

Randy Mayes · Daniel Rixen · Matt Allen *Editors*

Topics in Experimental Dynamic Substructuring, Volume 2

Proceedings of the 31st IMAC, A Conference on Structural
Dynamics, 2013



Conference Proceedings of the Society for Experimental Mechanics Series

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ISSN 2191-5644 ISSN 2191-5652 (electronic)
ISBN 978-1-4614-6539-3 ISBN 978-1-4614-6540-9 (eBook)
DOI 10.1007/978-1-4614-6540-9
Springer New York Heidelberg Dordrecht London

Library of Congress Control Number: 2013939185

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Preface

Topics in Experimental Dynamic Substructuring, Volume 2: Proceedings of the 31st IMAC, A Conference on Structural Dynamics, 2013 represents one of the seven volumes of technical papers presented at the 31st IMAC, a conference and exposition on structural dynamics, 2013, organized by the Society for Experimental Mechanics and held in Garden Grove, California, from February 11 to 14, 2013. The full proceedings also include volumes on nonlinear dynamics; dynamics of bridges; dynamics of civil structures; model validation and uncertainty quantification; special topics in structural dynamics; and modal analysis.

Each collection presents early findings from experimental and computational investigations on an important area within structural dynamics. Substructuring is one of these areas.

Substructuring is a general paradigm in engineering dynamics where a complicated system is analyzed by considering the dynamic interactions between subcomponents. In numerical simulations, substructuring allows one to reduce the complexity of parts of the system in order to construct a computationally efficient model of the assembled system. A subcomponent model can also be derived experimentally, allowing one to predict the dynamic behavior of an assembly by combining experimentally and/or analytically derived models. This can be advantageous for subcomponents that are expensive or difficult to model analytically. Substructuring can also be used to couple numerical simulation with real-time testing of components. Such approaches are known as hardware-in-the-loop or hybrid testing.

Whether experimental or numerical, all substructuring approaches have a common basis, namely the equilibrium of the substructures under the action of the applied and interface forces and the compatibility of displacements at the interfaces of the subcomponents. Experimental substructuring requires special care in the way the measurements are obtained and processed in order to assure that measurement inaccuracies and noise do not invalidate the results. In numerical approaches, the fundamental quest is the efficient computation of reduced order models describing the substructure's dynamic motion. For hardware-in-the-loop applications difficulties include the fast computation of the numerical components and the proper sensing and actuation of the hardware component. Recent advances in experimental techniques, sensor/actuator technologies, novel numerical methods, and parallel computing have rekindled interest in substructuring in recent years, leading to new insights and improved experimental and analytical techniques.

The organizers would like to thank the authors, presenters, session organizers, and session chairs for their participation in this track.

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Chapter 1

Integrating Biodynamic Measurements in Frequency-Based Substructuring to Study Human-Structure Interaction

Sébastien Perrier, Yvan Champoux, and Jean-Marc Drouet

Abstract The mechanical behavior of the human body has long been characterized using biodynamic measurements on various human body parts in several positions and postures. Generally, these measurements are gathered as close as possible to the skin-mechanical structure interface for best results understanding how the body reacts when in contact with a vibrating structure. Substructuring methods have been widely used on mechanical structures to study and improve the dynamic behavior of complex assemblies. In the case of interactions between a human body and a vibrating structure, the dynamics involved in the structure alone is as important as the dynamics of the human body. Thus, the use of Frequency-Based Substructuring (FBS) to combine biodynamic measurements with the structure's dynamic behavior is essential to understanding the vibration transmission phenomena in this complex assembly.

This article presents the advantages of this approach as well as the challenges when performing FBS between a mechanical structure and biodynamic measurements. The study focuses on a vibrating handlebar in conjunction with 3 different holding positions of the hand-arm system. The FBS assemblies are gathered and the results are compared with experimental measurements on the entire assembled structure for each position over a frequency range between 1 and 100 Hz.

Keywords Biodynamics • Substructuring • Human-structure interaction • Experimental measurements • Structural dynamics

1.1 Introduction

The mechanical behavior of the human body under vibration has long been characterized and studied in the context of health and safety, as well as in the context of dynamic comfort evaluation. Among them, whole-body vibration and hand-transmitted vibration have been studied the most in order to minimize the undesirable effects of vibration [1–19]. Hand-transmitted vibration is of great interest since the hands are highly sensitive in capturing vibration discomfort and high levels of hand vibration can even be detrimental to overall health.

The characterization of hand-transmitted vibration (HTV) depends on several parameters such as vibration frequency and magnitudes, axis of vibration, coupling forces, frequency weighting, and others. For example in ISO-5349 [13], which documents the measurement, the dose–response relationship, and the weighting filter for assessment of HTV, the current weighting function proposed significantly suppresses the magnitude of vibration at frequencies above 100 Hz. The dynamic response behavior of the hand-arm system can be described as *through-the-hand-arm* and *to-the-hand* response functions. The *through-the-hand-arm* response function describes the transmission of vibration as the ratio of the motion magnitude at a specific segment of the hand-arm to that at the hand-handle interface. The biodynamic response in terms of the *to-the-hand* function relates the vibration in the vicinity of the hand to the force at the driving point [7]. The *to-the-hand* dynamic response behavior is the one to use when studying humans in contact with vibrating structures. This biodynamic response behavior has been extensively investigated, but the majority of the studies are based upon either the absorbed power or the driving point mechanical impedance Z (DPMI). These two expressions are relatively similar. The DPMI is computed from

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the force F – velocity v relationship at the driving point, as described in Eq. (1.1), and the vibration power at the driving point is computed from the parameters used in calculating the DPML, as described in Eq. (1.2).

$$Z(j\omega) = \frac{F(j\omega)}{v(j\omega)} \quad (1.1)$$

$$P(\omega) = \text{Re}[Z(j\omega)] |v(j\omega)|^2 \quad (1.2)$$

where v is the root-mean-square value of the velocity at the same frequency.

The biodynamic response characteristics of the human hand-arm have been measured on human subjects under carefully controlled conditions to control several parameters such as posture, hand position, type of excitation, excitation direction, push and grip forces [2–6]. Nonetheless, considerable differences are known to exist in the measured data reported by investigators. These differences have been partially attributed to variations in intrinsic and extrinsic variables, test conditions, methodologies employed in the various studies [1], and the dynamic behavior of the handle itself [8, 15].

With difficulties to obtain repetitive measurements involving representative human-subject samples and test conditions due to inter- and intrasubject variabilities, biodynamic models of the hand and arm have been proposed to characterize vibration amplitude and power flow in the coupled hand, structure, and workpiece system. The majority of the reported models are mechanical models that comprise lumped mass, stiffness, and damping elements in which the lumped parameter values are identified upon the curve fitting of the measured data.

The models, therefore, do not adequately represent the biomechanical properties of the human hand and arm [7]. Furthermore, the models characterize the uncoupled biodynamic behavior of the hand and arm along the three independent orthogonal axes of vibration. All the models thus neglect to consider the dynamic coupling effects of the hand and arm.

Owing to the complex nature of the structure vibration and coupled hand-structure system dynamics, the approach developed in this work is to consider the hand-arm system as a “black box”. Its dynamic response with measured DPML can be described, and these characteristics assembled with those of the structure by a substructuring method to develop a coupling model for the study of human–structure interactions.

Substructuring methods have become a focus of research in structural dynamics [20–30] and have been used on mechanical structures to study and improve the dynamic behavior of complex assemblies. Whereas knowledge of the interactions between structure assemblies has been a major concern in mechanical engineering since the 1960s, studies of human–structure interactions in mechanics are sparsely documented in comparison.

To the authors’ knowledge, the first attempt to study a substructuring coupling that involves both the human body and a mechanical structure was made by the authors themselves [31]. No human-structure interactions using the Frequency-Based Substructuring (FBS) method have been explored since that time. In contact with a human body part, the dynamic behavior of a lightweight structure changes radically from its original uncoupled condition. Therefore, the use of FBS to combine biodynamic measurements with the structure’s dynamic behavior is essential to understand vibration transmission phenomena.

In this work, we present the advantages of this approach as well as the challenges when performing FBS using computed characteristics of the mechanical structure and biodynamic measurements. The FBS method allows coupling between substructures through consideration of their interface set only [26]. This is a major advantage in linking a mechanical structure with the human body where only interface measurements can be gathered. The study focuses on the hand-arm in conjunction with a vibrating structure in 3 different positions. Although the human body is not linear, mechanical coupling using the FBS method can be used with the hand-arm system by controlling various parameters [31]. Each substructure is characterized in the frequency range [1, 100] Hz. The mechanical structure is characterized by the mechanical mobility Frequency Response Function (FRF) using a Finite Elements model and the hand-arm by measured mechanical impedance FRF. Finite Elements (FE) are used to characterize the mechanical structure since all the mechanical information, as well as the translational and rotational degrees of freedom, are intrinsically available. The hybrid FBS assemblies are then gathered and the results are compared with experimental measurements over the entire assembled structure for each position.

1.2 Methods

The methods used in this paper to join the human hand-arm with a mechanical structure include the generalized frequency domain substructure synthesis presented in [26]. This well-known method, also referred to as FBS (Frequency-Based Substructuring), combines the response FRF data of each substructure to analyze the dynamics of a complex assembled structure. It enables substructures to be coupled by taking into account the characteristics of the interface nodes only. It can

also utilize experimentally derived data effectively in combination with finite element results. This method is based on an implicit statement of the force and velocity continuity considerations at the connection nodes. It leads to the mathematical expression for the FRF coupling of two substructures in terms of mobility Y (velocity/force) shown in Eq. (1.3) where: a and b identify the two substructures involved; A is the set of internal degrees of freedom in substructure a ; I is the interface contact set between substructures a and b ; and B is the set of internal degrees of freedom in substructure b .

$$\begin{bmatrix} \mathbf{Y}_{AA} & \mathbf{Y}_{AI} & \mathbf{Y}_{AB} \\ \mathbf{Y}_{IA} & \mathbf{Y}_{II} & \mathbf{Y}_{IB} \\ \mathbf{Y}_{BA} & \mathbf{Y}_{BI} & \mathbf{Y}_{BB} \end{bmatrix}^{ab} = \begin{bmatrix} \mathbf{Y}_{AA}^a & \mathbf{Y}_{AI}^a & 0 \\ \mathbf{Y}_{IA}^a & \mathbf{Y}_{II}^a & 0 \\ 0 & 0 & \mathbf{Y}_{BB}^b \end{bmatrix} - \begin{Bmatrix} \mathbf{Y}_{AI}^a \\ \mathbf{Y}_{II}^a \\ -\mathbf{Y}_{BI}^b \end{Bmatrix} [\mathbf{Y}_{II}^a + \mathbf{Y}_{II}^b]^{-1} \begin{Bmatrix} \mathbf{Y}_{IA}^a \\ \mathbf{Y}_{II}^a \\ -\mathbf{Y}_{IB}^b \end{Bmatrix}^T \quad (1.3)$$

Substructures a and b represent the mechanical structure and the hand-arm, respectively. The coupling process using the FBS method is achieved by computing the mechanical structure mobility characteristics from the FE model and by measuring the impedance at the hands. These characteristics are assembled through Eq. (1.3) to predict the assembly's dynamic behavior. The mobility characteristics of the assembly are then compared with the measured mobility characteristics of the mechanical structure in contact with both hands. For this study, only interface data can be gathered for the hand-arm system, and therefore the equation can be simplified as shown in Eq. (1.4).

$$\begin{bmatrix} \mathbf{Y}_{AA} & \mathbf{Y}_{AI} \\ \mathbf{Y}_{IA} & \mathbf{Y}_{II} \end{bmatrix}^{ab} = \begin{bmatrix} \mathbf{Y}_{AA}^a & \mathbf{Y}_{AI}^a \\ \mathbf{Y}_{IA}^a & \mathbf{Y}_{II}^a \end{bmatrix} - \begin{Bmatrix} \mathbf{Y}_{AI}^a \\ \mathbf{Y}_{II}^a \end{Bmatrix} [\mathbf{Y}_{II}^a + \mathbf{Y}_{II}^b]^{-1} \begin{Bmatrix} \mathbf{Y}_{IA}^a \\ \mathbf{Y}_{II}^a \end{Bmatrix}^T \quad (1.4)$$

Since the measurements of the mobility characteristics are made on the mechanical structure in contact with both hands, we are focusing only on the mechanical mobility determined by an excitation force applied at the internal set A for the substructure a and a velocity response at the interface set I .

$$[\mathbf{Y}_{IA}^{ab}] = [\mathbf{Y}_{IA}^a] - [\mathbf{Y}_{II}^a] [\mathbf{Y}_{II}^a + \mathbf{Y}_{II}^b]^{-1} [\mathbf{Y}_{IA}^a] \quad (1.5)$$

Equation (1.5) can be developed to reveal the left and right hands interface points, respectively, $I1$ and $I2$.

$$\begin{bmatrix} \mathbf{Y}_{I1A}^{ab} \\ \mathbf{Y}_{I2A}^{ab} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{I1A}^a \\ \mathbf{Y}_{I2A}^a \end{bmatrix} - \begin{bmatrix} \mathbf{Y}_{I1I1}^a & \mathbf{Y}_{I1I2}^a \\ \mathbf{Y}_{I2I1}^a & \mathbf{Y}_{I2I2}^a \end{bmatrix} \left[\begin{bmatrix} \mathbf{Y}_{I1I1}^a & \mathbf{Y}_{I1I2}^a \\ \mathbf{Y}_{I2I1}^a & \mathbf{Y}_{I2I2}^a \end{bmatrix} + \begin{bmatrix} \mathbf{Y}_{I1I1}^b & \mathbf{Y}_{I1I2}^b \\ \mathbf{Y}_{I2I1}^b & \mathbf{Y}_{I2I2}^b \end{bmatrix} \right]^{-1} \begin{bmatrix} \mathbf{Y}_{I1A}^a \\ \mathbf{Y}_{I2A}^a \end{bmatrix} \quad (1.6)$$

Furthermore, assumption is made that left and right hands are dynamically independent.

$$\begin{bmatrix} \mathbf{Y}_{I1A}^{ab} \\ \mathbf{Y}_{I2A}^{ab} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{I1A}^a \\ \mathbf{Y}_{I2A}^a \end{bmatrix} - \begin{bmatrix} \mathbf{Y}_{I1I1}^a & \mathbf{Y}_{I1I2}^a \\ \mathbf{Y}_{I2I1}^a & \mathbf{Y}_{I2I2}^a \end{bmatrix} \left[\begin{bmatrix} \mathbf{Y}_{I1I1}^a & \mathbf{Y}_{I1I2}^a \\ \mathbf{Y}_{I2I1}^a & \mathbf{Y}_{I2I2}^a \end{bmatrix} + \begin{bmatrix} \mathbf{Y}_{I1I1}^b & 0 \\ 0 & \mathbf{Y}_{I2I2}^b \end{bmatrix} \right]^{-1} \begin{bmatrix} \mathbf{Y}_{I1A}^a \\ \mathbf{Y}_{I2A}^a \end{bmatrix} \quad (1.7)$$

1.2.1 Target Measurements

These measurements are used to evaluate the accuracy of the FBS prediction for the mechanical structure coupled with both hands (Eq. (1.7)). The mechanical structure is an aluminum assembly composed of a handlebar, a stem and a support. Figure 1.1 shows the structure installed on an excitation device used to gather target measurements with both hands on the structure. This excitation device is a hydraulic excitation system able to handle important loads and designed to produce a vertical excitation. Only the vertical Z-axis is presented in this paper.

To get the mobility of this assembly (structure with hands on it), the excitation is characterized using a six degrees of freedom (DOFs) forces/moments sensor (AMTI MC3A-500) installed between the excitation device and the support, and an accelerometer is fixed to the handlebar as close as possible to the hands. The velocity is obtained by integrating the acceleration in the frequency domain. A vibration signal reproducing the dynamic characteristics of a road is provided to the excitation system [32]. The acquisition system is a LMS SCADAS Recorder using Test.Lab 12A software.

Three postures of the hand-arm leaning on the structure are tested (Fig. 1.2). Each of them requires that the arms be kept straight. For each posture, the subject is asked to hold the handle without applying any grip force, just leaning on the structure. The push force applied by the subject on the structure is measured to control the posture during acquisition. The subject is asked to maintain a constant push force during the test using the DC values displayed by the 6DOFs forces/moments sensor in the X and Z axes. For each posture, the DC values along X and Z axes are recorded for further use during the measurement of the hand-arm mechanical impedance.

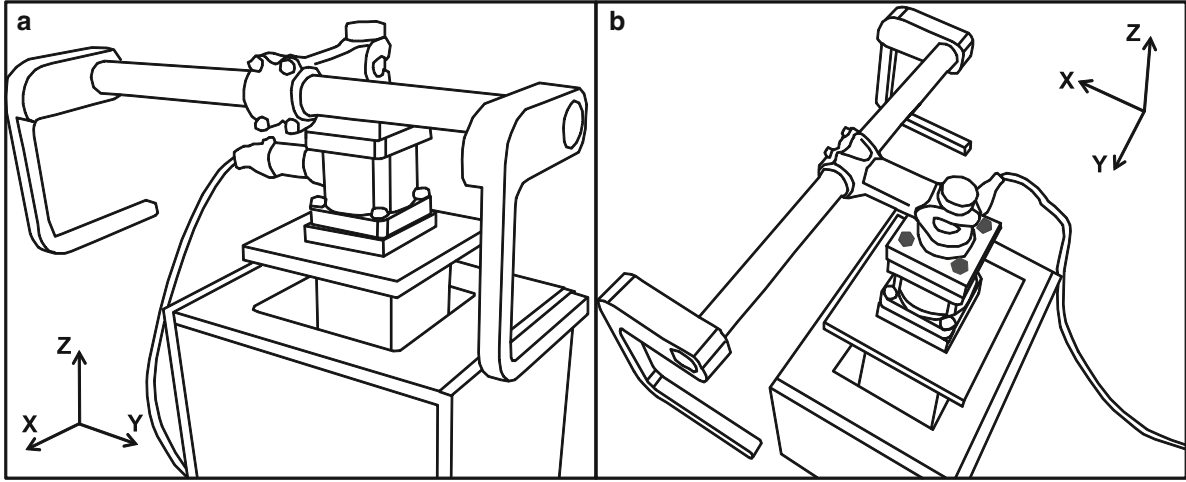


Fig. 1.1 Mechanical assembly with the 6DOFs forces/moments sensor installed on the excitation device. (a) Front view, (b) Top view

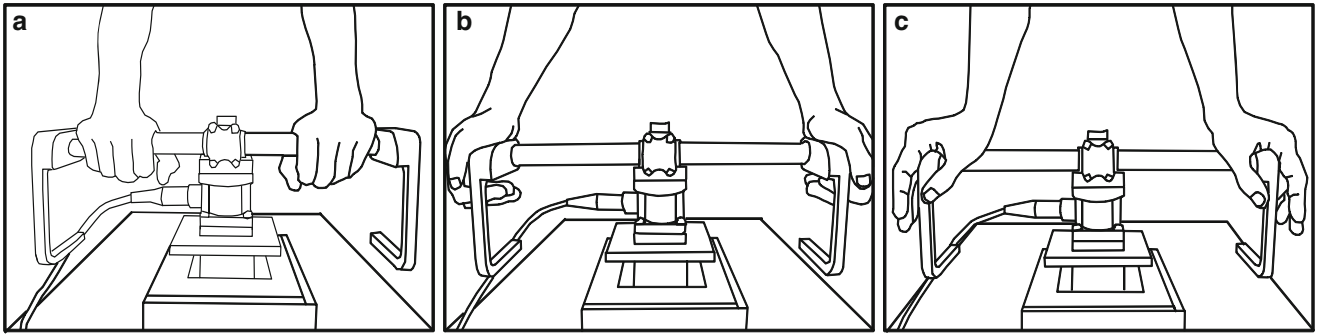


Fig. 1.2 Three test postures for the hands on the mechanical assembly. (a) Posture with DC values X: 10 N and Z: -70 N. (b) Posture with DC values X: 15 N and Z: -85 N. (c) Posture with DC values X: 25 N and Z: -105 N

1.2.2 Hand-Arm Mechanical Impedance

In Eq. (1.7), Y_{III}^b corresponds to the matrix mobility for the left hand-arm system and Y_{I2I2}^b corresponds to the matrix mobility for the right hand-arm system. In these two matrices, only the term corresponding to an excitation and response along the vertical Z-axis is retained.

Typically, the dynamic characteristics of the hand-arm are in the form of mechanical impedance. For this reason and in this study, the hand-arm is characterized in terms of measured impedance. Theoretically, the mobility also known as admittance is the inverse of the impedance ($[Y] = [Z]^{-1}$).

The mechanical impedance of the hand-arm is obtained using two specially designed handles. One has a cylindrical body and the other one has a shape that is as close as possible to the shape of the assembly handlebar. For each measurement, the handle is equipped with an accelerometer and is mounted on the 6 DOFs forces/moments sensor installed on the same excitation device as for the target measurements (Fig. 1.3). The same LMS system is used as a signal generator and the dynamic characteristics of the road are provided to the excitation system.

Measurements are performed with and without a hand on the handle. Because the handles are sufficiently rigid in the frequency range of interest, the handle impedance can then be subtracted from the total impedance (hand + handle) to obtain the hand-arm mechanical impedance [4, 9].

$$Z_{\text{Hand}}(\omega) = Z_{\text{Total}}(\omega) - Z_{\text{Handle}}(\omega) \quad (1.8)$$

The impedance data can then be inverted to get the mobility of the hand-arm system and used directly in Eq. (1.7).

Fig. 1.3 Diagram of the hand-arm impedance measurement system (1: LMS Test.Lab 12A software, 2: Excitation device, 3: 6DOFs forces/moments sensor model MC3-A-500 from AMTI, 4: Accelerometer 356B11 type ICP from PCB Piezotronics, 5: Handle)

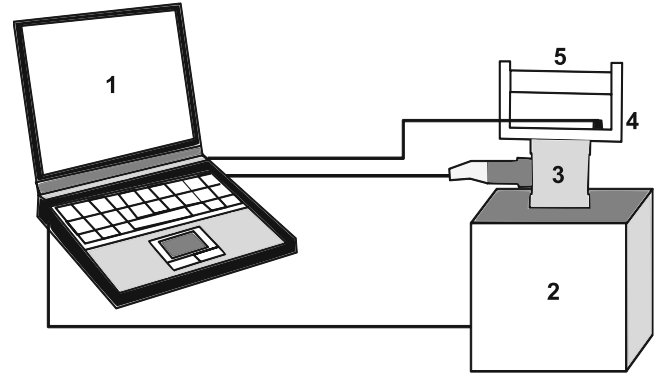
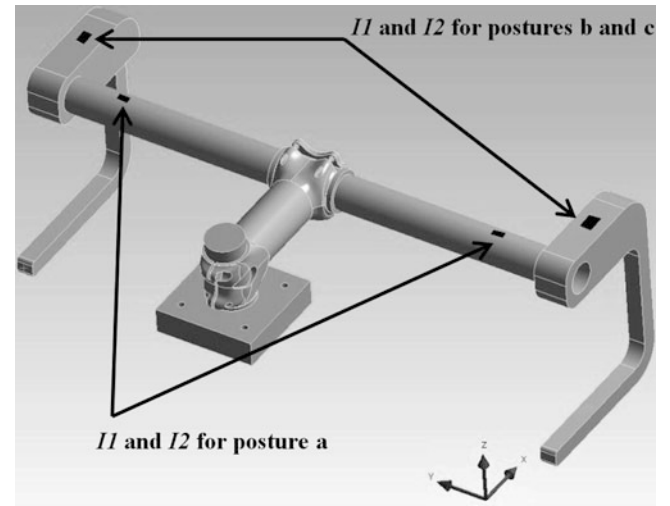


Fig. 1.4 Points location for the FE model of the assembly to obtain the needed FRFs for the different postures



The study is carried out on one subject to avoid inter-subject variability. The subject is asked to lean on the handle and to reproduce each posture to maintain the push forces along X and Z axes corresponding to the tested postures in the target measurements section. Since we are measuring each hand separately, we assume that the DC forces along X and Z axes should be half of the DC values retained when measuring on the structure assembly with both hands (target measurements).

1.2.3 Mechanical Structure Mobility

For the mechanical structure composed of a handlebar, a stem and a support, the FRFs needed for the FBS coupling process (Eq. 1.7) are obtained using an FE model (named FE FRFs). Translational DOFs (TDOFs) as well as rotational DOFs (RDOFs) are thus intrinsically available. This model of the mechanical assembly was previously updated to fit the experimental comparison in free – free conditions.

To perform the FBS coupling in Eq. (1.7), a total of 216 FE FRFs (TDOFs + RDOFs) are requested for the mechanical structure. These FRFs are obtained using the harmonic response section of the Ansys 14 Workbench software. The frequency resolution for the harmonic response of the structure is 1 Hz with a frequency span from 1 to 100 Hz. Moreover, specific commands need to be encoded to get rotational information at some points and to extract the results in *.txt files.

Considering the three different postures, the requested information in terms of FRFs are not the same for each one (Fig. 1.4)

FRFs for each posture are then computed. In Matlab, FE FRFs of the mechanical structure are coupled with experimental FRFs of both hand-arm dynamic characteristics to obtain an FBS prediction of the coupled structures.

1.3 Results

The FBS predictions for the hands coupled with the mechanical structure according to the three tested postures are compared to the target measurements corresponding to the hands holding the handlebar with the same postures and push forces. Comparisons are presented in terms of mobility along the Z-axis measured at the *I2* interface point corresponding to the right hand (Figs. 1.5, 1.6, and 1.7). In order to observe the influence of the hands touching the handlebar, the dynamic behavior of the mechanical structure alone is also shown (dotted line).

1.4 Discussion

Figures 1.5, 1.6, and 1.7 are presented with the same amplitude scale. It is possible to observe that the mobility of the structure alone at the point *I2* for posture a (Fig. 1.5) is lower than the mobility of the structure alone at the point *I2* for postures b and c (Figs. 1.6 and 1.7).

In terms of FBS prediction, the results for postures a and b (Figs. 1.5 and 1.6) demonstrate that the FBS model succeeds in providing reliable predictions for the influence of the hands on a mechanical structure with different postures and push forces applied by the subject on the structure. This is an indication that coupling between the hand-arm systems and a relatively complex mechanical structure is possible using this method.

Fig. 1.5 Comparison between target measurement (hands on the structure) and FBS prediction for posture a

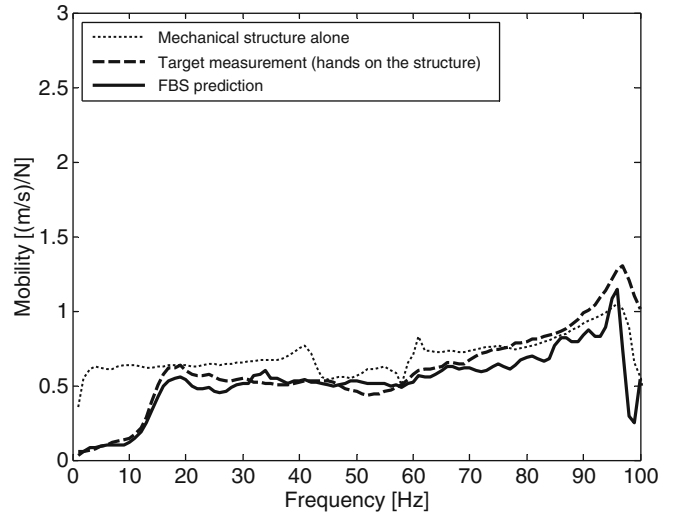


Fig. 1.6 Comparison between target measurement (hands on the structure) and FBS prediction for posture b

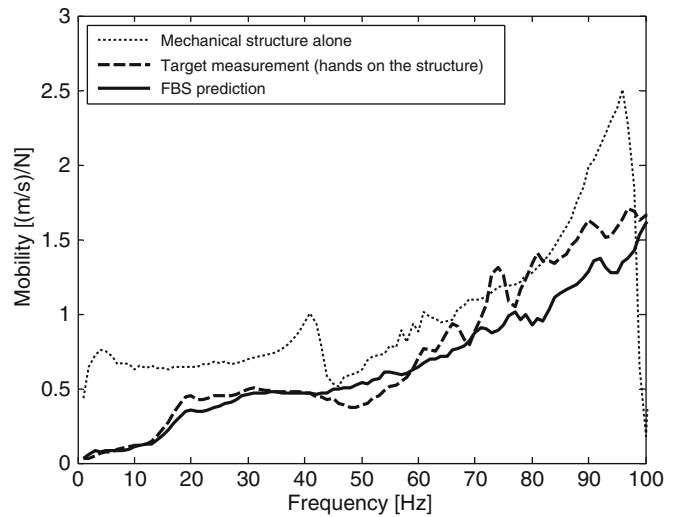
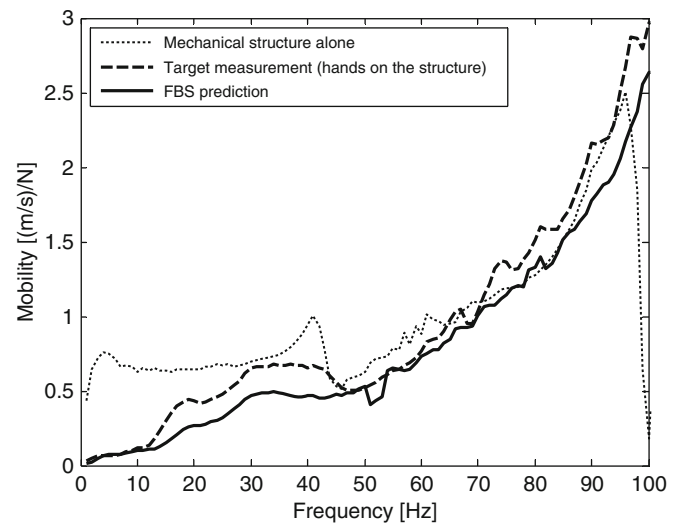


Fig. 1.7 Comparison between target measurement (hands on the structure) and FBS prediction for posture c



Nevertheless, results for posture c show some discrepancies (Fig. 1.7). The FBS model does not provide accurate results in the frequency span 1–50 Hz. This can be explained by the difficulty to reproduce the exact posture when measuring the dynamic characteristics of the hands separately. Furthermore, with such an extreme position of the wrists (see Fig. 1.2) there is an important contact surface between the hands and the mechanical structure. The hands were in contact at the two extremities of the handlebar but also with a part of the central tube while the FRFs from the FE model were obtained for a specific point. This alerts us to the fact that complex postures should be carefully reproduced during testing phases and also that the FE FRFs needed should be extracted for interface areas instead of interface points. The discrepancy between the FBS prediction and the target measurement for this specific posture may also be explained by the fact that, for hand measurement, only the vertical z-axis is considered in this work. This assumption could be too restrictive for the coupling of the hand-arm systems with the structure in this posture.

In terms of influence of the hands on the structure, we can easily identify that the posture of the subject has a clear influence on the dynamics of the assembly by looking at the target measurement of each figure from Figs. 1.5, 1.6, and 1.7. For example, with posture a, the subject provides a significant contribution to the structure dynamic up to 16 Hz while with posture b, the subject has a relatively important contribution up to 43 Hz. By integrating biodynamic measurements in the Frequency-Based Substructuring method, it is really important to control both the push force and the posture while carrying out measurements to obtain accurate predictions.

1.5 Conclusion

This study shows that reliable predictions can be obtained using the FBS method even with non-linear structures such as human body parts. Mechanical coupling predictions between the human body and a mechanical structure are thus possible using this method. Results show that it is important to control several parameters when performing the FBS coupling with a human body part because the mechanical behavior of the human body is sensitive to several factors such as position, orientation, and forces. This work also highlights the following merits of the FBS method: (1) Direct use of shaker test data, (2) Combination of substructures when only the data interfaces are known, (3) hybrid use of experimental and FE FRFs.

This work also highlights the possibility of predicting the influence of hand-arm systems on a structure using specific postures. The FBS method is a promising approach to study vibration interaction mechanisms between a mechanical structure and the human body. These results will be beneficial to further enhance interactions between humans and structures. This is a major breakthrough in human-structure interactions since it is possible to predict the influence of a human subject on a structure during the design phase of the structure in question. Limitations to this approach still remain due to the need to process a large amount of data and with regard to the computation time required for extracting FRFs.

Acknowledgements The authors gratefully acknowledge financial support from the National Science and Engineering Council of Canada (NSERC) and the participation of Cervélo and Vroomen – White Design.

References

1. Adewusi SA, Rakheja S, Marcotte P, Boileau P-E (2008) On the discrepancies in the reported human hand-arm impedance at higher frequencies. *Int J Indust Ergon* 38:703–714
2. Aldien Y, Marcotte P, Rakheja S, Boileau P-E (2006) Influence of hand-arm posture on biodynamic response of the human hand-arm exposed to z_h -axis vibration. *Int J Ind Ergon* 36:45–59
3. Aldien Y, Marcotte P, Rakheja S, Boileau P-E (2005) Mechanical impedance and absorbed power of hand-arm under x_h -axis vibration and role of hand forces and posture. *Ind Health* 43:495–508
4. Besa AJ, Valero FJ, Suñer JL, Carballeira J (2007) Characterization of the mechanical impedance of the human hand-arm system: the influence of vibration direction, hand-arm posture and muscle tension. *Int J Ind Ergon* 37:225–231
5. Burström L (1990) Measurements of the impedance of the hand and arm. *Int Arch Occup Environ Health* 62:431–439
6. Burström L (1997) The influence of biodynamic factors on the mechanical impedance of the hand and arm. *Int Arch Occup Environ Health* 69:437–446
7. Dong RG, Rakheja S, Schopper AW, Han B, Smutz WP (2001) Hand-transmitted vibration and biodynamic response of the human hand-arm: a critical review. *Crit Rev Biomed Eng* 29(4):391–441
8. Dong RG, Welcome DE, McDowell TW, Wu JZ (2008) Analysis of handle dynamics-induced errors in hand biodynamic measurements. *J Sound Vib* 318:1313–1333
9. Dong RG, Welcome DE, McDowell TW, Wu JZ (2006) Measurement of biodynamic response of human hand-arm system. *J Sound Vib* 294:807–827
10. Griffin MJ (1990) *Handbook of human vibration*. Academic, London
11. Griffin MJ, Whitham EM, Parsons KC (1982) Vibration and comfort, I. translational seat vibration. *Ergonomics* 25(7):603–630
12. International Standard Organization (ISO 2631) (1997) Mechanical vibration and shock—evaluation of human exposure to whole-body vibration. International Standard Organization
13. International Standard Organization (ISO/DIS 5349) (2001) Mechanical vibration—measurement and evaluation of human exposure to hand-transmitted vibration. International Standard Organization
14. Lundström R, Burström L (1989) Mechanical impedance of the human hand-arm system. *Int J Ind Ergon* 3:235–242
15. Marcotte P, Aldien Y, Boileau P-E, Rakheja S, Boutin J (2005) Effect of handle size and hand-handle contact force on the biodynamic response of the hand-arm system under z_h -axis vibration. *J Sound Vib* 283:1071–1091
16. Mansfield NJ (2005) *Human response to vibration*. CRC
17. Reynolds DD, Angevine EN (1977) Hand-arm vibration, Part II: vibration transmission characteristics of the hand and arm. *J Sound Vib* 51(2):255–265
18. Richard S (2005) Étude du comportement dynamique d'un vélo de route en lien avec le confort du cycliste. MScA Thesis, Université de Sherbrooke
19. Richard S, Champoux Y (2006) Development of a metric related to the dynamic comfort of a road bike. In: *Proceedings of the IMAC XXIV*, St. Louis, 2006
20. Avitabile P (2003) Twenty years of structural dynamic modification: a review. *Sound Vib* 14–25
21. Craig R, Bampton M (1968) Coupling of substructures for dynamic analysis. *AIAA J* 6(7):1313–1319
22. De Klerk D, Rixen D, De Jong J (2006) Frequency based substructuring (FBS) method reformulated according to the dual domain decomposition method. In: *Proceedings of the fifteenth international modal analysis conference*, Society for Experimental Mechanics, Paper 136, 2006
23. De Klerk D, Rixen DJ, Voormeeren SN, (2008) General framework for dynamic substructuring—history, review, and classification of techniques. *AIAA J* 46(5)
24. Ewins DJ (2000) *Modal testing: theory, practice and application*, 2nd edn. Research Studies Press
25. Hurty WC (1960) Vibrations of structural systems by component mode synthesis. *J Eng Mech/Am Soc Civil Eng* 86(4):51–69
26. Jetmundsen B (1986) On frequency domain methodologies for prescribed structural modification and subsystem synthesis. PhD Thesis, Rensselaer Polytechnic Institute, New York
27. Jetmundsen B, Bielawa R, Flannelly W (1988) Generalized frequency domain substructure synthesis. *J Am Helicop Soc* 33(1):55–65
28. Klosterman A (1971) On the experimental determination and use of modal representations of dynamic characteristics. Ph.D. Thesis, University of Cincinnati, Department of Mechanical Engineering
29. MacNeal R (1971) Hybrid method of component mode synthesis. *Comput Struct* 1(4):581–601
30. Mayes RL (2012) Tutorial on experimental dynamic substructuring using the transmission simulator method. In: *Proceedings of the 30th international modal analysis conference*. Society for experimental mechanics (Topics in experimental dynamics substructuring and wind turbine dynamics), pp 1–9
31. Perrier SS, Champoux Y, Drouet J-M (2012) Using substructuring to predict the human hand influence on a mechanical structure. In: *Proceedings of the 30th international modal analysis conference*. Society for Experimental Mechanics (Topics in experimental dynamics substructuring and wind turbine dynamics), pp 33–44
32. Lépine J, Champoux Y, Drouet J-M (2011) Excitation techniques for testing bike vibration transmission in the laboratory. In: *Proceedings of the 29th International modal analysis conference*. Society for experimental mechanics (Sensors, Instrumentation and Special Topics), p 35

Chapter 2

Investigation of Modal Iwan Models for Structures with Bolted Joints

Brandon J. Deaner, Matthew S. Allen, Michael J. Starr, and Daniel J. Segalman

Abstract Structures with mechanical joints are difficult to accurately model; even when the natural frequencies of the system remain essentially constant, the damping introduced by the joints is often observed to depend nonlinearly on amplitude. Although models for individual joints have been employed with some success, the modeling of a structure with many joints remains a significant obstacle. This work explores whether nonlinear damping can be applied in a modal framework, where instead of modeling each discrete joint within a structure, a nonlinear damping model is used for each mode of interest. This approach assumes that the mode shapes of the structure do not change significantly with amplitude and that there is negligible coupling between modes. The nonlinear Iwan joint model has had success in modeling the nonlinear damping of individual joints and is used as a modal damping model in this work. The proposed methodology is first evaluated by simulating a structure with a small number of discrete Iwan joints (bolted joints) in a finite element code. A modal Iwan model is fit to simulated measurements from this structure and the accuracy of the modal model is assessed. The methodology is then applied to actual experimental hardware with a similar configuration and a modal damping model is identified for the first few modes of the system. The proposed approach seems to capture the response of the system quite well in both cases, especially at low force levels when macro-slip does not occur.

Keywords Nonlinear damping • Bolted joints • Iwan model • Energy dissipation • Modal damping

2.1 Introduction

Mechanical joints are known to be a major source of damping in jointed structures. However, the physics at the interface are quite complex and the amplitude dependence of damping in mechanical joints has proven quite difficult to predict. For many systems, linear damping models seem to capture the response of a structure at calibrated force levels. However, that approach relies on testing the structure at large force levels to calibrate the model. Furthermore, that approach can be over conservative or even erroneous since a linear model does not capture the amplitude dependence of the damping. Thus, it is crucial to understand how mechanical joints behave at a range of force levels so that the response of a jointed structure can be modeled accurately.

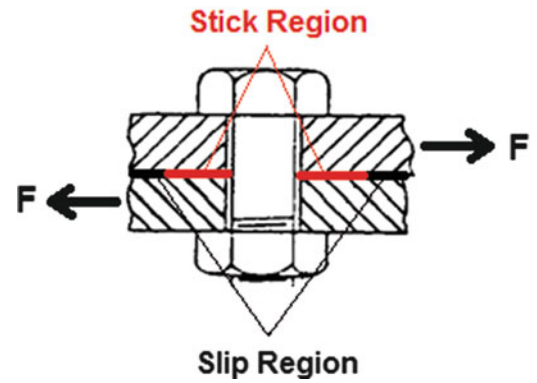
In this work, nonlinearities associated with mechanical joints will be classified into two different regions, micro-slip and macro-slip. Consider the joint shown in Fig. 2.1, where a bolt is used to connect two slabs of material. The preload in the bolt creates a contact region between the two slabs near the bolt. If a force F is applied to the slabs, slip will occur at the

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Fig. 2.1 Contact and Slip regions are shown for a mechanical joint undergoing micro-slip [1]



outskirts of the contact region. This means that there will be a region of stick and a region of slip as indicated in Fig. 2.1. For this case, the bolted joint is said to be undergoing micro-slip due to the small slip displacements that cause the frictional energy loss [1].

Macro-slip occurs when the stick region vanishes and larger slip displacements are possible. If a very large displacement is considered, the slabs of material will eventually come into contact with the bolt. This further complicates the system and is not considered in this work.

To capture the response of the joint in both the micro-slip and macro-slip regions, a 4-Parameter Iwan model was developed in [2]. This constitutive model has a small number of parameters and yet accounts for the key characteristics of the joint's response including the joint slip force (F_S), joint stiffness (K_T), and power law energy dissipation (χ , β). In the past decade, the 4-Parameter Iwan model has been implemented to predict the vibration of structures with a few discrete joints [3, 4]. However, when modeling individual joints, each joint requires a unique set of parameters $\{F_S, K_T, \chi, \beta\}$, which means that hundreds or even thousands of joint parameters may need to be deduced for the systems of interest. On the other hand, when a small number of modes are active in a response, some measurements have suggested that a simpler model may be adequate. Segalman investigated the idea of applying the 4-Parameter Iwan model in a modal framework in [5]. He used simulated measurements to deduce the modal Iwan parameters for two simple spring mass systems and compared this to the more rigorous approach where each discrete joint in the system was treated separately.

This work builds on Segalman's work [5] by simulating a more realistic finite element structure. First, a structure with four discrete Iwan joints is modeled using finite elements and the simulated response data is used to deduce a set of modal Iwan parameters. Particular attention is given to the degree to which the modal Iwan model captures the response of the finite element model that includes discrete Iwan joints. The methodology is then applied experimentally to an actual beam with a small link attached through two bolted joints. The beam was tested with free-free conditions, and great care had to be taken to assure that the suspension system did not dominate the measured damping. The initial results are promising, revealing that the modal Iwan approach does capture the response of the actual structure quite well in the micro-slip regime.

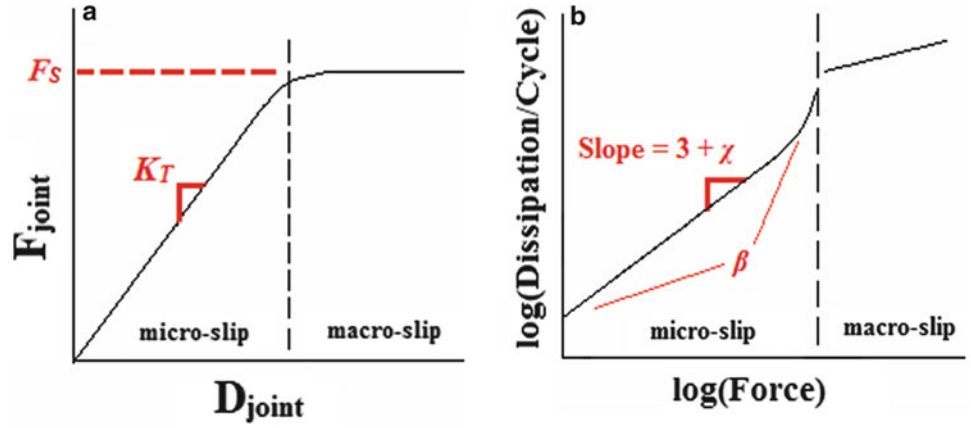
2.2 Nonlinear Energy Dissipation Model

The 4-Parameter Iwan model that is used in this work was initially presented in [2]. A historical review of the major contributors prior to Iwan's work led to the Iwan model being referred to as the Bauschinger-Prandtl-Ishlinskii-Iwan (BPII) model in a more recent work [6]. However, for simplicity and consistency with previous works, in this work the constitutive model will be referred to as the Iwan model.

2.2.1 Parallel-Series Iwan Model

A parallel arrangement of elements, each composed of a spring and frictional damper in series, is referred to as a parallel-series model in [7]. The physical representation of a parallel-series Iwan model is shown in [2]. The force in the joint, $F(t)$, is shown to have the following form,

Fig. 2.2 (a) Both macro-slip force (F_S) and joint stiffness (K_T) can be found from the force-displacement relationship of the joint. (b) The χ value is found from the slope of the dissipation and the β value is a measure of dissipation level and shape of the dissipation curve



$$F(t) = \int_0^{\infty} \rho(\phi) [u(t) - x(t, \phi)] d\phi \quad (2.1)$$

where

$$\dot{x}(t, \phi) = \begin{cases} \dot{u} & \text{if } \|u - x(t, \phi)\| = \phi \text{ and } \dot{u}[u - x(t, \phi)] > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.2)$$

and $u(t)$ is the extension of the joint, $x(t, \phi)$ is the displacement of the frictional damper with strength ϕ and $\rho(\phi)$ is the population density of the spring and frictional damper elements with strength ϕ .

The response properties of the Iwan joint are characterized by the population density $\rho(\phi)$. Various population densities and their limitations are discussed in [6, 8]. Experiments at small force levels have revealed that the energy dissipated by the joint over one vibration cycle tends to be a power of the applied force. Analytically, the energy dissipation associated with pure material damping yields a power of 2.0, while the energy dissipation associated with friction between two bodies when the contact pressure is uniform yields a power of 3.0 [1, 2, 9, 10]. Experimentally, the dissipation tends to have a power-law slope that ranges between 2.0 and 3.0 [10]. One explanation for why experimental results often dissipate energy at a power less than 3.0 is due to the presumption that the contact pressure is nonuniformly distributed in joints [9].

In any event this means that at small force levels, a population density that dissipates energy in a power-law type fashion is desired. At large force levels, joints are known to exhibit a discontinuous nonlinearity associated with the initiation of macro-slip. This characteristic of joints must also be accounted for by the population density.

2.2.2 4-Parameter Iwan Model

To accommodate the behavior of joints at a large range of forces, a 4-parameter population density was developed in [2] with the form:

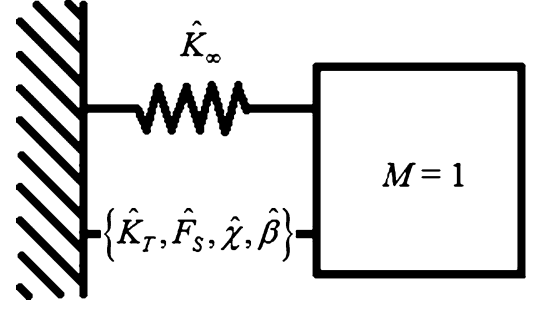
$$\rho(\phi) = R\phi^\chi [H(\phi) - H(\phi - \phi_{\max})] + S\delta(\phi - \phi_{\max}) \quad (2.3)$$

where $H(\cdot)$ is the Heaviside step function, $\delta(\cdot)$ is the Dirac delta function and the four parameters that characterize the joint include: R , which is associated with the level of energy dissipation, χ , which is directly related to the power law behavior of energy dissipation, $\phi_{\max}$, which is equal to the displacement at macro-slip, and the coefficient S , which accounts for a potential discontinuous slope of the force displacement plot when macro-slip occurs.

The four parameters $\{R, \chi, \phi_{\max}, S\}$ are converted to more physically meaningful variables $\{K_T, F_S, \chi, \beta\}$ in [2, 4]. F_S is the joint force necessary to initiate macro-slip, K_T is the stiffness of the joint, χ is directly related to the slope of the energy dissipation in the micro-slip regime, and β relates to the level of energy dissipation and the shape of the energy dissipation curve as the macro-slip force is approached.

This new set of parameters, $\{K_T, F_S, \chi, \beta\}$, can be deduced from the two plots seen in Fig. 2.2. Given a set of simulation or experimental data, there are a variety of methods that could be used to find the energy dissipation and joint force. A few of

Fig. 2.3 Schematic of the model for each modal degree of freedom. Each mode has a unique set of Iwan parameters that characterize its nonlinear damping (and to a lesser extent stiffness)



these methods will be highlighted in Sect. 2.2.4. Once these are known, the Iwan parameters can be determined graphically, or using nonlinear optimization as discussed in [2], or through some combination of the two. This work focuses on the graphical approach as it lends more insight into the importance of each parameter and its validity.

If we consider a system where the damping is strictly due to the 4-Parameter Iwan model, the governing equation becomes

$$M\ddot{x} + K_\infty x = F^X + F^J \quad (2.4)$$

where M and K_∞ are the linear mass and stiffness matrices of a finite element model, F^X is the vector of external forces, and F^J is a vector of nonlinear joint forces that the joint applies to the structure. Note that F^J is the 4-parameter Iwan model defined in Eq. (2.1) with opposite sign convention. F^J has nonzero entries corresponding to the two ends where the joint is attached to the finite element model and depends on the displacement at those points.

As discussed previously, the 4-parameter Iwan model has been implemented to predict the response of structures with a small number of discrete joints [3, 4]. However, each joint requires a unique set of parameters $\{K_T, F_S, \chi, \beta\}$, so many joint parameters need to be deduced and it is not likely that they can all be determined uniquely from a set of experimental measurements.

2.2.3 Modal Iwan Model

In order to circumvent this limitation, Segalman proposed that energy dissipation be applied on a mode-by-mode basis, using the 4-parameter Iwan constitutive model [5]. The Iwan model is assumed to be applicable to each mode, although the parameters of each mode are tuned to match experimental measurements and hence aren't necessarily the same as the parameters of any individual joint in the structure. In general, the nonlinearity that joints introduce can couple the modes of a system so that modes in the traditional linear sense can not be defined. However, experiments have often shown that structures with joints are typically quite linear, suggesting that one might be able to model the structure as a collection of uncoupled linear modes, each with nonlinear damping characteristics. This would allow one to model a structure with a relatively small number of modes that capture its performance in the frequency band of interest, and the response of each mode could be found through a nonlinear, single degree-of-freedom simulation. Specifically, each modal degree-of-freedom is modeled by a single degree-of-freedom oscillator, as shown in Fig. 2.3, with a 4-parameter Iwan model placed between the modal degree of freedom and the ground. A second spring is placed in parallel with the 4-parameter Iwan model, representing the residual stiffness of that mode when the Iwan model is in macro-slip.

Again, the modal Iwan parameters $\{\hat{K}_T, \hat{F}_S, \hat{\chi}, \hat{\beta}\}$ in Fig. 2.3 will not be the same as the $\{K_T, F_S, \chi, \beta\}$ parameters of any discrete Iwan joint parameters in the structure.

The response for each mode r is then governed by the following differential equation,

$$\ddot{q}_r + \omega_{0,r}^2 q_r = \Phi_r^T F^X + \hat{F}_{J,r} + \hat{K}_{T,r} q_r \quad (2.5)$$

where ϕ_r and ω_0 are found by solving the eigenvalue problem $[[K_0] - \omega_{0,r}^2] \Phi_r = 0$ and $\hat{F}_{J,r}$ is the modal joint force and $\hat{K}_{T,r}$ is the modal joint stiffness for the r^{th} mode. The low force mode shapes are used to define the modal parameters, so the stiffness matrix used in the eigenvalue problem above must include the low-force stiffness of the joints, i.e. $[K_0] = [K_\infty] + [K_T]$, where $[K_T]$ is a matrix that captures the stiffness that the joints contribute to the structure when the load is infinitesimal. This presumes that the stiffness of the joints has been included when calculating the mode shapes ϕ_r and

natural frequencies ω_0 of the structure (i.e. the modal Iwan model exhibits no slip). In practice, these modal parameters will come from a low-level modal test and so this assumption should be valid. On the other hand, if the structure is modeled in finite elements then the stiffness of the joints should be included when calculating the mode shapes; this functionality is built into the Sierra/SD (Salinas) finite element package [11, 12].

2.2.4 Extracting Frequency and Energy Dissipation Data

In this work, the modal Iwan parameters $\{\hat{K}_T, \hat{F}_S, \hat{\chi}, \hat{\beta}\}$ will be extracted by estimating the frequency and energy dissipation from the free response of each mode of interest. Two methods for calculating frequency and energy dissipation were explored in this work.

2.2.4.1 Peak-Picking and Zero-Crossing Approach

The first method was developed by Segalman in an unpublished document. The time history of the acceleration is approximated as

$$a(t) = e^{P(t)} \cos(\omega(t)t) \quad (2.6)$$

where $e^{P(t)}$ is the envelope of the response, and $\omega(t)$ is the instantaneous frequency. The envelope $e^{P(t)}$ is found by identifying the acceleration peaks and fitting a cubic spline to the logarithm of those peaks. The frequency $\omega(t)$ is found by calculating the zero-crossings of a response. The energy dissipation per cycle is calculated using the change in peak kinetic energy and is given by

$$KE_P = \frac{M}{2} v_P^2 = \frac{M}{2} \frac{e^{2P(t)}}{\omega^2} \quad (2.7)$$

where v_P is the peak velocity of each cycle. The change in peak kinetic energy is calculated by taking the derivative of the peak kinetic energy which is then multiplied by the period ($2\pi/\omega$) to estimate the energy dissipation per cycle D . This assumes that kinetic energy changes slowly relative to the period so that it can be approximated as constant over each cycle.

$$D \approx \frac{dKE_P}{dt} \frac{2\pi}{\omega} = \frac{4\pi}{\omega} \frac{dP(t)}{dt} KE_P \quad (2.8)$$

2.2.4.2 Hilbert Transform with Polynomial Smoothing Approach

The second method uses the Hilbert transform along with curve fitting to smooth the instantaneous phase and amplitude found from a Hilbert transform [13, 14]. An analytic signal is formed when the Hilbert transform, $\tilde{a}(t)$, is added to the acceleration response [15]

$$A(t) = a(t) + i\tilde{a}(t) \quad (2.9)$$

The magnitude of the analytic signal is the envelope of the response and is approximated by

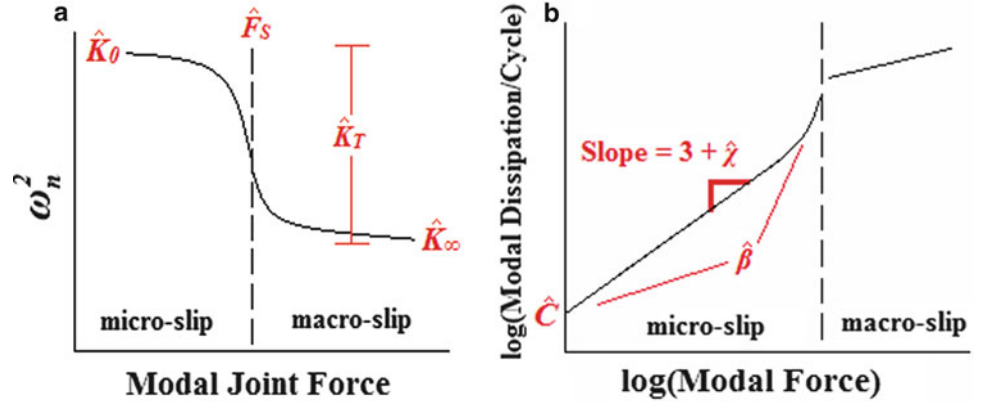
$$Z(t) = |A(t)| = A_0 \exp(-\zeta(t)\omega_n(t)) \quad (2.10)$$

where A_0 is the initial amplitude, $\omega_n(t)$ is the frequency, and $\zeta(t)$ is the coefficient of critical damping. The instantaneous phase is obtained from the analytic signal using the following equation.

$$\varphi(t) = \tan^{-1} \left(\frac{\tilde{a}(t)}{a(t)} \right) \quad (2.11)$$

In other works, for example those by Feldman [15], the instantaneous natural frequency is apparently found by low-pass filtering and then differentiating the phase. However, those works give little detail regarding the optimal low-pass filter and

Fig. 2.4 (a) Modal macro-slip force ($\hat{F}_S$) and modal joint stiffness ($\hat{K}_T$) can be found from the softening seen in the natural frequency. (b) The $\hat{\chi}$ value is found from the slope of the modal dissipation and the $\hat{\beta}$ value is a measure of modal dissipation level and shape of the modal dissipation curve



the results depend heavily on the filter used. If the response is not filtered then noise in the measured response is amplified by the differentiation making the results meaningless, even when the noise is quite small (e.g. integration error in the simulations presented here). In this work, the measured phase is smoothed by fitting a polynomial to the data before differentiating [13]. In addition, the beginning and end of the phase data are deleted since they tend to be contaminated by end effects in the Hilbert Transform.

Both methods for extracting the frequency and energy dissipation data were explored in this work. In general, the Hilbert Transform with Curve Fitting approach was found to smooth out the data more effectively than the peak-picking/zero-crossing approach and will be used in the plots shown in this paper.

2.2.5 Deducing Modal Iwan Parameters

This section explains how modal Iwan parameters are deduced from energy dissipation versus force measurements. The procedure closely follows that used by Segalman in [5] and the work done by Guthrie, which was described in an unpublished memo. The methods described in the previous section provide a set of frequency and energy dissipation data such as that shown schematically in Fig. 2.4. Note that these are quite similar to those shown in Fig. 2.2 for a discrete joint, except that all of the quantities, such as the slip force, are now in a modal form rather than physical. Also, since the modal Iwan model has a linear spring in parallel, its stiffness does not go to zero at macro-slip but instead it simply drops to K_∞ .

The $\hat{\chi}$ parameter is found by fitting a line to the data for the log of energy dissipation versus log of the modal force at low force levels. Then the $\hat{\chi}$ parameter for each mode r is given by the slope of that line minus three:

$$\hat{\chi}_r = \text{Slope}_r - 3 \quad (2.12)$$

In order to deduce the Iwan modal stiffness $\hat{K}_T$, the natural frequencies of each mode are plotted versus modal joint force. A softening of the system, characterized by a drop in frequency, illustrates the amount of modal stiffness associated with all the relevant joints of the system. Assuming that the mode shapes used in the modal filtering process are mass normalized, the equation for modal joint stiffness for each mode becomes

$$\hat{K}_{T,r} = \hat{K}_{0,r} - \hat{K}_{\infty,r} = \omega_{0,r}^2 - \omega_{\infty,r}^2 \quad (2.13)$$

where ω_0 is the natural frequency corresponding to the case when all the joints in the structure exhibit no slipping, and ω_∞ is the natural frequency when all of the joints are slipping. This points to the major advantage of using a modal Iwan implementation as opposed to a discrete Iwan implementation. For a modal Iwan implementation, only the modal stiffness and modal slip force of each kept mode needs to be considered as opposed to calculating the stiffness and slip force associated with each joint individually.

The modal joint slip force, $\hat{F}_S$, can be estimated from Fig. 2.4. To find the last parameter, $\hat{\beta}$, all of the previous parameters found are needed along with the y-intercept, $\hat{C}$, of the line that was fit in order to find the $\hat{\chi}$ parameter. Then, the following equation was formed from [2] that can be used to solve for $\hat{\beta}$ numerically.

$$\hat{F}_{s,r} = \left[\frac{4(\hat{\chi}_r + 1) \hat{K}_{T,r}^{\hat{\chi}_r+2} \left(\hat{\beta}_r + \frac{\hat{\chi}_r+1}{\hat{\chi}_r+2} \right)^{\hat{\chi}_r+1}}{\hat{C}_r K_{\infty,r}^{(3+\hat{\chi}_r)} (2 + \hat{\chi}_r) (3 + \hat{\chi}_r) (1 + \hat{\beta}_r)^{\hat{\chi}_r+2}} \right]^{\frac{1}{\hat{\chi}_r+1}} \quad (2.14)$$

Once all of these parameters have been determined, one can reconstruct the modal energy dissipation versus force curve for the modal Iwan model and compare it to the measurements. This is helpful, since experience has revealed that it is sometimes necessary to adjust the parameters $\{\hat{K}_T, \hat{F}_S, \hat{\beta}\}$ so that the dissipation versus force curve accurately fits the measurements. In the micro-slip region, the modal energy dissipation per cycle, $\hat{D}_r$, is adapted from [2] and is given by,

$$\hat{D}_r = 4\hat{r}_r^{\hat{\chi}_r+3} \left(\frac{\hat{F}_{S,r}}{\hat{K}_{T,r}} \right) \left(\frac{(\hat{\beta}_r + 1)(\hat{\chi}_r + 1)}{\left(\hat{\beta}_r + \frac{\hat{\chi}_r+1}{\hat{\chi}_r+2} \right)^2 (\hat{\chi}_r + 2) (\hat{\chi}_r + 3)} \right) \quad (2.15)$$

where $\hat{r}_r$ is found by iteratively solving:

$$\frac{\hat{F}_{J,r}}{\hat{F}_{S,r}} = \hat{r}_r \left(\frac{(\hat{\beta}_r + 1) - \frac{\hat{r}_r^{\hat{\chi}_r+1}}{\hat{\chi}_r+2}}{\hat{\beta}_r + \frac{\hat{\chi}_r+1}{\hat{\chi}_r+2}} \right) \quad (2.16)$$

For the macro-slip region, modal energy dissipation is given by:

$$\hat{D}_r = 4q_{p,r} \hat{F}_{S,r} \quad (2.17)$$

where $q_{p,r}$ is the peak modal displacement over a cycle of the response.

2.3 Finite Element Simulations

Both the discrete 4-parameter Iwan model and the modal Iwan model have been implemented into Sierra/SD (Salinas), a structural dynamics finite element code developed by Sandia National Laboratories [11, 12]. Sierra/SD was used to simulate the response of a structure with several discrete Iwan joints, in order to generate response data that was then fit to a modal Iwan model. Then the response of the modal Iwan model could be compared to that of the truth model to evaluate the proposed procedure.

2.3.1 Finite Element Model

The model of interest is a beam with a link attached to the center of the beam. The general dimensions of the beam ($20'' \times 2'' \times 0.25''$) and link ($3.5'' \times 0.5'' \times 0.125''$) are shown in Fig. 2.5. The beam is meshed with fairly coarse hexahedron elements (approximate edge length varies between $0.125''$ and $0.25''$) with a total of 1,554 nodes to keep the computational cost reasonable. The first five natural frequencies of this model were compared to the natural frequencies of a finely meshed model containing 37,553 nodes, and it was found that the error was less than 1%.

The representative model has four square washers in contact with the beam as seen in Fig. 2.6. Four Iwan elements are defined for the two in-plane shear directions at the contact patches between the washers and the beam. Note that no other damping is included in this model, so for the discrete Iwan simulations, all of the energy dissipation is due to the Iwan model.

In addition to the Iwan elements defined in the in-plane directions, constraints are placed on all out-of-plane rotations and displacements to prevent penetration and separation between the washers and the beam. Two of the washers are merged to the link which adds stiffness to the beam. The beam, link, and washers are given the linear elastic material properties of Stainless Steel 304.

Fig. 2.5 Computational model of a beam with several discrete Iwan joints

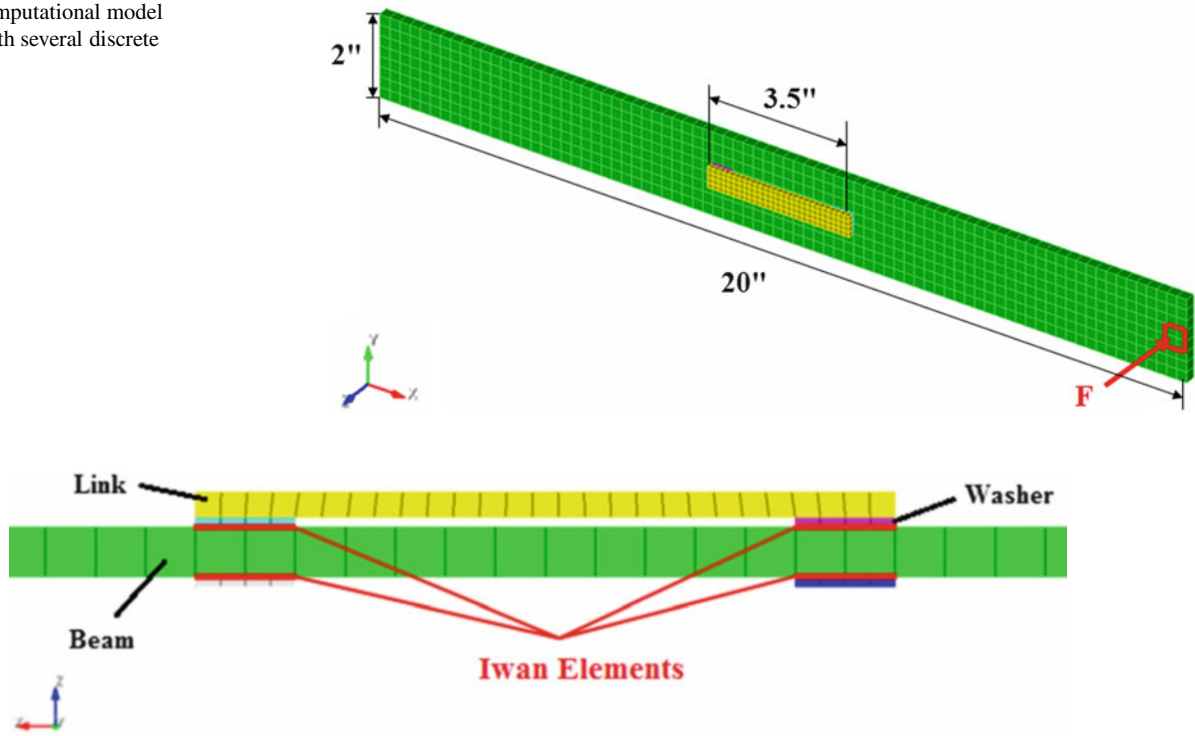


Fig. 2.6 Location of discrete Iwan joints. An Iwan joint is used at each surface where the washers come into contact with the beam

Table 2.1 Iwan parameters used for the discrete Iwan simulations

K_T (lb/in)	F_S (lb)	χ	β
$2.75 \cdot 10^5$	300	-0.3	0.02

2.3.2 Discrete Iwan Simulations

An experiment was simulated by exciting the finite element model with an impulsive force. The force was applied to nine nodes at the tip of the beam as shown in Fig. 2.5. A haversine forcing function was used with a duration of 1.6 ms. The duration of the force was chosen to primarily excite the first three elastic modes of vibration which will be analyzed in this work.

The acceleration time histories at all nodes were extracted and filtered using the pseudo inverse of the mode shape matrix including the first 50 mode shapes from a linear eigenanalysis. This was accomplished using a modal filter [16], i.e. by approximating the response as follows,

$$\ddot{x} = \Phi \ddot{q} \quad (2.18)$$

where ϕ is the mode shape matrix including the first 50 linear modes, and then pre-multiplying both sides of Eq. (2.18) with the pseudo-inverse of the mode shape matrix.

The discrete Iwan parameters used for these simulations were those found from previous experimental studies of a lap joint [2]. The parameters for all four discrete Iwan elements are shown in Table 2.1.

2.3.3 Modal Iwan Simulations

The methodology described in Sect. 2.2.5 was then used to deduce the modal Iwan parameters of the first three elastic bending modes. The modal Iwan model was then used to predict the response of the system to the impulsive force used for the discrete Iwan simulations. This was done using the implementation of the modal Iwan model in the Sierra/SD finite element code. Hence, it was fairly straightforward to obtain the response of the modal model at every node of the finite element model. The modal coordinate amplitudes were not provided directly, so they were estimated from the acceleration time histories using

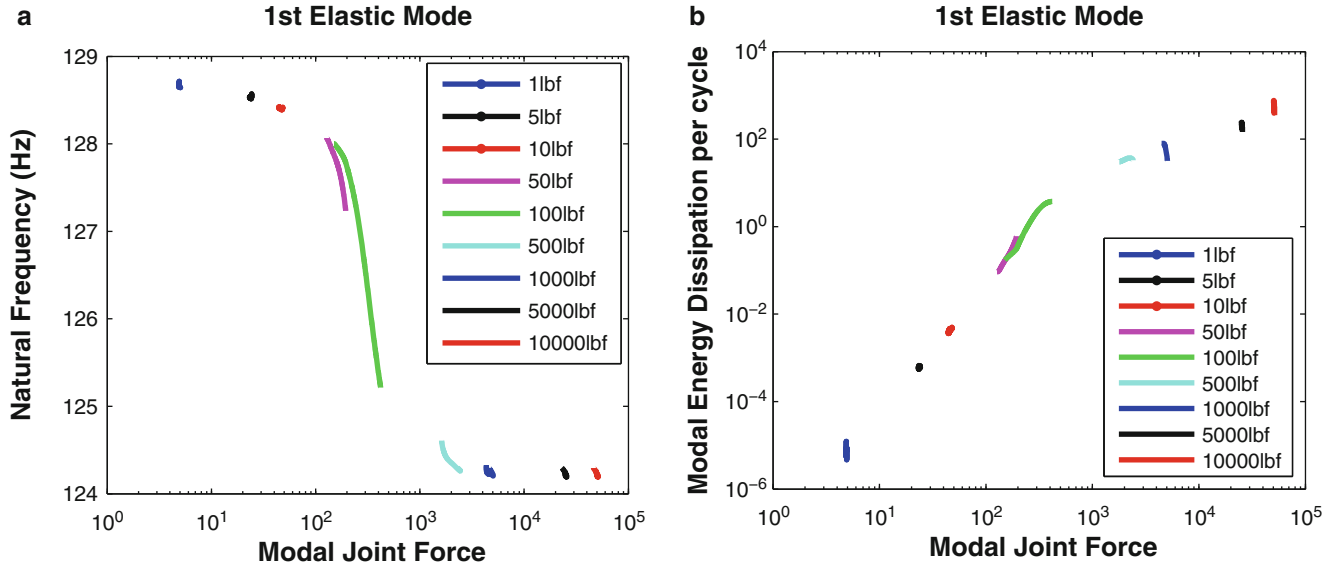


Fig. 2.7 Natural frequency (a) and energy dissipation (b) versus modal joint force for the 1st elastic mode of the finite element model. The change in the natural frequency observed in (a) is used to find the modal joint stiffness ($\hat{K}_T$) and the sharp change in slope in (b) is used to find the modal macro-slip force ($\hat{F}_S$)

Table 2.2 Iwan parameters used for the discrete Iwan simulations

Mode	$\hat{K}_T$	$\hat{F}_S$	$\hat{\chi}$	$\hat{\beta}$
1 st Elastic	$4.45 \cdot 10^4$	239.4	-0.183	0.3
2 nd Elastic	$2.0 \cdot 10^4$	$1.0 \cdot 10^5$	-0.999	3.0
3 rd Elastic	$8.73 \cdot 10^5$	534.6	-0.048	0.38

the same process described previously. Both simulations were integrated using a Newmark-Beta integration scheme with an iterative Newton loop to solve the residual force equation. The step size was set to $1 \cdot 10^{-5}$ seconds with 20,000 time steps for a total time history of 0.2 seconds.

2.3.4 Simulation Results

The response of the truth model (i.e. that in Sect. 2.3.2) was modally filtered and the modal energy dissipation versus force was found using the procedure described in Sect. 2.2.4. The modal joint stiffness $\hat{K}_T$ and modal macro-slip force $\hat{F}_S$ were then estimated from this data, shown in Fig. 2.7. Figure 2.7 shows the frequency versus modal joint force, $\hat{F}_J$ in Eq. (2.5), was estimated from the simulated measurements. Figure 2.7 shows the modal energy dissipation, $\hat{D}$ in Eq. (2.8), versus modal joint force. Notice that the joint transitions to macro-slip over a range of forces so the modal slip force cannot be determined precisely. The modal Iwan parameters were deduced using the procedure described in Sect. 2.2.5, and the parameters obtained are shown in Table 2.2 for the first three elastic bending modes.

2.3.4.1 Comparison of Modal Time Responses

Now that the modal model has been identified, one can compare its response to that of the finite element truth model in Sect. 2.3.2. To simplify the comparison, the modal responses of the first mode were first assessed with impact force levels of 10 lbf and 1000 lbf, and the comparison is shown in Fig. 2.8. The modal responses agree very precisely at the 10lbf level. On the other hand, when an impact force with an amplitude of 1000lbf is applied to the model, the modal Iwan model seems to damp the response more than the discrete Iwan model. It is also interesting to note the sharp initial increase in modal acceleration due to the impact force, which seems to be well captured at both force levels.

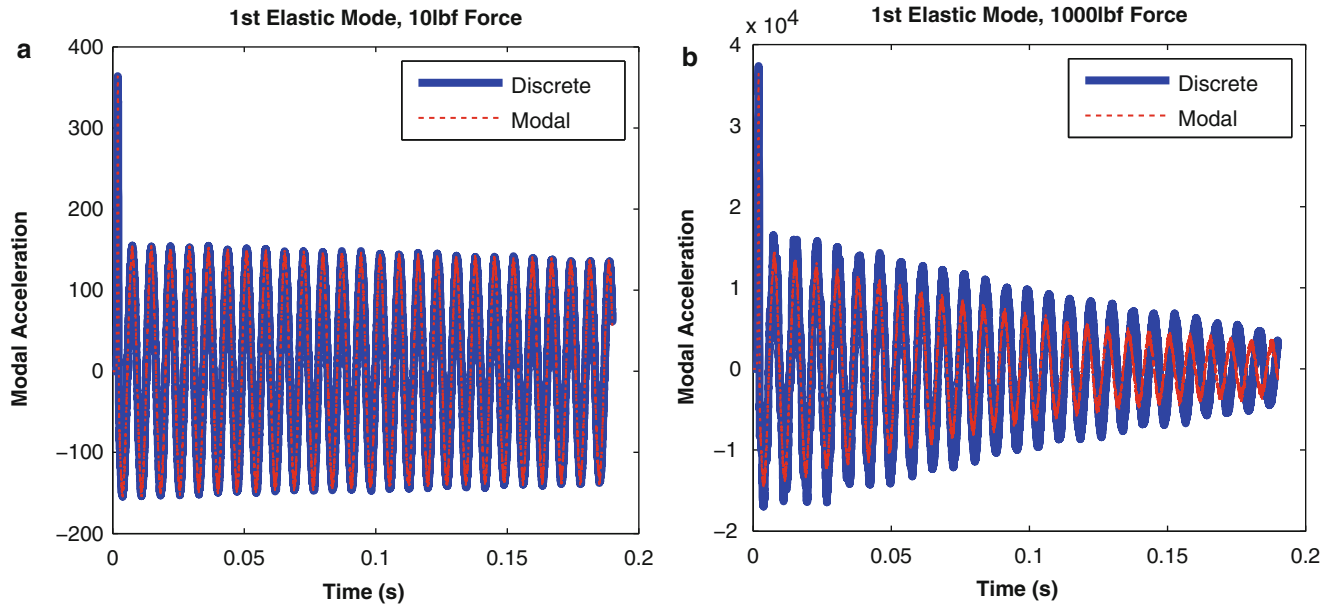
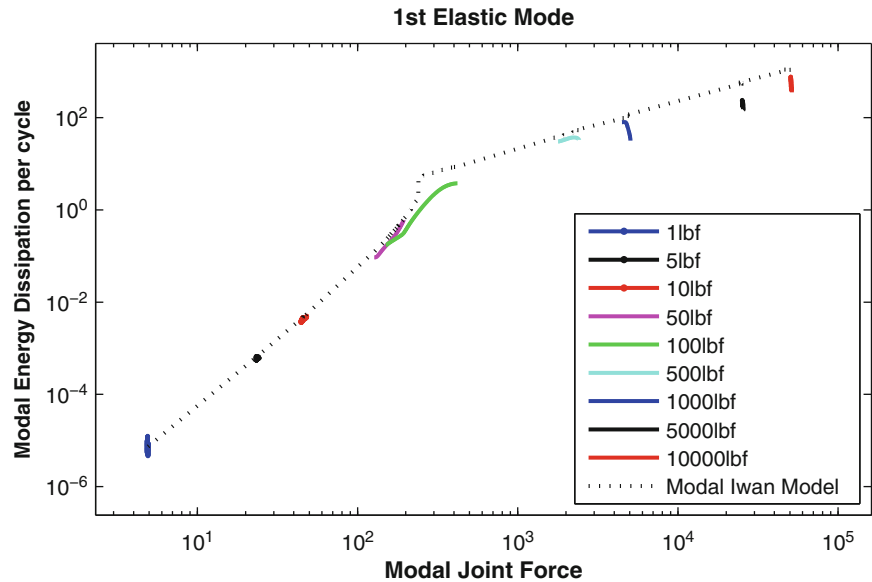


Fig. 2.8 Comparison of the response of the first elastic mode for both the Modal and Discrete Iwan models for impact forces of 10 lbf (a) and 1000 lbf (b)

Fig. 2.9 Modal energy dissipation versus modal force for the 1st elastic mode. The *colored lines* show simulations of the finite element truth model (with discrete Iwan joints) and the *black dashed line* shows the dissipation for the modal Iwan model. The two compare well for a range of impact forces from 1 lbf to 10,000 lbf (color figure online)



It becomes clear that one would have to compare a large number of time histories to assess the performance of the modal Iwan model over the force range of interest. It is more effective to compare the modal energy dissipation that each model gives over the entire range of modal force. The energy dissipation of the modal Iwan model was computed using Eq. (2.15), (2.16), and (2.17) and compared to that used to construct the model, as shown in Fig. 2.9.

2.3.4.2 Comparison of Modal Energy Dissipation

The comparison for the 1st elastic mode, in Fig. 2.9, reveals that the modal Iwan model does a very good job of predicting the energy dissipation for a range of impact loads that span the micro-slip and macro-slip regions. However, the data at higher force levels shows considerable scatter and the agreement is not as good at those force levels. This is why the comparison in Fig. 2.8 showed the modal Iwan model over-predicting the damping in the system at high force levels. Based on Fig. 2.9, one would expect a similar level of disagreement at 5,000 and 10,000 lbf.

Fig. 2.10 Modal energy dissipation versus modal force for the 2nd elastic mode. The 2nd bending mode is lightly damped and the fit reveals that this mode can be well approximated as linear

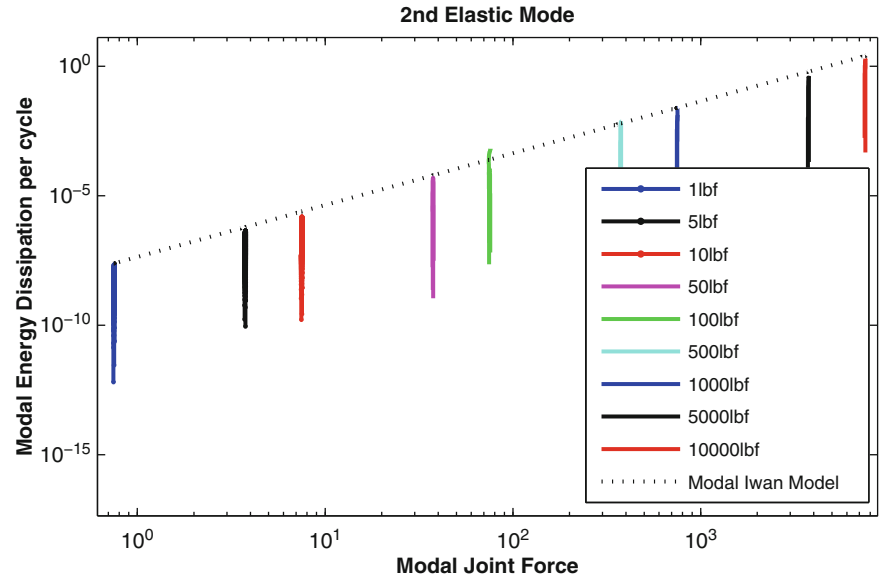
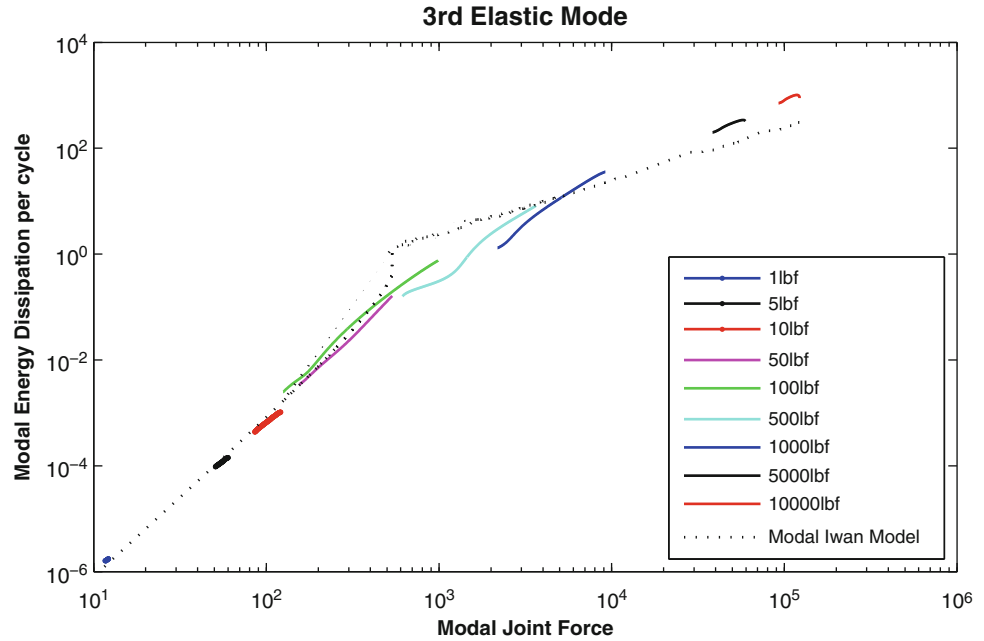


Fig. 2.11 Modal energy dissipation versus modal force for the 3rd elastic mode. The modal model agrees well with the discrete simulations for this mode in the micro-slip region, but not in the macro-slip region



The modal energy dissipation versus force for the second elastic mode is shown in Fig. 2.10. There are several interesting features in this plot. First notice how each simulation's energy dissipation seems to drop to essentially zero towards the end of each time record. Each time history was limited to 0.2 seconds due the small time step required and the large volume of data that was generated when each nodal time history was written to a file. Since there is an extremely small amount of damping in the 2nd bending mode, the fitted envelope has very little decay. Hence, the estimate of the energy dissipation is very sensitive to envelope fit by the Hilbert Transform algorithm. The slope of the modal Iwan model in Fig. 2.10 has a value of approximately 2 corresponding to a $\hat{\chi}$ value of -1 which is characteristic of a linearly damped system. Thus, the dissipation in this mode could be modeled with a linear mode and a linear modal damping ratio instead of a modal Iwan model. Apparently, the location of the link is such that the discrete Iwan joints are not exercised by this mode.

The comparison in Fig. 2.11 shows that the deduced modal Iwan model for the 3rd elastic mode does a very good job of predicting the energy dissipation in the micro-slip region, for which a $\hat{\chi}$ value of $\hat{\chi}_3 = -0.048$ was obtained. This value indicates quite a strong dependence on force (recall that each discrete joint had $\chi = -0.3$). On the other hand, the modal Iwan model does a poor job of capturing the macro-slip region. The slope of the discrete Iwan simulations seems to be greater

than the slope of the modal Iwan model in macro-slip. This suggests that Eq. (2.17) may not entirely describe the energy dissipation in the macro-slip region for this system. A similar effect was observed for some of the modes of the system studied in [5], the cause of which will be explored further in the future.

2.3.4.3 Discussion

These results validate the use of a modal Iwan model to capture the effect of discrete Iwan joints in the micro-slip region. On the other hand, the results suggest that the modal Iwan model may need to be modified to capture the response of a system at higher force levels corresponding to the macro-slip region. This is to be expected, perhaps, since the mode shapes used for the macro-slip region include joint stiffness; however, when the joints are in the macro-slip region the mode shapes should no longer include the joint stiffness. Also, the assumption of negligible modal interactions could very well be violated when the joints slip significantly.

2.4 Beam Experiments

The modal Iwan methodology was also assessed experimentally using experimental measurements from a free-free beam with a bolted link attached. The experimental setup was designed to minimize the effect of damping associated with the boundary conditions. Free boundary conditions were used because any other choice, e.g. clamped, would add significant damping to the system. In Sect. 2.4.5, the energy dissipation characteristics of the modal Iwan model are compared with those of the actual experiment.

2.4.1 Test Structure

The structure tested in this work was first used by Sumali in [17]. The beam was designed to have the following characteristics: the structure had numerous modes from 0–2000 Hz, the modes are well separated in frequency and the modes do not switch order when the links are attached. Previously, a washer was inserted between the link and the beam as was done for the finite element model in Fig. 2.6. Initial tests were done in this configuration but the damping in the joint seemed linear, indicating that there was no slip present in the joint for the force levels tested. The washers were then removed in order to spread out the clamping force between the beam and the connecting element. The resulting configuration is shown in Fig. 2.12. This change seemed to cause the joint to slip more easily, so a greater degree of nonlinearity was observed. Also, the bolt torque for the original experiments was set between 80 and 110 inch-pounds which results in bolt preload force of approximately 1600–2200 lbf. The bolt was tightened to a much lower torque of only 5 inch-pounds in these experiments.

The dimensions of the beam and link are the same as in the finite element model described in Sect. 2.3.1. However, there is no washer that separates the beam ($20'' \times 2'' \times 0.25''$) from the link ($3.5'' \times 0.5'' \times 0.125''$) as shown in Fig. 2.12. The bolts used to attach the link to the beam were $1/4''$ -28 fine-threaded bolts. All components were made of AISI 304 stainless steel.

2.4.2 Lab Setup Challenges

The damping ratios of a freely supported structure are sensitive to the support conditions, as was explored in detail in [18]. Therefore, special attention must be given to the support conditions to assure that the damping that they add does not contaminate the results.

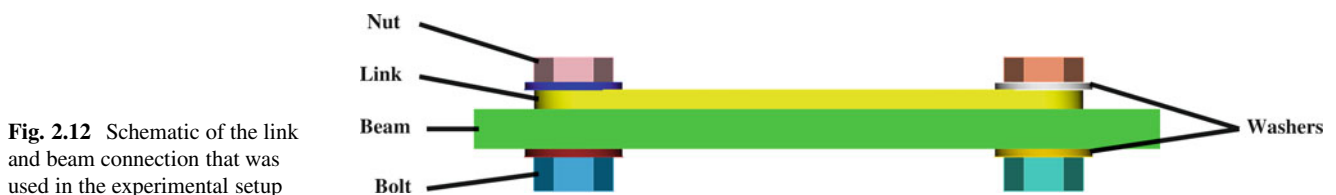


Fig. 2.12 Schematic of the link and beam connection that was used in the experimental setup

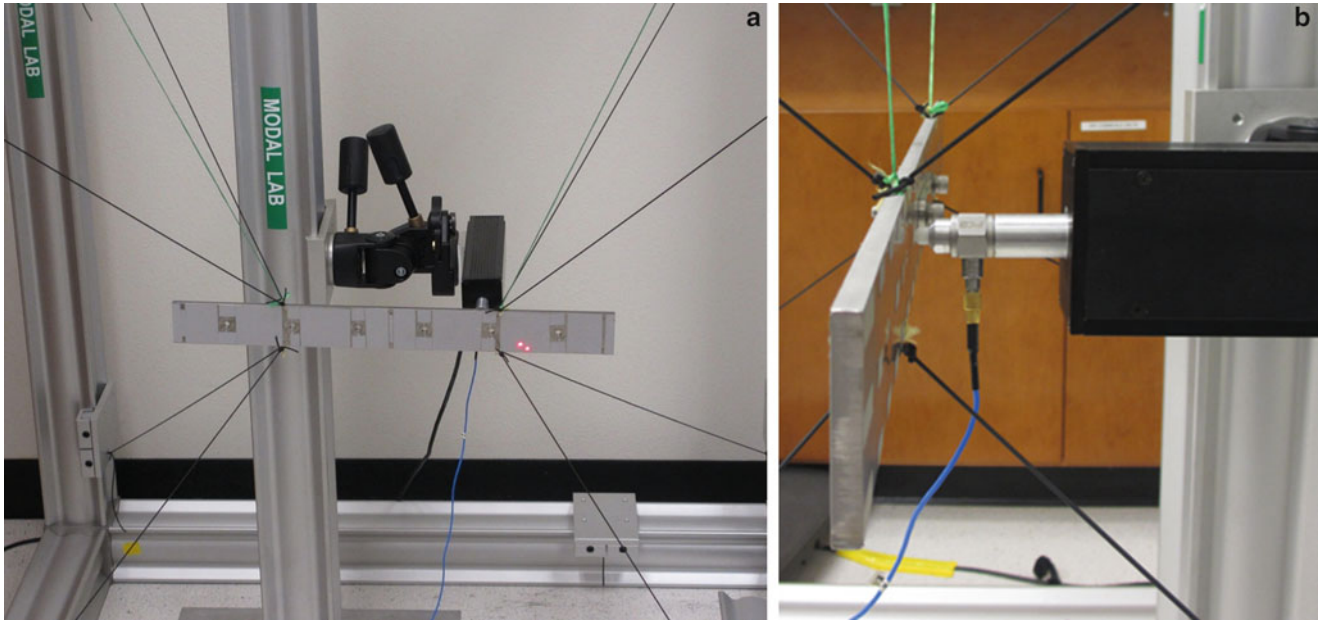


Fig. 2.13 Photograph showing how the beam was suspended (a). Automatic hammer positioned behind the impact point. One link is attached to the beam and can be seen behind the hammer (b)

Initially, the beam was suspended by two strings that act as pendulum supports as was done in [18]. To prevent the rigid body modes from contaminating the elastic mode data, the pendulum motion of the beam should have a low natural frequency compared to the first elastic mode. The general rule of thumb states that the error in the measured natural frequencies will be less than 0.5% when the rigid body natural frequencies are less than one tenth of the elastic natural frequencies of interest. However, that same design theoretically produces 10% error in the measured damping ratios. A frequency ratio of approximately 20 was obtained when the beam was supported by only these two strings. Although this test provided a good baseline that was used to evaluate the damping of the boundary conditions in subsequent tests, there were practical considerations that prohibited its use for most of the testing.

Specifically, the velocity of the beam was measured with a scanning laser Doppler vibrometer in order to eliminate any damping associated with the cables of contact sensors. Hence, if the beam swings significantly in its pendulum mode, the point which the laser is measuring may change significantly during the measurement. Also, an automated hammer was used to excite the beam, but the hammer only retracts about one inch after impact. As a result, the pendulum motion of the beam caused almost unavoidable double hits with this setup. Finally, in the processing described subsequently, it is important for the automatic hammer to apply a highly consistent impact force. Any swinging of the beam caused the impact forces to vary from test to test and it was extremely difficult and time consuming to try to manually eliminate the swinging. For these reasons, soft bungee cords were used to suppress the rigid body motion of the beam while attempting to add as little damping as possible to the system. The final set up was similar to that used in [17] and is shown in Fig. 2.13. This setup was used for all of the measurements shown in this paper.

2.4.3 Experimental Setup

The dynamic response of the beam was measured using a Polytec Scanning Laser Doppler Vibrometer (PSV-400) to measure the response at 54 points on the beam. A Polytec single point Laser Doppler Vibrometer (OFV-534) was used to measure a reference point to verify that the hammer hits were consistent. The reference laser was positioned close to the impact force location. An Alta Solutions Automated Impact Hammer with a nylon hammer tip was used to supply the impact force, which is measured by a force gauge attached between the hammer and the hammer tip.

The beam is suspended by 2 strings that support the weight of the beam and the 8 bungee cords prevent excessive rigid-body motion. The bungees and strings were connected to the beam at locations where the odd bending modes have little motion in order to minimize damping in those modes. However, even after all of the care that was taken, some damping was added by the supports. A preliminary analysis was done in which the damping ratios of the first few modes of the beam were measured with and without bungees, and the results are shown in Table 2.3.

Table 2.3 Damping ratios for the beam with and without bungees and with and without the link attached

Mode	ζ without bungees and without link (%)	ζ without bungees with link at 50 in-lbf of torque (%)	ζ with bungees without link (%)	ζ with bungees with link at 5 in-lbf of torque (%)
1 st Elastic	0.01	0.025	0.016	0.15
2 nd Elastic	0.025	0.03	0.055	0.06
3 rd Elastic	0.02	0.025	0.045	0.18

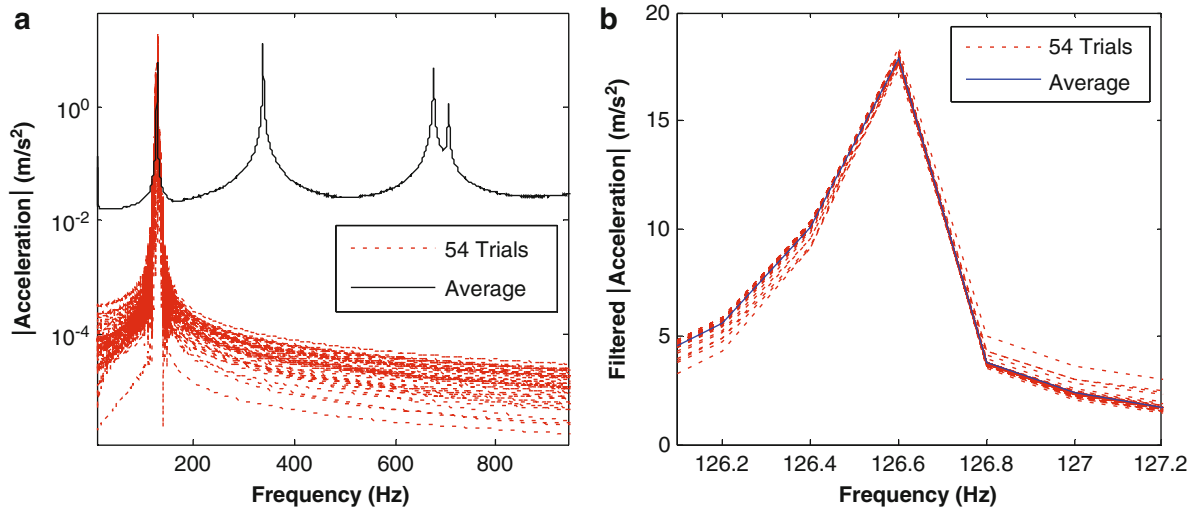


Fig. 2.14 (a) Filtered frequency responses of all 54 data points compared with the average of the unfiltered frequency response. (b) Filtered frequency responses after dividing by the mass normalized mode shape value; the trials are remarkably consistent

The damping of all of the modes is light, as one would expect for a monolithic structure with only a few bolted joints; the largest measured damping ratio was $\zeta = 0.0006$ for the second mode with the link and bungees attached. For the setup without bungees, the damping ratios of the first three elastic modes were found to increase slightly when the link was attached and the bolts were tightened to 50 in-lbf of torque. As discussed, a high torque will cause a large preload in the bolts which inhibits micro-slip. When the bungees were added to the setup, the damping ratios for all modes increased by a factor of between 1.6 and 2.3. For the first mode this is comparable to the increase in damping that was observed when the link was attached at 50 in-lb of torque, and it is considerably larger than the damping caused by the joint for modes 2 and 3. For this reason, the bolt torque was reduced to 5 in-lbf, causing the damping in the first mode to increase by an order of magnitude. The damping in the 2nd and 3rd modes increased to above what was observed for the beam without a link but with the bungees attached.

2.4.4 Lab Data Processing

A couple of approaches were explored to extract modal acceleration ring-downs from the laboratory data. Mass normalized mode shapes were found by fitting a linear modal model with the Algorithm of Mode Isolation (AMI) [19]. Then the mode shapes were used in a modal filter as in Eq. (2.18), as was done in Sect. 2.3.4. However, the modal responses obtained still showed clear evidence of frequency content due to other modes. Another approach that was explored was to band pass filter the data in the frequency domain in order to isolate a mode as was done in [14]. Although this approach seemed to work, each measurement point had to be filtered and analyzed separately, and no averaging could be performed for all the measurement points since each point has a different magnitude associated with its mode shape value. In the end the measurements were band pass filtered in order to isolate each mode, as was done in [14], and then scaled by the corresponding mode shape value. A Butterworth filter was used in the band pass filtering process. The cut-off frequencies used for the first elastic mode were 120 and 135 Hz, and the FFT of the resulting filtered responses is shown in Fig. 2.14. The measurements at each point were then scaled by the corresponding mass normalized mode shape value in order to obtain a single average modal acceleration for each mode from the set of 54 measurement points. The resulting time responses are shown in Fig. 2.15, as well as the average time response that was used to identify a modal Iwan model.

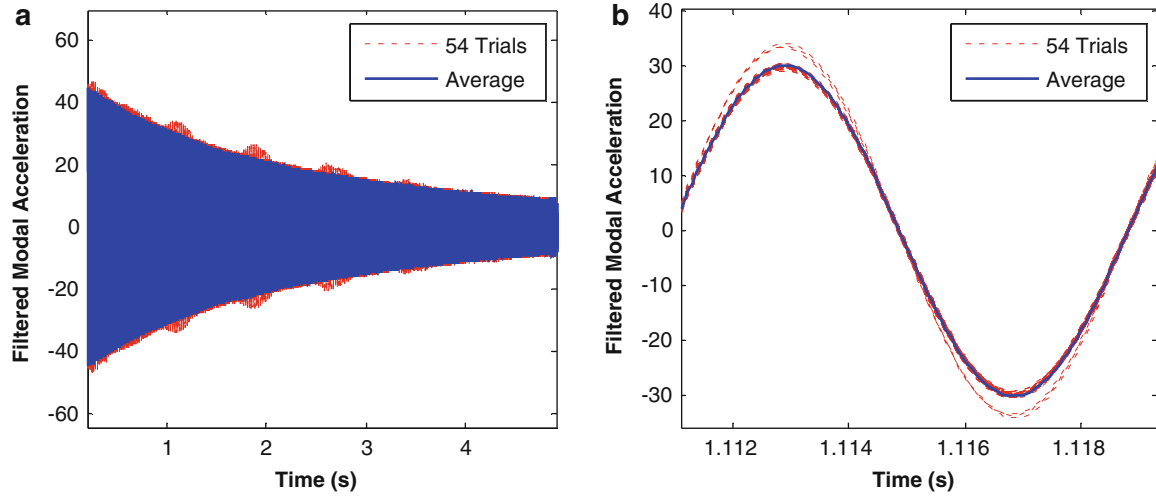
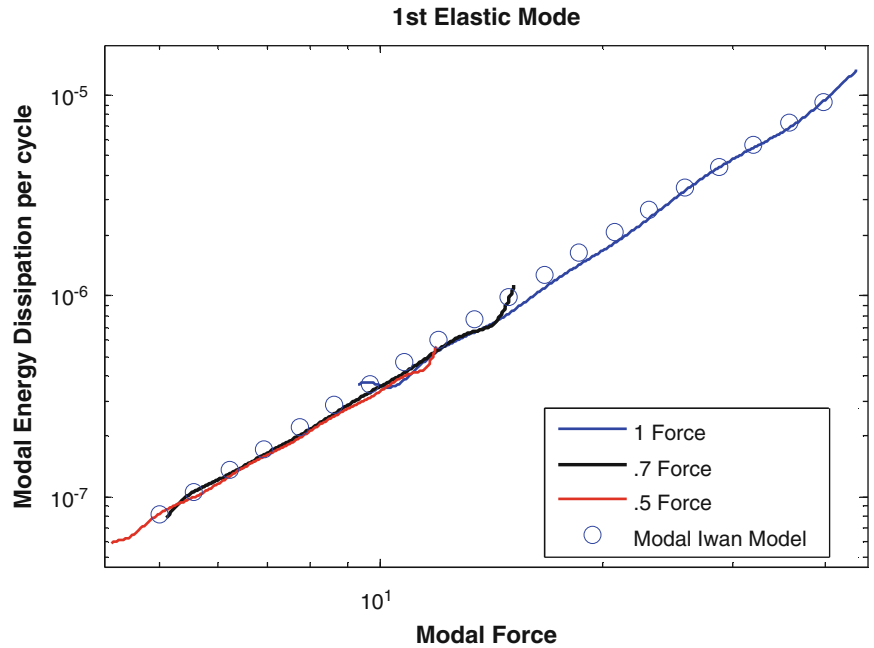


Fig. 2.15 (a) Modal time responses for the first mode and the averaged time response. (b) Detail view showing that the measurements were highly consistent except for three measurement points

Fig. 2.16 Modal energy dissipation per cycle versus modal force for the 1st elastic mode



The responses are remarkably repeatable suggesting that the impact load provided by the automatic hammer was essentially the same from trial to trial. On the other hand, there are a few trials that differed from the others by 10% or more. For the data shown in the figures above, the amplitude of the impact force ranged from approximately 155 to 160 Newtons.

The voltage applied to the automatic hammer was adjusted so that a range of impact forces were applied to the beam. Then, the response was measured at all 54 points and this process was repeated at each impact force level.

2.4.5 Lab Results

Modal Iwan models were deduced from laboratory data in a similar fashion as was done in the simulation data. However, macro-slip was not clearly observed at the force levels that were used, so the parameters $\{\hat{K}_T, \hat{F}_S, \hat{\beta}\}$ were estimated and then adjusted until the modal Iwan model reproduced the measured dissipation versus force curve as well as possible. The energy dissipation per cycle for the 1st elastic mode is shown in Fig. 2.16.

Table 2.4 Slope of the dissipation versus force curves for different impact force levels

Impact force level	Slope	$\hat{\chi}$
1 Force	2.398	−0.602
0.7 Force	2.168	−0.832
0.5 Force	2.096	−0.904

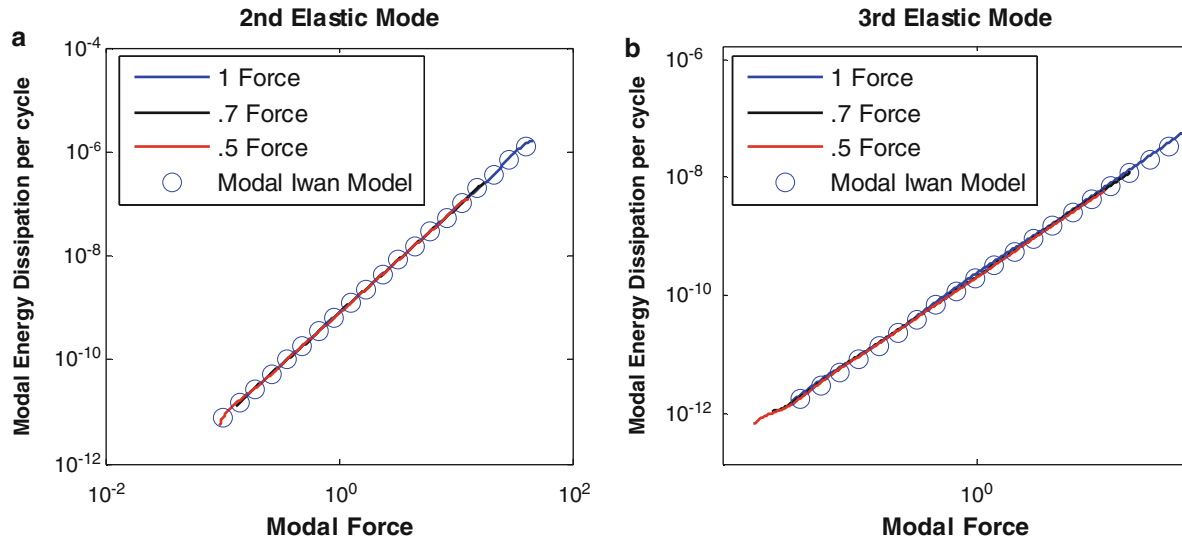


Fig. 2.17 Modal energy dissipation per cycle versus modal force for the 2nd (a) and 3rd (b) elastic modes

Three different force levels were used. The notation “1 Force” indicates that the maximum voltage was given to the automatic hammer, while the 0.7 and 0.5 Force cases correspond to 70% and 50% of the maximum voltage, respectively. The measurements seem to fit the dissipation curve for the modal Iwan model quite well; however, this is a bit of an artifact of the log scale. An even better fit is obtained if each response is taken separately, but when this is done the slope of the curve changes with force level. The slope of each dissipation versus force curve is given in Table 2.4. At the 50% force level, the slope is 2.1 indicating a $\hat{\chi}$ value of -0.9 and a nearly linear response. The modal Iwan model used in Sect. 2.3.4 does not exhibit this phenomenon, but it is readily explained. Simulations of a single modal Iwan response reveal a very similar phenomenon if a linear viscous damping element is added in parallel with the Iwan joint. At low force levels the damping is dominated by the viscous damper, while at higher force levels the Iwan joint increases the dissipation and hence increases the value of $\hat{\chi}$. For this system, the viscous damping provided by the suspension system could be responsible for the linear damping seen at low force levels, although it was thought to be negligible at these force levels.

Figure 2.17 shows the modal energy dissipation versus modal force for the 2nd and 3rd elastic modes. Again, the modal Iwan model seems to fit the data quite well, on a log scale at least. The vertical axis also reveals that the energy dissipation per cycle associated with each mode seems to decrease with an increase in mode number. The dissipation curve for the 2nd elastic mode has a slope of 2.01 corresponding to a $\hat{\chi}$ value of -0.99 which is characteristic of a linear system. The $\hat{\chi}$ value for the 3rd elastic mode is -0.97 , also suggesting that this mode behaves linearly. Thus, the dissipation in these modes could be modeled with a linear mode and a linear modal damping ratio instead of a modal Iwan model.

The fact that the 3rd mode exhibits linear damping is quite different from what was observed in the simulations in Sect. 2.3.4. There the damping in the 3rd mode seemed to be more highly nonlinear than that of the 1st mode. Note however, that in the simulation the third elastic mode did reach macro-slip at a force that was about an order of magnitude higher than what was required for the first mode, so perhaps the force levels used were insufficient to excite nonlinearity in the third mode.

2.5 Conclusion

This work applied the modal Iwan model developed by Segalman [5] to simulation data from a finite element model with several discrete Iwan joints and to laboratory data. The 4-parameter Iwan model was used to model the nonlinear damping associated with each mode. This model was found to fit the simulation data very well in the micro-slip region, and it agreed

qualitatively well into macro-slip. This finding is encouraging; the time response of the modal Iwan model can be found in a tiny fraction of the time required to find the response of the full system with discrete Iwan joints. Further work is needed to determine whether this approach could be extended beyond macro-slip, but the results to date are fairly encouraging. Our experience revealed that while the $\hat{\chi}$ value and the joint stiffness $\hat{K}_T$ could be readily determined, the slip force $\hat{F}_S$ and $\hat{\beta}$ value were more ambiguous even when the dissipation curve was available over a wide range of forces. This suggests that it may not be advisable to extrapolate the behavior of the system to very high force levels.

On the other hand, the experimental study presented several challenges. In order to maintain similarity with the finite element simulations, a beam with a single link was tested. However, the link is small relative to the beam and seems to affect its dynamics very little. As a result, the damping effects of the joints were difficult to observe. However, the damping nonlinearity that was observable did fit the modal Iwan framework very well. This could prove highly advantageous, as the modal Iwan framework allows one to condense a very large quantity of measured data into a few physically meaningful parameters.

Acknowledgements This work was conducted at Sandia National Laboratories. Sandia is a multi-program laboratory operated under Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94-AL85000. The authors would especially like to thank Michael Guthrie for a helpful discussion regarding the simulation work. They are also indebted to Jill Blecke, Hartono Sumali, Randall Mayes, Brandon Zwink and Patrick Hunter for the help that they provided with the laboratory setup and testing.

References

1. Segalman DJ (2001) An initial overview of Iwan modelling for mechanical joints. Sandia National Laboratories, Albuquerque, New Mexico SAND2001-0811
2. Segalman DJ (September 2005) A four-parameter Iwan model for lap-type joints. *J Appl Mech* 72:752–760
3. Allen MS, Mayes RL (2010) Estimating the degree of nonlinearity in transient responses with zeroed early-time fast Fourier transforms. *Mech Syst Signal Process* 24:2049–2064
4. Segalman DJ, Holzmam W (2005) Nonlinear response of a lap-type joint using a whole-interface model. Presented at the 23rd international modal analysis conference (IMAC-XXIII), Orlando, 2005
5. Segalman DJ (2010) A modal approach to modeling spatially distributed vibration energy dissipation. Sandia National Laboratories, Albuquerque, New Mexico and Livermore, California SAND2010-4763
6. Segalman DJ, Starr MJ (2012) Iwan models and their provenance. Presented at the ASME 2012 international design engineering technical conference, Chicago, 2012
7. Quinn DD, Segalman DJ (2005) Using series-series Iwan-type models for understanding joint dynamics. *J Appl Mech* 72:666–673
8. Segalman DJ, Starr MJ (2004) Relationship among certain joints constitutive models. Sandia National Laboratories, Albuquerque, New Mexico SAND2004-4321
9. Song Y, McFarland DM, Bergman L, Vakakis AF (2005) Effect of pressure distribution on energy dissipation in a mechanical lap joint. *AIAA J* 43:420–425
10. Smallwood DO, Gregory DL, Coleman RG (2000) Damping investigations of a simplified frictional shear joint. Sandia National Laboratories, Albuquerque, New Mexico SAND2000-1929C
11. Reese GM, Bhardwaj MK, Segalman DJ, Alvin K, Driessen B (1999) Salinas—user’s notes. Sandia National Laboratories, Albuquerque, New Mexico SAND99-2801
12. Reese GM, Walsh TF, Bhardwaj MK (2011) Salinas—theory manual. Sandia National Laboratories, Albuquerque, New Mexico SAND2011-8272
13. Sumali H, Kellogg RA (2011) Calculating damping from ring-down using hilbert transform and curve fitting. Presented at the 4th international operational modal analysis conference (IOMAC), Istanbul, 2011
14. Sracic MW, Allen MS, Sumali H (2012) Identifying the modal properties of nonlinear structures using measured free response time histories from a scanning laser Doppler vibrometer. Presented at the international modal analysis conference XXX, Jacksonville, 2012
15. Feldman M (1993) Non-linear system vibration analysis using hilbert transform—I. Free vibration analysis method ‘freevib’. *Mech Syst Signal Process* 8:119–127
16. Zhang Q, Allemang RJ, Brown DL (1990) Modal filter: concept and application. Presented at the 8th international modal analysis conference (IMAC VIII), Kissimmee, 1990
17. Sumali H (2011) An experiment setup for studying the effect of bolt torque on damping. Presented at the 4th international conference on experimental vibration analysis for civil engineering structures (EVACES), Varenna, 2011
18. Carne TG, Griffith DT, Casias ME (2007) Support conditions for experimental modal analysis. *Sound Vib* 41:10–16
19. Allen MS, Ginsberg JH (2006) A global, single-input-multi-output (SIMO) implementation of the algorithm of mode isolation and applications to analytical and experimental data. *Mech Syst Signal Process* 20:1090–1111

Chapter 3

Identification of Nonlinear Joint Characteristic in Dynamic Substructuring

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Abstract Assembled structures often include local nonlinear behavior depending on the interface properties which can strongly change the overall dynamics. A typical example is a high local damping due to friction in bolted joints. A further example are bushing elements, like those which are widely-used in the automotive industry and which often show complicated characteristics like nonlinear stiffness and frequency dependency. In this contribution, measured and simulated FRFs of different systems are considered in order to identify nonlinear joint characteristics. The computation of the FRFs is established by the Harmonic Balance Method using a substructure formulation. A detection of the influence of the local damping properties is done by the Hilbert Transformation. For the computation of FRFs of systems with progressive stiffness, the Harmonic Balance Method is extended by a Continuation Method in order to be able to capture multiple solutions and is applied to a numerical example system. As an application an assembly consisting of two beam-like substructures coupled by a bolted joint is used in order to identify the friction characteristic of a real system. The results are used for further investigations of coupled axle parts of a wheel brake.

3.1 Introduction

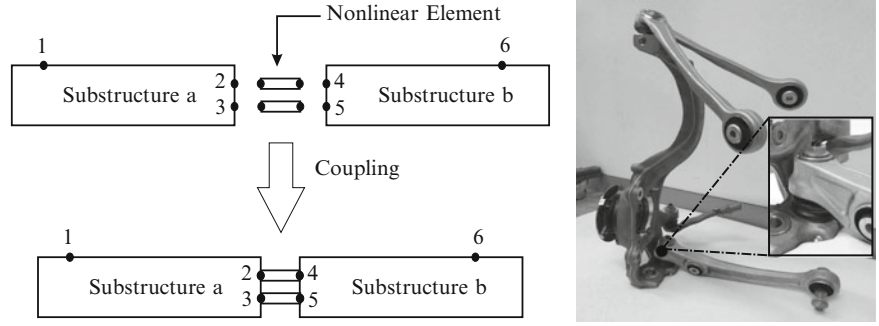
Substructuring methods offer the possibility to model high order finite element models in an efficient way. A separated representation of the dynamics of the participated substructures enables the application of model reduction methods like the Craig-Bampton method and a later assembly of the different parts by Component Mode Synthesis (CMS). Therefore, the degrees of freedom (DoF) can be reduced drastically. Alternatively, frequency response functions (FRF) can be used to characterize the dynamics of an assembly for a specific input-output behavior and the coupling is done in the frequency domain by a Frequency Based Substructuring Method [3].

For real life structures like the wheel brake in Fig 3.1, the coupling of the substructures often introduces nonlinear behavior to the overall structure due to additional dynamics within the joint like e.g. friction, clearance or complicated types of stiffness. The nonlinearities have to be considered in the modeling and in the substructuring formulation which leads schematically to systems like can be seen in Fig. 3.1 on the left. Since the joints are local nonlinearities, the influence on the overall dynamics differs depending on the intensity of the motion in the interface at each mode. Therefore, a nonlinear contribution in FRFs is not always obvious and for complex assemblies is a predictive statement about the influence of the nonlinearity a challenging task. In order to get a more systematic approach to detect nonlinearities, the Hilbert Transformation (HT) is a useful tool to detect nonlinear influences in a FRF. In this paper the Hilbert Transformation is used to detect and identify the characteristics of the nonlinearities in coupled systems. The paper is organized as follows. In the first section the nonlinearities which are considered in this papers are presented. Then the Hilbert transformation is briefly introduced and its usage to detect nonlinearities is shown. The method is applied to a numerical example in the fourth section. An extension of the computation method of FRFs is presented in section five and applied to a similar numerical example. In section seven an application of the method is given. The paper closes with a short conclusion.

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Fig. 3.1 *Left:* Coupling of two substructures with nonlinear elements. *Right:* Automotive wheel brake assembly



3.2 Types of Nonlinearity

In this section the different types of nonlinearities considered in this paper are presented. Important nonlinear behavior in joints often show stiffening effects or friction occurs between two coupling interfaces. A stiffening effect can be modeled by a progressive polynomial stiffness, e.g. a cubic stiffness is used in the well-known Duffing oscillator. Friction can be modeled by a Jenkins friction element [4] which consists of a spring and a Coulomb element in series. The behavior of a Jenkins element for high amplitudes is similar to a degressive stiffness but with an additional saturation which leads to damping. The force-displacement relations for these nonlinearities are schematically shown in Fig. 3.2. Within this paper FRFs are used which are established for these types of nonlinearities by a substructuring method using the Harmonic Balance Method [4]. The detection of the nonlinearities within the FRFs is done by the Hilbert Transformation which is briefly introduced in the next section.

3.3 Hilbert Transformation

The Hilbert Transformation (HT) is an integral transformation as well as the Fourier Transformation. In contrast to the Fourier Transformation which transforms a time domain signal in the frequency domain and vice versa, the Hilbert Transformation is a transformation within the same domain. For an application of the HT on complex FRF, the principle of the transformation is that the real part of the FRF $G(\omega)$ can be computed by its imaginary part. After some mathematics [6], this relation can be expressed by

$$\text{Re } G(\omega) = i \text{Im } G(\omega) * -\frac{i}{\pi\omega} = \mathcal{H} \{ \text{Im } G(\omega) \}, \quad (3.1)$$

$$\text{Im } G(\omega) = \text{Re } G(\omega) * -\frac{i}{\pi\omega} = \mathcal{H} \{ \text{Re } G(\omega) \}. \quad (3.2)$$

Here, $\mathcal{H}$ denotes the Hilbert operator and $*$ describes the convolution integral. For more details to the derivation of these relations, see [2, 5, 6]. The Hilbert Transformation uses the property of causality of an impulse response function which is established e.g. by an Inverse Fourier Transformation of the FRF $G(\omega)$. For linear systems, the causality of the derived impulse response of a linear FRF $G(\omega)$ is always given [6], so that $G(\omega)$ is invariant under the Hilbert Transformation. This means that the Hilbert transform $H(\omega)$ is equal to the FRF $G(\omega)$,

$$H(\omega) = G(\omega) = \mathcal{H} \{ G(\omega) \}. \quad (3.3)$$

For nonlinear systems this relation does not hold. The Hilbert transform is not equal to the FRF, $H(\omega) \neq G(\omega)$. This fact can be used to detect nonlinearities within FRFs which is done in this paper. Applied to the previously presented nonlinearities, a consideration of the Nyquist plots in the complex plane nearby a mode displays this fact, Fig. 3.3. Beside the detection of a nonlinear influence within a mode, an interpretation of the distortion of the Hilbert transform admits a statement of the kind of nonlinearity which is in the system. For example, the distortion of a progressive and degressive stiffness, Fig. 3.3, show in different directions.

Fig. 3.2 Force-displacement relation for different types of nonlinearities

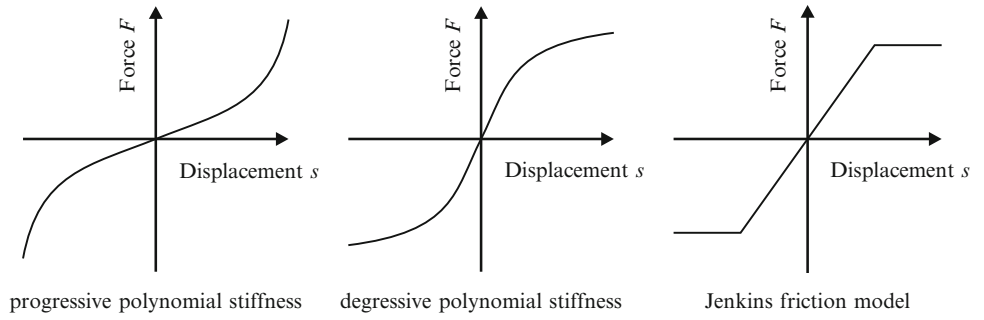
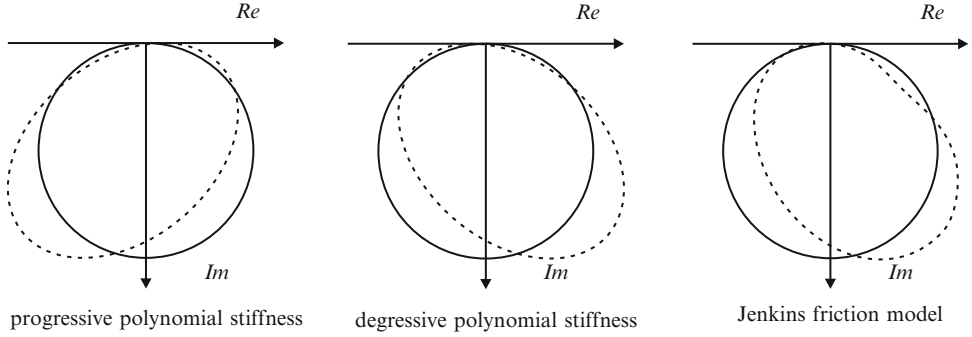


Fig. 3.3 Nyquist plot of the three types of nonlinearities. The FRF $G(\omega)$ (solid) and the Hilbert transform $H(\omega)$ (dashed) is shown



Corehence Function

Since the interpretation of the distortion of the Hilbert transform in a Nyquist plot strongly depend on the quality of the underlying data, a detection or actually an identification of the nonlinearity is sometimes not that clear. Therefore, in [5, 6] the so called Corehence Function is introduced which is defined by

$$\lambda(\omega)^2 = \frac{|E \{H(\omega)G(\omega)^*\}|^2}{E \{|H(\omega)|^2\} E \{|G(\omega)|^2\}}. \quad (3.4)$$

Hereby, the operator $E \{ \dots \}$ denotes the mean value of several FRFs and appropriate Hilbert functions. Similar to the well known coherence function this function has a value of one if the FRFs are equal and decreases in regions where the function values differ. Applied to FRFs which are measured for different excitation amplitudes this function can be used to detect which frequency range or mode is influenced by the nonlinearity.

3.4 Numerical Example: Jenkins Friction

In this section the proposed method is applied to an example system consisting of two equal two-mass oscillators coupled by an element containing a Jenkins friction element, see Fig. 3.4. Parameters of the system are given in Table 3.1. The excitation force F_{exc} is applied at substructure a and the displacement output x_4 at substructure b is considered. FRFs are computed using the Harmonic Balance Method with a Frequency Based Substructuring formulation, see [4] for details.

Since the nonlinearity represents a local property not all four modes of the system are effected by it. Depending on the modeshape of the underlying linear system only the second and fourth mode show damping due to the friction, Fig. 3.5. A computation of the Corehence function of two FRFs of this system with two different excitation amplitude $\hat{F}_{\text{exc}} = 15 \text{ N}$ and $\hat{F}_{\text{exc}} = 30 \text{ N}$, Fig. 3.5, verifies this result. Additionally, the intensity of the nonlinear influence is indicated by the value of the Corehence function. The Nyquist plot of the second mode, Fig. 3.6 on the left, offers the possibility to identify the characteristic of the nonlinearity since the distortion of the Hilbert transform goes to the right, like expected. The Nyquist plot of the third mode, Fig. 3.6 on the right, shows that the Hilbert transform is unaffected by the nonlinearity which matches the results of the FRF and the Corehence.

Fig. 3.4 Example system consisting of two substructures *a* and *b* coupled by a Jenkins friction element

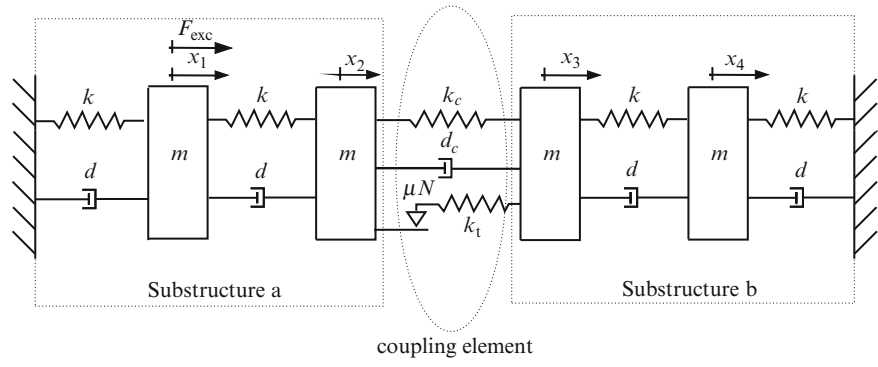


Table 3.1 Parameters of the system with Jenkins friction element

Parameter	Value	Unit	Parameter	Value	Unit
m	1	kg	k_c	$4 \cdot 10^4$	N/m
d	4	Ns/m	k_t	$4 \cdot 10^4$	N/m
k	$4 \cdot 10^4$	N/m	μ	1	–
d_c	4	Ns/m	F_{exc}	15, 30	N

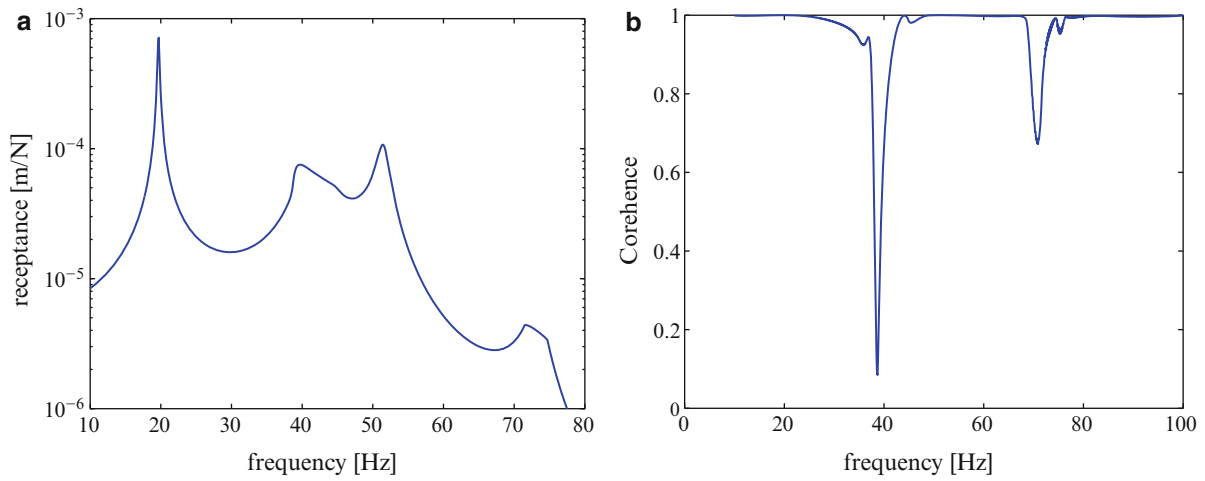


Fig. 3.5 Left: FRF $\hat{x}_4/\hat{F}_{rmexc}$ for the coupled system. Right: Corehence function of two FRFs with different excitation amplitudes

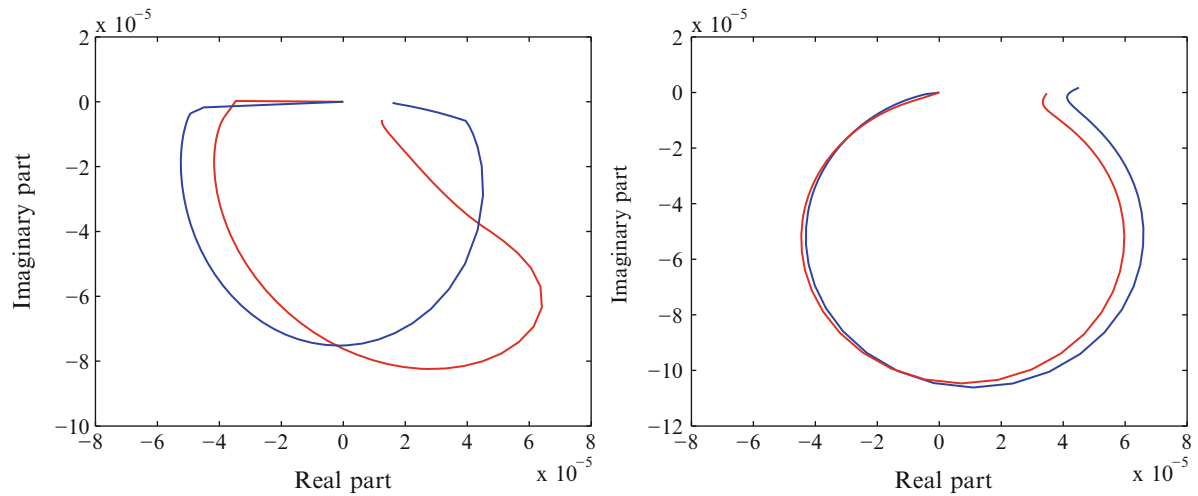


Fig. 3.6 Left: Nyquist plot of the FRF at the second mode (blue) and appropriate Hilbert transform (red). Right: Nyquist plot of the FRF at the third mode (blue) and appropriate Hilbert transform (red) (color figure online)

3.5 Continuation Method

In the previous section a nonlinearity of friction type was investigated. The FRFs of these types of nonlinearities are calculated using the Harmonic Balance Method and can be solved with standard Newton-Raphson method. Therefore, the equations of motion are transformed into the implicit form $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ and are solved iteratively at predefined frequencies with a frequency increment δf . For a consideration of systems with a progressive stiffness additional numerical effort has to be made due to the occurrence of multiple solutions, like well known from the Duffing oscillator. A standard Newton-Raphson method is only able to compute one solution at each frequency step depending on the initial value of the iteration. This leads to jumps in the FRF, see Fig. 3.7.

To not jump over the frequency range with multiple solutions, the solution algorithm has to be extended by a constraint g guiding the direction along the curve. Hence, an additional parameter has to be introduced to control the frequency suiting to the actual course of the curve. This also leads to the possibility of calculating multiple solutions for the same frequency. Generalized, this leads according to [7] to a problem of the form

$$\tilde{\mathbf{f}}(\mathbf{x}, \lambda) = \begin{pmatrix} \mathbf{f}(\mathbf{x}, \lambda) \\ g(\mathbf{x}, \lambda) \end{pmatrix} = \mathbf{0} \quad (3.5)$$

describing a function $\mathbf{f}$ and an additional path-following constraint g which are not only dependent on $\mathbf{x}$ but also dependent on a parameter λ parameterizing the curve. For example this parameter can be chosen as the frequency for the calculation of FRFs. There exist several different concepts to solve such systems that commonly lead to Prediction-Correction Methods [1]. In this work a Prediction-Correction Method is used which consists of a Tangent Prediction step and a Gauss-Newton Correction. Here, a prediction of the next point $\mathbf{x}_{p(n+1)}$ is done based on the derivative $\mathbf{v}_n$ and a distance h of a found solution $\mathbf{x}_n$ at a frequency step n . The solution is searched in a hyperplane perpendicular to the tangent vector $\mathbf{v}_n$ through $\mathbf{x}_{p(n+1)}$. Corrections with the Gauss-Newton are done at each iteration step k updating the tangent vector until the solution on the curve $\mathbf{x}_{n+1}$ is found. Additionally, a step size control for the frequency is implemented. A schematic diagram of the used algorithm is shown in Fig. 3.8.

3.6 Numerical Example: Cubic Stiffness

In this section the proposed algorithm is applied to the four mass oscillator with a cubic stiffness within the coupling element. The nonlinear force is calculated by the following law

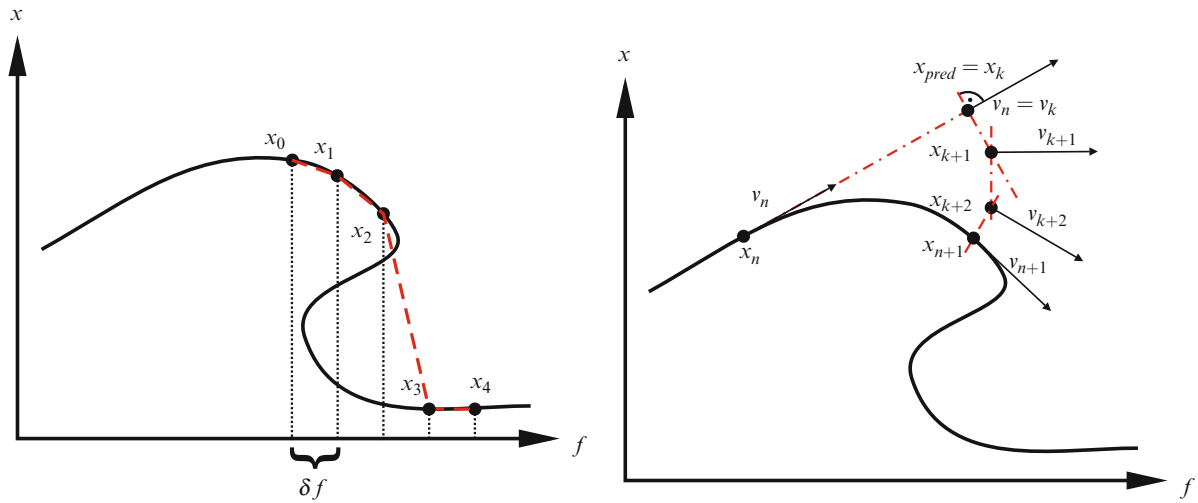
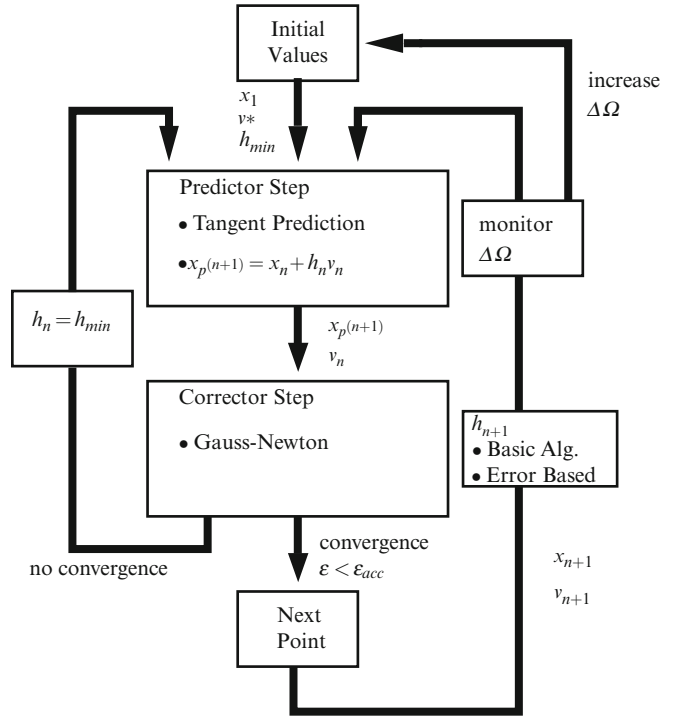


Fig. 3.7 Left: Problems with multiple solutions when using a standard Newton-Raphson algorithm. Right: Prediction correction method

Fig. 3.8 Predictor-corrector algorithm overview



$$F_{nl} = \beta(x_3 - x_2)^3. \quad (3.6)$$

Excitation is done at the second degree of freedom x_2 and FRFs are given also for the second DOF, see Fig. 3.9. The parameters of the system are given in Table 3.2.

The results of the FRF computation with the continuation method show that again not all modes are effected by the nonlinearity. Like expected, the second and fourth mode exhibit nonlinear behavior. For high enough excitation forces there arise multiple solutions, Fig. 3.10. The algorithm is able to indicate the different branches. In Fig. 3.10 the nonlinear influence is obvious in the FRF. For testing the detection method, FRFs for a small excitation amplitude are used where the nonlinear influence is hardly visible, see Fig. 3.11. The Corehence function of these FRFs is again able to detect the nonlinear influence, Fig. 3.12 on the left. Even the Hilbert transform in the Nyquist plot of the fourth mode hints the kind of nonlinearity, Fig. 3.12 on the right.

3.7 Application

The proposed joint characteristic identification method is applied in this section to a two beam experiment where the beams are coupled by a bolted joint, Fig. 3.13. The excitation is done by a shaker at the left beam and the acceleration is measured at the right beam. The dimensions of the system are given in Fig. 3.14. The clamping force of the bolted joint is measured by a force transducer.

Fig. 3.9 Example system consisting of two substructures a and b coupled by a cubic stiffness

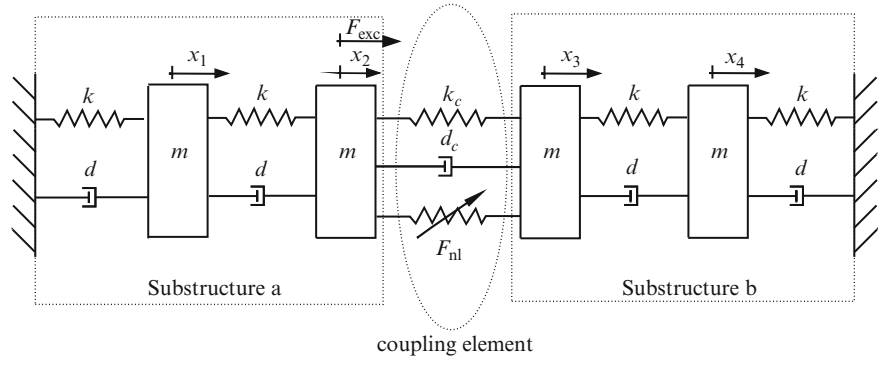


Table 3.2 Parameter of the numerical example system with a cubic stiffness

Parameter	Value	Unit	Parameter	Value	Unit
m	1	kg	d_c	0.02	Ns/m
k	1	N/m	β	0.05	N/m
d	0.02	Ns/m	F_{exc}	0.1, 0.2, 0.3	N
k_c	1	N/m	—	—	—

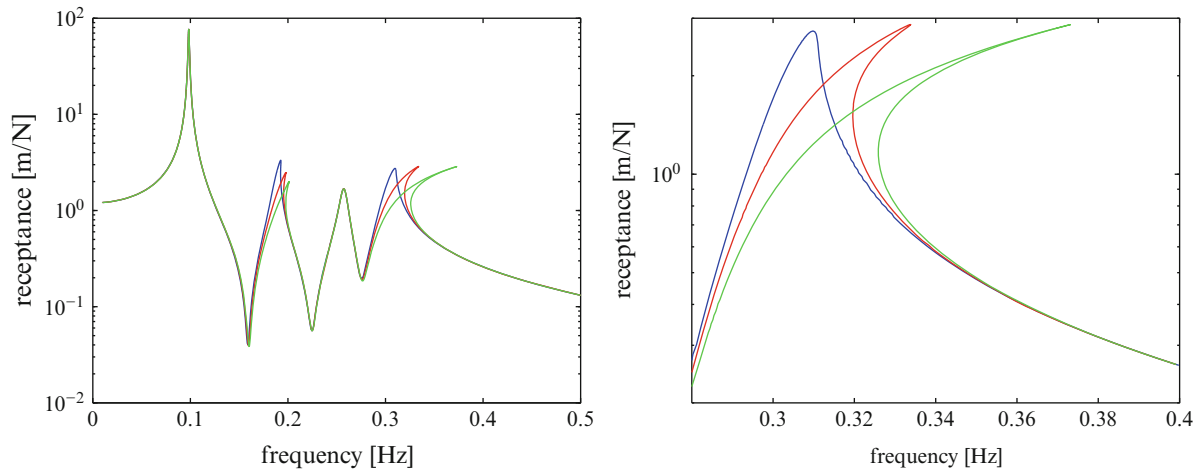


Fig. 3.10 Left: FRF of DoF x_2 for different excitation forces $F_{exc}=[0.1, 0.2, 0.3]$ (blue, red, green). Right: Zoom on fourth mode (color figure online)

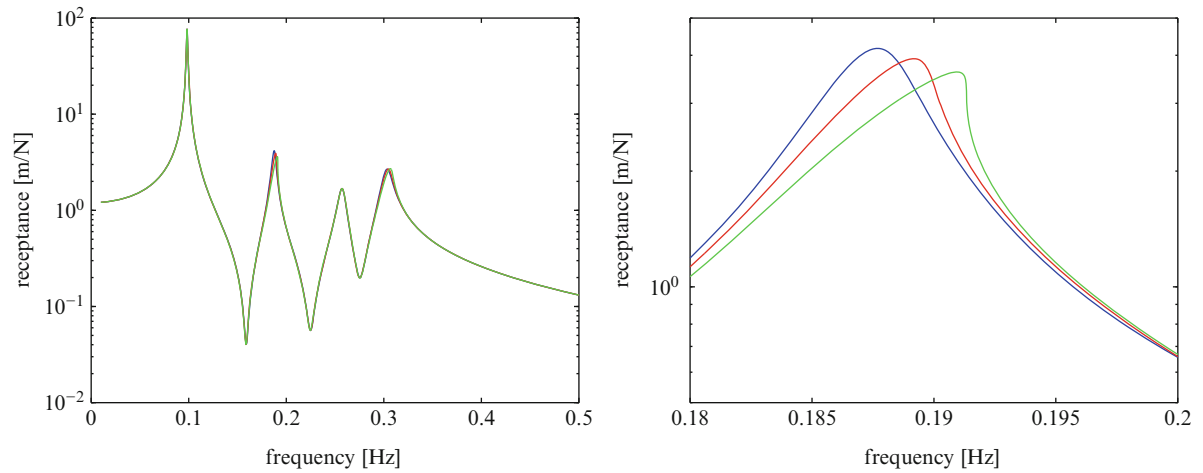


Fig. 3.11 Left: FRF of DoF x_2 for different excitation forces $F_{exc}=[0.025, 0.05, 0.075]$ (blue, red, green). Right: Zoom on fourth mode (color figure online)

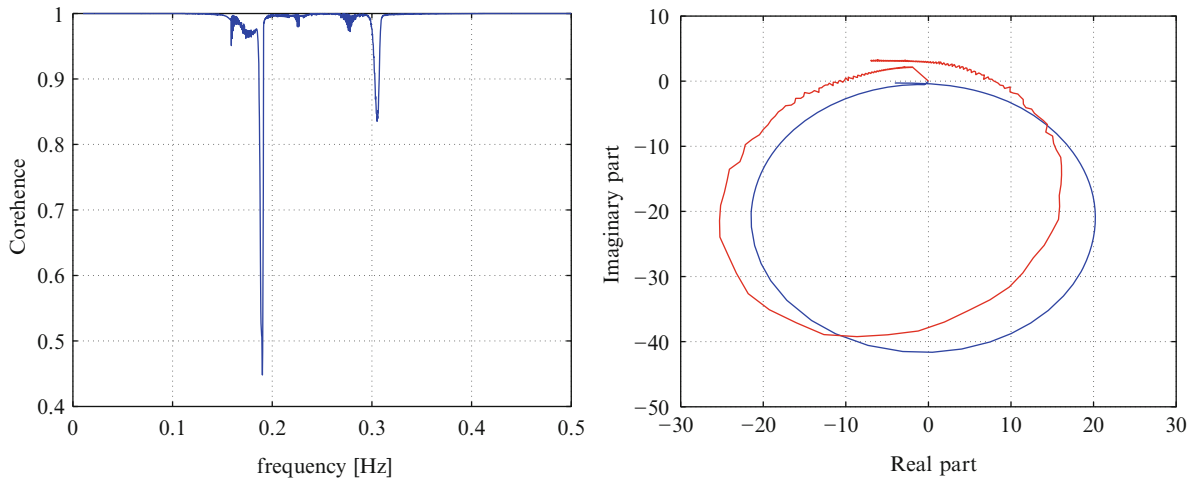


Fig. 3.12 Left: Corehence function for the FRFs with the three excitations. Right: Nyquist plot of the fourth mode

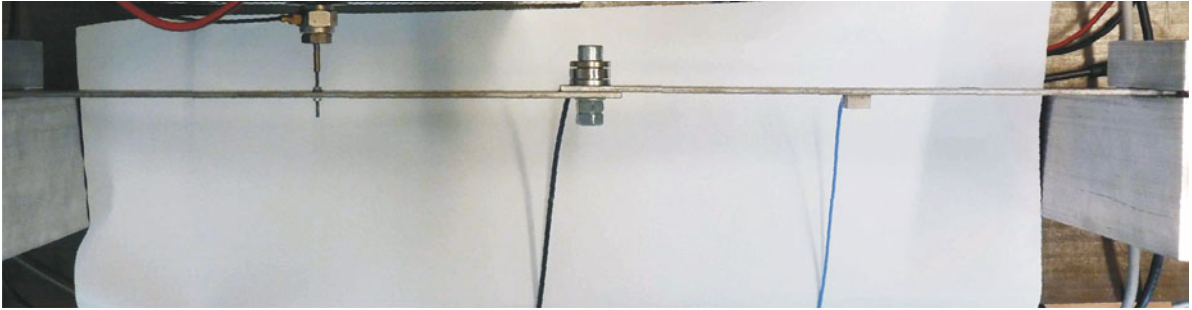
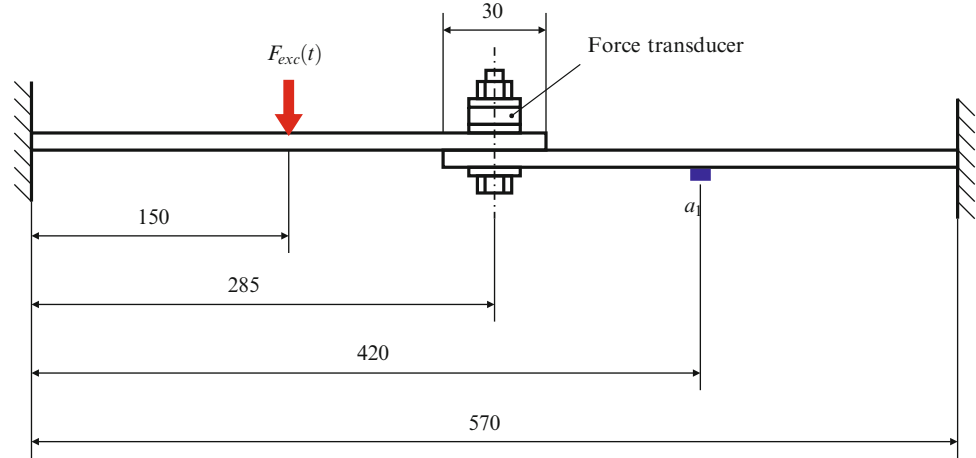


Fig. 3.13 Double beam experiment

Fig. 3.14 Sketch of the double beam experiment. Dimensions are given in *mm*



FRF measurements are done for several clamping forces. In Fig. 3.15 the results of the first and third mode are shown. It can be seen that the damping and also the eigenfrequencies change with the applied normal force. The excitation force is kept constant by a control loop with an amplitude of $\hat{F}_{exc}=1.75$ N. For an application of the Corehence function, a FRF measurement with a different excitation amplitude is needed. For an equal clamping force $F_N=100$ N two FRFs with excitation force $\hat{F}_{exc}=1.5$ N and $\hat{F}_{exc}=1.75$ N are measured and their Hilbert transforms are computed, Fig. 3.16 on the left. The Corehence function of these two functions are given in Fig. 3.16 on the right for the third mode. It can be seen that the Corehence function detects the nonlinear influence within this mode.

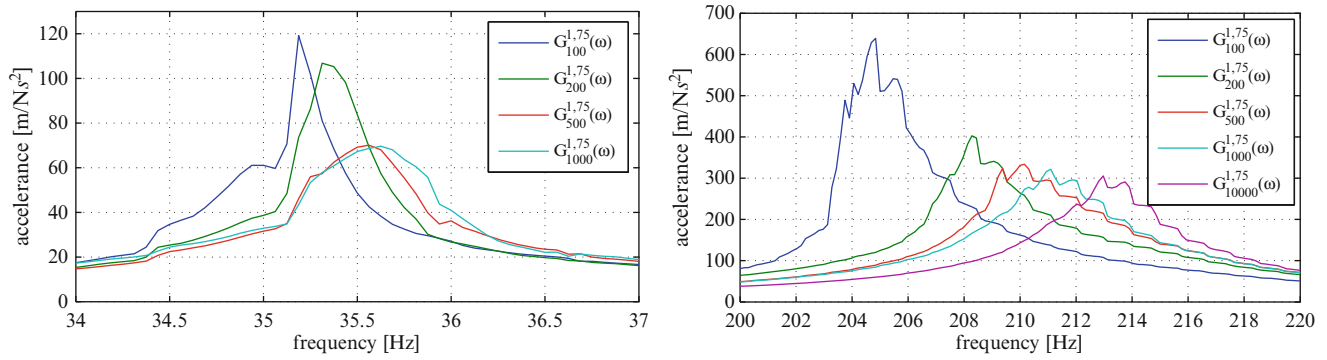


Fig. 3.15 Left: FRF of the first mode with excitation force $\hat{F}_{exc} = 1,75$ N for different normal forces $F_N = [100, 200, 500, 1000]$ N. Right: FRF of the second mode with excitation force $\hat{F}_{exc} = 1,75$ N for different normal forces $F_N = [100, 200, 500, 1000, 10000]$ N

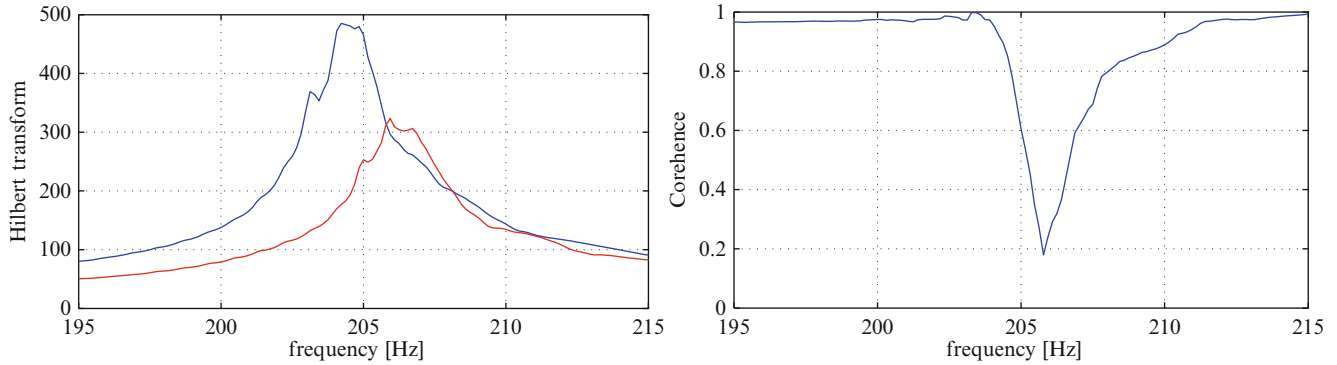


Fig. 3.16 Left: Hilbert transform for $F_N = 100$ N and excitation amplitude $\hat{F}_{exc} = 1,75$ N (blue) and $\hat{F}_{exc} = 1,5$ N (red). Right: Corehence function of the two FRFs (color figure online)

3.8 Conclusion

In this work, a method to detect nonlinearities within FRFs is presented. The Hilbert Transformation is used to compute Nyquist plots and the Corehence functions for this purpose. The method is applied to two numerical example systems, one with a friction type nonlinearity and the other with a cubic stiffness. Using an extension of the Harmonic Balance Method with a substructuring formulation FRFs are simulated. The Corehence function is able to detect the modes with nonlinear influence of the coupled systems within the FRF. Additionally, the Nyquist plot of the Hilbert transform allows an conclusion of the implemented nonlinearity. Furthermore, the method is applied to measured data of a two-beam experiment where the nonlinear influence of the friction can also be detected. Further effort will be made on better measurement data and on the detection and identification of the nonlinearities within the wheel brake assembly. Moreover, a parameter identification by a comparison of measured and simulated data is thinkable.

References

1. Allgower EL, Georg K (1997) Numerical Path Following. In: Handbook of Numerical Analysis. North-Holland, editors, P. G. Ciarlet and J. L. Lions 5:3–207
2. Bruns JU (2004) Detektion und Identifikation von Nichtlinearitäten in mechanischen Schwingungssystemen. PhD Thesis, University of Hannover
3. De Klerk D, Rixen D, Voormeeren S (2008) General framework for dynamic substructuring: history, review and classification of techniques. AIAA J 46(5):1169–1181. doi: 10.2514/1.33274
4. Reuss P, Zeumer B, Herrmann J, Gaul L (2011) Consideration of interface damping in dynamic substructuring. In: Proceedings of the IMAC-XXX, Jacksonville, Florida

5. Vakakis AF, Golinval JC, Kerschen G, Worden K (2006) Past, present and future of nonlinear system identification in structural dynamics. *J Sound Vib* 20:505–592
6. Worden K, Tomlinson GR (2001) Nonlinearity in structural dynamics - detection, identification and modeling. Institute of Physics Publishing, Bristol, UK
7. Wriggers P (2008) Nonlinear finite element methods. Springer, Berlin

Chapter 4

Structural Modification of Nonlinear FEA Subcomponents Using Nonlinear Normal Modes

Robert J. Kuether and Mathew S. Allen

Abstract Structural modification procedures are widely used to predict changes in the dynamics of a structure based on the addition of stiffeners, reinforcements, modifications of bolted joints, and payloads. A truncated modal basis of a linear dynamic system can be used to predict the changes of the mode shapes and frequencies due these modifications. This work proposes an extension of modal structural modification for geometrically nonlinear structures, by representing the structure with a set of nonlinear normal modes. An approximate quasi-linear modal model is defined from the fundamental frequency and maximum deformation shape of the nonlinear normal mode solutions. The resulting quasi-linear model has energy dependent mode shapes and natural frequencies. An iterative algorithm then applies modal structural modification to the quasi-linear modal model such that the modal parameters used in the substructuring routine are appropriate for each response level of interest. The method is demonstrated on a finite element model of a geometrically nonlinear beam with a variable elastic boundary condition. The model is meant to mimic the uncertain boundary conditions of a substructure in a hypersonic air vehicle. The nonlinear normal modes are computed for the unmodified beam, and a torsion spring is added to the boundary using the proposed method. It is found to give accurate predictions of the nonlinear modes of the assembly, potentially at a greatly reduced computational cost.

Keywords Nonlinear normal modes • Structural modification • Substructuring • Reduced order modeling • Nonlinear dynamics

4.1 Introduction

Substructuring approaches in structural dynamics have been widely used to predict the changes in dynamics caused by the addition of a structural element, or even after coupling another substructure. Substructuring methods can be classified as to whether the system is represented in either the physical, modal or frequency domain. An example of physical domain coupling is the finite element method, where simple geometric elements are coupled to predict the behavior of an assembly [1]. The geometry of the assembly can be incredibly complicated if enough elements are coupled together, allowing for realistic structures to be modeled. Modal substructuring methods allow one to reduce the model order at the subcomponent level. A truncated modal basis can provide an accurate reduced order model of the system, allowing for an assembly of subcomponents to have significantly fewer degrees-of-freedom (DOF) than would be required for an accurate finite element model. Linear normal modes have been used to couple linear systems for over 50 years, one of the most popular techniques being the Craig-Bampton method [2]. In this work, a modal structural modification technique is extended to nonlinear subcomponent models represented by a truncated set of nonlinear normal modes (NNM). An iterative algorithm is developed in order to quickly estimate the NNMs of a nonlinear structure after the addition of a lumped mass or spring element.

Nonlinearities in structural models are introduced by a variety of phenomena such as large deformations, nonlinear material constitutive laws, buckling, and friction in joints. These physics must be accounted for to accurately predict the behavior of a structure, especially when the linearity assumption is no longer valid. For example, large deformation analysis of a thin, flat, planar structure (such as the stiffened skin panel of a hypersonic air vehicle) shows that the coupling of the

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axial and bending motions actually reduces the maximal amplitude of the response. Using linear analysis for such predictions would actually over estimate the response amplitude and the dynamic stress, resulting in an overly conservative design. On the other hand, other nonlinear phenomena, such as localization and internal resonance, can drive a structure to far larger stresses than would be predicted by linear theory. In situations such as these, it is important to accurately model nonlinearity to avoid damaging response levels under certain operating conditions. Unfortunately, time simulations of large scale nonlinear finite element models are very computationally expensive and tend to drive design engineers away from nonlinear analysis, if possible. The nonlinear normal mode provides an interpretation of the nonlinear behavior of a structure that can be used as a design tool for models that exhibit nonlinearity, without the need for repeated transient analysis simulations. It is with this motivation that modal analysis techniques are extended to nonlinear systems using NNMs as a basis.

A nonlinear normal mode has been defined as a “not necessarily synchronous periodic response to the conservative nonlinear equations of motion”, as developed by Vakakis et al. [3, 4]. This definition is an extension of the pioneering work of Rosenberg [5]. The NNM describes how the resonant frequency and deformation shape change with energy. Many key nonlinear phenomena can be described with this approach, such as internal resonance, localization and frequency-energy dependence. In many structural dynamic systems, an NNM can be thought of as an extension of a linear mode shape and frequency at low energy levels. One numerical method for computing NNMs is the shooting and pseudo-arclength continuation technique developed by Peeters et al. [6]. Other numerical methods have been developed as well [7–10]. The nonlinear normal mode describes the resonant conditions of a nonlinear structure, as they form the backbone to the nonlinear forced response curves. Although the property of superposition and orthogonality do not apply, NNMs still provide tremendous insight into the behavior of the system.

Several prior works have been put forth to extend substructuring techniques to nonlinear systems in the frequency and modal domain. A substructuring approach was developed by Cömert and Özgüven in [11], where two linear systems in the modal or frequency domain were coupled by a nonlinear spring element. The nonlinear spring element was approximated by a describing function, satisfying a single frequency harmonic balance model. The nonlinear assembly resulted in a nonlinear receptance matrix, which is used to predict the monoharmonic, steady state forced response. In later publications, the method was extended to the attachment of a nonlinear element to a linear system [12] using the frequency based substructuring (FBS) approach presented in [13]. The method was also demonstrated by coupling a linear system to a localized nonlinear structure, represented by a nonlinearity matrix derived from the describing function method [14]. Other frequency domain methods were presented in [15–17], coupling a localized nonlinear element represented by a describing function to the linear FRF of a structural model.

Other substructuring techniques have been developed for nonlinear systems based on the concept of the nonlinear normal mode. Chong and Imregun [18] developed a modal substructuring approach for structures with global nonlinearities. They defined a nonlinear mode as either a parametric modal model fit to nonlinear frequency response measurements, or as the eigenvalues and eigenvectors of the mass and tangent stiffness matrix about a given static configuration. They used an iterative algorithm based on the subcomponent nonlinear modes to satisfy a balance between the modal amplitudes in the assembly and subcomponent models. Another approach developed by Apiwattanalungarn et al. [19] was based on an NNM defined as a two-dimensional invariant manifold in phase space. The nonlinear subcomponent was represented by a single fixed interface NNM and a set of linear constraint modes. The method worked well for a SDOF subcomponent, but the manifolds required to describe higher order nonlinear models are very costly to compute so the method was deemed to be impractical. Recently, Allen and Kuether [20] have proposed a substructuring approach based on the NNM definition of Rosenberg, Vakakis and Kerschen mentioned earlier. The deformation shapes and fundamental frequencies from the exact NNM solutions were used to define a quasi-linear modal model, in which the mode shapes and natural frequencies were dependent on energy. The method of Lagrange multipliers was used to couple two nonlinear structures represented by the quasi-linear modal model. The full set of NNMs were used in this previous publication. The nonlinear structural modification procedure presented here is an extension of this work.

This work presents a nonlinear structural modification procedure that can quickly and accurately predict how the NNMs of the system will change due to the addition of a lumped mass or spring element. This work is motivated by the need to predict the response of realistic structures that are modeled in commercial finite element packages. The nonlinear modification procedure uses a truncated set of NNMs computed from a nonlinear finite element model, allowing for order reduction at the subcomponent level. The NNMs of the unmodified structure are computed using the algorithm in [6], which relies on numerical time integration and the computation of the Jacobian matrix of the equations of motion. Recently, the authors found that one can accurately predict the NNMs of a structure by combining this algorithm with a nonlinear reduced order model (NLRom) of the structure [7]. This work uses a truncated set of NNMs computed using this approach, to predict how those nonlinear modes would change due to a modification; this avoids the need to re-run the finite element model to numerically recompute the reduced order model and then the nonlinear modes. The proposed method uses the deformations and fundamental frequencies from the exact NNM solution to define a quasi-linear modal model. This model is treated as a

linear modal model, although while recognizing that the quasi-linear natural frequencies and deformation shapes vary with energy. An iterative approach is then used with the method of Lagrange multipliers to adjust the modes so they reflect the properties of the structure at the energy level of interest. The result is an estimate of the nonlinear modes of the structure at a certain energy level, and the procedure is easily repeated over the energy range of interest.

The remainder of this paper is outlined as follows. Section 4.2.1 discusses how the nonlinear normal modes of a geometrically nonlinear finite element model are found using nonlinear reduced order models. A description of the quasi-linear modal model used to approximate the nonlinear system is defined in Sect. 4.2.2, and the iterative nonlinear structural modification algorithm is explained in detail in Sect. 4.2.3. Section 4.3 demonstrates the proposed algorithm by modifying the boundary stiffness of a geometrically nonlinear beam. A discussion of the accuracy and limitations of the results are accompanied with a comparison between the modified and exact NNMs.

4.2 Theoretical Development

4.2.1 Representing Nonlinear Subcomponents with Nonlinear Normal Modes

A nonlinear normal mode describes the periodic free response of the autonomous, conservative nonlinear equations of motion for a system. In general, the discretized equations for an N -DOF system can be written as

$$M\ddot{x} + Kx + f_{NL}(x) = \{0\} \quad (4.1)$$

where M is the $N \times N$ mass matrix, K is the $N \times N$ linear stiffness matrix, and $f_{NL}(x)$ is the $N \times 1$ nonlinear restoring force vector. The displacement, velocity and acceleration are represented by the $N \times 1$ vectors x , $\dot{x}$, and $\ddot{x}$, respectively. The NNMs computed from the full order equations of motion exactly satisfy the periodicity condition, allowing for strong nonlinearities to be captured. The exact multi-harmonic NNM response can be written for the system in Eq. (4.1) as a complex Fourier Series of the form

$$x_{NNM}(t) = \sum_{k=-\infty}^{\infty} X_{NNM,k} e^{ik\omega_{NNM}t} \quad (4.2)$$

where $X_{NNM,k}$ is the complex amplitude of the k^{th} harmonic, ω_{NNM} is the fundamental frequency, and t is time. The energy dependence of the solution comes from the fact that the fundamental frequency and complex amplitudes evolve as the amplitude of the response increases. At low energy, the periodic solution branches are in the neighborhood of the linear normal mode solutions of the linearized system. For an N -DOF system, there exists N nonlinear normal modes that are nonlinear extensions of linear modes.

In general, the closed form equations of motion are not defined explicitly for the nonlinear finite element models of interest, so a numerical approach is required to compute the NNMs. One recently developed numerical approach is based on a shooting and pseudo-arclength continuation technique, and works by integrating the conservative nonlinear equations of motion and iteratively adjusting the initial conditions until a periodic response is found [6]. While this method is very accurate and effective, it is too computationally expensive to be practical for high fidelity finite element models, especially since the closed form equation of motion is not known. The numerical shooting and continuation techniques rely on the computation of the Jacobian matrix of the equations of motion, and must be computed using finite difference schemes. The Jacobian matrix requires $2N$ finite difference computations, and may need to be computed several times for a single solution on the NNM branch.

In order to overcome these difficulties, in [7] the authors proposed to use a nonlinear reduced order model to decrease the size of the nonlinear equations of motion, and therefore the size of the Jacobian matrix required by the continuation algorithm. This NLROM can be used with the algorithm in [6] then to calculate the NNMs of the structure. A geometrically nonlinear finite element model with N -DOF can be reduced to a low-order system of nonlinear modal equations, based on a linear modal coordinate transformation. The low frequency linear modes are computed from the linear (or linearized) mass and stiffness matrices, M and K , from the equations of motion in Eq. (4.1). A truncated modal basis, shown in Eq. (4.3), is used to transform the full order system from N physical DOF to a reduced set of m generalized modal coordinates. In general, the low frequency modes in the bandwidth of interest are selected for the coordinate transformation, although higher frequency modes may be included.

$$x = [\Phi]q \quad (4.3)$$

Φ is the $N \times m$ mass normalized mode matrix with each column being a linear mode shape, and q is the $m \times 1$ modal coordinate vector. Applying the coordinate transformation to the full nonlinear system of equations in Eq. (4.1), and premultiplying by Φ^T , a nonlinear modal equation can be written as

$$\ddot{q}_r + \omega_r^2 q_r + \theta_r(q_1, q_2, \dots, q_m) = 0 \quad (4.4)$$

where $\theta_r(q_1, q_2, \dots, q_m)$ is approximated as a polynomial form as

$$\theta_r(q_1, q_2, \dots, q_m) = \sum_{i=1}^m \sum_{j=i}^m B_r(i, j) q_i q_j + \sum_{i=1}^m \sum_{j=i}^m \sum_{k=j}^m A_r(i, j, k) q_i q_j q_k \quad (4.5)$$

The nonlinear coefficients A_r and B_r are unknown for each modal equation. The resulting equations are nonlinear and fully coupled, but the order is much less than that of the original model ($m \ll N$). The nonlinearity is approximated by a quadratic and cubic polynomial function of the modal amplitude, and captures stiffening and softening behavior caused by geometric nonlinearities. Several methods exist for determining these nonlinear coefficients from a nonlinear finite element model, as reviewed in [21]. The method chosen for this work is the Implicit Condensation and Expansion (ICE) technique [22]. The ICE method determines the nonlinear coefficients A_r and B_r from a set of nonlinear static solutions to a series of applied loads. The resulting static deformations are used to form a least squares problem that can be solved to fit the nonlinear coefficients in Eq. (4.5). For flat structures such as planar beams and plates, an expansion of the membrane displacements caused by the bending displacements is needed to account for axial-bending coupling. The ICE method uses the reduced modal basis and static solutions to compute a set of membrane modes that are orthogonal to the bending modes. The details of the approach are found in [23]. Once the coefficients A_r and B_r are determined, the nonlinear Eqs. (4.4) and (4.5) used with the continuation algorithm in [6] in order to compute the NNMs of the structure.

4.2.2 Quasi-Linear Modal Models

The proposed structural modification method is closely related to the nonlinear substructuring work of Özgüven and his collaborators [11, 12, 14]. In their works the nonlinear subcomponent model is approximated by a fundamental frequency harmonic balance model. Consider the nonlinear equations of motion of the form in Eq. (4.1). It is often the case that a single frequency harmonic force causes a response that is well approximated by a single harmonic at the same forcing frequency. The steady state harmonic response can then be written as

$$x = \text{Re}(X e^{i\omega t}) \quad (4.6)$$

where X is the complex amplitude of the response and ω is the fundamental response frequency. Under this assumption, the nonlinear restoring force vector is also assumed to be harmonic and can be written as

$$f_{NL}(x) = \text{Re}(F_{NL} e^{i\omega t}) \quad (4.7)$$

Budak and Özgüven [24] have shown that the harmonic nonlinear restoring force amplitude vector, F_{NL} , can be written as a function of the response amplitude, X , as

$$F_{NL} = [K_{NL}(X)] X \quad (4.8)$$

where $K_{NL}(X)$ is an $N \times N$ nonlinear stiffness matrix that is a function of the unknown complex amplitudes, X . The Describing Function Method can be used to generate the nonlinear stiffness matrix for various types of nonlinearities, as described in [25]. Combining Eqs. (4.6), (4.7), and (4.8) with Eq. (4.1), the fundamental harmonic balance model becomes

$$[-\omega^2 M + K + K_{NL}(X)] X = F \quad (4.9)$$

where F is the complex amplitude vector of the externally applied force. The nonlinear algebraic equations in Eq. (4.9) can be iteratively solved for the steady state response X to a harmonic external load F .

This fundamental harmonic approximation to the nonlinear equations of motion can be used with a variety of substructuring techniques, either in the physical or frequency domain. Here, the modification procedure for the addition of a discrete, linear spring element is shown to demonstrate the use of the nonlinear harmonic balance model. The method

of Lagrange multipliers is used to couple the two systems; the details of the approach are outlined in [26]. The equation of motion for a discrete spring element attached to point “c” is expressed as

$$k_{spring}x_c = f_{spring} \quad (4.10)$$

The unconstrained equations of motion for the two uncoupled systems becomes

$$\left[\begin{bmatrix} -\omega^2 M & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & k_{spring} \end{bmatrix} + \begin{bmatrix} K_{NL}(X) & 0 \\ 0 & 0 \end{bmatrix} \right] \begin{Bmatrix} \{X\} \\ x_c \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ f_{spring} \end{Bmatrix} \quad (4.11)$$

In order to assemble the two models, force equilibrium and compatibility must be satisfied. The compatibility constraints for the assembly are expressed as

$$[a] \begin{Bmatrix} \{X\} \\ x_c \end{Bmatrix} = 0 \quad (4.12)$$

The $1 \times (N + 1)$ vector a consists of an entry of 1 for the connection point on the unmodified structure, and -1 at the last entry related to x_c . The remaining entries are zeros. Using the compatibility condition, the constrained and unconstrained coordinates are related by

$$\begin{Bmatrix} \{X\} \\ x_c \end{Bmatrix} = [B] \{X_u\} \text{ where } B = \text{null}(a) \quad (4.13)$$

Inserting Eqs. (4.13) into (4.11), and premultiplying by B^T , the unconstrained equations of motion for the modified system becomes

$$[B]^T \left[\begin{bmatrix} -\omega^2 M & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & k_{spring} \end{bmatrix} + \begin{bmatrix} K_{NL}(X) & 0 \\ 0 & 0 \end{bmatrix} \right] [B] \{X_u\} = [B]^T \begin{Bmatrix} \{F\} \\ f_{spring} \end{Bmatrix} \quad (4.14)$$

This constitutes a set of nonlinear algebraic equations that must be solved iteratively to obtain the monoharmonic, steady state forced response of the modified nonlinear system. Note that a similar procedure can be applied to add a lumped mass or a nonlinear spring element, or it may even be used to couple two nonlinear systems together.

One should note that this procedure presumes that the equations of motion are known explicitly, or at least that the harmonic balance model is known. For the nonlinear finite element models that are of interest in this work, the harmonic balance model $K_{NL}(X)$ is not easy to form, and the system matrices M , K and $K_{NL}(X)$ may be prohibitively large. Furthermore, this approach does not provide a mechanism for order reduction at the subcomponent level; each nonlinear subcomponent is represented by a physical model and equilibrium must be satisfied for every node in the assembly.

On the other hand, it is interesting to note that the harmonic balance model has the same form as a linear model, only it is tuned to a specific amplitude of response. This prompts the question: Can an amplitude dependent linear modal model be used to predict the amplitude dependent frequency and deformation shape of a nonlinear structure in a similar way? If this were the case, then perhaps the number of subcomponent modes could also be truncated to reduce the number of generalized coordinates needed to represent the system, reducing the computational cost further. That is the impetus for the methodology proposed in this work. Although a proof for the proposed procedure has not yet been developed, the idea behind the method is intuitive and quite similar to what was outlined above.

Specifically, we propose to represent the substructure of interest using a quasi-linear (QL) modal form that is based on its nonlinear normal modes. This is similar, in spirit at least, to the amplitude dependent harmonic balance model in Eq. (4.9), only in the proposed the energy level of each NNM is used as the adjustable parameter rather than the response amplitude of each node. Hence, the quasi-linear modal parameters can be written as

$$\Phi^{QL}(E) = \left[\phi_1^{QL}(E_1) \ \phi_2^{QL}(E_2) \ \dots \ \phi_P^{QL}(E_P) \right] \quad (4.15)$$

$$\omega^{QL}(E) = \begin{bmatrix} \omega_1^{QL}(E_1) & 0 & 0 & 0 \\ 0 & \omega_2^{QL}(E_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \omega_P^{QL}(E_P) \end{bmatrix} \quad (4.16)$$

where P is the number of NNMs used in the QL model, $\phi_r^{QL}(E_r)$ is the mass normalized QL mode shape of the r^{th} mode, and $\omega_r^{QL}(E_r)$ is the fundamental frequency of the r^{th} mode. The QL mode shape is defined from the initial physical displacement (having a strain energy E_r) that initiates the NNM response of the nonlinear equations of motion. This can be thought of as the maximum displacement field in the NNM response. For a given solution along the NNM curve, the QL mode shape is defined as

$$\phi_r^{QL}(E_r) = \frac{x_{NNM,r}(E_r)}{\sqrt{x_{NNM,r}^T(E_r)[M]x_{NNM,r}(E_r)}} \quad (4.17)$$

where E_r is the strain energy of the initial physical displacement $x_{NNM,r}(E_r)$. There are many discrete energy solutions for each NNM, each having a different QL mode shape and fundamental frequency. With the QL modal model, modal substructuring approaches (e.g. method of Lagrange multipliers) are used based on the modal equations of motion of the form

$$[I]\ddot{q}^{QL} + [\omega^{QL}(E)]q^{QL} = 0 \text{ where } x(t) = [\Phi^{QL}(E)]q_r^{QL} \quad (4.18)$$

An iterative algorithm is proposed in the following subsection that seeks to satisfy the energy dependence in the QL modes.

4.2.3 Structural Modification Algorithm Using NNMs

The quasi-linear modal model defined in the previous subsection approximates a nonlinear model as a linear system with mode shapes and frequencies that depend on modal energy. The modes are then assembled using the method of Lagrange multipliers, exactly as would be done when assembling linear substructures based on their modal models. This paper focuses on structural modification, so the algorithm is specialized to that case to a certain extent, as detailed below.

1. Compute truncated set of NNMs:

Compute the NNMs of the nonlinear equations of motion for the number of modes desired.

2. Define QL modal model at a fixed energy level:

Take the displacements that initiate the NNM solutions, $x_{NNM,r}(E_r)$, in the truncated basis set at the same energy level E_j . From each displacement $x_{NNM,r}(E_r)$, define the mass normalized quasi-linear mode shape $\phi_r^{QL}(E_r)$ as shown in Eq. (4.17). The modal amplitude for the r^{th} mode is then defined as

$$q_r^{QL} = \sqrt{x_{NNM,r}^T(E_r)[M]x_{NNM,r}(E_r)} \quad (4.19)$$

such that the relation between the physical and modal response is $x_{NNM,r}(E_r) = \phi_r^{QL}(E_r)q_r^{QL}$. Next, specify the natural frequency $\omega_r^{QL}(E_r)$ of each QL mode from the NNM fundamental frequency at E_j , and assemble the QL modal model in Eq. (4.18).

3. Apply structural modification procedure to QL modal model:

With the QL modal model, the method of Lagrange multipliers can now be used to couple either a linear spring or lumped mass to the system. For example, attaching a linear spring to the x_c DOF results a set of differential-algebraic equations given by

$$\begin{bmatrix} [I] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{q}^{QL} \\ \ddot{x}_c \end{Bmatrix} + \begin{bmatrix} [\omega^{QL}(E)] & 0 \\ 0 & k \end{bmatrix} \begin{Bmatrix} q^{QL} \\ x_c \end{Bmatrix} = \begin{Bmatrix} [\Phi^{QL}(E)]^T \{F\} \\ F_c \end{Bmatrix} \quad (4.20)$$

$$[a] \begin{Bmatrix} q^{QL} \\ x_c \end{Bmatrix} = 0 \quad (4.21)$$

A number of methods exist for solving this set of equations. For structural modification problems, the constrained and unconstrained generalized coordinates can be related as

$$\begin{Bmatrix} q^{QL} \\ x_c \end{Bmatrix} = \begin{bmatrix} I \\ \Phi_c^{QL}(E) \end{bmatrix} \{q^{QL}\} = [B] \{q_u^{QL}\} \quad (4.22)$$

where q_u^{QL} are the unconstrained coordinates of Eq. (4.20). Using the $\mathbf{B}$ matrix, the new unconstrained equations of motion satisfying compatibility and force equilibrium become

$$[\mathbf{B}]^T \begin{bmatrix} [\mathbf{I}] & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{B}] \{q_u^{QL}\} + [\mathbf{B}]^T \begin{bmatrix} [\omega^{QL}(E)] & 0 \\ 0 & k \end{bmatrix} [\mathbf{B}] \{q_u^{QL}\} = [\mathbf{B}]^T \left\{ \begin{bmatrix} \Phi^{QL}(E) \end{bmatrix}^T \{F\} \\ F_c \right\} \quad (4.23)$$

Next, the eigenvalues and eigenvectors are computed from the equations of motion in Eq. (4.23). Resulting from this will be a set of mode shapes Φ^{mod} and frequencies ω^{mod} of the modified QL modal model.

4. Compute the energy distribution among the modes of the unmodified structure:

The generalized coordinates of the unconstrained, modified equations can be related to the generalized coordinates of the unmodified coordinates as

$$\begin{Bmatrix} q^{QL} \\ x_c \end{Bmatrix} = [\mathbf{B}] \{q_u^{QL}\} \quad (4.24)$$

The generalized coordinates for the modified QL mode shapes are the same as the generalized coordinates of the unmodified QL modes, based on the form of the $\mathbf{B}$ matrix in Eq. (4.22). Therefore, the physical deformation of each modal response of the modified structure can be written as

$$x_r^{\text{mod}} = \phi_r^{\text{mod}} q_r^{QL} \quad (4.25)$$

Using each physical deformation x_r^{mod} , the total strain energy in the unmodified structure is computed. The energy distribution of each physical response in Eq. (4.25) is defined as

$$\mathbf{E}^{\text{mod}} = [E_1^{\text{mod}} \ E_2^{\text{mod}} \ \dots \ E_p^{\text{mod}}] \quad (4.26)$$

5. Check whether the strain energy of the modified deformation matches the energy in the unmodified modal model:

Define a tolerance condition as

$$\frac{\|\mathbf{E} - \mathbf{E}^{\text{mod}}\|_2}{\|\mathbf{E}^{\text{mod}}\|_2} < \varepsilon \quad (4.27)$$

Ideally, the difference between the energy used in the QL model and the energy of the modified deformation should be equal to zero. Since this is a numerical approach, the algorithm proceeds until the difference in energy is below some tolerance, ε .

- 5a. If the tolerance is satisfied, then store the modified modal parameters. The approximate NNM of the modified structure for the r^{th} mode is then given by $\phi_r^{QL, \text{mod}}(E_r^{\text{mod}})$, $\omega_r^{QL, \text{mod}}(E_r^{\text{mod}})$ and q_r^{QL} . Increase the energy level that was used to initiate the algorithm and repeat steps 2 through 5.
- 5b. If the tolerance is not satisfied, then return to step 2 and update the QL modal model parameters $\phi_r^{NNM}(E_r)$ and $\omega_r^{NNM}(E_r)$ at the discrete modal energies $\mathbf{E}^{\text{mod}}$. Note that for each iteration, the modal amplitudes q_r^{QL} are not updated. In general, the distribution of energy between each mode will not be the same. Repeat steps 2–5 until the tolerance in Eq. (4.27) is satisfied.

Based on the observed performance, the iterative algorithm converges to a balanced solution within 3–8 iterations for an ε value of 1e-4. A schematic of the algorithm is shown below in Fig. 4.1.

4.3 Numerical Results

4.3.1 Structural Modification of Geometrically Nonlinear Beam

The nonlinear structural modification technique is now applied to a planar, geometrically nonlinear beam that is modeled in Abaqus finite element software. This problem was motivated by prior studies [27, 28] which showed that the nonlinear dynamic response of an inlet ramp panel is sensitive to the stiffness of the structure which it is attached to. A torsion spring is

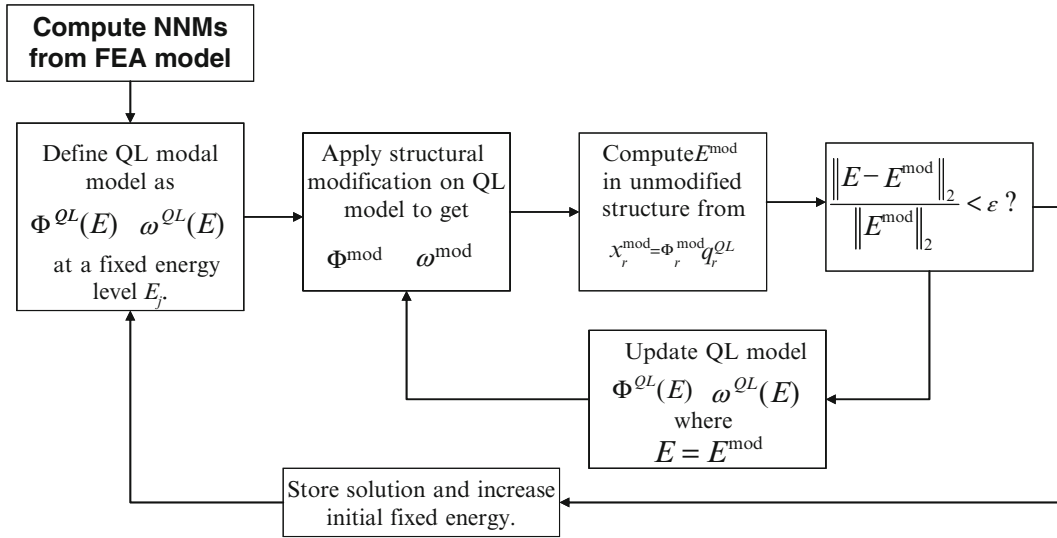


Fig. 4.1 Schematic of structural modification algorithm to predict NNMs of a modified structure

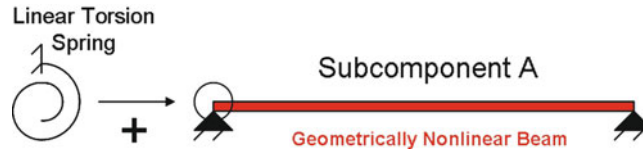


Fig. 4.2 Structural modification case study to understand the effects of an uncertain elastic boundary condition. A torsion spring is coupled to a geometrically nonlinear beam

coupled one end, as shown in Fig. 4.2, to represent the uncertain elastic boundary condition. The model is used to understand the effects of the boundary stiffness on the nonlinear normal modes of the structure. The beam is known to exhibit stiffening nonlinearities due to the coupling between the transverse and axial motions for large deformations. The beam under study is 9 inches in length, with cross sectional dimensions of 0.5 inches wide by 0.031 inches thick. It is modeled with forty B31 beam elements in Abaqus, resulting in 123 DOF. It is constructed of steel with a Young's modulus of 29,700 ksi, a shear modulus of 11,600 ksi and a mass density of 7.36×10^{-4} lb-s²/in⁴. The dimensions of the beam come from a benchmark structure used to validate a variety of reduced order modeling techniques [29]. The torsion spring stiffness is denoted as K_t .

The simply supported beam in Fig. 4.2 represents the unmodified structure to which the torsion spring is attached. A nonlinear ROM is built of this model and used to compute the NNMs of the structure to provide a basis for the quasi-linear modal model used in the structural modification procedure. The NNMs of the modified configuration (e.g. the simply supported beam with the torsion spring attached at the left boundary) are also directly computed by creating an NLROM of a second finite element model that includes the torsion spring. These results are considered the exact solutions for comparison with the structural modification procedure. Here, a torsion spring with a stiffness value of $K_t = 25$ in-lbf/rad is studied.

4.3.2 Linear Structural Modification Results

The nonlinear normal modes of a geometrically nonlinear structure converge to the linear normal modes at low energy levels. As the response amplitude becomes small, the nonlinear effects due to large deformations become negligible and the periodic solutions to the conservative equations of motion simply become the linear modes. This is consistent with the assumptions made with linear vibration theory. The first 10 linear normal modes of the finite element models of the simply supported beam and spring-beam assembly are described in Table 4.1.

A modal convergence study is initially performed on the linear modes of the simply supported beam when a torsion spring with $K_t = 25$ in-lbf/rad is coupled to the left end. The modal convergence provides insight into the number of NNM solutions to use as a basis to the nonlinear structural modification procedure. The relative percent error of linear natural frequencies from the modal structural modification are presented in Table 4.2. The predicted frequencies are compared to

Table 4.1 Exact linear normal modes of unmodified and modified beam

Mode	Simply supported		Modified with $K_t = 25$ in-lbf/rad	
	Type	Frequency (Hz)	Type	Frequency (Hz)
1	Bending I	34.85	Bending I	45.08
2	Bending II	139.4	Bending II	153.4
3	Bending III	313.8	Bending III	329.7
4	Bending IV	558.2	Bending IV	575.3
5	Bending V	872.7	Bending V	890.7
6	Bending VI	1258	Bending VI	1273
7	Bending VII	1714	Bending VII	1733
8	Bending VIII	2241	Bending VIII	2260
9	Bending IX	2840	Bending IX	2860
10	Bending X	3511	Bending X	3531

Table 4.2 Percent error of natural frequency

Modified beam mode number	Number of linear modes in modal basis of simply supported beam										
	—	1	2	3	4	5	6	7	8	9	10
1	—	15.6	7.0	4.6	3.4	2.7	2.3	1.9	1.7	1.5	1.3
2	—	—	4.8	2.7	1.9	1.5	1.3	1.1	0.9	0.8	0.7
3	—	—	—	2.0	1.3	1.0	0.8	0.7	0.6	0.5	0.4
4	—	—	—	—	1.0	0.7	0.5	0.5	0.4	0.3	0.3
5	—	—	—	—	—	0.6	0.4	0.3	0.3	0.2	0.2
6	—	—	—	—	—	—	0.4	0.3	0.2	0.2	0.2
7	—	—	—	—	—	—	—	0.3	0.2	0.2	0.1
8	—	—	—	—	—	—	—	—	0.2	0.1	0.1
9	—	—	—	—	—	—	—	—	—	0.1	0.1
10	—	—	—	—	—	—	—	—	—	—	0.1

the exact solutions in Table 4.1. As the number of modes in the truncated modal basis is increased, the relative error in the predicted natural frequency decreases. In order for the frequency error to be less than 2% for the linear modes, the first 7 bending modes are required. The largest error comes in the first mode of the modified structure, as the low frequency modes have the greatest shift in frequency due to the modification.

4.3.3 Nonlinear Structural Modification Results

Based on the results of the linear modal convergence study, the first 8 NNMs of the geometrically nonlinear beam were used as a basis for the nonlinear structural modification procedure. Hence, the error in the linear modes and the low energy NNMs will be less than 2%. The frequency-energy dependence of each mode in the NNM basis is presented in Fig. 4.3. The NNMs of the finite element model are computed from two separate nonlinear reduced order models. To minimize the computational cost, the inherent uncoupling due to the symmetry in the beam was exploited and the even modes (NNMs 2, 4, 6, 8) were computed from an NLROM with linear modes 2, 4, 6 and 8. Similarly with the odd modes, an NLROM with linear modes 1, 3, 5 and 7 are used to compute NNMs 1, 3, 5 and 7.

This set of nonlinear modes was used to define a quasi-linear modal model, which was used in the modification procedure to predict the NNMs of the system when a torsion spring with $K_t = 25$ in-lbf/rad was added to the left end of the beam. The frequency-energy dependence of the predicted NNMs is shown in Fig. 4.5.

The frequency-energy plot shows that the predicted NNMs of the modified structure are in excellent agreement with the exact NNMs. At low energy, where the NNMs converge to the linear normal mode, the error between each of the frequencies is due to modal basis truncation error, as discussed previously. The error in frequencies along the entire solution branch is less than 2% for the first three modes (except where there is an internal resonance). It is expected that the error would decrease further if additional modes were used. By analogy with linear substructuring, one would also expect the error to decrease if the modal basis were improved. For example, we could mass load the interface so that the modes would have nonzero curvature (nonzero moment) at the end. This would constitute a nonlinear extension of the method presented in [30].

To further explore the comparison between the predicted NNMs and the actual ones, we examine the predicted maximum deformation shape of the NNM at a discrete location along the 2nd NNM branch. The maximum deformation shape of the modified structure is compared in Fig. 4.4 for a solution at the highest energy considered on the 2nd NNM branch (marked

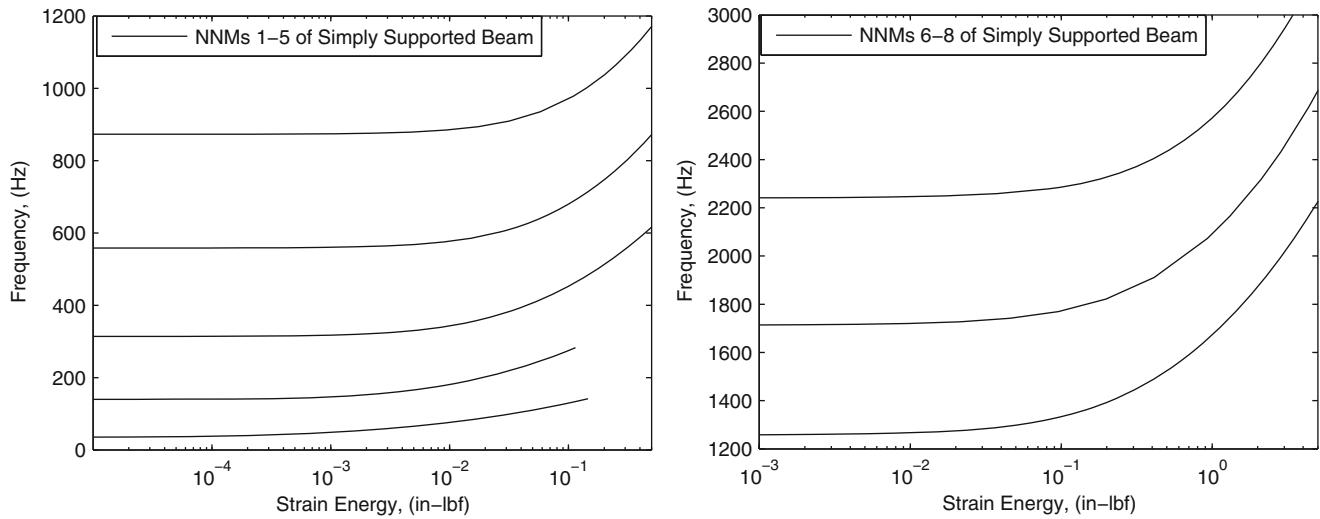
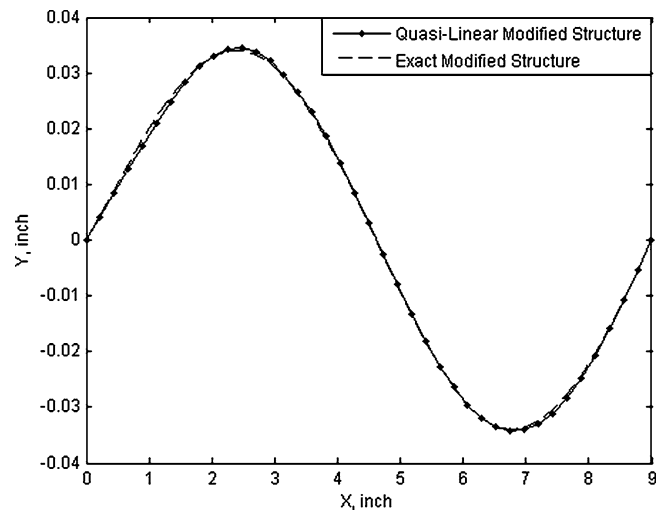


Fig. 4.3 Nonlinear normal modes 1 through 8 for unmodified simply supported beam. This set of modes constitutes the modal basis used in the nonlinear structural modification procedure

Fig. 4.4 Comparison of the maximum deformation the modified beam for the 2nd NNM at the highest energy considered



with black circle in Fig. 4.5). The red line represents the response predicted by using the structural modification procedure, and the blue line is the exact solution. There is excellent agreement between these deformation shapes, even when the frequency has shifted over 75% from 154.8 to 274.6 Hz. On the other hand, the deformation shape of this NNM is still dominated by the second linear bending mode shape, with small contributions from other linear modes.

The time histories can also be compared. For example, Fig. 4.6 shows the transverse displacement at $x = 2.25''$ over one period of the response from the NNM initiated by the initial deformation in Fig. 4.4. The blue line is the NNM found using the response integrated with the nonlinear reduced order model, and the red line is the quasi-linear response predicted using the structural modification procedure. The QL response is only capable of estimating the fundamental frequency of the NNM, and cannot predict higher harmonics. The exact response is dominated by the fundamental frequency, and the higher order harmonics are relatively small. Therefore, the quasi-linear response is a good approximation to the exact response at this energy level. If the higher harmonics had a greater contribution to the exact response, the QL response would not be as accurate.

On the other hand, one can see in Fig. 4.5 that there are energies at which the actual frequency-energy plot contains features that are not captured by the QL model. Since the QL modal model is based on the fundamental harmonic of the response, the modification procedure is only capable of predicting the fundamental harmonic of the assembly. Figures 4.7 and 4.8 show expanded views of NNMs 1 and 2, which reveal that there are folds in the frequency-energy curve where there

Fig. 4.5 Plot of frequency-energy dependence of first three NNMs of the simply supported beam for the (solid) unmodified NNMs used with QL modal model, (dotted) NNMs predicted by the modal structural modification procedure, and (dashed) exact NNMs computed from a finite element model of the beam-spring structure

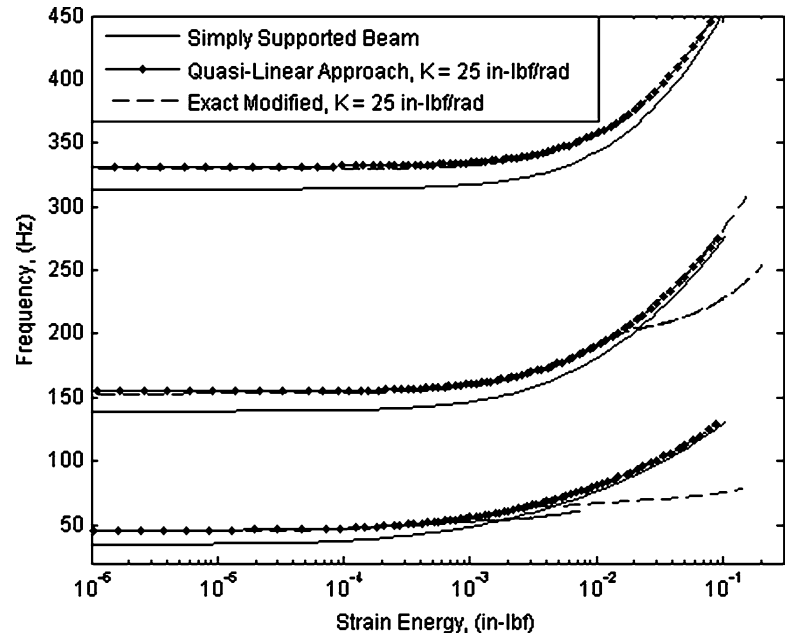
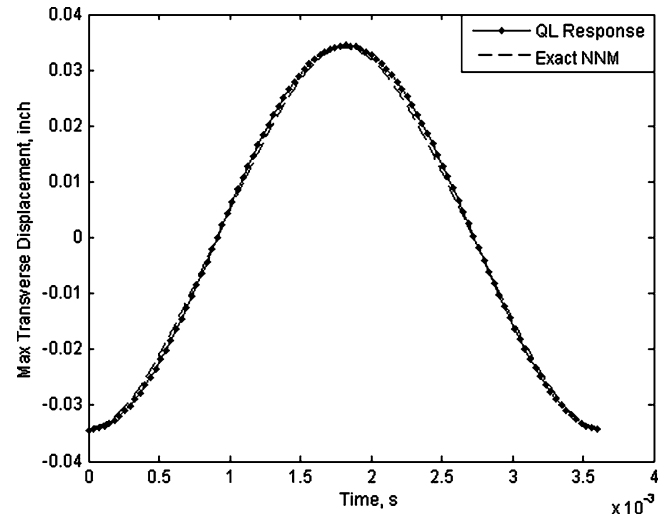


Fig. 4.6 Time history of (dotted) QL NNM transverse displacement at $x = 2.25''$ compared to the (dashed) exact NNM of the modified beam at $x = 2.25''$



is more than one solution at a discrete energy level. These are known as internal resonances, and occur when two NNMs interact with one another. Typically, these occur when two NNMs have commensurate NNM frequencies at a given energy level.

The predicted NNMs from the modification procedure follow the main solution branch quite well when there is no internal resonance present. One can actually predict candidate locations along the NNM branch where an internal resonant condition is possible. For example, the harmonics of the first predicted NNM are overlaid on the frequency-energy plot in Fig. 4.9. One can see that the 3x harmonic of NNM 1 intersects the predicted frequency-energy curve of NNM 2. One would expect a 3:1 internal resonance in the NNM 1 branch at the energy level where these two solutions intersect. In fact, the first fold in the frequency-energy curve of the exact solution for NNM 1 is a 3:1 internal resonance with the NNM 2.

To further illustrate this internal resonance, the time history of the modal amplitudes are plotted in Fig. 4.10, taken from the point marked with a black circle in Fig. 4.9. The response of the modal coordinates q_1 and q_2 from the NLROM of the spring-beam finite element model demonstrates the interaction between the underlying linear modal coordinates. Clearly, the second modal coordinate is dominant and oscillates at 3 times the frequency of the first modal coordinate. The QL model cannot capture this type of response, as discussed previously. However, several investigators [31, 32] have suggested multi-harmonic balance approaches that might be able to capture such features.

Fig. 4.7 Frequency-energy plot of 1st NNM for (solid) unmodified beam (dashed) exact modified beam and (dotted) modified using the QL modification procedure

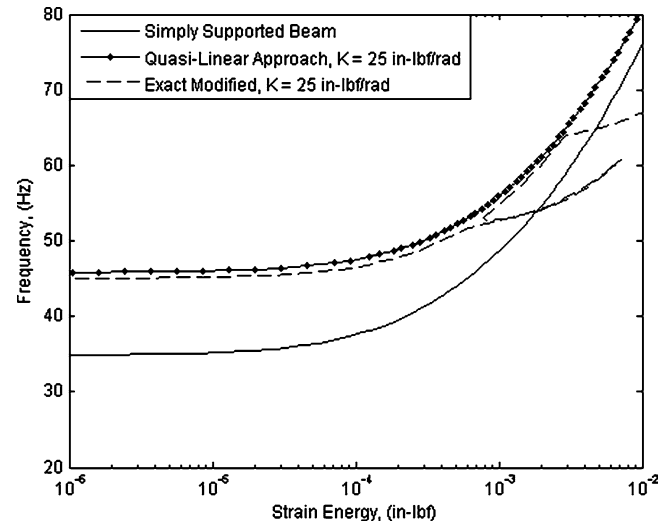


Fig. 4.8 Frequency-energy plot of 2nd NNM for (solid) unmodified beam (dashed) exact modified beam and (dotted) modified using the QL modification procedure

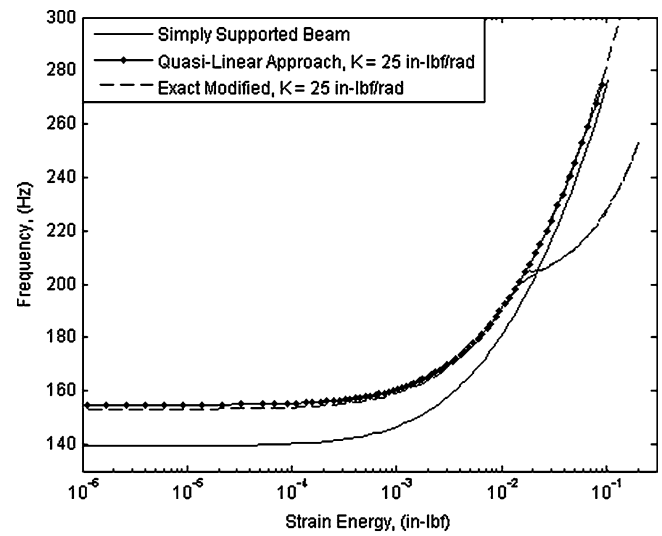


Fig. 4.9 Plot of frequency-energy curves for (solid) harmonics of first predicted NNM, (dashed) exact NNM 1, and (dotted) modified NNMs

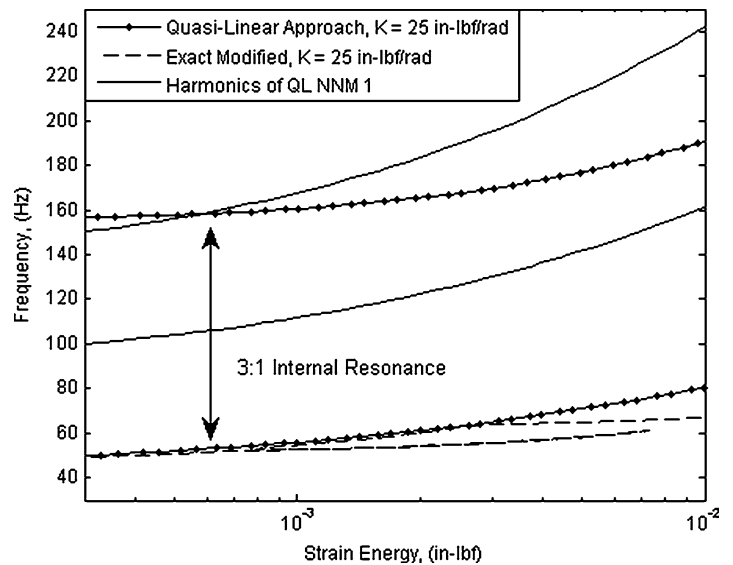
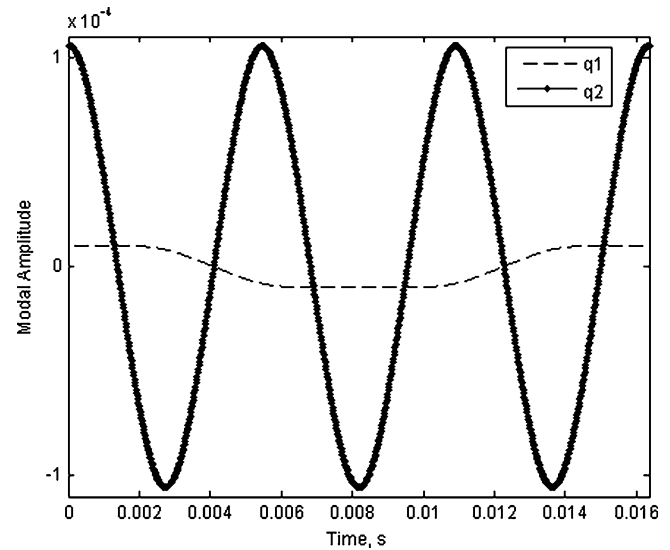


Fig. 4.10 Time history of modal coordinates in NLROM of 3:1 internal resonance



The main advantage of this approach is the ability to quickly iterate on a nonlinear structural model to predict the nonlinear dynamic behavior due to a modification. At this point the authors are unable to comment on the exact computational savings that would be achieved using this approach, as there were some issues regarding the manner in which the algorithm was implemented. However, our experiences with other similar algorithms suggest that the reduction would be dramatic. This approach avoids the need to recompute the nonlinear reduced order model (using the FEA software) and the need to perform repeated time integrations to predict the NNMs. During early stages of design, when many design configurations are proposed, this approach could provide a means to quickly study the changes to the system.

4.4 Conclusion

A new structural modification procedure based on the nonlinear normal modes of a structure is presented in this work. The method is validated by applying it to a geometrically nonlinear finite element model modified by the addition of a torsional spring. The results show that the proposed method is capable of accurately predicting the nonlinear modes due to this structural modification, presumably at a reduced computational cost. The discrepancies between the predictions and the truth model at low energy were shown to be due to the truncation error of the basis set used in the QL modal model. Experience with linear systems suggest that the results could be improved by using a richer modal basis for the unmodified structure.

The results also demonstrated that the modification procedure does not capture multi-harmonic responses such as internal resonances. This is due to the fact that the QL modal model only accounts for the fundamental frequency of the response. However, the results did demonstrate that one can use the predicted NNM curves to determine candidate locations of an internal resonance. This method is entirely suitable for design studies where one often wishes to rapidly probe the design space to determine an optimal design. A higher fidelity model could then be used to check the results.

In future work, the method will be extended to a substructuring approach where the NNMs of two finite element model subcomponents will be used to predict the NNMs of the assembly. This approach would allow one to replace computationally expensive subcomponent models with simpler models so the system level behavior can be studied quickly, and it provides the designer with certain insights regarding what types of changes should be made to the subcomponents to produce the desired responses. This type of analysis has been a cornerstone to structural dynamics for over half a decade.

References

1. Cook RD, Malkus DS, Plesha ME, Witt RJ (2002) Concepts and applications of finite element analysis, 4th edn. John Wiley & Sons Inc., Hoboken, New Jersey.
2. Craig RRJ, Bampton MCC (1968) Coupling of substructures for dynamic analysis. AIAA J 6:1313–1319

3. Kerschen G, Peeters M, Golinval JC, Vakakis AF (2009) Nonlinear normal modes. Part I. A useful framework for the structural dynamicist. *Mech Syst Signal Process* 23:170–94
4. Vakakis AF (1997) Non-linear normal modes (NNMs) and their applications in vibration theory: an overview. *Mech Syst Signal Process* 11:3–22
5. Rosenberg RM (1960) Normal modes of nonlinear dual-mode systems. *J Appl Mech* 27:263–268
6. Peeters M, Vigu   R, S  randour G, Kerschen G, Golinval JC (2009) Nonlinear normal modes, Part II: toward a practical computation using numerical continuation techniques. *Mech Syst Signal Process* 23:195–216
7. Allen MS, Kuether RJ, Deaner BJ, Sracic MW (2012) A numerical continuation method to compute nonlinear normal modes using modal coordinates. In: 53rd AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics, and materials conference, Honolulu, Hawaii, 2012
8. Burton T (2007) Numerical calculation of nonlinear normal modes in structural systems. *Nonlinear Dyn* 49:425–441
9. Kuether RJ, Allen MS (2012) Computing nonlinear normal modes using numerical continuation and force appropriation. Presented at the ASME 2012 international design engineering technical conferences IDETC/CIE, Chicago, 2012
10. Slater JC (1996) A numerical method for determining nonlinear normal modes. *Nonlinear Dyn* 10:19–30
11. C  mert MD, Ozguven HN (1995) A Method for forced harmonic response of substructures coupled by nonlinear elements. In 13th international modal analysis conference (IMAC XIII), Nashville, 1995
12. Kalaycioglu T, Ozguven HN (2011) Harmonic response of large engineering structures with nonlinear modifications. In 8th international conference on structural dynamics, EURO-DYN 2011, 2012, July 4, 2011–July 6, 2011, Leuven, 2011, pp 3623–3629
13. Nevzat   zg  ven H (1990) Structural modifications using frequency response functions. *Mech Syst Signal Process* 4:53–63
14. Kalaycioglu T, Ozguven HN (2012) Nonlinear structural modification and nonlinear coupling. In: 30th IMAC, a conference on structural dynamics, 2012, January 30, 2012–February 2, 2012, Jacksonville, 2012, pp 225–237
15. Ferreira JV, Ewins DJ (1996) Nonlinear receptance coupling approach based on describing functions. In 14th international modal analysis conference (IMAC XIV), Dearborn, 1996
16. Watanabe K, Sato H (1988) Development of nonlinear building block approach. *J Vib Acoust Stress Reliab Design* 110:36–41
17. Ferreira JV, Ewins DJ (1997) Multi-harmonic nonlinear receptance coupling approach. In: 15th international modal analysis conference (IMAC XV), Orlando, 1997, pp 27–33
18. Chong YH, Imregun M (2000) Coupling of non-linear substructures using variable modal parameters. *Mech Syst Signal Process* 14:731–746
19. Apiwattanalungarn P, Shaw SW, Pierre C (2005) Component mode synthesis using nonlinear normal modes. *Nonlinear Dyn* 41:17–46
20. Allen MS, Kuether RJ (2012) Substructuring with nonlinear subcomponents: a nonlinear normal mode perspective. Presented at the 30th international modal analysis conference (IMAC XXX), Jacksonville, 2012
21. Hollkamp JJ, Gordon RW, Spottswood SM (2005) Nonlinear modal models for sonic fatigue response prediction: a comparison of methods. *J Sound Vib* 284:1145–63
22. Hollkamp JJ, Gordon RW (2008) Reduced-order models for nonlinear response prediction: implicit condensation and expansion. *J Sound Vib* 318:1139–1153
23. Gordon RW, Hollkamp JJ (2011) Reduced-order models for acoustic response prediction. Air Force Research Laboratory (AFRL), Wright-Patterson Air Force Base, OH AFRL-RB-WP-TR-2011-3040
24. Budak E, Nevzat   zg  ven H (1993) Iterative receptance method for determining harmonic response of structures with symmetrical non-linearities. *Mech Syst Signal Process* 7:75–87
25. Tanrikulu O, Kuran B, Ozguven HN, Imregun M (1993) Forced harmonic response analysis of nonlinear structures using describing functions. *AIAA J* 31:1313–1320
26. Ginsberg JH (2001) *Mechanical and structural vibrations*, 1st edn. Wiley, New York
27. Culler AJ, McNamara JJ (2010) Studies on fluid-thermal-structural coupling for aerothermoelasticity in hypersonic flow. *AIAA J* 48:1721–1738
28. Culler AJ, McNamara JJ (2011) Impact of fluid-thermal-structural coupling on response prediction of hypersonic skin panels. *AIAA J* 49:2393–2406
29. Gordon RW, Hollkamp JJ, Spottswood JJ (2003) Nonlinear response of a clamped-clamped beam to random base excitation. Presented at the eighth international conference on recent advances in structural dynamics, University of Southampton, Southampton, 2003
30. Allen MS, Mayes RL, Bergman EJ (2010) Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections. *J Sound Vib* 329:4891–4906
31. Petrov EP (2009) Analysis of sensitivity and robustness of forced response for nonlinear dynamic structures. *Mech Syst Signal Process* 23:68–86
32. Petrov EP, Ewins DJ (2003) Analytical formulation of friction interface elements for analysis of nonlinear multi-harmonic vibrations of bladed disks. *J Turbomachinery* 125:364–371

Chapter 5

Modeling and Calibration of Small-Scale Wind Turbine Blade

Anders T. Johansson, Carl-Johan Lindholm, Khorsand Vakilzadeh, and Thomas Abrahamsson

Abstract The SEM Substructuring Focus Group has chosen an Ampair 600W wind turbine to be used as a test bed in the continued efforts to further experimental and experimental-analytical substructure coupling techniques. To assess such coupling techniques, validated models of the parts, the substructures considered, play a crucial role. This paper describes the modeling, calibration and validation of a Finite Element (FE) model of a blade for the test bed turbine. Orthotropic composite material modeling is used to set up the model, which is calibrated and validated based on results from an ambitious measurement campaign including both non-destructive testing for dynamic properties and dedicated destructive tests for deduction of material properties. The measurement campaign is carefully described in the paper.

Keywords Model calibration • Wind turbines • Mechanical testing • Vibration testing • Modal analysis

5.1 Introduction

IMAC XXX saw the introduction of the A600 wind turbine as a benchmark structure, as well as initial efforts in both experimental and numerical modeling, e.g. [1–4]. These first efforts were predominantly focused on blade modeling. Characterization of A600 blade models has carried over to IMACXXXI, where efforts on blade modeling and calibration are reported also in [5, 6].

Likewise, the present study aims to build a high-quality Finite Element model of an A600 blade. To support the modeling, a number of material tests have been performed, ranging from thickness measurements of the blade's skin through chemical analysis for determination of the polymer materials in the blade to destructive mechanical testing for estimation of material properties. Furthermore, vibration data has been collected over a large frequency range through stepped-sine testing. From this data, frequency-domain state-space subspace (N4SID) system identification [7] is used to build a test model to which the nominal model is calibrated (updated) [8]. In a parallel study, blade variability is investigated [9]. While it is concluded in that study that the variability of the A600 blades is fairly large, this paper is concerned only with calibration to a specific blade; serial number 963.

The remainder of this paper is structured in the following way: In section 5.2, the material testing and Finite Element modeling of the blade is described. Section 5.3 describes the calibration of the model to measured data. Finally, section 5.4 holds concluding remarks.

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5.2 FE Model

FE modeling of the blade was performed using Siemens' pre- and postprocessor FEMAP v.10.3 with the MD NASTRAN v.2011.1 solver. The outer geometry of the blade was provided by the SEM Substructuring Focus Group. In order to decide on the modeling strategy, a series of tests were performed to determine the material composition of the blade. Thereafter, a model was built and verified based on the stability of the first flexible eigenfrequencies upon mesh refinement.

5.2.1 Material Testing

First, in order to identify the type of materials in the blade, shavings of the black skin and the white core (see Fig. 5.1a) were chemically analyzed by infrared spectroscopy and differential scanning calorimetry. The analysis indicated that both the core and the skin of the blade consisted of polypropylene (PP) [10], with signs indicating that there might be mineral material in the skin material. To verify that there was indeed mineral material in the skin and to investigate its structure, the PP resin was burned off in an oven. The resulting glass fibres, in the form woven rowing, are seen in Fig. 5.1b.

Thus concluding that the blade consists of a skin with a woven glass fibre in a PP resin set on a PP core, some educated guesses on the material data for FE modeling could be formed. To substantiate the material parameters further, mechanical testing of the coupons cut out of the blade according to Fig. 5.1a was performed. The tensile stiffness of the coupons was evaluated by tensile testing. The results are shown in Table 5.1. Because the blade does not present any planar surfaces, the preparation of uniform coupons for the skin presented some difficulties. The thickness of the samples was also subject to some variation, both between samples and within the same sample. This serves as an explanation for the large spread in data seen for the skin material.

In addition to the tensile testing, compressive tests of the core was performed (on part c1 and c2). The results of the two tests had a mean value of about 800MPa; however, experience has shown that bulk data estimates from this type of testing are to be viewed as lower limits. Since the compression coupons also were very small out of necessity, the results were deemed less reliable.

The compression coupons were also used for estimation of the core material density, found to be 818kg/m³. The weight of the full blade was 798.4g.

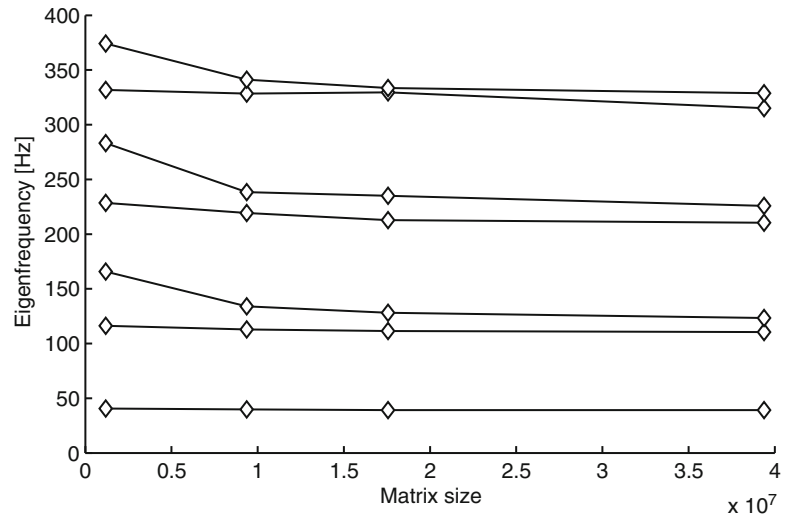


Fig. 5.1 (a) Cut blade with part labels. Parts c1 and c2 were used for compression testing of the core, 7,9 and 11 for tensile testing of the core and 13b, 13f, 14b, 14f and 15f for tensile testing of the skin, where f and b denotes front and back surface, respectively. (b) Glass fibre content of blade

Table 5.1 Results from tensile testing. The presented quantities are: Tested specimen identification code, thickness t and estimated modulus E (directional). For definition of the different specimens, see Fig. 5.1a

Core			Skin		
ID	t [mm]	E [MPa]	ID	t [mm]	E [MPa]
7	6.0	1590.4	13b	2.7	6432.9
9	6.0	1374.5	13f	2.9	7732.9
11	6.0	2270.7	14b	2.8	8510.9
			14f	2.9	10237.7
			15f	3.0	3280.6
Mean	6.0	1745.2	Mean	2.86	7239.0
St. dev.	0.0	467.7	St. dev.	0.11	2606.3

Fig. 5.2 Verification of mesh size. The first seven eigenfrequencies analyzed using four different models. The mesh was deemed converged after one refinement. Note that the frequencies in this figure do not correspond to the nominal values



5.2.2 Modeling

Drawing on the results from the testing stage, the FE model was built as a laminate skin on an isotropic core. The core was modeled by linear four-noded tetrahedral elements (CTETRA) using an isotropic material model with the mean measured Young's modulus, $E = 1745 \text{ MPa}$, and an estimated Poisson's ratio of $\nu = 0.3$ and density 818 kg/m^3 .

The skin was modeled predominantly by four-noded rectangular plate elements (CQUAD4), with a few triangular (CTRIA). Common practice to model glass-reinforced plastic materials is to use composite shell elements. Because of this, the measured values are recalculated to an equivalent unidirectional ply. This also makes it easier to estimate the shear properties – shear in a unidirectional ply can be estimated based on the shear modulus of the isotropic resin property, thus $G = E/2(1 + \nu)$. The material model of the skin was prepared using the built in laminate theory of FEMAP. The layup consists of four layers, each 7mm thick (to yield a total thickness of 2.8mm), stacked in the order $0^\circ - 90^\circ - 0^\circ - 90^\circ$. Compare this to the weave of Fig. 5.1b; four layers are used since experience has shown that it represents a weave more accurately.

The orthotropic material model used for the layers was prepared to match the measured stiffness; the Young's modulus in the direction perpendicular to the fibre direction was set to that of PP at $E_2 = 1745 \text{ MPa}$, whereas in the fibre direction the stiffness was estimated to $E_1 = 14.5 \text{ GPa}$. The shear modulus in the different directions were set to 700 MPa in all three directions and $\nu_{12} = 0.3$. This ensures a bulk stiffness of 7228 MPa in the $0^\circ - 90^\circ$ -directions of the plane, corresponding to the measured mean. For details, confer [11].

The skin of the model was furthermore modeled using an offset to ensure that the outer surfaces of the geometry are also the outer surfaces of the model. This will however mean that there is an overlap of skin and core material in the skin. To match the measured weight of the model, the density of the skin was tuned in at 272 kg/m^3 , yielding a model weight of 798.6 g , differing by only 0.2 g from the measured value. Because of the overlap, the skin material density estimate arrived at in this manner is 1090 kg/m^3 .

In this study, all computations have been eigensolutions made in free-free conditions. Thus, six rigid body modes are included in each result set; these will however not be presented. When the first mode is referred, the first flexible mode is implied.

5.2.3 Verification

To ensure convergence of the mesh, a convergence analysis of the first seven eigenfrequencies with respect to four models of increasing order was performed. The solution times varied from a few seconds to hours from coarsest to finest mesh. The results of the convergence study is shown in Fig. 5.2. Note that at the time of the convergence analysis, the material data was not fully analysed and so the eigenvalues do not correspond to those of the nominal model. It was decided that the

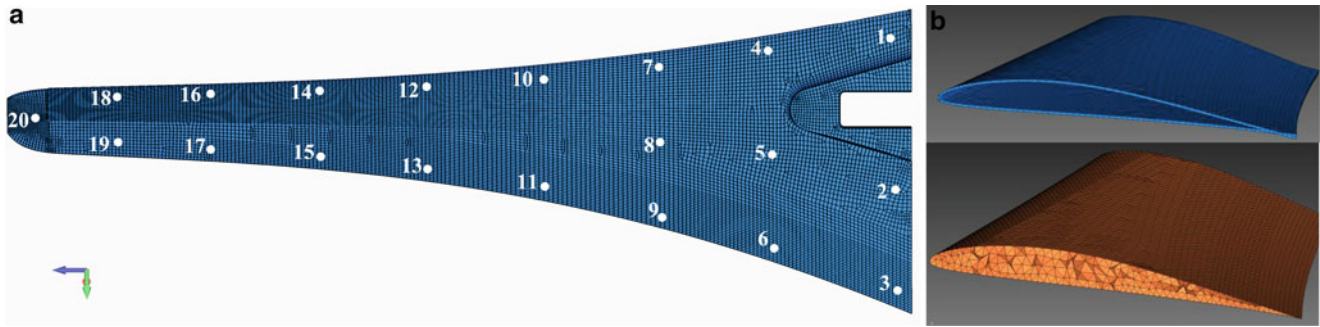


Fig. 5.3 Finite element model, dynamic measurement set and skin and core mesh

convergence was good enough for the second coarsest model; this model is used henceforth. It consists of 317390 nodes and 209714 elements, of which 25347 are linear laminate plates and the remaining 184367 are solid tetras. The model is seen in Fig. 5.3.

5.3 Model Calibration

This section describes a model calibration campaign to improve the predictive quality of the A600 blade model over a large frequency range.

5.3.1 Vibration Measurements

To validate the blade model described in the previous section, a stepped sine test was performed in the frequency range from 30 to 900Hz with 0.2Hz increments, exciting 12 of the first flexible modes of the system. 20 accelerometers were used to record the accelerance of the system in the direction perpendicular to the blade at the same locations used in [4, 9]. The measurement set is depicted in Fig. 5.3a. The system was excited perpendicular to the blade surface at node 1, and eigenvalues and eigenvectors were extracted from the data by identifying a linear time invariant model of the proper order using frequency-domain system identification by the system identification toolbox in Matlab [7]. By using the relation $A(\omega) = j\omega Y(\omega)$, where A is the accelerance and Y the mobility, the system identification could be carried out on mobility data. This ensures that the high-frequency modes are comparatively less influential on the response of the system.

5.3.2 Initial Correlation

A comparison of the measured eigenvalues and eigenvectors to the nominal FE modes was conducted, seen in Table 5.2 and Fig. 5.4a. The results show a fairly large frequency deviation, with a maximum of 5.5% relative error for the first five flexible modes, where the MAC correlation between measured and analytical eigenvectors is good. Beyond the fifth mode, no modes can be said to correlate well for the nominal model. Thus, the model needs calibration.

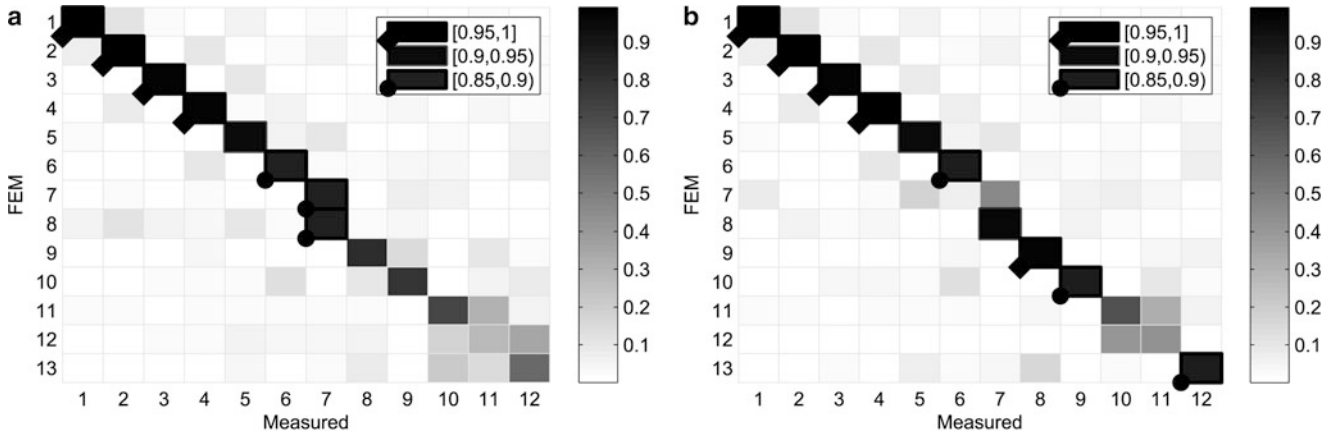
5.3.3 Calibration

There are many different techniques for model calibration, many of them described in the standard textbook by Friswell and Mottershead [8]. In this study, the linear error in eigenfrequency has been used as the cost function, calculated as:

$$J(\theta) = \varepsilon(\theta)^T \varepsilon(\theta) \quad (5.1)$$

Table 5.2 Comparison between measured and finite element eigenfrequencies in [Hz]

	Frequency comparison [Hz]			Relative comparison [%]	
	Measured	FEM nominal	FEM calibrated	Meas.-FEM nom.	Meas.-FEM cal.
Mode 1	46.5	48.1	45.2	3.5	2.7
Mode 2	129.4	136.5	128.9	5.5	0.4
Mode 3	191.7	181.9	188.1	5.1	1.8
Mode 4	250.7	264.1	250.2	5.3	0.2
Mode 5	324.0	317.7	326.1	2.0	0.7
Mode 6	404.5	426.4	405.0	5.4	0.1
Mode 7	–	455.4	452.2	–	–
Mode 8	481.3	476.2	474.2	1.1	1.5
Mode 9	576.7	609.7	584.5	5.7	1.4
Mode 10	647.6	622.0	631.2	4.0	2.5
Mode 11	750.4	746.2	734.7	0.6	2.1
Mode 12	769.1	797.0	779.0	3.6	1.3
Mode 13	857.6	831.2	823.4	3.1	4.0

**Fig. 5.4** MAC correlation plots. (a) MAC matrix for correlation between nominal model modes and measurement modes. (b) MAC matrix for correlation between calibrated model modes and measurement modes

where

$$\varepsilon(\theta) = \begin{bmatrix} \vdots \\ f_i^{Meas} - f_j(\theta)^{FEM} \\ \vdots \end{bmatrix} \quad (5.2)$$

Here, f_i^{Meas} and f_j^{FEM} are the i :th measured and j :th Finite Element eigenfrequencies in Hz, respectively. i and j are matched indices, resulting from a mode pairing scheme aimed at ensuring that the eigenfrequency of the FE mode which most accurately represents a given measured mode is compared to that mode. Mode pairing is described further in [12]. Because of the poor correlation beyond the fifth mode, it was decided that only the first five modes were to be used in the model calibration scheme. The argument (θ) is used to indicate dependence of the updating parameters.

Because of the large spread in the static measurements, it was found that the bulk data parameters were suitable for updating. Thus, a parameter vector consisting of the six main bulk data entries was set up, i.e:

$$\theta = \ln[E_{1,skin} \ E_{2,skin} \ G_{12,skin} \ G_{1z,skin} \ G_{2z,skin} \ E_{core}]^T \quad (5.3)$$

Table 5.3 Comparison between nominal and calibrated material parameter values

	$E_{1,skin}$	$E_{2,skin}$	$G_{12,skin}$	$G_{1z,skin}$	$G_{2z,skin}$	E_{core}	[GPa]
Nominal value	14.5	1.75	0.7	0.7	0.7	1.75	
Calibrated value	11.6	2.84	0.90	0.72	0.72	1.67	

Where the natural logarithm was used in the parameter definitions to contract the parameter space. A single starting point was generated, from the parameters of the nominal model described above. Constraints were introduced approximating the level of uncertainty in the modeling, and the Active Set algorithm in Matlab's optimization toolbox [13] was used to find a local minima of the cost function. Gradients were approximated numerically by Matlab.

The resulting parameters are shown in Table 5.3. No constraints were active for the final result. The resulting eigenfrequencies and MAC correlations can be seen in Table 5.2 and Fig. 5.4b, respectively.

5.3.4 Validation

Since the model was calibrated using only the first five eigenfrequencies as parameters, seven more eigenfrequencies are available for validation purposes. After updating, it is found that all of the modes have a relative error below 3% except for the highest mode. In a parallel study, it was shown that the coefficient of variation describing the variability between blades was also at around 3%. Since the MAC correlation of eigenvectors has also improved dramatically after calibration, the model can be deemed validated.

If one is very generous in defining correlation (as $MAC > 85\%$), three modes remain uncorrelated, with FE mode number 7 standing out the most in that it seemingly does not have a measurement counterpart at all. It may not be fully clear in Fig. 5.5g, but this is an edge-wise bending mode, and so the main deflection of the mode is perpendicular to both the sensing and the actuating of the measurements, and hence it is both unobservable and uncontrollable. Thus, this is an effect of the measurement setup rather than the model. Mode 10, Fig. 5.5j, is a mode dominated mainly by deflection in the weak corner of the blade's base, and Mode 11, Fig. 5.5k, seems to have vibrational node lines at about the same length-direction intervals as we are sensing, making it largely unobservable and sensitive to slight misalignments of the FE output to the Measurement positions.

Taking all these factors into account, the model presented in this study seems to be a good working model of an Ampair 600W blade.

5.4 Conclusions

This paper presents a FE model of Ampair A600W wind turbine blades. The model is built using laminate theory in the skin and linear isotropic material modeling in the core. Several tests were performed to identify and characterize the materials of the blade. The model was successfully verified, calibrated and validated based on dynamic testing in the frequency range from 30 to 900Hz.

Using the built-in laminate theory in FEMAP to compute the stiffness parameters of the skin layup based on the calibrated parameters results in 6772MPa, which is well within the standard deviation of the tensile testing results at 7239.0 ± 2606.3 MPa. Likewise, the calibrated core stiffness was 1666MPa which is within the bound at 1745.2 ± 467.7 MPa.

After calibration, all eigenfrequencies but one are within 3% of the measured values and the MAC matrix is diagonal-heavy. The relative lack of correlation for some modes can however be explained in terms of observability and controllability.

Acknowledgements The authors would like to acknowledge Jerry Stigsson and Håkan Lindqvist at DIAB AB for their respective contributions to the laboratory testing of this paper. The FE modeling of the paper has been assisted by Johan Hedlund at CCG Composites Consulting Group.

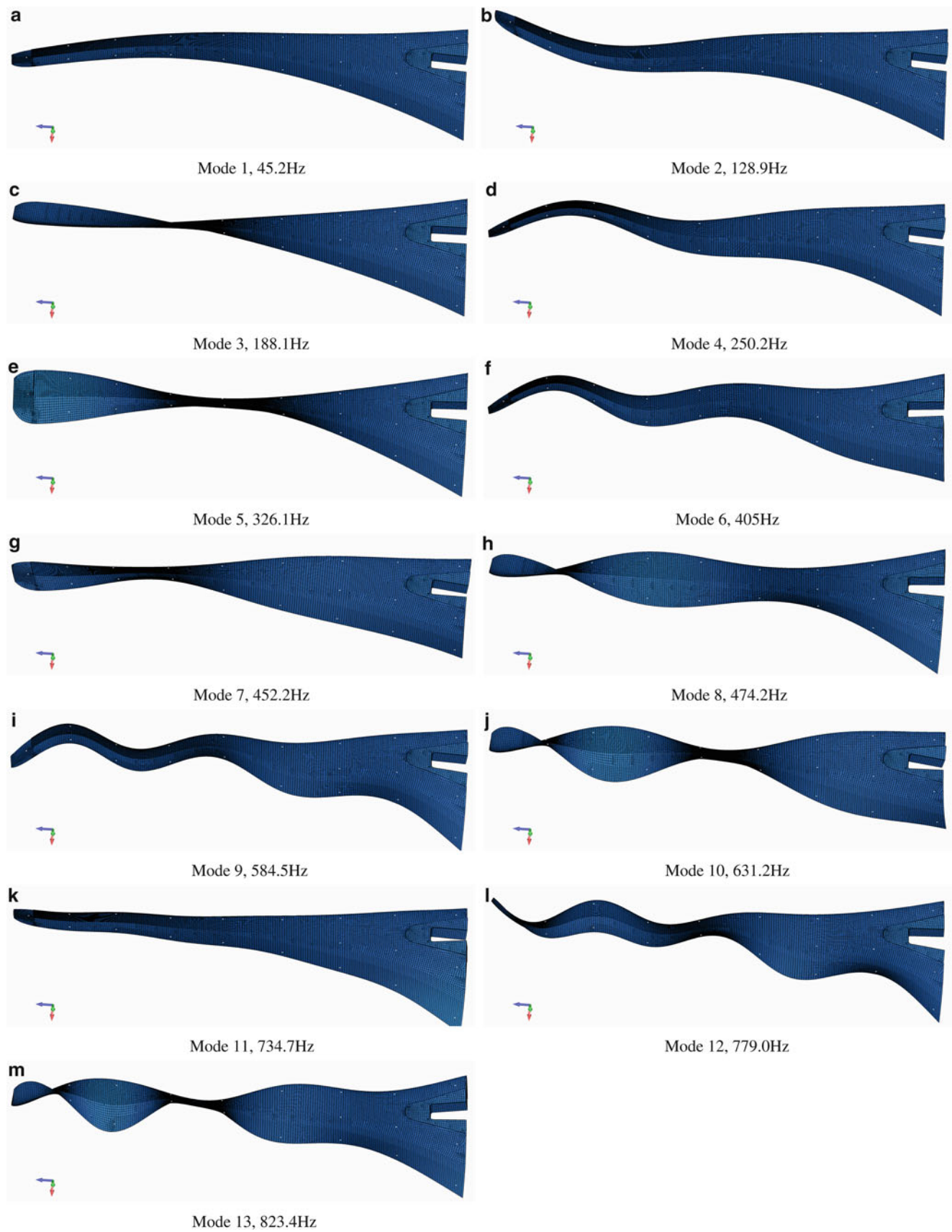


Fig. 5.5 Flexible vibrational modes of calibrated FE model

References

1. Mayes R (2012) An introduction to the SEM substructures focus group test bed the Ampair 600 wind turbine. In: Proceedings of the SEM IMAC XXX conference, Jacksonville, FL, USA, 30 Jan–2 Feb 2012
2. Mayes R (2012) Wind turbine experimental dynamic substructure development. In: Proceedings of the SEM IMAC XXX conference, Jacksonville, FL, USA, 30 Jan–2 Feb 2012
3. Nurbhai MS, Macknelly DJ (2012) Modal assessment of wind turbine blade in preparation of experimental substructuring. In: Proceedings of the SEM IMAC XXX conference, Jacksonville, FL, USA, 30 Jan–2 Feb 2012
4. Harvie J, Avitabile P (2012) Comparison of some wind turbine blade tests in various configurations. In: Proceedings of the SEM IMAC XXX conference, Jacksonville, FL, USA, 30 Jan–2 Feb 2012
5. Linderholt A, Chen YC (2013) Model calibration of an A600 wind turbine blade. In: Proceedings of the SEM IMAC XXXI conference, Los Angeles, CA, USA, 11–14 Feb 2013
6. Nurbhai M, Macknelly D (2013) Dynamic structuring turbine blade FE model update and correlation with test. In: Proceedings of the SEM IMAC XXXI conference, Los Angeles, CA, USA, 11–14 Feb 2013
7. McKelvey T, Akcay H, Ljung L (1996) Subspace-based multivariable system identification from frequency response data. *IEEE Trans Automat Contr* 41:960–979
8. Friswell MI, Mottershead JE (1995) Finite element model updating in structural dynamics. Kluwer, Dordrecht
9. Gibanica M et al (2013) Spread in modal data obtained from wind turbine blade testing. In: Proceedings of the SEM IMAC XXXI conference, Los Angeles, CA, USA, 11–14 Feb 2013
10. Brydson JA (1999) *Plastics materials, referex engineering*. Elsevier Science, Amsterdam
11. Zenkert D (1997) *The Handbook of Sandwich Construction*. EMAS Ltd, Cradley Heath
12. Möller P, Friberg O (1998) An approach to the mode pairing problem. *Mech Syst Signal Process* 12:515–523
13. Gill PE et al (1984) Procedures for optimization problems with a mixture of bounds and general linear constraints. *ACM Trans Math Software* 10:282–298

Chapter 6

Ranking Constituents of Coupled Models for Improved Performance

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Abstract In partitioned analysis, constituent models representing different scales or physics are routinely coupled to simulate complex physical systems. Such constituent models are invariably imperfect and thus, yield a degree of disagreement with reality, known as model form error. This error propagates through coupling interfaces and degrades the accuracy of the coupled system. To efficiently improve the coupled system, resources must be allocated to systematically improve the constituent models. This study proposes a tool and an associated metric that exploits the availability of experimental data to prioritize constituent models. This metric is useful in tracing the error of coupled systems to their origins and to quantify the contribution of constituent error to the overall error of coupled systems. The proposed metric is used to rank constituents based on (i) the relative model form error of the constituents, (ii) the sensitivity of the model form error of the coupled system to the model form error in the constituents, and (iii) the cost to improve their performance. The applicability of the proposed metric is demonstrated through a proof-of-concept structural example, by coupling individual frame elements to model a portal frame. Coupling and uncertainty inference of the inexact constituent models are achieved using optimization, where both separate-effect and integral-effect experiments are employed to train the model form error of the constituents and coupled system.

Keywords Code development • Code prioritization • Model calibration • Optimization • Partitioned analysis

6.1 Introduction

Computational tools and computer models are being implemented more frequently to simulate the response of physical phenomena. Recently, simulation of such phenomena has been achieved by merging several constituent models that represent distinct domains to generate a general system with an accurate representation of reality [1]. However, some limitations, such as simplified physics or lack of knowledge of the true behavior and equations of the physical phenomena, cause some level of disagreement between simulation models and reality. This disagreement between the predicted simulations and true responses of the physical system is known as *model form error* (MFE). In coupled models, the MFE of the constituents contribute to the predictive ability of the coupled system [2, 3]. Thus, to improve the accuracy of predictions in the coupled system, the constituent models should be enhanced. Constituents should be ranked based on their impact on the reduction of MFE so that resources may be dedicated more effectively for model improvement. This study proposes a prioritization metric for identifying the critical constituents of the coupled system for improvement.

Limited studies on prioritization of code development activities in coupled structural systems exist in literature. A few studies on coupled systems with different applications are reported here. In software engineering for example, continued code development is required for enhancing existing software. For this purpose, various approaches have been proposed to aid code developers in locating potential defects in different components of the software and updating and improving its performance [4–8]. Dynamic analysis along with verification and validation of fully-coupled mechanical systems are discussed by Alvin and Reese [9], in which they propose a plan for validation of separate physics and fully-coupled effects.

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Improving a coupled turbulence model by improving each of the constituents is discussed by Kumar and Ghoniem [10, 11]. Döscher et al. [12] used experimental data for verification and validation of strongly coupled ocean-atmosphere-ice models for general circulation in Northern Europe. Li et al. [13] used a validated coupled hydrologic-crop growth model to manage water resources in agriculture and also predict crop production.

Terejanu et al. [14] used a Bayesian framework to select constituent models to improve the predictive capability of the coupled model. The constituent models are selected from their respective suite of models with each representing a different set of physics for which a set of experimental data is available. The optimal selection of the constituent models is the one that, upon coupling, will best predict the quantity of interest.

Hegenderfer and Atamturktur [15] proposed a novel code prioritization metric based on Phenomena Identification and Ranking Table (PIRT). They used three criteria to rank the constituent models for further improvement: current knowledge level (uncertainty), importance (sensitivity), and error analysis (initial error). They implemented this approach on a steel structural frame with semi-rigid bolted connections. Hegenderfer and Atamturktur [15] however, did not consider cost in their defined metric.

In the current study, a ranking metric for code prioritization of the constituents in the coupled systems is defined such that the accuracy of the coupled system is most efficiently improved. Three components are used to define this metric: ratio of the MFE in the constituents, sensitivity of the coupled system to the response of each constituent, and cost of improvement. This study is organized into the following sections. In Section 6.2, the mathematical basis of the coupling and the validation of the constituents are discussed by defining three terms: MFE index, sensitivity index, and cost index. The section continues by defining the code prioritization metric as a function of these indices. Then, the defined metric is applied to a structural case study in Section 6.3. Section 6.4 includes the results and shortcomings of the study and reviews suggestions for future work.

6.2 Methodology

The schematic diagram of the two constituents, A and B, of a strongly coupled model considering their MFEs are shown in Fig. 6.1. Symbols Ψ_A , Ψ_B , and $\Psi_{Coupled}$ represent the MFE of constituent A, constituent B, and the coupled system, respectively. Y_A^{sim} and Y_B^{sim} represent simulation results of the constituents. Y_A^{obs} , Y_B^{obs} , and Y_{Int}^{obs} represent separate-effect experiments for constituents A and B, and integral-effect experiment of the coupled system, respectively. In this research, separate-effect and integral-effect experiments are used to train the MFE of the constituents and coupled system simultaneously.

Parameters x , θ , and z represents input values for simulation models (x represents control parameters, θ represents calibration parameters, and z are all remaining input values that are not considered control or calibration parameters). Here, superscripts ‘*dep*’ and ‘*indep*’ stand for dependent and independent parameters in the input/output of the simulation models, respectively.

To solve this coupled system, several methods are proposed [17]; for instance, Newton-like coupling methods [18–20], Block-Jacobi and Block-Gauss-Seidel methods [21], and an optimization based method [22]. In optimization based coupling (OBC), coupling conditions are employed to form an objective function. In this method, the dependent input parameters of the constituents are considered as optimization variables and their solution is achieved through an optimization process. The OBC approach that is adopted in this study eliminates the divergence issues of classical coupling techniques and is suitable for parallel computation [22]. If we assume the MFE of the constituents will be zero, then the response of the coupled system will represent reality. However, this assumption is impractical and, some level of MFE in the constituents and coupled system will certainly exist. To account for the effect of the inexactness of the simulation models, their MFEs should be considered in the coupling process.

The response of the physical system can be represented in terms of inexact simulation models, Y_A^{sim} , Y_B^{sim} and the corresponding MFE, $\psi(x)$, as expressed in the following equation [23, 24].

$$Y_{\otimes}^{obs}(x_A) = Y_{\otimes}^{sim}(x_{\otimes}, \theta_{\otimes}, z_{\otimes}) + \psi_{\otimes}(x_{\otimes}, \alpha_{\otimes}); \otimes \in \{A, B\} \quad (6.1)$$

In this equation, α represents the non-physical coefficients that should be calculated to represent the MFEs. For the coupled system, Eq. (6.1) can be written in the following form:

$$Y_{int}^{obs}(x_A, x_B) = M_{Coupled}(x_A^{indep}, \theta_A, z_A^{indep}, x_B^{indep}, \theta_B, z_B^{indep}) + \psi_{Coupled}(x_A, x_B, \alpha_{Coupled}) \quad (6.2)$$

where $M_{Coupled}$ is the inexact coupled model, and $\psi_{Coupled}$ is the MFE for the coupled system which is a function of MFEs in the constituents. In this study, to represent MFEs, a two variable polynomial surface function, as shown in Eq. (6.3), is used.

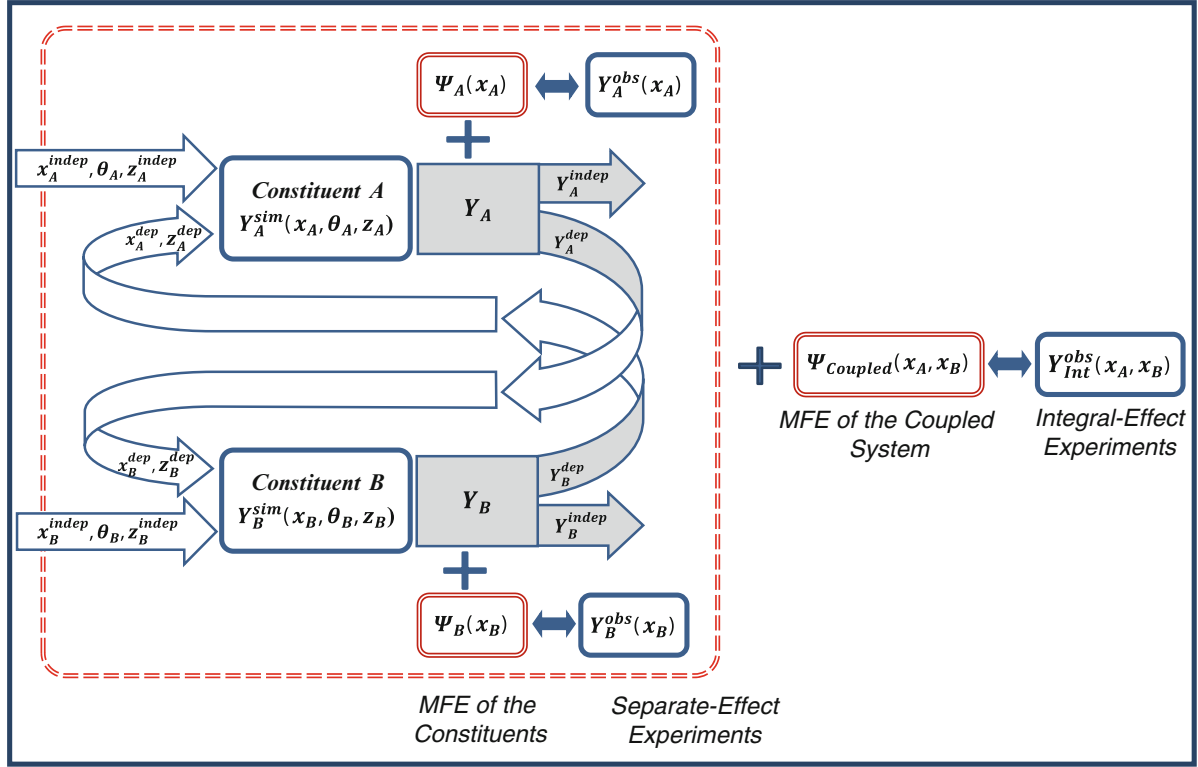


Fig. 6.1 MFE of the coupled model [16]

The orders of the two control parameters, x_1 and x_2 , are p and q , which are selected based on the number of experiments. Continuity and differentiability of the polynomial functions make them appropriate candidates for approximations of MFE [25, 26].

$$\psi(x_1, x_2, \alpha) = \begin{bmatrix} 1 & x_1 & \cdots & x_1^p \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1(q+1)} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2(q+1)} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{(p+1)1} & \alpha_{(p+1)2} & \cdots & \alpha_{(p+1)(q+1)} \end{bmatrix} \begin{bmatrix} 1 \\ x_2 \\ \vdots \\ x_2^q \end{bmatrix} \quad (6.3)$$

To solve the coupling problem, an appropriate numerical algorithm is required. In the current study, the previously developed coupling-uncertainty quantification approach proposed by Farajpour and Atamturktur [27] is employed. The solution process of this coupled system involves calculating the calibration parameters, θ , and also training the MFE of the constituents and coupled system (finding α values). Finding these values will help us in uncertainty quantification, bias correction of inexactness of constituents, and the response of the coupled system, all of which can be used for prioritization of the constituents. Thus, two objectives in the uncertainty quantification process include: (i) training the MFEs by calculating α values for constituents and the coupled system, and (ii) reducing the uncertainty by calibrating the θ values. Therefore, the objective function of the uncertainty quantification is defined as follows:

$$\begin{aligned} Z_{uq}(\alpha, \theta) = & \sum_{j=1}^{s_{Int}} \sum_{i=1}^{r_{Int}} \left(y_{Int}^{obs}((x_1)_i, (x_2)_j) - y_{Coupled}^{sim}((x_1)_i, (x_2)_j, \theta) - \psi_{Coupled}((x_1)_i, (x_2)_j, \alpha) \right)^2 \\ & + \sum_{j=1}^{s_A} \sum_{i=1}^{r_A} \left(y_A^{obs}((x_1)_i, (x_2)_j) - y_A^{sim}((x_1)_i, (x_2)_j, \theta) - \psi_A((x_1)_i, (x_2)_j, \alpha) \right)^2 \\ & + \sum_{j=1}^{s_B} \sum_{i=1}^{r_B} \left(y_B^{obs}((x_1)_i, (x_2)_j) - y_B^{sim}((x_1)_i, (x_2)_j, \theta) - \psi_B((x_1)_i, (x_2)_j, \alpha) \right)^2 \end{aligned} \quad (6.4)$$

where r_* and s_* ($*$ $\in \{Int, A, B\}$) indicate the number of available experiments in each direction of the control parameters. The decision variables of the objective function are θ and α . By minimizing the objective function in Eq. (6.4), calibrating uncertain calibration values and training the discrepancy bias will be achieved simultaneously [28]. The trained MFEs are used to rank critical components in the coupled systems that require the most improvement. To define an appropriate code prioritization metric, the foundation behind this concept should be considered which will aid in the definition of an appropriate metric. Three essential factors of the code prioritization metric are considered in the upcoming sections, including ratio of the MFE in the constituents, sensitivity analysis, and economic concerns.

6.2.1 MFE Index

One of the prioritization criteria that should be considered in the prioritization metric is the relative value of the MFE in the constituents. In the following equation, we define the relative MFE of the constituents, $\bar{\psi}^{Rlt}$, as the ratio of the average MFE, $\bar{\psi}$, to the average simulation response within the entire range of the control parameter:

$$\bar{\psi}_{\otimes}^{Rlt} = \frac{|\overline{\psi_{\otimes}(x_{\otimes}, \alpha_{\otimes})}|}{|\overline{Y_{\otimes}^{sim}}|}; \quad \otimes \in \{A, B\} \quad (6.5)$$

where $\bar{\psi}^{Rlt}$ is the unitless value which represents the inexactness of the constituent model. In the ideal case for a constituent model that exactly represents the reality, this value is zero. By increasing this value, the relative value of the MFE increases, which indicates the prediction of the constituent is inaccurate, and the simulation model requires improvement. The normalized *MFE index* as a function of relative MFE in each constituent is defined as follows:

$$MI_{\otimes}^N = \frac{\bar{\psi}_{\otimes}^{Rlt}}{\max(\bar{\psi}_A^{Rlt}, \bar{\psi}_B^{Rlt})}; \quad \otimes \in \{A, B\} \quad (6.6)$$

This MFE index is a value between zero and one, where zero represents higher predictive capability, and one represents lower predictive capability of the constituents.

6.2.2 Sensitivity Index

The next factor that is considered in the definition of the code prioritization metric is the effect of the MFE in the constituents on the response of the coupled system, which is known as *sensitivity analysis*. For instance, one constituent may have a large ratio of MFE but at the same time have a negligible effect in the MFE of the coupled system. Thus, improvement of this constituent would not be of much benefit. To study the effect of MFE of the constituents on the MFE of the coupled system, the variation of the MFE of the coupled system with respect to variation of those in each of the constituents should be evaluated. The most common approach for conducting this sensitivity analysis is based on a statistical concept known as *coefficient of determination* or *R-squared* [29, 30]. This value determines the proportion of variance of one variable that is predictable from the other variable. The value of R^2 is such that $0 < R^2 < 1$. If the value of R^2 is one, then the regression line passes exactly through every scatter point. If this value is close to zero, the line is further away from the scatter points. Therefore, the R^2 value for a metric shows how well the regression line represents the data, and how certain the model developer can be in making predictions from a certain model. The R^2 values for MFE of the constituents are calculated as follows:

$$R_{\otimes}^2 = \frac{\left(n \sum_{i=1}^n \overline{\psi_{\otimes}}_i \overline{\psi_{Coupled}}_i - \left(\sum_{i=1}^n \overline{\psi_{\otimes}}_i \right) \left(\sum_{i=1}^n \overline{\psi_{Coupled}}_i \right) \right)^2}{\left(n \left(\sum_{i=1}^n \overline{\psi_{\otimes}}_i^2 \right) - \left(\sum_{i=1}^n \overline{\psi_{\otimes}}_i \right)^2 \right) \left(n \left(\sum_{i=1}^n \overline{\psi_{Coupled}}_i^2 \right) - \left(\sum_{i=1}^n \overline{\psi_{Coupled}}_i \right)^2 \right)}; \quad \otimes \in \{A, B\} \quad (6.7)$$

where n is the number of design of experiments that are used for the sensitivity analysis. To obtain the design of experiments, the value of the MFE in each constituent is varied within a full range of 0% to 100 % considering three levels. For each level, the average MFEs in the other constituents and coupled system are calculated. A similar approach is repeated for all constituents to obtain a series of data for all possible combinations (3-level full factorial design of experiments). By substituting these values in Eq. (6.7), the R^2 value for each constituent is obtained. The higher the R^2 value, the more influential that constituent is on the MFE of the coupled system, and a higher priority for code improvement activities should be placed on that constituent.

6.2.3 Cost Index

Software cost estimation has been an active area of research since the emergence of the software and coding industry. Numerous estimation techniques, such as Analogy costing, Expert judgment, Parkinson's method, Price-to-Win, and COCOMO have been proposed and evaluated [31–33].

Most cost estimation models attempt to first estimate an effort, then convert that effort into duration of the project and finally a cost [31]. For simulation codes that are developed to simulate a physical reality, the average MFE of the constituents may be used as a criterion for the required resources and amount of effort that should be dedicated for improvement. A higher MFE in a code represents more effort needed to improve the code and consequently requires higher costs. In the current study, to estimate the development cost of the constituents, all of the missing physics that will be implemented for improvement activity are defined in a systematic manner. Then, the required time for implementation of each of the physics is estimated. Finally, based on the required time and wage of the experts, the development cost for each of the constituents is calculated. Based on calculated costs, a *cost index* for each constituent is defined as follows:

$$CI_{\otimes}^N = \frac{C_{\otimes}}{\max(C_A, C_B)}; \quad \otimes \in \{A, B\} \quad (6.8)$$

where $C_{\otimes}$ is the development cost of each constituent, and $CI_{\otimes}^N$ represents *normalized cost indices*. The cost index is a positive value less than or equal to one. By decreasing the cost index for one constituent, the priority for improvement of that constituent increases because of its lower cost for improvement.

6.2.4 Code Prioritization Metric (CPM)

Through combining the three indices that are previously defined, a metric is proposed to prioritize the code development of the constituents in physics-based coupled systems. This metric provides the sequence of code development based on the maturity of each constituent. This metric is defined as a function of the relative MFEs in the constituents, sensitivity of the MFE in the coupled system to those in the constituents, and development cost for improvement of each constituent. *Code prioritization metrics* (CPM) are defined based on the three normalized indices as follows:

$$CPM_{\otimes} = \frac{MI_{\otimes}^N \times R_{\otimes}^2}{CI_{\otimes}^N}; \quad \otimes \in \{A, B\} \quad (6.9)$$

Based on the definition of CPM provided above, the range of the CPM will be $0 < CPM < +\infty$. Increasing the MFE and sensitivity indices of the constituents, or reducing the cost index, increases the value of the CPM. By increasing the CPM value for each constituent, the priority of these constituents for code development activity increases.

6.3 Case Study

The methodology introduced earlier is implemented on a structural example of a portal frame as shown in Fig. 6.2 [34–36]. The frame is decomposed into three members (substructures), and each member is analyzed in a separate constituent model; then the results of are coupled together. The exchanged data includes the displacement and internal forces of the end nodes in each member, which are then transferred to the corresponding node in the adjacent member (Fig. 6.2).

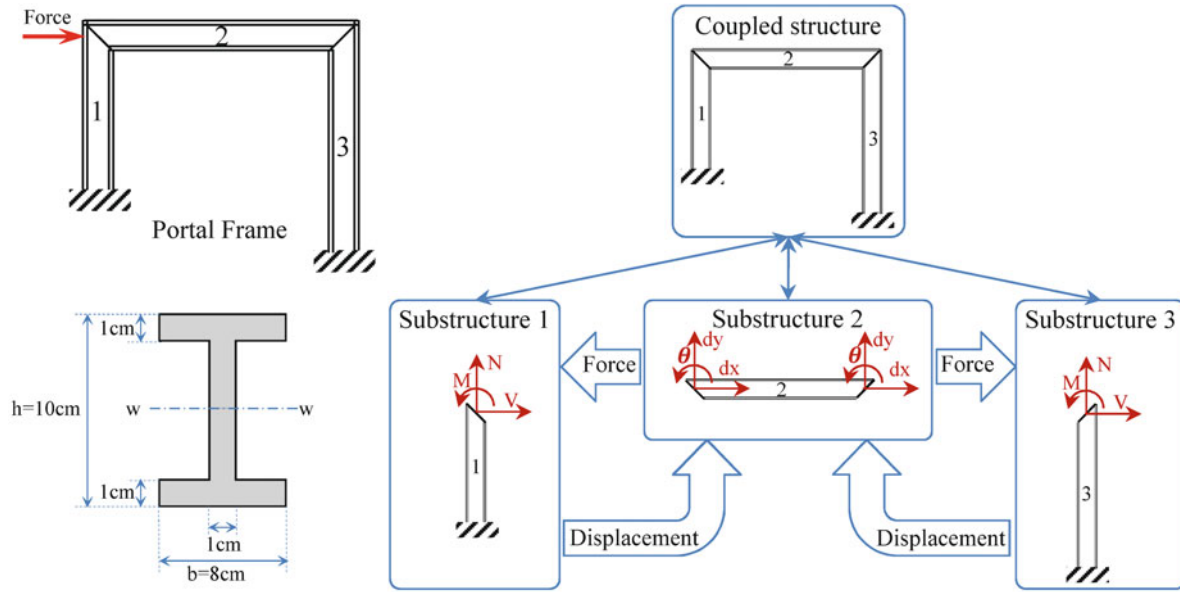


Fig. 6.2 Coupled portal frame [16]

Table 6.1 Input data of the portal frame

Description (unit)	Substructure 1	Substructure 2	Substructure 3
Length of the member (cm)	250.0	300.0	300.0
Young's modulus (Pa)	200×10^9	200×10^9	200×10^9
Shear modulus (Pa) $[E/(2 \times (1 + \nu))]$	7.6746×10^{10}	7.6746×10^{10}	7.6746×10^{10}
Poisson's ratio	0.303	0.303	0.303

6.3.1 Exact Model and Synthetic Experimental Data

The frame consists of three members constructed of steel with uniform I-cross sections as shown in Fig. 6.2. Vertical members are fixed to a solid base with the assumption that connections between beams and columns are rigid. A lateral load is applied to the connection of members 1 and 2 with bending of the I-cross sections happening over centroid axis (w-w) (Fig. 6.2). All loading is assumed to be in the elastic range, and the structural members are assumed not to yield under bending or shear stresses. The Timoshenko beam theory [37], considering axial, shear, and flexural deformations of frame elements, is employed to construct the finite element model. Material properties and geometrical data of the model frame are given in Table 6.1. The shear correction factor of the I-cross sections for the Timoshenko beam theory is approximated using Eq. (6.10) [38].

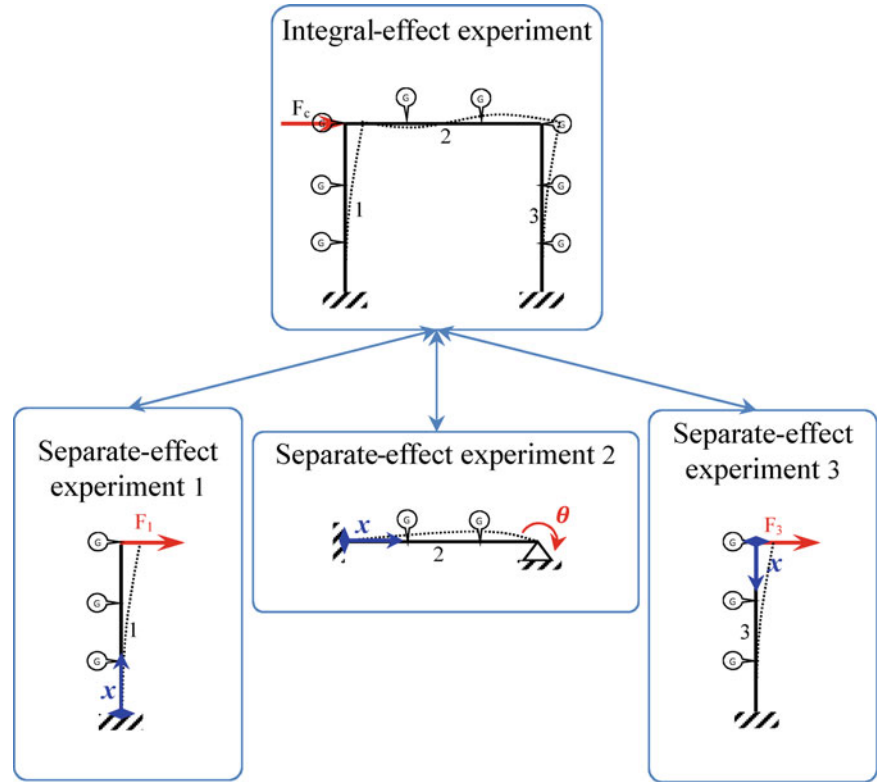
$$\kappa = \frac{A}{I_w^2} \int_A \left(\frac{Q_w}{b} \right)^2 dA \quad (6.10)$$

where κ is the shear correction factor, A is the cross sectional area, Q_w is the first moment of area, and b is the width of the member.

A series of analysis on the complete frame and each of the constituents is carried out to generate the integral-effect and separate-effect experimental data. Generated data includes the displacement of the nodes on the members. Locations of the nodes that are used to obtain the data are shown in Fig. 6.3. The nodes on the columns give horizontal displacements, and the nodes on the beam give the vertical displacements.

To obtain the experimental data of the complete frame (integral-effect experimental data), the frame is subjected to two levels of static force (50kN, 100kN), and the displacement of the nodes are calculated. To generate separate-effect experimental data, the members of the frame are analyzed separately. In columns, two different levels of forces (50kN, 100kN) are imposed in the lateral direction to the top of the columns while the bottom is fixed. To generate experimental data for the middle beam, support conditions shown in Fig. 6.3 are imposed on the beam. Unlike constituents 1 and 3, for which the input parameters are the imposed forces, the input parameters of constituent 2 are imposed deformations

Fig. 6.3 Location of the nodes in the portal frame and constituents to acquire integral-effect and separate-effect experimental data [16]



at the ends of the beam. Herein, to generate experimental data for this beam, we set all deformations, other than the rotation of the right hand side of the beam, to zero, and displacement data is collected for the two known rotations in the right end (0.02Rad , 0.04Rad). In all members, the displacements of the unloaded structures are assumed to be zero. To generate synthetic experimental data, 1 % experimental error is considered as the zero-mean Gaussian random variable.

In this case study, one of the control parameters is the distance of the nodes from the beginning of the member, as indicated by the arrow in Fig. 6.3. For constituents 1 and 3 and for the coupled system, the second control parameter is the applied force. For constituent 2, the second control parameter of the trained MFE is the rotation imposed on the right hand side of the beam.

6.3.2 Inexact Model

Analysis of the constituents is carried out using the finite element approach, and stiffness matrices are generated using Euler-Bernoulli beam theory [37]. Shear deformations are not considered in the inexact simulation models. This simplification is the source of MFE in this case study which introduces some deviation between simulation results and synthetic experimental data.

The results of analysis of inexact constituents are coupled together to obtain the response of the coupled system. The horizontal point load is applied to the connection of members 1 and 2, as shown in Fig. 6.3. In the analysis of the coupled frame, the lateral load is applied to constituent 1. The resulting deformation of constituent 1 is imposed on constituent 2. Then, the reaction forces of constituent 2 are compared with the imposed forces on constituents 1 and 3. The process is repeated until internal forces and node deformations of the connections converge.

The calibration parameters are the Young's modulus of the three members. MFEs of the constituents and coupled system are trained using the polynomial function with two variables, as shown in Eq. (6.3). In this example, one polynomial function is trained for each of the constituents, and three more are trained for the coupled system. The order of polynomials in the trained MFE for the first control parameter (coordinate of the node) is chosen to be ($p = 3$), and for the second variable (imposed force or rotation), it is chosen to be ($q = 2$). The decision variables of the optimization procedure are Young's modulus of the three members and non-physical coefficients of the trained MFEs. The optimization problem is solved using Nelder-Mead Simplex Method through the *fminsearch* function provided in MATLAB [39]. The initial values of

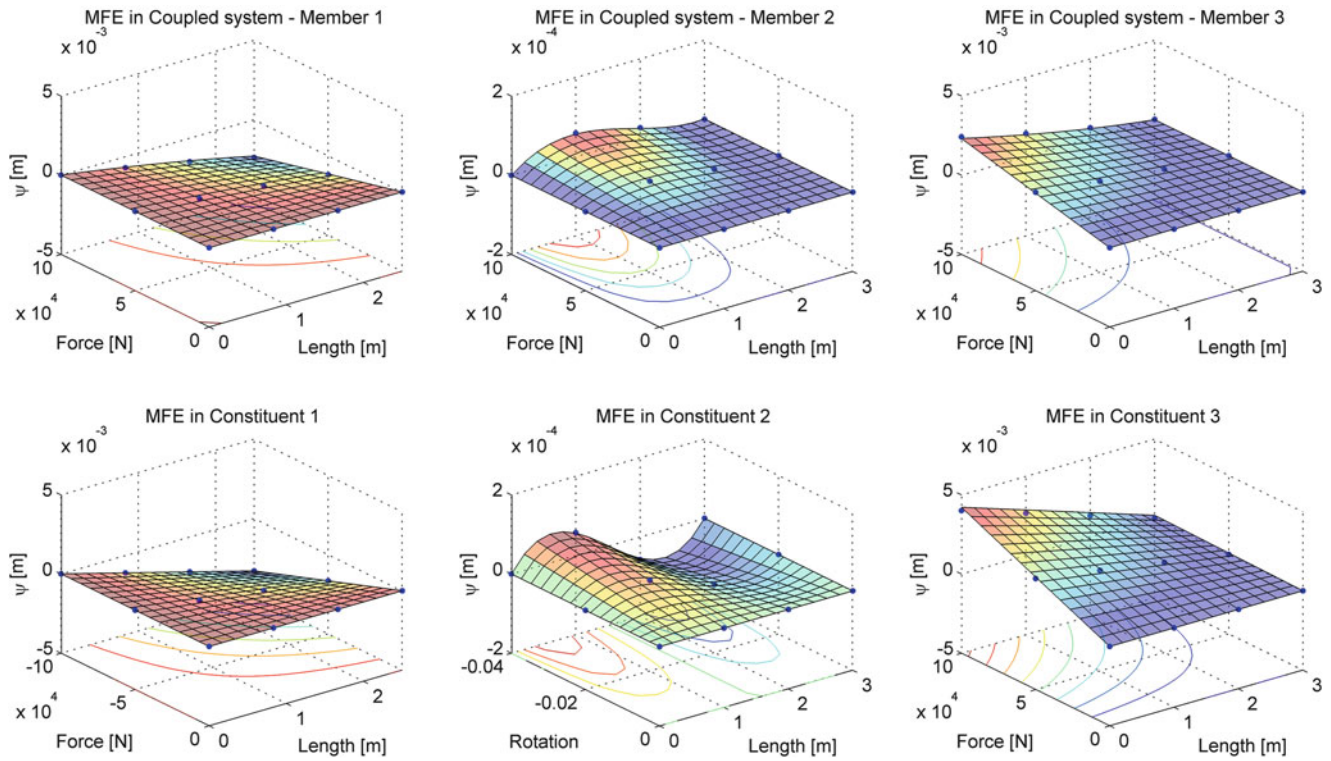


Fig. 6.4 Trained MFE of the coupled system and constituents

Table 6.2 Code prioritization metric (CPM)

Description (unit)	Substructure 1	Substructure 2	Substructure 3
Model form error index (MI^N)	1.0000	0.7617	0.6944
Sensitivity index (R^2)	0.7089	0.0947	0.1964
Cost index (CI^N)	1.0000	1.0000	1.0000
Code prioritization metric (CPM)	0.7089	0.0721	0.1364

the calibration parameters are taken as random numbers within the range of $\pm 25\%$ of the exact values provided in Table 6.1, and the initial values of the nonphysical coefficients are considered to be zero. The threshold value of the optimization process is set at 10^{-7} . Through the optimization, we can simultaneously calculate the values of calibration parameters and nonphysical coefficients of trained MFEs.

6.3.3 Prioritization of the Constituents

Synthetic experimental data is used to calibrate the Young's modulus of the member materials and also train the MFEs. The calibrated values of Young's modulus for members 1, 2, and 3 are 200.00×10^9 , 200.33×10^9 , and 199.35×10^9 , respectively. The trained MFEs are provided in Fig. 6.4. As shown, the values of the MFEs, where there is no imposed force or deformation, are equal to zero. By increasing the absolute value of the imposed forces and deformations, the absolute values of the MFEs increase.

The MFE indices of the constituents are calculated and provided in Table 6.2. The results show that the ratio of the error in constituent 1 is higher than in constituents 2 and 3. The sensitivity analyses of the constituents are conducted, and the results are provided in Table 6.2. The MFE of the coupled system is more sensitive to variation of the MFEs of constituent 1 than of constituents 2 and 3. In this case study, the improvement cost of all three constituents is assumed to be equal. Thus, the values of the cost indices of the constituents are equal to one. The values of the CPM for the three constituents are calculated and provided in Table 6.2. The results show that constituent 1 should have a higher priority for code development efforts than constituents 2 and 3 (Fig. 6.5).

Fig. 6.5 MFE, sensitivity, and cost indices in the constituents

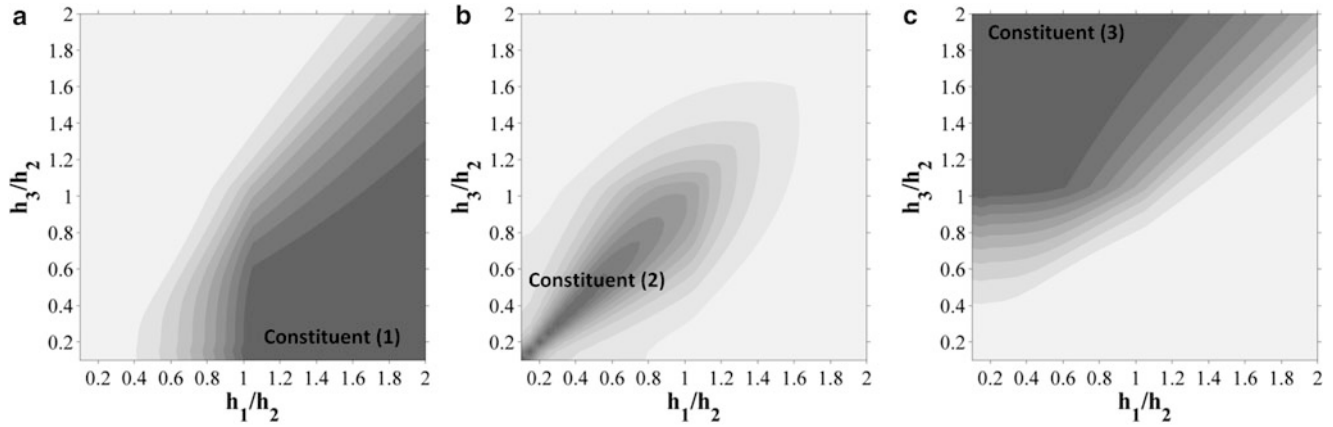
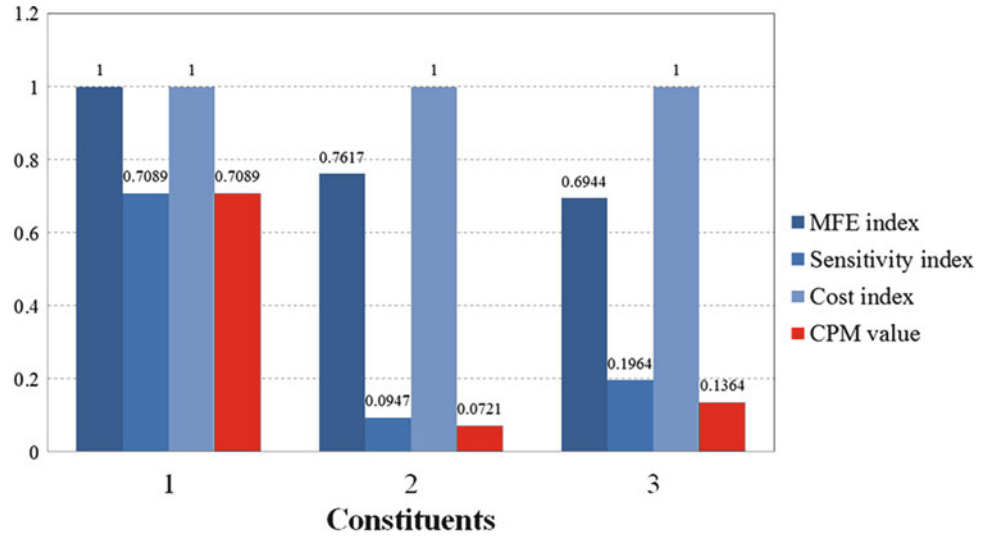


Fig. 6.6 Value of the CPM versus relative thickness of the members. (a) CPM in constituent (1), (b) CPM in constituent (2), (c) CPM in constituent (3)

6.3.4 Effect of the Thickness of the Members on Prioritization of the Constituents

Any parameter that influences the MFE of the constituents or coupled system can change the CPM value and code improvement priority. Herein, the effect of the thickness of cross sections (h in Fig. 6.2) on the CPM value of the constituents is studied. The lengths of all the members are considered to be 300 cm. According to Eq. (6.10), variation of the thickness of the cross sections (h) will change the shear correction factors of the cross sections. The CPM values of the three constituents are calculated based on various h values in the portal frame; the results are shown in Fig. 6.6. In Fig. 6.6a-c, the x and y axes represent the relative thickness of the members. Dark regions represent a higher priority, and bright regions represent a lower priority for code improvement. For instance, in Fig. 6.6a, by increasing the thickness of member (1), the priority of that member for development activity increases. Similarly, a trend can be observed for member (3) in Fig. 6.6c, where increasing the thickness of member (3) increases the priority of that member for development activity. Increasing the thickness of member (2) will increase the priority of constituent (2) for development activity (Fig. 6.6b). The source of MFE in this case study is shear deformation. These results confirm that by increasing the thickness of a member (leading to reduction of length/thickness), the MFE and sensitivity of that constituent increases and brings higher priority for that constituent for improvement activity. This confirms success of the defined CPM value in selection of the critical constituents. The results of Fig. 6.6 are combined to obtain Fig. 6.7. In Fig. 6.7a-c, each region represents which constituent should be improved. The three steps of development are provided in Fig. 6.7. Figure 6.8 is a summarized form of Fig. 6.7a-c, representing the order of all three constituents.

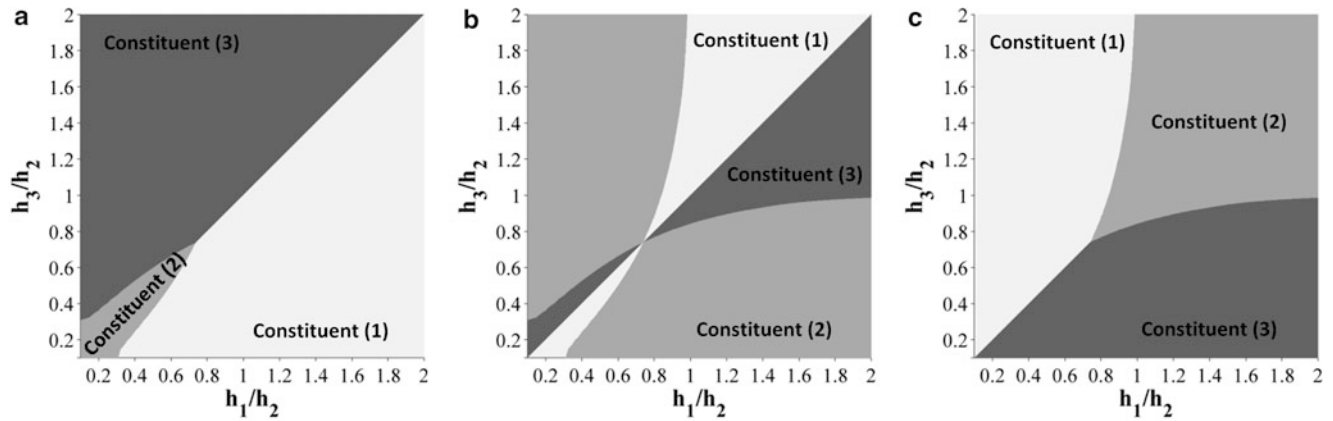
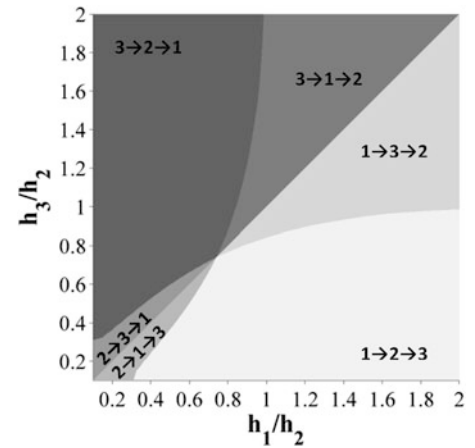


Fig. 6.7 Priority of code development based on member thickness ratio. (a) First priority, (b) second priority, (c) third priority

Fig. 6.8 Order of the code development as a function of member thickness ratio



6.4 Conclusion

This research proposes a new prioritization metric to aid in resource allocation for code improvement in the constituents of complex systems. The proposed metric incorporates three major and influential factors: (i) the ratio of the MFE to the simulation response in each constituent, (ii) the sensitivity of the MFE of the coupled system to the MFE of the constituents, and (iii) the development cost of the constituents. Coupling and uncertainty quantification problems are solved using optimization techniques. Separate-effect and integral-effect experiments are used to train the MFEs of the constituents and the coupled system simultaneously. The MFEs of the constituents are trained using polynomial surfaces. The proposed prioritization metric is successfully applied to a proof-of-concept structural frame. Analysis of the frame is achieved through coupling of the three substructures, and the CPM is used to rank the three constituents based on the amount of MFE, the sensitivity to the coupled system, and the cost associated with making improvements.

According to the definition of the MFE and sensitivity indices, these values are dependent on the value of calibration and other input parameters. Therefore, the value of the CPM will change by changing each of these parameters. This implies that other than the governing equation and solution algorithm of the constituents, the input values will also affect the CPM of the constituents. For instance, in the case study discussed in this manuscript, governing equations for solving all constituents are similar; however, based on their input parameters (such as dimensions) and the role of each of the constituents in the coupled system (being beam or column), the code development priority of constituents will vary. Though they are using the same formulation, they have a different impact on the response of interest. Thus, this technique for prioritizing the constituents is plausible only for specific input parameters, and changing these parameters may change the priority of the constituents. For instance, changing the dimensions, materials, and other input data will change the CPM value of the constituents.

In some cases, code development efforts or the experimental campaigns are ongoing processes. In such cases, to increase the efficiency of the proposed constituent ranking process, the new experimental data and earlier development activities should be considered to obtain the most updated results.

One of the challenges of any study in the context of code development is realistic cost estimation of the constituents. Even though we simply assumed that the code developer could predict the required time for code improvement activity; due to the complexity of the constituents, precise estimations will be very difficult.

The other weakness of this study is the computational cost for calculating MFEs in the constituents and in the coupled system. This issue is more problematic while conducting sensitivity analysis. The large number of non-physical coefficients in the MFEs has an immense impact on the number of decision variables in the optimization problem and computational cost. The number of non-physical coefficients for all the trained functions is dependent on the order of the control parameters, and increasing the order will increase the number of non-physical coefficients. Also, increasing the number of control parameters will also increase the number of independent variables in the polynomials and number of non-physical coefficients. This shows that increasing the order of control parameters or the number of control parameters will increase the number of decision variables of the optimization procedure which will increase the computational cost of the algorithm. Limiting these variables can go a long way to reducing the computational cost of the procedure.

References

- Lieber M, Wolke R (2008) Optimizing the coupling in parallel air quality model systems. *Environ Model Softw* 23:235–243
- Bunya S, Dietrich JC, Westerink JJ, Ebersole BA, Smith JM, Atkinson JH, Jensen R, Resio DT, Luettich RA, Dawson C, Cardone VJ, Cox AT, Powell MD, Westerink HJ, Roberts HJ (2010) A high resolution coupled riverine flow, tide, wind, wind wave and storm surge model for Southern Louisiana and Mississippi: part I—model development and validation. *Mon Weather Rev* 138:345–377
- Dietrich JC, Bunya S, Westerink JJ, Ebersole BA, Smith JM, Atkinson JH, Jensen R, Resio DT, Luettich RA, Dawson C, Cardone VJ, Cox AT, Powell MD, Westerink HJ, Roberts HJ (2010) A high-resolution coupled riverine flow, tide, wind, wind wave, and storm surge model for Southern Louisiana and Mississippi. Part II: synoptic description and analysis of hurricanes Katrina and Rita. *Mon Weather Rev* 138:378–404
- Schneidewind NF (1991) Setting maintenance quality objectives and prioritizing maintenance work by using quality metrics. *Proceedings of the conference on software maintenance, Sorrento, Italy*, pp 240–249
- Menzies T, Greenwald J, Frank A (2007) Data mining static code attributes to learn defect predictors. *IEEE Trans Softw Eng* 33(1):2–13
- Turhan B, Kocak G, Bener A (2009) Data mining source code for locating software bugs: A case study in telecommunication industry. *Expert Syst Appl* 36:9986–9990
- Melouk SH, Raja U, Keskin BB (2010) Managing resource allocation and task prioritization decisions in large scale virtual collaborative development projects. *Inf Resour Manage J* 23(2):53–76
- Ray M, Kumawat KL, Mohapatra DP (2011) Source code prioritization using forward slicing for exposing critical elements in a program. *J Comput Sci Technol* 26(2):314–327
- Alvin KF, Reese GM (2000) A plan for structural dynamics code and model verification and validation. *Proceedings of the 18th international modal analysis conference—IMAC XVIII, San Antonio TX*, pp 342–348
- Kumar M, Ghoniem AF (2012) Multiphysics simulations of entrained flow gasification. Part I: validating the nonreacting flow solver and the particle turbulent dispersion model. *Energy Fuel* 26:451–463
- Kumar M, Ghoniem AF (2012) Multiphysics simulations of entrained flow gasification. Part II: constructing and validating the overall model. *Energy Fuel* 26:464–479
- Döscher R, Willén U, Jones C, Rutgersson A, Meier HEM, Hansson U, Graham LP (2002) The development of the regional coupled ocean-atmosphere model RCAO. *Boreal Environ Res* 7:183–192
- Li Y, Kinzelbach W, Zhou J, Cheng GD, Li X (2012) Modelling irrigated maize with a combination of coupled-model simulation and uncertainty analysis, in the northwest of China. *Hydrol Earth Syst Sci* 16:1465–1480
- Terejanu G, Oliver T, Simmons C (2011) Application of predictive model selection to coupled models. *Proceedings of the world congress on engineering and computer science 2011, vol II, WCECS 2011, San Francisco, USA*
- Hegenderfer J, Atamturktur S (2013) Prioritization of code development efforts in partitioned analysis. *Comput-Aided Civ Inf Eng* 28(4):289–306
- Atamturktur S, Farajpour I (2012) Prioritization of code development efforts in partitioned analysis of coupled systems. *Struct Eng Mech* (under review)
- Matthies HG, Niekamp R, Steindorf J (2006) Algorithms for strong coupling procedures. *Comput Methods Appl Mech Eng* 195:2028–2049
- Matthies HG, Steindorf J (2002) Fully coupled fluid-structure interaction using weak coupling. *Proc Appl Math Mech* 1(1):37–38
- Matthies HG, Steindorf J (2003) Partitioned strong coupling algorithms for fluid-structure interaction. *Comput Struct* 81:805–812
- Fernandez MA, Moubachir M (2005) A Newton method using exact jacobians for solving fluid–structure coupling. *Comput Struct* 83:127–142
- Joosten MM, Dettmer WG, Peric D (2009) Analysis of the block Gauss–Seidel solution procedure for a strongly coupled model problem with reference to fluid–structure interaction. *Int J Numer Meth Eng* 78:757–778
- Farajpour I, Atamturktur S (2012) Optimization-based strong coupling procedure for partitioned analysis. *ASCE J Comput Civ Eng* 26(5):648–660
- Higdon D, Gattiker J, Williams B, Rightley M (2007) Computer model validation using high-dimensional outputs. In: Bernardo J, Bayarri MJ, Berger JO, Dawid AP, Heckerman D, Smith AFM, West M (eds) *Bayesian statistics, vol 8*. Oxford University Press, Oxford, UK
- Higdon D, Nakhleh C, Gattiker J, Williams B (2008) A Bayesian calibration approach to the thermal problem. *Comput Method Appl Mech Eng* 197(29–32):2431–2441
- Atkinson K, Han W (2009) *Theoretical numerical analysis: a functional analysis framework*, 3rd edn. Springer, New York, pp 116–118
- Mastroianni G, Milovanovic GV (2008) *Interpolation processes: basic theory and applications*. Springer, Berlin, pp 3–4

27. Farajpour I, Atamturktur S (2012) Partitioned analysis of coupled numerical models considering imprecise parameters and inexact models. *ASCE J Comput Civil Eng*. doi:10.1061/(ASCE)CP.1943-5487.0000253
28. Farajpour I, Atamturktur S (2012) Error and uncertainty analysis of inexact and imprecise computer models. *ASCE J Comput Civil Eng*. doi:10.1061/(ASCE)CP.1943-5487.0000233
29. Casella G, Berger RL (2002) *Statistical inference*, 2nd edn. Duxbury Press, Belmont, CA, p 556
30. Janke SJ, Tinsley FC (2005) *Introduction to linear models and statistical inference*. Wiley, Hoboken, p 230
31. Leung H, Fan Z (2002) *Software cost estimation, vol II. Handbook of software engineering*. Department of Computing, The Hong Kong Polytechnic University, Hong Kong
32. Jørgensen M, Shepperd M (2007) A systematic review of software development cost estimation studies. *IEEE Trans Softw Eng* 33(1):33–53
33. Malik AA, Boehm B (2011) Quantifying requirements elaboration to improve early software cost estimation. *Inform Sci* 181:2747–2760
34. Sobieszczanski-Sobieski J, James BB, Dovi AR (1985) Structural optimization by multilevel decomposition. *AIAA J* 23(11):1775–1782
35. Vadde S, Krishnamachari RS, Allen JK, Mistree F (1991) The Bayesian compromise decision support problem for hierarchical design involving uncertainty. *Advances in Design Automation*, G.A. Gabriele, ed., vol. 32-1, ASME, New York, pp 209–218
36. Allen JK, Mistree F (1993) Design of hierarchical and non-hierarchical systems. *Proceedings of the NSF grantees conference on design and manufacturing systems research*, Charlotte, NC, pp 443–447
37. Hartmann F, Katz C (2007) *Structural analysis with finite elements*, 2nd edn. Springer, Berlin, p 295
38. Pilkey WD (2005) *Formulas for stress, strain, and structural matrices*, 2nd edn. Wiley, Hoboken, NJ, p 29
39. Lagarias JC, Reeds JA, Wright MH, Wright PE (1998) Convergence properties of the Nelder-Mead simplex method in low dimensions. *SIAM J Optim* 9(1):112–147

Chapter 7

Numerical Substructuring Methods in Finite Element Analysis

Joshua Mendoza and A. Keith Miller

Abstract The proper coupling of substructures into system assemblies is a key part of achieving acceptable model fidelity and accuracy. Component model reduction leads to the system becoming stiffer and thus residual flexibilities at load and boundary interfaces must be included in the system-level model. The condition of the substructure system matrices can offer insight into properly re-conditioning the system to reduce the residual error associated with model reduction. Numerical experimentation will demonstrate how different substructure matrix conditioning affects the entire system assembly and solution. By taking the simulated results of the substructure, corrupting the mode shapes, and assembling the new model, the differences in solutions can be evaluated and then compared. The methods of numerical experimentation are presented and their results will be used to gain insight into ways to recondition the matrices associated with the residual structural model to minimize model errors. The methods of reconditioning the matrices associated with the residual structural model are being developed to apply to system assemblies where some of the components are defined entirely from experimental data and where it is difficult to pre-assess the condition of the parameters of the matrices associated with experimentally defined components.

Keywords Substructuring • Component mode synthesis • Model reduction • Numerical experimentation

7.1 Introduction

Finite element modeling and analysis (FEM & FEA) are essential tools for design and analysis of designs. FEA programs are tools that allow the user to solve for a system's behavior under different loading and boundary conditions. Design analysis using FEMs can be a very efficient and effective tool on simple structures. Proper modeling techniques combined with intuitive modeling practices can yield highly accurate models with reasonable computational and time costs. However, large and complex structural systems often demand computationally intensive models that require a large investment of an analyst's time. Such large system models, although complex, are sometimes necessary for the proper analysis of vital systems. The aviation and aerospace industries are two examples of organizations that have a need for high fidelity full scale system models. There are several different methods of reducing the computational and time costs of FEMs. A currently practiced method is to perform analysis on substructures of the system which can undergo individual analysis. This is often beneficial when a specific response is sought, such as internal stress or acceleration levels. A problem with substructure assembly to system-level models is appropriately capturing the boundary conditions that actually exist in the field. The analysis of the substructures can be combined to approximate a solution for the entire assembly [1]. Two commercially available finite element codes, Ansys® (version 13.0) and Comsol® (version 4.2a), were evaluated for providing the analytical substructures and the simulated test data for the "experimental" modal component. Ultimately, the Comsol® code was chosen because of

This work was performed at New Mexico Tech and supported in part by the Federal Aviation Authority Center of Excellence for Commercial Space Transportation under Research Grant 293, 2012.

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the open and easily accessible interface with the Matlab[®] code. Matlab[®] is the code chosen do to the system assembly and the error and sensitivity evaluation. Eventually, code modifications are envisioned for either of these commercial codes, or other finite element packages to do the system assembly and dynamic analyses.

Another method to reduce computational and time costs is to conduct modal reduction on system-level models and/or substructure level models. The modal reduction disregards the elements that contribute least to the solution of the model. Once the substructure models are reduced they are then assembled into a complete system-level model which is often termed the residual structure. The system now has fewer degrees of freedom and repeated, computationally efficient analysis can be performed on the reduced order model.

A reduced order model will have inherent residual errors that must be mitigated. Some reduced-order modeling methods, such as the Craig-Bampton method which uses interface constraint modes, are currently accepted in industry and can yield accurate results. Another method of reducing computational requirements and time, while simultaneously increasing model accuracy, is to insert experimentally measured modal component data into a residual finite element model. Properly coupling physical test data into system models is currently an open area of research where little progress in the development of a commercially available robust model has been made. There are steps to begin the process of inserting physical data into models: first, the system is generally divided into multiple substructures; a substructure or group of substructures can then be chosen to be represented by their physical data; the system can then be assembled from the FEM components and the measured components. Precise physical testing methods must also be implemented in order to properly identify the dynamic properties of each substructure. There is a gap in the methods to precisely measure responses and extract system parameters from experimental tests [2]. If the physical data can be properly inserted into the model, the system model can be greatly improved. The physical data aids in driving the solution using the real properties. This method reduces computational time, and when performed properly, provides an accurate analysis of the system. The method is highly specialized and the codes tend to be limited in the systems that they can represent.

7.2 Numerical Experimentation

The goal of numerical experimentation is to determine the proper procedures for coupling reduced order physical system properties to global system FEMs. The goal is to determine the best way in which experimental data can be processed so that it can be properly implemented into the FEMs. There are many components that will need to be studied and evaluated in order to accomplish this main goal. The first step is to create a truth model and verify that model against experimental results. The next step is to substructure the truth model. One substructure will be used to simulate a modal test. The response of the simulation will be used to extract the modal parameters of the model. The extracted modal parameters from the simulated test will then be assembled with a computational FEM substructure to create the assembled system-level model. Results from the analysis of the assembled system will be compared to the truth model. Systematic adjustment of the modal parameters which define the “modal” component is performed and the newly assembled system responses are assessed against the truth model.

The first step in the research is to establish a truth model. The truth model is a high fidelity model that is a standard for comparison of experimental data and simulated data. The structure chosen for numerical experimentation is a simple beam with embedded concentrated masses, shown in Fig. 7.1. The truth model was built in both Ansys[®] and Comsol[®], but ultimately the choice was made to move forward with Comsol[®]. The eigen properties of the truth model of the beam were compared to available experimental results. The truth model was found using the eigenfrequency study in Comsol[®] under 3D Solid Mechanics [3, 4]. The truth model can then be used for comparison when experimental data is not available for the

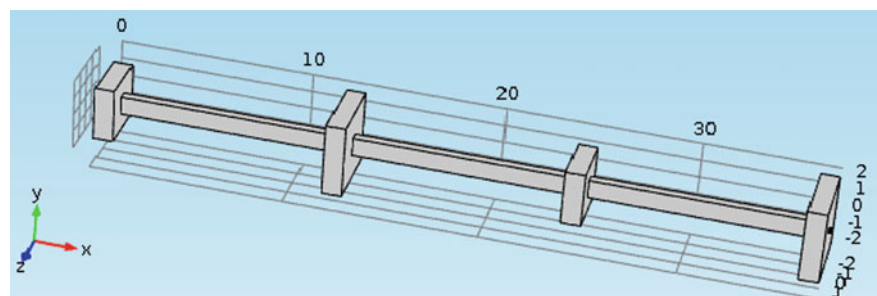


Fig. 7.1 Truth model consists of beam with masses

Fig. 7.2 Computational FEM model comprised of beam elements



Fig. 7.3 Computational model for substructure assembly

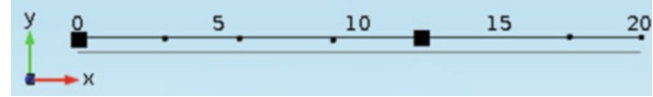


Fig. 7.4 Substructure of truth model to simulate modal test

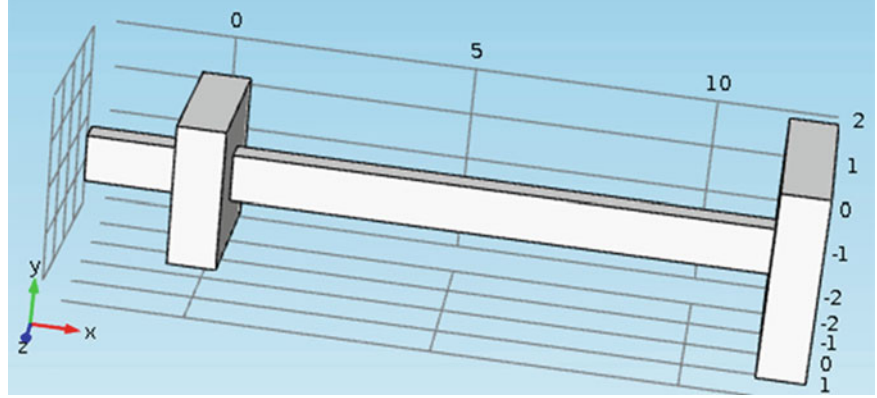
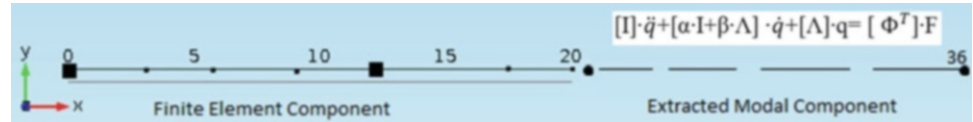


Fig. 7.5 FEM component assembled to extracted modal model



specific substructures. Truth models of each of the substructures were also created. Once the truth model was established, the finite element substructure was also modeled and the modeling of the experimentally derived modal substructure can begin.

To simplify the model and reduce the complexity, a computational model of the beam was created using beam elements as well, seen in Fig. 7.2. The computational model is modeled using beam physics in the Structural Mechanics Module in Comsol [4]. The computational model was significantly less complex and much faster to solve. A substructure of the computational model will be one component of the assembled beam and represent the finite element component. The substructured computational model will be referred to as Substructure A of the beam, seen in Fig. 7.3. The substructured computational model will serve as the FEM to which the modal parameters (extracted from the simulated modal test) will be coupled to. The computational model will be much faster and easier to experiment with, as well as easier to assemble due to its reduced complexity. The other substructure component will consist of the model built from simulated modal test data.

A substructure of the truth model, seen in Fig. 7.4, will be used to simulate modal tests. The substructure derived from modal parameters will be Substructure B. The truth model simulations serve as a stand-in for experimental hardware responses. Simulated data were chosen for numerical experimentation so that experimental noise and measurement errors could be avoided during early experimentation. The modal test simulation consists of using a non-ideal (real) impulse and measuring the acceleration response at various points. Several points were chosen to be loaded with a simulated impulse in the x, y, and z directions. A fast Fourier transform (FFT) analysis of the accelerometer data demonstrated that the simulated system was behaving as expected. The peak frequencies matched the natural frequencies from the experimental setup. The next phase will be to use curve fitting techniques on the frequency response functions to determine modal parameters. The extracted model can then be assembled to the FEM substructure and the system-level model can be compared to the truth model.

Once the modal parameters have been extracted from the simulated test model, the computational model and the modally derived model can be assembled. The assembled system is depicted in Fig. 7.5. The system will be assembled by coupling the computational model (Substructure A) to the modal derived component (Substructure B). The modal system will be found using the equation in the image, where; I is the identity matrix, q is the principal coordinate system, α and β are the proportional damping coefficients, Λ is the diagonal matrix consisting of the system's modal frequencies, Φ is the modal matrix, and F is the force on the system. Initially all modes and mode shapes will be kept and the complete simulated

Table 7.1 Natural frequencies of truth model

Method	1st Mode frequency (Hz)	3rd Mode frequency (Hz)
Experimental	32.3	89.8
Truth model	32.5	91.8
Percent error	0.61	2.22

Table 7.2 Natural frequencies of beam FEM

Method	1st Mode frequency (Hz)	3rd Mode frequency (Hz)
Computational model	30.4	85.9
Matlab model	30.9	85.3
Percent error	1.6	0.7

Table 7.3 Natural frequencies comparison between methods and structures

	Experimental	Truth model	Computational model	Simulated results
Full beamFrequency 1 (Hz)	32.3	32.8	30.4	32
Full beamFrequency 2 (Hz)	89.8	92.3	86	92
Substructure AFrequency 1 (Hz)	135	130	130	N/A
Substructure AFrequency 2 (Hz)	385	378	361	N/A
Substructure BFrequency 1 (Hz)	N/A	242	N/A	237
Substructure BFrequency 2 (Hz)	N/A	590	N/A	584

substructure will be assembled to the finite element model. The initial assembled system should, barring numerical errors, match exactly with the truth models. Corruption of the modal parameters and comparison to the condition of the assembled system should offer insight into a relationship and possible corrective measures.

7.3 Initial Results

A solids model of the entire, assembled, beam with distributed mass was developed in Comsol®. The solids model served as a truth model for comparison of experimental systems. The eigenvalues were compared to the natural frequencies determined from a Sandia report [5]. The results of the natural frequency comparison can be seen in Table 7.1. The resulting error was less than 5% and deemed acceptable. The difference seen in the frequencies was attributed to assumed material properties. To simplify the initial analyses and assembly procedures, a model using beam elements was established and eigenvalues were compared to a solids model developed in Comsol®. The natural frequencies of the beam model had greater error when compared to the experimental than the frequencies of the solids model/experimental data. This is due to the driving functions of the chosen elements as well as ideal point mass vs. real masses. Comparisons were also made of eigenvalues found from Matlab's "eig" function and the Comsol beam model and judged to be acceptable. The mass and stiffness matrices were extracted into Matlab using Comsol's "LiveLink" with Matlab feature. Results produced by Matlab's eig function correlate very well with Comsol's eigenfrequency studies. The errors between Matlab and Comsol can be found in Table 7.2. The eigen response of the computational model was verified against the truth model and was shown to have less than 5% error in modal frequencies. The next step for results was to compare the responses of the experimental data, truth models, computational models, and simulated responses.

A modal test was simulated using the entire truth model. An FFT of the time responses of the accelerations at all the points showed dominant frequencies around 32 Hz and 92 Hz. The dominant frequencies from the first rounds of simulated data showed a strong agreement with the truth model and experimental data for the entire, assembled, beam. The agreement between the simulated model and the truth and experimental models paved the way for the simulations of the substructured model. The next phase was to develop the substructured models and simulations. Substructure A is a computational model using beam elements.

Substructure A will comprise the finite element aspect of the numerical experimentation and substructure B will be derived from simulated experimental data. Table 7.3 below shows the initial results of the models, simulations, and experimental data. For the entire beam, the models and simulated data all show a strong agreement with the experimental results. In general the truth model agreed with the experimental data that was obtained, thus allowing the truth models to be used for comparisons against the computational models and simulated results. Substructure B will be the modally derived component.

The dominant frequencies, determined from an FFT of the acceleration response, from the substructure B modal simulations were checked against the natural frequencies of the truth model; they were within 2.5% and thus the response data was deemed suitable for parameter extraction.

7.4 Future Work

Extracting modal parameters from substructures and assembling them into system-level models is the end goal of the research. The future work will be comprised of many of the same components described in the numerical experimentation section. The first step is to collect simulated modal responses from a substructure of the truth models. Then the parameter extraction routines will be applied to the simulated data. The extracted parameters will then be assembled to the finite element substructure to build the system-level model. The assembled model will be analyzed and the results will be compared to the truth models.

Numerical experimentation and simulation will continue and different methods will be developed to corrupt and correct system-level models. Assembly methods will need to be examined. Experimental modal testing will be conducted and the first round parameter estimation will be compared to the truth models. Techniques for handling noisy or corrupted modal parameters will be implemented. The corrected modal parameters will then be assembled to the FEM. The global system will be evaluated and compared to the truth model. A completely new structure will be tested. The parameters will be estimated from the responses. The new structure's parameters will be uploaded into a FEM and the results of the analysis will be compared to published experimental data on the system.

Modal testing on a physical component will be conducted. Using the methods developed during the numerical experimentation, the modal parameters will be extracted from the physical test data. The experimentally derived substructure will be assembled into the system-level model. Analysis will be performed on the system-level model and compared to published work on the same structure.

The final component will be the assembly of a computational package that allows for faster and smoother implementation of experimentally derived components into finite element packages.

References

1. Craig RR Jr (1981) Structural dynamics: an introduction to computer methods. Wiley, New York
2. de Klerk D et al (2008) General framework for dynamic substructuring: history, review, and classification of techniques. AIAA J 46(5):1169–1181
3. Comsol Multiphysics User's Guide Version 4.2a [PDF] (2011) Comsol, CM020002
4. Structural Mechanics Module User's Guide V 4.2a [PDF File] (2011) Comsol, CM021101
5. Martinez DR et al (1984) Combined experimental/analytical modeling using component mode synthesis. Sandia National Labs, SAND83-1889

Chapter 8

Substituting Internal Forces for Blocked Forces or Free Interface Displacements in Substructured Simulations

P.L.C. van der Valk and D.J. Rixen

Abstract An efficient method for computing responses of large and complex structures is Dynamic Substructuring. In Dynamic Substructuring methods one decomposes the large and complex structure in several substructures, which are individually modeled. The obtained knowledge of individual subsystems, allows one to create an efficient and compact model of the global system. When the forces can not be measured in the substructure from where the excitation originates, it can however be characterized on the interface either as a blocked force or a free interface motion. In this contribution the idea of blocked forces and equivalent free interface displacement, originally developed in the frequency domain, is extended to the time domain. The method transforms internal force and displacement excitation of linear substructures, into a set of equivalent forces or displacements on the boundary degrees of freedom. The method has a number of advantages; Firstly, it simplifies the reduction of force loaded structures, as it requires no special treatment for the forces acting on the internal parts of the structure. Also, it allows for groups working on different components to couple the dynamics by only sharing the interface excitations and (reduced) component models. Finally, it should be noted that using time-domain approaches compared to the previously published frequency-domain approaches for equivalent forces and displacements, allows also considering coupling with non-linear receiving substructures. It will be analytically proven that for all other substructures the exact same response will be found. In addition it will be shown how to recover the correct response for the force loaded substructure in the case the internal forces are known. The methods will be demonstrated using the model of an offshore wind turbine that is loaded with (linearized) wave forces on its support structure.

Keywords Structural dynamics • Substructuring • Internal forces • Blocked forces • Free interface displacements • Time domain

Nomenclature

A	– Block matrix related to Newmark scheme	β	– Newmark parameter
B	– Compatibility matrix (Boolean)	γ	– Newmark parameter
C	– Damping matrix	λ	– Vector of Lagrange multipliers
K	– Stiffness matrix	$\star^{(s)}$	– Pertaining to substructure s
M	– Mass matrix	$\star_n$	– Pertaining to time step n
N	– Block matrix related to Newmark scheme	$\star_{[i]}$	– Denotes internal DoF
Q	– Block matrix related to Newmark scheme	$\star_{[b]}$	– Denotes boundary DoF
Z	– Dynamic stiffness matrix	$\tilde{\star}$	– Equivalent or Alternative to
f	– External force vector	CMS	– Component Mode Synthesis
q	– Array of degrees of freedom	DoF	– Degrees of Freedom
u	– Array of degrees of freedom	IBS	– Impulse Based Substructuring

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8.1 Introduction

Dynamic substructuring denotes a family of methods to obtain the structural dynamic behavior of large and/or complex structures by dividing them into several smaller, simpler substructures (or components) of which the dynamic behavior is generally easier to determine. The knowledge of the component dynamics is subsequently used to reduce the complexity of the substructure by either approximating or pre-computing its dynamic behavior [2]. A problem often encountered when applying dynamic substructuring (DS) is to properly determine and represent forces acting on the internal degrees of freedom (DoF) of the structure. If one uses experimental models, these forces are often not measurable as one can only measure their effects as accelerations at the discrete measurement points, or as reaction forces if the substructure is constrained in a test stand.

8.1.1 *Applying Equivalent Loading in Vibration Transmission Analysis in the Frequency Domain*

If one would measure the constraint forces or displacements at the interfaces to other substructures, one in fact measures the excitation that the substructure on the “source” side applies on the other substructures (the “receivers”). It is shown that these constraint forces (or blocked forces) can be used, for instance, in a transfer path analysis to determine the effect of running gears on the noise in a vehicle [3]. On the other hand, one can also use the free interface displacements in a frequency based substructuring (FBS) analysis to determine the effect of factory floor vibrations on high precision machinery [11]. Hence, by using these methods there is no need to determine the true internal forces of the source structure, which is practically impossible.

Note that also for the numerical substructures in the FBS analysis one can encounter difficulties with forces acting on the internal degrees of freedom (DoF). As we will probably have a time series of loading available, the problem is not in obtaining these force, but in applying them in a substructuring analysis. If a large part of the DoF are force loaded, one has to generate the frequency response functions, describing the relationship between all these DoF for the entire frequency range of interest. This will result in a tremendous computational effort in order to compute and store these large 3-dimensional matrices with FRFs. If one could also replace all the internal forces by an equivalent loading on the interfaces, only the FRFs of the relatively few DoF on the interface are required.

8.1.2 *Applying Equivalent Loading in Vibration Transmission Analysis in the Time Domain*

As vibration transmission analysis in the frequency domain requires all substructures to be linear, one could encounter cases where one or more substructures are in fact nonlinear, hence one will have to perform the analysis in the time domain. The problem of forces on internal DoF is also encountered in substructuring methods in the time domain, such as component mode synthesis (CMS) [1] and the more recent impulse based substructuring (IBS) [13], used to reduce the computational effort of the linear substructures. Here, one usually distinguishes between internal DoF and boundary DoF, which are the more interesting ones, as they usually serve as the interface to other substructures or act as an input for external forces. Hence, when reducing a substructure, one generally wants to retain the boundary DoF and capture the internal DoF in either a modal basis (in the case of CMS) or implicitly include their behavior in an impulse response function (in the case of IBS). These methods are able to significantly reduce the number of DoF, while leading to accurate simulation results, if the external forces are only working on the boundary DoF. If a large part of the structure is force loaded, one can no longer include all these DoF in the list of boundary DoF, as this leads to unacceptably large (reduced) substructure models, which cripples the effectiveness of the CMS and IBS methods. On the other hand, if one includes the (large) set of force loaded DoF in the internal DoF, one has no guaranty that the reduction basis captures the external forces correctly.

Methods have been proposed to cope with internal forces in both the CMS methods, as well as in the IBS method. For the CMS methods, one enriches the reduction basis with so called Modal Truncation Vectors (MTVs), which enrich the reduction space with static deformation shapes, that result from the external loading [5]. An effective method for finding an optimal set of MTVs, which uses a proper orthogonal decomposition (POD) to find the dominant *force modes* from a time series of external forces, was proposed recently [9]. In the IBS method, one can also use the POD analysis to find the dominant force modes and from these, compute the impulse response functions due to a distributed load, as was shown in [12]. As, in these methods, the applied load is projected on a spatial reduction basis, this leads to an approximation of the applied load.

Therefore it can be imagined that using equivalent interface forces or displacements will also have a number of advantages in the time domain. Firstly, it simplifies the reduction of the dynamics of the source substructure since the internal excitation location need no specific treatment and no load case dependent reduction basis' have to be build. The only additional step that is required is to obtain the equivalent interface source (force or gap) by solving a dynamic problem of the source substructure alone, in its non-reduced form, as will be shown in this paper. Secondly, the response of the receiver substructures, which can be nonlinear, to this equivalent loading is exact with respect to the original loading. Finally, if different project teams or companies are working on different components, it allows them to couple the dynamics by sharing only the interface excitations and (reduced) component substructures.

8.1.3 Paper Outline

Firstly, in section 8.2 the existing force transformation methods in frequency domain, which are based on the same idea as their time domain counterparts, are introduced. In section 8.3, the time domain method of using *equivalent interface forces* and the associated analytical proof is presented. Its dual counterpart, which uses an *equivalent interface gap*, is derived in section 8.4. Both methods are applied in section 8.5 on the numerical test case of an offshore wind turbine, supported by an offshore jacket structure, which is excited by sea waves. Finally, in section 8.6, the conclusions are presented.

8.2 Blocked Forces and Free Displacements Approaches for Vibration Transmission Analysis in the Frequency Domain

In order to solve the problem of unmeasurable internal forces for vibration transmission analysis, methods have been proposed that either utilize blocked forces at the interfaces or free interface displacements. Like most experimental techniques, these methods were developed for the frequency domain. As the goal of this work is to extend these methods to the time domain, which would allow for efficient handling of internal forces in numerical substructuring analysis, as was presented in the introduction (section 8.1). Therefore, the original frequency domain method will be introduced in this section. As mentioned in the introduction the example given in this paper (see section 8.5) will be that of an offshore wind turbine. The illustrations used for clarification throughout this paper will also be that of an offshore wind turbine supported by an offshore jacket structure, which is excited by wave forces.

8.2.1 Method Using Blocked Forces in the Frequency Domain

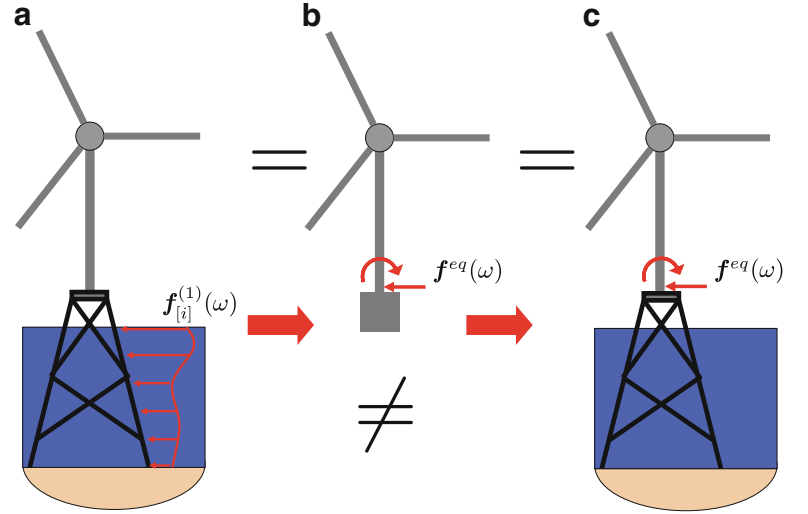
For the sake of simplicity, it is assumed here that the total structure consists of only two substructures, substructures 1, the jacket structure and substructure 2, the wind turbine, and that only substructure 1 is loaded on its internal DoF, as can be seen in Eq. (8.1) and Fig. 8.1a. Note however, that the developments are also valid for multiple structures and that all of them can have forces working on both the internal and boundary DoF. Here, the variables associated with the internal DoF are denoted by a subscript $[i]$ and the variables associated with the boundary DoF are denoted by a subscript $[b]$,

$$\begin{bmatrix} \mathbf{Z}_{[ii]}^{(S)} & \mathbf{Z}_{[ib]}^{(S)} & \mathbf{0} \\ \mathbf{Z}_{[bi]}^{(S)} & \mathbf{Z}_{[bb]}^{(S)} & \mathbf{Z}_{[bi]}^{(R)} \\ \mathbf{0} & \mathbf{Z}_{[ib]}^{(R)} & \mathbf{Z}_{[ii]}^{(R)} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{[i]}^{(S)} \\ \mathbf{u}_{[b]}^{(S)} \\ \mathbf{u}_{[i]}^{(R)} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{[i]}^{(S)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (8.1)$$

where, $\mathbf{Z}_{[bb]} = \mathbf{Z}_{[bb]}^{(S)} + \mathbf{Z}_{[bb]}^{(R)}$. By condensing substructure 1 (see Fig. 8.1b) on the interface, one obtains

$$\begin{bmatrix} \mathbf{Z}_{[bb]} - \mathbf{Z}_{[bi]}^{(S)} \mathbf{Z}_{[ii]}^{(S)-1} \mathbf{Z}_{[ib]}^{(S)} & \mathbf{Z}_{[bi]}^{(R)} \\ \mathbf{Z}_{[ib]}^{(R)} & \mathbf{Z}_{[ii]}^{(R)} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{[b]}^{(S)} \\ \mathbf{u}_{[i]}^{(R)} \end{bmatrix} = \begin{bmatrix} -\mathbf{Z}_{[bi]}^{(S)} \mathbf{Z}_{[ii]}^{(S)-1} \mathbf{f}_{[i]}^{(S)} \\ \mathbf{0} \end{bmatrix}, \quad (8.2)$$

Fig. 8.1 (a) Original load case;
(b) Condensation of substructure 1 on the interface;
(c) Re-expanding substructure 1



and it can be easily verified that one finds the correct response for both the interface DoF ($\mathbf{u}_{[b]}$) and $\mathbf{u}_{[i]}^{(R)}$. By only expanding the dynamic stiffness matrix $\mathbf{Z}^{(S)}$, but maintaining the internal forces in substructure 1 as condensed on the interface in Eq. (8.3) and Fig. 8.1c, we are back at the original structural problem, but now with a condensed force on the interface. Note that after the expansion, the set of DoF is also changed from $\mathbf{u}_{[i]}^{(S)}$ into $\tilde{\mathbf{u}}_{[i]}^{(S)}$.

$$\begin{bmatrix} \mathbf{Z}_{[ii]}^{(S)} & \mathbf{Z}_{[ib]}^{(S)} & \mathbf{0} \\ \mathbf{Z}_{[bi]}^{(S)} & \mathbf{Z}_{[bb]}^{(S)} & \mathbf{Z}_{[bi]}^{(R)} \\ \mathbf{0} & \mathbf{Z}_{[ib]}^{(R)} & \mathbf{Z}_{[ii]}^{(R)} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}_{[i]}^{(S)} \\ \mathbf{u}_{[b]}^{(R)} \\ \mathbf{u}_{[i]}^{(R)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{Z}_{[bi]}^{(S)} \mathbf{Z}_{[ii]}^{(S)-1} \mathbf{f}_{[i]}^{(S)} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{f}^{(eq)} \\ \mathbf{0} \end{bmatrix} \quad (8.3)$$

Note that to write Eq. (8.2) and define the equivalent interface force, we have used the fact that the internal forces are independent of the dynamic response and that substructure 1 (the source side) is linear. The other substructures (the receiving side) have been supposed to be linear as well since we use impedance formulations in the frequency domain. Nevertheless the concept of equivalent forces by itself does not require the receiving substructures to be linear as will be clear later when the time domain is discussed. If one would now compute the response from Eq. (8.1) and compare that to the response found from (8.3), one finds that the same response is obtained on the interface and for substructure 2, but that the response of substructure 1 is altered (as is visualized in Fig. 8.1), as can be seen in Eq (8.4).

$$\tilde{\mathbf{u}}_{[i]}^{(S)} = -\mathbf{Z}_{[ii]}^{(S)-1} \mathbf{Z}_{[ib]}^{(S)} \mathbf{u}_{[b]} = \mathbf{u}_{[i]}^{(S)} - \mathbf{Z}_{[ii]}^{(S)-1} \mathbf{f}_{[i]}^{(S)} \quad (8.4)$$

Obviously, if one is unable to determine or measure the internal forces of substructure 1, one is also unable to condense these to the interface, as is done in Eq. (8.2). But, if one is able to rigidly clamp the individual substructure at its interfaces, as is done in Eq. (8.5), one can measure the reaction forces $\mathbf{f}_{[b]}^{(S),con}$.

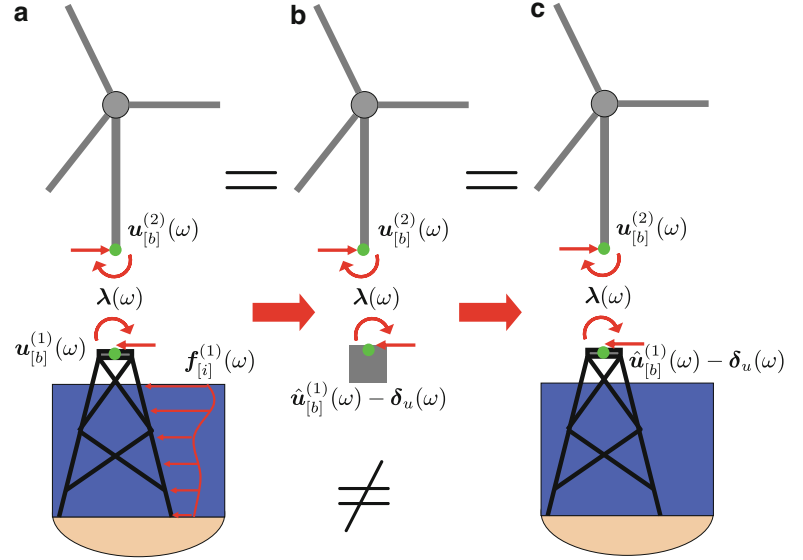
$$\begin{bmatrix} \mathbf{Z}_{[ii]}^{(S)} & \mathbf{Z}_{[ib]}^{(S)} \\ \mathbf{Z}_{[bi]}^{(S)} & \mathbf{Z}_{[bb]}^{(S)} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{[i]}^{(S),con} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{[i]}^{(S)} \\ \mathbf{f}_{[b]}^{(S),con} \end{bmatrix} \quad (8.5)$$

These reaction forces are then the exact opposite of the equivalent interface force, that was used in Eq. (8.3), as can be seen in Eq. (8.6).

$$\mathbf{f}_{[b]}^{(S),con} = \mathbf{Z}_{[bi]}^{(S)} \mathbf{Z}_{[ii]}^{(S)-1} \mathbf{f}_{[i]}^{(S)} = -\mathbf{f}^{(eq)} \quad (8.6)$$

As in reality, one is not able to create a measurement stand that is infinitely stiff the method has also been extended to account for a non-rigid test stands, as is presented in [3]. Note that this clamping strategy may not always be feasible in reality, as some structures may simply be too big to be restrained at the interface.

Fig. 8.2 (a) Original load case using dual assembly; (b) Condensation of substructure 1 on the interface forces, resulting in the applied gap (δ_u) in the compatibility; (c) Re-expanding substructure 1



8.2.2 Methods Using Free Interface Displacements in the Frequency Domain

As one can imagine, (rigidly) clamping any offshore structure, or in general any other immense structure, at the interfaces to other substructures is impossible. Therefore, another approach which could work here is to measure the free interface displacements instead of blocking forces [11]. Starting from the dual assembled counterpart of Eq. (8.1) and Fig. 8.2a, we have

$$\begin{bmatrix} \mathbf{Z}_{[ii]}^{(S)} & \mathbf{Z}_{[ib]}^{(S)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{Z}_{[bi]}^{(S)} & \mathbf{Z}_{[bb]}^{(S)} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_{[ii]}^{(R)} & \mathbf{Z}_{[ib]}^{(R)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_{[bi]}^{(R)} & \mathbf{Z}_{[bb]}^{(R)} & -\mathbf{I} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{[i]}^{(S)} \\ \mathbf{u}_{[b]}^{(S)} \\ \mathbf{u}_{[i]}^{(R)} \\ \mathbf{u}_{[b]}^{(R)} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{[i]}^{(S)} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (8.7)$$

where, $\boldsymbol{\lambda}$ are Lagrange multipliers that represent the connecting force intensities, such that $\boldsymbol{\lambda} = \mathbf{g}_{[b]}^{(S)} = -\mathbf{g}_{[b]}^{(R)}$. Note that the Lagrange multipliers satisfy the equilibrium condition *a priori* and that compatibility is explicitly enforced by the last line in Eq. (8.7), where it has been assumed for simplicity that the dofs are numbered identically on either side of the interface. By defining a signed Boolean operator

$$\mathbf{B} = [\mathbf{B}^{(S)} \ \mathbf{B}^{(R)}] = [\mathbf{0} \ \mathbf{I} \ \mathbf{0} \ -\mathbf{I}], \quad (8.8)$$

that locates the interface DoF ($\mathbf{u}_{[b]}^{(*)}$) in the full set of DoF ($\mathbf{u}^{(*)}$) and enforces the compatibility, Eq. (8.7) can be rewritten.

$$\begin{bmatrix} \mathbf{Z}^{(S)} & \mathbf{0} & \mathbf{B}^{(S)T} \\ \mathbf{0} & \mathbf{Z}^{(R)} & \mathbf{B}^{(R)T} \\ \mathbf{B}^{(S)} & \mathbf{B}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(S)} \\ \mathbf{u}^{(R)} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(S)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (8.9)$$

By condensing substructure 1 on the interface

$$\begin{bmatrix} \mathbf{Z}^{(R)} & \mathbf{B}^{(R)T} \\ \mathbf{B}^{(R)} & -\mathbf{B}^{(S)} \mathbf{Z}^{(S)^{-1}} \mathbf{B}^{(S)T} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(R)} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{B}^{(S)} \mathbf{Z}^{(S)^{-1}} \mathbf{f}^{(S)} \end{bmatrix}, \quad (8.10)$$

one can clearly see that the applied force is condensed to an imposed free interface displacement on the interface DoF, that can be interpreted as a gap (δ_u) or an error on the interface compatibility, as is shown in Fig. 8.2b. Re-expanding the left hand side of (8.10) again, gives us the original structural model again, where the loading is now applied as an equivalent interface gap (as shown in Fig. 8.2c). Again, note that after the expansion, the set of DoF is also changed from $\mathbf{u}^{(S)}$ into $\hat{\mathbf{u}}^{(S)}$.

$$\begin{bmatrix} \mathbf{Z}^{(S)} & \mathbf{0} & \mathbf{B}^{(S)T} \\ \mathbf{0} & \mathbf{Z}^{(R)} & \mathbf{B}^{(R)T} \\ \mathbf{B}^{(S)} & \mathbf{B}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}^{(S)} \\ \mathbf{u}^{(R)} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ -\mathbf{B}^{(S)} \mathbf{Z}^{(S)-1} \mathbf{f}^{(S)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \delta_u \end{bmatrix}. \quad (8.11)$$

Again, note that Eq. (8.10) and Eq. (8.11) are only valid if the applied loading is independent of the global dynamics of the system. If one now reviews the free interface response of substructure 1

$$\mathbf{Z}^{(S)} \mathbf{u}^{(S),free} = \mathbf{f}^{(S)}, \quad (8.12)$$

it is clear that by applying the (signed) Boolean matrix to locate the interface DoF ($\mathbf{u}_{[b]}^{(S)}$), one obtains the negative error on the interface compatibility condition.

$$\mathbf{u}_{[b]}^{(S),free} = \mathbf{B}^{(S)} \mathbf{Z}^{(S)-1} \mathbf{f}^{(S)} = -\delta_u, \quad (8.13)$$

Due to using the substructure Boolean operator $\mathbf{B}^{(S)}$, the sign of applied interface gap will always be consistent with Eq. (8.11).

8.3 Using Blocked Forces for Simulations in the Time Domain

The same ideas, as those introduced in section 8.2, can also be applied in the time domain. There is however, a fundamental difference here: whereas in the frequency domain the forced response at $\omega + \Delta\omega$ is not dependent on the response at ω , the response at $t + \Delta t$ is highly dependent on the response at t . Hence, in order to show that the method of applying a blocked force is allowed, one must prove that this is valid for $t = 0$ until $t = \infty$.

The idea of applying an equivalent interface force, instead of a set of internal forces, is very attractive when applying dynamic substructuring methods. As one is certain that the effect of the force is transmitted correctly to the rest of the structure, one needs not to worry about whether the reduction basis is able to capture the loading correctly. In addition, if one applies the modal truncation augmentation (MTA) methods, the size of the reduction basis will increase and it could result in the need for custom made superelements for different load cases, which is computationally expensive.

8.3.1 Newmark Time Integration

As a starting point we therefore take the linear equations of motion of a discretized dynamic component model s:

$$\mathbf{M}^{(s)} \ddot{\mathbf{u}}^{(s)}(t) + \mathbf{C}^{(s)} \dot{\mathbf{u}}^{(s)}(t) + \mathbf{K}^{(s)} \mathbf{u}^{(s)}(t) = \mathbf{f}^{(s)}(t), \quad (8.14)$$

Here, $\mathbf{M}^{(s)}$, $\mathbf{C}^{(s)}$ and $\mathbf{K}^{(s)}$ are the components mass, damping and stiffness matrix, respectively. For the sake of simplicity, the explicit time dependency and component notation are omitted in subsequent equations. In order to find the response of the system due to external loading, one has to apply time integration. The most common time integration method used in the field of structural dynamics, is the family of Newmark time integration methods [8]. In the Newmark scheme, a Taylor expansion is used to deduce the velocities and displacements of the next time station ($t_n = t_{n-1} + h$), according to:

$$\begin{aligned} \dot{\mathbf{u}}_n &= \dot{\mathbf{u}}_{n-1} + (1 - \gamma)h\ddot{\mathbf{u}}_{n-1} + \gamma h\ddot{\mathbf{u}}_n = \hat{\dot{\mathbf{u}}}_n + \gamma h\ddot{\mathbf{u}}_n \\ \mathbf{u}_n &= \mathbf{u}_{n-1} + h\dot{\mathbf{u}}_{n-1} + h^2\left(\frac{1}{2} - \beta\right)\ddot{\mathbf{u}}_{n-1} + h^2\beta\ddot{\mathbf{u}}_n = \hat{\mathbf{u}}_n + h^2\beta\ddot{\mathbf{u}}_n, \end{aligned} \quad (8.15)$$

where γ and β are parameters of the method and $\hat{\dot{\mathbf{u}}}_n$ and $\hat{\mathbf{u}}_n$ are referred to as the *predictors* and are depending on the displacements, velocities and accelerations of the previous time step. This time-stepping method can also be rewritten into a matrix vector operation (8.16).

$$\begin{bmatrix}
\mathbf{I} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{I} & \mathbf{0} \\
\mathbf{K} & \mathbf{C} & \mathbf{M} \\
-\mathbf{I} & -h\mathbf{I} & -h^2(\frac{1}{2} - \beta)\mathbf{I} & \mathbf{I} & \mathbf{0} & -h^2\beta\mathbf{I} \\
\mathbf{0} & -\mathbf{I} & -h(1 - \gamma)\mathbf{I} & \mathbf{0} & \mathbf{I} & -h\gamma\mathbf{I} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K} & \mathbf{C} & \mathbf{M} \\
-\mathbf{I} & -h\mathbf{I} & -h^2(\frac{1}{2} - \beta)\mathbf{I} & \mathbf{I} & \mathbf{0} & -h^2\beta\mathbf{I} \\
\mathbf{0} & -\mathbf{I} & -h(1 - \gamma)\mathbf{I} & \mathbf{0} & \mathbf{I} & -h\gamma\mathbf{I} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K} & \mathbf{C} & \mathbf{M} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_0 \\
\dot{\mathbf{u}}_0 \\
\ddot{\mathbf{u}}_0 \\
\mathbf{u}_1 \\
\dot{\mathbf{u}}_1 \\
\ddot{\mathbf{u}}_1 \\
\mathbf{u}_2 \\
\dot{\mathbf{u}}_2 \\
\ddot{\mathbf{u}}_2 \\
\vdots
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{u}_0 \\
\dot{\mathbf{u}}_0 \\
\mathbf{f}_0 \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{f}_1 \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{f}_2 \\
\vdots
\end{bmatrix}, \quad (8.16)$$

where $\mathbf{I}$ is the identity matrix. If one carefully breaks down (8.16), it can be seen that the first two lines can be read as constraints on the displacements and velocities, as they have to be identical to the given initial conditions. The third line is the equation of motion, with the only unknowns being the initial accelerations. The fourth and fifth lines, now represent the Newmark time discretization and thus make the displacements and velocities of the next time steps dependent on the past displacements, velocities and accelerations and on the current (unknown) accelerations. These unknown accelerations are computed using the equation of motion given in the sixth line. Putting Eq. (8.16) into a block form further simplifies the expression and results in:

$$\begin{bmatrix}
\mathbf{A} \\
\mathbf{N} & \mathbf{Q} \\
& \mathbf{N} & \mathbf{Q} \\
& & \ddots & \ddots
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_0 \\
\mathbf{q}_1 \\
\mathbf{q}_2 \\
\vdots
\end{bmatrix}
=
\begin{bmatrix}
\hat{\mathbf{f}}_0 \\
\hat{\mathbf{f}}_1 \\
\hat{\mathbf{f}}_2 \\
\vdots
\end{bmatrix}, \quad (8.17)$$

here,

$$\mathbf{q}_i = \begin{bmatrix} \mathbf{u}_i \\ \dot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_i \end{bmatrix}, \quad \hat{\mathbf{f}}_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{f}_i \end{bmatrix}, \quad \hat{\mathbf{f}}_0 = \begin{bmatrix} \mathbf{u}_0 \\ \dot{\mathbf{u}}_0 \\ \mathbf{f}_0 \end{bmatrix} \quad (8.18)$$

From Eq. (8.17), one can easily identify the repeating pattern of the same blocks ($\mathbf{Q}$ & $\mathbf{N}$), where the only exception is the block ($\mathbf{A}$) corresponding to the initial time (t_0). It should be noted that for conciseness it is assumed that the system is initially at rest ($\mathbf{u}_0 = \mathbf{0}$, $\dot{\mathbf{u}}_0 = \mathbf{0}$), but all the developments throughout this section are also valid for systems with non-zero initial conditions. The short hand notation introduced in this subsection will be used throughout this section in order to present the method of equivalent blocked forces in the time domain.

8.3.2 Condensing the Internal Force on the Boundary Degrees of Freedom

Similar to Eq. (8.2) in section 8.2.1, we want to condense internal forces onto the boundary DoF. Hence, Eq. (8.17) is split according to the internal and boundary degrees of freedom. Note that for clarity and conciseness, we have chosen to omit all other forces except the forces on the internal DoF, that need to be condensed to the interface.

$$\begin{bmatrix}
\mathbf{A}_{[ii]} & \mathbf{A}_{[ib]} & \mathbf{0} & \mathbf{0} \\
\mathbf{A}_{[bi]} & \mathbf{A}_{[bb]} & \mathbf{0} & \mathbf{0} \\
\mathbf{N}_{[ii]} & \mathbf{0} & \mathbf{Q}_{[ii]} & \mathbf{Q}_{[ib]} \\
\mathbf{0} & \mathbf{N}_{[bi]} & \mathbf{Q}_{[bi]} & \mathbf{Q}_{[bb]}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_{[i],0} \\
\mathbf{q}_{[b],0} \\
\mathbf{q}_{[i],1} \\
\mathbf{q}_{[b],1}
\end{bmatrix}
=
\begin{bmatrix}
\hat{\mathbf{f}}_{[i],0} \\
\mathbf{0} \\
\hat{\mathbf{f}}_{[i],1} \\
\mathbf{0}
\end{bmatrix}, \quad (8.19)$$

It should be noted however, that the method is not limited to cases with only loading on the internal DoF.

8.3.2.1 Response at the Initial Time Step

In order to compute the response of the initial time step, Eq. (8.20) is taken from Eq. (8.19).

$$\begin{bmatrix} \mathbf{A}_{[ii]} & \mathbf{A}_{[ib]} \\ \mathbf{A}_{[bi]} & \mathbf{A}_{[bb]} \end{bmatrix} \begin{bmatrix} \mathbf{q}_{[i],0} \\ \mathbf{q}_{[b],0} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_{[i],0} \\ \mathbf{0} \end{bmatrix}, \quad (8.20)$$

Note that Eq. (8.20) is referred to as being the reference problem at the initial time (t_0). Hence, the response can be found by solving the given set of equations, although this is not explicitly given here. A second approach can be taken by starting from Eq. (8.20) and condensing the internal forces onto the boundary DoF, which is similar to the step taken in the frequency domain in Eq. (8.2), after this one obtains:

$$\left(\mathbf{A}_{[bb]} - \mathbf{A}_{[bi]} \mathbf{A}_{[ii]}^{-1} \mathbf{A}_{[ib]} \right) \mathbf{q}_{[b],0} = \bar{\mathbf{A}}_{[bb]} \mathbf{q}_{[b],0} = -\mathbf{A}_{[bi]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} = \hat{\mathbf{f}}_{[b],0}^{eq}, \quad (8.21)$$

where we define $\bar{\mathbf{A}}_{[bb]}$ as the Schur complement of matrix $\mathbf{A}$. If one expands the left hand side of Eq. (8.21), we obtain the original *structural* model again, but with a different loading, as can be seen in Eq. (8.22). It can clearly be seen that the original set of loads on the internal DoF has been replaced by a set of alternative loads on the boundary DoF.

$$\begin{bmatrix} \mathbf{A}_{[ii]} & \mathbf{A}_{[ib]} \\ \mathbf{A}_{[bi]} & \mathbf{A}_{[bb]} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}_{[i],0} \\ \tilde{\mathbf{q}}_{[b],0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{A}_{[bi]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hat{\mathbf{f}}_{[b],0}^{eq} \end{bmatrix}, \quad (8.22)$$

We have now found the *equivalent* problem, which is different from the original problem given in Eq. (8.20), hence the name of the variables is changed from $\mathbf{q}_{[\star],0}$ to $\tilde{\mathbf{q}}_{[\star],0}$, where the $\tilde{\star}$ denotes the fact that we're now working with the equivalent problem. Hence, by comparing the reference result, obtained from Eq. (8.20), with the results obtained with the equivalent loading in Eq. (8.23), it is clear that for the case with the equivalent forces on the boundary DoF, the exact same response is found for the boundary DoF ($\tilde{\mathbf{q}}_{[b],0}$), but that the response of the internal DoF ($\tilde{\mathbf{q}}_{[i],0}$) differs from the reference result.

$$\begin{bmatrix} \tilde{\mathbf{q}}_{[i],0} \\ \tilde{\mathbf{q}}_{[b],0} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{[ii]}^{-1} \mathbf{A}_{[ib]} \bar{\mathbf{A}}_{[bb]}^{-1} \mathbf{A}_{[bi]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \\ -\bar{\mathbf{A}}_{[bb]}^{-1} \mathbf{A}_{[bi]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_{[i],0} - \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \\ \mathbf{q}_{[b],0} \end{bmatrix} \quad (8.23)$$

As the response at t_n is highly dependent on the response at t_{n-1} , one has to be sure, that the error made on the response of the internal DoF, doesn't affect the response of the boundary DoF in the next time step.

8.3.2.2 Response at the Next Time Steps

The set of equations that need to be solved in the first time step (Eq. (8.19)), write:

$$\begin{bmatrix} \mathbf{Q}_{[ii]} & \mathbf{Q}_{[ib]} \\ \mathbf{Q}_{[bi]} & \mathbf{Q}_{[bb]} \end{bmatrix} \begin{bmatrix} \mathbf{q}_{[i],1} \\ \mathbf{q}_{[b],1} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_{[i],1} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{N}_{[ii]} & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_{[bb]} \end{bmatrix} \begin{bmatrix} \mathbf{q}_{[i],0} \\ \mathbf{q}_{[b],0} \end{bmatrix}. \quad (8.24)$$

The reference response of the first time step is shown in Eq. (8.24). Here, one can easily see that the response of the boundary DoF ($\mathbf{q}_{[b],1}$) is a function of the response of the internal DoF in the previous time step(s) ($\mathbf{q}_{[i],0}$). Obviously, the next step is to start from Eq. (8.24) and, again, condense the internal DoF of the interface DoF.

$$\bar{\mathbf{Q}}_{[bb]} \mathbf{q}_{[b],1} = -\mathbf{Q}_{[bi]} \mathbf{Q}_{[ii]}^{-1} \left(\hat{\mathbf{f}}_{[i],1} - \mathbf{N}_{[ii]} \mathbf{q}_{[i],0} \right) - \mathbf{N}_{[bb]} \mathbf{q}_{[b],0} \quad (8.25)$$

As the next step is to re-expand the left hand side of Eq. (8.25), it requires that the set of variables is also changes from the original DoF ($\mathbf{q}$) to the alternative DoF ($\tilde{\mathbf{q}}$). Therefore, it is needed to substitute Eq. (8.23) into Eq. (8.25).

$$\bar{\mathbf{Q}}_{[bb]} \tilde{\mathbf{q}}_{[b],1} = -\mathbf{Q}_{[bi]} \mathbf{Q}_{[ii]}^{-1} \left(\hat{\mathbf{f}}_{[i],1} - \mathbf{N}_{[ii]} \left(\tilde{\mathbf{q}}_{[i],0} + \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \right) \right) - \mathbf{N}_{[bb]} \tilde{\mathbf{q}}_{[b],0} \quad (8.26)$$

It can easily be verified that re-expansion of the left hand side of Eq. (8.26) results in Eq. (8.27).

$$\begin{bmatrix} \mathbf{Q}_{[ii]} & \mathbf{Q}_{[ib]} \\ \mathbf{Q}_{[bi]} & \mathbf{Q}_{[bb]} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}_{[i],1} \\ \tilde{\mathbf{q}}_{[b],1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{Q}_{[bi]} \mathbf{Q}_{[ii]}^{-1} \left(\hat{\mathbf{f}}_{[i],1} - \mathbf{N}_{[ii]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \right) \end{bmatrix} - \begin{bmatrix} \mathbf{N}_{[ii]} & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_{[bb]} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}_{[i],0} \\ \tilde{\mathbf{q}}_{[b],0} \end{bmatrix}. \quad (8.27)$$

Hence, one can determine the equivalent interface force $\hat{\mathbf{f}}_{[b],1}^{eq}$ for this time step.

$$\hat{\mathbf{f}}_{[b],1}^{eq} = -\mathbf{Q}_{[bi]} \mathbf{Q}_{[ii]}^{-1} \left(\hat{\mathbf{f}}_{[i],1} - \mathbf{N}_{[ii]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \right) \quad (8.28)$$

After solving Eq. (8.27) and comparing this solution, obtained from the equivalent loading case, to the solution of the original problem, which can be obtained from Eq. (8.24), one finds

$$\begin{bmatrix} \tilde{\mathbf{q}}_{[i],1} \\ \tilde{\mathbf{q}}_{[b],1} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_{[i],1} - \mathbf{Q}_{[ii]}^{-1} \left(\hat{\mathbf{f}}_{[i],1} - \mathbf{N}_{[ii]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \right) \\ \mathbf{q}_{[b],1} \end{bmatrix}, \quad (8.29)$$

that the response of the boundary DoF is again equal to the reference solution ($\mathbf{q}_{[b],1}$), but the response of the internal DoF differs from the reference solution ($\mathbf{q}_{[i],1}$). As the Newmark method in Eq. (8.17) shows that each new time step is a repetition of the same blocks, the results obtained in this section can be generalized to:

$$\hat{\mathbf{f}}_{[b],n}^{eq} = -\mathbf{Q}_{[bi]} \left(\sum_{j=0}^{n-1} \left(-\mathbf{Q}_{[ii]}^{-1} \mathbf{N}_{[ii]} \right)^{n-(j+1)} \mathbf{Q}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],j+1} + \left(-\mathbf{Q}_{[ii]}^{-1} \mathbf{N}_{[ii]} \right)^n \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \right), \quad (8.30)$$

for the equivalent interface force, which will result in the following responses at any moment in time:

$$\begin{bmatrix} \tilde{\mathbf{q}}_{[i],n} \\ \tilde{\mathbf{q}}_{[b],n} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_{[i],n} - \sum_{j=0}^{n-1} \left(-\mathbf{Q}_{[ii]}^{-1} \mathbf{N}_{[ii]} \right)^{n-(j+1)} \mathbf{Q}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],j+1} - \left(-\mathbf{Q}_{[ii]}^{-1} \mathbf{N}_{[ii]} \right)^n \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \\ \mathbf{q}_{[b],n} \end{bmatrix} \quad (8.31)$$

8.3.3 Usage of Constraint Forces as Equivalent External Forces

Although Eq. (8.30) has a clear recursive behavior from which we could easily compute $\hat{\mathbf{f}}_{[b],n}^{eq}$, it can not be directly related to any physical interpretation. In this section it will be shown that the expression found for the equivalent force on the interface is equal to the force we would find by restraining the substructure, that is loaded on its internal DoF, on the boundary DoF. The associated time integration equations would then write:

$$\begin{bmatrix} \mathbf{A}_{[ii]} & \mathbf{A}_{[ib]} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{[bi]} & \mathbf{A}_{[bb]} & \mathbf{0} & \mathbf{0} \\ \mathbf{N}_{[ii]} & \mathbf{0} & \mathbf{Q}_{[ii]} & \mathbf{Q}_{[ib]} \\ \mathbf{0} & \mathbf{N}_{[bb]} & \mathbf{Q}_{[bi]} & \mathbf{Q}_{[bb]} \end{bmatrix} \begin{bmatrix} \mathbf{q}_{[i]}^{con,0} \\ \mathbf{0} \\ \mathbf{q}_{[i]}^{con,1} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_{[i],0}^{con} \\ \hat{\mathbf{f}}_{[b],0}^{con} \\ \hat{\mathbf{f}}_{[i],1}^{con} \\ \hat{\mathbf{f}}_{[b],1}^{con} \end{bmatrix}, \quad (8.32)$$

where, $\hat{\mathbf{f}}_{[b],\star}^{con}$ are the constraint forces on the clamped interface points. Hence, from the initial time step, we directly find that

$$\hat{\mathbf{f}}_{[b],0}^{con} = \mathbf{A}_{[bi]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \quad (8.33)$$

which is equal to the negative equivalent force at the interface in Eq. (8.21). The constraint force at t_1 is again computed using Eq. (8.32).

$$\hat{\mathbf{f}}_{[b],1}^{con} = \mathbf{Q}_{[bi]} \mathbf{Q}_{[ii]}^{-1} \left(\hat{\mathbf{f}}_{[i],1} - \mathbf{N}_{[ii]} \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \right) \quad (8.34)$$

Thus, one finds that the this can again be generalized

$$\hat{\mathbf{f}}_{[b],n}^{con} = \mathbf{Q}_{[bi]} \left(\sum_{j=0}^{n-1} \left(-\mathbf{Q}_{[ii]}^{-1} \mathbf{N}_{[ii]} \right)^{n-(j+1)} \mathbf{Q}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],j+1} + \left(-\mathbf{Q}_{[ii]}^{-1} \mathbf{N}_{[ii]} \right)^n \mathbf{A}_{[ii]}^{-1} \hat{\mathbf{f}}_{[i],0} \right) = -\hat{\mathbf{f}}_{[b],n}^{eq}, \quad (8.35)$$

such that one can find the equivalent force (given in Eq. (8.30)) on the interface by simply computing the constraint forces of the stand-alone substructure. From a practical point of view, this is easy to implement, as most of the finite element packages are able to compute the time response of a model. By substituting the result from Eq. (8.35) into Eq. (8.31), it can be observed that the response computed using the blocked forces can be written according to Eq. (8.36).

$$\begin{bmatrix} \tilde{\mathbf{q}}_{[i],n} \\ \tilde{\mathbf{q}}_{[b],n} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_{[i],n} - \mathbf{q}_{[i],n}^{con} \\ \mathbf{q}_{[b],n} \end{bmatrix} \quad (8.36)$$

Note that, as the procedure is only valid for linear systems, the time integration itself is relatively cheap. The time stepping matrix (used in the Newmark's method) only needs to be factorized once and allows to find the time response at each time step by simply applying forward and backward substitutions.

Note that the method discussed in this section can only be applied if the “source” structure (which is force loaded on its internal DoF) has to be linear in order to be able to replace the internal force by an equivalent force on the interface. Though, a fundamental difference with the corresponding method in the frequency domain is that the connected substructures (the “receivers”) can be nonlinear.

8.3.4 Practical Implementation of the Method

Up to now, only the theoretical proof of the method has been given and it is not presented yet how one would apply the method in practise. Firstly, it is important here to emphasis the difference between the actual force vector $\mathbf{f}$ and the array $\hat{\mathbf{f}}$, as was given in Eq. (8.18) and used throughout the proof. Note that $\hat{\mathbf{f}}$ contains the force vector, but also the initial conditions (here assumed to be zero for concisiveness), for the initial time step, and a large array filled with zeros, for the other time steps. As already shown in section 8.3.1, these lines are related to the Newmark time discretization and enforce the Newmark time relations (given in Eq. (8.15)) at each time step. Therefore, it is important to proof whether the method described in this section only modifies the force vector $\mathbf{f}$, or also the entries related to the Newmark time discretization. As we know that the equivalent force are equal to the constraint forces with the opposit sign, we will write out Eq. (8.33), which gives:

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{f}_{[b],0}^{con} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{[bi]} & \mathbf{C}_{[bi]} & \mathbf{M}_{[bi]} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{K}_{[ii]} & \mathbf{C}_{[ii]} & \mathbf{M}_{[ii]} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{f}_{[i],0} \end{bmatrix} \quad (8.37)$$

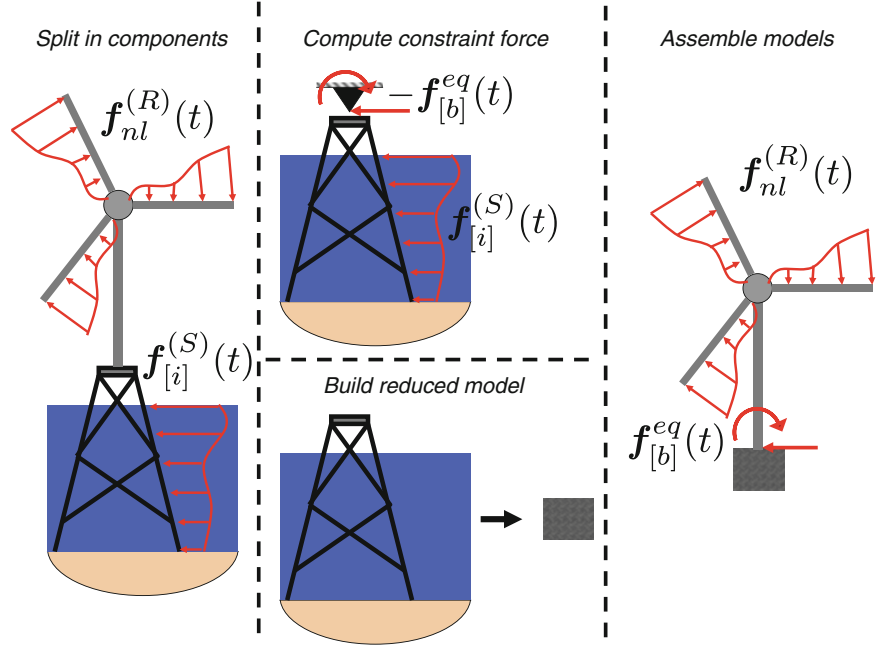
One can clearly see that due to the construction of $\mathbf{A}_{[bi]}$, the entries besides the force vector will always be zero. For obtaining the equivalent blocked forces for all other time steps (Eq. (8.35)), it can be shown that

$$\mathbf{Q}_{[bi]} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{[bi]} & \mathbf{C}_{[bi]} & \mathbf{M}_{[bi]} \end{bmatrix}, \quad (8.38)$$

which assures that one only needs the force vector $\mathbf{f}_{[b],n}^{con}$ in order to build $\hat{\mathbf{f}}_{[b],n}^{eq}$.

From a practical point of view, this is an important result, as implementing the method in practice will be quite straightforward now. The procedure is shown in Fig. 8.3, where the different steps are shown from left to right. Firstly, the original problem is split into substructures (or components), which are in this case the offshore jacket structure loaded by linear wave loads (substructure S) and the wind turbine, loaded by nonlinear wind forces (substructure R). As we are in this

Fig. 8.3 Steps taken when using the methods of equivalent blocked forces in the time domain



case only interested in the response of the turbine, we could apply the method of equivalent forces on the interface on the offshore jacket. Hence, in the next step (*Compute constraint force*) the constrained response of the jacket is computed by time integrating:

$$\mathbf{M}_{[ii]}^{(S)} \ddot{\mathbf{u}}_{[i]}^{(S)} + \mathbf{C}_{[ii]}^{(S)} \dot{\mathbf{u}}_{[i]}^{(S)} + \mathbf{K}_{[ii]}^{(S)} \mathbf{u}_{[i]}^{(S)} = \mathbf{f}_{[i]}^{(S)} \quad (8.39)$$

From the resulting accelerations, velocities and displacements, one can obtain the constraint forces at the interface for each time step and as it was shown that the constraint forces are equal to (but opposite in sign) the equivalent interface forces needed, we can directly obtain those from solving:

$$-\mathbf{f}_{[b]}^{eq} = \mathbf{M}_{[bi]}^{(S)} \ddot{\mathbf{u}}_{[i]}^{(S)} + \mathbf{C}_{[bi]}^{(S)} \dot{\mathbf{u}}_{[i]}^{(S)} + \mathbf{K}_{[bi]}^{(S)} \mathbf{u}_{[i]}^{(S)} \quad (8.40)$$

As the jacket model is linear, it can be reduced using one of the classical CMS or IBS methods, thereby significantly reducing the computational effort needing for solving the final nonlinear problem. By finally assembling the nonlinear wind turbine model with the reduced jacket model and applying the equivalent interface force, we can compute the correct response of the turbine in a far more efficiently then without the reduction. Note that the time stepping method and time steps used, should be identical to those used in Eq. (8.39) and that the used superelement should be able to describe the dynamics of the jacket accurately.

8.4 Applying Free Interface Displacements in the Time Domain

The time equivalent of the method presented in section 8.2.2, which is based on the idea of an *equivalent interface gap*, can be found by starting from the equations of motion of two substructures that are dually assembled

$$\begin{bmatrix} \mathbf{M}^{(S)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{(R)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}^{(S)} \\ \ddot{\mathbf{u}}^{(R)} \\ \boldsymbol{\lambda} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^{(S)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^{(R)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}^{(S)} \\ \dot{\mathbf{u}}^{(R)} \\ \boldsymbol{\lambda} \end{bmatrix} + \begin{bmatrix} \mathbf{K}^{(S)} & \mathbf{0} & \mathbf{B}^{(S)T} \\ \mathbf{0} & \mathbf{K}^{(R)} & \mathbf{B}^{(R)T} \\ \mathbf{B}^{(S)} & \mathbf{B}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(S)} \\ \mathbf{u}^{(R)} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(S)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (8.41)$$

where the superscript (S) denotes the *source* substructure and superscript (R) denotes the *receiver* substructure. By writing Eq. (8.41) into the set of equations governing the Newmark method in Eq. (8.17), we obtain for the first two time steps the following matrix with Newmark equations

$$\begin{bmatrix} \mathbf{A}^{(S)} & \mathbf{0} & \hat{\mathbf{B}}^{(S)T} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{(R)} & \hat{\mathbf{B}}^{(R)T} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hat{\mathbf{B}}^{(S)} & \hat{\mathbf{B}}^{(R)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{N}^{(S)} & \mathbf{0} & \mathbf{0} & \mathbf{Q}^{(S)} & \mathbf{0} & \hat{\mathbf{B}}^{(S)T} \\ \mathbf{0} & \mathbf{N}^{(R)} & \mathbf{0} & \mathbf{0} & \mathbf{Q}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{B}}^{(S)} & \hat{\mathbf{B}}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_0^{(S)} \\ \mathbf{q}_0^{(R)} \\ \boldsymbol{\lambda}_0 \\ \mathbf{q}_1^{(S)} \\ \mathbf{q}_1^{(R)} \\ \boldsymbol{\lambda}_1 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_0^{(S)} \\ \mathbf{0} \\ \mathbf{0} \\ \hat{\mathbf{f}}_1^{(S)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (8.42)$$

where $\hat{\mathbf{B}}^{(*)} = [\mathbf{B}^{(*)} \mathbf{0} \mathbf{0}]$, which is a signed Boolean operator that selects the displacements on the boundary DoF, from the total set of displacements, velocities and accelerations ($\mathbf{q}_n^{(*)}$) and enforces the compatibility condition.

8.4.1 Condensing the Internal Force on the Boundary Degrees of Freedom

The next step is to, similar to what is done in section 8.3, condense substructure 1 on the interface. Note that, as the interface is described here in terms of Lagrange multipliers, one condenses in fact the entire set of accelerations, velocities and displacements of the source subsystem ($\mathbf{q}_0^{(S)}$), and not just its internal set of DoF, onto the interface.

8.4.1.1 Response at the Initial Time Step

Again, starting at t_0 , the following set of equations are obtained from Eq. (8.42).

$$\begin{bmatrix} \mathbf{A}^{(S)} & \mathbf{0} & \hat{\mathbf{B}}^{(S)T} \\ \mathbf{0} & \mathbf{A}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \hat{\mathbf{B}}^{(S)} & \hat{\mathbf{B}}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_0^{(S)} \\ \mathbf{q}_0^{(R)} \\ \boldsymbol{\lambda}_0 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_0^{(S)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (8.43)$$

Solving this set results in the reference response of both substructures ($\mathbf{q}_0^{(*)}$) and the associated interface forces ($\boldsymbol{\lambda}_0$) at t_0 . Similar to the previous sections, the source substructure is condensed on the interface, results in:

$$\begin{bmatrix} \mathbf{A}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \hat{\mathbf{B}}^{(R)} & -\hat{\mathbf{B}}^{(S)} \mathbf{A}^{(S)-1} \hat{\mathbf{B}}^{(S)T} \end{bmatrix} \begin{bmatrix} \mathbf{q}_0^{(R)} \\ \boldsymbol{\lambda}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\hat{\mathbf{B}}^{(S)} \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \end{bmatrix}, \quad (8.44)$$

By expanding the left hand side of Eq. (8.44), the original block matrix is obtained again as in Eq. (8.43),

$$\begin{bmatrix} \mathbf{A}^{(S)} & \mathbf{0} & \hat{\mathbf{B}}^{(S)T} \\ \mathbf{0} & \mathbf{A}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \hat{\mathbf{B}}^{(S)} & \hat{\mathbf{B}}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}_0^{(S)} \\ \tilde{\mathbf{q}}_0^{(R)} \\ \tilde{\boldsymbol{\lambda}}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ -\hat{\mathbf{B}}^{(S)} \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \delta_{u,0} \end{bmatrix}, \quad (8.45)$$

but one can clearly see that the external load on the source substructure is replaced by a term that “excites” the compatibility condition. This can be physically interpreted as a gap in the equilibrium condition, that states that $\tilde{\mathbf{q}}_{[b],0}^{(S)} \neq \tilde{\mathbf{q}}_{[b],0}^{(R)}$. Note that the *equivalent* problem is found now, which is different from the original problem given in Eq. (8.43), hence the name of the

variables is changed from $\mathbf{q}^{(*)}$ to $\tilde{\mathbf{q}}^{(*)}$, where the $\tilde{\star}$ denotes the fact that we are now working with the equivalent problem. Solving the responses of the equivalent problem stated in Eq. (8.45), gives:

$$\begin{bmatrix} \tilde{\mathbf{q}}_0^{(S)} \\ \tilde{\mathbf{q}}_0^{(R)} \\ \tilde{\lambda}_0 \end{bmatrix} = \begin{bmatrix} -\mathbf{A}^{(S)-1} \hat{\mathbf{B}}^{(S)T} \tilde{\lambda}_0 \\ -\mathbf{A}^{(R)-1} \hat{\mathbf{B}}^{(R)T} \tilde{\lambda}_0 \\ (\hat{\mathbf{B}} \mathbf{A}^{(tot)-1} \hat{\mathbf{B}}^T)^{-1} \mathbf{B}^{(S)} \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_0^{(S)} - \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \\ \mathbf{q}_0^{(R)} \\ \lambda_0 \end{bmatrix}, \quad (8.46)$$

where,

$$\hat{\mathbf{B}} \mathbf{A}^{(tot)-1} \hat{\mathbf{B}}^T = \begin{bmatrix} \hat{\mathbf{B}}^{(S)} & \hat{\mathbf{B}}^{(R)} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{(S)} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{(R)} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{B}}^{(S)T} \\ \hat{\mathbf{B}}^{(R)T} \end{bmatrix}.$$

By comparing the result obtained from Eq. (8.46) with the reference result, which can be computed from Eq. (8.43), one will find that the original response for both the interface force and the receiver substructure (R) are obtained, but that an erroneous response for the source substructure is found. This directly explains the role of the applied *interface gap* in Eq. (8.45), as this in fact corrects for the incorrect response of the source substructure (S) in the compatibility equation.

8.4.1.2 Response at the Next Time Steps

Obviously, we can not validate the method by only discussing the first time step, hence also next time steps are discussed in this section. Starting from the response at t_1 , the result will be generalized such that it holds for all time steps. The set of equations for the first time step can be extracted from Eq. (8.42).

$$\begin{bmatrix} \mathbf{Q}^{(S)} & \mathbf{0} & \hat{\mathbf{B}}^{(S)T} \\ \mathbf{0} & \mathbf{Q}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \hat{\mathbf{B}}^{(S)} & \hat{\mathbf{B}}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_1^{(S)} \\ \mathbf{q}_1^{(R)} \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_1^{(S)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{N}^{(S)} \mathbf{q}_0^{(S)} \\ \mathbf{N}^{(R)} \mathbf{q}_0^{(R)} \\ \mathbf{0} \end{bmatrix}, \quad (8.47)$$

It is easily found that the substructure responses and interface forces at t_1 are given by:

$$\begin{bmatrix} \mathbf{q}_1^{(S)} \\ \mathbf{q}_1^{(R)} \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^{(S)-1} \left(\hat{\mathbf{f}}_1^{(S)} - \mathbf{N}^{(S)} \mathbf{q}_0^{(S)} - \hat{\mathbf{B}}^{(S)T} \lambda_1 \right) \\ \mathbf{Q}^{(R)-1} \left(-\mathbf{N}^{(R)} \mathbf{q}_0^{(R)} - \hat{\mathbf{B}}^{(R)T} \lambda_1 \right) \\ (\hat{\mathbf{B}} \mathbf{Q}^{(tot)-1} \hat{\mathbf{B}}^T)^{-1} \left(\hat{\mathbf{B}}^{(S)} \mathbf{Q}^{(S)-1} \left(\hat{\mathbf{f}}_1^{(S)} - \mathbf{N}^{(S)} \mathbf{q}_0^{(S)} \right) - \hat{\mathbf{B}}^{(R)} \mathbf{Q}^{(R)-1} \mathbf{N}^{(R)} \mathbf{q}_0^{(R)} \right) \end{bmatrix}. \quad (8.48)$$

In order to find the equivalent externally applied *interface gap* as was done for the first time step in Eq. (8.45), the accelerations, velocities and displacements of substructure 1 are condensed on the interface problem.

$$\begin{bmatrix} \mathbf{Q}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \hat{\mathbf{B}}^{(R)} & -\hat{\mathbf{B}}^{(S)} \mathbf{Q}^{(S)-1} \hat{\mathbf{B}}^{(S)T} \end{bmatrix} \begin{bmatrix} \mathbf{q}_1^{(R)} \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} -\mathbf{N}^{(R)} \mathbf{q}_0^{(R)} \\ -\hat{\mathbf{B}}^{(S)} \mathbf{Q}^{(S)-1} \left(\hat{\mathbf{f}}_1^{(S)} - \mathbf{N}^{(S)} \mathbf{q}_0^{(S)} \right) \end{bmatrix}, \quad (8.49)$$

Note that after the expansion, the set of DoF is also changed from $\mathbf{q}_n^{(S)}$ into $\tilde{\mathbf{q}}_n^{(S)}$, hence substituting the relation between $\mathbf{q}_0^{(*)}$ and $\tilde{\mathbf{q}}_0^{(*)}$, as given in Eq. (8.46), into the right hand side of Eq. (8.49) gives:

$$\begin{bmatrix} \mathbf{Q}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \hat{\mathbf{B}}^{(R)} & -\hat{\mathbf{B}}^{(S)} \mathbf{Q}^{(S)-1} \hat{\mathbf{B}}^{(S)T} \end{bmatrix} \begin{bmatrix} \mathbf{q}_1^{(R)} \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} -\mathbf{N}^{(R)} \tilde{\mathbf{q}}_0^{(R)} \\ -\hat{\mathbf{B}}^{(S)} \mathbf{Q}^{(S)-1} \left(\hat{\mathbf{f}}_1^{(S)} - \mathbf{N}^{(S)} \left(\tilde{\mathbf{q}}_0^{(S)} + \mathbf{A}^{(S)} \hat{\mathbf{f}}_0^{(S)} \right) \right) \end{bmatrix}, \quad (8.50)$$

After re-expanding the left hand side of Eq. (8.50), the original block form matrix is obtained again. Note that after the expansion, the set of DoF on the left hand side is also changed from $\mathbf{q}_n^{(S)}$ into $\tilde{\mathbf{q}}_n^{(S)}$.

$$\begin{bmatrix} \mathbf{Q}^{(S)} & \mathbf{0} & \hat{\mathbf{B}}^{(S)T} \\ \mathbf{0} & \mathbf{Q}^{(R)} & \hat{\mathbf{B}}^{(R)T} \\ \hat{\mathbf{B}}^{(S)} & \hat{\mathbf{B}}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}_1^{(S)} \\ \tilde{\mathbf{q}}_1^{(R)} \\ \tilde{\lambda}_1 \end{bmatrix} = \begin{bmatrix} -\mathbf{N}^{(S)} \tilde{\mathbf{q}}_0^{(S)} \\ -\mathbf{N}^{(R)} \tilde{\mathbf{q}}_0^{(R)} \\ -\hat{\mathbf{B}}^{(S)} \mathbf{Q}^{(S)-1} (\hat{\mathbf{f}}_1^{(S)} - \mathbf{N}^{(S)} \mathbf{A}^{(S)} \hat{\mathbf{f}}_0^{(S)}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \delta_{u,1} \end{bmatrix}, \quad (8.51)$$

Note here that the right hand side is also altered due to the substitution of Eq. (8.46) and the re-expansion of the left hand side and that we have now found the equivalent externally applied *interface gap* ($\delta_{u,1}$).

$$\begin{bmatrix} \tilde{\mathbf{q}}_1^{(S)} \\ \tilde{\mathbf{q}}_1^{(R)} \\ \tilde{\lambda}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^{(S)-1} \left(-\mathbf{N}^{(S)} \mathbf{q}_0^{(S)} + \mathbf{N}^{(S)} \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} - \hat{\mathbf{B}}^{(S)T} \tilde{\lambda}_1 \right) \\ \mathbf{Q}^{(R)-1} \left(-\mathbf{N}^{(R)} \mathbf{q}_0^{(R)} - \hat{\mathbf{B}}^{(R)T} \tilde{\lambda}_1 \right) \\ \left(\hat{\mathbf{B}} \mathbf{Q}^{(tot)-1} \hat{\mathbf{B}}^T \right)^{-1} \left(\hat{\mathbf{B}}^{(S)} \mathbf{Q}^{(S)-1} (\hat{\mathbf{f}}_1^{(S)} - \mathbf{N}^{(S)} \mathbf{q}_0^{(S)}) - \hat{\mathbf{B}}^{(R)} \mathbf{Q}^{(R)-1} \mathbf{N}^{(R)} \mathbf{q}_0^{(R)} \right) \end{bmatrix} \quad (8.52)$$

From this result, one can deduce that for each time step it holds that the response of the receiver substructure (r) and the resulting interface force is identical to the reference solution. For the source substructure (S), one finds that the relation between the equivalent response and the reference response is given in Eq. (8.53).

$$\begin{bmatrix} \tilde{\mathbf{q}}_n^{(S)} \\ \tilde{\mathbf{q}}_n^{(R)} \\ \tilde{\lambda}_n \end{bmatrix} = \begin{bmatrix} \mathbf{q}_n^{(S)} - \sum_{j=0}^{n-1} \left(-\mathbf{Q}^{(S)-1} \mathbf{N}^{(S)} \right)^{n-(j+1)} \mathbf{Q}^{(S)-1} \hat{\mathbf{f}}_{j+1}^{(S)} - \left(-\mathbf{Q}^{(S)-1} \mathbf{N}^{(S)} \right)^n \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \\ \mathbf{q}_n^{(R)} \\ \lambda_n \end{bmatrix} \quad (8.53)$$

For the equivalent externally applied interface gap, one finds a similar recursive relationship.

$$\delta_{u,n} = -\hat{\mathbf{B}}^{(S)} \left(\sum_{j=0}^{n-1} \left(-\mathbf{Q}^{(S)-1} \mathbf{N}^{(S)} \right)^{n-(j+1)} \mathbf{Q}^{(S)-1} \hat{\mathbf{f}}_{j+1}^{(S)} + \left(-\mathbf{Q}^{(S)-1} \mathbf{N}^{(S)} \right)^n \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \right) \quad (8.54)$$

8.4.2 Substructure's Free Interface Response Due to External Loading

As in reality, we would never explicitly solve (8.54), it will be shown that this can be computed by simple time integration of the free interface substructure. Starting from the Newmark equations in block form for the source substructure (S) with free interfaces for the initial and first time step, we obtain:

$$\begin{bmatrix} \mathbf{A}^{(S)} & \mathbf{0} \\ \mathbf{N}^{(S)} & \mathbf{Q}^{(S)} \end{bmatrix} \begin{bmatrix} \mathbf{q}_0^{(S),free} \\ \mathbf{q}_1^{(S),free} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_0^{(S)} \\ \hat{\mathbf{f}}_1^{(S)} \end{bmatrix}, \quad (8.55)$$

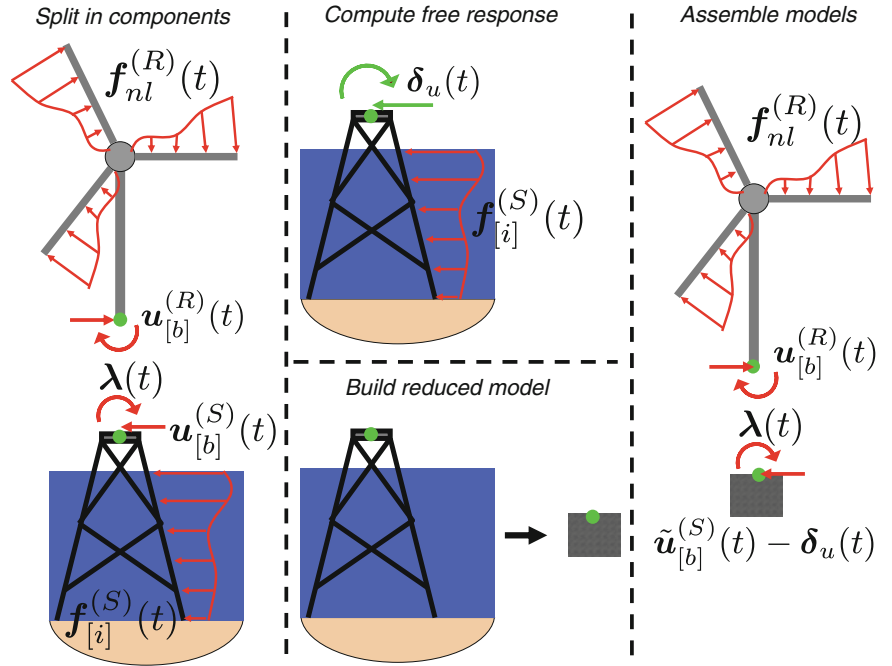
where the superscript *free* denotes the free interface response. The free response for these time steps is simply obtain from Eq. (8.55),

$$\begin{aligned} \mathbf{q}_0^{(S),free} &= \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \\ \mathbf{q}_1^{(S),free} &= \mathbf{Q}^{(S)-1} \left(\hat{\mathbf{f}}_1^{(S)} - \mathbf{N}^{(S)} \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \right), \end{aligned} \quad (8.56)$$

and can easily be generalized to all time steps.

$$\mathbf{q}_n^{(S),free} = \sum_{j=0}^{n-1} \left(-\mathbf{Q}^{(S)-1} \mathbf{N}^{(S)} \right)^{n-(j+1)} \mathbf{Q}^{(S)-1} \hat{\mathbf{f}}_{j+1}^{(S)} + \left(-\mathbf{Q}^{(S)-1} \mathbf{N}^{(S)} \right)^n \mathbf{A}^{(S)-1} \hat{\mathbf{f}}_0^{(S)} \quad (8.57)$$

Fig. 8.4 Steps taken when using the methods of equivalent interface gap in the time domain



Thus, one can obtain the equivalent interface gap, given in Eq.(8.54), from the free interface response, computed in Eq.(8.57).

$$\delta_{u,n} = -\hat{\mathbf{B}}^{(S)} \mathbf{q}_n^{(S),free} \quad (8.58)$$

Which is similar to what was found for the method which uses equivalent blocked forces (section 8.3), one can substitute Eq.(8.57) into Eq.(8.53) to find:

$$\begin{bmatrix} \tilde{\mathbf{q}}_n^{(S)} \\ \tilde{\mathbf{q}}_n^{(R)} \\ \tilde{\lambda}_n \end{bmatrix} = \begin{bmatrix} \mathbf{q}_n^{(S)} - \mathbf{q}_n^{(S),free} \\ \mathbf{q}_n^{(R)} \\ \lambda_n \end{bmatrix} \quad (8.59)$$

Which is similar to the result already found in Eq.(8.36), as also here the equivalent set of displacements, velocities and accelerations is equal to the reference result minus the free interface response, which is computed as a pre-processing step.

8.4.3 Practical Implementation of the Method

As the discussion was, up to now, only focussed on the analytical proof of the method, it will now be discussed how the method can be applied in practise. The different steps are illustrated in Fig. 8.4, starting from the original dual assembled problem, which is split into the different components. As the loading on the wind turbine is nonlinear, this model is to be retained and only the (linear) jacket structure can be reduced, using for instance the Dual Craig-Bampton method [10], which was developed for dual assembled systems. In addition to this, the equivalent interface gap needs to be computed, which can simply be obtained from the free interface response of the jacket, when loaded by the wave forces.

$$\begin{aligned} \mathbf{M}^{(S)} \ddot{\mathbf{u}}^{(S)} + \mathbf{C}^{(S)} \dot{\mathbf{u}}^{(S)} + \mathbf{K}^{(S)} \mathbf{u}^{(S)} &= \mathbf{f}^{(S)} \\ \delta_u &= -\mathbf{B}^{(S)} \mathbf{u}^{(S)} \end{aligned} \quad (8.60)$$

This response can be solved using the standard Newmark time integrations methods again, which are usually available in finite element packages. By using the Boolean operator for selecting the interface response, the equivalent interface gap is obtained. In the next step, the reduced source structure (the jacket) and the unreduced nonlinear receiver component (the wind turbine) are re-assembled and excited by the equivalent interface gap on the compatibility equation, as can be seen in Eq.(8.61).

$$\begin{bmatrix} \mathbf{M}^{(S)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{red}^{(R)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}^{(S)} \\ \ddot{\boldsymbol{\eta}}^{(R)} \\ \lambda \end{bmatrix} + \begin{bmatrix} \mathbf{C}^{(S)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{red}^{(R)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}^{(S)} \\ \dot{\boldsymbol{\eta}}^{(R)} \\ \lambda \end{bmatrix} + \begin{bmatrix} \mathbf{K}^{(S)} & \mathbf{0} & \mathbf{B}^{(S)T} \\ \mathbf{0} & \mathbf{K}_{red}^{(R)} & \mathbf{B}^{(R)T} \\ \mathbf{B}^{(S)} & \mathbf{B}^{(R)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(S)} \\ \boldsymbol{\eta}^{(R)} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \delta_u \end{bmatrix}, \quad (8.61)$$

where, the subscript *red* denotes the reduced structural matrices and $\boldsymbol{\eta}^{(R)}$ is the reduced set of DoF. As the linear subsystem is reduced, the response of the global system will now be much cheaper to compute. Note again that the pre-processing step in Eq. (8.60) needs to be performed with the same time integration method and parameters as the final coupled problem, given in Eq. (8.61).

8.5 Application to an Offshore Wind Turbine

In order to show the effectiveness of the method, a case study has been selected. In this case study, the structural model of the NREL 5MW turbine [6] has been taken and coupled to the UpWind reference jacket model [4], which are shown in Fig. 8.5. In the given test case, only the jacket is force loaded by (linearized) wave forces. These wave forces are computed using Morison's equation [7]

$$f = \rho V \ddot{v} + \rho C_a V (\ddot{v} - \ddot{u}) + \frac{1}{2} C_d A (\dot{v} - \dot{u}) |\dot{v} - \dot{u}|, \quad (8.62)$$

where ρ is the water density, A and V are respectively the area and volume of the submerged body, C_a and C_d are the added mass and drag coefficients and $\ddot{v}$, $\dot{v}$ are the flow acceleration and velocity. The first term in Eq. (8.62) is the Froude-Krylov force, the second term the hydrodynamic mass force and the last term is the drag force. By assuming that the effect of the structural motion ($\dot{u}$, $\ddot{u}$) is negligible in the drag force, the equation is linearized,

$$f = (1 + C_a) \rho V \ddot{v} - \rho C_a V \ddot{u} + \frac{1}{2} C_d A \dot{v} |\dot{v}|, \quad (8.63)$$

by adding the second term in Eq. (8.63), which is only dependent on the structural accelerations, to the equation of motion of the structural system, the remainder of the wave force can be computed directly from the geometry of the offshore structure and the flow field of the waves.

$$f_w = (1 + C_a) \rho V \ddot{v} + \frac{1}{2} C_d A \dot{v} |\dot{v}|, \quad (8.64)$$

The wave force are computed for the case of linear irregular waves with a significant wave height of 3.75 m and a period of 7.2 s and currents up to 0.5 ms^{-1} . Due to this linearization, both the structural model of the jacket and the applied loading are

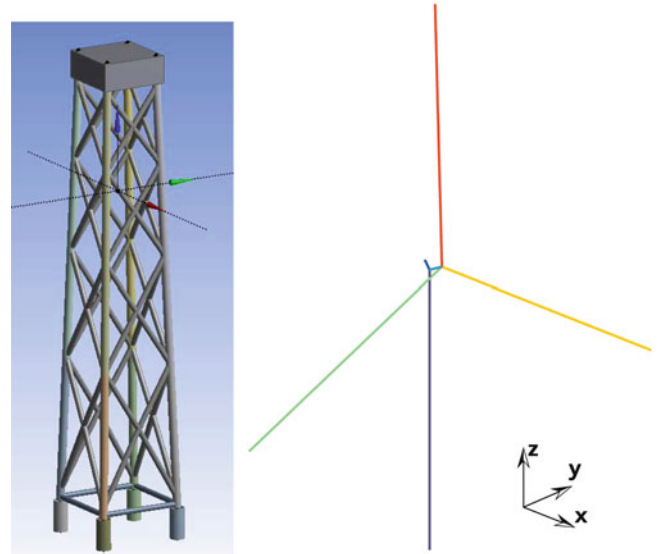
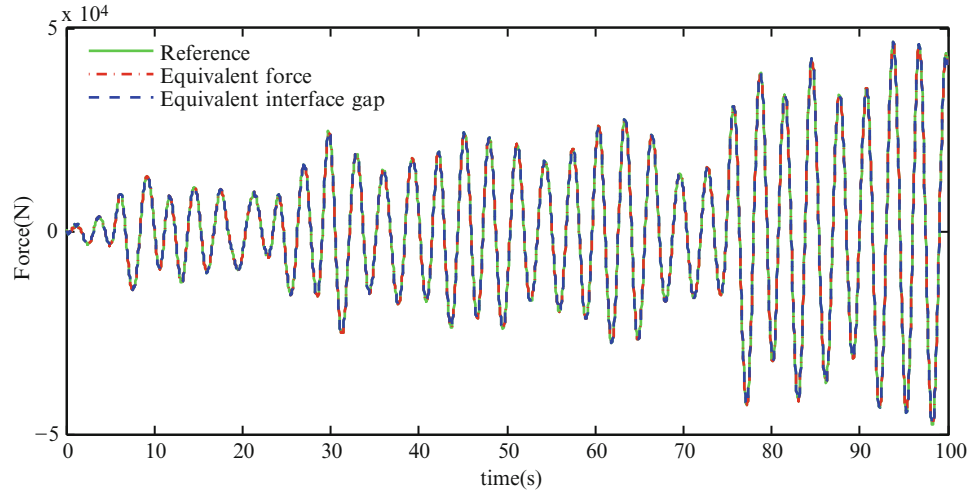


Fig. 8.5 Models of the Upwind reference jacket (*left*) and the NREL 5MW turbine (*right*)

Fig. 8.6 Coupling force between the jacket and wind turbine in the direction of the waves (x-component)



linear, hence the methods presented in section 8.3 and section 8.4 can be applied here. In order to (numerically) demonstrate the accuracy, which was already proven analytically, the equivalent wave loading is computed and applied to the models. The results are then compared to the full reference solution, computed from the original wave loads.

8.5.1 Coupling Forces Due to the Equivalent Wave Loading

As was stated in previous sections, the underlying interface problem remains exact. Hence, by applying the equivalent forces on the interface, the response of the interface, and the associated coupling forces between the substructures, will not be altered. This can also be seen from the results of the simulations in Fig. 8.6. It is clear that the coupling force computed by both of the equivalent loading methods perfectly matches the interface forces computed in the reference case.

8.5.2 Response of the Wind Turbine Substructure

As the underlying coupling problem between the turbines remains exact, as was shown both analytically and numerically, the response of the neighboring substructures, in this case the wind turbine, should remain identical to the original problem as well.

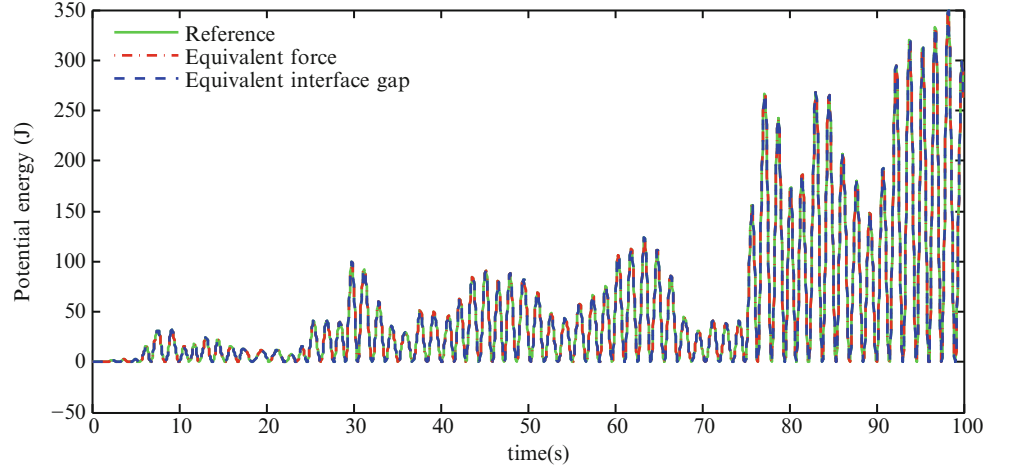
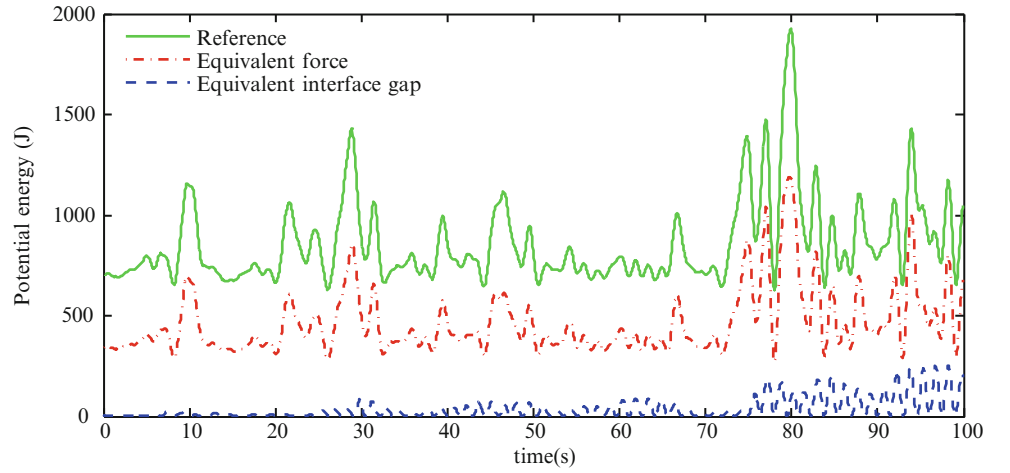
The comparison will be made using the potential energy of the wind turbine substructure ($\mathcal{V}^{(2)}$) of the system at each time step,

$$\mathcal{V}_n^{(2)} = \frac{1}{2} \mathbf{u}_n^{(2)T} \mathbf{K}^{(2)} \mathbf{u}_n^{(2)}, \quad (8.65)$$

where superscript 2 denotes the wind turbine substructure. The potential energy can be seen as a quantitative measure for the total deformation of the foundation and is thus a good parameter for comparing the results. The potential energy of the wind turbine, for both the reference case and cases with equivalent loading, is given in Fig. 8.7. One can clearly see that the potential energies for all three cases are identical, which numerically shows that the response of the turbine is identical to the reference case for both equivalent loading methods. Note that the potential energy starts from zero, as the loading is only applied on the jacket structure, the turbine will initially not be deformed.

8.5.3 Response of the Jacket Substructure

It was also proven that the response of substructure 1, whose internal forces are transferred to equivalent forces or an equivalent interface gap, will not remain identical to the original reference solution.

Fig. 8.7 Potential energy of the wind turbine substructure**Fig. 8.8** Potential energy of the jacket substructure

$$\tilde{\mathcal{V}}_n^{(1)} = \frac{1}{2} \tilde{\mathbf{u}}_n^{(1)T} \mathbf{K}^{(1)} \tilde{\mathbf{u}}_n^{(1)}, \quad (8.66)$$

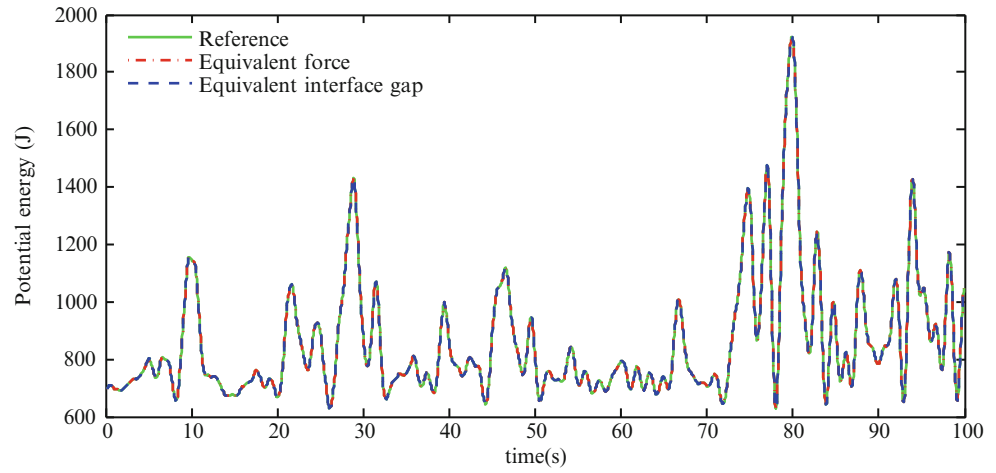
Hence, the potential energy $\tilde{\mathcal{V}}_n^{(1)}$, computed from the set of displacements $\tilde{\mathbf{u}}_n^{(1)}$, will be erroneous as well, as can be seen in Fig. 8.8. It can also be observed that the potential energy, in the case of an equivalent interface gap, is significantly lower than for the case of the equivalent forces that are applied on the interface. Recalling Eq. (8.36) and Eq. (8.59), it is clear that the response of substructure that is excited by the equivalent load, has an error that is equal to, in the first case, the constrained response and in the later case, to the free response.

As computing the equivalent interface force, or the equivalent compatibility gap, from the internal forces can be regarded as a pre-processing step, one can store the full set of displacements computed in this pre-processing step. By adding the result of the coupled simulations, performed with the equivalent loading, to the result of the associated pre-processing step one can obtain the correct response of the force loaded substructure. Hence one can compute the potential energy of the corrected response from:

$$\mathcal{V}_n^{(1)} = \frac{1}{2} \left[\tilde{\mathbf{u}}_n^{(1)} + \mathbf{u}_n^{(1),pp} \right]^T \mathbf{K}^{(1)} \left[\tilde{\mathbf{u}}_n^{(1)} + \mathbf{u}_n^{(1),pp} \right], \quad (8.67)$$

where, the superscript *pp* denotes the set of displacements computed in the pre-processing step. The result of Eq. (8.67) is given in Fig. 8.9, one can clearly observe that the exact same response is found, as was already shown analytically by Eq. (8.36) and Eq. (8.59).

Fig. 8.9 Potential energy of the jacket substructure; after adding the response of the pre-processing step



8.5.4 Discussion

It should be noted that the current example was selected in order to, numerically, show the relations derived analytically in sections 8.3 & 8.4. Therefore, applying the methods on the current example poses no computational advantage in comparison to directly obtaining the reference solution.

The advantages of the methods will become evident, when one applies them to substructured time analysis, where one would apply CMS or IBS methods to reduce the computational expense of obtaining the dynamic response of complex structural systems. Especially if one is dealing with a system, consisting of a number of nonlinear components and a (large) number of (large) linear components, substructuring methods are essential for limiting the computational effort needed for obtaining time responses. One should also keep in mind that simulations with nonlinear subcomponents can only be performed in the time domain, whereas one has a choice between time and frequency domain methods for obtaining the response of linear systems. By applying the methods presented in this paper, one is able to transform the internal forces of all linear substructures to the interfaces with their neighbors. Therefore, one needs not to worry about designing a specific reduction basis which captures all the internal loading, but one can create one reduced substructure, model using the classical CMS methods or IBS methods, that can be used in all different load cases. In addition to this, it also allows combining substructures and loading from different groups, while they only need to share the excitations on the interface and corresponding (reduced) substructure models.

Finally, it should be noted that obtaining the equivalent loading is, even for large systems, relatively cheap. As the time stepping matrix in the Newmark method only needs to be factorized once, and one only applies subsequent back and forward substitutions and linear matrix-vector operations in order to obtain either the blocked forces or free interface response.

8.6 Conclusions

In this work it was shown that one can transform the internal forces of a linear substructure to its interfaces with neighboring structures, such that the response of the interface and the neighboring substructures remains exactly identical. It was shown that if one applies primal assembly, one has to apply a set equivalent forces, that can be obtained by computing the constraint forces as a pre-processing step, which are obtained by constraining the force loaded substructure at its interface. Therefore, these forces are referred to as *equivalent blocked forces*. If one starts from the dually assembled global structure, one needs to apply a gap in the interface compatibility condition. This type of equivalent loading can be obtained from the free response of the force loaded substructure at its interfaces. Hence, this type of equivalent loading is referred to as an *equivalent interface gap* and is applied in the compatibility equation. Due to this transformation of the forces, the response found of the internal DoF of the force loaded substructure is incorrect with respect to the original problem. It is shown that, using the results from the pre-processing step, where one obtains the equivalent loading, one can obtain the correct response of the force loaded substructure.

The presence of forces on the internal DoF of substructure poses an extra difficulty for most of the substructuring methods, as they require custom designed spatial reduction basis' and/or effort to ensure that the force is captured well by the reduction basis. Hence, the methods proposed in this paper could serve as an simple pre-processing step, such that one can apply the classical CMS methods or IBS methods, without having to worry about internal forces on the reduced substructures.

References

1. Craig R (2000) Coupling of substructures for dynamic analyses – an overview. In: Proceedings of AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics, and materials conference and exhibit, April 2000, pp 1573–1584
2. de Klerk D, Rixen DJ (2010) Component transfer path analysis method with compensation for test bench dynamics. *Mech Syst Signal Proces* 24:1693–1710
3. de Klerk D, Rixen D, Voormeeren S (2008) General framework for dynamic substructuring: history, review and classification of techniques. *AIAA J* 46(5):1169–1181
4. de Vries W, Vemula NK, Passon P, Fischer T, Kaufer D, Matha D, Schmidt B, Vorpahl F (2011) Final report WP 4.2 - support structure concepts for deep water sites. Tech. rep., Project UpWind
5. Dickens JM, Stroeve A (2000) Modal truncation vectors for reduced dynamic substructure models. In: Structures, structural dynamics and material conference (Atlanta, 3–6 April 2000). 41st AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics, and materials conference and exhibit. AIAA-2000-1578
6. Jonkman J, Butterfield S, Musial W, Scott G (2009) Definition of a 5-MW reference wind turbine for offshore system development. Tech. rep., National Renewables Energy Laboratory
7. Morison JR, O'Brien MP, Johnson JW, Schaaf SA (1950) The force exerted by surface waves on piles. *Petroleum Trans (American Institute of Mining Engineers)* 189:149–154
8. Newmark N (1959) A method of computation for structural dynamics. In: Proceedings of the American society of civil engineers. *Journal of the Engineering Mechanics Division* (1959), pp 67–94
9. Nortier BP, Voormeeren SN, Rixen DJ (2012) Application of residual vectors to superelement modeling of an offshore wind turbine foundation. In: Proceedings of the SEM IMAC XXX conference, Jacksonville, FL, USA, 30 Jan–2 Feb 2012
10. Rixen DJ (2004) A dual Craig-Bampton method for dynamic substructuring. *J Comput Appl Math* 168:383–391
11. Rixen DJ, van der Valk PLC (XXXX) An impulse based substructuring approach for impact analysis and load case simulations. *J Sound Vib*
12. Rixen DJ, Boogaard MA, van Schothorst G, van der Poel GW (XXXX) Blocked forces and free displacements approaches for vibration transmission analysis. *Mech Syst Signal Process* (submitted)
13. van der Valk PLC, Rixen DJ (2012) Impulse based substructuring for coupling offshore structures and wind turbines in aero-elastic simulations. In: Proceedings of 53rd AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics and materials conference, Honolulu, HI, 23–26 April 2012

Chapter 9

Error Estimation and Adaptive Model Reduction Applied to Offshore Wind Turbine Modeling

S.N. Voormeeren, B.P. Nortier, and D.J. Rixen

Abstract Recently developed error estimation methods provide a powerful tool for the efficient creation of componentwise reduced models. Error estimation methods consist in estimating the contribution of each component to the reduction error of the assembled system, without evaluation of the full solution. This allows applying component model reduction in an adaptive manner, meaning that component models are refined based on their error contribution. By doing so, one can obtain optimal reduced models of the assembled structure in terms of accuracy with respect to model size. In this paper these methods are applied to the modeling of an offshore wind turbine.

In practice the dynamics of offshore wind turbines are analyzed using aero-elastic codes based on geometrically simplified structural models. As thousands of simulations are required per wind farm for verification purposes, such coarse models allow reasonable computation times while still capturing the overall dynamic behavior. However, the wind industry is moving towards more complex offshore foundation structures, such as jackets and tripods. Even the simplest models of such structures have many more DoF than the complete wind turbine model, leading to excessive computational cost. In this paper it is therefore proposed to apply error estimation and adaptive model reduction to obtain compact models of the total offshore wind turbine. Through application on a realistic model of a wind turbine and complex offshore foundation, it will be shown that this approach gives very compact yet accurate models of the combined structure.

Keywords Component mode synthesis • Error estimation • Adaptive model reduction • Offshore wind turbine modeling

9.1 Introduction

9.1.1 Dynamic Substructuring and Reduced Component Modeling

Dynamic substructuring is an approach that is used more and more often in the structural dynamic analysis of large and complex systems. Using this methodology, the dynamic behavior of the built-up structure is determined by the assembly of dynamic models of its components. Several advantages are associated to this approach, such as exchanging models between different project groups, fast reanalysis when components are changed, tailoring of the models to the components' characteristics, etc. Substructuring techniques can be applied to numerical finite element models, experimentally identified models, and combinations of numerical and experimental data (often called *hybrid* analysis). In this paper the focus is on the former.

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When applying substructuring to numerical finite element (FE) analysis, a powerful approach is to combine the componentwise analysis with component model reduction. This approach is often referred to as *component mode synthesis* (CMS). Such methods have already been developed in the 1960's [7, 8] and are still often used today. The typical workflow is to divide the system into components, obtain reduced component models by modal truncation, assemble these, and, if needed, further reduce the model size using interface reduction. With this approach very compact numerical models can be created for structural dynamic analysis of complex structures.

However, one important question that often remains unaddressed is how accurate these reduced models actually are. Indeed, to confidently use these models in practice requires insight in their accuracy, as both the component and interface reduction incur an error on the total model. In these procedures, physical DoF are replaced by generalized DoF through a truncated set of modes, leading to an approximation of the dynamic behavior. The accuracy of this approximation depends on the number of modes used to generate the model: the more modes the higher the accuracy. The computational efficiency of the reduced model is inversely related to the number of modes: more modes means more degrees of freedom (DoF).

Finding the right balance between accuracy and efficiency is not trivial and is in practice often done using simple rules of thumb. One often applied method is frequency selection where all modes up to a certain frequency are included. In order to test the accuracy of the so obtained reduced system, its response can be compared to that of the unreduced system. When the accuracy proves to be unsatisfactory, more modes can be iteratively included. However, two difficulties are associated with this approach. Firstly, obtaining the reference solution from the unreduced system is in general (very) computationally expensive. Secondly, no insight is gained in which components to enrich in order to improve the accuracy in the most effective way.

9.1.2 Error Estimation of Reduced Models

In this paper a more systematic approach for quantification of the accuracy of the reduced model is therefore developed. This is done on the basis of *error estimation* methods. Thereby the goal is to quantify the error arising from the component and interface reduction, such that effective enrichment schemes can be developed. This problem is very similar to the one arising one step earlier in the modeling process, namely in the construction of an adequate finite element model. Due to the discretization of the continuous problem a difference between the analytical and discrete solutions results, called the *discretization error*. Assuming an appropriate element type is chosen, this error is governed by the mesh size. Refining the mesh decreases the error, but also increases the computational effort and required storage space.

Various error estimation techniques have been developed in the field of FE modeling to predict and control the discretization error, based on which (local) mesh refinement strategies have been derived (see e.g. [2, 3]). An important requirement for these methods is that they provide a conservative estimate of the error, i.e. the error should never be underestimated. In this context one therefore usually speaks of error bounds instead of the actual error, since the latter generally remains unknown.

Two classes of error estimation methods are defined in the literature: *a priori* and *a posteriori* methods, referring to when in the modeling and analysis process the error estimation is performed. *A priori* error estimation is used to predict a certain error before the actual computation is performed. The error however is expressed in terms of constants which depend on the exact solution. The resulting estimates do not quantify error bounds but only provide information on the convergence and the stability properties of the approximate model and the asymptotic behavior of the error if the refinement parameters are changed [2].

On the other hand, *a posteriori* error estimation is used to give both qualitative and quantitative measures of the error after the actual computation, using only the approximate solutions obtained from the reduced system. This type of estimation is able to provide both global and local information. An example of the former is an error bound expressed in terms of some global norm (e.g. energy), while local information can for instance be the error in displacements or stresses at a specific target location in the model. From the advantages described above it is understood that a *a posteriori* estimation has become a popular method in error estimate and refinement strategies [3].

Given the discussion above, the focus of this chapter is the development of an *a posteriori* error estimation method for reduced substructured models. Indeed, error estimation techniques used for mesh refinement are seldom applied to component mode synthesis problems; in the literature only a few examples were found [9–11, 20]. The main ideas put forward in this paper are based on these publications, but the applied mathematics are adapted and simplified. More specifically, the original functional analysis notation is translated to algebraic notation as typically used in the structural dynamics community. Using the proposed concepts, error estimates can be derived for different types of dynamic analysis, based on which refinement strategies can be formulated. These allow to iteratively refine the component reduction bases until a satisfactory result is obtained (i.e. the error estimate drops below a certain threshold), a process that will be referred to as *adaptive model reduction*.

9.1.3 Paper Outline

The remainder of this paper is structured as follows. Next, section 9.2 derives the well-known Craig-Bampton method in a way that is suitable for error estimation. In section 9.3 error estimates are derived for systems excited by an external harmonic force. Thereafter, error estimates for approximations of eigenfrequencies and -modes of the assembled system are developed in section 9.4. Refinement schemes based on the error estimates are proposed in section 9.5. It is noted that in these two sections only a concise version of the derivation is given; for reasons of both compactness and clarity the full mathematical details are deemed out of the scope of this paper. Instead, for details of the algebraic derivation the reader is referred to [21].

In order to illustrate the use of error estimation and adaptive model reduction methods, a case study is performed in the second part of this paper in which the methods are applied to the modeling of an offshore wind turbine on a complex jacket foundation. In practice the dynamics of offshore wind turbines are analyzed using strongly simplified structural models. As thousands of simulations are required per wind farm for verification purposes, such coarse models allow reasonable computation times while still capturing the overall dynamic behavior. However, for a complex offshore foundation type such as a jacket, even the simplest models have many more DoF than the wind turbine model, leading to excessive computational cost. In the case study, error estimation and adaptive model reduction are therefore applied to obtain compact yet accurate models of the total offshore wind turbine. To this end, the wind turbine model is described in section 9.6; the results of a number of numerical experiments are subsequently presented in section 9.7. The paper is ended with conclusions in section 9.8.

9.2 Component Model Transformation, Assembly and Reduction

As was explained in the introduction, a technique that is often applied in the structural dynamic analysis of large or complex systems is component mode synthesis (CMS). This methodology combines the ideas of a componentwise “divide and conquer” approach and model reduction techniques as follows:

1. The complete system is decomposed into components or substructures.
2. Each component is modeled and reduced separately, using a suitable reduction basis. This basis usually consists of a set of static modes describing the interaction with neighboring components, and some vibration modes to account for the dynamics.
3. The reduced component models are assembled to form the model of the complete system.
4. Finally, in case the resulting model still has too many degrees of freedom arising from the component interfaces, interface reduction can be applied.

One very popular CMS method is the Craig-Bampton method [5]. This is the method considered in this work and will be explained hereafter. However, in order to enable error estimation and adaptive reduction the method is derived in a slightly different way than usual, as will be explained next.

9.2.1 Model Transformation Using Static Modes

As a starting point we take the equations of motion of a discretized dynamic component model s :

$$\mathbf{M}^{(s)}\ddot{\mathbf{u}}^{(s)}(t) + \mathbf{C}^{(s)}\dot{\mathbf{u}}^{(s)}(t) + \mathbf{K}^{(s)}\mathbf{u}^{(s)}(t) = \mathbf{f}^{(s)}(t) + \mathbf{g}^{(s)}(t) \quad (9.1)$$

Here $\mathbf{M}^{(s)}$, $\mathbf{C}^{(s)}$ and $\mathbf{K}^{(s)}$ are the components mass, damping and stiffness matrix, respectively. Vector $\mathbf{u}^{(s)}(t)$ denotes the degrees of freedom, and $\dot{\mathbf{u}}^{(s)}(t)$ and $\ddot{\mathbf{u}}^{(s)}(t)$ its respective time derivatives, $\mathbf{f}^{(s)}(t)$ the external force vector and $\mathbf{g}^{(s)}(t)$ the vector of the connection forces with other component models. To simplify the expressions we will drop the explicit time dependency. Furthermore, damping will not be considered here, but it can be included in the developments as long as the structure is lightly damped.

To allow assembly with other component models we partition the DoF vector $\mathbf{u}$ into a part belonging the interface (boundary) DoF, denoted by $\mathbf{u}_b$, and the internal DoF, denoted by $\mathbf{u}_i$. This gives the partitioned component equations of motion:

$$\begin{bmatrix} \mathbf{M}_{ii}^{(s)} & \mathbf{M}_{ib}^{(s)} \\ \mathbf{M}_{bi}^{(s)} & \mathbf{M}_{bb}^{(s)} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_i^{(s)} \\ \ddot{\mathbf{u}}_b^{(s)} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii}^{(s)} & \mathbf{K}_{ib}^{(s)} \\ \mathbf{K}_{bi}^{(s)} & \mathbf{K}_{bb}^{(s)} \end{bmatrix} \begin{bmatrix} \mathbf{u}_i^{(s)} \\ \mathbf{u}_b^{(s)} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_i^{(s)} \\ \mathbf{f}_b^{(s)} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{g}_b^{(s)} \end{bmatrix} \quad (9.2)$$

Note that no connection forces $\mathbf{g}$ are applied at the internal DoF. This partitioning can be made for all components $s = 1 \dots n$ of the total system, and their submatrices can subsequently be collected and expressed using block notation as:

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{bmatrix} = \begin{bmatrix} \mathbf{f}_i \\ \mathbf{f}_b \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{g}_b \end{bmatrix} \quad (9.3)$$

Here the following block diagonal matrices have been defined:

$$\begin{aligned} \mathbf{M}_{ii} &= \text{diag} \left(\mathbf{M}_{ii}^{(1)}, \dots, \mathbf{M}_{ii}^{(n)} \right) & \mathbf{K}_{ii} &= \text{diag} \left(\mathbf{K}_{ii}^{(1)}, \dots, \mathbf{K}_{ii}^{(n)} \right) \\ \mathbf{M}_{ib} &= \mathbf{M}_{bi}^T = \text{diag} \left(\mathbf{M}_{ib}^{(1)}, \dots, \mathbf{M}_{ib}^{(n)} \right) & \mathbf{K}_{ib} &= \mathbf{K}_{bi}^T = \text{diag} \left(\mathbf{K}_{ib}^{(1)}, \dots, \mathbf{K}_{ib}^{(n)} \right) \\ \mathbf{M}_{bb} &= \text{diag} \left(\mathbf{M}_{bb}^{(1)}, \dots, \mathbf{M}_{bb}^{(n)} \right) & \mathbf{K}_{bb} &= \text{diag} \left(\mathbf{K}_{bb}^{(1)}, \dots, \mathbf{K}_{bb}^{(n)} \right) \end{aligned} \quad (9.4)$$

Furthermore, the vectors are written in block form as:

$$\mathbf{q}_i = \begin{bmatrix} \mathbf{q}_i^{(1)} \\ \vdots \\ \mathbf{q}_i^{(n)} \end{bmatrix} \triangleq \text{col} \left(\mathbf{q}_i^{(1)}; \dots; \mathbf{q}_i^{(n)} \right) \quad \begin{aligned} \mathbf{u}_b &= \text{col} \left(\mathbf{u}_b^{(1)}; \dots; \mathbf{u}_b^{(n)} \right) & \mathbf{g}_b &= \text{col} \left(\mathbf{g}_b^{(1)}; \dots; \mathbf{g}_b^{(n)} \right) \\ \mathbf{f}_i &= \text{col} \left(\mathbf{f}_i^{(1)}; \dots; \mathbf{f}_i^{(n)} \right) & \mathbf{f}_b &= \text{col} \left(\mathbf{f}_b^{(1)}; \dots; \mathbf{f}_b^{(n)} \right) \end{aligned} \quad (9.5)$$

The equations of motion in (9.3) thus contain all the information from the components, but are still uncoupled. Next, these uncoupled equations will be reduced using the Craig-Bampton method, which consists in retaining the boundary and finding a good approximation for the internal DoF. This is done by splitting the response of the internal DoF into a quasi-static and a dynamic part:

$$\mathbf{u}_i = \mathbf{u}_{i,\text{stat}} + \mathbf{u}_{i,\text{dyn}} \quad (9.6)$$

The static response $\mathbf{u}_{i,\text{stat}}$ is obtained from the first line of (9.3) by setting the force on the internal DoF to zero (i.e. $\mathbf{f}_i = \mathbf{0}$) and neglecting the inertia forces by taking $\ddot{\mathbf{u}}_i$ and $\ddot{\mathbf{u}}_b$ to zero. By doing so we obtain:

$$\mathbf{u}_{i,\text{stat}} = -\mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} \mathbf{u}_b = \Psi_{C,i} \mathbf{u}_b \quad (9.7)$$

Here $\Psi_{C,i}$ are the so-called static constraint modes; note that this is a block-diagonal matrix containing the static constraint modes of the individual components. Each column of $\Psi_{C,i}$ represents the static response of $\mathbf{u}_i$ to a unit displacement of an interface DoF, while the other interface DoF are constrained. The number of static modes thus equals the number of boundary DoF. For components with large or refined interfaces this could limit the effectiveness of the reduction process. To overcome this issue interface reduction can be applied, as will be explained later.

The static modes are used to devise the following DoF transformation:

$$\begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \Psi_c \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}_i \\ \mathbf{u}_b \end{bmatrix} \quad (9.8)$$

Application of this transformation to the original equations of motion in eq. (9.3) gives:

$$\begin{bmatrix} \mathbf{M}_{ii} & \bar{\mathbf{M}}_{ib} \\ \bar{\mathbf{M}}_{bi} & \bar{\mathbf{M}}_{bb} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_{i,\text{dyn}} \\ \ddot{\mathbf{u}}_b \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{K}}_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{i,\text{dyn}} \\ \mathbf{u}_b \end{bmatrix} = \begin{bmatrix} \mathbf{f}_i \\ \bar{\mathbf{f}}_b \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{g}_b \end{bmatrix}, \quad (9.9)$$

with the (block diagonal) submatrices defined as¹:

$$\begin{aligned} \bar{\mathbf{K}}_{bb} &= \mathbf{K}_{bb} - \mathbf{K}_{bi} \mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} & \bar{\mathbf{M}}_{bb} &= \mathbf{M}_{bb} + \Psi_c^T \mathbf{M}_{ii} \Psi_c + \Psi_c^T \mathbf{M}_{ib} + \mathbf{M}_{bi} \Psi_c \\ \bar{\mathbf{M}}_{ib} &= \bar{\mathbf{M}}_{bi}^T = \mathbf{M}_{ii} \Psi_c + \mathbf{M}_{ib} & \bar{\mathbf{f}}_b &= \mathbf{f}_b + \Psi_c^T \mathbf{f}_i \end{aligned} \quad (9.10)$$

Note that the $\mathbf{u}_{i,\text{dyn}}$ correspond to the dynamic response of the internal DoF relative to the static displacements caused by static constraint modes; furthermore since all static modes are used the above transformation is exact.

¹Note that these matrices correspond to the reduced matrices of the Guyan approach.

9.2.2 Model Assembly

The transformed equations of motion in eq. (9.9) are still unassembled. Assembly is achieved by applying the so-called primal transformation. Using this transformation, the individual sets of component interface DoF are replaced by one unique interface displacement field, more details can be found in [21, 22]. This transformation can be written as:

$$\begin{bmatrix} \mathbf{u}_{i,\text{dyn}} \\ \mathbf{u}_b \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_b \end{bmatrix} \begin{bmatrix} \mathbf{u}_{i,\text{dyn}} \\ \mathbf{u}_\gamma \end{bmatrix}, \quad (9.11)$$

Here $\mathbf{L}_b$ is a Boolean assembly matrix comparable to the assembly matrices used to assemble individual finite elements.² This Boolean matrix is the block version of the component Booleans $\mathbf{L}_b^{(s)}$, which localize the component boundary DoF from the unique set of interface displacements, thereby satisfying the interface compatibility condition:

$$\mathbf{L}_b = \text{col}(\mathbf{L}_b^{(1)}; \dots; \mathbf{L}_b^{(n)}) \quad \text{where} \quad \mathbf{u}_b^{(s)} = \mathbf{L}_b^{(s)} \mathbf{u}_\gamma \quad (9.12)$$

Pre and post-multiplication of eq. (9.9) with this transformation leads to an assembled, unreduced set of equations with a block diagonal stiffness matrix:

$$\begin{bmatrix} \mathbf{M}_{ii} & \bar{\mathbf{M}}_{ib} \mathbf{L}_b \\ \mathbf{L}_b^T \bar{\mathbf{M}}_{bi} & \mathbf{L}_b^T \bar{\mathbf{M}}_{bb} \mathbf{L}_b \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_{i,\text{dyn}} \\ \ddot{\mathbf{u}}_\gamma \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_b^T \bar{\mathbf{K}}_{bb} \mathbf{L}_b \end{bmatrix} \begin{bmatrix} \mathbf{u}_{i,\text{dyn}} \\ \mathbf{u}_\gamma \end{bmatrix} = \begin{bmatrix} \mathbf{f}_i \\ \mathbf{L}_b^T \bar{\mathbf{f}}_b \end{bmatrix} \quad (9.13)$$

In the derivation of the error estimation methods, the above assembled equations will be considered as the reference system. The benefit of this formulation is that the stiffness matrix is block diagonal, a property that will be used in subsequent derivations. As a result, the interface can be considered as an additional domain to the internal component domains. Whereas in the CB method one usually considers a physical component (internal plus boundary DoF) as one domain, here only its internal DoF are defined as one domain. An additional domain is thereby defined, namely the global interface domain (i.e. the “sum” of all interfaces). As a result, an assembly of n components can now be seen as consisting of $n + 1$ domains, which is illustrated by expanding the block notation of the stiffness matrix:

$$\bar{\mathbf{K}} = \begin{bmatrix} \mathbf{K}_{ii}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{ii}^{(2)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{ii}^{(n)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_b^T \bar{\mathbf{K}}_{bb} \mathbf{L}_b \end{bmatrix} \quad \text{with} \quad \mathbf{L}_b^T \bar{\mathbf{K}}_{bb} \mathbf{L}_b = \sum_{s=1}^n \mathbf{L}_b^{(s)T} \bar{\mathbf{K}}_{bb}^{(s)} \mathbf{L}_b^{(s)} \quad \text{and} \quad \bar{\mathbf{u}} = \begin{bmatrix} \mathbf{u}_{i,\text{dyn}}^{(1)} \\ \mathbf{u}_{i,\text{dyn}}^{(2)} \\ \vdots \\ \mathbf{u}_{i,\text{dyn}}^{(n)} \\ \mathbf{u}_\gamma \end{bmatrix} \quad (9.14)$$

The interface is simply regarded as domain $n + 1$. Similarly, the mass matrix can be expanded to reveal the coupling terms between the internal and boundary domains:

$$\bar{\mathbf{M}} = \begin{bmatrix} \mathbf{M}_{ii}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{M}}_{ib}^{(1)} \mathbf{L}_b^{(1)} \\ \mathbf{0} & \mathbf{M}_{ii}^{(2)} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{M}}_{ib}^{(2)} \mathbf{L}_b^{(2)} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{ii}^{(n)} & \bar{\mathbf{M}}_{ib}^{(n)} \mathbf{L}_b^{(n)} \\ \mathbf{L}_b^{(1)T} \bar{\mathbf{M}}_{bi}^{(1)} & \mathbf{L}_b^{(2)T} \bar{\mathbf{M}}_{bi}^{(2)} & \dots & \mathbf{L}_b^{(n)T} \bar{\mathbf{M}}_{bi}^{(n)} & \mathbf{L}_b^T \bar{\mathbf{M}}_{bb} \mathbf{L}_b \end{bmatrix} \quad \text{with} \quad \mathbf{L}_b^T \bar{\mathbf{M}}_{bb} \mathbf{L}_b = \sum_{s=1}^n \mathbf{L}_b^{(s)T} \bar{\mathbf{M}}_{bb}^{(s)} \mathbf{L}_b^{(s)} \quad (9.15)$$

In compact notation this transformed version of the equations of motion will be written as:

$$\bar{\mathbf{M}} \ddot{\bar{\mathbf{u}}} + \bar{\mathbf{K}} \bar{\mathbf{u}} = \bar{\mathbf{f}} \quad (9.16)$$

²This assembly procedure can also be used when the interface discretizations are non-matching, but in that case $\mathbf{L}_b$ is no longer Boolean.

9.2.3 Model Reduction

In the last step of the derivation of the reduced model, the actual reduction is performed by means of modal truncation. In the current formulation, both the internal domains and the boundary DoF are reduced simultaneously. The former are reduced using the fixed interface modes traditionally used in the CB method. For each component s , these fixed interface vibration modes are obtained from solving the eigenproblem of the internal DoF:

$$\left(\mathbf{K}_{ii}^{(s)} - \omega_{i,n}^{2(s)} \mathbf{M}_{ii}^{(s)} \right) \Phi_{i,n}^{(s)} = \mathbf{0} \quad \text{for } s = 1 \dots n \quad (9.17)$$

Here $\Phi_{i,n}^{(s)}$ represents a single fixed interface vibration mode with unit modal mass and $\omega_{i,n}^{2(s)}$ its corresponding eigenfrequency. Combining these modes in a blockwise fashion for all internal domains, an approximation for the dynamic response of the internal DoF is obtained:

$$\mathbf{u}_{i,\text{dyn}} \approx \Phi_i \boldsymbol{\eta}_i \quad (9.18)$$

Next, the interface vibration modes are found by solving the statically condensed interface problem

$$\left(\mathbf{L}_b^T \bar{\mathbf{K}}_{bb} \mathbf{L}_b - \omega_{\gamma,j}^2 \mathbf{L}_b^T \bar{\mathbf{M}}_{bb} \mathbf{L}_b \right) \phi_{\gamma,j} = \mathbf{0}, \quad (9.19)$$

with $\phi_{\gamma,j}$ the j^{th} interface vibration mode and $\omega_{\gamma,j}^2$ its associated frequency. Again, the reduction is obtained by truncating the number of modes. Taking the n_γ interface modes with the lowest frequency, the interface DoF are approximated by:

$$\mathbf{u}_\gamma \approx \Phi_\gamma \boldsymbol{\eta}_\gamma \quad (9.20)$$

Combining the approximations for the internal and boundary DoF of eqs. (9.18) and (9.20) in a block diagonal matrix gives the following reduction basis:

$$\begin{bmatrix} \mathbf{u}_{i,\text{dyn}} \\ \mathbf{u}_\gamma \end{bmatrix} \approx \begin{bmatrix} \Phi_i & \mathbf{0} \\ \mathbf{0} & \Phi_\gamma \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_i \\ \boldsymbol{\eta}_\gamma \end{bmatrix} = \mathbf{R} \mathbf{q} \quad (9.21)$$

Application of this reduction finally leads to the Craig-Bampton reduced assembled system:

$$\begin{bmatrix} \mathbf{I} & \Phi_i^T \bar{\mathbf{M}}_{ib} \mathbf{L}_b \Phi_\gamma \\ \Phi_\gamma^T \mathbf{L}_b^T \bar{\mathbf{M}}_{bi} \Phi_i & \mathbf{I} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\eta}}_i \\ \ddot{\boldsymbol{\eta}}_\gamma \end{bmatrix} + \begin{bmatrix} \Omega_i^2 & \mathbf{0} \\ \mathbf{0} & \Omega_\gamma^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_i \\ \boldsymbol{\eta}_\gamma \end{bmatrix} = \begin{bmatrix} \Phi_i^T \mathbf{f}_i \\ \Phi_\gamma^T \mathbf{L}_b^T \tilde{\mathbf{f}}_b \end{bmatrix} \quad (9.22)$$

The above equation can be cast in a more compact notation as:

$$\tilde{\mathbf{M}} \ddot{\mathbf{q}} + \tilde{\mathbf{K}} \mathbf{q} = \tilde{\mathbf{f}} \quad (9.23)$$

Finally it is noted that if one chooses to leave the interface unreduced, i.e. $\Phi_\gamma = \mathbf{I}$, the reduced matrices in (9.22) will be equal to those obtained using the standard CB method.

After performing any type of structural dynamic analysis on the assembled reduced system of equation (9.23), the solution can be expanded to the space of the physical DoF using the reduction basis in (9.21). Since the reduction is an approximation the dynamic equilibrium is never satisfied exactly, giving rise to a force residual. This force residual can be calculated by substitution of the expanded reduced solution into the unreduced, but transformed equations of motion in (9.16):

$$\mathbf{r} = \tilde{\mathbf{f}} - \tilde{\mathbf{M}} \mathbf{R} \ddot{\mathbf{q}} - \tilde{\mathbf{K}} \mathbf{R} \mathbf{q} \quad (9.24)$$

Physically, the force residual can be interpreted as the constraining forces needed to restrict the system to its reduced displacement space. The magnitude of the residual depends on how well the contained modes describe the full system response.

9.3 Error Estimation for Harmonic Solutions

One type of analysis that is regularly encountered in structural dynamics is the harmonic analysis. For a system excited by a harmonic force it holds for the steady state solution that $\ddot{\mathbf{q}} = -\omega_e^2 \mathbf{q}$. Hence, one can define the dynamic stiffness of the unreduced system as

$$\bar{\mathbf{Z}} = \bar{\mathbf{K}} - \omega_e^2 \bar{\mathbf{M}}, \quad (9.25)$$

such that the force residual can be written as:

$$\mathbf{r} = \bar{\mathbf{f}} - \bar{\mathbf{Z}} \mathbf{R} \mathbf{q} \quad (9.26)$$

This residual can be allocated to the different domains:

$$\mathbf{r} = \text{col} \left(\mathbf{r}_i^{(1)}; \dots; \mathbf{r}_i^{(n)}; \mathbf{r}_\gamma \right) \quad (9.27)$$

An important property of this residual is its orthogonality with respect to the reduction basis, both on domain and assembly level:

$$\mathbf{R}^T \mathbf{r} = \mathbf{0} \quad \text{and} \quad \mathbf{R}^{(s)T} \mathbf{r}^{(s)} = \mathbf{0} \quad (9.28)$$

This property is referred to as Galerkin orthogonality, which simply means that the force residual exists only in the space spanned by the discarded modes. By definition the residual will therefore be zero if all the modes are retained; in that case $\mathbf{R}$ is merely a transformation. The fact that the residual vector can be split into domain contributions hints that it might be used as an indicator for the relative error produced by each domain. However, knowledge of the domain force residual does not directly relate to the size of the domain error.

The error is defined as the difference between the full and expanded reduced solution:

$$\mathbf{e} = \bar{\mathbf{u}} - \mathbf{R} \mathbf{q} = \bar{\mathbf{u}} - \tilde{\mathbf{u}} \quad (9.29)$$

Since the full solution creates no force residual, the residual of the reduced system can also be written as:

$$\mathbf{r} = \bar{\mathbf{Z}} \bar{\mathbf{u}} - \bar{\mathbf{Z}} \mathbf{R} \mathbf{q} = \bar{\mathbf{Z}} (\bar{\mathbf{u}} - \tilde{\mathbf{u}}) = \bar{\mathbf{Z}} \mathbf{e} \quad (9.30)$$

Clearly, there is a relation between the error and the residual; to compute the error however still requires the full solution. In the remainder of this section an upper bound will therefore be derived for the error on reduced solutions for the harmonic problem. To this end, the next subsection introduces the Dual Weighted Residual (DWR) method, which enables error estimates for quantities of interest. This approach is generalized in subsection 9.3.2 to obtain a global error bound in terms of the energy norm. As explained earlier only the main steps in the derivations are shown in this paper, for more details see [21]

9.3.1 Goal Oriented Error Estimates: The Dual Weighted Residual Method

The dual-weight-residual (DWR) method is often used to estimate the error resulting from discretization of a continuous domain by means of finite elements, which is then used as input for mesh refinement strategies. Less common is its use in model reduction, for one of the few examples in the literature see [15]. Here the DWR method will be employed to estimate the reduction error by finding a relation between the known force residual $\mathbf{r}$ and the unknown error $\mathbf{e}$.

When performing structural dynamic analyses one is often interested in a certain output quantity, e.g. the displacement or stress at a particular location in the structure. To measure the contribution of the reduction error to these quantities one can define the so-called linear goal oriented function, or target functional, as:

$$J(\bar{\mathbf{u}}) = \mathbf{a}^T \bar{\mathbf{u}} \quad (9.31)$$

Vector $\mathbf{a}$ is a selection vector which selects the predefined quantities of interest. Equally, this goal oriented function can be evaluated for the approximate reduced solution:

$$J(\tilde{\mathbf{u}}) = \mathbf{a}^T \tilde{\mathbf{u}} \quad (9.32)$$

Since the target functional is linear, subtracting the two gives the linear target functional of the error, also called goal oriented error:

$$J(e) = J(\bar{u}) - J(\tilde{u}) = \mathbf{a}^T (\bar{u} - \tilde{u}) = \mathbf{a}^T e \quad (9.33)$$

This goal oriented error cannot be evaluated without knowledge of the full solution. Recalling that $\bar{u}$ is the solution to the harmonic problem with the dynamic stiffness matrix $\bar{Z}$, one may rewrite the goal oriented error by:

$$J(e) = \mathbf{a}^T (\bar{u} - \tilde{u}) = (\bar{Z}^{-1} \mathbf{a})^T \mathbf{r} = \mathbf{d}^T \mathbf{r} \quad (9.34)$$

Here use was made of the symmetry of the dynamic stiffness matrix and $\mathbf{d}$ is referred to as the solution of the *adjoint problem*³:

$$\bar{Z} \mathbf{d} = \mathbf{a} \quad (9.35)$$

The adjoint solution $\mathbf{d}$ can be interpreted as the response of the system excited at the quantities of interest, defined in vector $\mathbf{a}$. Since the goal oriented error in the last line of eq. (9.34) is a vector product, it may be expressed as a summation over the individual domains:

$$J(e) = \sum_{s=1}^{n+1} \mathbf{d}^{(s)T} \mathbf{r}^{(s)} \quad \text{denoted by } e_{\Sigma} \quad (9.36)$$

Here $\mathbf{r}^{(s)}$ is the force residual of domain s (note that for $s = n + 1$ one has $\mathbf{r}_{\gamma}$), weighting by the trace of the adjoint solution $\mathbf{d}^{(s)}$ on the same domain this gives the domain error; their summation yields the total goal oriented error e_{Σ} . Hence, the value of the domain error gives an indication of its relative contribution to the total goal oriented error.

The domain errors can be positive and negative, such that in the summation the error in one domain can counteract the error in another. An upper bound for the error can therefore be obtained by using the triangle inequality:

$$|J(e)| \leq \sum_{s=1}^{n+1} \left| \mathbf{d}^{(s)T} \mathbf{r}^{(s)} \right| \quad \text{denoted by } e_{\leq \Sigma} \quad (9.37)$$

Here the notation $e_{\leq \Sigma}$ is introduced to denote the upper bound for the goal oriented error. However, to evaluate the goal oriented error one needs the solution to the adjoint problem, which requires solving the full system. Obviously this would take similar computational effort as solving for the full solution $\bar{u}$ which would allow the error e to be computed directly. Still, using an adjoint problem formulation is beneficial in case many load cases need to be solved. One would then want a reduced model to run all the load cases, but one could afford a full solution for the adjoint problem to estimate the accuracy.

Furthermore, in practice the adjoint solution is often approximated using a reduced model that is more refined than the one used to obtain $\tilde{u}$. As a result the inequality in expression (9.37) becomes an approximation:

$$|J(e)| \leq \sum_{s=1}^{n+1} \left| \mathbf{d}^{(s)T} \mathbf{r}^{(s)} \right| \approx \sum_{s=1}^{n+1} \left| \tilde{\mathbf{d}}^{(s)T} \mathbf{r}^{(s)} \right| = \sum_{s=1}^{n+1} m_J^{(s)} \quad \text{denoted by } e_{\approx \Sigma} \quad (9.38)$$

In this expression $\tilde{\mathbf{d}}$ is the approximate adjoint solution and $m_J^{(s)}$ an error indicator, collected in the vector $\mathbf{m}_J$, that is introduced to unify notations in the coming sections. It is emphasized that the approximate adjoint solution must be computed using a more refined model than the “nominal” model used to compute $\tilde{u}$.

9.3.2 Global Energy Norm Based Error Estimate

The main difficulty associated with the goal oriented error estimates derived above is that they require solving the adjoint problem. In this subsection the DWR method is therefore generalized to obtain a global error estimate that does not require any adjoint solution. In order to do so, first the following terms are defined to unify the notation for the internal and interface domains:

³In the literature this problem is also referred to as the *dual problem*.

$$\Phi_l^{(s)} = \begin{cases} \Phi_i^{(s)} & \text{for } s = 1 \dots n \\ \Phi_\gamma & \text{for } s = n + 1 \end{cases} \quad \text{and} \quad M^{(s)} = \begin{cases} M_{ii} & \text{for } s = 1 \dots n \\ L_b^T \bar{M}_{bb} L_b & \text{for } s = n + 1 \end{cases} \quad (9.39)$$

Here it is assumed that $\Phi_l^{(s)}$ contains the k_s lowest eigenmodes of each domain and normalized with respect to their corresponding mass matrix $M^{(s)}$. Note that $\Phi_l^{(s)}$ are the eigenmodes of $(K^{(s)}, M^{(s)})$ as used earlier for the Craig-Bampton method and interface reduction. Using these definitions, an operator $P^{(s)}$ can be defined for each domain as:

$$P^{(s)} = \Phi_l^{(s)} \Phi_l^{(s)T} M^{(s)} \quad (9.40)$$

Next, use is made of the fact that the domain residual is orthogonal to the associated domain reduction basis (Galerkin orthogonality), such that the goal oriented error in eq. (9.37) can be formulated as:

$$|J(e)| \leq \sum_{s=1}^{n+1} \left| d^{(s)T} r^{(s)} \right| = \sum_{s=1}^{n+1} \left| \left((I - P^{(s)}) d^{(s)} \right)^T r^{(s)} \right| \quad (9.41)$$

The above expression can be rewritten using the so-called Cauchy-Schwarz inequality. With this inequality, one can write the goal oriented error in eq. (9.41) as follows (for details see [21]):

$$|J(e)| \leq \sum_{s=1}^{n+1} \left\| (I - P^{(s)}) d^{(s)} \right\| \left\| \hat{r}^{(s)} \right\| \quad (9.42)$$

Here $\|\star\|$ is the L^2 norm $\sqrt{\star^T M^{(s)} \star}$, and it has been defined that $\hat{r}^{(s)} = M^{(s)-1} r^{(s)}$. Hence, the force residual is translated to an acceleration residual by pre-multiplication with the inverse mass matrix. In practice it might however be undesirable to factorize the domain mass matrix. Instead one could employ either use a lumped mass approximation, leading to diagonal domain mass matrices, or evaluate the acceleration residual using a more refined reduced model, similar to the approximate evaluation of the goal oriented error in the previous section.

Next, the projector in the first term on the right hand side is rewritten using the spectral expansion of the identity matrix (see e.g. [6] for the derivation of the spectral expansions). Furthermore, applying the definition of the L^2 norm then gives:

$$|J(e)| \leq \sum_{s=1}^{n+1} \left(d^{(s)T} M^{(s)} \Phi_h^{(s)} \Phi_h^{(s)T} M^{(s)} d^{(s)} \right)^{1/2} \left\| \hat{r}^{(s)} \right\| \quad (9.43)$$

Here $\Phi_h^{(s)}$ contains the domain's higher (i.e. discarded) modes; to obtain this result it was assumed that the domain eigenfrequencies are ordered in a monotonically increasing sequence. Hence one can conservatively scale the goal oriented error using the first discarded eigenvalue as follows:

$$|J(e)| \leq \sum_{s=1}^{n+1} \left(d^{(s)T} M^{(s)} \Phi_h^{(s)} \Omega_h^{(s)2} \Phi_h^{(s)T} M^{(s)} d^{(s)} \right)^{1/2} \frac{1}{\omega_{k_{s+1}}^{(s)}} \left\| \hat{r}^{(s)} \right\| \quad (9.44)$$

Here Ω_h is a diagonal matrix containing the eigenfrequencies associated to the discarded modes in Φ_h . With the spectral expansion of the domain stiffness matrix the bracketed term in (9.44) can be rewritten to obtain:

$$|J(e)| \leq \sum_{s=1}^{n+1} \left\| d^{(s)} \right\|_K \frac{1}{\omega_{k_{s+1}}^{(s)}} \left\| \hat{r}^{(s)} \right\| \quad (9.45)$$

Using the block-diagonality of the stiffness matrix of the transformed unreduced system, see eq. (9.13), and applying once more the Cauchy-Schwarz and triangle inequalities to separate the sum of products, the goal oriented error is found as (see [21]):

$$|J(e)| \leq \|d\|_K \left(\sum_{s=1}^{n+1} \frac{1}{\omega_{k_{s+1}}^{(s)2}} \left\| \hat{r}^{(s)} \right\|^2 \right)^{1/2} \quad (9.46)$$

In order to obtain an estimate for the global error $\mathbf{e}$, the selection vector for the adjoint problem is chosen as $\mathbf{a} = \bar{\mathbf{K}}\mathbf{e}$. Due to this choice, the goal oriented error $J(\mathbf{e})$ becomes a global error estimate in terms of the energy norm:

$$|J(\mathbf{e})| = |\mathbf{a}^T \mathbf{e}| = |\mathbf{e}^T \bar{\mathbf{K}} \mathbf{e}| = \|\mathbf{e}\|_K^2 \quad (9.47)$$

Consequently applying eq. (9.46), the global error estimate is found as:

$$\|\mathbf{e}\|_K^2 \leq \|\mathbf{d}\|_K \left(\sum_{s=1}^{n+1} \frac{1}{\omega_{k_{s+1}}^{(s)^2}} \|\hat{\mathbf{r}}^{(s)}\|^2 \right)^{1/2} \quad (9.48)$$

Here $\mathbf{d}$ is now the solution to the adjoint problem $\bar{\mathbf{Z}}\mathbf{d} = \bar{\mathbf{K}}\mathbf{e}$. However, this estimate still depends on the full adjoint solution. The final step in this derivation is therefore to obtain a bound for the energy norm of the adjoint solution $\|\mathbf{d}\|_K$ in terms of the error $\mathbf{e}$. Therefore, the adjoint solution computed from eq. (9.35) is written using the spectral expansion of the global matrices:

$$\mathbf{d} = \bar{\mathbf{Z}}^{-1} \bar{\mathbf{K}} \mathbf{e} = \sum_{j=1}^{n_a} (\omega_j^2 - \omega_e^2)^{-1} \boldsymbol{\phi}_j \boldsymbol{\phi}_j^T \bar{\mathbf{K}} \mathbf{e} = \sum_{j=1}^{n_a} \omega_j^2 (\omega_j^2 - \omega_e^2)^{-1} \boldsymbol{\phi}_j \boldsymbol{\phi}_j^T \bar{\mathbf{M}} \mathbf{e} \quad (9.49)$$

In this expression n_a is the number of DoF of the assembled, unreduced model; *the eigenfrequencies ω_j and mode shapes $\boldsymbol{\phi}_j$ now also refer to this total model*. Next, using the spectral expansion of the identity matrix, the above expression is rewritten:

$$\mathbf{d} = \sum_{j=1}^{n_a} (\omega_j^2 - \omega_e^2)^{-1} (\omega_j^2 - \omega_e^2 + \omega_e^2) \boldsymbol{\phi}_j \boldsymbol{\phi}_j^T \bar{\mathbf{M}} \mathbf{e} = \mathbf{e} + \sum_{j=1}^{n_a} \omega_e^2 (\omega_j^2 - \omega_e^2)^{-1} \boldsymbol{\phi}_j \boldsymbol{\phi}_j^T \bar{\mathbf{M}} \mathbf{e} = \mathbf{e} + \mathbf{d}_1 \quad (9.50)$$

Using the above result and the triangle inequality, the energy norm of the adjoint solution can now be expressed as:

$$\|\mathbf{d}\|_K \leq \|\mathbf{e}\|_K + \|\mathbf{d}_1\|_K \quad (9.51)$$

The final step is to obtain a bound for the last term. This can be achieved by writing the energy norm of this term and maximizing the result (see [21] for details):

$$\|\mathbf{d}_1\|_K^2 = \mathbf{e}^T \left(\sum_{j=1}^{n_a} \omega_e^2 (\omega_j^2 - \omega_e^2)^{-1} \bar{\mathbf{M}} \boldsymbol{\phi}_j \boldsymbol{\phi}_j^T \right) \bar{\mathbf{K}} \left(\sum_{k=1}^{n_a} \omega_e^2 (\omega_k^2 - \omega_e^2)^{-1} \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T \bar{\mathbf{M}} \right) \mathbf{e} \leq \max_{\omega_j} \left(\frac{\omega_e^2}{\omega_j^2 - \omega_e^2} \right)^2 \|\mathbf{e}\|_K^2 \quad (9.52)$$

With this result the bound for the energy norm of the adjoint solution can be expressed as:

$$\|\mathbf{d}\|_K \leq (1 + S_\omega) \|\mathbf{e}\|_K \quad \text{with} \quad S_\omega = \max_{\omega_j} \left(\frac{\omega_e^2}{\omega_j^2 - \omega_e^2} \right) \quad (9.53)$$

Here S_ω is the so-called stability factor, i.e. an additional factor on the inequality in order for it to hold true. This stability factor requires knowing the eigenfrequency of the total system, that is closest to the excitation frequency. Finally, combining this result and equation (9.48), the error estimate in the energy norm becomes:

$$\|\mathbf{e}\|_K \leq (1 + S_\omega) \left(\sum_{s=1}^{n+1} \frac{1}{\omega_{k_{s+1}}^{(s)^2}} \|\hat{\mathbf{r}}^{(s)}\|^2 \right)^{1/2} \quad \text{denoted by} \quad e_{\leq K} \quad (9.54)$$

To determine the relative domain contributions for use in refinement schemes, the following error indicators are introduced based on this estimate:

$$m_K^{(s)} = \frac{1}{\omega_{k_{s+1}}^{(s)^2}} \|\hat{\mathbf{r}}^{(s)}\|^2 \quad (9.55)$$

For convenience all domain indicators are collected in the vector $\mathbf{m}_K$. The error estimate $e_{\leq K}$ in eq. (9.54) can be interpreted as follows. The energy norm represents the total elastic energy in the system caused by the displacement error. The above equation finds an upper bound for it by conservatively scaling the (acceleration) residual using the eigenfrequencies of the first modes outside the space in which the approximate solution is obtained. This scaling of the residual can be interpreted as taking into account that part of the inverse stiffness matrix which is outside the reduced space. Multiplication by the stability factor then gives the estimate for the error in the energy norm.

The stability factor can be recognized as the dynamic amplification factor of the harmonic response due to the global eigenfrequency that is closest to the excitation frequency. Obviously, this eigenfrequency is not calculated from the full model but is also approximated by the reduced model. In order for the error estimate to remain an upper bound, it is therefore important that this frequency is accurately predicted by the reduced model. Should this not be the case, for instance when the reduction bases are still very poor at the start of a refinement algorithm (see section 9.5), the error estimate no longer holds quantitatively but can still be used to estimate the relative domain contributions. Finally, remark that the error estimate may be scaled by the energy norm of the (approximate) solution to obtain relative energy norms of the error.

9.4 Error Estimation for Global Eigensolutions

In addition to harmonic analysis, in practice an often seen type of dynamic analysis is the modal analysis. By extending the theory of the previous section, an error estimate can be derived for a global eigensolution (eigenfrequency and -mode of the assembled system) obtained from the reduced model without knowledge of the solution for the full model. The next subsection therefore first defines the error on the eigensolution, thereafter the actual estimates are derived in subsections 9.4.2 and 9.4.3. Throughout this section references will again be made to the work in [21], where the derivation of the error estimates is worked out in detail.

9.4.1 Error and Force Residual

In section 9.3 the goal was to create a reduced model whose harmonic response converges towards that of the unreduced model. Hence the error was defined as the difference between those two responses. Here the goal is to iteratively improve a reduced model having an approximate eigensolution that should converge towards a selected/target eigensolution of the unreduced model. Since an eigensolution consists of an eigenfrequency and an eigenmode, two errors need to be defined.

The error on the eigenfrequency of eigensolution j , denoted by e_ω , is simply defined as the absolute difference between the exact eigenfrequency ω_j obtained from the unreduced model and the approximate one $\tilde{\omega}$ obtained from the reduced system:

$$e_\omega = \left| \tilde{\omega}^2 - \omega_j^2 \right| \quad (9.56)$$

The error on the eigenmode is defined some what differently, namely as the part of the approximate eigenmode $\tilde{\phi}$ that is (mass) orthogonal to the space of its exact counterpart ϕ_j , i.e.:

$$\mathbf{e}_\phi = \tilde{\phi} - \phi_j \phi_j^T \tilde{\phi} = (\mathbf{I} - \mathbf{P}_j) \tilde{\phi} \quad (9.57)$$

Here mass normalized modes are assumed and $\mathbf{P}_j$ is an operator on the global level, similar to the projector used on the domain level in section 9.3.2. Note that in practice one might include multiple eigenmodes in this projector, for instance when several modes are very closely spaced. Note that the error estimates that will be derived in this section are equally valid in that case.

Since the eigensolution of the reduced system is an approximation of the exact solution it does not satisfy the eigenproblem of the full system. Hence, when the approximate eigensolution is substituted in the full eigenproblem a force residual is found:

$$\mathbf{r} = \tilde{\mathbf{K}} \tilde{\phi} - \tilde{\omega}^2 \tilde{\mathbf{M}} \tilde{\phi} \quad (9.58)$$

Note that the force residual can again be split into domain contributions similar to eq. (9.27). Furthermore, in order to converge to the correct global eigensolution, both the error on the eigenvalue and -vector need to be below a certain tolerance.

For the derivation of the error estimates for the approximate eigensolution, the starting point is again the target functional introduced in section 9.3.1. This target functional can be evaluated for the approximate mode $\tilde{\phi}$ as:

$$\begin{aligned} J(\tilde{\phi}) &= \mathbf{a}^T \tilde{\phi} = \mathbf{d}^T (\bar{\mathbf{K}} - \omega_j^2 \bar{\mathbf{M}}) \tilde{\phi} \\ &= \mathbf{d}^T \bar{\mathbf{K}} \tilde{\phi} - \omega_j^2 \mathbf{d}^T \bar{\mathbf{M}} \tilde{\phi} = \mathbf{d}^T \mathbf{r} \end{aligned} \quad (9.59)$$

In this expression ω_j^2 is the exact eigenfrequency of the unreduced assembled system, and $\mathbf{d}$ is again the solution to the adjoint problem which for the eigenproblem becomes:

$$(\bar{\mathbf{K}} - \omega_j^2 \bar{\mathbf{M}}) \mathbf{d} = \mathbf{a} \quad (9.60)$$

Next, the projector $\mathbf{P}$ is defined as the block diagonal equivalent of the domain projectors $\mathbf{P}^{(s)}$ defined in eq. (9.40). This allows to split the adjoint solution $\mathbf{d}$ in a part that lives in the space of the domain modes and an orthogonal part. One can apply this split to the first term eq. (9.59) to write the goal oriented error as:

$$J(\tilde{\phi}) = ((\mathbf{I} - \mathbf{P}) \mathbf{d})^T \bar{\mathbf{K}} \tilde{\phi} + (\mathbf{P} \mathbf{d})^T \bar{\mathbf{K}} \tilde{\phi} - \omega_j^2 \mathbf{d}^T \bar{\mathbf{M}} \tilde{\phi} \quad (9.61)$$

After a number of manipulations (see [21]), this equation can be rewritten to:

$$J(\tilde{\phi}) = ((\mathbf{I} - \mathbf{P}) \mathbf{d})^T (\bar{\mathbf{K}} - \tilde{\omega}^2 \bar{\mathbf{M}}) \tilde{\phi} + (\tilde{\omega}^2 - \omega_j^2) \mathbf{d}^T \bar{\mathbf{M}} \tilde{\phi} \quad (9.62)$$

This expression for the goal oriented error forms the basis for the error estimates for the approximate eigensolution. In the next two subsections, these estimates will be derived for the eigenfrequency and eigenmode, respectively.

9.4.2 Error Estimate for Global Eigenfrequency

To come to an error estimate for an approximate global eigenfrequency $\tilde{\omega}^2$, the first step is to choose the selection vector in the adjoint problem simply as $\mathbf{a} = \mathbf{0}$. With no goal quantity selected, the goal oriented error in eq. (9.62) reduces to zero:

$$\mathbf{0} = ((\mathbf{I} - \mathbf{P}) \mathbf{d})^T (\bar{\mathbf{K}} - \tilde{\omega}^2 \bar{\mathbf{M}}) \tilde{\phi} + (\tilde{\omega}^2 - \omega_j^2) \mathbf{d}^T \bar{\mathbf{M}} \tilde{\phi}, \quad (9.63)$$

while the adjoint problem in eq. (9.60) now amounts to:

$$\bar{\mathbf{K}} \mathbf{d} - \omega_j^2 \bar{\mathbf{M}} \mathbf{d} = \mathbf{0} \quad (9.64)$$

Hence, a non-trivial adjoint solution can be found as:

$$\mathbf{d} = \mathbf{P}_j \tilde{\phi} \quad (9.65)$$

This solution is simply verified with the definition of the orthogonal projector $\mathbf{P}_j$ in eq. (9.57):

$$\bar{\mathbf{K}} \phi_j \phi_j^T \bar{\mathbf{M}} \tilde{\phi} - \omega_j^2 \bar{\mathbf{M}} \phi_j \phi_j^T \bar{\mathbf{M}} \tilde{\phi} = \omega_j^2 \bar{\mathbf{M}} \phi_j \phi_j^T \bar{\mathbf{M}} \tilde{\phi} - \omega_j^2 \bar{\mathbf{M}} \phi_j \phi_j^T \bar{\mathbf{M}} \tilde{\phi} = \mathbf{0} \quad (9.66)$$

Using this adjoint solution, the last term $\mathbf{d}^T \bar{\mathbf{M}} \tilde{\phi}$ on the right hand side of eq. (9.63) can be written as:

$$\mathbf{d}^T \bar{\mathbf{M}} \tilde{\phi} \geq 1 - \delta \quad (9.67)$$

Here $0 \leq \delta < 1$ is a so-called safety factor which is often introduced in the literature on error estimation to ensure the conservatism of error bounds [10, 13]. Details of this step are shown in [21]. Substitution of this result in (9.63) then leads to the following:

$$(\tilde{\omega}^2 - \omega_j^2) (1 - \delta) \leq -((\mathbf{I} - \mathbf{P}) \mathbf{d})^T (\bar{\mathbf{K}} - \tilde{\omega}^2 \bar{\mathbf{M}}) \tilde{\phi} \quad (9.68)$$

This expression can be transformed to an error estimate on the approximate eigenfrequency by:

$$e_\omega \leq \frac{1}{1-\delta} \left| ((I - P) d)^T (\bar{K} - \tilde{\omega}^2 \bar{M}) \tilde{\phi} \right| \quad (9.69)$$

Next, using the definition of the residual in eq. (9.58), one can write:

$$e_\omega \leq \frac{1}{1-\delta} \left| ((I - P) d)^T r \right| \quad (9.70)$$

As before, the adjoint solution and residual can be written as a summation of the domain contributions. As a result, the eigenfrequency error estimate becomes:

$$e_\omega \leq \frac{1}{1-\delta} \sum_{s=1}^{n+1} \left| \left((I - P^{(s)}) d^{(s)} \right)^T r^{(s)} \right| \quad (9.71)$$

Indeed, apart from the safety factor δ , this expression is exactly similar to eq. (9.41) obtained in the derivation of the error estimates for harmonic problems. Hence, one can proceed the derivation exactly as in section 9.3.2 to obtain the equivalent of eq. (9.48):

$$e_\omega \leq \frac{1}{1-\delta} \|d\|_K \left(\sum_{s=1}^{n+1} \frac{1}{\omega_{k_{s+1}}^{(s)^2}} \|\hat{r}^{(s)}\| \right) \quad (9.72)$$

The final step in obtaining the error estimate for the eigenfrequency is to find a bound for the energy norm of the adjoint solution $\|d\|_K$. Recalling the adjoint solution found with $a = 0$ from eq. (9.65), one can write this energy norm as:

$$\|d\|_K = \left(\tilde{\phi}^T P_j^T K P_j \tilde{\phi} \right)^{1/2} = \left(\tilde{\phi}^T \bar{M} \phi_j^T \phi_j \bar{K} \phi_j \phi_j^T \bar{M} \tilde{\phi} \right)^{1/2} = \omega_j \left(\tilde{\phi}^T M P_j \tilde{\phi} \right)^{1/2} \leq \omega_j \quad (9.73)$$

In the last step it was used that $\tilde{\phi}^T M P_j \tilde{\phi} \leq 1$, see [21]. Finally, the error estimate for the approximate eigenfrequency is thus obtained as:

$$e_\omega \leq \frac{\omega_j}{1-\delta} \left(\sum_{s=1}^{n+1} \frac{1}{\omega_{k_{s+1}}^{(s)^2}} \|\hat{r}^{(s)}\| \right)^{1/2} = \frac{\omega_j}{1-\delta} \left(\sum_{s=1}^{n+1} m_K^{(s)} \right)^{1/2} \quad \text{denoted by } e_{\leq \omega} \quad (9.74)$$

As before, the notation $e_{\leq \omega}$ is used to indicate the error estimate and distinguish it from the actual error e_ω . Furthermore, m_K are the same domain indicators as defined previously (eq. (9.55)), and can be used in adaptive refinement schemes such that the error on the eigenfrequency is decreased most efficiently. Note that a relative frequency error can be found by normalizing $e_{\leq \omega}$ by the (approximate) target eigenfrequency.

Quite paradoxically, the error estimate for the approximate global eigenfrequency $\tilde{\omega}$ contains the exact eigenfrequency ω_j for which an error bound is sought. In practice ω_j is obviously not calculated but simply replaced by the approximate frequency obtained from the reduced model. Indeed, due to the reduction this frequency will always exceed the exact frequency of the unreduced model, such that the error estimate remains a conservative bound. When applying the error estimate in an adaptive reduction scheme it could therefore be that at first this frequency is poorly approximated, leading to a (too) high error estimate, but as the refinement progresses the eigenfrequency will converge and the error bound will become more and more accurate.

9.4.3 Error Estimate for Global Eigenmode

In order to arrive at an error estimate for the approximate eigenmode $\tilde{\phi}$, one chooses the selection vector in the goal oriented error similarly as in section 9.3.2, namely $a = \bar{K} e_\phi$. As a result, the goal oriented error again becomes a global energy norm of the error on the eigenmode as defined in eq. (9.57) (for details see [21]):

$$J(\tilde{\phi}) = a^T \tilde{\phi} = e_\phi^T \bar{K} \tilde{\phi} = \|e_\phi\|_K^2 \quad (9.75)$$

With this choice, the goal oriented error from eq. (9.62) becomes the following global error estimate:

$$\|e_\phi\|_K^2 = ((I - P) d)^T (\bar{K} - \tilde{\omega}^2 \bar{M}) \tilde{\phi} + (\tilde{\omega}^2 - \omega_j^2) d^T \bar{M} \tilde{\phi} \quad (9.76)$$

As in the previous subsection, the last term $(\tilde{\omega}^2 - \omega_j^2) d^T \bar{M} \tilde{\phi}$ in the expression is first analyzed. As is shown in [21], the following bound for this term is found:

$$(\tilde{\omega}^2 - \omega_j^2) d^T \bar{M} \tilde{\phi} \leq \delta \|e_\phi\|_K^2 \quad (9.77)$$

In this case the safety factor δ , with $0 \leq \delta < 1$, has the following property:

$$\max_{i \neq j} \left(\frac{\tilde{\omega}^2 - \omega_j^2}{\omega_i^2 - \omega_j^2} \right) \leq \delta \quad (9.78)$$

Such that, after substitution in eq. (9.76), the error estimate becomes:

$$\|e_\phi\|_K^2 \leq \frac{1}{1 - \delta} \left(((I - P) d)^T (\bar{K} \tilde{\phi} - \tilde{\omega}^2 \bar{M} \tilde{\phi}) \right) \quad (9.79)$$

From this point onwards, the same steps can be followed as in the previous subsection. By doing so one obtains:

$$\|e_\phi\|_K^2 \leq \frac{1}{1 - \delta} \|d\|_K \left(\sum_{s=1}^{n+1} \frac{1}{\omega_{k_s+1}^{(s)^2}} \|\hat{r}^{(s)}\| \right)^{1/2} \quad (9.80)$$

Again, the final step in the derivation of the error estimate is to establish a bound for the energy norm of the adjoint solution. Indeed, for the current adjoint solution this problem is very similar to that of section 9.3.2. Applying the same reasoning as therein thus leads to the following bound:

$$\|d\|_K \leq (1 + S_\omega) \|e_\phi\|_K \quad \text{with} \quad S_\omega = \max_{i \neq j} \left(\frac{\omega_j^2}{\omega_i^2 - \omega_j^2} \right) \quad (9.81)$$

Here S_ω is again a stability factor that is similar to that for the harmonic problem, except that the frequency of external excitation ω_e is replaced by the frequency of the target eigensolution. With this bound for the adjoint solution, the error estimate $e_{\leq \phi}$ for the approximate eigenmode finally becomes:

$$\|e_\phi\|_K \leq \frac{1 + S_\omega}{1 - \delta} \left(\sum_{s=1}^{n+1} \frac{1}{\omega_{k_s+1}^{(s)^2}} \|\hat{r}^{(s)}\| \right)^{1/2} = \frac{1 + S_\omega}{1 - \delta} \left(\sum_{s=1}^{n+1} m_K^{(s)} \right)^{1/2} \quad \text{denoted by} \quad e_{\leq \phi} \quad (9.82)$$

Similar to what was remarked before regarding the stability factor for the harmonic problem, it is noted here that the above stability factor depends on the (exact) global eigenfrequency that is closest to the target eigenfrequency. Again, these frequencies are not calculated from the full model but are approximated by the reduced model. Therefore, when the stability factor is poorly predicted, the error estimate no longer holds quantitatively but can still be used to estimate the relative domain contributions. The further the domain reduction bases are refined, the better the error estimate becomes.

9.5 Refinement Schemes for Adaptive Model Reduction

In the previous sections error estimates were presented that give quantitative measures of the reduced model's accuracy for different types of dynamic analysis. All estimates are based on the summation of domain error contributions and can therefore be used in refinement schemes. These refinement schemes are applied in an iterative way to perform adaptive model reduction, by refining the reduced until it has sufficient accuracy for the given analysis. This process is illustrated for the general case in Fig. 9.1. In this section two basic refinement strategies are proposed for use in adaptive reduction algorithms.

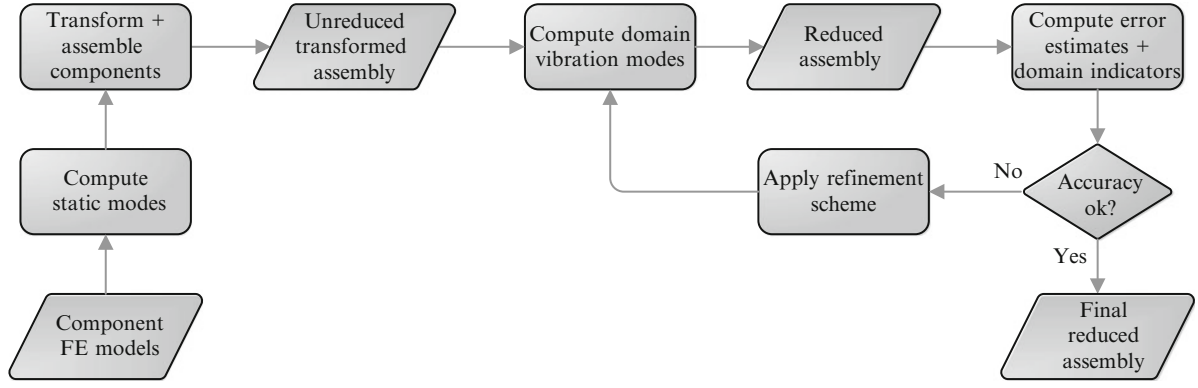


Fig. 9.1 Schematic representation of adaptive model reduction procedure using error estimation

Initially, the internal and interface domains are reduced with some number of modes k_s . From the resulting force residual the error estimates are evaluated to obtain the domain contributions to the overall error. Specifically, the domain error indicators in $\mathbf{m}_J$ or in $\mathbf{m}_K$ are computed, which serve as input for the refinement schemes.

The first refinement strategy is to obtain the maximum domain contribution and add a fixed number of modes k_{add} to all domains that contribute more than a certain fraction c of this maximum. This scheme will be referred to as “threshold refinement”. The parameter c is to be chosen beforehand with a value from zero to one. Using the generic notation $\mathbf{m}$ for the domain indicators, which depending on the type of estimate (goal oriented or global) that is used can either be $\mathbf{m}_J$ or $\mathbf{m}_K$, this refinement scheme can be expressed as in algorithm 1.

Algorithm 1 Refinement scheme 1 – threshold

```

Set threshold value for refinement  $0 \leq c \leq 1$ 
Select  $m_{\text{max}} = \max(\mathbf{m})$ 
for domain  $s = 1, \dots, n + 1$  do
    if  $m^{(s)} \geq c \cdot m_{\text{max}}$  then
         $k_s = k_s + k_{\text{add}}$ 
    end if
end for
  
```

In case one chooses $c = 0$ this scheme enriches all domains such that it effectively becomes a uniform refinement scheme, whereas with $c = 1$ only the most inaccurate domain is enriched.

The second refinement strategy computes a relative contribution for each domain based on the total error. This normalized distribution is then used to determine which share of the available enrichment modes k_{add} are to be added to each domain. Of course these values have to be rounded to the nearest integer. This refinement scheme is called “linear refinement” and is shown in algorithm 2.

Algorithm 2 Refinement scheme 2 – linear

```

Compute  $m_{\text{tot}} = \sum_{s=1}^{n+1} m^{(s)}$ 
for domain  $s = 1, \dots, n + 1$  do
    Compute  $\hat{m}^{(s)} = m^{(s)} / m_{\text{tot}}$ 
     $k_s = k_s + \hat{m}^{(s)} \cdot k_{\text{add}}$ 
end for
  
```

For both of the above refinement schemes it holds that they are easy to compute and implement. The benefit of the “linear” scheme is that the growth of the reduced model per iteration is known in advance and therefore can be better controlled. On the other hand, the “threshold” scheme offers more flexibility by adaptively choosing the value for c in the iterative model reduction algorithm. For instance one can apply quasi-uniform refinement (i.e. $c \rightarrow 0$) when the error is large, whereas targeted refinement steps can be taken when the error gets close to the tolerance ε ($c \rightarrow 1$).

Finally it is noted that in the adaptive model reduction schemes the domain reduction bases are iteratively refined with additional eigenmodes. Hence, from a numerical implementation perspective a suitable algorithm should be used for

obtaining these eigensolutions. Specifically, the numerical method should be capable of efficiently handling the computation of a limited number of eigenmodes when a number of lower eigenmodes has already been computed. Hence, a good choice would be a restarted iterative algorithm like the implicitly restarted Arnoldi (IRA) method [14, 19] or the implicitly restarted Lanczos method [4].

9.6 Case Study: Modeling an Offshore Wind Turbine on a Jacket Foundation

9.6.1 Offshore Wind Energy and Dynamic Analysis of Offshore Wind Turbines

Over the last decade, the offshore installed wind energy capacity in Europe has increased exponentially and many more offshore wind farms are currently planned or under construction. The current trends in offshore wind energy is to install ever larger turbines in increasingly deeper waters, having some profound implications for the foundations on which offshore wind turbines are installed. By far the most popular type of offshore foundation has up to now been the monopile due to its simplicity and robustness. However, the current trends render monopiles less and less attractive, since a disproportional amount of material is needed to withstand the more severe loading and fulfill the eigenfrequency requirements. More complex types of foundations then become economically attractive, with jacket structures currently as the most promising option. The different types of foundations are illustrated in Fig. 9.2.

Since the environmental conditions (water depth, soil properties) and ambient excitations (aero- and hydrodynamic loading) vary greatly across different offshore sites, each offshore wind farm is custom engineered. This means that a standard rotor-nacelle assembly (RNA) is combined with a specifically designed support structure. An important part of the engineering process is to perform dynamic simulations to assess whether the support structure can withstand the loads during its specified lifetime. For certification purposes, thousands of load cases need to be evaluated. So-called aero-elastic codes have been developed to efficiently perform these dynamic simulations, taking into account aerodynamic loads and coupling, the wind turbine controller and hydrodynamic loads. Given the number of load cases that need to be run, these models must be as coarse as possible to keep the computation times at an acceptable level. Therefore such aero-elastic models typically consist of 300–400 degrees of freedom (DoF), including the tower.

However, a jacket type of foundation is considerably more complex than a monopile and so is its dynamic behavior. Consequently, a more detailed model is needed: where a monopile can typically be represented with a few beam elements leading to a model of at maximum 50 DoF, a jacket structural model easily consists of more than 1000 DoF. Needless to say, this negatively impacts the computation times of the dynamic simulations. Simplifying or even neglecting the jacket dynamics cannot be done in general since dynamic coupling can occur between the jacket, tower and RNA [17, 18]. Fully coupled dynamic simulations of the integrated system are therefore essential.

Hence, the goal of this case study is to create “optimal” reduced models (in terms of their number of DoF versus accuracy) of an offshore wind turbine using error estimation and adaptive model reduction techniques. To this end, we use a reference wind turbine model described in the literature that consists of a 5 megawatt (MW) RNA on a tubular tower and jacket offshore foundation. The reference wind turbine model was created by the National Renewable Energy Laboratory (NREL) in the United States. This design, known as the “NREL 5 MW baseline wind turbine”, is a wind turbine of the conventional horizontal axis, three-bladed, upwind type on a tubular tower. Its detailed specifications can be found in [12]. The wind turbine is supported by the jacket structure initially designed for the UpWind project, details of this structure can be found in [24]. Next the modeling of the wind turbine and jacket foundation will be discussed in more detail.

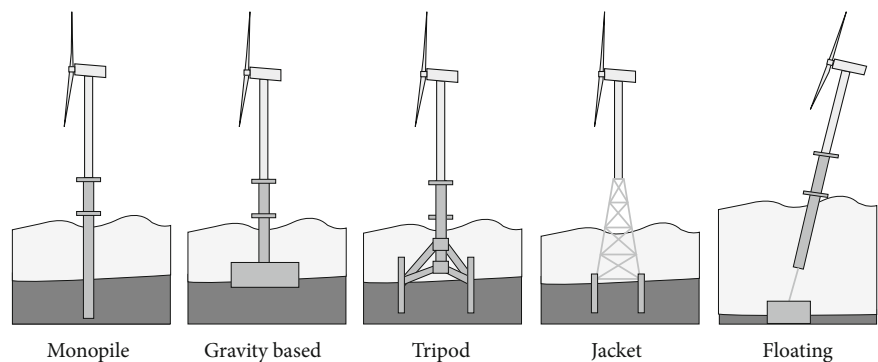


Fig. 9.2 Different foundation types for (from left to right) increasing water depths

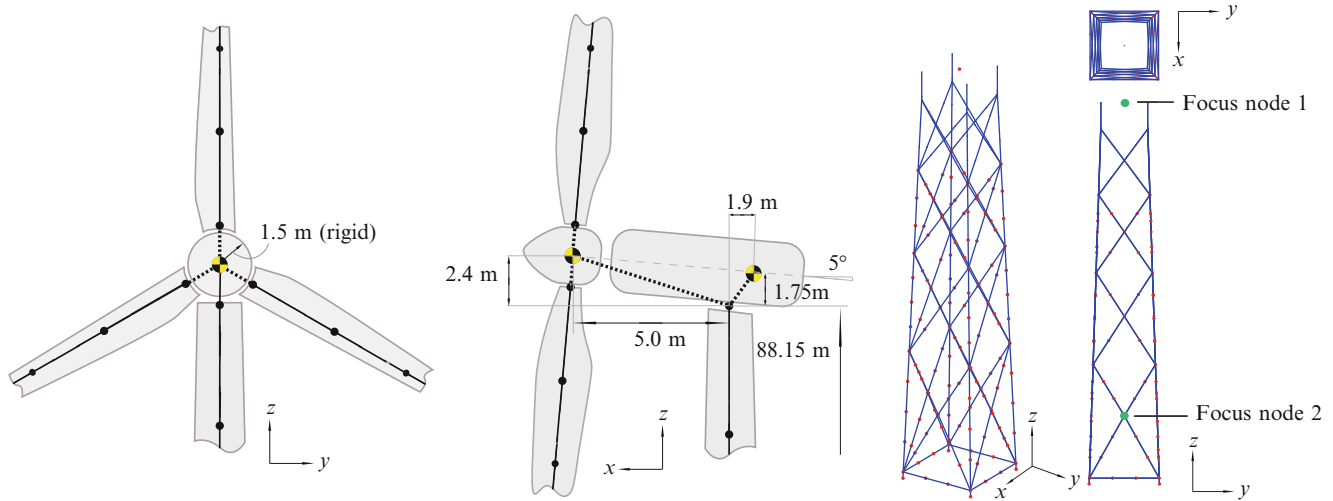


Fig. 9.3 RNA of the NREL 5 MW baseline turbine (a) and model of the OC4 jacket (b)

Table 9.1 Properties of RNA and tower component models and assembled wind turbine model

Part	No. of DoF	No. of elements
Rotor nacelle assembly	252	41
Tower	54	8
Assembled wind turbine	300	49

9.6.2 Implementation of NREL 5 MW Baseline Wind Turbine

The NREL wind turbine consists of a standard tubular tower, a nacelle, hub and three blades, where the latter three form the RNA. Figure 9.3a shows the main dimensions. Since the RNA and tower will be reduced separately in the next section, they are considered as two individual components. In this work these components are modeled as follows:

Tower A conical tower is created according to the specifications given in [24]. The tower is 68 meters in height, with a base diameter of 5.6 meter and top diameter of 4.0 meter, and is modeled using 8 three-dimensional linear Euler-Bernoulli beam elements. The tower has two 6 DoF interfaces, one at the bottom (tower bottom), which will be connected to the jacket structure and one at the top (tower top) which will be connected to the nacelle.

RNA – Nacelle The nacelle is modeled as a single stiff and massless element with on one end a point mass representing the nacelle's mass (240000 kg) and inertia, and on the other a 6 DoF interface which connects to the tower top. The large stiffness and low mass of the element are created using a high Young's modulus and a low density value. The parallel axis theorem was used to calculate the moments of inertia around its local axes. By modeling the nacelle in this way, the drivetrain dynamics are neglected and the rotor is implicitly assumed to be parked. Such a simplified model is justified by the fact that here the main interest lies in the influence of the support structure on the overall dynamics.

RNA – Hub Similar to the nacelle, the hub is modeled as a single stiff and massless element. The hub has a virtual distance of 1.5 meters between its center and the connection to the blades. The hub's mass (56780 kg) is added as a point mass at one end of the element together with its moments of inertia. The element has two 6 DoF interfaces, one end connects to the tower top, the other to the three blades.

RNA – Blades The blades are 61.5 meters in length and each has a mass of 17740 kg. They are modeled using 13 three-dimensional linear Euler-Bernoulli beam elements. Again, since here the influence of the support structure on the overall dynamics is investigated and no aerodynamic loads will be applied to the rotor, a linear model for the blades is deemed sufficiently accurate. A root-element, located at the base of each blade, is modeled as a 1.5 meter long stiff and massless element to account for the virtual distance to the hub center. The root-element has a 6 DoF interface which connects to the hub element.

The tower and RNA component models are created in Matlab; their numbers of DoF and elements are listed in Table 9.1.

Table 9.2 Mass comparison of turbine and jacket models

Model	Part	Calculated mass [tons]	OC4 mass [tons]
Turbine	Rotor nacelle assembly	350	350
	Tower	217	217
Jacket	Structure	676	675
	Transition piece	666	666
	Marine growth	180	180
	Water mass flooded legs	205	204
	Hydrodynamic added mass	672	675

9.6.3 Implementation of OC4 Jacket Foundation

The jacket foundation used in the OC4 project, hereafter called the OC4 jacket, consists of four main legs which are interconnected via four levels of cross-braces, see Fig. 9.3b. The jacket is designed for water depths of 50 meters and has a height of 68 meters. Based on the information in [24], the various parts of the jacket are modeled as follows:

Structure The main jacket structure, i.e. the four legs and cross-braces, is modeled in ANSYS using linear beam elements.

Their properties are defined according to the specifications given in [24].

Seabed connection As was described in [24], the stiffness of the connection to the seabed is assumed to be high and is therefore modeled as rigidly clamped; this excludes any soil effects from the model. Furthermore, the grouted connection between the main legs and soil piles is also modeled rigidly, leaving only the bottom 0.5 meters flexible.

Transition piece The connection between the turbine and jacket is realized using a concrete transition piece with a mass of 666 ton; its inertia was calculated using this mass and the specified dimensions. This transition piece rigidly connects the top elements of the jacket structure. The interface to the tower bottom is realized by a 6 DoF coupling.

Added mass A number of factors are described in [24] that add to the mass of the jacket structure, thereby influencing its dynamic behavior. These are the marine growth on the structure, the water contained in the free-flooded legs and the hydrodynamic mass felt from the surrounding water when the jacket structure vibrates. These added masses are calculated as nodal point masses (with directional properties) using the ASAS software package, which is an extension of ANSYS specialized for offshore and maritime applications. These masses are thereafter added to the structural model in ANSYS.

After completing the jacket model in ANSYS, it is exported to Matlab using the FEMLink toolbox (see [1]) for reduction and further calculations. The total jacket model consists of 229 linear beam elements and 1014 DoF. A plot is shown in Fig. 9.3b, where the point masses representing the added mass are shown as dots.

9.6.4 Verification of the Offshore Wind Turbine Model

By assembly of the RNA, tower and jacket models the total model of the offshore wind turbine is obtained, which consists of 1308 DoF. In order to verify the correctness of the implementation of the numerical model some checks are performed, namely:

Masses A straightforward but important check is to compare the mass contained in the models. An overview of the masses of the models created here is given in Table 9.2, along with (averaged) results reported by the OC4 participants [23]. It can be seen that the mass of the turbine model is in good agreement up to the level of accuracy available from the reported OC4 results. The same holds for the mass of the transition piece and marine growth on the jacket, while the minor differences for the structural, flooded water and hydrodynamic added mass are considered to be within modeling tolerances.

Eigenfrequencies Preferably an eigenfrequency comparison should be performed on the individual wind turbine and jacket models. Unfortunately, no such data was found in the literature. Therefore a modal analysis was performed on the total OWT model; the first ten eigensolutions are listed in Table 9.3. The results generally compare well to those reported by the OC4 participants (see [23]), but two remarks are in place. Firstly, since the drive-train is not modeled explicitly here, the drive-train mode is absent. Secondly, the rotor is fixed in the current simplified model of the nacelle. As a result the first side-side mode is shifted to a lower frequency than the first fore-aft mode.

Table 9.3 First 10 eigensolutions of total model

Mode no. [—]	Frequency [Hz]	Mode shape description
1	0.30	First global side-side
2	0.30	First global fore-aft
3	0.74	First asymmetric flapwise yaw
4	0.77	First asymmetric flapwise pitch
5	0.80	First flapwise collective
6	0.91	First edgewise collective
7	0.97	First asymmetric edgewise pitch
8	1.23	First asymmetric edgewise yaw
9	1.26	Second global fore-aft
10	1.29	Second global side-side

9.7 Case Study: Results of Adaptive Model Reduction

9.7.1 Numerical Experiments

Now the reference model is described, the next step is to apply model reduction. Indeed, with over 1300 DoF the full model is far larger than the models typically used in the wind industry and, given the number of load cases that need to be analyzed, ill-suited to perform load simulations. In this section the OWT model will therefore be adaptively reduced using the methods outlined in sections 9.3 and 9.4. Hereby the aim is to create models that have optimal accuracy, in terms of a specified quantity, with respect to their size.

As explained in the previous section, the total OWT model is divided into three components: the jacket foundation, the tower and the rotor-nacelle assembly (RNA). One could argue that the RNA could be further separated into individual blade, hub and nacelle models to allow more localized refinement of reduced component models. However, due to the simplicity of both the nacelle and hub models and the fact that in practice only a single reduced model for all three blades will be used, this separation is not made. The three component models are connected by two 6 DoF interfaces, see Fig. 9.4. The jacket structure and the RNA both have a single interface of 6 DoF which connects to the tower bottom and tower top, respectively.

In order to test the different aspects of the error estimation and adaptive model reduction methods, a series numerical experiments were conceived:

- Test 1 – Global reduced model for harmonic excitation. In this test a reduced assembly is created using a limited total number of DoF that are to be distributed in an optimal way over the domains based on their contributions to the total error.
- Test 2 – Goal oriented reduced model for harmonic excitation. For this test a quantity of interest is defined, for which a reduced assembled model is created such that the goal oriented error falls below a chosen threshold.
- Test 3 – Global reduced model for eigensolution convergence. In this test two target eigensolutions are chosen, for which a reduced assembled model is created such that the estimated error on the target eigenfrequency satisfies a preset tolerance.

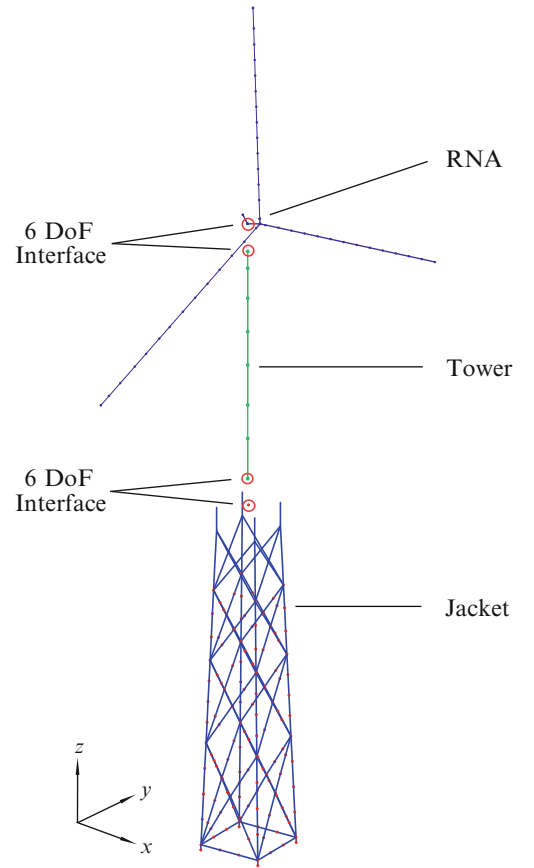
In line with the transformed model description introduced in section 9.2.1, the assembled model consists of four domains: the internal domains of the jacket structure, tower and RNA and a global interface domain. Furthermore, in all experiments the error estimates are evaluated using the exact acceleration residual $\hat{r}$ (see section 9.3.2).

In the first two experiments the assembled structure is excited by a harmonic force with a single fixed excitation frequency acting on the jacket model. These force vectors represent hydrodynamic loading from the waves and current, and are extracted as the dominant loading shapes (and frequencies) from a hydrodynamic load time series using the proper orthogonal decomposition method (for details see [16]). These hydrodynamic load time series were generated with the ASAS software package.

9.7.2 Test 1: Global Reduced Model for Harmonic Excitation

In this first test the aim is to create an optimal reduced model of the assembled OWT for a given maximum number of DoF. An optimal model is searched for in terms of the global energy norm of the error, while subjected to harmonic excitation. To do so, use is made of the error estimation method derived in section 9.3.2. In short, the following settings were used for this test:

Fig. 9.4 The three component models of the OWT model, connected through two interfaces



Type of problem:	Harmonic
Excitation vector:	Jacket domain loaded by hydrodynamic force vector obtained from a mixed sea state
Excitation frequency:	0.14 Hz
Type of error estimate:	Global energy norm
Refinement scheme:	Threshold refinement (alg. 1) with $c = 1$ and $k_{\text{add}} = 2$
Stopping criterion:	Total model size of 60 DoF

Next different results obtained from this test will be analyzed. Firstly, it is investigated how accurate the actual error estimate is through comparison with the exact error. The latter is obtained by solving the full problem, which in this case is still very cheap for a single load case, and computing the difference with the approximate solution. The result is shown in Fig. 9.5a, where both the exact and estimated error are plotted during the adaptive refinement of the OWT model. As a reminder, the exact error e_K is computed as the left hand side of eq. (9.54) while the estimated error $e_{\leq K}$ is the corresponding right hand side.

As can be seen, the error estimate forms a proper upper bound of the exact error. When the accuracy of the reduced model is still very poor it overestimates its error quite strongly, but for more refined models the estimate is rather close to the actual error (within one order of magnitude).

Secondly, the effect of adaptively refining the reduction basis is quantified. In this test refinement of the component models is performed using algorithm 1 with $c = 1$ and $k_{\text{add}} = 2$, meaning that at each iteration only the most inaccurate domain is enriched by 2 modes. This is continued until the total model size has reached 60 DoF. For the sake of comparison, the error estimate is also evaluated for the case where uniform refinement has been applied. That is, each domain is enriched by one mode until the total model again has 60 DoF. The results are shown in Fig. 9.5b. From this figure the benefit of adaptive refinement over uniform refinement can be clearly seen: the energy norm of the error is approximately a factor four times smaller for the adaptively reduced model. This is a significant difference that illustrates the power of adaptive model reduction.

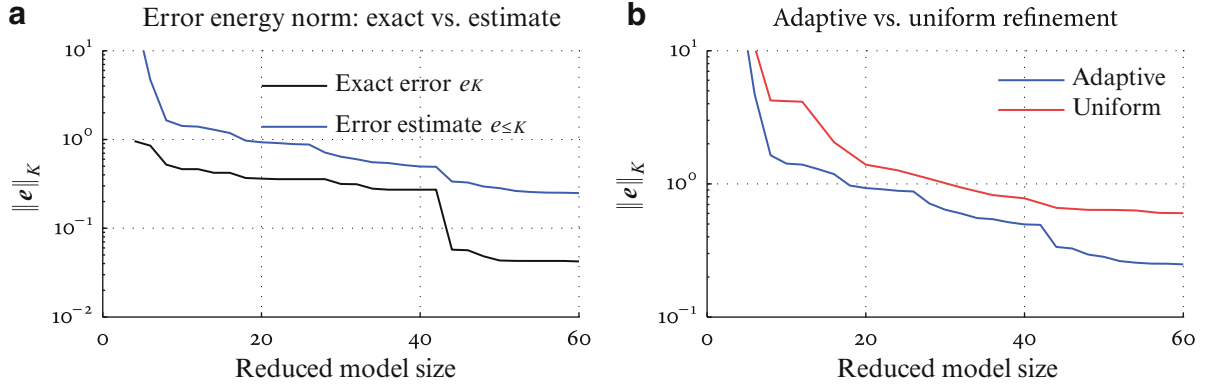


Fig. 9.5 Test 1: Exact vs. estimated error (a) and adaptive vs. uniform refinement (b)

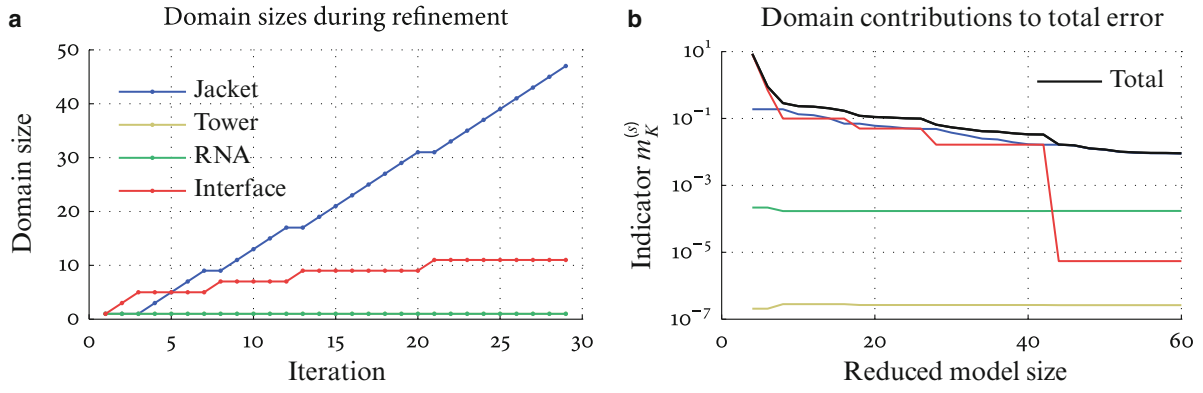


Fig. 9.6 Test 1: Domain sizes (a) and error contributions (a) during refinement

Furthermore, Fig. 9.6a shows the size of the different domains during adaptive refinement; the corresponding domain errors, expressed in terms of the domain indicators $m_K^{(s)}$ defined in eq. (9.55), can be seen in Fig. 9.6b. These plots show that in this case the model of the jacket is most refined, followed by the interface. This makes sense since only the jacket is subjected to external loading and hence shows localized deformation that requires more modes to be properly described. The response of the tower and RNA is already properly captured by only including their first mode. Of course, when the adaptive reduction would be continued these domains will at some point also be further refined, but the error in the jacket is dominant at first.

9.7.3 Test 2: Goal Oriented Reduced Model for Harmonic Excitation

The goal of the second test is to create a reduced model that is optimal with respect to a certain quantity of interest, when subjected to harmonic loading. To this end the goal oriented error estimation technique from section 9.3.1 is applied. The problem statement can be summarized as follows:

Type of problem:	Harmonic
Excitation vector:	Jacket domain loaded by hydrodynamic force vector obtained from a high-wave sea state
Excitation frequency:	0.15 Hz
Type of error estimate:	Goal oriented error
Quantity of interest:	Displacements at two focus nodes on jacket (see Fig. 9.3) and at tip nodes of each blade, all weighted equally
Size of adjoint problem:	Additional 10 modes per domain
Refinement scheme:	Linear refinement (alg. 2) with $k_{\text{add}} = 4$
Stopping criterion:	Estimated goal oriented error $e_{\approx \Sigma} < 10^{-4}$

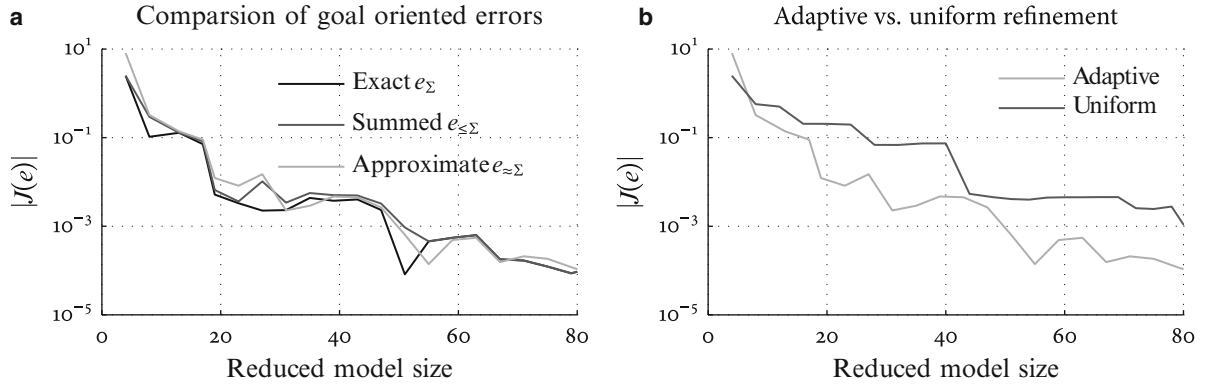


Fig. 9.7 Test 2: Comparison of exact and estimated goal oriented error (a) and adaptive vs. uniform model refinement (b)

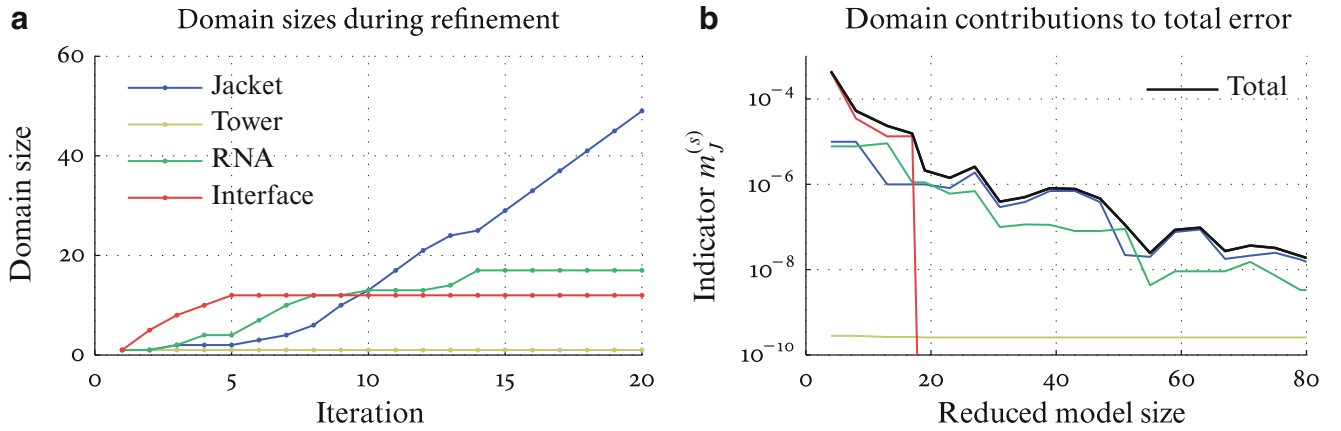


Fig. 9.8 Test 2: Domain sizes (a) and error contributions (b) during refinement

With the above settings the adaptive model reduction algorithm covered after 20 iterations, with a total model size of 79 DoF (due to round-off errors only 3 DoF were added in one iteration). As for the previous test, it is first investigated how well the goal oriented error estimate approximates the exact error. In the plot in Fig. 9.7a the exact goal oriented error e_Σ is shown (as computed from eq. (9.36)), as well as an upper bound $e_{\leq \Sigma}$ found by summation of the absolute domain contributions (i.e. eq. (9.37)) and the approximate goal oriented error $e_{\approx \Sigma}$ (the right hand side of eq. (9.38)) found from approximately solving the adjoint problem.

As expected, the summed error $e_{\leq \Sigma}$ forms a true upper bound for the exact error. This more or less also holds for the approximate goal oriented error $e_{\approx \Sigma}$, which only occasionally slightly underestimates the exact error. The quality of this approximate error estimator obviously depends on the accuracy with which the adjoint problem is solved; the more refined the adjoint model the better the error estimate and vice versa. In this case an additional 10 modes are used per domain, leading to a rather accurate adjoint model.

Figure 9.7b again shows the difference between adaptive and uniform refinement of the component reduction bases. In this test refinement is performed using the scheme in alg. 2 with $k_{\text{add}} = 4$, such that at each iteration 4 additional modes are distributed linearly over the domains according to their relative error contributions. Uniform refinement is again achieved by sequentially adding one mode to each domain until a total model size of 80 DoF is reached. For the goal oriented error and refinement strategy considered here, the benefit of adaptive refinement is clear: at a model size of only 80 DoF, the adaptively reduced model is a factor 10 more accurate in the quantities of interest than the uniformly refined model.

The sizes of the four domains during the refinement iterations is shown in Fig. 9.8a, the respective error contributions are indicated in (b). The latter are expressed in terms of the domain indicators $m_j^{(s)}$, see eq. (9.38). It can be seen that at first mainly the interface domain is enriched, until it consists of the full 12 DoF and its error contribution drops to (approximately) zero. This is caused by the fact that part of the quantities of interest are the displacements at the interface between the jacket and tower. Furthermore, the interface is crucial in transferring the response from the tower to the RNA, where the blade tip displacements are also quantities of interest.

Table 9.4 Distribution of DoF of adaptively reduced models for two target eigensolutions

Target eigensolution	Jacket	Tower	RNA	Interface	Total no. DoF
ϕ_5, ω_5 @ 0.80 Hz	9	3	33	10	55
ϕ_{10}, ω_{10} @ 1.29 Hz	84	8	62	10	164

After the interface is fully refined, both the RNA and jacket domains are further enriched. Especially the latter is heavily refined which, as in the previous test, is due to the localized loading applied to the jacket as well as the fact that the displacements at one of the jacket brace nodes are chosen as a quantity of interest.

9.7.4 Test 3: Global Reduced Model for Eigensolution Convergence

In this test adaptive model reduction is applied with the aim of converging to an eigensolution of the assembled system. Hence the error estimates for global eigensolutions, as derived in section 9.4, are applied to the OWT model. The following settings are used in the adaptive model reduction algorithm:

Type of problem:	Eigenproblem
Type of error estimate:	Eigenfrequency and eigenmode
Target eigensolution	Global mode 5 and mode 10
Refinement scheme:	Linear refinement (alg. 2) with $k_{\text{add}} = 4$
Stopping criterion:	Estimated eigenfrequency error $e_{\leq \omega} < 10^{-3}$

Two reduced models are created for the global eigensolution convergence, using respectively the fifth and tenth global mode as target eigensolution. As listed in Table 9.3, the fifth global eigenmode has a frequency of 0.80 Hz and is the first flapwise collective motion of the blades, also known as the coning mode; its motion is thus RNA-dominated. The tenth eigenmode, with a frequency of 1.29 Hz, is the second global side-side mode and hence involves all domains (jacket, tower, RNA and interfaces). Note that the stopping criterion is defined on the (relative) estimated error on the eigenfrequency $e_{\leq \omega}$; alternatively one can define a termination criterion based on the error estimate of the eigenmode, or a combination. Initially all domains are reduced with 3 vibration modes, i.e. the total model initially has 12 generalized DoF.

With the above settings and the fifth eigensolution as target, the adaptive algorithm converged in 13 iterations leading to a model with 55 DoF. Selecting the tenth mode as target eigensolution the algorithm needed 44 iterations to reach the required accuracy resulting in a total model of 164 DoF. The distribution of the DoF across the different domains is listed in Table 9.4. As speculated above, with the fifth mode as target eigensolution the RNA is much more refined than the other domains. Using the tenth mode as target indeed leads to further refinement of all domains; for instance, 8 modes are now required for the tower domain instead of the 3 modes that were sufficient to represent the fifth global mode. The evolution of the domain sizes and error contributions during the refinement iterations are shown in the plots in Fig. 9.9 for both target eigensolution 5 and 10. As before, the domain error contributions are expressed by the indicators $m_K^{(s)}$, see eq. (9.74).

Next, the error estimates are again compared to their exact counterparts, as shown in the plots in Fig. 9.10 for the two different target eigensolutions considered here. For the eigenfrequencies, the exact error e_ω as well as the estimated error $e_{\leq \omega}$, as computed from eq. (9.74), are shown in plots (a) and (c). Similarly, the exact and estimated errors for the eigenmodes (respectively e_ϕ and $e_{\leq \phi}$) are shown in plots (b) and (d), where the estimate is obtained from eq. (9.82). From these plots it is shown that the estimates form an upper bound for the exact errors.

Both error estimates however suffer from overestimation of the actual error, to a (much) larger extent than the global and goal oriented error estimates for harmonic problems applied in the two previous tests. This is especially true for the eigenfrequency error estimate, which overestimates the error by almost two orders of magnitude for target eigensolution 5. With increasing model size this overestimation seems to decrease as can be seen from the error estimate for target eigensolution 10. As explained in section 9.4.2, this overestimation is due to the fact that the estimate depends on the actual target eigenfrequency, see eq. (9.74), which is approximated by the reduced model and hence is quite badly overestimated when the model is coarse.

The error estimate on the eigenmode suffers from overestimation to a lesser extent, although still up to one order of magnitude. This due to the definition of the stability factor in eq. (9.81) used in the error estimate of eq. (9.82), which depends on the (exact) difference between the target eigenfrequency and its most closely spaced neighboring eigenfrequency. Again, this stability factor is evaluated with the reduced model, leading to overestimation of the true error.

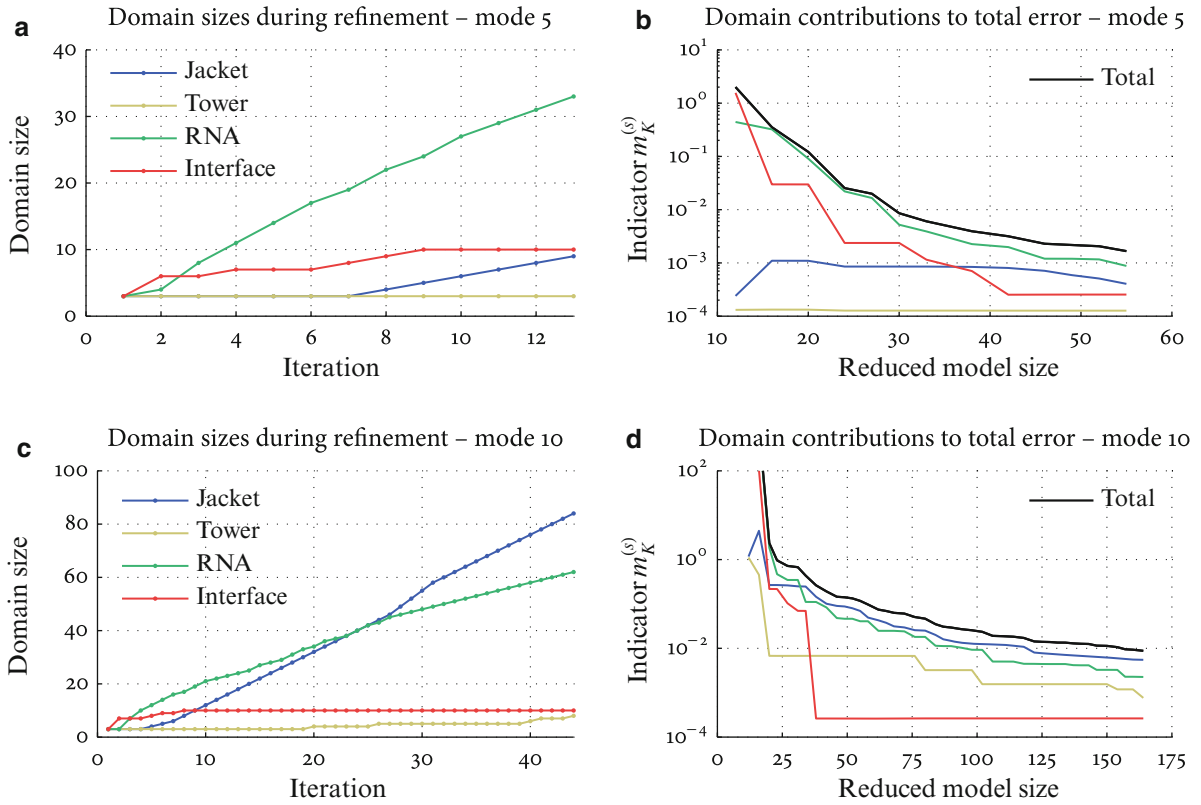


Fig. 9.9 Test 3: Domain sizes (a, c) and error contributions (b, d) during refinement iterations for target eigensolutions 5 and 10

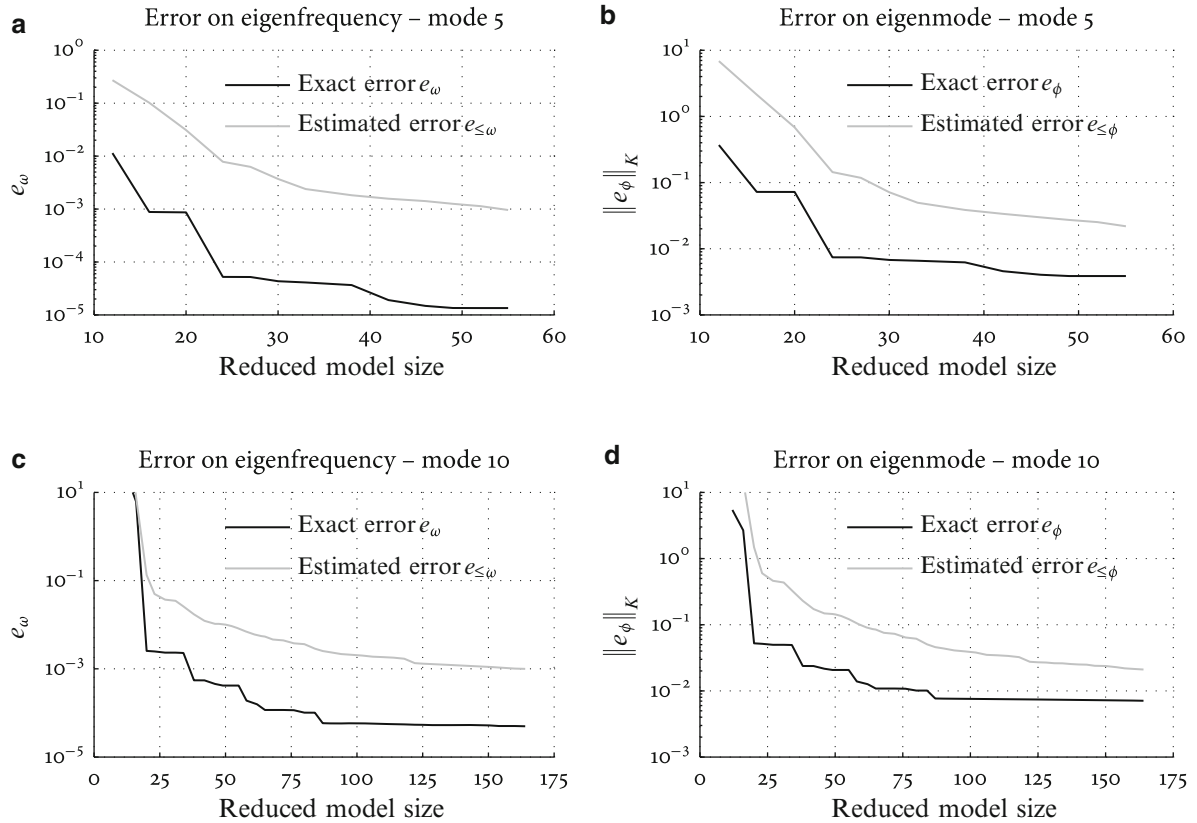


Fig. 9.10 Test 3: Comparison of exact and estimated error on eigenfrequency (a, c) and eigenmode (b, d) for target eigensolutions 5 and 10

9.8 Conclusions

This paper was concerned with error estimation for assembled reduced models. Since the reduced models only approximate the dynamic behavior of the full system, it is important to gain insight into their accuracy to use these models with confidence. Preferably, this should be achieved without knowledge of the full solution, which can be very computationally expensive, while at the same time providing information on the contribution of each component to the total error.

To this end, a posteriori error estimation methods were considered, which have recently been applied for the first time to the problem of estimating the reduction error. Based on that work, error estimates were derived in this paper in an algebraic model reduction setting. In order to do so, the Craig-Bampton reduced equations of motion were first written in an alternative format, resulting in a transformed system description with an uncoupled, block diagonal stiffness matrix. Thereafter, error estimates were derived for approximate solutions to harmonic problems. Goal oriented errors were considered first, and were subsequently generalized to a global error estimate in terms of the energy norm. Furthermore the problem of estimating the accuracy of approximate global eigensolutions was addressed. Using the same reasoning as for the harmonic solution, error estimates for an approximate eigenfrequency and -mode were obtained. Finally it was outlined how the error estimates may be used in an adaptive model reduction scheme.

To illustrate the use of these methods, a case study was performed on the modeling of an offshore wind turbine on a jacket foundation. Firstly, a reference offshore wind turbine (OWT) model from the literature was modeled, consisting of the NREL 5MW turbine placed on the OC4 jacket foundation structure. Using simple beam models, the wind turbine was modeled with 300 DoF while the jacket model consisted of over 1000 DoF, leading to an assembled model of the total OWT with 1308 DoF. Thereafter, this model was subjected to adaptive model reduction to create optimal models for three different combinations of problem types (harmonic, modal) and quantities of interest (local, global). The error estimates proved to be true upper bounds of the exact errors, although at some occasions the error was quite strongly overestimated. Nonetheless, in all cases the benefit of adaptive reduction could be clearly observed, leading to reduced models with up to ten times higher accuracy with respect to a uniformly reduced model of the same size.

In conclusion, this paper treated the algebraic derivation of methods to a posteriori estimate the reduction error for the Craig-Bampton CMS method. From the case study it can be concluded that these methods provide a powerful way of obtaining very compact as well as accurate dynamic models, enabling efficient integrated simulation of offshore wind turbines.

References

1. Balmès E, Bianchi J, Leclère J (2011) Structural dynamics toolbox & femlink – user’s guide, Paris, France. <http://www.sdtools.com/help/sdt.pdf>
2. Bangerth W, Rannacher R (2003) Adaptive finite element methods for differential equations. Birkhäuser, Boston
3. Bulgak H, Zenger C (eds) (1999) Error control and adaptivity in scientific computing. Kluwer, Dordrecht
4. Calvetti D, Reichel L, Sorensen DC (1994) An implicitly restarted Lanczos method for large symmetric eigenvalue problems. *Electron Trans Numer Anal* 2:1–21
5. Craig R, Bampton M (1968) Coupling of substructures for dynamic analysis. *AIAA J* 6(7):1313–1319
6. Géradin M, Rixen D (1997) Mechanical vibrations – theory and application to structural dynamics. Wiley, Chichester
7. Hurty W (1965) Dynamic analysis of structural systems using component modes. *AIAA J* 3(4):678–685
8. Hurty WC (1960) Vibrations of structural systems by component mode synthesis. *J Eng Mech, Division American Society of Civil Engineers* 86(4):51–69
9. Jakobsson H (2011) Adaptive reduction of finite element models in computational solid mechanics. Ph.D. thesis, Department of Mathematics and Mathematical Statistics, Umeå University, Sweden
10. Jakobsson H, Larson M (2011) A posteriori error analysis of component mode synthesis for the elliptic eigenvalue problem. *Comp Meth Appl Mech Eng* 200:2840–2847
11. Jakobsson H, Bengzon F, Larson MG (2011) Adaptive component mode synthesis in linear elasticity. *Int J Numer Meth Eng* 86(7):829–844. doi:10.1002/nme.3078. <http://dx.doi.org/10.1002/nme.3078>
12. Jonkman J, Butterfield S, Musial W, Scott G (2009) Definition of a 5 MW reference wind turbine for offshore system development. Tech. Rep. NREL/TP-500-38060, National Renewable Energy Laboratory
13. Larson M (2001) A posteriori and a priori error analysis for finite element approximations of self-adjoint elliptic eigenvalue problems. *SIAM J Numer Anal* 38(2):608–625
14. Lehoucq R, Sorensen DC (1996) Deflation techniques for an implicitly restarted Arnoldi iteration. *SIAM J Matrix Anal Appl* 17:789–821
15. Meyer M, Matthies HG (2003) Efficient model reduction in non-linear dynamics using the Karhunen-Loève expansion and dual-weighted-residual methods. *Comput Mech* 179–191 (2003)
16. Nortier BP, Voormeeren SN, Rixen DJ (2012) Application of residual vectors to superelement modeling of an offshore wind turbine foundation. In: Topics in experimental dynamics substructuring and wind turbine dynamics, vol 2. Conference proceedings of the society for experimental mechanics series, vol 27. Springer, New York, pp 149–163. http://dx.doi.org/10.1007/978-1-4614-2422-2_15

17. Schaumann P, Dubois J, Achmus M, Abdel-Rahman K, Seidel M (2011) Local dynamics of jacket support structures for offshore wind turbines. In: Proceedings of EWEA offshore 2011, Amsterdam, The Netherlands
18. Seidel M, Foss G (2006) Impact of different substructures on turbine loading and dynamic behaviour for the DOWNVInD project in 45m water depth. In: Proceedings of the European wind energy conference 2006, Athens, Greece
19. Sorensen D (1992) Implicit applications of polynomial filters in a k-step Arnoldi method. *SIAM J Matrix Anal Appl* 13:357–385
20. Troeng T (2010) Frequency response analysis using component mode synthesis. Master's thesis, Department of Mathematics and Mathematical Statistics, Umeå University, Sweden
21. Voormeeren S (2012) Dynamic substructuring methodologies for integrated dynamic analysis of wind turbines. Ph.D. thesis, Delft University of Technology, Delft, The Netherlands
22. Voormeeren S, van der Valk P, Rixen, D (2011) Generalized methodology for assembly and reduction of component models for dynamic substructuring. *AIAA J* 49(5):1010–1020. doi:10.2514/1.54302
23. Vorpahl F, Popko W (2011) IEA Wind Annex 30 – OC4 project. The offshore code comparison collaboration continuation. In: IEA wind side event at EWEA 2011 annual event, Brussels, Belgium
24. Vorpahl F, Popko W, Kaufer D (2011) Description of a basic model of the “Upwind reference jacket” for code comparison in the OC4 project under IEA Wind Annex XXX. Tech. rep.

Chapter 10

Coupling Experimental and Analytical Substructures with a Continuous Connection Using the Transmission Simulator Method

Randy L. Mayes and Daniel P. Rohe

Abstract The transmission simulator method of experimental dynamic substructuring has the capability to couple substructures with continuous connections. A hardware example with continuous connections is presented in which the method is used to couple an experimental substructure with a finite element substructure to predict full system response. The predicted response is compared with frequency response functions measured on the full system hardware. The experimental substructure captures the motion of a component packed in foam. This is coupled to a finite element model of a cylindrical metal case which contains the foam and is attached through a flange to a plate and beam structure.

Keywords Experimental dynamic substructuring • Transmission simulator

Nomenclature

DoF	Degree of freedom
FE	Finite element
FRF	Frequency response function
MCFS	Modal constraint for fixture and subsystem
TS	Transmission simulator
TSM	Transmission simulator method
f	Force
x	Displacement vector
q	Generalized coordinate
Φ	Mode shape matrix
ζ	Modal damping ratio
ω	Angular frequency (radians/second)
EXP	Subscript for the experimental model
$+$	Superscript for the pseudo-inverse of a matrix

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the U.S. Department of Energy under Contract DE-AC04-94AL85000.

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10.1 Introduction

The Transmission Simulator Method (TSM) overcomes many of the problems associated with classical experimental dynamic substructures, one of which is the attachment of the substructures over a continuous area. The experimentalist usually cannot measure every physical degree of freedom (DoF) of the experimental substructure that needs to be connected to the FE mesh for continuity and equilibrium if there is a connection over a continuous area. An application is described in which an experimental-based substructure is coupled with a finite element (FE) substructure with a continuous connection over a large area to predict full system response. The experimental-based substructure includes an internal foam mounted component and the FE substructure is the external case in which the component and foam are supported as well as another subassembly bolted to it. The transmission simulator (TS) is actually one of the subsystems of the full hardware set, instead of a separately machined fixture as in previous approaches. In both cases, frequency response functions are predicted with the coupled experimental-based and FE model and compared to measured responses on the actual hardware.

10.2 Overview of the Transmission Simulator Method

Previously, the method for Modal Constraints of Fixture and Subsystem (MCFS) which uses a transmission simulator for experimental dynamic substructures has been presented [1–6]. Some of the major advantages of using the transmission simulator are:

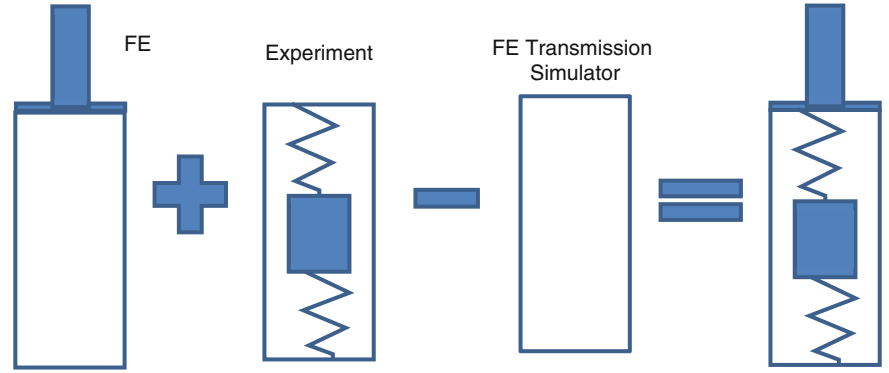
- It mass loads the boundaries which provide a better set of basis vectors than typical free modes.
- It inherently includes the compliance and damping of all the joints inside the case.
- It inherently includes rotational connection degrees of freedom (DoF) as well as translational DoF.
- The softer modal DoF constraints are very robust against normal experimental errors (unlike physical DoF constraints which generally cause the coupling to produce false modes and inaccurate results due to very small experimental errors).
- Measurement DoF do not have to be located directly at connections which are often inaccessible (as required when using physical DoF constraints).
- The method is just as applicable for continuous as well as for discrete connections (continuous connections are demonstrated in this work).

The transmission simulator is an instrumented fixture attached to the structure to be experimentally modeled. Displacement continuity and force equilibrium in the connection of the experimental substructure to another substructure are derived from experimental measurements made on the transmission simulator. The transmission simulator is usually modeled analytically so it can be removed from the experiment-based substructure. In some previous work [3, 4] the transmission simulator was a separate fixture that was not a part of the combined system. In that work, the analytical model of the transmission simulator was connected to the FE model, and then that model was connected to the transmission simulator of the experimental substructure through the modal DoF of the transmission simulator. Then both transmission simulators were subtracted. In another application [2], the transmission simulator was subtracted from the experimental substructure which was then coupled to the FE substructure through multi-point constraints, all within the FE code. Here another approach is taken as described below.

10.3 Coupling Scheme and Theory

The coupling scheme will be illustrated for the application. One desires to predict full system response of a cylinder that is bolted through a flange to a plate with a cantilevered beam attached. The cylinder has an internal component that is supported in foam. A retaining ring with the load plate at the end of the cylinder preloads the foam and component. In this scheme, the empty cylinder, plate and beam were modeled as the FE substructure. The experimental substructure included the internal component, surrounding foam, cylinder, load plate and retaining ring. The transmission simulator is the empty cylinder, and its model consists of modal DoFs that are truncated after a certain number of modes. The cylinder is actually part of the experimental-based substructure, which is necessary to hold all the internal parts together under preload. The FE substructure is coupled to the experimental substructure, each of which includes the transmission simulator.

Figure 10.1 represents the coupling process. The rectangle represents the cylinder. The blue box with a spring attached to each end represents the internal component, foam, load plate and retaining ring. In reality, there is a continuous connection between the FE, Experiment-based and TS substructures over the entire cylinder and its flange.

Fig. 10.1 Coupling scheme

Since both the FE substructure and experimental substructure contain the cylinder, which is the transmission simulator, one cylinder must be subtracted. The equations of motion to couple the FE and experimental based substructures (as well as subtracting off the extra cylinder) using mass normalized modes are

$$\begin{bmatrix} \omega_{FE}^2 & 0 & 0 \\ 0 & \omega_{EXP}^2 & 0 \\ 0 & 0 & -\omega_{TS}^2 \end{bmatrix} \begin{Bmatrix} q_{FE} \\ q_{EXP} \\ q_{TS} \end{Bmatrix} + j\omega \begin{bmatrix} 2\zeta\omega_{FE} & 0 & 0 \\ 0 & 2\zeta\omega_{EXP} & 0 \\ 0 & 0 & -2\zeta\omega_{TS} \end{bmatrix} \begin{Bmatrix} q_{FE} \\ q_{EXP} \\ q_{TS} \end{Bmatrix} - \omega^2 \begin{bmatrix} I_{FE} & 0 & 0 \\ 0 & I_{EXP} & 0 \\ 0 & 0 & -I_{TS} \end{bmatrix} \begin{Bmatrix} q_{FE} \\ q_{EXP} \\ q_{TS} \end{Bmatrix} = \Phi^T \{f\} \quad (10.1)$$

where the q vectors represent the modal DoF. Physical DoF, x , are estimated as $\Phi\{q\}$. The constraints in the physical DoF of the cylinder can be written as

$$\begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \begin{Bmatrix} x_{FE} \\ x_{EXP} \\ x_{TS} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (10.2)$$

However, with the transmission simulator approach one converts these displacement constraints to softer modal constraints by substituting modal DoF for the physical DoF and pre-multiplying by the pseudo-inverse of the transmission simulator mode shapes as

$$\begin{bmatrix} \Phi_{TS}^+ & 0 \\ 0 & \Phi_{TS}^+ \end{bmatrix} \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \begin{bmatrix} \Phi_{FE} & 0 & 0 \\ 0 & \Phi_{EXP} & 0 \\ 0 & 0 & \Phi_{TS} \end{bmatrix} \begin{Bmatrix} q_{FE} \\ q_{EXP} \\ q_{TS} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (10.3)$$

The modal constraint allows us to make discrete physical measurements and interpret them through the modal DoF as a continuous connection. Implicit in these equations are some requirements to use the method, which include:

1. A model that accurately represents the mode shapes and frequencies of the retained modes (usually a FE model) of the transmission simulator hardware is needed.
2. Transmission simulator hardware must be procured (in this work, it is actually a portion of the system hardware).
3. More sensor DoF than modes of the transmission simulator are required (over instrumentation makes the method more robust against experimental errors).
4. The mode shape matrix of the measurement DoF of the transmission simulator must be well conditioned since it must be inverted (this affects the choosing of the measurement DoF).
5. The mode shape matrix of the retained transmission simulator modes must span the space of the true motion well enough to be a sufficient approximation to the frequency of interest.
6. Desired response DoF in the experimental substructure, which are NOT connection DoF, must be instrumented and measured in the experimental substructure testing.
7. All the modes of the experimental hardware must be excited by appropriate selection and measurement of driving point FRFs on the transmission simulator.

In some situations, one must apply corrections to the coupled mass and stiffness matrices if they are indefinite, having negative eigenvalues (in this work, this was not a problem).

10.4 Hardware Description of Stuffed Cylinder/Plate/Beam

Finite element (FE) modeling to predict structural dynamics is readily applicable to certain structural materials and shapes and can be calibrated with linearized test data to take into account effects of some of the major joints. It is more difficult to represent preloaded foam assemblies with many interfaces, components, nonlinear compression pads and foams, especially if there are unknown preloads, tolerance stack-ups and resulting contacts. If the nonlinearities are modeled in an explicit FE code, then it usually takes too much computational time to predict random vibration time responses to aid in setting typical frequency domain environmental specifications. This application develops a linearized experimental-based substructure of the complex internals which will be coupled to the more tractable FE substructure of the rest of the system. Here we consider an application where the external case and attached bolted subassembly is modeled as a linear FE substructure, and the internal component with its preloaded foam, preloading retaining ring and load plate is modeled in the experimental substructure.

Here for the first time, the external case of the actual structure containing the internal parts is used as the transmission simulator. This allows development of an experimental substructure of the preloaded internal parts packed in foam. Here we compare the coupled FE and experimental-based substructure responses at key locations with measures of frequency response functions (FRFs) from the full system hardware. A load cell was built into the internal component (a steel block) so that its preload could be measured. Figure 10.2 shows the cutaway of the full system. A truth test was performed on the real hardware with the green plate/beam portion bolted at eight places to the blue cylinder. The experimental model consisted of the blue cylinder with gray threaded retaining ring and load plate (far right side of figure) compressing the three blocks of foam represented as red, brown, and tan as well as the gray internal steel blivet (representing an internal component). The blivet had three force gages at one end so that the preload in the component could be measured. The green plate and beam and blue cylinder portions were represented with the FE substructure. Figure 10.3 shows the 6061 T6 aluminum cylinder, plate and beam and the main foam components (actual hardware).

Fig. 10.2 Full system solid model

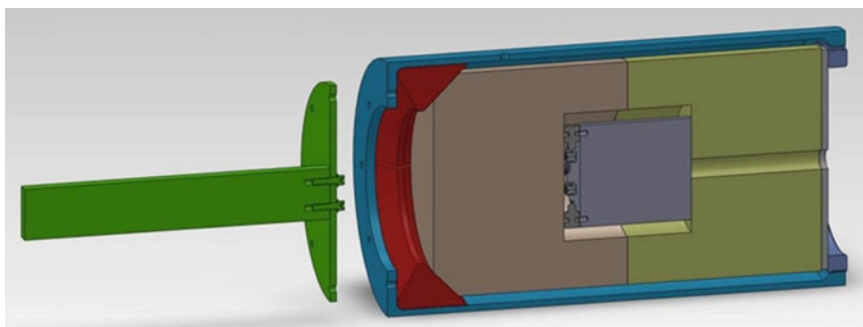


Fig. 10.3 Physical test hardware

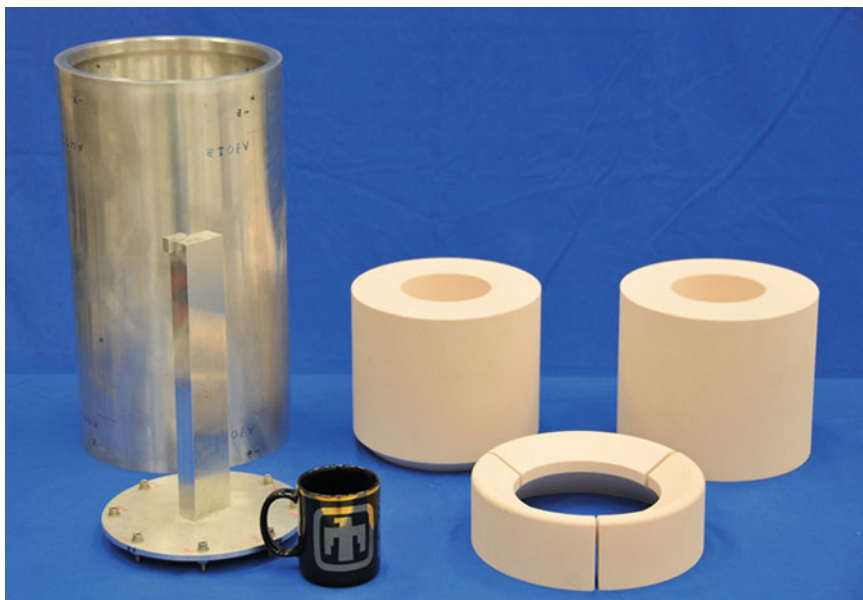
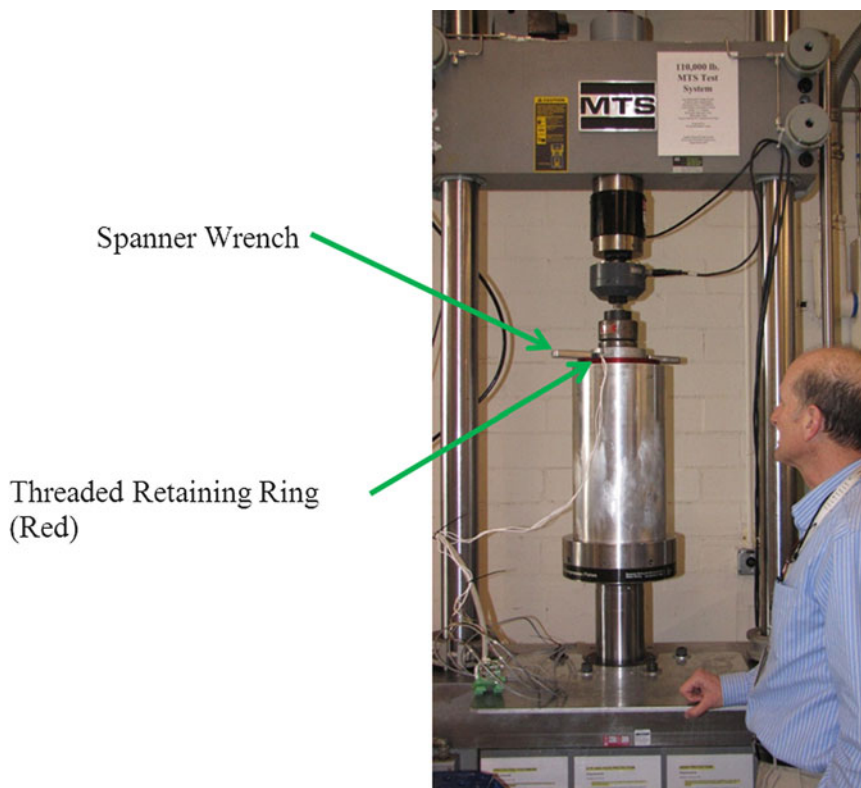


Fig. 10.4 Applying compressive preload to experimental substructure



The experimental hardware was assembled. The assembly was loaded by a hydraulic ram pressing on an aluminum load plate covering the end of the foam as shown in Fig. 10.4. After the load was applied by the hydraulic ram, the spanner wrench was used to engage and tighten the threaded retaining ring to maintain load on the plate. The original goal (not a requirement) was to have about 2000 pounds of preload in the foam parts and about 500 pounds of that preload would be experienced by the internal component. In the final loading of the experimental hardware, it was also found that a load of 4,530 lbf by the hydraulic ram provided a load of 1,090 lbf through the internal component. The retaining ring was tightened, the hydraulic ram removed, and the load through the internal component dropped to 484 lbf, due to elastic deflection of the aluminum load plate.

10.5 Transmission Simulator Finite Element Model and Sensor Location Algorithm

Accelerometers on the cylinder were utilized to provide the necessary information to couple the experimental substructure to the TS and FE substructure. Since the constraints are cast in the modal space of the TS, a FE model of the TS was utilized to optimize the accelerometer placement. The accelerometers must be placed so that the measured mode shapes of the TS modes retained for the coupling provide an accurate pseudo inverse for the constraint, Eq. (10.3). This is obtained by a low condition number of the TS measured mode shape matrix. (The condition number is the ratio of the largest eigenvalue to the smallest eigenvalue of the matrix). A candidate set of 80 tri-axial accelerometers was chosen at five axial stations at every 22.5° around the circumference. The sensor location algorithm eliminated tri-axes one at a time by determining which tri-ax, when eliminated, produced the lowest condition number of the remaining mode shape matrix. All rigid body and elastic modes were retained up to 2,230 Hz (22 modes) with the intent to provide an adequate model up to 2,000 Hz. This instrumentation set captured three sets of two lobed and three lobed ovaling modes as well as two sets of four lobed modes of the transmission simulator (bare cylinder without retaining ring). No bending modes of the bare cylinder are in this frequency band. The condition number of the retained mode shape matrix for all 80 tri-axes was approximately 3.5 and when optimized to 20 tri-axes was still about 3.5. Tri-axes were utilized because they provide easier visualization of the mode shapes for checking for errors in the experimental setup.

Table 10.1 Elastic modal frequencies of transmission simulator (bare cylinder)

Mode shape no lobes, no axial nodes	Test frequency (Hz)	FE frequency (Hz)	% Difference
2,0	394.7	384.6	−2.6
2,0	395.9	384.6	−2.9
3,0	951.9	941.0	−1.1
3,0	956.2	941.5	−1.5
2,1	1215.9	1225.5	0.8
2,1	1217.7	1225.6	0.6
3,1	1426	1406.6	−1.4
3,1	1436.1	1406.9	−2.0
4,0	1617.8	1603.4	−0.9
4,0	1624	1604.0	−1.2
4,1	2050	2036	−0.7
4,1	2057.8	2036.6	−1.0
2,2	2197.2	2224.2	1.2
2,2	2203.9	2224.4	0.9
3,2	2212.2	2227.9	0.7
3,2	2225.6	2228.4	0.1

A modal test of the TS cylinder was compared with the final FE model as shown in Table 10.1 below. All frequencies were within 3 % of test. The rigid body mode shapes were calculated based on the mass properties.

10.6 Force Locations Determined from the Transmission Simulator FE Model

As explained by Mayes and Arviso [7], since the TS mode shapes are assumed to adequately span the connection space motion, drive point forces that adequately excite the TS modes should also adequately excite the experimental substructure modal test. The algorithm used in [7] was also used here to select drive point DoF from the FE model mode shapes of the TS. First the drive point DoF was found which excited the most target modes with a mode shape at least 90% of the absolute value of the maximum value of the retained mode shapes at any candidate DoF. This was repeated for the additional remaining modes until drive points had been selected for all retained modes. Eight drive points were identified by this algorithm to excite the elastic modes. Two additional drive points were added to excite the axial and torsion rigid body modes. It should be mentioned here that these successfully excited all the desired bare cylinder modes well enough that they could be extracted. The SMAC [10] modal analysis software developed at Sandia was utilized for all the modal analysis.

When developing a modal experimental substructure, one needs very accurate drive point FRFs in order to ensure proper mode shape scaling. If the drive point scaling is wrong, it propagates into wrong scaling for the entire mode shape. This requires the force input to be co-linear with the accelerometer response. This co-linear requirement was met by actually impacting on the accelerometer, rather than next to the accelerometer. Care was taken to make sure the sensors were not overloaded. This required a rather light, but manageable hammer tap. Some trial drive point FRFs from impacts on the accelerometer were taken and compared to FRFs where a balanced number of hits to the aluminum cylinder at four places around the drive point accelerometer were applied. No big distortions appeared to occur with the impacts directly on the accelerometer, so tapping on the accelerometer was used for the rest of the impact testing in this work.

10.7 FE Substructure of Cylinder, Plate and Beam

The FE substructure of the cylinder, plate and beam was generated. The cylinder and beam were modeled with 20 node hex elements and the plate was modeled with four node shell elements. In the physical hardware at the eight bolted connections, a washer was installed between the forward flange of the cylinder and the plate. The washers were modeled with 20 node hex elements. Natural frequencies of the elastic modes were compared to modal frequencies from impact test data up to 1900 Hz as shown in Table 10.2. Rigid body modes and the orthogonal ovaling modes are not included in the table. All frequency errors were below three percent. Fifty FE modes up to 3500 Hz were utilized in the FE substructure calculations.

Table 10.2 FE substructure modal frequencies (Hz) compared to test

Test frequency	FE model frequency	% Difference
134.2	136.8	1.9
171.2	175.8	2.2
430	422.8	−1.7
511.2	525.9	2.9
975.7	960.6	−1.6
1027	1025	−0.2
1312	1312	0
1528	1509	−1.3
1637	1610	−1.7
1801	1774	−1.5
1833	1839	0.4

10.8 Experimental Substructure Modal Test

The experimental substructure hardware included the cylinder, retaining ring, load plate, foam and internal blivot with three force gages. The hardware was assembled and loaded as described previously. In addition to the TS instrumentation on the cylinder (previously described), one uniaxial (PCB 352A71) and three tri-axial accelerometers (PCB 356A32) were placed on one end of the internal steel blivot to obtain its rigid body motion in the foam. Four tri-axial accelerometers (PCB 356A32) were placed on the load plate 90° apart. In Fig. 10.5 one can see pictures that show the scale of the experimental substructure test hardware as well as many of the external tri-axial accelerometers (Endevco Model 65-100) and the impact hammer (PCB model 086C02). The accelerometers mounted on the cylinder used a cylindrical displacement coordinate system, and the internal accelerometers were all in a Cartesian displacement coordinate system. The geometry of the accelerometer node locations is shown in Fig. 10.6. Each colored trace line is drawn to connect all the accelerometer locations at a particular axial station in the Y direction. The center blue diamond with the 8000 series node numbers shows the accelerometer locations on the internal blivot. The 9000 series blue diamond shows the accelerometer locations on the loading plate. The experimental substructure was supported in an approximate free condition with bungee cords.

The same drive point locations were utilized in the experimental substructure modal test as were determined in the TS bare cylinder test. This is based on the assumption that the mode shapes of the transmission simulator span the space of the cylinder shapes of the experimental substructure as well as the full system when all components are added together. If the shapes of the TS span the space of motion, then the drive points that excite the TS shapes should excite the experimental substructure shapes. However, in the experimental substructure test with the internal foam and components, there were three ovaling modes that could not be extracted from these drive points, probably due to the higher damping of these closely spaced modes (they were extracted easily in the low damped bare cylinder test). Four additional input locations were attempted based on observing the modal coefficients of the FE model of the bare cylinder for the missing modes, and from two of the drive points, the missing ovaling modes were successfully extracted.

In these tests all the acceleration responses were gathered simultaneously from a particular driving point impact. Impact testing was performed in order to obtain accurate measures of FRFs out to a relatively high frequency. (It has been found in previous work for substructures of this size that a typical modal shaker force gage provides additional mass loading which is detrimental to the FRFs, causing frequencies to be as much as 20 percent too low on certain modes [9]. In addition, it becomes difficult to apply force at higher frequencies since the armature mass on our MB50 shakers has a resonance around 900 Hz, and they begin to roll off above that frequency.) FRFs were calculated from 10 averages. The frequency bandwidth was 3200 Hz and the frequency lines were spaced one Hz apart. No windows were applied to the time histories. A typical force autospectrum, FRF and coherence are shown for the 3006 radial direction in Fig. 10.7. The SMAC algorithm was used to extract the modes. In general, the ovaling modes had fairly low damping and the modes of the internal component had higher damping. Because the system was slightly nonlinear (particularly for the internal component modes), it was difficult to perform a multi-reference fit. This is because certain modes were excited more strongly from certain references, and would have a slightly lower frequency than they did for references which did not excite the mode as well. Consequently, the modes from each reference were extracted separately. Then a combined set of modes was generated by picking the mode with the highest peak in the Complex Mode Indicator Function (CMIF) across all the references for each modal frequency. To examine the credibility of the modal parameters from a particular reference, the FRFs were generated analytically from the modal parameters, and then the analytical CMIF was plotted against the experimental CMIF. A typical CMIF comparison for drive point 2014 radial is shown in Fig. 10.8.

Fig. 10.5 Experimental substructure



The desired modeling frequency bandwidth for this test was 2,000 Hz, based on traditional vibration test capabilities. From previous experience, if the substructure is modeled well up to and slightly beyond the frequency band of interest, it provides an adequate model for the full system. Forty elastic modes were extracted up to 2,200 Hz and were added to analytical modeling of the six rigid body modes to create the experimental substructure.

10.9 Truth Test

The experimental substructure hardware was attached to the plate and beam with eight bolts, and several sets of FRFs were collected using the same impact hammer and sampling parameters as before. Radial, circumferential and axial drive points were utilized on the cylinder. Lateral and axial drive points were utilized at the tip of the beam. The calculations using the substructures will be compared against the experimental CMIFs gathered from these hammer impacts. The truth test setup is shown in Fig. 10.9.

10.10 Predictions and Comparison with Experimental Truth Data

The desired modeling frequency band for this project was 2 kHz. The TS substructure utilized 22 free modes of the cylinder FE model to 2,230 Hz. Fifty modes of the FE model substructure (cylinder, plate and beam) were used up to 3,500 Hz with arbitrarily assigned damping of 0.5 percent. In addition to the six rigid body modes, there were 40 elastic modes extracted to develop the experimental substructure up to 2,200 Hz which included the experimental damping ratio.

Fig. 10.6 Geometry of experimental substructure accelerometer locations

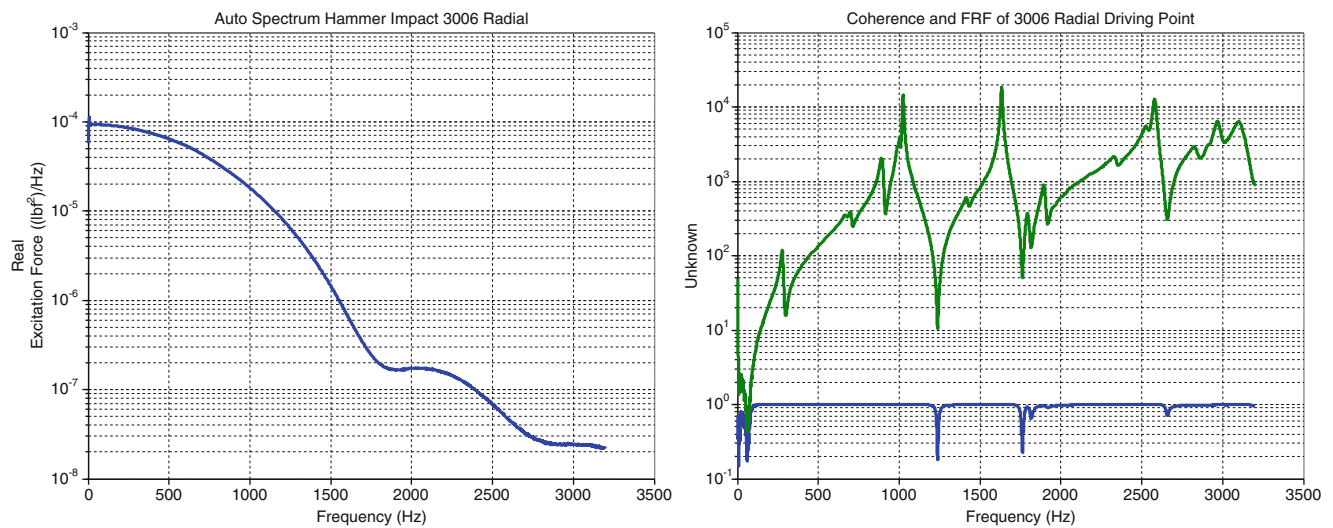
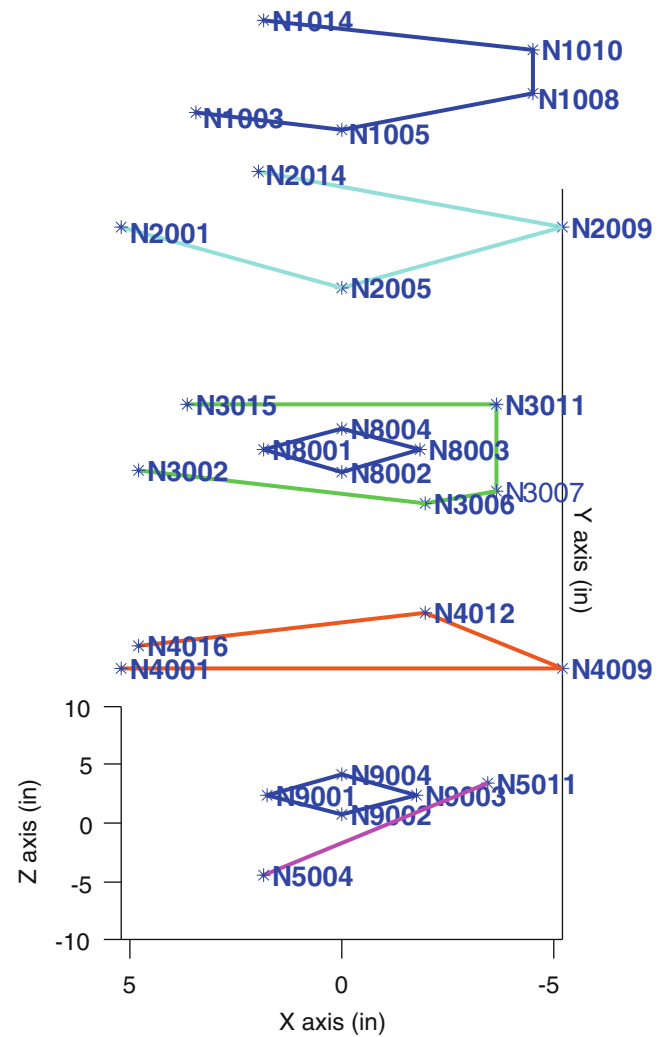


Fig. 10.7 Autospectrum of hammer force and FRF/coherence at 3006 radial direction

Fig. 10.8 Typical analytical/experimental CMIF comparison for FRFs from drive point 2014 radial

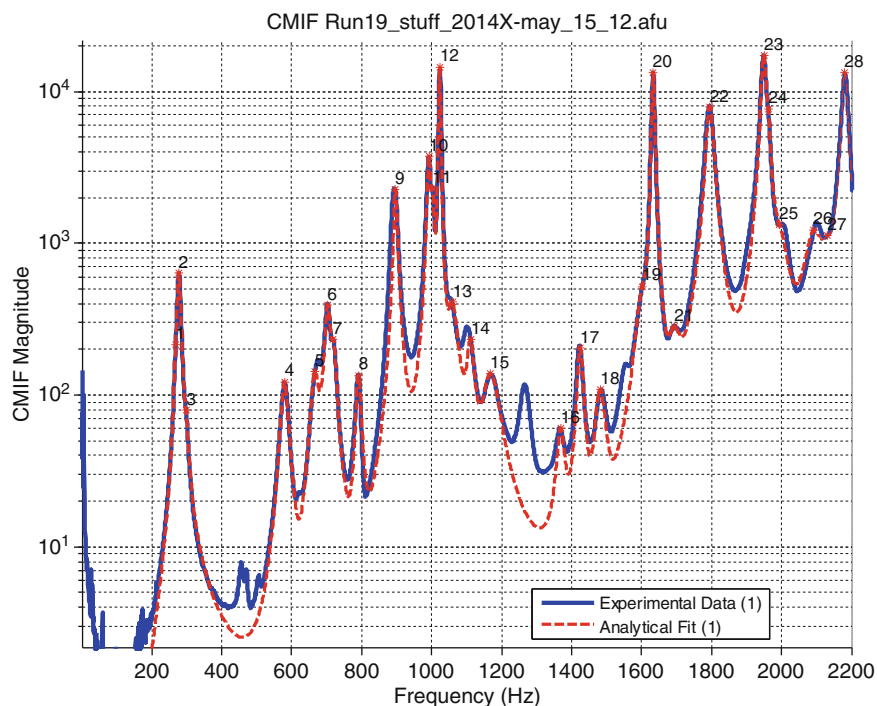


Fig. 10.9 Truth test setup



The comparisons will be presented in two side by side CMIF plots. The red plot is the substructure calculation and the black plot is the measured experimental (truth) CMIF. In Fig. 10.10, the left plot is the substructure result with only six constraints between each set of two substructures, i.e. the rigid body motions of the substructures are constrained together, while the right plot is the substructure result with 22 modal constraints between each set of two substructures. In the first set of plots, the CMIFs of the tip of the beam in three directions are chosen, due to a radial force applied at the middle station of the cylinder. Overall, it appears that the more constrained result on the right is a little better. For example, the extra 380 Hz mode marked by the arrow in the left plot is the first two lobed ovaling mode of the FE substructure. It can be seen that this mode is not present in the experimental data or in the more constrained right plot, where the two lobed mode actually moves up to nearly 900 Hz. These results would be useable for generating vibration specifications.

In Fig. 10.11 the comparison CMIFs for the responses of the end of the beam due to a lateral force at the end of the beam are shown. Either the 6 constraint or 22 constraint model would be useful.

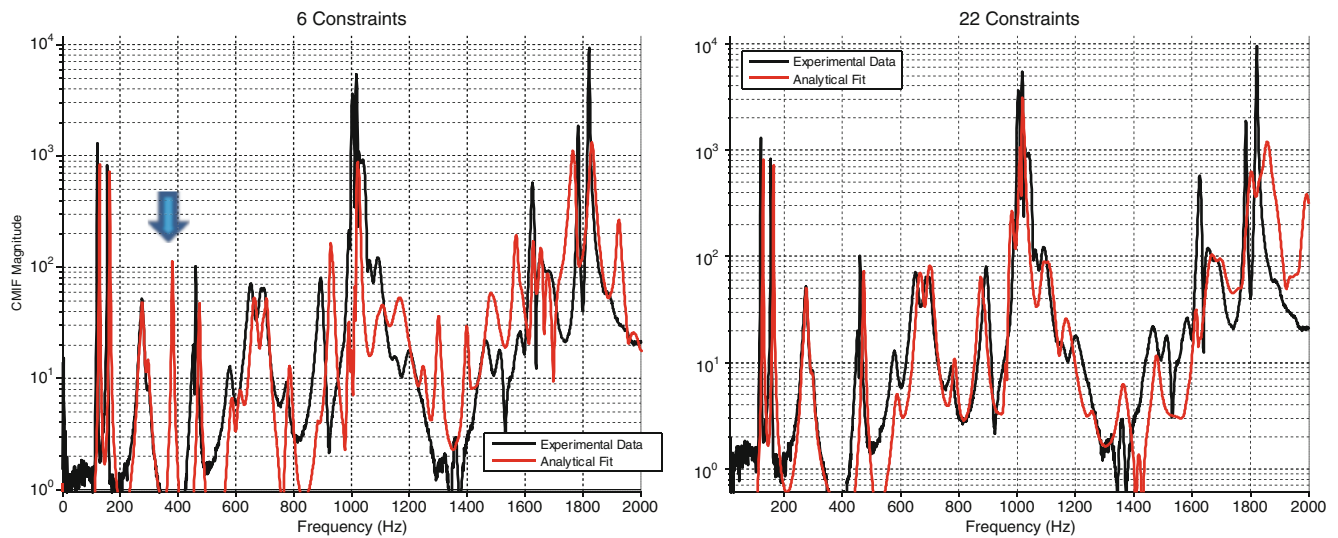


Fig. 10.10 CMIF comparison of substructure to experiment – tip of beam response from radial hammer excitation on cylinder

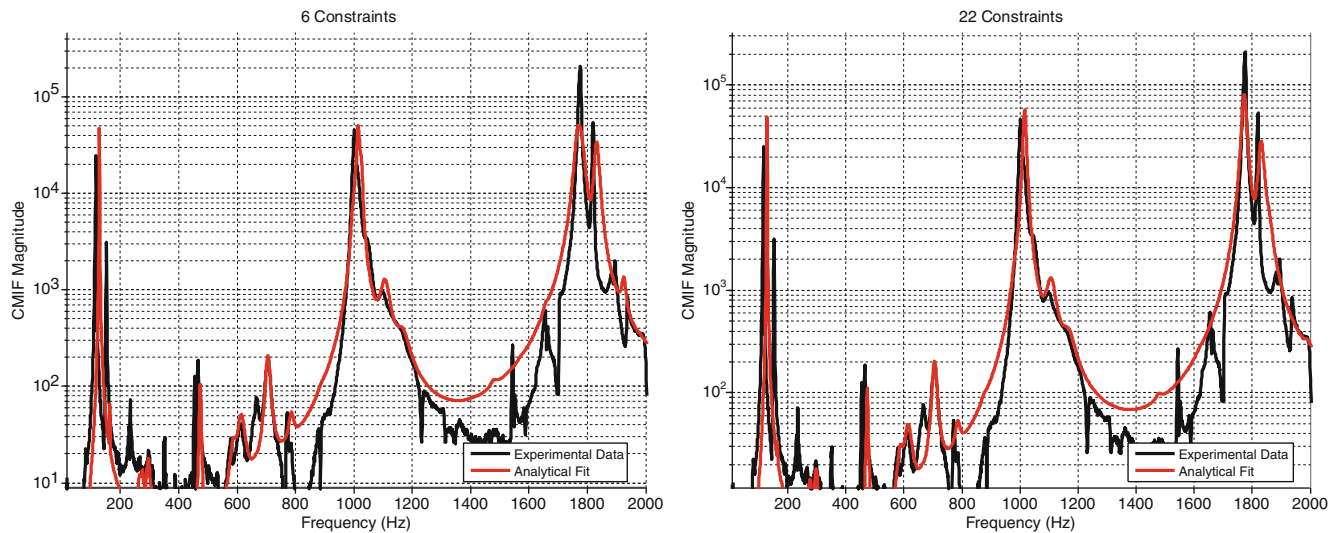


Fig. 10.11 CMIF comparison of substructure to experiment – tip of beam response from lateral hammer excitation on tip of beam

In Fig. 10.12 is the beam tip CMIF response to an axial impact on one corner of the tip of the beam. The results are very similar for both sets of constraints. The first axial mode of the beam on the system is shown by the arrow. Other resonances are due to lateral modes of the beam. These comparisons show the substructuring approach would be useful for predicting system level response for deriving specifications on a component modeled in the FE substructure (external to the experimental substructure).

Shifting the focus to the responses on the internal blivet, which represents an internal component, first consider an impact at the top edge of the cylinder in the axial direction. In Fig. 10.13 the plot arrangement is similar to the previous section. Again one sees in the six constraint result that the 380 Hz first two lobe oval mode and a mode around 1,300 Hz are not constrained properly as pointed out by the blue arrows. No other extra modes show up in the 2,000 Hz bandwidth, although the frequencies and amplitudes of the resonances are more accurate in the right hand 22 constraint result. Since there is not much response at the high frequency, this result is extremely pleasing out to 2,000 Hz. A “spike” at 775 Hz is highlighted by the red arrow in the experimental data. This modal frequency showed up in all FRF sets for only one accelerometer and is very lightly damped (0.1 %). A good drive point mode shape could not be extracted, because the resonance did not appear in the drive point with signal above the noise level. Consequently, the mode could not be scaled properly and was not included in the substructure.

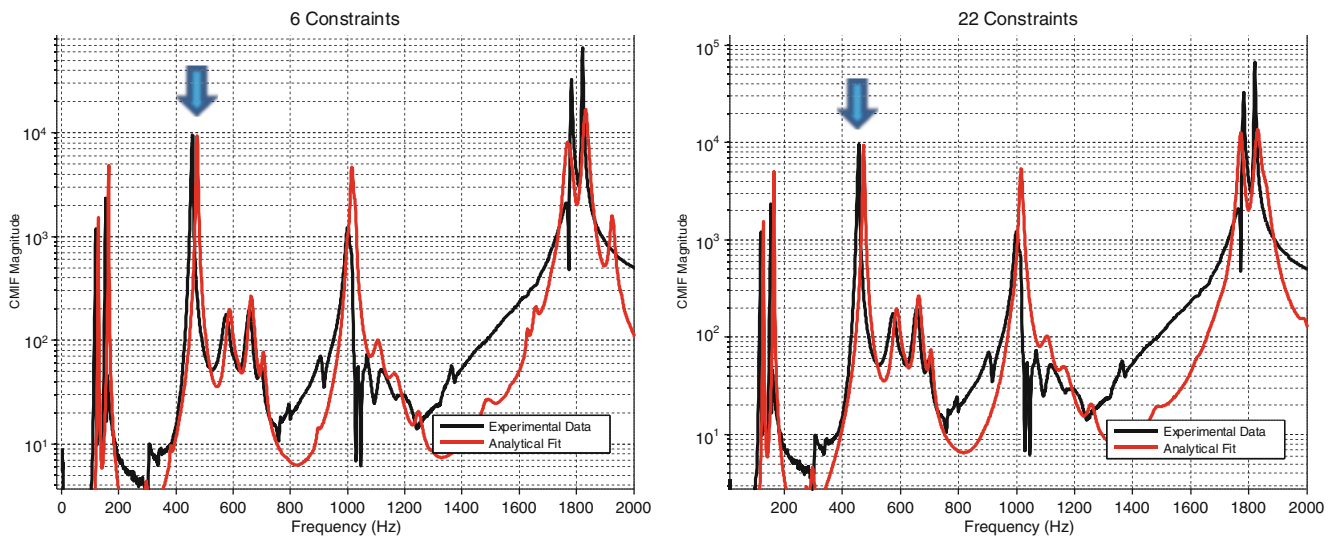


Fig. 10.12 CMIF comparison of substructure to experiment – tip of beam response from axial hammer excitation on corner tip of beam

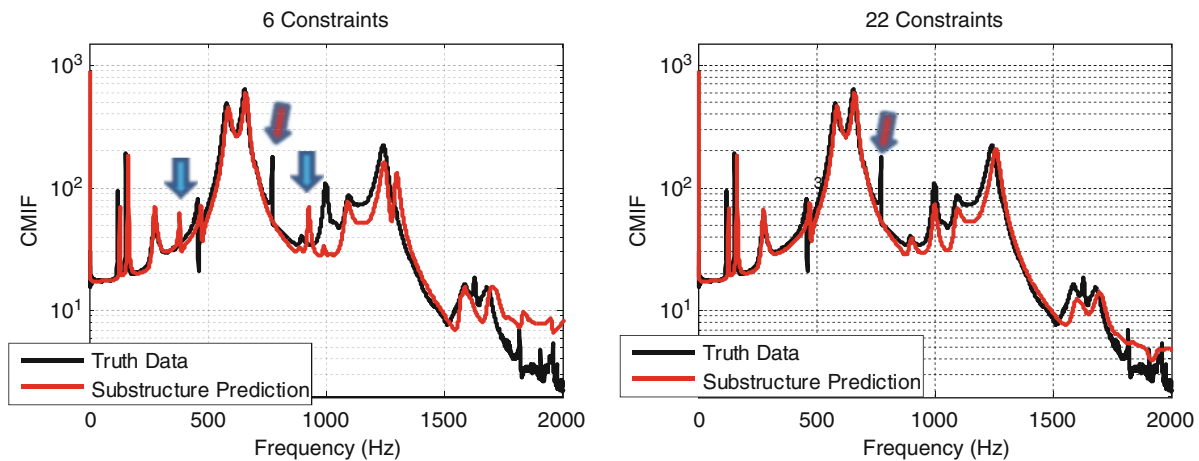


Fig. 10.13 CMIF comparison of substructure to experiment – internal blivet response from axial hammer excitation on cylinder at 1014

In Fig. 10.14 consider blivet response to a radial input mid span of the cylinder. In the 6 constraint case, the 380 Hz unconstrained FE substructure 2 lobe ovaling mode and a 1300Hz ovaling mode significantly pollute the substructure result at that frequency as pointed out by the arrow. The 22 constraint CMIF certainly is a much better comparison in both frequency and amplitude. However, even the better 22 constraint CMIF under predicts at 1000, 1600 and 1800Hz. The 1800Hz mode is due to a 2nd beam bending mode.

10.11 Conclusions

The TS method of dynamic substructuring was applied to a complex substructure. The complexities included continuous (not discrete) connections, preload, small nonlinearity and joints that would be difficult to model with FE. The method captures the damping of the experimental substructure. The method produced no spurious modes as are often identified with other methods. The experimental dynamic substructure was coupled to a FE substructure to produce a full system model for which the modal frequencies were all within a few percent of the experimental data on the full system. Although most of the frequency errors due to the constraint were less than a few percent, sometimes the mode shape errors were much more significant. CMIFs of the responses external to the experimental substructure on the beam were quite good out to 2000 Hz. CMIFs of the internal responses of the component in foam were excellent out to 1000 Hz, good to 1500 Hz and fair out to 2000 Hz.

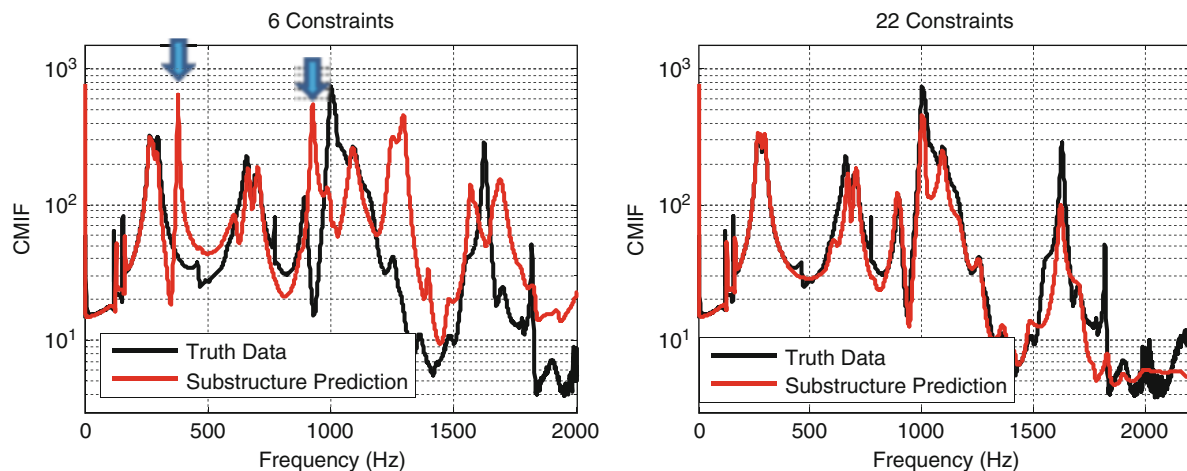


Fig. 10.14 CMIF comparison of substructure to experiment – internal blivot response from radial hammer excitation on cylinder at 3011 near mid bay

The TS mode shape basis was inadequate for a few of the ovaling modes, in particular. The mode shape basis might be improved in the future by applying the stiffness of the load plate and the plate on the cylinder-plate-beam to the TS FE model to more adequately represent the constraints applied to the ovaling by those plates. The mass of the plates could be left out. This would provide a mode shape basis much more similar to the experimental model and the full system model.

The method appears useable in its current form for predicting response of external components in the FE model. In future work, the plan is to extend this method for predicting internal component response in foam to address a system with many internal components, not just one, as performed here. One approach may be to statistically scatter several modes through the bandwidth and scale their damping and amplitude so that they predict the root mean square vibration response for a particular component to the rigid body motion of the external case.

References

1. Allen MS, Mayes RL, Bergman EJ (2010) Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections. *J Sound Vib* 329:4891–4906
2. Mayes RL, Ross MR (2012) Advancements in hybrid dynamic models combining experimental and finite element substructures. *Mech Syst Signal Process* 31:56–66
3. Mayes RL, Ross M, Hunter PS (2010) Examples of hybrid dynamic models combining experimental and finite element substructures. *Proceedings of the 28th international modal analysis conference*, paper 235, February
4. Mayes RL, Hunter PS, Simmermacher TW (2008) Combining experimental and analytical substructures with multiple connections. *Proceedings of the 26th international modal analysis conference*, paper 164, February
5. Allen MS, Mayes RL (2007) Comparison of FRF and modal methods for combining experimental and analytical substructures. *Proceedings of the 25th international modal analysis conference*, paper 269, February
6. Mayes RL, Stasiunas EC (2007) Combining lightly damped experimental substructures with analytical substructures. *Proceedings of the 25th international modal analysis conference*, paper 207, February
7. Mayes RL, Arviso M (2010) Design studies for the transmission simulator method of experimental dynamic substructuring. *Proceedings of the ISMA2010 conference*
8. Mayes RL, Allen MS, Kammer DC (2012) Eliminating indefinite mass matrices with the transmission simulator method of substructuring. *Proceedings of the 30th international modal analysis conference*, paper 163, February
9. Mayes RL, Ross M, Hunter PS (2010) Examples of hybrid dynamic models combining experimental and finite element substructures. *Proceedings of the 27th international modal analysis conference*, paper 235, February
10. Hensley DP, Mayes RL (2006) Extending SMAC to multiple references. *Proceedings of the 24th international modal analysis conference*, January, pp 220–230

Chapter 11

A New Structural Modification Method with Additional Degrees of Freedom for Dynamic Analysis of Large Systems

Burcu Sayin and Ender Cigeroglu

Abstract Structural modification techniques are widely used to predict the dynamic characteristics of modified systems by using the information of the original and modifying systems. Nowadays, due to the increased computational power, large finite element models are regularly used. During design optimization, modifications are done on the original structure in order to meet the design requirements which require reevaluation of the dynamic response of the structure. This is a time consuming process since for each modification, the whole structure needs to be analyzed. In this study, a new structural modification method is proposed in order to decrease the computational effort during the design optimization. Proposed method uses the modal data of the original system and system matrices of the modifying structure in order to predict the modal data of the modified system which can be used for the dynamic analysis. Initially, the application of the method is presented on a lumped parameter model. Later, the proposed method is applied on a finite element model of a cantilever beam which is modified by additional systems having different number of total degrees of freedom (DOFs) and different number of connection DOFs. Modal data of the original structure and the system matrices of the modifying part are extracted from a commercial finite element software. The modal information obtained from the proposed method and the one obtained from finite element software are compared and it is observed that they are in perfect agreement if all information of the original structure is used. The effect of reduction of the original structure is as well investigated and the performance of the developed method is studied for varying the size of the modifying system. The computation time required for the determination of frequency response function is compared with other structural modification methods where significant improvement in computation time is observed.

Keywords Structural modification • Ozguven's method • Additional degrees of freedom • Reanalysis • Direct modification

11.1 Introduction

When analyzing different design alternatives, structural modification techniques are useful at any time in the design stage which shorten the analysis time by a considerable amount. Especially for experimental studies or working with large scale Finite Element Models (FEM), it is difficult and time consuming to adapt every modification to the system in order to reanalyze it accurately. Direct structural modification, also called as reanalysis, is applied using original system information and dynamic structural modification matrix of the modifying part in order to predict the dynamic characteristics of modified systems. Structural modification techniques can be adopted for both systems when additional degrees of freedoms are present or not. These techniques predict the behavior of the modified system very accurately, when all information of the original system is included in the analysis. Structural modification techniques are divided into two as direct structural modification and inverse structural modification techniques [1]. Direct modification is based on determination of the dynamic

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characteristics of the modified system by modifying the original system. On the other hand, inverse structural modification is considered as an optimization process where the modifying structure is determined in order to obtain the desired modified system characteristics. Direct structural modification is studied by Crowley et al. [2] using experimentally obtained frequency response functions (FRFs) of the original structure; whereas, inverse structural modifications are studied by Kyprianou, et al. [3, 4], Mottershead et al. [5, 6] and Park et al. [7].

Modifications may or may not contribute additional DOFs to the original system. For structural modification without additional degrees of freedom, Sherman-Morrison [8] and Woodberry [9] formulas are employed. These methods find the dynamic matrix of the modified system by updating the dynamic matrix of the original system. Ozguven [10] proposed a general method for structural modification problems with or without additional degrees of freedom in the modified structures. This method is an FRF based method which obtains frequency response function of the modified structure by using the FRF of the original structure and dynamic structural modification matrix of the modifying structure. Sanliturk [11] presented a new method based on matrix inversion method combined with Sherman-Morrison formula and Woodberry formula for systems without additional degrees of freedoms. Tahtali et al. [12] applied this method to a plate and a propeller; whereas, Koksall et al. [13] used this method by using power series expansion in order to avoid matrix inversion. In addition to these, distributed modifications studied by D'Ambrogio and Sestieri [14] for systems without additional degrees of freedom and Hang et al. [15–17] worked on distributed modification for systems that their DOFs have changed after modification.

This paper presents a new structural modification method which can be applied to large systems having additional degrees of freedoms. This method uses the modal data of the original structure and system matrices of the modifying part in order to determine the modal data of the modified structure. Mode shapes and natural frequencies of the modified structure can be obtained exactly if all information of the original part is used. Computational efficiency of the method becomes much significant especially working with large scale models. In order to increase the computational efficiency of the method, largest possible system should be considered in design optimizations for the determination of the original structure; whereas, for the modifying part, system matrices can be extracted from a finite element software. Firstly, the developed method is presented on a lumped parameter model. Later, since reduction of the original structure is highly required for large scale systems, accuracy of the reduction process is investigated on a cantilever beam model. Developed method is compared to the Matrix Inversion Method, also called as OZGUVEN's Method [10], by comparing the computational time required for FRF calculation. Methods are applied on a simple cantilever beam model and comparisons are done for different sizes of the modifying system and different sizes of coupling DOFs.

11.2 New Structural Modification Method

Figure 11.1 shows two structures coupled by rigid connection, where subscript *A* corresponds to DOFs which belong only to the original structure, subscript *B* corresponds to DOFs which belong to both structures and subscript *C* corresponds to DOFs which belong only to the modifying part. For the coupled structure, system matrices of the original and modifying parts are added to obtain the equation of motion of the modified structure as seen in Eq. (11.1). Similarly, structural and/or viscous damping can also be included in the equation of motion, which is not considered here.

$$\left[\begin{bmatrix} M_0^{aa} & M_0^{ab} & 0 \\ M_0^{ba} & M_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix} \right] \{\ddot{x}\} + \left[\begin{bmatrix} K_0^{aa} & K_0^{ab} & 0 \\ K_0^{ba} & K_0^{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix} \right] \{x\} = \{F\}, \quad (11.1)$$

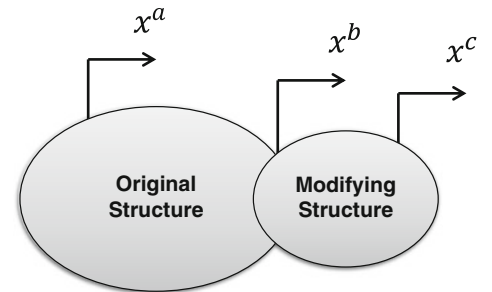


Fig. 11.1 Modified structure (rigid connection)

where $[M_o]$ and $[K_o]$ represent mass and stiffness matrices of the original structure while $[M_m]$ and $[K_m]$ represent mass and stiffness matrices of the modifying part. Let the response of the modified system be written as

$$\{x\} = \begin{Bmatrix} x^a \\ x^b \\ x^c \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} q_0 \\ x^c \end{Bmatrix} = [\Phi] \{q\}, \quad (11.2)$$

where $[\phi_o]$ is the mass normalized mode shape matrix of the original system. Identity matrix is added to the right bottom corner of the matrix in order to keep the information coming from the modifying part. Inserting this expression into Eq. (11.1) and pre-multiplying it by $[\Phi]^T$ gives the following mass and stiffness matrices of the modified structure as

$$[M_{\text{mod}}] = \begin{bmatrix} [I] & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & M_m^{bb} & M_m^{bc} \\ 0 & M_m^{cb} & M_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}, \quad (11.3)$$

$$[K_{\text{mod}}] = \begin{bmatrix} [\Omega_0] & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_m^{bb} & K_m^{bc} \\ 0 & K_m^{cb} & K_m^{cc} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \phi_0^a \\ \phi_0^b \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}, \quad (11.4)$$

The equation of motion becomes:

$$[M_{\text{mod}}] \{\ddot{q}\} + [K_{\text{mod}}] \{q\} = [\Phi]^T \{F\}, \quad (11.5)$$

where $[\Omega_0]$ is a diagonal matrix composed of squares of the natural frequencies of the original structure. For free vibrations, eigenvalue problem is defined as

$$([K_{\text{mod}}] - \lambda [M_{\text{mod}}]) \{q\} = \{0\}, \quad (11.6)$$

Solving the eigenvalue problem defined by Eq. (11.6) the modal data of the modified structure can be obtained. However, it should be noted eigenvectors obtained define $\{q\}$ rather than $\{x\}$; hence, they should be pre-multiplied by $[\Phi]$ which gives the mode shapes of the modified structure in $\{x\}$.

11.3 Case Study 1: Application on a Simple Lumped Parameter Model

In order to validate the developed structural modification method, it is applied on a simple lumped parameter model given in Fig. 11.2. As seen from the figure, the 4-DOF system, original structure, given at the top is modified by the 3-DOF structure, modifying part, given at the bottom by rigid connection. System parameters of the original structure are $M_1 = M_2 = M_3 = M_4 = 1$, $K_1 = K_2 = K_3 = K_4 = 10000$; whereas, for the modifying part they are $M_5 = M_6 = M_7 = 2$, $K_5 = K_6 = K_7 = 20000$.

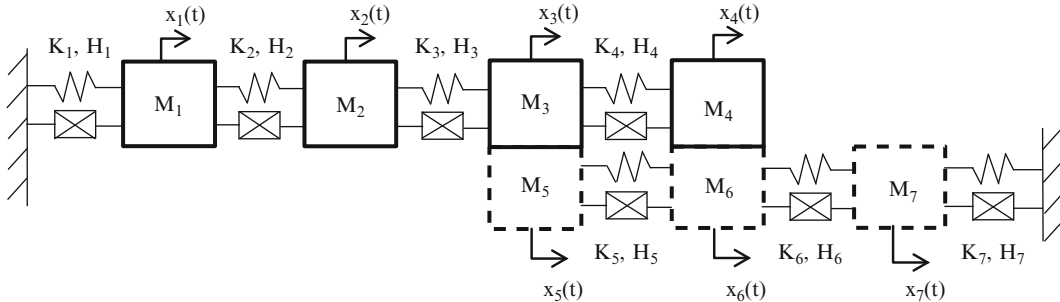


Fig. 11.2 Simple lumped parameter model: blue-original structure, red-modifying part (color figure online)

Table 11.1

Mode #	Natural frequencies found by the proposed method (rad/s)	Natural frequencies found by modal analysis of the modified system (rad/s)	Error %	MAC
1	42.34	42.34	0 %	1.00
2	100.00	100.00	0 %	1.00
3	132.75	132.75	0 %	1.00
4	168.77	168.77	0 %	1.00
5	179.17	179.17	0 %	1.00

Developed structural modification method is applied to this problem and the results obtained are presented in Table 11.1 which are compared with the exact ones obtained by solving the 5-DOF system. In the comparison, for natural frequencies error between the obtained and the exact one and for the mode shapes, the modal assurance criterion (MAC) is used which can be defined as

$$MAC(\{\psi\}_r, \{\psi\}_s) = \frac{(\{\psi\}_r^T, \{\psi\}_s)^2}{(\{\psi\}_r^T, \{\psi\}_r)(\{\psi\}_s^T, \{\psi\}_s)}, \quad (11.7)$$

which takes a value between 0 and 1, where 1 denotes that the compared vectors $\{\psi\}_r$ and $\{\psi\}_s$ are identical.

11.4 Case Study 2: Application on a FEM

11.4.1 Reduction of the Original Structure

For very large scale systems, reduction of the system is required in order to keep computational expenses at minimum or at a practical value. This can be achieved in the developed method by using a certain number of mode shapes which is much less than the number of total DOFs. In order to present the developed method, a cantilever beam modeled in a commercial finite element software is considered. FEM of the cantilever beam and the modifying part are shown in Fig. 11.3, where total numbers of DOFs on the original structure, modifying structure and the modified system are 216, 81 and 270, respectively. Assuming only the first 20 modes are of interest for the modified system, natural frequencies and mode shapes of the first 20 modes of the modified system are obtained by utilizing 20, 30 and 40 modes of the original structure. For the comparison of mode shapes obtained, modal assurance criterion (MAC) is used.

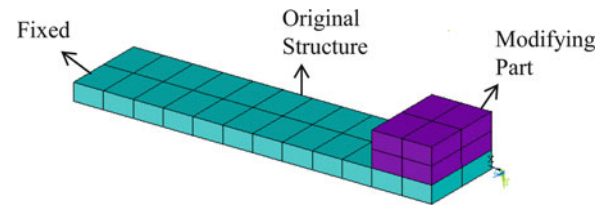
Comparison of the natural frequencies and the mode shapes of the modified structure for different number of modes retained in the original system are given in Tables 11.2 and 11.3, respectively.

11.4.2 Comparison of Computational Time Required

Developed structural modification method is compared with Ozguven's Method by means of computational time required to obtain receptance of the modified structure. For this comparison three cases are considered. In the first case, size of the original system and the number of common DOFs are kept constant in both methods. In the second case, sizes of the original and the modifying parts are kept constant while the number of common DOFs are changed. In the third case, sizes of the original structure, the modifying part and the number of common DOFs are changed in order to keep the size of the modified system constant. For the calculation of the computational time, two different procedures as described below are used:

For the developed model:

1. Reordering of the original system.
2. Reordering of mass and stiffness matrices of the modifying system.
3. Calculation of mass and stiffness matrices of the modified system
4. Solving the EVP for the modified system.
5. Calculation of the receptance of the defined DOF of the modified system in the range of frequency of interest.

Fig. 11.3 FEM of the cantilever beam**Table 11.2** Natural frequencies (Hz)

Developed method							
Mode #	Exact results	Using 20 modes		Using 30 modes		Using 40 modes	
		Nat. freq.	Err %	Nat. freq.	Err %	Nat. freq.	Err %
1	130.14	130.02	0.09	130.00	0.11	129.99	0.12
2	604.54	605.62	0.18	605.43	0.15	605.10	0.09
3	988.99	996.14	0.72	991.79	0.28	990.44	0.15
4	1104.65	1110.54	0.53	1109.59	0.45	1107.76	0.28
5	2799.18	2895.71	3.45	2838.28	1.40	2820.46	0.76
6	4158.96	4190.02	0.75	4185.60	0.64	4176.36	0.42
7	4409.29	4482.54	1.66	4469.44	1.36	4446.96	0.85
8	4793.14	4849.86	1.18	4821.67	0.60	4811.72	0.39
9	6292.99	6561.00	4.26	6390.09	1.54	6344.17	0.81
10	8803.87	9040.56	2.69	8998.08	2.21	8929.63	1.43
11	10058.67	10187.86	1.28	10143.49	0.84	10118.95	0.60
12	10877.85	11804.89	8.52	11206.78	3.02	11057.74	1.65
13	14492.40	15020.50	3.64	14898.24	2.80	14752.55	1.80
14	16458.13	16633.87	1.07	16563.70	0.64	16544.13	0.52
15	16562.25	17128.54	3.42	16841.34	1.69	16770.67	1.26
16	18108.22	21359.44	17.95	18735.07	3.46	18422.54	1.74
17	21922.63	23193.94	5.80	22835.13	4.16	22444.45	2.38
18	24152.82	24860.07	2.93	24344.07	0.79	24318.58	0.69
19	24288.33	37521.24	54.48	24630.32	1.41	24502.37	0.88
20	27322.41	44340.93	62.29	27570.93	0.91	27446.95	0.46

Table 11.3 MAC numbers

Developed method			
Mode #	Using 20 modes	Using 30 modes	Using 40 modes
1	1.00	1.00	1.00
2	1.00	1.00	1.00
3	1.00	1.00	1.00
4	1.00	1.00	1.00
5	1.00	1.00	1.00
6	0.96	0.97	0.99
7	0.98	0.99	0.99
8	0.99	1.00	1.00
9	0.98	1.00	1.00
10	0.99	0.99	1.00
11	0.99	0.99	1.00
12	0.94	0.99	1.00
13	0.98	0.98	0.99
14	0.95	0.98	0.99
15	0.96	0.98	0.99
16	0.85	0.97	0.99
17	0.94	0.95	0.98
18	0.98	1.00	1.00
19	0.10	0.95	0.99
20	0.05	0.98	0.99

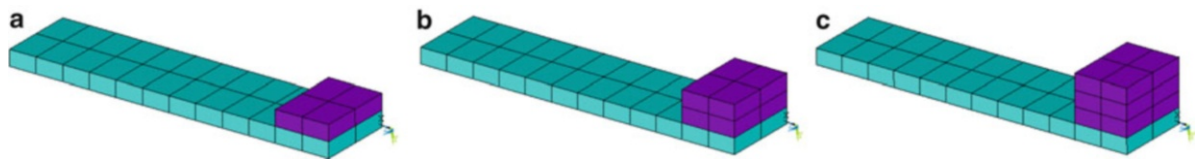


Fig. 11.4 Finite element models used in Case 1: *green*-original structure, *purple*-modifying part, size of the modifying part: (a) 54, (b) 81, (c) 108 (color figure online)

Table 11.4 Sizes of the models:
Case 1

Model #	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
1	216	54	243	27
2	216	81	270	27
3	216	108	297	27

Table 11.5 Comparison of calculation time: Case 1

Model #	Developed method (s)						Ozguven's method (s)					
	Including the modal solution			Excluding the modal solution			Including the FRF of the original system			Excluding the FRF of the original system		
	20	40	80	20	40	80	20	40	80	20	40	80
	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes
1	0.05	0.07	0.11	0.02	0.02	0.04	3.31	3.84	4.57	3.04	3.19	3.26
2	0.10	0.12	0.18	0.02	0.04	0.06	5.81	6.25	6.90	5.44	5.62	5.82
3	0.17	0.19	0.27	0.04	0.05	0.09	8.70	9.59	10.01	8.31	8.58	8.68

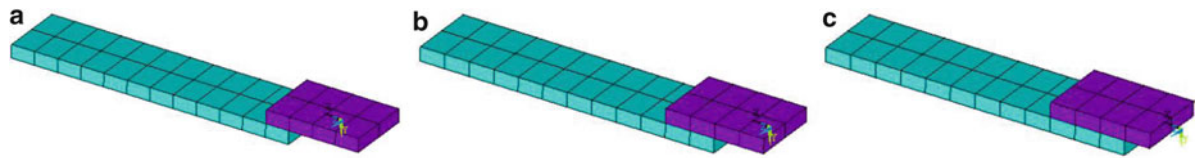


Fig. 11.5 Finite element models used in Case 2: *blue*-original Structure, *purple*-modifying part, size of the common DOFs: (a) 18, (b) 27, (c) 36 (color figure online)

For Ozguven's Method:

1. Reordering of the original system.
2. Reordering of mass and stiffness matrices of the modifying system.
3. For the frequency range of interest:
 - a. Calculation of the receptance of the original system.
 - b. Calculation of the dynamic structural modification matrix.
 - c. Calculate the receptance of the desired DOF of the modified system.

The calculations are done on a computer having a processor Intel Core i3-2310 M CPU @ 2.10 GHz with 4,00 GB of RAM. Finite element models of the original and modifying structures for the first case is given Fig. 11.4.

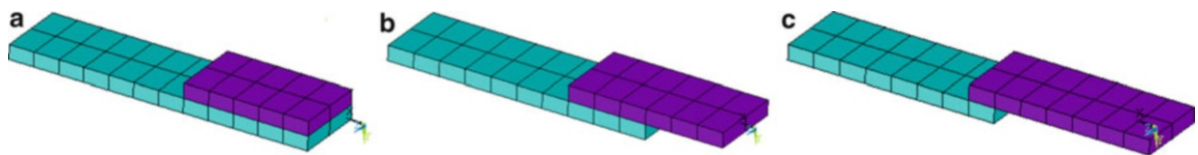
In this case, the effect of modifying system size (hence, the size of the modified system) on the computational time is studied. Common number of DOFs is 27 in all cases. The sizes of the FEM considered in this comparison are presented in Table 11.4. For the cases when first 20-40-80 modes of the original system are included in the calculations. In the computations, the frequency range of interest is considered as 500-1200 rad/s utilizing 351 frequency points. From the results (see Table 11.5) it can be observed that as the number of DOFs in the modifying part; hence, the modified structure, increases, time required to determine the receptance increases as well. However, it can be observed that proposed method decreases the computational time significantly compared to Ozguven's Method. On the other hand, it should be noted that developed method cannot be used if the experimental receptance of the original structure is at hand. In that case, Ozguven's Method is the fastest choice as indicated by [18] for systems without additional DOFs.

Table 11.6 Sizes of the models:
Case 2

Model #	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
1	216	90	288	18
2	216	90	279	27
3	216	90	270	36

Table 11.7 Comparison of calculation time: Case 2

Model #	Developed method (s)						Ozguven's method (s)					
	Including the modal solution			Excluding the modal solution			Including the FRF of the original system			Excluding the FRF of the original system		
	20	40	80	20	40	80	20	40	80	20	40	80
	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes
1	0.13	0.16	0.22	0.04	0.05	0.08	6.82	7.18	8.14	6.43	6.60	6.79
2	0.12	0.14	0.20	0.03	0.05	0.07	6.58	7.05	7.81	6.22	6.50	6.59
3	0.11	0.14	0.20	0.02	0.04	0.06	6.25	6.63	7.49	5.82	6.15	6.32

**Fig. 11.6** Finite element models used in Case 3: *green*-original structure, *purple*-modifying part, size of the modifying part: (a) 108, (b) 126, (c) 144 (color figure online)**Table 11.8** Sizes of the models:
Case 3

Model #	Original system DOF #	Modifying system DOF #	Modified system DOF #	Common DOF #
1	216	108	270	54
2	180	126	270	36
3	144	144	270	18

Table 11.9 Comparison of calculation time: Case 3

Model #	Developed method (s)						Ozguven's method (s)					
	Including the modal solution			Excluding the modal solution			Including the FRF of the original system			Excluding the FRF of the original system		
	20	40	80	20	40	80	20	40	80	20	40	80
	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes	modes
1	0.14	0.17	0.22	0.02	0.03	0.07	7.75	8.27	8.94	7.40	7.47	7.52
2	0.22	0.24	0.32	0.05	0.06	0.09	9.84	10.14	10.67	9.44	9.77	9.88
3	0.30	0.33	0.44	0.08	0.09	0.14	12.03	12.33	12.86	11.73	12.15	12.21

In the second case, sizes of modifying system and the original systems are kept constant, where the number of common DOFs is changed as 18, 27, and 36 as shown in Fig. 11.5. Table 11.6 gives the sizes of the models used; whereas, Table 11.7 presents the calculation time considering the first 20, 40 and 80 *modes* of the original system in both methods. The frequency range of interest is the same as the first case utilizing 351 frequency points. It can be seen that as the number of common DOFs increases, resulting in decrease in the total number of DOFs of the modified system, computation time required decreases slightly for both methods. However, if both methods are compared, the proposed method results in a drastic decrease in the computational time.

In the third case, the number of total DOFs of the modified system is kept constant by changing the sizes of the, original structure, modifying part and the common DOF. The finite element models used are presented in Fig. 11.6; whereas, the sizes of part are presented in Table 11.8. Results obtained by utilizing the first 20, 40 and 80 *modes* of the original structure are given in Table 11.9. Identical frequency range of interest is used as in the previous cases. It can be observed from the results

that as the number of DOFs in the modifying structure increases, solution time also increases for both methods, even though the number of common DOFs or the number of DOFs in the original structure decreases. Similar to the previous cases, the proposed requires significantly less computational time compared to Ozguven's method.

11.5 Conclusion

In this paper, a new structural modification method is presented which determines the natural frequencies and mode shapes of the modified structure using modal data of the original structure and mass and stiffness matrices of the modifying part. In order to demonstrate the method, a simple lumped parameter model is considered. If all modal information of the original structure is used in the analysis, it is seen that the exact mode shapes and natural frequencies of the modified structure can be obtained. For large scale models, using all information is not practical due to high computational time requirements. Therefore, reduction of the original structure is studied by considering FEM of a cantilever beam retaining 20, 40, and 80 modes of the original structure in the calculations. It is seen that, using more information provides more accurate results. Furthermore, the proposed method is compared to the Ozguven's method in terms of calculation time. Three different cases utilizing different DOFs in the original structure, modifying structure and modified system are studied. It is concluded that in all cases, proposed method provides a significant decrease in the computational time required compared to Ozguven's method. Therefore, the developed structural modification method can be employed in reanalysis of large finite element models, especially for design optimization purposes.

References

1. Stancioiu D, Ouyang H (2011) Structural modifications by the method of receptances. Session of the commission of acoustics, Bucharest, 25–26 May, SISOM 2011
2. Crowley JR, Rocklin GT, Klosterman AL, Vold H (1984) Direct structural modification using frequency response functions. Proceedings of international modal analysis conference, 2nd IMAC
3. Kyprianou A, Mottershead JE, Ouyang H (2005) Structural modification. Part 2: assignment of natural frequencies and antiresonances by an added beam. *J Sound Vib* 284(1–2):267–281
4. Kyprianou A, Mottershead JE, Ouyang H (2004) Assignment of natural frequencies by an added mass and one or more springs. *Mech Syst Signal Process* 18(2):263–289
5. Mottershead JE, Kyprianou A, Ouyang H (2004) Structural modification. Part 1: rotational receptances. *Mech Syst Signal Process* 18(2):263–289
6. Mottershead JE, Mares C, Friswell MI (2004) An inverse method for the assignment of vibration nodes. *Mech Syst Signal Process* 18(2):263–289
7. Park YH, Park YS (2000) Structural modification based on measured frequency response functions: an exact eigenproperties reallocation. *J Sound Vib* 237(3):411–426
8. Sherman J, Morrison WJ (1949) Adjustment of an inverse matrix corresponding to changes in the elements of a given column or given row of the original matrix. *Ann Math Statist* 20:621
9. Woodbury M (1950) Inverting modified matrices, Memorandum report 42, Statistical Research Group, Princeton University, Princeton, NJ
10. Ozguven HN (1990) Structural modifications using frequency response functions. *Mech Syst Signal Process* 4(1):53–63
11. Şanlıtürk KY (2002) An efficient method for linear and nonlinear structural modifications. Proceedings of ESDA 2002: 6th biennial conference on engineering systems design and analysis, ESDA2002/APM-028, Istanbul, Turkey
12. Tahtali M (1992) Vibration analysis of damped structures and structural reanalysis using a new structural modification method. *Mechanical Engineering*, Middle East Technical University, Ankara, Turkey, p 78
13. Köksal S, Cömert MD, Özgüven HN (2006) Reanalysis of dynamic structures using successive matrix inversion method. Proceedings of the international modal analysis conference, 24th IMAC, St. Louis, MO
14. D'Ambrogio W, Sestieri A (2001) Coupling theoretical data and translational FRFs to perform distributed structural modification. *Mech Syst Signal Process* 15(1):157–172
15. Hang H, Shankar K, Lai J (2008) Prediction of the effects on dynamic response due to distributed structural modification with additional degrees of freedom. *Mech Syst Signal Process* 22(8):1809–1825
16. Hang H, Shankar K, Lai J (2009) Effects of distributed structural dynamic modification with reduced degrees of freedom. *Mech Syst Signal Process* 23(7):2154–2177
17. Hang H, Shankar K, Lai J (2010) Effects of distributed structural dynamic modification with additional degrees of freedom on 3D structure. *Mech Syst Signal Process* 24:1349–1368
18. Canbaloglu G (2009) Structural modification with additional degrees of freedom in large systems. MSc Thesis, Mechanical Engineering Department, Middle East Technical University, Ankara

Chapter 12

IMAC XXXI: Additional Modal Testing of Turbine Blades and the Application of Transmission Simulator Substructuring Methodology for Coupling

David Macknelly, Mohsin Nurbhai, and Nicholas Monk

Abstract Building on work presented at IMAC XXX, the Structural Dynamics group at AWE have carried out additional modal tests on three Ampair 600 turbine blades, both in a free-free and fixed-base configuration. The results from these tests were correlated with experimental results published by the experimental substructuring focus group and also to an improved Finite Element Model.

Using the transmission simulator substructuring methodology, experimental results had a fixed boundary condition applied. The results of the substructuring calculations were correlated with equivalent FE simulations and experimental tests.

Keywords Dynamic substructuring • MCFS • Component mode synthesis • IMAC • Modal analysis • Fixed base

12.1 Introduction

Over the recent years there has been an increased interest in experimental dynamic substructuring which has led to the formation of the dynamic substructuring focus group in the Society for Experimental Mechanics. As part of the focus group, AWE has been conducting research based on the wind turbine test bed [1, 2], specifically focussing on dynamically characterising a set of turbine blades in preparation for experimental substructuring efforts. The turbine blades are ideal candidates for experimental substructuring due to their complex geometry and material properties, making them difficult to model analytically. Instead, if suitable substructuring techniques are utilised, an experimental model of the turbine blade can be coupled to a Finite Element (FE) model of the remaining wind turbine structure.

At IMAC XXX AWE presented [3] work detailing modal tests conducted on a series of Ampair 600 wind turbine blades and correlated with a finite element model. This paper builds on this work and details additional modal tests of the turbine blades attached to a circular aluminium plate, acting as a transmission simulator, in preparation for experimental substructuring calculations using the Modal Constraints for Fixture and Substructuring (MCFS) methodology developed by Allen et al. [4].¹

In addition, modal tests of the turbine blades in a fixed-base configuration were conducted. These were compared to results from tests on identical blades made by focus group members at the University of Massachusetts, Lowell (UML) [5]. The results were also used as truth-test to verify the accuracy of the MCFS method to apply a fixed-base condition to the turbine blade.

Lastly a new transmission simulator design was tested virtually using a finite element model of the turbine blade system and the MCFS framework in order to improve the accuracy of the application of a fixed-base condition.

* Atomic Weapons Establishment (AWE) is a multi-programme laboratory managed for the UK Ministry of Defence through AWE Management Limited.

¹It is beyond the scope of this paper to document the mathematical techniques used within the MCFS framework.

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12.2 Outline of Methodology

The main aim of this paper is to use additional modal test data of the Ampair 600 turbine blade attached to a transmission simulator fixture in a free-free configuration and a FE model of the same transmission simulator, as inputs to the MCFS substructuring framework [4] in order to apply a fixed-base boundary condition to the turbine blade.

This process is illustrated in Fig. 12.1, where Subsystem C represents the turbine blade (subsystem A) attached to the transmission fixture (subsystem B_e). Utilising the MCFS framework, a FE model of the transmission simulator that has been grounded at the blade attachment points (subsystem B_{an}) is removed from the test data to leave the turbine blade in a fixed-base configuration. This process is validated using truth-test data from the turbine blade that has been modal tested in a fixed-base configuration.

12.3 Transmission Simulator

To accomplish the substructuring methodology described in the previous section, it was necessary to design and manufacture a transmission simulator fixture that would replicate the impedance of the bolted connection between the turbine blade and hub. To expedite manufacture, an existing circular plate was modified to incorporate a series of blade mounting points, highlighted in red in Fig. 12.2. The layout of these mounting points allowed all three blades to be attached which may be required for future testing configurations.

Once the plate modifications had been completed, the finished transmission simulator was suspended via elastic cords and tested in a free-free condition using a Polytec 3D Scanning LDV (Fig. 12.3 left) and a modal shaker as an excitation

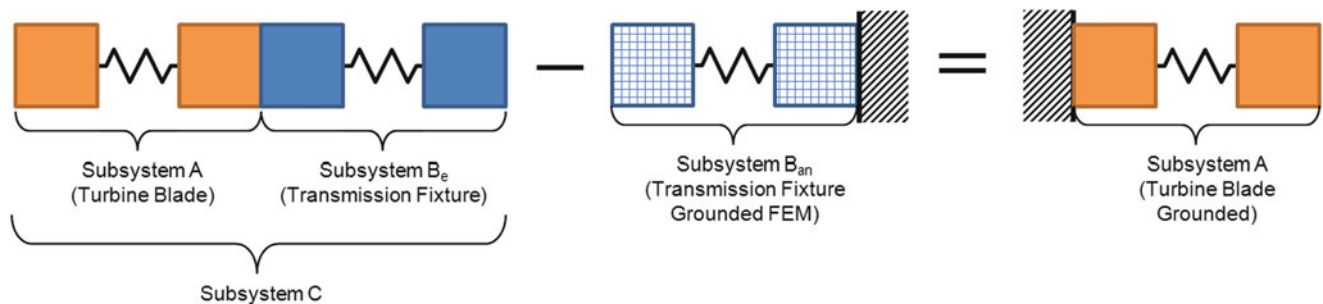


Fig. 12.1 Outline of the utilisation of the MCFS framework to apply a fixed-base condition to a turbine blade attached to a transmission simulator, tested in a free-free condition

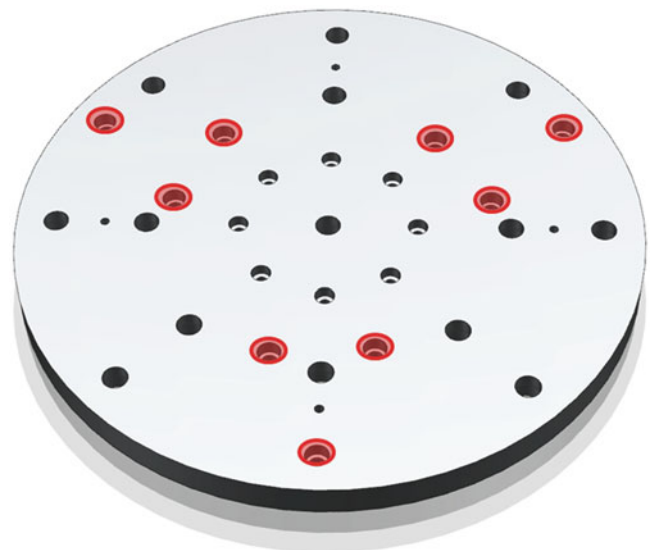


Fig. 12.2 Design of transmission simulator fixture, showing blade mounting points highlighted in red (color figure online)

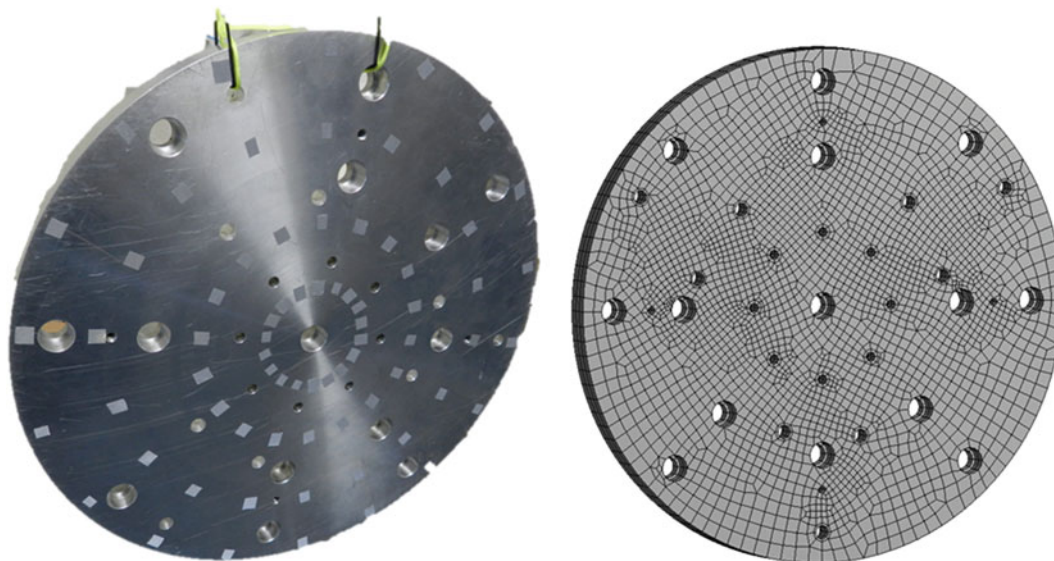
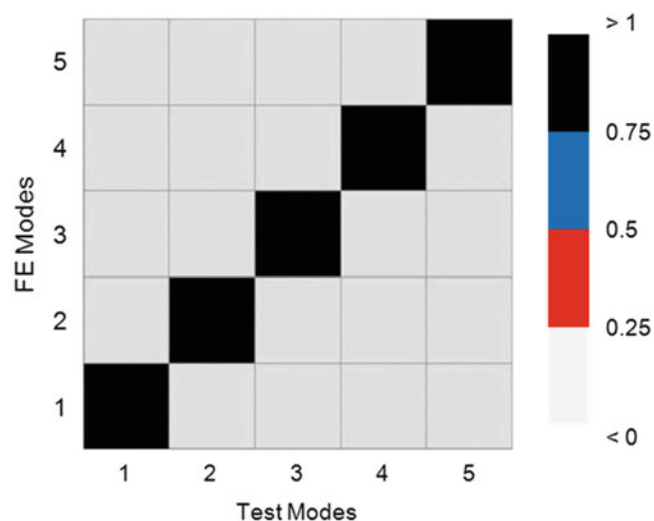


Fig. 12.3 Photo of the transmission simulator in free-free test configuration (*left*) and finite element model (*right*)

Fig. 12.4 Modal assurance criterion (MAC) plot between test and FE



source. The results from this test were compared with a fully featured Finite Element Model (Fig. 12.3 right) and achieved an excellent correlation, as demonstrated by the MAC in Fig. 12.4 and with frequency differences of less than 2%.

The high degree of correlation between test and FE provided confidence that results from the FE model could be successfully used to fulfil the transmission simulator function within the MCFS framework.

12.4 Turbine Blade Coupled to Transmission Simulator Test

After successfully correlating the transmission simulator FE model to the test hardware, each of the three turbine blades were connected to the transmission simulator (at the same mounting location each time) and tested in a free-free configuration (Fig. 12.5 Left). Each blade was bolted to the transmission fixture using nuts to act as a spacer between the blade and fixture (Fig. 12.5 Right).



Fig. 12.5 Photographs of the experimental setup for the turbine blade bolted to transmission simulator

Table 12.1 Extracted natural frequencies of turbine blade bolted to transmission simulator in a free-free configuration

Mode	AWE blade		
	726 Hz	733 Hz	754 Hz
1	47.5	48.7	48.9
2	134.4	132.5	135.0
3	251.7	251.1	258.5
4	326.5	326.5	339.1
5	417.1	417.3	422.5
6	490.8	496.8	508.4
7	788.6	790.3	801.6
8	944.6	940.2	972.0

Each blade was tested using a roving modal hammer,² with four single-axis accelerometers mounted in the Z direction; two on the turbine blade at points 18 & 9 and two on the transmission fixture at points 21 and 23. Point 18 was chosen as a consistent reference point for all tests, matching a reference point used during tests made by UML [5]. Other points were spread over the test structure using ‘engineering judgement’. The useful test bandwidth was 3200Hz with a frequency resolution of 0.78Hz.

The resulting data was analysed using the LMS Test.Lab PolyMAX algorithm to capture the first eight modes of the combined blade and transmission fixture system. Table 12.1 lists the natural frequencies of the extracted modes for each blade and transmission fixture assembly. Figure 12.6 shows the visualisation of mode two, including the test point node numbers and coordinate system used throughout all tests and FE models.

The resulting data was exported from the LMS software suite and converted into a format compatible with the MCFS framework within Matlab. In addition, the six rigid body modes of the free-free system were extracted from a FE model of the blade and transmission fixture in a free-free configuration (adapted from [6]) and integrated with the flexural modes at matching DoFs.

²A roving tri-axial accelerometer test was attempted but demonstrated mass-loading effects and so was not considered in this paper. A MIMO shaker test with a scanning 3D LDV will be utilised for future tri-axial testing.

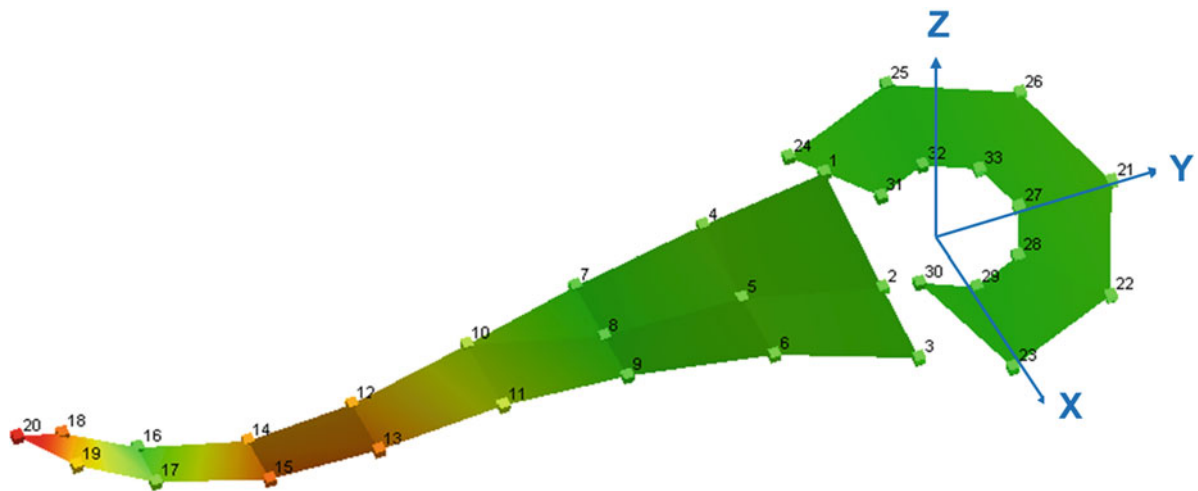


Fig. 12.6 Visualisation of mode two, showing the test point node numbers and coordinate system used

12.5 Turbine Blade Clamped Configuration

To provide truth-test data for the turbine blade in a fixed-base configuration, each blade was bolted to a surface table, via an adapter plate, at the hub connection points as shown in Figs. 12.7 and 12.8.

A roving modal hammer test was performed on each blade in the fixed-base configuration, using a single-axis accelerometer attached at point 18 (near the tip of the blade) as a reference. As with the free-free tests, the results were processed using LMS and an attempt was made to extract the first seven modes. This test aimed to replicate the setup from the fixed-base tests performed at the University of Massachusetts, Lowell [5] in order to be able to compare the results, which are shown in Table 12.2.

The results from the AWE blade tests compare well with those from UML. It is not currently understood why the third mode could not be extracted from blade 726 and 733 and will be tested again in future work.

12.6 Grounded Transmission Simulator Finite Element Model

The final data set input into the MCFS framework was the finite element model of the transmission simulator, fixed at the connection points with the turbine blade. Figure 12.9 shows the fixed boundary condition (highlighted in blue on the left image) applied to the transmission simulator model. It was important to ensure that the model shared the same coordinate system with the experimental tests, to avoid inaccuracies due to mismatching DoFs. This model was solved out to 20 modes (mode 6 is visualised on the right image of Fig. 12.9) and exported into a format compatible with the MCFS framework in Matlab.

12.7 Application of MCFS Framework

12.7.1 Application of Framework Using Modal Test Data

Once all of the experimental tests and finite element models were completed, each set of data was reduced down to the same common test points using the ICATS software package. Great care was taken to ensure that all experimental and finite element data was in the same coordinate system and shared common test points in order to minimise 'housekeeping' errors within the Matlab code to prepare the data for the MCFS framework.

Fig. 12.7 Photograph of an Ampair 600 turbine blade bolted to a surface table for a fixed-base modal test

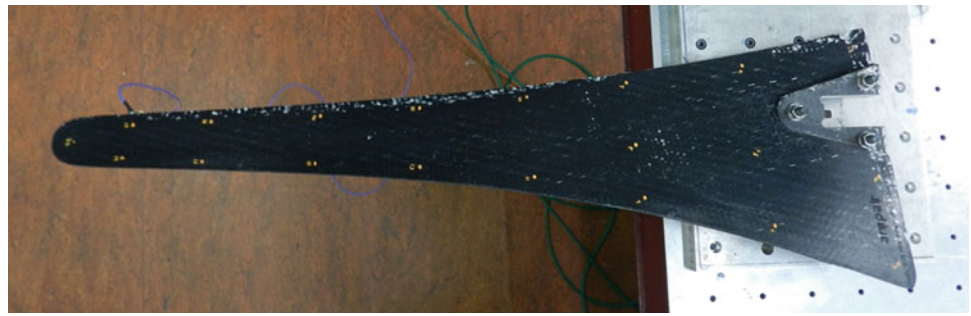


Fig. 12.8 Close-up photograph of the bolted connection between the Ampair 600 turbine blade and surface table adaptor plate

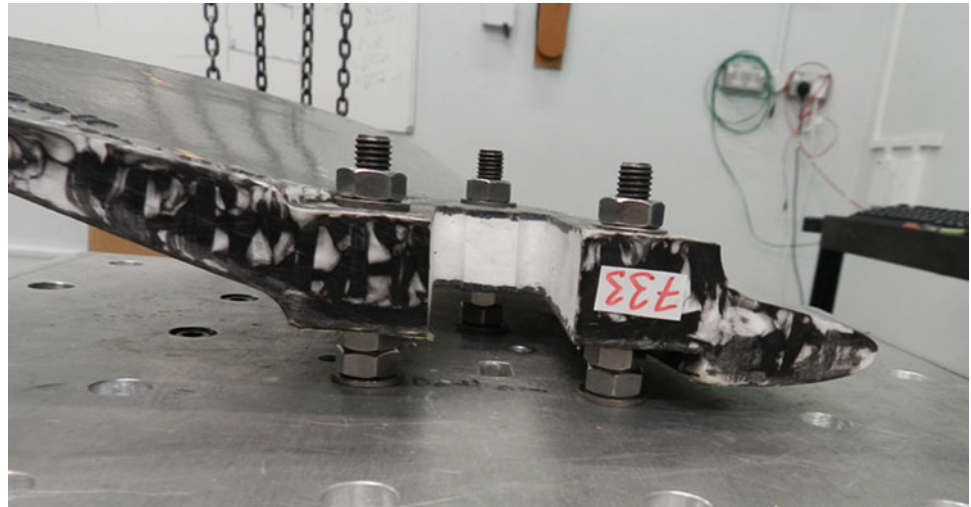


Table 12.2 Results of the turbine blades tested in a fixed-base configuration, including results from UML as reported at IMAC XXX [5]

Mode	AWE blades			UML blades					
	726	733	754	001	002	003	004	005	006
	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz
1	21.3	21.6	21.9	19.3	20.7	21.5	19.8	19.7	19.5
2	71.8	71.5	72.8	64.5	70	72.8	71.7	72	70
3	Not found	Not found	122.7	137.4	133.3	134.8	127	126.8	128.5
4	183.5	190.8	193.4						
5	267	279.3	282.1						
6	313.3	Not found	317.1						
7	368.3	385.5	382.6						

A set of connection points common to the experimental data and the finite element model were selected that spanned the surface of the transmission simulator. In addition, one location (point 18) was selected for drive point FRF comparison on the turbine blade which was compared to the fixed-base blade modal tests described in Section 12.5.

Once all of the data had been processed and converted into a format compatible with the MCFS framework, the Matlab script framework (provided by R. Mayes) were executed³ to produce a FRF of the response drive point 18z.

The application of the MCFS framework (as described in Fig. 12.1) using modal test data only considered one degree of freedom (Z direction) as the physical modal tests only utilised single axis accelerometers due to mass loading effects of the larger multi-axis accelerometer. Figure 12.10 shows a plot of the results of the substructuring calculation with a drive point FRF at 18z (near the tip of the blade – see Fig. 12.6) in dashed red, with the equivalent regenerated drive point from the verification hammer test in solid black.

Whilst the correlation between the two FRFs is reasonable for the first two modes, the agreement between two plots then starts to diverge significantly.

³The MCFS framework was utilised in a somewhat ‘black-box’ fashion, as the derivation and explanation of the framework is beyond the scope of this paper.

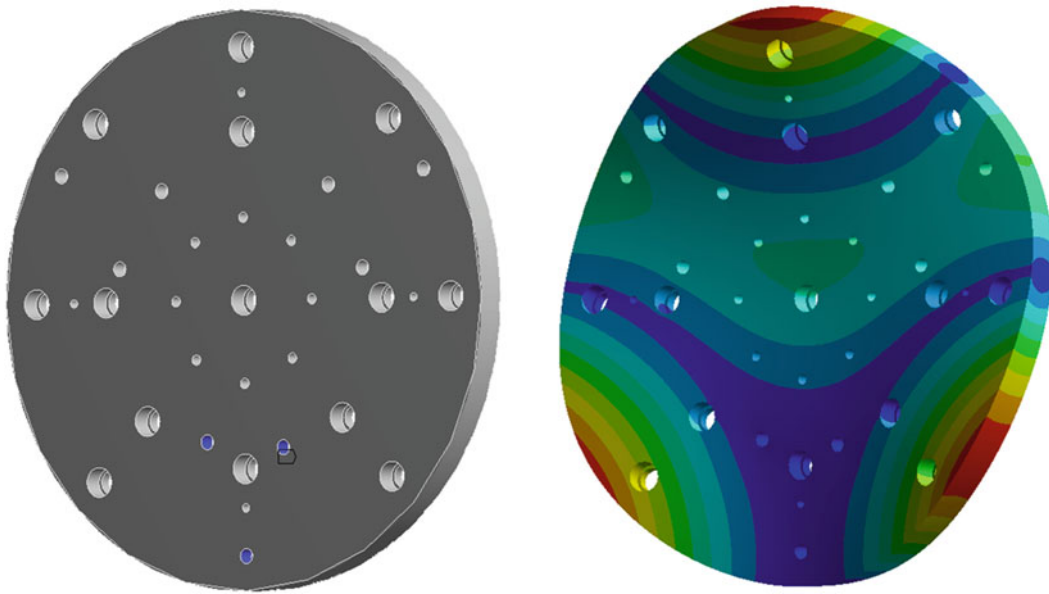


Fig. 12.9 Transmission simulator finite element model fixed at blade attachment points (*blue on left image*), with mode 6 visualised on *right image* (color figure online)

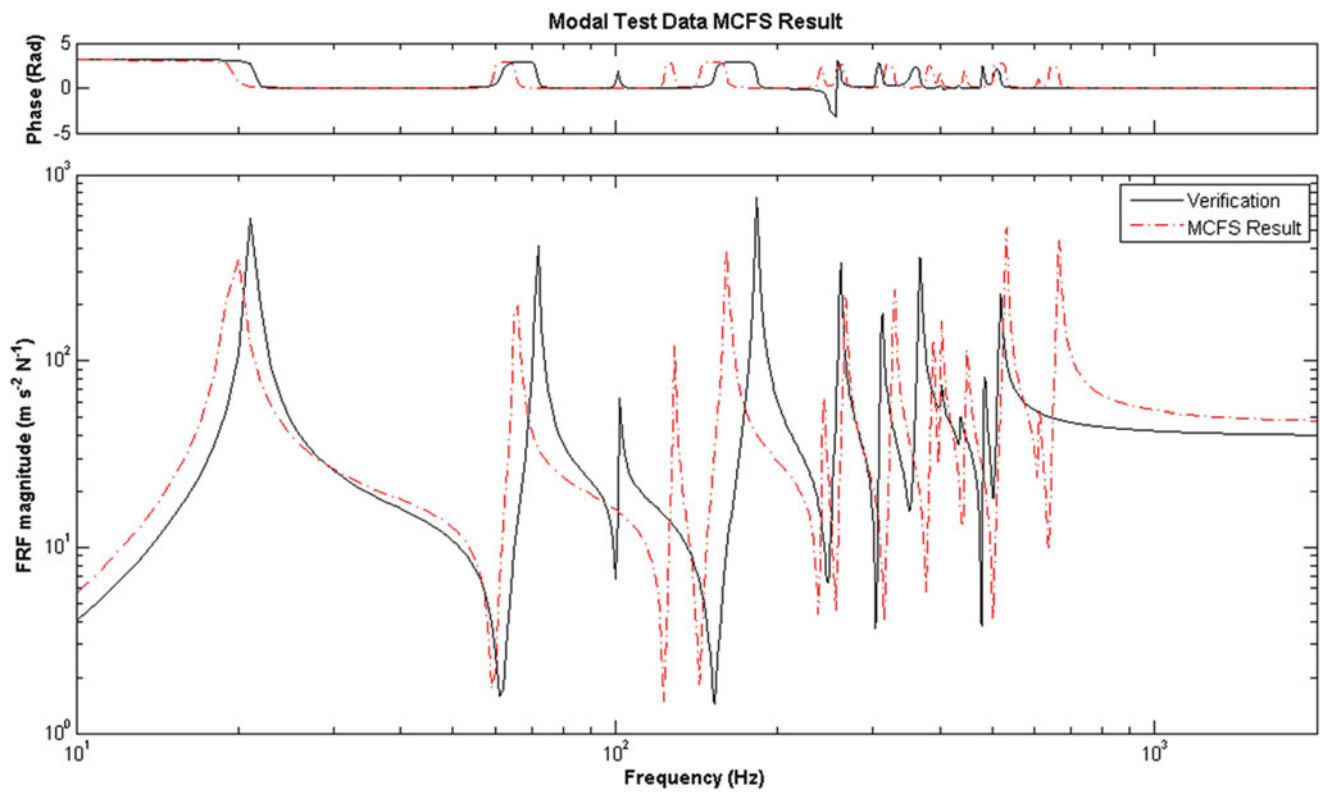


Fig. 12.10 Magnitude and phase FRFs of drive point 18z of the turbine blade in a fixed-base configuration as a result of the MCFS calculation (*dotted red line*) and the verification truth-test (*solid black line*) (color figure online)

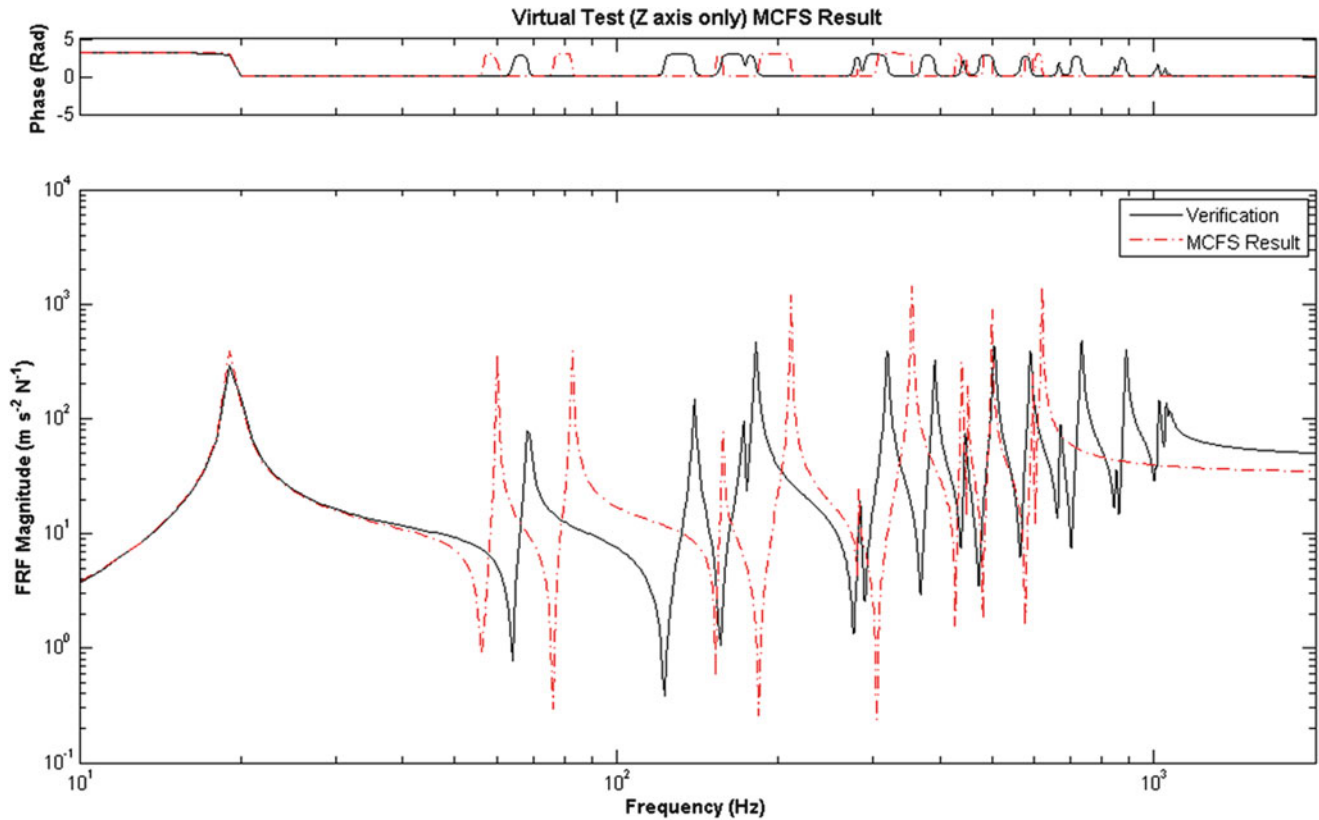


Fig. 12.11 Results from the MCFS calculation using only Z axis data from FE models of the turbine blade and transmission fixture and a fixed-base turbine blade as the verification system

12.7.2 Application of Framework Using Finite Element Results

In order to verify if experimental inaccuracies affected the quality of the MCFS calculations, the procedure was repeated using data purely from FE models of the turbine blade and transmission simulator. This included using a fixed-base simulation of the turbine blade as the verification model.

The first application of the framework with FE data used only Z axis DoFs, in order to replicate the physical tests. Figure 12.11 shows the drive point FRFs (at the same location on the tip of the turbine blade as Section 12.7.1) from the results of the substructuring calculation (in dashed red) and from the verification simulation (in solid black).

As with the physical test, the first mode correlated well, with subsequent modes failing to align with the verification model.

Both applications of the MCFS substructure procedure so far have only considered Z axis DoFs from each data set. To examine the effect of including extra DoFs in the calculations, the virtual test using FE data was repeated to include all three translational DoFs. Figure 12.12 shows the FRFs resulting from the MCFS calculation repeated with all translational DoFs included in the input data sets.

While the addition of the extra DoFs has improved the accuracy of the synthesised drive point FRF, it has also introduced some additional spurious spikes, possibly indicating that the substructuring calculations greatly overestimate the coupling between flexural modes of the turbine blade in each axis.

12.7.3 Application of Framework Using New Transmission Simulator

One of the greatest uncertainties of the MCFS methodology is the design of the transmission fixture, which acts to couple the physical test to the FE models. This fixture must accurately replicate the mounting interface, especially the impedance, between the component of interest and whatever it is to be attached to.

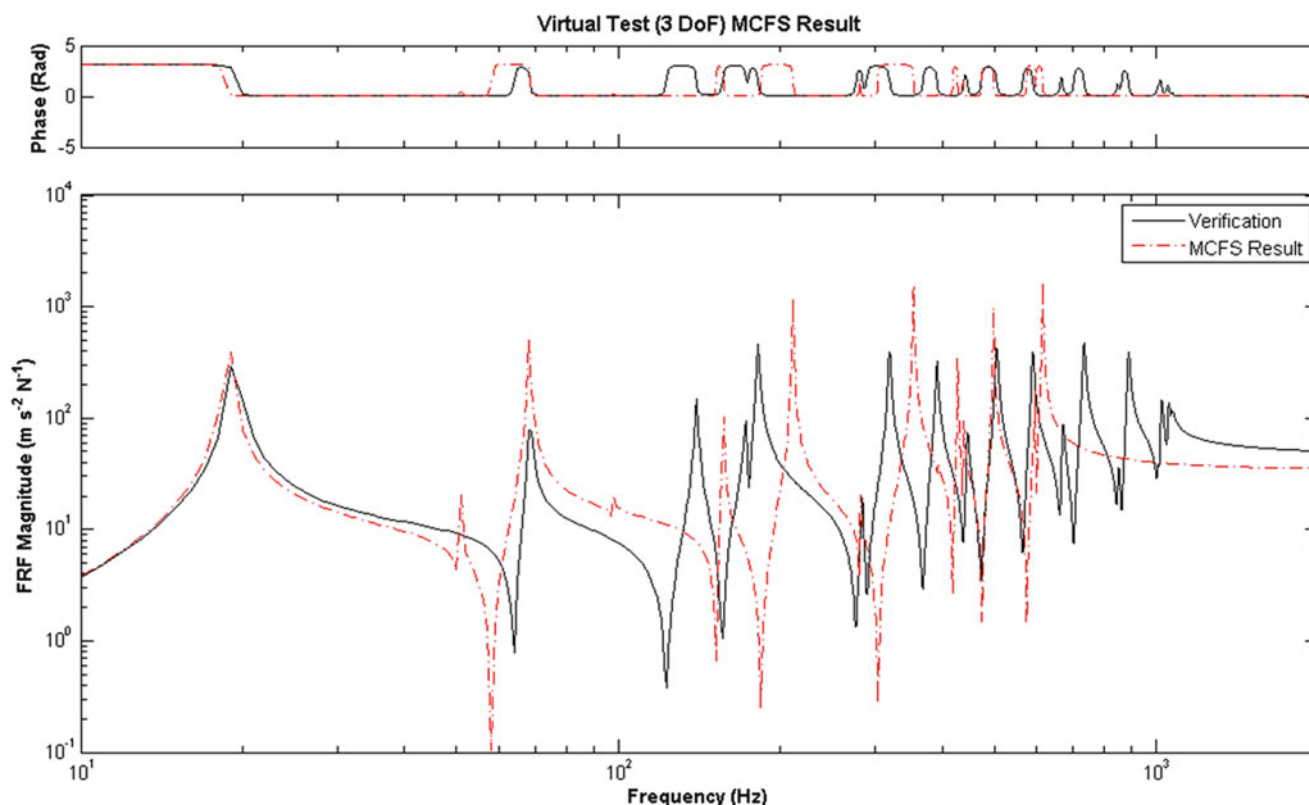
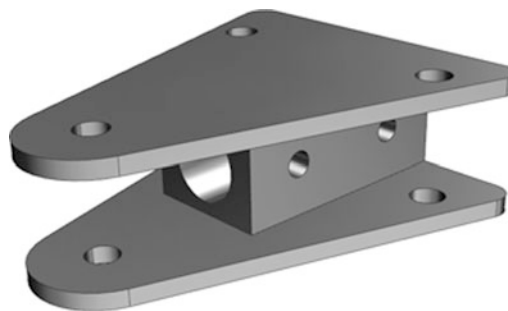


Fig. 12.12 Results from the MCFS calculation utilising all three translational DoFs from FE models of the turbine blade and transmission fixture and a fixed-base turbine blade as the verification system

Fig. 12.13 Image of the turbine blade attachment fixture provided by Ampair [7]



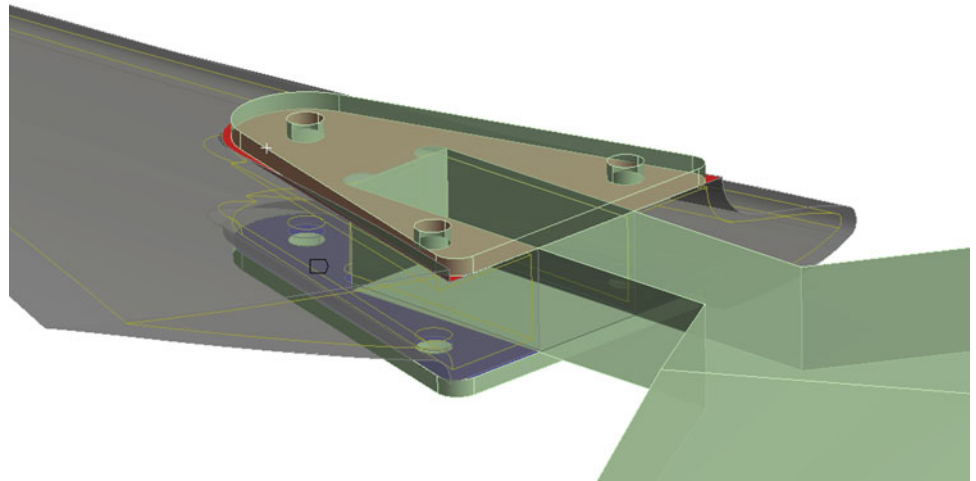
In order to investigate the sensitivity of the substructuring procedure to the fixture design, the 3DoF virtual test was repeated using an alternative transmission fixture design and mounting methodology. Figure 12.13 shows the official blade attachment fixture provided by Ampair [7] which mounts the turbine blade to the hub. An alternative transmission fixture was designed by using this geometry as a starting point and adding extra material. The resulting design is shown in Fig. 12.14 and uses the flat triangular inner surfaces as bonded contacts, shown in Fig. 12.15, replicating the physical mounting condition of the fixture surfaces clamped to the blade by the bolt force.

Figure 12.16 shows the results from the MCFS substructuring calculation (dashed red line) and the verification model (solid black line). The agreement between the two FRFs is substantially improved compared to the previous virtual tests, indicating that the substructuring calculations are sensitive to the design of the transmission fixture.

Fig. 12.14 Alternative transmission simulator design, utilising the blade attachment fixture from Ampair



Fig. 12.15 Alternative transmission simulator fixture contact definition to turbine blade component



12.8 Conclusions and Recommendations

This paper demonstrates that the MCFS framework [4] and transmission simulator methodology can be used to apply a fixed-base boundary condition to a turbine blade with varying degrees of success. Comparing the results of substructuring calculations using FE results with only one and three translational directions, show that the quality of the resulting synthesised drive point FRFs is improved with the more directions used.

In future work it is suggested to concentrate on acquiring high quality tri-axial modal results from the turbine blade and transmission simulator fixture, possibly utilising an automatic modal hammer and a tri-axial scanning LDV. In addition this paper has not investigated the selection of the DoFs chosen as the ‘connection’ points between the test hardware and the finite element model. Currently these points are chosen using ‘engineering judgement’, however a more qualitative method, such as QR-factorisation, could possibly be employed to select the most appropriate DoFs.

It has been shown that the quality of the synthesised FRFs is also sensitive to the design of the transmission simulator fixture and its blade mounting configuration. It is suggested that the mounting configuration and transmission simulator design is explored further in order to more accurately represent the true connection impedance between the turbine blade and the hub assembly.

Finally, a natural progression of the work reported in this paper would be to use the MCFS framework and transmission simulator methodology to couple the turbine blade to another substructure and validate the results with an identical physical hardware system that could be accurately modally tested.

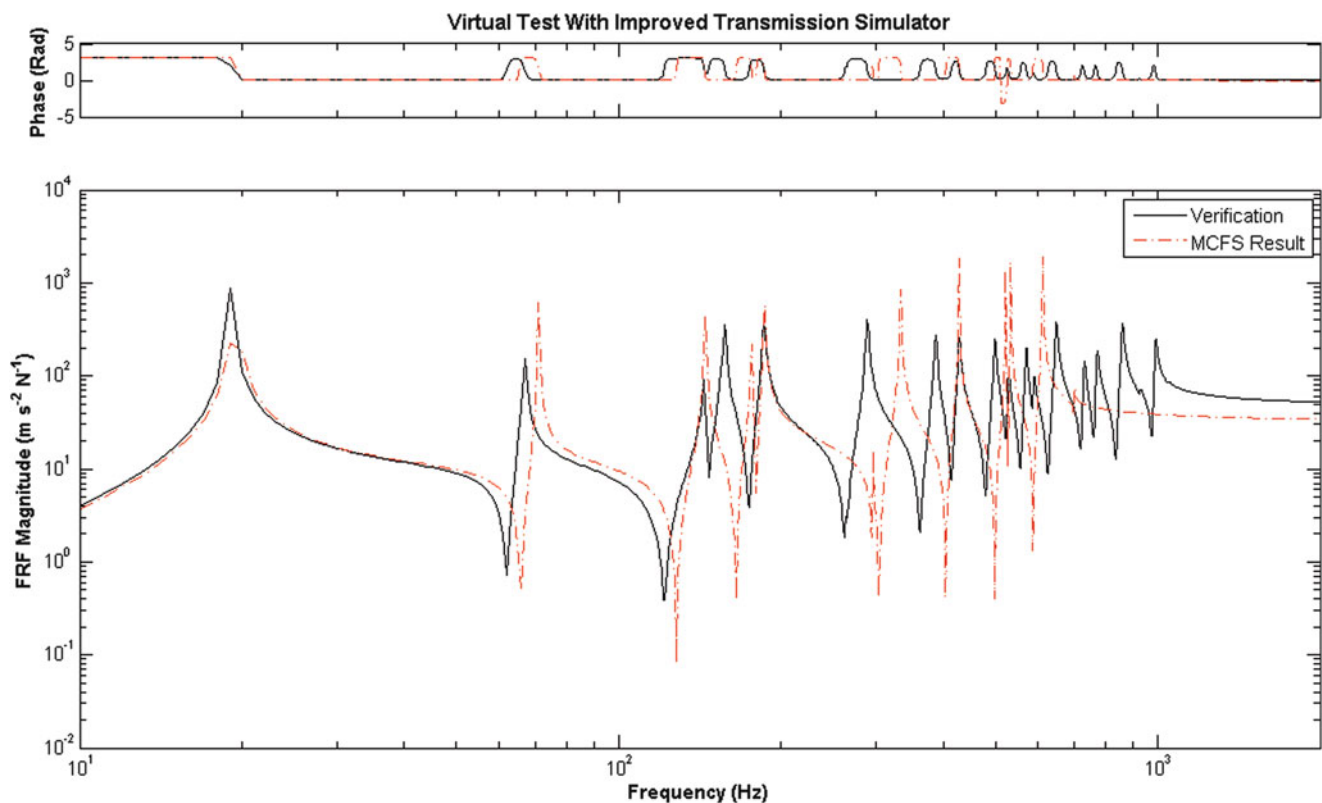


Fig. 12.16 Results from the MCFS calculation utilising all three translational DoFs from FE models of the turbine blade and alternative transmission fixture and a fixed-base turbine blade as the verification system

Acknowledgements

- P. Ind and P. Daborn – for extensive technical advice
- N. Monk – for provision of all modal test data
- M. Nurbhai – for provision of an improved turbine blade finite element model
- P. Harris and D. Sharman – for provision of blade attachment model and general advice
- R. Mayes – for providing the MCFS Matlab framework used for the substructuring calculations

References

1. Mayes R (2012) An introduction to the SEM substructures focus group test bed – The Ampair 600 wind turbine. Proceedings of the 30th international modal analysis conference, IMAC XXX, Jacksonville, FL, January
2. Mayes R (2012) Wind turbine experimental dynamic substructure development. Proceedings of the 30th international modal analysis conference, IMAC XXX, Jacksonville, FL, January
3. Nurbhai R, Macknelly D (2012) Modal assessment of wind turbine blade in preparation of experimental substructuring. Proceedings of the 30th international modal analysis conference, IMAC XXX, Jacksonville, FL, January
4. Allen M, Mayes R, Bergman E (2010) Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections. J Sound Vib 329:4891–4906
5. Harvie J (2011) Results from built in modal testing of six Ampair wind turbine blades. SDASL report #L111413. Structural Dynamics and Acoustic Systems Laboratory, University of Massachusetts at Lowell, August
6. Nurbhai M, Macknelly D (2012) Dynamic substructuring – wind turbine blade finite element model update and correlation with test, TN 48/12. AWE Internal Technical Note, October
7. Sharman D, Harris P. Ampair Energy Limited, Blandford, Dorset

Chapter 13

IMAC XXXI: Dynamic Substructuring

Mohsin Nurbhai and David Macknelly

Abstract Advancements in the field of substructuring has founded a renewed interest in the field of experimental dynamic substructuring resulting in the formation of the Dynamic Substructuring Focus Group within the Society for Experimental Mechanics.

This paper is in association with this focus group and looks at updating the finite element model of the test bed wind turbine blade, originally presented at IMAC XXX, based on intelligence obtained from manufactures and examination of cut up sections. A linear composite material model is utilised for the blade skin to help predict the modal response of the structure better, in comparison to past correlations. An iterative approach of optimising the FE model's element formulations and material model parameters was utilised.

A full modal correlation of this updated finite element model was carried out against available test data. Analysis of cut blade samples is also documented in this paper. The desire of having a best representative blade finite element model is ideally to have a benchmark which can be used for comparison of analytical substructures and experimental substructures. Also there is the added benefit of being able to apply environmental loading on structural features.

Keywords Dynamic substructuring • Wind turbine finite element model • Modal analysis • IMAC • Component mode synthesis

13.1 Introduction

Over the last several years there has been a revived interest in the field of experimental dynamic substructuring. This has led to the formation of the Dynamic Substructuring Focus Group within the Society for Experimental Mechanics. The International Modal Analysis Conference sees regular meetings of this focus group including a dynamic substructuring focussed session every year.

Modelling of substructures can be used in design development cycle and is particularly beneficial when the full design rights are not owned or available to the developer. Of course there is the added benefit of computational savings when utilising a reduced degree of freedom representation. In some cases where complexity of design is great, it is often cheaper, and more accurate, to develop an experimentally derived substructure rather than a purely analytical one.

This paper is in association with the Dynamic Substructuring Focus Group and looks at updating the Finite Element (FE) model of the test bed wind turbine blade, originally presented [1] at IMAC XXX. Within this test bed it is deemed that the complexity of the blade component justifies development of a representative experimental substructure. To support this endeavour there is a desire of having a best representative blade finite element model which can be initially used as

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a benchmark computational model for comparison of analytical Component Mode Synthesis (CMS) reduced substructures and experimentally derived substructures. Further advantages include ability to apply environmental loading on structural features. For details of the full wind turbine test bed please refer to the IMAC XXX substructuring test bed introduction papers [2, 3].

The update to the FE model is based predominantly on intelligence obtained from the manufactures and examination of cut up sections. The key ingredients to the development of a best representative FE model include an accurate illustration of the blade topological geometric form, together with good material properties. An elastic orthotropic composite material model is utilised for the blade skin to help predict the modal response of the structure better, in comparison to past correlations. An iterative approach of optimising the FE model's element formulations and material model parameters was utilised. A full modal correlation of this updated FE model was carried out against available test data together with visual analysis of cut blade samples to further understand material layout.

13.2 Modelling and Optimisation

13.2.1 Manufacturing

From discussions with the manufacturers, Ampair Energy Limited [4], the authors understanding of the topological layout, as illustrated in Fig. 13.1 below, is that the blade skin is formed of two ply thick reinforced glass fibre composite, where each fabric sheet is in a $0^\circ/90^\circ$ weave. The fibre content is E-glass and binding matrix is Polypropylene, which is applied as pre-impregnated before curing. The core of the blade is a thermoplastic based syntactic polymer composite exhibiting both plastic and foam like properties. This setup is then pressed and cured to produce the final product. The blade mounting flange and bolt holes are machined and drilled in later on.

13.2.2 FE Geometry and Element Formulation

The original geometry of the blade presented [1] at IMAC XXX was modified significantly. The outer skin was offset uniformly by 2 mm inwards, into the material, to produce the core geometry. The 2 mm outer 'shell' was used as the volume for the skin component. This revised blade geometry [8] was produced by AWE's reverse engineering team.

Three different topologies were investigated, and in total four different FE models were run using the commercially available FE code, ANSYS Version 14.0. Initially, three, free boundary modal models were created to investigate best topological build, as shown in Fig. 13.2 below.

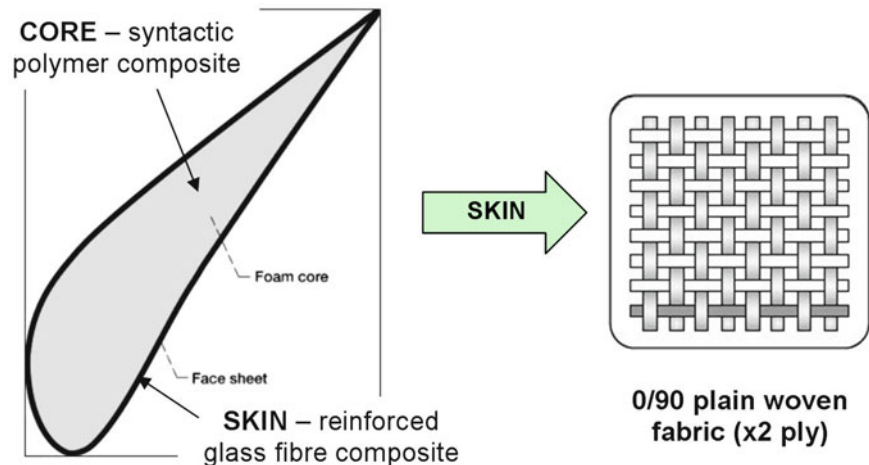


Fig. 13.1 Blade manufacturing configuration

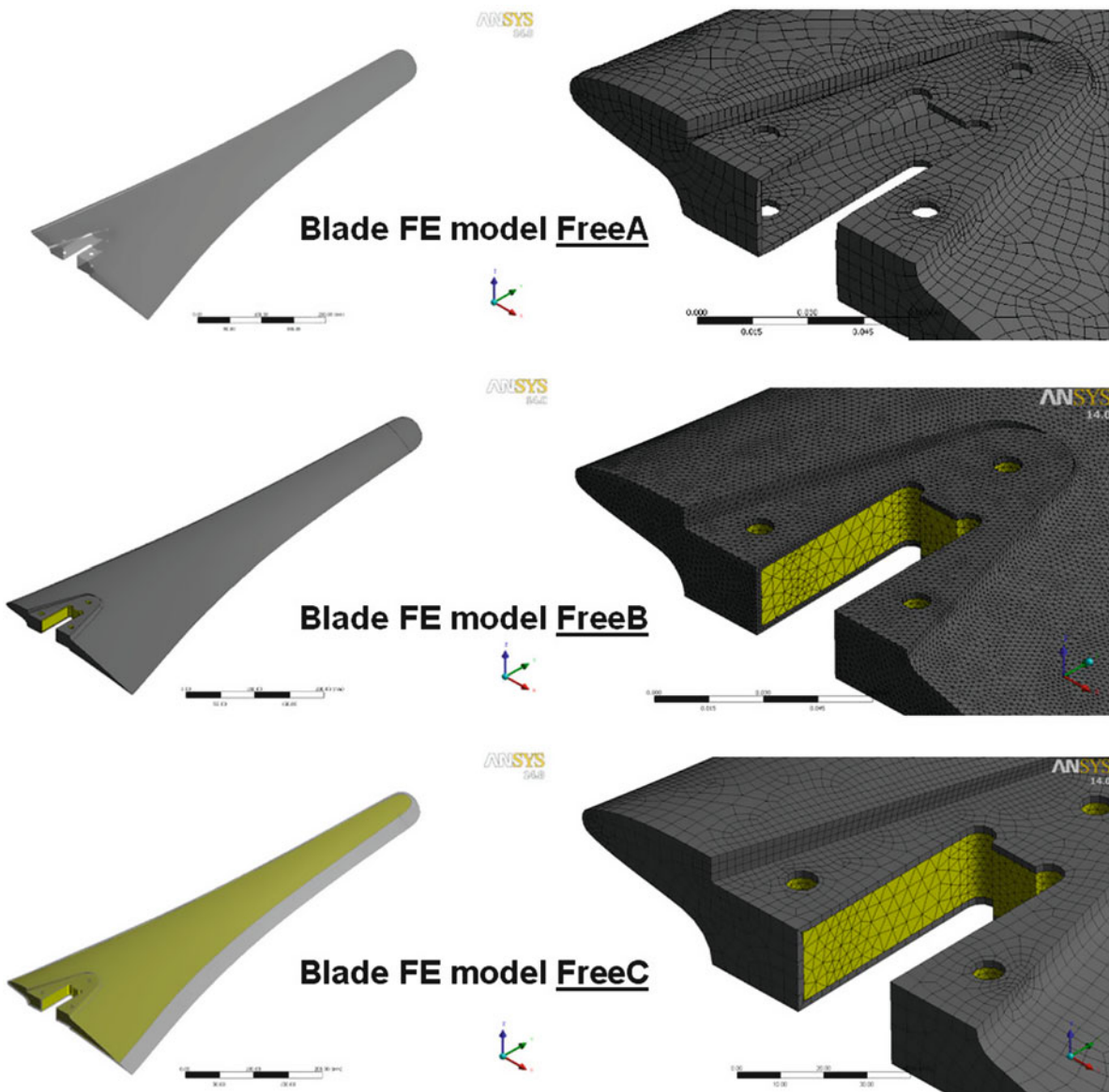


Fig. 13.2 Free boundary modal models – FreeA/B/C

1. Blade FE model FreeA – This was a skin only model with the mass of the core smeared across the skin. Quadratic shell elements were used.
2. Blade FE model FreeB – This was a fully featured solid model. Both skin and core were represented using a 10 noded tetrahedron element formulation. Tied contact using a multi-point-constraint algorithm was used to model the physics between the skin and core.
3. Blade FE model FreeC – This was hybrid model. The skin was modelled using quadratic shell elements and the core using 10 noded tetrahedron elements. Again, tied contact using a multi-point-constraint algorithm was used to model the physics between the skin and core.

As detailed in the ‘analysis of data’ section of this paper it was found that the shell/solid hybrid (FreeC) model showed the best correlation with test data and resultantly this topology was used to also generate a fourth fixed boundary modal model, namely FixedC, shown below in Fig. 13.3. This model was specifically generated to make correlations against fixed-base modal tests at AWE. However the analysis was done before any fixed-based modal tests were planned and the mounting configuration of the blade was assumed to be across the whole top and bottom face of the mounting region.

Fig. 13.3 Fixed boundary modal model – FixedC

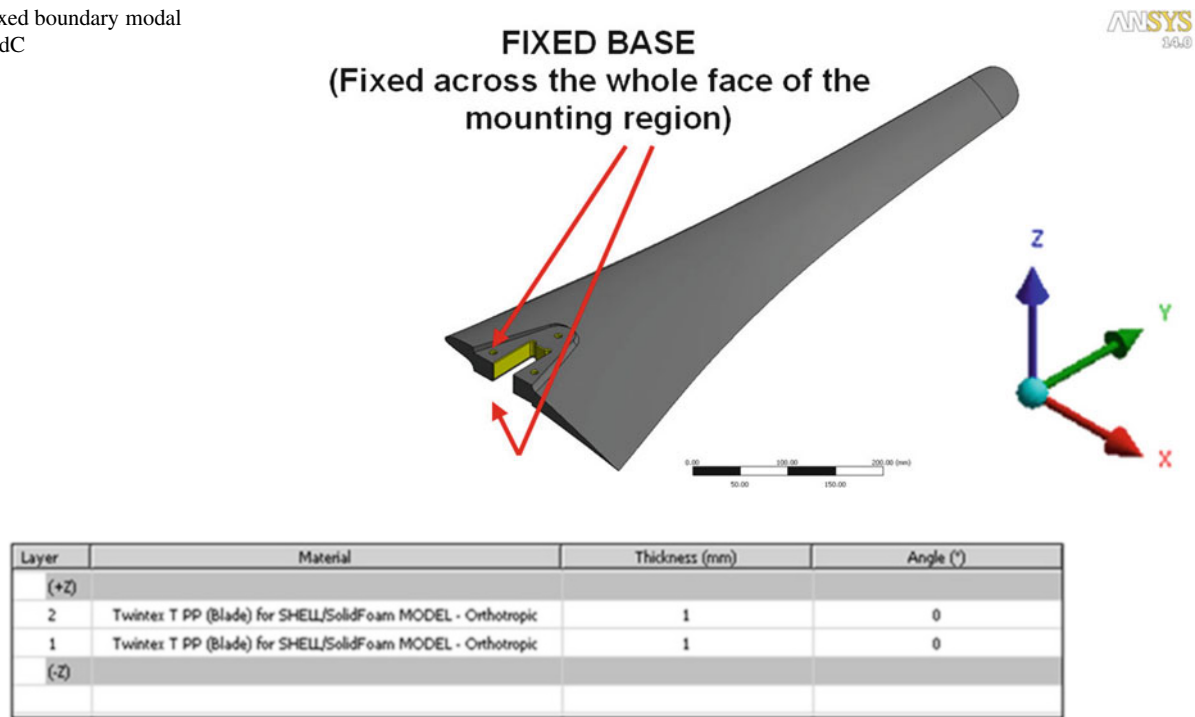


Fig. 13.4 Layered section approach for shell models

13.2.3 Mass Properties

A nominal set of total mass properties, for the blade, were used for all FE models. This value was derived as a mean from the measurement of actual blades procured by AWE for testing.

$$Mass_{Avg} = 785.9g$$

For the component featuring the core, the density of the syntactic polymer was always assumed to be fixed. Therefore, the density of the composite skin was artificially modified, model-to-model, to always give a total mass of ≈ 785.9 grams.

13.2.4 Material Properties

The material properties for the skin and core components were assumed to be linear elastic. The skin was modelled as orthotropic and the core as isotropic, see Fig. 13.3 for the global coordinate system of the models. The skin component in all cases is assumed as 2 mm uniformly thick across the blade, based on rough observations discussed later on in the ‘blade cut sample analysis’ section of this paper. Actual material properties were sourced both from the manufacturers and estimated from AWE’s materials databases. Information specified by the manufacturer does not supply original source as this is commercial in confidence.

Where the skin was modelled using shell element formulation a layered section approach was used. The shell surface was split into a layered section of two plies and each ply was supplied with an orientation (in this case 0° for both since each ply is effectively a $0^\circ/90^\circ$ weave) and orthotropic material properties, as shown in Fig. 13.4 below.

For the skin, the through thickness (transverse) moduli was not available and therefore had to be reverse engineered and tuned to fit available test data. In this instance both transverse moduli for tensile stiffness and shear stiffness were varied between 0.2–0.5 (in increments of 0.1) of the axial values, until best correlation against the first 5 free-free natural frequencies was achieved. One expects this transverse modulus to be very similar to that of pure Polypropylene resin, and actually this correlates well (within 20%) to a generic 50% glass fibre polypropylene composite [5] in AWE’s materials properties database. The final set of material properties used are shown below:

Skin (Reinforced glass fibre composite sheet)	
Density = 1290–1440 kg/m ³	<i>Reverse engineered from mass measurement</i>
Tensile modulus (X) = 14 GPa	<i>Provided by manufacturer [4]</i>
Tensile modulus (Y) = 14 GPa	<i>Provided by manufacturer [4]</i>
Tensile modulus (Z _{transverse}) = 4.2 GPa	<i>Reverse engineered from model tuning</i>
Poisson's ratio (XY) = 0.35	<i>Estimated from AWE materials DB [5]</i>
Poisson's ratio (YZ) = 0.38	<i>Estimated from AWE materials DB [6]</i>
Poisson's ratio (XZ) = 0.38	<i>Estimated from AWE materials DB [6]</i>
Shear modulus (XY) = 1.5 GPa	<i>Reverse engineered from model tuning</i>
Shear modulus (YZ) = 5 GPa	<i>Calculated using elastic theory</i>
Shear modulus (XZ) = 5 GPa	<i>Calculated using elastic theory</i>
Core (syntactic polymer composite alloy)	
Density = 700 kg/m ³	<i>Provided by manufacturer [4]</i>
Tensile modulus = 0.1 GPa	<i>Provided by manufacturer [4]</i>
Poisson's ratio = 0.35	<i>Estimated from AWE materials DB [6]</i>
Bulk modulus = 111 MPa	<i>Calculated using elastic theory</i>
Shear modulus = 37 MPa	<i>Calculated using elastic theory</i>

13.3 Analysis of Data

13.3.1 Blade Cut Sample Analysis

The three blades that AWE originally procured could not be used for cutting since modal testing was not fully complete. The manufacturer [4] kindly contributed a retired blade specifically for cut sample analysis; however it must be noted that for this specific blade the core has been dyed black (as opposed to naturally white), and this causes extra difficulty in inspection due to lack of contrast. Early visual analysis as shown in the bottom left hand photograph of Fig. 13.5 shows that generally speaking the main centre section of the blade has a skin thickness of approximately 2 mm. It is for this reason the blade FE geometry was produced to the 2 mm skin specification.

Further analysis, especially after cutting up the whole blade, shown in bottom right hand corner of Fig. 13.5, reveals a variation in the distribution of the skin thickness. Larger variance exists around the mounting region and some significant differences around the blade leading edge (thicker than 2 mm) and trailing edge (thinner than 2 mm). This suggests that the stiffness around the leading and trailing edges is different and unfortunately the FE model does not capture this behaviour.

This is also a major factor in explaining differences at higher frequencies where these asymmetrical features may induce localised mode shapes. There is also evidence of significant areas of porosity (one presumes will vary blade to blade) which may have a small effect on blade response.

13.3.2 Free Boundary Modal Analysis Comparison

Table 13.1 shows the modal free boundary comparisons between AWE free-free FE models and test data available from AWE and University of Massachusetts, LOWELL. It can be clearly seen that the closest natural frequency correlation against AWE test data is that of the FreeC model where the skin is represented as shell element formulation and the core with solids.

Figure 13.6 shows the modal assurance criterion (MAC) comparison between the AWE test data [1] and the FreeA FE model. The match is poor apart from the first two elastic modes. This may be due to the fact that the stiffness of the core is inherently not represented in this model.

Figure 13.7 shows the MAC comparison between the AWE test data and the FreeB FE model. This model shows very good mode shape correlation at lower frequency but the natural frequencies, as shown in Table 13.1, especially for the torsional modes do not compare well. Also note the step in MAC comparison for the missing 1st Torsion mode in the test data.

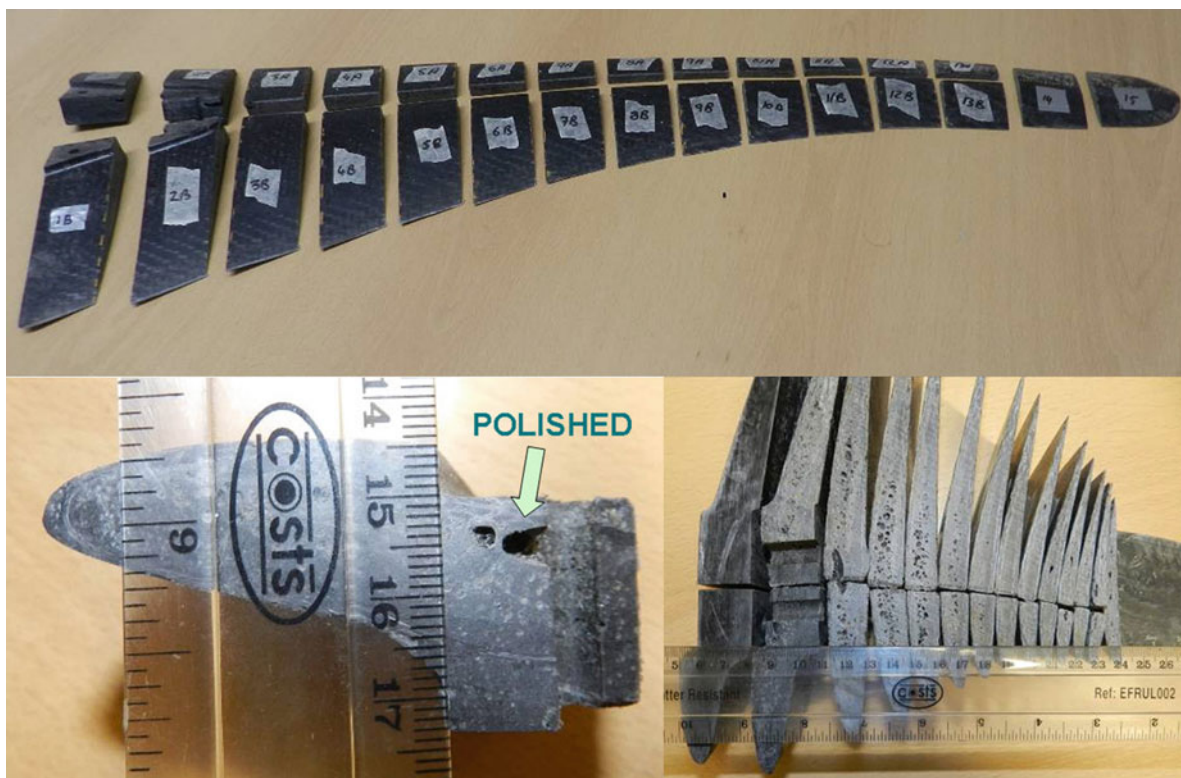


Fig. 13.5 Visual analysis of blade cut samples

Table 13.1 Modal free-free comparisons between AWE FE models and available test data

Mode shape	LOWELL test data blade #6 ^a [7] (Hz)	AWE Test data blade #726 ^b [1] (Hz)	FE model FreeA (Hz)	FE model FreeB (Hz)	FE model FreeC (Hz)
1st Bending	40	47	54	45	47
2nd Bending	122	134	122	122	133
1st Torsion	194	Missing ^c	147	159	198
3rd Bending	236	252	190	227	250
2nd Torsion	319	326	212	264	347

^aThis is 1 dataset out of 6 blades. The variance for the first 5 modes for the full data set is approximately 8%

^bThis is 1 dataset out of 3 blades. The variance for the first 4 modes for the full data set is approximately 3%

^cIt is believed that this mode was missing from the AWE test dataset as a result of the drive point location being placed on a nodal line

Figure 13.8 shows the MAC comparison between the AWE test data and the FreeC FE model. The correlation of mode shapes is very good and similar to the FreeB model. Looking at the MAC plot there is better shape correlation with the FreeB model than the FreeC, when correlated against the AWE test data. However, the natural frequency correlation between the FreeC model and LOWELL test data [7] compares really well for torsional modes.

The fact that the hybrid shell/solid FreeC model shows better correlation than the solid only FreeB model, especially for torsional stiffness, can be explained by the enhanced properties the shell element formulation have over solids for flexure and bending. Other solid element formulations may perform better but in this case all ANSYS defaults were used.

The reality is that the best FE representation for the blade probably lies somewhere between the FreeC and the FreeB model.

Further nodal line comparisons can also be made between the AWE test data and the FreeC FE model. These are illustrated in Fig. 13.9 below. In most cases the match is very good.

Both the FreeB and FreeC models have shown good correlation at lower frequencies. However, following the first 5-6 modes and at higher frequency, the correlation does begin to deteriorate. This could be as a result of various factors including accurate geometry representation, material properties, element formulation, mesh density, and other uncertainties in FE model. However, in the author's opinion, the driving factors for these differences are mainly to do with the precision of the blade model skin-thickness distribution and the accuracy of material properties.

Fig. 13.6 MAC comparison for AWE Blade 726 test data against FreeA FE model

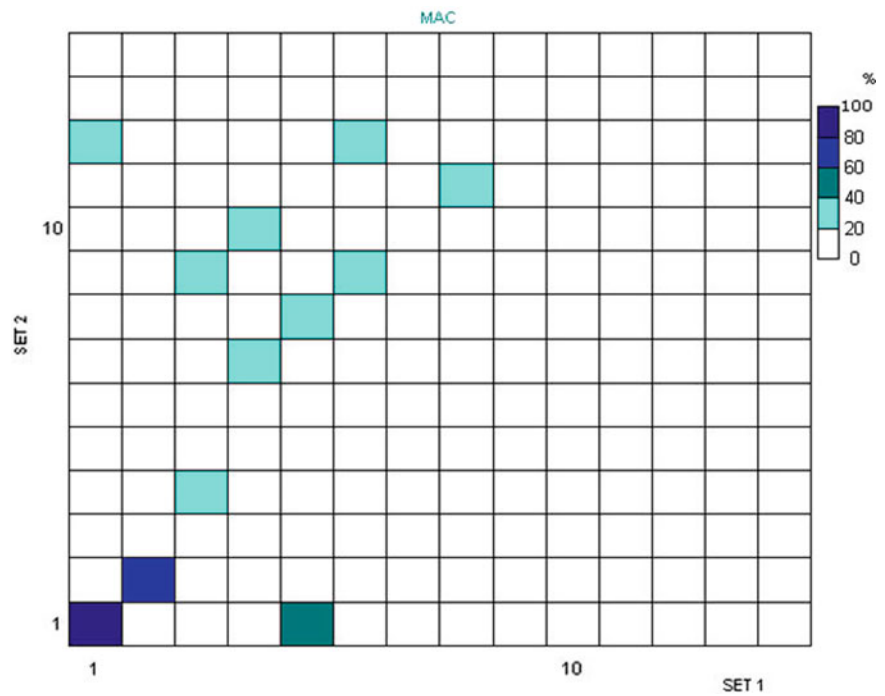
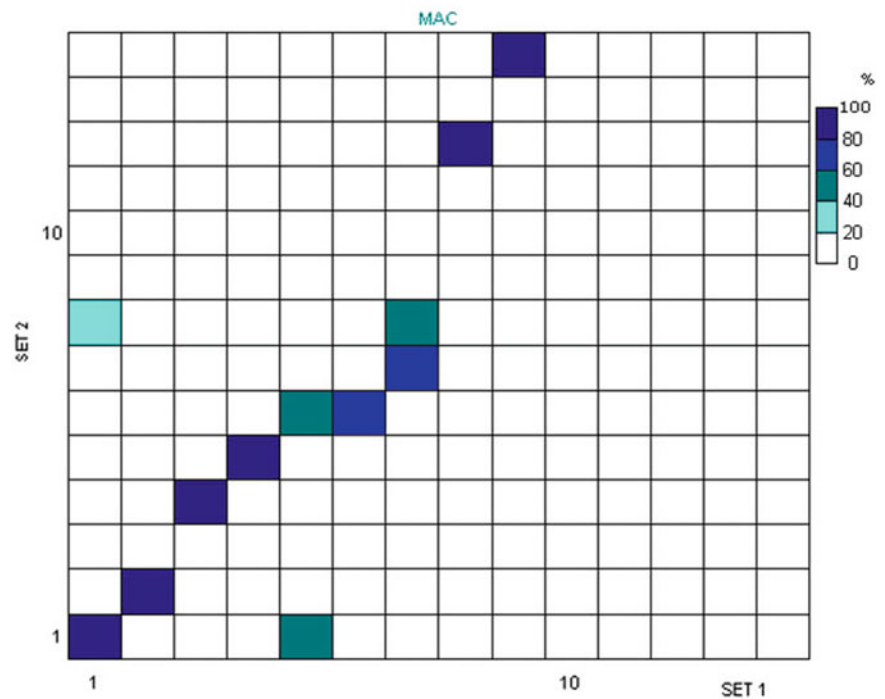


Fig. 13.7 MAC comparison for AWE Blade 726 test data against FreeB FE model



13.3.3 Fixed Boundary Modal Analysis Comparison

Table 13.2 shows the modal fixed boundary comparisons between the AWE fixed-base FE model and test data available from AWE and University of Massachusetts, LOWELL. It can be seen from the table that there is good correlation against test data for the first two bending modes. Poor correlation is seen for the higher natural frequencies. It is thought that the reason for this is mainly due to differences in how the blade has been clamped during test (shown in Fig. 13.10) compared to the fixed boundary condition in FE (shown in Fig. 13.3). Whilst the FE model is fixed along the full mounting face, in the test setup the contact area is defined by washers and the mounting bolts are torqued up to induce some preload in the blade.

Fig. 13.8 MAC comparison for AWE Blade 726 test data against FreeC FE model

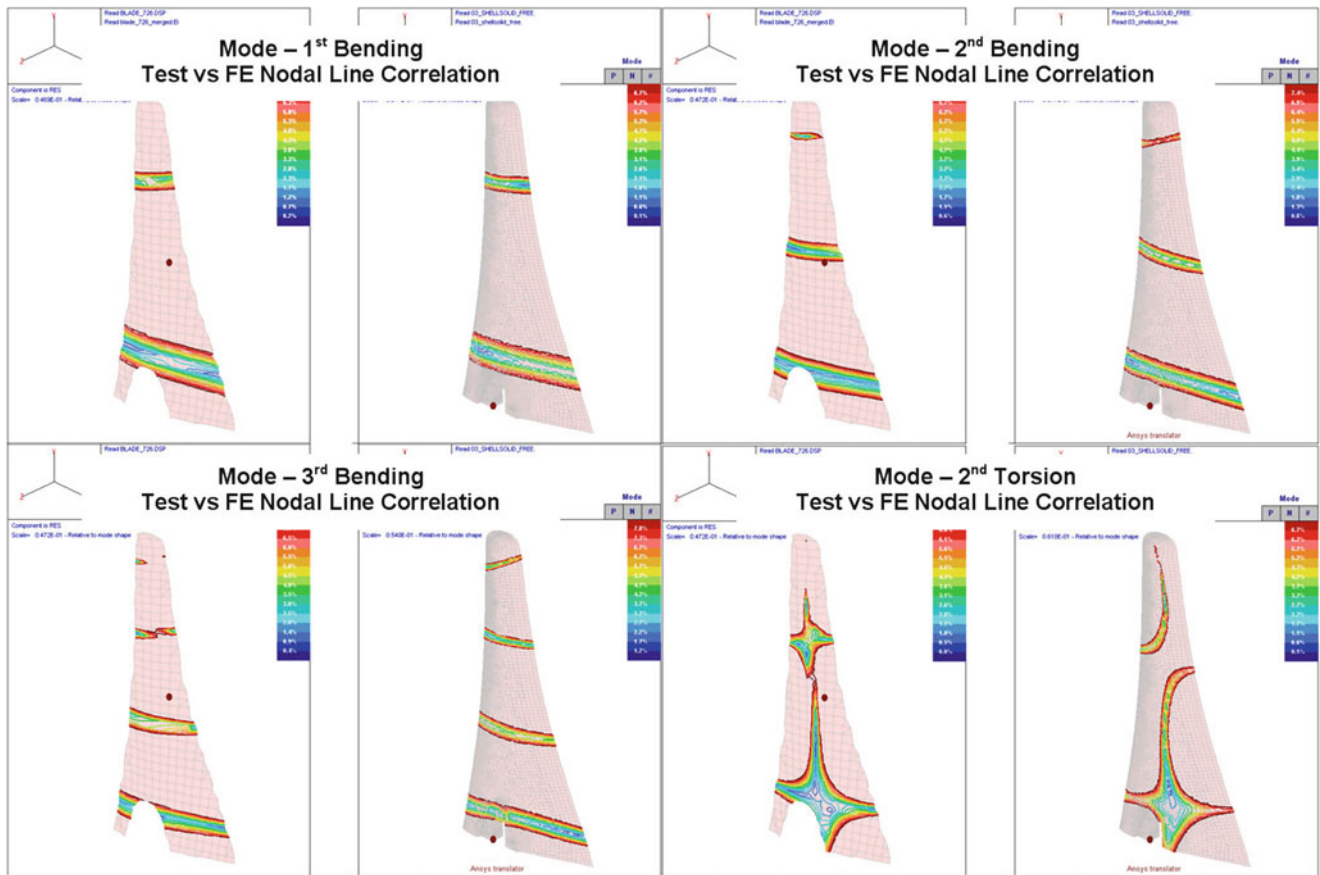
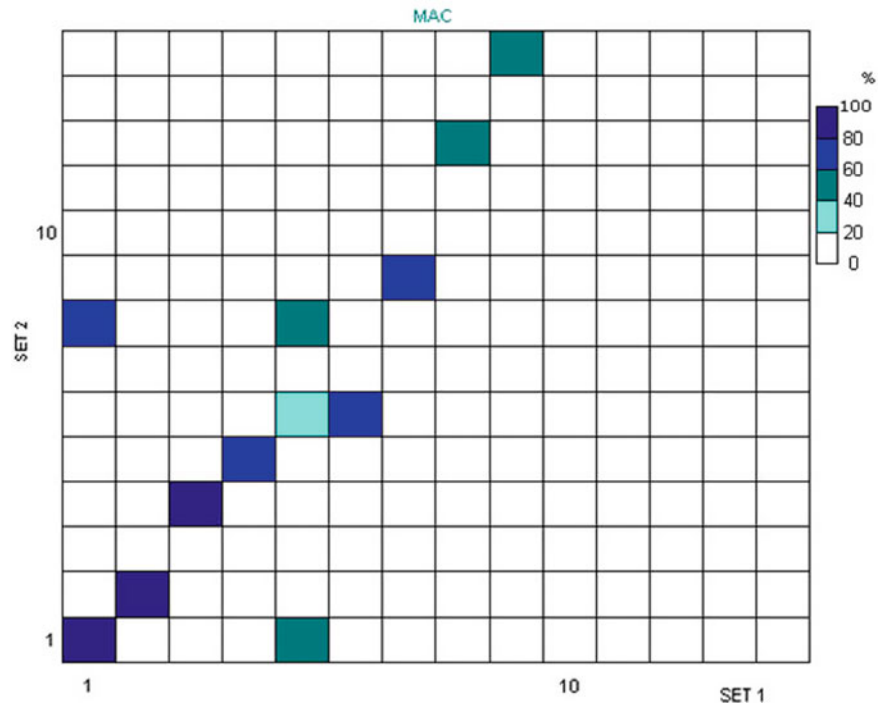


Fig. 13.9 Nodal line comparisons between Blade 726 test data and FreeC FE model

Table 13.2 Modal fixed-base comparisons between AWE FE model and available test data

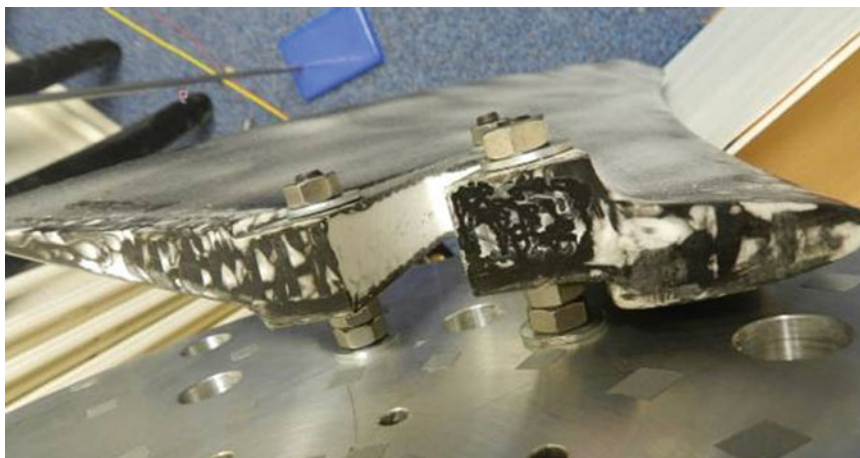
Mode shape	LOWELL test data blade #6 ^a [7] (Hz)	AWE test data blade # 726 ^b [9] (Hz)	FE model FixedC (Hz)
1st Bending	20	21	19
2nd Bending	70	72	67
Mode 3	129	Missing ^c	144
Mode 4	Not presented	184	158
Mode 5	Not presented	267	186
Mode 6	Not presented	313	266

^aThis is 1 dataset out of 6 blades. The variance for the first 3 modes for the full data set is approximately 8%

^bThis is 1 dataset out of 3 blades. The variance for the first 5 modes for the full data set is approximately 4%

^cThis missing mode in the AWE data set is yet to be investigated

Fig. 13.10 Clamping of blade for fixed base modal testing



13.4 Conclusions

The first consensus of the dynamic substructuring focus group was to characterise one blade with the efforts to develop the solid geometry and finite element model of this configuration. The results from this paper conclude that, thus far, the most representative geometric topology for the blade is that of hybrid element FE model where the skin is represented as a shell and the core as solid. The shell is further represented as a fibre lay using layered sections and applied with tuned orthotropic materials properties. There is little information available on representative material properties to fit FE material models. Therefore, without carrying out mechanical testing on skin and core samples the focus group will presently have to settle on some implied and estimated parameters.

It has been shown that at lower frequencies excellent correlation has been observed between FE and test. This model may be sufficient at this stage to help generate an analytically derived substructure for comparison against an experimentally derived substructure. However, with the inherent assumptions and uncertainties built into the analytical model we are still far away from a high fidelity (precise) representation.

Due to the geometric and material complexity of the blade it is understandable that generating a highly accurate substructure model analytically from first principles, would be most challenging and potentially very expensive. Perhaps it would be far more cost effective and more accurate to derive an experimental dynamic substructure of the blade that could be mapped back to the analytical domain.

13.5 Recommendations

To improve ones understanding of the geometric topology there may be further benefits in cutting up one of the AWE procured blades. The core of this blade is white in colour and will offer a much better contrast for inspection and measurement. The manufacturer has also provided AWE a cured sample of both skin and core material. In the future it will be extremely beneficial to generate mechanical test samples from these off-cuts and perform uni-axial tensile/compression tests.

A stochastic analysis investigating response verses varied input could also be carried out to scrutinise the uncertainties in geometric and material property variance.

Acknowledgements 1. D. Macknelly – for provision of free-free test data and general advice. 2. N. Gibbs – for provision of revised blade geometry. 3. D. Sharman – for provision of extra blade, manufacturing advice, and materials properties. 4. S. Chetwynd – for cutting up the blade. 5. N. Monk – for provision of fixed-base test data.

References

1. Nurbhai M, Macknelly D (2012) Modal assessment of wind turbine blade in preparation of experimental substructuring. IMAC XXX, Jacksonville, FL
2. Mayes R (2012) An introduction to the SEM substructures focus group test bed – The Ampair 600 wind turbine. Proceedings of the 30th international modal analysis conference, IMAC XXX, January
3. Mayes R (2012) Wind turbine experimental dynamic substructure development. Proceedings of the 30th international modal analysis conference, IMAC XXX, January
4. Sharman D. Ampair Energy Limited, Blandford, Dorset
5. AWE GRANTA MI (2012) Polypropylene (50% long glass fiber). Aero USA Database, October
6. AWE GRANTA MI (2012) Polypropylene (Homopolymer, 10% glass fiber). Aero USA Database, October
7. Harvie J (2011) Results from built in modal testing of six Ampair wind turbine blades. Structural Dynamics and Acoustic Systems Laboratory, University of Massachusetts at Lowell, SDASL Report #L111413-3, 9 August
8. Gibbs N (2012) Revised wind turbine blade geometry. AWE Team Centre Engineering, 3074873-D01, August
9. Monk N (2012) AWE RNET. email from: N. Monk, To: M. Nurbhai, Subject: Turbine Blade fixed base test, 17 Aug 2012

Chapter 14

Selection of Interface DoFs in Hub-Blade(s) Coupling of Ampair Wind Turbine Test Bed

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Abstract Substructure coupling is an important tool in several applications of modal analysis. It is particularly relevant in virtual prototyping of complex systems and responds to actual industrial needs, especially in an experimental context. Furthermore, the reverse problem, the decoupling of a substructure from an assembled system, arises when a substructure cannot be tested separately but only when coupled to neighboring substructures, a situation often encountered in practice. In this paper, the dynamic behavior of the Ampair test bed wind turbine rotor, made by three blades – each one bolted to the hub at three points – is analyzed. The aim is both to identify the dynamic behavior of the rotor starting from the frequency response functions (FRFs) of blades and hub, and to select a reduced set of relevant DoFs to represent the interface between blades and hub. FRFs to be used in the coupling procedure are obtained starting from FE model of each substructure, by using a super-element based computational approach. The decoupling problem, with the aim of identifying the dynamic behavior of each blade from the FRFs of the assembled rotor and of the hub, is also considered.

Keywords Experimental Dynamic Substructuring • Interface DoFs • Wind Turbine

14.1 Introduction

In experimental dynamic substructuring two main problems can be defined: addition of substructures (coupling) and subtraction of substructures (decoupling). Coupling is important to find out the dynamic behaviour of complex structures from a dynamic description of their components. It is particularly relevant in virtual prototyping of complex systems and responds to actual industrial needs, especially in an experimental context, i.e. when the model of at least one subsystem derives from experimental tests. Decoupling can be important in built-up structures where some components (critical subsystems or joints) cannot be removed or accessed easily. Decoupling involves the identification of the dynamic behaviour of a structural subsystem, starting from information about the remaining part of the structural system (residual subsystem) and from the known dynamic behaviour of the complete system.

Addition of substructures (coupling) can be seen as a structural modification problem [1]. Similarly, the decoupling problem can be seen as a structural modification problem with negative modification. Due to modal truncation problems, in experimental dynamic substructuring, the use of FRFs (Frequency Based Substructuring) is preferred with respect to the use of modal parameters. The main algorithm for frequency based substructuring is the improved impedance coupling [2] that involves just one matrix inversion with respect to the classical impedance coupling technique that requires three inversions. A general framework for dynamic substructuring is provided in [3,4]: in this context, the so called dual domain decomposition is very useful for experimental application, since it allows to retain the full set of global DoFs by ensuring equilibrium at the interface between substructures. A similar formulation for the decoupling problem is developed and discussed in [5–8].

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Whatever be the used approach, the dynamic behaviour (FRFs) at all the coupling DoFs must be determined. Therefore, if coupling involves transmission of moments, either rotational measurements or techniques involving a suitable selection of translational DoFs are required, such as the equivalent multi point connection [4,9] or the transmission simulator method [10, 11]. With regard to the decoupling problem, additional FRFs at some internal DoFs of the residual subsystem can be used [6, 12, 13] to avoid ill-conditioning at some particular frequencies.

In this paper, dynamic substructuring is applied to the rotor of the Ampair 600 wind turbine. This system has been proposed as a test bed by the Society of Experimental Mechanics focus group on experimental dynamic substructuring, to enable advancements in experimental dynamic substructuring technology and theory. Several specimens of the turbine have been bought. A description of the turbine with modifications made to the system to make it more linear is presented in [14] together with results from a rudimentary modal test on the whole turbine. The results of initial modal tests on six blades are reported in [15]. Geometry data about the blades and the hub have also been made available to members of the focus group.

The aim of this paper is to select a reduced set of relevant DoFs to describe the interface between each blade and the hub in order to identify the dynamic behavior of the rotor starting from the FRFs of the blades and the hub. FRFs to be used in the coupling procedure are obtained starting from the FE model of each substructure, by using a super-element based computational approach. The choice of using computational FRFs is dictated by the need of being aware of the effect of each simplifying assumption, by comparing the results obtained at each step with the reference ones. The decoupling problem, with the aim of identifying the dynamic behavior of each blade from the FRFs of the assembled rotor and of the hub, is also considered

14.2 Theoretical Background

The coupled structural system is assumed to be made by two (or more) subsystems joined through a number of couplings (see Fig. 14.1). The degrees of freedom (DoFs) of the coupled system can be partitioned into internal DoFs (not belonging to the couplings) and coupling DoFs (c).

14.2.1 Addition of Subsystems

If addition of subsystems (coupling problem) is considered, all subsystems are assumed to be known whilst the FRF of the coupled system is unknown. In the frequency domain, the equation of motion of a linear time-invariant subsystem r may be written as:

$$\mathbf{Z}^{(r)}(\omega)\mathbf{u}^{(r)}(\omega) = \mathbf{f}^{(r)}(\omega) + \mathbf{g}^{(r)}(\omega) \quad (14.1)$$

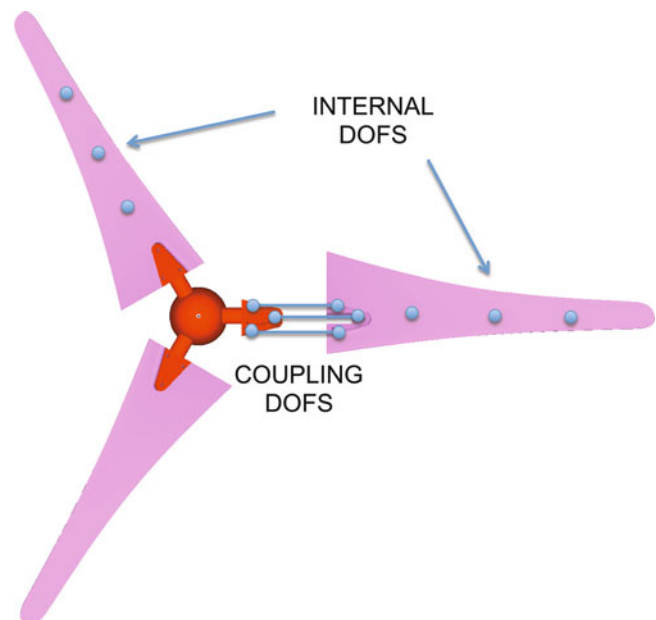


Fig. 14.1 Scheme of the substructuring problem

where:

$\mathbf{Z}^{(r)}$ is the dynamic stiffness matrix of subsystem r ;

$\mathbf{u}^{(r)}$ is the vector of degrees of freedom of subsystem r ;

$\mathbf{f}^{(r)}$ is the external force vector;

$\mathbf{g}^{(r)}$ is the vector of connecting forces with other subsystems (constraint forces arising from compatibility conditions).

For the sake of simplicity, the explicit frequency dependence will be omitted. Furthermore, the procedure will be developed with reference to two subsystems, bearing in mind that it can be easily extended to more subsystems.

The equation of motion of the subsystems to be coupled can be written in a block diagonal format as:

$$\mathbf{Z}\mathbf{u} = \mathbf{f} + \mathbf{g} \quad \text{i.e.} \quad \begin{bmatrix} \mathbf{Z}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}^{(2)} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{(1)} \\ \mathbf{u}^{(2)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \end{Bmatrix} + \begin{Bmatrix} \mathbf{g}^{(1)} \\ \mathbf{g}^{(2)} \end{Bmatrix} \quad (14.2)$$

The compatibility condition at the interface DoFs implies that any pair of matching DoFs $u_l^{(1)}$ and $u_m^{(2)}$, i.e. DoF l on subsystem 1 and DoF m on subsystem 2 must have the same displacement, that is $u_l^{(1)} - u_m^{(2)} = 0$.

This condition can be generally expressed as:

$$\mathbf{B}\mathbf{u} = \mathbf{0} \quad \text{i.e.} \quad \begin{bmatrix} \mathbf{B}^{(1)} & \mathbf{B}^{(2)} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{(1)} \\ \mathbf{u}^{(2)} \end{Bmatrix} = \mathbf{0} \quad (14.3)$$

where each row of $\mathbf{B}$ corresponds to a pair of matching DoFs. Note that $\mathbf{B}$ is, in most cases, a signed Boolean matrix and it can be written by distinguishing the contribution of the different subsystems.

The equilibrium condition for constraint forces associated with the compatibility conditions implies that, when the connecting forces are added for a pair of matching DoFs, their sum must be zero, i.e. $g_l^{(1)} + g_m^{(2)} = 0$: this holds for any pair of matching DoFs. Furthermore, if DoF k on subsystem 1 (or 2) is not a connecting DoF, it must be $g_k^{(1)} = 0$: this holds for any non-interface DoF.

Overall, the above conditions can be expressed as:

$$\mathbf{L}^T \mathbf{g} = \mathbf{0} \quad (14.4)$$

where the matrix $\mathbf{L}$ is a Boolean localisation matrix. Note that the number of rows of $\mathbf{L}^T$ is equal to the number of non-interface DoFs plus the number of pairs of interface DoFs.

Eqs. (14.2–14.4) can be put together to obtain the so called 3-field formulation [3]:

$$\begin{cases} \mathbf{Z}\mathbf{u} = \mathbf{f} + \mathbf{g} \\ \mathbf{B}\mathbf{u} = \mathbf{0} \\ \mathbf{L}^T \mathbf{g} = \mathbf{0} \end{cases} \quad (14.5)$$

14.2.1.1 Dual Formulation in the Frequency Domain [3]

In the dual formulation, the total set of DoFs is retained, i.e. each interface DoF is present as many times as there are substructures connected through that DoF. The equilibrium condition $g_l^{(1)} + g_m^{(2)} = 0$ at a pair of interface DoFs is ensured by choosing, for instance, $g_l^{(1)} = -\lambda$ and $g_m^{(2)} = \lambda$. Due to the construction of $\mathbf{B}$, the overall interface equilibrium can be ensured by writing the connecting forces in the form:

$$\mathbf{g} = -\mathbf{B}^T \boldsymbol{\lambda} \quad (14.6)$$

where $\boldsymbol{\lambda}$ are Lagrange multipliers corresponding to connecting force intensities. Since there is a unique set of connecting force intensities $\boldsymbol{\lambda}$, the interface equilibrium condition (14.4) is satisfied automatically for any $\boldsymbol{\lambda}$, i.e.

$$\mathbf{L}^T \mathbf{g} = -\mathbf{L}^T \mathbf{B}^T \boldsymbol{\lambda} = \mathbf{0} \quad (14.7)$$

Then $\mathbf{B}^T$ is the nullspace of $\mathbf{L}^T$, so Eq. (14.7) is always satisfied and the system of equations (14.5) becomes:

$$\begin{cases} \mathbf{Z}\mathbf{u} + \mathbf{B}^T\boldsymbol{\lambda} = \mathbf{f} \\ \mathbf{B}\mathbf{u} = \mathbf{0} \end{cases} \quad (14.8)$$

In matrix notation:

$$\begin{bmatrix} \mathbf{Z} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix} \quad (14.9)$$

that is:

$$\begin{bmatrix} \mathbf{Z}^{(1)} & \mathbf{0} & \mathbf{B}^{(1)T} \\ \mathbf{0} & \mathbf{Z}^{(2)} & \mathbf{B}^{(2)T} \\ \mathbf{B}^{(1)} & \mathbf{B}^{(2)} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{(1)} \\ \mathbf{u}^{(2)} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \\ \mathbf{0} \end{Bmatrix} \quad (14.10)$$

Note that $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ extract the coupling DoFs among the full set of DoFs.

By eliminating $\boldsymbol{\lambda}$, it is possible to obtain a relation in the form $\mathbf{u} = \mathbf{H}\mathbf{f}$, which provides the FRF of the coupled system [6]:

$$\mathbf{u} = \left(\mathbf{Z}^{-1} - \mathbf{Z}^{-1}\mathbf{B}^T (\mathbf{B}\mathbf{Z}^{-1}\mathbf{B}^T)^{-1} \mathbf{B}\mathbf{Z}^{-1} \right) \mathbf{f} \quad (14.11)$$

In expanded notation:

$$\begin{aligned} \begin{Bmatrix} \mathbf{u}^{(1)} \\ \mathbf{u}^{(2)} \end{Bmatrix} &= \left(\begin{bmatrix} \mathbf{Z}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}^{(2)} \end{bmatrix}^{-1} - \begin{bmatrix} \mathbf{Z}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}^{(1)T} \\ \mathbf{B}^{(2)T} \end{bmatrix} \times \right. \\ &\quad \left. \times \left(\begin{bmatrix} \mathbf{B}^{(1)} & \mathbf{B}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}^{(1)T} \\ \mathbf{B}^{(2)T} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{B}^{(1)} & \mathbf{B}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}^{(2)} \end{bmatrix}^{-1} \right) \begin{Bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \end{Bmatrix} \end{aligned} \quad (14.12)$$

i.e., by introducing the FRFs $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ at the full set of DoFs instead of $[\mathbf{Z}^{(1)}]^{-1}$ and $[\mathbf{Z}^{(2)}]^{-1}$:

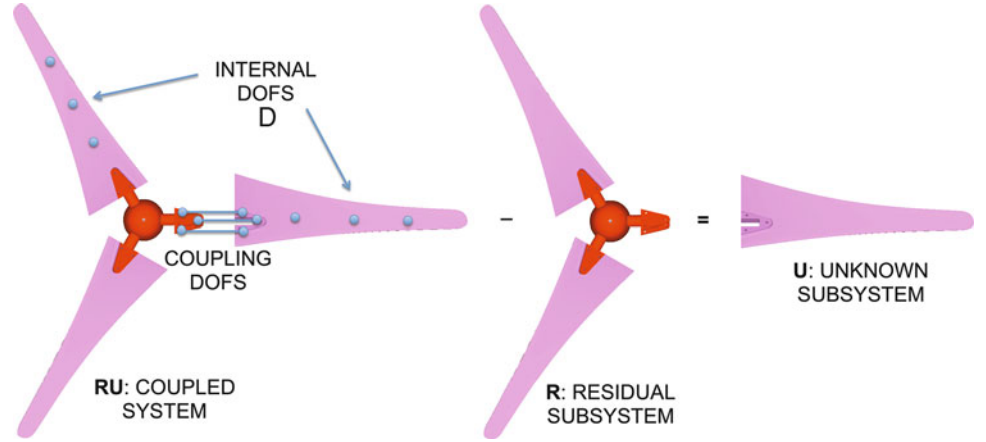
$$\mathbf{H}^{\text{RU}} = \begin{bmatrix} \mathbf{H}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{(2)} \end{bmatrix} - \begin{bmatrix} \mathbf{H}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{(1)T} \\ \mathbf{B}^{(2)T} \end{bmatrix} \left(\begin{bmatrix} \mathbf{B}^{(1)} & \mathbf{B}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{H}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{(1)T} \\ \mathbf{B}^{(2)T} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{B}^{(1)} & \mathbf{B}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{H}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^{(2)} \end{bmatrix} \quad (14.13)$$

With the dual formulation, the rows and columns corresponding to the coupling DoFs appear twice in $\mathbf{H}^{\text{RU}}$. Obviously, only independent entries are retained.

14.2.2 Subtraction of Subsystems Using the Dual Domain Decomposition [6]

If subtraction of subsystems (decoupling problem) is considered, the coupled structural system RU and a residual subsystem R are assumed to be known whilst the FRF of subsystem U is unknown. The unknown subsystem (U) and the residual subsystem (R) joined through a number of couplings (see Fig. 14.2). The degrees of freedom (DoFs) of the coupled system can be partitioned into internal DoFs (not belonging to the couplings) of subsystem U (u), internal DoFs of subsystem R (r), and coupling DoFs (c).

It is required to find the FRF of the unknown substructure U starting from the FRF of the coupled system RU . The subsystem U can be extracted from the coupled system RU by cancelling the dynamic effect of the residual subsystem R . This can be accomplished by adding to the coupled system RU a fictitious subsystem with a dynamic stiffness opposite to that of the residual subsystem R and satisfying compatibility and equilibrium conditions. According to this point of view, the interface between the coupled system RU and the fictitious subsystem should not only include the coupling DoFs between subsystems U and R , but should as well include the internal DoFs of subsystem R . However, it can be shown that the problem

Fig. 14.2 Scheme of the decoupling problem

can be solved by considering a number of interface DoFs at least equal to the number of coupling DoFs n_c . Therefore, three options for interface DoFs can be considered:

- standard interface, including only the coupling DoFs (c) between subsystems U and R ;
- extended interface, including also some internal DoFs ($i \subseteq r$) of the residual substructure;
- mixed interface, including some coupling DoFs ($d \subseteq c$) and some internal DoFs ($i \subseteq r$) of the residual substructure.

In the framework of the dual formulation in the frequency domain (see Section 14.2.1.1), the union between the coupled system RU and the fictitious subsystem can be written (see Eq. 14.10) as:

$$\begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} & \mathbf{B}^{RU^T} \\ \mathbf{0} & -\mathbf{Z}^R & \mathbf{B}^{R^T} \\ \mathbf{B}^{RU} & \mathbf{B}^R & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{RU} \\ \mathbf{f}^R \\ \mathbf{0} \end{Bmatrix} \quad (14.14)$$

Following the same procedure used in Section 14.2.1.1, it is possible to obtain the FRF of the unknown subsystem U .

$$\mathbf{H}^U = \begin{bmatrix} \mathbf{H}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^R \end{bmatrix} - \begin{bmatrix} \mathbf{H}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^R \end{bmatrix} \begin{bmatrix} \mathbf{B}^{RU^T} \\ \mathbf{B}^{R^T} \end{bmatrix} \left(\begin{bmatrix} \mathbf{B}^{RU} & \mathbf{B}^R \end{bmatrix} \begin{bmatrix} \mathbf{H}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^R \end{bmatrix} \begin{bmatrix} \mathbf{B}^{RU^T} \\ \mathbf{B}^{R^T} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{B}^{RU} & \mathbf{B}^R \end{bmatrix} \begin{bmatrix} \mathbf{H}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^R \end{bmatrix} \quad (14.15)$$

Note that $\mathbf{H}^{RU}$ and $\mathbf{H}^R$ are the FRFs at the full set of DoFs of the coupled system and the residual subsystem.

14.3 Case Study

In this paper the dynamic behavior of the rotor of the Ampair 600 wind turbine is analyzed. The rotor is made by three blades clamped to the hub by three bolts and two plates that sandwich the blade. The blades are made of glass reinforced polyester and they are coated with a white epoxy. The hub to which the blades are attached has an overspeed mechanism to vary the pitch of the blades: the mechanism is activated by centrifugal masses that are fixed through bolts in the holes at the greatest radial distance from the rotation axis of the hub.

One of the most interesting features in the study of coupling and decoupling problem is the possibility to use both experimental and numerical data. In this case a Finite Element Model of the whole rotor is developed to extract the Frequency Response Functions needed to apply the Frequency Based Substructuring methods. In this model all the alteration to the commercial turbine introduced to reduce the nonlinear behavior are considered. The overspeed mechanism was potted to reduce the rattling and the free play between the different parts of the mechanism and it is not considered in this modeling. Moreover, the centrifugal masses that activate the mechanism are not taken into account and lastly the internal shaft is considered to be directly bonded to the hub.

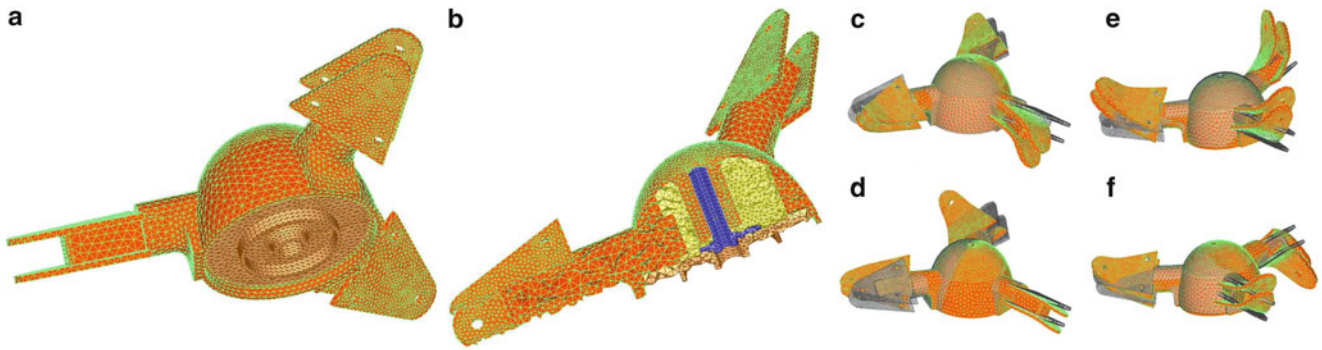
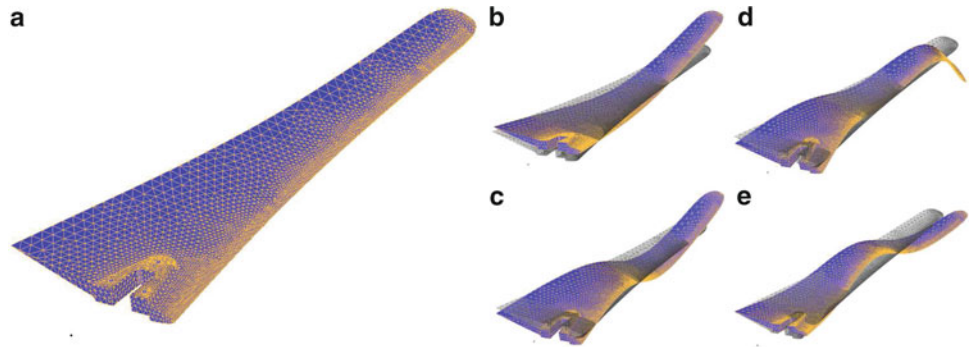


Fig. 14.3 Assembled Hub FE model (a), a cross section of the model showing the different parts of the assembly (b) and the first four modal shapes (c–f)

Fig. 14.4 FEM model of the blade (a) and first four modal shapes (b–e)



Four substructures are defined inside the whole rotor: the central hub and the three blades which are identical to each other, so only two different substructures have to be modeled to find the dynamic behavior of the whole system. The hub is made of isotropic material with a negligible damping. For the sake of simplicity it is modeled entirely in Aluminum Alloy, keeping the chance to eventually consider different materials for each component. The Finite Element Model of the hub (Fig. 14.3) is made of around 130000 tetrahedral elements with linear shape function. The blades are made of Fiber Reinforced Polyester but the mechanical properties are unknown. Here a linear material is considered and the Finite Element Model of each blade (Fig. 14.4) is made of around 50000 tetrahedral elements.

The whole rotor, composed by the three blades fixed on the central hub, is also modeled (Fig. 14.5) with the aim to compare its behavior to the coupling results and to approach the decoupling of the system. Although the connection between the substructures is ensured by contacting surfaces, efforts to simplify as much as possible the interface are undertaken, with the aim to minimize the DoFs involved in the coupling without losing important information about the dynamic behavior of the interface. After several analysis the connection between the substructures is modeled by enforcing compatibility only at the translational DoFs of the master nodes of the Rigid Body Elements (RBEs) which are realized on both edges of each hole (Fig. 14.6). This 6 points connection (18 DoF) introduces a relative error lower than 3% in the determination of the first modal frequencies in the range 0–250 Hz compared to the modal frequencies when the compatibility is enforced on each node on the contact surface. Moreover this highlights the low influence of the rotational DoFs in the connecting points considered.

To have an idea of the dynamic behavior of the system and of the subsystems, the natural frequencies of the blade subsystems, of the hub subsystem and of the coupled rotor system are shown in Table 14.1. For the coupled system, the values are referred to the simplified connection described above.

Receptance matrices to be used for the coupling with Frequency Based Substructuring techniques are obtained through a superelement based computational approach (Fig. 14.7). In this way, the receptance matrix is determined only on a subset of physical points that are relevant to solve the coupling problem. First, superelements are defined using a Craig-Bampton reduction of the Finite Element Model; then, the receptance matrix is directly computed using the mass and stiffness matrices of the reduced system. In the Craig-Bampton method, the reduced mass $\mathbf{M}$ and stiffness $\mathbf{K}$ matrices are defined for a subset of physical DoFs and a set of fictitious DoFs representing the modes of the structure constrained at the connecting points. After condensation, the reduced mass and stiffness matrices are extracted from the FEM and imported in Matlab environment to find the receptance matrix of the subsystem at each frequency of interest $\mathbf{H}(\omega) = [\mathbf{K} - \omega^2 \mathbf{M}]^{-1}$.

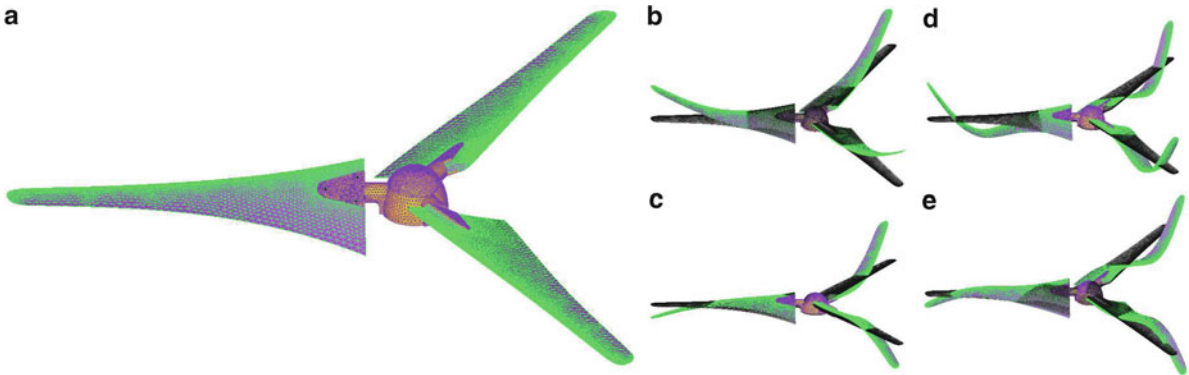


Fig. 14.5 FEM model of the rotor (a) and first four modal shapes (b–e)

Fig. 14.6 RBE at the interface

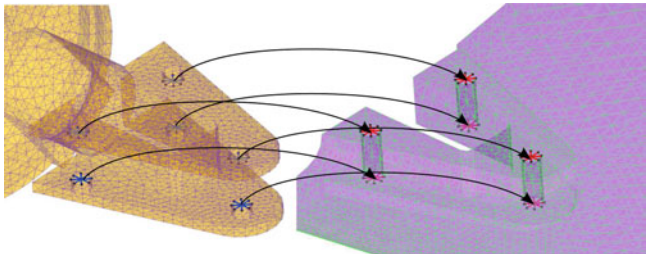


Table 14.1 Natural frequencies of the systems [Hz]

Mode	System		
	Blade	Hub	Rotor
1–6	0	0	0
7	42.64	1977.77	21.43
8	123.93	1977.77	28.77
9	238.6	2284.54	28.77
10	251.83	3091.28	73.34
11	329.22	3091.29	76.62
12	430.24	3424.74	76.63
13	532.22	3643.71	124.47
14	702.64	3643.79	124.47
15	781.74	3643.81	165.08
16	942.22	4863.77	175.97

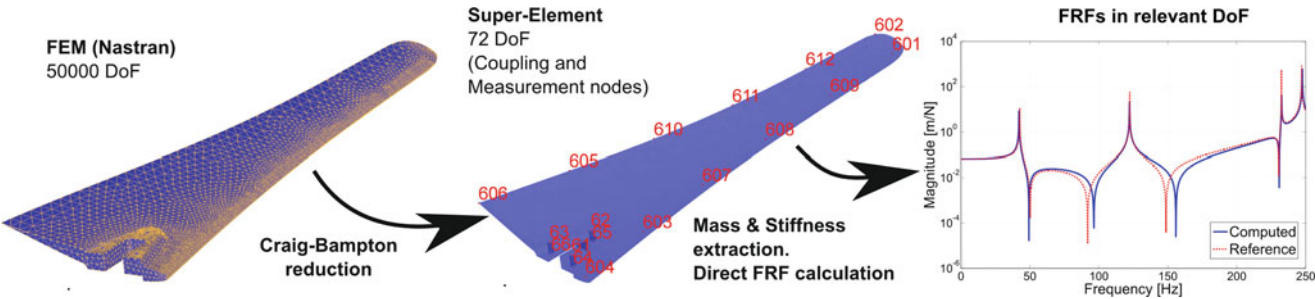


Fig. 14.7 SuperElement computational approach for the determination of the receptance matrix

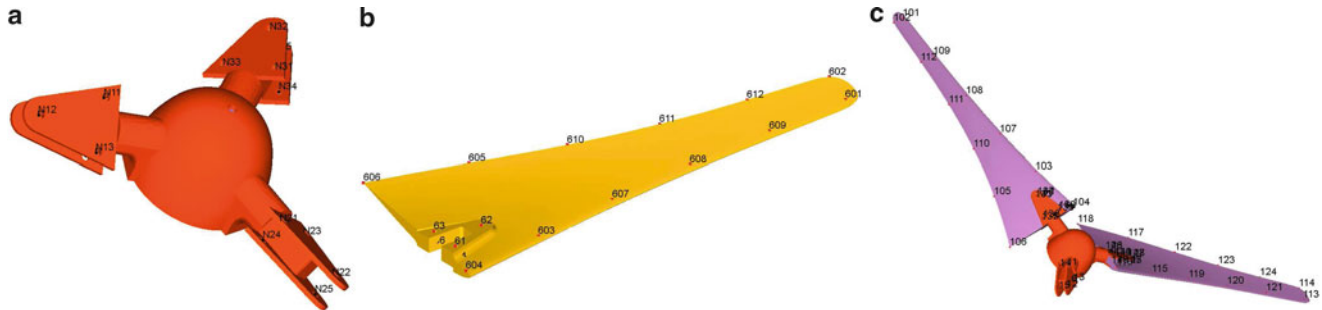


Fig. 14.8 Relevant nodes for coupling in the hub (a), in each blade (b) and in the subsystem composed by the hub and 2 blades (c)

14.3.1 Coupling One Blade with the Rest of the Rotor

Coupling between substructures is performed using the dual formulation. The coupled receptance matrix $\mathbf{H}_C$ is defined as:

$$\mathbf{H}_C = \mathbf{H} - \mathbf{H}\mathbf{B}^T(\mathbf{B}\mathbf{H}\mathbf{B}^T)^{-1}\mathbf{B}\mathbf{H} \quad (14.16)$$

where $\mathbf{H}$ is the receptance matrix of the whole system assembled using the dual approach, and $\mathbf{B}$ is the signed boolean matrix that defines the compatibility condition.

The receptance matrix of the hub substructure ($\mathbf{H}^H$) is determined at 18 coupling nodes (6 for each blade, see Fig. 14.8a), by considering 16 modes for the Craig Bampton reduction. The receptance matrix of the blade substructure ($\mathbf{H}^B$) is determined at 18 nodes (6 coupling nodes and 12 internal nodes, see Fig. 14.8b), by considering also in this case 16 modes for the CB reduction. To analyze the single coupling interface, a substructure composed by the hub and 2 blades is also considered, in this case the receptance matrix ($\mathbf{H}^{H+2B}$) is defined at 42 nodes (6 coupling nodes and 36 internal nodes, see Fig. 14.8c). In this case, due to the more complex behavior of the substructures, 20 modes are considered for the CB reduction.

The reference system of the blade is different from the reference system of the hub, so a rotation is applied to the receptance matrix of the blade, in order that the two reference systems are coincident and the matrix $\mathbf{B}$ is boolean.

The two substructures (hub with two blades + blade) are coupled using the dual approach. The receptance matrices of the substructures have size $\mathbf{H}^{H+2B} = [126 \times 126]$ and $\mathbf{H}^B = [54 \times 54]$. The interface among the two substructures is first assumed at the 18 translational DoF of the 6 coupling nodes. The receptance matrix assembled in the dual way is expressed as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}^{H+2B}_{[126 \times 126]} & \mathbf{0}_{[126 \times 54]} \\ \mathbf{0}_{[54 \times 126]} & \mathbf{H}^B_{[54 \times 54]} \end{bmatrix} \quad (14.17)$$

The matrix $\mathbf{B}$ that defines the compatibility condition is:

$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 114]} & -\mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} \end{bmatrix} \quad (14.18)$$

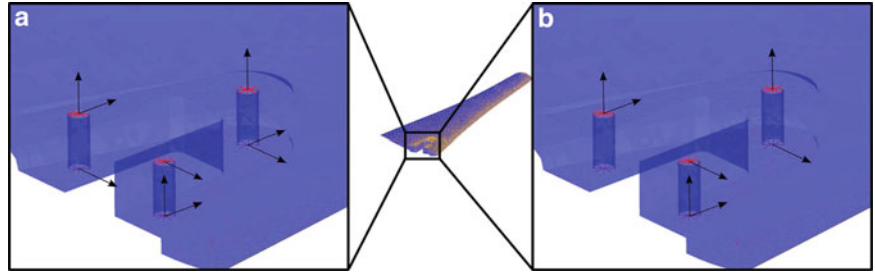
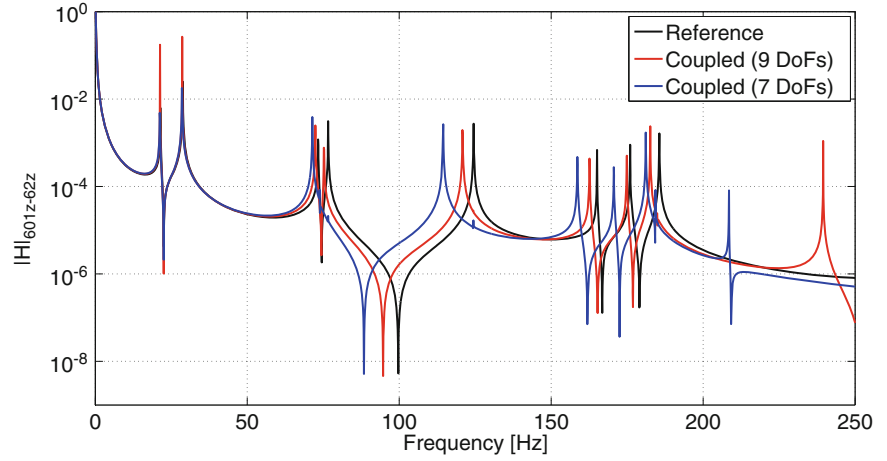
where matrices $\mathbf{I}_{[n \times n]}$ and $\mathbf{0}_{[n \times m]}$ are respectively the square diagonal unit matrix of size n and the null matrix of size $[n \times m]$.

The receptance matrix of the assembled system is computed using Eq. (14.17) with a frequency resolution of 0.1 Hz. The natural frequencies (estimated from peaks of the receptance modulus) are determined very accurately: in fact, the coupling procedure itself does not introduce any error in the natural frequencies, when compared to the natural frequencies obtained from the FEM with the simplified interface shown in Fig. 14.6.

However, a large number of measurements should be performed to determine the receptance matrices of the substructures at all the translational DoFs of the coupling nodes. Therefore, efforts are undertaken to reduce the number of DoFs involved in the coupling procedure. The aim is to find a subset of relevant DoFs to solve the coupling problem without losing information.

After several analyses, a subset of 9 relevant DoFs is found (see Fig. 14.9a). In order to have a limited error, the selected DoFs should be able to represent the type of connection that in this case is a rigid joint, enforcing constraints on 6 DoFs. So all the translational and rotational actions should be reproduced through forces acting on the selected DoFs.

Trying to further reduce the number of interface DoFs, using for instance the set of 7 DoFs shown in Fig. 14.9b, drastically worsens the results. The Frequency Response Function for the selected subset of 9 DoFs and for the further reduced subset

Fig. 14.9 Interface DoFs selected for coupling**Fig. 14.10** FRF of the coupled system: reference (black lines), after coupling 1 blade at 9 DoFs (red lines) or 7 DoFs (blue lines) (color figure online)**Table 14.2** Natural frequencies of the coupled system (1 blade with the rest of the rotor)

Mode number	Reference [Hz]	7 DoF		9 DoF	
		[Hz]	[Err%]	[Hz]	[Err%]
7	21.4	21.2	−0.93%	21.3	−0.47%
8–9	28.8	28.4	−1.39%	28.6	−0.69%
10	73.3	71.4	−2.59%	72.4	−1.23%
11–12	76.6	73.8	−3.66%	75.2	−1.83%
13–14	124.5	114.5	−8.03%	120.8	−2.97%
15	165.1	158.6	−3.94%	162.6	−1.51%
16	176	170.6	−3.07%	174.9	−0.62%
17–18	185.6	181.1	−2.42%	182.6	−1.62%

composed of 7 DoFs is shown in Fig. 14.10 compared to the reference model. The subset composed by 7 DoFs introduces a much larger error compared to the subset of 9 DoFs. Analyzing the FRFs and reading the values of the natural frequencies in Table 14.2, the whole FRF appears to be shifted to the left: this is an effect of a drop in the stiffness of the joint, due to the removal of some coupling DoFs.

14.3.2 Coupling the Three Blades with the Hub

The study carried out on a single blade is extended to the problem of coupling the three blades with the hub. Now, three coupling interfaces have to be considered. To manage all coupling interfaces in a similar manner, a different reference system is introduced for each coupling interface, having the y axis parallel to the longitudinal direction of the blade, the z axis perpendicular to the coupling surface, and the x axis defined accordingly. Therefore, the receptance matrix considered for any of the three blades is always the same, whereas the receptance matrix of the hub, rotated in these references systems, highlights the axial symmetry of the hub.

$$\mathbf{H}^H = \begin{bmatrix} \mathbf{H}^I & \mathbf{H}^{II} & \mathbf{H}^{III} \\ \mathbf{H}^{III} & \mathbf{H}^I & \mathbf{H}^{II} \\ \mathbf{H}^{II} & \mathbf{H}^{III} & \mathbf{H}^I \end{bmatrix} \quad (14.19)$$

Fig. 14.11 FRF of the coupled system: reference (black lines), after coupling 3 blades (red lines) (color figure online)

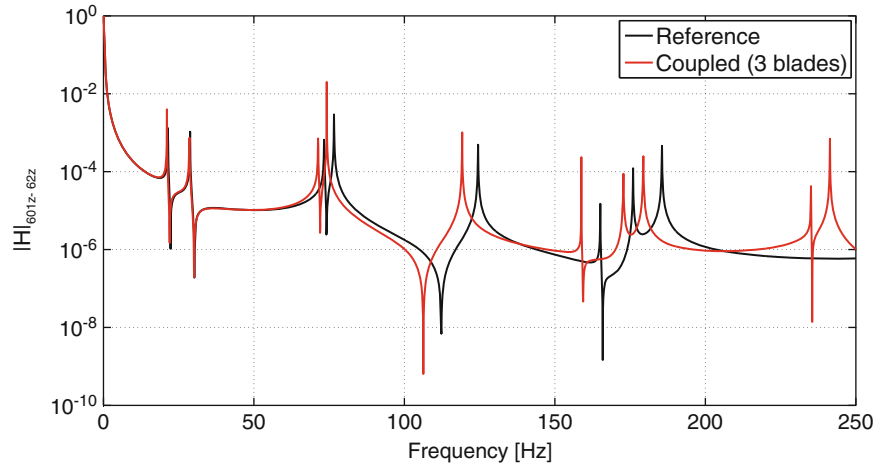


Table 14.3 Natural frequencies of the 3 blades coupled system [Hz]

Mode number	Reference [Hz]	Coupl. 3 blades	
		[Hz]	[Err%]
7	21.4	21.1	-1.40%
8–9	28.8	28.6	-0.69%
10	73.3	71.3	-2.73%
11–12	76.6	74.2	-3.13%
13–14	124.5	119.2	-4.26%
15	165.1	158.8	-3.82%
16	176	172.8	-1.82%
17–18	185.6	179.4	-3.34%

The receptance matrices of the hub and the blades have the the following sizes: $\mathbf{H}^H = [54 \times 54]$ and $\mathbf{H}^B = [54 \times 54]$. In the formulation of the coupling problem by considering all the coupling DoFs, the matrices $\mathbf{H}$ and $\mathbf{B}$ are as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{[54 \times 54]}^H & \mathbf{0}_{[54 \times 54]} & \mathbf{0}_{[54 \times 54]} & \mathbf{0}_{[54 \times 54]} \\ \mathbf{0}_{[54 \times 54]} & \mathbf{H}_{[54 \times 54]}^B & \mathbf{0}_{[54 \times 54]} & \mathbf{0}_{[54 \times 54]} \\ \mathbf{0}_{[54 \times 54]} & \mathbf{0}_{[54 \times 54]} & \mathbf{H}_{[54 \times 54]}^B & \mathbf{0}_{[54 \times 54]} \\ \mathbf{0}_{[54 \times 54]} & \mathbf{0}_{[54 \times 54]} & \mathbf{0}_{[54 \times 54]} & \mathbf{H}_{[54 \times 54]}^B \end{bmatrix} \quad (14.20)$$

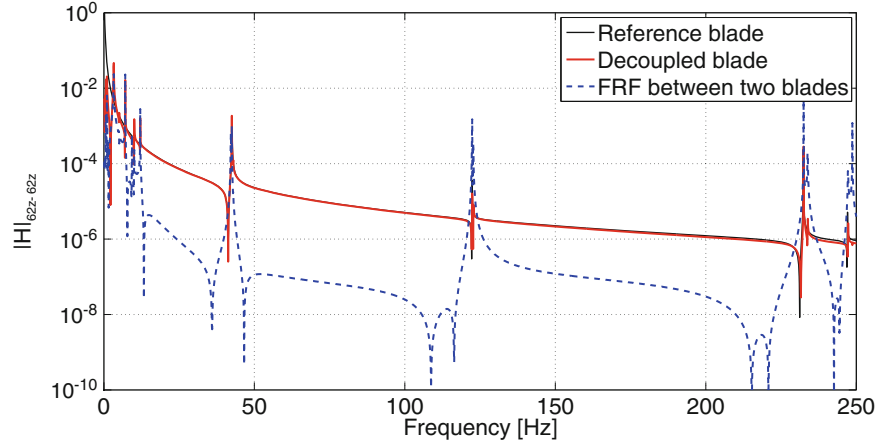
$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & -\mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} \\ \mathbf{0}_{[18 \times 18]} & \mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} & -\mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} \\ \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} & -\mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 36]} \end{bmatrix} \quad (14.21)$$

The FRF of the whole system coupled by means of the 9 DoFs selected above is shown in Fig. 14.11 compared to the reference FRF of the rotor. Moreover, natural frequencies of the coupled system are shown in Table 14.3, compared to the reference ones.

14.3.3 Decoupling

In this case, the dynamic behavior of the 3 blades is found starting from the dynamic behavior of the whole rotor (Fig. 14.5) and of the central hub (Fig. 14.3). The problem is tackled in the simplest way, by using the standard interface: only the DoFs that physically connect the residual substructure (the hub) and the unknown substructure (the 3 blades) are used as interface DoFs. In this way, all decoupling techniques bring to the same results. To decouple the systems in order to find the dynamic behavior of the blades, the dual decoupling technique is used. It is expressed by the following relation that allows to find the receptance matrix of the unknown system:

Fig. 14.12 FRF of a blade: reference (black lines), after decoupling (red lines); FRF between two different blades (blue lines) (color figure online)



$$\mathbf{H}^{3B} = \begin{bmatrix} \mathbf{H}^R & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^H \end{bmatrix} - \begin{bmatrix} \mathbf{H}^R & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^H \end{bmatrix} \begin{bmatrix} \mathbf{B}^{RT} \\ \mathbf{B}^{HT} \end{bmatrix} \left(\begin{bmatrix} \mathbf{B}^R & \mathbf{B}^H \end{bmatrix} \begin{bmatrix} \mathbf{H}^R & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^H \end{bmatrix} \begin{bmatrix} \mathbf{B}^{RT} \\ \mathbf{B}^{HT} \end{bmatrix} \right)^+ \begin{bmatrix} \mathbf{B}^R & \mathbf{B}^H \end{bmatrix} \begin{bmatrix} \mathbf{H}^R & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^H \end{bmatrix} \quad (14.22)$$

where $\mathbf{H}^R$ is the receptance matrix of the whole rotor and $\mathbf{H}^H$ is the receptance matrix of the hub. The previous matrices are both defined only for the standard interface DoFs, which connect the residual substructure and the unknown substructure to each other and their size is $[54 \times 54]$. In this analysis, 60 modes for the whole rotor and 16 modes for the hub are taken into account. Furthermore:

$$\begin{bmatrix} \mathbf{B}^R & \mathbf{B}^H \end{bmatrix} = \begin{bmatrix} \mathbf{B}^R & \mathbf{B}^H \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & -\mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} \\ \mathbf{0}_{[18 \times 18]} & \mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & -\mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} \\ \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{I}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & \mathbf{0}_{[18 \times 18]} & -\mathbf{I}_{[18 \times 18]} \end{bmatrix} \quad (14.23)$$

Also in this case, three different reference systems – one for each blade – are introduced to express the FRFs in coupling nodes. The three blades that represent the unknown structure are not connected together, so that the receptance matrix of the unknown system $\mathbf{H}^{3B}$ is expected to be a block diagonal matrix composed by three matrices nominally identical to each other, because of the radial symmetry:

$$\mathbf{H}^{3B} = \begin{bmatrix} \mathbf{H}^{B1} \approx \mathbf{0} \approx \mathbf{0} \\ \approx \mathbf{0} \mathbf{H}^{B2} \approx \mathbf{0} \\ \approx \mathbf{0} \approx \mathbf{0} \mathbf{H}^{B3} \end{bmatrix} \quad (14.24)$$

The decoupling problem is solved using all the translational DoFs of the reduced interface. The identified dynamic behavior (Fig. 14.12) of the blade reproduces exactly the reference behavior of the blade except for frequencies between 0 and 13 Hz. This is probably due to numerical problems related to the computation of rigid body modes, that are often troublesome for FE software. The dashed line represents the FRF between two DoFs belonging to different blades. As stated previously, it should be theoretically zero. Actually, due to the approximations involved in the use of the reduced interface, it is not zero, but it is several orders of magnitude lower than other FRFs, except at resonances, whose amplitude is not significant being the model undamped.

14.4 Summary and Discussion

In this paper, the rotor of the Ampair 600 wind turbine is considered as a test bed for dynamic substructuring, with the aim of selecting a reduced set of relevant DoFs to describe the interface between each blade and the hub. The connection between these two items is ensured by mechanical contact, and therefore compatibility should be enforced between all facing nodes on the contact surface. A first significant reduction is achieved by enforcing compatibility only at the translational DoFs of 6 points, representing the edges of the holes through which the blade is bolted to the hub. Such simplified connection

(18 DoFs for each blade) introduces a relatively low error in the natural frequencies in the range 0–250 Hz. A second reduction is achieved by enforcing compatibility only at 9 DoFs, selected so as to obtain a behavior similar to that of a rigid joint: the additional error introduced by this operation is quite low. Any further attempt to reduce the number of interface DoFs drastically increases the error.

The reduced 9 DoFs interface provides good results in both the considered coupling cases: (a) hub plus two blades coupled with one blade and (b) hub coupled with three blades. In the decoupling case, good results are obtained using the 18 DoFs interface, meaning that the implementation of the procedure is correct. The next step will consider the use of the reduced 9 DoFs interface.

Further developments are envisaged in several directions. First of all, the model of the blade is not very precise, since it considers a homogeneous and isotropic fiber reinforced polyester. On the contrary, it has been pointed out that the reinforcing glass fabric is placed only in the outer part of the blade, and therefore a non homogeneous orthotropic material should be considered. A second line of development is to consider noise polluted FRFs to check the robustness of the technique. Thirdly, experimentally measured FRFs should be used for sub structuring. In fact, the use of computational FRFs is justified by the need of checking the effect of each simplifying assumption, but in view of the use of experimental FRFs.

Acknowledgements This research is supported by grants from University of Rome La Sapienza and University of L'Aquila.

References

1. D'Ambrogio W, Sestieri A (2004) A unified approach to substructuring and structural modification problems. *Shock Vib* 11(3–4):295–310
2. Jetmundsen B, Bielawa R, Flannelly W (1988) Generalised frequency domain substructure synthesis. *J Am Helicopter Soc* 33(1):55–64
3. de Klerk D, Rixen DJ, Voormeeren S (2008) General framework for dynamic substructuring: history, review, and classification of techniques. *AIAA J* 46(5):1169–1181
4. de Klerk D (2009) Dynamic response characterization of complex systems through operational identification and dynamic substructuring. Ph.D. thesis, TU Delft
5. D'Ambrogio W, Fregolent A (2009) Decoupling procedures in the general framework of frequency based substructuring. In: *Proceedings of 27th IMAC, Orlando, USA, Feb 2009*
6. D'Ambrogio W, Fregolent A (2010) The role of interface DoFs in decoupling of substructures based on the dual domain decomposition. *Mech Syst Signal Process* 24(7):2035–2048. doi:10.1016/j.ymssp.2010.05.007; also in *Proceedings of ISMA 2010, Leuven, Belgium, Oct 2010*, pp 1863–1880
7. Voormeeren SN, Rixen DJ (2010) A dual approach to substructure decoupling techniques. In: *Proceedings of 28th IMAC, Jacksonville, USA, Feb 2010*
8. Voormeeren SN, Rixen DJ (2012) A family of substructure decoupling techniques based on a dual assembly approach. *Mech Syst Signal Process* 27:379–396. doi: 10.1016/j.ymssp.2011.07.028.
9. D'Ambrogio W, Fregolent A (2005) Prediction of substructure properties using decoupling procedures, In: Soize C, Schuëller G (eds) *Structural dynamics - Eurodyn 2005. Proceedings of the 6th international conference on structural dynamics*, Paris, France, Sep 2005, pp 1893–1898
10. Mayes RL, Arviso M (2010) Design studies for the transmission simulator method of experimental dynamic substructuring. In: Sas P, Bergen B (eds) *Proceedings of ISMA 2010 - international conference on noise and vibration engineering*, Leuven, Belgium, Sep 2010, pp 1929–1938
11. Mayes RL, Ross MR (2012) Advancements in hybrid dynamic models combining experimental and finite element substructures. *Mech Syst Signal Process* 31:56–66
12. D'Ambrogio W, Fregolent A (2008) Promises and pitfalls of decoupling procedures. In: *Proceedings of 26th IMAC, Orlando, USA, Feb 2008*
13. Sjövall P, Abrahamsson T (2008) Substructure system identification from coupled system test data. *Mech Syst Signal Process* 22(1):15–33
14. Mayes RL (2012) An introduction to the SEM substructures focus group test bed - the Ampair 600 wind turbine. In: Mayes R, Rixen D, Griffith D, De Klerk D, Chauhan S, Voormeeren S, Allen M, Proulx T (eds) *Topics in experimental dynamics substructuring and wind turbine dynamics*, vol 2. Conference proceedings of the society for experimental mechanics series, vol 27. Springer, New York, pp 61–70. 10.1007/978-1-4614-2422-2_7
15. Harvie J, Avitabile P (2012) Comparison of some wind turbine blade tests in various configurations. In: Mayes R, Rixen D, Griffith D, De Klerk D, Chauhan S, Voormeeren S, Allen M, Proulx T (eds) *Topics in experimental dynamics substructuring and wind turbine dynamics*, vol 2. Conference proceedings of the society for experimental mechanics series, vol 27. Springer, New York, pp 73–79. 10.1007/978-1-4614-2422-2_9

Chapter 15

The Ampair 600 Wind Turbine Benchmark: Results From the Frequency Based Substructuring Applied to the Rotor Assembly

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Abstract Prior to any structural system realization during the design phase, the structural dynamic should be characterized. Dynamic characterization provides the designers with local and global dynamic information which can be used to optimize the structures. To characterize the dynamic of too large and complex structures generally Dynamic Substructuring (DS) techniques are used. Experimental DS is one of these techniques and is recently more in use. Many researchers put effort in developing and evolving new concepts. The substructuring focus group at Society for Experimental Mechanics (SEM) uses a small-scale wind turbine, Ampair 600, in a combined effort to validate, classify and advance these techniques. In this paper the substructuring results, obtained with the LM FBS formulation applied to the wind turbines rotor are given. The Interface Deformation Mode (IDM) method is adopted and applied to overcome the lack of Rotational Degrees of Freedom (RDoF) and to minimize the measurement noise. To include the joint stiffness and damping a Substitute (Fixture) is used and two methods are proposed to model flexible and rigid Cyclic Symmetric Structures (CSS). The results obtained in this first substructure analysis of the rotor show that good results can be found in the lower frequency range.

Keywords Substructuring • Cyclic Symmetric structures • Substitute • Fixture • Coupling • Decoupling • Rotational DoFs

15.1 Introduction

Dynamic substructuring can be utilized to meet the increasing demand on reliability and life time of the large and complex structures. DS divides a structural system into subsystems and assembles their analytical or/and experimental models. As a result local optimization is more feasible and more insight can be obtained over the local dynamics of the substructures without loss of generality. The coupling is normally done in frequency or modal domain, and in time domain if Impulse Based Substructuring (IBS) is applied [15].

This work concentrates on the experimental structure modeling, particularly in the cases where no analytical models of structures exist and/or experimental models are preferable. Experimental models (frequency response functions) can be more favorable when a structure shows special frequency dependencies or has high modal density and large modal damping. Besides the experimental models might be easier and faster to obtain and time-consuming (analytical) modeling of the complex structures can be avoided.

Frequency Based Substructuring (FBS) can be adopted to couple the experimental and/or synthesized frequency response functions (FRF) in the frequency domain. Among the FBS formulations one can mention Impedance coupling, Admittance

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coupling or Lagrange Multiplier Frequency Based Substructuring (LM FBS). All these methods are mathematically equivalent but might differ in the numerical handlings needed for their application.

LM FBS is very straightforward to apply to coupling and decoupling problems and is used in this work to substructure the Ampair 600 rotor which consists of a hub and three blades. The Ampair 600 is an international Benchmark between research groups to improve substructuring techniques. Details of the benchmark activities can be found in [10]. The hub is first modeled as rigid body and later as flexible body. For the blades three models are considered. The first model is derived from blade-substitute assembly using decoupling techniques. The second model is the synthesized model of the first model and the third model is derived directly from measurement without substitute. It is shown that using a substitute (fixture) is highly recommended and using synthesized model can result in wrong prediction of the assembled rotor. Furthermore two methods are proposed which seem to contribute to consistent experimental modeling of the Cyclic Symmetric Structures.

15.2 Frequency Based Substructuring

If one divides a structural system into substructures, each substructure dynamics can be analyzed in physical, modal or frequency domain. In the physical domain each structure is represented with its mass, stiffness and damping matrix, in the modal domain with a linear combination of normal modes and their possible augmentations and in the frequency domain with its FRF's. To couple the substructures in any domain, two conditions must be met: *compatibility* and *equilibrium*. Compatibility ensures that the displacements at coupling points are equal and equilibrium equilibrates the forces at those points. The assembly can be *primal* or *dual*. Satisfying the compatibility *prior* to equilibrium yields the primal formulation. Interchanging this sequence and satisfying the equilibrium prior to compatibility yields the dual formulation. In the frequency domain, primal assembly is equivalent to Impedance Coupling and dual assembly leads to LM FBS where the Lagrange Multipliers correspond to unknown interface forces between the substructures [3, 4]. Following the LM FBS the assembled system matrix equals:

$$Y_{ass} = Y \left(I - B^T (BYB^T)^{-1} BY \right) \quad (15.1)$$

In Eq (15.1), Y is the block diagonal matrix with the substructures FRF's on the main diagonal. B is signed Boolean matrix which is formed on the coupling points. The coupled system matrix Y_{ass} is very straightforward to compute and needs just one matrix inversion. For detailed derivation and physical interpretation of Eq (15.1) see e.g. [3, 4]. LM FBS can also be used to decouple a structure if it should be measured in an assembly because it is too delicate or it is preloaded. Besides when the boundaries should be loaded during the measurement in order to mimic the characteristics of adjacent substructure a substitute (fixture), for instance lumped mass, can be attached to interface.¹ To remove the substitute from the substructure prior to the coupling process LM FBS can be utilized. Voormeeren and Rixen [18] have derived a family of methods to perform decoupling. Based on dual formulation the decoupling problem on the interface becomes²:

$$u^{AB} = \left(Y^{AB} - Y^{AB} B^{ABT} \left(B^{AB} Y^{AB} B^{ABT} - B^A Y^A B^{AT} \right) B^{AB} Y^{AB} \right) F^{AB} \quad (15.2)$$

In Eq. (15.2) Y_{AB} is the substructure-substitute assembly and Y_A is the model of the substitute which are known and can be measured or simulated. A very clear illustration of the decoupling problem in modal domain is given by Allen et al. [1] which is also applicable to models in frequency domain.

15.2.1 RDoF, Noise and Inconsistency

For the successful application of LM FBS to coupling and decoupling problems, Rotational Degrees of Freedom (RDoF) should be included in the models and noise and experimental errors should be minimized. RDoF are very difficult to excite and neglecting them provides meaningless result, if the coupling is performed through a single point. To tackle the RDoF

¹Due to attached mass the structure deforms locally near the interface in the lower structure modes, which is not possible in the usual free-interface configuration.

²In decoupling it is also possible to impose compatibility constraints on the internal DoF's. It would improve the decoupling when the substitute has also internal DoF's and will lead to slightly different formulations.

problem the Interface Deformation Modes (IDM) can be applied. IDM maps the measured translational coordinates, in a least square sense, to an admissible space spanned by local interface modes in order to attain the RDoF. See e.g. [5, 6]. It returns a virtual point with six DoF's including the DoF's.

The errors and inaccuracies are incorporated in the measured data set because of cross-axis sensitivity of sensors, inaccuracies in measurement chain, mass loading, sensor misalignment, etc. They can be divided in two groups; inconsistencies and random measurement noise. The inconsistencies are mainly caused by varying additional mass and stiffness effects, which result in incoherent modal content and slightly different entities of the response model such as discrepancy in the resonance frequencies, damping factors and shifting of the mode shapes. Rixen [14] has shown how these inconsistencies will cause spurious peaks in the assembled structure in the vicinity of substructure eigen frequency or will modify the resonances and anti-resonances in the assembled configuration.

Coupling and decoupling are also sensitive to experimental errors and noise. Around the resonances the structural dynamic is mostly dictated by one single mode and as consequences the condition numbers of the measured FRF's are very high around these frequencies. This makes the matrix inversion very sensitive to the uncertainties and measurement noise and amplifies them to the coupled system. Voormeeren et al. [19] have shown that decoupling is in particular sensitive around the anti-resonances of the known structure, while coupling is most sensitive around the resonance frequencies of the subsystems.

15.3 Cyclic Symmetric Structure

Certain structures exhibit translational or rotational periodicity and can be thought of as being composed of identical structures. Typical structures with rotational periodicity are; turbine disk, wheels or wind turbine hub. They are usually known as Cyclic Symmetric Structures (CSS) and if their symmetry properties are properly used they can be measured or computed in a very efficient manner.

In analytical terms these structures are first cut into identical cells with their own local coordinates. Just one cell is modeled and computed and the results are then post processed to attain the dynamic of the whole structure. See e.g. [13]. In experimental terms the structure is virtually divided into identical sections with their own local coordinates. Taking full advantage of the symmetric properties with minimum number of coordinates measurement the full model can be synthesized. The first method proposed in this section shows how this can be done.

In the field of DS sometimes just the rigid body dynamics are enough to make a model. The mass, location of centre of gravity and moments of inertia are necessary and sufficient information to calculate the mass normalized rigid body modes and with that the rigid body dynamic. In analytical terms these properties can be easily obtained but dealing with structures for which no analytical models are available they should be defined through experimental routine and appropriate methods. See e.g. [2, 8, 16, 17]. The second method given in this section explains how a rigid body model can be synthesized based on measured inertia properties around one point.

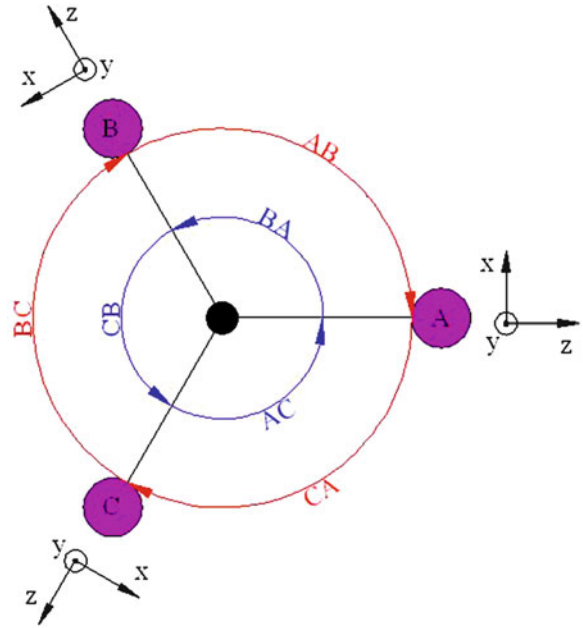
All the techniques are illustrated and described as they have been derived to be applied to a wind turbine hub but they can be generalized and used for any other CSS.

15.3.1 CSS Synthesizing

The periodic structures which are cyclic symmetric such as airplane propeller, bladed disk or wind turbine rotor exhibit geometrical and structural properties which are rotationally periodic. The nature of these structures can be easily understood if one considers the circulant matrix properties. See e.g. [11]. A matrix is said to be circulant if their columns are obtained from one vector and its cyclic permutations:

$$C = \text{circ}[c_1, c_2, \dots, c_n] = \begin{bmatrix} c_1 & c_n & \cdots & c_3 & c_2 \\ c_2 & c_1 & & c_4 & c_3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{n-1} & c_{n-2} & \cdots & c_1 & c_n \\ c_n & c_{n-1} & \cdots & c_2 & c_1 \end{bmatrix} \quad (15.3)$$

Fig. 15.1 Schematic representation of a cyclic symmetric structure



In Eq [15.3], C has n distinct elements. When C is symmetric and circulant, the number of distinct elements is $(n + 2)/2$ or $(n + 1)/2$ for even and odd n , respectively.

Imagine a structure as it is shown in Fig. 15.1, which consists of three identical sectors, A , B and C . The dynamic flexibility of the full structure is:

$$Y_{tot} = \begin{bmatrix} Y_{AA} & Y_{AB} & Y_{AC} \\ Y_{BA} & Y_{BB} & Y_{BC} \\ Y_{CA} & Y_{CB} & Y_{CC} \end{bmatrix} \quad (15.4)$$

Using the properties of the cyclic structure and taking into account that when the structure has been measured in the local direction $Y_{AB} = Y_{CA} = Y_{BC}$ and $Y_{BA} = Y_{CB} = Y_{AC}$, as indicated by the bowed arrows in Fig. 15.1, Eq. (15.4) becomes:

$$Y_{ABC} = \begin{bmatrix} Y_{AA} & Y_{AB} & Y_{BA} \\ Y_{BA} & Y_{AA} & Y_{AB} \\ Y_{AB} & Y_{BA} & Y_{AA} \end{bmatrix} \quad (15.5)$$

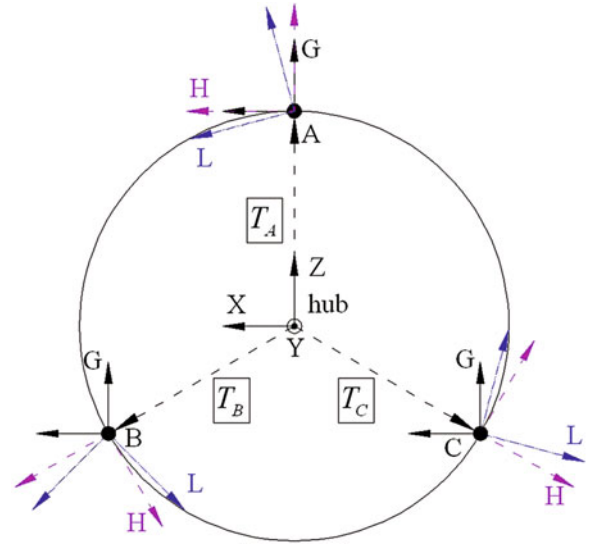
According to Eq. (15.5) the Inertance (Receptance, Mobility or Accelerance) of the given structure is circulant and just by measuring one driving point FRF and one cross FRF the full model can be constructed. This may shorten the measurement time and ensures the reciprocity of the full model.

15.3.2 Inertia Properties and Rigid CSS Model

Imagine a cyclic symmetric system as shown in Fig. 15.2. It represents schematically a hub with three joints A , B and C . The global coordinate system is named **hub** and is located at the geometric center of the hub. One of its axes is aligned with the hub rotational axis which is automatically a principal axis.

At the hub joints three other coordinate systems are defined. The first one is called **G** and is aligned with the global coordinate system. The second coordinate system is an intermediate coordinate system between the **G** and the local coordinate system of each joint and is indicated with **H**. The third and the last coordinate system is the local coordinate system of each joint and is labeled with **L**. These intermediate locale coordinate systems are handy to use and can simplify the end results. For instance due to cyclical properties one can consider the properties in the local reference frame **H** as being cyclic symmetric.

Fig. 15.2 Defined coordinates system in the hub



When the hub is modeled as a point mass with six DoFs along the rotational axis and the full receptance of the hub is required at the hub joints local coordinates, the response model of the point mass at that given point should be expanded to the joints. Because of periodicity and symmetry of the hub just two block matrices are sufficient and necessary to build up the full receptance matrix as shown in section 15.3.1. For instance let's consider Y_{AA} and Y_{AB} . In order to calculate Y_{AA} , first the applied forces at point A should be transformed and transported to the **hub**, whereafter the acceleration can be computed. Once this is done using the geometrical relations the computed accelerations at **hub** can be transported and transformed back to A and Y_{AA} can be synthesized. The transformation of the applied forces from the local coordinated at A to global forces at hub is given with:

$$\begin{aligned} F_A^H &= R_A^{HL^T} F_A^L \\ F_A^G &= R_A^{GH^T} F_A^H = R_A^{GH^T} R_A^{HL^T} F_A^L \\ F_A^{hub} &= T_A^T F_A^G = T_A^T R_A^{GH^T} R_A^{HL^T} F_A^L \end{aligned} \quad (15.6)$$

the accelerations due to these forces at the hub center, which is assumed rigid, are:

$$\alpha_A^{hub} = M^{-1} F_A^{hub} \quad (15.7)$$

using the same procedure used in Eq. (15.6), the acceleration at the joint local coordinates can be computed with:

$$\begin{aligned} \alpha_A^G &= T_A \alpha_A^{hub} \\ \alpha_A^H &= R_A^{GH} \alpha_A^G = R_A^{GH} T_A \alpha_A^{hub} \\ \alpha_A^L &= R_A^{HL} \alpha_A^H = R_A^{HL} R_A^{GH} T_A \alpha_A^{hub} \end{aligned} \quad (15.8)$$

substituting Eq. (15.6) and (15.7) into Eq. (15.8) yields:

$$\alpha_A^L = R_A^{HL} R_A^{GH} T_A M^{-1} T_A^T R_A^{GH^T} R_A^{HL^T} F_A^L \quad (15.9)$$

finally the block matrix Y_{AA} can be computed with:

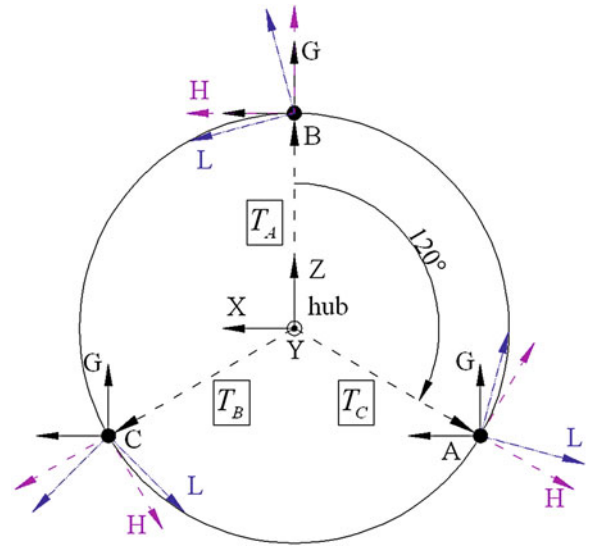
$$Y_{AA} = R_A^{HL} R_A^{GH} T_A M^{-1} T_A^T R_A^{GH^T} R_A^{HL^T} \quad (15.10)$$

Imagine the hub being rotated clockwise $-2\pi/3$ as shown in Fig. 15.3. The hub is invariant to this rotation around the rotational axis (cylindrical axis), i.e. $R_B^{GH^T} M^{-1} R_B^{GH} = R_C^{GH^T} M^{-1} R_C^{GH} = M^{-1}$. It emphasizes that B is equivalent to A , A to C and that Y_{AB} is the same as Y_{CA} . This is the same principle mentioned in section 15.3.1 but placed in another context.

Using the same procedure as used before one may write:

$$Y_{AB} = R_C^{HL} R_C^{GH} T_C M^{-1} T_A^T R_A^{GH^T} R_A^{HL^T} \quad (15.11)$$

Fig. 15.3 Hub being rotated -120°



T_A , T_B and T_C are matrices which are built on the vectors from point **hub** to points A , B and C , respectively and are also known as geometrical transformation matrix. It holds that:

$$\begin{aligned} T_B &= R_B^{GH^T} T_A R_B^{GH} \\ T_C &= R_C^{GH^T} T_A R_C^{GH} \end{aligned} \quad (15.12)$$

after substituting Eq. (15.12) into (15.11) the formulation of the block matrix Y_{AB} becomes:

$$Y_{AB} = R_C^{HL} T_A R_C^{GH} M^{-1} T_A^T R_A^{GH^T} R_A^{HL^T} \quad (15.13)$$

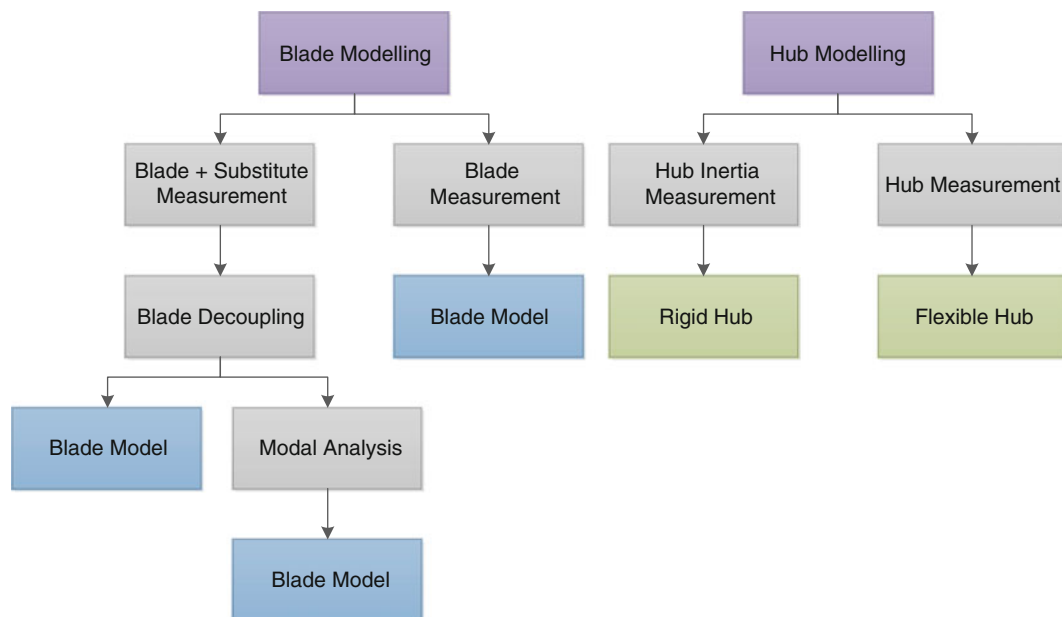
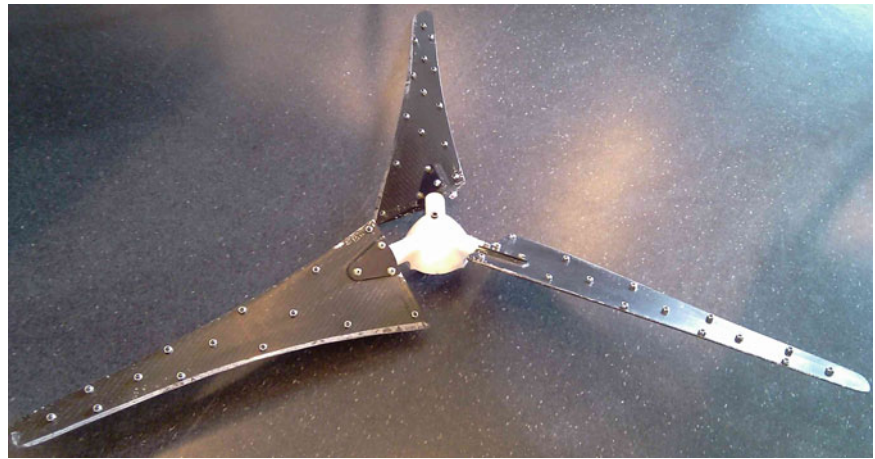
Eq. (15.13) is the simplified version of Eq. (15.11) and uses the geometrical information of cell A only. Thus from measured inertia properties around a point, using Eq. (15.5), (15.10) and (15.13) the rigid CSS model with three joint can be synthesized. M^{-1} is the dynamic flexibility of a rigid body. For the definition of M see e.g. [2, 16, 17]

15.4 Results

The Ampair 600 wind turbine consists of three blades, hub, nacelle, tower and the base which is supported on a trampoline freely. Some modifications are made to the nacelle to force it to behave more linearly. The pitch mechanism is fixed and the generator is replaced with a mock generator. In e.g. [12] it is shown how the pitch mechanism can be fixed. The rotor (hub-blades assembly) analysis consist of modeling the blades and the hub and substructuring them. For all three blades a unique model is used in the assembly. In Fig. 15.4 the rotor assembly is shown as it is used for the validation measurement.

15.4.1 Substructures Modeling

All blades are glass reinforced polyester structures with a complex and curved geometry and not homogeneous material properties. In total three blade models and two hub models are constructed and as a consequence six assembly variants are generated. Figure 15.5 illustrates how these models are established. By comparing all assembly results with the validation measurement the best modeling strategies can be defined. One could also have used the blade measurement to synthesized the blade model but this choice has been omitted. There was a strong believe that modeling the blade this way, will result in an inaccurate model.

Fig. 15.4 Ampair 600 rotor**Fig. 15.5** Substructures modeling steps

15.4.2 Blade

The blade is measured at six coupling and fourteen internal points. The internal points are distributed over the surface of the blade which catches the wind when the wind turbine is in operation. The sensors together with dummy masses are roved to measure the blades. Roving Hammer is not applied because of enormous blade tip flexibility which results easily in double hit and/or bad coherence. Besides, roving the sensors appears to be the only way to hit the blades in all three directions in the most easiest way. The measurements are relatively consistent in terms of added mass since dummy masses are used, which have the same mass as the used accelerometers.

In the rotor assembly each blade is clamped into one of the hub joints. The damping at the joints and the stiffness locally added to the blade are not directly measurable and a blade model without them doesn't describe the dynamics of the blade correctly. Thus a substitute is used to model the blade which is a chopped off version of the hub joint (bracket), as it is advised by Allen et al. [1]. The bracket sandwiches the blade at the root and is bolted to the blade using three $M8$, tightened with 16 [in-lb] as prescribed in [9]. To make excitation in all three directions possible, six aluminum blocks are screwed to the bolt ends. See Fig. 15.6b.

The bracket is assumed to behave rigidly in the frequency range of interest. It is made of aluminum and weights 0.173 [kg]. The principal axes of the bracket are aligned with the global axis of the blade and the moment of inertia about these axis

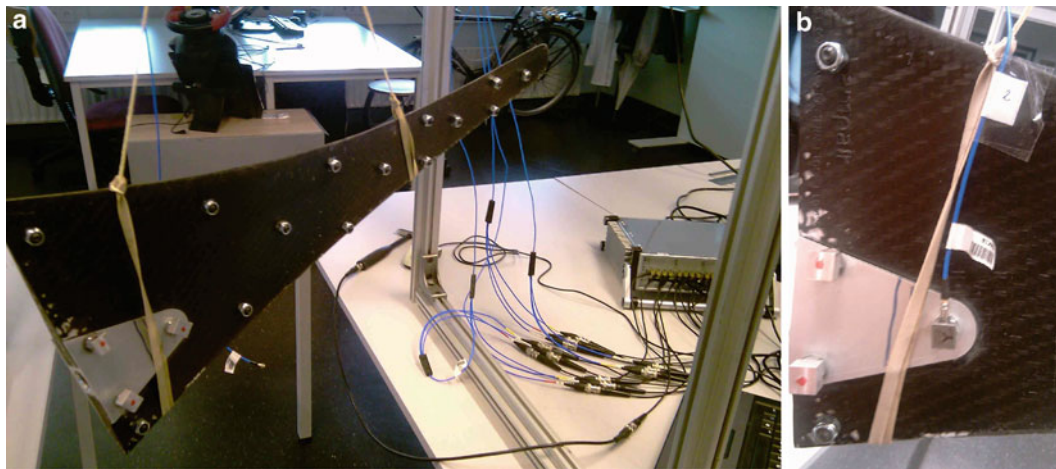


Fig. 15.6 (a) Blade hanging during measurement (b) Blade root

Table 15.1 The first eleven identified blade modes

#	Description	[Hz]
1	1 st flap	45.1
2	2 nd flap	125
3	1 st torsion	180.6
4	3 rd flap	241.4
5	2 nd torsion	310.8
6	4 th flap	383.1
7	1 st edge	397.1
8	3 rd torsion	476.2
9	5 th flap	567.8
10	4 th torsion	639.5
11	6 th flap	736.4

at the bracket center of gravity are $I_{xx} = 0.1244e^{-3}$, $I_{yy} = 0.14e^{-3}$ and $I_{zz} = 0.6523e^{-3}$ [kg.m²]. The first elastic mode of the bracket is at 1700 [Hz] but given the bracket is pre-stressed in the blade-bracket assembly due to the bolt fastening, the first elastic mode will be stiffer and its eigenfrequency will be far above 1700 [Hz]. For this reasons the bracket can be assumed perfectly rigid up to 1000 [Hz]. To decouple the blade from the bracket the following steps are taken:

- the six measured nodes at the blade root are mapped into a virtual point with six DoF using IDM [5,6]. This point coincide with the bracket center of gravity.
- the bracket mass matrix is inverted.
- bracket is subtracted form the blade using LM FBS technique

After applying decoupling the blade model can be synthesized using experimental modal analysis techniques.³ See e.g. [7] for more details over modal analysis. Eighteen modes are identified up to 1600 [Hz] and together with the lower and upper residuals the second blade model is generated. The first eleven identified modes are given in Table 15.1. The third blade model is obtained by measuring the blade in the free-interface condition without substitute.

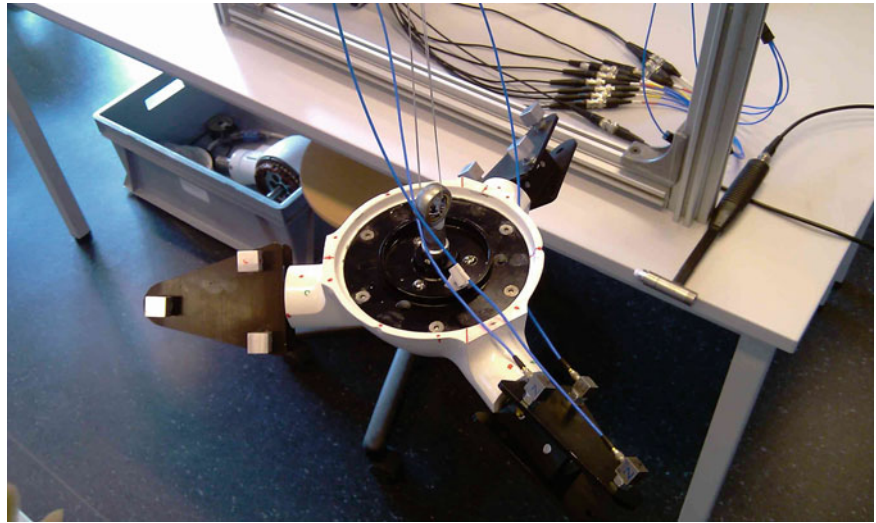
15.4.3 Hub

There is a belief that the low frequency dynamic of the hub-blades assembly is mainly influenced by the mass and inertia of the hub. Considering this the hub can be modeled as a rigid body. Thus after locking the hub mechanism as mentioned earlier, the inertia properties are measured and a rigid hub model is constructed based on the theory given in section 15.3.2. The measured values are given in Table 15.2. The second hub model stems from the hub interface measuring and will serve as flexible hub model. The modeling steps are as follows:

³SDtools is used as the modal analysis toolbox.

Table 15.2 Hub measured inertia properties

$m = 3.85[\text{kg}]$	$x_{og} = 0.0014[\text{m}]$	$y_{og} = -0.0386[\text{m}]$	$z_{og} = 0.0[\text{m}]$	$I_{xx} = 0.0201[\text{kg.m}^2]$
$I_{xy} = 0.0003[\text{kg.m}^2]$	$I_{xz} = -0.0002[\text{kg.m}^2]$	$I_{yy} = 0.0288[\text{kg.m}^2]$	$I_{yz} = 0.0004[\text{kg.m}^2]$	$I_{zz} = 0.0212[\text{kg.m}^2]$

Fig. 15.7 Hub hanging free**Table 15.3** Possible assemblies

Blade \ Hub	Rigid	Flexible
Synthesized	SR	SF
Decoupled	DR	DF
Blade	BR	BF

- according to the theory given in section 15.3.1 just two sets of FRF's, Y_{AA} and Y_{AB} are measured and used to synthesize the full hub model
- at each bracket the measured nodes are mapped into a virtual point using IDM, which coincide with the blade virtual point

Figure 15.7 indicates which interfaces are measured. It is worth mentioning that the brackets are considered as being part of the hub and that the inertia properties are measured at a point located at the hub rotational axis on the hub-disk outer plane.

15.4.4 Assembly and Validation

With the three blade and two hub models six assemblies are analyzed. All models are obtained through experimental routines and with exception for the synthesized blade model all models are based on the raw measured data. In the assembly the global coordinates of the blades coincide with the local coordinates of the hub joints. When coupling is done the results are transformed to the global direction and the modes are extracted. The possible assemblies are given in Table 15.3.

To validate the coupling results the rotor assembly is measurement and Complex Mode Indicator Function (CMIF) is employed to compare the results. CMIF indicates the existence of real or complex modes and the peaks in the CMIF plot represent modes and their corresponding frequency. Consequently it can be used to examine the quality of the obtained results and check if spurious peaks are present in the prediction. Number of CMIF plots equals the driving point measurements.

Figure 15.8 shows the sum of CMIF curves for all coupling variants together with that of the validation. By analyzing the CMIF plots one may conclude that:

- the synthesized blade results in remarkable inaccurate predictions. The assembly results are poor both in low and high frequency regions.
- using substitute returns better assembly result. If the blade is measured without substitute the local structural changes are not taken into consideration.

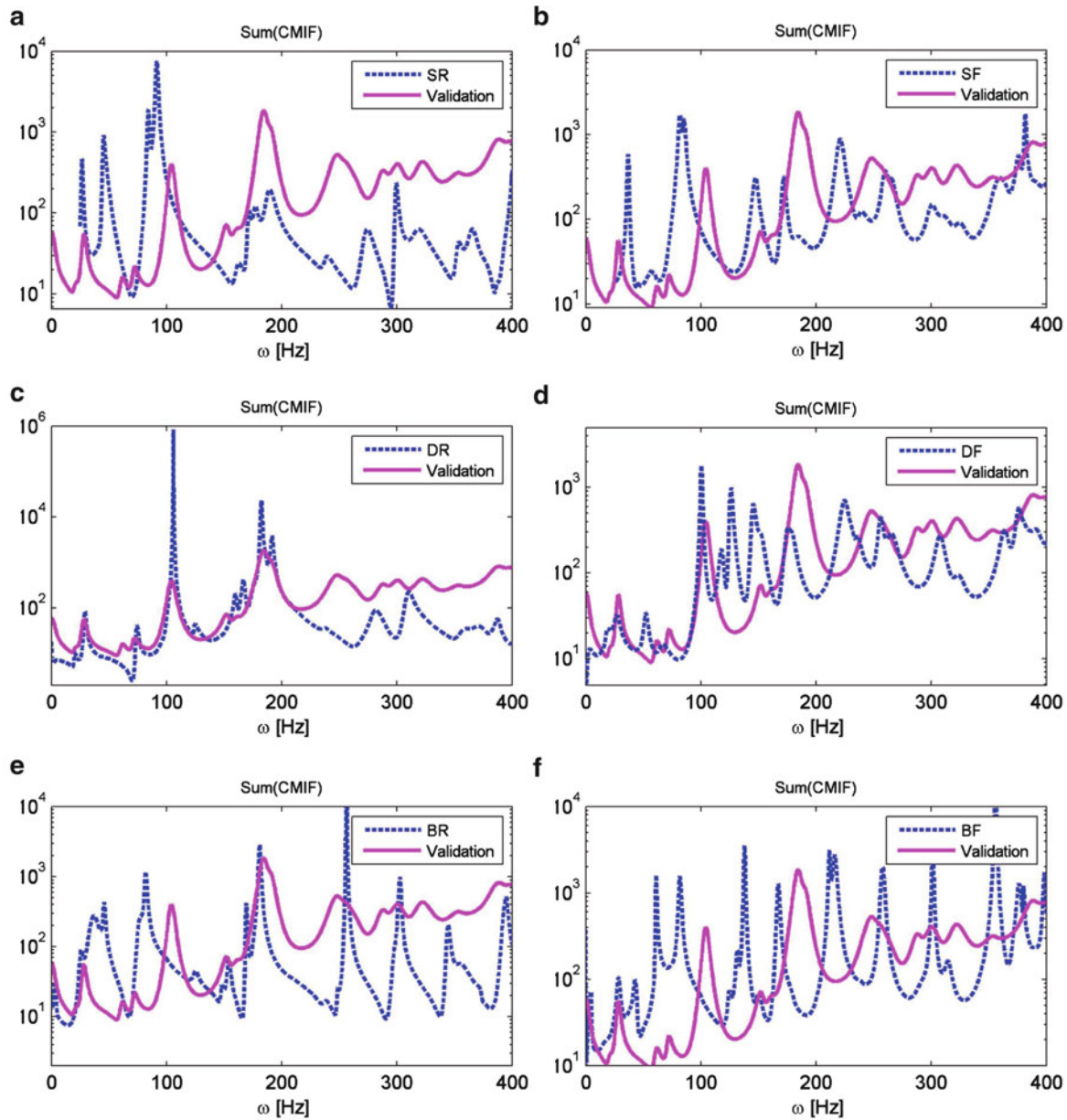


Fig. 15.8 Sum over the CMIF curves for each assembly

- the combination of the rigid hub and the decoupled blade (DR) gives a good result in the low frequency region.
- the combination of the flexible hub and the decoupled blade (DF) gives reasonable results in low and high frequency regions.

15.4.5 Discussion

In general one may state that the wrong prediction using synthesized blade might be caused by:

- damping which is distributed in a non proportional way, like the joint damping.
- close modes or/and highly damped modes
- the absence of proper lower and higher residual terms
- important modes for the assembly which are not dominant enough to be identified

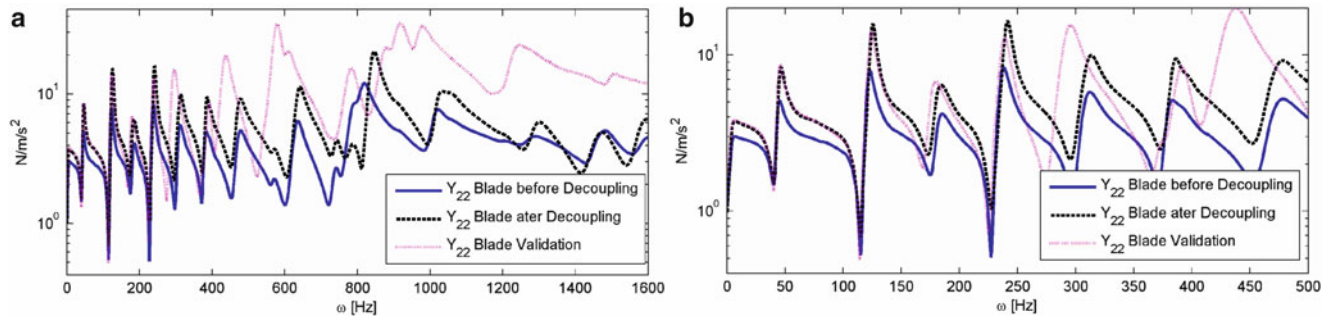


Fig. 15.9 Blade validation after decoupling

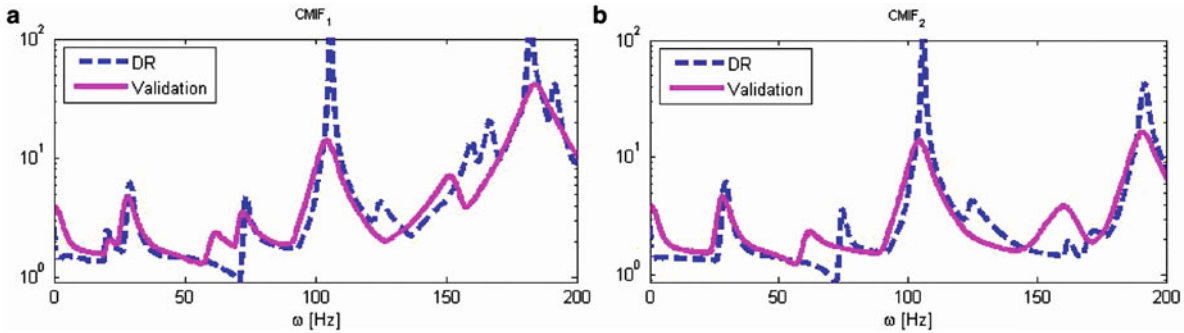


Fig. 15.10 1st and 2nd DR CMIF plots

In contrast to other blade models the decoupled blade gives more reliable results. In Fig. 15.9 an interface driving point is shown in the out of the plane direction of the blade-bracket assembly together with that of the decoupled blade and the directly measurement blade alone (validation). Subtracting the substitute from the blade changes the modal masses and with that all eigen frequencies. Comparing the solid line with the dashed line one sees that the average amplitude of the response is greater and the resonances are shifted to the right as a result of mass removal. Also more modes emerge and are identifiable. The difference between the dotted (validation) and the dashed (blade decoupled) line shows that the fixture doesn't just mass load the blade but also add stiffness. For instance the 1st torsion, the 1st edge and 3rd torsion are less stiff without the bracket and their resonance are shifted to left instead of being shifted to right in contrast to the flapping modes.

As it appears using substitute during experimental structure modeling will lead to a more realistic model as it resembles the actual joint mechanism and stiffness. Due to these facts DR and DF assemblies are most close to the validation. Furthermore the DR assembly, where the hub is rigid, estimates the lower frequency region very accurately but the overall response amplitude at higher frequencies is overestimated. It shows nine resonances up to 185 [Hz], from which some are in very good accordance with the validation but it misses a resonance at 60 [Hz]. BF assembly produces reliable results but prediction at the lower region is not as good as DR. However the average amplitude at the higher region improves and unlike the DR it misses no peaks. In the lower frequency range up to 185 [Hz], for DR 9 modes and for DF 10 modes have been identified. See Figs. 15.10 and 15.11. Not considering the missed mode by DF, 3rd mode, all modes exhibit approximately the same shape. See Table 15.4.⁴

At first glance the results both for DR and DF don't perfectly match the validation. The difference between the predictions and validation might be caused by the joint damping which is not included properly in the decoupled blade model. The damping mechanism and its contribution to each mode can be a function of adjacent structure mass. Since the used substitute mass is not comparable to the hub mass by redesigning the substitute and make it heavier, joint damping might be better included in the blade model. In spite of this in principle the assembly results are reasonable and show that FBS in general and LM FBS in specific have the potential to couple and decouple experimental models. In Fig. 15.12 typical out of plane interface driving points are shown for both assemblies. Figure 15.13 shows cross FRF's from one hub joint to another hub joint.

⁴In [12] all extracted modes are illustrated.

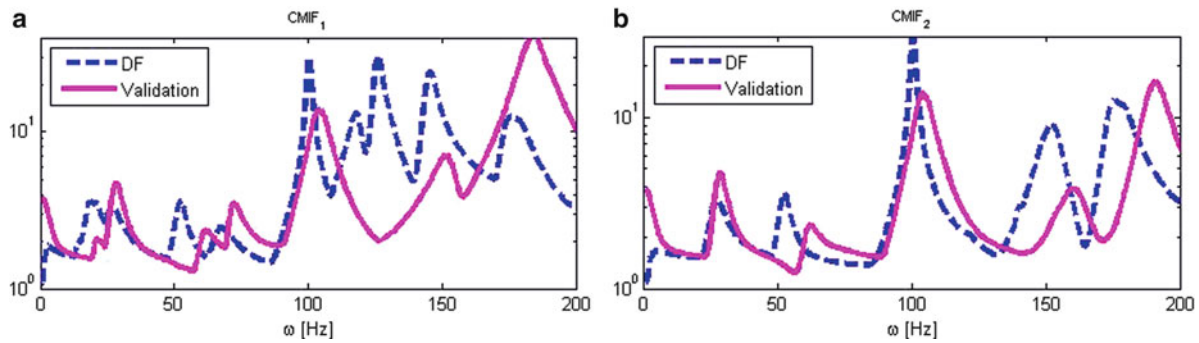


Fig. 15.11 1st and 2nd DF CMIF plots

Table 15.4 DR and DF extracted mode shapes

#	Mode shape	DR	DF
1 st	1 st flap mode, all blades in phase	18.8 [Hz]	20.9 [Hz]
2 nd	1 st flap mode, two blades in phase and one out of phase	25.4 [Hz]	29.0 [Hz]
3 rd	2 nd flap mode, two blades in phase and one out of phase	52.3 [Hz]	n/a
4 th	2 nd flap mode, for DR two blades in phase and one out of phase and for DF all blades in phase	65.5 [Hz]	74.3 [Hz]
5 th	3 rd flap mode, traveling waves for DR (blades bend after each other cyclically) and for DF two blades bend out of phase and the third blade stands still	98.3 [Hz]	106.4 [Hz]
6 th	3 rd flap mode, all blades in phase	117.0 [Hz]	118.0 [Hz]
7 th	3 rd flap mode, two blades in phase and one out of phase	125.0 [Hz]	123.8 [Hz]
8 th	1 st flap/torsion mode, all blades bend and twist in phase	144.3 [Hz]	159.4 [Hz]
9 th	1 st flap/torsion mode, for DR all blades bend and twist in phase and for DF one blade twists and two blades bend	151.6 [Hz]	167.6 [Hz]
10 th	1 st torsion mode, for the DF one blade twists out of phase and for DR all blades twist in phase	178.8 [Hz]	182.4 [Hz]

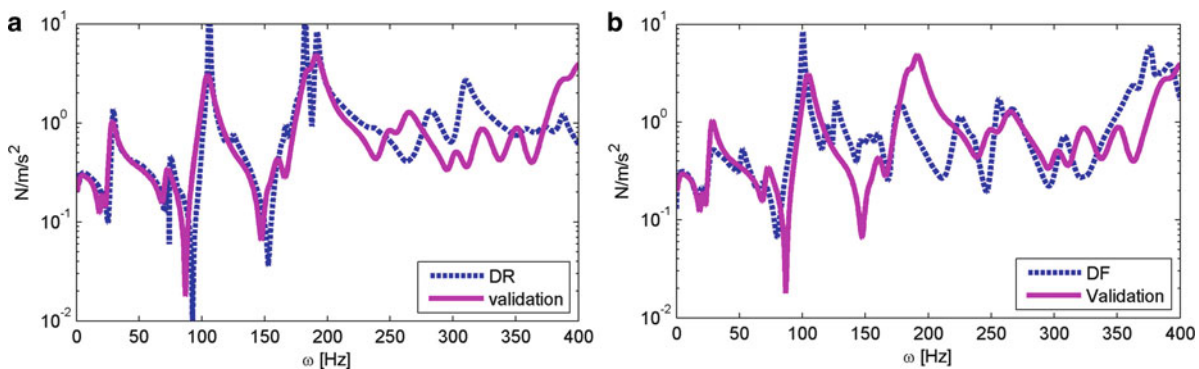


Fig. 15.12 DR and DF driving point FRF's

The rotor assembly of the Ampair 600 serves as a sufficient start point to examine different techniques but still some questions are left unanswered:

- would the blade model change if a much heavier fixture was used?
- were the results still the same if all three blades were modeled one by one and used in the assembly?
- could the rigid hub model augmented with elastic modes give better results?

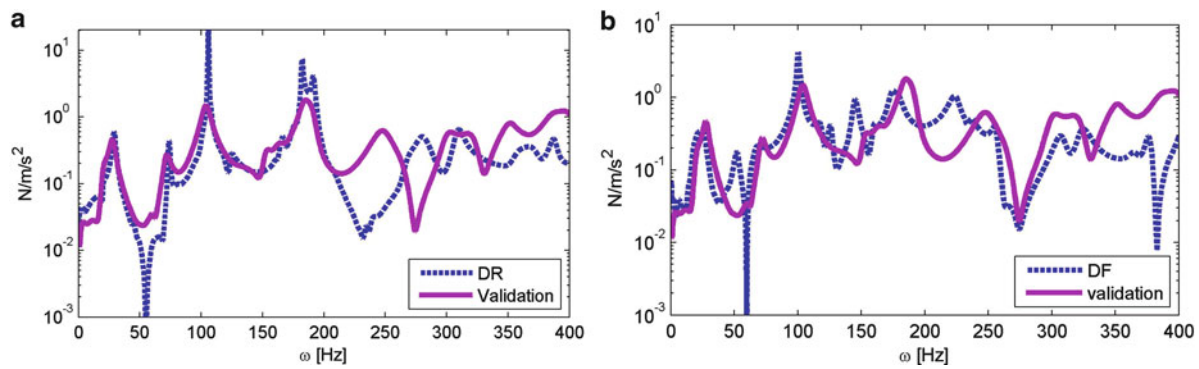


Fig. 15.13 DR and DF cross point FRF's

15.5 Conclusions

To Advance and categorize the experimental DS technology and theory and compare the existing methods the Ampair 600 test bed is initiated by substructuring subgroup at Society of Experimental Mechanics. The topics treated in this work are related to experimental DS, applied to the Ampair 600 rotor assembly. It is shown that:

- the joint stiffness and damping are not directly measurable and interface substitutes should be used. The substitute doesn't just mass loads the interface but also rigidifies the connection area which results in different modal characteristic
- due to substitute mass loading more modes are identifiable in the frequency range of interest
- the typical issues pertaining to experimental DS such as RDoF and noise can be handled by means of IDM
- all blade models except for the decoupled blade give inaccurate and poor results both in low and high frequency regions
- by assuming the hub to be rigid the lower frequency region is estimated very accurate but the higher region prediction is less accurate
- a flexible, experimentally measured, hub model produces reasonable results and at the high frequency region the prediction improves. However the rigid hub gives the best results in the low frequency region

Two methods are proposed to model the Cyclic Symmetric Structures (CSS). The first method is namely developed to synthesis the CSS full model from the minimum number of measured coordinates. It uses the fact that the Inertance (Receptance, Mobility or Accelerance) matrix is circulant and by cyclic permutation of one column the other columns can be obtained. The second method describes, knowing the inertia properties, how the CSS rigid body model can be constructed. It uses the inertance at a given point, with respect to which the inertia properties are given/calculated, and by means of geometrical relations calculates the inertance at any other point. Both methods produce experimental models which are defined in the local coordinates of the interface nodes.

Acknowledgements We would like to thank Sandia National Laboratories and especially Howard W. Arris for helping us podding the hub. Also We thank Ampair Energy Ltd. for providing us the brackets and the blade drawing.

References

1. Allen MS, Mayes RL, Bergman EJ (2010) Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections. *J Sound Vib* 329(23):4891–4906. ISSN 0022-460X. doi:10.1016/j.jsv.2010.06.007
2. Almeida RAB, Urgueira, APV, Maia NMM (2007) Identification of rigid body properties from vibration measurements. *J Sound Vib* 299(4–5): 884–899. ISSN 0022-460X. doi:10.1016/j.jsv.2006.07.043
3. de Klerk D, Rixen DJ, de Jong J (2006) The frequency based substructuring (fbs) method reformulated according to the dual domain decomposition method, IMAC-XXIV: A Conference & Exposition on Structural Dynamics, January 30–February 2, 2006, St. Louis, Missouri USA
4. de Klerk D, Rixen DJ, Voormeeren SN (2008) General framework for dynamic substructuring: history, review and classification of techniques. *AIAA* 46(5):1169–1181. doi:10.2514/1.33274
5. de Klerk D, Voormeeren SN, Petzsche T (2008) Solving the rdof problem in experimental dynamic substructuring, IMAC-XXVI, A Conference & Exposition on Structural Dynamics, February 4–7, 2008, Orlando, Florida USA

6. de Klerk D (2009) Dynamic response characterization of complex systems through operational identification and dynamic substructuring. PhD thesis, Delft University of Technology
7. Ewins DJ (2000) Modal testing: theory, practice, and application. Mechanical engineering research studies: engineering dynamics series. Research Studies Press. ISBN 9780863802188
8. Lee H, Lee Y-B, Park Y-S (1999) Response and excitation points selection for accurate rigid-body inertia properties identification. *Mech Syst Signal Process* 13(4):571–592. ISSN 0888-3270. doi:10.1006/mssp.1998.0190
9. Mayes RL (2012) Assembly instructions for Ampair 600 wind turbine test bed, unpublished manual
10. Mayes RL (2012) An introduction to the sem substructures focus group test bed the ampair 600 wind turbine. In: Mayes R, Rixen D, Griffith DT, De Klerk D, Chauhan S, Voormeeren SN, Allen MS (eds) *Topics in experimental dynamics substructuring and wind turbine dynamics*, vol 2. Conference proceedings of the society for experimental mechanics series, vol 27. Springer, New York, pp 61–70. ISBN 978-1-4614-2421-5. doi:10.1007/978-1-4614-2422-2_7
11. Ottarsson GS (1994) Dynamic modeling and vibration analysis of mistuned bladed disks. PhD thesis, University of Michigan
12. Rahimi S (2012) Experimental dynamic substructuring, coupling and decoupling: application to Ampair 600 wind turbine. Master's thesis, Delft University of Technology <http://repository.tudelft.nl/view/ir/uuid:d8f5dd50-c25b-4448-a278-c062d60254ce/>
13. Rixen D, Lohman R (2005) Efficient computation of eigenmodes of quasi-cyclic structures, IMAC-XXIII: A Conference & Exposition on Structural Dynamics, January 31–February 3, 2005, Orlando, Florida
14. Rixen DJ (2008) How measurement inaccuracies induce spurious peaks in frequency based substructuring, IMAC-XXVI, A Conference & Exposition on Structural Dynamics, February 4–7, 2008, Orlando, Florida USA
15. Rixen DJ, Boogaard MA, van Schothorst G, van der Poel GW (XXXX) Blocked forces and free displacements approaches for vibration transmission analysis. *Mech Syst Signal Process* (submitted)
16. Schedlinski C, Link M (2001) A survey of current inertia parameter identification methods. *Mech Syst Signal Process* 15(1):189–211. ISSN 0888-3270. doi:10.1006/mssp.2000.1345
17. Urgueira APV (1995) On the rigid body properties estimation from modal testing. IMAC-XIII, 13th international modal analysis conference
18. Voormeeren SN, Rixen DJ (2012) A family of substructure decoupling techniques based on a dual assembly approach. *Mech Syst Signal Process* 27(0):379–396. doi:10.1016/j.ymsp.2011.07.028
19. Voormeeren SN, de Klerk D, Rixen DJ (2010) Uncertainty quantification in experimental frequency based substructuring. *Mech Syst Signal Process* 24(1):106–118. doi:10.1016/j.ymsp.2009.01.016

Chapter 16

Coupling of a Bladed Hub to the Tower of the Ampair 600 Wind Turbine Using the Transmission Simulator Method*

Daniel P. Rohe and Randy L. Mayes

Abstract This paper presents an example of the transmission simulator method of experimental dynamic substructuring combining two substructures of the Substructures Focus Group's test bed, the Ampair 600 Wind Turbine. The two substructures of interest are the hub-and-blade assembly and the tower assembly that remains after the hub is removed. The hub-and-blade substructure was developed from elastic modes of a free-free test of the hub and blades, and rigid body modes were constructed from measured mass properties. Elastic and rigid body modes were extracted from experimental data for the tower substructure. A bladeless hub was attached to the tower to serve as the transmission simulator for this substructure. Modes up to the second bending mode of the blades and tower were extracted. Substructuring calculations were then performed using the transmission simulator method, and a model of the full test bed was derived. The combined model was compared to truth data from a test on the full turbine.

Keywords Experiment • MCFS • Substructuring • Transmission simulator • Wind turbine

16.1 Introduction

Dynamic substructuring gives an analyst the ability to predict the response of a built up system based on the responses of its subcomponents. This is particularly useful when the final system is too large or complex to test as one piece, or when design constraints require the redesign of one component while others remain the same, especially if the designing group has little control over the component to which their component attaches. This effort focuses on a wind turbine structure where a subsystem containing the blades is coupled to a subsystem containing the tower and nacelle. One can imagine a scenario in which a turbine tower is erected, but the boundary conditions of the tower are not well known due to uncertainties in soil properties or other factors. It may be easier to experimentally test the tower than to accurately model the interaction between the base of the tower and the ground, then couple the experimental model of the tower to a model containing the blades of the system. This way any problematic modes may be discovered before the system is assembled, allowing the potential for structural modification of either system.

The transmission simulator method of experimental dynamic substructuring is used in this effort. This method offers distinct advantages over other substructuring methods, but it can be more expensive to perform because it requires an additional component—the transmission simulator. However, in this case this expense is avoided by using a component of the turbine system as the transmission simulator, which reduces the cost of the test as the component should be readily available. This allows the transmission simulator method to be comparable in effort to other methods of substructuring.

* Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the U.S. Department of Energy under Contract DE-AC04-94AL85000.

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16.2 Test Objectives and Methods

The objective of these tests is to create a substructured model of the Ampair 600 Wind Turbine assembled from component models of the tower and a blades-and-hub structure. This model will be compared to an experimental ‘truth’ model from a modal test of the entire assembled structure. Of interest in these experiments were the modes up to the second bending modes of the tower and blades. Previous tests containing these structures in similar configurations [1, 2] showed that the highest frequencies of interest would occur near 100 Hz. The test range was set from 0 to 156.25 Hz to allow these modes (and a few higher) to be captured.

For each test, a 0.3 kg (0.7 lb) impact hammer with the softest available tip was used to excite the structures’ elastic modes. The modes were extracted with the SMAC algorithm [3].

The pitch, roll, and yaw rotations as used in this paper are defined as the rotations about the x , z , and y directions, respectively, as shown in Fig. 16.1. The origin of the coordinate system is defined as the tip of the hub, placing the majority of the structure in the $-z$ direction.

16.3 Test Structures

Tests were performed on two different substructures in order to assemble them into the complete Ampair turbine structure. The first structure was the hub, disconnected from the nacelle at the mock generator shaft, with the blades attached. The second substructure was the turbine with the hub locked to the parked shaft and the blades removed. Both structures are shown in Fig. 16.2. Since both structures include the hub, one will need to be subtracted as is common in the transmission simulator method.

16.3.1 Bladeless Tower Test

16.3.1.1 Test Setup

Mode shapes from previous tests revealed rough location for tower nodes, which would need to be avoided when placing accelerometers. The tower was instrumented such that the first and second bending shapes of the tower would be visible and distinguishable from one another. The hub and nacelle were both instrumented such that relative rigid body motions between the two components would be visible, and the tail was instrumented to distinguish flapping and torsion modes. The massive base of the tower was instrumented with four high-sensitivity (500 mV/g) accelerometers because its motion tended to be very small due to its large mass. Accelerometer locations and a line model are shown in Fig. 16.3.



Fig. 16.1 Global coordinate system and rotation definitions

Fig. 16.2 Structures tested to perform substructuring. (a) Bladeless tower (*left*) and (b) hub and blades (*right*)

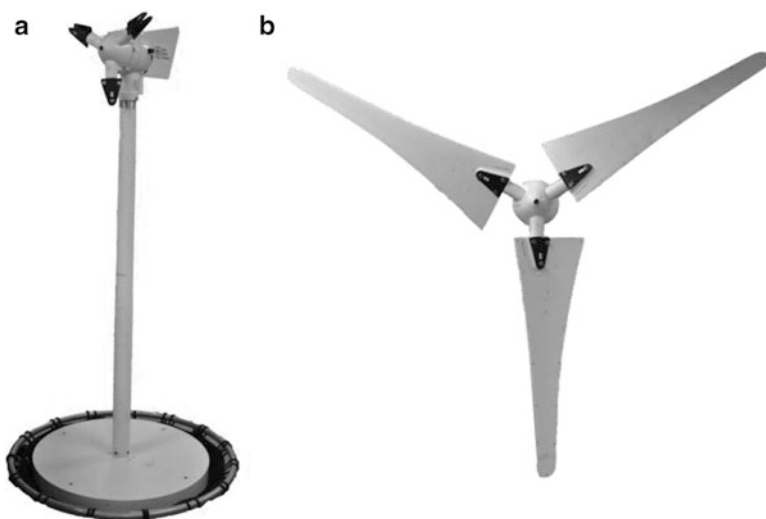
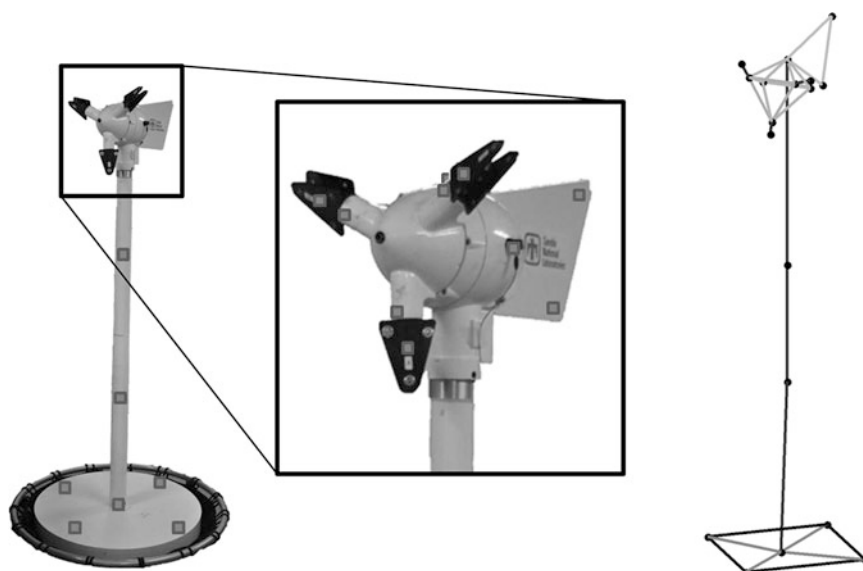


Fig. 16.3 Instrumentation locations for the tower structure



The structure was excited at a large number of drive points in order to ensure that all of the modes were excited adequately. The tower was impacted to excite pole bending modes. Several drive points were chosen on the transmission simulator, both on the arms (white) and on the clamps (black). The lower tail point was also excited.

The tower structure rests upon a commercial trampoline. Previous work [1] has shown that the lowest elastic modes near 20 Hz are only about 5 times higher in frequency than the highest rigid body modes near 4 Hz, whereas the desired ratio is typically 10 times or more. Therefore, considering this system to be a ‘free-free’ approximation may not be an adequate assumption. Also, precise mass properties for this structure, from which properly-scaled rigid body modes could be reconstructed, were not known. However, in many studies it has been seen that properly-scaled rigid body modes play an integral part in substructuring calculations [4, 5]. For these reasons, the rigid body modes of the structure were measured experimentally in a separate test.

The rigid body modes were excited via a stepped-sine shaker input to the base and tower of the structure (Fig. 16.4). From these inputs, all six rigid body modes could be excited. Since the frequencies were on the order of just a few Hz, the test was operating on the lower end of the accelerometers’ sensitivity. This led to mode shapes which were slightly noisy. To average out these small errors, rigid body modes were constructed from the three principal rotations and three principal translations and then fit to the experimental data in a least-squares fashion. This effort gave smooth, scaled rigid body shapes that matched the frequencies from [1] well. These modes were appended to the elastic modes from the impact test to derive a full modal model of the tower.

Fig. 16.4 Drive points to excite rigid body modes: tower for pitching and rolling modes (*left*) and base for yawing (*top right*) and y-direction bounce (*bottom right*)



Table 16.1 List of extracted modes from the tower test

Mode	Frequency (Hz)	Damping ratio (%)	Description
1	0.89	1.62	z-direction translation
2	0.95	3.64	x-direction translation
3	2.83	8.24	y-direction translation
4	3.04	2.90	Yaw mode
5	3.25	6.57	Pitch mode
6	3.93	6.69	Roll mode
7	20.53	1.01	1st Bending in yz-plane
8	20.55	0.95	1st Bending in xy-plane
9	46.84	0.25	Pole torsion mode
10	73.89	0.70	2nd Bending in yz-plane
11	84.59	0.70	2nd Bending in xy-plane
12	96.75	0.30	Tail flapping mode
13	148.87	1.50	Generator shaft torsion

16.3.1.2 Results

Six rigid body modes and seven elastic modes were extracted from the data in the test range. The natural frequencies and damping ratios for each mode are shown in Table 16.1.

The analytical model was a good fit to the experimental data. This can be attributed to the simplicity of the turbine tower system, which resembles a beam with large masses on both ends. Good data was generally taken from impacting the pole and transmission simulator. Data from tail impacts tended to not look as good. Since the tail is quite soft compared to the rest of the system, an adequate force level on the tail would result in large deformations, which resulted in slightly nonlinear behavior. Additionally, this large displacement made it difficult to impact the structure without double hits. Complex mode identifier functions (CMIFs) from pole and tail impacts are shown in Fig. 16.5.

Fig. 16.5 CMIFs from a pole reference (*top*) and the tail reference (*bottom*). Experimental data is shown in *black*, and the analytical fit is shown in *gray*

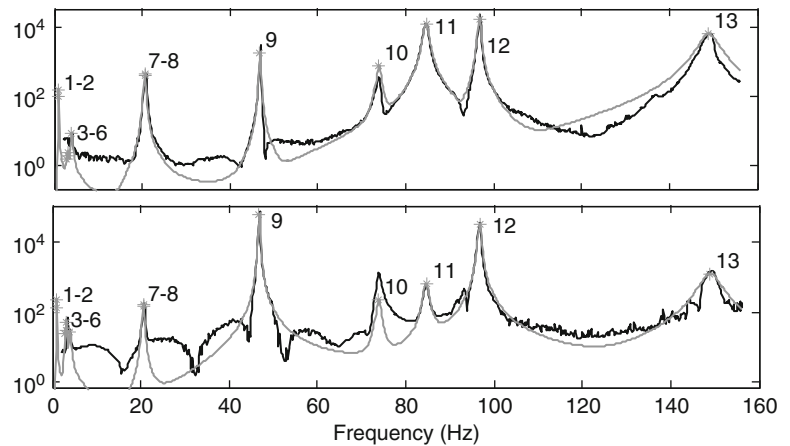
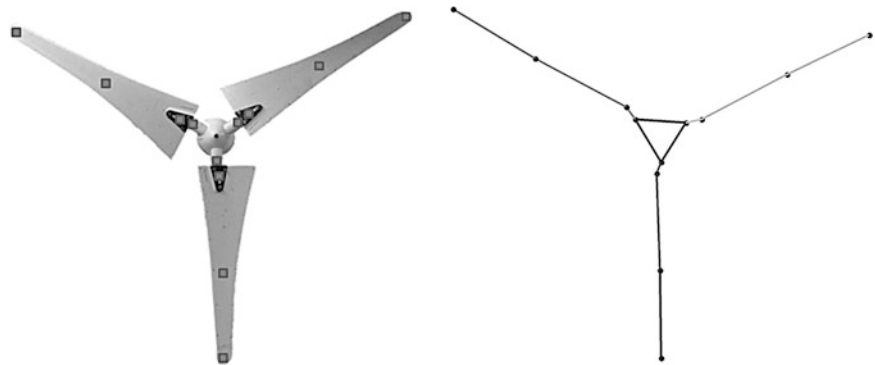


Fig. 16.6 Instrumentation locations for the hub and blades



16.3.2 Hub and Blades Test

16.3.2.1 Test Setup

For the hub and blade structure, the transmission simulator was instrumented identically as for the tower structure test. Each blade was instrumented with two triaxial accelerometers to identify first and second bending and edgewise modes. Torsion modes would not be easily identifiable in this configuration; however, recent tests on the blades [2] suggest that these modes will be out of the frequency range of interest. The accelerometer locations and a line model are shown in Fig. 16.6.

The structure was excited at drive points on the transmission simulator and near the midpoint of the blade. The blade tips did not work well as a drive point as the large deflection of the blade tip made it difficult not to double-hit the structure.

Because the hub was supported on soft springs, the rigid body modes were too low in frequency to measure via modal test with the current setup. The rigid body modes of the system needed to be constructed from mass properties. Mass properties from the hub were measured experimentally on a Space Electronics mass properties machine, and mass properties for the three blades were estimated from the CAD geometry presented in [6]. Three blades were arranged as they would be in the assembly, and mass properties were calculated by the Solidworks modeling software shown in Fig. 16.7. It was assumed that the blade's base (darker portion of the blade in Fig. 16.7) and the blade's span (lighter portion of the blade in Fig. 16.7) had unique but uniform densities, and each density was tuned so that the total mass and center of gravity along the span were consistent with a simple balancing experiment. The calculated densities of each portion of the blade are given in Table 16.2.

The mass properties of the blades and the hub were combined using the parallel axis theorem. Rigid body modes shapes were created from the three principal translations and rotations, and scaled using the calculated mass properties.

16.3.2.2 Results

The six analytically generated rigid body modes were combined with nine elastic modes extracted from the impact tests. These modes are enumerated in Table 16.3.

Fig. 16.7 Blade solid models based on measured blade geometry

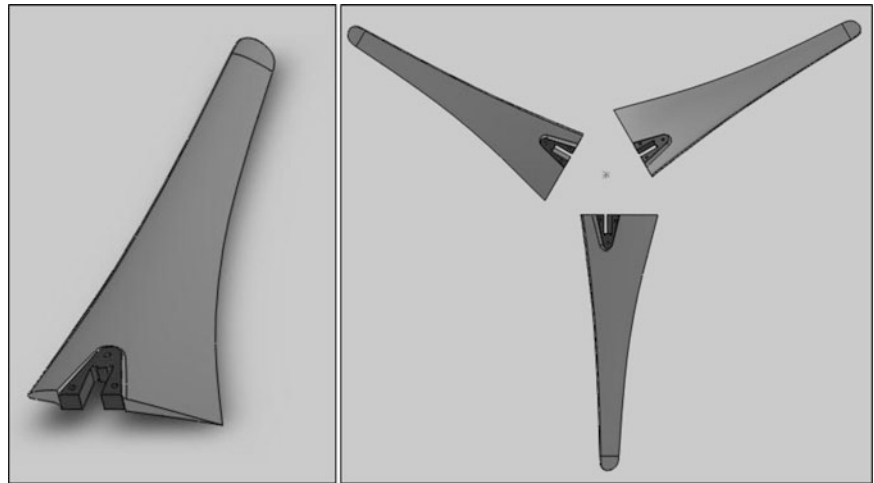


Table 16.2 Blade approximate densities

Blade span	940.5 kg/m ³
Blade base	2664.5 kg/m ³

Table 16.3 List of extracted modes from blade and hub test

Mode	Frequency (Hz)	Damping ratio (%)	Description
1	0.1 ^a	0	x-direction translation
2	0.1 ^a	0	y-direction translation
3	0.1 ^a	0	z-direction translation
4	0.1 ^a	0	Rigid body yaw
5	0.1 ^a	0	Rigid body pitch
6	0.1 ^a	0	Rigid body roll
7	20.36	2.17	1st Bending, 3 blades in phase
8	27.27	1.75	1st Bending, 1 blade out of phase
9	28.97	1.84	1st Bending, 1 blade out of phase
10	53.19	3.27	2 Blade edgewise mode
11	62.14	1.81	2 Blade edgewise mode
12	68.38	1.76	2nd Bending, 3 blades in phase
13	95.31	2.14	2nd Bending, 1 blade out of phase
14	100.05	1.50	2nd Bending, 1 blade out of phase
15	153.09	2.49	3 Blade edgewise mode

^aNote that the rigid body modes were given small positive frequencies to ensure that no negative mass or stiffness developed from the subtraction

Due to the nature of the connections of this substructure, it behaved less linearly than the tower structure tested previously. The blades are attached to the hub via features that clamp the blade when tightened. These joints, relying primarily on friction, will slip if a large enough excitation is applied. Similar to the tower, when the structure was excited on the stiffer transmission simulator, the smaller response gave more linear results. Exciting at the blades' midspan gave strong bending responses, but apparently more nonlinear response. The focus here was to fit the strongest resonances below 100 Hz (see Fig. 16.8).

16.4 Substructuring Methodology

The transmission simulator method was used to assemble the systems presented above with the turbine hub as the transmission simulator. The transmission simulator method offers distinct advantages over the conventional method of constraining individual degrees of freedom at the interface. Specifically, if one did not have a transmission simulator loading

Fig. 16.8 CMIFs of the blade and hub structure. References from *top to bottom*: *z*-direction on transmission simulator; *y*-direction on transmission simulator, edgewise modes are excited better; *z*-direction on blade, strong bending excitation, but nonlinear effects in high frequency range. Experimental data is *black* and the analytical fit is *gray*

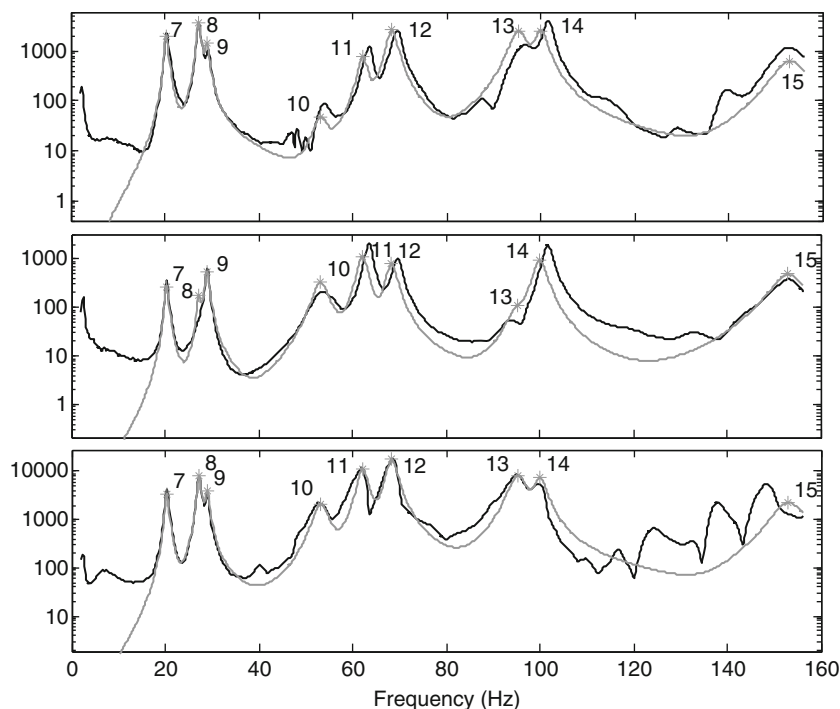


Table 16.4 Mass properties

Property	Hub	Three blades	Entire rotor
Mass	3.6 kg	2.6 kg	6.2 kg
$c_{g,z}$	−0.0615 m	−0.0673 m	−0.0637 m
I_{xx}	0.0150 kg-m ²	0.189 kg-m ²	0.204 kg-m ²
I_{yy}	0.0148 kg-m ²	0.189 kg-m ²	0.204 kg-m ²
I_{zz}	0.0276 kg-m ²	0.3769 kg-m ²	0.405 kg-m ²

Moments of inertia are measured from the center of mass of each object. Since all have rotational symmetry about *z*, all cross products of inertia are zero, and all c_g lie on the *z*-axis

up the free end of the shaft, there would be no tip loads or moments developed on that free end due to the free boundary condition. This, combined with its lack of length, would cause the shaft to behave rigidly in every mode in the band of interest. No joint flexibility or damping would be captured. Additionally, the analyst may have trouble capturing the rotations of the end of the shaft accurately. The transmission simulator method mass-loads the shaft and implicitly captures the rotations of the connection point providing a much better set of basis vectors than free modes without the hub.

For this experiment, the transmission simulator contained 15 degrees of freedom, 5 on each blade anchor. Through a quick tap test, it was found that the first elastic mode of the transmission simulator had a natural frequency near 1200 Hz, which was far above the frequency range of the test. Therefore, the transmission simulator was assumed to be rigid over the frequency band of interest. The transmission simulator model used in substructuring calculations contained only 6 rigid body modes—the three principal translations and rotations—created from the measured mass properties of the hub shown in Table 16.4.

The substructuring calculations then proceed. For the following, the subscript *t* represents the tower-and-hub structure; the subscript *b* represents the blades-and-hub structure; and the subscript *TS* represents the transmission simulator structure (hub). Φ represents the mode shapes of each system, ω represents the natural frequencies, and ζ represents the damping ratios. The $\mathbf{x}_i$ vectors represent physical degrees of freedom on subsystem *i*, where $\mathbf{q}_i$ represent modal degrees of freedom. A subscript *m* denotes the subset of measured physical degrees of freedom that exist on the transmission simulator in each substructure.

Starting with all three systems (tower, blades, and a *negative* transmission simulator) concatenated in the typical mass-normalized modal representation of the equations of motion

$$\begin{aligned}
& \begin{bmatrix} \mathbf{I}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_b \\ \mathbf{0} & -\mathbf{I}_{TS} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_t \\ \ddot{\mathbf{q}}_b \\ \ddot{\mathbf{q}}_{TS} \end{Bmatrix} + \begin{bmatrix} \begin{bmatrix} \ddots & 2\zeta_t \omega_t & \ddots \end{bmatrix} & \mathbf{0} \\ \begin{bmatrix} \ddots & 2\zeta_b \omega_b & \ddots \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & -\begin{bmatrix} \ddots & 2\zeta_{TS} \omega_{TS} & \ddots \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_t \\ \ddot{\mathbf{q}}_b \\ \ddot{\mathbf{q}}_{TS} \end{Bmatrix} \\
& + \begin{bmatrix} \begin{bmatrix} \ddots & \omega_t^2 & \ddots \end{bmatrix} & \mathbf{0} \\ \begin{bmatrix} \ddots & \omega_b^2 & \ddots \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & -\begin{bmatrix} \ddots & \omega_{TS}^2 & \ddots \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_t \\ \mathbf{q}_b \\ \mathbf{q}_{TS} \end{Bmatrix} = \begin{Bmatrix} \Phi_t^T \mathbf{F}_t \\ \Phi_b^T \mathbf{F}_b \\ \Phi_{TS}^T \mathbf{F}_{TS} \end{Bmatrix} \quad (16.1)
\end{aligned}$$

the constraints on the system can be assembled. These constraints equate the motion of the transmission simulator in each substructure.

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & -\mathbf{I} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{t,m} \\ \mathbf{x}_{b,m} \\ \mathbf{x}_{TS,m} \end{Bmatrix} = \mathbf{0} \quad (16.2)$$

Casting equation (16.2) into modal coordinates gives

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & -\mathbf{I} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \Phi_{t,m} & \mathbf{0} \\ \Phi_{b,m} & \Phi_{TS,m} \\ \mathbf{0} & \Phi_{TS,m} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_t \\ \mathbf{q}_b \\ \mathbf{q}_{TS} \end{Bmatrix} = \mathbf{0} \quad (16.3)$$

or

$$\begin{bmatrix} \Phi_{t,m} & \mathbf{0} & -\Phi_{TS,m} \\ \mathbf{0} & \Phi_{b,m} & -\Phi_{TS,m} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_t \\ \mathbf{q}_b \\ \mathbf{q}_{TS} \end{Bmatrix} = \mathbf{0} \quad (16.4)$$

The transmission simulator method transforms the physical constraints to softer modal constraints by pre-multiplying the equation by the pseudo-inverse of the transmission simulator mode shapes

$$\begin{bmatrix} \Phi_{TS,m}^+ & \mathbf{0} \\ \mathbf{0} & \Phi_{TS,m}^+ \end{bmatrix} \begin{bmatrix} \Phi_{t,m} & \mathbf{0} & -\Phi_{TS,m} \\ \mathbf{0} & \Phi_{b,m} & -\Phi_{TS,m} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_t \\ \mathbf{q}_b \\ \mathbf{q}_{TS} \end{Bmatrix} = \mathbf{0} \quad (16.5)$$

The two left-most matrices can be collected to form a single matrix that specifies the constraints on the modal degrees of freedom $\mathbf{q}_i$.

$$\mathbf{A} \begin{Bmatrix} \mathbf{q}_t \\ \mathbf{q}_b \\ \mathbf{q}_{TS} \end{Bmatrix} = \mathbf{0} \quad (16.6)$$

The uncoupled set of modal degrees of freedom $\mathbf{q} = \{\mathbf{q}_t \ \mathbf{q}_b \ \mathbf{q}_{TS}\}^T$ can be transformed into a coupled set through some transformation $\{\mathbf{q}\} = [\mathbf{L}] \{\mathbf{q}_u\}$. By substituting this transformation into (16.6) it becomes clear that $\mathbf{L}$ must reside in the null space of $\mathbf{A}$ since $\mathbf{q}_u$ is generally not zero. That is, $\mathbf{L}$ must be orthogonal to $\mathbf{A}$ so that

$$\mathbf{A} \mathbf{L} \mathbf{q}_u = \mathbf{0} \quad (16.7)$$

Substituting $\mathbf{L}\mathbf{q}_u$ for $\mathbf{q}$ in (16.1) and pre-multiplying by $\mathbf{L}^T$ gives the coupled equations of motion for the system, from which eigenvalues, eigenvectors, and damping properties can be found.

$$\begin{aligned} \hat{\mathbf{M}}\ddot{\mathbf{q}}_u + \hat{\mathbf{C}}\dot{\mathbf{q}}_u + \hat{\mathbf{K}}\mathbf{q}_u &= \mathbf{Q} \\ \hat{\mathbf{M}} &= \mathbf{L}^T \begin{bmatrix} \mathbf{I}_t & \mathbf{0} \\ & \mathbf{I}_b \\ \mathbf{0} & -\mathbf{I}_{TS} \end{bmatrix} \mathbf{L} \\ \hat{\mathbf{C}} &= \mathbf{L}^T \begin{bmatrix} \begin{bmatrix} \ddots & 2\zeta_t\omega_t & \ddots \end{bmatrix} & & \mathbf{0} \\ & \begin{bmatrix} \ddots & 2\zeta_b\omega_b & \ddots \end{bmatrix} & \\ \mathbf{0} & & -\begin{bmatrix} \ddots & 2\zeta_{TS}\omega_{TS} & \ddots \end{bmatrix} \end{bmatrix} \mathbf{L} \\ \hat{\mathbf{K}} &= \mathbf{L}^T \begin{bmatrix} \begin{bmatrix} \ddots & \omega_t^2 & \ddots \end{bmatrix} & & \mathbf{0} \\ & \begin{bmatrix} \ddots & \omega_b^2 & \ddots \end{bmatrix} & \\ \mathbf{0} & & -\begin{bmatrix} \ddots & \omega_{TS}^2 & \ddots \end{bmatrix} \end{bmatrix} \mathbf{L} \\ \mathbf{x} &= \begin{bmatrix} \Phi_t & \mathbf{0} \\ & \Phi_b \\ \mathbf{0} & \Phi_{TS} \end{bmatrix} \mathbf{L}\mathbf{q}_u \end{aligned} \quad (16.8)$$

16.5 Substructuring Results

Coupling the systems together as described in the previous section, a full turbine model is derived. This substructured model is now compared to the results from the truth test on the entire turbine. This truth test used accelerometers in exactly the same locations as on the substructures, allowing a point-by-point comparison of the substructured turbine and the measured turbine.

Since the modes of the turbine were found to be closely spaced in previous work [1], an ad hoc sorting algorithm was implemented that would assign equivalent modes between the truth and substructured models. This would allow a correct comparison of the modes of the systems in case errors caused the modes to appear out-of-order or spurious computational modes developed like in [4]. This sorting algorithm assigns a numerical weight based on the value of the Modal Assurance Criterion (MAC) and the frequency error between the truth mode and substructured mode. While a straight MAC comparison would perform adequately to find similar modes, experience in [4] has shown that sometimes high frequency substructured mode shapes can look the same as correct low frequency modes and in rare circumstances can have a better MAC. By weighting by frequency error as well, this miscorrelation can often times be avoided. This was useful for mode 8 of the truth system in Fig. 16.9. Based purely on MAC, substructured mode 13 would have been chosen, even though there is a substructured mode with a similar (though slightly worse) MAC near that frequency. The weight W_{ij} between the i th truth mode and the j th substructured mode is

$$W_{ij} = \text{MAC}(\Phi_{\text{Truth},i}, \Phi_{\text{Substr},j}) \cdot (1 - F \cdot |f_{\text{Truth},i} - f_{\text{Substr},j}|) \quad (16.9)$$

where F is a weight applied to the frequency error, in this case, $F = 0.025$. The highest weight W_{ij} for the i th truth mode is then said to correspond to that mode. The truth modes and their substructured complements are shown in Table 16.5. The MAC between the two systems is shown in Fig. 16.9.

Fig. 16.9 MAC between truth and substructured modes.
Black = 1, White = 0

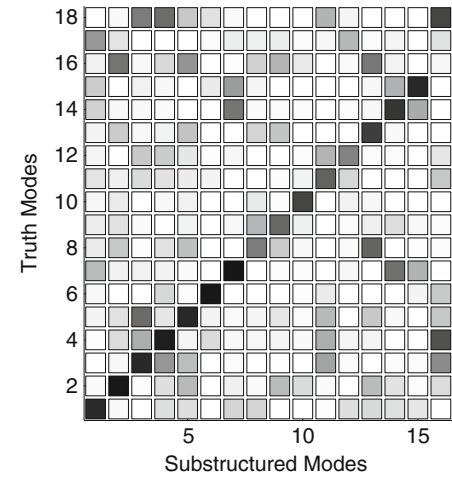


Table 16.5 Substructuring results compared to the truth modes

Truth mode	Frequency (Hz)	Substr match	Frequency (Hz)	Error	Truth damping	Substr damping	Error	MAC
Mode 1	17.24	Mode 1	17.86	3.58%	1.12%	0.99%	−12.08%	0.92
Mode 2	17.70	Mode 2	18.06	2.05%	0.78%	1.12%	44.53%	0.98
Mode 3	18.71	Mode 3	19.10	2.10%	0.81%	1.24%	52.68%	0.93
Mode 4	20.06	Mode 4	19.85	−1.02%	0.87%	1.32%	51.83%	0.97
Mode 5	21.46	Mode 5	21.21	−1.16%	0.92%	1.22%	32.93%	0.93
Mode 6	30.06	Mode 6	29.66	−1.32%	1.79%	0.44%	−75.37%	0.98
Mode 7	37.31	Mode 7	38.15	2.24%	1.00%	0.77%	−22.71%	0.99
Mode 8	48.38	Mode 8	51.27	5.98%	1.70%	2.65%	56.15%	0.62
Mode 9	54.87	Mode 9	56.92	3.74%	3.15%	1.40%	−55.52%	0.70
Mode 10	60.68	Mode 10	61.50	1.34%	1.96%	1.76%	−10.16%	0.83
Mode 11	66.16	Mode 11	66.31	0.21%	1.47%	1.70%	15.72%	0.72
Mode 12	68.60	Mode 12	75.41	9.93%	0.82%	0.77%	−6.58%	0.59
Mode 13	84.44	Mode 13	88.10	4.33%	0.92%	1.54%	66.43%	0.85
Mode 14	95.26	Mode 14	95.50	0.24%	1.14%	1.24%	9.15%	0.88
Mode 15	106.85	Mode 15	104.62	−2.09%	0.86%	0.92%	6.16%	0.93
Mode 16	121.28	—	—	—	1.15%	—	—	—
Mode 17	140.80	—	—	—	1.19%	—	—	—
Mode 18	149.85	Mode 16	152.36	1.68%	1.44%	2.47%	70.95%	0.83

Looking at Table 16.5, several trends are clear. The frequencies of the majority of the substructured modes are close to the truth model, within 5% error. Two notable examples are modes 8 and 12, each of which contributes significant error to the CMIFs in Fig. 16.10. Mode 12 is significantly high and appears well separated from mode 11 in the substructured model, whereas it is very close to mode 11 in the truth model. Overall, the frequencies are accurate out to about 100 Hz. Above this range two high frequency truth modes at 121 Hz and 141 Hz were missed by the substructuring calculations. This can be seen clearly in the MAC of Fig. 16.9 as the rows of the matrix that have low correlation to any of the columns. However, it is expected that the fidelity of the model would fall off in this range.

A comparison of the mode shapes shows good agreement as well. Over the majority of the frequency range, the MAC between the truth model and the substructured model is high (Fig. 16.12). Even when the value of the MAC is low (Fig. 16.11), the mode shapes bear a visual resemblance. Errors in the mode shapes yield scaling errors in the CMIFs of Fig. 16.10, which are more pronounced in the region between 40 and 80 Hz, where the MACs tend to be low.

The substructured damping ratios are on the same order of magnitude as the measured damping ratios, but they are as much as 75% off. The effects of this can be seen in the CMIFs of Fig. 16.13; mode 6 of the substructured model is significantly less damped than the corresponding measured model. There is a significant difference in the height of the two peaks even though the mode shapes are scaled similarly. Overall the damping ratios were worst aspect of the comparison, and were not predicted well at all. The turbine consists of many bolted joints and press fits. Notably, the blades are held in place primarily

Fig. 16.10 CMIFs from an x -direction pole reference (*top*) and a reference on the transmission simulator (*bottom*). The truth model shown in *black* and substructured model shown in *gray*

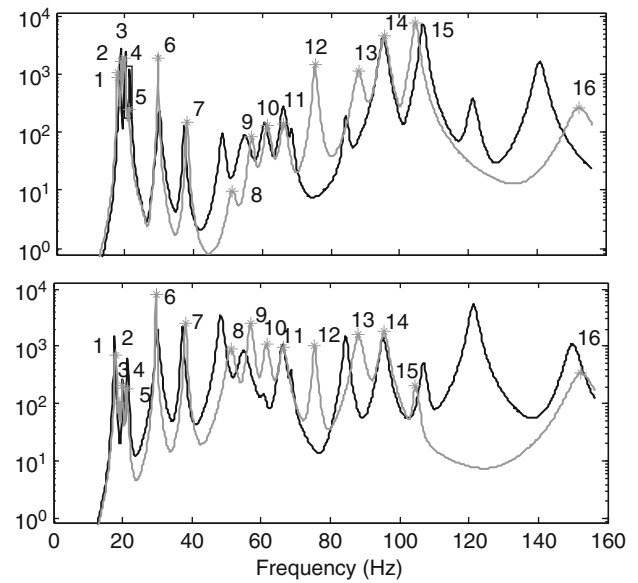
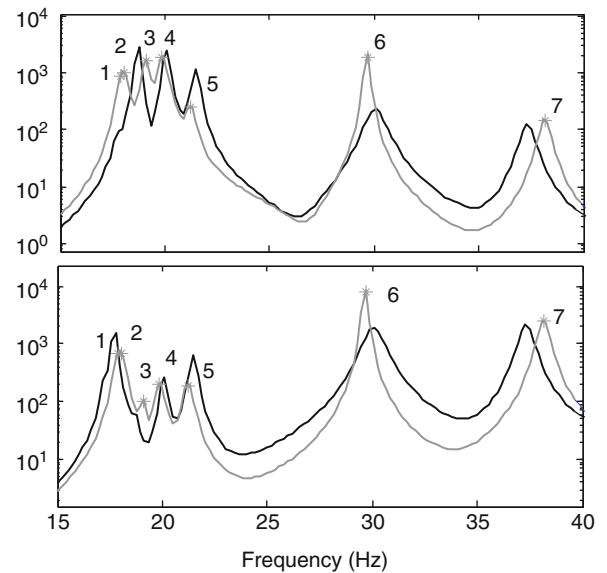


Fig. 16.11 Zoom of Fig. 16.10 showing the tight grouping of modes near 20 Hz. The CMIFs show an x -direction pole reference (*left*) and transmission simulator reference (*right*). Apart from damping errors, this frequency range is very well-behaved



by the friction generated by the two sides of the black clamp. The hub is also pressed onto the mock generator shaft. These joints are likely not modeled well by the viscous damping assumed in these calculations, so it is reasonable that these values would be in error.

16.6 Possible Error Sources

Although the results for most modes are quite good, there is still up to 10% error in the frequencies of the system and MACs as low as 0.6. There is always the potential that these differences are due to nonlinearities of the system. It was seen that for both the blades and the tower substructures that the magnitude of displacement could shift the frequencies of the system (note the peaks in Figs. 16.5 and 16.8). For this reason, modal data taken from ‘soft’ drive points was viewed with suspicion, and often a different drive point would be selected to extract the mode as it was typically more linear. Nonlinearities could certainly cause the assembled system to behave slightly differently than a combination of the two substructures—for an equal force input, the deflections will be slightly different. Several other possible sources of error are considered here.

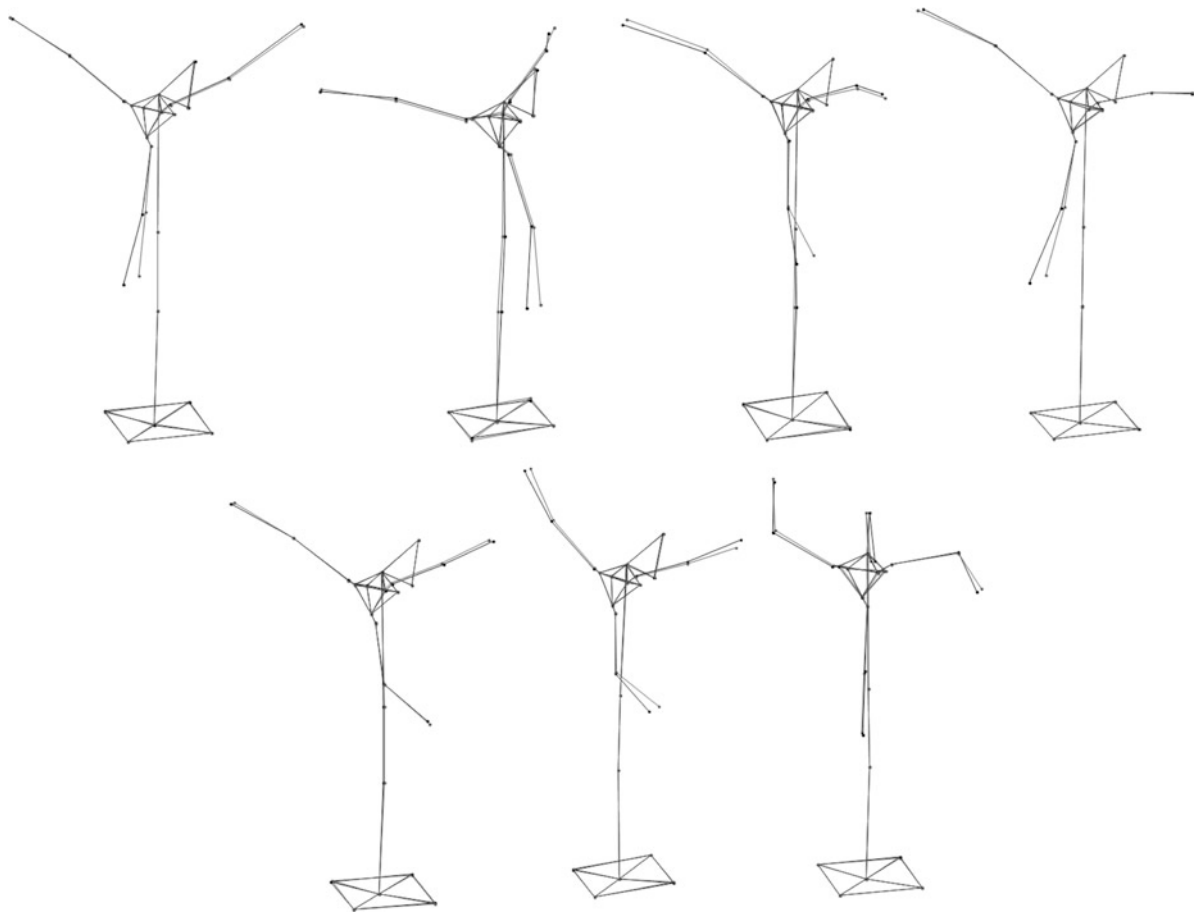


Fig. 16.12 Mode shapes with high MAC between substructured (*light gray*) and truth (*dark gray*) data. From *top left* to *bottom right*, elastic modes 1 through 7

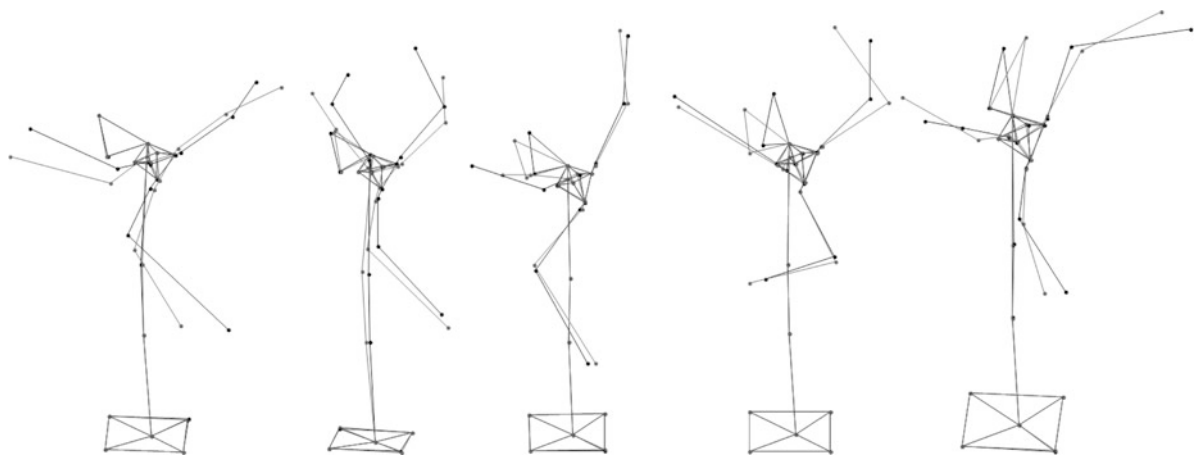
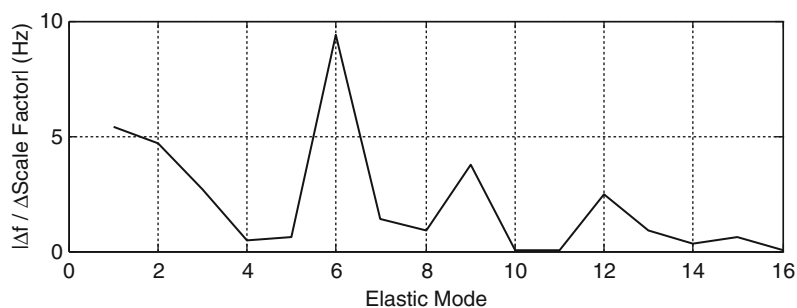


Fig. 16.13 From *left* to *right*, elastic modes 8 through 12. These shapes had the worst correlation to the truth data

Fig. 16.14 Sensitivity in the modes' natural frequencies to scaling the blades' rotational inertia



16.6.1 Accelerometer Mounting

Great care was taken in mounting the accelerometers so that they would be consistent throughout all the different measurements. Accelerometers were left in place between tests to ensure consistent mass-loading and orientation between tests. This should not be a significant source of error.

16.6.2 Mode Shape Scaling

Initially this effort utilized a very rough blade approximation as a triangular plate to estimate mass properties. It was seen that in this case, the first few elastic modes of the substructuring calculation were up to 15% off the truth data, which led to the creation of the more refined model appearing in this paper. To further investigate other modes that may be sensitive to the error, a brief sensitivity study was performed, which consisted of scaling the rotational inertia of the blades, and watching how the natural frequencies of each mode varied. The results are shown in Fig. 16.14. Not surprisingly, the lower modes were more strongly affected by the moment of inertia. This agrees with an example in [7], where the low frequency modes contain large contributions from the rigid body mode shapes of the individual substructures. There are also three distinct peaks at modes 6, 9, and 12. These modes show a high sensitivity to scaling the blades' inertia properties. It would seem that the majority of the sensitive natural frequencies were accurate in this study, which suggests that the mass properties used in the calculation are accurate.

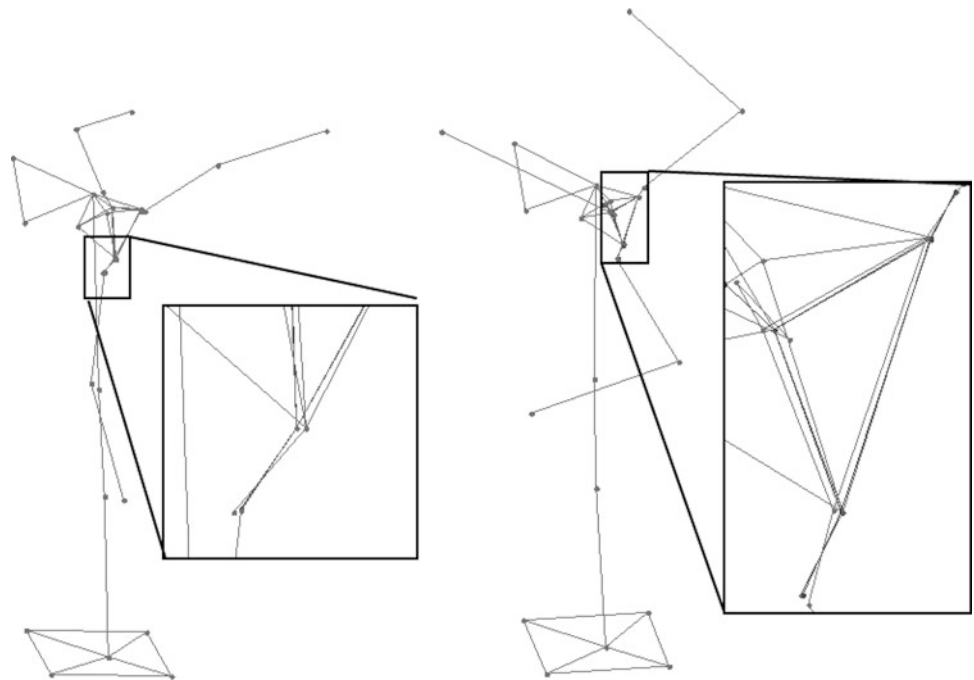
16.6.3 Modal Truncation

The frequency of interest of the test was up to approximately 100 Hz, so the test proceeded up to 156 Hz to try to capture some of the modes above the frequency band to include in substructuring calculations. However, in this extra range only one mode was found in each subsystem that could be reasonably extracted. It is reasonable that the substructured model should not be accurate above that frequency range if each substructure only contains one mode, though this will largely be problem dependent.

16.6.4 Constraint Satisfaction

One additional source for potential error which is unique to the transmission simulator method is the satisfaction of the constraint equations. In the transmission simulator method, these constraints are softened, and satisfied in a least-squares fashion. In this specific example, the transmission simulator was assumed to be rigid in the frequency band of interest, since its first mode is near 1200 Hz. If the constraints are satisfied perfectly, the three transmission simulator models (one negative model and one from each substructure) will be moving in unison. Selected mode shapes of the substructured system are shown in Fig. 16.15. The discrepancies in motion are obviously small compared to the overall motion of each mode shape, so there should not be significant error from the constraints.

Fig. 16.15 Selected mode shapes showing small errors in the transmission simulator constraints. Degrees of freedom are colored according to their original substructure. Elastic modes 6 and 11 are shown



16.7 Conclusions

This effort combined two experimental substructures consisting of components of the Ampair 600 Wind Turbine Test Bed. The blades were removed from the system to form the first substructure, and the second substructure consisted of the blades and the hub. A rigid transmission simulator consisting of only the hub was used to complete the substructuring. Rigid body modes of the tower system were measured experimentally, and the blade system rigid body modes were constructed analytically. The substructured model was compared to a truth model derived from a modal test of the entire turbine.

The rigid body modes and the first seven elastic modes were accurate by model correlation standards with frequency errors less than four percent and MACS above 0.92. Higher modes had frequency errors between 0.2 and 10 percent and MACS from 0.6 to 0.93. This range also demonstrated the most nonlinearity with obvious frequency shifts and damping changes from one impact reference to the next as shown in Fig. 16.8. Damping errors in the predictions were worst, with fifty percent errors common. Future work that should be performed is characterization of the effect of the nonlinear joints on the estimate of viscous damping and mode shape errors. Modal truncation errors are apparent in the very high frequency band, but it is not known whether this is the cause of some of the errors in the mid-frequency band.

References

1. Mayes RL (2012) An introduction to the SEM substructures focus group test bed – the Ampair 600 wind turbine. Proceedings of the 30th international modal analysis conference, Orlando, FL, February
2. Harvey J, Avitable P (2012) Comparison of some wind turbine blade tests in various configurations. Proceedings of the 30th international modal analysis conference, Orlando, FL, February
3. Hensley DP, Mayes RL (2006) Extending SMAC to multiple references. Proceedings of the 24th international modal analysis conference, St. Louis, Missouri, February, pp 220–230
4. Rohe DP, Allen MS (2012) Investigation into the effect of mode shape errors on validation experiments for experimental-analytical substructuring. Proceedings of the ASME 2012 int'l design engineering technical conferences & computers and information in engineering conference, Chicago, IL, August
5. Mayes RL, Rohe DP (2013) Coupling experimental and analytical substructures with a continuous connection using the transmission simulator method. Proceedings of the 31st international modal analysis conference, Garden Grove, CA, February
6. Moshin N, Macknelly D (2012) Modal assessment of wind turbine blade in preparation of experimental substructuring. Proceedings of the 30th international modal analysis conference, Orlando, FL, February
7. Allen MS, Kammer DC, Mayes RL (2010) Uncertainty in experimental/analytical substructuring predictions: a review with illustrative examples. Proceedings of the international conference on noise and vibration engineering, Leuven, Belgium

Chapter 17

Spread in Modal Data Obtained from Wind Turbine Blade Testing

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Abstract This paper presents a pre-study for an on-going research project in experimental dynamic substructuring, initiated by the SEM Substructuring Focus Group. The focus group has selected a small wind turbine, the Ampair 600W, to serve as test bed for the studies. The turbine blades are considered in this study. A total of 12 blades have been tested for modal properties in a free-free configuration. The data has been acquired and analysed by students participating in the undergraduate course “Structural Dynamics – Model Validation” at Chalmers University of Technology. Each blade was tested by different students as part of their required course work to account for spread in modal properties between the blades. A subset of the blades were tested independently multiple times to account for variability in the test setup. Furthermore, correlation analysis of test data was made with Finite Element model eigensolution data of the blade.

Keywords Substructuring • Wind Turbines • Vibration Testing • Modal Analysis • Statistics.

17.1 Introduction

Advances in computer technology have made computer simulations of complicated physical objects possible in industry. More and more companies are relying on relatively cheap Finite Element (FE) modelling rather than expensive prototyping. However, a production sequence will always contain uncertainties, causing a spread between individuals in a produced population. This spread stems from a variety of sources, such as material spread, production process variability and finite precision in the assembly of components. To create models that account for these differences requires data about the spread, in terms of characteristic properties of the population. Modal analysis can be used in order to quantify the spread in dynamical properties, see Friswell and Mottershead [1]. Also, static measurements are of interest to quantify mass and geometrical properties.

In recent years, interest in experimental dynamic substructuring have lead SEM to create a focus group with the intention to increase the knowledge in experimental substructuring and theory. The group have chosen a common testbed, the Ampair 600W wind turbine, for benchmarking, see [2].

This paper deals with the blades of said wind turbine, which have previously been studied by, e.g. Harvie and Avitabile [3] and Nurbhai and Macknelly [4]. Vibration testing, mass and geometric measurements are performed on 12 blades. Emphasis is put on the spread between individual blades and between groups of blades. The lab testers were mainly students attending the course Structural Dynamics Model Validation at Chalmers University of Technology during the spring of 2012. During the course, each student validated and calibrated a rough FE model with their dynamical measurements. In this paper, one of the correlated models is presented.

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17.2 Measurements

Vibration testing was performed on blades hung in a free-free like configuration. A total of 12 blades were tested, of which some were tested multiple times by different individuals. Multiple tests are structured in three rounds. All blades were tested in round 1, while only some were tested in round 2 and 3. The static measurements consisted of characterising the geometry by measuring pitch angles at three cross-sections of the blade and weighing the blades.

17.2.1 Vibrations Test

Each student would prepare their assigned blade individually by first weighing the blade and then set it up for vibration testing. The whole procedure was supervised by teaching assistants. The blades were hung in thin and light-weight cords through two of the three bolt mounts in order to obtain an almost free-free configuration. Light-wight accelerometers of IEPE (Integrated Electronics Piezo Electric) type were used (PCB 352C22, 0.5 gramme) and were attached by synthetic wax at predefined locations, see Fig. 17.1. The predefined accelerometer locations were specified in [3]. Additional mass-loading caused by the measurement equipment was neglected. In total 21 accelerometers were used, 20 at the predefined locations, and 1 behind the shaker. The accelerometer behind the shaker was used to verify that the sensor at location 3 served as a direct accelerance. The shaker was placed just beside location 3, see Fig. 17.2b. The input signal was a stepped sine signal ranging from 30 Hz to 500 Hz with a step size of 0.25 Hz. In order to obtain modal properties, system identification of the obtained data was performed in MATLAB using the System Identification Toolbox. A linear model identification, N4SID, which is based on a state-space subspace method, further explained in [5], was used to estimate a model from the measured data. During the course each student made an individual system identification of their data, where parameters of the identification method differed. In preparing this paper, a new system identification was performed so that the same parameters were used for all blades. Thus, only the measurements are dependent on the lab testers.

17.2.2 Geometry

To characterise the geometry, a fixture was made to ensure that the same points were measured for each blade. Each blade was clamped to the fixture by two bolts. The third hole (towards the tip) could not be clamped because of variability in the hole geometry. The tip of the blade rested on a solid position so that the tip did not move during the measurements and was consistent for all blades, see the point marked with T in Fig. 17.1. One blade was used to define the x and y coordinates for all 20 locations, shown in Fig. 17.1. These locations were assumed consistent for all blades, although slight variation in the

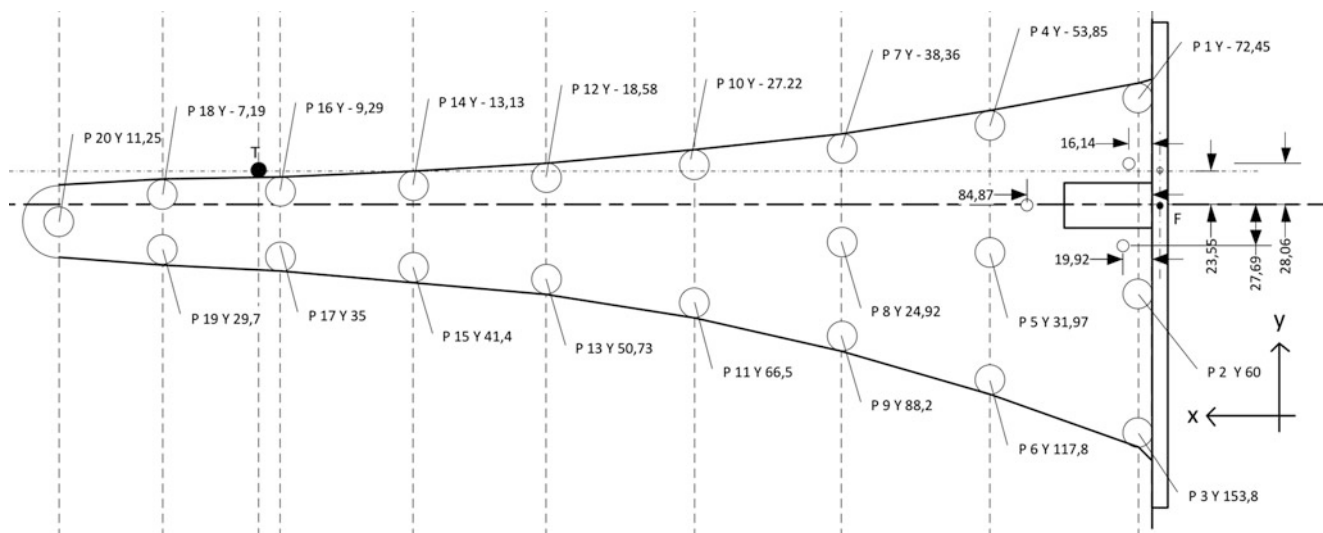


Fig. 17.1 Drawing of one blade with fixture (*rightmost* part) showing the predefined locations and their y coordinates. This was used to measure the relative height difference for seven points. Note that the z direction is perpendicular to the plane. The measurement locations were defined relative to the fixture by the point marked with F. The point marked with T is where the fixture's reference tip rested on the blade leading edge

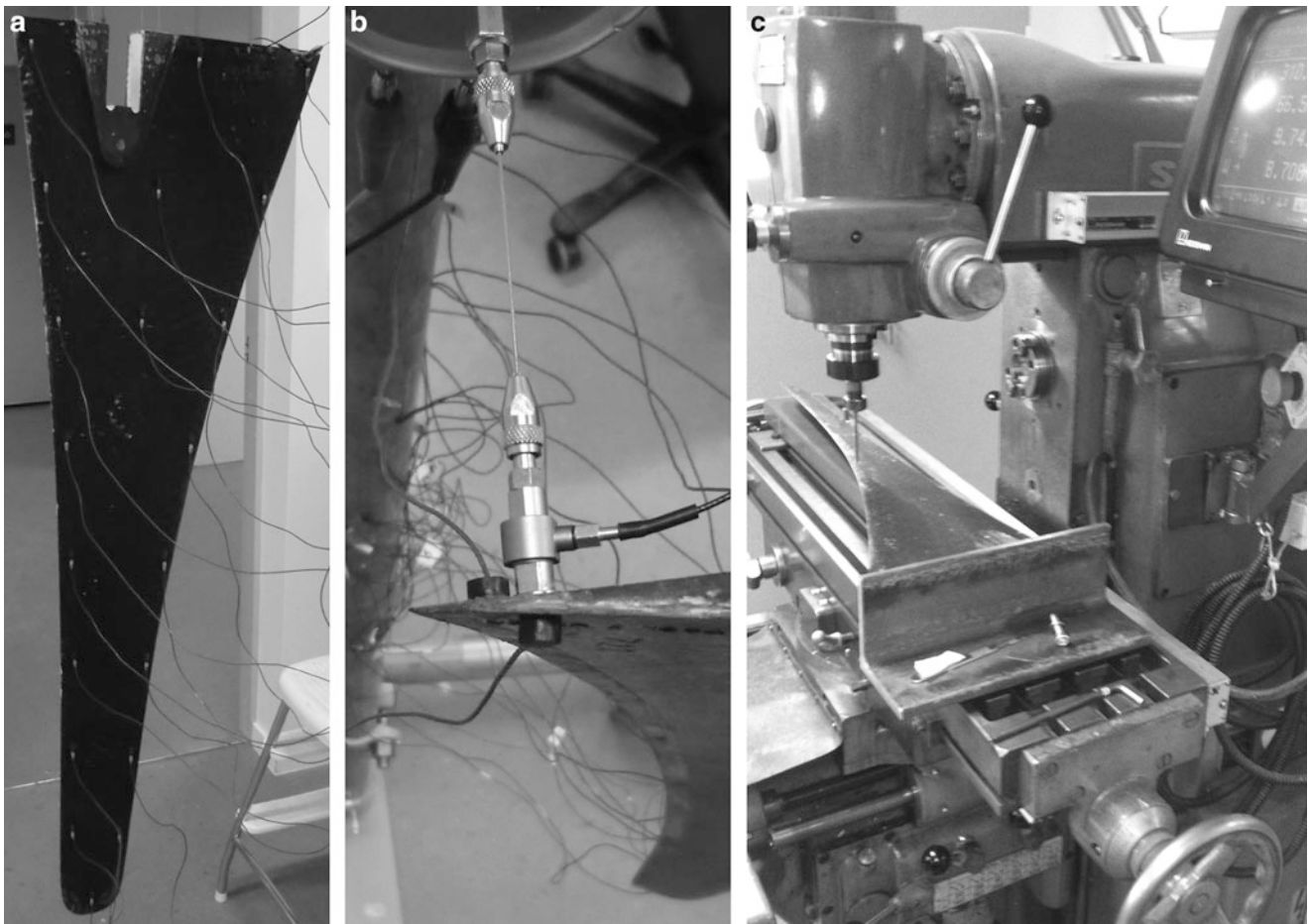


Fig. 17.2 (a) Vibration test setup with a blade mounted. (b) Close-up of direct acceleration sensors. The load cell is situated to the right of accelerometer 3, accelerometer 21 is behind the load cell. (c) Machine used to measure the geometry

location markings was observed during the measurements. Furthermore, a reference point was chosen on the fixture so that for each blade the same x and y coordinates were used to measure the relative z coordinate, see the point marked with F in 17.1. Seven points (P1, P2, P3, P10, P11, P18 and P19) were measured by the machine shown in Fig. 17.2c (which had an accuracy of approximately $1\ \mu\text{m}$). In order to be consistent throughout the measurements, a thin paper was put between the machine tip and the blade. When the tip was lowered on to the blade the paper served as an indicator of when to stop. From the measured points, an angle was calculated by knowing the distance between the two points on the same y coordinate and their relative height difference. One blade (serial number 790) was different from the other blades in that it could only be clamped by one bolt. This separated the blade from the others by giving it slightly different test characteristics, but the blade did not behave significantly different from the other blades and was included in the statistical analysis.

17.3 Measurement Results

The geometry and mass measurements were analysed to find out if there were significant deviations between the blades and whether the difference between the four sets was significant. Further, an investigation of the dynamical properties was conducted for each blade in round 1, focusing on eigenfrequencies and the Modal Assurance Criterion (MAC), see Ewins [6]. A MAC matrix is presented for all blades, shown in Fig. 17.3. Also, the impact of lab testers was investigated by comparing the eigenfrequencies of multiple tests on the same blade. Note that all blades were not tested multiple times; the ones that were are specified in Table 17.3. Blades were ordered in sets of three as each Ampair 600W wind turbine blade set consisted of three blades. Each set was classified by a roman numeral (I, II, III, IV). The blades were classified by their serial numbers. The following serial numbers constituted the four sets: I={828, 852, 790}, II={841, 722, 819}, III={878, 877, 881}, IV={863, 962, 948}.

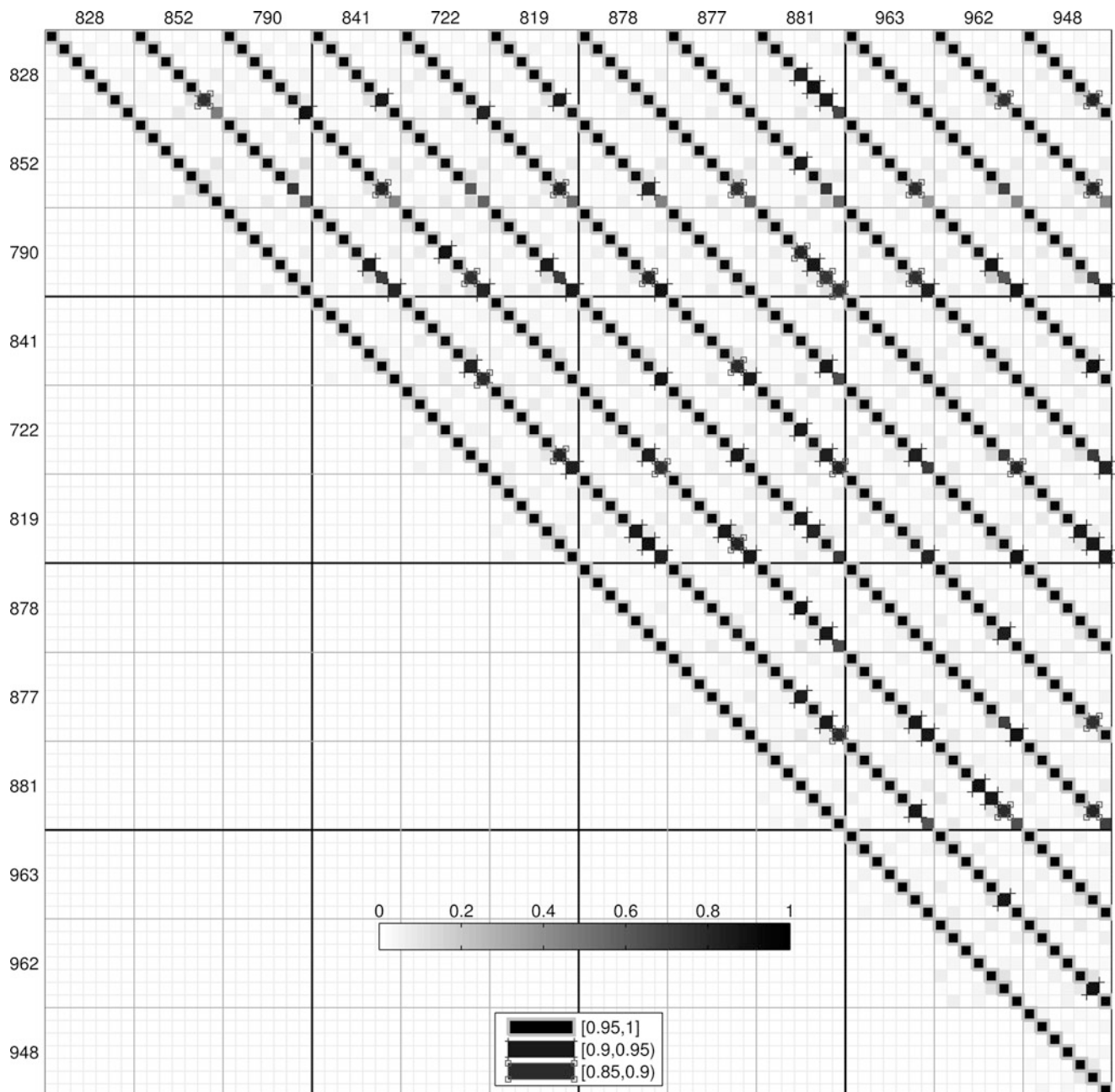


Fig. 17.3 MAC comparison for 7 modes based on tests of 12 individual blades. Blades are identified by their serial numbers. A cell with grey borders marks a correlation over 0.95 while plus signs at the corners marks a correlation over 0.9 but under 0.95. Squares at the corners marks a correlation between 0.85 and 0.9. The presented data is from tests performed in round 1

Note also that the six first modes (rigid body modes) are omitted and thus referring to the first mode implies the first *elastic* mode.

17.3.1 Geometry and Mass

In Table 17.1 the three cross-sectional angles for each blade are presented. Angle 1 in the table represent the angle between points P1 and P3, towards the blade base. Angles 2 and 3 represents angles between points P10, P11 and P18, P19, respectively. The measurements imply that the blades are curved more towards the tip which can also be seen by looking at

Table 17.1 Cross-sectional angles (degrees) and mass (gramme) for all blades, grouped in their sets. Also, mean (μ), standard deviation (σ) and the coefficient of variation (COV, %) are given for the angles and mass of all blades

Set	I			II			III			IV			Total		
Blade	828	852	790	841	722	819	878	877	881	963	962	948	μ	σ	COV
Angle 1	10.09	10.32	10.15	10.06	9.94	10.19	9.99	9.97	9.83	10.02	10.00	9.98	10.04	0.13	1.27
Angle 2	21.83	21.31	21.06	20.56	22.70	23.21	21.28	21.35	21.76	22.20	24.78	22.37	22.03	1.14	5.17
Angle 3	27.79	24.97	27.15	23.62	29.78	32.83	25.51	25.59	27.28	28.43	35.53	27.34	27.98	3.38	12.07
Mass	818.80	816.60	818.20	829.60	829.00	829.50	812.10	812.40	812.70	798.40	798.50	798.40	814.52	11.63	1.43

Table 17.2 Eigenfrequencies (Hz) for all the blades from test round 1 grouped in their sets. Also, mean (μ , Hz), standard deviation (σ , Hz) and the coefficient of variation (COV, %) are given for each mode of all blades

Set	I			II			III			IV			Total		
Blade	828	852	790	841	722	819	878	877	881	963	962	948	μ	σ	COV
Mode 1	45.60	46.48	48.43	45.16	45.18	43.58	46.66	47.61	45.48	46.39	44.31	46.82	45.97	1.37	2.97
Mode 2	129.32	128.86	128.10	128.78	124.61	128.78	128.23	134.11	126.95	129.63	124.46	130.27	128.51	2.54	1.98
Mode 3	191.18	190.76	198.77	189.26	203.54	185.17	191.34	192.34	199.99	191.90	193.57	198.47	193.86	5.25	2.71
Mode 4	248.80	251.42	240.84	248.27	241.02	247.72	245.67	255.43	244.14	251.01	241.21	249.22	247.06	4.61	1.87
Mode 5	323.99	313.92	318.07	321.68	331.27	311.74	316.07	315.15	328.25	324.69	331.45	326.72	321.92	6.85	2.13
Mode 6	399.28	388.73	394.61	392.72	388.09	393.95	391.05	401.59	393.44	401.39	385.84	393.43	393.67	5.04	1.28
Mode 7	467.65	434.33	464.76	461.95	471.34	454.03	469.43	467.00	464.02	478.61	482.29	478.40	466.15	12.76	2.74

the blades. More interesting, though, is that the angles vary more towards the tip of the blades, represented by the standard deviation (σ). This can be expected from the manufacturing process where residual stresses will affect the tip to bend out of shape. The largest difference between two single blades were for blade 841 and 962; they differ around 12 degrees towards the tip.

The mass for each blade, shown in Table 17.1, is seen to be uniform in the groups while it differs more between the groups. This indicates that the blades were grouped in sets of three by weighing the blades. While the mass is easily seen to be grouped, the angles are harder to distinguish. In order to quantify this study, an ANOVA analysis was performed to find out whether the angles were significantly different between groups. The analysis concluded that there was no significant correlation between the angles in the groups. In other words, the blades were not grouped by their geometrical properties.

17.3.2 Modal Data

The dynamical properties studied are eigenfrequencies, presented in Table 17.2, and correlations between modeshapes, quantified by the MAC. The MAC matrix for all blades in round 1 is presented in Fig. 17.3. In Table 17.2 it is seen that the variation between eigenfrequencies increase with higher modes, which can be expected. The COV (coefficient of variation) is almost constant, at between 2 and 3%. The highest standard deviation (σ) is held for the seventh mode. There are no outliers in the eigenfrequencies for data in round 1.

The MAC matrix between all 12 blades in Table 17.2 is shown in Fig. 17.3. In the figure, the four sets are separated by black bold lines. Correlations equal to or exceeding 0.95 are marked with a grey border while correlations between 0.9 and 0.95 are marked with plus signs at the corners. The square markings at the corners represents values between 0.85 and 0.9. There are no anomalies and every off-diagonal value is under 0.1. Note that some correlations are low for the last mode. In particular mode 6 and 7 for blade 852 correlates poorly. The correlation is generally lower towards the higher modes.

Some frequency response functions obtained from the vibration testing, where a stepped sine input signal was used, are shown in Fig. 17.4. Towards the higher frequencies, greater differences were observed, as expected from previous results. Also, it can be seen that almost all modes are clearly separated from each other.

Based on results from the mass measurements regarding differences between blades in and between the sets, an ANOVA analysis was conducted to find out whether the hypothesis holds for the dynamical properties as well. The ANOVA analysis concluded that there was no significant correlation between the blades in the sets. This means that the blades were not grouped in sets of three based on their dynamical properties.

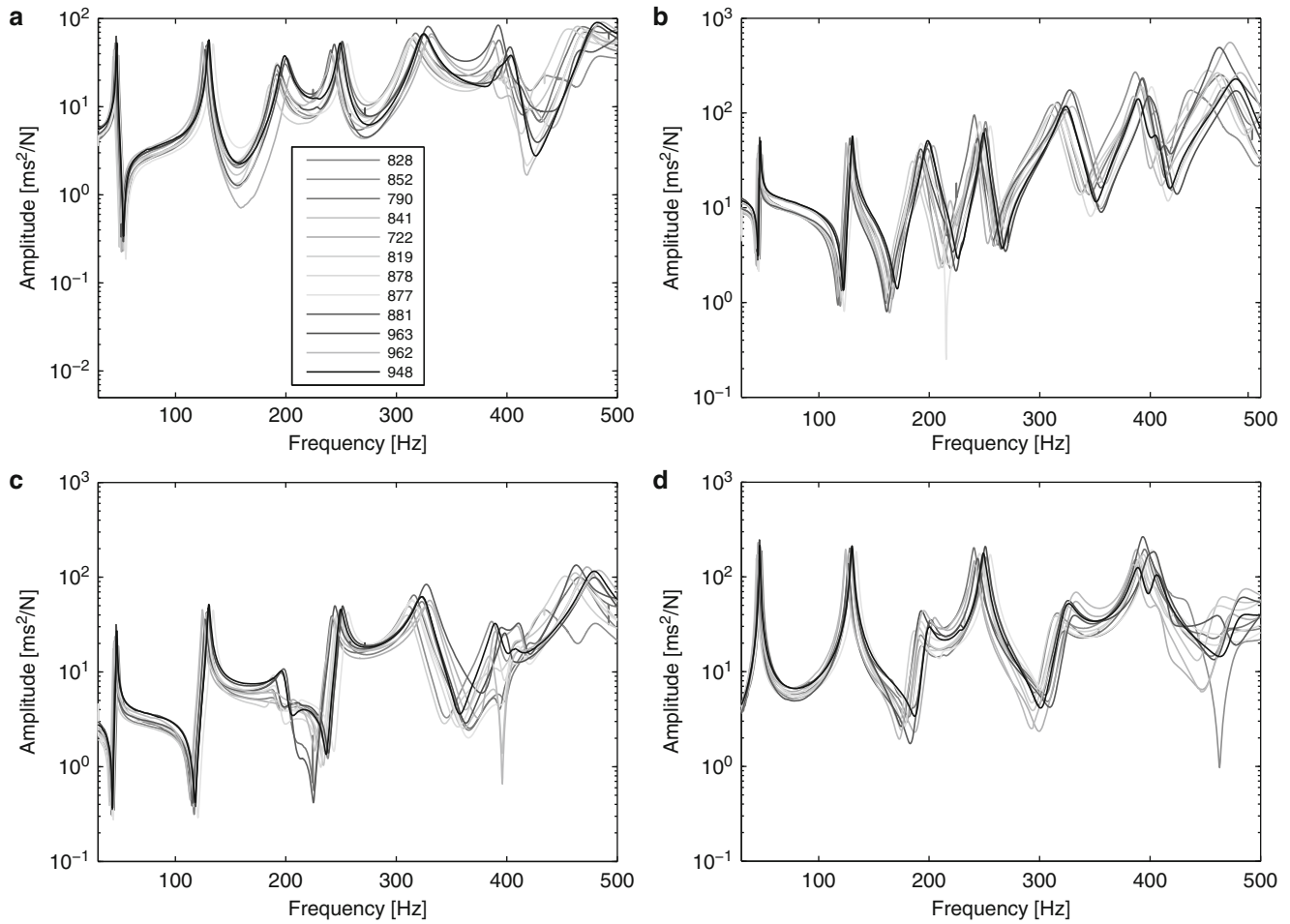


Fig. 17.4 Four frequency response functions from tests in round 1. The excitation is denoted (u_1) and the accelerometer outputs are denoted (y_1), (y_3), (y_7) and (y_{20}) where the number specifies the location on the blade, see Fig. 17.1. Note that the excitation is located just beside sensor 3 on the blade, see Fig. 17.2b for its physical location. Sensor 3 gives the direct acceleration and its frequency response function is shown in subfigure (b)

17.3.3 Multiple Tests

Some blades were tested two and three times by different lab testers in order to investigate the impact lab testers had on the measurements. The results from the tests are presented in Table 17.3 as the mean (μ) and standard deviation (σ) between multiple tests. Also presented in Table 17.3 are eigenfrequency statistics for all performed tests, i.e. for blades tested once and multiple times. Although the standard deviation for the blades tested multiple times is smaller than that of the population at large, it is not insignificant. For blade 790, 828 and 772 the variation in mode 7 is high, compared to the other blades in Table 17.3. The highest standard deviation is found for blade 828, for each mode. Note also that blade 948 has very small and uniform standard deviations for all modes and with a uniform distribution, compared to the other blades.

One should keep in mind that the tests were performed by inexperienced lab testers, mainly by students conducting their first vibration test. Also, depending on how the stinger, seen in Fig. 17.2b, was configured, there was a noticeable difference in test results.

Table 17.3 Eigenfrequency statistics from repeated testes, given on the form $\mu \pm \sigma$ in (Hz). Also, mean (μ , Hz), standard deviation (σ , Hz) and the coefficient of variation (COV, %) are given for all tests performed on every blade

Blade	Tested thrice							
	790	841	963	962	948			
Mode 1	48.35 $\pm$ 0.08	45.25 $\pm$ 0.09	46.36 $\pm$ 0.02	44.26 $\pm$ 0.05	46.83 $\pm$ 0.01			
Mode 2	128.02 $\pm$ 0.18	128.97 $\pm$ 0.23	129.59 $\pm$ 0.04	124.34 $\pm$ 0.13	130.26 $\pm$ 0.01			
Mode 3	198.44 $\pm$ 0.48	190.10 $\pm$ 0.92	191.75 $\pm$ 0.14	192.98 $\pm$ 0.64	198.42 $\pm$ 0.05			
Mode 4	240.89 $\pm$ 0.30	248.64 $\pm$ 0.39	250.78 $\pm$ 0.21	240.96 $\pm$ 0.26	249.26 $\pm$ 0.09			
Mode 5	317.12 $\pm$ 1.18	322.25 $\pm$ 0.85	324.37 $\pm$ 0.31	330.77 $\pm$ 0.72	326.69 $\pm$ 0.03			
Mode 6	394.24 $\pm$ 1.08	392.92 $\pm$ 0.61	401.26 $\pm$ 0.22	385.32 $\pm$ 0.60	393.47 $\pm$ 0.12			
Mode 7	463.24 $\pm$ 3.12	462.01 $\pm$ 1.76	478.18 $\pm$ 0.40	481.40 $\pm$ 1.03	478.32 $\pm$ 0.21			
Blade	Tested twice				Total			
	828	722	877	881	μ	σ	COV	
Mode 1	45.20 $\pm$ 0.55	45.09 $\pm$ 0.12	47.54 $\pm$ 0.10	45.39 $\pm$ 0.13	46.01	1.34	2.92	
Mode 2	126.55 $\pm$ 3.91	124.37 $\pm$ 0.34	133.92 $\pm$ 0.27	126.80 $\pm$ 0.21	128.18	2.73	2.13	
Mode 3	196.40 $\pm$ 7.39	202.91 $\pm$ 0.89	192.52 $\pm$ 0.26	199.71 $\pm$ 0.40	194.83	4.73	2.43	
Mode 4	244.16 $\pm$ 6.56	240.45 $\pm$ 0.80	255.12 $\pm$ 0.45	243.68 $\pm$ 0.66	246.28	4.89	1.98	
Mode 5	325.16 $\pm$ 1.67	330.70 $\pm$ 0.80	314.98 $\pm$ 0.24	328.18 $\pm$ 0.09	323.21	6.03	1.87	
Mode 6	392.82 $\pm$ 9.13	387.60 $\pm$ 0.70	401.22 $\pm$ 0.53	392.90 $\pm$ 0.75	393.25	5.26	1.34	
Mode 7	471.94 $\pm$ 6.06	473.75 $\pm$ 3.41	467.31 $\pm$ 0.44	464.48 $\pm$ 0.65	469.32	10.64	2.27	

17.4 FEM Correlation

The course, in which the work for this paper was conducted, included correlating and calibrating a FE model by MAC values and eigenfrequencies, which will be presented here. Apart from this, the course content consisted roughly of; pretest planing, where emphasis was put on the method of effective independence, see [7]. Vibration testing, where students used the available test equipment, and system identification, where students used the System Identification Toolbox in MATLAB.

The outer geometry of the blade was provided by the SEM Substructuring focus group, from which a FE model was built in FEMAP and analysed using NX NASTRAN. The FE model had to be simplified as the material of the blade was unknown and some standard initial material had to be guessed. The material parameters were chosen from from [8]. Further, simple isotropic shell elements were used to model the skin of the blade and no solid core was used. In order to find improved material parameters, latin hypercube sampling was used to find 10 sample locations. Latin hypercube sampling is further explained in [9]. These 10 samples were then mapped to a beta distribution which was assumed to describe the material properties. Thus, 10 new independent values of Young's modulus and shear modulus were obtained. The 10 new models were then correlated with the measurements and compared to the initial model. This method was used as there was no time to perform a more advanced optimisation. Also, the FE model was simplified so much that more advanced optimisation methods would probably not have yielded significantly better results.

In Fig. 17.5 the best fitted model is correlated with blade 772. It can be seen that quite good correlation is achieved for the first four modeshapes. The two first modes have MAC values over 0.95. The eigenfrequencies also correlated reasonably well for the first four modes as seen in the subfigures of Fig. 17.5.

17.5 Conclusions

The Ampair 600W wind turbine system has been chosen as a benchmark test bed for the SEM Substructuring focus group, with machines already set up in several countries. To ensure a meaningful comparison of these, a quantification of the expected variability of the turbine system's integral components is of great interest. This study, focuses on the variability of the blades.

A series of measurements on 12 blades from the wind turbine have been performed as part of the course, Structural Dynamics – Model Validation, given at Chalmers University of Technology. ANOVA results have been used to confirm that the blades come in weight-matched sets, but also that weight-matching does not imply a similar matching of geometrical or dynamical properties.

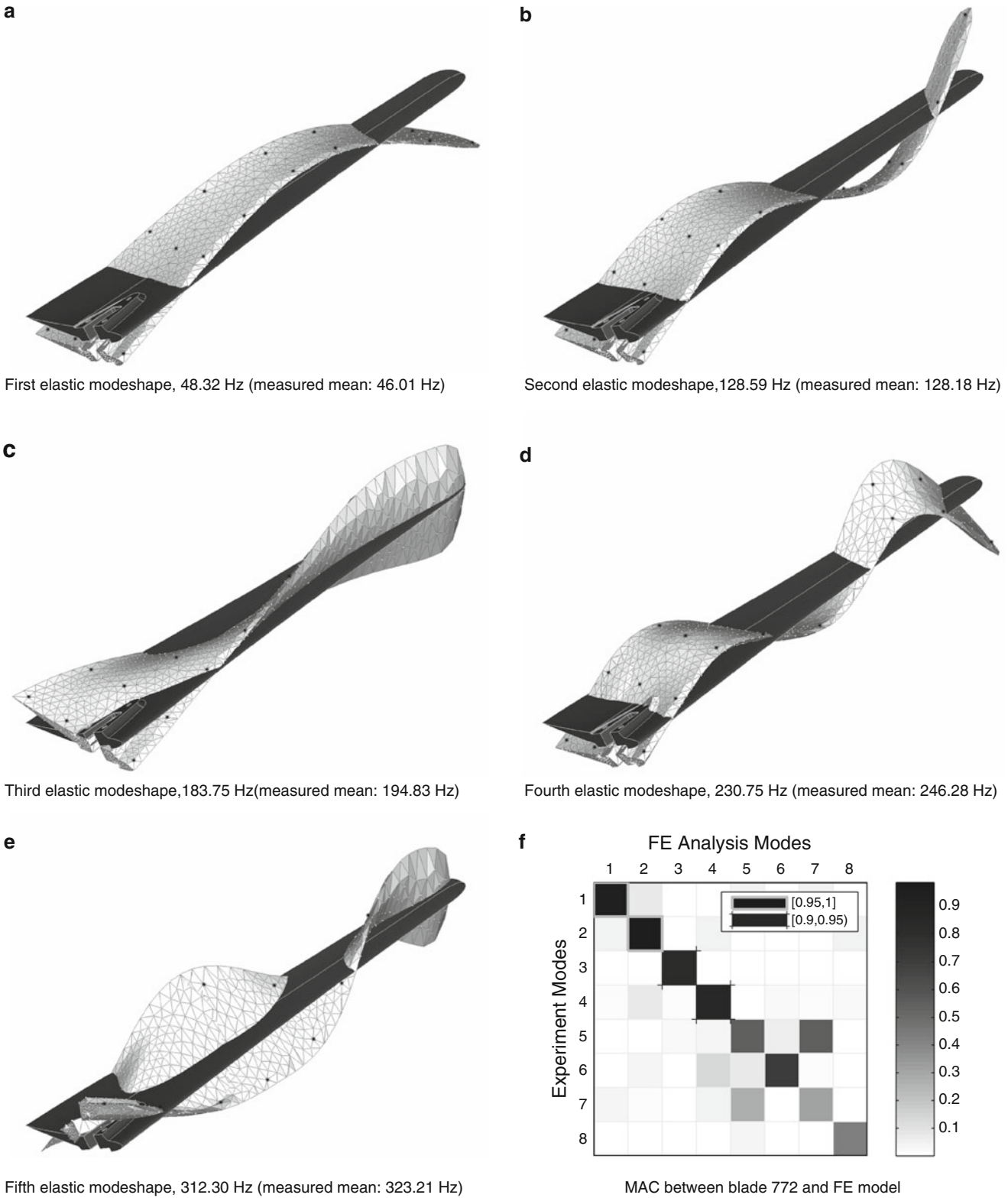


Fig. 17.5 The first five elastic modeshapes for the FE model are shown from (a) to (e). The measured mean is the same as in Table 17.3. The black dots are sensor locations as specified in Fig. 17.1. (f) The MAC matrix

The measured spread between the different blades was considerable. Measured pitch angles at different positions along the length of the blade showed increasing Coefficients of Variation (COV), from 1.27% at the base to 12% at the tip. The COV for the seven eigenfrequencies measured for each blade was stable, at around 3% for all modes.

Different lab testers were responsible for the vibration testing of different blades. To investigate the effect of this on the modal data, a few blades were tested multiple times, by different people. The spread resulting from a change of lab testers varied between the different blades, ranging from almost insignificant to standard deviations comparable to that between blades.

A spurious mode was identified in the measurements. The mode was not consistent with either the MAC correlation nor the eigenfrequencies, not even for the same blade tested multiple times in seemingly well-performed tests. This mode was removed as it was caused by either imperfect drive rod assembly, see [6], or by the system identification process.

Finally, a linear and isotropic FE model with shell elements was built. It was possible to calibrate the model so that the first four modes had a MAC-correlation exceeding 90%. To further improve the FE model, an orthotropic material model must be used and the core material must be modelled. Such a model is presented in [10].

References

1. Friswell MI, Mottershead JE (1995) Finite element model updating in structural dynamics. Kluwer, Dordrecht
2. Mayes R (2012) An introduction to the SEM substructures focus group test bed – the Ampair 600 wind turbine. In: Proceedings of the 30th IMAC, Jacksonville, 2012, pp 61–70
3. Harvie J, Avitabile P (2012) Comparison of some wind turbine blade tests in various configurations. In: Proceedings of the 30th IMAC, Jacksonville, 2012, pp 73–79
4. Nurbhai M, Macknelly D (2012) Modal assessment of wind turbine blade in preparation of experimental substructuring. In: Proceedings of the 30th IMAC, Jacksonville, 2012, pp 71–72
5. McKelvey T, Akcay H, Ljung L (1996) Subspace-based multivariable system identification from frequency response data. IEEE Trans Automat Contr 41(7):960–979
6. Ewins DJ (2000) Modal testing: theory and practice. Research Studies Press, Baldock
7. Kammer DC (2005) Sensor set expansion for modal vibration testing. Mech Syst Signal Process 17(4):700–713
8. Sundström B (1988) Handbok och formelsamling i hållfasthetslära [Handbook and formulas in solid mechanics]. Department of Solid Mechanics, KTH, Stockholm (in Swedish)
9. McKay MD, Beckman RJ, Conover WJ (1979) A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. Technometrics 21(2):239–245
10. Johansson AT, Lindholm C-J, Khorsand M, Abrahamsson T (2013) Modeling and calibration of small-scale wind turbine blade. In: Proceedings of the 31th IMAC, Garden Grove, 2013

Chapter 18

Implementation of Admittance Test Techniques for High-Precision Measurement of Frequency Response Functions

Timothy S. Edwards

Abstract Sandia National Laboratories has developed an experimental procedure for characterizing the input and transfer impedance functions of flight test units. Admittance relations that describe the force-acceleration relations can be derived from this procedure and used for a variety of purposes ranging from environmental response prediction to substructure coupling and numerical model validation. The theoretical developments described herein have been developed using substructure coupling methods. The method allows characterization in six degrees of freedom as well as the removal of fixture effects from the measured frequency response functions. This paper describes the basic theory, presents a numerical validation of the procedure, and illustrates a complete experimental example.

Keywords Admittance modeling • Model validation • Impedance • Substructure coupling

Nomenclature

B	Boolean selection matrix
M	Mass matrix
m	Mass
x	Physical coordinate
η	Modal coordinate
Φ	Orthonormal mode shape matrix
Φ_{ext}	Mode shape at external DOF
Φ_c	Mode shape at connection DOF
$[\bullet]^T$	Matrix transpose
$[\bullet]^H$	Hermetian matrix transpose
H	FRF matrix
$H_{c,c}$	Driving point FRF matrix at connections
$H_{ext,ext}$	Driving point FRF matrix at external DOF
$H_{i,c}$	Transfer FRF matrix, internal response, connection point input
$H_{i,ext}$	Transfer FRF matrix, internal response, external point input
I_{xx}	Rotational moment of inertia
I_{xy}	Rotational product of inertia
R	Rotation matrix
T	Coordinate transformation matrix

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

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18.1 Introduction

Experimental response modeling has been gaining in importance at Sandia National Laboratories. Experimental models are used for a variety of purposes including prediction of responses and finite element model validation. Experimentally derived models can also be coupled with finite element models to reduce model development time and capture dynamics that may be difficult to formulate analytically such as the behavior of bolted joints.

Specialized test methods are required to develop experimental response models due to the difficulty associated with accurate characterization of the dynamics of the attachment points and the rotational degrees of freedom. Traditional modal tests fail to exercise the attachment degrees of freedom of free bodies and therefore the fitted modal models do not accurately represent the attachment dynamics. Rotational degrees of freedom are not exercised in fixed-base modal tests or base-excited vibration tests. Our experience indicates that ignoring rotational degrees of freedom leads to gross inaccuracies in predicted responses because three quarters of the terms in the frequency response function (FRF) matrix are dependent on rotations.

To overcome these difficulties requires special test methods that are capable of exercising the attachment degrees of freedom as well as accounting for rotational degrees of freedom at the input locations. This paper presents a new method for obtaining experimental response models. This method uses special fixtures at the attachment points to excite all the degrees of freedom and implements substructure coupling techniques to remove the effects of these fixtures from the measured FRF matrixes. The theory and numerical codes are verified using simulated test data from a finite element model. An application example that compares the results from two complex systems is also presented.

18.2 Theory

The techniques illustrated in this work use modal transformations to map physical displacements and forces measured on a test fixture to un-measured displacements and forces at a connection point. Substructuring methods are then used to remove the dynamic effects of the test fixture from the FRF data.

In this work the test fixtures are designed to have a first resonance well above the frequency range of interest. This allows assumption that the test fixtures are rigid and simplifies the required instrumentation and subsequent calculations. More recent work by Mayes et al. [1–4] has included consideration of the elastic modes of the test fixture. These works also use a different formulation for removing the dynamic effects of the test fixtures. While including the elastic modes adds a degree of complexity, the theoretical development is a simple extension of that for rigid fixtures and provides a higher degree of fidelity.

For a rigid body, forces and accelerations are related by

$$[M]\{\ddot{x}\} = \{f\} \quad (18.1)$$

Using the transformation into modal coordinates,

$$[\Phi]\{\eta\} = \{x\} \quad (18.2)$$

where $[\Phi]$ are a column matrix of mass-normalized mode shapes,. Using Eq. (18.2) and premultiplying by $[\hat{\Phi}]^T$ yields

$$\{\ddot{\eta}\} = [\Phi]^T \{f\} \quad (18.3)$$

A set of external forces, $\{f_{ext}\}$ excites the test fixtures and produces responses, $\{\ddot{\eta}_{ext}\}$. But, an equivalent set of forces can be applied at another point to produce the same modal responses.

$$\{\ddot{\eta}_{ext}\} = \{\ddot{\eta}_c\} \quad (18.4)$$

Solving for the external motions in terms of the equivalent modal displacements yields

$$\{x_{ext}\} = [\Phi_{ext}][\Phi_c]^{-1}\{x_c\} \quad (18.5)$$

The equivalent modal forces are

$$[\Phi_{\text{ext}}]^T \{f_{\text{ext}}\} = [\Phi_c]^T \{f_c\} \quad (18.6)$$

Solving for the external forces in terms of the equivalent forces at location c, yields

$$\{f_{\text{ext}}\} = [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \{f_c\} \quad (18.7)$$

The driving-point admittance relation that has been measured experimentally is

$$\{x_{\text{ext}}\} = [H_{\text{ext,ext}}] \{f_{\text{ext}}\} \quad (18.8)$$

and the transfer admittance relation is

$$\{x_i\} = [H_{i,\text{ext}}] \{f_{\text{ext}}\} \quad (18.9)$$

Using Eqs. (18.6) and (18.7) we rewrite Eq. (18.8) as

$$[\Phi_{\text{ext}}] [\Phi_c]^{-1} \{x_c\} = [H_{\text{ext,ext}}] [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \{f_c\} \quad (18.10)$$

Rearranging Eq. (18.10) to put in the admittance form yields

$$\{x_c\} = [\Phi_c] [\Phi_{\text{ext}}]^{-1} [H_{\text{ext,ext}}] [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \{f_c\} \quad (18.11)$$

The new driving-point admittance matrix, which was not measured experimentally, but rather inferred from measured data on the test fixture, is given by

$$[H_{c,c}] = [\Phi_c] [\Phi_{\text{ext}}]^{-1} [H_{\text{ext,ext}}] [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \quad (18.12)$$

A similar development leads to the relation of forces at the connection point to responses elsewhere as

$$\{x_i\} = [H_{i,\text{ext}}] [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \{f_c\} \quad (18.13)$$

so that the transfer admittance related to the connection point is given by

$$[H_{i,c}] = [H_{i,\text{ext}}] [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \quad (18.14)$$

In general, not all external degrees of freedom will be observed at each node. Therefore, a Boolean matrix, B premultiplies the mode shape matrix to select the observed degrees of freedom. Substitution of $[B][\Phi_{\text{ext}}] = [\hat{\Phi}_{\text{ext}}]$

$$[H_{c,c}] = [\Phi_c] [\Phi_{\text{ext}}]^{-1} [B]^{-1} [H_{\text{ext,ext}}] [B]^{T-1} [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \quad (18.15)$$

$$[H_{i,c}] = [H_{i,\text{ext}}] [B]^{T-1} [\Phi_{\text{ext}}]^{T-1} [\Phi_c]^T \quad (18.16)$$

The inferred admittance functions, $[H_{c,c}]$, and $[H_{i,c}]$ represent force-acceleration relationships *with the test fixture still attached*. The next step is to analytically remove the effects of the test fixture to yield the admittance matrix of unit under test alone.

Although different techniques exist for removing the effects of the fixture, all fall under the heading of substructure coupling. In this work, we use the technique of Jetmundson et al. [5]. This formulation is particularly well-suited and is generally well-conditioned.

$$\begin{bmatrix} \tilde{H}_{i,i} & \tilde{H}_{i,c} \\ \tilde{H}_{c,i} & \tilde{H}_{c,c} \end{bmatrix} = \begin{bmatrix} H_{i,i} & H_{i,c} \\ H_{c,i} & H_{c,c} \end{bmatrix} - \begin{bmatrix} H_{i,c} \\ H_{c,c} \end{bmatrix} [H_{c,c} - \Delta H_{c,c}]^{-1} \begin{bmatrix} H_{i,c} \\ H_{c,c} \end{bmatrix}^T \quad (18.17)$$

All the driving point and transfer admittance terms for the unit under test, free from the effects of the test fixtures, can be found using Eq. (18.17).

If the test fixtures are assumed rigid, $\{x_{ext}\}$ must be at least 6×1 and each of the six observed DOF must be linearly independent. But there are certain advantages if more than six DOF are observed for each connection point. If $\{x_{ext}\}$ is of dimension larger than six the modal displacements, $\{\eta\}$, can be found in a least-squared-error sense using the pseudoinverse $\{\eta\} = [\Phi_{ext}]^+ \{x_{ext}\}$. This may result in a better estimate of the FRFs in the presence of measurement errors.

18.3 Implementation

These concepts have been implemented on several test bodies with the intent of measuring both driving-point and transfer FRFs. These FRFs can then be used for both model validation, coupled structure analysis and directly in response prediction calculations.

In these tests four admittance test fixtures, such as those shown in Fig. 18.1, are affixed to the attachment locations of the test bodies. Each block is equipped with six accelerometers and is excited in six, linearly independent degrees of freedom.

The accelerometers are precisely located on the test fixtures to minimize the errors associated with uncertainty in their locations. There are nine nodes on each test fixture. Eight of these are coincident with the vertices of the cube and one is located at the attachment location. As given in the Appendix 1, the mode shape for a given node is

$$\Phi_{Node} = \begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (18.18)$$

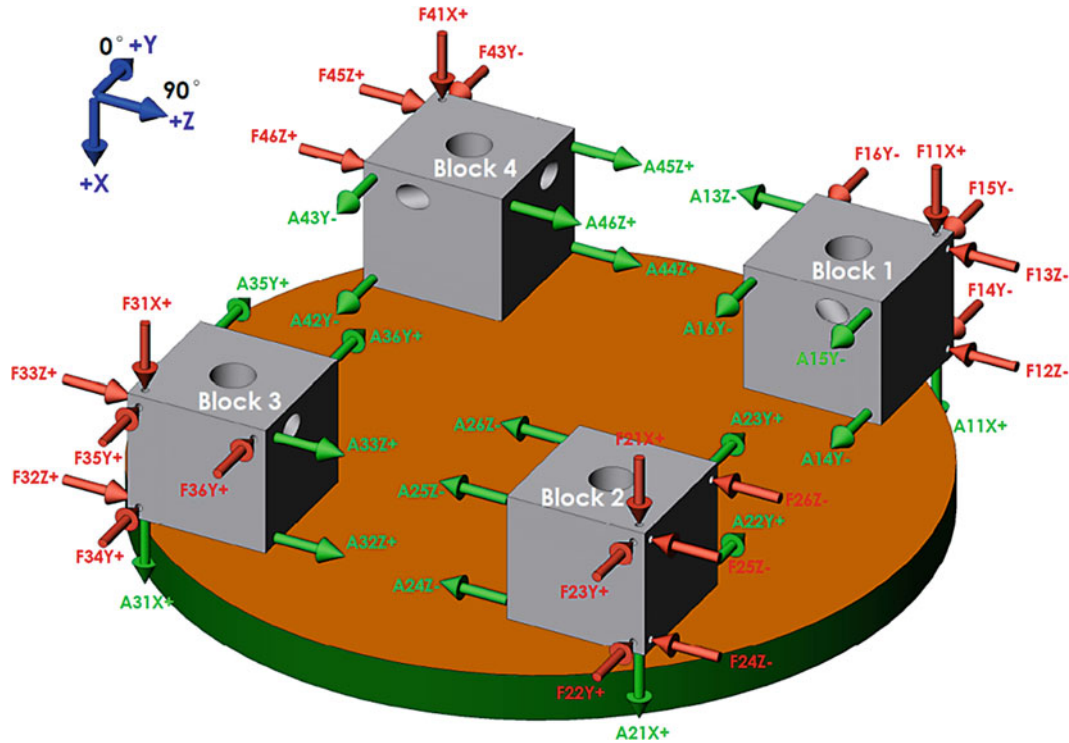


Fig. 18.1 Admittance test fixtures

where x , y , and z are the coordinates of the node with respect to an arbitrary reference location. In this work, the reference location is taken as the attachment node, although the reference location is eliminated in the subsequent mathematical manipulations. The nodes on a given test fixture block are coupled. Therefore, the mode shape matrix for a single block is of dimension 54×6 and takes the form

$$\Phi_{\text{Block}} = \begin{bmatrix} \Phi_{\text{Node 1}} \\ \vdots \\ \Phi_{\text{Node 9}} \end{bmatrix} \quad (18.19)$$

The blocks are not coupled between blocks. Therefore, the 144×24 mode shape matrix of all four admittance test blocks is block diagonal and given by

$$\Phi_{\text{Total}} = \begin{bmatrix} \Phi_{\text{Block 1}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Phi_{\text{Block 4}} \end{bmatrix} \quad (18.20)$$

Each node of the test fixture has six degrees of freedom, but in the configuration under consideration, only a single degree of freedom is observed at a node, and six, linearly independent degrees of freedom are observed on the entire block. The Boolean matrix previously described selects the observed degrees of freedom from the complete mode shape matrix.

Since six, linearly independent degrees of freedom are observed on each block, the Boolean matrix must be of dimension 24×144 . Because some of the transducers depicted in Fig. 18.1 are oriented so they are measuring in the negative sense, the Boolean matrix can be combined with the coordinate transform described in the Appendix 1. The Boolean matrix is not depicted here, but contains a 1 or -1 (depending on the sensing polarity of the transducer) in the (i,j) location where the i 'th row corresponds to the order that the observed DOF appears in the measured FRF matrix $H_{ext,ext}$ and the j 'th column corresponds to the row location of the observed DOF in the mode shape matrix, Φ_{Total} . The Boolean matrix can also be implemented at the lower levels of assembly Φ_{Node} or Φ_{Block} if so desired. For example, the Boolean matrix for a node at which the y degree of freedom is observed would be

$$B_{\text{Node}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (18.21)$$

The mode shapes at the connection degrees of freedom, Φ_c , are of dimension 6×6 and are uncoupled. The full matrix is thus

$$\Phi_c = \begin{bmatrix} \Phi_{\text{Bc 1}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Phi_{\text{c 4}} \end{bmatrix} \quad (18.22)$$

and is of dimension 24×24 . The sub-matrices take the form of Eq. (18.22), but because the mode shape matrices have been written with respect to these locations, Φ_c is simply the identity matrix.

The mass properties of each of the test fixtures is the same, to within a coordinate rotation. For test block 1,

$$M_1 = \begin{bmatrix} m & 0 & 0 & 0 & -mz & my \\ 0 & m & 0 & mz & 0 & -mx \\ 0 & 0 & m & -my & mx & 0 \\ 0 & mz & -my & I_{xx} & I_{xy} & I_{xz} \\ -mz & 0 & mx & I_{yx} & I_{yy} & I_{yz} \\ my & -mx & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

With the rotation matrix defined as

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The inertia matrices of the remaining blocks can be found using the relations

$$\begin{aligned} M_2 &= R^T M_1 R \\ M_3 &= R^T M_2 R \\ M_4 &= R^T M_3 R \end{aligned} \tag{18.23}$$

The full inertia matrix is then assembled from the inertia matrices of the individual test fixtures as

$$M = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & M_4 \end{bmatrix} \tag{18.24}$$

With $M^{-1} = \Delta H_{c,c}$, the FRF matrix *without* the effects of the test fixtures, $\tilde{H}$ can be found using Eq. (18.17).

18.4 Verification

To verify these theoretical formulations and to conduct sensitivity analyses, a finite element simulation was carried out to replicate the experimental process. The finite element simulation offered an opportunity to study this process in the absence of experimental noise that can add significant uncertainty to the process. A simple structure similar in character to the two test bodies described later in this paper was modeled and four admittance test fixtures were attached to its connection points. The driving-point and transfer admittance FRFs were then computed from six input locations on each admittance test block. The responses were computed at the six input locations and at a remote response location.

With six independent inputs on each of four test fixtures, the measured driving point admittance matrix was 24×6 . Only three translations were captured in the response measurement, replicating conventional, tri-axial experimental measurements. The transfer admittance matrix was therefore of dimension 3×24 . The as-measured (before translation or removal of inertia effects) partition of the transfer matrix representing the contributions of a single admittance test fixture to the response location is shown in Fig. 18.3.

Since the matrix $[H_{c,c} - \Delta H_{c,c}]$ must be inverted to remove the effects of the test fixtures, it is worthwhile to plot the condition number of this matrix as a function of frequency to ensure numerical stability in the matrix inversion. Since this data is relatively free from measurement noise, the magnitude of the condition number can serve as a benchmark for the experimental data that follows below (Fig. 18.4).

Several sources of error were studied in this exercise. Errors in rotational inertia terms can be quite large without affecting the accuracy of the estimated FRFs. On the other hand, the mass must be accounted-for quite accurately. Three quarters of the inertia matrix terms contain mass terms while only one fourth of the terms contain terms dependent on rotational inertia making the effects of uncertainty in the mass more significant than those in the inertial terms. While uncertainties in geometric quantities added some error to the final results, these effects were small compared to errors in the mass (Fig. 18.2).

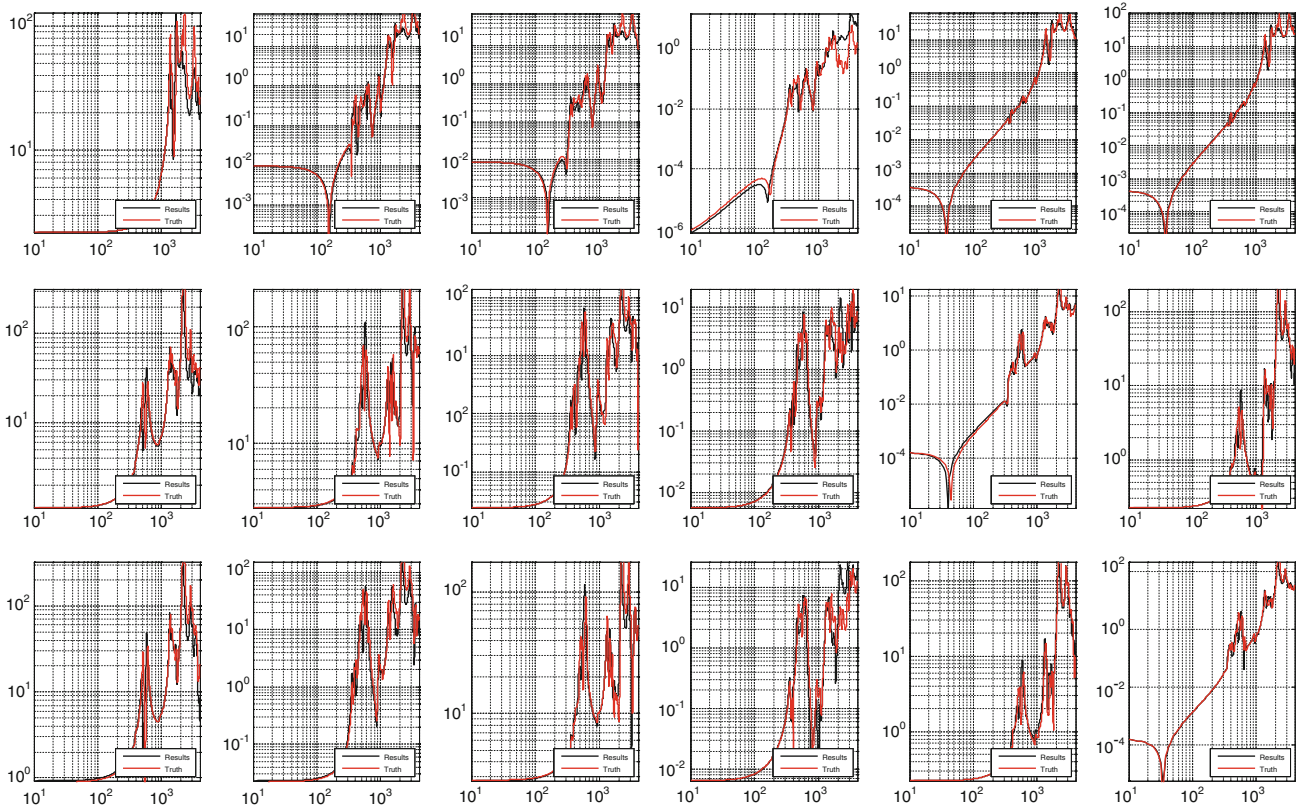


Fig. 18.2 Transfer admittance: truth data compared to derived results: *Black*: derived, *Red*: Truth Data (color figure online)

18.5 Data Cleansing

While the numerical verification discussed above was conducted in the relative absence of measurement noise. As discussed in Ref. [6], experimental frequency response functions can have very small anomalies. These errors are generally not observable, but can drastically affect the FRF's of the combined system. These anomalies can arise due to both measurement noise and subsequent mathematical processing of the FRF's.

One such anomaly is non-positive definiteness of the imaginary part of the driving point FRF matrix. Positive definiteness of the imaginary part is assured when the eigenvalues of the imaginary part of the FRF matrix are positive. Figure 18.5 shows the eigenvalues of the imaginary part of the measured driving point FRF matrix of Body #1. There are spurious negative eigenvalues indicating the presence of measurement noise as discussed above. The procedure developed in [6] was used to correct these errors by setting the negative eigenvalues to zero and reconstituting the FRF matrix using the modified eigenvalues. The corrected eigenvalues are shown in Fig. 18.6. The corrected FRF matrix was then used for all subsequent processing.

A second data correction is usually required to enforce reciprocity. Betti's Theorem requires that $H^H = H$ where $(\bullet)^H$ indicates the Hermitian transpose. Close examination of Fig. 18.7 shows that reciprocity does not hold in the as-measured data. This may be due to measurement errors or due to nonlinearities of the structure. In either case, the modal description used in substructure coupling theory requires that the data follow the principals of linear structural dynamics. Reciprocity can be enforced in the test data by making the FRF matrix conjugate symmetric. Each term was taken as the average of the real and imaginary parts along the corresponding elements using the relation $Average = \frac{1}{2} H H^H$.

An alternate approach for data cleansing involves fitting a modal model to the measured data which will ensure that linear properties such as those discussed above are enforced. Such a method was undertaken by Imregun et al. [7].

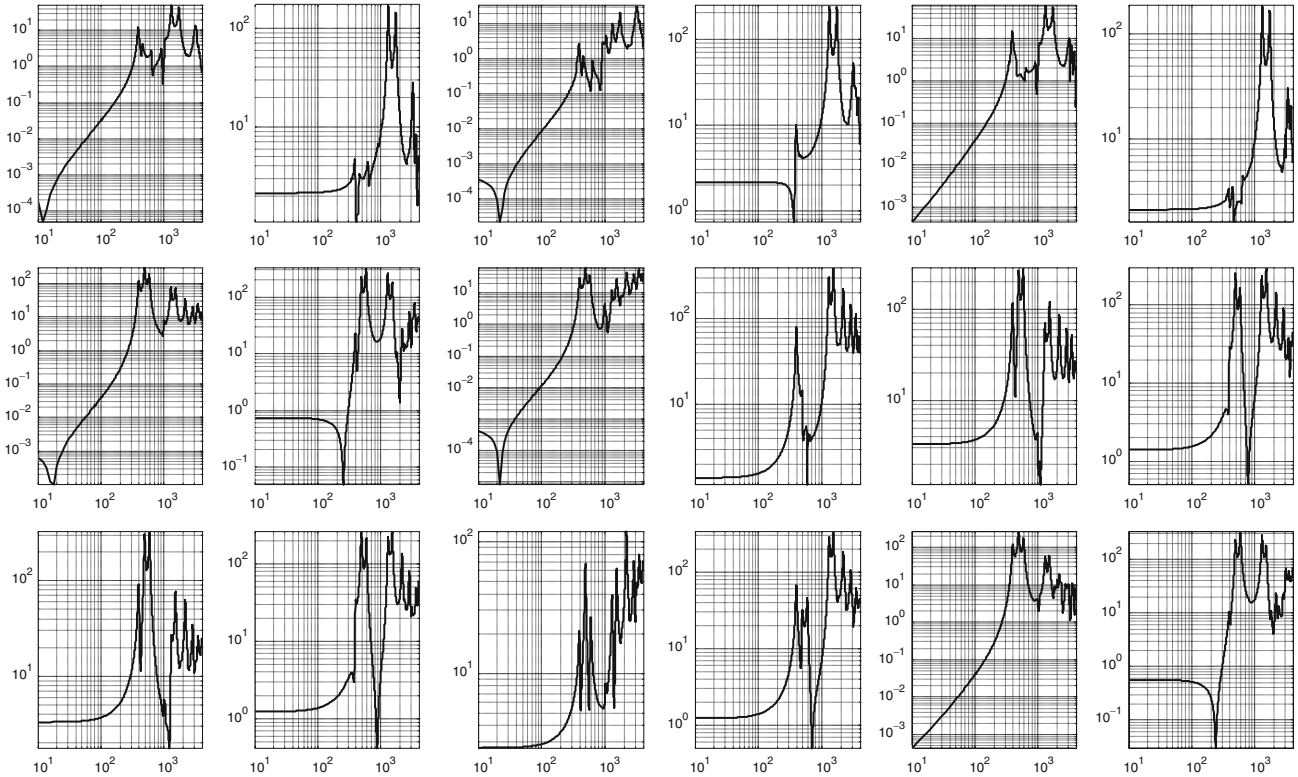
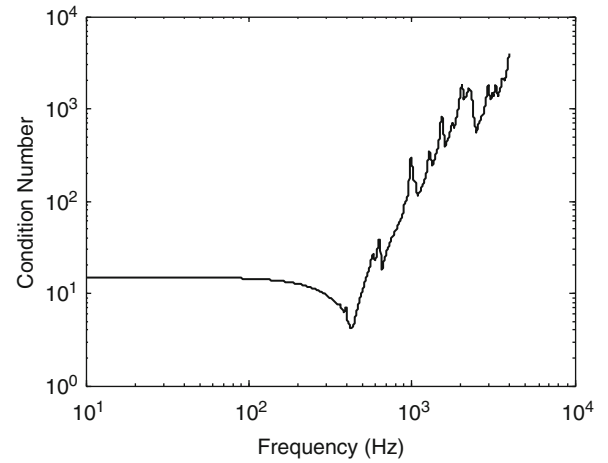


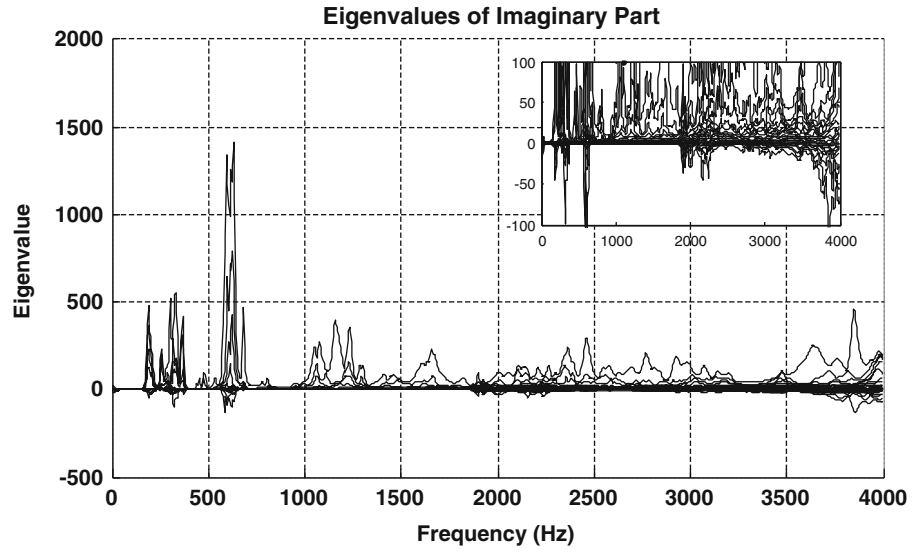
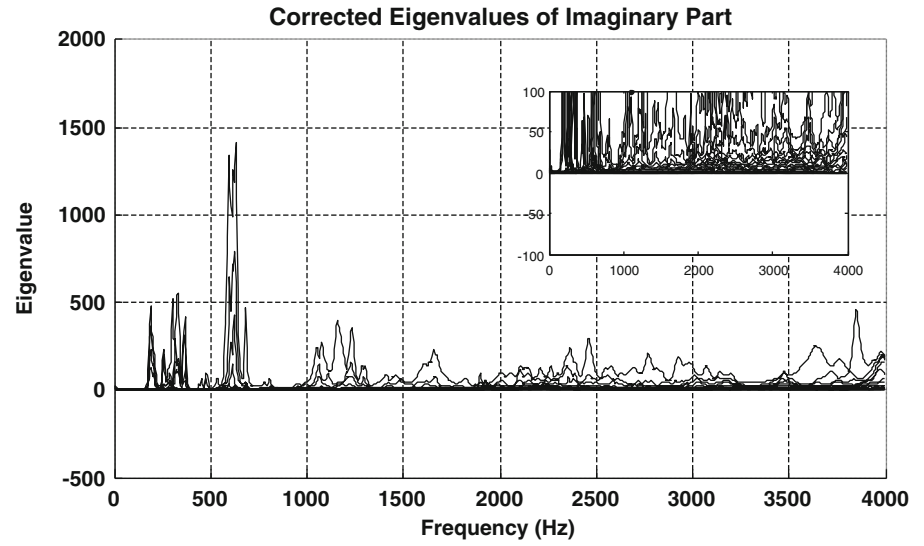
Fig. 18.3 As-measured transfer admittance matrix

Fig. 18.4 Condition number in numerical verification



18.6 Experimental Example

The experimental and data cleansing methods were carried out on two test bodies for purposes of comparing their driving point admittances. If the two bodies have very similar admittance functions at the attachment locations, then the inputs to the two bodies from the next-assembly will also be very similar. This information was needed to assess the effects of changes in the next-assembly and the resultant changes in response levels in the two bodies due to impedance effects. Each of the two bodies was fitted with the four admittance test fixtures depicted in Fig. 18.1.

Fig. 18.5 As-measured eigenvalues**Fig. 18.6** Corrected eigenvalues

Adding to the complex bookkeeping required to perform these calculations, the data collected in this experiment were collected and used by two separate agencies that use different coordinate systems. The two coordinate systems are related by the transformation shown in the equation below.

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{agency 1}} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\text{agency 2}}$$

This transformation matrix T is assembled into a block diagonal matrix, 24×24 in dimension, that relates the FRF matrix in the two coordinate systems as shown in the equations below

$$\{x\}_{\text{agency 1}} = [H] \{f\}_{\text{agency 1}}$$

$$\{x\}_{\text{agency 1}} = [T] \{x\}_{\text{agency 2}}$$

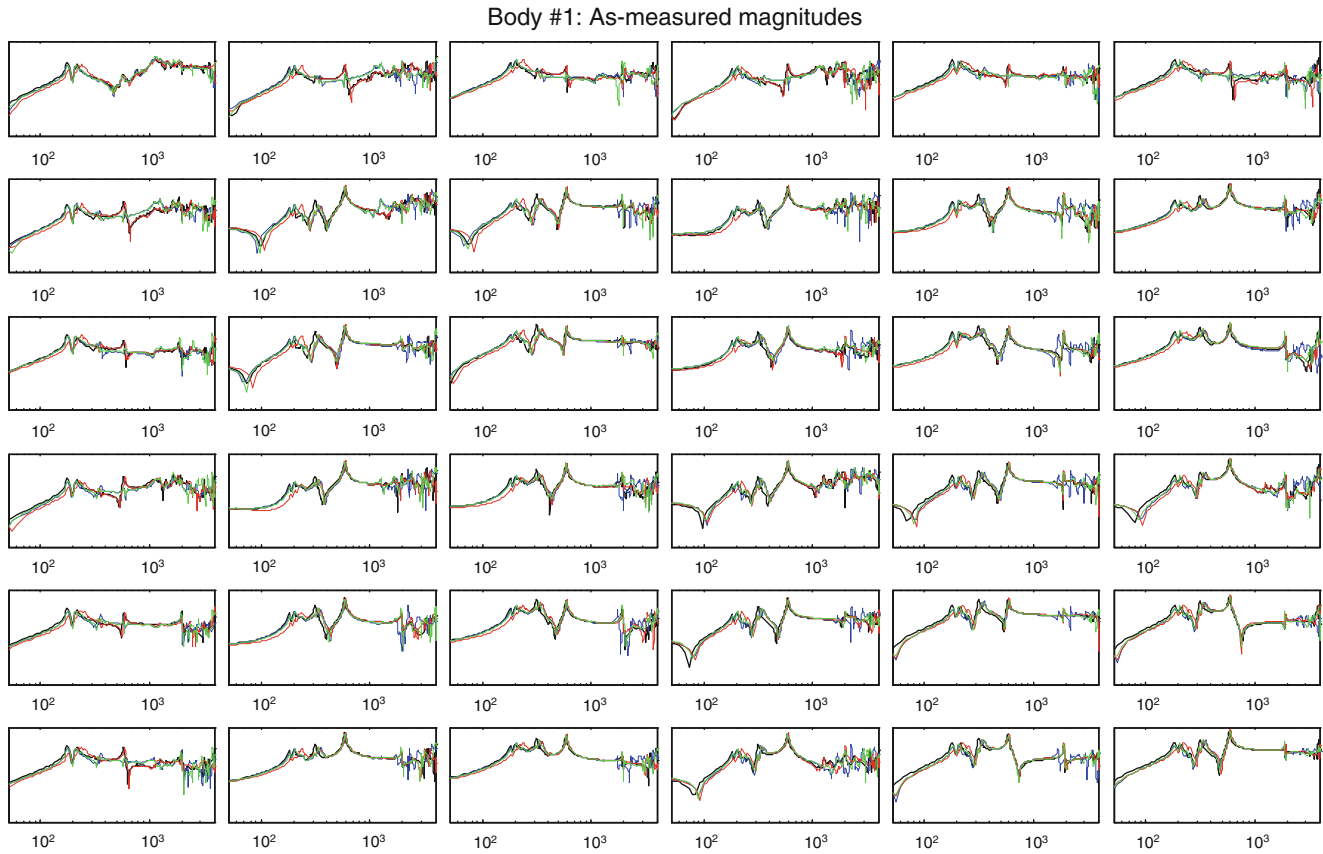


Fig. 18.7 As-measured driving point admittance functions

$$[T] \{x\}_{agency\ 2} = [H] \{f\}_{agency\ 1}$$

$$\{x\}_{agency\ 1} = [T]^T [H] \{f\}_{agency\ 1}$$

The raw data was first transformed into the proper coordinates before any further processing. Next, as can be seen in Fig. 18.1, some of the transducers measure in the negative polarity of their respective axes. There are 24 experimental measurements in the as-measured, driving point FRF matrix. The coordinate rotation matrix, T , was used to correct the measured FRF matrix for the measurement polarities of the transducers using the relation $H_B = T^T H_A T$. The full rotation matrix appears in the Appendix 3.

The measured, 6×6 driving-point subsets of the full FRF matrices for each test fixture on Body #1, after the coordinate transformations described above, appear in Fig. 18.7. For clarity, the FRFs from similar locations on each test fixture are plotted together in the same sub-plot. The data shows excellent repeatability on each of the four test fixture blocks. This is partially due to the symmetry of the unit under test. In the axial direction all four input locations have approximately the same admittance. Some asymmetry is apparent in the rotational admittance functions. Both of these observations are expected due to the symmetries of the test body. Data checks like this are an invaluable part of the testing process as finding and correcting polarity and channel labeling errors can consume significant time and effort.

Figures 18.8 and 18.9 show the driving point FRFs after enforcing reciprocity and removing the negative eigenvalues from the imaginary part of the FRFs, respectively. The next step is to transfer forces and displacements from the applied and measured locations to the connection point via Equation 18.15. This step requires the assembly of mode shape matrices for the test fixtures.

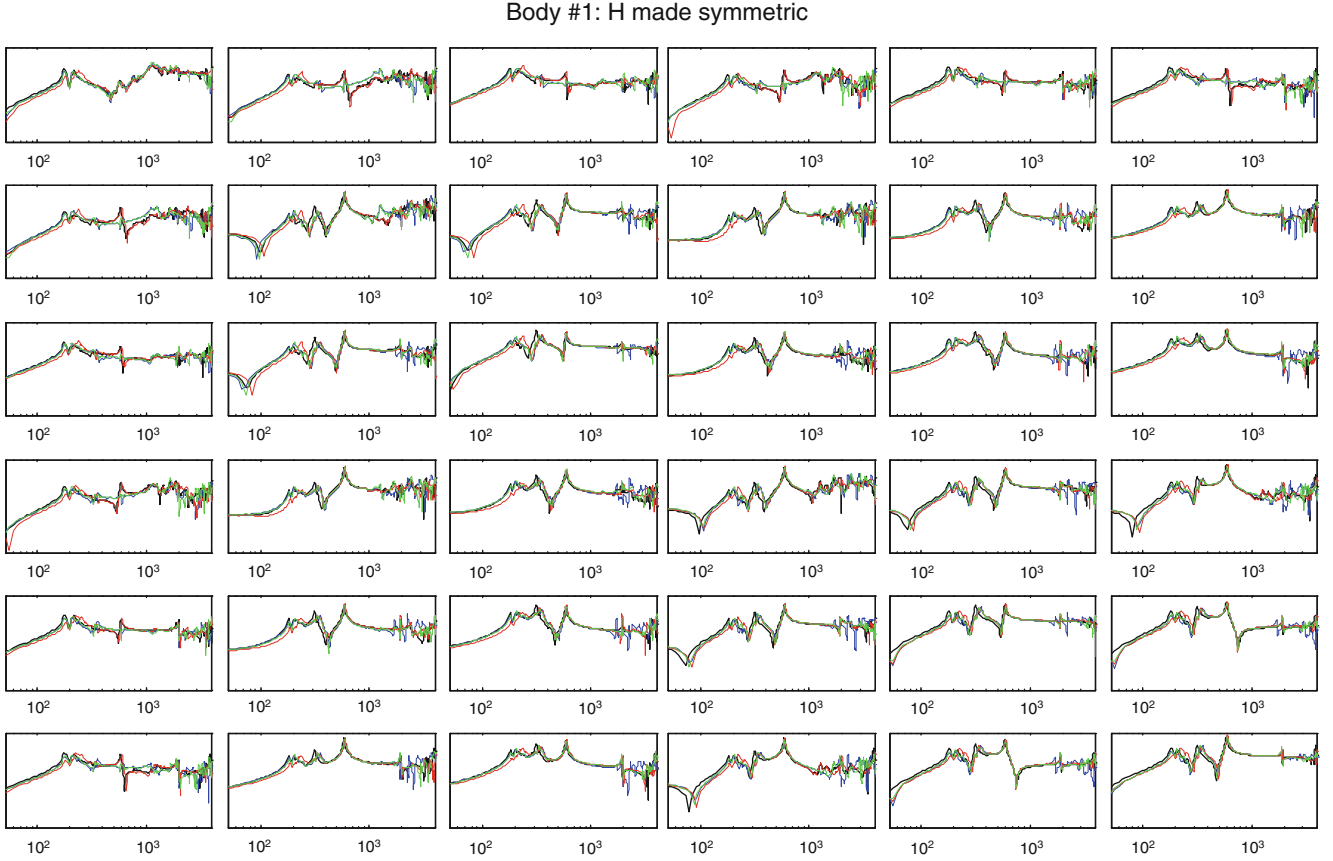


Fig. 18.8 Body #1 driving point admittance functions after enforcing reciprocity

The mode shape at the node A11X+ is given by

$$\Phi_{\text{Node A11X+}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1.5 & -1.5 \\ 0 & 1 & 0 & -1.5 & 0 & -0.25 \\ 0 & 0 & 1 & 1.5 & 0.25 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The full, 54×6 mode shape matrix for Block #1 is assembled from nodal matrices such as the one above, and is shown in the Appendix 1. For block #1, the observed degrees of freedom are 1X+, 2Z-, 3Z-, 4Y- 5Y- and 6Y-T the Boolean matrix which selects these degrees of freedom for Block #1 is 6×54 . The Boolean matrix is shown in the Appendix 1. After performing the selection operation $\Phi_{\text{Block 1}} = B \Phi_{\text{Block 1}}$, the resulting 6×6 matrix of observed mode shapes on block #1 is given by

$$\Phi_{\text{Block 1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1.5 & -1.5 \\ 0 & 0 & 1 & 1.5 & 0.25 & 0 \\ 0 & 0 & 1 & 1.5 & 3 & 0 \\ 0 & 1 & 0 & -1.5 & 0 & -0.25 \\ 0 & 1 & 0 & -1.5 & 0 & -3 \\ 0 & 1 & 0 & 1.5 & 0 & -3 \end{bmatrix}$$

The full mode shape matrix of all four admittance test blocks is assembled according to Eq. (18.22). This matrix also appears in the Appendix 1. The mode shapes at the connection node can also be selected from the full mode shape matrix

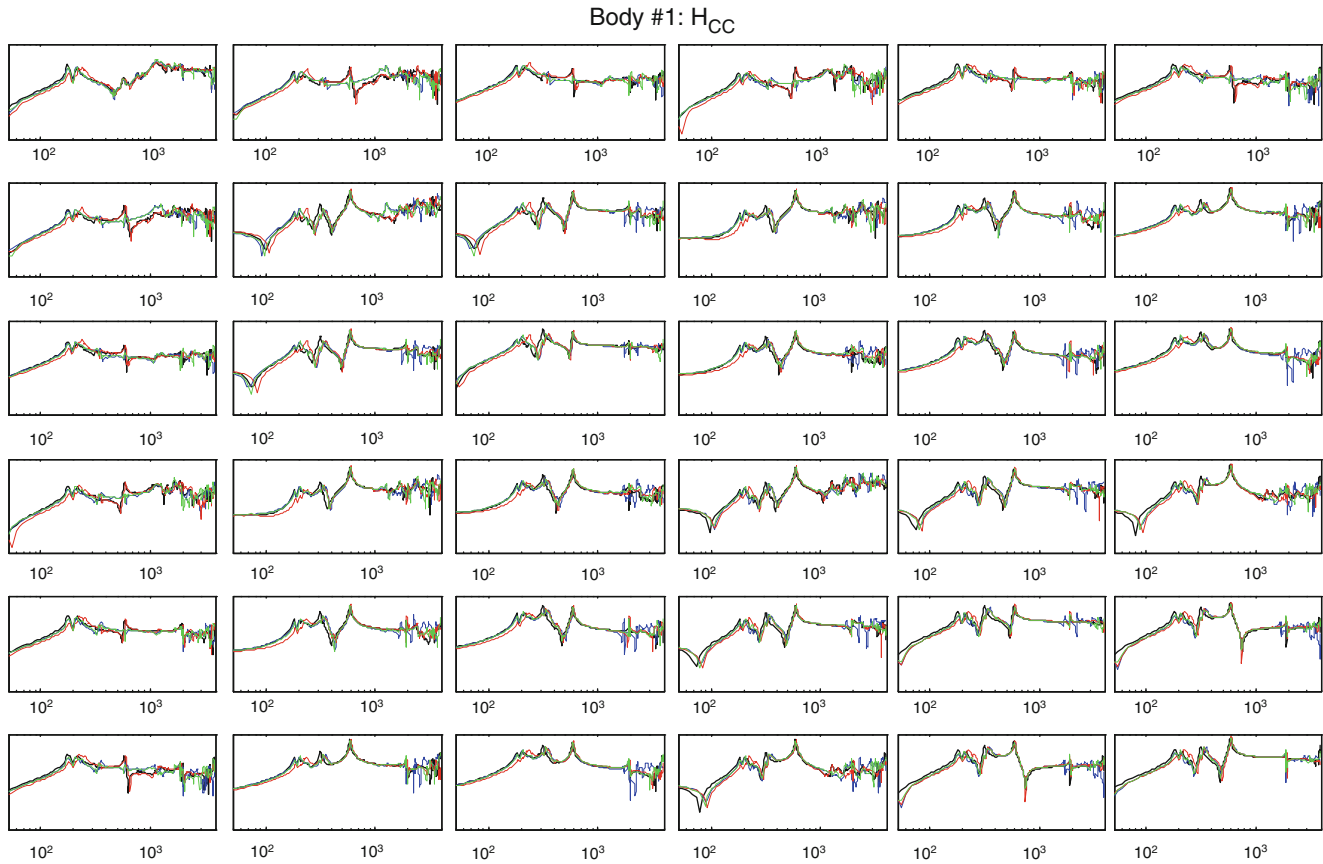


Fig. 18.9 Body#1 driving point admittance functions after removing negative eigenvalue of the imaginary part

using a Boolean matrix, but because the mode shapes are written with respect to the connection node, the mode shapes at the connection for a single block are simply the 6×6 identity matrix.

$$\Phi_{c1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The full 24×24 mode shape matrix for all connection points is assembled according to Eq. (18.22), and is also shown in the Appendix 2.

With these mode shape matrices in hand, the FRF matrix as measured on the admittance test fixtures are transferred to the connection point using Eq. (18.15). The inertia matrix for any single block is formed according to the approach contained in the Appendix 2. For example, the inertia matrix for Block #1 is given by

$$M_1 = \begin{bmatrix} 0.0831 & 0 & 0 & 0 & 0.023 & 0.0157 \\ 0 & 0.0831 & 0 & -0.023 & 0 & -0.1378 \\ 0 & 0 & 0.0831 & -0.0157 & 0.1378 & 0 \\ 0 & -0.023 & -0.0157 & 0.19353 & -0.0287 & 0.0488 \\ 0.023 & 0 & 0.1378 & -0.0287 & 0.3927 & 0.0209 \\ 0.0157 & -0.1378 & 0 & 0.0488 & 0.0209 & 0.3863 \end{bmatrix}$$

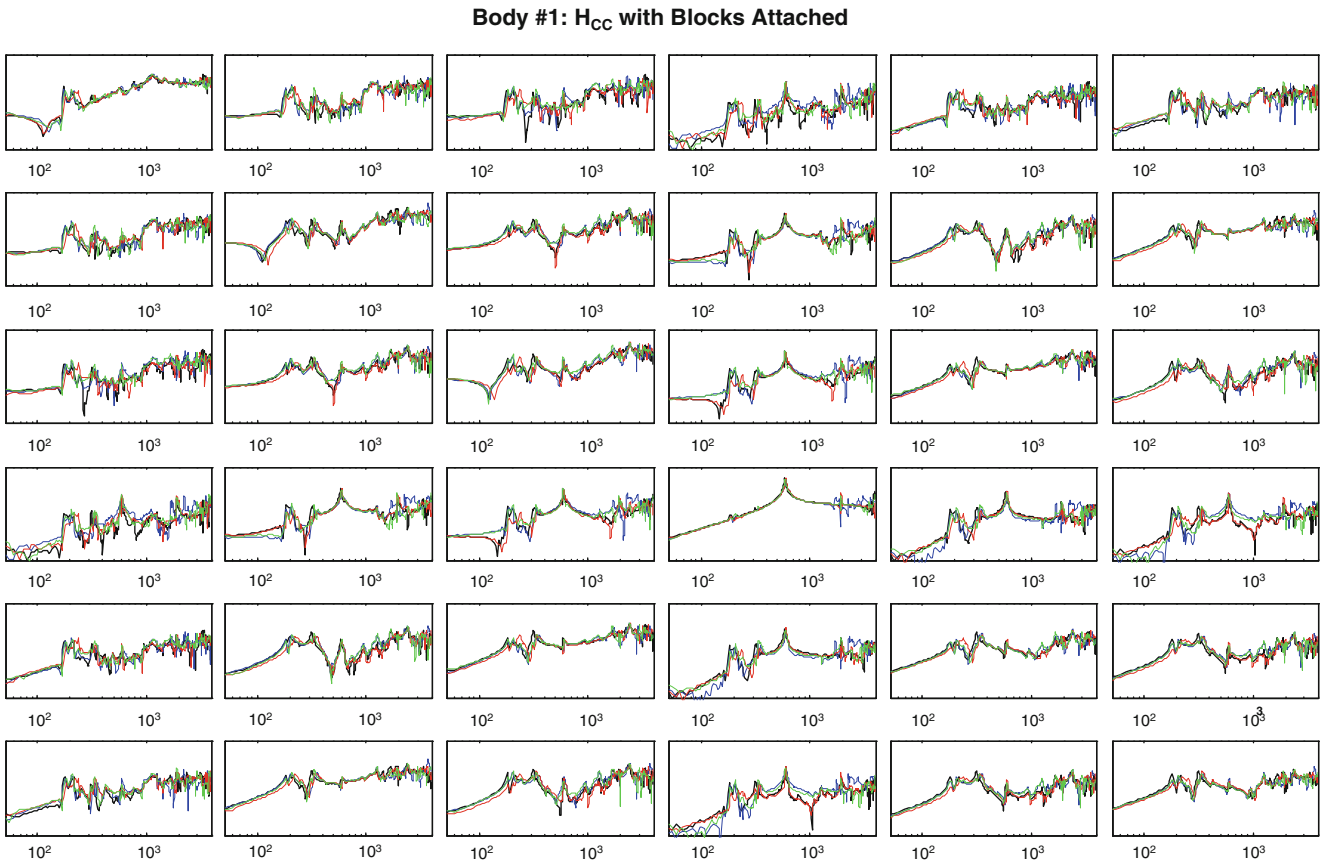


Fig. 18.10 Body#1 driving point admittance functions after transferring to the connection point

The inertia matrices for the remaining blocks are then found from a coordinate rotation according to Eq. (18.23), where the rotation matrix is given by

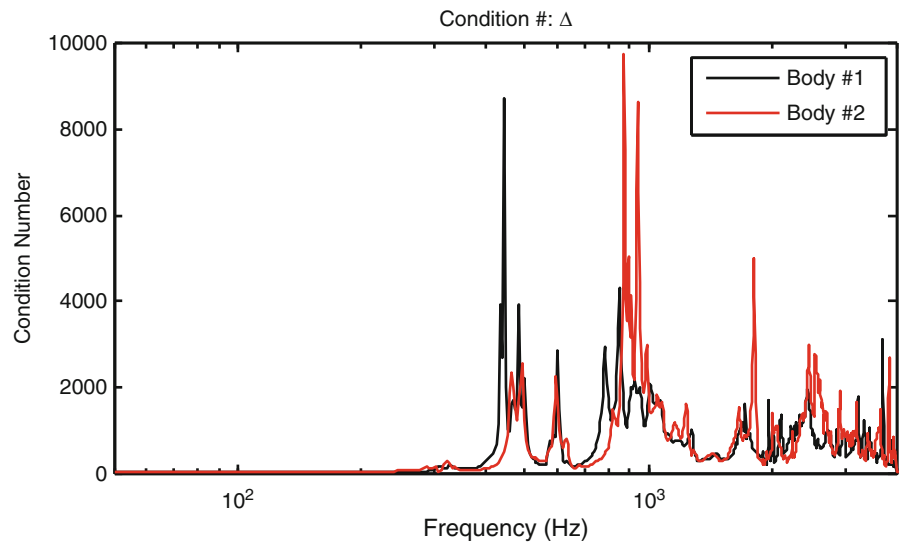
$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

Figure 18.10 shows the driving point FRF matrix after transference to the connection points. The next step is the removal of the effects of the test fixtures using Eq. (18.17). This involves finding the inverse of the matrix $[H_{c,c} - \Delta H_{c,c}]$. Numerical errors can plague this operation if this matrix is ill-conditioned. Figure 18.11 shows the matrix condition number for the two experimental bodies. The condition numbers shown are roughly twice the values found in the numerical verification above. But considering that the condition number can vary over many order of magnitude, it is believe that the matrix inversion is stable for both bodies and no numerical ill-conditioning effects are in play.

Figure 18.12 shows the final driving point FRF matrix for body #1 after the dynamic effects of the test fixtures have been removed. These FRFs can be compared to the Body #2 driving point FRF shown in Fig. 18.13. The FRFs are plotted together in Fig. 18.14.

The rotational asymmetry occurs in input pairs that are 180 degrees apart around the circumference of the aft flange. The same asymmetries can be observed in the Body #2 driving point admittance functions shown in Fig. 18.13. Although some

Fig. 18.11 Matrix condition numbers



Body #1: Hcc with blocks removed

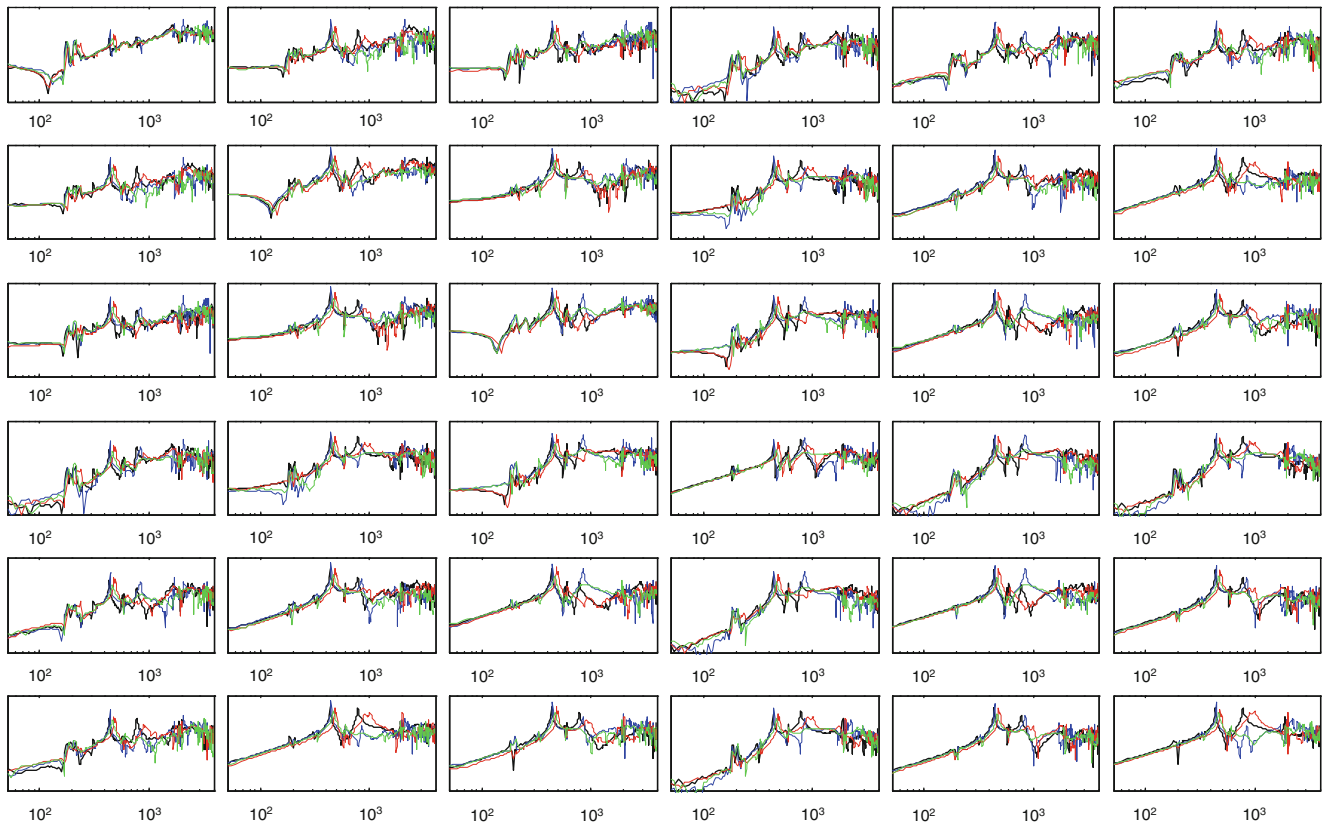


Fig. 18.12 Body#1 driving point admittance functions after removing the effects of the test fixtures

differences are observed, owing to the very different internal structure of the two units, the driving-point admittance functions are generally similar, with no large resonance differences. This indicates that the effect of these two bodies on their attachment locations is largely the same and that the inputs to these bodies in the field will be largely unaffected by differences in the impedance of the bodies.

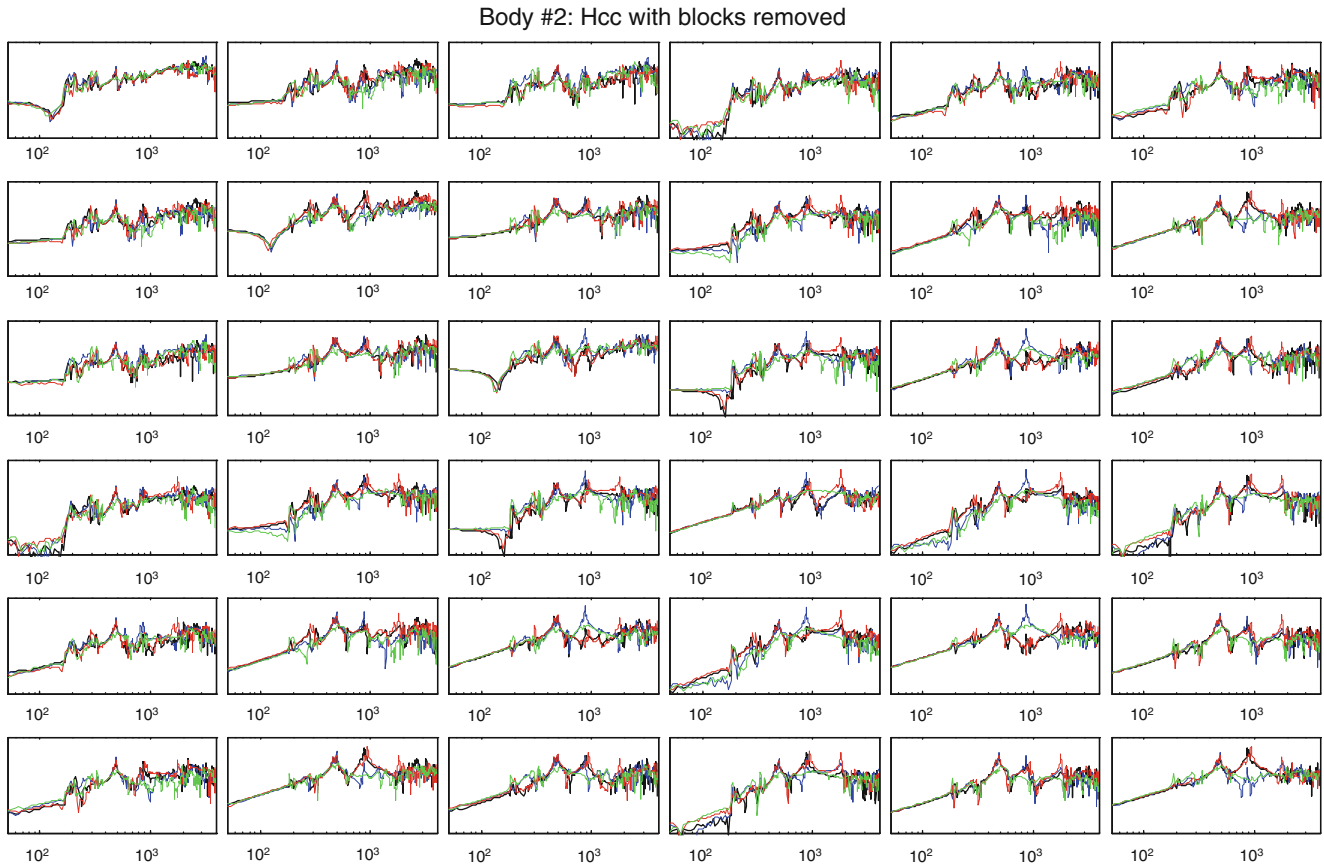


Fig. 18.13 Body#2 driving point admittance functions after removing the effects of the test fixtures

18.7 Experimental Observations and Improvements

During the course of these experiments several improvements were made to the process to enhance the quality of the measured data. Measurement data quality is of utmost importance in admittance modeling due to the demands and sensitivities of the processing that accompanies it.

Initially the experimental excitation was carried-out with a modal hammer. This produced unsatisfactory results due to several reasons. First, the lack of excitation energy propagating to remote parts of the test bodies made the accurate measurement of responses impossible. This led to very poor estimates of transfer FRFs. Second, the non-uniform spectrum of the input force led to numerical errors during data processing. The experimental results were improved by using a modal shaker driven by a closed-loop, feedback controller enforcing a flat spectrum of the input force for excitation.

During the course of the experiment the load cell measuring the excitation force is moved from location to location to excite the different degrees of freedom of the test fixture. If compensatory measures are not taken the movement of the load cell will cause the inertia matrix of the test fixture to vary with each movement of the excitation location. In an effort to hold the inertia of the test fixture as constant as possible between the different excitation locations a mock load cell was devised to replicate the mass and rotational inertia of the actual load cell and stinger attachment sleeve. The mock load cells were used at each excitation location on the test fixture while that excitation location was not in use. By exchanging the real load cell and mock load cell as the location of the excitation varied, the inertia of the test fixture was held as constant as reasonably possible (Fig. 18.15).

Although studies conducted using the numerical verification model indicated that uncertainty in the location of the accelerometers on the test fixture did not have a large contribution to the overall uncertainty, a locating jig was used to ensure that the accelerometers were precisely located. The locating jig and accelerometers are shown on the admittance test fixture in Fig. 18.16. The small effort required for this improvement was worthwhile even considering that the quality of the data can still be high without such measures.

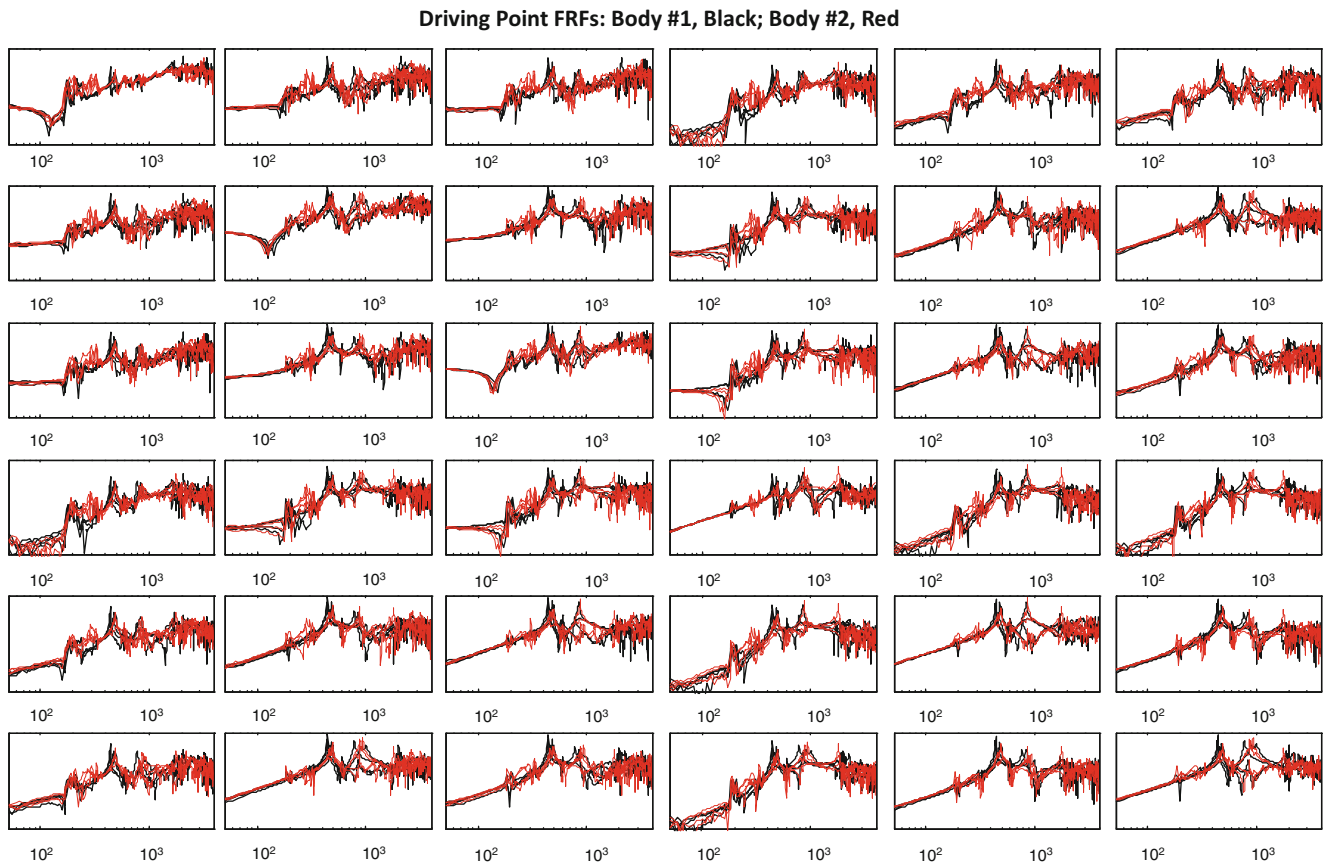
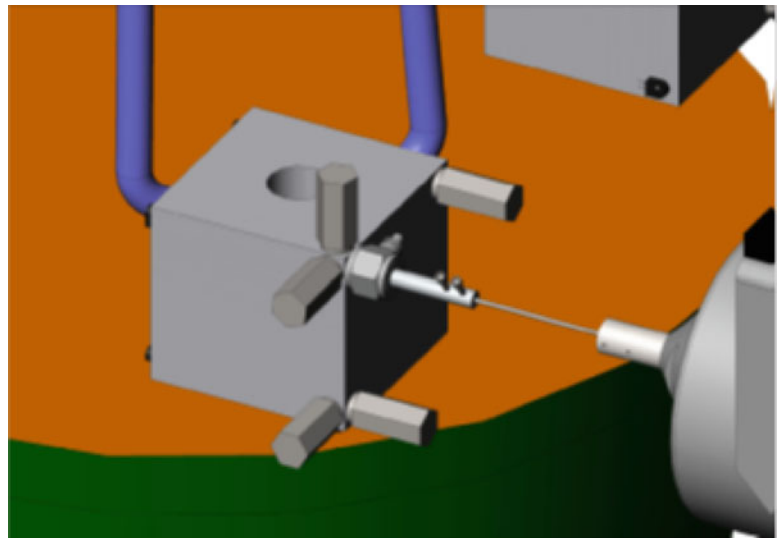


Fig. 18.14 Body#1 and Body #2 driving point admittance functions

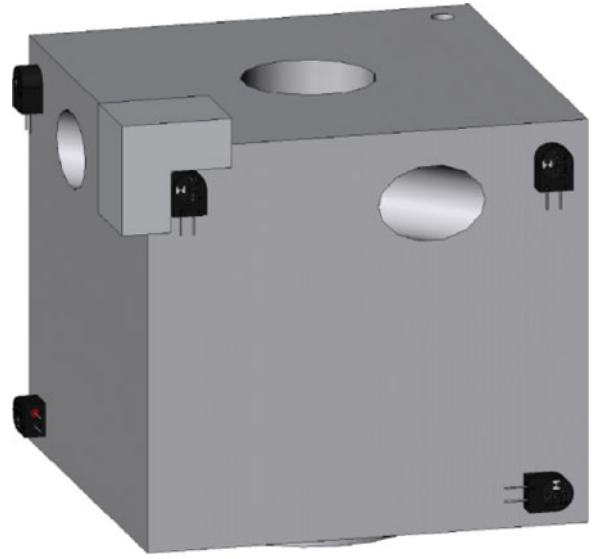
Fig. 18.15 Test fixture with load cell and mock load cells



In future implementation of these techniques, there may be advantages to increasing the number of transducers beyond six on each fixture to make the problem over-determined. It is expected that adding more transducers would significantly improve the quality of the processed data since least-squared error estimates of the modal forces and responses could be used in the calculations.

Acknowledgements The author wishes to acknowledge Greg Tipton, Todd Simmermacher, Randy Mayes, each of whom made substantial contributions to this work.

Fig. 18.16 Accelerometer
locating jig



A.1 Appendix 1: Mode Shape Matrices

In the special case presented in this work, the six degrees of freedom are observed on each admittance test fixture. This makes the formulation of the mode shape matrices and subsequent calculations quite easy.

Although all 6 DOF may not be observed at each node of the test fixture, each node possesses 6 DOF. *Considering only rigid body modes*, the mode shapes at a node are given by

$$[\Phi_{\text{node}}] = \begin{bmatrix} I & R \\ 0 & I \end{bmatrix} \quad (18.25)$$

where R is a 3×3 matrix that accounts for the translations of the node due to rotational motion. Note the implied assumption of small-angle rotation. R is given by

$$R_1 = \begin{bmatrix} \{\omega_x \times n_1\}^T & \{\omega_y \times n_1\}^T & \{\omega_z \times n_1\}^T \end{bmatrix} \quad (18.26)$$

where $\omega_x = [1 \ 0 \ 0]$, $\omega_y = [0 \ 1 \ 0]$, $\omega_z = [0 \ 0 \ 1]$ and n_1 is the vector location of the node of interest.

Since one degree of freedom is observed at each of six nodes on each test fixture, $[\Phi_{\text{ext}}]$ represents the 6×6 mode shapes at the observed degrees of freedom. The observed degrees of freedom are linearly independent.

The 24×24 mode shape matrix of the overall set of test fixtures is then assembled from the mode shape matrices of the uncoupled blocks as

$$[\Phi_{\text{ext}}] = \begin{bmatrix} \Phi_{\text{ext}}^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Phi_{\text{ext}}^4 \end{bmatrix} \quad (18.27)$$

The mode shapes evaluated at the connection point are given by

$$[\Phi_c] = \begin{bmatrix} I & R_c \\ 0 & I \end{bmatrix} \quad (18.28)$$

The Boolean matrix, $[B]$, in $N \times M$ (N observed DOF $\times M = 6 \times m$ nodes) has zeros everywhere, except a one in the column identifying the observed DOF in the row signifying the order of the experimental measurement, corresponding to the terms in H .

If elastic modes are included, then $[\Phi_{\text{node}}]$ would contain additional columns, with corresponding columns added to $[\Phi_{\text{ext}}]$. If more than 6 DOF are observed on a block, then $[\Phi_{\text{ext}}]$ would have additional rows.

B.1 Appendix 2: Inertia Matrices

The term $\Delta H_{c,c}$ represents the admittance of the test fixtures. This admittance matrix can be represented either in the modal domain [7] or in the spatial domain as

$$\Delta H_{c,c} = [K] - \omega^2 [M] + j [C]]^{-1} \quad (18.29)$$

In this work, we assume that the test fixtures are rigid, so that the admittance of the test fixtures is totally represented by $-\omega^2 [M]^{-1}$. Because we will accounting for the inertia effects of the block at the connection point, the inertia matrix *must be evaluated with respect to the connection point*.

$$[M_{BL}] = \begin{bmatrix} m & 0 & 0 & 0 & -mz & my \\ 0 & m & 0 & mz & 0 & -mx \\ 0 & 0 & m & -my & mx & 0 \\ 0 & mz & -my & I_{xx} & I_{xy} & I_{xz} \\ -mz & 0 & mx & I_{yx} & I_{yy} & I_{yz} \\ my & -mz & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \quad (18.30)$$

Inertia terms at the center of gravity may be related to inertia terms at the connection point through the relation

$$I_c = I_{CG} + m [R^T R I - R R^T] \quad (18.31)$$

where $R = [x \ y \ z]^T$, and represents the vector *from* the CG *to* the connection point.

The inertia matrices of the individual test fixtures is then assembled into a diagonal matrix whose terms are the individual matrices, $[M_{BL}]$. This inertia matrix is then inverted to find $\Delta H_{c,c}$.

In the case presented in this work, there are 4 test fixtures, each with 6 degrees of freedom (there would be additional degrees of freedom if elastic modes were included) so that the inertia matrix is 24×24 .

$$[M] = \begin{bmatrix} M_{BL} & 0 & 0 & 0 \\ 0 & M_{BL} & 0 & 0 \\ 0 & 0 & M_{BL} & 0 \\ 0 & 0 & 0 & M_{BL} \end{bmatrix} \quad (18.32)$$

$$\Delta H_{c,c} = M^{-1} \quad (18.33)$$

C.1 Appendix 3: Coordinate Rotations

Oftentimes, it is necessary to account for changes in coordinate axes. Consider two, right-hand coordinate axes $\mathbb{A}$ and $\mathbb{B}$. These two axes are related by a rotation about z followed by a rotation about x .

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\mathbb{A}} = \begin{Bmatrix} -x \\ -z \\ -y \end{Bmatrix}_{\mathbb{B}} \quad (18.34)$$

This rotation can be expressed in matrix form as

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\mathbb{A}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{\mathbb{B}} \quad (18.35)$$

Designate the matrix, T such that

$$T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad (18.36)$$

By definition,

$$x_{\mathbb{A}} = T x_{\mathbb{B}} \quad (18.37)$$

Since T is orthogonal $T^T = T^{-1}$. This allows the forces in the coordinate systems to be related though

$$F_{\mathbb{B}} = T^T F_{\mathbb{A}} \quad (18.38)$$

To illustrate the transformation of inertia and admittance matrices, consider the relation for rotational inertia, in coordinate axis $\mathbb{A}$

$$F_{\mathbb{A}} = I_{\mathbb{A}} \ddot{\omega}_{\mathbb{A}} \quad (18.39)$$

Substituting Eqs. (18.37) and (18.38) yields

$$T^{T^{-1}} F_{\mathbb{B}} = I_{\mathbb{A}} T \ddot{\omega}_{\mathbb{B}} \quad (18.40)$$

Rewriting the above in the form of Eq. (18.39) yields

$$F_{\mathbb{B}} = T^T I_{\mathbb{A}} T \ddot{\omega}_{\mathbb{B}} \quad (18.41)$$

From this relation, it is apparent that the two inertia matrices are related by

$$I_{\mathbb{B}} = T^T I_{\mathbb{A}} T \quad (18.42)$$

It should be noted that mode shape matrices cannot generally be rotated quite as simply. Rotation of mode shape matrices generally requires both a Boolean selection matrix and a rotation matrix.

D.1 Appendix 4: Matrices used in Calculations

Table D.1 T-matrix[illegible]

Table D.2 Full mode shape matrix for Block #1

1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	1.5	1.5
0	1	0	-1.5	0	-3
0	0	1	-1.5	3	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	1.5	-1.5
0	1	0	-1.5	0	-3
0	0	1	1.5	3	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	1.5	-1.5
0	1	0	-1.5	0	-0.25
0	0	1	1.5	0.25	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	1.5	1.5
0	1	0	-1.5	0	-0.25
0	0	1	-1.5	0.25	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	-1.5	1.5
0	1	0	1.5	0	-3
0	0	1	-1.5	3	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	-1.5	-1.5
0	1	0	1.5	0	-3
0	0	1	1.5	3	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	-1.5	-1.5
0	1	0	1.5	0	-0.25
0	0	1	1.5	0.25	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
1	0	0	0	-1.5	1.5
0	1	0	1.5	0	-0.25
0	0	1	-1.5	0.25	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

Table D.3 Boolean matrix for Block #1

[illegible]

Table D.5 Mode shape matrix at connection points

[illegible]

References

1. Mayes RL, Ross MR (August 2012) Advancements in hybrid dynamic models combining experimental and finite element substructures. *Mech Syst Signal Process* 31:56–66
2. Allen MS, Mayes RL, Bergman EJ (November 2010) Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections. *J Sound Vib* 329:4891–4906
3. Mayes RL, Hunter PS, Simmermacher TW (2008) Combining experimental and analytical substructures with multiple connections. In: *Proceedings of the 26th international modal analysis conference*, February 2008, Orlando, FL USA, p 164
4. Mayes RL, Stasiunas EC (2007) Combining lightly damped experimental substructures with analytical substructures. In: *Proceedings of the 25th international modal analysis conference*, February 2007, Orlando, FL, USA, p 207
5. Jetmundson B, Bielawa RL, Flannelly WG (1998) Generalized frequency domain substructure synthesis. *J Am Helicopter Soc* 33:55–64
6. Carne TG, Dohrmann CR (2006) Improving experimental frequency response function matrices for use in admittance modeling. In: *Proceedings of the 24th international modal analysis conference*, February 2006, #316 St. Louis, MO, USA
7. Imregun M, Robb DA, Ewins DJ (1987) Structural modification and coupling dynamic analysis using measured FRF data. In: *Proceedings of the 5th IMAC*, 1987. Society for Experimental Mechanics, Inc., London, pp 1136–1141

Chapter 19

Proposed Approach for Admittance Testing of a Complex Aerospace Structure*

Michael Arviso and Randall L. Mayes

Abstract It is common practice to utilize lower fidelity payloads to represent a complex aerospace payload during delivery system-with-payload, ground testing. Typically, the high fidelity payload hardware is not only costly but hard to acquire. Admittance testing can theoretically be used to experimentally model responses of a payload due to interface forces. In this paper we will consider the question, “Can the response of high fidelity payload hardware be predicted from an environmental test on a delivery system with low fidelity payloads and admittance data on all the substructures?” In theory, by acquiring the admittance models of the high and low fidelity payloads as well as the delivery system, one can adjust the measured interface responses and predict the response as if the high fidelity unit had been present in the ground test instead of the low fidelity payload. In this paper, work in progress is described to demonstrate the payload substitution capability for a specific complex aerospace structure.

Keywords Frequency based substructuring admittance modal

Nomenclature

FBS	Frequency based substructuring
DOF	Degree of freedom
H	Frequency response function
FRF	Frequency response function
PL	Payload
TS	Transmission simulator
DS	Delivery system
INT	Interim system
Φ_{SC}^{TS}	Transmission simulator mode shape matrix
SC	Sensor connection

Subscripts

O	Output response
C	Connection response

* Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

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I	Force input degree of freedom
T	Transpose
+	Pseudo-inverse
Q	Modal connection degree of freedom
X	Vector

19.1 Introduction

One of the tests in a current program applies a random environment to a delivery system with low fidelity payloads mounted on it. The purpose of this work is to use admittance modeling theory, which will now be referred to as frequency based substructuring (FBS), to analytically replace the low fidelity payload with a high fidelity payload in order to predict full system response as if the high fidelity payload had been attached to the delivery system in the environment.

As in any program, the first year has given greater insight into the magnitude of the testing that needs to be performed to investigate the frequency based substructuring theory. Logistics and availability of hardware play an important role in coordinating testing in stages that allows not only analysis of data but collaboration with finite element modeling in developing the process. Truth models need to be developed to which predictions with the FBS method can be compared to quantify its uncertainty.

This approach utilizes FBS with a fixture at the connection called the transmission simulator. The transmission simulator method can be used with either modal based substructures or FBS as has been previously published [3]. Typically, the transmission simulator is a specially made and easy to model fixture on which connection equilibrium and continuity between multiple substructures is satisfied. The transmission simulator is used as the common interface between the substructures. The transmission simulator is a substructure in itself, and inherently includes the effects of moments and rotations even though one may only measure translational accelerations and forces. In this approach the transmission simulator is actually a subset of the true mounting hardware. As depicted in Fig. 19.1, the three mounting pedestals which connect a payload to the delivery system will be used as the transmission simulator.

The objective of the work is to take measured acceleration responses from an environmental test of the delivery system with low fidelity payloads attached, and mathematically calculate what the response would have been if a high fidelity payload had been attached. One task is to measure the FBS substructures for the delivery system, high fidelity payload and low fidelity payload.

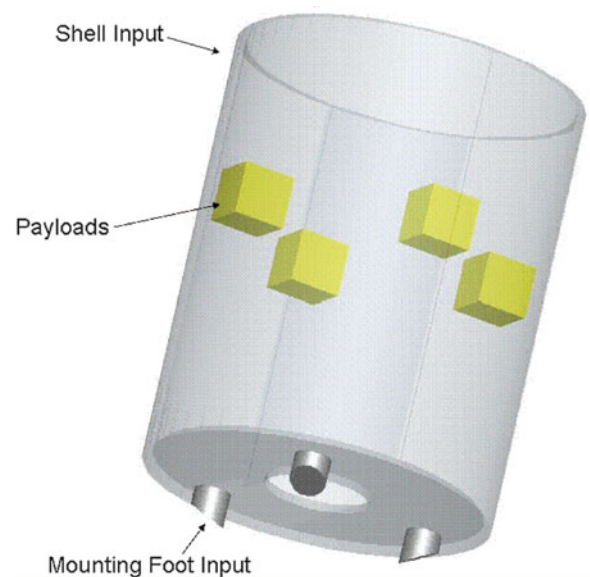


Fig. 19.1 Simplified illustration of payload

19.2 Payload, Delivery System and Transmission Simulator Substructures

The aerospace payload consists of an exterior shell structure surrounding interior bracing, which supports several brackets holding payloads. Figure 19.1 shows a simplified illustration of the payload. Rather than developing other fixtures for transmission simulators to mount to each substructure, we elected to use the three mounting pedestals as the transmission simulator for both the payload substructure and delivery system substructure.

Before any testing began, a finite element model of one pedestal was developed and its natural frequencies were compared to test data.

The modes and frequency response functions (FRFs) of the transmission simulator (pedestal) were calculated using Salinas, a massively parallel structural dynamics code developed at Sandia National Laboratories [1]. Ten locations were selected on each pedestal to mount accelerometers and apply input forces. This allowed a least squares fit to six rigid body modes and one elastic mode of each pedestal to provide seven modal constraints for each pedestal to connect one substructure to another.

19.3 FBS Testing

By all accounts, the FBS approach requires the care and attention similar to a very detailed modal test.

The basic approach is to develop a FBS of three different payloads with varying hardware fidelity and a FBS of the delivery system so that any combination of payload and delivery system could be modeled.

A truth test was performed which includes a low fidelity payload attached to a representative structure of the delivery system. In this test a hammer input was imparted at a location on the delivery system and FRFs were gathered on the payload system. Modes were extracted from the measured FRFs, up to 1000 Hz, using the SMAC algorithm [2] to give confidence that the FRFs were meaningful. In the future, FBS substructures will be developed for the representative delivery system and the low fidelity payload. These will be added together to see how well the truth FRFs can be represented from the combined FBS substructures.

For the program purposes, testing of a medium and high fidelity payloads with the transmission simulators attached have been conducted to develop FBS substructures for those payloads. In the future, data would also be collected on low and high fidelity delivery systems with the transmission simulators attached.

The following list describes the test series for documenting the FBS testing which will be performed in stages and Fig. 19.2 shows simplified illustrations of the payload and delivery system shown with the transmission simulators.

- Transmission simulator characterization
- Truth testing of low fidelity payload attached to representative delivery system

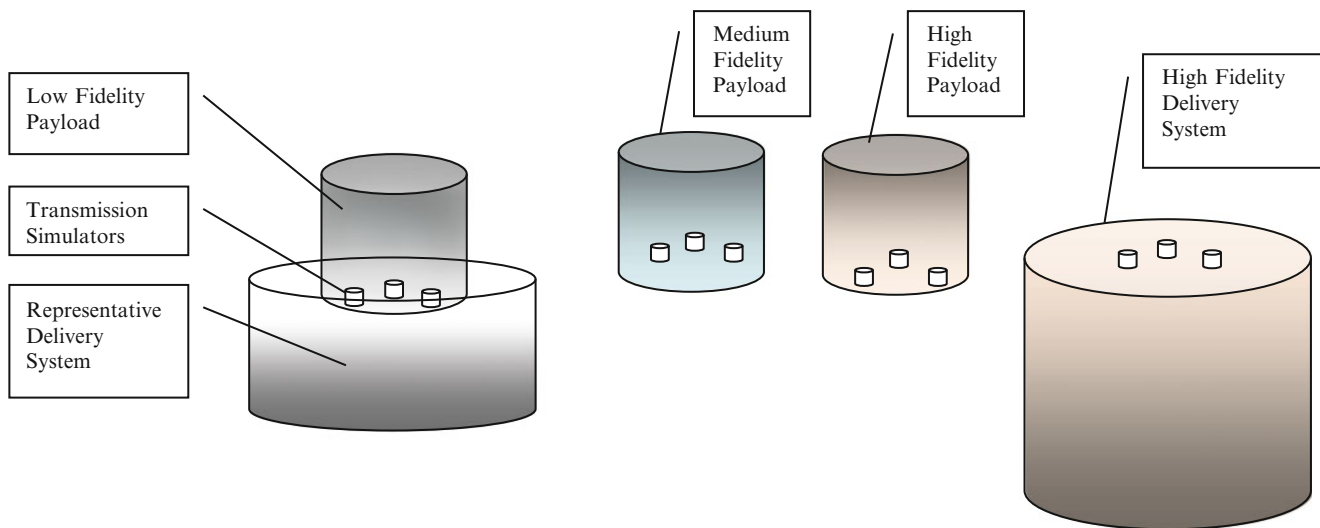


Fig. 19.2 Simplified illustration of payload and delivery system (truth hardware on left)

- Low fidelity payload characterization
- Representative delivery system characterization
- Medium fidelity payload characterization
- High fidelity payload characterization
- High fidelity delivery system characterization

The transmission simulators act as the common interface so they are instrumented with enough fidelity to capture motion and forces that can be represented with the modal motion and forces associated with the six rigid body modes and one elastic mode. One needs to have more sensors on the transmission simulator than modes of each transmission simulator in the bandwidth in order to reduce the effects of experimental errors. Co-linear force inputs to each accelerometer location with the use of the impact hammer at all simulator locations are required to obtain the connection FRF matrix on any substructure. Additional cross FRFs are gathered from accelerometers on components of interest inside a payload.

For testing up to this time on the payloads, data acquisition systems were set to measure FRFs in the 0 to 2000 Hz range. No window was used with the modal hammer force inputs. Six averages were collected at each input location.

19.4 Theory

The theory is developed in two sections with the first section addressing the FBS theory to connect two substructures and then subtract another substructure. The second section addresses the derivation of equations to determine the resulting connection degrees of freedom motion if an unknown force is applied on a payload and delivery system, measurements are made at the connection DOF on the transmission simulators, and then the payload is analytically removed and replaced by a different payload.

19.4.1 FBS Theory to Connect Two Substructures and Then Subtract Another Substructure

The FRFs for the payload (PL) will be designated as $\mathbf{H}^{PL}$ with the understanding that there is an FRF matrix for every frequency line in the FRF. The designation denoting these FRFs as a function of frequency will be omitted for brevity. The mounting feet will be a substructure which we denote as the transmission simulator (TS) has FRFs designated as $\mathbf{H}^{TS}$. The delivery system FRFs will be denoted as $\mathbf{H}^{DS}$. The responses of each substructure can be further subdivided into groups of degrees of freedom where subscript O is output response, subscript C is a connection response (on the mounting feet) and subscript I is a force input degree of freedom. There can be inputs and outputs on any substructure, but to limit the scope, here we will only have an input degree of freedom (DOF) on the delivery system substructure and outputs on the payload substructure.

The standard FBS equations will be modified with the free mode shapes of the transmission simulator (mounting feet). This matrix needs to have more measurement DOF than modes to provide the least squares averaging effect. A model or test data can be utilized to provide these mode shapes. As a minimum one would generally include all 6 rigid body mode shapes, or for 3 mounting feet this would be 18 rigid body mode shapes. If a mode shape is not exercised in the actual assembly motion, it is not necessary. One can visualize that the roll mode shape of each mounting foot might not be very important, so this may be one that could be neglected for each mounting foot. In addition as many elastic modes as are desired may be added in. In previous work, it appeared that any elastic modes up to the frequency of desired accuracy should be included. Let us denote the transmission simulator mode shapes as

$$\Phi_{SC}^{TS}$$

which will only be measured at specific DOFs that we will call the sensor connection (SC) DOF. With this approach, the transmission simulator, or mounting feet, will be installed on the payload to get $\mathbf{H}^{PL}$ and on the delivery system support substructure to get $\mathbf{H}^{DS}$. Then we will connect $\mathbf{H}^{PL}$ and $\mathbf{H}^{DS}$ together and subtract one set of mounting feet, $\mathbf{H}^{TS}$, to get the final system. The standard FBS equation for input on one system (A) with output on another system (B) giving response on total system (C) would be

$$\mathbf{H}_{OI}^C = \mathbf{H}_{OC}^B (\mathbf{H}_{CC}^B + \mathbf{H}_{CC}^A)^{-1} \mathbf{H}_{CI}^A \quad (19.1)$$

or for our system

$$\mathbf{H}_{OI}^{INT} = \mathbf{H}_{OC}^{PL} (\mathbf{H}_{CC}^{PL} + \mathbf{H}_{CC}^{DS})^{-1} \mathbf{H}_{CI}^{DS} \quad (19.2)$$

where the superscript INT denotes the interim system with the payload and delivery system connected together that has 2 sets of mounting feet, one of which still needs to be subtracted. In the subscripts, the first subscript represents an acceleration response DOF and the second subscript represents a force input DOF. Now we modify Eq. (19.2) to get modal connection DOF using the pseudo-inverse of the mode shape matrix giving

$$\mathbf{H}_{OI}^{INT} = \mathbf{H}_{OC}^{PL} \Phi_{SC}^{TS^+} (\Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^+} + \Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^+})^{-1} \Phi_{SC}^{TS^+} \mathbf{H}_{CI}^{DS} \quad (19.3)$$

where the superscript T represents transpose and the superscript + represents the pseudo-inverse. From this one can see that bookkeeping is paramount, e.g. if there are 30 connection measurements they MUST be in the same order in the mode shape matrix and the FRF matrices for ALL substructures. One will need several FRFs between inputs, outputs and modal connection DOF for the final step. Starting with

$$\mathbf{H}_{CC}^{INT} = (\mathbf{H}_{CC}^{PL^{-1}} + \mathbf{H}_{CC}^{DS^{-1}})^{-1} \quad (19.4)$$

and performing a similar transformation with the transmission simulator mode shapes yields

$$\mathbf{H}_{QQ}^{INT} = ((\Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^+})^{-1} + (\Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^+})^{-1})^{-1} \quad (19.5)$$

in which the sensor connection DOF have now been replace with modal connection DOF which are denoted with subscript Q. The output on the INT system with inputs at the Q DOF is analogous to Eq. (19.3) with

$$\mathbf{H}_{OQ}^{INT} = \mathbf{H}_{OC}^{PL} \Phi_{SC}^{TS^+} (\Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^+} + \Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^+})^{-1} \Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^+}, \quad (19.6)$$

and the modal connection response on the INT system with inputs at the I DOF is

$$\mathbf{H}_{QI}^{INT} = \Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^+} (\Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{PL} \Phi_{SC}^{TS^+} + \Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{DS} \Phi_{SC}^{TS^+})^{-1} \Phi_{SC}^{TS^+} \mathbf{H}_{CI}^{DS} \quad (19.7)$$

The transmission simulator modal connection FRF is

$$\mathbf{H}_{QQ}^{TS} = \Phi_{SC}^{TS^+} \mathbf{H}_{CC}^{TS} \Phi_{SC}^{TS^+} \quad (19.8)$$

The standard equation for adding two systems together with the input and output on system A is

$$\mathbf{H}_{OI}^C = \mathbf{H}_{OI}^A - \mathbf{H}_{OC}^A (\mathbf{H}_{CC}^A + \mathbf{H}_{CC}^B)^{-1} \mathbf{H}_{CI}^A \quad (19.9)$$

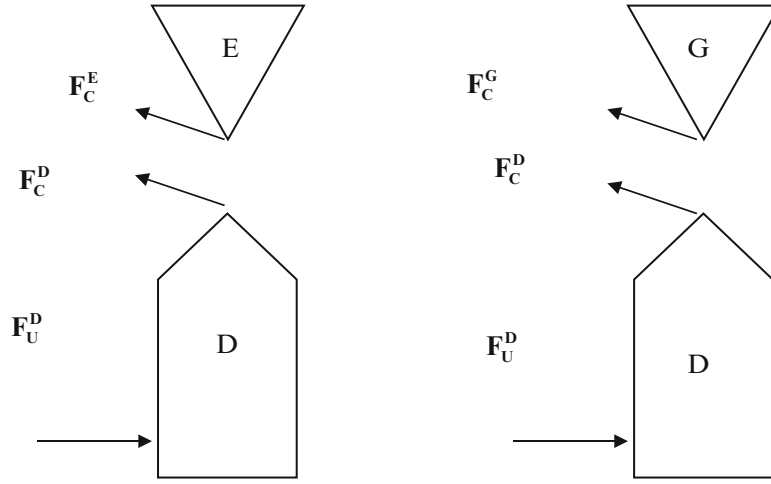
The final step is to subtract the transmission simulator (one set of mounting feet) using the form of Eq. (19.9) with both the input and output on INT and modal connection Q DOF as

$$\mathbf{H}_{OI}^{FINAL} = \mathbf{H}_{OI}^{INT} - \mathbf{H}_{OQ}^{INT} (\mathbf{H}_{QQ}^{INT} - \mathbf{H}_{QQ}^{TS})^{-1} \mathbf{H}_{QI}^{INT} \quad (19.10)$$

19.4.2 Predicting Connection Point Response Based on a Multi-substructure Test with Unknown but Constant Force Input on the Upstream Substructure After Analytically Changing the Downstream Substructure

Given substructure D which is excited by an unknown but constant force and attached to substructure E on which responses are desired, the connection point responses between D and E are measured. Remove substructure E and attach a new substructure G and predict the new connection point response using FBS. Here substructure D is denoted as the upstream

substructure on which force is applied and substructures E or G as downstream substructures on which responses are desired, but no external force is applied. For our acoustic environmental test, substructure D would represent the delivery system which is being excited and substructures E or G might be different payloads. In the figures below, $\mathbf{F}_U^D$ is the unknown applied force and $\mathbf{F}_C^D$, $\mathbf{F}_C^E$ and $\mathbf{F}_C^G$ are the connection forces between the substructures denoted by their superscript.



Consider the structure on the left side which will ultimately have substructure D coupled to substructure E, and will be excited by $\mathbf{F}_U^D$ in an environmental test. For this system the connection force vectors

$$\mathbf{F}_C^D = -\mathbf{F}_C^E \quad (19.11)$$

where we denote the connection DOF as subscript C. We can write the following equations of motion in terms of FRFs as

$$\mathbf{X}_C^D = \mathbf{H}_{CU}^D \mathbf{F}_U^D + \mathbf{H}_{CC}^D \mathbf{F}_C^D \quad (19.12)$$

and

$$\mathbf{X}_C^E = \mathbf{H}_{CC}^E \mathbf{F}_C^E \quad (19.13)$$

where the superscripts are related to the particular substructure and subscripts are the response or input DOF. The $\mathbf{H}$ systems are matrices and the $\mathbf{X}$ responses are vectors. When the system is connected

$$\mathbf{X}_C^E = \mathbf{X}_C^D \quad (19.14)$$

So equate the right hand side of Eqs. (19.12) and (19.13) to give

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D + \mathbf{H}_{CC}^D \mathbf{F}_C^D = \mathbf{H}_{CC}^E \mathbf{F}_C^E \quad (19.15)$$

Substituting Eq. (19.11) into Eq. (19.15) gives

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D - \mathbf{H}_{CC}^D \mathbf{F}_C^E = \mathbf{H}_{CC}^E \mathbf{F}_C^E \quad (19.16)$$

Gathering terms yields

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{F}_C^E \quad (19.17)$$

From Eq. (19.13) we can replace $\mathbf{F}_C^E$ to give

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{H}_{CC}^{E-1} \mathbf{X}_C^{DE} \quad (19.18)$$

where $\mathbf{X}_C^{DE}$ is the connection response of DOF C on either system D or E or the combined system DE. We specified in the beginning that the unknown force was constant. In this case $\mathbf{H}_{CU}^D$ is also unknown but constant. Applying exactly the same approach to the system on the right hand side of the figure which has a different substructure G yields a similar result with

$$\mathbf{H}_{CU}^D \mathbf{F}_U^D = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^G) \mathbf{H}_{CC}^{G-1} \mathbf{X}_C^{DG} \quad (19.19)$$

To eliminate the unknown force and unknown FRF on the left hand side, equating the right hand side of Eq. (19.18) to Eq. (19.19) gives

$$(\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{H}_{CC}^{E-1} \mathbf{X}_C^{DE} = (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^G) \mathbf{H}_{CC}^{G-1} \mathbf{X}_C^{DG} \quad (19.20)$$

where $\mathbf{X}_C^{DE}$ is known from measured test data and all other terms are measured FBS substructure matrices. Now we find the desired predicted response at the connection DOF as

$$\mathbf{X}_C^{DG} = \mathbf{H}_{CC}^G (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^G)^{-1} (\mathbf{H}_{CC}^D + \mathbf{H}_{CC}^E) \mathbf{H}_{CC}^{E-1} \mathbf{X}_C^{DE} \quad (19.21)$$

This theory is based on an assumption that no external forces (other than the connection force to substructure D) are applied to substructures E or G. We can now also predict the responses of various measured internal DOF in substructure G. First determine equivalent connection point forces on system G from

$$\mathbf{F}_C^G = \mathbf{H}_{CC}^{G-1} \mathbf{X}_C^{DG} \quad (19.22)$$

The final goal of determining output responses at DOF O on substructure G can be predicted from

$$\mathbf{X}_O^G = \mathbf{H}_{OC}^G \mathbf{F}_C^G \quad (19.23)$$

These can also be predicted with the modal responses of the transmission simulator applied to Eqs. (19.22) and (19.23). Equation (19.22) in transmission simulator modal coordinates is

$$\mathbf{F}_Q^G = (\Phi_{SC}^{T+} \mathbf{H}_{CC}^G \Phi_{SC}^{+})^{-1} \Phi_{SC}^{T+} \mathbf{X}_C^{DG} \quad (19.24)$$

$\mathbf{X}_C^{DG}$, which is needed in Eq. (19.24), can be calculated from the more easily inverted transmission simulator modal matrices and $\mathbf{X}_C^{DE}$ to modify Eq. (19.21) to give

$$\begin{aligned} \mathbf{X}_C^{DG} = & \mathbf{H}_{CC}^G \Phi_{SC}^{TS+} (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^D \Phi_{SC}^{TS+} + \Phi_{SC}^{TS+} \mathbf{H}_{CC}^G \Phi_{SC}^{TS+})^{-1} \times \\ & (\Phi_{SC}^{TS+} \mathbf{H}_{CC}^D \Phi_{SC}^{TS+} + \Phi_{SC}^{TS+} \mathbf{H}_{CC}^E \Phi_{SC}^{TS+}) (\mathbf{H}_{CC}^E \Phi_{SC}^{TS+})^+ \mathbf{X}_C^{DE} \end{aligned} \quad (19.25)$$

Equation (19.23) in modal connection coordinates is

$$\mathbf{X}_O^{DG} = \mathbf{H}_{OC}^G \Phi_{SC}^{+} \mathbf{F}_Q^G \quad (19.26)$$

19.5 Discussion

As one goes about planning, scheduling, and implementing a test series there are always hurdles that must be overcome before the actual data can even be analyzed.

Upon project initiation, came the reality of dealing with instrumentation issues and funding issues. As mentioned previously there must be a collaboration and availability of hardware in order to instrument, assemble, and perform these experiments that will then allow one to exercise the theory outlined in this paper. The funding, hardware and scheduling issues have delayed the completion of the project.

Preliminary data has been collected on the representative delivery system with the low fidelity payload which is to be used as a truth test as well as the medium and high fidelity payloads. Current work involves capturing data to characterize the high fidelity delivery system with the use of the transmission simulators. Further work involves the characterization of a low fidelity payload that was utilized to capture truth test data in the beginning of the program. Additional work revolves around the use of acoustical input as the excitation source and its impact upon the admittance testing theory being exercised in this paper.

19.6 Conclusion

Admittance testing of a complex aerospace structure or as we prefer the name frequency based substructuring continues to move into new areas of exploration. There are still many ideas or approaches that may be considered to perform additional experiments. As the model and structure move forward in areas of study, other areas of interest may arise.

Currently we have only gathered portions of the data to exercise the theory presented in this paper. Acoustic environmental tests in various configurations are being scheduled.

Consideration in areas such as instrumentation selection, setup, data acquisition setup, and noise levels should also be investigated as part of any large experiment as one relies on data being collected for supporting the admittance testing theory.

Future work will focus on implementing the theory outlined in the paper with more supporting data results from the original truth testing performed in order to verify the theory in a step by step fashion. The objective as testing moves forward in the program is to be able to predict output responses for each subsequent test in the series of tests for various excitation inputs with the use of admittance test theory for any payload or combination of payloads.

References

1. Bhardwaj M, Pierson K, Reese G, Walsh T, Day D, Alvin K, Peery J, Farhat C, Lesoinne M (2002) Salinas: a scalable software for high-performance structural and solid mechanics simulations. Supercomputing 2002, Baltimore, MD, November
2. Hensley DP, Mayes RL (2006) Extending SMAC to multiple references. Proceedings of the 24th international modal analysis conference, February, pp 220–230
3. Mayes RL (2012) Tutorial on experimental dynamic substructuring using the transmission simulator method. Proceedings of the 30th international modal analysis conference, paper 161, February

Chapter 20

Validation of Current State Frequency Based Substructuring Technology for the Characterisation of Steering Gear–Vehicle Interaction

M.V. van der Seijs, D. de Klerk, D.J. Rixen, and S. Rahimi

Abstract This paper presents a validation study of several experimental Frequency Based Substructuring (FBS) techniques that were developed recently. Advances in the techniques up to 2009 were already applied and validated using the rear axle differential - vehicle interaction as a test case. Since then several advances were achieved by various researchers, such as the Modal Constraints for Fixtures and Subsystems (MCFS) and the method of Equivalent Multi-Point Connections (EMPC). So far these advances have been tested on relatively simple academic problems. This validation study is conducted at BMW and focuses on the complex interaction between the steering gear and the vehicle. First, the response of the vehicle is obtained experimentally while a simple substitute beam is installed in place of the steering gear. This beam is then decoupled and replaced numerically by the actual steering gear using Frequency Response Functions (FRF) obtained from finite element models. The whole procedure of decoupling and coupling is evaluated by comparison with a validation measurement. Different to the MCFS method, the decoupling and coupling is performed using Lagrange Multiplier FBS instead of component mode synthesis. This paper also presents a different derivation for the virtual point transformation that extends and generalises the EMPC method.

Keywords Frequency based substructuring • Experimental substructuring • NVH • Virtual point transformation.

Nomenclature

Y	–	Measured receptance matrix
H	–	Virtual point receptance matrix
u, f	–	Measured displacements / forces
q, m	–	Virtual point displacements / forces
R, G	–	Transformation matrix for displacements / forces

DS	–	Dynamic Substructuring
FBS	–	Frequency Based Substructuring
FRF	–	Frequency Response Function
DoF	–	Degree of Freedom
EMPC	–	Equivalent Multi Point Connection

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20.1 Introduction

Dynamic Substructuring (DS) is widely used by industry to characterise the dynamic behaviour of complex structural systems. The component-wise approach of DS offers many advantages compared to a global method, such as the possibility to identify dynamic problems locally and the ability to couple numerically modelled parts to experimentally identified components [7]. A particular class of DS is the Frequency Based Substructuring (FBS) method that allows identification of important system properties (such as resonances and vibration levels) using the Frequency Response Functions (FRFs) of its subsystems. This method is often applied in the automotive industry for NVH¹ analysis and optimisation. Those studies typically focus on either reducing the overall noise to an acceptable level, or shaping the vibration levels in certain frequency bands such that a desired sound is experienced at the driver's ear.

20.1.1 Substructuring with Experimental and Numerical Components

In the transfer path from a certain excitation source to the receiver, numerous components can often be identified that exhibit complex dynamic behaviour. Some of those components may be too complex to model numerically, due to for instance high damping or non-linear effects. Others may be supplied by third parties that prefer not to share a detailed technical description. In such cases component FRFs should be measured in order to obtain a model that can be used for DS with other (numerically modelled) subsystems.

However up to today, the coupling of experimental components has proven to be one of the major challenges in DS. If the experimental components can be reasonably described by uncoupled modes, Component Mode Synthesis (CMS) techniques can be applied based on the classic Craig–Bampton method [4]. These modal-domain methods are efficient in use and can provide good physical insight in the dynamics of the coupled system. Also, the recently proposed Modal Constraints for Fixtures and Subsystems (MCFS) method offers a practical solution for measuring the modes of a subsystem at the interface correctly by introducing a so-called Transmission Simulator [1, 2, 10]. However in case of high-tech components as encountered in the automotive industry of today, it is often impractical or even impossible to decompose the dynamical behaviour into uncoupled modes. Therefore CMS methods cannot be applied and a frequency domain approach such as the Lagrange Multiplier Frequency Based Substructuring (LM-FBS) method [6, 9] is preferred.

Advances in experimental FBS up to 2009 have already been applied and validated successfully using the interaction between the rear axle differential and the vehicle as a test case [5]. Since then several advances have been made in the field of substructure coupling / decoupling and the coupling of experimental models to their numerical counterparts. So far these techniques have only been demonstrated on relatively simple academic problems.

20.1.2 Focus of Validation Study

This paper presents a validation of the current state of FBS techniques for the automotive industry. The study focuses on the complex interaction between the steering gear and the bodywork of a vehicle. In addition to traditional techniques, two concepts are particularly addressed:

1. *The transformation of measured FRFs into virtual point FRFs.* To be compatible with FE models, one should measure the FRFs for the exact connecting points. This is typically achieved by impact or shaker excitation at the interface points of the structure, while the translational accelerations are measured with 3D piezoelectric sensors. This way one neglects the coupling effects due to rotations at the interface, which can be a fair assumption for low frequency modes. For higher frequencies however (in this study above 1 kHz) it is known that the rotational Degrees of Freedom (DoFs) do play a significant role [8] and should be taken into account in the coupling equations.

Section 20.2 discusses the Equivalent Multi-Point Connection (EMPC) method to obtain a 6-DoF description for virtual points using the measured FRFs from multiple 3D sensors. A novelty is the use of excitations on additional measurement points on the structure instead of the sensor faces. This helps overcoming problems with cross-talk between the sensor channels and sensitivity to errors on the impact location.

¹Noise, Vibration and Harshness.

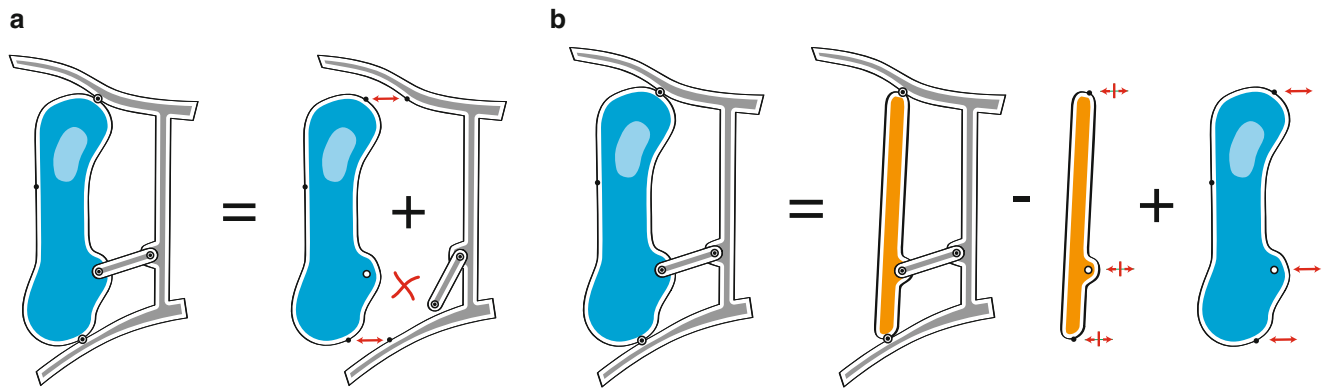


Fig. 20.1 Two approaches for subsystem coupling. Using a substitute beam during measurement, the subsystems can be measured in the correct configuration and damping effects in the coupling points are automatically taken into account. (a) Procedure of direct coupling. (b) Procedure of substitute coupling

2. *The use of a substitute beam in substructure coupling.* A strict requirement for substructure coupling and decoupling is to have coinciding interface DoFs. In practice it can be difficult to measure the subsystems in the exact same configuration as they are in the assembled situation, for instance when a connecting part is hinged as illustrated in Fig. 20.1a. To resolve this problem, a substitute part can be put in place that has the same connecting points as the original structure. This may later on be removed numerically by a decoupling step, after which the original structure can be coupled to obtain the desired configuration, as illustrated in Fig. 20.1b.

Another downside of direct coupling is the fact that damping introduced at the connecting points is neglected [3]. This generally causes the resonances and anti-resonances of the coupled receptance to be too prominent. If the substitute beam is designed to have the same geometry at the interface, one can assume that a similar damping effect is incorporated, resulting in a physically correct attenuation of the resonances and anti-resonances.

The substitute coupling as discussed in section 20.3 is conceptually very similar to the Transmission Simulator approach that was proposed by Allen and Mayes [2, 10]. In this work however, the method is put in the context of frequency based coupling and decoupling.

20.1.3 Test Case

In order to validate the aforementioned methods, a study has been conducted at the R&D facility of BMW in Munich. The aim of the study was to characterise the dynamic interaction between the steering gear and the vehicle through FBS with both measured and numerically modelled components. Accelerance FRFs have been obtained by impact hammer measurements for three subsystems: the *bodywork*, the *steering gear* and a so-called *substitute beam*. Substructuring has been applied to these subsystems in a similar fashion as illustrated in Fig. 20.1. The results of the test case are presented in section 20.4.

20.1.4 Paper Outline

This paper starts off in section 20.2 with the theory of the virtual point transformation that is used to transform the measured interface dynamics into a FEM compatible representation. Section 20.3 continues with a discussion of the substructure coupling and decoupling procedures. The results of the validation study are presented and discussed in section 20.4, followed by some conclusions in section 20.5.

20.2 Virtual Point Transformation

The dynamic substructuring algorithm formally requires that compatibility and equilibrium is explicitly satisfied on the interface points of the coupled substructures (see section 20.3). In case of FE models this implies that for every coinciding node three translations and three rotations² must be equal, while the forces and moments along these directions are equal and opposing in sign. Substructure coupling is therefore fully determined by 6 DoFs per coupling node.

In practice components are often coupled by bolted connections or welds (as illustrated in Fig. 20.2a), that physically show more resemblance to a line or surface connection than to a single point. Modelling a continuous contact surface or line is of course not feasible as one would theoretically need an infinite amount of DoFs. It is therefore common practice to reduce the interface problem to one or more connecting points, assuming that the structure surrounding the interface behaves rigidly, or at least in the frequency range of interest. The local rigidness of the interface is the essential observation that leads to the formulation of the EMPC method as discussed next.

The EMPC method proposes, as the name implies, to reduce the connectivity of substructures to a few translational directions at multiple points close to the contact area. Doing so, the rotational coupling is accounted for implicitly, as the structure near the contact area is locally rather rigid. This is illustrated in Fig. 20.2b. Measuring the FRFs for the translational directions can be straightforwardly done using 3D acceleration sensors and impact or shaker excitations. As a rule of thumb, $p = 3$ points (not in line) are generally sufficient to capture the coupling effects in all directions, leaving $3p = 9$ translational FRFs describing the interface.

It was however observed that the standard EMPC method results in a too strict definition of the interface problem. An extension of the method was proposed by [8]. By projecting the translational DoFs onto 6 rigid interface modes, one explicitly requires the interface points to obey rigidness. The 6 interface modes are described by the 3 translations and 3 rotations as seen from a single point, the so-called *virtual point* as illustrated in Fig. 20.2c. The *extended EMPC method* has some additional advantages:

- By projecting the overdetermined set of measured FRFs in a least-square sense (as will be shown), errors due to measurement noise and bad coherence are reduced.
- One can choose the location of the virtual point uniquely for different substructures. When substructures are physically different, it can be difficult to put the sensors in the exact same configuration relative to the interface location. By writing the coupling equations for the virtual point, this problem is immediately resolved.
- The virtual point described by 3 translational DoFs (T-DoFs) and 3 rotational DoFs (R-DoFs) is very easily coupled to FE models, as it completely describes the state (location and orientation) of a single node.

Additionally to the 6 rigid interface modes, one can choose to add flexible modes if more complex interface deformations are required. However the theory as discussed next is limited to rigid modes only.

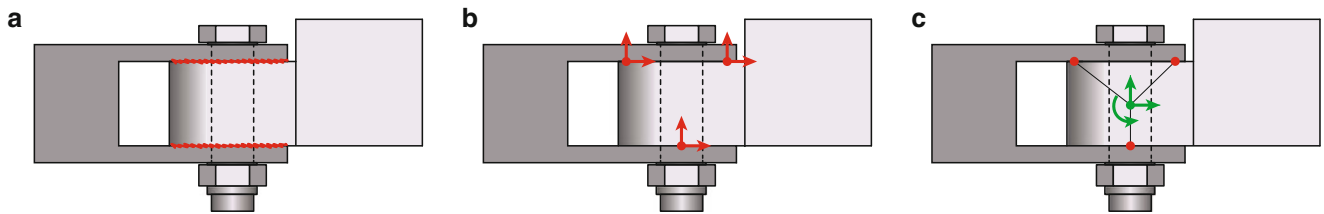
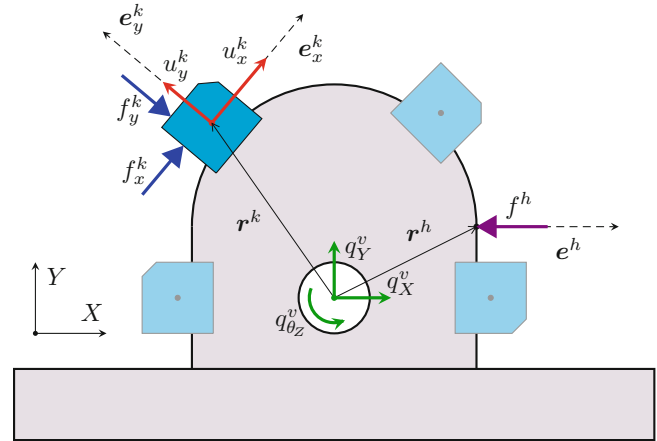


Fig. 20.2 The practice of substructure coupling: three ways to model the interface problem. (a) The full contact area. (b) Standard EMPC: multiple points, only translational directions. (c) Extended EMPC: One virtual point, three translations and three rotations

²Note that not all finite elements comprise both translational and rotational DoFs.

Fig. 20.3 Location $\mathbf{r}^k$ and orientation $\mathbf{e}^k$ of a sensor relative to the virtual point. The sensor displacements $\mathbf{u}^k$ are indicated by arrows and the impacts on the sensor $\mathbf{f}^k$ by thick arrows. The virtual point coordinates $\mathbf{q}^v$ are shown in the center. An additional excitation point on the structure is depicted by $\mathbf{f}^h$



20.2.1 Equivalent Multi-point Connection Method

In order to discuss the theory of the EMPC method, let us consider a single coupling point of a subsystem as illustrated in Fig. 20.3. The coupling point is surrounded by $n_k = 4$ triaxial acceleration sensors, measuring a total of 12 translational displacements.³ The derivations of the virtual point displacements and virtual point forces are very similar; the basic theory will be discussed along the derivation for the displacements.

Virtual Point Displacements

The displacements of sensor k in the local sensor frame (x, y, z) are denoted by the vector $\mathbf{u}^k = (u_x^k, u_y^k, u_z^k)$. The distance from the associated virtual point to the sensor is given by vector $\mathbf{r}^k$ and the orientation of the sensor with respect to the global frame (X, Y, Z) is determined by the three unit vectors $[\mathbf{e}_x^k, \mathbf{e}_y^k, \mathbf{e}_z^k] = \mathbf{E}^k$. The displacements in the global frame denoted by $\bar{\mathbf{u}}^k = (u_X^k, u_Y^k, u_Z^k)$ are then found using the components of the three unit vectors:

$$\bar{\mathbf{u}}^k = \begin{bmatrix} u_X^k \\ u_Y^k \\ u_Z^k \end{bmatrix} = \begin{bmatrix} e_{x,X}^k & e_{y,X}^k & e_{z,X}^k \\ e_{x,Y}^k & e_{y,Y}^k & e_{z,Y}^k \\ e_{x,Z}^k & e_{y,Z}^k & e_{z,Z}^k \end{bmatrix} \begin{bmatrix} u_x^k \\ u_y^k \\ u_z^k \end{bmatrix} = \mathbf{E}^k \mathbf{u}^k$$

The rotation procedure can be generalised for the complete set of displacements $\mathbf{u}$ using the block-diagonal rotation matrix $\mathbf{E} \triangleq \text{diag}(\mathbf{E}^{(1)}, \dots, \mathbf{E}^{(n_k)})$ with the property $\mathbf{E} \mathbf{E}^T = \mathbf{I}$, such that the following transformation holds:

$$\bar{\mathbf{u}} = \mathbf{E} \mathbf{u} \quad \text{and} \quad \mathbf{u} = \mathbf{E}^T \bar{\mathbf{u}} \quad (20.1)$$

Let us now express the sensor displacements in the global frame using the state of the virtual point. The displacements and (infinitesimal) rotations of virtual point v are contained in the DoF set $\mathbf{q}^v = (q_X^v, q_Y^v, q_Z^v, q_{\theta_X}^v, q_{\theta_Y}^v, q_{\theta_Z}^v)$. Knowing the geometric location $\mathbf{r}^k$ relative to the virtual point, one can set up the following relation for $\bar{\mathbf{u}}^k$:

$$\bar{\mathbf{u}}^k = \begin{bmatrix} u_X^k \\ u_Y^k \\ u_Z^k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & r_Z^k & -r_Y^k \\ 0 & 1 & 0 & -r_Z^k & 0 & r_X^k \\ 0 & 0 & 1 & r_Y^k & -r_X^k & 0 \end{bmatrix} \begin{bmatrix} q_X^v \\ q_Y^v \\ q_Z^v \\ q_{\theta_X}^v \\ q_{\theta_Y}^v \\ q_{\theta_Z}^v \end{bmatrix} + \begin{bmatrix} \mu_X^k \\ \mu_Y^k \\ \mu_Z^k \end{bmatrix} = \bar{\mathbf{R}}^{kv} \mathbf{q}^v + \bar{\boldsymbol{\mu}}^k \quad (20.2)$$

³ Actually the sensors measure accelerations, but for simplicity of notation displacements are considered in the derivation.

The virtual point displacements are directly related to the sensor displacements; the infinitesimal rotations are approximated linearly from the displacements and the “rotation arm” given by the components of $\mathbf{r}^k$. Vector $\bar{\boldsymbol{\mu}}^k$ contains the residuals of the sensor displacements that were not described by the coordinates of $\mathbf{q}^v$. Again, the transformation can be generalised for all sensors and virtual points by stacking the global displacement transformation matrices $\bar{\mathbf{R}}^{kv}$ row-wise for k and column-wise for v :

$$\bar{\mathbf{u}} = \bar{\mathbf{R}}\mathbf{q} + \bar{\boldsymbol{\mu}} \quad \text{with} \quad \bar{\mathbf{R}} \triangleq \begin{bmatrix} \bar{\mathbf{R}}^{(1,1)} & \dots & \bar{\mathbf{R}}^{(1,n_v)} \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{R}}^{(n_k,1)} & \dots & \bar{\mathbf{R}}^{(n_k,n_v)} \end{bmatrix}$$

The complete matrix $\bar{\mathbf{R}}$ typically consists of independent blocks, as each virtual point corresponds to an independent group of sensors. Finally, the sensor displacements can be obtained directly using a local frame rotation matrix:

$$\mathbf{u} = \mathbf{R}\mathbf{q} + \boldsymbol{\mu} \quad \text{with} \quad \mathbf{R} \triangleq \mathbf{E}^T \bar{\mathbf{R}} \quad (20.3)$$

Two remarks should be made on the transformation matrix:

- The transformation matrix $\mathbf{R}$ contains the n_q modes with admissible interface displacements for the n_u sensor channels, in other words: the modes that guarantee rigid interface behaviour. This implies that the displacements that do not obey rigidity end up in the residual displacement $\boldsymbol{\mu}$. The interpretation of this residual displacement is discussed in section 20.2.2.
- Throughout this derivation, the displacements have been grouped per sensor, i.e. per three channels. For programming purposes it may be easier to treat the sensor channels individually, such that a single displacement DoF u^i in the local direction $\mathbf{e}^i$ can be related to the associated set of virtual point coordinates $\mathbf{q}^v$. Combination and expansion of (20.1) and (20.3) then leads to the following direct relation:

$$u^i = \left[\mathbf{e}^{iT} \quad (\mathbf{r}^i \times \mathbf{e}^i)^T \right] \mathbf{q}^v + \mu^i = \mathbf{R}^{iv} \mathbf{q}^v + \mu^i \quad (20.4)$$

from which the full matrix $\mathbf{R}$ can be assembled row-wise per channel i .

In order to obtain an expression for the virtual point coordinates, (20.3) is pre-multiplied with the transposed transformation matrix. The residual displacements drop out as they are orthogonal to the space spanned by $\mathbf{R}$ such that $\mathbf{R}^T \boldsymbol{\mu} = \mathbf{0}$.

$$\mathbf{R}^T \mathbf{u} = \mathbf{R}^T (\mathbf{R}\mathbf{q} + \boldsymbol{\mu}) = (\mathbf{R}^T \mathbf{R}) \mathbf{q}$$

The expression can now be inverted to obtain the virtual point coordinates:

$$\mathbf{q} = \mathbf{T}_u \mathbf{u} \quad \text{with} \quad \mathbf{T}_u \triangleq (\mathbf{R}^T \mathbf{R})^{-1} \mathbf{R}^T \quad (20.5)$$

The inverse transformation matrix $\mathbf{T}_u$ defines the transformation and projection from $\mathbf{u}$ to $\mathbf{q}$; it has size $n_q \times n_u$ and rank n_q . The meaning of the rank and conditioning of $\mathbf{R}$ and $\mathbf{T}_u$ will be addressed in section 20.2.2.

Virtual Point Forces

Equations (20.1) to (20.5) comprise all the necessary steps for the virtual point transformation of the displacements. The transformation of the forces can be performed in a similar step-wise way. Virtual forces and moments of virtual point v are denoted by the set $\mathbf{m}^v = (m_X^v, m_Y^v, m_Z^v, m_{\theta_X}^v, m_{\theta_Y}^v, m_{\theta_Z}^v)$. However, the translational forces $\mathbf{f}$ are not uniquely defined by the virtual point forces and moments $\mathbf{m}$. In fact the other way around is true, such that the following relation can be started with for the virtual loads $\mathbf{m}^v$ as a result of excitations $\mathbf{f}^k$ on the faces of sensor k :

$$\mathbf{m}^v = \begin{bmatrix} m_X^v \\ m_Y^v \\ m_Z^v \\ m_{\theta_X}^v \\ m_{\theta_Y}^v \\ m_{\theta_Z}^v \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -r_Z^k & r_Y^k \\ r_Z^k & 0 & -r_X^k \\ -r_Y^k & r_X^k & 0 \end{bmatrix} \begin{bmatrix} f_X^k \\ f_Y^k \\ f_Z^k \end{bmatrix} = \bar{\mathbf{G}}^{vk} \bar{\mathbf{f}}^k \quad (20.6)$$

$\bar{\mathbf{G}}^{vk}$ can be stacked column-wise for the n_k sensors and row-wise for the n_v virtual points to obtain $\bar{\mathbf{G}}$. After rotation, the following relation is obtained:

$$\mathbf{m} = \mathbf{G} \mathbf{f} \quad \text{with} \quad \mathbf{G} \triangleq \bar{\mathbf{G}} \mathbf{E} \quad (20.7)$$

As far as the excitations on the sensor faces are concerned, $\mathbf{G}$ is simply the transposed of $\mathbf{R}$, although care should be taken to include the directions of the impact forces correctly. If additional excitation points are considered as illustrated in Fig. 20.3, the system of equations should be expanded as follows:

$$\mathbf{m}^v = \begin{bmatrix} \mathbf{e}^h \\ \mathbf{r}^h \times \mathbf{e}^h \end{bmatrix} \mathbf{f}^h = \mathbf{G}^{vh} \mathbf{f}^h \quad (20.8)$$

Let us now try to invert equation (20.7) to obtain an expression for the forces $\mathbf{f}$. $\mathbf{G}$ consists of $n_m < n_f$ equations, which means that there are multiple combinations of forces $\mathbf{f}$ possible that result in the same forces and moments $\mathbf{m}$. Mathematically speaking: the columns of $\mathbf{G}$ are linearly dependent. We choose to apply the *Moore-Penrose pseudo-inverse* or right generalised inverse $\mathbf{G}^+ = \mathbf{G}^T (\mathbf{G} \mathbf{G}^T)^{-1}$ which leads to a solution for $\mathbf{f}$ in a minimal form:

$$\mathbf{f} = \mathbf{G}^+ \mathbf{m} = \mathbf{G}^T (\mathbf{G} \mathbf{G}^T)^{-1} \mathbf{m} \quad (20.9)$$

To be consistent with the notation of the displacements, a transformation matrix $\mathbf{T}_f$ is introduced in a transposed notation such that:

$$\mathbf{f} = \mathbf{T}_f^T \mathbf{m} \quad \text{with} \quad \mathbf{T}_f^T \triangleq \mathbf{G}^T (\mathbf{G} \mathbf{G}^T)^{-1} \quad (20.10)$$

Note that residual forces did not appear in this derivation, which leads to the interesting observation: *the forces computed by (20.10) resulting from an arbitrary combination of $\mathbf{m}$ will only load the interface in a rigid (hence non-elastic) way.* This is analogue to the previous statement that $\mathbf{f}$ is determined in a minimal sense.

Virtual Point Receptance

The equations for the displacement and force transformation can now be used to define the virtual point receptance. Let us first state the dynamic equation for the interface problem in the frequency domain:

$$\mathbf{u} = \mathbf{Y} \mathbf{f} \quad (20.11)$$

Substitution of (20.5) and (20.10) in (20.11) gives the following:

$$\begin{aligned} \mathbf{q} &= \mathbf{T}_u \mathbf{Y} \mathbf{T}_f^T \mathbf{m} \\ \mathbf{q} &= \mathbf{H} \mathbf{m} \quad \text{with} \quad \mathbf{H} \triangleq \mathbf{T}_u \mathbf{Y} \mathbf{T}_f^T \end{aligned} \quad (20.12)$$

Matrix $\mathbf{H}$ is the $n_q \times n_m$ receptance matrix for the frequency response functions of the virtual displacements to the virtual forces. Note that for the interface DoFs this matrix must be square, i.e. $n_{q_{\text{int}}} = n_{m_{\text{int}}}$, so that $\mathbf{H}_{\text{int}}$ can be inverted to obtain the dynamic stiffness matrix for the virtual interface points. This is effectively what will be done for the substructure coupling / decoupling in section 20.3. It is also worth mentioning here that the interface described by the virtual points comprises less DoFs than the collection of sensor channels for the interface, i.e. $n_{q_{\text{int}}} < n_{u_{\text{int}}}$. This means that the interface problem is “weakened” by the EMPC method which is in practice often beneficial for coupling / decoupling.

20.2.2 Performance of the Virtual Point Transformation

The application of the EMPC theory as presented in section 20.2.1 can be summarised as follows:

$$\mathbf{Y}_{n_u \times n_f} \xrightarrow{\mathbf{R}_{n_u \times n_q}, \mathbf{G}_{n_m \times n_f}} \mathbf{H}_{n_q \times n_m}$$

This involves a double coordinate transformation: one for the sensor displacements (20.5) and one for the force excitations (20.10). For the transformation to be fully determined, both $(\mathbf{R}^T \mathbf{R})$ and $(\mathbf{G} \mathbf{G}^T)$ must be full rank. For the displacement transformation, there are two possible causes for a rank deficiency: either the amount of measured sensor displacements is insufficient ($n_u < n_q$), or the locations and directions are not independent enough to determine the 3 translations and 3 rotations of the virtual point. If only two 3D sensors are used, one can imagine that the rotation along the axis between those sensors cannot be determined. This demonstrates why a minimum of three sensors is desired in practice. With respect to the force transformation a similar reasoning holds.

From that perspective it can also be understood that the virtual point receptance is determined in a least-square sense. Technically $\mathbf{T}_u$ and $\mathbf{T}_f^T$ are the pseudo-inverses of respectively $\mathbf{R}$ and $\mathbf{G}$. In the likely case that $n_u > n_q$ and $n_f > n_m$, filtering is applied on both the sensor and excitation side of the measured receptance matrix $\mathbf{Y}$.

It is now interesting to evaluate the effect of the filtering on the measured interface receptance, hence the validity of the rigidness assumption. One can define a filtered set of displacements $\tilde{\mathbf{u}}$ by rearranging (20.3):

$$\tilde{\mathbf{u}} = \mathbf{u} - \boldsymbol{\mu} = \mathbf{R} \mathbf{q}$$

For some given excitation $\mathbf{f}$, it follows that:

$$\begin{cases} \mathbf{u}(\mathbf{f}) = \mathbf{Y} \mathbf{f} \\ \tilde{\mathbf{u}}(\mathbf{f}) = \mathbf{R} \mathbf{q}(\mathbf{f}) = \mathbf{R} \mathbf{T}_u \mathbf{Y} \mathbf{f} \end{cases} \quad (20.13)$$

By definition $\tilde{\mathbf{u}}$ will be smaller than $\mathbf{u}$. A measure of how much of the excitation of $\mathbf{f}$ contributes to a rigid interface displacement can be defined using the Cauchy–Schwarz inequality. It takes the same form as for example the coherence factor for measured FRFs:

$$\rho^2 = \frac{(\mathbf{u} \cdot \tilde{\mathbf{u}})^2}{(\mathbf{u} \cdot \mathbf{u})(\tilde{\mathbf{u}} \cdot \tilde{\mathbf{u}})} \quad (20.14)$$

The so-called *rigidness* ρ is a frequency dependent value between zero (no rigid behaviour) and one (perfect rigid behaviour) for a specific load case $\mathbf{f}$.

If the rigidness is evaluated for a single impact j , one obtains a quantitative measure for how much of the impact resulted in rigid interface behaviour. This value ρ_j can be used in judging whether to retain or reject certain measured FRFs for the virtual point transformation. It will be shown in section 20.4.1 that the FRFs from additional excitations on the structure generally show much higher rigidness values than the from the excitations on the sensor faces.

20.3 Dynamic Substructuring

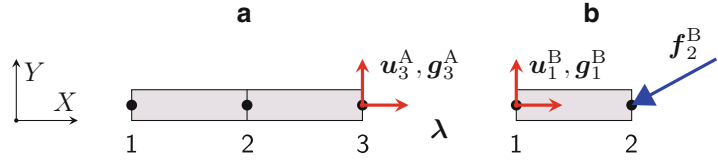
Many research has been devoted to dynamic substructuring (DS) with frequency response functions (FRFs) of the subsystems, commonly referred to as Frequency Based Substructuring (FBS). In this study we choose to apply Lagrange Multiplier Frequency Based Substructuring (LM-FBS) as described by for instance [6, 7], for its power to perform both coupling and decoupling in an organised and intuitive way. Before the coupling and decoupling variants are discussed in section 20.3.2, a short summary of LM-FBS is given below.

20.3.1 Lagrange Multiplier FBS

As for most DS methods LM-FBS requires two conditions to be satisfied explicitly, namely the compatibility of displacement and equilibrium of forces on the interface of connecting substructures. Essential in this technique is the definition of the signed Boolean matrix, that enables us to write the compatibility as well as the equilibrium condition in a convenient way.

Let us illustrate the use of the Boolean matrix by coupling two structures A and B as depicted in Fig. 20.4. For the purpose of illustration the problem is reduced to X and Y displacements only, such that the receptance FRFs of the structures are respectively denoted by the 6×6 matrix $\mathbf{Y}_A$ and 4×4 matrix $\mathbf{Y}_B$. Substructure coupling is performed at node 3 of A and node 1 of B. The compatibility conditions to realise the coupling requires the interface displacements $\mathbf{u}$ to be equal, while the interface forces $\mathbf{g}$ are equal and opposite in sign:

Fig. 20.4 Substructuring
example: two structures are
coupled at their interface point



$$\begin{cases} u_{3X}^A = u_{1X}^B \\ u_{3Y}^A = u_{1Y}^B \end{cases}, \quad \begin{cases} g_{3X}^A + g_{1X}^B = 0 \\ g_{3Y}^A + g_{1Y}^B = 0 \end{cases}$$

Using the signed boolean matrix $\mathbf{B}$ the two conditions can be written efficiently:

$$\mathbf{B}\mathbf{u} = \mathbf{0} \quad (20.15)$$

$$\mathbf{g} = -\mathbf{B}^T \boldsymbol{\lambda} \quad (20.16)$$

with:

$$\mathbf{B} = [\mathbf{0}_{2 \times 4} \quad \mathbf{I}_{2 \times 2} \quad -\mathbf{I}_{2 \times 2} \quad \mathbf{0}_{2 \times 2}], \quad \mathbf{u} = \begin{bmatrix} \mathbf{u}^A \\ \mathbf{u}^B \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} \mathbf{g}^A \\ \mathbf{g}^B \end{bmatrix}$$

$\boldsymbol{\lambda}$ is the set of Lagrange multipliers that represent the interface forces to keep the two structures together. Calling $\mathbf{Y} = \text{diag}(\mathbf{Y}^A, \mathbf{Y}^B)$ the block-diagonal receptance matrix of the uncoupled systems, the dynamic equation for the coupled system is then written as:

$$\mathbf{u} = \mathbf{Y}(\mathbf{f} + \mathbf{g}) = \mathbf{Y}(\mathbf{f} - \mathbf{B}^T \boldsymbol{\lambda}) \quad (20.17)$$

The Lagrange multipliers $\boldsymbol{\lambda}$ are however unknown. It can be shown [6] that the following relations hold for respectively the interface incompatibility, the dynamic interface stiffness and the Lagrange Multipliers:

$$\Delta \mathbf{u}_{\text{int}} = \mathbf{B}\mathbf{Y}\mathbf{f}$$

$$\mathbf{Z}_{\text{int}} = (\mathbf{B}\mathbf{Y}\mathbf{B}^T)^{-1}$$

$$\boldsymbol{\lambda} = \mathbf{Z}_{\text{int}} \Delta \mathbf{u}_{\text{int}}$$

such that the displacements for the coupled system are found by

$$\mathbf{u}^{\text{coupled}} = \mathbf{Y} \left(\mathbf{f} - \mathbf{B}^T (\mathbf{B}\mathbf{Y}\mathbf{B}^T)^{-1} \mathbf{B}\mathbf{Y}\mathbf{f} \right) \quad (20.18)$$

Equation (20.18) expresses the LM-FBS method in a single statement. In many cases one is interested in the system characteristics or FRF of the coupled system:

$$\mathbf{Y}^{\text{coupled}} = \mathbf{Y} - \mathbf{Y}\mathbf{B}^T (\mathbf{B}\mathbf{Y}\mathbf{B}^T)^{-1} \mathbf{B}\mathbf{Y} \quad (20.19)$$

This equation will be used for the analysis to obtain the coupled FRF.

20.3.2 FBS Coupling and Decoupling

The substructuring equation (20.19) can be used for both coupling and decoupling. The subsystem coupling steps are performed exactly as described by the above procedure. Decoupling can be performed using the same procedure, but in that case a minus sign is put before the receptance matrix of the subsystem to decouple. Let us illustrate what this means for the uncoupled receptance matrices:

- Coupling: Y^{A+B} follows from (20.19) with $Y = \begin{bmatrix} Y^A & \mathbf{0} \\ \mathbf{0} & Y^B \end{bmatrix}$.
- Decoupling: $Y^{(A|B)-B}$ follows from (20.19) with $Y = \begin{bmatrix} Y^{(A|B)} & \mathbf{0} \\ \mathbf{0} & -Y^B \end{bmatrix}$.

In case of coupling, subsystem A and B share only the interface points; the Boolean matrix is constructed such that the interface receptance is used for coupling. In case of decoupling, some internal points of subsystem B may also appear in the receptance matrix of (A|B). Those internal points, although physically not directly connected to A, can still be used to improve the conditioning of the interface stiffness. Some decoupling variants that are derived from this idea are described by [11]. For this study the *non-located overdetermined* method is used, which basically means that the compatibility condition is written on both the virtual coupling points and the virtual internal points, while the equilibrium condition is satisfied only on the virtual coupling points. Finally we can define the procedure of substitute coupling:

- Substitute coupling: $Y^{(A|C)-C+B}$ follows from:
 1. Decoupling using (20.19) with $Y = \begin{bmatrix} Y^{(A|C)} & \mathbf{0} \\ \mathbf{0} & -Y^C \end{bmatrix}$;
 2. Coupling using (20.19) with $Y = \begin{bmatrix} Y^{(A|C)-C} & \mathbf{0} \\ \mathbf{0} & Y^B \end{bmatrix}$.

Various substructuring variants have been considered in this study, of which the most interesting results will be discussed next.

20.4 Test Case

The FBS validation study at BMW was focused on the interaction between the steering gear and the vehicle. Let us shortly discuss the subsystems:

1. The bodywork (BW): the vehicle without the steering gear installed. Vibration transmission into the vehicle is measured using a 3D accelerometer on the steering wheel and a microphone at the position of the driver's ear.
2. The steering gear (SG). The SG has two bolted connections to the BW and a third hinged connection to the steering axis. The two connections to the front wheel suspension have been disconnected during the measurements.
3. The substitute beam (SB) for the purpose of substitute coupling. It was made out of steel and designed such that it has the same interface points as the SG and roughly the same geometry.

Impact hammer measurements have been performed to obtain FRFs for the following three different assemblies:

1. Validation measurement of the vehicle with SG installed (see Fig. 20.1a);
2. Measurement of the disassembled vehicle;
3. Measurement of the vehicle with SB in place (see Fig. 20.1b).

In order to obtain translations and rotations for the virtual coupling points, 4 sensors per coupling point are used. Furthermore, 4 internal points were measured on the SG or SB. In addition to these measurements, FRFs are synthesised for the SG and SB based on FEM models. Much care has been taken to achieve an exact match between the locations and coordinate systems of the virtual coupling points of the experimental and FE models.

20.4.1 Results of the Virtual Point Transformation

The virtual point transformation as discussed in section 20.2 has been applied to the measurement data. Per coupling points, 4 3D acceleration sensors have been used as shown in Fig. 20.5 (the fourth sensor is not visible). The sensor locations relative to the structure were identified with the aid of a photogrammetry system. With this system, the coordinates of three dots per sensor face were identified, such that the location and orientation of the sensors could be estimated very accurately.

Fig. 20.5 Three sensors are placed around the bolted coupling point (the fourth is not visible). The dotted stickers were identified using photogrammetry in order to determine the location of the sensors. The six points on the structure indicate the additional excitation points



The four sensors together yield twice as much acceleration channels as needed to determine the 6 DoFs of the virtual point. All sensor accelerations have been measured simultaneously, such that full columns of the receptance matrix were measured at once. With respect to the force excitations, we determine two groups:

1. Excitations on all three faces of the sensors;
2. Excitations on additional measurement points on the structure around the virtual point.

The first group of excitations enables construction of a full accelerance matrix for the sensor point FRFs; the diagonal terms of the matrix indeed correspond to driving-point FRFs. However, those FRFs often exhibit poor coherence, especially for the cross-directional FRFs within one sensor, e.g. $Y_{lx,ly}$. Possible reasons are cross-talk within the sensor (excessive responses of the Y and Z channel of a sensor when excited in the X direction) and locally a high sensitivity to errors on the impact location. The idea was therefore to introduce an additional set of excitations on measurement points near the sensors. Figure 20.5 shows 6 points that lie on the structure near the sensors, but a little bit further away from the virtual point. In contrast to the excitations on the sensor, those channels show much higher coherence and could therefore be of great value in estimating the virtual point FRFs.

Two sets of virtual point data have been generated. The first set only uses the 12 accelerance columns corresponding to the excitations on the sensor faces; the second set comprises the additional points and a selection of the (in terms of coherence) best on-sensor channels. The results are shown in Fig. 20.6. In order to evaluate the quality of the FRFs, pairs of reciprocal channels are shown. Clearly, the second set shows in general better reciprocity, suggesting a more truthful representation of the dynamics.

The question remains though to what extent those positions obey the local rigidity of the interface that is a requirement for application of the EMPC method. The rigidness value as defined by (20.14) is a good estimator for that purpose. Figure 20.7 shows two rigidness functions for an excitation around one of the virtual coupling points: the dark line corresponds to an excitation on the sensor face; the light line to an additional measurement point. The rigidness of the sensor excitation drops rapidly whereas the rigidness of the additional excitation point remains close to 1. This is another indication that the FRF of the sensor excitation is highly affected by the error on impact location, while the additional excitation points generally provide good FRFs.

20.4.2 Results of Dynamic Substructuring

Dynamic substructuring has been applied to the measured and modelled subsystem FRFs, in the full frequency range of 3 kHz. The subsystem connectivity can be understood from Fig. 20.1.

Direct Coupling

Let us first discuss the results of the direct coupling of the substitute beam to the disassembled vehicle. The FRFs of the beam were obtained from FE modelling, whereas the FRFs of the both the assembled vehicle were measured and transformed to

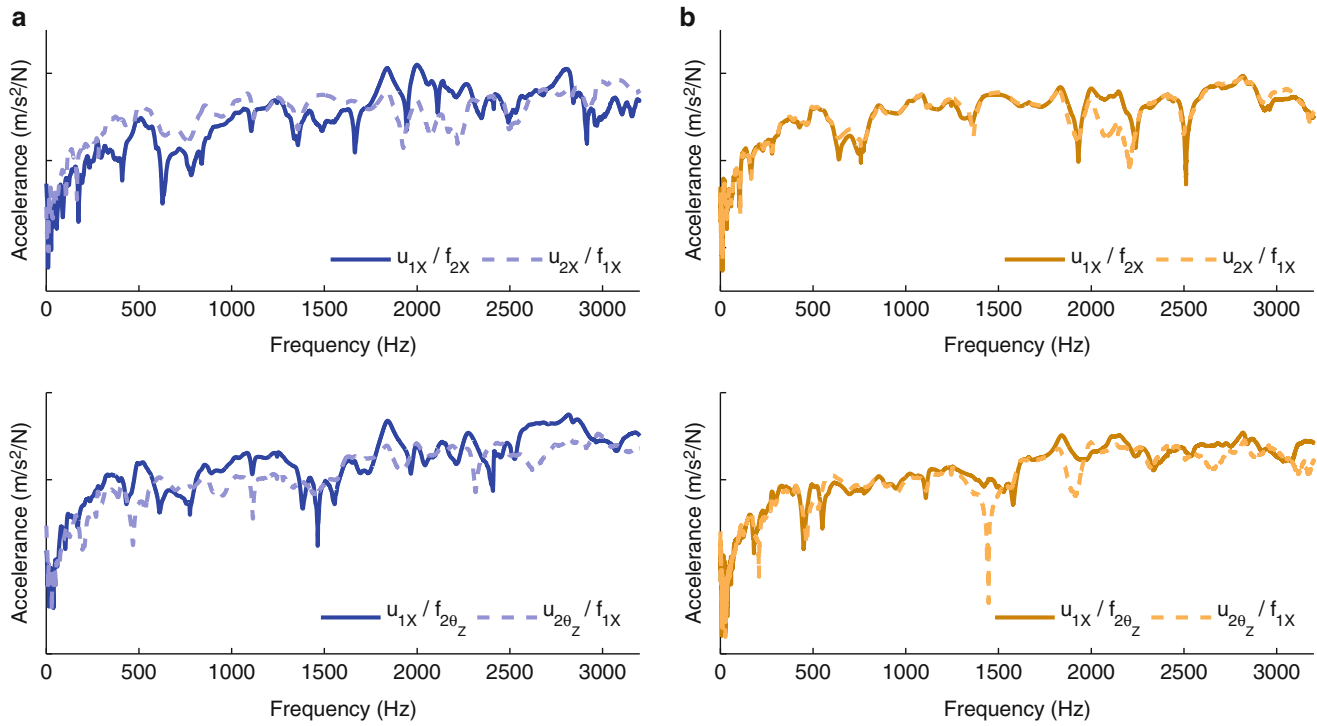
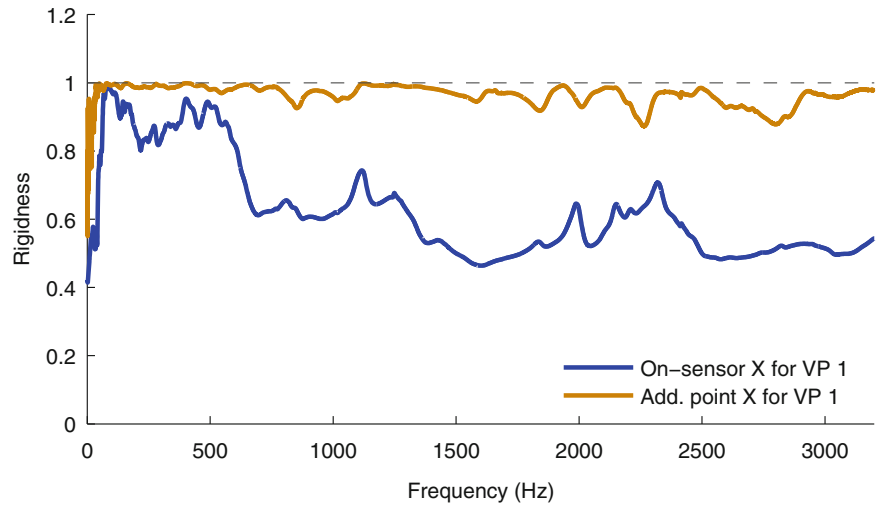


Fig. 20.6 Effect of using additional excitation points in the virtual point transformation. (a) Excitations only on sensor faces. (b) Additional excitations on the structure

Fig. 20.7 Rigidity of an excitation on the sensor and on an additional measurement point around VP 1



virtual points. Coupling was established for the two bolted connections only, as the coupling to the steering axis could not be achieved due to the difference in configuration. Figure 20.8 shows the results for direct coupling BW+SB through respectively the T-DoFs only (light) and both the T-DoFs and R-DoFs or 6-DoFs (dark). It is validated against the (BW—SB) measurement (black dashed) and the FRF of the substitute beam only (gray dashed). Some observations:

1. The effects of rotational coupling start to be significant above approximately 1 kHz. For higher frequencies the result of the 6-DoFs coupling seems to match the validation measurement best.
2. Direct coupling neglects the interface damping that is present in the validation measurement. It is seen that the resonances and anti-resonances of the coupling results are to prominent. It was indeed verified that the largest errors are made around the resonances and anti-resonances of the substitute beam model.

Fig. 20.8 Direct coupling of the substitute beam to the bodywork

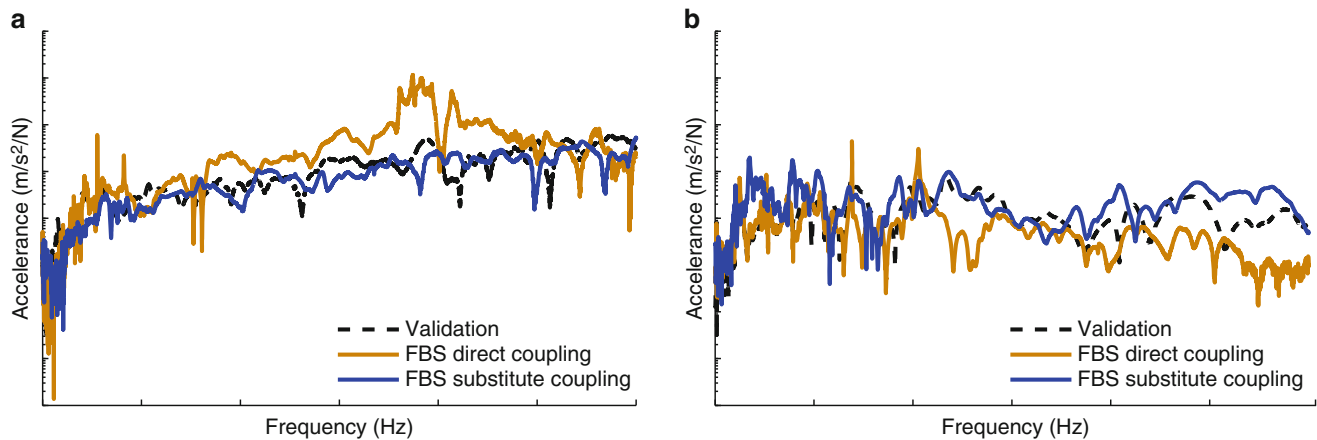
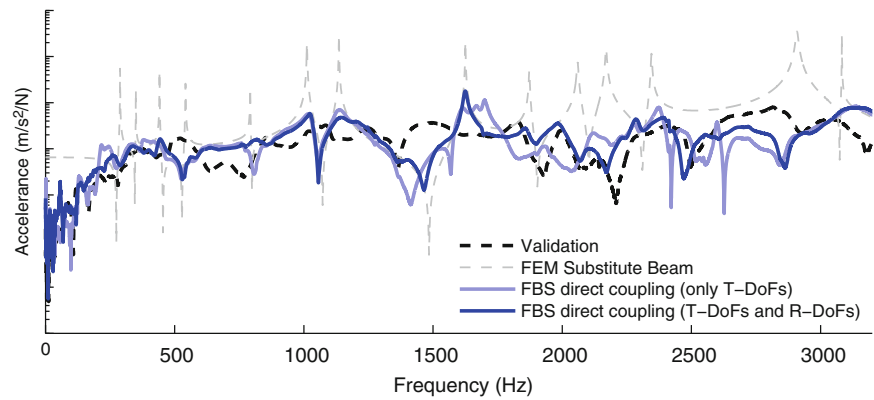


Fig. 20.9 Results of substitute coupling and direct coupling compared to a validation measurement of the assembled vehicle. (a) Transfer between the bolted connections points. (b) Transfer from the steering gear to the steering wheel

Direct coupling of the steering gear to the disassembled vehicle (BW—SG) provided similar insights; the results are omitted here.

Substitute Coupling

We can now evaluate the results of substitute coupling. Two transfer functions are considered: Fig. 20.9a shows the transfer from one of the bolted connections to the other; Fig. 20.9b shows the transfer from one of the coupling points to the steering wheel in the vehicle. Substitute coupling (BW—SB)—SB+SG and direct coupling BW+SG are compared with the validation measurement of the assembled vehicle (BW—SG). It is interesting to note that the model of the SB was obtained from FE modelling, whereas experimental models were used for both the BW and the SG.

In both cases the substitute coupling shows the best results compared to the validation measurements. It can be seen from the left figure that the direct coupling yields an overestimation of the accelerance within the steering gear. This may be due to the fact that the interface damping is neglected. The right figure on the other hand clearly shows the effect of the missing connection to the steering axis for direct coupling: the accelerance is underestimated.

In general it was found that substitute coupling provides more accurate results than direct coupling. Summarising, substitute coupling has offered the following advantages:

- The effects of damping at the interface has been taken into account;
- The coupling from the steering gear to the steering axis could be realised;
- The virtual point FRFs could be estimated more accurately for the (BW—SB) assembly than for the disconnected BW, due to easier accommodation of sensors for the first assembly.

20.5 Conclusions

This work presented a validation of some recently developed FBS techniques. Substructuring has been applied to models originating from FE models as well as experimental measurements. All substructuring procedures have been performed with FRFs for virtual (interface) points, obtained through a transformation based on the EMPC method. This method has been extended with excitations on additional measurement points on the structure near the sensors, leading to an improvement on the virtual point receptance.

Substitute coupling has been proposed as the FBS counterpart of the MCFS method. It has been shown that the method can overcome some problems of direct coupling, such as the inability to incorporate the interface damping and the loss of coupling dynamics at points that would traditionally be difficult to connect in a physically correct way.

Future work will be focused on extending the virtual point transformation and a further validation of the substitute coupling method.

Acknowledgements This study was performed in close collaboration with BMW, whom we greatly acknowledge for their support and organisation. We also like to thank Simon Mayer for performing the photogrammetry measurements and Daniël van den Bosch for his excellent work on the modelling of the experimental substructures. This research is supported and funded by the BMW AG.

References

1. Allen MS, Mayes RL (2007) Comparison of FRF and modal methods for combining experimental analytical substructures. In: Proceedings of the XXV international modal analysis conference. Society for Experimental Mechanics. Bethel, CT
2. Allen MS, Mayes RL, Bergman EJ (2010) Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections. *J Sound Vib* 329(23):4891–4906
3. Bakker M (2011) Identification of damping in joints using dynamic substructuring techniques. Master's thesis, Delft University of Technology
4. Craig RRJ, Bampton MCC (1968) Coupling of substructures using component mode synthesis. *AIAA J* 6:1313–1319
5. de Klerk D (2009) Dynamic response characterization of complex systems through operational identification and dynamic substructuring. PhD thesis, Delft University of Technology
6. de Klerk D, Rixen DJ, de Jong J (2006) The frequency based substructuring method reformulated according to the dual domain decomposition method. In: Proceedings of the XXIV international modal analysis conference. Society for Experimental Mechanics. Bethel, CT
7. de Klerk D, Rixen DJ, Voormeeren SN (2008) A general framework for dynamic substructuring – history, review and classification of techniques. *AIAA J* 46(5):1169–1181
8. de Klerk D, Rixen DJ, Voormeeren SN, Pasteuning F (2008) Solving the RDoF problem in experimental dynamic substructuring. In: Proceedings of the XXVI international modal analysis conference. Society for Experimental Mechanics. Bethel, CT
9. Jetmundsen B, Bielawa RL, Flannelly WG (1988) Generalized frequency domain substructure synthesis. *J Am Helicopter Soc* 33(1):55–64
10. Mayes RL (2012) Tutorial on experimental dynamic substructuring using the transmission simulator method. In: Proceedings of the XXX international modal analysis conference. Society for Experimental Mechanics. Bethel, CT
11. Voormeeren SN, Rixen DJ (2012) A family of substructure decoupling techniques based on a dual assembly approach. *Mech Syst Signal Process* 27(18):379

Chapter 21

Strategies to Exploit Test Data in Subsystem Subtraction

Walter D'Ambrogio and Annalisa Fregolent

Abstract In dynamic substructuring, the subtraction of subsystems (decoupling) is a very critical problem even in apparently trivial applications. Using simulated FRF data, it has been highlighted that - as predicted by the theory - the solution of decoupling problems is affected by ill-conditioning around a discrete number of frequencies, and is dependent on the choice of the 'measured DoFs'. In experimental dynamic substructuring, additional problems can arise (systematic errors, inconsistencies, etc.) that are strictly connected with the use of measured FRFs, and different strategies to obtain reliable results can be necessary. In this paper, experimental data, acquired on a test bed made by a plate and a rigid mass, are used to check the decoupling procedure and to look for additional issues that can not be observed from simulated data.

Keywords Decoupling • Experimental Dynamic Substructuring • Interface DoFs

21.1 Introduction

This paper deals with subsystem subtraction or substructure decoupling, i.e. how to get a substructure model from an assembled system in an experimental context. A typical yet simple application is mass cancellation, to correct for mass loading of accelerometers on FRF measurements. More generally, decoupling is a relevant issue for subsystems that cannot be measured separately, but only when coupled to their neighboring substructure(s) (e.g. a fixture needed for testing or subsystems that are very delicate or in operational conditions). To be more precise, the decoupling problem is defined as the identification of the dynamic behaviour of a structural subsystem, starting from the known dynamic behaviour of the assembled system, and of the residual subsystem, that is the remaining part of the structural system.

Subsystem subtraction is a special case of experimental dynamic substructuring, where experimental means that the model of at least one subsystem must derive from experimental tests. In fact, while for subsystem addition many well established techniques exist even when all substructures are modeled theoretically, in subsystem subtraction this would give no practical advantage. Due to modal truncation problems, FRFs (Frequency Based Substructuring) are preferred to modal parameters. The main algorithm for frequency based substructuring is the improved impedance coupling [1] that involves just one matrix inversion whereas the classical impedance coupling technique requires three inversions. A general framework for dynamic substructuring is provided in [2, 3], where primal and dual formulation are introduced.

Substructure decoupling techniques can be classified as inverse coupling techniques or direct decoupling techniques. In inverse coupling, the equations written for the coupling problem are rearranged to isolate (as unknown) one of the substructures instead of the assembled structure. Examples of inverse coupling are impedance and mobility approaches [4, 5]. Direct decoupling consists in adding to the assembled system a fictitious subsystem, which is the negative of the residual subsystem [6–8]. The technique starts from the 3-field formulation: one set of equations expressing the dynamic balance of

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the assembled system and, separately, of the fictitious subsystem; one set of equations enforcing compatibility at interface DoFs, one set of equations enforcing equilibrium of constraint forces at interface DoFs. To solve the problem, a dual approach [7], a primal approach [9] or an hybrid approach [10] can be used.

In experimental dynamic substructuring, the solution of decoupling problems is affected by ill-conditioning around a discrete number of frequencies [4, 5, 7]. Moreover, additional problems can arise (systematic errors, inconsistencies, etc.) that are strictly connected with the use of measured FRFs, and different strategies to obtain reliable results can be necessary. One of these strategies consists of smoothing measured FRFs using global curve fitting techniques. In this paper, experimental data, acquired on a test bed made by a plate and a rigid mass, are used to check the decoupling procedure and to look for additional issues that can not be observed from simulated data.

21.2 Direct Decoupling Techniques

The coupled structural system RU (N_{RU} DoFs) is assumed to be made by an unknown subsystem U (N_U DoFs) and a residual subsystem R (N_R DoFs) joined through a number of couplings (see Fig. 21.1). The residual subsystem (R) can be made by one or more substructures. The degrees of freedom (DoFs) of the coupled system can be partitioned into internal DoFs (not belonging to the couplings) of subsystem U (u), internal DoFs of subsystem R (r), and coupling DoFs (c).

It is required to find the FRF of the unknown substructure U starting from the FRF of the coupled system RU . The subsystem U can be extracted from the coupled system RU by cancelling the dynamic effect of the residual subsystem R . This can be accomplished by adding to the coupled system RU a fictitious subsystem with a dynamic stiffness opposite to that of the residual subsystem R and satisfying compatibility and equilibrium conditions. The dynamic equilibrium of the coupled system RU and of the fictitious subsystem can be expressed in block diagonal format as:

$$\begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{RU} \\ \mathbf{f}^R \end{Bmatrix} + \begin{Bmatrix} \mathbf{g}^{RU} \\ \mathbf{g}^R \end{Bmatrix} \quad (21.1)$$

where:

- $\mathbf{Z}^{RU}$, $\mathbf{Z}^R$ are the dynamic stiffness matrices of the coupled system RU and of the residual subsystem R , respectively;
- $\mathbf{u}^{RU}$, $\mathbf{u}^R$ are the vectors of degrees of freedom of the coupled system RU and of the residual subsystem R , respectively;
- $\mathbf{f}^{RU}$, $\mathbf{f}^R$ are the external force vectors on the coupled system RU and on the fictitious subsystem, respectively;
- $\mathbf{g}^{RU}$, $\mathbf{g}^R$ are the vectors of connecting forces between the coupled system and the fictitious subsystem, and viceversa (constraint forces associated with compatibility conditions).

According to this point of view, the interface between the coupled system RU and the fictitious subsystem should not only include all the coupling DoFs between subsystems U and R , but should as well include all the internal DoFs of subsystem R .

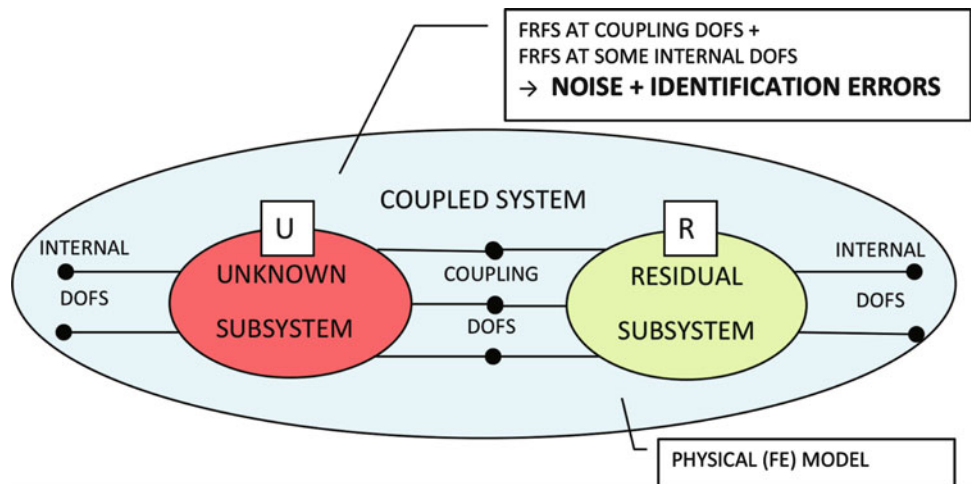


Fig. 21.1 Scheme of the decoupling problem

However, it can be shown that the problem can be solved by considering a number of interface DoFs at least equal to the number of coupling DoFs n_c . Therefore, three options for interface DoFs can be considered:

- standard interface, including only the coupling DoFs (c) between subsystems U and R ;
- extended interface, including also a subset of internal DoFs ($i \subseteq r$) of the residual subsystem R ;
- mixed interface, including subsets of coupling DoFs ($d \subseteq c$) and internal DoFs ($i \subseteq r$) of subsystem R .

The compatibility condition at the (standard, extended, mixed) interface DoFs implies that any pair of matching DoFs u_l^{RU} and u_m^R , i.e. DoF l on the coupled system RU and DoF m on subsystem R must have the same displacement, that is $u_l^{RU} - u_m^R = 0$. Let the number of interface DoFs on which compatibility is enforced be denoted as N_C .

The compatibility condition can be generally expressed as:

$$\begin{bmatrix} \mathbf{B}_C^{RU} & \mathbf{B}_C^R \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \end{Bmatrix} = \mathbf{0} \quad (21.2)$$

where each row of $\mathbf{B}_C = [\mathbf{B}_C^{RU} \ \mathbf{B}_C^R]$ corresponds to a pair of matching DoFs. Note that $\mathbf{B}_C$ has size $N_C \times (N_{RU} + N_R)$ and is, in most cases, a signed Boolean matrix.

Before deriving the equilibrium condition, it should be noted that the interface DoFs involved in the equilibrium condition need not to be the same as DoFs used to enforce compatibility, as long as controllability between equilibrium and compatibility DoFs is ensured. If the compatibility and the equilibrium DoFs are not the same, the approach is called non-collocated [6, 8]. Note that a non-collocated approach requires an extended or mixed interface and therefore it is only possible in the decoupling problems (in coupling problems only standard interface can be defined). Obviously, the traditional approach, in which compatibility and equilibrium DoFs are the same, is called collocated.

Let N_E denote the number of interface DoFs on which equilibrium is enforced. The equilibrium condition for constraint forces implies that their sum must be zero for any pair of matching DoFs, i.e. $g_r^{RU} + g_s^R = 0$. Furthermore, if DoF k on the coupled system RU (or on the residual subsystem R) does not belong to the equilibrium interface, it must be $g_k^{RU} = 0$ ($g_k^R = 0$): this holds for any DoF not involved in the equilibrium condition.

Overall, the above conditions can be expressed as:

$$\begin{bmatrix} \mathbf{L}_E^{RU} \\ \mathbf{L}_E^R \end{bmatrix}^T \begin{Bmatrix} \mathbf{g}^{RU} \\ \mathbf{g}^R \end{Bmatrix} = \mathbf{0} \quad (21.3)$$

where the matrix $\mathbf{L}_E = [\mathbf{L}_E^{RU} \ \mathbf{L}_E^R]$ is a Boolean localisation matrix. Note that the number of columns of $\mathbf{L}_E$ is equal to the number N_E of equilibrium interface DoFs plus the number N_{NE} of DoFs not belonging to the equilibrium interface. Note that $N_{NE} = N_{RU} + N_R - 2N_E$: in fact, the number of DoFs belonging to the equilibrium interface must be subtracted once from N_{RU} and once from N_R . Therefore, the size of $\mathbf{L}_E$ is $(N_{RU} + N_R) \times (N_{RU} + N_R - N_E)$.

Eqs. (21.1-21.3) can be put together to obtain the so-called 3-field formulation:

$$\begin{cases} \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{RU} \\ \mathbf{f}^R \end{Bmatrix} + \begin{Bmatrix} \mathbf{g}^{RU} \\ \mathbf{g}^R \end{Bmatrix} & (21.1) \\ \begin{bmatrix} \mathbf{B}_C^{RU} & \mathbf{B}_C^R \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \end{Bmatrix} = \mathbf{0} & (21.2) \\ \begin{bmatrix} \mathbf{L}_E^{RU} \\ \mathbf{L}_E^R \end{bmatrix}^T \begin{Bmatrix} \mathbf{g}^{RU} \\ \mathbf{g}^R \end{Bmatrix} = \mathbf{0} & (21.3) \end{cases}$$

Starting from the 3-field formulation, several assembly techniques can be devised. The classical ones are

- primal assembly [2, 9] where the compatibility condition is satisfied exactly by defining a unique set of interface DoFs;
- dual assembly [6, 7] where the equilibrium condition is satisfied exactly by defining a unique set of connecting force intensities.

In a previous work [10], the hybrid assembly is introduced aimed to satisfy both compatibility and equilibrium conditions.

21.2.1 Dual Assembly

In the dual assembly, the equilibrium condition $g_r^{RU} + g_s^R = 0$ at a pair of equilibrium interface DoFs is ensured by choosing, for instance, $g_r^{RU} = -\lambda$ and $g_s^R = \lambda$. If a Boolean matrix $\mathbf{B}_E$ is defined similarly to $\mathbf{B}_C$, but related to interface equilibrium DoFs, the overall interface equilibrium can be ensured by writing the connecting forces in the form:

$$\begin{Bmatrix} \mathbf{g}^{RU} \\ \mathbf{g}^R \end{Bmatrix} = - \begin{bmatrix} \mathbf{B}_E^{RU^T} \\ \mathbf{B}_E^{R^T} \end{bmatrix} \lambda \quad (21.4)$$

where λ are Lagrange multipliers corresponding to connecting force intensities and $\mathbf{B}_E$ is a $N_E \times (N_{RU} + N_R)$ matrix. Since there is a unique set of connecting force intensities λ , the interface equilibrium condition is satisfied automatically for any λ , i.e.

$$\begin{bmatrix} \mathbf{L}_E^{RU} \\ \mathbf{L}_E^R \end{bmatrix}^T \begin{Bmatrix} \mathbf{g}^{RU} \\ \mathbf{g}^R \end{Bmatrix} = - \begin{bmatrix} \mathbf{L}_E^{RU} \\ \mathbf{L}_E^R \end{bmatrix}^T \begin{bmatrix} \mathbf{B}_E^{RU^T} \\ \mathbf{B}_E^{R^T} \end{bmatrix} \lambda = \mathbf{0} \quad (21.5)$$

Then $\mathbf{B}_E^T$ is the nullspace of $\mathbf{L}_E^T$, and viceversa $\mathbf{L}_E$ is the nullspace of $\mathbf{B}_E$:

$$\begin{cases} \begin{bmatrix} \mathbf{B}_E^{RU} & \mathbf{B}_E^R \end{bmatrix} \begin{bmatrix} \mathbf{L}_E^{RU} \\ \mathbf{L}_E^R \end{bmatrix} = \mathbf{0} \\ \begin{bmatrix} \mathbf{L}_E^{RU} \\ \mathbf{L}_E^R \end{bmatrix}^T \begin{bmatrix} \mathbf{B}_E^{RU} & \mathbf{B}_E^R \end{bmatrix}^T = \mathbf{0} \end{cases} \quad (21.6)$$

In the dual assembly, the total set of DoFs is retained, i.e. each interface DoF is present as many times as there are substructures connected through that DoF. Since Eq. (21.5) is always satisfied, the 3-field formulation reduces to:

$$\begin{cases} \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \end{Bmatrix} + \begin{bmatrix} \mathbf{B}_E^{RU^T} \\ \mathbf{B}_E^{R^T} \end{bmatrix} \lambda = \begin{Bmatrix} \mathbf{f}^{RU} \\ \mathbf{f}^R \end{Bmatrix} \end{cases} \quad (21.1^*)$$

$$\begin{cases} \begin{bmatrix} \mathbf{B}_C^{RU} & \mathbf{B}_C^R \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \end{Bmatrix} = \mathbf{0} \end{cases} \quad (21.2)$$

To eliminate λ , Eq. (21.1*) can be written:

$$\begin{Bmatrix} \mathbf{u}^{RU} \\ \mathbf{u}^R \end{Bmatrix} = - \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_E^{RU^T} \\ \mathbf{B}_E^{R^T} \end{bmatrix} \lambda + \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix}^{-1} \begin{Bmatrix} \mathbf{f}^{RU} \\ \mathbf{f}^R \end{Bmatrix} \quad (21.7)$$

which substituted in Eq. (21.2) gives:

$$\begin{bmatrix} \mathbf{B}_C^{RU} & \mathbf{B}_C^R \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_E^{RU^T} \\ \mathbf{B}_E^{R^T} \end{bmatrix} \lambda = \begin{bmatrix} \mathbf{B}_C^{RU} & \mathbf{B}_C^R \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix}^{-1} \begin{Bmatrix} \mathbf{f}^{RU} \\ \mathbf{f}^R \end{Bmatrix} \quad (21.8)$$

from which λ , to be back-substituted in Eq. (21.1*), is found as:

$$\lambda = \left(\begin{bmatrix} \mathbf{B}_C^{RU} & \mathbf{B}_C^R \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_E^{RU^T} \\ \mathbf{B}_E^{R^T} \end{bmatrix} \right)^+ \begin{bmatrix} \mathbf{B}_C^{RU} & \mathbf{B}_C^R \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{RU} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^R \end{bmatrix}^{-1} \begin{Bmatrix} \mathbf{f}^{RU} \\ \mathbf{f}^R \end{Bmatrix} \quad (21.9)$$

To obtain a determined or overdetermined matrix for the generalized inversion operation, the following condition must be satisfied:

$$\text{number of rows of } \mathbf{B}_C \geq \text{number of rows of } \mathbf{B}_E$$

i.e.

$$N_C \geq N_E \geq n_c \quad (21.10)$$

Note that, if $N_C > N_E$, Eq. (21.8) is not satisfied exactly by vector λ given by Eq. (21.9), but only in the minimum square sense. This implies that also Eq. (21.2) is not satisfied exactly, i.e. compatibility conditions at interface are approximately satisfied. On the contrary, equilibrium is satisfied exactly due to the introduction of the connecting force intensities λ as in Eq. (21.4).

By substituting λ in Eq. (21.1*), it is obtained:

$$\begin{aligned} \begin{bmatrix} \mathbf{Z}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^{\text{R}} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{\text{RU}} \\ \mathbf{u}^{\text{R}} \end{Bmatrix} + \begin{bmatrix} \mathbf{B}_E^{\text{RU}T} \\ \mathbf{B}_E^{\text{R}T} \end{bmatrix} \left(\begin{bmatrix} \mathbf{B}_C^{\text{RU}} & \mathbf{B}_C^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^{\text{R}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_E^{\text{RU}T} \\ \mathbf{B}_E^{\text{R}T} \end{bmatrix} \right)^+ \\ \times \begin{bmatrix} \mathbf{B}_C^{\text{RU}} & \mathbf{B}_C^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^{\text{R}} \end{bmatrix}^{-1} \begin{Bmatrix} \mathbf{f}^{\text{RU}} \\ \mathbf{f}^{\text{R}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{\text{RU}} \\ \mathbf{f}^{\text{R}} \end{Bmatrix} \end{aligned} \quad (21.11)$$

Finally, $\mathbf{u}$ can be written in the form $\mathbf{u} = \mathbf{H}\mathbf{f}$, which provides the FRF of the unknown subsystem U :

$$\begin{aligned} \begin{Bmatrix} \mathbf{u}^{\text{RU}} \\ \mathbf{u}^{\text{R}} \end{Bmatrix} = \begin{bmatrix} \mathbf{Z}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^{\text{R}} \end{bmatrix}^{-1} - \begin{bmatrix} \mathbf{Z}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^{\text{R}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_E^{\text{RU}T} \\ \mathbf{B}_E^{\text{R}T} \end{bmatrix} \\ \times \left(\begin{bmatrix} \mathbf{B}_C^{\text{RU}} & \mathbf{B}_C^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^{\text{R}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_E^{\text{RU}T} \\ \mathbf{B}_E^{\text{R}T} \end{bmatrix} \right)^+ \begin{bmatrix} \mathbf{B}_C^{\text{RU}} & \mathbf{B}_C^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Z}^{\text{R}} \end{bmatrix}^{-1} \begin{Bmatrix} \mathbf{f}^{\text{RU}} \\ \mathbf{f}^{\text{R}} \end{Bmatrix} \end{aligned} \quad (21.12)$$

i.e., by noting that the inverted dynamic stiffness matrices $\mathbf{Z}^{\text{RU}-1}$ and $\mathbf{Z}^{\text{R}-1}$ are equal to the FRF matrices $\mathbf{H}^{\text{RU}}$ and $\mathbf{H}^{\text{R}}$ at the full set of DoFs:

$$\mathbf{H}^{\text{U}} = \begin{bmatrix} \mathbf{H}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^{\text{R}} \end{bmatrix} - \begin{bmatrix} \mathbf{H}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{B}_E^{\text{RU}T} \\ \mathbf{B}_E^{\text{R}T} \end{bmatrix} \left(\begin{bmatrix} \mathbf{B}_C^{\text{RU}} & \mathbf{B}_C^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{H}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{B}_E^{\text{RU}T} \\ \mathbf{B}_E^{\text{R}T} \end{bmatrix} \right)^+ \begin{bmatrix} \mathbf{B}_C^{\text{RU}} & \mathbf{B}_C^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{H}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^{\text{R}} \end{bmatrix} \quad (21.13)$$

With the dual assembly, the rows and the columns of $\mathbf{H}^{\text{U}}$ corresponding to compatibility and equilibrium DoFs appear twice. Furthermore, when using an extended or mixed interface, $\mathbf{H}^{\text{U}}$ contains some meaningless rows and columns: those corresponding to the internal DoFs of the residual substructure R . Obviously, only meaningful and independent entries are retained.

In Eq. (21.13), the product of the three matrices to be inverted can be defined as *interface flexibility matrix*. The interface flexibility matrix can be rewritten as:

$$\begin{bmatrix} \mathbf{B}_C^{\text{RU}} & \mathbf{B}_C^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{H}^{\text{RU}} & \mathbf{0} \\ \mathbf{0} & -\mathbf{H}^{\text{R}} \end{bmatrix} \begin{bmatrix} \mathbf{B}_E^{\text{RU}T} \\ \mathbf{B}_E^{\text{R}T} \end{bmatrix} = \mathbf{B}_C^{\text{RU}} \mathbf{H}^{\text{RU}} \mathbf{B}_E^{\text{RU}T} - \mathbf{B}_C^{\text{R}} \mathbf{H}^{\text{R}} \mathbf{B}_E^{\text{R}T} \quad (21.14)$$

It can be noticed that

$$\mathbf{B}_C^{\text{RU}} \mathbf{H}^{\text{RU}} \mathbf{B}_E^{\text{RU}T} = \hat{\mathbf{H}}^{\text{RU}}$$

where $\hat{\mathbf{H}}^{\text{RU}}$ is a subset of the FRF matrix of the coupled structure: pre-multiplication by $\mathbf{B}_C^{\text{RU}}$ extracts rows at compatibility DoFs, and post-multiplication by $\mathbf{B}_E^{\text{RU}}$ extracts columns at the equilibrium DoFs. Similarly,

$$\mathbf{B}_C^{\text{R}} \mathbf{H}^{\text{R}} \mathbf{B}_E^{\text{R}T} = \hat{\mathbf{H}}^{\text{R}}$$

where $\hat{\mathbf{H}}^{\text{R}}$ is the FRF of the residual structure at the same DoFs as above.

Therefore, the interface flexibility matrix becomes:

$$\mathbf{B}_C^{RU} \mathbf{H}^{RU} \mathbf{B}_E^{RU^T} - \mathbf{B}_C^R \mathbf{H}^R \mathbf{B}_E^{R^T} = \hat{\mathbf{H}}^{RU} - \hat{\mathbf{H}}^R \quad (21.15)$$

21.2.2 Singularity

The interface flexibility matrix (21.15) is strictly related to singularity. In fact, the choice of interface DoFs determines a set of frequencies at which the decoupling problem is ill conditioned. In [7, 9], it is shown that the interface flexibility matrix is singular:

- at the resonant frequencies of the residual substructure with coupling DoFs grounded, both for standard interface and for non collocated extended interface;
- at all frequencies for collocated extended interface, although this kind of singularity can be easily removed by smart inversion techniques.

21.3 Application

The experimental decoupling case study is performed using the test-bed structure shown in Fig. 21.2, which was especially designed for experimental coupling and decoupling studies [11]. To obtain an assembled system, an L-shaped solid steel block is bolted to a plate-like structure at three discrete connection points (nodes 5, 6 and 7 in Fig. 21.2). The block is assumed to behave as a rigid mass, which seems reasonable since its first flexible eigenfrequency is beyond the frequency range of interest, that is only up to 400 Hz. The plate/mass assembly is system RU , while the plate structure is the known subsystem R . The aim is therefore to identify the decoupled FRFs of the mass by subtracting the plate from the plate/mass assembly. Since the expected decoupled FRFs (inertances) are constants corresponding to the inertia of the block, any discrepancies due to decoupling errors can be readily seen.

The experimental FRFs of structure RU (plate/mass) and R (plate) are obtained by applying impact hammer excitation and measuring the resulting accelerations orthogonal to the structure at seven locations that are marked on the picture. The frequency range of the impact excitation using a plastic tip is up to 500 Hz. The structure is supported by inflated rubber tubes giving rigid body eigenfrequencies well separated from the first flexible mode around 60 Hz.

Three of the seven accelerometers are placed exactly at the interface locations (nodes 5, 6 and 7) at the underside of the plate for the plate structure R and at the underside of the mass for the assembled structure RU (see frame with detail in Fig. 21.2). It is therefore assumed that the FRFs at the interface locations are the same on both sides of the mass. The other

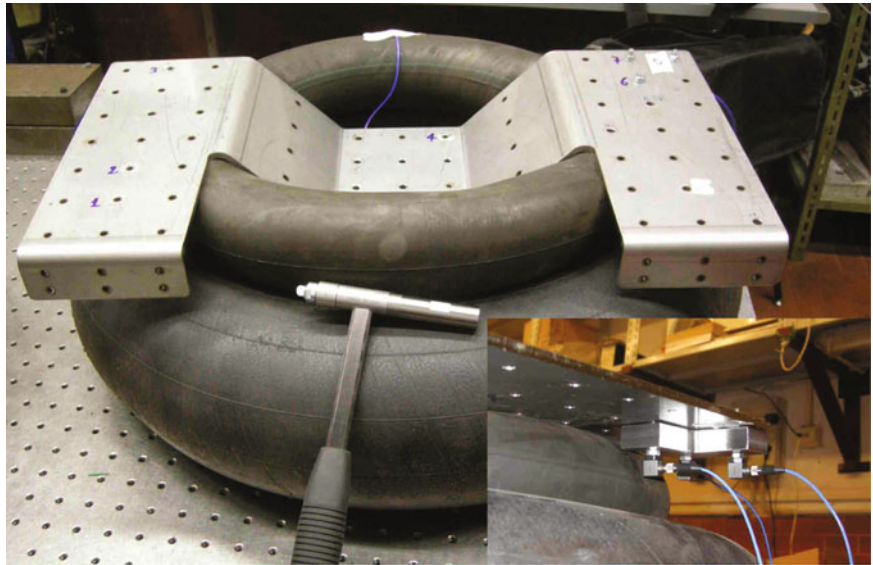


Fig. 21.2 Plate-like structure

Fig. 21.3 Reciprocity check on experimental FRFs among DoFs 1, 2 and 5, 6, 7

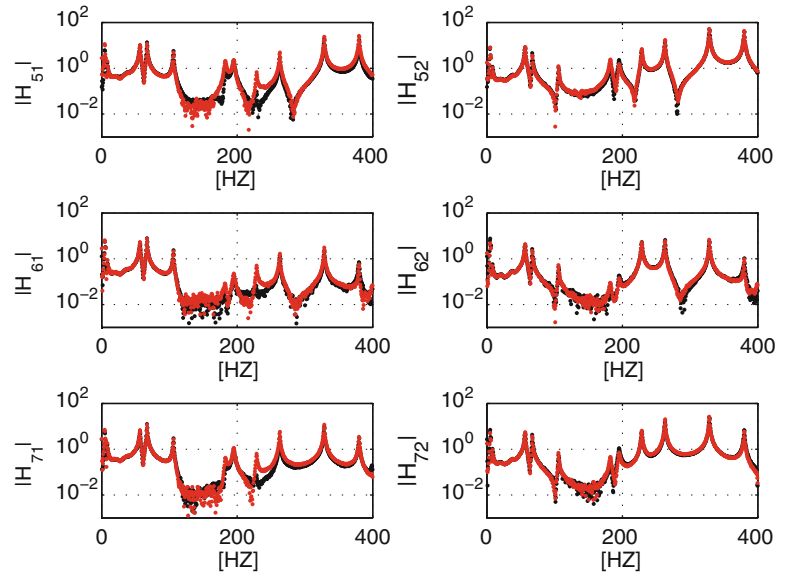
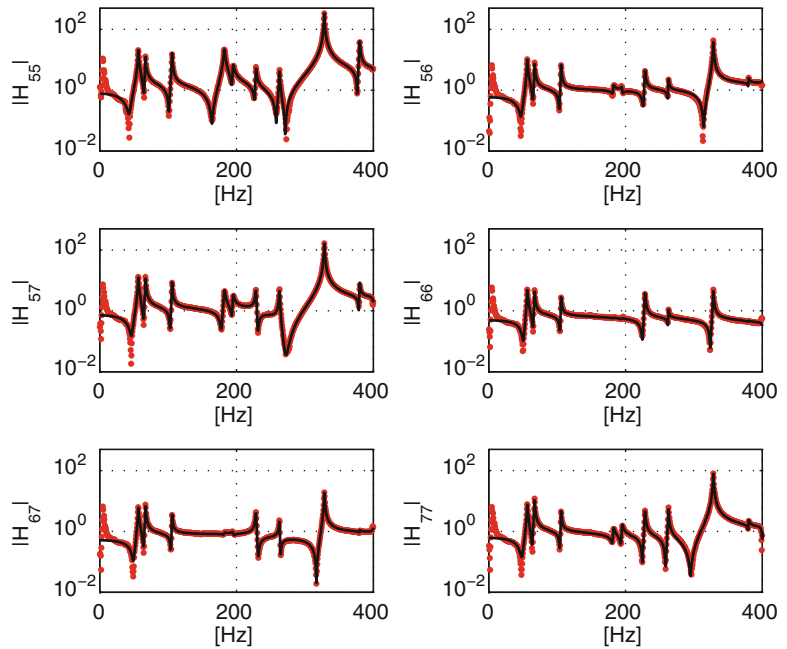


Fig. 21.4 Comparison between experimental (*dots*) and fitted FRFs (*lines*) of structure *RU* at the coupling DoFs

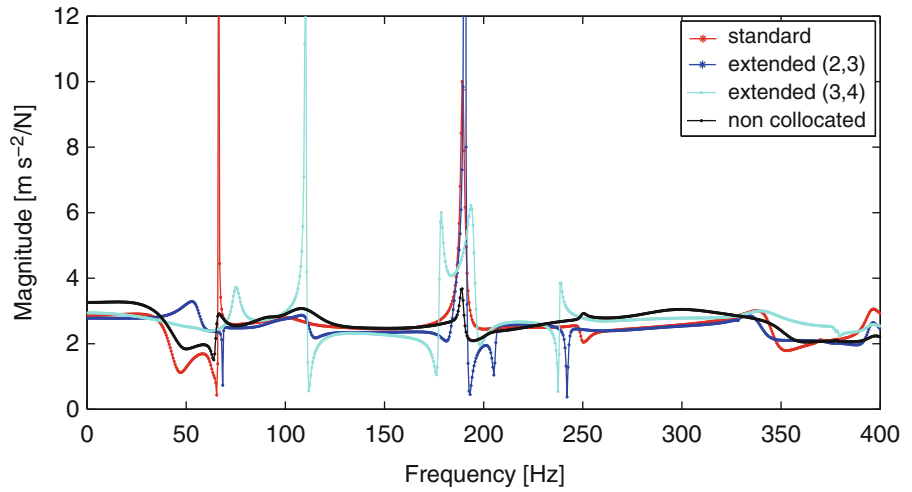


four accelerometers (nodes 1, 2, 3 and 4), placed at the underside of the plate, are used to measure internal DoFs of system *R* (plate). In order to obtain a complete FRF matrix, as required by the decoupling technique, impact hammer excitation is subsequently provided on all seven DoFs at the topside of the plate. A reciprocity check is performed on the experimental FRFs showing that reciprocity is not good for several FRFs, e.g. for all those involving DoF 1 and for some of those involving DoF 2. Among these FRFs, Fig. 21.3 shows those FRFs involving the coupling DoFs 5, 6, 7. Possibly, the use of such FRFs should be avoided in the subsequent steps.

In order to deal with a consistent FRF matrix, a curve fitting is performed using a poly-reference least square complex frequency domain technique [12] implemented in Matlab. The modal identification is performed up to 400 Hz, which is the frequency range of interest for decoupling. The curve fitting results for the FRFs at the coupling DoFs 5, 6 and 7 of the assembled structure *RU* show a good quality of the fitting (see Fig. 21.4). Similar results are obtained for the residual structure *R* at the same DoFs.

The FRFs (inertances) of subsystem *U* can be determined by using the procedure described previously where the compatibility and equilibrium DoFs are defined on a case by case basis.

Fig. 21.5 $|H_{77}^U|$ predicted using different interfaces



21.3.1 Decoupling

First of all, the case of standard interface (inertance known only at the coupling DoFs 5, 6 and 7) is considered. In this case, $N_C = 3$, $N_E = 3$ and

$$\mathbf{B}_C = \mathbf{B}_E = \begin{array}{c} \begin{array}{ccc|ccc} u_5^{RU} & u_6^{RU} & u_7^{RU} & u_5^R & u_6^R & u_7^R \\ \hline 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \\ \mathbf{B}_C^{RU} \quad \mathbf{B}_C^R \end{array} \quad (21.16)$$

In Fig. 21.5, the drive-point inertance at the coupling DoFs 7 of subsystem U , predicted using standard interface, is shown compared with results obtained in the sequel. The expected FRF should be constant with frequency. The FRF is badly identified between 50 and 65 Hz and spikes are observed around 65 and 190 Hz. As observed in [7], these ill-conditioned frequencies correspond to resonance frequencies of the residual system (plate) with the interface DoFs (5, 6, 7) grounded.

A way to raise the frequency range of the ill-conditioned frequencies is to use an extended interface, i.e. to assume that the FRF matrix of the coupled system is known not only at the coupling DoF but also at a subset of the four internal DoFs (1, 2, 3, 4) of the residual subsystem R .

A possibility is to use a collocated approach with some additional internal DoFs. The choice of the additional internal DoFs could be performed without considering the quality of the experimental FRFs. For instance, DoFs 2 and 3 could be chosen together with the coupling DoFs. In this case, $N_C = 5$, $N_E = 5$ and

$$\mathbf{B}_C = \mathbf{B}_E = \begin{array}{c} \begin{array}{ccccc|ccccc} u_2^{RU} & u_3^{RU} & u_5^{RU} & u_6^{RU} & u_7^{RU} & u_2^R & u_3^R & u_5^R & u_6^R & u_7^R \\ \hline 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{array} \\ \mathbf{B}_C^{RU} \quad \mathbf{B}_C^R \end{array} \quad (21.17)$$

In Fig. 21.5, the drive-point inertance at the coupling DoFs 7 of subsystem U , predicted using extended interface with additional internal DoFs 2 and 3, is shown. The FRF is improved with respect to the one obtained using standard interface: no significant spike is observed around 65 Hz and the value between 50 and 65 Hz is more constant, but a spike arises below 200 Hz.

A different choice of the additional internal DoFs is performed by considering the quality of the experimental FRFs. In this case, DoFs 1 and 2 should be discarded. Therefore, DoFs 3 and 4 are selected. Again, $N_C = 5$, $N_E = 5$ and

$$\mathbf{B}_C = \mathbf{B}_E = \begin{array}{c} \begin{array}{ccccc|ccccc} u_3^{RU} & u_4^{RU} & u_5^{RU} & u_6^{RU} & u_7^{RU} & u_3^R & u_4^R & u_5^R & u_6^R & u_7^R \\ \hline 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{array} \\ \mathbf{B}_C^{RU} \quad \mathbf{B}_C^R \end{array} \quad (21.18)$$

In Fig. 21.5, the drive-point inertance at the coupling DoFs 7 of subsystem U , predicted using extended interface with additional internal DoFs 3 and 4, is shown. The FRF is further improved in the frequency range between 0 and 65 Hz, where the value is almost constant as expected, but a spike arises above 100 Hz.

A final attempt is performed using a non collocated approach enforcing compatibility at DoFs 3, 4, 5, 6, 7 and equilibrium only at DoFs 5, 6, 7. In this case, $N_C = 5$, $N_E = 3$ and

$$\mathbf{B}_C = \begin{array}{c} \begin{array}{ccccc|ccccc} u_3^{RU} & u_4^{RU} & u_5^{RU} & u_6^{RU} & u_7^{RU} & u_3^R & u_4^R & u_5^R & u_6^R & u_7^R \\ \hline 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{array} \\ \mathbf{B}_C^{RU} \quad \mathbf{B}_C^R \end{array} \quad (21.19)$$

$$\mathbf{B}_E = \begin{array}{c} \begin{array}{ccc|ccc} u_5^{RU} & u_6^{RU} & u_7^{RU} & u_5^R & u_6^R & u_7^R \\ \hline 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \\ \mathbf{B}_E^{RU} \quad \mathbf{B}_E^R \end{array} \quad (21.20)$$

In Fig. 21.5, the drive-point inertance at the coupling DoFs 7 of subsystem U , predicted using non collocated approach, is shown. Results are very good above 65 Hz, but less precise at low frequencies.

In the considered application, the unknown FRF is expected to be constant and therefore it is quite easy to detect errors in the decoupling results. In any other practical case, the FRF of the unknown subsystem U is truly unknown, and therefore it is not so easy to detect errors by observing just one result. However, by comparing results obtained with different sets of interface DoFs, (e.g. Fig. 21.5), it can be noticed that spikes (related to ill conditioned frequencies) occur at frequencies that are different for each case. Therefore, a possible strategy to exploit the obtained results can be to split up the frequency axis in different frequency bands and to consider the best results in each frequency band.

21.4 Summary and Discussion

In this paper, a dual decoupling approach is applied to an experimental test-bed made by a plate-like structure and a solid steel mass. The aim is to predict the FRF of the solid mass (expected to be constant) starting from the FRFs measured on the assembled system and on the plate. Different compatibility and equilibrium conditions are imposed on the substructures: standard interface, extended interface and non collocated approach.

Results previously obtained using simulated (noise polluted) data on discrete systems showed that the use of additional internal DoFs always improves the accuracy of predicted FRFs. This is not anymore true when using experimental data where

the quality of the experimental FRFs affects the results. In the considered application, experimental FRFs involving DoFs 1 and 2 are not very satisfactory: therefore, the predicted FRF of the unknown subsystem U obtained using an extended interface including internal DoF 2 is worse than the one obtained using DoF 4 instead of DoF 2. Special attention should be placed on the selection of additional internal DoFs in order to use the experimental FRFs that appear to be more clean.

Acknowledgements This research is supported by grants from University of Rome La Sapienza and University of L'Aquila. The authors acknowledge the Substructuring group of TU Delft for providing a specimen of the test-bed structure.

References

1. Jetmundsen B, Bielawa R, Flannelly W (1988) Generalised frequency domain substructure synthesis. *J Am Helicopter Soc* 33(1):55–64
2. de Klerk D, Rixen DJ, Voormeeren S (2008) General framework for dynamic substructuring: history, review, and classification of techniques. *AIAA J* 46(5):1169–1181
3. de Klerk D (2009) Dynamic response characterization of complex systems through operational identification and dynamic substructuring. Ph.D. thesis, TU Delft
4. D'Ambrogio W, Fregolent A (2008) Promises and pitfalls of decoupling procedures. In: *Proceedings of 26th IMAC*, Orlando, USA, Feb 2008
5. Sjövall P, Abrahamsson T (2008) Substructure system identification from coupled system test data. *Mech Syst Signal Process* 22(1):15–33
6. Voormeeren SN, Rixen DJ (2010) A dual approach to substructure decoupling techniques. In: *Proceedings of 28th IMAC*, Jacksonville, USA, Feb 2010
7. D'Ambrogio W, Fregolent A (2010) The role of interface DoFs in decoupling of substructures based on the dual domain decomposition. *Mech Syst Signal Process* 24(7):2035–2048. doi:10.1016/j.ymssp.2010.05.007; also in *Proceedings of ISMA 2010*, 1863–1880, Leuven, Belgium
8. Voormeeren SN, Rixen DJ (2012) A family of substructure decoupling techniques based on a dual assembly approach. *Mech Syst Signal Process* 27:379–396. doi: 10.1016/j.ymssp.2011.07.028
9. D'Ambrogio W, Fregolent A (2011) Direct decoupling of substructures using primal and dual formulation. In: *Conference proceedings of the society of experimental mechanics series*, vol 4. *Linking models and experiments*, vol 2, Jacksonville, FL, USA, 31 Jan–3 Feb 2011. Springer, New York, pp 47–76
10. D'Ambrogio W, Fregolent A (2012) Direct hybrid formulation for substructure decoupling. In: *Conference proceedings of the society for experimental mechanics series*, vol 27. *Topics in experimental dynamics substructuring and wind turbine dynamics*, vol 2, Jacksonville, FL, USA, 30 Jan–2 Feb 2012, pp 89–107
11. van der Walk J, van Wuijckhuijse D, de Klerk D (2010) A benchmark test structure for experimental dynamic substructuring. In: *Proceedings of 28th IMAC*, Jacksonville, USA, Feb 2010
12. Culla A, D'Ambrogio W, Fregolent A (2012) Getting a symmetric residue matrix from the poly-reference least square complex frequency domain technique. In: Sas P (ed) *Proceedings of ISMA 2012 - international conference on noise and vibration engineering*, Leuven, Belgium, Sep 2012, pp 2755–2764

Chapter 22

Effects of Precise FRF Measurements for Frequency Based Substructuring

Julie Harvie and Peter Avitabile

Abstract Impedance modeling is often utilized to generate frequency response functions of an assembled system using modal characteristics of individual components. Sometimes slight inaccuracies are manifested in the characteristics of the components. Although those inaccuracies may seem minor in regards to the component for a traditional modal model, the inaccuracies can be amplified when impedance modeling is performed. Frequency based substructuring involves inversion of the frequency response functions and therefore requires considerable accuracy for the measurement, particularly in the area of anti-resonances.

For this study, some aspects of frequency based substructuring were explored, namely difficulties that might be encountered in experimental testing related to drive point measurements. The majority of studies were performed using analytical models to minimize contamination of data. The study focuses on connection point measurements where the actual location might be inaccessible due to physical constraints such as hardware, holes, etc. Hence the effect of introducing slight inaccuracies to the location of the drive point measurement was studied, along with other aspects related to the accuracy of the critical connection measurement.

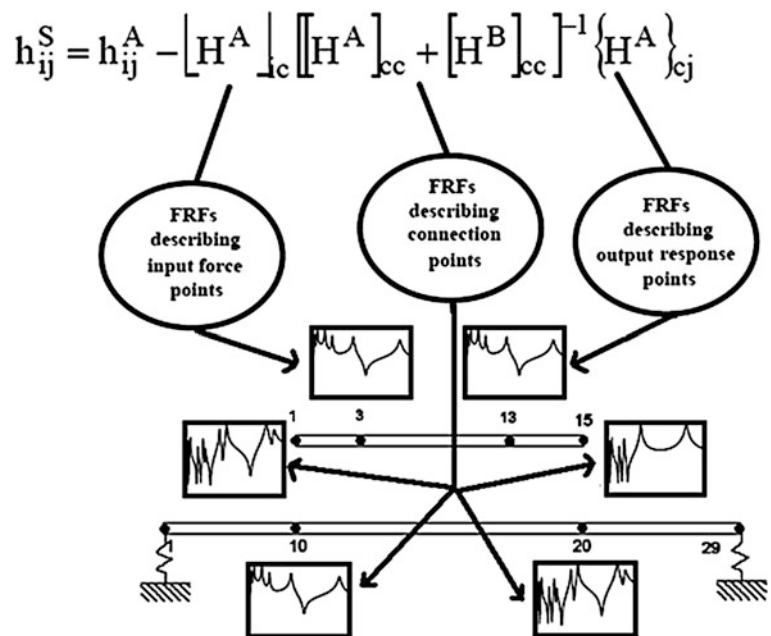
Keywords Substructuring • Experimental substructuring

22.1 Introduction

Frequency based substructuring (FBS) is one of several techniques that can be utilized to predict modal characterization of an assembled system without actually making measurements on the assembled system, but rather on the components. Details of the frequency based substructuring theory and some practical examples can be found in Refs [1, 2]. In order to perform frequency based substructuring, certain frequency response function (FRF) measurements are required—most notably, the drive point measurement. FRFs are needed at the locations where the structures will be connected as well as measurements relative to the desired input and output locations, as illustrated in Fig. 22.1. However it is sometimes nearly impossible to make measurements of FRFs at certain locations; for instance it could be difficult to mount accelerometers internally on a lengthy box beam to acquire a true drive point measurement. Another issue is generated when the substructures are connected using bolts because the connection locations can be holes, bolts, or other difficult measurements. To approximate the measurement at the immeasurable locations, a common approach is to make measurements close to the locations where attachment occurs. However, depending on how far the actual measurement is from the desired measurement locations, the measured FRFs can have some dissimilarities with the desired FRFs. The differences could likely be very minute at peaks of the FRF, but are more likely to be noticeable near the anti-resonances. With inversion of the FRFs necessary in the frequency based substructuring process, those differences in the FRF anti-resonances can propagate errors to the solution.

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Fig. 22.1 Illustration of FRFs needed for frequency based substructuring



22.2 Impetus for Study

For a directed study project, frequency based substructuring was performed on two aluminum beams both analytically and experimentally. A schematic is displayed in Fig. 22.2 to show the configuration. The test setup, shown in Fig. 22.3, involved bolting the beams together at two locations and making measurements next to the holes/rods due to the difficulty of making measurements exactly at the connection locations. During testing, there was uncertainty as to where the accelerometer and impact locations should be placed to best estimate the difficult FRFs. To approximate the measurement locations, accelerometers were placed slightly below the desired locations and impacts were made slightly above the desired locations. Although other approximations could have been made and investigated prior to testing, these points were merely intuitively chosen. When frequency based substructuring was performed using the obtained FRFs, extreme differences were seen between the analytical and experimental system FRFs. There was reason to believe that the main cause of these differences could be due to the offset of the measurements, so the following case studies were generated to study the necessity for precise FRF measurements in frequency based substructuring. The cases were executed using only analytical data to minimize data contamination and to illustrate a very basic problem devoid of any additional measurement issues.

22.3 Cases Studied

For this study, two simple aluminum beams were used as the test subjects to explore some possible difficulties that could be encountered when performing frequency based substructuring. The only difference between the cases was a 0.5 inch offset introduced in the FRF measurement location; all other parameters were kept constant. The cases are outlined in

Figure 22.4, where “A” denotes location of acceleration measurement and “F” denotes location of force measurement. The first case was generated to overcome the issue of the box beam structure, while the second and third cases were generated to overcome the issue of a thru-hole for assembly at the true measurement point. The cases can be summarized as:

- Case 1: Accelerometer on opposite face of box beam
- Case 2: Accelerometer below point, impact above point
- Case 3: Accelerometer to left of point, impact to right of point

Fig. 22.2 Basic schematic for tying two beams together

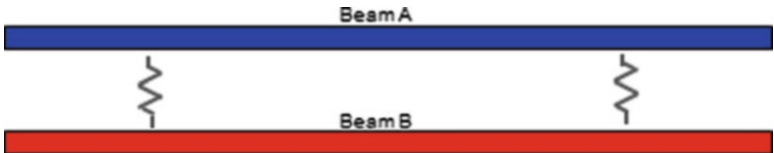
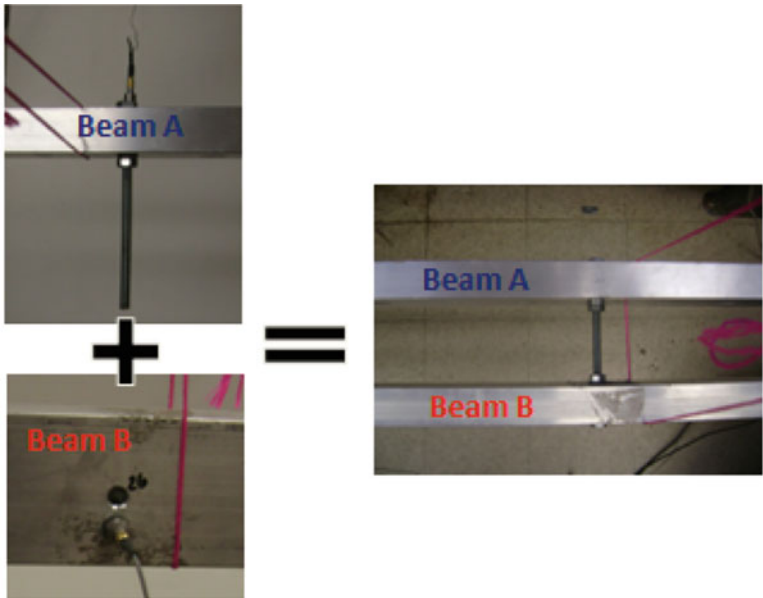


Fig. 22.3 Experimental test setup for directed study project



Case 1– Accelerometer on opposite face of box beam

Case 2– Accelerometer below point, impact above point

Case 3– Accelerometer to left of point, impact to right of point

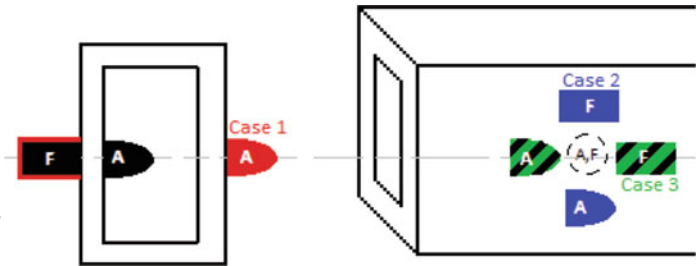
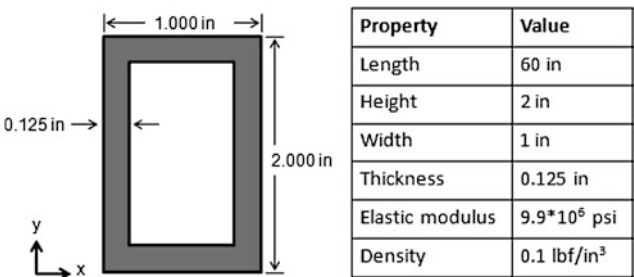


Fig. 22.4 Comparison of drive point measurements using various impacting techniques

Fig. 22.5 Approximate physical properties of aluminum box beams used for study



22.3.1 Structure Description and General Modeling/Testing Performed

The test subjects for the initial project were two physical aluminum box beams, with properties displayed in Fig. 22.5. The two box beams were assumed to be nearly identical and were tied together at two locations along the beams, as illustrated in Fig. 22.6. For this study, the connections were only assumed to have a transverse stiffness for simplicity; this was kept constant for all cases and is an acceptable approximation because all cases involve only analytical models where the properties of the connections are dictated. In the actual physical configuration rotation is also important, but that only further complicates the situation further and is not studied herein.

Fig. 22.6 Schematic of initial beam attachment used in analytical and experimental studies

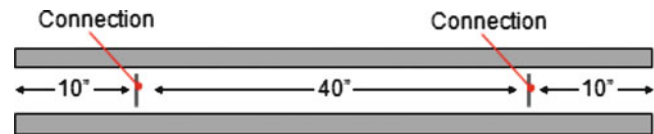
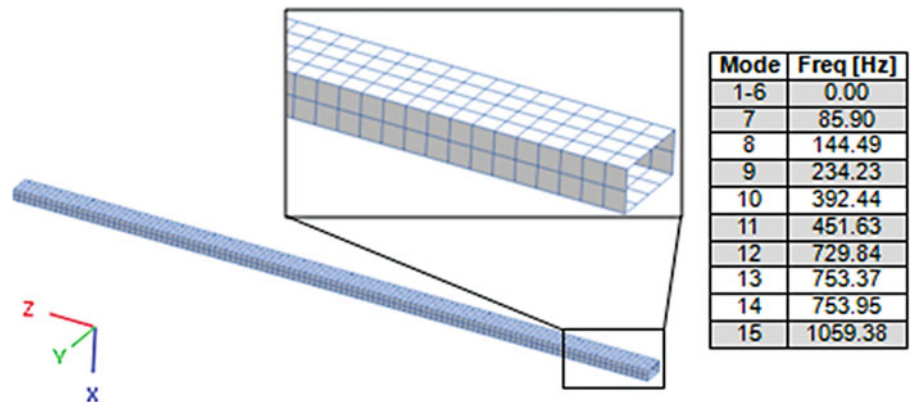


Fig. 22.7 Three dimensional model of beam used to study effects of measurement location



A model of the box beam was generated in FEMAP [1] using plate elements and processed in FEMTools [2]; the properties of this analytical model were set equal to the properties defined in Fig. 22.5. A screenshot of the model is shown in Fig. 22.7 along with the beam's first 15 natural frequencies. The planar modes are highlighted because those are the modes in the direction of interest; only planar motion was considered for these studies.

For each case, the appropriate FRFs of the model were synthesized in FEMTools using all modes to avoid truncation. The FRFs were synthesized up to 1300 Hz such that five modes could be observed. Frequency based substructuring was used to tie the beams together, and an FRF on one of the beams of the system was analyzed. The system FRF was chosen such that all system modes within the bandwidth would be included, and the same system FRF was evaluated for each case.

22.3.2 Case 1: Accelerometer on Opposite Face of Box Beam

The first case was introduced to study a possible solution to the difficulty of mounting accelerometers internally on a box beam to get a true drive point measurement. A very common way to combat this issue is to mount the accelerometers on the outer face of the box beam wall opposite from where the structure is impacted. This approximation is illustrated pictorially in Fig. 22.8 along with the resulting frequency response functions synthesized from the analytical model. Note that this FRF is at one of the connection degrees of freedom, as defined previously. These FRFs were identical for each connection degree of freedom due to symmetry. As seen, offsetting the measurement produces accurate response at peaks in the frequency response function, yet most of the anti-resonances contain substantial discrepancies. At low frequencies the FRFs are comparable, but the anti-resonances become farther apart as frequency increases. Note that the amplitudes at the resonances are essentially identical, and therefore this approximation may be acceptable for certain applications where only resonances are important (such as modal models where only shapes are used). However, as shown subsequently, frequency based substructuring involves inversion of FRFs where accuracy of the anti-resonances is very important.

Additionally, an experiment was performed to validate the characteristics observed in the analytical FRFs. An impact measurement was performed with an accelerometer mounted internally as well as an accelerometer mounted on the opposite face of the box beam. Note that this measurement was performed at the end of the beam for ease of internally mounting accelerometers and therefore should not match the analytical case exactly. The resulting FRFs obtained from this experiment are shown in Fig. 22.9 along with a schematic of the measurement setup. Once again the resonances compare agreeably while the anti-resonances contain noticeable discrepancies. This exercise supports the analytically discovered fact that measuring a frequency response function across a box beam is an acceptable approximation close to resonant frequencies, but a poor approximation at anti-resonances.

With experimental support of the trends being seen in the analytical FRFs, the analytical FRFs were then used to perform frequency based substructuring. All pertinent FRFs were synthesized for a true drive point measurement as well as measuring the output on the opposite box beam wall from the input. Figure 22.10 shows the resulting frequency response functions of the system using frequency based substructuring on both the true and approximate FRFs. In general, the resulting system

Fig. 22.8 Analytical FRFs showing difference between drive point measurement and having acceleration measurement on opposite wall of box beam

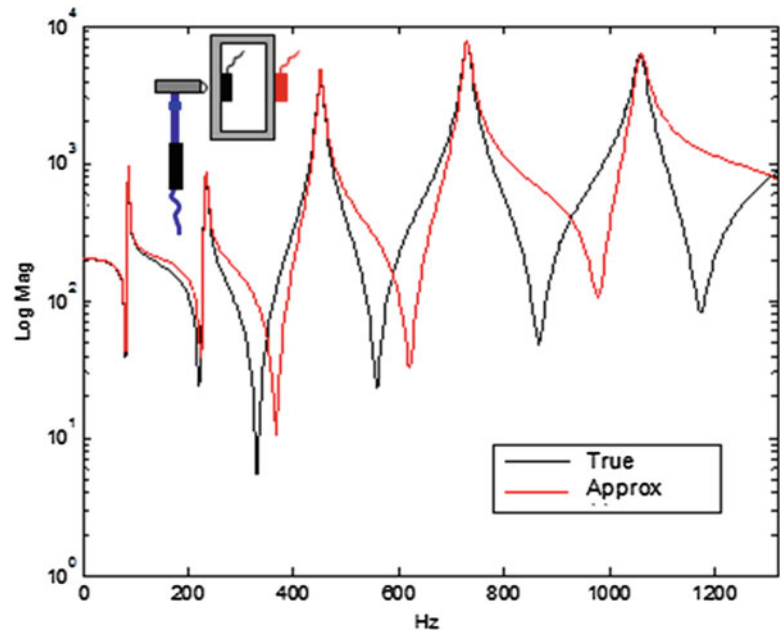
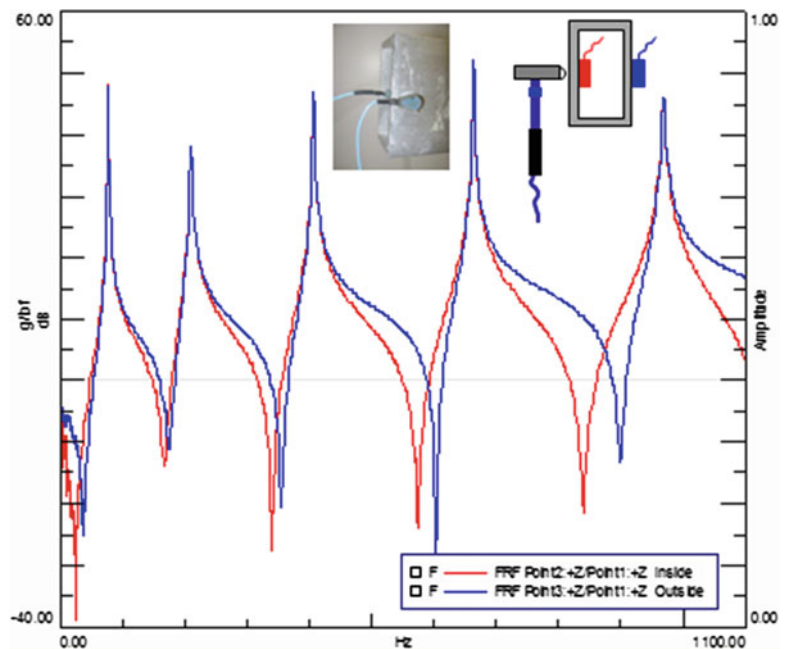


Fig. 22.9 Experimental FRFs obtained from placing accelerometer on opposite wall from impact and reference drive point measurement



FRFs do not compare very well. Looking closer, the resulting FRFs seem to compare agreeably up to about 250 Hz, which is also where the discrepancies in the individual substructures' FRFs become noticeable. This shows that, after frequency based substructuring, the system FRFs will only be accurate in regions where the individual component FRFs were accurate to the reference. When a certain frequency range of a substructure's FRF is initially discrepant from the true measurement, that discrepancy will be propagated to the system results. Although the differences in an FRF from measuring acceleration on the opposite wall of a box beam may seem insignificant, the approximation is not acceptable for those frequency ranges of a FBS system model.

In addition, a peak picking technique was used to estimate the natural frequencies of the system after performing frequency based substructuring on both the true FRFs and the estimates outlined for this case. As seen in Table 22.1, some of the system frequencies matched reasonably well while others did not. The first four frequencies were obtained accurately, yet some frequencies were estimated with up to a 26 percent difference from the true system frequencies. Also the results from the approximated FRFs did not predict the sixth mode of the true system. While some frequencies were predicted acceptably, there were also several errors predicted with this approximation.

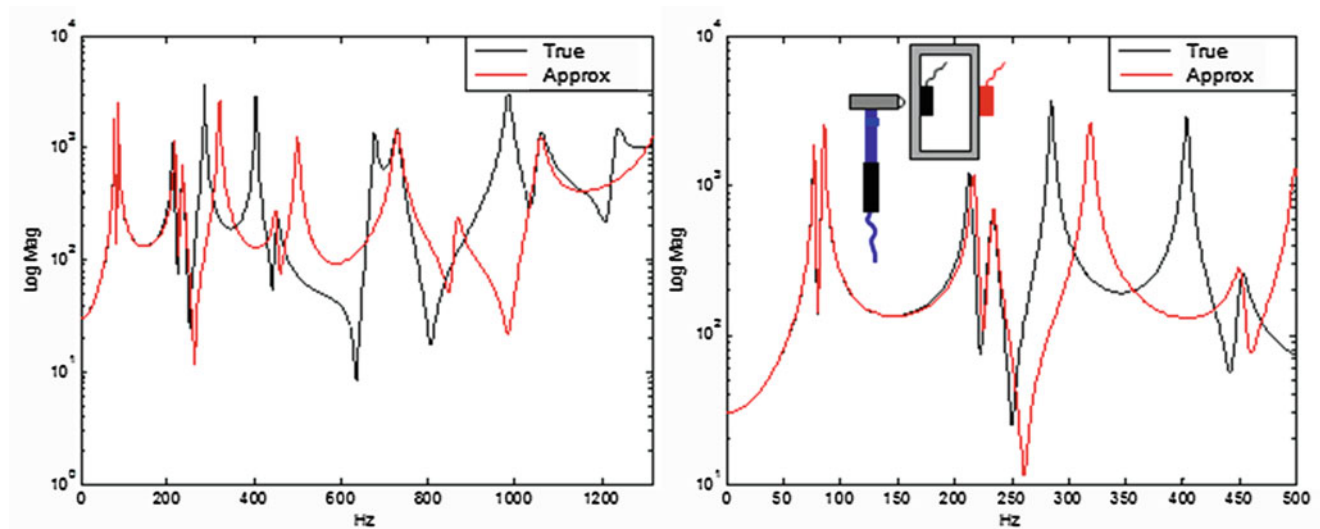


Fig. 22.10 Comparison of FBS between drive point measurement and having acceleration measurement on opposite wall of box beam for (a) up to 1300 Hz and (b) up to 500 Hz

Table 22.1 Frequency comparison for FBS using true measurements versus approximations for Case 1

Mode	Frequency (Hz)		
	True	Case 1	% Difference
1	76.90	76.69	0.27
2	85.69	85.69	0.00
3	213.1	216.8	−1.74
4	234.4	234.4	0.00
5	285.6	319.3	−11.80
6	403.6	—	—
7	453.4	449.7	0.82
8	677.5	498.8	26.38
9	728.7	729.5	−0.11
10	985.8	868.6	11.89
11	1063.0	1061.0	0.19

22.3.3 Case 2: Accelerometer Below Point, Impact Above Point

The second and third cases were generated to address another common measurement difficulty—holes at the locations where the substructures would be connected. Initially the measurement was approximated above/below the actual point because those points would move in line with the actual point for bending about the weak axis, which was the focus of this study. However as shown in Fig. 22.11, the resulting analytical FRF was not as similar to the drive point measurement as expected. Once again the resonances compare reasonably, but the anti-resonances contain major discrepancies. For this case, there were even differences between the FRFs at frequencies lower than the first resonant frequency. Furthermore, due to the off-axis nature of the setup, slight effects of a torsion mode were observed, as seen around 800 Hz in the FRF.

All pertinent FRFs were synthesized for a true drive point measurement as well as measuring the input/output above/below the true point, as detailed previously. Frequency based substructuring was performed on both data sets, with the resulting system FRFs shown in Fig. 22.12. Once again the resulting system FRFs do not compare very well overall. There are a few portions of the system FRFs that compare somewhat reasonably, but overall this approximation qualitatively seems worse than the first case. Because, in the component FRFs, the approximation contained differences from the true FRFs for the frequency range below the first peak, the system FRF also does not compare well for frequencies below the first peak. The irregularity noticed in the original FRF near 800 Hz was also seen to propagate through to the system FRF as an abnormality near 800 Hz. Very few portions of the system FRF match agreeably from this approximation. The approximation of taking measurements above/below a true point might seem somewhat adequate, but overall the approximation is unacceptable for use with frequency based substructuring.

Once again a peak picking technique was used to determine the frequencies of the system using both the true FRFs and the approximate FRFs, as shown in Table 22.2. Similar results were seen as the first case, where certain frequencies

Fig. 22.11 Analytical FRFs showing difference between drive point measurement and having measurement offset vertically from actual point

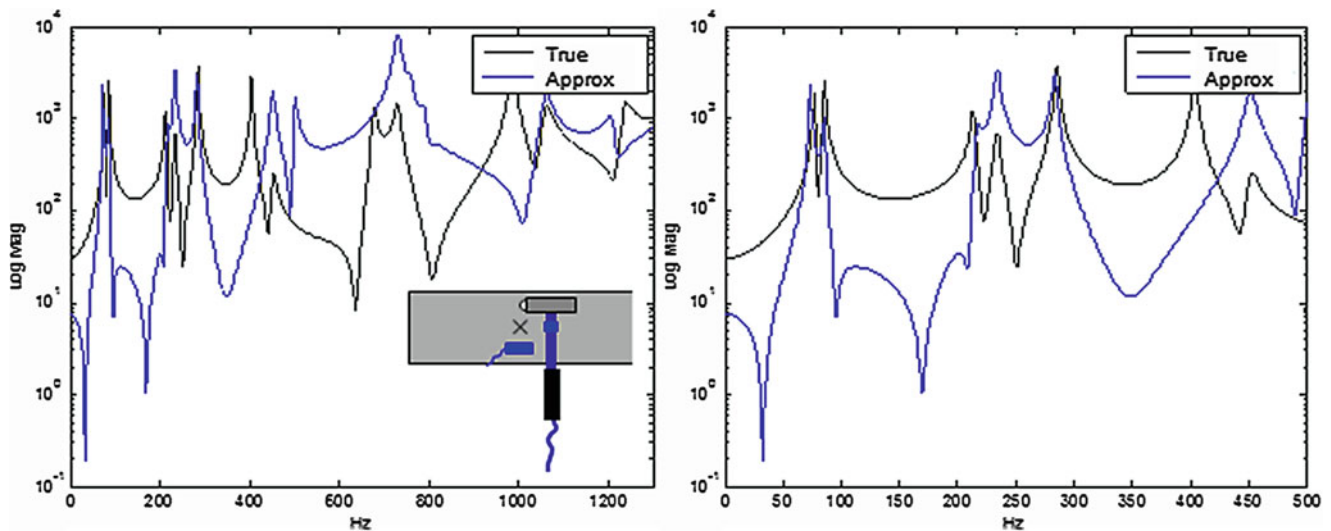
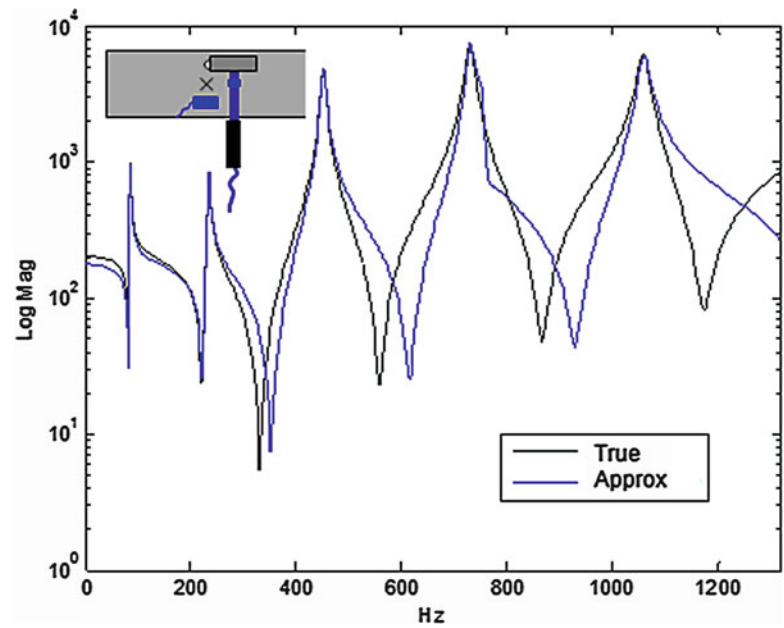


Fig. 22.12 Comparison of FBS between drive point measurement and having measurement offset vertically from actual point for (a) up to 1300 Hz and (b) up to 500 Hz

were predicted accurately while others were not. The sixth mode was also not predicted using this approximation. However this estimate caused an additional frequency to be predicted near 800 Hz, which, as mentioned previously, could possibly be attributed to the torsion mode seen in the component FRFs. Furthermore, when comparing Table 22.2 and Fig. 22.12, another difference is noted; although certain frequencies were predicted accurately, such as the 234 Hz mode, there is significant difference between the amplitude of the peaks. Depending on the application, the use of this approximation could produce significant issues.

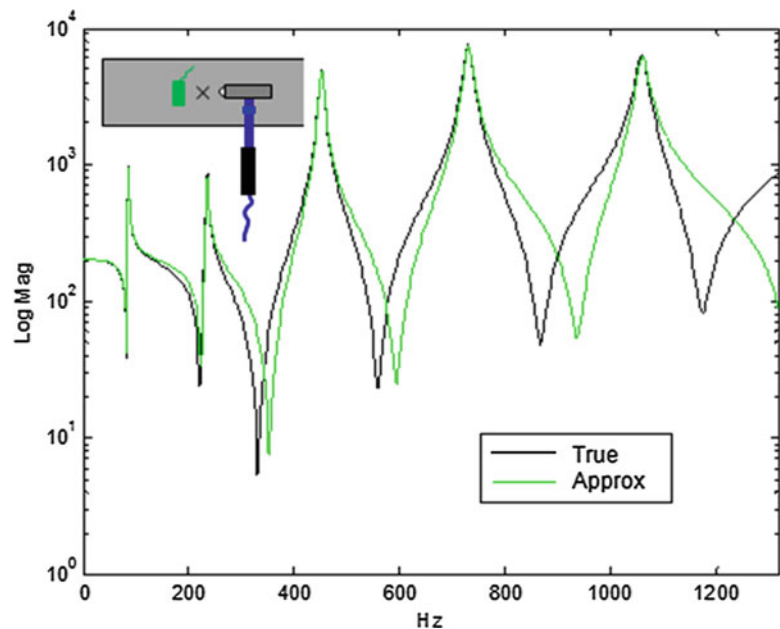
22.3.4 Case 3: Accelerometer to Left of Point, Impact to Right of Point

The third case, as mentioned previously, was also generated to combat an issue that can be presented when connection measurements are needed, but the connection location involves holes or rods. For this case, the measurements were kept along the centerline to avoid the issue of torsion modes appearing as seen in case 2. FRFs were synthesized to the left/right

Table 22.2 Frequency comparison for FBS using true measurements versus approximations for Case 2

Mode	Frequency (Hz)		% Difference
	True	Case 1	
1	76.90	73.97	3.81
2	85.69	85.69	0.00
3	213.1	217.5	-2.06
4	234.4	234.4	0.00
5	285.6	283.4	0.77
6	403.6	—	—
7	453.4	451.9	0.33
8	677.5	502.4	25.85
9	728.7	729.5	-0.11
10	—	758.0	—
11	985.8	787.3	18.67
12	1063.0	1061.0	0.19

Fig. 22.13 Analytical FRFs showing difference between drive point measurement and having measurement offset horizontally from actual point



of the actual point and compared to the true drive point, as shown in Fig. 22.13. Note that although there are not any detectable torsion modes as in the previous case, there are still major differences between the two FRFs at the anti-resonances. The peaks of both frequency response functions do match acceptably, although the regions between peaks are noticeably different.

The FRFs necessary for frequency based substructuring were synthesized using the horizontally offset approximations. FBS was performed using both the true measurements and the approximate measurements, with the resulting system FRFs shown in Fig. 22.14. Once again certain portions of the resulting FRF compare somewhat adequately, but overall the system FRF is not acceptable. Similar to the previous cases, slight discrepancies in component FRFs were transmitted to extreme inconsistencies in system FRFs after performing frequency based substructuring.

The estimated and exact system frequencies were once again determined using a peak picking technique, as shown in Table 22.3. Yet again, certain natural frequencies were predicted with greater accuracy than others. While the other two cases only missed the prediction of one natural frequency, this estimation also caused the eighth mode to be missed. The accuracy of this estimation was slightly better than the previous two cases, with only one mode with higher than ten percent difference from the true solution.

Clearly the estimations made in all three cases studied produced issues when used for frequency based substructuring. Extreme precision must be exercised when measuring frequency response functions for FBS because, as proven, slight differences in component FRFs can propagate to major inaccuracies in system FRFs.

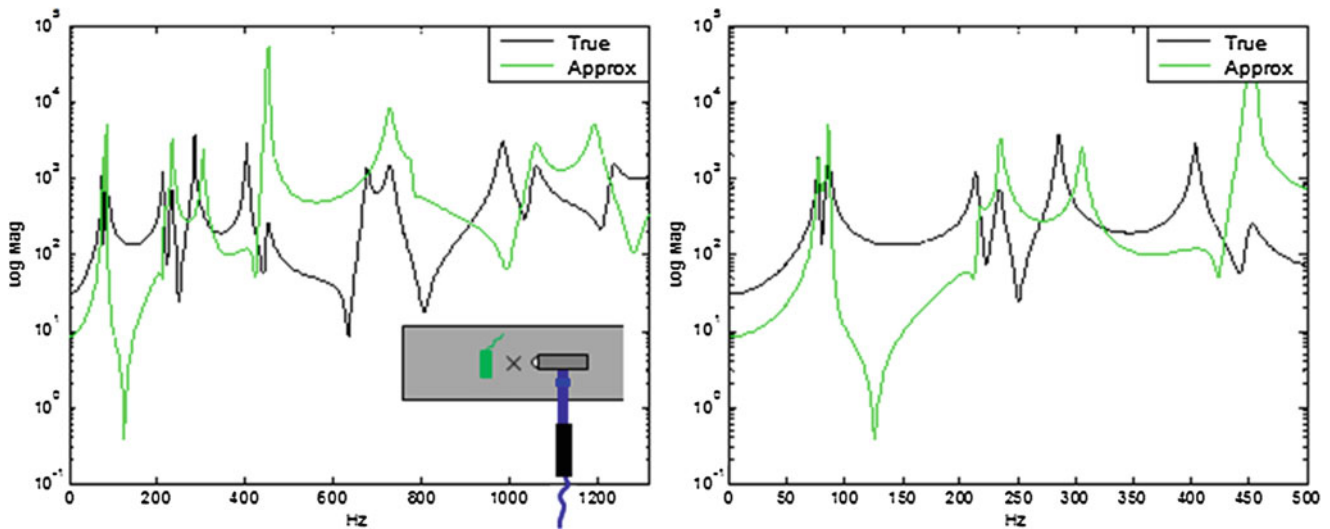


Fig. 22.14 Comparison of FBS between drive point measurement and having measurement offset horizontally from actual point for (a) up to 1300 Hz and (b) up to 500 Hz

Table 22.3 Frequency comparison for FBS using true measurements versus approximations for Case 3

Mode	Frequency (Hz)		% Difference
	True	Case 1	
1	76.90	77.63	−0.95
2	85.69	85.69	0.00
3	213.1	217.5	−2.06
4	234.4	235.1	−0.30
5	285.6	305.4	−6.93
6	403.6	—	—
7	453.4	453.4	0.00
8	677.5	—	—
9	728.7	729.5	−0.11
10	985.8	774.9	21.39
11	1063.0	1062.0	0.09

22.4 Conclusions

This paper presents some important information in regards to the need for very precisely measured frequency response functions. This is especially true for frequency based substructuring applications where the accuracy of the frequency response functions are of critical importance. Sometimes when certain frequency measurements are difficult to obtain, slight approximations are made. Although those approximations may not seem extremely different from the true measurements, those slight differences can have a substantial effect on the system results after performing frequency based substructuring. Several cases were explored to show how very minor effects, that may not be of a severe nature for mode shape extraction, become very critical in frequency based substructuring system modeling applications. Care should be taken when dealing with those difficulties by making the best possible approximations to ensure accurate system results.

References

1. Avitabile P, Piergentili F, Lown K (1997) Application of hybrid modeling techniques for computer related equipment. In: Proceedings of the fifteenth international modal analysis conference, Orlando, February 1997
2. Avitabile P (July 2001) A comparison of some system modeling approaches. *Shock Vib Dig* 33(4):281–291
3. FEMAP. Finite element modeling and postprocessing. Version 10.0.2, UGS Corporation
4. FEMTools dynamic design solutions. Version 3.1.0, Leuven

Chapter 23

Extending the Frequency Band for Fixed Base Modal Analysis on a Vibration Slip Table

Randy L. Mayes, Daniel P. Rohe, and Jill Blecke

Abstract In previous work, a modal test of a large beam like structure on a vibration slip table was analytically constrained to fixed base providing estimates of the first three bending modes active in the direction of slip table motion. This work extends the frequency band of the method to extract the first ten fixed base modes of the test article. All ten fixed base modal frequencies are within two percent of the truth test fixed base modes. When compared to the truth test, the estimated damping of the lower modes has large error, but at higher frequencies the estimated damping converges on the truth value.

Keywords Experimental • Fixed base • Modal test

Nomenclature

ζ	Modal damping ratio
Γ	Eigenvectors resulting from constrained eigenvalue problem
Ψ	Mass normalized real mode shapes
c	Subscript for degrees of freedom to be constrained to zero
DoF	Degree of freedom
f	Applied force
FRF	Frequency Response Function
L	Reduction matrix applying the constraint to equations of motion
q	Generalized coordinate
x	Physical DoF
Φ_c	The mode shape matrix of the slip table at DoF to be constrained
ω	Angular frequency (radians/second)

23.1 Motivation

At the highest level, there are applications for which one would like to characterize a structure fixed base, but the hardware is connected to a structure that has unknown dynamics. The method described here, allows one to perform a modal test attached to the base system with its dynamics and then analytically constrain the base to provide fixed base modes of the structure of interest. A more parochial interest is to determine fixed base modes on a vibration shaker table. This would allow one to use

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the U.S. Department of Energy under Contract DE-AC04-94AL85000.

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the fixed base modes of the test article for model validation. Since many systems, subsystems and components are tested on shaker tables in qualification procedures, this procedure can be performed at an incremental extra cost to provide fixed base model validation data. In some cases where budgets and time scales are tight and hardware is delicate, there are other driving concerns. First, executing a modal test in conjunction with a vibration test would save considerable time in that there would be no time lost in moving the object between facilities, and some duplication would be eliminated in tear down and setup. Second, because only one setup is required, handling is decreased thus reducing risk of damage to delicate test articles.

However, obtaining modal parameters for model validation from qualification tests is not trivial. Two primary difficulties are: (1) to determine the modal parameters, including modal mass, from data measured on the shaker table; and (2) to account for the dynamic effects of the vibration table so that the modal parameters can be related to the finite element model. In previous work Mayes and Bridgers [1] and Allen et al. [2] showed that fixed base modal frequencies and shapes can be approximated from modal tests on flexible fixtures. Mayes and Allen [3] also applied the method to a production vibration system instead of research hardware to see if there are any logistical obstacles. Some uncertainties were significant due to logistical difficulties associated with that real system, especially the damping. Some of the damping error in Mayes and Allen [3] was mitigated with a corrective damping technique applied by Mayes [4]. Napolitano et al. [5] applied their method to the same slip table and test article utilized in this work. They derived fixed base FRFs from a random vibration test using nine simultaneous shakers with uncorrelated input drive signals, and then extracted the modal parameters up to 415 Hz from the fixed base FRFs. Their method requires at least one shaker per active mode of the slip table.

In this work, the goal was to extend the capability from just under 500 Hz, as shown in [3, 4], to 1000 Hz. The work presented in this paper investigates almost the same system used in [3, 4], a test article mounted on a production shaker system at Sandia National Laboratories. The main difference in the production system is that the shaker is not attached to the slip table during the modal test. The slip table is a 1.07 m long by 0.91 m wide magnesium table, which is supported by a granite block. Oil forced from a pump lubricates the interface between the slip table and the granite block. A test article that is 1.2 m tall is utilized to attempt to provide some overturning moment to the slip table. Part of the challenge in this work was to extract truth shapes and frequencies up to the goal of 1000 Hz. A change in hardware for the truth test was utilized to achieve this goal.

The theoretical approach using component mode synthesis and the special constraint method will be reviewed in Sect. 23.2. Section 23.3 describes the method to obtain a “truth” model utilizing a seismic mass modal test of the test article with some analytical corrections to obtain fixed base results. The modal test on the slip table is described in Sect. 23.4. Section 23.5 addresses the attempt here to subtract the effects of external damping that do not come directly from the test article. The fixed base results are compared to the truth data in Sect. 23.6, and conclusions are declared in Sect. 23.7.

23.2 Review of Theory and Modal Constraints

The modal approach begins with the equations of motion based on modes extracted from the full system modal test on the shaker table as

$$\left[\omega_n^2 \right] \{q\} - \omega^2 [I] \{q\} = \Psi^T \{f\} \quad (23.1)$$

where ω_n represents the circular natural frequencies from the system modal test, q represents the modal coordinates, f is the vector of forces and Ψ provides the mass normalized mode shapes. A constraint for the table (subscript c) degrees of freedom can be written as

$$\{x_c\} = \{0\} \quad (23.2)$$

or using the modal substitution, then

$$\Psi_c \{q\} = \{0\}. \quad (23.3)$$

Using modal constraints (which are weaker than physical DoF constraints) produces

$$\Phi_c^+ \Psi_c \{q\} = \{0\} \quad (23.4)$$

where Φ_c comes from an estimate of the active shapes associated with the slip table during the test. The superscript $+$ indicates the pseudo-inverse. These shapes can come from mode shapes associated with the bare table, a Singular Value Decomposition of the table shapes from the actual modal test, or orthogonal mode shapes of the table during the test. This projects the constraint onto the vector space of Φ_c , which has the advantage of producing a least squares fit through any experimental modal analysis shape errors, assuming the matrix has more sensors (rows) than modes (columns). After the constraint is applied, the eigenvalue problem is solved to estimate the fixed base modes. Details of applying the constraint can be found many places including a previous tutorial [6]. The number of final modes will be reduced from the original number of modes in Eq. (23.1) by the number of constraints. As a practical consideration, any significant dynamics of the table which are active in the full system test need to be constrained. Modes of the table that are not active may be ignored. In the author's experience, it can actually be detrimental to the results to include them in the constraint mode shapes, Φ_c , particularly if there are no test article modes that exercise these inactive table modes.

23.3 Seismic Mass Truth Test

In order to evaluate the accuracy of the fixed base modes of the test article, a truth model was needed. Test hardware was devised which is basically a nylon beam with a base and some stiffening flanges. Previously in [3, 4], the test article was mounted on a 14,500 kg steel mass as seen in Fig. 23.1. The top surface of this large mass can be seen in the figure. The mass was supported on air springs. The article was tested in this configuration. The frequencies and damping of the modes from the SMAC algorithm [7] are given in Table 23.1. Although this was a massive boundary condition, it was not truly fixed base. Equations (23.1) and (23.4) were applied to estimate the fixed base frequencies, using only the six rigid body modes of the seismic mass in the Φ_c matrix. The fixed base modes and corresponding damping are also listed in the latter columns of the table. The rigid body modes on the seismic mass are not listed here, but had frequencies from 0.9 to 2.3 Hz. The rigid body modes were extracted with step sine testing and the elastic modes were extracted with impact or shaker testing. Long time blocks were required to get accurate damping since the damping is light. The results from the analytical fixed based columns are used as the best estimate for truth for the remainder of this work. The seismic mass elastic modal frequencies started at about 250 Hz. It was assumed in [3, 4] that they have negligible effect on the listed modes.

In this work, the desire was to achieve fixed base modes to at least 1000 Hz, so the truth model must be viable to the higher frequency. Since the large seismic mass has modes beginning at about 250 Hz, it was not clear that even modes 5 and 6 in the table above were accurate. Another set of hardware was utilized for the high frequency truth test.

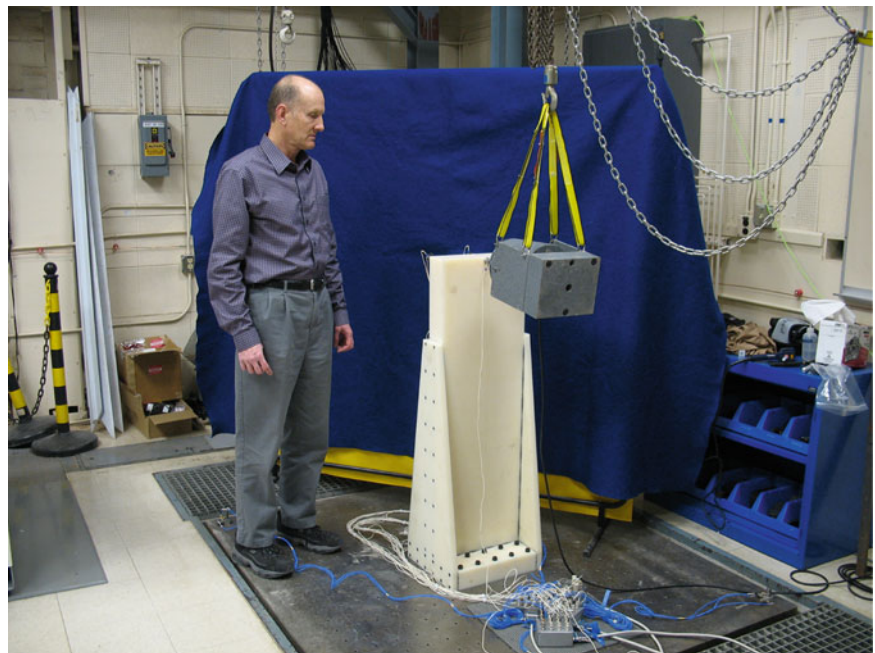
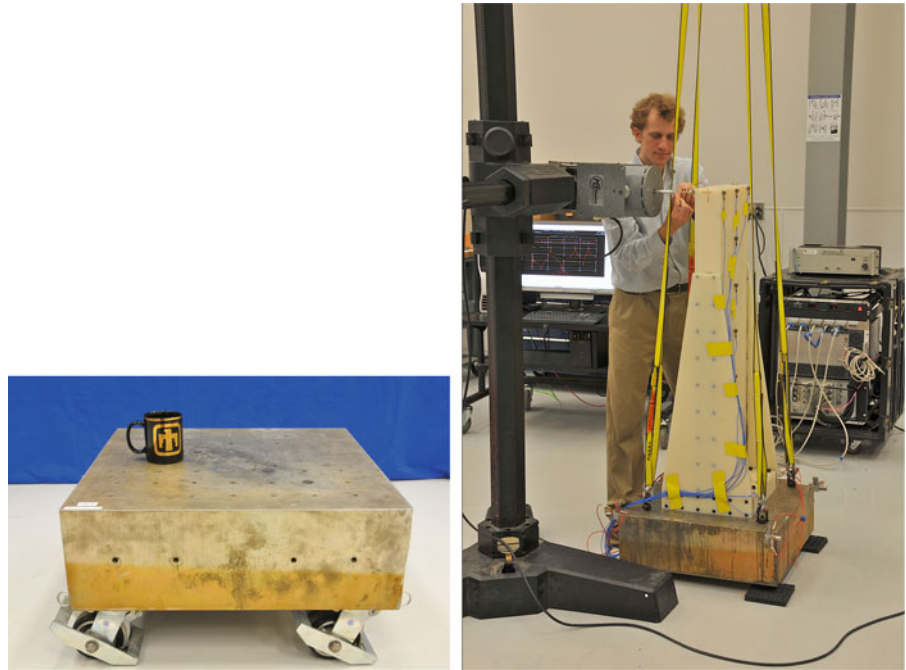


Fig. 23.1 Test article on seismic mass

Table 23.1 Large seismic mass truth test results

Mode #	Seismic mass test		Analytical fixed base		Mode description
	Frequency (Hz)	% Modal damping	Frequency (Hz)	% Modal damping	
1	38.77	0.22	38.54	0.22	First bend soft direction
2	91.78	0.20	91.13	0.19	First bend stiff direction
3	134.0	0.20	134.0	0.20	First torsion
4	164.2	0.25	164.0	0.24	2nd bend soft direction
5	342.7	0.25	342.3	0.25	2nd bend stiff direction
6	396.5	0.32	394.8	0.32	3rd bend soft direction

Fig. 23.2 Small seismic mass (*left*) and truth test setup (*right*)

Another fixture which is denoted as the small seismic mass for this paper was utilized as the base (shown in Fig. 23.2) for the test article. This steel mass was approximately $61 \times 61 \times 20.5$ cm with a weight of 605 kg. The raw data for the truth results came from a modal test of the test article mounted on this mass and suspended by straps as shown in Fig. 23.2.

The first elastic mode of the seismic mass was a classic plate mode at about 1370 Hz in which two corners that are diagonally opposite move up while the other two corners move down. The six rigid body modes of the plate and this elastic mode were included in the Φ_c matrix to constrain the base motion with the intent that analytically calculated fixed base modal parameters could be estimated up to 1370 Hz. A burst random modal test was conducted to 200 Hz and to 1600 Hz and elastic modes were extracted. Three different shaker locations were utilized, one to excite vertical motion, one to excite bending modes in the soft direction as well as torsion modes, and one to excite bending modes in the stiff direction of the beam. In the 200 Hz tests, a 7.6 cm long steel stinger was used, and in the 1600 Hz tests, a 1.9 cm long stinger was used. The diameter of the stinger was 1.5 mm. For each mode extracted, the response died out in the time window, so that accurate damping values might be obtained. The rigid body mode shapes were calculated analytically using calculated mass properties for the seismic mass and analytically generated mass properties of the test article from a solid model which matched the weight. The pendulum and torsion rigid body frequencies were calculated by timing the period of oscillation. Their damping was set arbitrarily at one percent of critical damping. The rigid body pitch and bounce frequencies and damping were extracted from a low frequency hammer impact modal test with time windows 32 seconds long. Accurate rigid body modal parameters are very important to achieve accurate results with this method. In the modal tests, one could observe resonances associated with string modes of the four support straps. These were neglected in these analyses.

Some example complex mode indicator functions are plotted in Fig. 23.3. The blue curve is the experimental data and the red curve is the analytical fit using SMAC [7]. One CMIF of the 200 Hz data in the stiff bending direction is shown, and one CMIF of the 1600 Hz data in the soft bending direction excitation is shown.

The results of the truth test are listed in Table 23.2. After the mode description, the first columns give the unconstrained frequencies and damping, the second set of columns give the constrained frequencies and damping, and the third set give the

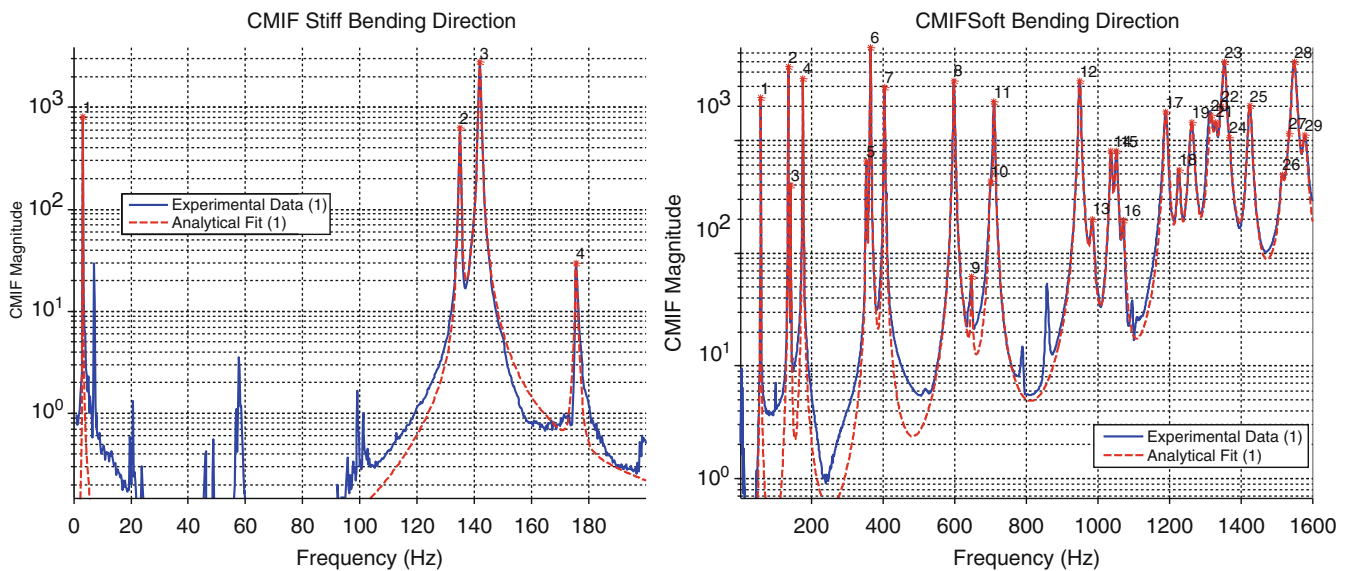


Fig. 23.3 CMIF of experimental and analytical modal parameter comparison for small seismic mass modal test

Table 23.2 Truth test with comparisons (truth results in green)

Description	Unconstrained small seismic mass		Constrained small seismic mass		Constrained large seismic mass	
	Frequency	Damping	Frequency	Damping	Frequency	Damping
1st bend soft direction	57.85 Hz	0.18%	38.99 Hz	0.16%	38.54 Hz	0.22%
1st bend stiff direction	135.18	0.21	89.05	0.18	91.13	0.19
1st torsion	142	0.25	134.31	0.21	134.0	0.20
2nd bend soft direction	175.75	0.24	163.61	0.22	164.0	0.24
2nd bend stiff direction	353.18	0.33	338.16	0.32	342.3	0.25
2nd torsion	364.91	0.26	364.24	0.26		
1 st Axial	409.31	0.26	392.4	0.25		
3rd bend soft direction	403.90	0.32	396.4	0.31	394.8	0.32
3 rd torsion	598.1	0.42	597.6	0.42		
3 rd bend stiff	647.3	0.61	643.8	0.61		
4 th torsion	700.5	0.47	700.4	0.47		
4 th bend soft	710.4	0.37	706.4	0.37		
4 th bend stiff	791.6	0.51	790.4	0.51		
5 th bend soft	859.2	0.29	859.2	0.29		
5 th torsion	949.5	0.41	949.2	0.41		
6 th torsion	984.5	0.51	984.3	0.51		
6 th bend soft	1037	0.43	1034.8	0.43		
7 th bend soft	1052.5	0.43	1048.2	0.43		
2 nd axial	1073	0.40	1062.3	0.39		
5 th bend stiff	1099.6	0.39	1097.2	0.39		
8 th bend soft	1190.7	0.49	1190.7	0.49		
7 th torsion	1262.7	0.60	1263.1	0.60		
9 th bend soft	1316.6	0.54	1316.1	0.54		
8 th torsion	1330	0.45	1330.7	0.45		
10 th bend soft	1344.3	0.51	1344.3	0.51		
11 th bend soft	1354	0.47	1352.3	0.47		

constrained frequencies and damping previously reported based on the large seismic mass. It is interesting to note that the mathematical constraint to fixed base for the small seismic mass reduces the first bending modal frequency by 35 percent. Because the large seismic mass constraint required a smaller change, i.e. it was already closer to a fixed base, the authors consider its results probably more accurate up to 200 Hz. However, above 200 Hz its elastic modes begin to impact the results, and they were not addressed in the constraint. Therefore the authors consider the results of the constrained small seismic mass more accurate above 200 Hz. The truth results are highlighted in green. The rows that correspond to bending

modes in the soft direction are highlighted in yellow. These will be utilized as the truth data in the final comparisons with the slip table results, which only excites the table in the soft bending direction. The maximum difference between the “truth” results between the constrained large seismic mass and constrained small seismic mass is about one percent, sometimes lower and sometimes higher. Damping results are virtually the same, since the uncertainty on the damping calculation is on the order of 0.05 percent of critical.

23.4 Modal Test on Slip Table

This test was similar to those performed in [3, 4] except for one major structural difference. The large qualification shaker and bullnose that connect the slip table to the drive system were not attached. The reason for this change is that the large shaker and bullnose are massive enough to introduce dynamics as low as 400 Hz into the system and a major extensional mode of the slip table and shaker armature around 900 Hz. By eliminating them, the first extensional mode of the table in the shaking direction is around 2035 Hz. The authors used one rigid body mode shape of the table in the shaking direction and the elastic mode at 2035 Hz in Φ_c . Also, the mass that the portable tip shaker has to move is reduced significantly by removing the bullnose/shaker attachment. Although one may excite some vertical modes of the table if one applies vertical forces to the slip table top, these modes are not active when the system is excited from the tip of the table in the direction the slip table is designed to move. With these dynamics, the authors had confidence they should be able to estimate fixed base modes up to at least 1000 Hz.

The shaker characterization accelerometers were triaxial gages placed at the four corners of the table and the mid-side locations at the back and on two sides (see Fig. 23.4). The test article was a nylon beam type structure also instrumented with triaxial accelerometers. The oil pump lubricating the interface between the magnesium slip table and the supporting granite block was operational during the testing to enable motion in the direction of slip. The system was driven from the tip of the slip table by an MB250 connected through a 23 cm long 6 mm diameter aluminum stinger to a PCB force gage which can be seen in Fig. 23.5. Both step sine and random excitation were explored. In general the FRFs overlaid from the two methods, except that higher force inputs decreased the damping causing higher peaks in the FRFs. The driving point FRF from the tip shaker can be seen in Fig. 23.6. The FRF shows five distinct modes, all of which are bending modes in the soft direction which is the direction of force input. Frequency spacing was 0.625 Hz over a bandwidth of 1600 Hz,



Fig. 23.4 Test article on slip table with MB250 tip shaker

Fig. 23.5 View of MB250 tip shaker, stinger, force gage

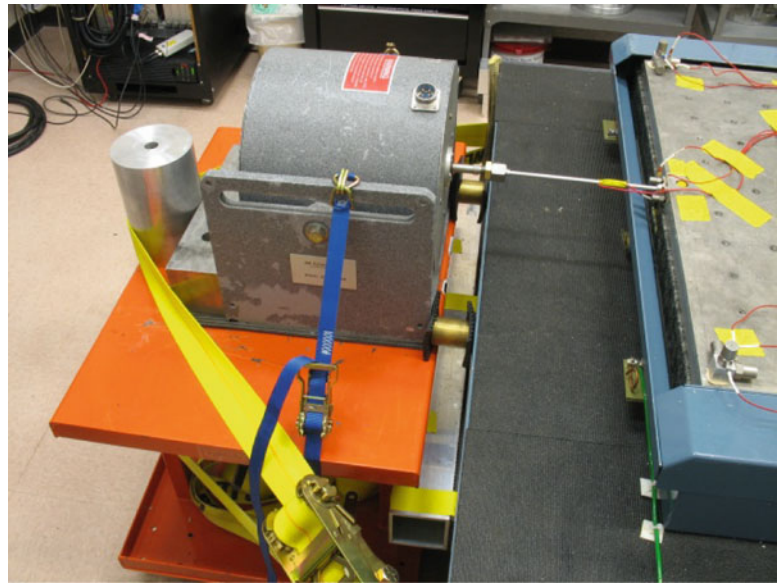
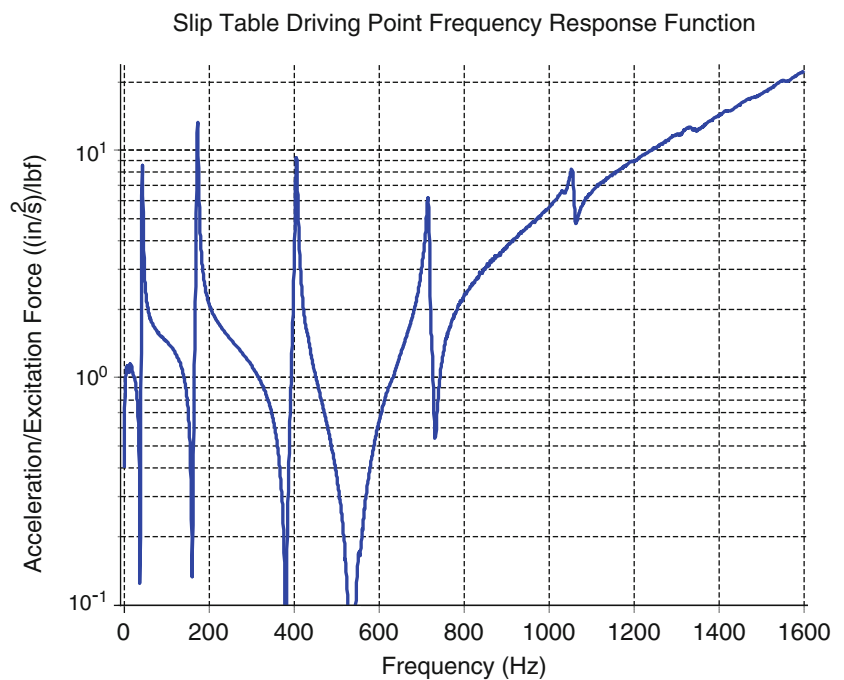


Fig. 23.6 Driving point FRF for slip table test



and a broad Hann window was utilized. The modal parameters extracted from this data set are listed in Table 23.3, and the corresponding constrained to fixed base parameters are shown as well. The modal parameters were extracted with the SMAC modal parameter algorithm [4] using a real modes assumption. The CMIF comparing the analytical construction from the extracted modal parameters with the experimental data is shown in Fig. 23.7. The highest level random data set, 392 N rms, was utilized for this work. It was noted that the damping of the lower modes decreased as the force amplitude increased. This indicates that some nonlinear damping effects are associated with the sliding of the slip table on the oil film and bearings.

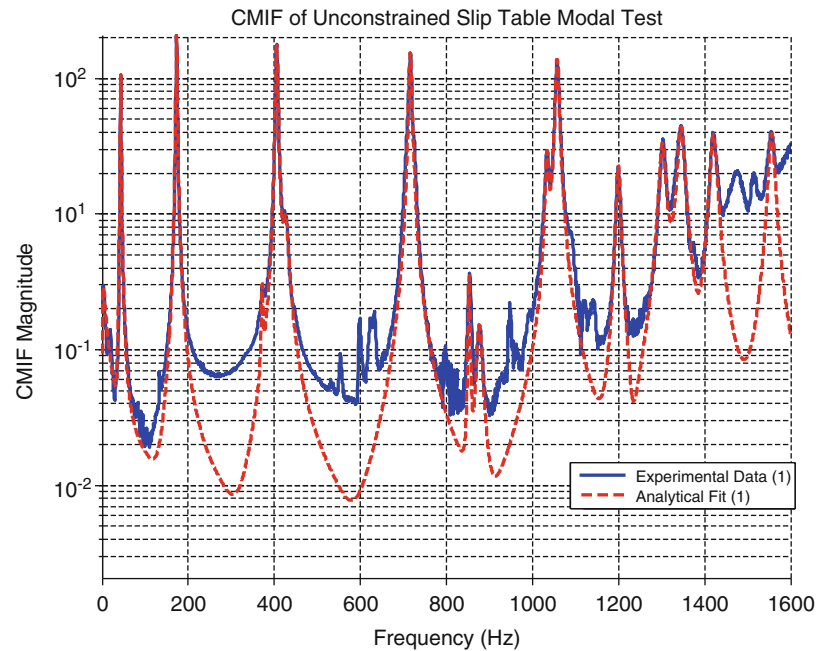
A few practical issues were experienced in the exercise. Two requirements are necessary to achieve an accurate constrained fixed base result. The first is that the modal mass of each mode must be accurately extracted. In SMAC, the modal mass is set to unity, and the driving point FRF is used to scale the drive point mode shape. This sets the scaling for the rest of the mode shapes. If a particular mode's driving point mode shape is below the noise floor of the experimental data, then the entire mode shape will probably be scaled poorly with possible poor results in the constrained result. The modes at 426, 852 and 1199 Hz cannot be detected by eye in the drive point FRF of Fig. 23.6. Therefore their scaling is suspect. The second consideration is that the table modes in Φ_c must span the space of the true table motion to constrain it properly. If there is zero table motion in

Table 23.3 Modal parameters from tip shaker, unconstrained and constrained

Description	Unconstrained slip table		Constrained to fixed base	
	Frequency	Damping	Frequency	Damping
1st bend soft direction	43.21 Hz	2.07%	38.19 Hz	0.95%
2nd bend soft direction	173.09	0.86	162.27	0.73
3rd bend soft direction	405.8	0.62	393.03	0.60
1 st Vertical ^a	426.29	1.69	426.33	1.68
4 th bend soft	716.2	0.51	702.5	0.49
5 th bend soft	852.9	0.25	852.9	0.25
6 th bend soft	1034.0	0.43	1029.3	0.43
7 th bend soft	1057.5	0.45	1041.5	0.44
8 th bend soft	1199.5	0.44	1199.5	0.44
9 th bend soft	1302	0.60	1301.5	0.60
10 th bend soft	1345.4	0.62	1344.2	0.62

^aThe vertical mode constrained result is suspect due to a weak drive point response and vertical motion of the table being unconstrained

Fig. 23.7 CMIF of unconstrained slip table test

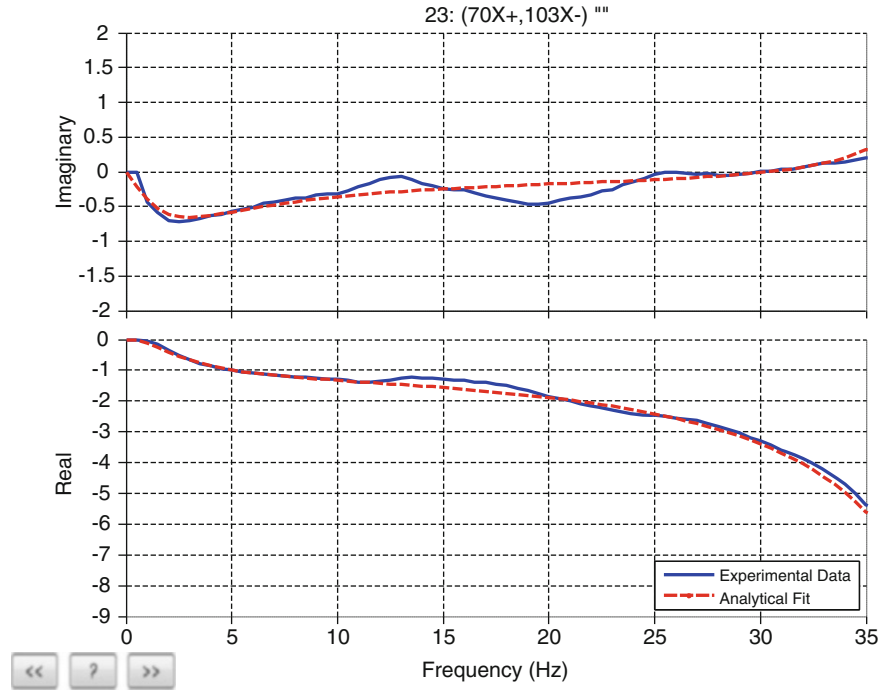


the first place (as is the case for the modes at 852 and 1199 Hz), then it may not make any difference. The modal frequency will not change due to base motion. But if there is table motion, the result may be inaccurate, if the chosen table shapes do not span the space of that motion. This occurs in the mode at 426 Hz, which turns out to be a vertical mode of the test article that is very weakly excited. In that mode the table has some pitch motion. This motion is not constrained by our lateral rigid body mode and the elastic extensional mode of the table, which we have chosen for Φ_c . The scaling is probably poor as well. In the constrained result, this mode hardly changes, and in addition it is clear that the table pitching motion is still evident. Therefore, the constrained result for the vertical mode should not be viewed with confidence. A couple of other modes that were weaker than the 426 Hz mode were thrown out of the data set since their scaling would be wrong.

23.5 Rigid Body Mode Damping Estimates and Subtraction

In this work, the rigid body modal frequency is actually zero Hz since the slip table is free to slip in the designed direction of motion. To keep the mathematics stable, the authors set the rigid body frequency to a low value (0.25 Hz), and the mode shape was analytically calculated from the mass of the slip table plus the mass of the test article (133.8 kg). The next step was to calculate the damping for the rigid body mode. The low frequency roll off of the FRF is associated with the damping. The authors used the equation of a mass with a viscous damper attached and fit it to the low frequency FRF. The analytical

Fig. 23.8 Low frequency damping fit to experimental FRF



fit for the optimized viscous damping and mass of the system is shown plotted in Fig. 23.8. For an assumed frequency of 0.25 Hz, this damping corresponded to a very large damping ratio of 6 (600 percent). One may note that there are additional highly damped dynamics in the 10–30 Hz range, which are not captured by the fit. These dynamics are more than two orders of magnitude smaller than the responses of the first four bending modes.

The damping that is applied to the system in the rigid body translation mode is beneath the slip table. This external damping pollutes the estimate of fixed base damping, which would not have any effects from damping beneath the slip table, since in a fixed base condition the table is not moving. It may be observed in Eq. (23.5) below that the fixed base damping results are a linear combination of the unconstrained damping results. The external damping effect should be removed. Mayes [4] developed an analytical method to remove the effect of viscous damping acting on the slip table. He showed that by changing the sign of the damping ratio on the rigid body modes that one could eliminate the external viscous damping effect in an analytical problem. This method was applied to obtain the fixed base damping from the unconstrained modal parameters of the slip table. The damping correction has a large effect on the first mode in Table 23.3 where the unconstrained damping is 2.07 percent and the constrained damping is 0.95 percent. The effect decreases to a negligible amount by the fifth mode. The theory for the damping calculation is repeated here for completeness.

$$\{\zeta_{new}\} = \text{diag}(\mathbf{\Gamma}' \mathbf{L}' [2\zeta\omega_n] \mathbf{L} \mathbf{\Gamma} ./ [2\omega_{new}]) \quad (23.5)$$

where $\{\zeta_{new}\}$ is the new vector of damping ratios, $[2\zeta\omega_n]$ is the diagonal damping matrix from the unconstrained modes from Table 23.3, $\mathbf{\Gamma}$ is the new eigenvector matrix when the constrained eigenvalue problem is solved, and $[2\omega_{new}]$ is the diagonal matrix with twice the new natural frequencies in radians/sec. $\mathbf{L}$ is the reduction matrix which applies the constraint to Eq. (23.1) with $\{q_c\}$ providing the new reduced number of constrained DoF as

$$\{q\} = \mathbf{L} \{q_c\}. \quad (23.6)$$

23.6 Comparison of Fixed Base Slip Table Calculations with Truth Data

In Table 23.4 the comparison of the fixed base frequency and damping calculations of the slip table (Table 23.3) are made with the “truth” results from the constrained seismic mass modes from the highlighted green results of Table 23.2.

The frequency results for all 10 modes are within 1.1 percent. It is also interesting to note that a fixed base finite element model of the test article was developed. The material properties of the nylon were not well known, but when the modulus

Table 23.4 Fixed base frequency/damping results and errors

Frequency-Hz	Truth frequency	Est frequency/error	Truth damping (%)	Est damping/error
1st bend soft direction	38.54 Hz	38.19 Hz/−0.9%	0.22	0.95%/332%
2nd bend soft direction	164	162.27/−1.1	0.24	0.73/204
3rd bend soft direction	396.4	393.03/−0.9	0.31	0.60/94
4 th bend soft	706.4	702.5/−0.6	0.37	0.49/32
5 th bend soft	859.2	852.9/−0.7	0.29	0.25/14
6 th bend soft	1034.8	1029.3/−0.5	0.43	0.43/0
7 th bend soft	1048.2	1041.5/−0.6	0.43	0.44/2
8 th bend soft	1190.7	1199.5/0.7	0.49	0.44/−10
9 th bend soft	1316.1	1301.5/−1.1	0.54	0.60/11
10 th bend soft	1344.3	1344.2/−0.07	0.51	0.62/22

of elasticity was calibrated to the “truth” test data, all the soft bending modes were within one percent of the truth test. The diagonal MAC values for the constrained slip table results vs. the constrained seismic mass results rounded to two decimal places are 1.0 1.0 .97 .95 .99 .91 .98 .95 .93 .78. The damping results have the greatest percentage error. Even with the subtraction for the external viscous damping, the damping from the fixed base slip table calculation has a large percentage error of 332 percent. Without the external viscous damping correction, the error would have been over twice as much. The damping error drops for higher modes so that by the fifth mode it appears to have converged to within 20 percent of the truth values, which is getting close to the damping algorithm resolution for such low damping ratios. It is not known if these errors are due to inaccuracies associated with the linear viscous damping model or the additional linear or nonlinear table dynamics (some of which are observable in the 10 to 30 Hz range of Fig. 23.8). For this slip table, if the test article had damping on the order of two percent, the effect of the damping beneath the slip table would be negligible, but for the low damping of the system here, it is not.

23.7 Conclusions

This work in conjunction with previous papers provides a refinement in estimating the fixed base modal parameters of a test article mounted on a vibration slip table from an unconstrained random vibration modal test where the force is measured from a portable shaker attached to the tip of the table with a modal force gage. Here the frequency band is extended by removing the dynamics of the bullnose and the large qualification shaker from the slip table, and adding the high frequency slip table extensional mode to the translational rigid body mode in the modal constraint set. A truth test, on a smaller seismic mass, allowed truth results to be calculated to a much higher frequency than in previous work. Fixed base frequencies for modes excited in the direction of the slip table motion are calculated to within about one percent of the truth values. Mode shape comparisons are also very good, with all diagonal MACs above 0.9 except the highest mode. The damping effects of the sliding of the slip table are modeled as viscous damping and fit to experimental low frequency FRFs. The effect of the sliding viscous damping is subtracted and improves the result, but the damping ratio error for the lower modes is still significant since the test article has very lightly damped modes (0.22 to 0.54 percent of critical damping). The sliding viscous damping effect decreases with frequency and almost disappears by the fifth mode.

References

1. Mayes RL, Bridgers LD (2009) Extracting fixed base modal models from vibration tests on flexible tables. In: Proceedings of the 27th international modal analysis conference, Orlando, February 2009, p 67
2. Allen MS, Gindlin HM, Mayes RL (2010) Experimental modal substructuring to extract fixed-base modes from a substructure attached to a flexible fixture. In: Proceedings of the 28th international modal analysis conference, Jacksonville, February 2010, p 164
3. Mayes RL, Allen MS (2011) Converting a driven base vibration test to a fixed base modal analysis. In: Proceedings of the 29th international modal analysis conference, Jacksonville, 2011, p 36
4. Mayes RL (2012) Refinements on estimating fixed base modes on a slip table. In: Proceedings of the 30th international modal analysis conference, February 2012, p 162
5. Napolitano KL, Yoder NC, Fladung WG (2013) Extraction of fixed base modes of a structure mounted on a shake table. In: Proceedings of the 31st international modal analysis conference, February 2013, p 232

6. Mayes RL (2012) Tutorial on experimental dynamic substructuring using the transmission simulator method. In: Proceedings of the 30th international modal analysis conference, February 2012, p 161
7. Hensley DP, Mayes RL (2006) Extending SMAC to multiple references. In: Proceedings of the 24th international modal analysis conference, February 2006, pp 220–230

Chapter 24

Extraction of Fixed-Base Modes of a Structure Mounted on a Shake Table

Kevin L. Napolitano, Nathanael C. Yoder, and William A. Fladung

Abstract Considerable savings in both cost and schedule could be realized if fixed-base modes, used to correlate fixed-base finite element models for coupled loads analyses, could be extracted while a structure is mounted on a flexible shake table before it undergoes environmental testing. Previously, a transmissibility-based methodology that uses multiple excitation sources mounted to the boundary support structure to help extract fixed-base frequency response functions was developed and applied to structures with isolated inertia supports. This paper presents results of applying the methodology to a structure mounted on a shake table at Sandia National Laboratory in New Mexico. The results demonstrate that the influence of the table's flexibility on the structure's dynamics could be largely eliminated for the first seven fixed-base modes.

Keywords Modal testing • Vibrations • Transmissibility • Coupled loads analysis • Satellite

24.1 Introduction

There has been much interest in extracting modal parameters from structures mounted on shake tables either by performing a conventional modal test while a structure is mounted to a shake table or by extracting modal parameters directly from shake table testing. In some cases the modal parameters extracted from these tests are nearly equivalent to modal parameters extracted from fixed-base modal tests, especially when the inertia of the shake table is much larger than the structure being tested or when the shake table is effectively constrained to moving in one degree of freedom (the driving direction).

However, in many cases, such as when large satellites are mounted to shake tables, the inertia of the satellite interacts with the shake table, resulting in motion at the base (such as rotation) in the extracted modes. Dynamics of the shake table can also be observed in the test data, and separating the effects of shake table dynamics from structure modes is often impossible.

Methods have been proposed to account for rigid-body base motion by using rigid-body motion of the shake table as a reference for calculating frequency response functions [1–4]. Using a coordinate transformation from absolute to relative motion, the resulting transfer functions are equivalent to fixed-base acceleration/force FRF. The challenges with this method are that (1) some modes of a structure may not be observable from a rigid-body base excitation, and (2) the method does not account for the potential flexible motion of the shake table.

Recently, two methods have been proposed to account for base flexibility in order to extract fixed-base modes from modal tests of structures mounted on flexible bases. The first method accounts for flexible motion of the shake table by introducing constraints based on measured deformations of the table [5]. When this is done, forces applied directly to the structure can be transformed into fixed-base FRF. The second method proposes that fixed-base modes could be extracted by exciting the table directly with as many or more independent references (shakers or impact locations) as independent deformations of the table over the frequency range of interest [6]. By using the independent shapes as references, corrected fixed-base frequency response functions could be calculated. Sandia National Laboratory invited ATA Engineering to apply this second method to a structure mounted on a slip table that was also being used to extend the first method to higher frequencies [7].

This paper describes the test setup, signal processing techniques, and modal extraction results of using slip table deformations as references to calculate fixed-base frequency response functions.

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24.2 Test Setup and Conduct

A picture of the test setup is presented in Fig. 24.1. The test article is a tapered I-beam, hereafter known as the “tombstone,” mounted to a slip table. For this test, the slip table was configured to move freely in the drive (“X”) direction by turning the oil lubrication on. A total of 94 channels of data were measured: 9 shaker load cells, 9 shaker drive point accelerometers, 31 accelerometer channels on the slip table, and 45 accelerometer channels on the tombstone.

The nine shakers are shown in Fig. 24.2. The first eight shakers were mounted on commercially available wire-frame shelves. Four of the shakers were connected to the horizontal plane of the slip table, with two (shakers 3 and 4) in the lateral direction and two (shakers 5 and 6) in the drive direction. Four other shakers (1, 2, 7, and 8) were mounted at an angle and positioned to drive the slip table vertically. The ninth shaker was mounted directly to the tombstone in the drive direction.

The test display model is presented in Fig. 24.3. The arrows correspond to locations and directions of measured accelerations. Test nodes 951, 953, 954, and 956 are labeled in this plot because they were used to create the generalized base acceleration references.

There were three additions to this test that were not part of the test conducted in the companion paper by Sandia [7]:

- (1) Twelve additional PCB 393A03 seismic accelerometers mounted on the slip table at test locations 951, 953, 954, and 956. These accelerometers were donated by the Modal Shop, Inc., for this test.
- (2) Additional accelerometers were mounted on the tombstone to help clearly observe torsion modes and differentiate them from translation modes of vibration.
- (3) The use of eight modal shakers to excite the slip table instead of using the slip table shaker directly.

The key to implementing the proposed methodology is to use the independent deformations of the shaker table as generalized references when calculating frequency response functions. To do this, the number of independent excitations on the shake table must be equal to or greater than the number of independent shapes in the frequency range of interest. Therefore, eight shakers were mounted to the shake table with the expectation that up to seven independent deformations of the table could potentially be observable—six rigid-body deformations that would be a linear combination of three rigid-body translations and three rigid-body rotations, and one potential elastic “potato-chip” mode of the slip table, where the table would appear to “twist.” Injecting enough excitation into the slip table to produce observable response of the accelerometers

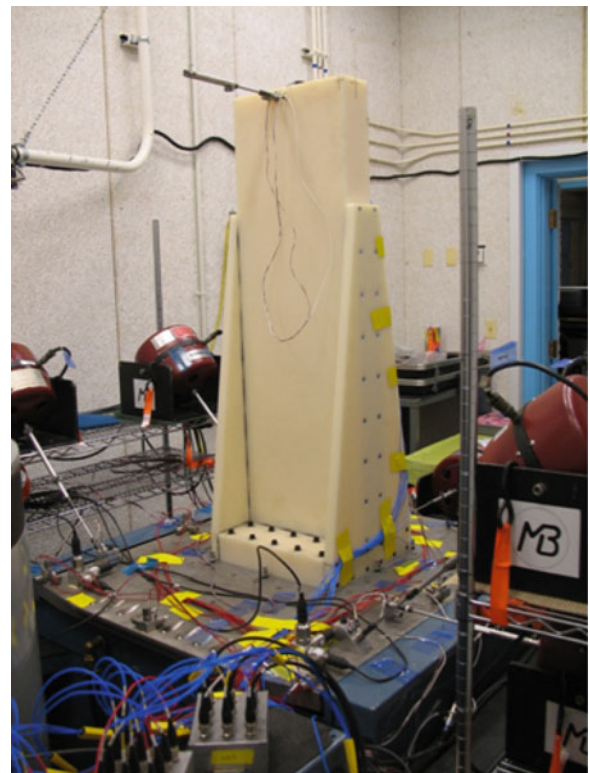


Fig. 24.1 Test article “tombstone” mounted on slip table

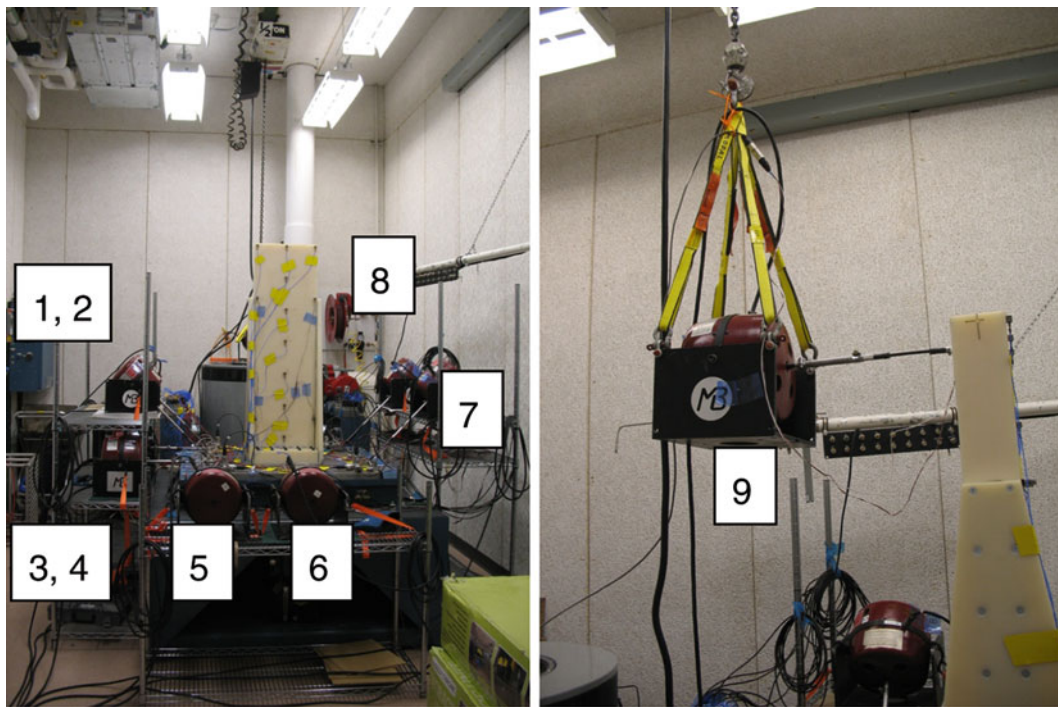
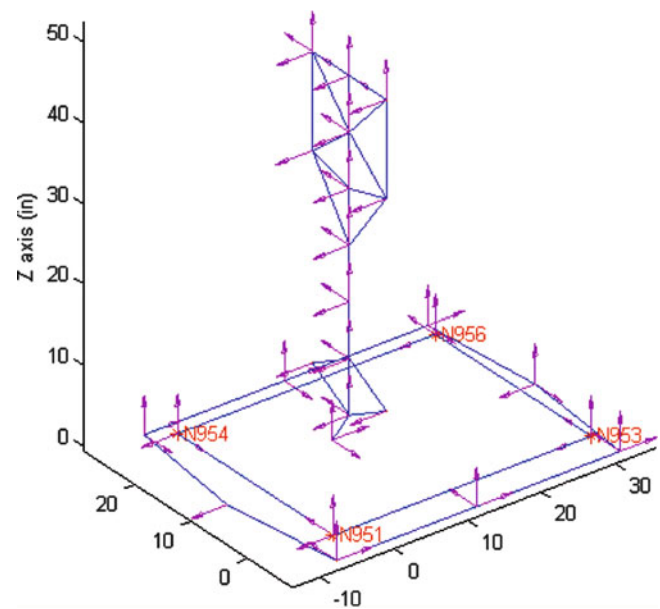


Fig. 24.2 Test setup. Eight modal shakers (1–8) were mounted directly to the slip table. A ninth shaker was mounted to the tombstone

Fig. 24.3 Test display model showing measured test degrees of freedom. Four accelerometers were used to estimate slip table base motion (951, 953, 954, and 956)



using only 110-lb modal shakers was anticipated to be the main challenge for this testing effort, and therefore a number of excitation techniques and excitation levels were investigated. A total of 54 test runs were conducted that included different excitation types (multiple reference burst random, multiple reference burst chirp, multisine sinusoidal sweep excitation [8, 9], and impact), excitation levels (low, medium, and high), and shaker stinger rod material (nylon and metal). This paper will present the results from one of these test runs (Test Run 47) that produced the highest-quality test data: multisine excitation, high level, with metal stinger rods.

24.3 Signal Processing

Frequency response functions using the nine shaker load cells as references were calculated first. Daniell's method of calculating frequency response functions was used because we have found that it produces FRF estimates with less bias than Welch's method when a window is required to process the data [10, 11]. A plot of the power spectrum mode indicator of the resulting frequency response functions for each of the nine shakers is presented in Fig. 24.4. Shaker 9 was able to excite the structure much more than the other shakers mounted directly to the slip table because the loads of the shaker weren't absorbed by the mass and inertia of the slip table. Shakers 5 and 6 were also able to cleanly excite the structure because they were mounted in the drive direction. The structure was not as easily excited by the other six shakers, and therefore FRF quality was not as high.

To generate fixed-base frequency response functions, the number of independent slip table deformations that are in the frequency range of interest must be estimated. In this paper, a singular value decomposition (SVD) of the time-history data was performed to estimate the number of independent shapes. Time-history data from four triaxial accelerometers (951, 953, 954, and 956) were assembled into a matrix, $[T]$, where the rows correspond to each time step and the columns correspond to the twelve channels. A singular value decomposition of the matrix product $[T]^T[T]$ was performed such that

$$[U][S][U]^T = [T]^T[T], \quad (24.1)$$

where $[U]^T[U] = [I]$, and the diagonal entries in the matrix $[S]$ are the singular values. The columns of the matrix $[U]$ correspond to shapes associated with each singular value. Table 24.1 presents the singular values scaled such that the summation of all singular values is equal to one. There are essentially three independent shapes that capture over 99% of the

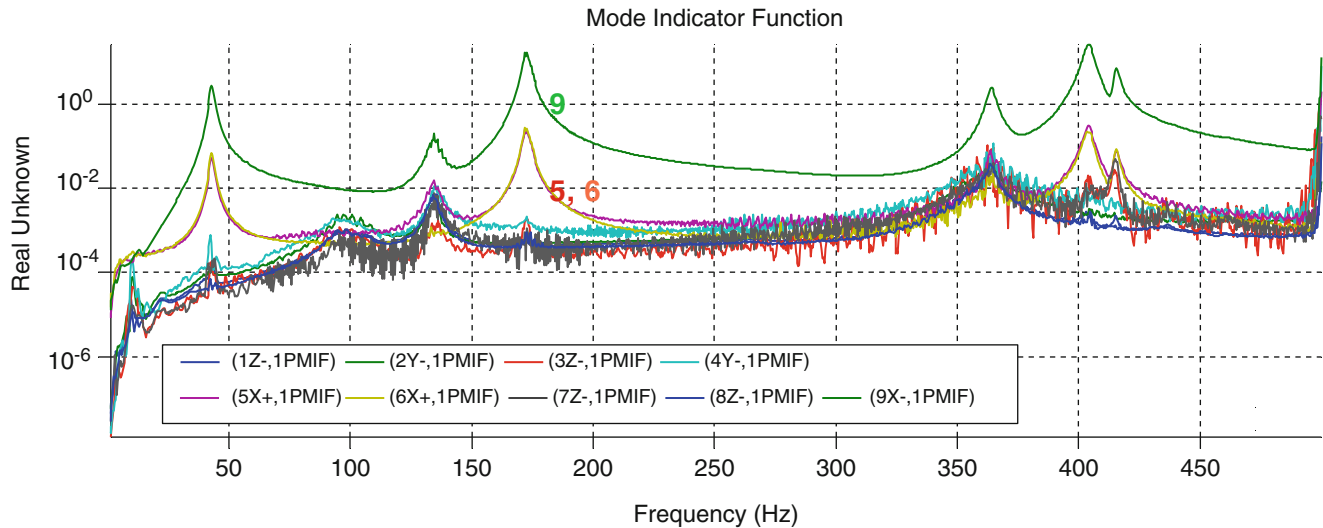


Fig. 24.4 Power spectrum mode-indicator function of all accelerometers due to each shaker

Table 24.1 Normalized singular values

Number	Singular value	Running sum
1	0.502	0.502
2	0.306	0.808
3	0.188	0.995
4	0.002	0.997
5	0.001	0.998
6	0.001	0.999
7	0.001	1.000
8	0.000	1.000
9	0.000	1.000
10	0.000	1.000
11	0.000	1.000
12	0.000	1.000

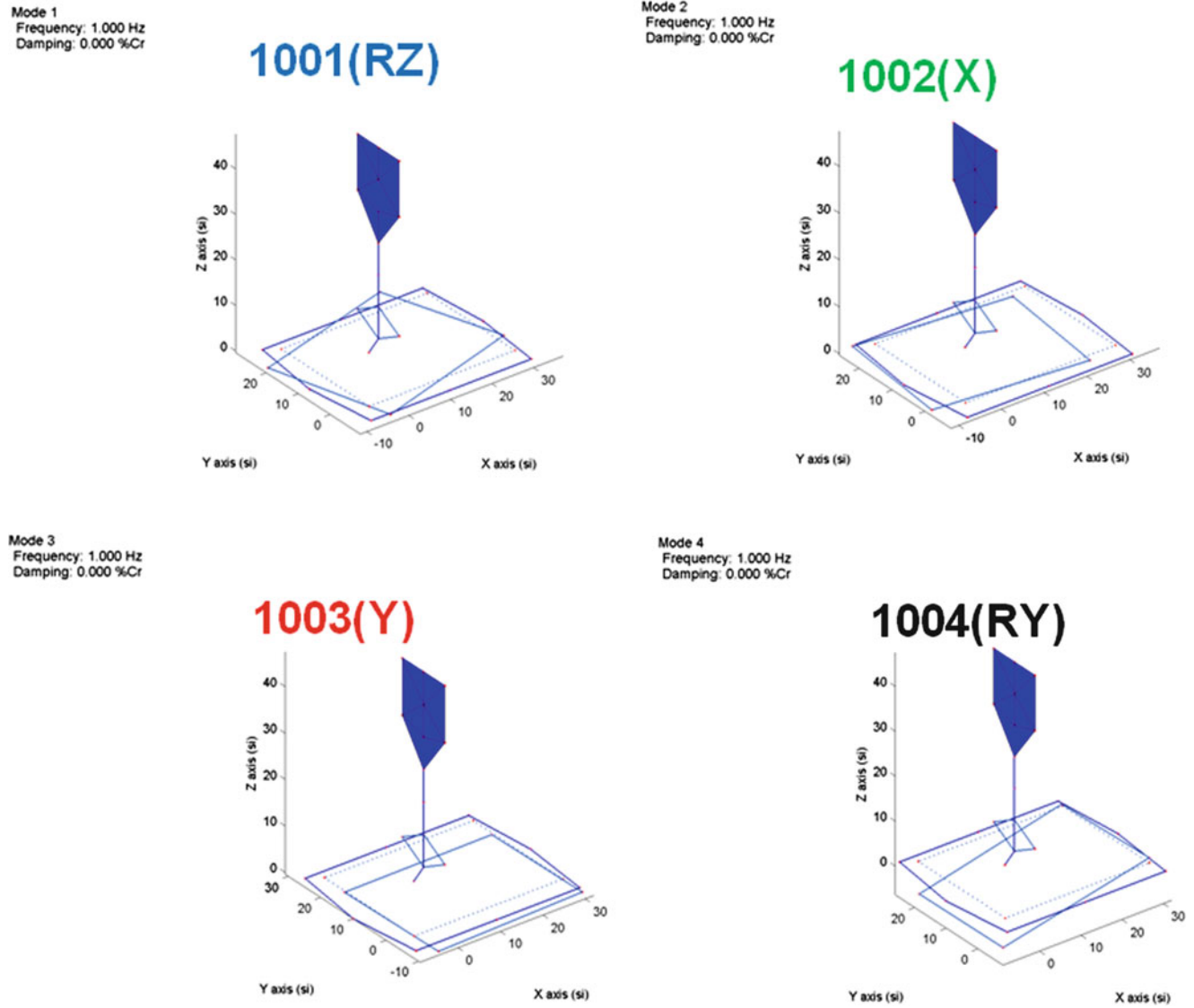


Fig. 24.5 First four singular value shapes of the slip table

slip table deformation. Plots of the first four singular value shapes are presented in Fig. 24.5. Note that only the deformation of the table was calculated using the SVD and therefore the tombstone shows no deformation and is simply included for reference. The majority of the independent motion corresponds to a linear combination of in-plane translation (X, Y) and in-plane rotation (RZ). The fourth shape contains less than 1% of the total motion and is represented by a plunging motion of the end of the slip table. Note that even though the number of independent loads on the slip table was eight, the number of independent deformations of the slip table seen by the structure is three.

A coordinate transformation from the twelve physical channels to generalized coordinates of the slip table was then calculated as

$$[T_{REF}] = [T][U] \quad (24.2)$$

The columns of $[T_{REF}]$ are the time history functions of the singular value shapes. The first three columns of $[T_{REF}]$ and the load cell associated with Shaker 9 on the structure were then used as references for calculating fixed-base frequency response functions. A summary plot of the resulting power spectrum mode-indicator function is presented in Fig. 24.6 below. Compared to the previous plot in Fig. 24.4, the quality of the frequency response functions is clearly better. Note also that

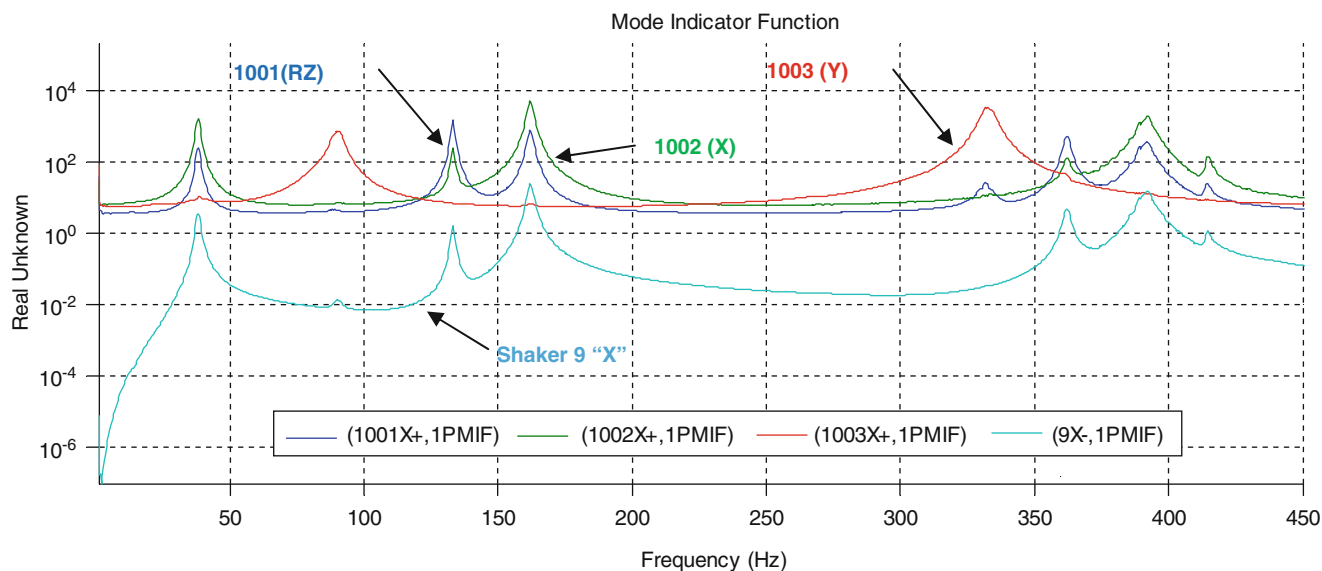


Fig. 24.6 Power spectrum mode-indicator function of all accelerometers due to Shaker 9 and three generalized slip table deformation

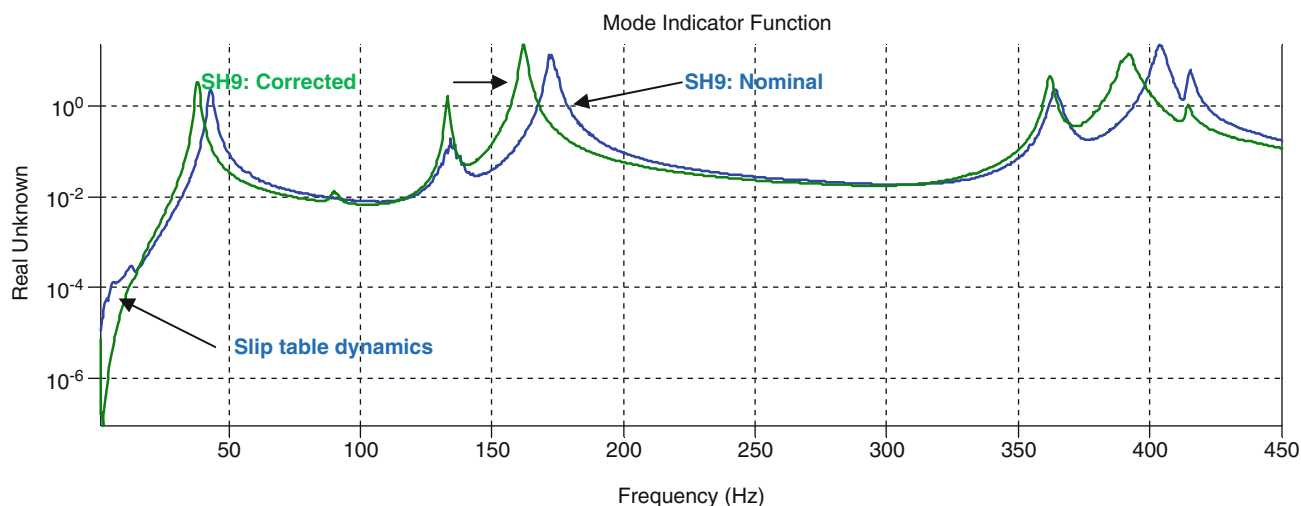


Fig. 24.7 Power spectrum mode-indicator function for Shaker 9: nominal and corrected

the resonant frequencies between the FRF associated with table motion (1001 through 1009) correspond to the resonant frequencies associated with Shaker 9 mounted on the tombstone, and that at least two modes driven by the third operating shape 1003 in the lateral (Y) direction are not driven by Shaker 9 on the tombstone because it is oriented in the “X” direction.

An overlay of the power spectrum mode-indicator function for Shaker 9 using the load cells and the slip table deformed shapes as references is shown in Fig. 24.7. The label “nominal” refers to FRF calculated using all load cells as references, and the label “corrected” refers to FRF calculated using Shaker 9 and the three generalized slip table deformations. Slip table dynamics are observable at low frequencies of the nominal data, but are not observable in the corrected data. Furthermore, the resonant frequencies of the corrected data are lower than the nominal data. This is an indication that the slip table acts more like an inertia restraint than a compliant restraint.

A close-up of Fig. 24.7 is presented below in Fig. 24.8. There is over a 10% difference in resonant frequency between the nominal and corrected FRF.

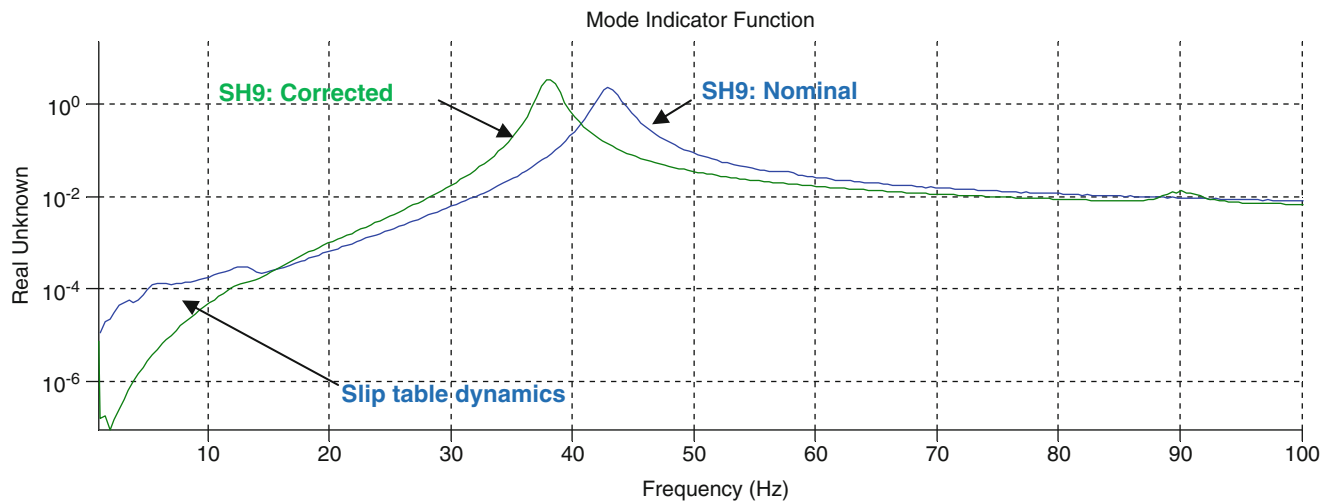


Fig. 24.8 Close-up of power spectrum mode-indicator function for Shaker 9: nominal and corrected

24.4 Error Estimation

One method of estimating how good the selected base accelerometer references were at fixing the base motion is to compare the FRF of the accelerometers on the slip table before and after the technique has been applied. Fig. 24.9 presents a summary power spectrum mode-indicator function plot of the slip table accelerometers and Shaker 9 in the nominal (blue) and fixed (green). The third (red) curve is the corrected power spectrum mode-indicator function for all of the measured accelerometers due to Shaker 9. A significant amount of base deformation was removed by using slip table deformation as a reference.

24.5 Shape Extraction

A plot of the complex mode-indicator function for the corrected FRF (Fig. 24.10) reveals eight independent modes below 450 Hz. Modal parameters for these eight shapes were extracted and are presented in Table 24.2.

The extracted mode shapes are presented in Fig. 24.11. The first seven mode shapes show no observable base motion, while the eighth shape (Fig. 24.12) shows visible motion of the slip table that is out of phase with the axial motion of the structure. This signifies that the vertical rocking of the slip table was not adequately constrained using the three primary shapes of the singular value decomposition. We attempted to use both the fourth singular value shape and the motion of the slip table in the eighth mode shape as references for further signal processing, but the resulting corrected FRF were not well behaved. The most likely reason is that this particular operating shape of the slip table was not adequately excited above the noise floor of the test.

Modal parameters for the eight independent shapes below 450 Hz were calculated and are compared to a test of the same tombstone on an isolated seismic mass in Table 24.2. The frequencies and shapes for the seismic mass test reported in this table were corrected using the method described in [7]. The frequencies extracted from the shake table testing are very reasonable compared to the corrected test on the little seismic mass. Percent differences in damping, however, are much higher. This is most likely due to not constraining the table fully with the measurements, allowing some damping of the oil bearing to “leak” into the fixed-base test. We suspect that the large percentage change in overall damping is due to the fact that higher-level slip table damping influences the very small amounts of damping in the tombstone itself. The overall difference between the two sets of modal parameters is quite small if they are viewed in the complex domain by measuring the change in magnitude and angle of the complex natural frequency. The error represented by percent change in magnitude and difference in angle is presented in Table 24.3. The percent difference in the magnitude of the complex natural frequency is equal to the percent difference in natural frequency since the magnitude of the complex frequency is the natural frequency. The maximum angular difference is 1.34 degrees for the first bending mode in the stiff direction. This small amount of angular error indicates that there is likely no way to reduce the error in the damping estimate.

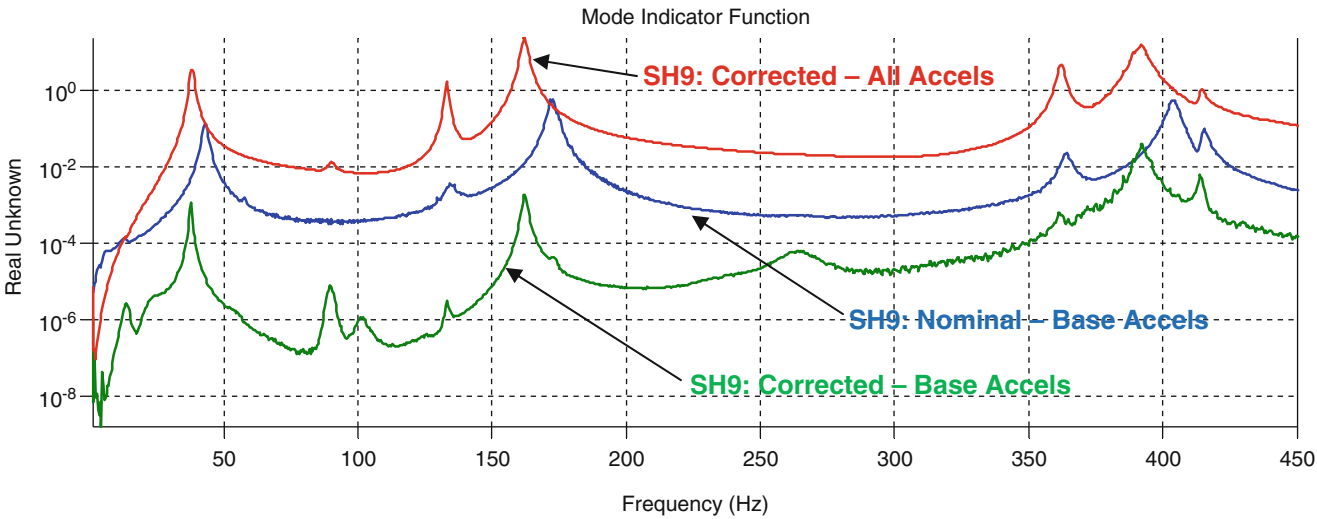


Fig. 24.9 Power spectrum mode-indicator function of slip table accelerometers for both nominal and corrected Shaker 9 FRF

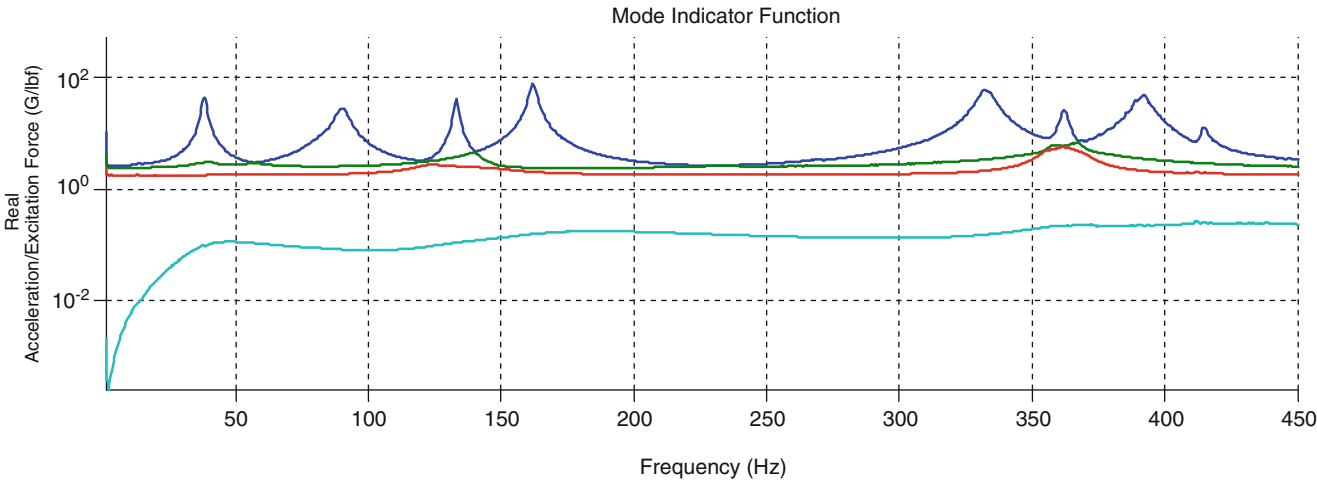


Fig. 24.10 Complex mode-indicator function for the corrected FRF

Table 24.2 Comparison of fixed-base modal parameters for conventional modal test (left) and using corrected FRF of tombstone mounted on slip table

Mode #	Mode description	Little seismic mass shaker test		ATA vibration slip table			
		Frequency (Hz)	Damping (%)	Frequency (Hz)	% Error	Damping (%)	% Error
1	First bend soft direction	38.99	0.16	37.97	−2.62	1.53	869.54
2	First bend stiff direction	89.05	0.18	89.48	0.49	2.52	1328.01
3	First torsion	134.31	0.21	133.08	−0.92	0.53	152.91
4	2nd Bend soft direction	163.61	0.22	161.90	−1.04	0.66	194.40
5	2nd Bend stiff direction	338.16	0.32	332.64	−1.63	0.58	83.26
6	2nd Torsion	364.24	0.26	362.04	−0.60	0.40	53.79
7	First axial	392.41	0.25	414.48	5.62	0.31	23.47
8	3rd Bend soft direction	396.44	0.31	391.49	−1.25	0.69	121.75

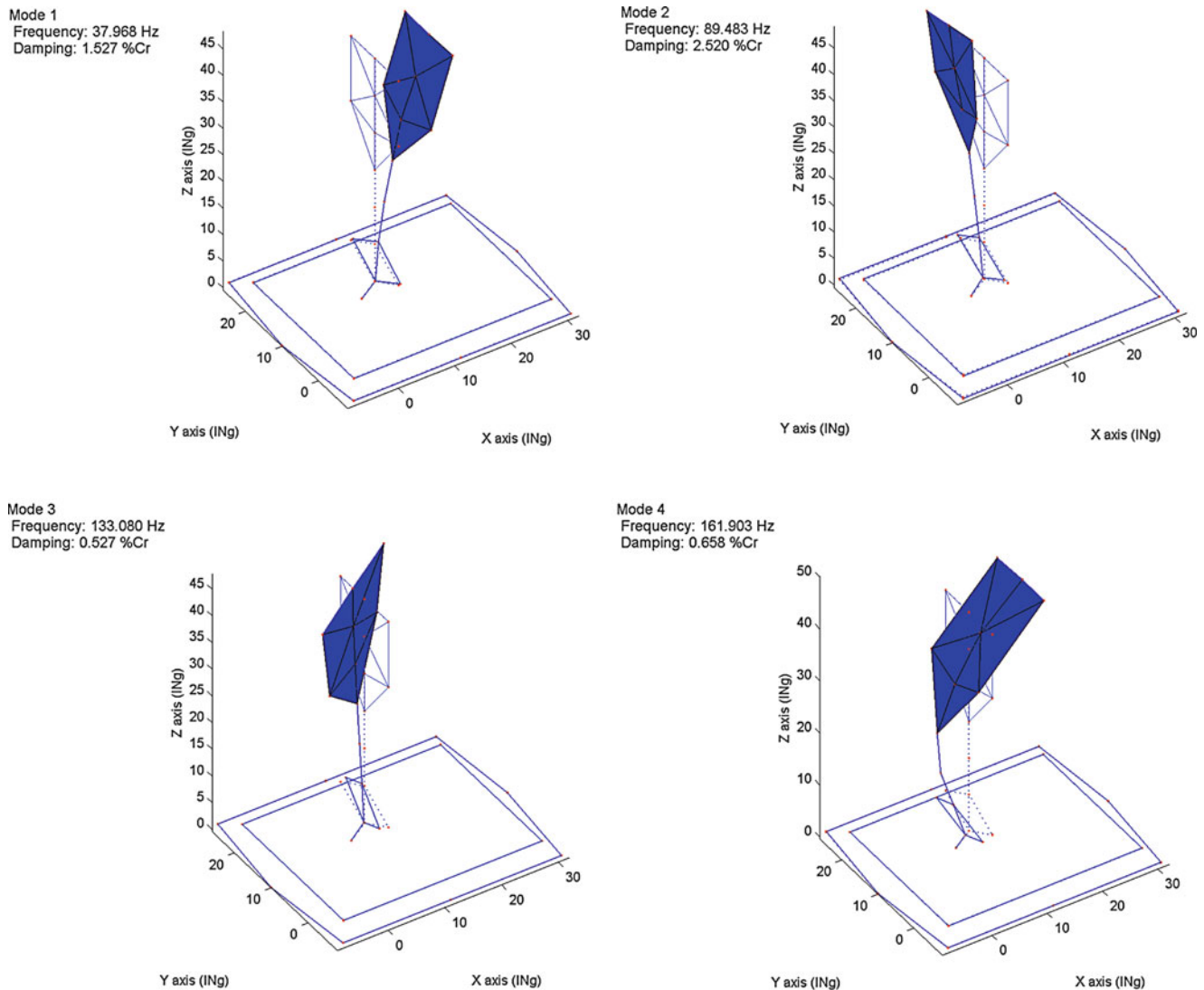


Fig. 24.11 continued

24.6 Discussion

The feasibility of using a transmissibility-based methodology that employs multiple exciters mounted to a slip table to help extract fixed-base frequency response functions and modes has been advanced in this paper. High-quality fixed-base modes were extracted for the first seven modes of the tombstone and compared very well with a fixed-base modal survey of the same structure. The lessons learned or verified from this effort are as follows:

- Sinusoidal excitation techniques are needed to be able to measure accelerations of the slip table.
- High-sensitivity accelerometers with low noise floors should be mounted to the slip table so that their motion can be cleanly measured.
- Singular value decomposition of the time-history data is an effective method for determining the number and shape of independent deformations of the boundary.
- The ability of the method to constrain the boundary can be viewed in near real time by viewing the FRF associated with shakers mounted on the structure to accelerometers mounted on the boundary before and after the FRF correction technique has been applied.
- Fixed-base shapes can be validated by viewing the mode shapes directly to determine how well the boundary is actually “fixed” to the structure.

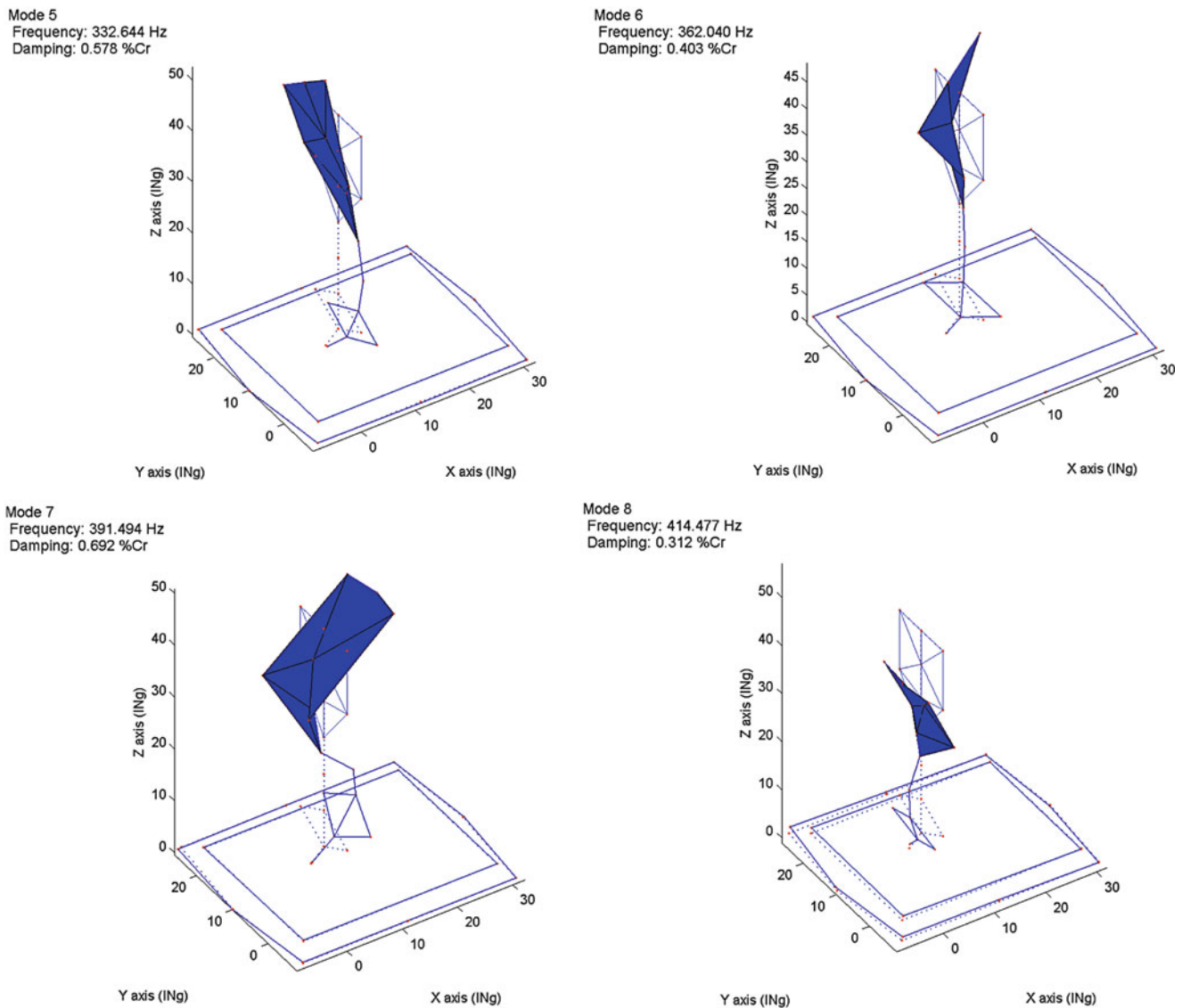


Fig. 24.11 Eight mode shapes below 450 Hz

- Fixed-base modal frequencies are observable from the corrected FRF associated with the generalized slip table deformations and can be combined with the FRF from the shakers mounted on the structure to extract modal parameters. In this test, the shaker mounted on the test article was not needed to extract the mode shapes.
- Estimates of modal parameters are very accurate when viewed in polar coordinates. However, if the test article damping is very low and the damping of the boundary condition is high, then there will likely be a large over-estimation of damping of the test article. In the future, the test setup may need to be designed to mitigate the effects of high levels of damping in the boundary structure.
- Estimation of the vertical mode of vibration was difficult because the vertical response of the slip table was small compared to the response of the slip table in the other directions. In the future, this may be avoided by performing this test while the structure is mounted in the vertical configuration during vibration testing.

ATA Engineering thanks Randall Mayes and Sandia National Laboratories for allowing us to demonstrate the transmissibility technique on a test article mounted to a shaker table. ATA also thanks Bruce Lachey and the Modal Shop, Inc., for providing the high-sensitivity accelerometers used in this test. The lessons learned during this effort will be used to extend the technique to help analysts receive a more accurate set of fixed-base modes for their use in model correlation.

Fig. 24.12 Eighth mode shape shows significant compliance of slip table. The motion of the slip table resembles the fourth shape from the singular value decomposition

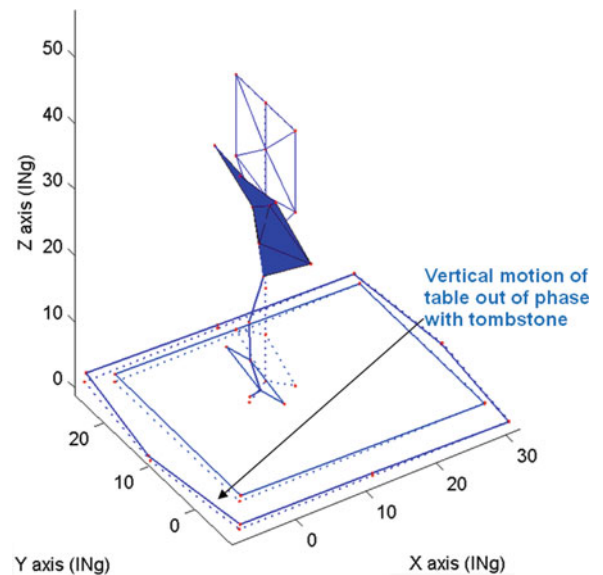


Table 24.3 Frequency error expressed in % error of complex frequency magnitude and angular difference

Mode #	Mode Description	Little Seismic Mass Shaker Test		ATA Vibration Slip Table			Difference (deg)
		Magnitude (Hz)	Angle from 90 (deg)	Magnitude (Hz)	% Error	Angle from 90 (deg)	
1	First bend soft direction	38.99	0.09	37.97	−2.62	0.88	−0.78
2	First bend stiff direction	89.05	0.10	89.48	0.49	1.44	−1.34
3	First torsion	134.31	0.12	133.08	−0.92	0.30	−0.18
4	2nd Bend soft direction	163.61	0.13	161.90	−1.04	0.38	−0.25
5	2nd Bend stiff direction	338.16	0.18	332.64	−1.63	0.33	−0.15
6	2nd Torsion	364.24	0.15	362.04	−0.60	0.23	−0.08
7	First axial	392.41	0.14	414.48	5.62	0.18	−0.03
8	3rd Bend soft direction	396.44	0.18	391.49	−1.25	0.40	−0.22

References

1. Carne TG, Martinez DR, Nord AR (1989) A comparison of fixed-base and driven base modal testing of an electronics package. In: Proceedings of the seventh international modal analysis conference, Las Vegas, February 1989, pp 672–679
2. Beliveau JG, Vigneron FR, Soucy Y, Draisey S (1986) Modal parameter estimation from base excitation. *J Sound Vib* 107:435–449
3. Fullekrug U (1996) Determination of effective masses and modal masses from base-driven tests. In: Proceedings of the 14th international modal analysis conference, Dearborn, 1996, pp 671–681
4. Sinapius JM (1996) Identification of fixed and free interface normal modes by base excitation. In: Proceedings of the 14th international modal analysis conference, Dearborn, February 1996, pp 23–31
5. Mayes RL, Bridgers LD (2009) Extracting fixed base modal models from vibration tests on flexible tables. In: Proceedings of the 27th international modal analysis conference, Orlando, February 2009
6. Napolitano K, Yoder N (2012) Fixed base FRF using boundary measurements as references—analytical derivation. In: Proceedings of the 30th international modal analysis conference, Jacksonville, February 2012
7. Mayes R, Rohe D, Blecke J (2013) Extending the frequency band for fixed base modal analysis on a vibration slip table. In: Proceedings of the 31st international modal analysis conference, Jacksonville, February 2013
8. Napolitano K, Linehan D (2009) Multiple sine sweep excitation for ground vibration tests. In: 27th international modal analysis conference, Orlando, February, 2009
9. Hoople G, Napolitano K (2010) Implementation of multi-sine sweep excitation on a large-scale aircraft. In: 28th international modal analysis conference, Jacksonville, February 2010
10. Napolitano K, Yoder N, Brillhart R (2012) A comparison of multiple impact testing methods. In: Proceedings of the 30th international modal analysis conference, Jacksonville, February 2012
11. Daniell P (□□□□) Discussion of ‘on the theoretical specification and sampling properties of autocorrelated time-series’. *J Roy Stat Soc* 8:88–90

Chapter 25

Efficient Method of Measuring Effective Mass of a System

Randy L. Mayes, Tyler F. Schoenherr, Jill Blecke, and Daniel P. Rohe

Abstract Effective mass is a system property that is used in the aerospace industry in predicting the forces of an elastic payload on the delivery system in a specific direction. Effective mass is usually calculated with the finite element model. Experimental effective mass can be used to validate calculated effective mass from a finite element model. Measuring the effective mass of a system, however, has been difficult and has been attempted by putting force sensors between the payload and a test base. A much more tractable method amenable to a payload mounted on a slip table is provided here. The method measures a driving point and a base frequency response function and uses a recently developed method to constrain the base response and calculate the effective mass. This theory will be demonstrated with an analytical system and example hardware.

Keywords Effective mass • Modal participation factor • Experimental

Nomenclature

$\ddot{x}$	Acceleration in one direction
Θ_c	Mode shapes of the fixture to be constrained
ζ	Modal damping ratio
Ψ	Mass normalized real mode shape matrix
Γ	Eigenvectors resulting from constraint equations
B	Subscript for the fixture or base
c	Subscript for degrees of freedom on fixture to be constrained to zero
DoF	Degree of freedom
f	Force
FRF	Frequency response function
i	Subscript for DoF number
k	Spring rate
k	Subscript for mode number
L	Reduction matrix applying the constraint to equations of motion
M_B	Mass of the fixture or base
m_i	Mass at DoF i
mm_k	Modal mass for mode k
P_k	Modal participation factor in one direction for mode k
$pmpf(k)$	Pseudo-modal participation factor for mode k

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the U.S. Department of Energy under Contract DE-AC04-94AL85000.

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q	Generalized coordinate
TA	Subscript for the test article
v	Velocity in one direction
x	Displacement in one direction
Φ	Rigid body mode shape vector with ones in the direction of interest
ω	Angular frequency (radians/second)

25.1 Motivation

Sandia National Laboratories has at least three motivations for having measured effective mass values for the modes of a base mounted payload in a specific direction. The first motivation comes from satellite payload applications. When performing loads analysis to determine the loads a payload puts back into the delivery bus during launch, the modes with high effective mass put high loads back into the bus. Classically, effective mass is calculated from the finite element model, but if the effective mass of the model is wrong, the loads analysis will be wrong. It would be of value to validate the model with test measured effective mass to greatly enhance the confidence in the loads analysis.

The second motivation is associated with the methodology now being utilized at Sandia to quantify margin. This methodology requires a modal model with the modal masses scaled so that the effective mass of each mode in the various shock or vibration test axes are known. Again, this information has always come from a finite element model in the past, but in some cases finite element models have not been generated. It would be of value to extract a modal model experimentally with the associated effective masses for each important mode in the various test axis directions. Then the margin quantification could proceed without the finite element model. Even if the finite element model is available, to check its effective masses against test data would provide a valuable check on the model for the margin calculation.

A third motivation is for vibration control. In certain cases force limiting is practiced to remove over-conservative effects of enveloping test environments. Knowing the effective mass of the test article would allow notching of acceleration inputs without installing force gages under the test article, because the effective mass and input accelerations could be used to limit the force into the structure.

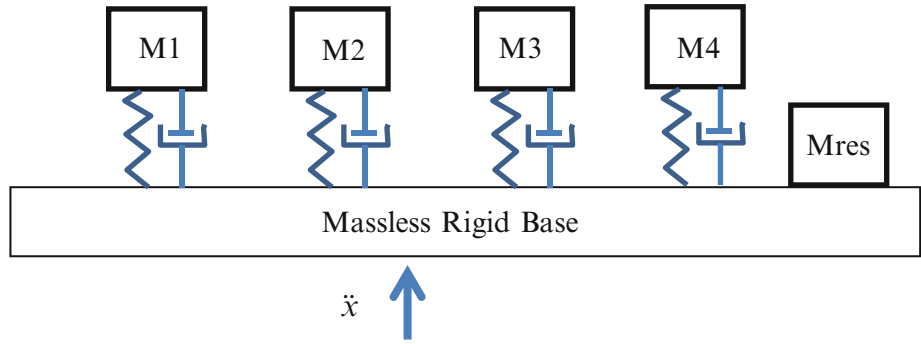
25.2 Effective Mass Concept and History

The effective mass offers a physical interpretation of a physical system with multiple modes of vibration being excited dynamically from a base, similar to testing that occurs for many systems. The concept was proposed in the early 1970s by Bamford [1] with others. For a base excited system, it is represented as attached to a massless base, which will be excited in only one direction with acceleration, $\ddot{x}$, with each mode represented by a single degree of freedom oscillator as shown in Fig. 25.1. The mass of each oscillator is valued so that it is the effective mass of the respective mode of vibration. The springs are scaled so that the mass vibrates at the appropriate modal frequency. In general, only the modes that have the significant effective masses are required to represent the response up to some desired frequency. In the figure we show four such modes. The other modes are truncated. The effective mass of all the truncated modes is added directly to the base as a residual mass. When the base is accelerated with some vibration specification, the various effective mass oscillators will impose the correct reaction force on the base in the direction of excitation. As can be seen from this illustration, effective mass is based on a system that can be represented as having a base input. It depends also on the assumption that the base is rigid.

Effective mass is related to modal participation factor, P_k . The derivations of modal participation factor and effective mass can be found in the FEMA 451B Topic 4 Notes; http://www.ce.memphis.edu/7119/fema_notes.htm. A major difference between effective mass and modal participation factor is that modal participation factor is different depending on the scaling of the modal mass, mm_k , whereas effective mass is a single defined value. In a fixed base eigenvalue problem of an analytical model of the system, the modal participation factor multiplied by $-\ddot{x}$ provides the modal force that will excite a particular mode for the rigid base acceleration, $\ddot{x}$. The effective mass, which provides the actual base reaction force associated with a particular mode, resolves any question about mode shape scaling and is calculated as

$$m_{eff,k} = P_k^2 mm_k \quad (25.1)$$

Fig. 25.1 Physical picture of effective mass concept



If Ψ_{fixed}^k is the fixed base mode shape vector for mode k and $\mathbf{M}$ is the mass matrix of the test article, the modal participation factor can be calculated from the rigid body mode shape vector, Φ , of the system released and translating in the direction of acceleration of the base as

$$P_k = \bar{\Phi}^T \mathbf{M} \bar{\Psi}_{fixed}^k / m m_k \quad (25.2)$$

where the rigid body shape values of vector Φ in the direction of acceleration are equal to one and in orthogonal directions are equal to zero. The modal participation factor and effective mass are related to direction. If vector Φ represents the rigid body mode shape in the y direction, as opposed to the x direction as shown in the figure, a different modal participation factor (and effective mass) will be calculated for mode k .

As can be seen, this standard approach requires a finite element model. If the finite element model is in error, the modal participation factor and effective mass will be in error. One method of attempting to extract the modal participation factor and effective mass is to sandwich force gages or a so-called force measurement device between the test article and the shaker table and measure those forces and extract modal parameters from the system during a vibration test [2–4]. Even such a measurement generally assumes the shaker table is rigid. Sedaghati, Soucy and Etienne [5] did this placing eight triaxial force gages beneath a payload during a vibration test. They checked their results by comparing with the effective mass of a finite element model. One of the larger effective mass measurements was within seven percent but two were off by greater than 40 percent and the effective mass of the first mode was off by hundreds of percent. They claimed the error was due to nonlinearity, but the FRF showing the apparent mass appeared to retain the shape one would expect for the first mode of a linear system. This also required an in situ re-calibration of the force sensors to match the measured mass of the test article, which demonstrates the difficulty of trying to obtain force measurements in a sandwich configuration.

25.3 Effective Mass Measurement Approach

Here the authors propose an approach logistically simpler than measuring all the forces of attachment of a structure to a slip table. It may be applied to any fixture which has at least one rigid body mode (a mode low enough in frequency that it can be considered to be represented purely by rigid body motion), as well as to a shaker table (which always has a rigid body mode in the direction it is designed to shake). The method does not require a rigid base assumption as with the sandwiched force measurement device. The natural dynamics of the shaker table or fixture are measured and then analytically constrained to provide a mathematical rigid base. The method builds on previous work which has been used to calculate fixed base mode shapes from experimental modal parameters extracted from a test article mounted to a shaker table or other dynamic fixture. Mayes and Bridgers [6] first showed that one could perform a modal analysis of a test article on a plate and analytically fix the modes of the plate so that the resulting constrained modes approximated the fixed base modes. Since then several refinements and studies have been performed [7–9]. This capability to analytically constrain the system inherently contains enough information to calculate the effective mass in directions well excited by the modal test, if one knows the mass properties of the fixture to which the test article is attached. From this point forward, the “fixture” refers to the base to which a test article is attached, whether it is a shaker table or some other system.

This approach will be demonstrated with a simple MATLAB spring mass model and an experimental system. The experimental system is shown set up on a slip table in Fig. 25.2. The test article is a nylon beam-like structure.

Fig. 25.2 Experimental test article mounted on slip table



25.4 Effective Mass Measurement Theory

Begin with a system that has one rigid body mode in the direction of the desired effective mass estimate. Let us consider an example multi-degree of freedom system as shown in Fig. 25.3.

The system has one rigid body mode in the x direction of motion. The mass of the base, M_B , is known. The multi-degree of freedom test article is represented by masses m_1 through m_n . Although the individual masses are not known, the test article total mass, which is the sum of m_1 through m_n , is known. The base cannot rotate, and is considered a rigid mass that may only move in the x direction. One performs a modal test on this system and at least determines the modal parameters and the mode shape of the base for the rigid body mode and several elastic modes, which we will call Ψ_B .

Recalling Eq. (25.2) as

$$P_k = \bar{\Phi}^T \mathbf{M} \bar{\Psi}_{fixed}^k / m m_k \quad (25.2)$$

the first question is how can we get Ψ_{fixed}^k ? In the appendix, the equations to constrain the experimental mode shapes obtained from the test of Fig. 25.3 are given providing

$$\Psi_{fixed} = \Psi \mathbf{L} \Gamma \quad (25.3)$$

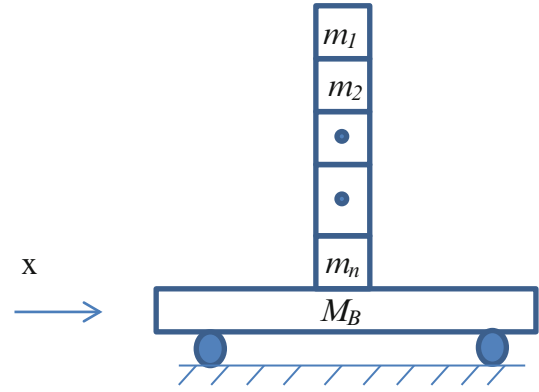
so that Ψ_{fixed}^k can be determined from

$$\Psi_{fixed}^k = \Psi \bar{\mathbf{a}}_k \quad (25.4)$$

where the vector $\bar{\mathbf{a}}_k$ is the k th column of $\mathbf{L} \Gamma$. Assume the mode shapes are scaled for unit modal mass and substitute Eqs. (25.4) into (25.2) to give

$$P_k = \bar{\Phi}^T \mathbf{M} \Psi \bar{\mathbf{a}}_k \quad (25.5)$$

Fig. 25.3 Simple analytical base mounted test article



Which can be expanded as

$$P_k = [\bar{\Phi}^T \mathbf{M} \bar{\Psi}_1 \quad \bar{\Phi}^T \mathbf{M} \bar{\Psi}_2 \quad \dots \quad \bar{\Phi}^T \mathbf{M} \bar{\Psi}_n] \bar{\mathbf{a}}_k \quad (25.6)$$

Let us define each term in the brackets in Eq. (25.6) as a pseudo-modal participation factor (*pmpf*). It is prefixed with “pseudo” because we obtain it with the base mass free to move, instead of fixed, as in the strict definition of modal participation factor. Each of the pseudo-modal participation factors considers the mode shape of the entire test article, but NOT the base. One must determine each of these *pmpf* s. For the first mode, which is the rigid body mode translating in x, we can note that

$$\bar{\Phi}^T \mathbf{M} = [m_1 \ m_2 \ \dots \ m_n]_{TA} \quad (25.7)$$

Which gives a row vector of all the masses of the test article, subscript TA, and

$$\bar{\Psi}_1 = \left\{ \begin{matrix} 1 \\ 1 \\ \dots \\ 1 \end{matrix} \right\} \Psi_B^1 \quad (25.8)$$

Combining Eqs. (25.7) and (25.8) yields the first *pmpf* from the rigid body mode as

$$pmpf(1) = \bar{\Phi}^T \mathbf{M} \bar{\Psi}_1 = [m_1 \ m_2 \ \dots \ m_n]_{TA} \left\{ \begin{matrix} 1 \\ 1 \\ \dots \\ 1 \end{matrix} \right\} \Psi_B^1 = m_{TA} \Psi_B^1 \quad (25.9)$$

This requires only two values, the known mass of the test article without base, m_{TA} , and Ψ_B^1 which is the measured base rigid body mode shape (mass normalized) in the desired direction. Though the individual elements of the mass matrix are not known in a hardware modal test, both terms are known on the right hand side if one extracts the mode shape of the base and weighs the test article.

The *pmpf* for any other elastic mode may be obtained from conservation of momentum. Assume we excite only one of the elastic modes, k , in an undamped sense. Then the momentum sum is

$$\sum_{i=1}^n m_i v_i + M_B v_B = 0 \quad (25.10)$$

where v is the velocity. The velocity can be replaced with the modal substitution to give

$$\left(\sum_{i=1}^n m_i \Psi_i^k + M_B \Psi_B^k \right) \dot{q} = 0 \quad (25.11)$$

with q as a generalized coordinate. Dividing by the generalized velocity and noting that the summation term is equal to the $pmpf(k)$ gives

$$pmpf(k) = \Phi^T \mathbf{M} \Psi^k = \sum_{i=1}^n m_i \Psi_i^k = -M_B \Psi_B^k \quad (25.12)$$

The $pmpf$ for each elastic mode comes simply from the mass of the base and the mode shape of the base. Now all quantities are known so Eqs. (25.9) and (25.12) are substituted into Eq. (25.6) to give the estimate of the true modal participation factor, P_k , from which effective mass can be obtained in Eq. (25.1). In review, $\tilde{\mathbf{a}}_k$ is the k th column of $\mathbf{L}\Gamma$, $pmpf(1)$ comes from the mass of the test article and the rigid body mode shape of the base, and the elastic $pmpf(k)$ are determined from the mass of the base and the test mode shape of the base. The resulting modal participation factor, P_k , is subject to errors of modal truncation.

The previous derivation shows how to estimate the effective mass from modal parameters for the described system which has only one unconstrained DoF for the base motion. However, most systems are not constrained with only one base DoF as the described system. They may have more than one rigid body mode of the fixture, and there may be other elastic modes of the fixture. Here we follow the previous work [6–9]. One performs a modal test on the system and fixture with enough accelerometers mounted on the fixture to uniquely identify every different shape active in the fixture. These fixture shapes are then captured as multiple independent vectors in Θ_c . One of those vectors, noted as vector j , should be rigid body motion that is purely in the x direction of Fig. 25.3. Equation (A4) provides the required constraint, except that row j should be removed from the constraint. This allows the system to have rigid body motion in the one desired direction as depicted in Fig. 25.3, while constraining all other rigid body and elastic motions. The appendix equations with the required adjustment to Eq. (25.A4) recasts the original dynamics to those of a problem constrained as defined in Fig. 25.3.

To review, one performs a forced modal test on the system. Enough accelerometers on the fixture must be placed so that the active rigid and elastic shapes may be captured with independent fixture shapes, Θ_c . The appendix equations are applied to obtain a system that is constrained similar to that shown in Fig. 25.3. Then use the partially constrained modal parameters, the mass of the base and the mass of the test article to calculate the pseudo-modal participation factors. Finally constrain the final rigid body DoF and determine the modal participation factors and resulting effective masses from Eqs. (25.1) and (25.6).

25.5 Analytical Example

Consider a spring mass system with three DoF as shown in Fig. 25.4. Here, M_B is the mass of the base and m_1 and m_2 are associated with the masses of the test article. So the total mass of the test article is $m_{TA} = m_1 + m_2$. For this configuration, the system has one rigid body mode. One might imagine M_B as the slip table and m_1 and m_2 as the test article mounted to the slip table.

One desires to perform a modal test of this system, with accelerometers mounted on the base. The mass of the base, M_B , and the mass of the test article, m_{TA} , are known. From the modal test three modes are extracted with mass normalized mode shapes of the base. The first mode is a rigid body mode. Then one desires to analytically fix the base, and get the modal participation factors of the test article and the effective mass for the two remaining modes in the x direction. The parameters used in this example are $M_B = 500$, $m_1 = 2$, $m_2 = 5$, $m_{TA} = 7$, $k_{B2} = 10,000$, $k_{12} = 10,000$. Using Eqs. (25.4), (25.7), (25.8–25.19) the effective mass for the first and second modes of the fixed base system are computed as 6.8577 and 0.1423, which match the true values calculated for a fixed base system from Eqs. (25.1), (25.2). Also, the sum of all the effective masses in one direction are equal to m_{TA} , which is seven for this case.

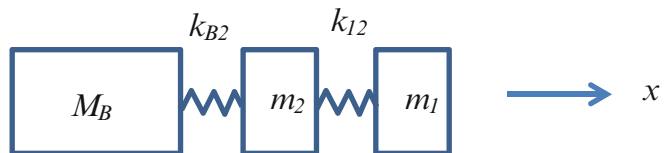


Fig. 25.4 Analytical example 3 DoF system

25.6 Hardware Example on Small Seismic Mass

The first hardware test setup is shown in Fig. 25.5. The test article is a 72 kg nylon beam and the fixture is a 605 kg steel seismic mass which was instrumented with four triaxes at the corners. The test article and seismic mass were suspended with straps, yielding rigid body modes below five Hz. A modal test was performed using a random shaker input and modes were extracted using SMAC [10]. Three input locations near the top of the beam were excited. Effective mass was extracted for the first ten modes to 1350 Hz in the softer bending direction of the beam. The seismic mass had one elastic classic plate mode at 1370 Hz, and it was constrained along with the rigid body modes of the mass for this analysis. One of the x direction accelerometers on the mass was found to have low amplitude, so it was excluded from the analysis. The beam was modeled with finite elements (FE) in SALINAS and the uncertain Young's modulus and Poisson's ratio were adjusted to best match the test data. All the first ten modes in the soft bending direction matched within two percent between the test and FE model. The effective masses are compared in Table 25.1, expressed as a fraction of the total test article mass. The difference column is the difference expressed as a percentage of the total test article mass. Most of the agreement is within two percent of the total test article mass. Interestingly, the very first mode is the worst comparison, with the values about eight percent different, which is an error that is a little over three percent of the total mass.

It was noted above that one of the accelerometers on the base that measured motion in the direction of the desired effective mass was excluded from the analysis. It appeared to be reading about 20 percent lower than the others for modes with fairly

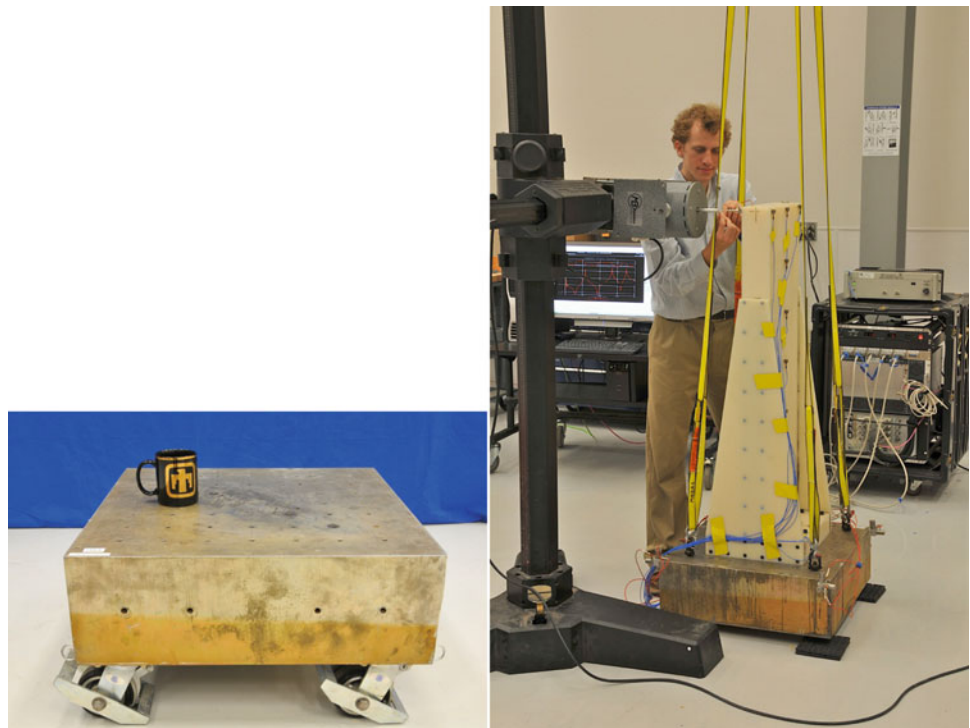


Fig. 25.5 Small seismic mass (left) and test article setup (right)

Table 25.1 Test effective mass from seismic mass test compared to FE model

Test frequency (Hz)	Effective mass from test	Effective mass from FE model	Difference as % of total test article mass
38.94	0.415	0.448	3.3
163.5	0.187	0.189	0.2
396.3	0.088	0.085	-0.4
706.4	0.037	0.047	1.0
859.2	0.004	0.000	-0.4
1034.7	0.011	0.000	-1.1
1048	0.003	0.023	2.0
1190.7	0.000	0.000	0.0
1316.1	0.027	0.004	-2.3
1344.3	0.020	0.004	-1.6

pure motion in that axis. When that accelerometer was included in the analysis, the difference from the FE calculation in the first test effective mass more than doubled. This result underscores the need for very accurate absolute measurements on the fixture structure.

25.7 Hardware Example on Slip Table

The same test hardware was used on a slip table with the same theory. In this case only two fixture modes were present. One was a rigid body mode of the slip table at 0 Hz and one was the first extensional mode of the slip table at 2035 Hz. In this work, a portable shaker was attached to the slip table with a force gage and the slip table was not connected to the large shaker that normally drives it. The test setup is shown in Fig. 25.6. Seven triaxial accelerometers were mounted on the slip table and used for the constraint analysis. The modal test utilized random input, and the modal extraction was performed with SMAC. Very similar test effective mass results were obtained from this separate setup and are given in Table 25.2.



Fig. 25.6 Test article mounted on slip table with portable shaker

Table 25.2 Effective mass extracted from slip table test compared to FE model

Test frequency (Hz)	Effective mass from test	Effective mass from FE model	Difference as % of total test article mass
38.2	0.413	0.448	3.5
162.3	0.177	0.189	1.2
393	0.080	0.085	0.5
701.5	0.051	0.047	−0.4
852.9	0.000	0.000	0.0
1028	0.033	0.000	−3.3
1040	0.018	0.023	0.5
1199	0.000	0.000	0.0
1301	0.001	0.004	0.3
1344	0.001	0.004	0.3

25.8 Conclusions

This paper develops the theory to calculate effective mass from a standard modal test of a test article on a fixture, when the fixture has at least one rigid body mode in the direction of the desired effective mass calculation. The strength of the theory is verified by an analytical example. It is then applied to two different test setups for the same test article. In one setup, the test article is mounted on a seismic mass and suspended softly with straps. In the other setup, the test article is mounted on a slip table for a shaker. The effective mass for each mode in one direction is extracted from the tests and compared to a FE model effective mass calculation for the test article. (The FE model Young's modulus and Poisson's ratio were calibrated so that the frequencies of the FE model match the test within two percent). The test effective mass calculations agree with the FE calculations within about four percent of the total mass of the test article. The first mode has the greatest test effective mass, and is the most different from the FE model calculation. Agreement for the test effective mass between the two tests is very consistent, being within about one percent of the total mass, even though the setups were quite different. It is not known whether the difference between the test effective mass and the FE effective mass in the first mode is due to test error or FE error.

A.1 Appendix – Constraining a System with One Rigid Body Mode to Fixed Base

The system of Fig. 25.3 can be modeled, neglecting damping, as

$$\left[\omega_n^2 \right] \{q\} - \omega^2 [I] \{q\} = \Psi^T \{f\} \quad (25.A1)$$

where ω_n represents the circular natural frequencies, q represents the modal coordinates, f is the vector of forces. A constraint for the fixture (subscript c) DoF can be written as

$$\{x_c\} = \{0\} \quad (25.A2)$$

or using the modal substitution, then

$$\Psi_c \{q\} = \{0\} \quad (25.A3)$$

Generally, with the authors' method, there will be many accelerometers mounted on the fixture represented by the lower mass, M_B . One may convert from many physical DoF constraints of Eqs. (25.A2) and (25.A3) to one modal constraint with

$$\bar{\Theta}_c^+ \Psi_c \{q\} = \{0\} \quad (25.A4)$$

Where the vector Θ_c comes from an estimate of the single translation rigid body mode shape vector of the fixture. The superscript $+$ indicates the pseudo-inverse. This projects the constraint onto the vector space of Θ_c , which has the advantage of producing a least squares fit through any experimental modal analysis shape errors, since Θ_c has more sensors (rows) than modes (columns). The unconstrained coordinates can be written in terms of a reduced number of unconstrained generalized coordinates as

$$\{q\} = \mathbf{L} \{q_L\} \quad (25.A5)$$

Substituting Eqs. (25.A5) into (25.A4) yields

$$\bar{\Theta}_c^+ \Psi_c \mathbf{L} \{q_L\} = \{0\}, \quad (25.A6)$$

where $\mathbf{L}$ is found as the matrix of vectors that are orthogonal to $[\bar{\Theta}_c^+ \Psi_c]^T$

$$\mathbf{L} = \text{null}(\bar{\Theta}_c^+ \Psi_c) \quad (25.A7)$$

which satisfies the constraint. Substituting Eqs. (25.A5) into (25.A1) and pre-multiplying by $\mathbf{L}^T$, gives the coupled eigenvalue problem

$$\mathbf{L}^T \left[\omega_n^2 \right] \mathbf{L} \{q_L\} - \omega^2 \mathbf{L}^T [I] \mathbf{L} \{q_L\} = \{0\} \quad (25.A8)$$

If we solve this eigenvalue problem and call the resulting generalized uncoupled coordinates p and the associated eigenvectors Γ , then from the modal substitution and Eqs. (25.A2), (25.A3) and (25.A5)

$$x = \Psi \mathbf{L} \Gamma \{p\} \quad (25.A9)$$

So the fixed base mode shapes are estimated as

$$\Psi_{fixed} = \Psi \mathbf{L} \Gamma \quad (25.A10)$$

Equation (A10) shows that the fixed base mode shapes are just a linear combination of the original mode shapes associated with the boundary condition of Fig. 25.3.

References

1. Wada BK, Bamford R, Garba JA (1972) Equivalent spring-mass system: a physical interpretation. In: The shock and vibration bulletin, US Naval Station, Key West, January 1972, pp 215–224
2. Fullekrug U (1996) Determination of effective masses and modal masses from base-driven tests. In: Proceedings of the 14th international modal analysis conference, Dearborn, February 1996, pp 671–681
3. Sinapius JM (2009) Identification of free and fixed interface normal modes by base excitation. In: Proceedings of the 14th international modal analysis conference, Orlando, February 2009, pp 23–31
4. Schedlinski C, Link M (1995) Identification of frequency response functions and modal data from base excitation tests using measured interface forces. In: Proceedings of the ASME conference on noise and vibration, Boston, 1995, ISBN 0-7918-1718-0
5. Sedaghati R, Soucy Y, Etienne N (2003) Experimental estimation of effective mass for structural dynamic and vibration applications. In: Proceedings of the 21st international modal analysis conference, Kissimmee, 2003, p 188
6. Mayes RL, Bridgers LD (2009) Extracting fixed base modal models from vibration tests on flexible tables. In: Proceedings of the 27th international modal analysis conference, Orlando, February 2009, p 67
7. Allen MS, Gindlin HM, Mayes RL (2010) Experimental modal substructuring to extract fixed-base modes from a substructure attached to a flexible fixture. In: Proceedings of the 28th international modal analysis conference, Jacksonville, February 2010, p 164
8. Mayes RL, Allen MS (2011) Converting a driven base vibration test to a fixed base modal analysis. In: Proceedings of the 29th international modal analysis conference, Jacksonville, 2011, p 36
9. Mayes RL (2012) Refinements on estimating fixed base modes on a slip table. In: Proceedings of the 30th international modal analysis conference, February 2012, p 162
10. Hensley DP, Mayes RL (2006) Extending SMAC to multiple references. In: Proceedings of the 24th international modal analysis conference, February 2006, pp 220–230

Chapter 26

Formulation of a Craig-Bampton Experimental Substructure Using a Transmission Simulator*

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Abstract Recently, a new experimental based substructure formulation was introduced, called Modal Constraint for Fixture and Subsystem (MCFS). This method reduces ill-conditioning by imposing constraints on substructure modal coordinates instead of the physical interface coordinates. The experimental substructure is tested in a free-free configuration, and the interface is exercised by attaching a flexible transmission simulator. An analytical representation of the fixture is then used to subtract its effects from the experimental substructure. The resulting experimental component is entirely modal based, and can be attached in an indirect manner to other substructures using MCFS. In contrast, this work presents a formulation in which the analytical representation of the transmission simulator is in the form of a Craig-Bampton (CB) substructure including fixed-interface modal coordinates and physical interface coordinates. The negative of the analytical representation of the transmission simulator is constrained to the experimental modal model using MCFS by eliminating the fixed-interface modal coordinates. The resulting experimental substructure contains a hybrid set of coordinates, including modal coordinates and the physical interface degrees of freedom, analogous to a CB representation. This new formulation offers the improved conditioning of the MCFS approach, but can be directly connected through the physical interface coordinates to other finite element based substructures.

Keywords Experimental substructures • Substructuring • Craig-Bampton

26.1 Introduction

Component mode synthesis (CMS) has been a fundamental tool for the structural analysis of large complex systems for years. Instead of the system being modeled as a whole, it is broken up into substructures that are then modeled and reduced. This approach is often a necessity due to sheer model size. In addition, individual system components are often constructed by different companies. This is especially true in the aerospace community. The Craig-Bampton (CB) substructure representation [1] has become the most popular and efficient approach within the aerospace industry. In recent years, there has been a renewed interest in combining analytical based and experimental based substructures using CMS and the imposition of constraints. Perhaps a specific substructure is too complicated to model analytically, or there is just not enough time available for modeling.

Over the years, a relatively large number of researchers have studied the feasibility of combining analytical and experimental substructures. For example, Martinez et al. investigated the coupling of an experimental beam like structure

* Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

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and a shell-payload structure [2]. Urgueira [3] considered the need for the inclusion of residual compliances when free interface substructure modes are used, and the difficulty in measuring rotations. Researchers have attempted to eliminate the need for interface residuals by mass loading the substructure interface during a free-free vibration test. For example, Ind [4] considered the problem of testing a delicate substructure by attaching a fixture, testing the combined system, and then removing the effects of the fixture from the measured responses using a variety of techniques. The direct approach to substructure coupling would be to then enforce compatibility in the physical connection degrees of freedom between substructures. Work in this area has resulted in a measured amount of success, but in most cases, significant difficulties were encountered due to lack of sufficient precision in the measurements, uncertainties, numerical ill-conditioning, inability to measure rotations, nonphysical results, etc. Large errors in system response can result from small errors in subcomponent models [5].

Recently, a new approach has been introduced, called Modal Constraint for Fixture and Subsystem (MCFS) [6]. This method reduces ill-conditioning by imposing constraints on substructure modal coordinates to connect substructures, instead of the physical interface coordinates. The experimental substructure is tested in a free-free configuration, and the interface is exercised by attaching a flexible fixture, or transmission simulator. A finite element (FEM) representation of the transmission simulator is then used to subtract its effects to produce the desired experimental based substructure representation. The modal constraint approach can be applied to both subtracting the FEM transmission simulator, and also connecting the resulting experimental substructure to other FEM based substructures. The approach has been used with success in several representative examples [7].

This paper presents a novel modification of the basic MCFS approach. Instead of using a free-free modal representation of the FEM transmission simulator in the subtraction process, it is represented by a Craig-Bampton substructure [1]. This approach has two significant advantages over the previous method. First, the fixed-interface modes can often better represent the shapes of the transmission simulator when it is attached to the substructure being tested. Second, after the subtraction process, the physical interface degrees of freedom are retained in the final experimental substructure representation. The experimental substructure can then be directly connected to other FEM based substructures within the system model.

26.2 Theory

As suggested, in order to properly exercise a substructure during a free-free vibration test, the interface is connected to a transmission simulator. Ultimately the transmission simulator must be subtracted in order to have an experimental representation of the desired substructure. A simple beam example, shown in Fig. 26.1, will be used to illustrate the process. Component *C* is the system tested to obtain experimental results, component *A* is the transmission simulator, and component *B* is the substructure for which an experimental representation is desired. The experimental representation of *B* is obtained by coupling *C* and a negative FEM representation of *A*.

The physical displacement vector for the transmission simulator can be partitioned into two sets, the *c*-set, corresponding to the interface degrees of freedom that connect to component *B*, and the *o*-set, corresponding to the interior degrees of freedom (dof). The resulting partitioned mass and stiffness matrices can be written as

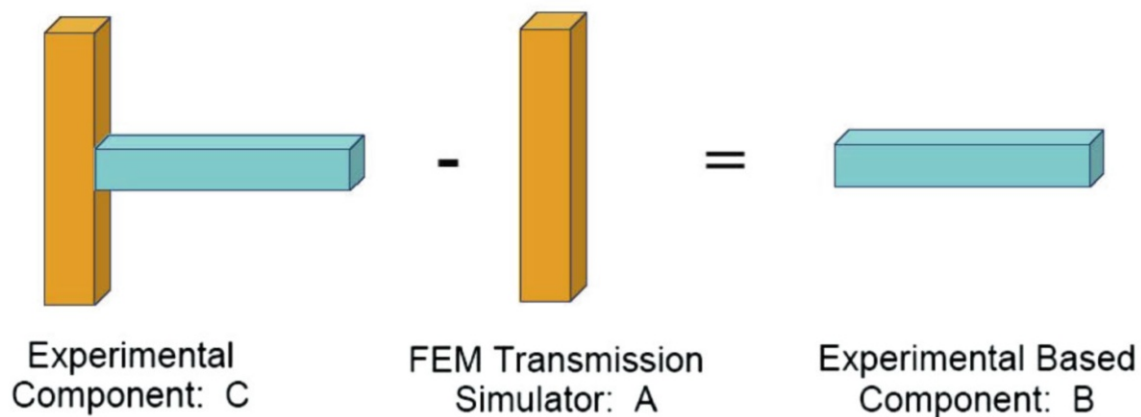


Fig. 26.1 Subtraction of TS from experimental substructure

$$M^A = \begin{bmatrix} M_{oo}^A & M_{oc}^A \\ M_{co}^A & M_{cc}^A \end{bmatrix} \quad K^A = \begin{bmatrix} K_{oo}^A & K_{oc}^A \\ K_{co}^A & K_{cc}^A \end{bmatrix} \quad (26.1)$$

The displacement vector for component A can be transformed into CB [1] coordinates using the relation

$$u^A = \begin{Bmatrix} u_o^A \\ u_c^A \end{Bmatrix} = \begin{bmatrix} \phi^A & \psi^A \\ 0 & I_c \end{bmatrix} \begin{Bmatrix} q^A \\ u_c^A \end{Bmatrix} = T_{CB}^A u_{CB}^A \quad (26.2)$$

where ϕ^A is a matrix of n_A fixed-interface modes, $\psi^A = -(K_{oo}^A)^{-1} K_{oc}^A$, I_c is an identity of order equal to the number of interface dof, q^A is the fixed mode displacement vector, and u_c^A represents the physical displacement of the interface. The CB representation describes the shape of the transmission simulator in terms of dynamic fixed-interface modes, and static shapes, called constraint modes $\Psi^A = [\psi^{AT} \ I_c]^T$, in which each column gives the static response of component A due to a unit deflection of one of the interface dof holding the others fixed. The corresponding CB mass and stiffness matrices are then given by

$$M_{CB}^A = (T_{CB}^A)^T M^A T_{CB}^A = \begin{bmatrix} I^A & M_{qc}^A \\ M_{cq}^A & M_S^A \end{bmatrix} \quad (26.3)$$

$$K_{CB}^A = (T_{CB}^A)^T K^A T_{CB}^A = \begin{bmatrix} (\omega^A)^2 & 0 \\ 0 & K_S^A \end{bmatrix} \quad (26.4)$$

where it has been assumed that the fixed-interface modes have been mass normalized, ω^A is a diagonal matrix of fixed-interface modal frequencies, and M_S^A and K_S^A are the component A mass and stiffness matrices statically reduced to the interface dof, respectively. Note that the stiffness matrix is uncoupled in CB coordinates, but mass is not. As mentioned previously, the advantage of this transmission simulator representation over a free-free modal representation is that the fixed-interface modes do a better job of representing the shape of the simulator in the coupled system being tested, and the physical interface dof are directly retained for connection to other substructures.

The CB representation of component A must now be subtracted from the experimental modal representation of the combined system C . The uncoupled equation of motion is given by

$$\begin{bmatrix} I^C & 0 \\ 0 & -M_{CB}^A \end{bmatrix} \begin{Bmatrix} \ddot{q}^C \\ \ddot{u}_{CB}^A \end{Bmatrix} + \begin{bmatrix} (\omega^C)^2 & 0 \\ 0 & -K_{CB}^A \end{bmatrix} \begin{Bmatrix} q^C \\ u_{CB}^A \end{Bmatrix} = 0 \quad (26.5)$$

where q^C is the displacement vector for the experimental modes, I^C is an identity matrix with dimension corresponding to the number of experimental modes retained, n_C , and ω^C is a diagonal matrix of the experimental combined system C frequencies.

It is assumed that the response of component C is measured at n_m locations on the transmission simulator during the vibration test. In order to couple component C and negative component A , the usual approach is to enforce compatible displacements at the interface degrees of freedom. However, in general, it is difficult to place sensors and measure all of the connecting degrees of freedom, especially if they are rotations. Following the MCFS approach [6], the next best coupling method is to enforce the physical displacement compatibility between the two components at the measured locations

$$u_m^A - u_m^C = 0 \quad (26.6)$$

where it is assumed that none of the interface dof are in the set of measured locations. Transforming to CB coordinates for component A and modal coordinates for experimental component C , Eq. (26.6) becomes

$$\phi_m^A q^A + \psi_m^A u_c^A - \phi_m^C q^C = 0 \quad (26.7)$$

in which the subscript m indicates a row partition corresponding to the measured dof. Equation (26.7) can be solved for the transmission simulator fixed-interface modal response

$$q^A = \phi_m^{A\dagger} [\phi_m^C q^C - \psi_m^A u_c^A] = \phi_m^{A\dagger} u_{m/c}^C \quad (26.8)$$

where $\phi_m^{A\dagger} = [\phi_m^{AT} \phi_m^A]^{-1} \phi_m^{AT}$ is the generalized inverse of the fixed-interface modes row partitioned to the measurement locations. The existence of this inverse requires that matrix ϕ_m^A is full column rank, implying that $n_m \geq n_A$. The term within the bracket

$$u_{m/c}^C = \phi_m^C q^C - \psi_m^A u_c^A \quad (26.9)$$

is the difference between the free response of the measured locations on component A during the vibration test of the combined structure C, and the static response of the same measured locations due to unit deflections of the interface dof predicted using the FEM representation of the simulator. It can therefore be considered as an estimate of the response of the measured locations with respect to the interface or connection dof.

Premultiplying through Eq. (26.8) by the component A fixed-interface modes at the measurement locations gives

$$u_{m/c}^A = \phi_m^A q^A = \phi_m^A \phi_m^{A\dagger} u_{m/c}^C = P_m^A u_{m/c}^C = \hat{u}_{m/c}^C \quad (26.10)$$

where P_m^A is an orthogonal projector onto the column space of ϕ_m^A , and $\hat{u}_{m/c}^C$ is then the orthogonal projection of the response of substructure C at the measurement locations relative to the interface onto this space. The modal constraint process used here is analogous to that used in the straightforward MCFS approach, but now it deals with motion of the transmission simulator relative to the interface dof, instead of their total response. The modal constraint approach does not strictly enforce the constraint in Eq. (26.6), but instead enforces the least-squares fit given in Eq. (26.10).

The modal constraint is applied by defining the coordinate transformation

$$u_G = \begin{Bmatrix} q^C \\ q^A \\ u_c^A \end{Bmatrix} = T u_r = \begin{bmatrix} I & 0 \\ \phi_m^{A\dagger} \phi_m^C & -\phi_m^{A\dagger} \psi_m^A \\ 0 & I \end{bmatrix} \begin{Bmatrix} q^C \\ u_c^A \end{Bmatrix} \quad (26.11)$$

Applying the constraint to the uncoupled system in Eq. (26.5) gives the approximation of the unforced equation of motion for the experimental based substructure B

$$\hat{M}^B \ddot{\hat{u}}^B + \hat{K}^B \hat{u}^B = 0 \quad (26.12)$$

in which the reduced displacement vector u_r in Eq. (26.11) has been renamed $\hat{u}^B$ where

$$\hat{u}^B = \begin{Bmatrix} \hat{q}^B \\ \hat{u}_c^B \end{Bmatrix} \quad (26.13)$$

and

$$\hat{M}^B = T^T \begin{bmatrix} I^C & 0 \\ 0 & -M_{CB}^A \end{bmatrix} T = \begin{bmatrix} I^C - \tau^T \tau & \tau^T \psi - \tau^T M_{qc}^A \\ \psi^T \tau - M_{cq}^A \tau & M_{cq}^A \psi + \psi^T M_{qc}^A - \psi^T \psi - M_S^A \end{bmatrix} \quad (26.14)$$

$$\hat{K}^B = T^T \begin{bmatrix} (\omega^C)^2 & 0 \\ 0 & -K_{CB}^A \end{bmatrix} T = \begin{bmatrix} (\omega^C)^2 - \tau^T (\omega^A)^2 \tau & \tau^T (\omega^A)^2 \psi \\ \psi^T (\omega^A)^2 \tau & -\psi^T (\omega^A)^2 \psi - K_S^A \end{bmatrix} \quad (26.15)$$

with $\tau = \phi_m^{A\dagger} \phi_m^C$ and $\psi = \phi_m^{A\dagger} \psi_m^A$. The substructure representation for component B given by Eqs. (26.14) and (26.15) is not precisely a CB representation because the stiffness matrix is not block diagonal. However, it is CB-like in that the displacement vector $\hat{u}^B$ contains modal degrees of freedom $\hat{q}^B$, and the physical degrees of freedom $\hat{u}_c^B$ at the points where component B will connect to other substructures. The experimental based representation for component B can then be easily connected to other FEM based substructures. However, unlike the CB representation, the modal degrees of freedom cannot be truncated based on frequency due to the stiffness coupling between the modal and interface dof in Eq. (26.15).

In general, a substructure's mass matrix must be positive definite, and the stiffness matrix must be positive definite, or semi-definite. However, in this particular situation, by design, this is not the case because of the retention of the physical interface dof. There is negative transmission simulator mass and stiffness associated with the interface dof partitions from Eqs. (26.14) and (26.15) given by

$$\hat{M}_{cc}^B = M_{cq}^A \psi + \psi^T M_{qc}^A - \psi^T \psi - M_S^A \quad (26.16)$$

and

$$\hat{K}_{cc}^B = -\psi^T (\omega^A)^2 \psi - K_S^A \quad (26.17)$$

In the corresponding modal dof partitions

$$\hat{M}_{qq}^B = I^C - \tau^T \tau \quad (26.18)$$

and

$$\hat{K}_{qq}^B = (\omega^C)^2 - \tau^T (\omega^A)^2 \tau \quad (26.19)$$

mass and stiffness associated with the transmission simulator fixed-interface modes is subtracted, but positive mass and stiffness associated with the motion of the interface dof in the free-free substructure C modes still remains. The partitions in Eqs. (26.16) and (26.17) make the substructure mass and stiffness matrices for component B indefinite, resulting in nonphysical negative eigenvalues. However, when the component is attached to another substructure of comparable, or larger, size than the transmission simulator, it is asserted that the coupling terms in $\hat{M}^B$ and $\hat{K}^B$ result in the proper cancellation of excess mass and stiffness due to the simulator at the interface. This explains the nonzero coupling in the stiffness matrix. This assertion is not proven here, but a simple example will illustrate this point later in the paper.

Another advantage of the proposed component B substructure representation is that the fixed-interface modes and frequencies can be determined directly by just crossing out the block row and column associated with the interface dof. The resulting constrained interface mass and stiffness matrices, $\hat{M}_c^B$ and $\hat{K}_c^B$, just correspond to the expressions given in Eqs. (26.18) and (26.19), respectively. Therefore, the fixed-interface modes and frequencies of component B can be derived from a free-free vibration test of the composite system C including the transmission simulator. If the proper number of free-free experimental modes for component C and fixed-interface modes from the transmission simulator are retained in the analysis, the matrices $\hat{M}_c^B$ and $\hat{K}_c^B$ should both be positive definite. Modal selection for each of the components is still a subject of research, but techniques presented in Ref. [8] can be used to adjust the fixed component matrices such that they are positive definite, if required.

Several observations can be made regarding the theory presented for this new experimental substructure representation. First, the experimental mode set, ϕ^C , is determined by the frequency range that is desired for experimental component B , and the response of B within the selected mode set. Experimental modes that contain only response of the transmission simulator and minimal response of component B should be avoided. Subtracting the transmission simulator from such modes results in a component B mass matrix that is either positive semi-definite, or worse, indefinite. Accurate representation of the transmission simulator over the experimental frequency range dictates the fixed-interface mode set ϕ^A containing n_A modes. Motion of C at the measurement locations relative to the motion of the interface is a subset of the space spanned by the experimental modes and FEM constraint modes at the measurement locations

$$R(u_{m/c}^C) \subset R(\phi_m^C \oplus \psi_m^A) = R^{n_m} \quad (26.20)$$

where $R(\cdot)$ denotes the range space, and R^{n_m} denotes n_m dimensional Euclidean space. In general, $R(\phi_m^A)$ is also a subset of R^{n_m} , but $R(u_{m/c}^C) \neq R(\phi_m^A)$. In order for the modal constraint in Eq. (26.10) to exactly enforce the desired constraint in Eq. (26.6), $R(u_{m/c}^C)$ must be a subspace of $R(\phi_m^A)$. This is guaranteed when $n_m = n_A$, however, in order to take advantage of the robustness of the modal constraint approach, the number of sensors should be selected such that $n_m > n_A$, providing a least squares solution.

Finally, it is important to note that there is currently not an a priori method for determining if a system synthesized by attaching the derived component B to another structure has negative eigenvalues, or inappropriate mass and stiffness matrix sign-definiteness. Instead, the system eigenvalues must be calculated to check their sign. Therefore, the proposed approach is not appropriate for situations that require the direct time integration of the system in physical coordinates. Instead, it is suggested that the proposed substructure representation should be limited to cases where modal superposition can be used. System modes with negative eigenvalues can then be removed from the modal model.

26.3 Numerical Example

A simple numerical example consisting of a planar T-beam, shown in Fig. 26.2, is used to demonstrate the proposed experimental substructure approach. As a first attempt, this paper simulates the experimental data using FEM representations consisting of simple beam elements. The T-beam represents component *C* that is tested in a free-free condition. Twenty-four component *C* modes were determined between 0.0 and 4780.0 Hz. The vertical member represents the transmission simulator, or component *A*. The horizontal member represents component *B*, which is the experimental based substructure to be determined. A CB representation of the transmission simulator was generated based on a simple FEM beam model. Ten fixed-interface modes were selected with frequencies between 456.0 and 15,944 Hz. The transmission simulator and component *B* connect at a single point through two translational and one rotational dof. In this simple case, the interface is statically determinate, therefore the constraint modes are merely rigid body modes. Effective independence [9] was used to locate 12 translational sensors on the transmission simulator such that the 10 fixed-interface modes, row partitioned to the sensors, were linearly independent.

The CB based transmission simulator was subtracted from the component *C* modal representation using the proposed modal constraint approach. The resulting free-free substructure representation for component *B* contained 24 modal dof, and 3 physical interface dof. Twenty-seven modes were then calculated for *B* with frequencies up to 8,670 Hz. Table 26.1 lists the first 15 computed frequencies compared with the first 15 frequencies computed using the corresponding FEM representation of a free component *B*. Three rigid body modes are correctly calculated, and the first 12 elastic modal frequencies agree well with corresponding FEM frequencies. Beyond the fifteenth mode, 3 pairs of complex conjugate frequencies appear, as well as 6 more real elastic modal frequencies. The 6 real higher modal frequencies are hard to match with the FEM frequencies, but this would also be the case with higher order frequencies computed with a straightforward CB representation for *B*. These higher order frequencies account for interface flexibility when connected to another structure. The appearance of the complex conjugate frequencies is due to the nonphysical negative mass and stiffness associated with the interface dof in the formulation of component *B* in Eqs. (26.14) and (26.15). Another contributor could be an over-subtraction of transmission simulator mass and stiffness during the synthesis process, as discussed in the previous section.

As mentioned, another advantage of this proposed formulation for component *B*, is the fact that the interface dof can be constrained to directly predict the fixed-interface natural frequencies, which can then also be compared with FEM results. Table 26.2 compares the predicted fixed-interface modes with those calculated by the corresponding FEM component *B*

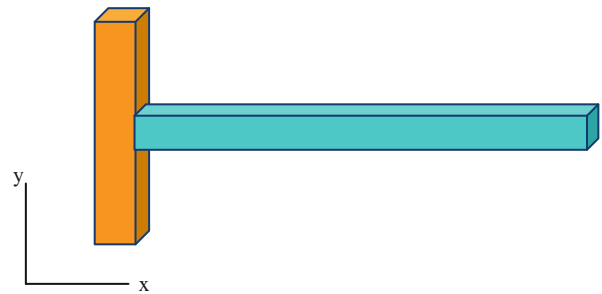


Fig. 26.2 Experimental component *C*

Table 26.1 Substructure *B* free-free modal frequencies

Mode	FEM	CB-MCFS	% Error
1	0.00	0.00	NA
2	0.00	0.04	NA
3	0.00	0.05	NA
4	45.40	45.40	−0.00
5	125.14	125.14	0.00
6	245.32	245.37	0.02
7	405.53	405.74	0.05
8	605.80	606.50	0.12
9	846.13	848.07	0.23
10	1126.52	1131.24	0.42
11	1447.00	1457.48	0.72
12	1807.58	1829.62	1.22
13	2118.66	2118.35	−0.01
14	2208.29	2253.54	2.05
15	2649.18	2741.28	3.48

Table 26.2 Substructure *B*
fixed-interface modal frequencies

Mode	FEM	Mode	CB-MCFS	% Error
1	7.13	1	7.14	0.12
2	44.71	2	44.71	0.00
3	125.19	3	125.19	0.00
4	245.32	4	245.32	0.00
5	405.53	5	405.54	0.00
6	605.80	6	605.82	0.00
7	846.12	7	846.17	0.01
8	1059.33	8	1059.33	0.00
9	1126.52	9	1126.60	0.01
10	1447.00	10	1447.12	0.01
11	1807.58	11	1807.76	0.01
12	2208.29	12	2208.55	0.01
13	2649.18	13	2649.52	0.01
14	3130.29	16	3130.73	0.01
15	3177.98	17	3178.04	0.00
16	3651.73	18	3652.26	0.01
17	4213.58	19	4214.15	0.01
18	4816.00	20	4815.46	-0.01
19	5296.63	21	5327.29	0.58
20	5459.16	22	5498.42	0.12

representation. The first 20 constrained modal frequencies are very accurately predicted, with all errors below 1%. The derived substructure representation for *B* produces 2 complex conjugate pairs of fixed-interface frequencies. This indicates that the corresponding constrained mass and stiffness matrices are indefinite due to subtracting too much mass and stiffness during the process of removing the transmission simulator. This can occur when there is a mismatch between the vector spaces spanned by the retained transmission simulator fixed-interface modes and the motion of the transmission simulator relative to the interface in the experimental component *C* modes. The authors have devised techniques for correcting this problem [8], but no attempt has been made to do so in this illustrative example.

An acceleration frequency response analysis was then performed for component *B* using modal expansion. In the case of the experimentally derived representation of *B*, the 3 pairs of complex conjugate modes were removed, but all the remaining modes were retained, including the high frequency modes not represented in the FEM. The results were then compared with those computed using the first 25 FEM free-free modes up to 7,636 Hz. Figure 26.3 shows a comparison between the two representations for the drive point frequency response at the left end transverse connection dof for 3% modal damping. The agreement between the two is very good in the low frequency range, but they diverge dramatically with increasing frequency. This is to be expected because the derived CB-like representation for component *B* contains information regarding the flexibility of the interface in its high frequency modes, while the free-free FEM modal representation does not.

The real demonstration of the accuracy of the experimental CB representation of component *B* is its performance when connected to another substructure. Therefore, the derived representation for *B* was directly connected to a FEM based CB representation of another beam substructure as shown in Fig. 26.4. The fact that the experimentally derived representation for *B* explicitly contains the interface dof makes it very easy to connect to other FEM based components. The first 29 modal frequencies of the synthesized system, up to 3,008 Hz, agree extremely well with the complete system FEM representation. The 30th and 31st modes represent a complex conjugate pair. It is believed that this pair appears due to the indefiniteness of the fixed interface partition of the mass and stiffness matrices for component *B*, mentioned earlier. The next 9 modes up to 4,836 Hz. also agree very well with the FEM representation. All frequencies have less than 1.0% error.

The accuracy of the synthesized system can also be demonstrated by performing a frequency response analysis using modal expansion with 3% modal damping. The first 29 modes with frequencies up to approximately 3,000 Hz. were used for both the synthesized and FEM beam systems. Figure 26.5 compares the drive point acceleration frequency response at the interface axial dof - *x*. Figure 26.6 illustrates a comparison of the response at transverse interface dof - *y*, due to input at the interface in the *x* direction. Both synthesized system responses are essentially exact.

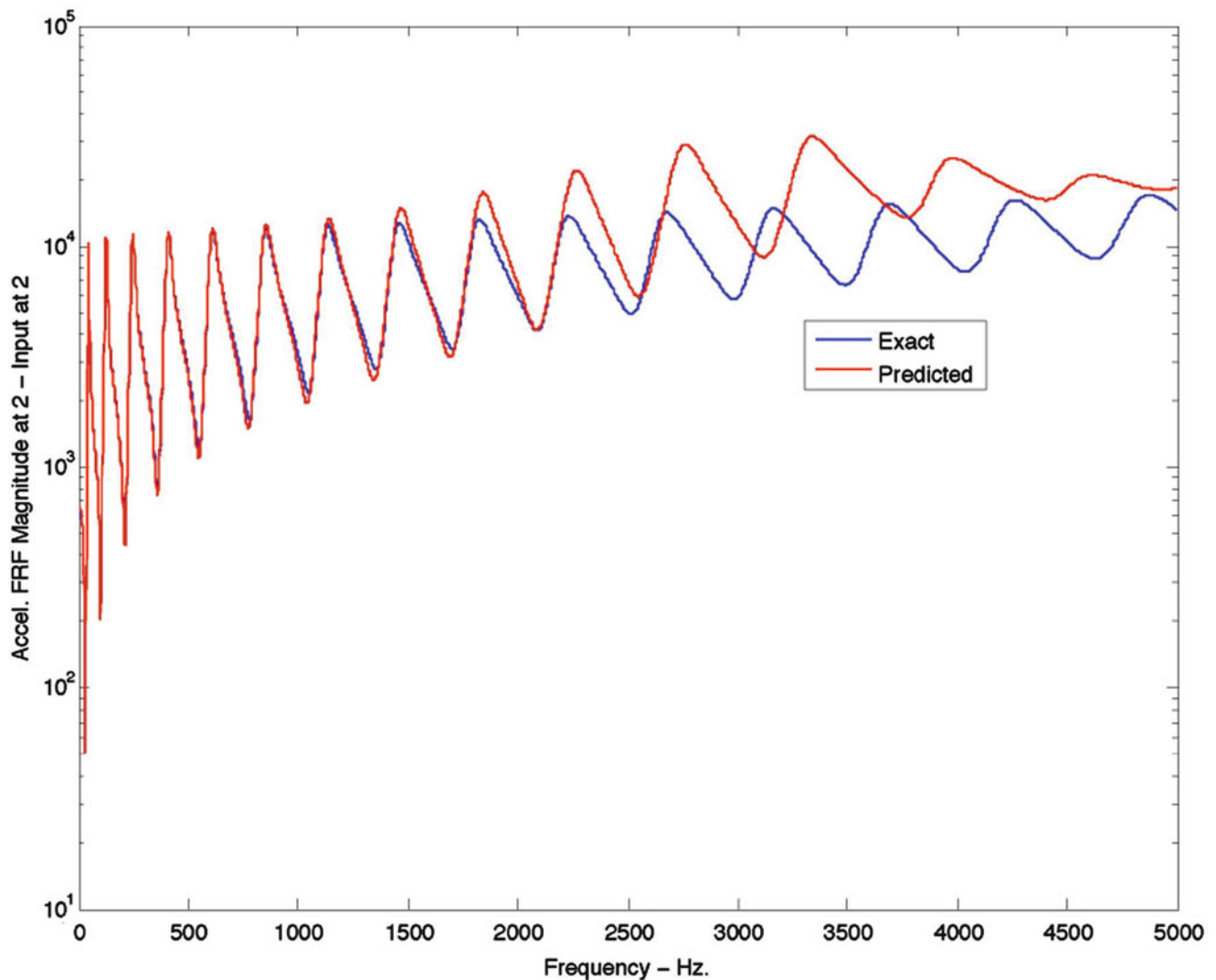


Fig. 26.3 Component *B* drive point acceleration frequency response at transverse interface dof - *y*

26.4 Conclusion

This paper has presented a novel modification of the basic Modal Constraint for Fixture and Subsystems approach for combining experimental based substructures with FEM substructure representations [7]. Instead of using a free-free modal representation of the FEM transmission simulator in the subtraction process, it is represented by a Craig-Bampton substructure representation [1]. This approach has two significant advantages over the original method. First, the fixed-interface modes can often better represent the shapes of the transmission simulator when it is attached to the substructure being tested. Second, after the subtraction process, the physical interface degrees of freedom are retained in the final experimental substructure representation. The experimental substructure can then be directly connected to other FEM based substructures within the system model, just like a usual Craig-Bampton representation. A simple numerical example consisting of beam representations was used to demonstrate the new method. For this simple example, extremely good results were obtained for system modal frequencies and frequency response when the experimental based substructure was connected to another beam substructure. In this case, the transmission simulator interface degrees of freedom provided a statically determinate connection. Therefore, in the Craig-Bampton representation of the transmission simulator, all of the elastic dynamics are contained in the fixed-interface modes. It is believed that this is the best-case scenario for the proposed method because the transmission simulator is subtracted by constraining the fixed-interface modes. In situations where the

Fig. 26.4 Experimental component *B* connected to FEM representation of a vertical beam

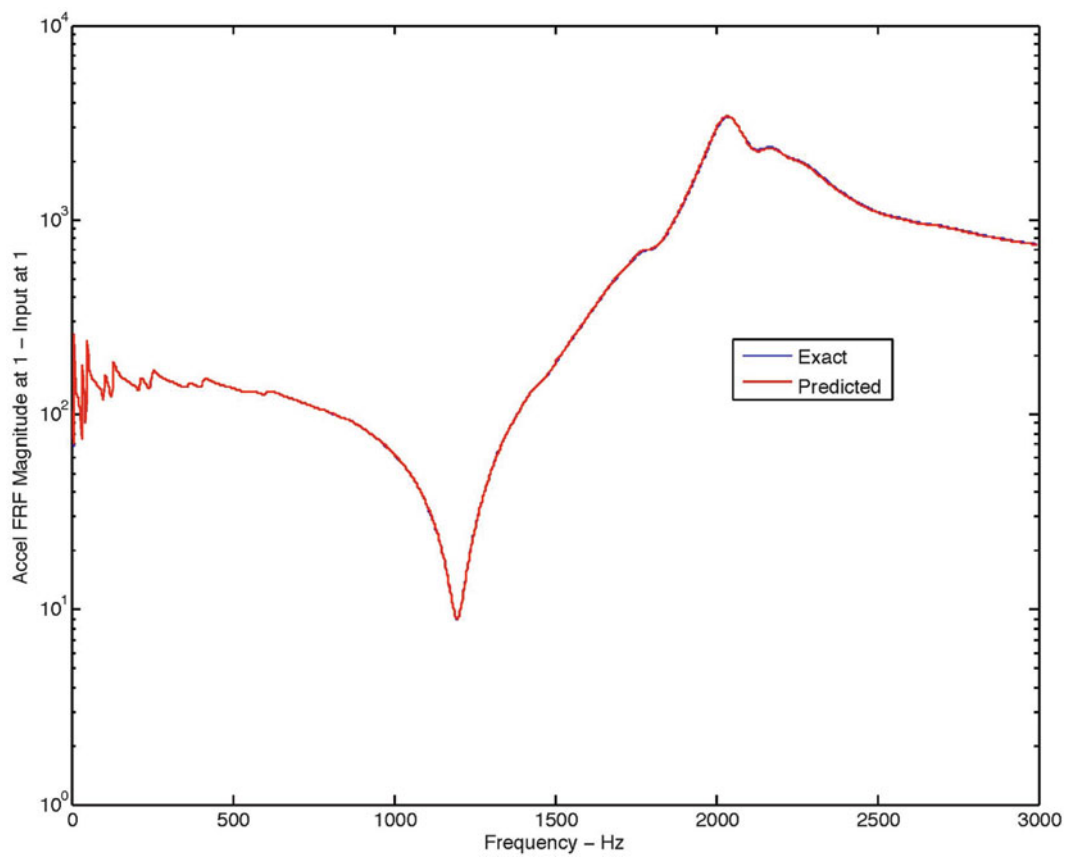
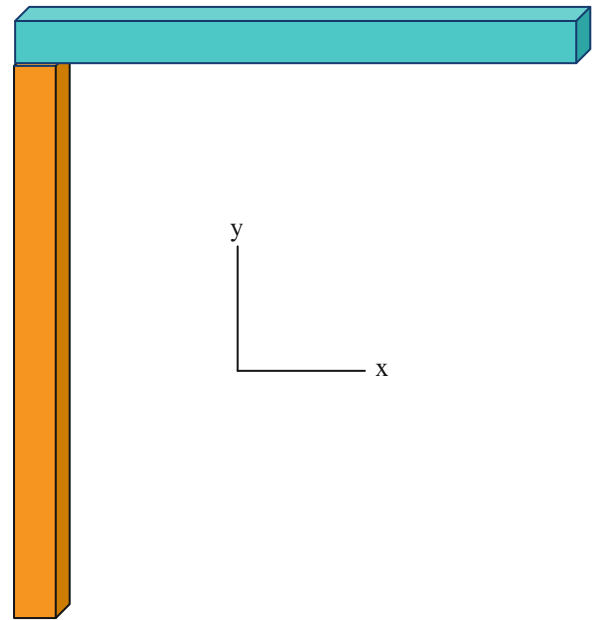


Fig. 26.5 System drive point acceleration frequency response at axial interface dof - *x*

interface is very redundantly connected, there will be less participation of the transmission simulator fixed modes within the frequency band of interest for the experimental component *C*. In this instance, the proposed approach will be less effective. Future work will focus on these more complicated multi-connected cases, and the modification of the proposed approach to handle them.

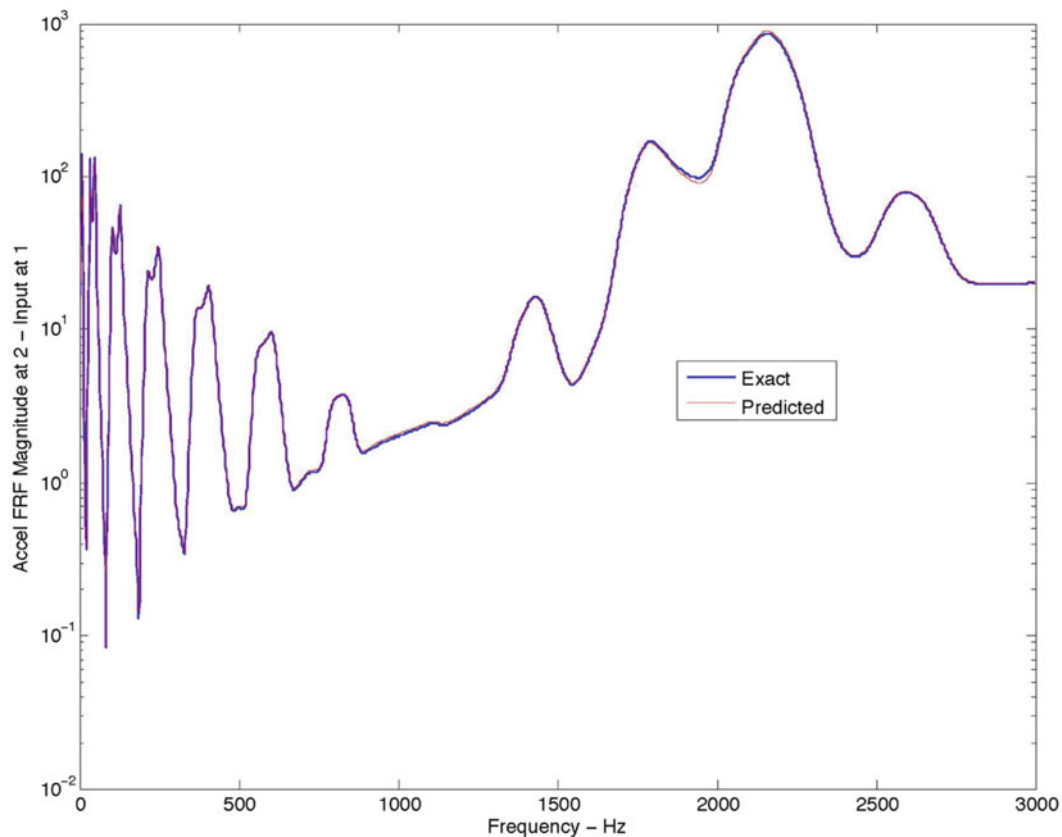


Fig. 26.6 System acceleration frequency response at interface dof - y, due to input in x

Finally, it is strongly suggested that the proposed experimental based substructure representation be limited in use to situations where modal analysis is possible for the synthesized system. Modes with nonphysical negative eigenvalues can then be easily removed from the system mode set. Future work will also focus on the development of methods to guarantee the proper sign definiteness for the system matrices.

Acknowledgements This work was supported by Sandia National Laboratories. Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

References

1. Craig RR, Bampton MCC (1968) Coupling of substructures for dynamic analysis. *AIAA J* 6(7):1313–1319
2. Martinez DR, Miller AK, Carne TG (1985) Combined experimental and analytical modeling of shell/payload structures. In: Joint ASCE/ASME mechanics conference, 1985, Albuquerque
3. Urgueira APV (1989) Dynamic analysis of coupled structures using experimental data. In: Imperial College of science, technology, & medicine, 1989, University of London, London
4. Ind P (2004) The non-intrusive modal testing of delicate and critical structures. In: Imperial College of science, technology, & medicine. 2004, University of London, London
5. Imregun M, Robb DA, Ewins DJ (1987) Structural modification and coupling dynamic analysis using measured FRF data. In: 5th international modal analysis conference, 1987, London
6. Allen MS, Mayes RL (2007) Comparison of FRF and modal methods for combining experimental and analytical substructures. In: 25th international modal analysis conference, 2007, Orlando

7. Mayes RL, Hunter PS, Simmermacher TW (2008) Combining experimental and analytical substructures with multiple connections. In: 26th international modal analysis conference, 2008, Orlando
8. Mayes RL, Allen MS, Kammer DC (2012) Eliminating indefinite mass matrices with the transmission simulator method of substructuring. In: Topics in experimental dynamics substructuring and wind turbine dynamics. In: Conference proceedings of the society of experimental mechanics, vol 2. 2012, pp 21–31
9. Kammer DC (1991) Sensor placement for on-orbit modal identification and correlation of large space structure. *J Guid Contr Dyn* 14(2):251–259