 0.052 \times (10.5,\ lb_m/gal) \times (12000\,ft) + 14.7\,psia = 6566.7\,psia$$

The pressure in the tubing at a depth of 12,000 *ft* can be determined using Eq. (4.43) as:

$$p_{12,000_t} = (1200\ psia) \times e^{\frac{(16)(12{,}000-0)}{1{,}544\times(1)\times(610)}} = 1{,}471.35\ psia$$

Thus the pressure difference can be calculated as:

$$\begin{aligned}\Delta p = p_{12,000_A} - p_{12,000_t} &= 6566.7 - 1{,}471.35 \\ &= \boldsymbol{5{,}095.35\ psia}\end{aligned}$$

This pressure difference between exterior and interior tubing is below the collapse tubing pressure, 8,500 *psi*. So there ***would not be any collapse of tubing string***.

However, let us verify the collapse based on density concept as explained earlier. In such case, the density of the gas in the tubing string at the surface can be calculated using Eq. (4.40) as:

$$\rho = \frac{(16)\times(1{,}200)}{80.3\times(1)\times(610)} = 0.3919\ \mathrm{lb_m/gal}$$

It is noted that if we use the Eq. (4.33), the pressure in the tubing of the well can be calculated at a depth of 12,000 *ft* as:

$$\begin{aligned}p_{12,000_t} &= 0.052 \times (0.3919,\ lb_m/gal) \times (12000\,ft) + 1{,}200\,psia \\ &= 1{,}444.55\,psia\end{aligned}$$

The above tubing pressure of the well is less by 26.8 *psia* comparing with the other calculation based on Eq. (4.43). Therefore the above decision for ***collapse is ok***.

4.2.6 Fluid Flow through Pipes

The mud pump total discharge pressure is defined as (Figure 4.2):

$$\Delta P_P = \Delta P_{sp} + \Delta P_{dP} + \Delta P_{dc} + \Delta P_{bn} + \Delta P_{ac} + \Delta P_{ap} \tag{4.44}$$

where:

ΔP_p = pump discharge pressure, *psi*
ΔP_{sp} = pressure loss in surface piping, stand pipe and mud hose, *psi*
ΔP_{dp} = pressure loss inside drill pipe, *psi*
ΔP_{dc} = pressure loss inside drill collar, *psi*
ΔP_{bn} = pressure loss across bit nozzle, *psi*
ΔP_{ac} = pressure loss in annulus in the drill collars, *psi*
ΔP_{ap} = pressure loss in annulus in the drill pipe, *psi*

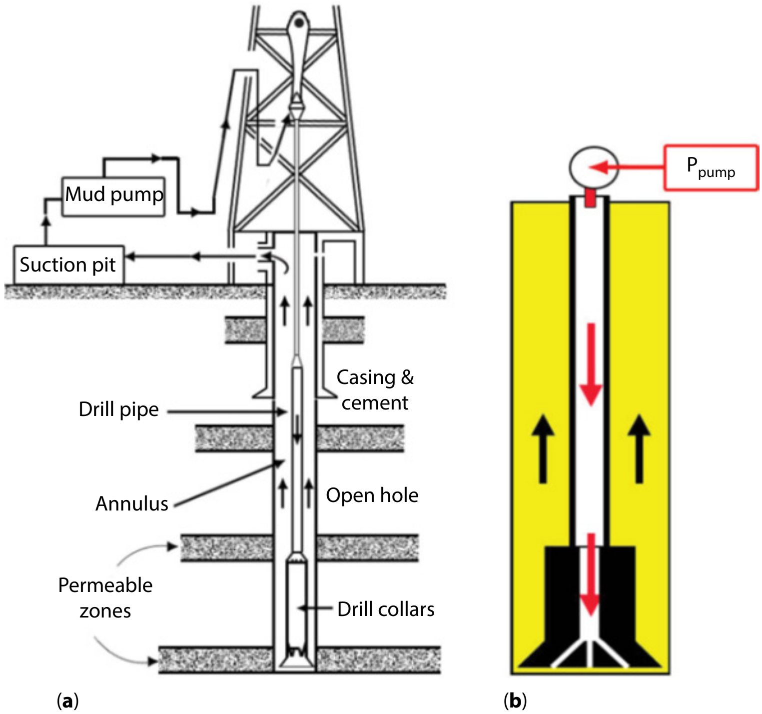

Figure 4.2 Well flow system

Example 4.18: Calculate the total pressure required to discharge a 10 *ppg* mud through the drilling circulating system. Use the following data for the mud pump pressure requirement. Here:

Pressure loss in surface piping, stand pipe and mud hose = 90 psi
Pressure loss inside drill pipe = 2000 *psi*
Pressure loss inside drill collar = 300 *psi*
Pressure loss across bit = 75 *psi*
Pressure loss in annulus in the drill collars = 350 *psi*
Pressure loss in annulus in the drill pipe = 2,700 *psi*

Solution:

Given data:

ΔP_{sp} = pressure loss in surface piping, stand pipe and mud hose = 90 *psi*
ΔP_{dp} = pressure loss inside drill pipe = 2,000 *psi*
ΔP_{dc} = pressure loss inside drill collar = 300 *psi*
ΔP_{bn} = pressure loss across bit = 75 *psi*
ΔP_{ac} = pressure loss in annulus in the drill collars = 350 *psi*
ΔP_{ap} = pressure loss in annulus in the drill pipe = 2,700 *psi*

Required data:

ΔP_p = pump discharge pressure

The total pump discharge pressure of the circulating system can be calculated using Eq. (4.44) as:

$$\Delta P_t = \Delta P_s + \Delta P_P + \Delta P_c + \Delta P_b + \Delta P_{ac} + \Delta P_{ap}$$
$$= 90 + 2{,}000 + 300 + 75 + 350 + 2{,}700 = \mathbf{5{,}515}\ \boldsymbol{psi}$$

The pressure drop in laminar flow is given by the Hagan-Poiseuille law which is given in field unit as:

$$\Delta P_{Lf} = \frac{\mu L \bar{v}}{1500\ d_i^2} \tag{4.45}$$

Here:
ΔP_{Lf} = Laminar flow pressure drop, *psi*
L = Length of the pipe, *ft*

For turbulent flow, Fanning's equation can be applied as

$$\Delta P_{tf} = \frac{f \rho L \bar{v}^2}{25.8\ d_i} \tag{4.46}$$

Here:
ΔP_{tf} = Turbulent flow pressure drop, *psi*
f = Fanning friction factor

The friction factor *f* of Eq. (4.46) can be obtained using Figure 4.12 in Hossain and Al-Majed (2015).

Example 4.19: Calculate the total pressure required to discharge a 12 *ppg* mud through the drilling circulating system. Use the following data for the mud pump pressure requirement. Here:

Pressure loss in surface piping, stand pipe and mud hose = 110 *psi*
Pressure loss inside drill pipe = 2500 *psi*
Pressure loss inside drill collar = 500 *psi*
Pressure loss across bit = 85 *psi*
Pressure loss in annulus in the drill collars = 400 *psi*
Pressure loss in annulus in the drill pipe = 3,000 *psi*

Solution:

Given data:

ΔP_{sp} = Pressure loss in surface piping, stand pipe and mud hose = 110 *psi*
ΔP_{dp} = Pressure loss inside drill pipe = 2500 *psi*
ΔP_{dc} = Pressure loss inside drill collar = 500 *psi*
ΔP_{bn} = Pressure loss across bit = 85 *psi*
ΔP_{ac} = Pressure loss in annulus in the drill collars = 400 *psi*
ΔP_{ap} = Pressure loss in annulus in the drill pipe = 3000 *psi*

Required data:

ΔP_p = Pump discharge pressure

The total pump discharge pressure of the circulating system can be calculated using Eq. (4.44) as:

$$\Delta P_t = \Delta P_{sp} + \Delta P_{dp} + \Delta P_{dc} + \Delta P_{bn} + \Delta P_{ac} + \Delta P_{ap}$$

$$\Delta P_t = 110 + 2500 + 500 + 85 + 400 + 3000 = \mathbf{6595\ \textit{psi}}$$

4.2.7 Fluid Flow through Drill Bits

A tri-cone bit has three nozzles.

$$\Delta p = \frac{1}{2}\rho_m v_n^2 \tag{4.47}$$

where:

ΔP = pressure drop or loss at any section, *psi*
ρ_m = mud density in *ppg*
v_n = mud velocity at the nozzle, *ft*/s

$$v_n = 33.36\sqrt{\frac{\Delta p_{bit}}{\rho_m}} \tag{4.48}$$

where:

ΔP_{bit} = pressure drop at the bit, *psi*

Nozzle area can be calculated as:

$$A_n = 0.3208\frac{q}{v_n} \tag{4.49}$$

where:

q = mud flow rate, gpm
A_n = total nozzle area at the bit, in^2

For tri-cone bit, there are three nozzles with equal sizes, therefore

$$A_n = 3A = 3\frac{\pi}{4}d_n^2 \tag{4.50}$$

where:

d_n = equivalent average diameter of individual nozzle at the bit, in

$$d_n = 32\sqrt{\frac{4A_n}{3\pi}} \tag{4.51}$$

Example 4.20: A drilling engineer was assigned to find out the nozzle sizes of a tri-cone bit which will be used for an immediate drilling operation. The supervisor asked the engineer to use 500 *gpm* of mud circulation at a pressure drop of 1200 *psi* through the bit. It is noted that the mud density is 10 *ppg*.

<u>Solution</u>:

Given data:

q = mud flow rate = 500 gpm
ΔP_{bit} = pressure drop at the bit = 1,200 psi
ρ_m = mud density = 10 ppg

Required data:

d_n = equivalent average diameter of individual nozzle at the bit, *in*

To calculate the mud velocity at the nozzle, Eq. (4.48) is used as:

$$v_n = 33.36\sqrt{\frac{\Delta p_{bit}}{\rho}} = 33.36 \times \sqrt{\frac{1200}{10}} = 365.44\ ft/s$$

Equivalent nozzle area can be calculated using Eq. (4.49) as:

$$A_n = 0.3208\frac{q}{v_n} = 0.3208 \times \frac{500}{365.44} = 0.4389\ in^2$$

Now equivalent average nozzle size at the bit can be calculated using Eq. (4.51) as:

$$d_n = 32\sqrt{\frac{4A_n}{3\pi}} = 32 \times \sqrt{\frac{4 \times 0.4389}{3\pi}} = 13.81\ in$$

Nozzle sizes are normally available in an integer value of $\frac{1}{32}$, i.e. $\frac{13}{32}$, $\frac{14}{32}$, and $\frac{15}{32}$. So we have to choose the sizes as $\frac{13}{32}$, $\frac{13}{32}$, and $\frac{14}{32}$

Cross-check for size:

If we use the sizes as $\frac{13}{32}$, $\frac{13}{32}$, and $\frac{14}{32}$

Equivalent nozzle area can be calculated as:

$$A_T = 2\times\left[\frac{\pi}{4}\times\left(\frac{13}{32}\right)^2\right] + 1\times\left[\frac{\pi}{4}\times\left(\frac{14}{32}\right)^2\right] = 0.4095\ \text{in}^2.$$

This design is ok because this area is less than the design area.

Again:

If we use the sizes as $\frac{13}{32}$, $\frac{14}{32}$, and $\frac{14}{32}$

Equivalent nozzle area can be calculated as:

$$A_T = 1\times\left[\frac{\pi}{4}\times\left(\frac{13}{32}\right)^2\right] + 2\times\left[\frac{\pi}{4}\times\left(\frac{14}{32}\right)^2\right] = 0.5598\ \text{in}^2.$$

This design is not ok because this area is greater than the design area.

Therefore the nozzle sizes of $\frac{13}{32}$, $\frac{13}{32}$, and $\frac{14}{32}$ **are ok**.

4.2.8 Pressure Loss Calculation of the Rig System

In the rig system, the total pressure loss includes surface, and connections pressure losses, pipe pressure losses (i.e., drillstring: drill pipe, drill collar), annular pressure losses, and pressure drop across the bit (Figure 4.3).

The total system pressure losses of the rig system can be calculated by the following equation which is similar to Eq. (4.44):

$$P_{rig} = P_{sp} + P_{dP} + P_{dc} + P_{bn} + P_{ac-h} + P_{ac-cas} + P_{adp-h} + P_{adp-cas} \tag{4.52}$$

Here:

P_{rig} = total pressure loss in the rig system, *psi*
P_{sp} = pressure loss in surface piping, stand pipe and mud hose, *psi*
P_{dp} = pressure loss inside drill pipe, *psi*
P_{dc} = pressure loss inside drill collar, *psi*
P_{bn} = pressure loss across bit nozzle, *psi*
$P_{ac\text{-}h}$ = pressure loss in annulus and the drill collars inside hole, *psi*
$P_{ac\text{-}cas}$ = pressure loss in annulus and the drill collars inside casing, *psi*
$P_{adp\text{-}h}$ = pressure loss around the drill pipe inside hole, *psi*
$P_{adp\text{-}cas}$ = pressure loss around the drill pipe inside casing, *psi*

i) Pipe Flow

The following equations are used to determine pressure loss while Bingham model is used:

The average velocity:

$$\bar{v} = \frac{24.5q}{d_i^2} \tag{4.53}$$

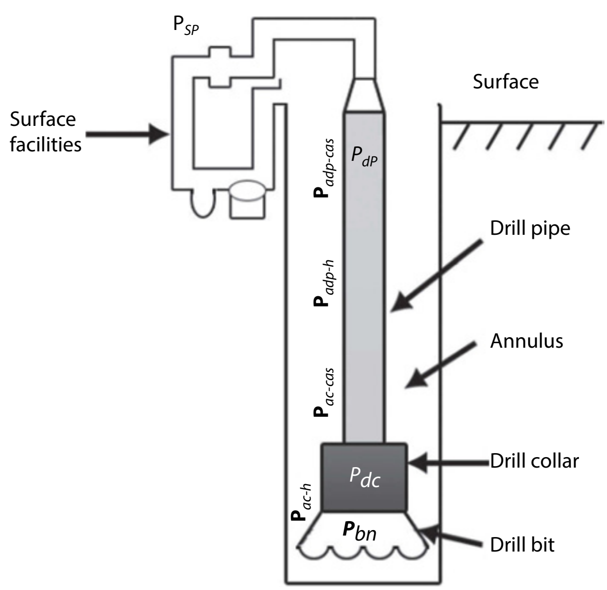

Figure 4.3 Schematic drawing of the circulating system.

where:
$\bar{v}$ = avg. fluid velocity, *ft/min*
q = mud pump rate, *gpm*
d_i = pipe inner diameter, *in*

Again, the critical velocity can be calculated using imperial units as:

$$V_{cB\_i} = \frac{97\mu_{p\_i} + 97\sqrt{\mu_{p\_i}^2 + 8.2\rho_m d_i^2 \tau_{y\_i}}}{\rho_m d_i} \tag{4.54}$$

where:
V_{cB_i} = critical velocity for the Bingham plastic model in imperial unit, *ft*/min
ρ_m = Mud density, $\frac{lb_m}{gal}$ (*ppg*)
d_i = Pipe inner diameter, *in*
τ_{y_i} = Yield point in imperial unit, $\frac{lb_f}{100\,ft^2}$
μ_{p_i} = Plastic viscosity in imperial unit, *cp*

If the flow is laminar (i.e. $\bar{v} < V_{cB\_i}$), the pressure drop can be calculated as:

$$P_{dp} = \frac{L}{300D}\left[\tau_{y\_i} + \frac{\mu_{p\_i}\bar{v}}{5d_i}\right] \tag{4.55}$$

where:
L = length of the drill pipe, *ft*

If the flow is turbulent (i.e., $\bar{v} > V_{cB\_i}$), the pressure drop at the drill pipe can be calculated as:

$$P_{dp} = \frac{8.91\times10^{-5}\rho_m^{\ 0.8} q^{1.8} \mu_{p\_i}^{\ 0.2} L}{d_i^{4.8}} \tag{4.56}$$

ii) Annular Flow

The following equations are used to determine pressure loss while Bingham model is used:

The average velocity:

$$\bar{v} = \frac{24.5q}{d_h^2 - d_{dpo}^2} \tag{4.57}$$

where:
d_h = Hole diameter, *in*
d_{dpo} = Outside diameter of drillpipe, *in*

Again, the critical velocity can be calculated using imperial units as:

$$V_{cB\_i} = \frac{97\mu_{p\_i} + 97\sqrt{\mu_{p\_i}^2 + 6.2\rho_m d_e^2 \tau_{y\_i}}}{\rho_m d_e} \tag{4.58}$$

where:
d_e = Annular distance, $in = d_h - d_{dpo}$ or $d_h - d_{dco}$
d_{dco} = Outside diameter of drill collar, in

If the flow is laminar (i.e., $\overline{v} < V_{cB\_i}$), the pressure drop at the annulus can be calculated as:

$$P_{an} = \frac{L\mu_{p\_i}\overline{v}}{60{,}000D_e^2} + \frac{L\tau_{y\_i}}{200D_e} \tag{4.59}$$

If the flow is turbulent (i.e. $\overline{v} > V_{cB\_i}$), the pressure drop at the annulus (drill pipe) can be calculated as:

$$P_{dp} = \frac{8.91\times10^{-5}\rho_m^{0.8}q^{1.8}\mu_{p\_i}^{0.2}L}{(d_h - d_{dpo})^3(d_h + d_{dpo})^{1.8}} \tag{4.60}$$

Equation (4.60) can be used for drill collar pressure loss with the change of drill pipe diameter (d_{dpo}) by drill collar outside diameter (d_{dco}).

iii) Bit Flow

The pressure drop across the bit nozzle can be calculated using the following equation which is similar to Eq. (4.52):

$$P_{bn} = P_{standpipe} - (P_{dP} + P_{dc} + P_{ac-hole} + P_{ac-cas} + P_{adp-hole} + P_{adp-cas}) \tag{4.61}$$

The pressure loss across the bit is calculated as:

$$P_b = \frac{8.311\times10^{-5}\rho_m q^2}{C_d^2 A_n^2} \tag{4.62}$$

where:
P_b = pressure drop or loss at drill bit, *psi*
ρ_m = mud density in *ppg*
C_d = discharge coefficient which is usually 0.95

The pressure drop across the bit can also be calculated with a 0.95 discharge coefficient as:

$$P_b = \frac{\rho_m q^2}{10858A_n^2} \tag{4.63}$$

The pressure drop across the bit is calculated with a nozzle velocity as:

$$P_b = \frac{\rho_m v_n^2}{1120} \tag{4.64}$$

Bit hydraulic horsepower (BHHP) can be calculated as:

$$HP_b = \frac{qP_b}{1714} \tag{4.65}$$

where:
HP_b = drill bit hydraulic horsepower, *hp*

Now, bit hydraulic horsepower per square inch of the bit can be calculated as:

$$HP_{si} = \frac{HP_b}{\frac{\pi}{4} d_b^2} = 1.273 \frac{HP_b}{d_b^2} \tag{4.66}$$

where:
HP_{si} = bit hydraulic horsepower per square inch of the bit, *hp/in*^{2}
d_b = diameter of the drill bit, *in*

Bit hydraulic power per square inch of the hole drilled can also be calculated as:

$$HP_{si\_h} = \frac{HP_b}{\frac{\pi}{4} d_h^2} = 1.273 \frac{HP_b}{d_h^2} \tag{4.67}$$

where:
HP_{si_h} = bit hydraulic horsepower per square inch of the hole drilled, *hp/in*^{2}
d_h = diameter of the hole, *in*

Example 4.21: While drilling a hole of 13¼" at a depth of 8,000 *ft*, the pump pressure drop is 5,500 *psi*, and total pressure loss is 3,200 *psi*. A 11.5 *ppg* mud is used to achieve a bit hydraulic horsepower per square inch of the hole of 1.2. Calculate the flow rate of the mud where it was assumed C_d = 0.95.

Solution:

Given data:

d_h = diameter of the hole = 13.25 *in*
TVD = Total vertical depth = 8,000 *ft*
P_p = pump pressure drop = 5500 *psi*
P_{fl} = friction pressure losses = 3200 *psi*
ρ_m = mud weight = 11.5 *ppg*
HP_{si_h} = bit hydraulic horsepower per square inch of the hole drilled = 1.2 *hp/in*^{2}

Required data:

q = mud flow rate, *gpm*

The drill bit pressure drop can be calculated using Eq. (4.65) as:

$$P_b = P_p - P_{fl} = 5500 - 3200 = 2300\ psi$$

Now, bit hydraulic horsepower (BHHP) can be calculated using Eq. (4.65) as:

$$HP_b = \frac{q \times 2300}{1714} = 1.342q$$

Bit hydraulic horsepower per square inch of the hole drilled is calculated using Eq. (4.67) as:

$$HP_{si\_h} = 1.273\frac{1.342q}{d_h^{\,2}} = 1.2 \Rightarrow q = 0.702 d_h^{\,2} = 0.702 \times (13.25)^2$$
$$= \mathbf{123.24}\ \boldsymbol{gpm}$$

Example 4.22: Calculate the pressure losses across the different sections of drill pipe and annulus. Use Bingham plastic fluid model where the following data are available. The total vertical depth (TVD) is 10,000 *ft*; the shoes of the casing diameter of 20" and 13-3/8" (ID = 12.565") are set at 1,200' and 4,500' respectively. The hole diameter is 26" and after the second casing shoe is 12.25". A 700' of drill collar with O.D. = 9" and I.D. = 2.875" was set at the bottom of the drillstring while the drill pipe O.D. = 5.5" and I.D. = 4.276". The mud weight is 10 *ppg* with a plastic viscosity of 12 *cp* and yield point is 13 $lb_m/100\,ft^2$. The pump rate for mud discharge was 750 *gpm* and the nozzle velocity is 22,200 *ft/min*.

<u>Solution</u>:

Given data:

Model = Bingham plastic fluid model
TVD = total vertical depth = 10,000 *ft*
$L_{cas_{20"}}$ = total casing length of 20" diameter = 1,200 *ft*
$L_{cas_{13-3/8"}}$ = total casing length of 13-3/8" diameter = 4,500 *ft*
d_{h1} = hole diameter = 26"
d_{h2} = hole diameter after second casing = 12.25"
$d_{cas\text{-}id}$ = second casing ID diameter after second casing = 12.565"
L_{dc} = total drill collar length = 700 *ft*
d_{dco} = outside diameter of drill collar = 9"
d_{dci} = inside diameter of drill collar = 2.875"
L_{dp} = total drill pipe length = (10,000 −700) = 9,300 *ft*
L_{dp-h} = total drill pipe length at the open hole = (10,000 − 4,500 −700) = 4,800 *ft*
d_{dpo} = outside diameter of drill pipe = 5.5"
d_{dpi} = inside diameter of drill pipe = 4.276"
ρ_m = mud density = 10 *ppg*
τ_{y_i} = yield point = $13.0\dfrac{lb_f}{100\,ft^2}$
μ_{p_i} = plastic viscosity = 12.0 *cp*
q = mud flow rate = 700 *gpm*
v_n = nozzle velocity = 22,200 *ft/min* = 370 *ft*/s

Required data:

P_{dp} = pressure loss inside drill pipe, *psi*
P_{dc} = pressure loss inside drill collar, *psi*

P_{bn} = pressure loss across bit nozzle, *psi*
$P_{ac\text{-}h}$ = pressure loss in annulus and the drill collars inside hole, *psi*
$P_{adp\text{-}h}$ = pressure loss around the drill pipe inside hole, *psi*
$P_{adp\text{-}cas}$ = pressure loss around the drill pipe inside casing, *psi*

Figure 4.4 presents the schematic view of the Example 4.22. To calculate the step by step pressure losses, it is necessary first to calculate the average velocity and critical velocity.

Inside drill pipe:
The average velocity for inside drill pipe can be calculated using Eq. (4.53) as:

$$\bar{v} = \frac{24.5q}{d_{dpi}^2} = \frac{24.5 \times (700\,gpm)}{(4.276\ in)^2} = 937.97\ ft/min$$

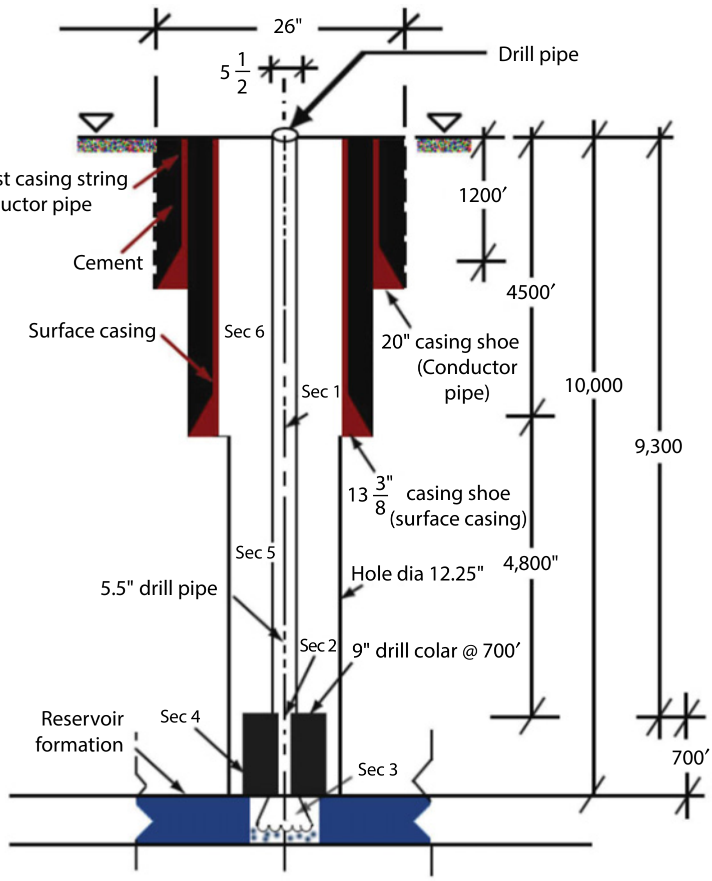

Figure 4.4 Schematic drawing of the casing and cementing system for pressure losses calculations for Example 4.22.

Again, the critical velocity can be calculated using Eq. (4.54) as:

$$V_{cB\_i} = \frac{97\mu_{p\_i} + 97\sqrt{\mu_p^2 + 8.2\rho_m d_{dpi}^2 \tau_{y\_i}}}{\rho_m d_{dpi}}$$

$$= \frac{= 97\times(12cp) + 97\sqrt{(12cp)^2 + 8.2\times(10ppg)\times(4.276\ in)^2 \times\left(13.0\dfrac{lb_f}{100ft^2}\right)}}{(10ppg)\times(4.276\ in)}$$

$$= 345\, ft/min$$

As $\bar{v} > V_{cB\_i}$, the flow is turbulent. Therefore, the pressure drop across the drill pipe (Sec 1 of Figure 4.4) can be calculated using Eq. (4.56) as:

$$P_{dp} = \frac{8.91\times10^{-5}\rho_m^{\ 0.8} q^{1.8} \mu_{p\_i}^{\ 0.2} L_{dp}}{d_{dpi}^{4.8}}$$

$$= \frac{8.91\times10^{-5}\times10^{0.8}\times700^{1.8}\times12^{0.2}\times9,300}{4.276^{4.8}} = \mathbf{1,062.66\ psi}$$

Inside drill collar:
The average velocity for inside drill collar can be calculated using same equation, Eq. (4.53), for drill pipe as:

$$\bar{v} = \frac{24.5q}{d_{dci}^2} = \frac{24.5\times(700gpm)}{(2.875\ in)^2} = 2074.86\, ft/min$$

Again, the critical velocity can be calculated using Eq. (4.54) as:

$$V_{cB\_i} = \frac{97\mu_{p\_i} + 97\sqrt{\mu_p^2 + 8.2\rho_m D_{dci}^2 \tau_{y\_i}}}{\rho_m d_{dci}}$$

$$= \frac{97\times(12cp) + 97\sqrt{(12cp)^2 + 8.2\times(10ppg)\times(2.875\ in)^2 \times\left(13.0\dfrac{lb_f}{100ft^2}\right)}}{(10ppg)\times(2.875\ in)}$$

$$= 359.77\, ft/min$$

As $\bar{v} > V_{cB\_i}$, the flow is turbulent. Therefore, the pressure drop across the drill collar (Sec 2 of Figure 4.4) can be calculated using Eq. (4.56) as:

$$P_{dc} = \frac{8.91\times10^{-5}\rho_m^{\ 0.8} q^{1.8} \mu_{p\_i}^{\ 0.2} L_{dc}}{d_{dci}^{4.8}}$$

$$= \frac{8.91\times10^{-5}\times10^{0.8}\times700^{1.8}\times12^{0.2}\times700}{2.875^{4.8}} = \mathbf{537.68\ psi}$$

Inside drill bit:
Equivalent nozzle area can be calculated using Eq. (4.49) as:

$$A_n = 0.3208\frac{q}{v_n} = 0.3208 \times \frac{700\ gpm}{370\ ft/s} = 0.6069\ in^2$$

Now if we consider $C_d = 0.95$, Eq. (4.63) is used to calculate the pressure loss across the bit nozzle (Sec 3 of Figure 4.4) as:

$$P_{bn} = \frac{\rho_m q^2}{10858\ A_n^2} = \frac{10\,ppg \times (700\,gpm)^2}{10858 \times 0.6069^2} = \mathbf{1225.25}\ \boldsymbol{psi}$$

Around outside drill collars and annulus:
The average velocity for outside drill collar and hole (i.e., annulus) can be calculated using Eq. (4.57) as:

$$\bar{v} = \frac{24.5q}{d_h^2 - d_{dco}^2} = \frac{24.5 \times (700\ gpm)}{(12.25\ in)^2 - (9\ in)^2} = 248.33\ ft/min$$

Again, the critical velocity can be calculated using Eq. (4.58) as:

$$\begin{aligned} V_{cB\_i} &= \frac{97\mu_{p\_i} + 97\sqrt{\mu_{p\_i}^2 + 6.2\rho_m (d_h - d_{dco})^2 \tau_{y\_i}}}{\rho_m (d_h - d_{dco})} \\ &= \frac{97 \times (12cp) + 97\sqrt{(12\,cp)^2 + 6.2 \times (10\,ppg) \times (12.25\ in - 9\ in)^2 \times \left(13.0\frac{lb_f}{100\,ft^2}\right)}}{(10\,ppg) \times (12.25\ in - 9\ in)} \\ &= 313.52\,ft/min \end{aligned}$$

Here, $\bar{v} < V_{cB\_i}$, therefore the flow is laminar. For this type of flow, the pressure drop at the annulus around the drill collar (Sec 4 of Figure 4.4) can be calculated using Eq. (4.59) as:

$$\begin{aligned} P_{ac-h} &= \frac{L_{dc}\mu_{p\_i}\bar{v}}{60{,}000\,D_e^2} + \frac{L_{dc}\tau_{y\_i}}{200\,D_e} \\ &= \frac{(700\,ft) \times (12\,cp) \times (248.33\,ft/min)}{60{,}000 \times (12.25\ in - 9\ in)^2} + \frac{(700\,ft) \times \left(13.0\frac{lb_f}{100\,ft^2}\right)}{200 \times (12.25\ in - 9\ in)} \\ &= \mathbf{17.29}\ \boldsymbol{psi} \end{aligned}$$

Around outside drill pipes and annulus:
There are two parts (Sec 5 and Sec 6) for this calculation.

The average velocity for outside drill pipe and hole (i.e. annulus) can be calculated for Sec 5 using Eq. (4.57) as:

$$\bar{v} = \frac{24.5q}{d_h^2 - d_{dpo}^2} = \frac{24.5 \times (700\ gpm)}{(12.25\ in)^2 - (5.5\ in)^2} = 143.14\ ft/min$$

Again, the critical velocity can be calculated using Eq. (4.58) as:

$$V_{cB\_i} = \frac{97\mu_{p\_i} + 97\sqrt{\mu_{p\_i}^2 + 6.2\rho_m (d_h - d_{dpo})^2 \tau_{y\_i}}}{\rho_m (d_h - d_{dpo})}$$

$$= \frac{97 \times (12cp) + 97\sqrt{(12cp)^2 + 6.2 \times (10ppg) \times (12.25\ in - 5.5\ in)^2 \times \left(13.0 \frac{lb_f}{100\ ft^2}\right)}}{(10ppg) \times (12.25\ in - 5.5 in)}$$

$$= 292.79\ ft/min$$

Here, $\bar{v} < V_{cB\_i}$, therefore the flow is laminar. For this type of flow, the pressure drop at the open hole annulus around the drill pipe (Sec 5 of Figure 4.4) can be calculated using Eq. (4.59) as:

$$P_{adp-h} = \frac{L_{dp-h}\mu_{p\_i}\bar{v}}{60{,}000 D_e^2} + \frac{L_{dp-h}\tau_{y\_i}}{200 D_e}$$

$$= \frac{(4800\ ft) \times (12cp) \times (143.14\ ft/min)}{60{,}000 \times (12.25\ in - 5.5\ in)^2} + \frac{(4800\ ft) \times \left(13.0 \frac{lb_f}{100\ ft^2}\right)}{200 \times (12.25\ in - 5.5\ in)}$$

$$= \mathbf{49.24\ psi}$$

Again, the outside drill pipe and cased hole (i.e., annulus) part (Sec 6) pressure loss can be calculated in the same fashion. The average velocity for outside drill pipe and cased hole (i.e., annulus) can be calculated for Sec 6 using Eq. (4.57) as:

$$\bar{v} = \frac{24.5q}{d_{cas-id}^2 - d_{dpo}^2} = \frac{24.5 \times (700\ gpm)}{(12.565\ in)^2 - (5.5\ in)^2} = 134.37\ ft/min$$

Again, the critical velocity can be calculated using Eq. (4.58) as:

$$V_{cB\_i} = \frac{97\mu_{p\_i} + 97\sqrt{\mu_{p\_i}^2 + 6.2\rho_m (d_{cas-h} - d_{dpo})^2 \tau_{y\_i}}}{\rho_m (d_{cas-h} - d_{dpo})}$$

$$= \frac{97 \times (12cp) + 97\sqrt{(12cp)^2 + 6.2 \times (10ppg) \times (12.565\ in - 5.5\ in)^2 \times \left(13.0 \frac{lb_f}{100\ ft^2}\right)}}{(10ppg) \times (12.565\ in - 5.5\ in)}$$

$$= 292.35\ ft/min$$

Here, $\bar{v} < V_{cB\_i}$, therefore the flow is laminar. For this type of flow, the pressure drop at the cased hole annulus around the drill pipe (Sec 6 of Figure 4.4) can be calculated using Eq. (4.59) as:

$$P_{adp-cas} = \frac{L_{adp-cas}\mu_{p\_i}\bar{v}}{60{,}000 D_e^2} + \frac{L_{adp-cas}\tau_{y\_i}}{200 D_e}$$

$$= \frac{(4500\,ft)\times(12cp)\times(134.37\,ft/min)}{60{,}000\times(12.565\ in - 5.5\ in)^2} + \frac{(4500\,ft)\times\left(13.0\dfrac{lb_f}{100\,ft^2}\right)}{200\times(12.565\ in - 5.5\ in)}$$

$$= \mathbf{43.82}\ \boldsymbol{psi}$$

iv) Pump Calculations

The pump pressure drop can be given as:

$$P_p = P_b + P_{fl} \tag{4.68}$$

where:

P_p = pump pressure drop, *psi*
P_{fl} = frictional pressure losses at different components of the circulating system, *psi*

Example 4.23: While drilling a hole of 12¼" at a depth of 8,000 *ft*, the pump pressure drop is 4,500 *psi*, and total pressure loss is 2,200 *psi*. A 10.5 *ppg* mud is used to achieve a bit hydraulic horsepower per square inch of the hole of 1.2. Calculate the flow rate of the mud where it was assumed $C_d = 0.95$.

Solution:

Given data:

d_h = diameter of the hole = 12.25 in
TVD = total vertical depth = 8,000 *ft*
P_p = pump pressure drop = 4,500 *psi*
P_{fl} = frictional pressure losses = 2,200 *psi*
ρ_m = mud weight = 10.5 *ppg*
HP_{si_h} = bit hydraulic horsepower per square inch of the hole drilled
= 1.2 hp/in^2

Required data:

q = mud flow rate, *gpm*

The drill bit pressure drop can be calculated using Eq. (4.68) as:

$$P_b = P_p - P_{fl} = 4500 - 2200 = 2{,}300\ psi$$

Now, bit hydraulic horsepower (BHHP) can be calculated using Eq. (4.65) as:

$$HP_b = \frac{q\times 2300}{1714} = 1.342\ q$$

Bit hydraulic horsepower per square inch of the hole drilled is calculated using Eq. (4.67) as:

$$HP_{si\_h} = 1.273\frac{1.342q}{d_h^2} = 1.2 \Rightarrow q = 0.702\ d_h^2 = 0.702 \times (12.25)^2$$
$$= \mathbf{105.34}\ \boldsymbol{gpm}$$

4.3 Multiple Choice Questions

1. Liquids where low molecular weight substances exist are called ______________.
   a) Newtonian fluid
   b) Non-Newtonian fluid
   c) Incompressible fluid
   d) None of the above

2. Which of the following is a Newtonian fluid?
   a) Water
   b) Light crude oil
   c) Gases
   d) All of the above

3. The shear stress is directly proportional to shear rate at a constant temperature and pressure for a ______________.
   a) Newtonian fluid
   b) Non-Newtonian fluid
   c) Compressible fluid
   d) In-compressible fluid

4. The linear relationship between shear stress and shear rate is valid only if the fluid moves in ______________.
   a) Confined layers
   b) Confined chambers
   c) Confined spaces
   d) None of the above

5. A fluid that flows in confined layers is said to be ______________.
   a) Laminar flow
   b) Turbulent flow
   c) Transitional flow
   d) All of the above

6. A fluid whose flow behavior or properties is not the same as Newtonian fluid is known as ______________.
   a) Compressible fluid
   b) Non-Newtonian fluid
   c) Incompressible fluid
   d) None of the above
7. A fluid in which the rate of shear is not proportional to the corresponding stress is known as ______________.
   a) Compressible fluid
   b) Non-Newtonian fluid
   c) Incompressible fluid
   d) Newtonian fluid
8. A fluid which cannot be described by a single constant value of viscosity is generally known as ______________.
   a) Compressible fluid
   b) Non-Newtonian fluid
   c) Incompressible fluid
   d) Newtonian fluid
9. Non-Newtonian fluids are classified as
   a) Shear-thickening
   b) Shear thinning
   c) Time-dependent
   d) All of the above
10. A fluid in which apparent viscosity increases with the increase of shear strain rate is known as
    a) Shear-thickening fluid
    b) Shear thinning fluid
    c) Visco-elastic fluid
    d) None of the above
11. A fluid where apparent viscosity decreases with the increase of the rate of shear strain is known as
    a) Visco-plastic fluid
    b) Visco-elastic fluid
    c) Shear thinning fluid
    d) None of the above
12. A shear thinning fluid is also generally known as
    a) Pseudoplastic fluid
    b) Pseudoelastic fluid
    c) Visco-elastic fluid
    d) None of the above

13. A fluid where apparent viscosity decreases with time after the shear rate is increased to a new constant value is known as
    a) Visco-plastic fluid
    b) Visco-elastic fluid
    c) Thixotropic fluid
    d) None of the above

14. A fluid where apparent viscosity increases with time after the shear rate is increased to a new constant value is known as
    a) Visco-plastic fluid
    b) Visco-elastic fluid
    c) Thixotropic fluid
    d) Rheopectic fluid

15. A fluid having a blend of viscous fluid behavior and of elastic solid-like behavior is called ______________.
    a) Visco-plastic fluid
    b) Visco-elastic fluid
    c) Thixotropic fluid
    d) Rheopectic fluid

16. Which of the following is a visco-elastic fluid?
    a) Honey
    b) Water
    c) Crude oil
    d) None of the above

17. Which of the following is a rheological model for non-Newtonian fluids?
    a) Bingham plastic model
    b) Power law model
    c) Shear thinning fluid model
    d) Herschel-Bulkley model
    e) All of the above

18. Which of the following is used to approximate the fluid behavior?
    a) Rheological models
    b) Simulation models
    c) Viscosity
    d) None of the above

19. Fluids that have a linear stress-strain relationship and which require a finite yield stress before they start to flow use ______________.
    a) Power-law model
    b) Bingham plastic model
    c) Shear thinning fluid model
    d) Herschel-Bulkley model

20. Which of the following is **not** an example of Bingham plastic fluid model?
    a) Salt water
    b) Clay suspension
    c) Toothpaste
    d) Mustard

21. Power-law fluids are also known as ______________.
    a) Ostwald-de Waele model
    b) Rutherford model
    c) Shear thinning fluid model
    d) None of the above

22. ______________ is the measure of the thickness of the fluid which is analogous to apparent viscosity of the fluid in power-law model.
    a) Flow consistency index
    b) Shear rate
    c) Flow behavior index
    d) All of the above

23. As the value of flow consistency index increases, the fluid becomes
    a) Thicker
    b) Thinner
    c) Denser
    d) Visco-plastic

24. The power-law model underestimates the ______________ at medium and low shear rate ranges.
    a) Shear stresses
    b) Flow consistency index
    c) Flow behavior index
    d) Viscosity

25. Herschel-Bulkley model is a combination of
    a) Bingham plastic & power law models
    b) Shear thinning & Bingham plastic models
    c) Shear thinning & power law models
    d) None of the above

26. Herschel-Bulkley model is also known as
    a) Yield power law model
    b) Visco-elastic model
    c) Thixotropic model
    d) None of the above

27. If yield point is equal to zero, Herschel-Bulkley model is reduced to
    a) Shear thinning model
    b) Power-law model
    c) Bingham plastic model
    d) None of the above

28. Flow regimes can be classified as
    a) Laminar flow
    b) Transition flow
    c) Turbulent flow
    d) All of the above

29. Laminar flow creates a ____________ flow.
    a) Steady state
    b) Transient flow
    c) Pseudo-steady state
    d) All of the above

30. Turbulent flow creates a ____________ flow.
    a) Steady state
    b) Transient flow
    c) Pseudo-steady state
    d) Unsteady state

31. All the following parameters need to be considered for designing mud flow rate except
    a) Mud weight
    b) Pump capacity
    c) Cuttings transport
    d) Bit hydraulic optimization

32. Gel strength of the mud is important when
    a) Mud is flowing
    b) Mud circulation is stopped
    c) Casing is running inside the well
    d) All of the above

33. Mud A and B have the same type of base fluid; if mud A has higher gel strength than mud B, which one should have poor cleaning efficiency?
    a) Mud with higher gel strength, i.e., mud A
    b) Mud with lesser gel strength i.e., mud B
    c) Both of them have poor cleaning efficiency
    d) Both of them have good cleaning efficiency

34. During the determination of flow regime, which one of the following parameters is not an important factor?
    a) Pipe length
    b) Pipe diameter
    c) Fluid flow rate
    d) Fluid viscosity

35. Which of the following parameters is important in determining the flow regime?
    a) Density of the fluid
    b) Mud type
    c) Mud pump type
    d) All of the above

36. For the same flow rate of a fluid, pressure losses inside the pipe increase when
    a) The pipe diameter decreases
    b) The pipe length increases
    c) The pipe diameter increases
    d) a and b

37. What is the relationship between ECD and EMW when circulation is stopped?
    a) ECD is always greater than EMW
    b) ECD is equal to EMW
    c) ECD is less than EMW
    d) Both a and b

38. At the same flow rate, pressure losses around the drill collars in the intermediate section of the well is
    a) Less than the pressure losses in the production section
    b) Greater than the pressure losses in the production section
    c) Similar to the pressure losses in the production section
    d) None of the above

39. For the same conditions, if the pipe diameter decreases, the tendency of flowing regime occurs from
    a) Laminar to turbulent
    b) Turbulent to laminar
    c) Laminar to transition
    d) All of the above

40. For the same conditions, if the fluid viscosity decreases, the tendency of flowing regime occurs from
    a) Laminar to turbulent
    b) Turbulent to laminar
    c) Turbulent to transition
    d) All of the above

Answers: 1a, 2d, 3a, 4a, 5a, 6b, 7b, 8b, 9d, 10a, 11c, 12a, 13c, 14d, 15b, 16a, 17e, 18a, 19b, 20a, 21a, 22a, 23a, 24a, 25a, 26a, 27b, 28d, 29a, 30d, 31a, 32b, 33b, 34a,35a, 36c, 37b, 38a, 39b, 40a.

4.4 Summary

Drilling hydraulics is one of the most important issues in drilling engineering. This chapter covers almost all aspects of hydraulics. The different types of fluids, models, and flow regimes are discussed elaborately. The pressure loss calculation shows the losses at different parts of a circulating system. The chapter presents almost all the formulas related to the drilling hydraulics. The workout examples and the MCQs are presented in a chronological manner. The exercise solutions are given in Appendix A. Moreover, this chapter added some more MCQs for the student's self-practice and the answers are given in Appendix B. The MCQs covered almost all the materials covered in this chapter.

4.5 Exercise and MCQs for Practice

4.5.1 Exercises (Solutions are in Appendix A)

Exercise 4.1: A moving plate is positioned 2.5 *cm* above a stationary plate. A force of 308 dynes is required to initiate the first movement, and forces of 587 and 673 dynes are required to move the plate at uniform velocities of 9.4 and 12.3 *cm/s*, respectively. Calculate the cross-sectional area of the upper plate and the plastic viscosity. ***Answers: 275 cp, 27 cm²***

Exercise 4.2: A moving plate of a cross-sectional area of 40 cm^2 is positioned above a stationary plate. The instrument read a force of 790 dynes when the plate was moving at a uniform speed of 11.25 *cm*/sec, and a force of 930 dynes when it was moving at a uniform speed of 14.75 *cm*/sec. If the force required to initiate the movement is 340 *dynes*, calculate the distance between the two plates and the plastic viscosity. ***Answers: 3.25 cp, 3.25 cm***

Exercise 4.3: A moving plate of a cross-sectional area "A" cm^2 is positioned "*L*" *cm* above a stationary plate. A force which is equal to 1.5 F_y is required to move the plate at a uniform velocity of "v_1" *cm/s*, and a force which is equal to 1.75 F_y is needed to move the plate at a uniform speed of "v_2" *cm/s*. Find the relation between the two speeds. F_y dynes is the force needed to initiate the first movement. ***Answer: 0.667***

Exercise 4.4: A power low fluid has a power low exponent of 0.86 reads shear rate of "γ" at shear stress of "τ". If the fluid reads another shear stress of "2τ", find out the relation of apparent viscosities for the two cases. ***Answer: 1.12***

Exercise 4.5: Two viscosity measuring instruments have moving plates of cross-sectional areas of 26 and 37 cm^2 and distance of 2.25 and 3.5 *cm* from their stationary plates, respectively. The first instrument reads a force of 390 dynes at speed of 10.27 *cm*/sec for a certain fluid. What should the second instrument read for the same fluid to come up with the same apparent viscosity? Assume negligible measuring errors for both instruments. ***Answers: 555 dynes, 15.98 cm/sec***

Exercise 4.6: Two apparent viscosity measurements were done for a power low fluid. If the ratio of the first apparent viscosity to the second one is 0.97 and the shear rate ratio is 1.2, calculate the flow behavior index of this fluid. ***Answer: 0.83***

Exercise 4.7: A power low fluid measured 61 and 105 degrees at speeds of 300 and 600 *rpm*, respectively. Viscosity of the same fluid was measured using another viscometer has an upper plate of cross-sectional area of 34 cm^2 and distance of 2.5 *cm* from the stationary plate. What force should the instrument reads if the upper plate speed was 8 *cm*/sec. ***Answer: 198 dynes***

Exercise 4.8: A power low fluid measured 34 and 66 degrees at speeds of 150 and 400 *rpm*, respectively. Viscosity of the same fluid was measured using another viscometer has an upper plate of cross-sectional area of 41 cm^2 and distance of 2 *cm* from the stationary plate. What is the speed of the upper plate that would give a force reading of 548 dynes? ***Answer: 11.4 cm/sec***

Exercise 4.9: An intermediate section is drilled with 12.25" bit and 5.5" drill pipes that have 5.0" inside diameter using drilling mud that has MW of 9.3 *ppg* and viscosity of 38 cp. If the rig has two pumps each pump deliver 7.5 gals/stroke and run at 75 *spm*, determine the flow regime inside the drill pipes. ***Answer: turbulent flow***

Exercise 4.10: Fresh water is planned to be pumped in a certain pipe at constant pumping rate of 6.5 *gpm*. If water density and viscosity are 8.34 *ppg* and 1.0 *cp*, what is the minimum pipe inside diameter that make the fluid flow behave as turbulent flow? ***Answer: 9.79***"

Exercise 4.11: A fluid is pumped in the annulus between two pipes at pumping rate of 300 *gpm*. Inside diameter of the outer pipe is 8.5", and outside diameter of the inner pipe is 7.0".If the fluid density is 8.8 *ppg* and fluid viscosity is 7.0 *cp*, what is the flow behavior? ***Answer: Turbulent flow***

Exercise 4.12: A well with a 9 5/8" casing of 8.88" inside diameter, and 5½" tubing of 5.0" inside diameter. KCl brine of 9.43 *ppg* was pumped inside the tubing and fresh water was in the annulus between the casing and the tubing. Down-hole valve was placed at depth of 9,250 *ft*. When the down-hole valve was opened and connected the tubing and annulus, some amount of water flowed out of the annulus before the annulus valve was closed. Casing shut in pressure was closed and stabilized at 495 *psi*. What is the level of the KCl brine that entered the annulus due to difference in hydrostatic pressure, and what is the volume of water that been replaced by the KCl brine? Note that tubing is always full of KCl brine and there is no pressure build up at the surface of the tubing. ***Answer: 516.8 ft, 23.7 bbls***

Exercise 4 13: A well drilled to a depth of 3000 *ft* using a certain drilling mud that gave a total annular pressure losses of 180 *psi* when circulated at a certain circulating condition. The drilling mud has been diluted with water, and a new length of 500 *ft* has been drilled using the new drilling mud. The total annular pressure losses at the current depth were estimated to be180 *psi*. If the difference between the old ECD to the current ECD is 0.36, what is the difference between the two mud densities? ***Answer: 0.2 ppg***

Exercise 4.14: An intermediate hole has been drilled using 12" bit that has 4 nozzles of $^{16}/_{32}$" each. Current depth of the well is 9,000 *ft* and drilling mud properties are mud weight of 11.8 *ppg*, yield point of 14.0 lb_f/ 100 ft^2 and viscosity of 15.0 cp. Last casing of 13 3/8" with inside diameter of 12.43 was set at depth of 6,500 *ft* all the way to the surface. Drill string consists of 450 *ft* of DCs of OD 9.0" and ID of 3.0; and the rest are DPs of OD 5.5" and ID of 4.13". If the pump flow rate is 750 *gpm*, calculate the minimum required pump pressure that can overcome all the system pressure losses. ***Answer: 3,041 psi***

Exercise 4.15: A production section of 8½" hole size is drilled down to the depth of 15,000 *ft*, and the last casing shoe was at 11,250 *ft* and inside diameter of 8.88". Drilling string consists of 725 *ft* drill collars of 6.5" OD and 2.5" ID; and the rest are drill pipes of 5.0" OD and 4.28" ID. Drilling fluid that is used has MW of 11.4 *ppg*, viscosity of 21.0 *cp* and yield point of 18.0 lb_f/100 ft^2. Drilling hydraulics calculations showed that pumping rate and pumping pressure should be 550 *gpm* and 3,600 *psi*, respectively. If it is planned to have 3 nozzles in the bit with equal sizes, what will be the size of each nozzle? ***Answer:*** $^{16}/_{32}$"

4.5.2 Exercises (Self-Practices)

E4.1: Calculate the shear stress of a fluid which has a viscosity of 60 *cp* and have a shear rate of 10 s^{-1}.

E4.2: Calculate the viscosity of a fluid if the shear stress is 15 *Pa* and the shear rate is 12 s^{-1}.

E4.3: A thin movable plate is set 2.5 *cm* above a stationary plate of having a cross-sectional area of 30 cm^2. If a force of 300 dynes is required to just initiate the upper plate and a force of 650 *dynes* is needed to move the plate with a uniform velocity of 7 *cm/s*, calculate the yield point and plastic viscosity of the fluid.

E4.4: A Fann V-G meter is used to measure the viscosity of the Bingham fluid and the following Fann data was found: $\theta_{300} = 35$; $\theta_{600} = 65$, and $\theta_{200} = 25$; $\theta_{400} = 55$. Calculate the plastic viscosity, and yield point of the fluid using Bingham plastic model.

E4.5: A moving plate is set 3 *cm* above a stationary plate which has a cross-sectional area of 20 cm^2. Calculate the consistency index and flow-behavior index if a force of 270 dyne is required to move the upper plate at a constant velocity of 9 *cm/s* and a force of 350 dyne is needed to move the plate with a uniform velocity of 12 *cm/s*.

E4.6: In the drilling fluid laboratory, a technician was observing the Fann V-G meter an experiment for the Bingham fluid where he was using the Fann V-G meter to measure the viscosity of the fluid. He found the following Fann data: $\theta_{300} = 30$; $\theta_{600} = 55$, and $\theta_{200} = 27$; $\theta_{400} = 49$. Calculate the consistency index and flow-behavior index for the power-law model.

E4.7: While drilling, a 12.0 lb_m/gal of mud having a viscosity of 1.3 *cp* was being circulated through drillstring at a rate of 600 *gal/min*. If the internal diameter of the drillpipe is 5.5 *in*, determine the type of flow in the drillpipe of the circulating system.

E4.8: While drilling, a 13.5 lb_m/gal of mud is used where Fann data was observed as $\theta_{300} = 26$; $\theta_{600} = 48$. The target depth was set at 10,500 *ft* (TVD). If the internal diameter of the drillpipe is 4.75 in, calculate the critical velocity inside the pipe and the critical flow rate.

E4.9: An intermediate casing string was cemented using the following muds: first section 9,000ft was filled by 10.5 lb_m/gal mud, second section of 1,300ft was filled by 13.3 lb_m/gal mud and the last section was filled by 15.5 lb_m/gal mud. Calculate the total hydrostatic pressure at 11,500 *ft*. Convert the pressure at 11,500 *ft* to an equivalent mud weight and determine if it will exceed the fracture gradient of 14.2 lb_m/gal. Also calculate the ECD for an annular pressure loss gradient of 0.046 *psi*/ft and an original mud weight of 12.0 *ppg*.

E4.10: The tubing of a well filled with sour gas (SO_2) to a vertical depth of 14,700 *ft*. The annular space is filled with a 11.5 lb_m/gal mud. Assume that the gas follows the ideal gas behavior. Calculate the amount by which the exterior pressure on the tubing exceeds the interior tubing pressure at 14,700 *ft*. The surface tubing pressure is 1,000 *psi*a and the mean temperature is 150^{0}F. If the collapse resistance of the tubing is 9,500 *psi*, will the tubing collapse due to the high external pressure? Justify your answer.

E4.11: Calculate the total pressure required to discharge a 12 *ppg* mud through the drilling circulating system. Use the following data for the mud pump pressure requirement. Here:

Pressure loss in surface piping, stand pipe and mud hose = 80 *psi*
Pressure loss inside drill pipe = 2200 *psi*
Pressure loss inside drill collar = 350 *psi*
Pressure loss across bit = 95 *psi*
Pressure loss in annulus in the drill collars = 460 *psi*
Pressure loss in annulus in the drill pipe = 3,800 *psi*

E4.12: You are drilling a 12-1/4" hole at 8,000 *ft* using 5.0" OD × 4.276" ID drill pipe and 800 *ft* of 8.0" × 2–3/4" drill collars. The well has 13–3/8", 68 *lbs/ft* casing (ID 12.415") set at 6000 *ft*. Mud is pumped down the drill pipe at the rate of 630 *gal/min*. Mud properties are: density = 12.0 *ppg*, viscosity = 14 *cp*, yield point = 20 *lb*/100 *sq-ft*. The bit has three 0.50" ID nozzles.

1. Calculate the pressure loss in the drill pipe.
2. Calculate the pressure loss in the drill collars.
3. Calculate the pressure loss across the bit.
4. Calculate the pressure loss across the drill collar and open hole annulus.
5. Calculate the pressure loss across the drill pipe and open hole annulus.
6. Calculate the pressure loss across the drill pipe and cased hole annulus.
7. Calculate the horse power of the mud pump required to pump the mud at 630 gpm.
8. Calculate the pressure at the inlet and the outlet of the pump.

E4.13: You need to find out the nozzle sizes of a tri-cone bit which will be used for an immediate drilling operation. The available data needs to be used are 550 *gpm* of mud circulation at a pressure drop of 1285 *psi* through the bit, and the mud density is 11 *ppg*.

E4.14: Calculate the pressure losses across the different sections of drill pipe and annulus. Use Bingham plastic fluid model where the following data are available. The total vertical depth (TVD) is 18,350 *ft*; the shoes of the casing diameter of 22 in and 13-3/8 in (ID = 12.565 in) are set at 1,500 *ft* and 5,500 *ft* respectively. The hole diameter is 30 *in* and after the second casing shoe is 12.25 in. A 900 *ft* of drill collar with O.D. = 9.5 *in* and I.D. = 2.875 *in* was set at the bottom of the drillstring while the drill pipe O.D. = 5.5 in and I.D. = 4.276 in. The mud weight is 10.5 *ppg* with a plastic viscosity of 11.8 *cp* and yield point is 13.5 $lb_m/100\ ft^2$. The pump rate for mud discharge was 860 *gpm* and the nozzle velocity is 21,860 *ft/min*.

E4.15: While drilling a hole of 9 7/8 *in* at a depth of 8,880 *ft*, the pump pressure drop is 3,800 *psi*, and total pressure loss is 2,450 *psi*. A 12.5 *ppg* mud is used to achieve a bit hydraulic horsepower per square inch of the hole of 1.3. Calculate the flow rate of the mud where it was assumed $C_d = 0.90$.

4.5.3 MCQs (Self-Practices)

1. Drilling hydraulics can be described as
   a) The static behavior of the fluids
   b) Mass transfer between fluids
   c) Dynamic behavior of the fluids
   d) Both a and c
2. Drilling hydraulics is important for
   a) Well safety
   b) Calculating well cost
   c) Calculating oil reserves
   d) All of the above
3. Incorrect hydraulics calculation may result in
   a) Slow ROP
   b) Improper hole cleaning
   c) Lost circulation
   d) All of the above
4. Most of the known fluids are
   a) Newtonian fluids
   b) Non-Newtonian fluids
   c) Low viscosity fluids
   d) None of the above

5. Example of Newtonian fluids is
   a) Crude oil
   b) Honey
   c) Water
   d) All of the above
6. Newtonian fluid is the fluid that has
   a) Constant apparent viscosity
   b) Constant density
   c) Constant gel strength
   d) All of the above
7. Which one of the following examples is the non-Newtonian fluid?
   a) Light crude oil
   b) Gases
   c) Water
   d) Heavy crude oil
8. Shear thinning fluid is the fluid where its apparent viscosity
   a) Decreases with the increase of shear strain rate
   b) Increases with the increase of shear strain rate
   c) Increases with the decrease of shear strain rate
   d) None of the above
9. If a fluid's apparent viscosity increases with time at a constant shear rate, it is called:
   a) Thixotropic fluid
   b) Reopectic fluid
   c) Viscoelastic fluid
   d) None of the above
10. If a fluid's apparent viscosity decreases with time at a constant shear rate, it is called:
    a) Thixotropic fluid
    b) Viscoelastic fluid
    c) Reopectic fluid
    d) None of the above
11. Bingham plastic fluid is a fluid that has
    a) Logarithmic stress strain relationship
    b) Curve stress strain relationship
    c) Linear stress-strain relationship and the line pass through the origin point
    d) Linear stress-strain relationship
12. The force that is required to move the Bingham fluids from static condition is called
    a) Gel strength
    b) Yield point
    c) Shear stress
    d) None of the above

13. All of the following are the types of power-law fluids except
    a) Pseudoplastic fluids
    b) Dilatant fluids
    c) Shear thickening fluids
    d) All of the above

14. The constant "*K*" in the power-law model is the measure of
    a) The fluid thickness
    b) The required force to start the flow
    c) The fluid carrying capacity
    d) All of the above

15. The constant "*n*" in the power-law model is the measure of
    a) The fluid thickness
    b) The fluid carrying capacity
    c) The degree of non-Newtonian behavior
    d) All of the above

16. When the fluid flows in a parallel layers, this flow is called
    a) Turbulent flow
    b) Laminar flow
    c) Transition flow
    d) Slug flow

17. The flow regime in which the velocity is not same in the center and walls of the pipe is called:
    a) Turbulent flow
    b) Laminar flow
    c) Transition flow
    d) None of the above

18. Which of the following flow regimes create more erosion to the walls of the well?
    a) Turbulent flow
    b) Transition flow
    c) Laminar flow
    d) All of the above

19. Turbulent flow got its name because
    a) The fluid moves fast but in layered form
    b) The fluid has high viscosity
    c) The fluid has high yield point
    d) The fluid flow in all directions

20. Transition flow can be defined as the
    a) Turbulent flow at the tubing walls and laminar at the center of the pipe
    b) Turbulent flow but uniform velocity
    c) Turbulent flow at the center of the pipe and laminar flow near the walls
    d) Laminar flow but uniform velocity

21. A fluid is flowing through pipe and it has the Reynolds number of 2,000. What is its flow regime?
    a) Laminar flow
    b) Transition flow
    c) Turbulent flow
    d) None of the above
22. A fluid is flowing through pipe and it has the Reynolds number of 9,500. What is its flow regime?
    a) Laminar flow
    b) Transition flow
    c) Turbulent flow
    d) None of the above
23. A fluid is flowing through pipe and it has the Reynolds number of 3,500. What is its flow regime?
    a) Laminar flow
    b) Transition flow
    c) Turbulent flow
    d) None of the above
24. For safety reasons during hydraulic calculations, transition flow is better classified as
    a) Laminar flow
    b) Turbulent flow
    c) Plug flow
    d) None of the above
25. The hydrostatic pressure of the mud is a function of
    a) Drilling mud density
    b) Well depth
    c) Well trajectory
    d) All of the above
26. A fluid flows at a velocity of 100 *ft*/sec; what is the flow regime?
    a) Laminar flow
    b) Turbulent flow
    c) Transition flow
    d) Not enough information to define the regime
27. A fluid flows at a velocity of 10 *ft*/sec: what is the flow regime?
    a) Laminar flow
    b) Turbulent flow
    c) Transition flow
    d) None of the above

28. What is the difference between ECD and EMW?
    a) There is no difference
    b) ECD considers the hydrostatic pressure while EMW does not
    c) ECD considers friction pressure due to flow while EMW does not
    d) ECD considers the depth of the well while EMW does not

29. What is the relationship between ECD and EMW during mud circulation?
    a) ECD is always greater than EMW
    b) ECD is equal to EMW
    c) ECD is less than EMW
    d) Both a and c

30. Most of the system pressure is consumed in
    a) Overcoming the pressure losses across the drill collars
    b) The bit nozzles
    c) Inside the drill string
    d) None of the above

31. Experimental studies show that ROP decreases with
    a) The decrease in the number of lobes of the motor
    b) The increase in the number of lobes of the motor
    c) Does not change at different number of lobes of the motor
    d) None of the above

32. Down-hole motors are used to
    a) Speed up the drilling rate of vertical wells
    b) Optimize drilling of deviated wells
    c) Drill the extended-reach wells
    d) All of the above

33. What is the main advantage of using aerated mud?
    a) Reduce mud cost
    b) Eliminates lost circulation
    c) Eliminate blow outs
    d) All of the above

34. Critical foam velocity is defined as the
    a) Maximum velocity to transport drill cuttings
    b) Minimum velocity inside the drill collars
    c) Minimum velocity to lift drill cuttings
    d) None of the above

35. What is the benefit of using foam mud?
    a) Increase well productivity
    b) Increase ROP
    c) Reduce stimulation requirements
    d) All of the above

36. For the same pipe specifications, what is the relationship between pressure losses due to friction for the normal drill pipes and coiled tubing drilling?
    a) Friction losses are higher in normal drill pipes
    b) Friction losses are higher in coiled tubing
    c) Friction is similar in both types
    d) All of the above
37. Dual gradient drilling is useful because it
    a) Reduces well cost
    b) Reduces hydrostatic pressure against the formation
    c) Reduces number of casings in the well
    d) Both b and c
38. What is the main benefit of Managed Pressure Drilling (MPD)?
    a) Reduce well cost
    b) Enlarge drilling window
    c) Increase ROP
    d) All of the above
39. What parameter should be considered when designing the mud flow rate?
    a) Mud density
    b) Well capacity
    c) Cuttings transport
    d) All of the above
40. At same conditions, increasing the pipe length tends to change the flow regime
    a) From laminar to turbulent
    b) From turbulent to laminar
    c) From transition to turbulent
    d) None of the above

4.6 Nomenclature

A	=	inner cross-sectional area of the fluid column, in^2
a	=	parameter in Carreau–Yasuda model, dimensionless
C	=	conversion constant
C_d	=	discharge coefficient which is usually 0.95
d_i	=	pipe inner diameter, *cm*, in.
d_h	=	hole diameter, in
d_{dpo}	=	outside diameter of drillpipe, in
d_e	=	annular distance, $in = d_h - d_{dpo}$ or $d_h - d_{dco}$
d_{dco}	=	outside diameter of drill collar, in
D_m	=	measured depth, *ft*
f	=	fanning friction factor
F_{down}	=	downward force on the fluid element applied by the fluid column above, lb_f
F_{up}	=	upward force on the fluid element applied by the below fluid column, lb_f
F_{self}	=	fluid element's self-weight acting as a downward force, lb_f

HP_b = drill bit hydraulic horse power, hp
K = flow consistency index, $Pa.s^n$
k = reservoir permeability, m_2
L = length of the drillpipe, ft
L_i = length for the section of interest (which is part of L_{tvd})
L_{tvd} = total vertical depth, ft
m = mass of gas
M = gas molecular weight
n = mole of gas = $\frac{m}{M}$
N_1, N_2 = the Fann rpm reading
p = pressure of the system, $psig$, N/m^2
p_a = Absolute pressure
P_b = pressure drop or loss at drill bit, psi
p_o = surface pressure at $L_{tvd} = 0$ which is also the constant of the integral.
p_t = total hydrostatic pressure
$P_{ac\text{-}h}$ = pressure loss in annulus and the drill collars inside hole, psi
$P_{ac\text{-}cas}$ = pressure loss in annulus and the drill collars inside casing, psi
$P_{adp\text{-}h}$ = pressure loss around the drill pipe inside hole, psi
$P_{adp\text{-}cas}$ = pressure loss around the drill pipe inside casing, psi
P_{bn} = pressure loss across bit nozzle, psi
P_{dc} = pressure loss inside drill collar, psi
P_{dp} = pressure loss inside drill pipe, psi
P_{rig} = total pressure loss in the rig system, psi
P_{sp} = pressure loss in surface piping, stand pipe and mud hose, psi
$\bar{v}$ = avg. fluid velocity, ft/min
$\frac{dp}{dL_{tvd}}$ = pressure gradient with respect to total vertical depth, $psig/ft$
q = circulating volume or mud pump rate, cc/s, gal/min
Q_{cB} = critical flow rate for the Bingham plastic model, gpm
R = universal gas constant
t = time, s
T = absolute temperature
u_x = fluid velocity in porous media in the direction of x axis, m/s
V = gas volume
$\bar{v}$ = avg. fluid velocity, cm/s
$\bar{v}$ = avg. fluid velocity, ft/s = $\frac{q}{2.448\ d_i^2}$
V_{cB} = critical velocity for the Bingham plastic model, ft/s
V_{cB_i} = critical velocity for the Bingham plastic model in imperial unit, ft/min
W_{sp} = specific weight of fluid, $lb_f/in^2\text{–}ft$
W_{sp} = specific weight of fluid, $lb_f/in^2\text{–}ft$
Z = universal gas constant
ϕ = porosity of fluid media, m^3/m^3
a = fractional order of differentiation, dimensionless
t = shear stress, Pa

τ_y = a minimum shear stress that needs to initiate fluid flow, *Pa*
μ_{p_i} = plastic viscosity in imperial unit, *cp*
τ_{y_i} = yield point in imperial unit, $\frac{lb_f}{100\,ft^2}$
r = fluid density, gm/cc, lb_m/gal
ρ_m = mud density, *ppg*
ρ_i = mud weight for the section of interest
ρ_{om} = original mud density, *ppg*
μ_d = dynamic viscosity of fluid, *Pa-s*
μ_p = Bingham plastic viscosity, *Pa-s*
$\frac{du_x}{dy}$ or γ = the velocity gradient perpendicular to the direction of shear, or equivalently the strain rate, s^{-1}
τ_0 = yield point stress at dial reading at 3 *rpm*
θ_{N_1}, θ_{N_2} = The Fann dial reading at N_1 and N_2 *rpm*
θ_3 = the Fann dial reading at 3 *rpm*
θ_{300} = the Fann dial reading at 300 *rpm*
θ_{600} = the Fann dial reading at 600 *rpm*
$\gamma = \frac{du_x}{dy}$ = shear rate or velocity gradient perpendicular the plan of shear, s^{-1}
n_p = power-law exponent or flow behaviour index, dimensionless
α_{SF} = shape factor which is medium-dependent
γ_{pm} = apparent shear rate within the porous medium, s^{-1}
n_c = power-law exponent for Carreau–Yasuda model, dimensionless
n_w = number of wellbore sections
μ_{eff} = fluid effective viscosity, *Pa–s*
μ_m = mud density, *ppg*
μ_0 = fluid dynamic viscosity at zero shear rate, *Pa–s*
μ_∞ = fluid dynamic viscosity at infinite shear rate, *Pa–s*
λ = time constant in Carreau–Yasuda model, *s*
η = ratio of the pseudopermeability of the medium with memory to fluid viscosity, $m^3 s^{1+\alpha}/kg$
ξ = a dummy variable for time i.e. real part in the plane of the integral, *s*
ΔL_{tvd} = differential total vertical depth, *ft*
ΔP_p = pump discharge pressure, *psi*
ΔP_{ac} = pressure loss in annulus in the drill collars, *psi*
ΔP_{an} = annular pressure loss, *psi*
ΔP_{ap} = pressure loss in annulus in the drill pipe, *psi*
ΔP_{bn} = pressure loss across bit nozzle, *psi*
ΔP_{dc} = pressure loss inside drill collar, *psi*
ΔP_{dp} = pressure loss inside drill pipe, *psi*
$\boldsymbol{\Delta P_{Lf}}$ = laminar flow pressure drop, *psi*
ΔP_{sp} = pressure loss in surface piping, stand pipe and mud hose, *psi*
$\boldsymbol{\Delta P_{tf}}$ = turbulent flow pressure drop, *psi*

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5

Well Control and Monitoring Program

5.1 Introduction

Well control and monitoring system is an integrated part of the drilling operations. Well control means an assurance of formation fluid (oil, gas or water) that does not flow in an uncontrolled way from the formations being drilled, into the borehole and eventually to the surface. It prevents the uncontrolled flow of formation fluids ("kick") from the wellbore. Hence, a kick can be defined as an unexpected entry of formation fluid(s) into the wellbore, causing a rise of mud-level in the mud pit. Therefore controlling of the well is an important issue in any drilling activity. This chapter addresses the different well control and monitoring program through sets of multiple choice question (MCQs). Workout examples related to well control are extensively covered. Workout examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

5.2 Different Mathematical Formulas and Examples

5.2.1 Control of Influx and Kill Mud

i) Shut-in Pressure:

Mathematically the drillpipe shut-in pressure can be interpreted as the amount by which bottom-hole pressure exceeds the hydrostatic mud pressure which is expressed as:

$$P_{sidp} + G_m H_{vc} = P_{bh} \tag{5.1}$$

where,

P_{sidp} = shut-in drillpipe pressure, *psi*
G_m = mud pressure gradient, *psi/ft*
H_{vc} = total vertical height of the mud column, *ft*
P_{bh} = bottom-hole (i.e., formation) pressure, *psi*

In terms of mud weight, formation pressure can be calculated as:

$$P_{bh} = P_{sidp} + 0.052\ \rho_{om} H_{vc} \tag{5.2}$$

where,

ρ_{om} = original mud weight, *ppg*

If we consider the annulus side, the bottom-hole pressure can be calculated as equal to the surface annulus pressure plus the combined hydrostatic pressure of the mud and influx. Mathematically the expression can be written as:

$$P_{siann} + G_i H_i + G_m H_m = P_{bh} \tag{5.3}$$

where,

P_{siann} = shut-in annulus pressure, *psi*
G_i = influx pressure gradient, *psi/ft*
H_i = vertical height of the influx or kick, *ft*
H_m = vertical height of mud in the annulus after influx, *ft* = $H_{vc} - H_i$

H_i can be calculated from the displaced volume of mud measured at surface (i.e., the pit gain) and the cross-sectional area of the annulus, i.e.:

$$H_i = \frac{V_{pit}}{A_{ann}} \tag{5.4}$$

where,

V_{pit} = pit gain volume, *bbls*
A_{ann} = cross-sectional area of the annulus, *bbls/ft*

Initial circulating pressure is calculated as:

$$P_{ic} = P_{sidp} + P_p + P_{ok} \tag{5.5}$$

where,

P_{ic} = initial circulating pressure, *psi*
P_p = slow circulating pump pressure, *psi*
P_{ok} = overkill pressure, *psi*

Final circulating pressure is calculated as:

$$P_{fc} = P_p \left(\frac{\rho_{km}}{\rho_{om}} \right) \tag{5.6}$$

where,

P_{fc} = final circulating pressure, *psi*
ρ_{km} = kill mud weight, *ppg*

Example 5.1: A vertical appraisal well is drilled to a target reservoir at 12,500 *ft* that has a formation gradient of 0.5500 *psi/ft*. Calculate the following:

i. Formation pressure in *psi* and formation gradient in *ppg*.
ii. If the mud weight must be greater than the formation gradient by 0.5 *ppg*, calculate the mud weight in *ppg* and mud hydrostatic pressure in *psi*.
iii. While drilling to the target depth, shut-in pressure inside the drill pipe was increased to 250 *psi*. The well was controlled, and later the driller realized that the formation has been shifted up due to tectonic movements with a new gradient of 0.5978. What is the new depth of the target formation at this area?

Solution:

Given data:

D_m = Well target depth = 12,500 *ft*
G_f = Formation gradient = 0.5500 *psi/ft*
P_{sidp} = Shut in annulus pressure = 250 *psi*
G_{f_new} = New formation gradient = 0.5978 *psi/ft*

Required data:

P_f = Formation pressure in *psi*
G_f = Formation gradient in *ppg*
ρ_m = Mud weight in *ppg*
P_m = Mud hydrostatic pressure in *psi*
D_{m_new} = New formation depth in *ft*

i. Formation pressure can be calculated using the equation:

$$P_f = G_f \times D_m = 0.55 \times 12{,}500 = \mathbf{6{,}875}\ \boldsymbol{psi}$$

Formation gradient in *ppg* can be calculated using the Eq. (4.36):

$$G_f = \frac{P_f}{0.052 \times D_m} = \frac{6{,}875}{0.052 \times 12{,}500} = \mathbf{10.58}\ \boldsymbol{ppg}$$

ii. The mud weight is equal to:

$$\rho_m = \rho_f + 0.5 = 10.58 + 0.5 = \mathbf{11.08}\ \boldsymbol{ppg}$$

(a factor of safety is considered as 0.5 *ppg*)
Now, the mud hydrostatic pressure can be calculated using Eq. (4.35):

$$P_m = 0.052 \times \rho_m \times D_m = 0.052 \times 11.08 \times 12{,}500 = \mathbf{7{,}200}\ \boldsymbol{psi}$$

iii. Shut-in drill pipe pressure increased due to shifting of the formation to a shallower depth. So the formation will have similar pressure of 6,875 *psi* as can be calculated by adding the shut-in pressure to the mud hydrostatic pressure. Using the new calculated formation gradient, new formation depth will be equal to:

$$D_{m\_new} = \frac{P_f}{G_{f_{new}}} = \frac{6,875}{0.5978} = \mathbf{11,500.5}\ \boldsymbol{ft}$$

So, the formation has been shifted up by 1,000 *ft*.

Example 5.2: A $8\frac{1}{2}''$ diameter hole is drilled up to 7,500 *ft* with a density of 12.5 *ppg*. If the formation pore pressure at this point is 4500 *psi*. Calculate i) mud pressure overbalance above the pore pressure, ii) if the mud density is 10.5 *ppg*, what would be the overbalance, and iii) if the fluid level in the annulus is dropped to 250 *ft* due to inadequate hole fill up during tripping, what would be the effect on bottom-hole pressure?

Solution:

Given data:

H_{vc} = total vertical height of the mud column = 7,500 *ft*

d_h = hole diameter = $8\frac{1}{2}''$

ρ_{om1} = original mud weight 1 = 12.5 *ppg*

P_f = formation pore pressure = 4500 *psi*

ρ_{om2} = original mud weight 2 = 10.5 *ppg*

H_{ann} = vertical height of the mud column in the annulus = 250 *ft*

Required data:

i. P_{ob1} = mud pressure overbalance at 7,500 *ft*

ii. P_{ob2} = mud pressure overbalance at 7,500 *ft* if mud density is 10.5 *ppg*

iii. Effect on bottom-hole pressure?

The overbalance at a depth of 7,500 *ft* can be calculated by Eq. (4.34a) which can be modified for overbalance as:

$$\begin{aligned} P_{ob1} &= 0.052\rho_{om1}H_{vc} - P_f \\ &= 0.052 \times (12.5\ ppg) \times (7500\ ft) - 4500\ psi = \mathbf{375}\ \boldsymbol{psi} \end{aligned}$$

The overbalance at a depth of 7,500 *ft* if mud density is 10.5 *ppg* as:

$$\begin{aligned} P_{ob2} &= 0.052\rho_{om2}H_{vc} - P_f \\ &= 0.052 \times (10.5\ ppg) \times (7500\, ft) - 4500\, psi = \mathbf{-405}\ \boldsymbol{psi} \end{aligned}$$

If the mud density is decreased, the negative sign implies that the well would be underbalanced by 405 *psi* with the consequent risk of an influx.

If the fluid level in the annulus is dropped by 250 *ft*, the effect would be to reduce the bottom-hole pressure by:

$$P_{bhp} = 0.052 \times (12.5\ ppg) \times (250\ ft) = 162.5\, psi$$

This result indicates that there would still be a net overbalance of **212.5** (i.e., 375–162.5) *psi*.

Example 5.3: An $8\frac{1}{2}''$ diameter hole is drilled up to 12,000 *ft* with a density of 12.0 *ppg*. If the formations pore pressure at this point is 5000 *psi*. Calculate i) mud pressure overbalance above the pore pressure, ii) if the mud density is 11 *ppg*, what would be the overbalance, and iii) if the fluid level in the annulus is dropped to 300 *ft* due to inadequate hole fill up during tripping, what would be the effect on bottom-hole pressure?

Solution:

Given data:

H_{vc} = total vertical height of the mud column = 12,000 *ft*

d_h = hole diameter = $8\frac{1}{2}''$

ρ_{om1} = original mud weight 1 = 12.0 *ppg*

P_f = formation pore pressure = 5000 *psi*

ρ_{om2} = original mud weight 2 = 11 *ppg*

H_{ann} = vertical height of the mud column in the annulus = 300 *ft*

Required data:

i. P_{ob1} = mud pressure overbalance at 12,000 *ft*

ii. P_{ob2} = mud pressure overbalance at 12,000 *ft* if mud density is 11 *ppg*

iii. Effect on bottom-hole pressure?

The overbalance at a depth of 10,000 *ft*. can be calculated by Eq. (4.34a) which can be modified for overbalance as:

$$P_{ob1} = 0.052\,\rho_{om1} H_{vc} - P_f = 0.052 \times (12.0\,ppg) \times (11000\,ft) - 5800\,psi$$

$$= \mathbf{1064}\,psi$$

The overbalance at a depth of 10,000 *ft*. if mud density is 11 *ppg* as:

$$P_{ob2} = 0.052\,\rho_{om2} H_{vc} - P_f = 0.052 \times (11.0\,ppg) \times (11000\,ft) - 5800\,psi$$

$$= \mathbf{492}\,psi$$

If the mud density is not decreased, the positive sign implies that the well would be overbalanced by 492 *psi* with no consequent risk of an influx.

If the fluid level in the annulus is dropped by 300 *ft*, the effect would be to reduce the bottom-hole pressure by:

$$P_{bhp} = 0.052 \times (12.0\,ppg) \times (300\,ft) = 187.2\,psi$$

This result indicates that there would still be a net overbalance of **876.8** (i.e. 1064–187.2) *psi*.

5.2.2 Type of Influx and Gradient Calculation

The influx gradient can be calculated as:

$$G_i = G_m - \frac{P_{siann} - P_{sidp}}{H_i} \tag{5.7}$$

It is noted that the above expression is given in this form because $P_{ann} > P_{dp}$, due to the lighter fluid being in the annulus. The type of fluid can be identified from the gradient calculated utilizing Eq. (5.7). Different references report different ranges of data for identifying the fluid types. However, the following are a guide.

A gas kick is recognized: $0.075 < G_i < 0.25$ *psi/ft*
An oil and gas mixture kick: $0.25 < G_i < 0.3$ *psi/ft*
An oil and condensate mixture kick: $0.3 < G_i < 0.4$ *psi/ft*
A water kick: $0.4 < G_i$ *psi/ft*

5.2.3 Kill Mud Weight Calculation

The mud weight required to kill the influx and would provide the overbalance while drilling ahead can be calculated from Eq. (5.1) as:

$$P_{bh} = P_{sidp} + G_m H_{vc} \tag{5.8}$$

If an overbalance is used the equation becomes:

$$G_k H_{vc} = P_{bh} + P_{ob} \tag{5.9}$$

where,
G_k = kill mud pressure gradient, *psi/ft*
P_{ob} = overbalance pressure, *psi*

Substituting Eq. (5.8) into Eq. (5.9), the final form of the above equation can be written as:

$$G_k = G_m + \frac{P_{sidp} + P_{ob}}{H_{vc}} \tag{5.10}$$

Formation pressure can be calculated in terms of mud weight as

$$P_{bh} = P_{sidp} + 0.052\rho_{om} H_{vc} \tag{5.11}$$

The kill mud weight can be calculated in terms of mud weight as

$$\rho_{km} = \frac{P_{sidp}}{0.052 H_{vc}} + \rho_{om} \tag{5.12}$$

If we consider overkill mud as a safety margin, Eq. (5.12) can be written as:

$$\rho_{km} = \rho_{om} + \frac{P_{sidp}}{0.052 H_{vc}} + \rho_{ok} \tag{5.13}$$

where,
ρ_{ok} = overkill mud weight for safety margin, *ppg*

The kill mud gradient can be calculated in terms of mud weight as

$$G_k = 0.052\rho_{om} + \frac{P_{sidp}}{H_{vc}} \tag{5.14}$$

Example 5.4: while drilling ahead at a target of 8,500 *ft*, the hole size was 7 *in*. The drilling crew noticed that there was a pit gain of 10 *bbls*. The well is shut-in and the drillpipe and annulus pressures were recorded as 650 *psi*, and 800 *psi* respectively. The bottom-hole assembly consists of 650 *ft* of $4\frac{3}{4}''$ OD collars and $3\frac{1}{2}''$ *m* drillpipe. The mud weight is 10.2 *ppg*. Assume a mud pressure gradient. Identify the influx and calculate the new mud weight, including an overbalance of 250 *psi*.

Solution:

Given data:

H_{vc}	=	total vertical height of the mud column	=	8500 *ft*	
d_h	=	hole diameter	=	7 *in*	
V_{pit}	=	pit gain volume	=	10 *bbls*	
P_{dp}	=	shut-in drillpipe pressure	=	650 *psi*	
P_{ann}	=	shut-in annulus pressure	=	800 *psi*	
H_{BHA}	=	bottom-hole assembly length	=	650 *ft*	
d_c	=	collar outer diameter	=	$4\frac{3}{4}''$	= 4.75"
d_{dp}	=	drillpipe diameter	=	$3\frac{1}{2}''$	= 3.5"
ρ_m	=	mud weight	=	10.2 *ppg*	
P_{ob}	=	overbalance pressure	=	250 *psi*	

Required data:

a) Type of influx
b) ρ_m = new mud weight in *ppg*

Nature of influx:

The vertical height of the influx can be calculated using Eq. (5.4) as

$$H_i = \frac{V_{pit}}{A_{ann}} = \frac{10\ bbls}{\pi(d_h^2 - d_c^2)/4} = \frac{(10 bbls)\times\left(\dfrac{ft^3}{0.178\ bbls}\right)}{\left\{\dfrac{\pi(7^2-4.75^2)}{4} in^2\right\}\times\dfrac{ft^2}{144\ in^2}} = 389.6\ ft$$

(Here, H_i is less than bottom-hxole assembly length, 650 *ft*)

Assuming a mud pressure gradient of 0.53 *psi/ft*, the type of influx can be calculated using Eq. (5.7) as:

$$G_i = G_m - \frac{P_{siann} - P_{sidp}}{H_i} = 0.53 - \frac{800-650}{389.6} = \mathbf{0.145\ psi/ft}$$

As long as the influx pressure gradient is within the range 0.075–0.25 *psi/ft*, the type of influx is probably **gas**.

New mud weight:

The new mud weight or kill mud weight can be calculated using Eq. (5.10) as:

$$G_k = G_m + \frac{P_{dp} + P_{ob}}{H_{vc}} = (0.53\ psi/ft) + \frac{(650\ psi) + (250\ psi)}{(8500\ ft)}$$
$$= 0.636\ psi/ft$$

Hence the new mud weight would be as:

$$\rho_m = \frac{0.636\ psi/ft}{0.052 \times 1\ ft} = \mathbf{12.23\ ppg}$$

Example 5.5: While drilling ahead at a target of 10,500 *ft*, the hole size was $8\frac{1}{2...}$. in. The drilling crew noticed that there was a pit gain of 15 *bbls*. The well is shut-in and the drillpipe and annulus pressures were recorded as 700 *psi*, and 800 *psi* respectively. The bottom-hole assembly consists of 850 *ft* of $4\frac{3}{4}''$ OD collars and $3\frac{1}{2}''$ drillpipe. The mud weight is 15.2 *ppg*.

Assume a mud pressure gradient. Identify the influx and calculate the new mud weight, including an overbalance of 450 *psi*.

Solution:

Given data:

H_{vc}	=	total vertical height of the mud column	=	10,500 *ft*	
d_h	=	hole diameter	=	$8\frac{1}{2}''$ in	
V_{pit}	=	pit gain volume	=	15 *bbls*	
P_{dp}	=	shut-in drillpipe pressure	=	700 *psi*	
P_{ann}	=	shut-in annulus pressure	=	800 *psi*	
H_{BHA}	=	bottom-hole assembly length	=	850 *ft*	
d_c	=	collar outer diameter	=	$4\frac{3}{4}''$	= 4.75"
d_{dp}	=	drillpipe diameter	=	$3\frac{1}{2}''$	= 3.5"
ρ_m	=	mud weight	=	15.2 *ppg*	
P_{ob}	=	overbalance pressure	=	450 *psi*	

Required data:

a) Type of influx
b) ρ_m = new mud weight in *ppg*

Nature of influx:

The vertical height of the influx can be calculated using Eq. (5.4) as

$$H_i = \frac{V_{pit}}{A_{ann}} = \frac{15\ bbls}{\pi(d_h^2 - d_c^2)/4}$$

$$= \frac{(15\ bbls) \times \left(\dfrac{ft^3}{0.178\ bbls}\right)}{\left\{\dfrac{\pi(8.5^2 - 4.75^2)}{4} in^2\right\} \times \dfrac{ft^2}{144\ in^2}} = 310.95\ ft$$

(Here, H_i is less than bottom-hole assembly length, 850 *ft*)

Assuming a mud pressure gradient of 0.53 *psi/ft*, the type of influx can be calculated using Eq. (5.7) as:

$$G_i = G_m - \frac{P_{siann} - P_{sidp}}{H_i} = 0.53 - \frac{800 - 700}{310.95} = \mathbf{0.208\ psi/ft}$$

As long as, the influx pressure gradient is within the range 0.075–0.25 *psi/ft*, the type of influx is probably **gas**.

New mud weight:

The new mud weight or kill mud weight can be calculated using Eq. (5.10) as:

$$G_k = G_m + \frac{P_{dp} + P_{ob}}{H_{vc}}$$

$$= (0.53\ psi/ft) + \frac{(700\ psi) + (450\ psi)}{(10500\ ft)} = 0.640\ psi/ft$$

Hence the new mud weight would be as:

$$\rho_m = \frac{0.640\ psi/ft}{0.052 \times 1\ ft} = \mathbf{12.30\ ppg}$$

Example 5.6: Determine the kill mud density and kill mud gradient for a shut-in-drill-pipe pressure of 600 *psi* at a depth of 12,000 *ft*. If the original mud weight is 14.5 *ppg* and the slow circulating pump pressure is 850 *psi*, find out also the initial and final circulating pressure of the system.

Solution:

Given data:

P_{sidp} = shut-in drillpipe pressure = 600 *psi*
H_{vc} = total vertical height of the mud column 12,000 *ft*
ρ_{om} = original mud weight = 14.5 *ppg*
P_p = slow circulating pump pressure = 850 *psi*

Required data:

ρ_{km} = kill mud weight, *ppg*
G_k = kill mud gradient, *psi/ft*
P_{ic} = initial circulating pressure, *psi*
P_{fc} = final circulating pressure, *psi*

The kill mud weight can be calculated using Eq. (5.12) as

$$\rho_{km} = \frac{P_{sidp}}{0.052\ H_{vc}} + \rho_{om} = \frac{(600\ psi)}{0.052 \times (12{,}000\ ft)} + (14.5\ ppg) = \mathbf{15.5}\ \boldsymbol{ppg}$$

If we consider an overkill mud weight of 0.5 *ppg* as a safety margin, Kill mud weight can be calculated using Eq. (5.13) as:

$$\rho_{km} = \rho_{om} + \frac{P_{sidp}}{0.052\ H_{vc}} + \rho_{ok} = 15.5 + 0.5 = \mathbf{16.0}\ \boldsymbol{ppg}$$

The kill mud gradient can be calculated using Eq. (5.14) as

$$G_k = 0.052\rho_{om} + \frac{P_{sidp}}{H_{vc}}$$
$$= 0.052 \times (14.5\ ppg) + \frac{(600\ psi)}{(12{,}000\ ft)} = \mathbf{0.804}\ \boldsymbol{psi/ft}$$

If we consider there is no overkill pressure, the initial circulating pressure is calculated using Eq. (5.5) as:

$$P_{ic} = P_{sidp} + P_p + P_{ok} = (600\ psi) + (850\ psi) + 0 = \mathbf{1450}\ \boldsymbol{psi}$$

Final circulating pressure is calculated is calculated using Eq. (5.6) as:

$$P_{fc} = P_p\left(\frac{\rho_{km}}{\rho_{om}}\right) = (850 psi) \times \left(\frac{15.5}{14.5}\right) = \mathbf{908}\ \boldsymbol{psi}$$

Example 5.7: Determine the kill mud density and kill mud gradient for a shut-in-drill-pipe pressure of 600 *psi* at a depth of 10,000 *ft*. If the original mud weight is 10.0 *ppg* and the slow circulating pump pressure is 700 *psi*, find out also the initial and final circulating pressure of the system.

Solution:

Given data:

P_{sidp} = shut-in drillpipe pressure = 600 *psi*
Hvc = total vertical height of the mud column 10,000 *ft*
ρ_{om} = original mud weight = 10.0 *ppg*
P_p = slow circulating pump pressure = 700 *psi*

Required data:

ρ_{km} = kill mud weight, *ppg*
G_k = kill mud gradient, *psi/ft*
P_{ic} = initial circulating pressure, *psi*
P_{fc} = final circulating pressure, *psi*

The kill mud weight can be calculated using Eq. (5.12) as

$$\rho_{km} = \frac{P_{sidp}}{0.052H_{vc}} + \rho_{om} = \frac{(600\ psi)}{0.052 \times (10{,}000\ ft)} + (10.0\ ppg) = \mathbf{11.2}\ \boldsymbol{ppg}$$

The kill mud gradient can be calculated using Eq. (5.14) as

$$G_k = 0.052\rho_{om} + \frac{P_{sidp}}{H_{vc}} = 0.052 \times (10.0\ ppg) + \frac{(600\ psi)}{(10{,}000\ ft)} = \mathbf{0.58}\ \boldsymbol{psi/ft}$$

If we consider there is no overkill pressure, the initial circulating pressure is calculated using Eq. (5.5) as:

$$P_{ic} = P_{sidp} + P_p + P_{ok} = (600\ psi) + (700\ psi) + 0 = \mathbf{1300}\ \boldsymbol{psi}$$

Final circulating pressure is calculated is calculated using Eq. (5.6) as:

$$P_{fc} = P_p\left(\frac{\rho_{km}}{\rho_{om}}\right) = (700\ psi) \times \left(\frac{11.2}{10.0}\right) = \mathbf{784}\ \boldsymbol{psi}$$

5.2.4 Kick Analysis

Mathematically, if $V_{pit} < V_{ann\_dc}$, the length of the kick can be calculated as (Figure 5.1):

$$L_k = \frac{V_{pit}}{C_{ann\_dc}} \tag{5.15}$$

where,

L_k = kick length (i.e., vertical height of influx, H_i), *ft*
C_{ann_dc} = the annulus capacity behind the drill collar, *bbl/ft*
V_{pit} = the pit gain volume, *bbl*
V_{ann_dc} = the annulus volume against drill collar, *bbl*

If $V_{pit} > V_{ann\_dc}$, the length of the kick is given by

$$L_k = L_{dc} + \frac{V_{pit} - V_{ann\_dc}}{C_{ann\_dp}} \tag{5.16}$$

where,

L_{dc} = length of the drill collar, *ft*
C_{ann_dp} = the annulus capacity behind the drillpipe, *bbl/ft*

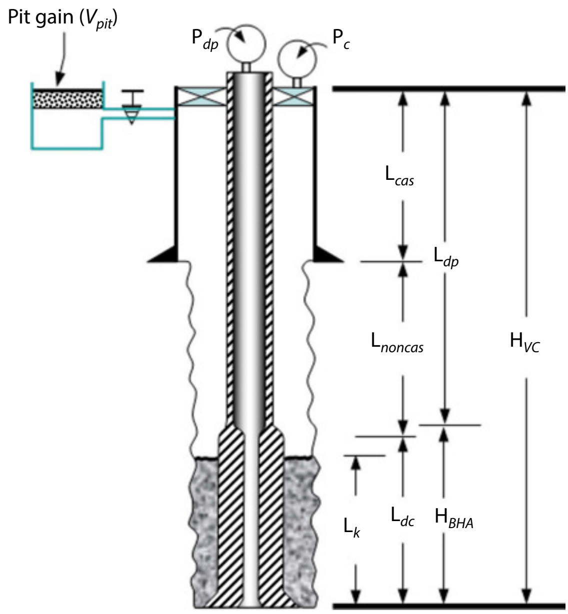

Figure 5.1 Schematic of initial well conditions during well control operations.

A pressure balance on the initial well system for a uniform mud density, ρ_m, is given by

$$P_{icp} + 0.052[\rho_{om}(H_{vc} - L_k) + \rho_k L_k - \rho_m H_{vc}] = P_{idp} \tag{5.17}$$

where,

P_{icp} = initial stabilized drill collar pressure, *psi*
P_{idp} = initial stabilized drillpipe pressure, *psi*
ρ_k = kick fluid (i.e., influx) density, *ppg*

Solving Eq. (5.17) for kick fluid density gives

$$\rho_k = \rho_{om} + \frac{P_{idp} - P_{icp}}{0.052\ L_k} \tag{5.18}$$

Example 5.8: A kick was detected while drilling a high-pressure zone. The depth of the formation was recorded 10,000 *ft* with a mud density of 9.0 *ppg*. The crew shut-in the well and recorded the pressure for drillpipe and drill collar as 350 *psi* and 430 *psi*, respectively. The observed total pit gain was 6.0 *bbl*. The annular capacity against 950 *ft* of drill collar is 0.028 *bbl/ft* and the overkill safety margin is 0.50 *ppg*. Compute the formation pressure, influx density, the type of fluid, required kill mud weight, and kill mud gradient.

Solution:
Given data:

H_{vc} = total vertical height of the mud column = 10,000 *ft*
ρ_{om} = original mud weight = 9.0 *ppg*
P_{sidp} = shut-in drillpipe pressure = 350 *psi*

P_{sidc} = shut-in drill collar pressure = 430 *psi*
V_{pit} = pit gain volume = 6*bbl*s
L_{dc} = length of drill collar = 950 *ft*
C_{ann_dc} = the annulus capacity behind the drill collar = 0.028 *bbl/ft*
ρ_{ok} = overkill mud as a safety margin = 0.5 *ppg*

Required data:
p_{bh} = formation pressure, *psi*
ρ_k = kick fluid or influx density, *ppg*
Type of fluid
ρ_{km} = kill mud weight, *ppg*
G_k = kill mud gradient, *psi/ft*

Formation pressure can be calculated using Eq. (5.11) as

$$P_{bh} = P_{sidp} + 0.052\rho_{om}H_{vc} = (350\ psi) + 0.052 \times (9.0\ ppg) \times (10000\ ft)$$
$$\boldsymbol{P_{bh} = 5030\ psi}$$

To calculate the kick density, we first need to calculate the length of the kick and therefore, the annular volume.

The annular volume against the drill collar,

$$V_{ann\_dc} = L_{dc} \times C_{ann\_dc} = 950\,ft \times 0.028\frac{bbl}{ft} = 26.6\ bbl$$

As long as $V_{pit} < V_{ann\_dc}$, the length of the kick can be calculated using Eq. (5.15) as:

$$L_k = \frac{V_{pit}}{C_{ann\_dc}} = \frac{(6.0\ bbl)}{(0.028\ bbl/ft)} = 214.29\,ft$$

The density of the kick fluid is calculated using Eq. (5.18) as

$$\rho_k = \rho_{om} + \frac{P_{idp} - P_{icp}}{0.052\ L_k} = (9.0\ ppg) + \frac{(350\ psi - 430\ psi)}{0.052 \times (214.29\,ft)} = \mathbf{1.82}\ \boldsymbol{ppg}$$

Therefore, the kick fluid is **gas.**

Consider overkill mud as a safety margin, the kill mud weight can be calculated using Eq. (5.13) as

$$\rho_{km} = \rho_{om} + \frac{P_{sidp}}{0.052\ H_{vc}} + \rho_{ok}$$
$$= (9.0\ ppg) + \frac{(350\ psi)}{0.052 \times (10{,}000\,ft)} + (0.5\ ppg) = \mathbf{10.17}\ \boldsymbol{ppg}$$

The kill mud gradient can be calculated using Eq. (5.14) as

$$G_k = 0.052\rho_{om} + \frac{P_{sidp}}{H_{vc}} = 0.052 \times (9.0\ ppg) + \frac{(350\ psi)}{(10{,}000\ ft)} = \mathbf{0.503\ psi/\ ft}$$

Example 5.9: A well was being drilled at a high-pressure zone of 12,000 *ft* vertical depth where 9.5 *ppg* mud was being circulated at a rate of 8.0 *bbl*/min. A pit gain of 95 *bbl* was noticed over a 3-minute period before the pump was stopped and the BOPs were closed. After the pressures stabilized, an initial drillpipe pressure of 500 *psi* and an initial casing pressure of 700 *psi* were recorded by the attendees at the rig side. The annular capacity against 950 *ft* of drill collar was 0.03 *bbl/ft* and the annular capacity against 850 *ft* of drill-pipe was 0.0775 *bbl/ft*. Compute the formation pressure, influx density.

Solution:

Given data:

H_{vc}	=	total vertical height of the mud column	=	12,000 *ft*
ρ_{om}	=	original mud weight	=	9.5 *ppg*
q_t	=	original mud circulation rate	=	8.0 *bbl/min*
V_{pit}	=	pit gain volume	=	95 *bbls*
t	=	time to stop the pump	=	3 *min*
P_{sidp}	=	shut-in drillpipe pressure	=	500 *psi*
P_{sidc}	=	shut-in drill collar pressure	=	700 *psi*
L_{dc}	=	length of drill collar	=	950 *ft*
C_{ann_dc}	=	the annulus capacity behind the drill collar	=	0.03 *bbl/ft*
L_{dp}	=	length of drillpipe	=	850 *ft*
C_{ann_dp}	=	the annulus capacity behind the drillpipe	=	0.0775 *bbl/ft*

Required data:

P_{bh}	=	formation pressure, *psi*
ρ_k	=	kick fluid or influx density, *ppg*

A schematic view of the example is shown in Figure 5.2. Formation pressure can be calculated using Eq. (5.11) as

$$P_{bh} = P_{sidp} + 0.052\rho_{om}H_{vc} = (500\ psi) + 0.052 \times (9.5\ ppg) \times (12000\ ft)$$

$$P_{bh} = \mathbf{6428\ psi}$$

To calculate the kick density, we first need to calculate the length of the kick and therefore, the annular volume.

The total annular volume against the drillpipe and drill collar,

$$V_{ann} = V_{ann\_dp} + V_{ann\_dc} = L_{dp} \times C_{ann\_dp} + L_{dc} \times C_{ann\_dc}$$

$$V_{ann} = \left(850\,ft \times 0.0775\frac{bbl}{ft}\right) + \left(950\,ft \times 0.03\frac{bbl}{ft}\right) = 94.37\ bbl$$

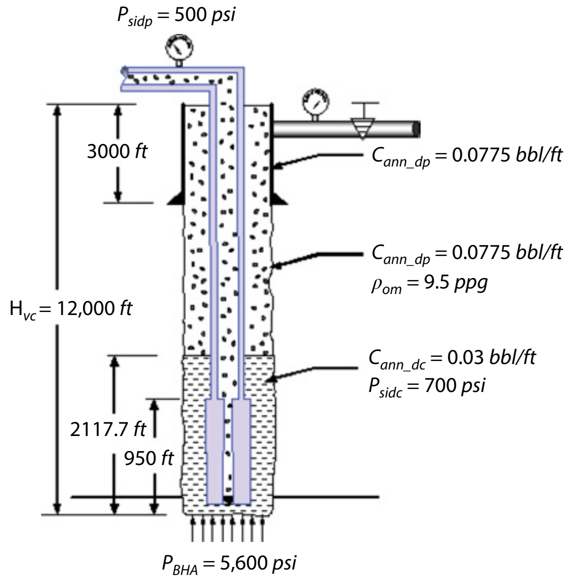

Figure 5.2 Illustration for Example 5.5.

However, kick length is determined based on the total annular volume against the drill collar only. So,

$$V_{ann\_dc} = L_{dc} \times C_{ann\_dc} = \left(950\,ft \times 0.03 \frac{bbl}{ft} \right) = 28.5\ bbl$$

If we assume that the kick fluids are mixed with the mud pumped while the well was flowing, so the total pit gain is

$$(V_{pit})_{total} = V_{pit} + q_t t = (95.0\ bbl) + (8.0\ bbl/min \times 3\ min) = 119.0\ bbl$$

As long as $(V_{pit})_{total} > V_{ann\_dc}$, the length of the kick can be calculated using Eq. (5.16) as

$$L_k = L_{dc} + \frac{V_{pit} - V_{ann\_dc}}{C_{ann\_dp}} = (950\ ft) + \frac{119 bbl - 28.5\ bbl}{0.0775\ bbl/ft} = 2{,}117.74\ ft$$

The density of the kick fluid is calculated using Eq. (5.18) as

$$\rho_k = \rho_{om} + \frac{P_{idp} - P_{icp}}{0.052\ L_k} = (9.5\ ppg) + \frac{(500\ psi - 700\ psi)}{0.052 \times (2{,}117.74\ ft)} = \mathbf{7.68\ ppg}$$

Example 5.10: A kick was detected while drilling a high-pressure zone. The depth of the formation was recorded 8,000 *ft* with a mud density of 14.3 *ppg*. The crew shut-in the well and recorded the pressure for drillpipe and drill collar as 200 *psi* and 400 *psi*, respectively.

The observed total pit gain was 30.0 *bbl*. The annular capacity against 950 *ft* of drill collar is 0.0836 *bbl/ft* and the overkill safety margin is 0.50 *ppg*. Compute the formation pressure, influx density, the type of fluid, required kill mud weight, and kill mud gradient.

Solution:

Given data:

H_{vc}	= total vertical height of the mud column	=	8,000 *ft*
ρ_{om}	= original mud weight	=	14.3 *ppg*
P_{sidp}	= shut-in drillpipe pressure	=	200 *psi*
P_{sidc}	= shut-in drill collar pressure	=	400 *psi*
V_{pit}	= pit gain volume	=	30 *bbls*
L_{dc}	= length of drill collar	=	950 *ft*
C_{ann_dc}	= the annulus capacity behind the drill collar	=	0.0836 *bbl/ft*
ρ_{ok}	= overkill mud as a safety margin	=	0.5 *ppg*

Required data:

P_{bh} = formation pressure, *psi*

ρ_k = kick fluid or influx density, *ppg*

Type of fluid

ρ_{km} = kill mud weight, *ppg*

G_k = kill mud gradient, *psi/ft*

Formation pressure can be calculated using Eq. (5.11) as

$$P_{bh} = P_{sidp} + 0.052\rho_{om}H_{vc} = (200) + 0.052 \times (14.3\ ppg) \times (8000\ ft)$$

$$P_{bh} = \mathbf{6148.8}\ \boldsymbol{psi}$$

To calculate the kick density, we first need to calculate the length of the kick and therefore, the annular volume.

The annular volume against the drill collar,

$$V_{ann\_dc} = L_{dc} \times C_{ann\_dc} = 950\,ft \times 0.0836\frac{bbl}{ft} = 79.42\ bbl$$

As long as $V_{pit} < V_{ann\_dc}$, the length of the kick can be calculated using Eq. (5.15) as:

$$L_k = \frac{V_{pit}}{C_{ann\_dc}} = \frac{30.0\ bbl}{0.0836\ bbl/ft} = 358.85\ ft$$

The density of the kick fluid is calculated using Eq. (5.18) as

$$\rho_k = \rho_{om} + \frac{P_{idp} - P_{icp}}{0.052L_k} = (14.3\ ppg) + \frac{(200\ psi - 400\ psi)}{0.052 \times (358.85\ ft)} = \mathbf{3.58}\ \boldsymbol{ppg}$$

Therefore, the kick fluid is **gas**.

Consider overkill mud as a safety margin, the kill mud weight can be calculated using Eq. (5.13) as

$$\rho_{km} = \frac{P_{sidp}}{0.052H_{vc}} + \rho_{om} + \rho_{ok}$$
$$= \frac{(200\ psi)}{0.052 \times (8{,}000\ ft)} + (14.3\ ppg) + (0.5\ ppg) = \mathbf{15.3\ ppg}$$

The kill mud gradient can be calculated using Eq. (5.14) as

$$G_k = 0.052\rho_{om} + \frac{P_{sidp}}{H_{vc}} = 0.052 \times (14.3\ ppg) + \frac{(200\ psi)}{(8{,}000\ ft)} = \mathbf{0.768\ psi/ft}$$

Example 5.11: A well was being drilled at a high-pressure zone of 10,000 *ft* vertical depth where 8.5 *ppg* mud was being circulated at a rate of 9.0 *bbl/min*. A pit gain of 65 *bbl* was noticed over a 3-minute period before the pump was stopped and the BOPs were closed. After the pressures stabilized, an initial drillpipe pressure of 400 *psi* and an initial casing pressure of 600 *psi* were recorded by the attendees at the rig side. The annular capacity against 950 *ft* of drill collar was 0.0386 *bbl/ft* and the annular capacity against 850 *ft* of drillpipe was 0.07 *bbl/ft*. Compute the formation pressure, influx density.

Solution:

Given data:

H_{vc}	=	total vertical height of the mud column	=	10,000 *ft*
ρ_{om}	=	original mud weight	=	8.5 *ppg*
q_t	=	original mud circulation rate	=	9.0 *bbl/min*
V_{pit}	=	pit gain volume	=	65 *bbls*
t	=	time to stop the pump	=	3 min
P_{sidp}	=	shut-in drillpipe pressure	=	400 *psi*
P_{sidc}	=	shut-in drill collar pressure	=	600 *psi*
L_{dc}	=	length of drill collar	=	950 *ft*
C_{ann_dc}	=	the annulus capacity behind the drill collar	=	0.0386 *bbl/ft*
L_{dp}	=	length of drillpipe	=	850 *ft*
C_{ann_dp}	=	the annulus capacity behind the drillpipe	=	0.07 *bbl/ft*

Required data:

P_{bh}	=	formation pressure, *psi*
ρ_k	=	kick fluid or influx density, *ppg*

Formation pressure can be calculated using Eq. (5.11) as

$$P_{bh} = P_{sidp} + 0.052\rho_{om}H_{vc} = (400) + 0.052 \times (8.5\ ppg) \times (10{,}000\ ft)$$
$$P_{bh} = \mathbf{4820\ psi}$$

To calculate the kick density, we first need to calculate the length of the kick and therefore, the annular volume.

The annular volume against the drill pipe and drill collar,

$$V_{ann} = V_{ann\_dp} + V_{ann\_dc} = L_{dp} \times C_{ann\_dp} + L_{dc} \times C_{ann\_dc}$$

$$V_{ann} = 850\ ft \times 0.07 \frac{bbl}{ft} + 950\ ft \times 0.0386 \frac{bbl}{ft} = 96.17\ bbl$$

However, kick length is determined based on the total annular volume against the drill collar only. So,

$$V_{ann\_dc} = L_{dc} \times C_{ann\_dc} = 950\ ft \times 0.0386 \frac{bbl}{ft} = 36.67\ bbl$$

If we assume that the kick fluids are mixed with the mud pumped while the well was flowing, so the total pit gain is

$$(V_{pit})_{total} = V_{pit} + q_t t = (65.0\ bbl) + \left(9.0 \frac{bbl}{min} \times 3\ min\right) = 92\ bbl$$

As long as $(V_{pit})_{total} > V_{ann\_dc}$, the length of the kick can be calculated using Eq. (5.16) as:

$$L_k = L_{dc} + \frac{V_{pit} - V_{ann\_dc}}{C_{ann\_dp}} = (950\ ft) + \frac{92.0\ bbl - 36.67\ bbl}{0.07\ bbl/ft} = 1740.42\ ft$$

The density of the kick fluid is calculated using Eq. (5.18) as

$$\rho_k = \rho_{om} + \frac{P_{idp} - P_{icp}}{0.052 L_k} = (8.5\ ppg) + \frac{(400\ psi - 600\ psi)}{0.052 \times (1740.42\ ft)} = \mathbf{6.29\ ppg}$$

***Example 5.12*:** A vertical development well was drilled, cased and cemented with 13 3/8" casing shoe at a depth of 7,500 *ft*. The well needs to be drilled at a target depth of 13,000 *ft* using the drilling fluid which has a maximum mud weight of 12.0 *ppg*. The fracture gradient at casing shoe is 12.5 *ppg* while the formation gradient is 11.5 *ppg*. The inside diameter of the last casing is 12.515" and the new hole diameter is 12.25". The standard surface temperature is 60 °F while the temperature gradient is 0.018 °F/*ft*. The outside drill pipe diameter is 5.0". If the expected gas kick gradient is 0.1 *psi/ft*, calculate the maximum volume of gas kick that can be handled in this case without fracturing the formations at the casing shoe.

Solution:

Given data:

d_{o_cas}	=	Casing outside diameter	=	13 3/8"
d_{i_cas}	=	Casing inside diameter	=	12.515"
L_{cas}	=	Casing length	=	7,500 *ft*
d_h	=	Hole diameter	=	12.25"
D_m	=	Total length of the well	=	13,000 *ft*

d_{dpo}	=	Drill pipe outside diameter	=	5.0"
T_s	=	Surface temperature	=	60 °F
dT	=	Temperature gradient	=	0.018 °F/*ft*
ρ_m	=	Maximum mud weight to be used	=	12.0 *ppg*
ρ_{fm}	=	Formation gradient in *ppg*	=	11.5 *ppg*
ρ_{fr}	=	Fracture gradient at shoe in *ppg*	=	12.5 *ppg*
G_i	=	Kick's pressure gradient	=	0.1 *psi/ft*

Required data:

V_k = Maximum kick volume

To calculate the kick volume at these conditions, first we need to calculate the kick height at the casing shoe because we know that the maximum fracture risk will be at the casing shoe. The following equation is used to calculate the kick height:

$$H_i = \frac{0.052\times\rho_m(D_m - L_{cas}) + 0.052\times L_{cas}\times\rho_{fr} - 0.052\times D_m\times\rho_f}{0.052\times\rho_m - G_i}$$
$$= \frac{0.052\times 12.0\times(13{,}000 - 7{,}500) + 0.052\times 7{,}500\times 12.5 - 0.052\times 13{,}000\times 11.5}{0.052\times 12.0 - 0.1}$$
$$= 1{,}017.2\ ft$$

The above height can be changed to a volume using the annulus capacity between casing and drill pipes as below:

$$V_k = H_i\times C_{ann\_dp} = 1{,}017.2\times\frac{\pi}{4}\times\frac{12.515^2 - 5.0^2}{144\times 5.615} = 130.1\ bbls$$

The volume at the bottom of the well can be calculated using Boyle's law of gases and assuming ideal gas behavior.

Pressure and temperature at the casing shoe are equal to:

$$P_{shoe} = 0.052\times L_{cas}\times\rho_{fr} = 0.052\times 7{,}500\times 12.5 = 4{,}875\ psi$$

$$T_{shoe} = T_s + dT\times L_{cas} + 460 = 60 + 0.018\times 7{,}500 + 460 = 655°R$$

For calculating shoe pressure, we used the fracture gradient because we are looking for maximum volume of the kick; and that volume will occur at the maximum allowed pressure which is the fracture pressure.

Pressure and temperature at the bottom of the well are equal to:

$$P_{TD} = 0.052\times TD\times\rho_f = 0.052\times 13{,}000\times 11.5 = 7{,}774\ psi$$

$$T_{TD} = T_s + dT\times TD + 460 = 60 + 0.018\times 13{,}000 + 460 = 754°R$$

Now, the volume at the bottom can be calculated using Boyle's law as follows:

$$\frac{P_{shoe}V_{K\_shoe}}{T_{shoe}} = \frac{P_{TD}V_{K\_TD}}{T_{TD}}$$

$$\frac{4{,}875 \times 130.1}{655} = \frac{7{,}774 \times V_{K\_TD}}{754}$$

$$V_{K\_TD} = \mathbf{93.9\ bbls}$$

The above volume is the maximum allowable kick volume to enter the well without breaking the casing shoe at the above conditions. If the kick volume is greater than the above volume, there will be a high risk of breaking the casing shoe and have loss circulation and at the same time severe entry of kick to the wellbore.

***Example 5.13*:** A vertical well is drilled at a depth of 11,500 *ft*, then 9 5/8" casing having inside diameter of 8.681" was set and cemented to the above said depth. A production section of 8.5" hole diameter is to be drilled to a target depth of 15,000 *ft*. The fracture gradient at casing shoe was estimated to be 13.65 *ppg* while the formation gradient is 12.4 *ppg*. The standard surface temperature is 75 °F while the temperature gradient is 0.02 °F/*ft*. The drill pipe outside diameter is 5.0". If the expected gas kick volume at casing shoe is 70 *bbls* and gas gradient is 0.09 *psi/ft*, calculate the maximum mud weight that can be used and handle the kick without fracturing the formations at the casing shoe.

Solution:

Given data:

d_{o_cas} = Casing outside diameter = 9 5/8"
d_{i_cas} = Casing inside diameter = 8.681"
L_{cas} = Casing length = 11,500 *ft*
d_h = Hole diameter = 8.5"
D_m = Total length of the well = 15,000 *ft*
d_{dpo} = Drill pipe outside diameter = 5.0"
T_s = Surface temperature = 75 °F
dT = Temperature gradient = 0.02 °F/*ft*
ρ_{fm} = Formation gradient in *ppg* = 12.4 *ppg*
ρ_{fr} = Fracture gradient at shoe in *ppg* = 13.65 *ppg*
G_i = Kick's pressure gradient = 0.09 *psi/ft*
V_k = Maximum kick volume = 70 *bbls*

Required data:

ρ_m = Maximum mud weight to be used in *ppg*

To calculate the maximum mud weight that can handle such a gas kick volume without breaking the casing shoe, first we need to calculate the length of the gas bubble at the casing shoe using the annulus capacity between casing and drill pipes as follows:

$$H_i = \frac{V_k}{C_{ca\_dp}} = \frac{70}{\frac{\pi}{4} \times \frac{8.681^2 - 5.0^2}{144 \times 5.615}} = 1431\ ft$$

Now, by knowing the kick height and other information we can calculate the maximum mud weight that can be used from the following equation:

$$H_i = \frac{0.052 \times \rho_m (D_m - L_{cas}) + 0.052 \times L_{cas} \times \rho_{fr} - 0.052 \times D_m \times \rho_f}{0.052 \times \rho_m - G_i}$$

$$1,431 = \frac{\begin{array}{c} 0.052 \times \rho_m \times (15,000 - 11,500) + 0.052 \times 11,500 \times 13.65 \\ - 0.052 \times 15,000 \times 12.4 \end{array}}{0.052 \times \rho_m - 0.09}$$

$$0.052 \times \rho_m - 0.09 = \frac{182\rho_m - 1509}{1,431}$$

$$0.1272\rho_m - 0.052\rho_m = 1.0547 - 0.09$$

$$\rho_m = \mathbf{12.83}\ \boldsymbol{ppg}$$

The above mud weight is the maximum mud weight that can be used to handle a 70 *bbls* of gas kick without breaking the formation at the casing shoe.

***Example 5.14*:** A well kick was encountered while drilling an intermediate section of 12.25" in a vertical well. The rig crew recorded a pit gain of 45 *bbls*. Shut-In Casing Pressure (SICP) was stabilized at 500 *psi* while Shut-In Drill Pipe Pressure (SIDPP) was zero because floating valve was used as part of BHA. Current depth was 5,000 *ft* and mud weight was 9.8 *ppg*. Drill collars outside diameter and length were 8.0" and 600 *ft* respectively, while drillpipe outside diameter was 5.0". The rig crew increased the drillpipe pressure in steps in order to determine the SIDPP. Table 5.1 shows the changes in the SIDPP and SICP.

If it is required to have mud hydrostatic pressure greater than the formation pressure by 100 *psi*, determine the nature of the influx and the new mud weight to control the well back.

Solution:

Given data:

d_h = Hole diameter = 12.25"

V_{pit} = Pit gain = 45 *bbls*

Table 5.1 Changes in the SIDPP and SICP for Example 5.14.

	P_{sidp} *psi*	P_{siann} *psi*
1	150	500
2	250	500
3	300	500
4	350	530
5	400	580

P_{siann}	=	Shut-in casing pressure	=	500 *psi*
P_{sidp}	=	Shut-in drill pipe pressure	=	0 *psi*
H_{vc}	=	Vertical height of the mud column	=	5,000 *ft*
d_{dp}	=	Drill pipe outside diameter	=	5.0"
d_c	=	Drill collar outside diameter	=	8"
L_{dc}	=	Length of the drill collar	=	600 *ft*
ρ_m	=	Mud weight	=	9.8 *ppg*
P_{ob}	=	Overbalance pressure required	=	100 *psi*

Required data:

Nature of the flux

ρ_K = Mud weight of the kill fluid in *ppg*.

To determine the nature of the flux and the required kill mud density, first we need to estimate SIDPP from the given data in the table. Since the float valve was used, SIDPP reading was zero. So the only way to know SIDPP is to apply pressure to the drill pipe in steps until we see the increase in SICP and then determine the SIDPP. From the table above, when the drillpipe pressure was increased up to 300 *psi*, there were no increases in SICP. However, when the drillpipe pressure was increased further to 350 *psi*, the SICP was increased to 550 *psi*. So from this information, SIDPP can be calculated as:

$$P_{dp} = 350 - (530 - 500) = 320\ psi$$

The volume of the annulus between the hole and drill collars is equal to:

$$V_{ann} = C_{ann\_dc} \times L_{dc} = \frac{d_h^2 - d_{dc}^2}{1029.4} \times L_{dc} = \frac{12.25^2 - 8.0^2}{1029.4} \times 600 = 50\ bbls$$

So, all of the flux was in the annulus between the drill collars and the hole. The height of the flux can now be calculated using Eq. (5.4):

$$H_i = \frac{V_{pit}}{C_{ann}} = \frac{45}{0.084} = 536\ ft$$

The mud gradient is equal to:

$$G_m = 0.052 \times \rho_m = 0.052 \times 9.8 = 0.51\ psi/ft$$

Now, the flux gradient can be calculated using Eq. (5.7):

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = 0.51 - \frac{500 - 320}{536} = 0.17\ psi/ft$$

Since the flux gradient is less than 0.25 *psi*/ft, **the flux is gas**.

The new mud gradient can be calculated using the Eq. (5.10):

$$G_k = G_m + \frac{P_{dp} + P_{ob}}{H_{vc}} = 0.51 + \frac{320 + 100}{5{,}000} = 0.594\ psi/ft$$

The kill mud weight is now equal to:

$$\rho_k = \frac{G_k}{0.052} = \frac{0.594}{0.052} = \mathbf{11.42}\ \boldsymbol{ppg}$$

Example 5.15: While drilling a 8.5" vertical hole using 11.7 *ppg* mud, 25 *bbls* of kick was recorded by rig crew at a depth of 12,000 *ft*. Later, the kick was removed and the estimated kick gradient was 0.13 *psi/ft*. Drilling string consists of 550 *ft* of 6" drill collars and 4 ¾" drill pipes. If the formation gradient was found to be 0.64 *psi/ft*, what were the shut-in annulus and drill pipe pressures? If the kill fluid should be 200 *psi* greater than the formation pressure, calculate the kill fluid density in *ppg*.

Solution:

Given data:

d_h = Hole diameter = 8.5"
V_{pit} = Pit gain = 25 *bbls*
H_{vc} = Vertical height of the mud column = 12,000 *ft*
d_{dp} = Drill pipe outside diameter = 4¾"
d_c = Drill collar outside diameter = 6.0"
L_{dc} = Length of the drill collar = 550 *ft*
ρ_m = Mud weight = 11.7 *ppg*
P_{ob} = Overbalance pressure required = 200 *psi*
G_i = Kick's pressure gradient = 0.13 *psi/ft*
G_f = Formation gradient = 0.64 *psi/ft*

Required data:

P_{sidp} = Shut-in drill pipe pressure in *psi*
P_{siann} = Shut-in casing pressure in *psi*
ρ_k = Mud weight of the kill fluid in *ppg*.

By knowing mud weight and formation gradient, shut-in drill pipe pressure can be determined using Eq. (5.1) after arrangement:

$$P_{sidp} = P_{bh} - G_m H_{vc} = 0.64 \times 12{,}000 - 0.052 \times 11.7 \times 12{,}000 = \mathbf{379}\ \boldsymbol{psi}$$

Now to calculate shut-in casing pressure, we need first to estimate the kick length in the bottom of the hole. Using drill collar capacity, the volume of the kick around the drill collars equal to:

$$V_{k\_dc} = L_{dc} \times C_{ann\_dc} = 550 \times \frac{8.5^2 - 6^2}{1029.4} = 19.36\ bbls$$

So, the rest of the kick volume will be around the drill pipes. The length of the kick around the drillpipe is equal to:

$$H_{i\_dp} = \frac{V_{pit} - V_{k_{dc}}}{C_{ann_{dp}}} = \frac{25 - 19.36}{\frac{8.5^2 - 4.75^2}{1029.4}} = 116.7\ ft$$

Total length of the kick is now equal to:

$$H_i = L_{dc} + H_{i\_dp} = 550 + 116.7 = 666.7\ ft$$

Casing shut-in pressure will be calculated using Eq. (5.7) and by using the available data:

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = 0.052 \times 11.7 - \frac{P_{siann} - 379}{666.7} = 0.13$$

$$P_{siann} = \mathbf{698\ \mathit{psi}}$$

Kill fluid gradient having 200 *psi* over the formation pressure can be calculated using Eq. (5.10):

$$G_k = G_m + \frac{P_{dp} + P_{ob}}{H_{vc}} = 0.052 \times 11.7 + \frac{379 + 200}{12{,}000} = 0.657\ psi/ft$$

Thus, the kill fluid density is equal to:

$$\rho_k = \frac{G_k}{0.052} = \frac{0.657}{0.052} = \mathbf{12.63\ \mathit{ppg}}$$

***Example 5.16*:** A pit gain of 60 *bbls* was noticed while drilling a 12.25" hole at depth of 9,000 *ft*, and shut-in annulus pressure was stabilized at 750 *psi*. The kick gradient was estimated to be 0.09 *psi/ft*, and based on that kill fluid of 11.69 *ppg* was prepared that would give a hydrostatic pressure greater than the formation pressure by 250 *psi*. The drillstring consisted of 700 *ft* of 8.0" drill collars and the rest was 5.0" drill pipes. What was the old drilling mud used before the kick occurred?

Solution:

Given data:

d_h	=	Hole diameter	=	12.25"
V_{pit}	=	Pit gain	=	60 *bbls*
H_{vc}	=	Vertical height of the mud column	=	9,000 *ft*
d_{dp}	=	Drill pipe outside diameter	=	5.0"
d_c	=	Drill collar outside diameter	=	8.0"
P_{siann}	=	Shut-in casing pressure	=	700 *psi*
L_{dc}	=	Length of the drill collar	=	750 *ft*

P_{ob} = Overbalance pressure required = 250 *psi*
G_i = Kick's pressure gradient = 0.09 *psi/ft*
ρ_k = Kill fluid density = 11.69 *ppg*

Required data:
ρ_m = Old mud weight in *ppg*

To calculate the old mud weight, we should know the formation pressure and shut-in drill pipe pressure. By knowing the kill fluid density and overbalance pressure, formation pressure can be estimated using Eq. (5.9):

$$P_{bh} = P_k - P_{ob} = 0.052 \times 11.69 \times 9{,}000 - 250 = 5{,}219\ psi$$

To calculate the shut-in drillpipe pressure, we should estimate the length of the kick in the bottom of the hole. Dividing the pit gain volume by the annulus capacity between the hole and the drill collars gives:

$$H_i = \frac{V_{pit}}{C_{ann_{dc}}} = \frac{60}{\frac{12.25^2 - 8.0^2}{1029.4}} = \frac{60}{0.084} = 717.7\ ft$$

Because the length of the flux is less than the length of the drill collars, all of the flux were around the drill collars. Now by using Eq. (5.7), we can estimate the shut-in drill pipe pressure as follows:

$$G_i = G_m - \frac{P_{siann} - P_{sidp}}{H_i} = \frac{5{,}219 - P_{sidp}}{9{,}000} - \frac{700 - P_{sidp}}{717.7} = 0.09$$
$$P_{siann} = 379\ psi$$

The hydrostatic pressure of the previous mud weight used before is equal to:

$$P_m = P_f - P_{sidp} = 5{,}219 - 379 = 4{,}840\ psi$$

Thus, the old mud weight is equal to:

$$\rho_m = \frac{P_m}{0.052 \times H_{vc}} = \frac{4{,}840}{0.052 \times 9{,}000} = \mathbf{10.34\ ppg}$$

***Example 5.17*:** A kick was encountered in a well while drilling a 6.0" production hole at depth of 16,000 *ft* using 12.2 *ppg* drilling fluid. Annulus and drillpipe pressures were stabilized at 850 and 500 *psi*. The kick gradient was estimated to be 0.335 *psi/ft*. Drillstring was consisted of 500 *ft* of 4.75" drill collars and the rest was 3.5" drill pipes. What was the kick volume?

Solution:

Given data:

d_h = Hole diameter = 6.0"
H_{vc} = Vertical height of the mud column = 16,000 *ft*
d_{dp} = Drill pipe outside diameter = 3.5"
d_c = Drill collar outside diameter = 4.75"
P_{siann} = Shut-in casing pressure = 850 *psi*
P_{sidp} = Shut-in drill pipe pressure = 500 *psi*
L_{dc} = Length of the drill collar = 500 *ft*
G_i = Kick's pressure gradient = 0.335 *psi/ft*
ρ_m = Mud weight = 12.2 *ppg*

Required data:

V_k = Kick volume in *bbls*

To find the kick volume, we should estimate the kick length at the bottom of the hole. From the given data, the length of the kick can be determined using Eq. (5.7):

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = 0.335 = 0.052 \times 12.2 - \frac{850 - 500}{H_i}$$

$$H_i = 1{,}169\ ft$$

Because the kick length is greater than the length of the drill collars, some volume of the kick was around the drill collars and the rest was around drill pipes. The volume around the drill collars is equal to:

$$V_{k\_dc} = L_{dc} \times C_{ann\_dc} = 500 \times \frac{6.0^2 - 4.75^2}{1029.4} = 6.5\ bbls$$

The volume of the rest of the kick length of 669 *ft* which is around the drill pipes is equal to:

$$V_{k\_dp} = (H_i - L_{dc}) \times C_{ann\_dp} = (1169 - 500) \times \frac{6.0^2 - 3.5^2}{1029.4} = 15.4\ bbls$$

So, the total kick volume = 6.5 + 15.4 = **21.9** *bbls*

***Example 5.18*:** A vertical production hole of 8.5" size was drilled using a drillstring consists of 700 *ft* of 6.0" drill collars and 5.0" drill pipes and a 11.1 *ppg* drilling fluid was used. A kick was encountered at a depth of 13,750 *ft* when the rig crew observed a pit gain of 30 *bbls*. Annulus and drill pipe pressures were stabilized at 1,000 and 595 *psi*. The kick gradient was estimated to be 0.081 *psi/ft*. Determine the formation gradient in *ppg*.

Solution:

Given data:

d_h = Hole diameter = 8.5"
H_{vc} = Vertical height of the mud column = 13,750 *ft*

d_{dp} = Drill pipe outside diameter = 5.0"
d_c = Drill collar outside diameter = 6.0"
P_{siann} = Shut-in casing pressure = 1,000 *psi*
P_{sidp} = Shut-in drill pipe pressure = 595 *psi*
L_{dc} = Length of the drill collar = 700 *ft*
G_i = Kick's pressure gradient = 0.081 *psi/ft*

Required data:
ρ_f = Formation gradient in *ppg*.

To calculate the formation gradient, first we need to estimate the kick length. The volume of the kick around the drill collars is equal to:

$$V_{k\_dc} = L_{dc} \times C_{ann\_dc} = 700 \times \frac{8.5^2 - 6.0^2}{1029.4} = 24.7 \; bbls$$

The remaining volume of the kick was around the drill pipes. The length of the kick around the drill pipe is equal to:

$$H_{i\_dp} = \frac{V_{pit} - V_{k_{dc}}}{C_{ann_{dp}}} = \frac{30 - 24.7}{\frac{8.5^2 - 5.0^2}{1029.4}} = 115.5 \, ft$$

So, the total length of the kick is equal to:

$$H_i = L_{dc} + H_{i\_dp} = 700 + 115.5 = 815.5 \, ft$$

We know that the formation pressure can be calculated using Eq. (5.1):

$$P_{bh} = P_{sidp} + G_m H_{vc}$$

So, the mud gradient is equal to:

$$G_m = \frac{P_{bh} - P_{sidp}}{H_{vc}}$$

Thus, formation pressure can be determined using Eq. (5.7) and substituting for the mud gradient as follows:

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = \frac{P_{bh} - P_{sidp}}{H_{vc}} - \frac{P_{ann} - P_{dp}}{H_i}$$

$$0.081 = \frac{P_{bh} - 595}{13{,}750} - \frac{1{,}000 - 595}{815.5}$$

$$P_{bh} = 8{,}537 \; psi$$

Now, formation gradient in *ppg* is equal to:

$$\rho_f = \frac{P_f}{0.052 \times H_{vc}} = \frac{8,537}{0.052 \times 13,750} = \mathbf{11.94}\ \boldsymbol{ppg}$$

(Here formation pressure will be the borehole pressure)

***Example 5.19*:** An intermediate hole of 8.5" size was drilled to a depth of 9,000 *ft* using 9.7 *ppg* mud. The Fracture gradient at the casing shoe at 5,500 *ft* was estimated to be 12.0 *ppg*. Drill pipe diameter was 5.0". If suddenly they started to penetrate a formation which has a gradient of 10.2 *ppg*, estimate the maximum kick volume that can be handled without fracturing the casing shoe, taking into consideration that the maximum allowable surface pressure at annulus is 650 *psi* and pressure gradient of the kick is 0.11 *psi/ft*.

Solution:

Given data:

L_{cas}	=	Casing length	=	5,500 *ft*
d_h	=	Hole diameter	=	8.5"
D_m	=	The current length of the well	=	9,000 *ft*
d_{dpo}	=	Drill pipe outside diameter	=	5.0"
ρ_m	=	Mud weight to be used	=	9.7 *ppg*
ρ_{fm}	=	Formation gradient in *ppg*	=	10.2 *ppg*
ρ_{fr}	=	Fracture gradient at shoe in *ppg*	=	12.0 *ppg*
G_i	=	Kick's pressure gradient	=	0.11 *psi/ft*
P_{ann}	=	Maximum allowable annulus pressure	=	650 *psi*

Required data:

V_k = Maximum kick volume can be handled in *bbls*

The worst-case scenario for fracturing the casing shoe occurred when the kick is just beneath the casing shoe. In this case, the surface pressure plus the mud hydrostatic pressure will be applied directly against the casing shoe. So it is very important that these two pressures must be less or at least equal to the fracture pressure of the casing shoe. So the maximum allowed kick length in this situation can be calculated using the following equation:

$$H_i = \frac{P_{ann} + 0.052 \times \rho_m \times L_{cas} + 0.052 \times \rho_m \times (D_m - L_{cas}) - 0.052 \times \rho_{fm} \times D_m}{0.052 \times \rho_m - G_i}$$

$$= \frac{650 + 0.052 \times 9.7 \times 5,500 + 0.052 \times 9.7 \times (9,000 - 5,500) - 0.052 \times 10.2 \times 9,000}{0.052 \times 9.7 - 0.11}$$

$$= 1,055\ ft$$

Now, by using the hole-drill pipe annulus capacity, the maximum allowable kick volume is equal to:

$$V_k = H_i \times C_{ann\_dp} = 1,055 \times \frac{8.5^2 - 5.0^2}{1029.4} = \mathbf{48.5}\ \boldsymbol{bbls}$$

So, if the volume of the gas kick is greater than the above volume, the tendency of breaking the casing shoe is higher.

Example 5.20**:** An intermediate hole of 12.25" size was drilled to a depth of 7,500 *ft* using 9.3 *ppg* mud. The fracture gradient of the casing shoe at 4,500 *ft* was estimated as 11.8 *ppg*. Drill pipe diameter was 5.0". If the maximum anticipated kick volume and gradient are 73.0 *bbls* and 0.12 *psi/ft*, determine the maximum allowable surface pressure that can be applied to the annulus without fracturing the casing shoe. Assume formation gradient is equal to 9.9 *ppg*.

Solution:

Given data:

L_{cas}	=	Casing length	=	4,500 *ft*
d_h	=	Hole diameter	=	12.25"
D_m	=	The current length of the well	=	7,500 *ft*
d_{dpo}	=	Drill pipe outside diameter	=	5.0"
ρ_m	=	Mud weight	=	9.3 *ppg*
ρ_{fm}	=	Formation gradient in *ppg*	=	9.9 *ppg*
ρ_{fr}	=	Fracture gradient at shoe in *ppg*	=	11.8 *ppg*
G_i	=	Kick's pressure gradient	=	0.12 *psi/ft*
V_k	=	Maximum kick volume	=	73.0 *bbls*

Required data:

P_{ann} = Maximum allowable annulus pressure at surface in *psi*

To calculate the maximum surface pressure to be applied in the annulus at this conditions, first the kick volume should be changed to length in the annulus between the drillpipe and the hole:

$$H_i = \frac{V_k}{C_{ann_{dp}}} = \frac{73}{\dfrac{12.25^2 - 5.0^2}{1029.4}} = 601\,ft$$

By knowing the kick length, we can use the following equation to calculate the maximum surface pressure:

$$H_i = \frac{P_{ann} + 0.052\times\rho_m\times L_{cas} + 0.052\times\rho_m\times(D_m - L_{cas}) - 0.052\times\rho_{fm}\times D_m}{0.052\times\rho_m - G_i}$$

$$= \frac{\begin{array}{l} P_{ann} + 0.052\times 9.3\times 4{,}500 + 0.052\times 9.3\times(7{,}500 - 4{,}500) \\ -0.052\times 9.9\times 7{,}500 \end{array}}{0.052\times 9.3 - 0.12} = 601$$

$$P_{ann} = \mathbf{453\ \boldsymbol{psi}}$$

5.2.5 Shut-in Surface Pressure

The maximum permissible shut-in surface pressure is given by the following equation:

$$P_{sifp} = P_{ann\_m} + G_m H_{cs} \tag{5.19}$$

Where,

P_{sifp} = $G_f H_{cs}$ = shut-in fracture pressure, *psi*
H_{cs} = vertical height of the casing shoe or depth to the casing shoe, *ft*
P_{ann_m} = maximum shut-in annulus pressure, *psi*
G_f = fracture pressure gradient, *psi/ft*

Example 5.21: The surface casing with an OD of $13\frac{3}{8}''$ set at a depth of 2,100 *ft*. The fracture gradient was found 0.68 *psi/ft*. The mud density was 10.6 *ppg* with a mud gradient of 0.6 *psi/ft*. Total depth of the well was 12,000 *ft* and the internal yield was 2,500 *psi*. Determine the maximum permissible surface pressure on the annulus. Assume that the casing burst is limited to 85% of design specification.

Solution:

Given data:

H_{cs} = depth to the casing shoe = 2,100 *ft* @ $13\frac{3}{8}''$
G_f = fracture pressure gradient = 0.68 *psi/ft*
ρ_m = mud weight = 10.6 *ppg*
G_m = mud pressure gradient = 0.6 *psi/ft*
H_{vc} = vertical height of the mud column 12,000 *ft*
Y_d = Internal yield = 2,500 *psi*
85% burst pressure

Required data:

P_{ann_m} = maximum shut-in annulus pressure, *psi*

Figure 5.10 illustrates the wellbore and casing set for the Example 5.6. If the casing burst is limited to 85% of the yield pressure, permissible pressure is then:

$$85\%\ \text{burst} = 0.85 \times Y_d = 0.85 \times (2500\ psi) = 2{,}125\ psi$$

The maximum permissible annulus pressure can be determined using Eq. (5.19) as

$$\begin{aligned} P_{ann\_m} &= G_f H_{cs} - G_m H_{cs} \\ &= (0.68\ psi/ft) \times (2100\ ft) - (0.6\ psi/ft) \times (2100\ ft) = \mathbf{168.0\ psi} \end{aligned}$$

Therefore, the maximum permissible annular pressure at the surface is 168.0 *psi*, which is that pressure which would produce formation fracturing at the casing seat.

Example 5.22: The surface casing with an OD of $13\frac{3}{8}''$ set at a depth of 3,100 *ft*. The fracture gradient was found 0.68 *psi/ft*. The mud density was 12.6 *ppg* with a mud gradient of 0.6 *psi/ft*. Total depth of the well was 15,000 *ft* and the internal yield was 3,500 *psi*. Determine the maximum permissible surface pressure on the annulus. Assume that the casing burst is limited to 90% of design specification.

Solution:

Given data:

H_{cs} = depth to the casing shoe = 3,100 *ft* @ $13\frac{3}{8}''$

G_f = fracture pressure gradient = 0.68 *psi/ft*

ρ_{om} = mud weight = 12.6 *ppg*

G_m = mud pressure gradient = 0.6 *psi/ft*

H_{vc} = vertical height of the mud column 15,000 *ft*

Y_d = Internal yield = 3,500 *psi*

90% burst pressure

Required data:

P_{ann_m} = maximum shut-in annulus pressure, *psi*

If the casing burst is limited to 90% of the yield pressure, permissible pressure is then:

$$90\% \text{ burst} = 0.90 \times Y_d = 0.90 \times (3500\ psi) = 3{,}150\ psi$$

The maximum permissible annulus pressure can be determined using Eq. (5.19) as

$$P_{ann\_m} = G_f H_{cs} - G_m H_{cs}$$

$$P_{ann\_m} = (0.68\ psi/ft) \times (3100\ ft) - (0.6\ psi/ft) \times (3100\ ft)$$
$$= 248.0\ psi$$

Therefore, the maximum permissible annular pressure at the surface is 248.0 *psi*, which is that pressure which would produce formation fracturing at the casing seat.

Example 5.23: An $8\frac{1}{2}''$ diameter hole is drilled up to 8,000 *ft* with a density of 8.6 *ppg*. If the formations pore pressure at this point is 3000 *psi*, calculate i) mud pressure overbalance above the pore pressure, ii) if the mud density is 7.0 *ppg*, what would be the overbalance, and iii) if the fluid level in the annulus is dropped to 250 *ft* due to inadequate hole fill up during tripping, what would be the effect on bottom-hole pressure?

Solution:

Given data:

H_{vc} = total vertical height of the mud column = 8,000 *ft*

d_h = hole diameter = $8\frac{1}{2}''$

ρ_{om1} = original mud weight 1 = 8.6 *ppg*

P_f = formation pore pressure = 3000 *psi*

ρ_{om2} = original mud weight 2 = 7 *ppg*

H_{ann} = vertical height of the mud column in the annulus = 250 *ft*

Required data:

P_{ob1} = mud pressure overbalance at 8,000 *ft*

P_{ob2} = mud pressure overbalance at 8,000 *ft* if mud density is 7 *ppg* and Effect on bottom-hole pressure?

The overbalance at a depth of 10,000 *ft*. can be calculated by Eq. (4.34a) which can be modified for overbalance as:

$$P_{ob1} = 0.052\,\rho_{om1}H_{vc} - P_f = 0.052 \times (8.0\ ppg) \times (8000\ ft) - 3000\ psi$$
$$= 328\ psi$$

The overbalance at a depth of 15,000 *ft*. if mud density is 12 *ppg* as:

$$P_{ob2} = 0.052\,\rho_{om2}H_{vc} - P_f = 0.052 \times (7.0\ ppg) \times (8000\ ft) - 3000\ psi$$
$$= -88\ psi$$

If the mud density is not increased, the negative sign implies that the well would be underbalanced by 88 *psi* with consequent risk of an influx.

If the fluid level in the annulus is dropped by 250 *ft*, the effect would be to reduce the bottom-hole pressure by:

$$P_{bhp} = 0.052 \times (7.0\ ppg) \times (250\ ft) = 91.0\ psi$$

This result indicates that there would still be a net overbalance of 237 (i.e. 328 – 91) *psi*.

Example 5.24: Determine the kill mud density and kill mud gradient for a shut-in drillpipe pressure of 800 *psi* at a depth of 15,000 *ft*. If the original mud weight is 13.6 *ppg* and the slow circulating pump pressure is 750 *psi*, find out also the initial and final circulating pressure of the system.

Solution:

Given data:

P_{sidp} = shut-in drillpipe pressure = 800 *psi*

Hvc = total vertical height of the mud column 15,000 *ft*

ρ_{om} = original mud weight = 13.6 *ppg*

P_p = slow circulating pump pressure = 750 *psi*

Required data:

ρ_{km} = kill mud weight, *ppg*

G_k = kill mud gradient, *psi/ft*

P_{ic} = initial circulating pressure, *psi*

P_{fc} = final circulating pressure, *psi*

The kill mud weight can be calculated using Eq. (5.12) as

$$\rho_{km} = \frac{P_{sidp}}{0.052H_{vc}} + \rho_{om} = \frac{(800\ psi)}{0.052 \times (15{,}000\ ft)} + (13.6\ ppg) = \mathbf{14.63\ ppg}$$

The kill mud gradient can be calculated using Eq. (5.14) as

$$G_k = 0.052\rho_{om} + \frac{P_{sidp}}{H_{vc}} = 0.052 \times (13.6\ ppg) + \frac{(800\ psi)}{(15{,}000\ ft)} = \mathbf{0.76}\ \boldsymbol{psi/ft}$$

If we consider there is no overkill pressure, the initial circulating pressure is calculated using Eq. (5.5) as:

$$P_{ic} = P_{sidp} + P_p + P_{ok} = (800\ psi) + (750\ psi) + 0 = \mathbf{1550}\ \boldsymbol{psi}$$

Final circulating pressure is calculated is calculated using Eq. (5.6) as:

$$P_{fc} = P_p\left(\frac{\rho_{km}}{\rho_{om}}\right) = (750\ psi) \times \left(\frac{14.63}{13.6}\right) = \mathbf{806.80}\ \boldsymbol{psi}$$

Example 5.25: A kick was detected while drilling a high-pressure zone. The depth of the formation was recorded 9,000 *ft* with a mud density of 11.3 *ppg*. The crew shut-in the well and recorded the pressure for drillpipe and drill collar as 300 *psi* and 600 *psi* respectively. The observed total pit gain was 25.0 *bbl*. The annular capacity against 950 *ft* of drill collar is 0.0836 *bbl/ft* and the overkill safety margin is 0.50 *ppg*. Compute the formation pressure, influx density, the type of fluid, required kill mud weight, and kill mud gradient.

Solution:

Given data:

H_{vc} = total vertical height of the mud column = 9,000 *ft*
ρ_{om} = original mud weight = 15.3 *ppg*
P_{sidp} = shut-in drillpipe pressure = 350 *psi*
P_{sidc} = shut-in drill collar pressure = 600 *psi*
V_{pit} = pit gain volume = 35 *bbls*
L_{dc} = length of drill collar = 950 *ft*
C_{ann_dc} = the annulus capacity behind the drill collar = 0.0836 *bbl/ft*
ρ_{ok} = overkill mud as a safety margin = 0.5 *ppg*

Required data:

P_{bh} = formation pressure, *psi*
ρ_k = kick fluid or influx density, *ppg*

Type of fluid

ρ_{km} = kill mud weight, *ppg*
G_k = kill mud gradient, *psi/ft*

Formation pressure can be calculated using Eq. (5.11) as

$$P_{bh} = P_{sidp} + 0.052\rho_{om}H_{vc} = (350) + 0.052 \times (15.3\ ppg) \times (9000\ ft)$$

$$P_{bh} = 7510.4\ \boldsymbol{psi}$$

To calculate the kick density, we first need to calculate the length of the kick and therefore, the annular volume.

The annular volume against the drill collar,

$$V_{ann\_dc} = L_{dc} \times C_{ann\_dc} = 950\ ft \times 0.0836 \frac{bbl}{ft} = 79.42\ bbl$$

As long as $V_{pit} < V_{ann\_dc}$, the length of the kick can be calculated using Eq. (5.15) as:

$$L_k = \frac{V_{pit}}{C_{ann\_dc}} = \frac{35.0\ bbl}{0.0836\ bbl/ft} = 418.66\ ft$$

The density of the kick fluid is calculated using Eq. (5.18) as

$$\rho_k = \rho_{om} + \frac{P_{idp} - P_{icp}}{0.052 L_k} = (15.3\ ppg) + \frac{(350\ psi - 600\ psi)}{0.052 \times (418.66\ ft)} = \mathbf{3.82\ ppg}$$

Therefore, the kick fluid is **gas**.

Consider overkill mud as a safety margin, the kill mud weight can be calculated using Eq. (5.13) as

$$\begin{aligned} \rho_{km} &= \frac{P_{sidp}}{0.052 H_{vc}} + \rho_{om} + \rho_{ok} \\ &= \frac{(350\ psi)}{0.052 \times (9{,}000\ ft)} + (15.3\ ppg) + (0.5\ ppg) = \mathbf{16.5\ ppg} \end{aligned}$$

The kill mud gradient can be calculated using Eq. (5.14) as

$$\begin{aligned} G_k &= 0.052 \rho_{om} + \frac{P_{sidp}}{H_{vc}} = 0.052 \times (15.3\ ppg) + \frac{(350\ psi)}{(9{,}000\ ft)} \\ &= \mathbf{0.83\ psi/ft} \end{aligned}$$

Example 5.26: The surface casing with an OD of $13\frac{3}{8}''$ set at a depth of 5,100 *ft*. The fracture gradient was found 0.68 *psi/ft*. The mud density was 15.6 *ppg* with a mud gradient of 0.6 *psi/ft*. Total depth of the well was 14,000 *ft* and the internal yield was 5,500 *psi*. Determine the maximum permissible surface pressure on the annulus. Assume that the casing burst is limited to 95% of design specification.

Solution:

Given data:

H_{cs} = depth to the casing shoe = 5,100 *ft* @ $13\frac{3}{8}''$

G_f = fracture pressure gradient = 0.68 *psi/ft*

ρ_{om} = mud weight = 15.6 *ppg*

G_m = mud pressure gradient = 0.6 *psi/ft*

H_{vc} = vertical height of the mud column 14,000 *ft*

Y_d = Internal yield = 5,500 *psi*

95% burst pressure

Required data:

P_{ann_m} = maximum shut-in annulus pressure, *psi*

If the casing burst is limited to 90% of the yield pressure, permissible pressure is then:

$$95\%\text{ burst} = 0.95 \times Y_d = 0.90 \times (5500\ psi) = 5{,}225\ psi$$

The maximum permissible annulus pressure can be determined using Eq. (5.19) as

$$P_{ann\_m} = G_f H_{cs} - G_m H_{cs}$$

$$P_{ann\_m} = (0.68\ psi/ft) \times (5100\ ft) - (0.6\ psi/ft) \times (5100\ ft) = 408.0\ psi$$

Therefore, the maximum permissible annular pressure at the surface is 408.0 *psi*, which is that pressure which would produce formation fracturing at the casing seat.

5.3 Multiple Choice Questions

1. Well control means assurance of formation fluids that does not flow in an ______________ way.
   a) Uncontrolled
   b) Controlled
   c) Semi-controlled
   d) None of the above
2. An unexpected entry of formation fluids into the wellbore is known as ______________.
   a) Punch
   b) Kick
   c) Tension
   d) None of the above
3. Technology used to control the fluid invasion and to maintain a balance between borehole pressure and formation pressure is known as ______________.
   a) Well control system
   b) Reservoir management system
   c) Well engineering system
   d) None of the above
4. Which of the following is not an option in well control system?
   a) Detect a kick
   b) Close the well at surface
   c) Remove formation fluid
   d) Make the well safe
   e) None of the above

5. The first line of defense in well control is to have sufficient __________ pressure in the wellbore.
   a) Drilling fluid
   b) Formation pressure
   c) Abnormal pressure
   d) None of the above
6. If the formation pressure is greater than the mud pressure, there is the possibility to have a __________.
   a) Oil
   b) Kick
   c) Gas
   d) None of the above
7. Equipment used to control blowouts is
   a) BOPs
   b) WOB
   c) Drilling rig
   d) None of the above
8. BOPs are referred to as the __________ component of well control system.
   a) Active
   b) Passive
   c) Auxiliary
   d) None of the above
9. Kick occurs due to the pressure __________.
   a) Transition
   b) Balance
   c) Imbalance
   d) None of the above
10. Which of the following causes pressure imbalance?
    a) Low mud density
    b) Low fluid level
    c) Lost circulation
    d) All of the above
11. The __________ the porosity and permeability of the formation are, the greater the potential for a severe kick is.
    a) Average
    b) Lower
    c) Higher
    d) None of the above
12. Formation fluid easily comes into the wellbore when there is __________ negative pressure differential.
    a) Greater
    b) Lower
    c) Intermediate
    d) None of the above

13. The ______________ of the well at all times must remain above the pore pressure of the formation to prevent additional influx of the formation fluids.
    a) Bottom-hole pressure
    b) Drilling mud
    c) Lost circulation
    d) None of the above

14. The most recent well control principle developed as blowout prevention is ______________.
    a) Primary control
    b) Secondary control
    c) Tertiary control
    d) None of the above

15. Which of the following is a well control principle?
    a) Primary control
    b) Secondary control
    c) Tertiary control
    d) All of the above

16. Which of the following well control principles exists for all drilling activities?
    a) Primary control
    b) Secondary control
    c) Tertiary control
    d) None of the above

17. Which of the following well control principles is the highest level of security and control?
    a) Primary control
    b) Secondary control
    c) Tertiary control
    d) None of the above

18. Which of the following well control principles is defined as the control by confirming that the borehole pressure is greater than the formation pressure?
    a) Primary control
    b) Secondary control
    c) Tertiary control
    d) None of the above

19. Primary control maintains a ______________ overbalance on the formation pressure.
    a) Negative
    b) Positive
    c) Pseudo
    d) None of the above

20. Which of the following is reason for low mud weight in wellbore?
    a) Overpressured zone
    b) Gas cutting of the mud
    c) Water contamination
    d) All of the above
21. The normal industry practice is to keep the overbalance pressure at around ________.
    a) 200–300 *psi*
    b) 100–200 *psi*
    c) 100–500 *psi*
    d) 300–500 *psi*
22. Large amount of overbalance reduces the ________.
    a) Amount of kick
    b) Rate of Penetration
    c) Mud density
    d) All of the above
23. Seepage of gas from the formation into the circulating mud produces a dramatic reduction in the ________ at surface.
    a) Mud weight
    b) Pore pressure
    c) Rate of penetration
    d) None of the above
24. A mud cut to 90% of its original weight will produce a decrease in bottom-hole pressure of only ________.
    a) 10 *psi*
    b) 20 *psi*
    c) 15 *psi*
    d) 5 *psi*
25. A process where drillpipe acts like a piston is known as
    a) Swabbing
    b) Tripping
    c) Weight on bit
    d) None of the above
26. The opposite effect of swabbing process is known as
    a) Surging
    b) Tripping
    c) WOB
    d) None of the above
27. When a fractured formation is drilled, ________ occurs.
    a) Lost circulation
    b) Kick
    c) Influx
    d) None of the above

28. Lost circulation can occur if too high a mud weight is used and the ____________ is exceeded.
    a) Temperature gradient
    b) Pressure gradient
    c) Fracture gradient
    d) All of the above
29. The pit gain indicates that the ____________ over the well has been lost.
    a) Primary control
    b) Secondary control
    c) Tertiary control
    d) All of the above
30. Purpose of secondary control is to
    a) Stop the flow of unexpected fluids
    b) Safely discharge the influx
    c) Prevent further influx
    d) All of the above
31. Pulling the drillstring too fast can cause
    a) Lost circulation
    b) Changes in the drilling mud properties
    c) Swabbing
    d) All of the above
32. High drilling fluid weight can
    a) Cause loss of circulation
    b) Break the formations
    c) Cause swabbing
    d) a and b
33. Underground blowouts can result in
    a) Swabbing
    b) Loss of circulation
    c) Drilling gas wells
    d) All of the above
34. All of the following are the reasons of lost circulation **except**
    a) Running the drillstring too fast
    b) High mud weight
    c) Pressure due to annular circulating frictions
    d) None of the above
35. Pressure of the annulus while circulation is ____________ than the annular pressure when the circulation is stopped.
    a) Greater
    b) Similar
    c) Less
    d) None of the above

36. Which of the following can be an indication of over-pressured zones during drilling?
    a) Increase in the hook load weight
    b) Increase in the pump pressure
    c) Long-shaped drill cuttings
    d) All of the above

37. During well kicks, the bit should be raised from the bottom of the well to
    a) Lessen the chances of getting stuck
    b) Let the Kelly cock above the rotary table to be closed if required
    c) Remove the Kelly if required
    d) All of the above

38. What is the consequence of not keeping the wellbore full of drilling fluid?
    a) Loss of circulation
    b) Swabbing
    c) Allowing formation fluids to enter the wellbore
    d) None of the above

39. What is the direct consequence of running the drillstring too fast?
    a) Break the weak formations
    b) Well kicks
    c) Swabbing
    d) a and b

40. What is the indirect consequence of running the drillstring too fast
    a) Loss of circulation
    b) Swabbing
    c) Well kicks
    d) a and c

Answers: 1a, 2b, 3a, 4e, 5a, 6b, 7a, 8b, 9c, 10d, 11c, 12a, 13a, 14c, 15d, 16a, 17c, 18a, 19b, 20d, 21a, 22b, 23a, 24a, 25a, 26a, 27a, 28c, 29a, 30d, 31c, 32d, 33b, 34d, 35a, 36c, 37d, 38c, 39a, 40c.

5.4 Summary

The chapter covered almost all aspects of well control and monitoring system in terms of MCQs and workout examples. The chapter presents almost all the formulas related to the well control and monitoring system. The exercise solutions are given in Appendix A. Moreover, this chapter added some more MCQs for the student's self-practice and the answers are given in Appendix B. The MCQs covered almost all the materials covered in this chapter.

5.5 Exercise and MCQs for Practice

5.5.1 Exercises (Solutions are in Appendix A)

Exercise 5.1: An oil zone in a certain field has been shifted up 750 *ft* in one part of that field. If the formation gradient in the deeper area is 0.5875 *psi/ft* and in the shallower area is 0.0.6279 *psi/ft*, determine the depth of the oil formation in the shallower and deeper areas assuming that the formation pressures are equal for both areas of the field. All depths and pressure gradients are measured at the center of the formation. ***Answer: 11,657ft and 10,907 ft***

Exercise 5.2: A vertical well drilled to a depth of 10,750 *ft*, and then 13 3/8" casing having inside diameter of 12.52" was set and cemented down to that depth. A new section of 12.25" size will be drilled to a depth of 14,900 *ft*. The fracture gradient at casing shoe was estimated to be 13.89 *ppg* while the formation gradient is 12.9 *ppg*. The outside diameter of drillpipe is 5.0". If the expected gas kick volume at casing shoe is 150 *bbls* and gas gradient is 0.13 *psi/ft*, calculate the maximum mud weight that can be used and handle the kick without fracturing the formations at the casing shoe? ***Answer: 13.42*** *ppg*

Exercise 5.3: A vertical well has been drilled, cased and cemented with 9 5/8" casing at depth of 13,000 *ft*. The well to be drilled to a target depth of 17,000 *ft* using mud has a maximum mud weight of 14.5 *ppg*. The fracture gradient at casing shoe is 15.7 *ppg* while the formation gradient is 14.1 *ppg*. The inside diameter of the last casing is 8.67" and the new hole diameter is 8.5". The standard surface temperature is 75 °F while the temperature gradient is 0.02 °F/*ft*. The drill pipe outside diameter is 5.0". If the expected gas kick gradient is 0.08 *psi/ft*, calculate the maximum volume of gas kick that can be handled in this case without fracturing the formations at the casing shoe. ***Answer: 78.9 bbls***

Exercise 5.4: A well kick was encountered while drilling a production section of 8.5" in a vertical well. Rig crew recorded a pit gain of 20 *bbls*. Shut-in casing pressure was stabilized at 900 *psi* while shut-in drillpipe pressure was zero because floating valve was used as part of BHA. Current depth was 14,500 *ft* and mud weight was 11.9 *ppg*. The outside diameter of drill collars and length were 6.0", and 450 *ft* respectively, while the outside diameter of the drillpipe was 4 ¾". Rig crew increased the drillpipe pressure in steps in order to determine the shut-in drillpipe pressure. When drillpipe pressure increased to 800 *psi*, annulus pressure increased to 975 *psi*. If it is required to have mud hydrostatic pressure greater than the formation pressure by 150 *psi*, determine the nature of the influx and the new mud weight to control the well back. ***Answer: 0.29 psi/ft, 13.06 ppg***

Exercise 5.5: A well kick of 35 *bbls* was noticed when drilling a 12.25" vertical hole using 10.54 *ppg* mud at depth of 6,250 *ft*. Later, the kick was removed and the rig crew found that the kick gradient was 0.16 *psi/ft*. Drilling string consists of 650 *ft* of 9.0" drill collars and 5.5" drill pipes. If the formation gradient was found to be 0.592 *psi/ft*, what were the shut-in annulus and drill pipe pressures? If it was required that the kill fluid hydrostatic pressure must be 100 *psi* higher than the formation pressure, calculate the kill fluid gradient in *psi/ft*. ***Answer: 275psi, 525 psi, 0.61 psi/ft***

Exercise 5.6: A pit gain of 15 *bbls* was noticed while drilling a 6.0" hole at a depth of 17,500 *ft*, and shut-in annulus pressure was stabilized at 950 *psi*. The kick gradient was later estimated to be 0.20 *psi/ft*. Based on the information, a kill mud of 13.33 *ppg* was prepared that would give a hydrostatic pressure greater than the formation pressure by 300 *psi*. The drillstring consisted of 600 *ft* of 4¾" drill collars and the rest was 3.5" drill pipes. Calculate the old drilling mud used to drill the hole before the kick was occurred. ***Answer: 12.4 ppg***

Exercise 5.7: While drilling a 12.25" intermediate hole at a depth of 10,000 *ft* using 9.9 *ppg* drilling mud, a kick was encountered in a well. Annulus and drill pipe pressures were stabilized at 625 and 395 *psi*. The kick gradient was estimated to be 0.122 *psi/ft*. Drillstring was consisted of 600 *ft* of 8.0" drill collars and the rest was 5.0" drill pipes. What was the kick volume? ***Answer: 49 bbls***

Exercise 5.8: A vertical production hole of 6.0" size was drilled using drillstring consisting of 800 *ft* of 4.75" drill collars and 3.5" drillpipe by using 13.8 *ppg* drilling fluids. A kick was encountered at a depth of 18,000 *ft* when the rig crew observed a pit gain of 18 *bbls*. Annulus and drillpipe pressures were stabilized at 1,200 *psi*, and 500 *psi* respectively. The kick gradient was estimated to be 0.097 *psi/ft*. Determine the formation gradient in *ppg* and the old mud weight. ***Answer: 14.32 ppg, 13.78 ppg***

Exercise 5.9: A production hole of 6.0" size was drilled to a depth of 17,250 *ft* using 13.3 *ppg* mud. The last casing shoe was at 14,000 *ft*, and the fracture gradient at the casing shoe was estimated to be 15.0 *ppg*. The diameter of the drillpipe was 3.5". If the formation gradient at the current depth was assumed to be 13.7 *ppg*, estimate the maximum kick volume that can be handled without fracturing the casing shoe, taking into consideration that the maximum allowable surface pressure in the annulus is 1,238 *psi* and the gradient of the kick is 0.14 *psi/ft*? ***Answer: 36.8 bbls***

Exercise 5.10: A production hole of 8.5" size was drilled to a depth of 12,500 *ft* using 11.6 *ppg* mud. The fracture gradient of the casing shoe, which is set at 10,500 *ft*, was estimated to be 13.0 *ppg*. The diameter of drillpipe was 5.0". If the maximum anticipated kick volume and gradients are 70.0 *bbls* and 0.19 *psi/ft*, determine the maximum allowable surface pressure that can be applied to the annulus without fracturing the casing shoe. Assume formation gradient is equal to 11.8 *ppg*. ***Answer: 760*** *psi*

5.5.2 Exercises (Self-Practices)

E5.1: A $7\frac{1''}{2}$ diameter hole is drilled up to 9,000 *ft* with a density of 11.5 *ppg*. If the formation pore pressure at this point is 4800 *psi*, calculate i) mud pressure overbalance above the pore pressure, ii) if the mud density is 10.5 *ppg*, what would be the overbalance, and iii) if the fluid level in the annulus is dropped to 300 *ft* due to inadequate hole fill up during tripping, what would be the effect on bottom-hole pressure?

E5.2: A $6\frac{1''}{2}$ diameter hole was drilled up to 7,000 *ft* where it encountered an overbalance of 350 *psi*. If the formation pore pressure at this point is 4800 *psi*, calculate i) mud density, ii) if the mud density is 9.5 *ppg*, what would be the overbalance, and iii) if the fluid level in the annulus is dropped to 200 *ft* due to inadequate hole fill up during tripping, what would be the effect on bottom-hole pressure?

E5.3: A target depth was set at 7,500 *ft* with a hole size of 6.5 in. The drilling crew noticed that there was a pit gain of 14 *bbls*. The well is shut-in and the drillpipe and annulus pressures were recorded as 600 *psi*, and 700 *psi* respectively. The bottom-hole assembly consists of 550 *ft* of $4\frac{3''}{4}$ OD collars and $3\frac{1''}{2}$ drillpipe. The mud weight is 10.4 *ppg* with a mud pressure gradient of 0.5 *psi/ft*. Identify the influx and calculate the new mud weight, including an overbalance of 200 *psi*.

E5.4: The surface casing with an OD of $8\frac{5''}{8}$ set at a depth of 2,000 *ft*. The fracture gradient was found 0.76 *psi/ft*. The mud density was 9.6 *ppg* with a mud gradient of 0.5 *psi/ft*. Total depth of the well was 10,000 *ft* and the internal yield was 2,470 *psi*. Determine the maximum permissible surface pressure on the annulus. Assume that the casing burst is limited to 80% of design specification.

E5.5: The surface casing with an OD of $8\frac{5''}{8}$ set at a depth of 2,000 *ft*. The fracture gradient was found 0.76 *psi/ft*. The mud density was 9.6 *ppg* with a mud gradient of 0.5 *psi/ft*. Total depth of the well was 10,000 *ft* and the internal yield was 2,470 *psi*. Determine the maximum permissible surface pressure on the annulus. Assume that the casing burst is limited to 80% of design specification.

E5.6: Determine the kill mud density and kill mud gradient for a shut-in-drillpipe pressure of 650 *psi* at a depth of 11,000 *ft*. If the original mud weight is 12.5 *ppg* and the slow circulating pump pressure is 800 *psi*, find out also the initial and final circulating pressure of the system.

E5.7: A kick was detected when drilling a high-pressure zone of a depth of the formation 10,000 *ft* with a mud density of 9 *ppg*. After the well was shut-in, the pressures recorded were for drillpipe and drill collar as 350 *psi* and 430 *psi* respectively. The total pit gain observed was 5 *bbl*. The annular capacity against 950 *ft* of drill collar is 0.0292 *bbl/ft*. the overkill safety margin is 0.50 *ppg*. Compute the formation pressure, kick density, the type of fluid, and required kill mud weight.

E5.8: A well was being drilled at a high-pressure zone of 9,500 *ft* vertical depth where 10.5 *ppg* mud was being circulated at a rate of 10.0 *bbl/min*. A pit gain of 30 *bbl* was noticed over a 2.5 minutes period before the pump was stopped and the BOPs were closed. After the pressures stabilized, an initial drillpipe pressure of 550 *psi* and an initial casing pressure of 750 *psi* were recorded by the driller. The annular capacity against 750 *ft* of drill collar was 0.029 *bbl/ft* and the annular capacity against 800 *ft* of drillpipe was 0.072 *bbl/ft*. Compute the formation pressure, influx density.

5.5.3 MCQs (Self-Practices)

1. The term "well control" means
   a) Controlling the circulation of drilling fluid in and out of the well
   b) Controlling the inclination of the well
   c) Controlling any undesired flow of hydrocarbons while drilling
   d) None of the above
2. A well kick is defined as
   a) An unexpected loss of drilling fluid into the formations
   b) An unexpected entry of formation fluids into the wellbore
   c) An expected entry of formation fluids into the wellbore
   d) All of the above
3. All of the following are the well kick signs **except**
   a) Increase of the mud weight
   b) Increase of mud volume at the mud pits
   c) Well flowing while pumps are shut off
   d) Increase in pumping flow rate
4. Severity of kick doesn't depend on
   a) Type of formation
   b) Formation pressure
   c) Nature of influx
   d) Type of drilling fluid
5. High porosity and low permeability formations have
   a) Greater potential for well kicks
   b) Lower potential for well kicks
   c) No potential for well kicks
   d) None of the above
6. High porosity and high permeability formations have
   a) Greater potential for well kicks
   b) Lower potential for well kicks
   c) No potential for well kicks
   d) None of the above
7. Tight formations have
   a) Greater potential for well kicks
   b) Lower potential for well kicks
   c) No potential for well kicks
   d) None of the above
8. All of the following are mandatory in well control system **except**
   a) Well kicks monitoring devices
   b) Special tanks to store the removed formation fluids
   c) Specific equipment to shut in the well at the surface
   d) Specific equipment to remove the formation fluids out of the wellbore

9. The active component of a well control system consists of
   a) Having blowout preventers
   b) Having emergency shut-down system
   c) Having a drilling fluid with a sufficient hydrostatic pressure
   d) All of the above
10. The passive component of well control consists of
    a) Having emergency shut-down system
    b) Having blowout preventers
    c) Having a drilling fluid with a sufficient hydrostatic pressure
    d) None of the above
11. In conventional drilling, bottom-hole pressure
    a) Must be less than the formation pressure
    b) Must be greater than the formation pressure
    c) Must be equal to the formation pressure
    d) Can be greater or less than the formation pressure
12. The primary well control is done by
    a) Making sure the borehole pressure is greater than the formation pressure
    b) Making sure blowout preventers are ready for any kicks
    c) Making sure kill fluid is ready to be pumped
    d) All of the above
13. All of the following can reduce the height of the mud column **except**
    a) Tripping the drill string out
    b) Swabbing the formation fluids into the borehole
    c) Lost circulation
    d) Decreasing the mud circulating pressure
14. The main purpose of the secondary control is to
    a) Stop the flow of formation fluids to the wellbore
    b) Circulate the formation fluids to the surface safely
    c) Prevent further influx to enter the borehole
    d) All of the above
15. All of the following are the primary well kick indicators **except**
    a) Mud pits volume increase
    b) Increase in the pumping flow rate
    c) Reduction in drillpipe weight
    d) Flow of the well while pumps are shut off
16. Formation pore pressure can be calculated using
    a) Drill pipe shut-in pressure
    b) Drill pipe flowing pressure
    c) Casing shut-in pressure
    d) a and c

17. After shut-in the well during well kick situations, casing shut-in pressure should be
    a) Similar to the drillpipe shut-in pressure
    b) Different than the drillpipe shut-in pressure
    c) Much less than the drillpipe pressure
    d) None of the above
18. The weight of the kill mud to be used during kick control operations should be
    a) Equal to the shut-in casing pressure
    b) Slightly less than the shut-in casing pressure
    c) Greater than the annular pressure
    d) Double the casing shut-in pressure
19. The calculated flux gradient was 0.2 *psi/ft*; what is the type of the fluid that entered the wellbore?
    a) Gas
    b) Oil
    c) Water
    d) a and b.
20. The calculated flux gradient was 0.45 *psi/ft*; what is the type of the fluid that entered the wellbore?
    a) Gas
    b) Oil
    c) Water
    d) b and c
21. Liquid kicks generally develop ______________ casing shut-in pressure when compared to gas kicks.
    a) Lower
    b) Higher
    c) Similar
    d) None of the above
22. Bag type preventers can close around
    a) A specific drillpipe sizes
    b) Any drillpipe sizes
    c) Tool joints of the drill pipes
    d) None of the above
23. Ram type preventers can close around
    a) Any drill collar sizes
    b) A specific drill collar size
    c) Tool joints of the drill collars
    d) None of the above
24. All of the following are the types of ram preventers except
    a) Shear rams
    b) Blind rams
    c) Circulating rams
    d) Pipe rams

25. Blind rams are used during well control when
    a) Drill pipes are in the well
    b) The drillstring is tripped in
    c) The drillstring is tripped out
    d) None of the above
26. Which of the following rams will be used if there is a blowout from inside the tubing?
    a) Blind rams
    b) Shear rams
    c) Annular rams
    d) Pipe rams
27. Which of the following rams will be used when the well is flowing only from the annulus?
    a) Pipe rams
    b) Shear rams
    c) Blind rams
    d) None of the above
28. Kill line is normally used to
    a) Circulate the kick fluid out
    b) Pump the kill fluid through inside the drill string
    c) Pump the kill fluid through the annulus
    d) None of the above
29. Kill line is connected to the well
    a) Above the BOP
    b) At the stand pipe
    c) Below the BOP
    d) All of the above
30. Diverter system is usually installed in the wellhead for drilling the
    a) Intermediate section
    b) Production section
    c) Liner section
    d) Surface section
31. Choke manifold is required to
    a) Control the flow of the flux out of the well
    b) Divert the fluids to the burning pit
    c) Allow remote flow and shut in the annulus
    d) All of the above
32. The main disadvantage of using float valves in terms of well control is
    a) Restrict the pumping rate of the drilling mud
    b) Increase the pressure loss inside the drill string
    c) Shut-in drillpipe pressure cannot be read
    d) None of the above

33. The drilling process which is precisely used to control the annular pressure during drilling is called
    a) Underbalanced drilling
    b) Managed pressure drilling
    c) Overbalanced drilling
    d) Conventional drilling
34. Managed pressure drilling (MPD) was designed to solve
    a) Lost circulation problems
    b) Stuck pipe
    c) Wellbore instability
    d) All of the above
35. In which of the following wells is the MPD not recommended to be used?
    a) Depleted reservoirs
    b) Wells have lots of lost circulation areas
    c) Wells penetrate abnormal formations
    d) All of the above
36. Which of the following is the main advantage of the wave processing system?
    a) Measuring the volume of the kick
    b) Detecting the entry of gases ahead of time
    c) Measuring the flow rate of the flux
    d) All of the above
37. Which of the following is the main disadvantage of the wave processing system?
    a) Does not work properly when drilling operations stop
    b) Does not work properly in offshore environment
    c) Does not work properly in conventional drilling
    d) None of the above
38. All of the following are the well kick indicators **except**
    a) Increase the flow rate
    b) Increase in the mud pits
    c) Flow while pumps are shut off
    d) None of the above
39. All of the following are the causes of a well kick except
    a) Abnormal pressure zones
    b) Swabbing
    c) Using oil-based muds in sandstone formations
    d) All of the above
40. Which of the following can cause swabbing?
    a) Balled bit
    b) Pumping the mud at low rates
    c) Pumping the mud at low pressures
    d) None of the above

5.6 Nomenclature

A_{ann} = cross-sectional area of the annulus, *bbls/ft*
C_{ann_dc} = the annulus capacity behind the drill collar, *bbl/ft*
C_{ann_dp} = the annulus capacity behind the drillpipe, *bbl/ft*
G_f = fracture pressure gradient, *psi/ft*
G_i = influx pressure gradient, *psi/ft*
G_k = kill mud pressure gradient, *psi/ft*
G_m = mud pressure gradient, *psi/ft*
H_i = vertical height of the influx or kick, *ft*
H_m = vertical height of mud in the annulus after influx, *ft* = $H_{vc} - H_i$
H_{vc} = total vertical height of the mud column, *ft*
H_{cs} = vertical height of the casing shoe or depth to the casing shoe, *ft*
L_{dc} = length of the drill collar, *ft*
L_k = kick length (i.e., vertical height of influx, H_i), *ft*
P_{ann_m} = maximum shut-in annulus pressure, *psi*
P_{bh} = bottom-hole (i.e., formation) pressure, *psi*
P_{fc} = final circulating pressure, *psi*
P_{ic} = initial circulating pressure, *psi*
P_{icp} = initial stabilized drill collar pressure, *psi*
P_{idp} = initial stabilized drillpipe pressure, *psi*
P_{ob} = overbalance pressure, *psi*
P_{ok} = overkill pressure, *psi*
P_p = slow circulating pump pressure, *psi*
P_{sifp} = $G_f H_{cs}$ = shut-in fracture pressure, *psi*
P_{sidp} = shut-in drillpipe pressure, *psi*
P_{siann} = shut-in annulus pressure, *psi*
V_{ann_dc} = the annulus volume against drill collar, bbl
V_{pit} = pit gain volume, *bbls*
ρ_k = kick fluid (i.e., influx) density, *ppg*
ρ_{km} = kill mud weight, *ppg*
ρ_{ok} = overkill mud weight for safety margin, *ppg*
ρ_{om} = original mud weight, *ppg*

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6

Formation Pore and Fractures Pressure Estimation

6.1 Introduction

The magnitude of the pressure in the pores of a formation is known as the formation pore pressure. It is sometime called simply formation pressure and is also designated as formation fluid pressure, or pressure in fluid contained in the pore spaces of the rock. This formation pressure is an essential consideration in many aspects of well planning and operations. This chapter discusses how to determine the formation fluid pressure and fracture pressure through workout examples. Understanding of the variation of these two parameters with depth is very important in planning and drilling of a well. Therefore, sets of multiple choice question (MCQs) are included which are related to the drilling fluid technology and their problems and solutions. Workout examples related to formation fluid pressure and fracture pressure are extensively covered. Workout examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

6.2 Different Mathematical Formulas and Examples

6.2.1 Underground Stresses

For an underground formation, if the density of the rock varies with depth, the vertical stress at depth D becomes can be expressed as:

$$\sigma_v = \int_0^D \rho g dD \tag{6.1}$$

6.2.2 Formation Pressure

i) Normal Pressure

Under normal compaction the pore fluid within the pore is treated as normal pore pressure which can be written at a depth, D as:

$$P_{fn} = \int_0^D \rho_{fn} g dD \tag{6.2}$$

where

P_{fn} = normal formation pore pressure

ρ_{fn} = formation fluid density at normal condition

D = total vertical depth

dD = vertical depth from a reference point (ground surface)

g = gravitational acceleration

In field unit hydrostatic pressure Eq. (6.2) can be written by recalling Eq. (4.34a) as:

$$P_{fn} = 0.052 \rho_m D \tag{6.3}$$

where

P_{fn} = normal formation pore pressure, *psi*

ρ_m = mud weight, *ppg*

D = total vertical depth, *ft*

Example 6.1: Find out the normal pore pressure at a depth of 5,000 *ft* below sea level. Assume that the drilling activities will be conducted in the California area. Also find out the mud weight for that area.

Solution:

Given data:

D = total vertical depth = 5,000 *ft*

G_{np} = normal pressure gradient for California
= 0.439 *psi/ft* (Table 6.1)

Required data:

P_{fn} = normal pore pressure, *psi*

ρ_m = mud weight, *ppg*

The normal pore pressure for the California area can be estimated as:

$$P_{fn} = G_{np} D = (5000\ ft) \times (0.439\ psi/ft) = \mathbf{2{,}195.0\ psi}$$

The mud weight can be calculated using Eq. (6.3) as:

$$\rho_m = \frac{P_{fn}}{0.052\ D} = \frac{(2195\ psi)}{0.052 \times (5000\ ft)} = \mathbf{8.44}\ \boldsymbol{ppg}$$

Table 6.1 Normal formation pressure gradients for several areas of active drilling.

	Pressure gradient (psi/ft)	Density (g/cm³)
Anadarko Basin	0.433	1
California	0.439	1.014
Gulf of Mexico	0.465	1.074
Mackezie Delta	0.442	1.021
Malaysia	0.442	1.021
North Sea	0.452	1.044
Rocky Mountain	0.436	1.007
West Africa	0.442	1.021
West Texas	0.433	1

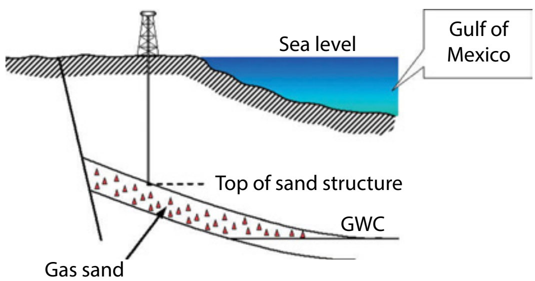

Figure 6.1 Differential density effects (Hossain and Al-Majed, 2015).

ii) Abnormal Pressure

Example 6.2: A gas sand reservoir is shown in Figure 6.1 where the average gas density was measured as 0.65 lb_m/gal. Assume that the water-filled portion of the sand is pressured normally and the gas/water contact is at a depth of 6,000 *ft*. Find out the mud weight that would be required to drill through the top of the sand structure safely at a depth of 4,500 *ft*.

Solution:

Given data:

ρ_g = gas density = 0.65 lb_m/gal
D_{gw} = total vertical depth of gas/water contact = 6,000 *ft*
D_s = total vertical depthof the sand structure = 4,500 *ft*
G_{np} = normal pressure gradient for gulf of Mexico = 0.465 *psi/ft* (Table 6.2)

Required data:

ρ_m = mud weight, *ppg*

The normal pressure at a depth of 6,000 *ft* of gas/water contact can be estimated using pressure gradient concept as:

$$P_{fn\_GWC} = G_{np}D = (6000\,ft)\times(0.465\,psi/ft) = 2{,}790.0 \text{ psi}$$

Again, the normal pressure at a depth of 4,500 *ft* where gas sand exists can be estimated using pressure gradient concept as:

$$P_{fn\_GS} = G_{np}D = (4500\,ft)\times(0.465\,psi/ft) = 2{,}092.5 \text{ psi}$$

However, the pressure in the gas sand at 4,500 *ft* can also be determined hydrostatic pressure concept as:

$$\begin{aligned} P_{fn\_GS} &= P_{pn\_GWC} - 0.052(\rho_g)\times(D_{GWC} - D_{GS}) \\ &= 2{,}790.0 - 0.052\times\left(0.65\frac{lbm}{ft}\right)\times(6{,}000\,ft - 4{,}500\,ft) \\ &= 2{,}739.3 \text{ psi} \end{aligned}$$

This pressure is higher than that calculated based on normal pressure gradient at 4,500 *ft*. therefore, the minimum mud weight can be calculated using Eq. (6.3) as:

$$\rho_m = \frac{P_{fn\_GS}}{0.052\ D} = \frac{2739.3\ psi}{0.052\times 4500\,ft} = \mathbf{11.7}\ \boldsymbol{ppg}$$

Example 6.3: A gas sand reservoir in the Gulf of Mexico has an average gas density of 0.70 lb_m/gal. Assume that the water-filled portion of the sand is pressured normally and the gas/water contact is at a depth of 7,000 *ft*. Find out the mud weight that would be required to drill through the top of the sand structure safely at a depth of 5,500 *ft*.

Solution:

Given data:

ρ_g	=	Gas density	=	0.7 lb_m/gal
D_{GWC}	=	Total vertical depth of gas/water contact	=	7,000 *ft*
D_{GS}	=	Total vertical depth of sand structure	=	5,500 *ft*
G_{np}	=	normal pressure gradient for Gulf of Mexico		
	=	0.465 *psi/ft* (Table 6.2)		

Required data:

ρ_m = Mud Weight, *ppg*

The normal pressure at a depth of 7,000 *ft* of gas/water contact can be estimated using pressure gradient concept as:

$$P_{fn\_GWC} = G_{np}D = (7000\,ft)\times(0.465\,psi/ft) = 3255\,psi$$

Table 6.2 A list of typical matrix and fluid densities.

Type	Substance	Density (gm/cc)
Rock Matrix	Sandstone	2.65
	Limestone	2.71
	Dolomite	2.87
	Anhydrite	2.98
	Halite	2.03
	Gypsum	2.35
Fluid	Freshwater	1.0
	Seawater	1.03–1.06
	Oil	0.6–0.7
	Gas	0.15

Again, the normal pressure at a depth of 5,500 *ft* where gas sand exists can be estimated using pressure gradient concept as:

$$P_{fn\_GS} = G_{np}D = (5500\,ft) \times (0.465\,psi/ft) = 2557.5\,psi$$

However, the pressure in the gas sand at 4,500 *ft* can also be determined hydrostatic pressure concept as:

$$\begin{aligned} P_{fn\_GS} &= P_{fn\_GWC} - 0.052(\rho_g) \times (D_{GWC} - D_{GS}) \\ &= 3255 - 0.052 \times (0.7) \times (7000 - 5500) \\ &= 3200.4\,psi \end{aligned}$$

This pressure is higher than that calculated based on normal pressure gradient at 4,500 *ft*. therefore, the minimum mud weight can be calculated using Eq. (6.3) as:

$$\rho_m = \frac{P_{fn\_GS}}{0.052D} = \frac{3200.4\,psi}{0.052 \times (5500\,ft)} = \mathbf{11.2\ ppg}$$

Example 6.4: A water aquifer is connected hydraulically to a fresh water lake. The aquifer is deviated at a certain angle (Figure 6.2). Two wells were drilled to the center of the aquifer. If the pressure difference between the two wells and the ratio between the pressures of the aquifer in the two wells are 110 *psi* and 0.91, respectively, what are the aquifer depths in both wells from the surface of each well?

Solution:

Given data:

P_d = Pressure difference = 110 psi

$\frac{P1}{P2}$ = Pressure ratio = 0.91 psi

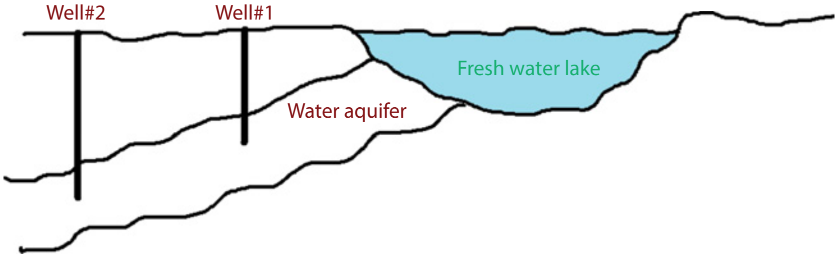

Figure 6.2 A fresh water lake for Example 6.4.

Required data:

D_{m1} = Aquifer depth in well 1

D_{m2} = Aquifer depth in well 2

Since the aquifer is connected to a fresh water lake, the pressure gradient of the aquifer will be equal to the gradient for fresh water having a density of 8.34 *ppg*. The aquifer's pressure gradient can be calculated using Eq. (6.3):

$$G_m = 0.052 \times \rho_m = 0.052 \times 8.34 = 0.434\ psi/ft$$

If we assume well 1 is shallower than well 2, the pressure in the second well will equal the pressure in the first well plus the pressure difference between the two wells. Mathematically:

$$\frac{P_1}{P_1 + P_{diff}} = \frac{P_1}{P_2} = \frac{P_1}{P_1 + 110} = 0.91$$

$$P_1 = \frac{110 * 0.91}{1.0 - 0.91} = 1{,}112\ psi$$

Thus, aquifer pressure in the second well is equal to:

$$P_2 = P_1 + P_{diff} = 1{,}112 + 110 = 1{,}222\,psi$$

The depth of the first well can be calculated using Eq. (6.3):

$$P_1 = G_m \times D_{m1} = 1{,}112 = 0.434 \times D_{m1}$$

$$D_{m1} = \mathbf{2{,}563}\ \boldsymbol{ft}$$

The depth of the second well can be calculated using Eq. (6.3):

$$P_2 = G_m \times D_{m2} = 1{,}222 = 0.434 \times D_{m1}$$

$$D_{m2} = \mathbf{2{,}816}\ \boldsymbol{ft}$$

Example 6.5: An aquifer is connected hydraulically to a water lake as shown in Figure 6.3. The aquifer is deviated at a certain angle. Two wells were drilled to the center of the aquifer. The depth of the shallowest well to the center of the aquifer is 3,200 *ft*. The ratio of the

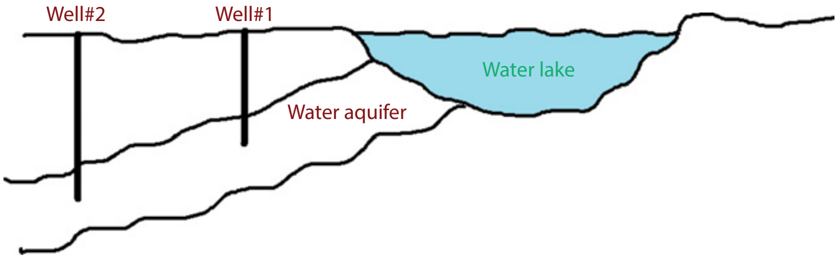

Figure 6.3 A fresh water lake for Example 6.5.

difference in pressure between the two wells (e.g., deepest to the shallowest well) is 0.141, whereas the pressure of the second well was 1,642 *psi*. Estimate the density of the water in the lake and the depth of the second well to the center of the aquifer.

Solution:

Given data:

D_{m_shall} = Depth of the shallowest well = 3,200 *ft*

$\frac{P_{deep}}{P_{shall}}$ = Pressure ratio for the shallowest well = 0.141

P_{deep} = Pressure of the deepest well = 1,642 *psi*

Required data:

ρ_m = Density of the water of the lake in *ppg*.

D_{m_deep} = Depth of the deepest well in *ft*.

Since the aquifer is connected hydraulically to the lake, they will share similar pressure gradient. If we assume the difference in vertical depth between the two wells is equal to D_{diff}, pressure in both wells can be calculated using Eq. (6.3):

$$P_{shall} = 0.052 \times \rho_m \times D_{m\_shall} = 0.052 \times 3{,}200 \times \rho_m = 166.4 \times \rho_m$$

$$P_{deep} = 0.052 \times \rho_m \times D_{m\_deep} = 0.052 \times (3{,}200 + D_{diff}) \times \rho_m$$

Now using the given pressure ratio between the two wells, we can calculate the water density and the depth of the second well as follows:

$$\frac{P_{diff}}{P_{shall}} = \frac{0.052 \times (D_{diff}) \times \rho_m}{166.4 \times \rho_m} = 0.141$$

$$\frac{0.052 \times D_{diff}}{166.4} = 0.141$$

$$D_{diff} = 452.2\, ft$$

So, the depth of the second well is equal to:

$$D_{m\_deep} = D_{m\_shall} + D_{diff} = 3{,}200 + 451 = 3{,}651\, \boldsymbol{ft}$$

The water density can now be calculated by knowing the pressure of the second well using Eq. (6.3): ft thick and the average

$$P_{Deep} = 0.052 \times \rho_m \times D_{m\_deep} = 0.052 \times 3{,}651 \times \rho_m = 1{,}642$$

$$\rho_m = \mathbf{8.65}\ \boldsymbol{ppg}$$

Form the above calculated density, this lake is not a fresh water lake but it has certain salinity.

Example 6.6: During grain density of rock matrix drilling an exploration well, a gas kick was encountered and safely circulated out. The well was in static condition and developed around 5,100 *psi* hydrostatic pressure. This pressure was 200 *psi* above the pressure at that depth. Drilling was resumed by drilling the gas formation. A water formation was there just below the gas formation. The water formation zone was 1,500 *ft* thick and the average pressure was 5,610 psi at the mid of water formation. Formation gradient of the water and gas formations were later estimated to be 0.52 and 0.14 *psi/ft*, respectively. What was the top depth of the gas formation?

Solution:

Given data:

P_h	=	Applied hydrostatic pressure at top of gas formation		
	=	5,100 *psi*		
P_{ob}	=	Applied overbalance pressure	=	200 *psi*
P_{w_mid}	=	Pressure of at the mid of water formation		
	=	5,610 *psi*		
h_w	=	Thickness of the water formation	=	1,500 *psi*
G_w	=	Pressure gradient of the water formation	=	0.52 *psi/ft*
G_g	=	Pressure gradient of the gas formation	=	0.14 *psi/ft*

Required data:

$D_{g\text{-}top}$ = Depth of the top of gas formation, *ft*

The depth of the mid of the water formation can be calculated using the pressure gradient and the formation pressure as follows:

$$D_{w\_mid} = \frac{P_{w_{mid}}}{G_w} = \frac{5{,}610}{0.52} = 10{,}789\,ft$$

So, the top depth of the water formation is equal to:

$$D_{w\_top} = D_{w\_mid} - \frac{h_w}{2} = 10{,}789 - \frac{1500}{2} = 10{,}039\ ft$$

Pressure at the bottom of the gas formation will be equal to the pressure at the top of the water formation. So,

$$P_{g\_bott} = P_{w\_top} = G_w \times D_{w\_top} = 0.52 \times 10{,}039 = 5{,}220\,psi$$

Pressure at the top of the gas formation is equal to:

$$P_{g\_top} = P_h - P_{ob} = 5{,}100 - 200 = 4{,}900\,psi$$

Now, by knowing the pressure at the top and bottom of the gas formation, we can calculate the thickness of the gas formation using the gas pressure gradient as follows:

$$h_g = \frac{P_{g_{bott}} - P_{g_{top}}}{G_g} = \frac{5{,}220 - 4{,}900}{0.14} = 2{,}286\,ft$$

Thus, the top depth of the gas formation is equal to:

$$D_{g\_top} = D_{w\_top} - h_g = 10{,}039 - 2{,}286 = \mathbf{7{,}753}\,\boldsymbol{ft}$$

Example 6.7: A gas field with a water formation beneath it. The pressures at the top of gas, gas water contact and bottom of the water formations are 3,695 *psi*, 3,740 *psi*, and 4,000 *psi* respectively. If the thickness of the gas formation is similar to that for water formation, determine the ratio between the pressure gradient of the gas formation and water formation. In addition, if the pressure gradient of the water formation is 0.46 *psi/ft*, calculate the thickness of each formation and the depth of the gas-water contact.

Solution:

Given data:

P_{g_top} = Pressure at top of gas formation = 3,695 *psi*

P_{gwc} = Pressure at the gas-water contact = 3,740 *psi*

P_{w_bott} = Pressure of at the bottom of water formation = 4,000 *psi*

G_w = Pressure gradient of the water formation = 0.46 *psi/ft*

Required data:

$\frac{G_g}{G_w}$ = Pressure gradient ratio between gas and water formations.

D_{gwc} = Depth of the gas-water contact, *ft*

h_w = thickness of the water formation

h_g = thickness of the gas formation

We know that at gas-water contact, the pressure at the bottom of the gas formation is equal to the pressure at the top of water formation. If we assume the thickness of gas formation equal to "*x*", the pressure of the bottom of gas formation is equal to:

$$P_{g\_bott} = P_{g\_top} + G_g x \Rightarrow 3{,}740 = 3{,}695 + G_g x$$

$$G_g = \frac{45}{x}$$

And the pressure at the top of the water formation is equal to:

$$P_{w\_top} = P_{w\_bott} - G_w x \Rightarrow 3{,}740 = 4{,}000 - G_w x$$

$$G_w = \frac{260}{x}$$

Thus, the ratio between the pressure gradient of the gas and water formations is equal to:

$$\frac{G_g}{G_w} = \frac{\frac{45}{x}}{\frac{260}{x}} = 0.173$$

Now, by knowing the pressure at the gas-water contact and the pressure gradient of the water formation we can determine the depth of the gas-water contact as follows:

$$P_{gwc} = G_w \times D_{gwc} \Rightarrow 3,740 = 0.46 \times D_{gwc}$$

$$D_{gwc} = \mathbf{8,130}\ \boldsymbol{ft}$$

The thickness of the water formation is equal to:

$$P_{w\_bott} - P_{gwc} = G_w \times h_w = (4,000 - 3,740) = 0.46 \times h_w$$

$$h_w = \mathbf{565}\ \boldsymbol{ft}$$

and the thickness of the gas formation will be equal to the water formation.

$$h_g = \mathbf{565}\ \boldsymbol{ft}$$

iii) Overburden Pressures

At equilibrium condition, the overburden pressure is the sum of vertical matrix stress and the formation pore pressure. Mathematically,

$$P_{ob} = \sigma_v + P_{fn} \tag{6.4}$$

where

P_{ob} = overburden pressure
σ_v = vertical matrix stress
P_{fn} = normal formation pore pressure

Example 6.8: Calculate the overburden pressure of an underground reservoir if the matrix stress is 8,500 *psi* and the formation pore pressure is 5000 *psi*.

Solution:

Given data:

σ_v = vertical matrix stress = 8,500 *psi*
P_{fn} = normal formation pore pressure = 5,000 *psi*

Required data:

P_{ob} = overburden pressure = ?

The overburden pressure can be estimated using the Eq. (6.4) as:

$$P_{ob} = \sigma_v + P_{fn} = 8,500\ psi + 5,000\ psi = \mathbf{13,000}\ \boldsymbol{psi}$$

Example 6.9: Calculate the formation pore pressure if the matrix stress is 9,500 *psi* and the overburden pressure of an underground reservoir is 13,500 *psi*.

Solution:

Given data:

σ_v = Vertical matrix stress = 9,500 *psi*
P_{ob} = Overburden Pressure = 13,500 *psi*

Required data:

P_{fn} = Normal formation pore pressure, *psi*

The normal formation pore pressure can be estimated using the Eq. (6.4) as:

$$P_{fn} = P_{ob} - \sigma_v = 13{,}500 - 9{,}500 = \mathbf{4000\ \textit{psi}}$$

Example 6.10: Calculate the overburden pressure of an underground reservoir if the matrix stress is 5,500 *psi* and the formation pore pressure is 2500 *psi*.

Solution:

Given data:

σ_v = Vertical matrix stress = 5,500 *psi*
P_{fn} = Normal formation pore pressure = 2,500 *psi*

Required data:

P_{ob} = Overburden Pressure, *psi*

The overburden pressure can be estimated using the Eq. (6.4) as:

$$P_{ob} = P_{fn} + \sigma_v = 5{,}500 + 2{,}500 = \mathbf{8000\ \textit{psi}}$$

Table 6.2 shows the typical matrix and fluid densities in general.

The bulk density at a given depth can be calculated as:

$$\rho_b = \rho_f \phi + \rho_r (1 - \phi) \tag{6.5}$$

where

ρ_b = bulk density of porous sediment
ρ_f = fluid density in the pore space
ρ_r = grain density of rock matrix
ϕ = porosity

The average bulk density data are expressed first in terms of average porosity. Then Eq. (6.5) for average porosity yields:

$$\phi_{avg} = \frac{\rho_r - \rho_b}{\rho_r - \rho_f} \tag{6.6}$$

In a straight-line trend, the equation is given by:

$$\phi = \phi_o e^{-K_\phi D_s} \tag{6.7}$$

where

ϕ_o = porosity at surface ($D = 0$)

K_ϕ = porosity decline constant at ϕ

D_s = the depth below the surface of the sediments

The porosity decline constant can be estimated from the Eq. (6.7) by taking ln in both sides of the equation and solving the same for K_ϕ as:

$$K_\phi = \frac{\dfrac{\ln\phi_o}{\ln\phi}}{D_s} \tag{6.8}$$

The vertical overburden stress (σ_{ob}) resulting from geostatic load at a depth can be written in terms of bulk density of the system in the same form of Eq. (6.1) as:

$$\sigma_{ob} = \int_0^D \rho_b g dD \tag{6.9}$$

So, the vertical overburden stress resulting from the geostatic load can be expressed in terms of average sediment porosity at a particular depth. Now substituting Eq. (6.5) into Eq. (6.9) yields:

$$\sigma_{ob} = g\int_0^D [\rho_f\phi + \rho_r(1-\phi)]dD \tag{6.10}$$

For offshore area, the total depth would be in two segments: i) from the surface to the ocean bottom (i.e., total depth of sea water: 0 to D_{sw}), and ii) from the mudline to the depth of interest (i.e. D_{sw} to D). In such case, Eq. (6.9) can be used for these two situations. Thus ρ_b would become seawater density, ρ_{sw} which is equal to 8.5 lb_m/gal up to the depth of D_{sw} and then from sea bed to the depth D would be same as mentioned in Eq. (6.10). So, Eq. (6.9) can be written as:

$$\sigma_{ob} = \rho_{sw} D_{sw} g + g\int_{D_{sw}}^{D} [\rho_r - (\rho_r - \rho_f)\phi]dD \tag{6.11}$$

$$\sigma_{ob} = \rho_{sw} D_{sw} g + g(D - D_{sw})\rho_r - g(\rho_r - \rho_f)\int_{D_{sw}}^{D} \phi dD \tag{6.12}$$

The variation of ϕ with depth D due to overburden stress can be estimated using Eq. (6.7). Therefore, substituting Eq. (6.7) into Eq. (6.12) yields:

$$\sigma_{ob} = \rho_{sw} D_{sw} g + g(D - D_{sw})\rho_r - g(\rho_r - \rho_f)\int_{D_{sw}}^{D} \phi_o e^{-K_\phi D} dD \tag{6.13}$$

Solving the Eq. (6.13) and applying the limits of the integration, the equation becomes as:

$$\sigma_{ob} = \rho_{sw} D_{sw} g + g(D - D_{sw})\rho_r - g(\rho_r - \rho_f)\phi_o \left[-\frac{1}{K_\phi}(e^{-K_\phi D} - e^{-K_\phi D_{sw}}) \right] \tag{6.14}$$

Let $D_s = D - D_{sw}$. Substituting this in Eq. 6(14) yields:

$$\sigma_{ob} = \rho_{sw} D_{sw} g + gD_s\rho_r - \frac{g(\rho_r - \rho_f)\phi_o}{K_\phi}[e^{-K_\phi D_{sw}} - e^{-K_\phi (D_s + D_{sw})}] \tag{6.15}$$

$$\sigma_{ob} = \rho_{sw} D_{sw} g + gD_s\rho_r - \frac{g(\rho_r - \rho_f)\phi_o}{K_\phi}[(e^{-K_\phi D_{sw}} - e^{-K_\phi D_s} e^{-K_\phi D_{sw}})] \tag{6.16}$$

$$\sigma_{ob} = \rho_{sw} D_{sw} g + gD_s\rho_r - \frac{g(\rho_r - \rho_f)\phi_o}{K_\phi} e^{-K_\phi D_{sw}}[1 - e^{-K_\phi D_s}] \tag{6.17}$$

In the right hand side of Eq. (6.17), within the range of D_{sw}, there is no existence of rock, so $\rho_r = 0$ and the porosity becomes 1. Therefore, the porosity decline constant will become zero. As a result, $e^{-K_\phi D_{sw}} = 1$. Thus, Eq. (6.17) can be written as:

$$\sigma_{ob} = \rho_{sw} D_{sw} g + gD_s\rho_r - \frac{g(\rho_r - \rho_f)\phi_o}{K_\phi}[1 - e^{-K_\phi D_s}] \tag{6.18}$$

For onshore area, Eq. (6.18) can be written as:

$$\sigma_{ob} = gD_s\rho_r - \frac{g(\rho_r - \rho_f)\phi_o}{K_\phi}[1 - e^{-K_\phi D_s}] \tag{6.19}$$

In field unit, Eqs. (6.18) and (6.19) can be written as:

$$\sigma_{ob} = 0.052 \left[\rho_{sw} D_{sw} + \rho_r D_s - \frac{(\rho_r - \rho_f)\phi_o}{K_\phi}(1 - e^{-K_\phi D_s}) \right] \tag{6.20}$$

$$\sigma_{ob} = 0.052 \left[\rho_r D_s - \frac{(\rho_r - \rho_f)\phi_o}{K_\phi}(1 - e^{-K_\phi D_s}) \right] \tag{6.21}$$

where

σ_{ob} = vertical overburden stress, *psi*
ρ_{sw} = density of sea water, lb_m/gal
D_{sw} = depth from surface to the ocean bottom, *ft*
ρ_r = grain density of rock matrix, lb_m/gal
D_s = the depth from the sea bed to up to a depth of interest, *ft*
ρ_f = density of fluid in the pore space, lb_m/gal
ϕ_o = porosity at surface ($D = 0$), fraction
K_ϕ = porosity decline constant at ϕ, ft^{-1}

Example 6.11: Determine porosity decline constant for North Sea area. It is noted that an average grain density of 2.55 g/cm^3, an average pore fluid density of 1.044 g/cm^3, and the value for surface porosity of 45% were recorded. Assume the average bulk density of the sediment is 2.35 g/cm^3 at a specified depth of 9,000 *ft*. Also compute the vertical overburden stress along the coast line of North Sea at the same depth.

Solution:

Given data:

ρ_r = average grain density of rock matrix = 2.55 g/cm^3
ρ_f = average density of fluid in the pore space = 1.044 g/cm^3
ϕ_o = porosity at surface ($D = 0$) = 0.45
D_s = the depth below the surface of the sediments = 9,000 *ft*
ρ_b = bulk density of porous sediment = 2.35 g/cm^3

Required data:

K_ϕ = porosity decline constant at ϕ = ?
σ_{ob} = vertical overburden stress = ?

Before calculating the porosity decline constant, we have to calculate the average porosity at the specified depth by using the Eq. (6.6) as:

$$\phi_{avg} = \frac{\rho_r - \rho_b}{\rho_r - \rho_f} = \frac{2.55 - 2.35}{2.55 - 1.044} = 0.133$$

Using Eq. (6.7), K_f can be calculated as:

$$\phi = \phi_o e^{-K_\phi D_s} \Rightarrow 0.133 = 0.45 e^{-K_\phi \times 9000} \Rightarrow K_\phi = \mathbf{0.000135}\ \boldsymbol{ft^{-1}}$$

As long as the vertical overburden stress is along the coast line of the North Sea, Eq. (6.19) can be used if we assume that $D_{sw} = 0$. Therefore,

$$\sigma_{ob} = (2.55\ gm/cm^3) \times (9000\,ft \times 25.4\ cm/ft) \times 981\ cm/s^2$$

$$-\frac{(2.55\ gm/cm^3 - 1.044\ gm/cm^3) \times 981 \dfrac{cm}{s^2} \times 0.45}{\dfrac{0.000135}{25.4\ cm/ft} ft^{-1}}$$

$$\times \left[1 - e^{\frac{-0.000135}{25.4\ cm/ft} ft^{-1} \times (9000\,ft \times 25.4\ cm/ft)} \right]$$

= 571854330 $gm/cm\ s^2$ – 87971272.6 $gm/cm\ s^2$

= 483,883,057.4 $gm/cm\ s^2$

= 483,883,057.4 $dynes/cm^2$

= 7,016.3 psi (since 1 dynes/cm^2 = 0.0000145 psi)

Example 6.12: A core sample of 3.81 *cm* in width and 15.20 *cm* in length from a limestone formation was being cleaned and dried. Dry weight was measured 415 *grams*. The core sample was completely saturated using 20 *grams* of fresh water. Determine the grain density and bulk density of this core sample.

Solution:

Given data:

d_{core}	=	width of the core sample	=	3.81 *cm*
l_{core}	=	length of the core sample	=	15.20 *cm*
W_{core_dry}	=	dry weight of the core sample	=	415 *gm*
W_{ws}	=	weight of that water saturated the core	=	20 *gm*

Required data:

ρ_b	=	bulk density of the core sample, *gm/cc*
ρ_r	=	grain density of the core sample, *gm/cc*

To calculate the porosity of the core sample, first we need to calculate the bulk volume and the pore volume. Because 20 grams were used to completely saturate the pores of the core sample, the pore volume is equal to:

$$V_{pore} = \frac{W_{ws}}{\rho_f} = \frac{20}{1.0} = 20\ cc$$

The bulk volume of the core sample is equal to the volume of the cylindrical shape:

$$V_b = \frac{\pi}{4} d_{core} \times l_{core} = \frac{\pi}{4} \times 3.81 \times 15.20 = 172.4\ cc$$

Now, porosity is equal to:

$$\phi = \frac{V_{pore}}{V_b} = \frac{20}{172.4} = 0.116$$

To calculate the grain density, first we need to calculate the grain volume using the following relation:

$$V_r = V_b \times (1 - \phi) = 172.4 \times (1 - 0.116) = 153.3\ cc$$

Thus, the grain density is equal to:

$$\rho_r = \frac{W_{core\_dry}}{V_r} = \frac{415}{153.3} = \mathbf{2.71\ gm/cc}$$

Bulk density can now be calculated using Eq. (6.5):

$$\rho_b = \rho_f \times \phi + \rho_r \times (1 - \phi) = 1.0 \times 0.116 + 2.71 \times (1 - 0.116)$$
$$= \mathbf{2.51\ gm/cc}$$

Example 6.13: A well is planned to be drilled to the depth of 11,000 *ft*. The last casing shoe was at 5,000 *ft* and the fracture gradient below the casing shoe was 0.66 *psi/ft*.

The designed mud weight to drill the new section is 10.4 *ppg*. What is the maximum pressure that can be applied at the surface? And if it is expected that the pressure at the depth of 11,000 *ft* is 7,450 *psi*, is it safe to drill through this depth? If not, what should be the recommendation?

Solution:

Given data:

D_m	=	bottom depth of the well	=	11,000 *ft*
D_{shoe}	=	depth of the casing shoe	=	5,000 *ft*
G_{fp}	=	fracture gradient at the casing shoe	=	0.66 *psi/ft*
ρ_m	=	mud weight	=	10.4 *ppg*
P	=	expected pressure at 11,000 *ft*	=	7,450 *psi*

Required data:

P_{max} = maximum pressure at the surface, *psi*

The maximum pressure to be applied at the surface is the difference between the maximum pressure at the weakest point "casing shoe" and the hydrostatic pressure of the mud at that point. The maximum pressure at the casing shoe can be calculated using Eq. (6.2) after modification:

$$P_f = G_{fg} \times D_{shoe} = 0.66 \times 5{,}000 = 3{,}300\ psi$$

The hydrostatic pressure of the mud at the casing shoe can also be calculated using Eq. (6.2):

$$P_m = 0.052 \times \rho_m \times D_{shoe} = 0.052 \times 10.4 \times 5{,}000 = 2{,}704\ psi$$

Thus, the maximum pressure is equal to:

$$P_{max} = P_f - P_m = 3{,}300 - 2{,}704 = 596 psi$$

To assess the situation when the pressure at 11,000 *ft* is 7,450 *psi*, we need to change this pressure to pressure gradient and then find out what will be the pressure at the casing shoe. Pressure gradient of this formation is equal to:

$$G = \frac{P}{D_m} = \frac{7{,}450}{11{,}000} = 0.68\ psi/ft$$

Now the pressure at the casing shoe will be:

$$P = G \times D_{shoe} = 0.68 \times 5{,}000 = \mathbf{3{,}400\ psi}$$

This is the maximum pressure that should be applied at the surface.

From the above calculations, it is not safe to drill through this formation because there is a possibility of fracturing the formation at the casing shoe. So the recommendation is to set another casing at the top of this formation before drilling it to avoid any well instability problems.

Example 6.14: A leak-off test was conducted to a well at a depth of 4,500 *ft* using 9.1 *ppg* drilling mud. When the mud leaked-off, the pump pressure was reading

725 *psi*. What is the fracture gradient of the formation at that depth? If a new mud of 9.6 *ppg* is to be used in this section, what will be the maximum allowable annulus pressure at surface?

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 4,500 *ft*
ρ_m = Mud weight used for leak-off test = 9.1 *ppg*
ρ_{m_new} = M=New mud weight to be used = 9.6 *ppg*
P = Leaked-off surface pressure = 725 *psi*

Required data:

G_{fp} = Fracture gradient at the casing shoe, *ppg*
P_{max} = Maximum annulus pressure at the surface, *psi*

To find the fracture gradient at the casing shoe, we will first determine the maximum pressure. From leak-off test data and the mud that used for the test, leaked-off pressure is equal to:

$$P_f = P_{h\_mud} + P_{surf} = 0.052 \times 9.1 \times 4{,}500 + 725 = 2{,}854.4\ psi$$

Fracture gradient is equal to:

$$G_{fp} = \frac{P_f}{D_{shoe}} = \frac{2{,}854.4}{4{,}500} = \mathbf{0.634\ psi/ft}$$

To determine the maximum annulus pressure at surface using the new mud of 9.6 *ppg*, hydrostatic pressure of the new mud at the casing shoe should be calculated using Eq. (6.3):

$$P_m = 0.052 \times \rho_m \times D_{shoe} = 0.052 \times 9.6 \times 4{,}500 = 2{,}246.4\ psi$$

The maximum allowable surface pressure is now equal to:

$$P_{max} = P_f - P_m = 2{,}854.4 - 2{,}246.4 = \mathbf{608\ psi}$$

Example 6.15: A producing gas field has an initial reservoir pore pressure of 3,750 *psi* and matrix stress of 8,010 *psi*. The decline of the pore pressure with time was estimated by the equation: $P_r = 3{,}271\ t^{-0.191}$ where P_r is the reservoir pore pressure and t is the time in years. What will be the overburden pressure after producing for 10 years assuming matrix stress remains the same?

Solution:

Given data:

P_{fn} = Initial pore pressure = 3,750 psi
σ_v = Vertical matrix stress = 8,010 psi

Required data:

P_{ob} = Overburden pressure in psi.

Initial overburden pressure of the above reservoir can be calculated using Eq. (6.4):

$$P_{ob} = P_{fn} + \sigma_v = 3{,}750 + 8{,}010 = 11{,}760\,psi$$

Because pore pressure is declining with production, the overburden pressure should decline too. The pore pressure after 10 years can be estimated using the above given equation:

$$P_r = 3{,}271\ t^{-0.191} = 3{,}271 \times 10^{-0.191} = 2{,}107\ psi$$

Thus, the overburden pressure after 10 years is equal to:

$$P_{ob} = P_{fn} + \sigma_v = 2{,}107 + 8{,}010 = \mathbf{10{,}117\ psi}$$

Example 6.16: A sedimentary formation has surface porosity of 0.4 and porosity decline constant of 0.00012. What is the estimated porosity at depth of 5,000 *ft*? And what will be the depth of the rock that has porosity of 0.24?

Solution:

Given data:

ϕ_o = surface porosity = 0.40
K_ϕ = porosity decline constant = 0.00012
D = rock depth = 5,000 *ft*
ϕ_D = porosity = 0.24

Required data:

D_s = depth at the given porosity, *ft*
ϕ_{5000} = porosity at 5,000 *ft*, %

Porosity at any depth can be calculated using Eq. (6.7):

$$\phi = \phi_o e^{-K_\phi D_s}$$

Porosity at 5,000 *ft* is equal to:

$$\phi_{5000} = 0.4 \times e^{-0.00012 \times 5{,}000} = \mathbf{0.22}$$

The depth of the given porosity can be determined by modifying the above equation as follows:

$$D_s = -\frac{\ln\left(\dfrac{\phi}{\phi_o}\right)}{K_\phi} = -\frac{\ln\left(\dfrac{0.24}{0.40}\right)}{0.00012} = \mathbf{4{,}257\ ft}$$

Example 6.17: A well was selected to be drilled in an area which is 350 *ft* below sea level near the coast of the sea (Figure 6.4). While drilling an intermediate section, a kick was encountered and shut-in drillpipe pressure was measured to be 140 *psi*. The density of the mud was 8.7 *ppg*. The kick was safely removed, and later they realized that the current formation and the sea water were in hydraulic communication. If the density of the sea water is 8.5 *ppg*, determine the depth at which the kick was occurred?

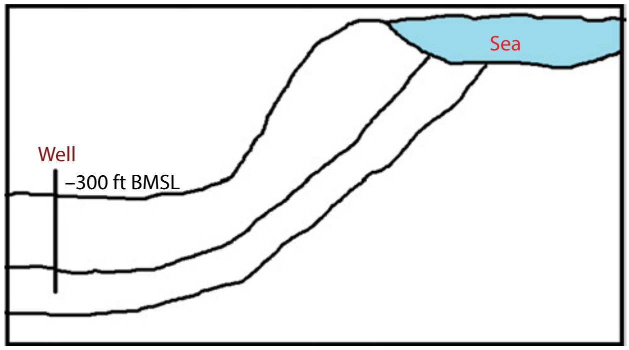

Figure 6.4 A well placement for Example 6.17.

<u>Solution</u>:

Given data:

D_o = top depth of the well from sea level = $-300\ ft$
p_{si_dp} = shut-in drill pipe pressure = $140\ psi$
ρ_m = mud weight = $8.7\ ppg$
ρ_{sw} = density of sea water = $8.5\ ppg$

Required data:

D = depth of the formation

Because the formation is connected hydraulically with the sea water, its pressure should be equal to the hydrostatic pressure of the sea water. The shut-in drill pipe pressure was the result of the formation pressure minus the mud hydrostatic pressure. The mud hydrostatic pressure can be calculated using Eq. (6.3):

$$P_m = 0.052 \times \rho_m \times D = 0.052 \times 8.7 \times D = 0.452\ D$$

The formation pressure can be estimated using the sea water density and the vertical distance between sea level and the depth of the formation. Formation pressure can also be estimated using Eq. (6.3):

$$P_f = 0.052 \times \rho_{sw} \times H = 0.052 \times 8.5 \times (D + 350) = 0.442(D + 300)$$

Shut-in drillpipe pressure is relating the mud hydrostatic pressure and the formation pressure with the following equation:

$$P_{si_{dp}} = P_f - P_m = 140 = 0.442D + 0.442 \times 350 - 0.452D$$

$$140 - 154.7 = (0.442 - 0.452) \times D$$

$$D = \frac{14.7}{0.01} = \mathbf{1{,}470}\ \boldsymbol{ft}$$

Example 6.18: At what depth below sea level will the normal pore pressure value be 2,200.0 *psi*? Assume that the drilling activities will be continued in Rocky Mountain area. Also, find out the mud weight for that area.

Solution:

Given data:

G_{np} = Normal pore pressure gradient for Rocky Mountain
= 0.436 *psi/ft*

P_{fn} = Normal pore pressure = 2200 *psi*

Required data:

D = Total vertical depth, *ft*

ρ_m = Mud weight, *ppg*

The formula for estimating normal pore pressure for Rocky Mountain is given as:

$$P_{fn} = G_{np}D \Rightarrow D = P_{fn}/G_{np} = \frac{2200}{0.436} = \mathbf{5045}\ \boldsymbol{ft}$$

The mud weight can be calculated using Eq. (6.3) as:

$$\rho_m = \frac{P_{fn}}{0.052D} = \frac{2200}{0.052 \times (5045\,ft)} = \mathbf{8.386}\ \boldsymbol{ppg}$$

6.2.3 Pore Pressure Estimation

i) Estimation using Seismic Data:

To estimate formation pore pressure from seismic data, the observed interval transit time is a porosity dependent parameter that varies with porosity according to the following relation:

$$t_t = t_f\phi + t_r(1-\phi) \tag{6.22}$$

where

t_t = the observed interval transit time, *s/ft*

t_f = the interval transit time in the pore fluid, *s/ft*

t_r = the interval transit time in rock matrix, *s/ft*

ϕ = porosity

In some cases, an acceptable straight line trend is not observed for any of the approaches. As a result a more complex model must be used. Such complex model can be derived using Eq. (6.7) where normal compaction process exists. Substituting Eq. (6.7) for the ϕ into Eq. (6.22) yields:

$$t_t = t_f\phi_o e^{-K_\phi D_s} + t_r(1-\phi_o e^{-K_\phi D_s}) \tag{6.23}$$

$$t_t = \phi_o(t_f - t_r)e^{-K_\phi D_s} + t_r \tag{6.24}$$

$$\frac{t_t}{\phi_o(t_f - t_r)} - \frac{t_r}{\phi_o(t_f - t_r)} = e^{-K_\phi D_s} \tag{6.25}$$

Taking ln in both sides of Eq. (6.25) yields:

$$\ln\left[\frac{t_r}{\phi_o(t_f - t_r)}\right] - \ln\left[\frac{t_t}{\phi_o(t_f - t_r)}\right] = K_\phi D_s \tag{6.26}$$

Equation (6.26) represents the normal pressure relationship of average observed sediment travel time and depth.

ii) Detection Techniques

Bingham Model: Bingham (1964) developed a model to detect overpressures which is written as:

$$R = AN^E\left(\frac{W}{d_b}\right)^{d_{exp}} \tag{6.27}$$

where

A = rock matrix strength constant or drillability constant
d_b = bit diameter, in
d_{exp} = bit weight exponent or *d*-exponent or formation drillability
E = rotary speed exponent
N = rotary speed, *rpm*
R = rate of penetration or drilling rate, *ft/hr*
W = weight on bit, lb_f

The model proposed by Bingham [i.e., Eq. (6.27)] is called a "drilling rate" equation.

Jordan and Shirley Model: Jordan and Shirley (1966) reorganized Eq. (6.27) for d_{exp} which is produced using Eq. (6.27) as:

$$\frac{R}{N} = \left(\frac{W}{d_b}\right)^{d_{exp}} \tag{6.28}$$

Taking log into the both sides of Eq. (6.28), the equation becomes as:

$$d_{exp} = \frac{\log\left(\frac{R}{N}\right)}{\log\left(\frac{W}{d_b}\right)} \tag{6.29}$$

A modified version of Eq. (6.29) can be written as:

$$d_{exp} = \frac{\log\left(\frac{R}{60N}\right)}{\log\left(\frac{12W}{10^6 d_b}\right)} \tag{6.30}$$

The model proposed by Jordan and Shirley [i.e., Eq. (6.30)] is called as "*d*-exponent" equation.

Example 6.19: Determine the value of the d_{exp} if the drilling rate is 35 ft/hr, the rotary RPM is 100, and the weight on the bit is 60,000 lb_f. Assume necessary data. Further calculate what will happen to d_{exp} if the drilling rate is increased to double of its original case. Make comments on the result.

Solution:

Given data:

R_1 = drilling rate = 35 *ft/hr*
N = rotary speed = 100 *rpm*
W = weight on bit = 60,000 lb_f
R_2 = drilling rate = 70 *ft/hr*
Assuming:
d_b = bit diameter = 12.25 *in*

Required data:

d_{exp1} = formation drillability
d_{exp2} = formation drillability

The d-exponent can be calculated by using the Eq. (6.30) as:

$$d_{exp1} = \frac{\log\left(\frac{R}{60N}\right)}{\log\left(\frac{12W}{10^6 d_b}\right)} = \frac{\log\left(\frac{35}{60\times 100}\right)}{\log\left(\frac{12\times 60{,}000}{10^6\times 12.25}\right)} = \frac{-2.2341}{-1.2308} = \mathbf{1.82}$$

Now, if the rate of penetration is doubled, then d-exponent can also be calculated in the same fashion as:

$$d_{exp2} = \frac{\log\left(\frac{R}{60N}\right)}{\log\left(\frac{12W}{10^6 d_b}\right)} = \frac{\log\left(\frac{70}{60\times 100}\right)}{\log\left(\frac{12\times 60{,}000}{10^6\times 12.25}\right)} = \frac{-1.9330}{-1.2308} = \mathbf{1.57}$$

It is showing that an increase in R resulted in a decrease in d_{exp}. In this case, doubling of the rate of penetration decreased the modified d-exponent from 1.82 to 1.57.

Example 6.20: Determine the value of the d_{exp} if the drilling rate is 66 *ft/hr*, the rotary RPM is 110 and the weight on the bit is 35,000 *lbf.* bit diameter is 8.5 *in*. Further calculate what will happen to d_{exp} if the drilling rate is decreased to half of its original case. Make comments on the result.

Solution:

Given data:

R_1 = Drilling rate = 66 *ft/hr*
N = Rotary Speed = 110 *rpm*
W = Weight on bit = 35,000 lb_f
R_2 = Drilling rate = 33 *ft/hr*

d_b = bit diameter = 8.5 *in*

Required data:

d_{exp1} = formation drill ability

d_{exp2} = formation drill ability

The d-exponent can be calculated by using the Eq. (6.30) as:

$$d_{exp1} = \frac{\log\left(\dfrac{R}{60N}\right)}{\log\left(\dfrac{12W}{10^6 d_b}\right)} = \frac{\log\left(\dfrac{66}{60\times 110}\right)}{\log\left(\dfrac{12\times 35{,}000}{10^6\times 8.5}\right)} = \frac{-2}{-1.3062} = \mathbf{1.531\,bb}$$

Now, if the rate of penetration is halved, then d-exponent can also be calculated in the same manner as:

$$d_{exp2} = \frac{\log\left(\dfrac{R}{60N}\right)}{\log\left(\dfrac{12W}{10^6 d_b}\right)} = \frac{\log\left(\dfrac{33}{60\times 110}\right)}{\log\left(\dfrac{12\times 35{,}000}{10^6\times 8.5}\right)} = \frac{-2.3011}{-1.3062} = \mathbf{1.762}$$

It is showing that a decrease in R resulted in an increase in d_{exp}. In this case, reducing the rate of penetration by half increased the modified d-exponent from 1.531 to 1.762.

Rehm and McClendon Model: Rehm and McClendon (1971) proposed modifying the d-exponent to correct for the effect of mud density changes as well as changes in weight on bit, bit diameter, and rotary speed as:

$$d_m = d_{exp}\frac{\rho_n}{\rho_e} \tag{6.31}$$

where

d_m = modified d-exponent

ρ_n = mud density equivalent to normal pore pressure gradient or normal mud weight, *ppg*

ρ_e = equivalent mud density at the bit while circulating or actual mud weight in use, *ppg*

As recommended by Rehm and McClendon (1971), Figure 6.2 shows the depth vs. d_m plot in a Cartesian coordinates for the normal pore pressure, and abnormal pressure trend line. The procedure for determining pore pressure from d_m can be explained as follows:

- Calculate d_m over 10–30 *ft* intervals
- Plot d_m vs depth (use only data from Clean shale sections)
- Determine the normal line for the d_m vs. depth plot.
- Establish where d_m deviates from the normal line to determine abnormal-pressure zone

Rehm and McClendon recommend using linear scales for both depth and d_m values when constructing a graph to establish formation pore pressure quantitatively (Figure 6.5). A straight-line normal pressure trend line having intercept with depth and slop is assumed such that

$$d_{mn} = d_{mo} + mD \tag{6.32}$$

where

d_{mn} = value of d_m read from the normal pressure trend line at a depth of interest (Figure 6.6)

d_{mo} = intercept of the normal trend line

m = slop of the normal trend line

D = depth

According to the authors, the value of slope m is fairly constant with changes in geologic age. The modified "*d*-exponent" correlation often is used for estimating the formation pressure gradient as well as the abnormal formation pressure. Rehm and

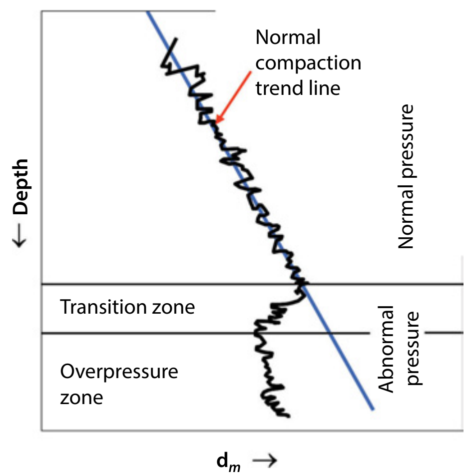

Figure 6.5 Depth verses d_m plotting in Cartesian coordinates.

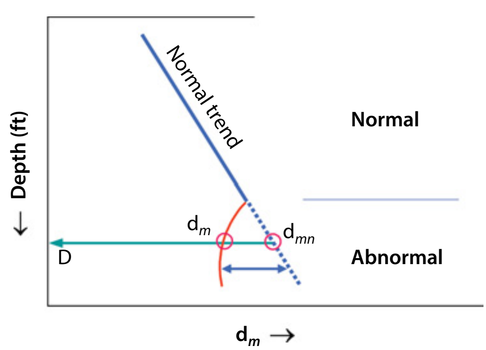

Figure 6.6 Depth verses d_m plotting showing d_{mn} on the trendline.

McClendon suggested the following empirical equation to calculate the equivalent mud density as

$$\rho_e = 7.56\log[d_{mn} - d_m] + 16.5 \tag{6.33}$$

Here, ρ_e is in lb_m/gal

The formation pressure gradient can be written as:

$$G_f = 0.052\ \rho_e \tag{6.34}$$

Here, G_f is in *psi/ft*

The formation pressure can be written as:

$$P_f = G_f D \tag{6.35}$$

Here, P_f is in *psi*

Example 6.21: For the Malaysia area, find the value of the weight on bit (W) if the modified *d*-exponent is 1.7, the drilling rate is 30 *ft/hr*, the rotary RPM is 90. In addition, an equivalent circulating density at the bit was 9.5 *lbm/gal*. Assume necessary data.

Solution:

Given data:

R = Drilling Rate = 30 *ft*/hr

N = Rotary Speed = 90 *rpm*

d_m = Modified *d*-exponent = 1.7

ρ_e = actual mud weight in use = 9.5 *ppg*

Additional Assumption:

d_b = bit diameter = 12.0 *in*

Required data:

W = Weight on bit =?

Modified *d*-exponent is given by using Eq. (6.31) as:

$$d_m = d_{exp}\frac{\rho_n}{\rho_e}$$

We know that for the Malaysia area, the normal formation pressure gradient is 0.442 *psi/ft*. So, the mud density equivalent to normal pore pressure gradient (ρ_n) can be calculated as:

$$\rho_n = \frac{0.442}{0.052} = 8.5\ ppg$$

Therefore,

$$1.7 = d_{exp}\frac{8.5}{9.5} \Rightarrow d_{exp} = 1.9$$

d-exponent can be given by using the Eq. (6.30) as:

$$d_{exp} = \frac{\log\left(\frac{R}{60N}\right)}{\log\left(\frac{12W}{10^6 d_b}\right)} \Rightarrow 1.9 = \frac{\log\left(\frac{30}{60\times 90}\right)}{\log\left(\frac{12W}{10^6 \times 12}\right)} \Rightarrow \boldsymbol{W = 65{,}000\ lb_f}$$

Example 6.22: For the Malaysia area, determine the value of the d_m if the drilling rate is 30 *ft/hr*, the rotary RPM is 90, and the weight on the bit is 65,000 lb_f. In addition, an equivalent circulating density at the bit was 9.5 *lbm/gal*. Assume necessary data.

Solution:

Given data:

R = drilling rate = 30 *ft/hr*
N = rotary speed = 90 *rpm*
W = weight on bit = 65,000 lb_f
ρ_e = actual mud weight in use = 9.5 *ppg*

Additional assumption:

d_b = bit diameter = 12.0 *in*

Required data:

d_m = modified d-exponent

Before finding out the dm, it is necessary to find out the d-exponent which can be calculated by using the Eq. (6.30) as:

$$d_{exp} = \frac{\log\left(\frac{R}{60N}\right)}{\log\left(\frac{12W}{10^6 d_b}\right)} = \frac{\log\left(\frac{30}{60\times 90}\right)}{\log\left(\frac{12\times 65{,}000}{10^6 \times 12.00}\right)} = \frac{-2.2553}{-1.1871} = 1.9$$

We know that for the area, the normal formation pressure gradient is 0.442 psi/ft (Table 6.2). So, the mud density equivalent to normal pore pressure gradient (ρ_n) can be calculated as:

$$\rho_n = \frac{0.442}{0.052} = 8.5\ ppg$$

Therefore, the modified d-exponent can be calculated using Eq. (6.31) as:

$$d_m = d_{exp}\frac{\rho_n}{\rho_e} = 1.9\times\frac{8.5}{9.5} = 1.7$$

Zamora Model: It is noted that Rehm and McClendon mentioned to use linear scales for plotting depth vs. d_m as shown in Figure 6.7. A straight-line normal pressure trend line having intercept d_{mo} and exponent m is assumed such that

$$d_{mn} = d_{mo}e^{mD} \tag{6.36}$$

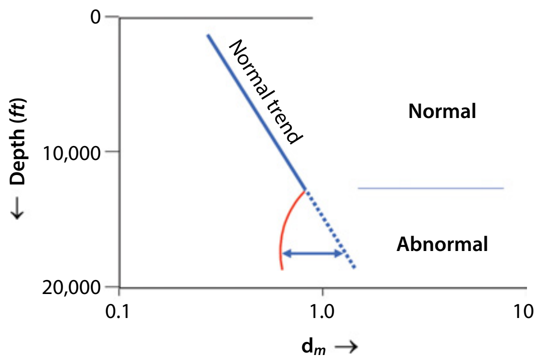

Figure 6.7 Depth verses d_m plotting in semi-logarithmic coordinates.

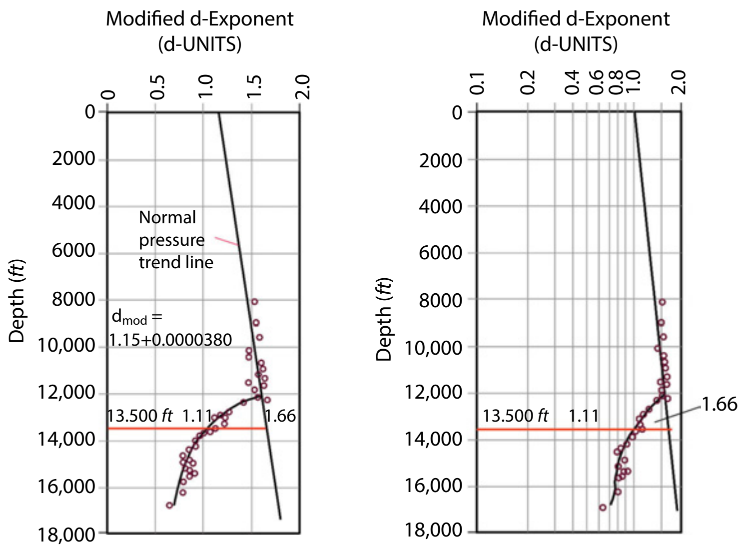

Figure 6.8 Depth verses d_{exp} and d_m plotting for Example 6.13.

Zamro introduced another empirical equation to calculate the formation pressure gradient as:

$$G_f = G_n \frac{d_{mn}}{d_m} \tag{6.37}$$

where

G_n = normal pressure gradient, lb_m/gal

Example 6.23: Figure 6.28 shows the depth vs. *d*-exponent and modified *d*-exponent plot. Estimate the formation pressure at 13,500 *ft* using Rehm and McClendon and the Zamora correlation. Assume that Figure 6.8 is constructed based on Gulf of Mexico data.

Solution:

Given data:

D = depth = 13,500 *ft*

From Figure 6.8, we can find out the d_{mn} and d_m at a depth of 13,500 *ft* as:

d_m = modified d-exponent = 1.11

d_{mn} = d_m from the normal pressure trend line at a depth of interest = 1.66

G_n = normal pressure gradient = 0.465 lb_m/gal (Table 6.2)

Required data:

P_f = formation pressure, *psi*

Rehm and McClendon Model:

Before finding out the formation pressure, it is necessary to find out the equivalent mud density based on Rehm and McClendon, which can be calculated by using the Eq. (6.33) as:

$$\rho_e = 7.56\log[d_{mn} - d_m] + 16.5 = 7.56\log[1.66 - 1.11] + 16.5$$
$$= 14.53\ ppg$$

The formation pressure gradient can be obtained using Eq. (6.34) as:

$$G_f = 0.052\rho_e = 0.052 \times 14.53 = 0.756\,psi/ft$$

Finally, the formation pressure can be calculated using Eq. (6.35) as:

$$P_f = G_f D = 0.756 \times 13{,}500 = \mathbf{10{,}206}\ \boldsymbol{psi}$$

Zamora Model:

The formation pressure gradient can be obtained directly using Eq. (6.37) as:

$$G_f = G_n \frac{d_{mn}}{d_m} = 0.465 \times \frac{1.66}{1.11} = 0.695\ psi/ft$$

Finally, the formation pressure can be calculated again using Eq. (6.35) as:

$$P_f = G_f D = 0.695 \times 13{,}500 = \mathbf{9{,}382.5}\ \boldsymbol{psi}$$

Eaton Model: The value of the formation pressure can be derived from the modified d-exponent, using the method proposed by Eaton (1976) as:

$$\frac{P_f}{D} = \frac{\sigma_{ob}}{D} - \left[\frac{\sigma_{ob}}{D} - \left(\frac{P_f}{D}\right)_n\right]\left(\frac{d_{mc}}{d_{mn}}\right)^{1.2} \tag{6.38}$$

where

σ_{ob} = overburden stress (i.e. $\sigma_{ob} = \sigma_v + P_{pn}$), *psi*

$\frac{P_f}{D}$ = formation pressure gradient, *psi/ft*

$\frac{\sigma_{ob}}{D}$ = overburden stress gradient, *psi/ft*

$\left(\frac{P_f}{D}\right)_n$ = normal pressure gradient, *psi/ft*

d_{mc} = calculated modified d-exponent at a given depth

d_{mn} = modified d-exponent from normal trend (i.e., extrapolated) at a given depth (Figure 6.9)

Eaton claims the relationship is applicable worldwide and is accurate to 0.5 *ppg*.

Example 6.24: What is the pore pressure at the point indicated on Figure 6.10? Assume Gulf Coast area where depth is 10,000 *ft*. Also assume that overburden stress gradient

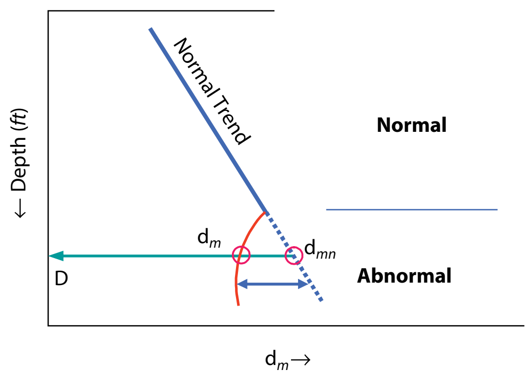

Figure 6.9 Depth verses d_m plotting showing d_{mn} on the trend line (Hossain and Al-Majed, 2015).

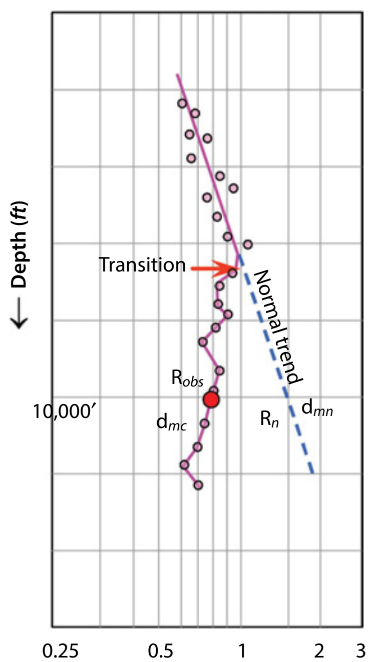

Figure 6.10 Depth verses R_{obs} and R_n plotting for Example 6.8.

is 0.95 *psi/ft*, and normal formation pressure gradient is 0.465 *psi/ft*. Use Eaton Equation. Find out the EMW of the formation too.

Solution:

Given data:

D = depth of the formation = 10,000 *ft*

$\frac{\sigma_{ob}}{D}$ = overburden stress gradient = 0.95 *psi/ft*

$\left(\frac{P_f}{D}\right)_n$ = normal pressure gradient = 0.456 *psi/ft*

Required data:

P_f = formation pore pressure at a depth of 10,000 *ft*, *psi*

From Figure 6.10, we can find out the R_{obs} and R_n at a depth of 10,000 *ft* as:

R_{obs} = observed shale resistivity of the formation = 0.8 *ohms-m*

R_n = resistivity of the formation at a normal trend = 1.55 *ohms-m*

The Eaton model can be expressed in terms of resistivity of the formation which is analogous with *d*-exponent as:

$$\frac{P_f}{D} = \frac{\sigma_{ob}}{D} - \left[\frac{\sigma_{ob}}{D} - \left(\frac{P_f}{D}\right)_n\right]\left(\frac{R_{obs}}{R_n}\right)^{1.2}$$

$$\frac{P_f}{D} = 0.95 - [0.95 - 0.456]\left(\frac{0.80}{1.55}\right)^{1.2} = 0.726624\, psi/ft$$

Therefore,

$$P_f = 0.726624 \times D = 0.726624 \times 10000 = \mathbf{7266.24\ psi}$$

The equivalent mud weight (EMW) can be calculated as (Eq. 4.36a):

$$EMW = \frac{P_f}{0.052 \times D} = \frac{7266.24}{0.052 \times 10000} = \mathbf{13.97}\ \boldsymbol{lb_m / gal}$$

Example 6.25: The mud engineer of an Arabian oil company designed the mud weight of 10 lb_m/gal for a formation needed to be drilled where the pressure gradient was found 0.52 *psi/ft*. The surface casing was set at a depth of 2,500 *ft*. It was noticed that fracture gradient below the surface casing was 0.73 *psi/ft*. The driller realized that he was passing a pressure transition zone while drilling at a depth of 10,000 *ft*. This new situation gave the impression that the designed mud weight might be less than pore pressure which results a kick. To avoid kick, determine the maximum safe underbalance between mud weight and pore pressure if well kicks from formation at a depth of 10,000 *ft*.

Solution:

Given data:

D = total vertical depth = 10,000 *ft*

ρ_m = mud weight = 10 *ppg*

G_{np} = normal pressure gradient = 0.52 *psi/ft*
D = depth at which surface casing is set = 2,500 *ft*
G_{fp} = fracture pressure gradient = 0.73 *psi/ft*

Required data:
EMW_{max} = maximum safe underbalance mud weight, *ppg*

Figure 6.11 shows the casing seat and pressure gradient where an elaboration is explained for this example. In general, when a well kicks, the well is shut in and the wellbore pressure increases until the new BHP equals the new formation pressure. At that point, influx of formation fluids enters into the wellbore ceases. Since the mud gradient in the wellbore has not changed, the pressure increases uniformly everywhere (Figure 6.11).

At 2,500 ft

The initial mud pressure can be estimated as:

$$P_{im} = G_{np}D = (0.52\,psi/ft)\times(2{,}500\,ft) = 1{,}300\ psi$$

The fracture pressure can also be estimated as:

$$P_{fp} = G_{fp}D = (0.73\,psi/ft)\times(2{,}500\,ft) = 1{,}825\ \text{psi}$$

Therefore, maximum allowable increase in pressure = (1,825 – 1,300) = 525 *psi*

At 10,000 ft

Since the pressure increases uniformly everywhere as shown in Figure 6.12, the maximum allowable increase in pressure at a depth of 10,000 *ft* will be 525 *psi*. This increase in pressure corresponds to an increase in mud weight which can be calculated using Eq. (4.36a) as:

$$EMW_{max} = \frac{525}{0.052\times10000} = \mathbf{1.01}\ \boldsymbol{lb_m/gal}$$

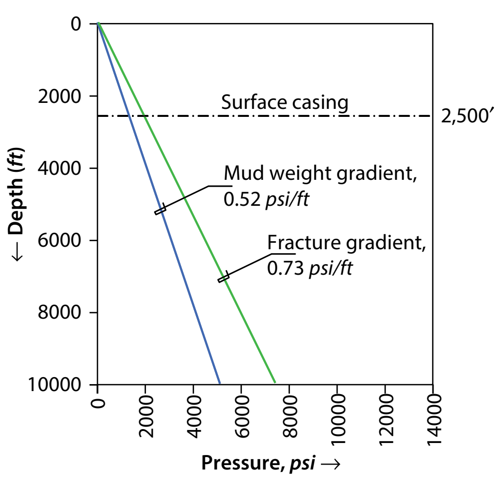

Figure 6.11 Casing set and pressure gradient for Example 6.9.

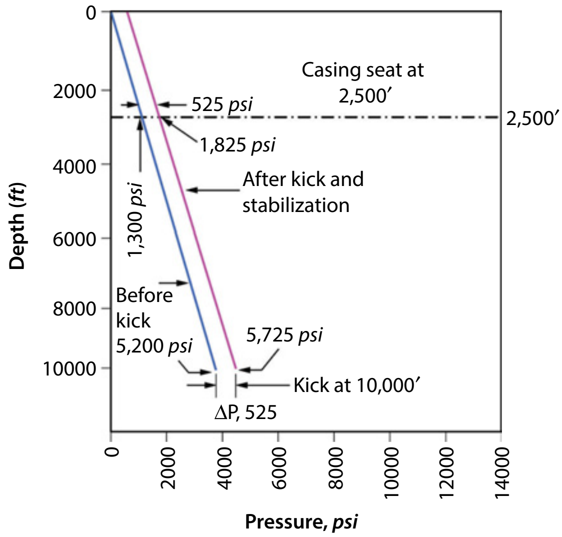

Figure 6.12 Wellbore pressures at different depth for Example 6.9.

This increase in EMW is the maximum which is the kick tolerance for a small kick.

Combs Model: In 1968, Combs presented a general equation for penetration rate which is proportional to weight on bit, rotary speed, and bit hydraulics. Each released to a fixed power as shown below:

$$R = R_d \left(\frac{W}{3{,}500 d_h} \right)^{a_W} \left(\frac{N}{200} \right)^{a_N} \left(\frac{q}{96 d_h d_n} \right)^{a_q} f(P_d) f(t_N) \tag{6.39}$$

where

f = function of

q = fluid circulation rate, *gpm*

R = shale drillability or rate of penetration, *ft/hr*

W = weight on bit, lb_f

a_W = bit weight exponent (= 1.0 for offshore Louisiana)

a_N = rotating speed exponent (= 0.6 for offshore Louisiana)

a_q = flow rate exponent (= 0.3 for offshore Louisiana)

d_h = borehole diameter, in

d_n = diameter of one bit nozzle, *in*

P_d = differential pressure, lb_f/gal/1000 *ft*.

R_d = shale drillability at zero differential pressure, *ft/hr*

t_N = bit wear index (equivalent to rotating hours),

$f(P_d)$ = function related to differential pressure

$f(t_N)$ = function related to bit wear

Density of Shale Cuttings: the following relation can be used:

$$\rho_{bs} V_s = \rho_w V_t \tag{6.40}$$

where
V_s = volume of shale cutting, ft^3
V_t = total volume of cup, ft^3
ρ_{bs} = bulk density of shale, lb_m/ft^3
ρ_w = density of water, lb_m/ft^3

Equation (6.40) can be written as:

$$\bar{\rho}_m V_t = \rho_{bs} V_s + \rho_w (V_t - V_s) \tag{6.41}$$

Substituting V_s from Eq. (6.40) into Eq. (6.41) and then solving for ρ_{bs} will give as:

$$\rho_{bs} = \frac{\rho_w^2}{2\rho_w - \bar{\rho}_m} \tag{6.42}$$

A mathematical model of the normal compaction trend for the bulk density of shale cuttings can be developed by using Eq. (6.5) as:

$$\rho_{bsn} = \rho_r - (\rho_r - \rho_f)\phi \tag{6.43}$$

Now, substituting Eq. (6.7) into Eq. (6.43) for porosity variation will give:

$$\rho_{bsn} = \rho_r - (\rho_r - \rho_f)\phi_o e^{-K_\phi D_s} \tag{6.44}$$

where
ρ_{bsn} = shale density for normally pressured shales

The grain density of pure shale is 2.65 g/cm^3. The average pore fluid density can be obtained from Table 6.2. Constants ϕ_0, and K_f can be based on shale-cutting bulk density measurements made in the normally pressured formations.

6.2.4 Methods for Estimating Fracture Pressure

i) Direct Method
The predicted surface leak-off pressure is given by:

$$P_{lo} = P_{fp} - 0.052\rho_m D + \Delta P_f \tag{6.45}$$

where
P_{lo} = surface leak-off pressure, psi
P_{fp} = observed fracture pressure, psi
ρ_m = mud density, lb_m/ft^3
D = total depth, ft
ΔP_f = friction pressure loss, psi

Frictional pressure loss can be calculated using the gel strength as:

$$\Delta P_f = \frac{\tau_g D}{300d} \tag{6.46}$$

where

τ_g = gel strength, $lb_m/100\,ft^2$
D = total depth, ft
d = inner diameter of the drill pipe, in

ii) Indirect Methods

The following equations and correlations are commonly used to determine the fracture pressure theoretically.

Hubbert and Willis Model: In 1957, Hubbert and Willis proposed a method for calculating fracture gradient based on the fact that fracturing occurs when the applied fluid pressure exceeds the sum of minimum effective stress and formation pressure which can be written as:

$$P_{fp} = \sigma_{min} + P_f \tag{6.47}$$

where

P_{fp} = observed fracture pressure at the point of interest, psi
σ_{min} = minimum effective stress at the point of interest, psi
P_f = formation pore pressure at the point of interest, psi

The fracture gradient determination can also be written as:

$$G_{fr\_min} = \frac{1}{3}\left[\frac{\sigma_{min}}{D} + \frac{2P_f}{D}\right] \tag{6.48a}$$

$$G_{fr\_max} = \frac{1}{2}\left[\frac{\sigma_{min}}{D} + \frac{P_f}{D}\right] \tag{6.48b}$$

where

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient at the point of interest, psi/ft
$\frac{\sigma_{min}}{D}$ = minimum effective stress gradient at the point of interest, psi/ft
$G_p = \frac{P_f}{D}$ = formation pore pressure gradient at the point of interest, psi/ft
D = depth, ft

If an overburden stress gradient or minimum effective stress gradient is assumed to be 1 psi/ft, Eq. (6.48) becomes as:

$$G_{fr\_min} = \frac{1}{3}\left[1 + \frac{2P_f}{D}\right] \tag{6.49a}$$

$$G_{fr\_max} = \frac{1}{2}\left[1 + \frac{P_f}{D}\right] \tag{6.49b}$$

Matthews and Kelly Model: Matthews and Kelly (1967) published a fracture gradient relationship which differs from the Hubbert and Willis model and was introduced.

$$\sigma_{min} = F_\sigma \sigma_z \tag{6.50}$$

where

F_σ = variable matrix stress coefficient for the depth at which the value of σ_z would be normal matrix stress, dimensionless

σ_z = matrix stress = $\sigma_{ob} - P_f$, *psi*

σ_{ob} = overburden pressure, *psi*

Substituting the Eq. (6.50) into Eq. (6.47), fracture pressure can be obtained as:

$$P_{fp} = F_\sigma \sigma_z + P_f \tag{6.51}$$

Equation (6.51) can be expressed in terms of fracture gradient. Matthews and Kelly developed the following equation for calculating fracture gradients in the sedimentary formations as:

$$G_{fr} = \frac{F_\sigma \sigma_z}{D} + \frac{P_f}{D} \tag{6.52}$$

Example 6.26: Calculate the minimum and maximum equivalent mud weight in *ppg* that can be used immediately below the casing seat at a depth of 14,000 *ft* for the pore pressure gradient of 0.45 *psi/ft* and an overburden gradient of 0.85 *psi/ft*. It is assumed that matrix stress coefficient is 0.612. Use Mathews and Kelly method.

<u>Solution</u>:

Given data:

D	=	Total vertical depth	=	14,000 *ft*
ρ_m	=	mud weight	=	11.0 *ppg*
G_p	=	Pore pressure gradient	=	0.45 *psi/ft*
G_o	=	Overburden gradient	=	0.85 *psi/ft*
F_σ	=	variable matrix stress coefficient	=	0.612

Required data:

EMW_{min} = maximum equivalent mud weight, *ppg*

EMW_{max} = maximum equivalent mud weight, *ppg*

The overburden pressure at a depth of

$$14{,}000\ ft = 0.85 \times 14000 = 11900\ psi$$

The pore pressure at a depth of

$$14{,}000\ ft = 0.45 \times 14000 = 6300\ psi$$

Using Mathews and Kelly method, the minimum stress can be calculated by Eq. (6.50) as:

$$\sigma_{min} = F_\sigma \sigma_z = 0.612 \times (11900 - 6300) = 3427.2\,psi$$

The fracture pressure can be obtained using Eq. (6.51) as:

$$P_{fp} = 3427.2 + 6300 = 9727.2\,psi$$

Therefore, maximum equivalent mud weight which can be calculated using Eq. (4.36a) as:

$$EMW_{max} = \frac{9727.2}{0.052 \times 14000} = \mathbf{13.36\ lb_m / gal}$$

And the minimum equivalent mud weight can be calculated as:

$$EMW_{min} = \frac{6300}{0.052 \times 14000} = \mathbf{8.65\ lb_m / gal}$$

Example 6.27: Calculate the minimum and maximum equivalent mud weight in ppg that can be used immediately below the casing seat at a depth of 12,000 *ft* for the pore pressure gradient of 0.58 *psi/ft* and an overburden gradient of 0.95 *psi/ft*. It is assumed that matrix stress coefficient is 0.712. Use Mathews and Kelly method.

Solution:

Given data:

D	=	total vertical depth	=	12,000 *ft*
ρ_m	=	mud weight	=	10 *ppg*
G_p	=	pore pressure gradient	=	0.58 *psi/ft*
G_o	=	overburden gradient	=	0.95 *psi/ft*
F_σ	=	variable matrix stress coefficient	=	0.712

Required data:

EMW_{min}	=	maximum equivalent mud weight, *ppg*
EMW_{max}	=	maximum equivalent mud weight, *ppg*

The overburden pressure at a depth of

$$12{,}000 \text{ ft} = 0.95 \times 12{,}000 = 11{,}400 \text{ psi}$$

The pore pressure at a depth of

$$12{,}000 \text{ ft} = 0.58 \times 12{,}000 = 6{,}960 \text{ psi}$$

Using Mathews and Kelly method, the minimum stress can be calculated by Eq. (6.50) as:

$$\sigma_{min} = F_\sigma \sigma_z = 0.712 \times (11400 - 6960) = 4{,}440\,psi$$

The fracture pressure can be obtained using Eq. (6.51) as:

$$P_{fp} = 4440 + 6960 = 11{,}400\ psi$$

Therefore, maximum equivalent mud weight which can be calculated using Eq. (4.36a) as:

$$EMW_{max} = \frac{11400}{0.052 \times 12000} = \mathbf{18.27}\ \boldsymbol{lb_m / gal}$$

And the minimum equivalent mud weight can be calculated as:

$$EMW_{min} = \frac{6960}{0.052 \times 12000} = \mathbf{11.15\ lb_m /gal}$$

Example 6.28: A well of 12,500 *ft* was drilled at Louisiana Gulf Coast area where the pore pressure gradient was found 0.695 *psi/ft*. Calculate the fracture gradient in units of *psi/ft* and lb_m/gal using Matthews and Kelly model.

<u>Solution</u>:

Given data:

D = total vertical depth = 12,500 *ft*

$G_p = \frac{P_f}{D}$ = pore pressure gradient = 0.695 *psi/ft*

Required data:

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient in *psi/ft*

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient in *ppg*

In this case P_f and D are known i.e. pore pressure gradient is known. The matrix stress, σ_z may be calculated using $\sigma_z = \sigma_{ob} - P_f$ where overburden gradient is considered as 1 *psi/ft*. Finally F_σ is determined graphically using Figure 6.13. We will use the above procedure to calculate the fracture gradient.

1. First, determine the pore pressure gradient.

$$G_p = \frac{P_f}{D} = 0.695\ psi/ft$$

2. Next, calculate the matrix stress.

$$\sigma_z = \sigma_{ob} \times D - P_f \times D = (1 - 0.695) \times 12500$$
$$= 0.305 \times 12500 = 3{,}812.5\,psi$$

3. Now determine the depth, D_i under normally pressured conditions. In such case, the rock matrix stress σ_z would be 3,812.5 *psi* and normal pore pressure gradient is 0.46 *psi/ft*.

$$\sigma_{zn} = \sigma_{obn} \times D_i - P_{fn} \times D_i$$

$$(1.0 - 0.46) \times D_i = 3{,}812.5$$

$$D_i = 7{,}060.19\,ft$$

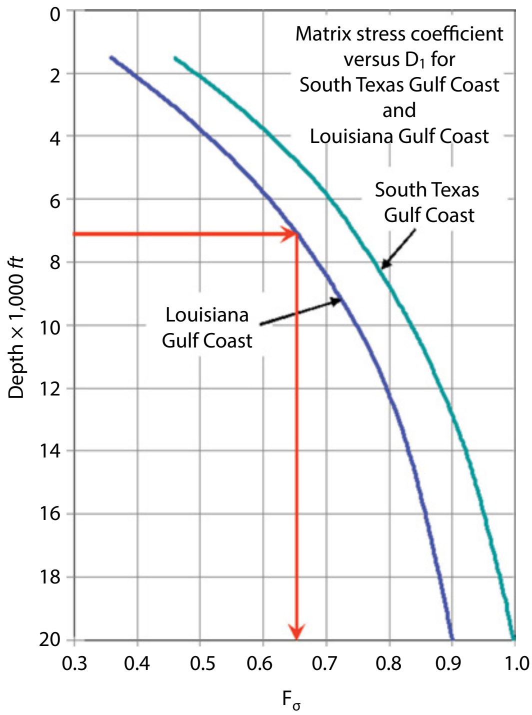

Figure 6.13 Matrix stress coefficients for Example 6.28 using Matthews and Kelly model.

4. Using $D_i = 7060\,ft$, Matthews and Kelly plot (Hossain and Al-Majed, 2015: Figure 6.40) is applied to construct Figure 6.10 and obtained the corresponding value of $F_z = 0.65$.
5. Finally to calculate the formation fracture gradient (G_f), Eq. (6.52) is applied.

$$G_{fr} = \frac{F_\sigma \sigma_z}{D} + \frac{P_f}{D} = 0.65 \times \frac{3{,}812.5}{12{,}500} + 0.695 = \mathbf{0.893\ psi/ft}$$

In terms of ppg, the formation fracture gradient is

$$G_{fr} = \frac{0.893 psi/ft}{0.052} = \mathbf{17.18\ ppg}$$

iv) Eaton Model: In 1969, Ben Eaton modified the Hubbert and Willis method which can be written as:

$$\sigma_x = \sigma_y = \sigma_h = \frac{\mu}{1-\mu}\sigma_z \tag{6.53}$$

where

σ_x = matrix stress in *x*-direction, *psi*
σ_y = matrix stress in *y*-direction, *psi*
μ = Poisson's ratio

Equation (6.53) is analogous with Eq. (6.51). Substituting $\sigma_z = \sigma_{ob} - P_f$ and applying the analogy, the fracture pressure can be obtained as:

$$P_{fp} = \frac{\mu}{1-\mu}(\sigma_{ob} - P_f) + P_f \tag{6.54}$$

The fracture gradient for any depth of interest can be written as:

$$G_{fr} = \frac{\mu}{1-\mu}\left(\frac{\sigma_{ob} - P_f}{D}\right) + \frac{P_f}{D} \tag{6.55}$$

v) Christmen Model: Christmen (1973) proposed a method to predict fracture gradient for offshore field application, which is shown below:

$$G_{ob} = \frac{1}{D}(\rho_w D_w + \rho_b D_{ml}) \tag{6.56}$$

where

G_{ob} = overburden gradient, *psi/ft*
ρ_w = density of seawater, lb_m/ft^3
ρ_b = average bulk density, lb_m/ft^3
D_w = seawater depth, *ft*
D_{ml} = depth below mud line, *ft*

If we assume the sea water density of 1.02 *g/cc*, Eq. (6.56) becomes as:

$$\frac{P_{fp}}{D} = \frac{1}{D}(0.44 D_w + \rho_b D_{ml}) \tag{6.57}$$

vi) Anderson *et al.*, Model: Anderson *et al.* (1973) developed a model based on Biot's stress/strain relationships for elastic porous media which can be written in form as:

$$P_{fp} = \alpha P_f + \frac{2\mu}{1-\mu}(P_{ob} - \alpha P_f) \tag{6.58}$$

where

P_{ob} = overburden pressure, *psi*
α = ratio of the compressibility of the porous rock matrix to the intrinsic compressibility of rock = $1 - \frac{c_r}{c_b}$
c_r = compressibility of the porous rock matrix, $1/psi$
c_b = bulk compressibility of the rock matrix, $1/psi$

vii) Belloti and Giacca Model: The following equations can be used to calculate fracture pressure gradient presented by Belloti and Giacca (1978).

$$G_{fr} = G_p + \frac{2\mu}{1-\mu}(G_{ob} - G_p) \tag{6.59}$$

where

G_{fr} = fracture gradient, *psi/ft*

G_{ob} = overburden gradient, *psi/ft*

G_p = pore pressure gradient, *psi/ft*

Equation (6.59) is used when the pressure is totally employed at the well bore, as in the case of filtration controlled by wall-building fluids. For the case of free formation invasion by drilling fluids where the pressure distribution creates a gradient inside the rock, the following equation can be used:

$$G_{fr} = G_p + 2\mu(G_{ob} - G_p) \tag{6.60}$$

The authors used a constant value of (μ) according to rock lithology.

Example 6.29: A well of 14,750 *ft* was drilled at South Texas Gulf Coast area where the pore pressure gradient was found 0.74 *psi/ft*. Calculate the fracture gradient in units of *psi/ft* and lb_m/gal using Hubbert and Willis model, Matthews and Kelly model, Eaton model, and Belloti and Giacca model. Summarize the results in tabular form, showing answers, in units of *lb/gal* and also in *psi/ft*.

Solution:

Given data:

D = total vertical depth = 14,750 *ft*

$G_p = \frac{P_f}{D}$ = pore pressure gradient = 0.74 *psi/ft*

Required data:

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient in *psi/ft*

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient in *ppg*

Hubbert and Willis model:

$$G_{fr\_min} = \frac{1}{3}\left[1 + \frac{2P_f}{D}\right] = \frac{1}{3}[1 + 2\times 0.74] = \mathbf{0.827}\ \boldsymbol{psi/ft}$$

$$G_{fr\_max} = \frac{1}{2}\left[1 + \frac{P_f}{D}\right] = \frac{1}{2}[1 + 0.74] = \mathbf{0.87}\ \boldsymbol{psi/ft}$$

In terms of ppg, the formation fracture gradient is

$$G_{fr\_min} = \frac{0.827\,psi/ft}{0.052\dfrac{psi/ft}{lb_m/gal}} = \mathbf{15.90}\ \boldsymbol{ppg} \text{ and}$$

$$G_{fr\_max} = \frac{0.87\,psi/ft}{0.052\dfrac{psi/ft}{lb_m/gal}} = \mathbf{16.73}\ \boldsymbol{ppg}$$

Matthews and Kelly model:

1. First, determine the pore pressure gradient.

$$G_p = \frac{P_f}{D} = 0.74 \text{ psi/ft}$$

2. Next, calculate the matrix stress.

$$\sigma_z = \sigma_{ob} \times D - P_f \times D = (1 - 0.74) \times 14{,}750$$
$$= 0.26 \times 14{,}750 = 3{,}835.0\,psi$$

3. Now determine the depth, D_i under normally pressured conditions. In such case, the rock matrix stress σ_z would be 3,812.5 *psi* and normal pore pressure gradient is 0.46 *psi/ft*.

$$\sigma_{zn} = \sigma_{obn} \times D_i - P_{fn} \times D_i$$
$$(1.0 - 0.46) \times D_i = 3{,}835.0$$
$$D_i = 7{,}101.85\,ft$$

4. Using D_i = 7101.85 *ft*, Matthews and Kelly plot (Hossain and Al-Majed, 2014: Figure 6.40) is applied to construct Figure 6.14 and obtained the corresponding value of F_z = 0.74.

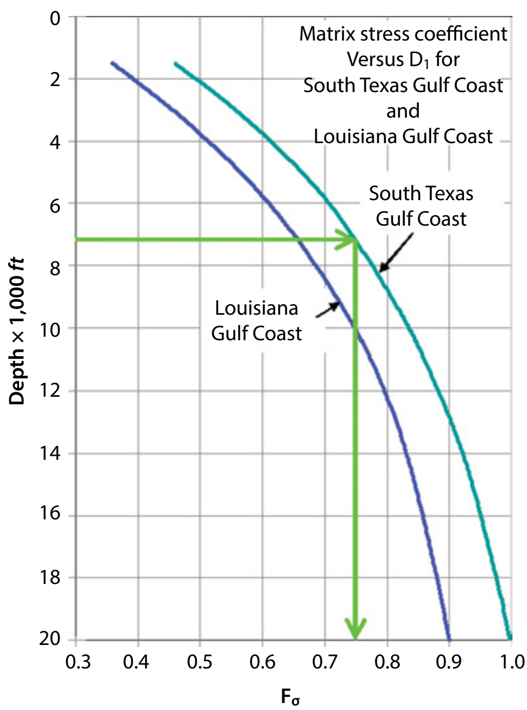

Figure 6.14 Matrix stress coefficients for Example 6.19 using Matthews and Kelly model

5. Finally to calculate the formation fracture gradient (G_f), Eq. (6.52) is applied.

$$G_{fr} = \frac{F_\sigma \sigma_z}{D} + \frac{P_f}{D} = 0.74 \times \frac{3,835.0}{14,750} + 0.74 = \mathbf{0.9324}\ \boldsymbol{psi/ft}$$

In terms of *ppg*, the formation fracture gradient is

$$G_{fr} = \frac{0.9324\, psi/ft}{0.052} = \mathbf{17.93}\ \boldsymbol{ppg}$$

Eaton model:
Equation (6.55) shows that overburden stress gradient and Poisson's ratio should be find out first from the graphs Figure 6.42 and Figure 6.43 as proposed by Eaton (Hossain and Al-Majed, 2015). Figures 6.15 and 6.16 show that the overburden stress gradient and Poisson's ratio respectively as:

$$\frac{\sigma_{ob}}{D} = 0.98\ psi/ft \text{ and } \mu = 0.48$$

Now applying Eq. (6.55), the fracture gradient can be calculated as:

$$G_{fr} = \frac{0.48}{1-0.48}(0.98-0.74) + 0.74 = \mathbf{0.962}\ \boldsymbol{\frac{psi}{ft}}$$

In terms of *ppg*, the formation fracture gradient is

$$G_{fr} = \frac{0.962\, psi/ft}{0.052} = \mathbf{18.49}\ \boldsymbol{ppg}$$

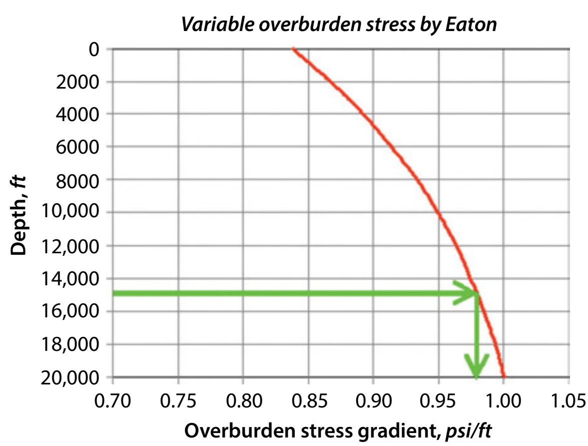

Figure 6.15 Variation of overburden stresses with depth for Example 6.19.

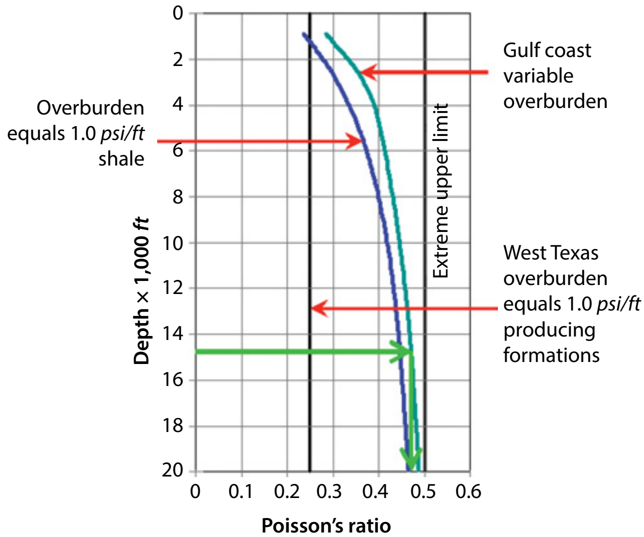

Figure 6.16 Variation of Poisson's ratio with depth for Example 6.19.

Belloti and Giacca model:

Applying Eq. (6.60), the fracture gradient can be calculated as:

$$G_{fr} = G_p + 2\mu(G_{ob} - G_p) = 0.74 + 2\times 0.48(0.98 - 0.74)$$
$$= \mathbf{0.9704\ psi/\,ft}$$

In terms of *ppg*, the formation fracture gradient is

$$G_{fr} = \frac{0.9704\,psi/\,ft}{0.052} = \mathbf{18.662\ ppg}$$

Example 6.30: A well of 14,000 *ft* was drilled at South Texas Gulf Coast area where the pore pressure gradient was found 0.64 *psi/ft*. Calculate the fracture gradient in units of *psi/ft* and *lbm/gal* using *Hubbert and Willis* model.

Solution:

Given data:

D = Total vertical depth = 14000 *ft*

$G_p = \frac{P_f}{D}$ = pore pressure gradient = 0.64 *psi/ft*

Required data:

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient, *psi/ft*

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient, *ppg*

Fracture Pressure gradient through *Hubbert and Willis* model can be calculated as:

$$G_{fr\_min} = \frac{1}{3}\left[1+\frac{2P_f}{D}\right] = \frac{1}{3}[1+2\times 0.64] = \mathbf{0.76}\ \boldsymbol{psi/ft}$$

$$G_{fr\_max} = \frac{1}{2}\left[1+\frac{P_f}{D}\right] = \frac{1}{2}[1+0.64] = \mathbf{0.82}\ \boldsymbol{psi/ft}$$

In terms of *ppg*, the formation fracture gradient is:

$$G_{fr\_min} = \frac{0.76\,psi/ft}{0.052\dfrac{psi/ft}{lb_m/gal}} = \mathbf{14.61}\ \boldsymbol{ppg} \text{ and}$$

$$G_{fr\_max} = \frac{0.82\,psi/ft}{0.052\dfrac{psi/ft}{lb_m/gal}} = \mathbf{15.77}\ \boldsymbol{ppg}$$

***Example 6.31*:** A well of 14,000 *ft* was drilled at South Texas Gulf Coast area where the pore pressure gradient was found 0.60 *psi/ft*. Calculate the fracture gradient in units of *psi/ft* and *lbm/gal* using *Eaton* Model.

Solution:

Given data:

D = Total vertical depth = 14000 *ft*

$G_p = \dfrac{P_f}{D}$ = pore pressure gradient = 0.60 *psi/ft*

Required data:

$G_{fr} = \dfrac{P_{fp}}{D}$ = fracture pressure gradient, *psi/ft*

$G_{fr} = \dfrac{P_{fp}}{D}$ = fracture pressure gradient, *ppg*

The fracture pressure gradient through *Eaton model* can be calculated as:

Equation (6.55) shows that overburden stress gradient and Poisson's ratio should be find out first from the graphs Figure 6.42 and Figure 6.43 as proposed by Eaton (Hossain and Al-Majed, 2015). Therefore,

$$\frac{\sigma_{ob}}{D} = 0.975\,psi/ft \text{ and } \mu = 0.46$$

Now applying Eq. (6.55), the fracture gradient can be calculated as:

$$G_{fr} = \frac{0.46}{1-0.46}(0.975-0.60)+0.60 = \mathbf{0.92}\ \boldsymbol{psi/ft}$$

In terms of *ppg*, the formation fracture gradient is,

$$G_{fr} = \frac{0.92\ psi/ft}{0.052\dfrac{psi/ft}{lb_m/gal}} = \mathbf{17.692\ ppg}$$

Example 6.32: A well of 13,000 *ft* was drilled at South Texas Gulf Coast area where the pore pressure gradient was found 0.55 *psi/ft*. Calculate the fracture gradient in units of *psi/ft* and *lbm/gal* using *Belloti and Giacca* model.

<u>Solution</u>:

Given data:

D = Total vertical depth = 13000 *ft*

$G_p = \frac{P_f}{D}$ = pore pressure gradient = 0.55 *psi/ft*

Required data:

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient, *psi/ft*

$G_{fr} = \frac{P_{fp}}{D}$ = fracture pressure gradient, *ppg*

Values of μ and G_{ob} are collected from Figure 6.43 and Figure 6.42 (Hossain and Al-Majed, 2015) which are shown below:

$$\mu = 0.45\ \&\ G_{ob} = 0.96$$

Applying Eq. (6.60), the fracture gradient can be calculated as:

$$G_{fr} = G_p + 2\mu(G_{ob} - G_p) = 0.55 + 2\times 0.45(0.96 - 0.55) = \mathbf{0.92\ psi/ft}$$

In terms of *ppg*, the formation fracture gradient is,

$$G_{fr} = \frac{0.92\ psi/ft}{0.052\dfrac{psi/ft}{lb_m/gal}} = \mathbf{17.692\ ppg}$$

6.3 Multiple Choice Questions

1. The magnitude of the pressure in the pores of a formation is known as the ______________.
   a) Formation pore pressure
   b) Bottom-hole pressure
   c) Differential pressure
   d) None of the above

2. ______________ increases the chances of stuck pipe and well control problems.
   a) Formation pore pressure
   b) Bottom-hole pressure
   c) Differential pressure
   d) None of the above
3. It is also very important to calculate the pressure at which the rocks will ______________.
   a) Fracture
   b) Puncture
   c) Rupture
   d) None of the above
4. Fractures can result in ______________ problems.
   a) Lost circulation
   b) Cross flow
   c) Stuck pipe
   d) None of the above
5. Theoretical and applied science of the mechanical behavior of rock and rock masses is known as ______________.
   a) Rock mechanics
   b) Geological engineering
   c) Geophysics
   d) None of the above
6. If the formation pressure is greater than the mud pressure, there is the possibility to have a ______________.
   a) Oil
   b) Kick
   c) Gas
   d) None of the above
7. Rock mechanics deals with the ______________ responses of all geological materials.
   a) Mechanical
   b) Electrical
   c) Geophysical
   d) None of the above
8. Knowledge of ______________ is very important in drilling engineering.
   a) Geological processes
   b) Geophysical processes
   c) Seismic surveys
   d) None of the above

9. The geological processes are often analogous with the occasions in ____________ laboratory testing.
   a) Rock mechanics
   b) Geological engineering
   c) Geophysics
   d) None of the above
10. Normally it is assumed that the mechanical properties of a rock are ____________.
   a) Constants
   b) Varying
   c) Average
   d) All of the above
11. Which of the following is a strength parameter of a rock?
   a) Friction angle
   b) Tensile strength
   c) Uniaxial compressive strength
   d) All of the above
12. Which of the following activity takes place in a variety of depositional environments?
   a) Transportation
   b) Deposition
   c) Sediment accumulation
   d) All of the above
13. The grain distribution may affect the ____________ of the rock.
   a) Mechanical properties
   b) Electrical properties
   c) Geophysical properties
   d) None of the above
14. ____________ is a measure of the diameter of the grain.
   a) Grain size
   b) Grain texture
   c) Grain color
   d) None of the above
15. Grain size determines the classes of ____________.
   a) Sedimentary rocks
   b) Igneous rocks
   c) Metamorphic rocks
   d) All of the above
16. Roundness and sphericity of grain determines ____________.
   a) Grain shape
   b) Grain size
   c) Grain texture
   d) None of the above

17. Grain sorting is a measure of the range of ______________.
    a) Grain shape
    b) Grain sizes
    c) Grain texture
    d) None of the above
18. A rock containing a wide range of grain sizes implies to be ______________ sorted.
    a) Poorly
    b) Efficiently
    c) Average
    d) None of the above
19. A rock containing a narrow range of grain sizes implies to be ______________ sorted.
    a) Poorly
    b) Well sorted
    c) Average
    d) None of the above
20. A basic difference between different rock types is ______________ of the rock.
    a) Grain shape
    b) Grain size
    c) Grain texture
    d) None of the above
21. Grain size is in ______________ range in shales.
    a) Millimeter
    b) Micro meter
    c) Nano meter
    d) None of the above
22. Grain size ranges between ______________ in sands.
    a) 0.1–1.0 mm
    b) 0.1–1.0 cm
    c) 0.1–1.0 nm
    d) All of the above
23. The difference in grain size effects ______________ of the rock.
    a) Petrophysical characteristics
    b) Geological Properties
    c) Geophysical properties
    d) None of the above
24. Resistance of the formation matrix to compaction is known as
    a) Underground stress
    b) Uniaxial Stress
    c) Deviatoric Stress
    d) None of the above

25. The stress arising from the weight of the rock overlying the zone under consideration is known as
    a) Underground stress
    b) Uniaxial Stress
    c) Deviatoric stress
    d) None of the above

26. The vertical stress increases downwards approximately at ______________.
    a) 0.8–1.0 *psi/ft*
    b) 0.1–1.0 *psi/ft*
    c) 0.5–1.0 *psi/ft*
    d) None of the above

27. The pore fluid remains in communication with the surface under ______________.
    a) Normal compaction
    b) Abnormal compaction
    c) Under compaction
    d) None of the above

28. Due to the communication with the formation and surface, the pore pressure gradient is a ______________.
    a) Straight line
    b) Polynomial curve
    c) Exponential curve
    d) All of the above

29. The gradient of the straight line in normal compaction is a representation of the ______________ of the fluid.
    a) Density
    b) Viscosity
    c) Shear stress
    d) All of the above

30. Any formation pressure above or below the hydrostatic gradient is called as ______________.
    a) Normal compaction
    b) Abnormal compaction
    c) Under compaction
    d) None of the above

31. All of the following are considered the causes of underpressured zones except:
    a) Thermal expansion
    b) Zone depletion
    c) Outcrop aquifer
    d) Rock compaction

32. What is the relationship between formation pore pressure and overburden pressure?
    a) Pore pressure is always less than overburden pressure
    b) Pore pressure is always greater than overburden pressure
    c) Pore pressure can be greater than overburden pressure
    d) None of the above
33. What will happen to the overburden pressure during production?
    a) It should increase
    b) It should decrease
    c) It will remain as it is
    d) No relation between production and overburden pressure
34. Which of the following is a direct indication of lost circulation?
    a) Decrease in mud pit levels
    b) Increase in mud flow rate
    c) Increase in flow line temperature
    d) None of the above
35. Which of the following is a direct indication of having an overpressured zone
    a) Decrease in mud pit levels
    b) Increase in mud weight of the return mud
    c) Decrease in mud flow rate
    d) Decrease in mud weight of the return fluid
36. Large drill cuttings are an indications of
    a) High pressure formation
    b) Normal pressure formation
    c) Low pressure formation
    d) All of the above
37. Inaccurate prediction of pore pressure
    a) Decreases the chances of well control problems
    b) Increases the chances of well control problems
    c) Has no relation with the well control problems
    d) None of the above
38. Well-sorted rocks are the rocks that contain
    a) Narrow range of grain sizes
    b) One grain size
    c) Many grain sizes
    d) None of the above
39. In which case will fracture pressure become similar to overburden pressure?
    a) Case of overpressured formations
    b) Case of compacted formations
    c) When all in-situ stresses are equal
    d) None of the above

40. Generally, fracture pressure increases as ______________.
    a) The depth increases
    b) The pore pressure decreases
    c) The overburden pressure decreases
    d) b and c

Answers: 1a, 2a, 3a, 4a, 5a, 6b, 7a, 8a, 9a, 10a, 11d, 12d, 13a, 14a, 15a, 16a, 17b, 18a, 19b, 20b, 21c, 22a, 23a, 24a, 25a, 26a, 27a, 28a, 29a, 30b, 31d, 32a, 33b, 34a, 35d, 36a, 37b, 38a, 39c, 40a

6.4 Summary

The chapter discusses the issues related to the formation pore pressure and fracture gradient. The different rock mechanical properties are discussed in detail. The development of underground stresses and the related formation pressure, fracture pressure are also outlined in this chapter. The importance of different types of pore pressures and their detail impact on the formation pore and fracture gradients are discussed thoroughly. The different causes of abnormal pressure with detailed detection and prediction techniques are the main focus of the chapter. Pore Pressure estimation and prediction techniques and correlations are well explained to understand the techniques. The same procedure is applied for fracture pressure and gradient calculation. The chapter presents almost all the formulas related to the drilling hydraulics. The workout examples and the MCQs are presented in a chronological manner. The exercise solutions are given in Appendix A. Moreover, this chapter added some more MCQs for the student's self-practice and the answers are given in Appendix B. The MCQs covered almost all the materials covered in this chapter.

6.5 Exercise and MCQs for Practice

6.5.1 Exercises (Solutions are in Appendix A)

Exercise 6.1: A water aquifer is connected hydraulically to a surface lake. The aquifer is deviated at a certain angle. Two wells were drilled to the center of the aquifer. The pressure difference between the two wells and the ratio between the pressure of the two wells are 220 *psi* and 0.8; respectively. If the vertical distance between the aquifer in the two wells is 500 *ft*, calculate the aquifer depth in both wells. ***Answers: 2,000 ft, 2,500 ft.***

Exercise 6.2: An aquifer is connected hydraulically to a surface lake of water. The aquifer is deviated at a certain angle. Two wells were drilled to the center of the aquifer. The depth of the deepest well to the center of the aquifer is 3,750 *ft*. The ratio of the difference in pressure between the two wells to the pressure of the deepest well is 0.20, whereas the pressure of the shallowest well is 1,342 *psi*. Estimate the density of the water in the lake and the depth of the shallowest well to the center of the aquifer. ***Answers: 8.6 ppg, 3,000 ft.***

Exercise 6.3: A 1,000 *ft* of gas formation above 750 *ft* of water formation. The top pressure of the gas formation is 4,022.5 *psi* and the bottom pressure of the water formation is 4,500 *psi*. If the pressure gradient of the water formation is 0.49 *psi/ft*, calculate the gas formation gradient and the depth of the oil water contact depth. ***Answers: 0.111 psi/ft, 9,184 ft.***

Exercise 6.4: A gas field with a water formation beneath it. The pressure difference between the top and the bottom of the gas formation is equal to that for the water formation. The gas and water formations pressure gradient are 0.19 and 0.46 *psi/ft*; respectively. If the pressure at the bottom of the water formation is 4,600 *psi*, calculate the ratio between the thickness of gas and water formations. And if the pressure at the top of the gas formation is 4,000 *psi*, calculate the gas and water formation thicknesses and the depth of the gas-water contact. ***Answers: 2.42, 1,579 ft, 652 ft, 9,348 ft.***

Exercise 6.5: A core sample of 2.54 cm in size and 7.62 cm in length from a sandstone formation. The core sample was cleaned and dried, and dry weight was measured to be 79.0 grams. The core sample was completely saturated using 9.8 grams of formation brine water that has a density of 1.09 gm/cc. Determine the grain density and bulk density of this core sample. ***Answers: 2.67 gm/cc, 2.30 gm/cc.***

Exercise 6.6: A well is planned to be drilled to the depth of 13,000 *ft*. The last casing shoe was at 7,500 *ft* and the fracture gradient below the casing shoe was 0.69 *psi/ft*. The designed mud weight to drill the new section is 11.8 ppg. What is the maximum pressure that can be applied at the surface? And if the expected pressure at the depth of 13,000 *ft* is 8,450 *psi*, what will be the maximum pressure at the surface when changing the mud to a new one has mud hydrostatic pressure 300 *psi* greater than the formation pressure? ***Answers: 573 psi, 127 psi.***

Exercise 6.7: A leak-off test was conducted to a well using the old drilling fluid. When the mud leaked-off, the pump pressure was reading 1,250 *psi*. The well was circulated with a new mud having a density greater than the old one by 0.5 ppg. The leak-off test was repeated, and surface pressure was recorded to be 944.5 *psi* when the mud was leaked-off. If the fracture pressure was found exactly same, what is the depth of the casing shoe? ***Answer: 11,750 ft.***

Exercise 6.8: An oil-producing field has an initial reservoir pore pressure of 4,650 *psi*. The decline of the pore pressure and overburden pressure with time was estimated to be following the equation: $P_{fn} = 4{,}150t^{-0.2011}$ and $P_{ob} = 13{,}104t^{-0.054}$, where P_r and P_{ob} are the reservoir pore pressure and overburden pressure; and t is the time in years. What was the original overburden pressure? ***Answer: 13,660 psi.***

Exercise 6.9: Two sedimentary formations that have surface porosities of 0.42 and 0.36, and porosity decline constants of 0.00018 and 0.00014, respectively. At which depth will both rocks have similar porosity? What is that porosity? ***Answers: 3,854 ft, 0.21.***

Exercise 6.10: While drilling the surface section in a well, a kick was encountered at depth of 1,000 *ft* and shut-in drill pipe pressure was measured to be 100 *psi*. The density of the mud that used was 8.6 ppg. The kick was safely removed, and later the rig crew realized that the current formation and the sea water were in hydraulic communication. If the density of the sea water is 8.62 ppg, determine the difference in vertical distance between the surface location of the well and the sea level. ***Answer: 221 ft***

6.5.2 Exercises (Self-Practices)

E6.1: Find out the normal pore pressure at a depth of 7,000 *ft* below sea level. Assume that the drilling activities will be continued in Malaysia. Also find out the mud weight for that area.

E6.2: Consider a gas sand reservoir as shown in Figure 6.1. If the water-filled portion of the sand is pressured normally and the gas/water contact occurred at a depth of 5,300 *ft*, what mud weight would be required to drill through the top of the sand structure safely at a depth of 4,100 *ft*? Assume the gas has an average density of 0.78 lb_m/gal.

E6.3: Determine the pore pressure of a normally pressured formation in the Gulf of Mexico at 9,000 *ft* depth.

E6.4: Calculate the matrix stress of an underground reservoir if the overburden pressure is 6,300 *psi* and the formation pore pressure is 4000 *psi*.

E6.5: Determine values for surface porosity of an area where an average grain density of 2.55 g/cm^3, an average pore fluid density of 1.02 g/cm^3, and the value for porosity decline constant is 0.00009 ft^{-1}. Assume the average bulk density of the sediment is 2.52 g/cm^3 at a specified depth of 8,500 *ft*.

E6.6: Determine porosity decline constant for West Texas area. It is noted that an average grain density of 2.50 g/cm^3, an average pore fluid density of 1.00 g/cm^3, and the value for surface porosity of 38% were recorded. Assume the average bulk density of the sediment is 2.25 g/cm^3 at a specified depth of 10,000 *ft*. Also compute the vertical overburden stress at the same depth.

E6.7: A penetration rate of 25ft/hr was observed while drilling in shale at a depth of 10,500ft using a 9.875-in bit in the Gulf of Mexico. The WOB was 26,000 lb_f and the rotary speed was 110rpm. The equivalent circulating density at the bit was 10.0 lb_m/gal. Compute the d_{exp} and the d_m. Assume the normal pressure gradient for the area as 0.465 *psi/ft*.

E6.8: Figure 6.28 shows the depth vs. *d*-exponent and modified *d*-exponent plot. Estimate the formation pressure at 14,000 *ft* using Rehm and McClendon and the Zamora correlation. Assume that Figure 6.28 is constructed based on North Sea data.

E6.9: What is the pore pressure at a depth of 12,500 *ft* if the formation is in the Gulf Coast area? Assume that overburden stress gradient is 0.85 *psi/ft*, and normal formation

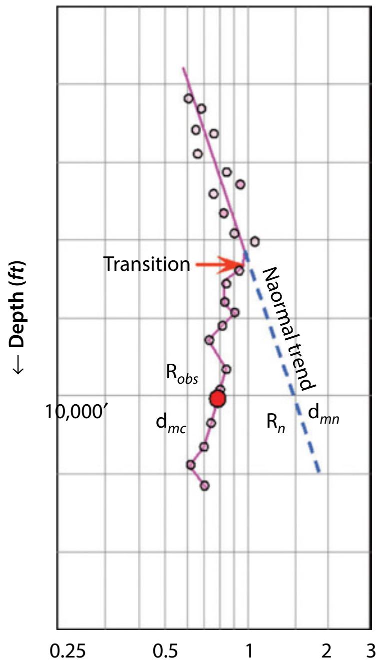

Figure 6.17 Depth verses R_{obs} and R_n plotting (Hossain and Al-Majed, 2015)

pressure gradient is 0.465 *psi/ft*. Use Eaton Equation. Use Figure 6.17. Also find out the EMW of the formation.

E6.10: The mud engineer of Schlumberger calculated the mud weight of 12 lb_m/gal for the North Sea area where the pressure gradient was found 0.452 *psi/ft*. The surface casing was set at a depth of 2,000 *ft*. The fracture gradient was calculated as 0.73 *psi/ft* and the transition zone was detected at a depth of 8,000 *ft* which results in a kick. To avoid kick, determine the maximum safe underbalance between mud weight and pore pressure if well kicks from formation at a depth of 8,000 *ft*.

E6.11: Calculate the minimum and maximum equivalent mud weight in ppg that can be used immediately below the casing seat at a depth of 10,000 *ft* for the pore pressure gradient of 0.57 *psi/ft* and an overburden gradient of 0.90 *psi/ft*. It is assumed that matrix stress coefficient is 0.70. Use Mathews and Kelly method.

E6.12: A well of 13,000 *ft* was drilled at Texas Gulf Coast area where the pore pressure gradient was found 0.735 *psi/ft*. Calculate the fracture gradient in units of *psi/ft* and lb_m/gal using Matthews and Kelly model.

E6.13: A well of 15,000 *ft* was drilled at South Louisiana Gulf Coast area where the pore pressure gradient was found 0.689 *psi/ft*. Calculate the fracture gradient in units of *psi/ft* and lb_m/gal using Hubbert and Willis model, Matthews and Kelly model, Eaton model, and Belloti and Giacca model. Summarize the results in tabular form, showing answers, in units of lb/gal and also in *psi/ft*.

6.5.3 MCQs (Self-Practices)

1. Formation pressure affects mainly
   a) Casing design
   b) Mud design
   c) Well control equipment
   d) All of the above

2. Accurate prediction of pore pressure
   a) Decreases the chances of well control problems
   b) Increases the chances of well control problems
   c) Has no relation with the well control problems
   d) None of the above

3. Improper estimation of fracture pressure may lead to
   a) Series of loss circulations-blowouts problems
   b) Increasing mud circulation rate
   c) Damaging the drill bit
   d) a and b

4. Mechanical properties of sedimentary rocks depend on
   a) The age of the rock
   b) Type of hydrocarbon fluid in their pores
   c) The depositional environment
   d) a and c

5. The process of creating a sedimentary rocks will mainly affect the
   a) Type of hydrocarbons
   b) Selection of well locations
   c) Distribution of in-situ stresses
   d) All of the above

6. Grain size of sedimentary rocks determines the
   a) In-situ stresses
   b) Rock class
   c) Rock permeability
   d) All of the above

7. Poorly sorted rocks are the rocks that contain
   a) Narrow range of grain sizes
   b) One grain size
   c) Many grain sizes
   d) None of the above

8. Vertical stress of rocks changes with
   a) Depth
   b) Rock type
   c) Rock's grain size
   d) All of the above
9. Formation pressure is created as a result of
   a) Compaction of the rock sediments
   b) Trapping of fluids in the rock pores
   c) Building of sediments on top of each other
   d) All of the above
10. Pore pressure is mainly dependent on the
    a) Grain size
    b) Pore size
    c) Pore throat geometry
    d) Pore fluids density
11. The hydrostatic gradient of normal pressured formations has the range from
    a) 0.0 to 0.5 *psi/ft*
    b) 0.43 to 0.5 *psi/ft*
    c) 0.5 to 0.65 *psi/ft*
    d) 0.65 to 1.0 *psi/ft*
12. All of the following formation gradients are considered abnormal pressure gradient except
    a) 0.65 *psi/ft*
    b) 0.55 *psi/ft*
    c) 0.35 *psi/ft*
    d) 0.45 *psi/ft*
13. Which of the following is not the reason of an abnormally pressure formation?
    a) Artesian system
    b) Thermal effect
    c) Biochemical effects
    d) None of the above
14. Overburden is defined as
    a) Weight of rocks below a specific formation
    b) Weight of rocks above a specific formation
    c) Weight of fluids in the pores of rock
    d) b and c
15. Which of the following overburden pressure is not depending on
    a) Rock type
    b) Minerals of the rock
    c) Fluid density
    d) None of the above

16. For a similar formation rock, porosity in the shallower depths is ____________ that in the deeper depths.
    a) Less than
    b) Greater than
    c) Equal to
    d) There is no relation

17. Which of the following is considered as one of the abnormal pressure predictive techniques?
    a) Geophysical measurements
    b) Analyzing offset wells data
    c) Wireline logs
    d) All of the above

18. Which of the following is not considered as one of the abnormal pressure predictive techniques?
    a) Drilling parameters
    b) Mud logging data
    c) Nearby wells data
    d) Wireline logs
19. All of the following are the detective techniques of abnormal pressure except?
    a) Changes in drag
    b) Pit volume changes
    c) Seismic data
    d) All of the above
20. Which of the following is an indication of abnormal pressured zone?
    a) Sudden ROP increase
    b) Sudden ROP decrease
    c) Constant ROP with WOB increase
    d) All of the above
21. In terms of mud parameters, which of the following is considered as an indication of abnormal pressured zones?
    a) Gas cut mud decrease
    b) Mud weight decrease
    c) Mud weight increase
    d) a and b
22. Which of the following is one of the confirmation techniques of abnormal pressured zones?
    a) Drill stem tests
    b) Seismic interpretation
    c) Wireline formation tests
    d) a and c

23. In which case will pore pressure become greater than the fracture pressure?
    a) Geopressured reservoirs
    b) Dry gas reservoirs
    c) Depleted reservoirs
    d) None of the above
24. Pore pressure is ______________ the fracture pressure.
    a) Sometimes greater than
    b) Sometimes less than
    c) Always less than
    d) Equal to
25. The fracture orientation is
    a) Always perpendicular to the minimum stress
    b) Always parallel to the minimum stress
    c) Always perpendicular to the maximum stress
    d) Unrelated to any of the rock stresses
26. All of the following are factors that affect the fracturing except
    a) Type of formation
    b) Formation pore pressure
    c) Type of fracturing fluid
    d) Degree of anisotropy
27. Which of the following is one of the direct methods of estimating fracture pressure?
    a) Leak-off test
    b) Mud integrity test
    c) Drill stem test
    d) Methylene blue test
28. When the LOT should be performed during drilling operations.
    a) Before starting to drill the well
    b) When drilling a new section
    c) Before running the casing
    d) All of the above
29. Which of the following is **not** one of the direct methods of fracture pressure estimations.
    a) PIT
    b) BOP
    c) DST
    d) b and c
30. Rock fracture gradient is important in
    a) Selecting casing seats
    b) Preventing loss circulations
    c) Planning the hydraulic fracturing jobs
    d) All of the above

31. Indirect methods of estimating fracture pressure are based on
    a) Cutting size analysis
    b) Mud gases analysis
    c) Stress analysis
    d) All of the above
32. As the pore pressure increases, the fracture pressure ______________
    a) Decreases
    b) Increases
    c) Remains same
    d) None of the above
33. What will happen to the fracture pressure if the mud hydrostatic pressure increases?
    a) Fracture pressure will increase
    b) Fracture pressure will decrease
    c) Fracture pressure will remain same as before
    d) None of the above
34. What will happen to the maximum allowable surface pressure (MASP) when the mud hydrostatic pressure increases?
    a) MASP will increase
    b) MASP will decrease
    c) MASP will remain same
    d) None of the above
35. Which of the following pressures is determining the density of the drilling fluid?
    a) Wellbore pressure
    b) Pore pressure
    c) Pump circulating pressure
    d) All of the following
36. Casing seat selection depends on
    a) Pore pressure
    b) Fracture pressure
    c) Both of them
    d) None of them
37. What is the major difference between sand and clay?
    a) Grain size of clays is greater than that of sand
    b) Grain size of clay is less than that of sand
    c) No difference in terms of grain size
    d) None of the above
38. A normal pressured formation has shifted up 300 *ft*, this formation will become
    a) Overpressured formation
    b) Underpressured formation
    c) Remains same
    d) None of the above

39. Abnormal pressured formation is the formation that has pressure gradient _____________ that of normal pressured formation.
    a) Less than
    b) Equal to
    c) Greater than
    d) a and c
40. None of the following is one of the problems of underpressured formations except
    a) Well kicks
    b) Differential sticking
    c) Well blowout
    d) Well caving

6.6 Nomenclature

A = rock matrix strength constant or drillability constant
D = total vertical depth, *ft*
d = inner diameter of the drill pipe, in
E = rotary speed exponent
f = function of
g = gravitational acceleration, *ft*/sec^2
m = slop of the normal trend line
N = rotary speed, rpm
q = fluid circulation rate, gpm
R = shale drillability or rate of penetration, *ft*/hr
W = weight on bit, lb_f
a_N = rotating speed exponent (= 0.6 for offshore Louisiana)
a_q = flow rate exponent (= 0.3 for offshore Louisiana)
a_W = bit weight exponent (= 1.0 for offshore Louisiana)
c_r = compressibility of the porous rock matrix, 1/*psi*
c_b = bulk compressibility of the rock matrix, 1/*psi*
d_b = bit diameter, in
d_h = borehole diameter, in
d_m = modified *d*-exponent
d_n = diameter of one bit nozzle, in
D_s = the depth from the sea bed to up to a depth of interest, *ft*
D_w = seawater depth, *ft*
d_{mc} = calculated modified *d*-exponent at a given depth
d_{mn} = modified *d*-exponent from normal pressure trend line (i.e., extrapolated) at a given depth (Figure 6.9)
d_{mo} = intercept of the normal trend line
D_{ml} = depth below mud line, *ft*
D_{sw} = depth from surface to the ocean bottom, *ft*
d_{exp} = bit weight exponent or *d*-exponent or formation drillability
dD = vertical depth from a reference point (ground surface)

F_s = variable matrix stress coefficient for the depth at which the value of σ_z would be normal matrix stress, dimensionless
$F(P_d)$ = function related to differential pressure
$F(t_N)$ = function related to bit wear
G_f = formation pressure gradient
G_n = normal pressure gradient, lb_m/gal
G_p = $\frac{P_f}{D}$ = formation pore pressure gradient at the point of interest, *psi/ft*
G_{fr} = $\frac{P_{fp}}{D}$ = fracture pressure gradient at the point of interest, *psi/ft*
G_{ob} = overburden gradient, *psi/ft*
P_d = differential pressure, lb_f/gal/1000 ft.
P_{fp} = observed fracture pressure at the point of interest, *psi*
P_f = formation pore pressure at the point of interest, *psi*
P_{fn} = normal formation pore pressure
P_{lo} = surface leak-off pressure, *psi*
P_{ob} = overburden pressure, *psi*
R_d = shale drillability at zero differential pressure, *ft*/hr
t_N = bit wear index (equivalent to rotating hours),
t_t = the observed interval transit time, s/ft
t_f = the interval transit time in the pore fluid, s/ft
t_r = the interval transit time in rock matrix, s/ft
V_s = volume of shale cutting, ft^3
V_t = total volume of cup, ft^3
K_ϕ = porosity decline constant at ϕ, ft^{-1}
$\frac{P_f}{D}$ = formation pressure gradient, *psi/ft*
$\left(\frac{P_f}{D}\right)_n$ = normal pressure gradient, *psi/ft*
α = ratio of the compressibility of the porous rock matrix to the intrinsic compressibility of rock = $1-\frac{c_r}{c_b}$
ρ = density of the rock
ρ_b = bulk density of porous sediment
ρ_w = density of seawater, lb_m/ft^3
ρ_b = average bulk density, lb_m/ft^3
ρ_e = equivalent mud density at the bit while circulating or actual mud weight in use, ppg
ρ_f = fluid density in the pore space
ρ_m = mud density, lb_m/ft^3
ρ_n = mud density equivalent to normal pore pressure gradient or normal mud weight, ppg
ρ_r = grain density of rock matrix, lb_m/gal
ρ_f = density of fluid in the pore space, lb_m/gal
ρ_{fn} = formation fluid density at normal condition

ρ_{sw} = density of sea water, lb_m/gal
ρ_{bs} = bulk density of shale, lb_m/ft^3
ρ_w = density of water, lb_m/ft^3
ϕ = porosity, dimensionless
ϕ_{avg} = average porosity
ϕ_o = porosity at surface ($D = 0$), fraction
σ_H = horizontal effective stress, *psi*
σ_H = horizontal effective stress, *psi*
σ_v = vertical matrix stress
σ_z = matrix stress = $\sigma_{ob} - P_f$, *psi*
σ_{ob} = overburden pressure, *psi*
σ_x = matrix stress in x-direction, *psi*
σ_y = matrix stress in y-direction, *psi*
σ_v = vertical stress
σ_{ob} = vertical overburden stress, *psi*
σ_{ob} = overburden stress (i.e., $\sigma_b = \sigma_v + P_{pn}$) , *psi*
σ_{min} = minimum effective stress at the point of interest, *psi*
$\frac{\sigma_{min}}{D}$ = minimum effective stress gradient at the point of interest, *psi/ft*
$\frac{\sigma_{ob}}{D}$ = overburden stress gradient, *psi/ft*
ΔP_f = friction pressure loss, psi
τ_g = gel strength, $lb_m/100ft^2$
μ = Poisson's ratio

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7

Basics of Drillstring Design

7.1 Introduction

The drillstring is an important part and a major component of the rotary drilling system. The drillstring is a pervasive term which is sometimes called drillstem. It is the connection between the rig and the drill bit. A typical drillstring consists of kelly, drillpipe, drill collars, tools and drill bit. The drillstring has two primary objectives: i) it provides a conduit for the drilling fluid to be pumped down through it, and circulates back up the annulus, ii) it provides torque to the drill bit for cutting the rock. The major functions of the drillstring are: i) to suspend the bit, ii) to transmit rotary torque from kelly to the drill bit (i.e., impart rotary motion to the bit), iii) to provide a conduit for circulating drilling fluid to the bit (i.e., provide fluid conduit from rig to bit), iv) to provide weight on bit (WOB), and v) to lower and raise the bit in the well. In addition, the drillstring may serve some of the following specialized services such as i) it allows formation evaluation and testing when logging tools cannot be run in the open hole, ii) it provides some stability to the bottom-hole assembly to minimize vibration and bit jumping, and iii) it allows formation fluid and pressure testing through the drillstring. This chapter addresses almost all mathematical formulas and related workout examples. Sets of multiple choice questions (MCQs) are also included which are related to the drillstring design and drill bit. Workout examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

7.2 Different Mathematical Formulas and Examples

7.2.1 Drillstring Design

i) Collapse Load

Collapse pressure can be defined as an external pressure required causing yielding of drillpipe or casing. The highest anticipated external pressure on the pipe can be written as:

$$P_C = 0.052 \times \rho_f \times L_{TVD} \tag{7.1a}$$

where,

P_C = collapse pressure, *psi*
ρ_f = density of fluid outside the drillpipe, *ppg*
L_{TVD} = total true vertical depth of the well at which P_C acts, *ft*

Equation (7.1a) can also be expressed as:

$$P_C = \frac{L_{TVD}\rho_f}{144} \tag{7.1b}$$

where,

P_C = collapse pressure, *psi*
ρ_f = density of fluid outside the drillpipe, lb_f/ft^3
L_{TVD} = true vertical depth at which P_C acts, *ft*

If there are different fluids inside and outside the drillpipe, the differential collapse pressures across the drillpipe prior to opening of the DST tool can be obtained as:

$$\Delta p_c = 0.052\ D\ \rho_{outside} - 0.052(D - X)\rho_{inside} \tag{7.2}$$

where,

D = total depth of fluid column or drillpipe, *ft*
X = depth of the empty drillpipe, *ft*
ρ_{inside} = density of fluid inside the drillpipe, *ppg*
$\rho_{outside}$ = density of fluid outside the drillpipe, *ppg*

When fluid density inside and outside drillpipe is the same, i.e., $\rho_{outside} = \rho_{inside} = \rho$

$$\Delta p_c = 0.052\ D\rho \tag{7.3}$$

When drillpipe is completely empty, Eq. (7.2) can be reformed as:

$$\Delta p_{c-max} = 0.052\ D\rho_{outside} \tag{7.4}$$

A safety factor in collapse can be determined by

$$SF = \frac{Collapse\ resistance}{Collapse\ pressure\ (\Delta p_c)} \tag{7.5}$$

Normally a safety factor of 1.125 is considered for collapse rating.

ii) Burst Pressure

Burst pressure develops when internal pressure is higher than that of external pressure. It can be rated as:

$$\Delta p_b = Internal\ pressure - External\ pressure \tag{7.6}$$

where,

Δp_b = burst load or pressure, *psi*

A safety factor in burst can be determined by

$$SF = \frac{Burst\ rating}{Allowable\ burst} \tag{7.7}$$

iii) Tension Load

The weight and length of the drillpipe can be calculated using the load balance of the drillstring as:

$$\begin{aligned} &0.9 \times \text{drillpipe yield strength} = \text{weight of DP} + \\ &\text{weight of DC} + \text{weight of HWDP} + \text{MOP} \end{aligned} \tag{7.8}$$

where,

MOP = margin of overpull or maximum overpull on the drillstring by the draw-works, lb_f

Mathematically, Eq. (7.8) can be written as:

$$0.9\ P_d = (L_{dp}W_{dp} + L_{dc}W_{dc} + L_{Hdp}W_{Hdp})B_f + MOP \tag{7.9}$$

where,

P_d = drillpipe yield strength or design weight, lb_f
L_{dp} = length of drillpipe, *ft*
L_{dc} = length of drill collar, *ft*
L_{Hdp} = length of heavy weight drillpipe, *ft*
W_{dp} = nominal weight of the drillpipe, lb_f/ft
W_{dc} = nominal weight of the drill collar, lb_f/ft
W_{Hdp} = nominal weight of the heavy weight drillpipe, lb_f/ft
B_f = buoyancy factor, fraction = $(1-\rho_m/\rho_s)$
ρ_m = mud density, lb_m/gal
ρ_s = density of steel, lb_m/ft^3

From Eq. (7.9), the total weight carried by the top joint of drillpipe is given by

$$P_a = (L_{dp}W_{dp} + L_{dc}W_{dc} + L_{Hdp}W_{Hdp})B_f \tag{7.10a}$$

If we use safety factor, Eq. (7.10a) can be written as:

$$P_a = (L_{dp}W_{dp} + L_{dc}W_{dc} + L_{Hdp}W_{Hdp})B_f \times SF \tag{7.10b}$$

where,

P_a = actual weight or total weight carried by the top joint, lb_f

To provide an added safety factor of 90%, the theoretical yield strength can be calculated as:

$$P_t = 0.9\ P_d \tag{7.11}$$

where,

P_t = theoretical yield strength, *psi*

If $P_a < P_d$, then pipe is ok for tension. In general, the difference between P_t and P_a gives the MOP.

The ratio of Eq. (7.11) and Eq. (7.10) gives the safety factor (SF) as:

$$SF = \frac{P_t}{P_a} = \frac{0.9\ P_d}{(L_{dp}W_{dp} + L_{dc}W_{dc})B_f} \tag{7.12}$$

Safety factor is normally in the range of 1.1–1.3. It is noted that SF is not applied for heavy weight drillpipe. In such case, Eq. (7.9) can be written in terms of SF as:

$$0.9P_d = (L_{dp}W_{dp} + L_{dc}W_{dc})B_f \times SF + L_{Hdp}W_{Hdp}B_f + MOP \tag{7.13}$$

Thus length of the drillpipe can be found by rearranging Eq. (7.13) as:

$$L_{dp} = \frac{0.9\ P_d - MOP}{SF \times W_{dp} \times B_f} - \frac{W_{dc}}{W_{dp}}L_{dc} - \frac{W_{Hdp}}{W_{dp}}\frac{L_{Hdp}}{SF} \tag{7.14a}$$

If we do not consider SF, length of the drillpipe can be found by rearranging Eq. (7.9) as:

$$L_{dp} = \frac{0.9P_d - MOP}{W_{dp} \times B_f} - \frac{W_{dc}}{W_{dp}}L_{dc} - \frac{W_{Hdp}}{W_{dp}}L_{Hdp} \tag{7.14b}$$

If dual-grade drillpipe is used at different section of drillstring, the length of drillpipe is calculated as:

$$L_{dp2} = \frac{0.9P_d - MOP}{SF \times W_{dp2} \times B_f} - \frac{W_{dp1}}{W_{dp2}}L_{dp1} - \frac{W_{dc}}{W_{dp2}}L_{dc} - \frac{W_{Hdp}}{W_{dp2}}\frac{L_{Hdp}}{SF} \tag{7.15}$$

where,

L_{dp1} = length of drillpipe grade 1, *ft*
L_{dp2} = length of drillpipe grade 2, *ft*
W_{dp1} = nominal weight of the drillpipegrade 1, lb_f/ft
W_{dp2} = nominal weight of the drillpipegrade 2, lb_f/ft

Example 7.1: A 5" drillpipe is planned to be used in drilling a well. The drillpipe has maximum collapse pressure of 10,000 *psi*. If the depth of the well is 7,500 *ft* and the mud weight used is 10.5 *ppg*, calculate the maximum collapse pressure and safety factor when inside the drillpipe is empty. Also determine the maximum depth at which drillpipes can be lowered down empty.

Solution:

Given data:

P_{c_res} = Collapse resistance = 10,000 *psi*
D_m = Well depth = 7,500 *ft*
MW = Mud weight = 10.5 *ppg*

Required data:

P_c = Collapse pressure
SF = Safety factor
D_{max} = Maximum depth

If the inside drillpipe is empty, no pressure is applied inside the drillpipe. To calculate safety factor for collapse, we should calculate the collapse pressure at the depth of 7,500 *ft*. Collapse pressure applied at the drillpipe is equal to Eq. (7.1a) as:

$$P_c = 0.052 \times \rho_m \times D = 0.052 \times 10.5 \times 7{,}500 = 4{,}095\ psi$$

Now safety factor is equal to, Eq. (7.5):

$$SF = \frac{P_{c_{res}}}{P_c} = \frac{10{,}000}{4{,}095} = \mathbf{2.44}$$

Using a safety factor of 1.13, the maximum depth can be determined by calculating the maximum allowable collapse pressure using Eq. (7.5):

$$SF = \frac{P_{c_{res}}}{P_c} = \frac{10{,}000}{P_c} = 1.13$$

$$P_c = \mathbf{8{,}850\ psi}$$

Maximum depth can be calculated using Eq. (7.1a):

$$P_{out} = 0.052 \times \rho_m \times D = 8{,}850 = 0.052 \times 10.5 \times D$$

$$D = \mathbf{16{,}208\ ft}$$

Example 7.2: A well was drilled to a depth of 11,500 *ft* using 11.0 *ppg* drilling mud. The drillstring has a float valve at the bottom of the string. When new drilling mud was pumped to a depth of 6,500 *ft*, collapse pressure at the bottom was calculated to be 500 *psi*. What was the density of the new mud?

Solution:

Given data:

D_m = Depth of the well = 11,500 *ft*
ρ_m = Mud weight of the current mud = 11.0 *ppg*
P_c = Collapse pressure = 500 *psi*

Required data:

ρ_{m_new} = Mud weight of the new mud, *ppg*

To determine the new mud weight, we have to calculate the hydrostatic pressure of the new mud. Float valve will prevent outside mud which is intended to enter inside the drillpipe. Therefore, collapse pressure was developed at the bottom of the drillstring. Outside pressure at the bottom can be calculated using Eq. (7.1a):

$$P_{out} = 0.052 \times \rho_m \times D = 0.052 \times 11.0 \times 11{,}500 = 6{,}578\ psi$$

Inside pressure can be calculated as follows:

$$P_{inside} = P_{out} - P_c = 6{,}578 - 500 = 6{,}078\ psi$$

Hydrostatic pressure of the old mud inside the drillstring can be calculated using Eq. (7.1a):

$$P_{h\_old} = 0.052 \times \rho_m \times D = 0.052 \times 11.0 \times (11{,}500 - 6{,}500) = 2{,}860\ psi$$

Hydrostatic pressure of the new mud can be calculated using Eq. (7.1a)

$$P_{h\_new} = P_{inside} - P_{h_{old}} = 6{,}078 - 2{,}860 = 3{,}218\ psi$$

Now the mud weight of the new mud is equal to:

$$P_{h\_new} = 0.052 \times \rho_{m\_new} \times D = 3{,}218 = 0.052 \times 6{,}500 \times \rho_{m\_new}$$

$$\rho_{m\_new} = \mathbf{9.52\ ppg}$$

Example 7.3: A production casing was planned to be cased and cemented from the top of the casing shoe at 18,000 *ft*. When casing was flushed with drilling fluid having MW of 13.0 *ppg*, burst safety factor was calculated to be 5.46. In addition, when the casing was full of cement slurry of 16.2 *ppg*, burst safety factor was calculated to be 2.23. What was the mud weight of the fluid in the annulus and the burst rating of the casing? Assume mud weight of the fluid in the annulus remains same.

Solution:

Given data:

D_m = Depth of the casing shoe = 18,000 *ft*
ρ_{in1} = Mud weight of the flushed mud = 13.0 *ppg*
ρ_{in2} = Mud weight of the cement slurry = 16.2 *ppg*
SF_{b1} = Burst safety factor, case#1 = 5.46
SF_{b2} = Burst safety factor, case#2 = 2.23

Required data:

ρ_{ann} = Mud weight of the fluid in the annulus, *ppg*
P_{burts} = Burst rating of the casing, *psi*

We know that burst pressure is the difference in pressure between inside and outside the pipe string. So burst pressure can be calculated using Eq. (7.6) as:

$$\Delta p_b = 0.052 \times D_m \times (\rho_{in} - \rho_{out})$$

For the first case of flushed fluid, burst pressure is equal to:

$$\Delta p_{b1} = 0.052 \times 18{,}000 \times (13.0 - \rho_{out})$$

And for the case of cement slurry, burst pressure is equal to:

$$\Delta p_{b2} = 0.052 \times 18{,}000 \times (16.2 - \rho_{out})$$

We also know that burst safety factor can be calculated using Eq. (7.7):

$$SF_b = \frac{\text{burst rating}}{\text{allowable burst}}$$

If we divide burst pressure at the first case to that at the second case we get:

$$\frac{SF_{b1}}{SF_{b2}} = \frac{16.2 - \rho_{out}}{13.0 - \rho_{out}} = \frac{5.46}{2.23} = 2.448$$

$$\rho_{out} = 10.8\ ppg \text{ i.e. } \rho_{ann} = \mathbf{10.8}\ \boldsymbol{ppg}$$

Now, burst rating can be calculated using Eq. (7.7):

$$SF_b = \frac{\text{burst rating}}{\text{allowable burst}} = \frac{\text{burst rating}}{0.052 \times 18{,}000 \times (13.0 - 10.8)} = 5.46$$

Therefore, Burst rating is

$$P_{burts} = \mathbf{11{,}245}\ \boldsymbol{psi\ OR}\ \mathbf{11{,}250}\ \boldsymbol{psi}$$

Example 7.4: A production casing was planned to be set at 20,000 *ft* with a drilling mud of 11.5 *ppg* at the annulus. When inside casing was filled with 13.5 *ppg* mud, burst safety factor was calculated to be 4.808. Burst safety factor was recalculated to be 2.945 when a certain length of cement slurry of 15.8 *ppg* was filled some part of the casing during pumping the cement. Determine the burst rating of the casing and the length of the cement slurry when the second safety factor was calculated. Assume that mud weight of the fluid in the annulus remains the same.

Solution:

Given data:

D_m = Depth of the casing shoe = 20,000 *ft*
MW_{in} = Mud weight of the drilling mud inside the casing = 13.5 *ppg*
MW_{out} = Mud weight of the drilling mud outside the casing = 11.5 *ppg*
MW_{cem} = Mud weight of the cement slurry = 15.8 *ppg*
SF_{b1} = Burst safety factor, case #1 = 4.808
SF_{b2} = Burst safety factor, case #2 = 2.945

Required data:

P_{burts} = Burst rating of the casing, *psi*
L_{cement} = Length of cement slurry inside the casing, *ft*

Burst rating of the casing can be calculated using Eq. (7.7), and the information for the first situation as follows:

$$SF_{b1} = \frac{\text{burst rating}}{\text{allowable burst}} = \frac{\text{burst rating}}{0.052 \times 20{,}000 \times (13.5 - 11.5)} = 4.808$$

$$\text{burst rating} = 10{,}000\ psi$$

For the second situation, a certain length of the casing was filled with cement slurry when the second safety factor was calculated. Assuming cement length equal to L_{cement}, Eq. (7.7) can be used to determine the length of the cement slurry inside the casing:

$$SF_{b2} = \frac{\text{burst rating}}{\text{allowable burst}}$$

$$2.945 = \frac{\text{burst rating}}{0.052 \times L_{cement} \times (15.8 - 11.5) + 0.052 \times (20{,}000 - L_{cement}) \times (13.5 - 11.5)}$$

$$L_{cement} = \mathbf{11{,}000}\ \boldsymbol{ft}$$

Example 7.5: A production casing was planned to be set in the well with drilling mud of 9.8 *ppg* at the annulus. When inside casing was filled with cement slurry of 15.8 *ppg* mud, burst safety factor was calculated to be 2.50. When cement slurry displaced and filled the annulus, mud weight of the drilling fluid inside the casing was 9.8 *ppg*. Collapse safety factor was calculated to be 2.27 when cement slurry was totally filled the annulus. Determine the ratio between burst and collapse ratings of the casing. In addition, if the burst rating of the casing is 11,300 *psi*, calculate collapse resistance of the casing and the casing setting depth.

<u>Solution</u>:

Given data:

MW_m = Mud weight of the drilling mud = 9.8 *ppg*
MW_m = Mud weight of the cement slurry = 15.8 *ppg*
SF_b = Burst safety factor = 2.50
SF_c = Collapse safety factor = 2.27
P_{burst} = Burst rating of the casing = 11,300 *psi*

Required data:

$\frac{P_b}{P_c}$ = burst/collapse pressure ratings ratio
P_{coll} = Collapse resistance of the casing
D_{shoe} = Casing setting depth

We know that collapse and burst safety factors can be calculated using Eq (7.5) and (7.7):

$$SF_c = \frac{\text{collapse resistance}}{\text{collapse pressure}}$$

and

$$SF_b = \frac{\text{burst rating}}{\text{Allowable burst}}$$

Collapse pressure and allowable burst are equal because in this specific case cement was inside the casing when drilling fluid was in the annulus, and cement was in the annulus when same drilling fluid was inside the casing. So the ratio between burst rating and collapse resistance can be determined simply by dividing burst safety factor by collapse safety factor as follows:

$$\frac{SF_b}{SF_c} = \frac{\text{burst rating}}{\text{collapse resistance}} = \frac{2.50}{2.27} = 1.102$$

If burst rating is equal to 11,300 *psi*, thus collapse resistance will be equal to:

$$\text{collapse resistance} = \frac{\text{burst rating}}{1.102} = \frac{11,300}{1.102} = \mathbf{10,254}\ \boldsymbol{psi}$$

To calculate casing setting depth, we can either determine burst pressure or collapse pressure from using burst of collapse safety factors. Thus, casing setting depth can be determined. Burst pressure can be calculated using Eq. (7.7) as:

$$SF_b = \frac{\text{burst rating}}{\text{Allowable burst}} = 2.50 = \frac{11,300}{\text{burst pressure}}$$

$$\text{burst pressure} = 4,520\ psi$$

Casing setting depth can now be calculated as follows:

$$\text{burst pressure} = 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_m)$$

$$4,520 = 0.052 \times D_{shoe} \times (15.8 - 9.8)$$

$$D_{shoe} = \mathbf{14,487}\ \boldsymbol{ft}$$

***Example 7.6*:** A drilling string consists of 750 *ft* of DCs have weight of 90 *ppf* and DPs have weight of 25 *ppf* was used to drill a well to a depth of 16,500 *ft* using 11.4 *ppg* drilling mud. If yield strength of drillpipe is 600,000 lb_f, calculate the safety factor at this situation. And if the maximum overpull that can be applied to the drillstring is 75,000 lb_f, to what depth can the current drillstring be used to drill this well?

Solution:

Given data:

L_{dc}	=	Length of drill collars	= 750 *ft*
D_m	=	Length of the well	= 16,500 *ft*
w_{dc}	=	Weight of drill collars	= 90 *ppf*
w_{dp}	=	Weight of drillpipes	= 25 *ppf*

P_d	=	Yield strength of drillpipes	=	600,000 lb_f
MW	=	Mud weight	=	11.4 ppg
MOP	=	Maximum overpull	=	75,000 lb_f

Required data:

SF = safety factor

D_{max} = Maximum depth that can be drilled with the current drillstring

To calculate safety factor, first total weight carried by the first drillpipe joint using Eq. (7.10a):

$$P_a = (L_{dp}w_{dp} + L_{dc}w_{dc}) \times B_f$$

Buoyancy factor can be calculated by knowing that steel density is 65 *ppg* as follows:

$$B_f = 1 - \frac{\rho_{st}}{\rho_m} = 1 - \frac{11.3}{65} = 0.825$$

Now, total weight is equal to:

$$P_a = (750 \times 90 + (16,500 - 750) \times 25) \times 0.825 = 380,531\ lbf$$

Theoretical yield strength can be calculated using Eq. (7.11):

$$P_t = 0.9P_d = 0.9 \times 600,000 = 540,000\ lbf$$

Thus, safety factor can now be determined using Eq. (7.12):

$$SF = \frac{P_t}{P_a} = \frac{540,000}{380,531} = \mathbf{1.42}$$

To calculate the maximum depth that can be drilled, we should first calculate the maximum weight that can be carried as below:

$$P_{a\_max} = P_t - MOP = 540,000 - 75,000 = 465,000\ lbf$$

Above weight is the buoyant weight, so we need to calculate the actual weight in air as follows:

$$P_{a\_air} = \frac{465,000}{0.825} = 563,636\ lbf$$

Part of the above weight will be the weight of the drill collars, so weight of drillpipe will be equal to:

$$W_{dp} = 563,636 - 90 \times 750 = 496,136\ lbf$$

Now length of the drillpipes is equal to:

$$L_{dp} = \frac{W_{dp}}{w_{dp}} = \frac{496,136}{25} = 19,845\ ft$$

Thus, maximum depth that can be drilled with the current drillstring is equal to:

$$D_{m\_max} = L_{dp} + L_{dc} = 19,845 + 750 = \mathbf{20,595\ ft}$$

Example 7.7: A drillstring consisting of 600 *ft* of DCs having weight of 80 *ppf* and DPs having weight of 20 *ppf* was used to drill a well to a depth of 12,000 *ft* using 15.8 *ppg* drilling mud. If the maximum allowable overpull was calculated to be 100,000 lb_f, calculate the safety factor at this situation.

Solution:

Given data:

L_{dc} = Length of drill collars = 600 *ft*
D_m = Length of the well = 12,000 *ft*
w_{dc} = Weight of drill collars = 80 *ppf*
w_{dp} = Weight of drillpipes = 20 *ppf*
MW = Mud weight = 15.8 *ppg*
MOP = Maximum overpull = 100,000 lb_f

Required data:

SF = safety factor

Because drillsting is submerged in the drilling fluid, buoyant weight should be calculated. Buoyancy factor can be calculated by knowing that steel density is 65 *ppg* as follows:

$$B_f = 1 - \frac{\rho_{st}}{\rho_m} = 1 - \frac{15.8}{65} = 0.757$$

To calculate safety factor, first total weight carried by the first drillpipe joint using Eq. (7.10a):

$$P_a = (L_{dp}w_{dp} + L_{dc}w_{dc}) \times B_f = (600 \times 80 + 11,400 \times 20) \times 0.757$$
$$= 208,932\ lbf$$

Theoretical yield strength can be calculated by knowing the MOP as follows:

$$P_t = P_a + MOP = 208,932 + 100,000 = 308,932\ lbf$$

Thus safety factor can be calculated using Eq. (7.12):

$$SF = \frac{P_t}{P_a} = \frac{308,932}{208,932} = \mathbf{1.48}$$

Example 7.8: A drillstring needs to be designed based on the information given here. It is noted that the outer diameter of the drillpipe is 5", total vertical depth is 12,000', mud weight is 75 lb_f/ft^3 (i.e., 10 *ppg*). Total MOP is 100,000 *lbs* and the design factor, SF = 1.3 (tension); SF = 1.125 (collapse). The bottom-hole assembly consists of 20 drill collars with an outer diameter of 6.25" and an inner diameter of 2.8125" where the weight of drill collar is 83 lb_f/ft and each collar is 30 *ft* long. In addition, you need to consider the length of slips is 12".

<u>Solution</u>:

Given data:

d_{odp}	=	outer diameter of drillpipe	$= 5\ in$
L_{TVD}	=	total vertical depth	$= 12{,}000\ ft$
ρ_m	=	mud weight	$= 75\ lb_f/ft^3$ (10 *ppg*)
MOP	=	margin of pull	$= 100{,}000\ lbs$
SF_T	=	design factor of safety for tension	$= 1.3$
SF_c	=	design factor of safety for collapse	$= 1.125$
N_{dc}	=	number of drill collar	$= 20$
d_{odc}	=	outer diameter of drill collar	$= 6.25\ in$
d_{idc}	=	inner diameter of drill collar	$= 2.8125\ in$
W_{dc}	=	weight of the drill collar	$= 83\ lb_f/ft$
L_{dc}	=	length of drill collar	$= 30\ ft$
L_{slips}	=	length of slips	$= 12\ in$

***Required data*:**
Design the drillstring

For Collapse Loading:
If total vertical depth is 12,000 *ft*, and the mud density is 10 *ppg*, then collapse pressure can be calculated using Eq. (7.1a) as:

$$P_C = 0.052\ L_{TVD}\rho_m = 0.052 \times 12{,}000\ ft \times 10 ppg = 6{,}240\ psi$$

If we use 75 lb_f/ft^3 mud, collapse pressure can be calculated using Eq. (7.1b) as:

$$P_C = \frac{L_{TVD}\rho_m}{144} = \frac{(12{,}000\ ft \times 75\ lb_f/ft^3)}{(144\ in^2/ft^2)} = 6{,}250\ psi$$

Applying SF for collapse, $P_C = 6{,}250\ psi \times 1.125 = 7{,}031\ psi$

Now from Table 7.1, choose 19.50 lb_f/ft for 5" and we select Grade D for which ID = 4.276".

For Tension Loading:

$$BF = 1 - \frac{\rho_f}{\rho_s} = 1 - \frac{75\ lb_f/ft^3}{490\ lb_f/ft^3} = 0.847$$

Table 7.1 Dimensions and strength of API seamless internal upset drillpipe.

Size of outer diameter (in.)	Weight per foot with coupling (lbf)	Internal diameter (in.)	Internal diameter at full upset (in.)	Collapse pressure*				Internal yield Pressure*				Tensile Strength			
				D	E	G**	S.135**	D	E	G**	S.135	D	E	G**	S.135**
				(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	1,000 (lbf)	1,000 (lbf)	1,000 (lbf)	1,000 (lbf)
2 3/8	4.85	1.995	1.437	6.850**	11.040	13.250	16.560	7.110**	10.500	14.700	18.900	70	98	137	176
2 3/8	6.65	1.815	1.125	11.440	15.600	18.720	23.400	11.350	15.470	21.660	27.850	101	138	194	249
2.7/8	6.85	2.441	1.875	-	10.470	12.500	15.700	-	9.910	13.870	17.830	-	136	190	245
2.7/8	10.40	2.151	1.187	12.770	16.510	19.810	24.760	12.120	16.530	23.140	29.750	157	214	300	386
3 1/2	9.50	2.992	2.250	-	10.040	12.110	15.140	-	9.520	13.340	17.140	-	194	272	350
3 ½	13.30	2.764	1.875	10.350	14.110	16.940	21.170	10.120	13.800	19.320	24.840	199	272	380	489
3 ½	15.50	2.602	1.750	12.300	16.770	20.130	25.160	12.350	16.840	23.570	30.310	237	323	452	581
4	11.85	3.476	2.937	-	8.410	10.310	12.820	-	8.600	12.040	15.470	-	231	323	415
4	14.00	3.340	2.375	8.330	11.350	14.630	17.030	7.940	10.830	15.160	19.500	290	285	400	514
4 1/2	13.75	3.958	3.156	-	7.200	8.920	10.910	-	7.900	11.070	14.230	-	270	378	486
4 ½	16.60	3.826	2.812	7.620	10.390	12.470	15.590	7.210	9.830	13.760	17.100	242	331	463	595
4 ½	20.00	3.640	2.812	9.510	12.960	15.560	19.450	9.200	12.540	17.560	15.500	302	412	577	742
5	16.25	4.408	3.750	-	6.970	8.640	10.500	-	7.770	10.880	13.960	-	328	459	591
5	19.50	4.276	3.687	7.390	10.000	12.090	15.110	6.970	9.500	13.300	17.100	290	396	554	712
5 ½	21.90	4.778	3.812	6.610	8.440	10.350	12.870	6.320	8.610	12.060	15.500	321	437	612	787
5 ½	24.70	4.670	3.500	7.670	10.460	12.560	15.700	7.260	9.900	13.860	17.820	365	497	696	895
5 9/16	19.00**	4.975	4.125	4.580	5.640	-	-	5.090	6.950	-	-	267	365	-	-
5 9/16	22.20**	4.859	3.812	5.480	6.740	-	-	6.090	8.300	-	-	317	432	-	-
5 9/16	25.25**	4.733	3.500	6.730	8.290	-	-	7.180	9.790	-	-	369	503	-	-
6 5/8	22.20**	6.065	5.187	3.260	4.020	-	-	4.160	5.530	-	-	307	418	-	-
6 5/8	25.20	5.965	5.000	4.010	4.810	6.160	6.430	4.790	6.540	9.150	11.770	359	489	685	881
6 5/8	31.90**	5.761	4.625	6.170	6.170	-	-	6.275	8.540	-	-	463	631	-	-

* Collapse, internal yield and tensile strengths are minimum values with no safety factor. D. F. G. S 135 are standard steel grades used in drillpipe.[5]

** Not API standard: shown for information only.

Now if we apply Eq. (7.10b) to calculate actual weight or total weight carried by the top joint, it becomes as:

$$\begin{aligned} P_a &= MOP + (L_{dp}W_{dp} + L_{dc}W_{dc}) \times BF \times SF_T \\ &= 100{,}000 + [(12{,}000 - 20\times 30)\times 19.5 + (20\times 30)\times 83]\times 0.847\times 1.3 \\ &= 400{,}000\ lb_f \end{aligned}$$

From Table 7.4, for 5" and 19.5 lb_f/ft drillpipe, P_t = 396,000 lb_f for Grade E and = 290,000 lb_f for Grade D

Decision: We need to select Grade E instead of Grade D because of huge difference of tensile strength. However as long as actual weight is greater than the theoretical yield strength (i.e., P > P_t), therefore the selected design of Grade **E is not OK** and needs to be verified again.

As the chosen grade is not ok, let us choose the next grade, which is 5½″ **outer diameters**. For this grade, let us choose the weight of the drillpipe is 21.90 lb_f/ft and grade E for which the tensile yield strength is 437,000 lb_f. Now, apply the chosen grade for the entire pipe.

For Tension and Compression Loading (Figure 7.1):
At 12,000 *ft*, i.e., the bottom of DC:

$$P_{dC\_bottom} = 0.052\ L_{TVD}\rho_m = 0.052\times 12{,}000\,ft\times 10\,ppg = 6{,}240\,psi$$

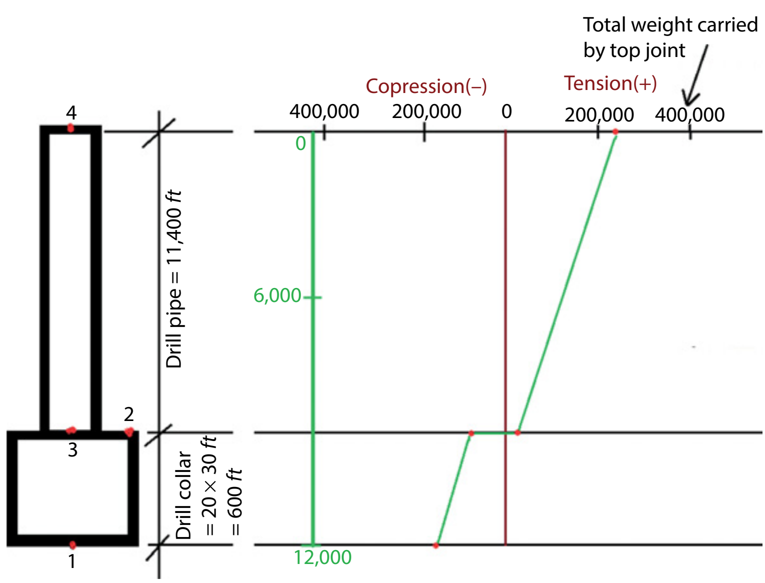

Figure 7.1 Axial Load distributions on the drillstring for Example 7.1.

Cross-sectional area of DC:

Referring to Figure 7.1,

$$A_{dC\_bottom} = \frac{\pi}{4}(d_{Od}^2 - d_{id}^2) = \frac{\pi}{4}(6.25^2 - 2.812^2) = 24.47\ in^2$$

$$F_{1\_bottom} = P_{dC\_bottom} \times A_{dC\_bottom} = 6{,}240 \times 24.47 = 152{,}692.8\ lb_s$$

$$W_{1\_dc} = L_{dC} \times \rho_{dc} = (20 \times 30) \times 83 = 49{,}800\ lb_s$$

So, tension at the bottom of the collar at point 1 = $-F_{1\_bottom}$ = −152,692.8 lb_s (Tension)

At 11,400 *ft* i.e. the top of DC:

$$A_{dC\_top} = \frac{\pi}{4}[(d_{Od}^2 - d_{id}^2)_{outer} + (d_{Od}^2 - d_{id}^2)_{inner}]$$

$$= \frac{\pi}{4}[(6.25^2 - 5.0^2) + (4.276^2 - 2.8125^2)] = 19.19\ in^2$$

$$P_{dC\_top} = 0.052\ L_{TVD}\rho_m = 0.052 \times 11{,}400\ ft \times 10\ ppg = 5{,}928\ psi$$

$$F_{2\_top} = P_{dC\_top} \times A_{dC\_top} = 5{,}928 \times 19.19 = 113{,}758\ lb_s$$

$$W_{2\_dc} = L_{dp} \times \rho_{dp} = (11{,}400\ ft \times 19.5) = 222{,}300\ lb_s$$

So, tension at the top of the collar

at point 2 = $-F_{1\_bottom} + W_{1\_dc}$ = (−152,692.8 + 49,800) lb_s
= −102,892.8 lb_s (Compression)

At 11,400 *ft* i.e., the bottom of the DP (Point 3):

$$A_{dp\_bottom} = \frac{\pi}{4}[(d_{Od}^2 - d_{id}^2)_{outer} + (d_{Od}^2 - d_{id}^2)_{inner}] = 19.19\ in^2$$

$$P_{dp\_bottom} = 0.052\ L_{TVD}\rho_m = 0.052 \times 11{,}400\ ft \times 10\ ppg = 5{,}928\ psi$$

$$F_{3\_bottom} = P_{dp\_bottom} \times A_{dp\_bottom} = 5{,}928 \times 19.119 = 113{,}758.0\ lb_s$$

$$W_{3\_dp} = L_{dp} \times \rho_{dp} = (11{,}400\ ft \times 19.5) = 222{,}300\ lb_s$$

So, tension at the bottom of the drillpipe

at point 3 = $-T_{2_{dp-dc}} + F_{3\_bottom}$ = (−102,892.8 + 113,758) lb_s
= 10,865.2 lb_s (Tension)

At the top of the DP (Point 4):

$$W_{4\_dp} = L_{dp} \times \rho_{dp} = (11{,}400\,ft \times 19.5) = 222{,}300\ lb_s$$

$$F_{4\_top} = \text{tension at the bottom of the drillpipe at point 3}$$

$$= T_3 = 10{,}865.2\ lb_s$$

So, tension at the top of the drillpipe at point 4 = $W_{4\_dp} + F_{4\_top}$ = 222,300 b_s + 10,865.2 lb_s = 233,165.2 lb_s (Tension)

Maximum allowable load:
If we assume that 85% of theoretical load can be allowed to carry by the drillstring, then the maximum allowable load is:

$$W_{4\_dp} = 0.85 \times P_t = 0.85 \times 396{,}000\ lb_s = 335{,}750\ lb_s$$

The total weight carried by the top joint, 400,000 lb_s and as the maximum allowable load is 335,750 *lbs*, therefore a different size of the drillpipe need to be selected for at least 1,200 *ft* (Figure 7.2). From Table 7.1, for 5.5" and 21.90 lb_f/ft drillpipe, P_t = 437,000 lb_f for Grade E. this grade can be selected up to 1,200 *ft*.

Decision:
We may choose the next grade for only the first 1,200'
0–1,200 *ft* : Grade E, 21.90 lb_f/ft
200–12,000 *ft* : Grade E, 19.5 lb_f/ft

Check the New Grade:
Now if we apply again Eq. (7.10b) to calculate actual weight or total weight carried by the top joint, it becomes as:

$$P_a = MOP + (L_{dp}W_{dp} + L_{dc}W_{dc}) \times BF \times SF_T$$

$$P_a = 100{,}000 + [1{,}200 \times 21.5 + (10{,}800 - 20 \times 30) \times 19.5 + (20 \times 30) \times 83] \times 0.847 \times 1.3 = 402{,}251.95\ lb_f$$

Table 7.1 shows, P_t = 437,000 lb_f/ft, and finally it shows that $P_a < P_t$. Therefore, the design is ok and this is the final design decision.

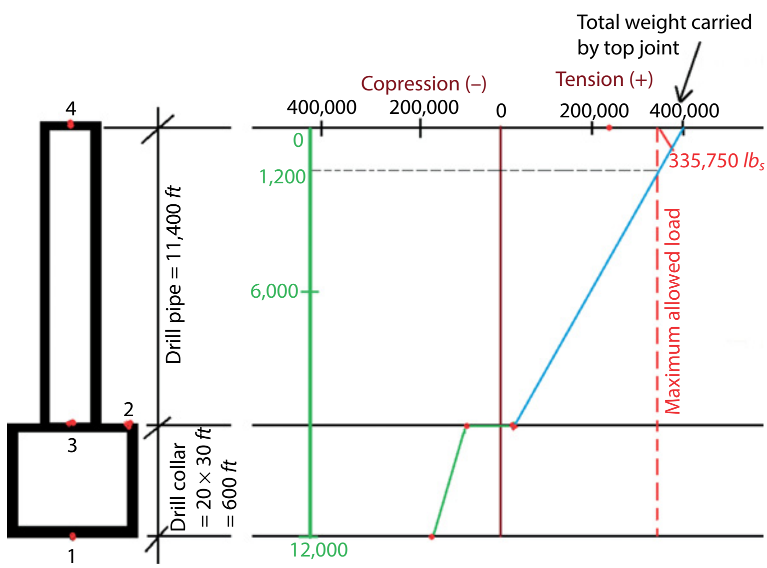

Figure 7.2 Axial Load and maximum load distributions on the drillstring for Example 7.1

iv) Other Design Factors

Shock Load: The additional tensile force generated due to this shock load can be obtained as

$$F_s = 3{,}200 \times W_{dp} \tag{7.16}$$

where,

W_{dp} = weight of drillpipe per unit length, lb_f/ft

Torsion: Torsion in a drillstring is produced by a twisting moment and can be calculated as:

$$\tau = \frac{T\rho}{I_p} \tag{7.17}$$

$$\frac{d\theta_t}{dz} = \frac{T}{E_s I_p} \tag{7.18}$$

where

τ = shear or torsional stress, *psi*

T = torque, *in-lb*$_f$

r = distance from the center of the drillpipe to a point under consideration ($d_i \leq 2r \leq d_o$), *in*

d_i = inside diameter of drillpipe, *in*

d_o = outside diameter of drillpipe, *in*

I_p = polar moment of inertia = $\frac{\pi}{32}(d_o^4 - d_i^4)$, in^4

E_s = shear modulus of elasticity = $\frac{E}{2(1+\upsilon)}$

E = Young's modulus of elasticity, *psi*

υ = Poisson's ratio, (the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force. $\upsilon = -\frac{\varepsilon_{trans}}{\varepsilon_{longitudinal}}$)

θ_t = angle of twist, *radian*

$\frac{d\theta_t}{dz}$ = differential angle of twist, in^{-1}

The maximum shear stress occurs at the outer fibre of the pipe, and for this case Eq. (7.17) can be written as

$$\tau_{max} = \frac{16 d_o T}{\pi(d_o^4 - d_i^4)} = \frac{T}{Z_p} \tag{7.19}$$

where

Z_p = polar sectional modulus, *psi*

Example 7.9: A drillstring has 3,500 *ft* long, and 5.5 in outer diameter drillpipe. While the pipe was moving, it was suddenly stopped. A torque of 225 lb_f-*in* is applied which develops torsional stress and angle at a distance of 6.124 from the center of the pipe. Assume that the Young's modulus of elasticity for steel is 29×10^6 *psi* and Poisson's ratio is 0.44. Find out the shock load, torsional stress, maximum shear stress and differential angle of twist.

Solution:

Given data:

d_{odp} = Outer diameter of drillpipe = 5.5 *in*
L_{dp} = Total drillpipe length = 3,000 *ft*
T = Torque = 200 *in*-lb_f
r = Distance from the center of the drillpipe to the point = 5.124 *in*
E = Young's Modulus of elasticity = 29,000000 *psi*
ν = Poisson's ratio = 0.44
W_{dp} = weight per feet = 21.9 lb_f/ft

From Table 7.1,
d_{idp} = inner diameter of drillpipe = 4.778 *in*

Required data:

τ = shear or torsional stress in *psi*
F_s = shock load in lb_f
$\frac{d\theta_t}{dz}$ = differential angle of twist, in^{-1}

Applying Eq. (7.16), shock load can be calculated as:

$$F_s = 3{,}200 \times W_{dp} = 3{,}200 \times (3{,}000 \times 21.9) = \mathbf{210.24 \times 10^6}\ \boldsymbol{psi}$$

The shear stress can be calculated using Eq. (7.17) as:

$$\tau = \frac{Tr}{I_p} = \frac{(200\ lb_f - in) \times 5.124\ in}{\frac{\pi}{32}(d_o^4 - d_i^4)} = \frac{(200\ lb_f - in) \times 5.124\ in}{\frac{\pi}{32}(5.5^4 - 4.778^4) \times in^4} = \mathbf{26.5} \frac{\boldsymbol{lb_f}}{\boldsymbol{in^2}}$$

The differential angle of twist can be calculated by applying Eq. (7.18) as:

$$\frac{d\theta_t}{dz} = \frac{T}{E_s I_p} = \frac{T}{\frac{E}{2(1+\nu)} \times \frac{\pi}{32}(d_o^4 - d_i^4)}$$

$$= \frac{(200\ lb_f - in)}{\frac{29{,}000000\ psi}{2(1+0.44)} \times \frac{\pi}{32}(5.5^4 - 4.778^4) \times in^4} = \mathbf{5.14 \times 10^{-7}}\ \boldsymbol{in^{-1}}$$

The maximum shear stress is calculated by Eq. (7.19) as:

$$\tau_{max} = \frac{16 d_o T}{\pi(d_o^4 - d_i^4)} = \frac{16 \times 5.5\ in \times (200\ lb_f - in)}{\pi(5.5^4 - 4.778^4) \times in^4} = \mathbf{17{,}600}\ \boldsymbol{psi}$$

Example 7.10: A drillstring has a 3,000 *ft* long, and 5.5 *in* outer diameter drillpipe. While the pipe was moving, it was suddenly stopped. A torque of 200 lb_f-*in* is applied which develops torsional stress and angle at a distance of 5.124 from the center of the pipe. Assume that the Young's modulus of elasticity for steel is 29 × 10⁶ *psi* and Poisson's ratio is 0.44. Find out the shock load, torsional stress, maximum shear stress and differential angle of twist.

Solution:

Given data:

d_{odp} = outer diameter of drillpipe = 5.5 *in*
L_{dp} = total drillpipe length = 3,000 *ft*
T = torque = 200 *in*-lb_f
r = distance from the center of the drillpipe to the point = 5.124 *in*
E = Young's modulus of elasticity = 29,000000 *psi*
υ = Poisson's ratio = 0.44
w_{dp} = weight per feet = 21.9 lb_f/ft

From Table 7.1,

d_{idp} = inner diameter of drillpipe = 4.778 *in*
w_{dp} = weight per feet = 21.9 lb_f/ft

Required data:

τ = shear or torsional stress in *psi*
F_s = shock load in lb_f
$\frac{d\theta_t}{dz}$ = differential angle of twist, in^{-1}

Applying Eq. (7.16), shock load can be calculated as:

$$F_s = 3{,}200 \times W_{dp} = 3{,}200 \times (3{,}000 \times 21.9) = \mathbf{210.24 \times 10^6}\ \boldsymbol{psi}$$

The shear stress can be calculated using Eq. (7.17) as:

$$\tau = \frac{Tr}{I_p} = \frac{(200\ lb_f - in) \times 5.124\ in}{\frac{\pi}{32}(d_o^4 - d_i^4)} = \frac{(200\ lb_f - in) \times 5.124\ in}{\frac{\pi}{32}(5.5^4 - 4.778^4) \times in^4}$$

$$= 26.5 \frac{lb_f}{in^2} = \mathbf{26.5}\ \boldsymbol{psi}$$

The differential angle of twist can be calculated applying Eq. (7.18) as:

$$\frac{d\theta_t}{dz} = \frac{T}{E_s I_p} = \frac{T}{\frac{E}{2(1+\upsilon)} \times \frac{\pi}{32}(d_o^4 - d_i^4)}$$

$$= \frac{(200\ lb_f - in)}{\frac{29{,}000000\ \text{psi}}{2(1+0.44)} \times \frac{\pi}{32}(5.5^4 - 4.778^4) \times in^4} = \mathbf{5.14 \times 10^{-7}}\ \boldsymbol{in^{-1}}$$

The maximum shear stress is calculated by Eq. (7.19) as:

$$\tau_{max} = \frac{16 d_o T}{\pi(d_o^4 - d_i^4)} = \frac{16 \times 5.5\ in \times (200\ lb_f - in)}{\pi(5.5^4 - 4.778^4) \times in^4} = \mathbf{17{,}600}\ \boldsymbol{psi}$$

The torque developed in the drillstring can be calculated if the horsepower required to rotate the string is obtained by recalling Eq. (2.1) for a given rpm as:

$$T = \frac{5252\ HP_{ds}}{N} \tag{7.20}$$

where
HP_{ds} = horsepower required to turn the rock bit and drillstring, *hp*
N = drillstring rotary speed, rev/min
T = torque, *ft-lb*$_f$
The horsepower required to rotate the drillpipe is given as

$$HP_{dp} = C_d d_o^2 N L_{dp} \gamma_m \tag{7.21}$$

where
HP_p = horsepower required to rotate the drillpipe, *hp*
C_d = an empirical factor that depends on hole inclination angle (0.000048–0.00000665 for hole angles ranging from 3 to 5°)
γ_m = specific gravity of mud

Example 7.11: While drilling, 250 *hp* was applied to rotate the drillstring and bit where 500 *rpm* was recorded from the rotary speed machine. In addition, 175 *hp* was applied to rotate 3,500 *ft* of drillpipe off 5 *in* OD with the same speed as drillstring. Assume that C_d = 0.000005. Calculate the required torque for drilling string and the specific gravity of mud.

Solution:
Given data:
HP_{ds} = horsepower required to turn the rock bit and drillstring = 250 *hp*
N = drillstring rotary speed = 500 *rev/min*
HP_p = horsepower required to rotate the drillpipe = 175 *hp*
C_d = an empirical factor that depends on hole inclination angle = 0.000005
L_{dp} = length of drillpipe = 3,500 *ft*
C_d = outer diameter of drillpipe = 5 *in*

***Required data*:**
T = torque in *ft-lb*$_f$
γ_m = specific gravity of mud

Applying Eq. (7.20), the torque developed in the drillstring can be calculated as:

$$T = \frac{5252\ HP_{ds}}{N} = \frac{5252 \times (250\ hp)}{500\ rpm} = \mathbf{2626\ lb_f\text{-}in}$$

The specific gravity of mud is calculated using Eq. (7.21) as:

$$\gamma_m = \frac{HP_{dp}}{C_d d_o^2 N L_{dp}} = \frac{(175\ hp)}{0.000005 \times 5^2 in^2 \times 500 \times 3{,}500\ ft} = \mathbf{0.8}$$

Example 7.12: While drilling, 250 *hp* was applied to rotate the drillstring and bit where 500 *rpm* was recorded from the rotary speed machine. In addition, 175 *hp* was applied to rotate 3,500 *ft* of drillpipe off 5 *in* OD with the same speed as drillstring. Assume that C_d = 0.000005. Calculate the required torque for drilling string and the specific gravity of mud.

Solution:

Given data:

HP_{ds}	=	horsepower required to turn the rock bit and drillstring	= 250 *hp*
N	=	drillstring rotary speed	= 500 *rev/min*
HP_p	=	horsepower required to rotate the drillpipe	= 175 *hp*
C_d	=	an empirical factor that depends on hole inclination angle	= 0.000005
L_{dp}	=	length of drillpipe	= 3,500 *ft*
C_d	=	outer diameter of drillpipe	= 5 *in*

Required data:

T = Torque in $ft\text{-}lb_f$

γ_m = Specific gravity of mud

Applying Eq. (7.20), the torque developed in the drillstring can be calculated as:

$$T = \frac{5252\ HP_{ds}}{N} = \frac{5252 \times (250\ hp)}{500\ rpm} = 2626\ lbf - in$$

The specific gravity of mud is calculated using Eq. (7.21) as:

$$\gamma_m = \frac{HP_{dp}}{C_d d_o^2 N L_{dp}} = \frac{(175\ hp)}{0.000005 \times 5^2 in^2 \times 500 \times 3{,}500\ ft} = 0.8$$

The following two equations can be used to calculate the maximum allowable makeup torque before the minimum torsional yield strength of the drillpipe body is exceeded. In such case, Eq. (7.19) can be rearranged and written for torsional yield strength due to pure tension as:

$$Q_{min} = \frac{0.096167\ I_p Y_{min}}{d_o} \tag{7.22}$$

where,

Q_{min} = minimum torsional yield strength, *ft-lb*$_f$

Y_{min} = minimum unit yield strength, *psi*

It is well established that during the normal drilling operations, the drillpipe is subjected to both torsion and tension. Thus Eq. (7.22) becomes as:

$$Q_{\min\_t} = \frac{0.096167\ I_p}{d_o}\sqrt{Y_{min}^2 - \frac{W_{tj}^2}{A^2}} \tag{7.23}$$

where,

Q_{min_t} = minimum torsional yield strength under tension, *lb*$_f$*-ft*

W_{tj} = total load in tension or total weight carried by the top joint, *lb*$_f$

A = cross-sectional area, *in*^{2}

Example 7.13: Find out the minimum torsional yield strength and torsional yield strength under tension for the following data: OD = 4.5 *in*, top joint load is 400,000 *lb*$_f$. Assume that the ID of the pipe is 3.958 *in*. Use E-grade pipe.

Solution:

Given data:

I_p = horsepower required to rotate the drillpipe = 175 *hp*

d_o = outer diameter of drillpipe = 4.5 *in*

W_{ij} = total load in tension carried by the top joint = 400,000 *lb*$_f$

d_i = inner diameter of drillpipe = 3.958 *in*

It is assumed that, Y_{min} of drillpipe = yield strength = 150,000 *psi*

Required data:

Q_{min} = minimum torsional yield strength in *ft-lb*$_f$

Q_{min_t} = minimum torsional yield strength under tension in *lb*$_f$*-ft*

The polar moment of inertia is calculated as:

$$I_p = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(4.5^4 - 3.958^4) = 16.16\ in^4$$

The cross-sectional area, A can be calculated as:

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(4.5^2 - 3.958^2) = 3.6\ in^2$$

Now, the minimum torsional yield strength is given applying Eq. (7.22) as:

$$Q_{min} = \frac{0.096167\ I_p Y_{min}}{d_o} = \frac{0.096167 \times (16.16\ in^4) \times (7{,}900\ psi)}{4.5\ in}$$
$$= \mathbf{2728.24}\ \boldsymbol{psi}$$

The minimum torsional yield strength under tension is also given by Eq. (7.23):

$$Q_{\min\_t} = \frac{0.096167\ I_p}{d_o}\sqrt{Y_{min}^2 - \frac{W_{tj}^2}{A^2}}$$

$$= \frac{0.096167 \times (16.16\ in^4)}{4.5\ in}\sqrt{150000^2 - \frac{400{,}000^2}{3.6^2}}$$

$$= \mathbf{3506.76\ lb_f - ft}$$

Example 7.14: Find out the minimum torsional yield strength and torsional yield strength under tension for the following data: OD = 4.5 *in*, top joint load is 400,000 lb_f. Assume that the ID of the pipe is 3.958 *in*. Use E-grade pipe.

<u>Solution</u>:

Given data:

I_p = horsepower required to rotate the drillpipe = 175 *hp*
d_o = outer diameter of drillpipe = 4.5 *in*
W_{ij} = Total load in tension carried by the top joint = 400,000 lb_f
d_i = inner diameter of drillpipe = 3.958 *in*

Required data:

Q_{min} = Minimum torsional yield strength in lb_f-ft
Q_{min_t} = Minimum torsional yield strength under tension in lb_f-ft

The polar moment of inertia is calculated as:

$$I_p = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(4.5^4 - 3.958^4) = 16.16\ in^4$$

The cross-sectional area, A can be calculated as:

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(4.5^2 - 3.958^2) = 3.6\ in^2$$

Now, the minimum torsional yield strength is given applying Eq. (7.22) as:

$$Q_{min} = \frac{0.096167\ I_p Y_{min}}{d_o}$$

$$= \frac{0.096167 \times (16.16\ in^4) \times (7{,}900\ psi)}{4.5\ in} = \mathbf{2728.24\ psi}$$

The minimum torsional yield strength under tension is also given by Eq. (7.23):

$$Q_{min} = \frac{0.096167\ I_p}{d_o}\sqrt{Y_{min}{}^2 - \frac{W_{tj}{}^2}{A^2}}$$

$$= \frac{0.096167 \times (16.16\ in^4)}{4.5\ in}\sqrt{150000^2 - \frac{400{,}000^2}{3.6^2}}$$

$$= \mathbf{3506.76\ lb_f - ft}$$

Example 7.15: While drilling, 350 *hp* was applied to rotate the drillstring and bit where 550 *rpm* was recorded from the rotary speed machine. In addition, 195 *hp* was applied to rotate 4,000 *ft* of drillpipe off 5.5 in OD with the same speed as drillstring. Assume that C_d = 0.000005. Calculate the required torque for drilling string and the specific gravity of mud.

Solution:

Given data:

HP_{ds}	=	horsepower required to turn the rock bit and drillstring	=	350 *hp*
N	=	drillstring rotary speed	=	550 *rev/min*
HP_p	=	horsepower required to rotate the drillpipe	=	195 *hp*
C_d	=	an empirical factor that depends on hole inclination angle	=	0.000005
L_{dp}	=	length of drillpipe	=	4,000 *ft*
C_d	=	outer diameter of drillpipe	=	5.5 *in*

Required data:

T	=	Torque in *ft-lb$_f$*
γ_m	=	Specfic gravity of mud

Applying Eq. (7.20), the torque developed in the drillstring can be calculated as:

$$T = \frac{5252\ HP_{ds}}{N} = \frac{5252 \times (350\ hp)}{550\ rpm} = 3342\ lbf - in$$

The specific gravity of mud is calculated using Eq. (7.21) as:

$$\gamma_m = \frac{HP_{dp}}{C_d d_o^2 N L_{dp}} = \frac{(195\ hp)}{0.000005 \times 5.5^2 in^2 \times 550 \times 4{,}000\ ft} = 0.6$$

Example 7.16: Find out the minimum torsional yield strength and torsional yield strength under tension for the following data: OD = 4.5 *in*, top joint load is 400,000 lb_f. Assume that the ID of the pipe is 3.958 *in*. Use E-grade pipe.

Solution:

Given data:

I_p	=	horsepower required to rotate the drillpipe	=	195 *hp*
d_o	=	outer diameter of drillpipe	=	5.5 *in*
W_{ij}	=	Total load in tension carried by the top joint	=	400,000 lb_f
d_i	=	inner diameter of drillpipe	=	3.958 *in*

Required data:

Q_{min}	=	Minimum torsional yield strength in lb_f-*ft*
Q_{min_t}	=	Minimum torsional yield strength under tension in lb_f-*ft*

The polar moment of inertia is calculated as:

$$I_p = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(5.5^4 - 3.958^4) = 65.71\ in^4$$

The cross-sectional area, A can be calculated as:

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(5.5^2 - 3.958^2) = 11.45\ in^2$$

Now, the minimum torsional yield strength is given applying Eq. (7.22) as:

$$\begin{aligned} Q_{min} &= \frac{0.096167\ I_p Y_{min}}{d_o} \\ &= \frac{0.096167 \times (65.71\ in^4) \times (7{,}900\ psi)}{5.5\ in} \\ &= \mathbf{9076.57}\ \boldsymbol{psi} \end{aligned}$$

The minimum torsional yield strength under tension is also given by Eq. (7.23):

$$\begin{aligned} Q_{min} &= \frac{0.096167\ I_p}{d_o}\sqrt{Y_{min}{}^2 - \frac{W_{tj}{}^2}{A^2}} \\ &= \frac{0.096167 \times (65.71\ in^4)}{5.5\ in}\sqrt{150000^2 - \frac{400{,}000^2}{11.45^2}} \\ &= \mathbf{167600.91}\ \boldsymbol{lbf - ft} \end{aligned}$$

The calculation of the approximate weight of a drillpipe includes the approximate weight of the tool joint assembly. The following equations can be used to calculate the adjusted weight as:

$$W_{dp-adj} = W_{dp-plain} + \frac{W_{dp-upset}}{29.4} \tag{7.24}$$

where,

$W_{dp\text{-}ad}$ = approx. adjusted weight of drillpipe, lb_f/ft
$W_{dp\text{-}plain}$ = plain end weight, lb_f/ft
$W_{dp\text{-}upset}$ = upset weight, lb_f/ft

Now the approximate adjusted weight of the tool joint can be calculated as:

$$W_{tool\ joint} = 0.222\ L(d_o^2 - d_i^2) + 0.167(d_o^3 - d_{TE}^3) - 0.501 d_i^2 (d_o - d_{TE}) \tag{7.25}$$

where,

$W_{tool\,joint}$ = approximate adjusted weight of the tool joint, lb_f/ft
L = combined length of pin and box, *in*
d_o = outside diameter of pin, *in*
d_i = inside diameter of pin, *in*
d_{TE} = diameter of box at elevator upset, *in*

Equation (7.24) can also be represented in terms of approximate adjusted weight of the tool joint and tool joint adjusted length as:

$$W_{dp-adj} \times 29.4 = \frac{W_{tool\ joint}}{29.4 + L_{tool\ joint}} \tag{7.26}$$

$$L_{tool\ joint} = \frac{L + 2.253(d_o - d_{TE})}{12} \tag{7.27}$$

where,

$L_{tool\ joint}$ = tool joint adjusted length, *ft*

Stretch of Pipe: the stretch of drillpipe develops due to the action of drill collars and its own weight carried out by the hook. So, drillpipe stretches under the action of drill collars and its own weight can be calculated separately as the elongation due to i) its own weight, and ii) drill collar's weight.

Due to its own weight:

In FPS system:

$$\varepsilon_o = \frac{L_{dp}^2 (65.44 - 1.44\ \rho_m)}{9.625 \times 10^7} \tag{7.28}$$

where:

ε_o = stretch due to own weight, *in*
L_{dp} = total length of drillpipe, *ft*
ρ_m = mud density, *ppg*

In MKS system, Eq. (7.28) can be written as:

$$\varepsilon_o = 2.346 \times 10^{-8} L_{dp}^2 (65.44 - 1.44\ \rho_m) \tag{7.29}$$

where:

ε_o = stretch due to own weight, *m*
L_{dp} = total length of drillpipe, *m*
ρ_m = mud density, *kg/lt*

Due to weight of drill collars:

In FPS system:

$$\varepsilon_{dc} = \frac{(L_{dc} + L_{dp}) \times W_{dc}}{735444\ W_{dp}} \tag{7.30}$$

where:

ε_o = stretch due to drill collar, in
L_{dp} = total length of drillpipe, *ft*
L_{dc} = total length of drill collar, *ft*
W_{dp} = weight of the drillpipe, lb_f/ft
W_{dc} = weight of the drill collar, lb_f/ft

In MKS system, Eq. (7.30) can be written as::

$$\varepsilon_{dc} = 373.8 \times 10^{-10} \frac{W_{dc}}{W_{dp}} (L_{dc} + L_{dp}) \tag{7.31}$$

where:

ε_{dc} = elongation due to drill collar, *m*
L_{dp} = total length of drillpipe, *m*
L_{dc} = total length of drill collar, *m*
W_{dp} = weight of the drillpipe, *kg/m*
W_{dc} = weight of the drill collar, *kg/m*

If tension is applied,

$$\varepsilon_t = \frac{L_{dp} P_{di-p}}{735294\ W_{dp}} \tag{7.32}$$

where:

ε_t = stretch due to tension, *ft*
P_{di-p} = differential pull, lb_f
W_{dp} = weight of the drillpipe, lb_f/ft

Example 7.17: A 10 *ppg* mud is circulated through a 5 *in* drillpipe assembly of 8,000 *ft*. If 50 drill collars of 30 *ft* long each are also used, calculate stretch for drillpipe and collar due to their own weight. Assume the OD and ID of drill collar as 6.25 *in* and 2.8125 *in*, respectively, and weight of drill collar is 93 lb_f/ft. In addition assume that a differential pull of 1,000 lb_f is applied on the drillpipe. Also, find out the stretch due to tension.

Solution:

Given data:

L_{dp} = total length of drillpipe = 8,000 *ft*
d_{odp} = outer diameter of drillpipe = 5 *in*
d_{idp} = outer diameter of drillpipe = 4.408 in (Table 7.4)
W_{dp} = weight of the drillpipe = 16.25 lb_f/ft (Table 7.4)
ρ_m = mud density = 10 *ppg*
L_{dc} = total length of drill collar = 30 *ft* × 50 = 1,500 *ft*
d_{odc} = outer diameter of drill collar = 6.25 *in*
d_{idc} = outer diameter of drill collar = 2.8125 *in*
W_{dc} = weight of the drill collar = 93.0 lb_f/ft
P_{di-p} = differential pull = 1,000 lb_f

Required data:

ε_o = stretch due to own weight in inch
ε_{dc} = stretch due to drill collar in inch
ε_t = stretch due to tension in *ft*

The stretch due to drillpipe own weight can be given using Eq. (7.28) as:

$$\varepsilon_o = \frac{L_{dp}^2(65.44 - 1.44\ \rho_m)}{9.625 \times 10^7} = \frac{8000^2(65.44 - 1.44 \times 10)}{9.625 \times 10^7} = \mathbf{33.93}\ \boldsymbol{in}$$

The stretch due to drill collar weight can be given using Eq. (7.30) as:

$$\varepsilon_{dc} = \frac{(L_{dc} + L_{dp}) \times W_{dc}}{735444\ W_{dp}} = \frac{(1500 + 8000) \times 93}{735444 \times 16.25} = \mathbf{0.074}\ \boldsymbol{in}$$

The stretch due to tension can be given using Eq. (7.32) as:

$$\varepsilon_t = \frac{L_{dp} P_{di-p}}{735294\ W_{dp}} = \frac{8000 \times 1000}{735294 \times 16.25} = \mathbf{0.67}\ \boldsymbol{ft}$$

Example 7.18: A 10 *ppg* mud is circulated through a 5 *in* drillpipe assembly of 8,000 *ft*. If 50 drill collars of 30 *ft* long each are also used, calculate stretch for drillpipe and collar due to their own weight. Assume the OD and ID of drill collar as 6.25 *in* and 2.8125 *in*, respectively, and weight of drill collar is 93 lb_f/ft. In addition assume that a differential pull of 1,000 lb_f is applied on the drillpipe. Also, find out the stretch due to tension.

Solution:

Given data:

L_{dp} = length of drillpipe = 8,000 *ft*
d_{odp} = outer diameter of drillpipe = 5 *in*
d_{idp} = inner diameter of drillpipe = 4.408 *in* (Table 7.4)
W_{dp} = weight of the drillpipe = 16.25 lb_f/ft (Table 7.4)
ρ_m = mud density = 10 *ppg*
L_{dc} = total length of drill collar = 30 *ft* × 50 = 1,500 *ft*
d_{odc} = outer diameter of drill collar = 6.25 *in*
d_{idc} = inner diameter of drill collar = 2.8125 *in*
W_{dc} = weight of the drill collar = 93.0 lb_f/ft
$P_{di\text{-}p}$ = differential pull = 1,000 lb_f

Required data:

ε_o = Stretch due to own weight in inch
ε_{dc} = Stretch due to drill collar in inch
ε_t = stretch due to tension in *ft*

The stretch due to drillpipe own weight can be given using Eq. (7.28) as:

$$\varepsilon_o = \frac{L_{dp}^2 (65.44 - 1.44\rho_m)}{9.625 \times 10^7} = \frac{8000^2 (65.44 - 1.44 \times 10)}{9.625 \times 10^7} = \mathbf{33.93}\, \boldsymbol{in}$$

The stretch due to drill collar weight can be given using Eq. (7.30) as:

$$\varepsilon_{dc} = \frac{(L_{dp} + L_{dc}) \times W_{dc}}{735444 W_{dp}} = \frac{(1500 + 8000) \times 93}{735444 \times 16.25} = \mathbf{0.074}\ \boldsymbol{in}$$

The stretch due to tension can be given using Eq. (7.32) as:

$$\varepsilon_t = \frac{L_{dp} P_{di-p}}{735294 W_{dp}} = \frac{8000 \times 1000}{735294 \times 16.25} = \mathbf{0.67}\ \boldsymbol{ft}$$

Critical Rotating Speed: The longitudinal vibration based on total length and drillpipe dimensions can be calculated as:

$$rpm_{Lc} = \frac{258{,}000}{L_{dp}} \tag{7.33}$$

where:

rpm_{Lc} = critical rpm for longitudinal vibration, *rev/min*

The transverse vibration can be calculated as:

$$rpm_{Tc} = \frac{4760,000}{L_{dp}}\sqrt{d_o^2 - d_i^2} \tag{7.34}$$

where:

rpm_{Tc} = critical rpm for transverse vibration, *rev/min*

Example 7.19: Find out the critical rpm for both longitudinal and transverse vibration if 5.5 *in* of 16.25 lb_f/ft, and 8,500 *ft* drillpipe.

<u>Solution</u>:

Given data:

L_{dp} = total length of drillpipe = 8,500 *ft*
d_{odp} = outer diameter of drillpipe = 5.5 *in*
d_{idp} = inner diameter of drillpipe = 4.408 *in* (Table 7.4)
W_{dp} = weight of the drillpipe = 16.25 lb_f/ft

Required data:

rpm_{Lc} = critical rpm for longitudinal vibration in *rev/min*
rpm_{Tc} = critical rpm for transverse vibration in *rev/min*

The critical rpm for longitudinal vibration can be given by Eq. (7.33) as:

$$rpm_{Lc} = \frac{258,000}{L_{dp}} = \frac{258,000}{8,500} = \mathbf{30.4\ \mathit{rpm}}$$

The critical rpm for transverse vibration can be given by Eq. (7.34) as:

$$rpm_{Tc} = \frac{476,000}{L_{dp}}\sqrt{(d_o^2 - d_i^2)} = \frac{476,000}{8,500}\sqrt{(5.5^2 - 4.408^2)}$$
$$= \mathbf{184.2\ \mathit{rpm}}$$

Example 7.20: Find out the critical rpm for both longitudinal and transverse vibration if 5 in of 16.25 lb_f/ft, and 7,500 *ft* drillpipe.

<u>Solution</u>:

Given data:

L_{dp} = total length of drillpipe = 7,500 *ft*
d_{odp} = outer diameter of drillpipe = 5 *in*
d_{idp} = outer diameter of drillpipe = 4.408 *in* (Table 7.4)
W_{dp} = weight of the drillpipe = 16.25 lb_f/ft

***Required data*:**

rpm_{Lc} = critical rpm for longitudinal vibration in *rev/min*
rpm_{Tc} = critical rpm for transverse vibration in *rev/min*

The critical rpm for longitudinal vibration can be given by Eq. (7.33) as:

$$rpm_{Lc} = \frac{258{,}000}{L_{dp}} = \frac{258{,}000}{7{,}500} = \mathbf{34.4}\ \boldsymbol{rpm}$$

The critical rpm for transverse vibration can be given by Eq. (7.34) as:

$$rpm_{Tc} = \frac{4760{,}000}{L_{dp}}\sqrt{d_o^2 - d_i^2} = \frac{4760{,}000}{7{,}500}\sqrt{5^2 - 4.408^2} = \mathbf{1498}\,\boldsymbol{rpm}$$

Example 7.21: Find out the critical rpm for both longitudinal and transverse vibration if 5 in of 16.25 lb_f/ft, and 7,500 *ft* drillpipe.

Solution:

Given data:

L_{dp} = total length of drillpipe = 7,500 *ft*
d_{odp} = outer diameter of drillpipe = 5 *in*
d_{idp} = inner diameter of drillpipe = 4.408 *in* (Table 7.4)
W_{dp} = weight of the drillpipe = 16.25 lb_f/ft

Required data:

rpm_{Lc} = critical rpm for longitudinal vibration in *rev/min*
rpm_{Tc} = critical rpm for transverse vibration in *rev/min*

The critical rpm for longitudinal vibration can be given by Eq. (7.33) as:

$$rpm_{Lc} = \frac{258{,}000}{L_{dp}} = \frac{258{,}000}{7{,}500} = \mathbf{34.4}\,\boldsymbol{rpm}$$

The critical rpm for transverse vibration can be given by Eq. (7.34) as:

$$rpm_{Tc} = \frac{476{,}000}{L_{dp}}\sqrt{(d_o^2 - d_i^2)} = \frac{476{,}000}{7{,}500}\sqrt{(5^2 - 4.408^2)} = \mathbf{149.8}\,\boldsymbol{rpm}$$

Example 7.22: A drillstring consisting of 800 *ft* of DCs having weight of 93 *ppf* and DPs having weight of 24.7 *ppf* was planned to drill a well to a depth of 15,000 *ft* using 10.9 *ppg* drilling mud. Calculate the drillpipe stretch when the drillstring is suspended in the rotary table. What will be the change in the drillpipe stretch if WOB of 18,000 lb_f is applied to the bit?

Solution:

Given data:

L_{dc} = Length of drill collars = 800 *ft*
D_m = Length of the well = 15,000 *ft*
w_{dc} = Weight of drill collars = 93 *ppf*
W_{dp} = Weight of drillpipes = 24.7 *ppf*
MW = Mud weight = 10.9 *ppg*
WOB = Applied weight on bit = 18,000 lb_f

Required data:

ε_{dp} = Drillpipe stretch

To calculate drillpipe stretch, first we should calculate the stretch of drillpipe due to its weight using Eq. (7.28):

$$\varepsilon_{dp} = \frac{L_{dp}^2(65.44 - 1.44\rho_m)}{9.625\times10^7} = \frac{14{,}200^2(65.44 - 1.44\times10.9)}{9.625\times10^7}$$
$$= 104.2\ in$$

To calculate drillpipe stretch due to weight of drill collars, first we should determine the weight of drill collars as follows:

$$W_{dc} = w_{dc}\times L_{dc}\times BF = 93\times800\times\left(1 - \frac{10.9}{65.44}\right) = 62{,}008\ lbf$$

Now, drillpipe stretch due to drill collars can be calculated using Eq. (7.30):

$$\varepsilon_{dc} = \frac{L_{dp}W_{dc}}{735{,}444w_{dp}} = \frac{14{,}200\times62{,}008}{735{,}444\times24.7} = 48.5\ in$$

Thus, total drillpipe stretch is equal to:

$$\varepsilon_t = \varepsilon_{dp} + \varepsilon_{dc} = 104.2 + 48.5 = 152.7\ in$$

When 18,000 lb_f applied to the bit, the weight of drill collars suspended in the drillstring will decrease to:

$$W_{dc\_new} = W_{dc} - WOB = 62{,}008 - 18{,}000 = 44{,}008\ lbf$$

Drillpipe stretch due to weight of drill collars can now be calculated to be:

$$\varepsilon_{dc\_new} = \frac{L_{dp}W_{dc}}{735{,}444w_{dp}} = \frac{14{,}200\times44{,}008}{735{,}444\times24.7} = 34.4\ in$$

The new drillpipe stretch is equal to:

$$\varepsilon_{t\_new} = \varepsilon_{dp} + \varepsilon_{dc\_new} = 104.2 + 34.4 = \mathbf{138.6\ \mathit{in}}$$

***Example 7.23*:** A drillstring consisting of 1,000 *ft* of DCs having weight of 44 ppf and DPs having weight of 13.3 *ppf* was planned to drill a well to a depth of 24,000 *ft* using 16.0 *ppg* drilling mud. Drillpipe stretch was calculated when drillstring was suspended. When certain weight was applied to the bit, calculated the WOB when drillpipe stretch was reduced by 16.5 *inches*.

Solution:

Given data:

L_{dc}	=	Length of drill collars	=	1,000 *ft*
D_m	=	Length of the well	=	24,000 *ft*
w_{dc}	=	Weight of drill collars	=	44 *ppf*
w_{dp}	=	Weight of drillpipes	=	13.3 *ppf*
MW	=	Mud weight	=	16.0 *ppg*
$\Delta\varepsilon_t$	=	Difference in drillpipe stretch	=	16.5 *inches*

Required data:

WOB= Applied weight on bit

To calculate WOB that applied to the bit and created reduction in drillpipe stretch, we should calculate the stretch when drillstring is in suspension. Drillpipe stretch due to the weight of the drillpipe can be calculated using Eq. (7.28) as:

$$\varepsilon_{dp} = \frac{L_{dp}^2(65.44 - 1.44\rho_m)}{9.625 \times 10^7} = \frac{23{,}000^2(65.44 - 1.44 \times 16.0)}{9.625 \times 10^7} = 233.0\ in$$

To calculate drillpipe stretch due to weight of drill collars, we should determine the weight of drill collars as:

$$W_{dc} = w_{dc} \times L_{dc} \times BF = 44 \times 1{,}000 \times \left(1 - \frac{16.0}{65.44}\right) = 33{,}236\ lbf$$

Now, drillpipe stretch due to drill collars can be calculated using Eq. (7.30):

$$\varepsilon_{dc} = \frac{L_{dp}W_{dc}}{735{,}444 w_{dp}} = \frac{23{,}000 \times 23{,}236}{735{,}444 \times 13.3} = 78.1\ in$$

Thus, total drillpipe stretch is equal to:

$$\varepsilon_t = \varepsilon_{dp} + \varepsilon_{dc} = 233.0 + 78.1 = 311.1\ in$$

When certain WOB was applied to the bit, the weight of drill collars suspended in the drillstring will decrease, and hence drillpipe stretch will decrease accordingly. Stretch due to drillpipe weight will not be affected, only the stretch due to weight of drill collars. Thus new stretch due to WOB can be calculated using Eq. (7.30):

$$\varepsilon_{dc\_new} = \frac{L_{dp}W_{dc}}{735{,}444 w_{dp}} = \frac{23{,}000 \times (33{,}236 - WOB)}{735{,}444 \times 13.3} = 78.1 - 16.5 = 61.6$$

$$WOB = \mathbf{7{,}040}\ \boldsymbol{lbf}$$

7.2.2 Bit Design

The design features of the most widely used bits are the roller cone bits, and PDC bits. The performance of bit can be evaluated based on the criteria such as how far it drilled, how fast it drilled (ROP), how much it costs to run per foot of the hole drilled.

i) Roller Cone Bits

Hydraulic Horsepower: The HP_H at the bit is given by:

$$HP_H = \frac{\Delta P_{bn} Q_{bn}}{1714} \tag{7.35}$$

where,

ΔP_{bn} = pressure drop across the nozzles of the bit, *psi*

Q_{bn} = flow rate through the bit, *gpm*

ii) PDC Bit

To estimate the bit performance of a PDC bit, WOB, RPM, mud properties, and hydraulic efficiency are important.

7.2.3 Drilling Optimization Techniques

i) Formation Characteristics

The formation characteristic is one of the most important parameters that influence the rate of penetration. The most important formation characteristics that affect the ROP are the elastic limit and ultimate strength of the formation. The shear strength predicted by the Mohr failure criteria sometimes is used to characterize the strength of the formation. To determine the shear strength from a single compression test, an average angle of internal friction varies from about 30 to 40^0 from the most rock. The following model has been used for a standard compression test:

$$\tau_0 = \frac{\sigma_1}{2} \cos\theta \tag{7.36}$$

where,

τ_0 = shear stress at failure, *psi*

σ_1 = compressive stress, *psi*

θ = angle of internal friction

The threshold force or bit weight $\left(\frac{W}{d}\right)_t$ required to initiate drilling was obtained by plotting drilling rate as a function of bit weight per bit diameter and then extrapolating back to a zero drilling rate.

ii) Combined Effect of Bit Weight and Rotary Speed on ROP

Maurer (1962) developed a theoretical equation for rolling cutter bits relating ROP to bit weight, rotary speed, bit size, and rock strength which can be written as:

$$ROP = \frac{K}{S_c^2}\left[\frac{W_b}{d_b} - \left(\frac{W_{tb}}{d_b}\right)_t\right]^2 N \tag{7.37}$$

where,

ROP = rate of penetration, ft/min
K = constant of proportionality
S_c = compressive strength of the rock
W_b = bit weight
W_{tb} = threshold bit weight
d_b = bit diameter
N = rotary speed
$\left(\frac{W_o}{d_b}\right)_t$ = threshold bit weight per inch of bit diameter

Bingham (1965) suggested the following drilling equation on the basis of considerable laboratory and field data. The equation can be written as:

$$ROP = K\left(\frac{W}{d_b}\right)^{a_5} N \tag{7.38}$$

where,

K = constant of proportionality that includes the effect of rock strength
a_5 = bit weight exponent

iii) Jet Velocity

Eckel (1968) studied micro bits in a laboratory drilling machine. He proposed the following model based on Reynolds number:

$$N_{R_e} = K_s \frac{\rho_f v d_{nz}}{\mu_a} \tag{7.39}$$

where,

K_s = a scaling constant
ρ_f = drilling fluid density
v = flow rate
d_{nz} = nozzle diameter
μ_a = apparent viscosity of drilling fluid at 10,000 s^{-1}

In Eq. (7.39), the shear rate of 10000 s^{-1} was chosen as representative of shear rates present in the bit nozzle. The scaling constant, K_s, is somewhat arbitrary, but a constant value of 1/1,976 was used by Eckel to yield a convenient range of Reynolds-number group.

iv) Effect of Mud Density (Overbalance) on Penetration Rate

Bourgoyne and Young (1974) observed that the relation between overpressure and penetration rate could be represented approximately by a straight line on semilog paper for the range of overbalance commonly used in field practice. In addition, they suggested normalizing the penetration rate data by dividing by penetration rate corresponding to zero overbalance (borehole pressure equal to formation pressure). Figure 7.3 shows the normalized ROP and overbalance for the data as suggested by Bourgoyne and Young. Note that a reasonably accurate straight-line representation of the data is possible for moderate values of overbalance.

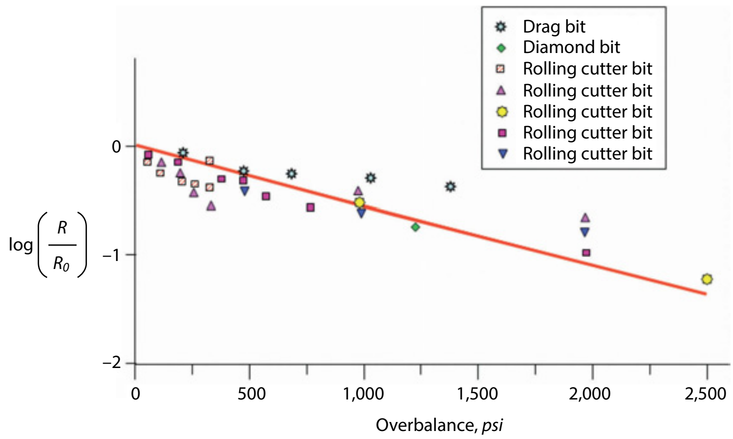

Figure 7.3 Exponential relation between penetration rate and overbalance for roller-cone bits.

From Figure 7.3, we see that the equation for the straight line is given by:

$$\log_{10}\left(\frac{R}{R_o}\right) = -m(P_{bh} - P_f) \tag{7.40}$$

where,

P_{bh} = circulating bottomhole pressure, *psi*
P_f = formation fluid pressure, *psi*
R = rate of penetration (ROP), *ft/hr*
R_o = ROP at zero overbalance, $(P_{bh} - P_f) = 0$, *ft/hr*
m = slope of the straight line in the plot, psi^{-1}

The circulating bottom-hole pressure (P_{bh}) can be expressed in terms of ECD (equivalent circulation density, ρ_c) as follows:

$$P_{bh} = 0.052\ \rho_c D \tag{7.41}$$

where,

D = total depth, *ft*

Also the formation fluid pressure (P_f) can be represented by pore pressure gradient, G_p as follows:

$$P_f = 0.052\ G_p D \tag{7.42}$$

Therefore, applying Eqs. (7.41) and (7.42), Eq. (7.40) can be rewritten as follows:

$$\log_{10}\left(\frac{R}{R_o}\right) = -0.052\ mD(\rho_c - G_p) \tag{7.43}$$

Equation (7.43) can be written as:

$$\log_{10}\left(\frac{R}{R_o}\right) = a_4 D(\rho_c - G_p) \tag{7.44}$$

where,

a_4 = overbalance exponent

This equation is useful for studying the effect of mud density changes on ROP. So, Eq. (7.44) can be rearranged as follows:

$$\frac{R_2}{R_1} = e^{2.304\ a_4 D(\rho_1 - \rho_2)} \tag{7.45}$$

where

R_1 = rate of penetration (ROP) corresponding to ρ_1, *ft/hr*
R_2 = rate of penetration (ROP) corresponding to ρ_2, *ft/hr*
ρ_1 = old mud weight, lb_f/gal
ρ_2 = new mud weight, lb_f/gal

Example 7.24: Estimate the change in penetration rate after the mud-weight is increased from 25 *ft/hr* to a certain rate using the following data: $a_4 = 3.56 \times 10^{-05}$, $D = 12{,}500\ ft$, $\rho_1 = 12.5\ lb_f/gal$, $\rho_2 = 13.5\ lb_f/gal$.

Solution:

Given data:

R_1	=	old rate of penetration	=	25 *ft/hr*
a_4	=	overbalance exponent	=	$3.56 \times 10^{-05}\ ft$
D	=	total depth	=	12,500 *ft*
ρ_1	=	old mud weight	=	12.5 lb_f/gal
ρ_2	=	new mud weight	=	13.5 lb_f/gal

***Required data*:**

R_2 = new rate of penetration in *ft/hr*

The new ROP can be given by Eq. (7.45) as:

$$\frac{R_2}{R_1} = e^{2.304\ a_4 D(\rho_1 - \rho_2)} = e^{2.304 \times 3.56 \times 10^{-05} \times 12{,}500\ (12.5 - 13.5)} = e^{-1.02528}$$

So,

$$R_2 = R_1 \times e^{-1.02528} = \left(25 \frac{ft}{hr}\right) \times 0.424 = \mathbf{10.6 \frac{ft}{hr}}$$

Therefore, new rate of penetration, $R_2 = 10.6 \frac{ft}{hr}$. If the mud weight is increased by 8.33%, ROP

ρ_m is increased by 8%, ROP decreases by 57.6%.

Example 7.25: Estimate the change in penetration rate after the mud-weight is increased from 25 *ft/hr* to a certain rate using the following data: $a_4 = 3.56 \times 10^{-05}$, $D = 12{,}500\ ft$, $\rho_1 = 12.5\ lb_f/gal$, $\rho_2 = 13.5\ lb_f/gal$

Solution:

Given data:

R_1 = old rate of penetration = 25 *ft/hr*
a_4 = overbalance exponent = 3.56×10^{-05}
D = total depth = 12,500 *ft*
ρ_1 = old mud weight = 12.5 lb_f/gal
ρ_2 = new mud weight = 13.5 lb_f/gal

Required data:

R_2 = new rate of penetration in *ft/hr*

The new ROP can be given by Eq. (7.45) as:

$$\frac{R_2}{R_1} = e^{2.304 a_4 D(\rho_1 - \rho_2)} = e^{2.304 \times 3.56 \times 10^{-05} \times 12{,}500 \times (12.5 - 13.5)} = e^{-1.02528}$$

So,

$$R_2 = R_1 \times e^{-1.02528} = \left(25 \frac{ft}{hr}\right) \times 0.424 = \mathbf{10.6} \frac{\boldsymbol{ft}}{\boldsymbol{hr}}$$

Therefore, new rate of penetration, $R_2 = 10.6 \frac{ft}{hr}$.

v) Bit Toot Wear

Several authors have published mathematical models for computing the effect of cutting-element wear on penetration rate for roller-cone bits. Galle and Woods (1963) published the following model:

$$ROP \propto \left(\frac{1}{0.928125\, h^2 + 6h + 1}\right)^{a_7} \tag{7.46}$$

where,

h = the fractional tooth height that has been worn away, *in*
a_7 = tooth wear exponent

A value of 0.5 was recommended for the exponent a_7 for self-sharpening wear of milled-tooth bits, the primary bit type discussed in Galle and Woods (1963). Bourgoyne and Young (1974) suggested a similar but less complex relationship:

$$ROP \propto exp(-a_7 h) \tag{7.47}$$

Bourgoyne and Young suggested that the exponent a_7 be determined on the basis of the observed decline of penetration rate with tooth wear for previous bit runs under similar conditions.

Example 7.26: The initial penetration rate of 30 *ft/hr* was recorded in shale at the beginning of a bit run. The previous bit was identical to the present bit and was operated under the same operating conditions (such as bit weight, rotary speed, mud density, and other factors). However, a drilling rate of 10 *ft/hr* was observed in the same shale formation just before pulling the bit. In addition, the previous bit was graded as T-6. Calculate the approximate value of a_7.

Solution:

Given data:

ROP_1 = initial rate of penetration = 30 *ft/hr*
ROP_2 = final rate of penetration = 10 *ft/hr*
h_1 = 0 *in*
h_2 = T-6 = $\frac{6}{8} = 0.75\,in$

Required data:

a_7 = exponent constant

The new ROP can be given by Eq. (7.47) as:

$$ROP_1 = Ke^{-a_7 h_1} = Ke^{-a_7(0)} = K$$

and

$$ROP_2 = Ke^{-a_7 h_2} = Ke^{-a_7(0.75)} = Ke^{-0.75a_7}$$

Now, dividing the first equation by the second equation as:

$$\frac{ROP_1}{ROP_2} = \frac{K}{Ke^{-0.75a_7}} = e^{0.75a_7}$$

Taking the natural logarithm of both sides and solving for a_7 gives:

$$a_7 = \frac{\ln\frac{ROP_1}{ROP_2}}{0.75} = \frac{\ln\frac{30}{10}}{0.75} = \mathbf{1.46}$$

Example 7.27: The initial penetration rate of 30 *ft/hr* was recorded in shale at the beginning of a bit run. The previous bit was identical to the present bit and was operated under the same operating conditions (such as bit weight, rotary speed, mud density, and other factors). However, a drilling rate of 10 *ft/hr* was observed in the same shale formation just before pulling the bit. In addition, the previous bit was graded as T-6. Calculate the approximate value of a_7.

Solution:

Given data:

ROP_1 = initial rate of penetration = 30 *ft/hr*
ROP_1 = final rate of penetration = 10 *ft/hr*
h_1 = 0 *in*
h_2 = T-6 = $\frac{6}{8}$ = 0.75 *in*

Required data:

a_7 = exponent constant

The new ROP can be given by Eq. (7.47) as:

$$ROP_1 = Ke^{-a_7 h_1} = Ke^{-a_7(0)} = K$$

and

$$ROP_2 = Ke^{-a_7 h_2} = Ke^{-a_7(0.75)} = Ke^{-0.75a_7}$$

Now, dividing the first equation by the second equation as:

$$\frac{ROP_1}{ROP_2} = \frac{K}{Ke^{-0.75a_7}} = e^{-0.75a_7}$$

Taking the natural logarithm of both sides and solving for a_7 gives:

$$a_7 = \frac{\ln\dfrac{ROP_1}{ROP_2}}{0.75} = \frac{\ln\dfrac{30}{10}}{0.75} = \mathbf{1.46}$$

7.2.4 Rate of Penetration Modeling

Maurer's Method: Maurer (1962) derived ROP equation for roller-cone type of bits considering the rock cratering mechanisms which is expressed as:

$$\frac{dF_D}{dt} = \frac{4}{\pi d_b^2}\frac{dV}{dt} \tag{7.48}$$

where:

F_D = distance drilled by bit, *ft*
t = time, *hr*
V = volume of rock removed,
d_b = bit diameter

***Example 7.28*:** A vertical hole was planned to be drilled using 12 ¼" bit. The bit managed to drill a volume of 125 ft^3 within 4 hours of drilling operations. If there was no hole enlargement, calculate the rate of penetration.

Solution:

Given data:

d_{bit} = Bit diameter = 12.25 *inches*
V = Volume drilled out = 125 ft^3
t = Time to drill the above volume = 4 *hrs*

Required data:

ROP = Rate of penetration

Rate of penetration can be estimated using Maurer's method; however, first rate of volume drilled should be calculated as follows:

$$\frac{dV}{dt} = \frac{125}{4} = 31.25\,ft^3/hr$$

Thus, rate of penetration can be calculated using Eq. (7.48):

$$ROP = \frac{dF_D}{dt} = \frac{4}{\pi d_{bit}^2}\frac{dV}{dt} = \frac{4}{\pi 12.25^2} \times 31.25 \times 144 = \mathbf{38.2\ ft/hr}$$

Galle and Woods' Method: Galle and Woods (1963) presented the drilling rate equation as given in (7.49) as a function of WOB and RPM.

$$\frac{dF_D}{dt} = C_{fd}\frac{\bar{W}^k}{a^p}r \tag{7.49}$$

where:

C_{fd} = formation drillability parameter
a = $0.028125h^2 + 6.0h + 1$
h = bit tooth dullness, fractional tooth height worn away, *in*
p = 0.5 (for self-sharpening or chipping type bit tooth wear)
k = 1.0 (for most formations except very soft formations), 0.6 (for very soft formations)
r = function of N which can be expressed as Eq. (7.50) and Eq. (7.51)
$\bar{W}$ = function of WOB and d_b, such that $\bar{W} = \frac{7.88\ WOB}{d_b}$

Now, r can be expressed for two types of formations.

For hard formation:

$$r = e^{\frac{-100}{N^2}} N^{0.428} + 0.2N\left(1 - e^{\frac{-100}{N^2}}\right) \tag{7.50}$$

For soft formation:

$$r = e^{\frac{-100}{N^2}} N^{0.750} + 0.5N\left(1 - e^{\frac{-100}{N^2}}\right) \tag{7.51}$$

where:

N = rotational speed

Galle and Woods (1963) also defined rate of dulling and bearing life equation respectively as shown in Eq. (7.52) and Eq. (7.53).

Rate of dulling equation:

$$\frac{dh}{dt} = \frac{1}{A_f}\frac{i}{am} \tag{7.52}$$

where:

$$i = N + 4.348 \times 10^{-5} N^3$$

$$m = 1359.1 - 714.19\log_{10}\bar{W}$$

Bearing life equation:

$$B = S\frac{L}{N} \tag{7.53}$$

where:

S = drilling fluid parameter
L = tabulated function of $\bar{W}$ used in bearing life equation

Bingham Model: Bingham (1965) proposed a rate of penetration equation based on laboratory data as stated in Eq. (7.54).

$$ROP = K\left(\frac{WOB}{d_b}\right)^{a_5} N \tag{7.54}$$

where:

ROP = rate of penetration
K = proportionality constant for rock strength effect
a_5 = bit weight exponent

Bourgoyne and Young's Method: Bourgoyne and Young's (1973 and 1974) method is given below.

Rate of penetration is expressed as:

$$\frac{d}{dt}(ROP) = e^{\left(a_1 + \sum_{i=2}^{8} a_i x_i\right)} \tag{7.55}$$

where:

a_1 = formation strength parameter
i = index number for i^{th} drilling rate of penetration equation coefficient or summation index for i^{th} data point
a_i = set of constants that relates with each of the drilling parameters considered
x_i = set of dimensionless drilling parameters calculated from the actual collected drilling data the dimensionless drilling parameters in Eq. (7.55) are described as following:

Formation Resistance:

$$x_1 = 1.0 \tag{7.56}$$

Consolidation Effects:

$$x_2 = 10{,}000 - TVD \tag{7.57}$$

Overpressure Effects:

$$x_3 = TVD^{0.69}(g_p - 9.0) \tag{7.58}$$

Differential Pressure:

$$x_4 = TVD(g_p - \rho_{ec}) \tag{7.59}$$

Bit diameter and WOB:

$$x_5 = \ln\left\{\frac{\dfrac{WOB}{d_b} - \left(\dfrac{WOB}{d_b}\right)_t}{4.0 - \left(\dfrac{WOB}{d_b}\right)_t}\right\} \tag{7.60}$$

Rotary Speed:

$$x_6 = \ln\left\{\frac{N}{100}\right\} \tag{7.61}$$

Tooth Wear:

$$x_7 = -h \tag{7.62}$$

Bit Hydraulic:

$$x_8 = \ln\left\{\frac{\rho_m Q}{350\ \mu d_n}\right\} \tag{7.63}$$

where:

TVD = total vertical depth, *ft*
g_p = pore pressure gradient of the formation, lb_f/gal
ρ_{ec} = equivalent circulating mud density at the bottom hole, lb_f/gal
$\dfrac{WOB}{d_b}$ = weight on bit per inch of bit diameter, 1,000 lb_f/in
$\left(\dfrac{WOB}{d_b}\right)_t$ = threshold weight on bit per inch of bit diameter, 1,000 lb_f/in
h = bit tooth dullness, fraction of original tooth height worn away
ρ_m = mud density, lb_f/gal
Q = flow rate, *gal/min*
μ = Viscosity
d_n = bit nozzle diameter, *in*

Combining the Eqs. (7.56–7.63), the open form of the general ROP Eq. (7.55) for roller cone bit types is given as:

$$\frac{d}{dt}(ROP) = e^{\left[\begin{array}{l} a_1 + a_2(10000 - TVD) + a_3 TVD^{0.69}(g_p - 9.0) + a_4 TVD(g_p - \rho_{ec}) + a_5 \\ \ln\left\{\dfrac{\dfrac{WOB}{d_b} - \left(\dfrac{WOB}{d_b}\right)_t}{4.0 - \left(\dfrac{WOB}{d_b}\right)_t}\right\} + a_6 \ln\left\{\dfrac{N}{100}\right\} + a_7(-h) + a_8 \ln\left(\dfrac{\rho_m Q}{350\,\mu d_n}\right) \end{array}\right]} \tag{7.64}$$

where:

a_1 = formation strength parameter
a_2 = exponent of the normal compaction trend
a_3 = under compaction exponent
a_4 = pressure differential exponent

a_5 = bit weight exponent
a_6 = rotary speed exponent
a_7 = tooth wear exponent
a_8 = hydraulic exponent

Bourgoyne and Young (1973) also expressed bit wear by using certain assumptions. Tooth wear model is defined as:

$$\frac{dh}{dt} = \frac{H_3}{\tau_H}\left(\frac{N}{100}\right)^{H_1}\left\{\frac{\left(\frac{WOB}{d_b}\right)_{max} - 4}{\left(\frac{WOB}{d_b}\right)_{max} - \frac{WOB}{d_b}}\right\} \times \left(\frac{1+\frac{H_2}{2}}{1+H_2 h}\right) \tag{7.65}$$

where:

H_1, H_2, H_3 = Constants that depend on bit type
τ_H = formation abrasiveness constant, *hrs*
$\left(\frac{WOB}{d_b}\right)_{max}$ = Bit weight per inch of bit diameter at which the bit teeth would fail instantaneously, 1,000 lb_f/in

Bearing wear model:

$$\frac{dB_{bw}}{dt} = \frac{1}{\tau_B}\left(\frac{N}{100}\right)\left(\frac{WOB}{4d_b}\right)^b \tag{7.66}$$

where:

B_{bw} = bearing wear fraction of the total life
τ_B = life of teeth at standard conditions, *hrs*
b = constant

Reza and Alcocer Method: Reza and Alcocer (1986a) developed a dynamic non-linear, multidimensional, dimensionless drilling model for deep drilling applications using Buckingham p-theorem. They defined the rate of penetration as given in Eq. (7.67) in a form of non-linear, multivariable equation.

$$\frac{ROP}{Nd_{bd}} = C_1\left[\frac{Nd_{bd}^2}{\upsilon}\right]^a\left[\frac{Nd_{bd}^3}{Q}\right]^b\left[\frac{Ed_{bd}}{WOB}\right]^c\left[\frac{\Delta p d_{bd}}{WOB}\right]^d \tag{7.67}$$

where:

ROP = rate of penetration, *ft/min*
C_1 = proportionality constant in penetration rate equation
d_{bd} = bearing diameter, in
υ = drilling fluid kinematic viscosity, *cp*
Q = volumetric flow rate, *gpm*
E = rock hardness, *psi*

In Eq. (7.67), C_1, a, b, c, and d are unknown parameters. In order to find the coefficients using the available data a linear regression analysis methodology was applied after taking the natural logarithm of both sides of the equation above. When the solution of the *ROP* equation was written the following relation was reported to investigate the deep well drilling problems, equation (7.68).

$$\frac{ROP}{Nd_{bd}} = 0.33\left[\frac{Nd_{bd}^2}{\upsilon}\right]^{0.43}\left[\frac{Nd_{bd}^3}{Q}\right]^{-0.68}\left[\frac{Ed_{bd}}{WOB}\right]^{-0.91}\left[\frac{\Delta pd_{bd}}{WOB}\right]^{-0.15} \tag{7.68}$$

The general equation for the rate of bit dulling was obtained as in equation (7.69).

$$\frac{d_{bt}}{Nd_b} = 0.001\left[\frac{Q}{Nd_b^3}\right]^{0.56}\left[\frac{WOB}{Ed_b^2}\right]^{0.26}\left[\frac{d_b}{Q}\right]^{-0.03} \tag{7.69}$$

where:

d_{bt} = bit tooth dullness, fraction of original tooth
d_b = bit diameter

The general equation for the *bit bearing life* was obtained as in equation (7.70).

$$\frac{B_{bw}}{N} = 0.05\left[\frac{Th_t d_{bd}}{NWOB}\right]^{0.51}\left[\frac{\upsilon}{Nd_{bd}^2}\right]^{0.4}\left[\frac{Q}{Nd_{bd}^3}\right]^{-0.5} \tag{7.70}$$

where:

B_{bw} = bearing wear fraction of the total life
T = temperature at the bottom of the hole, °F
h_t = heat transfer coefficient, $BTU/°F - ft^2\ hr$
d_{bd} = bearing diameter, in

Warren's Model: A model of the drilling process for tri-con bits called perfect-cleaning model was derived by Warren in 1987 and later modified by Hareland (Hareland and Hoberock, 1993). The perfect-cleaning model which is shown in the following equation is reviewed as a starting point for development of an imperfect-cleaning model.

$$ROP = \left[\frac{aS^2d_b^2}{N^bWOB^2} + \frac{c}{Nd_b}\right]^{-1} \tag{7.71}$$

where:

a, b, c = bit constant for Warren's constant
S = confined rock strength, *psi*

Unfortunately, ROP in most field cases is significantly inhibited by the rate of cuttings removal from under the bit. Thus Eq. (7.71) is not effective for predicting field ROP without modification to account for imperfect cleaning. Therefore, it is necessary to modify the ROP model for imperfect cleaning conditions which happen in a real situation. Thus the resultant expression for ROP is:

$$ROP = \left[\frac{aS^2 d_b^2}{NWOB^2} + \frac{b}{Nd_b} + \frac{cd_b \gamma_f \mu}{F_{jm}} \right]^{-1} \tag{7.72}$$

where:

γ_f = fluid specific gravity
μ = mud plastic viscosity, *cp*
F_{jm} = modified jet impact force, klb_f

The modified impact force is calculated from the following equation:

$$F_{jm} = [1 - A_v^{-0.122}]F_j \tag{7.73}$$

where:

A_v = ratio of jet velocity to return velocity
F_j = jet impact force, klb_f

If A_v is the ratio of the jet velocity to the fluid return velocity, the A_v (for three jets) is given by:

$$A_v = \frac{v_n}{v_f} = \frac{0.15\ d_b^2}{3\ d_n^2} \tag{7.74}$$

Modified Warren's Model: It is given by:

$$f_c(P_e) = c_c + a_c(P_e - 120)^{b_c} \tag{7.74}$$

where:

P_e = differential pressure
$f_c(P_e)$ = chip hold down function
a_c, b_c, c_c = lithology-dependent constant

Units of a_c, b_c, c_c are chosen such that $f_c(P_e)$ is dimensionless. Equation (7.71) can now be modified to include chip hold down effect and becomes:

$$ROP = \left[f_c(P_e)\left(\frac{aS^2 d_b^2}{N^b WOB^2} + \frac{b}{Nd_b} \right) + \frac{cd_b \gamma_f \mu}{F_{jm}} \right]^{-1} \tag{7.75}$$

Hareland (Hareland *et al.*, 1993) modified this ROP model for the effect of bit wear on ROP by introducing a wear function, W_f into the model:

$$ROP = W_f \left[f_c(P_e)\left(\frac{aS^2 d_b^2}{N^b WOB^2} + \frac{b}{Nd_b} \right) + \frac{cd_b \gamma_f \mu}{F_{jm}} \right]^{-1} \tag{7.76}$$

$$W_f = 1 - \frac{\Delta BG}{8} \tag{7.77}$$

where:

ΔBG = change in bit tooth wear

It can be calculated based on the WOB, ROP, relative rock abrasiveness and confined rock strength.

$$\Delta BG = W_c \sum_{i=1}^{n} WOB_i N_i (A_{r_{abr}})_i S_i \tag{7.78}$$

where:

W_c = bit wear coefficient
$A_{r_{abr}}$ = relative abrasiveness

Rock compressive strength is a function of pressure and lithology:

$$S = S_o(1 + a_s P_e^{b_s}) \tag{7.79}$$

where:

S = confined rock strength
S_o = unconfined rock strength
A_s, b_s = coefficient depends on formation permeability

Pessier and Fear Method: Pessier and Fear (1992) elaborated the mechanical specific energy methodology which was developed by Teale (1962). They performed simulator tests in the computer and conducted laboratory tests to quantify and develop an energy balanced model for drilling of boreholes under hydrostatically pressurized conditions. They derived an equation for mechanical specific energy, Eq. (7.80). They found better identification methodologies (than WOB and ROP concentrated evaluation) for bearing problems of the drill bits which are more quick and reliable by continuously monitoring *Es* and μ, Eq. (7.81).

$$E_s = WOB\left(\frac{1}{A_B} + \frac{13.33\mu_s N}{d_b ROP}\right) \tag{7.80}$$

$$\mu = 36\frac{T}{d_b WOB} \tag{7.81}$$

where:

E_s = bit specific energy, *psi*
A_B = borehole area, in^2
μ_s = bit specific coefficient of sliding friction,
μ = apparent viscosity at 10,000 sec^{-1}, *cp*

7.3 Multiple Choice Questions

1. The ______________ is an important part of the rotary drilling system.
   a) Drillstring
   b) Bottom-hole pressure
   c) Differential pressure
   d) None of the above
2. The drillstring is sometimes also called ______________ .
   a) Drillstem
   b) Drill weight
   c) Drill pressure
   d) None of the above
3. ______________ is a connection between the rig and the drill bit.
   a) Drillstring
   b) Drilling fluid
   c) Drill line
   d) None of the above
4. ______________ is often a source of problems such as washouts, twist-offs, and collapse failures.
   a) Drillstring
   b) Drilling mud
   c) Drill line
   d) None of the above
5. The drill bit is attached to the drill collars by means of a ______________.
   a) Bit sub
   b) Bit hub
   c) Bit tool
   d) None of the above
6. Drillstring provides ______________.
   a) Weight on rock
   b) Weight on bit
   c) Weight on mud
   d) None of the above
7. The drilling mud is circulated inside the ______________.
   a) Drillstring
   b) Drill line
   c) Drill crew
   d) None of the above

8. ______________ are placed above the bit to control the direction in which the drill bit penetrates the formation.
   a) Stabilizers
   b) Kelly
   c) Drill collars
   d) None of the above
9. Kelly fit into the device called ______________.
   a) Kelly bushing
   b) Kelly pushing
   c) Kelly fitting
   d) None of the above
10. Kelly bushing then fits into the ______________ which is mounted on the rotary table.
    a) Master bushing
    b) Master kelly
    c) Master swivel
    d) All of the above
11. Which of the following is a function of kelly?
    a) Transmit rotation and weight to the drill bit
    b) Carry the total weight of the drillstring
    c) A & B
    d) None of the above
12. ______________ forms the upper part of the drillstring.
    a) Drill collars
    b) Stabilizers
    c) Drillpipes
    d) None of the above
13. At the one end of the drillpipe there is the box, which has the ______________.
    a) Female threads
    b) Male threads
    c) Circular threads
    d) None of the above
14. At the one end of the drillpipe there is the pin, which has the ______________.
    a) Female threads
    b) Male threads
    c) Circular threads
    d) None of the above
15. Drillpipe is used in lengths known as ______________.
    a) Singles
    b) Doubles
    c) Multiples
    d) All of the above

16. Heavy wall drillpipe has a ______________ wall thickness than ordinary drillpipe.
    a) Greater
    b) Smaller
    c) Too high
    d) None of the above
17. Major functions of heavy wall drillpipe are
    a) To reduce failures at transition zone
    b) To reduce downhole torque
    c) To reduce differential sticking
    d) All of the above
18. ______________ is the component of the drillstring located directly above the drill bit and below the drillpipe.
    a) BHA
    b) WOB
    c) Drilling fluid
    d) None of the above
19. The primary component of the BHA is the ______________.
    a) Stabilizers
    b) Drill collars
    c) Drillpipes
    d) None of the above
20. Some classify ______________ as a part of the BHA.
    a) Drill bit
    b) Drillpipe
    c) Drill line
    d) None of the above
21. The weight of the collars ensures that the drillpipe is kept in ______________ to prevent buckling.
    a) Tension
    b) Friction
    c) Compression
    d) None of the above
22. Square drill collars provide ______________ stabilization to prevent buckling.
    a) 4 points
    b) 2 points
    c) 3 points
    d) All of the above
23. ______________ drill collars decrease the risk of differential pressure sticking of the BHA.
    a) Spiral
    b) Square
    c) Round
    d) None of the above

24. Monel drill collar is another name for ______________ drill collars.
    a) Non-magnetic
    b) Spiral
    c) Square
    d) None of the above

25. Non-magnetic drill collars are usually ______________.
    a) Non-spiral
    b) Non-square
    c) Non-circular
    d) None of the above

26. Which of the following is a type of jar?
    a) Hydraulic
    b) Mechanical
    c) Both
    d) None of the above

27. Jars are usually positioned at the ______________ of the drill collars.
    a) Top
    b) Bottom
    c) Middle
    d) None of the above

28. Drilling reamer is another name of ______________.
    a) Roller reamer
    b) Spiral reamer
    c) Square reamer
    d) All of the above

29. A ______________ is used between the drillstring and drill collars.
    a) Crossover sub
    b) Shock sub
    c) Bit sub
    d) All of the above

30. ______________ are also called as vibration dampener.
    a) Crossover sub
    b) Shock sub
    c) Bit sub
    d) None of the above

31. Kelly is always positioned at
    a) The top of drill collars
    b) The top of the drillpipe
    c) The top of the drillstring
    d) b and c

32. What is the primary function of the drill collars
    a) Transmit the rotation to the drill bit
    b) Provide the necessary weight on bit to drill the hole
    c) Conduit for mud circulation
    d) None of the above
33. What is the primary function of the slips
    a) Suspend the drillstring in the rotary table
    b) Suspend the drillstring in the hook while drilling
    c) Suspend the drillstring in the TDS while drilling
    d) All of the above
34. What is the primary function of the elevators
    a) Suspend the drillstring in the hook while drilling
    b) Suspend the drillstring in the hook while running in hole
    c) Suspend the drillstring in the hook while pulling out of the hole
    d) b and c
35. Which of the following is not a type of fixed cutter bits
    a) Insert bit
    b) Diamond bit
    c) PDC bit
    d) Impregnated bit
36. Short teeth bits are used to drill
    a) Hard rocks
    b) Medium rocks
    c) Soft rocks
    d) None of the above
37. In which case, the collapse load is equal at any point of the drillstring
    a) When drillstring is empty
    b) When fluid inside the drillstring is heavier than that outside
    c) When the fluid inside and outside the drillstring is the same
    d) None of the above
38. High ROP and low WOB mainly indicate
    a) Hard formation
    b) Soft formation
    c) Salt formation
    d) None of the above
39. Low ROP and high WOB mainly indicate
    a) Hard formation
    b) Medium formation
    c) Soft formation
    d) None of the above

40. Low RPM and high ROP mainly indicate
    a) Hard formation
    b) Sandstone formation
    c) Carbonate formation
    d) None of the above

Answers 1a, 2a, 3a, 4a, 5a, 6b, 7a, 8a, 9a, 10a, 11c, 12c, 13a, 14b, 15a, 16a, 17d, 18a, 19b, 20a, 21a, 22a, 23a, 24a, 25a, 26c, 27a, 28a, 29a, 30b, 31d, 32b, 33a, 34d, 35a, 36a, 37c, 38b, 39a, 40d.

7.4 Summary

The chapter presents almost all the formulas related to basic drillstring and BHA design, including drill bit. The different types of drill bit and their applications are outlined in detail. The ROP optimization and the factors that influence the ROP are also discussed. The existing ROP models are explained here. The workout examples and the MCQs are presented in a chronological manner. The exercise solutions are given in Appendix A. Moreover, this chapter added some more MCQs for the student's self-practice and the answers are given in Appendix B. The MCQs covered almost all the materials covered in this chapter.

7.5 Exercise and MCQs for Practice

7.5.1 Exercises (Solutions are in Appendix A)

Exercise 7.1: A drillpipe was planned to be used to drill a well to a depth of 9,000 *ft* using 10.72 *ppg* drilling fluid. If the collapse safety factor was calculated to be 1.445 when drillstring is empty, what is the collapse resistance of the drillpipes? And if the minimum allowable safety factor is 1.2, determine the maximum depth that the current drillstring can be lowered empty. ***Answers: 7,250 psi, 10,838 ft***

Exercise 7.2: A well was planned to be drilled to a depth of 15,000 *ft* using 12.5 *ppg* drilling mud. The drillstring has a float valve at the bottom of the string. When new drilling mud of 11.3 *ppg* was pumped to a certain depth, collapse pressure at the bottom was calculated to be 1,000 *psi*. What was the bottom depth of the new mud? ***Answer: 11,920 ft***

Exercise 7.3: A production casing of 13,350 *psi* burst rating was planned to be cased and cemented from the top to the casing shoe at 17,500 *ft*. When casing was flushed with a certain drilling fluid, burst safety factor was calculated to be 4.58. When cement slurry of 16.1 *ppg* was pumped to a depth of 8,750 *ft*, burst safety factor was calculated to be 3.06. What was the mud weights of the drilling fluid in the annulus and that used in flushing? Assume mud weight of the fluid in the annulus remains the same. ***Answers: 9.71 ppg, 12.92 ppg***

Exercise 7.4: A production casing was run to the depth of 15,000 *ft* with drilling mud of 9.9 *ppg* at the annulus. When inside casing was filled with 12.5 *ppg* mud, burst safety factor was calculated to be 4.32. Cement slurry of 16.4 *ppg* was pumped and displaced the fluid that was inside the casing to the annulus. If the fluid that was inside the casing totally displaced the fluid that was previously in the annulus, calculate the burst rating of the casing and also the burst safety factor when inside casing was full of cement slurry? ***Answers: 8,761 psi, 2.88***

Exercise 7.5: A production casing was planned to be set at 19,000 *ft*. When inside casing was filled with cement slurry, burst safety factor was calculated to be 2.02. And when cement slurry displaced and filled the annulus by using same drilling mud, collapse safety factor was calculated to be 1.89. Determine the ratio between burst and collapse ratings of the casing. And if the collapse resistance of the casing is 11,000 *psi*, calculate burst rating of the difference in mud weight between drilling mud and cement slurry. ***Answers: 1.068, 11,748psi, 5.89 ppg***

Exercise 7.6: A drilling string consisting of 725 *ft* of DCs having weight of 95 *ppf* and DPs having weight of 23 *ppf* was planned to drill a well to a depth of 19,750 *ft* using 14.0 *ppg* drilling mud. Safety factor was calculated to be 1.3 at this situation. Determine the yield strength of the drillpipe. Answer: ***573,977*** $\boldsymbol{lb_f}$

Exercise 7.7: A drilling string consisting of 800 *ft* of DCs having weight of 92 *ppf* and DPs having weight of 21 *ppf* was used to drill a well to a depth of 21,000 *ft* using 14.5 *ppg* drilling mud. If the maximum allowable overpull was calculated to be 135,000 lb_f, calculate the safety factor at this situation. ***Answer: 1.35***

Exercise 7.8: A drilling string consistsing of 1,100 *ft* of DCs having weight of 84 *ppf* and DPs having weight of 18.4 *ppf* was planned to drill a well to a depth of 18,250 *ft* using 12.6 *ppg* drilling mud. What will be the difference in the drillpipe stretch when drillstring is suspended and a WOB of 20,000 lbf is applied to the bit? ***Answer: 25.4 inches***

Exercise 7.9: A drilling string consisting of 700 *ft* of DCs that have weight of 95 *ppf* and DPs that have weight of 22 *ppf* was planned to drill a well to a depth of 9,500 *ft* using 10.4 *ppg* drilling mud. Two WOB values were applied to the bit, and the difference in drillpipe stretch between the two cases was calculated to be 6.0 inches. Determine the WOB difference between the first and second case. ***Answer: 11,032*** $\boldsymbol{lb_f}$

Exercise 7.10: A vertical hole was planned to be drilled using 17 ½" bit. The bit managed to drill volume of 1,000 *ft*3 within 7.13 hours of drilling. Calculate the rate of penetration. And if it was found that the hole was enlarged and the average hole diameter was found to be 17.86", recalculate the rate of penetration. ***Answers: 84 ft/hr, 80.6 ft/hr***

7.5.2 Exercise (Self-Practices)

E7.1: A drillstring needs to be designed based on the information given here. It is noted that the outer diameter of the drillpipe is 5.5", total vertical depth is 10,000', mud weight is 12 *ppg*. Total MOP is 150,000 lbs and the design factor, SF = 1.2 (tension); SF = 1.1

(collapse). The bottomhole assembly consists of 30 drill collars with an outer diameter of 6.625" where the weight of drill collar is 93 lb_f/ft and each collar is 30 *ft* long. In addition, you need to consider the length of slips is 10".

E7.2: Design a 5.5" and 24.7 lb_f/ft drillstring using a new pipe to reach a TVD of 12,500 *ft* in a vertical hole. The bottomhole assembly consists of 25 drill collars with an outer diameter of 6.625" and inner diameter of 2.929". The weight of drill collar is 93 lb_f/ft and each collar is 22 *ft* long. For design purpose, the additional information are: MW is 10.5 *ppg*, MOP is 140, 000 *lbs* and the design factors are 80% for tension and 1.125 for collapse. You need to consider the length of slips is 12".

E7.3: A drillstring has 5,000 *ft* long, and 5.0 in outer diameter drillpipe. While the pipe was moving, it was suddenly stopped. A torque of 270 lb_f-in is applied which develops torsional stress and angle at a distance of 4.15 from the center of the pipe. Assume that the Young's modulus of elasticity for steel is $29 \ ^\prime \ 10^6$ *psi* and Poisson's ratio is 0.64. Find out the shock load, torsional stress, maximum shear stress and differential angle of twist.

E7.4: During the drilling operation, 150 *hp* was applied to rotate the drillstring and bit where 800 *rpm* was recorded from the rotary speed machine. In addition, 105 *hp* was applied to rotate 2,900 *ft* of drillpipe, 5.5 *in* OD with the same speed as drillstring. Assume that $C_d = 0.0000043$. Calculate the required torque for drilling string and the specific gravity of mud.

E7.5: Find out the minimum torsional yield strength and torsional yield strength under tension for the following data: OD = 5.5 *in*, top joint load is 500,000 lb_f. Assume that the ID of the pipe is 4.67 *in*.

E7.6: A 12 *ppg* mud is circulated through a 5.5 *in* drillpipe assembly of 4,000 *ft*. If 30 drill collars of 32 *ft* long each are also used, calculate stretch for drillpipe and collar due to their own weight. Assume the OD and ID of drill collar as 6.25 in and 2.8125 *in*, respectively, and weight of drill collar is 93 lb_f/ft. In addition assume that a differential pull of 800 lb_f is applied on the drillpipe. Also find out the stretch due to tension.

E7.7: Estimate the change in penetration rate after the mud-weight is increased from 20 *ft*/hr to a certain rate using the following data: $a_4 = 3.46 \times 10^{-05}$, $D = 13{,}500\ ft$, $\rho_1 = 10.5\ lb_f/gal$, $\rho_2 = 11.0\ lb_f/gal$.

E7.8: An initial penetration rate of 20 *ft*/*hr* is observed in shale at the beginning of a bit run. The previous bit was identical to the current bit and was operated under the same conditions of the bit weight, rotary speed, mud density, and other factors. However, a drilling rate of 12 *ft*/*hr* was observed in the same shale formation just before pulling the bit. If the previous bit was graded T-6, compute the approximate value of a_7.

7.5.3 MCQs (Self-Practices)

1. Drillstring is ____________ of the drilling system
   a) An auxiliary component
   b) The main component
   c) Not part
   d) The less important component
2. All of the following are the drillstring main functions except
   a) Provide necessary torque to the bit
   b) Conduit for the drilling mud
   c) Suspend the bit
   d) None of the above
3. All of the following are **not** the drillstring main functions except
   a) Transmit rotary torque to the bit
   b) Increase the bit life
   c) Decrease the well cost
   d) All of the above
4. Among the following drillstring functions, which one is considered less important?
   a) Provide weight on bit
   b) Suspend the bit
   c) Allows pressure testing
   d) None of the above
5. Among the following drillstring functions, which one is considered more important?
   a) Suspend the bit
   b) Provide stability to the bottom-hole assembly
   c) Allows pressure testing
   d) All of the above
6. Which of the following is not part of the drillstring?
   a) Stand pipe
   b) Drillpipe
   c) Drill collar
   d) Kelly
7. Which of the following is one of the main parts of drillstring?
   a) Kelly
   b) Drill collars
   c) MWD
   d) a and b
8. Which of the following is part of the Bottom Hole Assembly (BHA)?
   a) Drillpipes
   b) Kelly
   c) Drill collars
   d) All of the above

9. Which of the following is a less important part of the BHA when drilling vertical wells?
   a) Drill collars
   b) MWD
   c) Near-bit stabilizer
   d) Drilling jar
10. Which of the following is a more important part of the BHA when drilling horizontal wells?
    a) Stabilizers
    b) MWD
    c) Centralizers
    d) None of the above
11. Which of the following is used to transmit the rotation from the rotary table to the drillstring?
    a) Drillpipes
    b) Drill collars
    c) Kelly
    d) All of the above
12. Which of the following drillstring is used to transmit the rotation from the rotary table to the drill bit?
    a) Kelly
    b) Drillpipes
    c) Drill collars
    d) All of the above
13. Which of the following is the common shape of the kelly?
    a) Pentagonal shape
    b) Hexagonal shape
    c) Heptagonal shape
    d) None of the above
14. Which of the following is used to suspend the drillpipe in the slips?
    a) Lifting sub
    b) Elevator
    c) Tool joint
    d) Suspending tool
15. Which of the following is used to suspend the drill collars in the elevator while running or pulling the drill collars?
    a) Lifting sub
    b) Slips
    c) Rotary table
    d) None of the above

16. Which of the following is the major function of heavy-weight drillpipes?
    a) Transmit the rotation to the bit
    b) Provide the necessary weight on the drill bit
    c) Suspend the drill bit
    d) Reduce failures in the drillstring transition zone

17. Which of the following is the primary component of the BHA?
    a) Drill collars
    b) Drilling jar
    c) MWD
    d) Hole openers

18. Which of the following is the function of the stabilizers?
    a) Control the hole deviation
    b) Reduce buckling and bending stresses
    c) Improve the bit performance
    d) All of the above

19. Drilling jars are usually positioned at
    a) The top of the drill bit
    b) The top of the drill collars
    c) The bottom of the kelly
    d) None of the above

20. Roller reamers can be used as ______________.
    a) Cross-overs
    b) Centralizers
    c) Stabilizers
    d) All of the above

21. What is the function of cross-over?
    a) Connect pipes with different thread types
    b) Connect pipes with different sizes and same thread types
    c) Connect pipes with different thread types and same sizes
    d) All of the above

22. Which of the following is used to reduce vertical vibration of the bit?
    a) Near-bit stabilizer
    b) Drilling jar
    c) Shock absorber
    d) None of the above

23. Which of the following is a type of fixed cutter bits?
    a) Insert bit
    b) Diamond bit
    c) Tri-cone bit
    d) None of the above

24. Which of the following is a type of roller cone bits?
    a) Insert bit
    b) Impregnated bit
    c) Steel-cutter bit
    d) All of the above
25. Which of the following is a type of diamond bits?
    a) PDC bit
    b) Natural diamond bit
    c) TSP bit
    d) All of the following
26. Long teeth bits are used to drill
    a) Hard rocks
    b) Medium rocks
    c) Soft rocks
    d) None of the above
27. Which of the following is the major factor in drillstring design?
    a) Mud weight
    b) Shape of the kelly
    c) Type of drilled formation
    d) None of the above
28. Which of the following does not affect drillstring design?
    a) Drill collars size
    b) Formation hardness
    c) Weight of drill collars
    d) Hole size
29. Which of the following situations has a maximum collapse?
    a) Same mud inside and outside the drillstring
    b) Heavier fluid inside the drillstring and lighter fluid outside
    c) Lighter fluid inside the drillstring and heavier fluid outside
    d) None of the above
30. The maximum collapse applied to the drillstring occurs when:
    a) Inside the drillstring is empty
    b) Outside the drillstring is empty
    c) Inside and outside drillstring are empty
    d) None of the above
31. Which of the following situations has the higher tension for the same drillstring?
    a) Hole is full of 10.0 *ppg* mud
    b) Hole is full of 11.0 *ppg* mud
    c) Hole is full of fresh water
    d) All of the above

32. Bit selection depends mainly on
    a) Expected formation pore pressure
    b) Formation hardness
    c) Rig type
    d) All of the above
33. What will generally happen to the ROP if the WOB is decreased?
    a) ROP will decrease
    b) ROP will increase
    c) ROP will not be affected
    d) None of the above
34. What will generally happen to the ROP if the RPM is increased?
    a) ROP will decrease
    b) ROP will increase
    c) ROP will not be affected
    d) None of the above
35. What will generally happen to the ROP if the MW is increased?
    a) ROP will increase
    b) ROP will remains same
    c) ROP will decrease
    d) None of the above
36. What will generally be the changes in the ROP if the mud viscosity is decreased?
    a) ROP will increase
    b) ROP will decrease
    c) ROP will be same
    d) No relation between viscosity and ROP
37. Which of the following is more important part of the BHA for all types of wells?
    a) Drill collars
    b) MWD
    c) Near-bit stabilizer
    d) Drilling jar
38. Which of the following is used to transmit the rotation from the rotary table to the kelly?
    a) Draw works
    b) TDS
    c) Kelly bushing
    d) All of the above
39. Which of the following is used to suspend the drillstring in hook while running in or pulling out of the hole?
    a) Lifting sub
    b) Elevator
    c) Tool joint
    d) Suspending tool

40. Which of the following is used to free the drillstring when it is stuck?
    a) Slip joint
    b) Reamers
    c) Stabilizers
    d) Drilling jar

7.6 Nomenclature

A	=	cross-sectional area, in^2
A_B	=	borehole area, in^2
a_4	=	overbalance exponent
a_5	=	bit weight exponent
a_6	=	rotary speed exponent
a_7	=	tooth wear exponent
B_f	=	buoyancy factor, fraction = $(1 - \rho_m/\rho_s)$
C_d	=	an empirical factor that depends on hole inclination angle (0.000048–0.00000665 for hole angles ranging from 3 to 5^0)
D	=	total depth of fluid column or drillpipe, *ft*
d_b	=	bit diameter
d_i	=	inside diameter of drillpipe, in
d_o	=	outside diameter of drillpipe, in
d_{nz}	=	nozzle diameter
d_{TE}	=	diameter of box at elevator upset, in
DSE	=	drilling specific energy, *psi*
E_s	=	shear modulus of elasticity = $\dfrac{E}{2(1+\upsilon)}$
E	=	Young's modulus of elasticity, *psi*
h	=	the fractional tooth height that has been worn away, *in*
HP_B	=	bit hydraulic horse power, *psi*
HP_{ds}	=	horsepower required to turn the rock bit and drillstring, *hp*
HP_p	=	horsepower required to rotate the drillpipe, *hp*
H_1	=	tooth geometry constant used to predict bit tooth wear
I_p	=	polar moment of inertia = $\dfrac{\pi}{32}(d_o^4 - d_i^4),\ in^4$
K	=	constant of proportionality that includes the effect of rock strength
K_s	=	a scaling constant
L	=	combined length of pin and box, *in*
L_{dc}	=	total length of drill collar, *ft*, *m*
L_{dp}	=	total length of drillpipe, *ft*, *m*
L_{dp1}	=	length of drillpipe grade 1, *ft*
L_{dp2}	=	length of drillpipe grade 2, *ft*
L_{Hdp}	=	length of heavy weight drillpipe, *ft*
L_{tdp}	=	total length of drillpipe, *ft*
$L_{tool\ joint}$	=	tool joint adjusted length = $\dfrac{L + 2.253(d_o - d_{TE})}{12}, ft$

m = slope of the straight line in the plot, psi^{-1}
N = drillstring rotary speed, rev/min
N_{Opt} = optimum rotary speed
P = actual weight or total weight carried by the top joint, lb_f
P_{bh} = circulating bottomhole pressure, psi
P_d = drillpipe yield strength or design weight, lb_f
P_f = formation fluid pressure, psi
P_t = theoretical yield strength, psi
Q_{bn} = flow rate through the bit, gpm
Q_{min} = minimum torsional yield strength, $ft\text{-}lb_f$
Q_{min_t} = minimum torsional yield strength under tension, $lb_f\text{-}ft$
R = rate of penetration (ROP), ft/hr, ft/min
r = distance from the center of the drillpipe to a point under consideration ($d_i \le 2r \le d_o$), in
R_o = ROP at zero overbalance, $(P_{bh} - P_f) = 0$, ft/hr
R_1 = rate of penetration (ROP) corresponding to ρ_1, ft/hr
R_2 = rate of penetration (ROP) corresponding to ρ_2, ft/hr
rpm_c = critical rpm, rev/min
S = compressive strength of the rock
T = torque, $in\text{-}lb_f$
t_b = bit drilling time
T_{or} = torque, $lb_f\text{-}ft$
TVD = total vertical depth of well, ft
v = flow rate
W = weight on bit or bit weight, lb_f
W_{dc} = weight of the drill collar, lb_f/ft, kg/m
W_{dp} = nominal weight of the drillpipe, lb_f/ft, kg/m
$W_{dc\text{-}adj}$ = approx. adjusted weight of drillpipe, lb_f/ft
$W_{dp\text{-}plain}$ = plain end weight, lb_f/ft
$W_{dp\text{-}upset}$ = upset weight, lb_f/ft
W_{Hdp} = nominal weight of the heavy weight drillpipe, lb_f/ft
W_o = threshold bit weight
$W_{tool\ joint}$ = approximate adjusted weight of the tool joint, lb_f/ft
W_v = vertical weight on bit component
W_{dp1} = nominal weight of the drillpipegrade 1, lb_f/ft
W_{dp2} = nominal weight of the drillpipegrade 2, lb_f/ft
X = depth of the empty drillpipe, ft
Y_{min} = minimum unit yield strength, psi
Z_p = polar sectional modulus, psi
$\frac{dF}{dt}$ = rate of penetration (ROP), ft/hr
$\left[\frac{W_v}{d_b}\right]_{opt}$ = optimum weight on bit and drill bit diameter

$\left(\dfrac{W_o}{d_b}\right)_t$	=	threshold bit weight per inch of bit diameter
ρ_f	=	density of fluid outside the drillpipe, *ppg*
ρ_{inside}	=	density of fluid inside the drillpipe, *ppg*
ρ_m	=	mud density, lb_m/gal, *kg/lt*
$\rho_{outside}$	=	density of fluid outside the drillpipe, *ppg*
ρ_s	=	density of steel, lb_m/ft^3
ρ_1	=	old mud weight, lb_f/gal
ρ_2	=	new mud weight, lb_f/gal
τ	=	shear or torsional stress, *psi*
υ	=	Poisson's ratio, (the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force. $\upsilon = -\dfrac{\varepsilon_{trans}}{\varepsilon_{longitudinal}}$)
ε_o	=	stretch due to own weight, *in, m*
ε_{dc}	=	stretch due to drill collar, *in, m*
ε_t	=	stretch due to tension, *ft*
τ_0	=	shear stress at failure, *psi*
σ_1	=	compressive stress, *psi*
θ	=	angle of internal friction
θ_t	=	angle of twist, *radian*
μ_a	=	apparent viscosity of drilling fluid at 10,000 s^{-1}
τ_H	=	formation abrasiveness constant
λ	=	bit hydraulic factor, dimensionless
γ_m	=	specific gravity of mud
ΔP_b	=	burst load or pressure, *psi*
ΔP_{bn}	=	pressure drop across the nozzles of the bit, *psi*
$\dfrac{d\theta_t}{dz}$	=	differential angle of twist, in^{-1}

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8

Casing Design

8.1 Introduction

Casing is defined as a heavy large diameter steel pipe which can be lowered into the well for some specific functions. Casing is strong steel pipe used in an oil or gas well to ensure a pressure-tight connection from the surface to the oil or gas reservoir. It is a steel pipe of approximately 40 *ft* in length that starts from the surface and goes down to the bottom of the borehole. It is rigidly connected to the rocky formation using cement slurry, which also guarantees hydraulic insulation. The space between the casing string and the borehole is then filled with cement slurry before drilling the subsequent hole section. The final depth of the well is completed by drilling holes of decreasing diameter and uses the same diameter protective casings in order to guarantee the borehole stability. In this chapter, sets of multiple choice question (MCQs) are included which are related to the casing technology. Workout examples related to the subject are extensively covered based on casing and drill bit design and analysis. Workout examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

8.2 Different Mathematical Formulas and Examples

8.2.1 Casing Design Process

The design process includes i) selection of casing sizes, ii) selection of setting depths, iii) definition of design properties, and iv) calculation of magnitude of properties.

Example 8.1: A well is being planned to drill where well completion requires the use of 7 *in*. production casing set at 15,000 *ft*. Determine the number of casing strings needed to reach this depth safely, and select the casing setting depth of each string. Pore pressure, fracture gradient, and lithology data from logs of nearby wells are given in Figure 8.1. Allow a 0.5 lb_m/gal trip margin, and a 0.5 lb_m/gal kick margin when making the casing-seat selections. The minimum length of surface casing required to protect the freshwater aquifers is 2,000 *ft*. Approximately 180 *ft* of conductor casing generally is required to prevent washout on the outside of the conductor. It is general practice in this area to cement the casing in shale rather than in sandstone.

Solution:

The planned-mud-density program first is plotted to maintain a 0.5 lb_m/gal trip margin at every depth. The design fracture line is then plotted to permit a 0.5 lb_m/gal kick margin at every depth. These two lines are shown in Figure 8.1 by dashed lines. To drill to a depth of 15,000 *ft*, a 17.6 lb_m/gal mud will be required (Point a). This, in turn, requires intermediate casing to be set at 11,400 *ft* (Point b) to prevent fracture of the formations above 11,400 *ft*. Similarly, to drill safely to a depth of 11,400 *ft* to set intermediate casing, a mud density of 13.6 lb_m/gal is required (Point c). This, in turn, requires surface casing to be set at 4,000 *ft* (Point d). Because the formation at 4,000 *ft* is normally pressured, the usual conductor-casing depth of 180 *ft* is appropriate. Only 2,000 *ft* of surface casing is needed to protect the freshwater aquifers. However, if this minimum casing length is used, intermediate casing would have to be set higher in the transition zone. An additional liner also would have to be set before the total depth is reached to maintain a 0.5 lb_m/gal kick margin. Because shale is the predominant formation type, only minor variations in casing-setting depth are required to maintain the casing seat in shale.

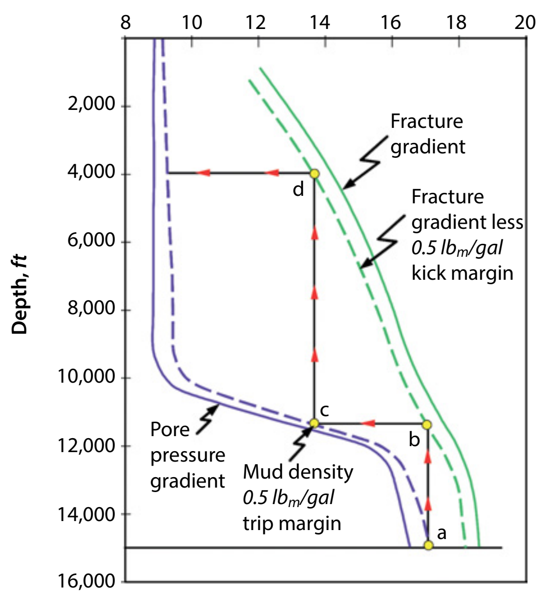

Figure 8.1 Setting depth example (Hossain and Al-Majed, 2015).

Example 8.2: A well is being planned to drill where well completion requires the use of 7-*in*. production casing set at 15,000 *ft*. Determine the casing size (i.e., OD) for each casing string needed to reach this depth safely. Pore pressure, fracture gradient, and lithology data from logs of nearby wells are given in Figure 8.1. Allow a 0.5 lb_m/gal trip margin, and a 0.5 lb_m/gal kick margin when making the casing-seat selections. The minimum length of surface casing required to protect the freshwater aquifers is 2,000 *ft*. Approximately 180 *ft* of conductor casing generally is required to prevent washout on the outside of the conductor. It is general practice in this area to cement the casing in shale rather than in sandstone.

Solution:

A 7 *in* production casing string is desired. An 8.625-*in* bit is needed to drill the bottom section of the borehole (Table 8.1). An 8.625-*in* bit will pass through most of the available 9.625 *in* casings (Table 8.2). However, a final check will have to be made after the required maximum weight per foot is determined. According to the data presented in Table 8.1, a 12.25-*in* bit is needed to drill to the depth of the intermediate casing. As shown in Table 8.2, a 12.25-*in* bit will pass through 13.375 in casing. A 17.5-*in* bit is needed to drill to the depth of the surface casing (Table 8.1). Finally, as shown in Table 8.2, a 17.5-*in* bit will pass through 18.625 in conductor casing, which will be driven into the ground.

Collapse pressure: it can be defined as the difference between external and internal pressure. Mathematically, it can be expressed as given by Eq. (8.1):

$$\Delta p_b = External\ pressure - Internal\ pressure \tag{8.1}$$

Burst pressure: it develops when internal pressure is higher than that of external pressure. It can be rated as

$$\Delta p_b = Internal\ pressure - External\ pressure \tag{8.2}$$

Table 8.1 Commonly used bit sizes for running API casing (Hossain and Al-Majed, 2014).

Casing size (OD) (in)	Coupling size (OD) (in)	Common bit sizes used (in)
4.5	5.0	6, 6.125, 6.25
5	5.563	6.5, 6.75
5.5	6.050	7.875, 8.375
6	6.625	7.875, 8.375, 8.5
6.625	7.390	7.875, 8.375, 8.5
7.0	7.656	8.625, 8.75, 9.5
7.625	8.500	9.875, 10.625, 11.0
8.625	9.625	11.0, 12.25
9.625	10.625	12.25, 14.75
10.75	11.750	15.0
13.375	14.375	17.5
16.0	17.0	20.0
20.0	21.0	24.0, 26.0

Table 8.2 Commonly used bit sizes that will pass through API casing (Hossain and Al-Majed, 2015).

Casing size (O.D., in.)	Weight per foot (IBM/ft)	Internal diameter (in.)	Drift diameter (in.)	Commonly used bit sizes (in.)
4½	9.5	4.09	3.965	3⅞
	10.5	4.052	3.927	
	11.6	4.000	3.875	
	13.5	3.920	3.795	3¾
5	11.5	4.560	4.435	4¼
	13.0	4.494	4.369	
	15.0	4.408	4.283	
	18.0	4.276	4.151	3⅞
5½	13.0	5.044	4.919	4¾
	14.0	5.012	4.887	
	15.5	4.950	4.825	
	17.0	4.892	4.764	
	20.0	4.778	4.653	4⅝
	23.0	4.670	4.545	4¼
6⅝	17.0	6.135	6.01	6
	20.0	6.049	5.924	5⅝
	24.0	5.921	5.796	
	28.0	5.791	5.666	
	32.0	5.675	5.55	4¾
7	17.00	6.538	6.413	6¼
	20.00	6.456	6.331	
	23.00	6.366	6.241	
	26.00	6.276	6.151	6⅛
	29.00	6.184	6.059	6
	32.00	6.094	5.969	
	35.00	6.006	5.879	
	38.00	5.920	5.795	5⅝
7⅝	20.00	7.125	7.000	6⅜
	24.00	7.025	6.900	
	26.40	6.969	6.844	

(*Continued*)

Table 8.2 Cont.

Casing size (O.D., in.)	Weight per foot (IBM/ft)	Internal diameter (in.)	Drift diameter (in.)	Commonly used bit sizes (in.)
7⅝	29.70	6.875	6.750	
	33.70	6.765	6.640	6½
	39.00	6.625	6.500	
8⅝	24.00	8.097	7.972	7⅞
	28.00	8.017	7.892	
	32.00	7.921	7.796	6¾
	36.00	7.825	7.700	
	40.00	7.725	7.600	
	44.00	7.625	7.500	
	49.00	7.511	7.386	
9⅝	29.30	9.063	8.907	8¾, 8½
	32.30	9.001	8.845	
	36.00	8.921	8.765	
	40.00	8.835	8.679	8⅝, 8½
	43.50	8.755	8.599	
	47.00	8.681	8.525	8½
	53.50	8.535	8.379	7⅞
10¾	32.75	10.192	10.036	9⅞
	40.50	10.05	9.894	
	45.50	9.950	9.794	9⅝
	51.00	9.850	9.694	
	55.00	9.760	9.604	
	60.70	9.660	9.504	8¾, 8½
	65.37	9.560	9.404	8¾, 8¾
11¾	38.00	11.154	10.994	11
	42.00	11.084	10.928	10⅝
	47.00	11.000	10.844	
	54.00	10.880	10.724	
	60.00	10.772	10.616	
13⅜	48.00	12.715	12.599	12¼
	54.5	12.615	12.459	

(*Continued*)

Table 8.2 Cont.

Casing size (O.D., in.)	Weight per foot (IBM/ft)	Internal diameter (in.)	Drift diameter (in.)	Commonly used bit sizes (in.)
13⅜	61	12.515	12.359	
	68	12.415	12.259	
	72	12.347	12.191	11
16	55	15.375	15.188	15
	65	15.250	15.062	
	75	15.125	14.939	14¾
	84	15.010	14.822	
	109	14.688	14.500	
18⅝	87.5	17.755	17.567	17½
20	94	19.124	18.936	17½

Example 8.3: Compute the burst requirement if the pore pressure is 7000 *psi* if the factor of safety is assumed as 1.05.

<u>Solution</u>:

Given data:

P_p = Pore Pressure = 7,000 *ft*

Required data:

P_{br} = Burst pressure in *psi*

The burst requirement based on the expected pore pressure with $SF = 1.05$ can be calculated as:

$$P_{br} = P_p \times SF = (7000\ psi) \times 1.05 = \mathbf{7350\ psi}$$

Example 8.4: Compute the burst requirement if the pore pressure is 7540 *psi* if the factor of safety is assumed as 1.1 and 1.2.

<u>Solution</u>:

Given data:

P_p = Pore Pressure = 7,540 *ft*

Required data:

P_{br} = Burst pressure in *psi*

The burst requirement based on the expected pore pressure with $SF = 1.1$ can be calculated as:

$$P_{br} = P_p \times SF = (7540\ psi) \times 1.1 = \mathbf{8294\ psi}$$

The burst requirement based on the expected pore pressure with $SF = 1.2$ can be calculated as:

$$P_{br} = P_p \times SF = (7540\ psi) \times 1.2 = \mathbf{9048}\ \boldsymbol{psi}$$

The whole casing string must be capable of withstanding this internal pressure without failing in burst.

8.2.2 Calculation of Magnitude of Design Properties

i) Collapse Strength:

Figure 8.2 shows the variation of collapse resistance with d_n/t for the four collapses. Five factors (F_1, F_2, F_3, F_4, and F_5) are used with the tube's d_n/t ratio to determine which of the four collapse-pressure formulas is applied. The factors are dependent on the yield strength of the tube. They are defined by the following equations:

$$F_1 = c_o + c_1\sigma_{yield} + c_2\sigma_{yield}^2 + c_3\sigma_{yield}^3 \tag{8.3}$$

$$F_2 = c_4 + c_5\sigma_{yield} \tag{8.4}$$

$$F_3 = c_6 + c_7\sigma_{yield} + c_8\sigma_{yield}^2 + c_9\sigma_{yield}^3 \tag{8.5}$$

$$F_4 = c_{10}\frac{\left[\dfrac{3R_F}{(2+R_F)}\right]^3}{\sigma_{yield}\left[\dfrac{3R_F}{(2+R_F)} - R_F\right]\left[1 - \dfrac{3R_F}{(2+R_F)}\right]^2} \tag{8.6}$$

$$F_5 = F_4 R_F \tag{8.7}$$

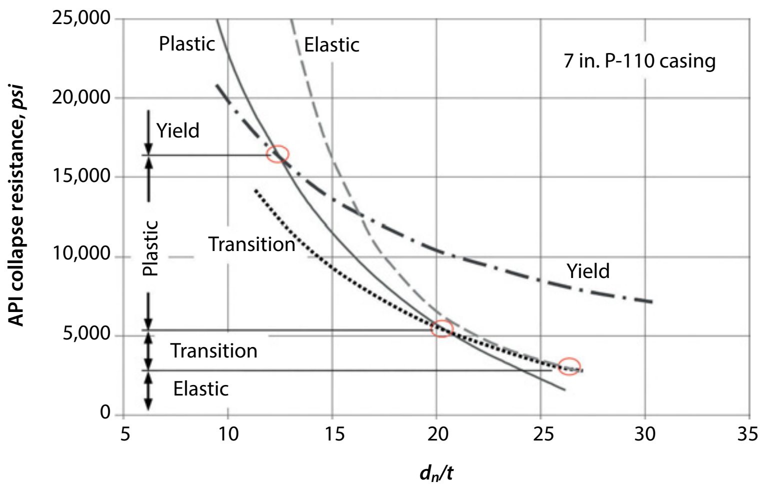

Figure 8.2 Collapse modes.

Here

c_o	=	2.8762	c_1	=	1.0679×10^{-6}
c_2	=	2.1302×10^{-11}	c_3	=	-5.3132×10^{-17}
c_4	=	0.026233	c_5	=	5.0609×10^{-7}
c_6	=	-465.93	c_7	=	3.0867×10^{-2}
c_8	=	-1.0483×10^{-8}	c_9	=	3.6989×10^{-14}
c_{10}	=	46.95×10^{6}	R_F	=	$\frac{F_2}{F_1}$

Example 8.5: A production casing was running to a depth of 12,000 *ft*. When casing was at bottom, inside casing was partially full of water up to a depth of 6,500 *ft*. Later, inside casing was filled with water up to the surface. If the mud weight in annulus is 14.5 *ppg*, calculate the collapse pressure at the casing shoe for both cases?

Solution:

Given data:

D_{shoe}	=	Depth of the casing shoe	=	12,000 *ft*
h_{wat}	=	Height of water inside casing	=	6,500 *ft*
MW_{wat}	=	Mud weight of water	=	8.34 *ppg*
MW_{ann}	=	Mud weight in the annulus	=	14.5 *ppg*

Required data:

P_c = Collapse pressure

To calculate the collapse pressure when the casing is partially full, first collapse pressure at the top of the water column must be calculated as follows:

$$P_c = 0.052 \times \rho_m \times D = 0.052 \times 14.5 \times (12{,}000 - 6{,}500) = 4{,}147\ psi$$

Now, collapse pressure at bottom is equal to the above pressure plus the difference in pressure in the water column due to the difference in mud weight inside and outside the casing.

$$\begin{aligned} P_c &= 4{,}147 + 0.052 \times h_w \times (\rho_m - \rho_w) \\ &= 4{,}147 + 0.052 \times 6{,}500 \times (14.5 - 8.34) = \mathbf{6{,}229}\ \boldsymbol{psi} \end{aligned}$$

Collapse pressure when the casing was full of water is equal to:

$$\begin{aligned} P_c &= 0.052 \times D_{shoe} \times (\rho_m - \rho_w) \\ &= 0.052 \times 12{,}000 \times (14.5 - 8.34) = \mathbf{3{,}844}\ \boldsymbol{psi} \end{aligned}$$

Example 8.6: A production casing with a collapse rating of 9,250 *psi* was planned to run in the production hole of a well. When casing was run empty to a certain depth, collapse pressure was measured to be half of the collapse rating. When the casing was at the bottom of 17,500 *ft*, safety factor of the collapse was calculated to be 1.3. If the mud weight of the fluid in the annulus was 16.8 *ppg*, determine the mud weight of the fluid inside the casing.

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 17,500 *ft*
P_{coll} = Casing collapse rating = 9,250 *psi*
ρ_{ann} = Mud weight in the annulus = 16.8 *ppg*
SF_{coll} = Collapse safety factor = 1.3

Required data:

D_{cas} = Depth of the casing
ρ_{in} = Density of the mud inside the casing

The casing was first run empty to a certain depth until the collapse pressure became half of the collapse rating of the casing or 4,625 *psi*. So by knowing the collapse pressure and no fluid inside the casing, we can determine the depth of the casing as follows:

$$P_c = 0.052 \times \rho_m \times D = 4{,}625 = 0.052 \times 16.8 \times D$$

$$D = \mathbf{5{,}294}\ \boldsymbol{ft}$$

When casing was at the bottom, inside casing was filled with certain fluid that developed collapse pressure against the casing with a safety factor of 1.3. So the collapse pressure is equal to:

$$P_c = \frac{P_{col}}{SF_{col}} = \frac{9{,}250}{1.3} = 7{,}115.4\ psi$$

Pressure developed by the annulus fluid is equal to:

$$P_{ann} = 0.052 \times \rho_m \times D = 0.052 \times 16.8 \times 17{,}500 = 15{,}288\ psi$$

Thus, the pressure inside the casing is equal to:

$$P_{in} = P_{ann} - P_c = 15{,}288 - 7{,}115.4 = 8{,}172.6\ psi$$

Now, the mud weight of the fluid inside the casing is equal to:

$$P_c = 0.052 \times \rho_m \times D = 8{,}172.6 = 0.052 \times 17{,}500 \times \rho_m$$

$$\rho_m = \mathbf{8.98}\ \boldsymbol{ppg}$$

Yield-Strength Collapse Pressure Formula: the yield-strength collapse-pressure formula calculates the external pressure that generates the minimum yield stress on the inside wall of a tube and can be derived theoretically using the Lam**é** equation. He formulated this equation for the thickest-walled tubulars used in oil wells. The Equation can be written as:

$$P_{cr} = 2\sigma_{yield}\left[\frac{\left(\frac{d_n}{t} - 1\right)}{\left(\frac{d_n}{t}\right)^2}\right] \tag{8.8}$$

This equation is applicable for $\frac{d_n}{t}$ values up to the value of the $\frac{d_n}{t}$ ratio where the plastic collapse formula becomes applicable. The $\frac{d_n}{t}$ ratio for this changeover point can be calculated as:

$$\frac{d_n}{t} = \frac{\sqrt{(F_1 - 2)^2 + 8\left[F_2 + \left(\frac{F_3}{\sigma_{yield}}\right)\right]} + (F_1 - 2)}{2\left[F_2 + \left(\frac{F_3}{\sigma_{yield}}\right)\right]} \tag{8.9}$$

where

d_n = nominal OD of pipe, *in*
t = thickness, *in*
P_{cr} = collapse pressure rating, *psi*
σ_{yield} = the minimum yield stress, *psi*

Plastic-Collapse Pressure Formula: the equation is based on 2,488 physical-collapse tests of K-55, N-80, and P-110 casings (API TR 5C3 2800). Statistical methods were used to analyze the results of the physical tests, and a plastic-collapse formula was developed to calculate a collapse value with a 95% probability that the actual collapse pressure will exceed the minimum stated with no more than a 0.5% failure rate:

$$P_{cr} = \sigma_{yield}\left[\frac{F_1}{\frac{d_n}{t}} - F_2\right] - F_3 \tag{8.10}$$

The $\frac{d_n}{t}$ ratio where the changeover from the plastic collapse formula to the transition formula can be calculated as:

$$\frac{d_n}{t} = \frac{2 + \frac{F_2}{F_1}}{\frac{3F_2}{F_1}} \tag{8.11}$$

Transition-Collapse Pressure Formula: the transition-collapse formula was developed to provide a transition from the plastic-collapse formula to the elastic-collapse formula:

$$P_{cr} = \sigma_{yield}\left[\frac{F_4}{\frac{d_n}{t}} - F_5\right] \tag{8.12}$$

The $\frac{d_n}{t}$ ratio where the changeover from the transition collapse formula to the elastic-collapse equation can be calculated as:

$$\frac{d_n}{t} = \frac{\sigma_{yield}(F_1 - F_4)}{F_3 + \sigma_{yield}\ (F_2 - F_5)} \tag{8.13}$$

Elastic-Collapse Pressure Formula: this equation was theoretically derived and was found to be an adequate upper bound for collapse pressures as determined by testing. API adopted this equation in 1968.

$$P_{cr} = \frac{46.95 \times 10^6}{\left(\frac{d_n}{t}\right)\left[\frac{d_n}{t} - 1\right]^2} \tag{8.14}$$

Collapse Resistance of Casing with Combined Loading Formula: API offers an equation to calculate the external pressure equivalent when both external and internal pressures are applied to a tubular:

$$P_{eq} = P_e - \left[1 - \frac{2}{\frac{d_n}{t}}\right] P_i \tag{8.15}$$

where

P_{eq} = external pressure equivalent in collapse due to external and internal pressure
P_e = external pressure, and P_i = Internal pressure

Collapse Pressure with Axial Stress: the current API formula accounts for the combined influence of tension and collapse loading on a casing by modifying the minimum yield strength to the yield strength of an axial-stress-equivalent grade. The equivalent yield-strength formula is:

$$\sigma_{pa} = \left[\sqrt{1 - 0.75\left(\frac{\sigma_a}{\sigma_{yield}}\right)^2} - 0.5\left(\frac{\sigma_a}{\sigma_{yield}}\right)\right]\sigma_{yield} \tag{8.16}$$

where

σ_{pa} = equivalent yield strength, *psi*
σ_a = total axial stress, not included bending due to hole deviation, doglegs, or buckling
σ_{yield} = minimum yield strength of pipe, psi

***Example 8.7*:** An intermediate section of 17.5″ size was planned to be drilled at a depth of 14,000 *ft*. A 13 3/8″ OD casing is to be set and cemented in this section. There are four casing types available in stocks which are shown in Table 8.3 below.

Table 8.3 Types of casing Example 8.7.

OD		ID	Yield
in	Grade	*in*	*psi*
13 3/8	N-80	12.159	80,000
		12.125	80,000
		12.031	80,000
		11.937	80,000

Maximum burst pressure has to be assumed when casing is full of 15.8 *ppg* cement slurry and fluid in the annulus is water. In addition, maximum collapse pressure has to be assumed when casing is full of fresh water and annulus is full of 15.8 *ppg* cement slurry. Safety factor for burst and collapse was designed to be 1.18 and 1.15 respectively. Design a casing that should be used in this section.

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 14,000 *ft*
ρ_{cement} = Density of cement slurry = 15.8 *ppg*
ρ_{wat} = Density of fresh water = 8.34 *ppg*
SF_c = Collapse safety factor = 1.15
SF_b = Burst safety factor = 1.18

Required data:

Casing to be used in the intermediate section

Selection of the casing type will depend mainly on the collapse and burst rating of each casing. So collapse and burst pressure for each casing should be calculated using Eq. (8.14) for collapse and Eq. (8.16).

$$P_{cr} = \frac{46.95\times10^6}{\left(\frac{d_n}{t}\right)\left(\frac{d_n}{t}-1\right)^2}$$

$$P_{br} = f\left[\frac{2\sigma_{yield}t}{d_n}\right]$$

Table 8.4 summarizes the collapse and burst pressure for the above casings using the above equations after applying safety design factors for collapse and burst:

From the well data, collapse pressure based on the assumed scenario is equal to:

$$\begin{aligned}P_c &= 0.052\times D_m\times(\rho_{cement}-\rho_{wat})\\ &= 0.052\times14,000\times(15.8-8.34)\\ &= 5,431\ psi\end{aligned}$$

Table 8.4 Details of casing collapse and burst for Example 8.7.

OD	ID	P_{cr}		P_{br}	$P_{br} \times SF_b$
in	*in*	*psi*	$P_{cr} \times SF_c$	*psi*	*psi*
13 3/8	12.159	4840	4209	6364	5393
	12.125	5272	4584	6542	5544
	12.031	6601	5740	7034	5961
	11.937	8146	7083	7526	6378

And burst pressure based on the assumed scenario is equal to:

$$P_b = 0.052 \times D_m \times (\rho_{cement} - \rho_{wat})$$
$$= 0.052 \times 14{,}000 \times (15.8 - 8.34) = 5{,}431\ psi$$

From the maximum anticipated collapse and burst pressures, and comparing these values with the values of collapse and burst rating calculated for each casing we suggest that the optimum casing to be used is 12.031 *in* ID casing. This casing has maximum allowable collapse and burst ratings of 5,740 *psi* and 5,961 *psi*.

Example 8.8: A production casing string is set at a depth of 15,750 *ft*. A cement slurry of 15.6 *ppg* has pumped and filled the casing from the top to the casing shoe while the annulus has 12.2 *ppg* drilling mud. The cement has been displaced by 11.7 *ppg* drilling mud until cement reached the top of the well. If the burst and collapse safety factors during pumping of cement have been estimated to be 3.5 for both cases, determine the collapse and burst rating of the casing.

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 15,750 *ft*
ρ_{cem} = Mud weight of cement slurry = 15.6 *ppg*
ρ_{ann} = Mud weight in the annulus = 12.2 *ppg*
ρ_{in} = Mud weight of displacing fluid = 11.7 *ppg*
SF_b = Burst safety factor = 3.5
SF_c = Collapse safety factor = 3.5

Required data:

P_{br} = Casing burst rating
P_{cr} = Casing collapse rating

When cement was pumped inside the casing, pressure inside the casing is greater than that in the annulus. So, the casing is subjected to burst pressure at the casing shoe. Burst pressure can be calculated as follows:

$$P_b = 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_{ann})$$
$$= 0.052 \times 15{,}750 \times (15.6 - 12.2) = 2{,}785\ psi$$

Using the job safety factor for burst, burst pressure rating of the casing is equal to:

$$P_{br} = P_b \times SF_b = 2{,}785 \times 3.5 = \mathbf{9{,}748}\ \boldsymbol{psi}$$

Now, when the cement is displaced out in the annulus, the casing is subjected to collapse pressure. Collapse pressure can be calculated using the following equation:

$$\begin{aligned} P_c &= 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_{in}) \\ &= 0.052 \times 15{,}750 \times (15.6 - 11.7) = 3{,}194\ psi \end{aligned}$$

Using the job safety factor for collapse, collapse pressure rating of the casing is equal to:

$$P_{cr} = P_c \times SF_c = 3{,}194 \times 3.5 = \mathbf{11{,}179}\ \boldsymbol{psi}$$

Example 8.9**:** A production casing string is set at a depth of 16,500 *ft*. Cement slurry of 16.4 ppg was pumped and displaced by 11.6 *ppg* drilling mud. When cement slurry was at the bottom of casing, cement column was about 7,000 *ft*. When cement was displaced in the annulus, cement column was about 11,000 *ft* and the rest was filled with a drilling mud of 12.6 *ppg*. Determine the depth at which the collapse and burst pressures are equal for both cases when cement inside the casing, and when the cement in the annulus.

Solution:

Given data:

D_{shoe}	=	Depth of the casing shoe	=	16,500 *ft*
ρ_{cem}	=	Mud weight of cement slurry	=	16.4 *ppg*
ρ_{ann}	=	Mud weight in the annulus	=	12.6 *ppg*
ρ_{in}	=	Mud weight of displacing fluid	=	11.6 *ppg*
L_{cem_cas}	=	Cement column inside casing	=	7,000 *ft*
L_{cem_ann}	=	Cement column in the annulus	=	11,000 *ft*

Required data:

Depths of zero pressures between the annulus and inside casing

Cement column will affect the pressure distributions inside the casing and in the annulus. Figure 8.3 shows the annulus and inside casing pressure plot. Annulus pressure was higher than that of inside the casing down to the top of cement column. Two pressure profiles matches each other at a depth of about 12,000 *ft*, which indicates similar pressure. So, the depth of equal pressure inside and outside the casing is around 12,000 *ft*.

Figure 8.4 shows inside and outside pressure profiles when cement was displaced to the annulus. In this case density of the mud inside the casing was less than the mud and cement slurry in the annulus. So pressure in the annulus was always greater than that of inside the casing. Therefore, the lines did not cross at any depth. The differences between pressures created collapse pressure of about 3,032 *psi* in the casing shoe.

Example 8.10: Compute the API collapse-pressure rating for 20-*in*, K-55 casing with a nominal wall thickness of 0.64 *in*. and a nominal weight per foot of 135 lb_f/ft.

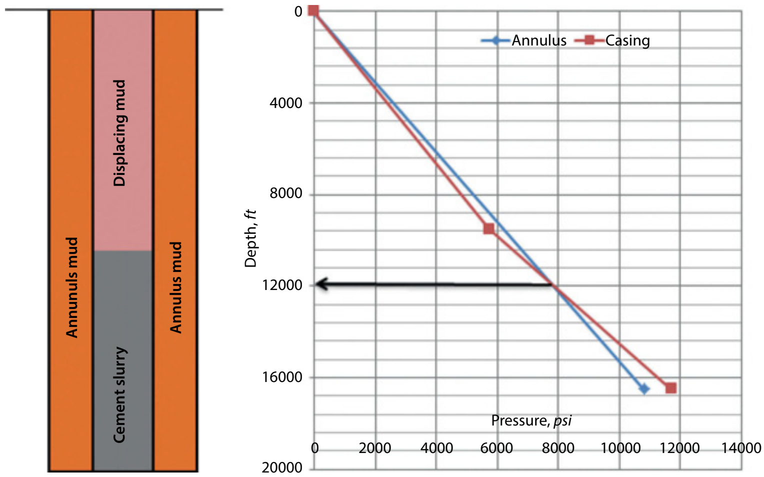

Figure 8.3 First case when cement slurry was inside the casing.

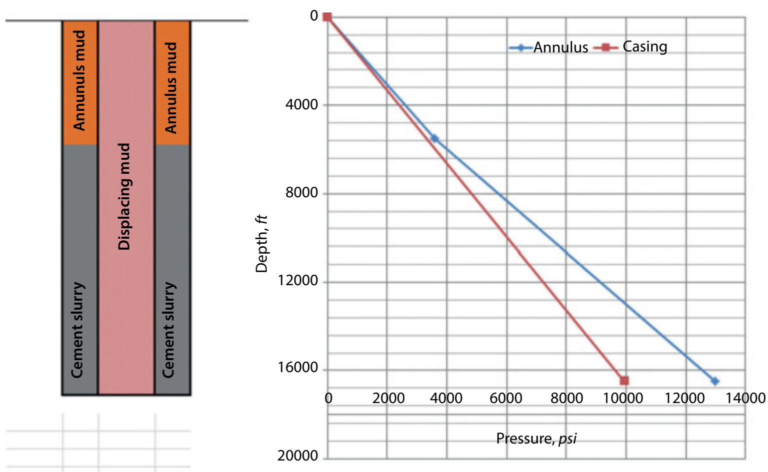

Figure 8.4 Second case when cement slurry was in the annulus.

Solution:

Given data:

d_n = nominal diameter of the casing pipe = 20 *in*

t = thickness of the casing pipe = 0.64 *in*

W_n = nominal weight per foot of the pipe = 135 lb_f/ft

Required data:

P_{cr} = Collapse pressure rating, *psi*

The $\frac{d_n}{t}$ ratio can be calculated as:

$$\frac{d_n}{t} = 20/0.64 = 31.25.$$

Now find $\frac{d_n}{t}$ using Eq. (8.13)

$$\frac{d_n}{t} = \frac{\sigma_{yield}(F_1 - F_4)}{F_3 + \sigma_{yield}(F_2 - F_5)}$$

Compare the two results. It is found that it falls in the range of transition collapse. Compute $F_1 - F_5$ using the following Equations.

$$F_1 = c_o + c_1\sigma_{yield} + c_2\sigma^2_{yield} + c_3\sigma^3_{yield}$$

$$F_2 = c_4 + c_5\sigma_{yield}$$

$$F_3 = c_6 + c_7\sigma_{yield} + c_8\sigma^2_{yield} + c_9\sigma^3_{yield}$$

$$F_4 = c_{10}\frac{\left[\frac{3R_F}{(2+R_F)}\right]^3}{\sigma_{yield}\left[\frac{3R_F}{(2+R_F)} - R_F\right]\left[1 - \frac{3R_F}{(2+R_F)}\right]^2}$$

$$F_5 = F_4 R_F$$

Here

c_o	=	2.8762	c_1	=	1.0679×10^{-6}
c_2	=	2.1302×10^{-11}	c_3	=	-5.3132×10^{-17}
c_4	=	0.026233	c_5	=	5.0609×10^{-7}
c_6	=	-465.93	c_7	=	3.0867×10^{-2}
c_8	=	-1.0483×10^{-8}	c_9	=	3.6989×10^{-14}
c_{10}	=	46.95×10^6	R_F	=	$\frac{F_2}{F_1}$

Eq. (18.5) is used to calculate collapse pressure rating.

$$P_{cr} = \sigma_{yield}\left[\frac{F_4}{\frac{d_n}{t}} - F_5\right]$$

$$P_{cr} = 55{,}000\left[\frac{1.989}{31.25} - 0.036\right] = \mathbf{1{,}520.64}\ \boldsymbol{psi}$$

Example 8.11: Compute the API collapse-pressure rating for 18-*in*, K-55 casing with a nominal wall thickness of 0.64 *in*. and a nominal weight per foot of 135 lb_f/ft.

Solution:

Given data:

d_n	=	nominal diameter of the casing pipe	=	18 *in*
t	=	thickness of the casing pipe	=	0.64 *in*
W_n	=	nominal weight per foot of the pipe	=	135 lb_f/ft

Required data:

P_{cr} = Collapse pressure rating, *psi*

The $\frac{d_n}{t}$ ratio can be calculated as:

$$\frac{d_n}{t} = \frac{18}{0.64} = 28.125$$

Now find $\frac{d_n}{t}$ using Eq. (8.13),

$$\frac{d_n}{t} = \frac{\sigma_{yield}(F_1 - F_4)}{F_3 + \sigma_{yield}(F_2 - F_5)}$$

Compare the two results. It is found that it falls in the range of transition collapse. Compute F_1–F_5 using the following Equations.

$$F_1 = c_o + c_1\sigma_{yield} + c_2\sigma^2_{yield} + c_3\sigma^3_{yield}$$

$$F_2 = c_4 + c_5\sigma_{yield}$$

$$F_3 = c_6 + c_7\sigma_{yield} + c_8\sigma^2_{yield} + c_9\sigma^3_{yield}$$

$$F_4 = c_{10}\frac{\left[\dfrac{3R_F}{2+R_F}\right]^3}{\sigma_{yield}\left[\dfrac{3R_F}{2+R_F} R_F\right]\left[1\dfrac{3R_F}{2+R_F}\right]^2}$$

$$F_5 = F_4 R_F$$

Here

c_o	=	2.8762	c_1	=	1.0679×10^{-6}
c_2	=	2.1302×10^{-11}	c_3	=	-5.3×10^{-17}
c_4	=	0.026233	c_5	=	5.0609×10^{-7}
c_6	=	-465.93	c_7	=	3.0867×10^{-2}
c_8	=	-1.0483×10^{-8}	c_9	=	3.6989×10^{-14}
c_{10}	=	46.95×10^{6}	R_F	=	$\frac{F_2}{F_1}$

Eq. (18.5) is used to calculate collapse pressure rating.

$$P_{cr} = \sigma_{yield}\left[\frac{F_4}{\frac{d_n}{t}} - F_5\right]$$

$$P_{cr} = 55{,}000\left[\frac{1.989}{28.125} - 0.036\right] = \mathbf{1909.6}\ \boldsymbol{psi}$$

ii) Burst Loading:

Barlow Model: API uses the Barlow model to determine the minimum internal yield pressure for tubular (API TR 5C3). The Barlow equation which is sometimes called an "API" burst as:

$$P_{br} = f\left[\frac{2\sigma_{yield} t}{d_n}\right] \tag{8.17}$$

where

f = wall-thickness correction factor = 0.875 for standard API tubulars when a 12.5% wall-thickness tolerance is specified.

P_{br} = burst pressure rating, *psi*

API recommends the use of Eq. (8.17) with wall thickness rounded to the nearest 0.001 in and the results rounded to the nearest 10 *psi*.

According to Lame equations, the burst loading can be estimated as:

$$P_{br} = \sigma_{yield}\left[\frac{(d_n^2 - d_m^2)}{(d_n^2 + d_m^2)}\right] \tag{8.18}$$

where

d_n = nominal OD of pipe, *in*

d_m = maximum pipe body ID based on minimum specific wall thickness, *in*

Example 8.12: Compute the burst requirement if the pore pressure is 6000 psi if the factor of safety is assumed as 1.1.

Solution:

Given data:

P_p = pore pressure = 6,000 *psi*

Required data:

P_{br} = burst pressure in *psi*

The burst requirement based on the expected pore pressure can be calculated as:

$$P_{br} = P_p \times SF = (6{,}000\ psi) \times 1.1 = \mathbf{6{,}600\ psi}$$

The whole casing string must be capable of withstanding this internal pressure without failing in burst.

Example 8.13: Compute the API burst resistance for 20-*in*, 133-lb_f/ft, *K*-55 casing with a nominal wall thickness of 0.64 *in*. Use Barlow model.

Solution:

Given data:

d_n = nominal OD of pipe = 20 *in*

σ_{yield} = minimum yield strength of pipe (*k*-55) = 55,000 *psi*

t = nominal wall thickness of pipe = 0.64 *in*

Required data:

P_{br} = burst pressure in *psi*

Using the API burst equation, Eq. (8.17), it can be calculated as:

$$P_{br} = f\left[\frac{2\sigma_{yield}t}{d_n}\right] = 0.875\left[\frac{2 \times 55{,}000\ psi \times 0.65\ in}{20\ in}\right] = \mathbf{3{,}128.13}\ \boldsymbol{psi}$$

Example 8.14: Compute the API burst resistance for 20-*in*, 133-lb_f/ft, *K*-55 casing with a nominal wall thickness of 0.64 *in*. Use Barlow model.

<u>Solution</u>:

Given data:

d_n = nominal OD of pipe = 20 *in*

σ_{yield} = minimum yield strength of pipe (*k*-55) = 55,000 *psi*

t = nominal wall thickness of pipe = 0.64 *in*

Required data:

P_{br} = Burst pressure in *psi*

Using the API burst equation, Eq. (8.17), it can be calculated as:

$$P_{br} = f\left[\frac{2\sigma_{yield}t}{d_n}\right] = 0.875\left[\frac{2 \times 55{,}000\ psi \times 0.64\ in}{20}\right] = \mathbf{3520}\ \boldsymbol{psi}$$

Example 8.15: Compute the API burst resistance for 15-in, 120-lb_f/ft, *K*-55 casing with a nominal wall thickness of 0.64 *in*. Use Barlow model.

<u>Solution</u>:

Given data:

d_n = nominal OD of pipe = 15 *in*

σ_{yield} = minimum yield strength of pipe (*k*-55) = 55,000 *psi*

t = nominal wall thickness of pipe = 0.64 *in*

Required data:

P_{br} = Burst pressure in *psi*

Using the API burst equation, Eq. (8.17), it can be calculated as:

$$P_{br} = f\left[\frac{2\sigma_{yield}t}{d_n}\right] = 0.875\left[\frac{2 \times 55{,}000\ psi \times 0.64\ in}{15}\right]$$
$$= \mathbf{4693}\ \boldsymbol{psi}$$

iii) Yield Strength:

Yield strength can be expressed as the ability of a metal to tolerate gradual progressive force without permanent deformation. It can be classified as tensile loading (i.e., pressure) and compressive loading. Axial tension loading results primarily from the weight of the casing string suspended below the joint of interest. Pipe body yield strength is the tension force that causes the pipe body to exceed its elastic limit. API defines the pipe body yield strength as the axial load in the tube, which results in the stress being equal to the material's minimum specific yield strength. For tension design, assume no buoyancy effect and thus pipe-body tensile strength can be expressed as:

$$F_{ten} = \frac{\pi}{4}\sigma_{yield}(d_{no}^2 - d_{ni}^2) \tag{8.19}$$

where

F_{ten} = pipe-body tensile strength, *psi*
d_{no} = nominal OD of pipe, *in*
d_{ni} = nominal ID of pipe, *in*

Equation (8.19) can be written in terms of cross-sectional area as

$$F_{ten} = \sigma_{yield} A_s \tag{8.20}$$

where, $A_s = \frac{\pi}{4}(d_{no}^2 - d_{ni}^2)$.

Example 8.16**:** Two types of 20″ casing are available to be used in a certain well as can be seen in Table 8.5 below. Determine which one of them can be used in deeper operations, and how deep the casing can be lowered. Assume tension safety design factor is 1.6 and the hole is full of 9.1 *ppg* mud.

Solution:

Given data:

Data for the two casings

ρ_m = Mud weight of the fluid in the hole = 9.1 *ppg*
SF_{ten} = Tension safety design factor = 1.6 *ppg*

Given data:

Casing type and difference in depth

To determine which one of the two casing types can be used in deep operations, we can calculate the weight for each one that can carry the load. First buoyancy factor can be calculated as follows:

$$BF = 1 - \frac{\rho_m}{64.5} = 1 - \frac{9.1}{64.5} = 0.859$$

Table 8.5 Casing details for Example 8.10.

Casing no	Grade	OD	ID	wt
		in	***in***	***ppf***
1	K-55	20	18.73	133
2	K-55	20	18.438	163

Minimum yield load can be calculated using E. (8.19):

$$F_{ten} = \frac{\pi}{4}\sigma_{yield}(d_{no}^2 - d_{ni}^2)$$

Minimum yield for casing 1 equal 2,127,730 lb_f, whereas for casing 2 is equal 2,593,549 lb_f. Thus **casing 2** can be used in deeper operations because it can carry more weights than casing 1. The difference in weight is 468,818 lb_f. We can use tension design factor and buoyancy factor to change this weight into length as follows:

$$\Delta F_{max} = \frac{\Delta F_{ten}}{SF_{ten}} = \frac{468{,}818}{1.6} = 293{,}011\ lbf$$

$$L = \frac{\Delta F_{max}}{BFw} = \frac{293{,}011}{0.859 \times 163} = \mathbf{2{,}093}\ \boldsymbol{ft}$$

Thus casing 2 can be lowered around 2,100 *ft* deeper than casing 1.

Example 8.17: A 13 3/8″, 12.515″, C-75, 61.0 *ppf* casing is planned to be run in an intermediate section of 7,000 *ft*. This section will be drilled using 9.8 *ppg* mud. During the design of casing, it is assumed that only shock effect is active and pressure testing shows the casing with pressure equal to 75% of the casing burst pressure. Tension safety factor is required to be 1.6. Determine whether the above casing can satisfy the required tension safety factor.

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 7,000 *ft*
OD_{cas} = Casing outside diameter = 13 3/8″
ID_{cas} = Casing inside diameter = 12.515″
w = Weight of one foot of casing = 61.0 *ppf*
SF_{ten} = Required tension safety factor = 1.6

Required data:

Above casing should pass the required tension safety factor

To determine whether the above casing can pass the required design factor, we should calculate the maximum casing load and the tension on the first casing joint. Maximum casing load or yield strength can be calculated using Eq. (8.19):

$$F_{ten} = \frac{\pi}{4}\sigma_{yield}(d_{no}^2 - d_{ni}^2)$$

$$F = \frac{\pi}{4} \times 75{,}000 \times (13.375^2 - 12.515^2) = 1{,}311{,}540\ lbf$$

Now, we will test the two cases of casing running and casing pressure testing separately:

Running the Casing:
During the casing run, casing is subject to tension and shock load. To calculate the tension at the first casing joint, first we should determine the buoyancy factor as follows:

$$BF = 1 - \frac{\rho_m}{64.5} = 1 - \frac{9.8}{64.5} = 0.848$$

Tension load can be calculated as follows:

$$F_{ten} = L_{cas} \times w \times BF = 7{,}000 \times 61.0 \times 0.848 = 362{,}096\ lbf$$

Shock load can be estimated using Eq. (7.16):

$$F_s = 3{,}200 \times w = 3{,}200 \times 61.0 = 195{,}200\ lbf$$

Total loads while running the casing is equal:

$$F_{run} = F_{ten} + F_s = 362{,}096 + 195{,}200 = 557{,}296\ lbf$$

Thus, safety factor while running the casing is equal to:

$$SF_{runing} = \frac{F}{F_{run}} = \frac{1{,}311{,}540}{557{,}296} = 2.35$$

Pressure Testing the Casing

While pressure testing, the casing is subject to two loads that are tension and pressure test which act as extra loads applied to the first joint of the casing. Tension on the first casing joint is similar to that calculated above which is 362,096 lb_f. To determine the pressure test rating, we should first calculate the burst pressure rating of the casing using Eq. (8.17):

$$P_{br} = f\left[\frac{2\sigma_{yield} t}{d_n}\right] = 0.875 \times \frac{75{,}000 \times \left(\frac{13.375 - 12.515}{2}\right)}{13.375}$$
$$= 4{,}220\ psi$$

Only 75% of the above pressure will be used for pressure testing. Thus, pressure test rating is equal to:

$$P = P_{br} \times 0.75 = 4{,}220 \times 0.75 = 3{,}165\ psi$$

Above pressure can be changed to load as follows:

$$F_p = P \times A_{cas} = 3{,}165 \times \frac{\pi}{4} \times 12.515^2 = 389{,}337\ lbf$$

Table 8.6 Types of casing for Example 8.18.

		Yield	OD	ID	t	Length	Pcr	Pbr
Casing no.	**Grade**	**psi**	**in**	**in**	**in**	**Ft**	**psi**	**psi**
1	J-55	55,000	20	19	0.500	40.0	770	1406
2	J-55	55,000	20	18.542	0.729	40.0	1500	3508

Now, the total loads applied while pressure testing on the casing is equal to:

$$F_{press} = F_{ten} + F_p = 362{,}096 + 389{,}337$$
$$= 751{,}433\ lbf$$

Thus, safety factor while running the casing is equal to:

$$SF_{runing} = \frac{F}{F_{press}} = \frac{1{,}311{,}540}{751{,}433} = 1.75$$

From the above calculations, both safety factors are greater than the required safety factor. **So it is safe to use the above casing.**

***Example 8.18*:** A surface section in a well was planned to be drilled at a depth of 3,000 *ft* using 25″ hole. A 20″ casing has to be run and cemented in this section. There are two casing types available as per Table 8.6 below. From previous experience, it is found that surface casing design was dependent mainly on collapse pressure rather than burst and tension. Design safety factor for collapse was set to be 1.2. Casing has to be cemented to the top using 15.8 *ppg* slurry. Cement slurry will be displaced using 9.0 *ppg* drilling mud. From the above information select the casing type to be used to insure economic selection. It is noted that multi-string selection is permitted.

Solution:

Given data:

Casing information in the table

D_{shoe} = Depth of the casing shoe = 3,000 *ft*
OD_{cas} = Casing outside diameter = 20.0″
SF_c = Collapse safety factor = 1.2
ρ_{cem} = Density of cement slurry = 15.8 *ppg*
SF_b = Density of the drilling mud = 9.0 *ppg*

Required data:

Above casing should pass the required tension safety factor

Selection of the casing string based on the collapse pressure will depend mainly on the expected collapse pressure which is developed due to the difference in mud density between the cement slurry and drilling mud. In this case, multi-string selection will be the optimum decision. So we should determine how many joints of low grade casing to be used. After applying collapse safety factor, maximum allowable collapse pressure for both casing types are listed in Table 8.7.

Table 8.7 Maximum allowable collapse pressure for Example 8.18.

Casing No.	Max. Pc
	psi
1	642
2	1250

Table 8.8 Collapse pressure versus depth for Example 8.18.

Depth	P_c	$P_{cr\_casing\ 1}$	$P_{cr\_casing\ 2}$
ft	*psi*	*psi*	*Psi*
0	0	642	1250
3000	1061	642	1250

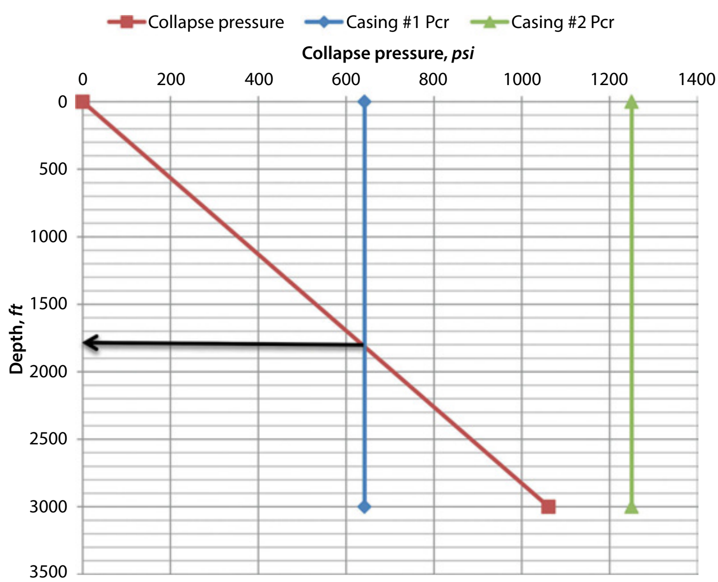

Figure 8.5 Collapse pressure versus depth.

Collapse pressure at any point in the casing can be calculated using the following equation:

$$P_c = 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_m)$$

Summary of collapse pressure versus depth are listed in Table 8.8.

Figure 8.5 shows the collapse pressure versus depth, and also collapse pressure rating for the two casings as described above.

From the above figure, casing 2 can be used for the whole section. However, as it is a high grade casing, it is not economically viable to use alone for the whole section. At depth of 1,800 *ft*, collapse pressure is equal to the collapse pressure rating for casing 1, hence casing 1 cannot be used beyond the depth of 1,800 *ft*. Casing 1 can be run down to the depth of 1,800 *ft*. From 1,800 *ft* to 3,000 *ft*, casing 2 is suggested to use. Thus, 30 joints for casing 2 will be run first in the hole and then 45 joints of casing 1 will be run accordingly.

Example 8.19: Compute the body-yield strength for 20-*in*., K-55 casing with a nominal wall thickness of 0.64 in. and a nominal weight per foot of 133 lb_f/ft.

Solution:

Given data:

d_n = nominal OD of pipe = 20 *in*
σ_{yield} = minimum yield strength of pipe (k-55) = 55,000 *psi*
t = nominal wall thickness of pipe = 0.64 *in*
W_n = nominal weight of pipe = 133 lb_f/ft

Required data:

F_{ten} = body-yield strength in *psi*

This pipe has minimum yield strength of 55,000 psi and an ID of (**K55)**

$$d = 20.00 - 2(0.64) = 18.72\ in$$

Thus, the cross-sectional area of steel can be calculated using the sub-equation of Eq. (8.20) as

$$A_s = \frac{\pi}{4}(20^2 - 18.72^2) = 38.93\ in^2$$

Now, a minimum pipe-body yield is predicted by Eq. 8.20 at an axial force of:

$$F_{ten} = \sigma_{yield} A_s = 55{,}000(38.93) = \mathbf{2{,}140{,}907.43\, lb_f}$$

Example 8.20: Compute the body-yield strength for 18-*in*., K-55 casing with a nominal wall thickness of 0.64 *in*. and a nominal weight per foot of 153 lb_f/ft.

Solution:

Given data:

d_n = nominal diameter of the casing pipe = 18 *in*
t = thickness of the casing pipe = 0.64 *in*
W_n = nominal weight per foot of the pipe = 135 lb_f/ft
σ_{yield} = minimum yield strength of pipe (k-55) = 55,000 *psi*

Required data:

F_{tem} = Body yield strength in *psi*

This pipe has minimum yield strength of 55,000 *psi* and an ID of (K55)

$$d = 18.00 - 2(0.64) = 16.72\ in$$

Thus, the cross-sectional area of steel can be calculated using the sub-equation of Eq. (8.20) as,

$$A_s = \frac{\pi}{4}(18^2 - 16.72^2) = 34.90\ in^2$$

Now, a minimum pipe-body yield is predicted by Eq. (8.20) at an axial force of:

$$F_{tem} = \sigma_{yield} A_s = 55{,}000(34.90) = \mathbf{1{,}919{,}500}\ \boldsymbol{lbf}$$

iv) Biaxial and Triaxial Loading:
The reduced equivalent yield strength is based on von Mises theory. The equivalent yield-strength formula is give as:

$$\sigma_a = \frac{Bouyant\ weight\ carried\ by\ weakest\ grade}{\frac{\pi}{4}(d_o^2 - d_i^2)} \tag{8.21}$$

$$\sigma_{pa} = \left[\sqrt{1 - 0.75\left(\frac{\sigma_a}{\sigma_{yield}}\right)^2} - 0.5\left(\frac{\sigma_a}{\sigma_{yield}}\right)\right]\sigma_{yield} \tag{8.22}$$

where:

σ_a = the axial stress due to tension, *psi*
σ_{pa} = the equivalent yield strength, *psi*
d_o = the outer casing diameter, *in*
d_i = the inner casing diameter, *in*

Biaxial effect is calculated using the following set of equations:

$$F = \frac{\left[46.95\times 10^6\left(\frac{3B/A}{2+B/A}\right)^3\right]}{\left[Y_{pa}\left(\frac{3B/A}{2+B/A} - \frac{B}{A}\right)\left(1 - \frac{3B/A}{2+B/A}\right)^2\right]} \tag{8.23}$$

$$G = \frac{F \times B}{A}$$

where A, B, C, F and G are empirical constants

$$A = 2.8762 + 0.10679\times 10^{-5}\sigma_{pa} + 0.21301\times 10^{-10}\sigma_{pa}^2 - 0.53132\times 10^{-16}\sigma_{pa}^3$$

$$B = 0.026233 + 0.50609 \times 10^{-6}\sigma_{pa}$$

$$C = -465.93 + 0.030867\sigma_{pa} - 0.10483 \times 10^{-7}\sigma_{pa}^2 + 0.36989 \times 10^{-13}\sigma_{pa}^3$$

Example 8.21: Determine the collapse strength for a 5 1/2″ O.D., 14.00 lb_f/ft, J-55 casing under axial load of 100,000 lb_f.

Solution:

Given data:

F_{ab} = equivalent axial force, lb_f = 100,000 lb_f
d_o = the outer casing diameter = 5.5 *in*
d_i = the inner casing diameter = 5.012 *in*
σ_{yield} = the minimum yield strength of the grade
= 55,000 *psi* (Grade J-55)

Required data:

σ_{pa} = the equivalent yield strength, *psi*

The axial tension will reduce the collapse pressure using Eq. (8.21) and Eq. (8.22) as:

$$\sigma_a = \frac{\textit{Bouyant weight carried by weakest grade}}{\frac{\pi}{4}(d_o^2 - d_i^2)}$$

$$= \frac{100{,}000\ lb_f}{\frac{\pi}{4}(5.5^2 - 5.012^2)in^2} = 24{,}820\ psi$$

$$\sigma_{pa} = \left[\sqrt{1 - 0.75\left(\frac{\sigma_a}{\sigma_{yield}}\right)^2} - 0.5\left(\frac{\sigma_a}{\sigma_{yield}}\right)\right]\sigma_{yield}$$

$$= \left[\sqrt{1 - 0.75\left(\frac{24{,}820}{55{,}000}\right)^2} - 0.5\left(\frac{24{,}820}{55{,}000}\right)\right]55{,}000$$

$$= \mathbf{38{,}216}\ \boldsymbol{psi}$$

Here the axial load decreased the J-55 rating to an equivalent "J-38.2" rating

Example 8.22: Determine the collapse strength for a 4 1/2″ O.D., 13.00 lb_f/ft, casing under axial load of 175,000 lb_f

Solution:

Given data:

F_{ab} = equivalent axial force = 175,000 lb_f

d_o = outer diameter of casing = 4.5 *in*

d_i = inner diameter of casing = 3.958 *in*

σ_{yield} = the minimum yield strength of the grade = 55,000 *psi*

Required data:

σ_{pa} = the equivalent yield strength, *psi*

The axial tension will reduce the collapse pressure using Eq. (8.21) and Eq. (8.22) as:

$$\sigma_a = \frac{Bouyant\ weight\ carried\ by\ weakest\ grade}{\frac{\pi}{4}(d_o^2 - d_i^2)}$$

$$= \frac{175{,}000\ lbf}{\frac{\pi}{4}(4.5^2 - 3.958^2)} = 48{,}605.028\ psi$$

$$\sigma_{pa} = \left[\sqrt{1 - 0.75\left(\frac{\sigma_a}{\sigma_{yield}}\right)^2} - 0.5\left(\frac{\sigma_a}{\sigma_{yield}}\right)\right]\sigma_{yield}$$

$$\sigma_{pa} = \left[\sqrt{1 - 0.75\left(\frac{48{,}605.028}{55{,}000}\right)^2} - 0.5\left(\frac{48{,}605.028}{55{,}000}\right)\right]55{,}000$$

$$= \mathbf{11{,}097\ psi}$$

Example 8.23: Determine the collapse strength for a 5 1/2″ O.D., 14.00 lb_f/ft, casing under axial load of 100,000 lb_f.

Solution:

Given data:

F_{ab} = equivalent axial force = 100,000 lb_f

d_o = outer diameter of casing = 5.5 *in*

d_i = inner diameter of casing = 5.012 *in*

σ_{yield} = the minimum yield strength of the grade = 55,000 *psi*

Required data:

σ_{pa} = the equivalent yield strength, *psi*

The axial tension will reduce the collapse pressure using Eq. (8.21) and Eq. (8.22) as:

$$\sigma_a = \frac{Bouyant\ weight\ carried\ by\ weakest\ grade}{\frac{\pi}{4}(d_o^2 - d_i^2)} = \frac{100{,}000\ lbf}{\frac{\pi}{4}(5.5^2 - 5.012^2)}$$

$$= 24{,}820\ psi$$

$$\sigma_{pa} = \left[\sqrt{1 - 0.75\left(\frac{\sigma_a}{\sigma_{yield}}\right)^2} - 0.5\left(\frac{\sigma_a}{\sigma_{yield}}\right) \right] \sigma_{yield}$$

$$\sigma_{pa} = \left[\sqrt{1 - 0.75\left(\frac{24{,}820}{55{,}000}\right)^2} - 0.5\left(\frac{24{,}820}{55{,}000}\right) \right] 55{,}000 = \mathbf{38{,}216}\ \boldsymbol{psi}$$

v) Effect of Bending:

In sections of the hole where there are severe doglegs (sharp bends) the bending stresses should be checked. The most critical sections are where dogleg severity exceeds 10° per 100'. So, stress can be expressed as:

$$\sigma_b = \pm \frac{1}{2} E d_n K_{ds} \tag{8.24}$$

where

σ_b = bending stress
E = Young modulus of elasticity
d_n = normal OD of pipe
K_{ds} = dogleg severity

In oilfield units where the dogleg severity, K_{ds}, is expressed as the change in angle in degrees per 100 *ft* of borehole length, and the pipe is assumed to be steel, the simplified form of Eq. (8.24) can be written as:

$$\sigma_b = \pm 218 d_n K_{ds} \tag{8.25}$$

In terms of an equivalent axial force, F_{ab} Eq. (8.25) can be expressed as:

$$F_{ab} = \pm \sigma_b A_s = \pm 218 d_n K_{ds} A_s \tag{8.26}$$

The area of steel, A_s can be expressed as the weight per feet of pipe divided by the density of steel. If we apply field unit, Eq. (8.26) becomes as:

$$F_{ab} = \pm 64\ d_n K_{ds} W_{dp} \tag{8.27}$$

where

F_{ab} = equivalent axial force, lb_f
d_n = normal OD of pipe, *in*
K_{ds} = dogleg severity, degrees/100 *ft*
W_{dp} = weight per foot of drillpipe in air, lb_f/ft

When the axial tension strength (F_{er}) divided by the cross-sectional area of the pipe wall under last perfect thread is greater than the minimum yield strength, the joint strength is given by:

$$F_{cr} = 0.95 A_{jp} \left\{ \sigma_{ult} - \left[\frac{140 K_{ds} d_n}{(\sigma_{ult} - \sigma_{yield})^{0.8}} \right]^5 \right\} \tag{8.28}$$

Here

σ_{ult} = ultimate strength, *psi*

$$\frac{F_{cr}}{A_{jp}} \geq \sigma_{yield},\ K_{ds} \text{ is in degrees/100 ft, and}$$

$$A_{jp} = \frac{\pi}{4}[(d_n - 0.1425)^2 - (d_n - 2t)^2]$$

When the axial tension strength divided by the cross-sectional area of the pipe wall under last perfect thread is less than the minimum yield strength, the joint strength is given by:

$$F_{cr} = 0.95 A_{jp} \left\{ \frac{\sigma_{ult} - \sigma_{yield}}{0.644} + \sigma_{yield} - 218.15\ K_{ds} d_n \right\} \tag{8.29}$$

It was developed from the experimental tests conducted with 5.5″, 17-lb_f/ft, K-55 casing with short round-thread coupling (STC)

Example 8.24: Determine the maximum axial stress for a 5 1/2″ O.D., 14.00 lb_f/ft, J-55 casing under axial load of 100,000 lb_f axial-tension load in a portion of a directional wellbore having a dogleg severity of 4°/100’. Compute the maximum axial stress assuming uniform contact between the casing and the borehole wall.

<u>Solution</u>:

Given data:

F_{ab} = equivalent axial force = 100,000 lb_f

d_{no} = outer diameter of casing = 5.5 *in*

d_{ni} = inner diameter of casing = 5.012 *in*

σ_{yield} = the minimum yield strength of the grade = 55,000 *psi*

Required data:

$\sigma_{pa\text{-}max}$ = the maximum axial stress, *psi*

The axial stress without bending can be calculated using Eq. (8.21):

$$\sigma_a = \frac{Bouyant\ weight\ carried\ by\ weakest\ grade}{\frac{\pi}{4}(d_{no}{}^2 - d_{ni}{}^2)}$$

$$= \frac{100{,}000\ lbf}{\frac{\pi}{4}(5.5^2 - 5.012^2)} = 24{,}820\ psi$$

The additional stress level on the convex side of the pipe caused by bending can be computed using Eq. (8.25) as:

$$\sigma_b = 218 d_n K_{ds} = 218 \times 5.5 \times 4 = 4{,}796\ psi$$

So, the total maximum axial stress will be:

$$\sigma_{pa-max} = \sigma_a + \sigma_b = 24{,}820 + 4{,}796 = \mathbf{29{,}616}\ \boldsymbol{psi}$$

Example 8.25: Determine the maximum axial stress for a 5 1/2″ O.D., 14.00 lb_f/ft, J-55 casing under axial load of 100,000 lb_f axial-tension load in a portion of a directional wellbore having a dogleg severity of 4°/100'. Compute the maximum axial stress assuming uniform contact between the casing and the borehole wall.

Solution:

Given data:

F_{ab} = equivalent axial force, lb_f = 100,000 lb_f
d_{no} = the nominal outer casing diameter = 5.5 *in*
d_{ni} = the nominal inner casing diameter = 5.012 *in*
σ_{yield} = the minimum yield strength of the grade = 55,000 *psi* (Grade J-55)

Required data:

$\sigma_{pa\text{-}max}$ = the maximum axial stress, *psi*

The axial stress without bending can be calculated using Eq. (8.21):

$$\sigma_a = \frac{Bouyant\ weight\ carried\ by\ weakest\ grade}{\frac{\pi}{4}(d_{no}^2 - d_{ni}^2)}$$

$$= \frac{100{,}000\ lb_f}{\frac{\pi}{4}(5.5^2 - 5.012^2)in^2} = 24{,}820\ psi$$

The additional stress level on the convex side of the pipe caused by bending can be computed using Eq. (8.25) as:

$$\sigma_b = 218 d_n K_{ds} = 218 \times 5.5 \times 4 = 4{,}796\ psi$$

So, the total maximum axial stress will be:

$$\sigma_{pa-max} = \sigma_a + \sigma_b = 24{,}820\ psi + 4{,}796\ psi = \mathbf{29{,}616}\ \boldsymbol{psi}$$

vi) Torsion:

For most casing strings, torque is seldom applied, and when it must be applied, it is limited to the connection makeup torque M_t. The torsional shear stress τ acting in the circumferential direction at a radius at some point in the pipe-body wall thickness is

$$\tau = \frac{M_t r}{J_p} \tag{8.30}$$

$$J_p = \frac{\pi t^4}{2}\left(\frac{d_n}{t} - 1\right)\left[\left(\frac{d_n}{t} - 1\right)^2 + 1\right] \tag{8.31}$$

where

τ = shear stress, *psi*

M_t = makeup torque,

J_p = polar moment of inertia

If we include internal and external pressures, axial force, bending, and torsion, the von Mises equivalent stress equation for torsion can be written as:

$$\sigma_{vm} = \sqrt{\frac{(\sigma_r - \sigma_t)^2 + [\sigma_t - (\sigma_a \pm \sigma_b)]^2 + [(\sigma_a \pm \sigma_b) - \sigma_r]^2 + 6\tau^2}{2}} \tag{8.32}$$

where

σ_{vm} = von Mises triaxial equivalent stress, *psi*

σ_a = total axial stress, not including bending due to hole deviation, doglegs, or buckling, *psi*

σ_b = bending stress, *psi*

σ_r = radial stress, *psi*

σ_t = tangential stress, *psi*

8.3 Multiple Choice Questions

1. For onshore drilling, the well is drilled in sections from ______________.
   a) Surface
   b) Sub-surface
   c) Seabed
   d) None of the above
2. For offshore drilling, the well is drilled in sections from ______________.
   a) Seabed
   b) Surface
   c) Sub-surface
   d) None of the above
3. The well is drilled in sections through all of the formations to the target depth because of ______________ limitations.
   a) Technological
   b) Manpower
   c) Resources
   d) None of the above
4. Once a certain length of hole is drilled, it has to be ______________.
   a) Cased
   b) Abandoned
   c) Shutoff
   d) None of the above

5. A heavy large diameter steel pipe which can be lowered into the well for some specific functions is called ______________.
   a) Casing
   b) Drill pipe
   c) Drill collars
   d) None of the above

6. ______________ ensures a pressure-tight connection from the surface to the oil or gas reservoir.
   a) Drilling mud
   b) Casing
   c) Drill pipe
   d) None of the above

7. Casing is rigidly connected to the rocky formation using ______________.
   a) Cement slurry
   b) Drilling fluid
   c) Hangers
   d) None of the above

8. The costs of the casing can constitute ______________ of the total cost of the well.
   a) 20–30%
   b) 10–20%
   c) 15–25%
   d) None of the above

9. The functions, and types or names of the various casings vary according to the ______________.
   a) Depth
   b) Pressure
   c) Formation
   d) None of the above

10. Marine riser is a special type of ______________ used in offshore drilling.
    a) Casing pipe
    b) Drill pipe
    c) Drill collars
    d) None of the above

11. The first casing used in the wellbore is called ______________
    a) Surface casing
    b) Production casing
    c) Conductor casing
    d) None of the above

12. ______________ is the largest and the uppermost casing used in wellbore.
    a) Surface casing
    b) Production casing
    c) Conductor casing
    d) None of the above

13. The final casing installed in the wellbore is called ______________.
    a) Production casing
    b) Intermediate casing
    c) Surface casing
    d) None of the above

14. The size of the ______________ varies according to the geographical locations.
    a) Production casing
    b) Conductor pipe
    c) Intermediate casing
    d) None of the above

15. ______________ is always cemented to surface.
    a) Conductor pipe
    b) Production casing
    c) Intermediate casing
    d) None of the above

16. ______________ is set at approximately 1,000–1,500 *ft* below the ground level or seabed.
    a) Production casing
    b) Conductor pipe
    c) Surface casing
    d) None of the above

17. ______________ are connected to the top of surface casing strings.
    a) Drill line
    b) Mud pumps
    c) Drill pipe
    d) BOPs

18. The length of the surface casing is normally ______________.
    a) 1,000–2,000 *ft*
    b) 300–5,000 *ft*
    c) 1,500–3,000 *ft*
    d) None of the above

19. ______________ is the first casing on which the BOPs are mounted.
    a) Production casing
    b) Surface casing
    c) Intermediate casing
    d) None of the above

20. ______________ is also called protection casing.
    a) Surface casing
    b) Intermediate casing
    c) Production casing
    d) None of the above

21. ______________ depends on well depth and geology in a specific area.
    a) Production casing
    b) Surface casing
    c) Intermediate casing
    d) None of the above

22. ______________ is usually set in the transition zone below or above an over-pressured zone.
    a) Surface casing
    b) Intermediate casing
    c) Production casing
    d) None of the above

23. The casing depth of the ______________ depends on the pore pressure profile of the underground fluids.
    a) Surface casing
    b) Production casing
    c) Intermediate columns
    d) None of the above

24. ______________ is the last casing string placed in the hole.
    a) Surface casing
    b) Intermediate casing
    c) Production casing
    d) None of the above

25. ______________ reaches the top of the pay formation.
    a) Surface casing
    b) Production casing
    c) Intermediate casing
    d) None of the above

26. ______________ can be used to produce fluid instead of tubing.
    a) Surface casing
    b) Intermediate casing
    c) Production casing
    d) None of the above

27. ______________ are hung on the intermediate casing by using a liner-hanger.
    a) Production casing
    b) Surface casing
    c) Liners
    d) None of the above

28. ______________ is a mechanism that locks into the casing head.
    a) Casing hanger
    b) Casing liner
    c) Casing pipe
    d) All of the above

29. ______________ is attached to the top of the casing which allows the casing to be suspended from the wellhead.
    a) Casing pipe
    b) Surface casing
    c) Casing hanger
    d) All of the above
30. The ______________ diameter of casing is recognized as the casing size.
    a) Inside
    b) Outside
    c) Average
    d) None of the above
31. What is the first step during handling of casing in the rig site?
    a) Proper protection
    b) Coupling check
    c) Pipe stacking
    d) None of the above
32. One method of landing casing in the casing hanger is that casing tension ______________
    a) should be similar to that applied to the casing during cementing
    b) should be less than that applied to the casing during cementing
    c) should be greater than that applied to the casing during cementing
    d) none of the above
33. All of the following factors are affecting casing design except:
    a) Stress analysis
    b) Strength of casing seat
    c) Availability of casing
    d) Landing procedure
34. Casing is subjected to many forces, which of the following is not one of these forces?
    a) Collapse force
    b) Shear force
    c) Burst force
    d) Tension force
35. Casing design process includes the following except
    a) Selection of setting depth
    b) Selection of casing sizes
    c) Defining design properties
    d) Selection of formation depth
36. Selection of casing setting depth depends on mainly ______________
    a) Formation fracture pressure
    b) Type of formation fluid
    c) Type of drill bits
    d) Casing size

37. If a 17½″ hole was drilled in a well, which of the following casing size will be run?
    a) 17 ½″ casing
    b) 17″ casing
    c) 13 3/8″ casing
    d) 18″ casing
38. If the required casing collapse pressure is 11,000 *psi*, what should be the design collapse pressure?
    a) 10,000 *psi*
    b) 12,375 *psi*
    c) 11,000 *psi*
    d) None of the above
39. If the designed casing burst is 15,000 *psi*, what is the required burst of the casing?
    a) 15,000 *psi*
    b) 16,500 *psi*
    c) 17,500 *psi*
    d) 13,650 *psi*
40. If the required casing tension is 150,000 lb_f, what should be the design casing tension?
    a) 300,000 lb_f
    b) 150,000 lb_f
    c) 75,000 lb_f
    d) 500,000 lb_f

Answers: 1a, 2a, 3a, 4a, 5a, 6b, 7a, 8a, 9a, 10a, 11c, 12c, 13a, 14b, 15a, 16c, 17d, 18b, 19b, 20b, 21c, 22b, 23c, 24c, 25b, 26c, 27c, 28a, 29c, 30b, 31c, 32a, 33d, 34b, 35d, 36a, 37c, 38b, 39d, 40a.

8.4 Summary

Casing technology is one of the important pillars of the oil industry operations. It is very important to give it a higher attention during the well construction. Failure to select the optimum casing will lead to losing the well. Therefore, the workout examples and MCQs are designed to cover the fundamental aspects of casing and its design criteria and selection procedure. To have a self-practice, different exercises and MCQs are set. The solutions of the exercises and MCQs are outlined in Appendix A and B, respectively.

8.5 Exercise and MCQs for Practice

8.5.1 Exercises (Solutions are in Appendix A)

Exercise 8.1: A production casing is planned to be set at a depth of 14,000 *ft*. the circulated mud weight in the annulus is 12.5 *ppg*. If the collapse rating of the casing is 9,875 *psi* and the minimum collapse safety factor is 1.25, calculate the casing depth which can be safely run inside the well without filling inside the casing with any fluid. If the collapse safety factor was estimated to be 1.55 and when the casing was at the bottom and full of a certain mud, what is the mud weight of the fluid inside the casing? ***Answers: 12,154 ft, 8.83 ppg***

Exercise 8.2: A production casing was planned to run in the production hole of a well with casing shoe at 15,000 *ft*. When casing was run empty to the mid depth, collapse safety factor was calculated to be 1.9. When the casing was at the bottom and full of a certain mud, safety factor of the collapse was calculated to be 2.75. If the mud weight of the fluid in the annulus was 14.0 *ppg*, determine the casing collapse rating and mud weight of the fluid inside the casing. ***Answers: 10,374 psi, 9.16 ppg***

Exercise 8.3: A surface section of 26.0″ size was planned to be drilled to a depth of 1,750 *ft*. A 20.0″ OD casing is to be set and cemented in this section. There are four casing types available in stocks which are shown in the table below.

Casing No.	**OD** ***in***	**Grade**	**ID** ***in***	**Yield** ***psi***
1	20.0	K-55	19.124	55,000
2			19.000	55,000
3			18.750	55,000
4			18.376	55,000

Maximum burst pressure has to be assumed when casing is full of 15.8 *ppg* cement slurry and annulus is empty. In addition, maximum collapse pressure has to be assumed when casing is full of fresh water and annulus is full of 15.8 *ppg* cement slurry. Safety factor for burst and collapse was designed to be 1.3 and 1.2, respectively. Which of the above casings should be used in this section? ***Answer: Casing #2***

Exercise 8.4: A production casing string is set at a depth of 18,750 *ft*. A cement slurry of 16.2 *ppg* was pumped and filled the casing from the top to the casing shoe while the annulus has 13.5 *ppg* drilling mud. The cement has been displaced by 12.0 *ppg* drilling mud until cement reached the top of the well. Burst pressure rating of the casing is 8,160 *psi*. If the burst and collapse safety factors during pumping the cement were equal, determine the collapse and burst rating of the casing. Also calculate the collapse pressure rating. ***Answers: 3.1, 12,695 psi***

Exercise 8.5: A production casing string is set at a depth of 13,000 *ft*. Cement slurry of 15.6 *ppg* was pumped and displaced by 10.5 *ppg* drilling mud. Mud weight in the annulus was 14.0 *ppg*. If the collapse pressure in the casing shoe is equal to the burst pressure at casing shoe when the cement was in the bottom of the casing, what was the cement column inside the casing before displacing it in the annulus? ***Answer: 8,928 ft***

Exercise 8.6: Two types of 5.5″ casing are available to be used in a certain well as can be seen in the table below:

Casing No.	Grade	OD	ID	Wt
		in	in	ppf
1	C-75	5.5	4.950	15.5
2	K-55	5.5	4.892	17.0

Determine which one of them can be used in deeper operations, and how deep the casing can be lowered? Assume design safety factor for tension is 2.0 and the hole is full of 13.2 *ppg* mud. ***Answer: number 2, and 1,244 ft.***

Exercise 8.7: A 9 5/8″, 8.755″, N-80, 43.5 ppf casing is planned to be run in a production section of 10,000 *ft*. This section will be drilled using 11.6 *ppg* mud. During the casing design, it was assumed only shock effect. It was also assumed that during pressure testing, the casing pressure was equal to 75% of the casing burst pressure. Tension safety factor is required to be 1.8. Determine whether the above casing can satisfy the required tension safety factor or not. ***Answer: doesn't meet the requirement***

Exercise 8.8: A production section in a well was planned to be drilled to a depth of 15,000 *ft* using 12.25″ hole size. 9 5/8″ casing has to be run and cemented in this section. There are two casing types available as per the specifications shown in the below table.

Casing No.	Grade	Yield	OD	ID	t	Length	P_{cr}	P_{br}
		psi	in	in	in	Ft	psi	psi
1	K-55	55,000	9.625	8.535	0.545	40.0	5450	5130
2	C-75	75,000	9.625	8.279	0.673	40.0	9177	7570

From previous experiences it is found that production casing design depends mainly on collapse pressure. Design safety factor for collapse was set to be 1.15. Casing has to be cemented to the top using 16.4 *ppg* slurry. Cement slurry will be displaced using fresh water. From the above information how many joints of each casing should be used to insure as an economic selection? ***Answers: 280 casing 1 & 95 casing 2***

8.5.2 Exercise (Self Practices)

E8.1: A well is being planned to drill where well completion requires the use of 7-*in*. production casing set at 12,000 *ft*. Determine the number of casing strings needed to reach this depth safely, and select the casing setting depth of each string. Pore pressure, fracture gradient, and lithology data from logs of nearby wells are given in Figure 8.18. Allow a 0.5 lb_m/gal trip margin, and a 0.5 lb_m/gal kick margin when making the casing-seat selections. The minimum length of surface casing required to protect the freshwater aquifers is 3,000 *ft*. Approximately 150 *ft* of conductor casing generally is required to prevent washout on the outside of the conductor. It is general practice in this area to cement the casing in shale rather than in sandstone.

E8.2: Compute the API collapse-pressure rating for 20-*in*, K-55 casing with a nominal wall thickness of 0.635 *in*. and a nominal weight per foot of 133 lb_f/ft.

E8.3: Compute the burst requirement if the pore pressure is 8,000 *psi* if the factor of safety is assumed as 1.3.

E8.4: Compute the API burst resistance for 22-in, 133-lb_f/ft, K-55 casing with a nominal wall thickness of 0.65 *in*. Use Barlow and Lame models.

E8.5: Compute the body-yield strength for 21-*in*., K-55 casing with a nominal wall thickness of 0.61 *in*. and a nominal weight per foot of 120 lb_f/ft.

E8.6: Determine the collapse strength for a 6 1/2″ O.D., 21.00 lb_f/ft, J-55 casing under axial load of 200,000 lb_f.

E8.7: Determine the maximum axial stress for a 5 1/2″ O.D., 14.00 lb_f/ft, J-55 casing under axial load of 300,000 lb_f axial-tension load in a portion of a directional wellbore having a dogleg severity of 5°/100'. Compute the maximum axial stress assuming uniform contact between the casing and the borehole wall.

8.5.3 MCQs (Self-Practices)

1. What is the worst condition for tension when designing the casing?
   a) Casing is full of liquid and heavier than that of annulus
   b) Casing is empty while annulus is full of fresh water
   c) No liquid inside the well
   d) Casing is full of liquid while annulus is empty
2. What is the worst condition for collapse when designing the casing?
   a) No liquid inside the well
   b) Casing is empty and annulus if full of heavier liquid
   c) Casing is empty and annulus if full of lighter liquid
   d) Casing is full of liquid while annulus is empty
3. What is the worst condition for burst when designing the casing?
   a) Casing is full of heavier liquid while annulus is empty
   b) Casing is full of natural gas while annulus is empty
   c) Casing is full of lighter liquid while annulus is empty
   d) Casing is empty while annulus is full of lighter liquid
4. Axial stress of a casing without bending is ________ the axial stress with bending.
   a) Greater than
   b) Similar to
   c) Less than
   d) None of the above

5. The design safety factors during casing design are ____________
   a) Always less than 1.0
   b) Always greater than 1.0
   c) Always equal to 1.0
   d) Any of the above
6. Sizes of the casing strings depend on mainly____________
   a) Well depth
   b) Well type
   c) Expected pore pressures
   d) Well size
7. How liner is installed in the well?
   a) It is hanged in the well head
   b) It is hanged in the surface casing
   c) It is hanged at the top of the last casing
   d) It is hanged at the bottom of the last casing
8. Which of the following casing strings should have the highest burst rating?
   a) Conductor casing
   b) Surface casing
   c) Production casing
   d) Liner
9. Which of the following casing strings should have the lowest collapse rating?
   a) Liner
   b) Surface casing
   c) Production casing
   d) Intermediate casing
10. Which of the following casing strings should have the highest tension rating?
    a) Liner
    b) Production casing
    c) Surface casing
    d) None of the above
11. An oil well has a total depth of 1,000 *ft*, how many casing strings should be used?
    a) One casing string
    b) Three casing strings
    c) Five casing strings
    d) Seven casing strings
12. A gas well has a total depth of 20,000 *ft*, how many casing strings should be used?
    a) Five casing strings
    b) Three casing strings
    c) Two casing strings
    d) One casing string

13. The shortest casing joint used for space out to set the casing in the hanger is called
    a) Tool joint
    b) Short joint
    c) Pup joint
    d) None of the above
14. The word "Casing" while drilling technology refers to the operation where
    a) hole is cased just after complete drilling
    b) casing is used in drilling instead of drill pipes
    c) casing is run while the well is drilled using drill pipes
    d) all of the above
15. Which of the following is the application of expandable casing technology?
    a) Gas injector wells
    b) Steam injector wells
    c) High pressure gas wells
    d) Casing repairs
16. Which one of the following casing grades can be chosen as surface casing?
    a) C-90
    b) K-55
    c) P-110
    d) All of the above
17. If a casing string of maximum yield strength of 130,000 *psi* is used, which of the following grades should be chosen?
    a) P-110
    b) Q-125
    c) V-150
    d) Any of the above grade
18. The casing accessory which is responsible for preventing cement from flowing back inside the casing is the ______________
    a) Casing head valve
    b) Casing shoe valve
    c) Casing hanger
    d) None of the above
19. Selection of casing weight will depend on ______________
    a) Type of drilled formation
    b) Type of formation fluid
    c) Expected pressures
    d) All of the above
20. Which joint of the casing string is subjected to the highest tension force?
    a) Last casing joint
    b) First casing joint
    c) All the casing string
    d) All of the above

21. The main purpose/s of casing the well is to
    a) Support walls of the borehole
    b) Prevent migration of fluids between layers
    c) Eliminates loss circulation issues
    d) All of the above
22. Failure of casing design can lead to ______________
    a) Loss of the well
    b) Loss of life
    c) Expensive repair
    d) All of the above
23. Number of casing strings in the well will depend on mainly______________
    a) Well budget
    b) Well depth
    c) Well type
    d) Well size
24. Which of the following casing type is used only in off-shore operations as initial casing?
    a) Stove pipe
    b) Conductor casing
    c) Liner
    d) None of the above
25. Riser string is connected between
    a) Subsea BOP and well head
    b) Surface BOP and subsea control valve
    c) Surface BOP and subsea BOP
    d) None of the above
26. Which one of the following casing strings has the largest diameter?
    a) Conductor casing
    b) Surface casing
    c) Intermediate casing
    d) Production casing
27. Which one of the following casing strings has the smallest diameter?
    a) Conductor casing
    b) Surface casing
    c) Intermediate casing
    d) Production casing
28. Which one of the following casing strings has the shortest length?
    a) Conductor casing
    b) Surface casing
    c) Intermediate casing
    d) Production casing

29. All of the followings are the purposes of conductor casing except
    a) Protect surface formations from erosion by mud circulation
    b) Provide protection against shallow gasses
    c) Isolate shallow gas formations
    d) Provide support for the BOP
30. Which of the following is the main purpose of conductor casing?
    a) Prevent well kicks
    b) Provide support to the well head
    c) Protect tubulars from corrosive fluids
    d) All of the above
31. What is the main purpose of surface casing?
    a) Provide support to the well head
    b) Prevent fresh water aquifers from contaminations
    c) Used as production casing to produce shallow zones
    d) a and b
32. Depth of surface casing depends mainly on the
    a) Presence of fresh water aquifers
    b) Presence of troublesome formations
    c) Thief zones
    d) All of the above
33. Which of the following is not the purpose of intermediate casing?
    a) Seal off troublesome formations
    b) Protect shallow water zones from contamination
    c) Contain abnormal pressure formations
    d) All of the above
34. The primary function of production casing is to
    a) Provide isolation for producing zones
    b) Seal off fresh water formations
    c) Seal of troublesome formations
    d) All of the above
35. What is the main advantage of using liners?
    a) Can be used to produce hydrocarbons
    b) Can be used to isolate troublesome formations
    c) Reduce the well cost
    d) All of the above
36. Which of the following casing strings can be ignored when the well is dry?
    a) Surface casing
    b) Production casing
    c) Intermediate casing
    d) Stove pipe

37. The size of the casing is referred to the
    a) Outer diameter of the casing
    b) Inner diameter of the casing
    c) Average between inner and outer diameters
    d) None of the above

38. The length of the casing is measured from the ______________ to the ______________.
    a) Top of the collar – top of the other collar
    b) Uppermost thread – uppermost thread
    c) Bottom of the collar – uppermost thread
    d) Uppermost thread – top of the collar

39. Which of the following casing grades has the highest strength?
    a) J-55
    b) S-95
    c) Q-125
    d) L-80

40. Which of the following casing grades is welded type?
    a) N-80
    b) H-40
    c) T-95
    d) V-150

8.6 Nomenclature

d_o = the outer casing diameter, *in*
d_i = the inner casing diameter, *in*
d_m = maximum pipe body ID based on minimum specific wall thickness, *in*
d_n = nominal OD of pipe, *in*
d_{ni} = nominal ID of pipe, *in*
d_{no} = nominal OD of pipe, *in*
E = Young modulus of elasticity
J_p = polar moment of inertia
K_{ds} = dogleg severity, degrees/100 *ft*
f = wall-thickness correction factor = 0.875 for standard API tubulars when a 12.5% wall-thickness tolerance is specified.
F_1, F_2, F_3, F_4, and F_5 = Five factors used in Eqs. (8.3–8.7)
F_{ab} = equivalent axial force, lb_f
F_{ten} = pipe-body tensile strength, *psi*
M_t = makeup torque,
P_{br} = burst pressure rating, *psi*
P_e = external pressure, *psi*
P_i = Internal pressure, *psi*
P_{cr} = collapse pressure rating, *psi*

P_{eq} = external pressure equivalent in collapse due to external and internal pressure
t = thickness, *in*
W_{dp} = weight per foot of drillpipe in air, lb_f/ft
σ_a = total axial stress, not including bending due to hole deviation, doglegs, or buckling, *psi*
σ_b = bending stress, *psi*
σ_r = radial stress, *psi*
σ_t = tangential stress, *psi*
σ_{pa} = equivalent yield strength, *psi*
σ_{ult} = ultimate strength, *psi*
σ_{yield} = the minimum yield stress or strength of pipe, *psi*
σ_{vm} = von Mises triaxial equivalent stress, *psi*
τ = shear stress, *psi*

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9

Cementing

9.1 Introduction

Oil well cement (OWC) is a powdery substance made of limestone and clay. Most cement used in the oil industry is a type of Portland cement. In the construction industry, cements are mixed with sand, gravel, and water to form concrete. In the oil industry, cements are mixed with water and special additives to form slurry, which is then pumped into the well. The slurry solidifies when it reaches the targeted place. The appropriate cement slurry design for well cementing is a function of many parameters, including the wellbore geometry, casing equipment, formation integrity, and drilling mud properties. Sets of multiple choice question (MCQs) related to cementing are included which are related to the drilling fluid technology and their problems and solutions. Workout examples related to mud engineering are extensively covered. Workout examples and MCQs are based on the writer's textbook, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

9.2 Different Mathematical Formulas and Examples

9.2.1 Cement Properties

Density

The density of neat cement slurry, i.e., mixture of water and cement, varies from 1773 kg/m^3 (110 lb_m/ft^3) to 1965 kg/m^3 (123 lb_m/ft^3) depending on the API Class of the cement and the water/cement ratio (w/c). The most common ones are listed in Table 9.1.

Table 9.1 Weighing materials.

Additives	Specific gravity	Color	Additional water (*gal/lbs*)
Barite	4.33	White	0.024
Hematite	4.95	Red	0.0023
Limenite	4.45	Black	0.00

The slurry density is calculated by adding the masses of all components and divided by the total of absolute volumes of all.

$$P_{slurry}\left(\frac{lbs}{gal}\right) = \frac{lb_{\text{cement}} + lb_{\text{water}} + lb_{\text{additives}}}{gal_{\text{cement}} + gal_{\text{water}} + gal_{\text{additives}}} \tag{9.1a}$$

Equation (9.1) can be written also as:

$$\begin{aligned}\text{Slurry Density, } lbs/gal &= \frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} V_i} \\ &= \frac{\text{weight of dry cement} + \text{weight of mixing water}}{\text{volume of dry cement} + \text{volume of mixing water}}\end{aligned} \tag{9.1b}$$

Example 9.1: A slurry is composed of a sack Class G cement, 35% silica flour and 44% water. Find the density of slurry.

Solution:

Given data:

Weight of Cement = 94 *lbs/sack*
Weight of Water = 44% of a cement sack
Weight of Silica Flour = 35% of cement sack

Required data:

W_t = Total weight (lb_s)
V_t = Total volume (*gal*)
V_{abst} = Absolute volumes (Halliburton cementing tables)

Weight of Water = $94 \times 0.44 = 41.36$ *lbs*

Weight of Silica Flour = $94 \times 0.35 = 32.9$ *lbs*

Total Weight = weight of cement+ weight of water + weight of silica flour
= $94 + 41.36 + 32.9 = 168.26$ *lb*

Volume of a cement = Absolute volume × Weight of cement
= $0.0382 \times 94 = 3.59$ *gal*

Volume of water = 0.1202×41.36
= 4.97 *gal*

Volume of silica flour = $0.454 \times 32.9 = 1.49$ *gal*

Total Volume $= V_{cement} + V_{water} + V_{silica\ flour}$
$= 3.59 + 4.97 + 1.49$
$= 10.05\ gal$

By applying the Eq. (9.1a), we get

$$\text{Density of slurry} = \frac{\text{total weight}}{\text{total volume}}$$

$$P_{slurry}\left(\frac{\text{lbs}}{\text{gal}}\right) = \frac{168.26}{10.05}$$

$$P_{slurry} = 16.74\ lbs/gal$$

Example 9.2: Slurry is composed of a sack Class G cement, 35% silica flour and 45% water. Find the density of slurry.

Solution:

Given data:

Weight of cement = 94 *lbs/sack*
Weight of water = 45% of a cement sack
Weight of silica flour = 35% of a cement sack
Weight of the drillpipe = $W_{dp} = 16.25\ lb_f/ft$

Required data:

W_t = Total weight (lb_s)
V_t = Total volume (*gal*)
V_{abst} = Absolute volumes (Halliburton cementing tables)

Weight of water $= 94 \times 0.45 = 42.3\ lbs$

Weight of silica flour $= 94 \times 0.35 = 32.9\ lbs$

Total Weight = Weight of cement + Weight of water + Weight of silica flour
$= 94+42.3+32.9 = 169.2\ lbs$

Volume of a cement = Absolute volume × Weight of cement
$= 0.0382 \times 94 = 3.59\ gal$

Volume of water $= 0.1202 \times 42.3 = 5.08446\ gal$

Volume of Silica flour $= 0.0454 \times 32.9 = 1.49366\ gal$

Total Volume $= V_{cement} + V_{water} + V_{silica\ flour}$
$= 3.59 + 5.08446 + 1.49366$
$= 10.16812\ gal$

By applying the Eq. (9.1), we get,

$$\text{Density of slurry} = \frac{\text{total weight}}{\text{total volume}}$$

$$\text{Density of slurry} = \frac{169.2}{10.16812}$$

$$P_{slurry}\left(\frac{lbs}{gal}\right) = 16.6402\ lbs/gal$$

Example 9.3: The cement slurry was blended using the following data: i) one sack of class G cement, and ii) 45% fresh water. Determine the slurry density and yield.

Solution:

Given data:

Weight of cement = 94 *lbs/sack*
Weight of water = 45% of a cement sack
Weight of silica flour = 35% of a cement sack
Weight of the drillpipe = $W_{dp} = 16.25\ lb_f/ft$

Required data:

W_t = Total weight (lb_s)
V_t = Total volume (*gal*)
P_{slurry} = Density of slurry

Weight of 1 sack of dry cement = 94 *lbs*

Weight of fresh water = 0.45×94 *lbs*
= 42.3 *lbs* per sack of cement

Volume of 1 sack of cement = 94 *lbs* / 26.18 *lbs/gal* = 3.59 *gal*

Volume of water per sack of cement = 0.1202 *gal/lbs* × 42.3 *lbs*
= 5.08446 *gal*

Applying Eq. (9.1b), we get,

$$\text{Slurry Density} = \frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} V_i} = \frac{94 + 42.3}{3.59 + 5.08446} = 15.71\ lbs/gal$$

Example 9.4: The cement slurry is composed of a sack Class G cement, 30% silica flour and 40% water. Find the density of slurry.

Solution:

Given data:

Weight of cement = 94 *lbs/sack*
Weight of water = 40% of a cement sack
Weight of silica flour = 30% of a cement sack
W_{dp} = weight of the drillpipe = $16.25\ lb_f/ft$

Required data:

W_t = Total weight (*lbs*)
V_t = Total volume (*gal*)
P_{slurry} = Density of slurry
V_{abst} = Absolute volumes (Halliburton cementing tables)

Weight of water $= 94 \times 0.40 = 37.6\ lbs$

Weight of silica flour $= 94 \times 0.30 = 28.2\ lbs$

Total Weight = Weight of cement + Weight of water + Weight of silica flour
$= 94 + 37.6 + 28.2$
$= 159.8\ lbs$

Volume of a cement = Absolute volume × Weight of cement
$= 0.0382 \times 94 = 3.59\ gal$

Volume of water $= 0.1202 \times 37.6 = 4.519\ gal$

Volume of Silica flour $= 0.0454 \times 28.2 = 1.280\ gal$

Total Volume $= V_{cement} + V_{water} + V_{silica\ flour}$
$= 3.59 + 4.519 + 1.280$
$= 9.389\ gal$

By applying the Eq. (9.1a), we get,

$$\text{Density of slurry} = \frac{\text{total weight}}{\text{total volume}}$$

$$\text{Density of slurry} = \frac{159.8}{9.389}$$

$$P_{slurry}\left(\frac{lbs}{gal}\right) = 17.02\ lbs/gal$$

Example 9.5: The cement slurry was blended using the following data: i) one sack of class G cement, and ii) 50% fresh water. Determine the slurry density and yield.

<u>Solution</u>:

Given data:

Weight of cement = 94 *lbs/sack*
Weight of water = 50% of a cement sack

Required data:

W_t = Total weight (*lbs*)
V_t = Total volume (*gal*)
P_{slurry} = Density of slurry

Weight of 1 sack of dry cement = 94 *lbs*

Weight of fresh water $= 0.50 \times 94\ lbs$
= **47 lbs per sack of cement**

Volume of 1 sack of cement = 94 *lbs* / 26.18 *lbs/gal*
= 3.59 *gal*

Volume of water per sack of cement $= 0.1202\ gal/lbs \times 47\ lbs$
= **5.6494 *gal***

$$\text{Slurry Density} = \frac{94+47}{3.59+5.6494} = \mathbf{15.261}\ \boldsymbol{lbs/gal}$$

Thickening Time

Thickening time is the time duration in which cement slurry remains pumpable. To determine the mixing and displacement times, equations applied:

$$T_m = \frac{\text{Volume of dry cement}}{\text{Mixing rate}} \tag{9.2}$$

$$T_d = \frac{\text{Amount of fluid required to displace plug}}{\text{Displacement rate}} \tag{9.3}$$

Viscosity and Yield Point

Rheological flow properties of cement slurry include plastic viscosity (μ_p), yield point (τ_y), frictional properties, gel strength, etc. The plastic viscosity and yield point are calculated by equations given below. The details of rheological properties are provided in Chapter 3rd of the book.

$$\mu_p = \theta_{600} - \theta_{300} \tag{9.4}$$

$$\tau_y = \theta_{300} - \mu_{\mathrm{p}} \tag{9.5}$$

Example 9.6: A rotational viscometer containing cement slurry gives a dial reading of 176 at a rotor speed of 300 *RPM* and a dial reading of 236 at a rotor speed of 600 *RPM***.** Calculate plastic viscosity and yield point.

Solution:

Given data:

θ_{600} = 236
θ_{300} = 176

Required data:

μ_p = Plastic viscosity, *cp*
τ_p = Yield Point, $lb_f/100\ ft^2$

Applying Eq. (9.4), we get,

$$\mu_p = \theta_{600} - \theta_{300} = 236 - 176 = 60\ cp$$

Now applying Eq. (9.5), we get,

$$\tau_p = \theta_{300} - \mu_p = 176 - 60 = 116\ lb_f/100\ ft^2$$

Example 9.7: A rotational viscometer containing cement slurry gives a dial reading of 196 at a rotor speed of 300 *RPM* and a dial reading of 250 at a rotor speed of 600 *RPM*. Calculate plastic viscosity and yield point.

Solution:

Given data:

$\theta_{600} = 250$

$\theta_{300} = 196$

Required data:

μ_p = Plastic viscosity, *cp*

τ_p = Yield point, $lb_f/100\ ft^2$

Applying Eq. (9.4), we get

$$\mu_p = \theta_{600} - \theta_{300} = 250 - 196 = 54\ cp$$

Now applying Eq. (9.5) for yield point calculations, we get

$$\tau_p = \theta_{300} - \mu_p = 196 - 54 = 142\ lb_f/100\ ft^2$$

Example 9.8: A rotational viscometer containing cement slurry gives a dial reading of 156 at a rotor speed of 300 *RPM* and a dial reading of 206 at a rotor speed of 600 *RPM*. Calculate plastic viscosity and yield point.

Solution:

Given data:

θ_{600} = 206

θ_{300} = 156

Required data:

μ_p = Plastic viscosity, *cp*

τ_p = Yield Point, $lb_f/100\ ft^2$

Applying Eq. (9.4), we get,

$$\mu_p = \theta_{600} - \theta_{300} = 206 - 156 = 50\ cp$$

Now applying Eq. (9.5), we get,

$$\tau_p = \theta_{300} - \mu_p = 156 - 50 = 106\ lb_f/100\ ft_2$$

Example 9.9: A slurry is composed of a sack Class G cement, 40% silica flour and 55% water. Find the density of slurry.

Solution:

Given data:

Weight of cement = 94 *lbs/sack*

Weight of water = 55% of a cement sack

Weight of silica flour = 40% of a cement sack

W_{dp} = weight of the drillpipe = 16.25 lb_f/ft

Required data:

W_t = Total weight (*lbs*)

V_t = Total volume (*gal*)

V_{abst} = Absolute volumes (Halliburton cementing tables)

Weight of water = 94×0.55 = 51.7 *lbs*

Weight of silica flour = 94×0.40 = 37.6 *lbs*

Total Weight = Weight of cement + Weight of water + Weight of silica flour
= 94+51.7+37.6 = 183.3 *lbs*

Volume of a cement = Absolute volume × Weight of cement
= 0.0382×94 = 3.59 *gal*

Volume of water = 0.1202×51.7 = 6.21434 *gal*

Volume of Silica flour = 0.0454×37.6 = 1.70704 *gal*

Total Volume = $V_{cement} + V_{water} + V_{silica\ flour}$
= 3.59 + 6.21434 + 1.70704
= 11.5114 *gal*

By applying the Eq. (9.1), we get,

$$\text{Density of slurry} = \frac{\text{total weight}}{\text{total volume}}$$

$$\text{Density of slurry} = \frac{183.3}{11.5114}$$

$$P_{slurry}\left(\frac{lbs}{gal}\right) = 15.92\ lbs/gal$$

Permeability

The permeability of cement slurry can be calculated by using the equation given below;

$$k = \frac{(2000 \times OP \times Q \times \mu \times L)}{(A \times (IP_2 - OP_2))} \tag{9.6}$$

where:

Q = Flow rate, *lit/s*

k = Permeability, *md*

A = Cross-Sectional Area, cm^2

OP = Outlet Pressure, *atm*

IP = Inlet Pressure, *atm*

μ = Viscosity, *cp*

L = Length, *cm*

Compressive Strength

The compressive strength of cement is the force that must be exerted to crush a mass of cement divided by the cross-sectional area of the mass. The support capability of the cement is given by:

$$F(lb_f) = 0.969 \times S_c \times d \times h \tag{9.7}$$

where:
F = support capability
S_c = comprehensive strength of cement, *psi*
d = casing outer diameter, *inch*
h = height of cement column behind casing.

9.2.2 Cement Volume Calculation

Number of Sacks

The number of sacks of cement required for the cementing operation can be calculated from the following Equation:

$$\text{No. of sacks required} = \frac{\text{total volume of slurry}}{\text{yield of cement}} \tag{9.12}$$

Mixwater Needed

The mixwater volume required for the cement slurry can be calculated from:

$$\text{Mixwater volume} = \text{water needed for a sack} \times \text{no. of sacks} \tag{9.13}$$

Additives Needed

The number of sacks of additive can be calculated from:

$$\text{Number of sacks of additive} = \text{No. of sacks of cement} \times \%\ \text{Additive} \tag{9.14}$$

Displacement Volume Required

The mud volume used to displace the cement from casing during the cementing operation is commonly known as the displacement volume. The displacement volume can be calculated by following mathematical relation:

$$\text{Displacement volume} = \text{Capacity of casing} \times \text{depth of float collar} \tag{9.15}$$

Duration of pumping

The pumping duration is used to determine the required setting time for the cement formulation. The duration of the operation can be calculated by following equation:

$$\text{Duration} = \frac{\text{vol. of slurry}}{\text{mixing rate}} + \frac{\text{vol. of slurry}}{\text{pumping rate}} + \frac{\text{displacement vol}}{\text{displacement rate}} + \text{safety factor} \tag{9.16}$$

Example 9.10: The cement slurry was blended using the following data: i) one sack of class G cement, and ii) 40% fresh water. Determine the slurry density and yield.

Solution:

$$\text{Slurry density, lbs/gal} = \frac{\sum_{i=1}^{n} W_i}{\sum_{i=1}^{n} V_i}$$

$$= \frac{\text{weight of dry cement+weight of mixing water}}{\text{volume of dry cement+volume of mixing water}}$$

Weight of 1 sack of dry cement	=	94 *lbs*
Weight of fresh water	=	0.04×94 *lbs*
	=	37.6 *lbs* per sack of cement
Volume of 1 sack of cement	=	94 *lbs*/26.18 *lbs*/*gal*
	=	3.59 *gal*
Volume of water per sack of cement	=	0.1202 *gal*/*lbs* $\times$ 37.6 *lbs*
	=	4.52 *gal*

$$\text{Slurry density} = \frac{94+37.6}{3.5+4.52} = 16.41\ lbs/gal$$

$$\text{Slurry yield, ft}^3\text{/sack} = \frac{8.9\ gal\,/\,sack}{7.48\,\dfrac{gal}{ft^3}} = \mathbf{1.08}\ \boldsymbol{ft^3\,/\,sack}$$

Example 9.11: Estimate the slurry volume used to cement the 500 feet cement Colum in the casing schematic below. How many sacks of cement were used if slurry yield was 1.12 $ft^3/sack$?

Solution:

$$\text{Annular capacity}, ft^3/ft = \frac{ID^2(inch) - OD^2(inch)}{183.55}$$

Production casing, 9 5/8" OD and 8.681" ID (61 *lb/ft*)
Intermediate casing, 13 3/8" OD and 12.347" (54.5 *lb/ft*)

$$\text{Annular capacity} = \frac{12.347^2(inch) - 9.625^2(inch)}{183.55} = 0.3258\ ft^3/ft$$

For a column height of 500 *ft*., cement volume used = 0.3258 ft^3/ft. $\times$ 500 = 162.92 ft^3

Number of sacks used

$$= \frac{\text{cement height }(ft.) \times \text{annular capacity}\left(\dfrac{ft^3}{ft}\right) + \text{ excess volume }(ft^3)}{\text{cement yield}\left(\dfrac{ft^3}{sack}\right)}$$

$$= \frac{162.92+0}{1.12} = 145.5\ sacks$$

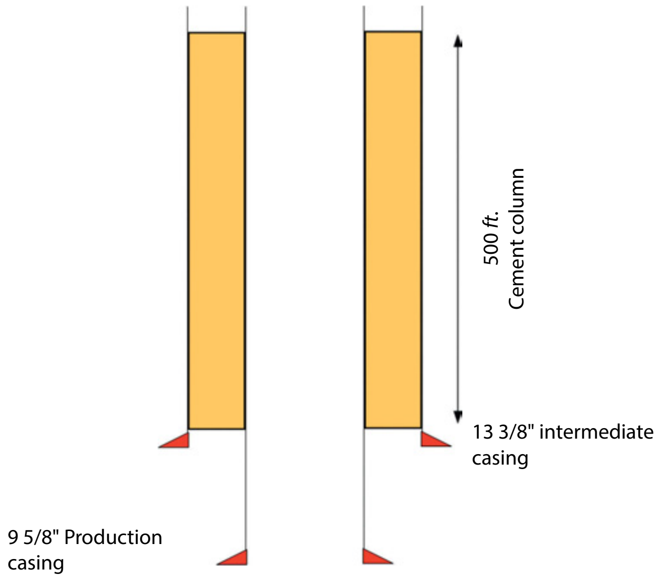


Figure 9.1 Schematic for Example 9.11.

Example 9.12: Estimate the slurry volume used to cement the 600 feet cement Colum in the casing schematic below. How many sacks of cement were used if slurry yield was 1.12 $ft^3/sack$?

Solution:

$$\text{Annular capacity}, ft^3/ft = \frac{ID^2(inch) - OD^2(inch)}{183.55}$$

Production casing, 9 5/8" OD and 8.681" ID (61 *lb/ft*)
Intermediate casing, 13 3/8" OD and 12.347" (54.5 *lb/ft*)

$$\text{Annular capacity} = \frac{12.347^2(inch) - 9.625^2(inch)}{183.55} = 0.3258\ ft^3/ft$$

For a column height of 600 *ft*., cement volume used = 0.3258 $ft^3/ft \times 600 = 195.48\ ft^3$

$$\frac{\text{Number of sacks used} = \text{cement height } (ft) \times \text{annular capacity} \left(\frac{ft^3}{ft}\right) + \text{excess volume } (ft^3)}{\text{cement yield}\left(\frac{ft^3}{sack}\right)}$$

$$\text{Number of sacks used} = \frac{195.48 + 0}{1.12} = 174.535\ sacks$$

Example 9.13: Estimate the slurry volume used to cement the 600 feet cement Colum in the casing schematic ow. How many sacks of cement were used if slurry yield was 1.12 $ft^3/sack$?

Solution:

$$\text{Annular capacity}, ft^3/ft = \frac{I^2(inch) - OD^2(inch)}{183.55}$$

Production casing, 9 5/8" OD and 8.681" ID (61 *lb/ft*)

Intermediate casing, 13 3/8" OD and 12.347" (54.5 *lb/ft*)

$$\text{Annular capacity} = \frac{12.347^2(inch) - 9.625^2(inch)}{183.55} = 0.3258\ ft^3/ft$$

For a column height of 600 *ft*., cement volume used = 0.3258 $ft^3/ft \times 600 = 195.48\ ft^3$

Number of sacks used

$$= \frac{\text{cement height }(ft) \times \text{annularcapacity}\left(\frac{ft^3}{ft}\right) + \text{excess volume }(ft^3)}{\text{cement yield}\left(\frac{ft^3}{sack}\right)}$$

$$\text{Number of sacks used} = \frac{195.48 + 0}{1.12} = 174.535\ sacks$$

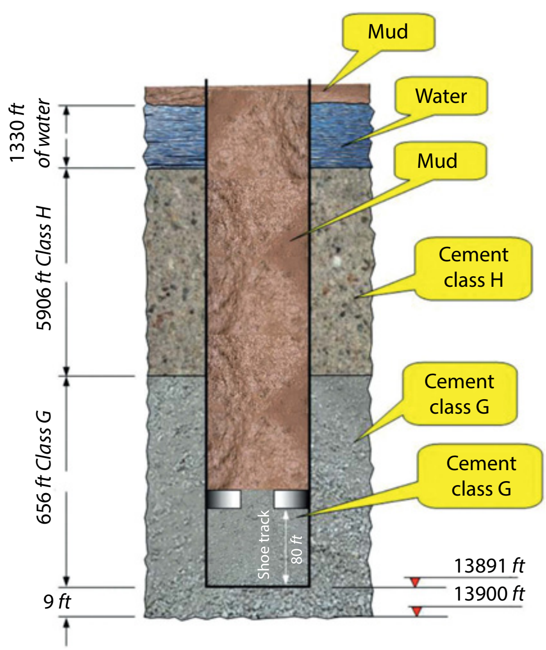


Figure 9.2 Schematic for Example 9.14.

Example 9.14: With the help of the given data and schematic in Figure 9.2, calculate the following: i) Quantity of cement of class G and H, and ii) Volume of mix water.

<u>Solution</u>:

Given data:

Hole depth	:	13,900 *ft*
Shoetrack	:	80 *ft*
Hole size	:	8.5 *inch*
Casing dimensions, *OD/ID*	:	7 *inch*/6.184
Mixwater required for Class G	:	5 *gallon/sack*
Slurry yield of Class G	:	1.15 $ft^3/sack$
Mixwater required for Class H	:	5.49 *gallon/sack*
Slurry yield of Class H	:	1.22 $ft^3/sack$

(i) Quantity of Cement

Class G:

Volume of class G slurry = volume of shoe track + volume of pocket + volume of 656 *ft* of annular space

$$= \frac{\pi}{4} \times (6.184)^2 \times \frac{1}{144} \times (80) + \frac{\pi}{4} \times (8.5)^2 \times \frac{1}{144} \times (9) + \frac{\pi}{4} \times (8.5^2 - 7^2) \times \frac{1}{144} \times (656)$$

$$= 16.7 + 3.5 + 83.2$$

$$= 103.4\, ft^3$$

$$\text{Number of sacks of Class G cement} = \frac{103.4\, ft^3}{1.15\, ft^3/sack} = 90$$

Class H:

Volume of class H slurry = (6562–656) × annular capacity

$$= 5906 \times \frac{\pi}{4}(8.5^2 - 7^2) \times \frac{1}{144} = 748.9\ \text{ft}^3$$

$$\text{Number of sacks of Class G cement} = \frac{748.9\ \text{ft}^3}{1.22\, ft^3/sack} = 614$$

(ii) Volume of mix water

= required for class G and class H cement

= (90 *sacks* × 5 *gallon/sack*) + (614 *sacks* × 5.49 *gallon/sack*)

= 3820.9 *gallon*

Example 9.15: The 9 5/8" casing of a well is to be cemented in place with a single stage cementing operation. The appropriate calculations are to be conducted prior to the operation. The details of the operation are as follows:

9 5/8" casing is set at : 13,800 *ft*

12¼" hole : 13,810 *ft*

13 3/8" 68 lbm/*ft* casing is set : 6,200 *ft*
TOC (top of cement) outside 9 5/8" casing : 3,000 *ft* above shoe
Slurry density = 15.9 *ppg*

Assume gauge hole, add 20% excess in open hole
The casing is to be cemented with class G cement with the following Additives:

0.2% D13R (retarder)

1.0% D65 (friction reducer)

Solution:

1. **Calculation of slurry volume between casing and hole**

$$9\frac{5}{8}'' \text{ casing and } 12\frac{1}{4}'' \text{ hole capacity} = \frac{\pi}{4}\times(d_{\text{hole}}^2 - d_{\text{casing}}^2)\times D$$

$$= \frac{\pi}{4}\times\frac{\{(12.25^2)-(9.625^2)\}}{144}\times 1,\ \frac{ft^3}{ft}$$

$$= 0.3132\frac{ft^3}{ft}$$

$$\text{Annular volume} = 3000\times 0.3132 = 939.6\ ft^3$$

$$\text{Add 20\% excess} = 0.20\times 939.6 = 187.9\ ft^3$$

$$\text{Total} = 1127.5\ ft^3 \cong 1128\ ft^3$$

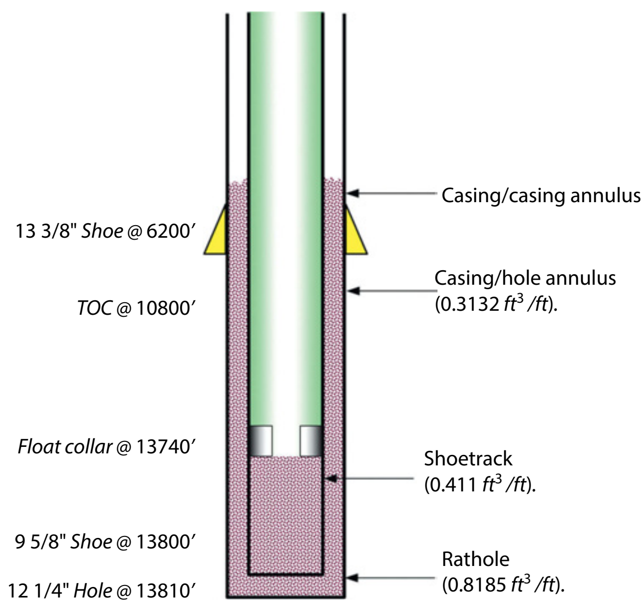


Figure 9.3 Schematic of Example 9.15.

2. **Calculation of slurry volume below the float collar:**

$$\text{Capacity of } 9\frac{5}{8}'',\ 47\frac{lb_m}{ft} = \frac{\pi}{4}\times(d^2_{\text{casing}})\times \text{D}$$
$$= \frac{\pi}{4}\times(9.625^2)\times 1, \frac{ft^3}{ft}$$
$$= 0.5050\frac{ft^3}{ft}$$

$$\text{Shoetrack volume} = (13{,}800 - 13740)\times 0.5050\ ft^3$$
$$\text{Total} = 30.8050\ ft^3$$

3. **Calculation of slurry volume in rathole:**

$$\text{Capacity of } 12\frac{1}{4}''\ \text{hole} = \frac{\pi}{4}\times(d^2_{\text{hole}})\times \text{D}$$
$$= \frac{\pi}{4}\times\frac{(12.25^2)}{144}\times 1, \frac{ft^3}{ft}$$
$$= 0.8185\frac{ft^3}{ft}$$

$$\text{Rathole volume} = (13{,}810 - 13800)\times 0.8185\ ft^3 = 8.2\ ft^3$$
$$\text{Add 20\% excess} = 0.20\times 8.2 = 1.64\ ft^3$$
$$\text{Total} = 9.84\ ft^3 \cong 10\ ft^3$$

$$\text{Total cement slurry volume} = 1128 + 31 + 10 = 1169\ ft^3$$

4. **Amount of cement and mixwater:**

Yield of Class *G* cement for density of 15.9 *ppg* = 1.14 ft^3/sk.

Mixwater requirements (From API Table) = 5.0 *gal / sk*.

$$\text{So, No. of Sacks of cement required} = \frac{1169}{1.14} = 1025.44 \cong 1026\ sk.$$

$$\text{Mixwater requirements} = 1026\times 5.0 = 5{,}130\ gal$$
$$= (5130\ gal)/(42\ gal/bbl.) = \mathbf{122\ \mathit{bbl}}.$$

5. **Amount of additives:**

$$\text{Retarder D13R (0.2\% by weight)} = 0.002\times 1026\ sks.\times 94\frac{lbm}{sks}$$
$$= 192.88\ lbm \cong 193\ lbm$$

$$\text{Friction reducer (1\% D65 by weight)} = 0.01\times 1026\ sks.\times 94\ lbm/sks.$$
$$= 964.44\ lbm \cong 965\ lbm$$

6. **Displacement volume:**

$$\begin{aligned}\text{Displacement volume} &= \text{Volume between cement head and float collar}\\ &= 0.5050 \times 13740 = 6938.7\,ft^3\\ &= \frac{6938.7\,ft^3}{5.615\,ft^3/bbl} = 1235.74\ bbl.\end{aligned}$$

Assume two bbl of cement for surface line

So, Total = 1238 *bbl*

Example 9.16: A class G cement with 30% silica flour and 44% water is planned to be used in cementing deep section in an exploration well. If the cement silo can hold 80% of the neat class G cement, determine the aerated density of the cement inside the silo and the cement slurry density? Bulk density of class G cement is 94 lbs/ft^3 and silica flour is 70 lbs/ft^3.

Solution:

Given data:

Bulk density of class G cement	=	94 lbs/ft^3
Bulk density of silica flour	=	70 lbs/ft^3
Water amount	=	44% of cement
Silica flour amount	=	30% of cement

Required data:

Aerated density of cement silo
Cement slurry density

To determine the total blend density of the cement and silica flour, we should calculate the total volume and weight of the blend based on 1 *sack* of class G cement as follows:

One sack of class G cement equals 1.0 ft^3 in volume and 94 *lbs* in weight. Weight of silica flour is equal to 30% of the weight of one sack of cement; or:

$$\text{Weight of silica flour} = 0.3 \times \text{weight of cement} = 0.3 \times 94 = 28.2\ lbs$$

Now, volume of silica flour can be determined using the bulk density of silica flour as follows:

$$\text{Bulk volume of silica} = \frac{\text{weight}}{\text{density}} = \frac{28.2}{70} = 0.403\,ft^3$$

Thus, total volume and weight of the blend is equal to:

$$\begin{aligned}\text{Total volume} &= \text{cement volume} + \text{silica volume}\\ &= 1.0 + 0.403 = 1.403\,ft^3\end{aligned}$$

$$\begin{aligned}\text{Total weight} &= \text{cement weight} + \text{silica weight}\\ &= 94 + 28.2 = 122.2\ lbs\end{aligned}$$

Blend bulk density is now equal to:

$$\text{Blend bulk density} = \frac{\text{total weight}}{\text{total volume}} = \frac{122.2}{1.403} = 87.108\ lbs/ft^3$$

When this blend transferred to the cement silo, aerated bulk density of the blend will become 80% of its original weight. So,

$$\text{Aerated bulk density} = 0.8 \times \text{blend bulk density}$$
$$= 0.8 \times 87.108 = \mathbf{69.686\ \mathit{lbs\ /\ ft^3}}$$

To calculate the cement slurry density, first weight and volume of the water should be calculated as follows:

$$\text{Weight of water} = 0.44 \times \text{weight of cement}$$
$$= 0.44 \times 94 = 41.36\ lbs$$

$$\text{Volume of water} = \frac{\text{weight of water}}{\text{water density}} = \frac{41.36}{62.4} = 0.663\ ft^3$$

Now total weight of the cement slurry is equal to:

$$\text{Total slurry weight} = \text{weight of blend cement} + \text{weight of water}$$
$$= 122.20 + 41.36 = 163.56\ lbs$$

Because cement and silica are powders, we are going to use the absolute volumes instead of bulk volume in calculating cement slurry density. Absolute volumes of class G cement and silica flour are 0.0382 and 0.0454 *gal/lbs*, respectively. Thus total slurry volume is now equal to:

$$\text{Total slurry volume} = \text{volume of blend cement} + \text{volume of water}$$
$$= 94 \times 0.0382 + 28.20 \times 0.0454 + 0.663 \times 7.481$$
$$= 9.831\,gal$$

Thus cement slurry density is equal to:

$$\text{Cement slurry density} = \frac{\text{slurry weight}}{\text{slurry volume}} = \frac{163.56}{9.831} = \mathbf{16.64\ \mathit{ppg}}$$

Example 9.17**:** Cement slurry is designed to be prepared using class G cement and fresh water. If the mud weight of the cement slurry is 13.0 *ppg*, what will be the water mass percentage based on the cement mass?

Solution:

Given data:

MW of cement slurry = 13.0 *ppg*

Required data:

Water percentage

To determine the water percentage, we should determine the mass and volume of cement and water. To simplify the calculation, the basis will be based on one sack of cement which has mass of 94 *lbs* and absolute volume of 3.591 *gallons*. If we assume the percentage mass of water is "*x*", thus the mass of water is equal to:

$$\text{Mass of water} = 94x \; lbs$$

Volume of water is equal to:

$$\text{Volume of water} = \frac{94x}{8.33} = 11.285x \; gallons$$

From the definition of cement slurry density, density is equal to:

$$\begin{aligned}
\text{Cement slurry MW} &= \frac{\text{mass of cement+mass of water}}{\text{volume of cement+volume of water}} \\
&= \frac{94+94x}{3.591+11.285x} = 13.0 \\
94+94x &= 46.68+146.71x \\
52.71x &= 47.32 \\
x &= \frac{47.32}{52.71} = 0.9
\end{aligned}$$

Thus, to prepare cement slurry with 13.0 *ppg* density, water amount should **90%** of the cement weight.

Example 9.18**:** Cement slurry is designed to be prepared using class G cement, Bentonite and fresh water. If the mud weight of the cement slurry is 14.77 *ppg* and water percentage is 60%, what will be the Bentonite mass percentage based on the cement mass?

Solution:

Given data:

MW of cement slurry = 14.77 *ppg*

Water percentage = 60%

Required data:

Bentonite percentage

To determine the Bentonite percentage, we should determine the mass and volume of cement and water. To simplify the calculation, the basis will be based on one sack of cement which has mass of 94 *lbs* and absolute volume of 3.591 *gallons*.

The mass of water is equal to:

$$Mass\ of\ water = 0.6 \times 94 = 56.4 \; lbs$$

The volume of water is equal to:

$$\text{Volume of water} = \frac{56.4}{8.33} = 6.77 \; gallons$$

If we assume the percentage mass of Bentonite is "x", thus the mass of water is equal to:

$$\text{Mass of Bentonite} = 94x\ lbs$$

Volume of Bentonite equal to:

$$\text{Volume of Bentonite} = 94x \times 0.0454\ gal/lbs = 4.27x\ gallons$$

From the definition of cement slurry density, density is equal to:

$$\begin{aligned}&\text{Cement slurry MW}\\ &= \frac{\text{mass of cement+mass of water+mass of Bentonite}}{\text{volume of cement+volume of water+volume of Bentonite}}\\ &= \frac{94 + 56.4 + 94x}{3.591 + 6.77 + 4.27x} = 14.77\end{aligned}$$

$$150.4 + 94x = 153.03 + 63.07x$$

$$30.93x = 2.63$$

$$x = \frac{2.63}{30.93} = 0.085$$

Thus, to prepare cement slurry with 13.0 *ppg* density, Bentonite amount should be 8.5% of the cement weight.

Example 9.19: An intermediate section in a well is planned to be cemented using 500 *bbls* of class G cement slurry with 35% by weight of silica flour. Cement slurry is designed to have mud weight of 15.83 *ppg* and water is 55% of cement weight. Determine the volume of water and the amount of cement and silica to be mixed in order to prepare the above cement slurry volume.

Solution:

Given data:
Volume of cement slurry = 500 *bbls*
MW of cement slurry = 15.83 *ppg*
Water amount = 55% of cement
Silica flour amount = 35% of cement

Required data:
Volume of water
Amount of cement
Amount of silica flour

To determine the amount of each material, we should calculate the volume percentage of each material based on the cement slurry density.

One sack of cement has 94 *lbs* and 3.591 *gallons* of absolute volume. The amount of silica flour based on one sack of cement is equal to:

$$\text{Mass of silica} = 94 \times 0.35 = 32.9\ lbs$$

$$\text{Absolute volume of silica} = 32.9 \times 0.0454 = 1.494 \; gallons$$

The amount of water based on one sack of cement is equal to:

$$\text{Mass of water} = 94.0 \times 0.55 = 51.7 \; lbs$$

$$\text{Volume of water} = \frac{51.7}{8.33} = 6.21 \; gallons$$

The slurry volume equal to:

$$\text{Slurry volume} = 3.591 + 1.494 + 6.21 = 11.295 \; gallons$$

The volume of water required is equal to:

$$\text{Total water volume} = \text{Total slurry volume} \times \frac{\text{water volume}}{\text{slurry volume}}$$

$$= 500 \; bbls \times \frac{6.21 \; gal}{11.295 \; gal} = 274.9 \; bbls$$

The amount of cement to be used is equal to:

$$\text{Mass of cement} = \text{Total slurry volume} \times \frac{\text{cement volume}}{\text{slurry volume}}$$

$$= \left(500 \; bbls \times \frac{42 \; gal}{bbl}\right) \times \frac{3.591 \; gal}{11.295 \; gal} \times \frac{1}{0.0382 \dfrac{gal}{lbs}}$$

$$= 174{,}777.3 \; lbs = 79.4 \; tons$$

The amount of silica flour to be used is equal to:

$$\text{Mass of silica} = \text{Total slurry volume} \times \frac{\text{silica volume}}{\text{slurry volume}}$$

$$= \left(500 \; bbls \times \frac{42 \; gal}{bbl}\right) \times \frac{1.494 \; gal}{11.295 \; gal} \times \frac{1}{0.0454 \dfrac{gal}{lbs}}$$

$$= 61{,}182.6 \; lbs = 27.8 \; tons$$

***Example 9.20*:** A production section in a well is planned to be cemented using 16.36 *ppg* class G cement slurry with 30% by weight of Bentonite. A 130 *bbls* of fresh water will be used in preparation of the cement slurry; which is about 47% of the mass of cement powder. Determine the amount of cement and Bentonite to be mixed in order to prepare the above cement slurry volume, and also determine the total volume of cement slurry.

Solution:

Given data:

Volume of fresh water	=	130 bbls
MW of cement slurry	=	16.36 *ppg*
Water amount	=	47% of cement
Bentonite amount	=	30% of cement

Required data:

Amount of cement
Amount of Bentonite
Volume of cement slurry

To determine the amount of each material and total volume of cement slurry, we should calculate the volume percentage of each material based on the cement slurry density. One sack of cement has 94 *lbs* and 3.59 *gallons* of absolute volume.

The amount of Bentonite based on one sack of cement is equal to:

$$\text{Mass of Bentonite} = 94 \times 0.30 = 28.2\ lbs$$

$$\text{Absolute volume of Bentonite} = 28.2 \times 0.0454 = 1.28\ gallons$$

The amount of water based on one sack of cement is equal to:

$$\text{Mass of water} = 94.0 \times 0.47 = 44.2\ lbs$$

$$\text{Volume of water} = \frac{44.2}{8.33} = 5.31\ gallons$$

The slurry volume is equal to:

$$\text{Slurry volume} = 3.59 + 1.28 + 5.31 = 10.18\ gallons$$

The volume ratios of each material are as follows:

$$\text{Water volume ratio} = \frac{\text{water volume}}{\text{slurry volume}} = \frac{5.31\ gal}{10.18\ gal} = 0.52$$

$$\text{Cement volume ratio} = \frac{\text{Cement volume}}{\text{slurry volume}} = \frac{3.59\ gal}{10.18\ gal} = 0.35$$

$$\text{Bentonite volume ratio} = \frac{\text{Bentonite volume}}{\text{slurry volume}} = \frac{61.28\ gal}{10.18\ gal} = 0.13$$

Total slurry volume can be calculated using the water volume ratio and water volume as follows:

$$\text{Total slurry volume} = \frac{\text{Water volume}}{\text{water volume ratio}} = \frac{130}{0.52} = 250\ bbls$$

Thus, the amount of cement based on total slurry volume is equal to:

$$\text{Mass of cement} = \text{Total slurry volume} \times \text{cement volume ratio}$$

$$= \left(250\ bbls \times \frac{42\ gal}{bbl}\right) \times 0.35 \times \frac{1}{0.0382 \frac{gal}{lbs}}$$

$$= 96{,}204\ lbs = 43.7\ tons$$

The amount of Bentonite to be used is equal to:

$$\text{Mass of Bentonite} = \text{Total slurry volume} \times \text{Bentonite volume ratio}$$

$$= \left(250\ bbls \times \frac{42\ gal}{bbl}\right) \times 0.13 \times \frac{1}{0.0454 \frac{gal}{lbs}}$$

$$= 30{,}066\ lbs = 13.7\ tons$$

Example 9.21: A well has 9 5/8" casing needs to be plugged by a balanced cement plug where the bottom depth of cement plug should be at 6,000 *ft*. 5.0" drillpipe of 3.75" inside diameter will be used to spot the cement plug in place. If the length of the cement plug is designed to be 150 *ft*, determine the volume of cement slurry needed. Also determine the top depth of cement plug of cement plug before removing the drill pipe out of cement plug, and the displacing fluid volume.

<u>Solution</u>:

Given data:

Casing ID	=	9 5/8"
Drill pipe OD	=	5.0"
Drill pipe ID	=	3.75"
Bottom depth of cement plug	=	6,000 *ft*
Cement plug length	=	150 *ft*

Required data:

Volume of cement slurry
Top depth of cement plug
Displacing fluid volume

The cement slurry volume can be calculated using the casing diameter and cement plug length as follows:

$$V_{slurry} = \frac{\pi}{4} ID_{cas}^2 L = \frac{\mu}{4} \times \left(\frac{9.625}{12}\right)^2 \times 150 = 75.8\ ft^3 = 13.5\ bbls$$

When pumping cement slurry, the drillpipe will be at the bottom of the cement plug at 6,000 *ft*. Thus, when cement plug is in place, the length of the cement plug will be greater than 150 *ft* due the volume of drillpipes. So, the top depth of cement plug must be calculated to insure proper displacement calculations. We know that after displacing the cement slurry, cement slurry volume of 13.5 bbls should be at the annulus between the casing and drillpipe, and also inside the drillpipe. The length of cement slurry inside and outside the drillpipes must be the same. If we assume the cement plug length before pulling the drillpipes equal to "L", the equation governing this situation is as follows:

$$V_{slurry} = V_{inside} + V_{outside} = (A_{inside} + A_{outside})L$$

$$L = \frac{V_{slurry}}{A_{inside} + A_{outside}} = \frac{75.8}{\frac{\pi}{4}\left(\left(\frac{3.75}{12}\right)^2 + \left(\left(\frac{9.625}{12}\right)^2 - \left(\frac{5.0}{12}\right)^2\right)\right)} = 170.1\,ft$$

Thus, the length of cement plug before pulling the drillpipe will be 170 *ft*. So, the displacement fluid will have a length of 6,000 *ft* minus 170 *ft*, which equals to 5,830 *ft*. displacement fluid can be determined using the following equation:

$$V_{disp} = \frac{\pi}{4} \times D^2 \times L = \frac{\pi}{4} \times \left(\frac{3.75}{12}\right)^2 \times 5{,}830 = 447.2\,ft^3 = 79.6\ bbls$$

***Example 9.22*:** A 100 *ft* cement plug was planned to be spotted in a 13 3/8" OD 12.5" ID casing. Calculate cement slurry density, yield and amount of each component with the help of the following information:

Sack of class G cement	=	94 *lbs*
Silica flour amount	=	35%
Fluid loss additives	=	1.3%
Liquid dispersant	=	0.3 *gal/sack*
Water amount	=	45%
Cement absolute volume	=	0.0382 *gal/lbs*
Silica flour absolute volume	=	0.0454 *gal/lbs*
Fluid loss absolute volume	=	0.093 *gal/lbs*
Liquid dispersant volume	=	0.101 *gal/lbs*

Solution:

Total weight and volume of each component of the cement slurry must be calculated in order to determine the mud weight and yield. For cement, one sack is equal to 94 *lbs* and occupies 3.59 *gal*s. Weight and volume of silica based on one sack of cement are equal to:

$$W_{silica} = 0.35 \times W_{cement} = 0.35 \times 94 = 32.9\ lbs$$

$$V_{silica} = 32.9 \times 0.0454 = 1.49\ gals$$

Weight and volume of fluid loss additives (FL) are as follows:

$$W_{FL} = 0.013 \times W_{cement} = 0.013 \times 94 = 1.2\ lbs$$

$$V_{FL} = 1.2 \times 0.093 = 0.11\ gal$$

Weight and volume of liquid dispersant (LD) are as follows:

$$V_{LD} = 0.3\ gal$$

$$W_{LD} = \frac{V_{LD}}{0.101} = \frac{0.3}{0.101} = 2.96\ lbs$$

Weight and volume of water are equal to:

$$W_{water} = 0.45 \times W_{cement} = 0.45 \times 94 = 42.3\ lbs$$

$$V_{FL} = \frac{42.3}{8.33} = 5.08\ gal$$

Now, cement slurry weight and volume are the summation of the above weights and volumes:

$$W_{slurry} = 94.0 + 32.9 + 1.22 + 2.96 + 42.3 = 173.38\ lbs$$

$$V_{slurry} = 3.59 + 1.49 + 0.11 + 0.30 + 5.08 = 10.58\ gals$$

Thus, slurry density and yield are equal to:

$$MW_{slurry} = \frac{W_{slurry}}{V_{slurry}} = \frac{173.38}{10.58} = 16.39\ ppg$$

$$Yield_{slurry} = \frac{V_{slurry}}{7.48} = \frac{10.58}{7.48} = 1.41 \frac{ft^3}{sack}$$

To calculate the amount of each component of cement slurry, first we should calculate the required volume of cement slurry as follows:

$$\text{Volume of slurry} = \frac{\pi}{4} D^2 \times L = \frac{\pi}{4} \times 12.5^2 \times 100 = 85.22\ ft^3$$

Amount of cement to be used is equal to:

$$\text{Sacks of cement} = \frac{\text{volume of slurry}}{\text{Yield}_{slury}} = \frac{85.22}{1.41} = 60.4\ sacks$$

Amount of silica flour is equals to:

$$\text{Amount of silica} = 0.35 \times 60.4 \times 94 = 1{,}988.5\ lbs$$

Amount of fluid loss additive is equal to:

$$\text{Amount of FL} = 0.013 \times 60.4 \times 94 \times 0.093 = 6.9\ gals$$

Amount of liquid dispersant additive is equal to:

$$\text{Amount of LD} = 0.3 \times 60.4 = 18.1\ gals$$

Amount of fresh water is equal to:

$$\text{Volume of water} = \frac{0.45 \times 60.4 \times 94}{8.33} = 306.7\ gals = 7.3\ bbls$$

***Example 9.23*:** A 1,500 *ft* surface section of 17½" hole size was planned to be cased and cemented using casing of 13 3/8" OD 12.5" ID and cement slurry of 16.0 *ppg* and 1.13 ft^3/Sk. Calculate the required amount of each component with the help of the following information:

Sack of class G cement	= 94 *lbs*
Retarder	= 0.04 *gal/sack*
Liquid dispersant	= 0.05 *gal/sack*
Water amount	= 4.8 *gal/sack*
Length of casing shoe	= 39.5 *ft*
Length of rathole	= 10.0 *ft*
Excess	= 25%
Depth of conductor	= 75 *ft*
Size of conductor	= 20" ID 19.25"

Solution:

To determine the amount of each component, the required cement slurry volume should be calculated first. The volume that should be filled with cement slurry is divided into four sections: annulus between casing and conductor, annulus between casing and open hole, float shoe, and rathole.

Slurry volume between casing and conductor:

In this section, no excess should be applied.

$$V_1 = \frac{\pi}{4}(ID_{cond}^2 - OD_{cas}^2) \times L_{cond} = \frac{\pi}{4} \times \frac{19.25^2 - 13.375^2}{144} \times 75 = 78.4\ ft^3$$

Slurry volume between casing and open hole:

In this section, excess should be applied.

$$V_2 = \frac{\pi}{4}(ID_{oh}^2 - OD_{cas}^2) \times L_{cas} \times 1.25$$

$$= \frac{\pi}{4} \times \frac{17.5^2 - 13.375^2}{144} \times (1{,}500 - 75 - 10) \times 1.25 = 1{,}228.6\ ft^3$$

Slurry volume in the casing shoe:

In this section, no excess should be applied.

$$V_3 = \frac{\pi}{4} ID_{cas}^2 \times L_{shoe} = \frac{\pi}{4} \times \frac{12.5^2}{144} \times 39.5 = 33.7\ ft^3$$

Slurry volume in the rathole:

In this section, excess should be applied.

$$V_4 = \frac{\pi}{4} D_{oh}^2 \times L_{rh} \times 1.25 = \frac{\pi}{4} \times \frac{17.5^2}{144} \times 10 \times 1.25 = 20.9\ ft^3$$

Total slurry volume is now equal to:

$$\text{Slurry volume} = 78.4 + 1{,}228.6 + 33.7 + 20.9 = 1{,}361.6\ ft^3$$

Cement amount can be estimated by using slurry yield as follows:

$$\text{Cement amount} = \frac{\text{slurry volume}}{\text{slurry yield}} = \frac{1{,}361.6}{1.13} = 1{,}204.9 \approx 1{,}205\ sks$$

Amount of dispersant equals:

$$\text{Dispersant amount} = 1{,}205 \times 0.05 = 60.3\ gals$$

Amount of retarder equals to:

$$\text{Retarder amount} = 1{,}205 \times 0.04 = 48.2\ gals$$

Amount of water equals to:

$$\text{Dispersant amount} = 1{,}205 \times 4.8 \approx 5{,}784\ gals \approx 138\ bbls$$

9.3 Multiple Choice Questions

1. Portland cement for the construction of oil and gas wells was introduced in ______________.
   a) 1920s
   b) 1930s
   c) 1900s
   d) None of the above

2. The cementing process involves mixing powder cement with water and some additives to prepare ______________.
   a) Drilling fluid
   b) Cement slurry
   c) Brine solution
   d) None of the above

3. There are ______________ main cementing stages in the drilling operations.
   a) Five
   b) Six
   c) Four
   d) None of the above

4. The cementing process is performed after the ______________ have been run in the wellbore.
   a) Casing strings
   b) Drilling fluid
   c) Drillpipe
   d) None of the above

5. ______________ is the first part of the completion process for a production well.
   a) Perforations
   b) Fracturing
   c) Cementing
   d) None of the above

6. ______________ is used most commonly to shut off water influx permanently into the well during the production phase.
   a) Packers
   b) Cementing
   c) Drillpipe
   d) None of the above

7. Oil well cement (OWC) is a powdery substance made of ______________.
   a) limestone and clay
   b) sodium chloride and clay
   c) limestone and potassium
   d) none of the above

8. Cements are mixed with sand, gravel and water to form concrete in the ______________.
   a) Oil industry
   b) Construction industry
   c) Ceramics industry
   d) None of the above

9. Cements are mixed with water and special additives to form slurry in the ______________.
   a) Ceramics industry
   b) Oil industry
   c) Construction industry
   d) None of the above

10. ______________ is the process of placing cement slurry in the annulus space between the well casing and the rock formations surrounding the wellbore.
    a) Fishing
    b) Hydraulic fracturing
    c) Oil well cementing
    d) None of the above

11. Placing a casing string is not sufficient to ensure wellbore stability; therefore ______________ is placed inside the wellbore.
    a) Drilling fluid
    b) Packers
    c) Cementing
    d) None of the above

12. ______________ is the process of injecting cement into a confined zone behind the casing such as casing leaks and flow channels in formations.
    a) Sidetracking
    b) Cementing liner strings
    c) Squeeze cementing
    d) None of the above

13. ______________ is a remedial job required to repair faulty primary cementing at a later age of well life.
    a) Squeeze cementing
    b) Sidetracking
    c) Cementing liner strings
    d) None of the above

14. Portland cement was patented in England in ______________.
    a) 1820
    b) 1824
    c) 1828
    d) 1832

15. ______________ is always cemented to surface.
    a) Production casing
    b) Intermediate casing
    c) Conductor pipe
    d) None of the above

16. ______________ production involves the transformation of the raw materials through a series of steps into a consistent powdered product.
    a) Additives
    b) Cement
    c) Bentonite
    d) None of the above

17. The API specifications for materials and testing for well cements include requirements for ______________ classes of oil-well-cements (OWCs).
    a) 5
    b) 6
    c) 7
    d) 8

18. Oil well cements are classified into grades based upon their ______________ content.
    a) Tricalcium aluminate
    b) Tricalcium silicate
    c) Dicalcium silicate
    d) None of the above

19. ______________ is used in milder, less demanding well conditions.
    a) Class B
    b) Class A
    c) Class C
    d) Class D

20. ______________ are specified for deeper, hotter and higher pressure well conditions.
    a) Class G & H
    b) Class B & C
    c) Class D & E
    d) Class A & F

21. The ______________ of OWC is slightly different from that of regular Portland cement.
    a) Chemical structure
    b) Chemical reaction
    c) Chemical composition
    d) None of the above

22. OWC usually have lower ______________ content.
    a) Tricalcium aluminate
    b) Tricalcium silicate
    c) Dicalcium silicate
    d) None of the above

23. The chemical composition of two classes of cement ______________ is similar.
    a) Class B & C
    b) Class D & E
    c) Class G & H
    d) Class A & F

24. The basic difference in Class G & Class H is ______________.
    a) Surface area
    b) Rate of reaction
    c) Compressive strength
    d) None of the above

25. Class H is ______________ than Class G cement.
    a) Finer
    b) Coarser
    c) Average
    d) None of the above

26. Class H cement has a ______________ water requirement than Class G cement.
    a) Average
    b) Higher
    c) Lower
    d) None of the above

27. Higher density cement column generate ______________ pressure on the formations which can result in fracturing the formations.
    a) Higher
    b) Lower
    c) Average
    d) None of the above

28. Lower density cement column generate ______________ pressure on the formations which can result in an influx from the formations.
    a) Higher
    b) Lower
    c) Average
    d) None of the above

29. Fluid loss control additive is,
    a) Bentonite
    b) Attapulgite
    c) CMHEC
    d) All of the above

30. ______________ occurs when the water in the cement slurry leaves it and invades the permeable formation.
    a) Compressive strength
    b) Fluid loss
    c) Thickening time
    d) None of the above

31. Which of the following is not a component of oil well cement?
    a) Cement powder
    b) Chemical additives
    c) Water
    d) Gravel

32. Cementing process can be performed
    a) After drilling the hole
    b) After running the casing string
    c) After installing the well head
    d) All of the above

33. Successful cement design depends mainly on
    a) Wellbore geometry
    b) Well depth
    c) Type of formation fluid
    d) All of the above

34. Which of the following is the major cement objectives?
    a) Protect casing from corrosion
    b) Protect surface water aquifers from contamination
    c) Eliminate shallow gas kicks
    d) All of the above

35. Which of the following is **not** one of the major cement objective?
    a) Support wellbore from collapse
    b) Provide zonal isolation
    c) Reduce shallow gas kicks
    d) None of the above

36. To abandon the well, cement should be set
    a) In the annulus between well and casing
    b) In certain cement plugs inside the casing
    c) Inside the casing from bottom to the top of the well
    d) None of the above

37. Oil well cementing can be used in many applications such as
    a) Side tracking
    b) Well abandonment
    c) Shut off water zones
    d) All of the above

38. Squeeze cementing is normally used to solve problems such as
    a) Unconsolidated formations
    b) High initial water saturation
    c) Casing leaks
    d) All of the above

39. In zonal isolation, ______________ should be considered to insure good isolation.
    a) Pressure and temperature changes during the well life
    b) Well depth
    c) Mud cake thickness
    d) All of the above

40. One of the raw materials of cement powder is:
    a) Calcium carbonate
    b) Sodium carbonate
    c) Barium carbonate
    d) All of the above

Answers 1a, 2b, 3c, 4a, 5c, 6b, 7a, 8b, 9b, 10c, 11c, 12c, 13a, 14b, 15c, 16b, 17d, 18a, 19b, 20a, 21c, 22a, 23c, 24a, 25b, 26c, 27a, 28b, 29d, 30b, 31d, 32b, 33d, 34a, 35c, 36b, 37d, 38c, 39a, 40a.

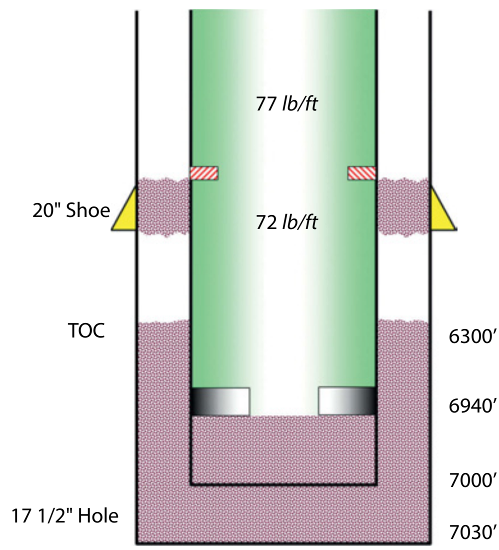
9.4 Summary

This chapter discusses different classes of cement and admixtures that have been designed for the different well conditions. Cementing technology is one of the important pillars of the oil industry operations. It is very important to give it full attention during the well construction. Failure to complete a successful cementing job will lead to a huge financial loss. Therefore, the workout examples and MCQs are designed to cover the fundamental aspects of cementing, cementing job and its design criteria and selection procedure. For self-practice, different exercises and MCQs are set. The solutions of the exercises and MCQs are outlined in Appendix A and B respectively.

9.5 Exercise and MCQs for Practice

9.5.1 Exercises (Solutions are in Appendix A)

Exercise 9.1: A class G cement with 15% Bentonite and 44% water is planned to be used in cementing deep section in a development well. If the cement silo can hold 85% of the neat class G cement, determine the aerated density of the cement inside the silo and the cement slurry density. Bulk density of class G cement is 94 *lbs/ft*³ and Bentonite is 60 *lbs/ft*³. ***Answers: 74.4 lbs/ft³, 16.3 ppg.***



Exercise 9.2: Cement slurry is designed to be prepared using class G cement and fresh water. If the mud weight of the cement slurry is 14.03 *ppg*, what will be the water mass percentage based on the cement mass? ***Answer: 68%.***

Exercise 9.3: Cement slurry is designed to be prepared using class G cement, silica flour and fresh water. If the mud weight of the cement slurry is 15.80 *ppg* and water percentage is 53%, what will be the silica flour mass percentage based on the cement mass? ***Answer: 28%.***

Exercise 9.4: A production section in a well is planned to be cemented using 100 bbls of class G cement slurry with 37% by weight of Bentonite. Cement slurry is designed to have mud weight of 15.73 *ppg* and water is 57% of cement weight. Determine the volume of water and the amount of cement and Bentonite to be mixed in order to prepare the above cement slurry volume. ***Answers: 55.4 bbls, 362 sacks, 5.7 tons.***

Exercise 9.5: A dry well is planned to be plugged using 16.0 *ppg* class G cement slurry with 12% by weight of silica flour and 46% by weight of fresh water. Class G cement of 9.66 tons will be used in preparation of the cement slurry. Determine the volume of water and the amount of silica flour to be added to the mixture in order to prepare the above cement slurry density. Also determine the total volume of cement slurry. ***Answers:108.5 sacks, 1.16 tons, 27.9 bbls, 50 bbls.***

Exercise 9.6: A well has 13 3/8" casing needs to be plugged by a balanced cement plug where the bottom depth of cement plug should be at 5,000 *ft*. A 5.0" drillpipe of 3.625" inside diameter will be used to spot the cement plug in place. If the length of the cement plug is designed to be 120 *ft*, determine the volume of cement slurry needed. Also determine the top depth of cement plug before removing the drillpipe out of cement plug, and the displacing fluid volume. ***Answers: 20.9 bbls, 4,871.5 ft, 62.2 bbls.***

Exercise 9.7: A 250 *ft* cement plug was planned to be spotted in a 7.0" OD and 6.18" ID casing. Calculate cement slurry density, yield and amount of each component with the help of the following information:

Sack of class G cement	=	94 *lbs*
Bentonite amount	=	11.5%
Fluid loss additives	=	1.6%
Liquid dispersant	=	0.7 *gal/sack*
Water amount	=	71%
Cement absolute volume	=	0.0382 *gal/lbs*
Silica flour absolute volume	=	0.0454 *gal/lbs*
Fluid loss absolute volume	=	0.093 *gal/lbs*
Liquid dispersant volume	=	0.101 *gal/lbs*

Answers: 13.9 ppg, 1.73 ft^3/sack, 30 sacks, 325.0 lbs, 4.2 gals, 21.0 gals, 5.75 bbls.

Exercise 9.8: A 3,500 *ft* intermediate section of 12¼" hole size was planned to be cased and cemented using casing of 95/8" OD 8.86" ID. The cement slurry of 16.0 *ppg* and 1.13 ft^3/sk was used to complete the job. Calculate the required amount of each component with the help of the following information:

Sack of class G cement	=	94 *lbs*
Retarder	=	0.04 *gal/sack*
Liquid dispersant	=	0.05 *gal/sack*
Water amount	=	4.8 *gal/sack*
Length of casing shoe	=	39.5 *ft*
Length of rathole	=	25.0 *ft*
Excess	=	15%
Depth of surface casing	=	700 *ft*
Size of surface casing	=	13 3/8" ID 12.5"

Answers: 1,301 sacks, 117.1 gals, 78.1 gals, 148.7 bbls

9.5.2 Exercise (Self-Practices)

E9.1: Determine the slurry density and yield for the following slurry composition:

Cement type	Class G cement
Bentonite	2%
Silica flour	35%
Water	45%

E9.2: What is the required volume of mix water that would be required to have same slurry density when we wish to add 10% of a liquid dispersant which has a specific volume of 0.1014 *gal*/lb? Also determine the new slurry yield.

E9.3: The new cement slurry in exercise 2 would be used to cement the annulus between a 16 inch casing and a 13-3/8" casing. Estimate the following: i) Volume of cement that will fill 650 *ft*. cement column in the annulus, ii) Number of sacks needed, and iii) Cement strength if laboratory result shows a slurry compressive strength of 2685 *psi*.

E9.4: A 200 *ft* plug is to be placed in 6-1/2 in hole at 9,000 *ft* using a 2-3/8 *in*, 4.6 *lb/ft* tubing. When 15 bbl of water are to be pumped ahead of the slurry that has a yield of 1.5 $ft^3/sack$, compute the:

a) Sacks of cement required,
b) Volume of water to be pumped behind the slurry,
c) Amount of mud required to displace the spacer to the balanced point.

E9.5: Calculate the amount of water per sack required to provide a slurry of 15.0 *ppg* for a slurry consisting of Class G cement + 8% BWOC Bentonite + 35% BWOC Silica flour.

E9.6: Cement Class G containing 3% Bentonite is to be mixed. The normal water content of Class A cement is 44%, for each added percent of Bentonite, 5.3% of water has to be added. The specific gravities of cement and Bentonite are found to be 3.13 and 2.65 respectively. The weight of Bentonite, the total percent of water to be added as well as the volume of water to be mixed with one sack of cement, the slurry yield and the slurry density have to be computed.

E9.7: A Class G cement core has a length of 3 *cm* and a diameter of 2.54 *cm*. It allows a water flow rate of 0.06 mL/s when placed under a pressure differential of 30 *psi*. Compute the permeability of cement core.

E9.8: A 9 5/8 inch casing is cemented in place with length of the cement column around the casing is 400 *ft*. The compressive strength of cement is 3000 *psi*. Compute the support capability of cement column.

E9.9: The following data are given:

Casing size = 13 3/8"
Hole size = 17 1/2"
Hole depth = 7030 *ft*
Mud Weight = 72 *lb/ft*

Shoe Track = 30 *ft*
This casing is to be cemented in two stages as follows:
Stage One: Shoe to 6300 *ft* (TOC) from surface
Stage Two: From 1500 to 1000 *ft* from surface. The stage collar is at 1500 *ft*.
Allow 20% excess of Class G cement for both stages.

Calculate: i) Calculate total no. of sacks of cement required in both stages, and ii) Calculate the total mixwater required in both stages.

Exercise 9.10: The 13 3/8" casing string of a well is to be cemented using class 'G' cement in two stages. The following data is provided:

20" Casing shoe : 1,600 *ft*
13 3/8" Casing 77 *lb/ft* : 0–1,000 *ft*
13 3/8" Casing 72 *lb/ft* : 1,000–7,000 *ft*.
17 1/2" open hole Depth : 7,030 *ft*.
Stage Collar Depth : 1,600 *ft*.
Shoetrack : 60 *ft*.
Cement stage 1 : (7,000–6,300 *ft*.)
Class 'G' Density : 15.9 *ppg*
Yield : 1.18 ft^3/sk
Mixwater Requirements : 0.67 ft^3/sk
Cement stage 2 : (1,600–1,000 *ft*.)
Class 'G' +5% Bentonite
Density : 13.3 *ppg*
Yield : 1.89 ft^3/sk
Mixwater Requirements : 1.37 ft^3/sk

Calculate the following:

a. The requiwred number of sacks of cement for a 1st stage of 600 *ft*. and a 2nd stage of 450 *ft* (Allow 30% excess in open hole)
b) The volume of mixwater required for each stage.
c) The total hydrostatic pressure exerted at the bottom of each stage of cement (assume a 10 *ppg* mud is in the well when cementing).

9.5.3 MCQs (Self-Practices)

1. Which of the following can be used as weighing material for the cement?
   a) Barite
   b) Bentonite
   c) Calcite
   d) All of the above

2. Thickening time of the cement can be controlled using
   a) Accelerators
   b) Retarders
   c) Weighing materials
   d) None of the above

3. In general, minimum cement compressive strength of ____________ has to be developed before staring any operations in the well.
   a) 50,000 *psi*
   b) 5,000 *psi*
   c) 500 *psi*
   d) 50 *psi*
4. The term "soundness" describes the ability of the hardened cement to
   a) Transmits sound waves
   b) Isolate sound noises
   c) Retain its volume after setting
   d) All of the above
5. The term "cement fineness" describes the
   a) Pore sizes inside the cement sheath
   b) Size of the cement particles
   c) Sizes of the gravels used
   d) None of the above
6. Which of the following compound is responsible for early strength of cement sheath?
   a) Tri-calcium silicate
   b) Di-calcium silicate
   c) Calcium silicate
   d) All of the above
7. Cement slurry can be prevented from contamination using
   a) Special chemicals
   b) Top plug
   c) Bottom plug
   d) b and c
8. Spacer fluid is pumped inside the casing
   a) Before releasing the top plug
   b) After releasing the bottom plug
   c) Before releasing the bottom plug
   d) None of the above
9. In which of the following cases multi-stage cementing should be considered?
   a) HPHT conditions
   b) Longer casing to be cemented
   c) Two casing strings
   d) All of the above
10. In high pressure squeeze operations, squeeze pressure should be
    a) Equal to the formation pressure
    b) Equal to the fracture pressure
    c) Greater than the fracture pressure
    d) Less than the fracture pressure

11. Accelerators are usually added to the cement slurry when hole depths are
    a) Shallow
    b) Deep
    c) All depths
    d) Accelerators are not required

12. ______________ is the most commonly used retarders in cement jobs.
    a) Calcium carbonate
    b) Calcium lignosulfonate
    c) Calcium chloride
    d) All of the above

13. Fluid loss agents are used to control the loss of
    a) Cement slurry to the formation
    b) Cement slurry to the fresh water aquifers
    c) Water to the formation
    d) All of the above

14. Which of the following is one of the extenders added to cement slurry?
    a) Sodium chloride
    b) Sodium carbonate
    c) Sodium hydroxide
    d) Sodium silicate

15. All of the following are **not** one of the lost circulation control agents except
    a) Gilsonite
    b) Cellophane
    c) Nylon
    d) All of the above

16. Dispersant additives are added to the cement slurry to
    a) Increase viscosity
    b) Decrease viscosity
    c) Increase density
    d) Decrease density

17. Silica sand is usually added to the cement slurry to
    a) Prevent strength retrogression
    b) Increase cement strength
    c) Increase cement density
    d) All of the above

18. What is the benefit of using centralizers?
    a) Centralize the casing string at the top of the well
    b) Insure uniform distribution of cement around the casing
    c) Centralize the cement inside the casing
    d) All of the above

19. What will happen if the top plug is put first during cementing?
    a) Cement job can be performed normally
    b) Cement job will fail
    c) Top plug will open when it reaches the casing shoe
    d) None of the above

20. To prevent back flow of cement slurry into the casing, float collar is equipped with
    a) Rupture dick valve
    b) Two way valve
    c) One way valve
    d) All of the above

21. When cement job evaluation is conducted?
    a) After displacing the cement
    b) After at least two weeks from cementing
    c) After two days from cementing
    d) After surface cement samples are hardened

22. Which of the following tools measures the bond between cement and casing?
    a) CBL
    b) CCL
    c) VDL
    d) All of the above

23. Which of the following tools measures the bond between cement and formation?
    a) CBL
    b) CCL
    c) VDL
    d) All of the above

24. Which of the following tools used for cement job evaluation?
    a) CBL
    b) CET
    c) VDL
    d) All of the above

25. Errors in cement volume calculations are mainly as a result of incorrect value of
    a) Depth
    b) Hole size
    c) Cement slurry density
    d) All of the above

26. How do you make sure that cement slurry filled all the annulus between casing and hole?
    a) Some of cement slurry must be displaced out at surface
    b) When pumped all the required displacement volume
    c) When top plug sat on top of bottom plug
    d) All of the above

27. Which of following information is not required during cement volume calculations?
    a) Formation fracture pressure
    b) Hole depth
    c) Formation fluid's type
    d) None of the above
28. Which of the following information is required during cement volume calculations?
    a) Formation porosity
    b) Formation fracture pressure
    c) Formation thickness
    d) All of the above
29. In which environments cement slurry design needs more attention?
    a) Deep wells
    b) HPHT
    c) Depleted reservoirs
    d) All of the above
30. Which of the following parameter limits the cement slurry density?
    a) Hole depth
    b) Formation fluid's type
    c) Formation fracture pressure
    d) All of the above

9.6 Nomenclature

μ_p	=	plastic viscosity
τ_y	=	yield point
API	=	American Petroleum institute
B_c	=	bearden consistency unit
BHT	=	bottom-hole temperature
BHST	=	bottom-hole static temperature
CBL	=	cement bond log
CET	=	cement evaluation tool
ft	=	*feet*
Gal	=	*gallon*
HPHT	=	high pressure high temperature
ID	=	inner diameter
lbs	=	pounds
OD	=	outer diameter
OWC	=	oilwell cement
P_{slurry}	=	slurry density, lb/gal
P_{si}	=	pound per square inch
RPM	=	revolution per minute
S_c	=	compressive strength, *psi*
SG	=	specific gravity

T_d	=	displacement time
T_m	=	mixing time
TOC	=	top of cement
T_t	=	thickening time, *hours*
VDL	=	variable density log
V_i	=	volume of slurry component *i*
w/c	=	water cement ratio
W_i	=	weight of cement component *i*

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10

Horizontal and Directional Drilling

10.1 Introduction

Directional drilling started as a result of the need to achieve goals that were not achievable by vertical wells. In the early times of oil well drilling, wells were drilled just above the target reservoir that were some distances below the drill rig, such that drilling was approximately one directional, i.e., in the vertical axis. However, with time and discovery of more oil and gas reservoirs at locations where it is either unsafe, or uneconomical, or impossible to erect a rig above such locations, there was then a need to devise a means of accessing such target reservoirs from other locations. Directional drilling can be one directional (1-D), two dimensional (2-D) or three dimensional (3-D). Different workout examples, MCQs, are presented in this chapter related to directional and horizontal drilling.

10.2 Different Mathematical Formulas and Examples

10.2.1 Horizontal Departure

The rectangular coordinates can be used to calculate the departure (horizontal displacement) between the surface location and the bottom hole target as follows:

$$\text{Horizontal departure} = \left[\left(\frac{\Delta E}{W}\right)^2 + \left(\frac{\Delta N}{S}\right)^2\right]^{1/2} \tag{10.1}$$

Polar coordinates are derived from the rectangular coordinates as follows:

$$\text{Azimuth} = \tan^{-1}\left\{\left(\frac{\Delta E}{W}\text{ coordinates}\right)\Bigg/\left(\frac{\Delta N}{S}\text{ coordinates}\right)\right\} \quad (10.2)$$

where Δ denotes difference in coordinates between E/W or N/S.

Example 10.1: A directional driller monitors the direction of a well from a reference location point *O* (Figure 10.1). The well has progressed 300 *meters* towards east and 500 *meters* towards south. What is the azimuth of the bottom of the well at this location? What is the horizontal departure (closure distance)?

Solution:

Given data:

ΔE = 300 *meters*
ΔS = 300 *meters*

Required data:

ε = Azimuth, *degree*
HD = Horizontal departure, *meter*

i. $$\text{Azimuth, } \varepsilon = 90° + \tan^{-1}\frac{500}{300}$$
$$= 149.04 \text{ degrees}$$
or
ε = S30.94E

ii. $$\text{Horizontal departure} = \sqrt{500^2 + 300^2}$$
$$= 583.1 \text{ meters}$$

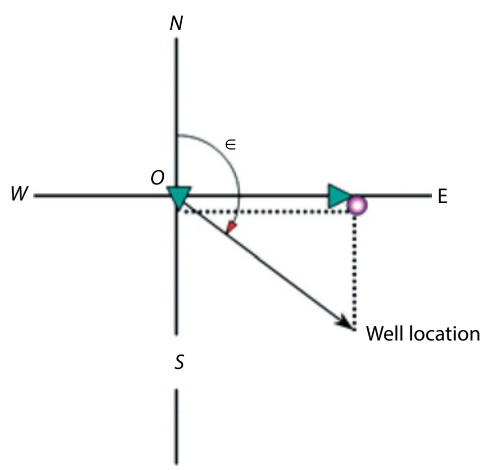

Figure 10.1 Well direction for Example 10.1.

Example 10.2: A directional driller monitors the direction of a well from a reference location point O. The well has progressed 500 *meters* towards east and 300 *meters* towards south. What is the azimuth of the bottom of the well at this location? What is the horizontal departure (closure distance)?

Solution:

Given data:

ΔE = 500 *meters*

ΔS = 300 *meters*

Required data:

ε = Azimuth, *degree*

HD = Horizontal departure, *meter*

1. Azimuth, $\varepsilon = 90^\circ + \tan^{-1}\dfrac{300}{500}$

   $= 120.96$ *degrees*

2. Horizontal departure $= \sqrt{300^2 + 500^2}$

   $= 583.1$ *meters*

Example 10.3: A directional driller monitors the direction of a well from a reference location point O. The well has progressed 550 *meters* towards east and 350 *meters* towards south. What is the azimuth of the bottom of the well at this location? What is the horizontal departure (closure distance)?

Solution:

Given data:

ΔE = 550 *meters*

ΔS = 350 *meters*

Required data:

ε = Azimuth, *degree*

HD = Horizontal departure, *meter*

3. Azimuth, $\varepsilon = 90^\circ + \tan^{-1}\dfrac{350}{550}$

   $= 122.46$ *degrees*

4. Horizontal departure $= \sqrt{350^2 + 550^2}$

   $= 651.92$ *meters*

Example 10.4: A directional driller monitors the direction of a well from a reference location point O. The well has progressed 250 *meters* towards east and 100 *meters* towards south. What is the azimuth of the bottom of the well at this location? What is the horizontal departure (closure distance)?

Solution:

Given data:

ΔE = 250 *meters*

ΔS = 100 *meters*

Required data:

ε = Azimuth, *degree*

HD = Horizontal departure, *meter*

5. Azimuth, $\varepsilon = 90^\circ + \tan^{-1}\frac{100}{250}$

$= 111.80$ *degrees*

6. Horizontal departure $= \sqrt{100^2 + 250^2}$

$= 269.26$ *meters*

Example 10.5: Another well drilled from the same location as the well above is at a direction of 195 *degrees* from the rig and at a horizontal displacement of 850 *yards* (Figure 10.2). Determine the well coordinates.

Solution:

Given data:

Azimuth = 195 *degrees*

Horizontal displacement = 850 *yards*

Angle SW = 195° – 180°

= 15°

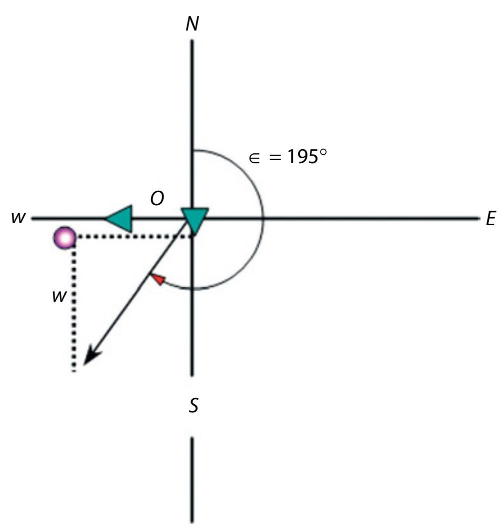

Figure 10.2 Well direction for Example 10.5.

$$\begin{aligned} SW &= OW \times \text{Sin } 15 \\ &= 850 \times 0.25882 \\ &= 220 \textit{ yards} \end{aligned}$$

$$\begin{aligned} OS &= OW \times \text{Cos } 15 \\ &= 850 \times 0.96593 \\ &= 821 \textit{ yards} \end{aligned}$$

Hence, the well coordinates are 220 *yards* west and 821 *yards* south

Example 10.6: Another well drilled from the same location as the well above is at a direction of 215 *degrees* from the rig and at a horizontal displacement of 650 *yards*. Determine the well coordinates.

Solution:

Given data:

Azimuth =145 *degrees*

Horizontal displacement =650 *yards*

$$\begin{aligned} \text{Angle } SW &= 215°-180° \\ &= 35° \end{aligned}$$

$$\begin{aligned} SW &= OW \times \text{Sin } 35 \\ &= 650 \times 0.5735 \\ &= 372.82 \textit{ yards} \end{aligned}$$

$$\begin{aligned} OS &= OW \times \text{Cos } 35 \\ &= 650 \times 0.81915 \\ &= 532.44 \textit{ yards} \end{aligned}$$

Hence, the well coordinates are 372.82 *yards* west and 532.44 *yards* south

Example 10.7: Another well drilled from the same location as the well above is at a direction of 195 *degrees* from the rig and at a horizontal displacement of 350 *yards*. Determine the well coordinates.

Solution:

Given data:

Azimuth = 195 *degrees*

Horizontal displacement = 350 *yards*

$$\begin{aligned} \text{Angle } SW &= 195°-180° \\ &= 15° \\ SW &= OW \times \text{Sin } 15 \\ &= 350 \times 0.2588 \\ &= 90.586 \textit{ yards} \end{aligned}$$

$$\begin{aligned} OS &= OW \times \text{Cos } 15 \\ &= 350 \times 0.9659 \\ &= 338.07 \textit{ yards} \end{aligned}$$

Hence, the well coordinates are 90.586 *yards* west and 338.07 *yards* south

Example 10.8: Another well drilled from the same location as the well above is at a direction of 266 *degrees* from the rig and at a horizontal displacement of 498 *yards*. Determine the well coordinates.

Solution:

Given data:

Azimuth = 276 *degrees*

Horizontal displacement = 498 *yards*

$$\text{Angle } SW = 266° - 180° = 86°$$

$$SW = OW \times \text{Sin } 86 = 498 \times 0.995 = 495.272 \textit{ yards}$$

$$OS = OW \times \text{Cos } 86 = 498 \times 0.0697 = 34.73 \textit{ yards}$$

Hence, the well coordinates are 495.272 *yards* west and 34.73 *yards* south

10.2.2 Buckling Models in Coiled Tubing

There are different buckling models that have been developed to investigate the buckling in coiled tubing. These models are given below:

Sinusoidal Buckling in a Horizontal Wellbore: When the axial compressive load along the coiled tubing reaches the following sinusoidal buckling load (F_{cr}), the initial (sinusoidal or critical) buckling of the coiled tube will occur in the horizontal wellbore as depicted by following equation.

$$F_{cr} = 2\left(\frac{EIW_e}{r}\right)^{0.5} \tag{10.3}$$

where,

E = Young's modulus of elasticity, *psi*

F_{cr} = sinusoidal buckling load, lb_f

I = moment of inertia of tubulars, in^4

W_e = tubular weight in mud, *lb/in*

r = radial clearance between wellbore and tubulars, *in*

A more general Sinusoidal Buckling Load equation for highly inclined wellbores (including the horizontal wellbore) is:

$$F_{cr} = 2\sqrt{\frac{EIW_e \sin\theta}{r}} \tag{10.4}$$

where,

θ = inclination angle, degree

Example 10.9: Consider a coiled tubing of 2.5" OD, 1.688" ID and 9.6 *ppg* mud. Calculate the sinusoidal buckling load. Assume that the hole size is 3.875".

Solution:

Given data:

Coiled tubing *OD* = 2.5 *inch*
Coiled tubing *ID* = 1.688 *inch*
Mud Weight = 9.6 *ppg*
Hole size = 3.875 *inch*

Required data:

E = Young's modulus of elasticity, *psi*
I = Moment of inertia of tubulars, in^4
We = Tubular weight in mud, *lb/in*
R = Radial clearance between wellbore and tubulars, *in*

$$W = \frac{\pi}{4}(2.5^2 - 1{,}688^2) \times \frac{12 \times 65.45}{231} = 9.08\ lb/ft$$

$$W_e = 9.08 - \left(1 - \frac{9.6}{65.45}\right) = \mathbf{8.226\ lb/ft}$$

$$E = 30{,}000{,}000\ \boldsymbol{psi}$$

$$I = \frac{\pi}{64}(2.5^2 - 1{,}688^2) = \mathbf{0.1669\ in^4}$$

$$R = \frac{3.875 - 2.5}{2.5} = \mathbf{0.55\,in}$$

Applying Eq. (10.4), we get,

$$F_{cr} = 2\left[\frac{30 \times 10^6 \times 0.1669 \times 8.226}{0.55}\right]^{0.5} = \mathbf{1.7307\ lbf}$$

Helical Buckling in a Horizontal Wellbore: When the axial compressive load reaches the following helical buckling load (F_{hel}) in the horizontal wellbore, the helical buckling of coiled tubing then occurs:

$$F_{hel} = 2(2\sqrt{2} - 1)\sqrt{\frac{EIW_e}{r}} \tag{10.5}$$

where,

F_{hel} = helical buckling load, lb_f

A more general helical buckling load equation for highly inclined wellbores (including the horizontal wellbore) is:

$$F_{hel} = 2(2\sqrt{2}-1)\sqrt{\frac{EIW_e \ \sin\theta}{r}} \tag{10.6}$$

Buckling in Vertical Wells: Lubinski (1950) derived the following buckling load equation for the initial buckling of tubulars in vertical wellbores:

$$F_{cr,b} = 1.94(EIW_e^2)^{\frac{1}{3}} \tag{10.7}$$

where

$F_{cr,b}$ = critical buckling load at tubular bottom in vertical wellbore, lb_f

Another initial buckling load equation for tubulars in vertical wellbores was also derived recently through an energy analysis:

$$F_{cr,b} = 2.55(EIW_e^2)^{\frac{1}{3}} \tag{10.8}$$

Helical Buckling in Vertical Wellbores: A helical buckling load for weighty tubulars in vertical wellbores was also derived recently through an energy analysis to predict the occurrence of the helical buckling:

$$F_{cr,b} = 5.55(EIW_e^2)^{\frac{1}{3}} \tag{10.9}$$

Example 10.10: Consider a coiled tubing of 2” *OD*, 1.688” *ID* and 8.6 *ppg* mud. Calculate the sinusoidal buckling load. Assume that that the hole size is 3.875”.

<u>Solution</u>:

Given data:

Coiled tubing *OD* = 2 *inch*
Coiled tubing *ID* = 1.688 *inch*
Mud Weight = 8.6 *ppg*
Hole size = 3.875 *inch*

Required data:

E = Young’s modulus of elasticity, *psi*
I = Moment of inertia of tubulars, in^4
W_e = Tubular weight in mud, *lb/in*
R = Radial clearance between wellbore and tubulars, *in*

$$W = \frac{\pi}{4}(2^2 - 1.688^2) \times \frac{12 \times 65.45}{231} = 3.07 \ lb/ft$$

$$W_e = 3.07 - \left(1 - \frac{8.6}{65.45}\right) = 0.2225 \ lb/ft$$

$$E = 30{,}000{,}000\ psi$$

$$I = \frac{\pi}{64}(2^2 - 1.688^2) = 0.3869\ in^4$$

$$R = \frac{3.875 - 2}{2} = 0.9375\ in$$

Applying Eq. (10.4), we get

$$F_{cr} = 2\left[\frac{30\times10^6 \times 0.3869\times 0.2225}{0.9375}\right]^{0.5} = 3.3171\ lbf$$

Example 10.11: Consider a coiled tubing of 2.25" *OD*, 1.688" *ID* and 10.6 *ppg* mud. Calculate the sinusoidal buckling load. Assume that that the hole size is 3.875".

Solution:

Given data:

Coiled tubing *OD* = 2.25 *inch*
Coiled tubing *ID* = 1.688 *inch*
Mud Weight = 10.6 *ppg*
Hole size = 3.875 *inch*

Required data:

E = Young's modulus of elasticity, *psi*
I = Moment of inertia of tubulars, in^4
W_e = Tubular weight in mud, *lb/in*
R = Radial clearance between wellbore and tubulars, *in*

$$W = \frac{\pi}{4}(2.25^2 - 1.688^2)\times\frac{12\times 65.45}{231} = 5.91\ lb/ft$$

$$W_e = 5.91 - \left(1 - \frac{10.6}{65.45}\right) = 5.00\ lb/ft$$

$$E = 30{,}000{,}000\ psi$$

$$I = \frac{\pi}{64}(2.25^2 - 1.688^2) = 0.108\ in^4$$

$$R = \frac{3.875 - 2.25}{2.25} = 0.722\ in$$

Applying Eq. (10.4), we get,

$$F_{cr} = 2\left[\frac{30\times10^6 \times 0.108\times 5}{0.722}\right]^{0.5} = \mathbf{9473.68}\ \boldsymbol{lbf}$$

10.2.3 Directional Patterns

Type 1: In this directional pattern, the kickoff is taken at the shallow depth and comes the buildup section and then tangent section that leads to target depth. This pattern applies in a deep well where there is a large horizontal displacement needed (Figure 10.3). Examination of Figure 10.4 reveals some geometric relationships;

$$VDT - KOP = ab + bd \tag{10.10}$$

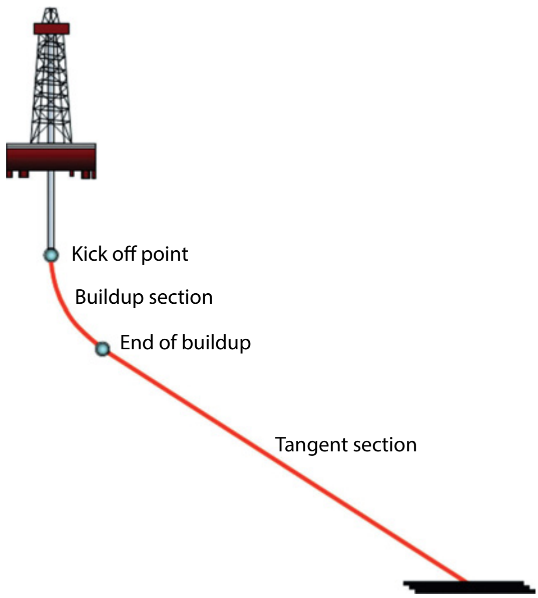

Figure 10.3 Type l (Build & Hold).

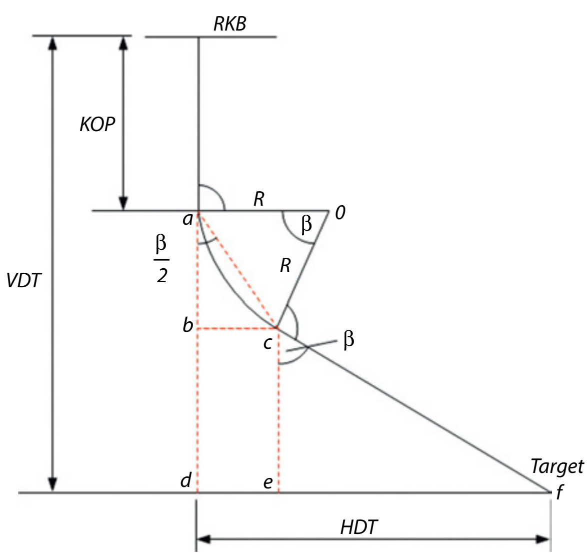

Figure 10.4 Slant type welll profile.

$$HDT = de + ef \tag{10.11}$$

The segments *ab*, *bc* and *bd* can be calculated as follow

$$ab = R\sin\beta \tag{10.12}$$

$$bc = R(1-\cos\beta) \tag{10.13}$$

$$bd = ce = \frac{ef}{\tan\beta} \tag{10.14}$$

Substituting Eq. (10.7), (10.8) and (10.9) to Eq. (10.5) and (10.6), we get

$$VDT - KOP = R\sin\beta + \frac{ef}{\tan\beta} \tag{10.15}$$

$$HDT = R(1-\cos\beta) + ef \tag{10.16}$$

Solving above equations for *ef*, we get

$$(VDT - KOP)\sin\beta + (R - HDT)\cos\beta = R \tag{10.17}$$

Eq. (10.17) describes the desired relationship between the departure of the tangent, the *VDT*, the *KOP* depth, the radius of curvature, and the inclination angle of the tangent section. If the target *VDT*, the *KOP* depth, the *HDT* and the radius of the build section are given, then Eq. (10.17) can be solved for the DL angle β.

$$\beta = arcsin\left[\frac{R}{\sqrt{(R-HDT)^2 + (VDT-KOP)^2}}\right] - arctan\left[\frac{R-HDT}{VDT-KOP}\right] \tag{10.18}$$

Type 2: This pattern is applied where multiple targets are hit with small horizontal displacements (Figure 10.5). Many kinds of pattern variations can be possible after taking kickoff. The disadvantage of this pattern is high torque, key seating and logging problem.

Type 3: In this directional pattern, the *KOP* is taken at very deep with buildup and tangent small section (Figure 10.6). This pattern is usually implemented in appraisal wells to measure the extent of well and to avoid the salt domes. Sometimes it is quite tough to take kickoff at deep because of harder formation.

***Example 10.12*:** Design the trajectory of a slant-type offshore well for the conditions stated below:

Elevation (above sea level) of the rotary Table	=	180 *ft*
Target depth (subsea)	=	–5,374 *ft*
Target south coordinate	=	2,147 *ft*
Target east coordinate	=	3,226 *ft*
Declination	=	6 °E
KOP depth	=	1,510 *ft*
Buildup rate	=	2°/100 *ft*

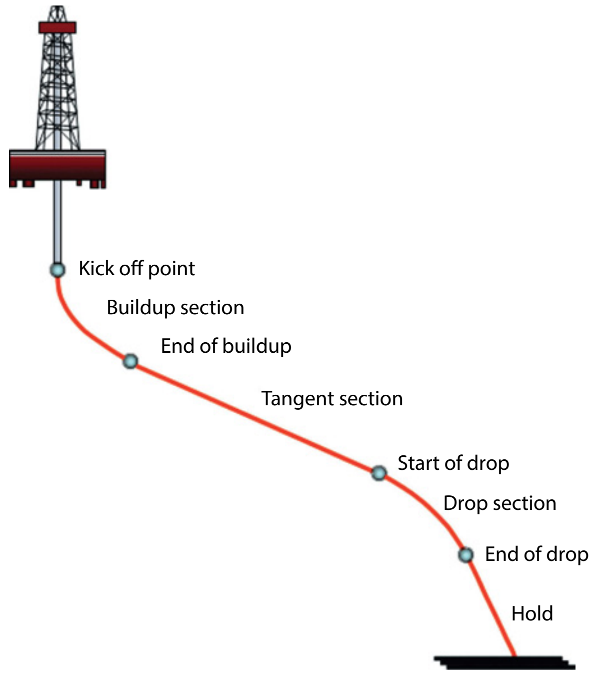

Figure 10.5 Type 2 (S Type Well).

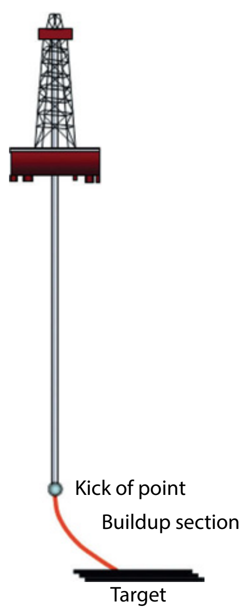

Figure 10.6 Type 3 (Deep Kick off and buildup).

A vertical section of this well is shown in Figure 10.7. And a horizontal view in Figure 10.8. Find the following:

1. Slant angle
2. Vertical depth at the beginning of the tangent part
3. Departure at the beginning of the tangent part
4. MD to the tangent

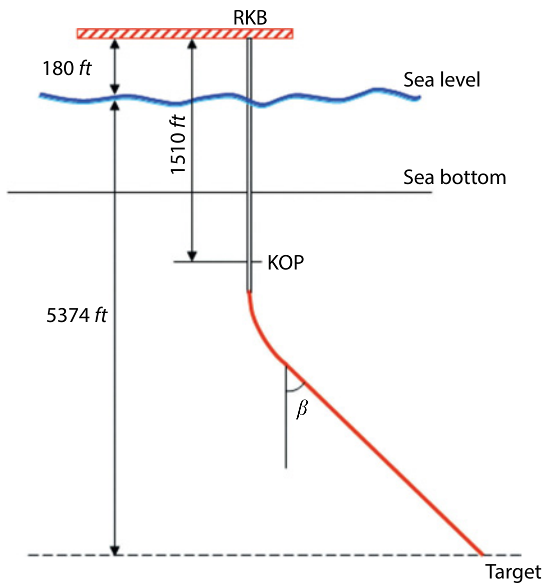

Figure 10.7 Offshore slant well profile of Example 10.3.

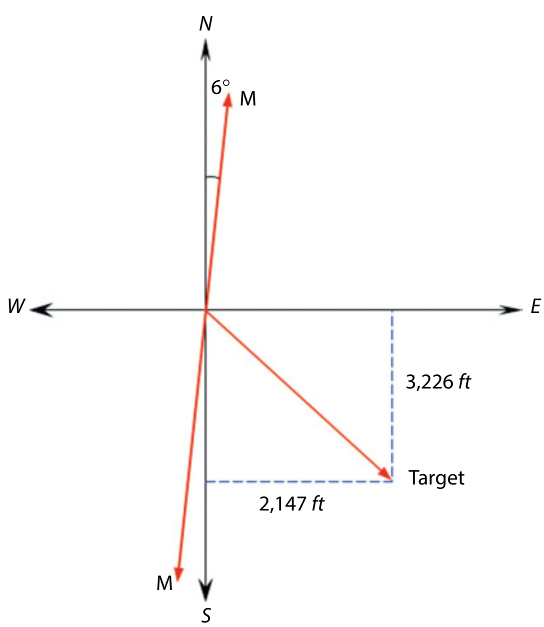

Figure 10.8 Horizontal view.

Solution:

1. **Target *VDT***

$$\text{Target } VDT = 180 + 5374 = 5554\, ft$$

2. ***HDT***

$$\text{HDT} = \sqrt{2147^2 + 3226^2} = 3875 \text{ ft}$$

3. **Target Direction**

$$\text{Target Direction} = \arctan\frac{3226}{2147} = \text{S56.35E}$$
$$(\text{Azimuth} = 180 - 56.35 = 123.65)$$

4. **Target Magnetic Direction**

$$\text{Target magnetic direction} = 56.35° + 6° = \text{S62.35E}$$

5. **Radius of Curvature**

$$\text{Radius of Curvature, } R = \frac{180}{(0.02)\pi} = 2865\, ft$$

6. **Slant Angle**

$$\beta = arcsin\left[\frac{2865}{\sqrt{(2865-3875)^2 + (5554-1510)^2}}\right] - arctan\left[\frac{2865-3875}{5554-1510}\right] = 57.4^{\circ}$$

Further calculations will be performed using the slant angle.

7. **Vertical Depth at the Beginning of Tangent Part**

$$VD_2 = 1510 + (2865)(\sin 57.4) = 3935\, ft$$

8. **Departure at the Beginning of Tangent Part**

$$HD_2 = 2865(1 - \cos 57.4) = 1321\, ft$$

9. **MD at the Beginning of Tangent Part**

$$S_2 = 1510 + 57.4/0.02 = 4380\, ft$$

10.2.4 Principles of Surveying

The basic principles of surveying can be illustrated by considering the two-dimensional system shown in Figure 10.9. The position (co-ordinates) of point B relative to the reference point A can be determined if the angle α and the distance AB is known. If the position of point A is defined as (0, 0) in the X, Y co-ordinate system the position of point B can be determined by the following equations:

$$Y_B = AB \sin\alpha \tag{10.19}$$

$$X_B = AB \cos\alpha \tag{10.20}$$

Hence the displacement of point B in the X and Y direction can be determined if the angle α and the linear distance between A and B are known. The position of a further point C can be determined by the same procedure. The X and Y displacement of C relative to the reference point A can be determined by adding together the X and Y displacement of Point B to A and those of Point C to B. This process of defining the position of a point relative to a specific reference point can be continued for any number of points.

Average Angle Method: In this method a straight line is assumed between Survey Stations 1, 2 (Figure 10.10).The inclinations and azimuth are averaged. The objective is to find out the location of Survey Station 2 with the help of the following parameters calculations:

1. North Co-ordinate
2. East Co-ordinate
3. Vertical Section (VS)

From Average Angle Method, the following values are obtained:

$$\Delta North = \Delta MD \times \sin\frac{(I_1 + I_2)}{2} \times cos\frac{(A_{z1} + A_{z2})}{2} \tag{10.21}$$

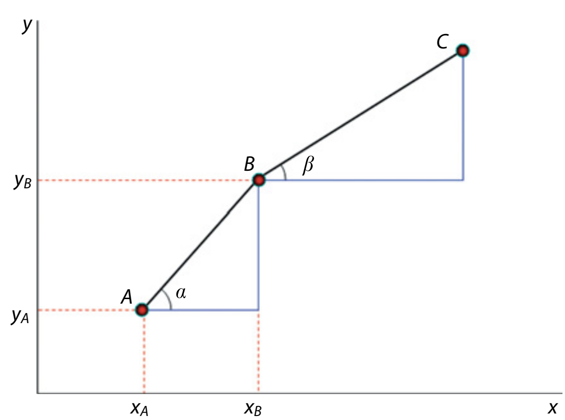

Figure 10.9 Basic principles of surveying.

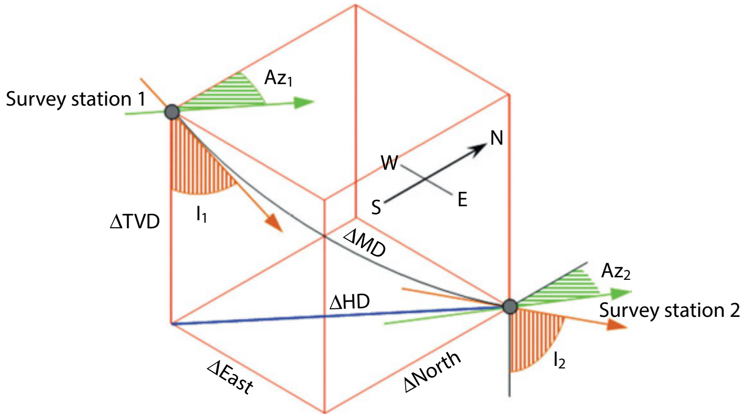

Figure 10.10 Schematic diagram for average angle method.

$$\Delta East = \Delta MD \times \sin\frac{(I_1 + I_2)}{2} \times sin\frac{(A_{z1} + A_{z2})}{2} \tag{10.22}$$

$$\Delta Vertical = \Delta MD \times cos\frac{(I_1 + I_2)}{2} \tag{10.23}$$

where

MD = measured depth between surveys in *ft*
I_1 = inclination (angle) at upper survey in *degrees*
I_2 = inclination (angle) at lower in *degrees*
A_{z1} = Azimuth direction at upper survey
A_{z2} = Azimuth direction at lower survey

Radius of Curvature: The curvature of the arc is determined by the survey inclinations and azimuths at the upper and lower survey stations as shown in Figure 10.10.

$$\Delta TVD = \frac{180 \times D(\sin\alpha_2 - \sin\alpha_1)}{\pi(\alpha_2 - \alpha_1)} \tag{10.24}$$

$$L = \frac{(360)^2 D(cos\alpha_1 - cos\alpha_2)(sin\varepsilon_2 - sin\varepsilon_1)}{4\pi^2(\alpha_2 - \alpha_1)(\varepsilon_2 - \varepsilon_1)} \tag{10.25}$$

$$M = \frac{(360)^2 D(cos\alpha_1 - cos\alpha_2)(cos\varepsilon_1 - cos\varepsilon_2)}{4\pi^2(\alpha_2 - \alpha_1)(\varepsilon_2 - \varepsilon_1)} \tag{10.26}$$

The North/South values from survey calculations are plotted versus East/West values. The plot represents the estimated well path on the horizontal plane (plan view) as shown in Figure 10.10. The TVD is plotted against the vertical sections to get the vertical view of the well (Figure 10.10). Closure distance of a point is the shortest distance from a particular survey point to the reference point, and it is given by the equation:

$$\text{Closure direction} = \sqrt{North^2 + East^2} \tag{10.27}$$

$$\text{Closure direction} = \tan^{-1}\frac{East}{North} \tag{10.28}$$

Vertical section of a point, VS is

$$VS = \cos(\varepsilon_{target} - \varepsilon_{cl}) \times (closure\ distance) \tag{10.29}$$

Example 10.13: A team of directional drillers are monitoring the progress of a horizontal well whose objective is to intersect a target reservoir at N55S from the well surface. The current location of the drill bit according to the last survey measurements is given below:

TVD	=	703 *feet*
MD	=	703 *feet*
Vertical section	=	3.0 *feet*
Coordinates: *E/W*	=	–1.2; *N/S* =5.0
Inclination	=	1.5 *degrees*
Direction	=	N 7o E

The team decided to run another survey after 24 hours and the following survey data were obtained from the downhole *MWD* tools:

Inclination	=	1.750
Direction	=	N42E
MD	=	1, 245 *feet*

Determine the location of this well 24 *hours* after the last survey.

Solution:
Using the method of radius of curvature,

$$\Delta TVD = \frac{180 \times D(\sin\alpha_2 - \sin\alpha_1)}{\pi(\alpha_2 - \alpha_1)}$$
$$= \frac{180 \times 342(\sin 1.5 - \sin 1.75)}{\pi(1.5 - 1.75)} = 342\ feet$$

$$L = \frac{(360)^2 D(\cos\alpha_1 - \cos\alpha_2)(\sin\varepsilon_2 - \sin\varepsilon_1)}{4\pi^2(\alpha_2 - \alpha_1)(\varepsilon_2 - \varepsilon_1)}$$
$$= \frac{(360)^2 \times 342 \times (\cos 1.75 - \cos 1.5)(\sin 55 - \sin 7)}{4\pi^2(1.5 - 1.75)(55 - 7)} = 8.072\ feet$$

$$M = \frac{(360)^2 D(\cos\alpha_1 - \cos\alpha_2)(\cos\varepsilon_1 - \cos\varepsilon_2)}{4\pi^2(\alpha_2 - \alpha_1)(\varepsilon_2 - \varepsilon_1)}$$
$$= \frac{(360)^2 \times 342 \times (\cos 1.75 - \cos 1.5)(\cos 7 - \cos 55)}{4\pi^2(1.5 - 1.75)(55 - 7)} = 4.85\ feet$$

Hence,

$$TVD = TVD1 + \Delta TVD$$
$$= 703 + 342$$
$$= 1045\ feet$$
$$N/S = N/S1 + L$$
$$= 5.0 + 8.072$$
$$= 13.072\ feet$$
$$E/W = E/W1 + M$$
$$= -1.2 + 4.85$$
$$= 3.65\ feet$$
$$\text{Closure Distance} = \sqrt{\text{North}^2 + \text{East}^2}$$
$$= \sqrt{13.072^2 + 3.65^2} = 13.57\ feet$$
$$\text{Closure direction} = \tan^{-1}\frac{\text{East}}{\text{North}}$$
$$= \tan^{-1}\frac{3.65}{13.072}$$
$$= \mathbf{15.61}\ \boldsymbol{degree}$$

Minimum Curvature: Minimum curvature is considered to be the most accurate method, but it does not lend itself easily to normal, hand-calculation procedures. Figure 10.11 shows minimum curvature method. The minimum curvature formulas for calculating directional parameters using diagram given in Figure 10.12:

$$\Delta TVD = \frac{D}{2} \times (\cos\alpha_1 + \cos\alpha_2) RF \tag{10.30}$$

$$L = \frac{D}{2} \times [(\sin\alpha_2 \times \cos\varepsilon_2) + (\sin\alpha_1 \times \cos\varepsilon_1)] RF \tag{10.31}$$

$$M = \frac{D}{2} \times [(\sin\alpha_2 \times \sin\varepsilon_2) + (\sin\alpha_1 \times \sin\varepsilon_1)] RF \tag{10.32}$$

$$RF = \frac{2 \times 180}{\beta \times \pi} \times \tan\frac{\beta}{2} \tag{10.33}$$

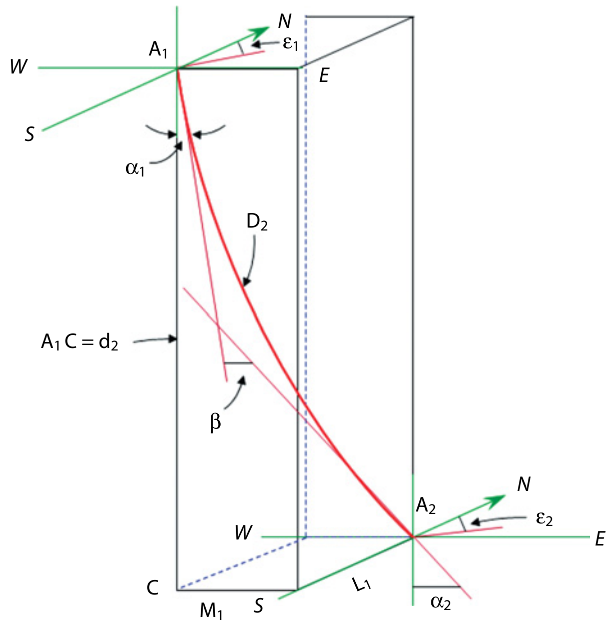

Figure 10.11 Minimum curvature.

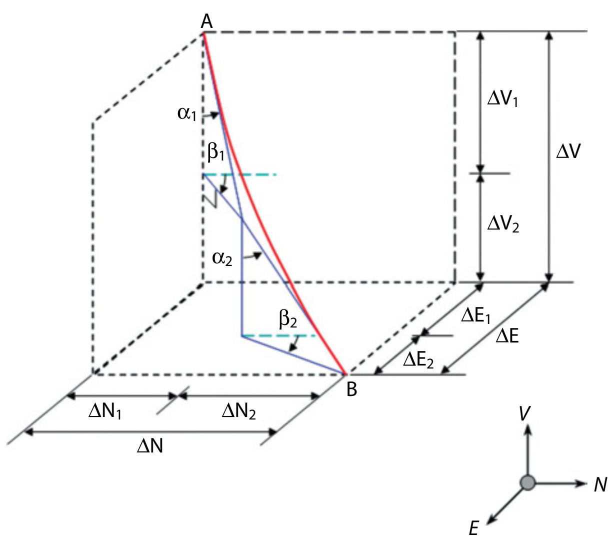

Figure 10.12 Balanced tangential method.

$$\cos\beta = \cos(\alpha_2 - \alpha_1) - \{(\sin(\alpha_1)\sin(\alpha_2)) \times [1 - \cos(\varepsilon_2 - \varepsilon_1)]\} \tag{10.34}$$

$$\text{DLS} = \frac{100}{D} a\ \text{rccos}[\cos(\alpha_2 - \alpha_1) - \{(\sin(\alpha_1)\sin(\alpha_2)) \times [1 - \cos(\varepsilon_2 - \varepsilon_1)]\}] \tag{10.35}$$

$$TVD_2 = TVD_1 + \Delta TVD \tag{10.36}$$

$$N_2 = N_1 + L_1 \tag{10.37}$$

$$E_2 = E_1 + M_1 \tag{10.38}$$

Balanced Tangential Method: The balanced tangential method assumes that the actual well path between two adjacent measurement stations can be approximated by two straight lines of equal length, $L/2$, as shown in Figure 10.12. This leads to the following expressions for the incremental distances between adjacent survey stations in the vertical direction (ΔV), in the direction of the northing (ΔN), and in the direction of the easting (ΔE):

$$\Delta V = \frac{L}{2}(\cos\alpha_1 + \cos\alpha_2) \tag{10.39}$$

$$\Delta N = \frac{L}{2}\sin\alpha_1 \cos\beta_1 + \sin\alpha_2 \cos\beta_2) \tag{10.40}$$

$$\Delta E = \frac{L}{2}(\sin\alpha_1 \sin\beta_1 + \sin\alpha_2 \sin\beta_2) \tag{10.41}$$

where the subscripts 1 and 2 denote the upper and lower survey stations, respectively.

10.2.5 Survey Calculations and Plotting Results

Calculate the dogleg severity of the section: Another parameter that is always calculated is the dogleg severity. The dogleg severity is the total three-dimensional angular changes between stations and can be calculated as shown in Figure 10.13.

The dogleg angle and dogleg severity can be calculated by following equations:

$$Dogleg\ angle = \cos^{-1}\{\cos\alpha_A \cos\alpha_B + \sin\alpha_A \sin\alpha_B \cos(\beta_A - \beta_B)\} \tag{10.42}$$

$$Dogleg\ severity = \frac{dogleg\ angle}{MD\ between\ two\ stations} \times 100 \tag{10.43}$$

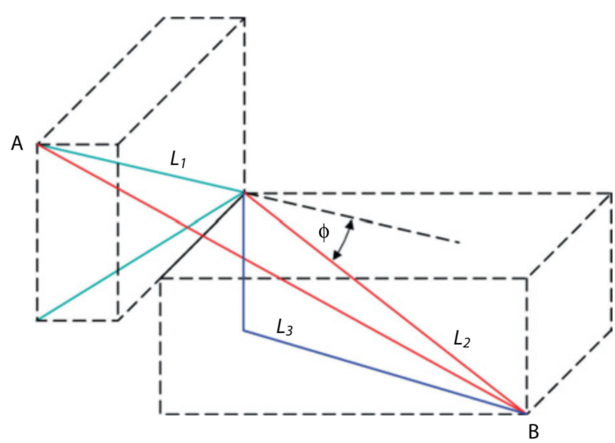

Figure 10.13 Dogleg angle.

Example 10.14: A team of directional drillers are monitoring the progress of a horizontal well whose objective is to intersect a target reservoir at N35S from the well surface. The current location of the drill bit according to the last survey measurements is given below:

TVD	=	903 *feet*
MD	=	903 *feet*
Vertical section	=	3.9 *feet*
Coordinates: *E*/*W*	=	–0.7; *N*/*S* = 4.7
Inclination	=	1.5 *degrees*
Direction	=	N 6° E

The team decided to run another survey after 24 hours and the following survey data were obtained from the downhole MWD tools:

Inclination	=	1.25^0
Direction	=	N52E
MD	=	1, 245 *feet*

Determine the location of this well 24 hours after the last survey.

Solution:

Given data:

TVD	=	903 *feet*
MD	=	903 *feet*
Vertical section	=	3.9 *feet*
Coordinates: *E*/*W*	=	–0.7; *N*/*S* = 4.7
Inclination	=	1.5 *degrees*
Direction	=	N 6° E

Using the method of radius of curvature

$$\Delta TVD = \frac{180 \times D(\sin a_2 - \sin a_1)}{\pi(a_2 - a_1)}$$

$$= \frac{180 \times 342(\sin 1.25 - \sin 1.5)}{3.142(1.25 - 1.5)}$$

$$= 342 \text{ feet}$$

$$L = \frac{(360)^2 D(cosa_1 - cosa_2)(sin\varepsilon_2 - sin\varepsilon_1)}{4\pi^2(a_2 - a_1)(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{(360)^2 \times 342(cos1.5 - cos1.25)(sin52 - sin6)}{4 \times 3.142^2(1.25 - 1.5)(52 - 6)}$$

$$= 6.98 \text{ feet}$$

$$M = \frac{(360)^2 D(cosa_1 - cosa_2)(cos\varepsilon_1 - cos\varepsilon_2)}{4\pi^2(a_2 - a_1)(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{(360)^2 \times 342(cos1.5 - cos1.25)(\cos 6 - \cos 52)}{4 \times 3.142^2(1.25 - 1.5)(52 - 6)}$$

$$= 3.87 \text{ feet}$$

Hence,

$$\begin{aligned} TVD &= TVD1 + \Delta TVD \\ &= 903 + 342 \\ &= 1,245 \text{ feet} \end{aligned}$$

$$\begin{aligned} N/S &= N/S1 + L \\ &= 4.7 + 6.98 \\ &= 11.68 \text{ feet} \end{aligned}$$

$$\begin{aligned} E/W &= E/W1 + M \\ &= -0.7 + 3.87 \\ &= 3.2 \text{ feet} \end{aligned}$$

$$\begin{aligned} \text{Closure direction} &= \sqrt{\text{North}^2 + \text{East}^2} \\ &= \sqrt{11.68^2 + 3.2^2} \\ &= 12.23 \text{ feet} \end{aligned}$$

$$\begin{aligned} \text{Closure direction} &= \tan^{-1}\frac{\text{East}}{\text{North}} \\ &= \tan^{-1}\frac{3.2}{11.68} \\ &= 15.3 \text{ degree} \end{aligned}$$

$$\begin{aligned} \text{VS} &= \cos(\varepsilon_{target} - \varepsilon_{cl}) \times (\text{closure distance}) \\ &= \text{Cos}\ (35 - 15.3) \times 12.23 \\ &= 11.51 \text{ feet} \end{aligned}$$

***Example 10.15*:** A well is planned to be drilled as a directional well. The well is kicked off from vertical section at point "*O*", and the drilling team found that the well has progressed 43.5 *meters* towards the west and 65 *meters* towards the north at point "*A*". After some time, the drilling team found that the well has progressed 72 *meters* towards the west and 100 *meters* towards the north point "*B*" from the previous measuring point, "*A*". Determine the azimuth and horizontal departure at the first and second measuring points from the kick off point "*O*".

Solution:

Given data:

$\Delta N_A = 65.0\ m$

$\Delta W_A = 43.5\ m$

$\Delta N_B = 100\ m$

$\Delta W_B = 72.0\ m$

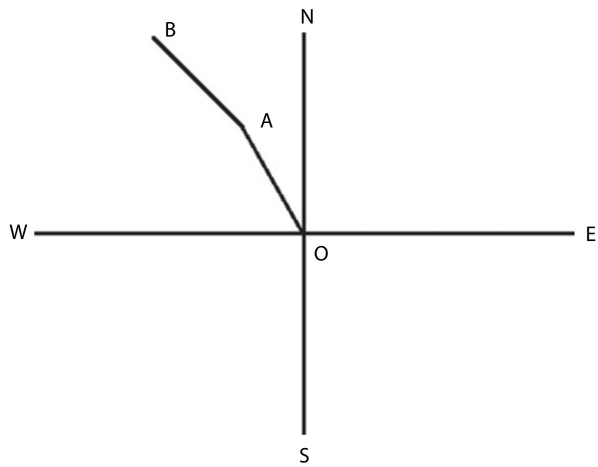

Figure 10.14 Example 10.15.

Required data:

ε = Azimuth in *degrees*
HD = Horizontal departure in *meters*

As can be seen from figure 1, the well was progressed at certain azimuth and that azimuth was deviated. So to know the azimuth to the point "*A*" and the final azimuth, we can use Eq. (10.2) as follows:

$$\varepsilon = 360 - \tan^{-1}\frac{\Delta W}{\Delta N}$$

The angle was subtracted from 360 because the angle in the N-W quarter. Horizontal departure can be calculated using Eq. (10.1) as follows:

$$HD = \sqrt{\Delta W^2 + \Delta N^2}$$

Azimuth and horizontal departure of the well from the kick off point to point "A" are equal to:

$$\varepsilon_{O-A} = 360 - \tan^{-1}\frac{\Delta W_{O-A}}{\Delta N_{O-A}} = 360 - \tan^{-1}\frac{43.5}{65.0}$$
$$= 360 - 33.8 = 326.2°$$

$$HD_{O-A} = \sqrt{\Delta W^2{}_{O-A} + \Delta N^2{}_{O-A}} = \sqrt{43.5^2 + 65.0^2} = 78.2\ m$$

Azimuth and horizontal departure of the well from point to point "A" to point "B" are equal to:

$$\varepsilon_{A-B} = 360 - \tan^{-1}\frac{\Delta W_{A-B}}{\Delta N_{A-B}} = 360 - \tan^{-1}\frac{72}{100} = 360 - 35.8 = 324.2°$$

$$HD_{A-B} = \sqrt{\Delta W^2{}_{A-B} + \Delta N^2{}_{A-B}} = \sqrt{72.0^2 + 100^2} = 123.2\ m$$

Azimuth and horizontal departure of the well from kick off point to point "*B*" are equal to:

$$\varepsilon_{O-B} = 360 - \tan^{-1}\frac{\Delta W_{O-A} + \Delta W_{A-B}}{\Delta N_{O-A} + \Delta N_{A-B}}$$

$$= 360 - \tan^{-1}\frac{115.5}{165} = 360 - 35.0 = 325.0°$$

$$HD_{O-B} = \sqrt{(\Delta W_{O-A} + \Delta W_{A-B})^2 + (\Delta N_{O-A} + \Delta N_{A-B})^2}$$
$$= \sqrt{115.5^2 + 165^2} = 201.4\ m$$

Example 10.16: A well is planned to be drilled as a directional well. The well azimuth and horizontal departure were designed to be 222° and 710 *meters*, respectively. After kick off, the well from vertical section, measurements were conducted and found that the well progressed 50 *meters* towards the west and 27.5 *meters* towards the south. What is the azimuth at the current point? If the azimuth is not the same as designed, what should be the new degree from the current point to be used until completing the well to reach the designed target?

Solution:

Given data:

HD = 710 m
ε = 222°
ΔW_A = 50 m
ΔS_A = 27.5 m

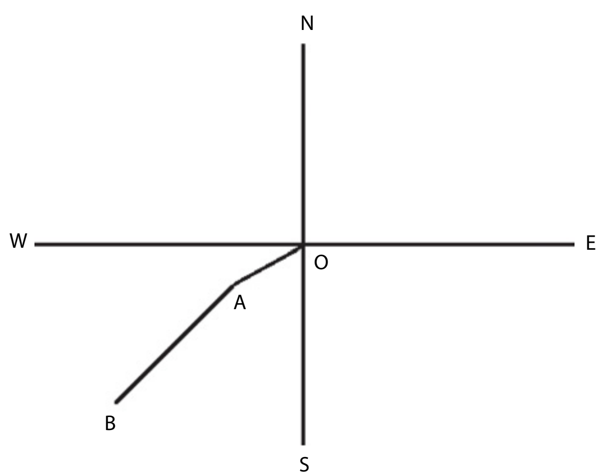

Figure 10.15 Example 10.16.

Required data:
ε_A = Azimuth in degrees at point A
ε = New azimuth to be adopted to the target

We can use the coordinates of the first measuring point to identify whether the well azimuth is maintained. Azimuth can be determined using Eq. (10.2):

$$\varepsilon_{O-A} = 180 + \tan^{-1}\frac{\Delta W_{O-A}}{\Delta S_{O-A}} = 180 - \tan^{-1}\frac{25.0}{27.5} = 180 + 61.2 = 241.2^{\circ}$$

From the above calculation, well azimuth should be adjusted in order to reach the target with final azimuth of 222 *degrees*. To do that, first we should determine the designed well coordinates based on the designed azimuth and horizontal displacement using Eqs. (10.1) and (10.2):

$$\Delta W = HD \times \sin(\varepsilon - 180) = 710 \times \sin(241.2 - 180) = 475.1\ m$$

$$\Delta S = HD \times \cos(\varepsilon - 180) = 710 \times \cos(241.2 - 180) = 527.6\ m$$

Now we can subtract the coordinates of the first measuring point from this coordinates to determine the new azimuth that should be adopted to reach the well target.

$$\Delta W_{A-B} = \Delta W_{A-O} - \Delta W_{O-A} = 475.1 - 50.0 = 425.1\ m$$

$$\Delta S_{A-B} = \Delta S_{A-O} - \Delta S_{O-A} = 527.6 - 27.5 = 500.1\ m$$

Thus, the new azimuth should be equal to:

$$\varepsilon_{A-B} = 180 + \tan^{-1}\frac{\Delta W_{A-B}}{\Delta S_{A-B}} = 180 - \tan^{-1}\frac{425.1}{500.1} = 180 + 40.4 = 220.4^{\circ}$$

***Example 10.17*:** A well is planned to be drilled as a directional well. The well azimuth and horizontal departure were measured at a point to be 168° and 96 *meters* from kick off point, respectively. Another measurement was obtained at the target point. Azimuth and horizontal departure were measured to be 152° and 429 *m* from the previous measuring point, respectively. Determine the equivalent well azimuth and horizontal departure of the target from the kick off point.

Solution:
Given data:
ε_{O-A} = 168°
ε_{A-B} = 152°
HD_{O-A} = 96 m
HD_{A-B} = 429 m

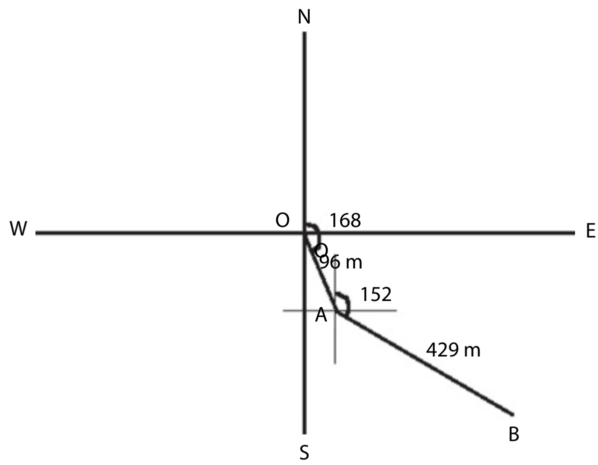

Figure 10.16 Example 10.17.

Required data:

ε_{O-B} = Well Azimuth

HD_{O-B} = Horizontal departure

To determine the equivalent well azimuth and horizontal departure, we should first calculate the well coordinates for the above two points. Well coordinates for point A are as follows:

$$\Delta E_{O-A} = HD_{O-A} \times \sin(180 - \varepsilon_{O-A})$$
$$= 96 \times \sin(180 - 168) = 20.0\ m$$

$$\Delta S_{O-A} = HD_{O-A} \times \cos(180 - \varepsilon_{O-A})$$
$$= 96 \times \cos(180 - 168) = 93.9\ m$$

Well coordinates for point B is as follows:

$$\Delta E_{A-B} = HD_{A-B} \times \sin(180 - \varepsilon_{A-B}) = 429 \times \sin(180 - 152) = 201.4\ m$$

$$\Delta S_{A-B} = HD_{A-B} \times \cos(180 - \varepsilon_{A-B}) = 429 \times \cos(180 - 152) = 378.8\ m$$

Now, well coordinates of the target can be determined by simply adding the east and south coordinates together as follows:

$$\Delta E_{O-B} = 20.0 + 201.4 = 221.4\ m$$

$$\Delta S_{O-B} = 93.9 + 378.8 = 472.7\ m$$

Equivalent well azimuth and horizontal departure of the well from the kick off point are equal to:

$$\varepsilon_{O-B} = 180 - \tan^{-1}\frac{\Delta E_{O-B}}{\Delta S_{O-B}} = 180 - \tan^{-1}\frac{221.4}{472.7}$$
$$= 180 - 25.1 = 154.9^{\circ}$$

$$HD_{O-B} = \sqrt{\Delta E^2_{O-B} + \Delta S^2_{O-B}} = \sqrt{221.4^2 + 472.7^2} = 522.0\ m$$

Example 10.18: A well is planned to be drilled as a directional well. The well coordinates were designed to be 341.5 *m* towards the east and 197 *m* towards the north, respectively. After kick off the well, measurements were conducted and found that the current azimuth and horizontal departure were 67° and 86.9 *m*. Is this azimuth the same as the designed one? And if the azimuth is not the same as designed, what should be the new degree from the current point to be used until completing the well to reach the designed target?

Solution:

Given data:

ΔE = *341.5 m*

ΔN = *197.0 m*

ε_{O-A} = 67°

HD_{O-A} = 86.9 *m*

Required data:

ε_{A-B} = Azimuth in degrees from point A to B

To know whether the current azimuth is same as the designed one, we should calculate the designed well azimuth using Eq. (10.2):

$$\varepsilon_{O-B} = \tan^{-1}\frac{\Delta E_{O-B}}{\Delta N_{O-B}} = \tan^{-1}\frac{341.5}{197.0} = 60^{\circ}$$

Thus, the current azimuth is not as per the design. So we need to adjust the azimuth to reach the target. First we should determine the current well coordinates as follows:

$$\Delta E_{O-A} = HD_{O-A} \times \sin(\varepsilon_{O-A}) = 86.9 \times \sin(67) = 80.0\ m$$

$$\Delta N_{O-A} = HD_{O-A} \times \cos(\varepsilon_{O-A}) = 86.9 \times \cos(67) = 34.0\ m$$

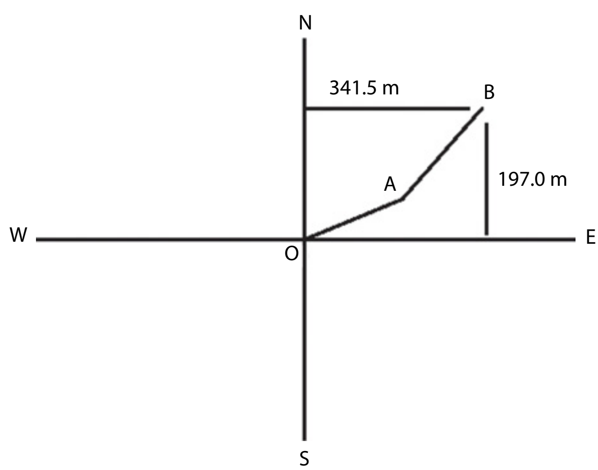

Figure 10.17 Example 10.18.

To determine the new azimuth that should be adopted, we should subtract the current well coordinates from the designed well coordinates as follows:

$$\Delta E_{A-B} = 341.5 - 80 = 261.5\ m$$

$$\Delta N_{A-B} = 197.0 - 34.0 = 163.0\ m$$

Thus, the new azimuth that should be adopted from the current point to the well target which is equal to:

$$\varepsilon_{A-B} = \tan^{-1}\frac{\Delta E_{A-B}}{\Delta N_{A-B}} = \tan^{-1}\frac{261.5}{163.0} = 58.1^{\circ}$$

***Example 10.19*:** Calculate the dogleg "*DL*" and the dogleg severity "*DLS*" of a section in a well that has the following information (Table 10.1):

<u>Solution</u>:
Given data:
Information in the Table above

Required data:
DL = Dogleg in *degrees*
DLS = Dogleg severity in degrees per 100 *ft*

Dogleg of the above section can be calculated using the following equation:

$$DL = cos^{-1}(\cos(I_2 - I_1) - (sin\ I_1 \times sin\ I_2 \times (1 - \cos(A_2 - A_1)))$$

$$DL = cos^{-1}(\cos(44 - 36) - (\sin 36 \times \sin 44 \times (1 - \cos(312 - 310)))$$
$$= 8.1^{\circ}$$

Dogleg severity for the above section can be calculated using the following equation:

$$DLS = DL \times \frac{100}{MD} = 8.1 \times \frac{100}{2,450 - 2,200} = 3.24^{\circ}$$

***Example 10.20*:** A well is designed as a deviated well to reach a certain target with the maximum *DLS* of 3.0. If the design is to have *KOP* at 1,000 *ft*, an inclination angle of 55° at and end of curvature "*EOC*" depth of 2,600 *ft*, can the well be drilled with the above information? If the answer is no, to which depth the *KOP* should be set? If based on geological

Table 10.1 Description of well for Example 10.19.

Point	Measured depth	Azimuth	Inclination
	feet	degrees	degrees
1	2200	310	36
2	2450	312	44

information *KOP* depth must be at 1,000 *ft*, what will be the maximum inclination angle at the *EOC* depth?

Solution:

Given data:

DLS = 3.0°
KOP = 1,000 *ft*
I = 55°
EOC = 2,600 *ft*

Required data:

KOP in feet
Maximum inclination angle in *degrees*

To know whether the above data of the well is suitable to drill the well to the target without having *DLS* greater than the designed one, we can calculate the *DLS* at the current data as follows:

$$DLS = DL \times \frac{100}{EOC - KOP} = 55 \times \frac{100}{2{,}600 - 1{,}000} = 3.44°$$

In the above equation, the dogleg was set to be 55° because the section will start from zero inclination at *KOP* to 55° *degrees* at the *EOC*. From above calculation, if we need to reach the target with inclination of 55°, we need to shift the *KOP* depth shallower than the designed depth. The new *KOP* depth can be calculated by setting the *DLS* to be 3.0° and looking for the *KOP* depth as follows:

$$KOP = EOC - 100 \times \frac{DL}{DLS} = 2{,}600 - 100 \times \frac{55}{3.0} = 767\ ft$$

Because geologically, the *KOP* depth cannot be less than 1,000 *ft*, in this case we should keep the *KOP* at 1,000 *ft* and change the inclination angle to meet the *DLS* design requirement as follows:

$$I = DL = \frac{DLS \times (EOC - KOP)}{100} = \frac{3.0 \times (2{,}600 - 1{,}000)}{100} = 48°$$

Thus, to keep the *KOP* depth at 1,000 *ft*, inclination angle should be 48° in order to reach the top of the target at measured depth of 2,600 *ft*.

Example 10.21: The following data refer to a directionally drilled well:

KOP = 3,500 *ft*
Northing coordinates of surface location = 1,750 *ft*
Easting coordinates of surface location = 2,800 *ft*

It is assumed that the co-ordinates of *KOP* are exactly similar to the co-ordinates of the surface. Five survey data were performed after the *KOP* which are shown in the below Table 10.2:

Table 10.2 Description of well KOP for Example 10.21.

	Azimuth	Inclination	MD
Point	**Degrees**	**Degrees**	**ft**
KOP	284.0	0.0	3,500
1	285.0	7.3	3,800
2	282.0	14.5	4,100
3	278.0	21.8	4,400
4	284.0	29.0	4,700
5	280.0	36.3	5,000

Using the radius of curvature method, calculate the well path between the above points?

Solution:

Given data:

$KOP = 3{,}500\ ft$
$N = 1{,}750\ ft$
$E = 2{,}800\ ft$
Table of the survey points

Required data:

Well path including true vertical depth, horizontal departure, and well co-ordinates

Well path calculation using radius of curvature method for the first measuring point is based on Eq. (10.24) through Eq. (10.29) as follows:

True vertical depth calculation, Eq. (10.24):

$$\Delta TVD_1 = \frac{180 \times D \times (\sin \propto_2 - \sin \propto_1)}{\pi(\propto_2 - \propto_1)}$$

$$= \frac{180 \times (3{,}800 - 3{,}500) \times (\sin 7.3 - \sin 0)}{\pi(7.3 - 0)} = 299.2\ ft$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta TVD_1 = 3{,}500 + 299.2 = 3{,}799.2\ ft$$

The northing distance can be calculated using Eq. (10.25):

$$\Delta N = \frac{180^2 \times D \times (\cos \propto_1 - \cos \propto_2)(\sin \varepsilon_2 - \sin \varepsilon_1)}{\pi^2(\propto_2 - \propto_1)(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{180^2 \times (3{,}800 - 3{,}500) \times (\cos 0 - \cos 7.3)(\sin 285 - \sin 284)}{\pi^2(7.3 - 0)(285 - 284)}$$

$$= 4.7\ ft$$

The easting distance can be calculated using Eq. (10.26):

$$\Delta E = \frac{180^2 \times D \times (\cos \propto_1 - \cos \propto_2)(\cos \varepsilon_1 - \cos \varepsilon_2)}{\pi^2 (\propto_2 - \propto_1)(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{180^2 \times (3{,}800 - 3{,}500) \times (\cos 0 - \cos 7.3)(\cos 285 - \cos 284)}{\pi^2 (7.3 - 0)(285 - 284)}$$

$$= -18.4\ ft$$

Horizontal departure can be calculated using Eq. (10.27):

$$HD_1 = \sqrt{\Delta N^2 + \Delta E^2} = \sqrt{4.7^2 + (-18.4)^2} = 19.0\ ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = 1{,}750 + 4.7 = 1{,}754.7\ ft$$

$$E_1 = KOP + \Delta E_1 = 2{,}800 + (-18.4) = 2{,}781.6\ ft$$

Similarly, we can calculate the well path for the rest of the points. Table 10.3 summarizes the results for all the points:

Example 10.22: Using the data of example 10.7 and minimum curvature method, calculate the well path between the above points. Compare the results with that obtained in example 10.7

Solution:

Given data:

KOP = 3,500 *ft*
N = 1,750 *ft*
E = 2,800 *ft*
Table of the survey points

Table 10.3 Summary of *KOP* for Example 10.21.

	Δ*TVD*	***TVD***	***HD***	**Cum. *HD***	**Δ*N***	**Northing**	**Δ*E***	**Easting**
Point	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**
KOP	3500	3500	0	0.0	1750	1750	2800	2800
1	299.2	3799.2	19.0	19.0	4.7	1754.7	−18.4	2781.6
2	294.4	4093.6	56.6	75.5	13.2	1767.9	−55.0	2726.7
3	284.9	4378.5	93.3	168.8	16.2	1784.1	−91.8	2634.8
4	270.9	4649.4	128.5	297.3	24.5	1808.6	−126.1	2508.8
5	252.5	4901.9	161.6	458.9	33.6	1842.2	−158.1	2350.7

Required data:
Well path including true vertical depth, horizontal departure, and well co-ordinates

Well path calculation using minimum curvature method for the first measuring point is based on Eq. (10.30) through Eq. (10.38) as follows:
True vertical depth calculation, Eq. (10.30), Eq. (10.33) and Eq. (10.34) as follows:

$$\begin{aligned}\beta_1 &= \cos^{-1}[\cos(\alpha_2-\alpha_1)-(\sin\alpha_1\times\sin\alpha_2\times(1-\cos(\varepsilon_2-\varepsilon_1))] \\ &= \cos^{-1}[\cos(7.3-0)-(\sin 0\times\sin 7.3\times(1-\cos(285-284))] \\ &= 7.3\end{aligned}$$

$$RF_1 = \frac{2\times180}{\beta\pi}\times\tan\frac{\beta}{2} = \frac{2\times180}{\pi\times7.3}\times\tan\frac{7.3}{2} = 1.0$$

$$\begin{aligned}\Delta TVD_1 &= \frac{D}{2}\times(\cos\propto_1+\cos\propto_2)\times RF \\ &= \frac{(3{,}800-3{,}500)}{2}\times(\cos 0+\cos 7.3)\times1.0 = 299.2\ ft\end{aligned}$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta TVD_1 = 3{,}500 + 299.2 = 3{,}799.2\ ft$$

The northing distance can be calculated using Eq. (10.31):

$$\begin{aligned}\Delta N &= \frac{D}{2}[(\sin\propto_2\ \cos\varepsilon_2)+(\sin\alpha_1\cos\varepsilon_1)] \\ &= \frac{3{,}800-3{,}500}{2}[(\sin 7.3\cos 284)+(\sin 0\cos 285)] = 4.7\ ft\end{aligned}$$

The easting distance can be calculated using Eq. (10.32):

$$\begin{aligned}\Delta N &= \frac{D}{2}[(\sin\propto_2\ \sin\varepsilon_2)+(\sin\alpha_1\sin\varepsilon_1)] \\ &= \frac{3{,}800-3{,}500}{2}[(\sin 7.3\sin 284)+(\sin 0\cos 285)] = -18.4\ ft\end{aligned}$$

Horizontal departure can be calculated using Eq. (10.27):

$$HD_1 = \sqrt{\Delta N^2+\Delta E^2} = \sqrt{4.7^2+(-18.4)^2} = 19.0\ ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = 1{,}750 + 4.7 = 1{,}754.7\ ft$$

$$E_1 = KOP + \Delta E_1 = 2{,}800 + (-18.4) = 2{,}781.6\ ft$$

Similarly, we can calculate the well path for the rest of the points. Table 10.4 summarizes the results for all the points. By comparing the results of calculations of radius of curvature and minimum curvature methods, we can find that there are very small differences in the results. In general, we can see that both methods gave exactly same results.

Example 10.23: The data below refer to a deviated drilled well:

KOP = 2,000 *ft*
Northing coordinates of surface location = 1,050 *ft*
Easting coordinates of surface location = −1,450 *ft*

The co-ordinates of KOP were exactly similar to the co-ordinates of the surface. Five survey data were performed after the KOP which are shown in Table 10.5.

Using the average angle method, calculate the well path between the above points.

Solution:
Given data:
KOP = 2,000 *ft*
N = 1,050 *ft*
E = −1,450 *ft*

Table of the survey points

Required data:
Well path including true vertical depth, horizontal departure, and well co-ordinates

Well path calculation using average angle method for the first measuring point is based on Eq. (10.21) through Eq. (10.23) as follows:

True vertical depth calculation, Eq. (10.23) as follows:

$$\Delta TVD_1 = \Delta MD_1 \times \cos\left(\frac{\propto_1 + \propto_2}{2}\right) = (2{,}400 - 2{,}000) \times \cos\left(\frac{0+8}{2}\right)$$
$$= 399\ ft$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta VD_1 = 2{,}000 + 399 = 2{,}399\ ft$$

The northing distance can be calculated using Eq. (10.21):

$$\Delta N_1 = \Delta MD_1 \times \sin\left(\frac{\propto_1 + \propto_2}{2}\right)\cos\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right)$$
$$= (2{,}400 - 2{,}000) \times \sin\left(\frac{0+8}{2}\right) \times \cos\left(\frac{61+58}{2}\right) = 24.0\ ft$$

Table 10.4 Summary of KOP for Example 10.22.

Point	β	*RF*	ΔTVD	*TVD*	*HD*	Cum. HD	ΔN	Northing	ΔE	Easting
	Degrees	fraction	ft	ft	ft	ft	ft	ft	ft	ft
KOP	0.00			3500	0	0.0	1,750	1750	2800	2800
1	7.25	1.00	299.2	3799.2	19.0	19.0	4.7	1754.7	−18.4	2781.6
2	7.27	1.00	294.4	4093.6	56.6	75.5	13.2	1767.9	−55.0	2726.6
3	7.35	1.00	284.9	4378.6	93.3	168.8	16.2	1784.1	−91.9	2634.7
4	7.68	1.00	270.9	4649.5	128.5	297.3	24.5	1808.6	−126.2	2508.5
5	7.56	1.00	252.5	4902.0	161.6	458.9	33.6	1842.2	−158.2	2350.3

Table 10.5 Summary of *KOP* for Example 10.23.

	Azimuth	Inclination	*MD*
Point	**Degrees**	**Degrees**	**ft**
KOP	61.0	0.0	2000
1	58.0	8.0	2400
2	60.0	18.0	2800
3	61.5	26.0	3200
4	60.5	37.0	3600
5	60.0	47.0	4000

Table 10.6 Summary of *KOP* for Example 10.23.

Point	Δ*TVD*	*TVD*	*HD*	Cum. *HD*	Δ*N*	Cum. *N*	Δ*E*	Cum. *E*
	ft	ft	ft	ft	ft	ft	ft	ft
KOP		2000	0	0.0	1050	1050	−1450	−1450
1	399.0	2399.0	27.9	27.9	14.2	1064.2	24.0	−1426.0
2	389.7	2788.8	90.0	117.9	46.3	1110.5	77.1	−1348.8
3	370.9	3159.6	149.8	267.7	73.2	1183.7	130.7	−1218.1
4	341.1	3500.7	209.0	476.7	101.3	1285.0	182.8	−1035.3
5	297.3	3798.0	267.7	744.4	132.8	1417.9	232.4	−802.9

The easting distance can be calculated using Eq. (10.22):

$$\Delta E_1 = \Delta MD_1 \times \sin\left(\frac{\propto_1 + \propto_2}{2}\right)\sin\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right)$$

$$= (2,400 - 2,000) \times \sin\left(\frac{0+8}{2}\right) \times \sin\left(\frac{61+58}{2}\right) = 24.0\ ft$$

Horizontal departure can be calculated using the following equation:

$$HD_1 = \sqrt{\Delta N^2 + \Delta E^2} = \sqrt{14.2^2 + 24^2} = 27.9\ ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = 1,050 + 14.27 = 1,064.2\ ft$$

$$E_1 = KOP + \Delta E_1 = (-1,450) + 24.0 = 1,426.0\ ft$$

Similarly, we can calculate the well path for the rest of the points. Table 10.6 summarizes the results for all the points.

Example 10.24: Using the data of example 10.9 and balanced tangential method, calculate the well path between the above points. Compare the results with that obtained in example 10.9.

Solution:

Given data:

KOP = 2,000 *ft*
N = 1,050 *ft*
E = −1,450 *ft*
Table of the survey points

Required data:

Well path including true vertical depth, horizontal departure, and well co-ordinates

Well path calculation using average angle method for the first measuring point is based on Eq. (10.39) through Eq. (10.41) as follows:

True vertical depth calculation, Eq. (10.39) as follows:

$$\Delta TVD_1 = \frac{\Delta MD_1}{2} \times (\cos \propto_1 + \cos \propto_2)$$
$$= \frac{2,400 - 2,000}{2} \times (\cos 0 + \cos 8) = 398.1\,ft$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta TVD_1 = 2,000 + 399 = 2,399\,ft$$

The northing distance can be calculated using Eq. (10.40):

$$\Delta N_1 = \frac{\Delta MD_1}{2} \times (\sin \propto_1 \cos \beta_1 + \sin \propto_2 \cos \beta_2)$$
$$= \frac{2,400 - 2,000}{2} \times (\sin 0 \cos 61 + \sin 8 \cos 58) = 14.8\,ft$$

The easting distance can be calculated using Eq. (10.41):

$$\Delta E_1 = \frac{\Delta MD_1}{2} \times (\sin \propto_1 \sin \beta_1 + \sin \propto_2 \sin \beta_2)$$
$$= \frac{2,400 - 2,000}{2} \times (\sin 0 \sin 61 + \sin 8 \sin 58) = 23.6\,ft$$

Horizontal departure can be calculated using the following equation:

$$HD_1 = \sqrt{\Delta N^2 + \Delta E^2} = \sqrt{14.8^2 + 23.6^2} = 27.8\,ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = (-1,450) + 14.8 = 1,426.4\,ft$$
$$E_1 = KOP + \Delta E_1 = (-1,450) + 23.6 = 1,426.0\,ft$$

Table 10.7 Summary of KOP for Example 10.24.

Point	Δ*TVD*	*TVD*	*HD*	Cum. HD	Δ*N*	Cum. *N*	Δ*E*	Cum. *E*
	ft	ft	ft	ft	ft	ft	ft	ft
KOP	2000	2000	0	0.0	1050	1050	−1450	−1450
1	398.1	2398.1	27.8	27.8	14.8	1064.8	23.6	−1426.4
2	388.3	2786.3	89.6	117.5	45.7	1110.4	77.1	−1349.3
3	370.0	3156.3	149.5	266.9	72.7	1183.1	130.6	−1218.7
4	339.5	3495.8	208.0	475.0	101.1	1284.2	181.8	−1036.9
5	296.1	3791.9	266.6	741.6	132.4	1416.6	231.4	−805.5

Similarly, we can calculate the well path for the rest of the points. Table 10.7 summarizes the results for all the points:

By comparing the results of calculations of average angle and balanced tangential methods, we can find that there are small differences in the results. The difference in the true vertical depth between the two methods is around 6 *ft*, whereas the difference in horizontal departure is around 3.0 ft.

10.3 Multiple Choice Questions

1. ______ started as a result of the need to achieve goals that were not achievable by vertical wells.
   a) Directional drilling
   b) Underbalance drilling
   c) Over balance drilling
   d) None of the above
2. In early days, the wells were mostly drilled in one direction which is in ______.
   a) Horizontal axis
   b) No need to drill
   c) Vertical axis
   d) None of the above
3. The position of a drill bit during directional drilling or any point in a directional well can be described by its ________.
   a) TVD
   b) MDT
   c) BHP
   d) None of the above

4. A multilateral well is a form of _______.
   a) Directional drilling
   b) Under balance drilling
   c) Over balance drilling
   d) None of the above
5. There is no _______ that a directional well can be assumed.
   a) Fixed target
   b) Fixed shape
   c) Fixed bit
   d) None of the above
6. Horizontal drilling involves drilling a well through a formation at well inclination of _____ from vertical.
   a) 30 degree
   b) 90 degree
   c) 120 degree
   d) None of the above
7. Horizontal wells ______ the exposure of pay zone.
   a) Increase
   b) Decrease
   c) nothing
   d) None of the above
8. Horizontal wells _____ the drawdown pressure in the well.
   a) Increases
   b) Averages
   c) Lowers
   d) None of the above
9. _______ is the angle by which the wellbore deviates from the vertical.
   a) Well inclination
   b) Well trajectory
   c) Well angle
   d) None of the above
10. ______ is the angle that occurs where the inclination of the borehole is held constant.
    a) Constant angle
    b) Stop angle
    c) Hold angle
    d) None of the above
11. ______ is the location where the borehole starts dropping inclination.
    a) Dipping
    b) Start of drop
    c) Declining
    d) None of the above

12. ______ is the distance between two points along a wellbore projected into a horizontal plane.
    a) TVD
    b) MD
    c) HD
    d) None of the above
13. ______ refers to the North Pole.
    a) Geographic north
    b) Grid north
    c) Magnetic north
    d) None of the above
14. The distance from the well surface reference point to the station of interest along the actual well path is called _______.
    a) TVD
    b) MD
    c) HD
    d) None of the above
15. The angle in the horizontal plane measured from a fixed reference direction is called _______.
    a) Azimuth
    b) Hold angle
    c) Well inclination
    d) None of the above
16. Horizontal drilling is usually applied to thin formation with good _______.
    a) Vertical permeability
    b) Horizontal permeability
    c) Porosity
    d) None of the above
17. In ________, high stresses in equipment and tubulars are common.
    a) Directional drilling
    b) Vertical drilling
    c) Horizontal drilling
    d) None of the above
18. ______ is difficult and less successful in horizontal drilling.
    a) Fishing
    b) Perforation
    c) Fracturing
    d) None of the above
19. _____ involves drilling of multiple branches of wellbores from a single wellbore.
    a) Directional drilling
    b) Multilateral drilling
    c) Vertical drilling
    d) Horizontal drilling

20. Drilling of a multilateral well consists of _______.
    a) Drilling of the main wellbore
    b) Sidetracking from the main wellbore
    c) Drilling the laterals
    d) All of the above
21. _______ is used for interventions in oil and gas wells.
    a) Coiled tubing drilling
    b) Extended reach drilling
    c) Foam drilling
    d) None of the above
22. ______ enables operators to more effectively find hydrocarbon pockets still untapped in the reservoir.
    a) Foam drilling
    b) Extended reach drilling
    c) Coiled tubing drilling
    d) None of the above
23. ________ has also been used as a cheaper version of work-over operations.
    a) Coiled tubing drilling
    b) Extended reach drilling
    c) Foam drilling
    d) None of the above
24. _______ utilizes a small rig and less voluminous surface equipment.
    a) Coiled tubing drilling
    b) Extended reach drilling
    c) Foam drilling
    d) None of the above
25. In _______, drilling is not interrupted for pipe connections.
    a) Extended reach drilling
    b) Coiled tubing drilling
    c) Foam drilling
    d) None of the above
26. In _______, the buckling of coiled tubing will occur if it becomes axially compressed.
    a) Horizontal wellbore
    b) Multilateral wellbore
    c) Vertical wellbore
    d) None of the above
27. Buckling in coiled tubing will happen if the axial compressive load ________ the buckling load in the vertical section.
    a) Exceeds
    b) Minimize
    c) Equals
    d) None of the above

28. _______ is the section of the drillstring below the drillpipe which helps maintain the well trajectory.
    a) MDT
    b) DST
    c) BHA
    d) None of the above
29. ________ are heavy and stiff steel tubular which are used at the bottom of a BHA to provide weight on bit.
    a) Heavy weight drillpipes
    b) Stabilizers
    c) Jars
    d) Drill collars
30. Key-seat Wiper can be run between the top drill collar and the bottom joint of HDWP where there is a problem of _______.
    a) Differential sticking
    b) Key seat
    c) Fluid loss
    d) None of the above
31. Which of the following is not the function of directional drilling?
    a) Drill many wells from one platform
    b) Drill in inaccessible locations
    c) Intercept many pay zones from single well
    d) None of the above
32. Which of the following is main advantage of directional drilling?
    a) Drill along thin reservoirs
    b) Reach deeper targets
    c) Drill HPHT reservoirs
    d) All of the above
33. If the pay zone is thin and long, which of the following well types is recommended?
    a) Vertical well
    b) Deviated well
    c) Horizontal well
    d) All of the above
34. If the pay zone is thick and beneath a village, which of the following well types is recommended?
    a) Vertical well
    b) Deviated well
    c) Horizontal well
    d) All of the above
35. If the pay zone is thick, which of the following well types is recommended?
    a) Vertical well
    b) Deviated well
    c) Horizontal well
    d) All of the above

36. Kick off point is defined as the point where
    a) The wellbore is deviated from the horizontal
    b) The wellbore is deviated to be vertical
    c) The wellbore is deviated from vertical
    d) None of the above
37. Long radius horizontal wells are the wells have radii of:
    a) Over 5000 *ft*
    b) 1000–3000 *ft*
    c) 250 – 500 *ft*
    d) Less than 250 *ft*
38. Ultra-short radius wells are the wells that have radii of
    a) Over 750 *ft*
    b) 500 – 750 *ft*
    c) 250 – 500 *ft*
    d) Less than 200 *ft*
39. Drag forces are higher in
    a) Long radius wells
    b) Medium radius wells
    c) Short radius wells
    d) All of the above
40. Hole cleaning is better in:
    a) Long radius wells
    b) Medium radius wells
    c) Short radius wells
    d) All of the above

Answers 1a, 2c, 3a, 4a, 5b, 6b, 7a, 8c, 9a, 10c, 11b, 12c, 13a, 14b, 15a, 16a, 17c, 18a, 19b, 20d, 21a, 22c, 23a, 24a, 25b, 26c, 27a, 28c, 29d, 30b, 31d, 32a, 33c, 34b, 35a, 36c, 37b, 38d, 39c, 40a.

10.4 Summary

In the past, directional drilling was difficult and costly and as a result failed to achieve wide industry acceptance, while today, it is relatively easier and more cost effective, and is the choice of production wells in many countries. Different workout examples and MCQs are presented in this chapter related to different concepts, definitions of directional drilling. The exercises are presented to enable readers to have more self-practice, and they can compare their results with the solutions in Appendix A. The answers to the self-practice MCQs exercises are presented in Appendix B.

10.5 Exercise and MCQs for Practice

10.5.1 Exercises (Solutions are in Appendix A)

Exercise 10.1: A well is planned to be drilled as a directional well. The well is kicked off from vertical section at point "O", and the drilling team found that the well has progressed 105 *meters* towards the west and 25 *meters* towards the south at point "A". After some time, the drilling team found that the well has progressed 178 *meters* towards the west and 99 *meters* towards the south at point "B" from the previous measuring point, "A". Determine the azimuth and horizontal departure at the first and second measuring points from the kick off point "O"? ***Answers: 256.6°, 107.9 m, 246.3°, 293.7 m.***

Exercise 10.2: A well is planned to be drilled as a directional well. The well azimuth and horizontal departure were designed to be 144.5° and 721 *meters*, respectively. After kick off the well from vertical section, measurements were conducted and found that the well progressed 511 *meters* towards the east and 338 *meters* towards the south. What is the azimuth at the current point? And if the azimuth is not the same as designed, what should be the new degree from the current point to be used until completing the well to reach the designed target? ***Answer: 136.8°.***

Exercise 10.3: A well is planned to be drilled as a directional well. The well azimuth and horizontal departure were measured at a point to be 63° and 600 *meters* from kick off point, respectively. Another measurement was obtained at the target point and azimuth and horizontal departure were measured to be 52° and 221 *m* from the previous measuring point, respectively. Determine the equivalent well azimuth and horizontal departure of the target from the kick off point. ***Answers: 60.0o, 818.0 m.***

Example 10.4: A well is planned to be drilled as a directional well. The well coordinates were designed to be 413 m towards the west and 688 *m* towards the north, respectively. After kick off the well, measurements were conducted and found that the current azimuth and horizontal departure were 324.5° and 341.3 *m*. Is this azimuth the same as the designed one? If the azimuth is not the same as designed, what should be the new degree from the current point to be used until completing the well to reach the designed target? ***Answer: 332.3°.***

Exercise 10.5: Calculate the dogleg "DL" and the dogleg severity "DLS" of a section in a well that has the following information:

	Measured depth	**Azimuth**	**Inclination**
Point	**ft**	**degrees**	**degrees**
1	2950	310	44
2	2300	312	58

Answers: 14.5°, 4.15°.

Exercise 10.6: A well is designed as a deviated well to reach a certain target with the maximum DLS of 2.5. If the design is to have KOP at 3,000 *ft*, an inclination angle of 48° at the end of curvature (EOC) depth of 4,750 *ft*, can the well be drilled with the above information? If the answer is no, at which depth should the KOP be moved? In addition, if based on geological information KOP depth must be at 3,000 *ft*, what will be the maximum inclination angle at the EOC depth? ***Answers: 2,830 m, 43.75°***.

Exercise 10.7: The following data refer to a directionally drilled well:

KOP = 2,750 *ft*
Northing coordinates of surface location = −4,350 *ft*
Easting coordinates of surface location = −5,550 *ft*

It is assumed that the co-ordinates of KOP are exactly similar to the co-ordinates of the surface. Five survey data were performed after the KOPs which are shown in the Table below:

Point	Azimuth	Inclination	MD
	Degrees	Degrees	ft
KOP	310.0	0.0	2,750
1	312.0	9.0	3,250
2	314.0	18.0	3,750
3	317.0	27.0	4,250
4	315.5	36.0	4,750
5	315.0	45.0	5,250

Using the radius of curvature method, calculate the well path between the above points?

Exercise 10.8: Using the data of exercise 10.7 and minimum curvature method, calculate the well path between the above points. Compare the results with that obtained in exercise 10.7.

Exercise 10.9: The data below refer to a deviated drilled well:

KOP = 5,250 *ft*
Northing coordinates of surface location = 900 *ft*
Easting coordinates of surface location = 500 *ft*

The co-ordinates of KOP were exactly similar to the co-ordinates of the surface. Five survey data were performed after the KOP which are shown in the Table below:

Point	Azimuth	Inclination	MD
	Degrees	Degrees	ft
KOP	156.0	0.0	5250
1	152.0	11.0	5900
2	154.0	25.0	6550
3	157.0	41.0	7200
4	156.0	58.0	7850
5	155.0	69.0	8500

Using the average angle method, calculate the well path between the above points.

Exercise 10.10: Using the data of exercise 10.9 and balanced tangential method, calculate the well path between the above points. Compare the results with that obtained in exercise 10.9.

10.5.2 Exercise

E10.1: The surface location of a rig is 10,125,000 *mN* and 3,050,000 *mW*. A reservoir rock is located 10,124,100 *mN* and 3,049,300 *mW*. The rig and reservoir have the same reference location. Determine the azimuth, horizontal departure, and coordinates of a well drilled from the surface location to the reservoir rock taking the rig location as a local reference point.

E10.2: Two wells drilled from the same surface location (point O) have the following well data:

Well A:	Azimuth	=	30 *degrees*
	Closure distance	=	831 feet
Well B:	Azimuth	=	105.5 *degrees*
	Closure distance	=	458 *meters*

Determine the following:

i. The shortest distance between the two wells
ii. The azimuth of well B (referenced to well A)
iii. If a third well C is to be drilled S25^{0}W from the surface location O such that it falls on same N/S coordinate with well B, determine the E/W coordinate and horizontal departure of this new well.

E10.3: Consider a coiled tubing of 2” OD, 1.688” ID and 8.6 *ppg* mud. Calculate the sinusoidal buckling load. Assume that that the hole size is 3.875”, at $\theta = 45°$

E10.4: Consider a coiled tubing of 2” OD, 1.688” ID and 8.6 *ppg* mud. Calculate the critical buckling load. Assume that that the hole size is 3.875”.

E10.5: Calculating the Position of a Survey Station Whilst drilling a deviated well, the Measured Depth, Inclination and Azimuth of the well are measured at station 23 (See survey data below). Calculate: 1. North and East co-ordinates 2. TVD vertical section 3. Dogleg severity of the next station according to the average angle method. The target bearing is 095°.

Station	**MD**	**INC.**	**AZI**	**N**	**E**	**TVD**	**VS**
22	3135	24.5	92	–30.78	344.60	3086.95	345.02
23	3500	25.5	92.5				

E10.6: A team of directional drillers are monitoring the progress of a horizontal well whose objective is to intersect a target reservoir at N35S from the well surface. The current location of the drill bit according to the last survey measurements is given below:

TVD	=	903 *feet*
MD	=	903 *feet*
Vertical section	=	3.9 *feet*
Coordinates: E/W	=	–0.7; N/S = 4.7
Inclination	=	1.5 *degrees*
Direction	=	N 6° E

The team decided to run another survey after 24 hours and the following survey data were obtained from the downhole MWD tools:

Inclination	=	1.25^0
Direction	=	N52E
MD	=	1, 245 feet

Determine the well location of this well 24 hours after the last survey using minimum curvature method and average angle method.

E10.7: It has been decided to sidetrack a well from 1,500 *ft*. The sidetrack will be a build and hold profile with the following specifications:

Target Depth: 10,000 *ft*.
Horizontal departure: 3,500 *ft*.
Build up Rate: 1.5° per 100 *ft*.

Calculate the following:

a. The drift angle of the well.
b) The TVD and horizontal deviation at the end of the build up section.
c) The total measured depth to the target.

E10.8: Using minimum curvature method, fill in the blank spaces in the Table below:

MD	Incl.	Azimuth	TVD	N/S	W/E
	1.5	202.50	1400	–1.21	–0.5
1500	3	202.50	1500		
	4.5	202.50		–10.88	–4.51

10.5.3 MCQs (Self-Practices)

1. Which of the following is not a major advantage of multilateral drilling technology?
   a) Decrease initial drilling cost
   b) Minimize drilling and completion equipment
   c) Control water production
   d) None of the above
2. Which of the following is the major challenge of multilateral drilling technology?
   a) Well intervention is difficult
   b) Well placement is difficult
   c) Well logging is difficult
   d) All of the above
3. What is the main challenge of extended reach drilling technology?
   a) Well log evaluation
   b) Excess drag and torque forces
   c) Well control
   d) None of the above
4. Which of the following is not the advantage of coiled tubing drilling technology?
   a) Live well intervention
   b) Quick mobilization and rig-up
   c) Reduce trip time
   d) None of the above
5. Which of the following is the application of coiled tubing technology?
   a) Squeeze cementing
   b) Drilling
   c) Fishing
   d) All of the above
6. What is the use of whipstock?
   a) Maintain the deviation angle
   b) Create window hole in the casing for sidetracking
   c) Plug the main hole for sidetracking
   d) All of the above

7. Which of the following equipment is used in directional drilling?
   a) Jet bit
   b) Whipstock
   c) Bent sub
   d) All of the above
8. Degree of curvature in directional drilling depends on mainly
   a) Type of downhole mud motor
   b) BHA length
   c) Bent sub angle
   d) All of the above
9. Which of the following is not part of the steerable drilling system?
   a) Drill collars
   b) Directional surveying tools
   c) Stabilizer
   d) Mud motor
10. Which of the following is part of the geosteering system?
   a) Survey system
   b) Drill collars
   c) Whipstock
   d) None of the above
11. When building the angle in directional drilling, drillstring should be
   a) Rotating at low speed
   b) Rotating at high speed
   c) Not rotating
   d) None of the above
12. When maintaining the angle in directional drilling, drillstring should be
   a) Rotating
   b) Off-bottom
   c) Not rotating
   d) None of the above
13. Which of the following techniques can be used to increase the rate of build of the angle?
   a) Increase the distance between stabilizers
   b) Use drill collars with small OD size
   c) Reduce rotary speed
   d) All of the above
14. Which of the following is one of the direction control principles?
   a) Fulcrum principle
   b) Stabilization principle
   c) Pendulum principle
   d) All of the above

15. In pendulum principle, the size of the near-bit stabilizer should be
    a) Overgauge
    b) Undergauge
    c) Same size as the drill bit
    d) None of the above

16. Selection of deviation survey tools doesn't depend on
    a) Maximum inclination
    b) Total well depth
    c) Limitation of the tool
    d) Formation fluid's type

17. All of the following are used as deviation survey tools except
    a) Magnetic survey tools
    b) LWD
    c) MWD
    d) None of the above

18. Which of the following survey tools is free of magnetic interference?
    a) Gyroscope tool
    b) Single-shot magnetic survey tool
    c) Multi-shot magnetic survey tool
    d) All of the above

19. Which of the following deviated sections need more directional survey readings in every short drilled lengths?
    a) Build section
    b) Tangent section
    c) Horizontal section
    d) All of the above

20. Which of the following data is not directly measured using survey tools?
    a) Inclination
    b) Azimuth
    c) TVD
    d) None of the above

21. Which of the following is the most accurate survey calculation method?
    a) Balanced tangential method
    b) Radius of curvature method
    c) Average angle method
    d) All of the above

22. Which of the following is the less accurate survey calculation method?
    a) Average angle method
    b) Radius of curvature method
    c) Minimum curvature method
    d) None of the above

23. Average angle method assumes ________ between the two survey stations.
    a) Smooth curve
    b) Zigzag line
    c) Straight line
    d) None of the above
24. The balanced tangential method assumes that the well path between the two survey stations can be approximated by ________
    a) Two lines of equal lengths
    b) Smooth curve
    c) One line tangent to the two survey stations
    d) Arc
25. Which of the following survey calculation methods assume smooth curve between the two survey stations?
    a) Average angle method
    b) Minimum curvature method
    c) Balanced tangential method
    d) Radius of curvature method
26. Which of the following survey calculation methods assume smooth arc between the two survey stations?
    a) Average angle method
    b) Minimum curvature method
    c) Balanced tangential method
    d) Radius of curvature method
27. What will happen when the drillstring is rotated during directional drilling?
    a) Inclination angle will increase
    b) Inclination angle will decrease
    c) Inclination angle will remains same
    d) None of the above
28. What will happen when mud circulation stops during directional drilling?
    a) Inclination angle will increase
    b) Inclination angle will decrease
    c) Inclination angle will remains same
    d) None of the above
29. Usually welded blade stabilizers are not recommended to be used in ________
    a) Hard formations
    b) Salt formations
    c) Soft formations
    d) All of the above
30. How multilateral wells minimize the initial well costs?
    a) Minimize the cost of vertical section
    b) Minimize surface facility costs
    c) Minimize the completion costs
    d) All of the above

10.6 Nomenclature

$A_{1,2}$	=	Measured Depth (the subscripts represent survey stations 1 and 2 respectively)
BHA	=	Bottom-hole assembly
BUR	=	Build up rate
C	=	Direction
CTD	=	Coiled tubing drilling
D	=	Course length between two survey stations, *ft*
DC	=	Drill collar
DLS	=	Dogleg severity, degrees/100 *ft*
DOR	=	Drop off rate
E	=	East total coordinate, *ft*
EOB	=	End of build
ERD	=	Extended Reach Drilling
HD	=	Horizontal displacement, *ft*
HWDP	=	Heavy weight drill pipe
KOP	=	Kick off point, *ft*
L	=	North-South course coordinate sometimes denoted by ΔNorth.
LWD	=	Logging while drilling
M	=	East-west course coordinate sometime denoted by ΔEast. Negative value means west
MD	=	Measured depth, *ft*
MWD	=	Measurement while drilling
N	=	North total coordinate
NMDC	=	Nonmagnetic drill collar
RF	=	Ratio factor
S	=	South total coordinate, *ft*
SNMDC	=	Short Nonmagnetic drill collar
TVD	=	True vertical depth, *ft*
VS	=	Vertical section, *ft*
W	=	West total coordinate, *ft*
β	=	Dogleg angle, degrees
α	=	Inclination, degrees
ε	=	Survey azimuth, direction of a course measured in a clockwise direction.

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11

Well Drilling Costs Analysis

11.1 Introduction

Well drilling costs analysis is needed in order to obtain the necessary authority for expenditures during the drilling phase. A systematic drilling cost analysis is done which reflects the different approximate itemized costs, guideline for costs, and information about the drilling project. In addition there are several reasons for producing a well cost which includes budgetary control, economics, partners recharging, and shareholders. The *Authorization for Expenditure* (AFE) is then used as a document for partners recharging, paying contractors and an overall control on well spending. This chapter discusses the factors affecting the drilling costs, types of costs, variables.ome typical examples are set to enhance the drilling costs estimation.ets of multiple choice question (MCQs) are also included which are related to drilling fluid technology and their problems and solutions. Workout examples related to mud engineering are extensively covered. Workout examples and MCQs are based on the writer's textbook, *Fundamentals ofustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

11.2 Different Mathematical Formulas and Examples

11.2.1 Authorization for Expenditure

Due to the huge financial involvement and high risk of having dry wells, the operator must prepare a spreadsheet, which is called the AFE. Normally each company will have its own customized AFE form. An example of an AFE for an offshore well in the Gulf of Mexico is shown in Table 11.1 (Hossain and Al-Majed, 2015).

Table 11.1 AFE for an offshore well in the Gulf of Mexico.

							AFE No.:	
							Original#:	
Lease:			Well No.:				Supplement#:	
Field:			County: Offshore		State: Louisiana			
Proposed TD:			Objective:					
Legal Location:						Budget period:		
Code		DHC to drill well to 24,000' md/tvd				Dry Hole Cost (BCP)	Completion Cost (ACP)	Total
BCP	ACP							
Estimated Intangible Drilling Cost								
		Surveys and permits				$0		$0
		Surface Damages						
		Location				$0		$0
		Location clean up	Rate	Days	Days			$0
		Rig move (mob & demob.)				$0		$0
		Drilling Cost – Turnkey						$0
		Drilling Cost – Day work	$219000	45		$9,855,000		$9,855,000
		Fuel/water (Rig/Boat)	$19,100	45		$859,500		$859,500
		Transportation (Boats/Air/Trucking)				$1,966,800		$1,966,800
		Rental tools (Equipment/Rental Repair)				$360,550		$360,550
		Bits				$226,880		$226,880
		Drilling mud/chemicals/mud engineer				$2,742,050		$2,742,050
		Mud and sample logging				$201,150		$201,150
		Dir. Drilling service/tools/motors /surveys				$511,815		$511,815
		MWD/LWD/PWD				$447,000		$447,000

		Cement and cementing services				$668,500		$668,500
		Casing crews and tools				$450,000		$450,000
		Fishing operations				$50,000		$50,000
		Logging – open hole				$0		$0
		Completion rig cost				$0		$0
		Perforating, case hole logging TCP, CBL				$0		$0
		Acidizing and fracturing (frac pack)				$0		$0
		Sand control GP				$0		$0
		Testing, BHP surveys, etc.				$0		$0
		Completion fluid and filtering				$0		$0
		Contract labour				$1,500		$1,500
		Rig supervisor +drilling engr.+opers./geol.				$268,200		$268,200
		Dock/dispatcher/communication/catering				$248,085		$248,085
		P&A/T&A				$0		$0
		Pipe inspection				$130,000		$130,000
		Overhead				$547,870		$547,870
		Insurance/Taxes				$0		$0
		Misc. (disposal/boat cleaning/ROV/true training/others)				$892,950		$892,950
		Contingencies				$561,567		$561,567
		Total Intangibles				**$20,989,398**	**$0**	**$20,989,398**
Estimated Tangible Drilling Cost								
		Drive pipe	369	36	$200.00	$73,730		$73,730
		Conductor	3,545	20	$71.40	$253,120		$253,120
		Surface casing	3,030	16	$71.00	$215,130		$215,130
		Intermediate casing	10,353	13 5/8	$60.41	$625,395		$625,395
		Drilling Liner	0	9 5/8	$36.56	$0		$0

(*Continued*)

Table 11.1 Cont.

Estimated Tangible Drilling Cost					
		Production liner		$0	$0
		Production casing			
		Production tubing		$0	$0
		Casing equipment/service/contingencies	$120,000		$120,000
		Wellhead Equipment/MLS Equipment	$660,000		$660,000
		Subsurface production equip. (Packers &CSSV & GP)			$0
		Pumping unit and installation			$0
		Rods and downhole pump			$0
		Tank Batteries			$0
		Separators, heaters, dehydrator etc.			$0
		Flow lines, fittings and connections			$0
		Caisson and/or protective structure			$0
		Contingencies	$0	$2,000	$2,000
		Total Tangibles	$1,949,375	$0	$1,949,375
Total Drilling and Completion Costs			$22,938,773		$22,938,773
Percent Working Interest			100.00%	100.00%	100.00%
Total Working Interest Well Cost			**$22,938,773**		**$22,938,773**
Approved:					

11.2.2 Drilling Cost Estimation

The overall well cost excluding the production is calculated as

$$C_{owc} = C_f + C_o \tag{11.1}$$

Here

C_{owc} = overall well cost excluding the production, *\$/ft*

C_f = drilling cost per unit depth, *\$/ft*

C_o = all other costs of making a foot of hole, such as casings, mud, cementing services, logging services, coring services, site preparation, fuel, transportation, completion, etc., *\$*

The drilling cost per foot for a bit is defined by the following formula:

$$C_f = \frac{C_b + C_r(t_d + t_c + t_t)}{\Delta D} \tag{11.2}$$

where

C_b = bit cost, *\$*

C_r = rig cost or fixed operating cost of the rig per unit time, *\$/hr*

ΔD = formation interval drilled or drilled footage, *ft*

t_d = drilling time or rotating time during the bit run, *hrs*

t_c = connection time or non-rotating time during the bit run, *hrs*

t_t = trip time, *hrs*

Equation (11.2) has several assumptions, such as, it ignores risk factors associated with drilling operations, inflation rate, costs of environmental effects, and the results of the cost analysis sometimes must be tempered with engineering judgement. Moreover, reducing the cost of a bit run will not necessarily result in lower well costs if the risk of encountering drilling problems such as stuck pipe, hole deviation, hole washout, etc., is increased greatly.

Example 11.1: The following table shows the bit performance of three bits for a sandstone formation at 10,000 *ft* depth. Determine which bit gives the lowest drilling cost if the fixed operating cost of the rig is \$500/hr, and the trip time is 8 hours.

Bit	Bit Cost (\$)	Total rotating time (*hrs*)	Total non-rotating time (*hrs*)	*ROP* (*ft/hr*)
A	950	25.5	0.25	12.0
B	1500	42.0	0.40	10.5
C	2000	70.5	0.7	8.5

Soultion:

Given data:

D = total depth = 10,000 *ft*

C_r = fixed operating cost of the rig per unit time = *\$500/hr*

t_t = trip time = 8 *hrs*

For bit A:

C_b = cost of bit = *$950.00*
t_d = rotating time during the bit run = 25.5 *hrs*
t_c = non-rotating time during the bit run = 0.25 *hr*
ROP = rate of penetration = 12.0 *ft/hr*

For bit B:

C_b = cost of bit = *$1,500.00*
t_d = rotating time during the bit run = 42.0 *hrs*
t_c = non-rotating time during the bit run = 0.40 *hr*
ROP = rate of penetration = 10.5 *ft/hr*

For bit C:

C_b = cost of bit = *$2,000.00*
t_d = rotating time during the bit run = 70.5 *hrs*
t_c = non-rotating time during the bit run = 0.70 *hr*
ROP = rate of penetration = 8.5 *ft/hr*

Required data:

C_{f_Lowest} = drilling cost per unit depth, *$/ft*

To calculate the lowest drilling cost, first of all we need to calculate the drilling footage for each bit which would be equal to individual ROP multiplied by rotating time. Now, Eq. (11.2) is used to calculate the cost per foot drilled for each type of bit. Thus

For bit A:

$$C_f = \frac{950 + 500(25.5 + 0.25 + 8)}{12 \times 25.5} = \mathbf{58.25\ \$/\mathit{ft}}$$

For bit B:

$$C_f = \frac{1500 + 500(42 + 0.40 + 8)}{10.5 \times 42} = 60.54\ \$/ft$$

For bit C:

$$C_f = \frac{2000 + 500(70.5 + 0.70 + 8)}{8.5 \times 70.5} = 69.42\ \$/ft$$

The lowest drilling cost is obtained using **Bit A**. This bit has lowest bit cost and lowest bit life with highest *ROP*.

Example 11.2: The following table shows the bit performance of three bits for a sandstone formation at 12,000 *ft* depth. Determine which bit gives the lowest drilling cost if the fixed operating cost of the rig is *$500/hr*, and the trip time is 8 hours.

Bit	Bit Cost ($)	Total rotating time (*hrs*)	Total non-rotating time (*hrs*)	ROP (*ft/hr*)
A	1000	24	1	15.0
B	1350	40	0.8	12.5
C	1750	65.5	0.5	9.5

Soultion:

Given data:

D = total depth = 12,000 *ft*
C_r = fixed operating cost of the rig per unit time = $500/*hr*
t_r = trip time = 8 *hrs*

For bit A:

C_b = cost of bit = $1,000.00
t_d = rotating time during the bit run = 24.0 *hrs*
t_c = non-rotating time during the bit run = 1 *hr*
ROP = rate of penetration = 15.0 *ft*/*hr*

For bit B:

C_b = cost of bit = $1,350.00
t_d = rotating time during the bit run = 40 *hrs*
t_c = non-rotating time during the bit run = 0.80 *hr*
ROP = rate of penetration = 12.5 *ft*/*hr*

For bit C:

C_b = cost of bit = $1,750.00
t_d = rotating time during the bit run = 65.5 *hrs*
t_c = non-rotating time during the bit run = 0.50 *hr*
ROP = rate of penetration = 9.5 *ft*/*hr*

Required data:

C_{f_lowest} = drilling cost per unit depth, *$/ft*

Now, Eq. (11.2) is used to calculate the cost per foot drilled for each type of bit. Thus,

For bit A:

$$C_f = \frac{1000 + 500(24 + 1 + 15)}{15.0 \times 24} = 58.33 \text{ \$/ft}$$

For bit B:

$$C_f = \frac{1350 + 500(40 + 0.8 + 12.5)}{12.5 \times 40} = \mathbf{56.0\ \$\,/\,ft}$$

For bit C:

$$C_f = \frac{1750 + 500(65.5 + 0.5 + 9.5)}{9.5 \times 65.5} = 64.0 \text{ \$/ft}$$

The lowest drilling cost is obtained using **Bit B**.

Drilling costs tend to increase exponentially with depth. It is a good strategy for drilling engineers to be dependent on previous data to estimate drilling time and cost for future operations. When enough data are available for a certain region, curve-fitting drilling cost data can be generated. Thus, it is often convenient to assume a relationship between total well cost, C_{dc}, and depth, D, given by

$$C_{dc} = a_{dc} e^{b_{dc} D} \tag{11.3}$$

where
C_{dc} = drilling cost, $
a_{dc} = constant depend on well location, $
b_{dc} = constant depend on well location, ft^{-1}
D = total depth, *ft*

Example 11.3: While analyzing the historical well cost data of 20 wells from Gulf of Mexico, it was estimated that a*dc* is USD 735,000 and *bdc* is 0.000035 ft^{-1}. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 12,000 *ft* and 15,000 *ft*, respectively. Use the exponential cost estimation formula.

Soultion:

Given data:
a_{dc} = constant depend on well location = $735,000
b_{dc} = constant depend on well location = 0.000035 ft^{-1}
D = total depth = 12,000 *ft* and 15,000 *ft*

Required data:
C_{dc} = drilling cost, $

To calculate the drilling cost, Eq. (11.3) is used to calculate for each depth as,

$$C_{dc} = 735000 e^{0.000035 D}$$

For D = 12,000 *ft*:

$$C_{dc} = 735000 e^{0.000035 \times 12000} = 1.11 \ million \ USD$$

For D = 15,000 *ft*:

$$C_{dc} = 735000 e^{0.000035 \times 15000} = 1.57 \ million \ USD$$

Example 11.4: While analyzing the historical well cost data of 20 wells from Gulf of Mexico, it was estimated that a_{dc} is USD 800,000 and b_{dc} is 0.000045 ft^{-1}. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 15,000 *ft* and 18,000 *ft* respectively. Use the exponential cost estimation formula.

<u>Soultion</u>:

Given data:

a_{dc} = constant depend on well location = $800,000

b_{dc} = constant depend on well location = 0.000045 ft^{-1}

D = total depth = 15,000 *ft* and 18,000 *ft*

Required data:

C_{dc} = drilling cost, $

To calculate the drilling cost, Eq. (11.3) is used to calculate for each depth as

$$C_{dc} = 800000e^{0.000045D}$$

For D = 15,000 *ft*:

$$C_{dc} = 800000e^{0.000045\times15000} = \boldsymbol{1.57\ million\ USD}$$

For D = 18,000 *ft*:

$$C_{dc} = 800000e^{0.000045\times18000} = \mathbf{1.80}\ \boldsymbol{million\ \ USD}$$

Example 11.5: While analyzing the historical well cost data of 20 wells from Gulf of Mexico, it was estimated that a_{dc} is USD 735,000 and b_{dc} is 0.000035 ft^{-1}. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 12,000 *ft* and 15,000 *ft*, respectively. Use the exponential cost estimation formula.

<u>Soultion</u>:

Given data:

a_{dc} = constant depend on well location = $735,000

b_{dc} = constant depend on well location = 0.000035 ft^{-1}

D = total depth = 10,000 *ft* and 12,000 *ft*

Required data:

C_{dc} = drilling cost, $

To calculate the drilling cost, Eq. (11.3) is used to calculate for each depth as,

$$C_{dc} = 735000e^{0.000035D}$$

For D = 10,000 *ft*:

$$C_{dc} = 735000e^{0.000035\times10000} = 1.04\ million\ USD$$

For D = 15,000 *ft*:

$$C_{dc} = 735000e^{0.000035\times12000} = 1.11\ million\ USD$$

11.2.3 Well Drilling Time Estimation

The estimation of drilling and completion time is a dependent variable which is governed by different activities while drilling. An accurate estimate of the time is necessary to drill the well before preparing an AFE. Well drilling time is estimated based on the basis of rig-up and rig down time, drilling time, trip time, casing placement time, formation evaluation and borehole survey time, completion time, non-productive time, and trouble time. A typical example of an actual time distribution for operations in a deep-water well in Gulf of Mexico is shown in Table 11.2 (Hossain and Al-Majed, 2015). The following example gives an idea of how a drilling time estimate is prepared.

Example 11.6: Find out the time required to drill a hole where the following data are assumed and taken from the three wells, Gulf of Mexico area:

36″ Hole for a *30″* Conductor which is 500 *ft* long from surface
26″ Hole for a *20″* Casing which is 1,500 *ft* long from *30″* casing
17.*5″* Hole for a 13.*375″* Casing which is 3,000 *ft* long from *20″* casing
12.*25″* Hole for a 9.*625″* Casing which is 6,000 *ft* long from 13.*375″* casing
8.*5″* Hole for a *7″* Casing which is 3,000 *ft* long from 9.*625″* casing
Total depth is 14,000 *ft* long

From three offset wells, the following data was established for average ROP for each hole section: *36″* hole 15 *ft*/*hr*, *26″* hole 15 *ft*/*hr*, 17.*5″* hole 27.5 ft/*hr*, 12.*25″* hole 14 *ft*/*hr*, and 8.*5″* hole 9 *ft*/*hr*. The expected flat times for this well are shown in Table 11.3. Calculate the total drilling time and plot the depth-time curve.

Solution:

Given data:

As stated in the question and in Table 11.3.

Table 11.2 Time Distribution for Gulf of Mexico Deep-water Well

Operation Description	Days	Percentage
Normal operation (except drilling)	44.40	37
Drilling	34.80	29
Lost time – operation problems	14.40	12
Lost time – service company equipment	3.60	3
Lost time – rig equipment	3.60	3
Weather-related problems	9.60	8
Plugging and abandoning	3.60	3
Rig moving and positioning	6.00	5
Total	**120.00**	**100.00**

Table 11.3 Estimated Flat Time

Casing size	Running and cementing	NU (days)	Total (days)
30″	1.0	2.5	3.5
20″	1.5	2.0	3.5
13 3/8″	1.5	2.5	4.0
9 5/8″	1.5	1.5	3.0
7″	2.5	3.5	6.0
Total			**20.0**

Table 11.4 Calculations of planned drilling time for Example 11.1

Hole size	Feets to drill (A)	Offset ROP (B) (ft/hr)	Planned hours (A/B)	Planned drilling days
36″	500	15.0	33.33	1.39
26″	1500	15.0	100.0	4.17
17.5″	3000	27.5	109.09	4.55
12.25″	6000	14.0	428.57	17.86
8.5″	3000	9.0	333.33	13.89
Total			**1004.32**	**41.86**

Required data:

i) total drilling time, *days*
ii) depth vs. time curve

First of all, we need to calculate the times required to drill each hole section using *ROP*s from the three offset wells data. At this stage it is wise to use the best *ROP* values from offset wells. This is because it is always possible to match or exceed previous performance if similar or better equipment is used. Indeed, some engineers may increase the possible *ROP* for the new well if it is known that high quality and up-to-date equipment will be used on the new well. Hence at this stage all drilling time estimates must not include allowance for down time. Therefore, using the given raw data, Table 11.4 is constructed for the planned drilling days.

Table 11.5 is developed using the data from Table 11.4 and the flat times from Table 11.3. A time-depth curve is constructed using data from Table 11.5 which is shown in Figure 11.1. The graph shows the planned time-depth curve as estimated through this example. However, during drilling, actual drilling times can be plotted on the same graph to compare actual performance against planned performance.

Table 11.5 Calculations of planned drilling time for Example 11.1

Operation Description	**Measured Depth**	**Activity (days)**	**Cumulative days**
Rig up to drilling operation	–	1.0	1.0
Drill *36″* hole up to 500 *ft* from the surface	500	1.39	2.39
Run cmt *30″* conductor casing/NU diverter	–	3.5	5.89
Drill *26″* hole up to 2,000 *ft* from the surface	2000	4.17	10.06
Run cmt *20″* casing and/NU wellhead	–	3.5	13.56
Drill 17.*5″* hole up to 5,000 *ft* from the surface	5000	4.55	18.11
Log the hole	–	1.0	19.11
Run cmt 13 3/*8″* casing and/ NU	–	4.0	23.11
Drill 12.*25″* hole up to 11,000 *ft* from the surface	11000	17.86	40.97
Log the 12 ½" hole	–	1.0	41.97
Run cmt 9 5/*8″* casing and/NU	–	2.5	44.47
Drill 8.*5″* hole up to 14,000 *ft* from the surface	14000	13.89	58.36
Log the 8 ½" hole (full open hole logging)	–	5.0	63.36
Run cmt *7″* liner, run CBL/VDL	–	4.0	67.36
Displace hole to completion fluids, prepare well for testing	–	2.0	69.36
Total days			**70.00**

It is noted that Example 11.3 shows the time estimate based on major operations such as drilling a hole section where drilling time, casing, tripping, circulating, making BHA, etc., are taken together. A detailed time estimate can be prepared for each hole section by considering the individual operation involved. This exercise requires experience of the engineer and also detailed knowledge of previous drilling experience in the area. The consultant or expertise of the drilling contractor normally performs the drilling operation. A new drilling engineer or crew will be on a learning level only.

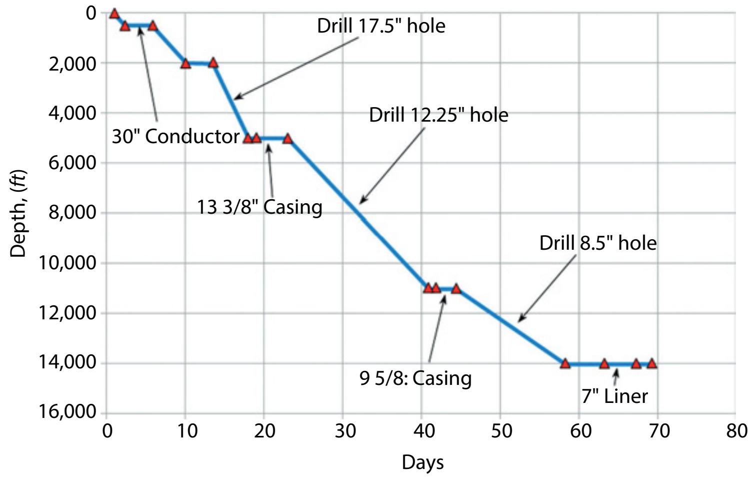

Figure 11.1 Time versus depth curve showing different stages of drilling activities.

Drilling Time Estimation

An estimation of drilling time can be based on historical *ROP* data where drilling program will be set for the area of interest. Under these conditions, *ROP* can be related to depth, *D* as:

$$\frac{dD}{dt} = Ke^{-AD} \tag{11.4}$$

where

$\frac{dD}{dt}$ = rate of penetration, *ft*/*hr*

K = constant, *ft*/*hr*

A = constant, ft^{-1}

D = total depth, *ft*

It is noted that constant A and K of Eq. (11.4) must be determined from the previous field data. Now, the drilling time can be obtained by integrating and solving the Eq. (11.4) for a given depth as:

$$K\int_0^{t_d} dt = \int_0^D e^{AD}\, dD \tag{11.5}$$

Equation (11.5) is a definite integral which is solved for t_d and the final form of the equation becomes as:

$$t_d = \frac{1}{AK}(e^{AD} - 1) \tag{11.6}$$

Equation (11.4) can also be used to determine for a given bit, *i* to drill from a depth of D_i to a depth of D_{i+L} which may be given as:

$$K\int_0^{t_{di}} dt = \int_{D_i}^{D_{i+L}} e^{AD} dD \tag{11.7}$$

Equation (11.7) can be solved for drilling time of a given bit, *i* as

$$t_{di} = \frac{1}{AK}(e^{AD_{i+L}} - e^{AD_i}) \tag{11.8}$$

where

t_{di} = drilling time for a given bit *i*, *hrs*

D_i = depth of interest from where drilling time would be measured for a given bit, *ft*

D_{i+L} = depth of interest up to where drilling time would be measured for a given bit, *ft*

The drilling time required to drill from D to $(D + 1000)$ can be obtained using Eq. (11.6) as:

$$t_{d'} = \frac{1}{AK}[\{e^{A(D+1000)} - 1\} - \{e^{AD} - 1\}] \tag{11.9}$$

where

$t_{d'}$ = drilling time required to drill from D to $(D + 1000)$, *hr*

Equation (11.9) reduces to:

$$t_{d'} = \frac{e^{AD}}{AK}[e^{1000A} - 1] \tag{11.10}$$

Trip Time Estimation

Well drilling time can be approximated using the following relation as

$$t_t = 2\left(\frac{\bar{t}_s}{\bar{l}_s}\right)\bar{D}_t \tag{11.11}$$

where

t_t = trip time required to change a bit and resume drilling operations, *hrs*

$\bar{t}_s$ = the average time required to handle one stand of drillstring, *hrs*

$\bar{l}_s$ = the average length of one stand of drillstring, *ft*

$\bar{D}_t$ = the mean depth where the trip was made (i.e., mean depth at the trip level), *ft*

It is noted that the time required to handle the drill collars is greater than for the rest of the drillstring, but this difference usually does not warrant the use of an additional term in Eq. (11.11). Historical data for the rig of interest are needed to determine, $\overline{t_s}$.

Number of Bit Estimation

Mathematically, the number of bits required for a given depth can be expressed as:

$$N_b = \frac{t_d}{t_{bl}} \tag{11.12}$$

where
N_b = numbers of bit, nos.
t_{bl} = average bit life for a particular depth, *hrs*

Connection Time Estimation

The third important time required for drilling operations is the connection time which can be calculated as:

$$t_c = N_s \times \overline{t_s} \tag{11.13}$$

where
N_s = numbers of average stands of drillstring, nos.

The number of average stands of the drillstring can be calculated as the total drill length over the average length of one stand of drillstring, i.e.,

$$N_s = \frac{\overline{D}}{\overline{l_s}} \tag{11.14}$$

where
$\overline{D}$ = mean depth of the well, *ft*

Total cost of the bit used for the planned well can be calculated as:

$$C_{Tb} = C_b \times N_b \tag{11.15}$$

where
C_{Tb} = total cost of the bits used for the whole drilling operations, *\$*

Total cost of the rig paid as rent for the whole drilling operation can be calculated as:

$$C_{Tr} = C_r \times t_d \tag{11.16}$$

where
C_{Tr} = total cost of the rig paid as rent for the whole drilling operation, *\$*

Coring Cost Estimation

While drilling, one of the objectives is to get the core sample for inspection.o, it is important to find out the coring cost per foot and the total coring time. Core recovery is given as the ratio length of the core recovered to the length of the core cut. It is usually expressed as a percentage.imilar to drilling costs, coring costs per foot can be given as:

$$C_{cf} = \left\{ \frac{C_{cb} + C_r(t_{ct} + t_{cr} + t_{cc} + t_{crl} + t_{co})}{\Delta D_{ROP}} \right\} \frac{1}{R_c} \tag{11.17}$$

where

C_{cf} = coring costs per foot, *\$/ft*
C_{cb} = core bit cost, *\$*
t_{ct} = core trip time, *hrs*
t_{cr} = core rotating time, *hrs*
t_{cc} = core connection time, *hrs*
t_{crl} = core recovery, and laying down of core barrel time, *hrs*
t_{co} = coring time, *hrs*
R_c = core recovery factor, %
ΔD_{ROP} = the formation depth where the coring will be done (a function of rate of penetration), *ft*

ΔD_{ROP} can be calculated as:

$$\Delta D_{ROP} = \int_0^{t_{cr}} ROP\ dt \tag{11.18}$$

where
ROP = rate of penetration, *ft/hr*

Example 11.7: A well is planned to drill up to 20,000 *ft* where casing settings are designed at 800 *ft*, 5,000 *ft*, 8,000 *ft* 15,500 *ft* and 18,000 *ft*. The rental cost of the rig is \$15,000/day and tripping time per stand is considered as 3.0 min. The bits used at different phases are given at Table 11.6 and the control well data for this region is given at Table 11.7. The formation constants for the third phase are given by $K = 320$ *ft/hr* and $A = 0.00034\ ft^{-1}$. Find out the cost to drill between 9,500 *ft* and 10,500 *ft* in *\$/ft* and the total rig rental cost.

Soultion:

Given data:

D_T = total vertical depth = 20,000 *ft*
D_{cs} = casing setting depth = 800 *ft*, 5,000 *ft*, 15,500 *ft*, and 18,000 *ft*
C_r = rig rental costs = \$15,000/day = *\$625/hr*
$\bar{t}_s$ = average time required to handle one stand of drillstring = 3.0 min = 0.05 *hrs*

K = Constant = 320 ft/hr
A = Constant = 0.00034 ft^{-1}

$$\bar{D}_t = \text{mean depth at the trip level} = \frac{(D+\Delta D)+D}{2}$$

$$= \frac{(9500+1000)+9500}{2} = 10{,}000 \text{ ft}$$

Other data are given in Table 11.6 and Table 11.7.

Required data:
C_{dc} = drilling cost, $

First of all, we need to calculate the drilling time required to drill from 9,500 ft to 10,500 ft by using Eq. (11.10) as:

$$t_{d'} = \frac{e^{AD}}{AK}[e^{1000A} - 1] = \frac{e^{0.00034\times 9500}}{0.00034\times 320}[e^{1000\times 0.00034} - 1]$$
$$= 94.09 \; hrs$$

Table 11.6 Bit use description.

Phase	Size (in)	Average life (hrs)	Cost ($)
1	22	45	4,500
2	17.5	36	3,400
3	13.5	26	2,600
4	8.75	22	2,250
5	5.75	12	1,700

Table 11.7 Well data.

D	9,500
ΔD	1,000
$\bar{t}_s$	3.0 min
$\bar{l}_s$	94 ft
t_{bl}	26 hrs
C_r	$15,000/day
C_b	$2,200

So, number of bits can be calculated using Eq. (11.12) where drilling time is considered as the time required to drill from 9,500 *ft* to 10,500 *ft* as:

$$N_b = \frac{t_{d'}}{t_{bl}} = \frac{94.09}{26} = 3.62 \text{ bits} \approx 4 \text{ bits}$$

(i.e., we have to use 4 bits)

Now the trip time per bit can be calculated using Eq. (11.11) as:

$$t_t = 2\left(\frac{\bar{t}_s}{\bar{l}_s}\right)\bar{D}_t = 2\left(\frac{0.05}{94.0}\right) \times 10{,}000 = 10.6 \text{ hrs}$$

Therefore total trip time will be = 10.6 *hrs* × 3.62 bits = 38.37 *hrs*

To calculate the connection time, first of all we need to determine the average numbers of stands needed for the operations which can be calculated using Eq. (11.14) as:

$$N_s = \frac{\bar{D}}{\bar{l}_s} = \frac{10{,}000}{94.0} = 106.38 \text{ nos.}$$

So, the connection time is obtained using Eq. (11.13) as:

$$t_c = N_s \times \bar{t}_s = 106.38 \times 0.05 = 5.32 \text{ hrs}$$

The total cost of the bits used for this planned are obtained using Eq. (11.15) as:

$$C_{Tb} = C_b \times N_b = 2{,}600 \times 4 = 10{,}400 \text{ dollars}$$

Now the cost per foot to drill from 9,500 *ft* to 10,500 *ft* (i.e., 1,000 *ft*) is calculated by using Eq. (11.2) where all three bits are being used as:

$$\begin{aligned} C_f &= \frac{C_b + C_r(t_d + t_c + t_t)}{\Delta D} \\ &= \frac{10400 + 625(94.09 + 38.37 + 5.32)}{1000} \\ &= \mathbf{96.51\ \$/\mathit{ft}} \end{aligned}$$

Total cost of the rig paid as rent for the whole drilling operation can be calculated using Eq. (11.16) where only 1,000 *ft* drilling time is used as:

$$C_{Tr} = C_r \times t_d = 625 \times 94.09 = \mathbf{58{,}806.25\ dollars}$$

Example 11.8: A coring job was planned from 15,600 *ft* to 15, 620 *ft*. Calculate the coring cost per foot with the following data:

Core bit cost	=	\$3,000
Rig cost	=	\$14,000 /*day*
Core trip time	=	12 *hrs*
Core rotating time	=	8 *hrs*
Connection time while coring	=	1 *hrs*
Core recovery, and laying down of core barrel time	=	1*hr*
Coring time	=	1.5 *hrs*
Core recovery factor	=	90%

Soultion:

Given data:

C_{cb} = core bit cost = \$3,000
C_r = rig cost = \$14,000/day = \$583.33/*hr*
t_{ct} = core trip time = 12 *hrs*
t_{cr} = core rotating time = 8 *hrs*
t_{cc} = core connection time = 1 *hr*
t_{crl} = core recovery, and laying down of core barrel time = 1 *hr*
t_{co} = coring time = 1.5 *hr*
R_c = core recovery factor = 0.90

Required data:

C_{cf} = coring cost, *\$/ft*

Equation (11.17) is used to calculate the core drilling cost as

$$C_{cf} = \left\{ \frac{C_{cb} + C_r(t_{ct} + t_{cr} + t_{cc} + t_{crl} + t_{co})}{\Delta D_{ROP}} \right\} \frac{1}{R_c}$$

$$= \left\{ \frac{3000 + 583.33(12 + 8 + 1 + 1 + 1.5)}{20} \right\} \times \frac{1}{0.90} = \mathbf{928.24\ \$/\,ft}$$

Example 11.9: A coring job was planned from 14,600 *ft* to 14,650 *ft*. Calculate the coring cost per foot with the following data:

Soultion:

Given data:

C_{cb} = core bit cost = \$3,500
C_r = rig cost = \$16,000/day = \$666.67/*hr*
t_{ct} = core trip time = 12 *hrs*
t_{cr} = core rotating time = 8 *hrs*
t_{cc} = core connection time = 1 *hr*
t_{crj} = core recovery, and laying down of core barrel time = 1 *hr*
t_{co} = coring time = 1.5 *hr*
R_c = core recovery factor = 0.90

Required data:
C_{cf} = coring cost, $

Equation (11.17) is used to calculate the core drilling cost as,

$$C_{cf} = \frac{C_{cb} + C_r(t_{ct} + t_{cr} + t_{cc} + t_{crj} + t_{co})}{\Delta D_{ROP}} \frac{1}{R_c}$$
$$= \frac{3500 + 666.67(12 + 8 + 1 + 1 + 1.5)}{50} \times \frac{1}{0.90} = \mathbf{425.92\ \$/\,ft}$$

Example 11.10: A coring job was planned from 14,600 *ft* to 14,650 *ft*. Calculate the coring cost per foot with the following data:

<u>Soultion</u>:

Given data:
C_{cb} = core bit cost = $2,500
C_r = rig cost = $12,000/day = $500/*hr*
t_{ct} = core trip time = 12 *hrs*
t_{cr} = core rotating time = 8 *hrs*
t_{cc} = core connection time = 1 *hr*
t_{crj} = core recovery, and laying down of core barrel time = 1 *hr*
t_{co} = coring time = 1.5 *hr*
R_c = core recovery factor = 0.90

Required data:
C_{cf} = coring cost, $

Equation (11.17) is used to calculate the core drilling cost as,

$$C_{cf} = \frac{C_{cb} + C_r(t_{ct} + t_{cr} + t_{cc} + t_{crj} + t_{co})}{\Delta D_{ROP}} \frac{1}{R_c}$$
$$= \frac{2500 + 500(12 + 8 + 1 + 1 + 1.5)}{50} \times \frac{1}{0.90} = \mathbf{316.667\ \$/\,ft}$$

11.2.4 Future Value Estimation

Future value is the value of an asset or cash at a specified date in the future that is equivalent in value to a specified sum today. The future value is determined as:

$$V_f = V_p \left(1 + \frac{i}{m}\right)^{nm} \tag{11.18}$$

where
V_f = future value, $
V_p = present value, $

i = nominal interest rate per year or growth rate in fraction, $
m = number of interest compounding per year or the number of payments per year, nos.
n = number of years, nos.

For the case of continuous compounding, which is commonly used within the petroleum producing industry, the relationship between the present and the future values becomes:

$$V_f = V_p(e^{in}) \tag{11.19}$$

The present value of a future payment is the core for the time value of money. The mathematical relationships for the other formulas are derived from this concept. For example, the annuity formula is the sum of a series of present value calculations. The present value formula can be written as:

$$V_p = V_f(1+i)^{-m} \tag{11.20}$$

The expected value is calculated as

$$V_{ex} = \sum_j^k P_j C_j \tag{11.21}$$

where
V_{ex} = expected value, $
P_j = provability of the j^th event
C_j = cost of the j^th event, $

Example 11.11: Marathon Oil Company invested money for buying some drillpipes six years ago for $8,000 and did not use it. What would its present value be today? Assume that the average rate of return for the past six years is 12% and the interest period is 4 times per year.

Soultion:

Given data:
V_p = present value = $8,000
i = interest rate per year = 12% = 0.12
m = the number of payments per year = 4 nos.
n = number of years = 6 yrs.

Required data:
V_f = future value, $

Equation (11.18) is used to calculate the future value (i.e., the present costs of the drillpipe as of today which was bought six years ago) as

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm} = 8000\left(1+\frac{0.12}{4}\right)^{6\times 4} = \mathbf{\$16,262.35}$$

Example 11.12: Marathon Oil Company invested money for buying some drillpipes six years ago for $9,500 and did not use it. What would its present value be today? Assume that the average rate of return for the past six years is 12% and the interest period is 4 times per year.

Soultion:

Given data:

V_p = present value = $9,500
i = interest rate per year = 12% = 0.12
m = the number of payments per year = 4 nos.
n = number of years = 6 yrs.

Required data:

V_f = Future Value, $

Equation (11.18) is used to calculate the future value (i.e., the present costs of the drill-pipe as of today which was bought six years ago) as,

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm} = 9500\times\left(1+\frac{0.12}{4}\right)^{6\times 4} = \$19311.54$$

Example 11.13: Marathon Oil Company invested money for buying some drill pipes six years ago for $7,500 and did not use it. What would its present value be today? Assume that the average rate of return for the past six years is 16% and the interest period is 2 times per year.

Soultion:

Given data:

V_p = present value = $7,500
i = interest rate per year = 16% = 0.16
m = the number of payments per year = 2 nos.
n = number of years = 6 yrs.

Required data:

V_f = Future Value, $

Equation (11.18) is used to calculate the future value (i.e., the present costs of the drill-pipe as of today which was bought six years ago) as,

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm} = 7500\times\left(1+\frac{0.16}{2}\right)^{6\times 2} = \mathbf{\$18886.275}$$

11.2.5 Price Elasticity

Price elasticity is used to measure the effect of economic variables such as demand or supply of rigs or wells drilled with respect to change in the crude oil price. It enables one to find out how sensitive one variable is with the other one, and it is also independent of units of measurement. It is the ratio of the percentage of the change of wells and footage drilled to the percentage change in the crude price. It describes the degree of responsiveness of the rig in demand or rig in supply to the change in the crude price.o drilling price elasticity can be obtained as

$$E_d = \frac{R}{P_{oil}} \tag{11.22}$$

where
E_d = drilling price elasticity
R = percentage change in drilling wells
P_{oil} = percentage change in the crude price

Example 11.14: The following table (Table 11.8) shows the bit performance of three bits for a sandstone formation at 12,000 *ft* depth. Determine which bit gives the lowest drilling cost if the fixed operating cost of the rig is *$500/hr*, and the trip time is 8 *hrs*.

Soultion:

Given data:
D = total depth = 12,000 *ft*
C_r = fixed operating cost of the rig per unit time = *$500/hr*
t_r = trip time = 8 *hrs*

For bit A:
C_b = cost of bit = *$*1,000.00
t_d = rotating time during the bit run = 24.0 *hrs*
t_c = non-rotating time during the bit run = 1 *hr*
ROP = rate of penetration = 15.0 *ft/hr*

For bit B:
C_b = cost of bit = *$*1,350.00
t_d = rotating time during the bit run = 40 *hrs*
t_c = non-rotating time during the bit run = 0.80 *hr*
ROP = rate of penetration = 12.5 *ft/hr*

Table 11.8 Data for Example 11.14.

Bit	**Bit Cost** *$*	**Total rotating time (hrs)**	**Total non-rotating time (hrs)**	***ROP*** **(ft/hr)**
A	1000	24	1	15.0
B	1350	40	0.8	12.5
C	1750	65.5	0.5	9.5

For bit C:
C_b = cost of bit = $ 1750.00
t_d = rotating time during the bit run = 65.5 *hrs*
t_c = non-rotating time during the bit run = 0.50 *hr*
ROP = rate of penetration = 9.5 *ft/hr*

Required data:
C_{f_lowest} = drilling cost per unit depth, *$/ft*

Now, Eq. (11.2) is used to calculate the cost per foot drilled for each type of bit. Thus,

For bit A:

$$C_f = \frac{1000 + 500(24 + 1 + 15)}{15.0 \times 24} = 58.33 \text{ \$/ft}$$

For bit B:

$$C_f = \frac{1350 + 500(40 + 0.8 + 12.5)}{12.5 \times 40} = 56 \text{ \$/ft}$$

For bit C:

$$C_f = \frac{1750 + 500(65.5 + 0.5 + 9.5)}{9.5 \times 65.5} = 64 \text{ \$/ft}$$

Example 11.15: While analyzing the historical well cost data of 20 wells from the Gulf of Mexico, it was estimated that a_{dc} is *$735,000* and b_{dc} is 0.000035 ft^{-1}. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 12,000 *ft* and 15,000 *ft* respectively. Use the exponential cost estimation formula.

<u>Soultion</u>:

Given data:
a_{dc} = constant depend on well location = *$735,000*
b_{dc} = constant depend on well location = 0.000035 ft^{-1}
D = total depth = 12,000 *ft* and 15,000 *ft*

Required data:
C_{dc} = drilling cost, *$*

To calculate the drilling cost, Eq. (11.3) is used to calculate for each depth as,

$$C_{dc} = 735000e^{0.000035D}$$

For D = 12,000 *ft*:

$$C_{dc} = 735000e^{0.000035 \times 12000} = 1.11 \; million \; USD$$

For D = 15,000 *ft*:

$$C_{dc} = 735000e^{0.000035\times 15000} = 1.57\ million\ USD$$

Example 11.16: A coring job was planned from 14,600 *ft* to 14,650 *ft*. Calculate the coring cost per foot with the following data:

<u>Soultion</u>:

Given data:

C_{cb} = core bit cost = \$3,500
C_r = rig cost = \$16,000/day = \$666.67/*hr*
t_{ct} = core trip time = 12 *hrs*
t_{cr} = core rotating time = 8 *hrs*
t_{cc} = core connection time = 1 *hr*
t_{crj} = core recovery, and laying down of core barrel time = 1 *hr*
t_{co} = coring time = 1.5 *hr*
R_c = core recovery factor = 0.90

Required data:

C_{cf} = coring cost, \$

Equation (11.17) is used to calculate the core drilling cost as,

$$\begin{aligned} C_{cf} &= \frac{C_{cb} + C_r(t_{ct} + t_{cr} + t_{cc} + t_{crj} + t_{co})}{\Delta D_{ROP}} \frac{1}{R_c} \\ &= \frac{3500 + 666.67(12 + 8 + 1 + 1 + 1.5)}{50} \times \frac{1}{0.90} \\ &= \mathbf{425.92\ \$/\mathit{ft}} \end{aligned}$$

Example 11.17: Marathon Oil Company invested money for buying some drill pipes six years ago for \$9,500 and did not use it. What would be the present value as of today? Assume that the average rate of return for the past six years is 12% and the interest period is 4 times per year.

<u>Soultion</u>:

Given data:

V_p = present value = \$9,500
i = interest rate per year = 12% = 0.12
m = the number of payments per year = 4 nos.
n = number of years = 6 yrs.

Required data:

V_f = Future Value, \$

Equation 11.18 is used to calculate the future value (i.e., the present costs of the drill-pipe as of today which was bought six years ago) as,

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm} = 9500\times\left(1+\frac{0.12}{4}\right)^{6\times 4} = \mathbf{\$19311.54}$$

Example 11.18: The following table (Table 11.9) shows the bit performance of three bits for a sandstone formation at 10,000 *ft* depth. Determine which bit gives the lowest drilling cost if the fixed operating cost of the rig is *$300/hr*, and the trip time is 8 hours.

Soultion:

Given data:

D = total depth = 10,000 *ft*
C_r = fixed operating cost of the rig per unit time = *$300/hr*
t_r = trip time = 8 *hrs*

For bit A:
C_b = cost of bit = *$700.00*
t_d = rotating time during the bit run = 20 *hrs*
t_c = non-rotating time during the bit run = 0.2 *hr*
ROP = rate of penetration = 11.0 *ft/hr*

For bit B:
C_b = cost of bit = *$1,050.00*
t_d = rotating time during the bit run = 38.5 *hrs*
t_c = non-rotating time during the bit run = 0.65 *hr*
ROP = rate of penetration = 12.5 *ft/hr*

For bit C:
C_b = cost of bit = *$1,450.00*
t_d = rotating time during the bit run = 62.5 *hrs*
t_c = non-rotating time during the bit run = 0.45 *hr*
ROP = rate of penetration = 10.5 *ft/hr*

Required data:

C_{f_lowest} = drilling cost per unit depth, *$/ft*

Now, Eq. (11.2) is used to calculate the cost per foot drilled for each type of bit. Thus,

Table 11.9 Data for Example 11.18.

Bit	Bit Cost *$*	Total rotating time (hrs)	Total non-rotating time (hrs)	ROP (ft/hr)
A	700	20	0.2	11.0
B	1050	38.5	0.65	12.5
C	1450	62.5	0.45	10.5

For bit A:

$$C_f = \frac{700+300(20+0.2+11)}{11.0\times 20} = 45.72 \ \$/ft$$

For bit B:

$$C_f = \frac{1050+300(38.5+0.65+12.5)}{12.5\times 38.5} = 34.38 \ \$/ft$$

For bit C:

$$C_f = \frac{1450+300(62.5+0.45+10.5)}{10.5\times 62.5} = 35.79 \ \$/ft$$

Example 11.19: While analyzing the historical well cost data of 20 wells from Gulf of Mexico, it was estimated that a_{dc} is $735,000 and b_{dc} is 0.000035 ft^{-1}. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 12,000 *ft* and 15,000 *ft* respectively. Use the exponential cost estimation formula.

Soultion:

Given data:

a_{dc} = constant depend on well location = *$735,000*
b_{dc} = constant depend on well location = 0.000035 ft^{-1}
D = total depth = 10,000 *ft* and 12,000 *ft*

Required data:

C_{dc} = drilling cost, *$*

To calculate the drilling cost, Eq. (11.3) is used to calculate for each depth as,

$$C_{dc} = 735000e^{0.000035D}$$

For D = 10,000 *ft*:

$$C_{dc} = 735000e^{0.000035\times 10000} = 1.04 \ million \ USD$$

For D = 15,000 *ft*:

$$C_{dc} = 735000e^{0.000035\times 12000} = 1.11 \ million \ USD$$

Example 11.20: A coring job was planned from 14,600 *ft* to 14,650 *ft*. Calculate the coring cost per foot with the following data:

Soultion:

Given data:

C_{cb} = core bit cost = *$2,500*
C_r = rig cost = *$12,000*/day = *$500/hr*

t_{ct} = core trip time = 12 *hrs*
t_{cr} = core rotating time = 8 *hrs*
t_{cc} = core connection time = 1 *hr*
t_{crj} = core recovery, and laying down of core barrel time = 1 *hr*
t_{co} = coring time = 1.5 *hr*
R_c = core recovery factor = 0.90

Required data:
C_{cf} = coring cost, *$*

Equation (11.17) is used to calculate the core drilling cost as,

$$C_{cf} = \frac{C_{cb} + C_r(t_{ct} + t_{cr} + t_{cc} + t_{crj} + t_{co})}{\Delta D_{ROP}} \frac{1}{R_c}$$
$$= \frac{2500 + 500(12 + 8 + 1 + 1 + 1.5)}{50} \times \frac{1}{0.90} = 316.667 \ \$ / ft$$

Example 11.21: Marathon Oil Company invested money for buying some drill pipes six years ago for $7,500 and did not use it. What would be the present value? Assume that the average rate of return for the past six years is 16% and the interest period is 2 times per year.

Soultion:
Given data:
V_p = present value = $7,500
i = interest rate per year = 16% = 0.16
m = the number of payments per year = 2 nos.
n = number of years = 6 yrs.

Required data:
V_f = Future Value, *$*

Equation (11.18) is used to calculate the future value (i.e., the present costs of the drill-pipe as of today which was bought six years ago) as,

$$V_f = V_p \left(1 + \frac{i}{m}\right)^{nm} = 7500 \times \left(1 + \frac{0.16}{2}\right)^{6 \times 2} = \mathbf{\$18886.275}$$

Example 11.22: A section of 5,000 *ft* length and 12 ¼" size in a well has to be drilled. There are three types of bits available with the following information in Table 11.10. If the trip time is 1 hour for each 1,000 *ft* depth, total non-rotating time for any bit that run in hole is 0.15 day and the fixed operating cost of the rig is $60,000/day; calculate the following:

Table 11.10 Data for Example 11.22.

Property	Bit type I	Bit type II	Bit type III
ROP, *ft*/hr	35.0	44.0	52
Longest the bit can drill, *ft*	1500	1350	1250
Cost, USD/bit	95,000	105,000	115,000

a. Number of bits you are going to use from each type, and also the time to drill the section?
b. The drilling cost per each foot drilled for each bit type? Which bit should be selected, and why?

Soultion:

Given data:

Data in the table

t_t = 1.0 hr/1000ft
t_c = 0.15 day/bit
ΔD = 5,000 ft
C_r = $60,000/day

Required data:

N_{bit} = Number of bit for each bit
t_d = Drilling time for each bit
C_f = Cost per foot for each bit

Number of bits to be used from each type to drill above section can be estimated simply by dividing the well section by the depth length that can be drilled by each bit. Mathematically:

$$N_{bit} = \frac{\Delta D}{l_{bit}}$$

Based on the length that can be drilled by each bit, to drill this section we need 4 bits from each bit type. For the first and second type, the last bit run will be used to complete the section and it can be reused again.

To calculate the time to drill above section using any type of the above bits, we should know that there is fixed time of 0.15 day for each well. In addition to that, trip in time is applied for the bits, but trip out time will not be applied to the first bit run, but for the rest of the bit runs of the same type. Mathematically drilling time for the first bit run is equal to:

$$t_{b1\_run1} = rotating\ \ time + non\ \ rotating\ \ time + trip\ \ in\ \ time$$
$$= \frac{l_{bit\_1}}{ROP_{b1}} + t_c + t_t \times \frac{\Delta D_1}{1000}$$

Table 11.11 Results for Bit type I.

	Time	Drilled depth	Remaining Depth	Comm. Depth
Bit type I	**hrs**	**ft**	**ft**	**ft**
Bit 1	48.0	1500	3500	1500
Bit 2	50.96	1500	2000	3000
Bit 3	53.96	1500	500	4500
Bit 4	27.39	500	0	5000
Total time for Bit type I is 152.87 hrs				

Table 11.12 Results for Bit type II.

	Time	Drilled depth	Remaining Depth	Comm. Depth
Bit type II	**hrs**	**ft**	**ft**	**ft**
Bit 1	35.63	1350	3650	1350
Bit 2	38.33	1350	2300	2700
Bit 3	41.03	1350	950	4050
Bit 4	34.24	950	0	5000
Total time for Bit type II is 149.2 hrs				

Drilling time for the rest of the bit runs equals to:

$$\begin{aligned} t_{b1\_run2} &= rotating\ \ time + non\ rotating\ \ time + trip\ in\ time \\ &\quad + trip\ \ out\ \ time \\ &= \frac{l_{bit\_1}}{ROP_{b1}} + t_c + t_t \times \frac{\Delta D_1}{1000} + t_t \times \frac{\Delta D_2}{1000} \end{aligned}$$

where ΔD_1 is the depth drilled by previous bit, and ΔD_2 is the new depth drilled by the new bit. Tables below summarize the calculation results for each bit type. Now, drilling cost per foot can be calculated using Eq. (11.2) as:

$$C_f = \frac{C_b + C_r(t_d + t_c + t_t)}{\Delta D}$$

Table 11.14 shows the drilling cost of the above section using any of the above bit types. From this table, although bit type I has the lowest *ROP* among them, it can drill the section with the minimum possible cost as compared to the other two bit types.

Table 11.13 Results for Bit type III.

Bit type III	Time	Drilled depth	Remaining Depth	Comm. Depth
	hrs	ft	ft	ft
Bit#1	28.89	1250	3750	1250
Bit#2	31.39	1250	2500	2500
Bit#3	33.89	1250	1250	3750
Bit#4	36.39	1250	0	5000
Total time for Bit type II is 130.6 hrs				

Table 11.14 Cost per foot.

Bit type	Cost
	$/foot
Type I	139.8
Type II	152.4
Type III	157.3

Example 11.23: Table 11.15 shows depths and costs recorded of 15 wells drilled in a certain area:

Use the above information to estimate well cost of a well that planned to be drilled to a depth of 18,500 *ft*.

<u>Soultion</u>:

Given data:

Data in the table

$D = 18{,}500\ ft$

Required data:

C_{dc} = Well cost

First step is to plot well costs versus depth in Cartesian coordinates to find out their relationship. Figure 11.2 below shows the relationship between the well cost and well depth.

It is convenient to use exponential trend between cost and depth.o from the above plot we can estimate the constants of the exponential equation that can be used to estimate the cost of the well that is planned to be drilled to a depth of 18,500 *ft*. Well cost can be estimated using Eq. (11.3) as follows:

$$C_{dc} = a_{dc} e^{b_{dc} D} = 1{,}783{,}156.4 e^{0.000053 \times 18{,}500} = \mathbf{4{,}753{,}518}\ \boldsymbol{USD}$$

Table 11.15 Results for Bit type III.

Well No.	Depth	Cost
	ft	USD
Well 1	10000	3094467
Well 2	12500	3466440
Well 3	15000	4232474
Well 4	11500	3274379
Well 5	14000	3664806
Well 6	14900	3507015
Well 7	13750	3393986
Well 8	12500	3201360
Well 9	16000	4057553
Well 10	17500	4663771
Well 11	14800	4091509
Well 12	13900	3666060
Well 13	14100	3752768
Well 14	12300	3648904
Well 15	12800	3546969

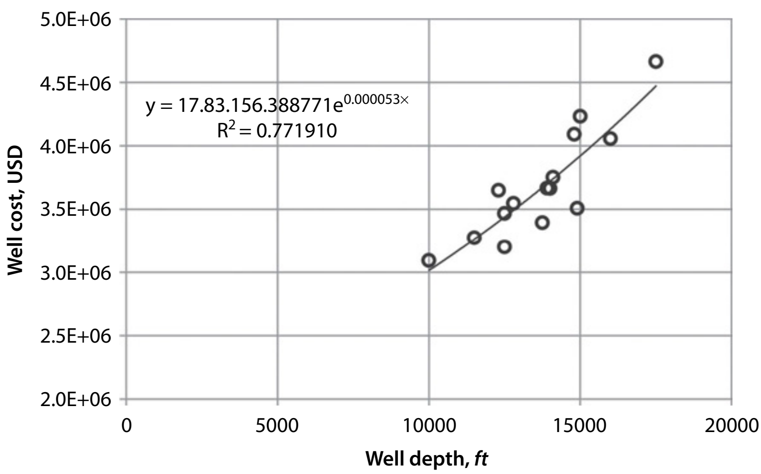

Figure 11.2 Well depth vs. well cost plot for Example 11.23.

So, from the above information, the estimated well cost is around USD 4,753,518. This can give us a rough estimate of what should be the expected range of well cost of that well.

Example 11.24: Two wells at the same area drilled to depths of 13,900 *ft* and 15,250 *ft* respectively. Their costs were found to be $3,992,049 and $4,435,341, respectively. If it is found that depth and cost relationship follow exponential relation, calculate the constants of the equation and the well cost of well that was planned to be drilled to a depth of 13,000 *ft*.

Soultion:

Given data:

D_{w1} = 13,900 *ft*
D_{w2} = 15,250 *ft*
D_{w3} = 13,000 *ft*
C_{dcw1} = 3,992,049 USD
C_{dcw2} = 4,435,341 USD

Required data:

a_{dc} = "*a*" constant in the equation
b_{dc} = "*b*" constant in the equation
C_{dcw3} = Well cost of the third well

From the information of the two wells, we can determine the values of the constants of Eq. (11.3). From the information of well 1, we can create the following equation:

$$C_{dcw1} = a_{dc} e^{b_{dc} D_{w1}}$$

$$3{,}992{,}049 = a_{dc} e^{b_{dc} \times 13{,}900} \tag{11.23}$$

And from the information of well 2, we can create the following equation:

$$C_{dcw2} = a_{dc} e^{b_{dc} D_{w2}}$$

$$4{,}435{,}341 = a_{dc} e^{b_{dc} \times 15{,}250} \tag{11.24}$$

By solving the above two equations simultaneously, we can determine the values of the constants. Dividing Eq. (11.24) over (11.23) gives:

$$\frac{4{,}435{,}341}{3{,}992{,}049} = e^{b_{dc}(15{,}250-13{,}900)}$$

$$1.111 = e^{b_{dc} \times 1{,}350}$$

$$b_{dc} = 0.000078\ ft^{-1}$$

Substituting the value of "*b*" constant in Eq. (1), we can get "*a*" constant as follows:

$$3{,}992{,}049 = a_{dc} e^{0.000078 \times 13{,}900}$$

$$a_{dc} = \$1{,}350{,}002$$

Thus, after we determined the values of the constants, we can easily estimate the cost of the third well using Eq. (11.3) as follows:

$$C_{dcw3} = a_{dc} e^{b_{dc} \times D_{w3}} = 1{,}350{,}002 \times e^{0.000078 \times 13{,}000} = \mathbf{\$3{,}721{,}423}$$

Example 11.25: Table 11.16 summarizes the planned depths and days for a development well need to be drilled to a depth of 11,000 *ft*. All operation durations remain as planned except average *ROP* for conductor, surface, intermediate and production holes were calculated to be 10, 40.0, 20.5, and 14.0 *ft*/*hr*; respectively. Calculate the actual drilling days, and then plot the planned drilling days versus the actual.

Soultion:

Given data:

Data in the table and the values of *ROP* for all the drilled sections

Required data:

Actual drilling days

As rig moves and rig up, evaluation, casing and cementing operations remain as per the plan, we need to calculate drilling days for each section from the average *ROP*. Drilling days for each section can be calculated using the following equation:

$$t_{section} = \frac{D_{section}}{ROP_{section}}$$

Drilling days for conductor section equal to:

$$t_{conductor} = \frac{D_{conductor}}{ROP_{conductor}} = \frac{100}{10 \times 24} = 0.42\ day$$

Table 11.16 Detail data for Example 11.25.

	Depth	Duration	Cumm. days
Operations	**ft**	**days**	**days**
Start of operations	0	0.00	0.00
Rig move and rig up	0	5.00	5.00
Drilling *26″* hole	100	0.69	5.69
Evaluation, Casing & cementing of *20″*conductor	100	1.01	6.71
Drilling 17.*5″* surface hole	1000	1.19	7.90
Casing & cementing 13 3/*8″* surface casing	1000	1.13	9.02
Drilling 12 ¼ “ Intermediate hole	6000	10.00	19.02
Casing & cementing 9 5/*8″* intermediate casing	6000	1.75	20.77
Drilling8 ½ “ production hole	11000	28.65	49.42
Casing & cementing *7″* production casing	11000	2.38	51.79

Drilling days for surface section equal to:

$$t_{surface} = \frac{D_{surface}}{ROP_{surface}} = \frac{1{,}000 - 100}{40 \times 24} = 0.94\ day$$

Drilling days for intermediate section equal to:

$$t_{intermediate} = \frac{D_{intermediate}}{ROP_{intermediate}} = \frac{6{,}000 - 1{,}000}{20.5 \times 24} = 10.16\ days$$

Drilling days for production section equal to:

$$t_{production} = \frac{D_{production}}{ROP_{production}} = \frac{11{,}000 - 6{,}000}{14 \times 24} = 14.88\ days$$

Thus actual drilling days equal to:

$$t_{drilling} = 5.0 + 0.42 + 1.01 + 0.94 + 1.63 + 10.16 + 3.25 + 14.88 + 4.38$$
$$= \mathbf{41.66}\ \boldsymbol{days}$$

Figure 11.3 shows the planned and actual drilling days of this well:

Example 11.26: An oil company invested certain money to be used next year for drilling a well. The required future money was estimated to be $3,000,000, and the market interest rate is 13% compounded 2 times. If the company decided not to use the money next year and continued investing the same amount and used it two years from now to drill the same

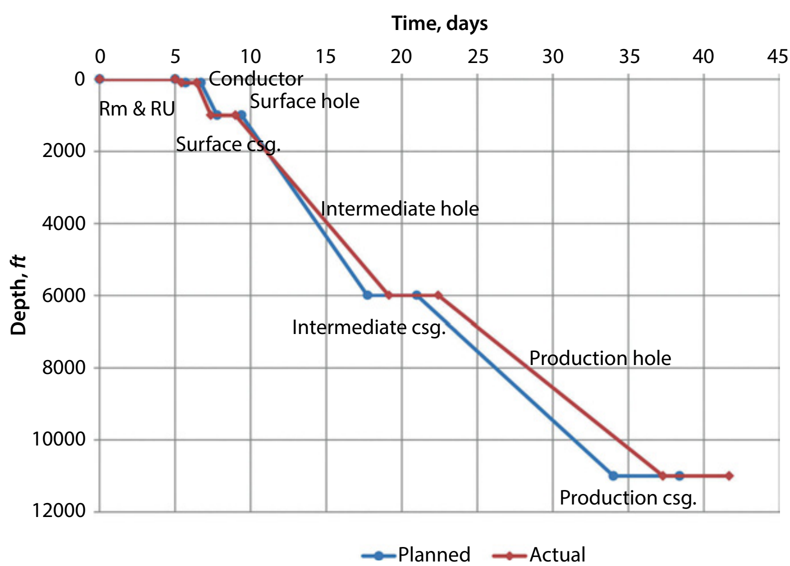

Figure 11.3 Time vs. well depth plot for Example 11.25

well, what will be the value of the money two years from now if the interest rate remains the same?

Soultion:

Given data:

V_f = Value of money one year from now = USD 3,000,000
i = Interest rate = 13%
m = Compounding = 2
n = Number of years = 1

Required data:

V_p = Present value of money
V_f = Value of money two years from now

To estimate the value of the money two years from today, we must first calculate the money now. We can use Eq. (11.18) and the value of money one year from now as follows:

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm}$$

$$3{,}000{,}000 = V_p\left(1+\frac{0.13}{2}\right)^{1\times 2}$$

$$V_p = 2{,}644{,}978\ USD$$

Thus, the value of money two years from now can be estimated using the same equation and the money at the present as follows:

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm} = 2{,}644{,}978\left(1+\frac{0.13}{2}\right)^{2\times 2} = \mathbf{\$3{,}402{,}675}$$

Example 11.27: An oil company is planning to invest money to drill two exploration wells in a four-year period. After two years of investment, the company took half of the money and drilled the first exploration well, and continued investing the rest of the money. After another two years, the company used the rest of the money which was 4,464,033 USD to drill the second well. If the interest rate is 11% compounded continuously, what is the amount of money invested by the company four years from now?

Soultion:

Given data:

V_f = Value of money after four years = USD 4,464,033
i = Interest rate = 11%

Required data:

V_p = Amount of money invested

To determine the amount of money invested at the beginning to drill these two wells, we should first determine the value of money after two years of investment using the value of money after four years for $4,464,033. The value of this money can be estimated using Eq. (11.19):

$$V_p = V_f e^{-in} = 4,464,033 \times e^{(-0.11\times 2)} = \$3,582,470$$

Because the company used half of the invested money after two years, the money after two years was double for the above money; or equal to $7,164,940. Thus the amount of money that invested is equal to:

$$V_p = V_f e^{-in} = 7,164,940 \times e^{(-0.11\times 2)} = \mathbf{\$5,750,000}$$

Example 11.28: An oil operator is planning to drill an oil well. The estimated cost for drilling, completing, and surface facilities were estimated to be 5,500,000 USD. The well is expected to produce 250 bbls/day for the first year, and follows an exponential decline trend with a nominal decline rate of 0.3 year^{-1}. The average price of one barrel is 45 USD and the cost of producing one barrel is 14 USD, and they are assumed to be constant throughout the production life of the well. Economic production rate is set to be 100 bbls/d. If the operator decided that the feasible profit must be 15% of the invested amount, determine the feasibility of investing on this well. Use interest rate of 16% compounded continuously.

Soultion:

Given data:

C_w	=	Total well cost	=	USD 5,500,000
q	=	Production rate	=	250 bbls/d
D_i	=	Production decline rate	=	0.3 year^{-1}
Pr_{oil}	=	Oil price	=	45 USD/bbl
c_{op_bbl}	=	Cost of producing one barrel	=	14 USD/bbl
q_a	=	Economic production rate	=	100 bbls/d
i	=	Interest rate	=	16%

Required data:

Feasibility of drilling the well

To check the feasibility of drilling the well based on the designed percentage, first we need to know how long the well can be produced. After that we need to bring back the revenue from oil production to present money in order to make comparison. To determine how long the well can be produced, we should calculate the time to reach the economic rate by using exponential decline rate equation as follows:

$$q_n = q_i \exp D_i t$$

So by using the above equation, we can determine the production life of the well. Table 11.17 shows the production for each year:

Thus, from the above table, the well can produce for 4 years before it reaches the economic production rate. Now we should do calculate the future revenues of oil

Table 11.17 Detail data for Example 11.28.

Year	rate bpd	Yearly production bbls
1	250.0	91,313
2	185.2	67,646
3	137.2	50,113
4	101.6	37,125

Table 11.18 Revenue and present value for Example 11.28.

Year	Revenue USD	*Vp*
1	2,830,688	2,830,688
2	2,097,025	1,786,967
3	1,553,514	1,128,083
4	1,150,872	712,140

production for each year, and then bring this money to the present value of money to make a proper comparison. Eq. (11.19) is used for continuous compounding as follows:

$$V_p = V_f \times e^{-in}$$

Now, we should calculate the future revenue for each year and then we should change the revenue of each year to the present value. The net profit of one barrel of oil equals:

$$Net\ profit = oil\ price - oil\ cost = 45 - 14 = \$31$$

The revenue of the first year will not be affected by the interest rate, only the revenue of the rest of years. The table below shows the revenue of each year and the present value of each year:

The total amount of present revenue is equal to 6,457,878 USD. The net profit of the project is equal to:

$$\begin{aligned} Net\ profit\ of\ project &= revenue - investment \\ &= 6,457,878 - 5,500,000 = 957,877\ USD \end{aligned}$$

The percentage of the profit based on the invested money equals to:

$$Profit\ in\ \% = \frac{profit}{investment} \times 100 = \frac{957,877}{5,500,000} \times 100 = 17.4\%$$

As the expected profit percentage is greater than the designed, **drilling the well is feasible.**

Example 11.29: A rig contractor bought a new service rig which will be rented to an oil operator for long-term completion and workover campaign. The purchasing price of the service rig is 6,500,000 USD, and it will be rented for 6,800 USD per day. The rig contractor assumed daily consumables and personnel fees of 1,000 and 1,503 USD; respectively. If the interest rate is assumed to be 11.1% compounded continuously and rig daily rate, consumables and personnel fees remains without any changes, how many years required by the rig contractor to payback their invested money? And what will be the contractor's net profit after 10 years?

Soultion:

Given data:

Pr_{rig}	=	Purchasing cost of the rig	=	USD 6,500,000
r_{rig}	=	Daily rig rate	=	6,800 USD/d
c_{rig}	=	Daily rig consumables	=	1,000 USD/d
$pers_{rig}$	=	Daily rig personnel fees	=	1,503 USD/d
i	=	Interest rate	=	11.1%

Required data:

Payback period

Net profit after 10 years

To determine the payback period, we are going to calculate the yearly revenue and then we should change it to present value amount using the specified interest rate. In this example interest rate, daily rig rate, consumables and personnel fees are assumed to be constant. In fact these three values can change with time depending on the market, but we assumed them to be constant for simplicity of calculations. To determine the yearly profit, we should first calculate the daily profit as follows:

$$\text{Daily profit} = \text{daily rig rate} - \text{daily consumables} - \text{personnel fees}$$
$$= 6{,}800 - 1{,}000 - 1{,}503 = \$4{,}297$$

Yearly profit is equal to:

$$\text{Yearly profit} = \text{daily profit} \times 365.25 = 4{,}296 \times 365.25$$
$$= \$1{,}569{,}114$$

The above amount will be the profit that the contractor will gain every year. However, to determine the payback period of the invested money, these amounts should be changed to present value amount in order to have one basis. Thus, the yearly profit of each year should be converted to present amount using Eq. (11.19) as follows:

$$V_p = V_f \times e^{-in}$$

Table 11.19 summarizes the yearly profit converted to present value amount:

From the above table, we can see that the rig contractor will pay back their invested money after six years from now. To calculate the net profit after 10 years of operations, we will calculate the yearly profit and then convert to present value. Table 11.20 shows the yearly profit staring from the sixth year up to the tenth year:

From the above Table 11.20, the rig contractor can earn about **$2,463,279** after 10 years of operations based on present value amount.

Table 11.19 Calculated yearly profit for Example 11.29

Year	Yearly future profit	Present value of future profit	Payback amount	Remaining amount
	USD	USD	USD	USD
1	1,569,114	1,404,588	1,404,588	5,095,412
2	1,569,114	1,257,020	2,661,608	3,838,392
3	1,569,114	1,124,956	3,786,564	2,713,436
4	1,569,114	1,006,767	4,793,330	1,706,670
5	1,569,114	900,995	5,694,325	805,675
6	1,569,114	806,335	6,500,660	–660

Table 11.20 Calculated yearly profit from sixth year for Example 11.29

Year	Yearly future profit	Present value of future profit	Cumm. profit
	USD	USD	USD
7	1,569,114	721,620	722,280
8	1,569,114	645,806	1,368,086
9	1,569,114	577,957	1,946,043
10	1,569,114	517,236	2,463,279

11.3 Multiple Choice Questions

1. Drilling costs can represent _______ of the exploration costs.
   a) About 40%
   b) Less than 10%
   c) More than 75%
   d) 100%

2. The document that is used to request money from partners to drill a certain well is called.
   a) Well budget
   b) PR
   c) AFE
   d) All of the above

3. What is the relation between well costs and well depth?
   a) Costs increase linearly with depth
   b) Cost decrease non-linearly with depth
   c) Cost increase non-linearly with depth
   d) None of the above

4. Which of the following factors has less effect on drilling costs?
   a) Well depth
   b) Hole diameter
   c) Formation fluid's type
   d) All of the above

5. Which of the following factors has a high effect on the drilling cost?
   a) Formation fluid's type
   b) Well depth
   c) Drilling fluid type
   d) All of the above

6. Which of the following factors can more independently affect drilling costs?
   a) Well depth
   b) Casing costs
   c) Cementing costs
   d) None of the above

7. Which of the following factors controls the drilling rig type?
   a) Well depth
   b) Well location
   c) Well size
   d) All of the above

8. Which of the following factors controls the drilling rig capacity?
   a) Well size
   b) Well location
   c) Well depth
   d) None of the above

9. Well pre-spud costs include all the following costs except
   a) Rig moveite preparation
   b) Rig up
   c) Installing BOP

10. Drilling the cement at the casing shoe is considered as part of
    a) Cementing cost
    b) Casing cost
    c) Drilling-rotating costs
    d) All of the above

11. Cost of running survey tool for well inclination is considered as part of
    a) Cementing cost
    b) Casing cost
    c) Drilling-rotating costs
    d) Drilling-none rotating costs

12. Fishing of a cone of tri-cone bit out of the hole is considered as part of
    a) Pre-spud costs
    b) Trouble costs
    c) Drilling-rotating costs
    d) Drilling-none rotating costs
13. Which of the following factors depend on bit selection?
    a) Running casing
    b) Well completion
    c) Drilling rate
    d) All of the above
14. Which of the following factors depends on the well depth?
    a) Trip time
    b) Lost circulation
    c) Directional drilling
    d) None of the above
15. Running casing string depends mainly on
    a) Casing size
    b) Well depth
    c) Crew efficiency
    d) All of the above
16. Proper well cost estimation depends mainly on proper estimation of
    a) Well depth
    b) Drilling time
    c) Hole problems prediction
    d) All of the above
17. Which of the following factors controls the drilling time estimations?
    a) Rock compressive strength
    b) Formation fluid type
    c) Rock porosity
    d) All of the above
18. Which of the following factors has no control on the trip time estimation?
    a) Well depth
    b) Drilling rig
    c) ROP
    d) None of the above
19. Number of drill bits to be used depends mainly on
    a) Bit life
    b) Rock hardness
    c) Well depth
    d) All of the above

20. Among the below factors, which is considered as a less important factor in well cost estimations?
    a) Drilling time
    b) Trip time
    c) Connection time
    d) All of the above

21. Future value estimation of the money is an expression of
    a) How to expend the money for the future of any project
    b) The value of the present money in the future
    c) The value of the future money in the future
    d) None of the above

22. Which of the following factors is controlling the future value estimations of money?
    a) Number of years
    b) Interest rate
    c) Number of compounding per year
    d) All of the above

23. The number of drilling rigs responds strongly to the oil prices when price elasticity
    a) Less than unity
    b) Equal to unity
    c) Greater than unity
    d) Equals to zero

24. The number of drilling rigs does not respond strongly to the oil prices when price elasticity
    a) Less than unity
    b) Equals to unity
    c) Greater than unity
    d) Equals to zero

25. The number of drilling rigs does not respond to the oil prices when price elasticity
    a) Less than unity
    b) Equal to unity
    c) Greater than unity
    d) Equals to zero

Answers 1a, 2c, 3c, 4c, 5b, 6a, 7b, 8c, 9d, 10c, 11d, 12b, 13c, 14a, 15d, 16b, 17a, 18c, 19d, 20c, 21b, 22d, 23c, 24a, 25d.

11.4 Summary

The chapter covers various methods that have been proposed over the past several decades to evaluate drilling cost and complexity. However, because of the large number of factors and events that impact drilling performance, predictive models are difficult to construct. Quantifying well costs and complexity is challenging, due either to restrictions on data collection and availability, constraints associated with modeling, or combinations of these factors. Drilling rates are often controlled by factors that the driller does not control and in ways that cannot be documented. The purpose is to review the primary methods used to assess drilling cost and complexity. This chapter discusses the list factors affecting well costs, estimated drilling time, list elements of well costing, calculation of well costs, understanding NPT and risks associated with well cost estimation in the form of MCQs and workout examples. The exercises are presented so that readers can have more self-practice; they can compare their results with the solutions in Appendix A. The answers of the self-practice MCQs exercises are presented in Appendix B.

11.5 Exercise and MCQs for Practice

11.5.1 Exercises (Solutions are in Appendix A)

Exercise 11.1: A section of 4,000 *ft* length and 8 ½" size in a well has to be drilled. There are three types of bits available with the following information.

Property	**Bit type I**	**Bit type II**	**Bit type III**
ROP, *ft*/*hr*	40.0	47.0	49.0
Longest the bit can drill, *ft*	1,300	1,400	1,100
Cost, $/bit	45,000	60,000	50,000

If the trip time is 1.5 hours for each 1,000 *ft* depth, total non-rotating time for any bit that run in hole is 0.2 day and the fixed operating cost of the rig is $80,500/day; calculate the following:

a. Number of bits you are going to use from each type, and also the time to drill the section?
b. Which bit should be selected, and why? ***Answers: Bit type II***

Exercise 11.2: Table below shows depths and costs recorded of 13 wells drilled in a certain area:

Use the above information to estimate well cost of a well that is planned to be drilled to a depth of 9,150 *ft*.

Well No.	Depth ft	Cost USD
Well 1	7,850	2,279,249
Well2	8,100	2,303,077
Well 3	6,950	2,059,568
Well 4	7,100	2,066,272
Well 5	8,500	2,411,976
Well 6	7,250	2,102,645
Well 7	7,400	2,108,857
Well 8	7,000	2,110,545
Well 9	6,500	2,049,193
Well 10	8,800	2,319,487
Well 11	8,100	2,247,787
Well 12	7,800	2,201,076
Well 13	7,750	2,227,648

Exercise 11.3: Two wells drilled to depths of 9,500 and 9,100 *ft*. Their costs were calculated to be 2,879,030 and 2,773,897 USD, respectively. If it is found that depth and cost relationship were follow exponential relation. Calculate the well cost of a well that is planned to be drilled to a depth of 10,100 *ft*.

Exercise 11.4: Table below summarizes the planned depths and days for a development well need to be drilled to a depth of 12,500 *ft*:

Operations	Depth ft	Duration days	Cumm. days
Start of operations	0	0.00	0.00
Rig move and rig up	0	3.00	3.00
Drilling *26″* hole	50	0.14	3.14
Evaluation, Casing & cementing of *20″* conductor	50	1.01	4.15
Drilling 17.*5″* surface hole	1500	1.47	5.62
Casing & cementing 13 3/*8″* surface casing	1500	1.75	7.37
Drilling 12 ¼ “ Intermediate hole	7250	7.26	14.63
Operations	**Depth ft**	**Duration days**	**Cumm. days**
Casing & cementing 9 5/*8″* intermediate casing	7250	3.71	18.34
Drilling 8 ½ “ production hole	12500	9.11	27.45
Casing & cementing *7″* production casing	12500	5.08	32.54

All operation durations remain as planned except average ROP for conductor, surface, intermediate and production holes were calculated to be 11, 44, 30, and 18 *ft*/*hr*; respectively. Calculate the actual drilling days, and then plot the planned drilling days versus the actual.

Exercise 11.5: An oil operator invested certain money to be used three years from now to complete and install surface facilities for four wells. The required future money was estimated to be USD 10,000,000 and the market interest rate is 8% compounded 4 times. If the company decided not to use the money at the third year and continue investing the same amount and use it in the fourth year from now, what is the amount that is invested by the operator, and the value of the money four years from now if the interest rate remains the same?

Exercise 11.6: An oil company is planning to invest money to drill 5 exploration wells. After two years of investment, the company took two-fifths of the money and drilled two exploration wells, and continued investing the rest of the money. After another one year, the company used the rest of the money which was 12,899,965 USD to drill the last three wells. If the interest rate is 12% compounded continuously, what is the amount of money that is invested by the company three years from now?

Exercise 11.7: An oil operator is planning to drill an oil well. The estimated cost for drilling, completing, and surface facilities were estimated to be 7,000,000 USD. The well is expected to produce 360 bbls/day for the first year, and follows an exponential decline trend with a nominal decline rate of 0.32 year^{-1}. The average price of one barrel is 50 USD and the cost of producing one barrel is 21 USD, and these are assumed to be constant throughout the production life of the well. Economic production rate is set to be 100 bbls/d. If the operator decided that the feasible profit must be 40% of the invested amount, determine the feasibility of investing on this well. Use interest rate of 20% compounded continuously.

Exercise 11.8: A rig contractor bought a new drilling rig to be rented to an oil operator for a long-term drilling campaign. The purchasing price of the rig is 14,000,000 USD, and it will be rented for 16,500 USD per day. The rig contractor assumed daily consumables and personnel fees of 2,500 and 2,867 USD; respectively. If interest rate is assumed to be 13% compounded continuously and rig daily rate, consumables and personnel fees remains without any changes, how many years required by the rig contractor to pay back their invested money. And what will be the contractor's net profit after 8 years?

11.5.2 Exercise

E11.1: The following table shows the bit performance of three bits for a sandstone formation at 14,000 *ft* depth. Determine which bit gives the lowest drilling cost if the fixed operating cost of the rig is *$550/hr*, and the trip time is 8 hours.

Bit	Bit Cost ($)	Total rotating time (hrs)	Total non-rotating time (hrs)	ROP (ft/hr)
1	850	24.5	0.20	11.0
2	1400	32.0	0.30	8.5
3	2200	40.5	0.60	6.5

E11.2: While analyzing the historical well cost data of 30 wells from the Arabian Peninsula, it was estimated that a_{dc} is USD 700,000 and b_{dc} is 0.000035 ft^{-1}. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 13,500 *ft* and 16,000 *ft* respectively. Use the exponential cost estimation formula.

E11.3: A recommended bit program is being prepared for a new well using bit performance records from nearby wells. Drilling performance records for three bits are shown for a thick limestone formation at 9,000 *ft*. Determine which bit gives the lowest drilling cost if the operating cost of the rig is *$400/hr*, the trip time is 7 hours, and connection time is 1 minute per connection.

Assume that each of the bits was operated at near the minimum cost per foot attainable for that bit.

Bit	Bit Cost ($)	Total rotating time (hrs)	Total non-rotating time (hrs)	ROP (ft/hr)
A	800	14.8	0.1	13.8
B	4900	57.7	0.40	12.6
C	4500	95.8	0.5	10.2

E11.4: It was estimated that a_{dc} is USD 850,000 and b_{dc} is 0.000047 ft^{-1} while analyzing the historical well cost data of 20 wells from the Gulf of Mexico. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 17,000 *ft* and 20,000 *ft* respectively by using the exponential cost estimation formula.

E11.5: Find out the time required to drill a hole where the following data are assumed and taken from the three wells in the Gulf of Mexico area:

36″ Hole for a *30″* Conductor which is 300 *ft* long from surface
26″ Hole for a *20″* Casing which is 1,400 *ft* long from *30″* casing
17.*5″* Hole for a 13.*375″* Casing which is 3,500 *ft* long from *20″* casing
12.*25″* Hole for a 9.*625″* Casing which is 6,500 *ft* long from 13.*375″* casing
8.*5″* Hole for a *7″* Casing which is 2,500 *ft* long from 9.*625″* casing
Total depth is 15,000 *ft* long

From three offset wells, the following data was established for average ROP for each hole section: *36″* hole 13 *ft/hr*, *26″* hole 14 *ft/hr*, 17.*5″* hole 25.5 ft/*hr*, 12.*25″* hole 12 *ft/hr*, and 8.*5″* hole 7 *ft/hr*. The expected flat times for this well are shown in below Table. Calculate the total drilling time and plot the depth-time curve.

Casing size	**Running and cementing**	**NU (days)**	**Total (days)**
30″	1.0	2.0	3.0
20″	1.5	2.5	4.0
13 3/8″	1.5	3.5	5.0
9 5/8″	1.5	2.5	4.0
7″	2.5	3.0	5.5
Total			**21.5**

Exercise 11.6: Determine the drilling cost per foot (CT) using the following data:

Bit cost (B) = \$27,000
Drilling time (t) = 50 hours
Rig cost (CR) = \$3,500/hour
Round trip time (T) = 12 hours
Footage per bit (F) = 5,000 *ft*

Exercise 11.7: Determine the bit cost using the following data:

Drilling cost (Cf) = \$50 1/ft
Drilling time (t) = 50 hours
Rig cost (CR) = \$3,500/hour
Round trip time (T) = 12 hours
Footage per bit (F) = 5,000 *ft*

E11.8: A well is planned to drill up to 14,000 *ft* where casing settings are designed at 750 *ft*, 4,000 *ft*, 7,500 *ft* 14,500 *ft* and 16,000 *ft*. The rental cost of the rig is \$16,000/day and tripping time per stand is considered as 2.5 min. The bits used at different phases are given at Table 11.6 and the control well data for this region is given in Table 11.7. The formation constants for the third phase are given by $K = 260\ ft/hr$ and $A = 0.00044\ ft^{-1}$. Find out the cost to drill between 8,200 *ft* and 10,700 *ft* in \$/*ft* and the total rig rental cost.

E11.9: A well is planned to drill up to 22,000 *ft* where casing settings are designed at 800 *ft*, 5,000 *ft*, 8,000 *ft*, 15,500 *ft*, and 20,000 *ft*. The rental cost of the rig is \$16,000/day and tripping time per stand is considered as 3.0 min. The bits used at different phases and well data are given below. The formation constants for the third phase are given by $K = 320\ ft/hr$ and $A = 0.00034$ ft^{-1}. Find out the cost to drill between 9,500 *ft* and 11,500 *ft* in \$/*ft*.

D	:	9,500	t_{bl}	:	26 *hrs*
ΔD	:	1,000	C_r	:	\$15,000/day
$\overline{t}_s$	:	3.0 min	C_b	:	\$2,200
$\overline{l}_s$	:	94 *ft*			

Phase	Size (in)	Average life (*hrs*)	Cost (*$*)
1	22	45	4,500
2	17.5	36	3,400
3	13.5	26	2,600
4	8.75	22	2,250
5	5.75	12	1,700

E11.10: A coring job was planned from 17,300 *ft* to 17, 330 *ft*. Calculate the coring cost per foot with the following data:

Core bit cost = *$*4,500
Rig cost = *$*13,900/day
Core trip time = 11 *hrs*
Core rotating time = 9.5 *hrs*
Connection time while coring = 1 *hr* 30 min
Core recovery, and laying down of core barrel time = 45 min
Coring time = 2.5 *hrs*
Core recovery factor = 87%

E11.11: A coring job was planned from 15,600 *ft* to 15, 620 *ft*. Calculate the core bit cost with the following data:

Coring cost = 928.24 *$/ft*
Rig cost = *$*14,000/day
Core trip time = 12 *hrs*
Core rotating time = 8 *hrs*
Connection time while coring = 1 *hrs*
Core recovery, and laying down of core barrel time = 1 *hr*
Coring time = 1.5 *hrs*
Core recovery factor = 90%

E11.12: Marathon Oil Company invested money for buying some drillpipes eight years ago for *$*8,600 and did not use it. What would its present value be today? Assume that the average rate of return for the past eight years is 13.5% and the interest period is 4.5 times per year.

E11.13: The following are the drilling performance records for three bits for a thick carbonate formation at 9,000 *ft* depth. Determine which bit gives the lowest drilling cost based on the information given in the following Table. Other data: $C_r = \$400/hr$; $t_t = 7\ hrs$.

Bit	Bit Cost ($)	t_b (hrs)	t_c (hrs)	ROP (ft/hr)
A	800	14.8	0.1	13.8
B	4900	57.7	0.4	12.6
C	4500	95.8	0.5	10.2

Ex11.14: The historical well cost data of 30 wells from Middle East area shows that a_{dc} is USD 900,000 and b_{dc} is 0.000067 ft^{-1}. Assuming all external conditions remained the same, estimate the cost for the two new wells with depths of 12,000 *ft* and 14,000 *ft*, respectively. Use the exponential cost estimation formula.

Ex 11.15: A well is planned to drill up to 15,000 *ft* where casing settings are designed at 200 *ft*, 3,000 *ft*, 7,000 *ft*, 12,500 *ft*, and 14,000 *ft*. The rental cost of the rig is $20,000/day and tripping time per stand is considered as 2.5 min. The bits used at different phases are given in Table 12.6 and the control well data for this region is given in the Table below. The formation constants for the fourth phase are given by $K = 290$ *ft/hr* and $A = 0.0003$ ft^{-1}. Find out the cost to drill between 11,500 *ft* and 12,500 *ft* in *$/ft* and the total rig rental cost.

D	11,500
ΔD	1,000
$\bar{t}_s$	2.5 min
$\bar{l}_s$	90 *ft*
t_{bl}	22 *hrs*
C_r	$20,000/day
C_b	$2,250

E11.16: A coring job was planned from 16,280 *ft* to 16,320 *ft*. Calculate the coring cost per foot if core bit cost is $2,600, rig rental cost is $12,000/day, core trip time is 10 *hrs*, core rotating time is 6 *hrs* and miscellaneous time is 2 *hrs*. Assume the core recovery factor is 85%.

E11.17: Arabian Oil Company invested money for buying an external casing packer (*ECP*) three years ago for $9,000 and did not use it. What would its present value be today? Assume that the average rate of return for the past three years is 10% and the interest period is 3 times per year.

Exercise 11.18: Marathon Oil Company invested money for buying some drill collars five years ago for $10,000 and did not use it. What would its present value be today? Assume that the average rate of return for the past six years is 14% and the interest period is 3 times per year.

E11.19: A well is planned to drill up to 18,000 *ft* where casing settings are designed at 300 *ft*, 3,000 *ft*, 7,000 *ft* 14,500 *ft* and 18,000 *ft*. The rental cost of the rig is $20,000/day and tripping time per stand is considered as 3.5 min. The bits used at different phases are given in Table 12.6 and the control well data for this region is given in the Table below. The formation constants for the fourth phase are given by $K = 294$ *ft/hr* and $A = 0.0004$ ft^{-1}. Find out the cost to drill between 11,500 *ft* and 12,500 *ft* in *$/ft* and the total rig rental cost.

D	11,500
ΔD	1,000
$\bar{t}_s$	2.5 min
$\bar{l}_s$	90 *ft*
t_{bl}	22 *hrs*
C_r	*$20,000*/day
C_b	*$2,250*

11.5.3 MCQs (Self Practices)

1. What will happen to the total well cost if the depth of the well has increased?
   a) It will increase
   b) It will decrease
   c) It will remain same
   d) None of the above

2. Which of the following operations has the lowest cost during exploration?eismic operations
   a) Drilling operations
   b) Well-pad construction
   c) None of the above

3. Which of the following operations has the highest cost during exploration?eismic operations
   a) Well-pad construction
   b) Drilling operations
   c) All of the above

4. If the average ROP of a well is more than that of another well in the same area, what will happen to the well cost?
   a) It will increase
   b) It will decrease
   c) It will not change
   d) None of the above

5. If the well depth has increased, what will happen to the cost per foot?
   a) It will increase
   b) It will decrease
   c) It will not change
   d) None of the above

6. All of the following are not considered as formation evaluation time except:
   a) Coring
   b) Drilling the holeide tracking
   c) All of the above
7. What is the effect of increasing the bit cost on the cost per foot for a specific section of the well?
   a) It will decrease
   b) It will not change
   c) It will increase
   d) It has no effect
8. Under which category should the cost of freeing the stuck drill string be classified?
   a) Rotating cost
   b) Non-productive time cost
   c) Reaming cost
   d) All of the above
9. All of the following are considered as drilling time except:
   a) Circulation
   b) Wiper trips
   c) Formation evaluation
   d) None of the above
10. All of the above are considered as formation evaluation times except:
    a) Coring
    b) Well loggingide-wall coring
    c) All of the above
11. All of the following are considered as drilling non-rotating costs except:
    a) Trip costsliding time costs
    b) Jarring time costs
    c) All of the above
12. All of the following are not considered as drilling rotating costs except:
    a) Mud displacement cost
    b) Cement displacement cost
    c) Inclination angle building cost
    d) None of the above
13. All of the following are considered as drilling rotating costs except:
    a) Trip to change the bit
    b) Drilling vertical hole
    c) Reaming the drilled section
    d) None of the above
14. All of the following are not considered as drilling non-rotating costs except:
    a) Running casing
    b) Cementing
    c) Changing BOP rams
    d) None of the above

15. The rig daily rate depends mainly on the:
    a) Type of rig
    b) Market conditions
    c) Length of contract
    d) All of the above

11.6 Nomenclature

A = constant, ft^{-1}
a_{dc} = constant depend on well location, $\$$
b_{dc} = constant depend on well location, ft^{-1}
C_b = bit cost, $\$$
C_f = drilling cost per unit depth, $\$/ft$
C_o = all other costs of making a foot of hole, such as casings, mud, cementing services, logging services, coring services, site preparation, fuel, transportation, completion, etc., $\$$
C_j = cost of the j[th] event, $\$$
C_r = rig cost or fixed operating cost of the rig per unit time, $\$/hr$
C_{dc} = drilling cost, $\$$
C_{cb} = core bit cost, $\$$
C_{cf} = coring costs per foot, $\$/ft$
C_{Tb} = total cost of the bits used for the whole drilling operations, $\$$
C_{Tr} = total cost of the rig paid as rent for the whole drilling operation, $\$$
C_{owc} = overall well cost excluding the production, $\$/ft$
D = total depth, ft
D_i = depth of interest from where drilling time would be measured for a given bit, ft
D_{i+L} = depth of interest up to where drilling time would be measured for a given bit, ft
$\bar{D}$ = mean depth of the well, ft
$\bar{D}_t$ = the mean depth where the trip was made (i.e., mean depth at the trip level), ft
E_d = drilling price elasticity
i = nominal interest rate per year or growth rate in fraction, $\$$
K = constant, ft/hr
$\bar{l}_s$ = the average length of one stand of drillstring, ft
m = number of interest compounding per year or the number of payments per year, nos.
n = number of years, nos.
N_b = numbers of bit, nos.
N_s = numbers of average stands of drillstring, nos.
P_j = provability of the j[th] event
P_{oil} = percentage change in the crude price

R = percentage change in drilling wells
R_c = core recovery factor, %
ROP = rate of penetration, *ft/hr*
t_c = connection time or non-rotating time during the bit run, *hrs*
t_d = drilling time or rotating time during the bit run, *hrs*
t_{di} = drilling time for a given bit *i*, *hrs*
t_d' = drilling time required to drill from D to $(D + 1000)$, *hr*
t_s = the average time required to handle one stand of drillstring, *hrs*
t_t = trip time required to change a bit and resume drilling operations, *hrs*
t_{bl} = average bit life for a particular depth, *hrs*
t_{ct} = core trip time, *hrs*
t_{cr} = core rotating time, *hrs*
t_{cc} = core connection time, *hrs*
t_{crl} = core recovery, and laying down of core barrel time, *hrs*
t_{co} = coring time, *hrs*
V_f = future value, *\$*
V_p = present value, *\$*
V_{ex} = expected value, *\$*
$\dfrac{dD}{dt}$ = rate of penetration, *ft/hr*
ΔD = formation interval drilled or drilled footage, *ft*
ΔD_{ROP} = the formation depth where the coring will be done (a function of rate of penetration), *ft*

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12

Well Completion

12.1 Introduction

Once a new hole is drilled up to the target level, a decision is made whether to set production casing and complete the well or to plug and abandon it. This important evaluation is made after a careful interpretation and consideration of well test data (coring, logging etc.). It is considered to be one of the most critical parts in the preparation for oil production. In petroleum engineering, well completion is the process of making a drilled well ready for production or injection. Well completion is defined as "the design, selection and installation of equipment and the specification of treatment and procedures necessary to allow a safe and controlled flow of hydrocarbon from the well and thereafter to bring the well into production which satisfies the operator's objectives for the field development". Therefore, completions are the interface between the reservoir and surface production which is a combination of physics, chemistry, mathematics, engineering, geology, hydraulics, material science and practical hands-on well-site experience. The definition of well completion shows that there are three basic requirements of any completion. A completion system must provide a means of hydrocarbon production or injection which is safe, efficient and/or economic, and reliable. It represents a large percentage of the expenditure in the development of an oil or gas field. It is of utmost importance that the well be "completed" correctly at the onset, in order that maximum overall productivity of the field may be obtained. The ideal completion is the lowest cost completion which will meet the demands placed on it during its producing lifetime. In simple terms, the term "well completion" refers to the methods by which a newly drilled well can be finalized so that reservoir fluids can be produced to surface production facilities efficiently and safely. In general, the process of completing a well includes i) a method of providing satisfactory communication between the reservoir and the borehole, ii) the design of the tubulars (casing and tubing) which will be installed in the well, iii) an appropriate method of raising reservoir fluids to the surface, iv) the design and the installation in the well of the various components used to allow efficient production, pressure integrity testing, emergency containment of reservoir fluids, reservoir monitoring, barrier placement, well maintenance and well kill, and v) the installation of safety devices and equipment which will automatically shut a well in the event of a blowout. In this chapter, sets of multiple choice question (MCQs) are included which are related to the drilling well completions. MCQs are based on the writer's text-book, *Fundamentals of Sustainable Drilling Engineering* – ISBN: 978-0-470-87817-0.

12.2 Multiple Choice Questions

1. A decision to run the production casing for exploration wells should be made directly
   a) After complete drilling
   b) After running and evaluating logging tests
   c) Before drilling the well
   d) None of the above
2. The main objective of completing the well is
   a) Maximum recovery
   b) Less cost
   c) Safe way
   d) All of the above
3. Which of the following is considered as smart completion?
   a) Tubing with sliding sleeves
   b) Dual packer completion
   c) Three tubing string completion
   d) None of the above
4. Which of the following is considered as conventional completion?
   a) Open hole completion
   b) Multi-completion with real time pressure recording
   c) Multi-completion with downhole control equipment
   d) All of the above
5. In which situations open hole completion is preferred?
   a) Pay zone is thin
   b) Formation is strong and consolidated
   c) Formation fluid is gas
   d) All of the above

6. What is the main advantage of open hole completion?
   a) Decrease formation damage
   b) No need for well cleanout
   c) Easy to isolate any portion of the pay zone
   d) All of the above
7. What is the main disadvantage of open hole completion?
   a) Cannot be converted to liner perforation
   b) Selective stimulation is difficult
   c) Rig time is greater than other types
   d) None of the above
8. All of the following are the advantages of uncemented liner completion except
   a) No perforation is required
   b) Help in control sand production
   c) Production control is easy
   d) None of the above
9. All of the following are the disadvantages of uncemented liner completion except
   a) Completion time is more
   b) Selective stimulation is difficult
   c) Perforation is required
   d) All of the above of the above
10. Which of the following completion is suitable when formation has fine and abrasive sand?
    a) Wire wrapped screen
    b) Slotted liner
    c) Screen liner
    d) External gravel pack
11. Cased and cemented completion is proposed when
    a) Formation rock is not consolidated
    b) Formation fluid is natural gas
    c) Pay zone is very thick
    d) All of the above
12. All of the following are not the advantages of perforated completion except
    a) Perforation cost is significant
    b) Selective stimulation can be difficult
    c) Can be easily dependent
    d) All of the above
13. Tubingless completion should be proposed when:
    a) High production rates are required
    b) High surface pressures are required
    c) Low production rates are required
    d) None of the above

14. What is the limitation of using multiple string completion?
    a) Packers cannot be used
    b) Cannot produce oil and gas from the same well
    c) Limitations in tubing string diameter
    d) All of the above
15. The type that allows more than one zone to produce from one tubing string is
    a) Commingled completion
    b) Sequential zonal completion
    c) Multi-zone completion
    d) All of the above
16. The completion type that allows more than one zone to produce from one tubing string selectively is
    a) Commingled completion
    b) Single string multi-zone completion
    c) Multi-zone completion
    d) None of the above
17. The completion type that deplete the pay zones in a well one after one is
    a) Commingled completion
    b) Multi-zone completion
    c) Sequential zonal completion
    d) None of the above
18. All of the following are the functions of packers except
    a) Protect casing from formation fluids
    b) Isolate damaged areas
    c) Provide selective production
    d) None of the above
19. Which of the following is used as a seat for gas lift valves?
    a) Side pocket valve
    b) Sliding sleeve
    c) Expansion joint
    d) None of the above
20. Which of the following landing nipples is frequently used in well completion?
    a) No-go nipples
    b) Selective-landing nipples
    c) Safety valve nipples
    d) All of the above
21. Tubing string is supported at the top of the well by
    a) Well head
    b) Tubing hanger
    c) Permanent packer
    d) Slips

22. Which of the following equipment is used to control well pressures?
    a) Christmas tree
    b) Downhole safety valve
    c) Surface safety valve
    d) All of the above
23. Down hole safety valves can be operated by
    a) Applying pressure to the annulus
    b) Applying pressure inside the tubing string
    c) Applying pressure through a pressure line to the valve
    d) All of the above
24. If sand production is noticed in a well, what should be the temporary solution to reduce it?
    a) Decrease production rate
    b) Increase production rate
    c) Install sand screen
    d) All of the above
25. If sand production is noticed in a well, what should be the permanent solution to reduce it?
    a) Decrease production rate
    b) Increase production rate
    c) Install sand screen
    d) All of the above
26. Which of the following acid solutions is good in stimulating sand formations?
    a) HCl acid
    b) HF acid
    c) EDTA chelating agent
    d) Acid mud
27. Which of the acid solutions is good in stimulating carbonate formations?
    a) HCl acid
    b) HF acid
    c) EDTA chelating agent
    d) Acid mud
28. Which of the acid solutions is go-5od in stimulating carbonate formations with low permeability and low fracture pressure?
    a) HCl acid
    b) HF acid
    c) EDTA chelating agent
    d) Acid mud
29. If produced water is the result of water coming from a zone in a well, what should be the temporary solution?
    a) Shut off that zone
    b) Produce below critical flow rate
    c) Increase production to get more oil
    d) All of the above

30. If produced water is coming from a zone in a well, what should be the permanent solution?
    a) Shut off that zone
    b) Produce below critical flow rate
    c) Increase production to get more oil
    d) All of the above

31. What is the best way of preventing well casings from corrosion?
    a) Add anti-corrosion additives in the casing fluids
    b) Install tubing string with packer
    c) Use anti-corrosion coating
    d) All of the above

32. Which of the following is considered as conventional completion?
    a) Tubing with sliding sleeves
    b) Dual packer completion
    c) Three tubing string completion
    d) All of the above

33. After removing the permanent packer from the well, can it be reused?
    a) It can be reused after maintenance
    b) It can be reused without any maintenance
    c) It cannot be reused
    d) None of the above

34. After removing retrievable packer from the well, can it be reused?
    a) It can be reused after maintenance
    b) It cannot be reused
    c) It can be reused without any maintenance
    d) None of the above

35. How can you make sure there is a corrosion in the tubing string?
    a) There is decrease in well head pressure
    b) There is increase in annulus pressure
    c) There is decrease in production rate
    d) All of the above

36. What will be the best completion option to produce heavy oils?
    a) Gas lifting
    b) ESP pumping
    c) Natural production
    d) None of the above

37. An oil formation has low productivity index—which of the following will be the best depletion option?
    a) Drill vertical well and inject acids to increase productivity
    b) Increase contact length by drilling horizontal well
    c) Drill vertical well and create fractures
    d) All of the above

38. For a well having four oil zones with the same properties, which of the following will be the best completion option?
    a) Four tubing string
    b) Three tubing strings and produce top zone through casing
    c) One tubing string and four packers and sliding sleeves
    d) All of the above
39. For a well having oil and sour gas zones, which of the following will be the best completion option?
    a) Two tubing string
    b) One tubing string and produce all zone through it
    c) One tubing string and produce top zone through casing
    d) None of the above
40. For a well having oil and gas zones, which of the following will be the best completion option?
    a) Two tubing string
    b) One tubing string and produce all zone through it
    c) One tubing string and produce top zone through casing
    d) None of the above
41. All of the following are main objectives of well completion except:
    a) Maximum recovery
    b) Identifying the pay-zone
    c) Less cost
    d) Safe operation
42. Which of the following is not considered as conventional completion?
    a) Multi-lateral completion
    b) Multi-completion with real time pressure recording
    c) Multi-completion with downhole control equipment
    d) All of the above
43. All of the following situations are suitable for open hole completion except
    a) Thin formation
    b) Unconsolidated formation
    c) Oil formation with gas cap on top
    d) All of the above
44. Which of the following completion is suitable for consolidated reservoirs?
    a) Open hole completion
    b) Sand screen completion
    c) External gravel pack
    d) All of the above
45. All of the following are the advantages of perforated completion except:
    a) Control gas production
    b) Can be easily deepened
    c) Selective stimulation is difficult
    d) None of the above

46. Packer completion should be proposed when:
    a) Two pay zones are present
    b) High production rates are required
    c) High surface pressures are required
    d) None of the above

47. The type that allows more than one zone to produce separately is:
    a) Packer completion
    b) Commingled completion
    c) Multi-zone completion
    d) a and b.

48. Tubing string is fixed at the bottom of the well by:
    a) Tubing hanger
    b) Tubing hanger
    c) Tubing anchor
    d) Slips

49. Which of the acid solutions is good in stimulating carbonate formations with good permeability and high fracture pressure?
    a) HF acid
    b) HCl acid
    c) EDTA chelating agent
    d) Acid mud

50. If a zone in a well produces gas as a result of gas conning, what should be the temporary solution?
    a) Shut off that zone
    b) Produce above critical flow rate
    c) Increase production to get more oil
    d) None of the above

Answer: 1b, 2d, 3d, 4a, 5b, 6a, 7b, 8c, 9c, 10a, 11a, 12c, 13a, 14c, 15a, 16b, 17c, 18d, 19a, 20d, 21b, 22d, 23c, 24a, 25c, 26d, 27a, 28c, 29b, 30a, 31b, 32d, 33c, 34a, 35d, 36b, 37b, 38c, 39a, 40c, 41b, 42d, 43d, 44a, 45c, 46a, 47d, 48c, 49b, 50d.

12.3 Summary

Well completion is the key factor in maintaining good well condition to deliver the required oil rate with minimum cost for the producing and surface processing units. It also takes into account the long-term benefit of reservoir total recovery. This chapter discusses all the drilling completion concepts through MCQs to test the student's understanding in the subject matter. The answers to the self-practiced MCQs exercises are presented in Appendix B.

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13

Additional Workout Examples

13.1 Introduction

This chapter covers all the chapters in the book with selected workout examples. The aim of this chapter is to give insight into some more difficult examples for the student. The chapter does not have any MCQs.

13.2 Drilling Fluids

Example 13.1: It is required to prepare 600 *bbls* of drilling fluid that have mud weight of 15.4 *ppg* using hematite with a specific gravity of 5.1. How many barrels of water and tons of hematite are needed to prepare the complete mud?

Solution:

Given data:

V_m = Required volume of the mud = 600 *bbls*
MW = Required mud weight = 15.4 *ppg*
SG_{Hem} = specific gravity of hematite = 5.1

Required data:

Volume of water in barrels.
Amount of hematite in tons.

To calculate the amount of each material, the mass balance equation can be used. The mass balance states that:

$$V_f \rho_f = V_w \rho_w + V_{hem} \rho_{hem}$$

In the above equation, the water volume equals the total mud volume minus the hematite volume.

$$V_f \rho_f = (V_f - V_{hem}) \rho_w + V_{hem} \rho_{hem}$$

By arranging the above equation and solving for the hematite volume, the equation becomes:

$$V_{hem} = \frac{V_f(\rho_f - \rho_w)}{\rho_{hem} - \rho_w} = \frac{600 \times (15.4 - 8.33)}{5.1 \times 8.33 - 8.33} = 124.21\ bbls$$

Because solids are not measured in barrels. It is measured by its mass. Therefore, we should change its volume to pounds or tons. So, hematite specific gravity will be used as follows:

$$Mass_{hem} = 124.21\ bbls \times \left(42 \frac{gals}{bbl}\right) \times \left(42.48 \frac{lbm}{gal}\right)$$
$$= 221{,}626 lbm = \mathbf{100}\ \boldsymbol{tons}$$

The required water volume is equal to:

$$V_w = V_f - V_{hem} = 600 - 124.21 = 475.8\ bbls\ OR\ \mathbf{476\ bbls}$$

Example 13.2: A new section in a well is planned to be drilled using 10.4 *ppg* mud. There is 500 *bbls* of 9.5 *ppg* mud in the mud tanks. The mud engineer also prepared 700 *bbls* of 10.4 *ppg* and stored it in another mud tank. The new mud volume to be prepared for a special purpose which is 1500 *bbls*. Calculate the following: a) if the mud engineer mixed all the mud that are available in the mud tanks, what should be the mud weight of the new mud, and b) calculate the amount of water and Barite to be added to the mud in the tanks to prepare 1500 *bbls* of 10.4 *ppg* mud. Barite specific gravity is 4.3.

Solution:

Given data:

V_{m1}	=	Volume of the first mud	=	500 *bbls*
MW_1	=	Mud weight of the first mud	=	9.5 *ppg*
V_{m2}	=	Volume of the second mud	=	700 *bbls*
MW_2	=	Mud weight of the second mud	=	10.4 *ppg*
V_{mf}	=	The required mud volume	=	1,500 *bbls*
SG_{Bar}	=	specific gravity of Barite	=	4.3

Required data:
MW_{mix} = Mud weight of the mixed mud
Amount of Barite
Amount of water

The mud weight of the new mixed mud can be calculated using the volume ratio of the mix muds. The volume ratio of the two muds is equal to:

$$R_1 = \frac{V_{m1}}{V_T} = \frac{500}{1,200} = 0.417$$

$$R_2 = \frac{V_{m2}}{V_T} = \frac{700}{1,200} = 0.583$$

The mud weight of the mixed fluid can be calculated using the following equation:

$$MW_{mix} = MW_1 \times R_1 + MW_2 \times R_2$$

$$MW_{mix} = 9.5 \times 0.417 + 10.4 \times 0.583 = \mathbf{10.03}\ \boldsymbol{ppg}$$

To prepare 1500 *bbls* of 10.4 *ppg* mud from the above mixed mud, we need to determine the amount of Barite and water to be added in order to raise the mud weight to 10.4 *ppg*. The mass balance equation is going to be used to determine the amount of Barite and water to be added to the mixed mud. First we will determine the MW of 300 *bbls* that should be mixed with 1,200 *bbls* of 10.03 *ppg* to result in 1500 *bbls* of 10.4 *ppg*. The above equations can again be used to find out that MW.

$$R_1 = \frac{V_{m1}}{V_T} = \frac{1,200}{1,500} = 0.8$$

$$R_2 = \frac{V_{m2}}{V_T} = \frac{300}{1,500} = 0.2$$

Now the MW of the new mud to be prepared is equal to:

$$MW_{mix} = MW_1 \times R_1 + MW_2 \times R_2$$

$$10.4 = 10.03 \times 0.8 + MW_2 \times 0.2$$

$$MW_2 = 11.88\ ppg$$

The amount of water Barite to be added to prepare 300 *bbls* of 11.88 *ppg* mud can be calculated as below. The Barite volume can be calculated using the following equation:

$$V_{Bar} = \frac{V_f(\rho_f - \rho_w)}{\rho_{Bar} - \rho_w} = 300 \times \frac{10.4 - 8.33}{4.3 \times 8.33 - 8.33} = \mathbf{22.6}\ \boldsymbol{bbls}$$

Changing the volume of Barite to pounds gives:

$$Mass_{Bar} = 22.6\ bbls \times \left(42 \frac{gals}{bbl}\right) \times \left(35.8 \frac{lbm}{gal}\right)$$
$$= 33{,}981\ lbm = \mathbf{15.45\ \mathit{tons}}$$

The amount of water is equal to 277.4 *bbl*. Thus if we add 15.45 tons of Barite to 277.4 *bbl*. of water and mixed this mud with the mud that we have in the tanks, we should have 1,500 *bbl*. of 10.4 *ppg* of mud that can be used for the new section.

Example 13.3**:** Four mud tanks of 30 *ft* × 7 *ft* × 7 *ft* have 14.5 *ppg* mud prepared using 10% low SG solid. The mud engineer decided to have the same mud volume in the tanks but increase the mud weight to 16.0 *ppg* and decrease the volume fraction of the low-SG-solids to 7%. How many barrels of original mud should be discarded? And what will be the amount of water and Barite that should be added to the original mud? The density of Barite is 35 *ppg*

Solution:

Given data:

n	=	Number of the mud tanks	=	4
L	=	Length of the mud tank	=	30 *ft*
W	=	Width of the mud tank	=	7 *ft*
H	=	Height of the mud in each tank	=	7 *ft*
MW_o	=	Mud weight of the original mud	=	14.5 *ppg*
MW_f	=	Mud weight of the final mud	=	16.0 *ppg*
f_1	=	The original volume fraction of low-SG-solids	=	10%
f_2	=	The new volume fraction of low-SG-solids	=	7%

Required data:

The discarded mud volume
Amount of Barite
Amount of water

The volumetric balance equation states that:

$$V_1 \times f_2 = V_2 \times f_1$$

If we need to have same mud volume but decrease the percentage of low-SG-solids, above equation should be used. The volume of the mud in the tanks equals:

$$V_1 = n \times L \times W \times H = 4 \times 30 \times 7 \times 7 = 5{,}880\, ft^3 = 1{,}047\ bbls$$

Thus,

$$V_1 = V_2 \times \frac{f_1}{f_2} = 1{,}047 \times \frac{0.07}{0.10} = 733\ bbls$$

Now, the volume of water to be added can be calculated using equation:

$$V_W = \frac{(\rho_B - \rho_2)\times V_2 - (\rho_B - \rho_1)\times V_1}{\rho_B - \rho_w}$$

$$V_W = \frac{(35.0-16.0)\times 1{,}047 - (35.0-14.5)\times 733}{35.0-8.33}$$
$$= 182.5\ bbls$$

The amount of Barite to be added is equal to:

$$m_B = (V_2 - V_1 - V_w)\rho_B = (1047-733-182.5)\times(35\times 42) = \mathbf{193{,}305}\ \boldsymbol{lbm}$$

Example 13.4: While drilling 12.25" section in a well at a depth of 7,500 *ft*, drilling was stopped to repair a problem in the mud pumps. The gel strength was measured to be 22 $lb_f/100\ ft^2$. Calculate the pressure required to break the gel of the mud to start pumping again. Drillstring consists of 540 *ft* of drill collar 8" OD and 2.5" ID, and 5.0" OD and 4.27" ID drill pipes.

Solution:

Given data:

D	=	Well depth	=	7,500 *ft*
d_h	=	Hole diameter	=	12.25"
τ_g	=	Gel strength	=	22 $lb_f/100\ ft^2$
OD_{DC}	=	Outside diameter of drill collars	=	8.0"
ID_{DC}	=	Inside diameter of drill collars	=	2.5"
L_{DC}	=	Length of drill collars	=	540 *ft*
OD_{DP}	=	Outside diameter of drill pipes	=	5.0"
ID_{DP}	=	Inside diameter of drill pipes	=	4.27"
L_{DP}	=	Length of drill pipes	=	9,500 *ft*

Required data:

Gel-breaking pressure

The required pressure to break the gel of the mud is applied inside the tubulars as well as in the annulus. The breaking pressure inside the tubulars can be calculated using the following equation:

$$P = \frac{\tau_g \times L}{300\times d_i}$$

The pressure required to break the mud gel inside the drill pipes is equal to:

$$P_{dp} = \frac{22\times 9{,}500}{300\times 4.27} = 119.5\ psi$$

The pressure required to break the mud gel inside the drill collars is equal to:

$$P_{dc} = \frac{22\times 540}{300\times 2.5} = 15.8\ psi$$

The pressure required to break the mud gel in the annulus between the pipes and the hole is equal to:

$$P = \frac{\tau_g \times L}{300 \times (d_h - d_i)}$$

The pressure required to break the mud gel in the annulus between the drill pipes and the hole is equal to:

$$P_{ann\_dp} = \frac{22 \times 9{,}500}{300 \times (12.25 - 5.0)} = 70.4\ psi$$

The pressure required to break the mud gel in the annulus between the drill collars and the hole is equal to:

$$P_{ann\_dc} = \frac{22 \times 540}{300 \times (12.25 - 8.0)} = 9.3\ psi$$

Thus, the pressure required to break the mud gel in the well is equal to:

$$P = 119.5 + 15.8 + 70.4 + 9.3 = 215\ psi$$

13.2 Drilling Hydraulics

Example 13.5: An intermediate section in a well was drilled using 9.6 *ppg* mud at a pumping rate of 650 gpm. The pump pressure was 2,250 *psi* while the total system pressure losses in the tubing and annulus were 975 *psi*. What is the horsepower of the bit? If the density of the mud is changed to 11.3 *ppg*, what will be the available horsepower assuming that the pumping pressure, rate and pressure losses remain the same?

Solution:

Given data:

ρ_o	=	Original mud weight	=	9.6 *ppg*
ρ_n	=	New mud weight	=	11.3 *ppg*
P_{pump}	=	Pump pressure	=	2,250 *psi*
Q_{pump}	=	Pumping rate	=	650 gpm
P_{losses}	=	System pressure losses	=	975 *psi*

Required data:

Bit hydraulic horsepower

The pressure across the bit can be calculated using the following equation:

$$\Delta P_{bit} = P_{pump} - P_{losses} = 2{,}250 - 975 = 1{,}275\ psi$$

The bit hydraulic horsepower can be calculated using the following equation:

$$HHP_{bit} = \frac{\Delta P_{bit} \times Q_p}{1714} = \frac{1{,}275 \times 650}{1714} = 484\ hp$$

To calculate the bit hp after changing the mud weight to 11.3 *ppg*, the pressure loss across the bit should be calculated first. The new pressure loss can be calculated using the following equation:

$$\frac{\Delta P_1}{\Delta P_2} = \frac{\rho_1 \times Q_1^2}{\rho_2 \times Q_2^2}$$

Since the flow rate is same, the new pressure loss is equal to:

$$\Delta P_2 = \Delta P_1 \times \frac{\rho_2}{\rho_1} = 1{,}275 \times \frac{11.3}{9.6} = 1{,}501\ psi$$

The new bit hydraulic available for drilling is equal to:

$$HHP_{bit} = \frac{\Delta P_{bit} \times Q_p}{1714} = \frac{1{,}501 \times 650}{1714} = 569\ hp$$

Example 13.6: A mud pump of a hydraulic horsepower of 1,150 hp and maximum pump pressure of 4,000 *psi* is used to pump a 12.8 *ppg* mud at 84% volumetric efficiency. The frictional pressure losses are 1,900 *psi* at pumping rate of 325 *gpm*. The flow index exponent is estimated to be 1.76. Calculate the optimum bit nozzles area and the optimum flow rate using the maximum bit hydraulic horsepower criterion. Use C_d = 0.95.

Solution:

Given data:

ρ	=	Mud weight	=	12.8 *ppg*
hp	=	Pump horsepower	=	1,150 *hp*
P_{pump}	=	Pump pressure	=	4,000 *psi*
Q_{pump}	=	Pump flow rate	=	325 *gpm*
P_{losses}	=	Friction pressure losses	=	1,900 *psi*
η_V	=	Pump volumetric efficiency	=	84%
m	=	Flow index exponent	=	1.76
C_d	=	Discharge coefficient	=	0.95

Required data:

A_n = Area of the bit nozzles in *inches*.
Q_{opt} = Optimum pump flow rate in *gpm*.

By using the maximum bit hydraulic horsepower criterion, the optimum friction losses pressure can be calculated using the following equation:

$$P_{loss,opt} = \frac{1}{1+m} \times P_{p,max} = \frac{1}{1+1.76} \times 4{,}000 = 1{,}449\ psi$$

Consequently, the optimum pressure loss at the bit is equal to:

$$P_{bit,opt} = P_{p,max} - P_{loss,opt} = 4{,}000 - 1{,}449 = 2{,}551\ psi$$

The optimum flow rate can be estimated using the following equation:

$$Q_{opt} = Q \times 10^{\left[\frac{1}{m}\log\frac{P_{loss,opt}}{P_{loss}}\right]} = 325 \times 10^{\left[\frac{1}{1.76}\times\log\left(\frac{1,449}{1,800}\right)\right]} = 279\ gpm$$

And the optimum area of the bit nozzles can be calculated using the following equation:

$$A_n = \sqrt{\left[\frac{8.311\times10^{-5}\times\rho\times Q_{opt}^2}{C_d^2\times\Delta P_{b,pt}}\right]} = \sqrt{\left[\frac{8.311\times10^{-5}\times12.8\times279^2}{0.95^2\times2,551}\right]}$$
$$= \mathbf{0.189}\ \boldsymbol{in^2}$$

Example 13.7: A section in a well is drilled using a bit that has 3 × 13 nozzles. The rig engineer noticed that if 11.3 *ppg* mud was pumped at a rate of 650 gpm, a pump pressure and pressure difference across the bit were 3,500 *psi*, and 2,250 *psi*, respectively. When the pumping was slowed down to a rate of 350 *gpm*, the pump pressure and pressure difference across the bit were 1,000 *psi* and 600 *psi*, respectively. The power of the pump is 1,700 *hp*, whereas the pump volumetric efficiency is 87%. The maximum allowable pump pressure is 4,500 *psi*. Determine the pump optimum operating conditions, the nozzles area of the new bit for the maximum bit horsepower performance, and the new bit horsepower for the selected conditions.

Solution:

Given data:

ρ	=	Mud weight	=	121.3 *ppg*
hp	=	Pump horsepower	=	1,700 *hp*
P_{p_max}	=	Maximum pump pressure	=	4,500 *psi*
Q_{p1}	=	Pump flow rate	=	650 *gpm*
P_{p1}	=	Corresponding pump pressure	=	3,500 *psi*
P_{b1}	=	Corresponding pressure loss across the bit	=	2,250 *psi*
Q_{p2}	=	Pump flow rate	=	350 *gpm*
P_{p1}	=	Corresponding pump pressure	=	1,000 *psi*
P_{b1}	=	Corresponding pressure loss across the bit	=	600 *psi*
η_V	=	Pump volumetric efficiency	=	87%
C_d	=	Discharge coefficient	=	0.95

Required data:

Pump operating conditions

A_n = Area of the bit nozzles in *inches*.

hp = The bit *hp*

To find out the pump operating conditions, we should calculate the maximum pumping rate and the optimum rate, then we compare them. First, the frictional pressure losses for the two pumping rates are:

$$P_{loss1} = P_{p1} - P_{b1} = 3,500 - 2,250 = 1,250\ psi$$

$$P_{loss2} = P_{p2} - P_{b2} = 1{,}000 - 600 = 400\ psi$$

The flow index exponent can be calculated using the equation:

$$m = \frac{\log \frac{P_{loss1}}{P_{loss2}}}{\log \frac{Q_1}{Q_2}} = \frac{\log \frac{1{,}250}{400}}{\log \frac{650}{350}} = 1.84$$

The maximum pumping rate that the pump can deliver is:

$$Q_{max} = 1714 \times \eta_v \times \frac{hp}{P_{max}} = 1714 \times 0.87 \times \frac{1{,}700}{4{,}500} = 563.3\ gpm$$

The optimum pressure losses are equal to:

$$P_{loss,opt} = \frac{1}{1+m} \times P_{max} = \frac{1}{1+1.84} \times 4{,}500 = 1{,}584.1\ psi$$

And the optimum pumping rate is equal to:

$$Q_{opt} = Q \times 10^{\left[\frac{1}{m}\log\frac{P_{loss,opt}}{P_{loss}}\right]} = 350 \times 10^{\left[\frac{1}{1.84}\times\log\left(\frac{1{,}584.1}{400}\right)\right]} = \mathbf{739.3\ gpm}$$

As the optimum pumping rate is greater than the maximum rate that the pump can deliver, we are going to consider the maximum rate in calculating the nozzles area. Nozzles area can be calculated using the following equation:

$$A_n = \sqrt{\left[\frac{8.311 \times 10^{-5} \times \rho \times Q_{opt}^2}{C_d^2 \times \Delta P_{b,opt}}\right]} = \sqrt{\left[\frac{8.311 \times 10^{-5} \times 11.3 \times 563.3^2}{0.95^2 \times 2{,}916}\right]}$$
$$= \mathbf{0.338\ in^2}$$

We can then find that the new bit should have three nozzles, each one has a size of 12/32".

The hydraulic horsepower of the new bit is equal to:

$$HP_b = \frac{\Delta P_b \times Q}{1714} = \frac{2{,}916 \times 563.3}{1{,}714} = \mathbf{958\ hp}$$

Example 13.8: A surface section of 17.5" in a well is planned to be drilled using 5.5" drill pipes. The drilling fluid has mud weight of 9.4 *ppg*, plastic viscosity of 42 *cp*, and yield point of 17 $lb_f/100\ ft^2$. The estimated cuttings size and density are 0.35" and 23 *ppg*, respectively. If the planned pumping rate is 800 *gpm*, calculate the annular velocity, cutting slip velocity and the transport ratio.

Solution:

Given data:

d_h	=	Hole diameter	=	17.5"
PV	=	Plastic viscosity	=	42 *cp*
YP	=	Yield point	=	17 lbf/100 ft^2
OD_{DP}	=	Outside diameter of drill pipes	=	5.5"
MW	=	Mud weight	=	9.4 *ppg*
$d_{cutting}$	=	Cutting size	=	0.35"
$\rho_{cutting}$	=	Allowable puling force	=	23 *ppg*
Q	=	Pumping rate	=	800 *gpm*

Required data:

V_a	=	Annular velocity
V_s	=	Slip velocity
R_t	=	Transport ratio

Annular velocity can be calculated using the following equation:

$$v_a = \frac{60 \times Q}{2.448 \times (d_h^2 - OD_{DP}^2)} = \frac{60 \times 800}{2.448 \times (17.5^2 - 5.5^2)}$$
$$= \mathbf{71.04}\ \boldsymbol{ft\ /\ min}$$

To calculate the slip velocity, the value of β should be determined. Apparent viscosity of the mud can be calculated using the following equation:

$$\mu_a = \mu_p + \frac{300 \times YP \times d_{cutting}}{v_a} = 42 + \frac{300 \times 17 \times 0.3}{71.04} = 67.1\ cp$$

β value can be calculated using the following equation:

$$\beta = \frac{\mu_a}{MW \times d_{cutting}} = \frac{67.1}{9.4 \times 0.35} = 20.4$$

Since β value is greater than 10, the slip velocity can be calculated using the following equation:

$$v_s = 86.4 \times d_{cutting} \times \sqrt{\frac{\rho_{cutting} - \rho_m}{\rho_m}} = 86.4 \times 0.35 \times \sqrt{\frac{23 - 9.4}{9.4}}$$
$$= \mathbf{36.37}\ \boldsymbol{f\ /min}$$

Transport ratio is given by the following equation:

$$R_t = 1 - \frac{v_s}{v_a} = 1 - \frac{36.37}{71.04} = 0.49$$

13.3 Well Control

Example 13.9: A kick was encountered while drilling a well at depth of 6,500 *ft* using 9.8 *ppg* mud. The stabilized shut in drill-pipe pressure was 450 *psi*. The mud tanks have 750 *bbls* of 9.8 *ppg* mud, which is enough to displace the well. Determine the amount of Barite to be added in order to control the well if the required over balance pressure is 300 *psi*. In addition, determine the final mud volume.

Solution:

Given data:

D	=	Well's current depth	=	6,500 *psi*
ρ_o	=	Mud weight of the original mud	=	9.8 *ppg*
$SIDPP$	=	Shut-in drill pipe pressure	=	450 *psi*
V	=	Mud volume in the mud tanks	=	750 *bbls*
P_{ob}	=	Required overbalance pressure	=	300 *psi*

Required data:

Amount of Barite

Final mud volume

To determine the amount of Barite to be added to the already prepared mud, we need to know the required MW of the kill fluid. The mud weight of the kill fluid can be calculated using the following equation:

$$\rho_{KF} = \frac{0.052 \times \rho_o \times D + SIDPP + P_{ob}}{0.052 \times D} = \frac{0.052 \times 9.8 \times 10{,}000 + 450 + 300}{0.052 \times 10{,}000}$$

$$\rho_{KF} = 12.02\ ppg$$

Thus, the Barite amount is equal to:

$$m_{Bar} = \frac{\rho_{Bar} \times V_f (\rho_f - \rho_o)}{\rho_{Bar} - \rho_f} = \frac{33.5 \times 42 \times 750 \times (12.02 - 9.80)}{33.5 - 12.02}$$

$$m_{Bar} = \mathbf{109{,}004}\ \boldsymbol{lbm}$$

The Barite volume is equal to:

$$V_{Bar} = \frac{m_{Bar}}{\rho_{Bar}} = \frac{109{,}004}{42 \times 33.5} = 77.5\ bbls$$

The final mud volume is equal to:

$$V_f = V_o + V_{Bar} = 750 + 75.5 = \mathbf{825.5}\ \boldsymbol{bbls}$$

Example 13.10: An intermediate section in a well is planned to be drilled from 7,500 *ft* to 11,750 *ft*. The drillstring consists of 900 *ft* drill collars and the rest are drill pipes. The maximum anticipated friction losses in the annulus due to mud flow are 0.045 *psi/ft* behind the drill collar and 0.01 *psi/ft* behind the drill pipes. The maximum anticipated surge and swab pressure during tripping are 0.03 *psi/ft* and 0.025 *psi/ft*, respectively. The required overbalanced pressure is 300 *psi*. The estimated pore and fracture pressure gradients at the above depths are 12.8 *ppg* and 14.2 *ppg*, 16.3 *ppg* and 17.8 *ppg*. Select the optimum mud weight to drill the above section under the above anticipated conditions, and determine whether the mud weight can make any problem for the well with regards to blow-out or mud losses.

Solution:

Given data:

L_{DC}	=	Length of drill collars	=	900 *ft*
D_{cas}	=	Last casing depth	=	7,500 *ft*
TD	=	Well total depth	=	11,750 *ft*
PL_{DP}	=	Pressure losses around the drill pipes	=	0.045 *psi/ft*
PL_{DC}	=	Pressure losses around the drill collars	=	0.010 *psi/ft*
P_{surge}	=	Surge pressure during tripping	=	0.030 *psi*
P_{swab}	=	Swab pressure during tripping	=	0.025 *psi*
P_{ob}	=	Over balanced pressure	=	300 *psi*
$G_{P@7,500}$	=	Pore pressure gradient at 7,500 *ft*	=	12.8 *ppg*
$G_{P@11,750}$	=	Pore pressure gradient at 11,750 *ft*	=	14.2 *ppg*
$G_{F@7,500}$	=	Fracture pressure gradient at 7,500 *ft*	=	16.3 *ppg*
$G_{P@11,750}$	=	Fracture pressure gradient at 11,750 *ft*	=	17.8 *ppg*

Required data:

MW = required mud weight in *ppg*.

To determine the required MW, we should calculate the two depths, then we should select the mud weight.

At depth of 7,500 ft:

The required MW based on the pore pressure gradient and over balance is equal to:

$$MW = G_p + \frac{P_{ob}}{0.05 \times D_{cas}} = 12.8 + \frac{300}{0.052 \times 7,500} = 13.57\ ppg$$

The ECD during pumping is equal to:

$$ECD = MW + \frac{PL_{DC} \times L_{DC} + PL_{DP} \times L_{DP}}{0.052 \times D_{cas}}$$

$$= 13.57 + \frac{0.045 \times 900 + 0.01 \times 6,600}{0.052 \times 7,500} = 13.84\ ppg$$

The surge and swab conditions should be checked as follows:

$$G_{swab} = MW - \frac{P_{swab} \times D_{cas}}{0.052 \times D_{cas}} = 13.57 + \frac{0.025 \times 7,500}{0.052 \times 7,500} = 13.09\ ppg$$

$$G_{surge} = MW + \frac{P_{surge} \times D_{cas}}{0.052 \times D_{cas}} = 13.57 + \frac{0.03 \times 7{,}500}{0.052 \times 7{,}500} = 14.15\ ppg$$

Since the ECD is between the surge and swab gradients, **it is safe to drill** with this ECD at depth of 7,500 *ft*.

At depth of 11,750 ft:

The required MW based on the pore pressure gradient and over balance is equal to:

$$MW = G_p + \frac{P_{ob}}{0.05 \times D_{cas}} = 14.2 + \frac{300}{0.052 \times 11{,}750} = 14.69\ ppg$$

The ECD during pumping is equal to:

$$ECD = MW + \frac{PL_{DC} \times L_{DC} + PL_{DP} \times L_{DP}}{0.052 \times D_{cas}}$$

$$= 14.69 + \frac{0.045 \times 900 + 0.01 \times 10{,}850}{0.052 \times 11{,}750} = 14.93\ ppg$$

The surge and swab conditions should be checked as follows:

$$G_{swab} = MW - \frac{P_{swab} \times D_{cas}}{0.052 \times D_{cas}} = 14.69 + \frac{0.025 \times 11{,}750}{0.052 \times 11{,}750} = 14.21 ppg$$

$$G_{surge} = MW + \frac{P_{surge} \times D_{cas}}{0.052 \times D_{cas}} = 14.69 + \frac{0.03 \times 11{,}750}{0.052 \times 11{,}750} = 15.27 ppg$$

Since the ECD is between the surge and swab gradients, **it is safe to drill with this ECD at depth of 7,500 ft**. However, we cannot drill all the way to the TD with 13.57 *ppg*, because this will make well to encounter kick at depth 11,750 *ft*. We know that when ECD is less than swab pressure gradient, there is a possibility to have a kick. Similarly, if we use MW of 14.69 *ppg* from 7,500 *ft* this will make the well under surge condition. Thus, this section cannot be drilled using conventional drilling methods.

***Example 13.11*:** While drilling a production section at depth of 12,500 *ft*, a kick was encountered and 25 *bbls* of kick volume was recorded. The density of the mud before kick was 12.0 *ppg* while the kick fluid density was estimated to be 1.5 *ppg*. The last casing shoe was at 11,500 *ft* while its fracture gradient was estimated to be 16.1 *ppg*. The drill collars length was 1,000 *ft*. The annulus-DC, annulus-DP and casing-DP capacities were 0.0292 *bbl/ft*, 0.0459 *bbl/ft*, and 0.0520 *bbl/ft*, respectively. Determine the kick tolerance for the above conditions.

Solution:

Given data:

TD	=	Well total depth	=	12,500 *ft*
D_{cas}	=	Last casing depth	=	11,500 *ft*
L_{DC}	=	Length of drill collars	=	1,000 *ft*

V_k	=	Kick volume	=	25 *bbls*
MW	=	Mud weight	=	12.0 *ppg*
ρ_k	=	Kick fluid density	=	1.5 *ppg*
$G_{F@11,500}$	=	Fracture pressure gradient at 11,500 *ft*	=	16.1 *ppg*
$C_{ann\text{-}DC}$	=	Annulus-DC volume capacity	=	0.0292 *bbl/ft*
$C_{ann\text{-}DP}$	=	Annulus-DP volume capacity	=	0.0459 *bbl/ft*
$C_{ann\text{-}DC}$	=	Casing-DP volume capacity	=	0.0520 *bbl/ft*

Required data:

ρ_T = Kick tolerance in *ppg*

To calculate the kick tolerance, we should first calculate the length of the kick in the well. To know whether the kick is only around the drill collars or not, we can calculate the volume of the annulus around the drill collars by the following equation:

$$V_{ann,DC} = C_{ann,DC} \times L_{DC} = 0.0292 \times 1{,}000 = 29.2 \ bbls$$

Since the volume around the drill collars is greater that the kick volume, all the kick fluid in the annulus between the hole and the drill collars.

The length of the kick fluid can be calculated using the following equation:

$$L_k = \frac{V_k}{C_{ann,DC}} = \frac{25.0}{0.0292} = 856.2 \ ft$$

Since the kick fluid didn't reach the casing shoe, the kick tolerance can be calculated using the following equation:

$$\rho_T = (\rho_f - \rho)\frac{D_{cs}}{TD} + (\rho - \rho_k)\frac{L_k}{TD}$$
$$= (16.1 - 12.0) \times \frac{11{,}500}{12{,}500} + (12.0 - 1.5) \times \frac{856.2}{12{,}500}$$
$$\rho_T = \mathbf{4.49\ ppg}$$

Thus, drilling mud can be increased by 4.49 *ppg* before breaking the casing shoe.

***Example 13.12*:** A well kick was encountered while drilling an intermediate section at depth of 10,250 *ft*. The kick volume was measured to be about 23 *bbl*. The density of the original mud was 10.4 *ppg* while the kick fluid density was estimated to be 1.63 *ppg*. The last casing shoe was at 9,550 *ft*, and the fracture gradient at the casing shoe was estimated to be 13.8 *ppg*. The drill collars length was 660 *ft*. The annulus-DC, annulus-DP and casing-DP capacities were 0.0259 *bbl/ft*, 0.0459 *bbl/ft*, and 0.0520 *bbl/ft*, respectively. Determine the kick tolerance for the above conditions.

Solution:

Given data:

TD	=	Well total depth	=	10,250 *ft*
D_{cas}	=	Last casing depth	=	9,550 *ft*

L_{DC}	=	Length of drill collars	=	660 *ft*
V_k	=	Kick volume	=	23 *bbls*
MW	=	Mud weight	=	10.4 *ppg*
ρ_k	=	Kick fluid density	=	1.63 *ppg*
$G_{F@9,550}$	=	Fracture pressure gradient at 9,550 *ft*	=	13.8 *ppg*
$C_{ann\text{-}DC}$	=	Annulus-DC volume capacity	=	0.0259 *bbl/ft*
$C_{ann\text{-}DP}$	=	Annulus-DP volume capacity	=	0.0459 *bbl/ft*
$C_{ann\text{-}DC}$	=	Casing-DP volume capacity	=	0.0520 *bbl/ft*

Required data:

ρ_T = Kick tolerance in *ppg*

We have to find out whether the kick fluid is only around the drill collars or also around the drill pipes. The volume of the annulus around the drill collars can be calculated as follows:

$$V_{ann,DC} = C_{ann,DC} \times L_{DC} = 0.0259 \times 660 = 17.1\ bbls$$

Since the volume around the drill collars is less than the kick volume, we need to determine the kick fluid volume in the annulus around the drill pipes and the hole and the annulus around the casing and the drill pipes. The volume around the hole and the drill pipes is equal to:

$$\begin{aligned} V_{ann,DP} &= C_{ann,DP} \times L_{DP,oh} = 0.0459 \times (10{,}250 - 9{,}550 - 660) \\ &= 1.8\ bbls \end{aligned}$$

The rest of the kick fluid volume will be around the casing and the drill pipes. The kick fluid length can now be calculated using the following equation:

$$\begin{aligned} L_k &= \frac{V_k - V_{ann,DC} - V_{ann,DP}}{C_{cas,DP}} + L_{oh} = \frac{23 - 17.1 - 1.8}{0.052} + (10{,}250 - 9{,}550) \\ &= 778.3\ ft \end{aligned}$$

The depth of the top of kick fluid is equal to:

$$TD - L_k = 10{,}250 - 778.3 = 9{,}471.7\ ft$$

Since the kick fluid reached the casing shoe, the kick tolerance can be calculated using the following equation:

$$\begin{aligned} \rho_T &= (\rho_f - \rho_k)\frac{D_{cs}}{TD} + \rho_k - \rho = (13.8 - 1.63) \times \frac{9{,}550}{10{,}250} + 1.63 - 10.4 \\ &= 2.57\ ppg \end{aligned}$$

Thus, drilling mud can be increased by 2.57 *ppg* before breaking the casing shoe.

13.4 Pore and Fracture Pressure Estimation

***Example 13.13*:** A drillstring got stuck in an intermediate hole of 12.25". An estimated length of the stuck is 60 *ft* by 8" OD drill collars, and the point of stuck is estimated to be at 6,500 *ft*. The mud cake thickness is estimated to be 2/16". The mud weight is 12.2 *ppg*, whereas the pore pressure gradient is estimated to be 11.46 *ppg*. If the friction coefficient is 0.135, determine the pulling force that need to be applied in order to free the drillstring.

Solution:

Given data:

D_{stuck}	=	Depth of stuck	=	6,500 *ft*
d_h	=	Hole diameter	=	12.25"
L_{stuck}	=	Length of the stuck	=	60 *ft*
OD_{DC}	=	Outside diameter of drill collars	=	8.0"
t_{mc}	=	Thickness of mud cake	=	2/16"
MW	=	Mud weight	=	12.2 *ppg*
$G_{P@6,500}$	=	Pore pressure gradient at 6,500 *ft*	=	11.46 *ppg*
Rfr	=	Friction coefficient	=	0.135

Required data:

F_{pull} = Puling force in *lbf*

The required pulling force can be estimated using the following equation:

$$F_{pull} = R_{fr} \times \Delta p \times A_c$$

where R_{fr} is the friction coefficient, ΔP is the pressure differential is *psi*, and A_c is the contact surface area in inches.

The pressure differential can be calculated using the following equation:

$$\Delta p = 0.052 \times D_{stuck} \times (\rho_m - G_p) = 0.052 \times 6{,}500 \times (12.2 - 11.46)$$
$$= 250.1\ psi$$

The contact surface area can be estimated using the following equation:

$$A_c = 2 \times 12 \times L_{stuck} \times \left\{ \left(\frac{d_h}{2} - t_{mc} \right)^2 - \left[\frac{d_h}{2} - t\_mc \times \frac{d_h - t_{mc}}{d_h - OD_{DC}} \right]^2 \right\}^{\frac{1}{2}}$$

$$A_c = 2 \times 12 \times 60 \times \left\{ \left(\frac{12.25}{2} - 0.125 \right)^2 - \left[\frac{12.25}{2} - 0.125 \times \frac{12.25 - 0.125}{12.25 - 8.0} \right]^2 \right\}^{\frac{1}{2}}$$
$$= 2377.4\ in^2$$

The pulling force is equal to:

$$F_{pull} = 0.135 \times 250.1 \times 2377.4 = \mathbf{80{,}277\ lb_f}$$

Example 13.14: An 8 ½" production section is being drilled using 6" OD drill collars and 5" drill pipes. Pore pressure gradient at 9,500 *ft* was estimated to be 8.81 *ppg*. Mud tests show that the mud cake thickness is about 1/16". If the maximum allowable over pull is 85,000 lb_f and estimated drill collars stuck length is 120 *ft*, determine the maximum allowable mud weight? Use friction coefficient of 0.11.

<u>**Solution**</u>:

Given data:

D_{stuck}	=	Depth of stuck	=	9,500 *ft*
d_h	=	Hole diameter	=	8.5"
L_{stuck}	=	Length of the stuck	=	120 *ft*
OD_{DC}	=	Outside diameter of drill collars	=	6.0"
t_{mc}	=	Thickness of mud cake	=	1/16"
$G_{P@6,500}$	=	Pore pressure gradient at 5,300 *ft*	=	8.81 *ppg*
Rfr	=	Friction coefficient	=	0.11
F_{pull}	=	Allowable puling force	=	85,000 *lbf*

Required data:

MW = Mud weight in *ppg*

To calculate the maximum allowable mud weight for the above situation, the differential pressure should be determined. The contact surface area can be calculated using the following equation:

$$A_c = 2\times 12\times L_{stuck}\times\left\{\left(\frac{d_h}{2}-t_{mc}\right)^2-\left[\frac{d_h}{2}-t\_mc\times\frac{d_h-t_{mc}}{d_h-OD_{DC}}\right]^2\right\}^{\frac{1}{2}}$$

$$A_c = 2\times 12\times 120\times\left\{\left(\frac{8.5}{2}-0.0625\right)^2-\left[\frac{8.5}{2}-0.0625\times\frac{8.5-0.0625}{8.5-6.0}\right]^2\right\}^{\frac{1}{2}}$$

$$= 3{,}182.5\ in^2$$

The differential pressure can be estimated using the following equation:

$$\Delta p = \frac{F_{pull}}{R_{fr}\times A_c} = \frac{85{,}000}{0.11\times 3{,}182.5} = 242.8\ psi$$

Thus, the allowable mud weight is equal to:

$$MW_{allowable} = G_P + \frac{\Delta p}{0.052\times D_{stuck}} = 8.81+\frac{242.8}{0.052\times 9{,}500} = \mathbf{9.30\ ppg}$$

So under the above conditions, the maximum allowable mud weight is 9.30 *ppg* that can allow to free the drillstring stuck if it is being differentially stuck.

13.5 Drillstring Design

***Example 13.15*:** A drilling rig drilled at a depth of 15,500 *ft* using drillstring consisting of bit, 660 *ft*, 90 ppf DC, and 15 ppf DP to the surface. The decision was made to change the bit. Using the following information, calculate the time required to pull out the first stand if the stand length is 90 *ft*. Note that the rig can handle three drill pipes in one stand:

Draw-works power : 650 *hp*
Draw-works efficiency : 72%
Number of drilling lines : 8
Block and tackle efficiency : 86%
Traveling block weight : 25,750 *lbs*
MW : 10.2 *ppg*

Solution:

Given data:

P_{DW}	=	Draw-works power	=	650 *hp*
η_{DW}	=	Draw-works efficiency	=	72%
N	=	Number of drilling lines	=	8
η_{BT}	=	Block and tackle efficiency	=	86%
W_{TB}	=	Traveling block weight	=	25,750 *lbs*
MW	=	Mud weight	=	10.2 *ppg*
D	=	Current depth of the well	=	15,500 *ft*
L_{DC}	=	Length of drill collars	=	660 *ft*
W_{DC}	=	Weight of drill collars	=	90 *ppf*
W_{DP}	=	Weight of drill pipes	=	15 *ppf*
L_{st}	=	Length of one stand	=	90 *ft*

Required data:

t = Time to pull out the first stand of the drillstring.

To determine the time to pull the first stand, first we need to calculate the total hook load. Because the drillstring is immersed in the mud, its load will be affected by the buoyancy force. Thus, the buoyancy factor must be calculated first using the following equation:

$$BF = 1 - \frac{\rho_m}{\rho_s} = 1 - \frac{10.2}{65.4} = 0.844$$

The drillstring load is equal to:

$$F_{ds} = F_{DC} + F_{DP} = (660 \times 90 + (15{,}500 - 660) \times 15) \times 0.844$$
$$= 238{,}008 \ lbs$$

The total hook load is equal to:

$$F_h = 238{,}008 + 25{,}750 = 263{,}758 \ lbs$$

Now, the fast line tension is equal to:

$$F_f = \frac{F_h}{n\eta_{bt}} = \frac{263{,}758}{8 \times 0.86} = 38{,}337\ lbs$$

Hence, the fast line speed is equal to:

$$v_f = \frac{P_o}{F_f} = \frac{650 \times 0.72 \times 33{,}000}{38{,}337} = 402.8\ ft/m$$

Thus, the time to pull the first stand is equal to:

$$t = \frac{L_s n}{v_f} = \frac{90 \times 8}{402.8} = 1.79\ mins$$

Example 13.16: Use the data below to calculate the side force that developed by resting the drillstring in the low side of a directional well:

Mud weight (ρ_m)	=	10.9 *ppg*
Top point depth (D_1)	=	1,500 *ft*
Top point inclination (θ_1)	=	13.25°
Top point azimuth (ε_1)	=	325°
Bottom point depth (D_2)	=	1,557 *ft*
Bottom point inclination (θ_2)	=	14.20°
Bottom point azimuth (ε_2)	=	326°
Weight of string section	=	24 *ppf*
Effective tension (F_e)	=	145,000 lb_f

Solution:

Given data:

The above data

Required data:

Side force at the above section.

The difference in inclination and azimuth between the points is:

$$\Delta\theta = \theta_2 - \theta_1 = 14.20 - 13.25 = 0.95° = 0.0166\ rad$$

$$\Delta\varepsilon = \varepsilon_2 - \varepsilon_1 = 326 - 325 = 1.0° = 0.0175\ rad$$

The average inclination is equal to:

$$\theta_{avg} = \frac{\theta_1 + \theta_2}{2} = \frac{13.25 + 14.20}{2} = 13.725° = 0.240\ rad$$

The buoyancy factor is equal to:

$$BF = 1 - \frac{\rho_m}{\rho_{st}} = 1 - \frac{10.9}{65.4} = 0.833$$

The effective weight of the string section is equal to:

$$W_b = w_{ds} \times BF \times \Delta l = 25.0 \times 0.833 \times 57 = 1{,}187\ lb_f$$

The side force can be calculated using the following equation:

$$F_s = \sqrt{(F_e \Delta \varepsilon \sin \theta_{avg})^2 + (F_e \Delta \theta + \mathrm{W_b} \sin \theta_{avg})^2}$$

$$F_s = \sqrt{(145{,}000 \times 1.0 \times \sin 13.725)^2 + (145{,}000 \times 0.95 - 1{,}187 \times \sin 13.725)^2}$$

$$F_s = \mathbf{2{,}206}\ \boldsymbol{lb_f}$$

The negative sign because the section is building section.

Example 13.17: A drillstring is rotated and pulled simultaneously inside a directional well during a normal back-reaming operation. The pulling speed is 90 *ft/min*, while the rotational speed is 45 *RPM*. The side force that is acting on 8" drill collars is estimated to be 5,000 lb_f. Estimate the torque and drag that are acting against the drill collar joints assuming the friction factor is 0.25.

Solution:

Given data:

v	=	Pulling speed	=	90 *ft/min*
ω	=	Rotational speed	=	45 *RPM*
OD_{DC}	=	Outside diameter of drill collars	=	8.0"
F_s	=	Side force	=	5,000 *ft*
E	=	Friction factor	=	0.25

Required data:

Torque and drag.

First, rotational speed should be changed to linear speed as follows:

$$\omega = \pi \times OD \times RPM = \frac{\pi \times 8.0 \times 45}{12} = 94.25\ ft/min$$

The resultant drill collars speed can be calculated using the following equation:

$$V_{rs} = \sqrt{v^2 + \omega^2} = \sqrt{90^2 + 94.25^2} = 130.3\ ft/min$$

The torque can be estimated using the following equation:

$$T = \varepsilon \times F_s \times r \times \frac{\omega}{V_{rs}} = 0.25 \times 5{,}000 \times \frac{8}{2 \times 12} \times \frac{94.25}{130.3} = 301.4\ lb_f - ft$$

Drag force can be calculated using the following equation:

$$F_D = \varepsilon \times F_s \times \frac{v}{V_{rs}} = 0.25 \times 5{,}000 \times \frac{90}{130.3} = 862\ lb_f$$

Example 13.18: A casing string was stuck while running in a production section of a well. The driller is planning to determine the approximate stuck point. The casing is 7", 29 *ppf*. The following data were observed:

#	**Pull (T), Kips**	**Stretch (e), cm**
1	165	–
2	190	25.5
3	215	50.5

Estimate the casing stuck point.

Solution:

Given data:

Data in the table

OD_{cas} = Outside diameter of casing = 7.0"

wt = Weight of the casing = 29 ppf

Required data:

The casing stuck point.

The stuck point can be estimated using the following equation:

$$L = \frac{735{,}294 \times e \times W}{\Delta T}$$

Because we have three readings, three values of stuck points will be calculated. Then an average value may be used as an estimated stuck point.

$$L_1 = \frac{735{,}294 \times 25.5 \times 29}{(190 - 165) \times 1{,}000} = 8{,}563\ ft$$

$$L_2 = \frac{735{,}294 \times 50.5 \times 29}{(215 - 165) \times 1{,}000} = 8{,}479\ ft$$

$$L_3 = \frac{735{,}294 \times 25 \times 29}{(215 - 190) \times 1{,}000} = 8{,}395\ ft$$

$$L = \frac{L_1 + L_2 + L_3}{3} = \frac{8{,}563 + 8{,}479 + 8{,}395}{3} = \mathbf{8{,}479\ ft}$$

Note that the differences in the stuck point depth is due to the errors in the measurements.

***Example 13.19*:** A vertical well was drilled, cased and cemented in place. A 5" drill pipe string was run to the bottom of the well to replace the existing drilling mud of 15.0 *ppg* with the suspension fluid. When the drill pipes were at the bottom of the well, the weight indicator reads a certain value. When the old mud was displaced by the suspension fluid, the weight indicator reads a value, 1.125 which is greater than the previous record. What is the mud weight of the suspension fluid?

Solution:

Given data:

OD = Outside diameter of the drill pipes = 13 3/8"

MW_{old} = Mud weight of the old mud = 15.0 *ppg*

R_{wt} = Ratio of the weight indicator readings = 1.125

Required data:

MW_2 = Mud weight of the suspension fluid

Buoyancy force will make the weight indicator reading changes as the fluid inside the well changes. Thus, when the drillstring reaches the bottom of the well the net weight that reads in the indicator equals:

$$W_1 = W_{dp} \times BF = W_{dp} \times \left(1 - \frac{15.0}{65.4}\right) = 0.771\ W_{dp} \tag{1}$$

The weight after displacing the suspension fluid equals:

$$W_2 = W_{dp} \times BF = W_{dp} \times \left(1 - \frac{\rho_2}{65.4}\right)$$

We know that the indicator reads a value of 1.125 which is greater than the previous reading. Thus:

$$W_2 = 1.125\ W_1 = W_{dp} \times \left(1 - \frac{\rho_2}{65.4}\right) \text{ OR}$$

$$W_1 = 0.889 W_{dp} \times \left(1 - \frac{\rho_2}{65.4}\right) \tag{2}$$

From Eqs. (1) and (2), we find that:

$$0.771 W_{dp} = 0.889 W_{dp} \times \left(1 - \frac{\rho_2}{65.4}\right)$$

$$\rho_2 = \mathbf{8.68}\ \boldsymbol{ppg}$$

13.6 Casing Design

Example 13.20: A casing-while-drilling (CWD) string of 17.5" bit and 13 3/8" OD, 12.615" ID, and 54.5 ppf weight casing was run in a surface section of a well. A 75 *bbls* of drilling mud was displaced out the well when the drill bit tagged at the bottom of the well. If the mud weight of the drilling fluid is 9.2 *ppg*, determine the following: a) the depth of the well, and b) the reading of the weight indicator.

<u>Solution</u>:

Given data:

OD	=	Outside diameter of the casing	=	13 3/8"
ID	=	Inside diameter of the casing	=	12.615"
Wt_{cas}	=	Weight of the casing	=	54.5 *ppf*
MW	=	Mud weight	=	9.2 *ppg*

Required data:

D	=	Well current depth in feet
W	=	Current reading of the weight indicator

a. To estimate the current well depth, the cross-sectional area of the casing must be calculated using the following equation:

$$A_{cas} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(13.375^2 - 12.615^2) = 15.513\ in^2 = 0.1077\ ft^2$$

Now, the current depth of the well can be calculated by using the displacement definition as follows:

$$Depth = \frac{Displacement}{Cross\ sectional\ area\ of\ the\ casing}$$

$$= \frac{75\ bbls \times 5.615 \frac{ft^3}{bbl}}{0.1077} = \mathbf{3{,}910\ \mathit{ft}}$$

b) The current reading of the rig's weight indicator can be determined using the buoyancy definition. First, the buoyancy factor should be calculated using 65.4 ppf as steel density as follows:

$$BF = 1 - \frac{\rho_m}{\rho_{st}} = 1 - \frac{9.2}{65.4} = 0.859$$

Thus, the weight indicator reading is equal to:

$$W = W_{air} \times BF = 3{,}910 \times 54.5 \times 0.859 = 183{,}056\ lbf$$

***Example 13.21*:** A 13 3/8" OD, 12.415" ID, and 68 ppf casing was run in a 2,000 *ft* surface section of a well. Cement slurry having a density of 15.8 *ppg* was pumped inside the casing, and then displaced out the casing to fill the annulus between the hole and the casing. The mud weight of the drilling mud which was in the well and used to displace the cement was 9.5 *ppg*. If the casing was hung on the hook, estimate the following: a) the reading of the weight indicator when all of the cement was inside the casing, and b) the reading of the weight indicator when all of the cement was in the annulus.

Solution:

Given data:

OD = Outside diameter of the casing = 13 3/8"
ID = Inside diameter of the casing = 12.415"
Wt_{cas} = Weight of the casing = 68.0 ppf
D = Depth of the surface section = 2,000 *ft*
MW_{mud} = Mud weight = 9.5 *ppg*
MW_{cem} = Cement weight = 15.8 *ppg*

Required data:

W_1 = Reading of weight indicator when cement was inside the casing.
W_2 = Reading of weight indicator when cement was in the annulus.

Due to the buoyancy force of the well fluids, the weight indicator will give a reading which is different than the reading of the casing in air. Also if there is more than one fluid in the well, the buoyancy force will change accordingly. The buoyancy factor if there are two fluids in the well will be calculated using the following equation:

$$BF = \frac{A_o\left(1-\dfrac{\rho_o}{\rho_s}\right) - A_i\left(1-\dfrac{\rho_i}{\rho_s}\right)}{A_o - A_i}$$

where:

A_o = External area of the casing or any tubular.
A_i = Internal area of the casing or any tubular.
r_o = Mud weight of the annular fluid.
r_i = Mud weight of the internal fluid.

a. External and internal areas are equal to:

$$A_o = \frac{\pi}{4}\times d_o^2 = \frac{\pi}{4}\times 13.375^2 = 140.50\ in^2$$

$$A_i = \frac{\pi}{4}\times d_i^2 = \frac{\pi}{4}\times 12.415^2 = 121.06\ in^2$$

Now, when the cement slurry was inside the casing, the buoyancy factor is equal to:

$$BF = \frac{140.50 \times \left(1 - \frac{15.8}{65.4}\right) - 121.06 \times \left(1 - \frac{9.5}{65.4}\right)}{140.50 - 121.06} = 1.454$$

Thus, the weight indicator should read:

$$W_1 = W_{cas} \times BF = 68.0 \times 2{,}000 \times 1.454 = \mathbf{197{,}804}\ \boldsymbol{lb_f}$$

b) When cement slurry was in the annulus, the buoyancy factor is equal to:

$$BF = \frac{140.50 \times \left(1 - \frac{9.5}{65.4}\right) - 121.06 \times \left(1 - \frac{15.8}{65.4}\right)}{140.50 - 121.06} = 0.159$$

Thus, the weight indicator read:

$$W_2 = W_{cas} \times BF = 68.0 \times 2{,}000 \times 0.159 = \mathbf{21{,}585}\ \boldsymbol{lb_f}$$

13.7 Cementing

Example 13.22: A cement job was planned to be performed using class H cement with slurry density of 16.4 *ppg*. The total cement slurry volume is 250 *bbls* with cement yield of 1.25 ft^3/sack. The amount of water to be used is 5.1 gal/sack. Calculate the amount of Barite to be added, and the number of sacks, amount of Barite, and the amount of water to be used to prepare the above cement slurry.

Solution:

Given data:

MW	=	Density of cement slurry	=	16.4 *ppg*
Y	=	Cement yield	=	1.25 ft^3/sack
V_{cem}	=	Volume of cement slurry	=	250 *bbls*
V_{wat}	=	Volume of water	=	5.1 gal/sack

Required data:

M_{bar}	=	Amount of Barite per sack of cement
No_{sacks}	=	Amount of cement in sacks
M_{bar}	=	Amount of Barite in tons
V_{wat}	=	Volume of water in barrels

The volume of cement slurry based on one sack of cement is equal to:

$$V = Y \times 7.48 = 1.25 \times 7.48 = 9.35\ gal/sack$$

The mass of water based on one sack of cement is equal to:

$$m_{wat} = V_{wat} \times \rho_w = 5.1 \times 8.33 = 42.48\ lbm$$

The rest of the mass will be the mass of cement plus the mass of the Barite. The solids mass can be calculated using the following equation:

$$\rho_{slurry} = \frac{m_{slurry}}{V_{slurry}} = \frac{42.48 + m_{solids}}{9.35} = 16.4$$

$$m_{solids} = 110.9\ lbm$$

We know that one sack of cement is equal to 94 lb_m. Thus the amount of Barite is equal to **16.9 lb_m (110.9 – 94.0)**. Now, by knowing the total volume of cement slurry to be prepared we can determine the amount of each component of the slurry as follow:

The number of sacks of cement can be determined using the cement yield as follows:

$$No_{sacks} = \frac{V_{cem}}{Y_{slurry}} = \frac{250 \times 42}{9.35} = \mathbf{1{,}123\ sacks}$$

To get the amount of Barite, the total mass of cement slurry needs to be calculated. Using the mass percentage of Barite, we can calculate the amount of Barite. The total mass of cement slurry is equal to:

$$m_{slurry} = V_{slurry} \times \rho_{slurry} = 250 \times 42 \times 16.4 = 172{,}200\ lbm$$

And the Barite mass ratio is equal to:

$$Barite\ mass\ ratio = \frac{barite\ mass}{total\ mass} = \frac{16.9}{16.9 + 94 + 42.8} = 0.11$$

The amount of Barite is now equal to:

$$M_{bar} = 0.11 \times 172{,}200 = 18{,}974\ lbm = \mathbf{9.4\ US\ tons}$$

The volume of water can be calculated using the following equation:

$$V_{wat} = \frac{V_{wat\ in\ 1.0\ sack}}{V_{tot\ in\ 1.0\ sack}} \times V_{slurry} = \frac{5.1}{9.35} \times 250 = \mathbf{136.4\ bbls}$$

Example 13.23: A cementing operation was designed in such a way that the gas migration potential "GMP" was less than 3.75 to cement a 4.5" casing. The casing setting depth was planned to be at 21,500 *ft* in a hole size of 6.0". The mid depth of the production zone is at 21,000 *ft* with a pore pressure of 15,000 *psi*. The density of the mud in the hole is 14.2 *ppg* and the expected top of cement will be at 18,000 *ft*. Calculate the minimum density of the cement slurry that fulfill the GMP requirement.

Solution:

Given data:

GMP	=	Gas migration potential	=	3.75
d_{cas}	=	Casing outside diameter	=	4.5"
d_{hole}	=	Hole size	=	6.0"
D_{shoe}	=	Casing setting depth	=	21,500 *ft*

P_f	=	Formation pressure	=	15,000 *psi*
MW	=	Mud weight	=	14.2 *ppg*
D_{fm}	=	Mid depth of the pay zone	=	21,000 *ft*
TOC	=	Top of cement	=	18,000 *ft*

Required data:

MW_{cem} = Density of cement slurry

The density of cement slurry that can meet the GMP requirement can be determined after calculating the pressure reduction and the pressure difference between the formation pressure and hydrostatic pressure. The pressure reduction can be calculated using the following equation:

$$P_r = 1.67 \times \frac{L_{cem}}{d_h - d_{cas}} = 1.67 \times \frac{21{,}000 - 18{,}000}{6.0 - 4.5} = 3{,}340\,psi$$

Using the GMP equation, we can determine the pressure difference as follows:

$$GMP = \frac{P_r}{P_{ob}} = 2.75 = \frac{3{,}340}{P_{ob}}$$

$$P_{ob} = 891\,psi$$

From the above pressure and the formation pressure, we can calculate the hydrostatic pressure when the cement is displaced as follows:

$$P_h = P_{ob} + P_{fm} = 891 + 15{,}000 = 15{,}891\ psi$$

We know that the annulus of the well will contain 18,000 *ft* of drilling mud and 3,000 *ft* of cement slurry. Thus, the density of cement slurry is equal to:

$$P_h = 0.052 \times (L_m \times \rho_m + L_{cem} \times \rho_{cem})$$

$$15{,}891 = 0.052 \times (18{,}000 \times 14.2 + 3{,}000 \times \rho_{cem})$$

$$\rho_{cem} = 16.67\,ppg$$

13.8 Horizontal and Directional Drilling

Example 13.24: Below is the deviation data taken from a directional well:

MD ft	Inclination °	azimuth °
9775	81.25	325
10150	86.75	321

Calculate the bending angle and the dog-leg severity for this section.

Solution:

Given data:

Data in the table

Required data:

β in °/100 *ft*

DLS in °/100 *ft*

The bending angle can be calculated using the following equation:

$$\beta = \arccos(\cos\theta_1 cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos\Delta\varepsilon)$$

$$\beta = \arccos(\cos 81.25 \cos 86.75 + \sin 81.25 \sin 86.75 \cos(321 - 325))$$

$$\beta = 6.79°$$

The dog-leg severity can be calculated using the equation:

$$DLS = \frac{\beta}{\Delta MD} \times 100$$

$$DLS = \frac{6.79}{10{,}150 - 9{,}775} \times 100 = 1.81°$$

Example 13.25: During drilling a directional well, a survey data shows an inclination of 21° and an azimuth of 167°. The inclination and azimuth were designed to be maintained at 3.0°/100 *ft* and 2.5°/100 *ft*, respectively, for the measured depth between 5,350 and 5,500 *ft*. Calculate the new inclination, new azimuth, the bore-hole bending angle, vertical and horizontal curvature, and the tool face angle.

Solution:

Given data:

θ_1	=	Inclination angle	=	21°
ε_1	=	Azimuth ngle	=	167°
$\Delta\theta$	=	Inclination increment	=	3.0°/100 *ft*
$\Delta\varepsilon$	=	Azimuth increment	=	2.5°/100 *ft*
ΔMD	=	Drilled section	=	150 *ft*

Required data:

θ_2	=	Final inclination angle
ε_2	=	Final azimuth angle
κ_θ	=	Vertical curvature angle
κ_ε	=	Horizontal curvature angle
β	=	Bending angle
γ	=	Tool face angle

Using the inclination and azimuth increments, we can determine the new angles at the MD of 5,500 *ft* as follows:

$$\theta_2 = \theta_1 + \Delta\theta \times \Delta MD = 21 + \frac{3.0}{100} \times 150 = 25.5°$$

$$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon \times \Delta MD = 167 + \frac{2.5}{100} \times 150 = 170.75°$$

Vertical curvature is similar to the inclination increment, or 3.0°. To get the horizontal curvature angle, the average inclination angle should be calculated as follows:

$$\bar{\theta} = \frac{\theta_1 + \theta_2}{2} = \frac{21 + 25.5}{2} = 23.25°$$

Horizontal curvature is equal to:

$$\kappa_\varepsilon = \frac{\Delta\varepsilon}{\sin\bar{\theta}} = \frac{2.5}{\sin 23.25} = 6.33°$$

Bending angle can be calculated using the following equation:

$$\beta = \arccos(\cos\theta_1 cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos\Delta\varepsilon)$$

$$\beta = \arccos(\cos 21 cos 25.5 + \sin 21 \sin 25.5 \cos(170.75 - 167))$$

$$\beta = 4.74$$

The tool face angle can be estimated using the following equation:

$$\gamma = arc\cos\left(\frac{\cos\theta_1 \cos\beta}{\sin\theta_2 \sin\beta}\right) = arc\cos\left(\frac{\cos 21 \times \cos 4.74}{\sin 25.5 \times \sin 4.74}\right) = 38.55°$$

Example 13.26: A directional survey was taken on a well at depth of 6,500 *ft*, and it was found that the inclination was 37° and the azimuth was 297°. The decision was made to made correction in the well path in such a way that the tool face angle should be 25°, and the dog leg severity should be 4.5°/100 *ft*. If a new survey is taken at depth of 6,625 *ft*, what will be the inclination and the azimuth?

Solution:

Given data:

θ_1	=	Inclination angle	=	37°
ε_1	=	Azimuth angle	=	297°
γ	=	Tool face angle	=	25
DLS	=	Dogleg severity	=	4.5°/100 *ft*
ΔMD	=	Drilled section	=	125 *ft*

Required data:

θ_2	=	Final inclination angle
θ_2	=	Final azimuth angle

The new inclination and azimuth can be calculated using the following steps:
First, the hole bending angle can be calculated using the following equation:

$$\beta = \frac{DLS}{100} \times \Delta MD = \frac{4.5}{100} \times 125 = 5.625°$$

The above angle can be used to calculate the new inclination with the help of the following equation:

$$\theta_2 = arc\cos(\cos\theta_1 \cos\beta - \sin\theta_1 \sin\beta \cos\gamma)$$
$$\theta_2 = arc\cos(\cos 37 \cos 5.625 - \sin 37 \sin 5.625 \cos 25) = \boldsymbol{42.16°}$$

The new azimuth can be calculated using the following equation:

$$\varepsilon_2 = \varepsilon_1 + arc\tan\left(\frac{\tan\beta \sin\gamma}{\sin\theta_1 + \tan\beta \cos\theta_1 \cos\gamma}\right)$$
$$\varepsilon_2 = \varepsilon_1 + arc\tan\left(\frac{\tan 5.625 \sin 25}{\sin 37 + \tan 5.625 \cos 37 \cos 25}\right) = 300.54°$$

Example 13.27: A horizontal well is planned to be placed in a hydrocarbon zone at vertical depth of 5,000 *ft*. The well TD has a horizontal departure of 3,500 *ft* and azimuth of 72° from the KOP. The well is designed to kick-off the vertical section at depth of 2,500 *ft* from the well top depth. Calculate the build-up rate that should be adopted to fulfill the above requirement. Also calculate the well total measured length.

Solution:

Given data:

TVD	=	Vertical depth of the target	=	5,000 *ft*
KOP	=	Kick-off depth of the well	=	2,500 *ft*
ε	=	Well azimuth	=	72°
D_H	=	Horizontal departure	=	3,500 *ft*
LH	=	Length of horizontal section	=	1,000 *ft*

Required data:

BUR = Build up rate in °/100 *ft*.
TMD = True measured depth of the well.

Because it is a horizontal well, the final inclination angle is 90°. The last drilled point of the well is 3,500 ft far horizontally. For the horizontal section length of 1,000 ft, the end of curvature should be 2,500 ft far from the KOP. To connect between the KOP and EOC with a curve that has a constant BUR, we are going to choose the radius of curvature to be 2,500 ft. The BUR can be calculated using the following equation:

$$BUR = \frac{180 \times 100}{\pi \times R} = \frac{180 \times 100}{\pi \times 2{,}500} = 2.29°$$

The curved section length can be calculated by knowing that this section is a quarter of a circle circumstance with a radius of 2,500 *ft*. The curved section is equal to:

$$Curved\ section = \frac{2\pi R}{4} = \frac{2\pi \times 2{,}500}{4} = 3{,}927\ ft$$

Thus, the well's true measured depth is equal to:

$$TMD = 2{,}500 + 3{,}927 + 1{,}000 = \mathbf{7{,}427\ \mathit{ft}}$$

13.9 Cost Analysis

Example 13.28: A single-acting triplex pump is used to deliver pressure of 1,750 *psi*. Stroke length of the pump is 20" and liner size is 6.5". The pump speed is designed to be 75 *spm* with a pump volumetric efficiency of 84%. The mechanical efficiency is 82% while the diesel engine efficiency is 60%. The engine is using diesel that has density of 7.2 *ppg* and heating value of 19,000 BTU/lb. If the cost of one gallon of diesel is equal to USD 0.75, calculate the cost of pumping 1,000 *bbls* of mud and the daily running cost of the pump.

Solution:

Given data:

P = Pumping pressure = 1,750 *psi*
d_l = Liner diameter = 6.5"
L_s = Liner length = 20"
N = Pump speed = 75 *spm*

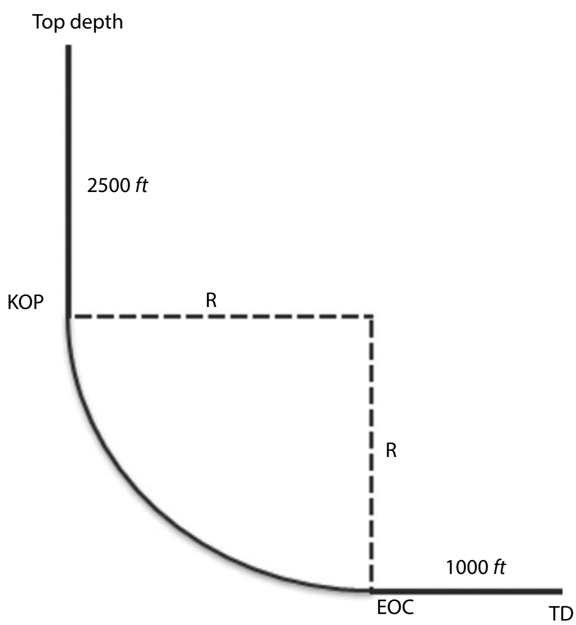

Figure 13.1 Well trajectory schematic.

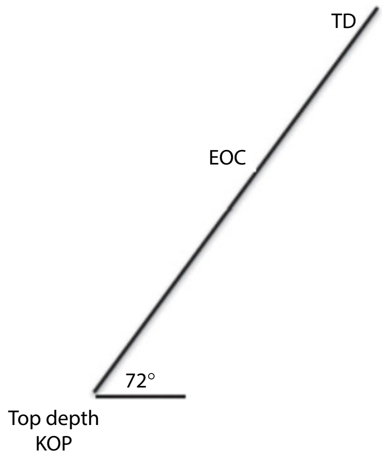

Figure 13.2 Well departure.

η_P	=	Pump displacement eff.	= 0.84
η_M	=	Pump Mechanical eff.	= 0.82
η_{eng}	=	Diesel engine eff.	= 0.60
ρ_{dies}	=	Density of diesel	= 7.2 *ppg*
HV_{dies}	=	Heating value of diesel	= 19,000 BTU/lb

Required data:

Cost of pumping 1,000 *bbls* and daily cost of the pump.

To determine the cost of pumping, we must estimate the engine input. The theoretical flow rate for a single-acting triplex pump is equal to:

$$Q_{th} = \frac{n \times N \times L \times D_i^2}{294} = \frac{3 \times 75 \times 20 \times 6.5^2}{294} = 646.4\,pgpm$$

So, the actual flow rate which can be pumped using this pump, is equal to:

$$Q_{actual} = Q_{th} \times \eta_p = 646.4 \times 0.84 = 543.0\ pgm$$

Now, the hydraulic horsepower of the pump can be estimated using the equation:

$$hp_H = \frac{P \times Q}{1714} = \frac{1{,}750 \times 543.0}{1714} = 544.4\ hp$$

The above is the output horsepower. The input horsepower can be calculated using the mechanical efficiency as follows:

$$hp_{in} = hp_H \times \eta_M = \frac{544.4}{0.82} = 676.1\ hp$$

Thus, the engine horsepower is equal to:

$$hp_{engine} = \frac{hp_{in}}{\eta_{eng}} = \frac{676.1}{0.6} = 1{,}126.9\ hp$$

The engine's fuel consumption can be estimated using the equation as:

$$Q_f = \frac{2{,}545 \times hp_{engine}}{HV} = \frac{2{,}545 \times 1{,}126.9}{19{,}000} = 150.9\ lb/hr$$

OR

$$Q_f = \left(150.9 \frac{lb}{hr}\right) \times \left(24 \frac{hr}{d}\right) \times \left(\frac{1}{7.2 \frac{lb}{gal}}\right) = 503.1\ gal/d$$

The daily cost of the mud pump is equal to:

$$Daily\ cost = 503.1 \times 0.75 = 377.3\ USD/d$$

To estimate the cost of pumping for 1,000 *bbls*, we should calculate the time required to pump this volume as follow:

The actual pumping rate in barrels per minutes is equal to:

$$Q_{actual} = 543.0\ gpm \times \frac{1}{42\ gal/bbl} = 12.9\ bbls/min$$

The time required to pump 1,000 barrels is:

$$Time = \frac{1{,}000}{12.9} = 77.5\ mins = 0.054\ day$$

Thus, the cost of pumping for 1,000 barrels is equal to:

$$C_{1000bbls} = 377.3 \times 0.054 = \mathbf{20.4\ USD}$$

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Appendix A

(Solutions of Exercises)

Chapter 2: Drilling Methods

Exercise 2.1: An internal combustion engine in a drilling rig is running at a speed of 1250 *rpm*. The engine is developing a torque of 1600 lb_f-ft and consuming about 27 *gal/hr* of diesel fuel that has a density of 7.2 *ppg*. Calculate the output power and the overall efficiency of the engine.

<u>Solution</u>:

Given data:

N = Engine running speed in *rpm* = 1,250 *rpm*

T = Engine torque in = 1,600 ft-lb_f

ρ_{diesel} = Diesel density = 7.2 *ppg*

w_f = Engine fuel consumption = 27 *gal/hr*

Required data:

Ps = Engine power

η_{ps} = Efficiency of the engine

The engine angular speed can be calculated using the following equation:

$$\omega = 2\pi N = 2\pi \times 1{,}250 = 7{,}854\ rad/min$$

The engine output power can be calculated from Eq. (2.1) as below:

$$P_s = T\omega = \frac{1{,}600(ft - lb_f) \times 7{,}854(rad/min)}{33{,}000\left(\dfrac{ft - lb_f}{min}/hp\right)} = 381\ hp$$

To calculate the efficiency of the engine, first we need to calculate the total heat energy consumed by the engine.

$$Q_i = w_f h_f$$

Heating value of diesel (H_f) is 19,000 $\frac{Btu}{lb_m}$

$$w_f = \frac{27\left(\frac{gal}{hr}\right) \times 7.2(ppg)}{60(min/hr)} = 3.24\frac{lb_m}{\text{min}}$$

So the total heat energy is equal to:

$$Q_i = \frac{3.24\left(\frac{lb_m}{min}\right) \times 20{,}000\left(\frac{Btu}{lb_m}\right) \times 779\left(ft - \frac{lb_f}{Btu}\right)}{33{,}000\frac{ft - lb_f}{min}/hp} = 1{,}453\ hp$$

Engine efficiency can now be calculated from Eq. (2.5):

$$\eta_{ps} = \frac{Power\ output}{Power\ input} = \frac{P_s}{Q_i} = \frac{381}{1{,}453} = 26.2\%$$

Exercise 2.2: A drilling rig is pulling a drillstring at velocity of 30 *ft/min*. If the weight of the drillstring is 450,000 lb_f, calculate the output power and the developed torque if the rig engines are running at a speed of 1350 *rpm*.

Solution:

Given data:

$\bar{v}$ = Pulling velocity = 30 *ft/min*

W = Drillstring weight = 450,000 lb_f

N = Engine running speed = 1,350 *rpm*

Required data:

P_s = Engine output power in *hp*

T = Engine torque in $ft\text{-}lb_f$

Engine output power can be calculated from Eq.(2.4):

$$P_s = W\bar{v} = \frac{450{,}000(lb_f) \times 30\left(\frac{ft}{min}\right)}{33{,}000(ft - lb_f/min/hp)} = 409.1\ hp$$

To calculate the engine torque, first we need to calculate the engine angular speed using the equation:

$$\omega = 2\pi N = 2\pi \times 1{,}350 = 8{,}482\ rad/min$$

Engine torque can now be calculated from Eq. (2.1):

$$T = \frac{P_s}{\omega} = \frac{409.1\,(hp)\times 33{,}000\left(\dfrac{ft-lb_f}{min}\Big/ hp\right)}{8{,}482\ (rad/min)} = 1{,}592\ lb_f - ft$$

Exercise 2.3: An internal combustion engine in a drilling rig has a flywheel diameter of 2 *feet* is running at a speed of 1200 *rpm*. The engine is developing a torque of 1,550 lb_f-ft. Calculate the flywheel velocity in *ft/min*, engine horsepower, and the maximum drillstring weight that the rig can be pulled if the required pulling velocity is 25*ft/min*.

Solution:

Given data:

d_{FW} = Flywheel diameter = 2.0 *ft*
N = Engine running speed = 1,200 *rpm*
T = Engine torque = 1,550 lb_f-ft
$\bar{v}$ = Maximum pulling velocity = 25 *ft/min*

Required data:

v = Flywheel velocity in *ft/min*
P_s = Engine output power in *hp*
W = Drillstring weight in lb_f

Flywheel velocity can be calculated from Eq. (2.3):

$$v = 2\pi\frac{d_{FW}}{2}N = \pi\times 2\times 1{,}200 = 7{,}540\ ft/min$$

To calculate the engine torque, first we need to calculate the engine angular speed using the equation:

$$\omega = 2\pi N = 2\pi\times 1{,}200 = 7{,}540\ rad/min$$

So, the engine power output can be calculated from Eq. (2.1) as:

$$P_s = T\times\omega = \frac{1{,}550\times 7{,}540}{33{,}000} = 2{,}422\times 0.29 = 354.2\ hp$$

Maximum load that can be pulled at velocity of 25*ft/min* is calculated from Eq. (2.4):

$$W = \frac{P_s}{\bar{v}} = \frac{354.2\times 33{,}000}{25} = 467{,}544\ lb_f$$

Exercise 2.4: A drilling rig engine that produces an output power of 450 hp at a torque of 2,500 lb_f-ft is used to pull a drilling string of a weight of 350,000 lb_f. Calculate the engine running speed in *rpm* and the maximum pulling velocity of the drilling string.

Solution:

Given data:

T = Engine torque = 2,500 lb_f-ft
Ps = Engine output power = 450 hp
W = Maximum drillstring weight = 350,000 lb_f

Required data:

N = Engine running speed in *rpm*
$\bar{v}$ = Drillstring pulling velocity in *ft/min*

To calculate the engine running speed, first we should calculate the engine angular speed by knowing the engine power and torque using Eq. (2.1) as below:

$$\omega = \frac{P_s}{T} = \frac{450 \times 33{,}000}{2{,}500} = 5{,}940\ rad/min$$

Now, engine speed can be calculated as below:

$$N = \frac{\omega}{2\pi} = \frac{5{,}940}{2\pi} = 945.4\ rpm$$

Maximum velocity of the drillstring is calculated using Eq. (2.4):

$$\bar{v} = \frac{P_s}{W} = \frac{450 \times 33{,}000}{350{,}000} = 42.4\ ft/min$$

Exercise 2.5: A 15,000 *ft* steel sand line cable weighing 1.1 lb_f/ft was suspended inside a well filled with 8.7 *ppg* brine. If an engine managed to reel the cable in 3.5 hours, what is the engine output power?

Solution:

Given data:

l = Cable length = 15,000 *ft*
w = Weight of cable = 1.1 lb_f/ft
MW = Mud weight = 8.7 *ppg*
t = Reeling time = 3.5 *hrs*

Required data:

P = Engine output power in *hp*

Engine output power can be calculated using Eq. (2.15):

$$P_{out} = Wv$$

The average reeling speed can be calculated as below:

$$v = \frac{l}{t} = \frac{15{,}000}{2.5hrs \times 60\dfrac{mins}{hr}} = 71.4\ ft/min$$

To calculate the weight of the cable, first we need to calculate the buoyancy factor (BF):

$$BF = 1 - \frac{\rho_{mud}}{\rho_{st}} = 1 - \frac{8.7}{64.5} = 0.865$$

where steel density is 65.05 *ppg*.

So, weight of the cable is equal to:

$$W = BF \times W_{air} = 0.865 \times 15{,}000 \times 1.1 = 14{,}274\ lb_f$$

As the cable is reeled, the cable weight will reduce. So average weight will be used to calculate the output power of the engine:

$$W = \frac{14{,}274}{2} = 7{,}137\ lb_f$$

So, the average output power used to reel the cable is:

$$P_{out} = \frac{7{,}137 \times 71.4}{33{,}000} = 15.44\ hp$$

Exercise 2.6: A drilling rig is designed to have 8 *lines* to be strung between crown and travelling blocks. Approximate length of each line is 175 *ft* plus extra length of 300ft. The draw works drum diameter is 36" and drilling line diameter is 1.25". If the plan is to have maximum of 4 laps in the drum, estimate the approximate length of the drum required to reel that drilling line.

<u>Solution</u>:

Given data:

n	=	Number of *lines*	=	8 *lines*
L	=	Length of each line	=	175 *ft*
l_{extra}	=	Extra length	=	300 *ft*
d_d	=	Diameter of the drum	=	36 inches
d_{dl}	=	Diameter of drilling line	=	1.25 inches
$laps_{max}$	=	Max. number of laps	=	4 laps

Required data:

L_{drum} = Length of the drum

The total length of the drilling line required is equal to:

$$Total\ length = L \times (n+2) + l_{extra} = 175 \times (8+2) + 300$$
$$= 2{,}050\ ft$$

Not all of this length will be reeled in the draw works drum as the whole lengths of the fast line and dead line will not be reeled. So, the reeled length will be equal to:

$$Reeled\ length = 2{,}050 - 175 \times 2 = 1{,}700\ ft$$

As the design is to have maximum of 4 laps in the draw works drum, the length of the drilling line in each lap will be equal to:

$$Length\ of\ the\ lap = number\ of\ reels \times lap\ perimeter$$

Number of reels in the first and third laps will be greater than that in the second and fourth laps by one reel. So, the total number of reels is equal to:

$$Total\ number\ of\ reels = n + (n-1) + n + (n-1) = 4n - 2$$

The lap perimeter is increasing as the drilling line is reeled in the drum. So we should first calculate the diameter of each lap

$$d_{lap1} = d_{drum} + d_{dl} = 36 + 1.25 = 37.25\ inches$$

For the rest of the laps, the increase in the diameter will be equal to:

$$d_{lap(x)} = d_{lap(x-1)} + 2 \times d_{dl} \times sin60$$

where x is the number of lap

Lap perimeter is equal to:

$$Lap\ perimeter = \pi d_{lap}$$

So, from all the above equations we can calculate each lap diameter and perimeter. The table below shows this information:

	Lap diameter	**Perimeter**	**Total length of the drilling line in each lap**
Lap #	**inches**	**ft**	**ft**
0	36.00	9.4	
1	37.25	9.8	$9.8n$
2	39.42	10.3	$10.3(n-1)$
3	41.58	10.9	$10.9n$
4	43.72	11.5	$11.5(n-1)$

The total length of the drilling line reeled in the drum is now equal to:

$$1{,}700 = 9.8n + 10.0(n-1) + 10.3n + 10.6(n-1)$$

From the above equation, the number of reels in the first lap (n) will be equal to:

$$n = \frac{1{,}700 + 10.3 + 11.5}{9.8 + 10.3 + 10.9 + 11.5} = 40.5\ reels\ OR\ 41\ reels$$

Now we can calculate the length of the drum as:

$$L_{drum} = n \times d_{dl} = 41 \times 1.25 = 51.25\ inches\ OR\ 4.27\ ft$$

Exercise 2.7: A hoisting system in a drilling rig has 2 *ft* diameter draw works drum that can rotate at a maximum speed of 159 *rpm* and output power of 450 *hp*. Ten *lines* are strung between crown and traveling blocks. The rig is drilling a vertical well using 9.7 *ppg* drilling fluid. Drilling string consists of 6 stands drill collars and 150 stands of drill pipes. Weight of one stand of drill collar in air is 8,650 lb_f and for the drill pipe is 2,380 lb_f, and all stands have same length that is 93 *ft*. Drilling engineers planned to pull out the drillstring to change the bit. If the drilling foreman planned to pull every 30 stands at the same pulling speed, calculate the following:

a. Maximum speed available for the traveling block system to pull the drill-string in *ft/min*.
b. How many stands need to be pulled before the hoisting system can be able to use his maximum pulling speed if the weight of the traveling block is 35,000 lb_f?
c. The time required to pull all the drillstring out of the hole if disconnecting each stand takes approximately 0.75 *minutes*.

<u>Solution</u>:

Given data:

N	=	Drum speed	=	159 *rpm*
P_{out}	=	Drum output power	=	450 *hp*
n	=	Number of *lines*	=	10 *lines*
d_d	=	Diameter of the drum	=	2 *feet*
MW	=	Mud weight of the drilling fluid	=	9.7 *ppg*
l_{stand}	=	Length of one stand	=	93 *ft*
Wt_{DC}	=	Dry weight of stand of DC	=	8,650 lb_f
Wt_{DP}	=	Dry weight of stand of DP	=	2,380 lb_f
t_{disc}	=	Disconnecting time of one stand	=	0.75 *mins*
W_{TB}	=	Weight of travelling block	=	35,000 *lbs*

Required data:

v_{BT}	=	Maximum speed of the block and tackle in *ft/min*
n_{stands}	=	Number of stands to pull before using the maximum speed.
$t_{tripout}$	=	Trip out time for changing the bit

a. Because the draw work drum has a maximum rotation of 159 *rpm*, this speed will be the maximum speed that the fast line can has. So, changing the rotation speed to linear speed will give the maximum speed for the fast line:

$$v_{fl} = \pi N d = \pi \times 159 \times 2 = 999\ ft/min$$

Now maximum speed for the block and tackle will be equal to:

$$v_{BT} = \frac{v_{fl}}{n} = \frac{999}{10} = 99.9\ OR\ 100\ ft/min$$

b. To calculate number of stands to be pulled before running the system at the maximum speed, we can first calculate what will be the weight taken by the hook in order to use the maximum speed using Eq. (2.15):

$$W = \frac{P_{out}}{v} = \frac{450 \times 33{,}000}{100.03} = 148{,}455.5\ lbf$$

Above weight is the weight of the remaining drillstring that can be pulled with the maximum speed plus the weight of the traveling block. So, the weight of the traveling block must be deducted to get the weight of the drillstring in hole:

$$W = 148{,}455.5 - 35{,}000 = 113{,}455.5\ lbf$$

Above weight is the weight of the remaining drillstring in mud, so we must calculate the actual weight of the sting in air using the buoyancy factor.

$$BF = 1 - \frac{MW}{64.5} = 1 - \frac{9.7}{64.5} = 0.85$$

where 64.5 is the approximate density of steel in *ppg*.

Now the weight of the remaining drillstring in air is equal to:

$$W_{air} = \frac{W_{mud}}{BF} = \frac{113{,}455.5}{0.85} = 133{,}477\ lbf$$

This weight includes weight of 6 stands of drill collars. So the weight of the remaining drill pipe stands is equal to:

$$W_{DP} = 133{,}477 - 8{,}650 \times 6 = 81{,}577\ lbf$$

Now number of drill pipe stands remaining in the hole is equal to:

$$stands_{Dp} = \frac{81{,}577}{2{,}380} = 34.3\ OR\ 34\ stands$$

So, the hoisting system will be able to pull the drilling string at the maximum pulling speed of 100 *ft/min* after pulling out 116 stands of drill pipes.

c. As the pulling speed remains constant during pulling each 30 stands, we should calculate the weight of drillstring after pulling each 30 stands in order to calculate the pulling speed that can be used. Note that after pulling about 116 stands of drill pipes, the pulling speed will remains the same at 100 *ft/min*. Pulling speed will be calculated using Eq. (2.15). Time required to pull one stand will be equal to:

$$t_{stand} = \frac{l_{stand}}{v}$$

The time required to pull the entire string will be equal to:

$$Pulling\ time = (t_{stand} + t_{disconnection,stand})number\ of\ stands$$

The table below summarizes the calculation of the speeds and total time required to pull the string out of the hole:

Current string Wt. in mud	Number of Stands to pull	Max. pulling speed	Time to pull one stand	Pulling and disconnecting time	Time for each 30 stands	Cumulative time
lb_f	#	ft/min	mins/stand	mins	mins	mins
347508	30	43	2.18	2.93	87.8	87.8
286826	30	52	1.80	2.55	76.4	164.2
226143	30	66	1.42	2.17	65.0	229.2
165460	30	90	1.04	1.79	53.6	282.8
104778	30	100	0.93	1.68	50.4	333.2
44095	6	100	0.93	1.68	10.1	343.2

So to pull the string out of the hole, it requires about 343.2 *minutes* or 5.72 hours.

Exercise 2.8: A hoisting system in a rig that is able to pull a maximum weight of 300,000 lb_f at a hook speed of 45 *ft/min* and drum speed of 51 *rpm*. Eight *lines* are strung between crown and traveling blocks. Calculate the minimum torque developed against the draw work drum due to the tension in the fast line. Assume block and tackle efficiency of 0.84.

Solution:

Given data:

N = Drum speed = 51 *rpm*
W = Maximum hook load = 300,000 lb_f
n = Number of *lines* = 8 *lines*
v_{BT} = Block and tackle speed = 45 *ft/min*
N = Drum speed = 51 *rpm*
η_{BT} = Block and tackle efficiency = 0.84

Required data:
T_{min} = Minimum torque on the drum

To calculate the drum diameter, we will first calculate the fast line tension from Eq. (2.19):

$$T_f = \frac{W}{n \times \eta_{BT}} = \frac{300{,}000}{8 \times 0.84} = 44{,}643\ lbf$$

Minimum torque developed in the draw work drum occurred when the block and tackle is in the nearest point to the rig floor. At that point, there will be only one lap of drilling line in the draw work drum. So, the torque will be equal to the tension multiplied by the radius of the drum. Radius of the drum can be calculated using the hook speed and drum rotation speed as follows:

$$d_d = \frac{v_{fl}}{\pi N} = \frac{45 \times 8}{\pi \times 51} = 2.25\ ft\ \ OR\ 27\ inches$$

Now the torque is equal to:

$$T_{min} = T_f \times \frac{d_d}{2} = 44{,}643 \times \frac{2.25}{2} = 51{,}154\ lbf - ft$$

Exercise 2.9: A hoisting system has an output power of 550 *hp* being used in a drilling rig that has 10 *lines* strung between crown and traveling blocks. When the rig was pulling the drillstring at speed of 41.5 *ft/min*, the fast line tension read 54,130 lb_f. If the rig is using all the available output power of the draw work to pull the drillstring, calculate the block and tackle efficiency and the pulling speed when the fast line tension reads 44,045 lb_f.

Solution:

Given data:

n	=	Number of *lines*	=	10 *lines*
P_{out}	=	Draw work output power	=	550 *hp*
v_{BT1}	=	Block and tackle speed for case#1	=	41.5 *ft/min*
T_{f1}	=	Fast line tension for case#1	=	54,130 lb_f
T_{f2}	=	Fast line tension for case#2	=	44,045 lb_f

Required data:
η_{BT} = Block and tackle efficiency.
W_{HL} = Hook load for the second speed in lb_f.

Block and tackle efficiency can be calculated using Eq. (2.19):

$$\eta_{BT} = \frac{W}{n \times T_f}$$

Now, the load carried by the hook at speed of 41.5 *ft/min* can be calculated using Eq. (2.15):

$$W = \frac{P_{out}}{v} = \frac{550 \times 33{,}000}{41.5} = 437{,}350\ lbf$$

Thus, block and tackle efficiency is equal to:

$$\eta_{BT} = \frac{437{,}350}{10 \times 54{,}130} = 0.808\ OR\ 0.81$$

To calculate the pulling speed when the fast line tension is 44,045 lb_f, we need first to calculate the hook load using Eq. (2.19):

$$W = n \times T_f \eta_{BT} = 10 \times 44{,}045 \times 0.81 = 355{,}867\ lbf$$

Now, the pulling speed can be calculated using Eq. (2.15):

$$v_{BT} = \frac{P_{out}}{W} = \frac{550 \times 33{,}000}{355{,}867} = 51\ ft/min$$

Exercise 2.10: A draw work has a drum diameter of 2.5 *ft* is been used to pull a drill-string using 8 *lines* strung between crown and traveling blocks. When the hook was at the nearest point to the rig floor, the torque developed at the draw work drum was 66,813 lb_f-*ft* and only 8 reels of drilling line was in the drum. And when the hook at the farthest point from the rig floor, the torque developed at the draw work drum was 93,300 lb_f-*ft* and 5 laps of drilling line was in the drum. Calculate the diameter of the drilling line used in this rig and the maximum pulling speed if the output power of the draw work is 500 *hp*. Assume block and tackle efficiency is 0.84.

Solution:

Given data:

n	=	Number of *lines*	=	8 *lines*
d_{drum}	=	Diameter of the drum	=	2.5 *ft*
P_{out}	=	Draw work output power	=	500 *hp*
T_1	=	Torque developed for case#1	=	66,813 lb_f-*ft*
T_{BT}	=	Torque developed for case#2	=	93,300 lb_f-*ft*
η_{BT}	=	Block and tackle efficiency	=	0.84

Required data:

d_{dl}	=	diameter of the drilling line in inches
v_{BT}	=	Block and tackle speed or hook speed in *ft/min*

To calculate the drilling line diameter, first we should calculate the fast line tension from the torque developed when the hook at the nearest point to the rig floor.

$$Torque = T_f \times r_{drum}$$

$$T_f = \frac{66{,}813}{\frac{2.5}{2}} = 53{,}450\ lbf$$

The change in the torque was due to the laps of drilling line that increase the distance between the center of the drum and the fast line tension. So, this distance is equal to:

$$r = \frac{T_2}{T_f} = \frac{93,300}{53,450} = 1.75\ ft$$

Above distance is equal to the radius of the drum plus the distance created by reeling the drilling line when the hook at the farthest point from the rig floor. At that point, there were 5 laps of drilling line reeled in the drum. The diameter of the laps can be approximated using equations:

$$d_{lap1} = d_{drum} + d_{dl}$$

For the rest of the laps, the increase in the diameter will be equal to:

$$d_{lap(x)} = d_{lap(x-1)} + 2 \times d_{dl} \times sin60$$

where x is the number of lap

So we should calculate the increase in the distance from the last lap to the first lap in order to get the drilling line diameter:

$$d_{lap(5)} = d_{lap(4)} + 2 \times d_{dl} \times sin60 = 3.5\ ft$$

$$d_{lap(4)} = 3.5 - 2 \times d_{dl} \times sin60$$

$$d_{lap(3)} = 3.5 - 4 \times d_{dl} \times sin60$$

$$d_{lap(2)} = 3.5 - 6 d_{dl} sin60$$

$$d_{lap(1)} = 3.5 - 8 d_{dl} sin60 = 2.5 + d_{dl}$$

$$7.93 d_{dl} = 3.5 - 2.5 = 1.0$$

$$d_{dl} = \frac{1.0}{7.93} = 0.126\ ft\ \ OR = 1.5\ inches$$

To calculate the maximum speed, we first need to calculate the hook load using Eq. (2.19):

$$W = n \times T_f \eta_{BT} = 8 \times 53,450 \times 0.84 = 359,185\ lbf$$

Now, we can use Eq. (2.15) to get the pulling speed:

$$v_{BT} = \frac{P_{out}}{W} = \frac{500 \times 33,000}{359,185} = 45.9\ ft/min$$

Exercise 2.11: A double-acting duplex pump has a liner size and length of 7.5 and 25 inches, respectively. If the pump requires 1,400 *hp* output power to be run at 75 *strokes/min* and give a pressure of 2,000 *psi*, calculate the diameter of the piston rod that satisfy the above required conditions. Assume displacement efficiency of 0.88.

Solution:

Given data:

d_l = Liner diameter = 7.5"
L_s = Liner length = 25"
P_{out} = Pump hydraulic output power = 1,400 *hp*
P = Pump pressure = 2,000 *psi*
η_p = Pump displacement eff. = 0.88
N = Pump speed in *strokes/min* = 75 *stroke/min*

Required data:

d_{pr} = Rod diameter

First, we need to calculate the output rate that the pump can deliver with that pressure using Eq. (2.33):

$$q = \frac{P_{out} \times 1714}{P} = \frac{1,400 \times 1714}{2,000} = 1,099.8 \; gals/min$$

Second we should calculate displacement per one strokes follows:

$$q_D = \frac{q}{N} = \frac{1,099.8}{75} = 15.99 \; gal/stroke$$

Now, by using the displacement equation (2.26), we can find the rod diameter:

$$q_D = \frac{\pi L_S}{2 \times 231 (in^3 / gal)} \times (2d_l^2 - d_{pr}^2)\eta_P$$

$$= \frac{\pi \times 25}{2 \times 231} \times (2 \times 7.5^2 - d_{pr}^2) \times 0.88 = 15.99$$

$$d_{pr}^2 = 2 \times 7.5^2 - 106.89 = 5.61$$

$$d_{pr} = 2.37 \; inches$$

Exercise 2.12: A new double-acting duplex pump is being installed in a drilling rig. The pump has 6.75" liner size, 25" liner length and 1.75" piston rod diameter. The pump was tested for 5 *minutes* at the maximum allowable pump speed of 100 *strokes/min* using fresh water. The actual volume the pump displaced during the test duration was estimated to be 7,050 *gallons*. What is the displacement efficiency of this pump?

Solution:

Given data:

d_l = Liner diameter = 6.75"
L_s = Liner length = 25"
d_{pr} = Rod diameter = 1.75"
N = Pump speed in *strokes/min* = 100 *stroke/min*
t_d = Test duration = 5 *minutes*
V_d = The actual volume displaced = 7,050 *gallons*

Required data:
η_P = Pump displacement eff.

To calculate the displacement efficiency, we should calculate the actual displacement, then we divide it by the theoretical displacement. Actual displacement can be calculated as follows:

$$q_{D,act} = \frac{V_d}{N \times t_d} = \frac{7{,}050}{100 \times 5} = 14.1\ gal/stroke$$

The theoretical displacement can be calculated using Eq. (2.26):

$$\begin{aligned} q_{D,theo} &= \frac{\pi L_S}{2 \times 231 (in^3/gal)} \times (2d_l^2 - d_{pr}^2) \\ &= \frac{\pi \times 25}{2 \times 231} \times (2 \times 6.75^2 - 1.75^2) \\ &= 14.97\ gal/stroke \end{aligned}$$

Now, the displacement efficiency of this pump will be equal to:

$$\eta_P = \frac{q_{D,act}}{q_{D,theo}} = \frac{14.1}{14.97} = \mathbf{0.942\ \textit{OR}\ 94.2\%}$$

Exercise 2.13: A single-acting triplex pump is required to deliver a pumping rate of 1,250 *gpm* at pressure of 1,750 *psi*. The size of each pump liner is 7.75", whereas the length is 24". If the displacement efficiency is 0.89, at what speed should the pump be run to deliver that rate? Also, what is the power output required to run the pump to achieve that pumping rate?

Solution:

Given data:
Q = Required pumping rate = 1,250 *gpm*
d_l = Liner diameter = 7.75"
L_s = Liner length = 24"
P = Pumping pressure = 1,750 *psi*
η_P = Pump displacement eff. = 0.89

Required data:
N = Pump speed in *strokes/min*
P_{out} = Pump hydraulic output power

To calculate the pumping speed, first we should calculate the pump displacement per stroke using Eq. (2.26):

$$\begin{aligned} q_D &= \frac{3 \times \frac{\pi}{4} \times d_l^2 \times l \times \eta_P}{231\ in^3/gal} = \frac{3 \times \frac{\pi}{4} \times 7.75^2 \times 24 \times 0.89}{231\ in^3/gal} \\ &= 13.09\ gal/stroke \end{aligned}$$

The pump speed can now be calculated as below:

$$N = \frac{q}{q_D} =, \frac{1250}{13.09} = 95.5 \; OR \; 96 \; strokes/minute$$

The required output power to run the pump at the specified conditions is equal to:

$$P_{out} = \frac{qP}{1714} = \frac{1,250 \times 1,750}{1714} = 1,276 \; hp$$

Exercise 2.14: A rig pump is planned to pump a drilling fluid at a pump speed of 84 *strokes/min* to give a pumping rate of 1,005 *gpm* at a stand pipe pressure of 1,750 *psi*. The pump displacement efficiency was previously estimated to be 0.92. When the pump was run at the planned speed, the driller noticed that the stand pipe pressure reads 1,694 *psi*. Later, they realized that the pump displacement efficiency was not correct. Calculate the correct pump displacement efficiency, and the new pumping speed that would give the planned pumping rate.

Solution:

Given data:

q	=	Planned pumping rate	=	1,005 *gpm*
P_{pl}	=	Planned pumping pressure	=	1,750 *psi*
P_{act}	=	Actual pump pressure	=	1,694 *psi*
$\eta_{P,old}$	=	Old pump displacement eff.	=	0.92
N	=	Previous pump speed	=	84 *strokes/min*

Required data:

$\eta_{P,corr}$	=	The correct pump displacement eff.
N	=	The new pump speed

First, we need to calculate the pump displacement volume per one stroke without deducting the losses due to the pump efficiency using the following equation:

$$Pump \; displacement \; volume = \frac{q_D}{\eta_{P,old}} = \frac{q}{N \times \eta_{P,old}} = \frac{1,005}{84 \times 0.92}$$
$$= 13.0 \; gals/stroke$$

Second, we should calculate the pump output power that was planned using Eq. (2.33):

$$P_{out} = \frac{qP}{1,714} = \frac{1,005 \times 1,750}{1714} = 1,026.1 \; hp$$

The above power was used at the actual situation, so we can use it to calculate the actual pumping rate using the actual pumping pressure as follows:

$$q = \frac{P_{out} \times 1714}{P} = \frac{1,026.1 \times 1,714}{1,694} = 1,038.2 \; gals/min$$

Now, the actual pump displacement is equal to:

$$q_{D,act} = \frac{q_{act}}{N} = \frac{1038.2}{84} = 12.35\ gals/stroke$$

Thus, the correct displacement efficiency is equal to:

$$\eta_{P,corr} = \frac{q_{D,act}}{Pump\ displacement\ volume} = \frac{12.35}{13.0} = 0.95$$

The pump speed that required to deliver the same pumping rate can be calculated as follows:

$$N_{new} = \frac{q}{q_{D,act}} = \frac{1{,}005}{12.35} = 81.4\ OR\ 81$$

Exercise 2.15: A rig company is planning to install new rig pumps that will be used in drilling a well. Hydraulics studies showed that it is required to have a total pumping rate of 1,850 *gpm*. A single-acting triplex pump of 6.25" liner size and 14" liner length is planned to be used. The maximum tested pump speed was 114 *strokes/min*, but the maximum allowable speed should only be 90% of the maximum tested speed. The pump displacement efficiency was estimated to be 0.93 How many pumps should they install to achieve the required pumping rate, and what will be the pumping speed if all the pumps are going to be run at the same speed?

Solution:

Given data:

q_{total}	=	Required pumping rate	=	1,850 *gpm*
d_l	=	Liner diameter	=	6.25"
L_s	=	Liner length	=	14"
η_P	=	Pump displacement eff.	=	0.93
N	=	Max. tested pump speed	=	114 *stroke/min*
$N_{allowable}$	=	Maximum allowable speed	=	90% of N

Required data:

Number of pumps

N = Pump speed in *strokes/min*

First we will calculate the displacement volume for one stroke per each pump using Eq. (2.26):

$$q_D = \frac{3\times\frac{\pi}{4}\times d_l^2\times l\times\eta_P}{231\ in^3/gal} = \frac{3\times\frac{\pi}{4}\times 6.25^2\times 14\times 0.93}{231\ in^3/gal} = 5.19\ gal/stroke$$

The maximum allowable pump speed is equal to 103 *strokes/min*, which is equivalent to 90% of the maximum tested pump speed. So, each pump can deliver the following rate per minute:

$$q = q_D \times N = 5.19 \times 103 = 534.6\ gals/min$$

Number of pumps can now be calculated as below:

$$Number\ of\ pumps = \frac{required\ rate}{the\ rate\ per\ one\ pump} = \frac{1{,}850}{534.6} = 3.5\ pumps$$

So, based on the current situation four pumps are required. If the four pumps are going to be run at the same pump speed, each pump will give the following rate per minute:

$$q_{pump} = \frac{q_{total}}{4} = \frac{1{,}850}{4} = 462.5\ gals/min$$

And the pumping speed will equal to:

$$N = \frac{q_{pump}}{q_D} = \frac{462.5}{5.19} = 89.1\ OR\ 89\ strokes/min$$

Exercise 2.16: For a drilling rig having a maximum hook load of 250,000 lb_f, and 8 *lines*. Calculate i) fast line load (F_f), ii) dead-line load (F_s), and iii) static derrick load (F_d).

Solution:

Given Data:

Hook Load (W) = 250,000 lb_f
Number of *lines* (n) = 8

Required Data:

Fast line Load (F_f)
Dead-line Load (F_s)
Static derrick load (F_d)

i) Fast Line Load

$$F_f = \frac{W}{n} = \frac{250000}{8} = 31{,}250\ lb_f$$

ii) Dead line load:

$$F_s = \frac{W}{n} = \frac{250000}{8} = 31{,}250\ lb_f$$

iii) Static Derrick Load:

$$F_d = F_f + W + F_s = 31{,}250 + 250{,}000 + 31{,}250 = 312{,}500\ lb_f$$

Exercise 2.17: For problem 2.16, if hoisting system efficiency is 0.84, recalculate the derrick and *lines* loads.

<u>Solution:</u>

Given Data:

Hook Load (W) = 250,000 bf
Number of *lines* (n) = 8
Hoisting system efficiency (E) = 0.84

Required Data:

Fast line Load (F_f)
Dead-line Load (F_s)
Static derrick load (F_d)

a. Calculating Fast line load:

$$F_f = \frac{W}{nE} = \frac{250{,}000}{(8 \times 0.84)} = 37{,}202.4\ lb_f$$

b. Dead line load:

$$F_s = \frac{W}{n} = \frac{250000}{8} = 31{,}250\ lb_f$$

c. Static Derrick Load:

$$F_d = F_f + W + F_s = 37{,}202.4 + 250{,}000 + 31{,}250$$
$$= 318{,}452.4\ lb_f$$

Exercise 2.18: if a rig has the following data: rig hoist load (W) = 200,000 lb_f, maximum draw works input power (P_i) = 800 hp, number of *lines* (n) = 10. Calculate i) static fast line tension (F_f), ii) maximum hook horsepower (P_h), iii) derrick load (F_d), iv) distribute the derrick load on its legs (A, B, C & D), v) maximum equivalent derrick load (F_e), vi) derrick efficiency factor (E_d).

1. For the following data:
   Rig Hoist load (W) = 200,000 lb_f
   Maximum draw works input power (P_i) = 800 hp
   Number of *lines* (n) = 10

Calculate:

a. Static fast line tension (F_f)?
b. Maximum hook horsepower (P_h)?
c. Derrick load (F_d)?
d. Distribute the derrick load on its legs (A, B, C & D)?
e. Maximum equivalent derrick load (F_e)?
f. Derrick efficiency factor (E_d)?

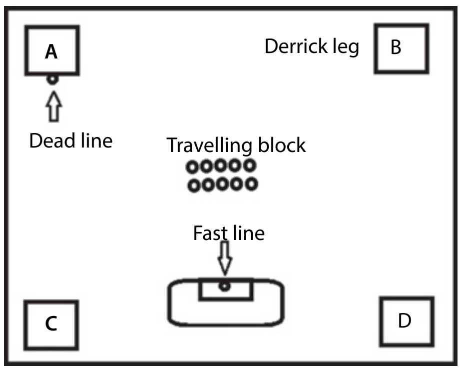
Solution:

Given data:

W = $200{,}000 lb_f$
P_i = $800\ hp$
Number of *lines* = 10
Hoisting efficiency = 0.81

Required data:

Static fast line tension (F_f)
Maximum hook horsepower (P_h)
Derrick load (F_d)
Loads on the derrick legs
Maximum equivalent derrick load (F_e)
Derrick efficiency factor (E_d)



a. Fast line tension:

$$F_f = \frac{W}{nE} = \frac{200{,}000}{(10 \times 0.8)} = 24{,}691\ lb_f$$

b. Maximum Hook horsepower:

$$P_h = P_i E = 800 \times 0.81 = 648\ hp$$

c. Derrick Load:

$$F_d = F_f + W + F_s = 24{,}691 + 200{,}000 + 20{,}000 = 244{,}691\ lb_f$$

d. Loads for the derrick legs:

Each leg will take part of the hook load; or

Leg A = Leg B = Leg C = Leg D = $W/4$

$$Leg\ B\ Load = \frac{W}{4} = \frac{200000}{4} = 50{,}000\ lbf$$

On top of that, Leg A will take the load of the dead line; or

$$Leg\ A\ Load = \frac{W}{4} + F_d = \frac{200000}{4} + 20{,}000 = 70{,}000\ lbf$$

Legs C & D will share the load of the fast line; or

$$Leg\ C\ Load = \frac{W}{4} + \frac{F_f}{2} = \frac{200000}{4} + \frac{24{,}691}{2} = 62{,}346\ lbf$$

$$Leg\ D\ Load = \frac{W}{4} + \frac{F_f}{2} = \frac{200000}{4} + \frac{24{,}691}{2} = 62{,}346\ lbf$$

e. Maximum equivalent derrick load:

$$F_{de} = 4 \times Leg\ A\ load = 4 \times 70{,}000 = 280{,}000\ lbf$$

f. Derrick efficiency factor:

$$E_d = \frac{F_d}{F_{de}} = \frac{244{,}691}{280{,}000} = 0.874$$

2. For the following data:
   Hole depth = 10,000 *ft.*
   Pipe info = 7" *OD* / 6.18"*ID*, 29 lb_f/ft
   Mud weight = 10.0 *ppg.*
   Number of *lines* = 10

Calculate:

a. Weight of casing in air and mud.
b. Hook load assuming the weight of traveling block is 23,500 lb_f.
c. Design factor when running 7" casing if the breaking strength of wire is 228,000 *lb* & efficiency factor is 0.81.

<u>Solution:</u>

Given data:

Hole depth = 10,000 *ft.*
Pipe info = 7" *OD* / 6.18"*ID*, 29 lb_f/ft
Mud weight = 10.0 *ppg.*
Number of *lines* = 10
Weight of traveling block = 23,500lb_f
Breaking strength of wire = 228,000 lb_f
Efficiency factor = 0.81

Required data:

Weight of casing in air and mud
Hook load
Design factor of drilling line

a. Weight of casing:

Weight of casing in air = 29 × 10000 = 290,000 lb_f
Weight of casing in mud = F × weight in air = (1 – 10/65.5) × 290,000 = 245,630 lb_f

b. Hook Load:

W = Wt. of travelling block + Wt. of casing string in mud = 23,500 + 245630 = 269,130 lb_f

c. Design factor:

$$Design\ factor = \frac{Breaking\ strength}{F_f}$$

$$F_f = \frac{269130}{10 \times 0.81} = 33641\ lbf$$

$$Design\ factor = \frac{228000}{33641} = 6.78$$

Chapter 3: Drilling Fluid

Exercise 3.1: A production hole is being drilled in an off-shore area using 8.5 inches bit at an average drilling rate of 17 *ft*/hr. The formation cuttings are stored in containers of 45 cubic *feet* volume to be dumped later in a safe manner. If the formation porosity was estimated to be 0.14, calculate the time required to fill one mud container. Assume the formation is consolidated and no hole enlargement.

Solution:

Given data:

d_{bit}	=	Bit diameter	=	8.5 inches
ROP	=	Drilling rate	=	17 ft/hr
ϕ	=	Formation porosity	=	0.14
V	=	mud container volume	=	45 ft^3

Required data:

t = Time to fill the mud container.

From the bit diameter and the mud container volume, we can calculate the length to be drilled in order to have cuttings volume similar to the volume of the mud container. We can use eq. (3.1):

$$V = \frac{\pi}{4} d_{bit}^2\ (1-\phi)\ L = \frac{\pi}{4} \times \frac{8.5^2}{144} (1-0.14)\ L = 45$$

$$L = 132.78\ ft$$

Thus, the time required to fill the mud container is equal to:

$$t = \frac{L}{ROP} = \frac{132.78}{17.0} = 7.81\ hrs$$

Example 3.2: An intermediate hole is been drilled using 12.25 inches bit. The interval was described to be a consolidated rock with an estimated gravity and porosity of 2.9 and 0.1; respectively. The interval took 10.5 *hrs* to be drilled and produced about 16.5 *tons*. If there was no hole enlargement and no swelling of cuttings, calculate the rate of drilling at this section.

Solution:

Given data:

d_{bit}	=	Bit diameter	=	12.25 inches
$\gamma_{cuttings}$	=	Cutting's specific gravity	=	2.9
ϕ	=	Formation porosity	=	0.1
t	=	drilling time	=	10.5 *hrs*
$M_{cuttings}$	=	Actual cutting mass	=	16.5 *tons*

Required data:

ROP = Drilling rate

If there was no enlargement and no swelling occurred due to the water in the mud, so the hole diameter will be exactly similar to the bit diameter. So by knowing the volume of the produced cuttings, we can calculate the length drilled, Hence the ROP can be calculated. The volume of cuttings is equal to:

$$V = \frac{mass}{density} = \frac{16.5 \times 2{,}200 \dfrac{lbm}{tons}}{2.9 \times 62.4 \dfrac{lbm}{cu\ ft} \times 5.615 \dfrac{cu\ ft}{bbl}} = 35.73\ bbls$$

Now, we can use Eq.(3.2a) to get the length of the section drilled:

$$V_s = \frac{(1-\phi_A) d_B^2 L}{1029} = \frac{(1-0.1) \times 12.25^2 \times L}{1029} = 35.73$$

$$L = 272.36\ ft$$

The rate of penetration can now be calculated as below:

$$ROP = \frac{L}{t} = \frac{272.36}{10.5} = 25.94\ ft/hr$$

Exercise 3.3: Two wells, A and B, have been drilled in the same area. Both wells drilled the same formation in the surface section. Surface section for well *A* was drilled using 17.5 inches bit while well *B* drilled using 20 inches bit. Both wells produced same cuttings volume at the same drilling time. If the hole size was assumed same as the bit size, what is the ratio of the average drilling rate of the two bits at the specified drilling time?

Solution:

Given data:

d_{bitA} = Bit diameter for well A = 17.5 inches
d_{bitB} = Bit diameter for well B = 20.0 inches

Required data:

ROP ratio

Because both wells are drilled at the same area, we can assume the drilled formations have the same properties. So if we assume the produced volume "V", the formation porosity "ϕ", and the drilling time "t". The volume produced in well A is equal to:

$$V_A = \frac{\pi}{4}\frac{d_{bitA}^2}{144}L_A(1-\phi)$$

And the volume produced for well B is equal to:

$$V_B = \frac{\pi}{4}\frac{d_{bitB}^2}{144}L_B(1-\phi)$$

Because $V_A = V_B$

$$\frac{\pi}{4}\times\frac{17.5^2}{144}L_A(1-\phi) = \frac{\pi}{4}\times\frac{20^2}{144}L_B(1-\phi)$$

$$L_A = 1.31L_B$$

If we now divide both lengths by the drilling time we get the *ROP* for each well as follows:

$$\frac{L_A}{t} = 1.31\frac{L_B}{t}$$

$$ROP_A = 1.31ROP_B \;\; OR \;\; \frac{ROP_A}{ROP_B} = 1.31$$

Exercise 3.4: Two fluids have densities of 9.6 pgg and 8.9 *ppg*, respectively. If both fluids read exactly the same reading in the March funnel at the same conditions, find out the relation between the viscosities of both fluids. And if the first fluid read 61 *seconds*, calculate the viscosity of both fluids.

Solution:

Given data:

ρ_{M1} = Density of the first fluid = 9.6 *ppg*
ρ_{M2} = Density of the second fluid = 8.9 *ppg*
t_{M1} = Reading of the first fluid = 61 *seconds*

Required data:

μ_{e1} = Viscosity of the first fluid.
μ_{e2} = Viscosity of the second fluid.

To determine the relation between viscosities of the fluids, we can use Eq. (3.8):

$$\mu_{e1} = \rho_{M1}(t_{M1} - 25)\ OR\ t_{M1} = \frac{\mu_{e1}}{\rho_{M1}} + 25$$

$$\mu_{e2} = \rho_{M2}(t_{M2} - 25)\ OR\ t_{M2} = \frac{\mu_{e2}}{\rho_{M2}} + 25$$

It is given that t_{M1} is equal to t_{M2}, so:

$$\frac{\mu_{e1}}{\rho_{M1}} + 25 = \frac{\mu_{e2}}{\rho_{M2}} + 25\ OR\ \frac{\mu_{e1}}{9.6} = \frac{\mu_{e2}}{8.9}$$

$$\frac{\mu_{e1}}{\mu_{e2}} = 1.079$$

Viscosity of the first fluid is equal to:

$$\mu_{e1} = \rho_{M1}(t_{M1} - 25) = \frac{9.6}{8.34} \times (61 - 25) = 41.44\ cp$$

And viscosity of the second fluid is equal to:

$$\mu_{e2} = \frac{\mu_{e1}}{1.079} = \frac{41.44}{1.079} = 38.4\ cp$$

Exercise 3.5: A drilling fluid has a density of 9.3 *ppg* read 66 *seconds* in the March funnel. A viscosifying additive was added to the fluid that did not make any changes to its density. If the viscosity of the new fluid was increased by 1.12 of the old viscosity, what should be the March funnel reading of the new fluid?

Solution:

Given data:

ρ_M = Density of the fluid = 9.3 *ppg*
t_{M1} = Reading of the old fluid = 66 *seconds*

Required data:

t_{M2} = Reading of the new fluid

From the given data of the fluid before the viscosifying additive we can calculate the viscosity using Eq. (3.8):

$$\mu_{e1} = \rho_M(t_{M1} - 25) = \frac{9.3}{8.34} \times (66 - 25) = 45.72\ cp$$

Now, the viscosity of the same fluid after adding the viscosifying additives is equal to:

$$\mu_{e2} = 1.12\ \mu_{e1} = 1.12 \times 45.72 = 51.21\ cp$$

Thus, if we put the fluid again in the March funnel after increasing the viscosity, the reading will be equal to:

$$t_{M2} = \frac{\mu_{e2}}{\rho_M} + 25 = \frac{51.21}{\frac{9.3}{8.34}} + 25 = 70.9 \ seconds$$

Exercise 3.6: A standard fluid that has a plastic viscosity of 28.5 *cp* and yield point of 8.9 $lb_f/100\ ft^2$ was used to calibrate a certain rotational viscometer. When the instrument ran at 600 *rpm* speed, it read 65.5 degrees. Calculate the instrument average error.

Solution:

Given data:

μ_p = Plastic viscosity = 28.5 *cp*
τ_B = Yield point = 8.9 $lb_f/100\ ft^2$
ϕ_{600} = Reading at speed of 600 *rpm* = 65.5°

Required data:

e = Viscometer's average error

From Eq. (3.10) we can calculate the instrument reading at the speed of 300 *rpm*:

$$\Phi_{300} = \Phi_{600} - \mu_p = 65.5 - 28.5 = 37°$$

We can now calculate the yield using the above value using Eq. (3.11):

$$\tau_B = \Phi_{300} - \mu_p = 37 - 28.5 = 8.5\ lb_f/ft^2$$

Now the instrument's average error can be calculated using the following equation:

$$\begin{aligned} Average\ error &= \frac{|Expected\ reading - actaula\ reading|}{expected\ reading} \times 100 \\ &= \frac{|8.9 - 8.5|}{8.9} \times 100 = 4.5\% \end{aligned}$$

So this rotational viscometer has an average error of 4.5%

Exercise 3.7: A drilling fluid has a plastic viscosity of 22 *cp* and yield point of 6.25 $lb_f/100\ ft^2$ using a rotational viscometer instrument. If another fluid reads 0.85 of the reading of the first fluid at speed of 300 *rpm* and 42° at speed of 600 *rpm*, calculate the viscosity and yield point of the second fluid.

Solution:

Given data:

μp_1 = Plastic viscosity of fluid 1 = 22.0 *cp*
τ_{B2} = Yield point of fluid 1 = 6.25 $lb_f/100\ ft^2$

ϕ_{600} = Reading at speed of 600 *rpm* of fluid 2 = 65.5°
ϕ_{300} = Reading at speed of 300 *rpm* of fluid 2 = 42°

Required data:
μp_2 = Plastic viscosity of fluid 2.
τ_{B2} = Yield point of fluid 2.

To calculate the plastic viscosity and yield point of the second fluid, we should first calculate the reading at 300 *rpm* speed from the given date. First we will calculate the reading at speed of 300 *rpm* of the first fluid using Eq. (3.11):

The reading at speed of 300 *rpm* for the first fluid can be calculated using Eq. (3.11):

$$\tau_B = \Phi_{300} - \mu_p = 6.25 = \Phi_{300} - 22.0$$

$$\Phi_{300} = 28.25°$$

From the given data, the reading at 300 speed for the second fluid is equal to 0.85 of the reading of the first fluid. Thus:

$$\Phi_{300f2} = 0.85 \times \Phi_{300f1} = 0.85 \times 28.25 = 24.01°$$

Now, plastic viscosity can be calculated using Eq. (3.10):

$$\mu_p = \Phi_{600} - \Phi_{300} = 42.0 - 24.01 = 18\ cp$$

And yield point can be calculated using Eq. (3.11):

$$\tau_B = \Phi_{300} - \mu_p = 24 - 18 = 6\ lb_f/ft^2$$

Exercise 3.8: A 54 *tons* of Bentonite 2.51 *gm/cc* was used to prepare a drilling mud by mixing it with water only. If the final density of the mud is 9.7 *ppg*, calculate the volume of water to be used and the volume of the drilling fluid that can be prepared using the above information.

Solution:

Given data:
ρ_{SC} = density of Bentonite = 2.51*gm/cc*
m_{Bent} = amount of Bentonite = 54 *tons*
ρ_{m1} = Original mud weight = 8.34 *ppg*
ρ_{m2} = Required mud weight = 9.7 *ppg*

Required data:
V_{water} = Volume of water in *barrels*
V_{m2} = volume of the mud in *barrels*.

First we should calculate the volume and weight percentages using Eq. (3.30) and (3.31):

$$\frac{V_{SC}}{V_{m2}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{SC} - \rho_{m1}} = \frac{9.7 - 8.34}{2.51 \times 8.34 - 8.34} \times 100 = 10.8\%$$

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} = 10.8\% \times \frac{2.51 \times 8.34}{9.7} = 23.31\%$$

From the weight percentage we can calculate the weight of the mud using the known weight of the Bentonite as follows:

$$mass_{m2} = \frac{mass_{SC}}{\left(\dfrac{mass_{SC}}{mass_{m2}}\right)} = \frac{54 \times 2{,}200}{0.2331} = 509{,}652.5\ lbm$$

Now by using the density of the mud we can calculate the volume of the prepared mud:

$$V_{m2} = \frac{mass_{m2}}{\rho_{m2}} = \frac{509{,}652.5}{\dfrac{9.7}{8.34} \times 62.4} = 7{,}022.4\ ft^3 = 1250.6\ bbls$$

The volume of water required to prepare the above mud is equal to:

$$V_{m1} = \frac{mass_{m2} - mass_{SC}}{\rho_{m1}} = \frac{509{,}652.5 - 54 \times 2{,}200}{62.4}$$
$$= 6{,}263.7\ ft^3 = 1{,}115.5\ bbls$$

Exercise 3.9: A 29.55 *tons* 2.49 *gm/cc* of Bentonite was mixed with an old mud used in the previous section of a well. A 1,500 *bbls* of a new mud having mud density of 11.2 *ppg* was prepared. What were the density and the volume of the old mud?

Solution:

Given data:

ρ_{SC} = density of Bentonite = 2.49 *gm/cc*
m_{Bent} = amount of Bentonite = 29.55 *tons*
ρ_{m2} = Required mud weight = 11.2 *ppg*
V_{m2} = volume of the new mud = 1,500 *bbls*

Required data:

ρ_{m1} = Original mud weight
V_{m1} = Volume of the old mud

To calculate the mud weight of the old mud, we first need to calculate the weight percent using the given data. The mass of the Bentonite is equal to:

$$mass_{SC} = 29.55 \times 2{,}200 = 65{,}010\ lbm$$

The mass of the new mud is equal to:

$$mass_{m2} = V_{m2} \times \rho_{m2} = (1{,}500 \times 5.615) \times \left(\frac{11.2}{8.34} \times 62.4\right) = 705{,}793.4\ lbm$$

Now, the mass percent is equal to:

$$\frac{mass_{SC}}{mass_{m2}}\% = \frac{65{,}010}{705{,}793.4}\% = 9.21\%$$

We can now use Eq. (3.31) to get the density of the old mud:

$$\frac{mass_{SC}}{mass_{m2}} \times \frac{\rho_{m2}}{\rho_{SC}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{SC} - \rho_{m1}};\ 0.0921 \times \frac{11.2}{2.49 \times 8.34}$$

$$= \frac{11.2 - \rho_{m1}}{2.49 \times 8.34 - \rho_{m1}}$$

$$\rho_{m1} - 0.0497\rho_{m1} = 11.2 - 0.0497 \times 20.77 = 9.29$$

$$\rho_{m1} = \frac{10.168}{0.9503} = 10.7\ ppg$$

To get the volume of the old mud, we first calculate the mass of the old mud:

$$mass_{m1} = mass_{m2} - mass_{SC} = 705{,}793.4 - 65{,}010 = 640{,}783.4\ lbm$$

The volume of the old mud is equal to:

$$V_{m1} = \frac{mass_{m1}}{\rho_{m1}} = \frac{640{,}783.4}{\frac{10.7}{8.34} \times 62.4} = 8{,}004\ ft^3 = 1425.5\ bbls$$

Exercise 3.10: A drilling fluid is planned to be prepared using water and Bentonite that has a density of 2.5 *gm/cc*. If it is planned to prepare 800 *bbls* of drilling fluid having a mud weight of 8.8 *ppg*, calculate the amount of Bentonite to be used in *tons* and the volume of water required to prepare this mud.

Solution:

Given data:

V_{m2} = Required volume of the mud = 800 *bbls*

ρ_{SC} = density of Bentonite = 2.50 *gm/cc*

ρ_{m2} = Required mud weight = 8.8 *ppg*

Required data:

M_{SC} = Amount of Bentonite to be mixed in *tons*

V_{water} = Volume of water in *barrels*.

From the given data, we can calculate the volume percentage of solids using Eq. (3.31):

$$\frac{V_{SC}}{V_{m2}} = \frac{\rho_{m2} - \rho_{m1}}{\rho_{SC} - \rho_{m1}} \times 100 = \frac{8.8 - 8.34}{2.50 \times 8.34 - 8.34} \times 100 = 3.68\%$$

Weight percentage can be calculated using Eq. (3.30):

$$\frac{m_{SC}}{m_{m2}} = \frac{V_{SC}}{V_{m2}} \times \frac{\rho_{SC}}{\rho_{m2}} = 3.68\% \times \frac{2.50 \times 8.34}{8.8} = 8.71\%$$

To calculate the amount of Bentonite in *tons*, first we need to calculate the mass of the final fluid:

$$\begin{aligned} mass\ of\ mud &= density\ of\ mud \times volume\ of\ mud \\ &= \frac{8.8}{8.34} \times 62.4 \times (800 \times 5.615) = 295{,}761\ lbm \end{aligned}$$

Now, the amount of Bentonite required can be calculated from the weight percentage and the mass of the mud as follows:

$$\begin{aligned} mass_{bent} &= Weight\% \times mass\ of\ mud = 0.0871 \times 295{,}761 \\ &= 25{,}760.8\ lbm \end{aligned}$$

By dividing the above value by 2,200, we can convert the amount to *tons* as follows:

$$mass_{bent} = \frac{25{,}760.8}{2{,}200} = 11.71\ tons$$

To calculate the required volume of water for preparing the above said mud, we first calculate the amount of water in pounds using the difference between the mass of the final fluid and the mass of Bentonite as below:

$$mass_{water} = mass_{mud} - mass_{bent} = 295{,}761 - 25{,}760.8 = 270{,}000\ lbm$$

Now by using the water density of 62.4 lbm/ft^3, we can calculate the volume of water as below:

$$V_w = \frac{mass_w}{\rho_w} = \frac{270{,}000}{62.4} = 4{,}327\ ft^3\ \ OR\ 770.6\ bbls$$

Exercise 3.11: If the maximum mud weight that can be achieved by mixing sodium chloride with water is 12.0 *ppg*, calculate the sodium chloride volume fraction that can be used to prepare the above mud weight. Sodium chloride density is 2.15 *gm/cc*.

Solution:

Given data:

ρ_{mud} = Maximum mud weight = 12.0 *ppg*

ρ_{Attap} = Density of sodium chloride = 2.15 *gm/cc* OR 17.93 *ppg*

Required data:

f_{sc} = Sodium chloride volume fraction

If we assume sodium chloride volume fraction is "*x*", so the water volume fraction is"1-*x*". Using Eq. (3.35):

$$\rho_{mix} = \rho_{wf_w} + \rho_{scf_{sc}}$$

$$12.0 = 8.34(1-x) + 17.93x$$

$$x = \frac{12.0 - 8.34}{17.93 - 8.34} = 0.382$$

So, the maximum volume fraction that can be achieved by sodium chloride is 0.382.

Exercise 3.12: An oil-based mud is prepared using a diesel that has a density of 54.5 *lbm/ft*^{3}, 1,500 *bbls* of water and 115.5 *tons* of Barite that has a density of 262.08 *lbm/ft*^{3}. If the mud weight of the prepared mud is 10.5 *ppg*, calculate the volume of diesel that used in preparing the above mud.

<u>Solution</u>:

Given data:

V_w	=	Water volume	=	1,500 *barrels*
P_{Diesel}	=	Diesel density	=	54.5 *lbm/ft*^{3}
ρ_{Bar}	=	Density of barite	=	262.08 *lbm/ft*^{3}
ρ_{mud}	=	Mud weight	=	10.5 *ppg*
$mass_{Bent}$	=	Mass of Barite	=	115.5 *tons*

Required data:

V_o = Diesel volume in *barrels*

Mass of water used is equal to:

$$mass_w = \rho_w \times V_w = 62.4 \times (1{,}500 \times 5.615) = 525{,}564\ lbm$$

Volume of water used is equal to:

$$V_w = 1{,}500 \times 5.615 = 8{,}422.5\ ft^3$$

Mass of Barite used is equal to:

$$mass_{sc} = 115.5 \times 2{,}200 = 254{,}100\ lbm$$

Volume of Barite used is equal to:

$$V_{sc} = \frac{mass_{sc}}{\rho_{sc}} = \frac{254{,}100}{262.08} = 969.6\ ft^3$$

From the definition of density:

$$\rho_{mix} = \frac{mass_o + mass_w + mass_{sc}}{V_o + V_w + V_{sc}}$$

$$= \frac{54.5 \times V_o + 525{,}564 + 254{,}100}{V_o + 8{,}422.5 + 969.6} = \frac{10.5}{8.34} \times 62.4$$

$$78.56V_o - 54.5V_o = 779{,}664 - 9{,}392 \times 78.56$$

$$V_o = \frac{81{,}423}{24.06} = 1{,}738\ ft^3 = 309.6\ bbls$$

Exercise 3 13: A 60 *tons* of Barite was used together with water to develop 1,050 *bbls* of a drilling fluid. If the density of Barite is 4.2 *gm/cc*, calculate the volume of water used and the density of the mud developed.

Solution:

Given data:

V_{mix} = Volume of the mud = 1,050 *barrels*

ρ_{Barite} = Density of Barite = 4.2 *gm/cc* = 262.08 *lbm/ft*^{3}

$Mass_{Barite}$ = Mass of Barite = 60 *tons* = 132,000 *lbm*

Required data:

V_w = Volume of water in *barrels.*

ρ_{mud} = Mud weight

Volume of Barite can be calculated as follows:

$$V_{sc} = \frac{mass_{sc}}{\rho_{sc}} = \frac{132{,}000}{262.08} = 503.8\ ft^3$$

Volume water that used in developing the above mud is equal to:

$$V_w = V_{mix} - V_{sc} = 1{,}050 \times 5.615 - 503.8 = 5{,}392\ ft^3\ \ OR\ 960\ bbls$$

The mass of the used water is equal to:

$$mass_w = \rho_w V_w = 62.4 \times 5{,}654 = 352.809\ lbm$$

The mud weight can be calculated using the following equation:

$$\rho_{mix} = \frac{mass_w + mass_{sc}}{V_w + V_{sc}} = \frac{352{,}809 + 132{,}000}{5{,}654 + 503.8} \times \frac{8.34}{62.4} = 11.0\ ppg$$

Exercise 3.14: A drilling fluid was developed using water and Hematite that has a density of 5.05 *gm/cc*. Mud engineer took 1,350 *bbls* of water to prepare 1,425 *bbls* of the mud. Calculate the amount of Hematite used to prepare the above mud and the mud weight.

Solution:

Given data:

V_{mix} = Volume of the mud = 1,425 *barrels*

V_w = Volume of water = 1,350 *barrels*

ρ_{Hem} = Density of Hematite = 5.05 *gm/cc* = 315.12 *lbm/ft*^{3}

Required data:

$Mass_{Hem.}$ = Mass of Hematite in *tons*

ρ_{mud} = Mud weight

Using volumes of water and mud, we can calculate the volume of Hematite as follows:

$$V_{sc} = V_{mix} - V_w = 5.615(1,425 - 1,350) = 421.13\ ft^3$$

Now the mass of Hematite is equal to:

$$mass_{sc} = \rho_{sc} V_{sc} = 315.12 \times 421.13 = 132,705\ lbm = 60.3\ tons$$

The mass of 7,580 ft^3 of water is equal to:

$$mass_w = \rho_w V_w = 62.4 \times 7,580 = 473,008\ lbm$$

The mud weight is calculated using the equation below:

$$\rho_{mix} = \frac{mass_w + mass_{sc}}{V_w + V_{sc}} = \frac{473,008 + 132,705}{7,580 + 421.13} \times \frac{8.34}{62.4} = 10.1\ ppg$$

Exercise 3.15: A 1,125 *bbls* of drilling mud that has mud weight of 10.4 *ppg* was diluted using 250 *bbls* of water. What will be the new mud weight after dilution?

Solution:

Given data:

V_{mix} = Volume of the mud = 1,125 *barrels*

V_w = Volume of water = 250 *barrels*

P_{mud_old} = Current mud weight = 10.4 *ppg*

Required data:

P_{mud_new} = New mud weight after dilution.

To calculate the new mud weight, we need to calculate the total mass and total volume of the mud after dilution. Mass of the current mud is equal to:

$$mass_{mud} = \rho_{mud} V_{mud} = \left(\frac{10.4}{8.34} \times 62.4 \right) \times (1,125 \times 5.615) = 491,535\ lbm$$

Mass of the water used in dilution is equal to:

$$mass_w = \rho_w V_w = 62.4 \times 250 \times 5.615 = 87,594\ lbm$$

The new mud weight is now equal to:

$$\rho_{mix} = \frac{mass_{mud} + mass_w}{V_{mud} + V_w} = \frac{491,535 + 87,495}{5.615(1,125 + 250)} \times \frac{8.34}{62.4} = 10.0\ ppg$$

Chapter 4: Drilling Hydraulics

Exercise 4.1: A moving plate is positioned 2.5 *cm* above a stationary plate. A force of 308 *dynes* is required to initiate the first movement, and forces of 587 and 673 *dynes* are required to move the plate at uniform velocities of 9.4 and 12.3 *cm*/s, respectively. Calculate the cross-sectional area of the upper plate and the plastic viscosity.

Solution:

Given data:

L = Distance between the plates = 2.5 *cm*
F_y = Force to initiate the movement = 308 *dynes*
F_1 = Force required for first speed = 587 *dynes*
v_1 = First speed of the plate = 9.4 *cm*/s
F_2 = Force required for second speed = 673 *dynes*
v_2 = Second speed of the plate = 12.3 *cm*/s

Required data:

μ_p = Plastic viscosity.
A = Cross-sectional area of the upper plate

Eq. (4.2a) can be used to develop a relation between plastic viscosity and the area of the plate using the given information:

$$\tau_y = \frac{F_y}{A} = \frac{308}{A}\ dynes/cm^2$$

$$\tau_1 = \frac{F_1}{A} = \frac{587}{A}\ dynes/cm^2$$

$$\tau_2 = \frac{F_2}{A} = \frac{673}{A}\ dynes/cm^2$$

$$\gamma_1 = \frac{v_1}{L} = \frac{9.4}{2.5} = 3.76\ s^{-1}$$

$$\gamma_2 = \frac{v_2}{L} = \frac{12.3}{3} = 4.92\ s^{-1}$$

Now two equations can be developed as follows:

$$\tau = \mu_p\gamma + \tau_y$$

$$\frac{587}{A} = \mu_p 3.76 + \frac{308}{A} \text{.......(1)}$$

$$\frac{673}{A} = \mu_p 4.92 + \frac{308}{A} \text{.......(2)}$$

Solving the above two equations together will give:

$$\mu_p = 2.75\ poise$$

$$\mu_p = 275\ cp$$

$$A = 27.0\ cm^2$$

Exercise 4.2: A moving plate of a cross-sectional area of 40 cm^2 is positioned above a stationary plate. The instrument read a force of 790 *dynes* when the plate was moving at a uniform speed of 11.25 *cm*/sec, and a force of 930 *dynes* when it was moving at a uniform speed of 14.75 *cm*/sec. If the force required to initiate the movement is 340 *dynes*, calculate the distance between the two plates and the plastic viscosity.

Solution:

Given data:

A	=	Cross-sectional area of the upper plate	=	40 cm^2
F_y	=	Force to initiate the movement	=	340 *dynes*
F_1	=	Force required for first speed	=	790 *dynes*
v_1	=	First speed of the plate	=	11.25 *cm*/s
F_2	=	Force required for second speed	=	930 *dynes*
v_2	=	Second speed of the plate	=	14.75 *cm*/s

Required data:

μ_p = Plastic viscosity.
L = Distance between the plates

Again, Eq. (4.2a) will be used to calculate the plastic viscosity and the distance between the two plates. First, shear stresses and yield point can be calculated as follows:

$$\tau_y = \frac{F_y}{A} = \frac{340}{40} = 8.5\ dynes/cm^2$$

$$\tau_1 = \frac{F_1}{A} = \frac{790}{40} = 19.75\ dynes/cm^2$$

$$\tau_2 = \frac{F_2}{A} = \frac{930}{40} = 23.25\ dynes/cm^2$$

Now two equations can be developed as follows:

$$\tau = \mu_p \gamma + \tau_y$$

$$19.75 = \mu_p \times \frac{11.25}{L} + 8.5 \ldots\ldots\ldots\ldots(1)$$

$$23.25 = \mu_p \times \frac{14.75}{L} + 8.5 \ldots\ldots\ldots\ldots(2)$$

Solving the above two equations together will give:

$$\mu_p = 3.25cp$$

$$L = 3.25\ cm$$

Exercise 4.3: A moving plate of a cross-sectional area "*A*" cm^2 is positioned "*L*" *cm* above a stationary plate. A force which is equal to 1.5 F_y was required to move the plate at a uniform speed of "v_1" *cm*/s, and a force which is equal to 1.75 F_y was needed to move the plate at a uniform speed of "v_2" *cm*/s. What is the relation between the two speeds? F_y is the force needed to initiate the first movement?

Solution:

Given data:

L	=	Distance between the plates	=	$L\ cm$
A	=	Cross-sectional area of the upper plate	=	$A\ cm^2$
F_1	=	Force required for first speed	=	$1.5\ F_y\ dynes$
v_1	=	First speed of the plate	=	$v_1\ cm$/s
F_2	=	Force required for second speed	=	$1.75\ F_y\ dynes$
v_2	=	Second speed of the plate	=	$v_2\ cm$/s

Required data:

"*c*" in the equation: $v_1 \ = \ cv_2$

Yield point is equal to:

$$\tau_y = \frac{F_y}{A}\ OR\ F_y = \tau_y A$$

Now shear stress for the first speed can be calculated as follows:

$$\tau_1 = \frac{1.5F_1}{A} = \frac{1.5\tau_y A}{A} = 1.5\tau_y$$

Similarly, shear stress for the second speed is equal to:

$$\tau_2 = \frac{1.75F_2}{A} = \frac{1.75\tau_y A}{A} = 1.75\tau_y$$

Using Eq. (4.2a) gives:

$$\tau = \mu_p\gamma + \tau_y$$

$$1.5\tau_y = \mu_p \times \frac{v_1}{L} + \tau_y\ OR\ 0.5\tau_y = \mu_p \times \frac{v_1}{L}\ldots\ldots\ldots\ldots(1)$$

$$1.75\tau_y = \mu_p \times \frac{v_1}{L} + \tau_y\ OR\ 0.75\tau_y = \mu_p \times \frac{v_2}{L}\ldots\ldots\ldots\ldots(2)$$

Dividing equation (1) over (2) gives:

$$\frac{v_1}{v_2} = \frac{0.50}{0.75} = 0.667$$

$$v_1 = 0.667 v_2$$

Exercise 4.4: A power low fluid has a power low exponent of 0.86 reads shear rate of "γ" at shear stress of "τ". If the fluid reads another shear stress of "2τ", find out the relation of apparent viscosities for the two cases.

Solution:

Given data:

n_p = The exponent = 0.86
τ_1 = Shear stress at run 1 = τ
γ_1 = Shear rate at run 1 = γ
τ_2 = Shear stress at run 2 = τ

Required data:

"m" in the equation: $\mu_{a1} = m\mu_{a2}$

The power low equation is; Eq. (4.7): $\tau = K\gamma^{n_p}$

We can create an equation for the first case as follows:

$$\tau_1 = K\gamma_1^{0.86} \ldots\ldots\ldots\ldots(1)$$

Similarly, we can create an equation for the second case as follows: $\tau_2 = 2\tau_1$; so:

$$2\tau_1 = K\gamma_2^{0.86} \ldots\ldots\ldots\ldots(2)$$

By dividing eq. (2)/eq. (1) gives:

$$\left(\frac{\gamma_2}{\gamma_1}\right)^{0.86} = 2$$

$$\gamma_2 = 2.239\ \gamma_1$$

The apparent viscosity equation is; Eq. (4.1):

$$\mu_a = \frac{\tau}{\gamma}, \text{ so}$$

$$\mu_{a1} = \frac{\tau_1}{\gamma_1} \text{ and } \mu_{a2} = \frac{\tau_2}{\gamma_2} = \frac{2\tau_1}{2.239\ \gamma_1} = \frac{0.893\ \tau_1}{\gamma_1}$$

So that: $\mu_{a1} = 1.12\ \mu_{a2}$

Exercise 4.5: Two viscosity measuring viscometers have moving plates of cross-sectional areas of 26 and 37 cm^2 and distance of 2.25 and 3.5 *cm* from their stationary plates, respectively. The first instrument reads a force of 390 *dynes* at speed of 10.27 *cm*/sec for a certain fluid. What should the second instrument read for the same fluid to come up with the same apparent viscosity? Assume negligible measuring errors for both instruments.

Solution:

Given data:

L_1 = Distance between the plates = 2.25 *cm*
L_2 = Distance between the plates = 3.5 *cm*
A_1 = Cross-sectional area of the upper plate = 26 cm^2
A_2 = Cross-sectional area of the upper plate = 37 cm^2
F_1 = Force required for first instrument = 390 *dynes*
v_1 = Speed of the plate for first instrument = 10.27 *cm*/s

Required data:

F_2 = Force of second instrument
v_2 = Speed of the second instrument

Because both instruments measure the apparent viscosity of the same fluid, shear stress and shear rate developed by both instruments must be equal. So, shear stress is equal to:

$$\tau = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

The force that should be developed to get the same apparent viscosity is equal to:

$$F_2 = A_2 \frac{F_1}{A_1} = 37 \times \frac{390}{26} = 555 \ dynes$$

Similarly, the speed that will be developed at the second instrument is equal to:

$$\gamma = \frac{v_1}{L_1} = \frac{v_2}{L_2} \ OR \ v_2 = L_2 \times \frac{v_1}{L_1} = 3.5 \times \frac{10.27}{2.25} = 15.98 \ cm/sec$$

Exercise 4.6: Two apparent viscosity measurements were done for a power low fluid. If the ratio of the first apparent viscosity to the second one is 0.97 and the shear rate ratio is 1.2, calculate the flow behavior index of this fluid.

Solution:

Given data:

$\frac{\mu_{a1}}{\mu_{a2}}$ = Apparent viscosities ratio = 0.97

$\frac{\gamma_1}{\gamma_2}$ = Shear rate ratio = 1.2

Required data:
n = Flow behavior index

K and n values will be calculated using Eq. (4.7):

$$\tau = K\gamma^{n_p}$$

So to get n and K values, we should create two equations as follows:

$$\tau_1 = K\gamma_1^{n_p}$$

$$\tau_2 = K\gamma_2^{n_p}$$

$$n_p = \frac{\ln\left(\dfrac{\tau_1}{\tau_2}\right)}{\ln\left(\dfrac{\gamma_1}{\gamma_2}\right)} \ldots\ldots\ldots\ldots(1)$$

Now, we need to find the relations of shear stress and shear rate in order to calculate n_p.

$$\frac{\mu_{a1}}{\mu_{a2}} = 0.97 \ OR \ \frac{\tau_1}{\gamma_1} = 0.97\frac{\tau_2}{\gamma_2}$$

$$\frac{\tau_1}{\tau_2} = 0.97\frac{\gamma_1}{\gamma_2} = 0.97 \times 1.2 = 1.164$$

Now we can calculate n and K as follows:

$$n_p = \frac{\ln\left(\dfrac{\tau_1}{\tau_2}\right)}{\ln\left(\dfrac{\gamma_1}{\gamma_2}\right)} = \frac{\ln(1.164)}{\ln(1.200)} = 0.83$$

Exercise 4.7: A power low fluid measured 61 and 105 degrees at speeds of 300 and 600 *rpm*, respectively. Viscosity of the same fluid measured using another viscometer has an upper plate of cross-sectional area of 34 cm^2 and distance of 2.5 *cm* from the stationary plate. What force should the instrument read if the upper plate speed was 8 *cm*/sec?

Solution:

Given data:

θ_{300} = *V-G* reading at 300 *rpm* = 61°
θ_{600} = *V-G* reading at 600 pm = 105°
L = Distance between the plates = 2.5 *cm*
A = Cross-sectional area of the upper plate = 34 cm^2

Required data:
F = Force at speed of 8 *cm*/sec

From the readings of 300 and 600 *rpm*, we can first calculate *n* and *K* using Eq. (4.9) and (4.11):

$$np = 3.322\log\left(\frac{\theta_{600}}{\theta_{300}}\right) = 3.322 \times \log\left(\frac{105}{61}\right) = 0.78$$

$$K = \frac{5.11 \times \theta_{300}}{511^{np}} = \frac{5.11 \times 61}{511^{0.78}} = 2.35\ eq.\ poise$$

Now, we can use Eq. (4.7) to calculate the force at speed of 8 *cm*/sec as follows:

$$\tau = K\gamma^{n_p}$$

$$\frac{F}{34} = 2.35 \times \left(\frac{8.0}{2.5}\right)^{0.78}$$

$$F = 198\ dynes$$

Exercise 4.8: A power low fluid measured 34 and 66 degrees at speeds of 150 and 400 *rpm*, respectively. Viscosity of the same fluid measured using another viscometer has an upper plate of cross-sectional area of 41 cm^2 and distance of 2 *cm* from the stationary plate. What is the speed of the upper plate that would give a force reading of 548 *dynes*?

Solution:

Given data:
θ_{150} = *V-G* reading at 150 *rpm* = 34°
θ_{400} = *V-G* reading at 400 pm = 66°
F = Force generated = 548 *dynes*

Required data:
v = Speed of the plate

From the readings of 150 and 400 *rpm*, we can first calculate *n* and *K* using Eq. (4.13) and (4.14):

$$n_p = \frac{\log\left(\frac{\theta_{400}}{\theta_{150}}\right)}{\log\left(\frac{400}{150}\right)} = \frac{\log\left(\frac{66}{34}\right)}{\log\left(\frac{400}{150}\right)} = 0.68$$

$$K = \frac{5.11 \times \theta_{N_1}}{(1.703 \times N_1)^{n_p}} = \frac{5.11 \times 34}{(1.703 \times 150)^{0.68}} = 4.09\ eq.\ poise$$

Now, we can use Eq. (4.7) to calculate the speed that would give a force of 343 *dynes* as follows:

$$\tau = K\gamma^{n_p}$$

$$\frac{548}{41} = 4.09 \times \left(\frac{v}{2}\right)^{0.68}$$

$$v = 11.4\ cm/sec$$

Exercise 4.9: An intermediate section is drilled with 12.25" bit and 5.5" drill pipes that have 5.0" inside diameter using drilling mud that has *MW* of 9.3 *ppg* and viscosity of 38 *cp*. If the rig has two pumps each pump delivers 7.5 *gals/stroke* and run at 75 *spm*, determine the flow regime inside the drill pipes.

<u>Solution</u>:

Given data:

d_h	=	Intermediate hole diameter	=	12.25"
d_{dp_i}	=	Inside diameter of the drill pipes	=	5.0"
MW	=	Mud weight	=	9.3 *ppg*
μ	=	Mud viscosity	=	38 *cp*
q_D	=	Pump displacement	=	7.5 *gals/stroke*
N	=	Pump speed	=	75 *spm*

Required data:

Determine the flow regime

Flow regime can be determined by calculating the Reynolds number using Eq. (4.21):

$$N_{Re} = \frac{928\ \rho \bar{v} d}{\mu}$$

First, we need to calculate mud flow rate as follows:

$$q = 2(two\ pumps) \times q_D \times N = 2 \times 7.5 \times 75 = 1{,}125\ gpm$$

Second, the mud average flow velocity will be calculated as below:

$$\bar{v} = \frac{q(gpm)}{2.448\ d^2(inches)^2} = \frac{1{,}125}{2.448 \times 5.0^2} = 18.38\ ft/sec$$

Reynolds number is now equal to:

$$N_{Re} = \frac{928 \times 9.3 \times 18.38 \times 5.0}{38} = 20{,}872$$

Thus, flow is turbulent

Exercise 4.10: Fresh water is planned to be pumped in a certain pipe at constant pumping rate of 6.5 *gpm*. If water density and viscosity are 8.34 *ppg* and 1.0 *cp*, what is the minimum pipe inside diameter that make the fluid flow behaves as turbulent flow?

Solution:

Given data:

ρ = Water density = 8.34 *ppg*
μ = Water viscosity = 1.0 *cp*
q = Flow rate = 6.5 *gpm*

Required data:

d = Minimum pipe diameter in inches

To calculate the minimum pipe diameter that should be used to have turbulent flow, we are going to use Reynolds number of 2,100 as minimum value to have turbulent flow. So, we can use Eq. (4.21) as below:

$$N_{Re} = \frac{928\ \rho \bar{v} d}{\mu} = \frac{928\ \rho q d}{2.448\ \mu d^2} = \frac{928 \times 8.34 \times 6.5}{2.448 \times 1.0 d} = 2{,}100$$

$$d_{min} = 9.79\ inches$$

If we use pipe has inside diameter less than 9.79, we will have laminar flow. So, to have turbulent flow the diameter of the pipe should be equal to or greater than 9.79 inches.

Exercise 4.11: A fluid is pumped in the annulus between two pipes at pumping rate of 300 *gpm*. Inside diameter of the outer pipe is 8.5", and outside diameter of the inner pipe is 7.0". If the fluid density is 8.8 *ppg* and fluid viscosity is 7.0 *cp*, what is the flow behavior?

Solution:

Given data:

ρ = Water density = 8.8 *ppg*
μ = Water viscosity = 7.0 *cp*
Q = Flow rate = 300 *gpm*
d_{inner_OD} = *OD* of the inner pipe = 7.0"
d_{outer_ID} = *ID* of the outer pipe = 8.5"

Required data:

Flow regime

Flow regime or behavior can be determined by calculating Reynolds number. For the case of annulus flow, the equivalent diameter is equal to:

$$d_e = d\_outer_i - d_{inner\,o} = 8.5 - 7.0 = 1.5''$$

Velocity is now equal to:

$$\bar{v} = \frac{q}{2.448\ d^2} = \frac{300}{2.448 \times 1.5^2} = 54.47\ ft/sec$$

Reynolds number can now be determined using Eq. (4.21) as below:

$$N_{Re} = \frac{928\ \rho \bar{v} d}{\mu} = \frac{928 \times 8.8 \times 54.47 \times 1.5}{7.0} = 95{,}312$$

Thus, flow is behaving as turbulent flow because Reynolds number is greater than 2,100

Exercise 4.12: A well with a 9 5/8" casing of 8.88" inside diameter, and 5½" tubing of 5.0" inside diameter. *KCl* brine of 9.43 *ppg* was pumped inside the tubing and fresh water was in the annulus between the casing and the tubing. Down hole valve was placed at depth of 9,250 *ft*. When the down-hole valve was opened and connected the tubing and annulus, some amount of water flowed out of the annulus before the annulus valve was closed. Casing shut-in pressure was closed and stabilized at 495 *psi*. What is the level of the *KCl* brine that entered the annulus due to difference in hydrostatic pressure, and what is the volume of water that has been replaced by the *KCl* brine? Note that tubing is always full of *KCl* brine and there is no pressure build up at the surface of the tubing.

<u>**Solution**</u>**:**

Given data:

D_{valve}	=	Valve depth	=	9,250 *ft*
d_{cas}	=	Casing inside diameter	=	8.88"
OD_{tub}	=	Outside diameter of tubing	=	5.5"
ID_{tub}	=	Inside diameter of tubing	=	5.0 "
MW_{KCl}	=	Mud weight of KCl brine	=	9.43 *ppg*
MW_{water}	=	Mud weight of water	=	8.34 *ppg*
$SITP$	=	Shut in tubing pressure	=	495 *psi*

Required data:

Level of *KCl* in the annulus
Amount of water replaced out

To calculate the above required data, first we calculate the hydrostatic pressures before opening the down-hole valve using Eq. (4.31):

$$P_{KCl} = 0.052 \times MW_{KCl} \times D_{valve} = 0.052 \times 9.43 \times 9{,}250 = 4{,}535.8\ psi$$

$$P_{water} = 0.052 \times MW_{water} \times D_{valve} = 0.052 \times 8.34 \times 9{,}250 = 4{,}011.5\ psi$$

After opening the down-hole valve, some amount of water in the annulus will be displaced out by *KCl* brine in the tubing due to the difference in hydrostatic pressures. But after closing the annulus at surface, a 495 *psi* pressure was built. After equilibrium, hydrostatic pressure in the tubing will be equal to the hydrostatic pressure in the annulus plus the surface casing pressure. So if we assume that "x" *ft* of *KCl* brine was entered the annulus, the equilibrium equation will be written as follows:

$$P_{KCl} = SITP + 0.052\{MW_{KCl} \times x + MW_{water}(9{,}250 - x)\}$$

$$4{,}535.8 = 495 + 0.052\{9.43 \times x + 8.34(9{,}250 - x)\}$$

So, from the above equation we can solve for the level of the *KCl* brine in the annulus:

$$x = 516.8\ ft$$

Thus the amount of water displaced out of the tubing is equal to:

$$\begin{aligned} V &= \frac{\pi}{4}(d_{cas\_id}{}^2 - d^2_{tub\_od})l \\ &= \frac{\pi}{4} \times \left(\left(\frac{8.88}{12}\right)^2 - \left(\frac{5.5}{12}\right)^2\right) \times 516.8 = 133.0\ ft^3\ OR\ 23.7\ bbls \end{aligned}$$

Exercise 4.13: A well drilled to a depth of 3,000 *ft* using a certain drilling mud that gave a total annular pressure losses of 180 *psi* when circulated at a certain circulating condition. The drilling mud has been diluted with water, and a new length of 500 *ft* has been drilled using the new drilling mud. The total annular pressure losses at the current depth were estimated to be180 *psi*. If the difference between the old *ECD* to the current *ECD* is 0.36, what is the difference between the two mud densities?

Solution:

Given data:

D	=	Previous depth of the well	=	3,000 *ft*
ΔD	=	The new drilled section	=	500 *ft*
ΔP_1	=	Annular pressure losses for case 1	=	180 *psi*
ΔP_2	=	Annular pressure losses for case 2	=	180 *psi*
$ECD_1 - ECD_2$	=	*ECD* difference	=	0.36

Required data:

Difference in the mud densities

We know that the equivalent circulating density is equal to, Eq. (4.37a):

$$ECD = \rho_{om} + \frac{\Delta p_{ann}}{0.052 \times L_{tvd}}$$

Now, we can create two equations for the two cases as follows:

$$ECD_1 = \rho_{om1} + \frac{180}{0.052 \times 3{,}000} = \rho_{om1} + 1.154 \qquad (1)$$

$$ECD_2 = \rho_{om2} + \frac{180}{0.052 \times 3{,}500} = \rho_{om2} + 0.989 \tag{2}$$

We know that:

$$ECD_1 - ECD_2 = 0.36 \tag{3}$$

Subtracting equation (1) from equation (2) gives:

$$ECD_1 - ECD_2 = \rho_{om1} - \rho_{om2} + 0.165 = 0.36 \tag{4}$$

$$\rho_{om1} - \rho_{om2} = 0.195$$

So the old mud has been diluted by 0.195 or 0.2 *ppg* as compared to the old mud density.

Exercise 4.14: An intermediate hole is being drilled using 12" bit that has 4 nozzles of $^{16}/_{32}$" each. Current depth of the well is 9,000 *ft* and drilling mud properties are mud weight of 11.8 *ppg*, yield point of 14.0 $lb_f/100ft^2$ and viscosity of 15.0 *cp*. Last casing of 13 3/8" with inside diameter of 12.43 was set at depth of 6,500 *ft* all the way to the surface. Drillstring consists of 450 *ft* of *DC*s of *OD* 9.0" and *ID* of 3.0; and the rest are *DP*s of *OD* 5.5" and *ID* of 4.13". If the pump flow rate is 750 *gpm*, calculate the minimum required pump pressure that can overcome all the system pressure losses.

<u>Solution</u>:

Given data:

D	=	Current depth	=	9,000 *ft*
d_h	=	Hole diameter	=	12.0"
D_{cas}	=	Last casing setting depth	=	6,500
L_{DC}	=	Length of drill collars	=	450 *ft*
OD_{DC}	=	Outside diameter of drill collars	=	9.0"
ID_{DC}	=	Inside diameter of drill collars	=	3.0 "
OD_{DP}	=	Outside diameter of drill pipes	=	5.5"
ID_{DP}	=	Inside diameter of drill pipes	=	4.13 "
MW	=	Mud weight	=	11.8 *ppg*
μ	=	Viscosity of mud	=	15 *cp*
τ_y	=	Mud yield point	=	14.0 $lb_f/100\,ft^2$
q	=	Pumping rate	=	750 *gpm*

Required data:

P_{pump} = Pump pressure in *psi*

To calculate pressure of the pump that can overcome all the system pressure losses, first system pressure losses should be calculated as below:

Pressure losses inside drill pipes:

Average velocity inside drill pipes should first be calculated using Eq. (4.53):

$$\bar{v} = \frac{24.5\ q}{d_{dpi}^2} = \frac{24.5 \times 750}{4.13^2} = 1{,}077\ ft/min$$

Critical velocity for the case of inside drill pipes can be calculated using Eq. (4.54):

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m d_{dpi}^2 \tau_{y-i}}}{\rho_m d_{dpi}}$$

$$= \frac{97 \times 15 + 97\sqrt{15^2 + 8.2 \times 11.8 \times 4.13^2 \times 14}}{10.7 \times 4.13} = 295\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss inside drill pipes can be calculated using Eq. (4.56):

$$P_{dc} = \frac{8.91 \times 10^{-5} \rho_m^{0.8} q^{1.8} \mu_{p_i}^{0.2} L_{dp}}{d_{dpi}^{4.8}}$$

$$= \frac{8.91 \times 10^{-5} \times 11.8^{0.8} \times 750^{1.8} 15^{0.2} \times (9{,}000 - 450)}{4.13^{4.8}}$$

$$= 1{,}560\ psi$$

Pressure loss inside drill collars:

Average and critical velocities for drill collars can be calculated as we did for the drill pipes using Eqs. (4.53) and (4.54) as below:

$$\bar{v} = \frac{24.5\ q}{d_{dci}^2} = \frac{24.5 \times 750}{3.0^2} = 2{,}042\ ft/min$$

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\ \rho_m d_{dci}^2 \tau_{y-i}}}{\rho_m d_{dci}}$$

$$= \frac{97 \times 15 + 97\sqrt{15^2 + 8.2 \times 11.8 \times 3.0^2 \times 14}}{11.8 \times 3.0}$$

$$= 307\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss inside drill pipes can be calculated using Eq. (4.56):

$$P_{dc} = \frac{8.91 \times 10^{-5} \rho_m^{0.8} q^{1.8} \mu_{p_i}^{0.2} L_{DC}}{d_{dci}^{4.8}}$$

$$= \frac{8.91 \times 10^{-5} \times 11.8^{0.8} \times 750^{1.8} 15^{0.2} \times 450}{3.0^{4.8}} = 381\ psi$$

Pressure drop across the bit:
For the bit, first we should calculate the equivalent nozzles:

$$A_n = n_{nozzles} \times \frac{\pi}{4} \times d^2_{nozzles} = 4 \times \frac{\pi}{4} \times \left(\frac{16}{32}\right)^2 = 0.785\ in^2$$

Now, pressure drop across the bit can be calculated using Eq. (4.63) and considering $C_d = 0.95$:

$$P_{bn} = \frac{\rho_m q^2}{10{,}858 A_n^2} = \frac{11.8 \times 750^2}{10{,}858 \times 0.785^2} = 991\ psi$$

Pressure loss in the annulus between hole and drill collars:
The average velocity for the annulus between hole and drill collars can be calculated using Eq. (4.57):

$$\bar{v} = \frac{24.5\ q}{d_h^2 - d_{dco}^2} = \frac{24.5 \times 750}{12.0^2 - 9.0^2} = 292\ ft/min$$

Critical velocity for the case of annulus velocity between drill collars and hole can be calculated using Eq. (4.58):

$$\begin{aligned} V_{cB-i} &= \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m (d_h - d_{dco})^2 \tau_{y-i}}}{\rho_m (d_h - d_{dci})} \\ &= \frac{97 \times 15 + 97\sqrt{15^2 + 8.2 \times 11.8 \times (12.0 - 9.0)^2 \times 14}}{11.8 \times (12.0 - 9.0)} \\ &= 307\ ft/min \end{aligned}$$

Because $\bar{v} < V_{cB-i}$, pressure loss in the annulus between drill collars and hole can be calculated using Eq. (4.59):

$$\begin{aligned} P_{dc\_ann} &= \frac{L_{dc} \mu_{p\_i} \bar{v}}{60{,}000\ D_e^2} + \frac{L_{dc} \tau_{y\_i}}{200\ D_e} \\ &= \frac{450 \times 15 \times 292}{60{,}000 (12.0 - 9.0)^2} + \frac{450 \times 14}{200 (12.0 - 9.0)} = 14\ psi \end{aligned}$$

Pressure loss in the annulus between hole and drill pipes:
The average velocity for the annulus between hole and drill pipes can be calculated using Eq. (4.57):

$$\bar{v} = \frac{24.5\ q}{d_h^2 - d_{dpo}^2} = \frac{24.5 \times 750}{12.0^2 - 5.5^2} = 162\ ft/min$$

Critical velocity for the annulus between hole and drill pipes can be calculated using Eq. (4.58):

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m (d_h - d_{dpo})^2 \tau_{y-i}}}{\rho_m (d_h - d_{dpi})}$$

$$= \frac{97 \times 15 + 97\sqrt{15^2 + 8.2 \times 11.8 \times (12.0 - 5.5)^2 \times 14}}{11.8 \times (12.0 - 5.5)} = 283\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss for the annulus between hole and drill pipes can be calculated using Eq. (4.59):

$$P_{dc\_ann} = \frac{L_{dc}\mu_{p\_i}\bar{v}}{60{,}000 D_e^2} + \frac{L_{dc}\tau_{y\_i}}{200 D_e}$$

$$= \frac{2{,}050 \times 15 \times 162}{60{,}000(12.0 - 5.5)^2} + \frac{2{,}050 \times 14}{200(12.0 - 5.5)} = 24\ psi$$

Pressure loss in the annulus between casing and drill pipes:
The average velocity for the annulus between casing and drill pipes can be calculated using Eq. (4.57):

$$\bar{v} = \frac{24.5\ q}{d_h^2 - d_{dpo}^2} = \frac{24.5 \times 750}{12.43^2 - 5.5^2} = 148\ ft/min$$

Critical velocity for the annulus between casing and drill pipes can be calculated using Eq. (4.58):

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m (d_h - d_{dpo})^2 \tau_{y-i}}}{\rho_m (d_h - d_{dpi})}$$

$$= \frac{97 \times 15 + 97\sqrt{15^2 + 8.2 \times 11.8 \times (12.43 - 5.5)^2 \times 14}}{11.8 \times (12.43 - 5.5)}$$

$$= 281\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss for the annulus between casing and drill pipes can be calculated using Eq. (4.59):

$$P_{dc\_ann} = \frac{L_{dc}\mu_{p\_i}\bar{v}}{60{,}000\ D_e^2} + \frac{L_{dc}\tau_{y\_i}}{200\ D_e}$$

$$= \frac{6{,}500 \times 15 \times 148}{60{,}000(12.43 - 5.5)^2} + \frac{6{,}500 \times 14}{200(12.43 - 5.5)} = 71\ psi$$

Thus, the pump pressure required to overcome all the above pressure losses is equal to:

$$P_{pump} = 1{,}560 + 381 + 991 + 14 + 24 + 71 = 3{,}041\ psi$$

Exercise 4.15: A production section of 8½" hole size is drilled down to the depth of 15,000 *ft*, and the last casing shoe was at 11,250 *ft* and inside diameter of 8.88". Drilling string consists of 725 *ft* drill collars of 6.5" OD and 2.5" ID; and the rest are drill pipes of 5.0" OD and 4.28" ID. Drilling fluid that is used has MW of 11.4 *ppg*, viscosity of 21.0 *cp* and yield point of 18.0 $lb_f/100ft^2$. Drilling hydraulics calculations showed that pumping rate and pumping pressure should be 550 *gpm* and 3,600 *psi*, respectively. If it is planned to have 3 nozzles in the bit with equal sizes, what will be the size of each nozzle?

Solution:

Given data:

D	=	Current depth	=	15,000 *ft*
d_h	=	Hole diameter	=	8.5"
D_{cas}	=	Last casing setting depth	=	11,250"
d_{cas}	=	Casing inside diameter	=	8.88"
L_{DC}	=	Length of drill collars	=	725 *ft*
OD_{DC}	=	Outside diameter of drill collars	=	6.5"
ID_{DC}	=	Inside diameter of drill collars	=	2.5"
OD_{DP}	=	Outside diameter of drill pipes	=	5.0"
ID_{DP}	=	Inside diameter of drill pipes	=	4.28"
MW	=	Mud weight	=	11.4 *ppg*
μ	=	Viscosity of mud	=	21.0 *cp*
τ_y	=	Mud yield point	=	18.0 $lb_f/100\ ft^2$
q	=	Pumping rate	=	550 *gpm*
P_p	=	Pumping pressure	=	3,600 *psi*

Required data:

d_{nozzle} = Nozzle diameter

To calculate the size of each nozzle, we should first calculate the pressure drop across the bit. And to calculate this pressure drop, we should calculate the pressure loss in the circulating system.

Pressure losses inside drill pipes:

Average velocity inside drill pipes should first be calculated using Eq. (4.53):

$$\bar{v} = \frac{24.5\ q}{d_{dpi}^2} = \frac{24.5 \times 550}{4.28^2} = 737\ ft/min$$

Critical velocity for the case of inside drill pipes can be calculated using Eq. (4.54):

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\ \rho_m d_{dpi}^2 \tau_{y-i}}}{\rho_m d_{dpi}}$$

$$= \frac{97 \times 21 + 97\sqrt{21^2 + 8.2 \times 11.4 \times 28 \times 18}}{11.4 \times 4.28} = 348\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss inside drill pipes can be calculated using Eq. (4.56):

$$P_{dc} = \frac{8.91 \times 10^{-5}\ \rho_m^{0.8} q^{1.8} \mu_{p_i}^{0.2} L_{dp}}{d_{dpi}^{4.8}}$$

$$= \frac{8.91 \times 10^{-5} \times 11.4^{0.8} \times 550^{1.8} 21^{0.2} \times (15{,}000 - 725)}{4.28^{4.8}} = 1{,}314\ psi$$

Pressure loss inside drill collars:
Average and critical velocities for drill collars can be calculated as we did for the drill pipes using Eqs. (4.53) and (4.54) as below:

$$\bar{v} = \frac{24.5\ q}{d_{dci}^2} = \frac{24.5 \times 550}{2.5^2} = 2{,}156\ ft/min$$

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m d_{dci}^2 \tau_{y-i}}}{\rho_m d_{dci}}$$

$$= \frac{97 \times 21 + 97\sqrt{21^2 + 8.2 \times 11.4 \times 2.5^2 \times 18}}{11.4 \times 2.5} = 383\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss inside drill pipes can be calculated using Eq. (4.56):

$$P_{dc} = \frac{8.91 \times 10^{-5}\ \rho_m^{0.8} q^{1.8} \mu_{p_i}^{0.2} L_{DC}}{d_{dci}^{4.8}}$$

$$= \frac{8.91 \times 10^{-5} \times 11.4^{0.8} \times 550^{1.8} 21^{0.2} \times 725}{2.5^{4.8}} = 876\ psi$$

Pressure loss in the annulus between hole and drill collars:
The average velocity for the annulus between hole and drill collars can be calculated using Eq. (4.57):

$$\bar{v} = \frac{24.5q}{d_h^2 - d_{dco}^2} = \frac{24.5 \times 550}{8.5^2 - 6.5^2} = 449\ ft/min$$

Critical velocity for the case of annulus velocity between drill collars and hole can be calculated using Eq. (4.58):

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m (d_h - d_{dco})^2 \tau_{y-i}}}{\rho_m (d_h - d_{dci})}$$

$$= \frac{97 \times 21 + 97\sqrt{21^2 + 8.2 \times 11.4 \times (8.5 - 6.5)^2 \times 18}}{11.4 \times (8.5 - 6.5)}$$

$$= 406\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss in the annulus between drill collars and hole can be calculated using Eq. (4.60):

$$P_{dp\_ann} = \frac{8.91 \times 10^{-5} \rho_m^{0.8} q^{1.8} \mu_{p_i}^{0.2} L_{dp\_ann}}{(d_h - d_{dco})^3 (d_h + d_{dco})^{1.8}}$$

$$= \frac{8.91 \times 10^{-5} \times 11.4^{0.8} \times 550^{1.8} \times 21^{0.2} \times 725}{(8.5 - 6.5)^3 (8.5 + 6.5)^{1.8}} = 68\ psi$$

Pressure loss in the annulus between hole and drill pipes:
The average velocity for the annulus between hole and drill pipes can be calculated using Eq. (4.57):

$$\bar{v} = \frac{24.5q}{d_h^2 - d_{dpo}^2} = \frac{24.5 * 550}{8.5^2 - 5.0^2} = 285\ ft/min$$

Critical velocity for the annulus between hole and drill pipes can be calculated using Eq. (4.58):

$$V_{cB-i} = \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m (d_h - d_{dpo})^2 \tau_{y-i}}}{\rho_m (d_h - d_{dpi})}$$

$$= \frac{97 \times 21 + 97\sqrt{21^2 + 8.2 \times 11.4 \times (8.5 - 5.0)^2 \times 18}}{11.4 \times (8.5 - 5.0)} = 359\ ft/min$$

Because $\bar{v} < V_{cB-i}$, pressure loss for the annulus between hole and drill pipes can be calculated using Eq. (4.59):

$$P_{dc\_ann} = \frac{L_{dc} \mu_{p\_i} \bar{v}}{60{,}000 D_e^2} + \frac{L_{dc} \tau_{y\_i}}{200 D_e}$$

$$= \frac{3{,}025 \times 21 \times 285}{60{,}000(8.5 - 5.0)^2} + \frac{3{,}025 \times 18}{200(8.5 - 5.0)} = 102\ psi$$

Pressure loss in the annulus between casing and drill pipes:
The average velocity for the annulus between casing and drill pipes can be calculated using Eq. (4.57):

$$\overline{v} = \frac{24.5q}{d_h^2 - d_{dpo}^2} = \frac{24.5 \times 550}{8.875^2 - 5.0^2} = 251\, ft/min$$

Critical velocity for the annulus between casing and drill pipes can be calculated using Eq. (4.58):

$$\begin{aligned} V_{cB-i} &= \frac{97\mu_p + 97\sqrt{\mu_p^2 + 8.2\rho_m (d_h - d_{dpo})^2 \tau_{y-i}}}{\rho_m (d_h - d_{dpi})} \\ &= \frac{97 \times 21 + 97\sqrt{21^2 + 8.2 \times 11.4 \times (8.875 - 5.0)^2 \times 18}}{11.4 * (8.875 - 5.0)} \\ &= 353\, ft/min \end{aligned}$$

Because $\overline{v} < V_{cB-i}$, pressure loss for the annulus between casing and drill pipes can be calculated using Eq. (4.59):

$$\begin{aligned} P_{dc\_ann} &= \frac{L_{dc}\mu_{p\_i}\overline{v}}{60{,}000 D_e^2} + \frac{L_{dc}\tau_{y\_i}}{200 D_e} \\ &= \frac{11{,}250 \times 21 \times 251}{60{,}000(8.88 - 5.0)^2} + \frac{11{,}250 \times 18}{200(8.88 - 5.0)} \\ &= 327\, psi \end{aligned}$$

Now, pressure drop across the bit is equal to the pumping pressure minus pressure losses inside and outside tubulars:

$$\Delta P_{bit} = 3{,}600 - 1{,}314 - 876 - 68 - 102 - 327 = 913\ psi$$

Nozzle area can be calculated using Eq. (4.63) and considering $C_d = 0.95$:

$$P_{bn} = \frac{\rho_m q^2}{10{,}858 A_n^2} = \frac{11.4 \times 550^2}{10{,}858 \times A_n^2} = 913 \text{ and } A_n = 0.589\ in^2$$

Nozzle size can be calculated as below:

$$A_n = n_{nozzles} \times \frac{\pi}{4} \times d_{nozzle}^2 = 3 \times \frac{\pi}{4} \times d_{nozzles}^2 = 0.589$$

$$d_{nozzle} = 0.5'' \ OR \ \frac{16}{32}''$$

Chapter 5: Well Control and Monitoring Program

Exercise 5.1: An oil zone in a certain field has been shifted up 750 *ft* in one part of that field. If the formation gradient in the deeper area is 0.5875 psi/ft and in the shallower area is 0.0.6279 *psi*/ft, determine the depth of the oil formation in the shallower and deeper areas assuming that the formation pressures are equal for both areas of the field. All depths and pressure gradients are measured at the center of the formation.

Solution:

Given data:

X = Shifting length of the formation = 750 *ft*

G_{f_deeper} = Formation gradient in the deeper area = 0.5875 *psi*/ft

G_{f_shall} = Formation gradient in the shallower area = 0.6279 *psi*/ft

Required data:

D_{m_deeper} = Depth of the formation in the deeper area in *ft*.

D_{m_shall} = Depth of the formation in the shallower area in *ft*.

Tectonic movement will only change the depth of the formation. So the formation pressure will remain the same. If we assume the depth of the formation in the deeper area is "*D*" *ft*, so the depth in the shallower area is "*D*-750" *ft*. Now the formation pressure in the deeper area is equal to:

$$P_{f\_deeper} = G_{f\_deeper} \times D_m = 0.5875D$$

And the formation pressure in the shallower area is equalto:

$$P_{f\_shall} = G_{f\_shall} \times D_m = 0.6279 \times (D - 750) = 0.6279D - 470.93$$

But we know that the formation pressure remains the same for both areas. Thus:

$$P_{f\_deeper} = P_{f\_shall}$$

$$0.5875D = 0.6279D - 470.93$$

$$0.0404D = 470.93$$

$$D = \frac{470.93}{0.0404} = 11{,}657\ ft$$

Hence, the depth of the formation in the deeper area is 11,657 *ft* and the depth in the shallower area is 10,907 *ft*.

Exercise 5.2: A vertical well drilled to a depth of 10,750 *ft*, and then 13 3/8" casing having inside diameter of 12.52" was set and cemented down to that depth. A new section of 12.25" size will be drilled to a depth of 14,900 *ft*. The fracture gradient at casing shoe was estimated to be 13.89 *ppg* while the formation gradient is 12.9 *ppg*. The drill pipe outside diameter is 5.0". If the expected gas kick volume at casing shoe is 150 *bbls* and gas gradient is 0.13 *psi*/ft, calculate the maximum mud weight that can be used and handle the kick without fracturing the formations at the casing shoe.

Solution:

Given data:

d_{o_cas} = Casing outside diameter = 13 3/8"
d_{i_cas} = Casing inside diameter = 12.52"
L_{cas} = Casing length = 10,750 *ft*
d_h = Hole diameter = 12.25"
D_m = Total length of the well = 14,900 *ft*
d_{dpo} = Drill pipe outside diameter = 5.0"
ρ_{fm} = Formation gradient in *ppg* = 12.9 *ppg*
ρ_{fr} = Fracture gradient at shoe in *ppg* = 13.89 *ppg*
G_i = Kick's pressure gradient = 0.13 *psi*/ft
V_k = Maximum kick volume = 150 *bbls*

Required data:

ρ_m = Maximum mud weight to be used in *ppg*

To calculate the maximum mud weight that can handle such a gas kick volume without breaking the casing shoe, first we need to calculate the length of the gas bubble at the casing shoe using the annulus capacity between casing and drill pipes as follows:

$$H_i = \frac{V_k}{C_{cas\_dp}} = \frac{150}{\frac{\pi}{4} \times \frac{12.52^2 - 5.0^2}{144 \times 5.615}} = 1{,}172\ ft$$

Now, by knowing the kick height and other information we can calculate the maximum mud weight that can be used from the following equation:

$$H_i = \frac{0.052 \times \rho_m (D_m - L_{cas}) + 0.052 \times L_{cas} \times \rho_{fr} - 0.052 \times D_m \times \rho_f}{0.052 \times \rho_m - G_i}$$

$$1{,}172 = \frac{\begin{array}{c}0.052 \times \rho_m \times (14{,}900 - 10{,}750) + 0.052 \times 10{,}750 \times 13.89 \\ -0.052 \times 14{,}900 \times 12.9\end{array}}{0.052 \times \rho_m - 0.13}$$

$$0.052 \times \rho_m - 0.13 = \frac{215.8\rho_m - 2230.41}{1{,}172}$$

$$0.1841\rho_m - 0.052\rho_m = 1.9028 - 0.13$$

$$\rho_m = 13.42\ ppg$$

The above mud weight is the maximum mud weight that can be used to handle a 70 *bbls* of gas kick without breaking the formation at the casing shoe.

Exercise 5.3: A vertical well been drilled, cased and cemented with 9 5/8" casing at depth of 13,000 *ft*. The well to be drilled to a target depth of 17,000 ft using mud has a maximum mud weight of 14.5 *ppg*. The fracture gradient at casing shoe is 15.7 *ppg* while the formation gradient is 14.1 *ppg*. The inside diameter of the last casing is 8.67" and the new hole diameter is 8.5". The standard surface temperature is 75 °F while the temperature gradient is 0.02 °F/ft. The drill pipe outside diameter is 5.0". If the expected gas kick gradient is 0.08 *psi*/ft, calculate the maximum volume of gas kick that can be handled in this case without fracturing the formations at the casing shoe.

Solution:

Given data:

d_{o_cas}	=	Casing outside diameter	=	9 5/8"
d_{i_cas}	=	Casing inside diameter	=	8.67"
L_{cas}	=	Casing length	=	13,000 *ft*
d_h	=	Hole diameter	=	8.5"
D_m	=	Total length of the well	=	17,000 *ft*
d_{dpo}	=	Drill pipe outside diameter	=	5.0"
T_s	=	Surface temperature	=	75 °F
d_T	=	Temperature gradient	=	0.02 °*F*/*ft*
ρ_m	=	Maximum mud weight to be used	=	14.5 *ppg*
ρ_{fm}	=	Formation gradient in *ppg*	=	14.1 *ppg*
ρ_{fr}	=	Fracture gradient at shoe in *ppg*	=	15.7 *ppg*
G_i	=	Kick's pressure gradient	=	0.08 *psi*/ft

Required data:

V_k = Maximum kick volume

To calculate the kick volume at these conditions, first we need to calculate the kick height at the casing shoe "since the maximum fracture risk will be at the casing shoe". The following equation will be used to calculate the kick height:

$$H_i = \frac{0.052\times\rho_m(D_m - L_{cas}) + 0.052\times L_{cas}\times\rho_{fr} - 0.052\times D_m\times\rho_f}{0.052\times\rho_m - G_i}$$

$$= \frac{\begin{array}{l}0.052\times 14.5\times(17{,}000-13{,}000)+0.052\times 13{,}000\times 15.7\\ \quad -0.052\times 17{,}000\times 14.1\end{array}}{0.052\times 14.5-0.08}$$

$$= 1{,}728.2\ ft$$

The above height can be changed to a volume using the annulus capacity between casing and drill pipes as below:

$$V_k = H_i\times C_{ann\_dp} = 1{,}728.2\times\frac{\pi}{4}\times\frac{8.67^2-5.0^2}{144\times 5.615} = 84.2\ bbls$$

The volume at the bottom of the well can be calculated using Boyle's law of gases and assuming ideal gas behavior. Pressure and temperature at the casing shoe are equal to:

$$P_{shoe} = 0.052 \times L_{cas} \times \rho_{fr} = 0.052 \times 13,000 \times 15.7 = 10,613\ psi$$

$$T_{shoe} = T_{surf} + dT \times L_{cas} + 460 = 75 + 0.02 \times 13,000 + 460 = 795°R$$

For calculating shoe pressure, we used the fracture gradient because we are looking for maximum volume of the kick; and that volume will occur at the maximum allowed pressure which is fracture pressure. Pressure and temperature at the bottom of the well are equal to:

$$P_{TD} = 0.052 \times D_m \times \rho_f = 0.052 \times 17,000 \times 14.1 = 12,464\ psi$$

$$T_{TD} = T_{surf} + dT \times D_m + 460 = 75 + 0.02 \times 17,000 + 460 = 875°R$$

Now, the volume at bottom can be calculated using Boyle's low as follows:

$$\frac{P_{shoe} V_{K\_shoe}}{T_{shoe}} = \frac{P_{TD} V_{K\_TD}}{T_{TD}}$$

$$\frac{10,613 \times 84.2}{795} = \frac{12,464 \times V_{K\_TD}}{875}$$

$$V_{K\_TD} = 78.9\ bbls$$

The above volume is the maximum allowed kick volume to enter the well without breaking the casing shoe at the above conditions. If the kick volume is greater than the above volume, there will be a greater risk of breaking the casing shoe and have loss circulation and at the same time sever entry of kick to the wellbore.

Exercise 5.4: A well kick was encountered while drilling a production section of 8.5" in a vertical well. Rig crew recorded a pit gain of 20 *bbls*. Shut-in casing pressure was stabilized at 900 *psi* while shut-in drill pipe pressure was zero because floating valve was used as part of BHA. Current depth was 14,500 *ft* and mud weight was 11.9 *ppg*. Drill collars outside diameter and length were 6.0" and 450 *ft*, while drill pipe outside diameter was 4 ¾". Rig crew increased the drill pipe pressure in steps in order to determine the shut-in drill pipe pressure. When drill pipe pressure increased to 800 *psi*, annulus pressure increased to 975 *psi*. If it is required to have mud hydrostatic pressure greater than the formation pressure by 150 *psi*, determine the nature of the influx and the new mud weight to control the well back.

Solution:

Given data:

d_h	=	Hole diameter	=	8.5"
V_{pit}	=	Pit gain	=	20 *bbls*
P_{siann}	=	Shut in casing pressure	=	900 *psi*
P_{sidp}	=	Shut in drill pipe pressure	=	0 *psi*

H_{vc} = Vertical height of the mud column = 14,500 *ft*
d_{dp} = Drill pipe outside diameter = 4.75"
d_c = Drill collar outside diameter = 6"
L_{dc} = Length of the drill collar = 450 *ft*
ρ_m = Mud weight = 11.9 *ppg*
P_{ob} = Overbalance pressure required = 150 *psi*

Required data:
Nature of the flux
ρ_K = Mud weight of the kill fluid in *ppg*.

To determine the nature of the flux and the required kill fluid density, first we need to estimate the shut-in drill pipe pressure from the given data. Because the float valve was used, SIDPP was reading zero. So the only way to know SIDPP is to apply pressure to the drill pipe in steps until you see increase in the SICP. Then you will determine the SIDPP. From the data above, when the drill pipe pressure increased to 800 *psi*, the SICP increased to 975 *psi*. So from this information, SIDPP will be equal to:

$$P_{dp} = 800 - (975 - 900) = 725\ psi$$

The volume of the annulus between the hole and drill collars is equal to:

$$V_{ann} = C_{ann\_dc} \times L_{dc} = \frac{d_h^2 - d_{dc}^2}{1029.4} \times L_{dc} = \frac{8.5^2 - 6.0^2}{1029.4} \times 450 = 15.85\ bbls$$

So, the rest of the flux volume will be around the drill pipes. The length of the flux around the drill pipes is equal to:

$$H_{i\_dp} = \frac{V_{pit} - V_{k_{dc}}}{C_{ann_{dp}}} = \frac{20 - 16.85}{\frac{8.5^2 - 4.75^2}{1029.4}} = 86.05\ ft$$

So, all of the flux length in the bottom of the hole is equal to:

$$H_i = L_{dc} + H_{i\_dp} = 450 + 86.05 = 536.05\ ft$$

The mud gradient is equal to:

$$G_m = 0.052 \times \rho_m = 0.052 \times 11.9 = 0.619\ psi/ft$$

Now, the flux gradient can be calculated using Eq. (5.7):

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = 0.619 - \frac{900 - 725}{536.05} = 0.292\ psi/ft$$

Since the flux gradient is greater than 0.25 and less than 0.45 *psi*/ft, the flux is oil and gas mixture.

The new mud gradient can be calculated using the Eq. (5.10):

$$G_k = G_m + \frac{P_{dp} + P_{ob}}{H_{vc}} = 0.619 + \frac{725 + 150}{14{,}500}$$
$$= 0.679\ psi/ft$$

The kill mud weight is now equal to:

$$\rho_k = \frac{G_k}{0.052} = \frac{0.679}{0.052} = 13.06\ ppg$$

Exercise 5.5: A well kick of 35 *bbls* was noticed when drilling 12.25" vertical hole using 10.54 *ppg* mud at depth of 6,250 *ft*. Later, the kick was removed and they found that the kick gradient was 0.16 *psi*/ft. Drilling string consists of 650 *ft* of 9.0" drill collars and 5.5" drill pipes. If the formation gradient was found to be 0.592 *psi*/ft, what were the shut-in annulus and drill pipe pressures? If it was required that the kill fluid hydrostatic pressure must be 100 *psi* higher than the formation pressure, calculate the kill fluid gradient in *psi*/ft.

<u>Solution</u>:

Given data:

d_h	=	Hole diameter	=	12.25"
V_{pit}	=	Pit gain	=	25 *bbls*
H_{vc}	=	Vertical height of the mud column	=	6,250 *ft*
d_{dp}	=	Drill pipe outside diameter	=	5.5"
d_c	=	Drill collar outside diameter	=	9.0"
L_{dc}	=	Length of the drill collar	=	650 *ft*
ρ_m	=	Mud weight	=	10.54 *ppg*
P_{ob}	=	Overbalance pressure required	=	100 *psi*
G_i	=	Kick's pressure gradient	=	0.07 *psi*/ft
G_f	=	Formation gradient	=	0.592 *psi*/ft

Required data:

P_{siann}	=	Shut in casing pressure in *psi*
P_{sidp}	=	Shut in drill pipe pressure in *psi*
ρ_k	=	Mud weight of the kill fluid in *ppg*.

Shut-in drill pipe pressure can be determined using Eq. (5.1) after arrangement:

$$P_{sidp} = P_{bh} - G_m H_{vc} = 0.592 \times 6{,}250 - 0.052 \times 10.54 \times 6{,}250 = 275\ psi$$

Now, to calculate shut-in casing pressure we need first to estimate the kick length in the bottom of the hole. Using drill collar capacity, the length of the kick around the drill collars equal to:

$$H_i = \frac{V_{pit}}{C_{ann_{dc}}} = \frac{35}{\dfrac{12.25^2 - 9.0^2}{1029.4}} = 522\ ft$$

Shut-in casing in pressure will be calculated using Eq. (5.7) and by using the available data:

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = 0.052 \times 10.54 - \frac{P_{siann} - 275}{522} = 0.07$$

$$P_{siann} = 525\ psi$$

Kill fluid gradient having 100 *psi* over the formation pressure can be calculated using Eq. (5.10):

$$G_k = G_m + \frac{P_{dp} + P_{ob}}{H_{vc}} = 0.052 \times 10.54 + \frac{275 + 100}{6{,}250} = 0.6081\ psi/ft$$

Exercise 5.6: A pit gain of 15 *bbls* was noticed while drilling a 6.0" hole at depth of 17,500 *ft*, and shut in annulus pressure was stabilized at 950 *psi*. The kick gradient was later estimated to be 0.20 *psi*/ft, and based on that kill fluid of 13.33 *ppg* was prepared that would give a hydrostatic pressure greater than the formation pressure by 300 *psi*. Drilling string consisted of 600 *ft* of 4¾" drill collars and the rest was 3.5" drill pipes. Calculate the old drilling mud used to drill the hole before the kick was occurred.

Solution:

Given data:

d_h	=	Hole diameter	=	6.0"
V_{pit}	=	Pit gain	=	15 *bbls*
H_{vc}	=	Vertical height of the mud column	=	17,500 *ft*
d_{dp}	=	Drill pipe outside diameter	=	3.5"
d_c	=	Drill collar outside diameter	=	4.75"
P_{siann}	=	Shut in casing pressure	=	950 *psi*
L_{dc}	=	Length of the drill collar	=	600 *ft*
P_{ob}	=	Overbalance pressure required	=	300 *psi*
G_i	=	Kick's pressure gradient	=	0.20 *psi*/ft
ρ_k	=	Kill fluid density	=	13.33 *ppg*

Required data:

ρ_m = Old mud weight in *ppg*

To calculate the old mud weight used previously, we should know the formation pressure and shut in drill pipe pressure. By knowing the kill fluid density and overbalance pressure, formation pressure can be estimated using Eq. (5.9):

$$P_{bh} = P_k - P_{ob} = 0.052 \times 13.33 \times 17{,}500 - 300 = 11{,}830\ psi$$

To calculate the shut in drill pipe pressure, first we should estimate the length of the kick in the bottom of the hole. Volume of the kick around the drill collars is equal to:

$$V_{k\_dc} = C_{ann\_dc} \times L_{dc} = \frac{d_h^2 - d_{dc}^2}{1029.4} \times L_{dc} = \frac{6.0^2 - 4.75^2}{1029.4} \times 600 = 7.83\ bbls$$

The remaining volume of the kick will be around the drill pipes, so the length of the kick around the drill pipes is equal to:

$$H_{i\_dp} = \frac{V_{pit - V_{k\_dc}}}{C_{ann_{dc}}} = \frac{15 - 7.83}{\frac{6.0^2 - 3.5^2}{1029.4}} = 310.7\ ft$$

Thus, the total length of the kick is equal to:

$$H_i = L_{dc} + H_{i\_dp} = 600 + 310.7 = 910.7\ ft$$

Now by using Eq. (5.7) we can estimate the shut-in drill pipe pressure as follows:

$$G_i = G_m - \frac{P_{siann} - P_{sidp}}{H_i} = \frac{11{,}830 - P_{sidp}}{17{,}500} - \frac{950 - P_{sidp}}{910.7} = 0.20$$

$$P_{siann} = 545\ psi$$

The hydrostatic pressure of the previous mud weight used before is equal to:

$$P_m = P_f - P_{sidp} = 11{,}830 - 545 = 11{,}285\ psi$$

Thus, the old mud weight is equal to:

$$\rho_m = \frac{P_m}{0.052 \times H_{vc}} = \frac{11{,}285}{0.052 \times 17{,}500} = 12.4\ ppg$$

Exercise 5.7: While drilling 12.25" intermediate hole at a depth of 10,000 *ft* using 9.9 *ppg* drilling mud, a kick was encountered in a well. Annulus and drill pipe pressures were stabilized at 625 and 395 *psi*. The kick gradient was estimated to be 0.122 *psi*/ft. Drilling string was consisted of 600 *ft* of 8.0" drill collars and the rest was 5.0" drill pipes. What was the kick volume?

Solution:

Given data:

d_h = Hole diameter = 12.25"
H_{vc} = Vertical height of the mud column = 10,000 *ft*
d_{dp} = Drill pipe outside diameter = 5.0"
d_c = Drill collar outside diameter = 8.0"
P_{siann} = Shut in casing pressure = 625 *psi*
P_{sidp} = Shut in drill pipe pressure = 395 *psi*
L_{dc} = Length of the drill collar = 600 *ft*
G_i = Kick's pressure gradient = 0.122 *psi*/ft
ρ_m = Mud weight = 9.9 *ppg*

Required data:

V_k = Kick volume in *bbls*

To find the kick volume, we should estimate the kick length at the bottom of the hole. From the given data, the length of the kick can be determined using Eq. (5.7):

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = 0.052 \times 9.9 - \frac{625 - 395}{H_i} = 0.122$$

$$H_i = 586\ ft$$

Because the kick length is less than the length of the drill collars, All of the kick volume was around the drill collars. The volume around the drill collars is equal to:

$$V_{k\_dc} = H_i \times C_{ann\_dc} = 586 \times \frac{12.25^2 - 8.0^2}{1029.4} = 49\ bbls$$

So, the total kick volume was 49 *bbls*

Exercise 5.8: A vertical production hole of 6.0" size was drilled using drilling string consisted of 800 *ft* of 4.75" drill collars and 3.5" drill pipes by using 13.8 *ppg* drilling fluids. A kick was encountered at a depth of 18,000 *ft* when the rig crew observed a pit gain of 18 *bbls*. Annulus and drill pipe pressures were stabilized at 1,200 and 500 *psi*. The kick gradient was estimated to be 0.097 *psi*/ft. Determine the formation gradient in *ppg* and the old mud weight.

Solution:

Given data:

d_h = Hole diameter = 6.0"
H_{vc} = Vertical height of the mud column = 18,000 *ft*
d_{dp} = Drill pipe outside diameter = 3.5"
d_c = Drill collar outside diameter = 4.75"
P_{siann} = Shut-in casing pressure = 1,200 *psi*
P_{sidp} = Shut-in drill pipe pressure = 500 *psi*
L_{dc} = Length of the drill collar = 800 *ft*
G_i = Kick's pressure gradient = 0.097 *psi*/ft

Required data:

ρ_f = Formation gradient in *ppg*.
ρ_m = Old mud weight in *ppg*.

To calculate the formation gradient, first we need to estimate the kick length. The volume of the kick around the drill collars is equal to:

$$V_{k\_dc} = L_{dc} \times C_{ann\_dc} = 800 \times \frac{6.0^2 - 4.75^2}{1029.4} = 10.4\ bbls$$

The remaining volume of the kick was around the drill pipes. The length of the kick around the drill pipe is equal to:

$$H_{i\_dp} = \frac{V_{pit} - V_{k_{dc}}}{C_{ann_{dp}}} = \frac{18 - 10.4}{\frac{6.0^2 - 3.5^2}{1029.4}} = 329.5\ ft$$

So, the total length of the kick is equal to:

$$H_i = L_{dc} + H_{i\_dp} = 800 + 329.5 = 1{,}129.5\ ft$$

We know that the formation pressure can be calculated using Eq. (5.1):

$$P_{bh} = P_{sidp} + G_m H_{vc}$$

So, the mud gradient is equal to:

$$G_m = \frac{P_{bh} - P_{sidp}}{H_{vc}}$$

Thus, formation pressure can be determined using Eq. (5.7) and substituting for the mud gradient as follows:

$$G_i = G_m - \frac{P_{ann} - P_{dp}}{H_i} = \frac{P_{bh} - P_{sidp}}{H_{vc}} - \frac{P_{ann} - P_{dp}}{H_i}$$

$$0.097 = \frac{P_{bh} - 500}{18{,}000} - \frac{1{,}200 - 500}{1{,}129.5}$$

$$P_{bh} = 13{,}401\ psi$$

Now, formation gradient in *ppg* is equal to:

$$\rho_f = \frac{P_f}{0.052 \times H_{vc}} = \frac{13{,}401}{0.052 \times 18{,}000} = 14.32\ ppg$$

Mud hydrostatic pressure can now be calculated using Eq. (5.1):

$$P_m = P_{bh} - P_{sidp} = 13{,}401 - 500 = 12{,}901\ psi$$

Thus, the old mud weight used before the kick occurred was equal to:

$$\rho_m = \frac{P_m}{0.052 \times H_{vc}} = \frac{12{,}901}{0.052 \times 18{,}000} = 13.78\ ppg$$

Exercise 5.9: A production hole of 6.0" size was drilled to a depth of 17,250 *ft* using 13.3 *ppg* mud. The last casing shoe was at 14,000 *ft*, and the fracture gradient at the casing shoe was estimated to be 15.0 *ppg*. Drill pipes diameter was 3.5". If the formation gradient at the current depth was assumed to be 13.7 *ppg*, estimate the maximum kick volume that can be handled without fracturing the casing shoe taking into consideration that the maximum allowable surface pressure in the annulus is 1,238 *psi* and the gradient of the kick is 0.14 *psi*/ft.

Solution:

Given data:

L_{cas}	=	Casing length	=	14,000 *ft*
d_h	=	Hole diameter	=	6.0"
D_m	=	The current length of the well	=	17,250 *ft*
d_{dpo}	=	Drill pipe outside diameter	=	3.5"
ρ_m	=	Mud weight to be used	=	13.3 *ppg*
ρ_{fm}	=	Formation gradient in *ppg*	=	13.7 *ppg*
ρ_{fr}	=	Fracture gradient at shoe in *ppg*	=	15.0 *ppg*
G_i	=	Kick's pressure gradient	=	0.14 *psi*/ft
P_{ann}	=	Maximum allowable annulus pressure	=	1,238 *psi*

Required data:

V_k = Maximum kick volume can be handled in *bbls*

The worst-case scenario for fracturing the casing shoe occurred when the kick is just beneath the casing shoe. In this case, the surface pressure plus the mud hydrostatic pressure will be applied directly against the casing shoe. So it is very important that these two pressures must be less or at least equal to the fracture pressure of the casing shoe. So the maximum allowed kick length in this situation can be calculated using the following equation:

$$H_i = \frac{P_{ann} + 0.052 \times \rho_m \times L_{cas} + 0.052 \times \rho_m \times (D_m - L_{cas}) - 0.052 \times \rho_{fm} \times D_m}{0.052 \times \rho_m - G_i}$$

$$= \frac{1{,}238 + 0.052 \times 13.3 \times 14{,}000 + 0.052 \times 13.3 \times (17{,}250 - 14{,}000) - 0.052 \times 13.7 \times 17{,}250}{0.052 \times 13.3 - 0.14}$$

$$= 1{,}594\ ft$$

Now, by using the hole-drill pipe annulus capacity, the maximum allowable kick volume is equal to:

$$V_k = H_i \times C_{ann\_dp} = 1{,}594 \times \frac{6.0^2 - 3.5^2}{1029.4} = 36.8\ bbls$$

So, if the volume of the gas kick is greater than the above volume, the tendency of breaking the casing shoe is higher.

Exercise 5.10: A production hole of 8.5" size was drilled to a depth of 12,500 *ft* using 11.6 *ppg* mud. The fracture gradient of the casing shoe that at 10,500 *ft* was estimated to be 13.0 *ppg*. Drill pipes diameter was 5.0". If the maximum anticipated kick volume and gradient are 70.0 *bbls* and 0.19 *psi*/ft, determine the maximum allowable surface pressure that can be applied to the annulus without fracturing the casing shoe. Assume formation gradient is equal to 11.8 *ppg*.

Solution:

Given data:

L_{cas}	=	Casing length	=	10,500 *ft*
d_h	=	Hole diameter	=	8.5"
D_m	=	The current length of the well	=	12,500 *ft*
d_{dpo}	=	Drill pipe outside diameter	=	5.0"
ρ_m	=	Mud weight	=	11.6 *ppg*
ρ_{fm}	=	Formation gradient in *ppg*	=	11.8 *ppg*
ρ_{fr}	=	Fracture gradient at shoe in *ppg*	=	13.0 *ppg*
G_i	=	Kick's pressure gradient	=	0.19 *psi*/ft
V_k	=	Maximum kick volume	=	70.0 *bbls*

Required data:

P_{ann} = Maximum allowable annulus pressure at surface in *psi*

To calculate the maximum surface pressure to be applied in the annulus at this conditions, first the kick volume should be changed to length in the annulus between the drill pipe and the hole:

$$H_i = \frac{V_k}{C_{ann_{dp}}} = \frac{70}{\frac{8.5^2 - 5.0^2}{1029.4}} = 1{,}525\ ft$$

By knowing the kick length, we can use the following equation to calculate the maximum surface pressure:

$$H_i = \frac{P_{ann} + 0.052 \times \rho_m \times L_{cas} + 0.052 \times \rho_m \times (D_m - L_{cas}) - 0.052 \times \rho_{fm} \times D_m}{0.052 \times \rho_m - G_i}$$

$$= \frac{P_{ann} + 0.052 \times 11.6 \times 10{,}500 + 0.052 \times 11.6 \times (12{,}500 - 10{,}500) - 0.052 \times 11.8 \times 12{,}500}{0.052 \times 11.6 - 0.19}$$

$$= 1{,}525$$

$$P_{ann} = 760\ psi$$

Chapter 6: Formation Pore and Fractures Pressure Estimation

Exercise 6.1: A water aquifer is connected hydraulically to a surface lake. The aquifer is deviated at a certain angle. Two wells were drilled to the center of the aquifer (Figure 6.1). The pressure difference between the two wells and the ratio between the pressure of the two wells are 220 *psi* and 0.8; respectively. If the vertical distance between the aquifer in the two wells is 500 *ft*, calculate the aquifer depth in both wells.

<u>**Solution**</u>:

Given data:

P_d = Pressure difference = 220 *psi*

$\frac{P_1}{P_2}$ = Pressure ratio = 0.91 *psi*

D_{diff} = Difference in depth between the two wells = 500 *ft*

Required data:

D_{m1} = Aquifer depth in well#1

D_{m2} = Aquifer depth in well#2

Because the aquifer is connected to a water lake, the pressure gradient of the aquifer will be equal to the gradient water having the same density. The aquifer's pressure gradient can be calculated by knowing the pressure differences and vertical differences between the two wells using Eq. (6.3):

$$G_m = 0.052 \times \rho_m = \frac{P_{diff}}{D_{diff}} = \frac{220}{500} = 0.44 \ psi/ft$$

If we assume well #1 is shallower than well #2, the pressure in the second well will be equal to the pressure in the first well plus the pressure difference between the two wells. Mathematically:

$$\frac{P_1}{P_1 + P_{diff}} = \frac{P_1}{P_2} = \frac{P_1}{P_1 + 220} = 0.8$$

$$P_1 = \frac{220 \times 0.8}{1.0 - 0.8} = 880 \ psi$$

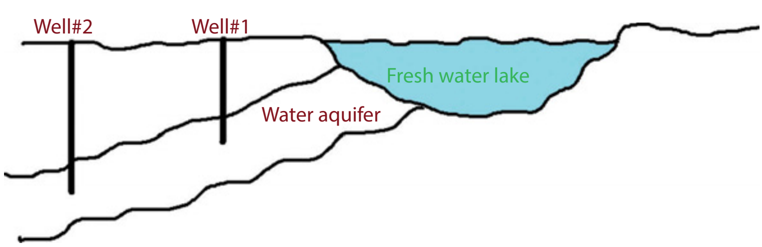


Figure 6.1 An aquifer for Example 6.1

Thus, aquifer pressure in the second well is equal to:

$$P_2 = P_1 + P_{diff} = 880 + 220 = 1{,}100\ psi$$

The depth of the first well can be calculated using Eq. (6.3):

$$P_1 = G_m \times D_{m1} = 880 = 0.44 \times D_{m1}$$
$$D_{m1} = 2{,}000\ ft$$

The depth of the second well can be calculated using Eq. (6.3):

$$P_2 = G_m \times D_{m2} = 1{,}100 = 0.44 \times D_{m1}$$
$$D_{m2} = 2{,}500\ ft$$

Exercise 6.2: An aquifer is connected hydraulically to a surface lake of water. The aquifer is deviated at a certain angle. Two wells were drilled to the center of the aquifer (Figure 6.1). The depth of the deepest well to the center of the aquifer is 3,750 *ft*. The ratio of the difference in pressure between the two wells to the pressure of the deepest well is 0.20, whereas the pressure of the shallowest well is 1,342 *psi*. Estimate the density of the water in the lake and the depth of the shallowest well to the center of the aquifer.

Solution:;

Given data:

D_{m_Deep}	=	Depth of the deepest well	=	3,750 *ft*
$\frac{P_{diff}}{P_{deep}}$	=	Pressure ratio for the deepest well	=	0.20
P_{Shall}	=	Pressure of the shallowest well	=	1,342 *psi*

Required data:

ρ_m	=	Density of the water of the lake in *ppg*.
D_{m_shall}	=	Depth of the shallowest well in *ft*.

Because the aquifer is connected hydraulically to the lake, they will share similar pressure gradient. If we assume the difference in vertical depth between the two wells is equal to D_{diff}, pressure in both wells can be calculated using Eq. (6.3):

$$P_{shall} = 0.052 \times \rho_m \times (D_{m_{shall}} + D_{diff}) \tag{1}$$

$$\begin{aligned} P_{deep} &= 0.052 \times \rho_m \times D_{m\_deep} = 0.052 \times 3{,}750 \times \rho_m \\ &= 195 \times \rho_m \end{aligned} \tag{2}$$

Now using the given ratio between the difference in the pressure between the two wells to the pressure in the deepest well, we can calculate the water density and the depth of the second well as follows:

$$\frac{Pdiff}{P_{deep}} = \frac{0.052 \times D_{diff} \times \rho_m}{195 \times \rho_m} = 0.20$$

$$D_{diff} = \frac{0.20 \times 195}{0.052} = 750\ ft$$

The depth of the shallowest well is equal to:

$$D_{m2} = D_{m1} - D_{diff} = 3{,}750 - 750 = 3{,}000\ ft$$

The water density can now be calculated by knowing the pressure of the shallowest well using Eq. (6.3):

$$P_{shall} = 0.052 \times \rho_m \times D_{m\_shall} = 0.052 \times 3{,}000 \times \rho_m = 1{,}342$$

$$\rho_m = 8.60\ ppg$$

Form the above calculated density, this lake is not a fresh water lake but it has certain salinity.

Exercise 6.3: A 1,000 *ft* of gas formation above 750 *ft* of water formation. The top pressure of the gas formation is 4,022.5 *psi* and the bottom pressure of the water formation is 4,500 *psi*. If the pressure gradient of the water formation is 0.49 *psi*/ft, calculate the gas formation gradient and the depth of the oil water contact depth.

Solution:

Given data:

P_{g_top}	=	The pressure of the top gas formation	=	4,022.5 *psi*
P_{w_bott}	=	The pressure of the bottom water formation	=	4,500 *psi*
h_w	=	Thickness of water formation	=	750 *psi*
h_g	=	Thickness of gas formation	=	1,000 *psi*
G_w	=	Pressure gradient of the water formation	=	0.49 *psi*/ft

Required data:

D_{gwc}	=	Depth of the oil water contact in *ft*.
G_g	=	Pressure gradient of the gas formation in *psi*/ft

The depth of the bottom of the water formation can be calculated using the pressure gradient and the formation pressure as follows:

$$D_{w\_bott} = \frac{P_{w_{bott}}}{G_w} = \frac{4{,}500}{0.49} = 9{,}184\ ft$$

So, the top depth of the water formation or the gas-water contact depth is equal to:

$$D_{gwc} = D_{w\_bott} - h_w = 9{,}184 - 750 = 8{,}434\ ft$$

Pressure at the bottom of the gas formation will be equal to the pressure at the top of the water formation. So:

$$\begin{aligned} P_{g\_bott} &= P_{w\_top} = G_w \times D_{w\_top} = 0.49 \times 8{,}434 \\ &= 4{,}133\ psi \end{aligned}$$

Now, by knowing the pressure at the top and bottom of the gas formation we can calculate the gas formation gradient using the gas formation thickness as follows:

$$G_g = \frac{P_{g_{bott}} - P_{g_{top}}}{h_g} = \frac{4{,}133 - 4{,}022.5}{1{,}000} = 0.111\ psi/ft$$

Exercise 6.4: A gas field with a water formation beneath it. The pressure difference between the top and the bottom of the gas formation is equal to that for the water formation. The gas and water formations pressure gradient are 0.19 and 0.46 *psi*/ft; respectively. If the pressure at the bottom of the water formation is 4,600 *psi*, calculate the ratio between the thickness of gas and water formations. And if the pressure at the top of the gas formation is 4,000 *psi*, calculate the gas and water formation thicknesses and the depth of the gas-water contact.

Solution:

Given data:

P_{g_top}	=	Pressure at top of gas formation	=	4,000 *psi*
P_{w_bott}	=	Pressure of at the bottom of water formation	=	4,600 *psi*
G_w	=	Pressure gradient of the water formation	=	0.46 *psi*/ft
G_g	=	Pressure gradient of the gas formation	=	0.19 *psi*/ft

Required data:

$\frac{h_g}{h_w}$	=	Formation thickness ratio between gas and water formations.
D_{gwc}	=	Depth of the gas-water contact in *ft*
h_g	=	Thickness of the gas formation in *ft*
h_w	=	Thickness of the water formation in *ft*

The ration between the thicknesses of gas and water formations can be calculated by knowing that the pressure difference in the gas formation is equal to the pressure differences in water formation. In other words, the gradient of the gas formation multiplied by the thickness is equal to the gradient of the water formation multiplied by its thickness. Mathematically:

$$h_g \times G_g = h_w \times G_w = 0.19 \times h_g = 0.46 \times h_w$$

$$\frac{g_g}{h_w} = \frac{0.46}{0.19} = 2.42$$

When the pressure at the top of the gas formation equal to 4,000 *psi*, so the pressure at the gas water contact will be equal to:

$$P_{gwc} = P_{g\_top} + \frac{P_{w_{bott}} - P_{g_{top}}}{2} = 4{,}000 + \frac{4{,}600 - 4{,}000}{2} = 4{,}300\ psi$$

The thickness of the gas formation is equal to:

$$h_g = \frac{P_{gwc} - P_{g_{top}}}{G_g} = \frac{4{,}300 - 4{,}000}{0.19} = 1{,}579\ ft$$

The thickness of the water formation will be equal to:

$$h_w = \frac{h_g}{2.42} = \frac{1{,}579}{2.42} = 652\ ft$$

The depth of the gas-water contact can be determined using Eq. (6.3) as follows:

$$D_{gwc} = \frac{P_{gwc}}{G_w} = \frac{4{,}300}{0.46} = 9{,}348\ ft$$

Exercise 6.5: A core sample of 2.54 *cm* in size and 7.62 *cm* in length from a sandstone formation. The core sample was cleaned and dried, and dry weight was measured to be 79.0 grams. The core sample was completely saturated using 9.8 grams of formation brine water that has a density of 1.09 *gm/cc*. Determine the grain density and bulk density of this core sample.

Solution:

Given data:

d_{core}	=	Size of the core sample	=	2.54 *cm*
l_{core}	=	Length of the core sample	=	7.62 *cm*
Wt_{core_dry}	=	Dry weight of the core sample	=	79 gm
Wt_f	=	Weight of that water saturated the core	=	9.8 gm
ρ_f	=	Density of the formation brine water	=	1.09 *gm/cc*

Required data:

ρ_b	=	bulk density of the core sample.
ρ_r	=	grain density of the core sample.

To calculate the porosity of the core sample, first we need to calculate the bulk volume and the pore volume. Because 9.8 grams were used to completely saturate the pores of the core sample, the pore volume is equal to:

$$V_{pore} = \frac{Wt_f}{\rho_f} = \frac{9.8}{1.09} = 8.99\ cc$$

The bulk volume of the core sample is equal to the volume of the cylindrical shape:

$$V_b = \frac{\pi}{4} d_{core} \times l_{core} = \frac{\pi}{4} \times 2.54 \times 7.62 = 38.61\ cc$$

Now, porosity is equal to:

$$\phi = \frac{V_{pore}}{V_b} = \frac{8.99}{38.61} = 0.233$$

To calculate the grain density, first we need to calculate the grain volume using the following relation:

$$V_r = V_b \times (1-\phi) = 38.61 \times (1-0.233) = 29.61\ cc$$

Thus, the grain density is equal to:

$$\rho_r = \frac{Wt._b}{V_r} = \frac{79}{29.61} = 2.67\ gm/cc$$

Bulk density can now be calculated using Eq. (6.5):

$$\begin{aligned}\rho_b &= \rho_f \times \phi + \rho_r \times (1-\phi) = 1.09 \times 0.233 + 2.67 \times (1-0.233) \\ &= 2.30\ gm/cc\end{aligned}$$

Exercise 6.6: A well is planned to be drilled to the depth of 13,000 *ft*. The last casing shoe was at 7,500 *ft* and the fracture gradient below the casing shoe was 0.69 *psi*/ft. The designed mud weight to drill the new section is 11.8 *ppg*. What is the maximum pressure that can be applied at the surface? And if the expected pressure at the depth of 13,000 *ft* is 8,450 *psi*, what will be the maximum pressure at the surface when changing the mud to a new one that has mud hydrostatic pressure 300 *psi* greater than the formation pressure?

Solution:

Given data:

D_m	=	Bottom depth of the well	=	13,000 *ft*
D_{shoe}	=	Depth of the casing shoe	=	7,500 *ft*
G_{fp}	=	Fracture gradient at the casing shoe	=	0.69 *psi*/ft
ρ_m	=	Mud weight	=	11.8 *ppg*
P	=	Expected pressure at 13,000 *ft*	=	8.450 *psi*
P_{ob}	=	Overbalance pressure	=	300 *psi*

Required data:
P_{max} = Maximum pressure at the surface

The maximum pressure to be applied at the surface is the difference between the maximum pressure at the weakest point "casing shoe" and the hydrostatic pressure of the mud at that point. The maximum pressure at the casing shoe can be calculated using Eq. (6.2) after modification:

$$P_f = G_{fg} \times D_{shoe} = 0.69 \times 7{,}500 = 5{,}175\ psi$$

The hydrostatic pressure of the mud at the casing shoe is can also be calculated using Eq. (6.2):

$$P_m = 0.052 \times \rho_m \times D_{shoe} = 0.052 \times 11.8 \times 7{,}500 = 4{,}602\ psi$$

Thus, the maximum pressure is equal to:

$$P_{max} = P_f - P_m = 5{,}175 - 4{,}602 = 573\ psi$$

To determine the new maximum pressure when we change the mud weight, first we should calculate the new mud weight. The required hydrostatic pressure of the mud is equal to:

$$P_m = P_f + P_{ob} = 8{,}450 + 300 = 8{,}750\ psi$$

Now the mud weight can be calculated using Eq. (6.2):

$$P_m = 0.052 \times \rho_m \times D_m = 0.052 \times \rho_m \times 13{,}000 = 8{,}750$$

$$\rho_m = 12.94\ ppg$$

Hydrostatic pressure of the new mud at the casing shoe is:

$$P_m = 0.052 \times \rho_m \times D_{shoe} = 0.052 \times 12.94 \times 7{,}500 = 5{,}048\ psi$$

The new maximum surface pressure with the new mud weight is equal to:

$$P_{max} = P_f - P_m = 5{,}175 - 4{,}048 = 127\ psi$$

Exercise 6.7: A leak-off test was conducted to a well using the old drilling fluid. When the mud leaked-off, the pump pressure was reading 1,250 *psi*. The well was circulated with a new mud having a density greater than the old one by 0.5 *ppg*. The leak-off test was repeated, and surface pressure was recorded to be 944.5 *psi* when the mud was leaked-off. If the fracture pressure was found exactly same, what is the depth of the casing shoe?

Solution:

Given data:

$\Delta\rho_m$	=	Difference in mud weights	=	0.5 *ppg*
P_1	=	Leaked-off surface pressure using the old mud	=	1,250 *psi*
P_2	=	Leaked-off surface pressure using the new mud	=	1,250 *psi*

Required data:
D_{shoe} = Depth of the casing shoe

Fracture pressure in both cases is the same, so we will create two equations for fracture pressure and equate them as follows:

$$P_f = P_{h\_mud1} + P_1 = 0.052 \times \rho_{m1} \times D_{shoe} + 1{,}250$$

$$P_f = P_{h\_mud2} + P_2 = 0.052 \times (\rho_{m1} + 0.5) \times D_{shoe} + 944.5$$

Equating Eq. (1) and (2) gives:

$$0.052 \times \rho_{m1} D_{shoe} + 1{,}250 = 0.052 \times \rho_{m1} D_{shoe} + 0.026 \times D_{shoe} + 944.5$$

$$D_{shoe} = \frac{1{,}250 - 944.5}{0.026} = 11{,}750\ ft$$

Exercise 6.8: An oil-producing field has an initial reservoir pore pressure of 4,650 *psi*. The decline of the pore pressure and overburden pressure with time was estimated to be following the equation:

P_{fn} = 4,150 t^{-2011} and P_{ob} = 13,104 t^{-0054},

where P_r and P_{ob} are the reservoir pore pressure and overburden pressure; and t is the time in years. What was the original overburden pressure?

Solution:
Given data:
P_{fn} = Initial pore pressure = 4,650 *psi*
Required data:
P_{ob} = Original overburden pressure in *psi*.

Original overburden pressure of the above reservoir can be calculated using Eq. (6.4):

$$P_{ob} = P_{fn} + \sigma_v$$

In the above equation, the missing data is the matrix stress of the reservoir rock. So, using the given equations we can calculate the pore pressure and overburden pressure at any time. Then we can calculate the matrix stress. Pore pressure after 5 years is equal to:

$$P_r = 4{,}150\ t^{-0.2011} = 3{,}271 \times 5^{-0.191} = 3{,}003\ psi$$

And overburden pressure after 5 years is equal to:

$$P_{ob} = 13{,}104\ t^{-0.054} = 13{,}104 \times 5^{-0.054} = 12{,}113$$

Because matrix stress can be assumed constant; and can now be calculated using Eq. (6.4):

$$\sigma_v = P_{ob} - P_{fn}\ 12{,}113 - 3{,}003 = 9{,}010\ psi$$

Thus, the original overburden pressure is equal to:

$$P_{ob} = P_{fn} + \sigma_v = 4{,}650 + 9{,}010 = 13{,}660\ psi$$

Exercise 6.9: Two sedimentary formations that have surface porosities of 0.42 and 0.36, and porosity decline constants of 0.00018 and 0.00014; respectively. At which depth will both rocks have similar porosity? What is that porosity?

Solution:

Given data:

ϕ_{o1} = Surface porosity for rock 1 = 0.42
$K_{\phi 1}$ = Porosity decline constant for rock 1 = 0.00018
ϕ_{o2} = Surface porosity for rock 2 = 0.36
$K_{\phi 2}$ = Porosity decline constant for rock 2 = 0.00014

Required data:

D = Rock depth
ϕ = Porosity

Porosity at any depth can be calculated using Eq. (6.7):

$$\phi = \phi_o e^{-K_{\varnothing} D_s}$$

Porosity equation for rock 1 equal to:

$$\phi = 0.42 \times e^{-0.00018 \times D_s}$$

Porosity equation for rock 2 equal to:

$$\phi = 0.36 \times e^{-0.00014 \times D_s}$$

Porosity of both rocks are similar at a certain depth, mean that both equations are equal. This leads to:

$$0.42 \times e^{-0.00018 \times D_s} = 0.36 \times e^{-0.00014 \times D_s}$$

$$e^{(0.00018 - 0.00014) D_s} = \frac{0.42}{0.36} = 1.1667$$

Taking the natural logarithm for both sides gives:

$$0.00004\ D_s = \ln(1.1667) = 0.1542$$

$$D_s = 3{,}854\ ft$$

The porosity at this depth for both rocks can be calculated using the inputs of each rock:

$$\phi = 0.42 \times e^{-0.00018 \times D_s} = 0.42 \times e^{-0.00018 \times 3{,}854} = 0.21$$

Exercise 6.10: While drilling the surface section in a well, a kick was encountered at depth of 1,000 *ft* and shut-in drill pipe pressure was measured to be 100 *psi*. The density of the mud that used was 8.6 *ppg*. The kick was safely removed, and later it was realized that the current formation and the sea water were in hydraulic communication. If the density of the sea water is 8.62 *ppg*, determine the difference in vertical distance between the surface location of the well and the sea level.

Solution:

Given data:

D	=	Depth of the kick formation	=	1,000 *ft*
P_{si_dp}	=	Shut in drill pipe pressure	=	100 *psi*
ρ_m	=	Mud weight	=	8.60 *ppg*
ρ_{sw}	=	Density of sea water	=	8.62 *ppg*

Required data:

H = Height of the well surface above sea level

Because the formation is connected hydraulically with the sea water, its pressure should be equal to the hydrostatic pressure of the sea water to that depth. The shut-in drill pipe pressure was the result of the formation pressure minus the mud hydrostatic pressure. The mud hydrostatic pressure can be calculated using Eq. (6.3):

$$P_m = 0.052 \times \rho_m \times D = 0.052 \times 8.6 \times 1{,}000 = 447.2\ psi$$

The formation pressure can be estimated using the sea water density and the vertical distance between the surface of the sea and the depth of the formation. Formation pressure can also be estimated using Eq. (6.3):

$$P_f = 0.052 \times \rho_{sw} \times H = 0.052 \times 8.62 \times (1{,}000 + H) = 0.448(1{,}000 + H)$$

Shut-in drill pipe pressure is relating the mud hydrostatic pressure and the formation pressure with the following equation:

$$P_{si_{dp}} = P_f - P_m = 100 = 0.448(1{,}000 + H) - 447.2$$

$$\frac{100 + 447.2}{0.448} = (1{,}000 + H)$$

$$H = 1{,}221 - 1{,}000 = 221\ ft$$

So the well surface location is 221 *ft* below the sea level, which is why a kick occurred.

Chapter 7: Basics of Drillstring Design

Exercise 7.1: A drill pipe was planned to be used to drill a well to a depth of 9,000 *ft* using 10.72 *ppg* drilling fluid. If the collapse safety factor was calculated to be 1.445 when drillstring is empty, what is the collapse resistance of the drill pipes? And if the minimum allowable safety factor is 1.2, determine the maximum depth that the current drillstring can be lowered empty.

Solution:

Given data:

SF = Collapse safety factor = 1.445
D_m = Well depth = 9,000 *ft*
MW = Mud weight = 10.72 *ppg*

Required data:

P_{c_res} = Collapse resistance
D_{max} = Maximum depth

If the inside drill pipes are empty, so no pressure is applied inside the drillstring. To calculate collapse resistance, we should calculate the collapse pressure at the depth of 9,000 *ft*. Collapse pressure applied at the bottom of the drillstring is equal to, Eq. (7.1a):

$$P_c = 0.052 \times \rho_m \times D = 0.052 \times 10.72 \times 9{,}000$$
$$= 5{,}017\ psi$$

Now collapse resistance of the drill pipes is equal to, Eq. (7.5):

$$SF = \frac{P_{c_{res}}}{P_c} = \frac{P_{c_{res}}}{5{,}017} = 1.445$$

$$P_{c_{res}} = 7{,}250\ psi$$

Maximum depth using safety factor of 1.2 can be determined by first calculating the maximum allowable collapse pressure using Eq. (7.5):

$$SF = \frac{P_{c_{res}}}{P_c} = \frac{7{,}250}{P_c} = 1.2$$

$$P_c = 6{,}042\ psi$$

Maximum depth can be calculated using Eq. (7.1a):

$$P_{out} = 0.052 \times \rho_m \times D = 6{,}042 = 0.052 \times 10.72 \times D$$

$$D = 10{,}838\ ft$$

Exercise 7.2: A well was planned to be drilled to a depth of 15,000 *ft* using 12.5 *ppg* drilling mud. The drillstring has a float valve at the bottom of the string. When new drilling mud of 11.3 *ppg* was pumped to a certain depth, collapse pressure at the bottom was calculated to be 1,000 *psi*. What was the bottom depth of the new mud?

Solution:

Given data:

D_m	=	Depth of the well	=	15,000 *ft*
MW_{old}	=	Mud weight of the old mud	=	12.5 *ppg*
MW_{new}	=	Mud weight of the new mud	=	11.29 *ppg*
P_c	=	Collapse pressure	=	750 *psi*

Required data:

D_{new} = Mud weight of the new mud

To determine the new mud weight, we have to calculate the hydrostatic pressure of the new mud. Float valve will prevent outside mud to enter inside the drill pipe, which is why collapse pressure was developed at the bottom of the drillstring. Outside pressure at the bottom can be calculated using Eq. (7.1a):

$$P_{out} = 0.052 \times \rho_m \times D = 0.052 \times 12.5 \times 15{,}000 = 9{,}750\ psi$$

Inside pressure can be calculated as follows:

$$P_{inside} = P_{out} - P_c = 9{,}750 - 750 = 9{,}000\ psi$$

If we assume the length of the old mud equal to "L", the length of the old mud will equal to:

$$P_{inside} = 0.052 \times \rho_{m\_old} \times L_{old} + 0.052 \times \rho_{m\_new} \times (15{,}000 - L)$$

$$9{,}000 = 0.052 \times 12.5 \times L_{old} + 0.052 \times 11.29 \times (15{,}000 - L)$$

$$L_{old} = 3{,}080\ ft$$

Thus, the length of the new mud was equal to:

$$L_{new} = 15{,}000 - L_{old} = 15{,}000 - 3{,}080 = 11{,}920\ ft$$

Exercise 7.3: A production casing of 13,350 *psi* burst rating was planned to be cased and cemented from the top to the casing shoe at 17,500 *ft* . When casing was flushed with a certain drilling fluid, burst safety factor was calculated to be 4.58. When cement slurry of 16.1 *ppg* was pumped to a depth of 8,750 *ft*, burst safety factor was calculated to be 3.06. What was the mud weight of the drilling fluid in the annulus and that used in flushing? Assume mud weight of the fluid in the annulus remains same.

Solution:

Given data:

D_m	=	Depth of the casing shoe	=	17,500 *ft*
P_b	=	Burst rating of the casing	=	13,350 *psi*
SF_{b1}	=	Burst safety factor, case#1	=	4.58

SF_{b1} = Burst safety factor, case#2 = 3.06
MW_{cem} = Mud weight of the cement slurry = 16.1 *ppg*

Required data:
MW_{ann} = Mud weight of the fluid in the annulus
MW_{in} = Mud weight of the fluid inside the casing

Eq. (7.7) will be used to determine the mud weight of the drilling fluids as well as cement slurry. In the first case, the following equation can be written:

$$SF_{b1} = \frac{burst\ rating}{allowable\ burst} = \frac{13{,}350}{0.052 \times 17{,}500 \times (\rho_{in} - \rho_{out})} = 4.58$$

$$\rho_{in} - \rho_{out} = 3.20$$

In the second situation, 7,500 *ft* of cement slurry was pumped inside the casing. Following equation can be written:

$$SF_{b2} = \frac{burst\ rating}{allowable\ burst}$$
$$= \frac{13{,}350}{0.052 \times 8{,}750 \times ((16.1 - \rho_{out}) + (\rho_{in} - \rho_{out}))} = 3.06$$

$$2\rho_{out} - \rho_{in} = 6.51$$

Solving the above three equations simultaneously, we find that mud weight is equal to:

$$\rho_{out} = 9.71\ ppg$$

$$\rho_{in} = 12.92\ ppg$$

Exercise 7.4: A production casing was run to the depth of 15,000 *ft* with drilling mud of 9.9 *ppg* at the annulus. When inside casing was filled with 12.5 *ppg* mud, burst safety factor was calculated to be 4.32. Cement slurry of 16.4 *ppg* was pumped and displaced the fluid that was inside the casing to the annulus. If the fluid that was inside the casing totally displaced the fluid that was previously in the annulus, calculate the burst rating of the casing and also the burst safety factor when inside casing was full of cement slurry.

Solution:

Given data:
D_m = Depth of the casing shoe = 15,000 *ft*
MW_{in} = Mud weight of the drilling mud inside the casing = 12.5 *ppg*
MW_{out} = Mud weight of the drilling mud outside the casing = 9.9 *ppg*
MW_{cem} = Mud weight of the cement slurry = 16.4 *ppg*
SF_{b1} = Burst safety factor, case#1 = 4.32

Required data:

P_{burts} = Burst rating of the casing
SF_{b2} = Burst safety factor for the case cement

Burst rating of the casing can be calculated using Eq. (7.7), and the information for the first situation as follows:

$$SF_{b1} = \frac{burst\ rating}{allowable\ burst} = \frac{burst\ rating}{0.052 \times 15{,}000 \times (12.5 - 9.9)} = 4.32$$

$$burst\ rating = 8{,}761\ psi$$

For the second situation, inside casing was filled with cement slurry displacing the mud that was inside the casing to the annulus. Eq. (7.7) can be used to determine the burst safety factor for the case of cement slurry:

$$SF_{b2} = \frac{burst\ rating}{allowable\ burst} = \frac{8{,}761}{0.052 \times 15{,}000 \times (16.4 - 12.5)} = 2.88$$

Exercise 7.5: A production casing was planned to be set at 19,000 *ft*. When inside casing was filled with cement slurry, burst safety factor was calculated to be 2.02. And when cement slurry displaced and filled the annulus by using same drilling mud, collapse safety factor was calculated to be 1.89. Determine the ratio between burst and collapse ratings of the casing. And if the collapse resistance of the casing is 11,000 *psi*, calculate burst rating of the difference in mud weight between drilling mud and cement slurry.

Solution:

Given data:

D_{shoe} = casing setting depth = 19,000 *ft*
SF_b = Burst safety factor = 2.02
SF_c = Collapse safety factor = 1.89
$P_{collapse}$ = Collapse resistance of the casing = 11,000 *psi*

Required data:

$\frac{P_b}{P_c}$ = burst/collapse pressure ratings ratio
P_{burst} = Collapse resistance of the casing
ΔMW = Difference in mud weight

We know that collapse and burst safety factors can be calculated using Eq. (7.5) and (7.7):

$$SF_c = \frac{collapse\ resistance}{collapse\ pressure} \text{ and } SF_b = \frac{burst\ rating}{Allowable\ burst}$$

Collapse pressure and allowable burst are equal because in this specific case because cement was inside the casing when drilling fluid was in the annulus, and cement was in the annulus when same drilling fluid was inside the casing. So the ratio between burst rating and collapse resistance can be determined simply by dividing burst safety factor by collapse safety factor as follows:

$$\frac{SF_b}{SF_c} = \frac{burst\ rating}{collapse\ resistance} = \frac{2.02}{1.89} = 1.068$$

If burst rating is equal to 11,300 *psi*, thus collapse resistance will be equal to:

$$burst\ rating = collapse\ resistance \times 1.068 = 11{,}000 \times 1.068$$
$$= 11{,}748\ psi$$

To calculate casing setting depth, we can either determine burst pressure or collapse pressure from using burst of collapse safety factors. Thus, casing setting depth can be determined. Burst pressure can be calculated using Eq. (7.7):

$$SF_b = \frac{burst\ rating}{Allowable\ burst} = 2.02 = \frac{11{,}748}{burst\ pressure}$$

$$burst\ pressure = 5{,}816\ psi$$

Casing setting depth can now be calculated as follows:

$$burst\ pressure = 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_m)$$

$$5{,}816 = 0.052 \times 19{,}000 \times (\rho_{cem} - \rho_m)$$

$$\rho_{cem} - \rho_m = 5.89\ ppg$$

Exercise 7.6: A drilling string consists of 725 *ft* of *DC*s have weight of 95 *ppf* and *DP*s have weight of 23 *ppf* was planned to drill a well to a depth of 19,750 *ft* using 14.0 *ppg* drilling mud. Safety factor was calculated to be 1.3 at this situation. Determine the yield strength of the drill pipe.

Solution:

Given data:

L_{dc}	=	Length of drill collars	=	725 *ft*
D_m	=	Length of the well	=	19,750 *ft*
w_{dc}	=	Weight of drill collars	=	95 *ppf*
w_{dp}	=	Weight of drill pipes	=	23 *ppf*
MW	=	Mud weight	=	14.0 *ppg*
SF	=	Safety factor	=	1.30

Required data:

P_d = Yield strength

To calculate yield strength of the pipe, first total weight carried by the first drill pipe joint using Eq. (7.10a):

$$P_a = (L_{dp} w_{dp} + L_{dc} w_{dc}) \times B_f$$

Buoyancy factor can be calculated by knowing that steel density is 65 *ppg* as follows:

$$B_f = 1 - \frac{\rho_{st}}{\rho_m} = 1 - \frac{14.0}{65} = 0.785$$

Now, total weight is equal to:

$$P_a = (725 \times 95 + (19{,}750 - 725) \times 23) \times 0.785 = 397{,}369\ lbf$$

Theoretical yield strength can be calculated using the known safety factor using Eq. (7.12):

$$SF = \frac{P_t}{P_a} = \frac{P_t}{397{,}369} = 1.30 \text{ and } P_t = 516{,}580\ lbf$$

Thus, yield strength of the drill pipe can be calculated using Eq. (7.11):

$$P_t = 0.9 P_d = 516{,}580$$

$$P_d = 573{,}977\ lbf$$

Exercise 7.7: A drilling string consisting of 800 *ft* of *DC*s having weight of 92 *ppf* and *DP*s having weight of 21 *ppf* was used to drill a well to a depth of 21,000 *ft* using 14.5 *ppg* drilling mud. If the maximum allowable overpull was calculated to be 135,000 lb_f, calculate the safety factor at this situation.

Solution:

Given data:

L_{dc}	=	Length of drill collars	=	800 *ft*
D_m	=	Length of the well	=	21,000 *ft*
w_{dc}	=	Weight of drill collars	=	92 *ppf*
w_{dp}	=	Weight of drill pipes	=	21 *ppf*
MW	=	Mud weight	=	14.5 *ppg*
MOP	=	Maximum overpull	=	135,000 lb_f

Required data:

SF = safety factor

Because drillstring is submerged in the drilling fluid, buoyant weight should be calculated. Buoyancy factor can be calculated by knowing that steel density is 65 *ppg* as follows:

$$B_f = 1 - \frac{\rho_{st}}{\rho_m} = 1 - \frac{14.5}{65} = 0.78$$

To calculate safety factor, first total weight carried by the first drill pipe joint using Eq. (7.10a):

$$P_a = (L_{dp}w_{dp} + L_{dc}w_{dc}) \times B_f = (800 \times 92 + 20{,}200 \times 21) \times 0.78$$
$$= 386{,}752\ lbf$$

Theoretical yield strength can be calculated by knowing the MOP as follows:

$$P_t = P_a + MOP = 386{,}752 + 135{,}000 = 521{,}752\ lbf$$

Thus safety factor can be calculated using Eq. (7.12):

$$SF = \frac{P_t}{P_a} = \frac{521{,}752}{386{,}752} = 1.35$$

Exercise 7.8: A drilling string consisting of 1,100 *ft* of *DC*s having weight of 84 *ppf* and *DP*s having weight of 18.4 *ppf* was planned to drill a well to a depth of 18,250 *ft* using 12.6 *ppg* drilling mud. What will be the difference in the drill pipe stretch when drill-string is suspended and a *WOB* of 20,000 lb_f is applied to the bit?

<u>Solution</u>:

Given data:

L_{dc}	=	Length of drill collars	=	1,100 *ft*
D_m	=	Length of the well	=	18,250 *ft*
w_{dc}	=	Weight of drill collars	=	84 *ppf*
w_{dp}	=	Weight of drill pipes	=	18.4 *ppf*
MW	=	Mud weight	=	12.6 *ppg*
WOB	=	Applied weight on bit	=	20,000 lb_f

Required data:

$\Delta\varepsilon_{dp}$ = Difference in drill pipe stretch

To calculate drill pipe stretch, first we should calculate the stretch of drill pipe due to its weight using Eq. (7.28):

$$\varepsilon_{dp} = \frac{L_{dp}^2(65.44 - 1.44\rho_m)}{9.625 \times 10^7} = \frac{17.150^2(65.44 - 1.44 \times 12.6)}{9.625 \times 10^7}$$
$$= 144.5\ in$$

To calculate drill pipe stretch due to weight of drill collars, first we should determine the weight of drill collars as follows:

$$W_{dc} = w_{dc} \times L_{dc} \times BF = 84 \times 1{,}100 \times \left(1 - \frac{12.6}{65.44}\right) = 74{,}598\ lbf$$

Now, drill pipe stretch due to drill collars can be calculated using Eq. (7.30):

$$\varepsilon_{dc} = \frac{L_{dp} W_{dc}}{735,444 w_{dp}} = \frac{17,150 \times 74,598}{735,444 \times 18.4} = 94.5\ in$$

Thus, total drill pipe stretch is equal to:

$$\varepsilon_t = \varepsilon_{dp} + \varepsilon_{dc} = 144.5 + 94.5 = 239.1\ in$$

When 20,000 lb_f is applied to the bit, the weight of drill collars suspended in the drill-string will decrease to:

$$W_{dc\_new} = W_{dc} - WOB = 74,598 - 20,000 = 54,598\ lbf$$

Drill pipe stretch due to weight of drill collars can now be calculated to be:

$$\varepsilon_{dc\_new} = \frac{L_{dp} W_{dc}}{735,444 w_{dp}} = \frac{17,150 \times 54,598}{735,444 \times 18.4} = 69.2\ in$$

The new drill pipe stretch is equal to:

$$\varepsilon_{t\_new} = \varepsilon_{dp} + \varepsilon_{dc\_new} = 144.5 + 69.2 = 213.7\ in$$

Thus, the difference in drill pipe stretch between the two cases is equal to:

$$\Delta\varepsilon_{dp} = 239.1 - 213.7 = 25.4\ in$$

Exercise 7.9: A drilling string consisting of 700 *ft* of *DC*s that have weight of 95 *ppf* and *DP*s that have weight of 22 *ppf* was planned to drill a well to a depth of 9,500 *ft* using 10.4 *ppg* drilling mud. Two WOB values were applied to the bit, and the difference in drill pipe stretch between the two cases was calculated to be 6.0 inches. Determine the WOB difference between the first and second case.

Solution:

Given data:

L_{dc}	=	Length of drill collars	=	700 *ft*
D_m	=	Length of the well	=	9,500 *ft*
w_{dc}	=	Weight of drill collars	=	95 *ppf*
w_{dp}	=	Weight of drill pipes	=	22 *ppf*
MW	=	Mud weight	=	10.4 *ppg*
$\Delta\varepsilon_t$	=	Difference in drill pipe stretch	=	6.0inches

Required data:

ΔWOB = The difference in weight on bit

To calculate the difference in *WOB* applied to the bit that made difference in drill pipe stretch, first we should calculate the stretch when drillstring in suspension. To calculate drill pipe stretch due to weight of drill collars, first we should determine the weight of drill collars as follows:

$$W_{dc} = w_{dc} \times L_{dc} \times BF = 95 \times 700 \times \left(1 - \frac{10.4}{65.44}\right) = 55{,}925\ lbf$$

Now, drill pipe stretch due to drill collars and the first applied *WOB* can be calculated using Eq. (7.30):

$$\varepsilon_{dc1} = \frac{L_{dp}(W_{dc} - WOB_1)}{735{,}444 w_{dp}} = \frac{8{,}800 \times (W_{dc} - WOB_1)}{735{,}444 \times 22}$$

Drill pipe stretch due to drill collars and the second applied *WOB* can be calculated using Eq. (7.30):

$$\varepsilon_{dc2} = \frac{L_{dp}(W_{dc} - WOB_2)}{735{,}444 w_{dp}} = \frac{8{,}800 \times (W_{dc} - WOB_2)}{735{,}444 \times 22}$$

The difference in drill pipe stretch between the two cases is 6.0 inches. Thus:

$$\varepsilon_{dc1} - \varepsilon_{dc2} = 6.0 = \frac{8{,}800 \times (WOB_2 - WOB_1)}{735{,}444 \times 22}$$

$$WOB_2 - WOB_1 = \frac{6.0 \times 22 \times 735{,}444}{8{,}800} = 11{,}032\ lbf$$

Exercise 7.10: A vertical hole was planned to be drilled using a 17 ½" bit. The bit managed to drill volume of 1,000 ft^3 within 7.13 hours of drilling. Calculate the rate of penetration. And if it was found that the hole was enlarged and the average hole diameter was found to be 17.86", recalculate the rate of penetration.

Solution:

Given data:

d_{bit}	=	Bit diameter	=	17.5 inches
V	=	Volume drilled out	=	1,000 ft^3
t	=	Time to drill the above volume	=	7.13 *hrs*

Required data:

ROP = Rate of penetration

Rate of penetration can be estimated using Maurer's method, but first rate of volume drilled should be calculated as follows:

$$\frac{dV}{dt} = \frac{1{,}000}{7.13} = 140.3\ ft^3/hr$$

Thus, rate of penetration can be calculated using Eq. (7.48):

$$ROP = \frac{dF_D}{dt} = \frac{4}{\pi d_{bit}^2}\frac{dV}{dt} = \frac{4}{\pi 17.5^2} \times 140.3 \times 144 = 84\ ft/hr$$

If the hole enlarged to be 17.86", rate of penetration should be equal to:

$$ROP = \frac{dF_D}{dt} = \frac{4}{\pi d_{bit}^2}\frac{dV}{dt} = \frac{4}{\pi 17.86^2} \times 140.3 \times 144 = 80.6\ ft/hr$$

Chapter 8: Casing Design

Exercise 8.1: A production casing is planned to be set at a depth of 14,000 *ft*. the circulated mud weight in the annulus is 12.5 *ppg*. If the collapse rating of the casing is 9,875 *psi* and the minimum collapse safety factor is 1.25, calculate the casing depth which can be safely run inside the well without filling inside the casing with any fluid. If the collapse safety factor was estimated to be 1.55 and when the casing was at the bottom and full of a certain mud, what is the mud weight of the fluid inside the casing?

Solution:

Given data:

D_{shoe}	=	Depth of the casing shoe	=	14,000 *ft*
P_{cas_col}	=	Casing collapse rating	=	9,875 *psi*
ρ_{ann}	=	Mud weight in the annulus	=	12.5 *ppg*
SF_{min_col}	=	Minimum collapse safety factor	=	1.25
SF_{col}	=	Collapse safety factor	=	1.55

Required data:

D_{cas}	=	Casing depth
ρ_m	=	Density of the mud inside the casing

Collapse pressure is the factor that can determine at which depth the casing can become empty. Minimum collapse safety factor can be used to calculate the maximum collapse pressure which should be applied to the casing using the casing collapse rating as:

$$P_{c\_max} = \frac{P_{cas\_col}}{SF_{\min\_col}} = \frac{9,875}{1.25} = 7,900\ psi$$

Now, the maximum depth to which the casing can be lowered and at which it becomes empty is equal to:

$$P_{c\_max} = 0.052 \times D_{cas} \times \rho_m = 7,900 = 0.052 \times D_{cas} \times 12.5$$

$$\boldsymbol{D_{cas} = 12,154\ ft}$$

In the above calculation, we assumed that density of the air in the casing is negligible.

To determine the density of the mud which was used to fill inside the casing, first we should calculate the collapse pressure at the bottom using the new calculated collapse safety factor as:

$$P_{c\_bott} = \frac{P_{cas\_col}}{SF_{col}} = \frac{9{,}875}{3.7} = 2{,}669\ psi$$

Outside pressure at the bottom of the casing can be calculated as follows:

$$P_{out} = 0.052 \times D_{shoe} \times \rho_m = 0.052 \times 14{,}000 \times 12.5 = 9{,}100\ psi$$

Inside pressure at the bottom of the casing can now be calculated as:

$$P_{in} = P_{out} - P_{c_{bott}} = 9{,}100 - 2{,}669 = 6{,}431\ psi$$

Density of the mud inside the casing can now be calculated as:

$$P_{in} = 0.052 \times D_{shoe} \times \rho_m = 6{,}431 = 0.052 \times 14{,}000 \times \rho_m$$

$$\boldsymbol{\rho_m = 8.83\ ppg}$$

Exercise 8.2: A production casing was planned to run in the production hole of a well with casing shoe at 15,000 *ft*. When casing was run empty to the mid depth, collapse safety factor was calculated to be 1.9. When the casing was at the bottom and full of a certain mud, safety factor of the collapse was calculated to be 2.75. If the mud weight of the fluid in the annulus was 14.0 *ppg*, determine the casing collapse rating and mud weight of the fluid inside the casing.

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 15,000 *ft*
$SF1_{coll}$ = Collapse safety factor = 1.9
ρ_{ann} = Mud weight in the annulus = 14.0 *ppg*
$SF2_{coll}$ = Collapse safety factor = 2.75

Required data:

P_{cas_col} = Casing collapse rating
ρ_{in} = Density of the mud inside the casing

The casing was first run empty to the mid depth of the well. So by knowing the collapse safety factor and pressure in the annulus, we can determine the collapse rating of the casing. Pressure in the annulus is equal to:

$$P_{ann} = 0.052 \times \rho_m \times D = 0.052 \times 14.0 \times 7{,}500 = 5{,}460\ psi$$

Casing collapse rating is now equal to:

$$P_{cas\_col} = P_{ann} \times SF1_{col} = 5{,}460 \times 1.9 = \mathbf{10{,}374}\ \boldsymbol{psi}$$

When casing was at the bottom, inside casing was filled with certain fluid that developed collapse pressure against the casing with a safety factor of 2.75. So the collapse pressure against the casing is equal to:

$$P_c = \frac{P_{col}}{SF_{col}} = \frac{10,374}{2.75} = 3,772\ psi$$

Pressure developed by the annulus fluid is equal to:

$$P_{ann} = 0.052 \times \rho_m \times D = 0.052 \times 14.0 \times 15,000 = 10,920\ psi$$

Thus, the pressure inside the casing is equal to:

$$P_{in} = P_{ann} - P_c = 10,920 - 3,772 = 7,148\ psi$$

Now, the mud weight of the fluid inside the casing is equal to:

$$P_c = 0.052 \times \rho_m \times D = 7,148 = 0.052 \times 15,000 \times \rho_m$$

$$\boldsymbol{\rho_m = 9.16\ ppg}$$

Exercise 8.3: A surface section of 26.0" size was planned to be drilled to a depth of 1,750 *ft*. A 20.0" OD casing is to be set and cemented in this section. There are four casing types available in stocks which are shown in table below.

Casing No.	*OD*	Grade	*ID*	Yield
	in		in	psi
1	20.0	K-55	19.124	55,000
2			19.000	55,000
3			18.750	55,000
4			18.376	55,000

Maximum burst pressure has to be assumed when casing is full of 15.8 *ppg* cement slurry and annulus is empty. In addition, maximum collapse pressure has to be assumed when casing is full of fresh water and annulus is full of 15.8 *ppg* cement slurry. Safety factor for burst and collapse was designed to be 1.3 and 1.2, respectively. Which of the above casings should be used in this section?

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 1,750 *ft*
ρ_{cement} = Density of cement slurry = 15.8 *ppg*
ρ_{wat} = Density of fresh water = 8.34 *ppg*
SF_c = Collapse safety factor = 1.2
SF_b = Burst safety factor = 1.3

Required data:
Casing to be used in the intermediate section

Selection of the casing type will depend mainly on the collapse and burst rating of each casing. So collapse and burst pressure for each casing should be calculated using Eq. (8.13) for collapse and Eq. (8.16).

$$P_{cr} = \frac{46.95\times10^6}{\left(\frac{d_n}{t}\right)\left(\frac{d_n}{t}-1\right)^2}$$

$$P_{br} = f\left[\frac{2\sigma_{yield}t}{d_n}\right]$$

The table below summarizes the collapse and burst pressure for the above casings using the reported equations after applying design safety factors for collapse and burst:

	OD	**ID**	P_{cr}	$P_{cr}\times SF_c$	P_{br}	$P_{br}\times SF_b$
Casing No.	**in**	**in**	**psi**	**psi**	**psi**	**psi**
1	20.0	19.124	515	430	2108	1621
2		19.000	772	643	2406	1851
3		18.75	1527	1272	3008	2314
4		18.376	3414	2845	3908	3006

From the well data, collapse pressure based on the assumed scenario is equal to

$$P_c = 0.052\times D_m\times(\rho_{cement}-\rho_{wat}) = 0.052\times1{,}750\times(15.8-8.34)$$
$$= 679\ psi$$

And burst pressure based on the assumed scenario is equal to:

$$P_b = 0.052\times D_m\times\rho_{cement} = 0.052\times1{,}750\times15.8 = 1{,}438\ psi$$

The maximum anticipated collapse and burst pressures are analyzed and these values are compared with the values of collapse and burst rating calculated for each casing. It is observed that the optimum casing size needed to be used which has an ID of 19.0 inches. This casing has maximum allowable collapse and burst ratings of 643 *psi* and 1,851psi. Therefore, the selection is **Casing number 2**.

Exercise 8.4: A production casing string is set at a depth of 18,750 *ft*. A cement slurry of 16.2 *ppg* was pumped and filled the casing from the top to the casing shoe while the annulus has 13.5 *ppg* drilling mud. The cement has been displaced by 12.0 *ppg* drilling mud until cement reached the top of the well. Burst pressure rating of the casing is 8,160 *psi*. If the burst and collapse safety factors during pumping the cement were equal, determine the collapse and burst rating of the casing. Also calculate the collapse pressure rating.

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 18,750 *ft*
ρ_{cem} = Mud weight of cement slurry = 16.2 *ppg*
ρ_{ann} = Mud weight in the annulus = 13.5 *ppg*
ρ_{in} = Mud weight of displacing fluid = 12.0 *ppg*
P_{br} = Casing burst rating = 8,160 *psi*

Required data:

SF_b = Burst safety factor
SF_c = Collapse safety factor
P_{cr} = Casing collapse pressure rating

When cement was pumped inside the casing, pressure inside the casing is greater than that in the annulus. So the casing is subjected to burst pressure at the casing shoe. Burst pressure can be calculated as follows:

$$P_b = 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_{ann}) = 0.052 \times 18{,}750 \times (16.2 - 13.5)$$
$$= 2{,}632.5\ psi$$

Using the burst pressure rating of the casing, burst safety factor is equal to:

$$SF_b = \frac{P_{br}}{P_b} = \frac{8{,}160}{2{,}632.5} = 3.1$$

The above burst safety factor is similar for the collapse safety factor which is equal to 3.1. Now, when the cement is displaced out in the annulus, the casing is subjected to collapse pressure. Collapse pressure can be calculated using the following equation:

$$P_c = 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_{in}) = 0.052 \times 18{,}750 \times (16.2 - 12.0)$$
$$= 4{,}095\ psi$$

Thus collapse pressure rating of the casing is equal to:

$$P_{cr} = P_c \times SF_c = 4{,}095 \times 3.1 = \mathbf{12{,}695}\ \boldsymbol{psi}$$

Exercise 8.5: A production casing string is set at a depth of 13,000 *ft*. Cement slurry of 15.6 *ppg* was pumped and displaced by 10.5 *ppg* drilling mud. Mud weight in the annulus was 14.0 *ppg*. If the collapse pressure in the casing shoe is equal to the burst pressure at casing shoe when the cement was in the bottom of the casing, what was the cement column inside the casing before displacing it in the annulus?

Solution:

Given data:

D_{shoe} = Depth of the casing shoe = 13,000 *ft*
ρ_{cem} = Mud weight of cement slurry = 15.6 *ppg*
ρ_{ann} = Mud weight in the annulus = 14.0 *ppg*
ρ_{in} = Mud weight of displacing fluid = 10.5 *ppg*

Required data:

L_{cement} = Cement slurry column inside he casing

As collapse pressure at the casing shoe is equal to the burst pressure, or outside pressure is equal to the inside pressure of the casing, we can determine annulus pressure and then we can use this value to determine the cement column. Pressure at the annulus is equal to:

$$P_{ann} = 0.052 \times \rho_m \times D_{shoe} = 0.052 \times 14.0 \times 13{,}000 = 9{,}464\ psi$$

If we assume that the cement column is "*x*", pressure inside the casing is equal to:

$$P_{in} = P_{ann} = 0.052 \times \rho_{cem} \times L_{cem} + 0.052 \times \rho_{m\_in} \times (D_{shoe} - L_{cem})$$

$$9{,}464 = 0.052 \times 15.6 \times x + 0.052 \times 10.5 \times (13{,}000 - x)$$

$$0.811x - 0.546x = 9{,}464 - 7{,}098$$

$$x = \frac{2{,}366}{0.265} = 8{,}928\ ft$$

Thus, cement slurry column was about 8,928 *ft* before displace it in the annulus, which created equal pressure at the casing shoe.

Exercise 8.6: Two types of 5.5" casing are available to be used in a certain well as can be seen in the table below:

Casing No.	Grade	***OD*** in	***ID*** in	***Wt*** *ppf*
1	C-75	5.5	4.950	15.5
2	K-55	5.5	4.892	17.0

Determine which one of them can be used in deeper operations, and how deep the casing can be lowered. Assume design safety factor for tension is 2.0 and the hole is full of 13.2 *ppg* mud.

Solution:

Given data:

Data for the two casings
ρ_m = Mud weight of the fluid in the hole = 13.2 *ppg*
SF_{ten} = Tension safety design factor = 2.0 *ppg*

Given data:
Casing type and difference in depth

To determine which one of the two casing types can be used in deeper operations, we can calculate the weight of each casing that can carry. First buoyancy factor can be calculated as follows:

$$BF = 1 - \frac{\rho_m}{64.5} = 1 - \frac{13.2}{64.5} = 0.795$$

Minimum yield load can be calculated using Eq. (8.18):

$$F_{ten} = \frac{\pi}{4}\sigma_{yield}(d_{no}^2 - d_{ni}^2)$$

Minimum yield for casing 1 is equal to 338,556 lb_f, whereas for casing 2 is equal 372,181 lb_f. Thus casing 2 can be used in deeper operations because it can carry more weights than casing 1. The difference in weight is 33,625 lb_f. We can use tension design factor and buoyancy factor to change this weight into length as follows:

$$\Delta F_{max} = \frac{\Delta F_{ten}}{SF_{ten}} = \frac{33{,}625}{2.0} = 16{,}8125\ lbf$$

$$L = \frac{\Delta F_{max}}{BFw} = \frac{16{,}812.5}{0.795 \times 17.0} = 1{,}244\ ft$$

Thus casing 2 can be lowered around 1,244 *ft* deeper than casing 1.

Exercise 8.7: A 9 5/8", 8.755", N-80, 43.5 *ppf* casing is planned to be run in a production section of 10,000 *ft*. This section will be drilled using 11.6 *ppg* mud. During the casing design, it was assumed only shock effect. It was also assumed that during pressure testing, the casing pressure was equal to 75% of the casing burst pressure. Tension safety factor is required to be 1.8. Determine whether the above casing can satisfy the required tension safety factor or not.

Solution:

Given data:

D_{shoe}	=	Depth of the casing shoe	=	10,000 *ft*
OD_{cas}	=	Casing outside diameter	=	9 5/8"
ID_{cas}	=	Casing inside diameter	=	8.755"
w	=	Weight of one foot of casing	=	43.5 *ppf*
SF_{ten}	=	Required tension safety factor	=	1.8

Required data:
Above casing should pass the required tension safety factor

To determine whether the above casing can pass the required design factor, we should calculate the maximum casing load and the tension on the first casing joint. Maximum casing load or yield strength can be calculated using Eq. (8.18):

$$F_{ten} = \frac{\pi}{4}\sigma_{yield}(d_{no}^2 - d_{ni}^2)$$

$$F = \frac{\pi}{4}\times 80{,}000\times(9.625^2 - 8.755^2) = 1{,}004{,}719\ lbf$$

Now, we will test the two cases of casing running and casing pressure testing separately:

Running the Casing:
During running casing, casing is subjected only to the tension and shock load. To calculate the tension at the first casing joint, first we should determine the buoyancy factor as follows:

$$BF = 1 - \frac{\rho_m}{64.5} = 1 - \frac{11.6}{64.5} = 0.820$$

Tension load can be calculated as follows:

$$F_{ten} = L_{cas}\times w\times BF = 10{,}000\times 43.5\times 0.820 = 356{,}767\ lbf$$

Shock load can be estimated using *E.* (7.16):

$$F_s = 3{,}200\times w = 3{,}200\times 43.5 = 139{,}200\ lbf$$

Total loads while running the casing is equal to:

$$F_{run} = F_{ten} + F_s = 356{,}67 + 139{,}200 = 495{,}967\ lbf$$

Thus, safety factor while running the casing is equal to:

$$SF_{runing} = \frac{F}{F_{run}} = \frac{1{,}004{,}719}{495{,}967} = 2.03$$

Pressure testing the Casing:
While pressure testing, the casing is subjected to two loads that are tension and pressure test which act as extra loads applied to the first joint of the casing. Tension on the first casing joint is similar to that calculated above which is 362,096 lb_f. To determine the pressure test rating, we should first calculate the burst pressure rating of the casing using Eq. (8.16):

$$P_{br} = f\left[\frac{2\sigma_{yield}t}{d_n}\right] = 0.875\times\frac{80{,}000\times\left(\frac{9.625 - 8.755}{2}\right)}{9.625} = 6{,}327\ psi$$

Only 75% of the above pressure will be used for pressure testing. Thus, pressure test rating is equal to:

$$P = P_{br}\times 0.75 = 6{,}327\times 0.75 = 4{,}745\ psi$$

Above pressure can be changed to load as follows:

$$F_p = P \times A_{cas} = 4{,}745 \times \frac{\pi}{4} \times 8.755^2 = 285{,}680\ lbf$$

Now, the total loads applied while pressure testing for the casing is equal to:

$$F_{press} = F_{ten} + F_p = 356{,}767 + 285{,}680 = 642{,}448\ lbf$$

Thus, safety factor while running the casing is equal to:

$$SF_{runing} = \frac{F}{F_{press}} = \frac{1{,}004{,}719}{642{,}448} = 1.56$$

From the above calculations, pressure testing for the casing does not meet the required design safety factor of 1.8. So if it is important to pressure test the casing by 75% of the casing burst pressure, in such case we should use casing with a grade higher than the above. However, if it is not necessary to pressurize the casing with the above pressure, we can do pressure test for the casing by 60% of the burst pressure. By doing that, safety factor during pressure testing will be 1.71 which is greater than the required design factor. Therefore, it ***doesn't meet the requirement.***

Exercise 8.8: A production section in a well was planned to be drilled to a depth of 15,000 *ft* using 12.25" hole size. 9 5/8" casing has to be run and cemented in this section. There are two casing types available as per the specifications shown in the table below.

		Yield	***OD***	***ID***	***t***	**Length**	P_{cr}	P_{br}
Casing No.	**Grade**	**psi**	**in**	**in**	**in**	**Ft**	**psi**	**psi**
1	K-55	55,000	9.625	8.535	0.545	40.0	5450	5130
2	C-75	75,000	9.625	8.279	0.673	40.0	9177	7570

From previous experience it is found that production casing design depends mainly on collapse pressure. Design safety factor for collapse was set to be 1.15. Casing has to be cemented to the top using 16.4 *ppg* slurry. Cement slurry will be displaced using fresh water. From the above information how many joints of each casing should be used to insure as an economic selection?

Solution:

Given data:

Casing information in the table

D_{shoe} = Depth of the casing shoe = 15,000 *ft*

OD_{cas} = Casing outside diameter = 9.625"

SF_c = Collapse safety factor = 1.15
ρ_{cem} = Density of cement slurry = 16.4 *ppg*
ρ_w = Density of the fresh water = 8.34 *ppg*

Required data:
Above casing should pass the required tension safety factor

Selection of the casing string based on the collapse pressure will depend mainly on the expected collapse pressure developed due to the difference in mud density between the cement slurry and drilling mud. In this case, multi-string selection will be the optimum decision. So we should determine how many joints of low-grade casing to be used. After applying collapse safety factor, maximum allowable collapse pressure for both casing types are listed in the table below:

	Max. P_c
Casing No.	**psi**
1	4,739
2	7,980

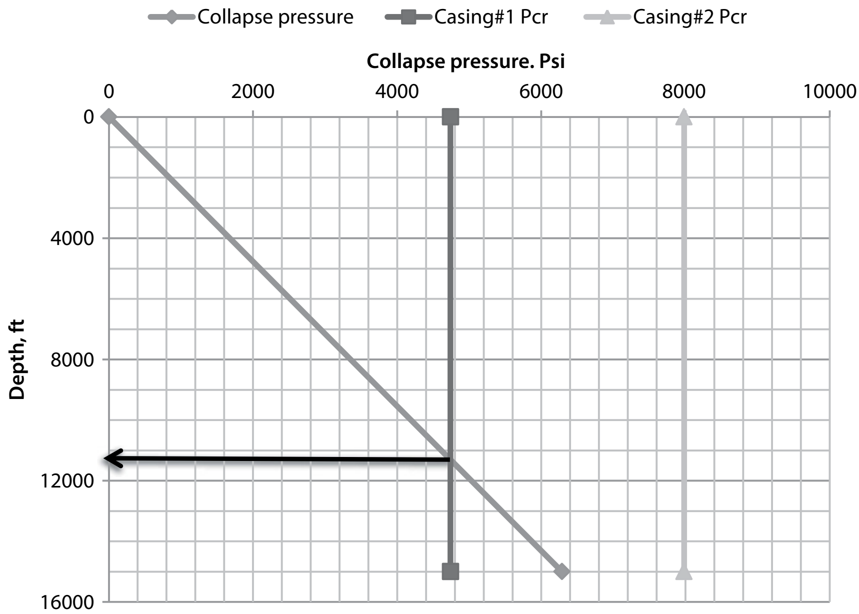
Collapse pressure at any point in the casing can be calculated using the following equation:

$$P_c = 0.052 \times D_{shoe} \times (\rho_{cem} - \rho_m)$$

Summary of collapse pressure versus depth are listed in the table below:

Depth	P_c	$P_{cr\_casing\ 1}$	$P_{cr\_casing\ 2}$
ft	**psi**	**psi**	**Psi**
0	0	4,739	7,980
15,000	6,287	4,739	7,980

The figure below shows the collapse pressure versus depth, and also collapse pressure rating for the two casings. From this figure, casing 2 can be used for the whole section. However, due to the high grade which normally costs more it is not economical to use alone. At the depth of 11,200 *ft*, collapse pressure is equal to the collapse pressure rating for casing 1, hence casing 1 cannot be deepened to depths deeper than 11,200 *ft*. Casing 1 can be run down to the depth of 11,200 *ft*. From 11,200 *ft* to 15,000 *ft*, casing 2 can be used. Thus, 95 joints of casing 2 will be run first in the hole. Then 280 joints of casing 1 will be run.



Chapter 9: Cementing

Exercise 9.1: A class *G* cement with 15% Bentonite and 44% water is planned to be used in cementing deep section in a development well. If the cement silo can hold 85% of the neat class *G* cement, determine the aerated density of the cement inside the silo and the cement slurry density. Bulk density of class *G* cement is 94 lbs/ft^3 and Bentonite is 60 lbs/ft^3.

Solution:

Given data:

Bulk density of class *G* cement	=	94 *lbs/cu ft*
Bulk density of Bentonite	=	60 *lbs/cu ft ft*
Water amount	=	44% of cement
Silica flour amount	=	15% of cement

Required data:

Aerated density of cement in silo
Cement slurry density

To determine the total blend density of the cement and Bentonite, we should calculate the total volume and weight of the blend based on 1 sack of class G cement as follows:

One sack of class G cement equals 1.0 cubic *ft* in volume and 94 *lbs* in weight. Weight of Bentonite is equal to 15% of the weight of one sack of cement; or:

$$Weight\ of\ bentonite = 0.15 \times weight\ of\ cement$$
$$= 0.15 \times 94 = 14.10\ lbs$$

Now, bulk volume of Bentonite can be determined using the bulk density of Bentonite as follows:

$$Volume\ of\ bentonite = \frac{weight}{density} = \frac{14.10}{60} = 0.235\ ft^3$$

Thus, total bulk volume and weight of the blend is equal to

$$\begin{aligned} Total\ volume &= cement\ volume + bentonite\ volume \\ &= 1.0 + 0.235 = 1.235\ ft^3 \end{aligned}$$

$$\begin{aligned} Total\ weight &= cement\ weight + bentonite\ weight \\ &= 94 + 14.10 = 108.10\ lbs \end{aligned}$$

Blend bulk density is now equal to

$$Blend\ bulk\ density = \frac{total\ weight}{total\ volume} = \frac{108.10}{1.235} = 87.53\ lbs/ft^3$$

When this blend is transferred to the cement silo, aerated bulk density of the blend will become 85% of its original weight. Or:

$$\begin{aligned} Aerated\ bulk\ density &= 0.8 \times blend\ bulk\ density \\ &= 0.8 \times 87.53 = \mathbf{74.40\ lbs/ft^3} \end{aligned}$$

To calculate the cement slurry density, first weight and volume of the water should be calculated as follows:

$$Weight\ of\ water = 0.44 \times weight\ of\ cement = 0.44 \times 94 = 41.36\ lbs$$

$$Volume\ of\ water = \frac{weight\ of\ water}{water\ density} = \frac{41.36}{62.4} = 0.663\ ft^3$$

Now total weight of the cement slurry is equal to:

$$\begin{aligned} Total\ slurry\ weight &= weight\ of\ blend\ cement + weight\ of\ water \\ &= 108.10 + 41.36 = 149.46\ lbs \end{aligned}$$

Because cement and silica are powders, we are going to use the absolute volumes instead of bulk volume in calculating cement slurry density. Absolute volumes of class *G* cement and silica flour are 0.0382 and 0.0454 *gal/lbs*, respectively. Thus total slurry volume is now equal to:

$$\begin{aligned} Total\ slurry\ volume &= volume\ of\ blend\ cement + volume\ of\ water \\ &= 94 \times 0.0382 + 14.10 \times 0.0454 + 0.663 \times 7.481 \\ &= 9.19\ gal \end{aligned}$$

Thus cement slurry density is equal to:

$$Cement\ slurry\ density = \frac{slurry\ weight}{slurry\ volume} = \frac{149.46}{9.19} = \mathbf{16.26\ ppg}$$

Exercise 9.2: Cement slurry is designed to be prepared using class *G* cement and fresh water. If the mud weight of the cement slurry is 14.03 *ppg*, what will be the water mass percentage based on the cement mass?

Solution:

Given data:

MW of cement slurry = 14.03 *ppg*

Required data:

Water percentage

To determine the water percentage, we should determine the mass and volume of cement and water. And to simplify the calculation, the basis will be based on one sack of cement which has mass of 94 *lbs* and absolute volume of 3.591 *gallons*. If we assume the percentage mass of water is "*x*", thus the mass of water equal to:

$$Mass\ of\ water = 94x\ lbs$$

Volume of water equal to:

$$Volume\ of\ water = \frac{94x}{8.33} = 11.285x\ gallon.$$

From the definition of cement slurry density, density is equal to:

$$Cement\ slurry\ MW = \frac{mass\ of\ cement + mass\ of\ water}{volume\ of\ cement + volume\ of\ water}$$

$$= \frac{94 + 94x}{3.591 + 11.285x} = 14.03$$

$$94 + 94x = 50.38 + 158.33x$$

$$64.33x = 43.62$$

$$x = \frac{47.32}{52.52} = 0.68$$

Thus, to prepare cement slurry with 14.03 *ppg* density, water amount should be 68% of the cement weight.

Exercise 9.3: Cement slurry is designed to be prepared using class G cement, silica flour and fresh water. If the mud weight of the cement slurry is 15.80 *ppg* and water percentage is 53%, what will be the silica flour mass percentage based on the cement mass?

Solution:

Given data:

MW of cement slurry = 15.80 *ppg*
Water percentage = 53%

Required data:

Bentonite percentage

To determine the silica percentage, we should determine the mass and volume of cement and water. And to simplify the calculation, the basis will be based on one sack of cement which has mass of 94 *lbs* and absolute volume of 3.591 *gallons*.

The mass of water is equal to:

$$Mass\ of\ water = 0.53 \times 94 = 49.82\ lbs$$

The volume of water is equal to:

$$Volume\ of\ water = \frac{49.82}{8.33} = 5.98\ gallons$$

If we assume the percentage mass of silica flour is "*x*", thus the mass of water equals to

$$Mass\ of\ silica = 94x\ lbs$$

Absolute volume of silica is equal to

$$Volume\ of\ silica = 94x \times 0.0454\ gal/lbs = 4.27x\ gallons$$

From the definition of cement slurry density, density is equal to:

$$\begin{aligned} &Cement\ slurry\ MW \\ &= \frac{mass\ of\ cement + mass\ of\ water + mass\ of\ silica}{volume\ of\ cement + volume\ of\ water + volume\ of\ silica} \\ &= \frac{94 + 49.82 + 94x}{3.591 + 5.98 + 4.27x} = 15.80 \end{aligned}$$

$$143.82 + 94x = 151.22 + 67.47x$$

$$26.53x = 7.4$$

$$x = \frac{7.4}{26.53} = \mathbf{0.279}$$

Thus, to prepare cement slurry with 15.80 *ppg* density, silica amount should be 27.9% of the cement weight.

Exercise 9.4: A production section in a well is planned to be cemented using 100 *bbls* of class G cement slurry with 37% by weight of Bentonite. Cement slurry is designed to have mud weight of 15.73 *ppg* and water is 57% of cement weight. Determine the volume of water and the amount of cement and Bentonite to be mixed in order to prepare the above cement slurry volume?

Solution:

Given data:

Volume of cement slurry	=	100 *bbls*
MW of cement slurry	=	15.73 *ppg*
Water amount	=	57% of cement
Bentonite amount	=	37% of cement

Required data:

Volume of water
Amount of cement
Amount of silica flour

To determine the amount of each material, we should calculate the volume percentage of each material based on the cement slurry density.

One sack of cement has 94 *lbs* and 3.591 *gallons* of absolute volume. Thus, the amount of Bentonite based on one sack of cement is equal to

$$Mass\ of\ Bentonite = 94 \times 0.37 = 34.8\ lbs$$

$$Absolute\ volume\ of\ Bentonite = 34.8 \times 0.0454 = 1.58\ gallons$$

The amount of water based on one sack of cement is equal to:

$$Mass\ of\ water = 94.0 \times 0.57 = 53.6\ lbs$$

$$Volume\ of\ water = \frac{53.6}{8.33} = 6.43\ gallons$$

The slurry volume equal to:

$$Slurry\ volume = 3.59 + 1.58 + 6.43 = 11.6\ gallons$$

The volume of water required is equal to:

$$Total\ water\ volume = Total\ slurry\ volume \times \frac{water\ volume}{slurry\ volume}$$

$$= 100\ bbls \times \frac{6.43\ gal}{11.6\ gal} = 55.4 bbls$$

The amount of cement to be used is equal to:

$$Mass\ of\ \ cement = Total\ slurry\ volume \times \frac{cement\ \ volume}{slurry\ volume}$$

$$= \left(100\ bbls \times \frac{42\ gal}{bbl}\right) \times \frac{3.59\ gal}{11.6\ gal} \times \frac{1}{0.0382\ \frac{gal}{lbs}}$$

$$= 34{,}027\ lbs = 15.5\ tons = \mathbf{362\ sacks}$$

The amount of Bentonite to be used is equal to:

$$Mass\ of\ \ silica = Total\ slurry\ volume \times \frac{Bentonite\ volume}{slurry\ volume}$$

$$= \left(100\ bbls \times \frac{42\ gal}{bbl}\right) \times \frac{1.58\ gal}{11.6\ gal} \times \frac{1}{0.0454\ \frac{gal}{lbs}}$$

$$= 12{,}600.6\ lbs = \mathbf{5.7\ tons}$$

Exercise 9.5: A dry well is planned to be plugged using 16.0 *ppg* class G cement slurry with 12% by weight of silica flour and 46% by weight of fresh water. Class G cement of 9.66 *tons* will be used in preparation of the cement slurry. Determine the volume of water and the amount of silica flour to be added to the mixture in order to prepare the above cement slurry density. Also determine the total volume of cement slurry.

<u>Solution</u>:

Given data:

Amount of class *G* cement	=	9.66 *tons*
MW of cement slurry	=	16.0 *ppg*
Water amount	=	46% of cement
Silica flour amount	=	12% of cement

Required data:

Volume of water in *bbls*
Amount of silica flour in *tons*
Volume of cement slurry in *bbls*

To determine the amount of each material and total volume of cement slurry, we should calculate the volume percentage of each material based on the cement slurry density.

One sack of cement has 94 *lbs* and 3.59 *gallons* of absolute volume. So, the amount of silica flour based on one sack of cement is equal to:

$$Mass\ of\ \ silica\ flour = 94 \times 0.12 = 11.28\ lbs$$

$$Absolute\ volume\ of\ \ flour = 11.28 \times 0.0454 = 0.51\ gallons$$

The amount of water based on one sack of cement is equal to:

$$Mass\ of\ water = 94.0 \times 0.46 = 43.24\ lbs$$

$$Volume\ of\ water = \frac{43.24}{8.33} = 5.18\ gallons$$

The slurry volume equal to:

$$Slurry\ volume = 3.59 + 0.51 + 5.18 = 9.28\ gallons$$

The volume ratios of each material are as follows:

$$Water\ volume\ ratio = \frac{water\ volume}{slurry\ volume} = \frac{5.18\ gal}{9.28\ gal} = 0.558$$

$$Cement\ volume\ ratio = \frac{Cement\ volume}{slurry\ volume} = \frac{3.59\ gal}{9.28\ gal} = 0.387$$

$$Bentonite\ volume\ ratio = \frac{Bentonite\ volume}{slurry\ volume} = \frac{0.51\ gal}{9.28\ gal} = 0.055$$

Mass of cement can be used to calculate the cement volume, silica flour volume and water volume used in preparing the above cement slurry. 9.66 *tons* of cement equals 21,252 *lbs* and by using the absolute cement volume, we can calculate the cement volume as follows:

$$Cement\ volume = cement\ mass \times 0.0382 = 21{,}252 \times 0.0382$$
$$= 811.8\ gals = \mathbf{108.5}\ \boldsymbol{sacks}$$

Mass of silica flour can be calculated as follows:

$$Mass\ of\ silica = Mass\ of\ cement \times 0.12 = 21{,}252 \times 0.12$$
$$= 2{,}550.2\ lbs = \mathbf{1.16}\ \boldsymbol{tons}$$

Volume of silica flour is now equal to:

$$Volume\ of\ silica = mass\ of\ silica \times 0.0454 = 2{,}550.2 \times 0.0454$$
$$= 115.8\ gal = 2.75\ bbls$$

Mass of water can be determined using the mass of cement as follows:

$$Mass\ of\ water = mass\ of\ cement \times 0.46 = 21{,}252 \times 0.46 = 9{,}775.9\ lbs$$

Volume of water is now equals to:

$$Volume\ of\ water = \frac{mass\ of\ water}{8.33} = \frac{9{,}775.9}{8.33}$$
$$= 1{,}173.6\ gal = \mathbf{27.94}\ \boldsymbol{bbls}$$

Thus, total slurry volume can be determined by adding the above three volumes as follows:

$$Slurry\ volume = cement\ volume + silica\ volume + water\ volume$$
$$= 19.33 + 2.75 + 27.94 = \mathbf{50.02}\ \boldsymbol{bbls}$$

Exercise 9.6: A well has 13 3/8" casing and needs to be plugged by a balanced cement plug where the bottom depth of cement plug should be at 5,000 *ft*. A 5.0" drill pipe of 3.625" inside diameter will be used to spot the cement plug in place. If the length of the cement plug is designed to be 120 ft, determine the volume of cement slurry needed. Also determine the top depth of cement plug before removing the drill pipe out of cement plug, and the displacing fluid volume.

Solution:

Given data:

Casing *ID*	=	13 3/8"
Drill pipe *OD*	=	5.0"
Drill pipe *ID*	=	3.625"
Bottom depth of cement plug	=	5,000 *ft*
Cement plug length	=	120 *ft*

Required data:

Volume of cement slurry
Top depth of cement plug
Displacing fluid volume

The cement slurry volume can be calculated using the casing diameter and cement plug length as follows:

$$V_{slurry} = \frac{\pi}{4} ID_{cas}^2 L = \frac{\mu}{4} \times \left(\frac{13.375}{12}\right)^2 \times 120 = 117.1\, ft^3 = \mathbf{20.9}\ \boldsymbol{bbls}$$

When pumping cement slurry, the drill pipe will be at the bottom of the cement plug at a depth of 5,000 *ft*. Thus, when cement plug is in place, the length of the cement plug will be greater than 120 *ft* due the volume of drill pipes. We know that after displacing the cement slurry, cement slurry volume of 20.9 *bbls* should be at the annulus between the casing and drill pipe, and also inside the drill pipe. In addition, the length of cement slurry inside and outside the drill pipes must be the same. If we assume that the cement plug length before pulling the drill pipes is equal to "*L*", the governing equation for this situation can be written as:

$$V_{slurry} = V_{inside} + V_{outside} = (A_{inside} + A_{outside})L$$

$$L = \frac{V_{slurry}}{A_{inside} + A_{outside}} = \frac{117.1}{\frac{\pi}{4}\left(\left(\frac{3.625}{12}\right)^2 + \left(\left(\frac{13.375}{12}\right)^2 - \left(\frac{5.0}{12}\right)^2\right)\right)}$$
$$= 128.5\, ft$$

Thus, the length of cement plug before pulling the drill pipe will be 128.5 *ft*. So, the displacement fluid will have a length of 5,000 *ft* minus 128.5 *ft*, which is equal to **4,871.5 ft**. Now, displacement fluid can be determined using the following equation:

$$V_{disp} = \frac{\pi}{4} \times D^2 \times L = \frac{\pi}{4} \times \left(\frac{3.625}{12}\right)^2 \times 4,871.5 = 349.1\ ft^3 = \mathbf{62.2\ bbls}$$

Exercise 9.7: A 250 *ft* cement plug was planned to be spotted in a 7.0" *OD* and 6.18" *ID* casing. Calculate cement slurry density, yield and amount of each component with the help of the following information:

Sack of class *G* cement	=	94 *lbs*
Bentonite amount	=	11.5%
Fluid loss additives	=	1.6%
Liquid dispersant	=	0.7 *gal/sack*
Water amount	=	71%
Cement absolute volume	=	0.0382 *gal/lbs*
Silica flour absolute volume	=	0.0454 *gal/lbs*
Fluid loss absolute volume	=	0.093 *gal/lbs*
Liquid dispersant volume	=	0.101 *gal/lbs*

Solution:

Total weight and volume of each component of the cement slurry must be calculated in order to determine the mud weight and yield. For cement, one sack is equal to 94 *lbs* and occupies 3.59 gals. Weight and volume of Bentonite based on one sack of cement is equal to:

$$W_{Bent.} = 0.11 \times W_{cement} = 0.115 \times 94 = 10.8\ lbs$$

$$V_{Bent.} = 10.8 \times 0.0454 = 0.49\ gals$$

Weight and volume of fluid loss additives (*FL*) are as follows:

$$W_{FL} = 0.016 \times W_{cement} = 0.016 \times 94 = 1.5\ lbs$$

$$V_{FL} = 1.5 \times 0.093 = 0.14\ gal$$

Weight and volume of liquid dispersant (*LD*) are as follows:

$$V_{LD} = 0.7\ gal$$

$$W_{LD} = \frac{V_{LD}}{0.101} = \frac{0.7}{0.101} = 6.9\ lbs$$

Weight and volume of water are equal to:

$$W_{water} = 0.71 \times W_{cement} = 0.71 \times 94 = 66.7\ lbs$$

$$V_{FL} = \frac{66.7}{8.33} = 8.02\ gal$$

Now, cement slurry weight and volume are the summation of the above weights and volumes:

$$W_{slurry} = 94.0 + 10.8 + 1.5 + 6.9 + 66.7 = 179.9\ lbs$$

$$V_{slurry} = 3.59 + 0.49 + 0.14 + 0.70 + 8.02 = 12.94\ gals$$

Thus, slurry density and yield are equal to:

$$MW_{slurry} = \frac{W_{slurry}}{V_{slurry}} = \frac{179.9}{12.94} = \mathbf{13.9\ \mathit{ppg}}$$

$$Yield_{slurry} = \frac{V_{slurry}}{7.48} = \frac{12.94}{7.48} = \mathbf{1.73\ \frac{\mathit{ft}^3}{\mathit{sack}}}$$

To calculate the amount of each component of cement slurry, first we should calculate the required volume of cement slurry as follows:

$$Volume\ of\ slurry = \frac{\pi}{4} D^2 \times L = \frac{\pi}{4} \times 6.18^2 \times 250 = 52.08\ ft^3$$

Amount of cement to be used is equal to:

$$Sacks\ of\ cement = \frac{volume\ of\ slurry}{Yield_{slury}} = \frac{52.08}{1.73} \approx \mathbf{30\ \mathit{sacks}}$$

Amount of Bentonite is equal to:

$$Amount\ of\ Bentonite = 0.115 \times 30.0 \times 94 \approx \mathbf{325.0\ \mathit{lbs}}$$

Amount of fluid loss additive is equal to:

$$Amount\ of\ FL = 0.016 \times 30.0 \times 94 \times 0.093 \approx \mathbf{4.2\ \mathit{gals}}$$

Amount of liquid dispersant additive is equal to:

$$Amount\ of\ LD = 0.7 \times 30.0 \approx \mathbf{21.0\ \mathit{gals}}$$

Amount of fresh water is equal to:

$$Volume\ of\ water = \frac{0.71 \times 30.0 \times 94}{8.33} = 241.4\ gals \approx \mathbf{5.75\ \mathit{bbls}}$$

Exercise 9.8: A 3,500 *ft* intermediate section of 12¼” hole size was planned to be cased and cemented using casing of 95/8" OD 8.86" ID. The cement slurry of 16.0 *ppg* and 1.13 ft^3/sk was used to complete the job. Calculate the required amount of each component with the help of the following information:

Sack of class *G* cement	=	94 *lbs*
Retarder	=	0.04 gal/sack
Liquid dispersant	=	0.05 gal/sack
Water amount	=	4.8 gal/sack
Length of casing shoe	=	39.5 *ft*
Length of rathole	=	25.0 *ft*
Excess	=	15%
Depth of surface casing	=	700 *ft*
Size of surface casing	=	13 3/8" ID 12.5"

Solution:
To determine the amount of each component, the required cement slurry volume should be calculated first. The volume that should be filled with cement slurry is divided into four sections: annulus between casing and surface casing, annulus between casing and open hole, float shoe, and rathole.

Slurry volume between casing and surface casing:
In this section, no excess should be applied.

$$V_1 = \frac{\pi}{4}(ID_{cond}^2 - OD_{cas}^2) \times L_{cond} = \frac{\pi}{4} \times \frac{12.5^2 - 9.625^2}{144} \times 750 = 352.8\ ft^3$$

Slurry volume between casing and open hole:
In this section, excess should be applied.

$$V_2 = \frac{\pi}{4}(ID_{oh}^2 - OD_{cas}^2) \times L_{cas} \times 1.15$$
$$= \frac{\pi}{4} \times \frac{12.25^2 - 9.625^2}{144} \times (3{,}500 - 750 - 25) \times 1.15 = 1{,}076.6\ ft^3$$

Slurry volume in the casing shoe:
In this section, no excess should be applied.

$$V_3 = \frac{\pi}{4} ID_{cas}^2 \times L_{shoe} = \frac{\pi}{4} \times \frac{8.86^2}{144} \times 39.5 = 16.9\ ft^3$$

Slurry volume in the rathole:
In this section, excess should be applied.

$$V_4 = \frac{\pi}{4} D_{oh}^2 \times L_{rh} \times 1.15 = \frac{\pi}{4} \times \frac{12.25^2}{144} \times 25 \times 1.15 = 23.5\ ft^3$$

Total slurry volume is now equal to:

$$Slurry\ volume = 352.8 + 1{,}076.6 + 16.9 + 23.5 = 1{,}469.8\ ft^3$$

Cement amount can be estimated by using slurry yield as follows:

$$Cement\ amount = \frac{slurry\ volume}{slurry\ yield} = \frac{1{,}469.8}{1.13} = 1{,}300.7 \approx \mathbf{1{,}301}\ \boldsymbol{sks}$$

Amount of dispersant equals:

$$Dispersant\ amount = 1{,}301 \times 0.09 \approx \mathbf{117.1}\ \boldsymbol{gals}$$

Amount of retarder equals to:

$$Retarder\ amount = 1{,}301 \times 0.06 \approx \mathbf{78.1}\ \boldsymbol{gals}$$

Amount of water equals to:

$$Dispersant\ amount = 1{,}301 \times 4.8 \approx 6{,}245\ gals \approx \mathbf{148.7}\ \boldsymbol{bbls}$$

Chapter 10: Horizontal and Directional Drilling

Exercise 10.1: A well is planned to be drilled as directional well. The well is kicked off from vertical section at point "*O*", and the drilling team found that the well has progressed 105 meters towards the west and 25 meters towards the south at point "*A*". After some time, drilling team found that the well has progressed 178 meters towards the west and 99 meters towards the south at point "*B*" from the previous measuring point, "*A*". Determine the azimuth and horizontal departure at the first and second measuring points from the kick off point "*O*."

Solution:

Given data:

ΔS_A = 25.0 m
ΔW_A = 105.0 m
ΔS_B = 99.0 m
ΔW_B = 178.0 m

Required data:

ε = Azimuth in degrees
HD = Horizontal departure in meters

As per the above information, the well was progressed a certain azimuth and that azimuth was deviated. So to know the azimuth to the point "*A*" and the final azimuth, we can use Eq. (10.2) as follows:

$$\varepsilon = 180 + \tan^{-1}\frac{\Delta W}{\Delta S}$$

The angle was added to 180 because the angle in the *S*-*W* quarter.

Horizontal departure can be calculated using Eq. (10.1) as follows:

$$HD = \sqrt{\Delta W^2 + \Delta S^2}$$

Azimuth and horizontal departure of the well from the kick off point to point "*A*" are equal to:

$$\varepsilon_{O-A} = 180 + \tan^{-1}\frac{\Delta W_{O-A}}{\Delta S_{O-A}} = 180 + \tan^{-1}\frac{105}{25} = 180 + 76.6 = 256.6°$$

$$HD_{O-A} = \sqrt{\Delta W^2{}_{O-A} + \Delta N^2{}_{O-A}} = \sqrt{105^2 + 25^2} = \mathbf{107.9\,\boldsymbol{m}}$$

Azimuth and horizontal departure of the well from point to point "*A*" to point "*B*" are equal to:

$$\varepsilon_{A-B} = 180 + \tan^{-1}\frac{\Delta W_{A-B}}{\Delta S_{A-B}} = 180 + \tan^{-1}\frac{178}{99} = 180 + 60.9 = 240.9°$$

$$HD_{A-B} = \sqrt{\Delta W^2{}_{A-B} + \Delta S^2{}_{A-B}} = \sqrt{178^2 + 99^2} = 203.7\ m$$

Azimuth and horizontal departure of the well from kick off point to point "*B*" are equal to:

$$\begin{aligned}\varepsilon_{O-B} &= 180 + \tan^{-1}\frac{\Delta W_{O-A} + \Delta W_{A-B}}{\Delta S_{O-A} + \Delta S_{A-B}} = 180 + \tan^{-1}\frac{283}{124} \\ &= 180 + 66.3 = 246.3°\end{aligned}$$

$$\begin{aligned}HD_{O-B} &= \sqrt{(\Delta W_{O-A} + \Delta W_{A-B})^2 + (\Delta S_{O-A} + \Delta S_{A-B})^2} \\ &= \sqrt{283^2 + 124^2} = \mathbf{293.7\,\boldsymbol{m}}\end{aligned}$$

Exercise 10.2: A well is planned to be drilled as directional well. The well azimuth and horizontal departure were designed to be 144.5° and 721 meters, respectively. After kick off the well from vertical section, measurements were conducted and found that the well progressed 511 meters towards the east and 338 meters towards the south. What is the azimuth at the current point? And if the azimuth is not same as designed, what should be the new degree from the current point to be used until completing the well to reach the designed target?

Solution:

Given data:

HD = 721 m

ε = 125.5°

ΔE_A = 511 m

ΔS_A = 338 m

Required data:

ε_A = Azimuth in degrees at point A
ε = New azimuth to be adopted to the target

We can use the coordinates of the first measuring point to identify whether the well azimuth is maintained. Azimuth can be determined using Eq. (10.2):

$$\varepsilon_{O-A} = 180 - \tan^{-1}\frac{\Delta E_{O-A}}{\Delta S_{O-A}} = 180 - \tan^{-1}\frac{511}{338} = 180 - 56.5 = 123.5°$$

From above calculation, well azimuth should be adjusted in order to reach the target with final azimuth of 144.5 degrees. To do that, first we should determine the designed well coordinates based on the designed azimuth and horizontal displacement using Equations (10.1) and (10.2):

$$\Delta E = HD \times \sin(180 - \varepsilon) = 721 \times \sin(180 - 123.5) = 587.0\ m$$

$$\Delta S = HD \times \cos(180 - \varepsilon) = 710 \times \cos(180 - 123.5) = 419.0\ m$$

Now we can subtract the coordinates of the first measuring point from this coordinates to determine the new azimuth that should be adopted to reach the well target.

$$\Delta E_{A-B} = \Delta E_{A-O} - \Delta E_{O-A} = 587 - 511 = 76\ m$$

$$\Delta S_{A-B} = \Delta S_{A-O} - \Delta S_{O-A} = 419 - 338 = 81\ m$$

Thus, the new azimuth should be equal to:

$$\varepsilon_{A-B} = 180 - \tan^{-1}\frac{\Delta S_{A-B}}{\Delta S_{A-B}} = 180 - \tan^{-1}\frac{811}{76} = 180 - 43.2 = 136.8°$$

Exercise 10.3: A well is planned to be drilled as directional well. The well azimuth and horizontal departure were measured at a point to be 63° and 600 meters from kick off point, respectively. Another measurement was obtained at the target point and azimuth and horizontal departure were measured to be 52° and 221 m from the previous measuring point, respectively. Determine the equivalent well azimuth and horizontal departure of the target from the kick off point.

Solution:

Given data:

ε_{O-A} = 63°
ε_{A-B} = 52°
HD_{O-A} = 600 m
HD_{A-B} = 221 m

Required data:

ε_{O-B} = Well Azimuth
HD_{O-B} = Horizontal departure

To determine the equivalent well azimuth and horizontal departure, we should first calculate the well coordinates for the above two points. Well coordinates for point A is as follows:

$$\Delta E_{O-A} = HD_{O-A} \times \sin(\varepsilon_{O-A}) = 600 \times \sin 63 = 534.6\ m$$

$$\Delta N_{O-A} = HD_{O-A} \times \cos(\varepsilon_{O-A}) = 600 \times \cos(63) = 272.4\ m$$

Well coordinates for point B is as follows:

$$\Delta E_{A-B} = HD_{A-B} \times \sin(\varepsilon_{A-B}) = 221 \times \sin 52 = 174.2\ m$$

$$\Delta N_{A-B} = HD_{A-B} \times \cos(\varepsilon_{A-B}) = 221 \times \cos 52 = 136.1\ m$$

Now, well coordinates of the target can be determined by simply adding the east and north coordinates together as follows:

$$\Delta E_{O-B} = 534.6 + 174.2 = 708.8\ m$$

$$\Delta N_{O-B} = 272.4 + 136.1 = 408.5\ m$$

Equivalent well azimuth and horizontal departure of the well from the kick off point are equal to:

$$\varepsilon_{O-B} = \tan^{-1}\frac{\Delta E_{O-B}}{\Delta N_{O-B}} = \tan^{-1}\frac{708.8}{408.5} = 60.0°$$

$$HD_{O-B} = \sqrt{\Delta E^2{}_{O-B} + \Delta N^2{}_{O-B}} = \sqrt{708.8^2 + 408.5^2} = \mathbf{818.0\,m}$$

Example 10.4: A well is planned to be drilled as directional well. The well coordinates were designed to be 413 m towards the west and 688 m towards the north, respectively. After kick off the well, measurements were conducted and found that the current azimuth and horizontal departure were 324.5° and 341.3 m. Is this azimuth the same as the designed one? If the azimuth is not the same as designed, what should be the new degree from the current point to be used until completing the well to reach the designed target?

Solution:

Given data:

ΔW = *413 m*
ΔN = *688 m*
ε_{O-A} = 324.5°
HD_{O-A} = 341.3 *m*

Required data:

ε_{A-B} = Azimuth in degrees from point A to B

To know whether the current azimuth is same as the designed one, we should calculate the designed well azimuth using Eq. (10.2):

$$\varepsilon_{O-B} = 360 - \tan^{-1}\frac{\Delta W_{O-B}}{\Delta N_{O-B}} = 360 - \tan^{-1}\frac{413}{688} = 329°$$

Thus, the current azimuth is not as per the design. So we need to adjust the azimuth to reach the target. First we should determine the current well coordinates as follows:

$$\Delta W_{O-A} = HD_{O-A} \times \sin(360 - \varepsilon_{O-A}) = 341.3 \times \sin(360 - 324.5) = 198\ m$$

$$\Delta N_{O-A} = HD_{O-A} \times \cos(360 - \varepsilon_{O-A}) = 86.9 \times \cos(360 - 324.5) = 278\ m$$

To determine the new azimuth that should be adopted, we should subtract the current well coordinates from the designed well coordinates as follows:

$$\Delta W_{A-B} = 413 - 198 = 215\ m$$

$$\Delta N_{A-B} = 688 - 278 = 410\ m$$

Thus, the new azimuth that should be adopted from the current point to the well target is equal to:

$$\varepsilon_{A-B} = 360 - \tan^{-1}\frac{\Delta W_{A-B}}{\Delta N_{A-B}} = \tan^{-1}\frac{215}{410} = 332.3°$$

Exercise 10.5: Calculate the dogleg "*DL*" and the dogleg severity "*DLS*" of a section in a well that has the following information:

Point	Measured depth	Azimuth	Inclination
	ft	**degrees**	**degrees**
1	2950	310	44
2	2300	312	58

Solution:

Given data:

Information in the table above

Required data:

DL = Dogleg in degrees
DLS = Dogleg severity in degrees per 100 *ft*

Dogleg of the above section can be calculated using the following equation:

$$DL = cos^{-1}(\cos(I_2 - I_1) - (sinI_1 \times sinI_2 \times (1 - \cos(A_2 - A_1)))$$

$$DL = cos^{-1}(\cos(58 - 44) - (\sin 44 \times \sin 58 \times (1 - \cos(130 - 135)))$$
$$= 14.5°$$

Dogleg severity for the above section can be calculated using the following equation:

$$DLS = DL \times \frac{100}{MD} = 8.1 \times \frac{100}{3,300 - 2,950} = 4.15°$$

Exercise 10.6: A well is designed as a deviated well to reach a certain target with the maximum *DLS* of 2.5. If the design is to have KOP at 3,000 *ft*, an inclination angle of 48° at the end of curvature (EOC) depth of 4,750 *ft*, can the well be drilled with the above information? If the answer is no, at which depth the KOP should be moved? In addition, if based on geological information KOP depth must be at 3,000 *ft*; what will be the maximum inclination angle at the EOC depth?

Solution:

Given data:

DLS = 2.5°
KOP = 3,000 *ft*
I = 48°
EOC = 4,750 *ft*

Required data:

KOP in *ft*
Maximum inclination angle in degrees

To know whether the above data of the well is suitable to drill the well to the target without having *DLS* greater than the designed one, we can calculate the *DLS* at the current data as follows:

$$DLS = DL \times \frac{100}{EOC - KOP} = 48 \times \frac{100}{4,750 - 3,000} = 2.74°$$

In the above equation, the dogleg was set to be 48° because the section will start from zero inclination at *KOP* to 48° degrees at the *EOC*. From the above calculation, if we need to reach the target with inclination of 48°, we need to shift the *KOP* depth shallower than the designed depth. The new *KOP* depth can be calculated by setting the *DLS* to be 2.5° and looking for the *KOP* depth as follows:

$$\begin{aligned} KOP &= EOC - 100 \times \frac{DL}{DLS} \\ &= 4,750 - 100 \times \frac{48}{2.5} = \mathbf{2,830}\ \boldsymbol{ft} \end{aligned}$$

Because geologically, the *KOP* depth cannot be less than 3,000 *ft*, in this case we should keep the *KOP* at 3,000 *ft* and change the inclination angle to meet the *DLS* design requirement as follows:

$$I = DL = \frac{DLS \times (EOC - KOP)}{100} = \frac{2.5 \times (4,750 - 3,000)}{100} = \mathbf{43.75°}$$

Thus, to keep the *KOP* depth at 3,000 *ft*, inclination angle should be 43.75° in order to reach the top of the target at measured depth of 4,750 *ft*.

Exercise 10.7: The following data refer to a directionally drilled well:

KOP	=	2,750 *ft*
Northing coordinates of surface location	=	–4,350 *ft*
Easting coordinates of surface location	=	–5,550 *ft*

It is assumed that the co-ordinates of *KOP* are exactly similar to the co-ordinates of the surface. Five survey data were performed after the *KOPs* which are shown in the table below:

	Azimuth	**Inclination**	***MD***
Point	**Degrees**	**Degrees**	**ft**
KOP	310.0	0.0	2,750
1	312.0	9.0	3,250
2	314.0	18.0	3,750
3	317.0	27.0	4,250
4	315.5	36.0	4,750
5	315.0	45.0	5,250

Using the radius of curvature method, calculate the well path between the above points?

Solution:

Given data:

KOP = 2,750 *ft*
N = –4,350 *ft*
E = –5,550 *ft*
Table of the survey points

Required data:

Well path including true vertical depth, horizontal departure, and well co-ordinates

Well path calculation using radius of curvature method for the first measuring point is based on Eq. (10.24) through Eq. (10.29) as follows:

True vertical depth calculation, Eq. (10.24):

$$\Delta TVD_1 = \frac{180 \times D \times (\sin \propto_2 - \sin \propto_1)}{\pi(\propto_2 - \propto_1)}$$
$$= \frac{180 \times (3{,}250 - 2{,}750) \times (\sin 9 - \sin 0)}{\pi(9-0)} = 497.9\ ft$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta TVD_1 = 2{,}750 + 497.9 = 3{,}247.9\ ft$$

The distance toward north can be calculated using Eq. (10.25):

$$\Delta N = \frac{180^2 \times D \times (\cos \propto_1 - \cos \propto_2)(\sin \varepsilon_2 - \sin \varepsilon_1)}{\pi^2 (\propto_2 - \propto_1)(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{180^2 \times (3{,}250 - 2{,}750) \times (\cos 0 - \cos 9)(\sin 312 - \sin 310)}{\pi^2 (9-0)(312-310)}$$

$$= 25.7\ ft$$

The distance toward east can be calculated using Eq. (10.26):

$$\Delta E = \frac{180^2 \times D \times (\cos \propto_1 - \cos \propto_2)(\cos \varepsilon_1 - \cos \varepsilon_2)}{\pi^2 (\propto_2 - \propto_1)(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{180^2 \times (3{,}250 - 2{,}750) \times (\cos 0 - \cos 9)(\cos 310 - \cos 312)}{\pi^2 (9-0)(312-310)}$$

$$= -29.6\ ft$$

Horizontal departure can be calculated using Eq. (10.27):

$$HD_1 = \sqrt{\Delta N^2 + \Delta E^2} = \sqrt{25.7^2 + (-29.6)^2} = 39.2\ ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = -4{,}350 + 25.7 = -4{,}324.3\ ft$$

$$E_1 = KOP + \Delta E_1 = -5{,}550 + (-29.6) = -5{,}579.6\ ft$$

Similarly, we can calculate the well path for the rest of the points. The below Table has summarized the results for all the points:

	Δ*TVD*	***TVD***	***HD***	**Cum. *HD***	**Δ*N***	**Northing**	**Δ*E***	**Easting**
Point	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**
KOP	2750	2750	0	0.0		–4350		–5550
1	497.9	3247.9	39.2	39.2	25.7	–4324.3	–29.6	–5579.6
2	485.7	3733.6	116.6	155.8	79.5	–4244.8	–85.3	–5664.8
3	461.5	4195.1	191.1	346.9	136.3	–4108.5	–134.0	–5798.8
4	425.9	4621.0	261.0	607.9	188.5	–3919.9	–180.5	–5979.3
5	379.8	5000.8	324.4	932.3	230.4	–3689.6	–228.4	–6207.6

Exercise 10.8: Using the data of exercise 10.7 and minimum curvature method, calculate the well path between the above points. Compare the results with that obtained in exercise 10.7.

Solution:

Given data:

KOP = 2,750 *ft*
N = –4,350 *ft*
E = –5,550 *ft*
Table of the survey points

Required data:
Well path including true vertical depth, horizontal departure, and well co-ordinates.

Well path calculation using minimum curvature method for the first measuring point is based on Eq. (10.30) through Eq. (10.38) as follows:

True vertical depth calculation, Eq. (10.30), Eq. (10.33) and Eq. (10.34) as follows:

$$\beta_1 = \cos^{-1}[\cos(\alpha_2 - \alpha_1) - (\sin\alpha_1 \times \sin\alpha_2 \times (1 - \cos(\varepsilon_2 - \varepsilon_1))]$$
$$= \cos^{-1}[\cos(9-0) - (\sin 0 \times \sin 9 \times (1 - \cos(312 - 310))] = 9$$

$$RF_1 = \frac{2\times 180}{\beta\pi} \times \tan\frac{\beta}{2} = \frac{2\times 180}{\pi \times 9} \times \tan\frac{9}{2} = 1.0$$

$$\Delta TVD_1 = \frac{D}{2} \times (\cos \propto_1 + \cos \propto_2) \times RF$$
$$= \frac{(3{,}250 - 2{,}750)}{2} \times (\cos 0 + \cos 9) \times 1.0$$
$$= 497.9\ ft$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta TVD_1 = 2{,}750 + 497.9 = 3{,}247.9\ ft$$

The northing distance can be calculated using Eq. (10.31):

$$\Delta N = \frac{D}{2}[(\sin \propto_2 \cos\varepsilon_2) + (\sin\alpha_1 \cos\varepsilon_1)]$$
$$= \frac{3{,}250 - 2{,}750}{2}[(\sin 9 \cos 312) + (\sin 0 \cos 310)]$$
$$= 25.7\ ft$$

The easting distance can be calculated using Eq. (10.32):

$$\Delta N = \frac{D}{2}[(\sin \propto_2 \sin\varepsilon_2) + (\sin\alpha_1 \sin\varepsilon_1)]$$
$$= \frac{3{,}250 - 2{,}750}{2}[(\sin 9 \sin 312) + (\sin 0 \cos 310)]$$
$$= -29.6\ ft$$

Horizontal departure can be calculated using Eq. (10.27):

$$HD_1 = \sqrt{\Delta N^2 + \Delta E^2} = \sqrt{25.7^2 + (-29.6)^2} = 39.2\ ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = -4{,}350 + 25.7 = -4{,}324.3\ ft$$

$$E_1 = KOP + \Delta E_1 = -5{,}550 + (-29.6) = -5{,}579.6\ ft$$

Similarly, we can calculate the well path for the rest of the points. The below Table is summarized the results for all the points:

Point	β	*RF*	Δ*TVD*	*TVD*	***HD***	Cum. *HD*	Δ*N*	Northing	Δ*E*	Easting
	Degrees	fraction	ft	ft	ft	ft	ft	ft	ft	ft
KOP	0.00	–	2750	2750	0	0.0	−4350	−4350	−5550	−5550
1	9.00	1.00	497.9	3247.9	39.2	39.2	25.7	−4324.3	−29.6	−5579.6
2	9.01	1.00	485.7	3733.6	116.6	155.8	79.5	−4244.8	−85.4	−5665.0
3	9.07	1.00	461.5	4195.1	191.1	346.9	136.3	−4108.5	−134.1	−5799.1
4	9.03	1.00	425.9	4621.0	261.0	607.9	188.5	−3919.9	−180.7	−5979.7
5	9.01	1.00	379.8	5000.8	324.4	932.3	230.4	−3689.6	−228.6	−6208.4

By comparing the results of calculations of radius of curvature and minimum curvature methods, we can find that there are very small differences in the results. In general we can see that both methods gave exactly same results.

Exercise 10.9: The data below refer to a deviated drilled well:

KOP = 5,250 *ft*
Northing coordinates of surface location = 900 *ft*
Easting coordinates of surface location = 500 *ft*

The co-ordinates of *KOP* were exactly similar to the co-ordinates of the surface. Five survey data were performed after the *KOP* which are shown in the table below:

	Azimuth	Inclination	*MD*
Point	**Degrees**	**Degrees**	**ft**
KOP	156.0	0.0	5250
1	152.0	11.0	5900
2	154.0	25.0	6550
3	157.0	41.0	7200
4	156.0	58.0	7850
5	155.0	69.0	8500

Using the average angle method, calculate the well path between the above points.

<u>Solution</u>:

Given data:

KOP = 5,250 *ft*
N = 900 *ft*
E = 500 *ft*
Table of the survey points

Required data:

Well path including true vertical depth, horizontal departure, and well co-ordinates.

Well path calculation using average angle method for the first measuring point is based on Eq. (10.21) through Eq. (10.23) as follows:

True vertical depth calculation, Eq. (10.23) as follows:

$$\Delta TVD_1 = \Delta MD_1 \times \cos\left(\frac{\propto_1 + \propto_2}{2}\right)$$

$$= (5{,}900 - 5{,}250) \times \cos\left(\frac{0+11}{2}\right) = 647\ ft$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta TVD_1 = 5{,}250 + 647 = 5{,}897\ ft$$

The northing distance can be calculated using Eq. (10.21):

$$\Delta N_1 = \Delta MD_1 \times \sin\left(\frac{\propto_1 + \propto_2}{2}\right)\cos\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right)$$

$$= (5{,}900 - 5{,}250) \times \sin\left(\frac{0+11}{2}\right) \times \cos\left(\frac{156+152}{2}\right) = -56.0\,ft$$

The easting distance can be calculated using Eq. (10.22):

$$\Delta E_1 = \Delta MD_1 \times \sin\left(\frac{\propto_1 + \propto_2}{2}\right)\sin\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right)$$

$$= (5{,}900 - 5{,}250) \times \sin\left(\frac{0+11}{2}\right) \times \sin\left(\frac{156+152}{2}\right) = 27.3\,ft$$

Horizontal departure can be calculated using the following equation:

$$HD_1 = \sqrt{\Delta N^2 + \Delta E^2} = \sqrt{(-56.0)^2 + 27.3^2} = 62.3\,ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = 900 + (-56.0) = 844.0\,ft$$

$$E_1 = KOP + \Delta E_1 = 500 + 27.3 = 527.3\,ft$$

Similarly, we can calculate the well path for the rest of the points. Table below summarize the results for all the points:

	Δ*TVD*	***TVD***	***HD***	**Cum. *HD***	**Δ*N***	**Cum. *N***	**Δ*E***	**Cum. *E***
Point	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**	**ft**
KOP	5250	5250	0	0.0	900	900	500	500
1	647.0	5897.0	62.3	62.3	–56.0	844.0	27.3	527.3
2	618.2	6515.2	200.9	263.2	–179.0	665.0	91.2	618.5
3	545.1	7060.3	354.0	617.2	–322.1	342.9	146.8	765.3
4	422.1	7482.5	494.3	1111.4	–453.3	–110.4	197.1	962.4
5	290.0	7772.5	581.7	1693.1	–529.3	–639.7	241.2	1203.6

Exercise 10.10: Using the data of exercise 10.9 and balanced tangential method, calculate the well path between the above points? Compare the results with that obtained in exercise 10.9.

<u>Solution</u>:

Given data:

KOP = 5,250ft
N = 900 *ft*
E = 500 *ft*
Table of the survey points

Required data:

Well path including true vertical depth, horizontal departure, and well co-ordinates. Well path calculation using average angle method for the first measuring point is based on Eq. (10.39) through Eq. (10.41) as follows:

True vertical depth calculation, Eq. (10.39) as follows:

$$\Delta TVD_1 = \frac{\Delta MD_1}{2} \times (\cos \propto_1 + \cos \propto_2)$$
$$= \frac{5{,}900 - 5{,}250}{2} \times (\cos 0 + \cos 11) = 644.0\,ft$$

True vertical depth of first measuring point is equal to:

$$TVD_1 = KOP + \Delta TVD_1 = 5{,}250 + 644.0 = 5{,}894.0\,ft$$

The northing distance can be calculated using Eq. (10.40):

$$\Delta N_1 = \frac{\Delta MD_1}{2} \times (\sin \propto_1 \cos \beta_1 + \sin \propto_2 \cos \beta_2)$$
$$= \frac{5{,}900 - 5{,}250}{2} \times (\sin 0 \cos 156 + \sin 11 \cos 152) = -54.8\,ft$$

The easting distance can be calculated using Eq. (10.41):

$$\Delta E_1 = \frac{\Delta MD_1}{2} \times (\sin \propto_1 \sin \beta_1 + \sin \propto_2 \sin \beta_2)$$
$$= \frac{5{,}900 - 5{,}250}{2} \times (\sin 0 \sin 156 + \sin 11 \sin 152) = 29.1\,ft$$

Horizontal departure can be calculated using the following equation:

$$HD_1 = \sqrt{\Delta N^2 + \Delta E^2} = \sqrt{(-54.8)^2 + 29.1^2} = 62.0\,ft$$

Northing and easting coordinates from the reference point is equal to:

$$N_1 = KOP + \Delta N_1 = 900 + (-54.8) = 845.2\,ft$$

$$E_1 = KOP + \Delta E_1 = 500 + 29.1 = 529.1\,ft$$

Similarly, we can calculate the well path for the rest of the points. The table below summarizes the results for all the points:

Point	ΔTVD ft	TVD ft	HD ft	Cum. HD ft	ΔN ft	Cum. N ft	ΔE ft	Cum. E ft
KOP	5250	5250	0	0.0	900	900	500	500
1	644.0	5894.0	62.0	62.0	–54.8	845.2	29.1	529.1
2	613.6	6507.6	199.3	261.4	–178.2	667.0	89.3	618.4
3	539.8	7047.4	350.5	611.8	–319.7	347.3	143.5	762.0
4	417.5	7464.9	488.8	1100.6	–448.1	–100.7	195.4	957.4
5	288.7	7753.6	579.0	1679.6	–526.8	–627.5	240.3	1197.7

By comparing the results of calculations of average angle and balanced tangential methods, we can find that there are noticeable differences in the results. The difference in the true vertical depth between the two methods is around 19 *ft*, whereas the difference in horizontal departure is around 13 *ft*.

Chapter 11: Well Drilling Costs Analysis

Exercise 11.1: A section of 4,000 *ft* length and 8 ½" size in a well has to be drilled. There are three types of bits available with the following information.

Property	Bit type I	Bit type II	Bit type III
ROP, *ft*/hr	40.0	47.0	49.0
Longest the bit can drill, *ft*	1,300	1,400	1,100
Cost, $/bit	45,000	60,000	50,000

If the trip time is 1.5 hours for each 1,000 *ft* depth, total non-rotating time for any bit that run in hole is 0.2 day and the fixed operating cost of the rig is $80,500/day; calculate the following:

a. Number of bits you are going to use from each type, and also the time to drill the section
b. Which bit should be selected, and why? ***Answers: Bit type II***

Solution:

Given data:

Data in the table

t_t = 1.5 hr/1000ft

t_c = 0.2 day/bit

ΔD = 4,000 *ft*

C_r = $80,500/day

Required data:

N_{bit} = Number of bit for each bit
t_d = Drilling time for each bit
C_f = Cost per foot for each bit

Number of bits to be used from each type to drill above section can be estimated simply by dividing the well section by the depth length that can be drilled by each bit. Mathematically:

$$N_{bit} = \frac{\Delta D}{l_{bit}}$$

Based on the length that can be drilled by each bit, to drill this section we need 4 bits from bit type I and III and 3 bits from the bit type II. For all the bit types, the last bit run will be used to complete the section and it can be reused.

To calculate the time to drill above section using any type of the above bits, we should know that there is fixed time of 0.2 day for each well. In addition to that, trip in time is applied for the bits. Trip out time will not be applied to the first bit run of each bit type, but for the rest of the bit runs of the same type. Mathematically drilling time for the first bit run is equal to:

$$\begin{aligned} t_{b1\_run1} &= \text{rotating time} + \text{non rotating time} + \text{trip in time} \\ &= \frac{l_{bit1}}{ROP_{b1}} + t_c + t_t \times \frac{\Delta D_1}{1000} \end{aligned}$$

Drilling time for the rest of the bit runs equals to:

$$\begin{aligned} & t_{b1\_run2} \\ &= \text{rotating time} + \text{non rotating time} + \text{trip in time} + \text{trip out time} \\ &= \frac{l_{bit1}}{ROP_{b1}} + t_c + t_t \times \frac{\Delta D_1}{1000} + t_t \times \frac{\Delta D_2}{1000} \end{aligned}$$

where ΔD_1 is the depth drilled by previous bit, and ΔD_2 is the new depth drilled by the new bit. Tables below summarize the calculation results for each bit type.

Table 1 Results for Bit type I.

	Time	Drilled depth	Remaining Depth	Comm. Depth
Bit type I	**hrs**	**ft**	**ft**	**ft**
Bit 1	39.3	1300	2700	1300
Bit 2	43.15	1300	1400	2600
Bit 3	47.05	1300	100	3900
Bit 4	19.15	100	0	4000
Total time for Bit type I is 148.6 hrs				

Drilling cost per foot can be calculated using Eq. (11.2):

$$C_f = \frac{C_b + C_r(t_d + t_c + t_t)}{\Delta D}$$

The table below shows the drilling cost of the above section using any of the above bit types:

From the above table, although bit type I has the lowest *ROP* among them, bit type II can drill the section with the minimum possible cost as compared to the other two bit types.

Table 2 Results for Bit type II.

	Time	**Drilled depth**	**Remaining Depth**	**Comm. Depth**
Bit type II	**hrs**	**ft**	**ft**	**ft**
Bit 1	35.69	1400	2600	1400
Bit 2	40.89	1400	1200	2800
Bit 3	40.53	1200	0	4000
Total time for Bit type II is 118.1 hrs				

Table 3 Results for Bit type III.

	Time	**Drilled depth**	**Remaining Depth**	**Comm. Depth**
Bit type III	**hrs**	**ft**	**ft**	**ft**
Bit#1	28.90	1100	2900	1100
Bit#2	32.20	1100	1800	2200
Bit#3	35.50	1100	700	3300
Bit#4	30.04	1100	0	700
Total time for Bit type II is 126.6 hrs				

Table 4 Cost per foot.

	Cost
Bit type	**USD/foot**
Type I	169.6
Type II	141.9
Type III	151.6

Exercise 11.2: Table below shows depths and costs recorded of 13 wells drilled in a certain area:

Well No.	Depth	Cost
	ft	USD
Well 1	7,850	2,279,249
Well2	8,100	2,303,077
Well 3	6,950	2,059,568
Well 4	7,100	2,066,272
Well 5	8,500	2,411,976
Well 6	7,250	2,102,645
Well 7	7,400	2,108,857
Well 8	7,000	2,110,545
Well 9	6,500	2,049,193
Well 10	8,800	2,319,487
Well 11	8,100	2,247,787
Well 12	7,800	2,201,076
Well 13	7,750	2,227,648

Use the above information to estimate well cost of a well that planned to be drilled to a depth of 9,150 *ft*.

Solution:

Given data:

Data in the table

$D = 9{,}150\ ft$

Required data:

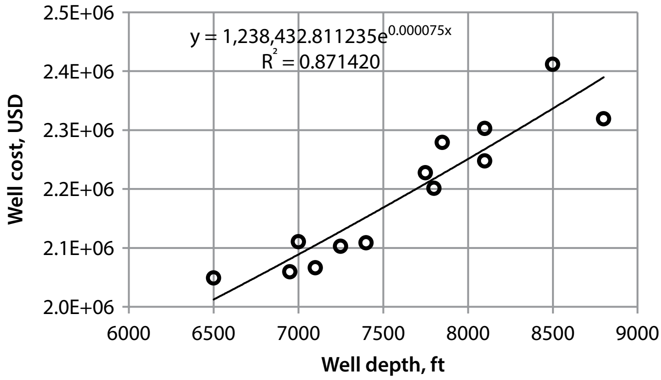
C_{dc} = Well cost

The first step is to plot well costs vs. depth in Cartesian coordinates to find out their relationship. The Figure below shows the relationship between the well cost and well depth.

It is convenient to use exponential trend between cost and depth. So from the above plot, we can estimate the constants of the exponential equation that can be used to estimate the cost of the well that is planned to be drilled to a depth of 9,150 *ft*. Well cost can be estimated using Eq. (11.3) as follows:

$$C_{dc} = a_{dc} e^{b_{dc} D} = 1{,}238{,}432.8 e^{0.000075 \times 9{,}150} = \mathbf{2{,}459{,}841}\, \boldsymbol{USD}$$

So, from the above information, the estimated well cost is around USD 2,459,841. This can give us a rough estimate of what should be the expected range of well cost of that well.



Exercise 11.3: Two wells drilled to depths of 9,500 and 9,100 *ft*. Their costs were calculated to be 2,879,030 and 2,773,897 USD, respectively. If it is found that depth and cost relationship were follow exponential relation. Calculate the well cost of a well that is planned to be drilled to a depth of 10,100 *ft*.

<u>**Solution**</u>**:**

Given data:

D_{w1} = 9,500 *ft*
D_{w2} = 9,100 *ft*
D_{w3} = 10,100 *ft*
C_{dcw1} = 2,879,030 USD
C_{dcw2} = 2,773,897 USD

Required data:

C_{dcw3} = Well cost of the third well

From the information of the two wells, we can determine the values of the constants of Eq. (11.3). From the information of well 1, we can create the following equation:

$$C_{dcw1} = a_{dc}e^{b_{dc}D_{w1}}$$

$$2,879,030 = a_{dc}e^{b_{dc}\times 9,500}$$

And from the information of well 2, we can create the following equation:

$$C_{dcw2} = a_{dc}e^{b_{dc}D_{w2}}$$

$$2,773,897 = a_{dc}e^{b_{dc}\times 9,100}$$

By solving the above two equations simultaneously, we can determine the values of the constants.

$$\frac{2,879,030}{2,773,897} = e^{b_{dc}(9,500-9,100)}$$

$$1.038 = e^{b_{dc}\times 400}$$

$$b_{dc} = 9.32\times 10^{-5}\ ft^{-1}$$

Substituting the value of "*b*", we can get "*a*" constant as follows:

$$2{,}879{,}030 = a_{dc}e^{0.0000932\times 9{,}500}$$

$$a_{dc} = 1{,}187{,}741\ USD$$

Thus, after we determined the values of the constants, we can easily estimate the cost of the third well using a similar approach with the above equation as follows:

$$C_{dcw3} = a_{dc}e^{b_{dc}\times D_{w3}} = 1{,}187{,}741\times e^{0.0000932\times 10{,}100} = \mathbf{3{,}044{,}246}\ \boldsymbol{USD}$$

Exercise 11.4: Table below summarizes the planned depths and days for a development well need to be drilled to a depth of 12,500 *ft*:

Operations	Depth ft	Duration days	Cumm. days
Start of operations	0	0.00	0.00
Rig move and rig up	0	3.00	3.00
Drilling 26" hole	50	0.14	3.14
Evaluation, Casing & cementing of 20" conductor	50	1.01	4.15
Operations	**Depth ft**	**Duration days**	**Cumm. days**
Drilling 17.5" surface hole	1500	1.47	5.62
Casing & cementing 13 3/8" surface casing	1500	1.75	7.37
Drilling 12 ¼" Intermediate hole	7250	7.26	14.63
Casing & cementing 9 5/8" intermediate casing	7250	3.71	18.34
Drilling 8 ½" production hole	12500	9.11	27.45
Casing & cementing 7" production casing	12500	5.08	32.54

All operation durations remains as planned except average *ROP* for conductor, surface, intermediate and production holes were calculated to be 11, 44, 30, and 18 *ft*/hr; respectively. Calculate the actual drilling days, and then plot the planned drilling days versus the actual.

Solution:

Given data:
Data in the table and the values of ROP for all the drilled sections

Required data:
Actual drilling days

Because rig move and rig up, evaluation, casing and cementing operations remains as per the plan, what we need is to calculate drilling days for each section from the average *ROP*. Drilling days for each section can be calculated using the following equation:

$$t_{section} = \frac{D_{section}}{ROP_{section}}$$

Drilling days for conductor section equal to:

$$t_{conductor} = \frac{D_{conductor}}{ROP_{conductor}} = \frac{50}{11 \times 24} = 0.19\ day$$

Drilling days for surface section equal to:

$$t_{surface} = \frac{D_{surface}}{ROP_{surface}} = \frac{1,500 - 50}{44 \times 24} = 1.37\ days$$

Drilling days for intermediate section equal to:

$$t_{intermediate} = \frac{D_{intermediate}}{ROP_{intermediate}} = \frac{7,250 - 1,500}{30 \times 24} = 7.99\ days$$

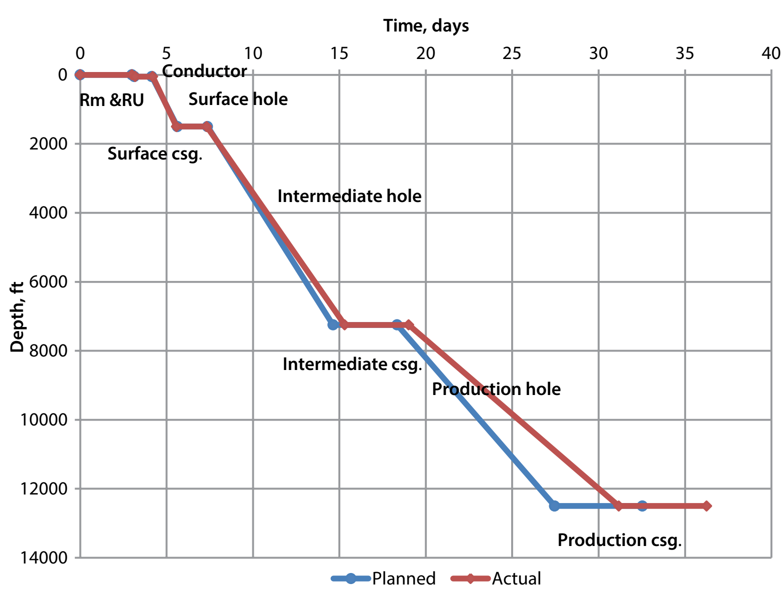
Drilling days for production section equal to:

$$t_{production} = \frac{D_{production}}{ROP_{production}} = \frac{12,500 - 7,250}{18 \times 24} = 12.15\ days$$

Thus actual drilling days equal to:

$$\begin{aligned} t_{drilling} &= 3.0 + 0.19 + 1.01 + 1.37 + 1.75 + 7.99 + 3.71 + 12.15 + 5.08 \\ &= 36.25\ days \end{aligned}$$

Figure below shows the planned and actual drilling days of this well:



Exercise 11.5: An oil operator invested certain money to be used three years from now to complete and install surface facilities for four wells. The required future money was estimated to be USD 10,000,000 and the market interest rate is 8% compounded 4 times. If the company decided not to use the money at the third year and continue investing the same amount and use it in the fourth year from now, what is the amount that is invested by the operator, and the value of the money four years from now if the interest rate remains the same?

Solution:

Given data:

V_f = Value of money one year from now = USD 10,000,000
i = Interest rate = 8%
m = Compounding = 4
n = Number of years = 3

Required data:

V_p = *Present value of money*
V_f = Value of money two years from now

To estimate the value of the money four years from today, we must first calculate the money now. We can use Eq. (11.18) and the value of the money three years from now as follow:

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm}$$

$$10{,}000{,}000 = V_p\left(1+\frac{0.08}{4}\right)^{3\times 4}$$

$$V_p = 7{,}884{,}932\ USD$$

Thus, the value of the money four years from now can be estimated using the same equation and the money at the present as follows:

$$V_f = V_p\left(1+\frac{i}{m}\right)^{nm} = 7{,}884{,}932\left(1+\frac{0.08}{4}\right)^{4\times 4} = 10{,}824{,}322\ USD$$

Exercise 11.6: An oil company is planning to invest money to drill five exploration wells. After two years of investment, the company took two-fifths of the money and drilled two exploration wells, and continued investing the rest of the money. After another one year, the company used the rest of the money which was 12,899,965 USD to drill the last three wells. If the interest rate is 12% compounded continuously, what is the amount of money that is invested by the company three years from now?

Solution:

Given data:

V_f = Value of money after three years = USD 12,899,965

i = Interest rate = 12%

Required data:

V_p = *Amount of money invested*

To determine the amount of money invested at the beginning to drill these five wells, we should first determine the value of the money after two years of investment using the value of the money after three years of 12,899,965. This value of money can be estimated using Eq. (11.19):

$$V_p = V_f e^{-in} = 12{,}899{,}965 \times e^{(-0.12\times1)} = 11{,}441{,}242\ USD$$

Because the company used two-fifths of the invested money after two years, the money after two years of investment was:

$$V_{2\,yrs} = 11{,}441{,}242 \times \frac{5}{3} = 19{,}068{,}737\ USD$$

Thus, the invested amount of money is equal to:

$$V_p = V_f e^{-in} = 19{,}068{,}737 \times e^{(-0.12\times2)} = 15{,}000{,}000\ USD$$

Exercise 11.7: An oil operator is planning to drill an oil well. The estimated cost for drilling, completing, and surface facilities were estimated to be 7,000,000 USD. The well is expected to produce 360 *bbls*/day for the first year, and follows an exponential decline trend with a nominal decline rate of 0.32 year^{-1}. The average price of one barrel is 50 USD and the cost of producing one barrel is 21 USD, and they are assumed to be constant throughout the production life of the well. The economic production rate is set to be 100 *bbls*/d. If the operator decided that the feasible profit must be 40% of the invested amount, determine the feasibility of investing on this well. Use an interest rate of 20% compounded continuously.

Solution:

Given data:

C_w	=	Total well cost	=	USD 7,000,000
q	=	Production rate	=	360 *bbls*/d
D_i	=	Production decline rate	=	0.2 year^{-1}
Pr_{oil}	=	Oil price	=	50 USD/bbl
c_{op_bbl}	=	Cost of producing one barrel	=	21 USD/bbl
q_a	=	Economic production rate	=	100 *bbls*/d
i	=	Interest rate	=	20%

Required data:

Feasibility of drilling the well

To check the feasibility of drilling the well based on the designed percentage, first we need to know how long the well can be produced. After that we need to bring back the revenue from oil production to present money in order to make comparison. To determine how long the well can be produced, we should calculate the time to reach the economic rate by using exponential decline rate equation as follows:

$$q_n = q_i \exp D_i t$$

So by using the above equation, we can determine the production life of the well. Table below shows the production for each year:

Year	rate	Yearly production
	bpd	*bbls*
1	360.0	131,490
2	261.4	95,481
3	189.8	69,334
4	137.8	50,347
5	100.1	36,559

Thus, from above table, the well can produce for five years before it reaches the economic production rate. Now we should calculate the future revenues of oil production for each year, and then bring this money to the present value of the money to make a proper comparison. We are going to use Eq. (11.19) of continuous compounding as follows:

$$V_p = V_f \times e^{-in}$$

What we should do is calculate the future revenue for each year. Then we should change the revenue of each year to the present value. The net profit of one barrel of oil equals:

$$\begin{aligned} \text{Net profit} &= oil\ price - oil\ cost \\ &= 50 - 21 = 29\ USD \end{aligned}$$

The revenue of the first year will not be affected by the interest rate, only the revenue of the rest of years. The table below shows the revenue of each year and the present value of each year:

Year	Revenue	V_p
	USD	
1	3,813,210	3,813,210
2	2,768,959	2,267,032
3	2,010,677	1,347,797
4	1,460,051	80,1293
5	1,060,215	476,385.1

The total amount of present revenue is equal to 8,988,937 USD. The net profit of the project is equal to:

$$\begin{aligned}\text{Net profit of project} &= revenue - investment\\ &= 8,988,937 - 7,000,000\\ &= 1,988,937\ USD\end{aligned}$$

The percentage of the profit based on the invested money equals to:

$$\text{Profit in \%} = \frac{\text{profit}}{\text{investment}} \times 100 = \frac{1,988,937}{7,000,000} \times 100 = 28.4\%$$

As the expected profit percentage is less than the designed, drilling the well is not feasible.

Exercise 11.8: A rig contractor bought a new drilling rig to be rented to an oil operator for a long-term drilling campaign. The purchasing price of the rig is 14,000,000 USD, and it will be rented for 16,500 USD per day. The rig contractor assumed daily consumables and personnel fees of 2,500 and 2,867 USD, respectively. If the interest rate is assumed to be 13% compounded continuously, and rig daily rate, consumables and personnel fees remains without any changes, how many years required by the rig contractor to payback their invested money? And what will be the contractor's net profit after eight years?

Solution:

Given data:

Pr_{rig}	=	Purchasing cost of the rig	=	USD 15,000,000
r_{rig}	=	Daily rig rate	=	16,500 USD/d
c_{rig}	=	Daily rig consumables	=	2,500 USD/d
$pers_{rig}$	=	Daily rig personnel fees	=	2,867 USD/d
i	=	Interest rate	=	13.0%

Required data:

Payback period
Net profit after 8 years

To determine the payback period, we are going to calculate the yearly revenue and then we should change it to present value amount using the specified interest rate. In this example interest rate, daily rig rate, consumables and personnel fees are assumed to be constant. In fact, these three values can change with time depending on the market, but we assumed them to be constant for simplicity of calculations. To determine the yearly profit, we should first calculate the daily profit as follows:

$$\begin{aligned}\text{Daily profit} &= \text{daily rig rate} - \text{daily consumables} - \text{personnel fees}\\ &= 16,500 - 2,500 - 2,867 = 11,133\ USD\end{aligned}$$

Yearly profit is equal to:

$$\text{Yearly profit} = \text{daily profit} \times 365.25$$
$$= 11{,}133 \times 365.25 = 4{,}066{,}328\ USD$$

The above amount will be the profit that the contractor will gain every year. But to determine the payback period of the invested money, these amounts should be converted to present value amount in order to have one basis. Thus, the yearly profit of each year should be converted to present amount using Eq. (11.19) as follows:

$$V_p = V_f \times e^{-in}$$

The table below summarizes the yearly profit converted to present value amount:

Year	Yearly future profit	Present value of future profit	Payback amount	Remaining amount
	USD	USD	USD	USD
1	4,066,328	3,570,624	3,570,624	10,429,376
2	4,066,328	3,135,349	6,705,973	7,294,027
3	4,066,328	2,753,135	9,459,109	4,540,891
4	4,066,328	2,417,516	11,876,624	2,123,376
5	4,066,328	2,122,809	13,999,434	566

From the above table, we can see that the rig contractor will pay back their invested money after about five years from now. To calculate the net profit after eight years of operations, we will calculate the yearly profit and then convert to present value. The table below shows the yearly profit staring from the sixth year up to the tenth year:

Year	Yearly future profit	Present value of future profit	Remaining amount
	USD	USD	USD
5	4,066,328	1,864,029	1,863,463
6	4,066,328	1,636,796	3,500,259
7	4,066,328	1,437,263	4,937,521
8	4,066,328	1,864,029	6,199,575

From the above table, the rig contractor can earn about 6,199,575 USD after eight years of operations based on present value amount.

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Appendix B

(MCQs Solutions)

Chapter 1: Drilling Methods

1.6 MCQs

Answers: 1c, 2d, 3a, 4c, 5b, 6b, 7c, 8d, 9b, 10d, 11a, 12b, 13c, 14a, 15d, 16d, 17d, 18d, 19b, 20a, 21d, 22d, 23d, 24d, 25c, 26d, 27c, 28c, 29d, 30a, 31c.

Chapter 2: Drilling Methods

2.4.3 MCQs

Answers: 1d, 2c, 3a, 4b, 5a, 6b, 7b, 8d, 9d, 10a, 11b, 12b, 13c, 14d, 15a, 16d, 17c, 18b, 19a, 20d, 21b, 22c, 23a, 24d, 25a, 26a, 27c, 28b, 29a, 30c, 31b, 32c, 33b, 34c, 35b.

Chapter 3: Drilling Fluid

3.5.3 MCQs

Answers: 1b, 2a, 3b, 4d, 5c, 6c, 7d, 8a, 9b, 10a, 11d, 12c, 13b, 14a, 15d, 16a, 17c, 18d, 19b, 20a, 21c, 22d, 23a, 24b, 25d, 26a, 27d, 28d, 29c, 30a, 31a, 32b, 33d, 34d, 35c, 36b, 37c, 38a, 39b, 40a, 41a.

Chapter 4: Drilling Hydraulics

4.4.3 MCQs

Answers: 1d, 2a, 3d, 4b, 5c, 6a, 7d, 8b, 9a, 10c, 11d, 12b, 13c, 14a, 15c, 16b, 17b, 18a, 19d, 20c, 21a, 22c, 23b, 24b, 25d, 26d, 27d, 28c, 29a, 30b, 31a, 32d, 33b, 34c, 35d, 36b, 37d, 38b, 39c, 40d.

Chapter 5: Well Control and Monitoring Program

5.4.3 MCQs

Answers: 1c, 2b, 3a, 4d, 5b, 6a, 7c, 8b, 9c, 10b, 11b, 12a, 13d, 14d, 15c, 16d, 17b, 18c, 19a, 20c, 21a, 22b, 23d, 24c, 25d, 26b, 27a, 28c, 29c, 30d, 31d, 32c, 33b, 34d, 35a, 36b, 37a, 38d, 39c, 40a.

Chapter 6: Formation Pore and Fractures Pressure Estimation

6.4.3 MCQs

Answers: 1d, 2a, 3a, 4d, 5c, 6b, 7c, 8a, 9b, 10d, 11b, 12d, 13d, 14d, 15b, 16b, 17d, 18a, 19c, 20a, 21b, 22b, 23d, 24c, 25a, 26c, 27a, 28b, 29d, 30d, 31c, 32b, 33c, 34b, 35b, 36c, 37b, 38a, 39d, 40b.

Chapter 7: Basics of Drillstring Design

Answers: 1b, 2d, 3a, 4c, 5a, 6a, 7d, 8c, 9b, 10b, 11c, 12d, 13b, 14c, 15a, 16d, 17a, 18d, 19b, 20c, 21d, 22c, 23b, 24a, 25d, 26c, 27a, 28b, 29c, 30a, 31c, 32b, 33a, 34b, 35c, 36a, 37a, 38c, 39b, 40d.

Chapter 8: Casing Design

8.4.3 MCQs

Answers: 1c, 2b, 3a, 4c, 5b, 6a, 7d, 8c, 9b, 10b, 11b, 12a, 13c, 14b, 15d, 16b, 17d, 18b, 19c, 20b, 21d, 22d, 23b, 24a, 25c, 26a, 27d, 28a, 29c, 30b, 31d, 32d, 33b, 34a, 35c, 36b, 37a, 38d, 39c, 40b.

Chapter 9: Cementing

9.4.3 MCQs

Answers: 1a, 2b, 3c, 4c, 5b, 6a, 7d, 8c, 9b, 10c, 11a, 12b, 13c, 14d, 15d, 16b, 17a, 18b, 19b, 20c, 21d, 22a, 23c, 24d, 25b, 26a, 27d, 28b, 29d, 30c.

Chapter 10: Horizontal and Directional Drilling

10.4.3 MCQs

Answers: 1c, 2a, 3b, 4d, 5d, 6b, 7d, 8c, 9a, 10a, 11c, 12a, 13d, 14d, 15b, 16d, 17b, 18a, 19a, 20c, 21b, 22a, 23c, 24a, 25d, 26b, 27c, 28d, 29a, 30d.

Chapter 11: Well Drilling Cost Analysis

11.4.3 MCQs

Answers: 1a, 2c, 3c, 4b, 5b, 6a, 7c, 8b, 9c, 10d, 11b, 12c, 13a, 14d, 15d.

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