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# Data Analysis in Management with SPSS Software

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*To my elder sister Sandhya Mohan for*
*having me introduced in statistics*
*Brother-in-law Rohit Mohan for his*
*helping gesture*
*And their angel daughter Saumya*

# Preface

While serving as a faculty of statistics for the last 30 years, I have experienced that the non-statistics faculty and research scholars in different disciplines find it difficult to use statistical techniques in their research problems. Even if their theoretical concepts are sound its troublesome for them to use statistical software. This book provides readers with a greater understanding of a variety of statistical techniques along with the procedure to use the most popular statistical software package SPSS.

The book strengthens the intuitive understanding of the material, thereby increasing the ability to successfully analyze data in the future. It enhances readers capability in using data analysis techniques to a broader spectrum of research problems.

The book is intended for the undergraduate and postgraduate courses along with pre-doctoral and doctoral course work on data analysis, statistics, and/or quantitative methods taught in management and other allied disciplines like psychology, economics, education, nursing, medical, or other behavioral and social sciences. This book is equally useful to the advanced researchers in the area of humanities and behavioural and social sciences in solving their research problems.

The book has been written to provide solutions to the researchers in different disciplines for using one of the powerful statistical software SPSS. The book will serve the students as a self-learning text of using SPSS for applying statistical techniques in their research problems.

In most of the research studies, data are analyzed using multivariate statistics which poses an additional problem for the beginners. These techniques cannot be understood without in-depth knowledge of statistical concepts. Further, several fields in science, engineering, and humanities have developed their own nomenclature assigning different names to the same concepts. Thus, one has to gather sufficient knowledge and experience in order to analyze their data efficiently. This book covers most of the statistical techniques including some of the most powerful multivariate techniques along with their detailed analysis and interpretation of the SPSS output that are required by the research scholars in different discipline to achieve their research objectives.

The USP of this book is that even without having the indepth knowledge of statistics, one can learn various statistical techniques and their applications on their own.

Each chapter is self-contained and starts with the topics like Introductory concepts, application areas, statistical techniques used in the chapter and step-by-step solved example with SPSS. In each chapter in depth interpretation of SPSS output has been made to help the readers in understanding the application of statistical techniques in different situations. Since the SPSS output generated in different statistical applications are raw and cannot be directly used for reporting hence model way of writing the results has been shown wherever it is required.

This book focuses on providing readers with the knowledge and skills needed to carry out research in management, humanities, and social and behavioral sciences by using SPSS. Looking at the contents and prospects of learning computing skills using SPSS, this book is a must for every researcher from graduate-level studies onward. Towards the end of each chapter, short answer questions, multiple-choice questions, and assignments have been provided as a practice exercise for the readers.

The common mistakes like using two-tailed test for testing one-tailed hypothesis, using the term "level of confidence" for defining level of significance or using the statement like "accepting the null hypothesis" instead of "not able to reject the null hypothesis" have been explained extensively in the text so that the readers may avoid such mistakes during organizing and conducting their research work.

The faculty who uses this book will find it very useful as it presents many illustrations with either real or simulated data to discuss analytical techniques in different chapters. Some of the examples cited in the text are from my own and my colleagues' research studies.

This book consists of 14 chapters. Chapter 1 deals with the data types, data cleaning, and procedure to start SPSS on the system. Notations used throughout the book in using SPSS commands have been explained in this chapter. Chapter 2 deals with descriptive study. Different situations have been discussed under which such studies can be undertaken. The procedure of computing various descriptive statistics has been discussed in this chapter. Besides computing procedure through SPSS, a new approach has been shown towards the end of the second chapter to develop the profile graph which can be used for comparing different domains of the populations.

Chapter 3 explains the chi-square and its different applications by means of solved examples. The step-by-step procedure of computing chi-square using SPSS has been discussed. Chi-square is the test of significance for association between the attributes, but it provides comparison of the two groups as well, in case of the responses being measured on the nominal scale. This fact has been discussed for the benefit of the readers.

Chapter 4 explains the procedure of computing correlation matrix and partial correlations using SPSS. The emphasis has been given on how to interpret the relationships.

In Chapter 5, computing multiple correlations and regression analysis have been discussed. Both the approaches of regression analysis in SPSS i.e. Stepwise and Enter methods have been discussed for estimating any measurable phenomenon.

In Chapter 6, application of t-test in testing the significance of difference between groups in all the three situations, that is, in one sample, two independent samples, and two dependent samples, has been discussed in detail. Procedures of using one-tailed and two-tailed tests have been thoroughly detailed.

Chapter 7 explains the procedure of applying one-way analysis of variance (ANOVA) with equal and unequal groups for testing the significance of variability among group means. The graphical approach has been discussed for post hoc comparisons of means besides using the *p*-value concept.

In Chapter 8, two-way ANOVA for understanding the causes of variation has been discussed in detail by means of solved examples using SPSS. The model way of writing the results has been shown, which the students should note. Procedure for doing interaction analysis has been discussed in detail by using the SPSS output.

In Chapter 9, the application of ANCOVA to study the role of covariate in experimental research has been discussed by means of a research example. Students can find the procedure of analyzing their data much easier after going through this chapter.

In Chapter 10, cluster analysis technique has been discussed in detail for market segmentation. The readers will come to know about the situations where cluster analysis can be used in their research studies. Discussions of all its basic concepts have been elaborated so that even a non-statistician can also appreciate and use it for their research data.

Chapter 11 deals with the factor analysis, one of the most widely used multivariate statistical techniques in management research. By going through this chapter, the readers can understand to study the characteristics of a group of data by means of few underlying structures instead of a large number of parameters. The procedure of developing the test battery using the factor analysis technique has also been discussed in detail.

In Chapter 12, we have discussed discriminant analysis and its application in various research situations. By learning this technique, one can develop classificatory model in classifying a customer into any of the two categories based on their relevant profile parameters. The technique is very useful in classifying a customer as good or bad for offering various services in the area of banking and insurance.

Chapter 13 explains the application of logistic regression for probabilistic classification of cases into one of the two groups. Basics of this technique have been discussed before explaining the procedure in solving logistic regression with SPSS. Interpretations of each and every output have been very carefully explained for easy understanding of the readers.

In Chapter 14, multidimensional scaling has been discussed to find the brand positioning of different products. This technique is especially useful if the popularity of products is to be compared on different parameters.

At each and every step, care has been taken so that the readers can learn to apply SPSS and understand minutest possible detail of analysis discussed in this book. The purpose of this book is to give a brief and clear description of how to apply variety of statistical analysis using any version of SPSS. We hope that this book will

provide students and researchers with a self-learning material of using SPSS to analyze their data.

Students and other readers are welcome to e-mail me their query related to any portion of the book at vermajp@sancharnet.in, to which timely reply will be sent.

Professor (Statistics) J.P. Verma

# Acknowledgements

I would like to extend my sincere thanks to my professional colleagues Prof. Y.P. Gupta, Prof. S. Sekhar, Dr. V.B. Singh, Prof. Jagdish Prasad and Dr. J.P. Bhukar for their valuable inputs in completing this text. I must thank to my research scholars who always motivated me to solve varieties of complex research problems which has contributed a lot in preparing this text. Finally I must appreciate the effort of my wife Hari Priya who not only provided me the peaceful environment in preparing this text but also helped me in correcting the manuscript language and format to a great extent. Finally I owe my loving gesture to my children Prachi and Priyam who have provided me the creative inputs in the preparation this manuscript.

Professor (Statistics) J.P. Verma

# Contents

# Chapter 1
# Data Management

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Explain different types of data generated in management research.
- Know the characteristics of variables.
- Learn to remove the outliers from the data by understanding different data cleaning methods before using in SPSS.
- Understand the difference between primary and secondary data.
- Know the formats used in this book for using different commands, subcommands, and options used in SPSS.
- Learn to install SPSS package for data analysis.
- Understand the procedure of importing data in other formats into SPSS.
- Prepare the data file for analysis in SPSS.

## Introduction

In today's world of information technology, enormous data is generated in every organization. These data can help in strategic decision-making process. It is therefore important to store such data in a warehouse so that effective mining can be done later for getting answers to many of the management issues. Data warehousing and data mining are therefore two important disciplines in the present-day scenario. Research in any discipline is carried out in order to minimize inputs and effectively utilizing the human resources, production techniques, governing principles, marketing policies, and advertisement campaigns to maximize outputs in the form of productivity. To be more specific, one may be interested to identify new forms of resources, devise organizational systems and practices to motivate culturally diverse set of individuals, and evaluate the existing organizations so as to make them more productive to the new demands on them. Besides, there may be any number of other issues like

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effective leadership, skill improvement, risk management, customer relationships, and guiding the evolution of technology, etc., where the researcher can make an effective contribution.

A researcher may use varieties of data analysis techniques in solving their research problems like: How to *motivate* people for work? How to make a television or FM channel more popular? How to enhance the *productivity* at work? Which strategy becomes more efficient? How organizational structure promotes *innovation*? How to *measure* training effectiveness? Due to cutthroat competition, the research issues have grown in number, scope, and complexity over the years. Due to availability of computer software for advanced data analysis, researcher has become more eager to solve many of these complex issues.

The purpose of data analysis is to study the characteristics of sample data for approximating it to the population characteristics. Drawing conclusion about the population on the basis of sample would be valid only if the sample is true representative of the population. This can be ensured by using the proper sampling technique. However, large sample need not necessarily improves the efficiency in findings. It is not the quantity but the quality of the sample that matters.

Data generated in management research may be analyzed by using different kinds of statistical techniques. These techniques differ as per the nature of the study which can be classified into any of the five categories; descriptive study, analytical study, inductive study, inferential study and applied study. Choosing statistical technique in data analysis depends upon nature of the problem. It is therefore important to know the situation under which these techniques are used.

Descriptive study is used if an organization or a group of objects needs to be studied about its different characteristics. In such studies, we usually tabulate and compile the data in a meaningful manner so that the statistics like mean, variance, standard error, coefficient of variance, range, skewness, kurtosis, percentiles, etc., can be computed in different groups.

Analytical studies are used for studying the functional relationships among variables. Statistics like product moment correlation, partial and multiple correlations are used in such study. Consider a study where it is required to explore the parameters on which the sale depends. One may like to find correlation between sales data and independent variables like incentives, salesman's IQ, number of marketing hours, and advertisement campaigns. Here, correlation between the sales data and other parameters may be investigated for their significance. Thus, in all those situations where relationships are investigated between the performance parameter and other independent parameters, analytical studies are used.

Inductive studies are those studies which are used to estimate some phenomenon of an individual or of an object on the basis of the sample data. Here, the phenomenon which we estimate does not exist at the time of estimation. One may estimate company's performance in the next 3 years on the basis of some of its present parameters like EPS, P/E ratio, cash reserves, demands, and production capacity.

In inferential study, inferences are drawn about the population parameters on the basis of sample data. Regression analysis is being used in such studies. The difference

between inferential and inductive studies is that the phenomenon which we infer on the basis of the sample exists in the inferential studies, whereas it is yet to occur in the inductive studies. Thus, assessing satisfaction level in an organization on the basis of a sample of employees may be the problem of inferential statistics.

Finally, applied studies refers to those studies which are used in solving the problems of real life. The statistical methods such as times series analysis, index numbers, quality control, and sample survey are included in this class of analysis.

## Types of Data

Depending upon the data types, two broad categories of statistical techniques are used for data analysis. For instance, parametric tests are used if the data are metric, whereas in case of nonmetric data, nonparametric tests are used. It is therefore important to know in advance the types of data which are generated in management research.

Data can be classified in two categories, that is, metric and nonmetric. Metric and nonmetric data are also known as quantitative and qualitative data, respectively. Metric data is analyzed using parametric tests such as $t$, $F$, $Z$, and correlation coefficient, whereas nonparametric tests such as sign test, median test, chi-square test, Mann-Whitney test, and Kruskal-Wallis test are used in analyzing nonmetric data.

Certain assumptions about the data and form of the distribution need to be satisfied in using parametric tests. Parametric tests are more powerful in comparison to that of nonparametric tests, provided required assumptions are satisfied. On the other hand, nonparametric tests are more flexible and easy to use. Very few assumptions need to be satisfied before using these tests. Nonparametric tests are also known as distribution-free tests.

Let us understand the characteristics of different types of metric and nonmetric data generated in research. Metric data is further classified into interval and ratio data. On the other hand, nonmetric data is classified into nominal and ordinal. The details of these four types of data are discussed below under two broad categories, namely, metric data and nonmetric data, and are shown in Fig. 1.1.

### *Metric Data*

Data is said to be metric if it is measured at least on interval scale. Metric data are always associated with a scale measure, and, therefore, it is also known as scale data or quantitative data. Metric data can be measured on two different types of scale, that is, *interval* and *ratio*.

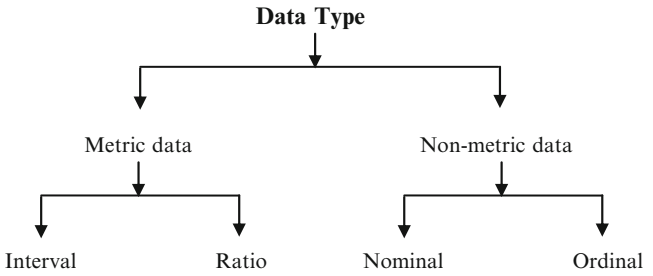



**Fig. 1.1** Types of data and their classification

### Interval Data

The interval data is measured along a scale where each position is equidistant from one another. In this scale, the distance between two pairs is equivalent in some way. In interval data, doubling principle breaks down as there is no zero on the scale. For instance, the 6 marks given to an individual on the basis of his IQ do not explain that his nature is twice as good as the person with 3 marks. Thus, interval variables measured on an interval scale have values in which differences are uniform and meaningful, but ratios are not. Interval data may be obtained if the parameters of job satisfaction or level of frustration is rated on scale 1–10.

### Ratio Data

The data on ratio scale has a meaningful zero value and has an equidistant measure (i.e., the difference between 30 and 40 is the same as the difference between 60 and 70). For example, 60 marks obtained on a test are twice of 30. This is so because zero can be measured on ratio scale. Ratio data can be multiplied and divided because of an equidistant measure and doubling principle. Observations that we measure or count are usually ratio data. Examples of ratio data are height, weight, sales data, stock price, advance tax, etc.

## *Nonmetric Data*

Nonmetric data is a categorical measurement and is expressed not in terms of numbers but rather by means of a natural language description. It is often known as "categorical" data. Examples of such data are like employee's category = "executive," department = "production," etc. These data can be measured on two different scales, that is, nominal and ordinal.

### Nominal Data

Nominal data is a categorical variable. These variables result from a selection in categories. Examples might be employee's status, industry types, subject specialization, race, etc. Data obtained on nominal scale is in terms of frequency. In SPSS,[1] nominal data is represented as "nominal."

### Ordinal Data

Variables on the ordinal scale are also known as categorical variables, but here the categories are ordered. The order of items is often defined by assigning numbers to them to show their relative position. Categorical variables that assess performance (good, average, poor, etc.) are ordinal variables. Similarly, attitudes (strongly agree, agree, undecided, disagree, and strongly disagree) are also ordinal variables. On the basis of the order of an ordinal variable, we may not know which value is the best or worst on the measured phenomenon. Moreover, the distance between ordered categories is also not measureable. No arithmetic can be done with the ordinal data as they show sequence only. Data obtained on ordinal scale is in terms of ranks. Ordinal data is denoted as "ordinal" in SPSS.

## Important Definitions

### *Variable*

A variable is a phenomenon that changes from time to time, place to place, and individual to individual. Examples of variable are salary, scores in CAT examination, height, weight, etc. The variables can further be divided into discrete and continuous. *Discrete variables* are those variables which can assume value from a limited set of numbers. Examples of such variables are number of persons in a department, number of retail outlets, number of bolts in a box, etc. On the other hand, *continuous variables* can be defined as those variables that can take any value within a range. Examples of such variables are height, weight, distance, etc.

[1] SPSS, Inc. is an IBM company which was acquired by IBM in October, 2009.

## *Attribute*

An attribute can be defined as a qualitative characteristic that takes sub-values of a variable, such as "male" and "female," "student" and "teacher," and married and unmarried.

## *Mutually Exclusive Attributes*

Attributes are said to be mutually exclusive if they cannot occur at the same time. Thus, in a survey, a person can choose only one option from a list of alternatives (as opposed to selecting as many that might apply). Similarly in choosing the gender, one can either choose male or female.

## *Independent Variable*

Any variable that can be manipulated by the researcher is known as independent variable. In planning a research experiment, to see the effect of low, medium, and high advertisement cost on sales performance, advertisement cost is an independent variable as the researcher can manipulate it.

## *Dependent Variable*

A variable is said to be dependent if it changes as a result of change in the independent variable. In investigating the impact on sales performance by the change in advertisement cost, the sales performance is a dependent variable, whereas advertisement cost is an independent variable. In fact, a variable may be a dependent variable in one study and independent variable in some other study.

## *Extraneous Variable*

Any additional variable that may provide alternative explanations or cast doubt on conclusions in an experimental study is known as extraneous variable. If the effect of three different teaching methods on the student's performance is to be compared, then the IQ of the students may be termed as extraneous variable as it might affect the learning efficiency during experimentation if the IQs are not same in all the groups.

## The Sources of Research Data

In designing a research experiment, one needs to specify the kind of data required and how to obtain it. The researcher may obtain the data from the reliable source if it is available. But if the required data is not available from any source, it may be collected by the researcher themselves. Several agencies collect data for some specified purposes and make them available for the other researchers to draw other meaningful conclusions as per their plan of study. Even some of the commercial agencies provide the real-time data to the users with cost. The data so obtained from other sources are referred as *secondary data,* whereas the data collected by the researchers themselves are known as *primary data*. We shall now discuss other features of these data in the following sections:

### *Primary Data*

The data obtained during study by the researchers themselves or with the help of their colleagues, subordinates, or field investigators are known as primary data. The primary data is obtained by the researcher in a situation where relevant data is not available from the reliable sources or such data do not exist with any of the agency or if the study is an experimental study where specific treatments are required to be given in the experiment. The primary data is much more reliable because of the fact that the investigator himself is involved in data collection and hence can ensure the correctness of the data. Different methods can be used to collect the primary data by the researcher. These methods are explained below:

#### By Observation

The data in this method is obtained by observation. One can ensure the quality of data as the investigator himself observes real situation and records the data. For example, to assess the quality of any product, one can see as to how the articles are prepared by the particular process. In an experimental study, the performance of the subjects, their behavior, and other temperaments can be noted after they have undergone a treatment. The drawback of this method is that sometimes it becomes very frustrating for the investigator to be present all the time for collecting the data. Further, if an experiment involves the human being, then the subjects may become conscious in the presence of an investigator, due to which performance may be affected which will ultimately result in inaccurate data.

### Through Surveys

This is the most widely used method of data collection in the area of management, psychology, market research, and other behavioral studies. The researcher must try to motivate respondents by explaining them the purpose of the survey and impact of their responses on the results of the study. The questionnaire must be short and must hide the identity of respondents. Further, the respondent may be provided reasonable incentives as per the availability of the budget. For instance, a pen, a pencil, or a notepad with print statements like "With best Compliments from. . ." or "With Thanks. . .," "Go Green," "Save Environment" may be provided before seeking their opinion on the questionnaire. You can print your organization name or your name as well if you are an independent researcher. The first two slogans may promote your company as well, whereas the other two convey the social message to the respondents. The investigator must ensure the authenticity of the collected data by means of cross-checking some of the sampled information.

### From Interviews

The data collected through the direct interview allows the investigator to go for in-depth questioning and follow-up questions. The method is slow and costly and forces an individual to be away from the job during the time of interview. During the interview, the respondent may provide the wrong information if certain sensitive issues are touched upon, and the respondent may like to avoid it on the premise that it might suffer their reputation. For instance, if the respondent's salary is very low and the questions are asked about his salary, it is more likely that you end up with the wrong information. Similarly in asking the question, as to how much you invest on sports for your children in a year, you might get wrong information due to the false ego of respondent.

### Through Logs

The data obtained through the logs maintained by the organizations may be used as primary data. Fault logs, error logs, complaint logs, and transaction logs may be used to extract the required data for the study. Such data provide valuable findings about system performance over time under different conditions if used well, as they are empirical data and obtained from the objective data sources.

Primary data can be considered to be reliable because you know how it was collected and what was done to it. It is something like cooking yourself. You know what went into it.

### *Secondary Data*

Instead of data obtained by the investigator himself if it is obtained from some other sources, it is termed as secondary data. Usually, companies collect the data for some specific purpose, and after that, they publish it for the use of the researchers to draw some meaningful conclusions as per their requirements. Many government agencies allow their real-time data to the researchers for using in their research study. For instance, census data collected by the National Sample Surveys Organization may be used by the researchers for getting several demographic and socio-economic information. Government departments and universities maintain their open-source data and allow the researchers to use it. Nowadays, many commercial agencies collect the data in different fields and make it available to the researchers with nominal cost.

The secondary data may be obtained from many sources; some of them are listed below:

- Government ministries through national informatics center
- Government departments
- Universities
- Thesis and research reports
- Open-source data
- Commercial organization

Care must be taken to ensure the reliability of the agency from which the data is obtained. One must ensure to take an approval of the concerned department, agency, organization, universities, or individuals for using their data. Due acknowledgment must be shown in their research report for using their data. Further, data source must be mentioned while using the data obtained from secondary sources.

In making comparison between primary and secondary data, one may conclude that primary data is expensive and difficult to acquire, but it is more reliable. Secondary data is cheap and easy to collect but must be used with caution.

## Data Cleaning

Before preparing the data file for analysis, it is important to organize the data on paper first. There are more chances that the data set may contain error or outlier. And if it is so, the results obtained may be erroneous. Analysts tend to waste lot of time in drawing the valid conclusions if data is erroneous. Thus, it is utmost important that the data must be cleaned before analysis. If data is clean, the analysis is straightforward and valid conclusions may be drawn.

In data cleaning, the invalid data is detected first and then it is corrected. Some of the common sources of errors are as follows:

- Typing errors in data entry
- Not applicable option or blank options are coded as "0"
- Data for one variable column is entered under the adjacent column
- Coding errors
- Data collection errors

## *Detection of Errors*

The wrongly fed data can be detected by means of descriptive statistics computed by SPSS. Following approaches may be useful in this regard.

### Using Minimum and Maximum Scores

By looking to the minimum and maximum scores of each variable in descriptive statistics, one can identify the error, if any, by knowing the acceptable limits of minimum and maximum scores of each variable. For instance, if the maximum score for the variable showing percentage of marks is 650, one must think of some typographical error while feeding the data as percentage of marks cannot be more than 100%.

### Using Frequencies

Frequencies of each score obtained in descriptive statistics may be used to identify the "dirty" data among the entered variables. For instance, most of the biometric data are normally distributed, and, therefore, if any variable shows large frequency for any values, it must be checked for any systematic error.

### Using Mean and Standard Deviation

Normally, the value of standard deviation is less than the mean except in case of certain distribution like negative binomial. Thus, if the standard deviation for any of the variables like age, height, or IQ is more than their mean, it can only be if some of the values of these variables are outliers. Such entries can easily be identified and removed.

### Logic Checks

Errors in data may also be detected by observing as to whether the responses are logical or not? For example, one would expect to see 100% of responses, not 110%. Similarly, if a question is asked to a female employee as to whether they have

availed maternity leave so far or not and if the reply is marked "yes" but you notice that the respondent is coded as male, such logical errors can be spotted out by looking to the values of the categorical variable. Logical approach should be used judiciously to avoid the embarrassing situation in reporting the finding like 10% of the men in the sample had availed the maternity leave during the last 10 years.

## Typographical Conventions Used in This Book

Throughout the book, certain convention has been followed in writing commands by means of symbol, bold words, italic words, and words in quotes to signify the special meaning. Readers should note these conventions for easy understanding of commands used in different chapters of this book.

| | |
|---|---|
| **Start ⇒ All Programs** | Denotes a menu command, which means choosing the command **All Program** from the **Start** menu. Similarly **Analyze ⇒ Correlate ⇒ Partial** means open the **Analyze** menu, then open the **Correlate** submenu, and finally choose **Partial**. |
| **Regression** | Any word written in bold refers to the main command of any window in the SPSS package. |
| *Prod_Data* | Any word or combination of words written in italics form during explaining SPSS is referred as variable. |
| "Name" | Any word or combination of words written in quotes refers to the subcommand. |
| 'Scale' | Any word written in single quote refers to one of the option under subcommand. |
| ***Continue*** | This refers to the end of selection of commands in a window and will take you to the next level of options in any computation. |
| **OK** | This refers to the end of selecting all the options required for any particular analysis. After pressing the OK invariably, SPSS will lead you to the output window. |

## How to Start SPSS

This book has been written by referring to the IBM SPSS Statistics 20.0 version; however, in all the previous versions of SPSS, procedure of computing is more or less similar.

The SPSS needs to be activated on your computer before entering the data. This can be done by clicking the left button of the mouse on SPSS tag by going through the SPSS directory in the **Start** and **All Programs** option (if the SPSS directory has been created in the Programs file). Using the following command sequence, SPSS can be activated on your computer system:

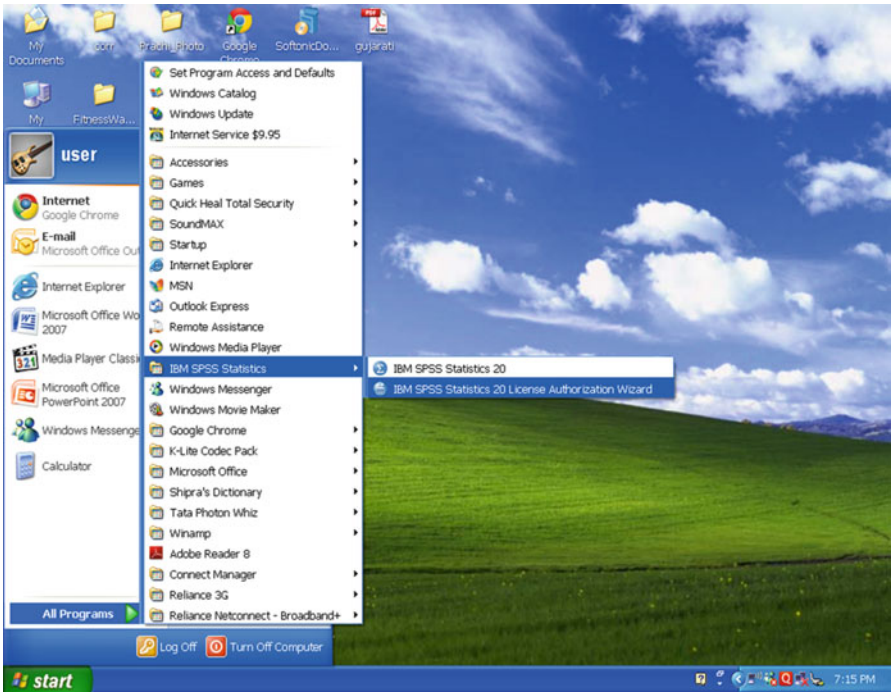



**Fig. 1.2** Commands for starting SPSS on your computer

Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 19

If you use the above-mentioned command sequence, the screen shall look like Fig. 1.2.

After clicking the tag SPSS, you will get the following screen to prepare the data file or open the existing data file.

If you are entering the data for new problem and the file is to be created for the first time, check the following option in the above-mentioned window:


And if the existing file is to be opened or edited, select the following option in the window:

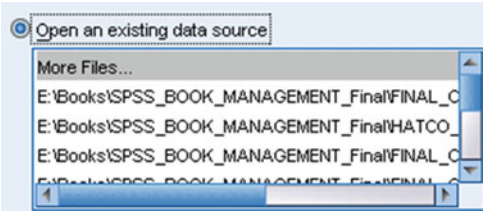


**Table 1.1** FDI inflows and trade (in percent) in different states

| S.N. | FDI | Exports inflows | Imports | Trade |
|---|---|---|---|---|
| 1 | 4.92 | 4.03 | 3.12 | 3.49 |
| 2 | 0.07 | 4.03 | 3.12 | 3.49 |
| 3 | 0.00 | 1.11 | 2.69 | 2.04 |
| 4 | 5.13 | 17.11 | 27.24 | 23.07 |
| 5 | 11.14 | 13.43 | 11.24 | 12.14 |
| 6 | 0.48 | 1.14 | 3.41 | 2.47 |
| 7 | 0.30 | 2.18 | 1.60 | 1.84 |
| 8 | 29.34 | 20.56 | 18.68 | 19.45 |
| 9 | 0.57 | 1.84 | 1.16 | 1.44 |
| 10 | 0.03 | 1.90 | 1.03 | 1.39 |
| 11 | 8.63 | 5.24 | 9.24 | 7.59 |
| 12 | 0.00 | 3.88 | 6.51 | 5.43 |
| 13 | 2.20 | 7.66 | 1.57 | 4.08 |
| 14 | 2.37 | 4.04 | 4.76 | 4.46 |
| 15 | 34.01 | 14.53 | 3.35 | 7.95 |
| 16 | 0.81 | 1.00 | 1.03 | 1.02 |

Click **OK** to get the screen to define the variables in the **Variable View**. Details of preparing data file are shown below.

## Preparing Data File

The procedure of preparing the data file shall be explained by means of the data shown in Table 1.1.

In SPSS, before entering data, all the variables need to be defined in the **Variable View**. Once **Type in data** option is selected in the screen shown in Fig. 1.3, click the **Variable View**. This will allow you to define all the variables in the SPSS. The blank screen shall look like Fig. 1.4.

Now you are ready for defining the variables row wise.

### *Defining Variables and Their Properties Under Different Columns*

Column 1: In first column, short name of the variables are defined. The variable name should essentially start with an alphabet and may use underscore and numerals in between, without any gap. There should be no space between any two characters of the variable name. Further, variable name should not be started with numerals or any special character.

Column 2: Under the column heading "Type," format of the variable (numeric or nonnumeric) and the number of digits before and after decimal are defined. This can be done by double-clicking the concerned cell. The screen shall look like Fig. 1.5.

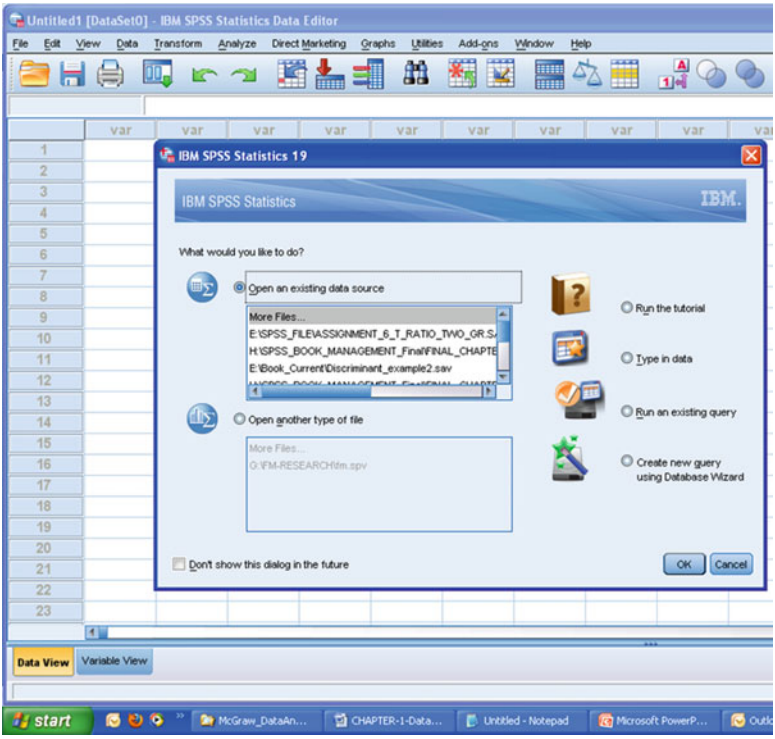



**Fig. 1.3** Screen showing the option for creating/opening file

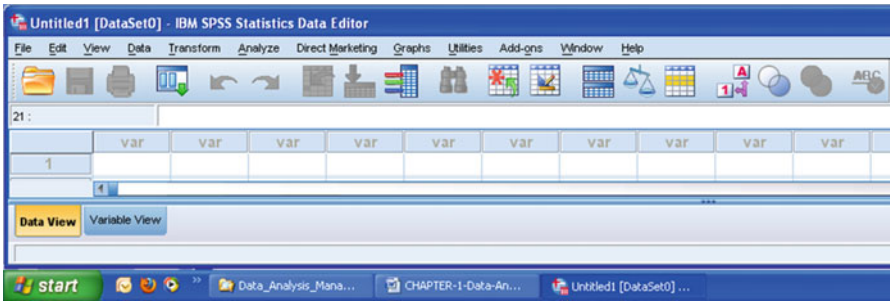



**Fig. 1.4** Blank format for defining the variables in SPSS

Column 3: Under the column heading "Width," number of digits a variable can have may be altered.

Column 4: In this column, number of decimal a variable can have may be altered.

Column 5: Under the column heading "Label," full name of the variable can be defined. The user can take advantage of this facility to write the expanded name of the variable the way one feels like.

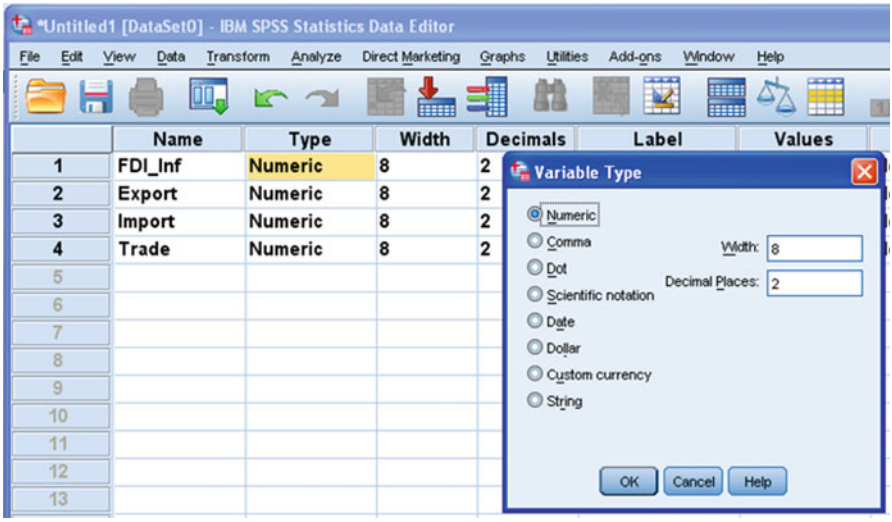



**Fig. 1.5** Option showing defining of variable as numeric or nonnumeric

Column 6: Under the column heading "Values," the coding of the variable may be defined by double clicking the cell. Sometimes, the variable is of classificatory in nature. For example, if there is a choice of choosing any one of the following four departments for training

(a) Production
(b) Marketing
(c) Human resource
(d) Public relation

then these departments can be coded as 1 = production, 2 = marketing, 3 = human resource, and 4 = public relation. While entering data into the computer, these codes are entered, as per the response of a particular subject. SPSS window showing the option for entering code has been shown in Fig. 1.6.

Column 7: In survey study, it is quite likely that for certain questions the respondent does not reply, which creates the problem of missing value. Such missing value can be defined under column heading "Missing."

Column 8: Under the heading "Columns," width of the column space where data is typed in Data View is defined.

Column 9: Under the column heading "Align," the alignment of data while feeding may be defined as left, right, or center.

Column 10: Under the column heading "Measure," the variable type may be defined as scale, ordinal, or nominal.

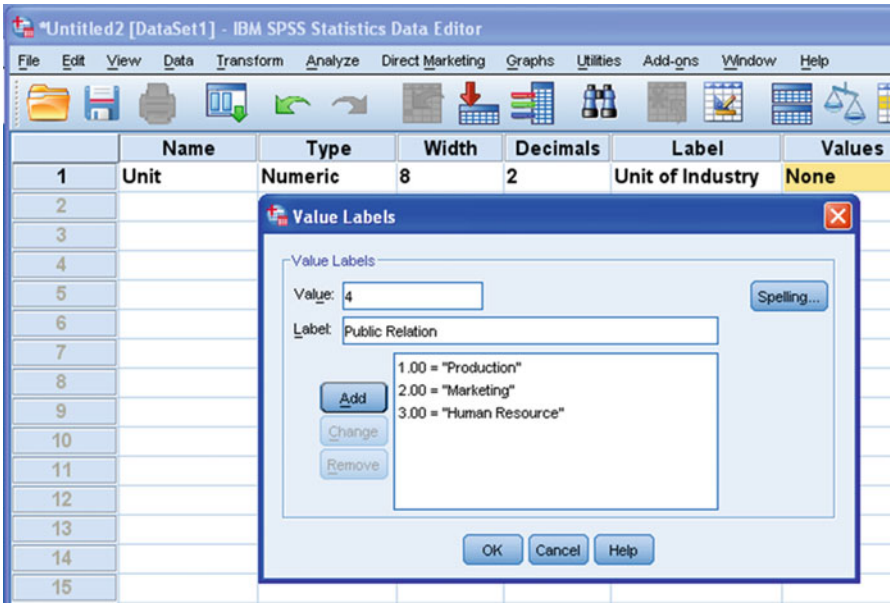



**Fig. 1.6** Screen showing how to define the code for the different labels of the variable

## *Defining Variables for the Data in Table 1.1*

1. Write short name of all the five variables as *States, FDI_Inf, Export, Import, and Trade* under the column heading "Name."
2. Under the column heading "Label," full name of these variables may be defined as *FDI Inflows, Export Data, Import Data,* and *Trade Data*. One can take liberty of defining some more detailed name of these variables as well.
3. Use default entries in rest of the columns.

After defining variables in the variable view, the screen shall look like Fig. 1.7.

## *Entering the Data*

After defining all the five variables in the **Variable View**, click **Data View** on the left bottom of the screen to open the format for entering the data. For each variable, data can be entered column wise. After entering the data, the screen will look like Fig. 1.8. Save the data file in the desired location before further processing.

After preparing the data file, one may use different types of statistical analysis available under the tag **Analyze** in the SPSS package. Different types of statistical analyses have been discussed in different chapters of the book along with their interpretations. Methods of data entry are different in certain applications; for

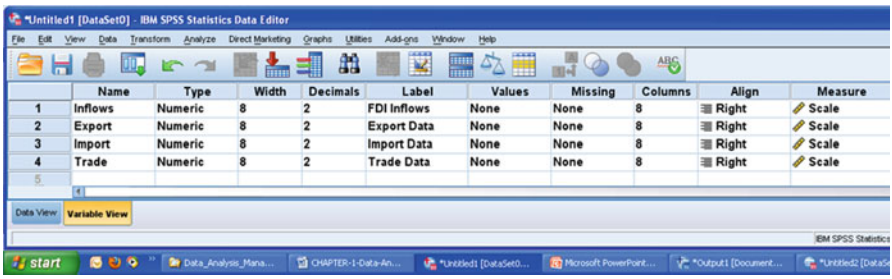
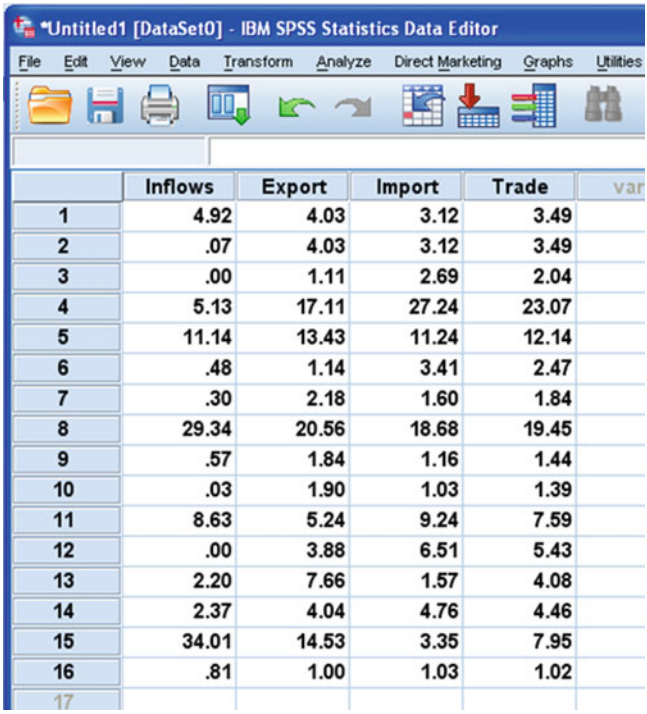



**Fig. 1.7** Variables along with their characteristics for the data shown in Table 1.1




| | Inflows | Export | Import | Trade | var |
|---|---|---|---|---|---|
| 1 | 4.92 | 4.03 | 3.12 | 3.49 | |
| 2 | .07 | 4.03 | 3.12 | 3.49 | |
| 3 | .00 | 1.11 | 2.69 | 2.04 | |
| 4 | 5.13 | 17.11 | 27.24 | 23.07 | |
| 5 | 11.14 | 13.43 | 11.24 | 12.14 | |
| 6 | .48 | 1.14 | 3.41 | 2.47 | |
| 7 | .30 | 2.18 | 1.60 | 1.84 | |
| 8 | 29.34 | 20.56 | 18.68 | 19.45 | |
| 9 | .57 | 1.84 | 1.16 | 1.44 | |
| 10 | .03 | 1.90 | 1.03 | 1.39 | |
| 11 | 8.63 | 5.24 | 9.24 | 7.59 | |
| 12 | .00 | 3.88 | 6.51 | 5.43 | |
| 13 | 2.20 | 7.66 | 1.57 | 4.08 | |
| 14 | 2.37 | 4.04 | 4.76 | 4.46 | |
| 15 | 34.01 | 14.53 | 3.35 | 7.95 | |
| 16 | .81 | 1.00 | 1.03 | 1.02 | |
| 17 | | | | | |

**Fig. 1.8** Screen showing entered data for all the variables in the data view

instance, readers are advised to note carefully the way data is entered for the application in Example 6.2 in Chap. 6. Relevant details have been discussed in that chapter.

## Importing Data in SPSS

In SPSS, data can be imported from ASCII as well as Excel file. The procedure of importing these two types of data files has been discussed in the following sections.

### *Importing Data from an ASCII File*

In ASCII file, data for each variable may be separated by a space, tab, comma, or some other character. The Text Import Wizard in SPSS facilitates you to import data from an ASCII file format. Consider the following set of data in ASCII file saved on the desktop by the file name Business data:

File name: *Business data*

| S.N. | FDI | Exports inflows | Imports | Trade |
|---|---|---|---|---|
| 1 | 4.92 | 4.03 | 3.12 | 3.49 |
| 2 | 0.07 | 4.03 | 3.12 | 3.49 |
| 3 | 0.00 | 1.11 | 2.69 | 2.04 |
| 4 | 5.13 | 17.11 | 27.24 | 23.07 |
| 5 | 11.14 | 13.43 | 11.24 | 12.14 |

The sequence of commands is as follows:

1. For importing the required ASCII file into SPSS, follow the below-mentioned sequence of commands in **Data View.**

$$\textbf{File} -> \textbf{Open} -> \textbf{Data} -> \textbf{Businessdata}$$

   – Choose "Text" as the "File Type" if your ASCII file has the .txt extension. Otherwise, choose the option "All files."
   – After selecting the file that you want to import, click **Open** as shown in Fig. 1.9.

2. After choosing the ASCII file from the saved location in Fig. 1.9, the Text Import Wizard will pop up automatically as shown in Fig. 1.10 that will take you for further option in importing the file. Take the following steps:

   – If your file does not match a predefined format, which is usually not, so click **Next**.

3. After clicking the **Next** option above, you will get the screen as shown in Fig. 1.11. Take the following steps:

   – Define delimiter and check the option "Delimited" as the data in the file is separated by either space or comma.
   – If variable names are written in the first row of your data file, check the header row option as "Yes," otherwise "No." In this, the option "Yes" will be selected because variable names have been written in the first row of the data file. Click **Next.**

4. After clicking the option **Next,** you will get the screen as shown in Fig. 1.12. Enter the line number where the first case of your data begins. If there is no

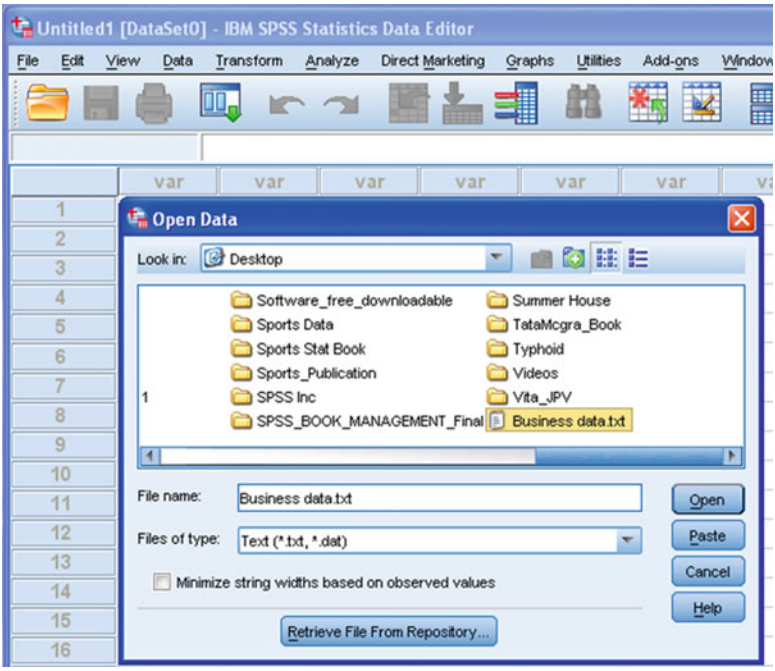



**Fig. 1.9** Selecting an ASCII file saved as text file for importing in SPSS

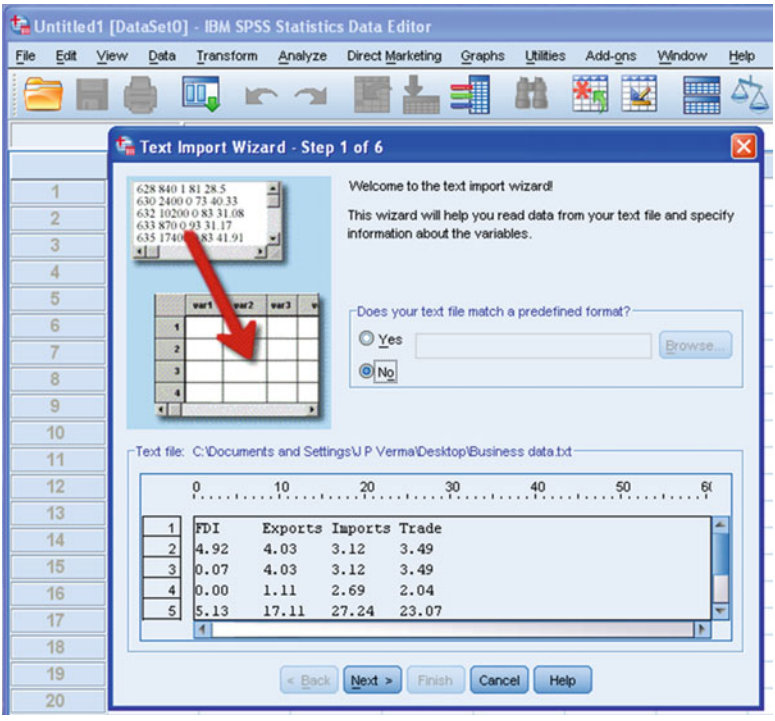



**Fig. 1.10** Import text wizard for opening an ASCII file in SPSS

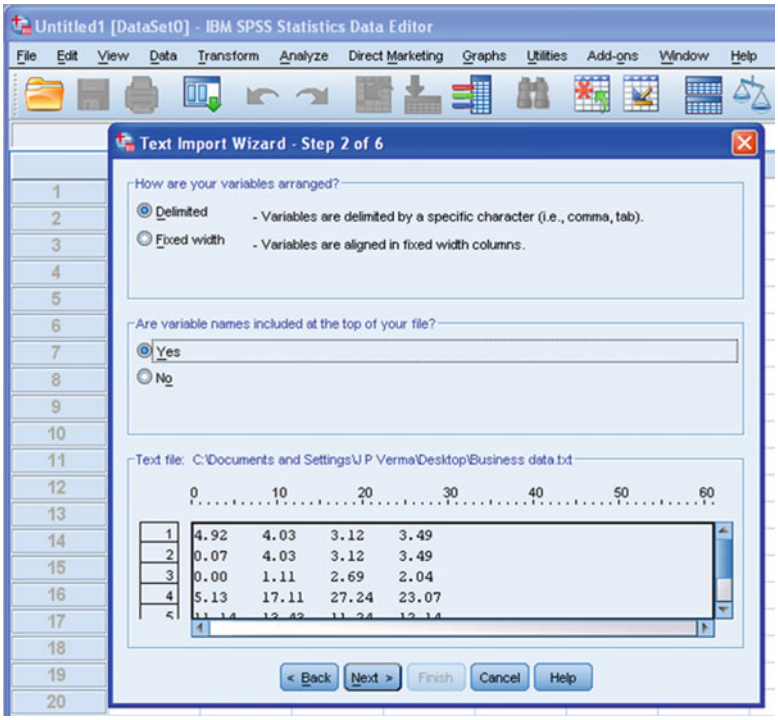



**Fig. 1.11** Defining option for delimiter and header row

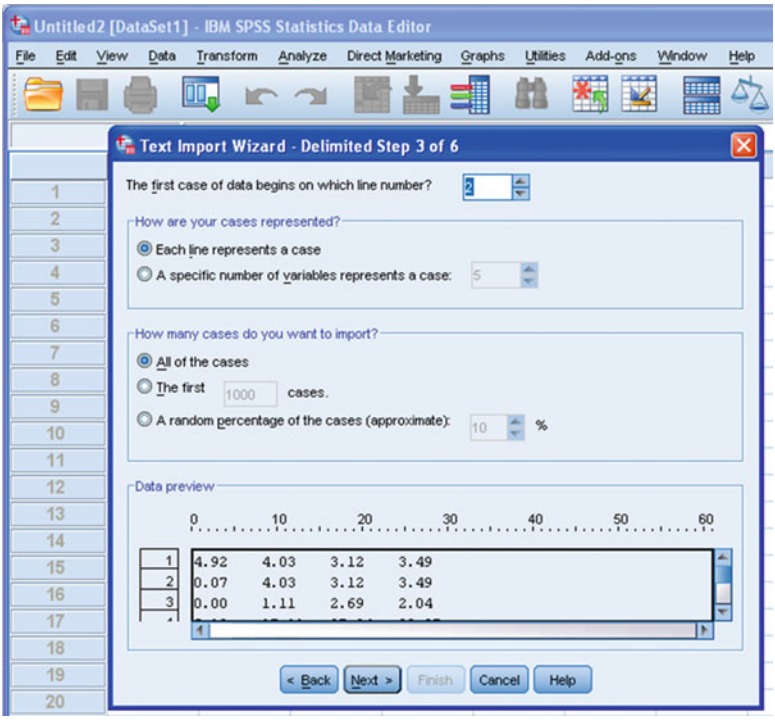



**Fig. 1.12** Defining option for beginning line of data and number of cases to be selected

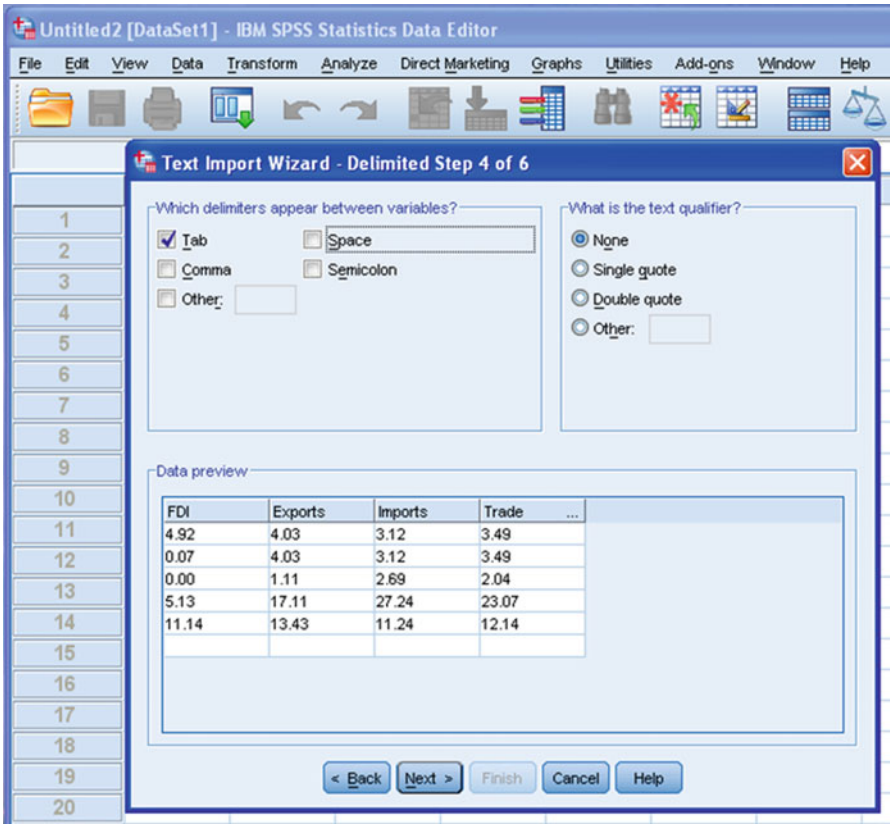



**Fig. 1.13** Defining option for delimiters and text qualifier

variable name in the first line of the data file, line 1 is selected; otherwise, line 2 may be selected as the data starts from line 2 in the data file. Take the following steps:

- Check the option "Each line represents a case." Normally in your data file, each line represents a case.
- Check the option "All of the cases." Usually, you import all the cases from the file. Other option may be tried if only few cases are imported from the file. Click **Next** to get the screen as shown in Fig. 1.13.

5. In Fig. 1.13, delimiters of the data file (probably comma or space) are set:

- Check the delimiters as "Coma" as the data is separated by comma. Other delimiters may be selected if used in the data file.
- Check the "Double quote" as text qualifier. Other options may be checked if the variables are flanked other than double quote.
- On the basis of the options chosen by you, SPSS formats the file in the small screen in the bottom. There you can check if everything is set correctly. Click **Next** when everything is ok to get the screen as shown in Fig. 1.14.

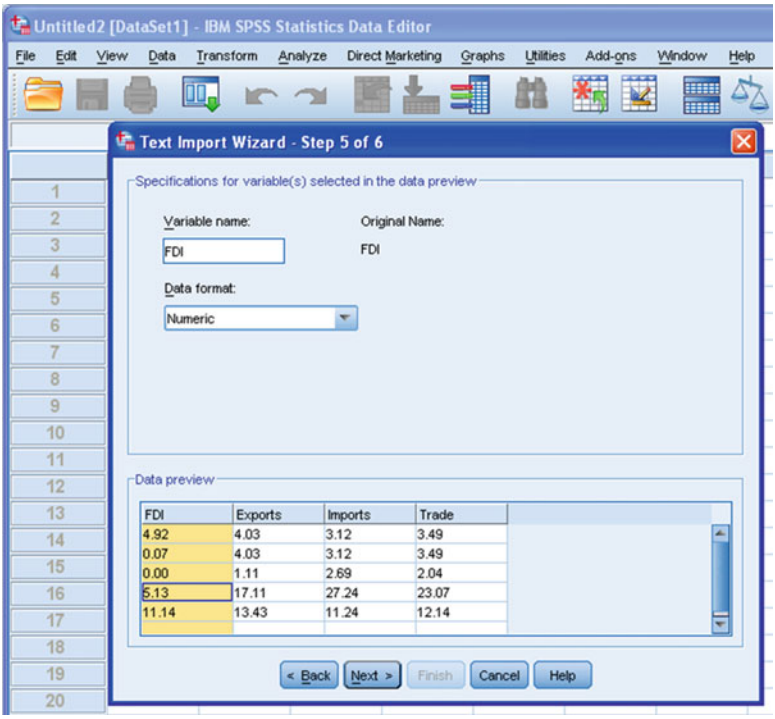



**Fig. 1.14** Defining specifications for the variables

6. In Fig. 1.14, you can define the specifications for the variables, but you may just ignore it if you have already defined your variables or want to do it later. Click **Next** to get the screen as shown in Fig. 1.15.
7. In Fig. 1.15, select all the default options and ensure that your actual data file has been shown in the window or not. Once your data is shown in the window, click **Finish**. This will import your file successfully in SPSS.

## *Importing Data File from Excel Format*

The data prepared in Excel file can be imported in SPSS by simple command. While importing Excel data file, one must ensure that it is not open. The sequence of commands for importing Excel data file is as follows:

1. **File** $->$ **Open** $->$ **Data** $->$ **requiredfile**
   - Choose "Excel" as the File Type if your ASCII file has the .xls extension. Otherwise, choose the option "All files."
   - After selecting the file that you want to import, click **Open** as shown in Fig. 1.16.

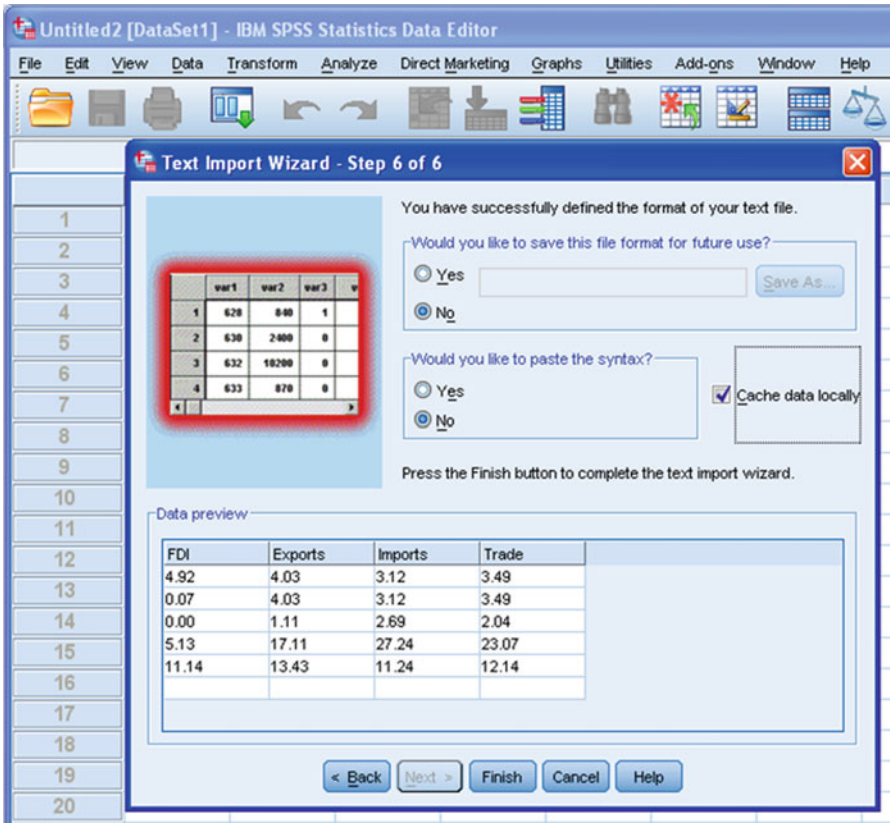



**Fig. 1.15** Option for saving the format

2. After choosing the required Excel file from the saved location in Fig. 1.16, you will get the pop screen called "Opening Excel Data Source" as shown in Fig. 1.17. Take the following steps:
   - Check the option "Read variable names from the first row of data" if you are using the header row in the data file.
   - Select the right worksheet from which you want to import the data. The screen will show you all the worksheets of the file containing data. If you have data only in the first worksheet, leave this option as it is.
   - If you want to use only a portion of data from the file, define the fields in "**Range**" option like A3:E8. This means that the data from the A3 row till column E8 shall be selected.
   - Press **Continue** to get the Excel file opened in SPSS.

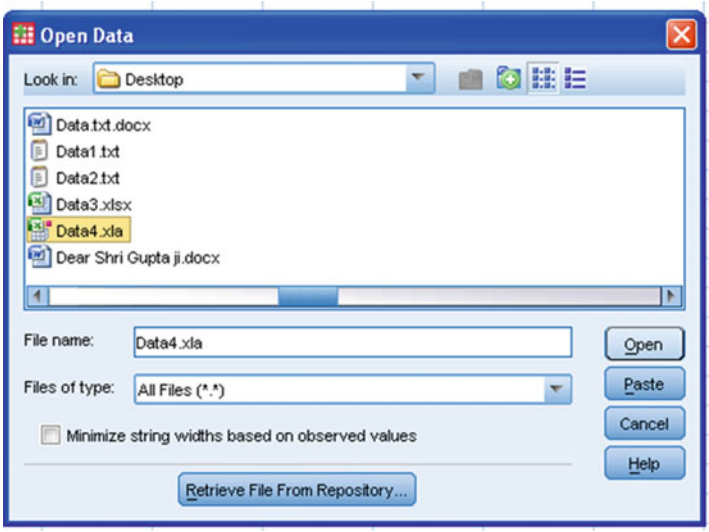



**Fig. 1.16** Selecting an Excel file for importing in SPSS

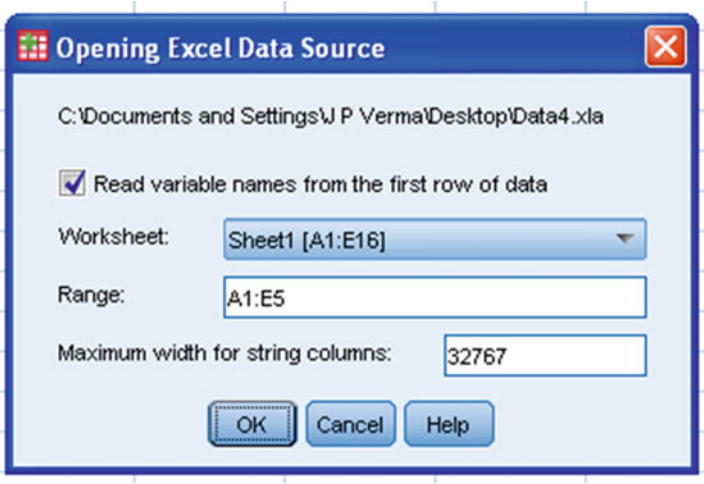



**Fig. 1.17** Option for defining the range of data in Excel sheet to be imported in SPSS

## Exercise

*Short-Answer Questions*

**Note:** Write the answer to each of the questions in not more than 200 words.

Q.1. What do you mean by inductive and inferential statistics? What is the difference between them? Explain by means of example.
Q.2. What do you mean by metric and nonmetric data? Discuss them by means of example.
Q.3. Under what situation analytical studies should be conducted? Discuss a situation where it can be used.
Q.4. What do you mean by mutually exclusive and independent attributes? Give two examples where the attributes are not mutually exclusive.
Q.5. What is an extraneous variable? How it affects the findings of an experiment? Suggest remedies for eliminating its effects.
Q.6. While feeding the data in SPSS, what are the possible mistakes that a user might commit?
Q.7. Explain in brief as to how an error can be identified in data feeding.

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. Given the following statements,

   I. Parametric tests do not assume anything about the form of the distribution.
   II. Nonparametric tests are simple to use.
   III. Parametric tests are the most powerful if their assumptions are satisfied.
   IV. Nonparametric tests are based upon the assumption of normality.

   choose the correct statements from the above-listed ones.

   (a) (I) and (II)
   (b) (I) and (III)
   (c) (II) and (III)
   (d) (III) and (IV)

2. If the respondents were required to rate themselves on emotional strength on a 9-point scale, what type of data would be generated?

   (a) Ratio
   (b) Interval
   (c) Nominal
   (d) Ordinal

3. The variable measured on which of the following scales are termed as categorical.

   (a) Ratio and interval
   (b) Interval and ordinal
   (c) Interval and nominal
   (d) Ordinal and nominal

4. In tossing an unbiased coin, one can get the following events:
   $E_1$: getting a head, $E_2$: getting a tail. Choose the correct statement.

   (a) $E_1$ and $E_2$ are independent.
   (b) $E_1$ and $E_2$ are mutually exclusive.
   (c) $E_1$ and $E_2$ are not equally likely.
   (d) $E_1$ and $E_2$ are independent and mutually exclusive.

5. While creating a new data file in SPSS, which option should be used?

   (a) Type in data
   (b) Open an existing data source
   (c) Open another type of file
   (d) None

6. Identify valid name of the variable.

   (a) SalesData
   (b) Cust No
   (c) Outlet " Center"
   (d) Sales-Data

7. While defining the types of the variable under the heading "Measure" in SPSS, what are the valid options out of the following?

   (i) Interval (ii) Scale
   (iii) Nominal (iv) Ordinal

   (a) (i),(ii), and (iii)
   (b) (i),(ii), and (iv)
   (c) (i),(iii), and (iv)
   (d) (ii),(iii), and (iv)

8. For analyzing the data, the commands are selected while being in the

   (a) Variable View
   (b) Data View
   (c) Data and Variable View
   (d) Neither in Data nor in Variable View

9. Runs scored in a cricket match is

   (a) Interval data
   (b) Ratio data
   (c) Nominal data
   (d) Ordinal data

10. In an experiment, effect of three types of incentives on satisfaction level has to be seen on the subjects. Choose the correct statement.

    (a) Incentive is a dependent variable, and satisfaction level is an independent variable.
    (b) Incentive is an independent variable, and satisfaction level is a dependent variable.
    (c) Incentive and satisfaction level are independent variables.
    (d) Incentive and satisfaction level both are dependent variables.

*Answers to Multiple-Choice Questions*

| | | | |
|---|---|---|---|
| Q.1 | c | Q.2 | b |
| Q.3 | d | Q.4 | b |
| Q.5 | a | Q.6 | a |
| Q.7 | d | Q.8 | b |
| Q.9 | b | Q.10 | b |

# Chapter 2
# Descriptive Analysis

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Learn the importance of descriptive studies.
- Know the various statistics used in descriptive studies.
- Understand the situations in management research for undertaking a descriptive study.
- Describe and interpret various descriptive statistics.
- Learn the procedure of computing descriptive statistics using SPSS.
- Know the procedure of developing the profile chart of a product or organization.
- Discuss the findings in the outputs generated by the SPSS in a descriptive study.

## Introduction

Descriptive studies are carried out to understand the profile of any organization that follows certain common practice. For example, one may like to know or be able to describe the characteristics of an organization that implement flexible working timing or that have a certain working culture. Descriptive studies may be undertaken to describe the characteristics of a group of employees in an organization. The purpose of descriptive studies is to prepare a profile or to describe interesting phenomena from an individual or an organizational point of view.

Although descriptive studies can identify sales pattern over a period of time or in different geographical locations but cannot ascertain the causal factors. These studies are often very useful for developing further research hypotheses for testing. Descriptive research may include case studies, cross-sectional studies, or longitudinal investigations.

Different statistics are computed in descriptive studies to describe the nature of data. These statistics computed from the sample provide summary of various measures. Descriptive statistics are usually computed in all most every experimental research

J.P. Verma, *Data Analysis in Management with SPSS Software*,
DOI 10.1007/978-81-322-0786-3_2, © Springer India 2013

study. The primary goal in a descriptive study is to describe the sample at any specific point of time without trying to make inferences or causal statements. Normally, there are three primary reasons to conduct such studies:

1. To understand an organization by knowing its system
2. To help in need assessment and planning resource allocation
3. To identify areas of further research

Descriptive studies help in identifying patterns and relationships that might otherwise go unnoticed.

A descriptive study may be undertaken to ascertain and be able to describe the characteristics of variables of interest, in a given situation. For instance, a study of an organization in terms of percentage of employee in different age categories, their job satisfaction level, motivation level, gender composition, and salary structure can be considered as descriptive study. Quite frequently descriptive studies are undertaken in organizations to understand the characteristics of a group or employees such as age, educational level, job status, and length of service in different departments.

Descriptive studies may also be undertaken to know the characteristics of all those organizations that operate in the same sector. For example, one may try to describe the production policy, sales criteria, or advertisement campaign in pharmacy companies. Thus, the goal of descriptive study is to offer the researcher a profile or to describe relevant aspects of the phenomena of interest in an organization, industry, or a domain of population. In many cases, such information may be vital before considering certain corrective steps.

In a typical profile study, we compute various descriptive statistics like mean, standard deviation, coefficient of variation, range, skewness, and kurtosis. These descriptive statistics explain different features of the data. For instance, mean explains an average value of the measurement, whereas standard deviation describes variation of the scores around their mean value; the coefficient of variation provides relative variability of scores; range gives the maximum variation; skewness explains the symmetricity; and kurtosis describes the variation in the data set.

In descriptive studies, one tries to obtain information regarding current status of different phenomena. Purpose of such study is to describe "What exists?" with respect to situational variables.

In descriptive research, the statement of problem needs to be defined first and then identification of information is planned. Once the objectives of the study are identified, method of data collection is planned to obtain an unbiased sample, and therefore, it is important to define the population domain clearly. Further, an optimum sample size should be selected for the study as it enhances the efficiency in estimating population characteristics.

Once the data is collected, it should be compiled in a meaningful manner for further processing and reporting. The nature of each variable can be studied by looking to the values of different descriptive statistics. If purpose of the study is

analytical as well, then these data may further be analyzed for testing different formulated hypotheses.

On the basis of descriptive statistics and graphical pictures of the parameters, different kinds of generalizations and predictions can be made. While conducting descriptive studies, one gets an insight to identify the future scope of the related research studies.

## Measures of Central Tendency

Researchers are often interested in defining a value that best describes some characteristics of the population. Often, this characteristic is a measure of central tendency. A measure of central tendency is a single score that attempts to describe a set of data by identifying the central position within that set of data. The three most common measures of central tendency are the mean, the median, and the mode. Measures of central tendency are also known as central location. Perhaps, you are more familiar with the mean (also known as average) as the measure of central tendency, but there are others, such as the median and the mode, which are appropriate in some specific situations.

The mean, median, and mode are all valid measures of central tendency, but, under different conditions, some measures of central tendency become more appropriate than other. In the following sections, we will look at the various features of mean, median, and mode and the conditions under which they are most appropriate to be used.

### *Mean*

The mean is the most widely used and popular measure of central tendency. It is also termed as average. It gives an idea as to how an average score looks like. For instance, one might be interested to know that on an average how much is the sale of items per day on the basis of monthly sales figure. The mean is a good measure of central tendency for symmetric distributions but may be misleading in case of skewed distribution. The mean can be computed with both discrete and continuous data. The mean is obtained by dividing the sum of all scores by the number of scores in the data set.

If $X_1, X_2, \ldots, X_n$ are the $n$ scores in the data set, then the sample mean, usually denoted by $\bar{X}$ (pronounced $X$ bar), is

$$\bar{X} = \frac{X_1 + X_2 + \ldots . X_n}{n}$$

This formula is usually written by using the Greek capital letter $\sum$, pronounced "sigma," which means "sum of. . .":

$$\bar{X} = \frac{1}{n} \sum X \tag{2.1}$$

In statistics, sample mean and population mean are represented in different manner, although the formulas for their computations are same. To show that we are calculating the population mean and not the sample mean, we use the Greek lower case letter "mu," denoted as $\mu$:

$$\mu = \frac{1}{n} \sum X$$

The mean is the model of your data set and explains that on an average, the data set tends to concentrate toward it. You may notice that the mean is not often one of the actual values that you have observed in your data set.

### Computation of Mean with Grouped Data

If $X_1, X_2, X_3, \ldots, X_n$ are $n$ scores with $f_1, f_2, f_3, \ldots, f_n$ frequencies respectively in the data set, then the mean is computed as

$$\bar{X} = \frac{\sum f_i X_i}{\sum f_i} = \frac{\sum f_i X_i}{n} \tag{2.2}$$

where

$\sum f_i X_i$ is the total of all the scores.

In case the data is arranged in class interval format, the $X$ will be the midpoint of the class interval. Let us see how to explain the data shown in the class interval form in Table 2.1. The first class interval shows that the ten articles are in the price range of Rs. 1–50 and that of six articles are in the range of Rs. 51–100 and so on. Here, the exact price of each article is not known because they have been grouped together. Thus, in case of grouped data, the scores lose its own identity. This becomes problematic as it is difficult to add the scores because their magnitudes are not known. In order to overcome this problem, an assumption is made while computing mean and standard deviation from the grouped data. It is assumed that the frequency is concentrated at the midpoint of the class interval. By assuming so, the identity of each and every score can be regained; this helps us to compute the sum of all the scores which is required for computing mean and standard deviation. But by taking this assumption, it is quite likely that the scores may be either underestimated or overestimated. For instance, in Table 2.1, if all the ten items

**Table 2.1** Frequency distribution of articles price

| Class interval (price range in Rs.) | Frequency ($f$) | Midpoint ($X$) | $fX$ |
|---|---|---|---|
| 1–50 | 10 | 25.5 | 255 |
| 51–100 | 6 | 75.5 | 453 |
| 101–150 | 4 | 125.5 | 502 |
| 151–200 | 4 | 175.5 | 702 |
| 201–250 | 2 | 225.5 | 451 |
| 251–300 | 2 | 275.5 | 551 |
| | $\sum f = n = 28$ | | $\sum fX = 2914$ |

would have had prices in the range of Rs. 1–50 but due to assumption they are assumed to have prices as Rs. 25.5, a negative error may be created which is added in the computation of mean. But it may be quite likely that the prices of other six items may be on the higher side, say Rs. 90, whereas they are assumed to have the price as 75.5 which creates the positive error. Thus, these positive and negative errors add up to zero in the computation of mean.

In Table 2.1, $\sum fX$ represents the sum of all the scores, and therefore,

$$\bar{X} = \frac{\sum f_i X_i}{n} = \frac{2914}{28} = 104.07$$

### Effect of Change of Origin and Scale on Mean

If the magnitude of data is large, it may be reduced by using the simple transformation

$$D = \frac{X - A}{i}$$

where "$A$" and "$i$" are origin and scale, respectively. Thus, any score which is subtracted from all the scores in the data set is termed as origin, and any score by which all the scores are divided is known as scale. The choice of origin and scale is up to the researcher, but the only criterion which one should always keep in mind is that the very purpose of using the transformation is to simplify the data and computation.

Let us see what is the effect of change of origin and scale on the computation of mean? If all the $X$ scores are transformed into $D$ by using the above-mentioned transformation, then taking summation on both sides,

$$\sum \mathrm{D} = \sum \left(\frac{X-A}{i}\right)$$

$$\Rightarrow \qquad \sum (X - A) = i \times \sum \mathrm{D}$$

**Table 2.2** Showing computation for mean

| Class interval (price range in Rs.) | Frequency ($f$) | Midpoint ($X$) | $D = \frac{X-175.5}{50}$ | $fD$ |
|---|---|---|---|---|
| 1–50 | 10 | 25.5 | −3 | −30 |
| 51–100 | 6 | 75.5 | −2 | −12 |
| 101–150 | 4 | 125.5 | −1 | −4 |
| 151–200 | 4 | 175.5 | 0 | 0 |
| 201–250 | 2 | 225.5 | 1 | 2 |
| 251–300 | 2 | 275.5 | 2 | 4 |
| | $\sum f = n = 28$ | | | $\sum fD = -40$ |

Dividing both side by $n$,

$$\frac{\sum (X - A)}{n} = \frac{i \times \sum D}{n}$$

$$\Rightarrow \quad \frac{1}{n}\sum X - \frac{nA}{n} = i \times \frac{1}{n}\sum D$$

$$\Rightarrow \quad \bar{X} = A + i \times \bar{D}$$

Thus, we have seen that if all the scores $X$ are transformed into $D$ by changing the origin and scales as $A$ and $i$, respectively, then the original mean can be obtained by multiplying the new mean $\bar{D}$ by the scale $i$ and adding the origin value into it. Thus, it may be concluded that the mean is not independent of change of origin and scale.

### Computation of Mean with Deviation Method

In case of grouped data, the mean can be computed by transforming the scores so obtained by taking the midpoint of the class intervals. Consider the data shown in Table 2.1 once again. After computing the midpoint of the class intervals, let us transform the scores by changing the origin and scale as 175.5 and 50, respectively. Usually, origin ($A$) is taken as the midpoint of the middlemost class interval, and the scale ($i$) is taken as the width of the class interval. The origin $A$ is also known as assumed mean (Table 2.2).

Here, width of the class interval $= i = 50$ and assumed mean $A = 175.5$

Since we know that

$$\overline{X} = A + i \times \overline{D} = A + i \times \frac{1}{n}\sum fD$$

$$\Rightarrow \quad \bar{X} = 175.5 + 50 \times \frac{1}{28} \times (-40)$$

$$= 175.5 - 71.43 = 104.07$$

In computing the mean, the factor $i \times (1/n) \sum fD$ can be considered as the correction factor. If the assumed mean is taken higher than the actual mean, the correction factor shall be negative, and, if it is taken as lower than the actual mean, the correction factor will become positive. One may take assume mean as the midpoint of the even lowest or highest class interval. But in that case, the magnitude of the correction factor shall be higher and the very purpose of simplifying the computation process shall be defeated. Thus, the correct strategy is to take the midpoint of the middlemost class interval as the assumed mean. However, in case the number of class intervals is even, midpoint of any of the two middle class intervals may be taken as the assumed mean.

## Properties of Mean

1. The mean is the most reliable measure of central tendency as it is computed by using all the data in the data set.
2. Mean is more stable than any other measures of central tendency because its standard error is least in comparison to median and mode. It simply means that if you compute mean of different samples that are drawn from the same population, then the fluctuation among these means shall be least in comparison to that of other measures of central tendencies like median and mode.
3. If $\bar{X}_1$ and $\bar{X}_2$ are the means of the two groups computed from the two sets of values $n_1$ and $n_2$, then the combined mean $\bar{X}$is given by the following formula:

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

4. The sum of the deviation of a set of values from their arithmetic mean is always 0. In other words,

$$\sum (X - \bar{X}) = 0$$

   To prove this, expand left-hand side of this expression

$$\begin{aligned} \sum (X - \bar{X}) &= \sum X - \sum \bar{X} \\ &= N\bar{X} - N\bar{X} = 0 \end{aligned}$$

5. The mean is highly affected by the outliers.
6. In the absence of even one observation, it is impossible to compute the mean correctly.
7. In case of open-ended class interval, the mean cannot be computed.

## *Median*

Median is the middlemost score in the data set arranged in order of magnitude. It is a positional average and is not affected by the extreme scores. If $X_1, X_2, \ldots, X_n$ are the $n$ scores in a data set arranged in the ascending or descending order, then its median is obtained by

$$M_d = \left(\frac{N+1}{2}\right)^{\text{th}} \text{score} \tag{2.4}$$

One should note that $(n+1)/2$ is not the median, but the score lying in that position is the median. Consider the weight of the following ten subjects: 56, 45, 53, 41, 48, 53, 52, 65, 38, 42.

After arranging the scores

| S.N.: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weight: | 38 | 41 | 42 | 45 | 48 | 52 | 53 | 53 | 56 | 65 |

Here, $n = 10$.

Thus, $M_d = \left(\frac{10+1}{2}\right)^{\text{th}} = 5.5^{\text{th}}\text{score} = \frac{(48+52)}{2} = 50$

In case of odd number of scores you will get a single score lying in the middle, but in case of even number of scores, the middlemost score is obtained by taking the average of the two middle scores as in that case there are two middle scores.

Median is used in case the effect of extreme scores needs to be avoided. For example, consider the marks of the students in a college as shown below:

| Student: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Marks: | 35 | 40 | 30 | 32 | 35 | 39 | 33 | 32 | 91 | 93 |

The mean score for these ten students is 46. However, the raw data suggests that this mean value might not be the best way to accurately reflect the typical performance of a student, as most students have marks in between 30 and 40. Here, the mean is being skewed by the two large scores. Therefore, in this situation, median gives better estimate of average instead of mean. Thus, in a situation where the effect of extreme scores needs to be avoided, median should be preferred over mean. In case the data is normally distributed, the values of mean, median, and mode are same. Moreover, they all represent the most typical value in the data set. However, as the data becomes skewed, the mean loses its ability to provide the best central location as the mean is being dragged in the direction of skew. In that case, the median best retains this position and is not influenced much by the skewed values. As a rule of thumb if the data is non-normal, then it is customary to use the median instead of the mean.

### Computation of Median for Grouped Data

While computing the median for grouped data, it is assumed that the frequencies are equally distributed in the class interval. This assumption is also used in computing the quartile deviation because median and quartile deviation both are nonparametric statistics and depend upon positional score. In case of grouped data, the median is computed by the following formula:

$$M_d = \| + \frac{\frac{n}{2} - F}{f_m} \times i \tag{2.5}$$

where

‖ : lower limit of the median class
$n$ : total of all the frequencies
$F$ : cumulative frequency of the class just lower than the median class
$f_m$ : frequency of the median class
$i$ : width of the class interval

The computation of the median shall be shown by means of an example. Consider the marks in mathematics obtained by the students as shown in Table 2.3.

In computing median, first of all we need to find the median class. Median class is the one in which the median is supposed to lie. To obtain the median class, we compute $n/2$ and then we look for this value in the column of cumulative frequency. The class interval for which the cumulative frequency includes the value $n/2$ is taken as median class.

Here, $n = 70$
and therefore, $\frac{n}{2} = \frac{70}{2} = 35$

Now, we look for 35 in the column of cumulative frequency. You can see that the class interval 31–35 has a cumulative frequency 48 which includes the value $n/2 = 35$. Thus, 31–35 is the median class. After deciding the median class, the median can be computed by using the formula (2.5).

Here, ‖ = Lower limit of the median class = 30.5
$f_m$ = Frequency of the median class = 20
$F$ = Cumulative frequency of the class just lower than the median class = 28
$i$ = Width of the class interval = 5
Substituting these values in the formula (2.5),

$$M_d = \| + \frac{\frac{n}{2} - F}{f_m} \times i$$
$$= 30.5 + \frac{35 - 28}{20} \times 5 = 30.50 + 1.75 = 32.25$$

In computing the lower limit of the median class, 0.5 has been subtracted from the lower limit because the class interval is discrete. Any value which is equal or

**Table 2.3** Frequency distribution of marks in mathematics

| Class interval (marks range) | Frequency ($f$) | Cumulative frequency ($F$) |
|---|---|---|
| 10 or less | 2 | 2 |
| 11–15 | 4 | 6 |
| 16–20 | 5 | 11 |
| 21–25 | 6 | 17 |
| 26–30 | 11 | 28 |
| 31–35 | 20 | 48 |
| 36–40 | 15 | 63 |
| 41–45 | 4 | 67 |
| 46–50 | 3 | 70 |
| | $\sum f = n = 70$ | |

Median class

F

$f_m$

greater than 30.5 shall fall in the class interval 31–35, and that is why actual lower limit is taken as 30.5 instead of 31. But in case of continuous class intervals, lower limit of the class interval is the actual lower limit, and we do not subtract 0.5 from it. In case of continuous class interval, it is further assumed that the upper limit is excluded from the class interval. This make the class intervals mutually exclusive.

In Table 2.3, the lowest class interval is truncated, and therefore, its midpoint can be computed; hence, the mean can not be computed in this situation. Thus, if the class intervals are truncated at one or both the ends, median is the best choice as a measure of central tendency.

## *Mode*

Mode can be defined as the score that occurs most frequently in a set of data. If the scores are plotted, then the mode is represented by the highest bar in a bar chart or histogram. Therefore, mode can be considered as the most popular option in the set of responses. Usually, mode is computed for categorical data where we wish to know as to which the most common category is. The advantage of mode is that it is not affected by the extreme scores (outliers). Sometime, there could be two scores having equal or nearly equal frequencies in the data set. In that case, the data set will have two modes and the distribution shall be known as bimodal. Thus, on the basis of the number of modes, the distribution of the scores may be unimodal, bimodal, or multimodal. Consider the following data set: 2, 5, 4, 7, 6, 3, 7, 8, 7, 9, 1, 7. Here, the score 7 is being repeated maximum number of times; hence, the mode of this data set is 7.

The mode can be used in variety of situations. For example, if a pizza shop sells 12 different varieties of pizzas, the mode would represent the most popular pizza. Mode may be computed to know as to which of the text book is more popular

than others, and accordingly, the publisher would print more copy of that book instead of printing equal number of all books.

Similarly, it is important for the manufacturer to produce more of the most popular shoes because manufacturing different shoes in equal numbers would cause a shortage of some shoes and an oversupply of others. Other applications of the mode may be to find the most popular brand of soft drink or biscuits to take the manufacturing decision accordingly.

### Drawbacks of Mode

1. Computing mode becomes problematic if the data set consists of continuous data, as we are more likely not to have any one value that is more frequent than the other. For example, consider measuring 40 persons' height (to the nearest 1 cm). It will be very unlikely that any two or more people will have the same height. This is why the mode is very rarely used with continuous data.
2. Mode need not necessarily be unique. There may be more than one mode present in the data set. In that case, it is difficult to interpret and compare the mode.
3. If no value in the data set is repeated, then every score is a mode which is useless.

### Computation of Mode for Grouped Data

In computing the mode with grouped data first of all one needs to identify the modal class. The class interval, for which the frequency is maximum, is taken as modal class. The frequency of the modal class is denoted by $f_0$, and that of frequencies before and after the modal class are represented by $f_1$ and $f_2$, respectively. Once these frequencies are identified, they can be used to compute the value of the mode. The formula for computing mode with the grouped data is given by

$$M_0 = || + \frac{\mathrm{f_m} - \mathrm{f_1}}{2\mathrm{f_m} - \mathrm{f_1} - \mathrm{f_2}} \times \mathrm{i} \tag{2.6}$$

where

‖ : lower limit of the modal class
$f_m$ : frequency of the modal class
$f_1$ : frequency of the class just lower than the modal class
$f_2$ : frequency of the class just higher than the modal class
$i$ : width of the class interval

Table 2.4 shows the distribution of age of bank employees. Let us compute the value of mode in order to find as to what is the most frequent age of employees in the bank.

Since the maximum frequency is 50 for the class interval 26–30, hence this will be the modal class here.

**Table 2.4** Frequency distribution of age

| Class interval C.I. | Frequency (*f*) |
|---|---|
| 21–25 | 25 |
| 26–30 | 50 |
| 31–35 | 10 |
| 36–40 | 5 |
| 41–45 | 4 |
| 46–50 | 2 |

Now, ‖ = lower limit of the modal class = 25.5
$f_m$ : frequency of the modal class = 50
$f_1$ : frequency of the class just lower than the modal class = 25
$f_2$ : frequency of the class just higher than the modal class = 10
$i$ : width of the class interval = 5
After substituting these values in the formula (2.6), we get

$$\begin{aligned} M_0 &= || + \frac{\mathrm{f_m} - \mathrm{f_1}}{2\mathrm{f_m} - \mathrm{f_1} - \mathrm{f_2}} \times \mathrm{i} \\ &= 25.5 + \frac{50 - 25}{2 \times 50 - 25 - 10} \times 5 \\ &= 25.5 + 1.92 = 27.42 \end{aligned}$$

Thus, one may conclude that mostly employees in the bank are of around 27 years of age.

## *Summary of When to Use the Mean, Median, and Mode*

Following summary table shows the suitability of different measures of central tendency for different types of data.

| Nature of variable | Suitable measure of central tendency |
|---|---|
| Nominal data (categorical) | Mode |
| Ordinal data | Median |
| Interval/ratio (symmetrical or nearly symmetrical) | Mean |
| Interval/ratio (skewed) | Median |

## Measures of Variability

Variability refers to the extent of scores that vary from each other. The data set is said to have high variability when it contains values which are considerably higher and lower than the mean value. The terms variability, dispersion, and spread are all synonyms and refer as to how much the distribution is spread out. Measure of central tendency refers to the central location in the data set, but the central location itself is not sufficient to define the characteristics of the data set. It may happen that the two data sets are similar in their central location but might differ in their variability. Thus, measure of central tendency and measure of variability both are required to describe the nature of the data correctly. There are four measures of variability that are frequently used, the range: interquartile range, variance, and standard deviation. In the following paragraphs, we will look at each of these four measures of variability in more detail.

### *The Range*

The range is the crudest measure of variability and is obtained by subtracting the lowest score from the highest score in the data set. It is rarely used because it is based on only two extreme scores. The range is simple to compute and is useful when it is required to evaluate the whole of a data set. The range is useful in showing the maximum spread within a data set. It can be used to compare the spread between similar data sets.

Using range becomes problematic if one of the extreme score is exceptionally high or low (referred to as outlier). In that case, the range so computed may not represent the true variability within the data set. Consider a situation where scores obtained by the students on a test were recorded and the minimum and maximum scores were 25 and 72, respectively. If a particular student did not appear in the exam due to some reason and his score was posted as zero, then the range becomes 72(72–0) instead of 47(72–25). Thus, in the presence of an outlier, the range provides the wrong picture about the variability within the data set. To overcome the problem of outlier in a data set, the interquartile range is often calculated instead of the range.

### *The Interquartile Range*

The interquartile range is a measure that indicates the maximum variability of the central 50% of values within the data set. The interquartile range can further be divided into quarters by identifying the upper and lower quartiles. The lower quartile ($Q_1$) is equivalent to the 25th percentile in the data set which is arranged in order of magnitude, whereas the upper quartile ($Q_3$) is equivalent to the 75th

percentile. Thus, $Q_1$ is a point below which 25% scores lie, and $Q_3$ refers to a score below which 75% scores lie. Since the median is a score below which 50% scores lie, hence, the upper quartile lies halfway between the median and the highest value in the data set, whereas the lower quartile lies halfway between the median and the lowest value in the data set. The interquartile range is computed by subtracting the lower quartile from the upper quartile and is given by

$$Q = Q_3 - Q_1 \tag{2.7}$$

The interquartile range provides a better picture of the overall data set by ignoring the outliers. Just like range, interquartile range also depends upon the two values. Statistically, the standard deviation is more powerful measure of variability as it is computed with all the values in the data set.

## *The Standard Deviation*

The standard deviation is the most widely used measure of variability, the value of which depends upon how closely the scores cluster around the mean value. It can be computed only for interval or ratio data. The standard deviation is the square root of the average squared deviation of the scores from its mean value and is represented by $\sigma$(termed as sigma):

$$\sigma = \sqrt{\frac{1}{N}\sum (X-\mu)^2}$$

After simplification,

$$\sigma = \sqrt{\frac{1}{N}\sum X^2 - \left(\frac{\sum X}{N}\right)^2} \tag{2.8}$$

where $\mu$ is the population mean. The term $\sigma$ is used for population standard deviation, whereas $S$ is used for sample standard deviation. The population standard deviation $\sigma$ can be estimated from the sample data by the following formula:

$$S = \sqrt{\frac{1}{n-1}\sum (X-\bar{X})^2}$$

After simplifying,

$$S = \sqrt{\frac{1}{n-1}\sum X^2 - \frac{(\sum X)^2}{n(n-1)}} \tag{2.9}$$

If $X_1, X_2, X_3, \ldots, X_n$ are the $n$ scores with $f_1, f_2, f_3, \ldots, f_n$ frequencies respectively the data set, then the standard deviation shall be given as

$$S = \sqrt{\frac{1}{n-1}\sum f(X - \bar{X})^2}$$

After simplification,

$$S = \sqrt{\frac{1}{n-1}\sum fX^2 - \frac{\left(\sum fX\right)^2}{n(n-1)}} \tag{2.10}$$

where $\bar{X}$ refers to the sample mean. The standard deviation measures the aggregate variation of every value within a data set from the mean. It is the most robust and widely used measure of variability because it takes into account every score in the data set.

When the scores in a data set are tightly bunched together, the standard deviation is small. When the scores are widely apart, the standard deviation will be relatively large. The standard deviation is usually presented in conjunction with the mean and is measured in the same units.

### Computation of Standard Deviation with Raw Data

The sample standard deviation of a series of scores can be computed by using the formula (2.9). Following are the data on memory retention test obtained on 10 individuals. The scores are the number of items recollected by individuals (Table 2.5).

**Table 2.5** Computation for standard deviation

| $(X)$ | $(X^2)$ |
|---|---|
| 4 | 16 |
| 5 | 25 |
| 3 | 09 |
| 2 | 04 |
| 6 | 36 |
| 8 | 64 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |
| 4 | 16 |
| $\sum X = 47$ | $\sum X^2 = 247$ |

Here $n = 10$, $\sum X = 47$, and $\sum X^2 = 247$.
Substituting these values in the formula (2.9),

$$\begin{aligned} S &= \sqrt{\frac{1}{n-1}\sum X^2 - \frac{(\sum X)^2}{n(n-1)}} \\ &= \sqrt{\frac{1}{10-1} \times 247 - \frac{(47)^2}{10 \times 9}} \\ &= \sqrt{27.44 - 24.54} = 1.7 \end{aligned}$$

Thus, the standard deviation of the test scores on memory retention is 1.7. Looking to this value of standard deviation, no conclusion can be drawn as to whether the variation is less or more. It is so because standard deviation is considered to be the absolute variability. This problem can be solved by computing coefficient of variability. It will be discussed later in this chapter

**Effect of Change of Origin and Scale on Standard Deviation**

Let us see what happens to the standard deviation if the origin and scale of the scores are changed in the data set. Let the scores transformed by using the following transformation:

$$D = \frac{X - A}{i}$$

$$\Rightarrow \qquad X = A + i \times D$$

where "$A$" is origin and "$i$" is the scale. One can choose any value of origin, but the value of scale is usually the width of the class interval.

Taking summation on both side and dividing both sides by $n$, we get

$$\bar{X} = A + i \times \bar{D}$$

(This has been proved above in (2.3))

$$S_X = \sqrt{\frac{1}{n-1}\sum f(X - \bar{X})^2}$$

*Since* $X - \bar{X} = A + iD - (A + i\bar{D}) = i(D - \bar{D})$
Substituting the value of $X - \bar{X}$, we get

$$S_X = \sqrt{\frac{1}{n-1}\sum i^2 f\left(D - \overline{D}\right)^2} = i \times \sqrt{\frac{1}{n-1}\sum f\left(D - \overline{D}\right)^2}$$

$$\Rightarrow \qquad S_X = i \times S_D \tag{2.11}$$

**Table 2.6** Computation of standard deviation

| Class interval (price range in Rs.) | Frequency ($f$) | Midpoint ($X$) | $D = \frac{X-175.5}{50}$ | $fD$ | $fD^2$ |
|---|---|---|---|---|---|
| 1–50 | 10 | 25.5 | −3 | −30 | 90 |
| 51–100 | 6 | 75.5 | −2 | −12 | 24 |
| 101–150 | 4 | 125.5 | −1 | −4 | 4 |
| 151–200 | 4 | 175.5 | 0 | 0 | 0 |
| 201–250 | 2 | 225.5 | 1 | 2 | 2 |
| 251–300 | 2 | 275.5 | 2 | 4 | 8 |
| | $n = 28$ | | | −40 | 128 |

Thus, it may be concluded that the standard deviation is free from change of origin but is affected by the change scale.

Let us compute the standard deviation for the data shown in Table 2.1. Consider the same data in Table 2.6 once again. After computing the midpoints of the class intervals, let us transform the scores by taking the origin and scale as 175.5 and 50, respectively. Usually, origin ($A$) is taken as the midpoint of the middlemost class interval, and the scale ($h$) is taken as the width of the class interval. The origin $A$ is also known as assumed mean.

Here, width of the class interval $= h = 50$ and assumed mean $A = 175.5$.

From the equation (2.11), $S_X = i \times S_D = i \times \sqrt{\frac{1}{n-1} \sum f\left(D - \overline{D}\right)^2}$

After simplification,

$$S_X = i \times \sqrt{\frac{1}{n-1} \sum fD^2 - \frac{\left(\sum fD\right)^2}{n(n-1)}}$$

Substituting the values of $n$, $\sum fD$ and $\sum fD^2$, we get

$$S_X = 50 \times \sqrt{\frac{1}{28-1} \times 128 - \frac{(-40)^2}{28 \times 27}}$$
$$= 80.93$$

## *Variance*

The variance is the square of standard deviation. It can be defined as the average of the squared deviations of scores from their mean value. It also measures variation of

the scores in the distribution. It shows the magnitude of variation among the scores around its mean value. In other words, it measures the consistency of data. Higher variance indicates more heterogeneity, whereas lower variance represents more homogeneity in the data.

Like standard deviation, it also measures the variability of scores that are measured in interval or ratio scale. The variance is usually represented by $\sigma^2$ and is computed as

$$\sigma^2 = \frac{1}{N}\sum (X - \mu)^2 \tag{2.12}$$

The variance can be estimated from the sample by using the following formula:

$$\begin{aligned}\sigma^2 &= \frac{1}{n-1}\sum (X - \bar{X})^2 \\ &= \frac{1}{n-1}\sum X^2 - \frac{(\sum X)^2}{n(n-1)}\end{aligned}$$

**Remark** Population mean and population standard deviation are represented by $\mu$ and $\sigma$, respectively, whereas sample mean and sample standard deviation are represented by $\bar{X}$ and $S$, respectively.

## *The Index of Qualitative Variation*

Measures of variability like range, standard deviation, or variance are computed for interval or ratio data. What if the data is in nominal form? In social research, one may encounter many situations where it is required to measure the variability of the data based on nominal scale. For example, one may like to find the variability of ethnic population in a city, variation in the responses on different monuments, variability in the liking of different sports in an institution, etc. In all these situations, an index of qualitative variation (IQV) may be computed by the following formula to find the magnitude of variability:

$$\text{IQV} = \frac{K\left(100^2 - \sum P^2\right)}{100^2(K-1)} \tag{2.13}$$

where

$K$ = The number of categories
$\Sigma P^2$ = Sum of squared percentages of frequencies in all the groups

**Table 2.7** Frequency distribution of the students in different community

| S.N. | Community | No. of students | % of students ($P$) | $P^2$ |
|---|---|---|---|---|
| 1 | Hindu | 218 | 68.1 | 4637.61 |
| 2 | Muslim | 55 | 17.2 | 295.84 |
| 3 | Christian | 25 | 7.8 | 60.84 |
| 4 | Sikh | 10 | 3.1 | 9.61 |
| 5 | Others | 12 | 3.8 | 14.44 |
| | | | | $\sum P^2 = 5018.34$ |

The IQV is based on the ratio of the total number of differences in the distribution to the maximum number of possible differences within the same distribution. This IQV can vary from 0.00 to 1.00. When all the cases are in one category, there is no variation and the IQV is 0.00. On the other hand, if all the cases are equally distributed across the categories, there is maximum variation and the IQV is 1.00.

To show the computation process, consider an example where the number of students belonging to different communities were recorded as shown in Table 2.7

Here, we have $K$ = number of categories = 5:

$$\begin{aligned} \text{IQV} &= \frac{K\left(100^2 - \sum P^2\right)}{100^2(K-1)} \\ &= \frac{5 \times \left(100^2 - 5,018.34\right)}{100^2 \times (5-1)} = \frac{24,908.3}{40,000} \\ &= 0.62 \end{aligned}$$

By looking to the formula (2.13), you can see that the IQV is partially a function of the number of categories. Here, we used five categories of communities. Had we used more number of categories, the IQV would have been quite less, and, on the other hand, if the number of categories would have been less than the value of IQV, it would have been higher than what we are getting.

## Standard Error

If we draw $n$ samples from the same population and compute their means, then these means will not be the same but will differ with each other. The variation among these means is referred as the standard error of mean. Thus, the standard error of any statistic is the standard deviation of that statistic in the sampling distribution. Standard error measures the sampling fluctuation of any statistic and is widely used in statistical inference. The standard error gives a measure of how well a sample is true

representative of the population. When the sample is truly representing the population, the standard error will be small.

Constructing confidence intervals and testing of significance are based on standard errors. The standard error of mean can be used to compare the observed mean to a hypothesized value. The two values may be different at 5% level if the ratio of the difference to the standard error is less than $-2$ or greater than +2.

The standard error of any statistics is affected by the sample size. In general, the standard error decreases with the increase in sample size. It is denoted by $\sigma$ with a subscript of a statistic for which it is computed.

Let $\bar{X}_1, \bar{X}_2, \bar{X}_3, \ldots, \bar{X}_n$ are the means of $n$ samples drawn from the same population. Then the standard deviation of these $n$ mean scores is said to be standard error of mean. The standard error of sample mean can be estimated by even one sample. If any sample consists of $n$ scores with population standard deviation $\sigma$, then the standard error of the mean is given by

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \tag{2.14}$$

Whereas the standard error of the standard deviation is given by

$$\sigma_{\sigma} = \frac{\sigma}{\sqrt{2n}} \tag{2.15}$$

Like standard error of the mean, the standard error of the standard deviation also measures the fluctuation of standard deviations among the samples.

**Remark** If population standard deviation $\sigma$ is unknown, it may be estimated by the sample standard deviation $S$.

## Coefficient of Variation (CV)

Coefficient of variation is an index which measures the extent of variability in the data set in relation to its mean value. It is free from unit and compensates with the value of mean in the data set. Coefficient of variation is also known as relative variability and is denoted by CV

$$\text{CV} = \frac{S}{\bar{X}} \times 100 \tag{2.16}$$

where $S$ and $\bar{X}$ represent sample standard deviation and sample mean respectively. Since coefficient of variation measures the relative variability and computes the variability in percentage, it can be used to know whether a particular parameter is more variable or less variable. Coefficient of variation can be used for comparing the variability of two groups in a situation when their mean values are not equal.

It may also be used to compare the variability of two groups of data having different units.

On the other hand, standard deviation is a measure of absolute variability, and therefore, it cannot be used to assess the variability of any data set without knowing its mean value. Further, standard deviation cannot be used to compare the variability of two sets of scores if their mean value differs.

Consider the following statistics obtained on the number of customers visiting the retail outlets of a company in two different locations in a month. Let us see what conclusions can be drawn with this information.

| Location | A | B |
|---|---|---|
| Mean | 40 | 20 |
| SD | 8 | 6 |
| CV | 20% | 30% |

The standard deviation of the number of customers in location A is larger in comparison to location B, whereas coefficient of variation is larger in location B in comparison to location A. Thus, it may be inferred that the variation among the number of customers visiting the outlet in location B is higher than that of location A.

## Moments

A moment is a quantitative value that tells us the shape of a set of points. The moment can be central or noncentral. Central moment is represented by $\mu_r$, whereas noncentral moment is denoted by $\mu_r'$. If the deviation of scores is taken around mean, then the moment becomes central, and if it is taken around zero or any other arbitrary value, it is known as noncentral moment. The *r*th central moment is given by

$$\mu_r = \frac{1}{n} \sum (X - \bar{X})^r \tag{2.17}$$

Different moments convey different meanings. For instance, second central moment $\mu_2$ is always equal to variance of a distribution. Similarly second, third, and fourth moments are used to compute skewness and kurtosis of the data set. On the other hand, *r*th noncentral moment around the origin zero is denoted by

$$\mu_r' = \frac{1}{n} \sum X^r \tag{2.18}$$

The first noncentral moment $\mu_1^{'}$ about zero always represents mean of the distribution. These noncentral moments are used to compute central moments by means of a recurrence relation.

## Skewness

Skewness gives an idea about the symmetricity of the data. In symmetrical distribution if the curve is divided in the middle, the two parts become the mirror image of each other. If the curve is not symmetrical, it is said to be skewed. The skewness of the distribution is represented by $\beta_1$ and is given by

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \tag{2.19}$$

where $\mu_2$ and $\mu_3$ are the second and third central moments. For a symmetric distribution, $\beta_1$ is 0. A distribution is positively skewed if $\beta_1$ is positive and negatively skewed if it is negative. In a positively skewed distribution, the tail is heavy toward the right side of the curve, whereas in a negatively skewed curve, the tail is heavy toward the left side of the curve. Further, in positively skewed curve, median is greater than mode, whereas in negatively skewed curve, the median is less than mode. Graphically both these curves can be shown by Fig. 2.1a, b:

The standard error of the skewness is given by

$$\text{SE(Skewness)} = \text{SE}(\beta_1) = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}} \tag{2.20}$$

where $n$ is the sample size. Some authors use $\sqrt{6/n}$ for computing standard error of the skewness, but it is a poor approximation for the small sample.

The standard error of skewness can be used to test its significance. In testing the significance of skewness, the following $Z$ statistic is used which follows a normal distribution.

$$Z = \frac{\sqrt{n(n-1)}}{n-2} \times \frac{\beta_1}{\text{SE}(\beta_1)} \tag{2.21}$$

The critical value of $Z$ is approximately 2 (for a two-tailed test with roughly at 5% level). Thus, if calculated value of $Z < -2$, we may interpret that the population is

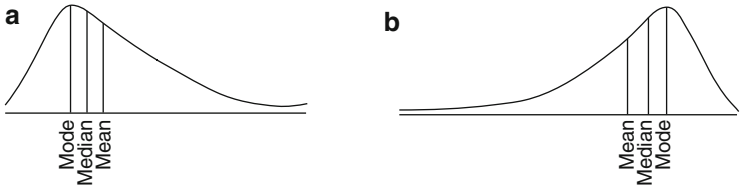



**Fig. 2.1** (**a** and **b**) Showing positively and negatively skewed curve

very likely to be skewed negatively. On the other hand, if calculated $Z > +2$, it may be concluded that the population is positively skewed.

However, in general, skewness values more than twice its standard error indicates a departure from symmetry. This gives a criterion to test whether skewness (positive or negative) in the distribution is significant or not. Thus, if the data is positively skewed, it simply means that majority of the scores are less than its mean value, and in case of negative skewness, most of the scores are more than its mean value.

## Kurtosis

Kurtosis is a statistical measure used for describing the distribution of observed data around the mean. It measures the extent to which the observations cluster around the mean value. It is measured by (Gamma) and is computed as

$$\gamma = \beta_2 - 3 \tag{2.22}$$

where $\beta_2 = \frac{\mu_4}{\mu_2^2}$, $\mu_2$ and $\mu_4$ represent the second and fourth central moments respectively.

For a normal distribution, the value of kurtosis $(\gamma)$ is zero. Positive value of kurtosis in a distribution indicates that the observations cluster more around its mean value and have longer tails in comparison to that of normal distribution, whereas a distribution with negative kurtosis indicates that the observations cluster less around its mean and have shorter tails.

Depending upon the value of kurtosis, the distribution of scores can be classified into any one of the three categories: leptokurtic, mesokurtic, and platykurtic.

If for any variable the kurtosis is positive, the curve is known as leptokurtic and it represents a low level of data fluctuation, as the observations cluster around the mean. On the other hand, if the kurtosis is negative, the curve is known as platykurtic and it means that the data has a larger degree of variance. In other words, if the value of kurtosis is significant and positive, it signifies less variability in the data set or we may say that the data is more homogenous. On the other hand, significant negative kurtosis indicates that there is more variability in the data set or we may conclude that the data is more heterogeneous. Further, if the kurtosis is 0,

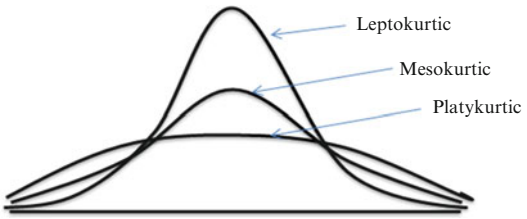



**Fig. 2.2** Classification of curve on the basis of kurtosis

the curve is classified as mesokurtic. Its flatness is equivalent to normal curve. Thus, a normal curve is always a mesokurtic curve. The three types of the curves are shown in Fig. 2.2

The standard error of kurtosis can be given by

$$\text{SE(Kurtosis)} = \text{SE}(\gamma) = 2\text{SE}(\beta_1)\sqrt{\frac{n^2-1}{(n-3)(n+5)}} \tag{2.23}$$

where $n$ is the sample size. Some author suggests the approximated formula for standard error of kurtosis as

$$\text{SE(Kurtosis)} = \sqrt{\frac{24}{n}} \tag{2.24}$$

but this formula is poor approximation for small samples.

The standard error of the kurtosis is used to test its significance. The test statistics $Z$ can be computed as follows:

$$Z = \frac{\gamma}{\text{SE}(\gamma)} \tag{2.25}$$

This $Z$ follows normal distribution. The critical value for $Z$ is approximately 2 for two-tailed test in testing the hypothesis that kurtosis $= 0$ at approximately 5% level.

If the value of calculated $Z$ is $< -2$, then the population is very likely to have negative kurtosis and the distribution may be considered as platykurtic. On the other hand, if the value of calculated $Z$ is $> +2$, then the population is very likely to have positive kurtosis and the distribution may be considered as leptokurtic.

## Percentiles

Percentiles are used to develop norms based on the performance of the subjects. A given percentile indicates the percentage of scores below it and is denoted by $P_X$. For example, $P_{40}$ is a score below which 40% scores lie. Median is also known as $P_{50}$, and it indicates that 50% scores lie below it. Percentiles can be computed to know the position of an individual on any parameter. For instance, 95th percentile obtained by a student in GMAT examination indicates that his performance is better than 95% of the students appearing in that examination.

Since 25th percentile $P_{25}$, 50th percentile $P_{50}$, and 75th percentile $P_{75}$ are also known as first, second, and third quartiles, respectively, hence procedure of computing other percentiles will be same as the procedure adopted in computing quartiles. Quartiles (the 25th, 50th, and 75th percentiles) divide the data into four groups of equal size. Percentiles at decile points and quartiles can be computed by using SPSS.

### *Percentile Rank*

A percentile rank can be defined as the percentage of scores that fall at or below a given score. Thus, if the percentile rank of a score $A$ is $X$, it indicates that $X$ percentage of scores lies below the score $A$. The percentile rank can be computed from the following formula:

$$\text{Percentile rank of the score } X = \frac{\text{CF} - 0.5 \times f_s}{n} \times 100 \tag{2.26}$$

where

CF: number of scores below $X$
$f_s$ : number of times the score $X$ occurs in the data set
$n$ : number of scores in the data set

## Situation for Using Descriptive Study

There may be varieties of situation where a descriptive study may be planned. One such situation has been discussed below to narrate the use of such study.

Nowadays Industries are also assuming social responsibilities toward society. They keep engage themselves in many of the social activities like adult literacy, slum development, HIV and AIDS program, community development, energy conservation drive, and go green campaign. One such organization has started its HIV and AIDS program in which it not only promotes the awareness but also provides treatments. This company provides antiretroviral therapy to anyone in the community who is a HIV-positive irrespective of whether that person is an employee of the company or not. The company also provides counseling, education, and training and disseminates information on nutrition, health, and hygiene. The target population of the company for this program is truck drivers, contract and migrant workers, employees of local organizations, and members of the local community. Descriptive study may be conducted to investigate the following issues:

(a) Number of programs organized in different sections of the society
(b) Number of people who attended the awareness program in different sections of the society

(c) Number of people who are affected with HIV/AIDS in different sections of the society
(d) The most vulnerable group affected by the HIV
(e) Details of population affected from HIV in different age and sex categories

To cater the above objectives, data may be processed as follows:

(i) Classify the data on HIV/AIDS-infected persons in different sections of the society like truck drivers, contract laborers, migrant's laborers, and local establishment members of the local community month wise in the last 5 years.
(ii) Classify the number of participants attending the HIV/AIDS awareness program in different sections of the society month wise in the last 5 years.
(iii) Compute the largest and smallest scores, mean, SD, coefficient of variation, standard error, skewness, kurtosis, and quartile deviation for the data in all the groups.

All these computations can be done by using SPSS, the procedure of which shall be explained later in this chapter by using the following example:

## Solved Example of Descriptive Statistics using SPSS

The procedure of computing various descriptive statistics including central tendency, dispersion, percentile values, and distribution parameters through SPSS has been explained in the solved Example 2.1.

**Example 2.1** In a study conducted by response of customers were obtained on various attributed of a company along with their satisfaction level. Apply descriptive analysis to compute various statistics and explain the findings (Table 2.8).

*Solution* In order to compute various descriptive statistics like mean, median, mode, SD, variance, skewness, SE of skewness, kurtosis, SE of kurtosis, range, minimum and maximum scores, and percentiles, a data file shall be made in SPSS and then steps shown below shall be followed to get the output. After getting the output, its interpretation shall be made.

### *Computation of Descriptive Statistics Using SPSS*

(a) ***Preparing Data File***
In order to use SPSS for computing descriptive statistics, a data file needs to be prepared. The data file can also be imported in SPSS from the ASCII or Excel files. The readers are advised to go through the first chapter of the book to learn

for starting the SPSS on the system, preparing the data file, and importing the file in SPSS from other sources. The following steps will help you to prepare the data file:

(i) *Starting the SPSS:* Use the following command sequence to start SPSS on your system:
**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**
After clicking **Type in Data** option, you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables*: In this example, there are eight variables that need to be defined along with their properties. Do the following:

1. Click **Variable View** in the left corner of the bottom of the screen to define variables and their properties.
2. Write short name of all the eight variables as *Del_Speed, Price_Lev, Price_Fle, Manu_Ima, Service, Salfor_Ima, Prod_Qua, and Sat_Lev* under the column heading **Name**.
3. Under the column heading **Label**, full name of these variables may be defined as *Delivery Speed, Price Level, Price Flexibility, Manufacturer Image, Service, Salesforce Image, Product Quality,* and *Satisfaction Level*.
4. Since all the variables were measured on an interval scale, hence select the option "Scale" under the heading **Measure** for each variable.
5. Use default entries in rest of the columns.

After defining variables in **Variable View**, the screen shall look like Fig. 2.3.

(iii) *Entering data*: After defining all the eight variables in the **Variable View**, click **Data View** on the left bottom of the screen to open the format for entering the data column wise. For each variable, enter the data column wise. After entering the data, the screen will look like Fig. 2.4. Save the data file in the desired location before further processing.

(b) **SPSS *Commands for Descriptive Analysis***
After entering the data in data view, do the following steps for computing desired descriptive statistics:

(i) *SPSS commands for descriptive statistics:* In data view, click the following commands in sequence:
**Analyze ⇨ Descriptive Statistics ⇨ Frequencies**
The screen shall look like as shown in Fig. 2.5.

(ii) *Selecting variables for computing descriptive statistics*: After clicking the **Frequencies** tag, you will be taken to the next screen for selecting variables

**Table 2.8** Response of customers on company's attributes

| S. N. | Delivery speed ($X_1$) | Price level ($X_2$) | Price flexibility ($X_3$) | Manufacturer image ($X_4$) | Service ($X_5$) | Salesforce image ($X_6$) | Product quality ($X_7$) | Satisfaction level ($X_8$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 4.1 | 0.6 | 6.9 | 4.7 | 2.4 | 2.3 | 5.2 | 4.2 |
| 2 | 1.8 | 3.0 | 6.3 | 6.6 | 2.5 | 4.0 | 8.4 | 4.3 |
| 3 | 3.4 | 5.2 | 5.7 | 6.0 | 4.3 | 2.7 | 8.2 | 5.2 |
| 4 | 2.7 | 1.0 | 7.1 | 5.9 | 1.8 | 2.3 | 7.8 | 3.9 |
| 5 | 6.0 | 0.9 | 9.6 | 7.8 | 3.4 | 4.6 | 4.5 | 6.8 |
| 6 | 1.9 | 3.3 | 7.9 | 4.8 | 2.6 | 1.9 | 9.7 | 4.4 |
| 7 | 4.6 | 2.4 | 9.5 | 6.6 | 3.5 | 4.5 | 7.6 | 5.8 |
| 8 | 1.3 | 4.2 | 6.2 | 5.1 | 2.8 | 2.2 | 6.9 | 4.3 |
| 9 | 5.5 | 1.6 | 9.4 | 4.7 | 3.5 | 3.0 | 7.6 | 5.4 |
| 10 | 4.0 | 3.5 | 6.5 | 6.0 | 3.7 | 3.2 | 8.7 | 5.4 |
| 11 | 2.4 | 1.6 | 8.8 | 4.8 | 2.0 | 2.8 | 5.8 | 4.3 |
| 12 | 3.9 | 2.2 | 9.1 | 4.6 | 3.0 | 2.5 | 8.3 | 5.0 |
| 13 | 2.8 | 1.4 | 8.1 | 3.8 | 2.1 | 1.4 | 6.6 | 4.4 |
| 14 | 3.7 | 1.5 | 8.6 | 5.7 | 2.7 | 3.7 | 6.7 | 5.0 |
| 15 | 4.7 | 1.3 | 9.9 | 6.7 | 3.0 | 2.6 | 6.8 | 5.9 |
| 16 | 3.4 | 2.0 | 9.7 | 4.7 | 2.7 | 1.7 | 4.8 | 4.7 |
| 17 | 3.2 | 4.1 | 5.7 | 5.1 | 3.6 | 2.9 | 6.2 | 4.4 |
| 18 | 4.9 | 1.8 | 7.7 | 4.3 | 3.4 | 1.5 | 5.9 | 5.6 |
| 19 | 5.3 | 1.4 | 9.7 | 6.1 | 3.3 | 3.9 | 6.8 | 5.9 |
| 20 | 4.7 | 1.3 | 9.9 | 6.7 | 3.0 | 2.6 | 6.8 | 6.0 |

for which descriptive statistics need to be computed. The screen shall look like as shown in Fig. 2.6. Do the following:

– Select the variables *Del_Speed, Price_Lev, Price_Fle, Manu_Ima, Service, Salfor_Ima, Prod_Qua,* and *Sat_Lev* from the left panel to the "Variable(s)" section of the right panel.

Here, all the eight variables can be selected one by one or all at once. To do so, the variable(s) needs to be selected from the left panel, and by arrow command, it may be brought to the right panel. The screen shall look like Fig. 2.6.

(iii) *Selecting option for computation:* After selecting the variables, options need to be defined for the computation of desired statistics. Do the following:

– Click the option **Statistics** on the screen as shown in Fig. 2.6. This will take you to the next screen that is shown in Fig. 2.7. Do the following:

  – Check the options "Quartiles" and "Cut points for *10* equal groups" in "Percentile Values" section.
  – Check the option "Mean," "Median," and "Mode" under "Central Tendency" section.
  – Check the option "Std. Deviation," "Variance," "Range," "Minimum," "Maximum," "Range," and "S.E. mean" under "Dispersion" section.

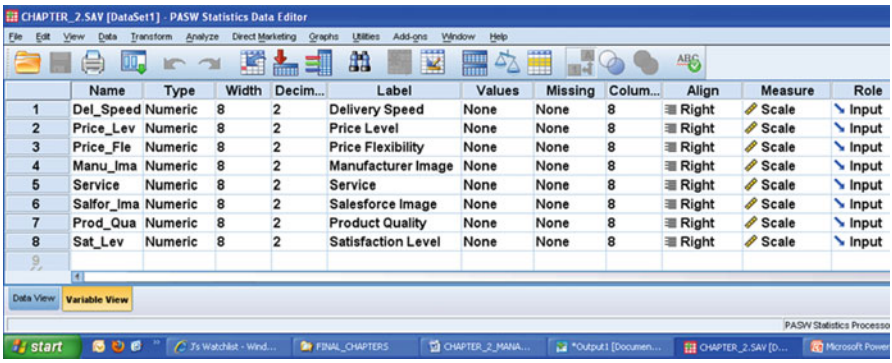



**Fig. 2.3** Defining variables along with their characteristics

– Check the option "Skewness" and "Kurtosis" under "Distribution" section.
– Click **Continue** for getting back to the screen shown in Fig. 2.6.

**Remarks**

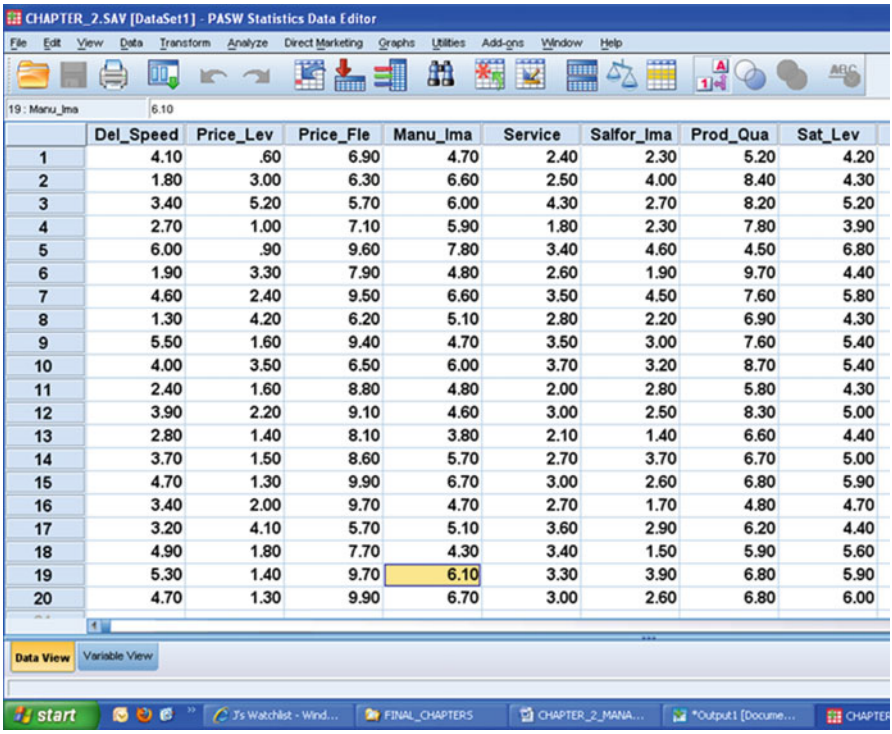
(a) You have four different classes of statistics like "Percentile Value," "Central Tendency," "Dispersion," and "Distribution" that can be computed. Any or all the options may be selected under these categories. Under the category "Percentile Values," quartiles can be checked (√) for computing $Q_1$ and $Q_3$. For computing percentiles at deciles points, cut points can be selected for 10 equal groups. Similarly, if the percentiles are required to be computed in the interval of 5, cut points may be selected as 5.

(b) In using the option cut points for the percentiles, output contains some additional information on frequency in different segments. If the researcher is interested, the same may be incorporated in the findings; otherwise, it may be ignored.

(c) *"Percentile"* option is selected if percentile values at different intervals are required to be computed. For example, if we are interested in computing $P_4$, $P_{16}$, $P_{27}$, and $P_{39}$, then these numbers are added in the "Percentile(s)" option.

(d) In this problem, only quartiles and cut points for "10" options have been checked under the heading "Percentile Values," whereas under the heading "Central Tendency," "Dispersion," and "Distribution," all the options have been checked.

(iv) *Option for graph:* The option **Chart** can be clicked in Fig. 2.6 if graph is required to be constructed. Any one of the option under this tag like bar charts, pie charts, or histograms may be selected. If no chart is required, then option "None" may be selected.




| | Del_Speed | Price_Lev | Price_Fle | Manu_Ima | Service | Salfor_Ima | Prod_Qua | Sat_Lev |
|---|---|---|---|---|---|---|---|---|
| 1 | 4.10 | .60 | 6.90 | 4.70 | 2.40 | 2.30 | 5.20 | 4.20 |
| 2 | 1.80 | 3.00 | 6.30 | 6.60 | 2.50 | 4.00 | 8.40 | 4.30 |
| 3 | 3.40 | 5.20 | 5.70 | 6.00 | 4.30 | 2.70 | 8.20 | 5.20 |
| 4 | 2.70 | 1.00 | 7.10 | 5.90 | 1.80 | 2.30 | 7.80 | 3.90 |
| 5 | 6.00 | .90 | 9.60 | 7.80 | 3.40 | 4.60 | 4.50 | 6.80 |
| 6 | 1.90 | 3.30 | 7.90 | 4.80 | 2.60 | 1.90 | 9.70 | 4.40 |
| 7 | 4.60 | 2.40 | 9.50 | 6.60 | 3.50 | 4.50 | 7.60 | 5.80 |
| 8 | 1.30 | 4.20 | 6.20 | 5.10 | 2.80 | 2.20 | 6.90 | 4.30 |
| 9 | 5.50 | 1.60 | 9.40 | 4.70 | 3.50 | 3.00 | 7.60 | 5.40 |
| 10 | 4.00 | 3.50 | 6.50 | 6.00 | 3.70 | 3.20 | 8.70 | 5.40 |
| 11 | 2.40 | 1.60 | 8.80 | 4.80 | 2.00 | 2.80 | 5.80 | 4.30 |
| 12 | 3.90 | 2.20 | 9.10 | 4.60 | 3.00 | 2.50 | 8.30 | 5.00 |
| 13 | 2.80 | 1.40 | 8.10 | 3.80 | 2.10 | 1.40 | 6.60 | 4.40 |
| 14 | 3.70 | 1.50 | 8.60 | 5.70 | 2.70 | 3.70 | 6.70 | 5.00 |
| 15 | 4.70 | 1.30 | 9.90 | 6.70 | 3.00 | 2.60 | 6.80 | 5.90 |
| 16 | 3.40 | 2.00 | 9.70 | 4.70 | 2.70 | 1.70 | 4.80 | 4.70 |
| 17 | 3.20 | 4.10 | 5.70 | 5.10 | 3.60 | 2.90 | 6.20 | 4.40 |
| 18 | 4.90 | 1.80 | 7.70 | 4.30 | 3.40 | 1.50 | 5.90 | 5.60 |
| 19 | 5.30 | 1.40 | 9.70 | 6.10 | 3.30 | 3.90 | 6.80 | 5.90 |
| 20 | 4.70 | 1.30 | 9.90 | 6.70 | 3.00 | 2.60 | 6.80 | 6.00 |

**Fig. 2.4** Screen showing entered data for all the variables in the data view

– Press **O.K.** for output.

(c) ***Getting the Output***
Clicking the option ***OK*** will lead you to the output window. The output panel shall have lots of results. It is up to the researcher to select the relevant outputs in their results. In the output window of the SPSS, the relevant output can be selected by pressing the right click of the mouse over it and may be copied in the word file. In this example, the output generated will look like as shown in Table 2.9

## *Interpretation of the Outputs*

Different interpretations can be made from the results in Table 2.9. However, some of the important findings that can be drawn are as follows:

1. Except price level, mean and median of all the variables are nearly equal.

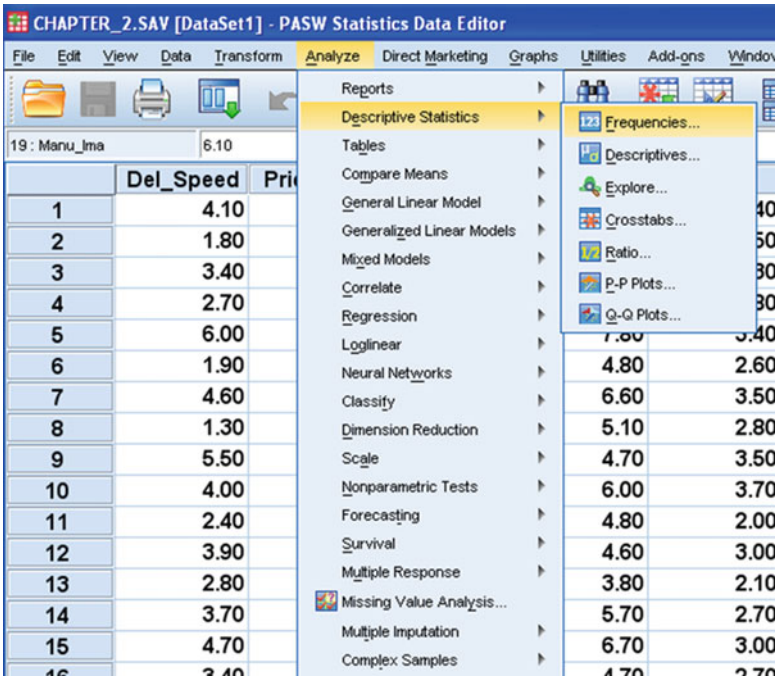



**Fig. 2.5** Screen showing the SPSS commands for computing descriptive statistics

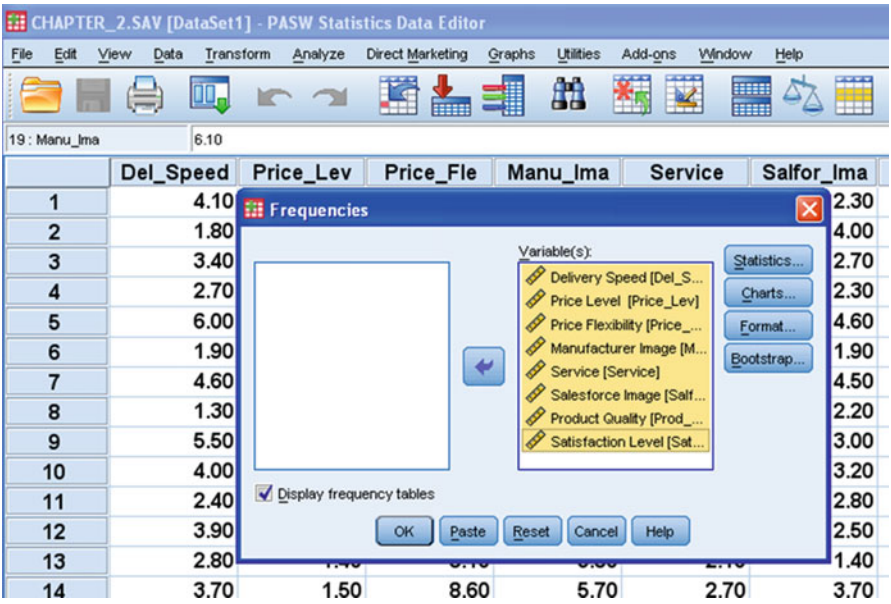



**Fig. 2.6** Screen showing selection of variables for descriptive analysis

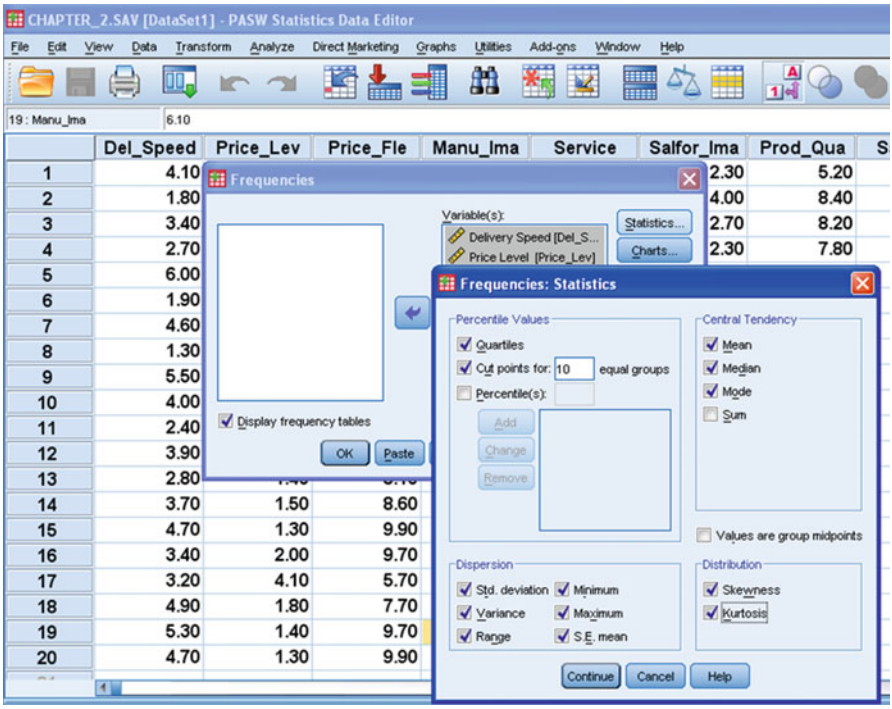



**Fig. 2.7** Screen showing option for different statistics to be computed

2. Standard error of mean is least for the service whereas maximum for the price flexibility.
3. As a guideline, a skewness value more than twice its standard error indicates a departure from symmetry. Since none of the variable's skewness is greater than twice its standard error ($2 \times .512$) hence all the variables are symmetrically distributed.
4. SPSS uses the statistic $\beta_2 - 3$ for kurtosis. Thus, for a normal distribution, kurtosis value is 0. If for any variable the value of kurtosis is positive, its distribution is known as leptokurtic, which indicates low level of data fluctuation around its mean value, whereas negative value of kurtosis indicates large degree of variance among the data and the distribution is known as platykurtic.

   Since the value of kurtosis for any of the variable is not more than twice its standard error of kurtosis hence none of the kurtosis values are significant. In other words the distribution of all the variables is mesokurtic.
5. Minimum and maximum values of the parameter can give some interesting facts and provide the range of variation. For instance, delivery speed of the products is in the range of 1.3–6 days. Thus, one can expect the delivery of any product in at the most 6 days time, and accordingly, one may try to place the order.

**Table 2.9** Output showing various statistics for different attributes of the company

| | | Del_Speed | Price_Lev | Price_Fle | Manu_Ima | Service | Salfor_Ima | Prod_Qua | Sat_Lev |
|---|---|---|---|---|---|---|---|---|---|
| N Valid | | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| Missing | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Mean | | 3.7150 | 2.2150 | 8.1150 | 5.5350 | 2.9650 | 2.8150 | 6.9650 | 5.0450 |
| SE of mean | | .29094 | .28184 | .33563 | .22933 | .14203 | .20841 | .30177 | .17524 |
| Median | | 3.8000 | 1.7000 | 8.3500 | 5.4000 | 3.0000 | 2.6500 | 6.8000 | 5.0000 |
| Mode | | 3.40[a] | 1.30[a] | 5.70[a] | 4.70 | 3.00 | 2.30[a] | 6.80 | 4.30[a] |
| Std. deviation | | 1.30112 | 1.26044 | 1.50097 | 1.02561 | .63518 | .93205 | 1.34957 | .78370 |
| Variance | | 1.693 | 1.589 | 2.253 | 1.052 | .403 | .869 | 1.821 | .614 |
| Skewness | | −.144 | .970 | −.338 | .380 | .010 | .459 | .001 | .490 |
| SE of skewness | | .512 | .512 | .512 | .512 | .512 | .512 | .512 | .512 |
| Kurtosis | | −.732 | .092 | −1.435 | −.467 | −.288 | −.500 | −.324 | −.566 |
| SE of kurtosis | | .992 | .992 | .992 | .992 | .992 | .992 | .992 | .992 |
| Range | | 4.70 | 4.60 | 4.20 | 4.00 | 2.50 | 3.20 | 5.20 | 2.90 |
| Minimum | | 1.30 | .60 | 5.70 | 3.80 | 1.80 | 1.40 | 4.50 | 3.90 |
| Maximum | | 6.00 | 5.20 | 9.90 | 7.80 | 4.30 | 4.60 | 9.70 | 6.80 |
| Sum | | 74.30 | 44.30 | 162.30 | 110.70 | 59.30 | 56.30 | 139.30 | 100.90 |
| Percentiles | 10 | 1.8100 | .9100 | 5.7500 | 4.3300 | 2.0100 | 1.5200 | 4.8400 | 4.2100 |
| | 20 | 2.4600 | 1.3000 | 6.3400 | 4.7000 | 2.4200 | 1.9600 | 5.8200 | 4.3000 |
| | 25 | 2.7250 | 1.3250 | 6.6000 | 4.7000 | 2.5250 | 2.2250 | 5.9750 | 4.3250 |
| | 30 | 2.9200 | 1.4000 | 6.9600 | 4.7300 | 2.6300 | 2.3000 | 6.3200 | 4.4000 |
| | 40 | 3.4000 | 1.5400 | 7.7800 | 4.9200 | 2.7400 | 2.5400 | 6.7400 | 4.5200 |
| | 50 | 3.8000 | 1.7000 | 8.3500 | 5.4000 | 3.0000 | 2.6500 | 6.8000 | 5.0000 |
| | 60 | 4.0600 | 2.1200 | 8.9800 | 5.9600 | 3.1800 | 2.8600 | 7.3200 | 5.3200 |
| | 70 | 4.6700 | 2.8200 | 9.4700 | 6.0700 | 3.4000 | 3.1400 | 7.7400 | 5.5400 |
| | 75 | 4.7000 | 3.2250 | 9.5750 | 6.4750 | 3.4750 | 3.5750 | 8.1000 | 5.7500 |
| | 80 | 4.8600 | 3.4600 | 9.6800 | 6.6000 | 3.5000 | 3.8600 | 8.2800 | 5.8800 |
| | 90 | 5.4800 | 4.1900 | 9.8800 | 6.7000 | 3.6900 | 4.4500 | 8.6700 | 5.9900 |

[a]Multiple modes exist. The smallest value is shown

*Del_Speed* delivery speed, *Price_Lev* price level, *Price_Fle* price flexibility, *Manu_Ima* manufacturer image, *Service* service, *Salfor_Ima* salesforce image, *Prod_Qua* product quality, *Sat_Lev* satisfaction level

6. Similarly, price flexibility of any product is in the range of 5.7–9.9%. This provides a feedback to the customers in taking a decision of buying an article in case of urgency.
7. Percentile scales can be used to draw various conclusions about different parameters. For instance, $P_{40}$ for the delivery speed is 3.40, which indicates that 40% customers get their product delivered in less than 3.4 days.

## Developing Profile Chart

In a descriptive study, a researcher generally computes different statistics that are described in Table 2.9. Based on these computations, meaningful interpretations can be made as shown above in the example. However, it would be more interesting to prepare a profile of the company using all its parameters investigated in the survey. The procedure of making a profile chart shall be explained by using the minimum score, maximum score, mean, and standard deviation of all the parameters shown in Table 2.9.

After manipulating data as per the following steps, the graphical functionality of Excel can be used to prepare the graphical profile of the company's parameters:

Step 1: Segregate the statistics like minimum score, maximum score, mean, and standard deviation of all the parameters in Table 2.9. The same has been shown in Table 2.10.

Step 2: Convert minimum and maximum scores for each of the variables into its standard scores by using the following transformation:

$$Z = \frac{X - \bar{X}}{S}$$

Thus, mean of all the variables will become same. The values so obtained are shown in Table 2.11.

Step 3: Convert these $Z$ values into its linear transformed scores by using the transformation $Z_l = 50 + 10 \times Z$. By using this transformation, the negative values of $Z$-scores can be converted into positive scores. Descriptive statistics shown in the form of linearly transformed scores are shown Table 2.12.

**Table 2.10** Selected descriptive statistics of all the variables

| Variables | Min | Max | Mean | S.D. |
|---|---|---|---|---|
| Delivery speed | 1.3 | 6.00 | 3.715 | 1.30 |
| Price level | 0.60 | 5.20 | 2.21 | 1.26 |
| Price flexibility | 5.70 | 9.90 | 8.12 | 1.50 |
| Manufacturer image | 3.80 | 7.80 | 5.54 | 1.03 |
| Service | 1.80 | 4.30 | 2.97 | 0.64 |
| Salesforce image | 1.40 | 4.60 | 2.82 | 0.93 |
| Product quality | 4.50 | 9.70 | 6.97 | 1.35 |
| Satisfaction level | 3.90 | 6.80 | 5.05 | 0.78 |

**Table 2.11** Standard scores of minimum, maximum, and average of all the variables

| | Min (*Z*) | Mean (*Z*) | Max (*Z*) |
|---|---|---|---|
| Delivery speed | −1.86 | 0 | 1.76 |
| Price level | −1.28 | 0 | 2.37 |
| Price flexibility | −1.61 | 0 | 1.19 |
| Manufacturer image | −1.69 | 0 | 2.19 |
| Service | −1.83 | 0 | 2.08 |
| Salesforce image | −1.53 | 0 | 1.91 |
| Product quality | −1.83 | 0 | 2.02 |
| Satisfaction level | −1.47 | 0 | 2.24 |

**Table 2.12** Transformed standard scores of minimum, maximum, and average of all the variables

| | Min | Mean | Max |
|---|---|---|---|
| Delivery speed | 31.4 | 50 | 67.6 |
| Price level | 37.2 | 50 | 73.7 |
| Price flexibility | 33.9 | 50 | 61.9 |
| Manufacturer image | 33.1 | 50 | 71.9 |
| Service | 31.7 | 50 | 70.8 |
| Salesforce image | 34.7 | 50 | 69.1 |
| Product quality | 31.7 | 50 | 70.2 |
| Satisfaction level | 45.3 | 50 | 72.4 |

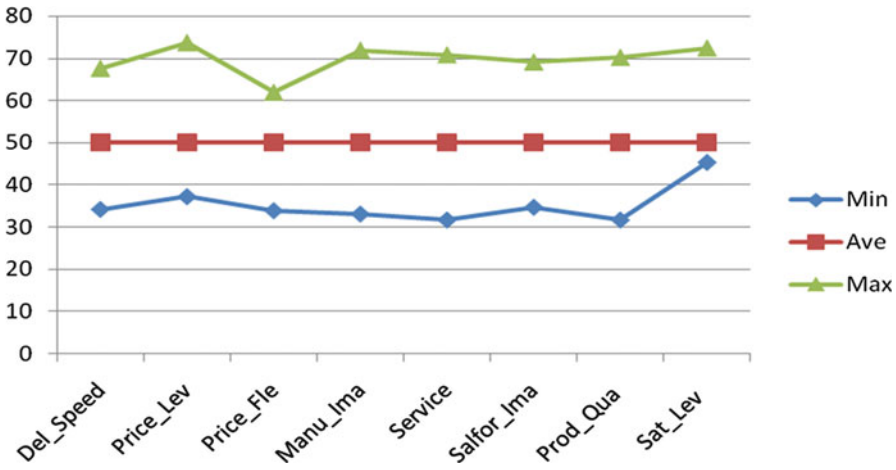



**Fig. 2.8** Profile chart of the company's performance indicators

Step 4: Use Excel graphic functionality for developing line diagram to show the company's profile on its various parameters. The profile chart so prepared is shown in Fig. 2.8.

## Summary of the SPSS Commands

1. Start SPSS by using the following commands:
   **Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

2. Click **Variable View** tag and define the variables *Del_Speed, Price_Lev, Price_Fle, Manu_Ima, Service, Salfor_Ima, Prod_Qua, and Sat_Lev* as a scale variable.
3. Once the variables are defined, then type the data for these variables by clicking **Data View**.
4. In the data view, follow the below-mentioned command sequence for computing descriptive statistics:

   **Analyze → Descriptive Statistics → Frequencies**
5. Select all the variables from left panel to the right panel for computing various descriptive statistics.
6. Click the tag **Statistics** and check the options under the headings "Percentile Values," "Central Tendency," "Dispersion," and "Distribution." Press ***Continue***.
7. Click the **Charts** option and select the required chart, if graph is required for all the variables.
8. Click **OK** to get the output for descriptive statistics.

## Exercise

*Short-Answer Questions*

**Note**: Write answer to each of the following questions in not more than 200 words:

Q.1. If average performance of two groups is equal, can it be said that both the groups are equally good?

Q.2. What do you mean by absolute and relative variability? Explain by means of examples.

Q.3. What is coefficient of variation? In what situation it should be computed? With the help of the following data on BSE quote during last trading sessions, can it be concluded that the group WIPRO's quotes were more variable than GAIL?

| | Group WIPRO | Group GAIL |
|---|---|---|
| Mean | 5400 | 170 |
| SD | 200 | 40 |

Q.4. Is there any difference between standard error of mean and error in computing the mean? Explain your answer.

Q.5. If skewness of a set of data is zero, can it be said to be is normally distributed? If yes, how? And if no, how it can be checked for its normality?

Q.6. If performance of a student is 96th percentile in a particular subject, can it be concluded that he is very intelligent in that subject? Explain your answer.

Q.7. What is a quartile measure? In what situation it should be used?

### *Multiple-Choice Questions*

**Note**: Question no. 1–10 has four alternative answers for each question. Tick mark the one that you consider the closest to the correct answer.

1. If a researcher is interested to know the number of employees in an organization belonging to different regions and how many of them have opted for club memberships, the study may be categorized as

   (a) Descriptive
   (b) Inferential
   (c) Philosophical
   (d) Descriptive and inferential both

2. Choose the correct sequence of commands to compute descriptive statistics.

   (a) Analyze -> Descriptive Statistics -> Frequencies
   (b) Analyze -> Frequencies -> Descriptive Statistics
   (c) Analyze -> Frequencies
   (d) Analyze -> Descriptive Statistics

3. Which pair of statistics are nonparametric statistics?

   (a) Mean and median
   (b) Mean and SD
   (c) Median and SD
   (d) Median and Q.D.

4. Standard error of mean can be defined as

   (a) Error in computing mean
   (b) Difference in sample and population mean
   (c) Variation in the mean values among the samples drawn from the same population
   (d) Error in measuring the data on which mean is computed

5. The value of skewness for a given set of data shall be significant if

   (a) Skewness is more than twice its standard error.
   (b) Skewness is more than its standard error.
   (c) Skewness and standard error are equal.
   (d) Skewness is less than its standard error.

6. Kurtosis in SPSS is assessed by

   (a) $\beta_2$
   (b) $\beta_2 + 3$
   (c) $\beta_2 - 3$
   (d) $2 + \beta_2$

7. In order to prepare the profile chart, minimum scores for each variable are converted into

   (a) Percentage
   (b) Standard score
   (c) Percentile score
   (d) Rank

8. While selecting option for percentile in SPSS, cut points are used for

   (a) Computing $Q_1$ and $Q_3$
   (b) Preparing the percentile at deciles points only
   (c) Cutting $Q_1$ and $Q_3$
   (d) Computing the percentiles at fixed interval points

9. If IQ of a group of students is positively skewed, what conclusions could be drawn?

   (a) Most of the students are less intelligent.
   (b) Most of the students are more intelligent.
   (c) There are equal number of high and low intelligent students.
   (d) Nothing can be said about the intelligence of the students.

10. If the data is platykurtic, what can be said about its variability?

    (a) More variability exists.
    (b) Less variability exists.
    (c) Variability is equivalent to normal distribution.
    (d) Nothing can be said about the variability.

*Assignment*

1. Following table shows the data on different abilities of employees in an organization. Compute various descriptive statistics and interpret its findings.

Data on different abilities of employees

| Define problems | Supervise others | Make decisions | Build consensus | Facilitate decision-making | Work on a team |
|---|---|---|---|---|---|
| .81 | .84 | .80 | .89 | .79 | .72 |
| .45 | .31 | .29 | .37 | .21 | .12 |
| .87 | .79 | .90 | .88 | .67 | .50 |
| .78 | .71 | .84 | .92 | .82 | .62 |
| .65 | .59 | .72 | .85 | .81 | .56 |
| .56 | .55 | .62 | .71 | .73 | .61 |

2. Following are the grades of ten MBA students in 10 courses. Compute various descriptive statistics and interpret your findings.

Data on grades of MBA students in ten courses

| S.N. | FACTG | MACTG | ECON | FIN | MKTG | ENVIR | MIS | QM | OPSM | OB |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 7 | 1 | 7 | 6 | 6 | 6 | 7 | 5 | 5 | 6 |
| 2 | 7 | 6 | 7 | 7 | 6 | 5 | 6 | 7 | 4 | 7 |
| 3 | 3 | 6 | 6 | 6 | 6 | 4 | 5 | 4 | 6 | 7 |
| 4 | 8 | 7 | 8 | 6 | 7 | 8 | 7 | 8 | 9 | 6 |
| 5 | 5 | 3 | 5 | 7 | 5 | 8 | 6 | 6 | 4 | 8 |
| 6 | 3 | 3 | 3 | 3 | 3 | 5 | 6 | 7 | 7 | 7 |
| 7 | 4 | 7 | 6 | 5 | 8 | 6 | 5 | 4 | 4 | 6 |
| 8 | 5 | 6 | 8 | 7 | 6 | 7 | 8 | 7 | 5 | 5 |
| 9 | 6 | 5 | 7 | 8 | 5 | 6 | 5 | 8 | 7 | 7 |
| 10 | 7 | 6 | 5 | 8 | 6 | 4 | 8 | 6 | 7 | 8 |

*FACTG* financial accounting for managers, *MACTG* management accounting, *ECON* economic environment of business, *FINE* managerial finance, *MKTG* marketing management, *NVIR* business environment, *MIS* management information systems, *QM* quantitative methods, *OPSM* operations management, *OB* organizational behavior

## Answers of Multiple-Choice Questions

| | | | |
|---|---|---|---|
| Q.1 | a | Q.2 | a |
| Q.3 | d | Q.4 | c |
| Q.5 | a | Q.6 | c |
| Q.7 | b | Q.8 | d |
| Q.9 | a | Q.10 | a |

# Chapter 3
# Chi-Square Test and Its Application

**Learning Objectives**

After completing this chapter you should be able to do the following:

- Know the use of chi-square in analyzing nonparametric data.
- Understand the application of chi-square in different research situations.
- Know the advantages of crosstabs analysis.
- Learn to construct the hypothesis in applying chi-square test.
- Explain the situations in which different statistics like contingency coefficient, lambda coefficient, phi coefficient, gamma, Cramer's V, and Kendall tau, for measuring an association between two attributes, can be used.
- Learn the procedure of data feeding in preparing the data file for analysis using SPSS.
- Describe the procedure of testing an equal occurrence hypothesis and testing the significance of an association in different applications by using SPSS.
- Interpret the output of chi-square analysis generated in SPSS.

## Introduction

In survey research, mainly two types of hypothesis are tested. One may test goodness of fit for a single attribute or may like to test the significance of association between any two attributes. To test an equal occurrence hypothesis, it is required to tabulate the observed frequency for each variable. The chi-square statistic in "nonparametric" section of SPSS may be used to test the hypothesis of equal occurrence.

The scores need to be arranged in contingency table for studying an association between any two attributes. A contingency table is the arrangement of frequency in rows and column. The process of creating a contingency table from the observed frequency is known as crosstab. The cross tabulation procedure provides tabulation of two variables in two-way table. A frequency distribution provides the distribution of one variable, whereas a contingency table describes the distribution of two or more variables simultaneously (Table 3.1).

J.P. Verma, *Data Analysis in Management with SPSS Software*,
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**Table 3.1** Preferences of male and female towards different incentives

| | | Incentives | | |
|---|---|---|---|---|
| | | Gift check (%) | Cash (%) | Gift article (%) |
| Gender | Male | 30 | 45 | 25 |
| | Female | 10 | 30 | 60 |

The following is an example of a 2 × 3 contingency table. The first variable "gender" has two options, male and female, whereas the second variable "incentives" has three options, gift check, cash, and gift article. Each cell gives the number of individuals who share the combination of traits.

The chi-square can be computed by using the Crosstabs option in "Descriptive Statistics" section of SPSS command. Besides chi-square, the Crosstabs option in SPSS provides the output showing magnitude of association along with summary statistics including percentage of frequency and expected frequency in each cell.

Two-way tabulation in Crosstabs can be used to establish an interdependent relationship between two tables of values but does not identify a causal relation between the values. Cross tabulation technique can be used to analyze the results of a survey study, for example, it may indicate a preference for certain types of jobs based on the respondent's gender.

## Advantages of Using Crosstabs

1. Crosstabs analysis is easy to understand and is good for the researchers, who do not want to use more sophisticated statistical techniques.
2. Crosstabs treats all data as nominal. In other words, the data is treated as nominal even if it is measured in interval, ratio, or ordinal form.
3. A table is more explanatory than a single statistics.
4. They are simple to conduct.

## Statistics Used in Cross Tabulations

In Crosstabs analysis, usually statistics like chi-square, contingency coefficient, lambda coefficient, phi coefficient, Kendall tau, gamma, or Cramer's V are used. These shall be discussed below:

### *Chi-Square Statistic*

If $X_1, \ldots, X_n$ are independent and identical $N(\mu, \sigma^2)$ random variables, then the statistics $\sum_{i=1}^{n} \left(\frac{X_i - \mu_.}{\sigma}\right)^2$ follows the chi-square distribution with $(n-1)$ degrees of freedom and is written as

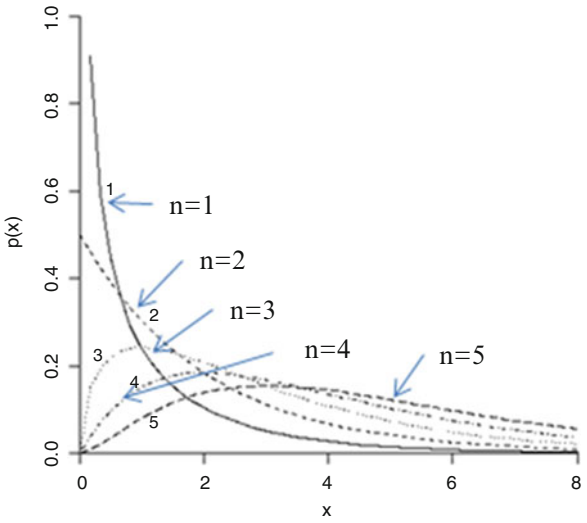



**Fig. 3.1** Probability distribution of chi-square for different degrees of freedom

$$\sum_{i=1}^{n}\left(\frac{X_i-\mu}{\sigma}\right)^2 \sim \chi^2(n-1)$$

The probability density function of the chi-square ($\chi^2$) random variable is

$$f(x)=\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2} \tag{3.1}$$

$$x>0 \text{ and } n=1,2,3,\ldots$$

The mean and variance of the chi-square statistics are $n$ and $2n$, respectively. The $\chi^2$ distribution is not unique but depends upon degrees of freedom. The family of distribution with varying degrees of freedom is shown in Fig. 3.1.

### Additive Properties of Chi-Square

If $\chi_1^2$ and $\chi_2^2$ are two independent chi-square variates with $n_1$ and $n_2$ degrees of freedom, respectively, then $\chi_1^2+\chi_2^2$ is also a chi-square variate with $n_1+n_2$ degrees of freedom. This property is used extensively in the questionnaire studies. Consider a study to compare the attitude of male and female consumers about a particular brand of car. The questionnaire may consist of questions under three factors, namely, financial consideration, driving comforts, and facilities. Each of these factors may have several questions. On each of the questions, attitude of male and female users may be compared using chi-square. Further, by using additive

properties, the chi-square of each question under a particular factor may be added to compare the attitude of male and female on that factor.

## *Chi-Square Test*

Chi-square test is the most frequently used nonparametric statistical test. It is also known as Pearson chi-square test and provides us the mechanism to test the independence of two categorical variables. The chi-square test is based upon a chi-square distribution just like the way a *t*-test is based upon *t*-distribution or an *F*-test is based upon an *F*-distribution. The results of the Pearson's chi-square test are evaluated by referencing to the chi-square distribution.

The chi-square statistic is denoted as $\chi^2$ and is pronounced as kai-square. The properties of chi-square were first investigated by Karl Pearson in 1900 and hence named after Karl Pearson chi-square test.

In using chi-square test, the chi-square ($\chi^2$) statistic is computed as

$$\chi^2 = \sum_{i=1}^{n} \frac{(f_o - f_e)^2}{f_e} \tag{3.2}$$

where $f_o$ and $f_e$ are the observed and expected frequencies for each of the possible outcome, respectively.

### Steps in the Chi-Square Test

The following steps are used in chi-square test:

1. Compute expected frequency for each of the observed frequency. The procedure for computing expected frequency is different in case of testing the goodness of fit and in testing the independence of attributes. This will be discussed later in the chapter while solving the example.
2. Calculate the value of chi-square statistic $\chi^2$ by using the formula (3.2).
3. Find degrees of freedom of the test. In testing the goodness of fit, the degrees of freedom is equal to $(r - 1)$, where $r$ is the number of categories in the population. On the other hand, in testing the independence of attributes, the degrees of freedom is obtained by $(r - 1) \times (c - 1)$, where $r$ and $c$ are the number of rows and columns, respectively.
4. Find the tabulated value of $\chi^2$ with required degrees of freedom and at a given level of significance from Table A.6 in the Appendix.
5. If the calculated $\chi^2$ is less than or equal to tabulated $\chi^2$, the null hypothesis is failed to be rejected, and if the calculated $\chi^2$ is greater than the tabulated $\chi^2$, the null hypothesis is rejected at the tested level of significance.

### Assumptions in Using the Chi-Square

While using chi-square test, following assumptions are made:

1. Sample must be random.
2. Frequencies of each attribute must be numeric and should not be in percentages or ratios.
3. Sample size must be sufficiently large. The chi-square test shall yield inaccurate findings if the sample size is small. In that case, the researcher might end up committing a type II error.
4. The observations must be independent of each other. In other words, the chi-square test cannot be used to test the correlated data. In that situation, McNemar's test is used.
5. Normally, all cell frequencies must be 5 or more. In large contingency tables, 80% of cell frequencies must be 5 or more. If this assumption is not met, the Yates' correction is applied.
6. The expected frequencies should not be too low. Generally, it is acceptable if 20% of the events have expected frequencies less than 5, but in case of chi-square with one degree of freedom, the conclusions may not be reliable if expected frequencies are less than 10. In all such cases, Yates' correction must be applied.

## *Application of Chi-Square Test*

The chi-square test is used for two purposes: first, to test the goodness of fit and, second, to test the independence of two attributes. In both the situations, we intend to determine whether the observed frequencies significantly differ from the theoretical (expected) frequencies. The chi-square tests in these two situations shall be discussed in the following sections:

### To Test the Goodness of Fit

In many decision-making situations, a marketing manager may like to know whether the pattern of frequencies that are observed fits well with the expected ones or not. The appropriate test in such situations is the $\chi^2$ test of goodness of fit. Thus, a chi-square test for goodness of fit is used to verify whether an observed frequency distribution differs from a theoretical distribution or not. This test can also be used to check whether the data is from any specific distribution like normal, binomial or Poisson. The chi-square test for goodness of fit can also be used to test an equal occurrence hypothesis. By using this test, one can test whether all brands are equally popular, or whether all the car models are equally preferred. In using the chi-square test for goodness of fit, only one categorical variable is involved.

Consider a situation in which a researcher is interested to know whether all the three specializations like finance, human resource, and marketing are equally popular among MBA students; an equal occurrence hypothesis may be tested by

**Table 3.2** Preferences of the college students about different brands of cold drinks

| Color | White | Orange | Brown |
|---|---|---|---|
| Frequencies | 50 | 40 | 30 |

**Table 3.3** Observed and expected frequencies of responses

| | Observed frequencies ($f_o$) | Expected frequencies ($f_e$) |
|---|---|---|
| White | 50 | 40 |
| Orange | 40 | 40 |
| Brown | 30 | 40 |

computing the chi-square. The "Nonparametric Tests" option in SPSS provides the computation of chi-square ($\chi^2$). In such situations, following set of hypotheses is tested:

$H_0$: All three specializations are equally popular.
$H_1$: All three specializations are not equally popular.

By using the procedure discussed above for applying chi-square test, the null hypothesis may be tested. The procedure would clear by looking to the following solved examples:

**Example 3.1** A beverages company produces cold drink with three different colors. One hundred and twenty college students were asked about their preferences. The responses are shown in Table 3.2. Do these data show that all the flavors were equally liked by the students? Test your hypothesis at .05 level of significance.

*Solution* Here it is required to test the null hypothesis of equal occurrence; hence, expected frequencies corresponding to each of the three observed frequencies shall be obtained by dividing the total of all the observed frequencies by the number of categories. Hence, expected frequency ($f_e$) for each category shall be (Table 3.3)

$$f_e = \frac{50 + 40 + 30}{3} = 40$$

Here, number of categories or rows ($r$) = 3 and number of columns ($c$) = 2.

$$\chi^2 = \sum_{i=1}^{r} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(50 - 40)^2}{40} + \frac{(40 - 40)^2}{40} + \frac{(30 - 40)^2}{40}$$

$$= \frac{100}{40} + 0 + \frac{100}{40} = 2.5 + 2.5$$

$$\Rightarrow \quad \text{Cal. } \chi^2 = 5.0$$

**Table 3.4** Observed and expected frequencies

| | Observed frequencies ($f_o$) | Expected frequencies ($f_e$) |
|---|---|---|
| Grade A | 90 | 75 |
| Grade B | 65 | 50 |
| Grade C | 60 | 75 |
| Grade D | 85 | 100 |

*Testing the Significance of Chi-Square*

The degrees of freedom $= (r-1) = 3-1 = 2$.

From Table A.6 in the Appendix, the tab $\chi^2_{.05}(2) = 5.991$.

Since Cal. $\chi^2 <$ Tab. $\chi^2_{.05}(2)$, the null hypothesis may not be rejected at .05 level of significance. Thus, it may be concluded that all the three colors of cold drinks are equally liked by the college students.

**Example 3.2** An examination was undertaken by 300 students, out of which 90 students had grade A, 65 got grade B, 60 got grade C, and the remaining had grade D. Do these figures commensurate with the final examination result which is in the ratio of 3:2:3:4 for various grades, respectively? Test the hypothesis at 5% level.

*Solution* The null hypothesis which is required to be tested here is

$H_0$: The students in the various grades were distributed in the ratio 3:2:3:4

The expected number of students ($f_e$) for each grade under the assumption that $H_0$ is true is as follows:

$$\text{Expected number of students getting grade A} = \frac{3}{3+2+3+4} \times 300 = 75$$

$$\text{Expected number of students getting grade B} = \frac{2}{12} \times 300 = 50$$

$$\text{Expected number of students getting grade C} = \frac{3}{12} \times 300 = 75$$

$$\text{Expected number of students getting grade D} = \frac{4}{12} \times 300 = 100$$

Thus, the observed and expected frequencies can be listed as shown in Table 3.4.

$$\chi^2 = \sum_{i=1}^{r} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(90-75)^2}{75} + \frac{(65-50)^2}{50} + \frac{(60-75)^2}{75} + \frac{(85-100)^2}{100}$$

$$= \frac{225}{75} + \frac{225}{50} + \frac{225}{75} + \frac{225}{100} = 3 + 4.5 + 3 + 2.25 = 12.75$$

$\Rightarrow$ Calculated $\chi^2 = 12.75$

*Testing the Significance of Chi-Square*

Degrees of freedom: $(r - 1)\ 4-1 = 3$.

From Table A.6 in the Appendix, the tab $\chi^2_{.05}(3) = 7.815$.

Since Cal. $\chi^2 >$ Tab. $\chi^2_{.05}(3)$, the null hypothesis may be rejected at .05 level of significance. It may thus be concluded that grades A, B, C, and D are not in proportion to 3:2:3:4.

## To Test the Independence of Attributes

The chi-square test of independence is used to know whether paired observations on two attributes, expressed in a contingency table, are independent of each other. There may be varieties of situations where chi-square test of independence may be used. For instance one may test the significance of association between income level & brand preference, family size & television size purchased or educational background & the type of job one does. Thus, chi-square test may be used to test the significance of an association between any two attributes.

Let us assume that the population can be classified into $r$ mutually exclusive classes $A_1$, $A_2$,..., $A_r$ on the basis of attribute A, and each of these $r$ classes are further classified into $c$ mutually exclusive classes, like $A_iB_1$, $A_iB_2$, ..., $A_iB_c$, etc.

If $f_{o_{ij}}$ is the observed frequency of $A_iB_j$, that is, $(A_iB_j) = f_{o_{ij}}$, the above classification can be shown in the following table known as contingency table.

| $B$ | | | | | |
|---|---|---|---|---|---|
| $A$ | $B_1$ | $B_2$ | ... | $B_c$ | Total |
| $A_1$ | $f_{o_{11}}$ | $f_{o_{12}}$ | ... | $f_{o_{1c}}$ | $(A_1)$ |
| $A_2$ | $f_{o_{21}}$ | $f_{o_{22}}$ | ... | $f_{o_{2c}}$ | $(A_2)$ |
| . | | | | | |
| . | | | | | |
| $A_r$ | $f_{o_{r1}}$ | $f_{o_{r2}}$ | ... | $f_{o_{rc}}$ | $(A_r)$ |
| Total | $(B_1)$ | $(B_2)$ | ... | $(B_c)$ | $N$ |

By assuming $A$ and $B$ as independent attributes, the expected frequencies of each cell can be computed as

$$E_{ij} = \frac{(A_i)(B_j)}{N}$$

Thus,

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{\left(f_{o_{ij}} - f_{e_{ij}}\right)^2}{f_{e_{ij}}} \tag{3.3}$$

shall be a $\chi^2$ variate with $(r - 1)(c - 1)$ degrees of freedom.

The value of the $\chi^2$ variate so obtained can be used to test the independence of two attributes.

Consider a situation where it is required to test the significance of association between Gender (male and female) and Response ("prefer day shift" and "prefer night shift"). In this situation, following hypotheses may be tested:

$H_0$: Gender and Response toward shift preferences are independent.
$H_1$: There is an association between the Gender and Response toward shift preferences.

The calculated value of chi-square ($\chi^2$) obtained from the formula (3.3) may be compared with that of its tabulated value for testing the null hypothesis.

Thus, if calculated $\chi^2$ is less than tabulated $\chi^2$ with $(r-1)(c-1)$ df at some level of significance, then $H_0$ may not be rejected otherwise $H_0$ may be rejected.

**Remark** If $H_0$ is rejected, we may interpret that there is a significant association between the gender and their preferences toward shifts. Here, significant association simply means that the response pattern of male and female is different. The readers may note that chi-square statistic is used to test the significance of association, but ultimately one gets the comparison between the levels of one attribute across the levels of other attribute.

**Example 3.3** Five hundred families were investigated to test the belief that high-income people usually prefer to visit private hospitals and low-income people often go to government hospitals whenever they fall sick. The results so obtained are shown in Table 3.5.

Test whether income and hospital preferences are independent. Compute the contingency coefficient to find the strength of association. Test your hypothesis at 5% level.

*Solution* The null hypothesis to be tested is

$H_0$: Income and hospital preferences are independent.

Before computing the value of chi-square, the expected frequencies for each cell need to be computed with the marginal totals and grand totals given in the observed frequency ($f_o$) table. The procedure is discussed in Table 3.6.

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(f_{o_{ij}} - f_{e_{ij}}\right)^2}{f_{e_{ij}}}$$

$$= \frac{(125-140.4)^2}{140.4} + \frac{(145-129.6)^2}{129.6} + \frac{(135-119.6)^2}{119.6} + \frac{(95-110.4)^2}{110.4}$$

$$= 1.69 + 1.83 + 1.98 + 2.15 = 7.65$$

$$\Rightarrow \text{Calculated } \chi^2 = 7.65$$

**Table 3.5** Observed frequencies ($f_o$) of responses

| Hospitals | | | |
|---|---|---|---|
| Income | Government | Private | Total |
| High | 125 | 145 | 270 |
| Low | 135 | 95 | 230 |
| Total | 260 | 240 | 500 |

**Table 3.6** Expected frequencies ($f_e$) of responses

| Hospitals | | | |
|---|---|---|---|
| Income | Government | Private | Total |
| High | $\frac{270\times260}{500} = 140.4$ | $\frac{270\times240}{500} = 129.6$ | 270 |
| Low | $\frac{230\times260}{500} = 119.6$ | $\frac{230\times240}{500} = 110.4$ | 230 |
| Total | 260 | 240 | 500 |

*Test of Significance*

Here, $r = 2$ and $c = 2$, and therefore degree of freedom is $(r - 1) \times (c - 1) = 1$.

From Table A.6 in the Appendix, the tab $\chi^2_{.05}(1) = 3.841$.

Since Cal. $\chi^2 >$ Tab. $\chi^2_{.05}(1)$, the null hypothesis may be rejected at .05 level of significance. It may therefore be concluded that there is an association between the income level and the types of hospital preferred by the people.

## Precautions in Using the Chi-Square Test

(a) While using chi-square test, one must ensure that the sample is random, representative, and adequate in size.
(b) Chi-square should not be calculated if the frequencies are in percentage form; in that case, these frequencies must be converted back to absolute numbers before using the test.
(c) If any of the cell frequencies is less than 5, then for each cell, .5 is subtracted from the difference of observed and expected frequency while computing chi-square. This correction is known as Yates' correction. SPSS automatically does this correction while computing chi-square.
(d) The sum of the observed frequencies should be equal to the sum of the expected frequencies.

## Testing the Significance of Chi-Square in SPSS

(a) In SPSS, the null hypothesis is not tested on the basis of the comparison between calculated and tabulated chi-square; rather, it uses the concept of *p* value. *p* value is the probability of rejecting the null hypothesis when actually it is true.

(b) Thus, the chi-square is said to be significant at 5% level if the $p$ value is less than .05 and is insignificant if it is more than .05.

## *Contingency Coefficient*

Contingency coefficient ($C$) provides the magnitude of association between the attributes in the cross tabulation. Its value can range from 0 (no association) to 1 (the theoretical maximum possible association). Chi-square simply tests the significance of an association between any two attributes but does not provide the magnitude of the association. Thus, if the chi-square value becomes significant, one must compute the contingency coefficient ($C$) to know the extent of association between the attributes. The contingency coefficient $C$ is computed by the following formula:

$$C = \sqrt{\frac{\chi^2}{\chi^2 + N}} \tag{3.4}$$

where $N$ is the sum of all frequencies in the contingency table.

## *Lambda Coefficient*

Lambda coefficient is used to test the strength of an association in the cross tabulation. It is assumed that the variables are measured at the nominal level. Lambda can have the value in the range 0 (no association) to 1 (the theoretical maximum possible association). *Asymmetric lambda* measures the percentage improvement in predicting the dependent variable. *Symmetric lambda* measures the percentage improvement when prediction is done in both directions.

## *Phi Coefficient*

In a situation when both the variables are binary, phi coefficient is used to measure the degree of association between them. This measure is similar to the correlation coefficient in its interpretation. Two binary variables are considered positively associated if most of the data falls along the diagonal cells. In contrast, two binary variables are considered negatively associated if most of the data falls off the diagonal.

The assumptions of normality and homogeneity can be violated when the categories are extremely uneven, as in the case of proportions close to .90, .95 or

.10, .05. In such cases, the phi coefficient can be significantly attenuated. The assumption of linearity cannot be violated within the context of the phi coefficient of correlation.

### *Gamma*

If both the variables are measured at the ordinal level, *Gamma* is used for testing the strength of association of the cross tabulations. It makes no adjustment for either table size or ties. The value of Gamma can range from −1 (100% negative association, or perfect inversion) to +1 (100% positive association, or perfect agreement). A value of zero indicates the absence of association.

### *Cramer's V*

It measures the strength of association between attributes in cross tabulations. It is a variant of the *phi coefficient* that adjusts for the number of rows and columns. Its value can range from 0 (no association) to 1 (the theoretical maximum possible association).

### *Kendall Tau*

*Tau b and Tau c* both test the strength of association of the cross tabulations in a situation when both variables are measured at the ordinal level. Both these tests *Tau b* and *Tau c* make adjustments for ties, but *Tau b* is most suitable for square tables whereas *Tau c* is most suitable for rectangular tables. Their values can range from −1 (100% negative association, or perfect inversion) to +1 (100% positive association, or perfect agreement). A value of zero indicates the absence of association.

## Situation for Using Chi-Square

Chi-square is one of the most popularly used nonparametric statistical tests used in the questionnaire study. Two different types of hypotheses, that is, testing the goodness of fit and testing the significance of association between two attributes, can be tested using chi-square.

Testing an equal occurrence hypothesis is a special case of goodness of fit. In testing an equal occurrence hypothesis, the observed frequencies on different

**Table 3.7** Observed frequencies in a contingency table

| | Category of preference | |
|---|---|---|
| Socioeconomic status | Prefer | Do not prefer |
| High | 80 | 15 |
| Low | 40 | 65 |

levels of a factor are obtained. The total of observed frequencies for all the levels is divided by the number of levels to obtain the expected frequencies for each level. Consider an experiment in which it is intended to test whether all the three locations, that is, Delhi, Mumbai, and Chennai, are equally preferred by the employees of an organization for posting. Out of 250 employees surveyed, 120 preferred Delhi, 80 preferred Mumbai, and 50 preferred Chennai. In this situation, the following null hypothesis may be tested using chi-square:

$H_0$: All the three locations are equally preferred.

Against the alternative hypothesis:

$H_1$: All the three locations are not equally preferred.

Here, the chi-square test can be used to test the null hypothesis of equal occurrence.

Another application of chi-square is to test the significance of association between any two attributes. Suppose it is desired to know as to whether preference of consumers for a specific brand of soap depends upon their socioeconomic status where the response of 200 customers is shown in Table 3.7.

The following null hypothesis may be tested by using the chi-square for two samples at 5% level to answer the question.

$H_0$: Socioeconomic status and soap preferences are independent.

Against the alternative hypothesis:

$H_1$: There is an association between the socioeconomic status and soap preferences.

If the null hypothesis is rejected, one may draw the conclusion that the preference of soap is significantly associated with the socioeconomic status of an individual. In other words, it may be concluded that the response patterns of the customers in high and low socioeconomic status are different.

The above two different kinds of application of chi-square have been discussed below by means of solved examples using SPSS.

## Solved Examples of Chi-square for Testing an Equal Occurrence Hypothesis

**Example 3.4** In a study, 90 workers were tested for their job satisfaction. Their job satisfaction level was obtained on the basis of the questionnaire, and the respondents were classified into one of the three categories, namely, low, average, and high. The observed frequencies are shown in Table 3.8. Compute chi-square in testing whether there is any specific trend in their job satisfaction.

**Table 3.8** Summary of responses of the workers about their job satisfaction levels

| Job satisfaction level | | |
|---|---|---|
| Low | Average | High |
| 40 | 30 | 20 |

*Solution* Here, the null hypothesis that is required to be tested is

$H_0$: All the three job satisfaction levels are equally probable.

Against the alternative hypothesis:

$H_1$: All the three job satisfaction levels are not equally probable.

The SPSS shall be used to compute the value of chi-square for testing the null hypothesis. Computation of chi-square for single sample using SPSS has been shown in the following steps:

## *Computation of Chi-Square Using SPSS*

(a) ***Preparing Data File***

Before using SPSS command to compute chi-square, a data file needs to be prepared. The following steps will help you prepare the data file:

(i) *Starting the SPSS:* Use the following command sequence to start SPSS:

**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

After checking the option **Type in Data** on the screen you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* There is only one variable *Job Satisfaction Level* that needs to be defined. Since this variable can assume any one of the three values, it is a nominal variable. The procedure of defining the variable in SPSS is as follows:

1. Click **Variable View** to define variables and their properties.
2. Write short name of the variable, that is, *Job_Sat* under the column heading **Name.**
3. For this variable, define the full name, that is, *Job Satisfaction Level* under the column heading **Label**.
4. Under the column heading **Values,** define "1" for low, "2" for medium, and "3" for high.
5. Under the column heading **Measure,** select the option "Nominal" because the variable Job_Sat is a nominal variable.
6. Use default entries in rest of the columns.

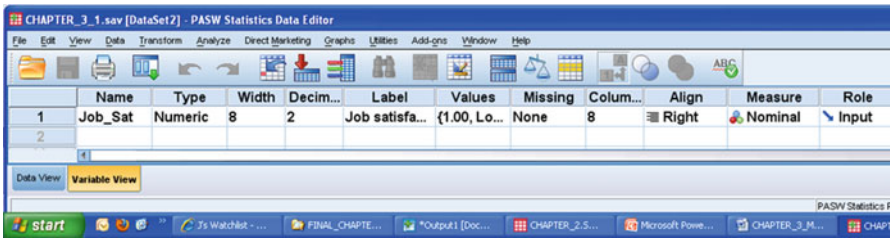



**Fig. 3.2** Defining variable along with its characteristics

After defining the variables in variable view, the screen shall look like Fig. 3.2.

(iii) *Entering data:* Once the variable *Job_Sat* has been defined in the **Variable View**, click **Data View** on the left bottom of the screen to open the format for entering data column wise.
In this example, we have only one variable *Job_Sat* with three levels as Low, Medium, and High. The Low satisfaction level was observed in 40 workers, whereas Medium satisfaction level was observed in 30 workers and High satisfaction level was observed in 20 workers. Since these levels have been defined as 1, 2, and 3, the data shall be entered under one variable *Job_Sat* as shown below:

Data feeding procedure in SPSS under **Data View**

| S.N. | Job_Sat | |
|---|---|---|
| 1 | 1 | |
| 2 | 1 | |
| 3 | 1 | Type "1" 40 times as Low satisfaction level was observed in 40 workers |
| . | . | |
| . | . | |
| . | . | |
| 40 | 1 | |
| 41 | 2 | |
| 42 | 2 | |
| . | . | Type "2" 30 times as Medium satisfaction level was observed in 30 workers |
| . | . | |
| . | . | |
| 70 | 2 | |
| 71 | 3 | |
| 72 | 3 | |
| 73 | 3 | Type "3" 20 times as Low satisfaction level was observed in 20 workers |
| . | . | |
| . | . | |
| . | . | |
| 90 | 3 | |

After entering data, the screen shall look like Fig. 3.3. Only the partial data has been shown in the figure as data set is long enough to fit in the window. Save the data file in the desired location before further processing.

(b) **SPSS *Commands for Computing Chi-Square***
After preparing the data file in data view, take the following steps to compute the chi-square:

(i) *Initiating the SPSS commands to compute chi-square for single variable:* In data view, click the following commands in sequence:

**Analyze → Nonparametric Tests → Legacy Dialogs → Chi – Square**

The screen shall look like Fig. 3.4.
**Note:** In other versions of SPSS, the command sequence is as follows:

Analyze → Nonparametric Tests → Chi-Square

(ii) *Selecting variable for computing chi-square*: After clicking the "Chi-Square" option, you will be taken to the next screen for selecting the variable for which chi-square needs to be computed. Since there is only one variable *Jobs satisfaction level* in the example, select it from the left panel by using the left click of the mouse and bring it to the right panel by clicking the arrow. The screen shall look like Fig. 3.5.

(iii) *Selecting the option for computation:* After selecting the variable, option needs to be defined for the computation of chi-square. Take the following steps:

- Click the **Options** in the screen shown in Fig. 3.5. This will take you to the next screen that is shown in Fig. 3.6.
  - Check the option "Descriptive."
  - Use default entries in other options.
  - Click ***Continue***. This will take you back to screen shown in Fig. 3.5
- Press **OK.**

(c) ***Getting the Output***
Pressing **OK** will lead you to the output window. The output panel shall have two results that are shown in Tables 3.9 and 3.10. These outputs can be selected by using right click of the mouse which may be copied in the word file.

## *Interpretation of the Outputs*

Table 3.9 shows the observed and expected frequencies of the different levels of job satisfaction. No cell frequency is less than 5, and, therefore, no correction is required while computing chi-square.

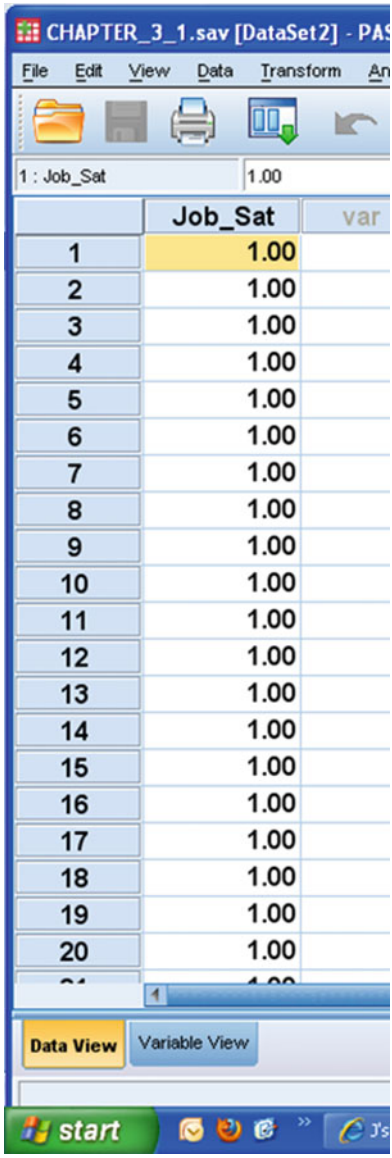



**Fig. 3.3** Screen showing entered data for the variable Job_Sat in the data view

The value of chi-square (=6.667) in Table 3.10 is significant at 5% level because its associated *p* value is .036 which is less than .05. Thus, the null hypothesis may be rejected. It may therefore be concluded that all the three job satisfaction levels are not equally probable.

So long the value of p is less than .05, the value of chi-square is significant at 5% level, and if the *p* value becomes more than .05, the chi-square becomes insignificant.

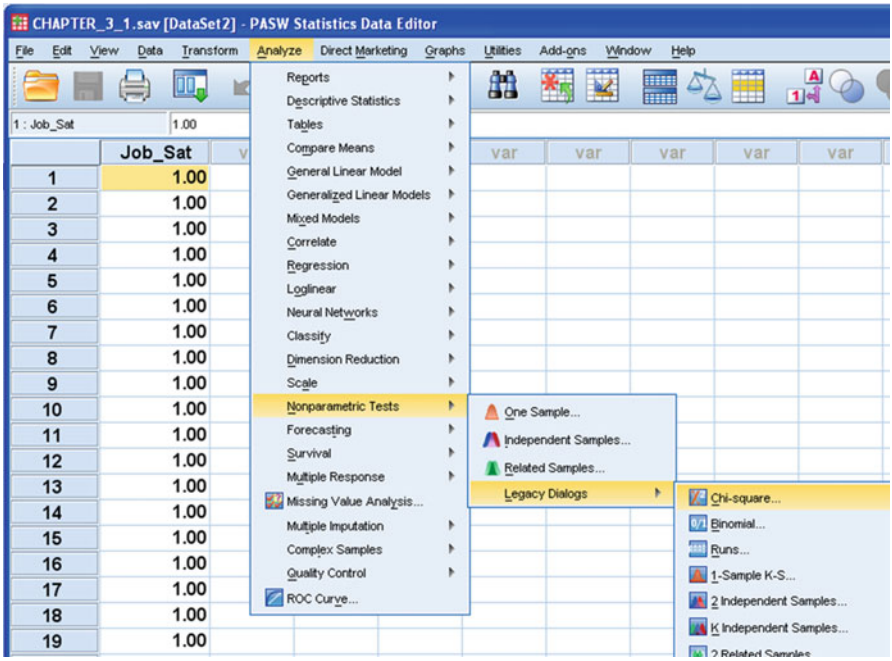



**Fig. 3.4** Screen showing the SPSS commands for computing chi-square

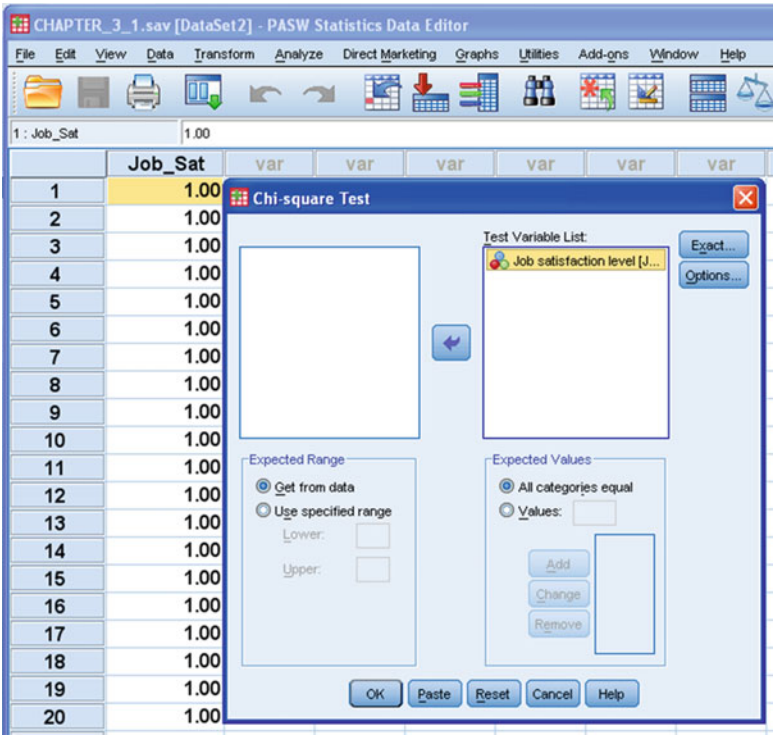



**Fig. 3.5** Screen showing selection of variable for chi-square

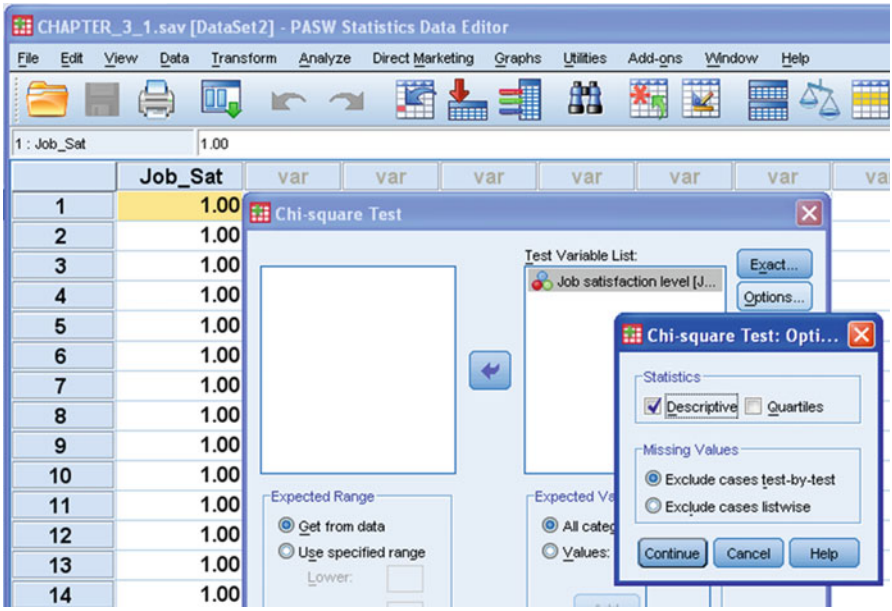



**Fig. 3.6** Screen showing option for chi-square computation

**Table 3.9** Observed and expected frequencies for different levels of job satisfaction

| | Frequencies | | |
|---|---|---|---|
| | Observed *N* | Expected *N* | Residual |
| Low | 40 | 30.0 | 10.0 |
| Medium | 30 | 30.0 | .0 |
| High | 20 | 30.0 | −10.0 |
| Total | 90 | | |

**Table 3.10** Chi-square for the data on job satisfaction level

| | Job satisfaction level |
|---|---|
| Chi-square | 6.667[a] |
| df | 2 |
| Asymp sig. | .036 |

[a]0 cells (0%) have expected frequencies less than 5. The minimum expected cell frequency is 30

## Solved Example of Chi-square for Testing the Significance of Association Between Two Attributes

**Example 3.5** Out of 200 MBA students, 40 were given an academic counseling throughout the semester, whereas other 40 did not receive this counseling. On the basis of their marks in the final examination, their performance was categorized as improved, unchanged, and deteriorated. Based on the results shown in Table 3.11, can it be concluded that the academic counseling is effective at 5% level?

**Table 3.11** Frequencies of the MBA students in a contingency table

| | Performance | | |
|---|---|---|---|
| Treatment | Improved | Unchanged | Deteriorated |
| Counseling group | 22 | 8 | 10 |
| Control group | 4 | 5 | 31 |

*Solution* In order to check whether academic counseling is effective, we shall test the significance of association between treatment and performance. If the association between these two attributes is significant, then it may be interpreted that the pattern of performance in the counseling and control groups is not same. In that case, it might be concluded that the counseling is effective since the number of improved cases is higher in counseling group than that of control group.
Thus, it is important to compute the chi-square first in order to test the null hypothesis.

$H_0$: There is no association between treatment and performance.

Against the alternative hypothesis:

$H_1$: There is an association between treatment and performance.

The commands for computing chi-square in case of two samples are different than that of one sample computed in Example 3.4.

In two-sample case, chi-square is computed using **Crosstabs** option in **Descriptive statistics** command of SPSS. The chi-square so obtained shall be used for testing the above-mentioned null hypothesis. Computation of chi-square for two samples using SPSS has been shown in the following steps:

## *Computation of Chi-Square for Two Variables Using SPSS*

(a) ***Preparing Data File***
As discussed in Example 3.4, a data file needs to be prepared for using the SPSS commands for computing chi-square. Follow the below-mentioned steps to prepare data file:

(i) *Starting the SPSS:* Use the following command sequence to start SPSS:

**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

By checking the option **Type in Data** on the screen you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* Here, two variables *Treatment* and *Performance* need to be defined. Since both these variables are classificatory in nature, they are treated as nominal variables in SPSS. The procedure of defining variables and their characteristics in SPSS is as follows:

1. Click **Variable View** to define variables and their properties.
2. Write short name of the variables as *Treatment* and *Performance* under the column heading **Name.**

3. Under the column heading **Label**, full name of the *Treatment* and *Performance* variables may be defined as *Treatment groups* and *Performance status,* respectively. There is flexibility in choosing full name of each variable.
4. In the *Treatment* row, double-click the cell under the column **Values** and add the following values to different labels:

| Value | Label |
|---|---|
| 1 | Counseling group |
| 2 | Control group |

5. Similarly in the *Performance* row, double-click the cell under the column **Values** and add the following values to different labels:

| Value | Label |
|---|---|
| 3 | Improved |
| 4 | Unchanged |
| 5 | Deteriorated |

There is no specific rule of defining the values of labels. Even "Improved," "Unchanged," and "Deteriorated" may be defined as 1, 2, and 3, respectively.

6. Under the column label **Measure,** select the option "Nominal" for both the variables *Treatment* and *Performance*.
7. Use default entries in rest of the columns. .
   After defining the variables in **Variable View,** the screen shall look like Fig. 3.7.

(iii) *Entering data*: Once the variables *Treatment* and *Performance* have been defined in the **Variable View**, click **Data View** on the left bottom of the screen to open the format for entering the data column wise.

In this example, there are two variables *Treatment* and *Performance*. *Treatment* has two value levels: "1" for counseling group and "2" for control group. Since there are 40 students in counseling group and 40 in control group in this example, under the *Treatment* column, write first 40 data as 1 and next 40 data as 2.

Since out of 40 students of counseling group, 22 showed "improved" (value = 3), 8 showed "unchanged" (value = 4), and 10 showed "deteriorated" (value = 5) performance, under the *Performance* column, type first 22 data as 3, next 8 data as 4, and subsequent 10 data as 5.

Similarly out of 40 students of control group, 4 showed "improved" (value = 3), 5 showed "unchanged" (value = 4), and 31 showed "deteriorated" (value = 5) performance; therefore, after typing the above data under the *Performance* column, type next 4 data as 3, 5 data as 4, and 31 data as 5.

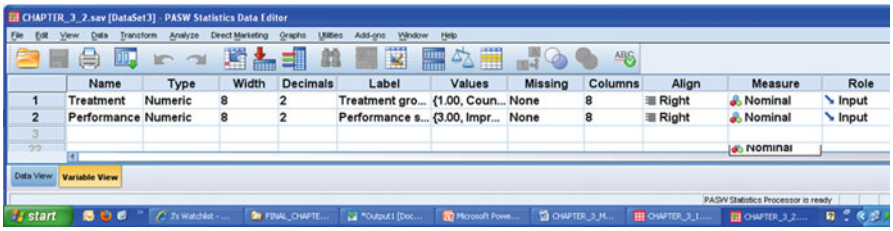



**Fig. 3.7** Defining variables along with their characteristics

Data feeding procedure for the data of Table 3.11 in SPSS under **Data View**

| S.N. | | Treatment | Performance | |
|---|---|---|---|---|
| 1 | | 1 | 3 | |
| 2 | | 1 | 3 | |
| 3 | | 1 | 3 | Type "3" twenty-two times as there |
| 4 | | 1 | 3 | are 22 students showed Improved |
| 5 | | 1 | 3 | performance in counseling group |
| 6 | | 1 | 3 | |
| 7 | | 1 | 3 | |
| 8 | | 1 | 3 | |
| 9 | | 1 | 3 | |
| 10 | | 1 | 3 | |
| 11 | | 1 | 3 | |
| 12 | Type "1" forty | 1 | 3 | |
| 13 | | 1 | 3 | |
| 14 | times as there are 40 | 1 | 3 | |
| 15 | students in the counseling group | 1 | 3 | |
| 16 | | 1 | 3 | |
| 17 | | 1 | 3 | |
| 18 | | 1 | 3 | |
| 19 | | 1 | 3 | |
| 20 | | 1 | 3 | |
| 21 | | 1 | 3 | |
| 22 | | 1 | 3 | |
| 23 | | 1 | 4 | |
| 24 | | 1 | 4 | Type "4" eight times as there are |
| 25 | | 1 | 4 | 8 students showed Unchanged |
| 26 | | 1 | 4 | performance in counseling |
| 27 | | 1 | 4 | group |
| 28 | | 1 | 4 | |
| 29 | | 1 | 4 | |
| 30 | | 1 | 4 | |
| 31 | | 1 | 5 | |
| 32 | | 1 | 5 | Type "5" ten times as there are 10 |
| 33 | | 1 | 5 | students showed Deteriorated |
| 34 | | 1 | 5 | performance in counseling |
| 35 | | 1 | 5 | group |

(continued)

(continued)

| S.N. | | Treatment | Performance | |
|---|---|---|---|---|
| 36 | | 1 | 5 | |
| 37 | | 1 | 5 | |
| 38 | | 1 | 5 | |
| 39 | | 1 | 5 | |
| 40 | | 1 | 5 | |
| 41 | | 2 | 3 | Type "3" four times as there are 4 students showed Improved performance in control group |
| 42 | | 2 | 3 | |
| 43 | | 2 | 3 | |
| 44 | | 2 | 3 | |
| 45 | | 2 | 4 | |
| 46 | | 2 | 4 | Type "4" five times as there are 5 students showed Unchanged performance in control group |
| 47 | Type "2" forty times as there are 40 students in the control group | 2 | 4 | |
| 48 | | 2 | 4 | |
| 49 | | 2 | 4 | |
| 50 | | 2 | 5 | |
| 51 | | 2 | 5 | |
| 52 | | 2 | 5 | Type "5" thirty-one times as there are 31 students showed Deteriorated performance in control group |
| 53 | | 2 | 5 | |
| 54 | | 2 | 5 | |
| 55 | | 2 | 5 | |
| 56 | | 2 | 5 | |
| 57 | | 2 | 5 | |
| 58 | | 2 | 5 | |
| 59 | | 2 | 5 | |
| 60 | | 2 | 5 | |
| 61 | | 2 | 5 | |
| 62 | | 2 | 5 | |
| 63 | | 2 | 5 | |
| 64 | | 2 | 5 | |
| 65 | | 2 | 5 | |
| 66 | | 2 | 5 | |
| 67 | | 2 | 5 | |
| 68 | | 2 | 5 | |
| 69 | | 2 | 5 | |
| 70 | | 2 | 5 | |
| 71 | | 2 | 5 | |
| 72 | | 2 | 5 | |
| 73 | | 2 | 5 | |
| 74 | | 2 | 5 | |
| 75 | | 2 | 5 | |
| 76 | | 2 | 5 | |
| 77 | | 2 | 5 | |
| 78 | | 2 | 5 | |
| 79 | | 2 | 5 | |
| 80 | | 2 | 5 | |

Treatment coding: 1 = Counseling group, 2 = Control group
Performance coding: 3 = Improved, 4 = Unchanged, 5 = Deteriorated

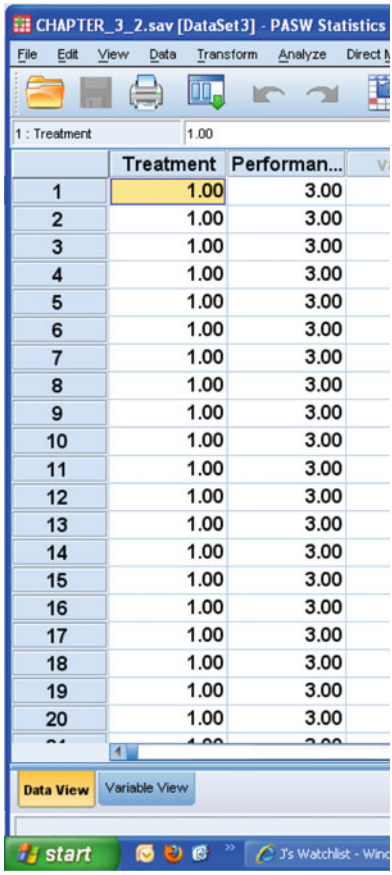



**Fig. 3.8** Screen showing entered data for the *Treatment* and *Performance* variables in the data view

After entering the data, the screen will look like Fig. 3.8. The screen shows only the partial data as the data is entered column wise which takes two-page-long entries. Save the data file in the desired location before further processing.

(b) **SPSS *Commands for Computing Chi-square with Two Variables***
After entering all the data by clicking the data view, take the following steps for computing chi-square:

(i) *Initiating the SPSS commands for computing chi-square:* In Data View, click the following commands in sequence:

**Analyze → Descriptive Statistics → Crosstabs**

The screen shall look like Fig. 3.9.

(ii) *Selecting variables for computing chi-square*: After clicking the "Crosstabs" option, you will be taken to the next screen for selecting variables for the crosstabs analysis and computing chi-square. Out of the two variables, one has to be selected in the Row(s) panel and the other in the Column(s) panel.

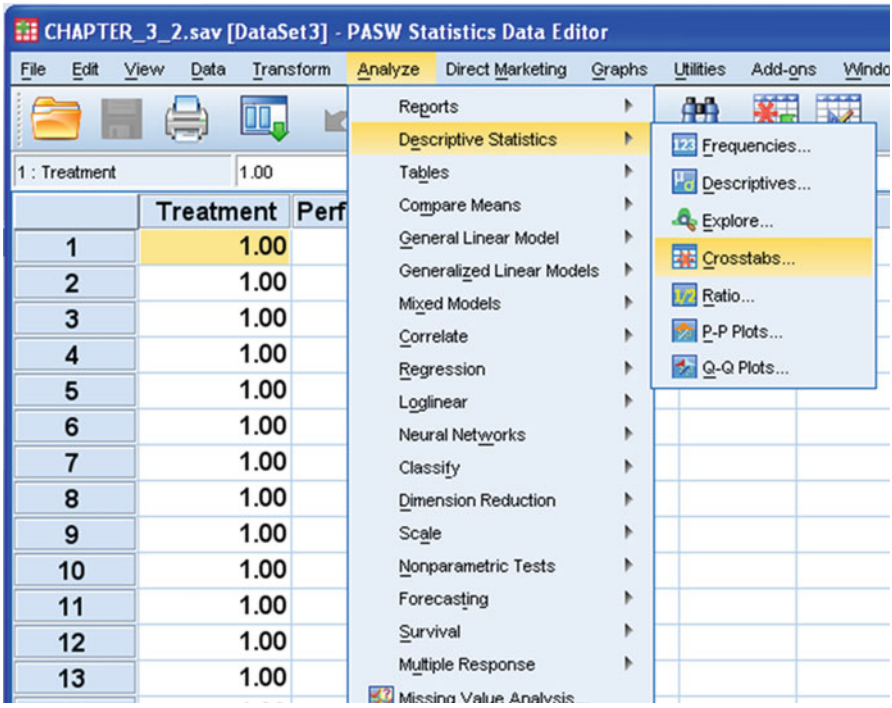



**Fig. 3.9** Screen showing the SPSS commands for computing chi-square in crosstabs

Select the variables *Treatment group* and *Performance status* from the left panel and bring them to the "Row(s)" and "Column(s)" sections of the right panel, respectively, by arrow button. The screen shall look like Fig. 3.10.

(iii) *Selecting option for computation:* After selecting variables, option needs to be defined for the crosstabs analysis and computation of chi-square. Take the following steps:

- Click **Statistics** option to get the screen shown in Fig. 3.11.
  - Check the options "Chi-square" and "Contingency coefficient."
  - Click ***Continue***.
- Click **Cells option to** get the screen shown in Fig. 3.12. Then,
  - Check the options "Observed" and "Expected" under the Counts section. Observed is checked by default.
  - Click ***Continue***. You will be taken back to the screen shown in Fig. 3.10.
  - Use default entries in other options. Readers are advised to try other options and see what changes they are getting.
  - Click **OK.**

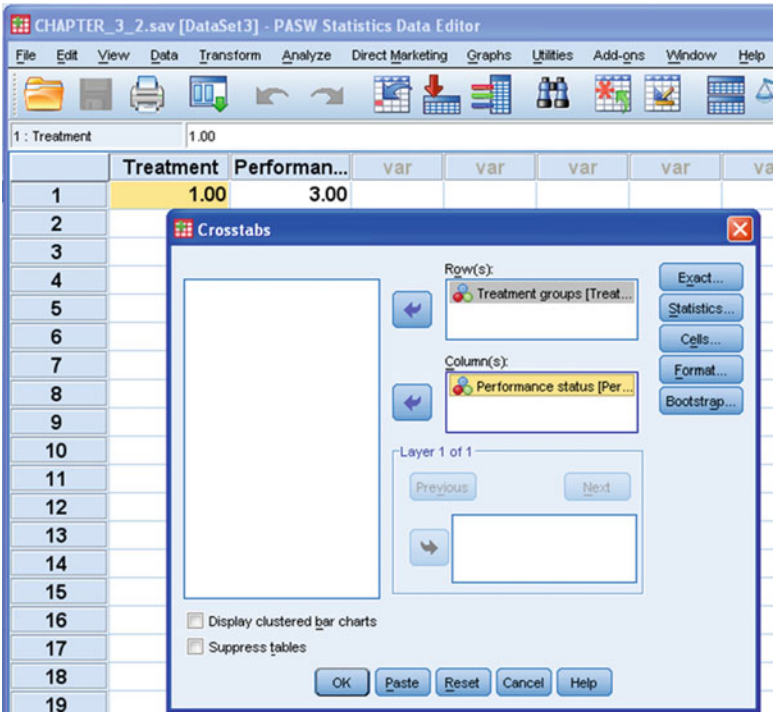



**Figure 3.10** Screen showing selection of variables for chi-square in crosstab

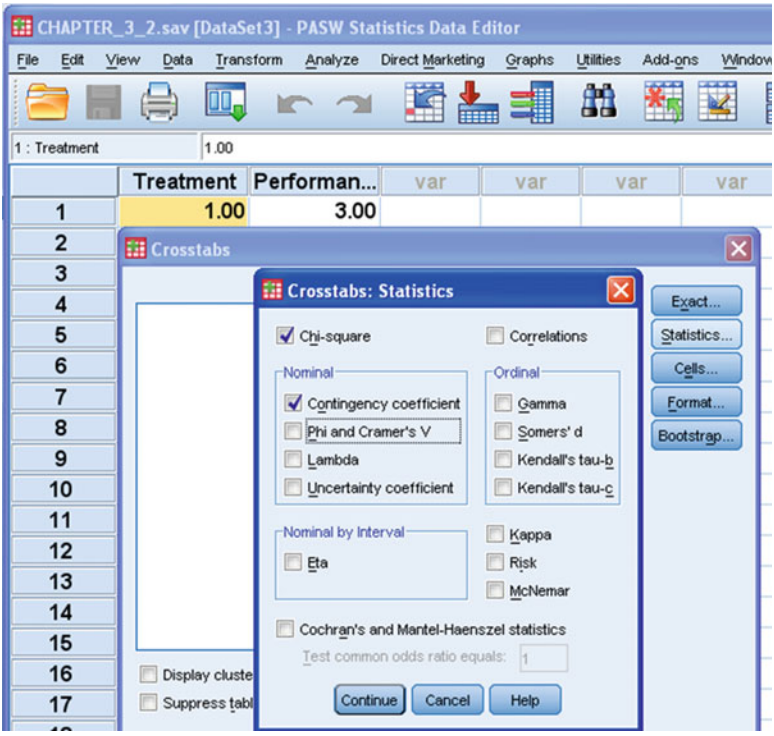



**Fig. 3.11** Screen showing option for computing chi-square and contingency coefficient

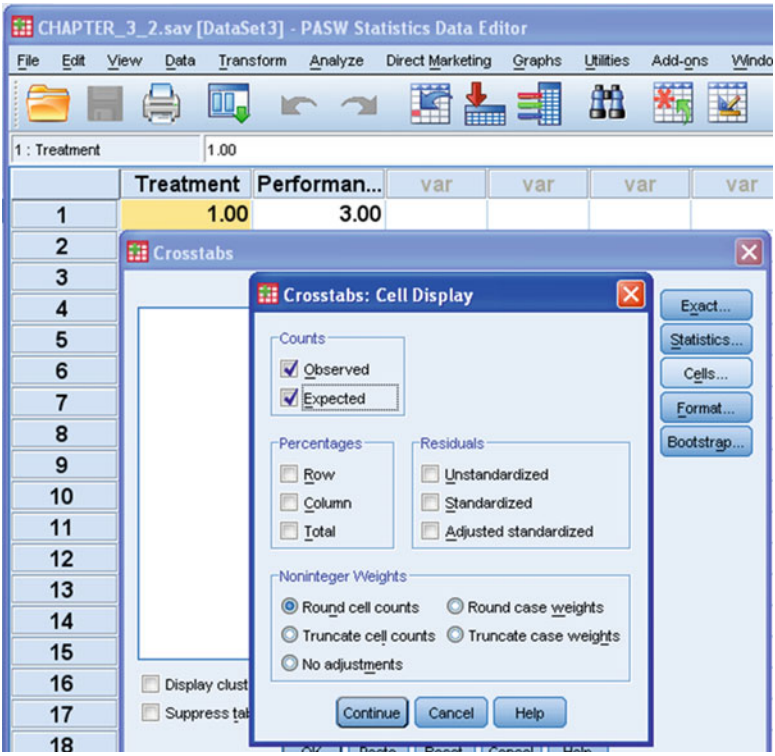



**Fig. 3.12** Screen showing option for computing observed and expected frequencies

**Table 3.12** Treatment groups × Performance status cross tabulation

| | | | Performance status | | | |
|---|---|---|---|---|---|---|
| | | | Improve | Unchanged | Deteriorated | Total |
| Treatment groups | Counseling Gp | Count | 22 | 8 | 10 | 40 |
| | | Expected count | 13.0 | 6.5 | 20.5 | 40.0 |
| | Control Gp | Count | 4 | 5 | 31 | 40 |
| | | Expected count | 13.0 | 6.5 | 20.5 | 40.0 |
| Total | | Count | 26 | 13 | 41 | 80 |
| | | Expected count | 26.0 | 13.0 | 41.0 | 80.0 |

(c) ***Getting the Output***

Clicking option **OK** will lead you to the output window. The output panel will have lots of results. It is up to the researcher to decide the relevant outputs to be shown in their report. The relevant output can be selected by using the right click of the mouse and copying in the word file. In this example, the output so generated by the SPSS will look like as shown in Tables 3.12, 3.13, and 3.14.

**Table 3.13** Chi-square for the data on Treatment × Performance

| | Value | df | Asymp. sig. (2-sided) |
|---|---|---|---|
| Pearson chi-square | 23.910[a] | 2 | .000 |
| Likelihood ratio | 25.702 | 2 | .000 |
| Linear-by-linear association | 23.400 | 1 | .000 |
| *N* of valid cases | 80 | | |

[a]0 cells (.0%) have expected count less than 5. The minimum expected count is 6.50

**Table 3.14** Contingency coefficient for the data on Treatment × Performance

| | | Value | Approx. sig. (*p* value) |
|---|---|---|---|
| Nominal by nominal | Contingency coefficient | 0.480 | 0.000 |
| *N* of valid cases | | 80 | |

### *Interpretation of the Outputs*

The observed and expected frequencies of the *Treatment group* × *Performance status* can be seen in Table 3.12. Since no cell frequency is less than 5, therefore, no correction is required while computing the chi-square. If any of the cell frequency had value 5 or less, then SPSS would have computed the chi-square after applying the correction.

Table 3.13 shows the value of chi-square ($\chi^2$) as 23.910, which is significant at 1% level as the p value is .000. Thus, we may reject the null hypothesis that "There is no association between *Treatment* and *Performance*." Hence, it may be concluded that there is a significant association between treatment and performance. In other words, it can be said that the pattern of academic performance is different in counseling and control group. Since the number of improved performance cases (22) is higher in counseling group than that of 4 in control group, it may be interpreted that the academic counseling is effective in improving the performance.

In Table 3.14, the value of contingency coefficient is 0.480. This is a measure of association between *Treatment* and *Performance*. This contingency coefficient can be considered to be significant as its p value is .000 which is less than .05. Thus, it may finally be concluded that counseling is significantly effective in improving the academic performance.

## Summary of the SPSS Commands

(a) **For computing chi-square statistic (for testing equal occurrence hypothesis):**

1. Start SPSS by using the following sequence of commands:

**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

2. Create data file by choosing the option **Type in Data**.
3. Click the tag **Variable View** and define the variable *Job_Sat* as "Nominal" variable.
4. For the variable *Job_Sat*, under the column heading **Values,** define "1" for low, "2" for medium, and "3" for high.
5. By clicking the **Data View,** enter first forty data of the variable *Job_Sat* as 1, next thirty as 2, and further 20 as 3 in the same column.
6. Click the following command sequence for computing chi-square:

**Analyze → Nonparametric Tests → Legacy Dialogs → Chi – Square**

7. Select the variable *Job_Sat* from left panel to the right panel.
8. Click the tag **Options** and check the box of "Descriptive." Press ***Continue***.
9. Click **OK** to get the output.

(b) **For computing chi-square statistic (for testing the significance of association between two attributes):**

1. Start SPSS by using the following command sequence:

**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

2. Click **Variable View** tag and define the variable *Treatment* and *Performance* as "Nominal" variables.
3. In the *Treatment* row, double-click the cell under the column **Values** and add the values "1" for Counseling group and "2" for Control group. Similarly, in the *Performance* row, define the value "3" for Improved, 4 for Unchanged, and 5 for Deteriorated.
4. Use default entries in rest of the columns.
5. Click **Data View** tag and feed first forty entries as 1 and next forty entries as 2 for the *Treatment* variable.
6. Similarly for the *Performance* variable, enter first twenty-two entries as 3, next eight entries as 4, and further ten entries as 5. These three sets of entries are for counseling group. Similarly for showing the entries of control group, enter first four entries as 3, next five entries as 4, and after that thirty-one entries as 5 in the same column.
7. Click the following command sequence for computing chi-square:

**Analyze → Descriptive Statistics → Crosstabs**

8. Select variables *Treatment group* and *Performance status* from the left panel to the "Row(s)" and "Column(s)" sections of the right panel, respectively.
9. Click the option **Statistics** and check the options "Chi-square" and "Contingency coefficient." Press ***Continue***.
10. Click **OK** to get the output.

## Exercise

*Short-Answer Questions*

**Note:** Write answer to each of the questions in not more than 200 words:

Q.1. Responses were obtained from male and female on different questions related to their knowledge about smoking. There were three possible responses Agree, Undecided, and Disagree for each of the questions. How will you compare the knowledge of male and female about smoking?

Q.2. Write in brief two important applications of chi-square.

Q.3. How will you frame a null hypothesis in testing the significance of an association between gender and IQ where IQ is classified into high and low category? Write the decision criteria in testing the hypothesis.

Q.4. Can the chi-square be used for comparing the attitude of male and female on the issue of "Foreign retail chain may be allowed in India" if the frequencies are given in 3 × 5 table below? If so or otherwise, interpret your findings. Under what situation chi-square is the most robust test?

Response on "Foreign retail chain may be allowed in India"

| | | Strongly agree | Agree | Undecided | Disagree | Strongly disagree |
|---|---|---|---|---|---|---|
| Gender | Male | 50 | 20 | 15 | 5 | 10 |
| | Female | 20 | 15 | 10 | 25 | 30 |

Q.5 If chi-square is significant, it indicates that the association between the two attributes exists. How would you find the magnitude of an association?

Q.6 What is phi coefficient? In what situation it is used? Explain by means of an example.

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. For testing the significance of association between Gender and IQ level, the command sequence for computing chi-square in SPSS is

   (a) Analyze -> Nonparametric Tests -> Chi-square
   (b) Analyze -> Descriptive Statistics -> Crosstabs
   (c) Analyze -> Chi-square -> Nonparametric Tests
   (d) Analyze -> Crosstabs -> Chi-square

2. Choose the most appropriate statement about the null hypothesis in chi-square.

   (a) There is an association between gender and response.
   (b) There is no association between gender and response.
   (c) There are 50−50% chances of significant and insignificant association.
   (d) None of the above is correct.

3. Response of the students on their preferences toward optional papers is as follows:

| | Response of the students | | |
|---|---|---|---|
| Subjects | Finance | Human resource | Marketing |
| No. of students | 15 | 25 | 20 |

The value of chi-square shall be

(a) 2
(b) 2.5
(c) 50
(d) 25

4. The value of chi-square for the given data shall be

| | | Gender | |
|---|---|---|---|
| | | Male | Female |
| Region | North | 30 | 20 |
| | South | 10 | 40 |

(a) 16.67
(b) 166.7
(c) 1.667
(d) 1667

5. Chi-square is used for

(a) Finding magnitude of an association between two attributes
(b) Finding significance of an association between two attributes
(c) Comparing the variation between two attributes
(d) Comparing median of two attributes

6. Chi-square is the most robust test if the frequency table is

(a) $2 \times 2$
(b) $2 \times 3$
(c) $3 \times 3$
(d) $m \times n$

7. While using chi-square for testing an association between the attributes, SPSS provides Crosstabs option. Choose the most appropriate statement.

(a) Crosstabs treats all data as nominal.
(b) Crosstabs treats all data as ordinal.
(c) Crosstabs treats some data as nominal and some data as ordinal.
(d) Crosstabs treats data as per the problem.

8. If responses are obtained in the form of the frequency on a 5-point scale and it is required to compare the responses of male and female on the issue "Marketing stream is good for the female students," which statistical test you would prefer?

   (a) Two-sample *t*-test
   (b) Paired *t*-test
   (c) One-way ANOVA
   (d) Chi-square test

9. If *p* value for a chi-square is .02, what conclusion you can draw?

   (a) Chi-square is significant at 95% confidence.
   (b) Chi-square is not significant at 95% confidence.
   (c) Chi-square is significant at .01 levels.
   (d) Chi-square is not significant at .05 levels.

10. The degree of freedom of chi-square in a $r \times c$ table is

    (a) $r + c$
    (b) $r + c - 1$
    (c) $rc$
    (d) $(r-1)(c-1)$

11. Phi coefficient is used if

    (a) Both the variables are ordinal.
    (b) Both the variables are binary.
    (c) Both the variables are interval.
    (d) One of the variables is nominal and the other is ordinal.

12. Gamma coefficient is used if

    (a) Both the variables are interval.
    (b) Both the variables are binary.
    (c) Both the variables are ordinal.
    (d) Both the variables may be on any scale.

*Assignments*

1. Following are the frequencies of students in an institute belonging to Low, Medium, and High IQ groups. Can it be concluded that there is a specific trend of IQ's among the students. Test your hypothesis at 5% level.

Frequencies of the students in different IQ groups

| IQ categories | Low IQ | Medium IQ | High IQ |
|---|---|---|---|
| Frequency | 20 | 65 | 35 |

2. In an organization following are the frequencies of male and female workers in the skilled and unskilled categories. Test whether nature of work is independent of the gender by computing chi-square. Also compute contingency coefficient

along with the expected frequency and percentage frequencies in the Crosstabs and interpret your findings. Test your hypothesis at 5% level.

Frequency of workers in different categories

| | | Workers | |
|---|---|---|---|
| | | Skilled | Unskilled |
| Gender | Male | 50 | 15 |
| | Female | 15 | 40 |

*Answers to Multiple-Choice Questions*

| | |
|---|---|
| Q.1 b | Q.2 b |
| Q.3 b | Q.4 a |
| Q.5 b | Q.6 a |
| Q.7 a | Q.8 d |
| Q.9 a | Q.10 d |
| Q.11 b | Q.12 c |

# Chapter 4
# Correlation Matrix and Partial Correlation: Explaining Relationships

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Learn the concept of linear correlation and partial correlation.
- Explore the research situations in which partial correlation can be effectively used.
- Understand the procedure in testing the significance of product moment correlation and partial correlation.
- Develop the hypothesis to test the significance of correlation coefficient.
- Formulate research problems where correlation matrix and partial correlation can be used to draw effective conclusion.
- Learn the application of correlation matrix and partial correlation through case study discussed in this chapter.
- Understand the procedure of using SPSS in computing correlation matrix and partial correlation.
- Interpret the output of correlation matrix and partial correlation generated in SPSS.

## Introduction

One of the thrust areas in the management research is to find the ways and means to improve productivity. It is therefore important to know the variables that affect it. Once these variables are identified, an effective strategy may be adopted by prioritizing it to enhance the productivity in the organization. For instance, if a company needs to improve the sale of a product, then its first priority would be to ensure its quality and then to improve other variables like resources available to the marketing team, their incentive criteria, and dealer's scheme. It is because of the fact that the product quality is the most important parameter in enhancing sale.

J.P. Verma, *Data Analysis in Management with SPSS Software*,
DOI 10.1007/978-81-322-0786-3_4, © Springer India 2013

To find how strongly a given variable is associated with the performance of an employee, an index known as product moment correlation coefficient "r" may be computed. The product moment correlation coefficient is also known as correlation coefficient, and it measures only linear relation between two variables.

When we have two variables that covary, there are two possibilities. First, the change in a thing is concomitant with the change in another, as the change in a child's age covaries with his weight, that is, the older, the heavier. When higher magnitude on one variable occurs along with higher magnitude on another and the lower magnitudes on both also occur simultaneously, then the things vary together positively, and we denote this situation as positive correlation.

In the second situation, two things vary inversely. In other words, the higher magnitudes of one variable go along with the lower magnitudes of the other and vice versa. This situation is denoted as negative correlation.

The higher magnitude of correlation coefficient simply indicates that there is more likelihood that if the value of one variable increases, the value of other variable also increases or decreases. However, correlation coefficient does not reveal the real relationship between the two variables until the effects of other variables are eliminated.

This fact can be well explained with the following example. John and Philip work for the same company. John has a big villa costing $540,000, whereas Philip owns a three-room apartment, costing $160,000. Which person has a greater salary?

Here, one can reasonably assume that it must be John who earns more, as he has a more expensive house. As he earns a larger salary, the chances are that he can afford a more expensive house. One cannot be absolutely certain; of course, it may be that John's villa was a gift from his father, or he could have gotten it in a contest or it might be a result of any legal settlement. However, most of the time, an expensive house means a larger salary.

In this case, one may conclude that there is a correlation between someone's salary and the cost of the house that he/she possesses. This means that as one figure changes, one can expect the other to change in a fairly regular way.

In order to be confident that the relationship exists between any two variables, it must be exhibited across some cases. A case is a component of variation in a thing. For example, different levels of IQ that go along with different marks obtained in the final examination may be perceived across students. If the correlation between IQ and marks of the students is positive, it indicates that a student with high IQ has high marks and the one with low IQ has low marks.

The correlation coefficient gives fair estimate of the extent of relationship between any two variables if the subjects are chosen at random. But in most of the situations, samples are purposive, and, therefore, correlation coefficient in general may not give the correct picture of the real relationship. For example, in finding correlation coefficient between the age of customers and quantity of moisturizer purchased, if the sample is collected from the high socioeconomic population, the result may not be valid as in this section of society, people understand the importance of the product and can afford to invest on it. However, to establish the relationship between sales and age of the users, one should collect the sample from all the socioeconomic status groups.

Even if the sample is random, it is not possible to find the real relationship between any two variables as it might be affected by other variables. For instance, if the correlation computed between height and weight of the children belonging to age category 12–18 years is 0.85, it may not be considered as the real relationship. Here all the subjects are in the developmental age, and in this age category, if the height increases, weight also increases; therefore, the relationship exhibited between height and weight is due to the impact of age as well. To know the real relationship between the height and weight, one must eliminate the effect of age. This can be done in two ways. First, all the subjects can be taken in the same age category, but it is not possible in the experimental situation once the data collection is over. Even if an experimenter tries to control the effect of one or two variables manually, it may not be possible to control the effect of other variables; otherwise one might end up with getting one or two samples only for the study.

In the second approach, the effects of independent variables are eliminated statistically by partialing out their effects by computing partial correlation. Partial correlation provides the relationship between any two variables after partialing out the effect of other independent variables.

Although the correlation coefficient may not give the clear picture of the real relationship between any two variables, it provides the inputs for computing partial and multiple correlations, and, therefore, in most of the studies, it is important to compute the correlation matrix among the variables. This chapter discusses the procedure for computing correlation matrix and partial correlation using SPSS.

## Details of Correlation Matrix and Partial Correlation

Matrix is an arrangement of scores in rows and column, and if its elements are correlation coefficients, it is known as correlation matrix. Usually in correlation matrix, upper diagonal values of the matrix are written. For instance, the correlation matrix with the variables $X_1$, $X_2$, $X_3$, and $X_4$ may look like as follows:

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 1 | 0.5 | 0.3 | 0.6 |
| $X_2$ | | 1 | 0.7 | 0.8 |
| $X_3$ | | | 1 | 0.4 |
| $X_4$ | | | | 1 |

The lower diagonal values in the matrix are not written because of the fact that the correlation between $X_2$ and $X_4$ is same as the correlation between $X_4$ and $X_2$.

Some authors prefer to write the above correlation matrix in the following form:

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | | 0.5 | 0.3 | 0.6 |
| $X_2$ | | | 0.7 | 0.8 |
| $X_3$ | | | | 0.4 |
| $X_4$ | | | | |

In this correlation matrix, diagonal values are not written as it is obvious that these values are 1 because correlation between the same two variables is always one.

In this section, we shall discuss the product moment correlation and partial correlation along with testing of their significance.

## *Product Moment Correlation Coefficient*

Product moment correlation coefficient is an index which provides the magnitude of linear relationship between any two variables. When we refer to correlation matrix, it is usually a matrix of product moment correlation coefficients. It is represented by "r" and is given by the following formula:

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}} \tag{4.1}$$

where $N$ is the number of paired scores. The limits of $r$ are from $-1$ to $+1$. The positive value of $r$ means higher scores on one variable tend to be paired with higher scores on the other, or lower scores on one variable tend to be paired with lower scores on the other. On the other hand, negative value of $r$ means higher scores on one variable tend to be paired with lower scores on the other and vice versa. Further, $r = +1$ indicates the perfect positive relationship between the two variables. This means that if there is an increase (decrease) in $X$ by an amount "a," the $Y$ will also be increased (decreased) by the same amount. Similarly $r = -1$ signifies the perfect negative linear correlation between the two variables. In this case, if the variable $X$ is increased (decreased) by an amount "b," then the variable $Y$ shall be decreased (increased) by the same amount. The three extreme values of the correlation coefficient $r$ can be shown graphically in Fig. 4.1.

**Example 4.1:** Following are the scores on age and memory retention. Compute the correlation coefficient and test its significance at 5% level (Table 4.1).

*Solution* In order to compute the correlation coefficient, first of all the summation $\sum X$, $\sum Y$, $\sum X^2$, $\sum Y^2$, and $\sum XY$ shall be computed in Table 4.2.

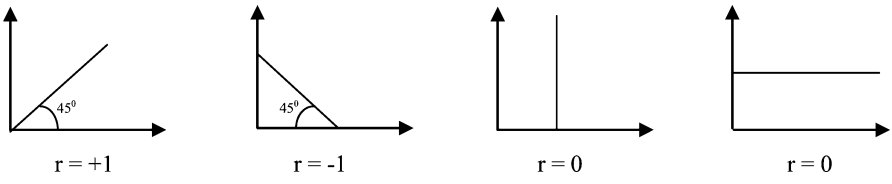



**Fig. 4.1** Graphical presentations of the three extreme cases of correlation coefficient

**Table 4.1** Data on age and memory retention

| S.N. | Age | Memory retention |
|---|---|---|
| 1 | 11 | 7 |
| 2 | 12 | 5 |
| 3 | 8 | 7 |
| 4 | 9 | 6 |
| 5 | 7 | 8 |
| 6 | 10 | 5 |
| 7 | 8 | 7 |
| 8 | 9 | 8 |
| 9 | 10 | 6 |
| 10 | 7 | 8 |

**Table 4.2** Computation for correlation coefficient

| S.N. | Age ($X$) | Memory retention ($Y$) | $X^2$ | $Y^2$ | $XY$ |
|---|---|---|---|---|---|
| 1 | 11 | 7 | 121 | 49 | 77 |
| 2 | 12 | 5 | 144 | 25 | 60 |
| 3 | 8 | 7 | 64 | 49 | 56 |
| 4 | 9 | 6 | 81 | 36 | 54 |
| 5 | 7 | 8 | 49 | 64 | 56 |
| 6 | 10 | 5 | 100 | 25 | 50 |
| 7 | 8 | 7 | 64 | 49 | 56 |
| 8 | 9 | 8 | 81 | 64 | 72 |
| 9 | 10 | 6 | 100 | 36 | 60 |
| 10 | 7 | 8 | 49 | 64 | 56 |
| Total | 91 | 67 | 853 | 461 | 597 |

Here $N$ is 10:

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$

Substituting the values in the equation,

$$r = \frac{10 \times 597 - 91 \times 67}{\sqrt{[10 \times 853 - 91^2][10 \times 461 - 67^2]}}$$
$$= \frac{-127}{\sqrt{249 \times 121}} = -0.732$$

*Testing the Significance*

To test whether the correlation coefficient −0.732 is significant or not, the tabulated value of $r$ required for significance at .05 level of significance and $N - 2(=8)$ degree of freedom can be seen from Table A.3 in the Appendix, which is 0.632. Hence, it may be concluded that there is a significant negative relationship between age and memory retention power. In other words, it may be inferred that as the age increases, the memory retention power decreases.

**Properties of Coefficient of Correlation**

1. The correlation coefficient is symmetrical with respect to the variables. In other words, correlation between height and weight is same as the correlation between weight and height. Mathematically $r_{xy} = r_{yx}$.
2. The correlation coefficient between any two variables lies in between $-1$ and $+1$. In other words, $-1 \leq r \leq 1$.
   Consider the following sum of the squares:

$$\sum \left[\frac{X-\bar{X}}{\sigma_x} \pm \frac{Y-\bar{Y}}{\sigma_y}\right]^2$$

$$\sigma_x^2 = \frac{1}{n}\sum (X-\bar{X})^2 \Rightarrow \sum (X-\bar{X})^2 = n\sigma_x^2$$

Since $\sigma_y^2 = \frac{1}{n}\sum (Y-\bar{Y})^2 \Rightarrow \sum (Y-\bar{Y})^2 = n\sigma_y^2$

and $r_{xy} = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{n\sigma_x\sigma_y} \Rightarrow \sum (X-\bar{X})(Y-\bar{Y}) = n\sigma_x\sigma_y r_{xy}$

Now

$$\begin{aligned}\sum \left[\frac{X-\bar{X}}{\sigma_x} \pm \frac{Y-\bar{Y}}{\sigma_y}\right]^2 &= \frac{\sum (X-\bar{X})^2}{\sigma_x^2} \pm 2\frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sigma_x\sigma_y} + \frac{\sum (Y-\bar{Y})^2}{\sigma_y^2} \\ &= \frac{n\sigma_x^2}{\sigma_x^2} \pm \frac{2n\sigma_x\sigma_y r_{xy}}{\sigma_x\sigma_y} + \frac{n\sigma_y^2}{\sigma_y^2} = 2n \pm 2nr \\ &= 2n(1 \pm r)\end{aligned}$$

Since the expression in the left-hand side is always a positive quantity,

$$\therefore \qquad 2n(1 \pm r) \ \geq \ 0 \ (n{>}0)$$

Taking positive sign

$$1 + r \geq 0 \qquad \therefore\ r \geq -1 \tag{4.2}$$

And if the sign is negative,

$$1 - r \geq 0 \quad \therefore\ r \leq 1 \tag{4.3}$$

Combining (4.2) and (4.3),

$$-1 \leq r \leq 1$$

3. The correlation coefficient is independent of origin and unit of measurement (scale), that is,
if the two variables are denoted as $X$ and $Y$ and $r_{xy}$, the correlation coefficient, then

$$r_{xy} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{(X - \bar{X})^2}\sqrt{(Y - \bar{Y})^2}} \tag{4.4}$$

Let us apply the transformation by shifting the origin and scale of $X$ and $Y$.
Let $U = \frac{X-a}{h}$ and $\mathrm{V} = \frac{\mathrm{Y-b}}{\mathrm{k}}$
where $a$, $b$, $h$, and $k$ are constants.

$$\therefore \mathrm{X} = \mathrm{a} + \mathrm{hU} \Rightarrow \sum X = \sum a + h \sum U$$
$$\Rightarrow \bar{X} = a + h\bar{U}$$

Thus, $$X - \bar{\mathrm{X}} = h(U - \bar{\mathrm{U}}) \tag{4.5}$$

Similarly,

$$\mathrm{Y} = \mathrm{b} + \mathrm{kV} \Rightarrow \sum Y = \sum b + k \sum V$$
$$\Rightarrow \bar{Y} = b + k\bar{V}$$

Thus, $$Y - \bar{\mathrm{Y}} = k(V - \bar{\mathrm{V}}) \tag{4.6}$$

Substituting the values of $(X - \bar{X})$and $(Y - \bar{Y})$from The Equations (4.5) and (4.6) into (4.4),

$$r_{x,y} = \frac{\sum h(U - \bar{U}) \times k(V - \bar{V})}{\sqrt{h^2 \sum (U - \bar{U})^2}\sqrt{k^2 \sum (V - \bar{V})^2}}$$
$$= \frac{hk \sum (U - \bar{U})(V - \bar{V})}{hk\sqrt{\sum (U - \bar{U})^2}\sqrt{\sum (V - \bar{V})^2}}$$
$$= \mathrm{r_{u,v}}$$
$$\therefore \mathrm{r_{x,y}} = \mathrm{r_{u,v}}$$

Thus, it may be concluded that the coefficient of correlation between any two variables is independent of change of origin and scale.

4. Correlation coefficient is the geometrical mean between two regression coefficients. If $b_{yx}$ and $b_{xy}$ are the regression coefficients, then

$$r_{xy} = \pm\sqrt{b_{yx} \times b_{xy}}$$

**Correlation Coefficient May Be Misleading**

As per the definition, correlation coefficient indicates the linear relationship between the two variables. This value may be misleading at times. Look at the following three situations:

1. Researchers often conclude that a high degree of correlation implies a causal relationship between the two variables, but this is totally unjustified. For example, both events, represented by $X_1$ and $X_2$, might simply have a common cause. If in a study, $X_1$ represents the gross salary of the family per month and $X_2$ is the amount of money spent on the sports and leisure activities per month, then a strong positive correlation between $X_1$ and $X_2$ should not be concluded that the people spend more on sports and leisure activities if their family income is more. Now, if a third variable $X_3$, the socioeconomic status, is taken into account, it becomes clear that, despite the strong positive value of their correlation coefficient, there is no causal relationship between "sports and leisure expenditure" and "family income," and that both are in fact caused by the third variable "socioeconomic status." It is not the family income which encourages a person to spend more on sports and leisure activities but the socioeconomic status which is responsible for such a behavior. To know the causal relationship, partial correlation may be used with limitations.
2. A low or insignificant value of the correlation coefficient may not signify the lack of a strong link between the two variables under consideration. The lower value of correlation coefficient may be because of the other variables affecting the relationships in a negative manner. And, therefore, the effect of those variables eliminated may increase the magnitude of the correlation coefficient. Path Analysis may provide the insight in this direction where a correlation coefficient may be split into direct and indirect relationships.
3. The ecological fallacy is another source of misleading correlation coefficient. It occurs when a researcher makes an inference about the correlation in a particular situation based on correlation of aggregate data for a group. For instance, if a high degree of relationship exists between height and performance of athletes in the USA, it does not indicate that every tall athlete's performance is excellent in the USA. And if we conclude so, it will be an ecological fallacy.
4. Correlation does not explain causative relationship. High degree of correlation between two variables does not indicate that one variable causes another. In other words, correlation does not show cause and effect relationship. In a distance learning program, if there is a high degree of correlation between the student's performance and the number of contact classes attended, it does not necessarily indicate that one gets more marks because he learns more during contact classes. Neither does it necessarily imply that the more classes you attend, the more intelligent you become and get good marks. Some other explanation might also explain the correlation coefficient. The correlation means that the one who attends more contact classes gets higher marks and those who attend less classes get less marks. It does not explain why it is the case.

5. One must ensure that the result of correlation coefficient should be generalized only for that population from which the sample was drawn. Usually for a specific small sample, correlation may be high for any two variables, and if it is so, then it must be verified with the larger representative and relevant sample.

### Limitations of Correlation Coefficients

One of the main limitations of the correlation coefficient is that it measures only linear relationship between the two variables. Thus, correlation coefficient should be computed only when the data are measured either on interval scale or ratio scale. The other limitation of the correlation coefficient is that it does not give the real relationship between the variables. To overcome this problem, partial correlation may be computed which explains the real relationship between the variables after controlling for other variables with certain limitations.

### Testing the Significance of Correlation Coefficient

After computing the correlation coefficient, the next question is to find as to whether it actually explains some relationship or it is due to chance.

The following mutually exclusive hypotheses are tested by using the statistical test for testing the significance of correlation coefficient:

$H_0$: $\bar{r} = 0$ (There is no correlation between the two variables in the population.)
$H_1$: $\bar{r} \neq 0$ (There is a correlation between the two variables in the population.)

In fact, $\bar{r}$ indicates the population correlation coefficient, and we test its significance on the basis of the sample correlation coefficient. To test the above set of hypotheses, any of the following three approaches may be used.

#### First Approach

The easiest way to test the null hypothesis mentioned above is to look for the critical value of $r$ with $n - 2$ degrees of freedom at any desired level of significance in Table A.3 in the Appendix. If the calculated value of $r$ is less than or equal to the critical value of $r$, null hypothesis would fail to be rejected, and if calculated $r$ is greater than critical value of $r$, null hypothesis may be rejected. For instance, if the correlation coefficient between height and self-esteem of 25 individuals is 0.45, then the critical value of $r$ required for significance at .05 level of significance and $N - 2(=23)$ df from Table A.3 in the Appendix can be seen as 0.396. Since calculated value of $r$, that is, 0.45 is greater than the critical value of $r$ (=0.396), the null hypothesis may be rejected at .05 level of significance, and we may conclude that there is a significant correlation between height and self-esteem.

Second Approach

Significance of correlation coefficient may be tested by using *t*-test as well. In this case, *t*-statistic is given by the following formula:

$$t = \frac{r}{\sqrt{1 - r^2}} \sqrt{n - 2} \tag{4.7}$$

Here $r$ is the observed correlation coefficient and $n$ is the number of paired sets of data.

The calculated value of $t$ is compared with that of tabulated value of $t$ at .05 level and $n-2$ df ($=t_{.05}(n-2)$). The value of tabulated $t$ can be obtained from Table A.2 in the Appendix.

| | | |
|---|---|---|
| Thus, if | Cal $t \leq t_{.05}(n-2)$, | null hypothesis is failed to be rejected at .05 level of significance |
| and if | Cal $t > t_{.05}(n-2)$, | null hypothesis may be rejected at .05 level of significance |

Third Approach

In this approach, significance of correlation coefficient is tested on the basis of its $p$ value. $p$ value is the probability of wrongly rejecting the null hypothesis. If $p$ value is .04 for a given correlation coefficient, it indicates that the chances of wrongly rejecting the null hypothesis are only 4%. Thus, so long $p$ value is less than .05 the correlation coefficient is significant and the null hypothesis may be rejected at 5% level. On the other hand, if $p$ value is more than or equal to .05, the correlation coefficient is not significant and the null hypothesis may not be rejected at 5% level.

**Note:** The SPSS output follows third approach and provides $p$ values for each of the correlation coefficient in the correlation matrix.

## *Partial Correlation*

Partial correlation is the measure of relationship between two variables after partialing out the effect of one or more independent variables. In computing partial correlation, the data must be measured either on interval or on ratio scale. For example, one may compute partial correlation if it is desired to see the relationship of age with stock portfolio after controlling the effect of income. Similarly to understand the relationship between price and demand would involve studying the relationship between price and demand after controlling the effect of money supply, exports, etc.

The partial correlation between $X_1$ and $X_2$ adjusted for $X_3$ is given by

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}} \tag{4.8}$$

The limits of partial correlation are −1 to +1.

The order of partial correlation refers to the number of independent variables whose effects are to be controlled. Thus, first-order partial correlation controls the effect of one variable, second-order partial correlation controls the effect of two variables, and so on.

The generalized formula for $(n - 2)$th order partial correlation is given by

$$r_{12.34.....n} = \frac{r_{12.345....(n-1)} - r_{1n.345...(n-1)}r_{2n.345....(n-1)}}{\sqrt{1 - r_{1n.345....(n-1)}^2}\sqrt{1 - r_{2n.345....(n-1)}^2}} \tag{4.9}$$

### Limitations of Partial Correlation

1. Since partial correlation is computed by using product moment correlation coefficient, it also assumes the linear relationship. But generally, this assumption is not valid especially in social sciences, as linear relationship rarely exists in such parameters.
2. The reliability of partial correlation decreases if its order increases.
3. Large number of data is required to draw the valid conclusions from the partial correlations.
4. In spite of controlling the effect of many variables, one cannot be sure that the partial correlation explains the real relationship.

### Testing the Significance of Partial Correlation

The significance of partial correlation is tested in a similar way as has been discussed above in case of product moment correlation.

In SPSS, significance of partial correlation is tested on the basis of $p$ value. The partial correlation would be significant at 5% level if its $p$ value is less than .05 and will be insignificant if the $p$ value is equal to or more than .05.

### Computation of Partial Correlation

**Example 4.2:** The following correlation matrix shows the correlation among different academic performance parameters. Compute partial correlations $r_{12.3}$ and $r_{12.34}$ and test their significance. Interpret the findings also (Table 4.3).

**Table 4.3** Correlation matrix among different paramters

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 1 | 0.7 | 0.6 | 0.4 |
| $X_2$ | | 1 | 0.65 | 0.3 |
| $X_3$ | | | 1 | 0.5 |
| $X_4$ | | | | 1 |

$X_1$: GMAT scores, $X_2$: Mathematics marks in high school, $X_3$: IQ scores, $X_4$: GPA scores

*Solution*

(i) Computation of $r_{12.3}$

Since we know that $$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}}$$

Substituting the values of correlation coefficients from the correlation matrix, we get

$$\begin{aligned} r_{12.3} &= \frac{0.7 - 0.6 \times 0.65}{\sqrt{1 - 0.6^2}\sqrt{1 - 0.65^2}} \\ &= \frac{0.70 - 0.39}{\sqrt{0.64}\sqrt{0.5775}} \\ &= 0.51 \end{aligned}$$

(ii) Computation of $r_{12.34}$

Since we know that $r_{12.34} = \dfrac{r_{12.3} - r_{14.3}r_{24.3}}{\sqrt{1 - r_{14.3}^2}\sqrt{1 - r_{24.3}^2}}$

We shall first compute the first-order partial correlations $r_{12.3}$, $r_{14.3,}$ and $r_{24.3}$ which are required to compute the second-order partial correlation $r_{12.34}$. Since $r_{12.3}$ has already been computed above, the remaining two shall be computed here.

Thus,

$$r_{14.3} = \frac{r_{14} - r_{13}r_{43}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{43}^2}} = \frac{0.4 - 0.6 \times 0.5}{\sqrt{1 - 0.6^2}\sqrt{1 - 0.5^2}} = \frac{0.1}{0.69} = 0.14$$

and

$$r_{24.3} = \frac{r_{24} - r_{23}r_{43}}{\sqrt{1 - r_{23}^2}\sqrt{1 - r_{43}^2}} = \frac{0.3 - 0.65 \times 0.5}{\sqrt{1 - 0.65^2}\sqrt{1 - 0.5^2}} = \frac{-0.025}{0.66} = -0.04$$

After substituting the values of $r_{12.3}$,$r_{14.3}$, and $r_{24.3}$, the second-order partial correlation becomes

$$r_{12.34} = \frac{r_{12.3} - r_{14.3}r_{24.3}}{\sqrt{1 - r_{14.3}^2}\sqrt{1 - r_{24.3}^2}} = \frac{0.51 - 0.14 \times (-0.04)}{\sqrt{1 - 0.14^2}\sqrt{1 - (-0.04)^2}}$$
$$= \frac{0.51 + 0.0056}{\sqrt{0.98}\sqrt{0.998}} = \frac{0.5156}{0.989}$$
$$= 0.521$$

## Situation for Using Correlation Matrix and Partial Correlation

Employee's performance largely depends upon their work environment, and, therefore, organizations give more emphasis to improve the working environment of their employees by means of different programs and policies. In order to know the various parameters that are responsible for job satisfaction, statistical techniques like correlation coefficient and partial correlation can be used. With the help of these statistics, one can understand the extent of multicollinearity among independent variables besides understanding the pattern of relationship between job satisfaction and independent variables.

In order to develop an effective strategy to improve the level of job satisfaction of employees, one should know as to what parameters are significantly associated with it. These variables can be identified from the correlation matrix. All those variables which show significant relation with the job satisfaction may be identified for further investigation. Out of these identified variables, it may be pertinent to know as to which variable is the most important one. Simply looking to the magnitude of the correlation coefficient, it is not possible to identify the most important variable responsible for job satisfaction because high correlation does not necessarily mean real relationship as it may be due to other independent variables. Thus, in order to know as to which variable is the most important one, partial correlation may be computed by eliminating the effect of other variables so identified in the correlation matrix.

The application of correlation matrix and partial correlation can be understood by considering the following research study:

Consider a situation where an organization is interested in investigating the relationship of job satisfaction with certain environmental and motivational variables obtained on its employees. Besides finding the relationships of job satisfaction with environmental and motivational variables, it may be interesting to know the relationships among the environmental and motivational variables as well. The following variables may be taken in the study:

*Dependent variable*

1. Job satisfaction ($X_1$)

*Independent variables*

1. Autonomy ($X_2$)
2. Organizational culture ($X_3$)
3. Compensation ($X_4$)
4. Upward communications ($X_5$)
5. Job training opportunity ($X_6$)
6. Management style ($X_7$)
7. Performance appraisal ($X_8$)
8. Recognition ($X_9$)
9. Working atmosphere ($X_{10}$)
10. Working relationships ($X_{11}$)

The following computations may be done to fulifil the objectives of the study:

1. Compute product moment correlation coefficient between Job satisfaction and each of the environmental and motivational variables.
2. Identify few independent variables that show significant correlations with the Job satisfaction for further developing the regression model. Say these selected variables are $X_3$, $X_9$, $X_6$, and $X_2$.
3. Out of these identified variables in step 2, pick up the one having the highest correlation with the dependent variable ($X_1$), say it is $X_6$.
4. Then find the partial correlation between the variables $X_1$ and $X_6$ by eliminating the effect of variables $X_3$, $X_9$, and $X_2$ in steps. In other words, find the first-order partial correlation $r_{16.3}$, second-order partial correlation $r_{16.39}$, and third-order partial correlation $r_{16.392}$.
5. Similarly find the partial correlation between other identified variables $X_3$, $X_9$, and $X_2$ with that of dependent variable ($X_1$) in steps. In other words, compute the following three more sets of partial correlation:

(i) $r_{13.9}$, $r_{13.96}$, and $r_{13.962}$
(ii) $r_{19.3}$, $r_{19.36}$, and $r_{19.362}$
(iii) $r_{12.3}$, $r_{12.39}$, and $r_{12.396}$

## *Research Hypotheses to Be Tested*

*By computing product moment correlation and partial correlation, the hypotheses that can be tested are as follows:*

(a) To test the significance of relationship between Job satisfaction and each of the environmental and motivational variables
(b) To test the significance of relationship among independent variables
(c) Whether few environmental and motivational variables are highly related with Job satisfaction

### *Statistical Test*

To address the objectives of the study and to test the listed hypotheses, the following computations may be done:

1. Correlation matrix among all the independent variables and dependent variable
2. Partial correlations of different orders between the Job satisfaction and identified independent variables

Thus, we have seen how a research situation requires computing correlation matrix and partial correlations to fulfill the objectives.

## Solved Example of Correlation Matrix and Partial Correlations by SPSS

**Example 4.3** To understand the relationships between patient's loyalty and other variables, a study was conducted on 20 patients in a hospital. The following data was obtained. Construct the correlation matrix and compute different partial correlations using SPSS and interpret the findings (Table 4.4).

**Table 4.4** Data on patient's loyalty and other determinants

| S.N. | Trust of patient | Service quality | Customer satisfaction | Patient loyalty |
|---|---|---|---|---|
| 1 | 37 | 69 | 52 | 25 |
| 2 | 35 | 69 | 50 | 20 |
| 3 | 41 | 94 | 70 | 26 |
| 4 | 33 | 50 | 41 | 7 |
| 5 | 54 | 91 | 66 | 25 |
| 6 | 41 | 69 | 56 | 20 |
| 7 | 44 | 68 | 53 | 23 |
| 8 | 45 | 95 | 71 | 32 |
| 9 | 49 | 95 | 68 | 21 |
| 10 | 42 | 75 | 57 | 28 |
| 11 | 35 | 82 | 70 | 28 |
| 12 | 37 | 80 | 57 | 22 |
| 13 | 47 | 82 | 61 | 23 |
| 14 | 44 | 74 | 59 | 26 |
| 15 | 54 | 100 | 78 | 29 |
| 16 | 35 | 82 | 54 | 24 |
| 17 | 39 | 63 | 36 | 16 |
| 18 | 32 | 57 | 38 | 15 |
| 19 | 53 | 99 | 74 | 32 |
| 20 | 49 | 98 | 63 | 25 |

*Solution* First of all, correlation matrix shall be computed using SPSS. Option shall be selected to show the significant correlation values. After selecting the variables that shows significant correlation with the customer loyalty, partial correlation shall be computed between customer loyalty and any of these selected variables after controlling the effect of the remaining variables. The correlation coefficients and partial correlations so obtained in the output using SPSS shall be tested for their significance by using the *p* value.

## *Computation of Correlation Matrix Using SPSS*

(a) **Preparing Data File**
Before using the SPSS commands for computing correlation matrix, the data file is required to be prepared. The following steps will help you prepare the data file:

(i) *Starting the SPSS:* Use the following command sequence to start SPSS:

**Start ⟶ All Programs ⟶ IBM SPSS Statistics ⟶ IBM SPSS Statistics 20**

After checking the option **Type in Data** on the screen you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* There are four variables, namely, *Customer trust, Service quality, Customer satisfaction*, and *Customer loyalty* that need to be defined. Since these variables were measured on interval scale, they will be defined as Scale variable in SPSS. Any variable measured on interval or ratio scale is defined as Scale variable in SPSS. The procedure of defining the variable in SPSS is as follows:

1. Click **Variable View** to define variables and their properties.
2. Write short name of these variables, that is, *Trust, Service, Satisfaction,* and *Loyalty* under the column heading **Name.**
3. Full names of these variables may be defined as *Customer trust, Service quality, Customer satisfaction*, and *Customer loyalty* under the column heading **Label**.
4. Under the column heading **Measure**, select the option "Scale" for all these variables.
5. Use default entries in rest of the columns.

After defining all the variables in Variable View, the screen shall look like Fig. 4.2.

(iii) *Entering data* After defining these variables in the **Variable View**, click **Data View** on the left bottom of the screen to open the format for entering data. For each variable, enter the data column wise. After entering data, the screen will look like Fig. 4.3. Save the data file in the desired location before further processing.

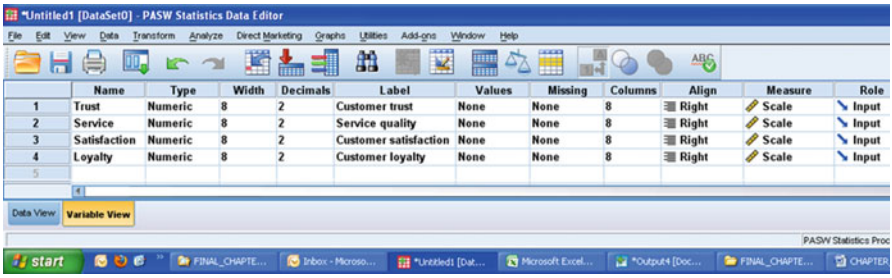



**Fig. 4.2** Defining variables along with their characteristics

(b) **SPSS *Commands for Computing Correlation Coefficient***

After preparing the data file in data view, take the following steps to prepare the correlation matrix:

(i) *Initiating the SPSS commands to compute correlations:* In Data View, click the following commands in sequence:

**Analyze ⟶ Correlate ⟶ Bivariate**

The screen shall look like Fig. 4.4.

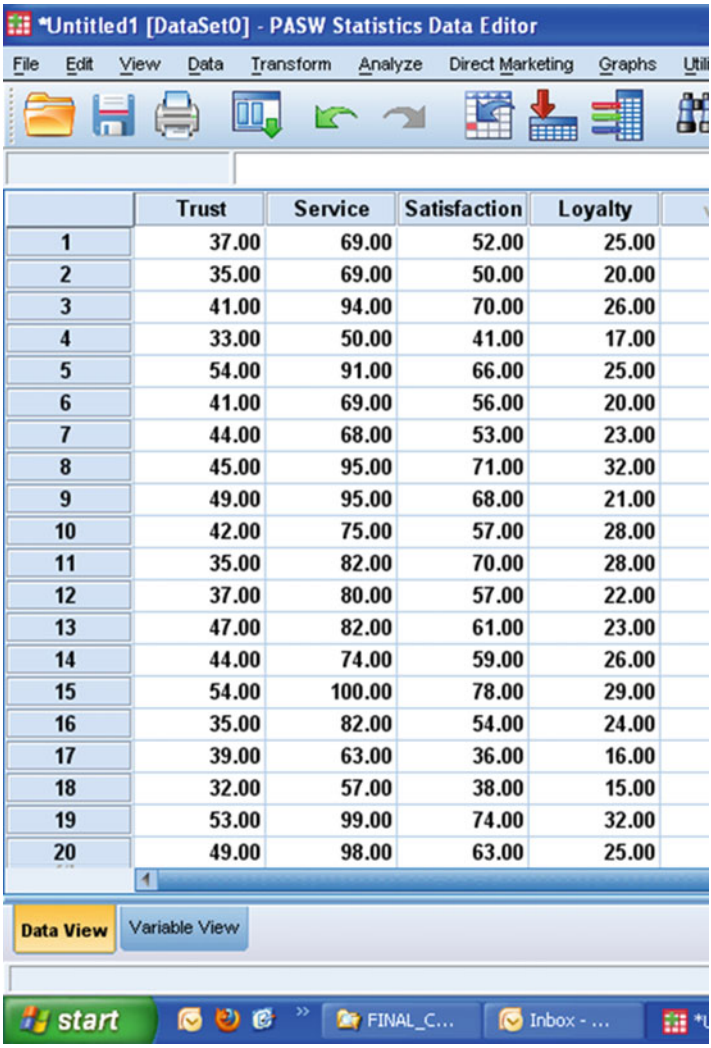
(ii) *Selecting variables for correlation matrix:* Clicking **Bivariate** option will take you to the next screen for selecting variables for the correlation matrix. Select all the variables from left panel to the right panel by using the arrow key. The variable selection may be made one by one or all at once. After selecting the variables, the screen shall look like Fig. 4.5.

(iii) *Selecting options for computation* After selecting the variables, option need to be defined for the correlation analysis. Take the following steps:

- In the screen shown in Fig. 4.5, ensure that the "Pearson," "Two-tailed," and "Flag significant correlations" options are checked. By default they are checked.
- Click the tag **Options**. This will take you to the screen shown in Fig. 4.6.
  - Check the option "Means and standard deviation."
  - Use default entries in other options. Readers are advised to try other options and see what changes they are getting in their output.
  - Click ***Continue***. This will take you back to the screen shown in Fig. 4.5.
- Click **OK.**

(c) ***Getting the Output***

After clicking **OK,** output shall be generated in the output windows. The two outputs generated in the form of descriptive statistics and correlation matrix are shown in Tables 4.5 and 4.6.




| | Trust | Service | Satisfaction | Loyalty |
|---|---|---|---|---|
| 1 | 37.00 | 69.00 | 52.00 | 25.00 |
| 2 | 35.00 | 69.00 | 50.00 | 20.00 |
| 3 | 41.00 | 94.00 | 70.00 | 26.00 |
| 4 | 33.00 | 50.00 | 41.00 | 17.00 |
| 5 | 54.00 | 91.00 | 66.00 | 25.00 |
| 6 | 41.00 | 69.00 | 56.00 | 20.00 |
| 7 | 44.00 | 68.00 | 53.00 | 23.00 |
| 8 | 45.00 | 95.00 | 71.00 | 32.00 |
| 9 | 49.00 | 95.00 | 68.00 | 21.00 |
| 10 | 42.00 | 75.00 | 57.00 | 28.00 |
| 11 | 35.00 | 82.00 | 70.00 | 28.00 |
| 12 | 37.00 | 80.00 | 57.00 | 22.00 |
| 13 | 47.00 | 82.00 | 61.00 | 23.00 |
| 14 | 44.00 | 74.00 | 59.00 | 26.00 |
| 15 | 54.00 | 100.00 | 78.00 | 29.00 |
| 16 | 35.00 | 82.00 | 54.00 | 24.00 |
| 17 | 39.00 | 63.00 | 36.00 | 16.00 |
| 18 | 32.00 | 57.00 | 38.00 | 15.00 |
| 19 | 53.00 | 99.00 | 74.00 | 32.00 |
| 20 | 49.00 | 98.00 | 63.00 | 25.00 |

**Fig. 4.3** Scheme of data feeding for all the variables

## *Interpretation of the Outputs*

The values of mean and standard deviation for all the variables are shown in Table 4.5. The user may draw the conclusions accordingly, and the findings may be used for further analysis in the study.

The actual output shows the full correlation matrix, but only upper diagonal values of the correlation matrix are shown in Table 4.6. This table shows the magnitude of correlation coefficients along with their $p$ values and sample size. The product moment correlation coefficient is also known as Pearson correlation as

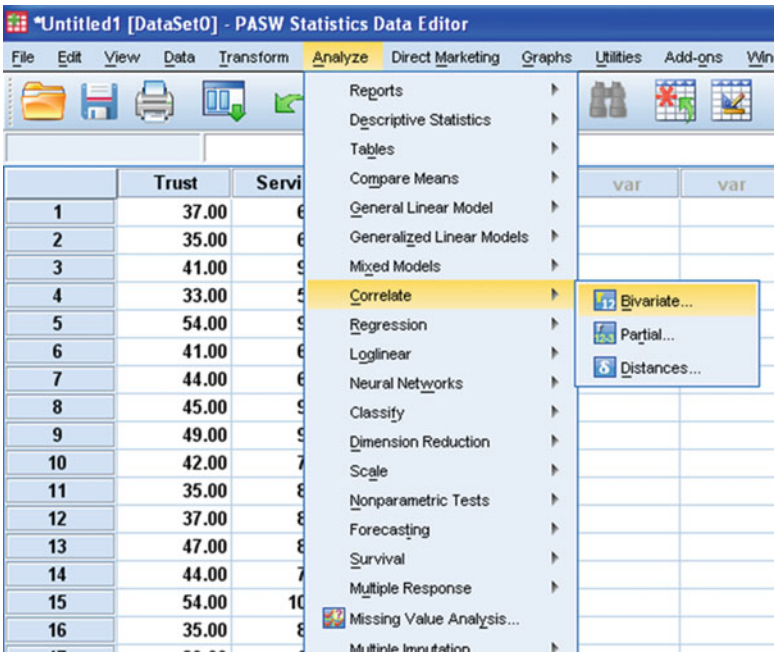



**Fig. 4.4** Screen showing SPSS commands for computing correlation matrix

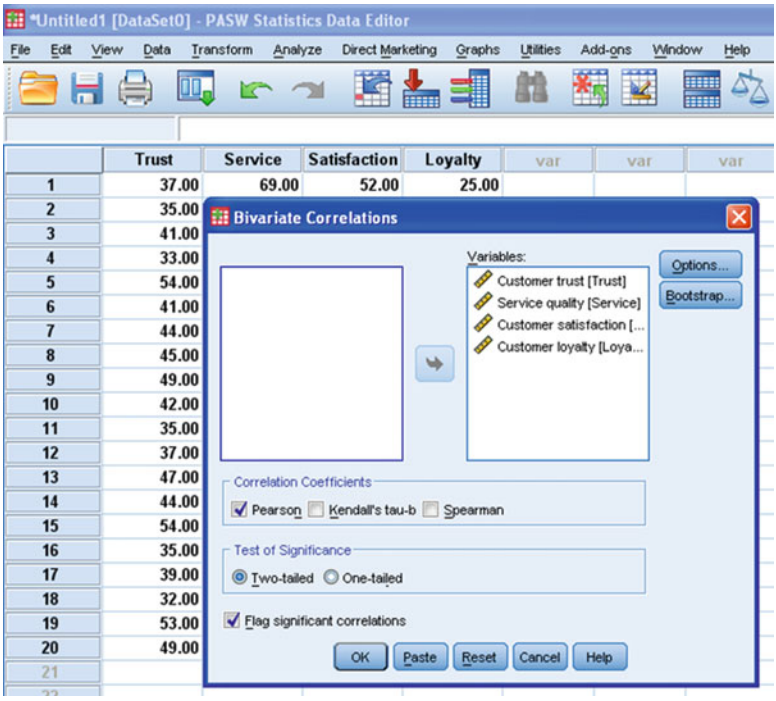



**Fig. 4.5** Screen showing selection of variables for computing correlation matrix

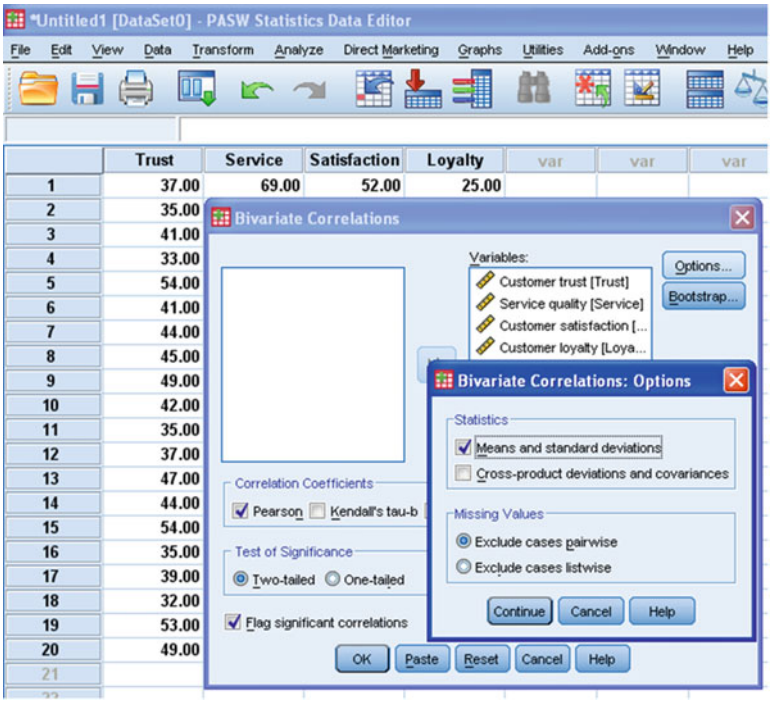



**Fig. 4.6** Screen showing option for computing correlation matrix and other statistics

it was developed by the British mathematician Karl Pearson. The value of correlation coefficient required for significance (known as critical value) at 5% as well as at 1% level can be seen from Table A.3 in the Appendix. Thus, at 18 degrees of freedom, the critical values of $r$ at 5 and 1% are 0.444 and 0.561, respectively. The correlation coefficient with one asterisk (*) mark is significant at 5% level, whereas the one with two asterisk (**) marks shows the significance at 1% level. In this example, the research hypothesis is two-tailed which states that "There is a significant correlation between the two variables." The following conclusions may be drawn from the results in Table 4.6:

(a) The Customer loyalty is significantly correlated with customer trust at 5% level, whereas it is significantly correlated with Service quality and Customer satisfaction at 1% level.
(b) Customer satisfaction is highly correlated with service quality. This is rightly so as only satisfied customers would be loyal to any hospital.
(c) All those correlation coefficients having $p$ value less than .05 are significant at 5% level. This is shown by asterisk (*) mark by the side of the correlation coefficient. Similarly correlations having $p$ value less than .01 are significant at 1% level, and this is indicated by two asterisk (**) marks by the side of correlation coefficient.

**Table 4.5** Descriptive statistics for the data on customer's behavior

| Variables | Mean | SD | *N* |
|---|---|---|---|
| Customer trust | 42.3000 | 7.01952 | 20 |
| Service quality | 79.6000 | 14.77338 | 20 |
| Customer satisfaction | 58.7000 | 11.74779 | 20 |
| Customer loyalty | 23.8500 | 4.79336 | 20 |

**Table 4.6** Correlation matrix for the data on customer's behavior along with p values

| | | Customer trust | Service quality | Customer satisfaction | Customer loyalty |
|---|---|---|---|---|---|
| | | ($X_1$) | ($X_2$) | ($X_3$) | ($X_4$) |
| Customer trust | Pearson correlation | 1 | .754** | .704** | .550* |
| ($X_1$) | sig. (2-tailed) | | .000 | .001 | .012 |
| | *N* | 20 | 20 | 20 | 20 |
| Service quality | Pearson correlation | | 1 | .910** | .742** |
| ($X_2$) | sig. (2-tailed) | | | .000 | .000 |
| | *N* | | 20 | 20 | 20 |
| Customer satisfaction | Pearson correlation | | | 1 | .841** |
| ($X_3$) | sig. (2-tailed) | | | | .000 |
| | *N* | | | 20 | 20 |
| Customer loyalty | Pearson correlation | | | | 1 |
| ($X_4$) | sig. (2-tailed) | | | | |
| | *N* | | | | 20 |

**Correlation is significant at the 0.01 level (2-tailed); *Correlation is significant at the 0.05 level (2-tailed)

## *Computation of Partial Correlations Using SPSS*

The decision of variables among which partial correlation needs to be computed depends upon objective of the study. In computing partial correlation, one of the variables is usually a criterion variable, and the other is the independent variable having the highest magnitude of correlation with the criterion variable. Criterion variable is the one in which the variation is studied as result of variation in other independent variables. Usually criterion variable is known as dependent variable. Here the criterion variable is the Customer loyalty because the effect of other variables needs to be investigated on it. Depending upon the situation, the researcher may choose any variable other than the highest correlated variable for computing partial correlation with that of dependent variable. In this example, partial correlation shall be computed between Customer loyalty ($X_4$) and Customer satisfaction ($X_3$) after eliminating the effect of Service quality ($X_2$) and Customer trust ($X_1$). This is because $X_3$ is highly correlated with the criterion variable $X_4$.

The decision of eliminating the effect of variables $X_2$ and $X_1$ has been taken because both these variables are significantly correlated with the criterion variable. However, one can investigate the relationship between $X_4$ vs. $X_2$ after eliminating the effect of the variables $X_3$ and $X_1$. Similarly partial correlation between $X_4$ vs. $X_1$ may also be investigated after eliminating the effect of the variables $X_3$ and $X_2$. The procedure of computing these partial correlations with SPSS has been discussed in the following sections:

(a) ***Data File for Computing Partial Correlation***
The data file which was prepared for computing correlation matrix shall be used for computing the partial correlations. Thus, procedure for defining the variables and entering the data for all the variables is exactly the same as was done in case of computing correlation matrix.

(b) **SPSS *Commands for Partial Correlation***
After entering all the data in the data view, take the following steps for computing partial correlation:

(i) *Initiating the SPSS commands for partial correlation:* In Data View, go to the following commands in sequence:

**Analyze ⟶ Correlate ⟶ Partial**

The screen shall look like Fig. 4.7.

(ii) *Selecting variables for partial correlation:* After clicking the **Partial** option, you will get the next screen for selecting variables for the partial correlation.

- Select the two variables *Customer loyalty* ($X_4$) and *Customer satisfaction* ($X_3$) from the left panel to the "Variables" section in the right panel. Here, relationship between the variables $X_4$ and $X_3$ needs to be computed after controlling the effects of *Service quality* ($X_2$) and *Customer trust* ($X_1$).
- Select the variables *Service quality* ($X_2$) and *Customer trust* ($X_1$) from the left panel to the "Controlling for" section in the right panel. $X_2$ and $X_1$ are the two variables whose effects are to be eliminated.

The selection of variables is made either one by one or all at once. To do so, the variable needs to be selected from the left panel, and by arrow command, it may be brought to the right panel. The screen shall look like Fig. 4.8.

(iii) *Selecting options for computation:* After selecting the variables for partial correlation and identifying controlling variables, option needs to be defined for the computation of partial correlation. Take the following steps:

- In the screen shown in Fig. 4.8, ensure that the options "Two-tailed" and "Display actual significance level" are checked. By default they are checked.
- Click the tag **Options**; you will get the screen as shown in Fig. 4.9. Take the following steps:
  - Check the box of "Means and standard deviations."

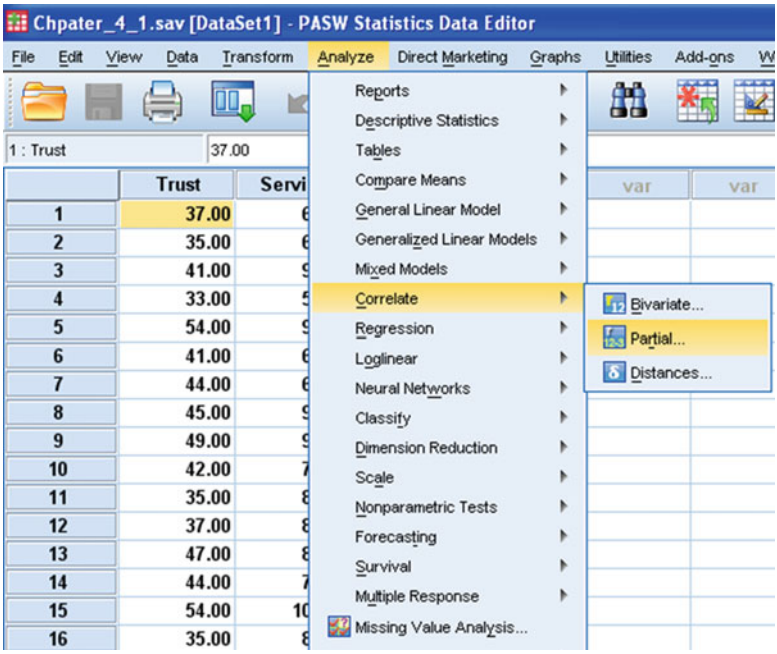



**Fig. 4.7** Screen showing SPSS commands for computing partial correlations

- Use the default entries in other options. Readers are advised to try other options and see what changes they are getting in their outputs.
- Click ***Continue***.

– Click **OK.**

(c) ***Getting the Output***

After clicking **OK**, outputs shall be generated in the output panel. The output panel shall have two tables: one for descriptive statistics and the other for partial correlation. These outputs can be selected by right click of the mouse and may be pasted in the word file. In this example, the output so generated by the SPSS will look like as shown in Tables 4.7 and 4.8.

## *Interpretation of Partial Correlation*

Table 4.7 shows the descriptive statistics of all the variables selected in the study. Values of mean and standard deviations may be utilized for further analysis. Readers may note that similar table of descriptive statistics was also obtained while computing correlation matrix by using SPSS.

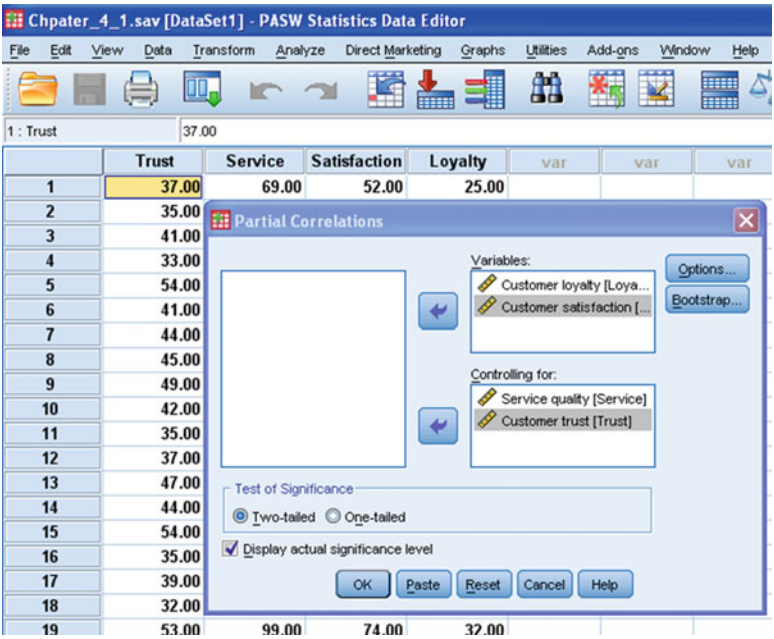



**Fig. 4.8** Screen showing selection of variables for partial correlation

In Table 4.8, partial correlation between Customer loyalty ($X_4$) and Customer satisfaction ($X_3$) after controlling the effect of Service quality ($X_2$) and Customer trust ($X_1$) is shown as 0.600. Since $p$ value for this partial correlation is .009, which is less than .01, it is significant at 1% level. It may be noted that the correlation coefficient between Customer loyalty and Customer satisfaction in Table 4.6 is 0.841 which is highly significant, but when the effects of Service quality and Customer trust are eliminated, the actual correlation dropped down to 0.600. But this partial correlation of 0.600 is still highly correlated in the given sample, and, hence, it may be concluded that within the framework of this study, there exists a real relationship between Customer loyalty and Customer satisfaction. One may draw the conclusion that at all cost, Customer satisfaction is the most important factor for maintaining patient's loyalty towards the hospital.

## Summary of the SPSS Commands

(a) For Computing Correlation Matrix

1. Start the SPSS by using the following commands:

**Start⟶ All Programs ⟶ IBM SPSS Statistics ⟶ IBM SPSS Statistics 20**

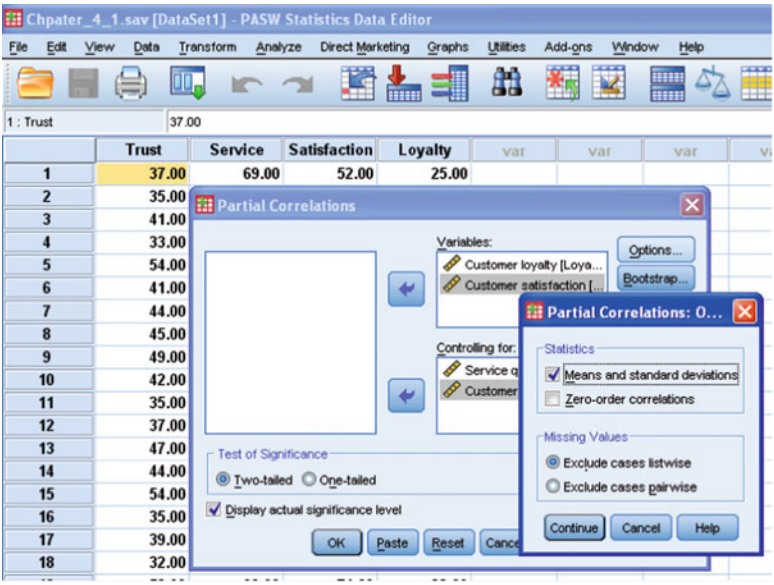



**Fig. 4.9** Screen showing option for computing partial correlation and other statistics

**Table 4.7** Descriptive statistics for the variables selected for partial correlations

| Variables | Mean | SD | *N* |
|---|---|---|---|
| Customer loyalty | 23.8500 | 4.79336 | 20 |
| Customer satisfaction | 58.7000 | 11.74779 | 20 |
| Service quality | 79.6000 | 14.77338 | 20 |
| Customer trust | 42.3000 | 7.01952 | 20 |

**Table 4.8** Partial correlation between Customer loyalty ($X_4$) and Customer satisfaction ($X_3$) after controlling the effect of Service quality ($X_3$) and Customer trust ($X_1$)

| Control variables | | | Customer loyalty ($X_4$) | Customer satisfaction ($X_3$) |
|---|---|---|---|---|
| Service quality ($X_2$) and Customer trust ($X_1$) | Customer loyalty ($X_4$) | Correlation | 1.000 | .600 |
| | | significance (2-tailed) | | .009 |
| | | df | 0 | 16 |
| | Customer satisfaction ($X_3$) | Correlation | .600 | 1.000 |
| | | significance (2-tailed) | .009 | |
| | | df | 16 | 0 |

Note: Readers are advised to compute partial correlations of different orders with the same data

2. Click **Variable View** tag and define the variables *Trust, Service, Satisfaction,* and *Loyalty* as Scale variables.
3. Once the variables are defined, type the data column wise for these variables by clicking **Data View**.
4. In Data View, click the following commands in sequence for correlation matrix:

**Analyze ⟶ Correlate ⟶ Bivariate**

5. Select all the variables from left panel to the "Variables" section of the right panel.
6. Ensure that the options "Pearson," "Two-tailed," and "Flag significant correlations" are checked by default.
7. Click the tag **Options** and check the box of "Means and standard deviations." Click ***Continue***.
8. Click **OK** for output.

(b) **For Computing Partial Correlation**

1. Follow steps 1–3 as discussed above.
2. With the same data file, follow the below-mentioned commands in sequence for computing partial correlations:

**Analyze ⟶ Correlate ⟶ Partial**

3. Select any two variables between which the partial correlation needs to be computed from left panel to the "Variables" section of the right panel.
   Select the variables whose effects are to be controlled, from left panel to the "Controlling for" section in the right panel.
4. After selecting the variables for computing partial correlation, click the caption **Options** on the screen. Check the box "Means and standard deviation" and press ***Continue***.
5. Click **OK** to get the output of the partial correlation and descriptive statistics.

## Exercise

*Short-Answer Questions*

**Note:** Write the answer to each of the questions in not more than 200 words.

Q.1. "Product moment correlation coefficient is a deceptive measure of relationship, as it does not reveal anything about the real relationship between two variables." Comment on this statement.

Q.2. Describe a research situation in management where partial correlation can be used to draw some meaningful conclusions.

Q.3. Compute correlation coefficient between $X$ and $Y$ and interpret your findings considering that $Y$ and $X$ are perfectly related by equation $Y = X^2$.

| $X$: | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $Y$: | 9 | 4 | 1 | 0 | 1 | 4 |

Q.4. How will you test the significance of partial correlation using $t$-test?

Q.5. What does the $p$ value refer to? How is it used in testing the significance of product moment correlation coefficient?

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. In testing the significance of product moment correlation coefficient, degree of freedom for $t$-test is

   (a) $N - 1$
   (b) $N + 2$
   (c) $N + 1$
   (d) $N - 2$

2. If the sample size increases, the value of correlation coefficient required for its significance

   (a) Increases
   (b) Decreases
   (c) Remains constant
   (d) May increase or decrease

3. Product moment correlation coefficient measures the relationship which is

   (a) Real
   (b) Linear
   (c) Curvilinear
   (d) None of the above

4. Given that $r_{12} = 0.7$ and $r_{12.3} = 0.28$, where $X_1$ is academic performance, $X_2$ is entrance test score, and $X_3$ is IQ, what interpretation can be drawn?

   (a) Entrance test score is an important contributory variable to the academic performance.
   (b) IQ affects the relationship between academic performance and entrance test score in a negative fashion.
   (c) IQ has got nothing to do with the academic performance.
   (d) It seems there is no real relationship between academic performance and entrance test score.

5. If $p$ value for a partial correlation is 0.001, what conclusion can be drawn?

   (a) Partial correlation is not significant at 5% level.
   (b) Partial correlation is significant at 1% level.

(c) Partial correlation is not significant at 1% level.
(d) Partial correlation is not significant at 10% level.

6. Partial correlation is computed with the data that are measured in

(a) Interval scale
(b) Nominal scale
(c) Ordinal scale
(d) Any scale

7. In computing correlation matrix through SPSS, all variables are defined as

(a) Nominal
(b) Ordinal
(c) Scale
(d) Any of the nominal, ordinal, or scale option depending upon the nature of variable

8. In computing correlation matrix through SPSS, the following command sequence is used:

(a) Analyze -> Bivariate -> Correlate
(b) Analyze -> Correlate -> Bivariate
(c) Analyze -> Correlate -> Partial
(d) Analyze -> Partial -> Bivariate

9. While selecting variables for computing partial correlation in SPSS, in "Controlling for" section, the variables selected are

(a) All independent variables except the two between which partial correlation is computed.
(b) Any of the independent variables as it does not affect partial correlation.
(c) Only those variables whose effects need to be eliminated.
(d) None of the above is correct.

10. The limits of partial correlation are

(a) −1 to 0
(b) 0−1
(c) Sometimes more than 1
(d) −1 to +1

*Assignments*

1. In a study, Job satisfaction and other organizational variables as perceived by the employees were assessed. The data were obtained on interval scale and are shown in the below mentioned table. Compute the following:

(a) Correlation matrix with all the seven variables
(b) Partial correlations : $r_{12.3}$, $r_{12.35}$, and $r_{12.356}$
(c) Partial correlations : $r_{13.2}$, $r_{13.25}$, and $r_{13.256}$
(d) Partial correlations : $r_{16.2}$, $r_{16.23}$, and $r_{16.235}$

Data on Job satisfaction and other organizational variables as perceived by the employees

| S. N. | Job satisfaction ($X_1$) | Autonomy ($X_2$) | Organizational culture ($X_3$) | Compensation ($X_4$) | Job training opportunity ($X_5$) | Recognition ($X_6$) | Working atmosphere ($X_7$) |
|---|---|---|---|---|---|---|---|
| 1 | 75 | 45 | 34 | 23 | 35 | 44 | 23 |
| 2 | 65 | 41 | 31 | 24 | 32 | 34 | 32 |
| 3 | 34 | 27 | 25 | 14 | 28 | 38 | 25 |
| 4 | 54 | 38 | 28 | 25 | 37 | 32 | 27 |
| 5 | 47 | 32 | 26 | 26 | 28 | 37 | 31 |
| 6 | 33 | 28 | 32 | 14 | 25 | 23 | 32 |
| 7 | 68 | 41 | 38 | 23 | 38 | 32 | 28 |
| 8 | 76 | 42 | 45 | 28 | 29 | 42 | 42 |
| 9 | 68 | 37 | 42 | 26 | 29 | 37 | 24 |
| 10 | 45 | 29 | 35 | 27 | 19 | 30 | 22 |
| 11 | 36 | 32 | 23 | 31 | 18 | 26 | 31 |
| 12 | 66 | 33 | 44 | 28 | 38 | 39 | 43 |
| 13 | 72 | 41 | 45 | 24 | 35 | 36 | 27 |
| 14 | 58 | 41 | 38 | 25 | 26 | 36 | 28 |
| 15 | 26 | 23 | 25 | 26 | 24 | 18 | 19 |
| 16 | 61 | 45 | 42 | 22 | 36 | 42 | 39 |

2. The data in the following table shows the determinants of US domestic price of copper during 1966–1980. Compute the following and interpret your findings:

   (a) Correlation matrix with all the six variables
   (b) Partial correlations: $r_{12.3,}$ $r_{12.34}$, and $r_{12.346}$
   (c) Partial correlations: $r_{13.2,}$ $r_{13.24}$, and $r_{13.246}$

   Determinants of US domestic price of copper

| Year | Avg. domestic copper price ($X_1$) | GNP ($X_2$) | Industrial production ($X_3$) | Exchange copper price ($X_4$) | No. of housing/year ($X_5$) | Aluminum price ($X_6$) |
|---|---|---|---|---|---|---|
| 1966 | 36.60 | 753.00 | 97.8 | 555.0 | 1,195.8 | 24.50 |
| 1967 | 38.60 | 796.30 | 100.0 | 418.0 | 1,321.9 | 24.98 |
| 1968 | 42.20 | 868.50 | 106.3 | 525.2 | 1,545.4 | 25.58 |
| 1969 | 47.90 | 935.50 | 111.1 | 620.7 | 1,499.5 | 27.18 |
| 1970 | 58.20 | 982.40 | 107.8 | 588.6 | 1,469.0 | 28.72 |
| 1971 | 52.00 | 1,063.4 | 109.6 | 444.4 | 2,084.5 | 29.00 |
| 1972 | 51.20 | 1,171.1 | 119.7 | 427.8 | 2,378.5 | 26.67 |
| 1973 | 59.50 | 1,306.6 | 129.8 | 727.1 | 2,057.5 | 25.33 |
| 1974 | 77.30 | 1,412.9 | 129.3 | 877.6 | 1,352.5 | 34.06 |
| 1975 | 64.20 | 1,528.8 | 117.8 | 556.6 | 1,171.4 | 39.79 |
| 1976 | 69.60 | 1,700.1 | 129.8 | 780.6 | 1,547.6 | 44.49 |
| 1977 | 66.80 | 1,887.2 | 137.1 | 750.7 | 1,989.8 | 51.23 |
| 1978 | 66.50 | 2,127.6 | 145.2 | 709.8 | 2,023.3 | 54.42 |

(continued)

(continued)

| Year | Avg. domestic copper price ($X_1$) | GNP ($X_2$) | Industrial production ($X_3$) | Exchange copper price ($X_4$) | No. of housing/year ($X_5$) | Aluminum price ($X_6$) |
|---|---|---|---|---|---|---|
| 1979 | 98.30 | 2,628.8 | 152.5 | 935.7 | 1,749.2 | 61.01 |
| 1980 | 101.40 | 2,633.1 | 147.1 | 940.9 | 1,298.5 | 70.87 |

Note: The data were collected by Gary R. Smith from sources such as American Metal Market, Metals Week, and US Department of Commerce publications

*Answers to Multiple-Choice Questions*

| | | | |
|---|---|---|---|
| Q.1 | d | Q.2 | b |
| Q.3 | b | Q.4 | d |
| Q.5 | b | Q.6 | a |
| Q.7 | c | Q.8 | b |
| Q.9 | c | Q.10 | d |

# Chapter 5
# Regression Analysis and Multiple Correlations: For Estimating a Measurable Phenomenon

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Explain the use of regression analysis and multiple correlation in research.
- Interpret various terms involved in regression analysis.
- Learn to use SPSS for doing regression analysis.
- Understand the procedure of identifying the most efficient regression model.
- Know the method of constructing the regression equation based on the SPSS output.

## Introduction

Regression analysis deals with estimating the value of dependent variable on the basis of one or more independent variables. To do so, an equation is developed between dependent and independent variables by means of least square method. When the estimation is done on the basis of one independent variable, the procedure is known as simple regression, and if the estimation involves more than one independent variable, it is referred to as multiple regression analysis.

In multiple regression analysis, the dependent variable is referred to as $Y$, whereas independent variables are denoted as $X$. The dependent variable is also known as criterion variable. The goal is to develop an equation that will determine the $Y$ variable in a linear function of corresponding $X$ variables. The regression equation can be either linear or curvilinear, but our discussion shall be limited to linear regression only.

In regression analysis, a regression model is developed by using the observed data obtained on dependent variable and several independent variables. During the process, only those independent variables are picked up for developing the model which shows significant relationship with dependent variable. Therefore, the researcher must be careful in identifying the independent variables in regression

J.P. Verma, *Data Analysis in Management with SPSS Software*,
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analysis study. It may be quite possible that some of the important independent variables might have been left in the study, and, therefore, in spite of the best possible effort, the regression model so developed may not be reliable.

Multiple regression analysis can be used in many applications of management and behavioral researches. Numerous situations can be listed where the use of this technique can provide an edge to the decision makers for optimum solutions.

For example, in order to evaluate and reform the existing organization and make them more responsive to the new challenges, the management may be interested to know; the factors responsible for sale to plan the business strategy, the estimated number of inventory required in a given month, and the factors affecting the job satisfaction of the employees. They may also be concerned in developing the model for deciding the pay packets of an employee, factors that motivate people to work, or parameters that affect the productivity of work. In all these situations, regression model may provide the input to the management for strategic decision-making. The success of the model depends upon the inclusion of relevant independent variables in the study. For instance, a psychologist may like to draw up variables that directly affect one's mental health causing abnormal behavior. Therefore, it is important for the researchers to review the literature thoroughly for identifying the relevant independent variables for estimating the criterion variable.

Besides regression analysis, there are other quantitative and qualitative methods used in performance forecasting. But the regression analysis is one of the most popularly used quantitative techniques.

In developing a multiple regression equation, one needs to know the efficiency in estimating the dependent variable on the basis of the identified independent variables in the model. The efficiency of estimation is measured by the coefficient of determination ($R^2$) which is the square of multiple correlation. The coefficient of determination explains the percentage of variance in the dependent variable by the identified independent variables in the model. The multiple correlation explains the relationship between the group of independent variables and dependent variable. Thus, high multiple correlation ensures greater accuracy in estimating the value of dependent variable on the basis of independent variables. Usually multiple correlation, $R$ is computed during regression analysis to indicate the validity of regression model. It is necessary to show the value of $R^2$ along with regression equation for having an idea about the efficiency in prediction.

Any regression model having larger multiple correlation gives better estimates in comparison to that of other models. We will see an explanation of the multiple correlation while discussing the solved example later in this chapter.

## Terminologies Used in Regression Analysis

In order to use the regression analysis effectively, it is essential to know different terminologies involved in it. These terms are discussed in the following sections.

## *Multiple Correlation*

Multiple correlation is a measure of relationship between a group of independent variables and a dependent variable. Since multiple correlation provides the strength of relationship between dependent variable and independent variables, it is used to determine the power of regression models also. The multiple correlation is represented by "*R*" and is computed by the following formula:

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{13}\, r_{23}}{1 - r_{23}^2}} \tag{5.1}$$

If the number of independent variables is more than two, then the multiple correlation is computed from the following formula:

$$R_{1.2345....n} = \sqrt{1 - (1 - r_{12}^2)(1 - r_{13.2}^2)(1 - r_{14.23}^2).....(1 - r_{1n.23...(n-1)}^2)} \tag{5.2}$$

where $r_{13.2}$, $r_{14.23}$, and $r_{1n.234...(n\ -\ 1)}$ are partial correlations.

The multiple correlation $R$ can have the value in between 0 and +1. Since multiple correlation is computed with the help of product moment correlation coefficients, therefore it also measures the linear relationship only. Further, the order of the multiple correlation is defined by $n - 2$, where $n$ is the number of variables involved in the computation of multiple correlation. Thus, the order of the multiple correlation $R_{12.345.....n}$ is $n - 2$. The value of $R$ closer to 1 indicates that the independent variables explain most of the variations in the dependent variable. On the other hand, if the value of $R$ is closer to 0, it signifies that independent variables are not capable of explaining the variation in the dependent variable. Thus, multiple correlation can be considered to be the yardstick of efficiency in estimating the value of dependent variable on the basis of the values of independent variables.

### Properties of Multiple Correlation

1. The multiple correlation can never be lower than the highest correlation between dependent and any of the independent variables. For instance, the value of $R_{1.234}$ can never be less than the value of any of the product moment correlations $r_{12}$, $r_{13}$, or $r_{14}$.
2. Sometimes, an independent variable does not show any relationship with dependent variable, but if it is combined with some other variable, its effect becomes significant. Such variable is known as suppression variable. These suppression variables should be handled carefully. Thus, if the independent variables are identified on the basis of their magnitude of correlations with the dependent variable for developing regression line, some of the suppression variable might

**Table 5.1** Correlation matrix of psychological variables

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ | 1 | −0.5 | 0.3 | 0.6 |
| $X_2$ | | 1 | 0.4 | 0.3 |
| $X_3$ | | | 1 | 0.4 |
| $X_4$ | | | | 1 |

$X_1$: Memory retention, $X_2$: Age, $X_3$: IQ, $X_4$: Stress level

be ignored. To handle this problem, SPSS provides the stepwise regression method.

3. In using the stepwise regression, the variables are picked up one by one depending upon their relative importance. Every time one variable is included, there is an increase in multiple correlation. But the increase in multiple correlation keeps on decreasing with the inclusion of every new variable. This is known as *law of diminishing return*.

**Example 5.1** Following is the correlation matrix obtained on the psychological variables. Compute $R_{1.23}$ and $R_{1.234}$ and interpret the findings (Table 5.1):

*Solution*

(i)

$$\therefore R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}}$$

Substituting values of these correlations from the correlation matrix,

$$R_{1.23} = \sqrt{\frac{(-0.5)^2 + 0.3^2 - 2 \times (-0.5) \times 0.3 \times 0.4}{1 - 0.4^2}}$$
$$= \sqrt{\frac{0.46}{0.84}} = 0.74$$

(ii)

$$\therefore R_{1.234} = \sqrt{1 - \left[\left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)\left(1 - r_{14.23}^2\right)\right]}$$

To compute $R_{1.234}$, we need to first find the values of first-order partial correlations: $r_{13.2}$, $r_{14.2,}$ and $r_{43.2}$.

$$r_{13.2} = \frac{r_{13} - r_{12}r_{32}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{32}^2}} = \frac{0.3 - (-0.5) \times 0.4}{\sqrt{1 - (-0.5)^2}\sqrt{1 - 0.4^2}} = \frac{0.50}{0.79} = 0.63$$

$$r_{14.2} = \frac{r_{14} - r_{12}r_{42}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{42}^2}} = \frac{0.6 - (-0.5) \times 0.3}{\sqrt{1 - (-0.5)^2}\sqrt{1 - 0.3^2}} = \frac{0.75}{0.83} = 0.90$$

$$r_{43.2} = \frac{r_{43} - r_{42}r_{32}}{\sqrt{1 - r_{42}^2}\sqrt{1 - r_{32}^2}} = \frac{0.4 - 0.3 \times 0.4}{\sqrt{1 - 0.3^2}\sqrt{1 - 0.4^2}} = \frac{0.28}{0.87} = 0.32$$

After substituting these partial correlations, the second-order partial correlation $r_{14.23}$ shall be computed; this in turn shall be used to compute the value of $R_{1.234}$.

$$r_{14.23} = \frac{r_{14.2} - r_{13.2}r_{43.2}}{\sqrt{1 - r_{13.2}^2}\sqrt{1 - r_{43.2}^2}} = \frac{0.90 - 0.63 \times 0.32}{\sqrt{1 - 0.63^2}\sqrt{1 - 0.32^2}} = \frac{0.6984}{0.7358} = 0.95$$

Thus, substituting the values of $r_{12}$, $r_{13.2,}$ and $r_{14.23}$ in the following equation:

$$\begin{aligned} R_{1.234} &= \sqrt{1 - \left[\left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)\left(1 - r_{14.23}^2\right)\right]} \\ &= \sqrt{1 - \left[\left(1 - (-0.5)^2\right)\left(1 - 0.63^2\right)\left(1 - 0.95^2\right)\right]} \\ &= \sqrt{1 - [0.75 \times 0.603 \times 0.10]} \\ &= 0.976 \end{aligned}$$

**Interpretation**

Taking $n$ equals to 2 and substituting the value of $r_{12}$ in Eq. (5.2),

$$R_{1.2} = \sqrt{1 - \left[1 - (-0.5)^2\right]} = \sqrt{0.25} = 0.5$$

Now let us have a look on the following values:

$R_{1.2} = 0.5$
$R_{1.23} = 0.74$
$R_{1.234} = 0.976$

It can be seen that the multiple correlation increases with the increase in the independent variable. Further, the increase in multiple correlation is larger when the third independent variable ($X_3$) is included in the model and after that the increase has reduced when one additional independent variable ($X_4$) is introduced.

## *Coefficient of Determination*

It can be defined as the variance explained in the dependent variable on the basis of the independent variables selected in the regression model. It is the square of multiple correlation and is represented by $R^2$. Thus, in regression analysis $R^2$ is

used for assessing the efficiency of the regression model. If for a particular regression model $R$ is 0.8, it means that 64% of the variability in the dependent variable can be explained by the independent variables selected in the model.

## *The Regression Equation*

The equation is said to be simple regression if the value of dependent variable is estimated on the basis of one independent variable only. If $Y$ is the dependent variable and $X$ is the independent variable, then the regression equation of $Y$ on $X$ is written as

$$(Y - \overline{Y}) = b_{yx}(X - \overline{X}) \tag{5.3}$$

Equation 5.3 can be used to predict the value of $Y$ if the value of $X$ is known. Similarly to estimate the value of $X$ from the value of $Y$, the regression equation of $X$ on $Y$ shall be used which is shown in Eq. 5.4.

$$(X - \overline{X}) = b_{xy}(Y - \overline{Y}) \tag{5.4}$$

where $\overline{X}$ and $\overline{Y}$ are the sample means of $X$ and $Y$, respectively, and $b_{yx}$ and $b_{xy}$ are the regression coefficients. These regression coefficients can be computed as

$$b_{yx} = r\frac{\sigma_Y}{\sigma_X} \tag{5.5}$$

$$\mathrm{b}_{xy} = r\frac{\sigma_X}{\sigma_Y} \tag{5.6}$$

After substituting the value of $b_{yx}$ in Eq. (5.3) and solving, we get

$$Y = r\frac{\sigma_Y}{\sigma_X}X + (\overline{Y} - r\frac{\sigma_Y}{\sigma_X}\overline{X}) \tag{5.7}$$

$$\Rightarrow Y = BX + C \tag{5.8}$$

where $B$ is equal to $r\frac{\sigma_Y}{\sigma_X}$ and C is $(\overline{Y} - r\frac{\sigma_Y}{\sigma_X}\overline{X})$. The coefficients B and C are known as unstandardized regression coefficient and regression constant respectively.

**Remark** Reproduce $r\frac{\sigma_Y}{\sigma_X}$ and $(\overline{Y} - r\frac{\sigma_Y}{\sigma_X}\overline{X})$ in equation format. $\overline{Y}$ is the mean of Y and $\overline{X}$ is the mean of X

After substituting the values of $b_{yx}$ and $b_{xy}$ in the regression equations (5.3) and (5.4), we get

$$(Y - \overline{Y}) = r\frac{\sigma_y}{\sigma_x}(X - \overline{X})$$

$$(X - \overline{X}) = r\frac{\sigma_x}{\sigma_y}(Y - \overline{Y})$$

After rearranging the above equations,

$$\frac{(Y - \overline{Y})}{\sigma_y} = r\frac{(X - \overline{X})}{\sigma_x}$$

$$\frac{(X - \overline{X})}{\sigma_x} = r\frac{(Y - \overline{Y})}{\sigma_y}$$

The above two equations can be rewritten as

$$Z_y = \beta_x Z_x \tag{5.9}$$

$$Z_x = \beta_y Z_y \tag{5.10}$$

The Eqs. (5.9) and (5.10) are known as regression equations in standard score form, and the coefficients $\beta_x$ and $\beta_y$ are known as beta coefficients and are referred to as standardized regression coefficients.

**Conditions of Symmetrical Regression Equations**

The two regression equations (5.3) and (5.4) are different. Equation (5.3) is known as regression equation of $Y$ on $X$ and is used to estimate the value of $Y$ on the basis of $X$, whereas Eq. (5.4) is known as regression equation of $X$ on $Y$ and is used for estimating the value of $X$ if $Y$ is known. These two equations can be rewritten as follows:

$$(Y - \overline{Y}) = b_{yx}(X - \overline{X})$$

$$(Y - \overline{Y}) = \frac{1}{b_{xy}}(X - \overline{X})$$

These two regression equations can be same if the expressions in the right-hand side of these two equations are same.

That is,

$$
\begin{aligned}
& b_{yx}(X-\overline{X}) = \frac{1}{b_{xy}}(X-\overline{X}) \\
& \Rightarrow b_{yx} \times b_{xy} = 1 \\
& \Rightarrow r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y} = 1 \\
& \Rightarrow r^2 = 1 \\
& \Rightarrow r = \pm 1
\end{aligned}
$$

Hence, the two regression equations shall be similar if there is a perfect positive or perfect negative correlation between them. In that situation, same regression equation can be used to estimate the value of $Y$ or value of $X$.

**Computation of Regression Coefficient**

The regression coefficient can be obtained for the given set of data by simplifying the formula:

$$
\begin{aligned}
\therefore B &= r\frac{\sigma_Y}{\sigma_X} \\
&= \frac{N\sum XY - \sum X \sum Y}{\sqrt{N\sum X^2 - (\sum X)^2}\sqrt{N\sum Y^2 - (\sum Y)^2}} \times \frac{\sqrt{\frac{1}{N}\sum Y^2 - \left(\frac{\sum Y}{N}\right)^2}}{\sqrt{\frac{1}{N}\sum X^2 - \left(\frac{\sum X}{N}\right)^2}}
\end{aligned}
$$

After solving,

$$
B = \frac{N\sum XY - \sum X \sum Y}{N\sum X^2 - (\sum X)^2} \tag{5.11}
$$

From Eqs. (5.7) and (5.8),

$$
C = \overline{Y} - r\frac{\sigma_Y}{\sigma_X}\overline{X}
$$

After substituting the value of $r\frac{\sigma_Y}{\sigma_X} = B$ from Eq. (5.11), we get

$$
C = \frac{\sum Y}{N} - \frac{N\sum XY - \sum X \sum Y}{N\sum X^2 - (\sum X)^2} \times \frac{\sum X}{N}
$$

After simplification,

$$C = \frac{\sum Y \sum X^2 - \sum X \sum XY}{N \sum X^2 - (\sum X)^2} \tag{5.12}$$

Thus, by substituting the value of $B$ and $C$ in Eq. (5.8), regression equation can be developed.

**Example 5.2** Consider the two sets of scores on job satisfaction ($X$) and autonomy ($Y$) as shown below. Compute the regression coefficient "$B$" and constant "$C$" and develop regression equation.

| | |
|---|---|
| Autonomy ($X$) | : 15 13 7 11 9 |
| Job satisfaction ($Y$) | : 9 8 5 8 6 |

*Solution* The regression equation given by $Y = BX + C$ can be constructed if the regression coefficient $B$ and constant $C$ are known. These can be obtained by the following formula (Table 5.2):

$$\therefore B = \frac{N \sum XY - \sum X \sum Y}{N \sum X^2 - (\sum X)^2} \quad C = \frac{\sum Y \sum X^2 - \sum X \sum XY}{N \sum X^2 - (\sum X)^2}$$

To compute "$B$" and "$C$," we shall first compute $\sum X$, $\sum Y$, $\sum X^2$, and $\sum XY$.

$$B = \frac{N \sum XY - \sum X \sum Y}{N \sum X^2 - (\sum X)^2} = \frac{5 \times 416 - 55 \times 36}{5 \times 645 - 55 \times 55} = 0.5$$

$$C = \frac{\sum Y \sum X^2 - \sum X \sum XY}{N \sum X^2 - (\sum X)^2} = \frac{36 \times 645 - 55 \times 416}{5 \times 645 - 55 \times 55} = 1.7$$

Substituting the values of $B$ and $C$, the regression equation becomes

$$Y(\text{Job satisfaction}) = 0.5X(\text{Autonomy}) + 1.7$$

These values can be obtained from the SPSS output discussed in the solved Example 5.1. The SPSS produces these outputs on the basis of least square methods. The method of least square has been discussed later in this chapter.

### Properties of Regression Coefficients

1. The square root of the product of two regression coefficients is equal to the correlation coefficient between $X$ and $Y$. The sign of the correlation coefficient is equal to the sign of the regression coefficients. Further, the signs of the two regression coefficients are always same.

**Table 5.2** Computation for regression coefficients

| Scores on | | | |
|---|---|---|---|
| Autonomy | Job satisfaction | | |
| ($X$) | ($Y$) | $X^2$ | $XY$ |
| 15 | 9 | 225 | 135 |
| 13 | 8 | 169 | 104 |
| 7 | 5 | 49 | 35 |
| 11 | 8 | 121 | 88 |
| 9 | 6 | 81 | 54 |
| $\sum X = 55$ | $\sum Y = 36$ | $\sum X^2 = 645$ | $\sum XY = 416$ |

$$\therefore b_{yx} \times b_{xy} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y} = r^2$$
$$\Rightarrow r = \pm\sqrt{b_{yx} \times b_{xy}}$$

To prove that the sign of the correlation coefficient between $X$ and $Y$ and both the regression coefficients are same, consider the following formula:

$$r_{xy} = \frac{\mathrm{Cov(X, Y)}}{\sigma_X \sigma_Y} \tag{5.13}$$

$$b_{yx} = \frac{\mathrm{Cov(X, Y)}}{\sigma_x^2} \tag{5.14}$$

$$b_{xy} = \frac{\mathrm{Cov(X, Y)}}{\sigma_y^2} \tag{5.15}$$

Since $\sigma_x$ and $\sigma_y$ are always positive, the values of $r_{xy}$, $b_{yx}$, and $b_{xy}$ will be same and will depend upon the sign of Cov($X$,$Y$).

2. If one of the regression coefficients is greater than 1, the other will have to be less than 1. Thus, in other words, both the regression coefficients can be less than 1 but can never be greater than 1.

We have

$$b_{yx} \times b_{xy} = r^2 \text{and} -1 \leqslant \mathrm{r} \leqslant 1$$
$$\therefore b_{yx} \times b_{xy} \leqslant 1$$
$$\Rightarrow \text{If} \quad \mathrm{b_{yx}} > 1, \text{ then } \mathrm{b_{xy}} < 1$$
$$\text{Or if} \quad \mathrm{b_{xy}} > 1, \text{then } \mathrm{b_{yx}} < 1$$

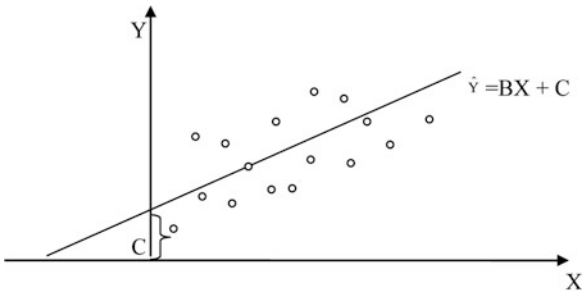



**Fig. 5.1** Plotting of data and the line of best fit

Hence, the two regression coefficients cannot be simultaneously greater than one.

3. The average of the two regression coefficients is always greater than the correlation coefficient.

$$\frac{b_{yx} + b_{xy}}{2} > r$$

## Least Square Method for Regression Analysis

The simple linear regression equation (5.8) is also known as least squares regression equation. Let us plot the paired values of $X_i$ and $Y_i$ for $n$ sets of data; the scattergram shall look like Fig. 5.1.

The line of best fit can be represented as

$$\hat{Y} = BX + C$$

where $B$ is the slope of the line and $C$ is the intercept on $Y$ axis. There can be many lines passing through these points, but the line of best fit shall be the one for which the sum of the squares of the residuals should be least. This fact can be explained as follows:

Each sample point has two dimensions $X$ and $Y$. Thus, for $i$th point, $Y_i$ is the actual value and $\hat{Y}_i$ is the estimated value obtained from the line. We shall call the line as the line of best fit if the total sum of squares is least for all these points.

$$\sum \left(Y_i - \hat{Y}_i\right)^2 = \left(Y_1 - \hat{Y}_1\right)^2 + \left(Y_2 - \hat{Y}_2\right)^2 + \left(Y_3 - \hat{Y}_3\right)^2 + \ldots + \left(Y_n - \hat{Y}_n\right)^2$$

Since the criterion used for selecting the best fit line is based upon the fact that the squares of the residuals should be least, the regression equation is known as least square regression equation. This method of developing regression equation is known as ordinary least square method (OLS) or simply least square method.

**Computation of Regression Coefficients by Least Square Methods**

Least square means that the criterion used to select the best fitting line is that the sum of the squares of the residuals should be least.

In other words, the least squares regression equation is the line for which the sum of squared residuals $\sum \left(Y_i - \hat{Y}_i\right)^2$ is least.

The line of best fit is chosen on the basis of some algebra based on the concept of differentiation and solving the normal equations. We can compute the regression coefficient $B$ and regression constant $C$ so that the sum of the squared residuals is minimized. The procedure is as follows:

Consider a set of $n$ data points $(X_1,Y_1)$, $(X_2,Y_2)$, ..., $(X_n,Y_n)$, then the regression line is

$$\hat{Y}_i = BX_i + C \tag{5.16}$$

and the actual value of $Y_i$ can be obtained by the model

$$Y_i = \hat{Y}_i + \varepsilon_i \tag{5.17}$$

where $Y_i$ is the actual value and $\hat{Y}_i$ is the estimated value obtained from the regression line shown in Eq. (5.16). The $\varepsilon_i$is the amount of error in estimating $Y_i$.

Our effort is to minimize the error $\varepsilon_i \forall_i$, so as to get the best fit of the regression line. This can be done by minimizing the sum of the squared deviation $S^2$ as shown below:

$$S^2 = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n} \left(Y_i - BX_i - C\right))^2 \tag{5.18}$$

The coefficients $B$ and $C$ are so chosen that $S^2$ is minimized. This can be done by differentiating equation (5.18) first with respect to $B$ and then with respect to $C$ and equating the results to zero.

Thus,

$$\frac{\partial S^2}{\partial B} = -2\sum_{i=1}^{n} X_i(Y_i - BX_i - C) = 0$$

and $\frac{\partial S^2}{\partial C} = -2\sum_{i=1}^{n} (Y_i - BX_i - C) = 0$

Solving these equations, we get

$$\sum_{i=1}^{n} X_i(Y_i - BX_i - C) = 0$$

and $\sum_{i=1}^{n}(Y_i - BX_i - C) = 0$

Taking the summation inside the bracket, the equations become

$$B\sum_{i=1}^{n}X_i^2 + C\sum_{i=1}^{n}X_i = \sum_{i=1}^{n}X_iY_i \tag{5.19}$$

$$B\sum_{i=1}^{n}X_i + nC = \sum_{i=1}^{n}Y_i \tag{5.20}$$

The above two equations are known as normal equations having two unknowns $B$ and $C$.

After solving these equations for $B$ and $C$,

we get $$B = \frac{N\sum XY - \sum X\sum Y}{N\sum X^2 - (\sum X)^2}$$

and

$$C = \frac{\sum Y\sum X^2 - \sum X\sum XY}{N\sum X^2 - (\sum X)^2}$$

**Assumptions Used in Linear Regression**

In using the linear regression model, the following assumptions must be satisfied:

1. Both the variables $X$ and $Y$ must be measured on either interval or ratio scale.
2. The regression model is linear in nature.
3. Error terms in estimating the dependent variable are independent and normally distributed.
4. Error distribution in predicting the dependent variable is constant irrespective of the values of $X$.

## *Multiple Regression*

Estimating a phenomenon is always a complex procedure and depends upon numerous factors. Therefore, complex statistical techniques are needed which can deal with interval or ratio data and can forecast for future outcomes. Ordinary least square method which is widely used in case of simple regression is also most widely used in case of predicting the value of dependent variable from the values of two or more independent variables. Regression equation in which dependent variable is estimated by using two or more independent variables is known as

multiple regression. Multiple regression equation having four independent variables looks like

$$Y = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$$

where

$Y$ is a dependent variable
$X_1$, $X_2$, $X_3$, and $X_4$ are the independent variables
$a$ represents regression constant
$b_1$, $b_2$, $b_3$, and $b_4$ are the unstandardized regression coefficients

**Procedure in Multiple Regression**

In developing the multiple regression equation, independent variables must be carefully chosen. Only those variables should be included in the study which is supposed to explain some variation in the dependent variable. Further, one must ensure that high degree of correlations does not exist among the independent variables so as to avoid the multicollinearity.

The following steps are used in multiple regression analysis:

1. Compute descriptive statistics like mean, standard deviation, skewness, kurtosis, frequency distribution, etc., and check the distribution of each variable by testing the significance of skewness and kurtosis.
2. Assess the linearity of each independent variable with the dependent variable by plotting the scatter diagram.
3. Check for multicollinearity among the independent variables by computing the correlation matrix among the independent variables. If multicollinearity exists between the independent variables then one of the independent variables must be dropped as it does not explain additional variability in the dependent variable.
4. Develop a regression equation by using the unstandardized regression coefficients ($B$ coefficients).
5. Test the significance of the regression coefficients by using the $t$-test. As a rule of thumb, a $t$-value greater than 2.0 is usually statistically significant but one must consult a $t$-table to be sure.
6. Test the significance of the regression model by using the $F$-test. The $F$-value is computed by dividing the explained variance by the unexplained variance. In general, an $F$-value of greater than 4.0 is usually statistically significant, but one must consult an $F$-table to be sure.
7. Compute $R^2$ and adjusted $R^2$ to know the percentage variance of the dependent variable as explained by all the independent variables together in the regression model.

### Limitations of Multiple Regression

There are certain limitations of multiple regression which are as follows:

1. Like simple regression, multiple regression also will not be efficient if the independent variables are not linearly related with dependent variable.
2. Multiple regression can be used only if the variables are either measured on interval or ratio scale. In case the data is measured on some other scale, other methods should be used for estimation.
3. Simple regression having one dependent and one independent variable usually requires a minimum of 30 observations. In general, add minimum of at least 10 observations for each additional independent variable added in the study.

### What Happens If the Multicollinearity Exists Among the Independent Variables?

While doing multiple regression if multicollinearity exists, the following things may happen:

1. The *F*-test for the multiple regression equation shows significance, but none of the *t*-ratios for the regression coefficients will be statistically significant.
2. By adding any additional variable in the equation, the size or the sign of the regression coefficients of other independent variables may radically change.

In case the multicollinearity is noted between any two independent variables, one may either drop one of the two independent variables or simply show in their findings that the multicollinearity is present.

### Unstandardized and Standardized Regression Coefficients

Unstandardized regression coefficients are usually known as *B* coefficients, whereas standardized regression coefficients are denoted as $\beta$(beta) coefficients. *B* coefficient explains the slopes of the regression lines. It indicates the *amount of change* in the dependent variable (*Y*) that is associated with a change in one unit of the independent variable (*X*). All *B* coefficients are known as unstandardized coefficients because the magnitude of their values is relative to the means and standard deviations of the independent and dependent variables in the equation. In other words, the slopes can be interpreted *directly* in terms of the raw values of *X* and *Y*. Because the value of a *B* coefficient depends on the scaling of the raw data, therefore it varies if the unit of the independent variable varies. For example, the magnitude of *B* coefficient keeps changing if the unit of the independent variable time changes as days, hours, minutes, etc. Since *B* coefficients depend upon the units of the independent variables, it cannot be easily compared within a regression equation. Thus, unstandardized regression coefficients cannot be used to find the relative importance of the independent variables in explaining the variability of the dependent variable in the model.

In order to compare the relative contribution of the independent variables in the regression model, another type of regression coefficient, beta ($\beta$), is used. These beta coefficients are standardized coefficients such that it adjusts for the different means and variances of the independent variables in the regression model. The standardized regression coefficients in any regression equation are measured on the same scale on 0 to 1. Thus, these standardized regression coefficients can be directly compared to one another, with the largest coefficient indicating the corresponding independent variable having the maximum influence on the dependent variable.

## Procedure of Multiple Regression in SPSS

In using SPSS for regression analysis, the regression coefficients are computed in the output. Significance of these regression coefficients are tested by means of *t*-test. The regression coefficient becomes significant at 5% level if its significance value (*p* value) provided in the output is less than .05. Significance of regression coefficient indicates that the corresponding variable significantly explains the variation in the dependent variable and it contributes to the regression model. *F*-test is computed in the output to test the significance of overall model whereas $R^2$ and adjusted $R^2$ show the percentage variability in the dependent variable as explained by all the independent variables together in the model. Further, standardized regression coefficients are computed in the output to find the relative predictability of the independent variables in the model.

## Methods of Regression Analysis

While doing regression analysis, the independent variables are selected either on the basis of literature or some known information. In conducting a regression study, a large number of independent variables are selected, and, therefore, there is a need to identify only those independent variables which explain the maximum variation in the dependent variable. This can be done by following any of the two methods, namely, "stepwise regression" or "Enter" method in SPSS.

### Stepwise Regression Method

This method is used in exploratory regression analysis where a larger number of independent variables are investigated and the researcher does not have much idea about the relationship of these variables with that of the dependent variable. In stepwise regression analysis, the independent variables are selected one by one depending upon the relative importance in the regression model. In other words, the first entered variable in the model has the largest contribution in explaining variability in the dependent variable. A variable is included in the model if its regression coefficient is significant at 5% level. Thus, if the stepwise regression method is selected for regression analysis, the variables are selected one by one and finally the regression coefficients of the retained variables are generated in the output. These regression coefficients are used to develop the required regression equation.

Enter Method

This method is used in confirmatory regression analysis in which an already developed regression model is tested for its validity on the similar sample group for which it was earlier developed. In this procedure a regression model is developed by selecting all the independent variables in the study. The computed value of $R^2$ is used to assess whether the developed model is valid for the population for which it is tested.

## Application of Regression Analysis

The main focus of any industry is to maximize the profits by controlling different strategic parameters. Optimum processes are identified, employees are motivated, incentives are provided to sales force, and human resources are strengthened to enhance the productivity and improve profit scenario. All these situations lead to an exploratory study where the end result is estimated on the basis of certain independent parameters. For instance, if one decides to know what all parameters are required to boost the sales figure in an organization, then a regression study may be planned. The parameters like employee's incentives, retailer's margin, user's schemes, product info, advertisement expenditure, and socioeconomic status may be studied to develop the regression model. Similarly, regression analysis may be used to identify the parameters responsible for job satisfaction in the organization. In such case, parameters like employee's salary, motivation, incentives, medical facility, family welfare incentives, and training opportunity may be selected as independent variables for developing regression model for estimating the job satisfaction of an employee. Regression analysis may identify independent variables which may be used for developing strategies in production process, inventory control, capacity utilization, sales criteria, etc. Further, regression analysis may be used to estimate the value of dependent variable at some point of time if the values of independent variables are known. This is more relevant in a situation where the value of dependent variable is difficult to know. For instance, in launching a new product in a particular city, one cannot know the sales figure, and accordingly it may affect the decision of stock inventory. By using the regression model on sales, one can estimate the sales figure in a particular month.

## Solved Example of Multiple Regression Analysis Including Multiple Correlation

**Example 5.3** In order to assess the feasibility of a guaranteed annual wage, the Rand Corporation conducted a study to assess the response of labor supply in terms of average hours of work($Y$) based on different independent parameters. The data were drawn from a national sample of 6,000 households with male head earnings less than $15,000 annually. These data are given in Table 5.3. Apply regression

analysis by using SPSS to suggest a regression model for estimating the average hours worked during the year based on identified independent parameters.

*Solution* To develop the regression model for estimating the average hours of working during the year for guaranteed wages on the basis of socioeconomic variables, do the following steps:

(i) Choose the "stepwise regression" method in SPSS to get the regression coefficients of the independent variables identified in the model for developing the regression equation.
(ii) Test the regression coefficients for its significance through *t*-test by using its significance value (*p* value) in the output.
(iii) Test the regression model for its significance through the *F*-value by looking to its significance value (*p* value) in the output.
(iv) Use the value of $R^2$ in the output to know the amount of variance explained in the dependent variable by the identified independent variables together in the model.

Steps involved in getting the output of regression analysis by using SPSS have been explained in the following sections.

## *Computation of Regression Coefficients, Multiple Correlation, and Other Related Output in the Regression Analysis*

(a) ***Preparing Data File***

Before using the SPSS commands for different output of regression analysis, the data file needs to be prepared.

The following steps will help you to prepare the data file:

(i) *Starting SPSS:* Use the following command sequence to start SPSS:

**Start ⟶ All Programs ⟶ IBM SPSS Statistics ⟶ IBM SPSS Statistics 20**

After clicking the **Type in Data**, you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* There are nine variables in this exercise which need to be defined in SPSS first. Since all these variables were measured on interval scale, they will be defined as "Scale" variable in SPSS. The procedure of defining the variables in SPSS is as follows:

1. Click **Variable View** to define variables and their properties.
2. Write short name of these variables, that is, *Hours, Rate, Ersp, Erno, Nein, Assets, Age, Dep,* and *School* under the column heading **Name.**
3. Full name of these variables may be defined as *Average hours worked during the year, Average hourly wage in dollars, Average yearly*

**Table 5.3** Data on average yearly hour and other socioeconomic variables

| | Hours | Rate | ERSP | ERNO | NEIN | Assets | Age | DEP | School |
|---|---|---|---|---|---|---|---|---|---|
| S.N. | ($X_1$) | ($X_2$) | ($X_3$) | ($X_4$) | ($X_5$) | ($X_6$) | ($X_7$) | ($X_8$) | ($X_9$) |
| 1 | 2,157 | 2.905 | 1,121 | 291 | 380 | 7,250 | 38.5 | 2.340 | 10.5 |
| 2 | 2,174 | 2.970 | 1,128 | 301 | 398 | 7,744 | 39.3 | 2.335 | 10.5 |
| 3 | 2,062 | 2.350 | 1,214 | 326 | 185 | 3,068 | 40.1 | 2.851 | 8.9 |
| 4 | 2,111 | 2.511 | 1,203 | 49 | 117 | 1,632 | 22.4 | 1.159 | 11.5 |
| 5 | 2,134 | 2.791 | 1,013 | 594 | 730 | 12,710 | 57.7 | 1.229 | 8.8 |
| 6 | 2,185 | 3.040 | 1,135 | 287 | 382 | 7,706 | 38.6 | 2.602 | 10.7 |
| 7 | 2,210 | 3.222 | 1,100 | 295 | 474 | 9,338 | 39.0 | 2.187 | 11.2 |
| 8 | 2,105 | 2.493 | 1,180 | 310 | 255 | 4,730 | 39.9 | 2.616 | 9.3 |
| 9 | 2,267 | 2.838 | 1,298 | 252 | 431 | 8,317 | 38.9 | 2.024 | 11.1 |
| 10 | 2,205 | 2.356 | 885 | 264 | 373 | 6,789 | 38.8 | 2.662 | 9.5 |
| 11 | 2,121 | 2.922 | 1,251 | 328 | 312 | 5,907 | 39.8 | 2.287 | 10.3 |
| 12 | 2,109 | 2.499 | 1,207 | 347 | 271 | 5,069 | 39.7 | 3.193 | 8.9 |
| 13 | 2,108 | 2.796 | 1,036 | 300 | 259 | 4,614 | 38.2 | 2.040 | 9.2 |
| 14 | 2,047 | 2.453 | 1,213 | 297 | 139 | 1,987 | 40.3 | 2.545 | 9.1 |
| 15 | 2,174 | 3.582 | 1,141 | 414 | 498 | 10,239 | 40.0 | 2.064 | 11.7 |
| 16 | 2,067 | 2.909 | 1,805 | 290 | 239 | 4,439 | 39.1 | 2.301 | 10.5 |
| 17 | 2,159 | 2.511 | 1,075 | 289 | 308 | 5,621 | 39.3 | 2.486 | 9.5 |
| 18 | 2,257 | 2.516 | 1,093 | 176 | 392 | 7,293 | 37.9 | 2.042 | 10.1 |
| 19 | 1,985 | 1.423 | 553 | 381 | 146 | 1,866 | 40.6 | 3.833 | 6.6 |
| 20 | 2,184 | 3.636 | 1,091 | 291 | 560 | 11,240 | 39.1 | 2.328 | 11.6 |
| 21 | 2,084 | 2.983 | 1,327 | 331 | 296 | 5,653 | 39.8 | 2.208 | 10.2 |
| 22 | 2,051 | 2.573 | 1,194 | 279 | 172 | 2,806 | 40.0 | 2.362 | 9.1 |
| 23 | 2,127 | 3.262 | 1,226 | 314 | 408 | 8,042 | 39.5 | 2.259 | 10.8 |
| 24 | 2,102 | 3.234 | 1,188 | 414 | 352 | 7,557 | 39.8 | 2.019 | 10.7 |
| 25 | 2,098 | 2.280 | 973 | 364 | 272 | 4,400 | 40.6 | 2.661 | 8.4 |
| 26 | 2,042 | 2.304 | 1,085 | 328 | 140 | 1,739 | 41.8 | 2.444 | 8.2 |
| 27 | 2,181 | 2.912 | 1,072 | 304 | 383 | 7,340 | 39.0 | 2.337 | 10.2 |
| 28 | 2,186 | 3.015 | 1,122 | 30 | 352 | 7,292 | 37.2 | 2.046 | 10.9 |
| 29 | 2,188 | 3.010 | 990 | 366 | 374 | 7,325 | 38.4 | 2.847 | 10.6 |
| 30 | 2,077 | 1.901 | 350 | 209 | 951 | 370 | 37.4 | 4.158 | 8.2 |
| 31 | 2,196 | 3.009 | 947 | 294 | 342 | 6,888 | 37.5 | 3.047 | 10.6 |
| 32 | 2,093 | 1.899 | 342 | 311 | 120 | 1,425 | 37.5 | 4.512 | 8.1 |
| 33 | 2,173 | 2.959 | 1,116 | 296 | 387 | 7,625 | 39.2 | 2.342 | 10.5 |
| 34 | 2,179 | 2.971 | 1,128 | 312 | 397 | 7,779 | 39.4 | 2.341 | 10.5 |
| 35 | 2,200 | 2.980 | 1,126 | 204 | 393 | 7,885 | 39.2 | 2.341 | 10.6 |

Source: D. H. Greenberg and M. Kosters, Income Guarantees and the Working Poor, The Rand Corporation, R-579-OEO, December 1970.
Hours($X_1$): average hours worked during the year
Rate($X_2$): average hourly wage (dollars)
ERSP($X_3$): average yearly earnings of spouse (dollars)
ERNO($X_4$): average yearly earnings of other family members (dollars)
NEIN($X_5$): average yearly non-earned income
Assets($X_6$): average family asset holdings (bank account) (dollars)
Age($X_7$): average age of respondent
Dep($X_8$): average number of dependents
School($X_9$): average highest grade of school completed

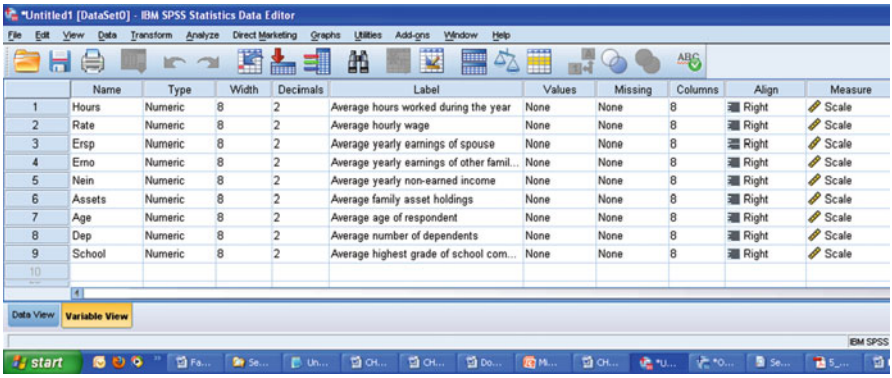



**Fig. 5.2** Defining variables along with their characteristics

*earnings of spouse in dollars, Average yearly earnings of other family members in dollars, Average yearly non-earned income, Average family asset holdings (bank account, etc.) in dollars, Average age of respondent, Average number of dependents,* and *Average highest grade of school completed* under the column heading **Label**.

4. Under the column heading **Measure,** select the option "Scale" for all these variables.
5. Use default entries in all other columns.

After defining these variables in variable view, the screen shall look like Fig. 5.2.

(iii) *Entering data:* After defining these variables in the **Variable View**, click **Data View** on the left bottom of the screen to enter data. For each variable, enter the data column wise. After entering data, the screen will look like Fig. 5.3. Save the data file in the desired location before further processing.

(b) **SPSS *Commands for Computing Correlation Coefficient***

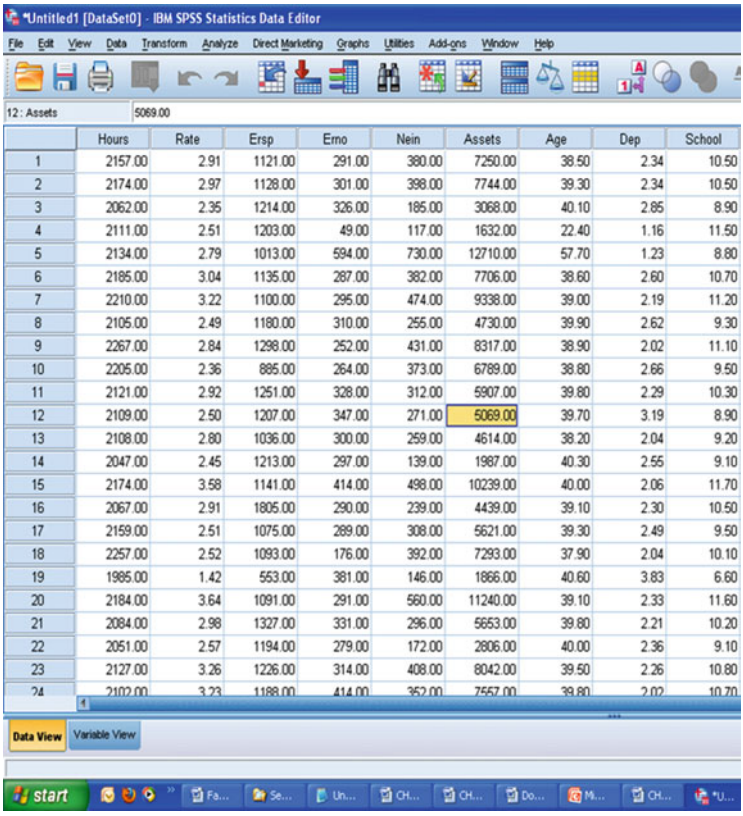
After preparing the data file in data view, take the following steps for regression analysis. Data file in SPSS can also be prepared by transporting the data from the other format like EXCEL or ASCII. The procedure of transporting data from other formats has been explained in Chap. 1.

(i) *Initiating the SPSS commands for regression analysis:* In data view, choose the following commands in sequence:

**Analyze⟶ Regression⟶Linear**

The screen shall look like Fig. 5.4.

(ii) *Selecting variables for regression analysis:* After clicking the **Linear** option, you will be taken to the next screen as shown in Fig. 5.5 for selecting the variables for regression analysis. Select the variable *Average hours worked during the year* (dependent variable) from left panel to the "Dependent" section of the right panel. Select all independent variables from left panel to the "Independent(s)" section of the right panel.




| | Hours | Rate | Ersp | Erno | Nein | Assets | Age | Dep | School |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2157.00 | 2.91 | 1121.00 | 291.00 | 380.00 | 7250.00 | 38.50 | 2.34 | 10.50 |
| 2 | 2174.00 | 2.97 | 1128.00 | 301.00 | 398.00 | 7744.00 | 39.30 | 2.34 | 10.50 |
| 3 | 2062.00 | 2.35 | 1214.00 | 326.00 | 185.00 | 3068.00 | 40.10 | 2.85 | 8.90 |
| 4 | 2111.00 | 2.51 | 1203.00 | 49.00 | 117.00 | 1632.00 | 22.40 | 1.16 | 11.50 |
| 5 | 2134.00 | 2.79 | 1013.00 | 594.00 | 730.00 | 12710.00 | 57.70 | 1.23 | 8.80 |
| 6 | 2185.00 | 3.04 | 1135.00 | 287.00 | 382.00 | 7706.00 | 38.60 | 2.60 | 10.70 |
| 7 | 2210.00 | 3.22 | 1100.00 | 295.00 | 474.00 | 9338.00 | 39.00 | 2.19 | 11.20 |
| 8 | 2105.00 | 2.49 | 1180.00 | 310.00 | 255.00 | 4730.00 | 39.90 | 2.62 | 9.30 |
| 9 | 2267.00 | 2.84 | 1298.00 | 252.00 | 431.00 | 8317.00 | 38.90 | 2.02 | 11.10 |
| 10 | 2205.00 | 2.36 | 885.00 | 264.00 | 373.00 | 6789.00 | 38.80 | 2.66 | 9.50 |
| 11 | 2121.00 | 2.92 | 1251.00 | 328.00 | 312.00 | 5907.00 | 39.80 | 2.29 | 10.30 |
| 12 | 2109.00 | 2.50 | 1207.00 | 347.00 | 271.00 | 5069.00 | 39.70 | 3.19 | 8.90 |
| 13 | 2108.00 | 2.80 | 1036.00 | 300.00 | 259.00 | 4614.00 | 38.20 | 2.04 | 9.20 |
| 14 | 2047.00 | 2.45 | 1213.00 | 297.00 | 139.00 | 1987.00 | 40.30 | 2.55 | 9.10 |
| 15 | 2174.00 | 3.58 | 1141.00 | 414.00 | 498.00 | 10239.00 | 40.00 | 2.06 | 11.70 |
| 16 | 2067.00 | 2.91 | 1805.00 | 290.00 | 239.00 | 4439.00 | 39.10 | 2.30 | 10.50 |
| 17 | 2159.00 | 2.51 | 1075.00 | 289.00 | 308.00 | 5621.00 | 39.30 | 2.49 | 9.50 |
| 18 | 2257.00 | 2.52 | 1093.00 | 176.00 | 392.00 | 7293.00 | 37.90 | 2.04 | 10.10 |
| 19 | 1985.00 | 1.42 | 553.00 | 381.00 | 146.00 | 1866.00 | 40.60 | 3.83 | 6.60 |
| 20 | 2184.00 | 3.64 | 1091.00 | 291.00 | 560.00 | 11240.00 | 39.10 | 2.33 | 11.60 |
| 21 | 2084.00 | 2.98 | 1327.00 | 331.00 | 296.00 | 5653.00 | 39.80 | 2.21 | 10.20 |
| 22 | 2051.00 | 2.57 | 1194.00 | 279.00 | 172.00 | 2806.00 | 40.00 | 2.36 | 9.10 |
| 23 | 2127.00 | 3.26 | 1226.00 | 314.00 | 408.00 | 8042.00 | 39.50 | 2.26 | 10.80 |
| 24 | 2102.00 | 3.23 | 1188.00 | 414.00 | 352.00 | 7557.00 | 39.80 | 2.02 | 10.70 |

**Fig. 5.3** Screen showing entered data for all the variables in the data view

Either the variable selection is made one by one or all at once. To do so, the variable needs to be selected from the left panel, and by arrow command, it may be brought to the right panel. After choosing the variables for analysis, the screen shall look like Fig. 5.5.

(iii) *Selecting the options for computation:* After selecting the variables, option needs to be defined for the regression analysis. Take the following steps:

- In the screen shown in Fig. 5.5, click the tag **Statistics**; you will get the screen as shown in Fig. 5.6.
  - Check the box "*R* squared change," "Descriptive," and "Part and partial correlations."
  - By default, the options "Estimates" and "Model fit" are checked. Ensure that they remain checked.
  - Click ***Continue.*** You will now be taken back to the screen shown in Fig. 5.5.

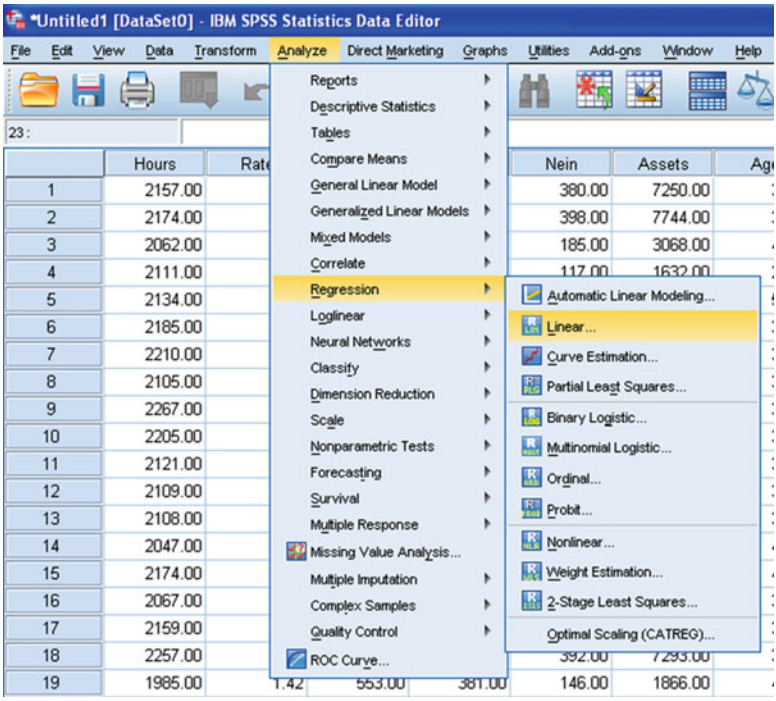



**Fig. 5.4** Screen showing SPSS commands for regression analysis

By checking the option "*R* squared change" the output shall include the values of $R^2$ and adjusted $R^2$. Similarly by checking the option "Descriptive" the output will provide the values of mean and standard deviations along with correlation matrix of all the variables, whereas checking the option "Part and partial correlations" shall provide the partial correlations of various orders between *Average hours worked during the year* and other variables. Readers are advised to try other options and see what changes they are getting in their outputs.

– In the option **Method** shown in Fig. 5.5, select "Stepwise."
– Click **OK**.

(c) ***Getting the Output***

Clicking the **OK** tag in Fig. 5.5 will lead you to the output window. In the output window of SPSS, the relevant outputs can be selected by using the right click of the mouse and may be copied in the word file. The output panel shall have the following results:

1. Mean and standard deviation
2. Correlation matrix along with significance value
3. Model summary along with the values of $R$, $R^2$ and adjusted $R^2$
4. ANOVA table showing $F$-values for all the models

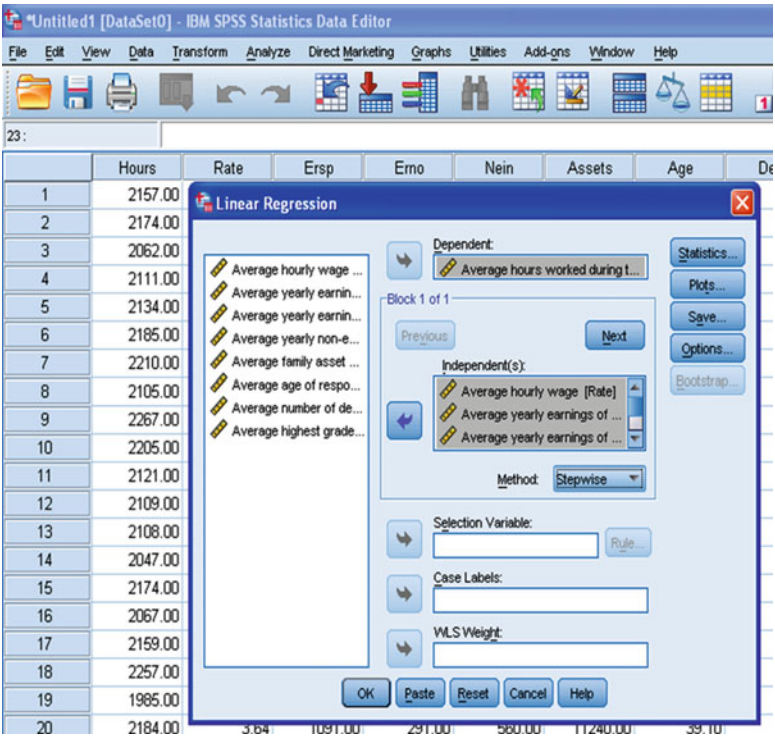



**Fig. 5.5** Screen showing selection of variables for regression analysis

5. Standardized and unstandardized regression coefficients of selected variables in different models along with their *t*-values and partial correlations

In this example, all the outputs so generated by the SPSS have been shown in Tables 5.4, 5.5, 5.6, 5.7, and 5.8.

## *Interpretation of the Outputs*

Different outputs generated in the SPSS are shown below along with their interpretations.

1. The values of mean and standard deviation for all the variables are shown in Table 5.4. These values can be used for further analysis in the study. By using the procedure discussed in Chap. 2, a profile chart may be prepared by computing other descriptive statistics for all the variables.
2. The correlation matrix in Table 5.5 shows the correlations among the variables along with their significance value (*p* value). Significance of these correlations has been tested for one-tailed test. The correlation coefficient with one asterisk

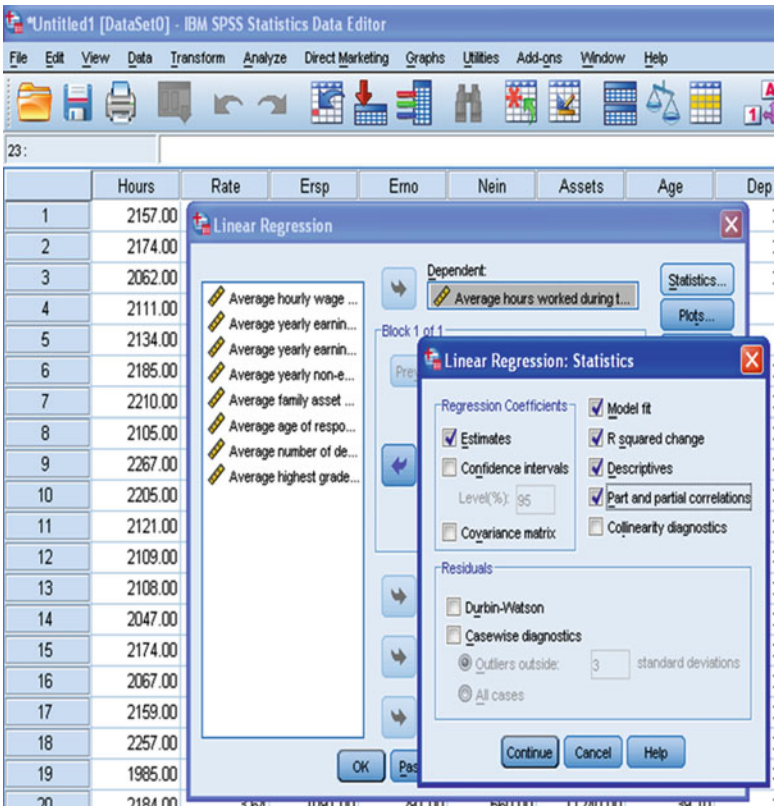



**Fig. 5.6** Screen showing options for computing various components of regression analysis

**Table 5.4** Descriptive statistics for different variables

| Variables | Mean | SD | *N* |
|---|---|---|---|
| Average hours worked during the year | 2,137.09 | 64.12 | 35 |
| Average hourly wage | 2.74 | .46 | 35 |
| Average yearly earnings of spouse | 1,083.66 | 256.78 | 35 |
| Average yearly earnings of other family members | 298.23 | 94.99 | 35 |
| Average yearly non-earned income | 348.23 | 167.59 | 35 |
| Average family asset holdings | 6,048.14 | 2,921.40 | 35 |
| Average age of respondent | 39.24 | 4.40 | 35 |
| Average number of dependents | 2.49 | .66 | 35 |
| Average highest grade of school completed | 9.92 | 1.17 | 35 |

mark (*) indicates its significance at 5% level. The asterisk mark (*) is put on the correlation coefficient if its value is more than the required value of correlation coefficients for its significance at 5% level which is .284. For one-tailed test, the required value of "*r*" for significance with 33 ($N - 2$) df can be seen from Table A.3 in the Appendix.

**Table 5.5** Correlation matrix for different variables along with significance level

| | Hours | Rate | Ersp | Erno | Nein | Assets | Age | Dep | School |
|---|---|---|---|---|---|---|---|---|---|
| *Pearson correlation* | | | | | | | | | |
| *Hours* | 1.000 | .556** | .124 | −.245 | .413** | .716** | −.077 | −.339* | .681** |
| *Rate* | .556** | 1.000 | .572** | .059 | .297* | .783** | .044 | −.601** | .881** |
| *Ersp* | .124 | .572** | 1.000 | −.041 | −.238 | .298* | −.015 | −.693** | .549** |
| *Erno* | −.245 | .059 | −.041 | 1.000 | .152 | .296* | .775** | .050 | −.299* |
| *Nein* | .413** | .297* | −.238 | .152 | 1.000 | .512** | .347* | −.045 | .219 |
| *Assets* | .716** | .783** | .298* | .296* | .512** | 1.000 | .414** | −.530** | .634** |
| *Age* | −.077 | .044 | −.015 | .775** | .347* | .414** | 1.000 | −.048 | −.331 |
| *Dep* | −339* | −.601** | −.693** | .050 | −.045 | −.530** | −.048 | 1.000 | −.603** |
| *School* | .681** | .881** | .549** | −.299* | .219 | .634** | −.331* | −.603** | 1.000 |
| *Sig. (1-tailed)* | | | | | | | | | |
| *Hours* | | .000 | .239 | .078 | .007 | .000 | .330 | .023 | .000 |
| *Rate* | .000 | . | .000 | .368 | .041 | .000 | .401 | .000 | .000 |
| *Ersp* | .239 | .000 | . | .408 | .084 | .041 | .465 | .000 | .000 |
| *Erno* | .078 | .368 | .408 | . | .192 | .042 | .000 | .387 | .041 |
| *Nein* | .007 | .041 | .084 | .192 | . | .001 | .021 | .398 | .103 |
| *Assets* | .000 | .000 | .041 | .042 | .001 | . | .007 | .001 | .000 |
| *Age* | .330 | .401 | .465 | .000 | .021 | .007 | . | .391 | .026 |
| *Dep* | .023 | .000 | .000 | .387 | .398 | .001 | .391 | . | .000 |
| *School* | .000 | .000 | .000 | .041 | .103 | .000 | .026 | .000 | . |

*Hours:* Average hours worked during the year
*Rate:* Average hourly wage
*Ersp:* Average yearly earnings of spouse
*Erno:* Average yearly earnings of other family members
*Nein:* Average yearly non-earned income
*Assets:* Average family asset holdings
*Age:* Average age of respondent
*Dep:* Average number of dependents
*School:* Average highest grade of school completed
*Significant at 0.05 level (1-tailed) Significant value of $r$ at .05 level with 33 df (1-tailed) = 0.284; **Significant at 0.01 level (1-tailed) Significant value of $r$ at .01 level with 33 df (1-tailed) = 0.392

Similarly for one-tailed test, the significance value for the correlation coefficient at .01 level with 33 (=$N - 2$) df can be seen as 0.392. Thus, all those correlation coefficients having values more than 0.392 are significant at 1% level. Such correlation coefficients have been shown with two asterisk marks (**).

Readers may also show the correlation matrix by writing the upper diagonal values as has been done in Chap. 4.

3. From Table 5.5, it can be seen that *Hours* (Average hours worked during the year) is significantly correlated with Rate (Average hourly wage), *Nein* (Average yearly non-earned income), *Assets* (Average family asset holdings), and *School* (Average highest grade of school completed) at 1% level, whereas with *Dep* (Average number of dependents) at 5% level.

**Table 5.6** Model summary along with the values of *R* and *R* square

| | | | | | Change statistics | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Model | *R* | *R* square | Adj *R* square. | SE of the estimate | *R* square change | *F* change | df1 | df2 | Sig. *F* change |
| 1 | .716[a] | .512 | .498 | 45.44102 | .512 | 34.687 | 1 | 33 | .000 |
| 2 | .861[b] | .742 | .726 | 33.58681 | .229 | 28.405 | 1 | 32 | .000 |
| 3 | .879[c] | .773 | .751 | 32.00807 | .031 | 4.235 | 1 | 31 | .048 |

[a]Predictors: (Constant), Average family asset holdings
[b]Predictors: (Constant), Average family asset holdings, Average yearly earnings of other family members
[c]Predictors: (Constant), Average family asset holdings, Average yearly earnings of other family members, Average number of dependents

4. The three regression models generated by the SPSS have been presented in Table 5.6. In the third model, the value of $R^2$ is .773, which is maximum, and, therefore, third model shall be used to develop the regression equation. It can be seen from Table 5.6 that in the third model, three independent variables, namely, *Assets* (Average family asset holdings), *Erno* (Average yearly earnings of other family members), and *Dep* (Average number of dependents), have been identified, and, therefore, the regression equation shall be developed using these three variables only. The $R^2$ value for this model is 0.773, and, therefore, these three independent variables explain 77.3% variations in *Hours* (Average hours worked during the year) in the USA. Thus, this model can be considered appropriate to develop the regression equation.
5. In Table 5.7, *F*-values for all the models have been shown. Since *F*-value for the third model is highly significant, it may be concluded that the model selected is highly efficient also.
6. Table 5.8 shows the unstandardized and standardized regression coefficients in all the three models. Unstandardized coefficients are also known as "B" coefficients and are used to develop the regression equation whereas standardized regression coefficients are denoted by "β" and are used to explain the relative importance of independent variables in terms of their contribution toward the dependent variables in the model. In the third model, *t*-values for all the three regression coefficients are significant as their significance values (*p* values) are less than .05. Thus, it may be concluded that the variables *Assets* (Average family asset holdings), *Erno* (Average yearly earnings of other family members), and *Dep* (Average number of dependents) significantly explain the variations in the *Hours* (Average hours worked during the year).

## Regression Equation

Using unstandardized regression coefficients (*B*) of the third model shown in Table 5.8, the regression equation can be developed which is as follows:

$$\text{Hours} = 2064.285 + 0.22 \times (\text{Assets}) - 0.371 \times (\text{Erno}) + 20.816 \times (\text{Dep})$$

**Table 5.7** ANOVA table showing *F*-values for all the models[a]

| Model | | Sum of squares | df | Mean square | *F* | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 71,625.498 | 1 | 71,625.498 | 34.687 | .000[b] |
| | Residual | 68,141.245 | 33 | 2,064.886 | | |
| | Total | 139,766.743 | 34 | | | |
| 2 | Regression | 103,668.390 | 2 | 51,834.195 | 45.949 | .000[c] |
| | Residual | 36,098.353 | 32 | 1,128.074 | | |
| | Total | 139,766.743 | 34 | | | |
| 3 | Regression | 108,006.736 | 3 | 36,002.245 | 35.141 | .000[d] |
| | Residual | 31,760.007 | 31 | 1,024.516 | | |
| | Total | 139,766.743 | 34 | | | |

[a]Dependent variable: Average hours worked during the year
[b]Predictors: (Constant), Average family asset holdings
[c]Predictors: (Constant), Average family asset holdings, Average yearly earnings of other family members
[d]Predictors: (Constant), Average family asset holdings, Average yearly earnings of other family members, Average number of dependents

where

*Hours:* Average hours worked during the year
*Assets:* Average family asset holdings
*Erno:* Average yearly earnings of other family members
*Dep:* Average number of dependents

Thus, it may be concluded that the above regression equation is quite reliable as the value of $R^2$ is 0.773. In other words, the three variables selected in this regression equation explain 77.3% of the total variability in the *Hour* (Average hours worked during the year), which is quite good. Since the *F*-value for this regression model is highly significant, the model is reliable. At the same time, all the regression coefficients in this model are highly significant, and, therefore, it may be interpreted that all the three variables selected in the model, namely, *Assets* (Average family asset holdings), *Erno* (Average yearly earnings of other family members), and *Dep* (Average number of dependents), have significant predictability in estimating the value of the *Hour* (Average hours worked during the year) in the USA.

## Summary of the SPSS Commands For Regression Analysis

1. Start SPSS by using the following commands:
   **Start ⟶ All Programs ⟶ IBM SPSS Statistics ⟶ IBM SPSS Statistics**
2. Create data file by clicking the tag **Type in Data**. Define all the variables and their characteristics by clicking the **Variable View**. After defining the variables, type the data for these variables by clicking **Data View**.

**Table 5.8** Regression coefficients of selected variables in different models along with their *t*-values and partial correlations[a]

| Model | | Unstandardized coefficients | | Standardized coefficients | | | Correlations | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *B* | Std. error | Beta | *t* | Sig. | Zero-order | Partial | Part |
| 1 | (Constant) | 2,042.064 | 17.869 | | 114.280 | .000 | | | |
| | Average family asset holdings | .016 | .003 | .716 | 5.890 | .000 | .716 | .716 | .716 |
| 2 | (Constant) | 2,123.257 | 20.162 | | 105.308 | .000 | | | |
| | Average family asset holdings | .019 | .002 | .864 | 9.190 | .000 | .716 | .852 | .826 |
| | Average yearly earnings of other family members | −.338 | .063 | −.501 | −5.330 | .000 | −.245 | −.686 | −.479 |
| 3 | (Constant) | 2,064.285 | 34.503 | | 59.828 | .000 | | | |
| | Average family asset holdings | .022 | .002 | .993 | 9.092 | .000 | .716 | .853 | .778 |
| | Average yearly earnings of other family members | −.371 | .063 | −.550 | −5.933 | .000 | −.245 | −.729 | −.508 |
| | Average number of dependents | 20.816 | 10.116 | .215 | 2.058 | .048 | −.339 | .347 | .176 |

[a]Dependent variable: Average hours worked during the year

3. Once the data file is ready, use the following command sequence for selecting the variables for analysis.

**Analyze → Regression → Linear**

4. Select the dependent variable from left panel to the "Dependent" section of the right panel. Select all other independent variables from left panel to the "Independent(s)" section of the right panel.
5. After selecting the variables for regression analysis, click the tag **Statistics** on the screen. Check the box "*R* squared change," "Descriptive," and "Part and partial correlations." Press ***Continue***.
6. In the **Method** option, select "Stepwise," then press **OK** to get the different outputs for regression analysis.

## Exercise

*Short-Answer Questions*

**Note:** Write answer to each of the questions in not more than 200 words.

Q.1. Describe regression analysis. Explain the difference between simple regression and multiple regression models.
Q.2. What is the difference between stepwise regression and backward regression?
Q.3. Discuss the role of $R^2$ in regression analysis. Explain multiple correlation and its order.
Q.4. Explain an experimental situation where regression analysis can be used.
Q.5. How will you know that the variables which are selected in the regression analysis are valid?
Q.6. What is the difference between Stepwise and Enter method in developing multiple regression equation?

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. The range of multiple correlation *R* is

   (a) −1 to 0
   (b) 0 to 1
   (c) −1 to 0
   (d) None of the above

2. SPSS commands for multiple regression analysis is

   (a) Analyze -> Linear -> Regression
   (b) Analyze -> Regression -> Linear
   (c) Analyze -> Linear Regression
   (d) Analyze -> Regression Linear

3. Choose the most appropriate statement

   (a) $R^2$ is a measure of multiple correlation.
   (b) $R^2$ is used for selecting the variables in the regression model.
   (c) $R^2$ is the amount of variability explained in the dependent variable by the independent variables.
   (d) All above are correct.

4. If $p$ value for the correlation between college GPA and GMAT score is .008, what conclusion can be drawn?

   (a) Correlation is not significant at 1% level.
   (b) Correlation is not significant at 5% level.
   (c) Correlation is significant at 1% level.
   (d) All above statements are wrong.

5. Following are the two statements about the significance of value of $r$:

   Statement I: Correlation coefficient required for significance at 1% is 0.462.
   Statement II: Correlation coefficient required for significance at 5% is 0.337.

   Choose the most appropriate alternative.

   (a) Statement I is right, but II is wrong.
   (b) Statement I is wrong, but II is right.
   (c) Both statements I and II are wrong.
   (d) Both statements are right.

6. In regression analysis, four models have been developed. Which model in your opinion is the most appropriate?

| Models | No. of independent variables | $R^2$ |
|---|---|---|
| (a) Model I: | 5 | 0.88 |
| (b) Model II: | 4 | 0.87 |
| (c) Model III: | 3 | 0.86 |
| (d) Model IV: | 2 | 0.65 |

7. In a regression analysis to estimate the sale of a particular product, the regression coefficients of independent variables were as follows:

| Independent variables | $B$ coefficient | $p$ value |
|---|---|---|
| Customer's incentive | 1.5 | .06 |
| Dealer's incentive | 2.2 | .009 |
| Hours of marketing | 3.1 | .32 |
| Product price | 1.2 | .006 |

   Choose the most appropriate statement.

   (a) Both Customer's incentive and Hours of marketing are significant at .05 level in the model.
   (b) Both Hours of marketing and Product price are significant at .05 level in the model.

(c) Both Dealer's incentive and Product price are significant at .01 level in the model.
(d) Both Dealer's incentive and Product price are not significant at .05 level in the model.

8. Choose correct statement about B and $\beta$ coefficients.

(a) "B" is an unstandardized coefficient and "β" is a standardized coefficient.
(b) "β" is an unstandardized coefficient and "B" is a standardized coefficient.
(c) Both "B" and "β" are standardized coefficients.
(d) Both "B" and "β" are unstandardized coefficients.

*Assignments*

1. The data on copper industry and its determinants in the US market during 1951–1980 are shown in the following table. Construct a regression model and develop the regression equation by using the SPSS. Test the significance of regression coefficients and explain the robustness of the regression model to predict the price of the copper in the US market.

| Determinants of US domestic price of copper | | | | | |
|---|---|---|---|---|---|
| DPC | GNP | IIP | MEPC | NOH | PA |
| 21.89 | 330.2 | 45.1 | 220.4 | 1,491.00 | 19.00 |
| 22.29 | 347.2 | 50.9 | 259.5 | 1,504.00 | 19.41 |
| 19.63 | 366.1 | 53.3 | 256.3 | 1,438.00 | 20.93 |
| 22.85 | 366.3 | 53.6 | 249.3 | 1,551.00 | 21.78 |
| 33.77 | 399.3 | 54.6 | 352.3 | 1,646.00 | 23.68 |
| 39.18 | 420.7 | 61.1 | 329.1 | 1,349.00 | 26.01 |
| 30.58 | 442.0 | 61.9 | 219.6 | 1,224.00 | 27.52 |
| 26.30 | 447.0 | 57.9 | 234.8 | 1,382.00 | 26.89 |
| 30.70 | 483.0 | 64.8 | 237.4 | 1,553.70 | 26.85 |
| 32.10 | 506.0 | 66.2 | 245.8 | 1,296.10 | 27.23 |
| 30.00 | 523.3 | 66.7 | 229.2 | 1,365.00 | 25.46 |
| 30.80 | 563.8 | 72.2 | 233.9 | 1,492.50 | 23.88 |
| 30.80 | 594.7 | 76.5 | 234.2 | 1,634.90 | 22.62 |
| 32.60 | 635.7 | 81.7 | 347.0 | 1,561.00 | 23.72 |
| 35.40 | 688.1 | 89.8 | 468.1 | 1,509.70 | 24.50 |
| 36.60 | 753.0 | 97.8 | 555.0 | 1,195.80 | 24.50 |
| 38.60 | 796.3 | 100.0 | 418.0 | 1,321.90 | 24.98 |
| 42.20 | 868.5 | 106.3 | 525.2 | 1,545.40 | 25.58 |
| 47.90 | 935.5 | 111.1 | 620.7 | 1,499.50 | 27.18 |
| 58.20 | 982.4 | 107.8 | 588.6 | 1,469.00 | 28.72 |
| 52.00 | 1,063.4 | 109.6 | 444.4 | 2,084.50 | 29.00 |
| 51.20 | 1,171.1 | 119.7 | 427.8 | 2,378.50 | 26.67 |
| 59.50 | 1,306.6 | 129.8 | 727.1 | 2,057.50 | 25.33 |
| 77.30 | 1,412.9 | 129.3 | 877.6 | 1,352.50 | 34.06 |
| 64.20 | 1,528.8 | 117.8 | 556.6 | 1,171.40 | 39.79 |
| 69.60 | 1,700.1 | 129.8 | 780.6 | 1,547.60 | 44.49 |
| 66.80 | 1,887.2 | 137.1 | 750.7 | 1,989.80 | 51.23 |

(continued)

(continued)

| Determinants of US domestic price of copper | | | | | |
|---|---|---|---|---|---|
| DPC | GNP | IIP | MEPC | NOH | PA |
| 66.50 | 2,127.6 | 145.2 | 709.8 | 2,023.30 | 54.42 |
| 98.30 | 2,628.80 | 152.5 | 935.7 | 1,749.20 | 61.01 |
| 101.40 | 2,633.10 | 147.1 | 940.9 | 1,298.50 | 70.87 |

DPC = 12-month average US domestic price of copper (cents per pound)
GNP = annual gross national product ($, billions)
IIP = 12-month average index of industrial production
MEPC = 12-month average London Metal Exchange price of copper (pounds sterling)
NOH = number of housing starts per year (thousands of units)
PA = 12-month average price of aluminum (cents per pound)
Note: The data are from the sources such as American Metal Market, Metals Week, and US Department of Commerce publications

**Note:** The data were collected by Gary R. Smith from sources such as American Metal Market, Metals

2. Data in the following table shows the crime rate in 47 states in the USA in 1960. Develop a suitable regression model for estimating the crime rate depending upon identified socioeconomic variables.

| US crime data for 47 states | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| S.N. | *R* | Age | ED | EX0 | LF | *N* | NW | U1 | U2 | *X* |
| 1 | 79.1 | 151 | 91 | 58 | 510 | 33 | 301 | 108 | 41 | 261 |
| 2 | 163.5 | 143 | 113 | 103 | 583 | 13 | 102 | 96 | 36 | 194 |
| 3 | 57.8 | 142 | 89 | 45 | 533 | 18 | 219 | 94 | 33 | 250 |
| 4 | 196.9 | 136 | 121 | 149 | 577 | 157 | 80 | 102 | 39 | 167 |
| 5 | 123.4 | 141 | 121 | 109 | 591 | 18 | 30 | 91 | 20 | 174 |
| 6 | 68.2 | 121 | 110 | 118 | 547 | 25 | 44 | 84 | 29 | 126 |
| 7 | 96.3 | 127 | 111 | 82 | 519 | 4 | 139 | 97 | 38 | 168 |
| 8 | 155.5 | 131 | 109 | 115 | 542 | 50 | 179 | 79 | 35 | 206 |
| 9 | 85.6 | 157 | 90 | 65 | 553 | 39 | 286 | 81 | 28 | 239 |
| 10 | 70.5 | 140 | 118 | 71 | 632 | 7 | 15 | 100 | 24 | 174 |
| 11 | 167.4 | 124 | 105 | 121 | 580 | 101 | 106 | 77 | 35 | 170 |
| 12 | 84.9 | 134 | 108 | 75 | 595 | 47 | 59 | 83 | 31 | 172 |
| 13 | 51.1 | 128 | 113 | 67 | 624 | 28 | 10 | 77 | 25 | 206 |
| 14 | 66.4 | 135 | 117 | 62 | 595 | 22 | 46 | 77 | 27 | 190 |
| 15 | 79.8 | 152 | 87 | 57 | 530 | 30 | 72 | 92 | 43 | 264 |
| 16 | 94.6 | 142 | 88 | 81 | 497 | 33 | 321 | 116 | 47 | 247 |
| 17 | 53.9 | 143 | 110 | 66 | 537 | 10 | 6 | 114 | 35 | 166 |
| 18 | 92.9 | 135 | 104 | 123 | 537 | 31 | 170 | 89 | 34 | 165 |
| 19 | 75 | 130 | 116 | 128 | 536 | 51 | 24 | 78 | 34 | 135 |
| 20 | 122.5 | 125 | 108 | 113 | 567 | 78 | 94 | 130 | 58 | 166 |
| 21 | 74.2 | 126 | 108 | 74 | 602 | 34 | 12 | 102 | 33 | 195 |
| 22 | 43.9 | 157 | 89 | 47 | 512 | 22 | 423 | 97 | 34 | 276 |
| 23 | 121.6 | 132 | 96 | 87 | 564 | 43 | 92 | 83 | 32 | 227 |
| 24 | 96.8 | 131 | 116 | 78 | 574 | 7 | 36 | 142 | 42 | 176 |

(continued)

(continued)

| US crime data for 47 states | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| S.N. | *R* | Age | ED | EX0 | LF | *N* | NW | U1 | U2 | *X* |
| 25 | 52.3 | 130 | 116 | 63 | 641 | 14 | 26 | 70 | 21 | 196 |
| 26 | 199.3 | 131 | 121 | 160 | 631 | 3 | 77 | 102 | 41 | 152 |
| 27 | 34.2 | 135 | 109 | 69 | 540 | 6 | 4 | 80 | 22 | 139 |
| 28 | 121.6 | 152 | 112 | 82 | 571 | 10 | 79 | 103 | 28 | 215 |
| 29 | 104.3 | 119 | 107 | 166 | 521 | 168 | 89 | 92 | 36 | 154 |
| 30 | 69.6 | 166 | 89 | 58 | 521 | 46 | 254 | 72 | 26 | 237 |
| 31 | 37.3 | 140 | 93 | 55 | 535 | 6 | 20 | 135 | 40 | 200 |
| 32 | 75.4 | 125 | 109 | 90 | 586 | 97 | 82 | 105 | 43 | 163 |
| 33 | 107.2 | 147 | 104 | 63 | 560 | 23 | 95 | 76 | 24 | 233 |
| 34 | 92.3 | 126 | 118 | 97 | 542 | 18 | 21 | 102 | 35 | 166 |
| 35 | 65.3 | 123 | 102 | 97 | 526 | 113 | 76 | 124 | 50 | 158 |
| 36 | 127.2 | 150 | 100 | 109 | 531 | 9 | 24 | 87 | 38 | 153 |
| 37 | 83.1 | 177 | 87 | 58 | 638 | 24 | 349 | 76 | 28 | 254 |
| 38 | 56.6 | 133 | 104 | 51 | 599 | 7 | 40 | 99 | 27 | 225 |
| 39 | 82.6 | 149 | 88 | 61 | 515 | 36 | 165 | 86 | 35 | 251 |
| 40 | 115.1 | 145 | 104 | 82 | 560 | 96 | 126 | 88 | 31 | 228 |
| 41 | 88 | 148 | 122 | 72 | 601 | 9 | 19 | 84 | 20 | 144 |
| 42 | 54.2 | 141 | 109 | 56 | 523 | 4 | 2 | 107 | 37 | 170 |
| 43 | 82.3 | 162 | 99 | 75 | 522 | 40 | 208 | 73 | 27 | 224 |
| 44 | 103 | 136 | 121 | 95 | 574 | 29 | 36 | 111 | 37 | 162 |
| 45 | 45.5 | 139 | 88 | 46 | 480 | 19 | 49 | 135 | 53 | 249 |
| 46 | 50.8 | 126 | 104 | 106 | 599 | 40 | 24 | 78 | 25 | 171 |
| 47 | 84.9 | 130 | 121 | 90 | 623 | 3 | 22 | 113 | 40 | 160 |

Source: W. Vandaele, "Participation in Illegitimate Activities: Erlich Revisited," in A. Blumstein, J. Cohen, and Nagin, D., eds., *Deterrence and Incapacitation,* National Academy of Sciences, 1978, pp. 270–335. 386

$R$ = crime rate, number of offenses reported to police per million population
Age = number of males of age 14–24 per 1,000 population
$S$ = indicator variable for southern states (0 = no, 1 = yes)
ED = mean number of years of schooling times 10 for persons age 25 or older
EX0 = 1,960 per capita expenditure on police by state and local government
LF = labor force participation rate per 1,000 civilian urban males age 14–24
$N$ = state population size in hundred thousands
NW = number of nonwhites per 1,000 population
U1 = unemployment rate of urban males per 1,000 of age 14–24
U2 = unemployment rate of urban males per 1,000 of age 35–39
$X$ = the number of families per 1,000 earnings 1/2 the median income

## *Answers to Multiple-Choice Questions*

| | | | |
|---|---|---|---|
| Q.1 | b | Q.2 | b |
| Q.3 | c | Q.4 | c |
| Q.5 | d | Q.6 | c |
| Q.7 | c | Q.8 | a |

# Chapter 6
# Hypothesis Testing for Decision-Making

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Understand the purpose of hypothesis testing.
- Learn to construct the hypotheses.
- Know the situations for using one- and two-tailed tests.
- Describe the procedure of hypothesis testing.
- Understand the *p* value.
- Learn the computing procedure manually in different situations by using *t*-tests.
- Identify an appropriate *t*-test in different research situations.
- Know the assumptions under which *t*-test should be used.
- Describe the situations in which one-tailed and two-tailed tests should be used.
- Interpret the difference between one-tailed and two-tailed hypotheses.
- Learn to compute *t*-statistic in different research situations by using SPSS.
- Learn to interpret the outputs of different *t*-tests generated in SPSS.

## Introduction

Human beings are progressive in nature. Most of our decisions in life are governed by our past experiences. These decisions may be subjective or objective. Subjective decisions are solely based upon one's own perception of viewing issues. These perceptions keep on changing from person to person. Same thing or situation can be perceived differently by different persons, and therefore, the decision cannot be universalized. On the other hand, if decisions are taken on the basis of scientific law, it is widely accepted and works well in the similar situations.

Decision makers are always engaged in identifying optimum decision in a given situation for solving a problem. Theory of statistical inference which is based on scientific principles provide optimum solution to these decision makers. Statistical inference includes theory of estimation and testing of hypothesis. In this chapter,

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different aspects of hypothesis testing regarding population parameter have been discussed. At times one may be interested to know as to whether the population mean is equal to the given value. In testing such hypothesis, a representative sample may be drawn to verify it by using statistical tests. Testing of hypothesis may be done for comparing two population averages on the basis of samples drawn from these populations. For instance, one may like to know whether memory retention power is more in girls or in boys in a particular age category or whether motivation level of employees in two different units of an organization is same or not. By comparing sample means of two groups, we intend to find out whether these samples come from the same population. In other words, we try to test whether their population means are equal or not. In this chapter, procedure of hypothesis testing in comparative studies has been discussed in detail. Besides comparative studies, the researcher may be interested to see the effect of certain treatment on dependent variable. Impact of advertisement campaign on the sale of a product, effect of training on employee's performance, and effect of stress management program on absenteeism are such examples where the posttesting mean may be compared with that of pretesting mean on the dependent variable.

For testing a hypothesis concerning mean, two statistical tests "$t$" and "$z$" are normally used. In case of small sample, $t$-test is used, whereas $z$-test is used in large-sample case. For all practical purposes, a sample is considered to be small if its size is less than or equal to 30 and large if it is more than 30. Since $t$-distribution approaches to $z$-distribution for $n > 30$, $t$-test can be considered as a specific case of $z$-test. The $t$-test is used in a situation where population is normally distributed and population variance is not known, and on the other hand $Z$ test is used when population variance is known. In this chapter, only $t$-tests in different situations have been discussed.

Usually testing of hypothesis is done for population mean and variance as these are the two indices which are used to describe the nature of data to a great extent. In this chapter, only testing of hypothesis concerning mean in different situations has been discussed in great detail. Plan of choosing a statistical test for testing a hypothesis has been shown graphically in Fig. 6.1.

This chapter describes the procedure of testing a hypothesis concerning single group mean and the difference between two group means for unrelated and related groups.

## Hypothesis Construction

Hypotheses are any assertion or statement about certain characteristics of the population. If the characteristics can be quantitatively measured by parameters such as mean or variance, then the hypothesis based on these parameters is said to be parametric. Whereas if the characteristics are qualitatively measured (e.g., assessment of quality, attitude, or perception), then the hypothesis so developed on these characteristics is known as nonparametric hypothesis. These parametric and

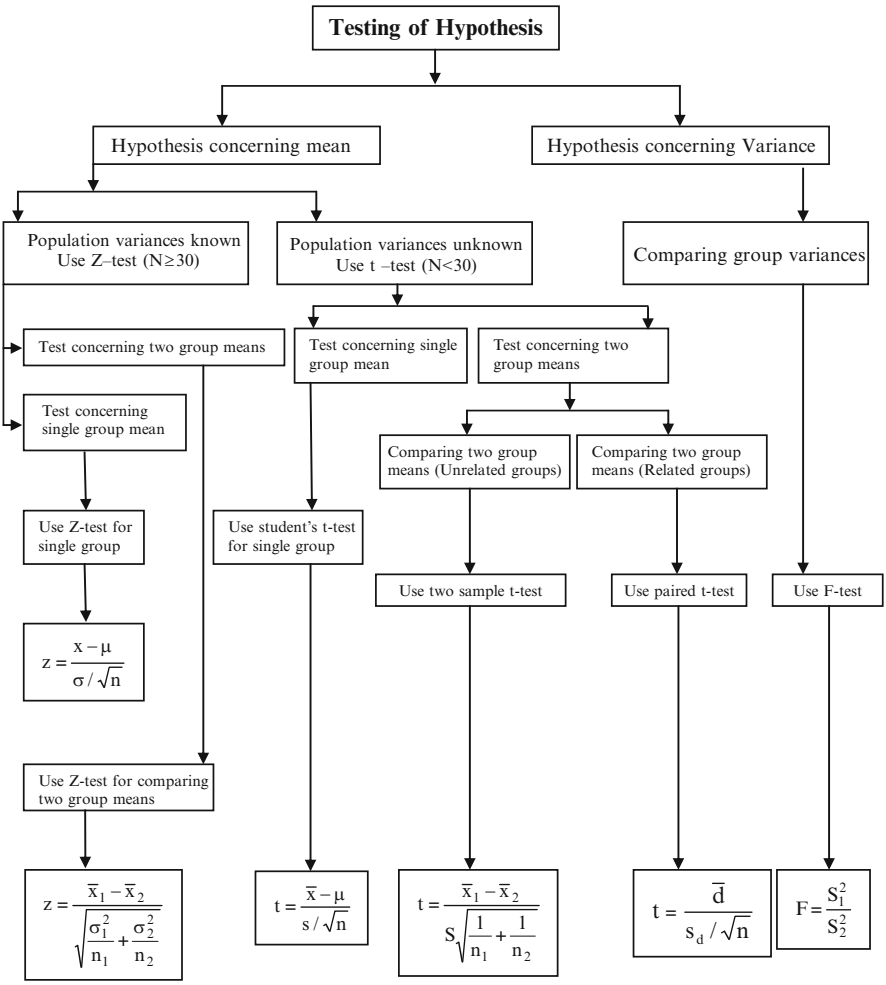



**Fig. 6.1** Scheme of selecting test statistic in hypothesis testing

nonparametric hypotheses are known as statistical hypotheses. A hypothesis is said to be statistical hypothesis if the following three conditions prevail:

1. The population may be defined.
2. Sample may be drawn.
3. The sample may be evaluated to test the hypothesis.

Statistical hypotheses are based on the concept of proof by contradiction. For example, consider that a hypothesis concerning population mean ($\mu$) is tested to see if an experiment has caused an increase or decrease in $\mu$. This is done by proof of contradiction by formulating a null hypothesis. Thus, in testing of hypothesis, one needs to formulate a research hypothesis which is required to be tested for some

population parameter. Based on research hypothesis, a null hypothesis is formulated. The null and the research (alternative) hypotheses are complementary to each other. In fact the null hypothesis serves as a means of testing the research hypothesis, and therefore, rejection of null hypothesis allows the researcher to accept the research hypothesis.

### *Null Hypothesis*

Null hypothesis is a hypothesis of no difference. It is denoted by $H_0$. It is formulated to test an alternative hypothesis. Null hypothesis is assumed to be true. By assuming the null hypothesis to be true, the distribution of the test statistic can be well defined. Further, null signifies the unbiased approach of the researcher in testing the research hypothesis. The researcher verifies the null hypothesis by assuming that it is true and rejects it in favor of research hypothesis if any contradiction is observed. In fact the null hypothesis is made for rejection. In case if the null hypothesis cannot be rejected on the basis of the sample data, it is said that the researcher fails to reject the null hypothesis. The sole purpose of the researcher is to try rejecting the null hypothesis in favor of research hypothesis in case the contradiction is observed on the basis of the sample.

### *Alternative Hypothesis*

Alternative hypothesis is also known as research hypothesis. In any research study, the researcher first develops a research hypothesis for testing some parameter of the population, and accordingly null hypothesis is formulated to verify it. The alternative hypothesis is denoted by $H_1$. Alternative hypothesis means that there is a difference between the population parameter and the sample value. In testing of hypothesis, the whole focus is to test whether research hypothesis can be accepted or not, and this is done by contradicting the null hypothesis.

## Test Statistic

In hypothesis testing, the decision about rejecting or not rejecting the null hypothesis depends upon the value of test statistic. A test statistic is a random variable $X$ whose value is tested against the critical value to arrive at a decision.

If a random sample of size $n$ is drawn from the normal population with mean, $\mu$ and variance, $\sigma^2$, then the sampling distribution of mean will also be normal with mean $\mu$ and variance $\sigma^2/n$. As per the central limit theorem even if the population from which the sample is drawn is not normal, the sample mean will still follow the normal distribution with mean, $\mu$, and variance $\sigma^2/n$ provided the sample size $n$ is large ($n > 30$).

Thus, in case of large sample ($n > 30$), for testing the hypothesis concerning mean, $z$-test is used. However, in cases of small sample ($n \leq 30$), the distribution of sample mean follows $t$-distribution if the population variance is not known. In such situation, $t$-test is used. In case population standard deviation ($\sigma$) is unknown, it is estimated by the sample standard deviation($S$). For different sample size, the $t$-curve is different, and it approaches to normal curve for sample size $n > 30$. All these curves are symmetrical and bell shaped and distributed around $t = 0$. The exact shape of the $t$-curve depends on the degrees of freedom.

In one-way ANOVA, the comparison between group variance and within-group variance is done by using the $F$-statistic. The critical value of $F$ can be obtained from the Table A4 or A5 in appendix for a particular level of significance and the degrees of freedom between and within the groups.

## Rejection Region

Rejection region is a part of the sample space in which if the value of test statistic falls, null hypothesis is rejected. Rejection region is also known as critical region. The value of the statistic in the distribution that divides sample space into acceptance and rejection region is known as critical value. These can be seen in Fig. 6.2.

The size of the rejection region is determined by the level of significance ($\alpha$). The level of significance is that probability level below which we reject the null hypothesis. The term statistical significance of a statistic refers only to the rejection of a null hypothesis at some level $\alpha$. It indicates that the observed difference between the sample mean and the mean of the sampling distribution did not occur by chance alone. So to conclude, if the test statistic falls in the rejection/critical region, $H_0$ is rejected, else $H_0$ is failed to be rejected.

## Steps in Hypothesis Testing

In experimental research, the inferences are drawn on the basis of testing a hypothesis on population parameters. The following steps are involved in decision-making process:

1. Formulate the null and alternative hypothesis for each of the parameters to be investigated. It is important to mention as to whether the hypothesis required to be tested is one tailed or two tailed.
2. Choose the level of significance at which the hypothesis needs to be tested. Usually in experimental research, the significance levels are chosen as 0.01, 0.05, or 0.10, but any value between 0 and 1 can be used.
3. Identify the test statistic (follow the guidelines shown in Fig. 6.1) that can be used to test the null hypothesis, and compute its value on the basis of the sample data.
4. Obtain the tabulated value of the statistic from the designated table. Care must be taken to obtain its value as its values are different at the same level of significance for one-tailed and two-tailed hypotheses.

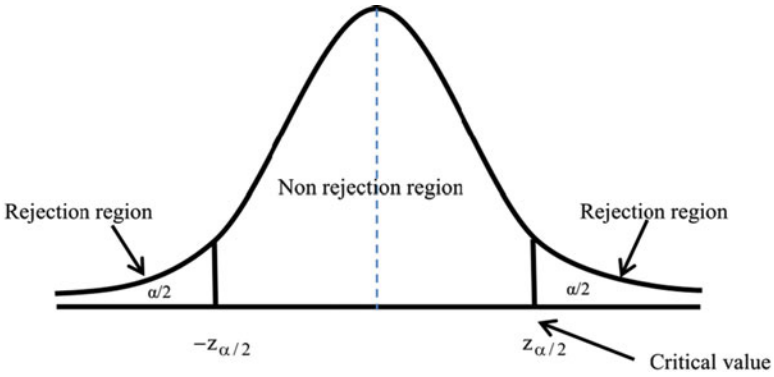



**Fig. 6.2** Different regions in two-tailed test

5. If calculated value of statistic is greater than tabulated value, null hypothesis is rejected, and if the calculated value of statistic is less than or equal to its tabulated value, null hypothesis is failed to be rejected. SPSS output provides *p* value against the computed value of statistic. If the *p* value is less than .05, the statistic is said to be significant and the null hypothesis may be rejected at significance level of .05; on the other hand, if the *p* value is more than .05, one would fail to reject the null hypothesis. If the null hypothesis is failed to be rejected, one may state that there is not enough evidence to suggest the truth of the alternative hypothesis.

## Type I and Type II Errors

We have seen that the research hypothesis is tested by means of testing the null hypothesis. Thus, the focus of the researcher is to find whether the null hypothesis can be rejected on the basis of the sample data or not. In testing the null hypothesis, the researcher has two options, that is, either to reject the null hypothesis or fail to reject the null hypothesis. Further, the true state of the null hypothesis may be true or false in either of these situations. Thus, the researcher has four courses of actions in testing the null hypothesis. The two actions, that is, rejecting the null hypothesis when it is false and fails to reject the null hypothesis when it is true, are correct decisions. Whereas the remaining two decisions, that is, rejecting the null hypothesis when it is true and fails to reject the null hypothesis when it is false, are the two wrong decisions. These two wrong decisions are known as two different kinds of errors in hypothesis testing. All the four courses of actions have been summarized in Table 6.1.

Thus, in hypothesis testing, a researcher is exposed to two types of errors known as type I and type II errors.

***Type I error*** can be defined as rejecting the null hypothesis, $H_0$, when it is true. The probability of type I error is known as level of significance and is denoted by $\alpha$. The choice of $\alpha$ determines the critical values. Looking to the relative importance of the decision, the researcher fixes the value of $\alpha$. Normally the level of significance is chosen as .05 or .01.

**Table 6.1** Decision options in testing of hypothesis

| | | True state of $H_0$ | |
|---|---|---|---|
| | | True | False |
| Researcher's decision about $H_0$ | Reject $H_0$ | Type I error | Correct decision |
| | Failed to reject $H_0$ | Correct decision | Type II error |

***Type II error*** is said to be committed if we fail to reject the null hypothesis ($H_0$) when it is false. The probability of type II error is denoted by the Greek letter $\beta$ and is used to determine the power of the test. The value of $\beta$ depends on the way the null hypothesis is false. For example, in testing the null hypothesis of equal population means for a fixed sample size, the probability of type II error decreases as the difference between population means increases. The term $1 - \beta$ is said to be the power of test. The power of test is the probability of rejecting the null hypothesis when it is wrong.

Often type I and type II errors are confused with $\alpha$ and $\beta$, respectively. In fact $\alpha$ is not the type I error but it is the probability of type I error and similarly $\beta$ is the probability of type II error and not the type II error. Since $\alpha$ is the probability, hence it can take any value in between 0 and 1, and one should write the statement like "null hypothesis may be rejected at .05 level of significance" instead of "null hypothesis may be rejected at 5% level of significance." Thus, the level of significance ($\alpha$) should always be expressed in fractions such as .05 and .01, or it may be written as 5 or 1% level. For fixed sample size, the reduction of type I and type II errors simultaneously is not possible because if you try to minimize one error, the other error will increase. Therefore, there are two ways to reducing these two errors.

The *first approach* is to increase the sample size. This is not always possible in research studies because once the data is collected, the same has to be used by the researcher for drawing the inferences. Moreover, by increasing the sample size, a researcher loses the control over experiment, due to which these errors get elevated.

The *second approach* is to identify the error which is more severe, fix it up at a desired level, and then try to minimize the other error to a maximum possible extent. In most of the research studies, type I error is considered to be more severe because wrongly rejecting a correct hypothesis forces us to accept the wrong alternative hypothesis. For example, consider an experiment where it is desired to test the effectiveness of an advertisement campaign on the sales performance. The null hypothesis required to be tested in this case would be, "Advertisement campaign either do not have any impact on sales or may reduce the sales performance." Now if the null hypothesis is wrongly rejected, an organization would go for the said advertisement campaign which in fact is not effective. These decisions will unnecessarily enhance the budget expenditure without any further appreciation in the revenue modal. Severity of type I error can also be seen in the following legal analogy. Convicts are presumed to be innocent until unless they are proved to be guilty. The purpose of the trial is to see whether the null hypothesis of innocence can be rejected based on the evidences. Here the type I error (rejecting a correct null hypothesis) means convicting the innocence, whereas type II error (failing to reject the false null hypothesis) means letting the guilty go free. Here the type I error is more severe than type II error because no innocent should be punished in comparison to guilty may get

no punishment. Type I error becomes more serious if the crime is murder and the person gets the punishment of death sentence. Thus, usually in research studies, the type I error is fixed at the desired level of, say, .05 or .01 and then type II error is tried to be minimized as much as possible.

The value of $\alpha$ and $\beta$ depends upon each other. For a fixed sample size, the only way to reduce the probability of making one type of error is to increase the other.

Consider a situation where it is desired to compare the means of two populations. Let us assume that the rejection regions have critical values $\pm \infty$. Using the statistical test, $H_0$ will never get rejected as it will exclude every possible difference in sample means. Since the null hypothesis will never be rejected, the probability of rejecting the null hypothesis when it is true will be zero. In other words, the value of $\alpha = 0$. Since the null hypothesis will never be rejected, the probability of type II error (failing to reject the null hypothesis when it is false) will be 1 or to say that $\beta = 1$.

Now consider the rejection regions whose critical values are 0,0. In this case, the rejection region includes every possible difference in sample means. This test will always reject $H_0$. Since the null hypothesis will always be rejected, the probability of type I error (rejecting $H_0$ when it is true) will be 1 or the value of $\alpha = 1$. Since the null hypothesis is always rejected, the probability of type II error (failing to reject $H_0$ when it is false) is 0, or the value of $\beta = 0$.

To conclude, a statistical test having rejection region bounded by the critical values $\pm \infty$ has $\alpha = 0$ and $\beta = 1$, whereas the test with a rejection region bounded by the critical values 0,0 has $\alpha = 1$ and $\beta = 0$. Consider a test having rejection region bounded by the critical values $\pm q$. As $q$ increases from 0 to $\infty$, $\alpha$ decreases from 1 to 0, while $\beta$ increases from 0 to 1.

## One-Tailed and Two-Tailed Tests

Consider an experiment in which null and alternative hypotheses are $H_0$ and $H_1$, respectively. We perform a test to determine whether or not the null hypothesis should be rejected in favor of the alternative hypothesis. In this situation, two different kinds of tests can be performed. One may either use a *one-tailed test* to see whether there is an increase or decrease in the parameter or may decide to use a *two-tailed test* to verify for any change in the parameter that can be increased or decrease. The word tail refers to the far left and far right of a distribution curve. These one-tailed and two-tailed tests can be performed at any of the two, 0.01 or 0.05, levels of significance.

***One-Tailed Test*** A statistical test is known as one-tailed test if the null hypothesis ($H_0$) is rejected only for the values of the test statistic falling into one specified tail of its sampling distribution. In one-tailed test, the direction is specified, that is, we are interested to verify whether population parameter is greater than some value. Or at times we may be interested to know whether the population parameter is less than some value. In other words, the researcher is clear as to what specifically he/she is interested to test. Depending upon the research hypothesis, one-tailed test can be classified as right-tailed or left-tailed tests. If the research hypothesis is to test whether

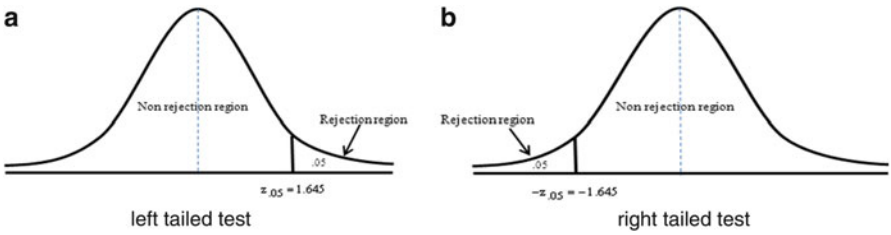
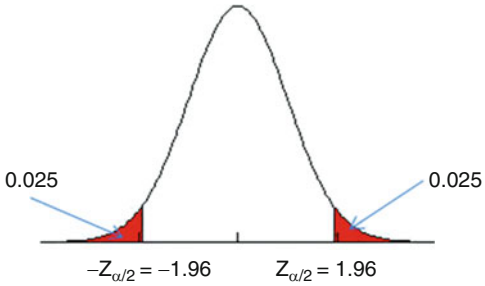



**Fig. 6.3** Critical regions at 5% level in (**a**) left-tailed test and (**b**) right-tailed test

**Fig. 6.4** Critical regions in two-tailed tests




the population mean is greater than some specified value, then the test is known as right-tailed test and the entire critical region shall lie in the right tail only. And if the test statistic falls into right critical region, the alternative hypothesis will be accepted instead of the null hypothesis. On the other hand, if the research hypothesis is to test whether the population mean is less than some specified value, then the test is known as left-tailed test and the entire critical region shall lie in the left tail only. The critical regions at 5% level in both these situations are shown in Fig. 6.3.

***Two-Tailed Test*** A statistical test is said to be a two-tailed test if the null hypothesis ($H_0$) is rejected only for values of the test statistic falling into either tail of its sampling distribution. In two-tailed test, no direction is specified. We are only interested to test whether the population parameter is either greater than or less than some specified value. If the test statistic falls into either of the critical regions, the alternative hypothesis will be accepted instead of the null hypothesis. In two-tailed test, the critical region is divided in both the tails. For example, if the null hypothesis is tested at 5% level, the critical region shall be divided in both the tails as shown in Fig. 6.4. Tables A.1 and A.2 in Appendix provide critical values for *z*-test and *t*-test, respectively.

## Criteria for Using One-Tailed and Two-Tailed Tests

A one-tailed test is used when we are quite sure about the direction of the difference in advance (e.g., exercise will improve the fitness level). With that assumption, the level of significance ($\alpha$) is only calculated from one tail of the distribution. However, in standard testing, the probability is calculated from both tails.

For instance if the significance of correlation is tested between age and medical expenses; one might hypothesize that medical expenses may increase or do not increase but will never decrease with age. In such case one-tailed hypothesis should be used. On the other hand, in testing the correlation between people's weights with their income, we may not have reasons to believe that the income will increase with increase in weights or the income will decreases with weights. Here we might be interested just to find out if there was any relationship at all and that is a two-tailed hypothesis.

The issue in deciding between one-tailed and two-tailed tests is not whether or not you expect a difference to exist. Had you known whether or not there was a difference, there is no reason to collect the data. Instead, the question is whether the direction of a difference can only go one way. One should only use a one-tailed test if there is an absolute certainty before data collection that in the overall populations, either there is no difference or there is a difference in a specified direction. Further, if you end up showing a difference in the opposite direction, you should be ready to attribute that difference to random sampling without bothering about the fact that the measured difference might reflect a true difference in the overall populations. If a difference in the "wrong" direction brings even little meaning to your findings, you should use two-tailed test.

The advantage of using one-tailed hypothesis is that you can use a smaller sample to test it. The smaller sample often reduces your cost of the experiment. But on the other hand, it is easier to reject the null hypothesis with a one-tailed test in comparison to two-tailed test. Thus, the level of significance increases in one-tailed test. Because of this reason, it is rarely correct to perform a one-tailed test; usually we want to test whether any difference exists.

## Strategy in Testing One-Tailed and Two-Tailed Tests

The strategy in choosing between one-tailed and two-tailed tests is to prefer a two-tailed test unless there is a strong belief that the difference in the population can only be in one direction. If the two-tailed test is statistically significant ($p < \alpha$), interpret the findings in one-tailed manner. Consider an experiment in which it is desired to test the null hypothesis that the average cure time of cold and cough by a newly introduced vitamin C tablet is 4 days against an alternative hypothesis that it is not. If a sample of 64 patients has an average recovery time of 3.5 days with $s = 1.0$ day, the $p$ value in this testing would be 0.0002 and therefore the null hypothesis $H_0$ will be rejected and we accept the alternative hypothesis $H_1$. Thus, in this situation, it is concluded that the recovery time is not equal to 4 days for the new prescription of vitamin C.

But we may conclude more than that in saying that the recovery time is less than 4 days with the new prescription of vitamin C. We arrive at this conclusion by combining the two facts: Firstly, we have proved that the recovery time is different than 4 days, which means it must be either less or more than 4 days, and secondly, the sample mean $\bar{X}$ (=3.5 days) in this problem is less than the specified value, that

is, 4 days(population mean). After combining these two facts, it may be concluded that the average recovery time (3.5 days) is significantly lower than the 4 days. This conclusion is quite logical because if we again test the null hypothesis $H_0: \mu \geq 4$ against the alternative hypothesis $H_1: \mu < 4$(one-tailed test), the $p$ value would be 0.0001 which is even smaller than 0.0002.

Thus, we may conclude first by answering the original question then going for writing about the directional difference such as "The mean recovery time in cold and cough symptom with the new prescription of vitamin C is different from 4 days"; in fact, it is less than 4 days.

## What Is *p* Value?

The $p$ value is the probability of wrongly rejecting the null hypothesis. It is analogous to the level of significance. Usually an experimenter decides to test the hypothesis at some desired level of significance. If the absolute value of test statistic increases, the probability of rejecting the correct null hypothesis decreases. Thus, if a null hypothesis is tested at the level of significance .05 and the value of test statistic is large so that its corresponding $p$ value is 0.004, in that case if we conclude that the null hypothesis is rejected at 5% level, it would not be logically correct as the error attached to this judgment is only 0.4%. In fact as the absolute value of test statistic increases, the $p$ value keeps on decreasing.

One may decide the level of significance in advance say, 0.05, but while explaining the decision, the concept of $p$ value should be used to report as to how much error is involved in the decision about rejecting or being unable to reject the null hypothesis. Thus, while testing a hypothesis, a $p$ value is calculated against the test statistic which is used to explain the error involved in the decision. In SPSS and other statistical packages, the $p$ values are automatically computed against each test statistic. Thus, if an experimenter decides to test the hypothesis at the significance level of 0.05, the test statistic shall be significant so long $p$ value is less than 0.05. The general practice is to write the $p$ value along with the value of test statistic. For instance, we may write as "Since the calculated $t = 4.0 (p = 0.0002)$ is significant, the null hypothesis may be rejected." The $p$ value may be calculated against the value of $t$-statistic by using the $t$-table or by using the free conversion software available on many sites such as http://faculty.vassar.edu/lowry/tabs.html#t.

## Degrees of Freedom

Any parameter can be estimated with certain amount of information or data set. The number of independent pieces of data or scores that are used to estimate a parameter is known as degrees of freedom and is usually abbreviated as df. In general, the degrees of freedom of an estimate are calculated as the number of independent scores that are required to estimate the parameter minus the number of parameters estimated as

intermediate steps in the estimation of the parameter itself. In general, each item being estimated costs one degree of freedom.

The *degrees of freedom* can be defined as the number of independent scores or pieces of information that are free to vary in computing a statistic.

Since the variance $\sigma^2$ is estimated by the statistic $S^2$ which is computed from a random sample of $n$ independent scores, let us see what the degrees of freedom of $S^2$ are. Since $S$ is computed from the sample of $n$ scores, its degrees of freedom would have been $n$, but because one degree of freedom is lost due to the condition that $\sum (X - \bar{X}) = 0$, the degrees of freedom for $S^2$ are $n - 1$. If we go by the definition, the degrees of freedom of $S^2$ are equal to the number of independent scores ($n$) minus the number of parameters estimated as intermediate steps (one, as $\mu$ is estimated by $\bar{X}$) and are therefore equal to $n - 1$.

In case of two samples, pooled standard deviation $S$ is computed by using $n_1 + n_2$ observations. In the computation of $S$, the two parameters $\mu_1$ and $\mu_2$ are estimated by $\bar{X}_1$, and $\bar{X}_2$ hence, the two degrees of freedom are lost and therefore the degrees of freedom for estimating $S$ are $n_1 + n_2 - 2$.

In computing chi-square in a $2 \times 2$ contingency table for testing the independence between rows and columns, it is assumed that you already know 3 pieces of information: the row proportions, the column proportions, and the total number of observations. Since the total number of pieces of information in the contingency table is 4, and 3 are already known before computing the chi-square statistic, the degrees of freedom are $4 - 3 = 1$. We know that the degrees of freedom for chi-square are obtained by $(r - 1) \times (c - 1)$; hence, with this formula, also the degrees of freedom in a $2 \times 2$ contingency table are 1.

## One-Sample *t*-Test

A *t*-test can be defined as a statistical test used for testing of hypothesis in which the test statistic follows a Student's *t*-distribution under the assumption that the null hypothesis is true. This test is used if the population standard deviation is not known and the distribution of the population from which the sample has been drawn is normally distributed. Usually *t*-test is used for small sample size ($n < 30$) in a situation where population standard deviation is not known. Even if the sample is large ($n \geq 30$) but if the population standard deviation is not known in that situation, also *t*-test should be used instead of *z*-test. A one-sample *t*-test is used for testing whether the population mean is equal to a predefined value or not. An example of a one-sample *t*-test may be to see whether population average sleep time is equal to 5 h or not.

In using *t*-test, it is assumed that the distribution of data is approximately normal. The *t*-distribution depends on the sample size. Its parameter is called the degrees of freedom (df) which is equal to $n - 1$, where $n$ is the sample size.

In one-sample test, *t*-statistic is computed by the following formula:

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} \tag{6.1}$$

Calculated *t* is compared with tabulated *t* at 0.05 level of significance and $n - 1$ degrees of freedom if the hypothesis is to be tested at 5% level. The value of tabulated *t* can be obtained from Table A.2 in Appendix. The *t*-statistic is tested for its significance by finding its corresponding *p* value. If *p* value is less than .05, the *t*-statistic becomes significant, and we reject the null hypothesis against the alternative hypothesis. On the other hand, if the *p* value is more than .05, the null hypothesis is failed to be rejected.

## *Application of One-Sample Test*

In the era of housing boom, everybody is interested to buy a home, and the role of banking institution is very important in this regard. Every bank tries to woe their clients by highlighting their specific features of housing loan like less assessment fee, quick sanctioning of the loans, and waving of penalty for prepayment. One particular bank was more interested to concentrate on loan processing time instead of other attributes and therefore made certain changes in their loan processing procedure without sacrificing the risk features so as to serve their clients with quick processing time. They want to test if their mean loan processing time differs from a competitor's claim of 4 h. The bank randomly selected a sample of few loan applications in their branches and noted the processing time for each cases. On the basis of this sample data, the authorities may be interested to test whether the bank's processing time in all their branches is equal to 4 h or not. One-sample *t*-test can provide the solution to test the hypothesis in this situation.

**Example 6.1** A professor wishes to know if his statistics class has a good background of basic math. Ten students were randomly chosen from the class and were given a math proficiency test. Based on the previous experience, it was hypothesized that the average class performance on such math proficiency test cannot be less than 75. The professor wishes to know whether this hypothesis may be accepted or not. Test your hypothesis at 5% level assuming that the distribution of the population is normal. The scores obtained by the students are as follows:

Math proficiency score: 71, 60, 80, 73, 82, 65, 90, 87, 74, and 72

*Solution* The following steps shall show the procedure of applying the *t*-test for one sample in testing the hypothesis, whether the students of statistics class had their average score on math proficiency test equal to 75 or not.

(a) Here the hypothesis which needs to be tested is

$$H_0 : \mu \geq 75$$

against the alternative hypothesis

$$H_1 : \mu < 75$$

(b) *The level of significance:* 0.05

(c) *Statistical test:* As per the test selection scheme shown in Fig. 6.1, the test applicable in this example shall be one-sample *t*-test.

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

To compute the calculated value of *t*, firstly it is required to compute the value of mean and standard deviation of the sample:

| $X$ | $X^2$ |
|---|---|
| 71 | 5,041 |
| 60 | 3,600 |
| 80 | 6,400 |
| 73 | 5,329 |
| 82 | 6,724 |
| 65 | 4,225 |
| 90 | 8,100 |
| 87 | 7,569 |
| 74 | 5,476 |
| 72 | 5,184 |
| $\sum X = 754$ | $\sum X^2 = 57,648$ |

Since $n = 10$, $\bar{X} = \frac{754}{10} = 75.4$ and

$$S = \sqrt{\frac{1}{n-1}\sum X^2 - \frac{(\sum X)^2}{n(n-1)}}$$

$$= \sqrt{\frac{1}{9} \times 57648 - \frac{754^2}{10 \times 9}} = \sqrt{6405.33 - 6316.84}$$

$$= 9.41$$

After substituting the value of mean and standard deviation,

$$\text{Calculated} \quad t = \frac{75.4 - 75}{9.41/\sqrt{10}} = \frac{0.4 \times \sqrt{10}}{9.41}$$

$$= 0.134$$

(d) *Decision criteria:* From Table A.2 in Appendix, the tabulated value of $t$ for one-tailed test at .05 level of significance with 9 degrees of freedom is $t_{.05}(9) = 1.833$.
Since calculated $t(= 0.134) < t_{.05}(9)$, hence the null hypothesis is failed to be rejected at 5% level.

(e) *Inference:* Since the null hypothesis is failed to be rejected, hence the alternative hypothesis that the average math proficiency performance of the students is less than 75 cannot be accepted. Thus, it may be concluded that the average students' performance on math proficiency test is equal or higher than 75.

## Two-Sample *t*-Test for Unrelated Groups

The two-sample $t$-test is used for testing the hypothesis of equality of means of two normally distributed populations. All $t$-tests are usually called *Student's t-tests*. But strictly speaking, this name should be used only if the variances of the two populations are also assumed to be equal. Two-sample $t$-test is based on the assumption that the variances of the populations $\sigma_1^2$ and $\sigma_2^2$ are unknown and population distributions are normal. In case the assumptions of equality of variances are not met, then the test used in such situation is called as Welch's $t$-test. Readers may read some other text for this test.

We often want to compare the means of two different populations, for example, comparing the effect of two different diets on weights, the effect of two teaching methodologies on the performance, or the IQ of boys and girls. In such situations, two-sample $t$-test can be used. One of the conditions of using two-sample $t$-test is that the samples are independent and identically distributed. Consider an experiment in which the job satisfaction needs to be compared among the bank employees working in rural and urban areas. Two randomly selected groups of 30 subjects each may be selected from rural and urban areas. Assuming all other conditions of the employees like salary structure, status and age categories to be similar, null hypothesis of no difference in their job satisfaction scores may be tested by using the two-sample $t$-test for independent samples. In this case, the two samples are independent because subjects in both the groups are not same.

### *Assumptions in Using Two-Sample* t*-Test*

The following assumptions need to be fulfilled before using the two-sample $t$-test for independent groups:

- The distributions of both the populations from which the samples have been drawn are normally distributed.
- The variances of the two populations are nearly equal.

- Population variances are unknown.
- The samples are independent to each other.

Since we assume that $\sigma_1^2$ and $\sigma_2^2$ are equal, we can compute a pooled variance $S^2$ of both the samples. The purpose of pooling the variances is to obtain a better estimate. The pooled variance is a weighted sum of variances. Thus, if the sample sizes $n_1$ and $n_2$ are equal, then $S^2$ is just an average of the individual variances. The overall degrees of freedom in that case will be the sum of the individual degrees of freedom of the two samples, that is,

$$\mathrm{df} = \mathrm{df} + \mathrm{df}_2 = (\mathrm{n}_1 - 1) + (\mathrm{n}_2 - 1) = \mathrm{n}_1 + \mathrm{n}_2 - 2$$

Computation of *t*-statistic is same irrespective of testing two-tailed or one-tailed hypotheses. The only difference in testing these hypotheses are in its testing criteria and critical values of "*t*." These cases shall be discussed in the following sections.

## *Application of Two-Sampled* t-*Test*

The situation where two-sample *t*-test is applicable can be easily understood by looking to the following case study. A pharmaceutical company decided to conduct an experiment to know as to whether high-protein diet or low-protein diets are more responsible in increasing the weights of male mouse in a controlled environment. Two groups of male mouse of similar age and weights may be selected randomly to serve as the two experimental groups. The number of mouse may be equal or unequal in both the groups. The first group may be fed with low-protein diet, whereas the other may be on the high-protein diet. To compare the average increase in their weights, two-sample *t*-test may be used to answer the research question.

Since one of the conditions of using the two-sample *t*-test is that the variance of the two groups must be equal, therefore *F*-test may be used to compare the variability. Only if the variability of the two groups is equal the two-sample *t*-test should be used. Here the null hypothesis of no difference in the increased weights of the high and low-protein groups is tested against the alternative hypothesis that the difference exists. In case the two-sample *t*-statistic is significant at some specified level of significance, the null hypothesis may be rejected, and it may be concluded that the effect of low-protein and high-protein diets on weights is different. On the other hand, if the *t*-statistic is not significant, we failed to reject the null hypothesis, and it may be concluded that it is not possible to find any significant difference in the rats' weight kept on high- and low-protein diets.

Further, if the null hypothesis is rejected, the mean weight of the high- and low-protein groups is seen, and if the average weight of the high-protein group is higher than that of the low-protein group, it may be concluded that the high-protein diet is more effective than the low-protein diet in increasing the weights of the rats.

### Case 1: Two-Tailed Test

Since we have already discussed the general procedure of testing of hypothesis and the situations under which the two-tailed tests should be used, here the working method for two-tailed test shall be discussed. In two-tailed test, null hypothesis is tested against the alternative hypothesis that the groups are different in their means. Acceptance of alternative hypothesis suggests that the difference exists between the two group means. Further, by looking to the mean values of the two groups, one may draw the conclusion as to which group means is greater than the other. There may be many situations where two-tailed test can be used. For example, consider an experiment where it is desired to see the impact of different kinds of music on the hours of sleep. The two groups of the subjects are randomly selected, and the first group is exposed to classical music, whereas the second group is exposed to Jazz music for 1 h before sleep for a week. To test whether average sleep hour remains same or different in two different kinds of music groups, a two-tailed test may be used. Here it is not known that a particular music may increase the sleep hour or not, and hence, two-tailed test would be appropriate. In case of two-tailed test, the testing protocol is as follows:

(a) *Hypotheses need to be tested*

$$\mathrm{H}_0 : \mu_1 = \mu_2$$
$$\mathrm{H}_1 : \mu_1 \neq \mu_2$$

(b) *Test statistic*

$$\text{Calculated} \quad t = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \tag{6.2}$$

where $S = \sqrt{\dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$

(c) *Degrees of freedom* $n_1 + n_2 - 2$

(d) *Decision criteria*

In two-tailed test, the critical region is divided in both the tails. If the level of significance is $\alpha$, then the area in each tail would be $\alpha/2$. If the critical value is $t_{\alpha/2}$ and

if calculated $|t| \leqslant t_{\alpha/2}$, $\mathrm{H}_0$ is failed to be rejected at $\alpha$ level of significance and if calculated $|t| > t_{\alpha/2}$, $\mathrm{H}_0$ may be rejected at $\alpha$ level of significance

**Note:** The value of calculated *t* is taken as absolute because the difference in the two means may be positive or negative.

**Case II: Right-Tailed Test (One-Tailed Test)**

We have already discussed the situations in which one-tailed test should be used. One-tailed test should only be used if an experimenter, on the basis of past information, is absolutely sure that the difference can go only in one direction. One-tailed test can be either right tailed or left tailed. In right-tailed test, it is desired to test the hypothesis, whether mean of first group is greater than that of the mean of the second group. In other words, the researcher is interested in a particular group only. In such testing, if the null hypothesis is rejected, it can be concluded that the first group mean is significantly higher than that of the second group mean. The situation where right-tailed test can be used is to test whether frustration level is less among those employees whose jobs are linked with incentives in comparison to those whose jobs are not linked with the incentives. Here the first group is the one whose jobs are linked with the incentives, whereas the second group's jobs are not linked with the incentives. In this situation, it is assumed that the employees feel happy in their jobs if it is linked with incentives. The testing protocol in testing the right-tailed hypothesis is as follows:

(a) *Hypotheses need to be tested*

$$H_0 : \mu_1 \leqslant \mu_2$$
$$H_1 : \mu_1 > \mu_2$$

(b) *Test statistic*

$$\text{Calculated} \quad t = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $S = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$

(c) *Degrees of freedom* $n_1 + n_2 - 2$

(d) *Decision criteria*

In one-tailed test, the entire critical region lies in one tail only. Here the research hypothesis is the right tailed; hence, the entire critical region would lie in the right tail only, and therefore, the sign of the critical value would be positive. If the critical value is represented by $t_\alpha$ and

if calculated $t \leqslant t_\alpha$, $H_0$ is failed to be rejected at $\alpha$ level of significance

and if calculated $t > t_\alpha$, $H_0$ may be rejected at $\alpha$ level of significance

**Case III: Left-Tailed Test (One-Tailed Test)**

At times the researcher is interested in testing whether a particular group mean is less than the second one. In this type of hypothesis testing, it is desired to test whether mean of first group is less than that of mean of the second group. Here if the null

hypothesis is rejected, it can be concluded that the first group mean is significantly smaller than that of the second group mean. Consider a situation where an exercise therapist is interested to know whether a 4-week weight reduction program is effective or not if implemented on the housewives. The two groups consisting 20 women each are selected for the study, and the first group is exposed to the weight reduction program, whereas the second group serves as a control and does not take part in any special activities except daily normal work. If the therapist is interested to know whether on an average first treatment group shows the reduction in their weight in comparison to those who did not participate in the program, the left-tailed test may be used. In this situation, as per the experience, it is known that any weight reduction program will always reduce the weight in general in comparison to those who do not participate in it, and therefore, one tailed test would be appropriate in this situation. The testing protocol in applying the left-tailed test is as follows:

(a) *Hypotheses need to be tested*

$$H_0 : \mu_1 \geq \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

(b) *Test statistic*

$$\text{Calculated} \qquad t = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$S = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

(c) *Degrees of freedom* $n_1 + n_2 - 2$

(d) *Decision criteria*

In one-tailed test, the entire critical region lies in one tail. Since this is a case of left-tailed test, hence the entire critical region lies in the left tail only and therefore the critical value would be negative. If the critical value is represented by $-t_\alpha$ and

if calculated $t \geq -t_\alpha$, $H_0$ is failed to be rejected at $\alpha$ level of significance

and if calculated $< -t_\alpha$, $H_0$ may be rejected at $\alpha$ level of significance

**Example 6.2** Counseling cell of a college keeps conducting sessions with the problematic students by using different methods. Since the number of visitors keeps increasing every day in the center, they have decided to test whether audiovisual-based counseling and personal counseling are equally effective in reducing the stress level. Eighteen women students were randomly chosen among those who visited the center. Nine of them were given the personal counseling, whereas the other nine were given the sessions with the audiovisual presentation. After the session, the students were tested for their stress level. The data so obtained are shown in Table 6.2.

**Table 6.2** Data on stress level for the students in both the counseling groups

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Personal counseling: | 27 | 22 | 28 | 21 | 23 | 22 | 20 | 31 | 26 |
| Audiovisual counseling: | 35 | 28 | 24 | 28 | 31 | 32 | 33 | 34 | 30 |

Test your hypothesis at 1% level, whether any one method of counseling is better than other. It is assumed that population variances are equal and both the populations are normally distributed.

**Solution** To test the required hypothesis, the following steps shall explain the procedure.

(a) *Here the hypothesis which needs to be tested is*

$$H_0 : \mu_{\text{Personal}} = \mu_{\text{Audi-visual}}$$

against the alternative hypothesis

$$H_1 : \mu_{\text{Personal}} \neq \mu_{\text{Audi-visual}}$$

(b) *The level of significance:* 0.05

(c) *Statistical test:* In this example, it is required to test a two-tailed hypothesis for comparing the means of two groups. Thus, as per the scheme of the test selection shown in Fig. 6.1, a two-sample *t*-test for independent groups shall be appropriate in this case which is given by

$$t = \frac{\bar{X} - \bar{Y}}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where the pooled standard deviation $S$ is computed as

$$S = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

In order to compute the value of *t*-statistic, the mean and standard deviation of both the groups along with the pooled standard deviation $S$ will have to be computed first (Table 6.3).

Since $n_1 = n_2 = 9$, $\bar{X} = \frac{220}{9} = 24.44$ and $\bar{Y} = \frac{275}{9} = 30.56$

$$\begin{aligned} S_x &= \sqrt{\frac{1}{n_1 - 1}\sum X^2 - \frac{(\sum X)^2}{n_1(n_1 - 1)}} \\ &= \sqrt{\frac{1}{8} \times 5488 - \frac{(220)^2}{9 \times 8}} = \sqrt{686 - 672.22} \\ &= 3.71 \end{aligned}$$

**Table 6.3** Computation for mean and standard deviation

| Personal counseling | | Audiovisual counseling | |
|---|---|---|---|
| $X$ | $X^2$ | $Y$ | $Y^2$ |
| 27 | 729 | 35 | 1,225 |
| 22 | 484 | 28 | 784 |
| 28 | 784 | 24 | 576 |
| 21 | 441 | 28 | 784 |
| 23 | 529 | 31 | 961 |
| 22 | 484 | 32 | 1,024 |
| 20 | 400 | 33 | 1,089 |
| 31 | 961 | 34 | 1,156 |
| 26 | 676 | 30 | 900 |
| $\sum X = 220$ | $\sum X^2 = 5,488$ | $\sum Y = 275$ | $\sum Y^2 = 8,499$ |

Similarly

$$S_Y = \sqrt{\frac{1}{n_2 - 1}\sum Y^2 - \frac{(\sum Y)^2}{n_2(n_2 - 1)}}$$
$$= \sqrt{\frac{1}{8} \times 8499 - \frac{(275)^2}{9 \times 8}} = \sqrt{1062.38 - 1050.35}$$
$$= 3.47$$

Further, pooled standard deviation $S$ is equal to

$$S = \sqrt{\frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}} = \sqrt{\frac{8 \times 3.71^2 + 8 \times 3.47^2}{9 + 9 - 2}}$$
$$= \sqrt{\frac{110.11 + 96.33}{16}} = 3.59$$

One of the conditions of using the two-sample $t$-test for independent groups is that the variance of the two populations must be same. This hypothesis can be tested by using the $F$-test.

Thus, $$F = \frac{S_X^2}{S_Y^2} = \frac{3.71^2}{3.47^2} = 1.14$$

From Table A.4 in Appendix, tabulated $F_{.05}(8,8) = 3.44$
Since calculated value of $F$ is less than the tabulated $F$, hence it may not be concluded that the variances of the two groups are different, and therefore, two-sample $t$-test for two independent samples can be applied in this example.

***Remark***: In computing $F$-statistic, the larger variance must be kept in the numerator, whereas the smaller one should be in the denominator.

After substituting the values of $\bar{X}$, $\bar{Y}$, and pooled standard deviation $S$, we get

$$\text{calculated t} = \frac{\bar{X} - \bar{Y}}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{24.44 - 30.56}{3.59\sqrt{\left(\frac{1}{9} + \frac{1}{9}\right)}}$$

$$= -\frac{6.12}{1.69}$$

$$= -3.62$$

$$\Rightarrow \quad \text{calculated} \quad |\text{t}| = 3.62$$

(d) *Decision criteria:* From Table A.2 in Appendix, the tabulated value of $t$ for two-tailed test at .05 level of significance with $16(=n_1 + n_2 - 2)$ degrees of freedom is $t_{.05}(16) = 2.12$.
Since calculated $t(= 3.62) > t_{.05}(16)$, the null hypothesis may be rejected at 5% level against the alternative hypothesis.
Further, since the mean stress score of the personal counseling group is lower than that of the audiovisual group, hence it may be concluded that the stress score of the personal counseling group is significantly less than that of the audiovisual group.

(e) *Inference:* Since the null hypothesis is rejected, hence the alternative hypothesis that the average stress scores of the personal counseling group as well as audiovisual counseling groups are not same is accepted. Further, since the mean stress score of the personal counseling group is significantly lower than that of the audiovisual group, it may be concluded that the personal counseling is more effective in comparison to that of the audiovisual counseling in reducing stress among women.

**Example 6.3** A researcher wishes to know whether girls' marriage age in metro cities is higher than that of class B cities. Twelve families from metro cities and 11 families from class B cities were randomly chosen and were asked about their daughter's age at which they got married. The data so obtained are shown in Table 6.4. Can it be concluded from the given data that the girls' marriage age was higher in metro cities in comparison to class B cities? Test your hypothesis at 5% level assuming that the population variances are equal and the distribution of both the populations from which the samples have been drawn are normally distributed.

*Solution* In order to test the hypothesis, the following steps shall be performed:

(a) *The hypothesis which needs to be tested is*

**Table 6.4** Marital age of the girls

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Metro city: | 29, | 28, | 27, | 31, | 32, | 25, | 28, | 24, | 27, | 30, | 35, | 26 |
| Class B city: | 28, | 25, | 24, | 28, | 22, | 24, | 23, | 21, | 25, | 24, | 28 | |

$$H_0 : \mu_{\text{Metro\_City}} \leqslant \mu_{\text{Class\_B\_City}}$$

against the alternative hypothesis

$$H_0 : \mu_{\text{Metro\_City}} > \mu_{\text{Class\_B\_City}}$$

(b) *The level of significance:* 0.05

(c) *Statistical test:* In this example, it is required to test one-tailed hypothesis for comparing means of the two groups. Thus, as per the scheme of the test selection shown in Fig. 6.1, a two-sample *t*-test for independent groups shall be appropriate in this case which is

$$t = \frac{\bar{X} - \bar{Y}}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where the pooled standard deviation $S$ is given by

$$S = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

To compute the value of $t$ statistic, the mean and standard deviation of both the groups along with the pooled standard deviation $S$ need to be computed first (Table 6.5).

Here $n_1 = 12$ and $n_2 = 11$ $\bar{X} = \frac{342}{12} = 28.5$ and $\bar{Y} = \frac{272}{11} = 24.73$

$$\begin{aligned} S_X &= \sqrt{\frac{1}{n_1 - 1}\sum X^2 - \frac{(\sum X)^2}{n_1(n_1 - 1)}} \\ &= \sqrt{\frac{1}{11} \times 9854 - \frac{(342)^2}{12 \times 11}} = \sqrt{895.82 - 886.09} \\ &= 3.12 \end{aligned}$$

**Table 6.5** Computation for mean and standard deviation

| Metro city | | Class B city | |
|---|---|---|---|
| $X$ | $X^2$ | $Y$ | $Y^2$ |
| 29 | 841 | 28 | 784 |
| 28 | 784 | 25 | 625 |
| 27 | 729 | 24 | 576 |
| 31 | 961 | 28 | 784 |
| 32 | 1,024 | 22 | 484 |
| 25 | 625 | 24 | 576 |
| 28 | 784 | 23 | 529 |
| 24 | 576 | 21 | 441 |
| 27 | 729 | 25 | 625 |
| 30 | 900 | 24 | 576 |
| 35 | 1,225 | 28 | 784 |
| 26 | 676 | | |
| $\sum X = 342$ | $\sum X^2 = 9,854$ | $\sum Y = 272$ | $\sum Y^2 = 6,784$ |

Similarly

$$S_Y = \sqrt{\frac{1}{n_2 - 1}\sum Y^2 - \frac{(\sum Y)^2}{n_2(n_2 - 1)}}$$
$$= \sqrt{\frac{1}{10} \times 6784 - \frac{(272)^2}{11 \times 10}} = \sqrt{678.4 - 672.58}$$
$$= 2.41$$

The pooled standard deviation $S$ is equal to

$$S = \sqrt{\frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}} = \sqrt{\frac{11 \times 3.12^2 + 10 \times 2.41^2}{12 + 11 - 2}}$$
$$= \sqrt{\frac{107.08 + 58.08}{21}} = 2.80$$

Since $t$-test can only be applied if the variance of both the populations is same, this hypothesis can be tested by using the $F$-test.

Thus, $F = \frac{S_X^2}{S_Y^2} = \frac{3.12^2}{2.41^2} = 1.67$

The tabulated value of $F$ can be seen from Table A.4 in Appendix.

Thus, tabulated $F_{.05}(11,10) = 2.85$

Since calculated value of $F$ is less than that of tabulated $F$, hence hypothesis of equality of variances in two groups may not be rejected, and therefore, the two-sample $t$-test for independent samples can be applied in this example.

After substituting the values of $\bar{X}$, $\bar{Y}$, and pooled standard deviation, $S$ we get

$$\text{calculated t} = \frac{\bar{X} - \bar{Y}}{S\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{28.5 - 24.73}{2.80\sqrt{\left(\frac{1}{12} + \frac{1}{11}\right)}}$$
$$= \frac{3.77}{2.80 \times 0.42}$$
$$= 3.21$$

(d) *Decision criteria:* From Table A.2 in Appendix, the tabulated value of *t* for one-tailed test at .05 level of significance with $21(=n_1 + n_2 - 2)$ degrees of freedom is $t_{.05}(21) = 1.721$. Similarly for one-tailed test, tabulated value of *t* at .01 level of significance is $t_{.01}(21) = 2.518$.
Since calculated $t(= 3.21) > t_{.05}(21)$, the null hypothesis may be rejected at 5% level. Further, calculated value of *t* is also less than that of tabulated value of *t* at 1% level as well; hence, *t*-value is also significant at 1% level.

(e) *Inference:* Since the null hypothesis is rejected, hence the alternative hypothesis that the marriage age of the girls in metro cities is higher than that of class B cities is accepted. It may thus be concluded that girls in metro cities prefers to marry late in age in comparison to that of class B cities.

### Paired *t*-Test for Related Groups

Paired *t*-test is used to test the null hypothesis that the difference between the two responses measured on the same experimental units has a mean value of zero. This statistical test is normally used to test the research hypothesis as to whether the posttreatment response is better than the pretreatment response. Paired *t*-test is used in all those situations where there is only one experimental group and no control group. The question which is tested here is to know whether the treatment is effective or not. This is done by measuring the responses of the subjects in the experimental group before and after the treatment. There can be several instances in which the paired *t*-test may be used. Such situations may be, for instance, to see the effectiveness of management development program on the functional efficiency, effectiveness of the weight training program in weight reduction, effectiveness of the psychological training in enhancing memory retention power, etc.

The paired *t*-test is also known as "repeated measures" *t*-test. In using the paired *t*-test, the data must be obtained in pair on the same set of subjects before and after the experiment.

While applying the paired *t*-test for two related groups, the pairwise differences, $d_i$, is computed for all *n* paired data. The mean, $\bar{d}$ and standard deviation, $S_d$, of the differences $d_i$ are calculated. Thus, paired *t*-statistic is computed as follows:

$$t = \frac{\bar{d}}{S_d/\sqrt{n}} \tag{6.3}$$

where "*t*" follows the Student's *t*-distribution with $n - 1$ degrees of freedom.

An assumption in using the paired $t$-test is that the difference $d_i$ follows the normal distribution. An experiment where paired difference is computed is often more powerful, since it can eliminate differences in the samples that increase the total variance $\sigma^2$. When the comparison is made between groups (of similar experimental units), it is called blocking. The paired difference experiment is an example of a randomized block experiment.

**Note:** The blocking has to be done before the experiment is performed.

## *Assumptions in Using Paired* t-*Test*

While using the paired $t$-test, the following assumptions need to be satisfied:

1. The distribution of the population is normal.
2. The distribution of scores obtained by pairwise difference is normal, and the differences are a random sample.
3. Cases must be independent of each other.

**Remark:** If the normality assumption is not fulfilled, you may use the nonparametric Wilcoxon sign rank test for paired difference designs.

## *Testing Protocol in Using Paired* t-*Test*

Testing protocol of using paired $t$-test is similar to that of two-sample $t$-test for independent groups discussed above. In applying paired $t$-test, the only difference is that the test statistic is

$$t = \frac{\bar{d}}{S_d/\sqrt{n}}$$

instead of the one used in two-sample $t$-test. Further, in paired $t$-test, the degrees of freedom are $n - 1$. While using paired $t$-test, one should normally construct the two-tailed test first, and if the difference is significant, then by looking to the values of the samples mean of the pre- and posttesting responses, one may interpret as to which group mean is higher than the other. In general using one-tailed test should be avoided until there is strong evidence that the difference can go only in one direction. In one-tailed test, the probability of rejecting the correct null hypothesis becomes more in comparison to two-tailed test for the same level of significance.

**Table 6.6** Calorie intake of the women participants before and after the nutrition educative program

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Before: | 2,900 | 2,850 | 2,950 | 2,800 | 2,700 | 2,850 | 2,400 | 2,200 | 2,650 | 2,500 | 2,450 | 2,650 |
| After: | 2,800 | 2,750 | 2,800 | 2,800 | 2,750 | 2,800 | 2,450 | 2,250 | 2,550 | 2,450 | 2,400 | 2,500 |

The trade-off using one- and two-tailed tests has been discussed in details while discussing the criteria for using one-tailed and two-tailed tests earlier in this chapter.

### Application of Paired *t*-Test

The application of paired *t*-test can be understood by considering the following situation. An herbal company has come out with a drug useful for lowering the cholesterol level if taken for a week. In order to claim its effectiveness, it has been decided to administer the drug on the patients with high cholesterol level. In this situation, the paired *t*-test can be used to test the hypothesis to know as to whether there is any difference in the cholesterol level between post- and pretest data after administering this new drug. In this situation, the null hypothesis of no difference may be tested to test the two-tailed hypothesis first. If the *t*-statistic is significant, we reject the null hypothesis in favor of the alternative, and it is concluded that the difference exists between the cholesterol levels of the patients before and after the administration of the drug. On the other hand, if the *t*-statistic is not significant, we may fail to reject the null hypothesis and we may end up in concluding that the claim of the drug being effective may not be proved with this sample. After having rejected the null hypothesis, by looking to the average cholesterol level of the patients before and after the administration of the drugs, one may conclude as to whether the drug is effective or not.

**Example 6.4** Twelve women participated in a nutritional educative program. Their calorie intake, before and after the program, was measured which are shown in Table 6.6.

Can you draw the conclusion that the nutritional educative program was successful in reducing the participant's calorie requirements? Test your hypothesis at 5% level assuming that the differences of the scores are normally distributed.

*Solution* In this example, data is paired. In other words, post- and pretest data belongs to the same person, and therefore, the groups may be called as related or paired. To test a hypothesis as to whether the nutritional educative program is effective or not in reducing the calorie intake, the following steps shall be performed:

(a) *Here the hypothesis which needs to be tested is*

$H_0 : \mu_D = 0$(Difference of means of the two groups is zero.)
against the alternative hypothesis
$H_1 : \mu_D \neq 0$(Difference of means of the two groups is not equal to zero.)

(b) *The level of significance:* 0.05

(c) *Statistical test:* In this example, it is required to test the effectiveness of the nutritional educative program in reducing the calorie consumption in diet. Here the null hypothesis that there is no difference in the means of the two groups is to be tested against the alternative hypothesis that there is a difference. Once the null hypothesis is rejected, then on the basis of mean values of the pre- and posttesting data of calorie consumption, the conclusion would be drawn as to whether the program was effective or not.
Thus, first a two-tailed test will be used, and if the null hypothesis is rejected against the alternative hypothesis, then the directional interpretation would be made by looking to the mean values. It is because of the fact that if the *t*-statistic is significant in two-tailed test, then it will also be significant at one-tailed test. This can be understood like this: for the two-tailed test, the critical value $t_{\alpha/2}$ at $\alpha$ level of significance will always be greater than that of the critical value $t_\alpha$ in one-tailed test. And therefore, if the calculated value of $t$ is greater than $t_{\alpha/2}$, it will also be greater than $t_\alpha$.
Since this example is a case of paired samples, hence as per the scheme of the test selection shown in Fig. 6.1, the paired *t*-test for related groups shall be appropriate in this case which is given by

$$t = \frac{\bar{d}}{S_d/\sqrt{n}}$$

where $\bar{d}$ is the mean of the difference between $X$ and $Y$, and $S_d$ is the standard deviation of these differences as given by

$$S_d = \sqrt{\frac{1}{n-1}\sum d^2 - \frac{(\sum d)^2}{n(n-1)}}$$

The $\bar{d}$ and $S_d$ shall be computed first to find the value of *t*-statistic (Table 6.7).

**Remark**: Difference $d$ can be computed by subtracting postdata from predata or vice versa because two-tailed test is being used here

Here number of paired data, $n = 12$,

$$\bar{d} = \frac{600}{12} = 50$$

and

**Table 6.7** Computation for $\bar{d}$and $S_d$ for paired $t$-ratio

| Before $X$ | After $Y$ | $d = X - Y$ | $d^2$ |
|---|---|---|---|
| 2,900 | 2,800 | 100 | 10,000 |
| 2,850 | 2,750 | 100 | 10,000 |
| 2,950 | 2,800 | 150 | 22,500 |
| 2,800 | 2,800 | 0 | 0 |
| 2,700 | 2,750 | −50 | 2,500 |
| 2,850 | 2,800 | 50 | 2,500 |
| 2,400 | 2,450 | −50 | 2,500 |
| 2,200 | 2,250 | −50 | 2,500 |
| 2,650 | 2,550 | 100 | 10,000 |
| 2,500 | 2,450 | 50 | 2,500 |
| 2,450 | 2,400 | 50 | 2,500 |
| 2,650 | 2,500 | 150 | 22,500 |
| | | $\sum d = 600$ | $\sum d^2 = 90,000$ |

$$S_d = \sqrt{\frac{1}{n-1}\sum d^2 - \frac{(\sum d)^2}{n(n-1)}} = \sqrt{\frac{1}{11} \times 90000 - \frac{(600)^2}{12 \times 11}}$$
$$= \sqrt{8181.82 - 2727.27}$$
$$= 73.85$$

After substituting the values of $\bar{d}$ and $S_d$, we get

$$\text{calculated t} = \frac{\bar{d}}{S_d/\sqrt{n}} = \frac{50}{73.85/\sqrt{12}}$$
$$= \frac{50 \times \sqrt{12}}{73.85}$$
$$= 2.345$$

(d) *Decision criteria:* From Table A.2 in Appendix, the tabulated value of $t$ for two-tailed test at .05 level of significance with 11($=n - 1$) degrees of freedom is $t_{.05/2}(11) = 2.201$.
Since calculated $t(= 2.345) > t_{.05/2}(11)$, the null hypothesis may be rejected at 5% level against the alternative hypothesis. It may therefore be concluded that the mean calorie intakes before and after the nutritional educative program are not same.
Since the mean calorie intake of the after-testing group is lower than that of the before-testing group, it may be concluded that the mean calorie score of the after-testing group is significantly less than that of the before-testing group.

(e) *Inference:* It is therefore concluded that the nutritional educative program is effective in reducing the calorie intake among the participants.

## Solved Example of Testing Single Group Mean with SPSS

**Example 6.5** The age of the 15 randomly chosen employees of an organization is shown in Table 6.8. Can it be concluded that the average age of the employees in the organization is 28 years? Test your hypothesis at 5% level and interpret your findings.

*Solution* The hypothesis that needs to be tested here is

$$H_0 : \mu = 28$$

against the alternative hypothesis

$$H_1 : \mu \neq 28$$

After using the SPSS commands as mentioned below for testing the population mean to be equal to 28, the output will generate the value of *t*-statistic along with its *p* value. If *p* value is less than .05, then the *t*-statistic will be significant and the null hypothesis shall be rejected at 5% level in favor of alternative hypothesis; otherwise, we would fail to reject the null hypothesis.

### *Computation of* t-*Statistic and Related Outputs*

(a) ***Preparing Data File***

Before using the SPSS commands for computing the value of *t*-statistic and other related statistics for single group, a data file needs to be prepared. The following steps will help you to prepare the data file:

**Table 6.8** Data on age

| |
|---|
| 27 |
| 31 |
| 34 |
| 29 |
| 28 |
| 34 |
| 33 |
| 36 |
| 26 |
| 28 |
| 29 |
| 36 |
| 35 |
| 31 |
| 36 |

(i) *Starting the SPSS:* Use the following command sequence to start SPSS:
**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**
After checking the option **Type in Data** on the screen you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* There is only one variable in this example which needs to be defined in SPSS along with its properties. Since the variable is measured on interval scale, hence will be defined as 'Scale' variable. The procedure of defining the variable in the SPSS is as follows:

1. Click **Variable View** to define the variable and its properties.
2. Write short name of the variable as *Age* under the column heading **Name.**
3. Under the column heading **Label**, define full name of the variable as *Employees' Age*.
4. Under the column heading **Measure**, select the option 'Scale' for the variable.
5. Use default entries in all other columns.

After defining the variable in variable view, the screen shall look like Fig. 6.5.

(iii) *Entering data:* After defining the variable in the **Variable View**, click **Data View** on the left bottom of the screen to enter the data. Enter the data for the variable column wise. After entering the data, the screen will look like Fig. 6.6. Save the data file in the desired location before further processing.

(b) **SPSS *Commands for Computing t-Statistic***
After entering the data in the data view, follow these steps for computing *t*-statistic:

(i) *Initiating the SPSS commands for one-sample t-test:* In data view, go to the following commands in sequence:

**Analyze ⇨ Compare Means ⇨ One-Sample *t* Test**

The screen shall look like Fig. 6.7 as shown below.

(ii) *Selecting variables for t-statistic:* After clicking the **One-Sample *t* Test** option you will be taken to the next screen for selecting variable for computing *t*-statistic. Select the variable *Age* from left panel and bring it to the right panel by clicking the arrow sign. In case of computing *t*-value for more than one variable simultaneously, all the variables can be selected together. The screen shall look like Fig. 6.8.

(iii) *Selecting the options for computation:* After selecting the variable, option needs to be defined for the one-sample *t*-test. Do the following:

- In the screen shown in Fig. 6.8, enter the 'test value' as 28. This is the assumed population mean age that we need to verify in the hypothesis.
- Click the tag **Options**, you will get the screen shown in Fig. 6.9. Enter the confidence interval as 95% and click ***Continue*** and then you will be taken back to the screen shown in Fig. 6.8.

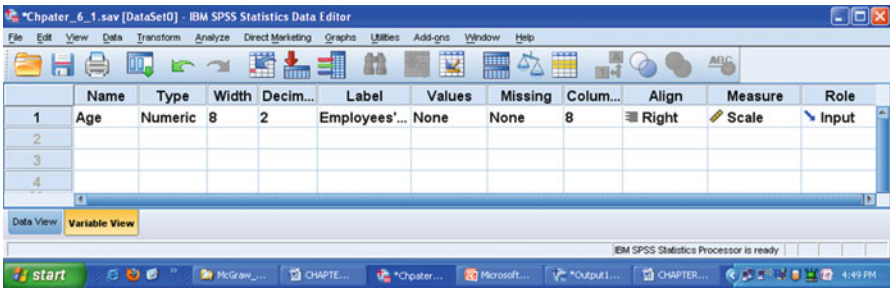
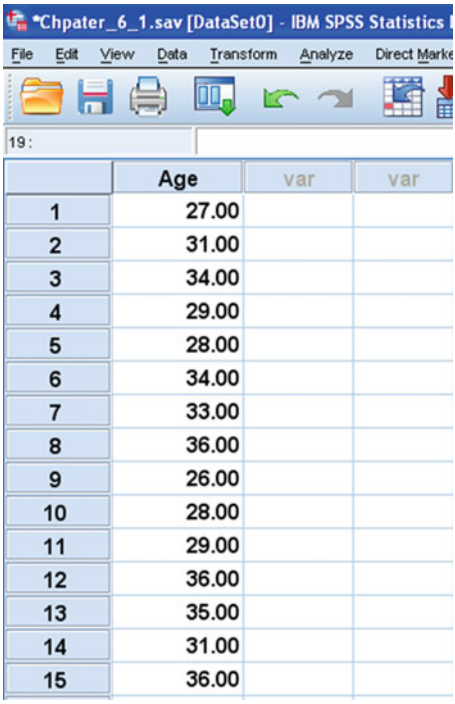



**Fig. 6.5** Defining variable and its characteristics for the data shown in Table 6.8

**Fig. 6.6** Screen showing entered data for the age variable in the data view




The confidence interval is chosen to get the confidence limits of mean based on sample data. Since in this example the null hypothesis needs to be tested at 5% level, choose the confidence interval as 95%.

– Click ***OK***.

(c) ***Getting the Output***

After clicking the **OK** tag in Fig. 6.8, you will get the output window. In the output window, the relevant outputs can be selected by using the right click of

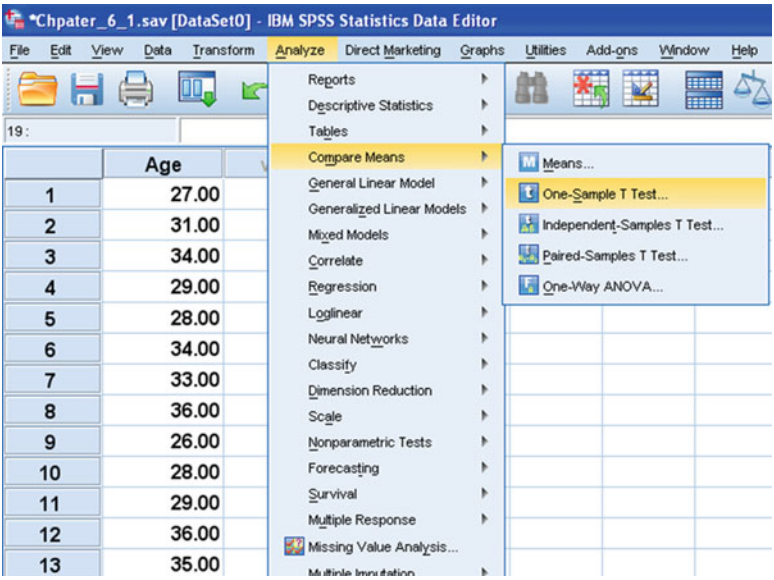



**Fig. 6.7** Screen showing SPSS commands for one sample *t*-test

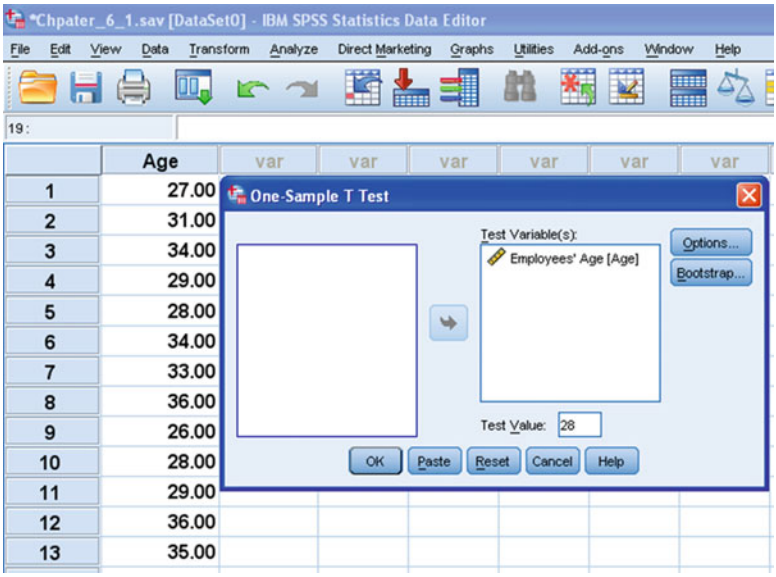



**Fig. 6.8** Screen showing selection of variable for one-sample *t*-test

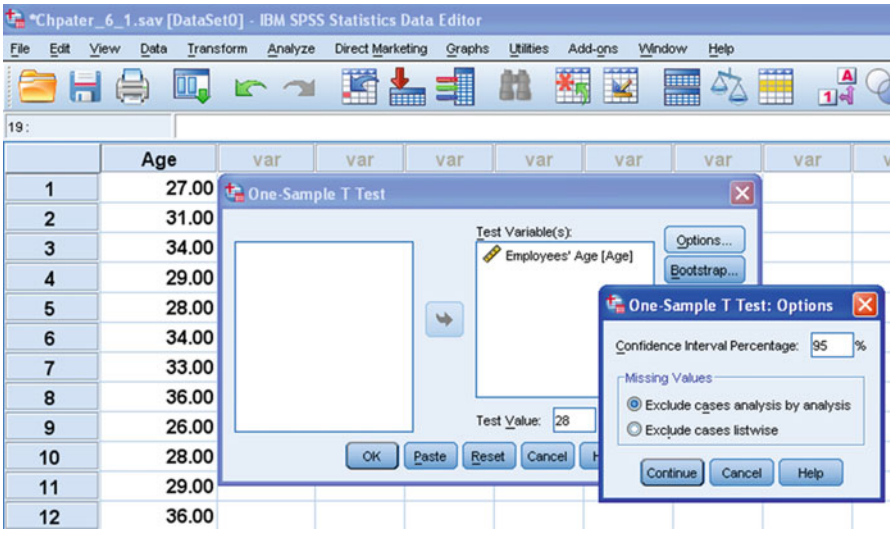



**Fig. 6.9** Screen showing options for computing one-sample *t*-test and selecting significance level

**Table 6.9** One-sample statistics

| | *N* | Mean | Std. deviation | Std. error mean |
|---|---|---|---|---|
| Employees' age | 15 | 31.5333 | 3.54293 | .91478 |

**Table 6.10** One-sample *t* test

| | Test value = 28 | | | | 95% confidence interval of the difference | |
|---|---|---|---|---|---|---|
| | *t* | df | Sig. (two-tailed) | Mean difference | Lower | Upper |
| Employees' age | 3.862 | 14 | .002 | 3.53333 | 1.5713 | 5.4953 |

**Table 6.11** *t*-table for the data on employees' age

| Mean | SD | Mean diff. | *t*-value | *p* value |
|---|---|---|---|---|
| 31.53 | 3.54 | 3.53 | 3.862 | .002 |

the mouse, and the content may be copied in the word file. The output panel shall have the following results:

1. Sample statistics showing mean, standard deviation, and standard error
2. Table showing the value of *t* and its significance level

In this example, all the outputs so generated by the SPSS will look like Tables 6.9 and 6.10. The model way of writing the results of one-sample *t*-test has been shown in Table 6.11.

### *Interpretation of the Outputs*

The mean, standard deviation, and standard error of mean for the data on age are given in Table 6.9. These values may be used for further analysis.

From Table 6.11, it can be seen that the *t*-value is equal to 3.862 along with its *p* value .002. Since *p* value is less than 0.05, it may be concluded that the *t*-value is significant and the null hypothesis may be rejected at 5% level. Further, since the average age of the employees in this problem is 31.5 which is higher than the assumed age of 28 years, hence it may be inferred that average age of the employees in the organization is higher than 28 years.

## Solved Example of Two-Sample *t*-Test for Unrelated Groups with SPSS

**Example 6.6** An experiment was conducted to assess delivery performance of the two pizza companies. Customers were asked to reveal the delivery time of the pizza they have ordered from these two companies. Following are the delivery time in minutes of the two pizza companies as reported by their customers (Table 6.12). Can it be concluded that the delivery time of the two companies is different? Test your hypothesis at 5% level.

*Solution* Here the hypothesis which needs to be tested is

$$H_0 : \mu_A = \mu_B$$

against the alternative hypothesis

$$H_1 : \mu_A \neq \mu_B$$

After computing the value of *t*-statistic for two independent samples by the SPSS, it will be tested for its significance. The SPSS output also gives the significance value (*p* value) corresponding to the *t*-value. The *t*-value would be significant if its corresponding *p* value is less than .05, and in that case, the null hypothesis shall be rejected at 5% level; otherwise, null hypothesis is failed to be rejected.

One of the conditions in using the two sample *t*-test is that the variance of the two groups must be equal or nearly equal. The SPSS uses Levene's *F*-test to test

**Table 6.12** Data of delivery time (in minutes) in two pizza companies

| S.N. | Company A | Company B |
|---|---|---|
| 1 | 20.5 | 20.5 |
| 2 | 24.5 | 17 |
| 3 | 15.5 | 18.5 |
| 4 | 21.5 | 17.5 |
| 5 | 20.5 | 20.5 |
| 6 | 18.5 | 16 |
| 7 | 21.5 | 17 |
| 8 | 20.5 | 18 |
| 9 | 19.5 | 18 |
| 10 | 21 | 18.5 |
| 11 | 21.5 | |
| 12 | 22 | |

this assumption. If the *p* value for *F*-test is more than .05, null hypothesis may be accepted, and this will ensure the validity of *t*-test.

Another important feature in this example is the style of feeding the data for SPSS analysis. The readers should note the procedure of defining the variables and feeding the data carefully in this example. Here there are two variables Pizza *Company* and *Delivery Time*. Pizza *Company* is a nominal variable, whereas *Delivery Time* is a scale variable.

## *Computation of Two-Sample* t*-Test for Unrelated Groups*

(a) ***Preparing Data File***

Before using the SPSS commands for computing the *t*-value and other related statistics for two independent groups, a data file needs to be prepared. The following steps will help you to prepare the data file:

(i) *Starting the SPSS:* Use the following command sequence to start SPSS:
**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**
After checking the option **Type in Data** on the screen you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* There are two variables in this example which need to be defined in SPSS along with their properties. Variable *Pizza Company* is defined as 'Nominal,' whereas *Delivery Time* is defined as 'Scale' as they are measured on nominal as well as interval scale, respectively. The procedure of defining the variables in SPSS is as follows:

1. Click **Variable View** to define the variables and their properties.
2. Write short name of the variables as *Company* and *Del_Time* under the column heading **Name.**

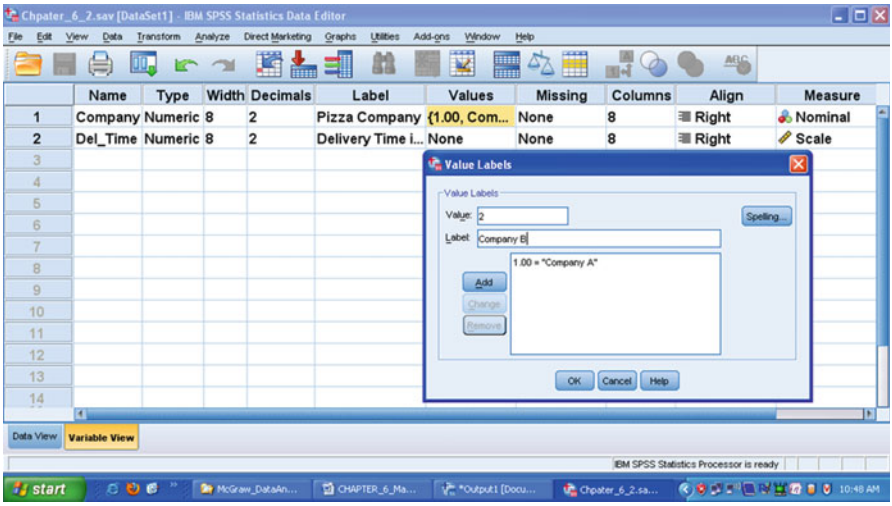



**Fig. 6.10** Defining variables along with their characteristics

3. Under the column heading **Label**, full names of these variables may be defined as *Pizza Company* and *Delivery Time,* respectively. Readers may choose some other names of these variables as well.
4. For the variable *Company,* double-click the cell under the column heading **Values** and add the following values to different levels:

| Value | Label |
|---|---|
| 1 | Company A |
| 2 | Company B |

The screen for defining the values can be seen in Fig. 6.10.

5. Under the column heading **Measure,** select the option 'Nominal' for the *Company* variable and 'Scale' for the *Del_Time* variable.
6. Use default entries in rest of the columns.

After defining the variables in variable view, the screen shall look like Fig. 6.10

(iii) *Entering data*

After defining both the variables in **Variable View,** click **Data View** on the left corner in the bottom of the screen shown in Fig. 6.10 to open the data entry format column wise. For the *Company* variable, type first twelve scores as 1 and the next ten scores as 2 in the column. This is because the value '1' denotes Company A and there are 12 delivery time scores reported by the customers. Similarly, the value '2' denotes Company B and there are 10 delivery time scores as reported by the customers. After entering the data, the screen will look like Fig. 6.11.

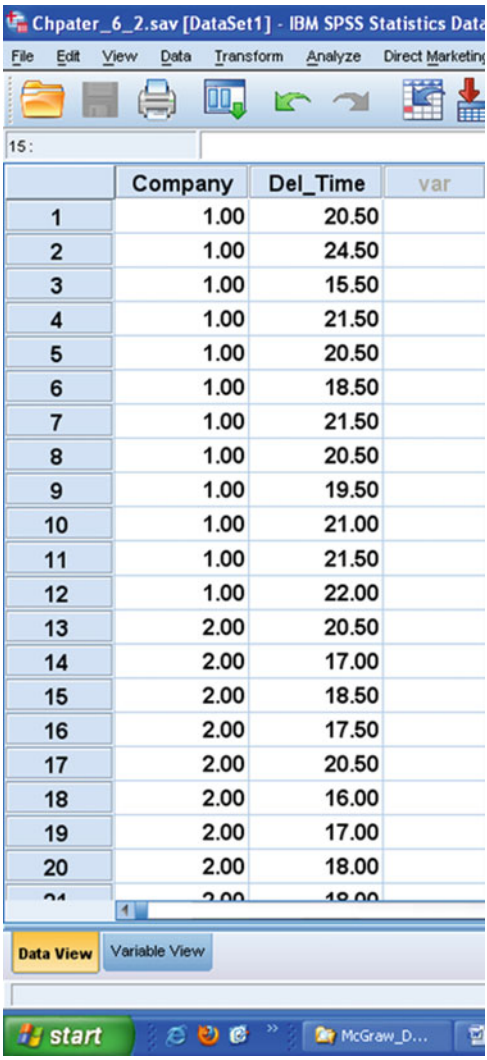



**Fig. 6.11** Screen showing entered data for company and delivery time in the data view

(b) **SPSS *Commands for Two-Sample t-Test***

After preparing the data file in data view, take the following steps for two-sample *t*-test:

(i) *Initiating the SPSS commands for two-sample t-test:* In data view, click the following commands in sequence:
**Analyze ⇨ Compare means ⇨ Independent-Samples *t* test**
The screen shall look like Fig. 6.12.

(ii) *Selecting variables for analysis:* After clicking the **Independent-Samples *t* test** option, you will be taken to the next screen for selecting variables for the two-sample *t*-test. Select the variable *Delivery Time* from left panel and

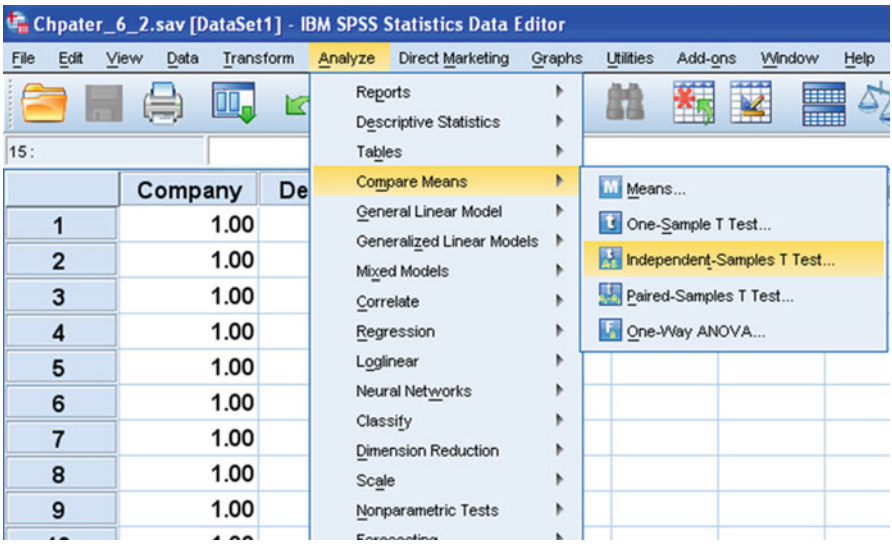



**Fig. 6.12** Screen showing SPSS commands for two-sample *t*-test

bring it in the "Test Variable" section of the right panel. Similarly, select the variable *Pizza Company* from the left panel and bring it to the "Grouping Variable" section of the right panel.

Select variable from the left panel and bring it to the right panel by using the arrow key. After selecting both the variables, the values '1' and '2' need to be defined for the grouping variable *Pizza Company* by pressing the tag 'Define Groups.' The screen shall look like Fig. 6.13.

**Note:** Many variables can be defined in the variable view in the same data file for computing several *t*-values for different independent groups.

(iii) *Selecting options for computation:* After selecting the variables, option needs to be defined for the two-sample *t*-test. Do the following:

- In the screen shown in Fig. 6.13, click the tag **Options** and you will get the screen shown in Fig. 6.14.
- Enter the confidence interval as 95% and click ***Continue*** to get back to the screen shown in Fig. 6.13. By default, the confidence interval is 95%; however, if desired, it may be changed to some other level.

  The confidence level is the one at which the hypothesis needs to be tested. In this problem, the null hypothesis is required to be tested at .05 level of significance, and therefore, the confidence level here shall be 95%. One can choose the confidence level as 90 or 99% if the level of significance for testing the hypothesis is .10 or .01, respectively.
- Click **OK** on the screen shown in Fig. 6.13.

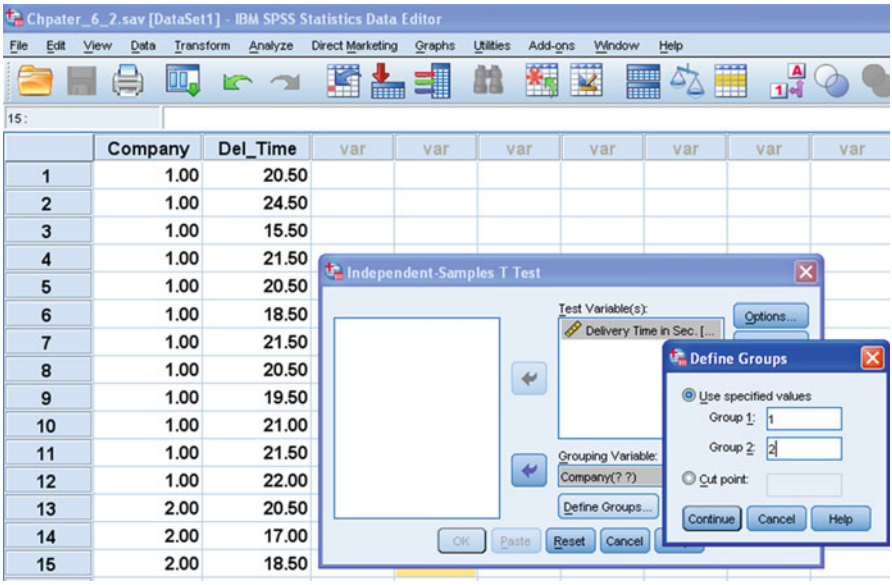



**Fig. 6.13** Screen showing selection of variable for two-sample *t*-test for unrelated groups

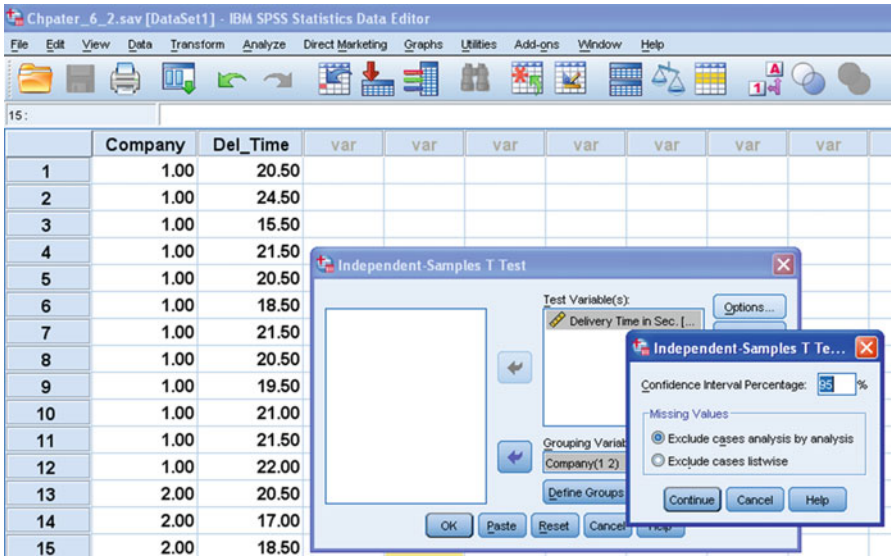



**Fig. 6.14** Screen showing the option for choosing the significance level

(c) ***Getting the Output***

Clicking the OK key in Fig. 6.14 will lead you to the output window. In the output window of the SPSS, the relevant outputs may be selected by using the

**Table 6.13** Descriptive statistics of the groups

| | Pizza company | *N* | Mean | Std. deviation | Std. error mean |
|---|---|---|---|---|---|
| Delivery time in sec | A | 12 | 20.58 | 2.16 | .62412 |
| | B | 10 | 18.15 | 1.45 | .45977 |

**Table 6.14** *F*- and *t*-table for testing the equality of variances and equality of means of two unrelated groups

| | Levene's test for equality of variance | | *t*-test for equality of means | | | | | 95% confidence interval of the difference | |
|---|---|---|---|---|---|---|---|---|---|
| | *F* | Sig. | *t* | df | Sig. (two-tailed) | Mean diff. | SE diff. | Lower | Upper |
| Delivery time in sec. | | | | | | | | | |
| **Equal variances assumed** | .356 | .557 | 3.028 | 20 | .007 | 2.43 | .804 | 0.76 | 4.11 |
| **Equal variances not assumed** | | | 3.139 | 19.3 | .005 | 2.43 | .775 | 0.81 | 4.05 |

right click of the mouse, and it may be copied in the word file. The output panel shall have the following results:

1. Descriptive statistics for the data in different groups
2. '*F*-' and '*t*-'values for testing the equality of variances and equality of means, respectively

(i) In this example, all the outputs so generated by the SPSS will look like Tables 6.13 and 6.14. The model way of writing the results of two-sample *t*-test for unrelated samples has been shown in Table 6.15.

## *Interpretation of the Outputs*

The following interpretations can be made on the basis of the results shown in the above outputs:

1. Table 6.13 shows the mean, standard deviation, and standard error of the mean for the data on delivery time of both the pizza companies. The mean delivery time of the company B is less than that of the delivery time of company A. However, whether this difference is significant or not shall be revealed by looking to the *t*-value and its associated *p* value. However, if the *t*-value is not significant, no one should draw the conclusion about the delivery time of the pizza companies by looking to the sample means.

**Table 6.15** *t*-table for the data on delivery time along with *F*-value

| Groups | Means | S.D. | Mean. diff | SE of mean diff | *t*-value | *p* value | *F*-value | *p* value |
|---|---|---|---|---|---|---|---|---|
| Company A | 20.58 | 2.16 | 2.43 | .804 | 3.028 | .007 | .356 | .557 |
| Company B | 18.15 | 1.45 | | | | | | |

2. One of the conditions for using the two-sample *t*-ratio for unrelated groups is that the variance of the two groups must be equal. To test the equality of variances, Levene's test was used. In Table 6.14, *F*-value is .356 which is insignificant as the *p* value is .557 which is more than .05. Thus, the null hypothesis of equality of variances may be accepted, and it is concluded that the variances of the two groups are equal.
3. It can be seen from Table 6.15 that the value of *t*-statistic is 3.028. This *t*-value is significant as its *p* value is 0.007 which is less than .05. Thus, the null hypothesis of equality of population means of two groups is rejected, and it may be concluded that the average delivery time of the pizza in both the companies is different. Further, average delivery time of the company B is less than that of the company A, and therefore, it may be concluded that the delivery of pizza by the company B to their customers is faster than that of the company A.

**Remark**: The readers can note that initially the two-tailed hypothesis was tested in this example, but the final conclusion has been made similar to the one-tailed test. This is because of the fact that if the *t*-statistic is significant in two-tailed test then it will also be significant at one-tailed test. To make it clearer, let us consider that for two-tailed test, the critical value is $t_{\alpha/2}$ at level of significance. This value will always be greater than that of the critical value of $t_{\alpha}$ in one-tailed test, and therefore, if the calculated value of $t$ is greater than $t_{\alpha/2}$, it will also be greater than $t_{\alpha}$.

## Solved Example of Paired *t*-Test with SPSS

**Example 6.7** An experiment was conducted to know the impact of new advertisement campaign on sale of television of a particular brand. The number of television units sold on 12 consecutive working days before and after launching the advertisement campaign in a city was recorded. The data obtained are shown in Table 6.16.

*Solution* Here the hypothesis which needs to be tested is

$H_0 : \mu_D = 0$ (Difference of average sales after and before the advertisement is zero.)

against the alternative hypothesis

**Table 6.16** Number of TV units sold in a city before and after the advertisement campaign

| Days | Before advertisement | After advertisement |
|---|---|---|
| 1 | 25 | 28 |
| 2 | 36 | 42 |
| 3 | 22 | 38 |
| 4 | 26 | 40 |
| 5 | 18 | 35 |
| 6 | 8 | 12 |
| 7 | 23 | 29 |
| 8 | 31 | 52 |
| 9 | 25 | 26 |
| 10 | 22 | 26 |
| 11 | 20 | 25 |
| 12 | 5 | 7 |

$H_1 : \mu_D \neq 0$ (Difference of average sales after and before the advertisement is not equal to zero.)

After getting the value of *t*-statistic for paired sample in the output of SPSS, it needs to be tested for its significance. The output so generated by the SPSS also gives the significance level (*p* value) along with *t*-value. The null hypothesis may be rejected if the *p* value is less than .05; otherwise, it is accepted. If the null hypothesis is rejected, an appropriate conclusion may be drawn regarding the effectiveness of the advertisement campaign by looking to the mean values of the sales before and after the advertisement.

In this problem, there are two variables *TV Sold before Advertisement* and *TV Sold after Advertisement*. For both these variables, data shall be entered in two different columns unlike the way it was entered in two-sample *t*-test for unrelated groups.

## *Computation of Paired* t*-Test for Related Groups*

(a) ***Preparing Data File***

The data file needs to be prepared first for using the SPSS commands for the computation of *t*-value and other related statistics. Follow the below-mentioned steps in preparing the data file.

(i) *Starting the SPSS:* Follow the below-mentioned command sequence to start SPSS on your computer:

**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

(ii) *Defining variables:* In this example, two variables *TV Sold before Advertisement* and *TV Sold after Advertisement* need to be defined along with their properties. Both these variables are scalar as they are measured on

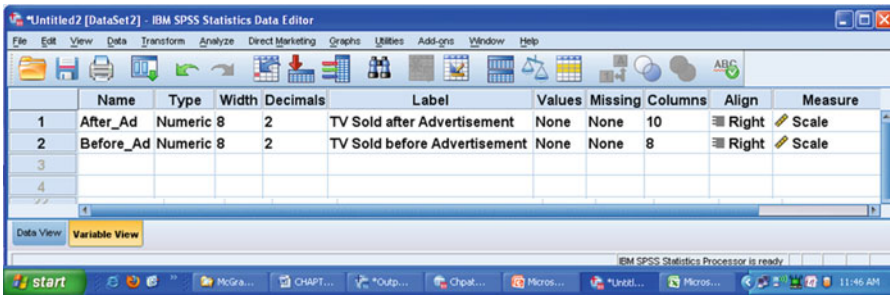



**Fig. 6.15** Variables along with their characteristics for the data shown in Table 6.16

ratio scale. These variables can be defined along with their properties in SPSS by using the following steps:

1. After clicking the Type in Data above, click the **Variable View** to define the variables and their properties.
2. Write short name of the variables as *After_Ad* and *Before_Ad* under the column heading **Name.**
3. Under the column heading **Label,** full name of these variables may be defined as *TV Sold before Advertisement* and *TV Sold after Advertisement,* respectively. Readers may choose some other names of these variables if so desired.
4. Under the column heading **Measure,** select the option 'Scale' for both the variables.
5. Use default entries in rest of the columns.

After defining the variables in variable view, the screen shall look like Fig. 6.15.

(iii) *Entering the data*

Once both these variables are defined in the **Variable View**, click **Data View** on the left corner in the bottom of the screen as shown in Fig. 6.15 to open the format for entering the data column wise. For both these variables, data is entered column wise. After entering the data, the screen will look like Fig. 6.16.

(b) **SPSS *Commands for Paired t-Test***

After entering all the data in the data view, take following steps for paired *t*-test.

(i) *Initiating SPSS commands for paired t-test:* In data view, click the following commands in sequence:

**Analyze → Compare means → Paired-Samples *t* Test**

The screen shall look like Fig. 6.17.

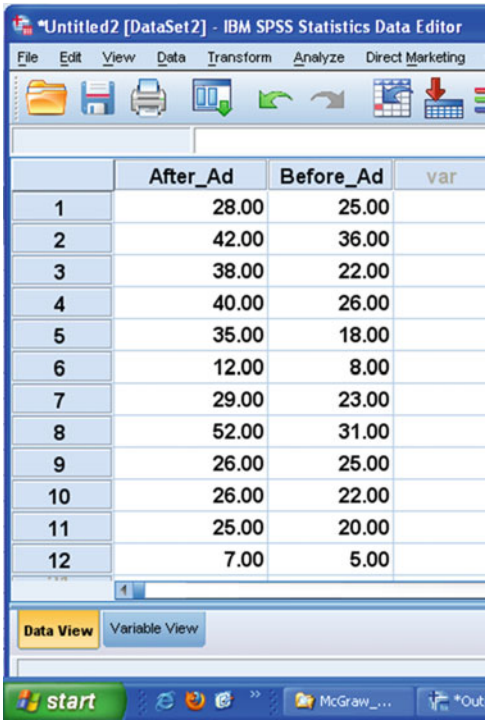



**Fig. 6.16** Screen showing entered data on TV sales before and after the advertisement campaign

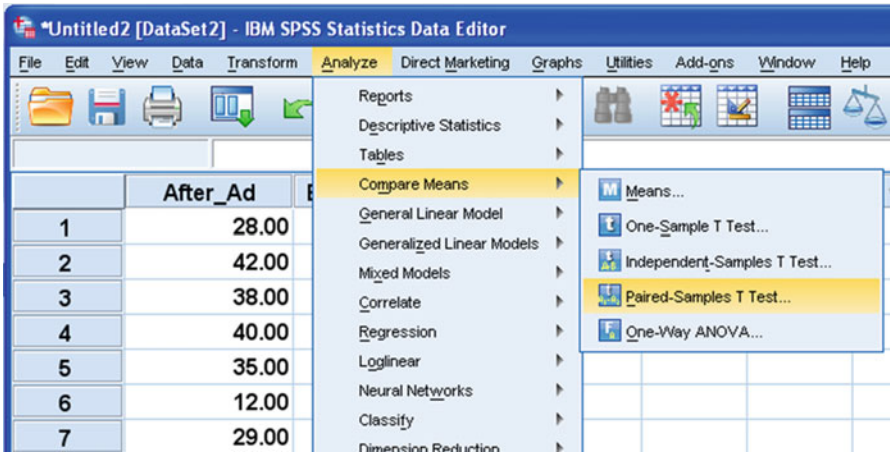



**Fig. 6.17** Screen showing SPSS commands for paired *t*-test

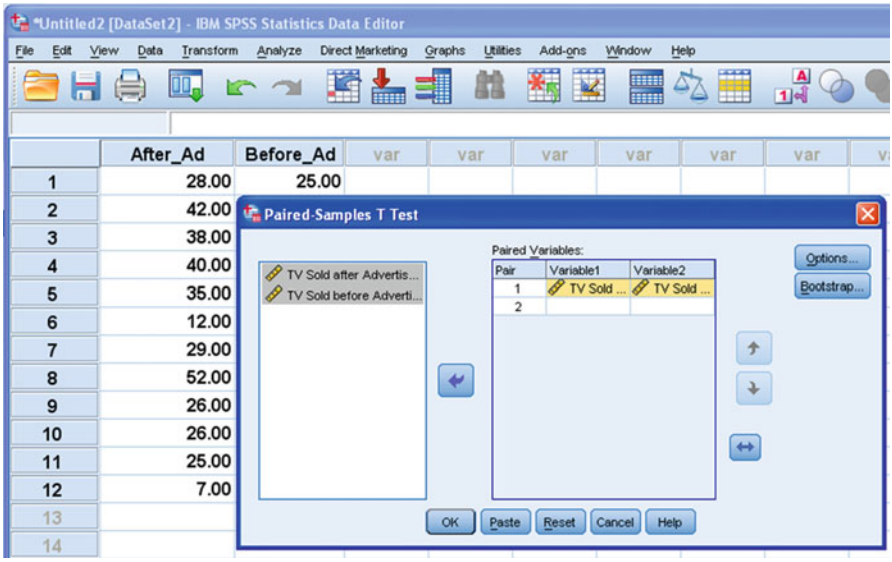



**Fig. 6.18** Screen showing selection of variables for paired *t*-test

(ii) *Selecting variables for analysis:* After clicking the **Paired-Samples *t* Test,** the next screen will follow for variable selection. Select the variable *TV Sold before Advertisement* and *TV Sold after Advertisement* from left panel and bring them to the right panel as variable 1 and variable 2 of pair 1. After selecting both the variables, the screen shall look like Fig. 6.18.
**Note:** Many pairs of variables can be defined in the variable view in the same data file for computing several paired *t*-tests. These pairs of variables can be selected together in the screen as shown in Fig. 6.18.

(iii) *Selecting options for computation:* After selecting the variables, option needs to be defined for computing paired *t*-test. Do the following:

- In the screen as shown in Fig. 6.18, click the tag **Options** and you will get the screen where by default confidence level is selected 95%. No need of doing anything except to click ***Continue***.
  One can define the confidence level as 90 or 99% if the level of significance for testing the hypothesis is .10 or .01, respectively.
- Click **OK** on the screen shown in Fig. 6.18.

(c) ***Getting the Output***
Clicking the **OK** tag in Fig. 6.18 will lead you to the output window. In the output window, the relevant results can be selected by using right click of the mouse and may be copied in the word file. The output panel shall have the following results:

**Table 6.17** Paired sample statistics

| | | Mean | *N* | SD | SE(mean) |
|---|---|---|---|---|---|
| Pair 1 | | | | | |
| | TV sold before advertisement | 21.75 | 12 | 8.61 | 2.49 |
| | TV sold after advertisement | 30.00 | 12 | 12.55 | 3.62 |

**Table 6.18** Paired *t*-test for the data on number of TV sold

| | Paired differences | | | 95% confidence interval of the difference | | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | SD | SE(M) | Lower | Upper | *t* | df | Sig.(two-tailed) |
| Pair 1 | | | | | | | | |
| Before advertisement After advertisement | 8.25 | 6.797 | 1.96 | 3.93 | 12.57 | 4.204 | 11 | .001 |

1. Paired samples statistics
2. Paired *t*-test table

In this example, all the outputs so generated by the SPSS will look like Tables 6.17 and 6.18.

## *Interpretation of the Outputs*

The following interpretations can be made on the basis of the results shown in the above output:

1. The values of the mean, standard deviation, and standard error of the mean for the data on TV sales before and after the advertisement are shown in Table 6.17. These values can be used to draw conclusion as to whether the advertisement campaign was effective or not.
2. It can be seen from Table 6.18 that the value of *t*-statistic is 4.204. This *t*-value is significant as the *p* value is 0.001 which is less than .05. Thus, the null hypothesis of equality of average TV sales before and after advertisement is rejected, and therefore, it may be concluded that the average sale of the TV units before and after the advertisement is not same.
   Further, by looking to the values of the mean sales of the TV units before and after advertisement in Table 6.17, you may note that the average sales have increased after the advertisement campaign. Since the null hypothesis has been rejected, it may thus be concluded that the increase in the TV units has been significantly increased due to advertisement campaign.

You may notice that we started with testing two-tailed test but ended up in testing one-tailed test. This is because of the fact that if the *t*-value is significant at 5% level in two-tailed test, then this will also be significant in one-tailed test.

## Summary of SPSS Commands for *t*-Tests

(a) **For One-Sample *t*-Test**

(i) Start the SPSS by using the following commands:

**Start → All Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

(ii) Click **Variable View** tag and define the variable *Age*.
(iii) Once the variables are defined, type the data for each variable by clicking **Data View.**
(iv) In the data view, follow the below-mentioned command sequence for computing one-sample *t*-test:

**Analyze ⟹ Compare Means ⟹ One-Sample *t* Test**

(v) Select the variable *Age* from left panel to the right panel by using the arrow command.
(vi) Enter the test value as 28. This is the population mean age which is required to be tested.
(vii) By clicking the tag **Options,** ensure that confidence interval is selected as 95% and click ***Continue***. Confidence level can be entered as 90 or 99% if the level of significance for testing the hypothesis is .10 or .01, respectively.
(viii) Press **OK** for output.

(b) **For Two-Sample *t*-Test for Unrelated Groups**

(i) Start the SPSS the way it is done in case of one-sample *t*-test.
(ii) In the variable view, define *Company* and *Del_Time* as a 'Nominal' and 'Scale' variables, respectively.
(iii) In the variable view under column heading **Values,** define the values '1' for Company A and '2' for Company B for the variable *Company*.
(iv) In the data view, feed the data of *Company A* as 1 for first twelve entries (as there are twelve scores in the Company A) and 2 as next ten entries (as ten scores are in Company B) column wise under the column *Company*. Under the column *Del_Time*, enter the first group of delivery time data and then in the same column, enter the second group of delivery time data.
(v) In the data view, follow the below-mentioned command sequence for computing the value of *t*:

**Analyze → Compare means → Independent-Samples *t* test**

(vi) Select the *Company* and *Del_Time* variables from left panel and bring them in the "Test Variable" and "Grouping Variable" sections of the right panel, respectively.
(vii) Define the values 1 and 2 as two groups for the grouping variable *Company*.
(viii) By clicking the tag **Options,** ensure that confidence interval is selected as 95% and click ***Continue***.
(ix) Press **OK** for output.

(c) **For Paired *t*-Test**

(i) Start the SPSS the way it is done in case of one-sample *t*-test.
(ii) In variable view, define the variables *After_Ad* and *Before_Ad* as scale variables.
(iii) In the data view, follow the below-mentioned command sequence for computing the value of *t* after entering the data for both the variables:
Analyze → Compare means → Paired-Samples *t* Test
(iv) Select the variables *After_Ad* and *Before_Ad* from left panel and bring them to the right panel as variable 1 and variable 2 of pair 1.
(v) By clicking the tag **Options,** ensure that confidence interval is selected as 95% and click ***Continue***.
(vi) Press **OK** for output.

## Exercise

*Short-Answer Questions*

**Note:** Write the answer to each of the following questions in not more than 200 words.

Q.1. What do you mean by pooled standard deviation? How will you compute it?
Q.2. Discuss the criteria of choosing a statistical test in testing hypothesis concerning mean and variances.
Q.3. What are the various considerations in constructing null and alternative hypotheses?
Q.4. What are the various steps in testing a hypothesis?
Q.5. Discuss the advantages and disadvantages of one- and two-tailed tests.
Q.6. Explain the situations in which one- and two-tailed tests should be used.
Q.7. Discuss the concept of one- and two-tailed hypotheses in terms of rejection region.
Q.8. What do you mean by type I and type II errors? Discuss the situations when type II error is to be controlled.
Q.9. What do you mean by *p* value? How it is used in testing the significance of test statistic?

Q.10. What do you mean by degrees of freedom? Discuss it in computing different statistics.
Q.11. Discuss a situation where one-sample *t*-test can be used. Explain the formula and procedure of testing the hypothesis.
Q.12. What are the various assumptions in using two-sample *t*-tests for unrelated groups? What is the solution if the assumptions are not met?
Q.13. Write the steps in testing a hypothesis in comparing the means of two unrelated groups.
Q.14. Discuss the procedure of testing a hypothesis by using paired *t*-test.
Q.15. Under what situations should paired *t*-test be used? Can it be used if sample sizes differ?

*Multiple-Choice Questions*

**Note:** Question no. 1–10 has four alternative answers for each question. Tick marks the one that you consider the closest to the correct answer.

1. If the value of *t*-statistic increases, then its associated *p* value

   (a) Increases
   (b) Decreases
   (c) Remains constant
   (d) Depends upon the level of significance chosen in the study

2. At a particular level of significance, if a null hypothesis is rejected in two-tailed test, then

   (a) It will be accepted in one-tailed test.
   (b) May be accepted or rejected in one-tailed test depending upon the level of significance.
   (c) It may be rejected in one-tailed test.
   (d) It will also be rejected in one-tailed test.

3. Choose the most appropriate statement.

   (a) *t*-test cannot be used for large sample.
   (b) *z*-test cannot be used for large sample.
   (c) *t*-test can be used for large sample.
   (d) Both *t*-test and *z*-test can be used for small sample.

4. Sample is said to be small if it is

   (a) 39
   (b) 31
   (c) 29
   (d) 32

5. In two-tailed hypothesis, the critical region is

   (a) Divided in both the tails in 1:4 proportion
   (b) Lying in right tail only

(c) Lying in left tail only
(d) Divided in both the tails

6. If a researcher wishes to test whether rural youth is less intelligent than the urban youth by means of the following hypotheses,

$$H_0 : \mu_{Rural} \geq \mu_{Urban}$$
$$H_1 : \mu_{Rural} < \mu_{Urban}$$

the critical region lies

(a) In the left tail only.
(b) In the right tail only.
(c) In both the tails.
(d) None of the above is correct.

7. In using two-sample *t*-test, which assumption is used?

(a) Variances of both the populations are equal.
(b) Variances of both the populations are not necessarily equal.
(c) No assumption is made on the population variance.
(d) Variance of one population is larger than other.

8. If Cal $t < t_\alpha$, choose the most appropriate statement.

(a) $H_0$ is failed to be rejected.
(b) $H_1$ may be rejected.
(c) $H_0$ may be rejected.
(d) $H_1$ is failed to be accepted.

9. If it is desired to compare the anxiety of male and female, which is the most appropriate set of hypotheses?

(a)
$$H_0 : \mu_{Male} = \mu_{Female}$$
$$H_1 : \mu_{Male} \neq \mu_{Female}$$

(b)
$$H_0 : \mu_{Male} = \mu_{Female}$$
$$H_1 : \mu_{Male} > \mu_{Female}$$

(c)
$$H_0 : \mu_{Male} = \mu_{Female}$$
$$H_1 : \mu_{Male} < \mu_{Female}$$

(d)
$$H_0 : \mu_{Male} \neq \mu_{Female}$$
$$H_1 : \mu_{Male} = \mu_{Female}$$

10. In testing the following set of hypotheses,

$$H_0 : \mu_1 \geq \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

choose the most appropriate statement.

(a) If Cal $t \leq t_\alpha$, $H_0$ may not be rejected.
(b) If Cal $t < -t_\alpha$, $H_0$ may be rejected.
(c) If Cal $t > -t_\alpha$, $H_0$ may be rejected.
(d) None of the above is correct.

11. If there are $N$ pairs of score and paired $t$-test is used for comparing the means of both the groups, what will be the degrees of freedom for $t$-statistic?

(a) $N$
(b) $2N - 2$
(c) $N + 1$
(d) $N - 1$

12. If attitude toward science is to be compared among 22 male and 18 women students of class XII by using $t$-ratio, what would be its degrees of freedom?

(a) 40
(b) 39
(c) 2
(d) 38

13. To see the effectiveness of observation method on learning skill, which of the SPSS command shall be used?

(a) One-sample $t$-test
(b) Independent-sample $t$-test
(c) Paired-sample $t$-test
(d) None of the above

14. Power of statistical test is given by

(a) $\beta$
(b) $1 + \beta$
(c) $\beta - 1$
(d) $1 - \beta$

*Assignments*

1. A random sample of 25 management students was tested for their IQ in a university. Their scores were as follows:

92, 101, 94, 93, 97, 98, 120, 104, 98, 96, 85, 121, 87, 96, 111, 102, 99, 95, 89, 102, 131, 107, 109, 99, 97

Can it be concluded that the management students in the university have a mean IQ score equal to 101? Test your hypothesis at 5% level.

2. The following data set represents the weight of the average daily household waste (kg/day/house) generated from 20 houses in a locality:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4.1 | 3.7 | 4.3 | 2.5 | 2.5 | 6.8 | 4.0 | 4.5 | 4.6 | 7.1 |
| 3.5 | 3.1 | 6.6 | 5.5 | 6.5 | 4.1 | 4.2 | 4.8 | 5.1 | 4.8 |

Can it be concluded that the average daily household waste of that community is 5.0 kg/day/house? Test your hypothesis at 1% level.

3. A feeding experiment was conducted with two random samples of pigs on the relative value of limestone and bone meal for bone development. The data so obtained on ash content are shown in the following table:

Ash contents (%) in the bones of pigs

| S.N. | Lime stone | Bone stone |
|---|---|---|
| 1 | 48.9 | 52.5 |
| 2 | 52.3 | 53.9 |
| 3 | 51.4 | 53.2 |
| 4 | 50.6 | 49.9 |
| 5 | 52 | 51.6 |
| 6 | 45.8 | 48.5 |
| 7 | 50.5 | 52.6 |
| 8 | 52.1 | 44.6 |
| 9 | 53 | 52.8 |
| 10 | 46.5 | 48.8 |

Test the significance of the difference between the mean ash content of the two groups at 5% level.

4. A company wanted to know as to which of the two pizza types, that is, fresh veggie and peppy paneer, was most popular among the people. An experiment was conducted in which 12 men were given two types of pizza, that is, fresh veggie pizza and pepper paneer pizza, to eat on two different days. Each pizza was carefully weighed at exactly 16 oz. After 20 min, the leftover pizzas were weighed, and the amount of each type of pizza remaining per person was calculated assuming that the subjects would eat more if they preferred the pizza type. The data so obtained is shown in the following table.

Weights of the leftover pizzas in both varieties

| S.N. | Fresh veggie (in oz.) | Pepper paneer (in oz.) |
|---|---|---|
| 1 | 12.5 | 15 |
| 2 | 5.87 | 7.1 |
| 3 | 14 | 14 |
| 4 | 12.3 | 13.7 |
| 5 | 3.5 | 14.2 |
| 6 | 2.6 | 5.6 |
| 7 | 14.4 | 15.4 |
| 8 | 10.2 | 11.3 |
| 9 | 4.5 | 15.6 |
| 10 | 6.5 | 10.5 |
| 11 | 4.3 | 8.5 |
| 12 | 8.4 | 9.3 |

Apply the paired *t*-test and interpret your findings. Do people seem to prefer fresh veggie pizza over pepper veggie pizza? Test your hypothesis at 5% level.

*Answers to Multiple-Choice Questions*

| | |
|---|---|
| Q.1 b | Q.2 d |
| Q.3 c | Q.4 c |
| Q.5 d | Q.6 a |
| Q.7 a | Q.8 a |
| Q.9 a | Q.10 b |
| Q.11 d | Q.12 d |
| Q.13 c | Q.14 d |

*Assignments*

1. Calculated value of $t = -0.037$; average IQ score of the students is 101.
2. Calculated value of $t = -1.286$; average daily household waste of the community is 5 kg/day/house.
3. Calculated value of $t = -0.441$; mean ash contents of both the groups are same.
4. Calculated value of $t = 3.193$ which is significant. People prefer fresh veggie pizza.

# Chapter 7
# One-Way ANOVA: Comparing Means of More than Two Samples

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Understand the basics of one-way analysis of variance (ANOVA).
- Learn to interpret the model involved in one-way analysis of variance.
- Learn the different designs of ANOVA.
- Describe the situations in which one-way analysis of variance should be used.
- Learn the manual procedure of applying one-way ANOVA in testing of hypothesis.
- Construct the null and research hypotheses to be tested in the research study.
- Learn what happens if multiple *t*-tests are used instead of one-way ANOVA.
- Understand the steps involved in one-way analysis of variance in equal and unequal sample sizes.
- Interpret the significance of *F*-statistic using the concept of *p* value.
- Know the procedure of making data file for analysis in SPSS.
- Understand the steps involved in using SPSS for solving the problems of one-way analysis of variance.
- Describe the output of one-way analysis of variance obtained in SPSS.

## Introduction

One-way analysis of variance is a statistical technique used for comparing means of more than two groups. It tests the null hypothesis that samples in different groups have been drawn from the same population. It is abbreviated as one-way ANOVA. This technique can be used in a situation where the data is measured either on interval or ratio scale. In one-way ANOVA, group means are compared by comparing the variability between groups with that of variability within the groups. This is done by computing an *F*-statistic. The *F*-value is computed by dividing the mean sum of squares between the groups by the mean sum of squares within the groups.

J.P. Verma, *Data Analysis in Management with SPSS Software*,
DOI 10.1007/978-81-322-0786-3_7, 

As per the central limit theorem, if the groups are drawn from the same population, the variance between the group means should be lower than the variance within the groups. Thus, a higher ratio ($F$-value) indicates that the samples have been drawn from different populations.

There are varieties of situations in which one-way analysis of variance can be used to compare the means of more than two groups. Consider a study in which it is required to compare the responses of the students belonging to north, south, west and east regions towards liking of mess food in the university. If the quality of mess food is rated on a scale of 1–10 (1 = "I hate the food," 10 = "Best food ever"), then the responses of the students belonging to different regions can be obtained in the form of the interval scores. Here the independent variable would be the student's region having four different levels namely north, south, east and west whereas the response of the students shall be the dependent variable. To achieve the objective of the study the null hypothesis of no difference among the mean responses of the four groups may be tested against the alternative hypothesis that at least one group mean differs. If the null hypothesis is rejected, a post hoc test is used to get the correct picture as to which group's liking is the best.

Similarly a human resource manager may wish to determine whether the achievement motivation differs among the employees in three different age categories (<25, 26–35, and >35 years) after attending a training program. Here, the independent variable is the employee's age category, whereas the achievement motivation is the dependent variable. In this case, it is desired to test whether the data provide sufficient evidence to indicate that the mean achievement motivation of any age category differs from other. The one-way ANOVA can be used to answer this question.

## Principles of ANOVA Experiment

There are three basic principles of design of experiments, that is, randomization, replication, and local control. Out of these three, only randomization and replication need to be satisfied by the one-way ANOVA experiments. Randomization refers to the random allocation of the treatment to experimental units. On the other hand, replication refers to the application of each individual level of the factor to multiple subjects. In other words, the experiment must be replicated in more than one subject. In the above example several employees in each age group should be selected in a random fashion in order to satisfy the principles of randomization and replication. This facilitates in drawing the representative sample.

## *One-Way ANOVA*

It is used to compare the means of more than two independent groups. In one-way ANOVA, the effect of different levels of only one factor on the dependent variable is investigated. Usually one-way ANOVA is used for more than two groups because

two groups may be compared using $t$-test. In comparing two group means, the $t$ and $F$ are related as $F = t^2$. In using one-way ANOVA, the experimenter is often interested in investigating the effect of different treatments on some subjects. Which may be people, animals, or plants, etc. For instance, obesity can be compared among the employees of three different departments: marketing, production, and human resource of an organization. Similarly anxiety of the employees can be compared in three different units of an organization. Thus, one-way ANOVA has a wide application in management sciences, humanities, and social sciences.

## *Factorial ANOVA*

A factorial design is the one in which the effect of two factors on the dependent variable is investigated. Here each factor may have several levels and each combination becomes a treatment. Usually factorial ANOVA is used to compare the main effect of each factor as well as their interaction effects across the levels of other factor on the criterion variable. But the situation may arise where each combination of levels in two factors is treated as a single treatment and it is required to compare the effect of these treatments on the dependent variable. In such situation one-way ANOVA can be used to test the required hypothesis. Consider a situation where the effect of different combination of duration and time on learning efficiency is to be investigated. The duration of interest is 30 and 60 minutes and the subjects are given training in the morning and evening sessions for a learning task. The four combinations of treatments would be morning time with 30 minutes duration, morning time with 60 minutes duration, evening time with 30 minutes duration and evening time with 60 minutes duration. In this case neither the main effect nor the interaction effects are of interest to the investigator rather just the combinations of these levels form four levels of the independent treatment.

If the number of factors and their levels are large, then lots of experimental groups need to be created which is practically not possible, and in that case fractional factorial design is used. In this design, only important combinations are studied.

## *Repeated Measure ANOVA*

Repeated measure ANOVA is used when same subjects are given different treatments at different time interval. In this design, same criterion variable is measured many times on each subject. This design is known as repeated measure design because repeated measures are taken at different time in order to see the impact of time on changes in criterion variable. In some studies of repeated measure design, same criterion variable is compared under two or more different conditions. For example, in order to see the impact of temperature on memory retention, a subject's memory might be tested once in an air-conditioned atmosphere and another time in a normal room temperature.

The experimenter must ensure that the carryover effect does not exist in administering different treatments on the same subjects. The studies in repeated measure design are also known as longitudinal studies.

### *Multivariate ANOVA*

Multivariate ANOVA is used when there are two or more dependent variables. It provides solution to test the three hypotheses, namely, (a) whether changes in independent variables have significant impact in dependent variables, (b) whether interaction among independent variables is significant, and (c) whether interaction among dependent variables is significant. Multivariate analysis of variance is also known as MANOVA. In this design, the dependent variables must be loosely related with each other. They should neither be highly correlated nor totally uncorrelated among themselves. Multivariate ANOVA is used to compare the effects of two or more treatments on a group of dependent variables. The dependent variables should be such so that together it conveys some meaning. Consider an experiment where the impact of educational background on three personality traits honesty, courtesy, and responsibility is to be studied in an organization. The subjects may be classified on the basis of their educational qualification; high school, graduation or post-graduation. Here the independent variable is the Education with three different levels: high school, graduation, and postgraduation, whereas the dependent variables are the three personality traits namely honesty, courtesy, and responsibility. The one-way MANOVA facilitates us to compare the effect of education on the personality as a whole of an individual.

## One-Way ANOVA Model and Hypotheses Testing

Let us suppose that there are $r$ groups of scores where first group has $n_1$ scores, second has $n_2$ scores, and so on, and $r$th group has $n_r$ scores. If $X_{ij}$ represents the $j$th score in the $i$th group ($i = 1, 2, \ldots, r$; $j = 1, 2, \ldots, n_i$), then these scores can be shown as follows:

| | | | | | | Total | Mean |
|---|---|---|---|---|---|---|---|
| | 1 | $X_{11}$ | $X_{12} \ldots ...$ | $X_{1j} ....$ | $X_{1n_1}$ | $R_1$ | $\overline{X}_1$ |
| | 2 | $X_{21}$ | $X_{22} \ldots ...$ | $X_{2j} \ldots .$ | $X_{2n_2}$ | $R_2$ | $\overline{X}_2$ |
| | . | . | . | . | . | | |
| | . | . | . | . | . | | |
| Samples | $i$ | $X_{i1}$ | $X_{i2} \ldots ...$ | $X_{ij} \ldots .$ | $X_{in_i}$ | $R_i$ | $\overline{X}_i$ |
| | . | . | . | . | . | | |
| | . | . | . | . | . | | |
| | $r$ | $X_{r1}$ | $X_{r2}. \ldots ...$ | $X_{rj} \ldots .$ | $X_{rn_r}$ | $R_r$ | $\overline{X}_r$ |
| | | | | | | $G = R_1 + R_2 + \ldots R_r$ | |

Here,

$N = n_1 + n_2 + \ldots n_r$, the total of all the scores
$G$ is the grand total of all $N$ scores
$R_i$ is the total of all the scores in $i$th group

The total variability among the above-mentioned $N$ scores can be attributed due to the variability between groups and variability within groups. Thus, the total variability can be broken into the following two components:

$$\begin{aligned} &\text{Total variability} = \text{Variability between groups} + \text{Variability within Groups} \\ &\text{or} \qquad \text{TSS} = \text{SS}_\text{b} + \text{SS}_\text{w} \end{aligned} \tag{7.1}$$

This is known as one-way ANOVA model where it is assumed that the variability among the scores may be due to the groups. After developing the model, the significance of the group variability is tested by comparing the variability between groups with that of variability within groups by using the $F$-test.

The null hypothesis which is being tested in this case is that whether variability between groups ($\text{SS}_\text{b}$) and variability within the groups ($\text{SS}_\text{w}$) are the same or not. If the null hypothesis is rejected, it is concluded that the variability due to groups is significant, and it is inferred that means of all the groups are not same. On the other hand, if the null hypothesis is not rejected, one may draw the inference that group means do not differ significantly. Thus, if $r$ groups are required to be compared on some criterion variable, then the null hypothesis can be tested by following the below mentioned steps:

(a) *Hypothesis construction:* The following null hypothesis is tested

$$\text{H}_0 : \mu_1 = \mu_2 = \ldots = \mu_r$$

against the alternative hypothesis that at least one mean differs.

(b) *Level of significance:* The level of significance may be chosen beforehand. Usually it is taken as .05 or .01.

(c) Statistical test: The $F$-test is used to test the above mentioned hypothesis. If $F$-value is significant, it indicates that the variability between groups is significantly higher than the variability within groups; in that case, the null hypothesis of no difference between the group means is rejected. $F$-value is obtained by computing the total sum of squares (TSS), sum of squares between groups ($\text{SS}_\text{b}$), and sum of squares within groups ($\text{SS}_\text{w}$):

   (i) *Total sum of squares (TSS):* It indicates the variation present in the whole data set around its mean value and is obtained by adding the sum of squares due to "between groups" and sum of squares due to "within groups." The total sum of squares can be defined as the sum of squared

deviations of all the scores from their mean value. It is usually denoted by TSS and is given by

$$\text{TSS} = \sum_i \sum_j \left( X_{ij} - \frac{G}{N} \right)^2$$

after solving

$$= \sum_i \sum_j X_{ij}^2 - \frac{G^2}{N} \tag{7.2}$$

Here $G$ is the grand total of all the scores. The degrees of freedom for total sum of squares is $N - 1$, and, therefore, mean sum of squares is computed by dividing TSS by $N - 1$.

(ii) *Sum of squares between groups ($SS_b$)*: The sum of squares between groups can be defined as the variation of group around the grand mean of the data set. In other words, it is the measure of variation between the group means and is usually denoted by $\text{SS}_\text{b}$. This is also known as the variation due to assignable causes. The sum of squares between groups is computed as

$$\text{SS}_\text{b} = \sum_i \frac{R_i^2}{n_i} - \frac{G^2}{N} \tag{7.3}$$

Since $r$ samples are involved in one-way ANOVA, the degrees of freedom for between groups is $r - 1$. Thus, mean sum of squares for between groups ($\text{MSS}_\text{b}$) is obtained by dividing $\text{SS}_\text{b}$ by its degrees of freedom $r - 1$.

(iii) *Sum of squares within groups* ($\text{SS}_\text{w}$): The sum of squares within groups is the residual variation and is referred as variation due to non-assignable causes. It is the average variation within the groups and is usually denoted by $\text{SS}_\text{w}$:

$$\text{SS}_\text{w} = TSS - SS_\text{b} \tag{7.4}$$

The degrees of freedom for the sum of squares within groups is given by $N - r$, and, therefore, mean sum of squares for within groups ($\text{MSS}_\text{w}$) is obtained by dividing $\text{SS}_\text{w}$ by $N - r$.

(iv) *ANOVA table:* After computing all sum of squares, these values are used in the analysis of variance(ANOVA) table for computing $F$-value as shown below.

ANOVA table

| Sources of variation | SS | df | MSS | $F$-value |
|---|---|---|---|---|
| Between groups | $\text{SS}_\text{b}$ | $r - 1$ | $\text{MSS}_\text{b} = \frac{\text{SS}_\text{b}}{r-1}$ | $F = \frac{\text{MSS}_\text{b}}{\text{MSS}_\text{w}}$ |
| Within groups | $\text{SS}_\text{w}$ | $N - r$ | $\text{MSS}_\text{w} = \frac{\text{SS}_\text{w}}{N-r}$ | |
| Total | TSS | $N - 1$ | | |

**Remark:** Sum of squares are additive in nature, but mean sum of squares are not

(v) *F-statistic*: Under the normality assumptions, the $F$-value obtained in the above table, that is,

$$F = \frac{\text{MSS}_\text{b}}{\text{MSS}_\text{w}} \tag{7.5}$$

follows an $F$-distribution with $(r-1, N-r)$ degrees of freedom.
This test statistic $F$ is used to test the null hypothesis of no difference among the group means.

(d) *Decision criteria*: The tabulated value of $F$ at .05 and .01 level of significance with $(r-1, N-r)$ degrees of freedom may be obtained from Tables A.4 and A.5, respectively, in the Appendix. If calculated value of $F$ is greater than tabulated $F$, the null hypothesis is rejected. And in that case it is concluded that at least one of the means will be different. Since ANOVA does not tell us where the difference lies, post hoc test is used to get the clear picture. There are several post hoc tests which can be used, but least significant difference (LSD) test is generally used in equal sample sizes, whereas Scheffe's test is most often used in unequal sample sizes.

In all the post hoc tests, a critical difference is calculated at a particular level of significance, and if the difference of any pair of observed means is higher than the critical difference, it is inferred that one mean is higher than the other; otherwise, group means are equal. By comparing all pair of means, conclusion is drawn as to which group mean is the highest. The procedure of such comparison can be seen in the solved Example 7.1.

LSD test provides the critical difference (CD) which is used for comparing differences in all the pair of means. The CD is computed as follows:

$$\text{Critical difference} = t_{.05}(N-r) \times \sqrt{\frac{2}{n}(\text{MSS})_\text{w}} \tag{7.6}$$

where the symbols have their usual meanings.
Since Scheffe's test is used in case of unequal sample size hence it also provides different critical difference for comparing different pair of group means. Here the critical difference (CD) is calculated as follows:

$$\text{CD for comparing } i^{\text{th}} \text{ and } j^{\text{th}} \text{ group means} = \sqrt{(r-1)F_{.05}(r-1, N-r)} \times \sqrt{\text{MSS}_\text{w}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} \tag{7.7}$$

where $n_i$ and $n_j$ represent the sample sizes of $i$th and $j$th groups, respectively, and other symbols have their usual meanings.
The SPSS output provides $p$ value (significant value) for each pair of means to test the significance of difference between them. If $p$ value for any pair

of means is less than .05, it is concluded that means are significantly different otherwise not. SPSS provides various options for post hoc tests. One may choose one or more options for analysis while using SPSS.

### *Assumptions in Using One-Way ANOVA*

While applying one-way ANOVA for comparing means of different groups, the following assumptions are made:

1. The data must be measured either on interval or ratio scale.
2. The samples must be independent.
3. The dependent variable must be normally distributed.
4. The population from which the samples have been drawn must be normally distributed.
5. The variances of the population must be equal.
6. The errors are independent and normally distributed.

**Remarks**

1. ANOVA is a relatively robust procedure in case of violations of the normality assumption.
2. In case the data is ordinal, a nonparametric alternative such as Kruskal-Wallis one-way analysis of variance should be used instead of parametric one-way ANOVA.

## Effect of Using Several *t*-tests Instead of ANOVA

Many times a researcher argues that what if I use three *t*-tests rather than using one-way ANOVA in comparing the means of three groups. One of the logics is that, why to use three times *t*-test if equality of means can be tested by using one-way ANOVA once. If the number of groups are more, then one needs to apply large number of *t*-tests. For example, in case of six groups, one needs to apply ${}^6C_2 = 15$, *t*-tests instead of one-time one-way ANOVA. This may be one of the arguments of the researcher in favor of using one-way ANOVA, but the main problem in using multiple *t*-tests instead of one-way ANOVA is that the type I error gets inflated.

If the level of significance has been chosen as $p_1$, then Fisher has showed that the type I error rate expands from $p_1$ to some larger value as the number of tests between paired means increases. The error rate expansion is constant and predictable which can be computed by the following equation:

$$p = 1 - (1 - p_1)^r \tag{7.8}$$

where $p$ is the new level of significance and $r$ is the number of *t*-tests used for comparing all the pair of group means.

For example, in comparing three group means, if *t*-tests are used instead of one-way ANOVA and if the level of significance is chosen as .05, then the total number of paired comparison would be ${}^3C_2 = 3$.

Here, $p_1 = 0.05$ and $r = 3$, and, therefore, the actual level of significance becomes

$$\begin{aligned} p &= 1 - (1 - p_1)^r \\ &= 1 - (1 - 0.5)^3 = 1 - 0.95^3 = 1 - 0.8574 \\ &= 0.143 \end{aligned}$$

Thus, in comparing three group means instead of using one-way ANOVA, if three *t*-tests are applied, then level of significance shall inflate from .05 to 0.143.

## Application of One-Way ANOVA

One-way ANOVA is used when more than two group means are compared. Such situations are very frequent in management research where a researcher may like to compare more than two group means. For instance, one may like to compare the mood state of the employees working in three different plants or to compare the occupational stress among three different age categories of employees in an organization.

Consider an experiment where a market analyst of a company is interested to know the effect of three different types of incentives on the sale of a particular brand of shampoo. Shampoo is sold to the customers with three schemes. In the first scheme 20% extra is offered in the same price, in the second scheme shampoo is sold with free bath soap, whereas in the third scheme it is sold to the customers with a free ladies' perfume. These three schemes are offered to the customers in the same outlet for 3 months. During the second month, sales of the shampoo are recorded in all three schemes for 20 days. In this situation, scheme is the independent variable having three different levels: 20% extra shampoo, shampoo with a bath soap, and shampoo with a ladies' perfume whereas, the sales figure is the dependent variable. Here the null hypothesis which is required to be tested would be

$H_0$ : Average sale of shampoo in all three incentive groups are same against the alternative hypothesis.

$H_1$ : At least one group mean is different.

The one-way ANOVA may be applied to compute *F*-value. If *F*-statistic is significant, the null hypothesis may be rejected, and in that case, a post hoc test may be applied to find as to which incentive is the most attractive in improving the sale of the shampoo. On the other hand, if *F*-value is not significant, one fails to reject the null hypothesis, and in that case, there would be no reason to believe that any one incentive is better than others to enhance the sale.

**Example 7.1** An audio company predicts that students learn more effectively with a constant low-tune melodious music in background, as opposed to an irregular loud orchestra or no music at all. To verify this hypothesis, a study was planned by dividing 30 students into three groups of ten each. Students were assigned to these three groups in a random fashion, and all of them were given a comprehension to read for 20 min. Students in group 1 were asked to study the comprehension with low-tune melodious music at a constant volume in the background. Whereas the students in group 2 were exposed to loud orchestra and group 3 to no music at all while reading the comprehension. After reading the comprehension, they were asked to solve few questions. The marks obtained are shown in the Table 7.1.

Do these data confirm that learning is more effective in particular background music? Test your hypothesis at 5% level.

*Solution* Following steps shall be taken to test the required hypothesis:

(a) *Hypotheses construction:* The researcher is interested in testing the following null hypothesis:

$$H_0 : \mu_{\text{Music}} = \mu_{\text{Orchestra}} = \mu_{\text{Without\_Music}}$$

against the alternative hypothesis that at least one mean is different.

(b) *Level of significance:* 0.05

(c) *Statistical test:* One-way ANOVA shall be used to test the null hypothesis. In order to complete the ANOVA table, first, all the sum of squares are computed. Here,

Number of groups $= r = 3$
Sample size in each group $= n = 10$
Total number of scores $= nr = 30$

The computation of group total, group means, and grand total has to be computed first which is shown in Table 7.2.

(i) $$\text{Correction factor(CF)} = \frac{G^2}{N} = \frac{135^2}{30} = 607.5$$

(ii) $$\begin{aligned}\text{Raw sum of squares(RSS)} &= \sum_{i}\sum_{j} X_{ij}^2 \\ &= \left(8^2 + 4^2 + 8^2 + \ldots 9^2 + 6^2\right) \\ &\quad + \left(4^2 + 6^2 + 3^2 + \ldots 4^2 + 3^2\right) \\ &\quad + \left(3^2 + 4^2 + 6^2 + \ldots 1^2 + 2^2\right) \\ &= 440 + 188 + 127 = 755\end{aligned}$$

**Table 7.1** Comprehension scores of the students under three different environments

| Music | Orchestra | Without music |
|---|---|---|
| 8 | 4 | 3 |
| 4 | 6 | 4 |
| 8 | 3 | 6 |
| 6 | 4 | 2 |
| 6 | 3 | 1 |
| 7 | 8 | 2 |
| 3 | 3 | 6 |
| 7 | 2 | 4 |
| 9 | 4 | 1 |
| 6 | 3 | 2 |

**Table 7.2** Computation of group total, group means, and grand total

| | Music | Orchestra | Without music | |
|---|---|---|---|---|
| | 8 | 4 | 3 | |
| | 4 | 6 | 4 | |
| | 8 | 3 | 6 | |
| | 6 | 4 | 2 | |
| | 6 | 3 | 1 | |
| | 7 | 8 | 2 | |
| | 3 | 3 | 6 | |
| | 7 | 2 | 4 | |
| | 9 | 4 | 1 | |
| | 6 | 3 | 2 | |
| Group total | $R_1 = 64$ | $R_2 = 40$ | $R_3 = 31$ | $G = R_1 + R_2 + R_3 = 135$ |
| Group mean | 6.4 | 4 | 3.1 | |

(iii) $$\text{Total sum of square(TSS)} = \text{RSS} - \text{CF} = \sum_i \sum_j X_{ij}^2 - \frac{G^2}{N}$$
$$= 755 - 607.5 = 147.5$$

(iv) $$\text{Sum of squares between groups } (\text{SS}_\text{b}) = \sum_i \frac{R_i^2}{n_i} - \frac{G^2}{N}$$
$$= \frac{64^2 + 40^2 + 31^2}{10} - 607.5$$
$$= 665.7 - 607.5 = 58.2$$

(v) $$\text{Sum of square within groups}(\text{SS}_\text{w}) = \text{TSS} - \text{SS}_\text{b}$$
$$= 147.5 - 58.2 = 89.3$$

**Table 7.3** ANOVA table for the data on comprehension test

| Sources of variation | SS | df | MSS | $F$-value |
|---|---|---|---|---|
| Between groups | 58.2 | $r - 1 = 2$ | $\frac{58.2}{2} = 29.1$ | 8.79 |
| Within groups | 89.3 | $N - r = 27$ | $\frac{89.3}{27} = 3.31$ | |
| Total | 147.5 | $N - 1 = 29$ | | |

**Table 7.4** Group means and their comparison

| Music | Orchestra | Without music | CD at 5% level |
|---|---|---|---|
| 6.4 | 4 | 3.1 | **1.67** |

"⌣" represents no significant difference between the means at 5% level

(d) *Decision criteria*
From Table A.4 in the Appendix, $F_{.05}(2,27) = 4.22$.
Since calculated $F(=8.79) > F_{.05}(2,27)$, the null hypothesis may be rejected. It is therefore concluded that learning efficiency in all the three experimental groups is not same. In order to find as to which group's learning efficiency is best, the least significance difference (LSD) test shall be applied. The critical difference in LSD test is given by

$$\text{CD} = t_{.05}(27) \times \sqrt{\frac{2 \times \text{MSS}_\text{w}}{n}}$$
$$= 2.052 \times \sqrt{\frac{2 \times 3.31}{10}} = 1.67$$

(e) *Results*
The group means may be compared by arranging them in descending order as shown in the Table 7.4
It is clear from Table 7.4 that the mean difference between "music" and "orchestra" groups as well as "music" and "without music" groups is greater than the critical difference. Since the mean difference between orchestra and without music groups is significant hence it is shown by clubbing their means by the line as shown in the Table 7.4.

(f) *Inference*
From the results, it is clear that the mean learning performance in music group is significantly higher than that of orchestra as well as nonmusic groups, whereas the mean learning of orchestra group is equal to that of nonmusic group. It is therefore concluded that melodious music improves the learning efficiency.

**Table 7.5** Data on psychological health

| S.N. | Banking | Insurance | Retail |
|---|---|---|---|
| 1 | 45 | 41 | 58 |
| 2 | 41 | 38 | 54 |
| 3 | 47 | 43 | 49 |
| 4 | 59 | 53 | 65 |
| 5 | 48 | 43 | 51 |
| 6 | 45 | 42 | 56 |
| 7 | 38 | 40 | 41 |
| 8 | 48 | 42 | 51 |
| 9 | 39 | 32 | 45 |
| 10 | 42 | 39 | 53 |
| 11 | 38 | 36 | 37 |
| 12 | 36 | 32 | 42 |
| 13 | 45 | 40 | 44 |
| 14 | 38 | 39 | 32 |
| 15 | 42 | 40 | 50 |

## Solved Example of One-Way ANOVA with Equal Sample Size Using SPSS

**Example 7.2** The data in the following table indicates the psychological health ratings of corporate executives in banking, insurance, and retail sectors. Apply one-way ANOVA to test whether the executives of any particular sector are healthier in their psychological health in comparison to other sectors. Test your hypothesis at 5% as well as 1% level (Table 7.5).

*Solution*

In this problem, it is required to test the following null hypothesis

$$H_0 : \mu_{\text{Banking}} = \mu_{\text{Insurance}} = \mu_{\text{Retail}}$$

against the alternative hypothesis that at least one mean differs.

The SPSS output provides $F$-value along with its significance value ($p$ value). The $F$-value would be significant if the $p$ value is less than .05. If $F$-value becomes significant, a post hoc test shall be used to compare the paired means. SPSS provides facility to choose one or more post hoc test for analysis.

In this example, since the sample sizes are equal, LSD test shall be used as a post hoc test for comparing the group means. However, one can choose other post hoc tests as well. The SPSS output provides the $p$ value for testing the significance of the difference between each pair of group means. Thus, by looking to the results of post hoc test, one can determine as to which group mean is higher. The procedure has been discussed while interpreting the output.

## *Computations in One-Way ANOVA with Equal Sample Size*

(a) ***Preparing data file***

A data file needs to be prepared before using the SPSS commands for one-way ANOVA with equal samples size. The following steps will help you prepare the data file:

(i) *Starting the SPSS:* Use the following command sequence to start SPSS:

**Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

After clicking the **Type in Data,** you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* There are two variables in this example which need to be defined along with their properties while preparing the data file. These variables are psychological health and sector. The psychological health is defined as scale variable, whereas sector is defined as nominal variable as they are measured on interval as well as nominal scales, respectively. The procedure of defining these variables in the SPSS is as follows:

1. Click the **Variable View** to define the variables and their properties.
2. Write short name of the variables as *Psy_Health* and *Sector* under the column heading **Name**.
3. Under the column heading **Label**, full names of these variables have been defined as *Psychological health rating* and *Different sector,* respectively. You may choose some other names of these variables as well.
4. For the variable *Sector,* double-click the cell under the column heading **Values** and add the following values to different levels:

| Value | Label |
|---|---|
| 1 | Banking |
| 2 | Insurance |
| 3 | Retail |

5. Under the column heading **Measure**, select the option "Scale" for the *Psy_Health* variable and "Nominal" for the *Sector* variable.
6. Use default entries in rest of the columns. The screen shall look like Fig. 7.1.

**Remark:** Many variables can be defined in the variable view simultaneously if ANOVA is to be applied for more than one variable.

(iii) *Entering data*

After defining both the variables in **Variable View,** click **Data View** on the left corner in the bottom of the screen as shown in Fig. 7.1 to open the

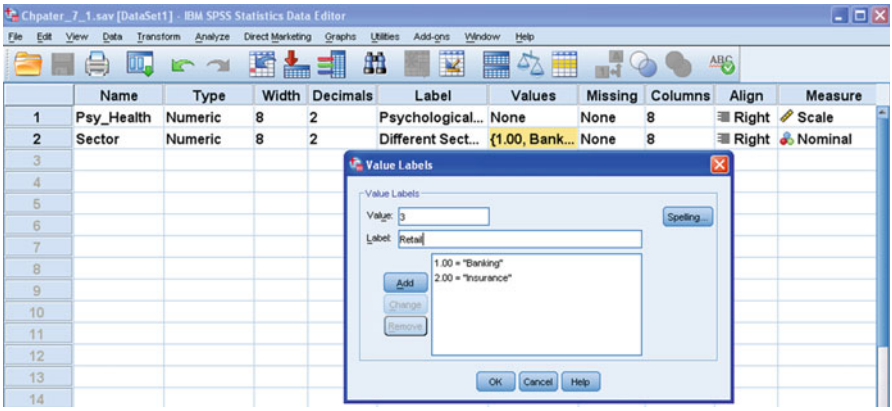



**Fig. 7.1** Defining variables along with their characteristics

data entry format column wise. After entering the data, the screen will look like as shown in Fig. 7.2. Since the data is large, only a portion of data is shown in the figure. Save the data file in the desired location before further processing.

(b) **SPSS *commands for one-way ANOVA***

After entering all the data in the data view, follow the below-mentioned steps for one-way analysis of variance:

(i) *Initiating the SPSS commands for one-way ANOVA:* In data view, click the following commands in sequence:

**Analyze ⇨ Compare Means ⇨ One-Way ANOVA**

The screen shall look like Fig. 7.3.

(ii) *Selecting variables for one-way ANOVA:* After clicking the **One-Way ANOVA** option, you will be taken to the next screen for selecting variables. Select the variables *Psychological health rating* and *Different sector* from left panel to the "Dependent list" section and "Factor" section of the right panel, respectively. The screen will look like Fig. 7.4.

(iii) *Selecting the options for computation:* After selecting the variables, option needs to be defined for generating the output in one-way ANOVA. Take the following steps:

– Click the tag **Post Hoc** in the screen shown in Fig. 7.4.

– Check the option "LSD." LSD test is selected because the sample sizes are equal. You may choose any other post hoc test if you so desire.
– Write "Significance level" as .05. By default, it is selected. However, you may select any other significance level like .01 or .10 as well.
– Click ***Continue***.

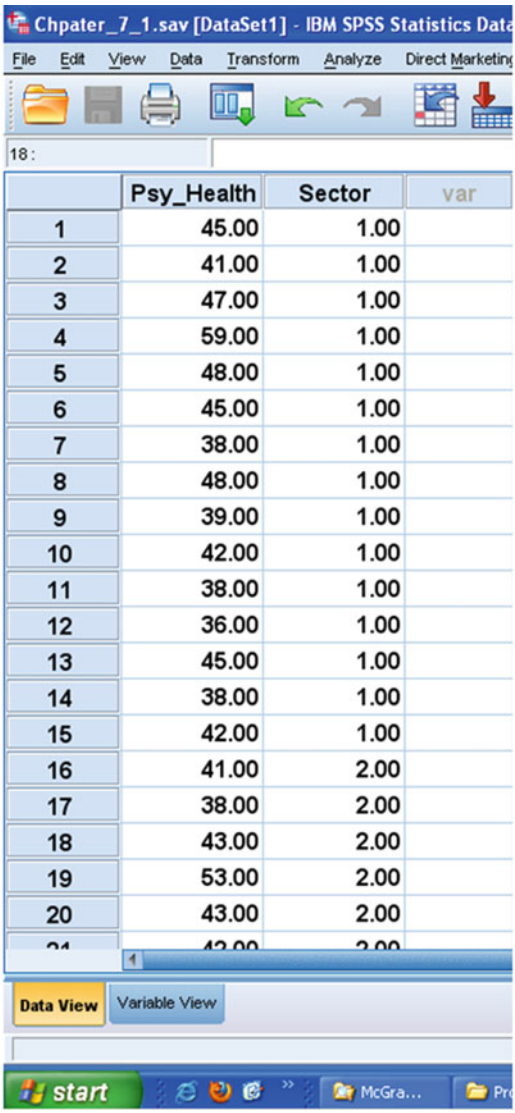



**Fig. 7.2** Screen showing entered data for the psychological health and sector in the data view

The screen will look like Fig. 7.5.

– Click the tag **Options** in the screen shown in Fig. 7.4 and then check 'Descriptive'.
– Click ***Continue***.

The screen for this option shall look like Fig. 7.6.

– After selecting the options, the screen shown in Fig. 7.4 shall be restored.
– Click **OK**.

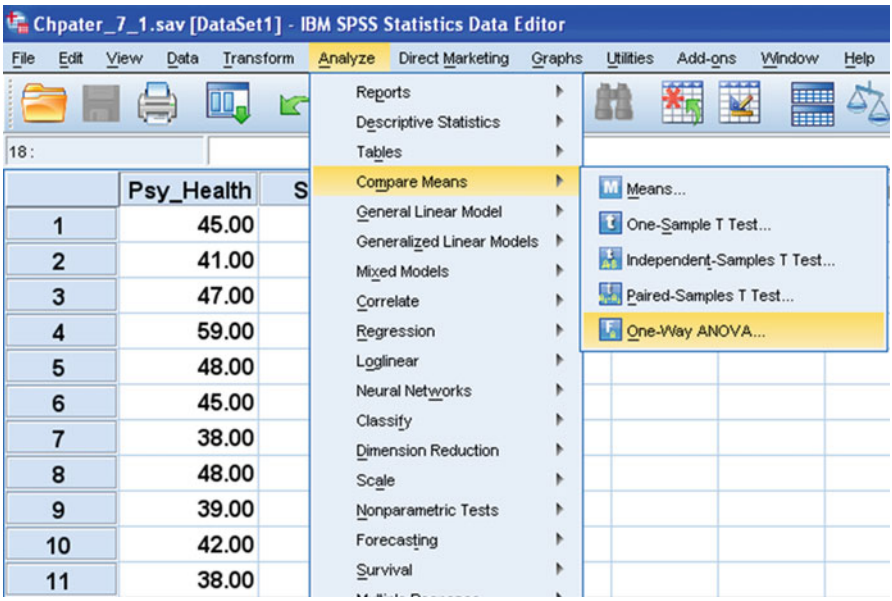



**Fig. 7.3** Screen showing SPSS commands for one-way ANOVA

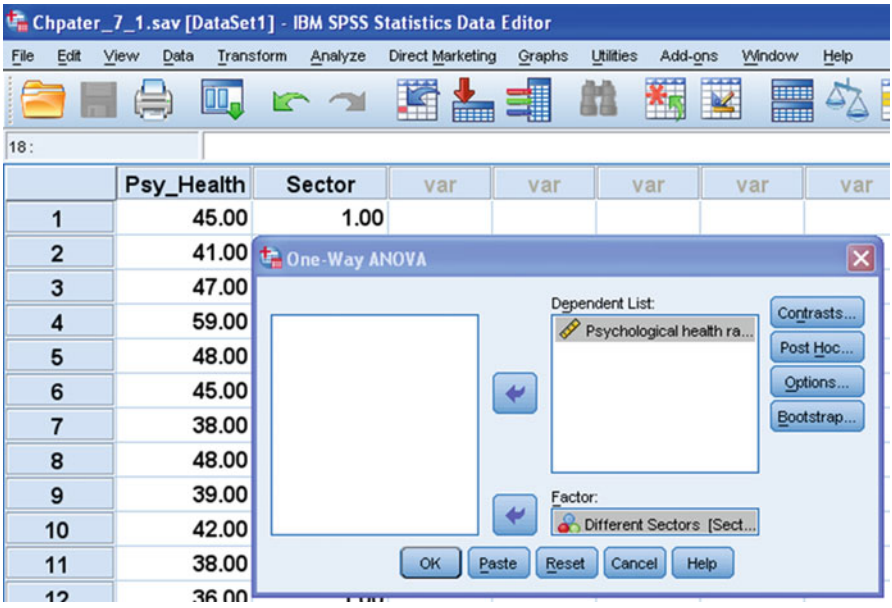



**Fig. 7.4** Screen showing selection of variables for one-way ANOVA

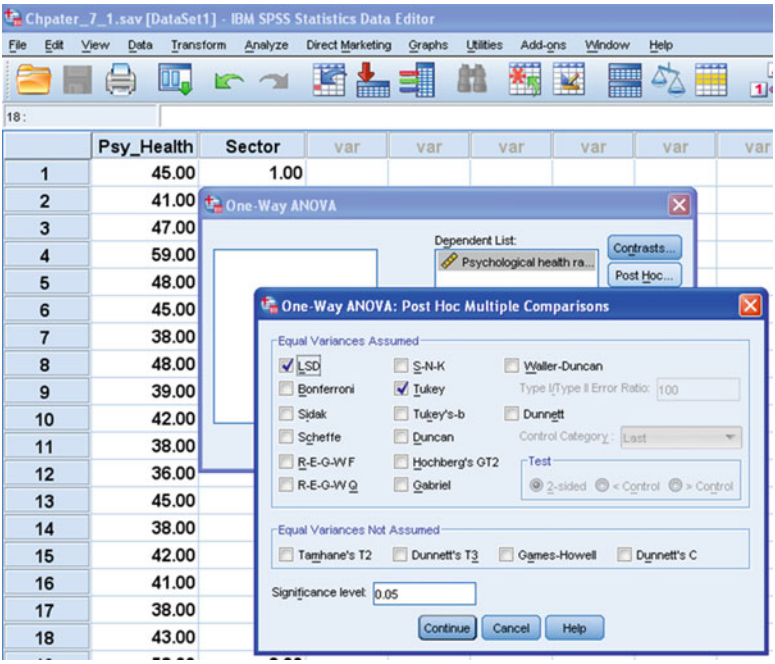



**Fig. 7.5** Screen showing options for post hoc test and significance level

(c) ***Getting the output***
After clicking **OK** in the screen shown in Fig. 7.4, the output shall be generated in the output window. The relevant outputs may be selected by using right click of the mouse and may be copied in the word file. Here, the following outputs shall be generated:

1. Descriptive statistics
2. ANOVA table
3. Post hoc comparison table

In this example, all the outputs so generated by the SPSS will look like as shown in Tables 7.6, 7.7, and 7.8.

## *Interpretations of the Outputs*

Different descriptive statistics have been shown in Table 7.6 which may be used to study the nature of the data. Further descriptive profiles of the psychological health rating for the corporate executives in different sectors can be developed by using the values of mean, standard deviation, and minimum and maximum scores in each groups. The procedure of developing such profile has been discussed in Chap. 2 of this book.

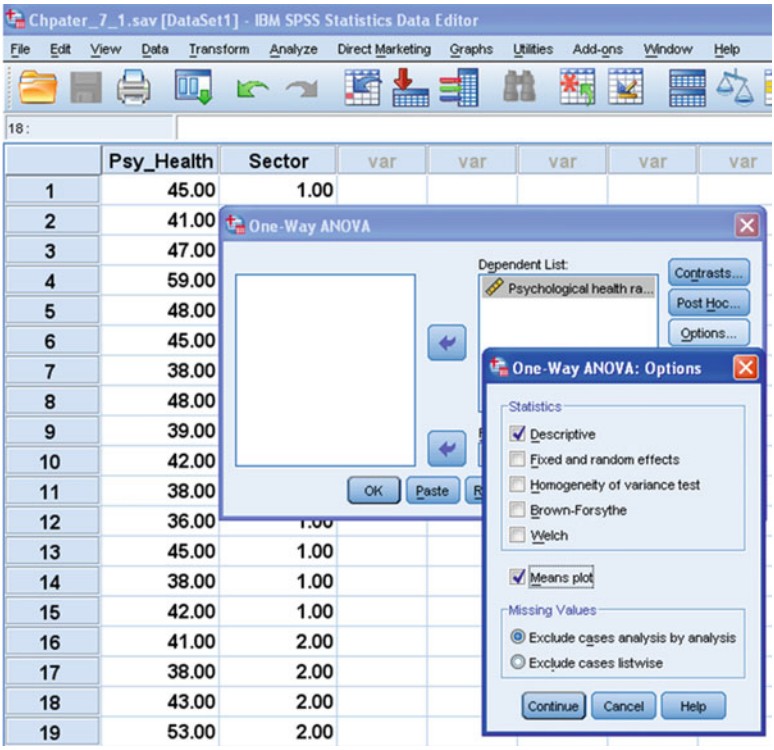



**Fig. 7.6** Screen showing options for descriptive statistics

**Table 7.6** Descriptive statistics for the data on psychological health among corporate executives in different sectors

| | | | | | 95% confidence interval for mean | | | |
|---|---|---|---|---|---|---|---|---|
| | *N* | Mean | SD | SE | Lower bound | Upper bound | Min. | Max. |
| Banking | 15 | 43.40 | 5.84 | 1.51 | 40.17 | 46.63 | 36.00 | 59.00 |
| Insurance | 15 | 40.00 | 4.97 | 1.28 | 37.25 | 42.75 | 32.00 | 53.00 |
| Retail | 15 | 48.53 | 8.53 | 2.20 | 43.81 | 53.26 | 32.00 | 65.00 |
| Total | 45 | 43.98 | 7.38 | 1.10 | 41.76 | 46.20 | 32.00 | 65.00 |

Note: Values have been rounded off nearest to the two decimal places

**Table 7.7** ANOVA table for the data on psychological health

| | Sum of squares | df | Mean square | *F* | Sig. (*p* value) |
|---|---|---|---|---|---|
| Between groups | 553.64 | 2 | 276.82 | 6.31 | .004 |
| Within groups | 1,843.33 | 42 | 43.89 | | |
| Total | 2,396.98 | 44 | | | |

Note: Values have been rounded off nearest to the two decimal places

**Table 7.8** Post hoc comparison of means using LSD test

| (*I*) Different sectors | (*J*) Different sectors | Mean difference ($I - J$) | Std. error | Sig. (*p* value) |
|---|---|---|---|---|
| Banking | Insurance | 3.40 | 2.42 | .167 |
| | Retail | −5.13* | 2.42 | .040 |
| Insurance | Banking | −3.40 | 2.42 | .167 |
| | Retail | −8.53** | 2.42 | .001 |
| Retail | Banking | 5.13* | 2.42 | .040 |
| | Insurance | 8.53** | 2.42 | .001 |

Note: The values of lower bound and upper bound have been omitted from the original output. The values have been rounded off nearest to the two decimal places
*The mean difference is significant at 5% level
**The mean difference is significant at 1% level

**Table 7.9** Mean scores on psychological health in different groups

| Retail | Banking | Insurance |
|---|---|---|
| 48.53 | 43.40 | 40.00 |

"⌣" represents no significant difference between the means

The mean of different groups in Table 7.6 and the results of Table 7.8 have been used to prepare the graphics shown in Table 7.9 which can be used to draw conclusions about post hoc comparison of means.

The *F*-value in Table 7.7 is significant at 5% level because its *p* value (=.004) is less than .05. Thus, the null hypothesis of no difference among the means of the three groups may be rejected at 5% level. Since the *p* value is also less than .01, the null hypothesis may be rejected at 1% level also.

Here, the *F*-value is significant; hence, the post hoc test needs to be applied for testing the significance of mean difference between different pairs of groups. Table 7.8 provides such comparison. It can be seen from this table that the difference between banking and retail groups on their psychological health rating is significant at 5% level because the *p* value for this mean difference is .04 which is less than .05.

Similarly, the difference between insurance and retail groups on their psychological health is also significant at 5% as well as 1% level because the *p* value attached to this mean difference is .001 which is less than .05 as well as .01.

There is no significant difference between the banking and insurance groups on their psychological health rating because the *p* value attached to this group is .167 which is more than .05.

All the above-mentioned three findings can be very easily understood by looking to the graphics in Table 7.9. From this table, it is clear that the mean psychological health rating score is highest among the executives in the retail sector in comparison to that of banking and insurance sectors. It may thus be concluded that the psychological health of the executives in the retail sector is best in comparison to that of banking and insurance sectors.

## Solved Example of One-Way ANOVA with Unequal Sample

**Example 7.3** A human resource department of an organization conducted a study to know the status of occupational stress among their employees in different age categories. A questionnaire was used to assess the stress level of the employees in three different age categories: <40, 40–55, and >55 years. The stress scores so obtained are shown in Table 7.10.
Apply one-way analysis of variance to test whether mean stress score of the employees in any two age categories are different. Test your hypothesis at 5% level.

*Solution* Solving problems of one-way ANOVA with equal and unequal samples through SPSS are almost similar. In case of unequal sample size, one should be careful in feeding the data. The procedure of feeding the data in this case shall be discussed below. Here, the SPSS procedure shall be discussed in brief as it is exactly similar to the one discussed in Example 7.2. Readers are advised to refer to the procedure mentioned in Example 7.2 in case of doubt in solving this problem of unequal sample size.

Here, the null hypothesis which needs to be tested is

$$H_0 : \mu_A = \mu_B = \mu_C$$

against the alternative hypothesis that at least one group mean differs.

If the null hypothesis is rejected, post hoc test will be used for comparing group means. Since the sample sizes are different, the Scheffe's test has been used for post hoc analysis.

**Table 7.10** Occupational stress scores among the employees in different age categories

| Group A (<40 years) | Group B (40–55 years) | Group C (>55 years) |
|---|---|---|
| 54 | 75 | 55 |
| 48 | 68 | 51 |
| 47 | 68 | 59 |
| 54 | 71 | 64 |
| 56 | 79 | 52 |
| 62 | 86 | 48 |
| 56 | 81 | 65 |
| 45 | 79 | 48 |
| 51 | 72 | 56 |
| 54 | 78 | 49 |
| 48 | 69 | |
| 52 | | |

## *Computations in One-Way ANOVA with Unequal Sample Size*

(a) ***Preparing data file:***

(i) *Starting the SPSS:* Start the SPSS the way it has been done in the above-mentioned example and click the **Type in Data** option. You will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables:* There are two variables in this example that need to be defined along with their properties while preparing the data file. The two variables are stress scores and age group. The stress score is defined as scale variable, whereas age group is defined as nominal variable as they are measured on interval as well as nominal scales, respectively. The procedure of defining these variables in the SPSS is as follows:

1. Click **Variable View** to define variables and their properties.
2. Write short name of the variables as *Stress* and *Age_Gp* under the column heading **Name**.
3. Under the column heading **Label**, full names of these variables may be defined as *Stress scores* and *Age group,* respectively. You may choose some other names of these variables as well.
4. For the variable *Age group,* double-click the cell under the column heading **Values** and add the following values to different levels:

| Value | Label |
|---|---|
| 1 | Group A (<40 years) |
| 2 | Group B (40–55 years) |
| 3 | Group C (>55 years) |

5. Under the column heading **Measure,** select the option "Scale" for the *Stress* variable and "Nominal" for the *Age_Gp* variable.
6. Use default entries in rest of the columns.

After defining all the variables in variable view, the screen shall look like Fig. 7.7.

**Remark:** More than one variable can be defined in the variable view for doing ANOVA for many variables simultaneously.

(iii) *Entering the data*: After defining the variables in the **Variable View**, enter the data, column-wise in **Data View**. The data feeding shall be done as follows:

Format of data feeding in Data View

| | S.N. | Stress | Age_Gp |
|---|---|---|---|
| Group A $n_1 = 12$ | 1 | 54 | 1 |
| | 2 | 48 | 1 |
| | 3 | 47 | 1 |
| | 4 | 54 | 1 |
| | 5 | 56 | 1 |
| | 6 | 62 | 1 |
| | 7 | 56 | 1 |
| | 8 | 45 | 1 |
| | 9 | 51 | 1 |
| | 10 | 54 | 1 |
| | 11 | 48 | 1 |
| | 12 | 52 | 1 |
| Group B $n_2 = 11$ | 13 | 75 | 2 |
| | 14 | 68 | 2 |
| | 15 | 68 | 2 |
| | 16 | 71 | 2 |
| | 17 | 79 | 2 |
| | 18 | 86 | 2 |
| | 19 | 81 | 2 |
| | 20 | 79 | 2 |
| | 21 | 72 | 2 |
| | 22 | 78 | 2 |
| | 23 | 69 | 2 |
| Group C $n_3 = 10$ | 24 | 55 | 3 |
| | 25 | 51 | 3 |
| | 26 | 59 | 3 |
| | 27 | 64 | 3 |
| | 28 | 52 | 3 |
| | 29 | 48 | 3 |
| | 30 | 65 | 3 |
| | 31 | 48 | 3 |
| | 32 | 56 | 3 |
| | 33 | 49 | 3 |

After feeding the data as mentioned above, the final screen shall look like Fig. 7.8.

(b) **SPSS *commands for one-way ANOVA for unequal sample size***
After entering all the data in data view, save the data file in the desired location before further processing.

(i) *Initiating the SPSS commands for one-way ANOVA:* In data view, go to the following commands in sequence:

**Analyze ⇨ Compare Means ⇨ One-Way ANOVA**

(ii) *Selecting variables for analysis:* After clicking the **One-Way ANOVA** option, you will be taken to the next screen for selecting variables. Select

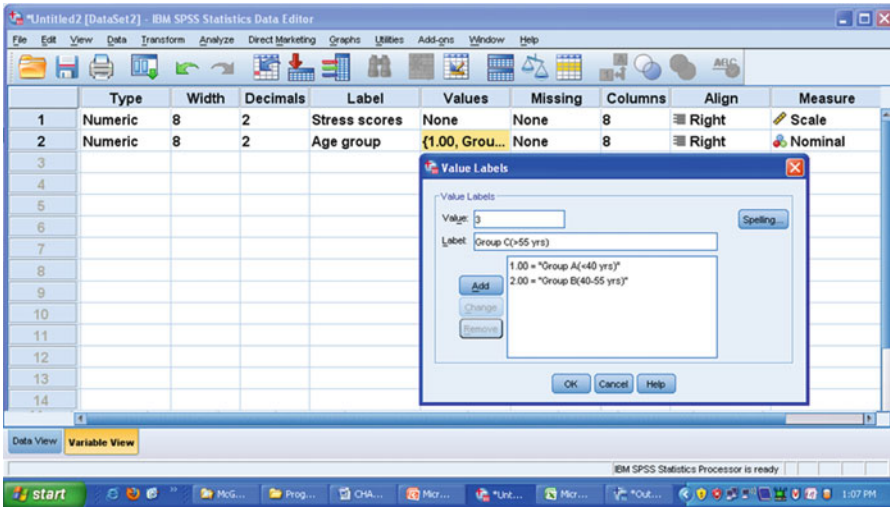



**Fig. 7.7** Defining variables along with their characteristics

the variables *Stress scores* and *Age group* from left panel to the "Dependent list" section and "Factor" section of the right panel, respectively. The screen shall look like Fig. 7.9.

(iii) *Selecting options for computation:* After variable selection, option needs to be defined for generating outputs in one-way ANOVA. This shall be done as follows:

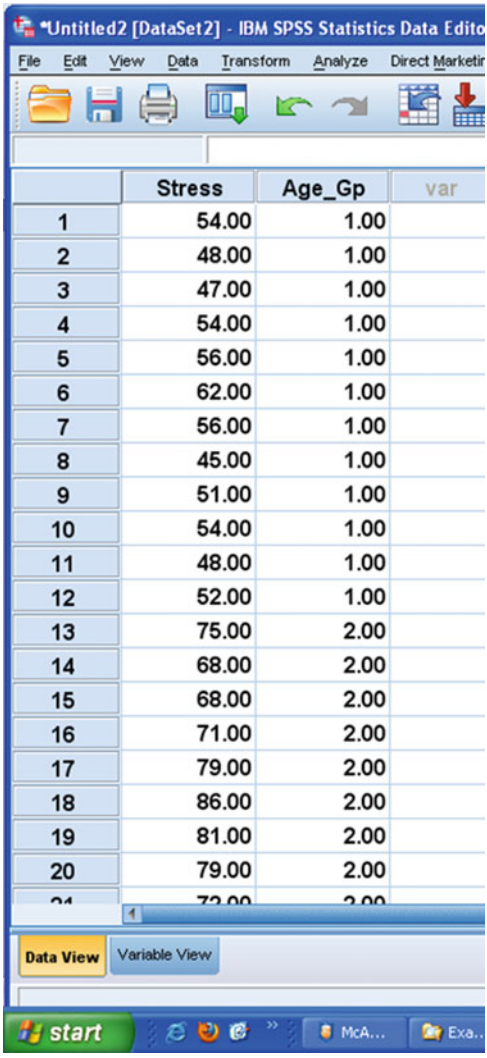
- Click the tag **Post Hoc** in the screen shown in Fig. 7.9.
  - Check the option "Scheffe." This test is selected because the sample sizes are unequal; however, you can choose any other test if you so desire.
  - If graph needs to be prepared, select the option "Means plot."
  - Write "Significance level" as .05. Usually this is written by default; however, you may write any other significance level like .01 or .10 as well.
  - Click ***Continue***.
- Click the tag **Options** and then check "Descriptive." Click ***Continue***.
- After selecting the options, click **OK**.

(c) ***Getting the output***

After clicking **OK** on the screen as shown in Fig. 7.9, the output shall be generated in the output window. The relevant outputs can be selected by using right click of the mouse and may be copied in the word file. The following output shall be generated in this example:

(a) *Descriptive statistics*

**Fig. 7.8** Showing data entry of stress scores for the employees in different age categories in data view




(b) *ANOVA table*
(c) *Post hoc comparison table*
(d) *Graph for means plot*

These outputs are shown in Tables 7.11, 7.12, and 7.13 and in Fig. 7.10. The Table 7.14 has been developed by using the descriptive statistics from the Table 7.11 and inputs from Table 7.13.

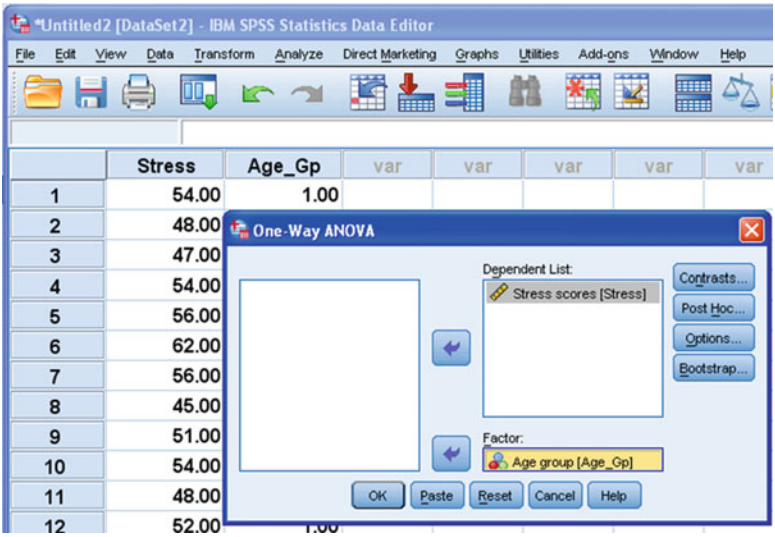



**Fig. 7.9** Screen showing selection of variables

**Table 7.11** Descriptive statistics for the data on occupational stress of employees in different age categories

| | | | | | 95% confidence interval for mean | | | |
|---|---|---|---|---|---|---|---|---|
| | *N* | Mean | SD | SE | Lower bound | Upper bound | Min. | Max. |
| Group A (<40 years) | 12 | 52.25 | 4.77 | 1.38 | 49.23 | 55.28 | 45.00 | 62.00 |
| Group B (40–55 years) | 11 | 75.09 | 5.97 | 1.80 | 71.08 | 79.10 | 68.00 | 86.00 |
| Group C (>55 years) | 10 | 54.70 | 6.29 | 1.99 | 50.20 | 59.20 | 48.00 | 65.00 |
| Total | 33 | 60.61 | 11.80 | 2.05 | 56.42 | 64.79 | 45.00 | 86.00 |

**Table 7.12** ANOVA table for the data on occupational stress

| | Sum of squares | df | Mean square | *F* | Sig. |
|---|---|---|---|---|---|
| Between groups | 3494.620 | 2 | 1747.310 | 54.42 | .000 |
| Within groups | 963.259 | 30 | 32.109 | | |
| Total | 4457.879 | 32 | | | |

## *Interpretation of the Outputs*

Table 7.11 shows the descriptive statistics of the data on occupational stress of employees in different age categories. These statistics can be used to develop a graphic profile of the employee's occupational stress in different age categories.

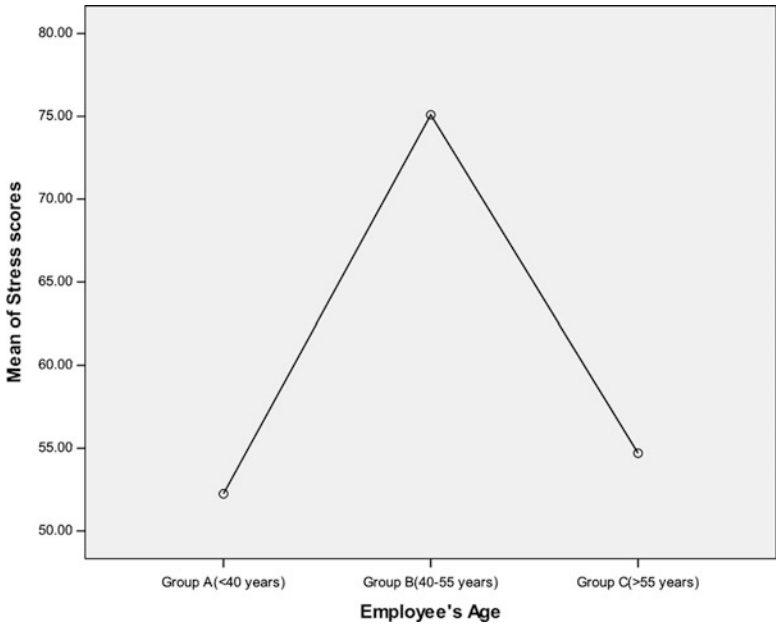



**Fig. 7.10** Graphical presentation of mean scores of occupational stress in three different age categories

The procedure of developing such profile has been discussed in detail in Chap. 2 of this book. Further, these descriptive statistics can be used to discuss the nature of data in different age categories.

Table 7.12 gives the value of calculated $F$. The $p$ value attached with the $F$ is .000 which is less than .05 as well as .01; hence, it is significant at 5% as well as 1% levels. Since the $F$-value is significant, the null hypothesis of no difference in the occupational stress among the employees in all the three age categories is rejected. The post hoc test is now used to compare the means in different pairs.

SPSS provides the option of choosing the post hoc test, and, therefore, one may choose any one or more test for post hoc analysis. In this example, the Scheffe's test was chosen to compare the means in different pairs. Table 7.13 provides such comparisons.

It can be seen that the difference between occupational stress of the employees in group A (<40 years) and group B (40–55 years) is significant at 5% as well as at 1% level both as the $p$ value for this mean difference is .000 which is less than .05 as well as .01. Similarly, the mean difference between occupational stress of the employees in group B (40–55 years) and group C (>55 years) is also significant at 5% as well as 1% level both as the $p$ value for this mean difference is .000 which is also less than .05 and .01. However, there is no significant difference between the occupational stress of the employees in group A (<40 years) and group C (>55 years) because the $p$ value is .606.

**Table 7.13** Post hoc comparison of group means using Scheffe's test

| (I) Age group | (J) Age group | Mean diff. (I − J) | SE | Sig. (*p* value) |
|---|---|---|---|---|
| Group A (<40 years) | Group B (40–55 years) | −22.84091* | 2.36531 | .000 |
| | Group C (>55 years) | −2.45000 | 2.42623 | .606 |
| Group B (40–55 years) | Group A (<40 years) | 22.84091* | 2.36531 | .000 |
| | Group C (>55 years) | 20.39091* | 2.47585 | .000 |
| Group C (>55 years) | Group A (<40 years) | 2.45000 | 2.42623 | .606 |
| | Group B (40–55 years) | −20.39091* | 2.47585 | .000 |

Note: The values of lower bound and upper bound have been omitted from the original output
*The mean difference is significant at 5% as well as 1% levels

**Table 7.14** Mean scores on occupational stress in different groups

| Group B (40–55 years) | Group C (>55 years) | Group A (<40 years) |
|---|---|---|
| 75.09 | 54.70 | 52.25 |

"⌣" represents no significant difference between the means

The above results can be easily understood by looking to the graphics in Table 7.14. This table has been obtained by combining the results of Tables 7.11 and 7.13.

Table 7.14 reveals that the mean occupational stress is highest among the employees in group B (40–55 years). Further, mean occupation stress is similar in group C (>55 years) and group A (<40 years). Since the option for mean plot was selected in the SPSS, Fig. 7.10 has been generated in the output which shows the mean plots of all the groups. The graph provides the conclusion at a glance.

*Inference*: On the basis of the results obtained above, it may be inferred that the occupational stress among the employees in the age category 40–55 years is maximum. The researcher may write their own reasons for these findings after studying the lifestyle and working environment of the employees in this age category. The results in the study provide an opportunity to the researcher to write their own reasoning or develop their theoretical concepts supported by the review of literatures.

## Summary of the SPSS Commands for One-Way ANOVA (Example 7.2)

(i) Start the SPSS by using the following commands:

**Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

(ii) Click **Variable View** tag and define the variables *Psy_Health* and *Sector* as scale and nominal variables, respectively.

(iii) Under the column heading **Values**, define "1" for banking, "2" for insurance, and "3" for retail.
(iv) After defining variables, type the data for these variables by clicking **Data View**.
(v) In the data view, follow the below-mentioned command sequence for the computation involved in one-way analysis of variance:

**Analyze ⇨ Compare Means ⇨ One-Way ANOVA**

(vi) Select the variables *Psychological health rating* and *Different sector* from left panel to the "Dependent list" section and "Factor" section of the right panel, respectively.
(vii) Click the tag **Post Hoc** and check the option "LSD" and ensure that the value of "Significance level" is written as .05. Click ***Continue***.
(viii) Click the tag **Options** and then check "Descriptive." Press ***Continue***.
(ix) Press **OK** for output.

## Exercise

*Short Answer Questions*

**Note**: Write answer to each of the following questions in not more than 200 words.

Q.1. In an experiment, it is desired to compare the time taken to complete a task by the employees in three age groups, namely, 20–30, 31–40, and 41–50 years. Write the null hypothesis as well as all possible types of alternative hypotheses.
Q.2. Explain a situation where one-way analysis of variance can be applied. Which variances are compared in one-way ANOVA?
Q.3. Define principles of ANOVA. What impact it will have if these principles are not met?
Q.4. In what situations factorial experiments are planned? Discuss a specific situation where it can be used.
Q.5. What is repeated measure design? What precaution one must take in framing such an experiment?
Q.6. Discuss the procedure of one-way ANOVA in testing of hypotheses.
Q.7. Write a short note on post hoc tests.
Q.8. What do you mean by different sum of squares? Which sum of square you would like to increase and decrease in your experiment and why?
Q.9. What are the assumptions in applying one-way ANOVA?
Q.10. If you use multiple $t$-tests instead of one-way ANOVA, what impact it will have on results?
Q.11. Analysis of variance is used for comparing means of different groups, but in doing so $F$-test is applied, which is a test of significance for comparing the variances of two groups. Discuss this anomaly.

Q.12. What do you mean by the post hoc test? Differentiate between LSD and Scheffe's test.
Q.13. What is $p$ value? In what context it is used?

*Multiple Choice Questions*

**Note**: For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. In one-way ANOVA experiment, which of the following is a randomization assumption that must be true?

   (a) The treatment must be randomly assigned to the subjects.
   (b) Groups must be chosen randomly.
   (c) The type of data can be randomly chosen to either categorical or quantitative.
   (d) The treatments must be randomly assigned to the groups.

2. Choose the correct statement.

   (a) Total sum of square is additive in nature.
   (b) Total mean sum of square is additive in nature.
   (c) Total sum of square is nonadditive.
   (d) None of the above is correct.

3. In one-way ANOVA, $X_{ij}$ represents

   (a) The sample mean of the criterion variable for the $i$th group
   (b) The criterion variable value for the $i$th subject in the $j$th group
   (c) The number of observations in the $j$th group
   (d) The criterion variable value for the $j$th subject in the $i$th group

4. In one-way ANOVA, TSS measures

   (a) The variability within groups
   (b) The variability between groups
   (c) The overall variability in the data
   (d) The variability of the criterion variable in any group.

5. In an experiment, three unequal groups are compared with total number of observations in all the groups as 31 (with some items missing). Calculate the test statistic for one-way ANOVA $F$-test.

| Source | df | SS | MS | $F$ |
|---|---|---|---|---|
| Between groups | | 7.5 | 3.75 | ? |
| Within groups | | | | |
| Total | | 20.8 | | |

(a) 17.89
(b) 789
(c) 7.89
(d) 78.9

6. Choose the correct statement.

(a) LSD may be used for unequal sample size.
(b) Scheffe's test may be used for unequal sample size.
(c) Scheffe's test may be used for comparing more than ten groups.
(d) None of the above is correct.

7. If two groups having 10 observations in each are compared by using one-way ANOVA and if $SS_w = 140$, then what will be the value of $MSS_w$?

(a) 50
(b) 5
(c) 0.5
(d) 50.5

8. In a one-way ANOVA, if the level of significance is fixed at .05 and if $p$ value associated with $F$-statistics is 0.062, then what should you do?

(a) Reject $H_0$, and it is concluded that the group population means are not all equal.
(b) Reject $H_0$, and it may be concluded that it is reasonable that the group population means are all equal.
(c) Fail to reject $H_0$, and it may be concluded that the group population means are not all equal.
(d) Fail to reject $H_0$, and it may be concluded that there is no reason to believe that the population means differ.

9. Choose the correct statement.

(a) If $F$-statistic is significant at .05 level, it will also be significant at .01 level.
(b) If $F$-statistic is significant at .01 level, it may not be significant at .05 level.
(c) If $F$-statistic is significant at .01 level, it will necessarily be significant at .05 level.
(d) If $F$-statistic is not significant at .01 level, it will not be significant at .05 level.

10. Choose the correct statement.

(a) If $p$ value is 0.02, $F$-statistic shall be significant at 5% level.
(b) If $p$ value is 0.02, $F$-statistic shall not be significant at 5% level.
(c) If $p$ value is 0.02, $F$-statistic shall be significant at 1% level.
(d) None of the above is correct.

11. In comparing the IQ among three classes using one-way ANOVA in SPSS, choose the correct statement about the variable types.

(a) IQ is a nominal variable and class is a scale variable.
(b) Both IQ and class are the scale variables.
(c) IQ is a scale variable and class is a nominal variable.
(d) Both IQ and class are the nominal variables.

12. If product sales are to be compared in three outlets, then choose the valid variable names in SPSS.

(a) Product_Sale and Outlet
(b) Product-Sale and Outlet
(c) Product_Sale and 3Outlet
(d) Product-Sale and 3_Outlet

13. If three groups of students are compared on their work efficiency and in each group there are 12 subjects, what would be the degrees of freedom for the within group in one-way ANOVA?

(a) 30
(b) 31
(c) 32
(d) 33

14. Choose the correct model in one-way ANOVA.

(a) $TSS = (SS)_b + (SS)_w$
(b) $TSS = (SS)_b - (SS)_w$
(c) $TSS = (SS)_b \times (SS)_w$
(d) $TSS = (SS)_b/(SS)_w$

15. In one-way ANOVA $F$-test, if $SS_w$ decreases (other sums of squares and degrees of freedom remain the same), then which of the following is true?

(a) The value of the test statistic increases.
(b) The $p$ value increases.
(c) Both (a) and (b).
(d) Neither (a) nor (b).

16. In a one-way ANOVA, the $p$ value associated with $F$-test is 0.100. If the level of significance is taken as .05, what would you do?

(a) Reject $H_0$, and it is concluded that some of the group population means may differ.
(b) Reject $H_0$, and it is reasonable to assume that all the group population means are equal.
(c) Fail to reject $H_0$, and it is concluded that some of the group population means differ.
(d) Fail to reject $H_0$, and it is reasonable to assume that all the group population means are equal.

17. In one-way ANOVA, four groups were compared for their memory retention power. These four groups had 8, 12, 10, and 11 subjects, respectively. What shall be the degree of freedom of between groups?

    (a) 41
    (b) 37
    (c) 3
    (d) 40

18. If motivation has to be compared among the employees of three different units using one-way ANOVA, then the variables Motivation and Units need to be selected in SPSS. Choose the correct selection strategy.

    (a) Motivation in "Factor" section and Plant in "Dependent list" section.
    (b) Motivation in "Dependent list" section and Plant in "Factor" section.
    (c) Both Motivation and Plant in "Dependent list" section.
    (d) Both Motivation and Plant in "Factor" section.

*Assignments*

1. A CFL company was interested to know the impact of weather on the life of the bulb. The bulb was lit continuously in hot humid and cold environmental conditions till it was fused. The following are the number of hours it lasted in different conditions:

Life of bulbs (in hours) in different environmental conditions

| S.N. | Humid | Hot | Cold |
|---|---|---|---|
| 1 | 400 | 450 | 520 |
| 2 | 425 | 460 | 522 |
| 3 | 423 | 480 | 529 |
| 4 | 465 | 490 | 521 |
| 5 | 422 | 540 | 529 |
| 6 | 435 | 580 | 540 |
| 7 | 444 | 598 | 579 |
| 8 | 437 | 589 | 595 |
| 9 | 437 | 540 | 510 |
| 10 | 480 | 598 | 530 |
| 11 | 475 | 578 | 567 |
| 12 | 430 | 549 | 529 |
| 13 | 431 | 542 | 523 |
| 14 | 428 | 530 | 510 |
| 15 | 412 | 532 | 570 |

   Apply one-way analysis of variance and test whether the average life of bulbs are same in all the weather conditions. Test your hypothesis at 5% level of significance as well as 1% level of significance.
2. It was experienced by a researcher that the housewives read local news with more interests in comparison to the news containing health information and read

health news with more interest in comparison to that of international news. To test this hypothesis, ten housewives were selected at random in each of the three groups. First group was given an article containing local news for reading, the second group read an article about health, whereas the third group was given an article related with international news. After an hour, subjects in each of these groups were tested for a recall measure test where they were asked true-false questions about the news story they read. The scores so obtained on the recall measure test in all the three groups are shown below:

Data on recall measure in three groups

| Local news | Health news | International news |
|---|---|---|
| 15 | 14 | 10 |
| 16 | 12 | 8 |
| 15 | 14 | 12 |
| 18 | 16 | 11 |
| 12 | 11 | 13 |
| 16 | 14 | 16 |
| 17 | 14 | 9 |
| 15 | 12 | 8 |
| 16 | 13 | 12 |
| 15 | 13 | 12 |

Apply one-way ANOVA and discuss your findings at 5% as well as 1% levels.

*Answers to Multiple-Choice Questions*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Q.1 | a | Q.2 | a | Q.3 | d | Q.4. | c |
| Q.5 | c | Q.6 | b | Q.7 | b | Q.8. | d |
| Q.9 | c | Q.10 | a | Q.11 | c | Q.12. | a |
| Q.13 | d | Q.14 | a | Q.15 | a | Q.16. | d |
| Q.17 | c | Q.18 | b | | | | |

# Chapter 8
# Two-Way Analysis of Variance: Examining Influence of Two Factors on Criterion Variable

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Explain the importance of two-way analysis of variance (ANOVA) in research.
- Understand different designs where two-way ANOVA can be used.
- Describe the assumptions used in two-way analysis of variance.
- Learn to construct various hypotheses to be tested in two-way analysis of variance.
- Interpret various terms involved in two-way analysis of variance.
- Learn to apply two-way ANOVA manually in your data.
- Understand the procedure of analyzing the interaction between two factors.
- Know the procedure of using SPSS for two-way ANOVA.
- Learn the model way of writing the results in two-way analysis of variance by using the output obtained in the SPSS.
- Interpret the output obtained in two-way analysis of variance.

## Introduction

A two-way analysis of variance is a design with two factors where we intend to compare the effect of multiple levels of two factors simultaneously on criterion variable. The two-way ANOVA is applied in two situations: first, where there is one observation per cell and, second, where there is more than one observation per cell. In a situation where there is more than one observation per cell, it is mandatory that the number of observations in each cell must be equal. Using two-way ANOVA with n observations per cell facilitates us to test if there is any interaction between the two factors.

Two-way analysis of variance is in fact an extension of one-way ANOVA. In one-way ANOVA, the effect of one factor is studied on the criterion variable, whereas in two-way ANOVA, the effect of two factors on the criterion variable is

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studied simultaneously. An additional advantage in two-way ANOVA is to study the interaction effect between the two factors. One of the important advantages of two-way analysis of variance design is that there are two sources of assignable causes of variation, and this helps to reduce the error variance and thus making this design more efficient.

Consider an experiment where a personal manger is interested to know whether the job satisfaction of the employees in different age categories is same or not irrespective of an employee being male or female. In testing this hypothesis, 15 employees may be randomly selected in each of three age categories: 20–30, 31–40, and 41–50 years, and one-way ANOVA experiment may be planned. Since in making the groups male and female employees were selected at random, and, therefore, if any difference in the satisfaction level is observed in different age categories, it may not be truly attributed due to the age category only. The variation might be because of their gender difference as well.

Now, the same experiment may be planned in two-way ANOVA with one factor as age and the second as gender. Here, the factor age has 3 levels and gender has 2 levels. By planning this experiment in two-way ANOVA, the total variability may be broken into two assignable causes, that is, age and gender, and, therefore, more variability among the employees' satisfaction level can be explained resulting in reduction of error variance. Thus, an experimenter is in a better position to explain the overall variability in the satisfaction level of the employees. Moreover, interaction effect, if any, between gender and age on the satisfaction level of the employees can also be studied in this design.

There may be several instances where two-way ANOVA experiment can be planned. For example, in studying the effect of three outlet locations on the sale of a particular facial cream, one may select another factor as age because it is assumed that the age is also responsible in the sale of this product besides the variation in the outlet location. In framing this experiment as a two-way ANOVA, the variation in the sale of this product due to difference in the outlet location can be efficiently explained by separating the variation due to age difference. The design has been shown in the following table.

| | District | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Outlet 1 | B | C | D | A |
| Outlet 2 | D | A | B | C |
| Outlet 3 | C | D | A | B |
| Outlet 4 | A | B | C | D |

## Principles of ANOVA Experiment

All the three basic principles of design, that is, randomization, replication, and local controls, are used in planning a two-way ANOVA experiment in order to minimize the error variance. In one-way ANOVA experiments only two principles

i.e. randomization and replication are used to control the error variance whereas in two-way ANOVA experiments all the three principles i.e. randomization, replication, and local control are used to control the error variance. The very purpose of using these three principles of design is to enable the researcher to conclude with more authority that the variation in the criterion variable is due to the identified level of a particular factor.

In two-way ANOVA experiment, the principle of randomization means that the samples in each group are selected in a random fashion so as to make the groups as homogeneous as possible. The randomization avoids biases and brings control in the experiment and helps in reducing the error variance up to a certain extent.

The principle of replication refers to studying the effect of two factors on more than one subject in each cell. The logic is that one should get the same findings on more than one subject. In two-way ANOVA experiment, the principle of replication allows a researcher to study the significance of interaction between the two factors. Interaction effect cannot be studied if there is only one observation in each cell.

The principle of local control refers to making the groups as homogeneous as possible so that variation due to one or more assignable causes may be segregated from the experimental error. Thus, the application of local control helps us in reducing the error variation and making the design more efficient.

In the example discussed above, in studying the effect of age on job satisfaction if the employees were divided only according to their age, then we would have ignored the effect of gender on job satisfaction which would have increased the experimental error. However, if the researcher feels that instead of gender if the job satisfaction varies as per their salary structure, then the subjects may be selected as per their salary bracket in different age categories. This might further reduce the experimental error. Thus, maximum homogeneity can be ensured among the observations in each cell by including the factor in the design which is known to vary with the criterion variable.

## Classification of ANOVA

By using the above-mentioned principles the two-way ANOVA can be used for different designs. Some of the most popular designs where two-way ANOVA can be used are discussed below.

### *Factorial Analysis of Variance*

Two-way ANOVA is the most widely used in factorial designs. The factorial design is used for more than one independent variable. The independent variables are also referred to as factors. In factorial design, there are at least two or more factors. Usually, two-way analysis of variance is used in factorial designs having two factors.

In this design, the effect of two factors (having different levels) is seen on the dependent variable. Consider an experiment where age ($A$) and gender ($B$) are taken as two factors whose effect has to be seen on the dependent variability, sincerity. Further, let the factor $A$ has three levels (20–30, 31–40, and 41–50 years) and the factor $B$ has two levels (Male and Female). Thus, in this design, $2 \times 3$, that is, six combination of treatment groups need to be taken. This design facilitates in studying the effect of both the factors $A$ and $B$ on the dependent variable. Further, in this design, significance of the interaction effect between the two factors can also be tested. The factorial design is very popular in the behavioral research, social sciences, and humanities. This design has a few advantages over single-factor designs. The most important aspect of the factorial design is that it can provide some information about how factors *interact* or combine in the effect they have on the dependent variable. The factorial design shall be discussed in detail while in solving two-way ANOVA problem later in this chapter.

## *Repeated Measure Analysis of Variance*

Another design where two-way ANOVA is used is the repeated measure design. This design is also known as a within-subject design. In this design, same subject is tested under repeated conditions over a time. The repeated measure design can be considered to be an extension of the paired-samples $t$-test because in this case, comparison is done between more than two repeated measures. The repeated measure design is used to eliminate the individual differences as a source of between-group differences. This helps to create a more powerful test. The only care to be taken in the repeated measure design is that while testing the same subject repeatedly, no carryover effect should be there.

## *Multivariate Analysis of Variance (MANOVA)*

In this design, effect of two factors is studied on more than one dependent variable. It is similar to the factorial design having two factors, but the only difference is that here we have more than one dependent variable. At times, it makes sense to combine the dependent variables for drawing the conclusion about the effects of two factors on it. For instance, in an experiment, if the effect of teaching methods and hostel facilities have to be seen on the overall academic performance (consisting four subjects: Physics, Chemistry, Math, and English) of the students, then it makes sense to see the effect of these two factors, that is, teaching methods and hostel facilities on all the subjects together. Once the effect of any of these factors is found to be significant, then the two-way ANOVA for each of the dependent variable is applied.

In using two-way MANOVA, the dependent variables should neither be highly correlated among themselves nor should they be totally uncorrelated. The benefit of using MANOVA is that one can study the effect of each factor and their interaction on the whole basket of dependent variables. It makes sense to study the effect of two factors on the group of dependent variables like personality, employees, students, etc. Personality is the sum total of many variables like honesty, sincerity, and positivity; similarly, employees may be classified as male and female, whereas students may be categories as undergraduate and postgraduate. At times, it may be interesting to see the impact of two factors like age and education on the personality of an individual. Once any of this factor's effect on the personality as a whole is significant, then the two-way analysis of variance may be applied for each of these dimensions of the personality separately to see how these dimensions are affected individually by the age and education.

## Advantages of Two-Way ANOVA over One-Way ANOVA

Two-way ANOVA design is more efficient over one-way ANOVA because of the following four reasons:

1. Unlike one-way ANOVA, the two-way ANOVA design facilitates us to test the effect of two factors at the same time.
2. Since in two-way ANOVA variation is explained by two assignable causes, it reduces the error variance. Due to this fact, two-way ANOVA design is more efficient than one-way ANOVA.
3. In two-way ANOVA, one can test for independence of the factors provided there is more than one observation per cell. However, number of observations in each cell must be equal. On the other hand, in one-way ANOVA, one may have the unequal number of scores in each group.
4. Besides reducing the error variance, two-way ANOVA also reduces the computation as it includes several one-way ANOVA.

## Important Terminologies Used in Two-Way ANOVA

### *Factors*

Independent variables are usually known as *factors*. In two-way ANOVA, the effect of two factors is studied on certain criterion variable. Each of the two factors may have two or more levels. The degrees of freedom for each factor is equal to the number of levels in the factor minus one.

### *Treatment Groups*

The number of treatments in two-way ANOVA experiment is equal to the number of combinations of the levels of the two factors. For example, if the factor $A$ has 2 levels, $A_1$ and $A_2$, and the factor $B$ has 3 levels, $B_1$, $B_2$, and $B_3$, then there will be $2 \times 3 = 6$ different treatment groups $A_1B_1$, $A_1B_2$, $A_1B_3$, $A_2B_1$, $A_2B_2$, and $A_2B_3$.

### *Main Effect*

The main effect is the effect of one independent variable (or factor) on the dependent variable across all the levels of the other variable. The interaction is ignored for this part. Just the rows or just the columns are used, not mixed. This is the part which is similar to one-way analysis of variance. Each of the variances calculated to analyze the main effects (rows and columns) is like between variances. The degrees of freedom for the main effect are one less than its number of levels. For example, if the factor $A$ has $r$ levels and factor $B$ has $c$ levels, then the degrees of freedom for the factor $A$ and $B$ would be $r - 1$ and $c - 1$, respectively.

### *Interaction Effect*

The joint effect of two factors on the dependent variable is known as interaction effect. It can also be defined as the effect that one factor has on the other factor. The degrees of freedom for the interaction is the product of degrees of freedom of both the factors. If the factors $A$ and $B$ have levels $r$ and $c$, respectively, then the degrees of freedom for the interaction would be $(r - 1) \times (c - 1)$.

### *Within-Group Variation*

The within-group variation is the sum of squares within each treatment groups. In two-way ANOVA, all treatment groups must have the same sample size. The total number of treatment groups is the product of the number of levels for each factor. The within variance is equal to within variation divided by its degrees of freedom. The within group is also denoted as error. The within-group variation is often denoted by SSE.

## Two-Way ANOVA Model and Hypotheses Testing

Let us suppose that there are two factors $A$ and $B$ whose effects have to be tested on the criterion variable $X$, and let the factors $A$ and $B$ have levels $r$ and $c$, respectively, with $n$ units per cell, then these scores can be written as follows:

| | | Factor $B$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | .. | $j$ | .. | $c$ | |
| Factor $A$ | 1 | $X_{111}$ | | $X_{1j1}$ | | $X_{1c1}$ | |
| | | $X_{112}$ | .. | $X_{1j2}$ | .. | $X_{1c2}$ | |
| | | . | | . | | . | |
| | | $\frac{X_{11n}}{T_{11}}$ | | $\frac{X_{1jn}}{T_{1j}}$ | | $\frac{X_{1cn}}{T_{1c}}$ | $R_1$ |
| | $i$ | $X_{i11}$ | | $X_{ij1}$ | | $X_{ic1}$ | |
| | | $X_{i12}$ | .. | $X_{ij2}$ | .. | $X_{ic2}$ | |
| | | . | | . | | . | |
| | | $\frac{X_{i1n}}{T_{i1}}$ | | $\frac{X_{ijn}}{T_{ij}}$ | | $\frac{X_{icn}}{T_{ic}}$ | $R_i$ |
| | $r$ | $X_{r11}$ | | $X_{rj1}$ | | $X_{rc1}$ | |
| | | $X_{r12}$ | .. | $X_{rj2}$ | .. | $X_{rc2}$ | |
| | | . | | . | | . | |
| | | $\frac{X_{r1n}}{T_{r1}}$ | | $\frac{X_{rjn}}{T_{rj}}$ | | $\frac{X_{rcn}}{T_{rc}}$ | $R_r$ |
| | | $C_1$ | | $C_j$ | | $C_c$ | $G = \sum R_i = \sum C_j$ |

where

$X_{ijk}$ represents the $k$th score in the $(i,j)$th cell
$T_{ij}$ represents the total of all the $n$ scores in the $(i,j)$th cell
$G$ is the grand total of all the scores
$R_i$ is the total of all the scores in $i$th level of the factor $A$
$C_j$ is the total of all the scores in $j$th level of the factor $B$
$N$ is the total of all the scores and is equal to $r \times c \times n$

In two-way ANOVA, the total variability among the above-mentioned $N$ scores can be attributed to the variability due to row (or factor $A$), due to column (or factor $B$), due to interaction (row $\times$ column ($A \times B$)), and due to error. Thus, the total variability can be broken into the following four components:

$$\begin{aligned}\text{Total variability} &= \text{Variability due to Row(SSR)} + \text{Variability due to Column(SSC)} \\ &\quad + \text{Variability due to Interaction(SSI)} + \text{Variability due to Error} \\ \text{or} \qquad \text{TSS} &= \text{SSR} + \text{SSC} + \text{SSI} + \text{SSE} \qquad (8.1)\end{aligned}$$

**Remark**: SSE is the variability within group which was represented by $SS_w$ in one-way ANOVA.

The above-mentioned model is a two-way ANOVA model where it is assumed that the variability among the scores may be due to row factor, column factor, interaction due to row and column, and the error factor. Since the variation in the group has been explained by the two factors instead of one factor in one-way ANOVA, reduction of error variance is more in two-way ANOVA in comparison to that of one-way ANOVA design. This makes this design more efficient than one-way ANOVA. After developing the above-mentioned model of variability, it is required to test whether the effects of these factors are significant or not in explaining the variation in the data. The significance of these components is tested by means of using $F$-test.

(a) *Hypotheses construction*: The hypotheses which are being tested in two-way ANOVA are as follows:

(i) $H_0 : \mu_{A_1} = \mu_{A_2} = \ldots = \mu_{A_r}$
(The population means of all the levels of the factor $A$ are equal. This is like the one-way ANOVA for the row factor.)

(ii) $H_0 : \mu_{B_1} = \mu_{B_2} = \ldots = \mu_{B_c}$
(The population means of all the levels of the factor $B$ are equal. This is like the one-way ANOVA for the column factor.)

(iii) $H_0$: There is no interaction between factors $A$ and $B$.
(This is similar to performing a test for independence with contingency table.)

The above-mentioned null hypotheses are tested against the alternative hypothesis that at least one mean is different.

If the null hypotheses mentioned in (i) and (ii) are rejected, then it is concluded that the variability due to factor is significant, and it is inferred that the means of all the groups in that factor is not same. On the other hand, if the null hypothesis is failed to be rejected, one may draw the inference that all group means are equal. If the hypothesis mentioned in (iii) is rejected, then one may conclude that there is a significant interaction between the factors $A$ and $B$. In other words, it may be concluded that the pattern of differences of group means in factor $A$ is not the same in different levels of factor $B$. This fact shall be discussed in detail while solving the Example 8.1. Thus, if the effects of factors $A$ and $B$ having levels $m$ and $n$, respectively, are to be seen on the criterion variable, then the following steps will explain the procedure of testing the hypotheses:

(b) *Level of significance*: The level of significance may be chosen beforehand. Usually, it is taken as .05 or .01.

(c) *Statistical test*: The $F$-test is used to compare the variability between levels of a factor with that of variability within groups. If $F$-value is significant, it indicates that variability between levels of the factor groups is significantly higher than the variability within groups; in that case, the null hypothesis of no difference between the group means is rejected. Before computing $F$-value, it is required to compute the total sum of squares (TSS), sum of squares due to row factor

$A$ (SSR), sum of squares due to column factor $B$ (SSC), sum of squares due to interaction $A \times B$ (SSI), and sum of squares due to error (SSE).

(i) Total sum of squares (TSS): It represents the total variation present in the data set and is usually represented by TSS. It is defined as the sum of the squared deviations of all the scores in the data set from their grand mean. The TSS is computed by the following formula:

$$\text{TSS} = \sum_{i=1}^{r} \sum_{j=1}^{c} \sum_{k=1}^{n} \left( X_{ijk} - \frac{G}{N} \right)^2$$

after solving

$$= \sum_{i} \sum_{j} \sum_{k} X_{ijk}^2 - \frac{G^2}{N} \tag{8.2}$$

Since the degrees of freedom for the TSS are $N - 1$, therefore mean sum of squares is computed by dividing the TSS by $N - 1$.

(ii) Sum of squares due to row factor (SSR): It is the measure of variation between the row group means and is usually denoted by SSR. This is also known as the variation due to row factor (one of the assignable causes). The sum of squares due to row is computed as follows:

$$\text{SSR} = \sum_{i=1}^{r} \frac{R_i^2}{nc} - \frac{G^2}{N} \tag{8.3}$$

Since $r$ levels of row factor $A$ is compared in two-way ANOVA, the degrees of freedom for SSR are given by $r - 1$. Thus, mean sum of squares for row factor is obtained by dividing the SSR by its degrees of freedom $r - 1$.

(iii) Sum of squares due to column factor (SSC): It is the measure of variation between the column group means and is usually denoted by SSC. This explains the variation due to column factor (one of the assignable causes). The sum of squares due to column factor is computed as

$$\text{SSC} = \sum_{j=1}^{c} \frac{C_j^2}{nr} - \frac{G^2}{N} \tag{8.4}$$

The degrees of freedom for SSC are given by $c - 1$ as there are $c$ column group means that are required to be compared. The mean sum of square for column factor is obtained by dividing SSC by $c - 1$.

(iv) Sum of squares due to interaction (SSI): It is the measure of variation due to the interaction of both the factors $A$ and $B$. It facilitates us to test whether or not there is a significant interaction effect. The sum of squares due to interaction is usually denoted by SSI. The SSI is computed as follows:

$$\text{SSI} = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{T_{ij}^2}{n} - \frac{G^2}{N} - \text{SSR} - \text{SSC} \tag{8.5}$$

The degrees of freedom for interaction are obtained by $(r-1)\times(c-1)$. The mean sum of squares due to interaction is obtained by dividing SSI by its degrees of freedom $(r-1)\times(c-1)$.

(v) *Sum of squares due to error (SSE)*: It is the measure of unexplained portion of the variation in the data and is denoted by SSE. This error is minimized in two-way ANOVA model because here the variability in the data is explained by the two factors besides interaction, in contrast to that of one factor in one-way ANOVA model. The degrees of freedom are given by $N - rc$. The SSE is computed as

$$\text{SSE} = \text{TSS} - \text{SSR} - \text{SSC} - \text{SSI} \tag{8.6}$$

The mean sum of square due to error is obtained by dividing SSE by its degrees of freedom $N - rc$.

(vi) ANOVA table: This is a summary table showing different sum of squares and mean sum of squares for all the components of variation. The computation of $F$-values is shown in this table. This table is popularly known as two-way ANOVA table. After computing all the sum of squares, the ANOVA table is prepared for further analysis which is shown as follows:

(vii) *F-statistic*: Under the normality assumptions, the $F$-value obtained in the ANOVA table, say, for row, follows a $F$-distribution with $(r-1, N-r)$ degrees of freedom. $F$-statistic is computed for each source of variation.

Two-way ANOVA table

| Sources of variation | SS | df | MSS | $F$ |
|---|---|---|---|---|
| Main effect $A$ (row) | SSR | $r-1$ | $S_R^2 = \text{SSR}/(r-1)$ | $F$ for row effect $= S_R^2/S_E^2$ |
| Main effect $B$ (column) | SSC | $c-1$ | $S_C^2 = \text{SSC}/(c-1)$ | $F$ for column effect $= S_C^2/S_E^2$ |
| Interaction effect $(A \times B)$ | SSI | $(r-1)\times(c-1)$ | $S_I^2 = \text{SSI}/(r-1)\times(c-1)$ | $F$ for interaction effect $= S_I^2/S_E^2$ |
| Error | SSE | $N-rc$ | $S_E^2 = \text{SSE}/(N-rc)$ | |
| Total | TSS | $N-1$ | | |

**Remark**: *The t*otal sum of squares is additive in nature

This test statistic $F$ is used to test the null hypothesis of no difference among the group means.

(d) *Decision criteria*: The tabulated value of $F$ at .05 or .01 level of significance with different degrees of freedom may be obtained from Tables A.4 or A.5, respectively in the Appendix. If the calculated value of $F$ is greater than the tabulated $F$, the null hypothesis is rejected, and in that case, at least one of the means is different than others. Since ANOVA does not tell us where the difference lies, post hoc test is used to get the clear picture. There are several post hoc tests which can be used but least significant difference (LSD) test is generally used in equal sample sizes. However, one may use other post hoc tests like Scheffe's, Tukey, Bonferroni, or Duncan as well.

In all the post hoc tests, a critical difference is computed at a particular level of significance, and if the difference of any pair of observed means is higher than the critical difference, it is inferred that the mean difference is significant otherwise insignificant. By comparing all pair of group means, conclusion is drawn as to which group mean is the highest. The detail procedure of applying the post hoc test has been discussed in the solved Example 8.1.

In LSD test, the critical difference (CD) is computed as

$$\text{CD} = t_{.05}(N - rc) \times \sqrt{\frac{2\text{MSSE}}{n}} \tag{8.7}$$

where the symbols have their usual meanings.

This critical difference (CD) is used for comparing differences in all the pair of means.

The SPSS output provides the significance value ($p$-value) for each of the $F$-statistics computed in two-way ANOVA table. If $p$-value is less than .05, $F$ would be significant. Post hoc test for comparing means is applied for those factors and interaction whose $F$-value is significant. The SPSS also provides $p$-values (significant value) for each pair of means in row, column, and interaction to test the significance of difference between them. If $p$-value for any pair of means is less than .05, it is concluded that means are significantly different otherwise not.

## *Assumptions in Two-Way Analysis of Variance*

Following assumptions need to be satisfied while using the two-way ANOVA design:

(a) The population from which the samples are drawn must be normally distributed.
(b) The samples must be independent.
(c) The population variances must be equal.
(d) The sample size must be same in each cell.

## Situation Where Two-Way ANOVA Can Be Used

In two-way ANOVA, we investigate the effect of main effects along with the interaction effect of two factors on dependent variable. The following situation shall develop an insight among the researchers to appreciate the use of this analysis.

Consider a situation where a mobile company is interested to examine the effect of gender and age of the customers on the frequency of short messaging service (sms) sent per week. Each person may be classified according to gender (men and women) and age category (16–25, 26–35, and 36–45 years). Thus, there will be six groups, one for each combination of gender and age. Random sample of equal size in each group may be drawn, and each person may be asked about the number of sms he or she sends per week. In this situation, there are three main research questions that can be answered:

(a) Whether the number of sms sent depends on gender
(b) Whether the number of sms sent depends on age
(c) Whether the number of sms sent depends on gender differently for different age categories of age, and vice versa

All these questions can be answered through testing of hypothesis in two-way ANOVA model. The first two questions simply ask whether sending sms depends on age and gender. On the other hand, the third question asks whether sending sms depends on gender differently for people in different age category, or whether sending sms depends on age differently for men and women. This is because one may think that men send more sms than women in 18–25 years age category, but women send more sms than men in 26–55 years age category. After applying the two-way ANOVA model, one may be able to explain the above-mentioned research questions in the following manner:

whether the factor gender has a significant impact on the number of sms sent irrespective of their age categories. And if it is so, one may come to know whether men send more sms than women or vice versa, irrespective of their age categories.

Similarly, one can test whether the factor age has a significant impact on the number of sms sent irrespective of gender. And if age factor is significant, one can know that in which age category people send more sms irrespective of their gender.

The most important aspect of two-way ANOVA is to know the presence of interaction effect of gender and age on sending the sms. One may test whether these two factors, that is, gender and age, are independent to each other in deciding the number of sms sent. The interaction analysis allows us to compare the average sms sent in different age categories in each of the men and women groups separately. Similarly, it also provides the comparison of the average sms sent by the men and women in different age categories separately.

The information provided through this analysis may be used by the marketing department to chalk out their promotional strategy for men and women separately in different age categories for the mobile users.

**Table 8.1** Data on toothpaste sales in different incentive groups

| | | Incentive | | |
|---|---|---|---|---|
| | | I | II | III |
| *Gender of sales manager* | Male | 15 | 15 | 18 |
| | | 13 | 14 | 16 |
| | | 14 | 10 | 10 |
| | | 11 | 9 | 12 |
| | | 9 | 8 | 16 |
| | Female | 10 | 13 | 11 |
| | | 7 | 14 | 10 |
| | | 9 | 16 | 13 |
| | | 7 | 17 | 12 |
| | | 8 | 14 | 11 |

**Example 8.1** An experiment was conducted by a utility company to study the effects of three sales incentives – toothpaste with 20% extra in the same price (incentive I), toothpaste along with traveling toothpaste (incentive II), and toothpaste along with bath soap (incentive III) – and to study the effects of gender of the sales manager (male and female) on the daily sales of a toothpaste. The daily sale of the toothpaste was recorded in each of the six groups for five continuous days. The data so obtained are shown in Table 8.1.

Apply two-way ANOVA and discuss your findings at 5% level.

*Solution* Here, the two factors that need to be investigated are gender and incentive and

Number of row factor (gender) $= r = 2$
Number of column factor (incentive) $= c = 3$
Number of scores in each cell $= n = 5$
Total number of scores $= N = n \times c \times r = 30$

In order to apply the two-way ANOVA, the following steps shall be performed:

(a) *Hypotheses construction*: The hypotheses that need to be tested are:

(i) $H_0 : \mu_{\text{Male}} = \mu_{\text{Female}}$
(The sales performance given by male and female sales manager is equal irrespective of the incentives.)

(ii) $H_0 : \mu_{\text{Incentive\_I}} = \mu_{\text{Incentive\_II}} = \mu_{\text{Incentive\_III}}$
(The sales performance under the three incentive schemes is equal irrespective of the gender of the sales manager.)

(iii) $H_0$: There is no interaction between the gender of the sales manager and the types of sales incentives.
(It is immaterial whether male or female are offering the sales incentives.)

**Table 8.2** Computation for two-way ANOVA

| | | Incentive | | | | |
|---|---|---|---|---|---|---|
| | | I | II | III | Row total ($R_i$) | Row mean |
| *Gender of sales manager* | Male | 15 | 15 | 18 | | |
| | | 13 | 14 | 16 | | |
| | | 14 | 10 | 10 | | |
| | | 11 | 9 | 12 | | |
| | | 9 | 8 | 16 | | |
| | | $T_{11} = 62$ | $T_{12} = 56$ | $T_{13} = 72$ | $R_1 = 190$ | 12.67 |
| | Female | 10 | 13 | 11 | | |
| | | 7 | 14 | 10 | | |
| | | 9 | 16 | 13 | | |
| | | 7 | 17 | 12 | | |
| | | 8 | 14 | 11 | | |
| | | $T_{21} = 41$ | $T_{22} = 74$ | $T_{23} = 57$ | $R_2 = 172$ | 11.47 |
| | Column total | $C_1 = 103$ | $C_2 = 130$ | $C_3 = 129$ | $G = 362$ | |
| | Column mean | 10.3 | 13.0 | 12.9 | | |

(b) *Level of significance*: .05

(c) *Test statistic*: In order to test the hypotheses, $F$-statistic shall be computed for row factor, column factor, and interaction in order to test their significance. After computing different sum of squares, the ANOVA table shall be prepared. The following computation shall be done for completing the ANOVA table:

*Computation*

Before computing components of different sum of squares, let us first compute the row, column, and cell total along with the grand total (Table 8.2).

1. Raw sum of square (sum of squares of all the scores in the study)

$$\begin{aligned}
\text{RSS} &= \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n} X_{ijk}^2 \\
&= \left(15^2 + 13^2 + \ldots 9^2\right) + \left(15^2 + 14^2 + \ldots 8^2\right) + \left(18^2 + 16^2 + \ldots 16^2\right) \\
&\quad + \left(10^2 + 7^2 + \ldots 8^2\right) + \left(13^2 + 14^2 + \ldots 14^2\right) + \left(11^2 + 10^2 + \ldots 11^2\right) \\
&= 792 + 666 + 1080 + 343 + 1106 + 655 \\
&= 4642
\end{aligned}$$

2. Correction factor = CF

$$= \frac{G^2}{N} = \frac{362^2}{30} = 4368.13$$

3. Total sum of squares = TSS

$$= \sum_i \sum_j \sum_k X_{ijk}^2 - \frac{G^2}{N} = \text{RSS} - \text{CF}$$

$$= 4642 - 4368.13 = 273.87$$

4. Sum of squares due to row factor(gender) = SSR

$$= \sum_{i=1}^{r} \frac{R_i^2}{nc} - \frac{G^2}{N} = \frac{190^2 + 172^2}{5 \times 3} - 4368.13$$

$$= 10.80$$

5. Sum of squares due to column factor(incentives) = SSC

$$= \sum_{j=1}^{c} \frac{C_j^2}{nr} - \frac{G^2}{N}$$

$$= \frac{103^2 + 130^2 + 129^2}{5 \times 2} - 4368.13 = 46.87$$

6. Sum of squares due to interaction = SSI

$$= \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{T_{ij}^2}{n} - \frac{G^2}{N} - \text{SSR} - \text{SSC}$$

$$= \frac{62^2 + 56^2 + 72^2 + 41^2 + 74^2 + 57^2}{5} - 4368.13 - 10.80 - 46.87$$

$$= 4514 - 4425.8 = 88.20$$

7. Sum of squares due to error = SSE

$$= \text{TSS} - \text{SSR} - \text{SSC} - \text{SSI}$$

$$= 273.87 - 10.80 - 46.87 - 88.20$$

$$= 128$$

Tabulated value of $F$ can be seen from Table A.4 in the Appendix. Thus, from Table A.4, the value of $F_{.05}$ (1,24) = 4.26 and $F_{.05}$ (2,24) = 3.40.

In Table 8.3, since the calculated value of $F$ for incentives and interaction is greater than their corresponding tabulated value of $F$, these two $F$-ratios are significant. However, $F$-value for gender is not significant.

**Table 8.3** Two-way ANOVA table for the data on sales of toothpaste

| Source of variation | Sum of squares (SS) | df | Mean sum of squares (MSS) | $F$ | Tab. $F$ at 5% level |
|---|---|---|---|---|---|
| Gender | 10.80 | $r - 1 = 1$ | 10.80 | 2.03 | 4.26 |
| Incentives | 46.87 | $c - 1 = 2$ | 23.44 | 4.398* | 3.40 |
| Interaction (gender* incentives) | 88.20 | $(r - 1) \times (c - 1) = 2$ | 44.10 | 8.27* | 3.40 |
| Error | 128.00 | $N - rc = 24$ | 5.33 | | |
| Corrected total | 273.87 | $N - 1 = 29$ | | | |

*Significant at 5% level

**Table 8.4** Mean sale in different incentive groups (both gender combined)

| Incentives | | | CD at 5% level |
|---|---|---|---|
| II | III | I | |
| 13.0 | 12.9 | 10.3 | 1.48 |

"_____/" Denotes no difference between the means at .05 level

Post hoc test shall be used to further analyze the column factor (incentives) and the interaction effect.

*Post Hoc Test for Column Factor (Incentives)*
LSD test shall be used to find the critical difference for comparing the means of the groups in the column factor. The critical difference (CD) is given by

$$\begin{aligned}\text{CD at .05 significance level} &= t_{.05}(24) \times \sqrt{\frac{2\text{MSSE}}{n \times r}} \\ &= 2.064 \times \sqrt{\frac{2 \times 5.33}{5 \times 2}} \text{[From Table A.2 in the Appendix, } t_{.05}(24) = 2.064] \\ &= 2.064 \times 1.03 \\ &= 2.13\end{aligned}$$

Table 8.4 shows that the mean difference between II and III incentive groups is less than the critical difference (=2.13); hence, there is no difference between these two incentive groups. To show this, a line has been drawn below these two group means. On the other hand, there is a significant difference between the means of II and I as well as III and I incentive groups. Thus, it may be concluded that the II and III incentives are equally effective and better than Ist incentive in enhancing the sale of the toothpaste irrespective of the gender of the sales manager.

**Table 8.5** Comparison of mean sale of toothpaste between male and female groups in each of the three incentive groups

| Gender | | | | |
|---|---|---|---|---|
| Incentives | Male | Female | Mean *diff.* | CD at 5% level |
| I | 12.4 | 8.2 | 4.2* | 2.10 |
| II | 11.2 | 14.8 | 3.6* | 2.10 |
| III | 14.4 | 11.4 | 3.0* | 2.10 |

*Significant at 5% level

**Table 8.6** Comparison of mean sale of toothpaste among different incentive groups in each of the two gender groups

| Gender | Incentives | | | CD at 5% level |
|---|---|---|---|---|
| Male | 14.4 (III) | 12.4 (I) | 11.2 (II) | 2.10 |
| Female | 14.8 (II) | 11.4 (III) | 8.2 (I) | 2.10 |

**Remark**: Arrange means of the groups in descending order

*Post Hoc Test for Interaction (Gender × Incentives)*

Since interaction effect is significant, comparison shall be made among the means of each incentive groups in each gender. Similarly, mean comparison shall also be made between male and female groups in each of the incentive groups. Since the cell size is similar, the critical difference for row comparison in each column and for column comparison in each row shall be same. The CD using LSD test shall be obtained by

$$\begin{aligned}\text{CD at .05 significance level} &= t_{.05}(24) \times \sqrt{\frac{2\text{MSSE}}{n}} \\ &= 2.064 \times \sqrt{\frac{2 \times 5.33}{5}} \\ &= 2.064 \times 1.46 \\ &= 3.01\end{aligned}$$

Table 8.5 provides the post hoc comparison of means of male and female groups in each of the three incentive groups. Since the mean difference between male and female in each of the three incentive groups is higher than the critical difference, these differences are significant at 5% level. Further, it may be concluded that the average sales of male group in I and III incentives groups are higher than that of female group whereas average sales of female is higher than that of male in II incentive group.

Table 8.6 shows the comparison of different incentive groups in each of the gender group. It can be seen from this table that in male section, average sales are significantly different in III and II incentive groups, whereas average sales in the III and I incentive groups as well as I and II incentive groups are same. On the other hand, in female section, the average sales in all the three incentive groups are significantly different from each other.

**Table 8.7** Data on sale of chocolates of different flavours and colours

| Chocolate color | | | |
|---|---|---|---|
| Sweetness | White | Milk | Dark |
| Semisweet chocolate | 25 | 20 | 35 |
| | 20 | 15 | 32 |
| | 22 | 17 | 31 |
| | 28 | 21 | 42 |
| | 23 | 25 | 30 |
| Bittersweet chocolate | 28 | 32 | 15 |
| | 24 | 35 | 22 |
| | 32 | 37 | 25 |
| | 32 | 38 | 12 |
| | 23 | 29 | 20 |
| Unsweetened chocolate | 26 | 20 | 15 |
| | 24 | 15 | 12 |
| | 32 | 17 | 10 |
| | 26 | 21 | 22 |
| | 31 | 25 | 13 |

Thus, on the basis of the given data, the result suggests that IIIrd incentive should be preferred if it is promoted by the male sales manager whereas the sales of the toothpaste would increase if it is promoted by the female sales manager using IInd incentive.

## Solved Example of Two-Way ANOVA Using SPSS

**Example 8.2** A chocolate manufacturing company wanted to know the impact of color and sweetness of its chocolates on buying decision of the customers. Data in Table 8.7 shows the number of units sold per day in a city for five consecutive days. Apply two-way analysis of variance to see whether the factors sweetness and color have significant effect on the sale of chocolates. Also test the significance of interaction between these two factors and discuss your findings at 5% level.

*Solution* Here, two main factors, namely, sweetness and color as well as interaction between sweetness and color, need to be investigated. Thus, following three hypotheses shall be tested:

(i) $H_0 : \mu_{Semi\_Sweet} = \mu_{Bitter\_Sweet} = \mu_{Un\_Sweet}$
(The sales of semisweet, bittersweet, and unsweetened chocolates are same irrespective of the color of the chocolate.)

(ii) $H_0 : \mu_{White} = \mu_{Milk} = \mu_{Dark}$
(The sales of white, milk, and dark chocolates are same irrespective of the sweetness of the chocolate.)

(iii) $H_0$: There is no interaction between the sweetness and color of the chocolate.

(It is immaterial whether any color of the chocolates is semisweet, bittersweet, or unsweetened.)

The SPSS output for two-way analysis of variance provides $F$-values for the sweetness factor, color factor, and interaction (sweetness × color) along with their significant values ($p$-values). The $F$-values for these factors and interaction shall be significant if their $p$-values are less than .05. For any factor or interaction if the $F$-value is significant, then a post hoc test shall be applied to compare the paired means. SPSS provides option to choose any of the post hoc tests for comparing the significance of mean difference.

In this example, since the sample sizes are equal, LSD test shall be used as a post hoc test for comparing group means. The SPSS output provides the significance values ($p$-values) for the difference of each pair of group means. The significance of difference between pair of group means is tested by using these $p$-values rather than computing the critical differences as is done in case of solving two-way ANOVA manually.

## *Computation in Two-Way ANOVA Using SPSS*

(a) ***Preparing data file***

(i) *Starting the SPSS*: Use the below-mentioned command sequence to start SPSS on your system:

**Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

After clicking the **Type in Data**, you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables*: There are three variables in this example, namely, sales, sweetness, and color. Since sales were measured on ratio scale, it has been defined as a scale variable, whereas sweetness and color were measured on nominal scale hence they have been defined as nominal variables. The procedure of defining the variables and their characteristics in SPSS is as follows:

1. Click **Variable View** to define variables and their properties.
2. Write short name of the variables as *Sale*, *Sweetness*, and *Colour* under the column heading **Name**.
3. Under the column heading **Label**, define full name of these variables as *Chocolate_Sale*, *Chocolate_Sweetness*, and *Chocolate_Colour*. Alternate names may also be chosen for describing these variables.
4. Under the column heading **Measure**, select the option "Scale" for the variable *Sale* and "Nominal" for the variables *Sweetness* and *Colour*.

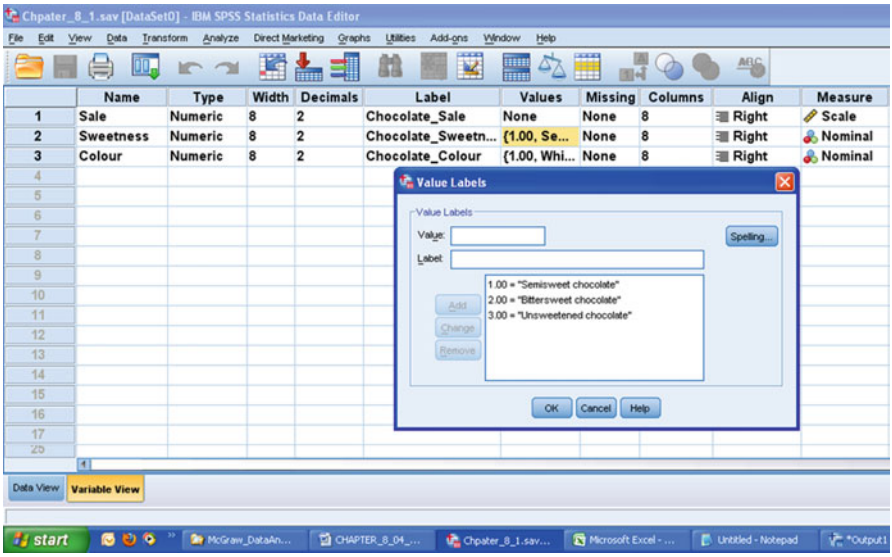


**Fig. 8.1** Defining variables along with their characteristics

5. For the variable *Sweetness*, double-click the cell under the column **Values** and add following values to different labels:

| Value | Label |
|---|---|
| 1 | Semisweet chocolate |
| 2 | Bittersweet chocolate |
| 3 | Unsweetened chocolate |

6. Similarly, for the variable *Colour*, add the following values to different labels:

| Value | Label |
|---|---|
| 1 | White |
| 2 | Milk |
| 3 | Dark |

7. Use default entries in rest of the columns.
   After defining the variables in variable view, the screen shall look like Fig. 8.1.

(iii) *Entering data*: After defining these variables in the **Variable View**, click **Data View** on the left bottom of the screen to enter the data. One should note the procedure of data feeding carefully in this example. First 15 sales data of semisweet chocolate of Table 8.6 are entered in the column of *Sales* after which next 15 data of bittersweet chocolate are entered in the same column, and thereafter the remaining 15 data of unsweetened chocolate are entered in the same column. Thus, in the column of *Sales* variable,

there will be 45 data. Under the column *Sweetness*, first 15 scores are entered as 1 (denotes semisweet chocolate), the next 15 scores are entered as 2 (denotes bittersweet chocolate), and the remaining 15 scores are entered as 3 (denotes unsweetened chocolate). Under the column *Colour*, first five scores are entered as 1 (denotes white color chocolate), next five scores as 2 (denotes milk color chocolate), and subsequent five scores as 3 (denotes dark color chocolate). These 15 data belong to semisweet chocolate group. Similarly, next 15 scores of bittersweet chocolate group and unsweetened chocolate groups can be just the repetition of the semisweet chocolate group. Thus, after feeding the first 15 data in the *Colour* column, repeat this set of 15 data twice in the same column.

After entering the data, the screen will look like Fig. 8.2. Save the data file in the desired location before further processing.

(b) **SPSS *commands for two-way ANOVA***

After entering the data in data view as per above-mentioned scheme, follow the below-mentioned steps for two-way analysis of variance:

(i) *Initiating the SPSS commands for two-way ANOVA*: In **Data View**, click the following commands in sequence:

**Analyze → General Linear Model → Univariate**

The screen shall look like Fig. 8.3.

(ii) *Selecting variables for two-way ANOVA*: After clicking the **Univariate** option, you will be taken to the next screen for selecting variables. Select the variables *Chocolate_Sale* from left panel to the "Dependent variable" section of the right panel. Similarly, select the variables *Chocolate_Sweetness* and *Chocolate_Colour* from left panel to the "Fixed Factor(s)" section of the right panel. The screen will look like Fig. 8.4.

(iii) *Selecting the option for computation*: After selecting the variables, various options need to be defined for generating the output in two-way ANOVA. Do the following:

- Click the tag **Post Hoc** in the screen shown in Fig. 8.4. Then,
  - Select the factors *Sweetness* and *Colour* from the left panel to the "Post Hoc Tests for" panel on the right side by using the arrow key.
  - Check the option "LSD." LSD test is selected as a post hoc because the sample sizes are equal in each cell.

The screen will look like Fig. 8.5.

- Click ***Continue***. This will again take you back to the screen as shown in Fig. 8.4.
  - Now click the tag **Options** on the screen and do the following steps:
    - Check the option "Descriptive."

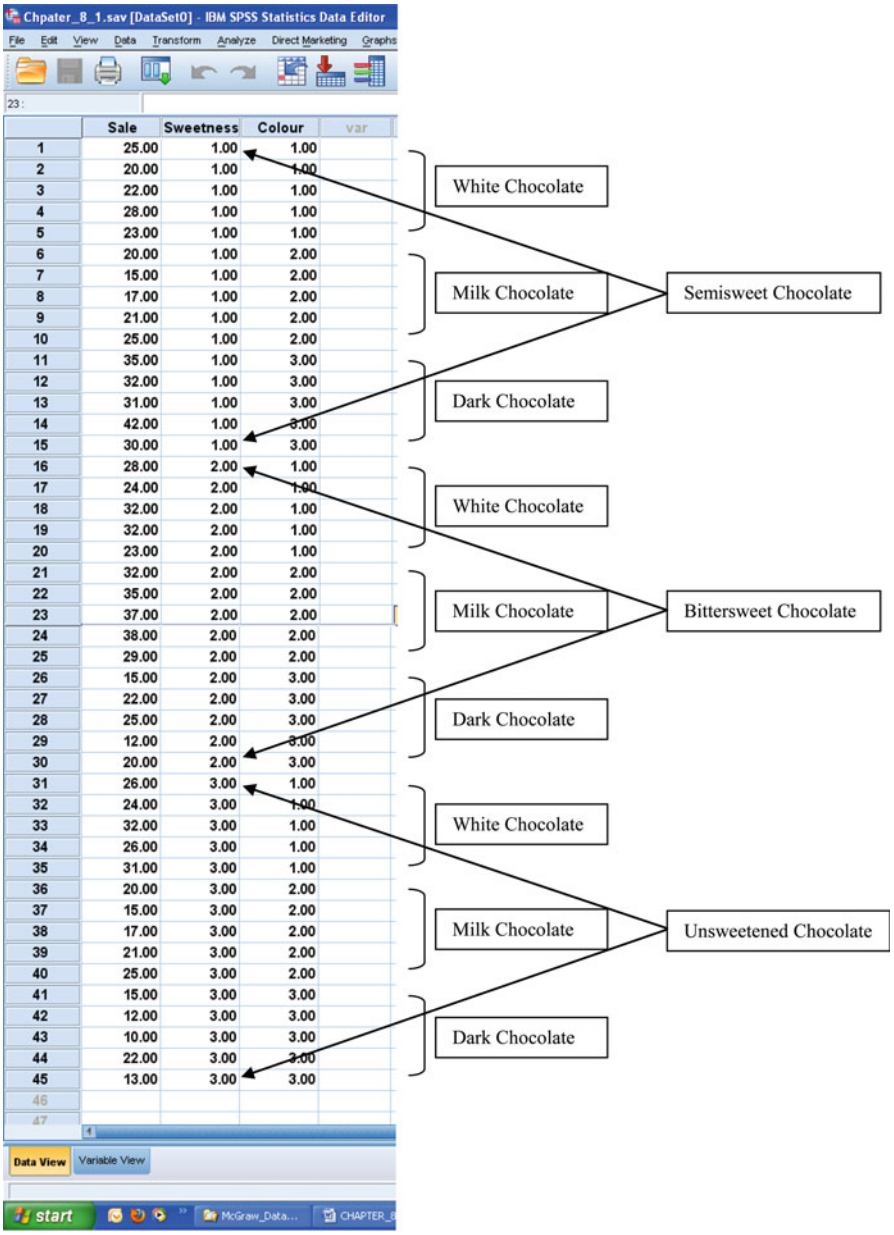


**Fig. 8.2** Screen showing data entry for different variables in the data view

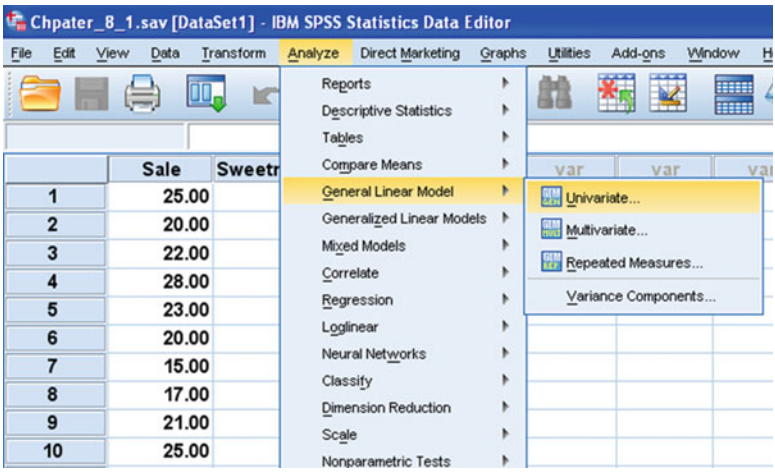


**Fig. 8.3** Screen showing SPSS commands for two-way ANOVA

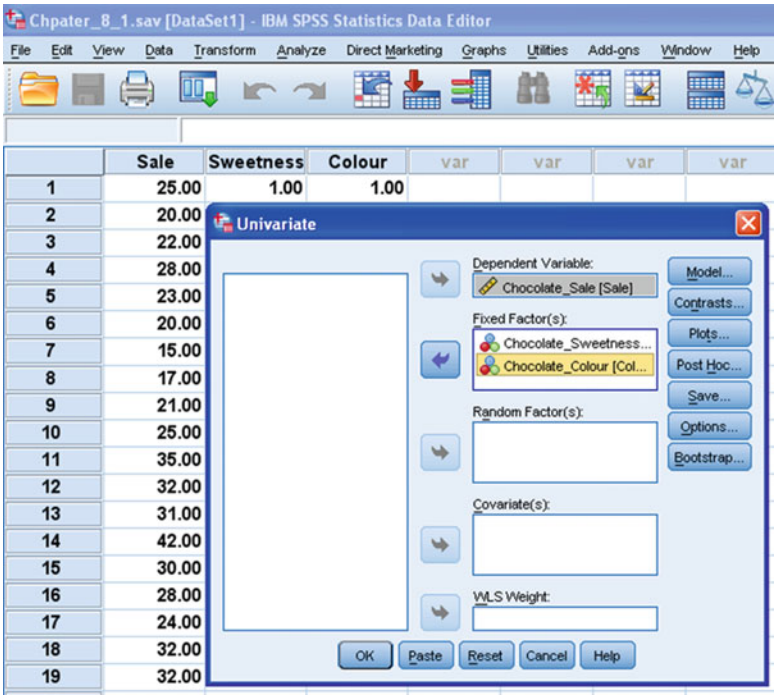


**Fig. 8.4** Screen showing selection of variables for two-way ANOVA

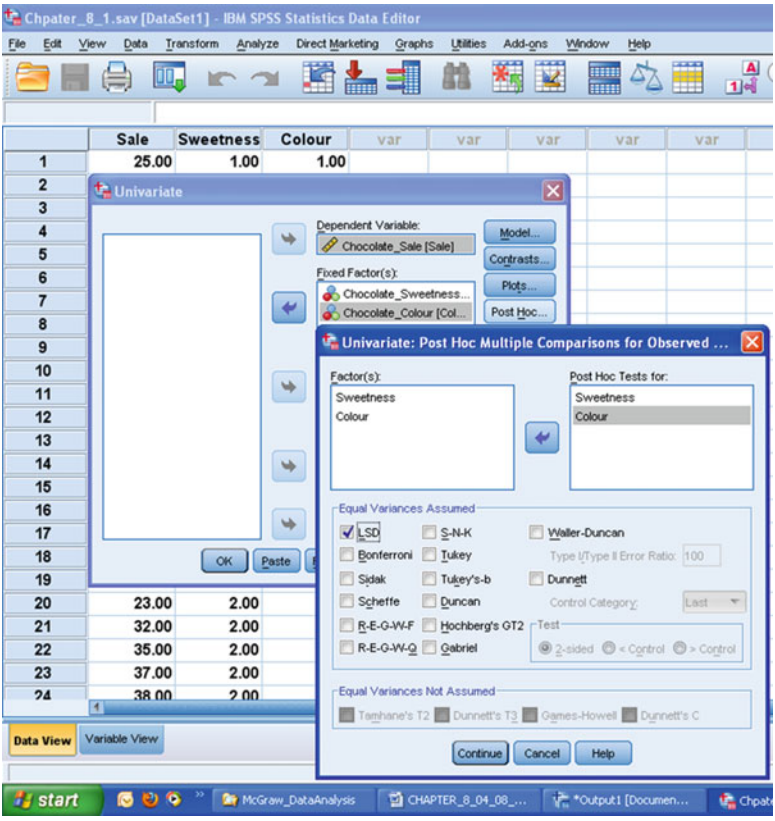


**Fig. 8.5** Screen showing options for post hoc test

- Select the variables *OVERALL, Sweetness, Colour*, and *Sweetness* × *Colour* from the left panel and bring them into the "Display Means for" section of the right panel.
- Check the option "Compare main effects."
- Ensure the value of Significance level as .05 in the box. The screen for these options shall look like as shown in Fig. 8.6.
- Click ***Continue*** to go back to the main screen.

After selecting the options, the screen shown in Fig. 8.4 shall be restored.

- Press **OK**.

(c) ***Getting the output***

After clicking **OK** in the screen shown in Fig. 8.4, various outputs shall be generated in the output window. Relevant outputs may be selected by using

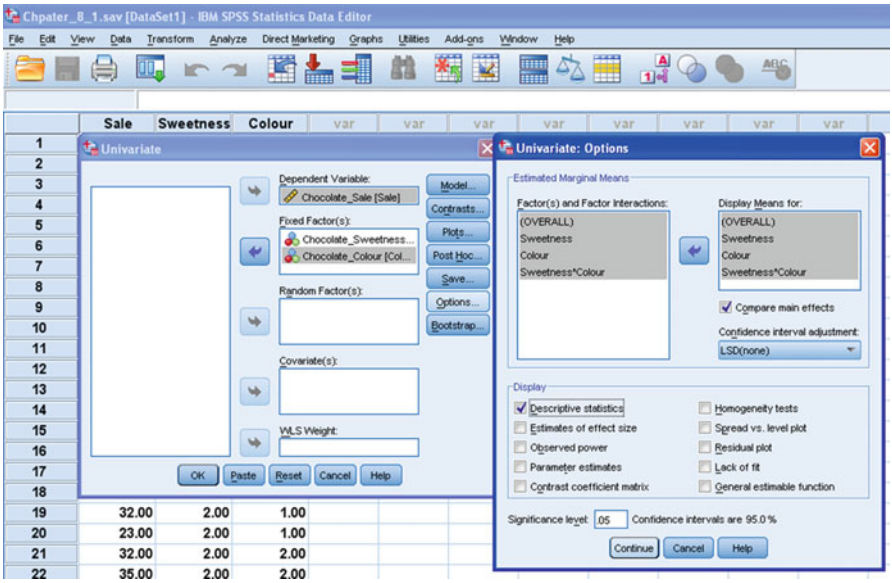


**Fig. 8.6** Screen showing options for descriptive statistics and comparison of main effects

right click of the mouse and may be copied in the word file. Here, the following outputs shall be selected:

1. Descriptive statistics
2. Two-way ANOVA table
3. Pairwise comparisons of sweetness groups (all color groups combined)
4. Pairwise comparisons of different color groups (all sweetness groups combined)

In this example, all the identified outputs so generated by the SPSS will look like as shown in Tables 8.8, 8.9, 8.10, and 8.11.
In order to interpret the findings, these outputs may be rearranged so that it can directly be used in your project. These rearranged formatted tables have been shown under the heading "**Model Way of Writing the Results**" in the next section.

## *Model Way of Writing the Results of Two-Way ANOVA and Its Interpretations*

The outputs so generated in the SPSS may be presented in user-friendly format by selecting the relevant details from Tables 8.8, 8.9, 8.10, and 8.11 and making some slight modifications. Further, if the interaction is significant, it is not possible to compare the cell means by using the outputs of SPSS. However, critical difference can be computed by using these outputs for testing the significance of mean

**Table 8.8** Descriptive statistics

| Sweetness | Color | Mean | Std. dev. | *N* |
|---|---|---|---|---|
| Semisweet chocolate | White | 23.60 | 3.04959 | 5 |
| | Milk | 19.60 | 3.84708 | 5 |
| | Dark | 34.00 | 4.84768 | 5 |
| | Total | 25.73 | 7.28469 | 15 |
| Bittersweet chocolate | White | 27.80 | 4.26615 | 5 |
| | Milk | 34.20 | 3.70135 | 5 |
| | Dark | 18.80 | 5.26308 | 5 |
| | Total | 26.93 | 7.73181 | 15 |
| Unsweetened chocolate | White | 27.80 | 3.49285 | 5 |
| | Milk | 19.60 | 3.84708 | 5 |
| | Dark | 14.40 | 4.61519 | 5 |
| | Total | 20.60 | 6.81175 | 15 |
| Total | White | 26.40 | 3.94244 | 15 |
| | Milk | 24.47 | 7.94505 | 15 |
| | Dark | 22.40 | 9.81107 | 15 |
| | Total | 24.42 | 7.64106 | 45 |

Dependent variable: Chocolate_Sale

**Table 8.9** Two-way ANOVA table generated by the SPSS

| Source | Type III sum | df | Mean square of squares | *F* | Sig. |
|---|---|---|---|---|---|
| Corrected model | 1946.978[a] | 8 | 243.372 | 14.086 | .000 |
| Intercept | 26840.022 | 1 | 26840.022 | 1553.44 | .000 |
| Sweetness | 339.511 | 2 | 169.756 | 9.825 | .000 |
| Color | 120.044 | 2 | 60.022 | 3.474 | .042 |
| Sweetness * color | 1487.422 | 4 | 371.856 | 21.522 | .000 |
| Error | 622.000 | 36 | 17.278 | | |
| Total | 29409.000 | 45 | | | |
| Corrected total | 2568.977 | 44 | | | |

Dependent variable: Chocolate_Sale
[a]*R* squared = .758 (adjusted *R* squared = .704)

difference among different groups. The procedure of comparing group means has been discussed later in this section.

The first important table consisting *F*-values for the factors and interaction can be reproduced by deleting some of the contents of Table 8.9. The information so reduced is shown in Table 8.12.

The *p*-values for the Sweetness, Color, and Interaction (Sweetness × Color) in Table 8.12 are less than .05; hence, all the three *F*-values are significant at 5% level. Thus, the null hypothesis for the Sweetness factor, Color factor, and Interaction (Sweetness × Color) may be rejected at .05 level of significance. Now the post hoc comparison analysis shall be done for these factors and interaction. These analyses are shown below.

**Table 8.10** Pairwise comparison of different sweetness groups

| (*I*) Chocolate_Sweetness | (*J*) Chocolate_Sweetness | Mean diff. (*I* – *J*) | Std. error | Sig.[a] (*p*-value) | 95% Confidence interval for difference[a] | |
|---|---|---|---|---|---|---|
| | | | | | Lower bound | Upper bound |
| Semisweet chocolate | Bittersweet chocolate | −1.200 | 1.518 | .434 | −4.278 | 1.878 |
| | Unsweetened chocolate | 5.133* | 1.518 | .002 | 2.055 | 8.212 |
| Bittersweet chocolate | Semisweet chocolate | 1.200 | 1.518 | .434 | −1.878 | 4.278 |
| | Unsweetened chocolate | 6.333* | 1.518 | .000 | 3.255 | 9.412 |
| Unsweetened chocolate | Semisweet chocolate | −5.133* | 1.518 | .002 | −8.212 | −2.055 |
| | Bittersweet chocolate | −6.333* | 1.518 | .000 | −9.412 | −3.255 |

Dependent variable: Chocolate_Sale
Based on estimated marginal means
*The mean difference is significant at the .05 level
[a]Adjustment for multiple comparisons: least significant difference (equivalent to no adjustments)

**Table 8.11** Pairwise comparison of different color groups

| (*I*) Chocolate_Colour | (*J*) Chocolate_Colour | Mean diff. (*I* – *J*) | Std. error | Sig.[a] | 95% Confidence interval for difference[a] | |
|---|---|---|---|---|---|---|
| | | | | | Lower bound | Upper bound |
| White | Milk | 1.933 | 1.518 | .211 | −1.145 | 5.012 |
| | Dark | 4.000* | 1.518 | .012 | .922 | 7.078 |
| Milk | White | −1.933 | 1.518 | .211 | −5.012 | 1.145 |
| | Dark | 2.067 | 1.518 | .182 | −1.012 | 5.145 |
| Dark | White | −4.000* | 1.518 | .012 | −7.078 | -.922 |
| | Milk | −2.067 | 1.518 | .182 | −5.145 | 1.012 |

Dependent variable: Chocolate_Sale
Based on estimated marginal means
*The mean difference is significant at the .05 level
[a]Adjustment for multiple comparisons: least significant difference (equivalent to no adjustments)

**Table 8.12** Two-way ANOVA table for the data on chocolate sale

| Source of variation | Sum of squares (SS) | df | Mean sum of squares (MSS) | *F* | *p*-value (sig.) |
|---|---|---|---|---|---|
| Sweetness | 339.51 | 2 | 169.76 | 9.83 | .000 |
| Color | 120.04 | 2 | 60.02 | 3.47 | .042 |
| Sweetness × color | 1,487.42 | 4 | 371.86 | 21.52 | .000 |
| Error | 622.00 | 36 | 17.28 | | |
| Corrected total | 2,568.977 | 44 | | | |

**Table 8.13** Comparison of mean chocolate sale in all the three sweetness groups (all colors combined)

| Bittersweet chocolate | Semisweet chocolate | Unsweetened chocolate | CD at 5% level |
|---|---|---|---|
| 26.93 | 25.73 | 20.60 | 3.08 |

"_____/"Denotes no difference between the means at 5% level
**Remark**: Arrange means of the group in descending order

### Row (Sweetness) Analysis

For row analysis, critical difference has been obtained by using the LSD test. The value of "$t$" at .05 level and 36 df (error degrees of freedom) can be obtained from Table A.2 in Appendix.

Thus,

$$\text{CD for row} = t_{.05}(36) \times \sqrt{\frac{2(\text{MSS})_{\text{E}}}{nc}} \quad [n = \text{number of scores in each cell} = 5]$$
$$[c = \text{number of column(colour groups)} = 3]$$
$$= 2.03 \times \sqrt{\frac{2 \times 17.28}{5 \times 3}} = 3.08$$

Table 8.13 has been obtained by using the contents from the Tables 8.8 and 8.10. Readers are advised to note the way this table has been generated.

If difference of any two group means is higher than the critical difference, the difference is said to be significant. Owing to this principle from Table 8.13, two conclusions can be drawn.

(a) Average sale of chocolate in bittersweet and semisweet categories is significantly higher than that of unsweetened category.
(b) Average sale of chocolate in bittersweet and semisweet categories is equal.

It may thus be inferred that bittersweet and semisweet chocolates are more preferred than unsweetened chocolates irrespective of the color of the chocolate.

**Remark** By looking at the $p$-values in Table 8.10, you can infer as to which group means differ significantly. If for any mean difference, significance value ($p$-value) is less than .05, then the difference is considered to be significant. In using this table, you can test the significance of mean difference, but it is difficult to find out as to which group mean is higher until and unless the results of Table 8.8 are combined. Hence, it is advised to construct Table 8.13 for post hoc analysis so as to get the clear picture in the analysis.

**Table 8.14** Comparison of mean chocolate sale in all the three Color groups (all sweetness types combined)

| White chocolate | Milk chocolate | Dark chocolate | CD at 5% level |
|---|---|---|---|
| 26.40 | 24.47 | 22.40 | 3.08 |

"_____/"Denotes no difference between the means at .05 level

### Column (Color) Analysis

For column analysis, critical difference has been obtained by using the LSD test as there are equal numbers of samples in each column.

Thus,

$$\text{CD for column} = t_{.05}(36) \times \sqrt{\frac{2(\text{MSS})_{\text{E}}}{nr}} \quad [n = \text{number of scores in each cell} = 5]$$

$$[r = \text{number of row(sweetness groups)} = 3]$$

$$= 2.03 \times \sqrt{\frac{2 \times 17.28}{5 \times 3}} = 3.08$$

Table 8.14 has been obtained from the contents in Tables 8.8 and 8.11.

From Table 8.14, the following two conclusions can be drawn:

(a) There is no difference in the average sale of white and milk chocolate. Similarly average sale of milk and dark chocolate is also same.
(b) Average sale of white chocolates is significantly higher than that of dark chocolates.

Thus, it may be inferred, in general, that the mean sale of the white chocolates is more in comparison to that of dark chocolate irrespective of the type of sweetness.

**Remark** You may note that critical difference for row and column analysis is same. It is so because the number of rows is equal to the number of columns.

### Interaction Analysis

Since $F$-value for the interaction is significant, it indicates that there is a joint effect of the chocolate sweetness and chocolate colors on the sale of chocolates. In other words, there is an association between sweetness and color of the chocolates. Thus, to compare the average chocolate sale among the three levels of sweetness in each of the color groups and to compare the average sales in all the three types of colored

**Table 8.15** Comparison of mean chocolate sale among different sweetness groups in each of the three colour groups

| Color | Sweetness | | | CD at 5% level |
|---|---|---|---|---|
| White | 27.80 (Bittersweet) | 27.80 (Unsweetened) | 23.60 (Semisweet) | 5.34 |
| Milk | 34.20 (Bittersweet) | 19.60 (Unsweetened) | 19.60 (Semisweet) | 5.34 |
| Dark | 34.00 (Semisweet) | 18.80 (Bittersweet) | 14.40 (Unsweetened) | 5.34 |

"_____"Denotes no difference between the means at .05 level

chocolates in each of the sweetness group, a critical difference (CD) has to be computed:

$$\text{CD for Interaction} = t_{.05}(36) \times \sqrt{\frac{2(\text{MSS})_{\text{E}}}{n}}$$
$$= 2.03 \times \sqrt{\frac{2 \times 17.28}{5}} = 5.34$$

Tables 8.15 and 8.16 have been generated with the help of the contents of Table 8.8. Readers are advised to note that CD is same for comparing all the three types of sweetness groups in each color group as well as for comparing all the colored groups in each of the sweetness group. It is so because the number of samples ($n$) in each cell is equal.

If the difference of group means is higher than that of the critical difference, it denotes that there is a significant difference between the two means; otherwise, group means are equal. If the mean difference is not significant, an underline is put against both the groups.

From Table 8.15, the following three conclusions can be drawn:

(a) The average sale of chocolates in all the three categories of sweetness groups is same for white chocolates.
(b) In milk chocolates, the average sale in bittersweet category is significantly higher than that of unsweetened and semisweet categories.
(c) In dark chocolates, the average sale in semisweet category is significantly higher than that of bittersweet and unsweetened categories.

It is thus concluded that in white chocolate, it hardly matters which sweetness flavor is being sold, whereas types of sweetness matters in case of milk and dark chocolates.

**Table 8.16** Comparison of mean chocolate sale among different colour groups in each of the three sweetness groups

| Sweetness | Color | | | CD at 5% level |
|---|---|---|---|---|
| Semisweet | 34.00 (Dark) | 23.60 (White) | 19.60 (Milk) | 5.34 |
| | | ╲_____╱ | | |
| Bittersweet | 34.20 (Milk) | 27.80 (White) | 18.80 (Dark) | 5.34 |
| Unsweetened | 27.80 (White) | 19.60 (Milk) | 14.40 (Dark) | 5.34 |
| | | ╲_____╱ | | |

"╲_____╱" Denotes no difference between the means at .05 level

From Table 8.16, the following three conclusions can be drawn:

(a) In semisweet category, the sale of dark chocolate is significantly higher than that of white and milk chocolates.
(b) In bittersweet category, the sale of milk chocolate is significantly higher than that of white and dark chocolates.
(c) In unsweetened category, the sale of white chocolate is significantly higher than that of milk and dark chocolates.

It may be inferred that in each of the sweetness flavor, it matters as to which color of chocolates is being sold.

## Summary of the SPSS Commands for Two-Way ANOVA

(i) Start the SPSS by using the following commands:

**Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

(ii) Click **Variable View** tag and define the variables *Sale* as a scale variable and *Sweetness* and *Colour as* nominal variables.
(iii) Under the column heading **Values**, define "1" for semisweet chocolates, "2" for bittersweet chocolates, and "3" for unsweetened chocolates for the variable *Sweetness*.
(iv) Similarly, for the variable *Colour*, under the column heading **Values**, define "1" for white chocolates, "2" for milk chocolates, and "3" for dark chocolates.
(v) Once the variables are defined, type the data for these variables by clicking **Data View**.

(vi) In the data view, follow the below-mentioned command sequence for two-way ANOVA:

**Analyze ⇨ General Linear Model ⇨ Univariate**

(vii) Select the variable *Chocolate_Sale* from left panel to the "dependent variable" section of the right panel. Similarly, select the variables *Chocolate_Sweetness* and *Chocolate_Colour* from left panel to the "Fixed Factor(s)" section of the right panel.

(viii) Click the tag **Post Hoc** and select the factors *Sweetness* and *Colour* from the left panel to the "Post Hoc test" panel on the right side. Check the option "LSD" and then click ***Continue***.

(ix) Click the tag **Options**, Select the variables *OVERALL, Sweetness, Colour*, and *Sweetness* × *Colour* from left panel to the right panel. Check the "Compare main effects" and "Descriptive" boxes and ensure the value of significance as .05. Click ***Continue***.

(x) Press **OK** for output.

## Exercise

*Short Answer Questions*

**Note**: Write answer to each of the following questions in not more than 200 words.

Q.1. What do you mean by main effects, interactions effects, and within-group variance? Explain by means of an example.

Q.2. Justify the name "two-way analysis of variance." What are the advantages of using two-way ANOVA design over one-way?

Q.3. While using two-way ANOVA, what assumptions need to be made about the data?

Q.4. Describe an experimental situation where two-way ANOVA can be used. Discuss different types of hypotheses that you would like to test.

Q.5. Discuss a situation where a factorial design can be used in market research. What research questions you would like to investigate?

Q.6. What is repeated measure design? Explain by means of an example. What precaution should be taken in planning such design?

Q.7. Explain MANOVA and discuss any one situation where it can be applied in management studies.

Q.8. Describe Latin square design. Discuss its layout. How is it different than factorial design?

*Multiple-Choice Questions*

**Note**: For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. In applying two-way ANOVA in an experiment, where "$r$" levels of factor $A$ and "$c$" levels of factor $B$ are studied. What will be the degree of freedom for interaction?

   (a) $rc$
   (b) $r + c$
   (c) $rc - 1$
   (d) $(r - 1)(c - 1)$

2. In an experiment "$r$" levels of factor $A$ are compared in "$c$" levels of factor $B$. Thus, there are $N$ scores in this experiment. What is the degree of freedom for within group?

   (a) $N - rc$
   (b) $N + rc$
   (c) $N - rc + 1$
   (d) $Nrc - 1$

3. In a two-way ANOVA, if the two factors $A$ and $B$ have levels 3 and 4, respectively, and the number of scores per cell is 3, what would be the degrees of freedom of error?

   (a) 36
   (b) 24
   (c) 12
   (d) 9

4. In order to apply two-way ANOVA

   (a) There should be equal number of observations in each cell.
   (b) There may be unequal number of observations in each cell.
   (c) There should be at least ten observations in each cell.
   (d) There is no restriction on the number of observations per cell.

5. Consider an experiment in which the Satisfaction levels of employees (men and women both) were compared in their plants located in three different cities. Choose the correct statement in defining the three variables Gender, City, and Satisfaction level in SPSS:

   (a) Gender and Satisfaction level are Scale variables and City is Nominal.
   (b) Gender and City are Nominal variables and Satisfaction level is Scale.
   (c) Gender and City are Scale variables and Satisfaction level is Nominal.
   (d) City and Satisfaction level are Scale variables and Gender is Nominal.

6. Command sequence in SPSS for starting two-way ANOVA is

   (a) Analyze -> General Linear Model -> Univariate
   (b) Analyze -> General Linear Model -> Multivariate
   (c) Analyze -> General Linear Model -> Repeated Measures
   (d) Analyze -> Univariate -> General Linear Model

7. While performing two-way ANOVA with SPSS, Fixed Factor(s) refers to

   (a) Dependent variable
   (b) Independent variables
   (c) Both dependent and independent variables
   (d) None of the above

8. If there are $N$ scores in a two-way ANOVA experiment, the total degree of freedom would be

   (a) $N + 1$
   (b) $N - 1$
   (c) $N$
   (d) $N - 2$

9. If 3 levels of factor $A$ are compared among the 4 levels of factor $B$, how many treatment groups will have to be created?

   (a) 7
   (b) 1
   (c) 12
   (d) 11

10. In an experiment, motivation level of employees was compared in three different organizations. Employees were categorized as per their gender to see its impact on motivation level. Interaction effect between plants and gender can be investigated only if there are

    (a) Equal number of observations in each cell
    (b) At least one cell must have five or more observations
    (c) Unequal number of observations in each cell
    (d) More than one and equal number of observations in each cell

11. What should be the minimum number of observations in order to perform two-way ANOVA?

    (a) 8
    (b) 6
    (c) 4
    (d) 2

12. What should be the minimum number of observations in order to perform two-way ANOVA with interaction effect?

    (a) 8
    (b) 6
    (c) 4
    (d) 2

*Assignments*

1. Four salesmen were appointed by a company to sell their products in door-to-door marketing. Their sales were observed in three seasons, summer, rainy, and winter, on month to month basis. The sales data so obtained (in lakhs of rupees) are shown in the following table:

Sales data (in lakhs of rupees) of the sales persons in different season

| | Salesmen | | | |
|---|---|---|---|---|
| Season | A | B | C | D |
| Summer | 36 | 36 | 21 | 25 |
| | 35 | 32 | 25 | 27 |
| | 32 | 30 | 28 | 24 |
| | 38 | 33 | 25 | 29 |
| Rainy | 26 | 28 | 29 | 29 |
| | 25 | 28 | 32 | 31 |
| | 27 | 31 | 33 | 34 |
| | 29 | 28 | 38 | 39 |
| Winter | 28 | 29 | 31 | 32 |
| | 27 | 32 | 35 | 31 |
| | 32 | 33 | 31 | 28 |
| | 29 | 35 | 41 | 33 |

   Discuss your findings by applying two-way ANOVA. Test your hypotheses at 5% level.
2. The management of a private bank was interested to know the stress level of their employees in different age categories and gender. A stress questionnaire was administered on the randomly selected employees in different age categories. The scores on their stress level are shown in the following table: Apply two-way ANOVA and discuss your findings at 5% level.

Stress scores of the employees in different age categories

| | Age category (years) | | |
|---|---|---|---|
| Gender | <35 | 35–50 | >50 |
| Male | 28 | 55 | 42 |
| | 29 | 51 | 39 |
| | 32 | 45 | 41 |
| | 25 | 48 | 42 |
| | 26 | 53 | 48 |
| Female | 28 | 51 | 55 |
| | 32 | 45 | 58 |
| | 35 | 49 | 61 |
| | 29 | 43 | 52 |
| | 31 | 48 | 50 |

*Answers to Multiple-Choice Questions*

| | | | |
|---|---|---|---|
| Q.1 d | Q.2 a | Q.3 b | Q.4 a |
| Q.5 b | Q.6 a | Q.7 b | Q.8 b |
| Q.9 c | Q.10 d | Q.11 c | Q.12 a |

# Chapter 9
# Analysis of Covariance: Increasing Precision in Comparison by Controlling Covariate

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Learn the concept of analysis of covariance.
- Know the application of analysis of covariance in different situation.
- Describe the concept of covariate and neutralize its effect from the treatment effect.
- Know the model involved in the analysis of covariance.
- Understand the concept of adjusting treatment means for covariate using linear regression.
- Understand the analysis of covariance graphically.
- Learn the method of using analysis of covariance.
- Understand as to why the analysis of covariance is efficient design in comparison to one-way analysis of variance.
- To be able to formulate the hypotheses in analysis of covariance.
- Understand the assumptions used in analysis of covariance.
- Know the method of preparing data file for analysis in SPSS.
- Learn the steps involved in using SPSS for analysis of covariance.
- Interpret the output obtained in analysis of covariance.
- Learn the model way of writing the results of analysis.

## Introduction

To compare the effectiveness of two or more treatments on certain criterion variable, we use one-way analysis of variance technique. This technique has been discussed in Chap. 7. In testing the comparative effectiveness of different treatments, the subjects are selected in each experimental group by using the

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principle of randomization. In a situation if the randomization is not possible, groups are equated on the basis of one or more known parameters. The randomization or matching is done in order to have the similar initial conditions so that whatever the changes in criterion variable occurs in the treatment groups can be attributed due to the treatments only. But in many situations, randomization of subjects or experimental units may not be possible as the experimenter may be forced to take the two or more intact samples from different locations due to administrative or financial constraints. For example, consider an experiment where it is desired to compare the effect of different types of tariff incentives on the mobile recharge revenue. In this situation, an experimenter does not have any choice to select the subjects randomly in different experimental groups. Samples will have to be drawn from the predefined clientele sets of different mobile companies. In such situations, groups are not homogeneous initially. These subjects in intact groups may differ in so many ways which might affect their behavior pattern. Thus, statistical control or indirect procedure is necessary to reduce the experimental error which causes due to such initial differences in the groups.

In experimental research, the individual variations that appear within the measures on the criterion variable are potentially correlated with some extraneous variable. If the criterion variable is a measure of how well subjects learn English speaking under one or the other of the two methods of instructions, the potential correlates are likely to include parameters such as prior knowledge of grammar, motivation level, aptitude, age, and intelligence. These potential correlates are known as covariates. Analysis of covariance can be used to compare the effectiveness of these instructional methods on learning English speaking after removing the effect of the identified covariates.

## Introductory Concepts of ANCOVA

Analysis of covariance (ANCOVA) is a statistical technique that may be considered as an extension of analysis of variance (ANOVA). Analysis of covariance combines features of one-way analysis of variance with simple linear regression. It is so because the treatment groups are compared like the way we do in analysis of variance and we adjust the measurement on criterion variable on the basis of covariate by using the concept of regression analysis.

In ANCOVA, we try to minimize the error variance by controlling the concomitant variable which varies along with the criterion variable in all the experimental groups. These concomitant variables are known as covariate. Typically, a covariate is highly correlated with a criterion variable; that is, the covariate contains information about the criterion variable and therefore possibly also explains the difference in the treatment groups. The purpose in the ANCOVA design is to isolate the variability component due to covariate so that group difference if any may be solely attributed to the treatments only. Analysis of covariance is used in a situation where there is at least one categorical factor and one or more continuous covariates.

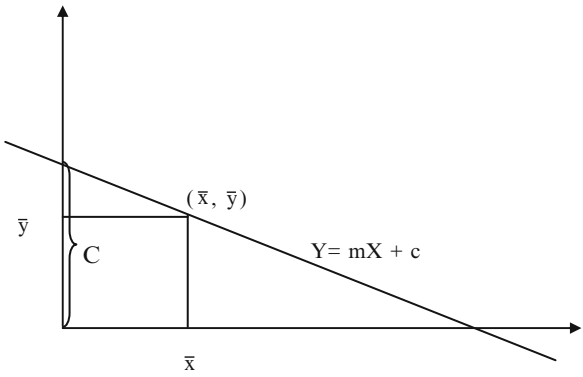


**Fig. 9.1** Regression equation of $Y$ on $X$

Since ANCOVA is the combination of ANOVA and regression analysis, this design can be used with any ANOVA model. One can do a completely randomized design, randomized block design, a Latin square design, or any other design if a covariate is put on it. All we need is one measurement for the covariate to go with every observation. In this chapter, only completely randomized design shall be discussed as an ANOVA model.

## Graphical Explanation of Analysis of Covariance

In order to understand the ANCOVA model, let us first refresh our concept of representing the line in the slope intercept form. You may recall that this line used to be represented by

$$\mathrm{Y} = \mathrm{mX} + \mathrm{c} \tag{9.1}$$

where $m$ is the slope and $c$ is the intercept of the line on $y$-axis. Graphically this equation can be represented by the Fig. 9.1.

Equation of line in any form may be converted to this slope intercept form for comparing their slopes and intercepts.

The line shown in Fig. 9.1 is a regression line for estimating the value of $Y$ if the value of $X$ is known. Now if you look at the vertical line over $\bar{x}$, it intersects the regression line at ($\bar{x}$ and $\bar{y}$). In other words, the point ($\bar{x}$, $\bar{y}$) actually lies on the regression line. This concept shall be used to explain the analysis of covariance.

To understand ANCOVA, let us consider $A$ and $B$ represent the two treatments. Further, $Y_A$ and $Y_B$ represent the value of criterion variable, whereas $X_A$ and $X_B$ represent the value of covariate in the two treatment groups A and B respectively. These two treatments are represented by the lines $A$ and $B$ in Fig. 9.2. If higher value

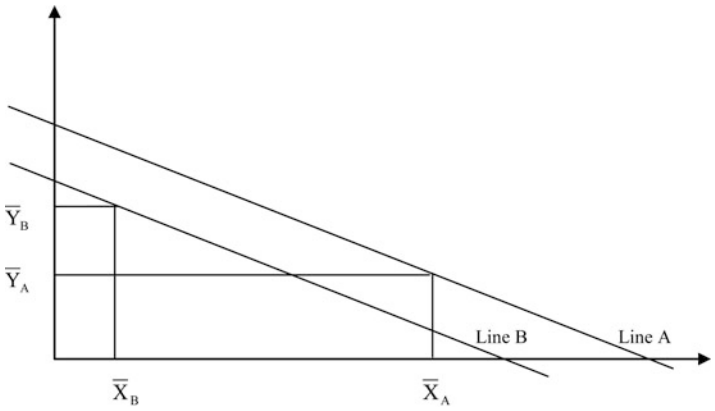


**Fig. 9.2** Geometrical representation of two treatments ($Y's$) with their covariates ($X's$)

of $Y$ indicates better performance, then obviously treatment $A$ would be better treatment because the line $A$ is higher everywhere than line $B$.

But sometimes corresponding to very low value of $\overline{X}_B$ and very high value of $\overline{X}_A$, the values of $\overline{Y}_B$ is higher than $\overline{Y}_A$. This can mislead the researcher. If line $B$ is always higher than line $A$, then the sample means end up reversed. The difference observed in $\overline{Y}'s$ is not because of the treatments but due to the covariate means ($\overline{X}'s$) which are so apart.

In analysis of covariance, what we do is to compare the values of $\overline{Y}_A$ and $\overline{Y}_B$ at $\bar{X}$ (overall grand mean of $\overline{X}_A$ and $\overline{X}_B$). Now the distance between the two curves is the same because the lines are parallel. What we do here is to adjust both means so that we are evaluating the points of the same $X$ on the curves. In this way, we get a more balanced comparison.

If treatments are compared without using ANCOVA, what may happen? Suppose the above-mentioned lines $A$ and $B$ are exactly the same, still $\overline{Y}_i$'s may be different. This may be because the effect of one treatment is observed at higher average covariate mean and the other treatment effect is measured with lower average covariate mean. This fact may not be revealed if the analysis of covariance is not done. Thus, in analysis of covariance, we compare the effect of treatments mean ($\overline{Y}_i$'s) by adjusting them with the average covariate means ($\bar{X}$).

## Analysis of Covariance Model

If $Y_{ij}$ represents the $j$th score of the criterion variable in the $i$th treatment group and $X_{ij}$ represents the $j$th score of the covariate in the $i$th treatment group, then the one-way ANCOVA model is represented as follows:

$$Y_{ij} = \mu + \beta(X_{ij} - \bar{X}) + \varepsilon_{ij} \tag{9.2}$$

where

$\mu$ is the overall population mean (on criterion variable)
$\beta$ is slope of the lines

$\bar{X}$ is combined mean of the covariate in all the treatment groups

$\varepsilon_{ij}$ is unexplained error terms which are independent and normally distributed with mean 0 and variance 1

One-way analysis of covariance fits a straight line to each treatment group of $X$-$Y$ data, such that the slopes of the lines are all equal. This fitted model may then be used to test the following null hypothesis:

$H_0$: The intercepts for each line are equal.

This hypothesis tests as to whether all the treatment group means are equal or not after making the adjustment for $X$ (covariate). Here, we assume that the slopes are equal. It is so because there is no point of comparing the treatments effect if one of the treatments produces positive effect whereas other induces negative effect.

Let us see how the treatment means are adjusted for covariate and are computed for comparison. Adding both sides of Eq. (9.2) for $j$ and dividing by $n$ (number of scores in each treatment group), we get

$$\frac{1}{n}\sum_j Y_{ij} = \mu + \beta\left(\frac{1}{n}\sum_j X_{ij} - \bar{X}\right) + \frac{1}{n}\sum_j \varepsilon_{ij}$$

$$\Rightarrow \quad \overline{Y}_i = \mu + \beta(\overline{X}_i - \bar{X}) + \bar{\varepsilon}_i$$

Since mean of $\varepsilon_i$ is zero, the equation becomes

$$\overline{Y}_i = \mu + \beta(\overline{X}_i - \bar{X}) \tag{9.3}$$

where

$\overline{Y}_i$ is mean of the criterion variable in the $i$th treatment group
$\mu$ is the overall population mean (on criterion variable)
$\overline{X}_i$ is mean of the covariate ($X$ data) in the $i$th treatment group

Other symbols have their usual meanings. If one-way ANCOVA model has two treatment groups, then the model (9.3) will give rise to two straight lines as shown in Fig. 9.2. Thus, by testing the hypothesis $H_0$ in ANCOVA, we actually compare the two treatments $\overline{Y}_A$ and $\overline{Y}_B$ after adjusting it for the covariates.

**Remark**

1. If the slope of line A and B is equal it indicates that the effect of both the treatments are in one direction only. Both the treatments will induce either positive or negative effect.
2. By comparing the intercepts of the lines, we try to compare whether the effect of all the treatments on the criterion variable is same or not.

## What We Do in Analysis of Covariance?

In analysis of covariance, the purpose of the analysis is to compare the posttreatment means of the groups by adjusting the initial variations in the grouping. The statistical control is achieved by including measures on supplementary or concomitant variate ($X$) in addition to the variate of primary interest ($Y$) after implementing the treatments. The concomitant variate that may not be of experimental interest is called covariate and designated as $X$. Let us designate the variable which is of interest in the experiment as $Y$, also known as criterion variable. Thus, in ANCOVA, we have two observations ($X$ and $Y$) from each subject. Measurements on $X$ (covariate) are obtained prior to the administration of treatments and are primarily to adjust the measurements on $Y$ (criterion). Covariate is the variable which is assumed to be associated with the criterion variable. When $X$ and $Y$ are associated, a part of the variability of $Y$ is due to the variation in $X$. If the value of covariate $X$ is constant over the experimental units, there would be corresponding reduction in the variance of $Y$.

Let us consider an example in which the analysis of covariance can be applied to reduce the estimate of experimental error. Suppose an experiment is conducted to study the effect of three different types of advertising campaign on the sale of a product. Further, the experimenter is forced to use three intact groups of outlets from three different states. However, there is a freedom to assign groups randomly to the treatment conditions (types of advertisement). Out of three groups, one may serve as control. Since the subjects cannot be assigned randomly to the treatment groups, the possibility of initial differences (before administration of treatment) among the groups always exist. Thus, one may decide to record the sale of each outlet ($X$) before applying the treatments, which serves as a measure of covariate. This measure of covariate is used to adjust the post advertisement sale figure ($Y$) in different outlets. The $Y$ is the sale of the product which is obtained after the implementation of the treatments (advertisements).

Thus, the variates $X$ and $Y$ can be defined as follows:

$X =$ sale of the product on 15 consecutive days in each experimental groups before treatments (advertisements).

$Y =$ sale of the product on 15 consecutive days in each experimental groups after treatments (advertisements).

In general, if $Y$ measures (criterion) are substantially correlated with $X$ measures (covariate), the analysis of covariance will result in similar estimate of experimental error than would be obtained from the analysis of variance.

Thus, in ANCOVA, the following null hypothesis is tested

$$H_0 : \mu_{\text{Adj\_Post\_Adv\_1}} = \mu_{\text{Adj\_Post\_Adv\_2}} = \mu_{\text{Adj\_Post\_Control}}$$

against the possible alternatives that at least one group mean differs
where

Adj_Post_Adv_1 is adjusted mean sale in the first treatment group (where first advertisement campaign was used)
Adj_Post_Adv_2 is adjusted mean sale in the second treatment group (where second advertisement campaign was used)
Adj_Post_Adv_3 is adjusted mean sale in the third treatment group (where no advertisement campaign was used)

ANCOVA table generated in the SPSS output contains the value of $F$-statistics along with their significance value. Thus, if $F$-value is significant, the null hypothesis is rejected and the post hoc test is used to compare the adjusted posttreatment means of different groups in pairs.

## When to Use ANCOVA

The analysis of covariance is used to test the comparative effectiveness of two or more treatments on the criterion variable after adjusting for their initial differences due to covariate. The covariate should be identified before the experiment, and its value should be measured on each of the experimental units. In many situations, it is not possible to identify single covariate which affects the measure on criterion variable during experimentation. In that case, initial testing ($X$) on the criterion variable in each of the treatment group may be considered as covariate, and the measure on the criterion variable after the treatment ($Y$) in all the treatment groups is the one in which we are interested to investigate. The analysis of covariance design should be used if the following things happen:

(a) The response on the criterion variable is continuous.
(b) There are one or more classification variables (treatment groups).
(c) There are one or more continuous independent variables (covariate).

## Assumptions in ANCOVA

In using the analysis of covariance design following assumptions are made:

1. The criterion variable must have the same variance in each of the treatment groups.
2. The data on criterion variable must have been obtained randomly.
3. The interaction between the criterion variable and covariate is negligible. The adjusted mean of the criterion variable in each of the treatment groups is computed owing to this assumption. If this assumption is violated, then the adjustment of the criterion variable to a common value of the covariate will be misleading.
4. Since ANCOVA uses the concept of linear regression, the assumption of linearity between independent and dependent variable must hold true.
5. The regression coefficients (slope) for each treatment groups must be homogeneous. If this assumption is violated, then the ANCOVA results will be misleading.

## Efficiency in Using ANCOVA over ANOVA

The ANCOVA design is more efficient in comparison to one-way ANOVA. It is because of the fact that in ANCOVA, a part of the variability due to error component is defined by the covariate and, hence, the error variance reduces comprehensively in comparison to one-way ANOVA design. In one-way ANOVA, the total variability is split into two components, that is, between groups and within groups. Here, the variability due to covariate is confounded into error component, and, hence, this design is inferior to ANCOVA in a situation where the covariate effects the measurement on criterion variable. In fact, one-way ANOVA should be used only in a situation where it is known that all the treatment groups are homogenous in all respect and perfect control is observed during the entire period of experimentation.

## Solved Example of ANCOVA Using SPSS

**Example 9.1** A study was planned to investigate the effect of different doses of vitamin C in curing the cold. Forty five subjects who were suffering from cold symptoms were divided into three groups. The first two groups were given a low dose and high dose of vitamin C every day whereas the third group was given a placebo. The number of days these subjects were suffering from cold before starting the treatment was taken as the covariate whereas the curing time in each treatment group was recorded as a dependent variable. The data so obtained on the subjects are shown in the Table 9.1.

**Table 9.1** Data on cold duration before and during implementation of vitamin C in different groups

| S.N. | Contents of vitamin C | | | | | |
|---|---|---|---|---|---|---|
| | High dose | | Low dose | | Placebo | |
| | Pre days | Post days | Pre days | Post days | Pre days | Post days |
| 1 | 0 | 2 | 14 | 12 | 1 | 10 |
| 2 | 10 | 3 | 16 | 13 | 10 | 8 |
| 3 | 11 | 5 | 5 | 8 | 5 | 14 |
| 4 | 15 | 9 | 12 | 10 | 6 | 9 |
| 5 | 6 | 3 | 0 | 1 | 10 | 13 |
| 6 | 12 | 8 | 8 | 4 | 5 | 11 |
| 7 | 9 | 7 | 12 | 9 | 12 | 15 |
| 8 | 13 | 7 | 5 | 10 | 13 | 15 |
| 9 | 1 | 6 | 19 | 10 | 6 | 10 |
| 10 | 8 | 13 | 14 | 8 | 19 | 20 |
| 11 | 7 | 12 | 6 | 11 | 8 | 12 |
| 12 | 6 | 10 | 8 | 11 | 8 | 14 |
| 13 | 4 | 3 | 5 | 8 | 6 | 12 |
| 14 | 3 | 2 | 2 | 6 | 5 | 9 |
| 15 | 4 | 3 | 4 | 6 | 8 | 14 |

Pre days: Cold duration before treatment
Post days: Cold duration during treatment

Apply analysis of covariance to see as to which dose of vitamin C is more effective in controlling cold. Test your hypothesis at 5% level of significance.

*Solution* In this example, the variables are as follows:

*Treatments:* The three treatments are as follows:
Treatment *A*: Administering high dose of vitamin C
Treatment *B*: Administering low dose of vitamin C
Treatment *C*: Administering placebo

*Covariate*: The number of days having cold symptoms before treatment.
*Dependent Variable*: *Curing time of cold symptoms*

Here, it is required to compare the average curing time among the three treatment groups, that is, high dose of vitamin C, low dose of vitamin C, and placebo, after adjusting for the covariate (average number of days having cold symptoms before treatments).

Thus, the following null hypothesis needs to be tested

$$H_0 : \mu_{\text{Adj\_Days\_in\_Treatment\_}A} = \mu_{\text{Adj\_Days\_in\_Treatment\_}B} = \mu_{\text{Adj\_Days\_in\_Treatment\_}C}$$

against the alternative hypothesis that at least one group mean (adjusted) is different

where

$\mu_{\text{Adj\_Days\_in\_Treatment\_}A}$ is adjusted mean curing time in treatment group $A$

$\mu_{\text{Adj\_Days\_in\_Treatment\_}B}$ is adjusted mean curing time in treatment group $B$

$\mu_{\text{Adj\_Days\_in\_Treatment\_}C}$ is adjusted mean curing time in treatment group $C$

The SPSS output provides ANCOVA table along with pairwise comparison of adjusted post means of different treatment groups. The pairwise comparison of means is done only when the $F$-ratio is significant.

The analysis of covariance table generated in the SPSS output looks similar to the one- way ANOVA table as only adjusted post means is compared here. In the ANCOVA table, $F$-value is shown along with its significance value ($p$-value). The $F$-value would be significant if its corresponding $p$-value is less than .05, and in that case null hypothesis would be rejected. Once the $F$-value is found to be significant, then a post hoc test is used to compare the paired means. SPSS provides the choice of post hoc test to be used in the analysis.

In this example, since the sample sizes are equal, LSD test shall be used as a post hoc test for comparing the group means. The SPSS output provides the significance value ($p$-value) for each pair of difference of group means. Thus, by looking at the values of means, the best treatment may be identified.

## *Computations in ANCOVA Using SPSS*

(a) ***Preparing data file***

Before using SPSS commands to solve the problem of analysis of covariance, a data file needs to be prepared. The following steps will help you prepare the data file:

(i) *Starting the SPSS*: Follow the below-mentioned command sequence to start SPSS:

**Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

After clicking the **Type in Data,** you will be taken to the **Variable View** option for defining variables in the study.

(ii) *Defining variables*

In this example, three variables, vitamin dose, cold duration before treatment and cold duration during treatment need to be defined. The procedure of defining these variables along with their characteristics is as follows:

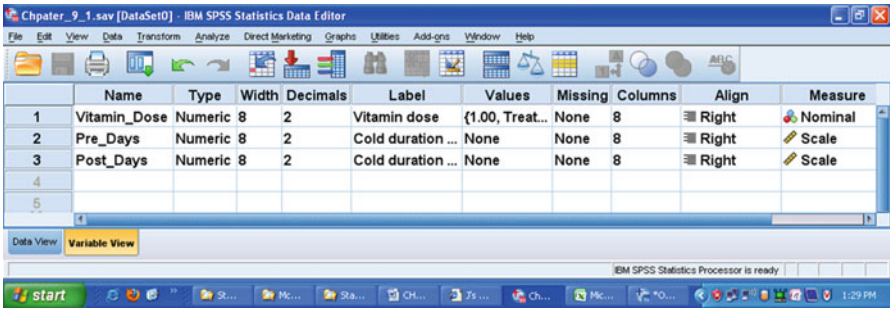


**Fig. 9.3** Defining variables and their characteristics

1. Click the **Variable View** to define the variables and their properties.
2. Write short name of the variables as *Vitamin_Dose*, *Pre_Days* and *Post_Days* under the column heading **Name**.
3. Under the column heading **Label**, define full name of these variables as *Vitamin dose*, *Cold duration before treatment*, and *Cold duration during treatment*. Other names may also be chosen for describing these variables.
4. Under the column heading **Measure,** select the option "Nominal" for the variable *Vitamin dose* and "Scale" for the variables *Cold duration before treatment* and *Cold duration during treatment*.
5. For the variable *Vitamin dose*, double-click the cell under the column **Values** and add the following values to different labels:

| Value | Label |
|---|---|
| 1 | Treatment *A* |
| 2 | Treatment *B* |
| 3 | Treatment *C* |

6. Use default entries in rest of the columns.

After defining the variables in variable view, the screen shall look as shown in Fig. 9.3.

(iii) *Entering data:* After defining all the variables in the **Variable View**, click **Data View** on the left corner in the bottom of the screen shown in Fig. 9.3 to open the format for entering the data column wise. After entering the data, the screen will look like Fig. 9.4. Save the data file in the desired location before further processing.

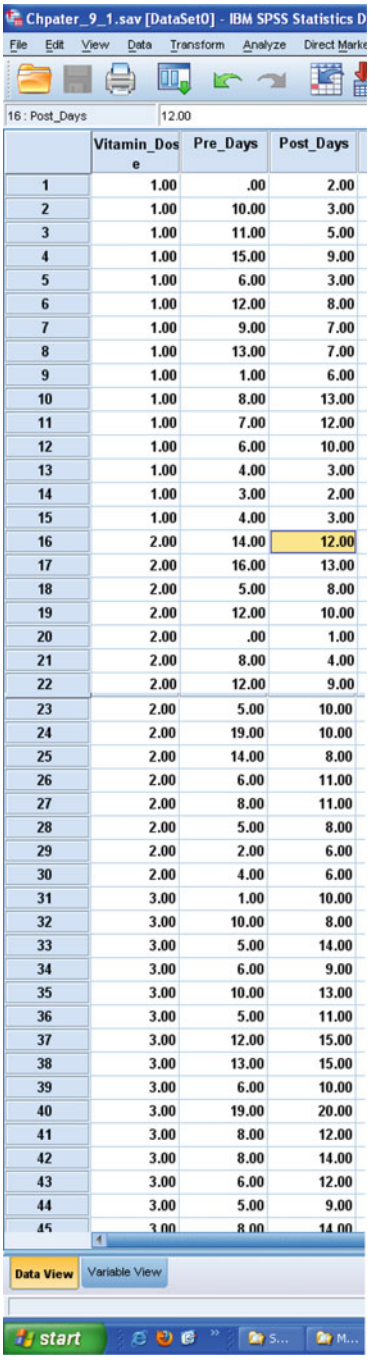


**Fig. 9.4** Screen showing entered data for all the variables in the data view

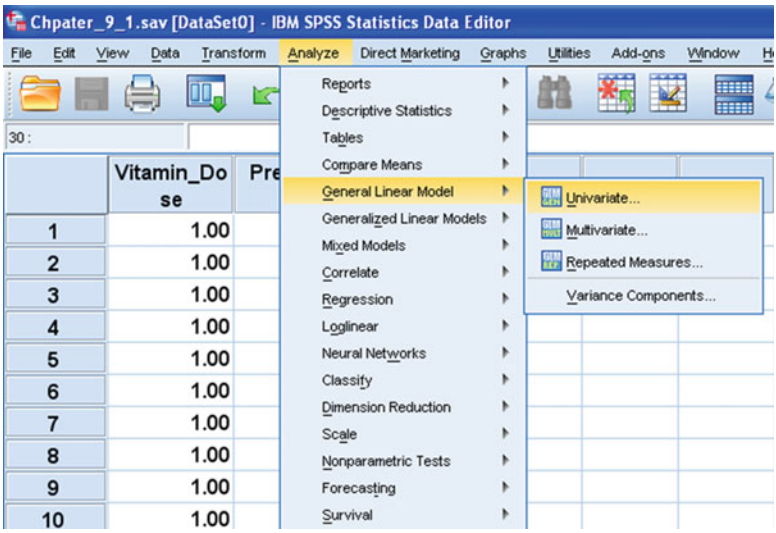


**Fig. 9.5** Sequence of SPSS commands for analysis of covariance

(b) ***SPSS commands for ANCOVA***

After preparing the data file, do the following steps for analysis of covariance:

(i) *Initiating the SPSS commands for ANCOVA:* In data view, click the following commands in sequence:

**Analyze ⇒ General Linear Model ⇒ Univariate**

The screen shall look like Fig. 9.5.

(ii) *Selecting variables for ANCOVA:* After clicking the **Univariate** option, you will be taken to the next screen as shown in Fig. 9.6 for selecting variables. Select the variables as follows:

- *Cold duration during treatment* from left panel to the "Dependent variable" section of the right panel.
- *Vitamin dose* from left panel to the "Fixed Factor(s)" section of the right panel.
- *Cold duration before treatment* from the left panel to the "Covariate(s)" section of the right panel.

(iii) *Selecting the options for computation:* After selecting the variables, different options need to be defined for generating the output in ANCOVA. This shall be done as follows:

- Click the tag **Model** on the screen shown in Fig. 9.6 and do the following:
  - Select the sum of squares option as "Type I."
  - Press ***Continue***.

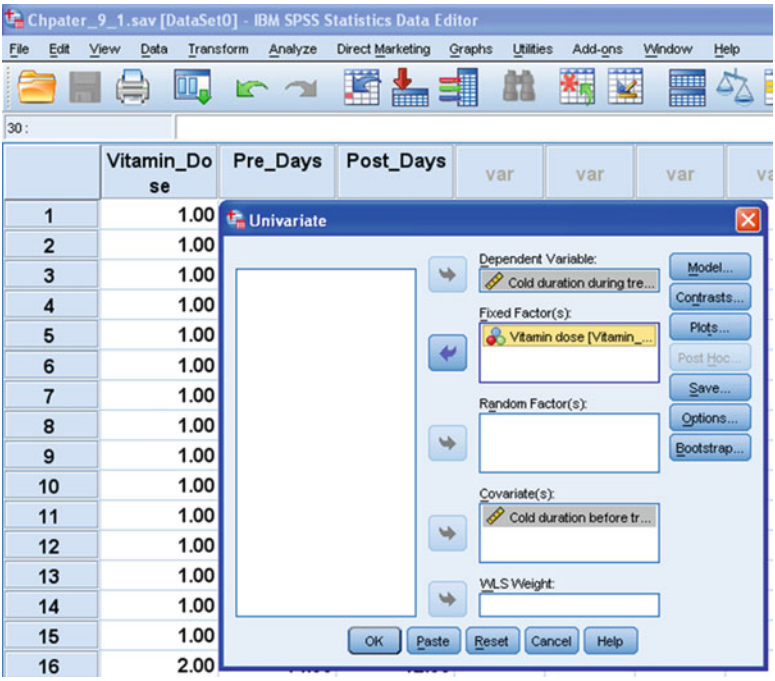


**Fig. 9.6** Selecting variables for ANCOVA

The screen will look like Fig. 9.7.

– Click the tag **Options** in the screen shown in Fig. 9.6 and do the following:

  – Select the variables *Overall* and *Vitamin_Dose* from the left panel to the "Display Means for" section of the right panel.
  – Check the option "Compare main effects."
  – Check the option "Descriptive statistics."
  – Ensure "Significance level" as .05. This value is written by default; however, you may write some other level of significance as .01 or .10, etc.
  – Click ***Continue***.

  The screen will look like Fig. 9.8.

  – Click **OK** on the screen shown in Fig. 9.6.

(c) ***Getting the output***

Clicking the option **OK** on the screen shown in Fig. 9.6 will take you to the output window. Relevant outputs can be selected by using the right click of the

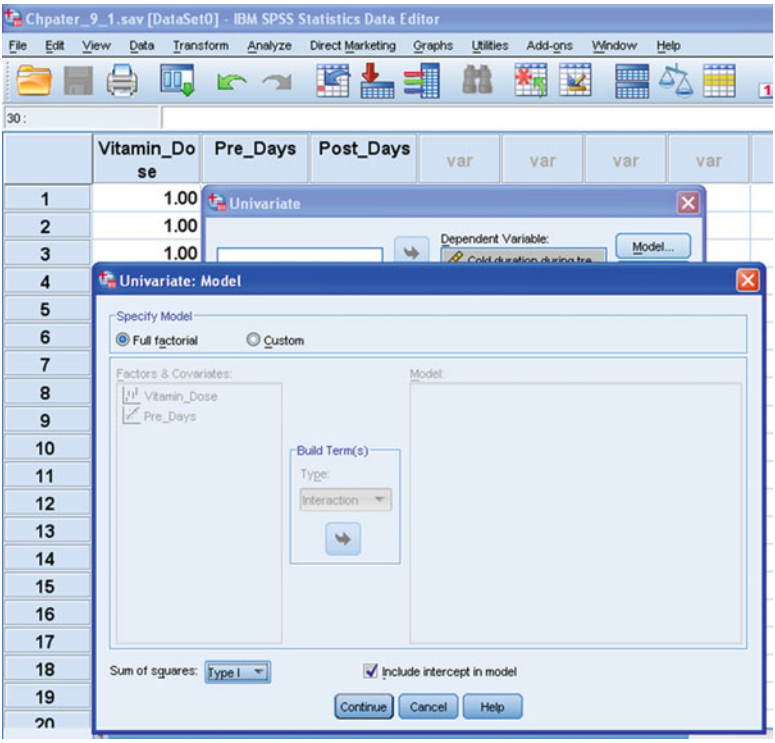


**Fig. 9.7** Screen showing option for choosing sum of squares and model type

mouse and may be copied in the word file. The identified outputs shall be rearranged for interpreting the findings. The details have been shown under the heading **Model Way of Writing the Results**.

(d) ***SPSS output***

The readers should note the kind of outputs to be selected from the output window of SPSS for explaining the findings. The following four outputs have been selected for discussing the results of ANCOVA:

1. Descriptive statistics
2. Adjusted estimates of the dependent variable
3. ANCOVA table
4. Post hoc comparison table

These outputs have been shown in Tables 9.2, 9.3, 9.4, and 9.5.

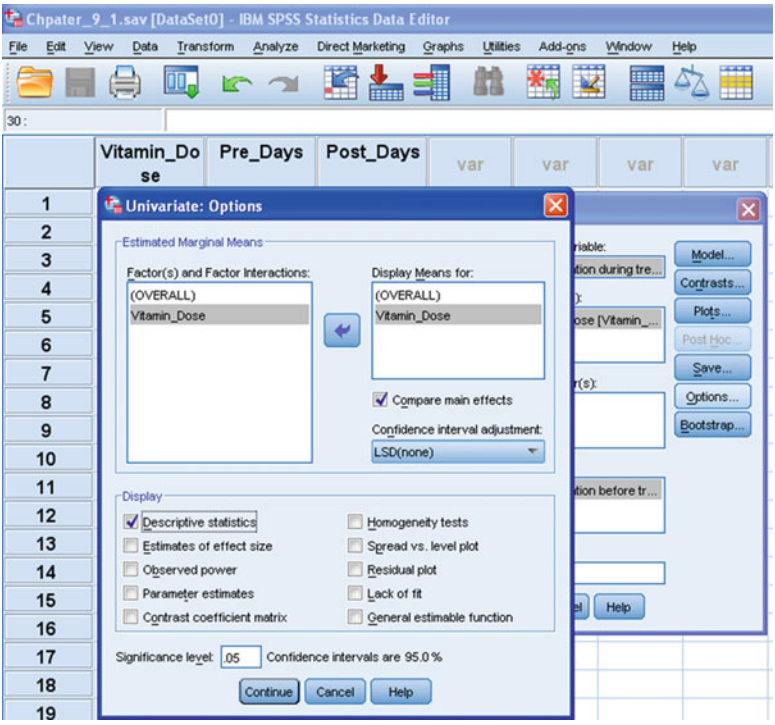


**Fig. 9.8** Selecting options for ANCOVA output

**Table 9.2** Descriptive statistics

| Vitamin dose | Mean | Std. deviation | *N* |
|---|---|---|---|
| Treatment *A* | 6.2000 | 3.62925 | 15 |
| Treatment *B* | 8.4667 | 3.18179 | 15 |
| Treatment *C* | 12.4000 | 3.11219 | 15 |
| Total | 9.0222 | 4.14778 | 45 |

Dependent variable: Cold duration during treatment

**Table 9.3** Adjusted estimates

| | | | 95% Confidence interval | |
|---|---|---|---|---|
| Vitamin dose | Mean | Std. error | Lower bound | Upper bound |
| Treatment *A* | 6.508[a] | .704 | 5.086 | 7.930 |
| Treatment *B* | 8.204[a] | .703 | 6.784 | 9.625 |
| Treatment *C* | 12.355[a] | .701 | 10.939 | 13.771 |

Dependent variable: Cold duration during treatment

[a]Covariates appearing in the model are evaluated at the following values: Cold duration before treatment = 8.0222

**Table 9.4** Tests "between-subjects" effects

| Source | Type I sum of squares | df | Mean square | *F* | Sig. |
|---|---|---|---|---|---|
| Corrected model | 454.761[a] | 3 | 151.587 | 20.565 | .000 |
| Intercept | 3,663.022 | 1 | 3,663.022 | 496.941 | .000 |
| Pre_Days | 183.993 | 1 | 183.993 | 24.961 | .000 |
| Vitamin_Dose | 270.768 | 2 | 135.384 | 18.367 | .000 |
| Error | 302.217 | 41 | 7.371 | | |
| Total | 4,420.000 | 45 | | | |
| Corrected total | 756.978 | 44 | | | |

Dependent variable: Cold duration during treatment
[a]R squared = .601 (adjusted R squared = .572)

**Table 9.5** Pairwise comparisons

| | | | | | 95% Confidence interval for difference[a] | |
|---|---|---|---|---|---|---|
| (I) Vitamin dose | (J) Vitamin dose | Mean diff. (I–J) | SE | Sig.[a] | Lower bound | Upper bound |
| Treatment *A* | Treatment *B* | −1.697 | .999 | .097 | −3.714 | .321 |
| | Treatment *C* | −5.847* | .994 | .000 | −7.855 | −3.839 |
| Treatment *B* | Treatment *A* | 1.697 | .999 | .097 | −.321 | 3.714 |
| | Treatment *C* | −4.151* | .992 | .000 | −6.155 | −2.146 |
| Treatment *C* | Treatment *A* | 5.847* | .994 | .000 | 3.839 | 7.855 |
| | Treatment *B* | 4.151* | .992 | .000 | 2.146 | 6.155 |

Dependent variable: Cold duration during treatment
Based on estimated marginal means
*The mean difference is significant at the .05 level
[a]Adjustment for multiple comparisons: Least significant difference (equivalent to no adjustments)

## Model Way of Writing the Results of ANCOVA and Their Interpretations

The above output generated by the SPSS can be shown in a much more user-friendly format by modifying the relevant contents of the Tables 9.2, 9.3, 9.4, and 9.5. The below-mentioned edited outputs can directly be shown in the project, dissertation, or thesis. These modified outputs shall be used to discuss the findings of ANCOVA.

(a) ***Descriptive Statistics of the Data Obtained on the Criterion Variable***
The mean and standard deviation of the criterion variable in different treatment groups have been shown in Table 9.6. Entries in this table have been copied from Table 9.2. If you are interested in computing different descriptive statistics for the covariate (Number of days having cold symptoms before treatment) also, the same be computed by using the procedure discussed in Chap. 2. However, the SPSS does not generate these statistics during ANCOVA analysis.

Look at the table heading which can be used in writing the final results in your study.

**Table 9.6** Mean and standard deviation of cold duration in different groups during treatment

| Vitamin dose | Mean | Std. deviation | *N* |
|---|---|---|---|
| Treatment *A* | 6.2 | 3.6 | 15 |
| Treatment *B* | 8.5 | 3.2 | 15 |
| Treatment *C* | 12.4 | 3.1 | 15 |
| Total | 9.0222 | 4.14778 | 45 |

Values have been rounded off

**Table 9.7** Adjusted mean and standard error for the data on cold duration in different groups during treatment

| | | | 95% Confidence interval | |
|---|---|---|---|---|
| Vitamin dose | Mean | Std. error | Lower bound | Upper bound |
| Treatment *A* | 6.5[a] | .70 | 5.09 | 7.93 |
| Treatment *B* | 8.2[a] | .70 | 6.78 | 9.63 |
| Treatment *C* | 12.4[a] | .70 | 10.94 | 13.77 |

[a]Covariates appearing in the model are evaluated at the following values:
Cold duration before treatment = 8.0222
Values have been rounded off

From Table 9.6, it can be seen that average time taken to cure the cold symptoms is highest in treatment group *C* whereas the least time is in treatment group *A*. Treatment C signifies the placebo, whereas treatment *A* is the high dose of vitamin C. The next question is to see whether this difference is significant or not after adjusting for the covariate (number of days having cold symptoms before treatment).

(b) ***Descriptive Statistics of the Data Obtained on the Criterion Variable after Adjusting for Covariate***

The adjusted mean and standard error of the criterion variable in different treatment groups have been shown in Table 9.7. The mean of criterion variable has been obtained in all the three treatment groups after adjusting for the covariate (Number of days having cold symptoms before treatment). These data have been taken from Table 9.3. Readers may note that these values are different from that of the unadjusted values shown in Table 9.6. The advantage of using the ANCOVA is that the differences in the posttesting means are compensated for the initial differences in the scores. In other words, it may be said that the effect of covariate is eliminated in comparing the effectiveness of treatments on the criterion variable.

Kindly note the heading of the table which may be used for writing the final results of ANCOVA.

(c) ***ANCOVA Table for the Data on Criterion Variable (Number of Days Having Cold Symptoms During Treatment)***

The main ANCOVA table may be reproduced by deleting some of the unwanted details of Table 9.4. The final results of ANCOVA have been shown in Table 9.8. The "significance" (Sig.) value has been named as *p*-value. In most of the scientific literature, *p*-value is used instead of term significance value.

**Table 9.8** ANCOVA table for the data on cold duration in different groups during treatment

| Source | Sum of squares | df | Mean square | *F* | (*p*-value) Sig. |
|---|---|---|---|---|---|
| Pre_Days | 183.993 | 1 | 183.993 | 24.961 | .000 |
| Vitamin_Dose | 270.768 | 2 | 135.384 | 18.367 | .000 |
| Error | 302.217 | 41 | 7.371 | | |
| Corrected total | 756.978 | 44 | | | |

**Table 9.9** Pairwise comparisons

| (I) Vitamin dose | (J) Vitamin dose | Mean diff. (I–J) | (*p*-value) Sig.[a] |
|---|---|---|---|
| Treatment *A* | Treatment *B* | −1.697 | .097 |
| | Treatment *C* | −5.847* | .000 |
| Treatment *B* | Treatment *A* | 1.697 | .097 |
| | Treatment *C* | −4.151* | .000 |
| Treatment *C* | Treatment *A* | 5.847* | .000 |
| | Treatment *B* | 4.151* | .000 |

Dependent variable: Cold duration during treatment
Based on estimated marginal means
*The mean difference is significant at the .05 level
[a]Adjustment for multiple comparisons: Least significant difference (equivalent to no adjustments)

Table 9.8 shows the *F*-value for comparing the adjusted means of the criterion variable in three Vitamin_Dose groups (treatment *A*, treatment *B*, and treatment *C*). You can note that *F*-statistic computed for Vitamin_Dose is significant because *p* -value associated with it is .000 which is less than .05. Thus, the null hypothesis of no difference among the adjusted means for the data on criterion variable (number of days having cold symptoms during treatment) in three treatment groups may be rejected at 5% level.

*Remark:* You can see that the *F*-value for Pre_Days (covariate) is also significant. It shows that the initial conditions of the experimental groups are not same, and that is why we are applying ANCOVA after adjusting mean values of the criterion variable for the covariate.

(d) ***Post Hoc Comparison for the Group Means in Post-measurement Adjusted with the Initial Differences***

Since *F*-statistic is significant, post hoc comparison has been made for the adjusted means of the three treatment groups, which is shown in Table 9.9. This, table has been obtained by deleting some of the information from Table 9.5. It may be noted here that *p*-value for the mean difference between treatments *A* and *C* as well between treatments *B* and *C* is .000. Since *p* value is less than .05, both these mean differences are significant at 5% level. Thus, the following conclusions can be drawn:

(i) There is a significant difference between the adjusted means of criterion variable (Number of days having cold symptoms during treatment) in treatment *A* (High vitamin C dose) and treatment *C* (Placebo).

**Table 9.10** Post hoc comparison of adjusted means in different groups for the data on cold duration during treatment with graphics

| Treatment *C* | Treatment *B* | Treatment *A* |
|---|---|---|
| 12.4 | 8.2 | 6.5 |

"____" represents no significant difference between the means

Treatment *A*: Administering high dose of vitamin C
Treatment *B*: Administering low dose of vitamin C
Treatment *C*: Administering placebo

(ii) There is a significant difference between the adjusted means of criterion variable (Number of days having cold symptoms during treatment) in treatment *B* (Low vitamin C dose) and treatment *C* (Placebo).
(iii) There is no significant difference between the adjusted means of criterion variable (Number of days having cold symptoms during treatment) in treatment *A* (High vitamin C dose) and treatment *B* (Low vitamin dose).

In order to find as to which treatment is the best, one can see the adjusted mean values of criterion variable in different treatment groups given in Table 9.7. Clubbing these adjusted means with the three conclusions mentioned above, one may get the answer. However, this task becomes much easier if Table 9.10 is developed. This table can be created by using the values of different adjusted group means from Table 9.7 and using *p*-values of mean differences from Table 9.9. In this table, the adjusted means of the criterion variable in different treatment groups have been shown in the descending order. If the difference between any two group means is significant (which can be seen from Table 9.10), nothing is done, and if the mean difference is not significant, an underline is put below both the group means.

Thus, it may be concluded that the average curing time in high and low vitamin groups was same. Further, the average curing time in both these groups was significantly less than that of placebo group.

Hence, it may be inferred that high vitamin dose as well as low vitamin dose are equally effective in curing the cold symptoms in comparison to that of placebo.

## Summary of the SPSS Commands

(i) Start the SPSS by using the following commands:

**Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20**

(ii) Click **Variable View** tag and define the variables *Vitamin_Dose* as nominal variable and *Pre_Days* and *Post_Days* as scale variables.
(iii) Under the column heading **Values** against the variable *Vitamin_Dose,* define "1" for Treatment *A*, "2" for Treatment *B*, and "3" for Treatment *C*.
(iv) After defining the variables, type the data for these variables by clicking **Data View**.
(v) In the data view, follow the below-mentioned command sequence for ANCOVA:

**Analyze ⇒ General Linear Model ⇒ Univariate**

(vi) Select the variables *Cold duration during treatment*, *Vitamin dose,* and *Cold duration before treatment from* left panel to the "Dependent variable" section, "Fixed Factor(s)" section, and "Covariate(s)" section of the right panel, respectively.
(vii) Click the tag **Model** and select the Sum of Squares option as "Type I." Press ***Continue***.
(viii) Click the tag **Options** and select the variables *Overall* and *Vitamin_Dose* from the left panel to the "Display Means for" section of the right panel. Check the option "Compare main effects" and "Descriptive statistics." Ensure the value of significance as .05 or .01 as the case may be. Press ***Continue***.
(ix) Click **OK** for output.

## Exercise

*Short Answer Questions*

**Note:** Write answer to each of the following questions in not more than 200 words.

Q1. What do you mean by the covariate? How it is controlled in ANCOVA? Give a specific example.
Q2. Describe an experimental situation where ANCOVA can be applied. Construct null hypothesis and all possible alternative hypotheses.
Q3. Thirty boys were selected for direct marketing of a vacuum cleaner in three similar cities. In each of the city, 10 boys were sent for direct marketing for a month. Three different kinds of incentives, namely, conveyance allowance, two percent bonus, and gifts were offered to these sales agents in these three cities on completing the target. To compare the effectiveness of three different incentives on sale, which statistical technique should be used?
Q4. If two treatment groups are to be compared on some criterion variable, how do you interpret if the slopes of the two regression lines are same? Further, if the intercepts are equal, what it conveys? Explain by means of graphical representation.

Q5. Explain the statement "the analysis of covariance is a mix of one-way ANOVA and linear regression."
Q6. Why the observed mean of criterion variable is adjusted in ANCOVA? How this adjustment is done?
Q7. What are the various assumptions used in analysis of covariance?
Q8. Which design is more efficient and why among one-way ANOVA and ANCOVA?

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. In designing an experiment, if the randomization is not possible, control is observed by matching the groups. This matching is done on the variable which is

   (a) Independent
   (b) Extraneous
   (c) Dependent
   (d) Any variable found suitable

2. Covariate is a variable which is supposed to be correlated with

   (a) Criterion variable
   (b) Independent variable
   (c) Dependent variable
   (d) None of the above

3. In ANCOVA, while doing post hoc analysis, which group means are compared?

   (a) Pretest group means
   (b) Posttest group means
   (c) Pretest adjusted group means
   (d) Posttest adjusted group means

4. In ANCOVA, if the slopes of the regression lines in different treatment groups are same, one can infer that

   (a) Some of the treatments will show the improvement where the other treatments may show the deterioration.
   (b) All the treatments will show either deterioration or improvement but with varying degrees.
   (c) One cannot tell about the improvement or deterioration due to different treatments.
   (d) All treatments will have the same amount of improvement in the criterion variable.

5. In ANCOVA, if intercepts of the regression lines in the two treatment groups are same, then it may be inferred that

(a) One treatment is better than other.
(b) One cannot say which treatment is more effective.
(c) Both the treatments are equally effective.
(d) No conclusion can be drawn.

6. In ANCOVA model, the error component is independently and identically normally distributed with

(a) Mean 0 and variance 1
(b) Mean 1 and variance 0
(c) Equal mean and variance 1
(d) Mean 0 and equal variance

7. In ANCOVA, the adjusted mean $\mu$ in the $i$th treatment group is obtained from the formula

(a) $\mu = \overline{Y}_i + \beta(\overline{X}_i - \bar{X})$
(b) $\mu = \overline{Y}_i - \beta(\overline{X}_i - \bar{X})$
(c) $\mu = \overline{Y}_i - \beta(\bar{X} - \overline{X}_i)$
(d) $\mu = \overline{Y}_i + \beta + (\overline{X}_i - \bar{X})$

8. In analysis of covariance, the criterion variable should be

(a) Continuous
(b) Nominal
(c) Ordinal
(d) Dichotomous always

9. One of the assumptions in using ANCOVA is

(a) The data on criterion variable must have been obtained by stratified sampling.
(b) The regression coefficients for each treatment groups must be heterogeneous.
(c) The interaction between the criterion variable and covariate is significant.
(d) The criterion variable must have the same variance in each of the treatment groups.

10. Choose the correct statement.

(a) The ANCOVA is more efficient than ANOVA because part of the error variance is explained by the covariate.
(b) ANOVA is more efficient than ANCOVA if the initial conditions are not same.
(c) ANOVA and ANCOVA are equally effective, and it is the matter of choice as to which analysis is to be used.
(d) All the above statements are correct.

11. In order to compare the effectiveness of three training programs on financial knowledge, an experiment was planned. Three groups of employees were

tested for their financial knowledge before and after the training program. While using SPSS for ANCOVA, three variables, namely, Pre_Knowledge Drib, Post_Knowledge, and Treatment_Group, need to be defined. Choose the correct types of each variable.

(a) Pre_Knowledge and Post_Knowledge are Scale and Treatment_Group is Ordinal.
(b) Pre_Knowledge and Post_Knowledge are Nominal and Treatment_Group is Scale.
(c) Pre_Knowledge and Treatment_Group are Scale and Post_Knowledge is Nominal.
(d) Pre_Knowledge and Post_Knowledge are Scale and Treatment_Group is Nominal.

12. While using SPSS for ANCOVA, the three variables, namely, Pre_Test, Post_-Test, and Treatment_Group, are classified as

(a) Post_Test as Dependent variable whereas Pre_Test and Treatment_Group as Fixed Factor(s)
(b) Post_Test as Dependent variable, Pre_Test as Covariate, and Treatment_Group as Fixed Factor
(c) Treatment_Group as Dependent variable, Pre_Test and Post_Test as Fixed Factor(s)
(d) Treatment_Group as Dependent variable, Post_Test as Covariate, and Pre_Test as Fixed Factor

13. Choose the correct sequence of commands in SPSS for starting ANCOVA.

(a) Analyze → Univariate → General Linear Model
(b) Analyze → General Linear Model → Multivariate
(c) Analyze → General Linear Model → Univariate
(d) Analyze → General Linear Model → Repeated Measures

*Assignments*

1. In a psychological experiment 60, subjects were randomly divided into three equal groups. These groups were taught with audiovisual aid, traditional method, and need-based methods. Prior to the treatments, learning motivation of all the subjects was assessed. After 4 weeks, improvement in academic achievements was noted. The data so obtained on academic achievements is shown in the Table A-1.
   Apply analysis of covariance to see as to which methodology of teaching is more effective for academic achievement. Test your hypothesis at .05 as well as .01 level of significance.
2. A study was conducted to know the impact of gender on life optimism. Since age is considered as factor effecting life optimism, it was considered as covariate.

**Table A-1** Scores on academic achievements and learning motivation in three types of teaching methods

| S.N. | Audiovisual group | | Traditional group | | Need-based group | |
|---|---|---|---|---|---|---|
| | Motivation | Achievement | Motivation | Achievement | Motivation | Achievement |
| 1 | 2 | 5 | 2 | 3 | 1 | 12 |
| 2 | 1 | 10 | 3 | 3 | 3 | 18 |
| 3 | 3 | 12 | 1 | 9 | 2 | 11 |
| 4 | 0 | 14 | 1 | 13 | 6 | 25 |
| 5 | 1 | 14 | 4 | 13 | 3 | 9 |
| 6 | 5 | 16 | 5 | 13 | 5 | 18 |
| 7 | 3 | 18 | 6 | 13 | 3 | 12 |
| 8 | 4 | 18 | 1 | 15 | 4 | 10 |
| 9 | 4 | 18 | 2 | 15 | 3 | 11 |
| 10 | 5 | 18 | 4 | 17 | 6 | 16 |
| 11 | 2 | 22 | 6 | 17 | 7 | 18 |
| 12 | 3 | 22 | 5 | 21 | 4 | 14 |
| 13 | 7 | 28 | 2 | 22 | 3 | 17 |
| 14 | 4 | 24 | 5 | 22 | 2 | 10 |
| 15 | 6 | 24 | 5 | 22 | 5 | 19 |
| 16 | 3 | 26 | 5 | 22 | 3 | 14 |
| 17 | 4 | 26 | 5 | 22 | 2 | 16 |
| 18 | 4 | 26 | 6 | 20 | 3 | 15 |
| 19 | 8 | 26 | 9 | 22 | 4 | 16 |
| 20 | 3 | 29 | 4 | 24 | 5 | 15 |

**Table A-2** Data on age and life optimism of the male and female

| | Male | | Female | |
|---|---|---|---|---|
| S.N. | Age | Life optimism | Age | Life optimism |
| 1 | 53 | 18 | 20 | 26 |
| 2 | 38 | 26 | 24 | 15 |
| 3 | 18 | 20 | 18 | 16 |
| 4 | 26 | 27 | 38 | 17 |
| 5 | 39 | 19 | 35 | 16 |
| 6 | 38 | 29 | 25 | 24 |
| 7 | 30 | 15 | 17 | 10 |
| 8 | 60 | 23 | 19 | 18 |
| 9 | 22 | 22 | 21 | 19 |
| 10 | 31 | 14 | 19 | 19 |
| 11 | 21 | 14 | 18 | 21 |
| 12 | 25 | 23 | 38 | 13 |
| 13 | 22 | 23 | 37 | 15 |
| 14 | 20 | 19 | 21 | 11 |
| 15 | 27 | 23 | 20 | 11 |
| 16 | 24 | 20 | 20 | 14 |
| 17 | 29 | 18 | 41 | 16 |
| 18 | 27 | 19 | 40 | 17 |
| 19 | 32 | 13 | 28 | 17 |
| 20 | 17 | 14 | 99 | 8 |

A questionnaire was administered on 20 male and 20 female subjects to know their life optimism. Their age was also noted. The data so obtained are listed in the Table A-2.

Apply analysis of covariance and discuss your findings to compare the life optimism among male and female adjusted for their age. Test your hypothesis at 5% as well as at 1% level.

*Answers to Multiple-Choice Questions*

| | | | | | |
|---|---|---|---|---|---|
| Q.1 | b | Q.2 | a | Q.3 | d |
| Q.4 | b | Q.5 | c | Q.6 | d |
| Q.7 | b | Q.8 | a | Q.9 | d |
| Q.10 | a | Q.11 | d | Q.12 | b |
| Q.13 | c | | | | |

# Chapter 10
# Cluster Analysis: For Segmenting the Population

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Understand the concept of cluster analysis.
- Know the different terminologies used in cluster analysis.
- Learn to compute different distances used in the analysis.
- Understand different techniques of clustering.
- Describe the assumptions used in the analysis.
- Explain the situations where cluster analysis can be used.
- Learn the procedure of using cluster analysis.
- Know the use of hierarchical cluster analysis and *K*-means cluster analysis.
- Describe the situation under which two-step cluster should be used.
- Understand various outputs of cluster analysis.
- Know the procedure of using cluster analysis with SPSS.
- Understand different commands and its outcomes used in SPSS for cluster analysis.
- Learn to interpret the outputs of cluster analysis generated by the SPSS.

## Introduction

Market analysts are always in search of strategies responsible for buying behavior. The whole lot of customers can be grouped on the basis of their buying behavior patterns. This segmentation of customers helps analysts in developing marketing strategy for different products in different segments of customers. These segments are developed on the basis of buying behavior of the customers in such a way so that the individuals in the segments are more alike but the individuals in different segments differ to a great extent in their characteristics. The concept of segmenting may be used to club different television serials into homogeneous categories on the basis of their characteristics. An archaeological surveyor's may like to cluster different idol excavated from archaeological digs into the civilizations from which

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they originated. These idols may be clustered on the basis of their physical and chemical parameters to identify their age and civilization to which they belong. Doctors may diagnose a patient for viral infection and determine whether distinct subgroups can be identified on the basis of a clinical checklist and pathological tests. Thus, in different fields several situations may arise where it is required to segment the subjects on the basis of their behaviour pattern so that an appropriate strategy may be formed for these segments separately. Segmenting may also be done for the objects based on their similarity of features and characteristics. Such segmenting of objects may be useful for making a policy decision. For instance, all the cars can be classified into small, medium and large segments depending upon their features like engine power, price, seating capacity, luggage capacity, and fuel consumption. Different policy may be adopted to promote these segments of vehicle by the authorities.

The problem of segmentation shall be discussed in this chapter by means of cluster analysis. The more emphasis has been given on understanding various concepts of this analysis and the procedure used in it. Further, solved example has been discussed by means of using SPSS for easy understanding of readers. The reader should note as to how different outputs generated in this analysis by the SPSS have been interpreted.

## What Is Cluster Analysis?

Cluster analysis is a multivariate statistical technique for grouping cases of data based on the similarity of responses to several variables/subjects. The purpose of cluster analysis is to place subjects/objects into groups, or clusters, suggested by the data, such that objects in a given cluster are homogenous in some sense, and objects in different clusters are dissimilar to a great extent. In cluster analysis, the groups are not predefined but are rather suggested on the basis of the data. The cluster analysis can also be used to summarize data rather than to find observed clusters. This process is sometimes called dissection.

## Terminologies Used in Cluster Analysis

### *Distance Measure*

In cluster analysis, cases/objects are clustered on the basis of dissimilarities (similarities) or distances between cases/objects. These distances (similarities) can be based on a single or multiple parameters where each parameter represents a rule or condition for grouping cases/objects. For example, if we were to cluster the songs, we may take into account the song length, singer, subjective ratings of the

**Table 10.1** Employees' profile

| | Age | Income | Qualification |
|---|---|---|---|
| Employee 1 | 2.5 | 2.4 | 2.4 |
| Employee 2 | 2.3 | 2.1 | 1.9 |
| Employee 3 | 1.2 | 1.9 | −0.9 |
| Employee 4 | 1.5 | −0.4 | 1.3 |

listeners, etc. The simplest way of computing distances between cases in a multidimensional space is to compute Euclidean distances. There are many methods available for computing distances and it is up to the researcher to identify an appropriate method according to the nature of the problem. Although plenty of methods are available for computing distances between the cases, we are discussing herewith the five most frequently used methods. These methods for computing the distances shall be discussed later in this chapter by using some data.

Consider the data in Table 10.1 where age, income, and qualification are the three different parameters on which employees need to be grouped into different clusters. We will see the computation of distances between the two employees using different distance method.

### Squared Euclidean Distance

A Euclidean distance is a geometric distance between two cases or objects. This is the most natural way of computing a distance between two samples. It computes the difference between two samples directly on the basis of the changes in magnitude in the sample levels. Euclidean distance is usually used in a situation where data sets are suitably normalized. It is computed by taking the square root of the sum of the squared difference on each of the variable measurements between the two cases. The formula for its computation is given by

$$\mathrm{de}_{ij} = \sqrt{\sum_{k=1}^{n} (X_{ik} - X_{jk})^2} \tag{10.1}$$

where

$X_{ik}$ is the measurement of $i$th cases on $k$th variable
$X_{jk}$ is the measurement of $j$th cases on $k$th variable
$n$ is number of variables

Let us compute the Euclidean distance between first and second employee by using their profile as shown in Table 10.1.

**Table 10.2** Computation of Euclidean space between employees 1 and 2

| | Age | Income | Qualification |
|---|---|---|---|
| Employee 1 | 2.5 | 2.4 | 2.4 |
| Employee 2 | 2.3 | 2.1 | 1.9 |
| Difference | 0.2 | 0.3 | 0.5 |
| Squared difference | 0.04 | 0.09 | 0.25 |

**Table 10.3** Computation of Manhattan distance between employees 1 and 2

| | Age | Income | Qualification |
|---|---|---|---|
| Employee 1 | 2.5 | 2.4 | 2.4 |
| Employee 2 | 2.3 | 2.1 | 1.9 |
| Absolute difference | 0.2 | 0.3 | 0.5 |

The squared Euclidean distance between employee 1 and employee 2 can be obtained by using the formula 10.1. The computation has been shown in the Table 10.2.

$$\text{Thus, Squared Euclidean space between first and second employee} = 0.04 + 0.09 + 0.25 = 0.38$$

Since Euclidean distance is the square root of the squared Euclidean distance,

Euclidean distance between first and second employee $= \text{de}_{12} = \sqrt{0.38} = 0.62$.

In computing the Euclidean distance, each difference is squared to find the absolute difference on each of the variables measured on both the employees. After adding all of the squared differences, we take the square root. We do it because by squaring the differences, the units of measurements are changed, and so by taking the square root, we get back the original unit of measurement.

If Euclidean distances are smaller, the cases are more similar. However, this measure depends on the units of measurement for the variables. If variables are measured on different scales, variables with large values will contribute more to the distance measure than the variables with small values. It is therefore important to standardize scores before proceeding with the analysis if variables are measured on different scales. In SPSS, you can standardize variables in different ways.

### Manhattan Distance

The Manhattan distance between the two cases is computed by summing the absolute distances along each variable. The Manhattan distance is also known as city-block distance and is appropriate when the data set is discrete. By using the data of Table 10.1 the Manhattan distance between first and second employee has been computed in the Table 10.3.

Thus, the Manhattan distance between first and second employees $= \text{dm} = 0.2 + 0.3 + 0.5 = 1.00$.

### Chebyshev Distance

The Chebyshev distance between the two cases are obtained by finding the maximum absolute difference in values for any variable. This distance is computed if we want to define two cases as "different" if they differ on any one of the dimensions. The Chebyshev distance is computed as

$$\text{Chebyshev distance } (x, y) = dc = \text{Max } |x_i - y_i| \tag{10.2}$$

In Table 10.1, the Chebyshev distance between the first and fourth employees would be 2.8 as this is the maximum absolute difference of these two employees on income variable.

### Mahalanobis (or Correlation) Distance

The Mahalanobis distance is based on the Pearson correlation coefficient which is computed between the observations of two cases or subjects. This correlation coefficient is used to cluster the cases. This is an important measure as it is a scale invariant. In other words, it is not affected by the change in units of the observations. Thus, the Mahalanobis distance (dm) between first and second employees can be obtained by computing the correlation coefficient between the observations 2.5, 2.4, 2.4 and 2.3, 2.1, 1.9.

### Pearson Correlation Distance

The Pearson distance (dp) is also based on the Pearson correlation coefficient between the observations of the two cases. This distance is computed as $dp = 1-r$ and lies between 0 and 2. Since the maximum and minimum values of $r$ can be +1 and $-1$, respectively, the range of the Pearson distance (dp) can be from 0 to 2. The zero value of dp indicates that the cases are alike, and 2 indicates that the cases are entirely distinct.

## *Clustering Procedure*

In cluster analysis, each case/object is considered to be a single cluster. The distances between these objects are computed by the chosen distance measure. On the basis of these distances computed in the proximity matrix, several objects are linked together. After having done so, how do we determine the distances between these new clusters? In other words, we need to have a linkage or amalgamation criteria to determine when two clusters are sufficiently similar to be linked together. There are various protocols: for example, we may link two clusters

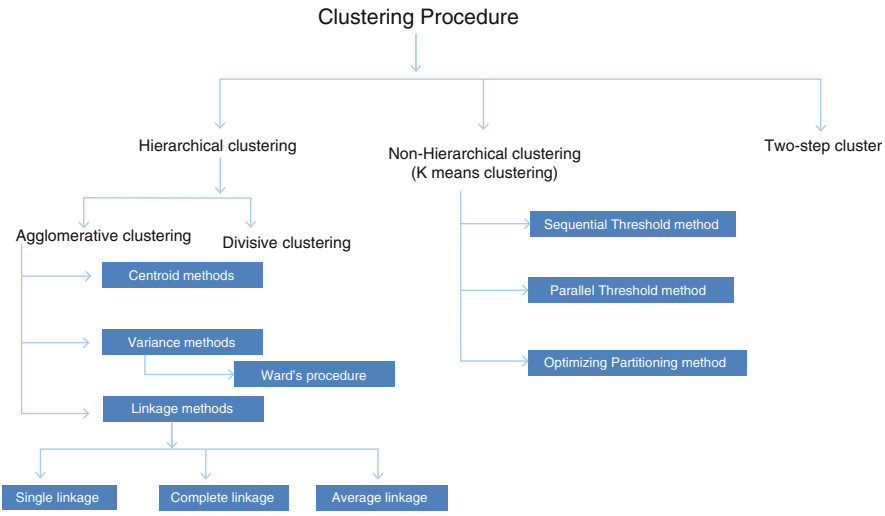



**Fig. 10.1** Different clustering procedures

together on the basis of the smallest distance between the two objects, one from each of the two different clusters. Similarly the two clusters may be linked together on the basis of the maximum distance between the two objects, one from each cluster. There are different ways the objects can be clustered together. The entire clustering procedures can be broadly classified in three different categories, that is, hierarchical clustering, nonhierarchical clustering, and two-step clustering. These procedures shall be discussed in detail under various headings in this section. The details of various classification procedures have been shown graphically in Fig. 10.1.

## Hierarchical Clustering

In hierarchical clustering, objects are organized into a hierarchical structure. It creates a hierarchy of clusters which may be represented in a treelike structure known as dendrogram. Objects are grouped into a tree of clusters by using the distance (similarity) matrix as clustering criteria. In this tree structure, the root consists of a single cluster containing all observations, whereas the leaves refer to the individual observations. *Hierarchical clustering* is the best for small data sets because in this procedure a proximity matrix of the distance/similarity is computed for each pair of cases in the data set.

Hierarchical clustering can be either agglomerative or divisive. In agglomerative clustering, one starts at the individual objects and successively merges clusters together. On the other hand, in the divisive clustering, one starts with all the objects as one cluster and recursively splits the clusters. We shall now discuss various types of clustering protocols of these two types of hierarchical clustering in detail.


**Fig. 10.2** Raw data showing the distances between the objects

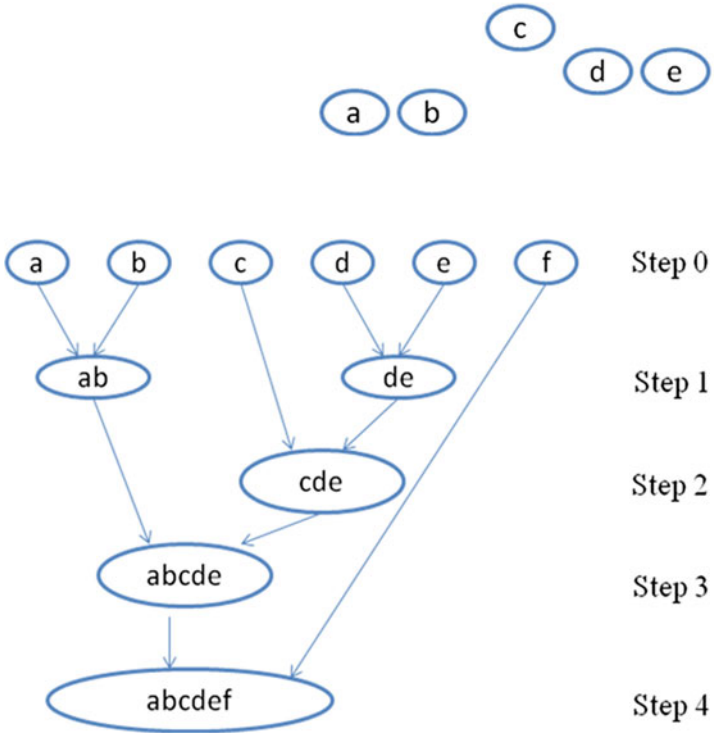



**Fig. 10.3** Formation of clusters at different stages in agglomerative clustering

## Agglomerative Clustering

In agglomerative clustering, all the individual objects/cases are considered as a separate cluster. These objects (atomic clusters) are successively merged into bigger and bigger clusters using specified measure of similarity between the pair of objects. The choice of which clusters to merge is determined by a linkage criteria. Thus, in agglomerative clustering, we start at the leaves and successively clusters are merged together to form the dendrogram. The clusters shall keep merging with each other until all of the objects are in a single cluster or until certain termination conditions are satisfied. The termination condition is decided by the researcher which depends upon the number of clusters required to be formed. One of the criteria in deciding the number of clusters to be formed depends upon whether some meanings can be attached to these clusters or not. Consider the following raw data in Fig. 10.2. Each data is a case/object and is considered to be the independent cluster. Depending upon the distances, these clusters are merged in different steps, and finally we get a single cluster. The formation of these clusters at different stages is shown in Fig. 10.3.

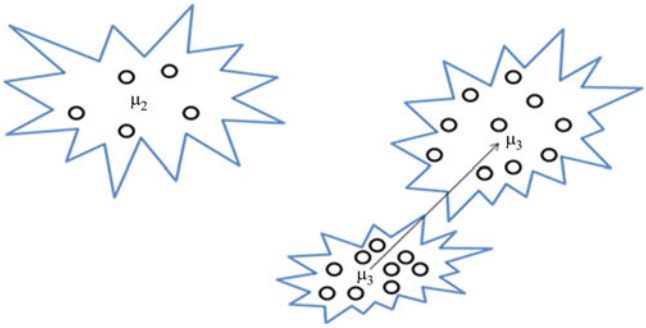



**Fig. 10.4** Linkage of clusters using centroid method

In agglomerative clustering, different methods are used to form the clusters. These methods are discussed below.

*Centroid Method*

In this method, clusters are merged on the basis of the Euclidean distance between the cluster centroids. Clusters having least Euclidean distance between their centroids are merged together. In this method, if two unequal sized groups are merged together, then larger of the two tends to dominate the merged cluster. Since centroid methods compare the means of the two clusters, outliers affect it less than most other hierarchical clustering methods. However, it may not perform well in comparison to Ward's method or average linkage method (Milligan 1980). Linkage of clusters using centroid method is shown in Fig. 10.4.

*Variance Methods*

In this method, clusters are formed that minimize the within cluster variance. In other words, clusters are linked if the variation within the two clusters is least. This is done by checking the squared Euclidean distance to the center mean. The method used in checking the minimum variance in forming clusters is known as Ward's minimum variance method. This method tends to join the clusters having small number of observations and is biased towards producing clusters with same shape and with nearly equal number of observations. The variance method is very sensitive to the outliers. If "a" to "g" represents seven clusters then cluster formation using Ward's method can be shown graphically in Fig. 10.5.

*Linkage Methods*

In agglomerative clustering, clusters are formed on the basis of three different types of linkage methods.

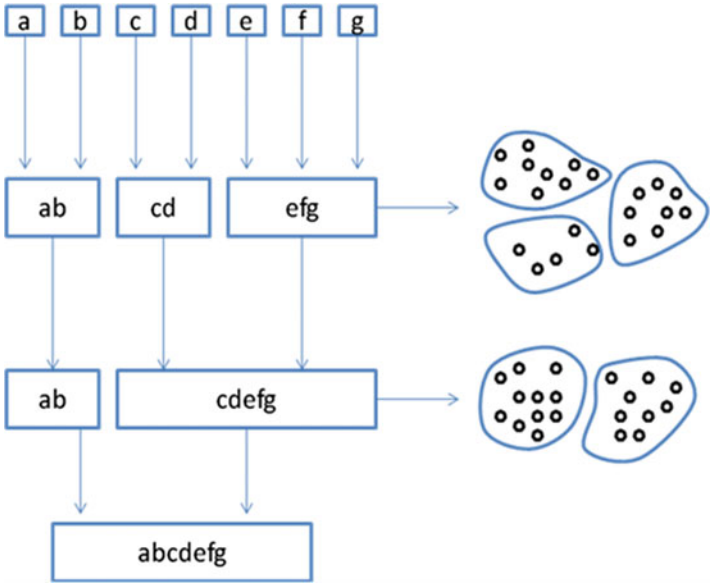
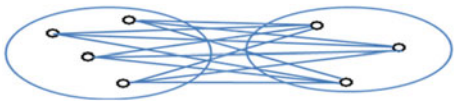



**Fig. 10.5** Linkage of clusters using variance method

**Fig. 10.6** Clusters based on single linkage

**Fig. 10.7** Clusters based on complete linkage

**Fig. 10.8** Clusters based on average linkage



1. *Single Linkage Method:* In this method, clusters are formed on the basis of minimum distance between the closest members of the two clusters. This is also known as nearest neighbor rule. This kind of linkage can be seen in Fig. 10.6.
2. *Complete Linkage Method:* In this method, clusters are formed on the basis of minimum distance between the farthest members of the two clusters. This is also known as furthest neighbor rule. Complete linkage can be shown by Fig. 10.7.
3. *Average Linkage Method*: This procedure uses the minimum average distance between all pairs of objects (in each pair one member must be from a different cluster) as the criteria to make the next higher cluster. Average linkage can be shown by Fig. 10.8.

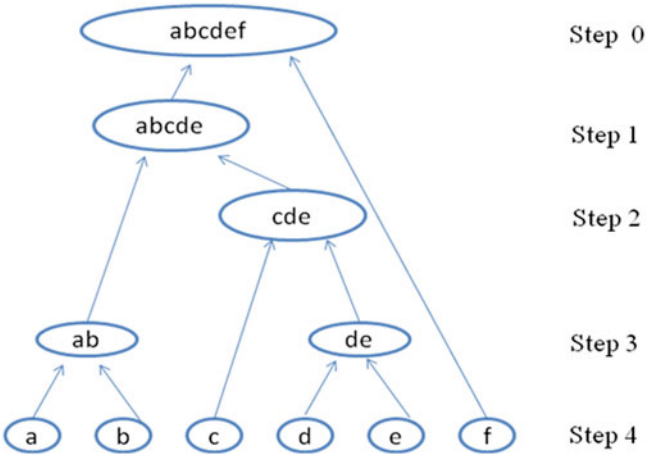



**Fig. 10.9** Formation of clusters at different stages in divisive clustering

Divisive Clustering

In divisive clustering, we start by considering all individual objects/cases as one cluster (called as root) and recursively splits into smaller and smaller clusters owing to any of the distance criteria, until each object forms a cluster on its own or until it satisfies certain termination conditions which depend upon the number of clusters to be formed. Here, data objects are grouped in a top down fashion. Thus, in divisive clustering, we start at the root and reaches to leaves. Divisive clustering is just the reverse of agglomerative clustering. Cluster formation in divisive clustering schedule can be seen in Fig. 10.9.

### Nonhierarchical Clustering (*K*-Means Cluster)

Unlike hierarchical clustering, in *K*-means clustering, a number of clusters are decided in advance. Solutions cannot be obtained for a range of clusters unless you rerun the analysis every time for different number of clusters. In *K*-means clustering, the first step is to find the *K*-centers. We start with an initial set of *K*-means and classify cases/objects based on their distances to the centers. Next, the cluster means are computed again using the cases/objects that are assigned to the cluster. After this, we reclassify all cases/objects based on the new set of means. This iterative process keeps going until cluster means do not change much between successive steps. Once the stability of cluster means is achieved, the means of the clusters are calculated once again and all the cases are assigned to their permanent clusters. If one can have a good guesses for the initial *K*-centers, those can be used as initial starting points; otherwise, let the SPSS find *K* cases that are well separated and use these values as initial cluster centers.

In hierarchical clustering, distance or similarity matrix between all pair of cases is required to be computed. This matrix becomes voluminous if the number of cases

is in thousands. Because of this, so much processing is required, and even with the modern computer, one needs to wait for some time to get the results. On the other hand, *k*-means clustering method does not require computation of all possible distances.

Nonhierarchical clustering solution has three different approaches, that is, sequential threshold method, parallel threshold, and optimizing partitioning method.

The *sequential threshold* method is based on finding a cluster center and then grouping all objects that are within a specified threshold distance from the center. Here, one cluster is created at a time.

In *parallel threshold* method, several cluster centers are determined simultaneously and then objects are grouped depending upon the specified threshold distance from these centers. These threshold distances may be adjusted to include more or fewer objects in the clusters.

The *optimizing partitioning* is similar to other two nonhierarchical methods except it allows for reassignment of objects to another cluster depending on some optimizing criterion. In this method, a nonhierarchical procedure is run first, and then objects are reassigned so as to optimize an overall criterion.

***Precautions***: *K*-means clustering is very sensitive toward the outliers because they will usually be selected as initial cluster centers. If outlier exists in the data, this will result in outliers forming clusters with small number of cases. Therefore, it is important for the researcher to screen the data for outliers and remove them before starting the cluster analysis.

### Two-Step Cluster

Two-step clustering procedure is an exploratory statistical tools used for identifying the natural grouping of cases/objects within a large data set. It is an efficient clustering procedure in a situation where the data set is very large. This procedure has an ability to create clusters if some of the variables are continuous and others are categorical. It provides automatic identification of number of clusters present in the data.

There are two assumptions in this analysis: first, the variables are independent, and, second, each continuous variable follows a normal distribution whereas each categorical variable has a multinomial distribution. The two-step cluster analysis procedure provides solution in two steps which are explained as follows:

**Step 1:** Pre-cluster Formation

Pre-clusters are the clusters of original cases/objects that are used in place of raw data to reduce the size of the distance matrix between all possible pair of cases. After completing the pre-clustering, the cases in the same pre-cluster are treated as a single entity. Thus, the size of the distance matrix depends upon the number of pre-clusters instead of cases. Hierarchical clustering method is used on these pre-clusters instead of the original cases.

**Step 2:** Clustering Solutions Using Pre-clusters

In the second step, the standard hierarchical clustering algorithm is used on the pre-clusters for obtaining the cluster solution. The agglomerative clustering algorithm may be used to produce a range of cluster solutions. To determine which number of clusters is the best, each of these cluster solutions may be compared using either Schwarz's Bayesian criterion (BIC) or the Akaike information criterion (AIC) as the clustering criterion. The readers are advised to read about these procedures from some other texts.

## *Standardizing the Variables*

Cluster analysis is normally used for the data measured on interval scale and rarely used for ratio data. In cluster analysis, distances are computed between the pair of cases on each of the variables. And if the units of measurement for these variables are different, then one must be worried about its impact on these distances.

Variables having larger values will have a larger impact on the distance compared to variables that have smaller values. In that case, one must standardize the variables to a mean of 0 and a standard deviation of 1.

If the variables are measured on interval scale and range of scale is same for each of the variable, then standardization of variables is not required, but if its range of measurement scale is different for different variables or if they are measured on ratio scale, then one must standardize the variables in some way so that they all contribute equally to the distance or similarity between cases.

## *Icicle Plots*

It is the plotting of cases joining to form the clusters at each stage. You can see in Fig. 10.10 what is happening at each step of the cluster analysis when average linkage between groups is used to link the clusters. The figure is called an *icicle plot* because the columns representing cases look like icicles hanging from eaves. Each column represents one of the case/object you are clustering. Each row represents a cluster solution with different numbers of clusters.

If you look at the figure from bottom up, the last row (not shown) is the first step of the analysis. Each of the cases is a cluster of its own. The number of clusters at that point is 6. The five-cluster solution arises when the cases "a" and "b" are joined into a cluster. It is so because they had the smallest distance of all pairs. The four-cluster solution results from the merging of the cases "d" and "e" into a cluster. The three-cluster solution is the result of combining the cases "c" with "de." Going similarly, for the one cluster solution, all of the cases are combined into a single cluster.

| | | a | | b | | c | | d | | e | | f | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of Clusters | 1 | X | X | X | X | X | X | X | X | X | X | X | X |
| | 2 | X | X | X | X | X | X | X | X | X | X | | X |
| | 3 | | X | X | X | | X | X | X | X | X | | X |
| | 4 | | X | X | X | | X | | X | X | X | | X |
| | 5 | | X | X | X | | X | | X | | X | | X |

**Fig. 10.10** Vertical icicle plot

*Remarks*

1. When pairs of cases are tied for the smallest distance to form a cluster, an arbitrary selection is made. And, therefore, if cases are sorted differently, you might get a different cluster solution. But that should not bother you as there is no right or wrong answer to a cluster analysis. Many groupings are equally viable.
2. In case of large number of cases in cluster analysis, icicle plot can be developed by taking cases as rows. You must specify the "Horizontal" on the Cluster Plots dialog box.

## *The Dendrogram*

The dendrogram is the graphical display of the distances on which clusters are combined. The dendrogram can be seen in Fig. 10.22 and is read from left to right. Vertical lines show joined clusters. The position of the line on the scale represents the distance at which clusters are joined. The observed distances are rescaled to fall into the range of 1–25, and hence you do not see the actual distances; however, the ratio of the rescaled distances within the dendrogram is the same as the ratio of the original distances. In fact, the dendrogram is the graphical representation of the information provided by the agglomeration schedule.

## *The Proximity Matrix*

Consider the data of four employees on three different parameters age, income, and qualification as shown in the Table 10.4. Let us see how the proximity matrix is developed on these data.

The proximity matrix is the arrangement of squared Euclidean distances in rows and columns obtained between all pairs of cases. The squared Euclidean distances shall be computed by adding the squared differences between the two employees on each of the three variables.

**Table 10.4** Employees' profile

| | Age | Income | Qualification |
|---|---|---|---|
| Employee 1 | 2.5 | 2.4 | 2.4 |
| Employee 2 | 2.3 | 2.1 | 1.9 |
| Employee 3 | 1.2 | 1.9 | −0.9 |
| Employee 4 | 1.5 | −0.4 | 1.3 |

**Table 10.5** Proximity matrix

| | Squared Euclidean Distance | | | |
|---|---|---|---|---|
| Cases | Employee 1 | Employee 2 | Employee 3 | Employee 4 |
| Employee 1 | 0 | 0.38 | 12.83 | 10.05 |
| Employee 2 | 0.38 | 0 | 2.25 | 7.25 |
| Employee 3 | 12.83 | 2.25 | 0 | 10.22 |
| Employee 4 | 10.05 | 7.25 | 10.22 | 0 |

The distance between employees 1 and 2 $= (2.5-2.3)^2 + (2.4-2.1)^2 + (2.4-1.9)^2 = .04 + .09 + .25 = 0.38$

The distance between employees 1 and 4 $= (2.5–1.5)^2 + (2.4 + 0.4)^2 + (2.4-1.3)^2 = 1.00 + 7.84 + 1.21 = 10.05$

The distance between employees 2 and 3 $= (2.3-1.2)^2 + (2.1-1.9)^2 + (1.9 + 0.9)^2 = 1.21 + 0.04 + 1.00 = 2.25$

This way, all distances can be computed which are shown in Table 10.5. This table is known as the proximity matrix.

All the entries in the diagonal are 0 because an employee does not differ with himself. The smallest difference between two employees is 0.38, the distance between the employee 1 and employee 2. The largest distance, 12.83, occurs between employee 1 and employee 3. The distance matrix is symmetric, and, therefore, you can see that the distance between the first and third employee is same as the distance between the third and first employee.

## What We Do in Cluster Analysis

In using cluster analysis, one needs to follow different steps to get the final results. You may not understand all the steps at this moment but use it as a blueprint of the analysis and proceed further, and I am sure by the time you finish reading the entire chapter, you will have a fairly good idea about its application. Once you understand different concepts of cluster analysis discussed in this chapter, you will be taken to a solved example by using SPSS, and this will give you practical knowledge of using this analysis to your data set with SPSS. Below are the steps which are used in cluster analysis:

1. Identify the variables on which subjects/objects need to be clustered.

2. Select the distance measure for computing distance between cases. One can choose any of the distance measures like squared Euclidean distance, Manhattan distance, Chebyshev distance, or Mahalanobis (or correlation) distance.
3. Decide the clustering procedure to be used from the wide variety of clustering procedure available in the hierarchical or nonhierarchical clustering sections.
4. Decide on the number of clusters to be formed. The sole criteria in deciding the number of clusters is based on the fact that one should be able to explain these clusters on the basis of their characteristics.
5. Map and interpret clusters using illustrative techniques like perceptual maps, icicle plots, and dendrograms and draw conclusions.
6. Assess reliability and validity of the obtained clusters by using any one or more of the following methods:

   (i) Apply the cluster analysis on the same data by using different distance measure.
   (ii) Apply the cluster analysis on the same data by using different clustering technique.
   (iii) Split the same data randomly into two halves and apply the cluster analysis separately on each part.
   (iv) Repeat cluster analysis on same data several times by deleting one variable each time.
   (v) Repeat cluster analysis several times, using a different order each time.

## Assumptions in Cluster Analysis

Following assumptions need to be satisfied in cluster analysis:

1. The cluster analysis is usually used for the data measured on interval scale. However, it can be applied for any type of data. If the variable set includes continuous as well as categorical, then two-step cluster should be used.
2. The variables in the cluster analysis should be independent with each other.
3. Inter-object similarity is often measured by Euclidean distance between pairs of objects.
4. The data needs to be standardized if the range or scale of measurement of one variable is much larger or different from the range of others.
5. In case of nonstandardized data, Mahalanobis distance is preferred as it compensates for intercorrelation among the variables.
6. In applying two-step cluster with continuous as well as categorical variables, it is assumed that the continuous variables are normally distributed whereas categorical variables have multinomial distribution.

## Research Situations for Cluster Analysis Application

Cluster analysis can be applied to a wide variety of research problems in the area of management, psychology, medicine, pharmaceuticals, social sciences, etc. Following are the situations where this technique can be applied:

1. Cluster analysis can be used to classify the consumer population into market segments for understanding the requirements of potential customers in different groups. Such studies may be useful in segmenting the market, identifying the target market, product positioning, and developing new products.
2. In a big departmental store, all inventories may be clustered into different groups for placing them in same location or giving the similar code for enhancing sale and easy monitoring of the products.
3. In the field of psychiatry, the cluster analysis may provide the cluster of symptoms such as paranoia and schizophrenia, which is essential for successful therapy.
4. In educational research, all schools of a district can be classified into different clusters on the basis of the parameters like number of children, teacher's strength, total grant, school area, and location to develop and implement the programs and policies effectively for each of these groups separately.
5. In the area of mass communication, television channels may be classified into homogenous groups based on certain characteristics like TRP, number of programs televised per week, number of artists engaged, coverage time, programs in different sectors, advertisements received, and turnover. Different policies may be developed for different groups of channels by the regulatory body.
6. In medical research, cluster analysis may provide the solution for clustering of diseases so that new drugs may be developed for different clusters of diseases. This analysis may also be useful in clustering the patients on the basis of symptoms for easy monitoring of drug therapy on mass scale.

## Steps in Cluster Analysis

By learning terminologies involved in cluster analysis and the guidelines discussed in the heading "What We Do in Cluster Analysis?", you are now in a better position to understand the procedure of its use for addressing your objectives. The cluster analysis is usually done in two stages. The whole analysis is carried out in two stages, the details of which have been discussed in the following steps:

Stage 1

1. The first step in cluster analysis is to apply the hierarchical cluster analysis in SPSS to find the agglomerative schedule and proximity matrix for the data obtained on each of the variables for all the cases. To form clusters, you need

to select a criterion for determining similarity or distance between cases and a linkage criterion for merging clusters at successive steps. After doing so, the SPSS output provides proximity matrix which shows the distances (similarity) between all the cases/objects and agglomerative schedule which is used to find the number of clusters present in the data on the basis of fusion coefficients. The detailed discussion as to how to do it shall be made while discussing the solved example of cluster analysis using SPSS.

2. Prepare icicle plot and dendrogram of the data. These two figures can be obtained by providing options in the SPSS. The icicle plot is the visual representation of the agglomerative schedule whereas the dendrogram plot shows how distant (or close) cases are when they are combined.

Stage 2

3. The second step in cluster analysis is to apply the *K*-means cluster analysis in SPSS. The process is not stopped in the first stage just because of the fact that *K*-means analysis provides much stable clusters due to interactive procedure involved in it in comparison to the single-pass hierarchical methods. The *K*-means analysis provides four outputs, namely, initial cluster centers, case listing of cluster membership, final cluster centers, and analysis of variance for all the variables in each of the clusters.
4. The case listing of cluster membership is used to describe as to which case belongs to which of the clusters.
5. The final cluster centers are obtained by doing iteration on the initial cluster solutions. It provides the final solution. On the basis of final cluster centers, the characteristics of different clusters are explained.
6. Finally, ANOVA table describes as to which of the variables is significantly different across all the identified clusters in the problem.

The detailed discussion of the above-mentioned outputs in cluster analysis shall be done by means of the results obtained in the solved example using SPSS.

## Solved Example of Cluster Analysis Using SPSS

**Example 10.1** A media company wants to cluster its target audience in terms of their preferences toward quality, contents, and features of FM radio stations. Twenty randomly chosen students were selected from a university who served the sample for the study. Below-mentioned 14 questions were finally selected by their research team after the content and item analysis which measured many of the variables of interest. The respondents were asked to mark their responses on a 5-point scale where 1 represented complete disagreement and 5 complete agreement. The responses of the respondents on all the 12 questions that measured different dimensions of FM stations are shown in Table 10.6.

**Table 10.6** Response of students on the questions related to quality, contents, and features of FM radio stations

| SN | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 | Q14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 4 | 2 | 5 | 1 | 3 | 5 | 2 | 1 | 4 | 3 | 4 | 3 | 4 |
| 2 | 1 | 4 | 4 | 2 | 5 | 2 | 3 | 5 | 2 | 3 | 4 | 2 | 2 | 3 |
| 3 | 2 | 2 | 3 | 3 | 2 | 4 | 2 | 3 | 4 | 4 | 2 | 2 | 4 | 4 |
| 4 | 5 | 3 | 3 | 4 | 4 | 4 | 5 | 3 | 2 | 5 | 2 | 5 | 3 | 5 |
| 5 | 4 | 1 | 2 | 4 | 1 | 1 | 5 | 4 | 2 | 4 | 3 | 4 | 2 | 4 |
| 6 | 4 | 2 | 3 | 4 | 2 | 5 | 2 | 1 | 5 | 2 | 1 | 3 | 5 | 3 |
| 7 | 2 | 3 | 2 | 2 | 3 | 4 | 3 | 4 | 4 | 3 | 4 | 3 | 5 | 2 |
| 8 | 5 | 2 | 2 | 5 | 2 | 4 | 5 | 2 | 2 | 4 | 1 | 5 | 1 | 4 |
| 9 | 2 | 4 | 4 | 2 | 5 | 3 | 4 | 4 | 3 | 3 | 5 | 2 | 2 | 2 |
| 10 | 3 | 4 | 4 | 2 | 4 | 3 | 2 | 5 | 2 | 4 | 3 | 3 | 2 | 2 |
| 11 | 4 | 5 | 4 | 3 | 5 | 4 | 4 | 4 | 1 | 1 | 5 | 4 | 3 | 2 |
| 12 | 2 | 4 | 5 | 1 | 4 | 2 | 2 | 4 | 2 | 4 | 3 | 4 | 4 | 3 |
| 13 | 2 | 5 | 4 | 2 | 5 | 3 | 3 | 5 | 3 | 2 | 5 | 3 | 3 | 2 |
| 14 | 1 | 5 | 4 | 5 | 4 | 3 | 2 | 5 | 3 | 3 | 5 | 4 | 4 | 2 |
| 15 | 2 | 5 | 5 | 3 | 4 | 2 | 3 | 4 | 4 | 3 | 4 | 3 | 3 | 3 |
| 16 | 5 | 3 | 2 | 4 | 5 | 2 | 4 | 4 | 3 | 5 | 2 | 5 | 2 | 5 |
| 17 | 5 | 2 | 3 | 5 | 2 | 3 | 5 | 2 | 4 | 5 | 3 | 4 | 4 | 4 |
| 18 | 5 | 2 | 2 | 2 | 2 | 4 | 4 | 3 | 4 | 2 | 2 | 2 | 4 | 1 |
| 19 | 4 | 3 | 3 | 3 | 4 | 5 | 2 | 3 | 5 | 4 | 3 | 2 | 5 | 2 |
| 20 | 3 | 4 | 4 | 1 | 2 | 4 | 4 | 2 | 4 | 2 | 3 | 4 | 4 | 3 |

*Questions on quality, contents, and features of FM stations*

1. The FM station should provide more old Hindi songs.
2. FM stations must help an individual in solving their personal problems.
3. The presentation style of RJs helps popularizing an FM station.
4. An FM station should provide some kind of prizes/incentives to its listeners.
5. The station must telecast latest songs more often.
6. The FM stations must contain more entertaining programs.
7. Popularity of RJs depends upon their humor and ability to make program interesting.
8. FM station should provide more opportunity to listeners to talk to celebrities.
9. RJs voice must be clear and melodious.
10. FM channels should play $24 \times 7$.
11. FM stations should give information for other sports along with cricket.
12. FM stations should provide information regarding educational/professional courses available in the city.
13. FM stations should provide information regarding different shopping offers available in the city.
14. RJs should speak in an understandable language, preferably in local language.

*Solution* In earlier chapters, you have seen the procedure of applying different statistical techniques by using SPSS. By now, you must have been well acquainted with the procedure of starting the SPSS on the system, defining variables and their

characteristics and preparing data file, and, therefore, these steps shall be skipped in this chapter. In case of any clarification, readers are advised to go through Chap. 1 for detailed guidelines for preparing the data file.

The steps involved in using SPSS for cluster analysis shall be discussed first, and then the output obtained from the analysis shall be shown and explained. The whole scheme of cluster analysis with SPSS is as follows:

### *Stage 1*

First of all, the hierarchical cluster analysis shall be done by using the sequence of SPSS commands. The following outputs would be generated in this analysis:

(a) Proximity matrix of distances (similarity) between all the cases/objects
(b) Agglomerative schedule
(c) Icicle plot
(d) Dendrogram

On the basis of fusion coefficients in the agglomerative schedule, the number of clusters (say $K$) is decided.

### *Stage 2*

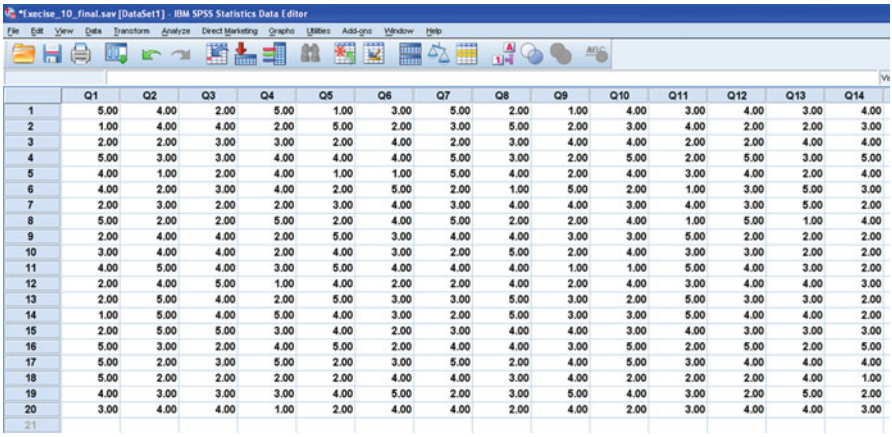
After deciding the number of clusters in the hierarchical cluster analysis, the data is again subjected to $K$-means cluster analysis in SPSS. Using this analysis, the following outputs would be generated:

(a) Initial cluster centers
(b) Case listing of cluster membership
(c) Final cluster centers
(d) Analysis of variance for comparing the clusters on each of the variables

### *Stage 1: SPSS Commands for Hierarchical Cluster Analysis*

(a) ***Data file*** After defining variable names and their labels, prepare the data file for the responses of the students on all the variables shown in Table 10.2. The data file shall look like as shown in Fig. 10.11.
(b) ***Initiating command for hierarchical cluster analysis:*** After preparing the data file, start the hierarchical analysis in SPSS by the following command sequence (Fig. 10.12):

**Analyze → Classify → Hierarchical Cluster**




| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | Q13 | Q14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5.00 | 4.00 | 2.00 | 5.00 | 1.00 | 3.00 | 5.00 | 2.00 | 1.00 | 4.00 | 3.00 | 4.00 | 3.00 | 4.00 |
| 2 | 1.00 | 4.00 | 4.00 | 2.00 | 5.00 | 2.00 | 3.00 | 5.00 | 2.00 | 3.00 | 4.00 | 2.00 | 2.00 | 3.00 |
| 3 | 2.00 | 2.00 | 3.00 | 3.00 | 2.00 | 4.00 | 2.00 | 3.00 | 4.00 | 4.00 | 2.00 | 2.00 | 4.00 | 4.00 |
| 4 | 5.00 | 3.00 | 3.00 | 4.00 | 4.00 | 4.00 | 5.00 | 3.00 | 2.00 | 5.00 | 2.00 | 5.00 | 3.00 | 5.00 |
| 5 | 4.00 | 1.00 | 2.00 | 4.00 | 1.00 | 1.00 | 5.00 | 4.00 | 2.00 | 4.00 | 3.00 | 4.00 | 2.00 | 4.00 |
| 6 | 4.00 | 2.00 | 3.00 | 4.00 | 2.00 | 5.00 | 2.00 | 1.00 | 5.00 | 2.00 | 1.00 | 3.00 | 5.00 | 3.00 |
| 7 | 2.00 | 3.00 | 2.00 | 2.00 | 3.00 | 4.00 | 3.00 | 4.00 | 4.00 | 3.00 | 4.00 | 3.00 | 5.00 | 2.00 |
| 8 | 5.00 | 2.00 | 2.00 | 5.00 | 2.00 | 4.00 | 5.00 | 2.00 | 2.00 | 4.00 | 1.00 | 5.00 | 1.00 | 4.00 |
| 9 | 2.00 | 4.00 | 4.00 | 2.00 | 5.00 | 3.00 | 4.00 | 4.00 | 3.00 | 3.00 | 5.00 | 2.00 | 2.00 | 2.00 |
| 10 | 3.00 | 4.00 | 4.00 | 2.00 | 4.00 | 3.00 | 2.00 | 5.00 | 2.00 | 4.00 | 3.00 | 3.00 | 2.00 | 2.00 |
| 11 | 4.00 | 5.00 | 4.00 | 3.00 | 5.00 | 4.00 | 4.00 | 4.00 | 1.00 | 1.00 | 5.00 | 4.00 | 3.00 | 2.00 |
| 12 | 2.00 | 4.00 | 5.00 | 1.00 | 4.00 | 2.00 | 2.00 | 4.00 | 2.00 | 4.00 | 3.00 | 4.00 | 4.00 | 3.00 |
| 13 | 2.00 | 5.00 | 4.00 | 2.00 | 5.00 | 3.00 | 3.00 | 5.00 | 3.00 | 2.00 | 5.00 | 3.00 | 3.00 | 2.00 |
| 14 | 1.00 | 5.00 | 4.00 | 5.00 | 4.00 | 3.00 | 2.00 | 5.00 | 3.00 | 3.00 | 5.00 | 4.00 | 4.00 | 2.00 |
| 15 | 2.00 | 5.00 | 5.00 | 3.00 | 4.00 | 2.00 | 3.00 | 4.00 | 4.00 | 3.00 | 4.00 | 3.00 | 3.00 | 3.00 |
| 16 | 5.00 | 3.00 | 2.00 | 4.00 | 5.00 | 2.00 | 4.00 | 4.00 | 3.00 | 5.00 | 2.00 | 5.00 | 2.00 | 5.00 |
| 17 | 5.00 | 2.00 | 3.00 | 5.00 | 2.00 | 3.00 | 5.00 | 2.00 | 4.00 | 5.00 | 3.00 | 4.00 | 4.00 | 4.00 |
| 18 | 5.00 | 2.00 | 2.00 | 2.00 | 2.00 | 4.00 | 4.00 | 3.00 | 4.00 | 2.00 | 2.00 | 2.00 | 4.00 | 1.00 |
| 19 | 4.00 | 3.00 | 3.00 | 3.00 | 4.00 | 5.00 | 2.00 | 3.00 | 5.00 | 4.00 | 3.00 | 2.00 | 5.00 | 2.00 |
| 20 | 3.00 | 4.00 | 4.00 | 1.00 | 2.00 | 4.00 | 4.00 | 2.00 | 4.00 | 2.00 | 3.00 | 4.00 | 4.00 | 3.00 |
| 21 | | | | | | | | | | | | | | |

**Fig. 10.11** Showing data file for all the variables in SPSS

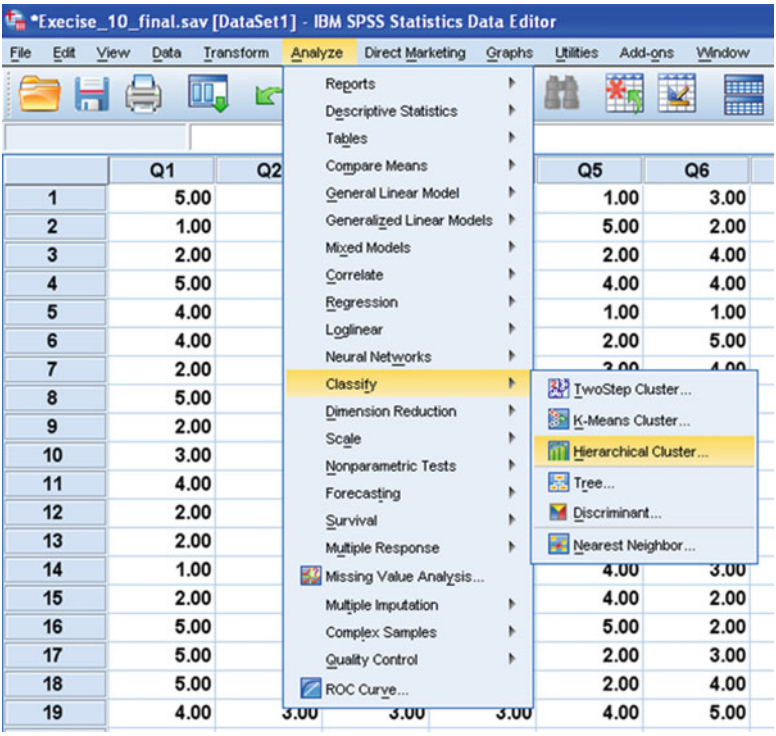



**Fig. 10.12** Sequence of SPSS commands for hierarchical cluster

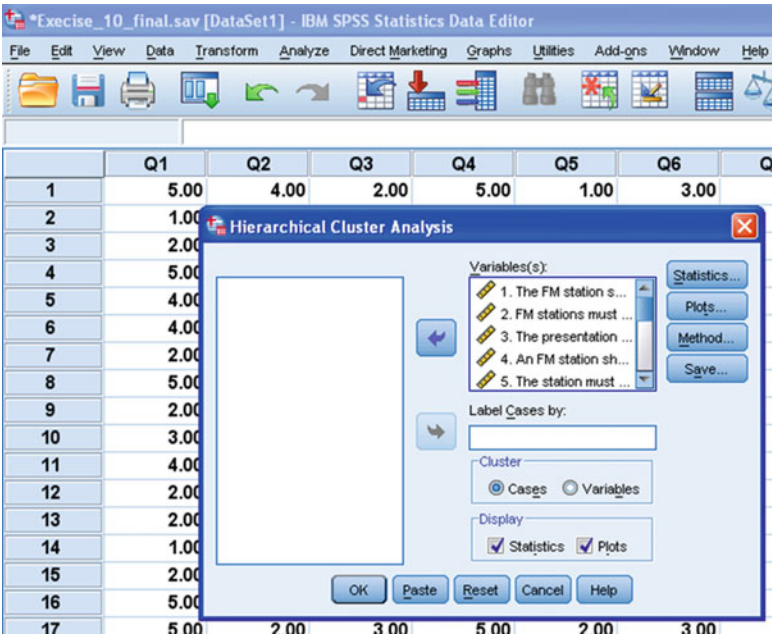



**Fig. 10.13** Selecting variables for hierarchical analysis

(i) *Selecting variables for analysis:* After clicking the **Hierarchical Cluster** option, you will be taken to the next screen for selecting variables. Select the variables as follows:

- Select all the variables and bring them in the "Variable(s)" section.
- Ensure that in the "Display" section, the options "Statistics" and "Plots" are checked. These are selected by default.
- In case if a variable denoting label of each cases is defined in the variable view while preparing the data file, then bring that variable under the section "Label Cases by." While defining the variable for label in the variable view, define its variable type as String under the column heading **Type.** However, for the time being, you can skip the process of defining the variable for label and leave the option "Label Cases by" blank.
  The screen will look like as shown in Fig. 10.13.

(ii) *Selecting options for computation:* After selecting the variables, you need to define different options for generating all the four outputs of hierarchical analysis. Take the following steps:

- Click the tag **Statistics** in the screen shown in Fig. 10.13 and take the following steps:
- Ensure that the "Agglomerative schedule" is checked. By default, it is checked.

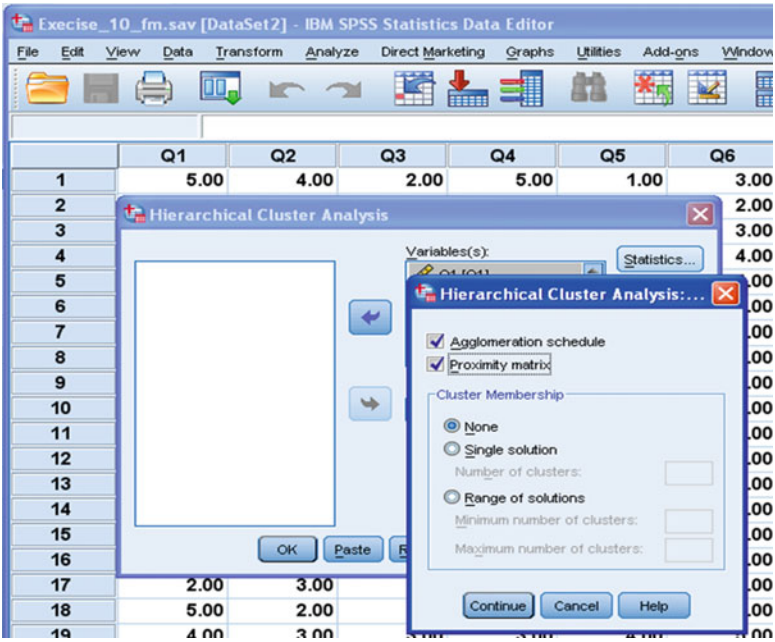



**Fig. 10.14** Screen showing option for generating agglomerative schedule and proximity matrix

- Check "Proximity matrix."
- Leave other options by default and click ***Continue***.

The screen will look like Fig. 10.14.

- Click the tag **Plots** in the screen shown in Fig. 10.13 and take the following steps:
  - Check the option "Dendrogram."
  - Ensure that the option "All clusters" is checked in the "icicle plot" section. This is checked by default.
  - Ensure that the option "Vertical" is checked in the "Orientation section." This is also checked by default. The option "Vertical" is selected if the number of cases is small. However, if the number of cases is large, then select "Horizontal."
  - Click ***Continue***.

The screen will look like Fig. 10.15.

- Click the tag **Method** in the screen shown in Fig. 10.13 and do the following steps:
  - Select the option "Ward's method" as cluster method. You can choose any other linkage method. For details read the methods under the heading

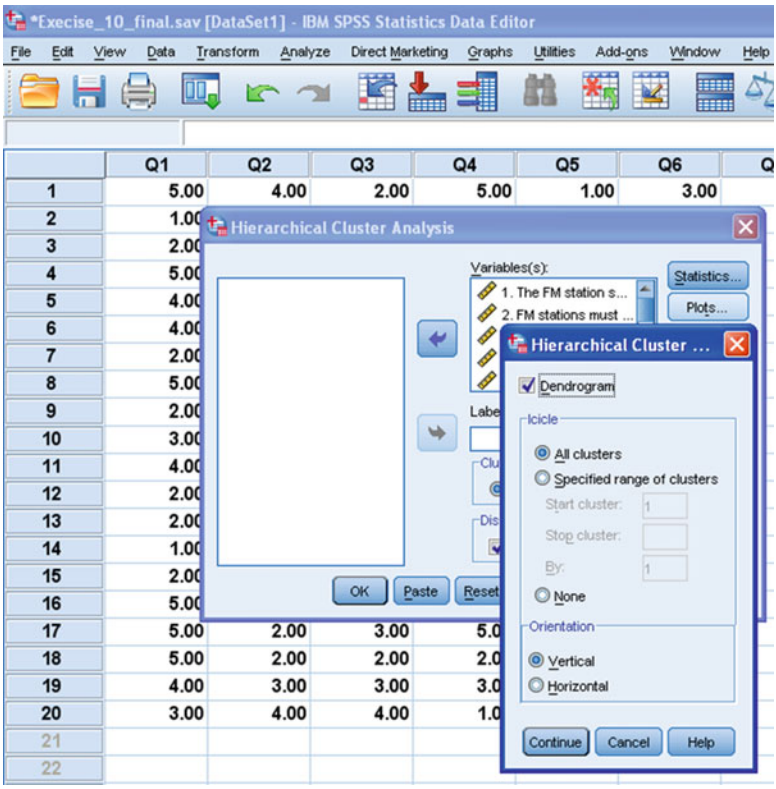



**Fig. 10.15** Selecting options for dendrogram and icicle plot

Distance Method discussed earlier. Personally I prefer the Ward's method as it depends upon the minimum variance concept and gives the clusters which are more homogenous within itself.

- Select the option "Squared Euclidean distance" as an interval measure. However, you can choose any other method as a distance measure like Euclidean distance, Pearson correlation method, or Chebyshev method. But generally squared Euclidean method is used to find the distance in the proximity matrix.
- Select the option "None" in the "Transform Values" section. This is so because in our example, the units of measurement for all the variables are same. However, if the units of measurements are different, one needs to standardize the variables. The most popular transformation is "Z-scores" which needs to be selected if the measurement units are different for all the variables.
- Check the option "Absolute values" in the "Transform Measures" option. This option transforms the values generated by the distance measure. This option is not required for squared Euclidean distance.

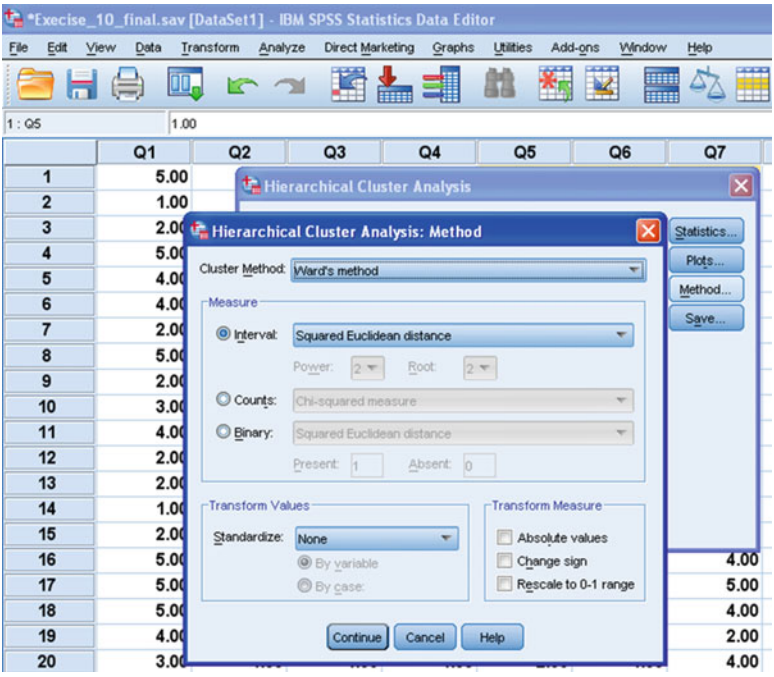



**Fig. 10.16** Selecting options for cluster method and distance measure criteria

- Click Continue. You will be taken back to the screen shown in Fig. 10.13. The screen will look like as shown in Fig. 10.16.

- Click **OK**

(c) ***Getting the output:*** Clicking the option **OK** shall generate lot of outputs in the output window. The four outputs that would be selected are Proximity matrix, Agglomerative schedule, Icicle plot, and Dendrogram. These outputs have been shown in Tables 10.7, 10.8 and Fig. 10.21, 10.22.

## *Stage 2: SPSS Commands for* K*-Means Cluster Analysis*

Stage 1 was the explorative process where number of initial clusters was identified. These initial clusters were identified on the basis of fusion coefficients in the agglomerative schedule. After deciding the number of clusters, apply the *K*-means cluster analysis in stage 2. In stage 1, three clusters were identified on the basis of the agglomeration schedule in Table 10.8 (for details, see Interpretation of Findings). This shall be used to find the final solution in the *K*-means cluster

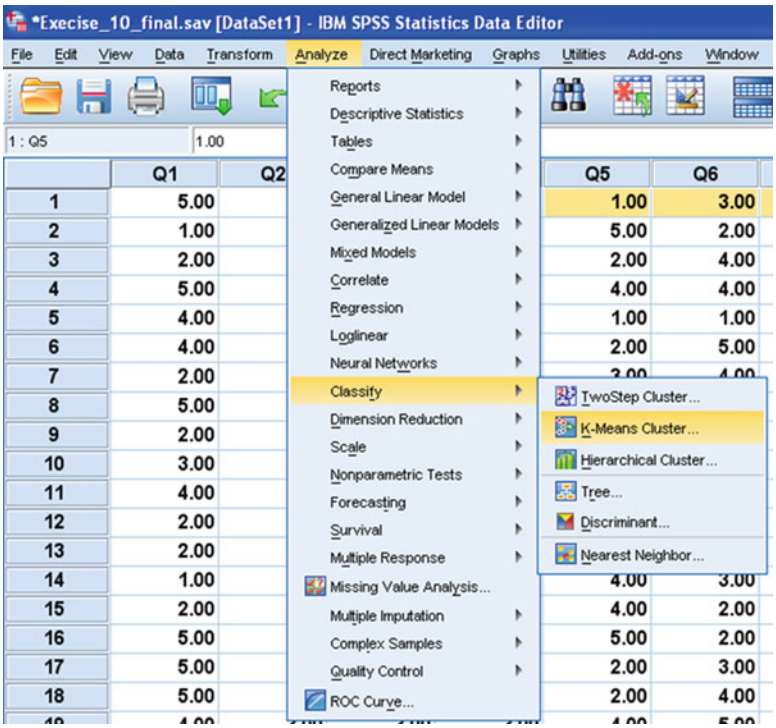



**Fig. 10.17** Sequence of SPSS commands for *K*-means cluster

analysis. The data file developed for the hierarchical analysis is also used for the *K*-means cluster analysis. Follow these steps in stage 2.

(i) *Initiating command for K-means cluster analysis:* Start the *K*-means analysis by using the following command sequence (Fig. 10.17):

**Analyze → Classify → K-Means Cluster Analysis**

(ii) *Selecting variables for analysis:* After clicking the ***K*-Means Cluster Analysis** option, you will be taken to the next screen for selecting variables. Select the variables as follows:

- Select all the variables and bring them in the "Variable(s)" section.
- Write number of clusters as 3. This is so because only three clusters were identified from the hierarchical analysis.
- Click the option **Iterate** and ensure that the minimum iteration is written as 10. In fact, this is done by default. If you want to have more than 10 maximum iterations, it may be mentioned here.
- Click ***Continue.***

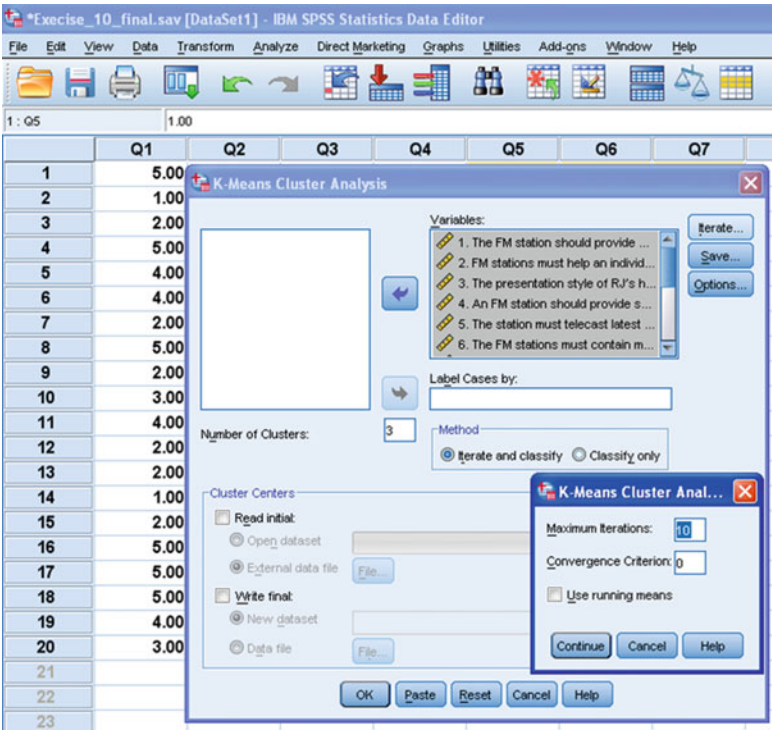



**Fig. 10.18** Screen showing selection of variables for *K*-means analysis

The screen shall look like Fig. 10.18.

– Click the tag **Save** and take the following steps:

  – Check the option "Cluster membership."
  – Check the option "Distance from cluster center."

– Click ***Continue.***

The screen shall look like Fig. 10.19.

– Click the tag **Options** and take the following steps:

  – Ensure that the option "Initial cluster centers" is checked. In fact, this is checked by default.
  – Check the option "ANOVA table."
  – Check the option "Cluster information for each case."

– Click ***Continue.***

  The screen would look like Fig. 10.20.

– Click **OK**.

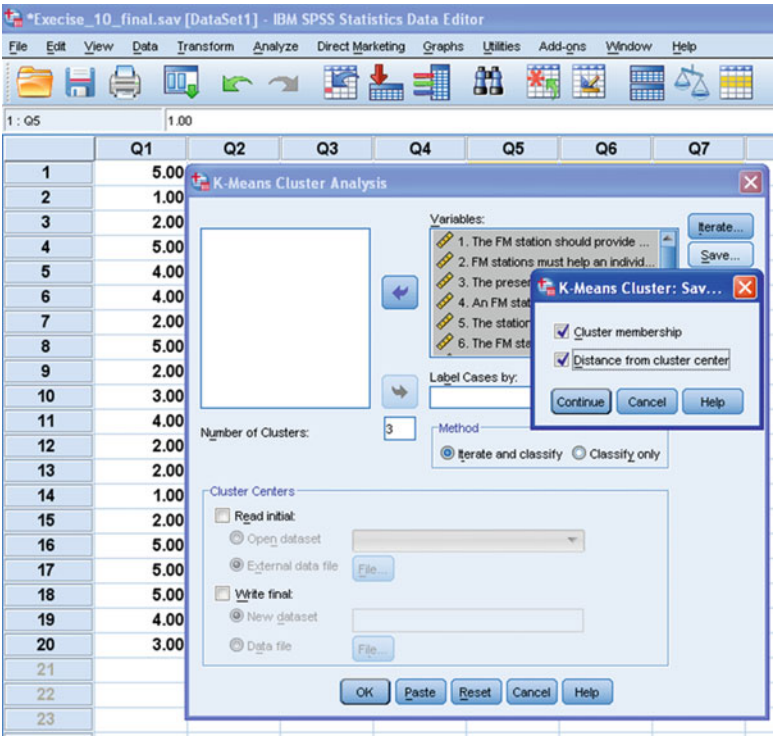



**Fig. 10.19** Screen showing option for getting cluster memberships and distance from cluster center

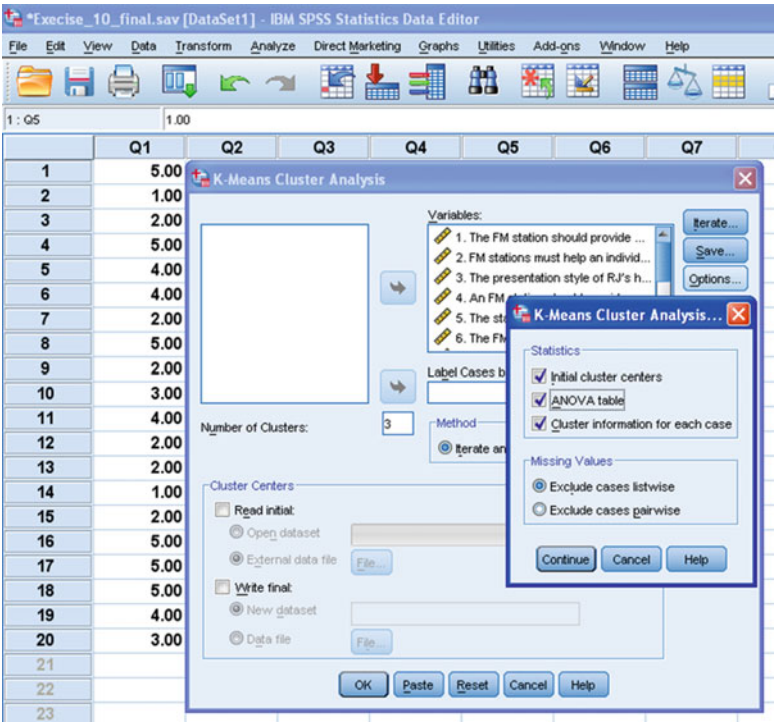



**Fig. 10.20** Screen showing options for cluster information and ANOVA

## *Interpretations of Findings*

***Stage 1:*** The agglomerative cluster analysis done in stage 1 provided the outputs shown in Tables 10.7 and 10.8 and in Figs. 10.21 and 10.22. The agglomerative analysis is explorative in nature. Its primary purpose is to identify the initial cluster solution. Therefore, one should take all possible parameters to identify the clusters so that important parameters are not left out. We shall now discuss the results generated in the agglomerative analysis in stage 1.

### Proximity Matrix: To Know How Alike (or Different) the Cases Are

Table 10.7 is a proximity matrix which shows distances between the cases. One can choose any distance criterion like squared Euclidean distance, Manhattan distance, Chebyshev distance, Mahalanobis (or correlation) distance, or Pearson correlation distance. In this example, the squared Euclidean distance was chosen as a measure of distance. The minimum distance exists between the 9th and 13th cases which is 6.00, whereas the maximum distance is observed between the 8th and 13th cases which is 87.00. The minimum distance means that these two cases would combine at the very first instance. This can be seen from Table 10.8 where 9th and 13th cases are combined into a single cluster in the very first stage. Similarly, the 8th and 13th cases are in the extreme clusters which can be seen in the dendrogram shown in Fig. 10.22.

### Agglomerative Schedule: To Know How Should Clusters Be Combined

Table 10.8 is an agglomerative schedule which shows how and when the clusters are combined. The agglomerative schedule is used to decide the number of clusters present in the data and one should identify the number of clusters by using the column labeled “Coefficients” in this table. These coefficients are also known as fusion coefficients. The values under this column are the distance (or similarity) statistic used to form the cluster. From these values, you get an idea as to how the clusters have been combined. In case of using dissimilarity measures, small coefficients indicate that those fairly homogenous clusters are being attached to each other. On the other hand, large coefficients show that the dissimilar clusters are being combined. In using similarity measures, the reverse is true, that is, large coefficients indicate that the homogeneous clusters are being attached to each other, whereas small coefficients reveal that dissimilar clusters are being combined.

The value of fusion coefficient depends on the clustering method and the distance measure you choose. These coefficients help you decide how many clusters you need to represent the data. The process of cluster formation is stopped when the increase (for distance measures) or decrease (for similarity measures) in

**Table 10.7** Proximity matrix

| | Squared Euclidean distance | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Case | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | .000 | 68.000 | 45.000 | 19.000 | 21.000 | 52.000 | 52.000 | 16.000 | 61.000 | 51.000 | 49.000 | 60.000 | 69.000 | 62.000 | 54.000 | 33.000 | 17.000 | 45.000 | 58.000 | 42.000 |
| 2 | 68.000 | .000 | 39.000 | 57.000 | 55.000 | 82.000 | 30.000 | 80.000 | 7.000 | 11.000 | 29.000 | 16.000 | 9.000 | 24.000 | 12.000 | 49.000 | 71.000 | 59.000 | 48.000 | 42.000 |
| 3 | 45.000 | 39.000 | .000 | 40.000 | 40.000 | 19.000 | 17.000 | 47.000 | 40.000 | 30.000 | 64.000 | 31.000 | 46.000 | 43.000 | 31.000 | 46.000 | 30.000 | 28.000 | 17.000 | 25.000 |
| 4 | 19.000 | 57.000 | 40.000 | .000 | 30.000 | 51.000 | 49.000 | 15.000 | 52.000 | 40.000 | 46.000 | 45.000 | 60.000 | 61.000 | 47.000 | 10.000 | 16.000 | 50.000 | 45.000 | 41.000 |
| 5 | 21.000 | 55.000 | 40.000 | 30.000 | .000 | 65.000 | 49.000 | 23.000 | 56.000 | 46.000 | 66.000 | 55.000 | 68.000 | 67.000 | 53.000 | 28.000 | 22.000 | 44.000 | 67.000 | 51.000 |
| 6 | 52.000 | 82.000 | 19.000 | 51.000 | 65.000 | .000 | 34.000 | 48.000 | 71.000 | 61.000 | 73.000 | 62.000 | 73.000 | 68.000 | 56.000 | 65.000 | 33.000 | 23.000 | 20.000 | 28.000 |
| 7 | 52.000 | 30.000 | 17.000 | 49.000 | 49.000 | 34.000 | .000 | 66.000 | 23.000 | 25.000 | 37.000 | 26.000 | 21.000 | 26.000 | 24.000 | 53.000 | 41.000 | 21.000 | 14.000 | 18.000 |
| 8 | 16.000 | 80.000 | 47.000 | 15.000 | 23.000 | 48.000 | 66.000 | .000 | 73.000 | 57.000 | 67.000 | 74.000 | 87.000 | 84.000 | 70.000 | 25.000 | 21.000 | 47.000 | 64.000 | 52.000 |
| 9 | 61.000 | 7.000 | 40.000 | 52.000 | 56.000 | 71.000 | 23.000 | 73.000 | .000 | 14.000 | 20.000 | 23.000 | 6.000 | 25.000 | 11.000 | 50.000 | 58.000 | 44.000 | 35.000 | 31.000 |
| 10 | 51.000 | 11.000 | 30.000 | 40.000 | 46.000 | 61.000 | 25.000 | 57.000 | 14.000 | .000 | 26.000 | 11.000 | 14.000 | 25.000 | 15.000 | 36.000 | 54.000 | 40.000 | 31.000 | 33.000 |
| 11 | 49.000 | 29.000 | 64.000 | 46.000 | 66.000 | 73.000 | 37.000 | 67.000 | 20.000 | 26.000 | .000 | 35.000 | 14.000 | 29.000 | 27.000 | 54.000 | 64.000 | 50.000 | 49.000 | 35.000 |
| 12 | 60.000 | 16.000 | 31.000 | 45.000 | 55.000 | 62.000 | 26.000 | 74.000 | 23.000 | 11.000 | 35.000 | .000 | 19.000 | 28.000 | 14.000 | 45.000 | 57.000 | 53.000 | 38.000 | 26.000 |
| 13 | 69.000 | 9.000 | 46.000 | 60.000 | 68.000 | 73.000 | 21.000 | 87.000 | 6.000 | 14.000 | 14.000 | 19.000 | .000 | 15.000 | 9.000 | 56.000 | 70.000 | 50.000 | 37.000 | 31.000 |
| 14 | 62.000 | 24.000 | 43.000 | 61.000 | 67.000 | 68.000 | 26.000 | 84.000 | 25.000 | 25.000 | 29.000 | 28.000 | 15.000 | .000 | 14.000 | 59.000 | 61.000 | 67.000 | 40.000 | 46.000 |
| 15 | 54.000 | 12.000 | 31.000 | 47.000 | 53.000 | 56.000 | 24.000 | 70.000 | 11.000 | 15.000 | 27.000 | 14.000 | 9.000 | 14.000 | .000 | 43.000 | 47.000 | 49.000 | 32.000 | 24.000 |
| 16 | 33.000 | 49.000 | 46.000 | 10.000 | 28.000 | 65.000 | 53.000 | 25.000 | 50.000 | 36.000 | 54.000 | 45.000 | 56.000 | 59.000 | 43.000 | .000 | 26.000 | 58.000 | 51.000 | 55.000 |
| 17 | 17.000 | 71.000 | 30.000 | 16.000 | 22.000 | 33.000 | 41.000 | 21.000 | 58.000 | 54.000 | 64.000 | 57.000 | 70.000 | 61.000 | 47.000 | 26.000 | .000 | 36.000 | 35.000 | 37.000 |
| 18 | 45.000 | 59.000 | 28.000 | 50.000 | 44.000 | 23.000 | 21.000 | 47.000 | 44.000 | 40.000 | 50.000 | 53.000 | 50.000 | 67.000 | 49.000 | 58.000 | 36.000 | .000 | 21.000 | 23.000 |
| 19 | 58.000 | 48.000 | 17.000 | 45.000 | 67.000 | 20.000 | 14.000 | 64.000 | 35.000 | 31.000 | 49.000 | 38.000 | 37.000 | 40.000 | 32.000 | 51.000 | 35.000 | 21.000 | .000 | 28.000 |
| 20 | 42.000 | 42.000 | 25.000 | 41.000 | 51.000 | 28.000 | 18.000 | 52.000 | 31.000 | 33.000 | 35.000 | 26.000 | 31.000 | 46.000 | 24.000 | 55.000 | 37.000 | 23.000 | 28.000 | .000 |

This is a dissimilarity matrix

**Table 10.8** Agglomeration schedule

| | Cluster combined | | | Stage cluster first appears | | |
|---|---|---|---|---|---|---|
| Stage | Cluster 1 | Cluster 2 | Coefficients | Cluster 1 | Cluster 2 | Next stage |
| 1 | 9 | 13 | 3.000 | 0 | 0 | 2 |
| 2 | 2 | 9 | 7.333 | 0 | 1 | 5 |
| 3 | 4 | 16 | 12.333 | 0 | 0 | 17 |
| 4 | 10 | 12 | 17.833 | 0 | 0 | 12 |
| 5 | 2 | 15 | 24.000 | 2 | 0 | 12 |
| 6 | 7 | 19 | 31.000 | 0 | 0 | 8 |
| 7 | 1 | 8 | 39.000 | 0 | 0 | 9 |
| 8 | 3 | 7 | 48.000 | 0 | 6 | 15 |
| 9 | 1 | 17 | 58.000 | 7 | 0 | 11 |
| 10 | 18 | 20 | 69.500 | 0 | 0 | 13 |
| 11 | 1 | 5 | 81.500 | 9 | 0 | 17 |
| 12 | 2 | 10 | 94.333 | 5 | 4 | 14 |
| 13 | 6 | 18 | 107.500 | 0 | 10 | 15 |
| 14 | 2 | 14 | 121.667 | 12 | 0 | 16 |
| 15 | 3 | 6 | 137.000 | 8 | 13 | 18 |
| 16 | 2 | 11 | 153.750 | 14 | 0 | 18 |
| 17 | 1 | 4 | 172.417 | 11 | 3 | 19 |
| 18 | 2 | 3 | 261.524 | 16 | 15 | 19 |
| 19 | 1 | 2 | 389.800 | 17 | 18 | 0 |

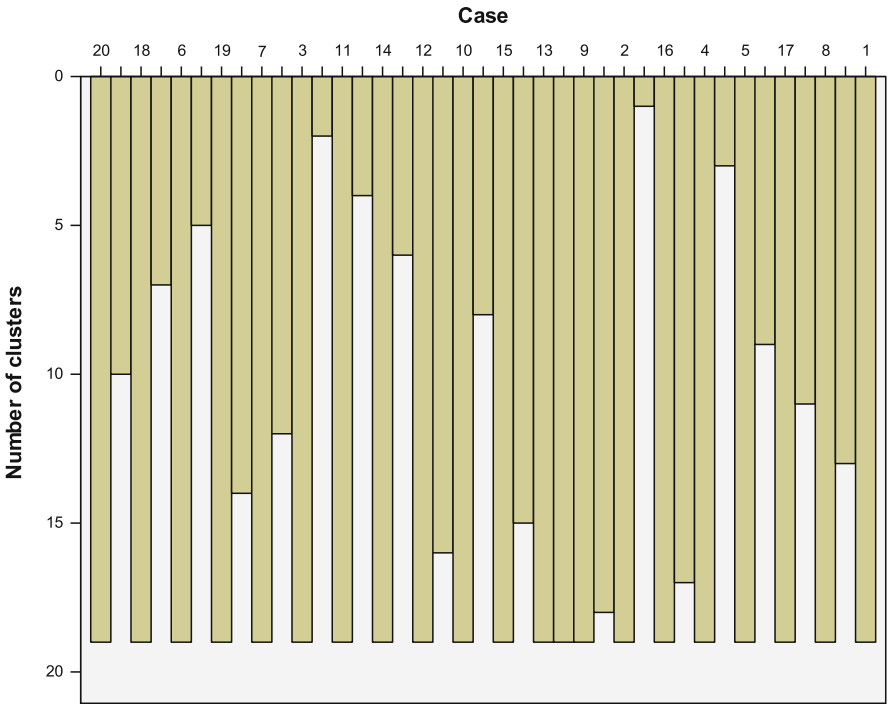



**Fig. 10.21** Vertical icicle plot using Ward's linkage

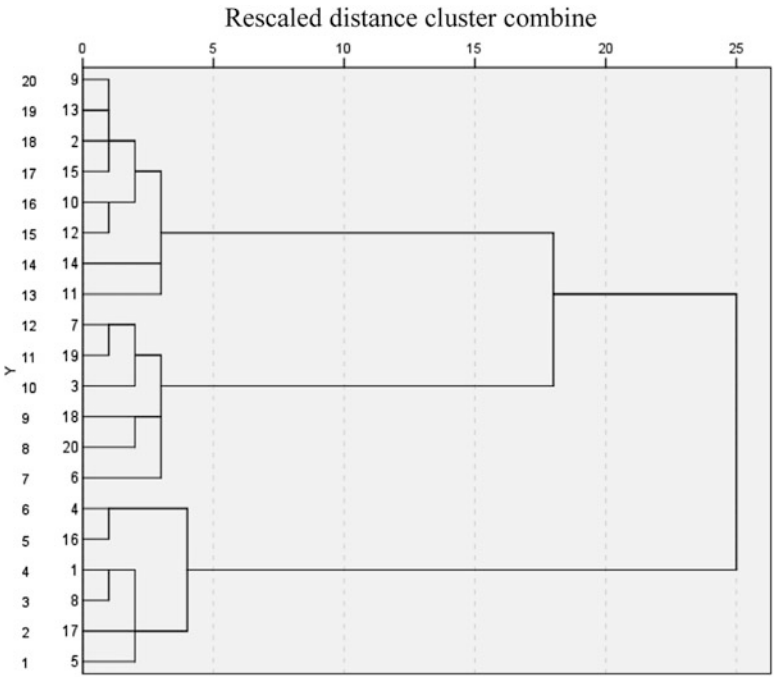



**Fig. 10.22** Dendrogram using Ward linkage

the coefficients between the two adjacent steps is large. In this example, the process can be stopped at the three cluster solution, after stage 17. Let us see how it is done?

We should look for the coefficients from the last row upward because we want the lowest possible number of clusters due to economy and its interpretability. Stage 20 represents a one cluster solution where all the cases are combined into one cluster, and, therefore, it is not shown in Table 10.8. The largest difference (389.800–261.524) exists in the coefficients between stages 18 and 19, which means we have to stop the process of cluster formation after stage 19; this would result in only two-cluster solution. However, we may not be interested to represent the data by two clusters only; therefore, we will look for the next larger difference of (261.524–172.417) which is equal to 89.107 (between stage 18, the three-cluster solution, and stage 17, the four-cluster solution). The next one after that is (172.417–153.750), only 18.667, between stages 17 and 16. Thereafter, the difference keeps decreasing. So we decide to stop the cluster formation at stage 18 which is a three-cluster solution.

Thus, in general, the strategy is to first identify the largest difference in the coefficients and identify the stage of the lowest coefficient as the cluster solution. However, it is up to the researcher to decide the number of clusters depending upon its interpretability. You can see from the dendrogram shown in Fig. 10.22 that three clusters are clearly visible in this case.

The agglomeration schedule starts off using the case numbers that has smallest distance as shown by the icicle plot in Fig. 10.21. The cluster is formed by adding cases. The number of the lowest case becomes the number of this newly formed cluster. For example, if a cluster is formed by merging cases 3 and 6, it would be known as cluster 3, and if the clusters are formed by merging cases 3 and 1, then it would be known as cluster 1.

The columns labeled "Stage Cluster First Appears" shows the step at which each of the two clusters that are being joined first appear. For example, at stage 9 when clusters 1 and 17 are combined, it tells you that cluster 1 was first formed at stage 7 and cluster 17 is a single case, and that the resulting cluster (known as 1) will see action again at stage 11 (under the column "Next stage"). If number of cases are small then the icicle plot explains step-by-step clustering summary better than the agglomeration schedule.

### The Icicle Plot: Summarizing the Steps

Figure 10.21 is the icicle plot which is a graphical representation of agglomerative schedule. It tells you how the clusters are formed at each stage. The figure is called an icicle plot because the columns look like icicles hanging from eaves. Each column represents one of the objects you are clustering. Each row shows a cluster solution with different number of clusters. You see the figure from the bottom up. The last row (not shown) is the first step of the analysis. Each of the cases is a cluster of itself. The number of clusters at this point is 20. The nineteen-cluster solution arises when cases 9 and 13 are joined into a cluster. It happened because they had the smallest distance among all pairs. The eighteen-cluster solution results from the merging of case 2 with cluster 9 into a cluster. This will go on till all the clusters are combined into a single cluster.

**Remark:** In case of large number of cases, icicle plot can be developed by showing cases as rows. For this, specify Horizontal in the Cluster Plots dialog box in SPSS.

### The Dendrogram: Plotting Cluster Distances

Figure 10.22 shows the dendrogram which is used to show the plotting of cluster distances. It provides a visual representation of the distance at which clusters are combined. We read the dendrogram from left to right. A vertical line represents the joined clusters. The position of the line on the scale shows the distance at which clusters are joined. The computed distances are rescaled in the range of 1–25, and, therefore, actual distances cannot be seen here; however, the ratio of the rescaled distances within the dendrogram is the same as the ratio of the original distances.

The first vertical line, corresponding to the smallest rescaled distance, is for the case 9 and case 13. The next vertical line is at the next smallest distance for the cluster 9 and case 2. It can be seen from Table 10.8 that the lowest coefficient is

**Table 10.9** Initial cluster centers

| Variables | Cluster | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1. The FM station should provide more old Hindi songs | 5.00 | 2.00 | 4.00 |
| 2. FM stations must help an individual in solving their personal problems | 3.00 | 5.00 | 2.00 |
| 3. The presentation style of RJs helps popularizing an FM station | 2.00 | 4.00 | 3.00 |
| 4. An FM station should provide some kind of prizes/incentives to its listeners | 4.00 | 2.00 | 4.00 |
| 5. The station must telecast latest songs more often | 5.00 | 5.00 | 2.00 |
| 6. The FM stations must contain more entertaining programs | 2.00 | 3.00 | 5.00 |
| 7. Popularity of RJs depends upon their humor and ability to make program interesting | 4.00 | 3.00 | 2.00 |
| 8. FM station should provide more opportunity to listeners to talk to celebrities | 4.00 | 5.00 | 1.00 |
| 9. RJs' voice must be clear and melodious | 3.00 | 3.00 | 5.00 |
| 10. FM channels should play 24 × 7 | 5.00 | 2.00 | 2.00 |
| 11. FM stations should give information for other sports along with cricket | 2.00 | 5.00 | 1.00 |
| 12. FM stations should provide information regarding educational/professional courses available in the city | 5.00 | 3.00 | 3.00 |
| 13. FM stations should provide information regarding different shopping offers available in the city | 2.00 | 3.00 | 5.00 |
| 14. RJs should speak in an understandable language, preferably in local language | 5.00 | 2.00 | 3.00 |

**Table 10.10** Iteration history[a]

| Iteration | Change in cluster centers | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1 | 3.375 | 1.753 | 2.953 |
| 2 | .000 | .480 | .566 |
| 3 | .000 | .000 | .000 |

[a]Convergence achieved due to no or small change in cluster centers. The maximum absolute coordinate change for any center is .000. The current iteration is 3. The minimum distance between initial centers is 7.483

3.000, which is for cases 9 and 13. The next smallest distance is shown by the coefficient as 7.333 which is for cluster 9 and case 2. Thus, what you see in this plot is what you already know from the agglomeration schedule.

**Remark:** While reading the dendrogram, one should try to determine at what stage the distances between clusters that are combined is large. You look for large distances between sequential vertical lines. In this case, large distance between the vertical lines suggests a three-cluster solution.

***Stage 2*** With the help of hierarchical cluster analysis, the number of cluster was decided to be three. After this, *K*-means cluster analysis was applied to get the final solution of the cluster means. The SPSS generated the outputs in the form of Tables 10.9, 10.10, 10.11, 10.12, 10.13, and 10.14. We shall now explain these outputs and discuss the cluster characteristics.

**Table 10.11** Final cluster centers

| Statements | Cluster 1 | 2 | 3 |
|---|---|---|---|
| 1. The FM station should provide more old Hindi songs | 4.83[a] | 2.13 | 3.33 |
| 2. FM stations must help an individual in solving their personal problems | 2.50 | 4.50[a] | 2.67 |
| 3. The presentation style of RJs helps popularizing an FM station | 2.33 | 4.25[a] | 2.83 |
| 4. An FM station should provide some kind of prizes/incentives to its listeners | 4.50[a] | 2.50 | 2.50 |
| 5. The station must telecast latest songs more often | 2.50 | 4.50[a] | 2.50 |
| 6. The FM stations must contain more entertaining programs | 2.83 | 2.75 | 4.33[a] |
| 7. Popularity of RJs depends upon their humor and ability to make program interesting | 4.83[a] | 2.88 | 2.83 |
| 8. FM station should provide more opportunity to listeners to talk to celebrities | 2.83 | 4.50[a] | 2.67 |
| 9. RJs' voice must be clear and melodious | 2.33 | 2.50 | 4.33[a] |
| 10. FM channels should play 24 × 7 | 4.50[a] | 2.88 | 2.83 |
| 11. FM stations should give information for other sports along with cricket | 2.33 | 4.25[a] | 2.50 |
| 12. FM stations should provide information regarding educational/ professional courses available in the city | 4.50[a] | 3.13 | 2.67 |
| 13. FM stations should provide information regarding different shopping offers available in the city | 2.50 | 2.88 | 4.50[a] |
| 14. RJs should speak in an understandable language, preferably in local language | 4.33[a] | 2.38 | 2.50 |

[a]Shows strong agreement toward response

### Initial Cluster Centers

The first step in *K*-means clustering was to find the *K*-centers. This is done iteratively. Here, the value of *K* is three because three clusters were decided on the basis of agglomerative schedule. We start with an initial set of centers and keep modifying till the changes between two iterations are small enough. Although one can also guess these centers which can be used as initial starting points, it is advisable to let SPSS find *K* cases that are well separated and use these values as initial cluster centers. In our example, Table 10.9 shows the initial centers.

Once the initial cluster centers are selected by the SPSS, each case is assigned to the nearest cluster, depending upon its distance from the cluster centers. After assigning all the cases to these clusters, the cluster centers are once again recomputed on the basis of its member cases. Again, all the cases are assigned by using the recomputed cluster centers. This process keeps on going till no cluster center changes appreciably. Since the number of iteration is taken as 10 by default in SPSS (see Fig. 10.18), this process of assigning cases and recomputing cluster centers will keep repeating to a maximum of ten times. In this example, you can see from Table 10.10 that the three iterations were sufficient.

**Table 10.12** ANOVA table

| | Cluster | | Error | | | |
|---|---|---|---|---|---|---|
| | Mean square | df | Mean square | df | *F* | Sig. *p*-value |
| 1. The FM station should provide more old Hindi songs | 12.579 | 2 | .885 | 17 | 14.217 | .000 |
| 2. FM stations must help an individual in solving their personal problems | 8.858 | 2 | .637 | 17 | 13.901 | .000 |
| 3. The presentation style of RJs helps popularizing an FM station | 7.042 | 2 | .333 | 17 | 21.125 | .000 |
| 4. An FM station should provide some kind of prizes/ incentives to its listeners | 8.400 | 2 | 1.000 | 17 | 8.400 | .003 |
| 5. The station must telecast latest songs more often | 9.600 | 2 | 1.118 | 17 | 8.589 | .003 |
| 6. The FM stations must contain more entertaining programs | 5.042 | 2 | .686 | 17 | 7.346 | .005 |
| 7. Popularity of RJs depends upon their humor and ability to make program interesting | 8.204 | 2 | .620 | 17 | 13.230 | .000 |
| 8. FM station should provide more opportunity to listeners to talk to celebrities | 7.392 | 2 | .716 | 17 | 10.328 | .001 |
| 9. RJs' voice must be clear and melodious | 7.667 | 2 | .745 | 17 | 10.289 | .001 |
| 10. FM channels should play 24 × 7 | 5.671 | 2 | .777 | 17 | 7.299 | .005 |
| 11. FM stations should give information for other sports along with cricket | 8.108 | 2 | .843 | 17 | 9.617 | .002 |
| 12. FM stations should provide information regarding educational/professional courses available in the city | 5.546 | 2 | .571 | 17 | 9.711 | .002 |
| 13. FM stations should provide information regarding different shopping offers available in the city | 6.938 | 2 | .699 | 17 | 9.932 | .001 |
| 14. RJs should speak in an understandable language, preferably in local language | 7.646 | 2 | .512 | 17 | 14.926 | .000 |

The *F*-tests should be used only for descriptive purposes because the clusters have been chosen to maximize the differences among cases in different clusters. The observed significance levels are not corrected for this and thus cannot be interpreted as tests of the hypothesis that the cluster means are equal

## Final Cluster Centers

Table 10.11 shows the final cluster centers after iteration stops, and cases are reassigned to the clusters. Using these final cluster centers, cluster characteristic are described.

Each question in this example is responded on a 1–5 scoring scale, where 5 stands for total agreement and 1 stands for total disagreement. Thus, if any score shown in Table 10.11 is more than 2.5, it indicates the agreement toward the statement, and if it is less than 2.5, it reflects disagreement. Thus, owing to these

**Table 10.13** Cluster membership

| Case number | Cluster | Distance |
|---|---|---|
| 1 | 1 | 2.953 |
| 2 | 2 | 2.378 |
| 3 | 3 | 2.887 |
| 4 | 1 | 2.461 |
| 5 | 1 | 3.424 |
| 6 | 3 | 3.367 |
| 7 | 3 | 2.828 |
| 8 | 1 | 2.779 |
| 9 | 2 | 2.325 |
| 10 | 2 | 2.580 |
| 11 | 2 | 3.828 |
| 12 | 2 | 3.226 |
| 13 | 2 | 1.705 |
| 14 | 2 | 3.487 |
| 15 | 2 | 2.215 |
| 16 | 1 | 3.375 |
| 17 | 1 | 2.838 |
| 18 | 3 | 3.162 |
| 19 | 3 | 2.708 |
| 20 | 3 | 3.317 |

**Table 10.14** Number of cases in each cluster

| | | |
|---|---|---|
| Cluster | 1 | 6.000 |
| | 2 | 8.000 |
| | 3 | 6.000 |
| Valid | | 20.000 |
| Missing | | .000 |

criteria, the characteristics of these three clusters of cases were as follows (refer to the question details in Example 10.1):

Cluster 1

FM listeners belonging to this cluster were of the strong opinion that channels should provide more old Hindi songs (Q.1) and provide some incentives to the listeners (Q.4). They strongly feel that the humor and ability to deliver interesting programs make RJs more popular (Q.7). The channel should play 24 × 7 (Q.10) and must air information regarding educational opportunity available in the city (Q.12), and the RJ must speak in local dialect (Q.14).

Further, listeners belonging to this cluster feel that FM channels should air more entertaining programs (Q.6) and should provide more opportunity to listeners to talk to the celebrities (Q.8).

Cluster 2

Listeners belonging to this cluster strongly felt that FM channels must provide solutions to personal problems (Q.2), RJs presentation skill to be important for the channels (Q.3), channels to provide more often the latest songs (Q.5), channels to arrange more dialogues between celebrities and their audience (Q.8), and should air information about sports other than cricket also (Q.11).

Further, listeners to this cluster also felt that FM channels should air more entertaining programs (Q.6). Humor and ability to deliver interesting programs make RJs more popular (Q.7). The channels must play 24 × 7 (Q.10) and should provide information regarding educational opportunity (Q.12) and shopping offers (Q.13) available in the city.

Cluster 3

Listeners in this cluster were strongly of the view that the FM channels must contain more entertaining programs (Q.6), RJs voice must be very clear and melodious (Q.9), and channels should provide information regarding shopping offers available in the city (Q.13).

Further, listeners in this cluster were also of the view that channels should air more old Hindi songs (Q.1), provide solution to the personal problems (Q.2), and believe RJs to be the key factor in popularizing the FM channels (Q.3). They were of the view that the humorous RJs make programs more interesting (Q.7). Channels should provide more opportunity to listeners to talk to the celebrities (Q.8), they should operate 24 × 7 (Q.10) and, at the same time, must air the information regarding educational opportunities available in the city (Q.12).

**ANOVA: To Know Differences Between Clusters**

Table 10.12 shows ANOVA for the data on all the 14 variables. The $F$-ratios computed in the table describe the differences between the clusters. $F$-ratio is significant at 5% level if the significance level ($p$-value) associated with it is less than .05. Thus, it can be seen in Table 10.12 that $F$-ratios for all the variables are significant at 5% level as their corresponding $p$-values are less than .05.

***Remark***

1. There is a divided opinion on the issue of using ANOVA analysis for comparing the clusters on each of the parameters. The footnote in Table 10.12 warns that the observed significance levels should not be interpreted in the usual fashion because the clusters have been selected to maximize the differences between clusters.

2. It is up to the researcher to decide about using ANOVA for determining the significance of variables. If ANOVA is used, then the interpretation of clusters should be made on the basis of those variables which are significantly different across clusters at any predefined level of significance.

### Cluster Membership

Table 10.13 shows the cluster membership of the cases. You can see that six cases belong to cluster 1, eight cases to cluster 2, and six cases to cluster 3.

Table 10.14 is a summary of Table 10.13. You do not like to see clusters with very few cases unless they are really different from the remaining cases.

## Exercise

*Short Answer Questions*

Q.1. Discuss a research situation where cluster analysis can be applied.
Q.2. Write the steps involved in cluster analysis.
Q.3. What is squared Euclidean distance? What impact it will have if the variables are measured on different units? Suggest the procedure in that situation.
Q.4. When should you use Chebyshev distance and Mahalanobis distance? How these distances are computed?
Q.5. How hierarchical clustering is different than *K*-means clustering?
Q.6. What is the difference between single linkage and average linkage method?
Q.7. What do you mean by Ward's minimum variance method?
Q.8. What is the difference between agglomerative and divisive clustering? Can both these clustering be shown in a single graph? If yes, how?
Q.9. In what situation two-stage clustering is done? Explain the steps in brief.
Q.10. Why hierarchical clustering is known as explorative technique? Explain briefly the advantage of using this method.
Q.11. In cluster analysis, when do we need to standardize the variable and why?
Q.12. What do you mean by icicle plot and what it conveys? Show it by sketch.
Q.13. What is the purpose of proximity matrix? Develop a proximity matrix by using any set of data on three cases measured on four variables.
Q.14. Discuss the assumptions used in cluster analysis.
Q.15. How the properties of clusters are explained?
Q.16. Would you agree to use ANOVA in cluster analysis? If yes, how clusters should be explained, and, if not, why?
Q.17. In using SPSS for *K*-means cluster analysis, what output would be generated if the option "Cluster interaction for each case" is chosen?

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

Answer Q.1 to Q.4 on the basis of the following information. In a cluster analysis, if the data on two cases are as follows:

| | | | |
|---|---|---|---|
| Case 1 | 14 | 8 | 10 |
| Case 2 | 10 | 11 | 12 |

1. The squared Euclidean distance shall be

   (a) 27
   (b) 29
   (c) 28
   (d) $\sqrt{29}$

2. The Manhattan distance shall be

   (a) 81
   (b) 9
   (c) 3
   (d) $\sqrt{9}$

3. The Chebyshev distance shall be

   (a) 4
   (b) 3
   (c) 2
   (d) 9

4. The Euclidean distance shall be

   (a) 29
   (b) 30
   (c) $\sqrt{29}$
   (d) 28

5. Cluster analysis is a($n$)

   (a) Explorative analysis
   (b) Descriptive analysis
   (c) Deductive analysis
   (d) Predictive analysis

6. When cluster is formed in agglomerative clustering by joining case 3 with case 7, then the resultant cluster would be known as

   (a) Cluster 7
   (b) Cluster 3
   (c) Cluster 3,7
   (d) Cluster 7,3

7. In Ward's method, any two clusters are joined to form a new cluster if

   (a) The variation within each cluster is same
   (b) The variation within one cluster is minimum than other
   (c) The variation within the two clusters is least
   (d) The variation between both the clusters is maximum

8. In complete linkage method, the clusters are formed on the basis of

   (a) Minimum distance between the closest members of the two clusters
   (b) Minimum average distance between all pairs of objects (in each pair, one member must be from a different cluster)
   (c) Minimum square Euclidean distances between any two clusters
   (d) Minimum distance between the farthest members of the two clusters

9. The number of clusters is decided on the basis of fusion coefficients in agglomerative schedule. In doing so, if distance matrix is considered, then

   (a) We look for the largest difference in the coefficients
   (b) We look for the smallest difference in the coefficients
   (c) We look for the equality of the coefficients
   (d) It is up to the researcher to decide any criteria

10. In cluster analysis, the characteristics of the clusters are decided on the basis of

    (a) Initial cluster solution
    (b) Final cluster solution
    (c) Cluster membership
    (d) ANOVA table

*Assignments*

1. The data on five nutritional contents of different food articles are shown in Table-A. Identify the suitable clusters of food articles based on these five nutritional contents. Use centroid clustering method and squared Euclidean distances to find the clusters. Apply hierarchical clustering and then *K*-means clustering method to find the final solution for clusters. Explain your findings and discuss the characteristics of different clusters.
2. Ratings were obtained on different brands of car for their six parameters shown in the Table-B. These cars are in specific prize range. The rating 1 indicates complete agreement and 5 indicates complete disagreement. Apply cluster analysis to discuss the characteristics of identified clusters of cars. Use Ward's method of clustering and squared Euclidean distance measure for cluster formation. Use the label cases option in SPSS.

**Table-A** Nutritional components of different food articles

| Food article | Carbohydrates | Protein | Fat | Iron | Vitamin |
|---|---|---|---|---|---|
| 1 | 354 | 20 | 28 | 10 | 2.4 |
| 2 | 89 | 13 | 3 | 38 | 1.7 |
| 3 | 375 | 20 | 33 | 8 | 2.6 |
| 4 | 192 | 23 | 11 | 17 | 3.7 |
| 5 | 116 | 21 | 13 | 12 | 1.8 |
| 6 | 169 | 24 | 8 | 12 | 1.5 |
| 7 | 160 | 18 | 10 | 115 | 2.5 |
| 8 | 320 | 24 | 16 | 9 | 2.9 |
| 9 | 220 | 8 | 31 | 12 | 2.5 |
| 10 | 158 | 25 | 6 | 11 | 5.9 |
| 11 | 202 | 19 | 15 | 7 | 2.5 |
| 12 | 263 | 21 | 21 | 9 | 2.8 |
| 13 | 280 | 21 | 29 | 10 | 2.8 |
| 14 | 72 | 12 | 3 | 83 | 6 |
| 15 | 46 | 8 | 6 | 74 | 5.4 |
| 16 | 415 | 14 | 40 | 7 | 2 |
| 17 | 132 | 18 | 4 | 14 | 3.5 |
| 18 | 204 | 20 | 12 | 6 | 1 |
| 19 | 125 | 11 | 40 | 12 | 2.3 |
| 20 | 342 | 21 | 27 | 8 | 2.5 |
| 21 | 189 | 22 | 10 | 9 | 2.7 |
| 22 | 136 | 23 | 5 | 22 | 2.8 |

(Hint: Transform your data into *Z*-scores by using the commands in SPSS)

**Table-B** Ratings on different cars on their characteristics

| S.N. | Car | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 1 | Logan | 4 | 2 | 2 | 4 | 4 | 4 |
| 2 | Renault Logan Edge | 4 | 3 | 3 | 4 | 3 | 3 |
| 3 | Mahindra-Renault Logan Edge | 3 | 2 | 4 | 2 | 4 | 3 |
| 4 | Mahindra Verito | 4 | 4 | 2 | 3 | 3 | 3 |
| 5 | Swift Dzire | 3 | 3 | 3 | 2 | 2 | 3 |
| 6 | Maruti Swift Dzire | 4 | 2 | 2 | 3 | 3 | 4 |
| 7 | Chevrolet Beat | 4 | 3 | 1 | 4 | 2 | 5 |
| 8 | Tata Venture | 5 | 2 | 2 | 3 | 4 | 2 |
| 9 | Chevrolet Aveo | 4 | 3 | 3 | 2 | 3 | 1 |
| 10 | Tata Sumo Spacio | 2 | 4 | 2 | 3 | 3 | 2 |
| 11 | Skoda New Fabia | 3 | 5 | 3 | 5 | 4 | 4 |
| 12 | Hyundai i10 | 4 | 4 | 3 | 4 | 3 | 3 |
| 13 | Tata Indigo e-CS | 3 | 3 | 4 | 3 | 3 | 2 |
| 14 | Maruti Suzuki Swift | 4 | 4 | 3 | 2 | 4 | 4 |
| 15 | Maruti Suzuki A-Star | 2 | 5 | 2 | 4 | 2 | 1 |
| 16 | Maruti Suzuki Ritz Genus | 3 | 4 | 3 | 5 | 5 | 2 |
| 17 | Premier Rio | 1 | 3 | 4 | 2 | 3 | 2 |
| 18 | Nissan Micra | 2 | 2 | 4 | 3 | 3 | 4 |
| 19 | Volkswagen Polo | 3 | 3 | 4 | 5 | 5 | 5 |
| 20 | Skoda Fabia | 4 | 1 | 5 | 4 | 4 | 5 |
| 21 | Mahindra-Renault Logan | 3 | 3 | 4 | 3 | 2 | 4 |
| 22 | Tata Sumo Victa | 2 | 2 | 5 | 3 | 3 | 3 |
| 23 | Tata Sumo Grande | 3 | 4 | 4 | 2 | 2 | 2 |
| 24 | Tata Indigo Marina | 4 | 2 | 5 | 3 | 3 | 1 |

*Parameters of the Car*

1. The leg space in the car is comfortable.
2. Car space is big enough to keep my luggage during outing.
3. The car is giving the same mileage as mentioned in the brochure.
4. Driving is very comfortable.
5. Security feature of the car is good.
6. Accessories provided in the car are of good quality.

Besides explaining the characteristics of clusters, also answer the following:

(a) What are the minimum and maximum distances between the cases?
(b) How many clusters you would like to identify and what is the maximum distance between the fusion coefficients?
(c) What criteria you would adopt to discuss the properties of the clusters?
(d) Explain cluster characteristics on the basis of ANOVA and see if it is different than what you have explained earlier.
(e) How many cases/cars are in each cluster?

*Answers to Multiple-Choice Questions*

| | | | |
|---|---|---|---|
| Q.1 b | Q.2 b | Q.3 a | Q.4 c |
| Q.5 a | Q.6 b | Q.7 c | Q.8 d |
| Q.9 a | Q.10 b | | |

# Chapter 11
# Application of Factor Analysis: To Study the Factor Structure Among Variables

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Understand the factor analysis and its application.
- Learn the difference between exploratory and confirmatory factor analysis.
- Know the use of factor analysis in developing test batteries.
- Interpret different terms involved in factor analysis.
- Explain the situations where factor analysis can be used.
- Know the procedure of retaining the factors and identifying the variables in it.
- Explain the steps involved in factor analysis.
- Understand the steps involved in using SPSS for factor analysis.
- Discuss the outputs obtained in factor analysis.
- Learn to write the results of factor analysis in standard format.

## Introduction

Buying decision of an individual depends upon large number of product characteristics. But the market strategy cannot be developed on the basis of all those parameters of a product that affect the buying behavior of an individual. The factor analysis, a multivariate technique, comes to our rescue in solving such problems. The factor analysis technique reduces the large number of variables into few underlying factors to explain the variability of the group characteristics. The concept used in factor analysis technique is to investigate the relationship among the group of variables and segregate them in different factors on the basis of their relationship. Thus, each factor consists of those variables which are related among themselves and explain some portion of the group variability. For example, personality characteristics of an individual can be assessed by the large number of parameters. The factor analysis may group these variables into different factors

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where each factor measure some dimension of personality characteristics. Factors are so formed that the variables included in it are related with each other in some way. The significant factors are extracted to explain the maximum variability of the group under study.

In marketing research, application of factor analysis provides very useful inputs to the decision makers to focus on few factors rather than a large number of parameters in making their products more acceptable in the market. For instance, consider an automobile company is interested to know as to what makes their customer to choose a particular model of the car. Several issues like mileage, easy loan facility, roof height, leg space, maintenance, road clearance, steering functionality, brakes, lighting, and luggage space may be investigated by taking the responses from the consumers. There may be endless issues on which the opinion of the customers can be taken. But by using the factor analysis, these variables may be clubbed in different factors like *economy* (mileage, easy loan facility), *comfort* (roof height, leg space, maintenance, luggage space), and *technology* (steering functionality, brakes, lighting, and road clearance). Thus, instead of concentrating on so many parameters, the authorities will make a strategy to optimize these three factors for the growth of their business. Further, these factors may be used to construct the perceptual maps and other product positioning.

Thus, in factor analysis, few factors are extracted out of the large set of variables. Since variables in each of the factors are associated among themselves, therefore they represent the same phenomenon. In this way, instead of studying all the parameters, few extracted factors are studied. These factors so extracted explain much of the variations of the group characteristics.

Factor analysis is used for both *exploratory* as well as *confirmatory* studies. In exploratory study, we do not know anything about the number of factors to be extracted, the number of variables included in each factor and percentage of variability explained by these extracted factors. The researcher takes all those variables under study which are suggested by the review studies or guided by the researchers own knowledge or experts opinion. Exploratory studies are just like mining important variables from a large number of variables to form factors. Unimportant variables do not figure in any of the identified factors. Such variables are excluded on the basis of their low communality. The process will be discussed later in the book. Thus, through exploratory study, a researcher can extract the factors underlying all prospective variables which have been selected on the pretext that they explain some or other dimension of the group behavior. Such study also reveals the number of variables which loads on each factor significantly and the percentage variability explained by each factor toward the group characteristics.

On the other hand in confirmative factor analysis, it is required to test the existing factor model. In other words, before starting the experiments, it is assumed that the factor analysis will produce only specified number of factors and specific number of variables are loaded on each factor and that the how much variability shall be explained by the identified factors. Thus, a factor model developed in an exploratory study is being tested in the confirmatory study to have its validity.

The factor analysis can be used to develop test battery for assessing group characteristics. To assess employee's performance, several variables like timeliness,

cost-effectiveness, absenteeism, tardiness, creativity, quality, adherence to policy, gossip and other personal habits, personal appearance, manager's appraisal, self-appraisal, and peer appraisal are usually measured. By using factor analysis, these variables can be clubbed into different factors. On the basis of variable's loading and their explainability, one or two variables from each factor can be selected to form the test battery. However, to validate the test battery, the confirmatory factor analysis must be done on similar but different sample groups.

Another application of factor analysis is in developing of a questionnaire. While doing item analysis, unimportant questions are removed from the questionnaire. Factor analysis may be used to indicate the loss in the measurement of variability in removing the unimportant questions from the final questionnaire. Further, it helps in classifying the questions into different parameters in the questionnaire.

## What Is Factor Analysis?

Factor analysis is a multivariate statistical technique used to identify the factors underlying the variables by means of clubbing related variables in the same factor. It is a dimension reduction technique which reduces the large number of variables into few factors without sacrificing much, the power of explained variability by the variables. Variables are clubbed into different factors on the basis of their interrelation. In initial solution of factor analysis, variables may belong to more than one factor. But by using the factor rotation technique, these factors may be made mutually exclusive. Thus, instead of defining the group characteristics by the large number of variables, a few factors may do this job. The number of factors is identified by means of the criterion known as eigenvalue. The magnitude of variable's loading on the factor is used as a criterion for retaining that variable in the factor. Sufficient number of data set is required to run the factor analysis. As a thumb rule, number of data set should be at least five per variable. Thus, if there are 15 variables in the problem, the sample must be approximately 75. However, there is a procedure of testing the adequacy of sample size in running the factor analysis. This is done by using the KMO test. We shall discuss it in detail later in this chapter.

## Terminologies Used in Factor Analysis

To understand the factor analysis technique, it is essential to know the meaning of various terms involved in it. It is assumed that the readers are familiar with the basic logic of statistical reasoning and the concepts of variance and correlation; if not, it is advised that they should read the basic statistics topic at this point, from the earlier chapters discussed in this book.

## *Principal Component Analysis*

Principal component analysis (PCA) is closely related to factor analysis. It is used to reduce the large number of variables into smaller number of principal components that will account for most of the variance in the observed variables. In this method, the factor explaining the maximum variance is extracted first.

Principal component analysis method is used when the data on large number of variables are obtained and some of the variables are redundant. Here, redundancy means that some of the variables are correlated with one another, possibly because they are measuring the same construct. Because of this redundancy, one believes that it should be possible to reduce the observed variables into a smaller number of principal components that will account for most of the variance in the observed variables. In fact, principal component analysis is similar to the procedure used in exploratory factor analysis

One must understand that the principal component analysis and factor analysis are not same. In PCA, one performs a variance-maximizing rotation of the variable space, and it takes into account all variability in the variables. On the other hand, factor analysis is the procedure of estimating the amount of variability explained due to common factors (communality). These two methods become same if the error terms in the factor analysis model (the variability not explained by common factors) can be assumed to have the same variance.

## *Factor Loading*

Factor loading can be defined as the correlation coefficient between the variable and factor. Just like Pearson's *r*, the squared factor loading of a variable indicates the percentage variability explained by the factor in that variable. As a rule of thumb, 0.7 or higher factor loading represents that the factor extracts sufficient variance from that variable. The percentage variance explained in all the variables accounted for by each factor can be computed by dividing the sum of the squared factor loadings for that factor divided by the number of variables and multiplied by 100.

## *Communality*

The communality can be defined as the sum of the squared factor loadings of a variable in all the factors. It is the variance in that variable accounted for by all the factors. The communality of variable is represented by $h^2$. It measures the percentage of variance in a given variable explained by all the factors jointly and may be considered as the reliability of the variable. Low communality of a variable indicates that the variable is not useful in explaining the characteristics of the

group and the factor model is not working well for that variable. Thus, variables whose communalities are low should be removed from the model as such variables are not related to each other. Any variable whose communality is $<.4$ should usually be dropped. However, the communalities must be interpreted in relation to the interpretability of the factors. For instance a communality of .80 may seem to be high but becomes meaningless, unless the factor on which the variable is loaded is interpretable, normally it usually will be. On the other hand a communality of .25 may look to be low but becomes meaningful if the variable can well define the factor.

Hence, it is not the value of communality of a variable that is important, but the variable's role in interpretation of the factor is the important consideration. However, the variable whose communality is very high usually explain the factor well. If the value of communality is more than 1, then one must expect that something is wrong with the solution. Such situation indicates that either sample is too small or the researcher has identified too many or too few factors.

## *Eigenvalue*

The eigenvalue for a given factor measures the variance in all the variables which is accounted for by that factor. It is also called as characteristics root. The sum of the eigenvalues of all the factors is equal to the number of variables. The decision about the number of factors to be retained in the factor analysis is taken on the basis of eigenvalues. If a factor has a low eigenvalue, then it is contributing little to the explanation of variances in the variables and may be dropped. Eigenvalues measure the amount of variation in the total sample accounted for by each factor.

## *Kaiser Criteria*

While applying the factor analysis one needs to decide as to how many factors should be retained. As per the Kaiser's criteria only those factors having eigenvalue $>1$ should be retained in the factor analysis. Initially each variable is supposed to have its eigenvalue 1. Thus, it may be said that unless a factor extracts at least as much as the equivalent of one original variable, it is dropped. This criterion was proposed by Kaiser and is the most widely used by the researchers.

## *The Scree Plot*

The scree plot is a graphical representation of the factors plotted along X-axis against their eigenvalues, on the Y-axis. As one moves toward the X-axis (factors), the eigenvalues drop. When the drop ceases and the curve makes an elbow toward

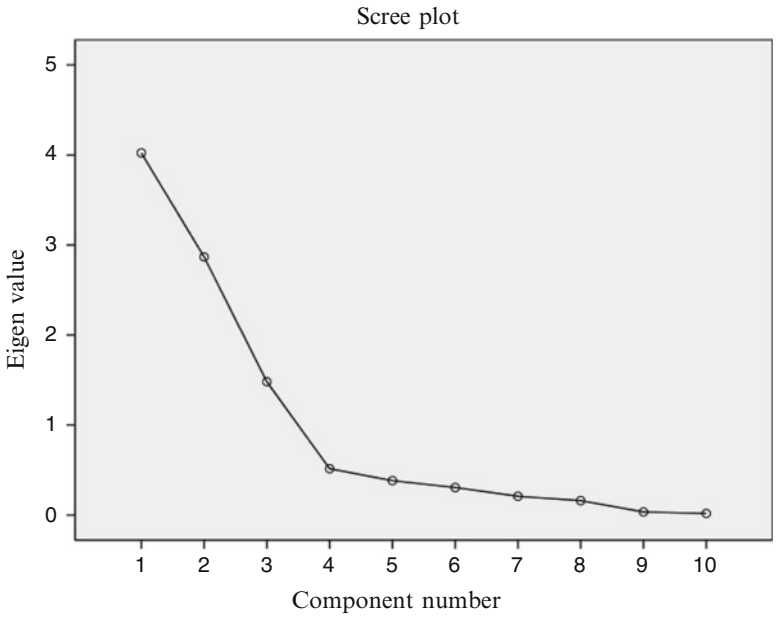



**Fig. 11.1** Scree plot for the factors

less steep decline, Cattell's scree test says to drop all further components after the one starting the elbow. Thus, the factors above the elbow in the plot are retained. The *scree* test was developed by Cattell. "Scree" is a term used in geology. The scree is the rubble at the bottom of a cliff. In scree test, if a factor is important, it will have a large variance. The scree plot may look like Fig. 11.1.

## *Varimax Rotation*

Unrotated factor solution obtained after applying the principal component analysis is rotated by using any of the rotational technique to enhance the interpretability of factors. The varimax rotation is the most widely used rotation technique in factor analysis. It is an orthogonal rotation of the factor axes to maximize the variance of the squared loadings of a factor on all the variables in a factor matrix, which has the effect of relocating the original variables into extracted factor. After varimax rotation, each factor will tend to have either large or small loadings of any particular variable and thus facilitates a researcher to identify each variable in one and only one factor. This is the most common rotation option. Other rotational strategies are quartimax, equamax, direct oblimin, and promax methods which are not much used by the researcher.

## What Do We Do in Factor Analysis?

The factor analysis involves different steps which are discussed below. You may not understand all the steps at a glance but do not lose heart and continue to read. After reading these steps, once you go through the solved example discussed later in this chapter, a full clarity of the concepts can be achieved. The steps discussed below cannot be done manually but may be achieved by using any statistical package. So try and relate these steps with the output of factor analysis.

1. Compute descriptive statistics for all the variables. Usually mean and standard deviation are provided by the standard statistical packages while running the factor analysis. However, you may run descriptive statistics program to compute other descriptive statistics like skewness, kurtosis, standard error, and coefficient of variability to understand the nature of the variables under study.
2. Prepare correlation matrix with all the variables taken in the study.
3. Apply KMO test to check the adequacy of data for running factor analysis. The value of KMO ranges from 0 to 1. The larger the value of KMO more adequate is the sample for running factor analysis. As a convention, any value of KMO more than .5 signifies the adequacy of sample for running the factor analysis. A value of 0 indicates that the distinct factors cannot be made and hence, the sample is not adequate. On the other hand, if its value is approaching 1, then the factor analysis yields distinct and reliable factors. Kaiser recommends accepting values $>0.5$ as acceptable (values below this should lead you to either collect more data or rethink which variables to include). Further, the values between 0.5 and 0.7 are mediocre, values between 0.7 and 0.8 are good, values between 0.8 and 0.9 are great, and values above 0.9 are superb (Hutcheson and Sofroniou 1999).
4. Apply Bartlett's test of sphericity for testing the hypothesis that the correlation matrix is not an identity matrix. If the correlation matrix is an identity matrix, the factor analysis becomes inappropriate. Thus, if the Bartlett's test of sphericity is significant, it is concluded that the correlation matrix is not an identity matrix and the factor analysis can be run.
5. Obtain unrotated factor solution by using principal component analysis. This will provide you the number of factors along with their eigenvalues. We retain only those factors whose along with their eigenvalues. This can also be shown graphically by scree plot. This solution also provides the factor loadings of the variables on different factors, percentage variability explained by each factor, and the total variability explained by all the factors retained in the model.
6. Thus, this primary factor analysis solution can tell you the percentage of variability explained by all the identified factors together. However, it is not possible to identify the variables included in each factor because some of the variables may belong to more than one factor. This problem is sorted out by choosing the appropriate rotation technique.

7. Obtain final solution by using the varimax rotation option, available in SPSS. This will solve the problem of redundancy of variables in different factors. As a rule of thumb, if the factor loading of any variable on a factor is equal or more than 0.7, then it should belong to that factor. The reason for choosing 0.7 factor loading as a cut off point is that because factor loading represents correlation coefficient hence at least 49% ($= 0.7^2$) variability of the variable must be explained by the factor to which it belongs. However, other variables whose loadings are $<0.7$ can also be identified in that factor on the basis of its explainability.
8. Identified factors in step 6 are given names depending upon the nature of variables included in it.
9. If the purpose of the factor analysis is to develop a test battery also, then one or two variables from each factor may be selected on the basis of their magnitude of loadings. These variables so selected may form the test battery. Each variable in the test battery is assigned weight. The weights assigned to the variable in the test battery depend upon the percentage variability explained by the factor from which it belongs. Usually, the first factor explains the maximum variance, and therefore two or three variables may be kept from it depending upon the nature of the variables and its explainability. From rest of the factors, normally one variable per factor is selected, as the sole purpose of the factor analysis is to reduce the number of variables so that the maximum variance in the group may be explained.

## *Assumptions in Factor Analysis*

While using the factor analysis, the following assumptions are made:

1. All the constructs which measure the concepts have been included in the study.
2. Sufficient sample size has been taken for factor analysis. Normally sample size must be equal to 5–20 times the number of variables taken in the study.
3. No outlier is present in the data.
4. Multicollinearity among the variables does not exist.
5. Homoscedasticity does not exist between the variables because factor analysis is a linear function of measured variables. The meaning of homoscedasticity between the variables is that the variance around the regression line is the same for all values of the predictor variable (X).
6. Variables should be linear in nature. Nonlinear variables may also be used after transforming it into linear variables.
7. Data used in the factor analysis is based on interval scale or ratio scale.

### *Characteristics of Factor Analysis*

Following are some of the important features of factor analysis:

- The variables used in the factor analysis may be objective or subjective provided subjective variables can be expressed into scores.
- The factor analysis extracts the hidden dimensions among the variables which may not be observable from direct analysis.
- This analysis is simple and inexpensive to perform.

### *Limitations of Factor Analysis*

Although the factor analysis is very useful multivariate statistical technique, however, it has some limitations as well.

- Much of the advantage of factor analysis technique can be achieved only if the researcher is able to collect a sufficient set of product attributes. If some of the important attributes are missed out, the results of factor analysis will not be efficient.
- If majority of the variables are highly related to each other and distinct from other items, factor analysis will assign a single factor to them. This will not reveal other factors that capture more interesting relationships.
- Naming the factors may require researcher's knowledge about the subject matter and theoretical concepts, because often multiple attributes can be highly correlated for no apparent reason.

## Research Situations for Factor Analysis

Since factor analysis is used to study the group characteristics by means of identified factors out of the large number of variables, it has tremendous application in management, social sciences, and humanities. Few research applications are discussed below:

1. To understand the buying behavior of a particular product, several parameters like customer's age, education, job status, salary, exposure to product advertisement, and availability are responsible. Factor analysis may help the market analysts to identify few factors instead of large number of parameters to develop the marketing strategy for launching the product.
2. In a mall, it is interesting to see the buying behaviour of the customers. On the basis of the customer's purchase history, the articles may be clubbed together and kept in nearby counters to enhance the sale.

3. In an educational institution, the administration may be interested to know the factors that are responsible for enhancing the status of the institution. Such accomplishment can be achieved by controlling a large number of parameters. The factor analysis may extract the underlying factors like academic curriculum, student's facilities, counseling procedure, and placement opportunity on which the administration may concentrate to improve its image instead of large number of parameters.
4. To evaluate a product, a survey technique can be used to identify various parameters. Based on it, factor analysis can identify few factors on which management can take decisions to promote their product.
5. Factor analysis may be used to create a lifestyle questionnaire for evaluating the quality of life. After dropping the questions from the questionnaire on the basis of item analysis, factor analysis provides the insight as to how much efficiency has been sacrificed due to it.

## Solved Example of Factor Analysis Using SPSS

### Example 11.1

An industrial researcher wanted to investigate the climate of an organization. A set of 12 questions were developed to measure different parameters of the climate. The subject could respond these questions on five-point scale with 5 indicating strongly agree and 1 strongly disagree attitude towards the question. The responses obtained on the questionnaire are shown in Table 11.1 along with the description of the questions. Apply factor analysis technique to study the factor structure and suggest the test battery that can be used for assessing the climate of any industrial organization. Also apply the scree test for retaining factors graphically and KMO test for testing the adequacy of data.

Statements

1. Employees are encouraged to attend training programs organized by outside agencies.
2. Any employee can reach up to the top level management position during their carrier.
3. Employees are praised by the immediate boss for doing something useful in the organization
4. Medical facilities for the employees and their families are excellent
5. Employees are given preference in jobs announced by the group of companies.
6. For doing some creative work or working creatively, employees get incentives
7. Employee's children are honored for their excellent performance in their education.
8. Employees are cooperative in helping each other to solve their professional problems
9. Fees of employees children are reimbursed during their schooling

**Table 11.1** Data on the parameters of organizational climate

| S.N | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 | S11 | S12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 1 | 3 | 5 | 4 | 2 | 2 | 4 | 3 | 4 |
| 2 | 3 | 3 | 5 | 2 | 3 | 4 | 5 | 1 | 3 | 5 | 4 | 5 |
| 3 | 5 | 3 | 4 | 2 | 2 | 5 | 5 | 2 | 3 | 4 | 3 | 4 |
| 4 | 3 | 4 | 4 | 1 | 3 | 4 | 4 | 1 | 2 | 5 | 3 | 4 |
| 5 | 4 | 3 | 4 | 2 | 2 | 5 | 4 | 2 | 3 | 4 | 2 | 4 |
| 6 | 3 | 2 | 5 | 2 | 3 | 4 | 5 | 1 | 4 | 5 | 5 | 5 |
| 7 | 5 | 3 | 4 | 2 | 2 | 4 | 4 | 2 | 3 | 4 | 2 | 4 |
| 8 | 4 | 4 | 5 | 1 | 1 | 4 | 4 | 1 | 2 | 4 | 3 | 2 |
| 9 | 5 | 3 | 4 | 2 | 3 | 4 | 4 | 3 | 3 | 3 | 2 | 4 |
| 10 | 5 | 3 | 5 | 2 | 2 | 5 | 5 | 2 | 4 | 4 | 4 | 5 |
| 11 | 4 | 4 | 4 | 1 | 3 | 5 | 5 | 2 | 3 | 4 | 3 | 3 |
| 12 | 4 | 2 | 5 | 2 | 4 | 5 | 4 | 1 | 2 | 5 | 2 | 2 |
| 13 | 3 | 2 | 4 | 1 | 3 | 4 | 4 | 1 | 3 | 2 | 3 | 3 |
| 14 | 5 | 3 | 5 | 2 | 2 | 5 | 4 | 2 | 3 | 5 | 2 | 4 |
| 15 | 4 | 3 | 4 | 1 | 3 | 4 | 5 | 1 | 2 | 4 | 3 | 4 |
| 16 | 4 | 2 | 4 | 3 | 2 | 4 | 5 | 2 | 3 | 3 | 2 | 5 |
| 17 | 2 | 3 | 4 | 2 | 3 | 4 | 4 | 2 | 4 | 4 | 3 | 3 |
| 18 | 4 | 2 | 5 | 2 | 4 | 5 | 4 | 3 | 3 | 5 | 2 | 4 |
| 19 | 3 | 3 | 4 | 1 | 3 | 3 | 4 | 2 | 2 | 4 | 3 | 4 |
| 20 | 4 | 3 | 4 | 2 | 2 | 4 | 5 | 3 | 3 | 3 | 2 | 5 |
| 21 | 5 | 4 | 4 | 1 | 3 | 4 | 5 | 2 | 2 | 4 | 3 | 3 |
| 22 | 4 | 3 | 5 | 2 | 4 | 5 | 4 | 1 | 3 | 5 | 4 | 4 |
| 23 | 3 | 2 | 4 | 1 | 3 | 5 | 4 | 2 | 2 | 5 | 3 | 5 |
| 24 | 4 | 3 | 4 | 2 | 2 | 4 | 4 | 1 | 3 | 4 | 2 | 4 |
| 25 | 5 | 2 | 5 | 3 | 3 | 5 | 5 | 2 | 4 | 5 | 3 | 3 |

10. Employees get fast promotion if their work is efficient and consistence
11. Senior managers are sensitive to the personal problems of their employees.
12. Employees get cheaper loan for buying vehicles.

*Solution*

By applying the factor analysis following issues shall be resolved:

1. To decide the number of factors to be retained and the total variance explained by these factors
2. To identify the variables in each factor retained in the final solution, on the basis of its factor loadings
3. To give names to each factor retained on the basis of the nature of variables included in it
4. To suggest the test battery for assessing the climate of any industrial organization
5. To test the adequacy of sample size used in factor analysis

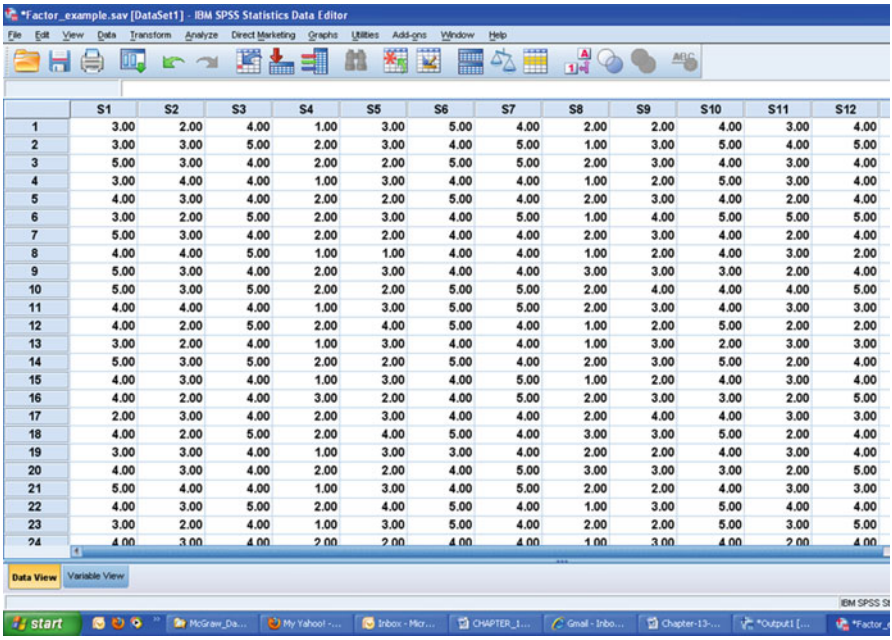
These objectives will be achieved by generating the output of factor analysis in SPSS. Thus, the procedure of using SPSS for factor analysis in the given example shall be discussed first, and thereafter the output shall be explained in the light of the objectives to be fulfilled in this study.

## *SPSS Commands for the Factor Analysis*

Before running the SPSS commands for factor analysis, a data file needs to be prepared. By now, you must have been familiar in preparing the data file. If not, you may go through the procedure discussed in Chap. 1 in this regard. Do the following steps for generating outputs in factor analysis:

(i) *Data file*: In this problem, all 12 statements are independent variables. These variables have been defined as 'Scale' variable because they were measured on interval scale. Variables measured on interval as well as ratio scales are treated as scale variable in SPSS. After preparing the data file by defining variable names and their labels, it will look like Fig. 11.2.

(ii) *Initiating command for factor analysis*: Once the data file is prepared, click the following command sequence in the Data View:




| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 | S11 | S12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.00 | 2.00 | 4.00 | 1.00 | 3.00 | 5.00 | 4.00 | 2.00 | 2.00 | 4.00 | 3.00 | 4.00 |
| 2 | 3.00 | 3.00 | 5.00 | 2.00 | 3.00 | 4.00 | 5.00 | 1.00 | 3.00 | 5.00 | 4.00 | 5.00 |
| 3 | 5.00 | 3.00 | 4.00 | 2.00 | 2.00 | 5.00 | 5.00 | 2.00 | 3.00 | 4.00 | 3.00 | 4.00 |
| 4 | 3.00 | 4.00 | 4.00 | 1.00 | 3.00 | 4.00 | 4.00 | 1.00 | 2.00 | 5.00 | 3.00 | 4.00 |
| 5 | 4.00 | 3.00 | 4.00 | 2.00 | 2.00 | 5.00 | 4.00 | 2.00 | 3.00 | 4.00 | 2.00 | 4.00 |
| 6 | 3.00 | 2.00 | 5.00 | 2.00 | 3.00 | 4.00 | 5.00 | 1.00 | 4.00 | 5.00 | 5.00 | 5.00 |
| 7 | 5.00 | 3.00 | 4.00 | 2.00 | 2.00 | 4.00 | 4.00 | 2.00 | 3.00 | 4.00 | 2.00 | 4.00 |
| 8 | 4.00 | 4.00 | 5.00 | 1.00 | 1.00 | 4.00 | 4.00 | 1.00 | 2.00 | 4.00 | 3.00 | 2.00 |
| 9 | 5.00 | 3.00 | 4.00 | 2.00 | 3.00 | 4.00 | 4.00 | 3.00 | 3.00 | 3.00 | 2.00 | 4.00 |
| 10 | 5.00 | 3.00 | 5.00 | 2.00 | 2.00 | 5.00 | 5.00 | 2.00 | 4.00 | 4.00 | 4.00 | 5.00 |
| 11 | 4.00 | 4.00 | 4.00 | 1.00 | 3.00 | 5.00 | 5.00 | 2.00 | 3.00 | 4.00 | 3.00 | 3.00 |
| 12 | 4.00 | 2.00 | 5.00 | 2.00 | 4.00 | 5.00 | 4.00 | 1.00 | 2.00 | 5.00 | 2.00 | 2.00 |
| 13 | 3.00 | 2.00 | 4.00 | 1.00 | 3.00 | 4.00 | 4.00 | 1.00 | 3.00 | 2.00 | 3.00 | 3.00 |
| 14 | 5.00 | 3.00 | 5.00 | 2.00 | 2.00 | 5.00 | 4.00 | 2.00 | 3.00 | 5.00 | 2.00 | 4.00 |
| 15 | 4.00 | 3.00 | 4.00 | 1.00 | 3.00 | 4.00 | 5.00 | 1.00 | 2.00 | 4.00 | 3.00 | 4.00 |
| 16 | 4.00 | 2.00 | 4.00 | 3.00 | 2.00 | 4.00 | 5.00 | 2.00 | 3.00 | 3.00 | 2.00 | 5.00 |
| 17 | 2.00 | 3.00 | 4.00 | 2.00 | 3.00 | 4.00 | 4.00 | 2.00 | 4.00 | 4.00 | 3.00 | 3.00 |
| 18 | 4.00 | 2.00 | 5.00 | 2.00 | 4.00 | 5.00 | 4.00 | 3.00 | 3.00 | 5.00 | 2.00 | 4.00 |
| 19 | 3.00 | 3.00 | 4.00 | 1.00 | 3.00 | 3.00 | 4.00 | 2.00 | 2.00 | 4.00 | 3.00 | 4.00 |
| 20 | 4.00 | 3.00 | 4.00 | 2.00 | 2.00 | 4.00 | 5.00 | 3.00 | 3.00 | 3.00 | 2.00 | 5.00 |
| 21 | 5.00 | 4.00 | 4.00 | 1.00 | 3.00 | 4.00 | 5.00 | 2.00 | 2.00 | 4.00 | 3.00 | 3.00 |
| 22 | 4.00 | 3.00 | 5.00 | 2.00 | 4.00 | 5.00 | 4.00 | 1.00 | 3.00 | 5.00 | 4.00 | 4.00 |
| 23 | 3.00 | 2.00 | 4.00 | 1.00 | 3.00 | 5.00 | 4.00 | 2.00 | 2.00 | 5.00 | 3.00 | 5.00 |
| 24 | 4.00 | 3.00 | 4.00 | 2.00 | 2.00 | 4.00 | 4.00 | 1.00 | 3.00 | 4.00 | 2.00 | 4.00 |

**Fig. 11.2** Screen showing data file for the factor analysis in SPSS

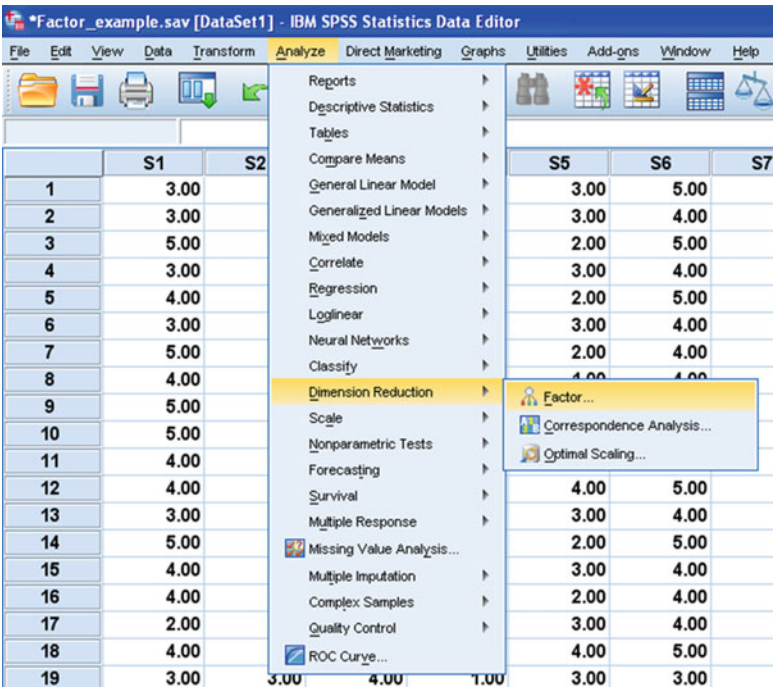



**Fig. 11.3** Screen showing SPSS commands for factor analysis

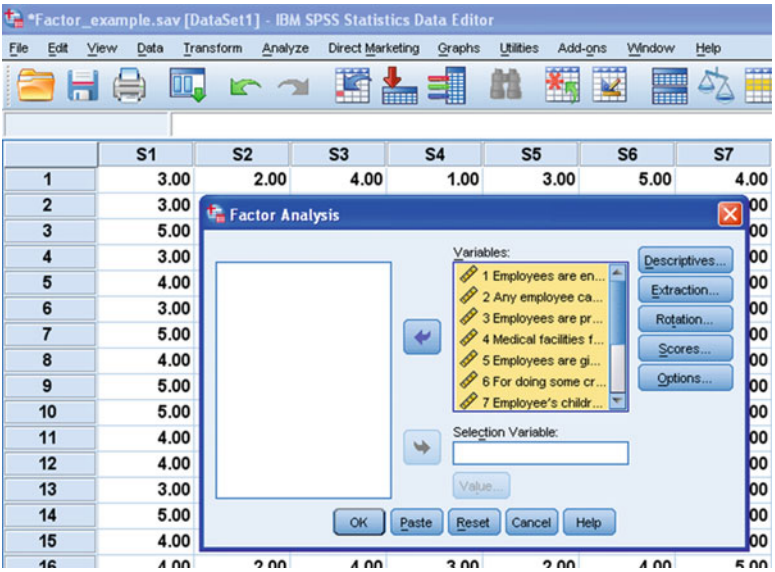



**Fig. 11.4** Screen showing selection of variables for factor analysis

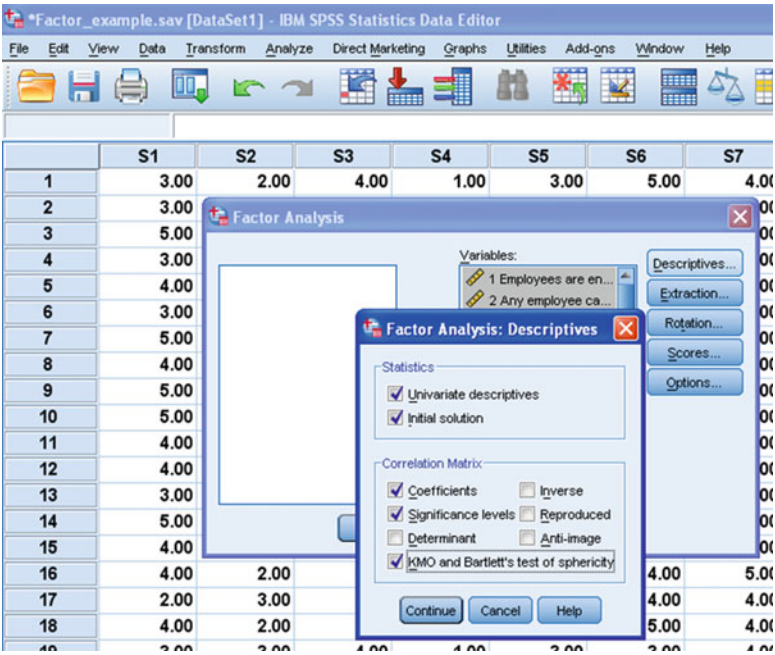



**Fig. 11.5** Screen showing option for correlation matrix and initial factor solution

**Analyze → Dimension Reduction → Factor**

The screen shall look like as shown in Fig. 11.3.

(iii) *Selecting variables for factor analysis*: After clicking the **Factor** option, the SPSS package will take you to the next screen for selecting variables. Select all the variables from left panel to the "Variables" section of the right panel. The screen will look like Fig. 11.4.

(iv) *Selecting options for computation*: After selecting the variables, various options need to be defined for generating the output in factor analysis. Do the following:

- Click the tag **Descriptives** in the screen shown in Fig. 11.5 and
  - Check the option "Univariate descriptive" and ensure that the option "Initial Solution" is checked in the Statistics section by default.
  - Check the option "Coefficients," "Significance levels," and "KMO and Bartlett's test of sphericity" in "Correlation Matrix" section.

    The screen will look like Fig. 11.5.
  - Press ***Continue***. This will again take you back to the screen shown in Fig. 11.4.
- Now click the tag **Extraction** and then check "Scree plot." Let other options remain as it is by default.

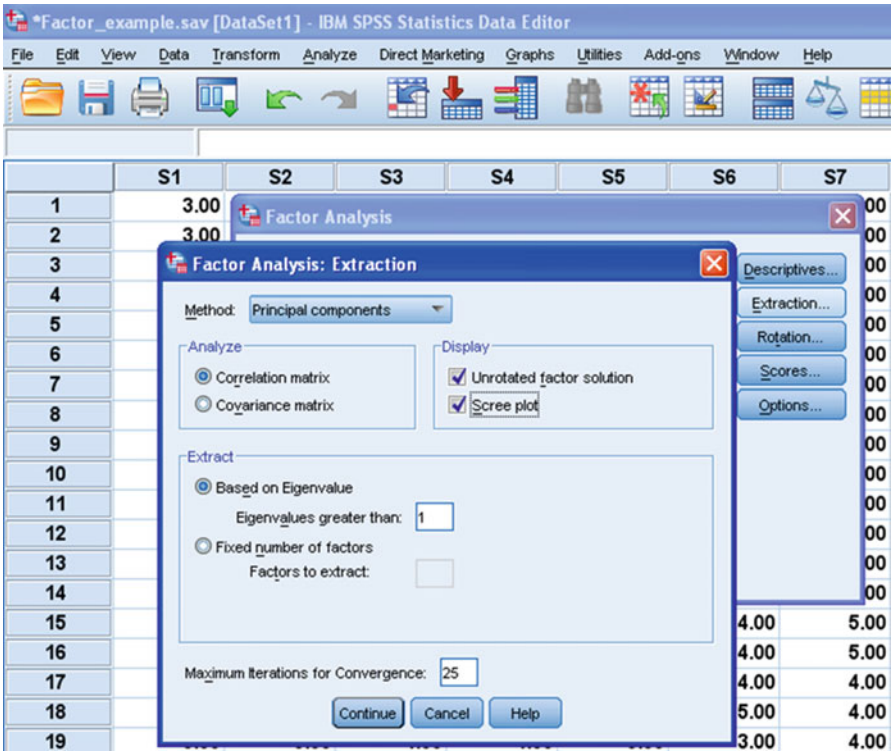



**Fig. 11.6** Screen showing option for unrotated factor solution and scree plot

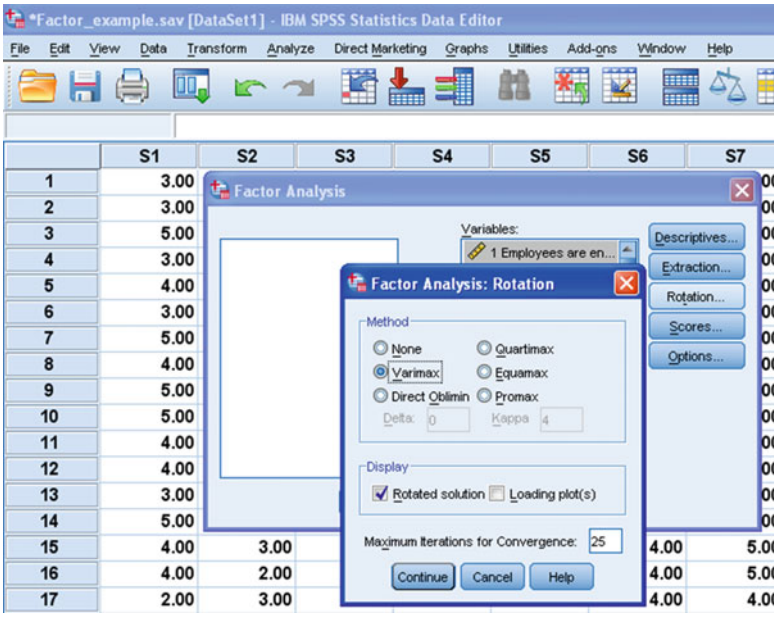



**Fig. 11.7** Screen showing option for factor rotation

The screen shall look like Fig. 11.6.

- Press ***Continue***. This will again take you back to the screen shown in Fig. 11.4.
- Now click the tag **Rotation** and then check "Varimax" rotation option. Let other options remain as it is by default.
  - The screen shall look like Fig. 11.7.
  - Press ***Continue*** to go back to the main screen.
- Press **OK** for output.

(v) *Getting the output*: After pressing **OK** in the screen shown in Fig. 11.4, the SPSS will generate the outputs in the output window. These outputs can be selected by using the right click of the mouse and may be pasted in the word file. The SPSS shall generate many outputs, but the following relevant outputs have been selected for the discussion:

(a) Descriptive statistics
(b) Correlation matrix
(c) KMO and Bartlett's test
(d) Communalities of all the variables
(e) Total variance explained
(f) Scree plot
(g) Component matrix: unrotated factor solution
(h) Rotated component matrix: varimax-rotated solution

In this example, all the outputs so generated by the SPSS are shown in Tables 11.2–11.7 and Fig. 11.8.

## *Interpretation of Various Outputs Generated in Factor Analysis*

The above-mentioned outputs generated in this example by the SPSS shall be discussed to provide the answers to various issues related to model developed in this study.

1. Table 11.2 shows the mean and SD for all the variables in the study. You may add some more statistics like coefficient of variation, skewness, kurtosis, and range to study the nature of variables. However, in that case, you have to use the SPSS option of **Descriptive** discussed in Chap. 2 of this book.
2. Table 11.3 is the correlation matrix of all the variables. This is the first step in factor analysis on the basis of which variables are grouped into factors. The SPSS provides significance value ($p$ value) for each correlation coefficient. However, values of correlation coefficient required for its significance at 1% as well as 5% can be seen from Table A.3 in the Appendix. Meaningful conclusions can be drawn from this table for understanding relationships among variables.

**Table 11.2** Descriptive statistics for the parameters of organizational climate

| | | Mean | Std. deviation | *N* |
|---|---|---|---|---|
| 1 | Employees are encouraged to attend training programs organized by outside agencies | 3.9200 | .86217 | 25 |
| 2 | Any employee can reach up to the top level management position during their carrier | 2.8400 | .68799 | 25 |
| 3 | Employees are praised by the immediate boss for doing something useful in the organization | 4.3600 | .48990 | 25 |
| 4 | Medical facilities for the employees and their families are excellent | 1.7200 | .61373 | 25 |
| 5 | Employees are given preference in jobs announced by the group of companies | 2.7200 | .73711 | 25 |
| 6 | For doing some creative work or working creatively, employees get incentives | 4.4000 | .57735 | 25 |
| 7 | Employee's children are honored for their excellent performance in their education | 4.4000 | .50000 | 25 |
| 8 | Employees are cooperative in helping each other to solve their professional problems | 1.7600 | .66332 | 25 |
| 9 | Fees of Employees children are reimbursed during their schooling | 2.8400 | .68799 | 25 |
| 10 | Employees get fast promotion if their work is efficient and consistence | 4.1600 | .80000 | 25 |
| 11 | Senior managers are sensitive to the personal problems of their employees | 2.8400 | .80000 | 25 |
| 12 | Employees get cheaper loan for buying vehicles | 3.8800 | .88129 | 25 |

3. Table 11.4 shows the result of KMO test, which tells whether sample size taken for the factor analysis was adequate or not. It tests whether the partial correlations among variables are small. The value of KMO ranges from 0 to 1. The closer the value of KMO to 1, the more adequate is the sample size to run the factor analysis. Usually the value of KMO more than 0.5 is considered to be sufficient for doing factor analysis reliably. In this case, KMO value is 0.408, which is <.5; hence, the sample size is not adequate, and more samples should be taken for the analysis. Since this is a simulated example developed to make the procedure clear, hence less number of data set was taken.
   Further, Bartlett's test of sphericity is used to test the null hypothesis that the correlation matrix is an identity matrix. Since significance value (*p* value) of Bartlett's test is .002 in Table 11.4, which is <.01, hence it is significant, and the correlation matrix is not an identity matrix. Thus, it may be concluded that the factor model is appropriate.
4. Table 11.5 shows the communalities of all the variables. Higher communality of a variable indicates that the major portion of its variability is explained by all the identified factors in the analysis. If communality of variable is <.4, it is considered to be useless and should normally be removed from the model. From Table 11.5, it can be seen that the communalities of all the variables are more than .4; hence, all the variables are useful in the model.

**Table 11.3** Correlation matrix for the parameters of the organizational climate

| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 | S11 | S12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S1 | 1 | 0.188 | 0.17 | 0.35 | −0.299 | 0.318 | 0.271 | 0.329 | 0.118 | −0.041 | −0.321 | −0.068 |
| S2 | 0.188 | 1 | −0.193 | −0.407* | −0.339 | −0.252 | 0.073 | −0.088 | −0.232 | −0.027 | 0.027 | −0.239 |
| S3 | 0.17 | −0.193 | 1 | 0.349 | 0.175 | 0.354 | 0.068 | −0.236 | 0.302 | 0.591** | 0.366 | −0.089 |
| S4 | 0.35 | −0.407* | 0.349 | 1 | −0.088 | 0.212 | 0.244 | 0.237 | 0.679** | 0.095 | −0.18 | 0.243 |
| S5 | −0.299 | −0.339 | 0.175 | −0.088 | 1 | 0.176 | −0.136 | −0.058 | −0.092 | 0.362 | 0.203 | −0.118 |
| S6 | 0.318 | −0.252 | 0.354 | 0.212 | 0.176 | 1 | 0 | 0.152 | 0.168 | 0.397* | −0.036 | −0.066 |
| S7 | 0.271 | 0.073 | 0.068 | 0.244 | −0.136 | 0 | 1 | 0.05 | 0.315 | −0.063 | 0.375 | 0.303 |
| S8 | 0.329 | −0.088 | −0.236 | 0.237 | −0.058 | 0.152 | 0.05 | 1 | 0.186 | −0.239 | −0.468 | 0.234 |
| S9 | 0.118 | −0.232 | 0.302 | 0.679** | −0.092 | 0.168 | 0.315 | 0.186 | 1 | −0.027 | 0.254 | 0.242 |
| S10 | −0.041 | −0.027 | 0.591** | 0.095 | 0.362 | 0.397* | −0.063 | −0.239 | −0.027 | 1 | 0.302 | 0.028 |
| S11 | −0.321 | 0.027 | 0.366 | −0.18 | 0.203 | −0.036 | 0.375 | −0.468 | 0.254 | 0.302 | 1 | 0.208 |
| S12 | −0.068 | −0.239 | −0.089 | 0.243 | −0.118 | −0.066 | 0.303 | 0.234 | 0.242 | 0.028 | 0.208 | 1 |

Value of "$r$" required for its significance at .05 level = 0.396, $d.f$ = N−2 = 23, * Significant at .05 level
Value of "$r$" required for its significance at .01 level = 0.505, $d.f$ = N−2 = 23, ** Significant at .01 level

**Table 11.4** KMO and Bartlett's test

| | | |
|---|---|---|
| Kaiser-Meyer-Olkin measure of sampling adequacy | | .408 |
| Bartlett's test of sphericity | Approx. chi-square | 105.281 |
| | df | 66 |
| | Sig. | .002 |

**Table 11.5** Communalities of all the variables

| | | Initial | Extraction |
|---|---|---|---|
| 1 | Employees are encouraged to attend training programs organized by outside agencies | 1.000 | .810 |
| 2 | Any employee can reach up to the top level management position during their carrier | 1.000 | .761 |
| 3 | Employees are praised by the immediate boss for doing something useful in the organization | 1.000 | .764 |
| 4 | Medical facilities for the employees and their families are excellent | 1.000 | .756 |
| 5 | Employees are given preference in jobs announced by the group of companies | 1.000 | .597 |
| 6 | For doing some creative work or working creatively, employees get incentives | 1.000 | .602 |
| 7 | Employee's children are honored for their excellent performance in their education | 1.000 | .635 |
| 8 | Employees are cooperative in helping each other to solve their professional problems | 1.000 | .613 |
| 9 | Fees of employees children are reimbursed during their schooling | 1.000 | .665 |
| 10 | Employees get fast promotion if their work is efficient and consistence | 1.000 | .680 |
| 11 | Senior managers are sensitive to the personal problems of their employees | 1.000 | .868 |
| 12 | Employees get cheaper loan for buying vehicles | 1.000 | .548 |

**Table 11.6** Total variance explained

| | Initial eigenvalues | | | Extraction sums of squared loadings | | | Rotation sums of squared loadings | | |
|---|---|---|---|---|---|---|---|---|---|
| Component | Total | % of variance | cumulative % | Total | % of variance | cumulative % | Total | % of variance | cumulative % |
| 1 | 2.721 | 22.677 | 22.677 | 2.721 | 22.677 | 22.677 | 2.312 | 19.266 | 19.266 |
| 2 | 2.360 | 19.668 | 42.345 | 2.360 | 19.668 | 42.345 | 2.232 | 18.601 | 37.867 |
| 3 | 1.728 | 14.403 | 56.748 | 1.728 | 14.403 | 56.748 | 2.083 | 17.355 | 55.222 |
| 4 | 1.488 | 12.397 | 69.144 | 1.488 | 12.397 | 69.144 | 1.671 | 13.923 | 69.144 |
| 5 | .951 | 7.929 | 77.073 | | | | | | |
| 6 | .740 | 6.165 | 83.238 | | | | | | |
| 7 | .589 | 4.907 | 88.145 | | | | | | |
| 8 | .558 | 4.646 | 92.791 | | | | | | |
| 9 | .371 | 3.089 | 95.880 | | | | | | |
| 10 | .278 | 2.320 | 98.200 | | | | | | |
| 11 | .143 | 1.191 | 99.390 | | | | | | |
| 12 | .073 | .610 | 100.000 | | | | | | |

Extraction method: Principal component analysis

**Table 11.7** Component matrix[a] unrotated factor solution

| | | Component | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| 1 | Employees are encouraged to attend training programs organized by outside agencies | 0.303 | 0.549 | −0.285 | 0.58 |
| 2 | Any employee can reach up to the top level management position during their carrier | −0.467 | 0.057 | 0.172 | 0.715 |
| 3 | Employees are praised by the immediate boss for doing something useful in the organization | 0.681 | −0.429 | −0.15 | 0.306 |
| 4 | Medical facilities for the employees and their families are excellent | 0.763 | 0.4 | −0.048 | −0.109 |
| 5 | Employees are given preference in jobs announced by the group of companies | 0.156 | −0.567 | −0.282 | −0.414 |
| 6 | For doing some creative work or working creatively, employees get incentives | 0.539 | −0.071 | −0.535 | 0.139 |
| 7 | Employee's children are honored for their excellent performance in their education | 0.386 | 0.21 | 0.601 | 0.285 |
| 8 | Employees are cooperative in helping each other to solve their professional problems | 0.131 | 0.689 | −0.25 | −0.243 |
| 9 | Fees of employees children are reimbursed during their schooling | 0.712 | 0.251 | 0.297 | −0.082 |
| 10 | Employees get fast promotion if their work is efficient and consistence | 0.444 | −0.615 | −0.25 | 0.205 |
| 11 | Senior managers are sensitive to the personal problems of their employees | 0.279 | −0.615 | 0.631 | 0.119 |
| 12 | Employees get cheaper loan for buying vehicles | 0.321 | 0.217 | 0.505 | −0.377 |

Extraction method: Principal component analysis
[a]Four components extracted

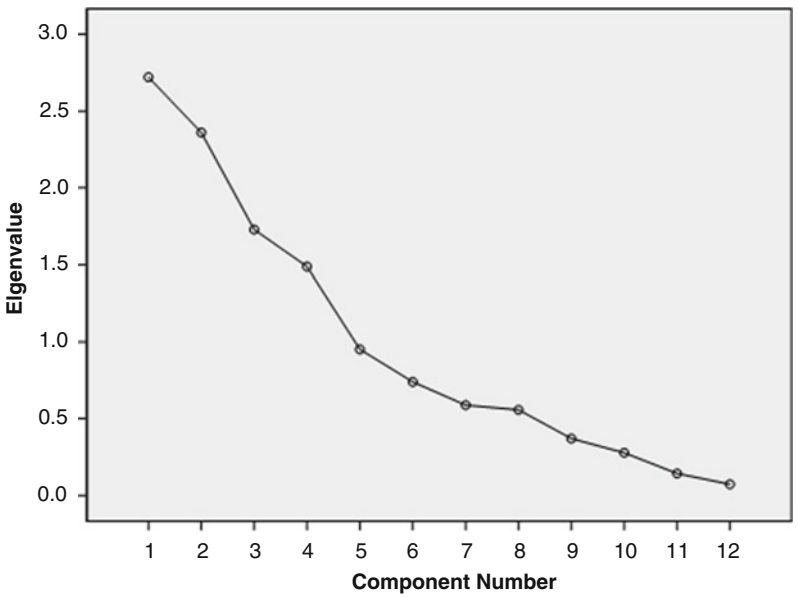



**Fig. 11.8** Scree plot for the factors

5. Table 11.6 shows the factors extracted and the variance explained by these factors. It can be seen that after rotation, the first, second, third, and fourth factors explain 19.266, 18.601, 17.355, and 13.923% of the total variance, respectively. Thus, all these four factors together explain 69.144% of the total variance.
   The eigenvalues for each of the factor are shown in Table 11.6. Only those factors are retained whose eigenvalues are 1 or more than 1. Here, you can see that the eigenvalue for the first four factors are >1; hence, only four factors have been retained in this study.
   Figure 11.8 shows the scree plot which is obtained by plotting the factors (along X-axis) against their eigenvalues (along Y-axis). This plot shows that only four factors have eigenvalues above elbow bent; hence, only four factors have been retained in this study.
6. Table 11.7 shows the first initial unrotated solution of the factor analysis. Four factors have been extracted in this study. The factor loadings of all the variables on each of the four factors have been shown in this table. Since this is an unrotated factor solution, and therefore some of the variables may show their contribution in more than one factor. In order to avoid this situation, the factors are rotated. Varimax rotation has been used in this example to rotate the factors, as this is the most popular method used by the researchers due to its efficiency.
7. After using the varimax rotation, the final solution so obtained is shown in Table 11.8. Clear picture emerges in this final solution about the variables explaining the factors correctly. The rotation facilitates the variable to appear in one and only factor.
   Variables are usually identified in a factor if their loading on that factor is 0.7 or more. This ensures that the factor extracts sufficient variance from that variable. However, one may reduce this threshold value if sufficient variables cannot be identified in the factor. In this problem, the variables have been retained in a factor in which its loadings are greater than or equal to 0.6. Owing to this criterion variables have been grouped in each of the four factors, namely, welfare, motivation, interpersonal relation, and career which are shown in Tables 11.9, 11.10, 11.11, and 11.12.
   Factor 1 in Table 11.9 contains variables that measure the welfare of employees in an organization, and therefore it may be termed as "*Welfare Factor*." On the other hand, all items mentioned in Table 11.10 measure the motivation of employees; hence, factor 2 is named as "Motivation Factor." Similarly the items in Tables 11.11 and 11.12 are related with measuring relationships among employees and career-related issues; hence, factor 3 and factor 4 may be termed as "*interpersonal relation factor*" and "*career factor,*" respectively.
   In order to develop a test battery to measure the climate of an organization, one may choose variables from these identified factors. Since percentage contribution of each factor in the measurement of total variability are more or less same, hence one variable from each factor having highest loadings on the factor may be picked up to develop the test battery for measuring the climate of an organization. Thus, the test battery so developed is shown in Table 11.13. One may

**Table 11.8** Rotated component matrix[a]: varimax-rotated solution

| | | Component | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| 1 | Employees are encouraged to attend training programs organized by outside agencies | 0.124 | 0.385 | −0.482 | 0.644 |
| 2 | Any employee can reach up to the top level management position during their carrier | −0.382 | −0.153 | 0.23 | 0.734 |
| 3 | Employees are praised by the immediate boss for doing something useful in the organization | 0.19 | 0.809 | 0.271 | −0.021 |
| 4 | Medical facilities for the employees and their families are excellent | 0.689 | 0.365 | −0.385 | −0.016 |
| 5 | Employees are given preference in jobs announced by the group of companies | −0.15 | 0.291 | 0.164 | −0.68 |
| 6 | For doing some creative work or working creatively, employees get incentives | 0.038 | 0.722 | −0.268 | −0.084 |
| 7 | Employee's children are honored for their excellent performance in their education | 0.621 | −0.01 | 0.256 | 0.429 |
| 8 | Employees are cooperative in helping each other to solve their professional problems | 0.252 | −0.114 | −0.732 | 0.012 |
| 9 | Fees of employees children are reimbursed during their schooling | 0.785 | 0.214 | −0.051 | 0.025 |
| 10 | Employees get fast promotion if their work is efficient and consistence | −0.079 | 0.725 | 0.339 | −0.185 |
| 11 | Senior managers are sensitive to the personal problems of their employees | 0.307 | 0.125 | 0.87 | −0.048 |
| 12 | Employees gets cheaper loan for buying vehicles | 0.671 | −0.261 | 0.055 | −0.159 |

Extraction method: Principal component analysis
Rotation method: Varimax with Kaiser normalization
[a]Rotation converged in seven iterations

**Table 11.9** Factor 1: *welfare*

| Items | | Loadings |
|---|---|---|
| 4 | Medical facilities for the employees and their families are excellent | 0.689 |
| 7 | Employee's children are honored for their excellent performance in their education | 0.621 |
| 9 | Fees of employees children are reimbursed during their schooling | 0.785 |
| 12 | Employees get cheaper loan for buying vehicles | 0.671 |

choose more than one variable from one or two factors also, depending upon their explainability.

Readers are advised to run the confirmatory factor analysis with more data set to these questions before using this instrument to measure the organizational climate because this was a simulated study.

**Table 11.10** Factor 2: *motivation*

| Items | | Loadings |
|---|---|---|
| 3 | Employees are praised by the immediate boss for doing something useful in the organization | 0.809 |
| 6 | For doing some creative work or working creatively, employees get incentives | 0.722 |
| 10 | Employees get fast promotion if their work is efficient and consistence | 0.725 |

**Table 11.11** Factor 3: *interpersonal relation*

| Items | | Loadings |
|---|---|---|
| 8 | Employees are cooperative in helping each other to solve their professional problems | −0.732 |
| 11 | Senior managers are sensitive to the personal problems of their employees | 0.87 |

**Table 11.12** Factor 4: *career*

| Items | | Loadings |
|---|---|---|
| 1 | Employees are encouraged to attend training programs organized by outside agencies | 0.644 |
| 2 | Any employee can reach up to the top level management position during their carrier | 0.734 |
| 5 | Employees are given preference in jobs announced by the group of companies | −0.68 |

**Table 11.13** Test battery for measuring the climate of an organization

| Items | | Loadings |
|---|---|---|
| 9 | Fees of employees children are reimbursed during their schooling | 0.785 |
| 10 | Employees get fast promotion if their working is efficient and consistence | 0.725 |
| 11 | Senior managers are sensitive to the personal problems of their employees | 0.87 |
| 2 | Any employee can reach up to the top level management position during their carrier | 0.734 |

## Summary of the SPSS Commands for Factor Analysis

(i) Start SPSS and prepare data file by defining the variables and their properties in **Variable View** and typing the data column-wise in **Data View**.

(ii) In the data view, follow the below-mentioned command sequence for factor analysis:

**Analyze → Dimension Reduction → Factor**

(iii) Select all the variables from left panel to the "Variables" section of the right panel.

(iv) Click the tag **Descriptives** and check the options "Univariate descriptives," "Initial Solution," "Coefficients," "Significance levels," and "KMO and Bartlett's test of sphericity." Press ***Continue***.

(v) Click the tag **Extraction** and then check "Scree plot." Let other options remain as it is by default. Press ***Continue***.

(vi) Click the tag **Rotation** and then check "Varimax" rotation option. Let other option remains as it is by default. Press ***Continue***.
(vii) Click **OK** for output.

## Exercise

*Short Answer Questions*

**Note**: Write answer to each of the following questions in not more than 200 words.

Q.1. What do you mean by a factor? What is the criterion of retaining a factor in a study and identifying the variables in it?
Q.2. How the factor analysis is useful in understanding the group characteristics
Q.3. Describe an experimental situation in which the factor analysis can be used.
Q.4. How factor analysis can be useful in developing a questionnaire?
Q.5. Discuss the procedure of developing a test battery to assess the lifestyle of employees of an organization.
Q.6. What is principal component analysis and how it is used in factor analysis?
Q.7. What do you mean by eigenvalue? How the Kaiser's criterion works in retaining factors in the model?
Q.8. What do you mean by scree test? How is it useful in identifying the factors to be retained through graph?
Q.9. What is the importance of communality in factor analysis?
Q.10. What is the significance of factor loadings? How it is used to identify the variables to be retained in the factors?
Q.11. Why the factors are rotated to get the final solution in factor analysis? Which is the most popular rotation method and why?

*Multiple-Choice Questions*

**Note**: Question no. 1–10 has four alternative answers for each question. Tick marks the one that you consider the closest to the correct answer.

1. Factor analysis is a technique for

   (a) Correlation analysis
   (b) Dimension reduction
   (c) Finding the most important variable
   (d) Comparing factors

2. Principal component analysis extracts the maximum variance in the

   (a) Last extracted factor
   (b) Second extracted factor
   (c) First extracted factor
   (d) Any extracted factor

3. In exploratory factor analysis,

   (a) The factors are identified among the large number of variables
   (b) The variables are clubbed into the factors
   (c) The variables that do not contribute to the factor model are removed
   (d) Factor model is tested

4. The sample is adequate in factor analysis if the value of KMO is

   (a) $<0.5$
   (b) $\geq 0.0.5$
   (c) 0
   (d) 1

5. The variable's variability is considered to be measured by the identified factors if its communality is

   (a) $\geq 0.3$
   (b) $\geq 0.6$
   (c) $\geq 0.4$
   (d) 1

6. Choose the correct sequence of SPSS commands for factor analysis

   (a) Analyze → Dimension Reduction → Factor
   (b) Analyze → Factor → Dimension Reduction
   (c) Factor → Dimension Reduction → Analyze
   (d) Dimension Reduction → Factor → Analyze

7. Owing to Kaiser's criteria the factor is retained if its eigenvalue is

   (a) Less than 1
   (b) Equal to 1
   (c) More than 2
   (d) More than 1

8. Scree test is the graph between

   (a) Eigenvalues and factors
   (b) Percentage variance explained and factors
   (c) Maximum factor loadings in the factors and factors
   (d) Communality and factor

9. Conventionally a variable is retained in a factor if its loading is greater than or equal to

   (a) 0.4
   (b) 0.5
   (c) 0.7
   (d) 0.2

10. Varimax rotation is used to get the final solution. After rotation

    (a) Factor explaining maximum variance is extracted first
    (b) All factors whose eigenvalues are more than 1 are extracted
    (c) Three best factors are extracted
    (d) Non overlapping of variables in the factors emerges

11. Eigen value is also known as

    (a) Characteristics root
    (b) Factor loading
    (c) Communality
    (d) None of the above

12. KMO test in factor analysis is used to test whether

    (a) Factors extracted are valid or not?
    (b) Variables identified in each factor are valid or not?
    (c) Sample size taken for the factor analysis was adequate or not?
    (d) Multicolinearity among the variables exists or not?

13. Bartlett's test in factor analysis is used for testing

    (a) Same adequacy
    (b) Whether correlation matrix is identity matrix
    (c) Usefulness of variable
    (d) Retaining the factors in the model

14. While using factor analysis certain assumptions need to be satisfied. Choose the most appropriate assumption

    (a) Data used in the factor analysis is based on interval scale or ratio scale
    (b) Multicolinearity among the variables exist
    (c) Outlier is present in the data
    (d) Size of the sample does not affect the analysis.

*Assignments*

1. It is decided to measure the personality profile of the senior executives in a manufacturing industry. Eleven personality characteristics were measured on 30 senior executives chosen randomly from an organization. Marks on each of these characteristics were measured on a ten-point scale. The meaning for each of these characteristics is described below the table. The data so obtained are shown in the following table. Apply factor analysis using varimax rotation. Discuss your findings and answer the following questions:

    (a) Whether data is adequate for factor analysis?
    (b) Whether sphericity is significant?
    (c) How many factors have been extracted?
    (d) In your opinion, what should be the name of the factors?
    (e) What factor loadings you suggest for a variable to qualify in a factor?
    (f) Can you suggest the test battery for screening the personality characteristics of an executive?

Data on personality characteristics obtained on senior executives

| S. N. | Friend | Achiev | Order | Auto | Domi | Sensit | Exhibit | End | Need | Help_Tem | Le_change |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 3 | 5 | 8 | 7 | 6 | 8 | 4 | 7 | 9 | 8 |
| 2 | 7 | 4 | 4 | 6 | 8 | 7 | 7 | 5 | 8 | 8 | 7 |
| 3 | 6 | 5 | 5 | 7 | 9 | 8 | 8 | 3 | 9 | 8 | 9 |
| 4 | 7 | 4 | 4 | 6 | 8 | 8 | 7 | 5 | 7 | 7 | 8 |
| 5 | 8 | 3 | 5 | 8 | 7 | 7 | 8 | 4 | 8 | 8 | 7 |
| 6 | 6 | 4 | 4 | 6 | 8 | 7 | 9 | 4 | 9 | 6 | 8 |
| 7 | 7 | 4 | 5 | 6 | 9 | 6 | 8 | 5 | 8 | 7 | 9 |
| 8 | 7 | 5 | 3 | 6 | 8 | 7 | 7 | 3 | 9 | 8 | 8 |
| 9 | 8 | 4 | 3 | 7 | 9 | 8 | 8 | 3 | 7 | 7 | 7 |
| 10 | 6 | 5 | 4 | 8 | 8 | 7 | 7 | 4 | 7 | 8 | 8 |
| 11 | 8 | 4 | 3 | 6 | 7 | 7 | 8 | 5 | 8 | 7 | 9 |
| 12 | 6 | 5 | 4 | 7 | 8 | 6 | 7 | 4 | 9 | 8 | 7 |
| 13 | 7 | 3 | 5 | 6 | 7 | 7 | 8 | 3 | 8 | 7 | 7 |
| 14 | 6 | 4 | 3 | 7 | 8 | 7 | 7 | 4 | 7 | 8 | 8 |
| 15 | 7 | 4 | 5 | 7 | 9 | 8 | 8 | 5 | 8 | 7 | 9 |
| 16 | 8 | 5 | 4 | 6 | 7 | 7 | 7 | 3 | 7 | 8 | 8 |
| 17 | 5 | 4 | 4 | 8 | 8 | 6 | 8 | 4 | 8 | 7 | 7 |
| 18 | 6 | 3 | 4 | 6 | 7 | 7 | 9 | 5 | 9 | 8 | 8 |
| 19 | 7 | 4 | 5 | 7 | 9 | 6 | 8 | 3 | 9 | 7 | 7 |
| 20 | 6 | 5 | 3 | 6 | 8 | 7 | 6 | 4 | 8 | 9 | 8 |
| 21 | 8 | 4 | 4 | 6 | 7 | 6 | 8 | 5 | 7 | 7 | 7 |
| 22 | 7 | 3 | 3 | 8 | 8 | 7 | 7 | 4 | 8 | 8 | 6 |
| 23 | 8 | 4 | 4 | 6 | 9 | 6 | 8 | 3 | 7 | 7 | 7 |
| 24 | 7 | 5 | 5 | 6 | 8 | 6 | 9 | 4 | 8 | 9 | 8 |
| 25 | 6 | 4 | 5 | 7 | 7 | 7 | 8 | 5 | 7 | 7 | 9 |
| 26 | 5 | 5 | 3 | 5 | 8 | 8 | 9 | 4 | 8 | 8 | 8 |
| 27 | 7 | 4 | 4 | 7 | 7 | 6 | 7 | 5 | 7 | 7 | 7 |
| 28 | 8 | 3 | 3 | 8 | 8 | 7 | 7 | 4 | 8 | 8 | 8 |
| 29 | 7 | 4 | 5 | 7 | 7 | 6 | 8 | 4 | 9 | 9 | 9 |
| 30 | 8 | 5 | 5 | 6 | 8 | 8 | 7 | 5 | 8 | 7 | 8 |

*Friend* Friendliness, *Achiev* Achievement, *Order* Orderliness, *Auto* Autonomy, *Domi* Dominance, *Sensit* Sensitiveness, *Exhibit* Exhibition, *End* Endurance, *Need* Neediness, *Help_Tem* Helping temperament, *Le_change* Learn to change

*Explanation of Parameters*

(a) *Friendliness*: Being friendly with others and try to be networked all the time.
(b) *Achievement*: Doing one's best or difficult tasks and achieving recognition
(c) *Orderliness*: Doing work systematically
(d) *Autonomy*: Lead your life the way you feel like.
(e) *Dominance*: Always ready to assume the leadership
(f) *Sensitiveness*: Understand the other's point of view in analyzing the situation.

(g) *Exhibition*: To showcase one's self by appearance, speech, and manner for attracting others.
(h) *Endurance*: Being focus toward work until it is completed and being able to work without being distracted.
(i) *Neediness*: Always ready to take support of others with grace and remains obliged for that.
(j) *Helping temperament*: Always ready to help the needy and less fortunate.
(k) *Learn to change*: Always ready to change due to change environment.

2. A researcher wants to know the factors that are responsible for people to choose the Rajdhani Express at different routes in India. Twenty respondents who recently traveled from this train were selected for getting their responses. These subjects were given a questionnaire consisting of ten questions mentioned below. They were asked to give their opinion on a seven-point scale where 1 indicates complete agreement and seven complete disagreements. The responses so obtained are shown in the following table.

Apply factor analysis and use varimax rotation to discuss your findings. Explain the factors so extracted in the study.

Questionnaire includes

1. The attendants are caring
2. The bedding provided in the train is neat and clean.

Response data obtained from the passengers on the services provided during journey in the train

| S. N. | 1. Caring | 2. Bedding | 3. Courteous | 4. Food | 5. Spray | 6. Toilets | 7. Timeliness | 8. Seats | 9. Clean | 10. Snacks |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 1 | 2 | 3 | 4 | 2 | 2 | 4 | 1 |
| 2 | 3 | 1 | 2 | 3 | 2 | 5 | 4 | 2 | 5 | 2 |
| 3 | 4 | 2 | 4 | 4 | 3 | 6 | 4 | 3 | 6 | 3 |
| 4 | 1 | 1 | 2 | 3 | 2 | 4 | 3 | 2 | 4 | 2 |
| 5 | 2 | 2 | 3 | 4 | 2 | 4 | 4 | 3 | 3 | 3 |
| 6 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 3 | 2 |
| 7 | 4 | 1 | 3 | 6 | 2 | 5 | 5 | 1 | 5 | 5 |
| 8 | 5 | 1 | 5 | 5 | 3 | 6 | 5 | 2 | 6 | 4 |
| 9 | 5 | 1 | 2 | 2 | 2 | 5 | 3 | 2 | 5 | 1 |
| 10 | 3 | 2 | 3 | 2 | 2 | 5 | 3 | 3 | 5 | 2 |
| 11 | 6 | 1 | 4 | 4 | 2 | 4 | 4 | 2 | 6 | 3 |
| 12 | 6 | 2 | 6 | 3 | 3 | 6 | 3 | 3 | 6 | 2 |
| 13 | 2 | 5 | 3 | 4 | 6 | 5 | 4 | 6 | 4 | 3 |
| 14 | 1 | 2 | 3 | 3 | 3 | 4 | 4 | 3 | 2 | 2 |
| 15 | 3 | 1 | 2 | 4 | 3 | 5 | 5 | 2 | 3 | 3 |
| 16 | 4 | 5 | 3 | 4 | 5 | 4 | 5 | 6 | 4 | 3 |
| 17 | 2 | 2 | 2 | 1 | 3 | 3 | 3 | 3 | 5 | 1 |
| 18 | 2 | 1 | 3 | 3 | 2 | 3 | 4 | 2 | 6 | 2 |
| 19 | 1 | 2 | 2 | 3 | 3 | 4 | 3 | 3 | 2 | 2 |
| 20 | 3 | 1 | 1 | 1 | 4 | 2 | 2 | 2 | 3 | 1 |

3. The *ticket* checkers are very courteous.
4. The quality of food is good.
5. To done away the foul smell fresheners are sprayed.
6. Toilets are always clean.
7. Foods are provided timely during the journey.
8. Seats are very comfortable.
9. Surroundings are clean all the time clean.
10. Vendors keep providing fresh and hot snacks all the time.

*Answers of Multiple-Choice Questions*

| | | | |
|---|---|---|---|
| Q.1 b | Q.2 c | Q.3 c | Q.4 b |
| Q.5 c | Q.6 a | Q.7 d | Q.8 a |
| Q.9 c | Q.10 d | Q.11 a | Q.12 c |
| Q.13 b | Q.14 a | | |

# Chapter 12
# Application of Discriminant Analysis: For Developing a Classification Model

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Understand the importance of discriminant analysis in research.
- List down the research situation where discriminant analysis can be used.
- Understand the importance of assumptions used in discriminant analysis.
- Know the different concepts used in discriminant analysis.
- Understand the steps involved in using SPSS for discriminant analysis.
- To interpret the output obtained in discriminant analysis.
- Explain the procedure in developing the decision rule using discriminant model.
- Know to write the results of discriminant analysis in standard format.

## Introduction

Often we come across a situation where it is interesting to know as to why the two naturally occurring groups are different. For instance, after passing the school, the students can opt for continuing further studies, or they may opt for some skill-related work. One may be interested to know as to what makes them to choose their course of action. In other words, it may be desired to know on what parameters these two groups may be distinct. Similarly one may like to identify the parameters which distinguish the liking of two brands of soft drink by the customers or which make the engineering and management students different. Thus, to identify the independent parameters responsible for discriminating these two groups, a statistical technique known as discriminant analysis (DA) is used. The discriminant analysis is a multivariate statistical technique used frequently in management, social sciences, and humanities research. There may be varieties of situation where this technique can play a major role in decision-making process. For instance, the government is very keen that more and more students should opt for the science stream in order to have the technological advancement in the country.

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Therefore, one may investigate the factors that are responsible for class XI students to choose commerce or science stream. After identifying the parameters responsible for discriminating a science and commerce student, the decision makers may focus their attention to divert the mindset of the students to opt for science stream.

Yet another application where discriminant analysis can be used is in the food industry. In launching the new food product, much of its success depends upon its taste, and, therefore, product formulation must be optimized to obtain desirable sensory quality expected by consumers. Thus, the decision maker may be interested to know the parameters that distinguish the existing similar product and new proposed product in terms of the product properties like sensory characteristics, percent of ingredients added, pricing, and contents. In this chapter, the discriminant analysis technique shall be discussed in detail along with its application with SPSS.

## What Is Discriminant Analysis?

Discriminant analysis is a multivariate statistical technique used for classifying a set of observations into predefined groups. The purpose is to determine the predictor variables on the basis of which groups can be determined. The discriminant model is built on the basis of a set of observations for which the groups are known. This set of observation is the past data on the basis of which discriminant analysis technique constructs a set of linear functions of the predictors, known as discriminant function, such that

$$Z = c + b_1X_1 + b_2X_2 + \ldots\ldots b_nX_n \tag{12.1}$$

where

$c$ is a constant
$b$'s are the discriminant coefficients
$X$'s are the predictor variables

Only those independent variables are picked up which are found to have significant discriminating power in classifying a subject into any of the two groups. The discriminant function so developed is used for predicting the group of a new observation set.

The discriminant analysis is actually known as discriminant function analysis but in short one may use the term discriminant analysis. In discriminant analysis, the dependent variable is a categorical variable, whereas independent variables are metric. The dependent variable may have more than two classes, but the discriminant analysis is more powerful if it has two classifications. In this text, the discriminant analysis shall be discussed only for two-group problem.

After developing the discriminant model, for a given set of new observation the discriminant function $Z$ is computed, and the subject/object is assigned to first group if the value of $Z$ is less than 0 and to second group if more than 0. This

criterion holds true if an equal number of observations are taken in both the groups for developing a discriminant function. However, in case of unequal sample size, the threshold may vary on either side of zero.

The main purpose of a discriminant analysis is to predict group membership based on a linear combination of the predictive variables. In using this technique, the procedure starts with a set of observations where both group membership and the values of the interval variables are known. The end result of the procedure is a model that allows prediction of group membership when only the interval variables are known.

A second purpose of the discriminant analysis is to study the relationship between group membership and the variables used to predict group membership. This provides information about the relative importance of independent variables in predicting group membership.

Discriminant function analysis is similar to the ordinary least square (OLS) regression analysis. The only difference is in the nature of dependent variable. In discriminant function analysis, the dependent variable is essentially a categorical (preferably dichotomous) variable, whereas in multiple regression it is a continuous variable. Other differences are in terms of the assumptions being satisfied in using discriminant analysis which shall be discussed later in this chapter.

## Terminologies Used in Discriminant Analysis

Discriminant analysis provides discriminant function which is used to classify an individual or cases into two categories on the basis of the observations on the predictor variables. If the discriminant model developed in the analysis is robust for a set of data, the percentage of correct classification of cases in the classification table increases. To understand the application of discriminant analysis using SPSS on any data set, it is essential to know its basics.

### *Variables in the Analysis*

In discriminant analysis, the dependent variable is categorical in nature. It may have two or more categories. The procedure used in discriminant analysis becomes very complicated if the dependent variable has more than two categories. Further the efficiency of the model also decreases in that case. The model becomes very powerful if the dependent variable has only two categories. The dependent variable is also known as criterion variable. In SPSS, dependent variable is known as grouping variable. It is the object of classification on the basis of independent variables.

The independent variables in the discriminant analysis are always metric. In other words, the data obtained on the independent variables must be measured

either on interval or ratio scale. The independent variables in discriminant analysis are also known as predictor variables.

## *Discriminant Function*

A discriminant function is a latent variable which is constructed as a linear combination of independent variables, such that

$$Z = c + b_1X_1 + b_2X_2 + \ldots + b_nX_n$$

where
$b_1, b_2, \ldots, b_n$ are discriminant coefficients
$X_1, X_2, \ldots, X_n$ are discriminating variables
$c$ is a constant

The discriminant function is also known as canonical root. This discriminant function is used to classify the subject/cases into one of the two groups on the basis of the observed values on the predictor variables.

## *Classification Matrix*

In discriminant analysis, the classification matrix serves as a yardstick in measuring the accuracy of a model in classifying an individual/case into one of the two groups. The classification matrix is also known as confusion matrix, assignment matrix, or prediction matrix. It tells us as to what percentage of the existing data points are correctly classified by the model developed in discriminant analysis. This percentage is somewhat similar to $R^2$ (percentage of variation in dependent variable explained by the model).

## *Stepwise Method of Discriminant Analysis*

Discriminant function can be developed either by entering all *independent variables together* or in s*tepwise* depending upon whether the study is confirmatory or exploratory. In confirmatory data analysis, discriminant function is developed on the basis of all the independent variables selected in the study, whereas in exploratory study the independent variables are selected one by one. In stepwise discriminant analysis, a variable is retained in the model if its regression coefficient is significant at 5% level and removed from the model if it is not significant at 10% level.

## *Power of Discriminating Variables*

After developing the model in discriminant analysis based on selected independent variables, it is important to know the relative importance of the variables so selected. This relative importance of the variable is determined by the coefficient of the discriminating variable in the discriminant function. SPSS provides these coefficients in the output and are named as standardized canonical discriminant function coefficients. The higher the value of coefficient, the better is the discriminating power.

## *Box's M Test*

While applying ANOVA, one of the assumptions was that the variances are equivalent for each group, but in DA the basic assumption is that the variance-covariance matrices are equivalent. By using Box's M tests, we test a null hypothesis that the covariance matrices do not differ between groups formed by the dependent variable. The researcher would not like this test to be significant so that the null hypothesis that the groups do not differ can be retained. Thus, if the Box's M test is insignificant, it indicates that the assumptions required for DA holds true.

However, with large samples, a significant result of Box's M is not regarded as too important. Where three or more groups exist, and Box's M is significant, groups with very small log determinants should be deleted from the analysis.

## *Eigenvalues*

Eigenvalue is the index of overall model fit. It provides information on each of the discriminant functions (equations) produced. In discriminant analysis, the maximum number of discriminant functions produced is the number of groups minus 1. In case dependent variable has two categories, only one discriminant function shall be generated. In DA, one tries to predict the group membership from a set of predictor variables. If the dependent variable has two categories and there are $n$ predictive variables, then a linear discriminant equation, $Z_i = c + b_1X_1 + b_2X_2 + \ldots + b_nX_n$, is constructed such that the two groups differ as much as possible on $Z$. Here, one tries to choose the weights $b_1$, $b_2$,..., $b_n$ in computing a discriminant score ($Z_i$) for each subject so that if an ANOVA on $Z$ is done, the ratio of the between groups sum of squares to the within groups sum of squares is as large as possible. The value of this ratio is known as eigenvalue.

Thus, eigenvalue is computed with the data on $Z$ and is a quantity maximized by the discriminant function coefficients.

$$\text{Eigenvalue} = \frac{\text{SS}_{\text{Between\_groups}}}{\text{SS}_{\text{Within\_groups}}} \tag{12.2}$$

The larger the eigenvalue, the better is the model in discriminating between the groups.

### *The Canonical Correlation*

The canonical correlation in discriminant analysis is equivalent to eta in an ANOVA and is equal to the point biserial correlation $r_b$ between group and $Z$. Square of the canonical correlation indicates the percentage of variation explained by the model in the grouping variable and is similar to $R^2$. The canonical correlation is computed on $Z$ which is as follows:

$$\text{Canonical correlation} = \sqrt{\frac{\text{SS}_{\text{Between\_groups}}}{\text{SS}_{\text{Total}}}} \tag{12.3}$$

### *Wilks' Lambda*

It is used to indicate the significance of discriminant function developed in the discriminant analysis. The value of Wilks' lambda provides the proportion of total variability not explained by the discriminant model. For instance, if the value of Wilks' lambda is 0.28, it indicates that 28% variability is not explained by the model. The value of Wilks' lambda ranges from 0 to 1, and low value of it (closer to 0) indicates better discriminating power of the model. Thus, the Wilks' lambda is the converse of the squared canonical correlation.

## What We Do in Discriminant Analysis

Different steps that are involved in discriminant analysis have been discussed in this section. Initially you may not understand all the steps clearly but continue to read this chapter, and once you complete reading the solved example using SPSS discussed in this chapter, your understanding level about this topic shall be enhanced. All the steps discussed below cannot be performed manually but may be achieved by using any statistical package. So go through these steps and try understanding the outputs of your discriminant analysis.

1. The first step in the discriminant analysis is to identify the independent variables having significant discriminant power. This is done by taking all the independent variables together in the model or one by one. The option for these two methods can be seen in SPSS as "*Enter independents together*" and "*Use stepwise method,*" respectively.
   In *stepwise method*, an independent variable is entered in the model if its corresponding regression coefficient is significant at 5% level and excluded at subsequent stages until and unless it is significant at 10% level. Thus, in developing discriminant function, the model will enter only significant independent variables. The model so developed is required to be tested for its robustness.
2. In the second step, a discriminant function model is developed by using the discriminant coefficients of the predictor variables and the value of constant shown in the "*Unstandardized canonical discriminant function coefficients*" table generated in the SPSS output. This is similar to developing of regression equation. This way, the function so generated may be used to classify an individual into any of the two groups. The discriminant function shall look like as follows:

$$Z = c + b_1X_1 + b_2X_2 + \ldots\ldots\ldots + b_nX_n$$

   where

   $Z$ is the discriminant function
   $X$'s are predictor variables in the model
   $c$ is the constant
   $b$'s are the discriminant constants of the predictor variables
3. After developing discriminant model, the Wilks' lambda is computed in the third step for testing the significance of discriminant function developed in the model. This indicates the robustness of discriminant model. The value of Wilks' lambda ranges from 0 to 1, and the lower value of it close to 0 indicates better discriminating power of the model. Further, significant value of chi-square indicates that the discrimination between the two groups is highly significant.
   After selecting independent variables as predictors in the discriminant model, the model is tested for its significance in classifying the subjects/cases correctly into groups. For this, SPSS generates a classification matrix. This is also known as confusion matrix. This matrix shows the number of correct and wrong classification of subjects in both the groups. High percentage of correct classification indicates the validity of the model. The level of accuracy shown in the classification matrix may not hold for all future classification of new subjects/cases.
4. In the fourth step, the relative importance of predictor variables in discriminating the two groups is discussed. The SPSS generates the "*Standardized canonical discriminant function coefficients*" table. The variable with higher coefficient in the table is the most powerful in discriminating the two groups, whereas the variable having least coefficient indicates low discriminating power.

5. Finally, a criterion for classification is developed on the basis of the midpoint of the mean value of the transformed groups obtained in the table titled "*Functions at group centroids*" generated in the SPSS output. If the value of the function $Z$ computed on the basis of the equation developed in step 2 is less than this midpoint, the subject is classified in one group and if it is more than it is classified in second group.

### *Assumptions in Using Discriminant Analysis*

While applying discriminant analysis, one should test the assumptions used in this analysis. Following are the assumptions which are required to be fulfilled while using this analysis:

1. Each of the independent variables is normally distributed. This assumption can be examined by the histograms of frequency distributions. In fact, violations of the normality assumption are usually not serious because in that case the resultant significance tests are still reliable. One may use specific tests like skewness and kurtosis for testing the normality in addition to graphs.
2. All variables have linear and homoscedastic relationships. It is assumed that the variance/covariance matrices of variables are homogeneous in both the groups. Box M test is used for testing the homogeneity of variances/covariances in both the groups. However, it is sensitive to deviations from multivariate normality and should not be taken too seriously.
3. Dependent variable is a true dichotomy. The continuous variable should never be dichotomized for the purpose of applying discriminant analysis.
4. The groups must be mutually exclusive, with every subject or case belonging to only one group.
5. All cases must be independent. One should not use correlated data like before-after and matched pair data.
6. Sample sizes of both the groups should not differ to a great extent. If the sample sizes are in the ratio 80:20, logistic regression may be preferred.
7. Sample size must be sufficient. As a guideline, there should be at least five to six times as many cases as independent variables.
8. No independent variables should have a zero variability in either of the groups formed by the dependent variable.
9. Outliers should not be present in the data. To solve this problem, inspect descriptive statistics.

## Research Situations for Discriminant Analysis

The discriminant analysis is used to develop a model for discriminating the future cases/objects into one of the two groups on the basis of predictor variables. Hence, it is widely used in the studies related to management, social sciences, humanities,

and other applied sciences. Some of the research situations where this analysis can be used are discussed below:

1. In a hospitality firm, the data can be collected on employees in two different job classifications: (1) customer support personnel and (2) back office management. The human resources manager may like to know if these two job classifications require different personality types. Each employee may be tested by a battery of psychological test which consists of a measure of socialization trait, extrovertness, frustration level, and orthodox approach.
   The model can be used to priorities the predictor variable which can be used to identify the employees in different category during selection process. Further, the model may be helpful in developing the training program for future employees recruited in different categories.
2. A college authority might divide a group of past graduate students into two groups: students who finished the economics honors program in 3 years and those who did not. The discriminant analysis could be used to predict successful completion of the honors program based on the independent variables like SAT score, XII maths score, and age of the candidates. Investigating the prediction model might provide insight as to how each predictor individually and in combination predicted completion or noncompletion of the economics honors program at the undergraduate level.
3. A marketing manager may like to develop a model on buying two different kinds of toothpaste on the basis of the product and customer profiles. The independent variables may consist of age and sex of the customer and contained quantity, taste, price of the products, etc. The insight from the developed model may provide the decision makers in the company to develop and market their products with success.
4. A social scientist may like to know the predictor variable which is responsible for smoking. The data on variables like the age at which the first cigarette was smoked and other reasons of smoking like self-image, peer pressure, and frustration level can be studied to develop a model for classifying an individual into smoker and nonsmoker. The knowledge so accrued from the developed model may be used to start the ad campaign against smoking.
5. In medical research, one may like to predict whether patient would survive from burn injury based on the combinations of demographic and treatment variables. The predictor variables might include burn percentage, body parts involved, age, sex, and time between incident and arrival at hospital. In such situations, the discriminant model so developed would allow a doctor to assess the chances of recovery based on predictor variables. The discriminant model might also give insight into how the variables interact in predicting recovery.

## Solved Example of Discriminant Analysis Using SPSS

**Example 12.1** The marketing division of a bank wants to develop a policy for issuing visa gold card to its customers through which one can shop and withdraw up

to Rs. 100,000 at a time for 30 days without any interest. Out of several customers, only a handful number of customers are required to be chosen for such facility. Thus, a model is required to be made on the basis of the existing practices for issuing similar card to the customers on the basis of the following data. The data was collected on 28 customers in the bank who were either issued or denied similar card earlier. Apply discriminant analysis to develop a discriminant function for issuing or denying the golden visa card to the customers on the basis of their profile. Also test the significance of the model so obtained. Discuss the efficiency of classification and relative importance of the predictor variables retained in the model (Table 12.1).

*Solution*

Here it is required to do the following:

1. To develop a discriminant function for deciding whether a customer be issued a golden credit card

**Table 12.1** Account details of the customers

| S. N. | Credit card | Average daily balance last 1 year | Number of days balance <50,000 last 1 year | Annual income in lakh | Family size | Average number of transaction/ month |
|---|---|---|---|---|---|---|
| 1 | Issued | 68,098 | 2 | 36.52 | 4 | 8 |
| 2 | Denied | 43,233 | 12 | 26.45 | 3 | 13 |
| 3 | Issued | 50,987 | 0 | 25.6 | 5 | 11 |
| 4 | Denied | 39,870 | 31 | 26.85 | 5 | 12 |
| 5 | Denied | 37,653 | 51 | 25.65 | 6 | 11 |
| 6 | Denied | 35,347 | 48 | 28.45 | 5 | 14 |
| 7 | Issued | 65,030 | 1 | 22.45 | 2 | 4 |
| 8 | Issued | 72,345 | 0 | 42.34 | 5 | 6 |
| 9 | Denied | 34,534 | 32 | 31.9 | 4 | 8 |
| 10 | Issued | 87,690 | 1 | 30.45 | 6 | 15 |
| 11 | Denied | 43,563 | 4 | 28.45 | 5 | 10 |
| 12 | Denied | 50,879 | 6 | 24.8 | 6 | 9 |
| 13 | Denied | 58,034 | 1 | 24.45 | 5 | 12 |
| 14 | Issued | 76,345 | 0 | 29.45 | 6 | 3 |
| 15 | Issued | 69,067 | 3 | 34.24 | 4 | 11 |
| 16 | Denied | 43,008 | 5 | 54.45 | 4 | 8 |
| 17 | Issued | 75,437 | 2 | 28.76 | 8 | 20 |
| 18 | Denied | 34,009 | 8 | 34.25 | 4 | 14 |
| 19 | Issued | 52,409 | 4 | 31.45 | 4 | 7 |
| 20 | Denied | 51,654 | 4 | 31.8 | 3 | 13 |
| 21 | Issued | 64,065 | 2 | 25.67 | 5 | 10 |
| 22 | Denied | 49,003 | 4 | 33.45 | 2 | 7 |
| 23 | Issued | 65,030 | 1 | 25.63 | 4 | 15 |
| 24 | Issued | 59,024 | 2 | 32.52 | 5 | 12 |
| 25 | Issued | 75,007 | 0 | 28.45 | 3 | 8 |
| 26 | Denied | 46,342 | 12 | 34.54 | 5 | 15 |
| 27 | Denied | 56,803 | 1 | 32.76 | 4 | 17 |
| 28 | Issued | 59,034 | 3 | 26.87 | 3 | 8 |

2. To identify the predictor variable in developing the model and find their relative importance
3. To test the significance of the model
4. To explain the efficiency of classification

These issues shall be discussed with the output generated by the SPSS in this example. Thus, the procedure of using SPSS for discriminant analysis in the given example shall be explained first, and thereafter the output shall be discussed in the light of the objectives of the study.

## *SPSS Commands for Discriminant Analysis*

In order to perform discriminant analysis with SPSS, a data file needs to be prepared first. Since the initial steps in preparing the data file has been explained in earlier chapters, it will not be repeated here again. In case of difficulty, you may go through the procedure discussed in Chap. 1 in this regard. Take the following steps for generating the outputs in discriminant analysis:

(i) *Data file:* Here, five independent variables and one dependent variable need to be defined. The dependent variable *Card_decision* is defined as a nominal variable, whereas all five independent variables as scale variables in SPSS. After preparing the data file by defining variable names and their labels, the screen will look like as shown in Fig. 12.1.

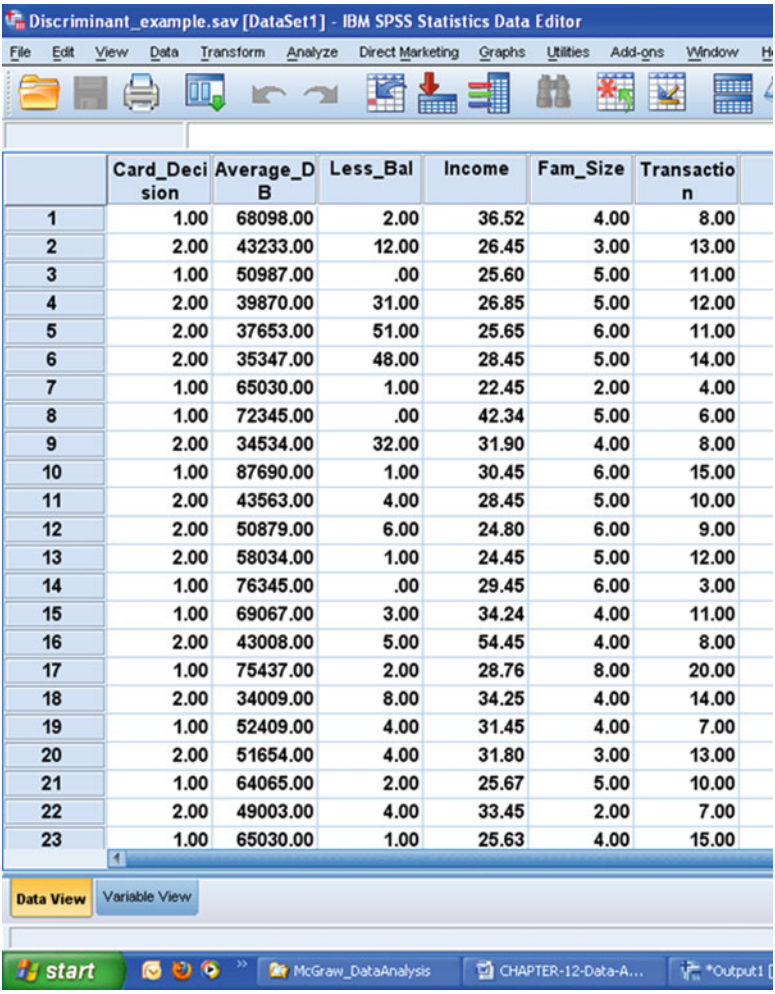
(ii) *Initiating command for discriminant analysis:* After preparing the data file, click the following command sequence in the Data View:

**Analyze → Classify → Discriminant**

The screen shall look like Fig. 12.2.

(iii) *Selecting variables for discriminant analysis:* After clicking the **Discriminant** option, the SPSS will take you to the window where variables are selected.

- Select the dependent variable *Card_Decision* from left panel to the "Grouping Variable" section of the right panel. Define minimum and maximum range of the grouping variable as "1" and "2" and click continue.
- Select all independent variables from left panel and bring them to the "Independents" section of the right panel.
- Check the option "Use stepwise method" if you have many independent variables and the effort is to identify the relevant predictive variables. Such studies are known as explorative studies. Whereas if you want to go for confirmatory analysis, check the option "Enter independents together." Here, the model is built on all the independent variables; hence, the option "Enter independents together" is checked. In this case, the effort is to test




| | Card_Decision | Average_DB | Less_Bal | Income | Fam_Size | Transaction |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 68098.00 | 2.00 | 36.52 | 4.00 | 8.00 |
| 2 | 2.00 | 43233.00 | 12.00 | 26.45 | 3.00 | 13.00 |
| 3 | 1.00 | 50987.00 | .00 | 25.60 | 5.00 | 11.00 |
| 4 | 2.00 | 39870.00 | 31.00 | 26.85 | 5.00 | 12.00 |
| 5 | 2.00 | 37653.00 | 51.00 | 25.65 | 6.00 | 11.00 |
| 6 | 2.00 | 35347.00 | 48.00 | 28.45 | 5.00 | 14.00 |
| 7 | 1.00 | 65030.00 | 1.00 | 22.45 | 2.00 | 4.00 |
| 8 | 1.00 | 72345.00 | .00 | 42.34 | 5.00 | 6.00 |
| 9 | 2.00 | 34534.00 | 32.00 | 31.90 | 4.00 | 8.00 |
| 10 | 1.00 | 87690.00 | 1.00 | 30.45 | 6.00 | 15.00 |
| 11 | 2.00 | 43563.00 | 4.00 | 28.45 | 5.00 | 10.00 |
| 12 | 2.00 | 50879.00 | 6.00 | 24.80 | 6.00 | 9.00 |
| 13 | 2.00 | 58034.00 | 1.00 | 24.45 | 5.00 | 12.00 |
| 14 | 1.00 | 76345.00 | .00 | 29.45 | 6.00 | 3.00 |
| 15 | 1.00 | 69067.00 | 3.00 | 34.24 | 4.00 | 11.00 |
| 16 | 2.00 | 43008.00 | 5.00 | 54.45 | 4.00 | 8.00 |
| 17 | 1.00 | 75437.00 | 2.00 | 28.76 | 8.00 | 20.00 |
| 18 | 2.00 | 34009.00 | 8.00 | 34.25 | 4.00 | 14.00 |
| 19 | 1.00 | 52409.00 | 4.00 | 31.45 | 4.00 | 7.00 |
| 20 | 2.00 | 51654.00 | 4.00 | 31.80 | 3.00 | 13.00 |
| 21 | 1.00 | 64065.00 | 2.00 | 25.67 | 5.00 | 10.00 |
| 22 | 2.00 | 49003.00 | 4.00 | 33.45 | 2.00 | 7.00 |
| 23 | 1.00 | 65030.00 | 1.00 | 25.63 | 4.00 | 15.00 |

**Fig. 12.1** Screen showing partial data file for the discriminant analysis in SPSS

the model. Such studies are known as confirmatory studies. In this example, all the variables have been selected to build the model. The screen will look like Fig. 12.3.

(iv) *Selecting the option for computation:* After selecting variables, different option needs to be defined for generating the output in discriminant analysis. Take the following steps:

- Click the tag **Statistics** in the screen shown in Fig. 12.3. and
- Check the option of "Means" and "Box's M" in the "Descriptives" section.

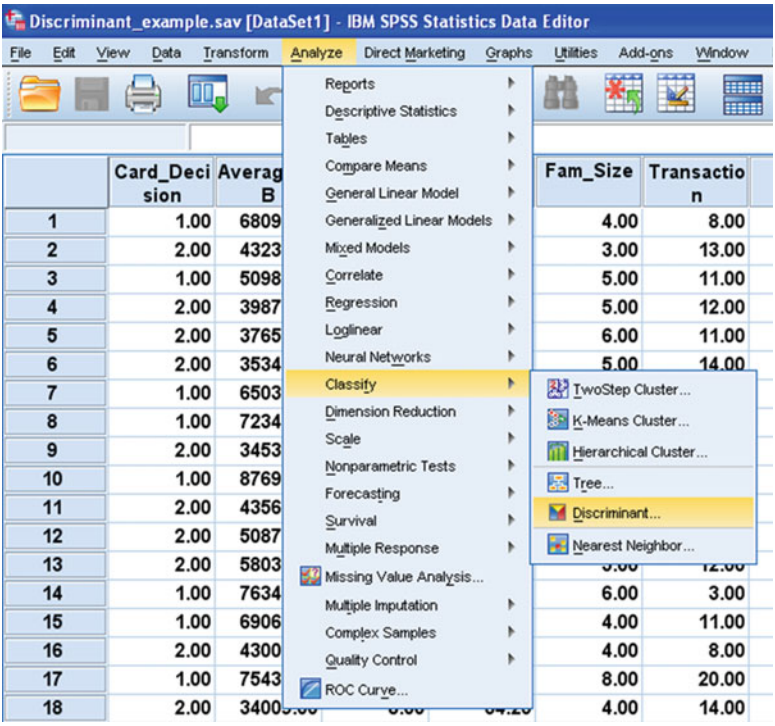



**Fig. 12.2** Screen showing SPSS commands for discriminant analysis

- Check the options "Fisher's" and "Unstandardized" in the "Function Coefficients" section. The screen showing these options shall look like as shown in Fig. 12.4.
- Press ***Continue***. This will take you back to the screen shown in Fig. 12.3.
- Click the tag **Classify** in the screen as shown in screen 12.3. and
- Check the option "Summary table" in the Display section.
- Check the option "Casewise results" if you want to know wrongly classified cases by the model.

The screen for these options shall look like Fig. 12.5.

- Click ***Continue***.
- Click **OK** for output.

(v) *Getting the output:* After clicking the **OK** option in Fig. 12.3, the output in the discriminant analysis shall be generated in the output window. Selected outputs can be copied in the word file by using the right click of the mouse over identified area of the output. Out of many outputs generated by the SPSS, the following relevant outputs have been picked up for discussion:

1. Group statistics including mean and standard deviation

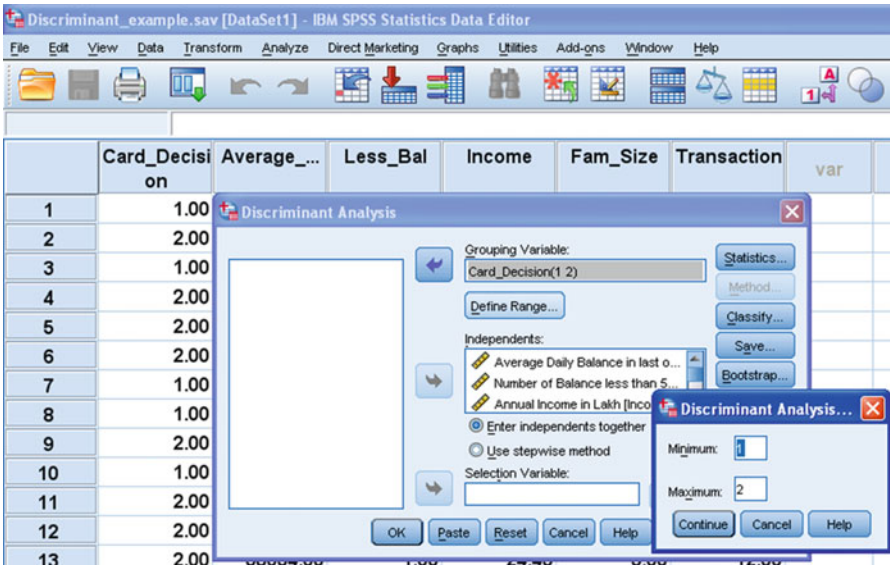



**Fig. 12.3** Screen showing selection of variables for discriminant analysis

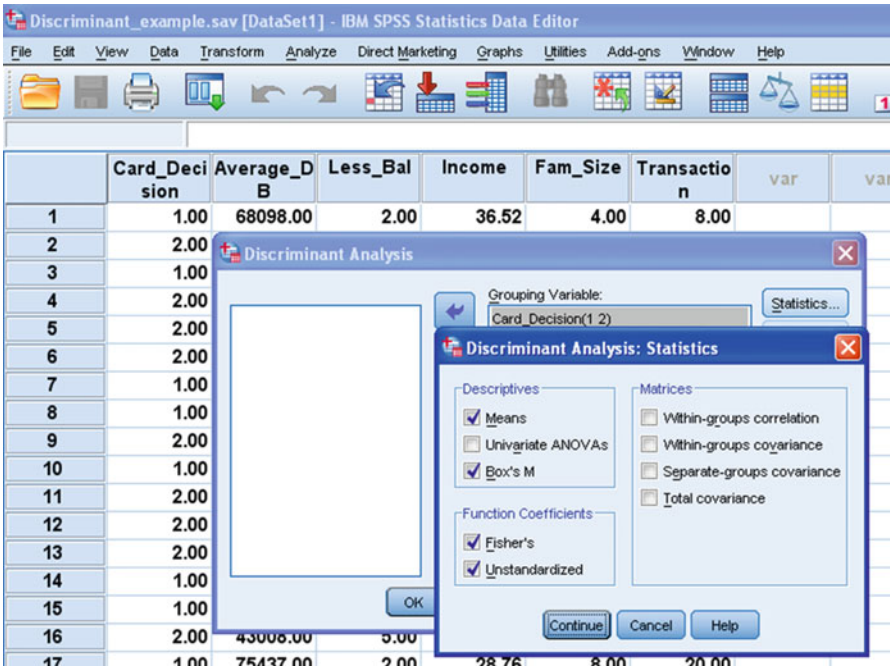



**Fig. 12.4** Screen showing the options for descriptive statistics and discriminant coefficients

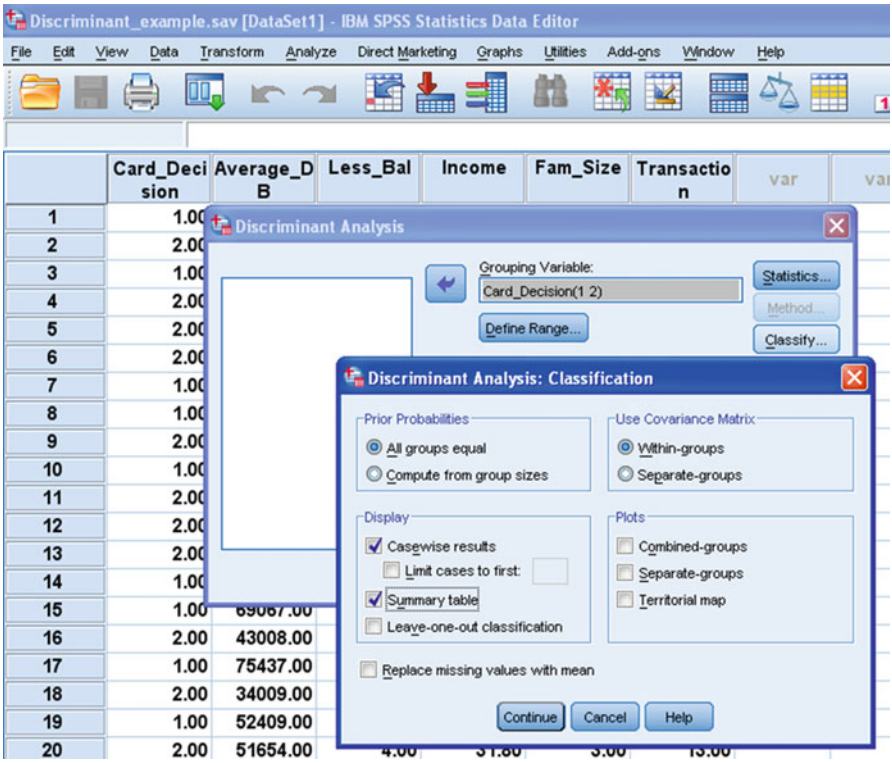



**Fig. 12.5** Screen showing the options for classification matrix

2. Unstandardized canonical discriminant function coefficients table
3. Eigen values and canonical correlation
4. Wilks' lambda and chi-square test
5. Classification matrix
6. Standardized canonical discriminant function coefficients
7. Functions at group centroids

These outputs so generated by the SPSS are shown in Tables 12.2–12.8 and Fig. 12.6.

## *Interpretation of Various Outputs Generated in Discriminant Analysis*

The above-mentioned output so generated by the SPSS will now be discussed to answer the issues raised in the example.

1. Table 12.2 shows descriptive statistics containing mean and standard deviation for all the variables in both the groups, that is, card issued group and card denied

**Table 12.2** Group statistics: mean and standard deviation of all independent variables in different groups

| Issue decision | | Mean | SD |
|---|---|---|---|
| Card issued | Average daily balance in last 1 year | 67,112.00 | 9989.74 |
| | Number of balance less than 5,000 in last 1 year | 1.509 | 1.29 |
| | Annual income in lakh | 30.03 | 5.19 |
| | Family size | 4.57 | 1.50 |
| | Average transaction per month | 9.86 | 4.62 |
| Card denied | Average daily balance in last 1 year | 44,566.57 | 7923.67 |
| | Number of balance less than 5,000 in last 1 year | 15.64 | 17.38 |
| | Annual income in lakh | 31.30 | 7.56 |
| | Family size | 4.36 | 1.15 |
| | Average transaction per month | 11.64 | 2.95 |
| Total | Average daily balance in last 1 year | 55,839.29 | 14,493.43 |
| | Number of balance less than 5,000 in last 1 year | 8.5714 | 14.08 |
| | Annual income in lakh | 30.67 | 6.39 |
| | Family size | 4.46 | 1.32 |
| | Average transaction per month | 10.75 | 3.91 |

**Table 12.3** Unstandardized canonical discriminant function coefficients

| Variables selected | Function 1 |
|---|---|
| Average daily balance in last 1 year ($X_1$) | .000 |
| Number of balance less than 5,000 in last 1 year ($X_2$) | −.002 |
| Annual income in lakh ($X_3$) | −.028 |
| Family size ($X_4$) | .017 |
| Average transaction per month ($X_5$) | −.099 |
| Constant | −4.253 |

group. The readers may draw relevant conclusions as per their objectives from this table.

2. Table 12.3 reveals the value of unstandardized discriminant coefficients which are used in constructing discriminant function. Since all independent variables were included to develop the model, the discriminant coefficients of all the five independent variables are shown in Table 12.3.
   Thus, discriminant function can be constructed by using the values of constant and coefficients of these five independent variables as shown in Table 12.3.

$$Z = -4.253 - .002 \times X_2 - .028 \times X_3 + .017 \times X_4 - .099 \times X_5$$

where

$X_2$ is number of balance less than 5,000 in last 1 year
$X_3$ is annual income in lakh
$X_4$ is family size
$X_5$ is average transaction per month

**Table 12.4** Eigenvalues

| Function | Eigenvalue | % of variance | Cumulative % | Canonical correlation |
|---|---|---|---|---|
| 1 | 1.975[a] | 100.0 | 100.0 | .815 |

[a]First 1 canonical discriminant functions were used in the analysis

**Table 12.5** Wilks' lambda and chi-square test

| Test of function(s) | Wilks' lambda | Chi-square | df | Sig. |
|---|---|---|---|---|
| 1 | .336 | 25.618 | 5 | .000 |

**Table 12.6** Classification results[a]

| | | Predicted group membership | | |
|---|---|---|---|---|
| | Issue decision | Card issued | Card denied | Total |
| Original count | Card issued | 12 | 2 | 14 |
| | Card denied | 1 | 13 | 14 |
| % | Card issued | 85.7 | 14.3 | 100.0 |
| | Card denied | 7.1 | 92.9 | 100.0 |

[a]89.3% of original grouped cases correctly classified

3. The canonical correlation is 0.815 as shown in Table 12.4. This indicates that approximately 66% of the variation in the two groups is explained by the discriminant model.
   Since the Wilks' lambda provides the proportion of unexplained variance by the model, the lesser its value, the better is the discriminant model. The value of Wilks' lambda lies in between 0 and 1. Its value here is 0.336 as shown in Table 12.5; hence, the model can be considered good because only 33.6% variability is not explained by the model. To test the significance of Wilks' lambda, the value of chi-square is calculated which is shown in Table 12.5. Since the *p* value associated with it is .000 which is less than .05, it may be inferred that the model is good.
4. Table 12.6 is a classification matrix which shows the summary of correct and wrong classification of cases in both the groups on the basis of the developed discriminant model. This table shows that out of 14 customers whom credit card was issued, 12 were correctly classified by the developed model and 2 were wrongly classified in the card denied group. On the other hand, out of 14 customers whom card was denied, 13 were classified by the model correctly in the card denied group and only 1 customer was wrongly classified in the card issued group. Thus, out of 28 cases, 25 (89.3%) cases were correctly classified by the model which is quite high; hence, the model can be considered as valid. Since this model is developed on the basis of a small sample, the level of accuracy shown in the classification matrix may not hold for all future classification of new cases.
5. Table 12.7 shows the standardized discriminant coefficients of the independent variables in the model. The magnitude of these coefficients indicates the discriminating power of the variables in the model. The variable having higher

**Table 12.7** Standardized canonical discriminant function coefficients

| Variables selected | Function 1 |
|---|---|
| Average daily balance in last 1 year | .988 |
| Number of balance less than 5,000 in last 1 year | −.019 |
| Annual income in lakh | −.184 |
| Family size | .023 |
| Average transaction per month | −.382 |

**Table 12.8** Functions at group centroids

| Issue decision | Function 1 |
|---|---|
| Card issued | 1.354 |
| Card denied | −1.354 |

Unstandardized canonical discriminant functions evaluated at group means

magnitude of the absolute function value is more powerful in discriminating the two groups. Since absolute function value of the variable *average daily balance in last one year* is .988 which is higher than that of the variable *average transaction per month (.382),* average daily balance is having more discriminating power than the average transaction per month. By careful examination, you may notice that the coefficient of average daily balance is the maximum, and in fact it is very large in comparison to other variables; hence, it may be concluded that this is the most important variable in taking a decision to issue or not to issue the golden visa card.

**Remark:** You may run the discriminant analysis on the same data by using the option "Use stepwise method" in order to ascertain the fact, whether the variable *average daily balance* gets selected in the model. You may also check as to how much accuracy is reduced in the model if some of the independent variables are dropped from the model.

6. One of the purposes of running discriminant analysis in this example was to develop a decision model for classifying a customer into two categories, that is, card issued and card denied. Table 12.8 shows the means for the transformed group centroids. Thus, the new mean for group 1 (card denied) is −1.354 and for group 2 (card issued) is +1.354. This indicates that the midpoint of these two is 0. These two means can be plotted on a straight line by locating the midpoint as shown in Fig. 12.6.

Figure 12.6 defines the decision rule for classifying any new customer into any of the two categories. If the discriminant score of any customer falls to the right of the midpoint ($Z > 0$), he/she is classified into the card issue category, and if it falls to the left of the midpoint ($Z < 0$), he/she is classified into card denied category.

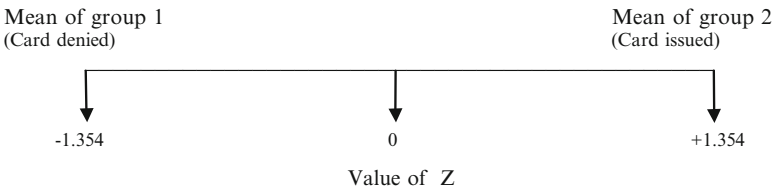



**Fig. 12.6** Means of the transformed group centroids

## Summary of the SPSS Commands for Discriminant Analysis

(i) After preparing the data file, follow the below-mentioned command sequence for discriminant analysis:

**Analyze** ⟶ **Classify** ⟶ **Discriminant**

(ii) Select the dependent variable *Card_Decision* from left panel to the "Grouping Variables" section of the right panel and define its minimum and maximum range as "1" and "2." Further, select all independent variables from the left panel to the "Independents" section of the right panel. Check the option "Enter independents together."
(iii) Click the tag **Statistics** and check options for "Means," "Fisher's," and "Unstandardized" in it. Click ***Continue***.
(iv) Click the tag **Classify** and check option for "Summary table." Press ***Continue***.
(v) Press **OK** for output.

## Exercise

*Short Answer Questions*

**Note:** Write answer to each of the following questions in not more than 200 words.

Q.1. Explain a research situation where discriminant analysis can be used and discuss its utility.
Q.2. In discriminant analysis, what dependent variable refers to? What is the data type of dependent variable in SPSS?
Q.3. Discuss situations in which the discriminant analysis uses the two different methods like "*Enter independents together*" and "*Use stepwise method*" for developing the discriminant model.
Q.4. What do you mean by discriminating variables? What is its significance in discriminant analysis?
Q.5. What is the significance of Box's M test in discriminant analysis? What does the magnitude of Box's M signify?
Q.6. What do you mean by eigenvalues? Explain its importance.
Q.7. Explain the significance of canonical correlation. What does it convey?

Q.8. Explain the role of Wilks' lambda in discriminant analysis. Comment on the models if its values are 0, 0.5, and 1 in three different situations.
Q.9. Explain the purpose of classification matrix in discriminant analysis. How the percentage of correct classification is similar to $R^2$?
Q.10. What is discriminant function and how it is developed? How this function is used in decision-making?
Q.11. One of the conditions in discriminant analysis is that "All variables have linear and homoscedastic relationships." Explain the meaning of this statement.
Q.12. What do you mean by the discriminating power of the variables? How will you asses it?

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. In discriminant analysis, independent variables are treated as

   (a) Scale
   (b) Nominal
   (c) Ordinal
   (d) Ratio

2. In discriminant analysis, dependent variable is measured on the scale known as

   (a) Grouping
   (b) Ordinal
   (c) Nominal
   (d) Criterion

3. Discriminant function is also known as

   (a) Eigenvalue
   (b) Regression coefficient
   (c) Canonical root
   (d) Discriminant coefficient

4. Confusion matrix is used to denote

   (a) Correctly classified cases
   (b) Discriminant coefficients
   (c) $F$-values
   (d) Robustness of different models

5. The decision criteria in discriminant analysis are as follows:
   Classify in first group if $Z < 0$
   Classify in second group if $Z > 0$
   The above criteria hold true

   (a) If size of the samples in both the groups are equal
   (b) If size of the samples in both the groups are nearly equal
   (c) If size of the samples in both the groups are in the proportion of 4:1
   (d) In all the situations

6. In stepwise method of discriminant analysis, a variable is included in the model if it is found significant at

   (a) 2% level
   (b) 1% level
   (c) 10% level
   (d) 5% level

7. The Wilks' lambda indicates

   (a) Percentage variability in the two groups explained by the model
   (b) Robustness of the model
   (c) Proportion of total variability not explained by the discriminant model
   (d) Significance of discriminant coefficients

8. One of the assumptions in discriminant analysis is that

   (a) All variables have curvilinear and homoscedastic relationships.
   (b) All variables have linear and non-homoscedastic relationships.
   (c) All variables have curvilinear and non-homoscedastic relationships.
   (d) All variables have linear and homoscedastic relationships.

9. Correct sequence of commands in SPSS for discriminant analysis is

   (a) Analyze → Discriminant → Classify
   (b) Analyze → Classify → Discriminant
   (c) Discriminant → Analyze → Classify
   (d) Discriminant → Classify → Analyze

10. Value of Wilks' lambda ranges from

    (a) −1 to +1
    (b) 0 to 1
    (c) −1 to 0
    (d) −2 to 2

11. Discriminant function is developed on the basis of

    (a) Standardized coefficients
    (b) Unstandardized coefficients
    (c) Classification matrix
    (d) Functions at group centroids

12. The power of discrimination of an independent variable is determined by

    (a) Unstandardized canonical coefficients
    (b) Wilks' lambda
    (c) Standardized canonical coefficients
    (d) Eigenvalues

13. In explorative discriminant analysis,

    (a) All the independent variables are taken in the study.
    (b) Only those variables which are known to have sufficient discriminating power are taken in the study.
    (c) Maximum number of relevant independent variables are taken in the study.
    (d) It is up to researcher to identify the independent variables in the study.

14. Choose the correct statement in discriminant analysis.

    (a) Dependent variable is an ordinal variable.
    (b) The groups should not be mutually exclusive.
    (c) Sample sizes should differ to a great extent.
    (d) No independent variables should have a zero variability in either of the groups formed by the dependent variable.

15. In discriminant analysis, the square of the canonical correlation is an indicator of

    (a) Relationship among the independent variables
    (b) Efficiency of the predictor variables
    (c) Discriminating power of the independent variables
    (d) The percentage of variability explained by the predictor variables in the two groups

*Assignments*

1. A study was conducted to know the variables responsible for selection in the bank probationary officers examination. Thirty candidates who appeared in the examination were identified for the study, and following data were obtained on them.

Results of the examination and subject's profile

| S.N. | Bank examination result | IQ | English | Numerical aptitude | Reasoning |
|---|---|---|---|---|---|
| 1 | Successful | 78 | 56 | 65 | 78 |
| 2 | Successful | 76 | 76 | 76 | 89 |
| 3 | Not successful | 74 | 52 | 63 | 93 |
| 4 | Not successful | 65 | 49 | 62 | 90 |
| 5 | Successful | 83 | 71 | 82 | 85 |
| 6 | Successful | 79 | 80 | 86 | 84 |
| 7 | Not successful | 91 | 54 | 52 | 89 |
| 8 | Not successful | 64 | 65 | 53 | 84 |
| 9 | Not successful | 53 | 69 | 54 | 85 |
| 10 | Successful | 60 | 78 | 75 | 92 |
| 11 | Not successful | 65 | 69 | 63 | 83 |
| 12 | Successful | 86 | 73 | 83 | 83 |
| 13 | Not successful | 53 | 65 | 67 | 83 |
| 14 | Successful | 74 | 69 | 80 | 78 |
| 15 | Successful | 60 | 68 | 81 | 74 |
| 16 | Successful | 75 | 75 | 78 | 85 |

(continued)

(continued)

Results of the examination and subject's profile

| S.N. | Bank examination result | IQ | English | Numerical aptitude | Reasoning |
|---|---|---|---|---|---|
| 17 | Not successful | 56 | 73 | 75 | 83 |
| 18 | Not successful | 65 | 64 | 56 | 84 |
| 19 | Not successful | 56 | 58 | 64 | 86 |
| 20 | Successful | 95 | 68 | 78 | 82 |
| 21 | Successful | 92 | 80 | 74 | 83 |
| 22 | Not successful | 45 | 73 | 71 | 91 |
| 23 | Successful | 85 | 56 | 89 | 74 |
| 24 | Successful | 68 | 45 | 83 | 85 |
| 25 | Not successful | 64 | 73 | 64 | 84 |
| 26 | Not successful | 70 | 71 | 56 | 86 |
| 27 | Successful | 78 | 74 | 84 | 94 |
| 28 | Not successful | 64 | 70 | 55 | 86 |
| 29 | Not successful | 42 | 67 | 51 | 76 |
| 30 | Successful | 82 | 67 | 90 | 83 |

Develop a discriminant model. Test the significance of the developed model and find the relative importance of the independent variables in the model. Compare the efficiency of the two discriminant function models obtained by taking all the variables at once and stepwise methods.

2. A branded apparel company wanted to reward its loyal customers by means of incentives in the form of 60% discount in the first week of New Year. The company had a loose policy of identifying a customer into loyal or disloyal on the basis of certain criterion which was more subjective. However, the management was interested to develop a more scientific approach to build up a model of classifying a customer into loyal and disloyal group. A sample of 30 customers were chosen from the database, and their purchase details were recorded which are shown in the following table:
   Apply discriminant analysis to build up a classification model which can be used for the existing and future customers to reward as per the company policy. Test

Purchase data of the customers of the apparel company

| S. N. | Customer classification | No. of purchases/ year in a year | Purchase amount in a year | No. of kids' wear apparel/year | No. of ladies apparel/year | No. of gents |
|---|---|---|---|---|---|---|
| 1 | Loyal | 6 | 109,870 | 23 | 12 | 2 |
| 2 | Disloyal | 8 | 27,000 | 4 | 8 | 18 |
| 3 | Loyal | 11 | 135,000 | 22 | 23 | 11 |
| 4 | Loyal | 15 | 12,340 | 12 | 5 | 4 |
| 5 | Disloyal | 9 | 54,000 | 20 | 23 | 8 |
| 6 | Disloyal | 4 | 34,000 | 12 | 8 | 20 |
| 7 | Loyal | 8 | 98,000 | 16 | 9 | 22 |
| 8 | Loyal | 8 | 80,002 | 23 | 25 | 3 |
| 9 | Disloyal | 4 | 71,000 | 25 | 15 | 19 |
| 10 | Loyal | 8 | 180,000 | 35 | 24 | 12 |

(continued)

(continued)

| S. N. | Customer classification | No. of purchases/ year in a year | Purchase amount in a year | No. of kids' wear apparel/year | No. of ladies apparel/year | No. of gents |
|---|---|---|---|---|---|---|
| 11 | Disloyal | 6 | 34,012 | 3 | 2 | 15 |
| 12 | Loyal | 12 | 67,000 | 12 | 8 | 5 |
| 13 | Loyal | 5 | 92,008 | 20 | 12 | 9 |
| 14 | Disloyal | 4 | 12,000 | 6 | 2 | 8 |
| 15 | Loyal | 10 | 71,540 | 6 | 15 | 8 |
| 16 | Disloyal | 4 | 13,450 | 1 | 2 | 15 |
| 17 | Loyal | 14 | 125,000 | 24 | 15 | 8 |
| 18 | Loyal | 20 | 80,000 | 5 | 20 | 7 |
| 19 | Disloyal | 5 | 56,021 | 15 | 10 | 15 |
| 20 | Loyal | 9 | 170,670 | 21 | 25 | 12 |
| 21 | Disloyal | 6 | 1,012 | 1 | 1 | 1 |
| 22 | Disloyal | 7 | 54,276 | 13 | 8 | 15 |
| 23 | Loyal | 15 | 100,675 | 25 | 25 | 5 |
| 24 | Loyal | 12 | 106,750 | 30 | 15 | 4 |
| 25 | Disloyal | 11 | 3,500 | 2 | 2 | 3 |
| 26 | Disloyal | 5 | 2,500 | 2 | 1 | 3 |
| 27 | Loyal | 10 | 89,065 | 14 | 21 | 8 |
| 28 | Loyal | 9 | 80,540 | 15 | 19 | 16 |
| 29 | Disloyal | 7 | 12,000 | 4 | 4 | 6 |
| 30 | Disloyal | 3 | 5,056 | 4 | 2 | 3 |

the significance of discriminant function, explain the percentage of correct classification by the model, and discuss the relative importance of independent variable. Find out the percentage of variability explained by the discriminant model in both the situations when all the variables are included in the model and when the variables are identified using stepwise procedure.

**Hint**: In Assignment 2, since the number of scores in loyal and disloyal customer groups are not same, you may not get mean of $Z$ as 0. In this example, you will get the new mean for group 1 (Disloyal group) as $-1.603$ and new mean for group 2 (Loyal group) as 1.403. Thus, midpoint of these groups would be $-.1$ instead of 0. A customer would be classified as disloyal or loyal depending upon $Z < -.1$ or $Z > -.1$.

*Answers to Multiple-Choice Questions*

Q.1 a Q.2 c
Q.3 c Q.4 a
Q.5 a Q.6 d
Q.7 c Q.8 d
Q.9 b Q.10 b
Q.11 b Q.12 c
Q.13 c Q.14 d
Q.15 d

# Chapter 13
# Logistic Regression: Developing a Model for Risk Analysis

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Learn the difference between logistic regression and ordinary least squares regression.
- Know the situation where logistic regression can be used.
- Describe the logit transformation used in the analysis.
- Understand different terminologies used in logistic regression.
- Explain the steps involved in logistic regression.
- Understand the assumptions used in the analysis.
- Know the SPSS procedure involved in logistic regression.
- Understand the odds ratio and its use in interpreting the findings.
- Interpret the outputs of logistic regression generated by the SPSS.

## Introduction

Logistic regression is a useful statistical technique for developing a prediction model for any event that is binary in nature. A binary event can either occur or not occur. It has only two states which may be represented by 1(occurrence) and 0 (nonoccurrence). Logistic regression can also be applied in a situation where the dependent variable has more than two classifications. The logistic regression can either be binary or multinomial depending upon whether the dependent variable is classified into two groups or more than two groups, respectively. In this chapter, the discussion shall be made only for binary logistic regression.

Logistic regression is useful in a situation where we are interested to predict the occurrence of any happening. It has vast application in the area of management, medical and social researches because in all these discipline occurrence of a phenomenon depends upon the independent variables that are metric as well as categorical in nature. Logistic regression can be used for developing model for

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financial prediction, bankruptcy prediction, buying behavior, fund performance, credit risk analysis, etc. We have witnessed the failure of high-profile companies in the recent past. This has generated an interest among the industrial researcher to develop a model for bankruptcy prediction. Such model can also be made for retail and other firms on the basis of the accounting variables such as inventories, liabilities, receivables, net income (loss), and revenue. On the basis of such model, one can estimate the risk of bankruptcy of any organization.

In hilly regions, there is always a fear of landslide which causes heavy damage to the infrastructure and human lives. The logistic regression model can be used to find out the landslide susceptibility in such areas. The model can identify more probable areas prone to landslides. On the basis of such information, appropriate measures may be taken to reduce the risk from potential landslide hazard. In developing the logistic model for landslide susceptibility, the remote sensing and geographic information system (GIS) data may be used as independent variables.

In product marketing, it is required to identify those customers on whom advertisement should be focused. Consider a situation in which a company has introduced an herbal cream costing Rs. 520 and wishes to identify the parameters responsible for the customers to buy this product. The data on the parameters like age, gender, income, and family size may be collected on the customers who have inspected the cream at the counter in few stores. Here the dependent variable is the buying decision of the customer (1 if the cream is purchased and 0 if not), whereas the independent variables are the mix of ratio (age, income, family size) and categorical variable (gender). Since the dependent variable is the dichotomous variable and independent variables have a combination of ratio and categorical variables, the logistic regression can be applied to identify the variables that are responsible for the buying behavior of the customers. Further, the relative importance of the independent variable can also be known by this analysis, and therefore, decision maker may focus on those variables which maximize the chances of buying the product.

In the financial sector, financial companies may be interested to find the attributes of the financial managers responsible for fund performance. One may investigate by using logistic model as to which of the independent variables out of educational background, gender, and seniority of the fund managers are related with the fund performance.

Due to large number of listed companies on the bourses, there is always a fear of credit issues and frequent credit crises. The logistic model may be developed for credit risk analysis which may provide the monitoring agency a system of identifying corporate financial risk which works as an effective indicator system. In developing such model, the past data is usually taken on the identified parameters.

## What Is Logistic Regression?

Logistic regression is a kind of predictive model that can be used when the dependent variable is a categorical variable having two categories and independent variables are either numerical or categorical. Examples of categorical variables are

buying/not buying a product, disease/no disease, cured/not cured, survived/not survived, etc. The logistic regression is also known as logit model or logistic model. The dependent variable in the logit model is often termed as outcome or target variable, whereas independent variables are known as predictive variables. A logistic regression model is more akin to nonlinear regression such as fitting a polynomial to a set of data values. By using the logistic model, the probability of occurrence of an event is predicted by fitting data to a logit function or a logistic curve.

## Important Terminologies in Logistic Regression

Before getting involved into serious discussion about the logistic regression, one must understand different terminologies involved in it. The terms which are required in understanding the logistic regression are discussed herewith.

### *Outcome Variable*

Outcome variable is that variable in which a researcher is interested. In fact it is a dependent variable which is binary in nature. The researcher is interested to know the probability of its happening on the basis of several risk factors. For example, the variables like buying decision (buying = 1, not buying = 0), survival (surviving = 1, not surviving = 0), bankruptcy (bankruptcy of an organization = 1, no bankruptcy = 0), and examination results (pass =1, fail = 0) are all outcome variables.

### *Natural Logarithms and the Exponent Function*

The natural log is the usual logarithmic function with base e. The natural log of $X$ is written as $\log(X)$ or $\ln(X)$. On the other hand, the exponential function involves the constant "$e$" whose value is equal to 2.71828182845904 ($\approx$2.72). The exponential of $X$ is written as $\exp(x) = e^x$. Thus, $\exp(4)$ equals to $2.72^4 = 54.74$.

Since natural log and exponential function are opposite to each other,

$$E^4 = 54.74$$

$$\Rightarrow \quad \ln(54.74) = 4$$

## *Odds Ratio*

If probability of success ($p$) of any event is 0.8, then the probability of its failure is $(1 - p) = 1-0.8 = 0.2$. The odds of the success can be defined as the ratio of the probability of success to the probability of failure. Thus, in this example, odds of success is $0.8/0.2 = 4$. In other words, the odds of success is 4 to 1. If the probability of success is 0.5, then the odds of success is 1 and it may be concluded that the odds of success is 1 to 1.

In logistic regression, odds ratio can be obtained by finding the exponential of regression coefficient, exp($B$), and is sometimes written as e$B$. If the regression coefficient $B$ is equivalent to 0.80, then the odds ratio will be 2.40 because exp (0.8) = 2.4.

The odds ratio of 2.4 indicates that the probability of $Y$ equals to 1 is 2.4 times as likely as the value of $X$ is increased by one unit. If an odds ratio is .5, it indicates that the probability of $Y = 1$ is half as likely with an increase of $X$ by one unit (here there is a negative relationship between $X$ and $Y$). On the other hand, the odds ratio 1.0 indicates that there is no relationship between $X$ and $Y$.

The odds ratio can be better understood if both variables $Y$ and $X$ are dichotomous. In that case, the odds ratio can be defined as the probability that $Y$ is 1 when $X$ is 1 compared to the probability that $Y$ is 1 when $X$ is 0. If the odds ratio is given, then $B$ coefficient can be obtained by taking the log of the odds ratio. It is so because log and exponential functions are opposite to each other.

The transformation from probability to odds is a monotonic transformation. It means that the odds increases as the probability increases or vice versa. Probability ranges from 0 to 1, whereas the odds ranges from 0 to positive infinity.

Similarly the transformation from odds to log of odds, known as log transformation, is also a monotonic transformation. In other words, the greater the odds, the greater is the log of odds and vice versa. Thus, if the probability of success increases, the odds ratio and log odds both increase and vice versa.

## *Maximum Likelihood*

Maximum likelihood is the method of finding the least possible deviation between the observed and predicted values using the concept of calculus specifically derivatives. It is different than ordinary least squares (OLS) regression where we simply try to find the best-fitting line by minimizing the squared residuals.

In maximum likelihood (ML) method, the computer uses different "iterations" where different solutions are tried for getting the smallest possible deviations or best fit. After finding the best solution, the computer provides the final value for the deviance, which is denoted as "−2 log likelihood" in SPSS. Cohen et al. (2003) called this deviance statistic as −2LL, whereas some other authors like Hosmer and Lemeshow (1989) called it D. This deviance statistic follows the chi-square distribution.

The likelihood ratio test, D, is used as goodness-of-fit. This test is referred in SPSS by "chi-square." The significance of this test can be seen by looking to its value in the chi-square table in the appendix using degrees of freedom equal to the number of predictors.

## *Logit*

The logit is a function which is equal to the log odds of a variable. If $p$ is a probability that $Y = 1$(occurrence of an event), then $p/(1 - p)$ is the corresponding odds. The logit of the probability $p$ is given by

$$\text{Logit}(\text{p}) = \log\left(\frac{\text{p}}{1-\text{p}}\right) \tag{13.1}$$

In logistic regression, logit is a special case of a link function. In fact, this logit serves as a dependent variable and is estimated from the model.

## *Logistic Function*

A logistic curve is just like sigmoid curve and is obtained by the logistic function given by

$$p = f(z) = \frac{\text{e}^Z}{1+\text{e}^Z} \tag{13.2}$$

The shape of the curve is like a letter "S." In logistic function, the argument $z$ is marked along horizontal axis and the value of the function $f(z)$ along the vertical axis (Fig. 13.1).

The main feature of this logistic function is that the variable $Z$ can assume any value from minus $-\infty$ to $+\infty$, but the outcome variable $p$ can have the values only in the range 0–1. This function is used in logistic regression model to find the probability of occurring the target variable for a given value of independent variables.

## *Logistic Regression Equation*

The logistic regression equation is similar to the ordinary least squares (OLS) regression equation with the only difference that the dependent variable here is

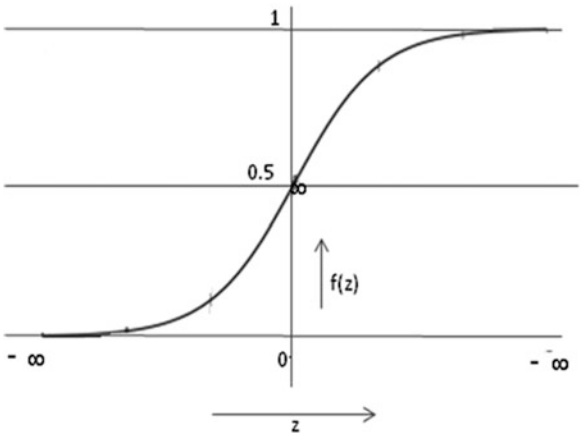



**Fig. 13.1** Shape of the logistic function

the log odds of the probability that the dependent variable $Y = 1$. It is written as follows:

$$\text{logit} = \ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = B_0 + B_1X_1 + B_2X_2 + \ldots\ldots\ldots + B_nX_n \tag{13.3}$$

where $B_0$ is an intercept and $B_1, B_2 \ldots B_n$ are the regression coefficients of $X_1, X_2 \ldots \ldots, X_n$, respectively. The dependent variable in logistic regression is log odds, which is also known as logit.

Since in logistic regression log odds acts as a dependent variable which is regressed on the basis of the independent variables, interpretation of regression coefficients is not as easy as in case of OLS regression. In case of OLS regression, the regression coefficient $b$ represents the change in $Y$ with one unit change in $X$. This concept is not valid in case of logistic regression equation; instead the regression coefficient $b$ is converted into odds ratio to interpret the happening of outcome variable. The interpretation of odds ratio has been discussed above in detail under the heading "Odds Ratio."

## *Judging the Efficiency of the Logistic Model*

In case of OLS regression equation, $R^2$ used to be the measure of efficiency in assessing the suitability of the model. But in case of logistic regression, this statistic is no longer valid indicator of model robustness, because of the fact that the dependent variable here is a binary variable. Thus, to assess the suitability of the logistic model, we use the concept of deviance. In logistic regression, the chi-square

is used as a measure of model fit instead of $R^2$. It tells you about the fit of the observed values ($Y$) to the expected values ($\hat{Y}$). If the difference between the observed values from the expected values increases, the fit of the model becomes poorer. Thus, the effort is to have the deviance as small as possible. If more relevant variables are added to the equation, the deviance becomes smaller, indicating an improvement in fit.

## Understanding Logistic Regression

In logistic regression, the approach of prediction is similar to that of ordinary least squares (OLS). However, in logistic regression, a researcher predicts the probability of an occurrence of a dependent variable which is binary in nature. Another difference in logistic regression is that the independent variables can be a mix of numerical and categorical. Due to dichotomous nature of the dependent variable, assumptions of OLS that the error variances (residuals) are normally distributed are not satisfied. Instead, they are more likely to follow a logistic distribution.

In using logistic distribution, one needs to make an algebraic conversion to arrive at usual linear regression equation. In logistic regression, no standard solution is obtained and no straightforward interpretation can be made as is done in case of OLS regression. Further, in logistic model, there is no $R^2$ to measure the efficiency of the model; rather a chi-square test is used to test how well the logistic regression model fits the data.

### *Graphical Explanation of Logistic Model*

Let us first understand the concept of logistic regression with one independent variable. Consider a situation where we try to predict whether a customer would buy a product($Y$) depending upon the number of days($X$) he saw the advertisement of that product. It is assumed that the customers who watch the advertisement for many days will be more likely to buy the product. The value of $Y$ can be 1 if the product is purchased by the customer and 0 if not.

Since the dependent variable is not a continuous, hence the goal of logistic regression is to predict the likelihood that $Y$ is equal to 1 (rather than 0) given certain values of $X$. Thus, if there is a positive linear relationship between $X$ and $Y$, then the probability that a customer will buy the product ($Y = 1$) will increase with the increase in the value of $X$ (number of days advertisement seen). Hence, we are actually predicting the probabilities instead of value of the dependent variable.

**Table 13.1** Mean score for each category

| No. of days advertisement viewing | Probability that $Y = 1$ (average of 0s and 1s in each category) |
|---|---|
| 0–3 | .17 |
| 4–6 | .40 |
| 7–9 | .50 |
| 10–12 | .56 |
| 13–15 | .96 |

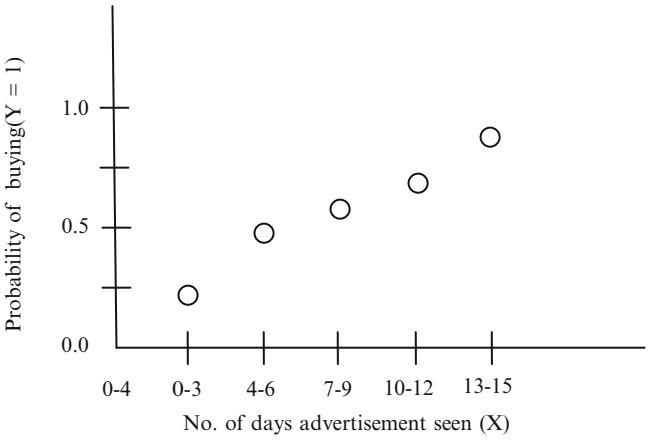



**Fig. 13.2** Graphical representation of the probability of buying versus number of days advertisement seen

In this simulated experiment in investigating the behavior of 100 customers in terms of buying the product of more than Rs. 1,000, their range of viewing the advertisement for the number of days was from 0 to 15 days. We may plot the probability that $Y = 1$ with the increase in the value of $X$ in terms of the graph. To make it more convenient, let us club the number of advertisement-viewing days into the categories 0–3, 4–6, 7–9, 10–12, and 13–15. Computing the mean score on $Y$ (taking the average of 0s and 1s) for each category, the data would look like as shown in Table 13.1.

If we plot these data, the graph would look like as shown in Fig. 13.2.

If we look at this graph, it looks like an S-shaped graph. If there is a strong relationship between $X$ and $Y$, the graph would be closer to perfect S-shaped unlike the OLS regression where you get the straight line.

### *Logistic Model with Mathematical Equation*

If $Y$ is the target variable (dependent) and $X$ is the predictive variable and if the probability that $Y = 1$ is denoted as $\hat{p}$, then the probability that $Y$ is 0 would be $1 - \hat{p}$. The logistic model for predicting $\hat{p}$ would be given by

$$\ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = B_0 + B_1 X \tag{13.4}$$

where $\ln\left(\frac{\hat{p}}{1-\hat{p}}\right)$ is the log of the odds ratio and is known as logit and $B_0$ is the constant and $B_1$ is the regression coefficient.

In effect, in logistic regression this logit, $\ln\left(\frac{\hat{p}}{1-\hat{p}}\right)$, is the dependent variable against which independent variables are regressed.

From Eq. (13.4), the probability ( $\hat{p}$ ) that $Y = 1$ can be computed for a given value of $X$.
Let us assume that

$$Z = \ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = B_0 + B_1 X \tag{13.5}$$

$$\Rightarrow \qquad \frac{\hat{p}}{1-\hat{p}} = e^Z$$

Or

$$\hat{p} = \frac{e^Z}{1+e^Z} = \frac{e^{B_0+B_1X}}{1+e^{B_0+B_1X}} \tag{13.6}$$

Thus, in the logistic regression, first a logit or log of odds ratio, that is, $\ln\left(\frac{\hat{p}}{1-\hat{p}}\right)$, is computed for a given value of $X$, and then the probability ($\hat{p}$) that $Y = 1$ is computed by using formula (13.6). In fact (13.6) gives the logistic function as

$$f(z) = \frac{e^Z}{1+e^Z} \tag{13.7}$$

This function if plotted by taking $z$ on horizontal axis and $f(z)$ on vertical axis looks like as shown in Fig. 13.3.

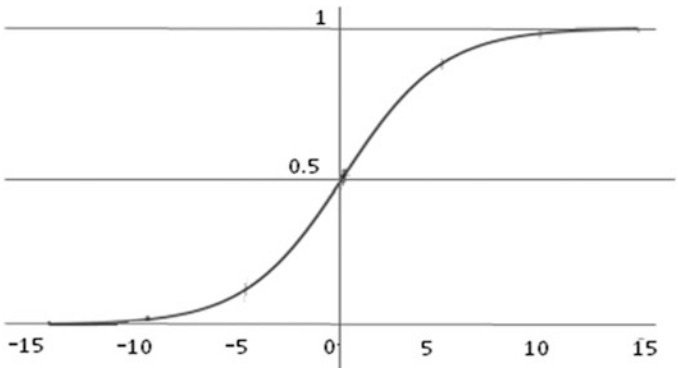



**Fig. 13.3** Logistic function for finding the probability of $Y = 1$

## *Interpreting the Logistic Function*

In logistic regression, the logistic function shown in Fig. 13.3 is used for estimating the probability of an event happening ($Y = 1$) for different values of $X$. Let us see how it is done.

In the logistic function shown in (13.7), the input is $z$ and output is $f(z)$. The value of $z$ is estimated by the logistic regression Eq. (13.5) on the basis of the value of $X$. The important characteristics of the logistic function are that it can take any value from negative infinity to positive infinitive, but the output will always be in the range of 0–1.

If there are $n$ independent variables, then the value of $z$ or logit or log of odds shall be estimated by the equation:

$$Z = \text{logit} = \ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = B_0 + B_1X_1 + B_2X_2 + \ldots\ldots\ldots + B_nX_n \qquad (13.8)$$

where $B_0$ is an intercept and $B_1$, $B_2$. . ..$B_n$ are the regression coefficients of $X_1$, $X_2$ . . ...,$X_n$, respectively.

The variable $Z$ is estimated from (13.8) for a given value of $Xs$. It is a measure of the total contribution of all the independent variables used in the model.

If the outcome variable is the risk factor for happening of an event say bankruptcy of an organization, then each of the regression coefficients shows the contribution toward the probability of that outcome. If the regression coefficient is positive, it indicates that the explanatory variable increases the probability

of the outcome, whereas in case of negative regression coefficient, it decreases the probability of that outcome. On the other hand, a large regression coefficient means that the corresponding variable is a high risk factor which strongly influences the probability of that outcome, whereas a near-zero regression coefficient indicates that the corresponding variable is not an important risk factor and has little influence on the probability of that outcome.

## *Assumptions in Logistic Regression*

Following are the assumptions used in the logistic regression:

1. The target variable is always binary. If by nature it is continuous, a criterion may be defined to convert it into binary.
2. The predictor variables can either be numerical or categorical. In case the categorical variable has more than two categories, a dummy variable $D$ (it may have the variable name as well) is created and different categories may be denoted by code 1, 2, 3, etc. Care should be taken that the highest code should refer the reference category. By default, the SPSS assumes the highest coding as reference category and marks it 0. For instance, if the qualification is taken as categorical variable, then this variable $D$ may be coded as follows:

   $D = 3$, if the subject's qualification is XII standard or less.
   $D = 2$, if the subject is graduate.
   $D = 1$, if the subject's qualification is postgraduation or more.

   In SPSS the highest coding is taken as the reference category by default, and therefore, you will find that in the output, XII or less qualification category is represented by 0 and the interpretation is made with reference to this category only. However, SPSS does provide the facility to change the reference category to the lowest code as well.
3. It is assumed that the logit transformation of the outcome variable has a linear relationship with the predictor variables.
4. Many authors suggested that a minimum of ten events per predictive variables should be taken in the logistic regression. For example, in a study where cure is the target variable of interest and 100 out of 150 patients get cured, the maximum number of independent variables one can have in the model is $100/10 = 10$.

## *Important Features of Logistic Regression*

1. The logistic regression technique is more robust because the independent variables do not have to be normally distributed or have equal variance in each group.

2. The independent variables are not required to be linearly related with the dependent variable.
3. It can be used with the data having nonlinear relationship.
4. The dependent variable need not follow normal distribution.
5. The assumption of homoscedasticity is not required. In other words, no homogeneity of variance assumption is required.

Although the logistic regression is very flexible and can be used in many situations without imposing so many restrictions on the data set, the advantages of logistic regression come at a cost. It requires large data set to achieve reliable and meaningful results. Whereas in OLS regression and discriminant analysis, 5 to 10 data per independent variable is considered to be minimum threshold, logistic regression requires at least 50 data per independent variable to achieve the reliable findings.

## Research Situations for Logistic Regression

Due to the flexibility about its various assumptions, the logistic regression is widely used in many applications. Some of the specific applications are discussed below:

1. A food joint chain may be interested to know as to what factors may influence the customers to buy big-size Pepsi in the fast-food center. The factors may include the type of pizza (veg. or non-veg.) ordered, whether French fries ordered, the age of the customer, and their body size (bulky or normal). The logistic model can provide the solution in identifying the most probable parameters responsible for buying big-size Pepsi in different food chains.
2. A study may investigate the parameters responsible for getting admission to MBA program in Harvard Business School. The target variable is a dichotomous variable with 1 indicating the success in getting admission, whereas 0 indicates failure. The parameters of interest may be working experience of the candidates in years, grades in the qualifying examination, TOEFL and GMAT scores, and scores on the testimonials. By way of logistic model, the relative importance of the independent variables may be identified and the probability of success of an individual may be estimated on the basis of the known values of the independent variables.
3. A market research company may be interested to investigate the variables responsible for a customer to buy a particular life insurance cover. The target variable may be 1 if the customer buys the policy and 0 if not. The possible independent variables in the study may be the age, gender, socioeconomic status, family size, profession (service/business), etc. By knowing the most likely causes for getting success in selling the policy, the company may target the campaign toward the target audience.

4. Incidence of HIV infection may be investigated by using the logistic model, where the independent variables may be identified as person's movement (frequent or less frequent), age, sex, occupation, personality type, etc. The strategy may be developed by knowing the most dominant causes responsible for HIV infection, and accordingly mass campaign may be initiated for different sections of the society in an efficient manner. One of the interesting facts in such studies may be to investigate the important factors of the HIV incidences in different sections of the society due to different dynamics.
5. The incidence of cardiac death may be investigated based on the factors like age, sex, activity level, BMI, and blood cholesterol level of the patients by fitting the logit model. The odds ratio will help you find the relative magnitude of risk involved with different factors.

## Steps in Logistic Regression

After understanding the concepts involved in logistic regression, now you are ready to use this analysis for your problem. The detailed procedure of this analysis using SPSS shall be discussed by using a practical example. But before that, let us summarize the steps involved in using the logistic regression:

1. Define the target variable and code it 1 if the event occurs and 0 otherwise. The target variable should always be dichotomous.
2. Identify the relevant independent variables responsible for the occurrence of target variable.
3. In case if any independent variable is categorical having more than two categories, define the coding for different categories as discussed in the "Assumptions" section.
4. Develop a regression model by taking dependent variable as log odds of the probability that target variable $Y = 1$. Logistic regression model can be developed either by using forward/backward step methods or by using all the independent variables in the model. Forward/backward step methods are usually used in explorative study where it is not known whether the independent variable has some effect on the target variable or not. On the other hand, all the independent variables are used in developing a model if the effect of independent variables is known in advance and one tries to authenticate the model. Several options for forward/backward methods are available in the SPSS, but "Forward:LR" method is considered to be the most efficient method. On the other hand, for taking all the independent variables in the model, the SPSS provides a default option with "Enter" command.
5. After choosing the method for binary logic regression, the model would look like as follows where $\hat{p}$ is the probability that the target variable $Y = 1$:

$$\ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = Z = B_0 + B_1X_1 + B_2X_2 + \ldots\ldots\ldots + B_nX_n$$

The variables have their usual meanings. The log odds $\ln\left(\frac{\hat{p}}{1-\hat{p}}\right)$ is also known as logit.

6. The estimated probability of occurring the target variable can be estimated for a given set of values of independent variables by using the following formula:

$$\widehat{p} = \frac{e^Z}{1+e^Z} = \frac{e^{B_0+B_1X_1+B_2X_2\ldots..B_nX_n}}{1+e^{B_0+B_1X_1+B_2X_2+\ldots...B_nX_n}}$$

The above-mentioned equation gives rise to the logistic curve which is S-shaped as shown in Fig. 13.3. The probability can also be computed from this curve by computing the value of $Z$.

7. Exponential of the regression coefficient is known as odds ratio. These odds ratios are used to find the relative contribution of all the independent variables toward the occurrence of target variable. Thus, the odds ratio corresponding to each of the regression coefficients is computed for investigating the relative contribution of independent variables toward the occurrence of dependent variable. For example, the odds ratio of 3.2 for the variable $X_1$ indicates that the probability of $Y$ (dependent variable) equals to 1 is 3.2 times as likely as the value of $X_1$ is increased by one unit. And if an odds ratio for the variable $X_3$ is .5, it indicates that the probability of $Y = 1$ is half as likely with an increase of $X_3$ by one unit (here there is a negative relationship between $X_3$ and $Y$). On the other hand, if the odds ratio for the variable $X_2$ is 1.0, it indicates that there is no relationship between $X_2$ and $Y$.

## Solved Example of Logistics Analysis Using SPSS

**Example 13.1** A researcher wanted to investigate the factors responsible for getting the job of coin note examiner in banks. The data was obtained by the recruitment agency that was responsible for appointment of bank employees. The investigator collected the data on the outcome variable (appointed or not appointed) and independent variables like education (number of years of college education), sex, experience, age, metro/nonmetro status, and marital status. These data are shown in Table 13.2. Apply logistic regression by using SPSS to develop a model for estimating the probability of success in getting the job on the basis of candidate's profiles. Further, discuss the comparative importance of these independent variables in getting success during an interview for the job. The coding for the categorical variables is shown below the table.

**Table 13.2** Data on the candidate's profile along with success status

| S.N. | Job success | Education | Sex | Experience in years | Age | Metro | Marital status |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 16 | 1 | 7 | 23 | 1 | 1 |
| 2 | 0 | 15 | 1 | 5 | 25 | 0 | 0 |
| 3 | 1 | 16 | 1 | 5 | 27 | 1 | 1 |
| 4 | 1 | 15 | 1 | 2 | 26 | 1 | 0 |
| 5 | 0 | 16 | 0 | 3 | 28 | 0 | 0 |
| 6 | 1 | 15 | 1 | 2 | 26 | 0 | 1 |
| 7 | 0 | 13 | 1 | 3 | 33 | 1 | 1 |
| 8 | 0 | 12 | 0 | 2 | 32 | 0 | 1 |
| 9 | 1 | 12 | 1 | 3 | 26 | 1 | 1 |
| 10 | 0 | 13 | 0 | 3 | 30 | 0 | 0 |
| 11 | 1 | 12 | 0 | 1 | 28 | 1 | 1 |
| 12 | 0 | 12 | 0 | 2 | 28 | 0 | 0 |
| 13 | 1 | 15 | 1 | 6 | 32 | 1 | 1 |
| 14 | 1 | 12 | 1 | 3 | 38 | 0 | 1 |
| 15 | 0 | 16 | 0 | 2 | 23 | 0 | 0 |
| 16 | 1 | 15 | 1 | 3 | 22 | 1 | 0 |
| 17 | 1 | 16 | 1 | 7 | 23 | 0 | 1 |
| 18 | 0 | 15 | 1 | 5 | 25 | 0 | 0 |
| 19 | 1 | 16 | 1 | 5 | 27 | 1 | 1 |
| 20 | 1 | 12 | 0 | 2 | 28 | 0 | 1 |
| 21 | 1 | 16 | 1 | 4 | 28 | 1 | 0 |
| 22 | 1 | 15 | 1 | 3 | 28 | 0 | 1 |
| 23 | 0 | 12 | 0 | 2 | 26 | 1 | 0 |
| 24 | 0 | 14 | 0 | 5 | 29 | 0 | 0 |

*Job success* : 0 : Failure 1 : Success
*Sex* : 0 : Female 1 : Male
*Metro* : 0 : Nonmetro resident 1 : Metro resident
*Marital status* : 0 : Unmarried 1 : Married

*Solution*

The above-mentioned problem can be solved by using SPSS. The steps involved in getting the outputs shall be discussed first and then the output so generated shall be explained to fulfill the objectives of the study.

The logistic regression in SPSS is run in two steps. The outputs generated in these two sections have been discussed in the following two steps:

## *First Step*

### Block 0: Beginning Block

The first step, called Block 0, includes no predictors and just the intercept. This model is developed by using only constant and no predictors. The logistic regression compares this model with a model having all the predictors to assess whether the later model is more efficient. Often researchers are not interested in this model. In this part, a "null model," having no predictors and just the intercept, is described.

Because of this, all the variables entered into the model will figure in the table titled "Variables not in the Equation."

## *Second Step*

### Block 1: Method = Forward:LR

The second step, called Block 1, includes the information about the variables that are included and excluded from the analysis, the coding of the dependent variable, and coding of any categorical variables listed on the categorical subcommand. This section is the most interesting part of the output in which generated outputs are used to test the significance of the overall model, regression coefficients, and odds ratios.

The above-mentioned outputs in two steps are generated by the SPSS through a single sequence of commands, but the outputs are generated in two different sections with the headings "Block 0: Beginning Block" and "Block 1." You have the liberty to use any method of entering independent variables in the model out of different methods available in SPSS. These will be discussed while explaining screen shots of logistic regression in the next section.

The procedure of logistic regression in SPSS shall be defined first and then relevant outputs shall be shown with explanation.

## *SPSS Commands for the Logistic Regression*

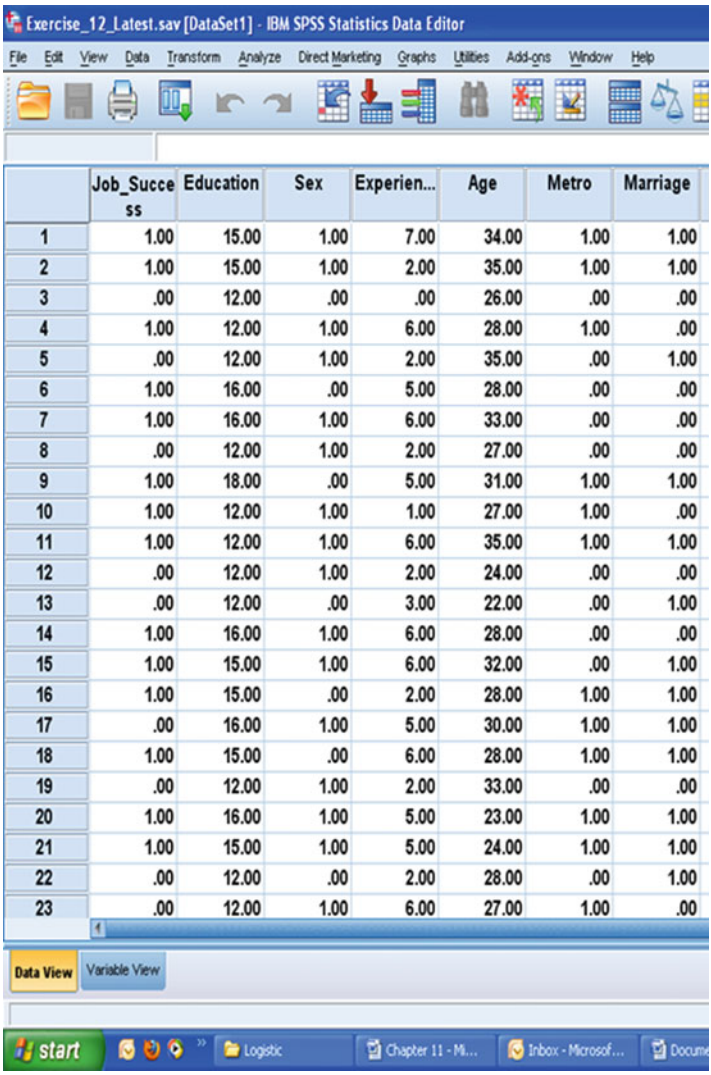
To run the commands for logistic regression, a data file is required to be prepared. The procedure for preparing the data file has been explained in Chap. 1. After preparing the data file, do the following steps for generating outputs in logistic regression:

(i) *Data file:* In this problem, job success is a dependent variable which is binary in nature. Out of six independent variables, three variables, namely, sex, metro, and marital status, are binary, whereas remaining three, education, experience, and age, are scale variables. In SPSS all binary variables are defined as nominal. After preparing the data file by defining variable names and their labels, it will look like as shown in Fig. 13.4.

(ii) *Initiating command for logistic regression:* After preparing the data file, click the following commands in sequence (Fig. 13.5):

**Analyze → Regression → Binary˜Logistic**

(iii) *Selecting variables for analysis:* After clicking the **Binary Logistic** option, you will get the next screen for selecting dependent and independent variables. After selecting all the independent variables, you need to select the binary independent variables included in it by clicking the option. The selection of variables can be made by following the below-mentioned steps:




| | Job_Success | Education | Sex | Experien... | Age | Metro | Marriage |
|---|---|---|---|---|---|---|---|
| 1 | 1.00 | 15.00 | 1.00 | 7.00 | 34.00 | 1.00 | 1.00 |
| 2 | 1.00 | 15.00 | 1.00 | 2.00 | 35.00 | 1.00 | 1.00 |
| 3 | .00 | 12.00 | .00 | .00 | 26.00 | .00 | .00 |
| 4 | 1.00 | 12.00 | 1.00 | 6.00 | 28.00 | 1.00 | .00 |
| 5 | .00 | 12.00 | 1.00 | 2.00 | 35.00 | .00 | 1.00 |
| 6 | 1.00 | 16.00 | .00 | 5.00 | 28.00 | .00 | .00 |
| 7 | 1.00 | 16.00 | 1.00 | 6.00 | 33.00 | .00 | .00 |
| 8 | .00 | 12.00 | 1.00 | 2.00 | 27.00 | .00 | .00 |
| 9 | 1.00 | 18.00 | .00 | 5.00 | 31.00 | 1.00 | 1.00 |
| 10 | 1.00 | 12.00 | 1.00 | 1.00 | 27.00 | 1.00 | .00 |
| 11 | 1.00 | 12.00 | 1.00 | 6.00 | 35.00 | 1.00 | 1.00 |
| 12 | .00 | 12.00 | 1.00 | 2.00 | 24.00 | .00 | .00 |
| 13 | .00 | 12.00 | .00 | 3.00 | 22.00 | .00 | 1.00 |
| 14 | 1.00 | 16.00 | 1.00 | 6.00 | 28.00 | .00 | .00 |
| 15 | 1.00 | 15.00 | 1.00 | 6.00 | 32.00 | .00 | 1.00 |
| 16 | 1.00 | 15.00 | .00 | 2.00 | 28.00 | 1.00 | 1.00 |
| 17 | .00 | 16.00 | 1.00 | 5.00 | 30.00 | 1.00 | 1.00 |
| 18 | 1.00 | 15.00 | .00 | 6.00 | 28.00 | 1.00 | 1.00 |
| 19 | .00 | 12.00 | 1.00 | 2.00 | 33.00 | .00 | .00 |
| 20 | 1.00 | 16.00 | 1.00 | 5.00 | 23.00 | 1.00 | 1.00 |
| 21 | 1.00 | 15.00 | 1.00 | 5.00 | 24.00 | 1.00 | 1.00 |
| 22 | .00 | 12.00 | .00 | 2.00 | 28.00 | .00 | 1.00 |
| 23 | .00 | 12.00 | 1.00 | 6.00 | 27.00 | 1.00 | .00 |

**Fig. 13.4** Screen showing data file for the logistic regression analysis in SPSS

- Select the dependent variable from the left panel to the "Dependent" section in the right panel.
- Select all independent variables including categorical variables from left panel to the "Covariates" section in the right panel.
- Click the command **Categorical** and select the categorical variables from the "Covariates" section to the "Categorical Covariates" in the right panel. The screen will look like Fig. 13.6.
- Click ***Continue.***

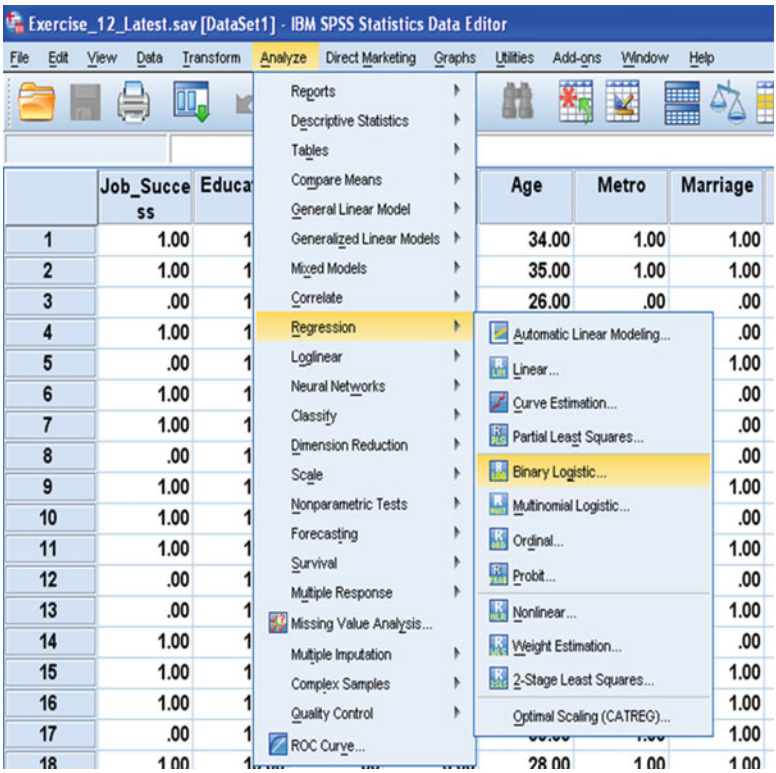



**Fig. 13.5** Screen showing of SPSS commands for logistic regression

(iv) *Selecting options for computation:* After selecting the variables, you need to define different options for generating the outputs in logistic regression. Do the following steps:

- Click the tag **Options** in the screen shown in Fig. 13.6 for selecting outputs related to statistics and plots. Do the following steps:
- Check "Classification Plots."
- Check "Hosmer-Lemeshow goodness-of-fit."
- Let all other default options be selected. The screen will look like Fig. 13.7.
- Click ***Continue***.

(v) *Selecting method for entering independent variables in logistic regression*: You need to define the method of entering the independent variables for developing the model. You can choose any of the options like Enter, Forward:LR, Forward:Wald, Backward:LR, or Backward:Wald. Enter method is usually selected when a specific model needs to be tested or the contribution of independent variables toward the target variable is known in advance. On the other hand, if the study is exploratory in nature, then any of the forward or backward methods can be used. In this study, the Forward:LR method shall be used because the study is exploratory in nature.

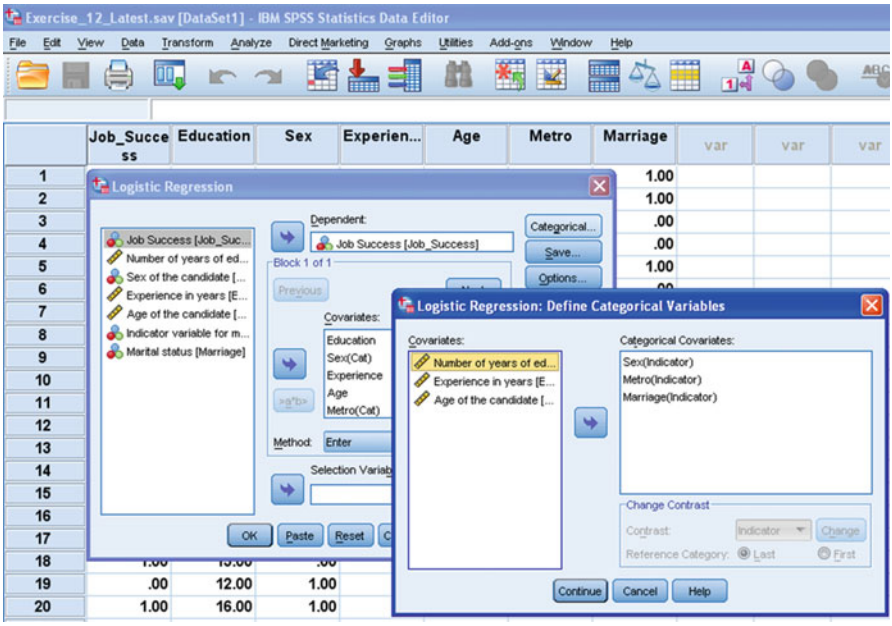



**Fig. 13.6** Screen showing selection of variables for logistic regression

The Forward:LR method is considered to be the most efficient method among different forward and backward methods. Here LR refers to the likelihood ratio method. In this method, the variables are selected in the model one by one based on their utility. To select this method in the SPSS, do the following steps:

- Select the option "Forward:LR" by using the dropdown menu of the command **Method** in the screen shown in Fig. 13.6.
- Click **OK.**

(vi) *Getting the output:* Clicking the option **OK** shall generate lots of output in the output window. These outputs may be selected from the output window by using right click of the mouse and may be copied in the word file. The relevant outputs so selected for discussion are shown in Tables 13.3–13.12. One must understand the meaning of these outputs so that while writing thesis or project report, they may be incorporated with proper explanation.

## *Interpretation of Various Outputs Generated in Logistic Regression*

### Descriptive Findings

Table 13.3 shows the number of cases (*N*) in each category (e.g., included in the analysis, missing, and total) and their percentage. In logistic regression, a listwise deletion of missing data is done by default in SPSS. Since there is no missing data,

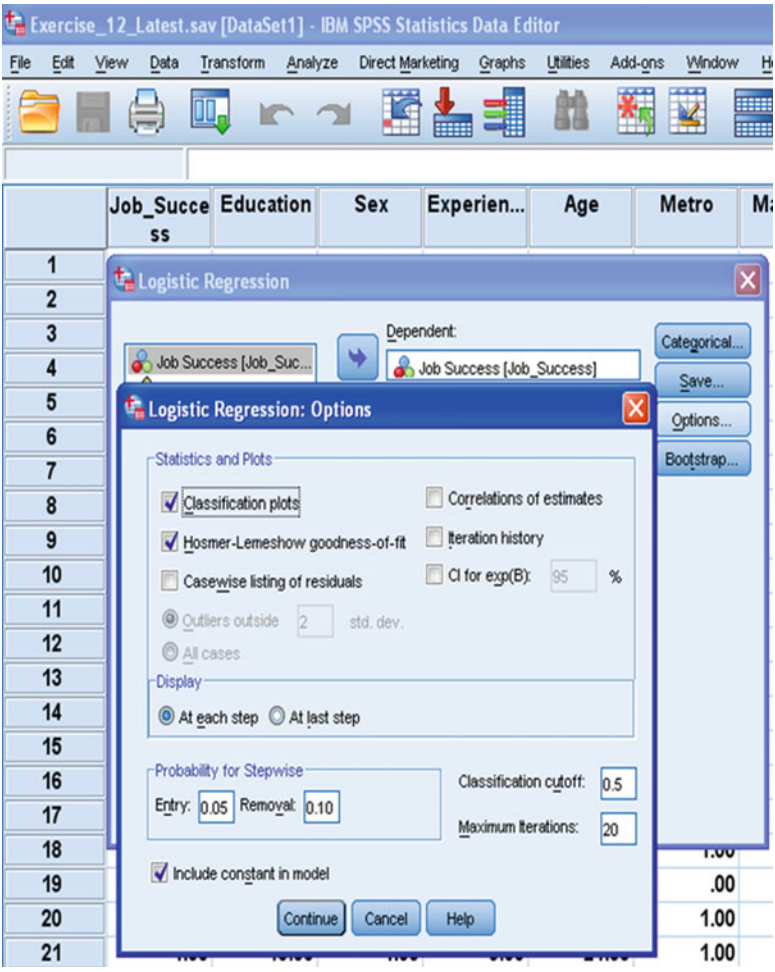



**Fig. 13.7** Screen showing option for generating classification plots and Hosmer-Lemeshow goodness-of-fit

the number of missing cases is shown as 0. Table 13.4 shows the coding of the dependent variable used in the data file, that is, 1 for success and 0 for failure in getting the job.

Table 13.5 shows the coding of all the categorical independent variables along with their frequencies in the study. While coding the categorical variables, highest number should be allotted to the reference category because by default SPSS considers the category with the highest coding as the reference category and gives the code as 0. For instance, if you define the coding of the variable sex as 0 for "female" and 1 for "male," then the SPSS will consider male as the reference category and convert its code to 0 and the other category female as 1.

**Table 13.3** Case processing summary

| Unweighted cases[a] | | *N* | Percent |
|---|---|---|---|
| Selected cases | Included in analysis | 23 | 100.0 |
| | Missing cases | 0 | .0 |
| | Total | 23 | 100.0 |
| Unselected cases | | 0 | .0 |
| Total | | 23 | 100.0 |

[a]If weight is in effect, see classification table for the total number of cases

**Table 13.4** Dependent variable encoding

| Original value | Internal value |
|---|---|
| Failure | 0 |
| Success in getting job | 1 |

**Table 13.5** Categorical variable coding

| | | | Parameter coding |
|---|---|---|---|
| | | Frequency | (1) |
| Marital status | Unmarried | 10 | 1.000 |
| | Married | 13 | .000 |
| Metro | Nonmetro | 11 | 1.000 |
| | Metro | 12 | .000 |
| Sex | Female | 7 | 1.000 |
| | Male | 16 | .000 |

If you look into the coding of the independent categorical variables in the Table 13.2, that is, sex (0:female, 1:male), metro (0:nonmetro resident, 1:metro resident), and marital status (0:unmarried and 1:married), these coding have been reversed by the SPSS as shown in the Table 13.5. It is because SPSS by default considers the highest coding as the reference category and converts it into 0. However, you can change the reference category as the lowest coding in SPSS screen as shown in the Fig. 13.6.

## Analytical Findings

The findings in this section are the most interesting part of the output. These findings include the test of the overall model, significance of regression coefficients, and the values of the odds ratios.

In this study, since the **Forward:LR** method has been chosen for logistic regression, you will get more than one model with different number of variables in it. The results of the logistic regression shall be discussed in two blocks. In the first block, the logistic regression model shall be developed by using the constant without using any of the independent variables. This model may be used to compare the utility of the model developed in block two by using the identified independent variables.

**Table 13.6** Classification table[a, b]

| | | | Predicted | | |
|---|---|---|---|---|---|
| | | | Job | | Percentage correct |
| Observed | | | Failure | Success | |
| Step 0 | Job | Failure | 0 | 10 | 0 |
| | | Success | 0 | 14 | 100.0 |
| | | Overall Percentage | | | 58.3 |

[a]Constant is included in the model
[b]The cut value is .500

**Table 13.7** Variables in the equation

| | | *B* | S.E. | Wald | df | Sig. | Exp(*B*) |
|---|---|---|---|---|---|---|---|
| Step 0 | Constant | .336 | .414 | .660 | 1 | .416 | 1.400 |

Block 0: Beginning Block

In Block 0, the results are shown for the model with only the constant included before any coefficients (i.e., those relating to education, sex, experience, age, metro, and marital) are entered into the equation. Logistic regression compares the model obtained in Block 0 with a model including the predictors to determine whether the latter model is more efficient. The Table 13.6 shows that if nothing is known about the independent variables and one simple guesses that a person would be selected for the job, we would be correct 58.3% of the time. Table 13.7 shows that the Wald statistics is not significant as its significance value is 0.416 which is more than 0.05. Hence, the model with constant is not worth and is equivalent to just guessing about the target variable in the absence of any knowledge about the independent variables.

Table 13.8 shows whether each independent variable improves the model or not. You can see that the variables sex, metro, and marital may improve the model as they are significant with sex and marital slightly better than metro. Inclusion of these variables would add to the predictive power of the model. If these variables had not been significant and able to contribute to the prediction, then the analysis would obviously be terminated at this stage.

Block 1 Method = Forward:LR

In this block, results of the different models with different independent variables shall be discussed.

Table 13.9 shows the value of −2 log likelihood (−2LL), which is a deviance statistic between the observed and predicated values of the dependent variable. If this deviance statistic is insignificant, it indicates that the model is good and there is no difference between observed and predicted values of dependent variable. This number in absolute term is not very informative. However, it can be used to compare different models having different number of predictive variables. For

**Table 13.8** Variables not in the equation

| | | | Score | df | Sig. |
|---|---|---|---|---|---|
| Step 0 | Variables | Education | 1.073 | 1 | .300 |
| | | Sex(1) | 7.726 | 1 | .005 |
| | | Experience | .728 | 1 | .393 |
| | | Age | .174 | 1 | .677 |
| | | Metro(1) | 4.608 | 1 | .032 |
| | | Marital(1) | 8.061 | 1 | .005 |
| | | Overall statistics | 14.520 | 6 | .024 |

**Table 13.9** Model summary

| Step | −2 log likelihood | Cox and Snell *R*-square | Nagelkerke *R*-square |
|---|---|---|---|
| 1 | 24.053[a] | .300 | .403 |
| 2 | 18.549[b] | .443 | .597 |

[a]Estimation terminated at iteration number 4 because parameter estimates changed by less than .001

[b]Estimation terminated at iteration number 5 because parameter estimates changed by less than .001

instance, in Table 13.9, the value of −2LL has reduced from 24.053 to 18.549. This indicates that there is an improvement in model 2 by including an additional variable, sex. In fact, the value of −2LL should keep on decreasing if you go on adding the significant predictive variables in the model.

Unlike OLS regression equation, there is no concept of $R^2$ in logistic regression. It is because of the fact that the dependent variable is dichotomous and $R^2$ cannot be used to show the efficiency of prediction. However, several authors have suggested pseudo *R*-squares which are not equivalent to the *R*-square that is calculated in OLS regression. Thus, this statistic should be interpreted with great caution. Two such pseudo *R*-squares suggested by Cox and Snell and Nagelkerke are shown in Table 13.9. As per Cox and Snell's $R^2$, 44.3% of the variation in the dependent variable is explained by the logistic model. On the other hand, Nagelkerke's $R^2$ explains 59.7% variability of the dependent variable by the independent variables in the model. Nagelkerke's $R^2$ is more reliable measure of relationship in comparison to Cox and Snell's $R^2$. Nagelkerke's $R^2$ will normally be higher than Cox and Snell's $R^2$.

In order to find whether the deviance statistic −2 log likelihood is insignificant or not, Hosmer and Lemeshow suggested the chi-square statistic which is shown in Table 13.10. In order that the model is efficient, this chi-square statistic should be insignificant. Since the *p* value associated with chi-square in Table 13.10 is .569 for the second model, which is greater than .05, it is insignificant and it can be interpreted that the model is efficient.

Table 13.11 is a classification table which shows the observed and predicted values of the dependent variable in both the models. In the second model, it can be seen that out of 10 candidates who did not get the success in getting the job, four were wrongly predicted to get the job. Similarly out of 14 candidates who succeeded to get the job, none was wrongly predicted to be failure. Thus, the model correctly classified 83.3% cases. This can be obtained by (20/24) × 100.

**Table 13.10** Hosmer and Lemeshow test

| Step | Chi-square | df | Sig. |
|---|---|---|---|
| 1 | .000 | 0 | . |
| 2 | 1.129 | 2 | .569 |

**Table 13.11** Classification table[a]

| | | | Predicted | | |
|---|---|---|---|---|---|
| | | | Job | | Percentage correct |
| Observed | | | Failure | Success | |
| Step 1 | Job | Failure | 8 | 2 | 80.0 |
| | | Success | 3 | 11 | 78.6 |
| | | Overall percentage | | | 79.2 |
| Step 2 | Job | Failure | 6 | 4 | 60.0 |
| | | Success | 0 | 14 | 100.0 |
| | | Overall percentage | | | 83.3 |

[a]The cut value is .500

**Table 13.12** Variables in the equation

| | | *B* | S.E. | Wald | df | Sig. | Exp(*B*) |
|---|---|---|---|---|---|---|---|
| Step 1[a] | Marital(1) | −2.686 | 1.024 | 6.874 | 1 | .009 | .068 |
| | Constant | 1.705 | .769 | 4.918 | 1 | .027 | 5.500 |
| Step 2[b] | Sex(1) | −2.666 | 1.278 | 4.352 | 1 | .037 | .070 |
| | Marital(1) | −2.711 | 1.253 | 4.682 | 1 | .030 | .066 |
| | Constant | 2.779 | 1.146 | 5.886 | 1 | .015 | 16.106 |

[a]Variable(s) entered on step 1: marital
[b]Variable(s) entered on step 2: sex

Table 13.12 is the most important table which shows the value of regression coefficients *B*, Wald statistics, its significance, and odds ratio exp(*B*) for each variable in both the models. The *B* coefficients are used to develop the logistic regression equation for predicting the dependent variable from the independent variables. These coefficients are in log-odds units. Thus, the logistic regression equation in the second model is given by $\log \frac{p}{1-p} = 2.779 - 2.666 \times \text{Sex}(1) - 2.711 \times \text{Marital}(1)$where *p* is the probability of getting the job. The dependent variable in the logistic regression is known as logit(*p*) which is equal to $\log(p/(1 - p))$.

The estimates obtained in the above logistic regression equation explain the relationship between the independent variables and the dependent variable, where the dependent variable is on the logit scale. These estimates tell the amount of increase (or decrease, if the sign of the coefficient is negative) in the estimated log odds of "job success" = 1 that would be predicted by a 1 unit increase (or decrease) in the predictor, holding all other predictors constant.

Because regression coefficients *B* are in log-odds units, they are often difficult to interpret; hence, they are converted into odds ratios which are equal to exp(*B*). These odds ratios are shown in the last column of Table 13.12.

Significance of the Wald statistics indicates that the variable significantly predicts the success in getting the bank job, but it should be used only in a situation

where the sample size is quite large, preferably more than 500. In case of small sample, the level of significance gets inflated and it does not give the correct picture. Since in this problem the value of chi-square in Hosmer and Lemeshow test as shown in Table 13.10 is insignificant, the model can be considered to be valid for predicting the success in getting the bank's job on the basis of the second model with two independent variables, that is, marital and sex.

### *Explanation of Odds Ratio*

In Table 13.12, the exp(*B*) represents the odds ratio for all the predictors. If the value of the odds ratio is large, its predictive value is also large. Since the second model is the final model in this study, the discussion shall be done for the variables in this model only. Here both the independent variables, that is, sex and marital, are significant. Since the sex(1) variable has a larger odds ratio .070, this is slightly a better predictor in comparison to marital(1) variable in getting the bank's job.

The value of exp(*B*) for the variable sex(1) is 0.070. It indicates that if the candidate appearing in the bank exams is female, then there would be decrease in the odds of 93% (.07–1.00 = −.93). In other words, if a female candidate is appearing in the bank examination, her chances of success would be 93% less than the men candidate if other variables are kept constant. Similarly the exp(*B*) value of the variable marital(1) is .066. This indicates that there would be decrease in the odds of 93.4% (.066–1.000 = −.934). It can be interpreted that if the candidate appearing in the bank examination is unmarried, his/her chances of success would be 93.4% less than the married candidate provided other variables are kept constant.

### *Conclusion*

To conclude, if the candidate is male and married, the chances of odds increases for getting selected for a bank job in comparison to female and unmarried candidate.

## Summary of the SPSS Commands for Logistic Regression

(i) Start SPSS and prepare the data file by defining the variables and their properties in **Variable View** and typing the data column-wise in Data View.
(ii) In the Data View, follow the below-mentioned command sequence for factor analysis:

**Analyze ⟶ Regression ⟶ Binary Logistic**

(iii) Select the dependent variable from the left panel to the "Dependent" section in the right panel and all independent variables including categorical variables from left panel to the "Covariates" section in the right panel.
(iv) By clicking the **Categorical command**, select the categorical variables from the "Covariates" section to the "Categorical Covariates" in the right panel and click ***Continue.***
(v) Click the tag **Options** and check "Classification Plots" and "Hosmer-Lemeshow goodness-of-fit" and click ***Continue***.
(vi) Ensure that the option **Forward:LR** is chosen by default and then click **OK** for output.

## Exercise

*Short Answer Questions*

**Note:** Write answer to each questions in not more than 200 words.

Q.1. What is logit and how is it used to interpret the probability of success?
Q.2. What do you mean by odds ratio? Explain the monotonic transformation in relation with odds ratio and log odds.
Q.3. Explain the logistic function and its characteristics.
Q.4. Why is the logit function used in logistic regression analysis?
Q.5. Explain the meaning of maximum likelihood and the significance of $-2$ log likelihood.
Q.6. What is the difference between logic regression and OLS regression?
Q.7. How are the dummy variables created in a situation where an independent categorical variable has more than two options?
Q.8. Write any four assumptions used in logistic regression.
Q.9. What are the advantages of using logistic regression analysis?
Q.10. Explain any one research situation in detail where logistic regression can be applied.
Q.11. Write in brief the various steps involved in logistic regression.
Q.12. What is Hosmer and Lemeshow test? How is it used and what does it indicate?

*Multiple-Choice Questions*

**Note:** For each of the question, there are four alternative answers for each question. Tick mark the one that you consider the closest to the correct answer.

1. Logistic regression is used when the dependent variable is

   (a) Continuous
   (b) Ordinal
   (c) Binary
   (d) Categorical

2. If $\exp(3) = 20.12$, then $\log(20.12)$ is

   (a) 20.12
   (b) 23.12

(c) 17.12
(d) 3

3. If the probability of success is 0.6, then the odds of success is

(a) 0.4
(b) 1.5
(c) 2.4
(d) 0.75

4. In a logistic regression, if the odds ratio for an independent variable is 2.5, then which of the following is true?

(a) The probability of the dependent variable happening is 0.25.
(b) The odds against the dependent variable happening is 2.5.
(c) The odds for the dependent variable happening is 2.5.
(d) The odds for the dependent variable happening is 2.5 against one unit increase in the independent variable.

5. If $p$ is the probability of success, then the logit of $p$ is

(a) $\ln \frac{1-p}{p}$
(b) $\ln \frac{1+p}{p}$
(c) $\log \frac{p}{1-p}$
(d) $\log \frac{p}{1+p}$

6. The logistic function $f(z)$ is equal to

(a) $\frac{e^z}{1+e^z}$
(b) $\frac{1+e^z}{e^z}$
(c) $\frac{e^z}{1-e^z}$
(d) $\frac{1-e^z}{e^z}$

7. In logistic regression, odds ratio is equivalent to

(a) Log($B$)
(b) Exp($B$)
(c) $B$ coefficient
(d) $\frac{p}{1-p}$

8. Choose the correct statement.

(a) The independent variable is required to be linearly related with the dependent variable.
(b) The independent variable is required to be linearly related with logit transformation of the outcome variable.
(c) The dependent variable is always continuous.
(d) Probability of success in the outcome variable is equivalent to the log odds.

9. Choose the correct command for starting logistic regression in SPSS.

   (a) Analyze → Regression → Binary Logistic
   (b) Analyze → Regression → Logistic Regression
   (c) Analyze → Binary Logistic → Regression
   (d) Analyze → Logistic → Binary Regression

10. In using the Hosmer-Lemeshow goodness-of-fit, model is considered to be good if

    (a) Chi-square is significant at any predefined level.
    (b) Chi-square is not significant at any predefined level.
    (c) Chi-square is equal to 100.
    (d) All the regression coefficients are significant.

*Assignments*

1. Following are the scores of 90 candidates in different subjects obtained in a MBA entrance examination. Apply the logistic regression to develop a model for predicting success in the examination on the basis of independent variables. Discuss the comparative importance of independent variables in predicting success in the examination. For the variable MBA, coding 1 represents success and 0 indicates failure in the examination. Similarly gender 1 indicates male and 2 indicates female.

| MBA | English | Reasoning | Math | Gender | MBA | English | Reasoning | Math | Gender |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 68 | 50 | 65 | 0 | 0 | 46 | 52 | 55 | 1 |
| 0 | 39 | 44 | 52 | 1 | 0 | 39 | 41 | 33 | 0 |
| 0 | 44 | 44 | 46 | 1 | 0 | 52 | 49 | 49 | 0 |
| 1 | 50 | 54 | 61 | 1 | 0 | 28 | 46 | 43 | 0 |
| 1 | 71 | 65 | 72 | 0 | 0 | 42 | 54 | 50 | 1 |
| 1 | 63 | 65 | 71 | 1 | 0 | 47 | 42 | 52 | 0 |
| 0 | 34 | 44 | 40 | 0 | 0 | 47 | 57 | 48 | 1 |
| 1 | 63 | 49 | 69 | 0 | 0 | 52 | 59 | 58 | 0 |
| 0 | 68 | 43 | 64 | 0 | 0 | 47 | 52 | 43 | 1 |
| 0 | 47 | 45 | 56 | 1 | 1 | 55 | 62 | 41 | 0 |
| 0 | 47 | 46 | 49 | 1 | 0 | 44 | 52 | 43 | 0 |
| 0 | 63 | 52 | 54 | 0 | 0 | 47 | 41 | 46 | 0 |
| 0 | 52 | 51 | 53 | 0 | 0 | 45 | 55 | 44 | 1 |
| 0 | 55 | 54 | 66 | 0 | 0 | 47 | 37 | 43 | 0 |
| 1 | 60 | 68 | 67 | 1 | 0 | 65 | 54 | 61 | 0 |
| 0 | 35 | 35 | 40 | 0 | 0 | 43 | 57 | 40 | 1 |
| 0 | 47 | 54 | 46 | 1 | 0 | 47 | 54 | 49 | 0 |
| 1 | 71 | 63 | 69 | 0 | 1 | 57 | 62 | 56 | 0 |
| 0 | 57 | 52 | 40 | 1 | 0 | 68 | 59 | 61 | 1 |
| 0 | 44 | 50 | 41 | 0 | 0 | 52 | 55 | 50 | 0 |
| 0 | 65 | 46 | 57 | 0 | 0 | 42 | 57 | 51 | 0 |
| 1 | 68 | 59 | 58 | 1 | 0 | 42 | 39 | 42 | 1 |

(continued)

| MBA | English | Reasoning | Math | Gender | MBA | English | Reasoning | Math | Gender |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 73 | 61 | 57 | 1 | 1 | 66 | 67 | 67 | 1 |
| 0 | 36 | 44 | 37 | 0 | 1 | 47 | 62 | 53 | 0 |
| 0 | 43 | 54 | 55 | 0 | 0 | 57 | 50 | 50 | 0 |
| 1 | 73 | 62 | 62 | 1 | 1 | 47 | 61 | 51 | 1 |
| 0 | 52 | 57 | 64 | 1 | 1 | 57 | 62 | 72 | 1 |
| 0 | 41 | 47 | 40 | 0 | 0 | 52 | 59 | 48 | 1 |
| 0 | 50 | 54 | 50 | 0 | 0 | 44 | 44 | 40 | 1 |
| 0 | 50 | 52 | 46 | 1 | 0 | 50 | 59 | 53 | 1 |
| 0 | 50 | 52 | 53 | 0 | 0 | 39 | 54 | 39 | 0 |
| 0 | 47 | 46 | 52 | 0 | 1 | 57 | 62 | 63 | 1 |
| 1 | 62 | 62 | 45 | 1 | 0 | 57 | 50 | 51 | 1 |
| 0 | 55 | 57 | 56 | 1 | 0 | 42 | 57 | 45 | 0 |
| 0 | 50 | 41 | 45 | 1 | 0 | 47 | 46 | 39 | 0 |
| 0 | 39 | 53 | 54 | 1 | 0 | 42 | 36 | 42 | 1 |
| 0 | 50 | 49 | 56 | 0 | 0 | 60 | 59 | 62 | 0 |
| 0 | 34 | 35 | 41 | 0 | 0 | 44 | 49 | 44 | 0 |
| 0 | 57 | 59 | 54 | 1 | 0 | 63 | 60 | 65 | 1 |
| 1 | 65 | 60 | 72 | 0 | 1 | 65 | 67 | 63 | 1 |
| 1 | 68 | 62 | 56 | 0 | 0 | 39 | 54 | 54 | 0 |
| 0 | 42 | 54 | 47 | 0 | 0 | 50 | 52 | 45 | 1 |
| 0 | 53 | 59 | 49 | 1 | 1 | 52 | 65 | 60 | 0 |
| 1 | 59 | 63 | 60 | 1 | 1 | 60 | 62 | 49 | 1 |
| 0 | 47 | 59 | 54 | 1 | 0 | 44 | 49 | 48 | 0 |

2. In an assembly election, victory of a candidate depends upon many factors. In order to develop a model for predicting the success of a candidate (1 if elected and 0 if not elected) on the basis of independent variables, the data on 30 contestants were obtained on the variables like candidate's age, sex (1 for male and 0 for female), experience in politics, status in politics (1 for full time and 0 for part time), education (in number of years), and elected history (1 if elected earlier

Profile data of the contestants in the assembly election

| Election result | Age (in years) | Sex | Experience (in years) | Status in politics | Education (no. of years) | Election history |
|---|---|---|---|---|---|---|
| 1.00 | 48.00 | 1.00 | 10.00 | 1.00 | 15.00 | 1.00 |
| 1.00 | 42.00 | 1.00 | 16.00 | 1.00 | 18.00 | 1.00 |
| 1.00 | 46.00 | 1.00 | 12.00 | 1.00 | 15.00 | .00 |
| .00 | 42.00 | .00 | 16.00 | .00 | 16.00 | 1.00 |
| .00 | 45.00 | .00 | 20.00 | 1.00 | 18.00 | 1.00 |
| 1.00 | 47.00 | 1.00 | 18.00 | .00 | 15.00 | .00 |
| .00 | 34.00 | .00 | 28.00 | .00 | 15.00 | .00 |
| .00 | 47.00 | 1.00 | 20.00 | .00 | 12.00 | 1.00 |
| .00 | 36.00 | 1.00 | 30.00 | 1.00 | 10.00 | 1.00 |
| 1.00 | 63.00 | .00 | 35.00 | .00 | 16.00 | .00 |
| .00 | 45.00 | 1.00 | 25.00 | 1.00 | 12.00 | .00 |
| 1.00 | 54.00 | .00 | 20.00 | 1.00 | 16.00 | 1.00 |

(continued)

| Election result | Age (in years) | Sex | Experience (in years) | Status in politics | Education (no. of years) | Election history |
|---|---|---|---|---|---|---|
| 1.00 | 58.00 | 1.00 | 34.00 | .00 | 18.00 | .00 |
| .00 | 54.00 | .00 | 38.00 | .00 | 12.00 | .00 |
| .00 | 56.00 | 1.00 | 35.00 | .00 | 10.00 | 1.00 |
| 1.00 | 55.00 | .00 | 30.00 | 1.00 | 15.00 | .00 |
| 1.00 | 54.00 | .00 | 31.00 | 1.00 | 16.00 | .00 |
| 1.00 | 58.00 | 1.00 | 34.00 | .00 | 15.00 | 1.00 |
| .00 | 37.00 | 1.00 | 35.00 | .00 | 10.00 | .00 |
| 1.00 | 45.00 | 1.00 | 22.00 | .00 | 15.00 | 1.00 |
| .00 | 34.00 | 1.00 | 5.00 | 1.00 | 12.00 | 1.00 |
| .00 | 47.00 | .00 | 9.00 | .00 | 12.00 | 1.00 |
| .00 | 42.00 | 1.00 | 8.00 | .00 | 12.00 | 1.00 |
| 1.00 | 45.00 | 1.00 | 6.00 | .00 | 15.00 | .00 |
| 1.00 | 28.00 | 1.00 | 2.00 | 1.00 | 16.00 | 1.00 |
| 1.00 | 43.00 | .00 | 12.00 | 1.00 | 16.00 | 1.00 |
| .00 | 35.00 | 1.00 | 11.00 | 1.00 | 15.00 | 1.00 |
| 1.00 | 43.00 | .00 | 18.00 | 1.00 | 15.00 | .00 |
| 1.00 | 45.00 | .00 | 17.00 | .00 | 16.00 | 1.00 |
| 1.00 | 41.00 | 1.00 | 13.00 | .00 | 15.00 | 1.00 |
| .00 | 42.00 | 1.00 | 15.00 | 1.00 | 15.00 | .00 |

and 0 if not elected earlier). Apply the logistic regression and develop the model for predicting success in assembly election.

*Answers to Multiple-Choice Questions*

Q.1 c Q.2 d Q.3 b Q.4 d
Q.5 c Q.6 a Q.7 b Q.8 b
Q.9 a Q.10 b

# Chapter 14
# Multidimensional Scaling for Product Positioning

**Learning Objectives**

After completing this chapter, you should be able to do the following:

- Know the use of multidimensional scaling in market research.
- Understand the different terms used in multidimensional scaling.
- Learn the procedures used in multidimensional scaling.
- Able to identify the research situations where multidimensional scaling can be used.
- Describe the SPSS procedure involved in multidimensional scaling.
- Explain the various outputs generated by the SPSS in this analysis.

## Introduction

Multidimensional scaling (MDS) is a series of statistical techniques used for identifying the key dimensions underlying respondents' evaluations of objects and keeping them in multidimensional space. MDS is widely used in marketing research for positioning of brands. It would be desired for any company to know as to how its brand of products is rated among other similar competing brands. While assessing the brand image of any product, the respondents may rate it on the basis of its overall image or on the basis of certain attributes. Thus, besides knowing the relative positioning of the products, one may like to know the strength of the product in comparison to other similar products on different dimensions. The MDS can be used to solve varieties of problems in management research. For example, it finds application in market segmentation, product life cycle, vendor evaluation, and advertising media selection.

Though it is possible to use MDS with quantitative variables (i.e., on the basis of price, aesthetics, color, size, shape, weight, etc.), but it is mostly used to compare objects in a situation where the bases of comparison are not known. This approach of the MDS is a philosophical perspective because every person experiences the

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world in their own way. From this perspective, MDS procedure based on the predefined attributes is not completely satisfactory as it fails to take the individual experience into account. One way to overcome this problem is to look at the constructs an individual use to construe the world. Since the MDS is often used to identify key dimensions underlying customer evaluations of products, services, or companies, therefore once the data is at hand, multidimensional scaling can help determine the following:

- While evaluating the objects, what dimensions are used by the respondents?
- The relative importance of each dimension.
- How the objects are placed in the perceptual map.

Thus, by using the multidimensional scaling methods, one can analyze their current level of consumer satisfaction in the market and modify the marketing mix based upon the current consumer preference and satisfaction.

## What Is Multidimensional Scaling

Multidimensional scaling is a sequence of techniques for exploring similarities or preferences among objects. These objects can be products, organizations, brands, outlets, etc. In this technique, similarities or preferences of objects are measured on some dimensions, and accordingly the objects are positioned in the multidimensional space for understanding the brand positioning. Through multidimensional technique, a researcher can get an idea about the respondent's perceived relative image of a set of objects. The multidimensional scaling is also known as perceptual mapping. In this technique, we transform consumer judgments of overall similarity or preferences into distances represented in multidimensional space.

## Terminologies Used in Multidimensional Scaling

### *Objects and Subjects*

In multidimensional scaling, the object refers to the products, organizations, opinions, or other choices to be compared and positioned in multidimensional space. The objects are also known as variables or stimuli. On the other hand, the subject refers to the respondents who rate the objects in multidimensional scaling. The subjects are the one who are picked up in the sample for conducting the research study. Sometimes the subjects are termed as the "source," and the objects are termed as "target."

## *Distances*

Distance refers to the difference in the two objects on any one or more dimension as perceived by a respondent. It is the fundamental measurement concept in MDS. Distance may also be referred as similarity, preferences, dissimilarity, or proximity. There exist many alternative distance measures, but all are functions of dissimilarity/similarity or preference judgments.

## *Similarity vs. Dissimilarity Matrices*

If the cells of matrix represent the degree of similarity between pairs represented by the rows and columns of the matrix, then the matrix is said to be similarity matrix. On the other hand, if cells of the matrix represent the extent to which one object is preferred over other in the pair, then the matrix is said to be dissimilarity matrix. Larger cell values represent greater distance. The algorithm used by SPSS in multidimensional scaling is more efficient with dissimilarity/preference measures than with similarity/proximity measures. For this reason, distance matrices are used in SPSS instead of similarity matrices.

## *Stress*

Stress (phi) is a goodness-of-fit test that measures the efficiency of the MDS models. The smaller the stress, the better is the fit. Stress measures the difference between interpoint distances in computed MDS space and the corresponding actual input distances. High stress indicates measurement error, and also it may reflect having too few dimensions. Stress is not much affected by sample size provided the number of objects is appreciably more than the number of dimensions.

## *Perceptual Mapping*

Perceptual mapping is a graphical representation of objects in multidimensional space. In perceptual map, points are shown for both, that is, column as well as row objects. In obtaining the perceptual map, the consumer's views about a product are plotted on a chart. Respondents are asked to give their preferences by showing each of the pair of the objects by asking about their experience with the product in terms of its performance, packaging, price, size, etc. These qualitative responses are shown on a chart (called a perceptual map) using a suitable scale (such as the Likert scale). The results of the perceptual mapping are used in improving the product or developing a new product.

### *Dimensions*

While preparing dissimilarity matrix, the respondent may be asked to rate the two objects/products on a particular characteristics such as color, look, energy efficiency, and cost. These characteristics are said to be the dimension on which the evaluation may take place. Usually the products are rated on two or more than two dimensions. These dimensions may be predefined or may be perceived by the respondents of their own.

## What We Do in Multidimensional Scaling?

The multidimensional scaling technique can be applied by either using dissimilarity-based approach or attribute-based approach. The methodologies adopted in these two approaches shall be discussed in detail below. However, solved example shall be discussed only for dissimilarity-based approach of multidimensional scaling. The detail working of this approach with SPSS has been shown in Example 14.1.

### *Procedure of Dissimilarity-Based Approach of Multidimensional Scaling*

The dissimilarity-based approach is very simple to understand and is very useful in understanding the consumer behavior. In this approach, the respondents are asked to rate different pairs of comparable objects on the basis of their experience. While evaluating the pair of objects, the dissimilarity measure is noted on the basis of some of the parameters that the respondents have in their mind. No predefined attributes or objective criteria are given on the basis of which the respondent can evaluate the two objects in the pair. Following steps are adopted in this approach:

#### Steps in Dissimilarity-Based Approach

1. Find the distance matrix among all the objects. It can be obtained by simply ranking of distances between an object and all other objects by a consumer. This matrix can be obtained by providing the consumer a card containing pair of objects written on it, and the candidate needs to specify a number indicating the difference between the two objects on any numerical scale which can represent distance between the two objects. This process is repeated for all pairs of brands being included in the study. In this process, no attributes are identified on which the consumer is asked to decide on the difference. The distance measure so

**Table 14.1** Matrix of dissimilarity scores

| | Alto | Estilo | Wagon R | Swift | Santro | I-1o | Ford Figo | Tata Indica |
|---|---|---|---|---|---|---|---|---|
| Alto | 0 | 1 | 3 | 7 | 4 | 2 | 4 | 1 |
| Estilo | 1 | 0 | 4 | 5 | 6 | 1 | 1 | 5 |
| Wagon R | 3 | 4 | 0 | 2 | 1 | 5 | 6 | 7 |
| Swift | 7 | 5 | 2 | 0 | 1 | 5 | 7 | 6 |
| Santro | 4 | 6 | 1 | 1 | 0 | 4 | 5 | 4 |
| I-10 | 2 | 1 | 5 | 5 | 4 | 0 | 1 | 6 |
| Ford Figo | 4 | 1 | 6 | 7 | 5 | 1 | 0 | 3 |
| Tata Indica | 1 | 5 | 7 | 6 | 4 | 6 | 3 | 0 |

obtained for all the pair of objects can be compiled into a matrix as shown in Table 14.1. This distance matrix serves the input data for the multidimensional scaling.

2. After obtaining the distance matrix for each consumer, take the average of these distances for each pair of objects to make the final distance matrix which is normally used as an input data. However, multidimensional scaling can be used for a single user as well.
3. Compute the value of "stress" for the solution in each dimension. Since the value of stress represents a measure of lack of fit, therefore the intension is to get the solution with an acceptably low value of a stress.
4. On the basis of the least value of the stress obtained in different solutions, obtained in step 3, the number of dimensions is decided.
5. After deciding the number of dimensions, the objects are plotted on a map for visual assessment of objects positioning.
6. Name these dimensions by keeping in mind the attributes of the brands like cost, features, and look. The procedure would be clear by looking to the solved Example 14.1.

## *Procedure of Attribute-Based Approach of Multidimensional Scaling*

In attribute-based approach, the respondents are required to assess each pair of objects on the basis of the predefined criteria (i.e., color, weight, look, features, cost, etc.). In this method, perceptual map of the objects is developed using discriminant analysis. This perceptual map can be developed using the factor analysis as well. However, there is a debate as to which method produces better perceptual maps. In this chapter, we shall discuss only discriminant analysis method for developing perceptual map. In Chap. 12, we have discussed the procedure of discriminant analysis in detail for categorizing the customer into two groups (issuing/not issuing the credit cards). In MDS, we may have as many groups as there are objects/brands. Thus, in this case, mostly we will get more than one

discriminant function. For example, in case of three objects/brands, you could get two functions, and with four objects, you may get up to three discriminant functions. The solution of discriminant analysis gives the value of eigenvalue for each discriminant function. This eigenvalue explains the amount of variance that is explained by the discriminant function. This percentage variance explained by the discriminant function is used to decide as to how many discriminant functions one should use. If two discriminant functions are used, then they form two axes of the perceptual map. Whereas if three discriminant functions are used, then you get three perceptual maps, that is, function 1 vs. function 2, function 1 vs. function 3, and function 2 vs. function 3. These discriminant functions represent the axes on which the objects are first located and thereafter the attributes are located.

To find the number of dimensions and the perceptual map of different objects, following steps are used:

1. Obtain consumers' perceptions on different attributes on the different competing brands. This serves as the input data for the discriminant analysis.
2. Run the discriminant analysis by taking all the independent variables together in the model. The option for this method can be seen in SPSS as "Enter independents together."
3. The SPSS output shall generate the following results:

   (a) Group statistics including mean and standard deviation
   (b) Unstandardized canonical discriminant function coefficients table
   (c) Eigen values and canonical correlation
   (d) Wilks' lambda and chi-square test
   (e) Classification matrix
   (f) Standardized canonical discriminant function coefficients
   (g) Functions at group centroids

   **Remark**: For generating the above-mentioned outputs for MDS, you can refer back the solved Example 12.1 in Chap. 12.

4. The eigenvalue would decide as to how many discriminant function you want to use.
5. Draw perceptual map (or maps) separately by using the standardized canonical discriminant coefficients. This can be done by using Excel or any other graphic package. The discriminant function denotes the axes on which the objects/brands are first located, and then attributes are placed on the same graph.

## *Assumptions in Multidimensional Scaling*

Following assumptions are made while performing the multidimensional scaling:

1. All respondents will rate the objects on the same dimensions.
2. Dimensions are orthogonal.

3. The respondents have the same perception about the dimensionality in assessing the distances among the objects.
4. Respondents will attach the same level of importance to a dimension, even if all respondents perceive this dimension.
5. There is no change in the judgments of a stimulus in terms of either dimensions or levels of importance over time.

### *Limitations of Multidimensional Scaling*

Although MDS is widely used for positioning the brand image and comparing the product characteristics, it has some limitations as well.

1. It is difficult to obtain the similarity and preferences of the respondents toward a group of objects because perceptions of the subjects may differ considerably.
2. Because every product has lots of variant model having different characteristics and therefore the group of objects taken for comparing their brand image may itself differ on many counts. Due to this fact, true positioning may not be possible.
3. Preferences change over time, place, and socioeconomic status and therefore brand positioning obtained in a particular study may not be generalized.
4. The bias exists in the data collection.
5. In case of nonmetric data, all the MDS techniques are subject to the problem of local optima and degenerate solutions.
6. Although metric MDS are more robust than nonmetric MDS and produce good maps but the dimension interpretation, the main work of MDS is highly subjective and depends upon the questioning of the interviewers.

## Solved Example of Multidimensional Scaling (Dissimilarity-Based Approach of Multidimensional Scaling) Using SPSS

**Example 14.1** Twenty customers were asked to rate 8 cars by showing the cards bearing the name of a pair of cars. All possible pair of cars were shown, and the customers were asked to rate their preferences of one car over other on an 8-point scale. If the customer perceived that the two cars were completely dissimilar, a score of 8 was given, and if the two cars were exactly similar, a score of 0 was given. Following dissimilarity scores were obtained and are shown in Table 14.1. Use multidimensional scaling to find the number of dimensions the consumers use in assessing different brands and name these dimensions. Develop perceptual map and position these eight brands of cars in a multidimensional space.

*Solution* In order to find the number of dimensions used by the consumers in assessing these eight brands of car, the multidimensional scaling option of SPSS shall be used to generate outputs showing stress value for the solutions of different dimensions. Simultaneously dimensions for stimulus coordinates in different solutions shall also be obtained which shall be used to place all the eight brands of cars in the multidimensional map.

## *SPSS Commands for Multidimensional Scaling*

The data file needs to be prepared before using SPSS commands to generate outputs in multidimensional scaling. Following steps would be performed to get the relevant outputs for further interpretation in the analysis.

(i) *Data file*: Here, eight variables need to be defined. All these variables shall be defined as ordinal as the scores are the dissimilarity ratings. After preparing the data file by defining variable names and their labels, it will look like Fig. 14.1.

(ii) *Initiating command for multidimensional analysis*: After preparing the data file, click the following command sequence in the Data View:
**Analyze → Scale → Multidimensional Scaling (ALSCAL)**
The screen shall look like Fig. 14.2.

(iii) *Selecting variables for discriminant analysis*: After clicking the **Multidimensional Scaling** option, the SPSS will take you to the window where variables are selected.

- Select all the variables from left panel to the "Variables" section of the right panel.
- Click the tag **Model** in the screen shown in Fig. 14.3.
  - Write minimum and maximum dimension for which the solution is required. Since, in this problem, there are eight brands, hence maximum of up to three-dimensional solution shall be obtained. In case of more number of brands, solutions of more dimensions may be investigated.
  - Let other options are checked by default.
  - Click ***Continue***.
- Click the tag **Option** in the screen as shown in screen 14.3.
  - Check the option "Group plots" in the Display section.
  - Let other options are checked by default.
  - Click ***Continue.***

  The screen for these options shall look like Fig. 14.4.
  - Click **OK** for output.

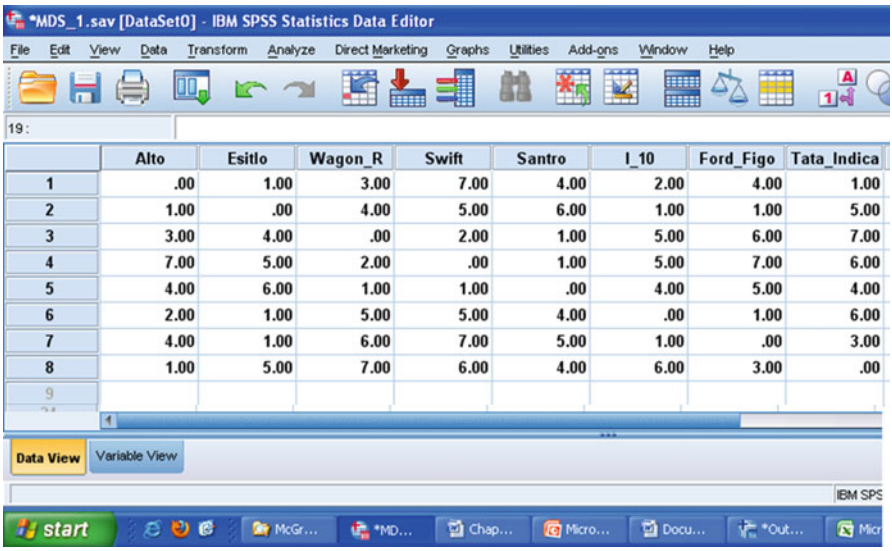



**Fig. 14.1** Screen showing data file for the multidimensional scaling in SPSS

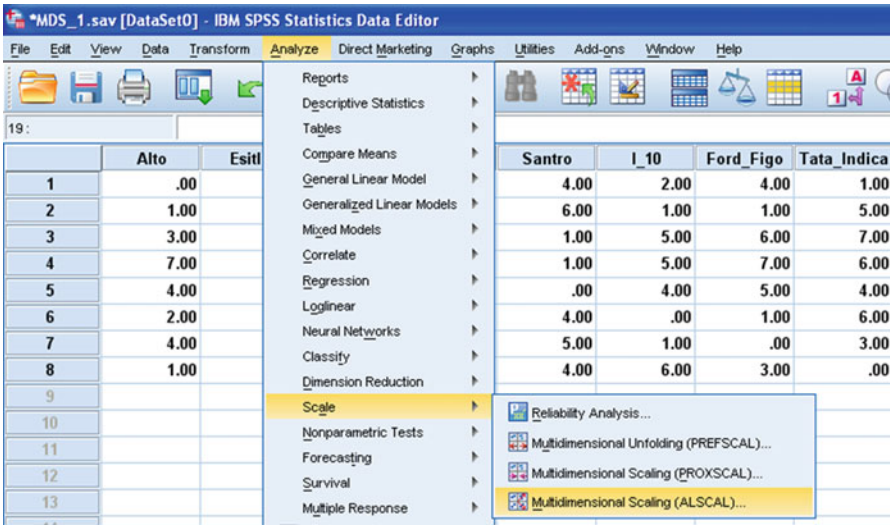



**Fig. 14.2** Screen showing SPSS commands for multidimensional scaling

(iv) *Getting the output*: After clicking the **OK** option in Fig. 14.3, the output in the multidimensional scaling shall be generated in the output window. Selected outputs can be copied in the word file by using the right click of the mouse over identified area of the output. Out of many outputs generated by the SPSS, the following relevant outputs have been picked up for discussion:

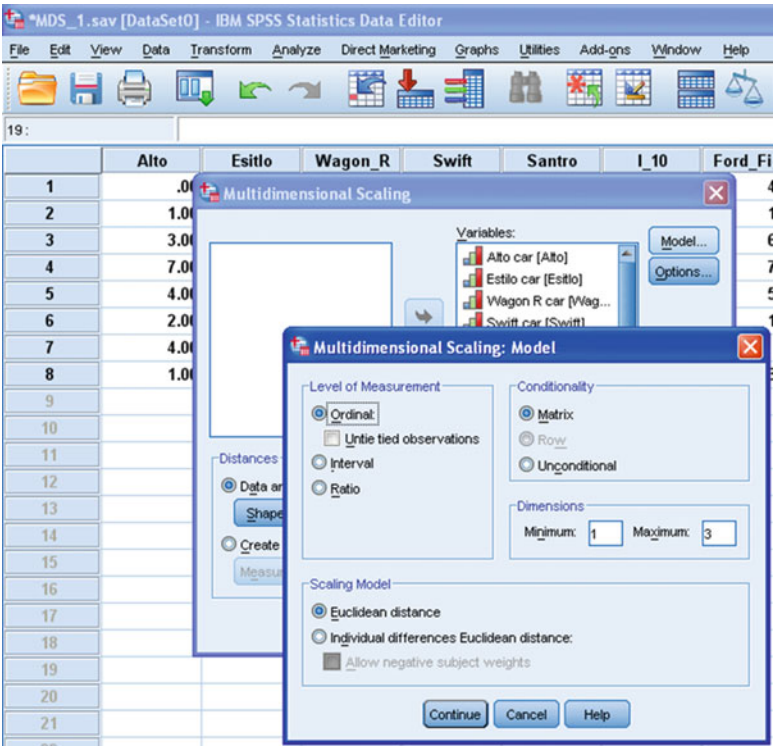



**Fig. 14.3** Screen showing selection of variables and dimensions

1. Iteration details for the three-dimensional solution, stress value of the matrix, and stimulus coordinates (Tables 14.2 and 14.3)
2. Iteration details for the two-dimensional solution, stress value of the matrix, and stimulus coordinates (Tables 14.4 and 14.5)
3. Iteration details for the one-dimensional solution, stress value of the matrix, and stimulus coordinates (Tables 14.6 and 14.7)
4. Perceptual map of all the eight brands (Fig. 14.5)

These outputs so generated by the SPSS are shown in Tables 14.2, 14.3, 14.4, 14.5, 14.6, and 14.7 and Fig. 14.5.

## *Interpretation of Various Outputs Generated in Multidimensional Scaling*

From these outputs, it is required to determine the number of dimensions in which you feel the best solution exists. This decision is based upon the stress value for the solutions in different dimensions. Tables 14.2 and 14.3 show the three-dimensional

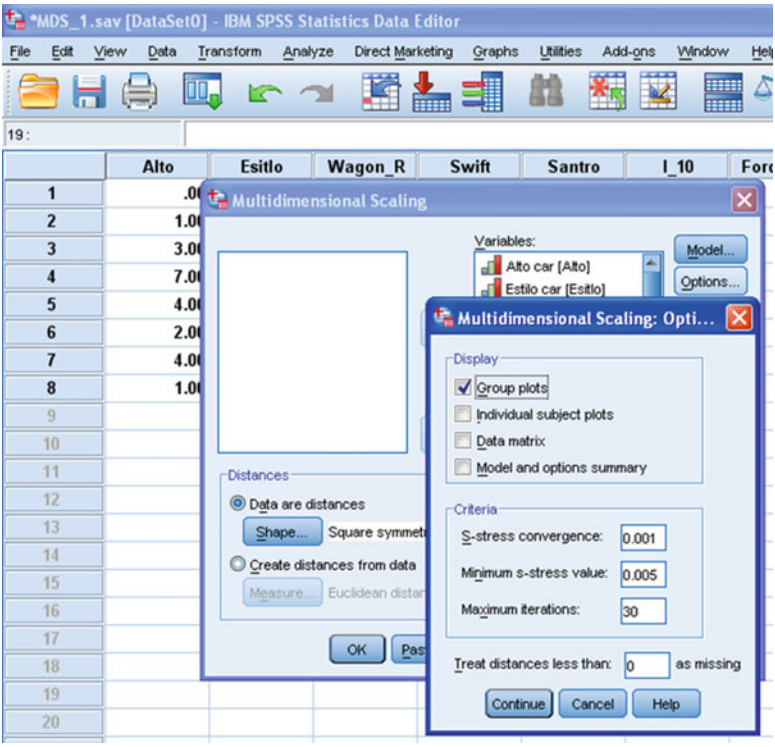



**Fig. 14.4** Screen showing the options for perceptual mapping

solution, and the stress value for these solutions is 0.07911. Tables 14.4 and 14.5 contain two-dimensional solutions along with the stress value as 0.16611. On the other hand, the one-dimensional solutions are shown in Tables 14.6 and 14.7 along with the stress value 0.42024.

Stress value shows the lack of fit, and therefore, it should be as close to zero as possible. Owing to these criteria, the one-dimensional solution is not good at all as this contains the maximum value of stress (0.42024). The two-dimensional solution looks better as it is close to zero, but the three-dimensional solution is the best because its stress value is the least.

Since in this problem there are only eight brands, therefore it is not possible to get a solution in more than three dimensions. If you have more than 14 or 15 brands, you may try some higher dimension solution. To find out the optimum solution, one needs to have the trade-off between stress value and the number of dimensions.

### Three-Dimensional Solution

Based on the stress value, the three-dimensional solution is the best as in that case the stress value is the least and closest to zero. Therefore, the next task is to define

the names of these three dimensions. These dimensions are the attributes of these brands drawn either through our experience or knowledge of the market through a survey of the customers or a combination of these methods. Thus, the three dimensions may be named as follows:

Dimension 1: Spacious
Dimension 2: Fuel economy
Dimension 3: Stylish

By looking to the scores on the three dimensions in Table 14.3, it may be concluded that the brands like Wagon R, Swift, and Santro are spacious than other brands of similar cars. Brands like Tata Indica and Alto are fuel economical cars, whereas the brands like Ford Figo and Swift are more stylish cars.

**Table 14.2** Iteration details for the three-dimensional solution Young's S-stress formula 1 is used

| Iteration | S-stress | Improvement |
|---|---|---|
| 1 | .14535 | |
| 2 | .12004 | .02531 |
| 3 | .11372 | .00632 |
| 4 | .11188 | .00184 |
| 5 | .11126 | .00062 |

Iterations stopped because
S-stress improvement is <.001000
Stress and squared correlation (RSQ) in distances RSQ values are the proportion of variance of the scaled data (disparities) in the partition (row, matrix, or entire data) which is accounted for by their corresponding distances. Stress values are Kruskal's stress formula 1.
For matrix,
**Stress = .07911 RSQ = .92211**
Configuration derived in three dimensions

**Table 14.3** Stimulus coordinates

| | | Dimension | | |
|---|---|---|---|---|
| Stimulus number | Stimulus name | 1 | 2 | 3 |
| 1 | Alto | .8774 | .6086 | −.9932 |
| 2 | Estilo | .9917 | −1.0586 | −.4867 |
| 3 | Wagon_R | −1.3459 | −.1183 | −1.2193 |
| 4 | Swift | −1.8536 | −.0029 | .8010 |
| 5 | Santro | −1.4590 | .6055 | .2352 |
| 6 | I_10 | .6751 | −1.3468 | .4928 |
| 7 | Ford_Figo | 1.2702 | −.5423 | .9944 |
| 8 | Tata_Indica | .8441 | 1.8548 | .1759 |

### Two-Dimensional Solution

For the sake of understanding, the perceptual map shall be discussed for two-dimensional solutions. If two-dimensional solutions would have been preferred instead of three-dimensional solutions, then the perceptual map would be shown by Fig. 14.5. Looking to this figure, the brands like Swift, Santro, and Wagon R are perceived to be similar (spacious). Similarly the brands like Tata Indica and Alto are perceived to be similar (fuel economy). In this case, we are losing information on the third dimension which was "stylishness" in the three-dimensional solution. This loss of information may be critical in some cases. It is therefore advisable to analyze the data from a three-dimensional solution instead of a two-dimensional, provided stress value warrants so.

**Table 14.4** Iteration details for the two-dimensional solution Young's S-stress formula 1 is used

| Iteration | S-stress | Improvement |
|---|---|---|
| 1 | .22053 | |
| 2 | .19234 | .02820 |
| 3 | .17623 | .01611 |
| 4 | .16411 | .01211 |
| 5 | .15461 | .00950 |
| 6 | .14791 | .00670 |
| 7 | .14367 | .00424 |
| 8 | .14159 | .00208 |
| 9 | .14139 | .00020 |

Iterations stopped because
S-stress improvement is <.001000
Stress and squared correlation (RSQ) in distances RSQ values are the proportion of variance of the scaled data (disparities) in the partition (row, matrix, or entire data) which is accounted for by their corresponding distances. Stress values are Kruskal's stress formula 1.
For matrix,
**Stress = .16611 RSQ = .87594**
Configuration derived in two dimensions

**Table 14.5** Stimulus coordinates

| | | Dimension | |
|---|---|---|---|
| Stimulus number | Stimulus name | 1 | 2 |
| 1 | Alto | 1.0933 | .7542 |
| 2 | Estilo | .9651 | −.7312 |
| 3 | Wagon_R | −1.4408 | .1261 |
| 4 | Swift | −1.4492 | .2257 |
| 5 | Santro | −1.4133 | .2052 |
| 6 | I_10 | .9121 | −.8085 |
| 7 | Ford_Figo | .6121 | −1.3142 |
| 8 | Tata_Indica | .7206 | 1.5429 |

**Table 14.6** Iteration details for the one-dimensional solution Young's S-stress formula 1 is used

| Iteration | S-stress | Improvement |
|---|---|---|
| 1 | .44444 | |
| 2 | .43243 | .01201 |
| 3 | .43185 | .00057 |

Iterations stopped because
S-stress improvement is <.001000
Stress and squared correlation (RSQ) in distances RSQ values are the proportion of variance of the scaled data (disparities) in the partition (row, matrix, or entire data) which is accounted for by their corresponding distances. Stress values are Kruskal's stress formula 1.
For matrix,
**Stress = .42024 RSQ = .57334**
Configuration derived in one dimension

**Table 14.7** Stimulus coordinates

| | | Dimension |
|---|---|---|
| Stimulus number | Stimulus name | 1 |
| 1 | Alto | −.8034 |
| 2 | Estilo | −.7738 |
| 3 | Wagon_R | 1.2367 |
| 4 | Swift | 1.4484 |
| 5 | Santro | 1.1694 |
| 6 | I_10 | −.6166 |
| 7 | Ford_Figo | −.8593 |
| 8 | Tata_Indica | −.8015 |

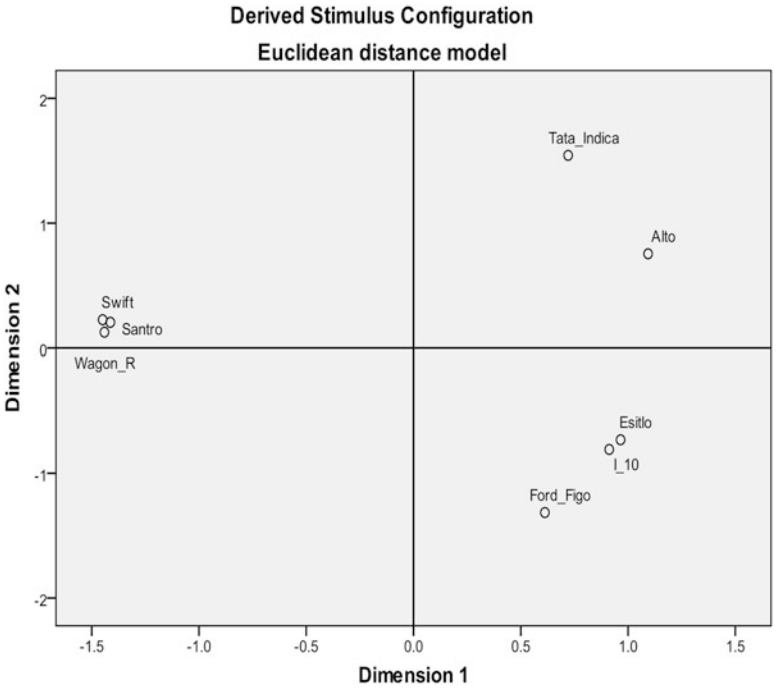



**Fig. 14.5** Perceptual map of different brands of car (two-dimensional output)

## Summary of the SPSS Commands for Multidimensional Scaling

(i) Start SPSS and prepare the data file by defining the variables and their properties in **Variable View** and typing the data column wise in Data View.
(ii) In the data view, follow the below-mentioned command sequence for multidimensional scaling:
**Analyze → Scale → Multidimensional Scaling (ALSCAL)**
(iii) Select all the variables from left panel to the "Variables" section of the right panel.
(iv) Click the tag **Model** and write minimum and maximum dimension for which the solution is required. Let other options are checked by default. Click ***Continue***.
(v) Click the tag **Option** in the screen and check the option "Group plots" in the Display section. Let other options are checked by default. Click ***Continue.*** Click **OK** for output.

## Exercise

*Short Answer Questions*

**Note**: Write answer to each of the following questions in not more than 200 words.

1. Define multidimensional scaling and explain a situation in marketing where this technique can be used.
2. Discuss the procedure used in dissimilarity-based approach of multidimensional scaling.
3. What are the steps used in attribute-based approach of multidimensional scaling?
4. What are the drawbacks of multidimensional scaling?
5. Explain the assumptions used in multidimensional scaling.
6. Describe any five terminologies used in multidimensional scaling.
7. What do you mean by stress score? What is its significance and how is it used in deciding the solution in multidimensional scaling?
8. What are the various considerations in deciding the name of the dimensions?
9. What do you mean by a perceptual map? Explain by means of an example.
10. Explain the difference in attribute-based approach and dissimilarity-based approach of multidimensional scaling.

*Multiple-Choice Questions*

**Note**: Question no. 1–10 has four alternative answers for each question. Tick mark the one that you consider the closest to the correct answer.

1. MDS refers to

   (a) Multidimensional spaces
   (b) Multidirectional spaces
   (c) Multidimensional perceptual scaling
   (d) Multidimensional scaling

2. Stress is a measure of

   (a) Distance between the two brands
   (b) Goodness of fit
   (c) Correctness of the perceptual map
   (d) Error involved in deciding the nomenclature of dimensions

3. Perceptual mapping is a

   (a) Graphical representation of the dimensions in multidimensional space
   (b) Graphical representation of objects in multidimensional space
   (c) Graphical representation of the distances of the objects
   (d) Graphical representation of brands in two-dimensional space

4. Dimensions refer to

   (a) The brands on which clustering is made
   (b) The characteristics of the brands which are clubbed for assessment
   (c) The brands which have some attributes common in them
   (d) The characteristics on which the evaluation may take place

5. In dissimilarity-based approach of multidimensional scaling, the input data are

   (a) Nominal
   (b) Ordinal
   (c) Scale
   (d) Ratio

6. The solution of multidimensional is accurate if the value of stress is

   (a) Less than 1
   (b) More than 1
   (c) Closer to 0
   (d) Closer to 0.5

7. In attribute-based approach of multidimensional scaling, the input data can be

   (a) Interval
   (b) Nominal
   (c) Ordinal
   (d) None of the above

8. One of the assumptions in multidimensional scaling is

   (a) The respondents will not rate the objects on the same dimensions.
   (b) Dimensions are orthogonal.

(c) Respondents will not attach the same level of importance to a dimension, even if all respondents perceive this dimension.
(d) Data are nominal.

9. Choose the correct sequence of commands in SPSS for multidimensional scaling.

(a) Analyze → Scale → Multidimensional Scaling (ALSCAL)
(b) Analyze → Multidimensional Scaling (ALSCAL) → Scale
(c) Analyze → Scale → Multidimensional Scaling (PROXSCAL)
(d) Analyze → Multidimensional Scaling (PROXSCAL) → Scale

10. Following solutions are obtained in the multidimensional scaling:

(i) One-Dimensional Solution with Stress score = 0.7659
(ii) Two-Dimensional Solution with Stress score = 0.4328
(iii) Three-Dimensional Solution with Stress score = 0.1348
(iv) Four-Dimensional Solution with Stress score = 0.0924

Which solution would you prefer?

(a) ii
(b) i
(c) iv
(d) iii

*Assignments*

1. A refrigerator company wanted to draw a perceptual map using its consumers' perceptions regarding its own brand and five competing brands. These six brands were Samsung, LG, Videocon, Godrej, Sharp, and Hitachi. The customers were shown a card containing a pair of names of these brands and were asked to rate in terms of dissimilarity between the two on an 8-point rating scale. The rating of 8 indicates that the two brands are distinctively apart, whereas 1 indicates that the two brands are exactly similar as perceived by the customers. This exercise was done on all the pair of brands. The average dissimilarity ratings obtained by all the customers are shown in the following table. Apply the multidimensional scaling and interpret your findings by plotting the perceptual map of these brands.

Dissimilarity ratings obtained by the customers on the six brands of the refrigerators

| | Samsung | LG | Videocon | Godrej | Sharp | Hitachi |
|---|---|---|---|---|---|---|
| Samsung | 0 | 4 | 3 | 7 | 4 | 3 |
| LG | | 0 | 3 | 8 | 3 | 2 |
| Videocon | | | 0 | 7 | 3 | 5 |
| Godrej | | | | 0 | 6 | 8 |
| Sharp | | | | | 0 | 4 |
| Hitachi | | | | | | 0 |

2. The authorities in a university wanted to assess its teachers as perceived by their students on a seven-point scale by drawing the perceptual map. Six teachers,

Smith, Anderson, Clark, Wright, Mitchell and Johnson were rated by 25 students. Score 7 indicated that the two teachers were distinctively apart, whereas the score 1 represented that they were exactly similar as perceived by the students. Following is the dissimilarity matrix obtained on the basis of the average dissimilarity scores obtained on all the 25 students. By using the multidimensional scaling technique, draw the perceptual map.

Dissimilarity ratings obtained by the students on the teachers in the college

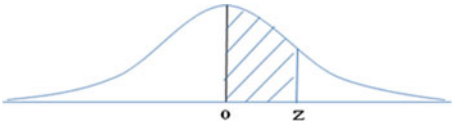
| | Smith | Anderson | Clark | Wright | Mitchell | Johnson |
|---|---|---|---|---|---|---|
| Smith | 0 | 4 | 3 | 1 | 5 | 6 |
| Anderson | | 3 | 2 | 5 | 3 | 2 |
| Clark | | | 0 | 4 | 3 | 4 |
| Wright | | | | 0 | 6 | 5 |
| Mitchell | | | | | 0 | 4 |
| Johnson | | | | | | 0 |

*Answers of Multiple-Choice Questions*

| | | | |
|---|---|---|---|
| Q.1 d | Q.2 b | Q.3 b | Q.4 d |
| Q.5 b | Q.6 c | Q.7 a | Q.8 b |
| Q.9 a | Q.10 d | | |

# Appendix: Tables

**Table A.1** The normal curve area between the mean and a given $z$ value




| $Z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

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**Table A.2** Critical values of '*t*'

| Level of significance (α) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| one-tail | 0.50 | 0.25 | 0.20 | 0.15 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 |
| two-tail df | 1.00 | 0.50 | 0.40 | 0.30 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.002 | 0.001 |
| 1 | 0.000 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 318.31 | 636.62 |
| 2 | 0.000 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| 3 | 0.000 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | 0.000 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 0.000 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | 0.000 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 0.000 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | 0.000 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 0.000 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | 0.000 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | 0.000 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 0.000 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | 0.000 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| 14 | 0.000 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | 0.000 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | 0.000 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | 0.000 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 | 0.000 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| 19 | 0.000 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | 0.000 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| 21 | 0.000 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | 0.000 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| 23 | 0.000 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| 24 | 0.000 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| 25 | 0.000 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| 26 | 0.000 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| 27 | 0.000 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| 28 | 0.000 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| 29 | 0.000 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| 30 | 0.000 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| 40 | 0.000 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 60 | 0.000 | 0.679 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 80 | 0.000 | 0.678 | 0.846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| 100 | 0.000 | 0.677 | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| 1,000 | 0.000 | 0.675 | 0.842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.330 | 2.581 | 3.098 | 3.300 |
| ∞ | 0.000 | 0.674 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |

**Table A.3** Critical values of the correlation coefficient

| Level of significance for two-tailed test | | | | |
|---|---|---|---|---|
| df ($n-2$) | 0.10 | 0.05 | 0.02 | 0.01 |
| 1 | 0.988 | 0.997 | 0.9995 | 0.9999 |
| 2 | 0.900 | 0.950 | 0.980 | 0.990 |
| 3 | 0.805 | 0.878 | 0.934 | 0.959 |
| 4 | 0.729 | 0.811 | 0.882 | 0.917 |
| 5 | 0.669 | 0.754 | 0.833 | 0.874 |
| 6 | 0.622 | 0.707 | 0.789 | 0.834 |
| 7 | 0.582 | 0.666 | 0.750 | 0.798 |
| 8 | 0.549 | 0.632 | 0.716 | 0.765 |
| 9 | 0.521 | 0.602 | 0.685 | 0.735 |
| 10 | 0.497 | 0.576 | 0.658 | 0.708 |
| 11 | 0.476 | 0.553 | 0.634 | 0.684 |
| 12 | 0.458 | 0.532 | 0.612 | 0.661 |
| 13 | 0.441 | 0.514 | 0.592 | 0.641 |
| 14 | 0.426 | 0.497 | 0.574 | 0.623 |
| 15 | 0.412 | 0.482 | 0.558 | 0.606 |
| 16 | 0.400 | 0.468 | 0.542 | 0.590 |
| 17 | 0.389 | 0.456 | 0.528 | 0.575 |
| 18 | 0.378 | 0.444 | 0.516 | 0.561 |
| 19 | 0.369 | 0.433 | 0.503 | 0.549 |
| 20 | 0.360 | 0.423 | 0.492 | 0.537 |
| 21 | 0.352 | 0.413 | 0.482 | 0.526 |
| 22 | 0.344 | 0.404 | 0.472 | 0.515 |
| 23 | 0.337 | 0.396 | 0.462 | 0.505 |
| 24 | 0.330 | 0.388 | 0.453 | 0.496 |
| 25 | 0.323 | 0.381 | 0.445 | 0.487 |
| 26 | 0.317 | 0.374 | 0.437 | 0.479 |
| 27 | 0.311 | 0.367 | 0.430 | 0.471 |
| 28 | 0.306 | 0.361 | 0.423 | 0.463 |
| 29 | 0.301 | 0.355 | 0.416 | 0.456 |
| 30 | 0.296 | 0.349 | 0.409 | 0.449 |
| 35 | 0.275 | 0.325 | 0.381 | 0.418 |
| 40 | 0.257 | 0.304 | 0.358 | 0.393 |
| 45 | 0.243 | 0.288 | 0.338 | 0.372 |
| 50 | 0.231 | 0.273 | 0.322 | 0.354 |
| 60 | 0.211 | 0.250 | 0.295 | 0.325 |
| 70 | 0.195 | 0.232 | 0.274 | 0.303 |
| 80 | 0.183 | 0.217 | 0.256 | 0.283 |
| 90 | 0.173 | 0.205 | 0.242 | 0.267 |
| 100 | 0.164 | 0.195 | 0.230 | 0.254 |
| df ($n-2$) | 0.05 | 0.25 | 0.01 | 0.005 |
| Level of significance for one tailed test | | | | |

**Table A.4** *F*-table: critical values at .05 level of significance

| $n_1/n_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.76 | 8.74 | 8.73 | 8.71 | 8.70 | 8.69 | 8.68 | 8.67 | 8.67 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.94 | 5.91 | 5.89 | 5.87 | 5.86 | 5.84 | 5.83 | 5.82 | 5.81 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.70 | 4.68 | 4.66 | 4.64 | 4.62 | 4.60 | 4.59 | 4.58 | 4.57 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.03 | 4.00 | 3.98 | 3.96 | 3.94 | 3.92 | 3.91 | 3.90 | 3.88 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.60 | 3.57 | 3.55 | 3.53 | 3.51 | 3.49 | 3.48 | 3.47 | 3.46 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.31 | 3.28 | 3.26 | 3.24 | 3.22 | 3.20 | 3.19 | 3.17 | 3.16 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.10 | 3.07 | 3.05 | 3.03 | 3.01 | 2.99 | 2.97 | 2.96 | 2.95 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 | 2.89 | 2.86 | 2.85 | 2.83 | 2.81 | 2.80 | 2.79 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.82 | 2.79 | 2.76 | 2.74 | 2.72 | 2.70 | 2.69 | 2.67 | 2.66 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.72 | 2.69 | 2.66 | 2.64 | 2.62 | 2.60 | 2.58 | 2.57 | 2.56 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.63 | 2.60 | 2.58 | 2.55 | 2.53 | 2.51 | 2.50 | 2.48 | 2.47 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.57 | 2.53 | 2.51 | 2.48 | 2.46 | 2.44 | 2.43 | 2.41 | 2.40 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.51 | 2.48 | 2.45 | 2.42 | 2.40 | 2.38 | 2.37 | 2.35 | 2.34 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.46 | 2.42 | 2.40 | 2.37 | 2.35 | 2.33 | 2.32 | 2.30 | 2.29 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.41 | 2.38 | 2.35 | 2.33 | 2.31 | 2.29 | 2.27 | 2.26 | 2.24 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.37 | 2.34 | 2.31 | 2.29 | 2.27 | 2.25 | 2.23 | 2.22 | 2.20 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.34 | 2.31 | 2.28 | 2.26 | 2.23 | 2.21 | 2.20 | 2.18 | 2.17 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.31 | 2.28 | 2.25 | 2.23 | 2.20 | 2.18 | 2.17 | 2.15 | 2.14 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.26 | 2.23 | 2.20 | 2.17 | 2.15 | 2.13 | 2.11 | 2.10 | 2.08 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.22 | 2.18 | 2.15 | 2.13 | 2.11 | 2.09 | 2.07 | 2.05 | 2.04 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.18 | 2.15 | 2.12 | 2.09 | 2.07 | 2.05 | 2.03 | 2.02 | 2.00 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.15 | 2.12 | 2.09 | 2.06 | 2.04 | 2.02 | 2.00 | 1.99 | 1.97 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.13 | 2.09 | 2.06 | 2.04 | 2.01 | 1.99 | 1.98 | 1.96 | 1.95 |
| 35 | 4.12 | 3.27 | 2.87 | 2.64 | 2.49 | 2.37 | 2.29 | 2.22 | 2.16 | 2.11 | 2.08 | 2.04 | 2.01 | 1.99 | 1.96 | 1.94 | 1.92 | 1.91 | 1.89 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.04 | 2.00 | 1.97 | 1.95 | 1.92 | 1.90 | 1.89 | 1.87 | 1.85 |
| 45 | 4.06 | 3.20 | 2.81 | 2.58 | 2.42 | 2.31 | 2.22 | 2.15 | 2.10 | 2.05 | 2.01 | 1.97 | 1.94 | 1.92 | 1.89 | 1.87 | 1.86 | 1.84 | 1.82 |
| 50 | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.99 | 1.95 | 1.92 | 1.89 | 1.87 | 1.85 | 1.83 | 1.81 | 1.80 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.95 | 1.92 | 1.89 | 1.86 | 1.84 | 1.82 | 1.80 | 1.78 | 1.76 |
| 70 | 3.98 | 3.13 | 2.74 | 2.50 | 2.35 | 2.23 | 2.14 | 2.07 | 2.02 | 1.97 | 1.93 | 1.89 | 1.86 | 1.84 | 1.81 | 1.79 | 1.77 | 1.75 | 1.74 |
| 80 | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 2.00 | 1.95 | 1.91 | 1.88 | 1.84 | 1.82 | 1.79 | 1.77 | 1.75 | 1.73 | 1.72 |
| 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.89 | 1.85 | 1.82 | 1.79 | 1.77 | 1.75 | 1.73 | 1.71 | 1.69 |
| 200 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.84 | 1.80 | 1.77 | 1.74 | 1.72 | 1.69 | 1.67 | 1.66 | 1.64 |
| 500 | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.81 | 1.77 | 1.74 | 1.71 | 1.69 | 1.66 | 1.64 | 1.62 | 1.61 |
| 1,000 | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.80 | 1.76 | 1.73 | 1.70 | 1.68 | 1.65 | 1.63 | 1.61 | 1.60 |
| >1,000 | 1.04 | 3.00 | 2.61 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.79 | 1.75 | 1.72 | 1.69 | 1.67 | 1.64 | 1.62 | 1.61 | 1.59 |
| $n_1/n_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |

| 20 | 22 | 24 | 26 | 28 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 | 100 | 200 | 500 | 1,000 | >1,000 | $n_1/n_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8.66 | 8.65 | 8.64 | 8.63 | 8.62 | 8.62 | 8.60 | 8.59 | 8.59 | 8.58 | 8.57 | 8.57 | 8.56 | 8.55 | 8.54 | 8.53 | 8.53 | 8.54 | 3 |
| 5.80 | 5.79 | 5.77 | 5.76 | 5.75 | 5.75 | 5.73 | 5.72 | 5.71 | 5.70 | 5.69 | 5.68 | 5.67 | 5.66 | 5.65 | 5.64 | 5.63 | 5.63 | 4 |
| 4.56 | 4.54 | 4.53 | 4.52 | 4.50 | 4.50 | 4.48 | 4.46 | 4.45 | 4.44 | 4.43 | 4.42 | 4.42 | 4.41 | 4.39 | 4.37 | 4.37 | 4.36 | 5 |
| 3.87 | 3.86 | 3.84 | 3.83 | 3.82 | 3.81 | 3.79 | 3.77 | 3.76 | 3.75 | 3.74 | 3.73 | 3.72 | 3.71 | 3.69 | 3.68 | 3.67 | 3.67 | 6 |
| 3.44 | 3.43 | 3.41 | 3.40 | 3.39 | 3.38 | 3.36 | 3.34 | 3.33 | 3.32 | 3.30 | 3.29 | 3.29 | 3.27 | 3.25 | 3.24 | 3.23 | 3.23 | 7 |
| 3.15 | 3.13 | 3.12 | 3.10 | 3.09 | 3.08 | 3.06 | 3.04 | 3.03 | 3.02 | 3.01 | 2.99 | 2.99 | 2.97 | 2.95 | 2.94 | 2.93 | 2.93 | 8 |
| 2.94 | 2.92 | 2.90 | 2.89 | 2.87 | 2.86 | 2.84 | 2.83 | 2.81 | 2.80 | 2.79 | 2.78 | 2.77 | 2.76 | 2.73 | 2.72 | 2.71 | 2.71 | 9 |
| 2.77 | 2.75 | 2.74 | 2.72 | 2.71 | 2.70 | 2.68 | 2.66 | 2.65 | 2.64 | 2.62 | 2.61 | 2.60 | 2.59 | 2.56 | 2.55 | 2.54 | 2.54 | 10 |
| 2.65 | 2.63 | 2.61 | 2.59 | 2.58 | 2.57 | 2.55 | 2.53 | 2.52 | 2.51 | 2.49 | 2.48 | 2.47 | 2.46 | 2.43 | 2.42 | 2.41 | 2.41 | 11 |
| 2.54 | 2.52 | 2.51 | 2.49 | 2.48 | 2.47 | 2.44 | 2.43 | 2.41 | 2.40 | 2.38 | 2.37 | 2.36 | 2.35 | 2.32 | 2.31 | 2.30 | 2.30 | 12 |
| 2.46 | 2.44 | 2.42 | 2.41 | 2.39 | 2.38 | 2.36 | 2.34 | 2.33 | 2.31 | 2.30 | 2.28 | 2.27 | 2.26 | 2.23 | 2.22 | 2.21 | 2.21 | 13 |
| 2.39 | 2.37 | 2.35 | 2.33 | 2.32 | 2.31 | 2.28 | 2.27 | 2.25 | 2.24 | 2.22 | 2.21 | 2.20 | 2.19 | 2.16 | 2.14 | 2.14 | 2.13 | 14 |
| 2.33 | 2.31 | 2.29 | 2.27 | 2.26 | 2.25 | 2.22 | 2.20 | 2.19 | 2.18 | 2.16 | 2.15 | 2.14 | 2.12 | 2.10 | 2.08 | 2.07 | 2.07 | 15 |
| 2.28 | 2.25 | 2.24 | 2.22 | 2.21 | 2.19 | 2.17 | 2.15 | 2.14 | 2.12 | 2.11 | 2.09 | 2.08 | 2.07 | 2.04 | 2.02 | 2.02 | 2.01 | 16 |
| 2.23 | 2.21 | 2.19 | 2.17 | 2.16 | 2.15 | 2.12 | 2.10 | 2.09 | 2.08 | 2.06 | 2.05 | 2.03 | 2.02 | 1.99 | 1.97 | 1.97 | 1.96 | 17 |
| 2.19 | 2.17 | 2.15 | 2.13 | 2.12 | 2.11 | 2.08 | 2.06 | 2.05 | 2.04 | 2.02 | 2.00 | 1.99 | 1.98 | 1.95 | 1.93 | 1.92 | 1.92 | 18 |
| 2.16 | 2.13 | 2.11 | 2.10 | 2.08 | 2.07 | 2.05 | 2.03 | 2.01 | 2.00 | 1.98 | 1.97 | 1.96 | 1.94 | 1.91 | 1.89 | 1.88 | 1.88 | 19 |
| 2.12 | 2.10 | 2.08 | 2.07 | 2.05 | 2.04 | 2.01 | 1.99 | 1.98 | 1.97 | 1.95 | 1.93 | 1.92 | 1.91 | 1.88 | 1.86 | 1.85 | 1.84 | 20 |
| 2.07 | 2.05 | 2.03 | 2.01 | 2.00 | 1.98 | 1.96 | 1.94 | 1.92 | 1.91 | 1.89 | 1.88 | 1.86 | 1.85 | 1.82 | 1.80 | 1.79 | 1.78 | 22 |
| 2.03 | 2.00 | 1.98 | 1.97 | 1.95 | 1.94 | 1.91 | 1.89 | 1.88 | 1.86 | 1.84 | 1.83 | 1.82 | 1.80 | 1.77 | 1.75 | 1.74 | 1.73 | 24 |
| 1.99 | 1.97 | 1.95 | 1.93 | 1.91 | 1.90 | 1.87 | 1.85 | 1.84 | 1.82 | 1.80 | 1.79 | 1.78 | 1.76 | 1.73 | 1.71 | 1.70 | 1.69 | 26 |
| 1.96 | 1.93 | 1.91 | 1.90 | 1.88 | 1.87 | 1.84 | 1.82 | 1.80 | 1.79 | 1.77 | 1.75 | 1.74 | 1.73 | 1.69 | 1.67 | 1.66 | 1.66 | 28 |
| 1.93 | 1.91 | 1.89 | 1.87 | 1.85 | 1.84 | 1.81 | 1.79 | 1.77 | 1.76 | 1.74 | 1.72 | 1.71 | 1.70 | 1.66 | 1.64 | 1.63 | 1.62 | 30 |
| 1.88 | 1.85 | 1.83 | 1.82 | 1.80 | 1.79 | 1.76 | 1.74 | 1.72 | 1.70 | 1.68 | 1.66 | 1.65 | 1.63 | 1.60 | 1.57 | 1.57 | 1.56 | 35 |
| 1.84 | 1.81 | 1.79 | 1.77 | 1.76 | 1.74 | 1.72 | 1.69 | 1.67 | 1.66 | 1.64 | 1.62 | 1.61 | 1.59 | 1.55 | 1.53 | 1.52 | 1.51 | 40 |
| 1.81 | 1.78 | 1.76 | 1.74 | 1.73 | 1.71 | 1.68 | 1.66 | 1.64 | 1.63 | 1.60 | 1.59 | 1.57 | 1.55 | 1.51 | 1.49 | 1.48 | 1.47 | 45 |
| 1.78 | 1.76 | 1.74 | 1.72 | 1.70 | 1.69 | 1.66 | 1.63 | 1.61 | 1.60 | 1.58 | 1.56 | 1.54 | 1.52 | 1.48 | 1.46 | 1.45 | 1.44 | 50 |
| 1.75 | 1.72 | 1.70 | 1.68 | 1.66 | 1.65 | 1.62 | 1.59 | 1.57 | 1.56 | 1.53 | 1.52 | 1.50 | 1.48 | 1.44 | 1.41 | 1.40 | 1.39 | 60 |
| 1.72 | 1.70 | 1.67 | 1.65 | 1.64 | 1.62 | 1.59 | 1.57 | 1.55 | 1.53 | 1.50 | 1.49 | 1.47 | 1.45 | 1.40 | 1.37 | 1.36 | 1.35 | 70 |
| 1.70 | 1.68 | 1.65 | 1.63 | 1.62 | 1.60 | 1.57 | 1.54 | 1.52 | 1.51 | 1.48 | 1.46 | 1.45 | 1.43 | 1.38 | 1.35 | 1.34 | 1.33 | 80 |
| 1.68 | 1.65 | 1.63 | 1.61 | 1.59 | 1.57 | 1.54 | 1.52 | 1.49 | 1.48 | 1.45 | 1.43 | 1.41 | 1.39 | 1.34 | 1.31 | 1.30 | 1.28 | 100 |
| 1.62 | 1.60 | 1.57 | 1.55 | 1.53 | 1.52 | 1.48 | 1.46 | 1.43 | 1.41 | 1.39 | 1.36 | 1.35 | 1.32 | 1.26 | 1.22 | 1.21 | 1.19 | 200 |
| 1.59 | 1.56 | 1.54 | 1.52 | 1.50 | 1.48 | 1.45 | 1.42 | 1.40 | 1.38 | 1.35 | 1.32 | 1.30 | 1.28 | 1.21 | 1.16 | 1.14 | 1.12 | 500 |
| 1.58 | 1.55 | 1.53 | 1.51 | 1.49 | 1.47 | 1.43 | 1.41 | 1.38 | 1.36 | 1.33 | 1.31 | 1.29 | 1.26 | 1.19 | 1.13 | 1.11 | 1.08 | 1,000 |
| 1.57 | 1.54 | 1.52 | 1.50 | 1.48 | 1.46 | 1.42 | 1.40 | 1.37 | 1.35 | 1.32 | 1.30 | 1.28 | 1.25 | 1.17 | 1.11 | 1.08 | 1.03 | >1,000 |
| 20 | 22 | 24 | 26 | 28 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 | 100 | 200 | 500 | 1,000 | >1,000 | $n_1/n_2$ |

**Table A.5** *F*-table: critical values at .01 level of significance

| $n_1/n_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.13 | 27.05 | 26.98 | 26.92 | 26.87 | 26.83 | 26.79 | 26.75 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.45 | 14.37 | 14.31 | 14.25 | 14.20 | 14.15 | 14.11 | 14.08 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.96 | 9.89 | 9.82 | 9.77 | 9.72 | 9.68 | 9.64 | 9.61 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.79 | 7.72 | 7.66 | 7.61 | 7.56 | 7.52 | 7.48 | 7.45 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.54 | 6.47 | 6.41 | 6.36 | 6.31 | 6.28 | 6.24 | 6.21 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.73 | 5.67 | 5.61 | 5.56 | 5.52 | 5.48 | 5.44 | 5.41 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.18 | 5.11 | 5.05 | 5.01 | 4.96 | 4.92 | 4.89 | 4.86 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.77 | 4.71 | 4.65 | 4.60 | 4.56 | 4.52 | 4.49 | 4.46 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.46 | 4.40 | 4.34 | 4.29 | 4.25 | 4.21 | 4.18 | 4.15 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.22 | 4.16 | 4.10 | 4.05 | 4.01 | 3.97 | 3.94 | 3.91 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 4.02 | 3.96 | 3.91 | 3.86 | 3.82 | 3.78 | 3.75 | 3.72 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.70 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.86 | 3.80 | 3.75 | 3.70 | 3.66 | 3.62 | 3.59 | 3.56 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.73 | 3.67 | 3.61 | 3.56 | 3.52 | 3.49 | 3.45 | 3.42 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.62 | 3.55 | 3.50 | 3.45 | 3.41 | 3.37 | 3.34 | 3.31 |
| 17 | 8.40 | 6.11 | 5.19 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.52 | 3.46 | 3.40 | 3.35 | 3.31 | 3.27 | 3.24 | 3.21 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.43 | 3.37 | 3.32 | 3.27 | 3.23 | 3.19 | 3.16 | 3.13 |
| 19 | 8.19 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.36 | 3.30 | 3.24 | 3.19 | 3.15 | 3.12 | 3.08 | 3.05 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.29 | 3.23 | 3.18 | 3.13 | 3.09 | 3.05 | 3.02 | 2.99 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.18 | 3.12 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 | 2.88 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.09 | 3.03 | 2.98 | 2.93 | 2.89 | 2.85 | 2.82 | 2.79 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 3.02 | 2.96 | 2.90 | 2.86 | 2.82 | 2.78 | 2.75 | 2.72 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.96 | 2.90 | 2.84 | 2.79 | 2.75 | 2.72 | 2.68 | 2.65 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.91 | 2.84 | 2.79 | 2.74 | 2.70 | 2.66 | 2.63 | 2.60 |
| 35 | 7.42 | 5.27 | 4.40 | 3.91 | 3.59 | 3.37 | 3.20 | 3.07 | 2.96 | 2.88 | 2.80 | 2.74 | 2.69 | 2.64 | 2.60 | 2.56 | 2.53 | 2.50 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.73 | 2.66 | 2.61 | 2.56 | 2.52 | 2.48 | 2.45 | 2.42 |
| 45 | 7.23 | 5.11 | 4.25 | 3.77 | 3.45 | 3.23 | 3.07 | 2.94 | 2.83 | 2.74 | 2.67 | 2.61 | 2.55 | 2.51 | 2.46 | 2.43 | 2.39 | 2.36 |
| 50 | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.79 | 2.70 | 2.63 | 2.56 | 2.51 | 2.46 | 2.42 | 2.38 | 2.35 | 2.32 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.56 | 2.50 | 2.44 | 2.39 | 2.35 | 2.31 | 2.28 | 2.25 |
| 70 | 7.01 | 4.92 | 4.07 | 3.60 | 3.29 | 3.07 | 2.91 | 2.78 | 2.67 | 2.59 | 2.51 | 2.45 | 2.40 | 2.35 | 2.31 | 2.27 | 2.23 | 2.20 |
| 80 | 6.96 | 4.88 | 4.04 | 3.56 | 3.26 | 3.04 | 2.87 | 2.74 | 2.64 | 2.55 | 2.48 | 2.42 | 2.36 | 2.31 | 2.27 | 2.23 | 2.20 | 2.17 |
| 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.43 | 2.37 | 2.31 | 2.27 | 2.22 | 2.19 | 2.15 | 2.12 |
| 200 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 | 2.41 | 2.34 | 2.27 | 2.22 | 2.17 | 2.13 | 2.09 | 2.06 | 2.03 |
| 500 | 6.69 | 4.65 | 3.82 | 3.36 | 3.05 | 2.84 | 2.68 | 2.55 | 2.44 | 2.36 | 2.28 | 2.22 | 2.17 | 2.12 | 2.07 | 2.04 | 2.00 | 1.97 |
| 1,000 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 | 2.27 | 2.20 | 2.15 | 2.10 | 2.06 | 2.02 | 1.98 | 1.95 |
| >1,000 | 1.04 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.25 | 2.19 | 2.13 | 2.08 | 2.04 | 2.00 | 1.97 | 1.94 |
| $n_1/n_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

| 19 | 20 | 22 | 24 | 26 | 28 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 | 100 | 200 | 500 | 1,000 | >1,000 | $n_1/n_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 26.72 | 26.69 | 26.64 | 26.60 | 26.56 | 26.53 | 26.50 | 26.45 | 26.41 | 26.38 | 26.35 | 26.32 | 26.29 | 26.27 | 26.24 | 26.18 | 26.15 | 26.13 | 26.15 | 3 |
| 14.05 | 14.02 | 13.97 | 13.93 | 13.89 | 13.86 | 13.84 | 13.79 | 13.75 | 13.71 | 13.69 | 13.65 | 13.63 | 13.61 | 13.58 | 13.52 | 13.49 | 13.47 | 13.47 | 4 |
| 9.58 | 9.55 | 9.51 | 9.47 | 9.43 | 9.40 | 9.38 | 9.33 | 9.29 | 9.26 | 9.24 | 9.20 | 9.18 | 9.16 | 9.13 | 9.08 | 9.04 | 9.03 | 9.02 | 5 |
| 7.42 | 7.40 | 7.35 | 7.31 | 7.28 | 7.25 | 7.23 | 7.18 | 7.14 | 7.11 | 7.09 | 7.06 | 7.03 | 7.01 | 6.99 | 6.93 | 6.90 | 6.89 | 6.89 | 6 |
| 6.18 | 6.16 | 6.11 | 6.07 | 6.04 | 6.02 | 5.99 | 5.94 | 5.91 | 5.88 | 5.86 | 5.82 | 5.80 | 5.78 | 5.75 | 5.70 | 5.67 | 5.66 | 5.65 | 7 |
| 5.38 | 5.36 | 5.32 | 5.28 | 5.25 | 5.22 | 5.20 | 5.15 | 5.12 | 5.09 | 5.07 | 5.03 | 5.01 | 4.99 | 4.96 | 4.91 | 4.88 | 4.87 | 4.86 | 8 |
| 4.83 | 4.81 | 4.77 | 4.73 | 4.70 | 4.67 | 4.65 | 4.60 | 4.57 | 4.54 | 4.52 | 4.48 | 4.46 | 4.44 | 4.42 | 4.36 | 4.33 | 4.32 | 4.32 | 9 |
| 4.43 | 4.41 | 4.36 | 4.33 | 4.30 | 4.27 | 4.25 | 4.20 | 4.17 | 4.14 | 4.12 | 4.08 | 4.06 | 4.04 | 4.01 | 3.96 | 3.93 | 3.92 | 3.91 | 10 |
| 4.12 | 4.10 | 4.06 | 4.02 | 3.99 | 3.96 | 3.94 | 3.89 | 3.86 | 3.83 | 3.81 | 3.78 | 3.75 | 3.73 | 3.71 | 3.66 | 3.62 | 3.61 | 3.60 | 11 |
| 3.88 | 3.86 | 3.82 | 3.78 | 3.75 | 3.72 | 3.70 | 3.65 | 3.62 | 3.59 | 3.57 | 3.54 | 3.51 | 3.49 | 3.47 | 3.41 | 3.38 | 3.37 | 3.36 | 12 |
| 3.69 | 3.66 | 3.62 | 3.59 | 3.56 | 3.53 | 3.51 | 3.46 | 3.43 | 3.40 | 3.38 | 3.34 | 3.32 | 3.30 | 3.27 | 3.22 | 3.19 | 3.18 | 3.17 | 13 |
| 3.53 | 3.51 | 3.46 | 3.43 | 3.40 | 3.37 | 3.35 | 3.30 | 3.27 | 3.24 | 3.22 | 3.18 | 3.16 | 3.14 | 3.11 | 3.06 | 3.03 | 3.01 | 3.01 | 14 |
| 3.40 | 3.37 | 3.33 | 3.29 | 3.26 | 3.24 | 3.21 | 3.17 | 3.13 | 3.10 | 3.08 | 3.05 | 3.02 | 3.00 | 2.98 | 2.92 | 2.89 | 2.88 | 2.87 | 15 |
| 3.28 | 3.26 | 3.22 | 3.18 | 3.15 | 3.12 | 3.10 | 3.05 | 3.02 | 2.99 | 2.97 | 2.93 | 2.91 | 2.89 | 2.86 | 2.81 | 2.78 | 2.76 | 2.75 | 16 |
| 3.19 | 3.16 | 3.12 | 3.08 | 3.05 | 3.03 | 3.00 | 2.96 | 2.92 | 2.89 | 2.87 | 2.83 | 2.81 | 2.79 | 2.76 | 2.71 | 2.68 | 2.66 | 2.65 | 17 |
| 3.10 | 3.08 | 3.03 | 3.00 | 2.97 | 2.94 | 2.92 | 2.87 | 2.84 | 2.81 | 2.78 | 2.75 | 2.72 | 2.71 | 2.68 | 2.62 | 2.59 | 2.58 | 2.57 | 18 |
| 3.03 | 3.00 | 2.96 | 2.92 | 2.89 | 2.87 | 2.84 | 2.80 | 2.76 | 2.73 | 2.71 | 2.67 | 2.65 | 2.63 | 2.60 | 2.55 | 2.51 | 2.50 | 2.49 | 19 |
| 2.96 | 2.94 | 2.90 | 2.86 | 2.83 | 2.80 | 2.78 | 2.73 | 2.69 | 2.67 | 2.64 | 2.61 | 2.58 | 2.56 | 2.54 | 2.48 | 2.44 | 2.43 | 2.42 | 20 |
| 2.85 | 2.83 | 2.78 | 2.75 | 2.72 | 2.69 | 2.67 | 2.62 | 2.58 | 2.55 | 2.53 | 2.50 | 2.47 | 2.45 | 2.42 | 2.36 | 2.33 | 2.32 | 2.31 | 22 |
| 2.76 | 2.74 | 2.70 | 2.66 | 2.63 | 2.60 | 2.58 | 2.53 | 2.49 | 2.46 | 2.44 | 2.40 | 2.38 | 2.36 | 2.33 | 2.27 | 2.24 | 2.22 | 2.21 | 24 |
| 2.69 | 2.66 | 2.62 | 2.58 | 2.55 | 2.53 | 2.50 | 2.45 | 2.42 | 2.39 | 2.36 | 2.33 | 2.30 | 2.28 | 2.25 | 2.19 | 2.16 | 2.14 | 2.13 | 26 |
| 2.63 | 2.60 | 2.56 | 2.52 | 2.49 | 2.46 | 2.44 | 2.39 | 2.35 | 2.32 | 2.30 | 2.26 | 2.24 | 2.22 | 2.19 | 2.13 | 2.09 | 2.08 | 2.07 | 28 |
| 2.57 | 2.55 | 2.51 | 2.47 | 2.44 | 2.41 | 2.39 | 2.34 | 2.30 | 2.27 | 2.25 | 2.21 | 2.18 | 2.16 | 2.13 | 2.07 | 2.03 | 2.02 | 2.01 | 30 |
| 2.47 | 2.44 | 2.40 | 2.36 | 2.33 | 2.31 | 2.28 | 2.23 | 2.19 | 2.16 | 2.14 | 2.10 | 2.07 | 2.05 | 2.02 | 1.96 | 1.92 | 1.90 | 1.89 | 35 |
| 2.39 | 2.37 | 2.33 | 2.29 | 2.26 | 2.23 | 2.20 | 2.15 | 2.11 | 2.08 | 2.06 | 2.02 | 1.99 | 1.97 | 1.94 | 1.87 | 1.83 | 1.82 | 1.81 | 40 |
| 2.34 | 2.31 | 2.27 | 2.23 | 2.20 | 2.17 | 2.14 | 2.09 | 2.05 | 2.02 | 2.00 | 1.96 | 1.93 | 1.91 | 1.88 | 1.81 | 1.77 | 1.75 | 1.74 | 45 |
| 2.29 | 2.27 | 2.22 | 2.18 | 2.15 | 2.12 | 2.10 | 2.05 | 2.01 | 1.97 | 1.95 | 1.91 | 1.88 | 1.86 | 1.82 | 1.76 | 1.71 | 1.70 | 1.69 | 50 |
| 2.22 | 2.20 | 2.15 | 2.12 | 2.08 | 2.05 | 2.03 | 1.98 | 1.94 | 1.90 | 1.88 | 1.84 | 1.81 | 1.78 | 1.75 | 1.68 | 1.63 | 1.62 | 1.60 | 60 |
| 2.18 | 2.15 | 2.11 | 2.07 | 2.03 | 2.01 | 1.98 | 1.93 | 1.89 | 1.85 | 1.83 | 1.78 | 1.75 | 1.73 | 1.70 | 1.62 | 1.57 | 1.56 | 1.54 | 70 |
| 2.14 | 2.12 | 2.07 | 2.03 | 2.00 | 1.97 | 1.94 | 1.89 | 1.85 | 1.82 | 1.79 | 1.75 | 1.71 | 1.69 | 1.65 | 1.58 | 1.53 | 1.51 | 1.50 | 80 |
| 2.09 | 2.07 | 2.02 | 1.98 | 1.95 | 1.92 | 1.89 | 1.84 | 1.80 | 1.76 | 1.74 | 1.69 | 1.66 | 1.63 | 1.60 | 1.52 | 1.47 | 1.45 | 1.43 | 100 |
| 2.00 | 1.97 | 1.93 | 1.89 | 1.85 | 1.82 | 1.79 | 1.74 | 1.69 | 1.66 | 1.63 | 1.58 | 1.55 | 1.52 | 1.48 | 1.39 | 1.33 | 1.30 | 1.28 | 200 |
| 1.94 | 1.92 | 1.87 | 1.83 | 1.79 | 1.76 | 1.74 | 1.68 | 1.63 | 1.60 | 1.57 | 1.52 | 1.48 | 1.45 | 1.41 | 1.31 | 1.23 | 1.20 | 1.17 | 500 |
| 1.92 | 1.90 | 1.85 | 1.81 | 1.77 | 1.74 | 1.72 | 1.66 | 1.61 | 1.58 | 1.54 | 1.50 | 1.46 | 1.43 | 1.38 | 1.28 | 1.19 | 1.16 | 1.12 | 1,000 |
| 1.91 | 1.88 | 1.83 | 1.79 | 1.76 | 1.73 | 1.70 | 1.64 | 1.59 | 1.56 | 1.53 | 1.48 | 1.44 | 1.41 | 1.36 | 1.25 | 1.16 | 1.11 | 1.05 | >1,000 |
| 19 | 20 | 22 | 24 | 26 | 28 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 | 100 | 200 | 500 | 1,000 | >1,000 | $n_1/n_2$ |

**Table A.6** Critical values of Chi-square

| Probability under $H_0$ that $\chi^2 \geqslant$ Chi-square | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| df | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | – | – | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

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