Farzad Hejazi
Keyhan Karimzadeh

Analysis Procedure for Earthquake Resistant Structures

Springer

Analysis Procedure for Earthquake Resistant Structures

Farzad Hejazi · Keyhan Karimzadeh

Analysis Procedure for Earthquake Resistant Structures

Springer

Farzad Hejazi
Department of Civil Engineering
University Putra Malaysia
Serdang, Selangor
Malaysia

Keyhan Karimzadeh
Department of Civil Engineering
University Putra Malaysia
Serdang, Selangor
Malaysia

ISBN 978-981-10-8838-4 ISBN 978-981-10-8839-1 (eBook)
https://doi.org/10.1007/978-981-10-8839-1

Library of Congress Control Number: 2018937354

Printed on acid-free paper

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
part of Springer Nature
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Preface

This book demonstrates the analysis procedure for structures subjected to the lateral loads such as earthquake and wind. The structural analysis includes the process for calculation and distribution of earthquake load into structural elements, and calculates the displacement due to applied lateral load for different types of structures such as reinforced concrete and steel framed structures. The purpose of preparing this book is to provide a clear guideline for civil engineers for the seismic analysis of structures with various building systems and distribute lateral load to the structural components. This book covers all necessary algorithms, equations, procedure, and illustration for the analysis of structure subjected to lateral load in a step-by-step manner.

The developed procedure is based on manual calculation which is formulated for realistic structures, and it provides suitable and clear techniques for the analysis of structures.

The book consists of four chapters: Chap. 1: Moment Resistance System, Chap. 2: Braced Steel Frame System, Chap. 3: Frames with Shear Wall, and Chap. 4: Dynamic and Nonlinear Static Analysis. Each chapter is divided into five parts to calculate the applied loads to the structure, determine the center of mass and stiffness, and distribute lateral load to the structure as well as calculate the structural displacement.

In order to demonstrate the application of developed procedure, all the explained theories and calculation procedures are implemented for the analysis of real building and all architectural drawings and structural details are provided in each chapter.

Therefore, by preparing this book, an attempt has been made to create an understandable and simplified procedure for the analysis of structures subjected to a lateral load for civil engineers.

Selangor, Malaysia

Farzad Hejazi
Keyhan Karimzadeh

Contents

List of Figures

List of Tables

List of Charts


SUBARU

Chapter 1
Moment Resistance System

In the recent two decades, the number of high intensity earthquake occurrence is incredibly increased which resulted many building damages and human lost. Inappropriate design of structures and construction process are the main reasons of buildings damage and collapse due to seismic load. The main challenge of structure design is determining the imposed earthquake load to the building and also the distribution of seismic load to the different structural components.

Therefore, the main focus of this chapter is about calculation of earthquake load and process of seismic load distribution between structural members in various type of buildings such as steel and concrete structures. However, the same procedure can be implemented for distribution of any lateral load effect on structural elements such as wind load.

In this chapter, the moment-resistance frame system is considered which is one of the most conventional earthquake resistant systems for buildings.

This chapter is divided into 5 main sections:

In the first section of this chapter, behavior of the moment-resistance frame system subjected to gravity and the lateral load is demonstrated. The approximate methods are implemented for analysis of the framed structure, and slope-deflection method is explained to calculate the displacement of moment resistant frame subjected to lateral load after determining of lateral stiffness of the frame.

In order to clear understanding the highlighted concepts in this book, it is tried to demonstrate the application of explained theories and methods in real buildings in each section and describe all calculations and computations process for real structure details. For this purpose, a 6 story building is considered, and in the second section of this chapter, the all architectural plans and structure detail are presented.

The third section of the first chapter is devoted to defining the gravity load and seismic force based on IBC Code and EURO CODE and determining of the center

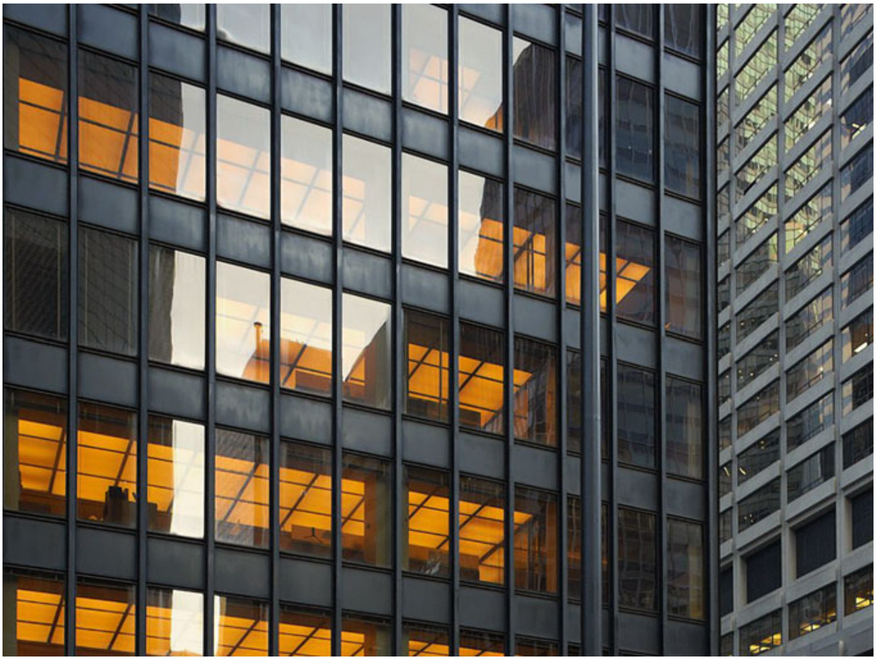
F. Hejazi and K. Karimzadeh, *Analysis Procedure for Earthquake Resistant Structures*,
https://doi.org/10.1007/978-981-10-8839-1_1

of mass, stiffness, shear rigidity and the eccentricity of each floor level of building by proposing the calculation charts. Furthermore, the distribution of the earthquake force in the structural members using the Portal and Cantilever methods are explained in details.

The fourth section illustrates the calculation procedure for determining displacements in the different frames in two perpendicular directions of the moment-resistance structure subjected to earthquake force.

Finally, the fifth section of this chapter presents the proposed charts and detail calculation of weight, stiffness, and displacement of the floor levels and frames of the considered structure.

1.1 Moment Resistance System



- Resistance System in Structure
- The Behavior of Moment Resistance Frame Under Gravity and Lateral Load
- Stiffness Concept
- Shear Rigidity

1.1.1 *Resistance System in Structure*

The loads applied to the structure are divided into three categories, gravity, lateral, and other loads. The different parts of the building structure transfers load to the foundation which in turn transfers it to the ground. The gravity resistance system uses gravitational load of the structure. Lateral résistance system is a part of the structure that should resist against lateral load and transfer this load to the foundation. Based on the review of existing literature and implementation, it is observed that most common of types lateral loads are wind and earthquake which impact the structure in different directions. For this reason, suitable lateral resistance systems should be provided in the two perpendiculars direction. Chart 1.1 shows the various types of lateral resistance structural systems.

This book focuses on the moment resistance frame, shear wall frame, and steel bracing systems.

1.1.2 *Moment Resistance Frame*

A moment resisting frame often consists of bents arranged in perpendicular or parallel bents manner and girders which are interconnected rigidly. The resistance for lateral loading is created by the bending resistance of the columns, girders, and connections. Another factor that dampens gravity loading is the use of continuity frame and by reducing the moments in the girders. Additionally, the continuity of

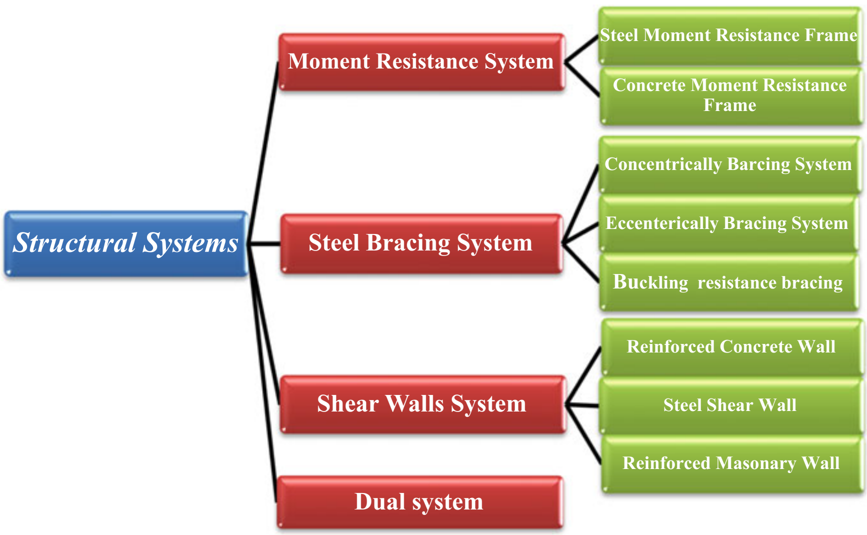


Chart 1.1 Different types of lateral resistance structural

the frame influences towards reducing gravity. Specifically, the capacity of loading in moment resistance frame shall decrease by increasing the height and distance of the columns. The reduction in moment resistance allows the moment resistance frame base to be ductile. The ductility of plastic deformation is divided into three categories:

1. Ordinary moment resisting frame
2. Intermediate moment resisting frame
3. Special moment resisting frame

A given frame can be used up to 25 stories due to the presence of ductility as a lateral load resistance system at the moment resting frame. The shear walls or cores are combined with a rigid frame. These results in a sturdier structure which allows the height to reach further and that can be extended up to 50 stories. One of the advantages of moment resisting frame is the simplicity and convenience of its rectangular shape. Because of this arrangement, the internal frame division offers improved support to the shear wall and bracing system. Figure 1.1 shows two types of moment resistance connection.

Despite these advantages, some disadvantages of moment resisting frame are:

It is important that rigid steel connections are appropriately designed and calibrated accurately since such structures are typically more expensive than the pin connection. Despite the economic advantages at the moment resting frame, there are some limitations when scaling it beyond 25 stories. The lateral displacement can be controlled by increasing the stiffness of the frame and the number of columns. The lateral displacement can be compensated in the building plan through the use column arrangements and ensuring the moment resting frame is also adequately designed. In Fig. 1.2, the moment resistance frame under lateral load illustrates the shape and offers a better understanding of moment resistance frame under lateral load. Furthermore,

1. The columns and beams create a bow because of continuity between members of the frame.
2. Based on the moment resistance connection between members of the frame, the angle between column and beam remains fixed (1.90).

Fig. 1.1 Two types of moment resistance connection

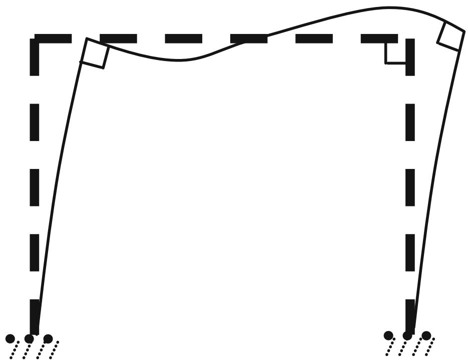


Fig. 1.2 Deformation of moment resistance frame

1.1.3 The Behavior of Moment Resistance Frame

This part shows the behavior of moment resistance structure under gravity and lateral load.

1.1.3.1 Gravity Load

Figure 1.3 shows moment resistances frame that is subjected to the gravity load. The load is withstood by the beam and finally transferred through the columns to the ground. The load has a tendency to create rotation at the ends of the beam. Moment resistance connection prevents free rotation at the end of the beam from occurring. The joint is such that the column tends to confine the beam end from the rotation. This limitation has some significant consequences. The most important subsequence is the fact that the column top is offering restraint to the rotation, which means that the column must be withstanding or picking up bending moments

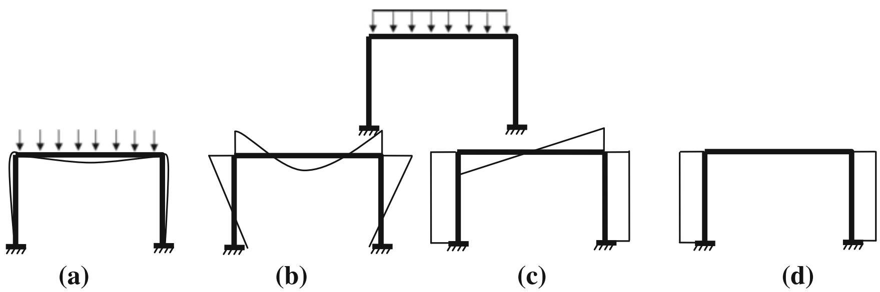


Fig. 1.3 Moment resistance frame under gravity load. **a** Deformation diagram, **b** moment diagram, **c** shear diagram, **d** axial diagram

in addition to axial forces, so making its design difficult. The moment resistance connection does not provide full end fixity for either the beam or the column. Because the load has a tendency to create rotation at the end of the beam, the connected top of the column rotates as well. The whole joint between the column and the beam will rotate as integrity. As a result, rigidity causes the angle between column and beam to remain fixed. Figure 1.3 shows deformation, moment, shear, and axial diagram of moment resistance frame subjected gravity load.

Analysis of the Moment Resistance Frame Under Gravity Load

The general procedure for making an approximate analysis of a frame carrying gravity loads is considered in this section. The moment resistance frame under gravity load is demonstrated in Fig. 1.4, which has fixed connections at the column bases. In this method acting, the following assumptions are made:

1. The contra flexure points in moment bending of the beam are located between 0.1L and 0.2L at the end of the beam. It is somewhat harder to locate contra flexure points affiliated with gravity loads than those associated with lateral loads. If the connection provided full fixity (and no end rotation occurred at all at beam ends), then the contra flexure points would be 0.21L from either end. Since some rotations do occur, but the joint does not freely rotate, the actual conditions lie somewhere between simple supported connections and fully fixed connections; the contra flexure points in the beam thus lie somewhere between 0L and 0.21L from the joints. For beam and columns of normal stiffness, the contra flexure points are typically found about 0.15L from either ends.
2. The middle part is reserved to be an assumption as a simple beam and is a computed reaction force and middle moment (Fig. 1.5).

$$\left\{ \textit{Reaction force}: \bar{V} = \frac{q \times 0.7L}{2} = 0.35qL \right. \tag{1.1}$$

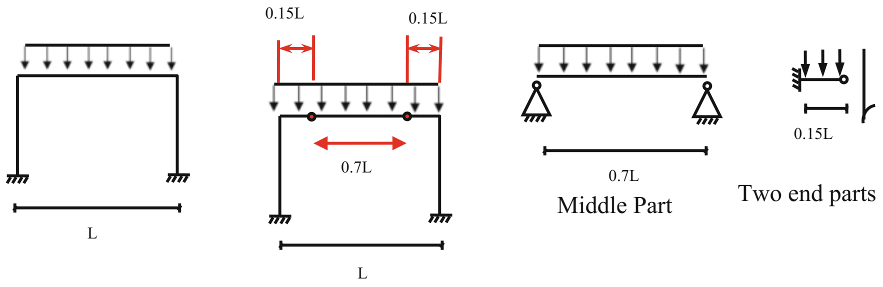


Fig. 1.4 The location of zero moments in resistance frame

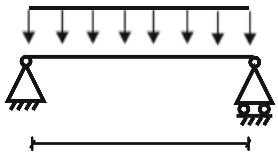


Fig. 1.5 Middle part of beam

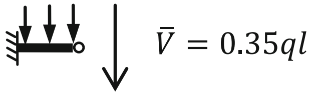


Fig. 1.6 End section of beam

$$\left\{ Moment: M^{+} = \frac{q \times 0.7L^2}{8} = 0.06125qL^2 \right. \tag{1.2}$$

3. Reaction force is applied to the end of beam using the shear and moment at point the beam is calculated (Figs. 1.5 and 1.6)

$$\bar{V} = 0.35ql \tag{1.3}$$

$$\{ Shear\,force\,at\,the\,end: V = q \times 0.15L + \bar{V} = 0.5qL \tag{1.4}$$

$$\{ Moment\,bending\,on\,end: M^{-} = \frac{q \times (0.15L)^2}{2} + (\bar{V} + 0.15L) = 0.06375qL^2 \tag{1.5}$$

1.1.3.2 Lateral Load

Moment resistance frame subjected to the lateral loads shown in Fig. 1.7. Based on the presence of a moment resistance connection, the beams confine the columns from freely rotating in a way that would lead to the total collapse of the structure. The stiffness of the beam contributes to the lateral load carrying resistance of a frame, as well as serving to transfer part of the lateral load from on columns to the other. The action of a lateral load on a frame produces bending moment, shear, and axial forces in all members. Bending moments induced by earthquake loads or wind loads are often the highest near the moment resistance joints. Deformations, moment, shear, and axial diagram of moment resistance frame is illustrated in Fig. 1.7.

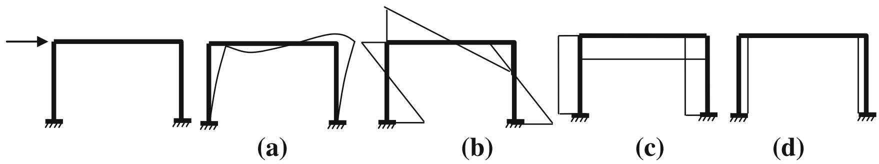
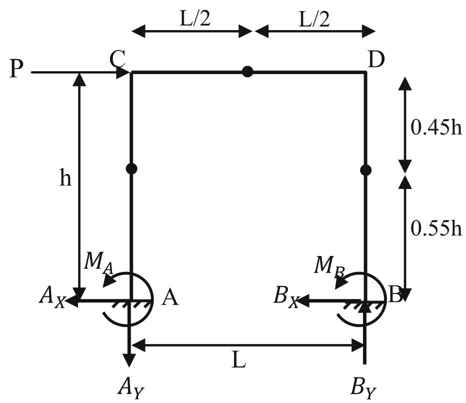


Fig. 1.7 Moment resistance frame under gravity load. **a** Deformation diagram, **b** moment diagram, **c** shear diagram, **d** axial diagram

- **Analysis of the moment resistance frame under lateral load**:

A single bay moment resistance frame under lateral load is illustrated in Fig. 1.8. The figure demonstrates the reactive forces and moments developed at the foundations. There are six unknown quantities and only three equations of statics available for use. Thus, the frame is statically indeterminate to the third degree. Consequently, three presumptions must be made if a static analysis is to be used. As before, in stiffness part, slop-deflection method is chosen to calculate the slopes and displacement. In this part, an approximate method to compute the reaction force in the single-bay frame is opted. As is evident, three points of contra flexure develop. One is in the mid-span of the beam. Two others are near the mid-height of the columns. They are not precisely at mid-height since the top joints rotate slightly. If the joints simply translated horizontally without rotation, the points of contra flexure would have to be identically at mid-height. The slight rotation of the upper joints causes the point of inflection to rise somewhat to the location indicated in Fig. 1.9. Fixing the location of the three points of inflection makes a static analysis possible. The frame can be decomposed or separated at these points of zero moments (Axial forces and shears can be found from the knowledge that the net rotational moments around these points for any piece must be zero). This analysis is illustrated in Fig. 1.9. Note that if the frame is decomposed into two sections at the contra flexure points in the column, the upper part is quite analogous to the frame, except that column heights differ.

Fig. 1.8 Moment resistant frame under lateral load



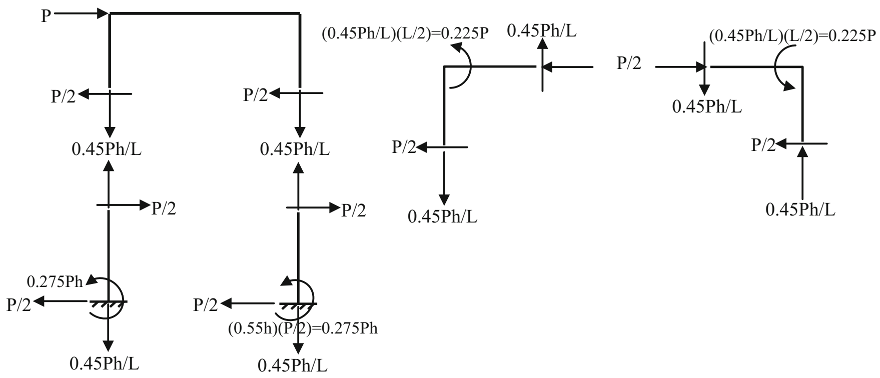


Fig. 1.9 Analysis of moment resistance frame under lateral load

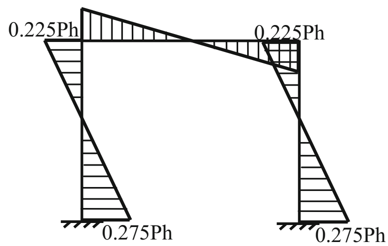


Fig. 1.10 Final moment diagram of single bay moment resistance frame under lateral load

The analysis technique is the same. This structure can then be imagined as simply resting on two vertical cantilever elements. The horizontal thrusts associated with the upper part produce moments in the lower elements. A final moment diagram is shown in Fig. 1.10.

1.1.4 Stiffness Concept

Figure 1.11 illustrates an understanding of the stiffness concept based on Hock low. The figure shows that the cantilever beam with force F is applied the end of the column and created the lateral displacement (Δ). According to Eq. (1.6), it is clearly seen the column acts as a spring with stiffness K = 3EI/L^3.

$$\begin{cases} Column: \Delta = \frac{FL^3}{3EI} \rightarrow F = \frac{3EI}{L^3} \times \Delta \\ spring: F = K\Delta \end{cases} \tag{1.6}$$

For a better understanding of the stiffness concept, consider one story moment resistance frame where force F is applied to the frame, which results in the lateral displacement (Δ) applied to the frame. If the force F is increased gradually from

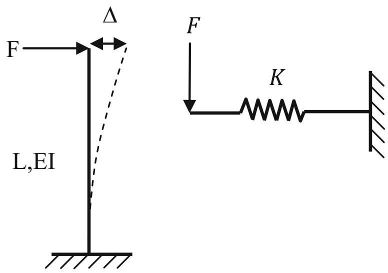


Fig. 1.11 Concept of stiffness

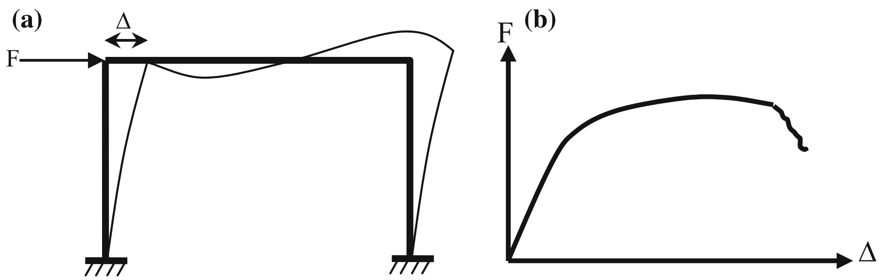
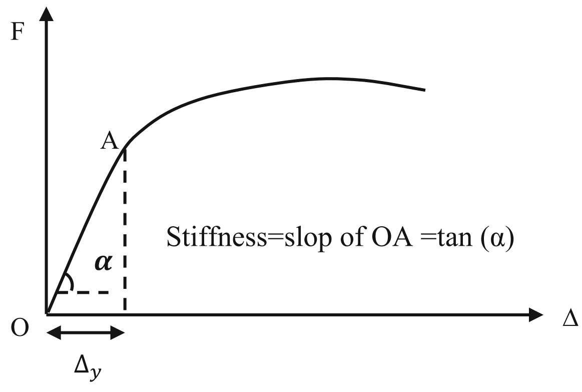


Fig. 1.12 **a** Moment resistance frame under lateral load, **b** pushover graph

Fig. 1.13 Concept of stiffness in force-displacement graph



zero until the collapse of the structure; a pattern is shown in the graph in Fig. 1.12b is observed. This is called the pushover graph.

The first part of the graph in Fig. 1.13 is linear, which shows the structure has elastic behavior until (A) and then structure behaves nonlinearly that the stiffness of the structure is obtained from the slope of the linear part. The stiffness of structure is defined after (A), but based on the graph in Fig. 1.13 force–displacement slope of nonlinear part respect to the linear part is decreased dramatically. It means that when the structure is pushed to the nonlinear phases, the stiffness of structure is reduced significantly, too.

1.1.4.1 The Stiffness of Moment Resistance Frame

The stiffness of a moment resistance frame can be calculated by applying Slop-Deflection method. Therefore, a concentrated unit force is applied to the single bay moment resistance frame. As explained by the Hock low, the stiffness can be calculated by dividing the force via displacement. However, when the force is equal to unity, the stiffness will be inversely proportional to the displacement. Next section will describe the Slop-Deflection method.

- **Slope-Deflection method**:

By applying slope deflection, the statically indeterminate frames that are undergoing sideways movements can be analyzed. It is important to note that the axial displacement of beams and columns are ignored in the analysis, simply because the values are insignificant. It is also found that a frame will not move sideways if the followings are applied:

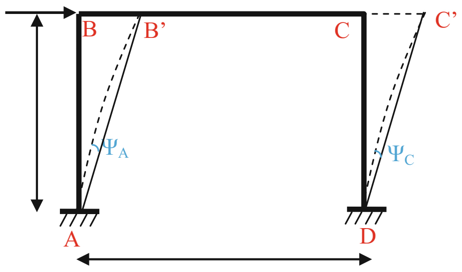
1. The frames are restrained from moving sideways.
2. Introducing symmetrical loading and frame geometries.

However, it is imperative to note that general loading is not often asymmetrical. Hence, sideways placed frames should be eliminated.

For example, as shown in Fig. 1.14 the frame is symmetrical under lateral loading. As a result, B and C joints are displaced and skewed towards the right direction, which is illustrated in the figure through an unknown amount Δ. By taking this into account, three unknown displacements can be located within this frame, namely rotations θ_B, θ_C, and the linear deformation Δ. The unknown joint rotations θ_B and θ_C are associated to joint moments by the moment equilibrium equations. Similarly, it is necessary to factor force-equilibrium experienced a linear displacement Δ with respect to the end A. Thus the slope-deflection equation for column AB is as same as the beam that undergoes the support settlement. However, in this case, Δ is unknown. With this observations and information, it is possible to write the following slope-deflection equations for the members. Equations when unknown linear displacement takes place. Furthermore, it is required that the column rotation be unknown while applying the slope-deflection equation to the columns located above the frame. It can be seen that in the column AB, the end B

$$M_{AB} = M_{AB}^{F} + \frac{2EI}{h}[(2\theta_A + \theta_B - 3\Psi_{AB})] \tag{1.7}$$

Fig. 1.14 Plane frame undergoing sway



where;

$$\Psi_{AB} = \frac{\Delta}{h} \tag{1.8}$$

Ψ is assumed to be positive as the chord to the elastic curve rotates in the counter-clockwise direction

$$M_{BA} = M_{BA}^F + \frac{2EI}{h}[(2\theta_B + \theta_A - 3\Psi_{AB})] \tag{1.9}$$

$$M_{BC} = M_{BC}^F + \frac{2EI}{h}[(2\theta_B + \theta_C)] \tag{1.10}$$

$$M_{CB} = M_{CB}^F + \frac{2EI}{h}[(2\theta_C + \theta_B)] \tag{1.11}$$

$$M_{CD} = M_{CD}^F + \frac{2EI}{h}[(2\theta_C + \theta_D - 3\Psi_{CD})] \tag{1.12}$$

$$\Psi_{CD} = \frac{\Delta}{h}$$

$$M_{DC} = M_{DC}^F + \frac{2EI}{h}[(2\theta_D + \theta_C - 3\Psi_{CD})] \tag{1.13}$$

The three unknown variables (θ_B, θ_C, and Δ) can be analyzed with three equations. Based on these equations, two equations can be derived by considering the moment equilibrium theory of joint B and C

$$\sum M_B = 0 \qquad M_{BA} + M_{BC} = 0 \tag{1.14}$$

$$\sum M_C = 0 \qquad M_{CD} + M_{CB} = 0 \tag{1.15}$$

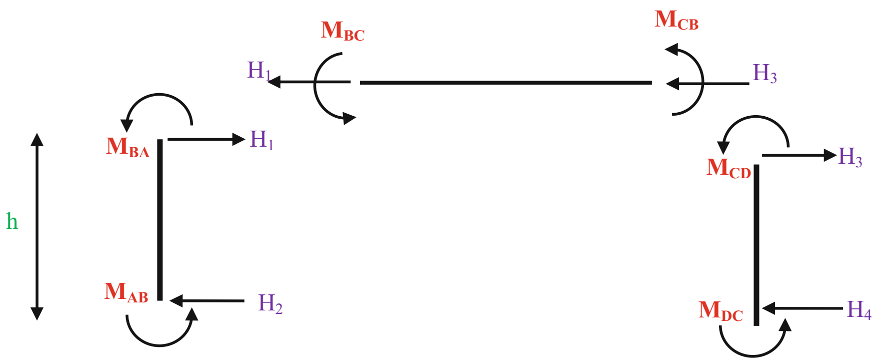


Fig. 1.15 Free body diagrams of columns and beam

Considering the free body diagram of the frame in Fig. 1.15, the horizontal shear force acting at A and B of the column AB is shown.

Similarly, for members, the shear force H_3 is given by:

$$H_3 = \frac{M_{DC} + M_{CD}}{h} \tag{1.16}$$

Based on the B and C, the required third equation can be obtained by considering the equilibrium of member, BC

$$\sum F_x = 0 \qquad H_1 + H_3 = 0 \tag{1.17}$$

$$\frac{M_{BA} + M_{AB}}{h} + \frac{M_{DC} + M_{CD}}{h} = 0 \tag{1.18}$$

It is now possible to get three simultaneous equations in three unknown's θ_B, θ_C and Δ by substituting the magnitude of beam end moments from Eq. (1.8) in Eqs. (1.14), (1.15) and (1.18). The combined equation can solve and evaluate the joint rotations and translations. By knowing joint rotations and translations, beam end moments are calculated from slope-deflection equations (IIT, 2008).

1.1.4.2 The Calculation Stiffness of Moment Resistance Frame

For calculating the stiffness of a moment resistance frame, a concentrated unit force is applied to the single bay moment resistance frame. Lateral displacement is calculated by the slope-deflection method. Figure 1.16 shows moment resistance frame under lateral load.

The ends A and B are fixed. Hence, θ_A, $\theta_B = 0$. Joints B and C translates by the same amount Δ. Hence, The displacement generated when joints D and C rotate is denoted by an amount Δ.

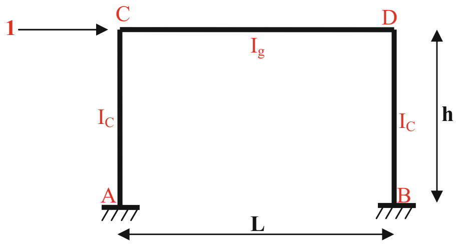


Fig. 1.16 Moment resistance frame under lateral load

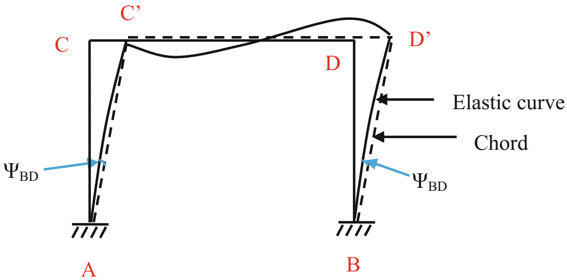


Fig. 1.17 Columns rotation

Along with this, tone translation and two rotation, which are unknown displacements need to be evaluated. Taking to account the kinematically evaluated structure, fixed end moments are calculated first. Thus:

$$M_{AC}^F = 0; \quad M_{CA}^F = 0; \quad M_{CD}^F = 0; \quad M_{DC}^F = 0; \quad M_{BD}^F = 0; \quad M_{DB}^F = 0 \tag{1.19}$$

Chord to the elastic curve AC and BD′ rotates by an amount $\Psi_{AC} = \Psi_{BD} = \frac{\Delta}{h}$, that it is shown in Fig. 1.17.

Six equations of slop-deflection based on Eq. (1.19) are as follows:

$$M_{AC} = \frac{2EI_c}{h}(2\theta_A + \theta_C - 3\Psi_{AC}), \quad \Psi_{AC} = \frac{3\Delta}{h}, \quad \theta_A = 0 \tag{1.20}$$

$$M_{AC} = \frac{2EI_c}{h}\left(\theta_C - \frac{3\Delta}{h}\right) \tag{1.21}$$

$$M_{CA} = \frac{2EI_c}{h}(2\theta_C + \theta_A - 3\Psi_{CA}), \quad \Psi_{AC} = \frac{3\Delta}{h}, \quad \theta_A = 0 \tag{1.22}$$

$$M_{CA} = \frac{2EI_c}{h}\left(2\theta_C - \frac{3\Delta}{h}\right) \tag{1.23}$$

$$M_{BD} = \frac{2EI_c}{h}\left(\theta_D - \frac{3\Delta}{h}\right)\Psi_{BD} = \frac{3\Delta}{h}, \quad \theta_B = 0 \tag{1.24}$$

$$M_{DB} = \frac{2EI_c}{h}\left(2\theta_D - \frac{3\Delta}{h}\right)\Psi_{BD} = \frac{3\Delta}{h}, \quad \theta_B = 0 \tag{1.25}$$

$$M_{CD} = \frac{2EI_g}{l}(2\theta_C + \theta_D) \tag{1.26}$$

$$M_{DC} = \frac{2EI_g}{l}(2\theta_D + \theta_C) \tag{1.27}$$

Then, joint equilibrium of C and D is considered based on Fig. 1.18.

$$\{ M_C = 0 \quad M_{CA} + M_{CD} = 0 \tag{1.28}$$

$$\{ M_D = 0 \quad M_{DB} + M_{DC} = 0 \tag{1.29}$$

Based on the horizontal equilibrium of the entire frame, the required third equation is expressed as (Fig. 1.19):

$$\sum f_x = 0 \quad 1 - \mathrm{Ax} - \mathrm{Bx} = 0 \tag{1.30}$$

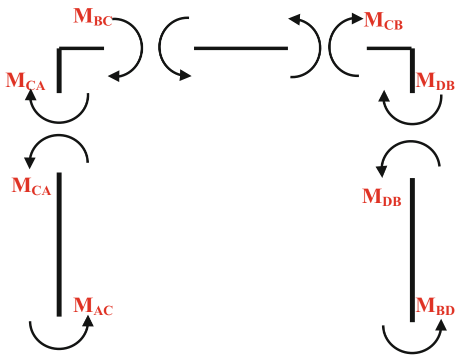


Fig. 1.18 Free body diagram of joint B and C

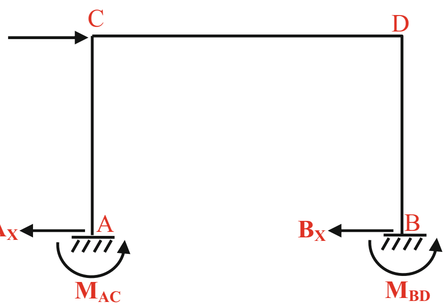


Fig. 1.19 Moment resistance frame under lateral load and its reaction

Reaction considering the equilibrium of the column AB and CD,

$$\begin{gathered} M_C^{AC} = 0 \rightarrow (-\mathrm{A_X} \times \mathrm{h}) + M_{AC} + M_{CA} = 0 \\ (-A_X \times h) + M_{AC} + M_{CA} = 0 \\ A_X = \frac{MAC + MCA}{h} \end{gathered} \tag{1.31}$$

$$\begin{gathered} M_D^{BD} = 0 \rightarrow (-\mathrm{B_X} \times \mathrm{h}\ ) + M_{BD} + M_{DB} = 0 \\ B_X = \frac{M_{BD} + M_{DB}}{h} \end{gathered} \tag{1.32}$$

The Eq. (1.16) is now,

$$h = M_{AC} + M_{CA} + M_{BD} + M_{DB} \tag{1.33}$$

Substituting the beam end moments from Eqs. (1.9)–(1.13) in Eqs. (1.14), (1.15) and (1.16)

$$\left\{ \frac{2\mathrm{EI_c}}{\mathrm{h}} \left(2\theta_\mathrm{C} - \frac{3\Delta}{\mathrm{h}} \right) + \frac{2\mathrm{EI_g}}{\mathrm{l}} (2\theta_\mathrm{C} + \theta_\mathrm{D}) = 0 \right. \tag{1.34}$$

$$\left\{ \begin{array}{l} \frac{2\mathrm{EI_c}}{\mathrm{h}} \left(2\theta_\mathrm{D} - \frac{3\Delta}{\mathrm{h}} \right) + \frac{2\mathrm{EI_g}}{\mathrm{l}} (2\theta_\mathrm{D} + \theta_\mathrm{C}) = 0 \\ \mathrm{h} = \mathrm{M_{AC}} + \mathrm{M_{CA}} + \mathrm{M_{BD}} + \mathrm{M_{DB}} \end{array} \right. \tag{1.35}$$

$$(I), (II) \Rightarrow \theta_C = \theta_D$$

Then substituted $\theta_C = \theta_D$ to the Eqs. (1.33) and (1.34)

$$\begin{gathered} \tfrac{2EI_c}{h}\left(\theta_C - \tfrac{3\Delta}{h}\right) + \tfrac{2EI_c}{h}\left(2\theta_C - \tfrac{3\Delta}{h}\right) + \tfrac{2EI_c}{h}\left(\theta_D - \tfrac{3\Delta}{h}\right) + \tfrac{2EI_c}{h}\left(2\theta_D - \tfrac{3\Delta}{h}\right) = h \\ \tfrac{2EI_c}{h}\left[\left(\theta_C - \tfrac{3\Delta}{h}\right) + \left(2\theta_C - \tfrac{3\Delta}{h}\right) + \left(\theta_D - \tfrac{3\Delta}{h}\right) + \left(2\theta_D - \tfrac{3\Delta}{h}\right)\right] = h \end{gathered} \tag{1.36}$$

$$\frac{2EI_c}{h}\left[6\theta_C - \frac{12\Delta}{h}\right] = h \Rightarrow \frac{12EI_c\theta_C}{h} - \frac{24EI_c\Delta}{h^2} = h \tag{1.37}$$

$$\frac{2EI_C}{h}\left(2\theta_C - \frac{3\Delta}{h}\right) + \frac{2EI_g}{l}(2\theta_C + \theta_D) = 0 \tag{1.34}$$

$$\theta_C = \theta_D \Rightarrow \frac{2EI_C}{h}\left(2\theta_C - \frac{3\Delta}{h}\right) + \frac{2EI_g}{l}(2\theta_C + \theta_C) = 0$$

$$\left(\frac{4EI_c}{h} + \frac{6EI_g}{l}\right)\theta_C - \frac{6EI_c\Delta}{h^2} = 0 \tag{1.38}$$

The calculation θ_C can be computed from Eqs. (1.37) and (1.38). For this reason, the Eq. (1.38) should be multiplied in the (−4) which is then summed with Eq. (1.37)

$$\begin{cases} \frac{12EI_c\theta_C}{h} - \frac{24EI_c\Delta}{h^2} = h \\ \times(-4)\left(\frac{4EI_c}{h} + \frac{6EI_g}{l}\right)\theta_C - \frac{6EI_c\Delta}{h^2} = 0 \end{cases}$$

$$+\begin{cases} \frac{12EI_c\theta_C}{h} - \frac{24EI_c\Delta}{h^2} = h \\ \left(\frac{-16EI_c}{h} + \frac{-24EI_g}{l}\right)\theta_C + \frac{24EI_c\Delta}{h^2} = 0 \end{cases}$$

$$\left(\frac{-4EI_c}{h} - \frac{24EI_g}{l}\right)\theta_C = h$$

$$\theta_C = -\frac{h}{\left(\frac{4EI_c}{h} + \frac{24EI_g}{l}\right)} = -\frac{h}{4E}\left(\frac{1}{\frac{I_c}{h} + \frac{6I_g}{l}}\right) \tag{1.38}$$

For calculation of Δ, θ_C should be included in Eq. (1.37), then

$$\frac{12EI_c}{h}\left[\left(\frac{-h}{4E}\right)\left(\frac{1}{\frac{I_c}{h}+\frac{6I_g}{l}}\right)\right] - \left[\left(\frac{24EI_c}{h^2}\right)\Delta\right] = h$$

$$\left[\left[\left(\frac{-3I_c}{\frac{I_c}{h}+\frac{6I_g}{l}}\right)\right] - h\right] = \left[\left(\frac{24EI_c}{h^2}\right)\Delta\right]$$

$$\left[\left[\left(\frac{-3I_c}{\frac{I_c}{h}+\frac{6I_g}{l}}\right)\right] - h\right] = \left[\left(\frac{24EI_c}{h^2}\right)\Delta\right] \Rightarrow \left[\left[\left(\frac{-3I_c}{\frac{I_c}{h}+\frac{6I_g}{l}}\right)\right] - h\right] = \left[\left(\frac{24EI_c}{h^2}\right)\Delta\right]$$

$$\left[\frac{(-4I_c)-\left(\frac{6hI_g}{l}\right)}{\left(\frac{I_c}{h}\right)+\left(\frac{6I_g}{l}\right)}\right] = \left[\left(\frac{24EI_c}{h^2}\right)\Delta\right] \Rightarrow \Delta = \left[\frac{\frac{(-4I_C)-\left(\frac{6hI_g}{l}\right)}{\left(\frac{I_C}{h}\right)+\left(\frac{6I_g}{l}\right)}}{\frac{24EI_c}{h^2}}\right] = \left[\frac{h^2\left(-4I_c-\frac{6hI_g}{l}\right)}{24EI_c\left(\frac{I_c}{h}+\frac{6I_g}{l}\right)}\right]$$

$$\Delta = \left[\frac{-\left(4I_ch^3l + 6h^4I_g\right)}{\left(144EI_gI_ch + 24EI_c^2l\right)}\right] \tag{1.39}$$

Based on the Hock low, the stiffness of one bay moment resistance frame is calculated and shown in Eq. (1.40).

$$K = \left[\frac{\left(144EI_gI_ch + 24EI_c^2l\right)}{\left(4I_ch^3l + 6I_gh^4\right)} \right]$$

$$K = \left[\frac{\left(144EI_gI_ch + 24EI_c^2l\right)}{\left(4I_ch^3l + 6I_gh^4 - 18\boldsymbol{I_g h^4} + 18\boldsymbol{I_g h^4}\right)} \right] = \left[\frac{\left(144EI_gI_ch + 24EI_c^2l\right)}{\left(4I_ch^3l + 24I_gh^4 - 18\boldsymbol{I_g h^4}\right)} \right]$$

$$= \frac{6EI_c}{h^3} \left[\frac{\left(24I_gh + 4I_cl\right)}{\left(4I_cl + 24I_gh - 18I_gh\right)} \right] = \frac{6EI_c}{h^3} \left[\frac{\frac{\left(24I_gh + 4I_cl\right)}{6hI_g}}{\frac{\left(4I_cl + 24I_gh - 18I_gh\right)}{\boldsymbol{6hI_g}}} \right] = \frac{6EI_c}{h^3} \left[\frac{\left(4 + \frac{4I_cl}{6hI_g}\right)}{\left(4 + \frac{4I_cl}{6hI_g} - 3\right)} \right]$$

$$\boldsymbol{k} = \frac{\boldsymbol{6EI_c}}{\boldsymbol{h^3}} \left[\frac{4\left(1 + \frac{\boldsymbol{I_c l}}{\boldsymbol{6hI_g}}\right)}{4\left(1 + \frac{\boldsymbol{I_c l}}{\boldsymbol{6hI_g}}\right) - 3} \right] \tag{1.40}$$

1.1.5 Shear Rigidity

In the process of analysis moment resistance frame, the first step is calculation and distribution of horizontal lateral load at each bent. For this reason, an assumption is made such that the floors are rigid in-plane, so the horizontal displacement of all vertical frames on the floor level is related by the horizontal translation and rotation of the floor slab. The total external shear at each level is distributed based on the shear rigidity (GA) at each bent. The bending resistance of the columns, girders, and connections provide resistance to the lateral loading, so the moment inertia of columns and girders play an important role in shear rigidity. Equation (1.41) shows the shear rigidity (GA) formulation (Smith, Coull, & Stafford-Smith, 1991)

$$GA = \frac{12E}{hi\left(\frac{1}{G} + \frac{1}{C}\right)i} \tag{1.41}$$

where;

E: module of elasticity

h_i: height of the story

G = $\sum \frac{I_g}{L}$ for all girders of span L in the floor i of the bent

C = $\sum \frac{I_c}{h}$ for all columns in story i of the bent

I_g = Moment inertia of girder

I_c = Moment inertia of column

h = height of column

l = length of be

1.2 Define Project

- Architectural Plan
- Material Property

Definition and Properties used in this project:

A multi-story steel structure building is considered. Site plan, floor plan, elevation, and section plans are given in Figs. 1.20, 1.21, 1.22, 1.23, 1.24 and 1.25. It has five

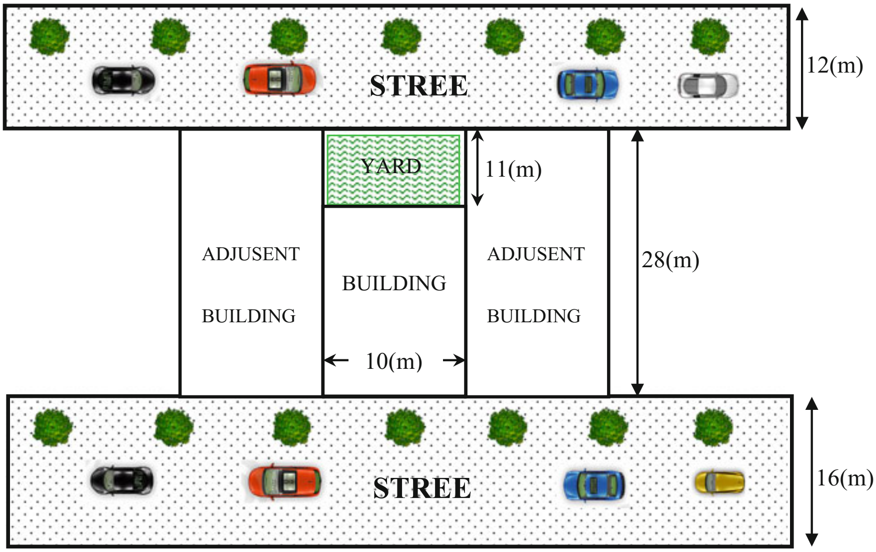


Fig. 1.20 Site plan of considered building

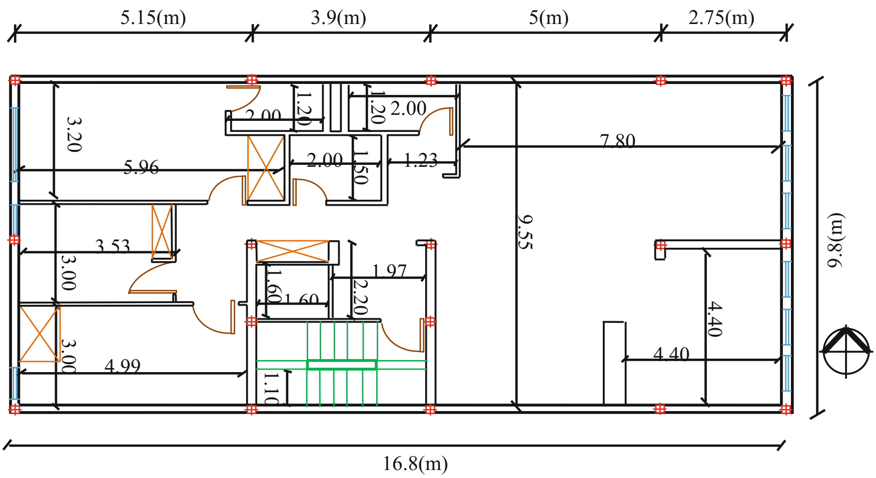


Fig. 1.21 Floorplan

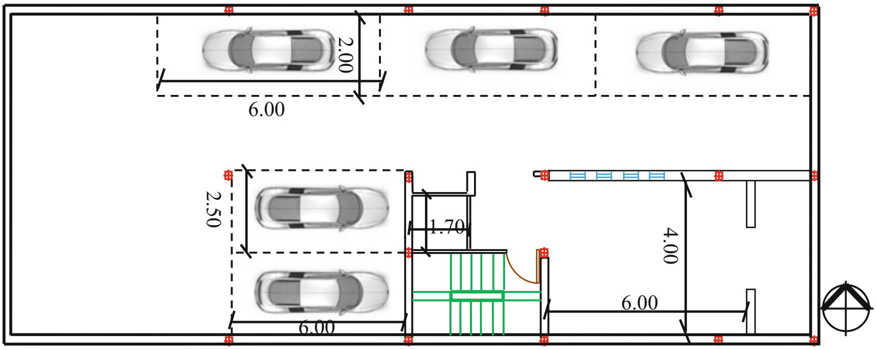


Fig. 1.22 Ground floor

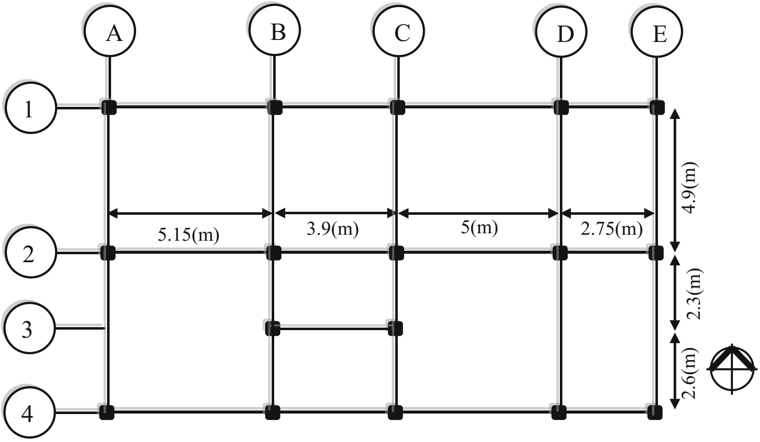


Fig. 1.23 Sketch illustrating the location of the columns, and beams on the floor plan

stories, a residential building, and the ground floor is occupied as a parking space. Total height of the building is 23.4 m, which includes the doom roof. The building is located in the high-risk zone of the earthquake on a site with type C soil and D in Euro code 8 and IBC 2012, respectively.

1.2.1 Architectural Plans

The architectural plans consist of the site, floor, section, and elevation plan that will be explored in this chapter.

Fig. 1.24 Section plan

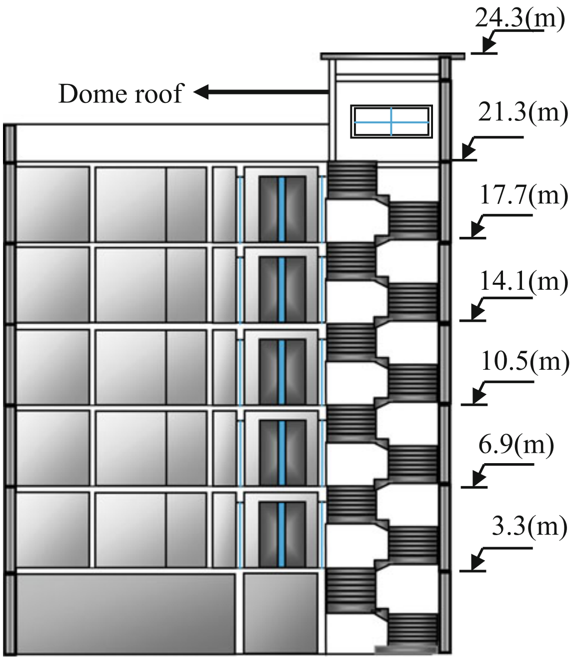
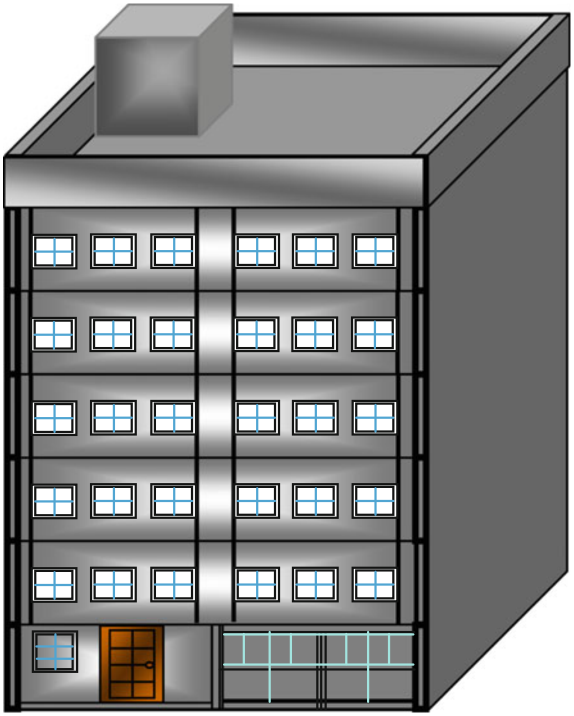


Fig. 1.25 Elevation plan



1.2.2 Site Plan

The proposed multi-story residential building is located in an urban area. The land lot measures 28 m × 10 m, while each floor measures 17 m × 10 m, with an allowable GFA of 60% from a total land of 170 m^2. The proposed GFA is 850 m^2. This land is surrounded by buildings to its east and west. It also includes two streets along its north side. There are two separation joints of 11 cm on the east and west sides of the building. Some detail of buildings shown in Table 1.1.

- **Separation Joint**:

Nowadays buildings are being constructed very close to each other in the metropolitan area. The separation distance is the space between two adjacent buildings, which prevents pounding each other during an earthquake. The minimum building separation distance is 0.5% of the structure height. The building has been surrounded by two buildings on both the east and the west side, as shown in the site plan. Therefore, the separation distance is considered 11 cm between the building and its adjacent constructions. Length of the separation distance = 0.005 H.

Where H is the height of the structure. In this problem, the height of the structure is equal to 21.3 m, then:

$$0.005 \times 21.3\text{ m} = 0.108\text{ m} \approx 11\,\text{cm}$$

1.2.2.1 Floor Plan

The structural system consists of the moment resistance frame. One of the typical floor and the ground floor are shown in Figs. 1.21 and 1.22, respectively. All the floors are used as residences, while the ground floor is used for parking space. The north-south direction of the plan has been divided into four bays by five grid lines (A–E), which is also the case in the east-west direction (1–4). The distances between the grid lines along the N–S direction are 5.15 (m), 3.9 (m), 5 (m), and 2.75 (m), respectively, whereas the distance between the grid lines along the E–W direction considered 4.9 (m), 2.3 (m), and 2.6 (m), respectively. Composite system is selected as a floor system with a 9 cm concrete slab.

Table 1.1 Plan requirements

Address	Zone (III)
Planning requirements:	
1. Site area	280 m^2
2. Allowable GFA	170 m^2
3. Proposed GFA	5 * 10 * 17 = 850 m^2
4. Number of floors	6 Floors
5. Number of units	5 Units
6. Building high	24.3
7. Parking	5

1.2.2.2 Elevation Plan

The elevation plans are demonstrating the configuration and detail of stories, as shown in Figs. 1.24 and 1.25. In Fig. 1.24a cross-section of the building is shown in which the height of each story and the thickness of the floors are given. The total height of the building above the ground floor is 21.3 m. The height of the first floor (between the levels 0 and 1) amounts to 3.3 m, while the height of the other floors are 3.6 m. Table 1.2 illustrates the architectural and structural levels and the other information of the structure.

1.2.3 Material Property

The steel and concrete material properties are shown in Table 1.3.

The ground type is determined by class C with $V_{s,30}$ = 350 as an average shear velocity.

$V_{s,30}$ = is Propagation's mean value of Swaves in 30 m topsoil profile at shear strain of 10^{-5} or less.

Table 1.2 Information on cross-section of structure

Level	Architecture level	Structural level	Height of floor from foundation	Height of floor	Height of wall
Roof	21	21	21.3	3.6	
5th floor	17.4	17.4	17.7	3.6	3.3
4th floor	13.8	13.8	14.1	3.6	3.3
Third floor	10.2	10.2	10.5	3.6	3.3
Second floor	6.6	6.6	6.9	3.6	3.3
First floor	3	3	3.3	3.3	3
Foundation	−0.3	−0.3			

Table 1.3 Material properties

Property	Material	
	Steel	Concrete
Mass per unit volume	$800\ \frac{kg}{m^3}$	$250\ \frac{kg}{m^3}$
Weight per unit volume	$7850\ \frac{kg}{m^3}$	$2500\ \frac{kg}{m^3}$
Young's modulus	$2\times10^6\ \frac{kg}{cm^2}$	$2\times10^5\ \frac{kg}{cm^2}$
Poisson's ratio	0.3	0.2

1.3 Loading

- Loading
- Center of Mass
- Eurocode8
- Shear Rigidity
- Torsional Force
- Distribution of Lateral Force
- Analysis of the Moment Resistance Frame Under Gravity Load
- The Approximate Method for Analysis of Moment Resistance Frame Under Lateral Load

1.3.1 Introduction

The loads applied to the structure are divided into three categories. These are gravity load, lateral load, and other loads. Within these categories, the gravity load is divided into four types, dead load, live load, snow load and rain load. Subsequent sections explain the Gravity load and lateral load.

1.3.2 Gravity Load

Gravity loads are those that are applied to the structure consisting of dead load, live load, snow load and rain load. In this book, to the focus is on two types of loads, namely the dead load and live load.

1.3.2.1 Dead Load

The total of all permanent parts of structure that consists of floor, wall, beam and column is referred as the dead load. Based on Fig. 1.26 dead load is applied to the structure in three different ways:

1. Distributed or surface load that acts over a surface area (measured in N/mm^2), such as floor load.
2. Line load is one that acts along a line (N/mm), such as weight of external wall beam.
3. Concentrated load (N).

For calculation of dead load, the following included are floor, wall, and staircase. The total weight of each part of the structure is defined as the density of the material and multiplied by the volume.

$$\text{Dead load} = \text{volume} * \text{density}$$

The dead load is the weight of floor, wall, staircase, beam, column and connection. The description of the factors in the calculation is described in the following sections.

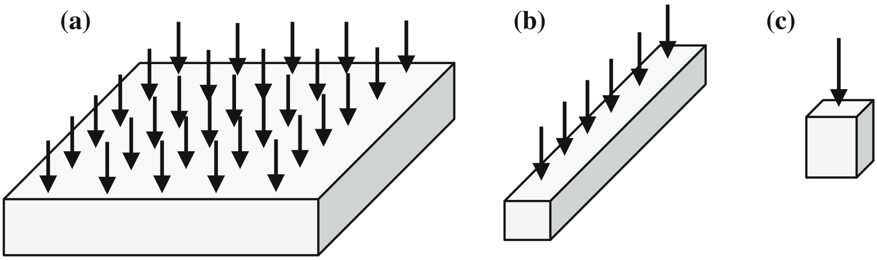


Fig. 1.26 Dead load **a** uniform load (N/mm^2), **b** uniform distributed load (N/mm), **c** concentrated load (N)

Floor System

One of the considerable functions of a floor, the system is to pick up and distribute the gravity load to a horizontal and vertical member of structure like beams, columns, and walls. This is fulfilled by the out of plane bending action of the floor system. The second structural function of floor system transfers the lateral load to different lateral load resistance systems. This action is called the diaphragm action. The lateral load such as an earthquake load, which strike the building are distributed to different lateral load resisting members through the diaphragm action of the floor slabs and roof. The floor system is connected to the building structural framing members equip an effective bracing system. As a bracing system, the diaphragm transfers lateral loads to lateral load resistance members such as shear wall, moment resistance frame and braced frame system. The composite steel-concrete floor system as a simple and convenient slab system is selected to be considered among all the floor systems.

Composite Floor

Among structures, concrete is used to cover the roof, which consists of steel girder and spans a cross different structures. In the concrete portion, and within the effective width, each plays a role in exerting certain force. In a cross-section which carries compression stress, and with the steel girder to carry tension stress, the two materials have to be structurally attached together. However, in the case of beams, this combination of tension is achieved using shear connectors, and these are attached to the upper flange of the steel beam. Due to the positioning of the thermal bars, specifically perpendicular to each other in the concrete, the bars prevent cracking. Figure 1.27 shows the composite floor with molding and shear stud.

Meanwhile, shear connectors are commonly referred to as stud or shear stud in the industry and is available in a broad range of sizes, material, and grades. Headed studs are most commonly used, while shear connectors including hooked studs, or pieces of C-channel are relatively less common. One advantage of the shear connectors is that it creates a strong bond between the steel beam and the concrete floor

Fig. 1.27 Composite floor

Fig. 1.28 Shear connectors. **a** Head stud **b** C-channel

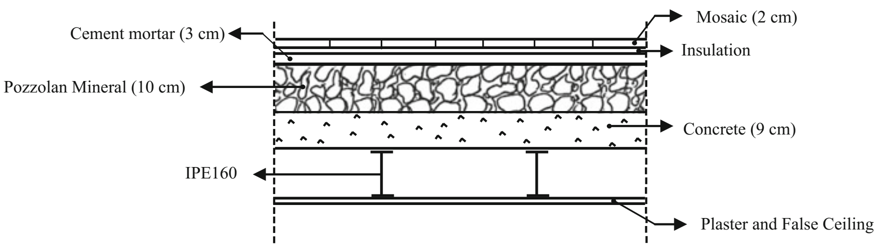


Fig. 1.29 Detail of roof

slab. This bond enables the concrete slab to interface with steel beams to reduce load deflection. Some types of the shear studs are shown in Fig. 1.28.

Some of the properties of the composite floor:

1. Molding is necessary for composite floor
2. Interstitial space between girders is 80–120 cm
3. Normal thickness of concrete is 8–10 cm
4. Inability of casting concrete all floors simultaneously
5. Shear stud is necessary (for more continuity)

- **Detail of composite roof**:

Figure 1.29 depicts the composite floor at roof level. A dead load of roof floor is calculated based on the details shown in Fig. 1.29. Thus, the total weight of floor is the summation of different parts of the floor. In order to calculate the weight of 1 m^2, density of each material should be multiplied by the thickness. Hence:

1. Weight of mosaic (kg/m^2) = (Unit weigh of mosaic (kg/m^3)) × (Thickness of mosaic (m) = 2400 (kg/m^3) × 0.02 (m) = 48 kg/m^2
2. Weight of cement mortar (kg/m^2) = (Unit weigh of cement mortar (kg/m^3)) × (Thickness of cement mortar (m)) = 2100 (kg/m^3) × 0.03 (m) = 63 kg/m^2

3. Weight of insulation $(\mathrm{kg/m^2}) = 15\ \mathrm{kg/m^2}$
4. Weight of mineral pozzolan $(\mathrm{kg/m^2}) = 600\ (\mathrm{kg/m^3}) \times 0.1\ (\mathrm{m}) = 60\ \mathrm{kg/m^2}$
5. Weight of concrete deck $(\mathrm{kg/m^2}) = 2500\ (\mathrm{kg/m^3}) \times 0.09\ (\mathrm{m}) = 225\ \mathrm{kg/m^2}$
6. Weight of IPE16 $(\mathrm{kg/m^2})$ = (Unit weigh of IPE160 $(\mathrm{kg/m^3})$) × (Area of cross section $(\mathrm{m^2})$)/1 (m) = $\left(7850\left(\frac{\mathrm{kg}}{\mathrm{m^3}}\right) \times 20.1 \times 10^{-4}(\mathrm{m^2})\right)/1\ (\mathrm{m}) = 15.8\ \mathrm{kg/m^2}$
7. Weight of plaster and false ceiling $(\mathrm{kg/m^2}) = 50\ \mathrm{kg/m^2}$

Total weight of the roof is the summation of seven different part of the floor, so:

Total weight of roof: $48\ \mathrm{kg/m^2} + 63\ \mathrm{kg/m^2} + 15\ \mathrm{kg/m^2} + 60\ \mathrm{kg/m^2} + 225\ \mathrm{kg/m^2} + 15.8\ \mathrm{kg/m^2} + 50\ \mathrm{kg/m^2} = 480\ \mathrm{kg/m^2}$.

Total weight of top most floor is the summation of the weight of its parts which is equal to 480 $(\mathrm{kg/m^2})$. Table 1.4 shows the total weight of the roof.

- **Detail of composite floor**:

Figure 1.30 shows the different segments of the composite floor at every story. Based on detail in Fig. 1.30, the total weight of one meter square of the floor is 435 $(\mathrm{kg/m^2})$. The calculation of the weight of floor is illustrated in Table 1.5.

Table 1.4 Calculation weight of the roof

Name	$A\ (\mathrm{m^2})/t\ (\mathrm{m})$	$\gamma\ (\mathrm{kg/m^3})$	$\mathrm{kg/m^2}$
Mosaic	0.02	2400	48
Cement mortar	0.03	2100	63
Insulation			15
Mineral pozzolana	0.1	600	60
Concrete	0.09	2500	225
IPE16	$20.1 \times 10^{-4}\ \mathrm{m^2}$	7850	15.8
Plaster and false ceiling			50
			$\simeq 480$

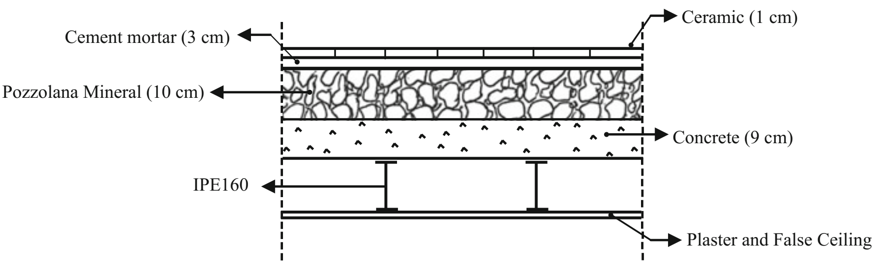


Fig. 1.30 Detail of floor

Table 1.5 Calculation weight of floor

Name	t (m)/A (m^2)	γ (kg/m^3)	kg/m^2
Ceramic	0.01	2100	21
Cement mortar	0.03	2100	63
Mineral pozzolana	0.1	600	60
Concrete	0.09	2500	225
IPE160	20.1×10^{-4} m^2	7850	15.8
Plaster and false ceiling			50
			435

Wall

The building walls function to support roofs, floors and ceiling, it encloses a space as part of the building with a roof which gives the buildings their form. In addition to the support provided by the walls, it also functions to provide shelter and security. The building wall in this book is divided two categories and is shown in Chart 1.2.

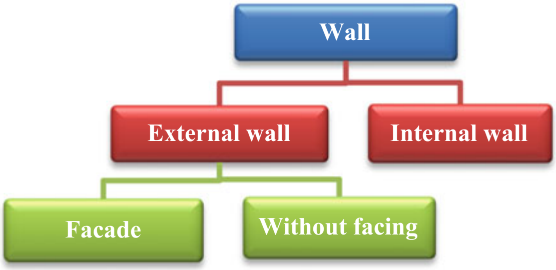
- **Internal wall (partition wall)**:

This is used to divide the interior space into rooms but does not support any weight. The interior wall of considered building consists of a brick wall (100, 200 mm), rendering and plaster. Brick wall with a 200 mm thickness is used in the parking space.

- **External wall**:

This is divided into two categories in this project namely, facade and without facing based, the exterior wall is located in the East–West direction is said to be façade external wall, while the external wall that located in the North–South is without facing. Figure 1.31 shows the position of the internal and external wall in the building as an example.

Chart 1.2 Building wall



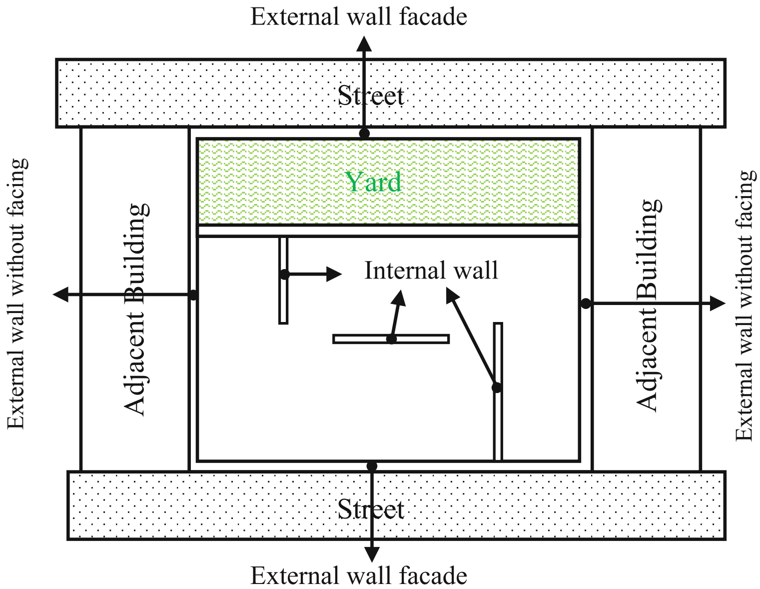


Fig. 1.31 The position of internal and external wall

Note:

As shown in Fig. 1.24, the East view consists of openings (windows) in the surrounding wall (façade wall). This can cause reduction of the total weight of surrounded wall (external wall). For this reason; the calculation weight should also include the total area of the opening with respect to the total area of surrounding the wall. Figure 1.32 shows the detail of location and size of the opening in the second story.

$$\frac{area\ of\ opening}{area\ of\ wall} = \frac{6 \times (1 \times 1.6)}{3.3 \times 9.8} = 0.296(29.6\%)$$

In this project, the size of the opening is taken to be 30%.

- **Details of Internal Wall**

Internal walls function as partitions which separate one space from another. An illustration of the separation of spaces using walls is shown in Fig. 1.33. The total weight a partition with 10 (cm) and 20 (cm) thickness are 175 $\left(\frac{\text{kg}}{\text{m}^2}\right)$ and 260 $\left(\frac{\text{kg}}{\text{m}^2}\right)$ respectively. The method to calculate each partition is shown in Tables 1.6 and 1.7.

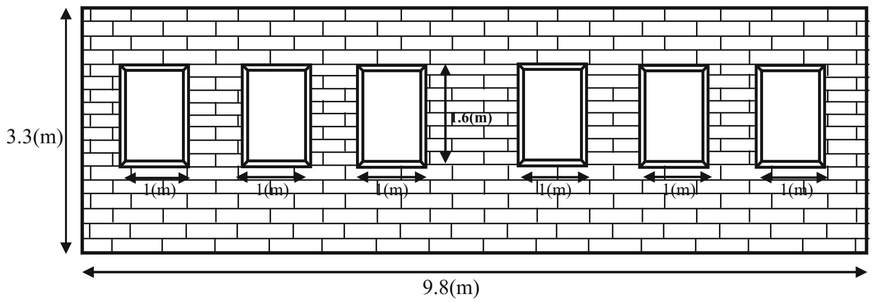


Fig. 1.32 Detail of location and size of opening in north view of building in the second story

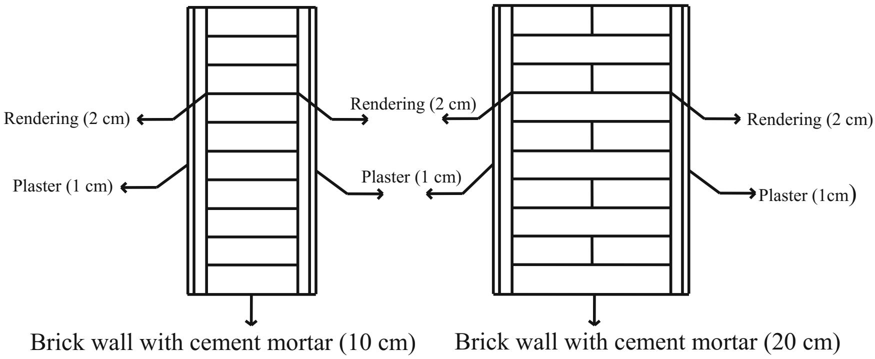


Fig. 1.33 Detail of internal wall

Table 1.6 Calculation weight of internal wall (10 cm)

Name	t(m)	γ(kg/m^3)	kg/m^2
plaster	0.02	1300	26
Rendering	0.04	1600	64
Brick wall	0.1	850	85
			175

Table 1.7 Calculation weight of internal wall (20 cm)

Name	t(m)	γ(kg/m^3)	kg/m^2
Plaster	0.02	1300	26
Rendering	0.04	1600	64
Brick wall	0.2	850	170
			260

Note:

The walls such as the partition around the toilet, bathroom, and kitchen may be had the details different with those in Fig. 1.33, For instance, in these partitions usually there is a layer of insulation with tiling which can change the total weight of the wall, but it is usually ignored for simplifying.

- **Equivalent distributed load of internal wall**:

The total weight of the internal wall (partition) is considered as a dead load and must be added to the dead load of the floor, so it should be calculated as an equivalent load. Equation (1.42) shows the equivalent partition load formula

$$Equivalent\,Distributed\,Load = \frac{W \times L \times h}{A} \tag{1.42}$$

where

W = weight of partition (kg/m^2)
L = Total length of partition (m)
h = Net-height of partition (m)
A = Area of story (m^2)

The floor plan in Fig. 1.21 shows that the net area of the building in each story is 154.5 m^2. The total length of the partition of thickness 20(cm) in the parking and other stories are 16.15 (m), and 17.55 (m), respectively and the total length of the partition of thickness 10 (cm) is 21.9 (m). Based on Eq. (1.2), the equivalent load of the partition in the parking and stories are 81.533 $\frac{\text{kg}}{\text{m}^2}$ and 179.32 $\frac{\text{kg}}{\text{m}^2}$, respectively.

The equivalent load of the partition in the Parking

$$\begin{cases} Area = 154.5\ \text{m}^2 \\ total\ length = 16.15\ \text{m} \\ netheight = 3\ \text{m} \\ the\ weight\ of\ partition = 260\ \frac{\text{kg}}{\text{m}^2} \end{cases} \overset{\text{Eq. (42)}}{\Rightarrow} \frac{w \times l \times h}{A} = \frac{260 \times 16.15 \times 3}{154.5} = 81.533\ \frac{\text{kg}}{\text{m}^2}$$

The equivalent load of the partitions of 10 (cm) thickness:

$$\begin{cases} Area = 154.5\ \text{m}^2 \\ total\ length = 21.9\ \text{m} \\ pureheight = 3.3\ \text{m} \\ weight\ of\ partition = 175\ \frac{\text{kg}}{\text{m}^2} \end{cases} \overset{\text{Eq. (42)}}{\Rightarrow} \frac{w \times l \times h}{A} = \frac{175 \times 21.9 \times 3.3}{154.5} = 81.86\ \frac{\text{kg}}{\text{m}^2}$$

The equivalent load of the partitions of 20 (cm) thickness:

$$\left\{\begin{array}{c} Area = 154.5\ \mathrm{m}^2 \\ total\ length = 17.55\ \mathrm{m} \\ pureheight = 3.3\ \mathrm{m} \\ weight\ of\ partition = 260\ \frac{\mathrm{kg}}{\mathrm{m}^2} \end{array}\right. \overset{\text{Eq. (42)}}{\Rightarrow} \frac{w \times l \times h}{A} = \frac{260 \times 17.55 \times 3.3}{154.5} = 97.46\ \frac{\mathrm{kg}}{\mathrm{m}^2}$$

The equivalent load of the partition in the stories = the equivalent load of the partitions of 10 (cm) thickness + the equivalent load of the partitions of 20 (cm) thickness.

The equivalent load of the partition in the stories = 81.86 + 97.46 = 179.32 $\frac{\mathrm{kg}}{\mathrm{m}^2}$.

- **Detail of External Wall**:

The external wall is referred to walls that surround the building with 20 (cm) thickness. Such walls can be divided into two types, facade and without facing. The detail of external wall is shown in Fig. 1.34. The total weight of facade wall is 307 $\left(\frac{\mathrm{kg}}{\mathrm{m}^2}\right)$ while the total weight of without facing is 236 $\left(\frac{\mathrm{kg}}{\mathrm{m}^2}\right)$. Tables 1.8 and 1.9 show the calculation of external wall.

As observed, a 30% opening is calculated for external façade wall in North–South of the building. For this reason, the total weight of external façade wall is reduced to 215. The calculation of the total weight of external façade wall is shown formulation (1.42):

$$\left(1 - \frac{\text{area of the opening}}{\text{area of the wall}}\right) \times \text{total weight of external wall withfacing}$$
$$= \text{total weight of wall with opening}$$

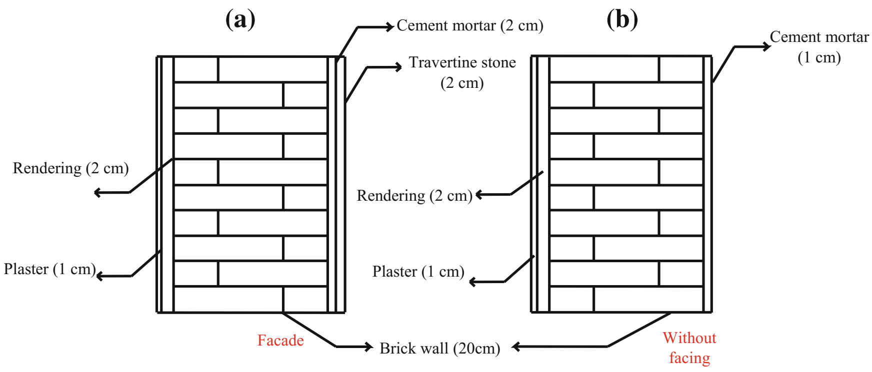


Fig. 1.34 The detail of the external wall **a** façade **b** without facing

Table 1.8 Calculation weight of external wall (20 cm) without facing

Name	t(m)	γ(kg/m^3)	kg/m^2
Plaster	0.01	1300	13
Rendering	0.02	1600	32
Brick wall	0.02	850	170
Cement mortar	0.01	2100	21
			236

Table 1.9 Calculation weight of external wall (20 cm) facade

Name	t(m)	γ(kg/m^3)	kg/m^2
Plaster	0.01	1300	13
Rendering	0.02	1600	32
Brick wall	0.02	850	170
Cement mortar	0.02	2100	42
Travertine	0.02	2500	50
			307

Hence

$$(1 - 0.3) \times 307 \frac{\text{kg}}{\text{m}^2} = 215 \frac{\text{kg}}{\text{m}^2}$$

- **Detail of parapet wall**:

A parapet is an external wall at the edge of a roof, terrace, balcony, walkway or other structure. A parapet may simply be the part of an exterior wall that continues above the line of the ceiling surface that offers inhabitants against a sudden drop. The calculation of the weight of this wall is shown in Fig. 1.35 and Table 1.10. The height of parapet wall in this module is 0.6 m, and total weight of parapet wall is 205 $\frac{\text{kg}}{\text{m}^2}$.

Staircase

Stairs are primarily a bridge for a large inclined space that is interrupted by small vertical distances. The interruptions can be of straight, round shape and consist of two or more straight pieces connected at right angles. Figure (1.36) shows a typical view of the staircase. Some parts of staircase are:

- **Step**

The step is composed of the tread and riser.

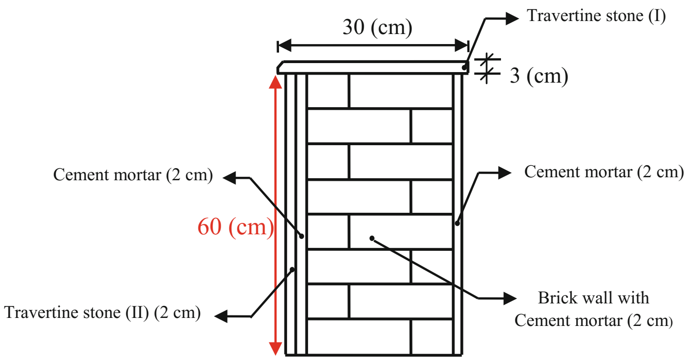
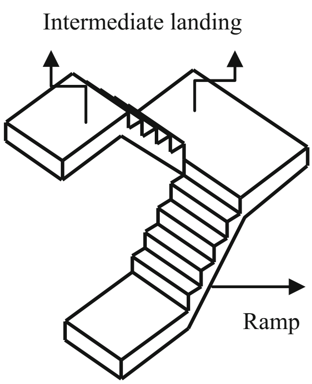


Fig. 1.35 Detail of parapet wall

Table 1.10 Calculation of the weight of the parapet wall of 0.6 (m) height

Name	t(m)	h (m)	γ(kg/m^3)	kg/m
Travertine (I)	0.03	0.3	2500	22.5
Cement mortar	0.04	0.6	2100	50.4
Brick wall	0.02	0.6	850	102
Travertine (II)	0.02	0.6	2500	30
				205

Fig. 1.36 Typical view of staircase



- **Tread**:

This is the part of the stair way where inhabitants of a building lay their first step. Often the thickness of the tread is constructed as same as any other floor. The tread "depth" is measured from the outer edge of the step to the vertical riser" between "the steps. Similarly, the "width" of the thread taken from both ends of the side.

- **Rise**

The riser is the vertical part between each treads on the stair, which may be absent for an “open” stair effect.

- **Stringer**

In majority of the staircase types, a stringer is a structural segment which supports the treads.

- **Landing or platform**

This is defined as the floor area at the top or bottom step in a staircase Also, an intermediate landing aids in cases where a change in the direction of the stairs is necessary. This is often a small platform that is built as part of the stair, but it is still in between main floor levels. The intermediate landing can also function as the area to pause while taking the stairs. The detail of staircase is shown in Fig. 1.37.

- **Calculation of staircase dead load**:

A dead load of stairs is consists of two parts:

1. A dead load of the ramp is further divided into:
   1.1 dead load of main part of the ramp
   1.2 dead load of steps
2. Dead load of landing

Chart 1.3 shows the main section of the staircase for calculation of dead load.
Figure 1.38 shows the method of calculating the dead load of a stair case. The figure shows two sectional views (AA, B-B) and illustrates the details of ramp and landing for calculating of the stair dead load.

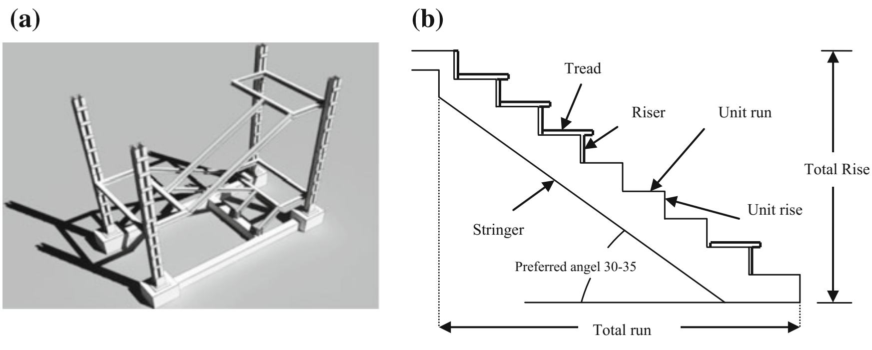


Fig. 1.37 Staircase. **a** Steel staircase, **b** concrete staircase, **c** detail of staircase

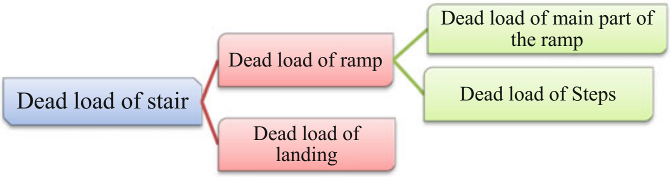


Chart 1.3 The main part of staircase for calculating

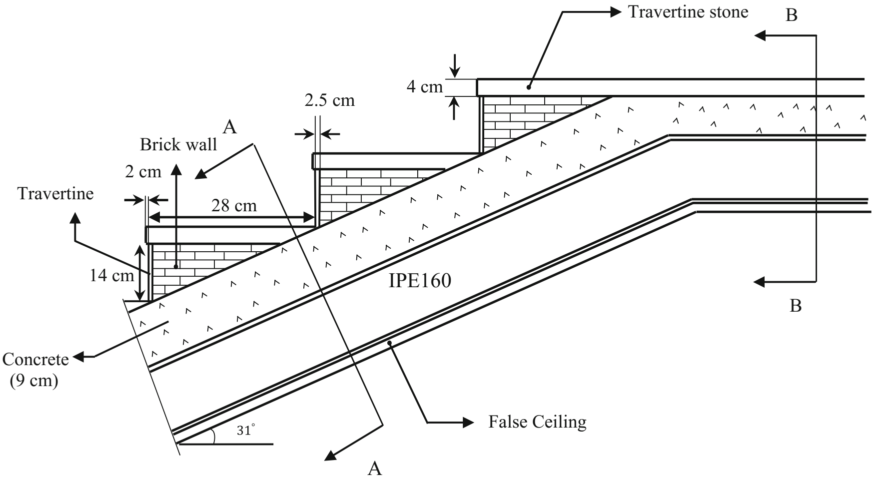


Fig. 1.38 Detail of stair

Step 1: **(Calculation of a dead load of the ramp)**:
Based on section A-A in Fig. 1.39, magnitude of dead load of the ramp is 307 $\frac{\text{kg}}{\text{m}^2}$ and detail of calculation is shown in Table 1.11.

Step 2: **Calculation of a dead load of stair's step**:
Detail of step is shown in Fig. 1.40. According to Fig. 1.40a–b, each step can be equivalent of a triangular part with 18, 30.5, 36 cm dimension, then the slope of stair should be calculated a computed number of steps in 1 m. Based on Eq. 1.43, slope of the staircase is equal to 31°. Equation 1.44 illustrated the number of steps in 1 m of the stair. The weight of one step is about 95 $\frac{\text{kg}}{\text{m}^2}$, as shown in Table 1.12.

$$\begin{cases} \sqrt{30.5^2 + 18^2} = 36 \\ \alpha = \tan^{-1}\left(\frac{18}{30.5}\right) = 31 \end{cases} \tag{1.43}$$

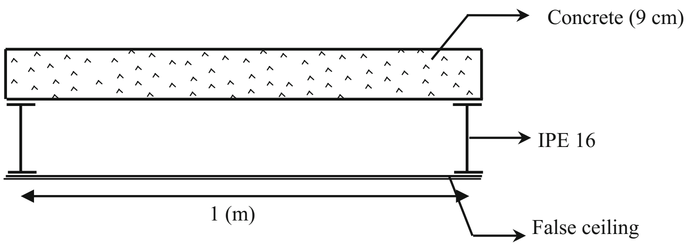


Fig. 1.39 Section A-A

Table 1.11 Calculation of ramp of section A-A

Name	Number	Thickness	$\gamma\,(kg/m)^3$	kg/m^2
Concrete	1	0.09	2500	225
IPE160	2		7850	31.6
False ceiling	1			50
				307

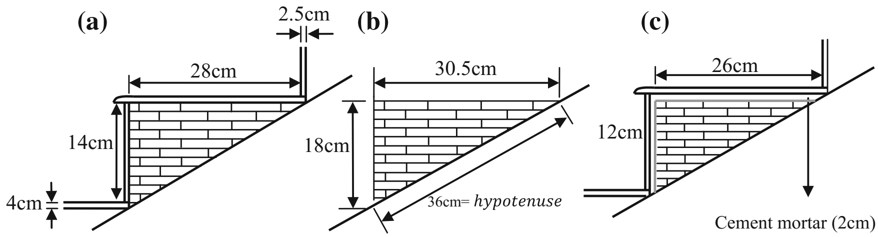


Fig. 1.40 Detail of step. **a** Dimension of step, **b** dimension of triangular brickwork, **c** dimension of mortar cement

Table 1.12 Calculation of step for 1 (m) of width of stair

Name	area (m^2)	$\gamma\,(kg/m^3)$	kg/m
Travertine (I)	0.345×0.04	2500	34.5
Cement mortar	$0.02 \times (0.3 + 0.2)$	2100	21
Brick wall	$(1/2) \times (0.26 \times 0.12)$	1850	28.86
Travertine (II)	0.025×0.16	2500	10
			94.36 = 95

$$Number\ of\ steps\ in\ 1\ \mathrm{m} = \frac{1}{dimention\ of\ hypotenuse} = \frac{1}{0.36} = 2.8 \quad (1.44)$$

Weight of steps in 1 (m^2) = Number of steps in 1 (m) × weight per meter of one step.
So:

$$\frac{1}{0.36} \times 95 = 264\ \mathrm{kg/m^2}$$

Step 3: **Calculation of total dead load of stair's ramp**:
Prior to the ramp, a stair is divided into two parts (the main part of the ramp, steps). Based on parts, the total dead load of the ramp is a summation of the weight of these two parts. Dead load of the ramp (that is associated with the staircase) is 307 $\frac{\mathrm{kg}}{\mathrm{m^2}}$ and dead load of stair case steps is 264 $\left(\frac{\mathrm{kg}}{\mathrm{m^2}}\right)$. The summation of these loads is equal to 571 $\left(\frac{\mathrm{kg}}{\mathrm{m^2}}\right)$ in the long side of the slope. Slope side of the stair is calculated by Eq. (1.43)

$$\text{Total weight of stair in a long of horizontal} = \frac{\text{Total weight of stair in the length of slope}}{\cos(\propto)} \quad (1.45)$$

Based on Eq. (1.45), total dead load of staircase ramp is 684 $\left(\frac{\mathrm{kg}}{\mathrm{m^2}}\right)$.

Total weight of stair in the length of slop = 307 + 280 = 571 $\left(\frac{\mathrm{kg}}{\mathrm{m^2}}\right)$

Total weight of stair in length of horizontal = 571/cos (31) = 666.1 $\left(\frac{\mathrm{kg}}{\mathrm{m^2}}\right)$

Step 4: **Calculation of a dead load of landing**:
Based on the section B-B and the details of the landing in Fig. 1.41, the dead load of the stairs landing is 399 $\left(\frac{\mathrm{kg}}{\mathrm{m^2}}\right)$. The detail of the calculation of the landing load is shown in Table 1.13.

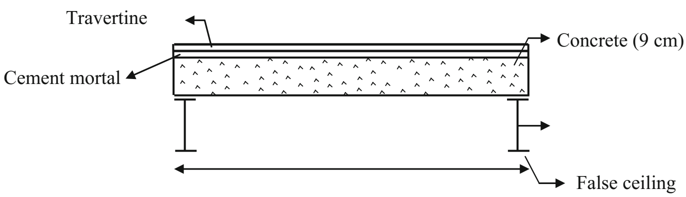


Fig. 1.41 Detail of stair's landing

Table 1.13 Calculation of landing part

Name	$t(\mathrm{m})/A(\mathrm{m}^2)$	$\gamma(\mathrm{kg/m}^3)$	$\mathrm{kg/m}^2$
Travertine	0.02	2500	50
Cement mortar	0.025	2100	52.5
Concrete	0.09	2500	225
IPE160	$20.1 \times 10^{-4}\ \mathrm{m}^2$	7850	15.8
False ceiling			55
			399

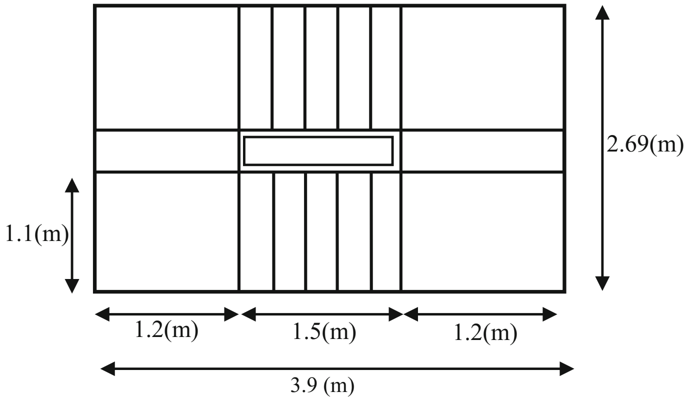


Fig. 1.42 Illustration of the dimensions of staircase

Step 5: **Calculation of the total weight of staircase**:

The dimension of the staircase is shown in Fig. 1.42. According to the floor plans, the total net area of the staircase is 10.14 (m^2). The landing area consists of two parts of 2.6 m × 1.2 m dimensions, and also the staircase ramp consists of two parts with dimensions of 1.5 m × 1.1 m Based on these dimensions, the area of landing and ramp area are 6.24 (m^2) and 3.3 (m^2) respectively. Total weight of stair is computed from Eq. (1.46)

$$Total\ weight\ of\ stair = \sum (area\ of\ stair\ parts \times weight\ of\ the\ stair\ parts) \tag{1.46}$$

Total area of landings = 2 × (2.6 × 1.2) = 6.24 m^2
Total area of ramps = 2 × (1.1 × 1.5) = 3.3 m^2
Based on Eq. (1.46);

$$
\begin{aligned}
\textit{Total weight of staircase} &= (\textit{area of landing} \times \textit{weight of landing}) \\
&\quad + (\textit{area of ramp} \times \textit{weight of ramp}) \\
&\quad (6.24\ \text{m}^2 \times 399\ \frac{\text{kg}}{\text{m}^2}) + (3.3 \times 666.15\ \frac{\text{kg}}{\text{m}^2}) = 4688\ \text{kg}
\end{aligned}
$$

The equivalent uniformly distributed dead load of the staircase is calculated by Eq. (1.47)

$$
\begin{aligned}
&\text{Equivalent uniformly} - \text{distributed dead load of staircase} \\
&\quad = \frac{\text{Total weight of staircase}}{\text{pure area of the staircase}}
\end{aligned} \tag{1.47}
$$

Therefore, the equivalent uniformly distributed dead load of the staircase will be:

$$
\frac{4688}{(2.6 \times 3.9)} = 462 \frac{\text{kg}}{\text{m}^2}
$$

- **The weight of steel structure**:

To compute the weight of steel structure frame, the total weight of columns, beams, and connection need to be calculated. Sections of column base on architectural drawing in each story and column properties are illustrated in Tables 1.14 and 1.15. Half of the height of higher floor and half of height of lower floor is used for calculation of the column weight. For instance, in the first story, half of the height of story (1) and another half of the height of story (2) are utilized for calculating the weight of columns. Equation 1.48 shows the method to calculate the column weights. The calculation of columns is illustrated in Tables 1.14 and 1.15. At the same time, the weight of the beam is computed based on the total length of the beam multiplied by the area of beam and the specific weight of the beam. This is captured in Eq. 1.49, while Table 1.16 shows the sections of the beam. The weight of connection is calculated according to 20% of summation of the total weight of

Table 1.14 Section of column in each story

Column	Size of story 1, 2	Size of story 3, 4	Size of story 5, 6
C-1	*Box* 300 × 10	*Box* 250 × 10	*Box* 250 × 10
C-2	*Box* 350 × 10	*Box* 300 × 10	*Box* 300 × 10
C-3	*Box* 400 × 10	*Box* 350 × 10	*Box* 300 × 10
C-4	*Box* 450 × 15	*Box* 400 × 12	*Box* 350 × 10

Table 1.15 Sections of beam

Beam	**Size**
B-1	Pg 350 × 150 × 10
B-2	Pg 350 × 150 × 8
B-3	Pg 500 × 200 × 10
B-4	Pg 350 × 200 × 10
B-5	IPE 140
B-6	IPE 160
B-7	IPE 200

Table 1.16 Column properties

Column size	Area cm^2	γ	$G_{kg/m}$
Box 250 × 10	96	7850	75.36
Box 300 × 10	116	7850	91.06
Box 350 × 10	136	7850	106.76
Box 400 × 10	156	7850	122.46
Box 400 × 12	186.24	7850	146.19
Box 450 × 15	261	7850	204.88

columns and a beam in each story. Figure 1.43 shows the location and section of beams and columns.

$$\begin{aligned}&\sum[NC[(Mass\,per\,unit\,C \times Half\ of\ the\ height\ of\ story\ top)\\&+(Mass\,per\,unit\,C \times Half\ of\ the\ height\ of\ story\,bottom)]]\end{aligned} \tag{1.48}$$

where this equation:
NC = Number of column

$$\begin{aligned}\text{Weight of beam} &= (\text{length of beam}) \times (\text{area of beam})\\&\times(\text{specific weight of steel})\end{aligned} \tag{1.49}$$

The weight of columns in the first story is calculated according to Eq. 1.48. Based on Fig. 1.43, there are four types of columns (C_1, C_2, C_3, and C4). As Fig. 1.43 shows, the number of columns in C_1 is 4, the number of column C_2, C_3, and C_4 are 3, 7, and 3, respectively. Area and mass per unit of each type of column are illustrated in Table 1.15. At least half of the height of story one is 1.65 m, and half of the height of story two is 1.8 m. Hence the total weight column in story one is 7439 kg. The total weight of the column in each story is shown in the Table 1.17.

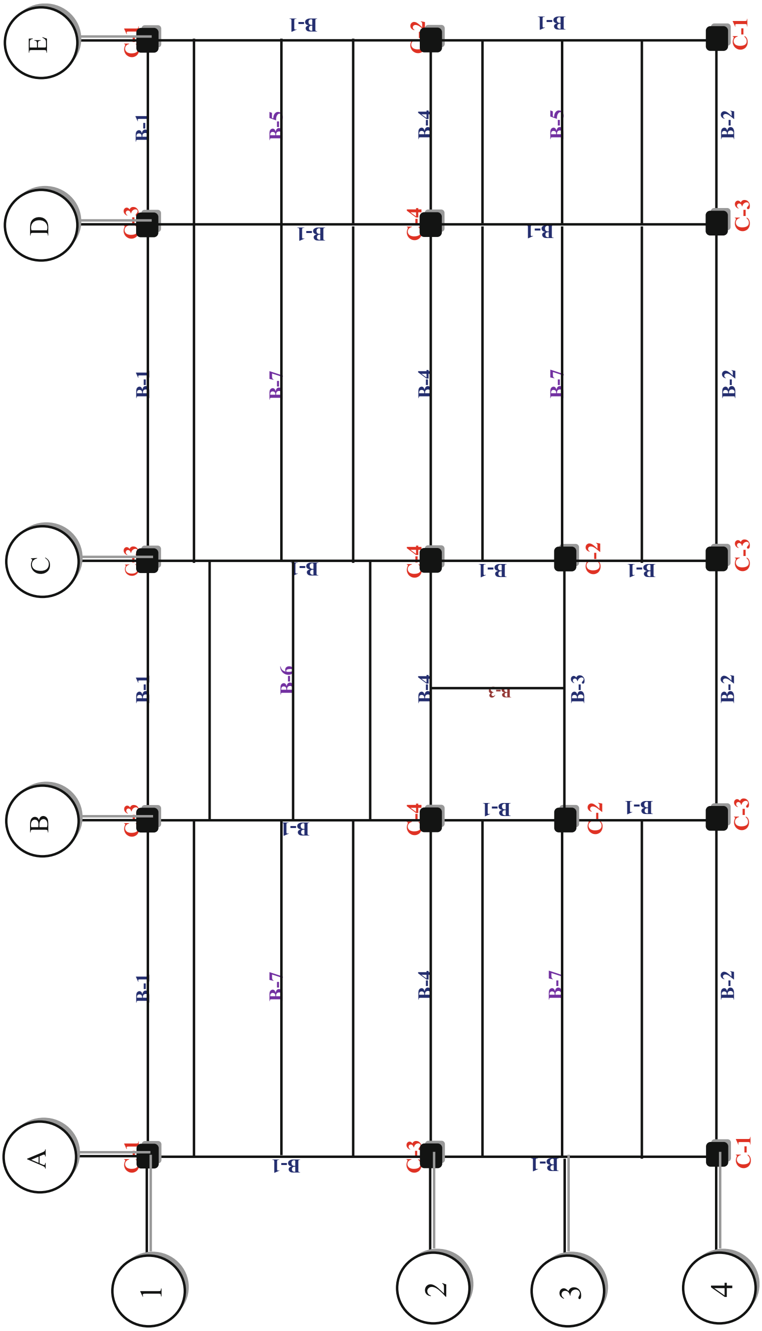


Fig. 1.43 Columns and beams situation

Table 1.17 Total weight of column in each story

Story	Column total weight
1	7439.511
2	7049.968
3	6337.836
4	5927.094
5	5516.352
6	3304.536
Dome roof	546.36

Table 1.18 Calculation of column's weight in each story

Story	Name of column	Number of columns	Size of box	$G_{kg/m}$	Half story height above and below	Weight of column
1	C-1	4	300 × 10	91.06	3.45	1256.628
7439.521	C-2	3	350 × 10	106.76	3.45	1104.966
	C-3	7	400 × 10	122.46	3.45	2957.409
	C-4	3	450 × 15	204.881	3.45	2120.5184
2	C-1	4	300 × 10	91.06	1.8	655.632
7049.94	C-1	4	250 × 10	75.36	1.8	542.592
	C-2	3	350 × 10	106.76	1.8	576.504
	C-2	3	300 × 10	91.06	1.8	491.724
	C-3	7	400 × 10	122.46	1.8	1542.996
	C-3	7	350 × 10	106.76	1.8	1345.176
	C-4	3	450 × 15	204.8	1.8	1105.92
	C-4	3	400 × 12	146.19	1.8	789.426
3	C-1	4	250 × 10	75.36	3.6	1085.184
6337.836	C-2	3	300 × 10	91.06	3.6	983.448
	C-3	7	350 × 10	106.76	3.6	2690.352
	C-4	3	400 × 12	146.19	3.6	1578.852
4	C-1	4	250 × 10	75.36	3.6	1085.184
5927.094	C-2	3	300 × 10	91.06	3.6	983.448
	C-3	7	350 × 10	106.76	1.8	1345.176
	C-3	7	300 × 10	91.06	1.8	1147.356
	C-4	3	400 × 12	146.19	1.8	789.426
	C-4	3	350 × 10	106.76	1.8	576.504
5	C-1	4	250 × 10	75.36	3.6	1085.184
5516.352	C-2	3	300 × 10	91.06	3.6	983.448
	C-3	7	300 × 10	91.06	3.6	2294.712
	C-4	3	350 × 10	106.76	3.6	1153.008
6	C-1	4	250 × 10	75.36	1.8	542.592
3304.536	C-2	2	300 × 10	91.06	3.3	600.996
	C-2	1	300 × 10	91.06	1.8	163.908
	C-3	2	300 × 10	91.06	3.3	600.996
	C-3	5	300 × 10	91.06	1.8	819.54
	C-4	3	350 × 10	106.76	1.8	576.504
Dome roof	C-2	2	300 × 10	91.06	1.5	273.18
546.36	C-3	2	300 × 10	91.06	1.5	273.18

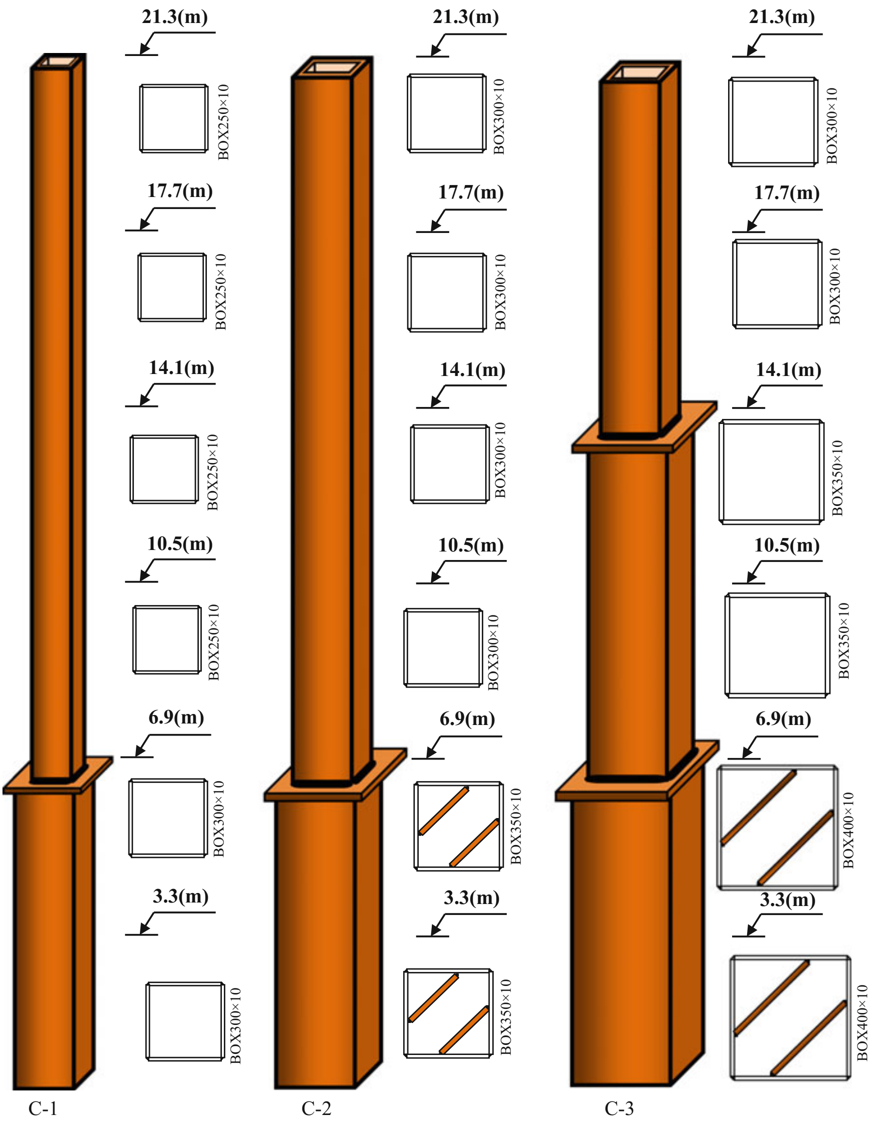


Fig. 1.44 Different types of columns of considered building

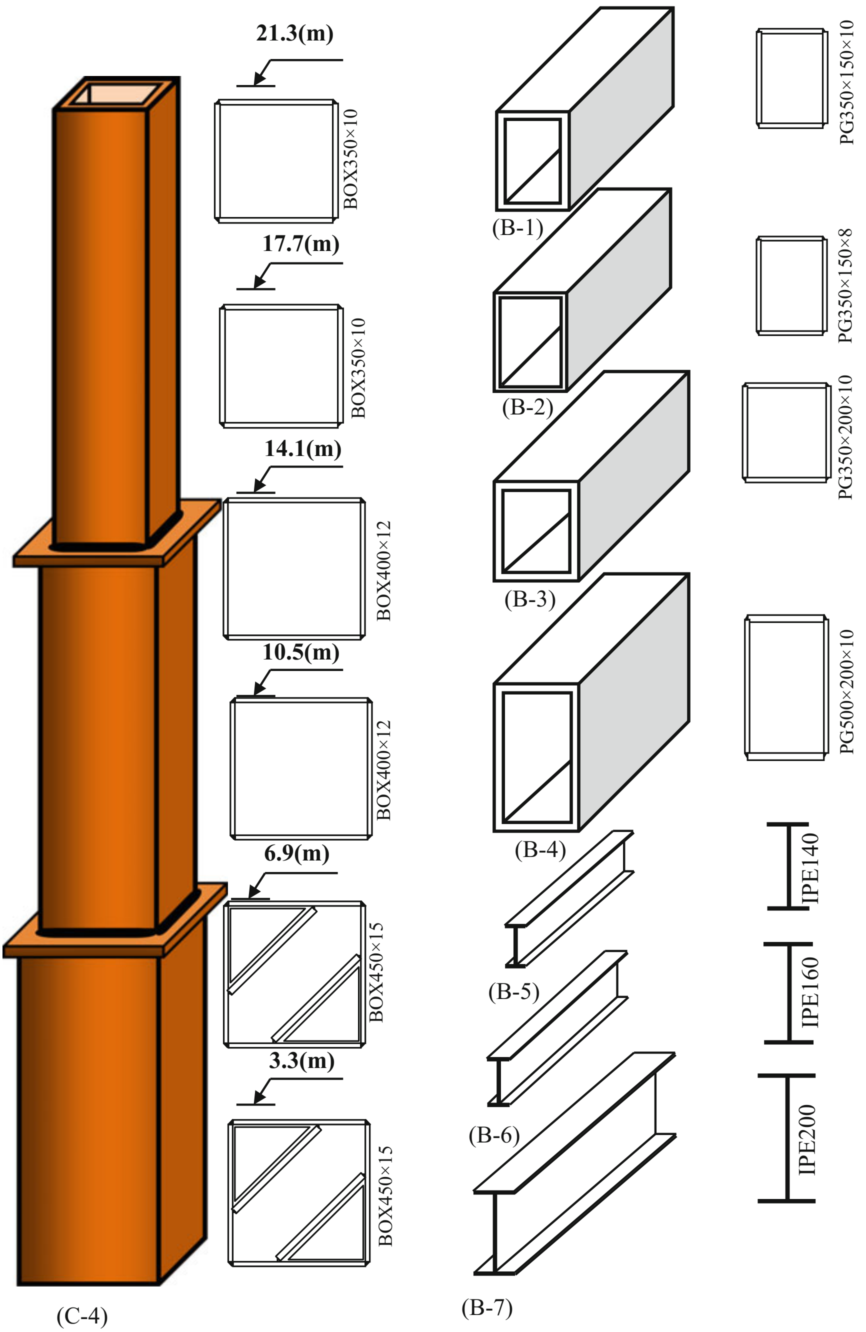


Fig. 1.45 Different types of columns and beams of considered building

But Table 1.18 shows the calculation of column in each story in details. The name of column, size, area mass per unit, plate (if added to the column), the volume of the plate and a particular weight of plate are outlined in Table 1.18. The table shows the total weight of every type of column separately. Figures 1.44 and 1.45 demonstrate different types of beam and columns sections that are used in the considered building.

- **The weight of columns in each story**:

Story 1: Half of the weight of first column story + half of the weight of second Colum

$$[4[(91.06 \times 1.65) + (91.06 \times 1.8)]] + [3[(106.76 \times 1.65) + (106.76 \times 1.8)]] \\ + [7[(122.46 \times 1.65) + (122.46 \times 1.8)]] + [3[(204.88 \times 1.65) + (204.88 \times 1.8)]] = 7439.511$$

So:

$NC_3 = 4$ Mass per unit of $C_1 = 91.06$ Half of the height of story $1 = 1.65\text{m}$

$NC_4 = 3$ MMass per unit of $C_2 = 106.76$ Half of the height of story $2 = 1.8\text{m}$

$NC_5 = 7$ Mass per unit of $C_3 = 122.46$

$NC_6 = 3$ Mass per unit of $C_4 = 204.88$

Story 2: Half of the weight of second column story + half of the weight of third column story

$$[4[(91.06 \times 1.8) + (75.36 \times 1.8)]] + [3[(106.76 \times 1.8) + (91.06 \times 1.8)]] \\ + [7[(122.46 \times 1.8) + (106.76 \times 1.8)]] + [3[(204.88 \times 1.8) + (146.19 \times 1.8)]] = 7049.968$$

Story 3: Half of the weight of third column story + half of the weight of fourth column story

$$[4[(75.36 \times 3.6) + (91.06 \times 3.6)]] + [7[(106.76 \times 3.6)]] + [3[(146.19 \times 3.6)]] \\ = 6337.836$$

Story 4: Half of the weight of fourth column story + half of the weight of fifth column story

$$[4[(75.36 \times 3.6)]] + [3[(91.06 \times 3.6)]] + [7[(106.76 \times 1.8) + (91.06 \times 1.8)]] \\ + [3[(146.19 \times 1.8) + (106.76 \times 1.8)]] = 5927.094$$

Story 5: Half of the weight of fifth column story + half of the weight of roof column story

$$[4[(75.36 \times 3.6)]] + [3[(91.06 \times 3.6)]] + [7[(91.06 \times 3.6)]] + [3[(106.76 \times 3.6)]] = 5516.352$$

Story 6: Half of the weight of roof column story + half of the weight of dome roof column story

$$[4[(75.36 \times 1.8)]] + [1[(91.06 \times 1.8)]] +] + [2[(91.06 \times (1.8 + 1.5))]] + [5[(91.06 \times 1.8)]] + [2[(91.06 \times (1.8 + 1.5))]] + [3[(106.76 \times 1.8)]] = 3304.536$$

Dome roof: $[2[(91.06 \times 1.5)]] + [2[(91.06 \times 1.5)]] = 546.36$

Total weight of column in story 1 is 7439.511 (kg). The weights for stories 2, 3, 4, 5 and six are 7049.968 (kg), 6337.836 (kg), 5927.094 (kg), 5516.352 (kg) and 3304.536 (kg) respectively. Total weight of the column of dome roof is 546.36 (kg).

Total weight of beams is computed according to the total length, area and specific gravity of each section and the multiplication between them. Tables 1.19 and 1.20 show the total weight of beam sections at each story and dome roof respectively. Total weight of beam at every story is 9943.4 kg, and dome roof is equal to 1052.06 kg. Based on these values, the total weight of steel structure is calculated in Table 1.21.

Table 1.19 Total weight of beam in each story

Beam section	Length (m)	Area (m^2)	γ	Total weight
$Pg(300 \times 150 \times 8)$	16.8	0.006944	7850	915.7747
$Pg(350 \times 200 \times 10)$	6.2	0.0106	7850	515.902
$Pg(350 \times 150 \times 10)$	65.8	0.0096	7850	4958.688
$Pg(500 \times 200 \times 10)$	16.8	0.0136	7850	1793.568
IPE20	30 + 30.9	0.00285	7850	1362.48
IPE16	11.7	0.00201	7850	184.608
IPE14	16.5	0.00164	7850	212.421
				9943.4417

Table 1.20 Total weight of beam in dome roof

Beam section	Length (m)	Area m^2	γ	Total weight
$Pg(350 \times 200 \times 10)$	3.9	0.0106	7850	324.52
$Pg(350 \times 150 \times 8)$	3.9	0.006944	7850	212.6
IPE16	7.8	0.00201	7850	123.0723
$Pg(350 \times 150 \times 10)$	5.2	0.0096	7850	391.87
				1052.0643

Table 1.21 Total weight of steel structure in each story

Story	Weight of beam	Weight of column	Total weight of beam/column	Net Area of plan	Weight	Weight of connection	Total weight
1	9943.4417	7439.511	17382.9527	154.5	112.511	22.50	135.011
2	9943.4417	7049.968	16993.409	154.5	109.99	22	132
3	9943.4417	6337.836	16281.2777	154.5	105.380	21.076	126.456
4	9943.4417	5927.094	15870.5357	154.5	102.722	20.544	123.266
5	9943.4417	5516.352	15459.7937	154.5	100.0633	20.01	120.0733
6	9943.4417	3304.536	13247.9777	154.5	85.747	17.15	102.897
Dome roof	1052.0643	546.36	1598.4243	10.14	157.635	31.527	189.162

Table 1.21 shows the total weight of steel structure frame for every story. According to this table, the total weight of steel frame consists of the total weight of column and beam for a specific story. As indicated, the weight of connection at every story is approximately 20% of total weight. For instance, if the total weight of column in story 3 is 6337.836 kg and the total weight of beam in this story is 9943.4(kg), then the summation of total weight is 16281.2 kg. The floor area of the plan is 154.5 m^2, so the total weight of summation of column and beam per area is 105.38 kg/m^2. As a result, the weight of connection at the third story is 20% of 105.38 kg/m^2 that it is equal to 21.07 kg/m^2. Hence, total weight of steel structure frame in story 3 is 105.38 + 21.07 = 126.45 kg/m^2. Total weight of steel structure will be calculated as the same as the third story.

1.3.2.2 Live Load

Live loads, also known as imposed loads, are temporary and are of short duration. They are also referred to as moving load or probabilistic loads. This particular type of load includes all forces that are variable within the object's normal operation cycle, not including construction or environmental loads. In addition, these dynamic loads may involve considerations such as impact, momentum, vibration,

Table 1.22 Summary of loading (kg/m^2)

Story	Roof dead load	Partition's load		External wall (without facing)	External wall (facade)	Live load
Floor	480	81.533	179.3	236 (kg/m^2)	215 (kg/m^2)	150
Roof	435	81.533	179.3	205 (kg/m)	205 (kg/m)	200
Stair	462	81.533	179.3			350

slosh dynamics of fluids and material fatigue. Moreover, roof and floor live loads are produced during maintenance by workers, equipment and materials. Such loads are also produced during the life of the structure by movable objects, such as planters and people. In Table 1.22, a summary of various loading is shown. It is noted that the weight of the steel structure is not interpreted as dead load arising from floor.

- **Total weight of structure**:

The total weight of the structure is determined based on the sum of the total weight of each story. Total weight of each story consists of:

1. The External Wall:

The weight of the external wall (façade or without facing) for each story is calculated by summing up half the weight of the upper story wall and half the weight of the lower story wall. Then, it is multiplied by the total length and weight per unit area of the wall.

2. The Staircase:

The sum of the weight per unit area of stairs and 20% of the live load is multiplied by the area of the stairs.

3. The floors, partition, and steel structure:

The dead load of the floor, an equivalent uniformly distributed load of the partition, the weight of the steel structure in each story, and 20% of the live load are summed up and multiplied with the total area of the plan. It should be noted that the weight of the partitions calculated based using half of the height of the upper story and half of the height of the lower story.

- **The weight of floor**:

The total weight of each story consists of different parts representing the calculation of the total weight of the floor. This is achieved by the following steps:

1. (Total Length of the external wall without facing) × ((the height of story down/2) + (the height of story up/2)) × (weight per unit area of external wall without facing)
2. (Length of external wall facing) × ((the height of story down/2) + (the height of story up/2)) × (weight per unit area of external wall with facing)
3. (Area of staircase) $[(\mathit{weight\ per\ unit\ area\ of\ stair}) + 0.2(\mathit{live\ load\ of\ staircase})]$

4.
$$(\text{Net area of plan})\left[\frac{\textit{the weight of partition in the story down}}{2} + \frac{\textit{the weight of partition in story up}}{2} + (\textit{the weight of the floor}) + (\textit{the weight of the structure}) + (0.2 \times \textit{live load of floor})\right]$$

Weight of the First Floor:

$$(33.6)\left(\frac{3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)] + (154.5)[\left(\frac{81.533}{2}+\frac{179.32}{2}+(435)+135+(0.2\times 200)\right] = 158044.182$$

Weight of the Second Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)] + (154.5)[\left(\frac{179.32}{2}+\frac{179.32}{2}+(435)+132+(0.2\times 200)\right] = 166954.8$$

Weight of the Third Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)] + (154.5)[\left(\frac{179.32}{2}+\frac{179.32}{2}+(435)+126.456+(0.2\times 200)\right] = 166098.252$$

Weight of the Fourth Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)] + (154.5)[\left(\frac{179.32}{2}+\frac{179.32}{2}+(435)+123.266+(0.2\times 200)\right] = 165605.4045$$

Weight of the Fifth Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)] + (154.5)[\left(\frac{179.32}{2}+\frac{179.32}{2}+(435)+120.0773+(0.2\times 200)\right] = 165112.1279$$

Weight of the Roof Floor:

$$(33.6)\left(\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}\right)(215)+(53.2.)[(205)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)[\left(\frac{179.32}{2}+(480)+102.897+(0.2\times 150)\right]=144882.48$$

Weight of the Doom Floor:

$$(10.14)[(480+189.162+(0.2\times 150)]+(13)(2.7\times 236)+(13\times 205)$$
$$=18038.102$$

The chart in Fig. 1.42 was designed to simplify the calculation of the total weight of each floor. It can be seen that each part of the structure (external wall, floor, etc.) contribute significantly to the weight of each floor.

(i) This table details the contribution of the external wall façade to the weight of the floor. This table consists of 4 parts. The first is the total length of external façade wall, the second is the height of the wall in each floor, the third is relevant to the weight of wall per meter square that was calculated in the loading chapter, and the four this the total weight of the external façade wall.
(ii) This table is related to the contribution of the external wall without facing to the weight of the floor. This table consists of 4 parts. The first is the total length of the external wall, the second is the height of the wall in each story, the third is relevant to the weight of the wall per meter square that was calculated in the loading chapter, and the last is the total weight of the external without a wall.
(iii) This table detail the contribution of the staircase to the total weight of the floor. It is classified into four parts. The first is the area of the staircase, the second is the weight of the staircase that was calculated in the loading chapter, the third is 20% of the staircase's live load, and the four this the total weight of staircase contributing to the weight of the floor.
(iv) This part of the chart illustrates the total weight of the floor, which forms the main part of the floor's weight. This table is classified into six parts. The first is the pure area of the floor, the second is the weight of internal wall, the third is the weight of floor, the fourth is related to the weight structures calculated in the loading part, the fifth is 20% of floor live load, and the six this the total weight of the floors.
(v) The total weight of the floor is tabulated in this table. It is clear that the total weight of the floors is the sum of the first four parts (i, ii, iii, iv).
(vi) This table shows percentages of the total building weight belonging to each part of the building (Fig. 1.46; Chart 1.4).

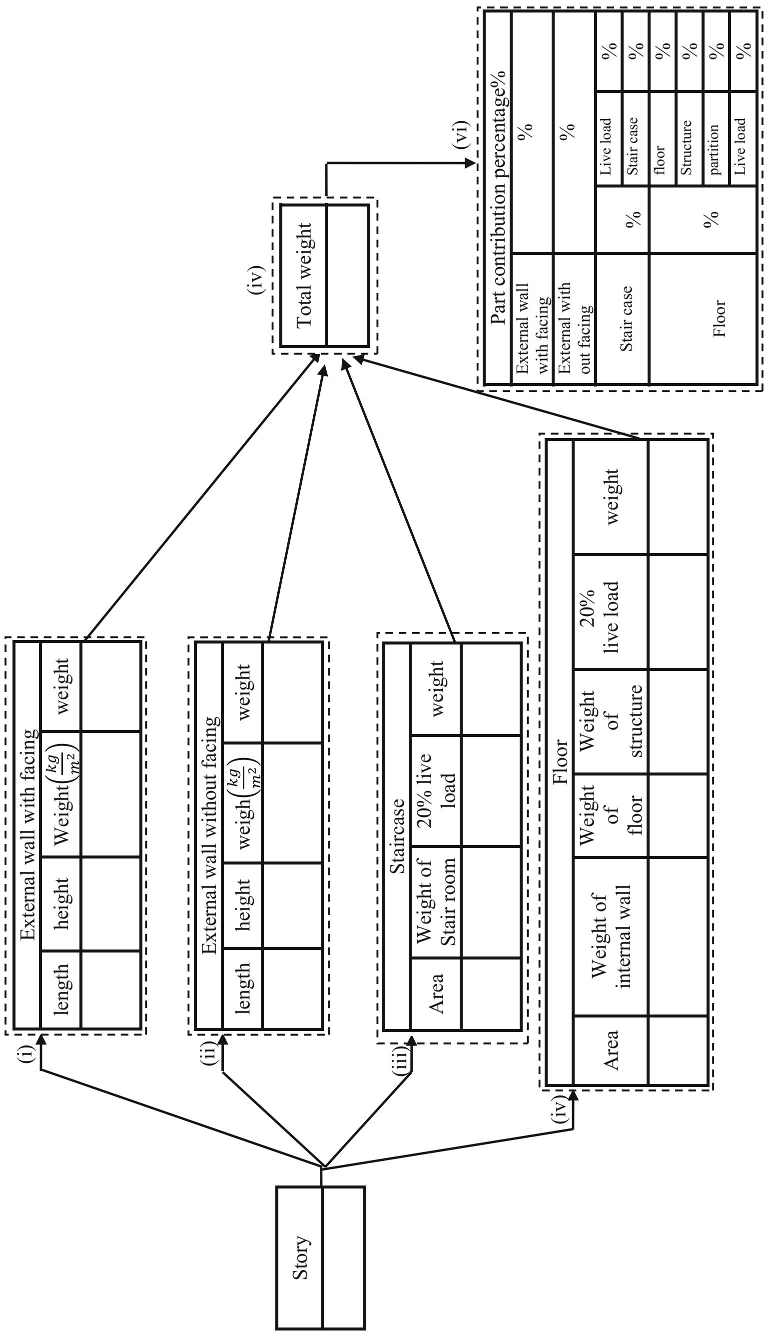


Fig. 1.46 Pattern chart of calculation total weight of story

Story
1

External wall with facing			
length	height	Weight$\left(\frac{kg}{m^2}\right)$	weight
19.6	$\frac{3}{2}+\frac{3.3}{2}$	215	13274.1

External wall without facing			
length	height	weigh$\left(\frac{kg}{m^2}\right)$	weight
33.6	$\frac{3}{2}+\frac{3.3}{2}$	236	24978.24

Staircase			
Area	Weight of Stair room	20% live load	weight
10.14	462	0.2×350	5394.48

Floor					
Area	Weight of internal wall	Weight of floor	Weight of structure	20% live load	weight
154.5	$\frac{81.53}{2}+\frac{179.32}{2}$	435	135.011	0.2 × 200	114397.36

Total weight
158044.41

Part contribution percentage%			
External wall with facing	8.39 %		
External with out facing	15.8 %		
Stair case	3.4%	Live load	0.48%
		Stair case	2.95%
Floor	72.4 %	floor	42%
		Structure	13%
		partition	13%
		Live load	3.9%

Chart 1.4 Total weight of story One

1.3.3 Center of Mass

The point of the structure where the resultant force acts on, it is the center of mass. Figure 1.47 shows a floor of the building. Assume that an earthquake force is applied to the structure in the Y direction. Force F_1, F_2,…, and F_8 is applied to the parts of the structure which have mass m_1, m2…, and m8 as shown in Fig. 1.47. Therefore, the forces are depicted in Fig. 1.47. Part one is staircase, part 2, 3 and 4 area floor, part 5 and six are external facade wall, and part 7 and 8 are external wall without facing.

Force is applied to the part 1 = $F_1 = m_1 \times a$ Force is applied to the part 2 = $F_2 = m_2 \times a$
Force is applied to the part 3 = $F_3 = m_3 \times a$ Force is applied to the part 4 = $F_4 = m_4 \times a$
Force is applied to the part 5 = $F_5 = m_5 \times a$ Force is applied to the part 6 = $F_6 = m_6 \times a$
Force is applied to the part 7 = $F_7 = m_7 \times a$ Force is applied to the part 8 = $F_8 = m_8 \times a$

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i} \qquad Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} \tag{1.49}$$

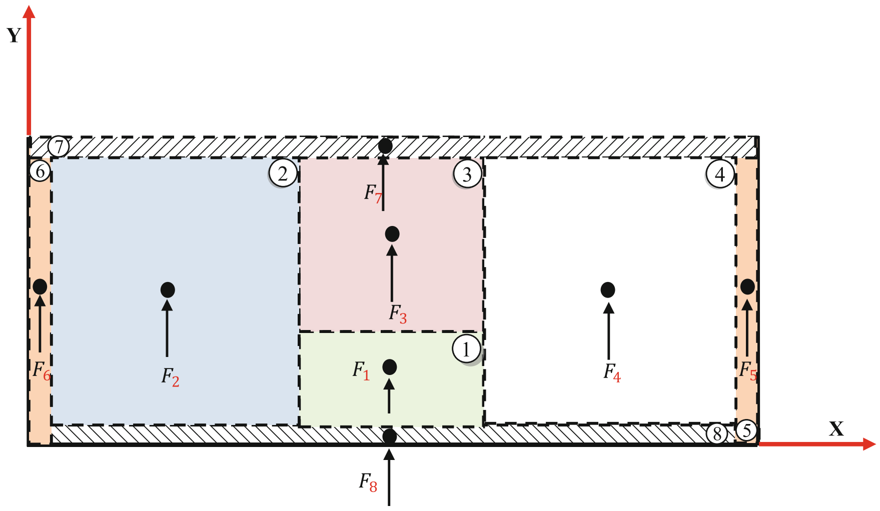


Fig. 1.47 Calculation center of mass

where these equations:

X_{CM}, Y_{CM}: coordinate of center of mass
m_i: effective earthquake weight of each part of the floor
X_i: distance of center of mass of each part of floor from the Y axis
Y_i: distance of center of mass of each part of floor from the X axis

Note:

Effective earthquake weight = dead load + 20% of live load

Centre of mass is calculated using Eq. (1.49), hence:

Moving on, the floor of each story should be divided into some parts. This is because the effective earthquake weight should be used to find the center of mass, and each part of the floor may have a different dead and live load. For instance, the live load of the staircase, balcony, and toilet are not the same. These parts may have slabs with different thickness, so there will be a different mass for each part. Hence, the floor of Story 1 is divided into eight parts. Figure 1.48 demonstrate the parts of Story 1 and the area of each part. Mass one (M_1) is the staircase, with the net area of 10.14 m^2. M_2, M_3, and M_4 are the other parts of the floor of there a with 50.47 (m^2), 28.08 (m^2), and 75.95 (m^2), respectively. Moreover, the façade external wall is represented by M_5 and M_6. M_7 and M_8 are correlated to the external wall without facing. The center of mass of Story (1) is determined through the following:

1. The dead load of Part 1 is 462 (kg/m^2), and the dead load of Part 2, 3 and 4 is 620.604 (kg/m^2). Meanwhile, Part 5 and 6 have the dead load of 677.25 (kg/m^2), and the dead load of Part 7 and 8 are equal to 743.4 (kg/m^2).

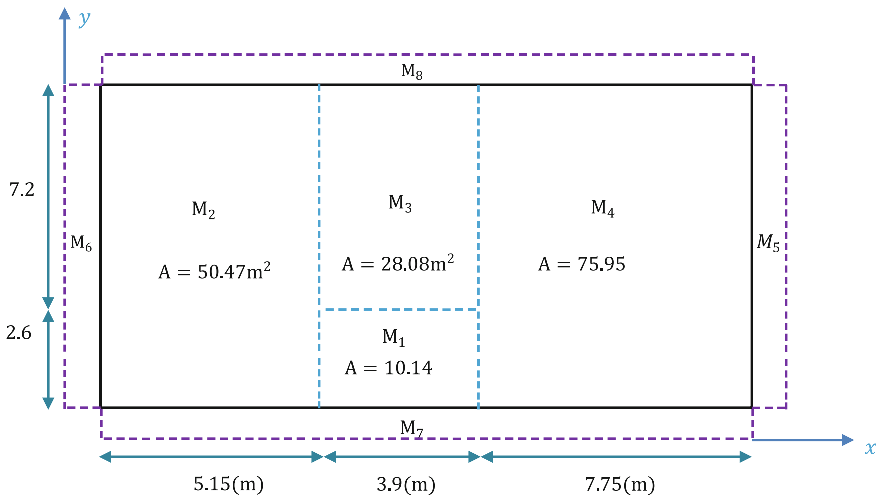


Fig. 1.48 Different part of floor plan

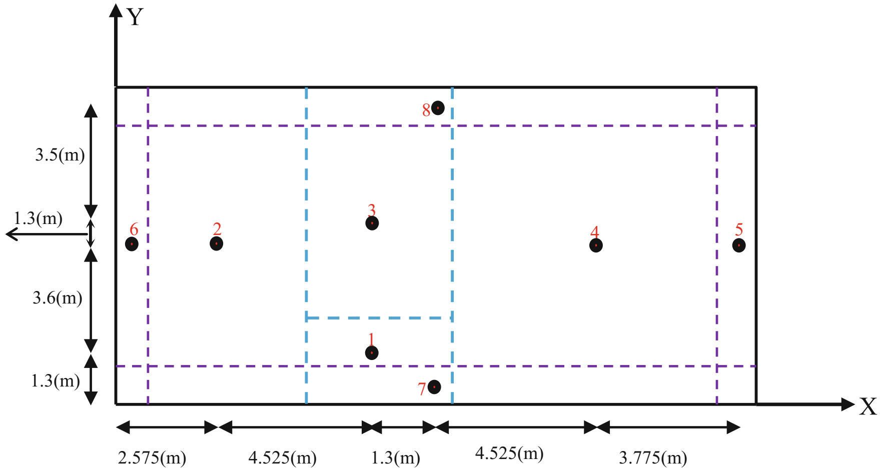


Fig. 1.49 Distance between center mass of each part to arbitrary origin

2. The live load of Part 1 is 350 (kg/m^2), and Part 2,3 and 4 have the same live load that equals to 200 (kg/m^2).
3. The effective weight of each component is calculated using the dead load and 20% of the live load.

Effective earthquake weight = (dead load × 0.2 live load) × (area/length)
Part 1: (462 + (0.2 × 350) × (10.14) = 5394.48
Part 2: $\left[435 + (0.2 \times 200) + \left(\frac{81.35}{2} + \frac{179.32}{2}\right) + 135.11\right] \times (50.47) = 37369.80$
Part 3: $\left[435 + (0.2 \times 200) + \left(\frac{81.35}{2} + \frac{179.32}{2}\right) + 135.11\right] \times (28.08) = 20791.442$
Part 4: $\left[435 + (0.2 \times 200) + \left(\frac{81.35}{2} + \frac{179.32}{2}\right) + 135.11\right] \times (75.95) = 56236.1142$
Part 5: (677.25) × (9.8) = 6637.05
Part 6: (677.25) × (9.8) = 6637.05
Part 7: (743.4) × (16.8) = 12489.12
Part 8: (743.4) × (16.8) = 12489.12

1. The coordinate of the center of mass for each part of the floor is determined with regard to the geometry of the part and shown in Fig. 1.49.

$$\begin{cases} X_1 = 7.1\ \text{m} \\ X_2 = 2.575\ \text{m} \\ X_3 = 7.1\ \text{m} \\ X_4 = 12.925\ \text{m} \\ X_5 = 16.7\ \text{m} \\ X_6 = 0.1\ \text{m} \\ X_7 = 8.4\ \text{m} \\ X_8 = 8.4\ \text{m} \end{cases} \begin{cases} Y_1 = 1.3\ \text{m} \\ Y_2 = 4.9\ \text{m} \\ Y_3 = 6.2\ \text{m} \\ Y_4 = 4.9\ \text{m} \\ Y_5 = 4.9\ \text{m} \\ Y_6 = 4.9\ \text{m} \\ Y_7 = 0.1\ \text{m} \\ Y_8 = 9.7\ \text{m} \end{cases}$$

6. The center of mass of Story 1 is determined using Eq. (1.10). Therefore:

$$\boldsymbol{X_{CM}} = \frac{\sum \mathrm{m_i x_i}}{\sum \mathrm{m_i}}$$

$$\begin{aligned} \sum m_i x_i = {} & [(5394.48 \times 7.1) + (37369.92351 \times 2.575) + (20791.50886 \times 7.1) \\ & + (56236.29266 \times 12.925) + (6636.05 \times 16.7) + (6636.05 \times 0.1) \\ & + (12489.12 \times 8.4) + (12489.12 \times 8.4)] = 1330301.913 \end{aligned}$$

$$\begin{aligned} \sum m_i = {} & (5394.48) + (37369.92351) + (20791.50886) + (56236.29266) + (6637.05) \\ & + (6637.05) + (12489.12) + (12489.12) = 158474.464 \end{aligned}$$

$$\mathbf{X_{cm}} = \frac{1330301.913}{158044.182} = 8.41\ (\text{m})$$

$$\boldsymbol{Y_{cm}} = \frac{\sum m_i y_i}{\sum m_i}$$

Table 1.23 Calculating of mass and center of mass for 1st floor

Number of mass	Length/area	kg/m − kg/m^2	M (kg)	X_i (m)	Y_i (m)	Mx (chg.)	$M_i y_i$ (chg.)
1	10.14	532	5394.48	7.1	1.3	38300.80	7012.82
2	50.47	740.436	37369.8	2.575	4.9	96227.235	183112.02
3	28.08	740.436	20791.442	7.1	6.2	147619.238	128906.94
4	75.95	740.436	56236.1142	12.925	4.9	726851.776	275556.95
5	9.8	677.25	6637.05	16.7	4.9	110838.73	32521.54
6	9.8	677.25	6637.05	0.1	4.9	663.70	32521.54
7	16.8	743.4	12489.12	8.4	0.1	104908.60	1248.91
8	16.8	743.4	12489.12	8.4	9.7	104908.60	121144.46
Sum			158044.182			1330301.913	782015.41

$\bar{x} = \frac{1330301.913}{158044.182} = 8.42$ $\bar{y} = \frac{782015.41}{158044.182} = 4.94$

Table 1.24 Calculating of mass and center of mass for 2nd floor

Number of mass	Length/area	kg/m − kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	786.32	39685.5704	2.575	4.9	102190.3438	194459.295
3	28.08	786.32	22079.865	7.1	6.2	156767.0415	136895.163
4	75.95	786.32	59721.004	12.925	4.9	771893.9767	292632.919
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.256	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.256	126913.248
Sum			166954.799			1405772.762	827362.213

$$\bar{x} = \frac{1405772.718}{166954.8} = 8.42 \qquad \bar{Y} = \frac{827362.204}{166954.8} = 4.95$$

Table 1.25 Calculating of mass and center of mass for 3rd floor

Number of mass	Length/area	kg/m − kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	780.77657	39405.793	2.575	4.9	101469.918	193088.4
3	28.08	780.77657	21924.206	7.1	6.2	155661.863	135930.1
4	75.95	780.77657	59299.980	12.925	4.9	766452.248	290569.9
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.256	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.256	126913.2
Sum			166098.34			1398505.43	822963.2

$$\bar{x} = \frac{1398505.43}{166098.34} = 8.42 \qquad \bar{Y} = \frac{822963.2}{166098.34} = 4.95$$

Table 1.26 Calculating of mass and center of mass for 4th floor

Number of mass	Length/area	kg/m − kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	777.586	39244.765	2.575	4.9	101055.27	192299.348
3	28.08	777.586	21834.614	7.1	6.2	155025.76	135374.60
4	75.95	777.586	59057.656	12.925	4.9	763320.2	289382.514
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.25	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.25	126913.248
Sum			165605.404			1394322.61	820431.298

$$\bar{x} = \frac{1394322.618}{165605.4045} = 8.42 \qquad \bar{Y} = \frac{820431.298}{165605.4045} = 4.95$$

Table 1.27 Calculating of mass and center of mass for 5th floor

Number of mass	Length/ area	kg/m − kg/ m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	774.4	39083.968	2.575	4.9	100641.217	191511.4432
3	28.08	774.4	21745.152	7.1	6.2	154390.58	134819.94
4	75.95	774.4	58815.68	12.925	4.9	760192.664	288196.832
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.25	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.25	126913.24
Sum			165112.127			1390145.85	817903.0432

$\bar{x} = \frac{1390145.85}{165112.1274} = 8.42 \qquad \bar{Y} = \frac{817903.0432}{165112.1274} = 4.95$

Table 1.28 Calculating of mass and center of mass for roof

Number of mass	Length/ area	kg/m − kg/ m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	702.557	35458.051	2.575	4.9	91304.48	173744.45
3	28.08	702.557	19727.8	7.1	6.2	140067.38	122312.36
4	75.95	702.557	53359.20	12.925	4.9	689667.66	261460.08
5	9.8	354.75	3476.55	16.7	4.9	58058.385	17035.095
6	9.8	354.75	3476.55	0.1	4.9	347.655	17035.095
7	16.8	389.4	6541.92	8.4	0.1	54952.128	654.192
8	16.8	389.4	6541.92	8.4	9.7	54952.128	63456.62
Parapet wall	9.8	205	2009	16.7	4.9	33550.3	9844.1
Parapet wall	9.8	205	2009	0.1	4.9	200.9	9844.1
Parapet wall	16.8	205	3444	8.4	0.1	28929.6	344.4
Parapet wall	16.8	205	3444	8.4	9.7	28929.6	33406.8
Sum			144882.48			1219261.027	716150.116

$\bar{x} = \frac{1219261.027}{144882.48} = 8.415 \qquad \bar{Y} = \frac{716150.116}{144882.48} = 4.94$

$$\sum m_i y_i = [(5394.48 \times 1.3) + (37369.92351 \times 4.9) + (20791.50886 \times 6.2) + (56236.29266 \times 4.9) + (6636.05 \times 4.9) + (6636.05 \times 4.9) + (12489.12 \times 0.1) + (12489.12 \times 9.7)] = 782015.41$$

$$\sum m_i = (5394.48) + (37369.92351) + (20791.50886) + (56236.29266) + (6637.05) + (6637.05) + (12489.12) + (12489.12) = 158044.182$$

$$Y_{cm} = \frac{782015.41}{158044.182} = 4.93\ (\text{m})$$

Tables 1.23, 1.24, 1.25, 1.26, 1.27 and 1.28 illustrate the calculation center mass from first floor to sixth floor respectively.

Total weight of roof and dome roof:
144882.48 + 18038.102 = 162920.582

$$\bar{x} = \frac{(144882.48)(8.415) + (18038.102)(7.1)}{162920.582} = 8.27$$
$$\bar{y} = \frac{(144882.48)(4.94) + (18038.102)(1.3)}{162920.582} = 4.537$$
$$135297.76 + 16990.7218 = 152288.4818$$

According to the results center of mass in roof, level is (x = 8.27, y = 4.537)

1.3.4 Lateral Load

After the earthquake in 1908 in Italy, a team of experienced civil engineers were commissioned to study the demolished buildings in this earthquake and find out the cause of the devastation. This team concluded that the earthquake had created a horizontal force in the structures that caused them to crash, and finally they suggested this force as 1/12 of the building weight. The view of these engineers got more complete, and the value of the earthquake force represented as the following relation:

$$V = CW$$

where V is the earthquake force, W is the building weight, and C is the earthquake coefficient. The earthquake coefficient was first considered as 1/12, and then that was considered as 1/10. After then, as the researchers developed and got more complete on the one hand, and the increase of the information achieved from the recorded earthquakes, on the other hand, the engineers found out that various factors affect determining the earthquake force and coefficient. These factors are gradually brought in the bylaws and while retaining the old format of the above relation, the effect of the effective factors on the earthquake force in determining the earthquake coefficient and the distribution of the earthquake force were implicitly included.

Note:

The equivalent static method, despite having weakness in modeling the dynamic and non-linear behavior of the structure, is counted as one of the most applicable methods to determine the earthquake force imposed to the structures due to its simplicity.

Earthquake loading has important ramifications in the design of buildings. In an earthquake, it involves sets of forces that arise as a result of earth seismic activity and act on the building mass. This distortion in the mass results in building movements. For this reason, the earthquake resistance design concentrates on the translational inertia forces and affect a building are more significantly than the vertical or rotational shaking components.

Meanwhile, other earthquake forces which are more severe may exist. This includes those due to land sliding, subsidence, active faulting below the foundation or liquefaction of the local sugared as a result of vibration. Such impact to the building by these factors and together with local factors can seriously impose constraints on the design of earthquake-proof buildings. This can be to such as extent that an alternative building site maybe required.

The underlying principle of earthquake is that the present intensity is inversely proportional to the earthquake frequency. However, major earthquakes does not both occur with moderate intensity and frequently, but the minor earthquakes do occur frequently. In principle, designing a building that can sustain against severe earthquakes is possible. However, the capital cost is rather high with short building lifetime. Hence, the usual specifications for an earthquake-resistant design of a building are as the followings:

1. Sustain against minor earthquakes without any form of damages;
2. Sustain again moderate earthquakes without structural damage. However non-structural damages are acceptable:
3. Sustain against higher intensity earthquake with non-structural damages. However, building should not collapse Withstand average intensity earthquakes with some level of non-structural and yet there should be no collapse of the building.

The earthquake loading is a relative response of the building to the ground vibration. Hence, the seismic loading can be estimated by applying two different methods, and most importantly both approaches will consider the structural properties on building and earthquake histories of the particular region.

The first method is known as the equivalent lateral force procedure, and the underlying principles of this method are to estimate the fundamental period of a structure and predictive ground acceleration or velocity. These principles are imperative in determining the maximum base shear. Thereafter, the base shear equivalent horizontal loading will be distributed in a standard procedure throughout the height of the building, and as a result, a static analysis can be performed on the structure. Based on the static analysis, the design forces are actually less than the forces induced upon the building due to an earthquake. In addition, lower design forces provided an improved strength to the structure due to the stress level imposed by the building load and associated components. Furthermore, the lower design forces is applicable due to the components' ductility, which is a reaction of the elastic limits. Therefore, the first method is simpler and quicker for high-rise buildings, which often have individual structural requirements.

Table 1.29 The value of C_t

C_t	Type of structure
0.085	Measured for steel frame's moment resistant space
0.075	Measured for concrete frame's moment resistant space
0.075	Measured for eccentrically braced frames
0.050	Measured for all the other parts of the building

In the second method, the procedure is highly refined and designed based on modal analysis, where a structure's modal frequencies are analyzed and applied in conjunction with the design spectra of earthquake to estimate the maximum modal responses. It is important to note that the second method requires longer processing period and highly complex compared to first method. However, it is a more precise method with the ability of carrying out nonlinear analysis of a structure.

1.3.4.1 EUROCODE 8 (Equivalent Lateral Force)

The expression $F_b = S_d(T) \times \lambda \times m$ is used to determine the seismic shear force denoted by F_b particularly for all the building's horizontal direction that are analysed in which:

S_d (T_1) is the design's ordinate spectrum at T_i period.
T_1 is the building vibration's fundamental period for lateral motion in the direction under consideration.
M is the building's total mass, above the rigid basement's top or above the building's foundation.
λ is the correction factor, which has a value equal to: $\lambda = 0.85$ if $T_1 \leq 2T_c$ where the building comprises of two or more storeys or a value of $\lambda = 1.0$ sometimes.

In order to determine the building's fundamental vibration period T_i, the structural dynamic's expression methods may be utilized such as the Rayleigh method. In buildings that have heights of 40 m or there about, the value of Ti may be determined by the following expression:

$$T = C_T \times H^{\left(\frac{3}{4}\right)} \tag{1.50}$$

In which, Table 1.29 shows the values of $C_{t.}$

H is the building's height measured in meters, right from the rigid basement's top or the foundation.

Table 1.30 Ground-type

Ground type	Description of stratigraphic profile	Parameters		
		vs, 30 (m/s)	NSPT (blows/ 30 cm)	cu (kPa)
A	Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface	>800	–	–
B	Deposits of very dense sand, gravel, or very stiff clay, at least several tens of meters in thickness, characterized by a gradual increase of mechanical properties with depth	360–800	>50	>250
C	Deep deposits of dense or medium dense sand, gravel or stiff clay with a thickness of several tens to many hundreds of meters	180–360	15–50	70–250
D	Deposits of loose-to-medium cohesionless soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil.	<180	<15	<70
E	A soil profile consisting of a surface alluvium layer with values of type C or D and thickness varying between about 5 and 20 m, underlain by stiffer material with vs. >800 m/s			
S_1	Deposits consisting, or containing a layer at least 10 m thick, of soft clays/silts with a high plasticity index (PI > 40) and high water content	<100 indicative		10–20
S_2	Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in types A–E or S1			

The Procedure for Determine the Equivalent Lateral Force

This part will try to show, in a step by step method, how to calculate the shear base by the help of Euro code 8. This procedure will also try give the sections and number of table in each step as per the Euro codes regulations (Table 1.30).

Step 1: Ground types

V_S, 30: refers to the S waves' propagation average value in the soil profile's upper 30 m occurring at 10^{-5} of shear strain or less as shown below

$$V_S, 30 = \frac{30}{\sum_{i=1,N} \frac{h_i}{v_i}} \tag{1.51}$$

where

h_i and v_i represent the thickness measured in metres and shear-wave velocity of 10^{-5} occurring at shear strain level or less in the i-th layer or formation when seen as N's total, in the top 30 m.

N_{SPT}: refers to the Standard Penetration Test blow count

C_u: underlined the soil's shear strength

Step 2: Components and spectrum of the appropriate type of elastic response

The largest amplitudes measured on the seismograph's standard long-period are usually produced by the surface waves having a period of 20 s or so and consequently, these waves' amplitude is based upon in the determination of Ms by aid of an equation.

Where

A is the ground motion's amplitude in microns; T is its corresponding period measured in seconds while Q(D, h) is the correction factor which is the function of distance, D (degrees) which represents the area between the earthquakes' epicentre, the station and the focal depth, h, measured in kilometers. The formula that represents the standard surface-wave is

$$M_s = \log_{10}\left(\frac{A}{T}\right) + 1.66\ \log_{10}(D) + 3.30 \quad (1.52)$$

The components and spectrum of elastic response are:
S = Soil Factor
T_B = The period's constant spectral acceleration part's lower limit
T_C = The period's constant spectral acceleration part's upper limit
T_D = The value defining the constant displacement response range's beginning of the period and spectrum
a_g = The type A ground design's ground acceleration ($a_g = \gamma_1 \cdot a_{gR}$)
The periods TC, TB and TD's values as well as the S or the soil factor, which is very critical in determining the elastic response spectrum's shape is dependent upon the ground type.

NOTE 1:
Every country has its own values that can be ascribed to the periods TC, TB, TD as well as for the S or the soil factor which can be found in the country's National Annex. But in cases where the deep geology is neither recommended nor allowed, the available choices to consider include the use of the two spectra types such as type 1 or type 2. Therefore, the earthquake that contributes more of the seismic hazards on any site can be defined, for the purpose of making probabilistic hazard assessment, by the magnitude of the surface-wave and if it has a surface-wave magnitude of not greater than 5.5, the type 2 spectrum is employed as shows. The parameters TB, S, TC and TD's recommended values resulting from ground types

Table 1.31 Value of the parameters describing Type 1 elastic response spectrum (M_s > 5.5)

Ground type	S	T_B(s)	T_C(s)	T_D(s)
A	1.0	0.15	0.4	2
B	1.2	0.15	0.5	2
C	1.15	0.20	0.6	2
D	1.35	0.20	0.8	2
E	1.4	0.15	0.5	2

Table 1.32 Value of the parameters describing Type 2 elastic response spectrum ($M_s < 5.5$)

Ground type	S	T_B(s)	T_C(s)	T_D(s)
A	1.0	0.05	0.25	1.2
B	1.35	0.05	0.25	1.2
C	1.5	0.10	0.25	1.2
D	1.8	0.10	0.30	1.2
E	1.6	0.05	0.25	1.2

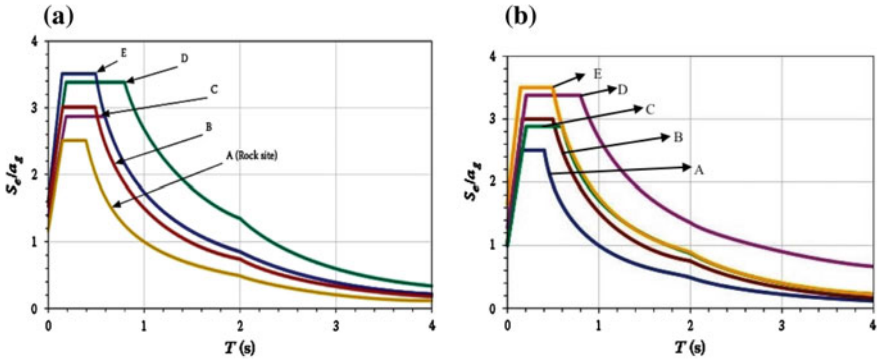


Fig. 1.50 **a** MS > 5.5, **b** MS < 5.5

Table 1.33 Different importance class of building

Importance class	Building
I	Buildings of minor importance for public safety, e.g., agricultural buildings, etc.
II	Ordinary buildings, not belonging to the other categories
III	Buildings whose seismic resistance is of importance in view of the consequences associated with a collapse, e.g., schools, assembly halls, cultural institutions etc.
IV	Buildings whose integrity during earthquakes is of vital importance for civil protection, e.g., hospitals, fire stations, power plants, etc.

A, B, C, D and E are given in Table 1.31 particularly for type 1 components and spectrum while Table 1.32 gives type 2 spectrum. Figure 1.50a, b demonstrate the recommended spectra of type 1 and type 2 respectively which are normalised by the ag, for 5% damping.

Step 3: Important factor

Buildings are categorised or classified based into four important classes based on the impact of the building's collapse on human life, public safety and civil protection especially in the post-earthquake period as well as the post-earthquake's socio-economic effects of the collapse. The different critical factors γ_1 characterizes the mentioned classes. Table 1.33 shows different categories of building importance.

Step 4: Fundamental period's determination

The Ti value, in S, especially for those buildings with heights of up to 40 m can be approximated by use of the following expression

$$T_1 = C_t \cdot H^{\left(\frac{3}{4}\right)} \tag{1.53}$$

where

C_t is 0,085 for steel frames' moment resistant space, 0.075 for both concrete moment resistant spaces and eccentrically braced steel frame and 0.050 for all other parts of the structure;
H is the building's height, in m, from the rigid basement's top or from the foundation.

Table 1.34 Consequences of structural regularity on seismic analysis and design

Regularity		Allowed simplification		Behaviour factor
Plan	Elevation	Model	Linear elastic analysis	(For linear analysis)
Yes	Yes	Planar	Lateral force	Reference value
Yes	No	Planar	Modal	Decreased value
No	Yes	Spatial	Lateral force	Reference value
No	No	Spatial	Modal	Decreased value

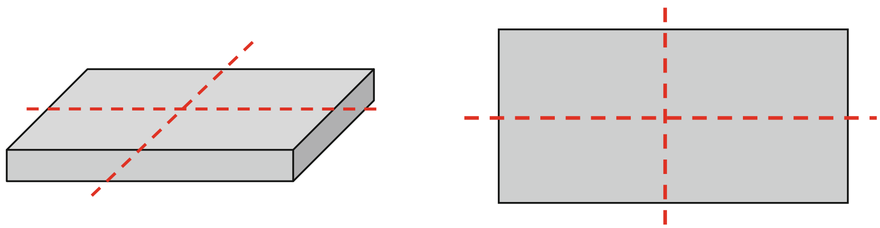
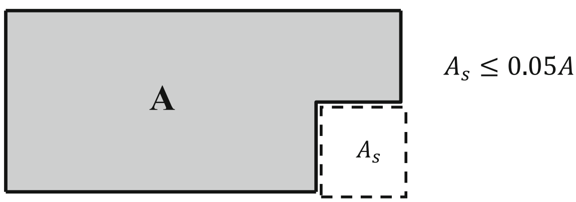


Fig. 1.51 Symmetrical plan

Fig. 1.52 Set-back in plan



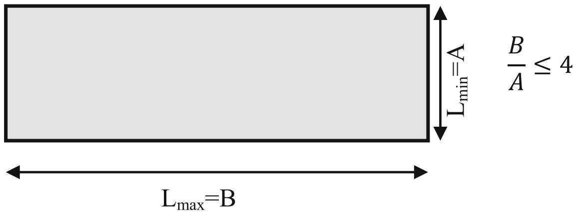


Fig. 1.53 Plan dimension

Step 5: Regularity and Horizontal seismic action's behaviour factor

For the purpose of seismic design, building structure are categorized into being regular and irregular. Regularity of the structure influences the required structural model, the required method of analysis and the value of the behaviour factor q (Table 1.34).

- **Criteria for regularity in Plan**:

1. With respect to the lateral stiffness and mass distribution, the building structure shall be approximately symmetrical in plan with respect to two orthogonal axes (Fig. 1.51).
2. The plan configuration shall be compact, i.e., each floor shall be delimited by a polygonal convex line. If in plan set-backs (re-entrant corners or edge recesses) exist, regularity in plan may still be considered as being satisfied, provided that these setbacks do not affect the floor in-plan stiffness and that, for each set-back, the area between the outline of the floor and a convex polygonal line enveloping the floor does not exceed 5% of the floor area (Fig. 1.52).

$$A_s \leq 0.05A$$

3. The in-plan stiffness of the floors shall be sufficiently large in comparison with the lateral stiffness of the vertical structural elements, so that the deformation of the floor shall have a small effect on the distribution of the forces among the vertical structural elements. In this respect, the L, C, H, I, and X plan shapes should be carefully examined, notably as concerns the stiffness of the lateral branches, which should be comparable to that of the central part, in order to satisfy the rigid diaphragm condition. The application of this paragraph should be considered for the global behaviour of the building.
4. The slenderness $\lambda = L_{max}/L_{min}$ of the building in plan shall be not higher than 4, where $L_{\max}$ and $L_{\min}$ are respectively the larger and smaller in plan dimension (Fig. 1.53) of the building, measured in orthogonal directions.

$$\frac{B}{A} \leq 4$$

5. At each level and for each direction of analysis x and y, the structural eccentricity e_o and the torsional radius r shall be in accordance with the two conditions below, which are expressed for the direction of analysis y:

$$e_{ox} \leq 0,30 \cdot r_x \tag{1.54}$$

$$r_x \geq l_s \tag{1.55}$$

where
e_{ox} is the distance between the centre of stiffness and the centre of mass, measured along the x direction, which is normal to the direction of analysis considered;
r_x is the square root of the ratio of the torsional stiffness to the lateral stiffness in the y direction ("torsional radius");
l_s is the radius of gyration of the floor mass in plan (square root of the ratio of (a) the polar moment of inertia of the floor mass in plan with respect to the centre of mass of the floor to (b) the floor mass).

- **Criteria for regularity in elevation**

1. All lateral load resisting systems, such as cores, structural walls, or frames, shall run without interruption from their foundations to the top of the building or, if setbacks at different heights are present, to the top of the relevant zone of the building (Fig. 1.54).
2. Both the lateral stiffness and the mass of the individual storeys shall remain constant or reduce gradually, without abrupt changes, from the base to the top of a particular building.
3. In framed buildings the ratio of the actual storey resistance to the resistance required by the analysis should not vary disproportionately between adjacent storeys.
4. When setbacks are present, the following additional conditions apply:

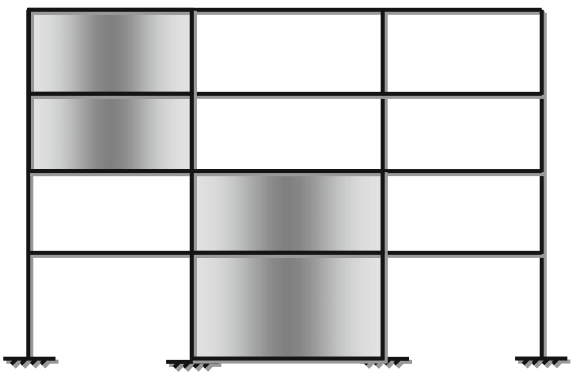


Fig. 1.54 Interruption structural wall in elevation

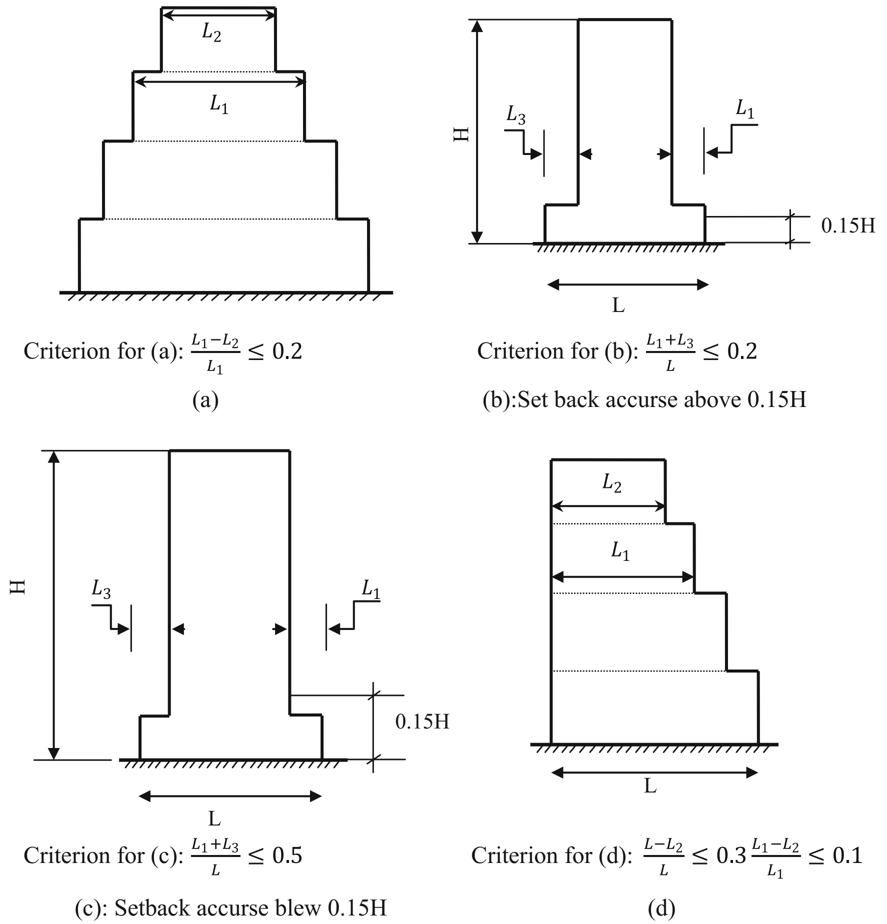


Fig. 1.55 Criteria for regularity of buildings with setbacks

(a) for gradual setbacks preserving axial symmetry, the setback at any floor shall be not greater than 20% of the previous plan dimension in the direction of the setback

(b) for a single setback within the lower 15% of the total height of the main structural system, the setback shall be not greater than 50% of the previous plan dimension In this case the structure of the base zone within the vertically projected perimeter of the upper storeys should be designed to resist at least 75% of the horizontal shear forces that would develop in that zone in a similar building without the base enlargement;

Table 1.35 Basic values of q_0

Structural type	Ductility class	
	DCM	DCH
(a) Moment resisting frames	4	$5\alpha_u/\alpha_1$
(b) Frame with concentric bracings	4	4
Diagonal bracings		
V-bracings	2	2.5
(c) Frame with eccentric bracings	4	$5\alpha_u/\alpha_1$
(d) Inverted pendulum	2	$2\alpha_u/\alpha_1$
(e) Structures with concrete cores or concrete walls	See Sect. 5	
(f) Moment resisting frame with concentric bracing	4	$4\alpha_u/\alpha_1$
(g) Moment resisting frames with infills unconnected concrete or masonry infills, in contact with the frame connected reinforced concrete infills. See Infills. Infills isolated from moment frame	2	2
	See Sect. 7	
	4	$5\alpha_u/\alpha_1$

(c) if the setbacks do not preserve symmetry, in each face the sum of the setbacks at all storeys shall be not greater than 30% of the plan dimension at the ground floor above the foundation or above the top of a rigid basement, and the individual setbacks shall be not greater than 10% of the previous plan dimension (Fig. 1.55).

In a building designed with a conventional analysis model, the behaviour factor q is taken as the seismic forces' approximation ratio approximating the forces that would be experienced by the building and if the building has a response that is totally within the 5% viscous damping, it will ensure a satisfactory response by the structure. The various parts of the EN 1998 provides the material and the structural systems' behavioural factor q values which determines the extent of influence by the viscous damping other than the known 5%. This behaviour factor q values may be very different for the structure's different horizontal directions even though the classification's ductility is the same in all directions. In addition, the behavioural factor q value for the upper limit can be calculated by use of the following expression

$$q = q_0 k_w > 1.5 \tag{1.56}$$

where

q_0 is the behaviour factor's basic value, which is dependent upon the structural system's type as well as its elevation regularity.
K_W is the factor showing the prevailing mode's failure particularly in the walls' structural systems.

Table 1.35 shows the various structural types' basic values of q_0.

However, when it comes to buildings with irregular elevation, the q_0's value should be reduced by 20% and thus α_1 and α_u will be defined as follows:
α_1 is the value that multiplies the horizontal seismic design action so that the flexural resistant is first reach in any part of the structure, other design actions kept constant;
α_u is the value that multiplies the horizontal seismic design action so that a number of section's plastic hinges are formed so that enough development is ensured that guarantees the overall stability of the structure while other design actions are kept constant. Moreover, this α_u factor may be obtained from a global analysis that is static and nonlinear.

1. For buildings, there are options of calculations such as the multiplication factor $\frac{\alpha_u}{\alpha_1}$, which if it is not calculated by way of explicit calculations that are regular in plan, the following values that are approximated values of, $\frac{\alpha_u}{\alpha_1}$ can be utilised.

   (a) Frame-equivalent dual systems or frames.

   - One-storeyed building $\frac{\alpha_u}{\alpha_1} = 1.1$
   - Multi-storeyed, one-bay frames building: $\frac{\alpha_u}{\alpha_1} = 1.2$
   - Multi-storeyed, multi-bay frame-equivalent dual systems or frames structure: $\frac{\alpha_u}{\alpha_1} = 1.3$

   (b) The Wall- or the wall-equivalent dual systems.

   - wall systems that have only two separate walls per horizontal direction: $\frac{\alpha_u}{\alpha_1} = 1.0$
   - other separate wall systems: $\frac{\alpha_u}{\alpha_1} = 1.1$
   - coupled wall- or wall-equivalent dual systems: $\frac{\alpha_u}{\alpha_1} = 1.2$

2. But in buildings that are irregular in plan, the approximate value of $\frac{\alpha_u}{\alpha_1}$ which can be used when calculations are not done for purposes of evaluation, can be the average of (a) 1.0 and that of (b) as is shown by the value given in(1) of this sub-clause.
3. Again, the values of $\frac{\alpha_u}{\alpha_1}$ which are higher than those provided in (1) and (2) as contained in this sub-clause can be used. This are further subject to confirmation by a global analysis that is both nonlinear and static.
4. The $\frac{\alpha_u}{\alpha_1}$ maximum value that may be used in the design is usually equal to 1.5, regardless of the result of the analysis mentioned in (3) of this sub-clause even when it results in higher values.
5. The given q_0's value for inverted pendulum systems can be increased, in a situation where it is ascertained that the correspondingly higher energy dissipation is a reality in the most important region of the structure.
6. In case of an application of a special and formal Quality System Plan to the design, the procurement as well as in the construction of the structure and addition of normal quality control plans, the values of q_0 may be increased as it

Table 1.36 Formula of design spectrum $S_d(T)$

T	$S_d(T)$
$0 \le T \le T_B$	$a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$
$T_B \le T \le T_C$	$a_g \cdot S \cdot \frac{2.5}{q}$
$T_C \le T \le T_D$	$\begin{cases} a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C}{T}\right] \\ \ge \beta \cdot a_g \end{cases}$
$T_D \le T$	$\begin{cases} a_g \cdot S \cdot \frac{2.5}{q} \left[\frac{T_C T_D}{T}\right] \\ \ge \beta \cdot a_g \end{cases}$

deem fit. However, the values increased are not allowed to exceed the values that are given in Table 1.5 by a margin bigger than 20%.

Therefore, the factor K_w which reflects the structural systems' prevailing failure mode in the walls will be represented by the following expression:

$$K_w = \begin{cases} 1.00, \ \textit{for the frame or the frame equivalent dual systems} \\ \frac{(1+\alpha_0)}{3} \leqslant 1, \ \textit{but not less than } 0.5, \textit{ for wall or wall-equivalent and torsionally flexible plans} \end{cases}$$

where

α_0 is the structural system walls' prevailing aspect ratio.

But in a case where the walls' aspect ratios $\frac{h_{wi}}{l_{wi}}$, i, of a structural system do not greatly differ, the prevailing aspect ratio α_0 may be determined by use of the following expression:

$$\alpha_0 = \sum h_{wi} / \sum l_{wi} \tag{1.57}$$

where

h_{wi} is the wall's i; and
l_{wi} is the section of wall i's length.

Step 6: Design spectrum

For the horizontal components of the earthquake action, the design spectrum $S_d(T)$ is considered by the following expressions in Table 1.36:

β represent the lower bound factor of the horizontal design spectrum

Step 7: Base shear force

Each building's horizontal direction's seismic shear force, F_b, used to evaluate each building can be worked out by using the following expression;

$$F_b = S_d(T_1) \times \lambda \times m \tag{1.58}$$

where

S_d (T_1) is the design spectrum's ordinate at period $T1$;
T_1 is the building vibration's fundamental period for the lateral motion in the direction under consideration;
m is the building's total mass, above the rigid basement's top or foundation.
λ is the correction factor, with its value equal to: λ = 0,85 in a case where $T1 < 2$ TC and the building has three or more stories, or λ = 1,0 otherwise.

Step 8: Vertical distribution of seismic force

In case of an approximation of the fundamental mode shape through horizontal displacements which increases linearly along the corresponding heights, the following expression will give the horizontal forces F_i:

$$F_i = F_b] \cdot \frac{z_i.m_i}{\sum z_j.m_j} \tag{1.59}$$

where

F_i is the horizontal force prevailing on storey i;
F_b is the seismic base shear force;
m_i, m_j are the storey or the structure's masses;
z_i, z_j are the masses of m_i, m_j heights above the seismic action application's level (Eurocode 8, 2004).

- **The calculation of equivalent lateral force using Eurocode**:

Step 1: Ground Type

There is no doubt that the average shear velocity is most important factor for the selection of the ground type. An average shear velocity is considered $V_S, 30 = 350\ \frac{\text{m}}{\text{s}}$ as a property of ground, according to the Table 1.30 the, ground type is considered as class C (Eurocode 8, 2004, Table 3.1).

Step 2: Appropriate type of elastic response spectrum and components

Table 1.37 Value of the parameters describing the recommended Type 1 elastic response

Ground type	S	T_B(s)	T_C(s)	T_D (s)
C	1.15	0.2	0.6	2

Depending on the characteristics of the most significant earthquake contributing to the local hazard this building is located in the high-risk zone of the earthquake. As a result, the magnitude of surface wave is considered more than 5.5 and the first type of spectrum is chosen as an elastic response spectrum. According to Table 3.2 of Euro code (8), the value of the parameters of elastic response spectrum is shown in Table 1.37 (Eurocode 8, 2004, 3.2.2.2(2)P).

Step 3: Important factor

Based on part (4.2.5) and Table 4.3 of Euro code eight the building is classified in class II and importance the factor is 1. ($\gamma = 1$) (Eurocode 8, 2004, 4.2.5).

Step 4: Determination of the fundamental period

The moment resistance system is selected for resisting the earthquake load in this building, so the fundamental period of the vibration of the structure for the lateral motion in the considered direction is calculated as fallowing (Eurocode 8, 2004, 4.3.3.2.2.3(P)):

$$T_1 = C_t.H^{\left(\frac{3}{4}\right)}$$
$$C_t = 0.085 \text{ Moment resistance steel frame}$$
$$\text{H} = 21.3\ (\text{m})$$
$$\text{T} = (0.085)(21.3)^{0.75} = 0.842759$$

Step 5: Behaviour factor of horizontal seismic action

The building is considered as a moment resistance frame in each two perpendicular directions, and the structure will be designed as a medium ductility class. the magnitude of behaviour factor is calculated by equation (Eurocode 8, 2004-(5-1)). The value of q_0 is determined by (Eurocode 8, 2004, Table 6.2). The magnitude of k_w is equal to 1 (Eurocode 8, 2004 5.2.2.2(11))

$$q = q_0 k_w > 1.5$$
$$q_0 = 4$$
$$k_w = 1$$

Step 6: Design spectrum

For the horizontal components of the earthquake action in considered structure, the design spectrum $S_d(T)$ is considered by (Eurocode 8, 2004/3.2.2.5(4)P]):

$$T_C \leq T \leq T_D\text{: } S_d(T) = \begin{cases} a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C}{T}\right] \\ \geq \beta . a_g \end{cases}$$

$$T_T = 0.84276$$
$$T_C = 0.6\mathrm{S_d(T)} \begin{cases} 0.3 \times 1.15 \times \frac{2.5}{4}\left[\frac{0.6}{0.84276}\right] = 0.15351 \\ \geq 0.2 \times 0.3 = 0.06 \end{cases}$$
$$T_D = 2$$

Step 7: Base shear force

The horizontal direction's seismic shear force, F_b, used to evaluate each building can be worked out by using the (Eurocode 8, 2004/4.3.3.2.2(1)P)

$$F_b = S_d(T_1) \times \lambda \times m$$

$$\mathrm{S_d(T_1)} = 0.15351$$

$$\lambda = 0.85 m = 984735.3479$$
$$F_b = 1284.947 \text{ kN}$$

In this structure, a moment resisting frame is used in X and Y direction, so the magnitude of the shear base in each direction will be the same $\left(F_{bx} = F_{by} = 1284.97 \text{ kN}\right)$.

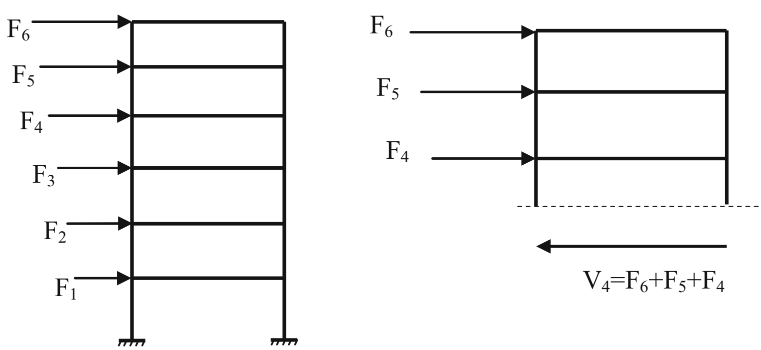


Fig. 1.56 Method of determination of shear in story

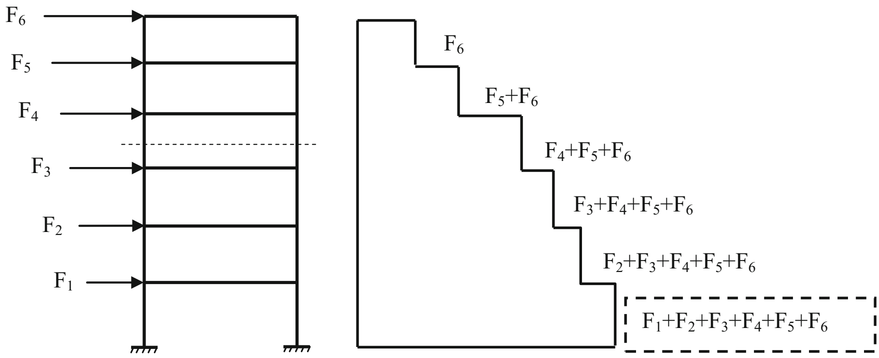


Fig. 1.57 Distribution of shear base in the height of structure

1.3.4.2 Distribution of Shear Base in Each Story

After calculation of base shear, the force of each floor is calculated accordance with Eq. (1.59). This means that the base shear is distributed between each floor. The followed can be deduced:

1. Calculation of actual earthquake weight
2. Lateral force that is applied to the floor should be calculated with Eq. (1.59)

Assuming the earthquake forces that are applied to each of the six stories of the building are calculated, these forces will be F1, F2,…, F6. The method mentioned in this section should be used to calculate the shear force in each story. Also, the horizontal equilibrium of the upper part of the section should be examined. Figure 1.56 illustrates the shear force in Story 4.

$$\textit{Shear in story } i = \sum_{j=i}^{n} (\textit{lateral force of story})_j \tag{1.59}$$

Equation (1.59) shows that the shear force generated in each story is equal to the sum of the forces resulted from the earthquake that is imposed to the floors above the story. Figure 1.57 shows the distribution of the shear force in each story.

In the structure under consideration, the base shear in each direction is 1284.95 kN, and the total weight of each story is calculated from previous parts. Table 1.38 shows the distribution of shear generated in each story and overturning of the moment. Lateral force and shear force in story 6 are equal 367.14 kN. Lateral force for story five is 309.19 (kN) which is essentially the summation of lateral force in story 5 and 6. These will be equal to shear in story five which has a magnitude of 676.33 kN. All the shear force in each floor is calculated by using Eq. (1.59).

Figure 1.58 shows the 3D distribution of shear base in X and Y direction in the height of the structure. As described, the moment resistance frames in both

Table 1.38 Distribution of shear base in each story

Story	h_i	w_i	w_ih_i	$\frac{w_ih_i}{\sum w_ih_i}$	$F_{i(\text{kN})}$	V_i(kN)	M_i(kN m)
6	21.3	162920.58	3470208.4	0.285724509	367.140828	367.1408275	660.8534896
5	17.7	165112.13	2922484.65	0.240626901	309.192795	676.3336229	2539.1075
4	14.1	165605.4	2335036.2	0.192258503	247.041972	923.3755945	5418.584092
3	10.5	166098.25	1744031.65	0.143597308	184.514919	1107.890514	9074.863087
2	6.9	166954.8	1151988.12	0.094850569	121.877946	1229.76846	13282.64924
1	3.3	158044.18	521545.801	0.04294221	55.1784605	1284.94692	17616.39488
		984735.347	12145294.8				

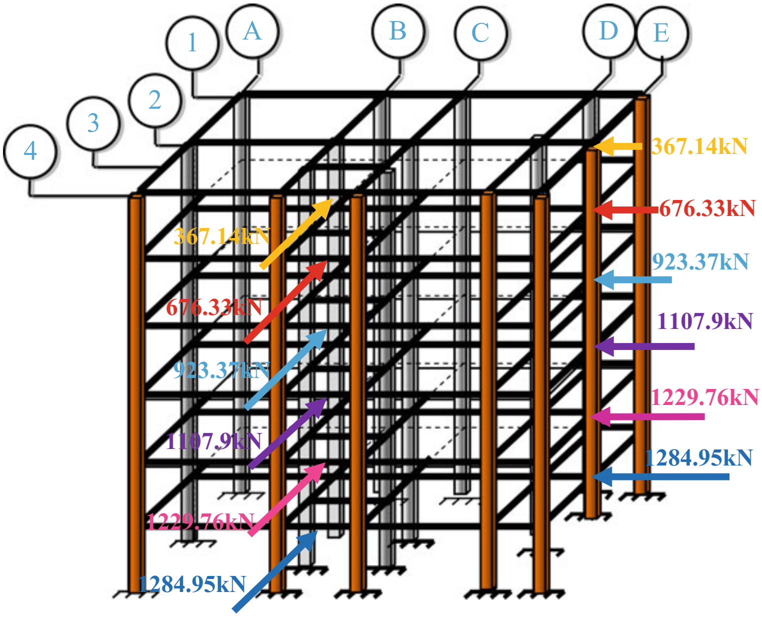


Fig. 1.58 Distribution of shear base in the height of structure in X and Y direction

directions from the shear base in both directions which will be same. The shear force in story 6 is 367.14 kN, and it increases to 1284.95 kN in the first story. This is needed to highlight that all lateral forces in each story are applied to the center of mass of their floors. Center of the mass of each floor is calculated in earlier sections.

1.3.4.3 IBC (Equivalent Lateral Load)

It is recommended by the IBC 2012 that the users should refer to the ASCE 7–10 specifications on the design based on the building and other structures' minimum design loads which is expressed by the formula:

$$V = C_S W \tag{1.60}$$

$$Cs = S_{DS}/(R/I) \tag{1.61}$$

Where C_s represents the seismic response coefficient; in which, in the short period (0.2 s), the S_{DS} corresponds to the design spectral acceleration; R refers to the ductility factor and I is the occupancy importance factor while W is the structure's weight.

Table 1.39 Site classification

Site class	Discretion	$\overline{V_s}$	$\bar{N}_{ch}$	$\bar{S}_u$
A	Hard rock	$>5000\ \frac{ft}{s}$	NA	NA
B	Rock	$2500–5000\ \frac{ft}{s}$	NA	NA
C	Very dense soil and soft rock	$1200–2500\ \frac{ft}{s}$	> 50	> 2000 psf
D	Stiff soil	$600–1200\ \frac{ft}{s}$	15–50	1000–2000 psf
E	Soft clay soil	$< 600\ \frac{ft}{s}$	<15	< 1000 psf
	Any profile with more than 10 ft of soil having the following characteristics: • Plasticity index PI > 20, • Moisture content w $\geq$ 40%, • Undrained shear strength $\bar{S}_u$ < 500 psf			
F	Soils requiring site response analysis in accordance with Sect. 21.1	See Sect. 20.3.1		
For SI: 1 ft/s = 0.3048 m/s; 1 lb/ft^2 = 0.0479 kN/m^2				

The Procedure for Determine the Equivalent Lateral Force

This section will explore, in a procedural manner and with the help of the IBC, the methods of calculating base shear. This procedure, additionally, will attempt to give the number of tables, equations and sections the each step captures and in accordance with the IBC specifications.

Step 1: How to determine the maximum considered earthquake and the design spectral response accelerations:

1.1 The mapped Maximum Considered Earthquake (MCE) spectral response acceleration is determined by the help of the spectral acceleration maps represented by Ss for short period and S1 for long period as shown in Figs. 1613.3.1(1) through 1613.3.1(6) in which S_1 is equal or less than 0.04 and S_S is equal or less than 0.15, and thus the structure is allowed to be assigned to Category A of the Seismic Design.

1.2 Depending on the properties of the soil, the site can be determined an can be categorised class A, B, C, D, E and F. as stipulated and expressed by chapter 20 of ASCE 7. However, in instances where the soil properties cannot be easily determined to know the site class, site class D shall be adopted unless the geotechnical data or the building officials confirm that there is site class E or F soils at the site. Table 1.39 shows the different categories class of the soil properties.

Table 1.40 Values of site coefficient F_a

Site class	Mapped spectral response acceleration at short period				
	$S_s \leq 0.25$	$S_s = 0.5$	$S_s = 0.75$	$S_s = 1$	$S_s \geq 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1	1	1	1	1
C	1.2	1.2	1.1	1	1
D	1.6	1.4	1.2	1.1	1
E	2.5	1.7	1.2	0.9	0.9
F	Note b	Note b	Note b	Note b	Note b

Table 1.41 Values of site coefficient F_v

Site class	Mapped spectral response acceleration at 1-s period				
	$S_a \leq 0.1$	$S_a = 0.2$	$S_a = 0.3$	$S_a = 0.4$	$S_s \geq 0.5$
A	0.8	0.8	0.8	0.8	0.8
B	1	1	1	1	1
C	1.7	1.6	1.5	1.4	1.3
D	2.4	2	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	2.4
F	Note b	Note b	Note b	Note b	Note b

1.3 Determine the MCE spectral response accelerations that are adjusted for site class effects, S_{MS} at short period as well as S_{M1} at long period as guided by the IBC 1613.3.3.

$$S_{MS} = F_a S_s \tag{1.62}$$

$$S_{M1} = F_v S_1 \tag{1.63}$$

where

F_a represents short-period site coefficient (at 0.2 period).
F_v represents long-period site coefficient (at 1.0 s period).

The value of F_a and F_v shall be evaluated in accordance with Tables 1.40 and 1.41 respectively.

1.4 As stipulated by the IBC 1613.3.4, Determine the damped design (5%) spectral response accelerations, usually the S_{DS} at short period and S_{D1} at long period

$$S_{DS} = \left(\frac{2}{3}\right) S_{MS} \tag{1.64}$$

Table 1.42 Risk category of buildings and other structures

Occupancy category	Nature of occupancy
I	Buildings and other structures that represent a low hazard to human life in the event of failure, including but not limited to: • Agricultural facilities • Certain temporary facilities • Minor storage facilities
II	Buildings and other structures except those listed in occupancy categories I, III and IV
III	Buildings and other structures that represent a substantial hazard to human life in the event of failure, including but not limited to: Covered structures whose primary occupancy is public assembly with an occupant load greater than 300 Buildings and other structures with elementary school, secondary school or day care facilities with an occupant load greater than 250 Buildings and other structures with an occupant load greater than 500 for colleges or adult education facilities Health care facilities with an occupant load of 50 or more resident patients, but not having surgery or emergency treatment facilities Jails and detention facilities Any other occupancy with an occupant load greater than 5000 Power-generating stations, water treatment for potable water, waste water treatment facilities and other public utility facilities not included in Occupancy Category IV. Buildings and other structures not included in Occupancy Category IV containing sufficient quantities of toxic or explosive substances to be dangerous to the public if released
IV	Buildings and other structures designated as essential facilities, including but not limited to: Hospitals and other health care facilities having surgery or emergency treatment facilities Fire, rescue and police stations and emergency vehicle garages Designated earthquake, hurricane or other emergency shelters Designated emergency preparedness, communication, and operation centers and other facilities required for emergency response Power-generating stations and other public utility facilities required as emergency backup facilities for Occupancy Category IV structures Structures containing highly toxic materials as defined by Section 307 where the quantity of the material exceeds the maximum allowable quantities of Table 307.1.(2) Aviation control towers, air traffic control centers and emergency aircraft hangars Buildings and other structures having critical national defense functions Water treatment facilities required to maintain water pressure for fire suppression

Table 1.43 Important factors by risk category of buildings and other structures for snow, Ice, and earthquake loads

Risk category	Snow importance factor I_s	Ice importance factor I_i	Ice importance factor I_w	Seismic importance factor I_e
I	0.80	0.80	1.00	1.00
II	1.00	1.00	1.00	1.00
III	1.10	1.25	1.00	1.25
IV	1.2	1.25	1.00	1.50

Table 1.44 Seismic design category based on short-period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
$S_{DS} < 0.167g$	A	A	A
$0.167g \leq S_{DS} < 0.33g$	B	B	C
$0.33\ g \leq S_{DS} < 0.50\ g$	C	C	D
$0.50\ g \leq S_{DS}$	D	D	D

Table 1.45 Seismic design category based on 1-s period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
$S_{D1} < 0.067g$	A	A	A
$0.067g \leq S_{D1} < 0.133g$	B	B	C
$0.133\ g \leq S_{D1} < 0.20g$	C	C	D
$0.20g \leq S_{D1}$	D	D	D

$$S_{D1} = \left(\frac{2}{3}\right) S_{M1} \tag{1.65}$$

where

S_{MS} represents the maximum considered earthquake spectral response accelerations for short period as determined in section 1613.3.3.

S_{D1} . represents maximum considered earthquake spectral response accelerations for long period as determined in section 1613.3.3.

Step 2: Seismic design category and Importance factor determination

Categories of risks for buildings as well as other structure are depicted in Table 1.42 (IBC Table 1604.5). Essential factors are described in Table 1.43 (ASCE 7-10 Table 1.5-2). Structures which are grouped into risk category 1, 2 or 3 that are positioned at the mapped spectral response speed parameter is at 1-s period where s1 is greater than or equal to 0.75 shall be allocated to Seismic Design

Table 1.46 Permitted analytical procedure

Seismic design category	Structural characteristics	Equivalent lateral force analysis Sect. 12.8	Modal response spectrum analysis Sect. 12.9	Seismic response history analysis Chap. 16
B, C	Occupancy category I or II buildings of light-framed construction not exceeding 3 stories in height	P	P	P
	Other occupancy category I or II buildings not exceeding 2 stories in height	P	P	P
	All other structures	P	P	P
D, E, F	Occupancy category I or II buildings of light-framed construction not exceeding 3 stories in height	P	P	P
	Other occupancy category I or II buildings not exceeding 2 stories in height	P	P	P
	Regular structures with T < 3.5Ts and all structures of light frame construction	P	P	P
	Irregular structures with T < 3.5Ts and having only horizontal irregularities Type 2, 3, 4, or 5 of Table 12.2-1 or vertical irregularities Type 4, 5a, or 5b of Table 12.3-1	P	P	P
	All other structures	NP	P	P

Notes: *P* indicates permitted, *NP* indicates not permitted

Category E. Structures that are grouped at Risk Category IV and located at mapped spectral response speed parameter of 1-s period, s1 equal to or greater than 0.75 shall be allocated to Seismic Design Category F. All other structures will be allocated to a seismic category design depending on their category of risk as well as the design spectral response parameters of speed, SD1 and SDs considered in relation to site-specific procedures of ASCE 7 or section 1613.3.4. Every structure and building will be allocated to the more intense seismic category design according to Tables 1.44 and 1.45 (Table 1613.3.5(2)), regardless of the major period of the structure vibration.

Step 3: Seismic Base Shear determination

Table 1.46 (ASCE7-10/Table 12.6-1) indicates one of the permitted types in the ASCE7-10 which comprises on of the major areas addressed in this structural analysis, which is predicated on the structural system, the seismic design category of

Table 1.47 Horizontal structural irregularities

	Irregularity type and description	Reference section	Seismic design category application
1a	**Torsional Irregularity** is defined to exist where the maximum story drift, computed including accidental torsion, at one end of the structure transverse to an axis is more than 1.2 times the average of the story drifts at the two ends of the structure. Torsional irregularity requirements in the reference sections apply only to structures in which the diaphragms are rigid or semi rigid	12.3.3.4 12.8.4.3 12.7.3 12.12.1 Table 12.6-1 Section 16.2.2	D, E, and F C, D, E, and F B, C, D, E, and F C, D, E, and F D, E, and F B, C, D, E, and F
1b	**Extreme Torsional Irregularity** is defined to exist where the maximum story drift, computed including accidental torsion, at one end of the structure transverse to an axis is more than 1.4 times the average of the story drifts at the two ends of the structure. Extreme torsional irregularity requirements in the reference sections apply only to structures in which the diaphragms are rigid or semi rigid	12.3.3.1 12.3.3.4 12.7.3 12.8.4.3 12.12.1 Table 12.6-1 Section 16.2.2	E and F D B, C, and D C and D C and D D B, C, and D
2	**Reentrant Corner Irregularity** is defined to exist where both plan projections of the structure beyond a reentrant corner are greater than 15% of the plan dimension of the structure in the given direction	12.3.3.4 Table 12.6-1	D, E, and F D, E, and F

Table 1.48 Horizontal structural irregularities

	Irregularity type and description	Reference section	Seismic design category application
3	**Diaphragm Discontinuity Irregularity** is defined to exist where there are diaphragms with abrupt discontinuities or variations in stiffness, including those having cut out or open areas greater than 50% of the gross enclosed diaphragm area, or changes in effective diaphragm stiffness of more than 50% from one story to the next	12.3.3.4 Table 12.6-1	D, E, and F D, E, and F
4	**Out-of-Plane Offsets Irregularity** is defined to exist where there are discontinuities in a lateral force-resistance path, such as out-of-plane offsets of the vertical elements	12.3.3.4 12.3.3.3 12.7.3 Table 12.6-1 16.2.2	D, E, and F B, C, D, E, and F B, C, D, E, and F D, E, and F B, C, D, E, and F
5	**Nonparallel Systems-Irregularity** is defined to exist where the vertical lateral force-resisting elements are not parallel to or symmetric about the major orthogonal axes of the seismic force–resisting system	12.5.3 12.7.3 Table 12.6-1 Section 16.2.2	C, D, E, and F B, C, D, E, and F D, E, and F B, C, D, E, and F

Table 1.49 Vertical structural irregularities

	Irregularity type and description	Reference section	Seismic design category application
1a	**Stiffness Soft Story Irregularity** is defined to exist where there is a story in which the lateral stiffness is less than 70% of that in the story above or less than 80% of the average stiffness of the three stories above	Table 12.6-1	D, E, and F
1b	**Stiffness-Extreme Soft Story Irregularity** is defined to exist where there is a story in which the lateral stiffness is less than 60% of that in the story above or less than 70% of the average stiffness of the three stories above	12.3.3.1 Table 12.6-1	E and F D, E, and F
2	**Weight (Mass) Irregularity** is defined to exist where the effective mass of any story is more than 150% of the effective mass of an adjacent story. A roof that is lighter than the floor below need not be considered	Table 12.6-1	D, E, and F
3	**Vertical Geometric Irregularity** is defined to exist where the horizontal dimension of the seismic force–resisting system in any story is more than 130% of that in an adjacent story	Table 12.6-1	D, E, and F
4	**In-Plane Discontinuity in Vertical Lateral Force-Resisting Element Irregularity** is defined to exist where an in-plane offset of the lateral force-resisting elements is greater than the length of those elements or there exists a reduction in stiffness of the resisting element in the story below	12.3.3.4 12.3.3.3 Table 12.6-1	D, E, and F B, C, D, E, and F D, E, and F
5a	**Discontinuity in Lateral Strength Weak Story Irregularity** is defined to exist where the story lateral strength is less than 80% of that in the story above. The story lateral strength is the total lateral strength of all seismic-resisting elements sharing the story shear for the direction under consideration	12.3.3.1 Table 12.6-1	E, and F D, E, and F
5b	**Discontinuity in Lateral Strength–Extreme Weak Story Irregularity** is defined to exist where the story lateral strength is less than 65% of that in the story above. The story strength is the total strength of all seismic-resisting elements sharing the story shear for the direction under consideration	12.3.3.1 12.3.3.2 Table 12.6-1	D, E, and F B and C D, E, and F

the structure, dynamic properties as well as the regularity with the concerned authority giving approval. However, the generally accepted procedure which is the best alternative can also be employed. The selected analysis procedure will be conducted as guided by the requirements contained in the corresponding section that is captured in Table 1.46 (ASCE7-10/Table 12.6-1) (Tables 1.47, 1.48 and 1.49).

- **Equivalent Lateral Force Analysis**:

Section 12.8 of ASCE 7-10 shall be utilized. As the Eq. (1.60) express, the seismic base shear V in a particular direction can be determined as:

$$V = C_S W$$

where:
W = Effective Seismic Weight.

Exceptions:

a. The situations where the storage loads inclusion can add a percentage of 5% or less seismic weight at that particular level, there is no need to be included in the effective seismic weight.
b. There is also no need to include floor live load in both the open parking structures and the public garages.

2. In situations where the floor load design requires provisions for partitions, the floor area's minimum weight of 0.48 kN/m^2 or actual partition weight depending on which is great.
3. Permanent equipment's total operating weight

C_s represents Seismic response coefficient

$$= \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} \tag{1.66}$$

R represents response modification factor, given in Table 1.52 (ASCE 7-10 Table 12.2-1).
I_e represents importance factor.

The Cs value shall not surpass the following:

$$C_S = \frac{S_{D1}}{T(R/I_e)} \quad for \quad T \le T_L \tag{1.67}$$

$$C_S = \frac{S_{D1} T_L}{T^2(R/I_e)} \quad for \quad T > T_L \tag{1.68}$$

The C_s value must not be less than:

$$C_S = 0.044 S_{DS} I_e \ge 0.01 \tag{1.69}$$

Table 1.50 Values of approximate period parameters Ct and x

Structure type	C_t	x
Moment-resisting frame systems in which the frames resist 100% of the required seismic force and are not enclosed or adjoined by components that are more rigid and will prevent the frames from deflecting where subjected to seismic forces		
Steel moment-resisting frames	0.028 (0.0724)[a]	0.8
Concrete moment-resisting frames	0.016 (0.0466)[a]	0.9
Eccentrically braced steel frames	0.03 (0.0731)[a]	0.75
All other structural systems	0.02 (0.0488)[a]	0.75

Table 1.51 Coefficient for upper limit on calculated period

Design spectral response, S_{D1}	Coefficient C_u
≥ 0.4	1.4
0.3	1.4
0.2	1.5
0.15	1.6
≤ 0.1	1.7

For structures that are located where S_1 is equal to or greater than 0.6 g, Cs must not be less than

$$= \frac{0.5S_1}{\left(\frac{R}{I_e}\right)} \tag{1.70}$$

where:

T = the structure's fundamental period

T_L = long-period transition period, (given in ASCE 7-10 Fig. 22), that is the transition period between the design spectrum's velocity and displacement-controlled portions An estimated value of T_a may be achieved from:

$$T_a = C_t \times h_n^x \tag{1.71}$$

where:

h_n = building's height above the base expressed in meters

C_t = building period coefficient as expressed in Table 1.50 (Table 12.8-2)

x = constant shown in Table 1.50 (Table 12.8-2)

Table 1.51 (Table 12.8-1), the worked out fundamental period, T, cannot be greater than the coefficient's product, C_u, times T_a, the approximate fundamental period.

In situations where the moment resisting frames do not go beyond the height of 12 stories and with a minimum of 3 m story height, the approximate period in seconds, T_a, can be given by the following forms:

$$T_a = 0.1\ \mathrm{N} \tag{1.72}$$

where

N = number of storeys beyond the base.

- **Seismic Forces' Vertical Distribution**:

The lateral seismic force (F_x) induced at any level will be expressed by the following equations:

$$F_x = C_{VX} \times V \tag{1.73}$$

And

$$C_{vx} = \frac{W_x \times h_x^k}{\sum_{i=1}^{n} W_i h_i^k} \tag{1.74}$$

where:

F_X = Lateral force at level x
C_{vx} = Vertical distribution factor
V = the building's shear base or the total design lateral force
W_x and W_i = the portions of W allocated to levels x and i
h_x and h_i = heights assigned to levels x and i
k = a building period related distribution exponent as follows:
k = 1 for buildings having the T less than or equal to 0.5 s
k = 2 for buildings having the T more than or equal to 2.5 s
For building with T between 0.5 and 2.5, interpolate between k = 1 and k = 2

The Calculation of Equivalent Lateral Force of Project Using IBC 2012

Step 1: Determination of maximum considered earthquake and design spectral response accelerations:

1.1 Depending on the characteristics of the most significant earthquake contributing to the local hazard this building is located in the high-risk zone of the earthquake. As a result, the value of S_1 and S_S is calculated 0.3 and 0.75 respectively.

Table 1.52 Design coefficient and factors foe seismic force-resisting system

Seismic force–resisting system	ASCE 7 section where detailing requirements are specified	Response modification coefficient, **Ra**	System over strength factor, $\mathbf{\Omega_0}$g	Deflection amplification factor, $\mathbf{C_d}$	Structural System limitations and building height (ft) limit				
					Seismic design category				
					B	C	D^d	E^d	F^0
A. *Bearing wall systems*									
1. Special reinforced concrete shear wall	14.2 and 14.2.3.6	5	$2\frac{1}{2}$	5	NL	NL	160	160	100
2. Ordinary reinforced concrete shear walls	14.2 and 14.2.3.4	4	$2\frac{1}{2}$	4	NL	NL	NP	NP	NP
3. Detailed plain concrete shear walls	14.2 and 14.2.3.2	2	$2\frac{1}{2}$	2	NL	NP	NP	NP	NP
4. Ordinary plain concrete shear walls	14.2 and 14.2.3.1	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$	NL	NP	NP	NP	NP
5. Intermediate precast shear walls	14.2 and 14.2.3.5	4	$2\frac{1}{2}$	4	NL	NL	40 [k]	40 [k]	40 [k]
6. Ordinary precast shear walls	14.2 and 14.2.3.3	3	$2\frac{1}{2}$	3	NL	NP	NP	NP	NP
7. Special reinforced masonry shear walls	14.4 and 14.4.3	5	$2\frac{1}{2}$	$3\frac{1}{2}$	NL	NL	160	160	100
8. Intermediate reinforced masonry shear walls	14.4 and 14.4.3	$3\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{4}$	NL	NL	NP	NP	NP
9. Ordinary reinforced masonry shear walls	14.4	2	$2\frac{1}{2}$	$1\frac{3}{4}$	NL	160	NP	NP	NP
10. Detailed plain masonry shear walls	14.4	2	$2\frac{1}{2}$	$1\frac{3}{4}$	NL	NP	NP	NP	NP
11. Ordinary plain masonry shear walls	14.4	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{4}$	NL	NP	NP	NP	NP
12. Prestressed masonry shear walls	14.4	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{3}{4}$	NL	NP	NP	NP	NP
13. Light-framed walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1, 14.1.4.2, and 14.5	$6\frac{1}{2}$	3	4	NL	NL	65	65	65
14. Light-framed walls with shear panels of all other materials	14.1, 14.1.4.2, and 14.5	2	$2\frac{1}{2}$	2	NL	NL	35	NP	NP
15. Light-framed wall systems using flat strap bracing	14.1, 14.1.4.2, and 14.5	4	2	$3\frac{1}{2}$	NL	NL	65	65	65

(continued)

Table 1.52 (continued)

Seismic force–resisting system	ASCE 7 section where detailing requirements are specified	Response modification coefficient, **Ra**	System over strength factor, $\boldsymbol{\Omega_0}$g	Deflection amplification factor, $\mathbf{C_d}$	Structural System limitations and building height (ft) limit				
					Seismic design category				
					B	C	D^d	E^d	F^0
B. *Building frame systems*									
1. Steel eccentrically braced frames, moment resisting connections at columns away from links	14.1	8	2	4	NL	NL	160	160	100
2. Steel eccentrically braced frames, non-moment-resisting, connections at columns away from links	14.1	7	2	4	NL	NL	160	160	100
3. Special steel concentrically braced frames	14.1	6	2	5	NL	NL	160	160	100
4. Ordinary steel concentrically braced frames	14.1	$3\frac{1}{4}$	2	$3\frac{1}{4}$					
5. Special reinforced concrete shear walls	14.2 and 14.2.3.6	6	$2\frac{1}{2}$	5	NL	NL	160	160	100
6. Ordinary reinforced concrete shear walls	14.2 and 14.2.3.4	5	$2\frac{1}{2}$	$4\frac{1}{2}$	NL	NL	NP	NP	NP
7. Detailed plain concrete shear wall	14.2 and 14.2.3.2	2	$2\frac{1}{2}$	2	NL	NP	NP	NP	NP
8. Ordinary plain concrete shear walls	14.2 and 14.2.3.1	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$	NL	NP	NP	NP	NP
9. Intermediate precast shear walls	14.2 and 14.2.3.5	5	$2\frac{1}{2}$	$4\frac{1}{2}$	NL	NL	40 [k]	40 [k]	40 [k]
10. Ordinary precast shear walls	14.2 and 14.2.3.3	4	$2\frac{1}{2}$	4	NL	NP	NP	NP	NP
11. Composite steel and concrete eccentrically braced frames	14.3	8	2	4	NL	NL	160	160	100
12. Composite steel and concrete concentrically braced frames	14.3	5	2	$4\frac{1}{2}$	NL	NL	160	160	100
13. Ordinary composite steel and concrete braced frames	14.3	3	2	3	NL	NL	NP	NP	NP
14. Composite steel plate shear walls	14.3	$6\frac{1}{2}$	$2\frac{1}{2}$	$5\frac{1}{2}$	NL	NL	160	160	100

(continued)

Table 1.52 (continued)

Seismic force–resisting system	ASCE 7 section where detailing requirements are specified	Response modification coefficient, **Ra**	System over strength factor, $\mathbf{\Omega_0}$g	Deflection amplification factor, $\mathbf{C_d}$	Structural System limitations and building height (ft) limit				
					Seismic design category				
					B	C	D^d	E^d	F^0
15. Special composite reinforced concrete shear walls with steel elements	14.3	6	$2\frac{1}{2}$	5	NL	NL	160	160	100
16. Ordinary composite reinforced concrete shear walls with steel elements	14.3	5	$2\frac{1}{2}$	$4\frac{1}{2}$	NL	NL	NP	NP	NP
17. Special reinforced masonry shear walls	14.4	$5\frac{1}{2}$	$2\frac{1}{2}$	4	NL	NL	160	160	100
18. Intermediate reinforced masonry shear walls	14.4	4	$2\frac{1}{2}$	4	NL	NL	NP	NP	NP
19. Ordinary reinforced masonry shear walls	14.4	2	$2\frac{1}{2}$	2	NL	160	NP	NP	NP
20. Detailed plain masonry shear walls	14.4	2	$2\frac{1}{2}$	2	NL	NP	NP	NP	NP
21. Ordinary plain masonry shear wall	14.4	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{4}$	NL	NP	NP	NP	NP
22. Prestressed masonry shear walls	14.4	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{3}{4}$	NL	NP	NP	NP	NP
23. Light-framed walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1, 14.1.4.2, and 14.5	7	$2\frac{1}{2}$	$4\frac{1}{2}$	NL	NL	65	65	65
24. Light-framed walls with shear panels of all other materials	14.1, 14.1.4.2, and 14.5	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	NL	NL	35	NP	NP
25. Buckling-restrained braced frames, non-moment-resisting beam-column connections	14.1	7	2	$5\frac{1}{2}$	NL	NL	160	160	100
26. Buckling-restrained braced frames, moment-resisting beam-column connections	14.1	8	$2\frac{1}{2}$	5	NL	NL	160	160	100
27. Special steel plate shear wall	14.1	7	2	6	NL	NL	160	160	100

(continued)

Table 1.52 (continued)

Seismic force–resisting system	ASCE 7 section where detailing requirements are specified	Response modification coefficient, **Ra**	System over strength factor, $\mathbf{\Omega_0}$g	Deflection amplification factor, $\mathbf{C_d}$	Structural System limitations and building height (ft) limit				
					Seismic design category				
					B	C	D^d	E^d	F^0
C. *Moment-resisting frame systems*									
1. Special steel moment frames	14.1 and 12.2.5.5	8	3	$5\frac{1}{2}$	NL	NL	NL	NL	NL
2. Special steel truss moment frames	14.1	7	3	$5\frac{1}{2}$	NL	NL	160	160	NP
3. Intermediate steel moment frames	12.2.5.6,12.2.5.7, 12.2.5.8,12.2.5.9, and 14.1	4.5	3	4					
4. Ordinary steel moment frames	12.2.5.6,12.2.5.7, 12.2.5.8, and 14.1	3.5	3	3					
5. Special reinforced concrete moment frames	12.2.5.5 and 14.2	8	3	$5\frac{1}{2}$	NL	NL	NL	NL	NL
6. Intermediate reinforced concrete moment frames	14.2	5	3	$4\frac{1}{2}$	NL	NL	NP	NP	NP
7. Ordinary reinforced concrete moment frames	14.2	3	3	$2\frac{1}{2}$	NL	NP	NP	NP	NP
8. Special composite steel and concrete moment frames	12.2.5.5and 14.3	8	3	$5\frac{1}{2}$	NL	NL	NL	NL	NL
9. Intermediate composite moment frames	14.3	5	3	$4\frac{1}{2}$	NL	NL	NP	NP	NP
10. Composite partially restrained moment frames	14.3	6	3	$5\frac{1}{2}$	160	160	100	NP	NP
11. Ordinary composite moment frames	14.3	3	3	$2\frac{1}{2}$	NL	NP	NP	NP	NP
D. *Dual systems with special moment frames capable of resisting at least 25% of prescribed seismic forces*	12.2.5.1								
1. Steel eccentrically braced frames	14.1	8	$2\frac{1}{2}$	4	NL	NL	NL	NL	NL

(continued)

Table 1.52 (continued)

Seismic force–resisting system	ASCE 7 section where detailing requirements are specified	Response modification coefficient, **Ra**	System over strength factor, $\mathbf{\Omega_0}$g	Deflection amplification factor, $\mathbf{C_d}$	Structural System limitations and building height (ft) limit				
					Seismic design category				
					B	C	D^d	E^d	F^0
2. Special steel concentrically braced frames	14.1	7	$2\frac{1}{2}$	$5\frac{1}{2}$	NL	NL	NL	NL	NL
3. Special reinforced concrete shear walls	14.2	7	$2\frac{1}{2}$	$5\frac{1}{2}$	NL	NL	NL	NL	NL
4. Ordinary reinforced concrete shear walls	14.2	6	$2\frac{1}{2}$	5	NL	NL	NP	NP	NP
5. Composite steel and concrete eccentrically braced frames	14.3	8	$2\frac{1}{2}$	4	NL	NL	NL	NL	NL
6. Composite steel and concrete concentrically braced frames	14.3	6	$2\frac{1}{2}$	5	NL	NL	NL	NL	NL
7. Composite steel plate shear walls	14.3	$7\frac{1}{2}$	$2\frac{1}{2}$	6	NL	NL	NL	NL	NL
8. Special composite reinforced concrete shear walls with steel elements	14.3	7	$2\frac{1}{2}$	6	NL	NL	NL	NL	NL
9. Ordinary composite reinforced concrete shear walls with steel element	14.3	6	$2\frac{1}{2}$	5	NL	NL	NP	NP	NP
10. Special reinforced masonry shear walls	14.4	$5\frac{1}{2}$	3	5	NL	NL	NL	NL	NL
11. Intermediate reinforced masonry shear walls	14.4	4	3	$3\frac{1}{2}$	NL	NL	NP	NP	NP
12. Buckling-restrained braced frame	14.1	8	$2\frac{1}{2}$	5	NL	NL	NL	NL	NL
13. Special steel plate shear walls	14.1	8	$2\frac{1}{2}$	$6\frac{1}{2}$	NL	NL	NL	NL	NL
E. *Dual systems with intermediate moment frames capable of resisting at least 25% of prescribed seismic forces*	12.2.5.1								
1. Special steel concentrically braced frames f	14.1	6	$2\frac{1}{2}$	5	NL	NL	35	NP	NP[h, k]
2. Special reinforced concrete shear walls	14.2	$6\frac{1}{2}$	$2\frac{1}{2}$	5	NL	NL	160	100	100

(continued)

Table 1.52 (continued)

Seismic force–resisting system	ASCE 7 section where detailing requirements are specified	Response modification coefficient, **Ra**	System over strength factor, $\boldsymbol{\Omega_0}$g	Deflection amplification factor, $\mathbf{C_d}$	Structural System limitations and building height (ft) limit				
					Seismic design category				
					B	C	D^d	E^d	F^0
3. Ordinary reinforced masonry shear walls	14.4	3	3	$2\frac{1}{2}$	NL	160	NP	NP	NP
4. Intermediate reinforced masonry shear walls	14.4	$3\frac{1}{2}$	3	3	NL	NL	NP	NP	NP
5. Composite steel and concrete concentrically braced frames	14.3	$5\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{2}$	NL	NL	160	100	NP
6. Ordinary composite braced frames	14.3	$3\frac{1}{2}$	$2\frac{1}{2}$	3	NL	NL	NP	NP	NP
7. Ordinary composite reinforced concrete shear walls with steel elements	14.3	5	3	$4\frac{1}{2}$	NL	NL	NP	NP	NP
8. Ordinary reinforced concrete shear walls	14.2	$5\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{2}$	NL	NL	NP	NP	NP
F. *Shear wall-frame interactive system with ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls*	12.2.5.1 and 14.2	$4\frac{1}{2}$	$2\frac{1}{2}$	4	NL	NP	NP	NP	NP
G. *Cantilevered column systems detailed to conform to the requirements for*	12.2.5.5								
1. Special steel moment frames	12.2.5.5 and 14.1	$2\frac{1}{2}$	$1\frac{1}{4}$	$2\frac{1}{2}$	35	35	35	35	35
2. Intermediate steel moment frames	14.1	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{2}$	35	35	35 [h]	NP[h, i]	NP[h, i]
3. Ordinary steel moment frames	14.1	$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{1}{4}$	35	35	NP	NP[h, i]	NP[h, i]
4. Special reinforced concrete moment frames	12.2.5.5 and 14.2	$2\frac{1}{2}$	$1\frac{1}{4}$	$2\frac{1}{2}$	35	35	35	35	35
5. Intermediate concrete moment frames	14.2	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{2}$	35	35	NP	NP	NP
6. Ordinary concrete moment frames	14.2	5	$1\frac{1}{4}$	1	35	NP	NP	NP	NP

(continued)

Table 1.52 (continued)

Seismic force–resisting system	ASCE 7 section where detailing requirements are specified	Response modification coefficient, **Ra**	System over strength factor, $\mathbf{\Omega_0}$g	Deflection amplification factor, $\mathbf{C_d}$	Structural System limitations and building height (ft) limit				
					Seismic design category				
					B	C	D^d	E^d	F^0
7. Timber frames	14.5	3	$1\frac{1}{2}$	$1\frac{1}{2}$	35	35	35	NP	NP
H. *Steel systems not specifically detailed for seismic resistance, excluding cantilever column systems*	14.1	3	3	3	NL	NL	NP	NP	NP

Table 1.53 Values of site coefficient F_a

Site class	Mapped spectral response acceleration at short period				
	$S_s \leq 0.25$	$S_s = 0.5$	$\boldsymbol{S_s = 0.75}$	$S_s = 1$	$S_s \geq 1.25$
D	1.6	1.4	**1.2**	1.1	1

Table 1.54 Values of site coefficient F_v

Site class	Mapped spectral response acceleration at 1-s period				
	$S_a \leq 0.1$	$S_a = 0.2$	$\boldsymbol{S_a} = 0.3$	$S_a = 0.4$	$S_s \geq 0.5$
D	2.4	2	**1.8**	1.6	1.5

1.2 There is no doubt that the averages shear velocity is most important factor for the selection of the site class. An average shear velocity is considered $V_S, 30 = 350\ \frac{\text{m}}{\text{s}}$ as a property of ground, according to the Table 1.39, the site class is considered as class D. ({ASCE7-10, 2010#31}//Table 20.3.1).

1.3 The maximum considered earthquake spectral response accelerations for short and long period are calculated according Eqs. (1.62) and (1.63) (IBC, 2012, section 1613.3.3). The value of site coefficients Fa and Fv are determined by Tables 1.40 and 1.41 (IBC, 2012, Table 1613.3.3(1) and Table 1613.3.3(2)) respectively (Tables 1.52, 1.53 and 1.54).

Hence;

$$S_{MS} = F_a S_s = 1.2 \times 0.75 = 0.9$$
$$S_{M1} = F_v S_1 = 1.8 \times 0.3 = 0.54$$

1.4 The value of S_{DS} and S_{D1} as damped design (5%) spectral response accelerations in at short and at long period are computed based on Eqs. (1.64) and (1.65) (IBC, 2012, section 1613.3.4.)

Table 1.55 SDC based on short-period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
$0.50g \leq S_{DS}$	D	D	D

Table 1.56 SDC based on 1-s period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
0.20g $\leq$ S_{D1}	D	D	D

$$S_{DS} = \left(\frac{2}{3}\right)S_{MS} = \left(\frac{2}{3}\right) \times (0.9) = 0.6g$$

$$S_{D1} = \left(\frac{2}{3}\right)S_{M1} = \left(\frac{2}{3}\right) \times (0.54) = 0.36g$$

Step 2: Determination of seismic design category and Importance factor

2.1 Based on Tables 1.42 and 1.43 the building is classified in class II and importance factor is evaluated (1). (I_e = 1) (IBC, 2012, IBC Table 1604.5 and ASCE 7-10 Table 1.5-2.)

2.2 Seismic design category (SDC) for $S_{DS} = 0.6g$ and $S_{D1} = 0.36g$ is evaluated "D" in accordance with Tables 1.44 and 1.45. (IBC, 2012, 1613.3.5(1) and Table 1613.3.5(2)) (Tables 1.55 and 1.56).

Step 3: Seismic Base Shear determination

3.1 For intermediate steel moment frame and according Table 1.52, response modification coefficient(R) is 4.5. ({ASCE7-10, 2010 #31}/ Table 12.2-1and ASCE 7-10)

3.2 The horizontal direction's seismic shear force, V, used to evaluate each building can be worked out by using the Eq. (1.60). ({ASCE7-10, 2010 #31}/Sect. 12.8 of ASCE 7-10)

$$V = Cs\ W$$

C_s is computed accordance with Eq. (1.61)

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{4.5}{1}} = 0.1333$$

3.3 Determination of the fundamental period

The moment resistance system is selected for resisting the earthquake load in this building, so the fundamental period of the vibration of the structure for the lateral motion in the considered direction is calculated based on Eq. (1.71) ({ASCE7-10, 2010 #31}/)

$$T_a = C_t \times h_n^x$$

The C_t and x in a given Eq. (1.71) is determined in accordance with Table 1.50 ({ASCE7-10, 2010 #31}//Table 12.8-2), hence;

$$C_t = 0.028, \quad x = 0.8$$

So,

$$T_a = 0.028 \times 21.3^{0.8} = 0.32349$$

According Table 1.51 ({ASCE7-10, 2010 #31}//Table 12.8-1) the value of C_u is equal with:

$$C_u = 1.4$$

Moving on, the calculated fundamental period, T, cannot exceed the product of the coefficient, C_u, in Table 1.51 ({ASCE7-10, 2010 #31}//Table 12.8-1) times the approximate fundamental period, T_a.

$$C_u \times T_a = 1.4 \times 0.32349 = 0.4528$$

1.4 Exceptions

- The value of C_s shall not exceed the value of Eq. (1.67), hence;

$$C_S = \frac{S_{D1}}{T(R/I_e)} \quad for \quad T \le T_L$$
$$C_S = \frac{S_{D1}}{T(R/I_e)} = \frac{0.36}{0.3234\left(\frac{4.5}{1}\right)} = 0.2473$$
$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{4.5}{1}} = 0.1333 < C_S = \frac{S_{D1}}{T(R/I_e)} = \frac{0.36}{0.3234\left(\frac{4.5}{1}\right)} = 0.2473(OK)$$

- The value of C_s shall not be less than the value of Eq. (1.69), so:

$$C_S = 0.044 S_{DS} I_e \ge 0.01$$
$$= 0.044 \times 0.6 \times 1 = 0.0264 \ge 0.01 \text{ (OK)}$$
$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{4.5}{1}} = 0.1333 > 0.0264 \ (OK)$$

As a result the value of C_s is computed according Eq. (1.61).

$$C_s = 0.1333$$

Table 1.57 Distribution of shear base in each floor

Story	Height	Weight	W_ih_i	$W_ih_i/\sum W_ih_i$	Force (kN)	Shear (kN)	Moment (kN m)
6	21.3	162920.58	3470208.35	0.2857	375.14	375.14	675.25
5	17.7	165112.13	2922484.7	0.2406	315.93	691.07	2594.43
4	14.1	165605.4	2335036.14	0.1922	252.42	943.49	5536.66
3	10.5	166098.25	1744031.63	0.1435	188.53	1132.03	9272.61
2	6.9	166954.8	1151988.12	0.0948	124.53	1256.56	13572.09
1	3.3	158044.18	521545.794	0.0429	56.380	1312.94	18000.27
		984735.347	12145294.7				

- The seismic base shear calculates in accordance with Eq. (1.60)

$$V = C_s \times W$$

$$V = 0.1333 \times 984735.3479 = 1312.65\ (\mathrm{kN})$$

Step 4: Vertical distribution of forces

The lateral seismic force (F_x) induced at any level shall be determined according Eqs. (1.73) and (1.74).

$$F_x = C_{VX} \times V$$

$$C_{vx} = \frac{W_x \times h_x^k}{\sum_{i=1}^{n} W_i h_i^k}$$

k = a distribution exponent related to the building period as follows:
k = 1 for buildings with T less than or equal to 0.5 s
k = 2 for buildings with T more than or equal to 2.5 s
Interpolate between k = 1 and k = 2 for buildings with T between 0.5 and 2.5

$$T_a = 0.32349 < 0.5\ \mathrm{s} \rightarrow k = 1$$

In the structure under consideration, the base shear in each direction is 1312.94 kN, and the total weight of each story is calculated from previous parts. The height of the story is illustrated in Fig. 1.24. Table 1.57 shows the distribution of shear generated in each story and overturning of the moment too. Lateral force and shear force in story 6 are equal 375.14 kN. Lateral force for story five is 315.93 kN. Shear force in story five is equal the summation of lateral force in stories five and six.

Table 1.58 Area and moment inertia of column and beam sections

Name	A	I_x	I_y
$Box \times 250 \times 10$	0.0096	9.232×10^{-5}	9.232×10^{-5}
$Box \times 300 \times 10$	0.0116	1.62786×10^{-4}	1.62786×10^{-4}
$Box \times 350 \times 10$	0.0136	2.62253×10^{-4}	2.62253×10^{-4}
$Box \times 400 \times 10$	0.0156	3.9572×10^{-4}	3.9572×10^{-4}
$Box \times 400 \times 12$	0.0186	4.67735×10^{-4}	4.67735×10^{-4}
$Box \times 450 \times 15$	0.0261	8.24107×10^{-4}	8.24107×10^{-4}
$Pg(300 \times 150 \times 8)$	0.006944	8.17129×10^{-5}	2.74305×10^{-5}
$Pg(350 \times 150 \times 10)$	0.0096	1.4662×10^{-4}	3.802×10^{-5}
$Pg(350 \times 200 \times 10)$	0.0106	1.75528×10^{-4}	7.2953×10^{-5}
$Pg(500 \times 200 \times 10)$	0.0136	4.2445×10^{-4}	1.00053×10^{-4}
IPE140	0.00164	5.41×10^{-6}	4.49×10^{-7}
IPE160	0.00201	8.69×10^{-6}	6.83×10^{-7}
IPE200	0.00285	91.32×10^{-5}	1.01×10^{-6}

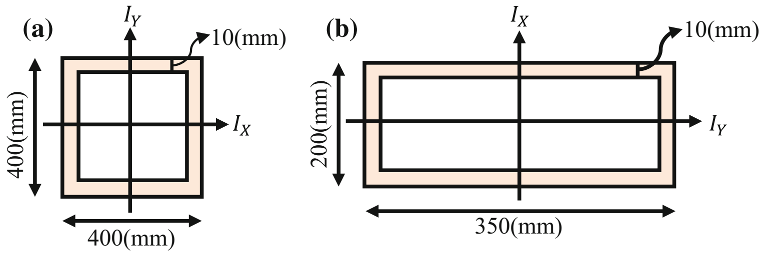


Fig. 1.59 **a** Details of column (C-3), **b** details of beam (B-4)

1.3.5 Shear Rigidity

In the process of analysis moment resistance frame, the first step is calculation and distribution of horizontal lateral load at each bent. For this reason, an assumption is made such that the floors are rigid in-plane, so the horizontal displacement of all vertical frames on the floor level is related by the horizontal translation and rotation of the floor slab. The total external shear at each level is distributed based on shear rigidity (GA) at each bent. The bending resistance of the columns, girders, and connections provide resistance to the lateral loading, so the moment inertia of columns and beams play a major role in shear rigidity. Equation (1.75) shows the shear rigidity (GA) formulation.

$$GA = \frac{12E}{hi\left(\frac{1}{G} + \frac{1}{C}\right)i} \tag{1.75}$$

Moment inertia and area of columns and beams of the models is shown in the Table 1.58. In the building under study, the moment inertia and area of the beam (B-4) and column (C-3) are calculated as samples. Figure 1.59 shows details of the beam (B-4) and column (C-3).

Area and moment of inertia for sections under study, (C-3) and (B-4) are determined by the following equation:

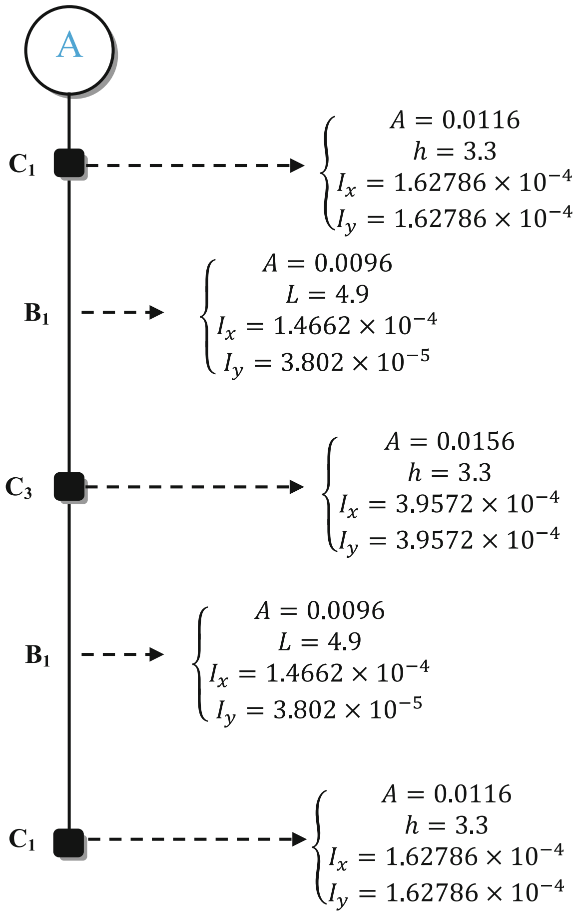


Fig. 1.60 Columns and beams' properties in bent (A)

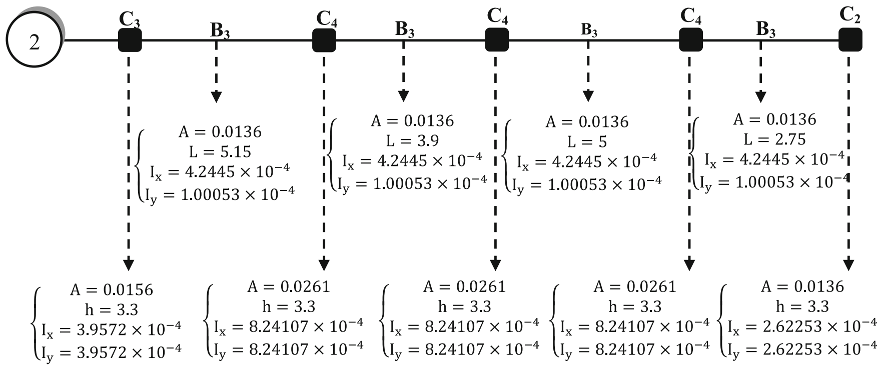


Fig. 1.61 Columns and beams' properties in bent (2)

$$C_3\begin{cases} A = [(400\times 400)-(380\times 380)]\times 10^{-6} = 0.0156\ \text{m}^2 \\ Ix = [\left(\frac{400^4}{12}-\frac{380^4}{12}\right)\times 10^{-12} = 0.0003957\ \text{m}^4 \\ Iy = [\left(\frac{400^4}{12}-\frac{380^4}{12}\right)\times 10^{-12} = 0.00039572\ \text{m}^4 \end{cases}$$

$$B_4\begin{cases} A = [(350\times 200)-(330\times 180)]\times 10^{-6} = 0.0106\ \text{m}^2 \\ Ix = [\left(\frac{350^3\times 200}{12}-\frac{330^3\times 180}{12}\right)\times 10^{-12} = 1.7552\times 10^{-4}\ \text{m}^4 \\ Iy = [\left(\frac{200^3\times 350}{12}-\frac{180^3\times 330}{12}\right)\times 10^{-12} = 7.2953\times 10^{-4}\ \text{m}^4 \end{cases}$$

The process to compute the shear rigidity is as follows:

1. Select story
2. Select bent
3. Calculate the moment inertia of columns
4. Calculate $C = \sum \frac{I_c}{h}$
5. Calculate moment inertia of girders in the same bent
6. Calculate $G = \sum \frac{I_g}{L}$
7. Calculate shear rigidity with Eq. (1.75)

The bents (A) and (2) of the story one in the considered model is selected as an example for calculating of shear rigidity and are shown in Figs. 1.60 and 1.61. This is the motivation in computing the area, moment inertia in the X and Y direction, length and height of beams and columns in the first story. Based on this, the area and moment inertia of sections are shown in Table 1.58.

Calculation of C and G is presented as:

$$C = \sum \frac{\mathrm{I_c}}{\mathrm{h}}$$

$$\begin{cases} C_1 = \frac{I_{c1}}{h} = \frac{1.62786\times10^{-4}}{3.3} = 4.9329 \times 10^{-5} \\ C_3 = \frac{I_{c1}}{h} = \frac{3.9572\times10^{-4}}{3.3} = 1.19915 \times 10^{-4} \\ C_1 = \frac{I_{c1}}{h} = \frac{1.62786\times10^{-4}}{3.3} = 4.9329 \times 10^{-5} \end{cases}$$

$$\mathrm{C} = 4.9329 \times 10^{-5} + 1.19915 \times 10^{-4} + 4.9329 \times 10^{-5} = 2.1857 \times 10^{-4}$$

$$\mathrm{G} = \sum \frac{\mathrm{I_g}}{\mathrm{L}}$$

$$\begin{cases} G_1 = \frac{I_{g1}}{l} = \frac{1.4662\times10^{-4}}{4.9} = 2.9922 \times 10^{-5} \\ G_1 = \frac{I_{g1}}{l} = \frac{1.4662\times10^{-4}}{4.9} = 2.9922 \times 10^{-5} \end{cases}$$

$$\mathrm{G} = 2.9922 \times 10^{-5} + 2.9922 \times 10^{-5} = 5.9844 \times 10^{-5}$$

After calculation of C and G, the shear rigidity should be calculated based on Eq. (1.75). Where

$$\begin{cases} E = 2 \times 10^8 \\ C = 2.1857 \times 10^{-4} \\ G = 5.9844 \times 10 - 5 \\ h_i = 3.3 \text{ m} \end{cases}$$

These magnitudes should be substituted in fallowing Equation. Then

$$GA = \frac{12E}{hi\left(\frac{1}{G} + \frac{1}{c}\right)i}$$

$$\mathrm{GA} = \frac{12 \times 2 \times 10^8}{3.3\left(\frac{1}{5.9844\times10-5} + \frac{1}{2.1857\times10^{-4}}\right)_1} = 34167.83007$$

Shear rigidity in bent A is equal to 34167.83007. Bent (2) consists of four columns and three beams. Based on properties of columns such as moment inertia, height, and area magnitude of C parameter is calculated. The value for G, according to the properties of beams like length, moment inertia, and area, are computed. Figure 1.61 shows the bent (2) with all properties of beams and column.

Calculation of C and G is shown as:

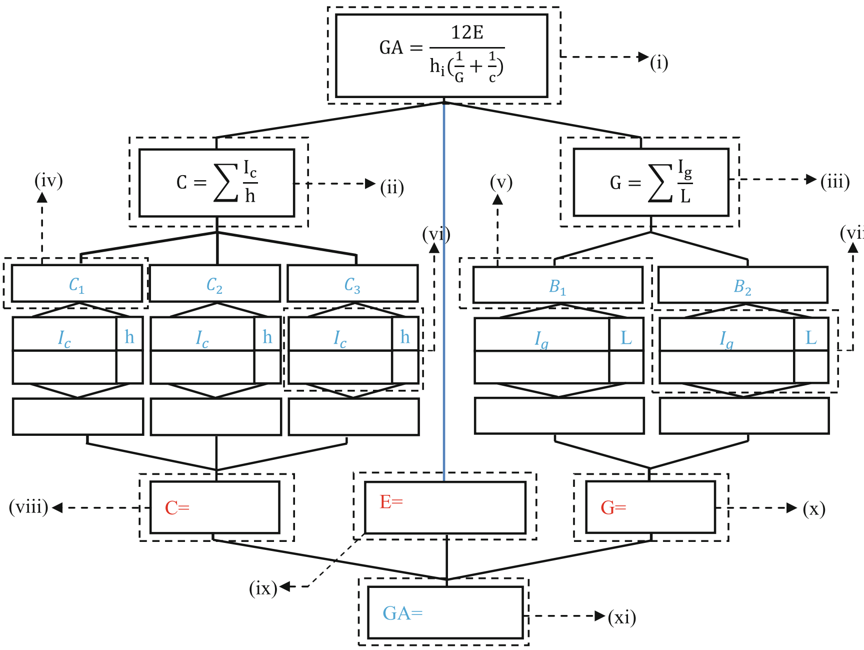


Fig. 1.62 Pattern chart of shear rigidity calculation

$$C = \sum \frac{I_c}{h}$$

$$\begin{cases} C_3 = \frac{I_{c_1}}{h} = \frac{3.9572\times10^{-4}}{3.3} = 1.19915 \times 10^{-4} \\ C_4 = \frac{I_{c_4}}{h} = \frac{8.24107\times10^{-4}}{3.3} = 2.4972 \times 10^{-4} \\ C_4 = \frac{I_{c_4}}{h} = \frac{8.24107\times10^{-4}}{3.3} = 2.4972 \times 10^{-4} \\ C_4 = \frac{I_{c_4}}{h} = \frac{8.24107\times10^{-4}}{3.3} = 2.4972 \times 10^{-4} \\ C_2 = \frac{I_{c_2}}{h} = \frac{2.62253\times10^{-4}}{3.3} = 7.94706 \times 10^{-5} \end{cases}$$

$$\mathrm{C} = 1.19915 \times 10^{-4} + 2.4972 \times 10^{-4} + 2.4972 \times 10^{-4} + 2.4972 \times 10^{-4} 7.94706 \times 10^{-5} = 9.48540 \times 10^{-4}$$

$$\mathrm{G} = \sum \frac{I_g}{L}$$

$$\begin{cases} B_3 = \frac{I_{c_1}}{l} = \frac{4.2445\times10^{-4}}{5.15} = 8.24174 \times 10^{-5} \\ B_3 = \frac{I_{c_4}}{l} = \frac{4.2445\times10^{-4}}{3.9} = 1.08833 \times 10^{-4} \\ B_3 = \frac{I_{c_4}}{l} = \frac{4.2445\times10^{-4}}{5} = 8.489 \times 10^{-5} \\ B_3 = \frac{I_{c_4}}{l} = \frac{4.2445\times10^{-4}}{2.75} = 1.54345 \times 10^{-5} \end{cases}$$

$$= 8.24174 \times 10^{-5} + 1.08833 \times 10^{-4} + 8.489 \times 10^{-5} + 1.54345 \times 10^{-5} = 4.30490 \times 10^{-4}$$

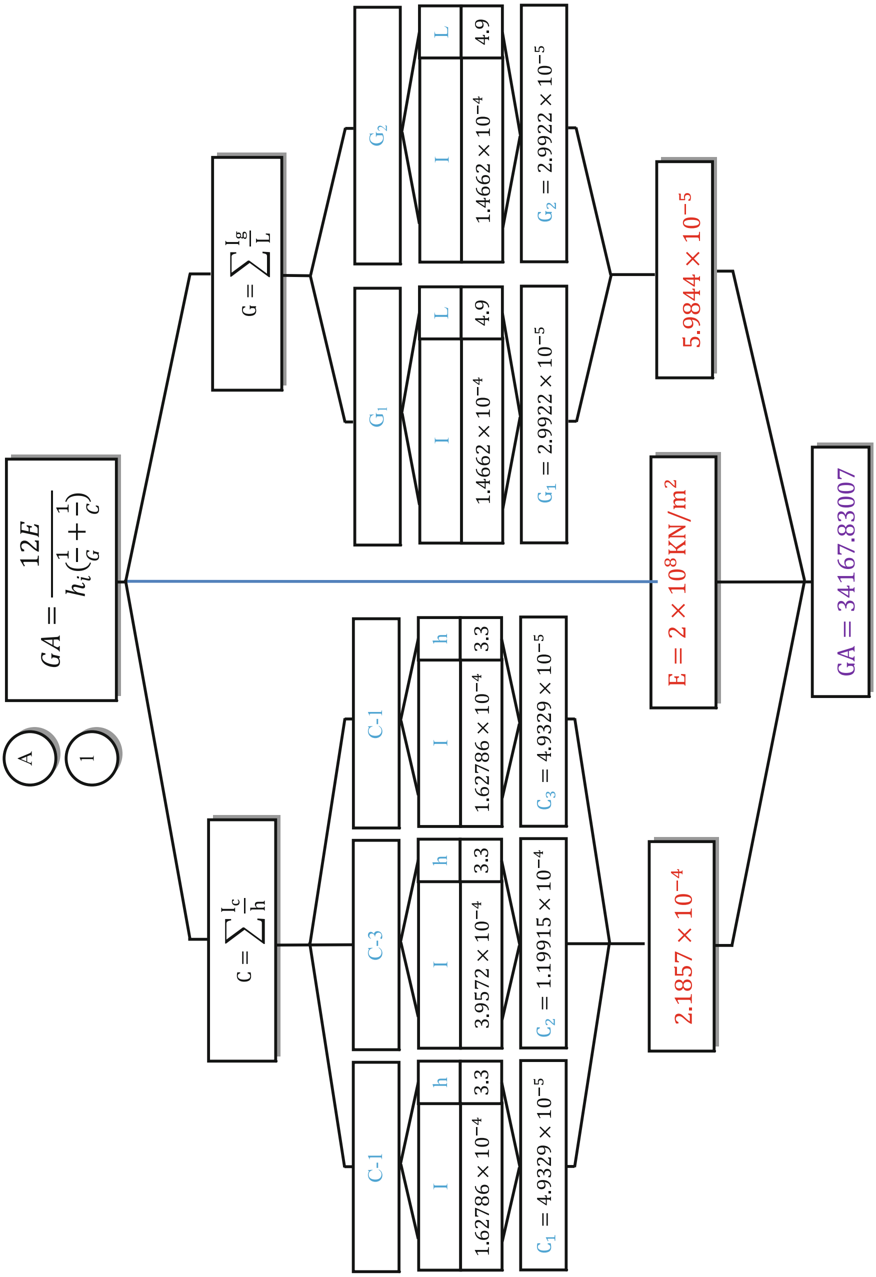


Chart 1.5 Shear rigidity of bent (A) story (1)

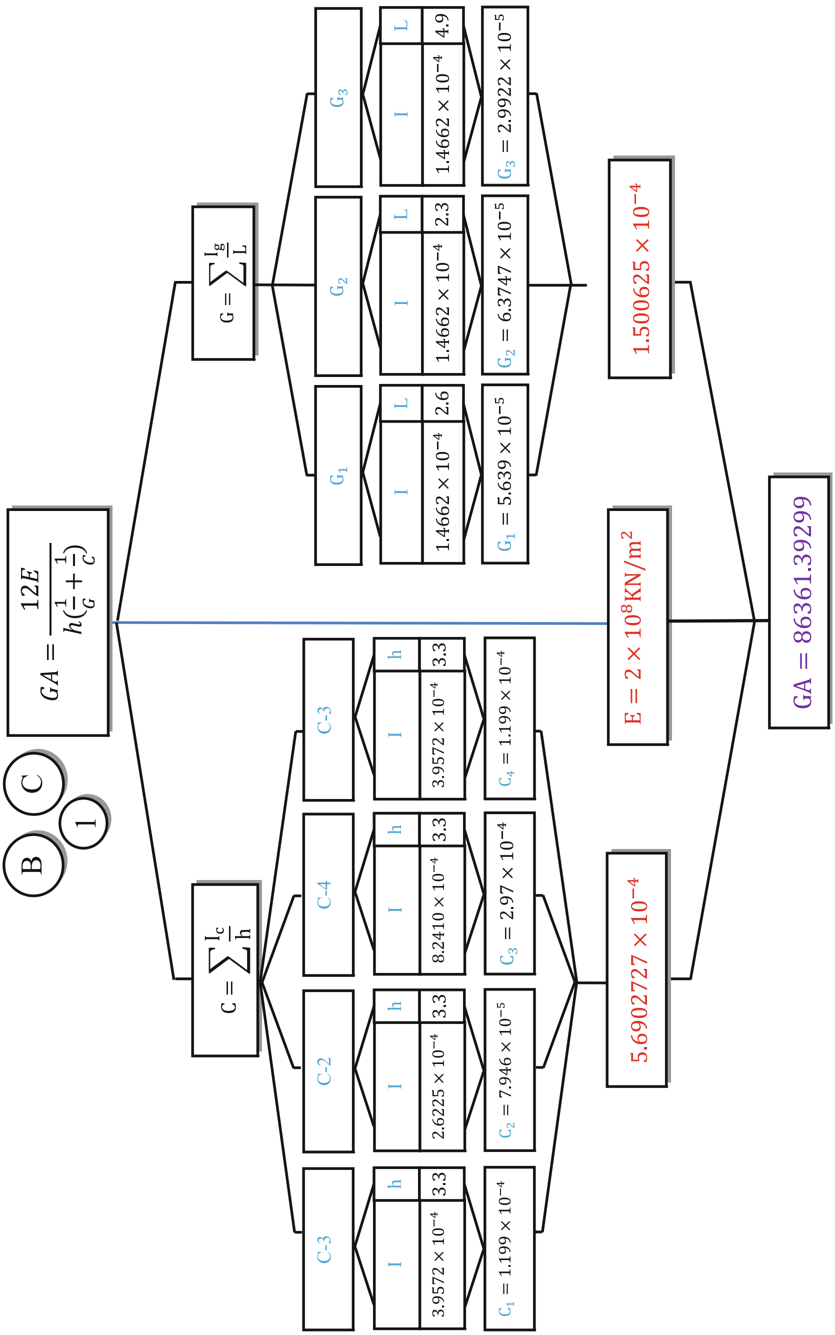


Chart 1.6 Shear rigidity of bent (B) and (C) story (1)

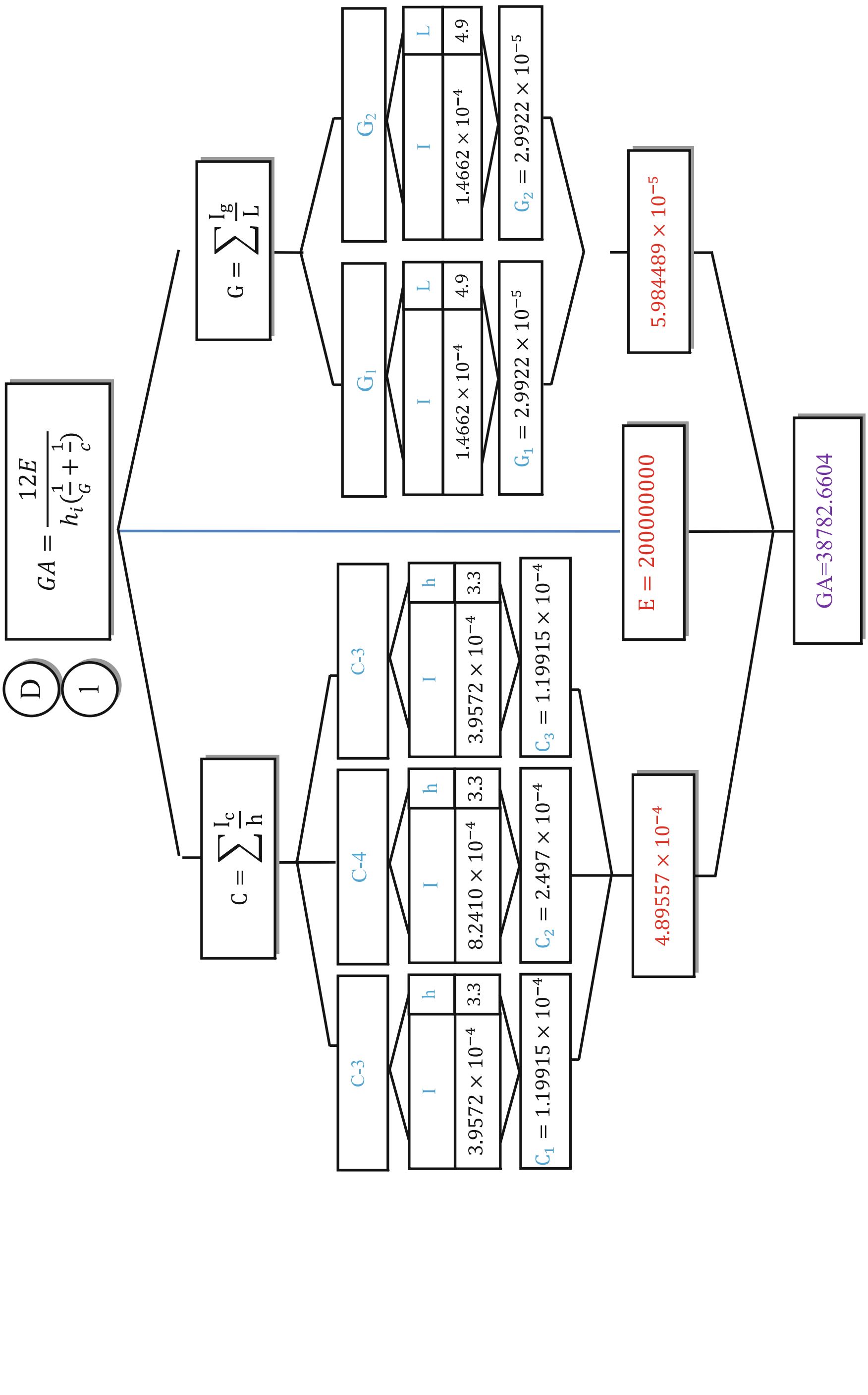


Chart 1.7 Shear rigidity of bent (D) story (1)

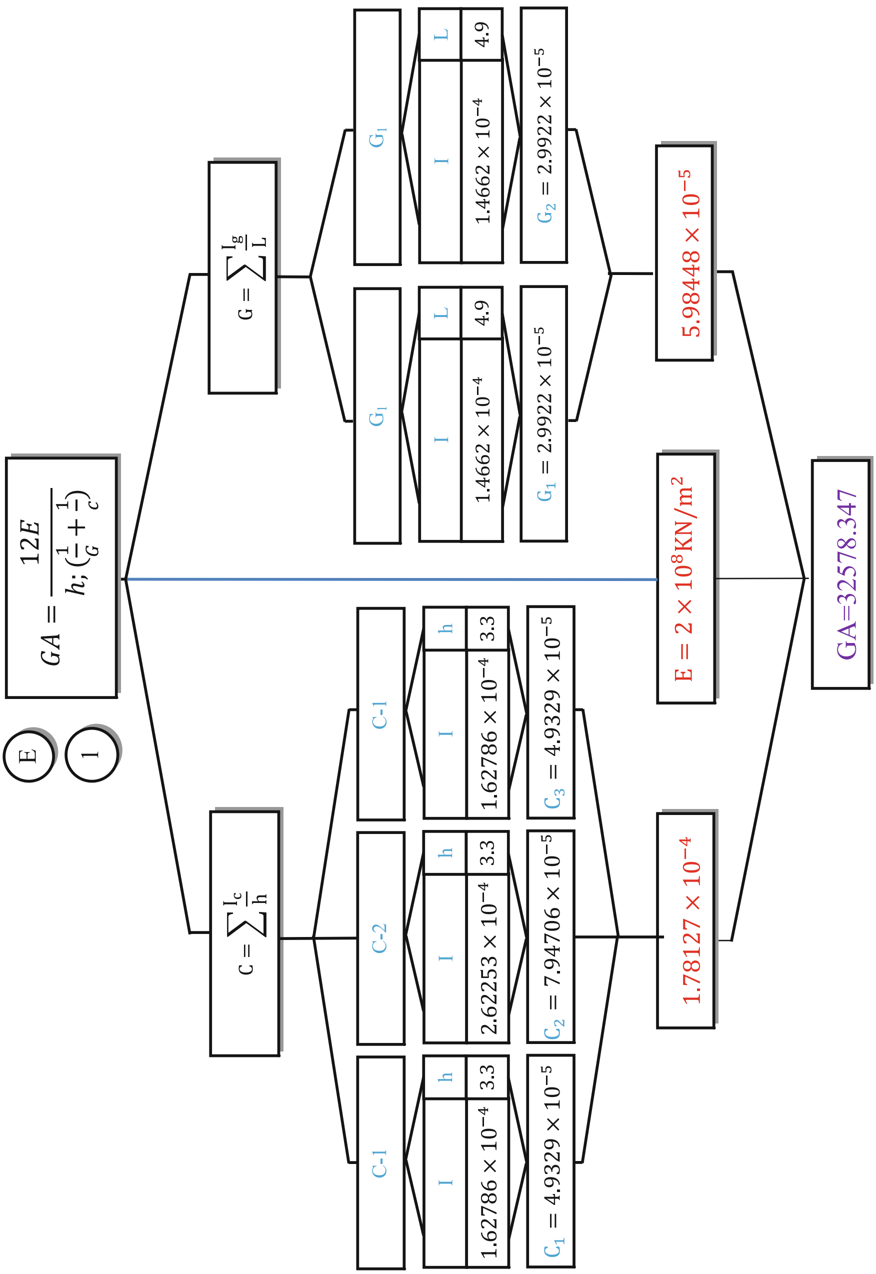


Chart 1.8 Shear rigidity of bent (E) story (1)

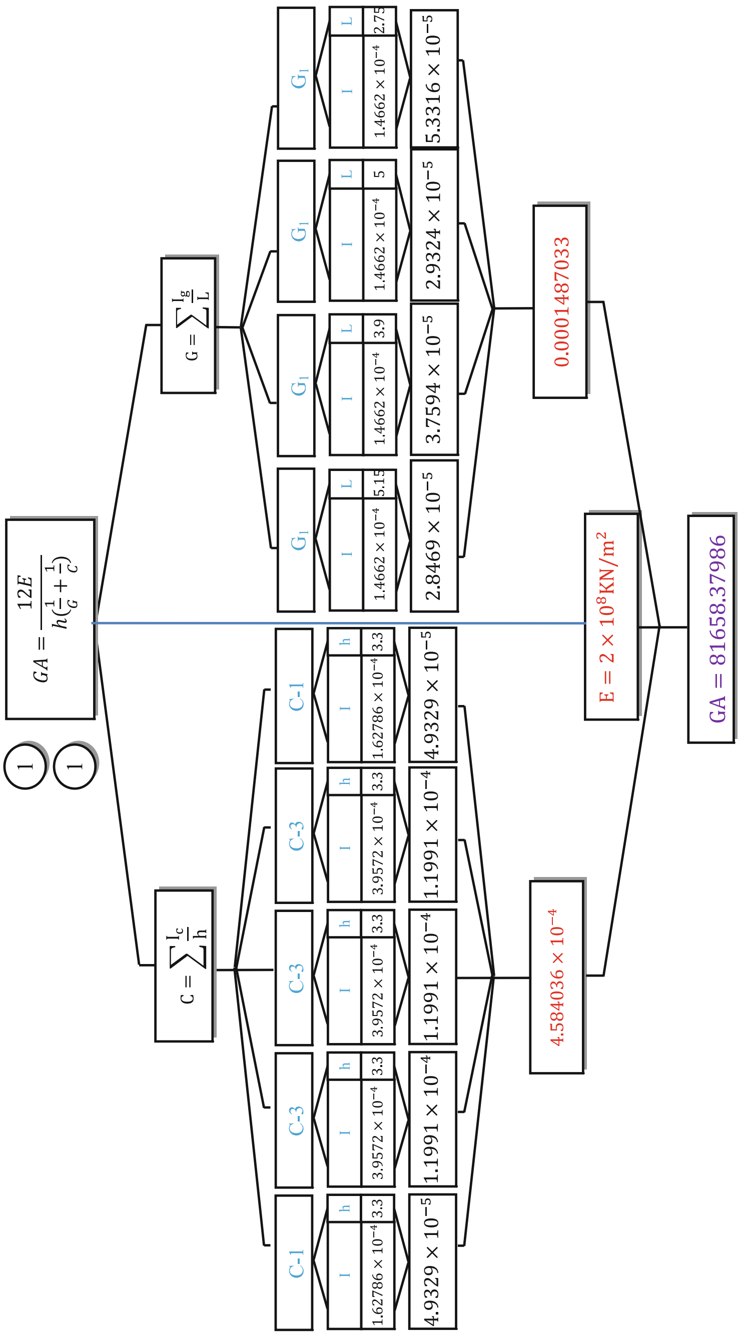


Chart 1.9 Shear rigidity of bent (1) story (1)

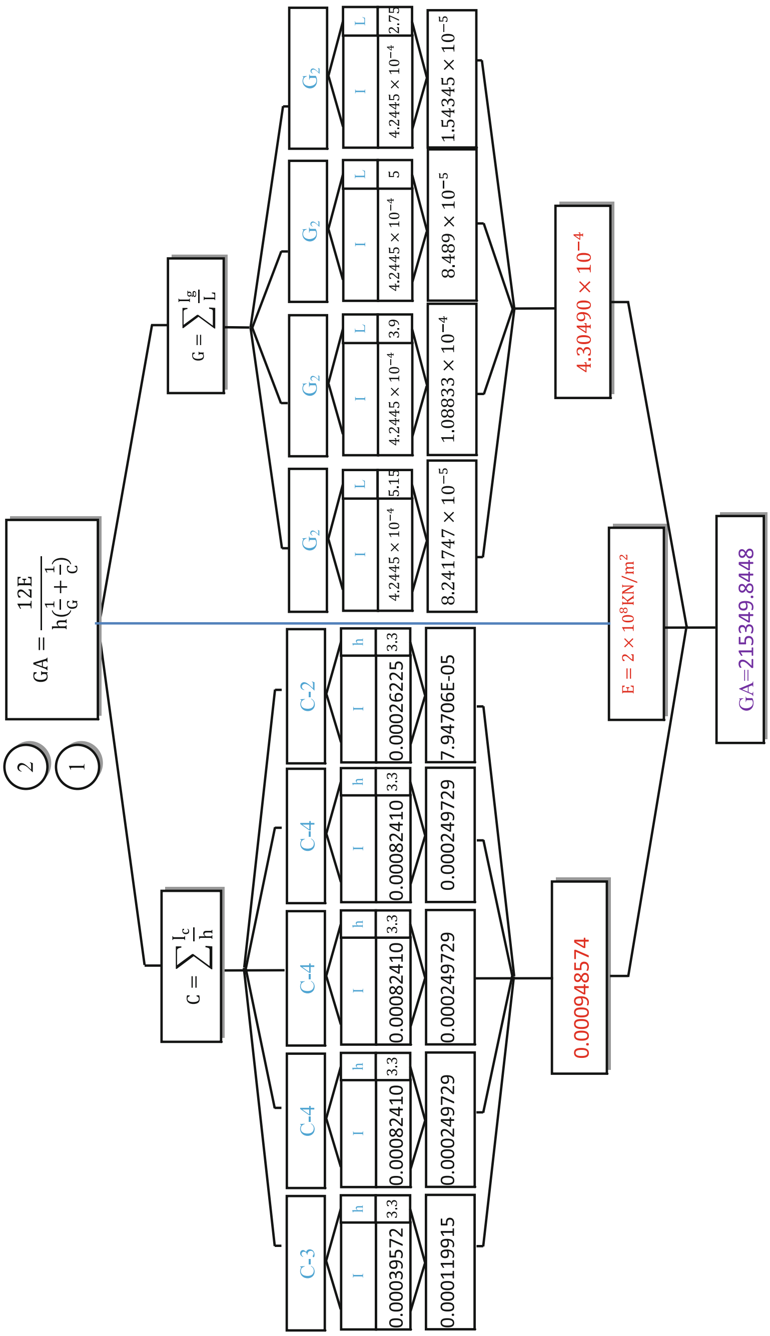


Chart 1.10 Shear rigidity of bent (2) story (1)

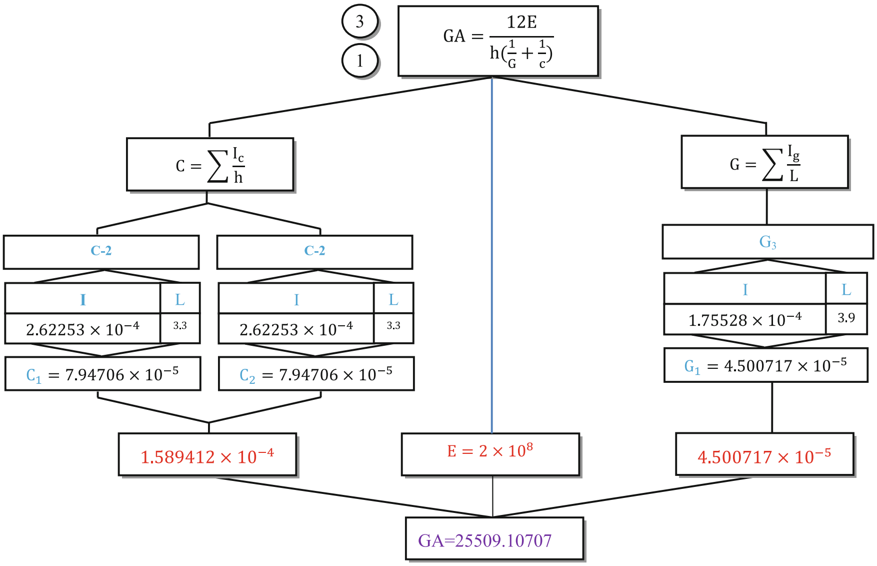


Chart 1.11 Shear rigidity of bent (3) story (1)

Upon calculation of C and G, the next step is in the calculation of shear rigidity based on Eq. (1.75). Here,

$$\begin{cases} E = 2 \times 10^8 \\ C = 9.48540 \times 10^{-4} \\ G = 4.30490 \times 10^{-4} \\ h_i = 3.3\ \text{m} \end{cases}$$

These magnitudes should be submitted in fallowing Equation, Then

$$GA = \frac{12E}{hi\left(\frac{1}{G} + \frac{1}{c}\right)i}$$

$$\text{GA} = \frac{12 \times 2 \times 10^8}{3.3\left(\frac{1}{4.30490 \times 10^{-4}} + \frac{1}{9.48540 \times 10^{-4}}\right)_1} = 215348.7251$$

The total magnitude of the shear rigidity of bent 2 is 215348.7251. The charts (1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11 and 1.12) show the calculation of shear rigidity of story 1 in bents (A)–(E) and bent (1)–(4). The chart shows a step-by-step process in the calculation of shear rigidity. Figure 1.62 shows the pattern graph of calculation of GA.

i: Shear rigidity formulation
ii: C parameter' formulation

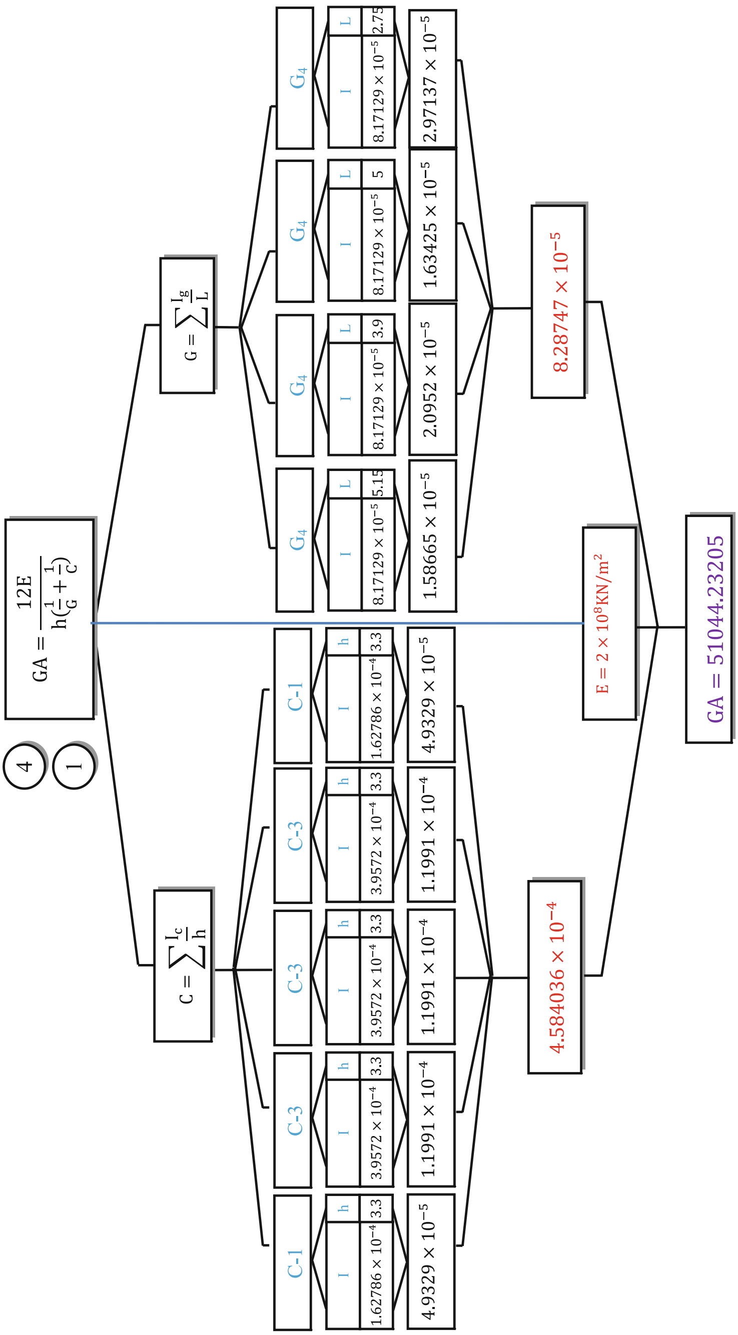


Chart 1.12 Shear rigidity of bent (3) story (1)

iii: G parameter's formulation
iv: Column's name
v: Beam's name
vi: Second-moment inertia and height of the column
vii: Second-moment inertia and length of the beam
viii: C value
ix: Module of elasticity
x: G value
xi: Shear rigidity value

1.3.5.1 Center of Rigidity (Stiffness)

The previous parts (A.4 and A.5) address the stiffness concept in a great detail. This part will focus on explaining the applicability of every location's resistance forces to each floor. The center of rigidity (CR), also called the structure's center of stiffness is usually the applied structure's location resultant lateral force in each story. In each story, the position of the center of rigidity may differ. However, the shear forces of the story may be applicable in any horizontal plane's direction. This force may be determined and shown in two components as Vx and Vy, where a simultaneous story translations are caused such as 'Δx' and 'Δy' respectively. The displacement caused by this story twist is usually proportional to the element's

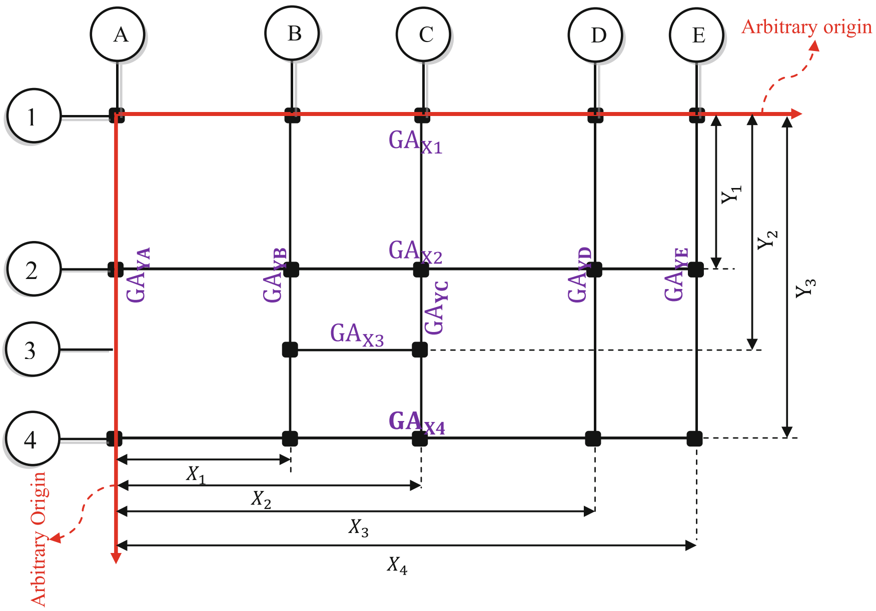


Fig. 1.63 The parameters necessary for calculation of center of rigidity

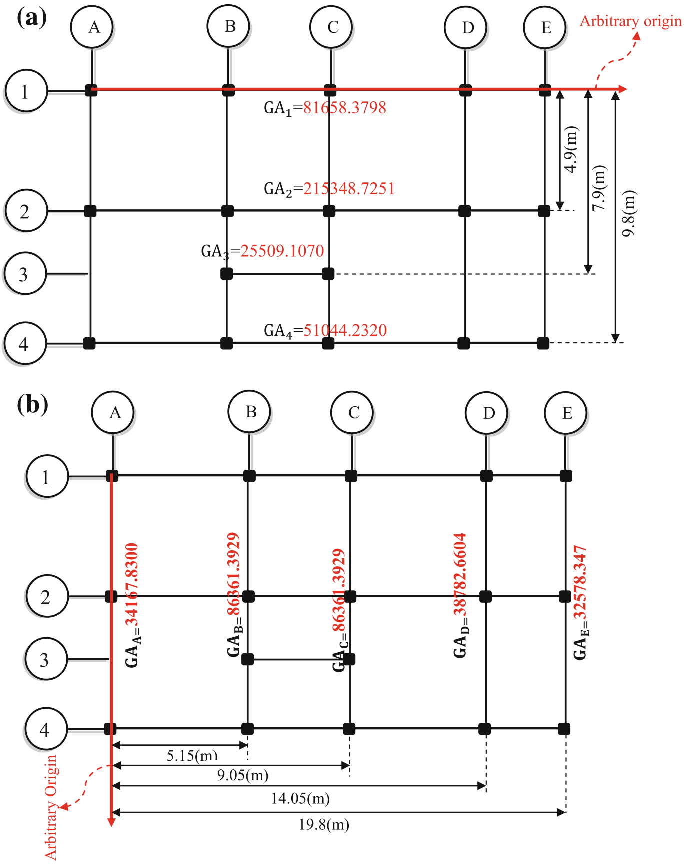


Fig. 1.64 **a** The shear rigidity in X direction and distance between bents and arbitrary origin. **b** Shear rigidity in Y direction and distance between bents and arbitrary origin

distance from the rotation's center or the CR. The center of rigidity's location is worked out based on the Eq. (1.76), which defines the center of rigidity's location in parallel bents' sets in story i, in a considered model, relative to an arbitrary origin, 0 as illustrated by Eq. (1.77).

$$X_{CR} = \frac{\sum k_{yi} \times x_i}{\sum k_{yi}}, \quad Y_{CR} = \frac{\sum k_{xi} \times y_i}{\sum k_{xi}} \tag{1.76}$$

where

X_{CR}, Y_{CR}: coordinates of center of stiffness

$k_{yi} = \textit{lateral stiffness of member i in the length of Y axis}$

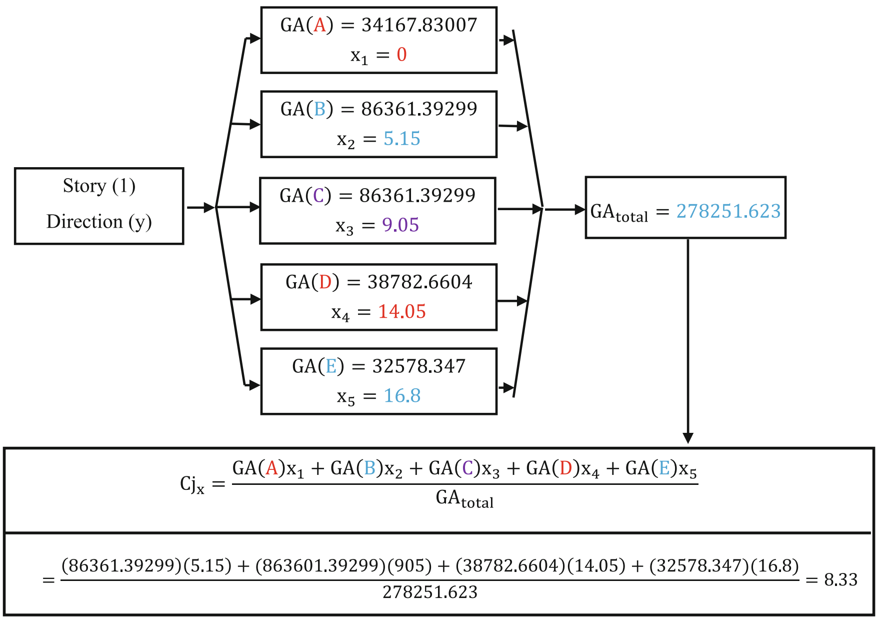


Chart 1.13 Calculation of coordinate of stiffness in Y direction

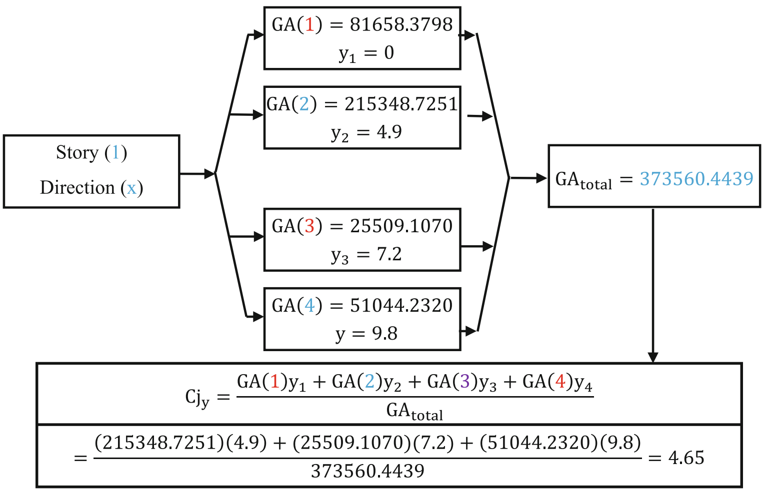


Chart 1.14 Calculation of coordinate of stiffness in X direction

$k_{xi} = lateral\ stiffness\ of\ member\ i\ in\ the\ length\ of\ X\ axis$

$x_i, y_i = coordinate\ of\ center\ of\ memebr\ i$

$$C_{jx} = \left[\frac{\sum(GA) \times X_i}{\sum(GA)}\right] C_{jy} = \left[\frac{\sum(GA) \times Y_i}{\sum(GA)}\right] \tag{1.77}$$

where;

GA_{yi} = lateral stiffness of bent I in the length of Y axis
GA_{xi} = lateral stiffness of bent I in the length of X-axis

Figure 1.63 shows the parameters are necessary to calculate the center of rigidity. It selects two arbitrary origins in the length of the X and Y direction. GA_{X_1}–GA_{X_4} are shear rigidity of bents 1–4 in the length of X axis, while GA_{Y_A}–GA_{Y_E} are shear rigidity of bent A to E in the length of the Y axis. X_1–X_4 are distance from the bent B to E to arbitrary origin in the length of Y axis respectively. Y_1 is distance of bent (2) from arbitrary axis in the length of the X direction, and Y_2 and Y_3 are the distance from the bent 3 and bent 4 to arbitrary axis respectively.

- The shear rigidity of first floor is calculated in the pervious part (C.4), this part of section is devoted the calculation center of rigidity. The location of the center of rigidity (CR) can be calculated using Eqs. (1.76–1.77). Accordingly, the Charts 1.13 and 1.14 illustrate the process of calculation of the center of rigidity. Figure 1.64a, b show the shear rigidity in the X and Y direction of the first story also the distance between each bent and arbitrary origin is demonstrated.

1.3.6 Torsional Forces

The point at which an object can achieve precise balance without any further rotation is referred to as the center of gravity. This point may also be referred to as

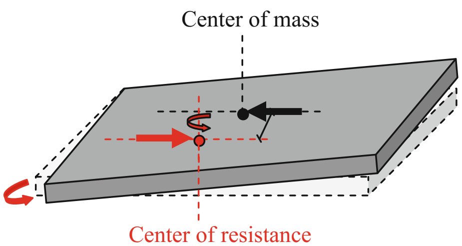


Fig. 1.65 Concept of torsion in structure

the center of mass. In any given building plan where the mass/weight is uniformly distributed, the geometric center overlaps the center of gravity. Based on the plan, the floor of the building has a geometric center through which the main lateral force is exerted. This force is made up of the weight of the roof, floors, as well as the walls. Similarly, the force resulting from the horizontal acceleration of particles is applied on the central point of the floor where mass is uniformly distributed. The building maintains a dynamic balance if the resulting force of resistance pushes back through the floor's geometric center. The resistance arises from moment/braced frames together with shear walls.

However, there are cases where imbalances occur between the positioning of the resisting components and the mass of the building. In such cases, torsional forces emerge, a phenomenon that engineers consider to be the eccentricity between the center of resistance and the center of gravity. Consequently, the building is likely to experience a ground motion rotate at the center of resistance leading to torsion in the plan. Here, torsion is used to describe a twisting action that causes undesirable concentrations of stress which may be dangerous. Such problems can be avoided by ensuring that the mass is well-distributed in the building plan. This entails uniform placement of walls, floors, and other column masses. The earthquake resistance elements should be arranged in a symmetrical manner to ensure that the building pushes back with appropriate force no matter the direction in which the floors move. A balanced resistance force prevents the building from rotating. This explains why it is necessary to choose the most symmetrical designs possible when bringing up structured in areas with seismic risk. However, it is worth noting that structures always have some level of torsion, which is accommodated in the building code (Fig. 1.65).

1.3.6.1 Calculation of Torsion

The concepts of center of mass and center of stiffness are explained in the previous section can be summarized as follows:

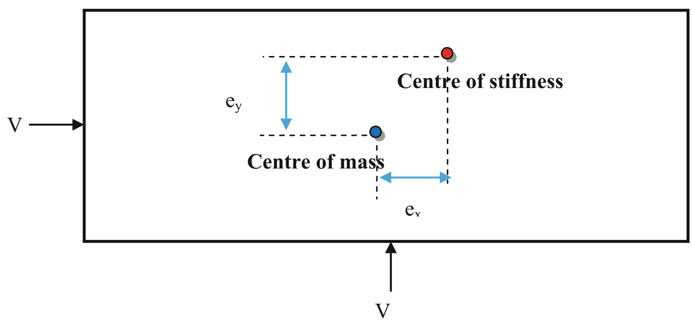


Fig. 1.66 Eccentricity in X and Y direction

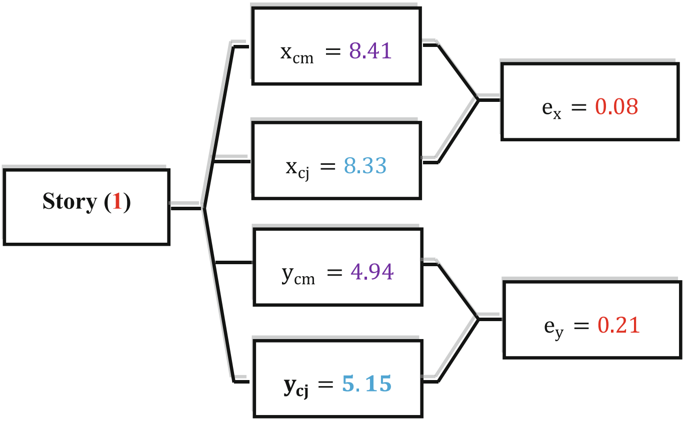


Chart 1.15 Eccentricity in the first floor

1. The resultant force that is caused by the earthquake is applied to a particular point called the center of mass in each story.
2. The resultant resistance force against the earthquakes produced in a special point called the center of stiffness.

Assuming that there is a distance between the center of mass and the center of stiffness in a story. Then, there will be an eccentricity between them, causing a torsion in the story. Therefore, the magnitude of the tensional moment depends on the magnitude of the eccentricity level. Figure 1.66 shows the eccentricity in the structure.

According to concept of torsion and Fig. 1.66, the eccentricity is calculated using Eq. (1.78)

$$\begin{cases} e_x = x_{cm} - x_{cj} \\ e_y = y_{cm} - y_{cj} \end{cases} \quad (1.78)$$

where:

$\mathbf{x_{cm}}, \mathbf{y_{cm}}$ = Coordinate of centre of mass

$\mathbf{x_{cj}}, \mathbf{y_{cj}}$ = Coordinate of centre of rigidity

1.3.7 Accidental Torsional Effects

Some uncertainties arise when obtaining masses or determining the spatial variation (for the seismic motion). These uncertainties are accounted for by considering the

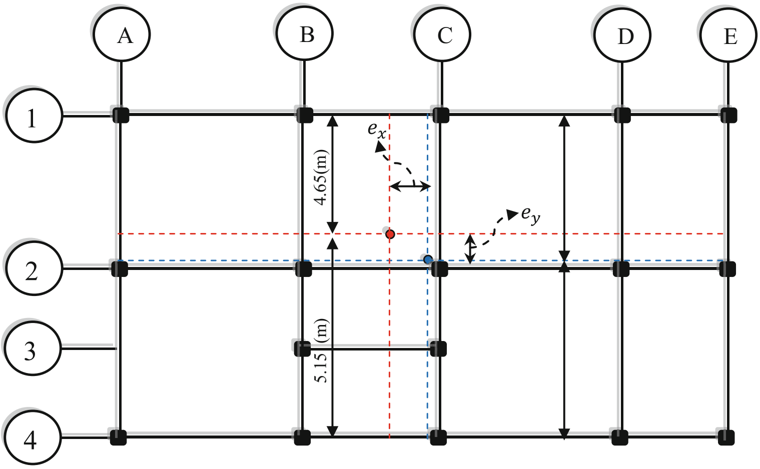


Fig. 1.67 Eccentricity in the first floor

centre of mass for each floor (i) as having been displaced from its usual location by an accidental eccentricity. The displacement occurs in each direction. This results to:

$$e_{ai} = \pm 0.05 \times Li \tag{1.79}$$

where,

e_{ai} = the accidental eccentricity for the building mass i from its nominal location. This is applied in one direction for all the floors
L_i = the floor measurement perpendicular to the direction displayed by the seismic action. (Code, 2005)

The eccentricity of the first floor of the considered building is calculated according to Eq. (1.78) and is shown in the chart (1.15) and Fig. 1.67.

According to the Eq. (1.79) the accidental eccentricity in X and Y direction are computed as follows:

$$e_{ai} = 0.05 \times 17 = 0.85\ (\mathrm{m})$$
$$e_{ai} = 0.05 \times 10 = 0.5\ (\mathrm{m})$$

1.3.8 Distribution of Lateral Shear in Bents

The total external shear at a level is distributed among the bents and in proportion to their shear-rigidity at that level. An estimate of the shear Q_{ji} carried by bent j at level (i) is given by

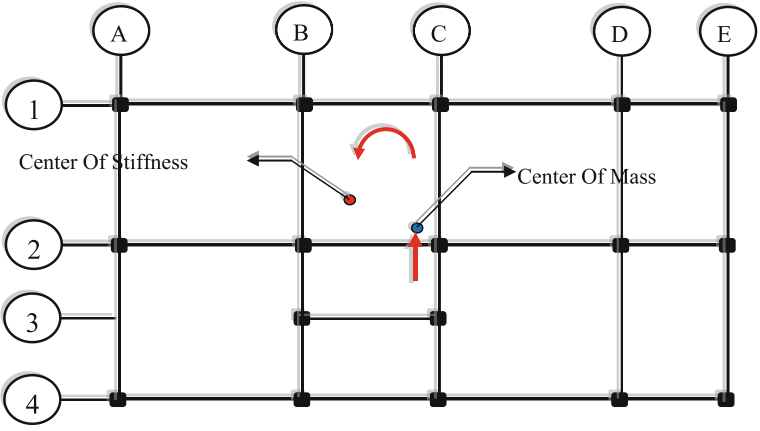


Fig. 1.68 Torsion bending in structure due to eccentricity

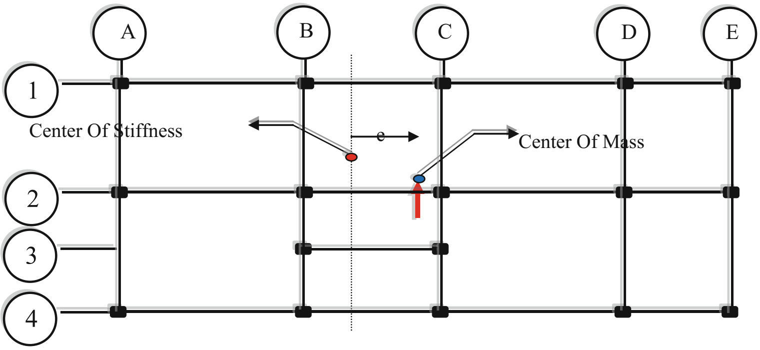


Fig. 1.69 Opposite and same side of center of twist

$$Q_{ji} = \left[\frac{(GA)_{ji}}{\sum(GA)_i} \pm \frac{e \times [(GA \times C)]_{ji}}{\sum[(GA)C^2]_i}\right] V_i \quad (1.80)$$

V_i is total shear of story i.
$(GA)_{ji}$ is the shear rigidity of bent j in story i.
e is the eccentricity of V_i from the center of sher rigidity in story i.
C is the distance of bent j from the center of shear rigidity.
$\sum(GA)_i$ is summation of the full set of bents parallel to the direction of loading.

As mentioned earlier, the center of the mass receives the applied force. if there is an eccentricity between the center of the mass and the center of rigidity leading to the production of torsion within the structure. Owing to this eccentricity, there will

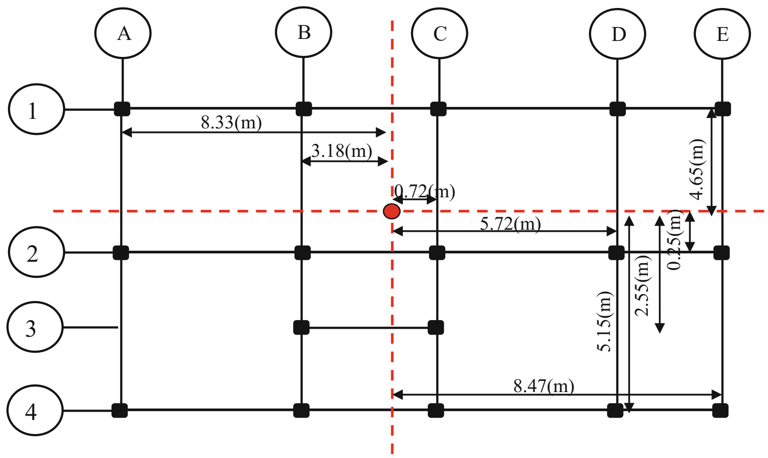


Fig. 1.70 Distance of each bent from center of stiffness

be tension applied to some bents while others will be under compression forces. Figure 1.68 demonstrates the resultant force, which the center of the mass receives as well as the produced torsion due to eccentricity of the structure. Based on the torsion's direction, it will be observed that bent A and B are under compression whereas bent C, D and F are under tension. The resultant shear in each bent at level i can be calculated in accordance with the combination of the shear, result from the resistance of the external torque's share and share of lateral shear at considered level. Through torsion moment, the shear forces and moments are decreased for the bents that can withstand the compression forces. However, for those that can withstand tension, the torsion moment increases the shear forces and moments.

As the discussion above illustrates, when C_{ji} is on the same side with the center twist, it is positive. And owing to the fact that they are on the same side, there will be an increase in the resultant loading's shear due to the eccentricity. This is often affected by the twisting behavior. Figure 1.69 shows that bent A and B are located on the opposite side while bent C, D and E are on the center twist's same side and thus the shear force in the story 1 is distributed as shown by Eq. (1.80) and is calculated as follows.

Step 1: According to Table 1.38 the shear force of story (1) is 1284.94 (kN) in X and Y direction.

Step 2: The shear rigidity in X and Y direction are calculated based on Eq. (1.60). Accordingly,
The shear rigidities and summation in Y direction are computed as:
$GA_{(A)} = 34167.83$, $GA_{(B)} = 86361.39$, $GA_{(C)} = 86361.39$, $GA_{(D)} = 38782.66$, $GA_{(E)} = 32578.34$,
$\sum \mathbf{GA} = 278251.623$
The shear rigidities and summation in X direction are computed:

Table 1.59 Distribution shear force in first stories 'bents

					Story (1)				
					Direction (y)				
Bent name	GA	$\frac{GA}{\sum GA}$	C_{jy}	C_j	$GA \times Cj^2$	e(x)	Q_2	$V(y_2)$	Q_{ij}
(A)	34167.83	0.122794	8.33	−8.33	2370868	0.93	−0.038	1284.94	108.456
(B)	86361.39	0.310371	8.33	−3.18	873321	0.93	−0.037	1284.94	351.214
(C)	86361.39	0.310371	8.33	0.72	44769.75	0.93	0.0083	1284.94	409.587
(D)	38782.66	0.139379	8.33	5.72	1268907	0.93	0.0299	1284.94	217.542
(E)	32578.35	0.117082	8.33	8.47	2337200	0.93	0.0372	1284.94	198.268
	278251.6				6203888.929				

Table 1.60 Distribution shear force in first stories 'bents

					Story (1)				
					Direction (x)				
Bent name	GA	$\frac{GA}{\sum GA}$	C_{jx}	C_j	$GA \times Cj^2$	e(y)	Q_2	$V(x_i)$	Q_{ij}
(1)	81658.3798	0.2185	4.65	−4.65	**1765658.31**	0.71	−0.081	1284.94	175.678
(2)	215348.725	0.5764	4.65	0.25	**13459.2953**	0.71	0.0115	1284.94	755.625
(3)	25509.1070	0.0682	4.65	2.55	**165872968**	0.71	0.0139	1284.94	105.657
(4)	51044.2320	0.1366	4.65	5.15	**1353820.64**	0.71	0.0564	1284.94	248.115
	373560.44				**3298811.22**				

$$GA_{(1)} = 81658.37, GA_{(2)} = 215348.72, GA_{(3)} = 25509.10, GA_{(4)} = 51044.23$$
$$\sum \boldsymbol{GA} = 373560.4439$$

Step 3: Eccentricities in X and Y direction are calculated using Eq. (1.78). Eccentricity in X direction (ex) is 0.08 (m) and this magnitude for y direction (ey) is 0.21 (m)). Moreover accidental eccentricity in X and Y direction is 0.85(0.05 * 17) and 0.5(0.05 * 10) respectively. These magnitudes must be add to the eccentricity in x and y direction.

Step 4: The distance of each bent respect to center of rigidity (C) in the X and Y direction is calculated and shows in Fig. 1.70.

C_j in X direction:

$$C_{j(A)} = 8.33, \quad C_{j(B)} = 8.33 - 5.15 = 3.18, \quad C_{j(C)} = 8.33 - (5.15 + 3.9) = 0.72,$$
$$C_{j(D)} = 8.33 - (5.15 + 3.9 + 5) = 5.72$$
$$C_{j(E)} = 8.33 - (5.15 + 3.9 + 5 + 2.75) = 8.47$$

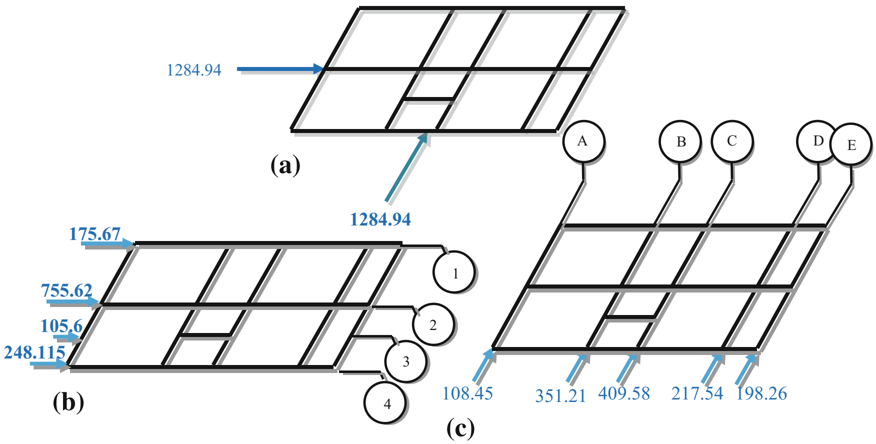


Fig. 1.71 **a** Shear force in x and y-direction, **b** distribution of shear force in y-direction, **c** distribution of shear force in x-direction

C_j in Y direction:

$$C_{j(1)} = 4.65, \quad C_{j(2)} = 4.9 - 4.65 = 0.25, \quad C_{j(3)} = 5.15 - 2.6 = 2.55,$$
$$C_{j(4)} = 4.9 - (4.65) + (2.3 + 2.6) = 5.15$$

Step 5: Substitute magnitude in Eq. (1.65). Tables 1.59 and 1.60 show the distribution of lateral shear in the story (1) in the Y and X direction.

$$Q_{ji} = \left[\frac{(GA)_{ji}}{\sum (GA)_i} \pm \underbrace{\frac{e \times [(GA \times C)]_{ji}}{\sum [(GA) C^2]_i}}_{Q_2} \right] V_i$$

Table 1.61 Distribution of shear force in story two in Y direction

					Story (2)				
					Direction (y)				
Bent name	GA	$\frac{GA}{\sum GA}$	C_{jy}	C_j	$GA \times Cj^2$	e(x)	Q_2	$V(y_2)$	$\boldsymbol{Q_{ij}}$
(A)	30720.6834	0.1226	8.336	−8.336	2134746.379	0.924	−0.038	1229.76	103.910
(B)	77690.7180	0.3101	8.336	−3.186	788607.0914	0.924	−0.036	1229.76	336.066
(C)	77690.7180	0.3101	8.336	0.714	39606.41727	0.924	0.0082	1229.76	391.563
(D)	35202.1832	0.1405	8.336	5.714	1149344.1	0.924	0.0299	1229.76	209.658
(E)	29196.0566	0.1165	8.336	9.464	2091584.941	0.924	0.0368	1229.76	188.592
	250500.36				6203888.929				

Table 1.62 Distribution of shear force in story two in X direction

					Story (2)				
					Direction (x)				
Bent Name	GA	$\frac{GA}{\sum GA}$	C_{jx}	Cj	$GA \times Cj^2$	e(y)	Q_2	$V(x_2)$	Q_{ij}
(1)	73222.9029	0.2190	4.66	−4.66	1590079.27	0.69	−0.0793	1229.76	171.854
(2)	191957.325	0.5742	4.66	0.24	11056.74197	0.69	0.01070	1229.76	719.413
(3)	22923.4896	0.0685	4.66	2.54	147893.1855	0.69	0.01353	1229.76	100.984
(4)	46148.1485	0.1380	4.66	5.14	1219215.624	0.69	0.05514	1229.76	237.596
	334251.836				2968244.821				

Table 1.63 Distribution of shear force in story 3 in Y direction

					Story (3)				
					Direction (y)				
Bent name	GA	$\frac{GA}{\sum GA}$	C_{jy}	Cj	$GA \times Cj^2$	e(x)	Q_2	$V(y_3)$	Q_{ij}
(A)	26919.1882	0.1220	8.348	−8.348	1875974.106	0.921	−0.0381	1107.89	92.94805
(B)	68161.4707	0.3089	8.348	−3.198	697101.2658	0.921	−0.0369	1107.89	301.2956
(C)	68161.4707	0.3089	8.348	0.702	33590.2454	0.921	0.00811	1107.89	351.2338
(D)	32779.3410	0.1485	8.348	5.702	1065748.292	0.921	0.03169	1107.89	199.7001
(E)	24625.8887	0.1116	8.348	8.452	1759182.471	0.921	0.03529	1107.89	162.7491
	220647.4				5431596.381				

Table 1.64 Distribution of shear force in story 3 in X direction

					Story (3)				
					Direction(x)				
Bent Name	GA	$\frac{GA}{\sum GA}$	C_{jx}	Cj	$GA \times Cj^2$	e(y)	Q_2	$V(x_3)$	Q_{ij}
(1)	63913.2212	0.2270	4.686	−4.686	1403444.604	0.664	−0.07525	1107.8	168.131
(2)	155326.187	0.5516	4.686	0.214	7113.31806	0.664	0.008352	1107.8	620.475
(3)	20034.37209	0.0711	4.686	2.514	126621.15783	0.664	0.012655	1107.8	92.8577
(4)	42267.87876	0.1501	4.686	5.114	1105431.664	0.664	0.054313	1107.8	226.500
	281541.658				2642610.744				

Table 1.65 Distribution of shear force in story 4 in Y direction

					Story (4)				
					Direction (y)				
Bent name	GA	$\frac{GA}{\sum GA}$	C_{jx}	Cj	$GA \times Cj^2$	e(x)	Q_2	$V(y_4)$	Q_{ij}
(A)	26919.1882	0.1220	8.348	−8.348	1878974.106	0.921	−0.0381	923.375	77.4679
(B)	68161.4707	0.3089	8.348	−3.198	697101.2658	0.921	−0.03696	923.375	251.116
(C)	68161.4707	0.3089	8.348	0.702	33590.2454	0.921	0.008113	923.375	292.737
(D)	32779.3468	0.1485	8.348	5.702	1065748.292	0.921	0.031693	923.375	166.440
(E)	24625.8887	0.1116	8.348	8.452	1759182.471	0.921	0.035293	923.375	135.643
	220647.4				5451596.381				

Table 1.66 Distribution of shear force in story 4 in X direction

					Story (4) Direction (x)				
Bent name	GA	$\frac{GA}{\sum GA}$	C_{jx}	Cj	$GA \times Cj^2$	e(y)	Q_2	$V(x_4)$	Q_{ij}
(1)	63913.22125	0.2270	4.686	−4.686	1403444.604	0.664	−0.0752	923.375	140.1295
(2)	155326.187	0.5516	4.686	0.214	7113.31806	0.664	0.00835	923.375	517.1373
(3)	20034.37209	0.0711	4.686	2.514	126621.15783	0.664	0.01265	923.375	77.39266
(4)	42267.87876	0.1501	4.686	5.114	1105431.664	0.664	0.05431	923.375	188.778
	281541.6589				2642610.744				

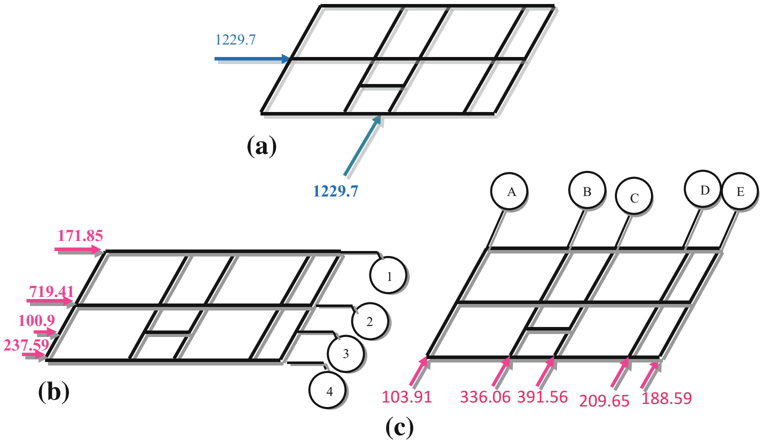


Fig. 1.72 **a** Shear force in x and y-direction, **b** distribution of shear force in y-direction, **c** distribution of shear force in x-direction

Based on Table 1.59, the proportion of bent (A) from the total external shear force (1284.94 kN) is 108.45 (kN). This magnitude representing bents (B)–(E) is 351.21 (kN), 409.58 (kN), 217.54 (kN) and 198.26 (kN) respectively. However, Table 1.60 shows the distribution of total lateral external shear force in the X-direction. According to this table, the proportion of bents (1)–(4) of total shear force is equal to 175.67 (kN), 755.62 (kN), 105.6 (kN) and 248.115 (kN) respectively. In Fig. 1.71, the distribution of total lateral external shear force in X and Y direction is illustrated.

It should be noted that calculation procedure of shear rigidity, the center of rigidity, and distribution of shear force for other stories are the same with the first story.

Distribution of shear force in the second story:

The calculation procedure of shear rigidity, cent of stiffness, the eccentricity of story 2 are in the appendix at the end of this chapter. Based on Eq. (1.80), the shear force is distributed in bents of story according to the proportion of shear rigidity at

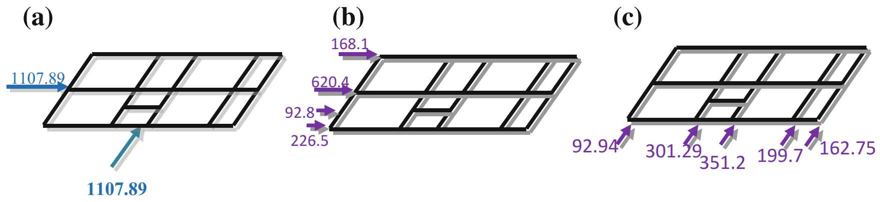


Fig. 1.73 **a** Shear force in x and y-direction in story (3), **b** distribution of shear force in y-direction, **c** distribution of shear force in the x-direction

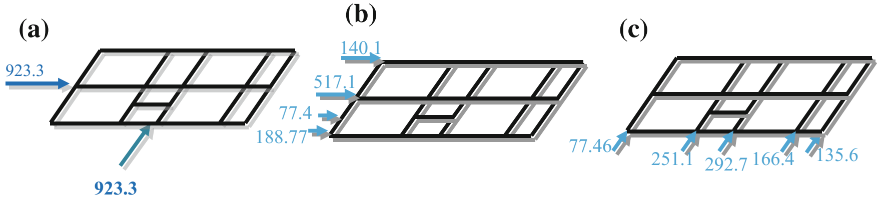


Fig. 1.74 **a** Shear force in x and y-direction in story (4), **b** distribution of shear force in y-direction, **c** distribution of shear force in the x-direction

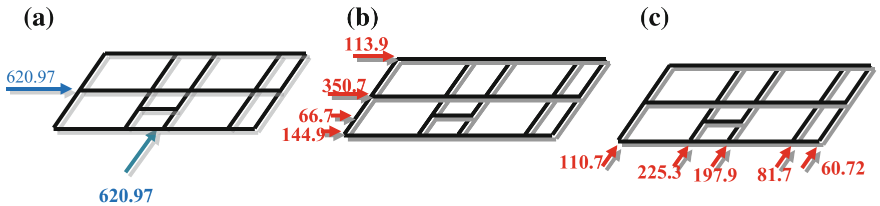


Fig. 1.75 **a** Shear force in x and y-direction in story (5), **b** distribution of shear force in y-direction, **c** distribution of shear force in x-direction

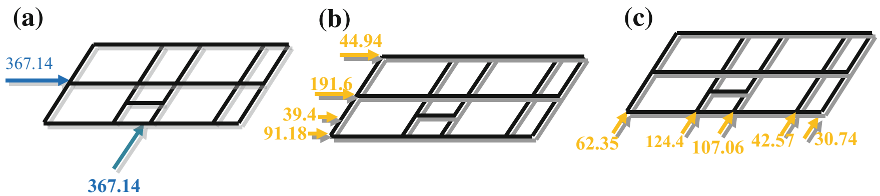


Fig. 1.76 **a** Shear force in the X and Y direction in story (6), **b** distribution of shear force in Y direction, **c** distribution of shear force in X direction

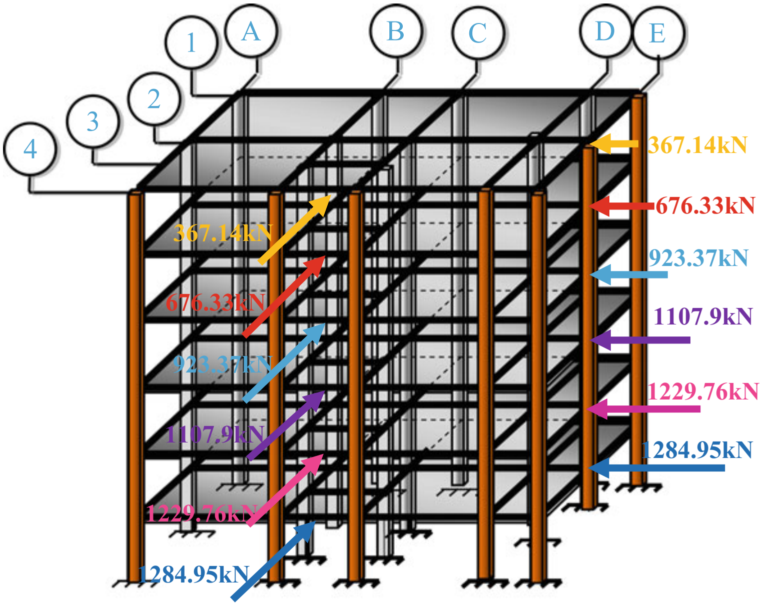


Fig. 1.77 Distribution of shear base in the height of structure

each bent. Tables 1.61 and (1.62) show the distribution of shear force in Y and X direction (Tables 1.63, 1.64, 1.65 and 1.66).

Based on Table 1.61 the proportion of bent (A) from the total external shear force (1229.76 kN) is 103.91(kN), this magnitude for bent (B)–(E) is 336.06 (kN), 391.56 (kN), 209.65 (kN) and 188.6 (kN) respectively. Table 1.62 shows the distribution of total lateral external shear force in the X-direction. According to this table, the proportion of bents (1)–(4) of total shear force is equal to 171.85 (kN), 719.41 (kN), 100.98 (kN) and 237.6 (kN) respectively. Figure 1.72 shows the distribution of total lateral external shear force in X and Y direction.

Distribution of shear force in the third and fourth story:

The calculation procedure of shear rigidity, cent of stiffness, the eccentricity of story 3, 4 are in the appendix at the end of this chapter. Based on Eq. (1.80), the shear force is distributed in bents of story 3 and four according to the proportion of shear rigidity at each bent. Figures 1.73 and 1.74 shows the distribution of total lateral external shear force in X and Y direction (Figs. 1.75, 1.76, 1.77, 1.78 and 1.79).

1.3.8.1 Distribution of Shear Force in the Fifth and Sixth Story

The calculation procedure of shear rigidity, center of stiffness, the eccentricity of story 5, 6 are in the appendix at the end of this chapter. Based on Eq. (1.80), the

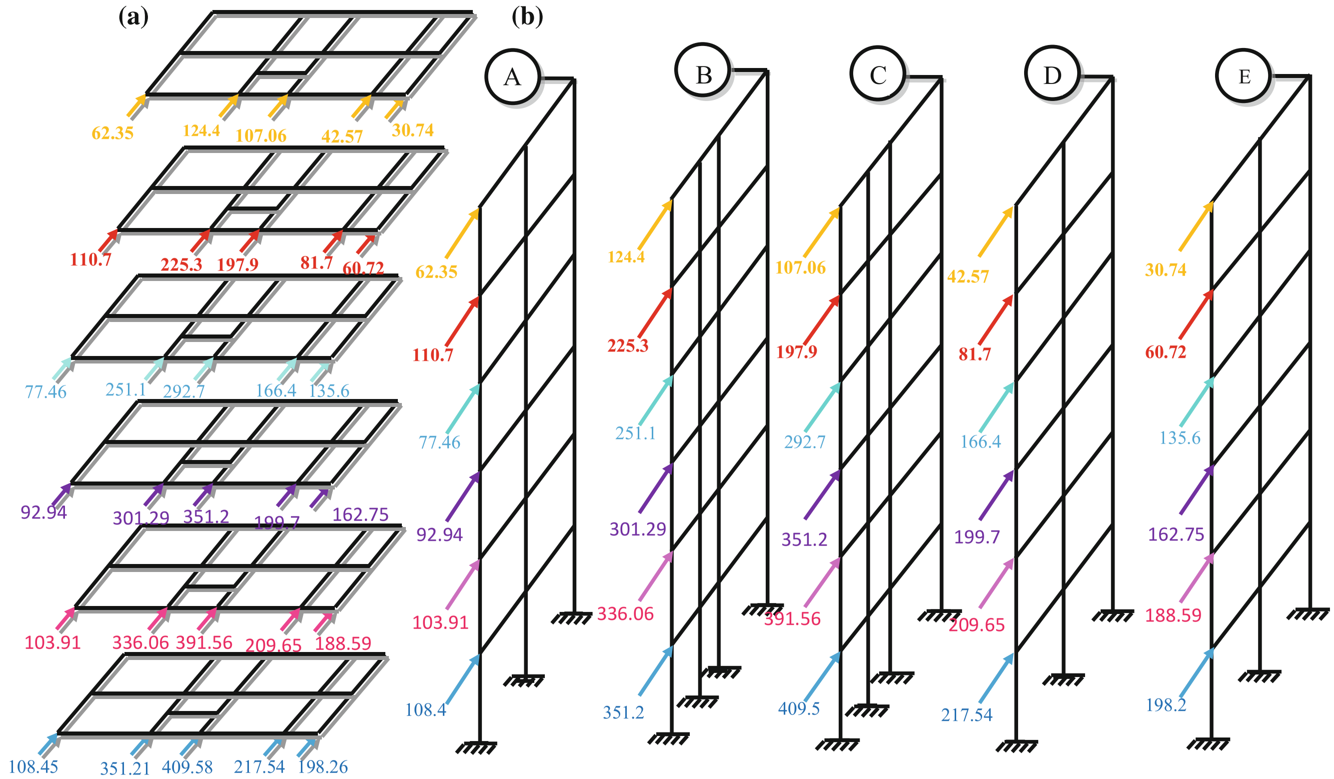


Fig. 1.78 **a** Distribution of shear in stories' bents, **b** shear force in each frame in Y direction

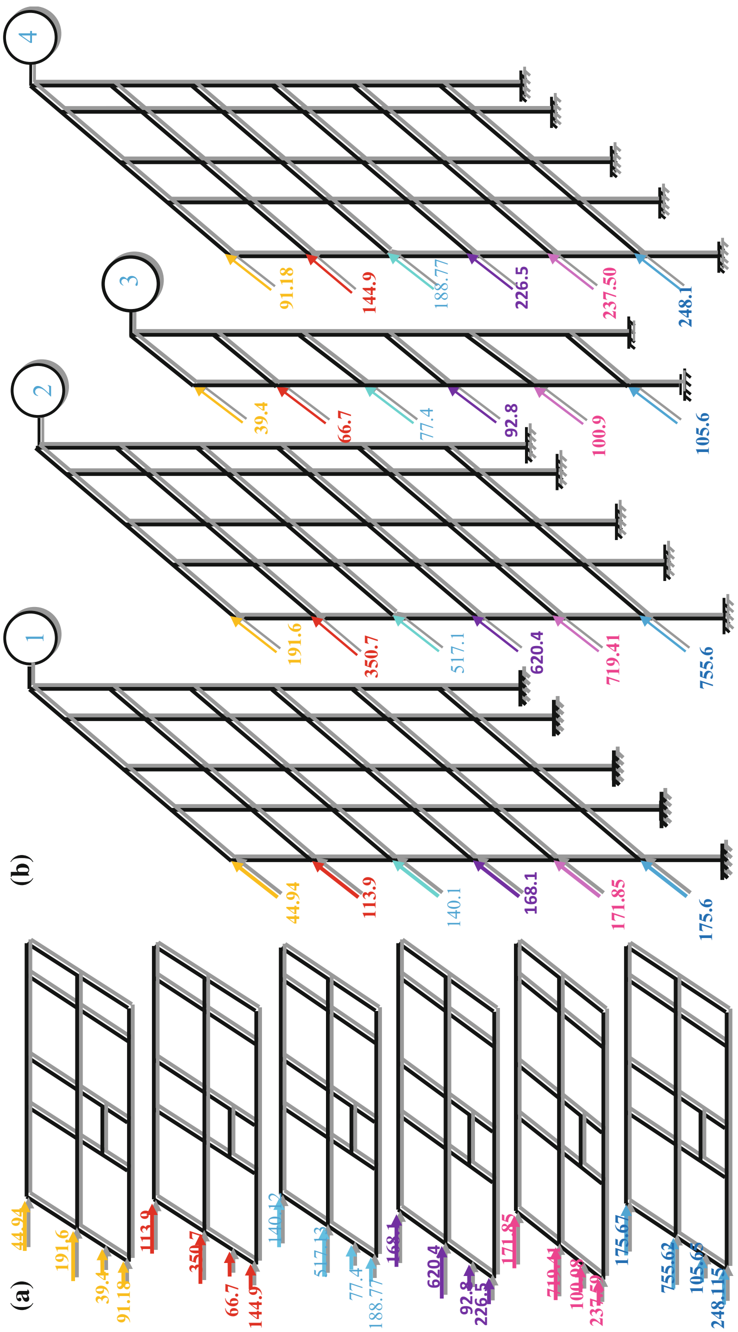


Fig. 1.79 **a** Distribution of shear in stories' bents, **b** shear force in each frame in X direction

Table 1.67 Distribution of shear force in story 5 in Y direction

					Story (5)				
					Direction (y)				
Bent Name	GA	$\frac{GA}{\sum GA}$	Cjy	Cj	$GA \times Cj^2$	e(x)	Q_2	$V(y_5)$	Q_{ij}
(A)	24625.8887	0.1264	8.47	8.47	1766683.419	0.901	0.0372	676.333	110.69459
(B)	58173.462	0.2986	8.47	3.32	641211.1675	0.901	0.0344	676.333	225.30733
(C)	58173.462	0.2986	8.47	−0.58	19569.55262	0.901	−0.0060	676.333	197.90488
(D)	29196.0566	0.1498	8.47	−5.58	909060.0967	0.901	−0.0290	676.333	81.692670
(E)	24625.8887	0.1264	8.47	−8.33	1708763.328	0.901	−0.0366	676.333	60.725554
	194794.7591				5045287.564				

Table 1.68 Distribution of shear force in story 5 in X direction

					Story (5)				
					Direction (x)				
Bent Name	GA	$\frac{GA}{\sum GA}$	Cjx	Cj	$GA \times Cj^2$	e(y)	Q_2	$V(x_5)$	Q_{ij}
(1)	55214.8201	0.2365	4.741	−4.741	1241067.984	0.609	−0.06798	676.3	113.98
(2)	119917.2382	0.5136	4.741	0.159	3031.627699	0.609	0.004952	676.3	350.76
(3)	20034.372	0.0858	4.741	2.459	121141.4564	0.609	0.012794	676.3	66.696
(4)	38279.72142	0.1639	4.741	5.059	979711.3228	0.609	0.050294	676.3	144.91
	233446.1517				2344952.391				

Table 1.69 Distribution of shear force in story 6 in Y direction

					Story (6)				
					Direction (y)				
Bent name	GA	$\frac{GA}{\sum GA}$	Cjy	Cj	$GA \times Cj^2$	e(x)	Q_2	$V(y_6)$	Q_{ij}
(A)	24625.8887	0.1264	8.47	8.47	1766683.419	1.05	0.04340	367.14	62.3509
(B)	58173.462	0.2986	8.47	3.32	641211.1675	1.05	0.04019	367.14	124.399
(C)	58173.462	0.298	8.47	−0.58	19869.5526	1.05	−0.0070	367.14	107.064
(D)	29196.0566	0.1498	8.47	−5.58	909060.0967	1.05	−0.0339	367.14	42.5796
(E)	24625.8887	0.1264	8.47	−8.33	1708763.328	1.05	s0.0426	367.14	30.7400
	19479.7591				5048287.564				

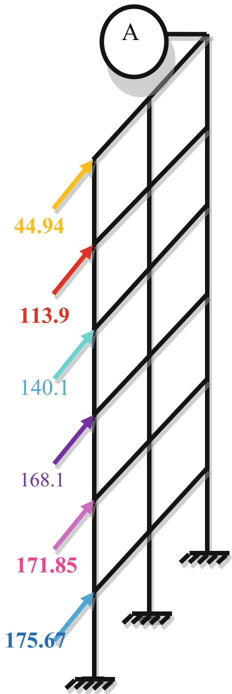
shear force is distributed in bents of story 5, 6 according to the proportion of shear rigidity at each bent. Tables 1.67, 1.68, 1.69 and 1.70 show the distribution of shear force in Y and X direction in fifth and sixth story. Figures 1.71 and 1.72 illustrates the distribution of total lateral external shear force in X and Y direction (Tables 1.69 and 1.70).

Figure 1.80 shows frame (A) under different shear distributed forces from story (1) until story (6). The shear force of frame (A) in the first story is 175.67 (kN) which is due to of distribution of total shear force of story (1) In grid line (A). The shear force of the second story in considered Building is 1129 (kN), so

Table 1.70 Distribution of shear force in story 5 in X direction

					Story (6)				
					Direction (x)				
Bent name	GA	$\frac{GA}{\sum GA}$	Cjx	Cj	$GA \times Cj^2$	e(y)	Q_2	$V(x_6)$	Q_{ij}
(1)	55214.8201	0.2365	4.741	−4.741	1241067.984	1.022	−0.1140	367.14	44.949
(2)	119917.238	0.5136	4.741	−0.159	3031.6276	1.022	0.00831	367.14	191.64
(3)	20034.372	0.0858	4.741	2.459	121141.4564	1.022	0.02147	367.14	39.390
(4)	38279.7214	0.1639	4.741	5.059	979711.3228	1.022	0.08440	367.14	91.189
	233446.151				2344952.391				

Fig. 1.80 Frame (A)



according to the shear rigidity of bent (A) in the story (2) (30720.68), 171.85 (kN) from total external shear force in story 2 (1129 kN) is distributed in gridline (A). Based on shear rigidity of bend(A) in story 3,4,5, and 6, the shear forces 168.1,140.1 (kN),113.9 (kN) and 44.94 (kN) from total external Shear force 1017 (kN), 923 (kN), 676 (kN) and 367 (kN) in story 3, 4, 5, and six are distributed respectively.

As indicated in the Euro code 8, the shear base in X and Y direction in the considered building is equal to 1284 (kN), which is because of the same lateral

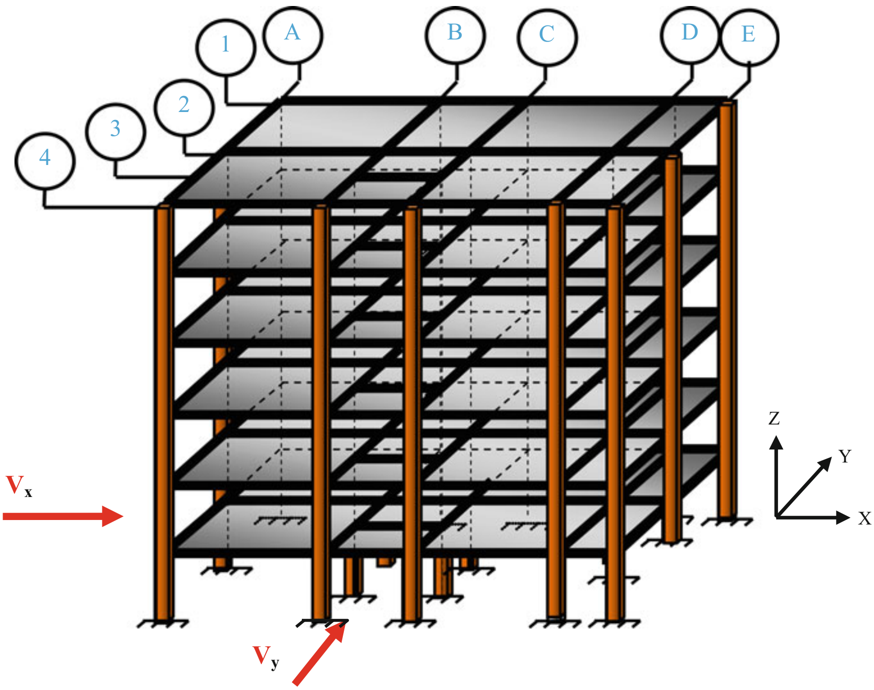


Fig. 1.81 The shear base in X and Y-direction

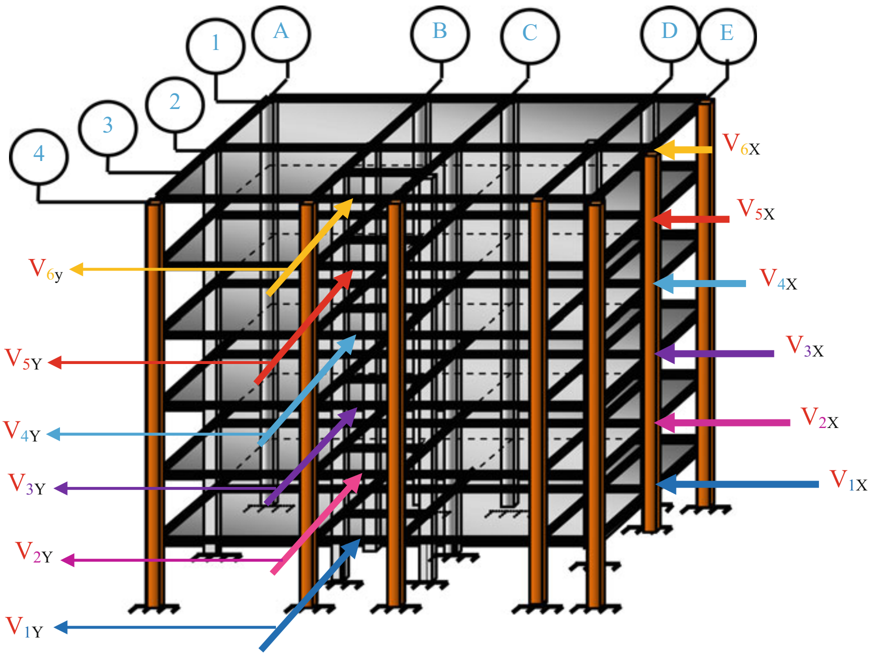
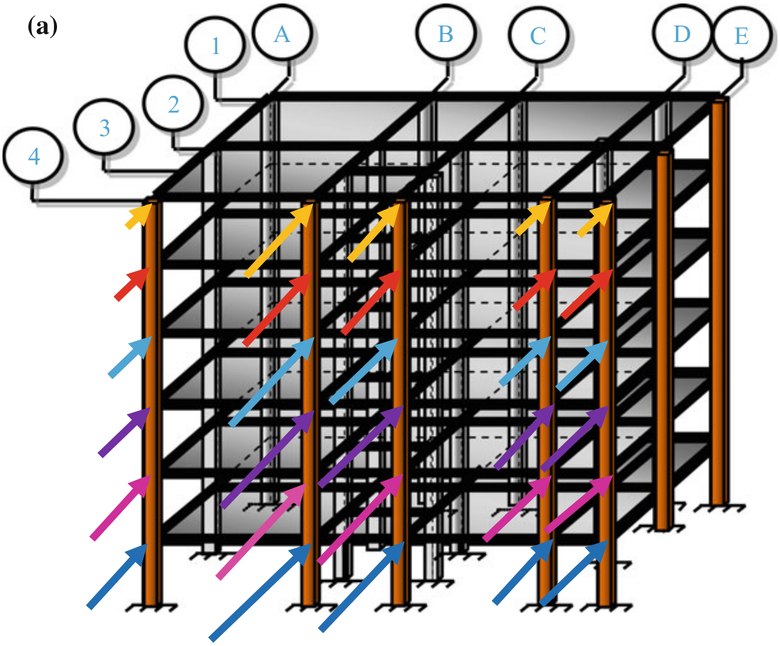


Fig. 1.82 Distribution of shear base in the height of structure in X and Y direction

(a)



(b)

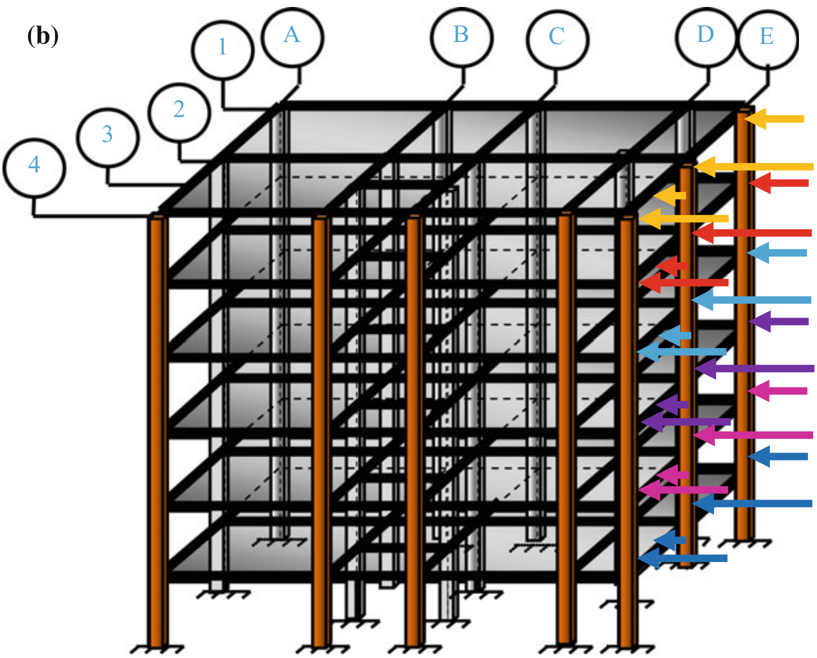


Fig. 1.83 **a** Distribution of shear force of Stories in their bents in Y direction, **b** distribution of shear force of Stories in their bents in X direction

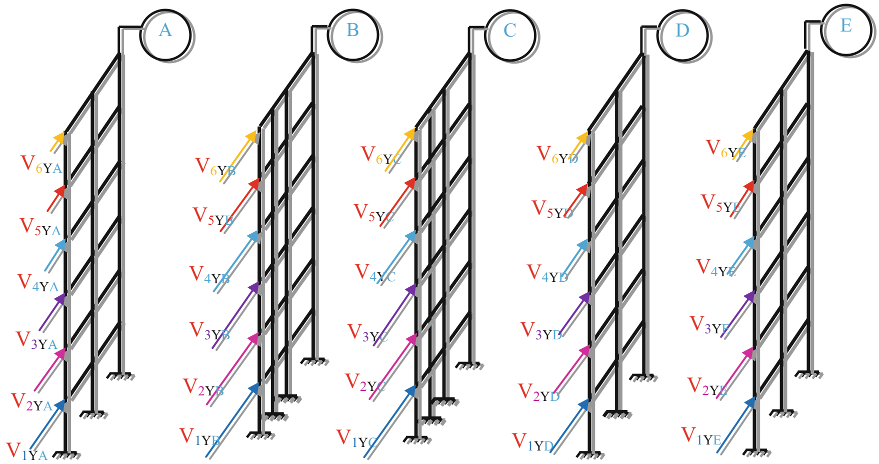


Fig. 1.84 Shear force distributed in Y direction in frame view

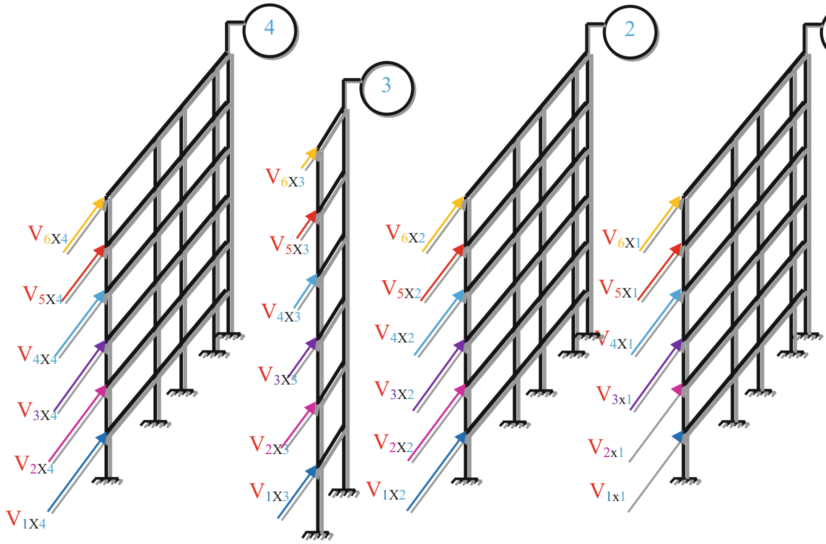


Fig. 1.85 Shear force distributed in X direction in frame view

resistance frames in both directions with the same ductility as well. Figure 1.81 shows the building with Vx and Vy shear base applied in X and Y direction. Meanwhile, earthquake shear force is distributed in the height of structure based on Eq. (1.59), and magnitude of the distributed shear load in each story is shown in Table 1.38. Figure 1.82 indicates the distribution of shear force in the height of the structure in X and Y directions.

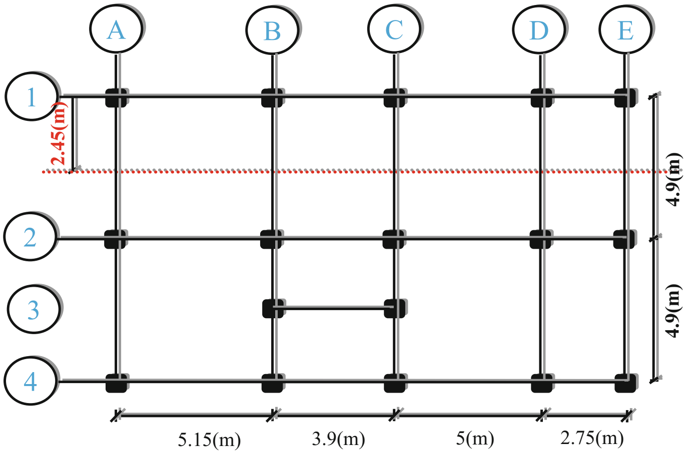


Fig. 1.86 Tributary width of beams in bent (1)

The magnitude of shear rigidity is calculated according to Eq. 1.76 in X and Y direction. Moreover, the shear force of each story distributed in each bent is due to the shear rigidity. According to Eq. 1.80, the shear force of each story is distributed in each bent. The considered model are shown in Fig. 1.83.

Figures 1.84 and 1.85 demonstrate that all frames of considered building in X and Y direction under distributed shear force, respectively.

1.3.9 Analysis of the Moment Resistance Frame Under Gravity Load

The method analysis of moment resistance frame under gravity load is explained in the first part of this chapter. Figure 1.86 shows a floor plan of the third story of considered building. The total load, which is applied to the beam in the bent (1) is calculated based on a dead load of the floor, partitions load, structure load, 20% of the live load; also, a total dead load of the external wall without facing. Equation (1.81) illustrates the procedure of calculation load which applied to gridline (1).

$$\begin{aligned}\text{Load applied bent}(1) = {} & [[(\text{floor load} + \text{partition load} + \text{structure load} + 20\%\ \text{of live load}) \\ & \times (\text{tributary width})] + (\text{extra wall without facing} \times \text{height of the wall})]\end{aligned} \quad (1.81)$$

According to the loading part, the floor load is 435 $(\frac{kg}{m^2})$, partition load, structure load and live load are 179.32 $(\frac{kg}{m^2})$, 129.246 $(\frac{kg}{m^2})$ and 200 $(\frac{kg}{m^2})$ respectively. The

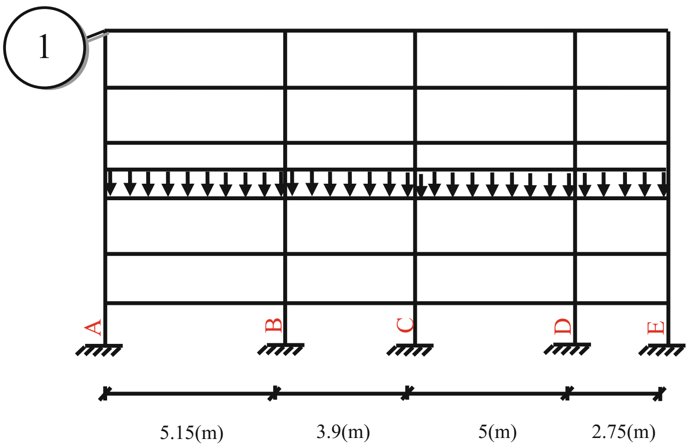


Fig. 1.87 Total gravity load in frame (1) story (3)

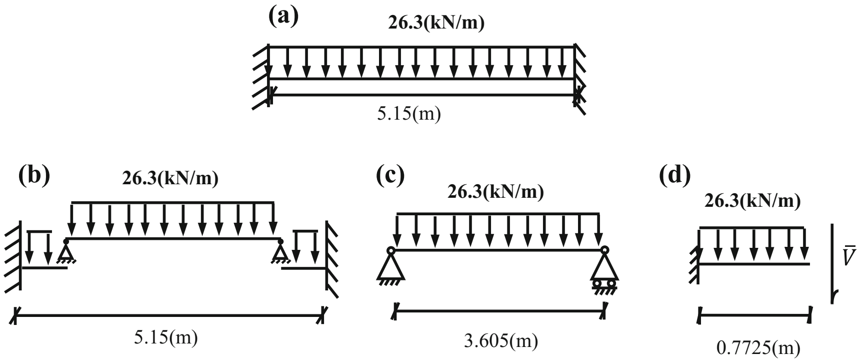


Fig. 1.88 **a** Moment resistance beam (A-B) under gravity load, **b** location of zero moments, **c** middle part of the beam, **d** end part of the beam

tributary width for bent 1 is half of the length of span (1–2) which is equal to 2.45 (m). The total load of external wall without facing is 215 $(\frac{kg}{m^2})$ and the height of the wall is 3.3 (m). Based on Eq. (1.66), the total load applied to the beams in bent (1) is:

$$\begin{aligned}\text{load in beam of bent}(1) &= [[(435 + 179.32 + 129.246 + (0.2 \times 200) \times (2.45)] + (215 \times 3.3)] \\ &= 26.3\ (\text{kN/m})\end{aligned}$$

The location and magnitude of loads that applied to the beams in a bent (1) are shown in Fig. 1.87.

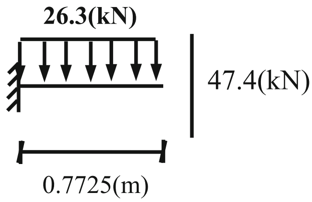


Fig. 1.89 Calculation of shear force and bending moment in end part of beam

Consider beam (A-B) with 5.15 (m) length. The total load applied to this beam is 26.3 (kN/m). Shear and moment of beam AB are calculated according to Eqs. (1.1), and (1.2). Figure 1.88 shows the process of analysis of rigid beam.

Reaction force and moment at the middle part are calculated according Eqs. (1.1) and (1.2).

$$\begin{cases} Reaction\,force: \bar{V} = \frac{q\times 0.7L}{2} = 0.35qL \qquad \bar{V} = \frac{26.3\times 3.605}{2} = 47.4\ (\text{kN}) \\ Moment: M^{+} = \frac{q\times 0.7L^{2}}{8} = 0.06125qL^{2} M^{+} = \frac{26.3\times 3.605^{2}}{8} = 42.72\ (\text{kN m}) \end{cases}$$

Figure 1.89 shows the end part of the beam AB under gravity load. Shear force and moment bending at the end of beam are calculated according to the Eq. (1.3) (Mohsen & Reza, 2015)

$$Shear\,force\,on\,the\,end: V = q\times 0.15L + \bar{V} = 0.5qL$$
$$V = 26.3\times 0.15(5.15) + 47.4 = 67.72(\text{kN})$$
$$Moment\,bending\,on\,end: M^{-} = \frac{q\times(0.15L)^{2}}{2} + (\bar{V} + 0.15L) = 0.06375qL^{2}$$
$$M^{-} = \frac{26.3\times(0.15\times 5.15)^{2}}{2} + (47.4 + (0.15\times 5.15)) = 44.489\ (\text{kN m})$$

1.3.10 The Approximate Methods for Analysis of Moment Resistance Frames Under Lateral Load

The calculation of lateral loads in a multi-story building is done using a technique known as the Assumption Method. It consists of two sub-methods: the Portal and Cantilever Method where each passes their unique advantages and conditions. The methods can be used to solve in lesser-known cases and reactions.

There are two assumptions in the approximate analysis of frames:

(1) Determination of the position of contra flexure point
(2) Determination of the distribution of forces between members

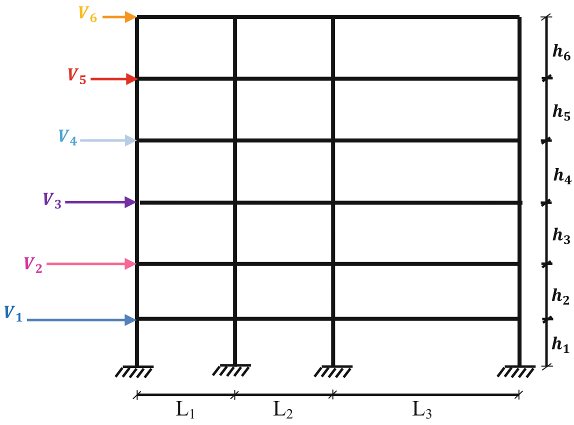


Fig. 1.90 Moment resistance frame under shear load

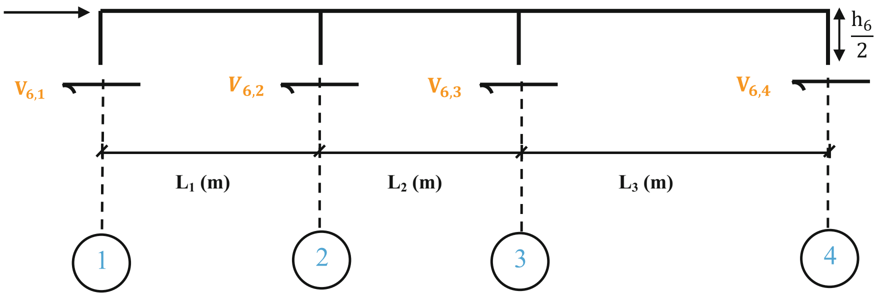


Fig. 1.91 Shear in columns is proportion of shear in

1.3.10.1 Portal Method

The portal method is an approximate analysis used for analyzing building frames that are subjected to lateral loading such as the one shown in Fig. 1.90. The portal method is especially suitable for low-rise building frames. (Applicable for structure with maximum of 25 stories and $h/b \leq 4$).

In this method two assumptions are considered:

(1) The contra flexure points are ones that are located in the middle span of girders and the height of columns.

(2) The horizontal shear at each story of frames is shared between the columns base that is proportional to the width of aisle at each column. The procedure for a whole frame analysis is as follows:

(1) At every story level, draw a line diagram of the frame and indicate the horizontal shear.
(2) At each story the shear to the column is proportional to the aisle width that support and indicate the value in the frames. Figure 1.91 shows shear of columns according to the shear in the Frame. The method to calculate shear in columns are shown in Eqs. (1.82–1.85)

1. Shear of column

$$V_{6,1} = \frac{half\ of\ lentgh\ of\ bay\ 1}{total\ length\ of\ span} \times horizontal\ shear = \frac{\frac{L_1}{2}}{L} \times V_6 \quad (1.82)$$

2. Shear of column

$$V_{6,2} = \frac{half\ of\ lentgh\ of\ bay\ 1 + bay\ 2}{total\ length\ of\ span} \times horizontal\ shear = \frac{\frac{L_1}{2} + \frac{L_2}{2}}{L} \times V_6 \quad (1.83)$$

3. Shear of column

$$V_{6,3} = \frac{half\ of\ lentgh\ of\ bay\ 2 + bay\ 3}{total\ length\ of\ span} \times horizontal\ shear = \frac{\frac{L_2}{2} + \frac{L3}{2}}{L} \times V_6 \quad (1.84)$$

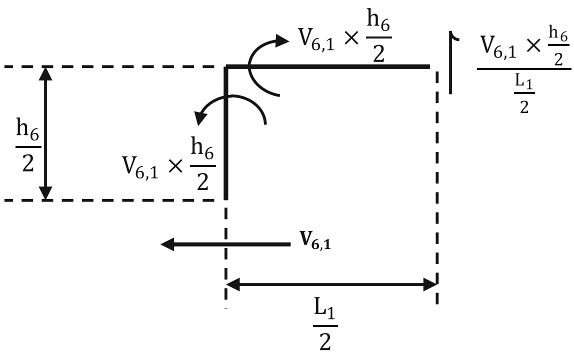


Fig. 1.92 Free diagram of top-left part

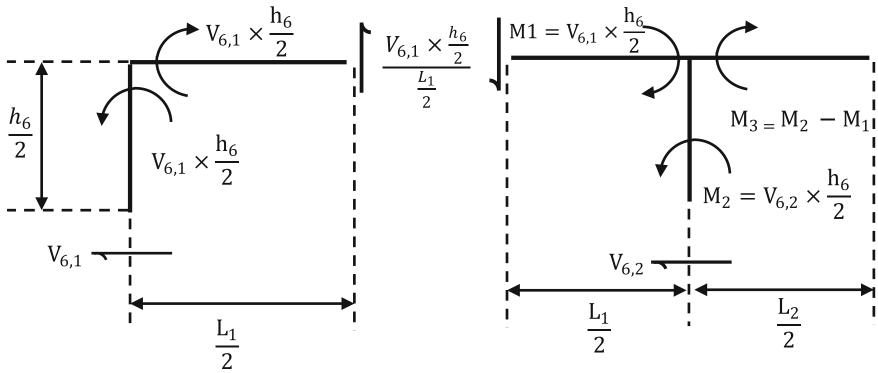


Fig. 1.93 Equilibrium of module

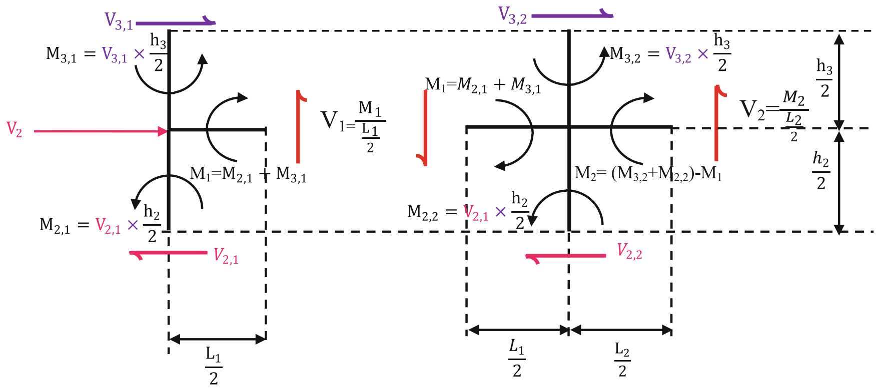


Fig. 1.94 Equilibrium of module in the second story

4. Shear of column

$$V_{6,4} = \frac{\textit{half of lentgh of bay 4}}{\textit{total lengthof span}} \times \textit{horizontal shear} = \frac{\frac{L_4}{2}}{L} \times V_6 \qquad (1.85)$$

(3) The process involves beginning with calculating the maximum moment from the top-left module. This is arrived by the product of column shear and half story height. Followed by this is the calculation of girder end moment located to the right of the joint. This side is also responsible for the moment at the other end of the girder is of the same magnitude but corresponds to the opposite curvature of evaluating the beam shear by dividing the girder-moment by half the span $\left(V_{beam} = \frac{V_{6,1} \times \frac{h_6}{2}}{\frac{L_1}{2}}\right)$. Figure 1.92 free body diagram shown in the top.

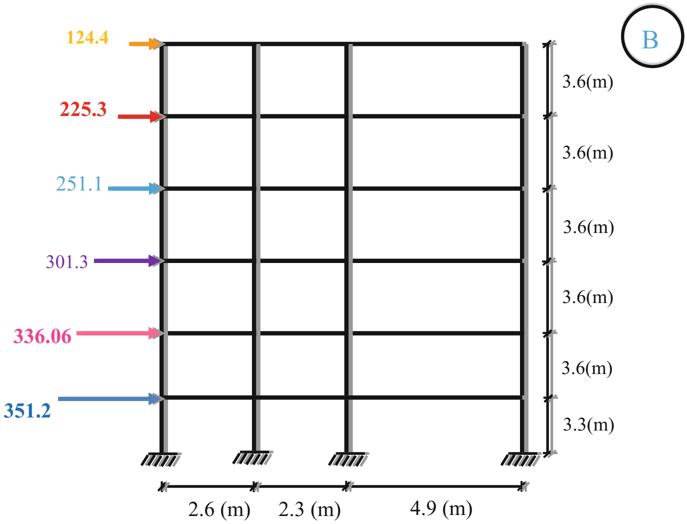


Fig. 1.95 Frame (B)

The equilibrium of the second joint is considered in this step. Step 3 is repeated in calculating the maximum moment in the second column, and this includes the moment and shear in the second girder from the left. The equilibrium of module and free body positions are shown in Fig. 1.93.

These steps are repeated for each successive module by working across and subsequently continued in the level below. The measurements are taken in the opposite direction: the shear and moment values are taken by starting from the left. The recorded bending moment are on the girders above the left-hand end and below the right-hand end, and similarly on the columns as viewed from the right. The shears are written perpendicular to the column and beam at the mid-height and midspans. When the member forces are applicable only to the particular segment in the structure, the horizontal row of modules in this level, in fact, consist of girders and hail columns in the above and below directions. The above can be used to analyze this separately without restarting from the top. Figure 1.94 illustrates the equilibrium of module in story 2.

- **Portal method for the frame (B)**:

Figure 1.88 shows the horizontal shear in the frame (B). The calculation of the forces used in the portal methods for each member is done through the analysis of the illustrated bent. For this reason, the procedural portal method is used and is

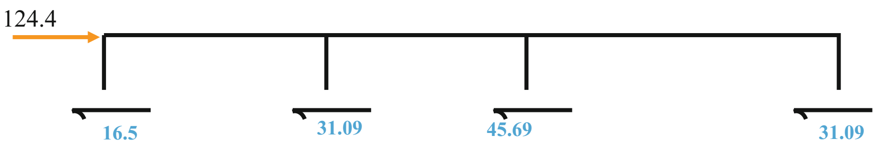
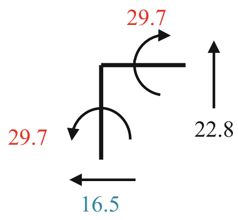


Fig. 1.96 The distribution of shear in columns

Fig. 1.97 Free body diagram



described in previous sections. The procedural portal method is performed using the following steps (Fig. 1.95):

(1) The horizontal shear in sixth floor of frame B is equal to 124.4 kN
(2) The distribution of shear between the top story columns is in proportion to the widths of the aisle supported, Eqs. (1.82)–(1.85) is used for calculation of shear in columns. Figure 1.96 illustrates the distribution of shear in columns.

$$V_{6,1} = \frac{\frac{L_1}{2}}{L} \times V_6 = \frac{\frac{2.6}{2}}{9.8} \times 124.4 = 16.5$$
$$V_{6,2} = \frac{\frac{L_1}{2} + \frac{L_2}{2}}{L} \times V_6 = \frac{\frac{2.6}{2} + \frac{2.3}{2}}{9.8} \times 124.4 = 31.09$$
$$V_{6,3} = \frac{\frac{L_2}{2} + \frac{L_3}{2}}{L} \times V_6 = \frac{\frac{2.3}{2} + \frac{4.9}{2}}{9.8} \times 124.4 = 45.69$$
$$V_{6,4} = \frac{\frac{L_4}{2}}{L} \times V_6 = \frac{\frac{4.9}{2}}{9.8} \times 124.4 = 31.09$$

After calculating the shear of each column in level 6, the shear can be computed from the top left of frame moment at the column and includes the girder while considering its free body equilibrium:

$$\text{Moment in the upper part of column} = \text{column shear} \times \text{half}-\text{story height of level 6}$$
$$= 16.51.8 = 29.7$$

For moment equilibrium of the joint, the moment at the left end of the first girded is equal to −29.7 kN m. The shear in girder is computed by dividing the moment in girder to half girder length. Figure 1.97 shows a moment in column and shear in the beam.

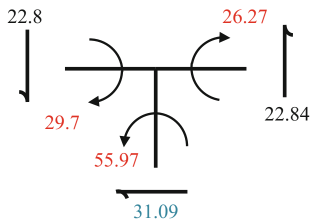


Fig. 1.98 Illustration of the free body diagram

$$\frac{29.7}{\frac{2.6}{2}} = 20.65\ \text{kN}$$

Due to the mid-length point of contra flexure, the moment at the right end of the girder has the same value as that of the left end. The column moments at the top and bottom of the story are equal. The sign convention for numerical values of the bending moment is that an anticlockwise moment (applied by a joint to the end of a member) is taken as positive.

The value of shear and moment are shown in Fig. 1.98. In the other step moment at the top of the second column, the shear and moment are calculated as follows:

$$\begin{aligned}\text{Moment in the upper part of column 2} &= \text{column 2 shear} \times \text{half-story height of level 6}\\ &= 31.091.8 = 55.97\ \text{kN m}\end{aligned}$$

$$\text{From moment equilibrium of joint, moment at the end of the second girder}$$
$$= -(50.97 - 29.7) = 26.27\ \text{kN m}$$

So shear amount in second girder is:

$$26.27/(2.3/2) = 22.84\ \text{kN}$$

The above procedure is repeated for successive modules to the right, and then continued to the floor below, and repeated from left. The steps taken in analyzing frame B with the portal method is shown in Fig. 1.99.

1.3.10.2 Cantilever Method

A. C Wilson described the cantilever method. The goal of this method is to determine an approximation of the effect of lateral forces experienced by tall buildings. It uses the assumption that tall buildings with a horizontal load such as the wind shall behave like a cantilevered beam.

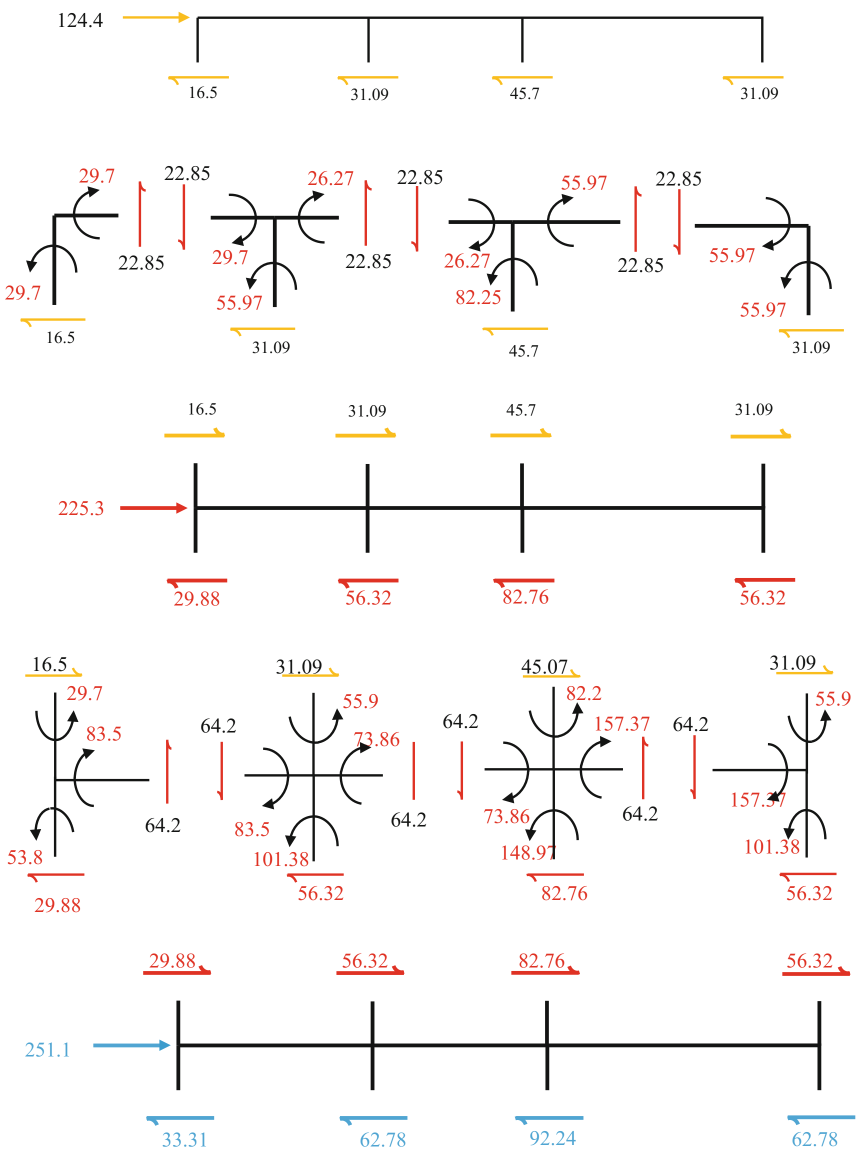


Fig. 1.99 The analysis of frame (B) with Portal method

The following are the assumptions that are commonly used in cantilever method:

1. Contra flexure is located in the middle of columns and mid-spans of girders.
2. The axial stress in a column is proportional to its distance from the centroid of the column areas.

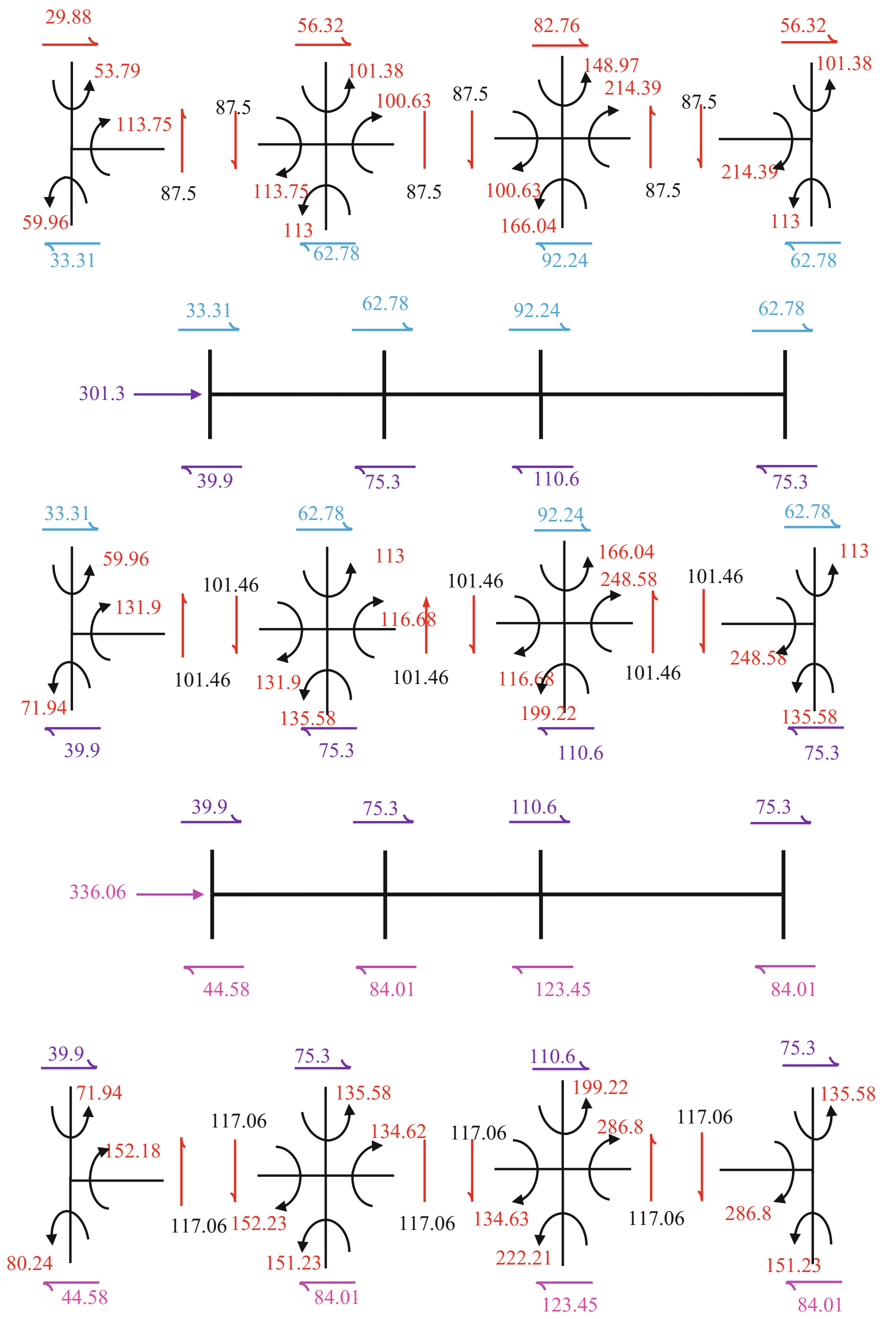


Fig. 1.99 (continued)

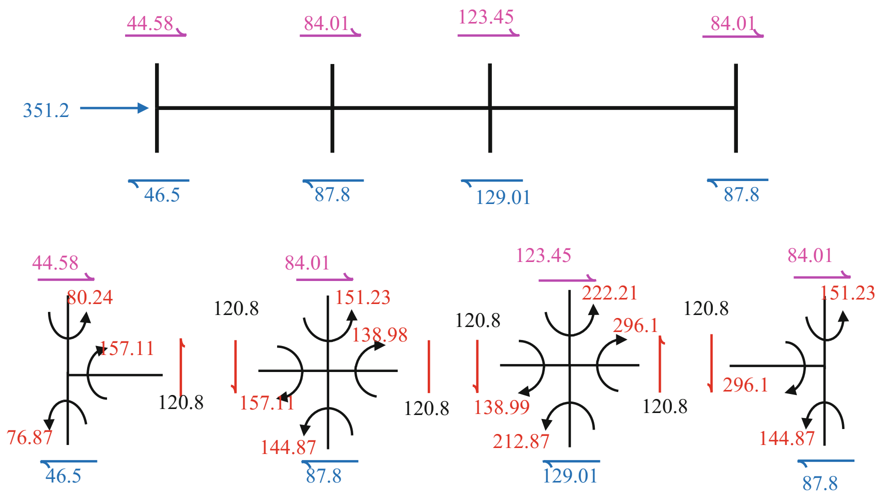


Fig. 1.99 (continued)

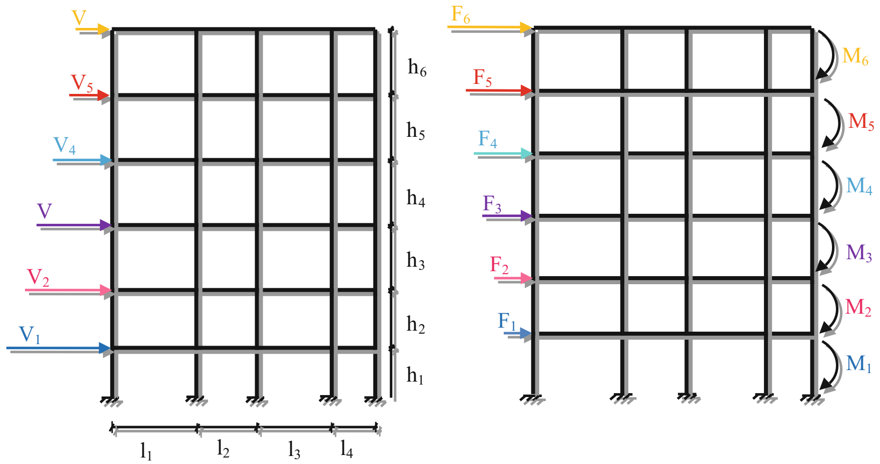


Fig. 1.100 The moment resistance of the six-story frame

The following steps are applied in the analysis:

1. The first step is to draw a line diagram of the frame and record the external moment at each mid-story level. Figure 1.100 shows the pattern observed in the moment frames under shear load and moment. The Eq. (1.86) below is used to calculate the magnitude of the moment in each level.

$$
\begin{aligned}
M_6 &= \left[F_6 \times \frac{h_6}{2}\right] \\
M_5 &= \left[\left[F_6 \times \left(h_6 + \frac{h_5}{2}\right)\right] + \left(F_5 \times \frac{h_5}{2}\right)\right] \\
M_4 &= \left[\left[F_6 \times \left(h_6 + h_5 + \frac{h_4}{2}\right)\right] + \left[F_5 \times \left(h_5 + \frac{h_4}{2}\right)\right] + \left(F_4 \times \frac{h_4}{2}\right)\right] \\
M_3 &= \left[\left[F_6 \times \left(h_6 + h_5 + h_4 + \frac{h_3}{2}\right)\right] + \left[F_5 \times \left(h_5 + h_4 + \frac{h_3}{2}\right)\right]\right. \\
&\quad \left. + \left[F_4 \times \left(h_4 + \frac{h_3}{2}\right)\right] + \left(F_3 \times \frac{h_3}{2}\right)\right] \\
M_2 &= \left[\left[F_6 \times \left(h_6 + h_5 + h_4 + h_3 + \frac{h_2}{2}\right)\right] + \left[F_5 \times \left(h_5 + h_4 + h_3 + \frac{h_2}{2}\right)\right]\right. \\
&\quad + \left[F_4 \times \left(h_4 + h_3 + \frac{h_2}{2}\right)\right] \\
&\quad \left. + \left[F_3 \times \left(h_3 + \frac{h_2}{2}\right)\right] + \left(F_2 \times \frac{h_2}{2}\right)\right] \\
M_1 &= \left[\left[F_6 \times \left(h_6 + h_5 + h_4 + h_3 + h_2 + \frac{h_1}{2}\right)\right] + \left[F_5 \times \left(h_5 + h_4 + h_3 + h_2 + \frac{h_1}{2}\right)\right]\right. \\
&\quad + \left[F_4 \times \left(h_4 + h_3 + h_2 + \frac{h_1}{2}\right)\right] + \left[F_3 \times \left(h_3 + h_2 + \frac{h_1}{2}\right)\right] + \left[F_2 \times \left(h_2 + \frac{h_1}{2}\right)\right] \\
&\quad \left. + \left(F_1 \times \left(\frac{h_1}{2}\right)\right)\right]
\end{aligned}
\tag{1.86}
$$

2. Next is to identify the centroid of the column area. This is done using Eq. (1.87). Hence:

$$
C = \frac{\sum A_i \bar{x}_i}{\sum A} \tag{1.87}
$$

where in the equation:

A_i: area of column
$\bar{x}_i$: Distance of column i from the selected axis

3. The Eq. (1.88) is used to calculate the second moment inertia of column areas.

$$
I = \sum A_i C_i^2 \tag{1.88}
$$

where in the equation:

A_i: area of column
C_i : Distance of column I from the centroid

4. In this step, the column axial F_i in each story is computed. This is done using Eq. (1.89) and is composed of parameters used to calculate the force of the columns at each story.
The equation is based on the cantilever method.

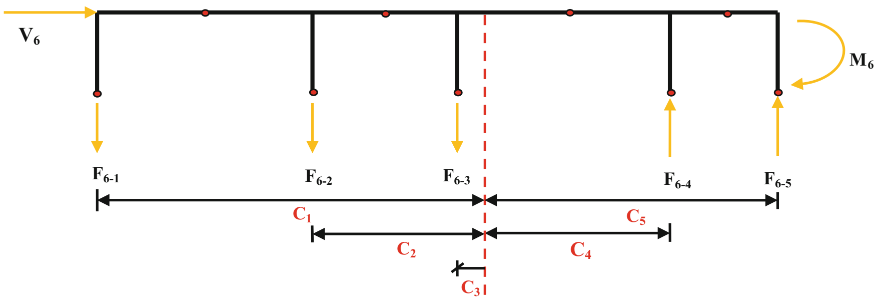


Fig. 1.101 Illustration of free body diagram of frame at level six

$$F_i = \frac{MC_i}{I} \times A_i \tag{1.89}$$

where in the equation:

M: External moment on story
C_i : Distance of column I from the centroid
I: Second-moment inertia
A_i: area of column

Figure 1.101 shows the free body diagram of story six which is also in accordance with the cantilever method.

5. Step 5 begins with the top-left module. This is done to find the vertical shear in the girder using the vertical equilibrium of the module. The girder-end moment is calculated by multiplying the beam shear and its half span. This needs to be calculated after first calculating the moment in the column just below the joint. This joint is the same as the equilibrium of girder and column moments at the joint. Upon doing this, the column shear is evaluated by dividing the column-top moment by half the story height. Then determine the shear and moment in the second girder and column. Figure 1.102 shows the equilibrium of the module.

- **Cantilever method for bent (1)**:

Frame (1) is selected as an example for analyzing using cantilever method. The steps in this analysis are as follows:

1. The external moment is calculated based on the forces in each story. Shows the frame (1) with earthquake force and external moment which is calculated according to of Eq. (1.86), hence (Fig. 1.103);

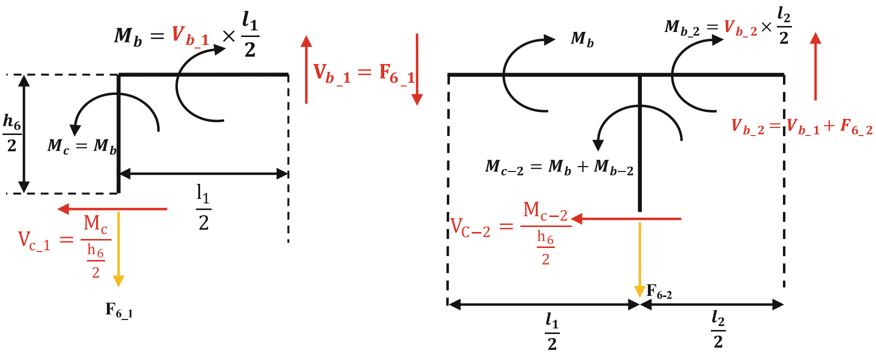


Fig. 1.102 Equilibrium of module

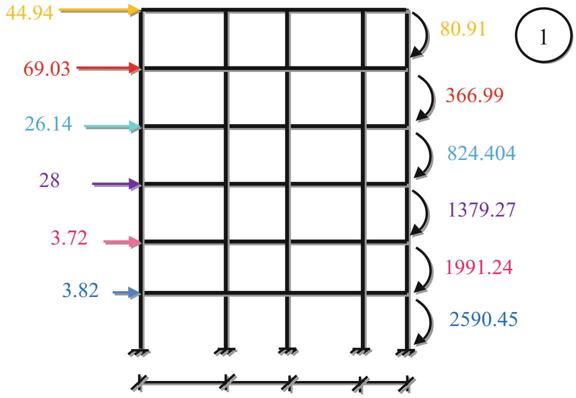


Fig. 1.103 The moment resistance frame (1) with external force and moment

$$M_6 = F_6 \times \frac{h_6}{2} = 44.94 \times \frac{3.6}{2} = 80.91$$

$$M_5 = \left[F_6 \times \left(h_6 + \frac{h_5}{2}\right)\right] + \left(F_5 \times \frac{h_5}{2}\right) = \left[44.94 \times \left(3.6 + \frac{3.6}{2}\right)\right] + \left(69.03 \times \frac{3.6}{2}\right) = 366.99$$

$$M_1 = \left[\left[44.94 \times \left(3.6 + 3.6 + 3.6 + 3.6 + 3.6 + \frac{3.3}{2}\right)\right] + \left[69.03 \times \left(3.6 + 3.6 + 3.6 + 3.6 + \frac{3.3}{2}\right)\right]\right.$$
$$+ \left[26.14 \times \left(3.6 + 3.6 + 3.6 + \frac{3.3}{2}\right)\right] + \left[28 \times \left(3.6 + 3.6 + \frac{3.3}{2}\right)\right]$$
$$\left. + \left[3.72 \times \left(3.6 + \frac{3.3}{2}\right)\right] + \left(3.82 \times \left(\frac{3.3}{2}\right)\right)\right] = 2590.45$$

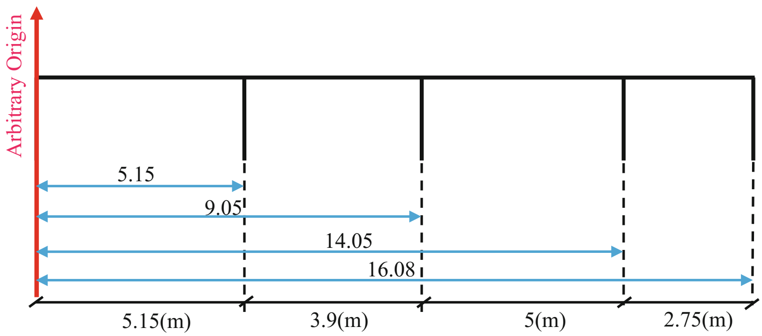


Fig. 1.104 Centroid of columns area of frame (1)

2. Location of the centroid of the area:
   Based on the description in cantilever method procedure, the centroid of column area is calculated using Eq. (1.87). Figure 1.104 shows the process of centroid calculation for the column area.

$$C_i = \frac{\sum A\bar{x}}{\sum(A)}$$

$$C_i = \frac{(0.0116 \times 5.15) + (0.0116 \times 9.05) + (0.0116 \times 14.05) + (0.0096 \times 16.8)}{(3 \times 0.0116) + (2 \times 0.0096)} = 9.055$$

   The centroid of column area is 9.055 (m) from the arbitrary origin. The distance of columns from the centroid area of columns is shown in Fig. 1.105.
3. The calculation of the second moment inertia is described in the previous step. This is reached using Eq. (1.88).

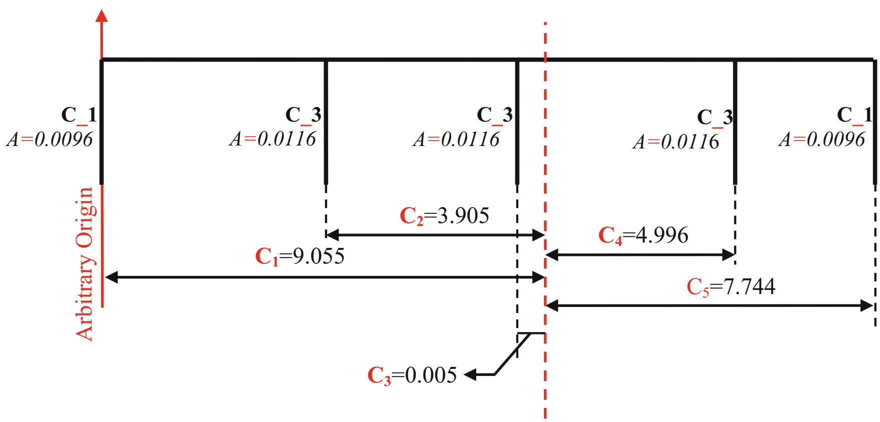


Fig. 1.105 The centroid of columns area of frame (1)

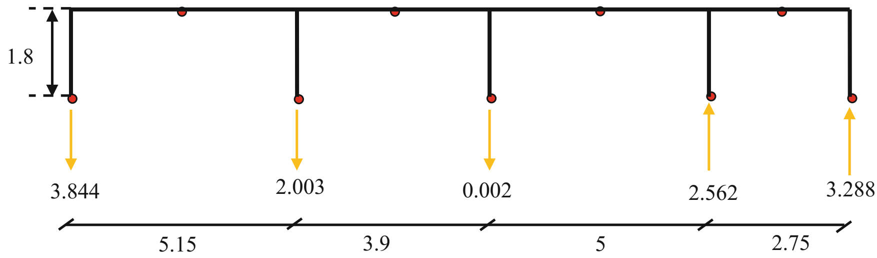


Fig. 1.106 The axial force of columns of story (1) cantilever method

$$I = \sum AC_i^2$$

$$I = \left(0.0096 \times 9.055^2\right) + \left(0.0116 \times 3.905^2\right) + \left(0.0116 \times 0.005^2\right) + \left(0.0116 \times 4.996^2\right) + \left(0.0096 \times 7.744^2\right) = 1.8292$$

4. Column' axial forces are calculated according to the Eq. (1.74). Figure 1.106 shows the contra flexure points and columns axial force. An external moment in story 6 is 117.8 (kN m) which is calculated in Step 1. The distance of each column with respect to the centroid column area is calculated in Step 2, and second-moment inertia is computed in Step 3. Area of columns is located in Table 1.58 as a Result all magnitudes should be substituted in Eq. (1.89)

$$F = \frac{MC}{I} \times A$$

$$F_{6\_1} = \frac{(80.91 \times 9.055)}{(1.8292)} \times 0.0096 = 3.844$$

$$F_{6\_2} = \frac{(80.91 \times 3.905)}{(1.8292)} \times 0.0116 = 2.003$$

$$F_{6\_3} = \frac{(80.91 \times 0.005)}{(1.8292)} \times 0.0116 = 0.002$$

$$F_{6\_4} = \frac{(80.91 \times -4.996)}{(1.8292)} \times 0.0116 = -2.562$$

$$F_{6\_5} = \frac{(80.91 \times -7.744)}{(1.8292)} \times 0.0096 = -3.288$$

5. **Starting with the top-left module**

The step by step analysis of the frame (1) is as follows: Fig. 1.107a shows the free body diagram frame (1). From vertical equilibrium of frame, shear in first girder = 3.844.

Moment at the left end of girder = shear × half-length of girder = 3.844 2.575 = 9.9 kN m

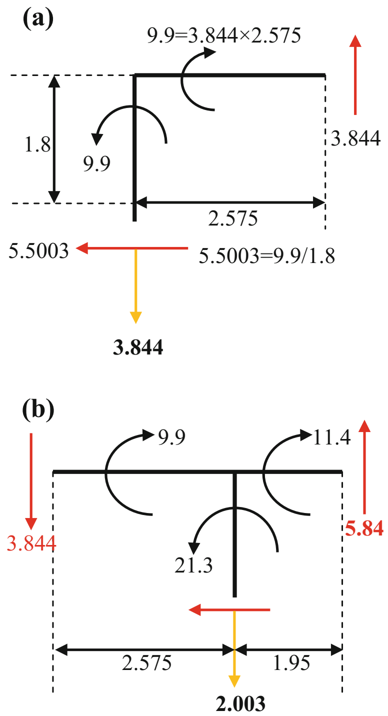


Fig. 1.107 **a** Free body diagram of top-left module, **b** free body diagram of next right module

From moment equilibrium of joint, moment at the top of Column = 9.9.

Shear in column = moment at top/half story height = 9.9/1.8 = 5.5003.

The moment at opposite ends of the girders and columns are of the same value. Figure 1.107b shows free body diagram of Next-right module. Considering the next right module, from the vertical equilibrium of module, shear in second girder = 3.844 + 2.003 = 5.84.

Moment in left end of second girder = shear × half-length of Girder = 5.84 × 1.95 = 11.4.

From moment equilibrium of joint, moment at top of column = −(−9.9 − 11.4) = 21.3.

Shear in column = moment at top/half story height = 21.3/1.8 = 11.8 (Fig. 1.107).

This procedure is repeated for successive modules to the right, then on the level below, working again from left to right. This is continued for analyzing frame (1), where all forces of each story are calculated with the same steps. The Charts 1.16, 1.17, 1.18, 1.19, 1.20 and 1.21 illustrates the calculation of axial force in columns

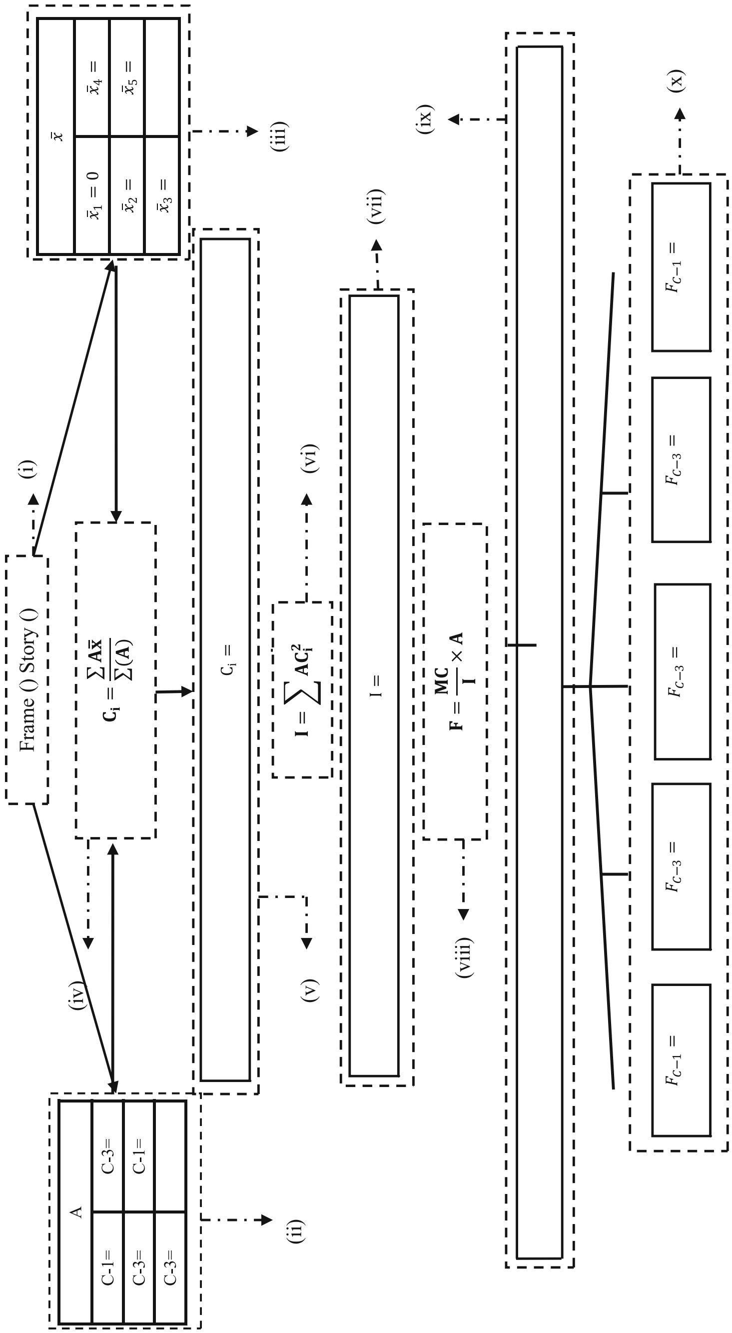


Fig. 1.108 Pattern chart of axial calculation force in the cantilever method

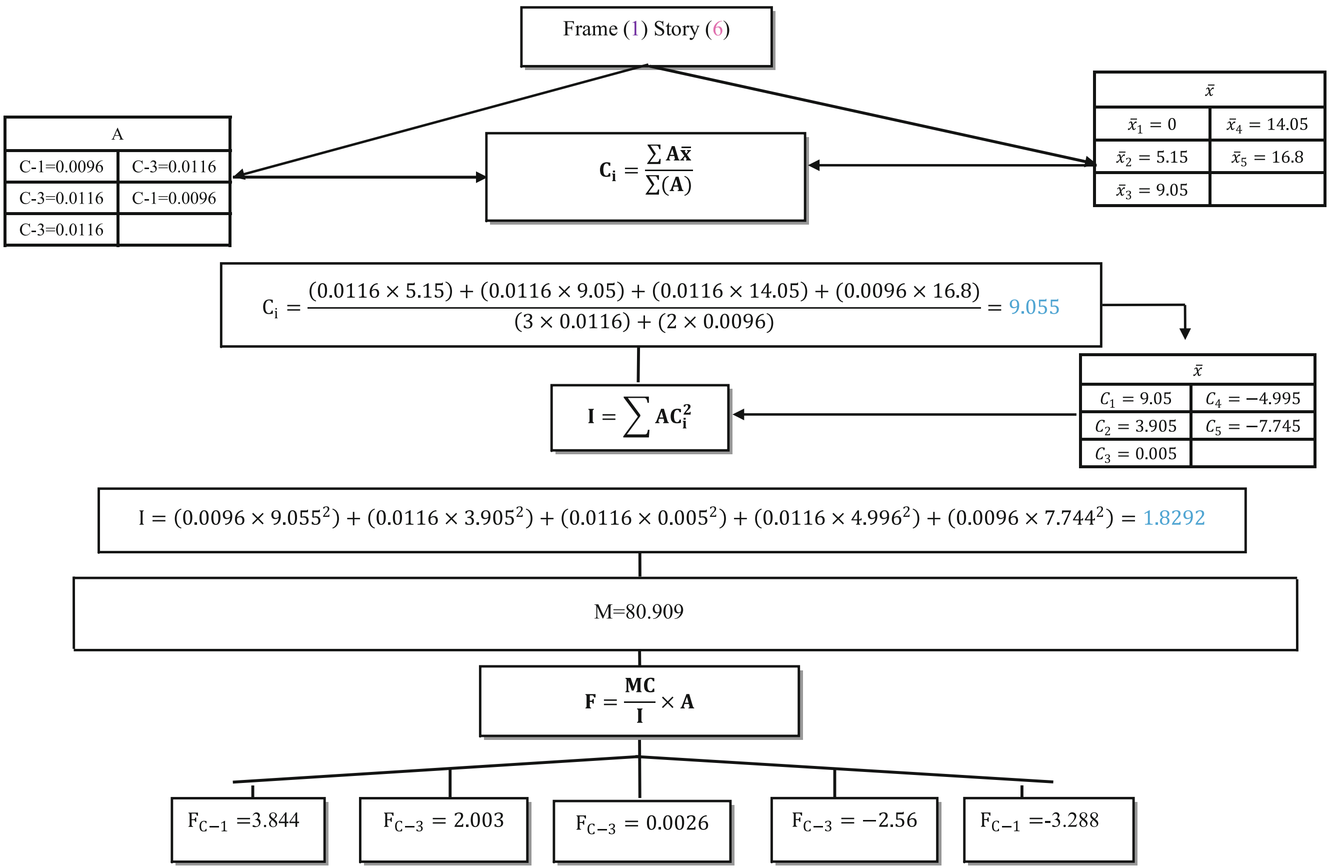


Chart 1.16 Procedure of calculation axial force in columns of story (6), frame (1); cantilever method

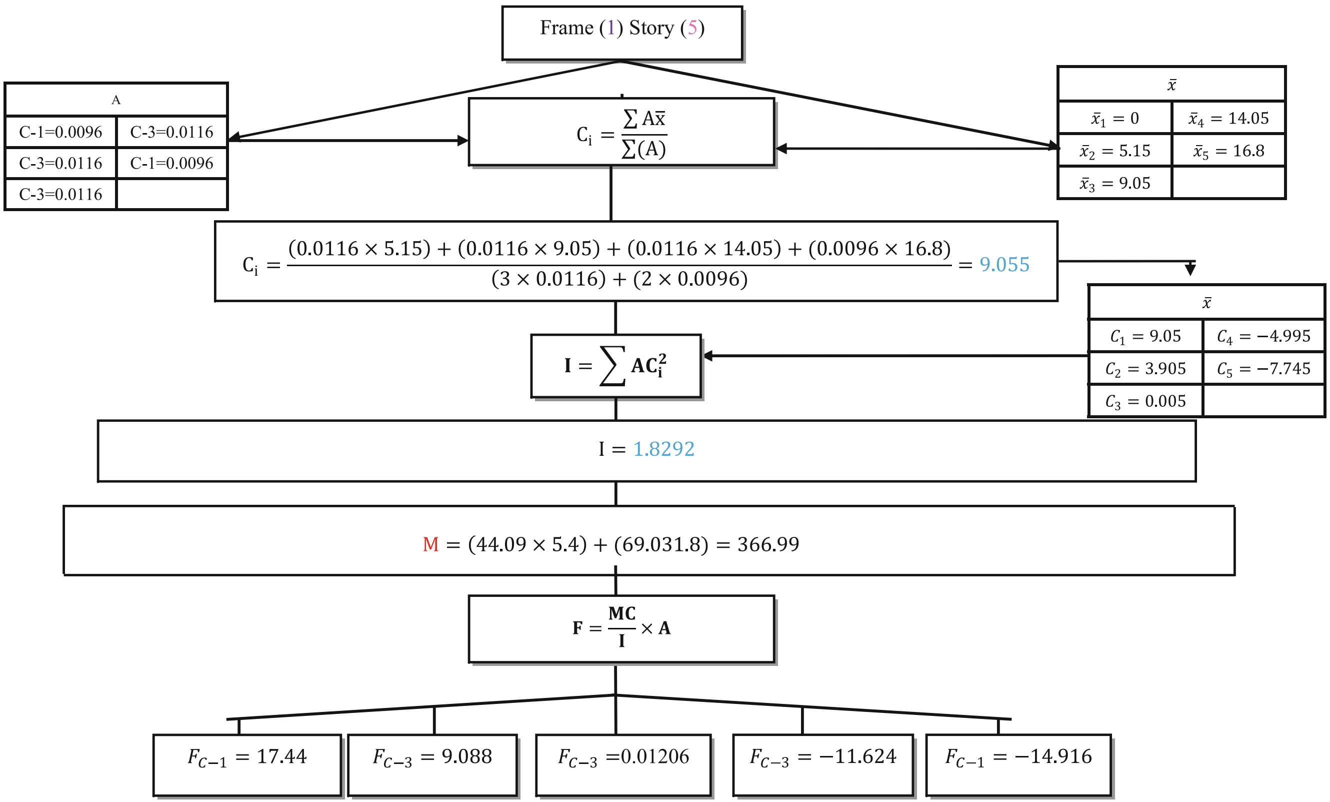


Chart 1.17 Procedure of calculation axial force in columns of story (5), frame (1); cantilever method

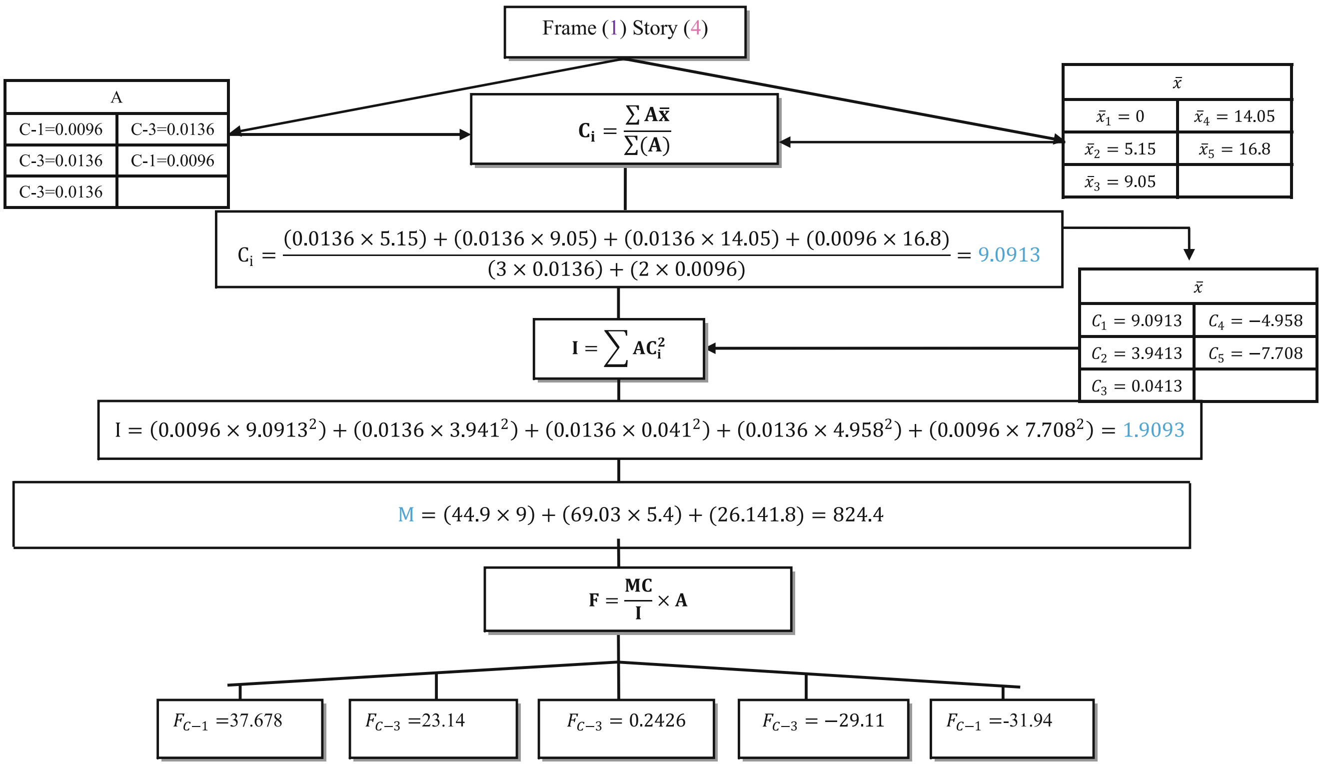


Chart 1.18 Procedure of calculation axial force in columns of story (4), frame (1); cantilever method

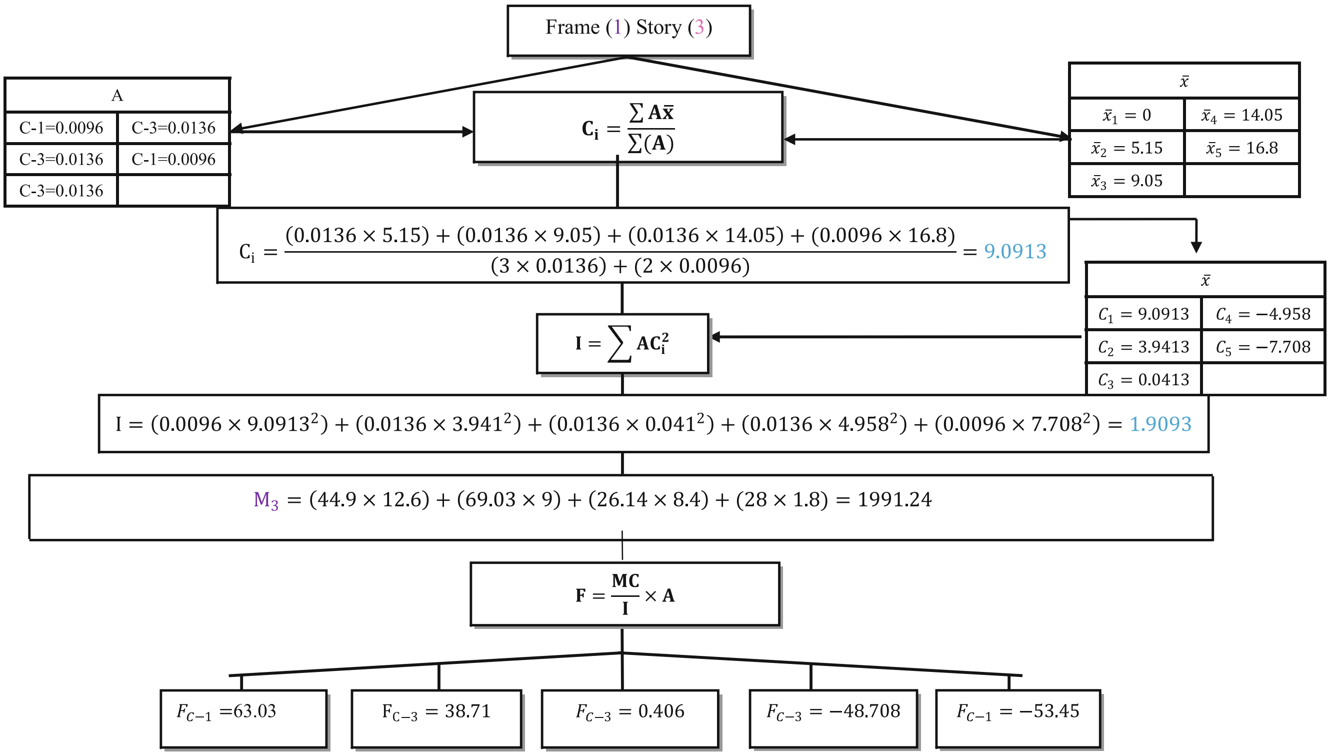


Chart 1.19 Procedure of calculation axial force in columns of story (3), frame (1); cantilever method

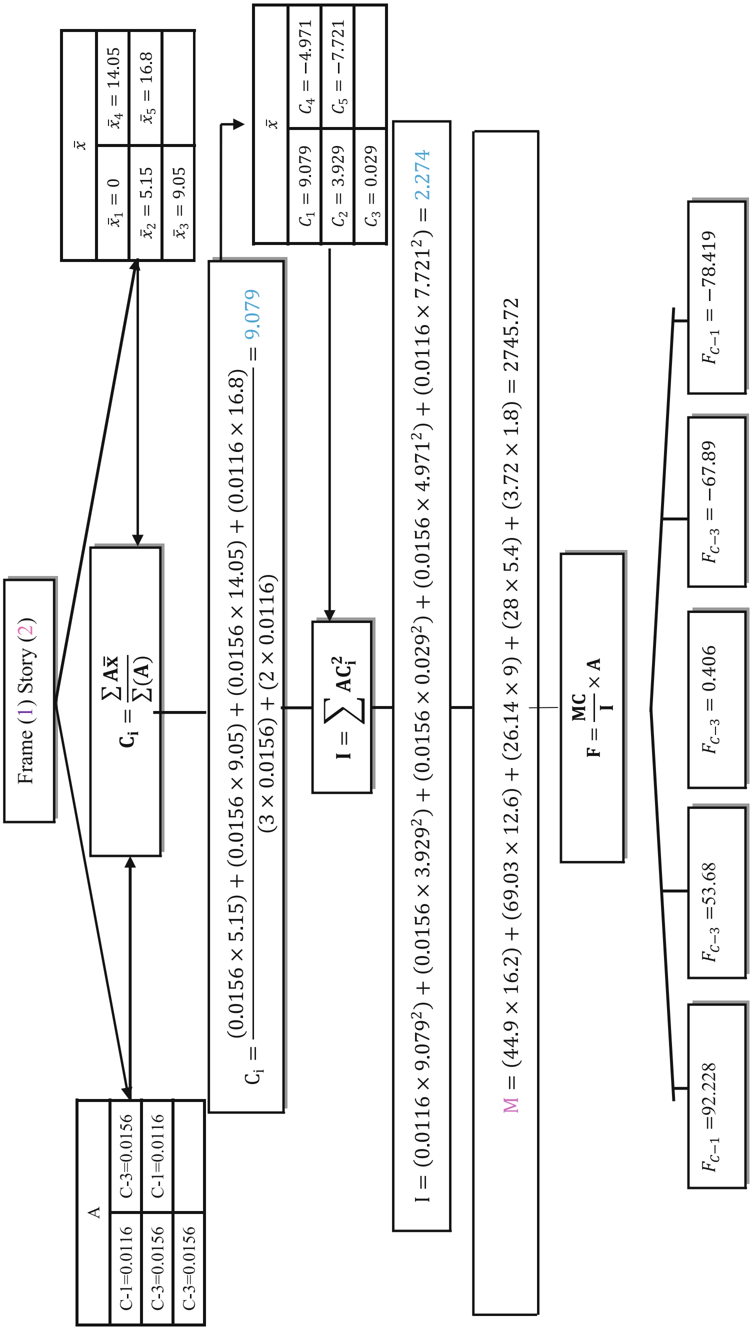


Chart 1.20 Procedure of calculation axial force in columns of story (2), frame (1); cantilever method

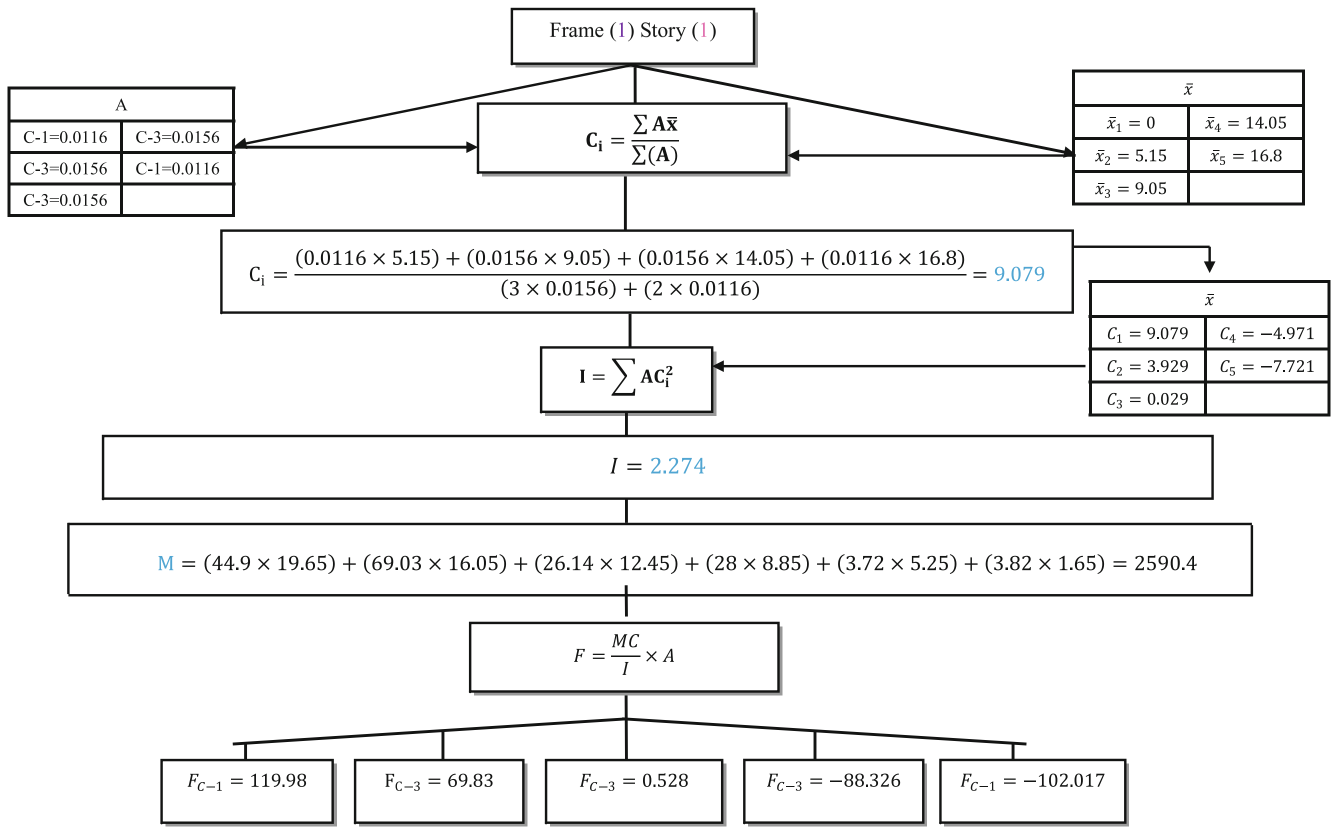


Chart 1.21 procedure of calculation axial force in columns of story (5), frame (1); cantilever method

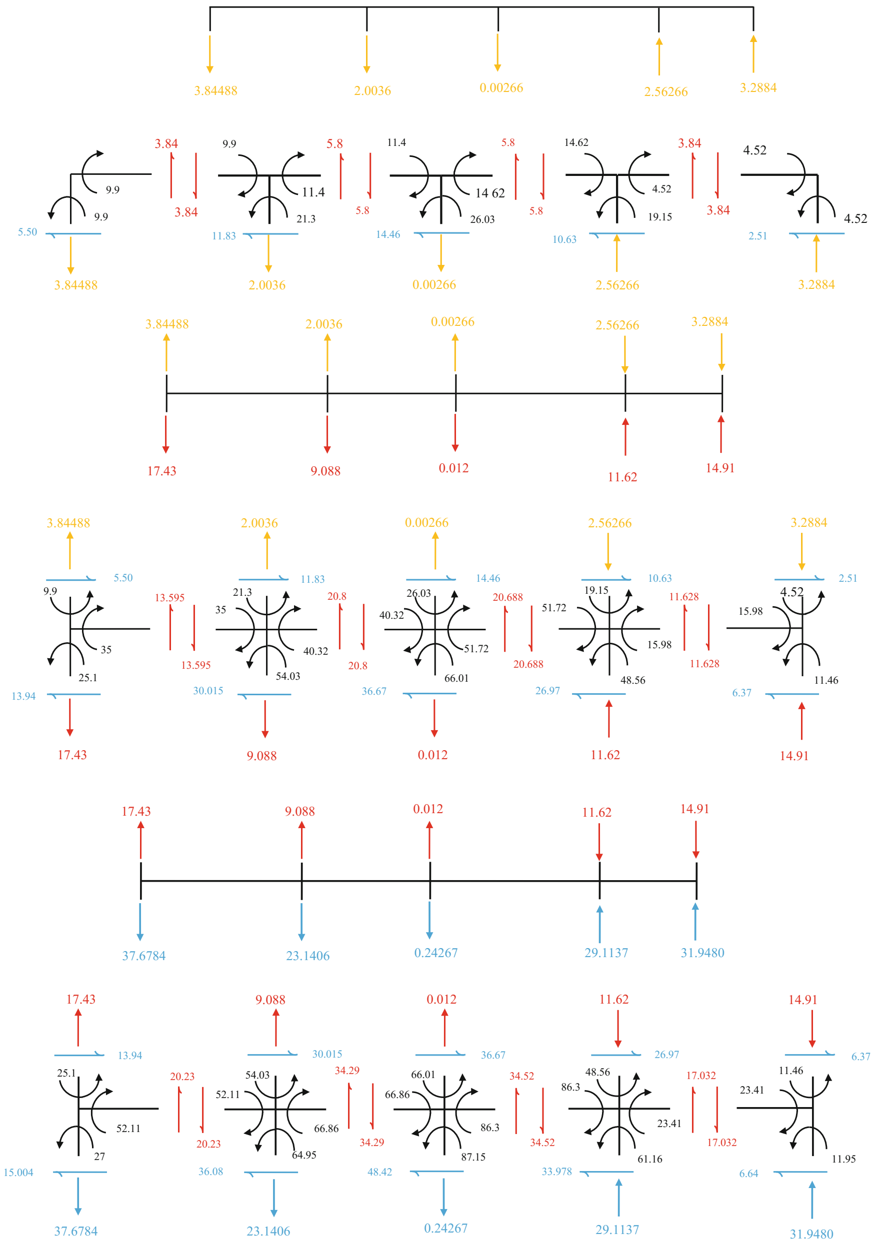


Fig. 1.109 Analysis of frame (1) with cantilever method

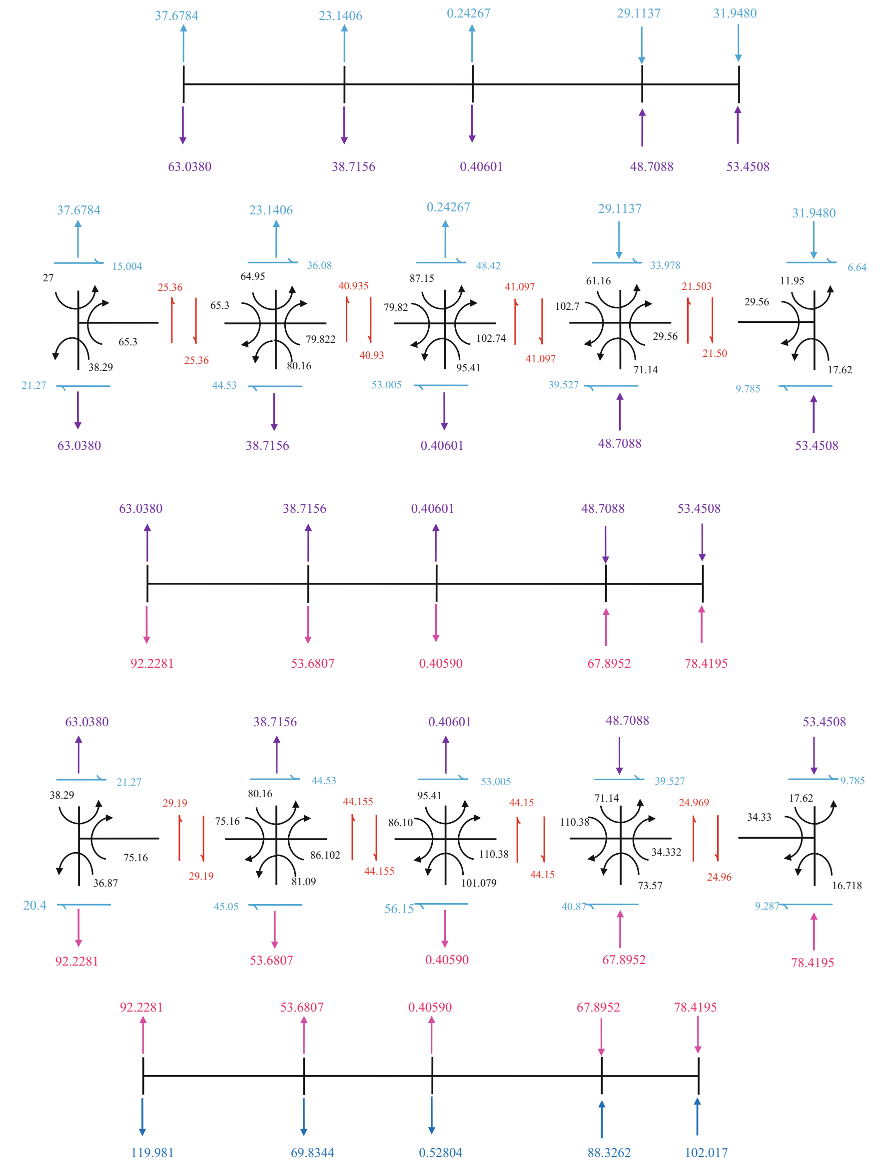


Fig. 1.109 (continued)

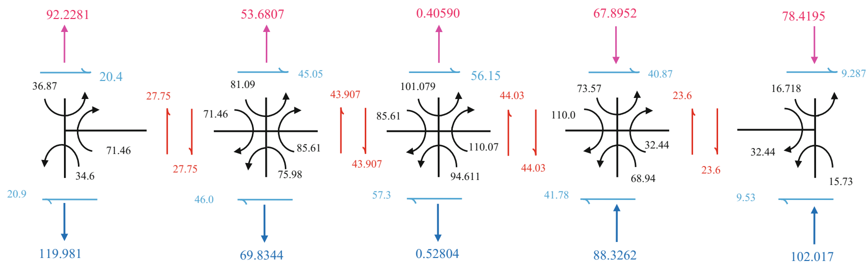


Fig. 1.109 (continued)

of the frame (1). Figure 1.108 shows the pattern chart of axial calculation force in the cantilever method. In addition, the procedure of analyzing of the frame (1) with cantilever method is shown in Fig. 1.109.

- (i): Frame and story's number
- (ii): area of columns
- (iii): distance from arbitrary origin
- (iv): formula of centroid column area
- (v): magnitude of centroid column area
- (vi): formula of second moment inertia
- (vii): magnitude of second moment inertia
- (viii): axial force of column's formulation
- (ix): calculation of external moment
- (x): axial force in each column

1.4 Displacement Analysis

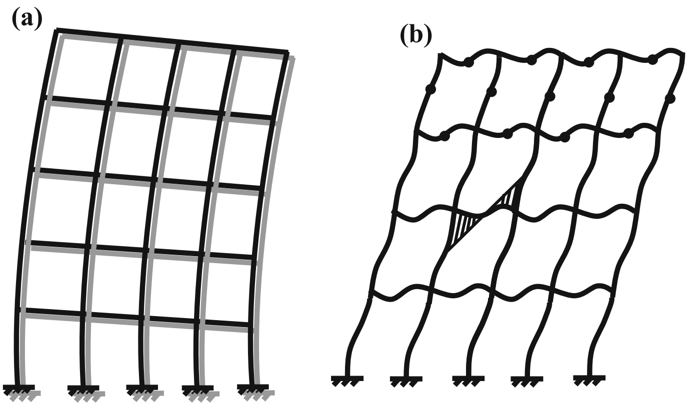


Fig. 1.110 **a** Overall bending displacement, **b** flexure displacement

1.4.1 Introduction

Upon selecting the initial sizes of the frame members, the horizontal drift of the structure can be roughly estimated. Based on the analysis, it is found that two factors can introduce the lateral deformation of moment resisting frame, namely:

1. Bending cantilever (overall bending) induced lateral deformation:
   This particular phenomenon is known as the Chord Drift. At each story, the complete moment of the external horizontal load (over turning) can be resisted at individual story levels. This is possible due to the axial tensile and compressive forces in the columns on opposite sides of the structure. The bending and the resulting horizontal displacement of the structure are the effect of the extension and shortening of columns. The rate of lateral displacement is 20% from the total lateral displacement of the entire structure, this is shown in Fig. 1.110a.
2. The beams' and columns' bending due to deformation:
   This particular phenomenon is widely known as the shear lag. The shear forces in horizontal and vertical modes are applied to the column and beam, respectively. Thereafter, the moment of bending induced within the members provides the lateral deformation in the beams and columns. As a result, the entire frame may be deformed and can be an equivalent to 80% of the total lateral displacement of the entire structure. In this 80, 65% is due to the bending in the beam, and the remaining 15% is related to the bending within the column.

It is assumed the drift analysis of contra flexure occurs in the frame at the mid-story level of the columns and the mid-span of the girders. This is a considered to be a reasonable assumption for high rise moment resistance frame for all stories except near the top and bottom. The deflection of moment resistance frame consists of three different parts: first is the story displacement due to girder flexure second is the story displacement due to column flexure while the third is the story displacement due to the overall bending. The Eq. (1.90) shows the total drift of moment resistance frame.

Total drift

$$\text{Total drift} \quad \delta_i = \delta_{ig} + \delta_{ic} + \delta_{if} \tag{1.90}$$

Where in the equation:

$$Girder\,flexure\,drift\ = \delta_{ig} = \frac{Q_i h_i^2}{12E}\left(\frac{1}{G}\right) \tag{1.91}$$

$$Column\,flexure\,drif\ = \delta_{ic} = \frac{Q_i h_i^2}{12E}\left(\frac{1}{C}\right) \tag{1.92}$$

$$Overall\,bending\,drift = \delta_{if} = \frac{M_i h_i^2}{EI_i} \tag{1.93}$$

Total displacement due to flexure is calculated according to Eq. (1.94).

$$Story\,displacement\,due\,to\,flexure = \delta_{ig} + \delta_{ic} = \frac{Q_i h_i^2}{12E}\left(\frac{1}{G} + \frac{1}{C}\right) \tag{1.94}$$

where in formulation:

$$Q_i = shear\,in\,story\,i$$

$$h_i = height\,of\,sory\,i$$

$$G = \frac{I_G}{L}, \quad C = \frac{I_C}{h}$$

I_c, I_g = second-moment inertia of column and girder.

- **Story displacement due to overall Bending**:

Even though the proportion of total drift due to overall bending may be small relative to the racking, the bending inclination increases cumulatively throughout the height. Consequently, in the upper stories, the story shear drift tends to be less than the lower region. In this region, the bending drift may become a significant part of the story drift. An estimate of the bending drift can be made by assuming the structure to behave as a flexural cantilever with a moment of inertia that is equal to the second moment of the column areas about their common centroid. If the moment diagram is used to construct the M/EI diagram, the area of the diagram between the base and the mid-height of story i gives the average slope of the story (i) due to bending action.

$$\theta_{if} = story\,inclination = area\,of\,the\,moment\,diagram = \frac{Mh}{EI} \tag{1.95}$$

Total displacement of overall bending is calculated based on Eq. (1.96)

$$\text{Story drift due to bending} = \delta_{if} = \frac{M_i h_i^2}{EI_i} \tag{1.96}$$

Where in the equation:

$$M_i = \text{Moment in story } i$$

$$h_i = \text{height of story } i$$

$$I_i = \sum \left(Ac^2\right) \text{Second-moment inertia of the column area about their comon centroid}$$

Computing of centroid and second-moment inertia in each story in frames:

$$I_i = \sum \left(Ac^2\right) \tag{1.97}$$

The assumption used in the mid-story-height point of contra flexure is not valid for the first story of a rigid frame because of the fully fixed or hinged conditions at the base of the columns. Therefore, special expressions should be used for the first story drift which is attributed to the column and girder bending. If the columns have rigid base connections, the first story displacement may be estimated by

$$\delta_{1,g+c} = \frac{Q_1 h_1^2}{12E} \frac{\left(\frac{2}{3G_1} + \frac{1}{C_1}\right)}{\left(1 + \frac{C_1}{6G_1}\right)} \tag{1.98}$$

If the columns have pinned base

$$\delta_{1,g+c} = \frac{Q_1 h_1^2}{12E} \left(\frac{3}{2G_1} + \frac{4}{C_1}\right) \tag{1.99}$$

Consider a six stories building that consists of four types of columns and seven types of beam. Table 1.71 illustrates the dimension of columns size in a different story of the building and Table 1.72 shows the properties of the columns. The analysis of the considered module displacement is made up of flexure displacement and overall bending displacement. For this reason, the Eq. (1.90) is used to calculate of total drift in all frames.

The frame (E) is selected as the first frame for displacement analysis. Shear force and external moment of the frame (E) is shown in Fig. 1.111. The analysis for displacement part indicates the need for analyzing the frame and the displacement should be calculated for the flexure displacement and overall bending displacement. In overall bending displacement analysis, second-moment inertia is calculated according to the centroid column area, so for this reason, centroid column area calculated as follows:

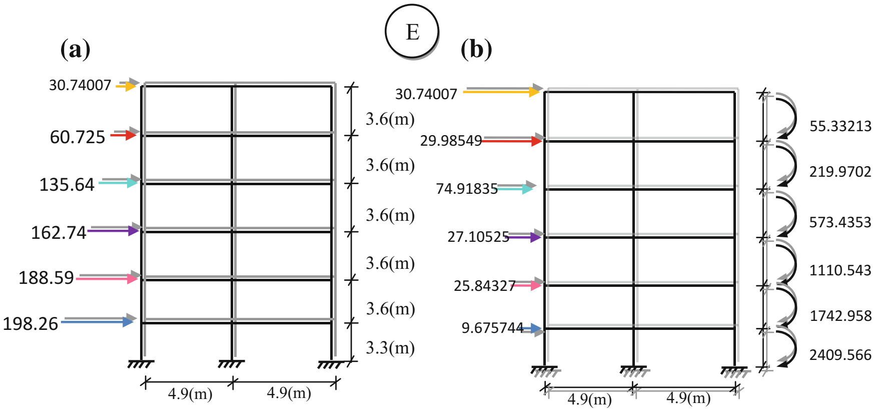


Fig. 1.111 **a** Shear force in frame (E), **b** force and moment in frame (E)

Table 1.71 Different types of columns

Column	Size of story 1, 2	Size of story 3, 4	Size of story 5, 6
C-1	$Box\,300 \times 10$	$Box\,250 \times 10$	$Box\,250 \times 10$
C-2	$Box\,350 \times 10$	$Box\,300 \times 10$	$Box\,300 \times 10$
C-3	$Box\,400 \times 10$	$Box\,350 \times 10$	$Box\,300 \times 10$
C-4	$Box\,450 \times 15$	$Box\,400 \times 12$	$Box\,350 \times 10$

Table 1.72 Area and mass per meter of columns

Column size	Area cm^2	γ	$G_{\text{kg/m}}$
$Box\,250 \times 10$	96	7850	75.36
$Box\,300 \times 10$	116	7850	91.06
$Box\,350 \times 10$	136	7850	106.76
$Box\,400 \times 10$	156	7850	122.46
$Box\,400 \times 12$	186.24	7850	146.19
$Box\,450 \times 15$	261	7850	204.88

$$C_{1,,2} = \frac{(0.0136 \times 4.9) + (0.0116 \times 9.8)}{(2 \times 0.0116) + (0.0136)} = 4.9$$

$$C_{3,4} = \frac{(0.0116 \times 4.9) + (0.0096 \times 9.8)}{(2 \times 0.0096) + (0.0116)} = 4.9$$

$$C_{5,6} = \frac{(0.0116 \times 4.9) + (0.0096 \times 9.8)}{(2 \times 0.0096) + (0.0116)} = 4.9$$

Then second-moment inertia is calculated according to Eq. (1.97) from story (1) to (6)

$$I_i = \sum \left(Ac^2\right)$$

Second-moment inertia in story 1 and 2 $I_{1,2} = (0.0116 \times 4.9^2) + (0.0116 \times 4.9^2) = 0.557032$

Second-moment inertia in story 3 and 4 $I_{3,4} = (0.0096 \times 4.9^2) + (0.0096 \times 4.9^2) = 0.460992$

Second-moment inertia in story 5 and 6 $I_{5,6} = (0.0096 \times 4.9^2) + (0.0096 \times 4.9^2) = 0.460992$

Next step is calculation of $\theta_i = \frac{Mh}{EI}$, it should mentioned that the θ_{if} is cumulative, hence;

$$\theta_{i,1} = \frac{M_1 \times h_1}{E \times I_1} = \frac{2409.56 \times 3.3}{2 \times 10^8 \times 0.55703} = 7.1374\text{E}{-05}$$

$$\theta_{i,2} = \frac{M_{2\times} h_2}{E \times I_2} = \frac{1742.95 \times 3.6}{2 \times 10^8 \times 0.55703} = 5.6322\text{E}{-05}$$

$$\theta_{if,2} = \theta_{i,2} + \theta_{i,1} = 0.000127697$$

$$\theta_{i,3} = \frac{M_{3\times} h_3}{E \times I_3} = \frac{1110.54 \times 3.6}{2 \times 10^8 \times 0.46099} = 4.3363\text{E}{-05}$$

$$\theta_{if,3} = \theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.000171059$$

$$\theta_{i,4} = \frac{M_{4\times} h_4}{E \times I_4} = \frac{573.43 \times 3.6}{2 \times 10^8 \times 0.46099} = 2.239\text{E}{-05}$$

$$\theta_{if,4} = \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.00019345$$

$$\theta_{i,5} = \frac{M_{5\times} h_5}{E \times I_5} = \frac{219.97 \times 3.6}{2 \times 10^8 \times 0.46099} = 8.589\text{E} - 06$$

$$\theta_{if,5} = \theta_{i,5} + \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.000202039$$

$$\theta_{i,6} = \frac{M_{6\times} h_6}{E \times I_6} \frac{55.33 \times 3.6}{2 \times 10^8 \times 0.46099} = 2.1605\text{E}{-06}$$

$$\theta_{if,6} = \theta_{i,6} + \theta_{i,5} + \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.000202039$$

Then displacement is calculated according to the Eq. (1.90). Table 1.73 shows the overall bending displacement frame (E).

Table 1.73 Overall bending drift

Story	h (m)	I_i (m^4)	E	Force	Moment	θ_i	θ_{if}	δ_{if}	Δ
6	3.6	0.460992	2E+08	30.74007	55.33213	2.1605E−06	0.000204199	0.000735	**0.00346993**
5	3.6	0.460992	2E+08	29.98549	219.9702	8.589E−06	0.000202039	0.000727	**0.002734814**
4	3.6	0.460992	2E+08	74.91835	573.4353	2.239E−05	0.00019345	0.000696	**0.002007475**
3	3.6	0.460992	2E+08	27.10525	1110.543	4.3363E−05	0.000171059	0.000616	**0.001311056**
2	3.6	0.557032	2E+08	25.84327	1742.958	5.6322E−05	0.000127697	0.00046	**0.000695243**
1	3.3	0.557032	2E+08	9.675744	2409.566	7.1374E−05	7.13744E−05	0.000236	**0.000235536**

Flexure displacement in frame (E) is calculated according to Eq. (1.94), hence;

$$\delta_{1,g+c} = \frac{Q_i h_i^2}{12E}\left(\frac{1}{G} + \frac{1}{C}\right)$$

Type (B-1) is chosen as a beam for all story of the frame (E). As a result, G is calculated as follow;

$$G = \sum \frac{I_G}{L} = \frac{1.4662 \times 10 - 4}{4.9} + \frac{1.4662 \times 10 - 4}{4.9} = 5.9844 \times 10^{-5}$$

Columns (C_1), (C_2) are selected for this frame, so C calculated as follow;

$$C_{1,2} = \sum \frac{I_C}{h} = \frac{1.62786 \times 10 - 4}{3.3} + \frac{2.62253 \times 10 - 4}{3.3} + \frac{1.62786 \times 10 - 4}{3.3} = 0.000178$$

$$C_{3,4} = \sum \frac{I_C}{h} = \frac{9.232 \times 10 - 5}{3.6} + \frac{1.62786 \times 10 - 4}{3.6} + \frac{9.232 \times 10 - 5}{3.6} = 9.65072 \times 10^{-5}$$

$$C_{5,6} = \sum \frac{I_C}{h} = \frac{9.232 \times 10 - 5}{3.6} + \frac{1.62786 \times 10 - 4}{3.6} + \frac{9.232 \times 10 - 5}{3.6} = 9.65072 \times 10^{-5}$$

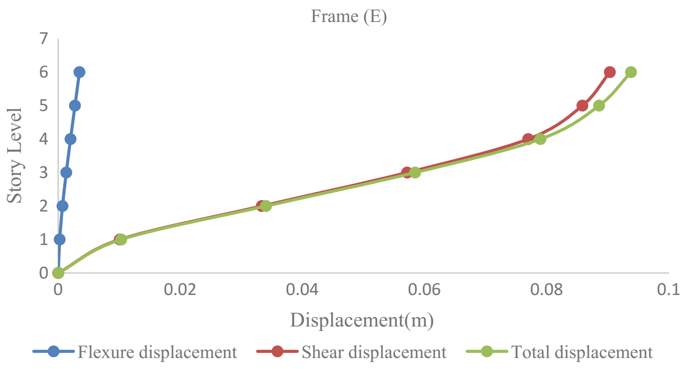
Table 1.74 shows the flexure displacement in the frame (E) and total displacement of frame (E) are calculated according to the Eq. (1.90) and is shown in Table 1.75. The displacement of story one is calculated based on Eq. (1.98) (Graph 1.1).

Table 1.74 Flexure displacement of frame (E)

Story	h (m)	E	V	G	C	δ	Δ
6	3.6	2E+08	30.74007	5.98E−05	9.65E−05	0.004494	0.090323
5	3.6	2E+08	60.72555	5.98E−05	9.65E−05	0.008877	0.085829
4	3.6	2E+08	135.6439	5.98E−05	9.65E−05	0.019829	0.076952
3	3.6	2E+08	162.7492	5.98E−05	9.65E−05	0.023792	0.057123
2	3.6	2E+08	188.5924	5.98E−05	0.000163	0.023256	0.033331
1	3.3	2E+08	198.2682	5.98E−05	0.000178	0.010075	0.010075

Table 1.75 Total displacement of frame (A)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.00347	0.090323	0.093793
5	0.002735	0.085829	0.088564
4	0.002007	0.076952	0.07896
3	0.001311	0.057123	0.058434
2	0.000695	0.033331	0.034026
1	0.000203	0.007853	0.008056



Graph 1.1 Displacement of frame (E)

$$\delta_{1,g+c} = \frac{Q_1 h_1^2}{12E} \frac{\left(\frac{2}{3G_1} + \frac{1}{C_1}\right)}{\left(1 + \frac{C_1}{6G_1}\right)} = \frac{198.2682(3.3^2)}{12 \times 2E+08} \frac{\left(\frac{2}{3\times5.98E-05} + \frac{1}{0.000178}\right)}{\left(1 + \frac{0.000178}{6\times5.98E-05}\right)} = 0.010075$$

Appendix

- Total weight of each story (Charts 1.22, 1.23, 1.24, 1.25, 1.26, 1.27, 1.28).
- Calculation of stiffness and center of stiffness in each story (Figs. 1.112, 1.114 and 1.116; Charts 1.28, 1.29, 1.30, 1.31, 1.32, 1.33, 1.34, 1.35, 1.36, 1.37, 1.38, 1.39, 1.40, 1.41, 1.42, 1.43, 1.44, 1.45, 1.46, 1.47, 1.48, 1.49, 1.50, 1.51, 1.52, 1.53, 1.54, 1.55, 1.56, 1.57, 1.58, 1.59, 1.60).

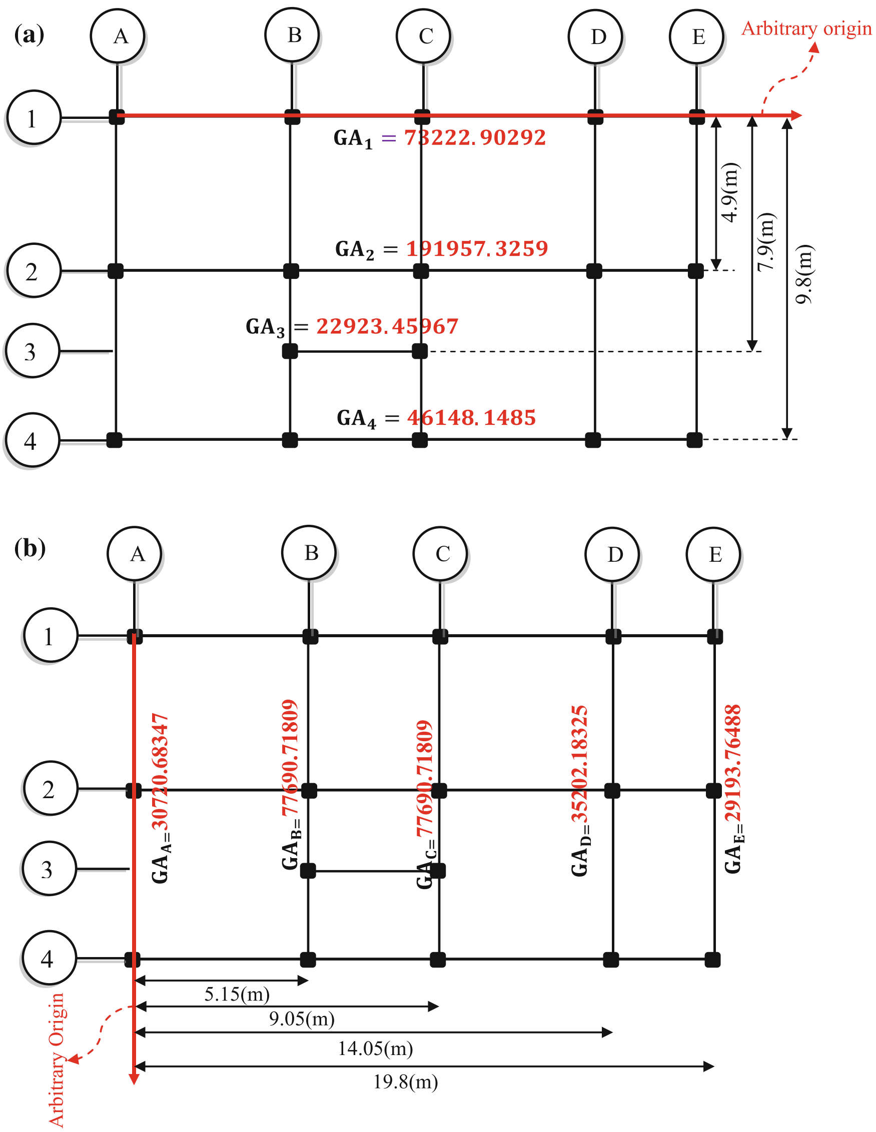


Fig. 1.112 **a** The shear rigidity in X direction and distance between bents and arbitrary origin. **b** Shear rigidity in Y direction and distance between bents and arbitrary origin

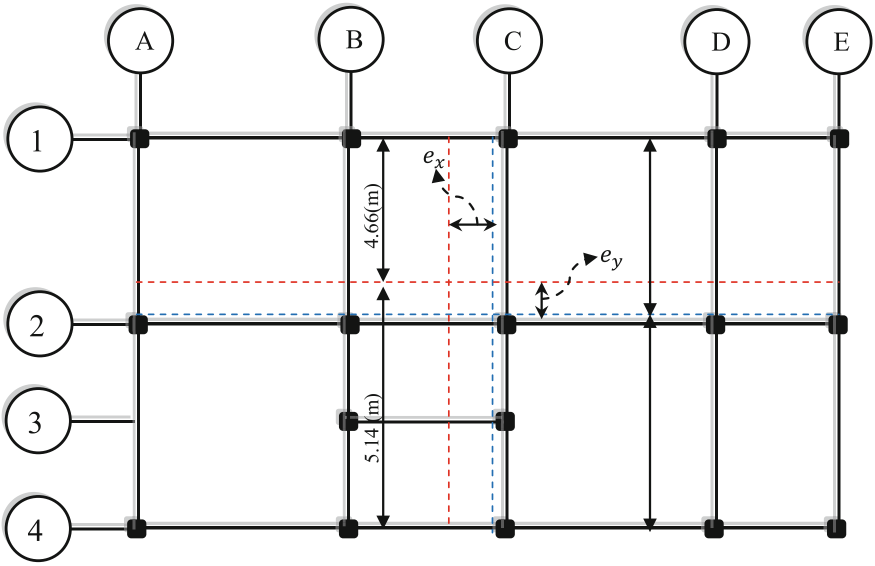


Fig. 1.113 Eccentricity in the second floor

- Calculation of Eccentricity in each story (Figs. 1.113, 1.115, 1.117, and 1.118; Charts 1.38, 1.39, 1.60 and 1.61).
- Calculation of Total displacement of frames (Figs. 1.119, 1.120, 1.121, 1.122, 1.123, 1.124, 1.125; Tables 1.76, 1.77, 1.78, 1.79, 1.80, 1.81, 1.82, 1.83, 1.84, 1.85, 1.86, 1.87, 1.88, 1.89, 1.90, 1.91, 1.92, 1.93, 1.94, 1.95 and 1.96; Graphs 1.2, 1.3, 1.4, 1.5, 1.6, 1.7 and 1.8).

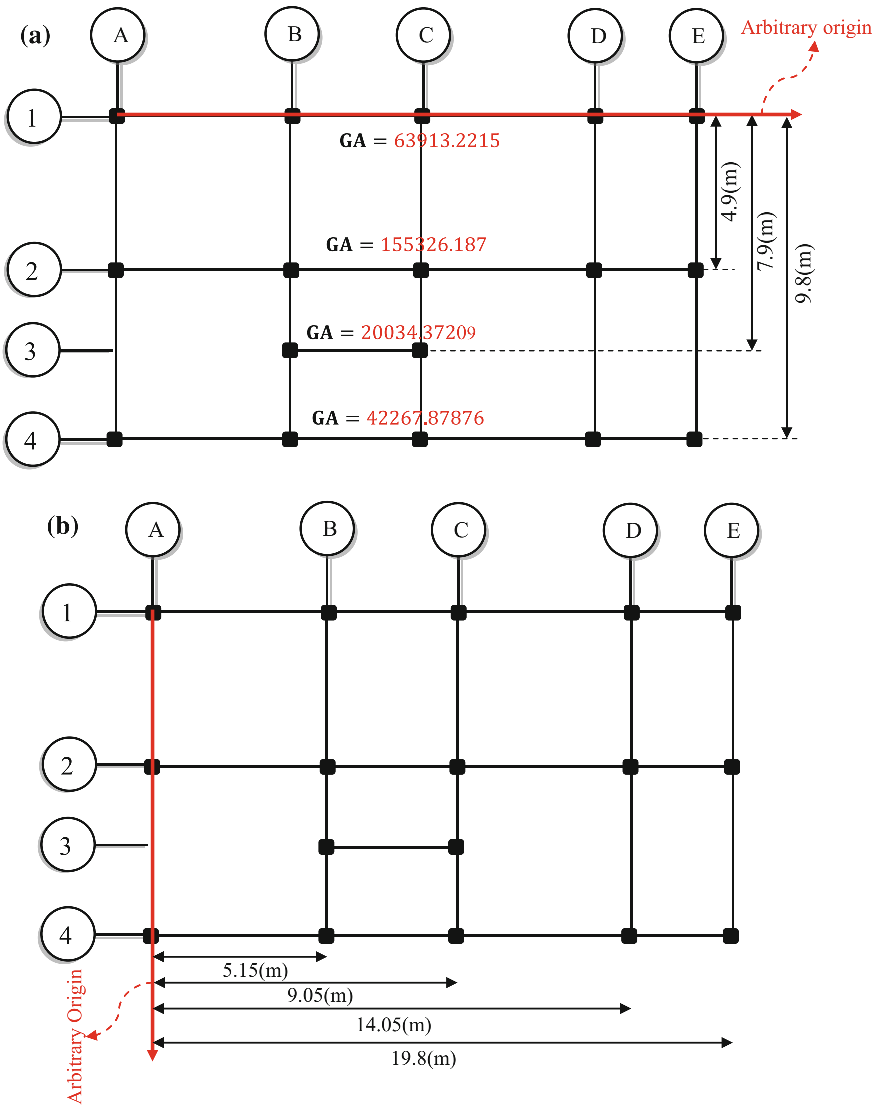


Fig. 1.114 **a** The shear rigidity in X direction and distance between bents and arbitrary origin. **b** Shear rigidity in Y direction and distance between bents and arbitrary origin

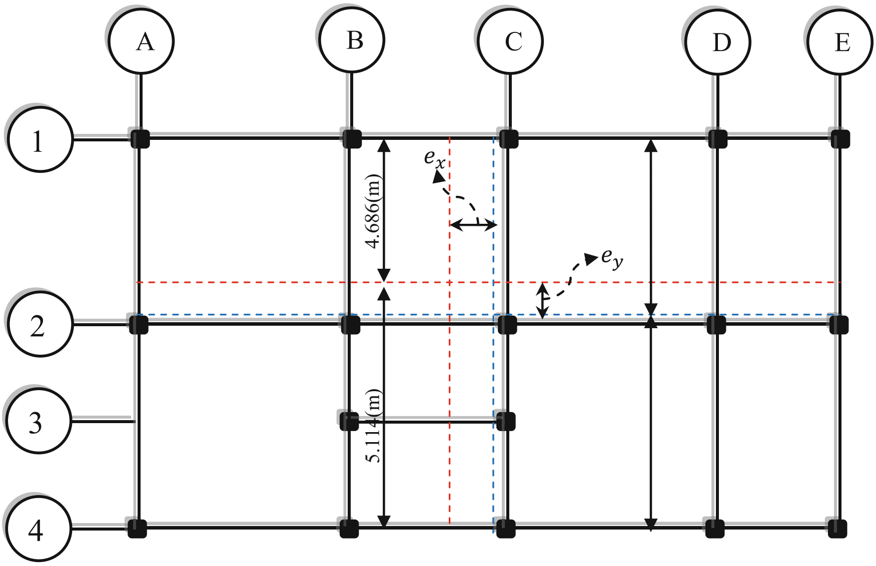


Fig. 1.115 Eccentricity In third and fourth floors

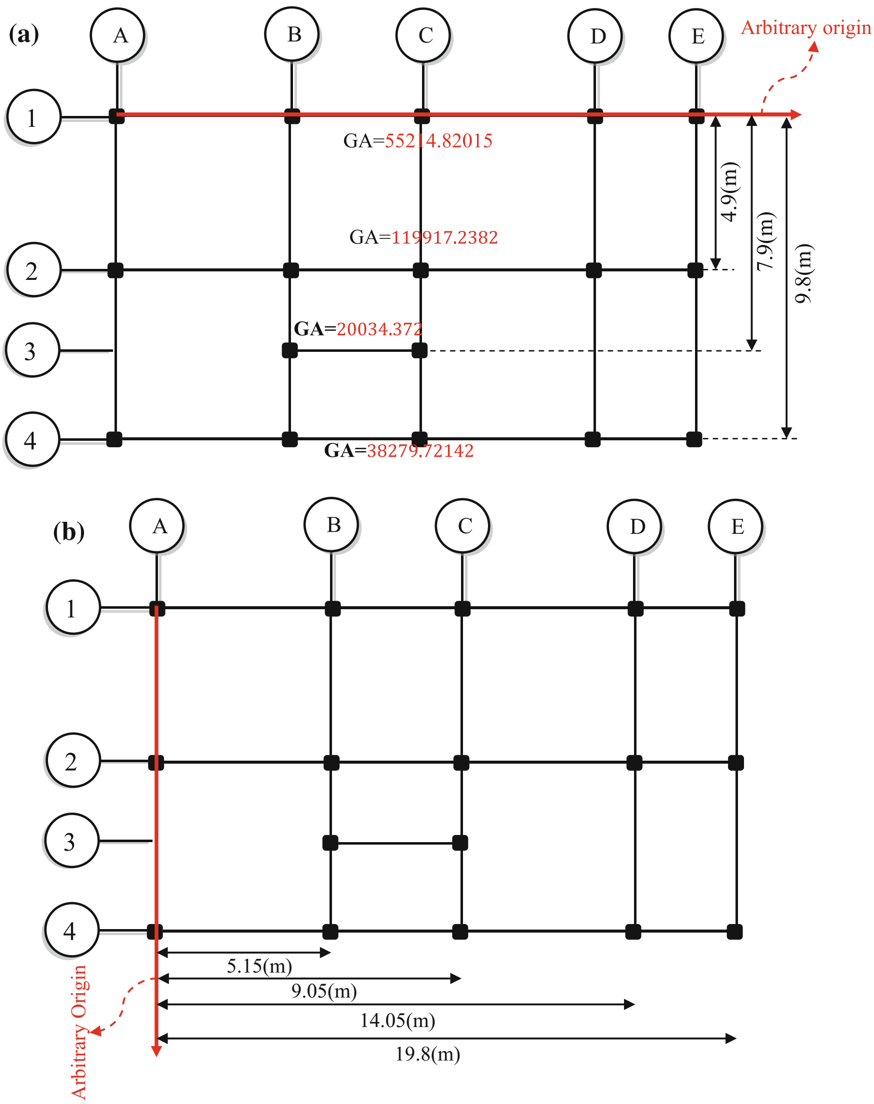


Fig. 1.116 **a** The shear rigidity in X direction and distance between bents and arbitrary origin. **b** Shear rigidity in Y direction and distance between bents and arbitrary origin

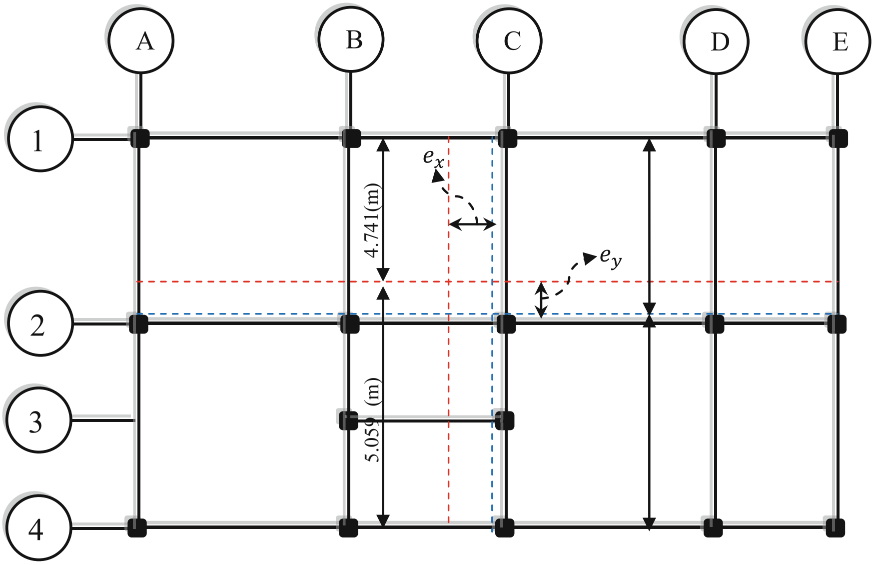


Fig. 1.117 Eccentricity in the fifth floor

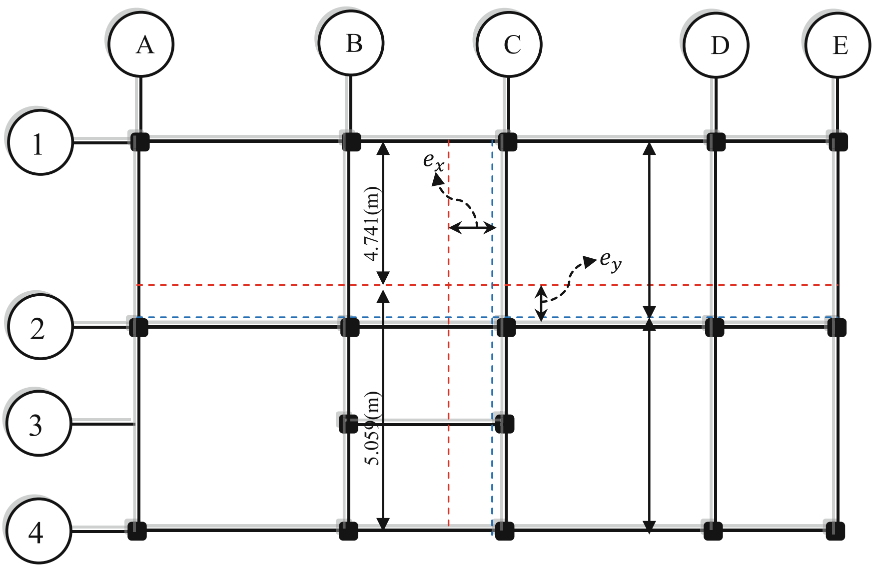


Fig. 1.118 Eccentricity in sixth floors

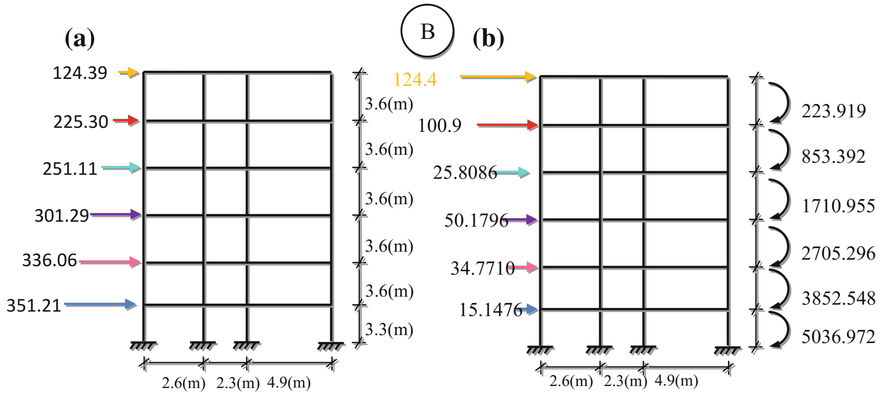


Fig. 1.119 **a** Shear force in frame (B), **b** force and moment in frame (B)

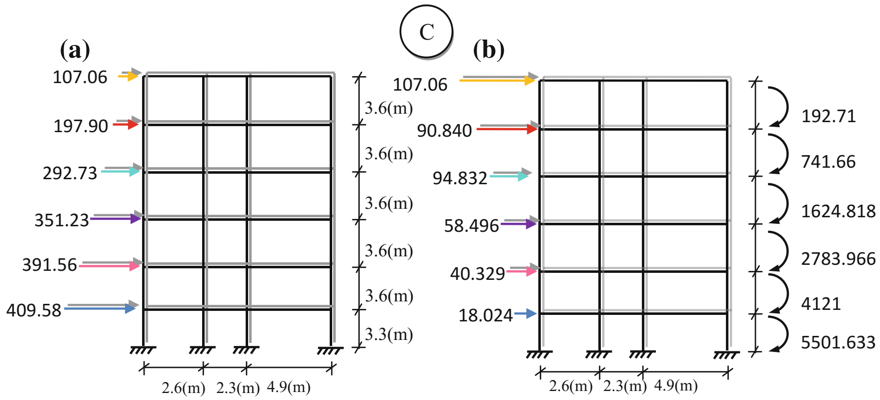


Fig. 1.120 **a** Shear force in frame (C), **b** force and moment in frame (C)

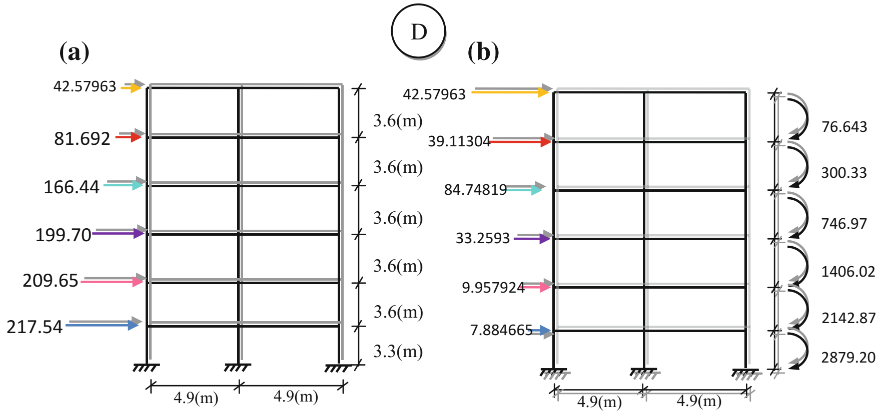


Fig. 1.121 **a** Shear force in frame (D), **b** force and moment in frame (D)

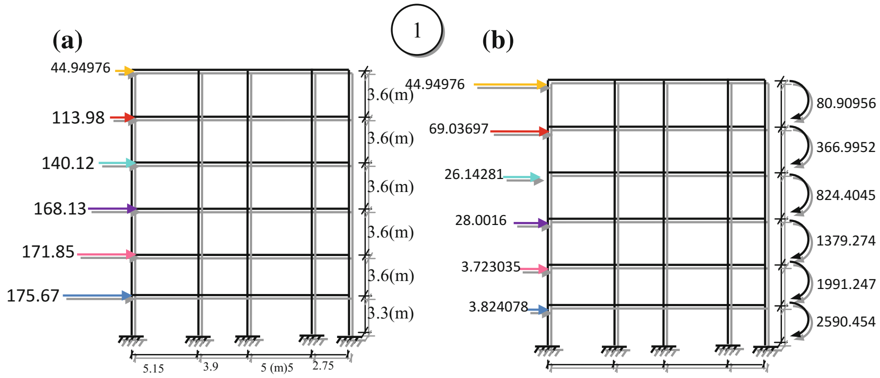


Fig. 1.122 **a** Shear force in frame (1), **b** force and moment in frame (1)

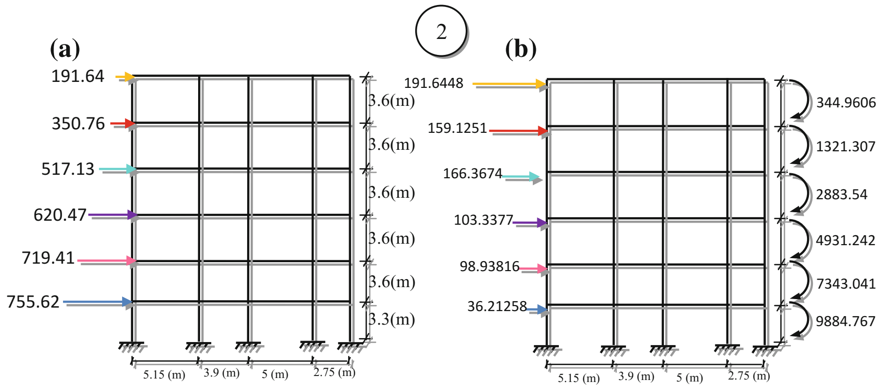


Fig. 1.123 **a** Shear force in frame (2), **b** force and moment in frame (2)

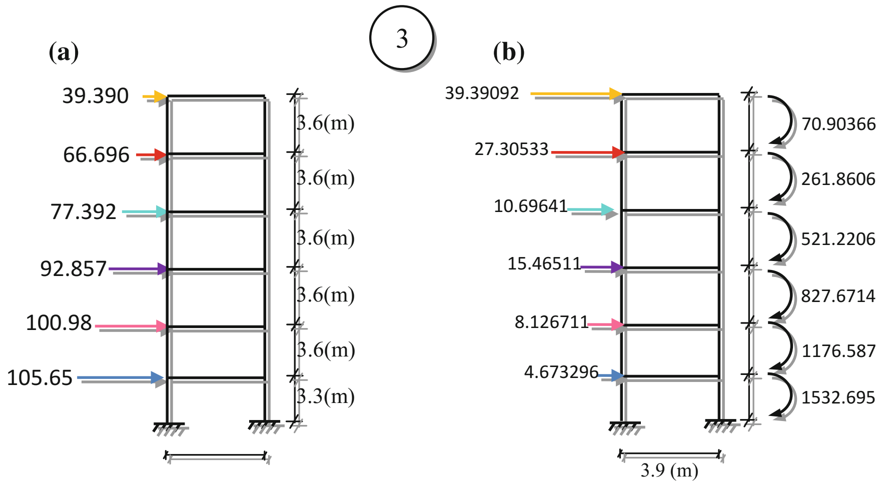


Fig. 1.124 **a** Shear force in frame (3), **b** force and moment in frame (3)

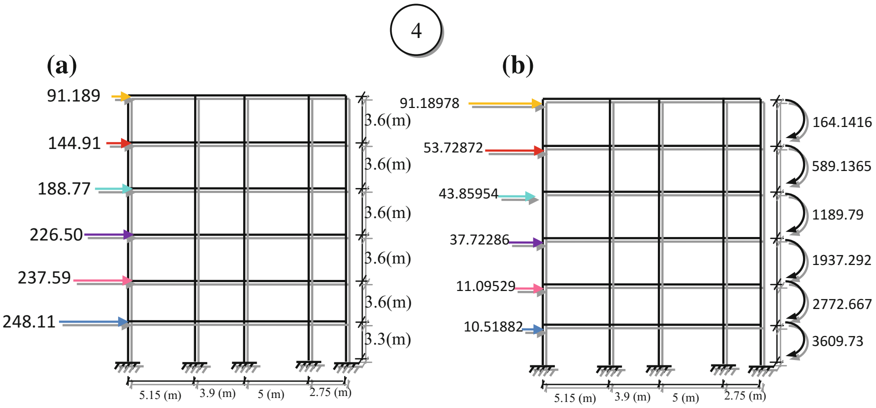


Fig. 1.125 **a** Shear force in frame (4), **b** force and moment in frame (4)

Table 1.76 Overall bending displacement of frame (B)

Story	h (m)	I_i (m^4)	E	Force	Moment	θ_i	θ_{if}	δ_{if}	Δ
6	3.6	0.60368	2E+08	124.399	223.9198	6.6766E−06	0.0003342	0.001203	0.005419
5	3.6	0.60368	2E+08	100.90	853.3928	2.5445E−05	0.0003275	0.001179	0.004215
4	3.6	0.70204	2E+08	25.8086	1710.955	4.3868E−05	0.0003020	0.001088	0.003036
3	3.6	0.70204	2E+08	50.1796	2705.296	6.9363E−05	0.0002582	0.00093	0.001949
2	3.6	0.80725	2E+08	34.7710	3852.548	8.5903E−05	0.0001888	0.00068	0.001019
1	3.3	0.80725	2E+08	15.1476	5036.972	0.00010295	0.0001029	0.00034	0.000339

Table 1.77 Flexure displacement of frame (B)

Story	h (m)	E	V	G	C	δ	Δ
6	3.6	2E+08	124.3999	0.00015	0.000209	0.007698	0.072154
5	3.6	2E+08	225.3073	0.00015	0.000209	0.013943	0.064456
4	3.6	2E+08	251.116	0.00015	0.000321	0.013263	0.050513
3	3.6	2E+08	301.2957	0.00015	0.000321	0.015913	0.03725
2	3.6	2E+08	336.0667	0.00015	0.000569	0.015283	0.021337
1	3.3	2E+08	351.2143	0.00015	0.000569	0.006054	0.006054

Table 1.78 Total displacement of frame (B)

Story	Overall bending	Flexure	Δ total
6	0.005419	0.072154	0.077573
5	0.004216	0.064456	0.068672
4	0.003037	0.050513	0.05355
3	0.001949	0.03725	0.039199
2	0.00102	0.021337	0.022356
1	0.00034	0.006054	0.006394

Table 1.79 Overall bending displacement of frame (C)

Story	h (m)	Ii (m^4)	E	Force	Moment	θi	θif	δif	Δ
6	3.6	0.60368	2E+08	107.06	192.7167	5.7462E−06	0.00034	0.001243	0.00570
5	3.6	0.60368	2E+08	90.840	741.6621	2.2114E−05	0.00033	0.001222	0.00446
4	3.6	0.70204	2E+08	94.832	1624.818	4.166E−05	0.00031	0.001143	0.00324
3	3.6	0.70204	2E+08	58.496	2783.966	7.138E−05	0.00027	0.000993	0.00209
2	3.6	0.80725	2E+08	40.329	4121	9.1889E−05	0.00020434	0.000736	0.00110
1	3.3	0.80725	2E+08	18.024	5501.633	0.00011245	0.00011245	0.000371	0.00037

Table 1.80 Flexure displacement of frame (C)

Story	h (m)	E	V	G	C	δ	Δ
6	3.6	2E+08	107.0648	0.00015	0.000209	0.006626	0.077751
5	3.6	2E+08	197.9049	0.00015	0.000209	0.012247	0.071126
4	3.6	2E+08	292.7372	0.00015	0.000321	0.015461	0.058879
3	3.6	2E+08	351.2338	0.00015	0.000321	0.018551	0.043418
2	3.6	2E+08	391.5631	0.00015	0.000569	0.017806	0.024867
1	3.3	2E+08	409.5876	0.00015	0.000569	0.007061	0.007061

Table 1.81 Total displacement of frame (C)

Story	Overall bending	Flexure	Δ total
6	0.005707	0.077751	0.083458
5	0.004464	0.071126	0.07559
4	0.003242	0.058879	0.062121
3	0.002099	0.043418	0.045517
2	0.001107	0.024867	0.025974
1	0.000371	0.007061	0.007432

Table 1.82 Overall bending displacement of frame (D)

Story	h (m)	Ii (m^4)	E	Force	Moment	θ_i	θ_{if}	δ_{if}	Δ
6	3.6	0.557032	2E+08	42.57963	76.643	2.4767E−06	0.00018643	0.000671	0.003136799
5	3.6	0.557032	2E+08	39.11304	300.33	9.705E−06	0.000183954	0.000662	0.00246565
4	3.6	0.653072	2E+08	84.74819	746.97	2.0588E−05	0.000174249	0.000627	0.001803417
3	3.6	0.653072	2E+08	33.2593	1406.02	3.8753E−05	0.00015366	0.000553	0.001176122
2	3.6	0.749112	2E+08	9.957924	2142.87	5.149E−05	0.000114907	0.000414	0.000622945
1	3.3	0.749112	2E+08	7.884665	2879.20	6.3418E−05	6.34175E−05	0.000209	0.000209278

Table 1.83 Flexure displacement of frame (D)

story	h (m)	E	V	G	C	δ	Δ
6	3.6	2E+08	42.57963	5.98E−05	0.000163	0.00525	0.082841
5	3.6	2E+08	81.69267	5.98E−05	0.000163	0.010073	0.077591
4	3.6	2E+08	166.4409	5.98E−05	0.000276	0.018279	0.067518
3	3.6	2E+08	199.7002	5.98E−05	0.000276	0.021932	0.049238
2	3.6	2E+08	209.6581	5.98E−05	0.000449	0.021441	0.027306
1	3.3	2E+08	217.5427	5.98E−05	0.000449	0.005865	0.005865

Table 1.84 Total displacement of frame (D)

Story	Overall bending	Flexure	Δ total
6	0.003137	0.082841	0.085978
5	0.002466	0.077591	0.080057
4	0.001803	0.067518	0.069321
3	0.001176	0.049238	0.050415
2	0.000623	0.027306	0.027929
1	0.000209	0.005865	0.006075

Table 1.85 Overall bending displacement of frame (1)

Story	h (m)	Ii (m^4)	E	Force	Moment	θi	θif	δif	Δ
6	3.6	1.8292	2E+08	44.94976	80.90956	7.9618E−07	5.97408E−05	0.000215	0.0009841
5	3.6	1.8292	2E+08	69.03697	366.9952	3.6114E−06	5.89447E−05	0.000212	0.0007690
4	3.6	1.9093	2E+08	26.14281	824.4045	7.7721E−06	5.53333E−05	0.000199	0.0005568
3	3.6	1.9093	2E+08	28.0016	1379.274	1.3003E−05	4.75612E−05	0.000171	0.0003576
2	3.6	2.274	2E+08	3.723035	1991.247	1.5762E−05	3.4558E−05	0.000124	0.0001864
1	3.3	2.274	2E+08	3.824078	2590.454	1.8796E−05	1.87962E−05	6.2E−05	6.20274E−05

Table 1.86 Flexure displacement of frame (1)

Story	h (m)	E	V	G	C	δ	Δ
6	3.6	2E+08	44.94976	0.000149	0.000187	0.002931	0.039685
5	3.6	2E+08	113.9867	0.000149	0.000187	0.007432	0.036754
4	3.6	2E+08	140.1295	0.000149	0.00027	0.007893	0.029322
3	3.6	2E+08	168.1311	0.000149	0.00027	0.00947	0.021429
2	3.6	2E+08	171.8542	0.000149	0.00042	0.008449	0.011959
1	3.3	2E+08	175.6782	0.000149	0.000458	0.00351	0.00351

Table 1.87 Total displacement of frame (1)

Story	Overall bending	Flexure	Δ total
6	0.000984	0.039685	0.040669
5	0.000769	0.036754	0.037523
4	0.000557	0.029322	0.029879
3	0.000358	0.021429	0.021787
2	0.000186	0.011959	0.012145
1	6.2E−05	0.00351	0.003572

Table 1.88 Overall bending displacement of frame (2)

Story	h (m)	Ii (m^4)	E	Force	Moment	θi	θif	δif	Δ
6	3.6	2.193649	2E+08	191.6448	344.9606	2.8306E−06	0.000171727	0.000618	0.002788419
5	3.6	2.193649	2E+08	159.1251	1321.307	1.0842E−05	0.000168896	0.000608	0.002170203
4	3.6	2.193649	2E+08	166.3674	2883.54	2.3661E−05	0.000158054	0.000569	0.001562176
3	3.6	2.193649	2E+08	103.3377	4931.242	4.0463E−05	0.000134393	0.000484	0.000993181
2	3.6	3.143545	2E+08	98.93816	7343.041	4.2046E−05	9.39301E−05	0.000338	0.000509364
1	3.3	3.143545	2E+08	36.21258	9884.767	5.1884E−05	5.18837E−05	0.000171	0.000171216

Table 1.89 Flexure displacement of frame (2)

Story	h (m)	E	V	G	C	δ	Δ
6	3.6	2E+08	191.6448	0.00043	0.000309	0.005753	0.062669
5	3.6	2E+08	350.7699	0.00043	0.000309	0.01053	0.056916
4	3.6	2E+08	517.1373	0.00043	0.000508	0.011986	0.046386
3	3.6	2E+08	620.475	0.00043	0.000508	0.014381	0.0344
2	3.6	2E+08	719.4132	0.00043	0.00087	0.013492	0.020019
1	3.3	2E+08	755.6257	0.00043	0.000949	0.006527	0.006527

Table 1.90 Total displacement of frame (2)

Story	Overall bending	Flexure	Δ total
6	0.002788	0.062669	0.065458
5	0.00217	0.056916	0.059086
4	0.001562	0.046386	0.047948
3	0.000993	0.0344	0.035393
2	0.000509	0.020019	0.020529
1	0.000171	0.006527	0.006699

Table 1.91 Overall bending displacement of frame (3)

Story	h (m)	Ii (m^4)	E	Force	Moment	θi	θif	δif	Δ
6	3.6	0.088218	2E+08	39.39092	70.90366	1.4467E−05	0.000792404	0.002853	0.012911116
5	3.6	0.088218	2E+08	27.30533	261.8606	5.343E−05	0.000777937	0.002801	0.010058461
4	3.6	0.088218	2E+08	10.69641	521.2206	0.00010635	0.000724507	0.002608	0.007257888
3	3.6	0.088218	2E+08	15.46511	827.6714	0.00016888	0.000618157	0.002225	0.004649663
2	3.6	0.103428	2E+08	8.126711	1176.587	0.00020477	0.000449279	0.001617	0.002424297
1	3.3	0.103428	2E+08	4.673296	1532.695	0.00024451	0.000244513	0.000807	0.000806892

Table 1.92 Flexure displacement of frame (3)

Story	h (m)	E	V	G	C	δ	Δ
6	3.6	2.00E+08	39.39092	4.5E−05	9.04E−05	0.007078	0.071884
5	3.6	2.00E+08	66.69625	4.5E−05	9.04E−05	0.011985	0.064805
4	3.6	2.00E+08	77.39266	4.5E−05	9.04E−05	0.013907	0.052821
3	3.6	2.00E+08	92.85777	4.5E−05	9.04E−05	0.016686	0.038914
2	3.6	2.00E+08	100.9845	4.5E−05	0.000146	0.015859	0.022228
1	3.3	2.00E+08	105.6578	4.5E−05	0.000159	0.006369	0.006369

Table 1.93 Total displacement of frame (B)

Story	Overall bending	Flexure	Δ total
6	0.012911	0.071884	0.084795
5	0.010058	0.064805	0.074864
4	0.007258	0.052821	0.060079
3	0.00465	0.038914	0.043564
2	0.002424	0.022228	0.024652
1	0.000807	0.006369	0.007176

Table 1.94 Overall bending displacement of frame (4)

Story	h (m)	Ii (m^4)	E	Force	Moment	θi	θif	δif	Δ
6	3.6	2.193649	2E+08	91.18978	164.1416	1.3469E−06	6.66636E−05	0.00024	0.0010633
5	3.6	2.193649	2E+08	53.72872	589.1365	4.8342E−06	6.53167E−05	0.000235	0.0008233
4	3.6	2.193649	2E+08	43.85954	1189.79	9.7628E−06	6.04826E−05	0.000218	0.0005882
3	3.6	2.193649	2E+08	37.72286	1937.292	1.5896E−05	5.07197E−05	0.000183	0.0003704
2	3.6	3.143545	2E+08	11.09529	2772.667	1.5876E−05	3.48233E−05	0.000125	0.0001878
1	3.3	3.143545	2E+08	10.51882	3609.73	1.8947E−05	1.89469E−05	6.25E−05	6.25249E−05

Table 1.95 Shear displacement frame (4)

Story	h (m)	E	V	G	C	δ	Δ
6	3.6	2.00E+08	91.18978	8.29E−05	0.000187	0.008576	0.082099
5	3.6	2.00E+08	144.9185	8.29E−05	0.000187	0.013629	0.073523
4	3.6	2.00E+08	188.778	8.29E−05	0.00027	0.016078	0.059895
3	3.6	2.00E+08	226.5009	8.29E−05	0.00027	0.019291	0.043816
2	3.6	2.00E+08	237.5962	8.29E−05	0.00042	0.018535	0.024525
1	3.3	2.00E+08	248.115	8.29E−05	0.000458	0.00599	0.00599

Table 1.96 Total displacement of frame (4)

Story	Overall bending	Flexure	Δ total
6	0.001063	0.082099	0.083163
5	0.000823	0.073523	0.074347
4	0.000588	0.059895	0.060483
3	0.00037	0.043816	0.044187
2	0.000188	0.024525	0.024713
1	6.25E−05	0.00599	0.006053

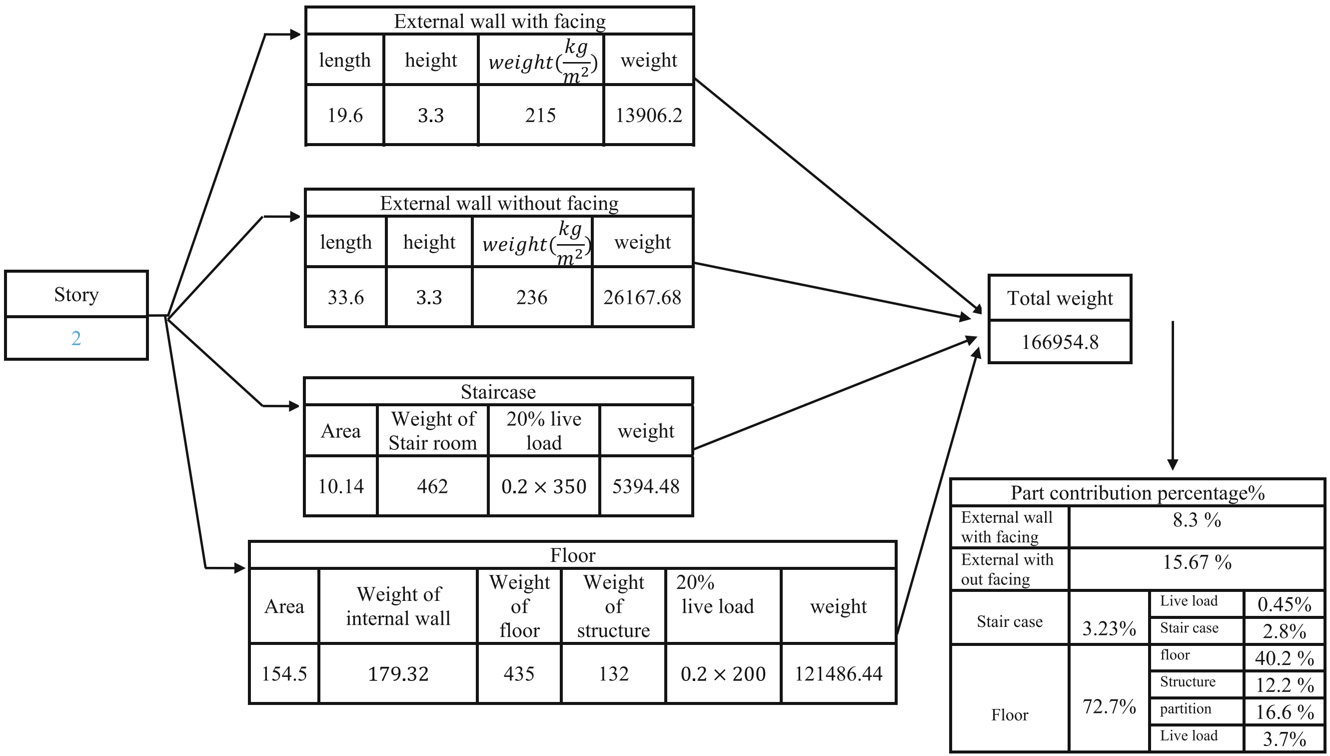


Chart 1.22 Total weight of story two

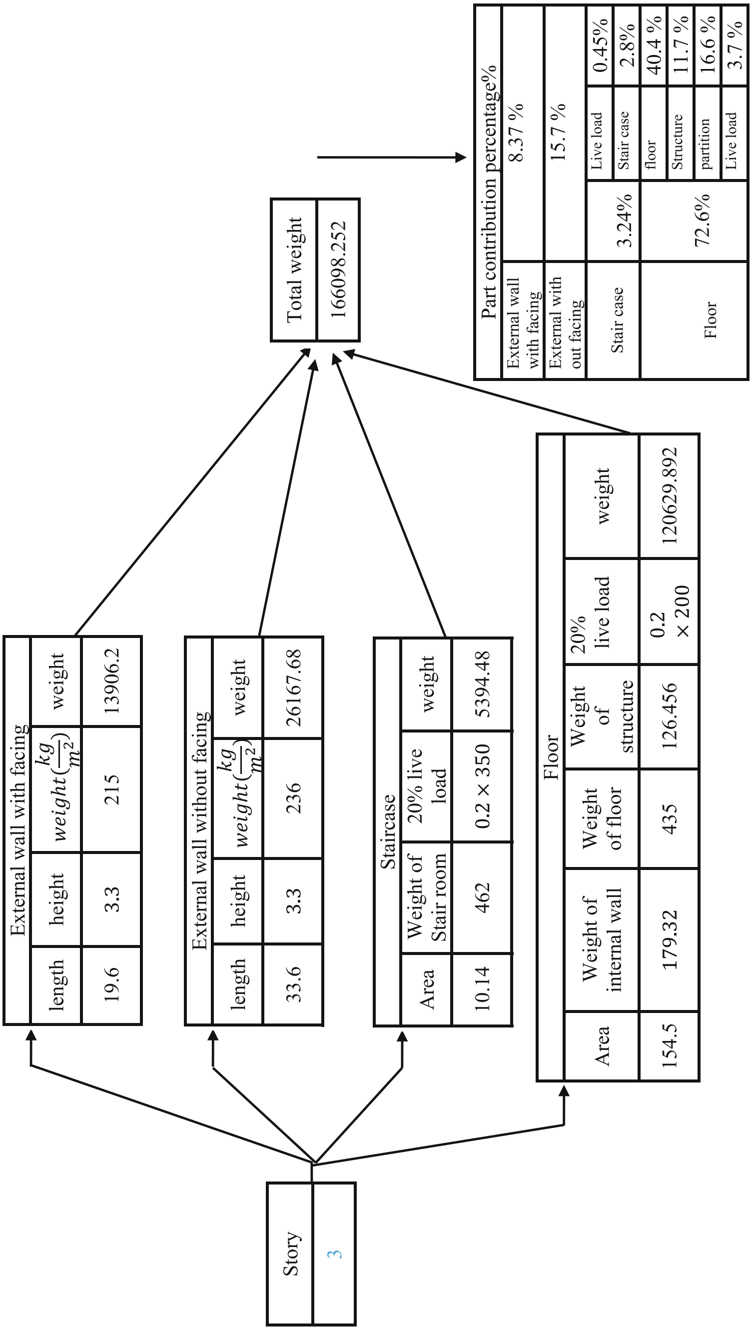


Chart 1.23 Total weight of story three

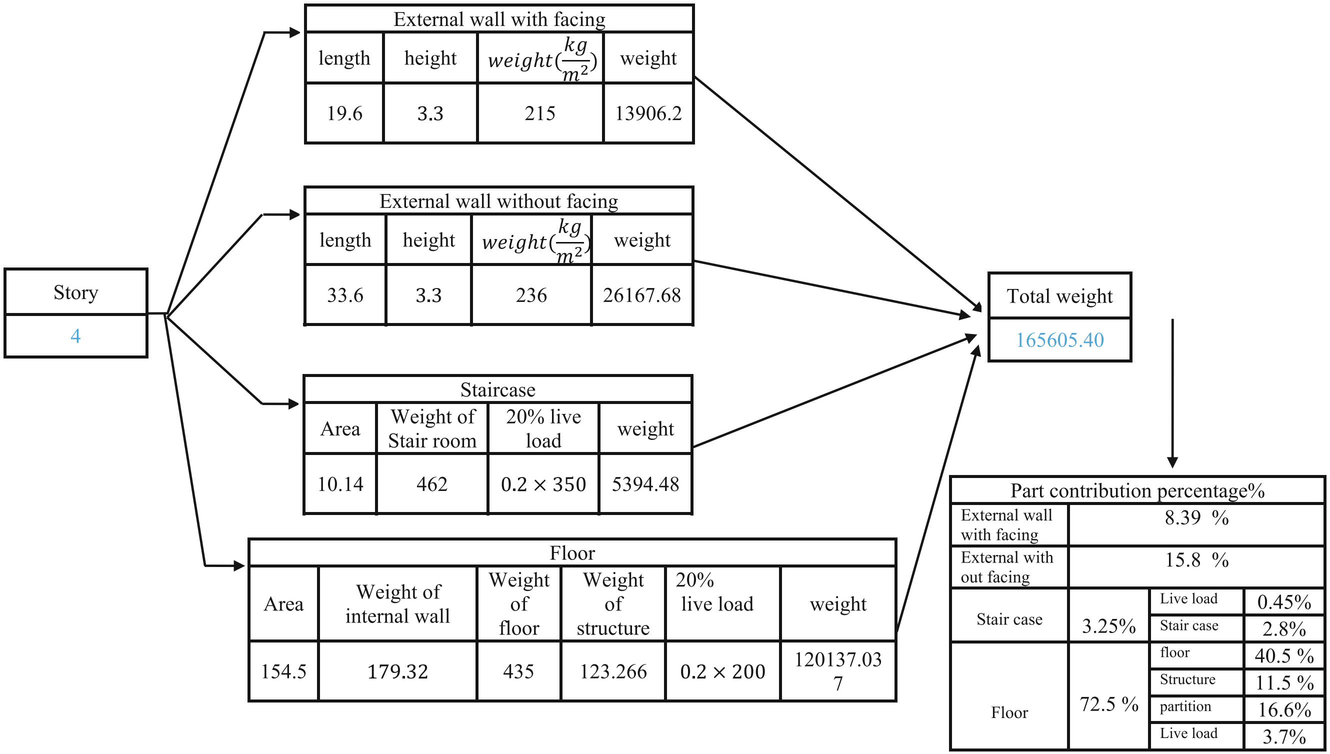


Chart 1.24 Total weight of story four in detail

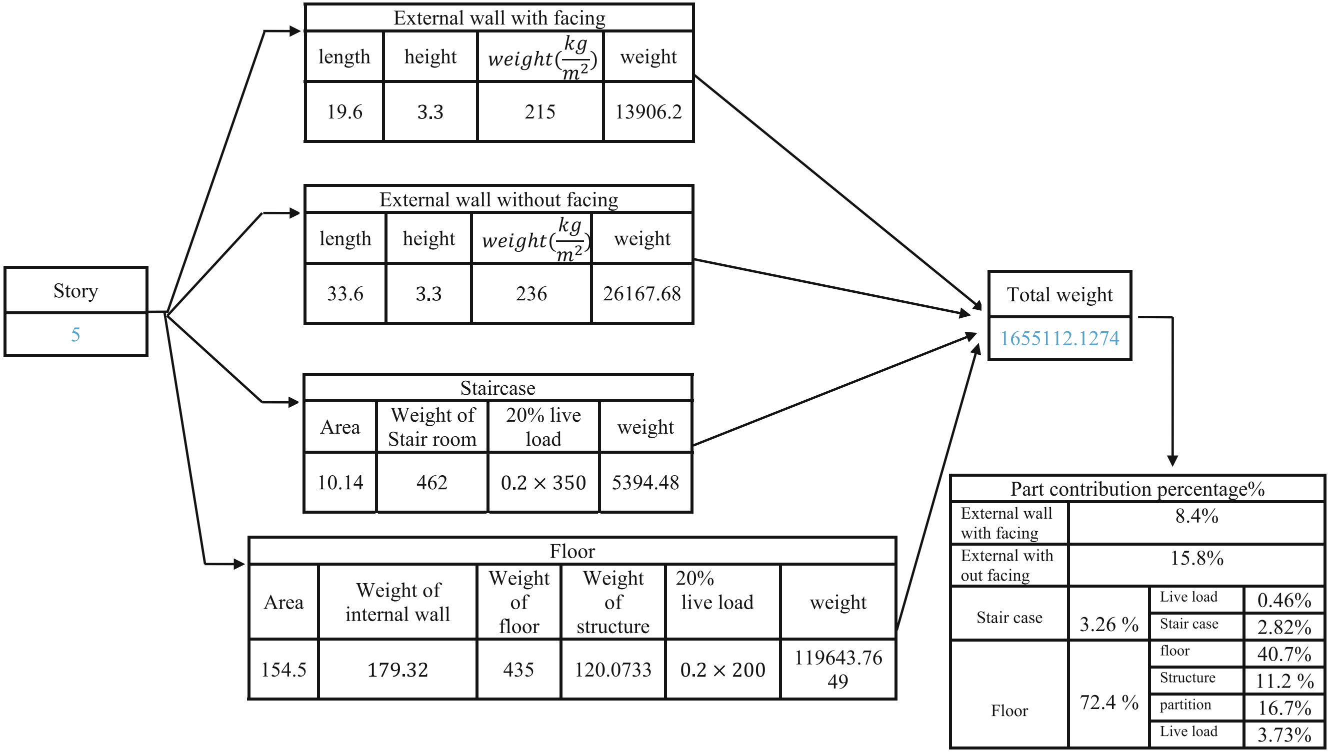


Chart 1.25 Total weight of story five

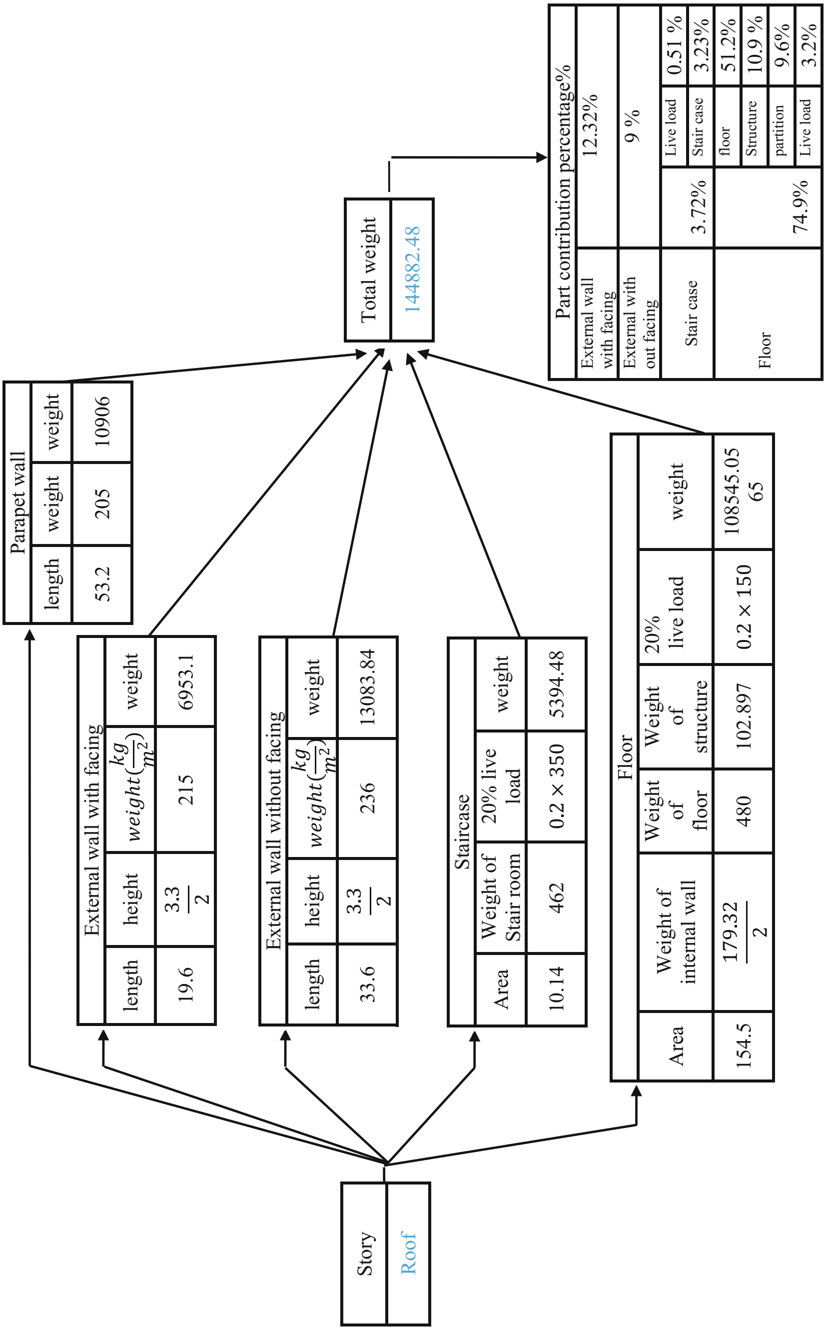


Chart 1.26 Total weight of story six

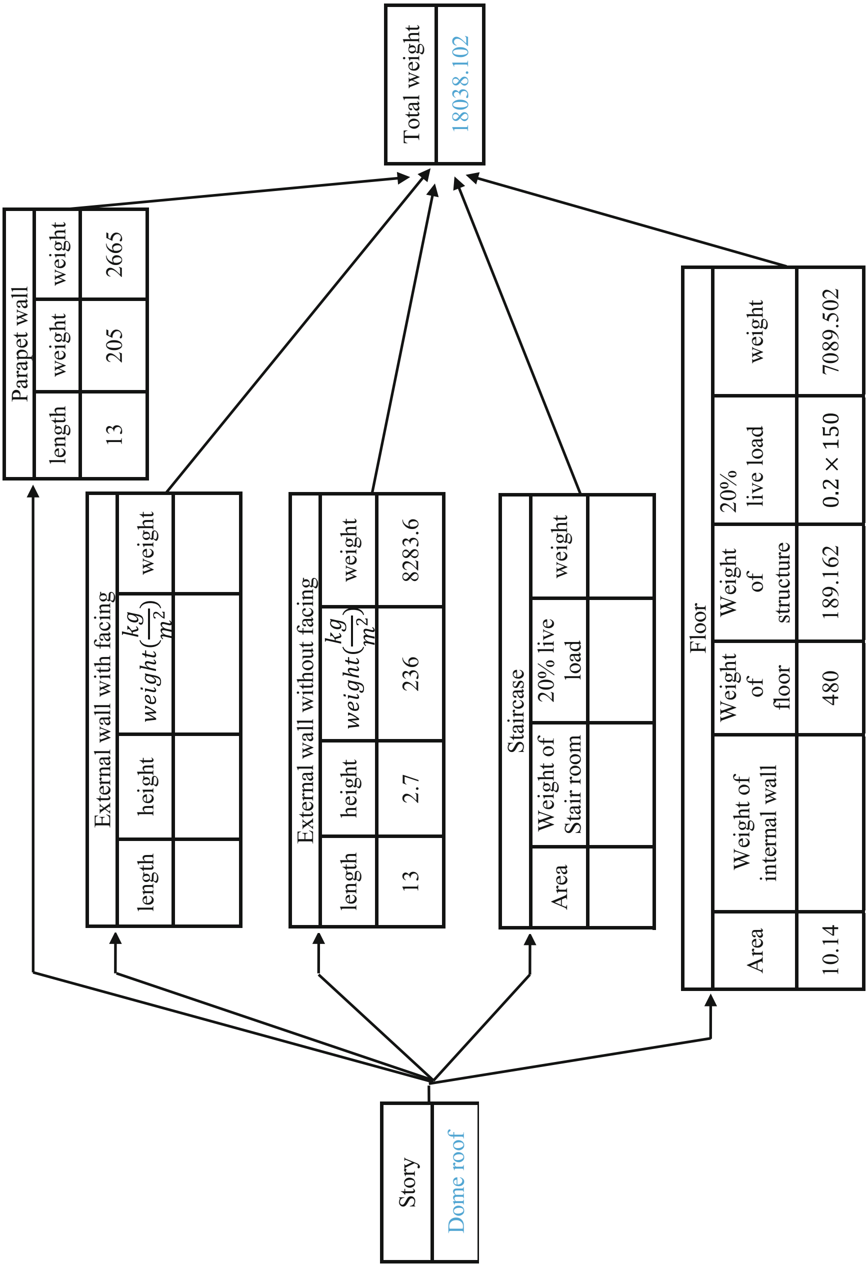


Chart 1.27 Total weight of dome roof

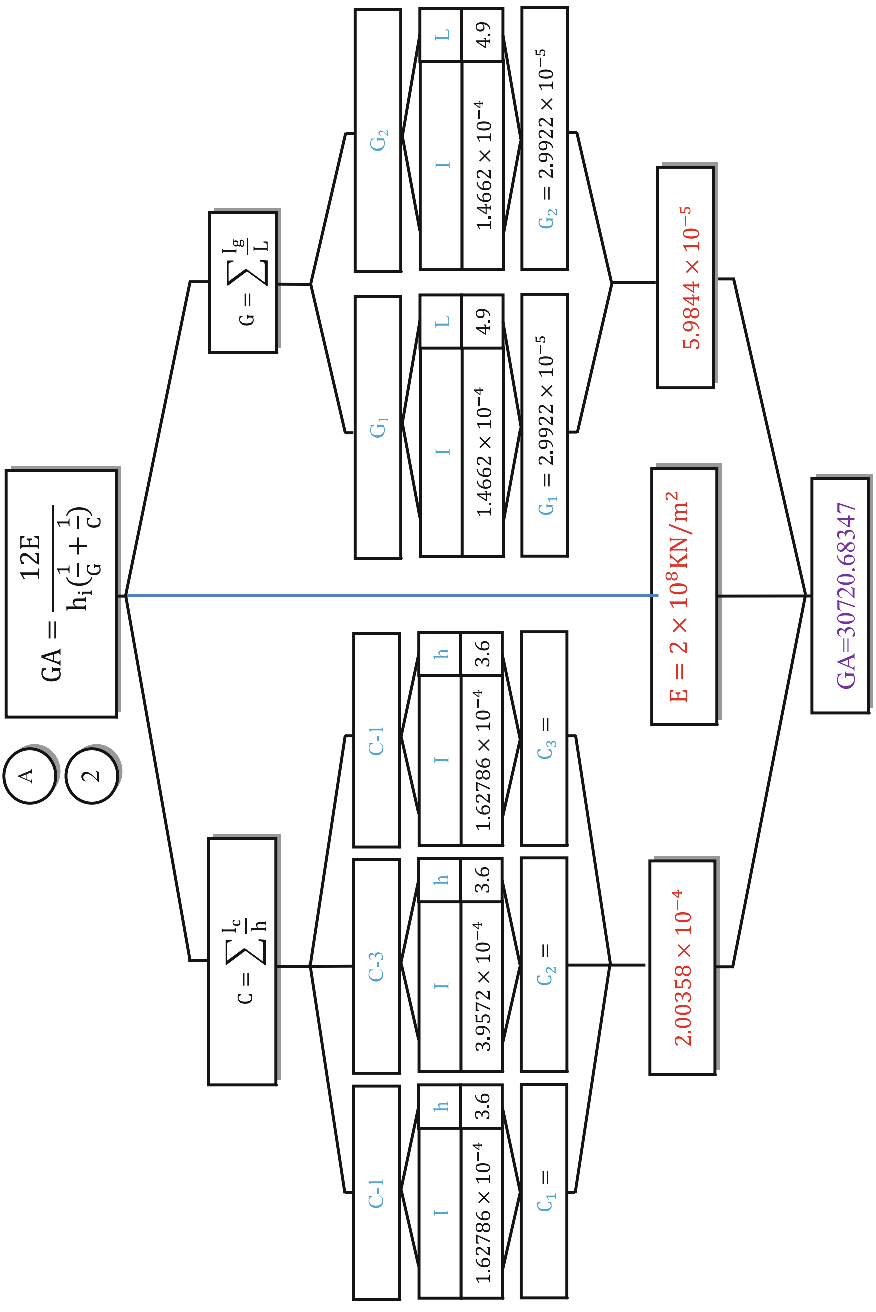


Chart 1.28 Shear rigidity of bent (A) story (2)

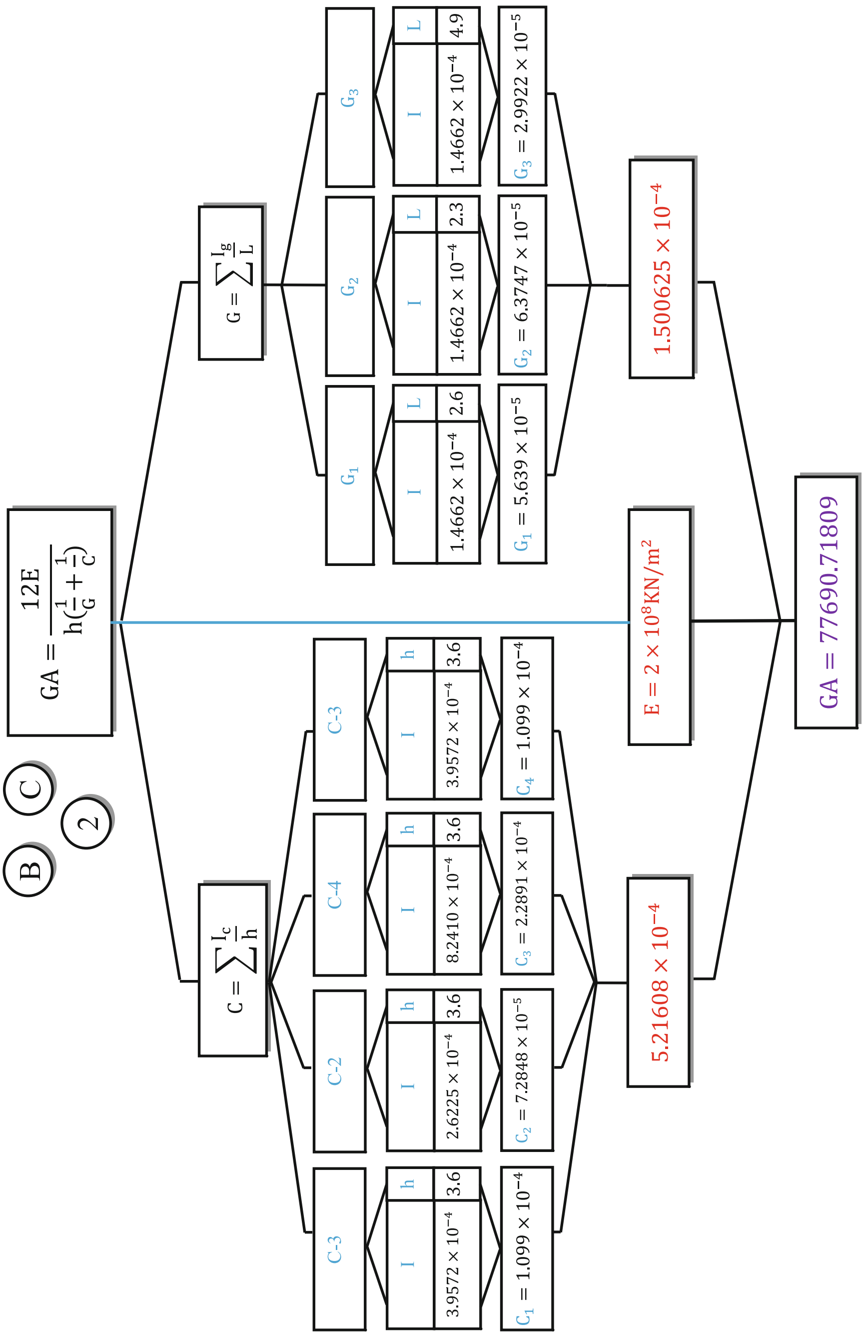


Chart 1.29 Shear rigidity of bent (B) and (C) story (2)

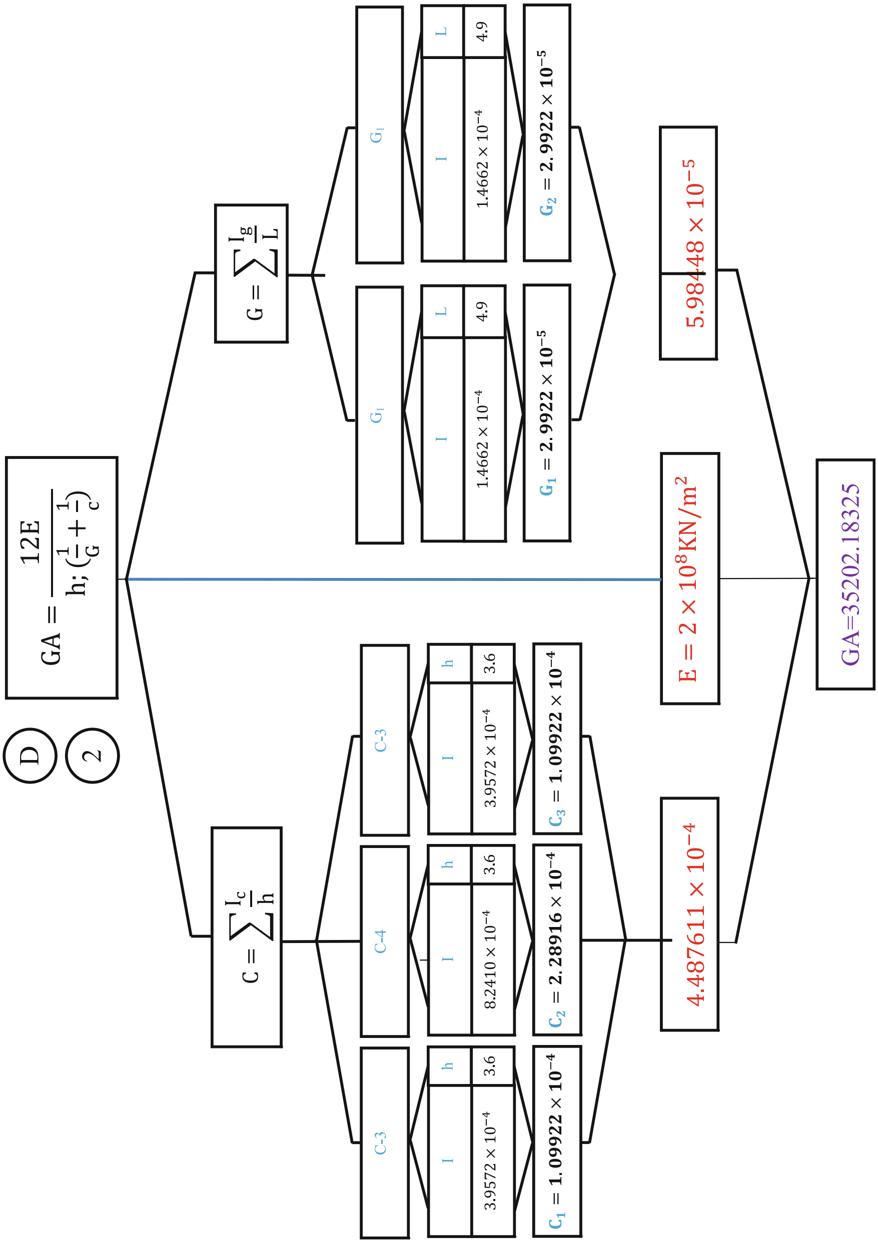


Chart 1.30 Shear rigidity of bent (D) story (2)

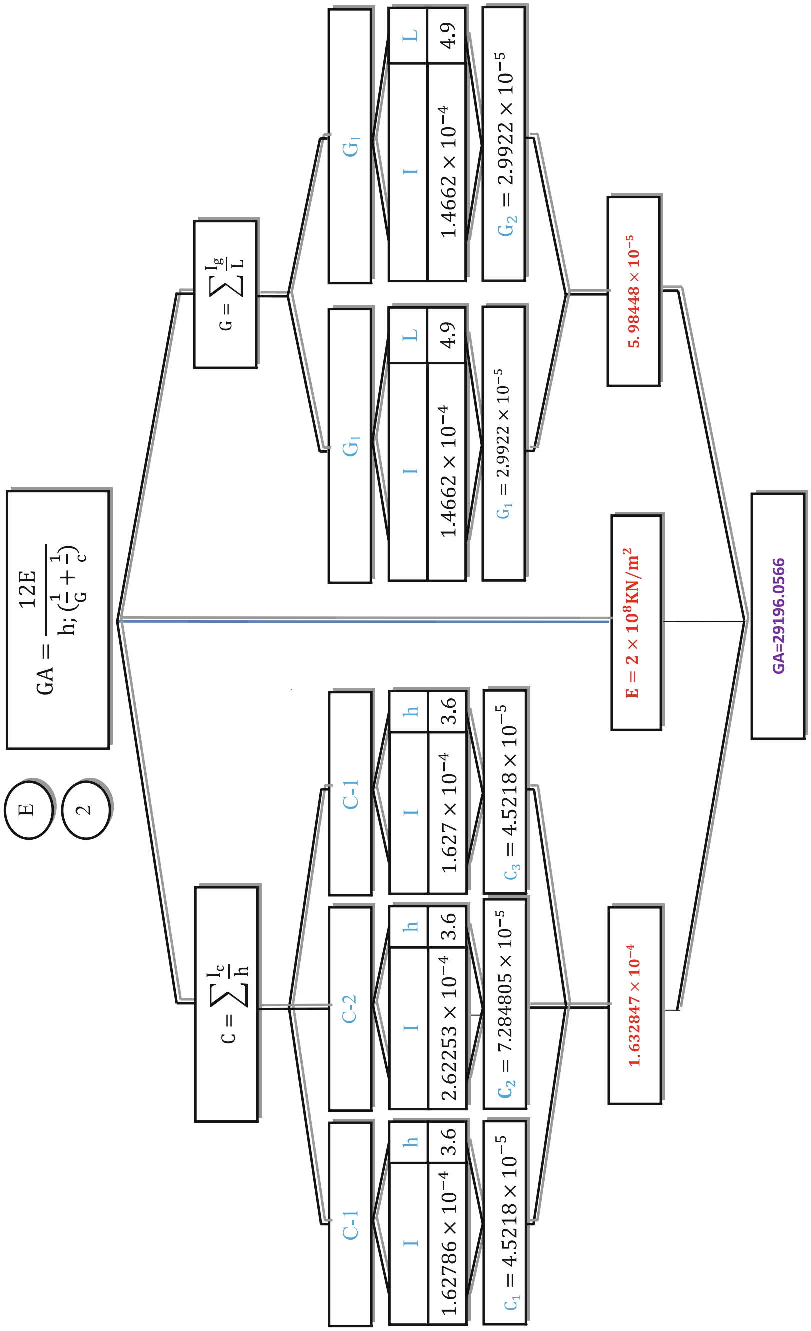


Chart 1.31 Shear rigidity of bent (E) story (2)

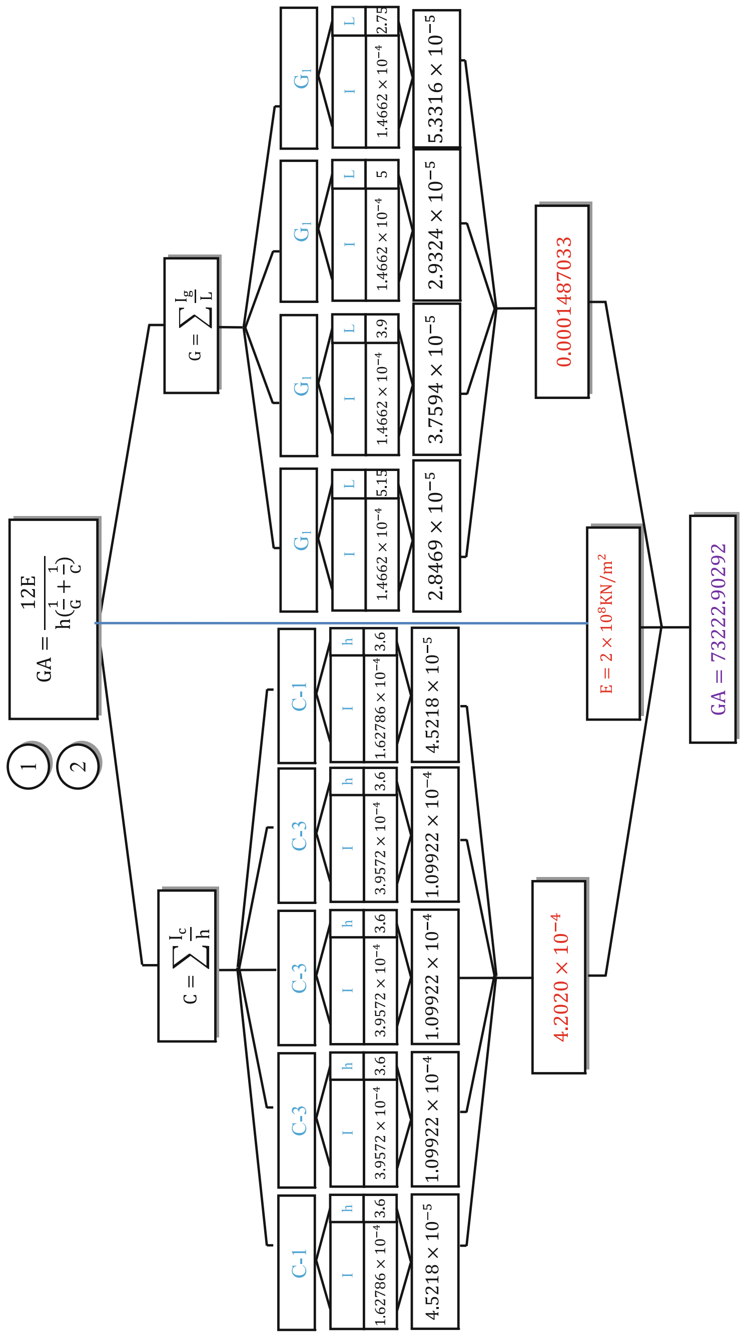


Chart 1.32 Shear rigidity of bent (1) story (2)

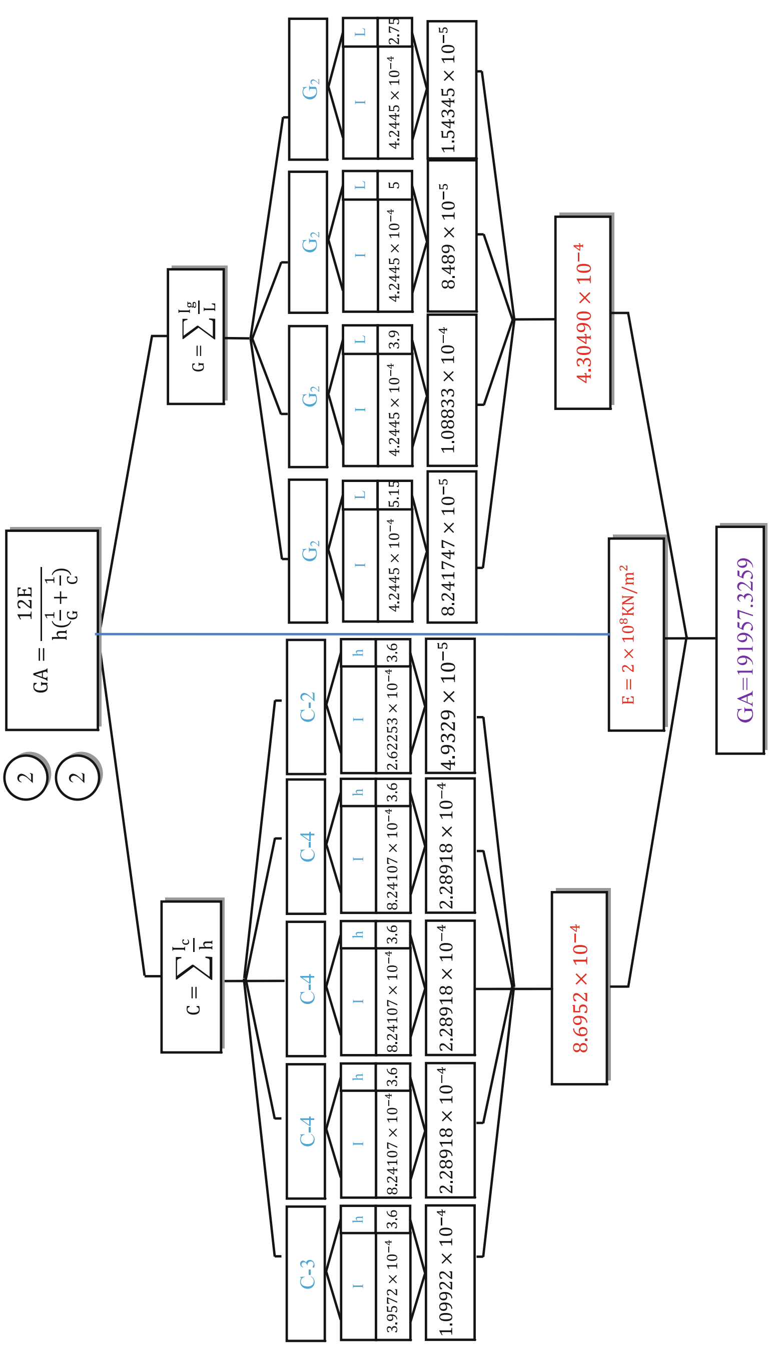


Chart 1.33 Shear rigidity of bent (2) story (2)

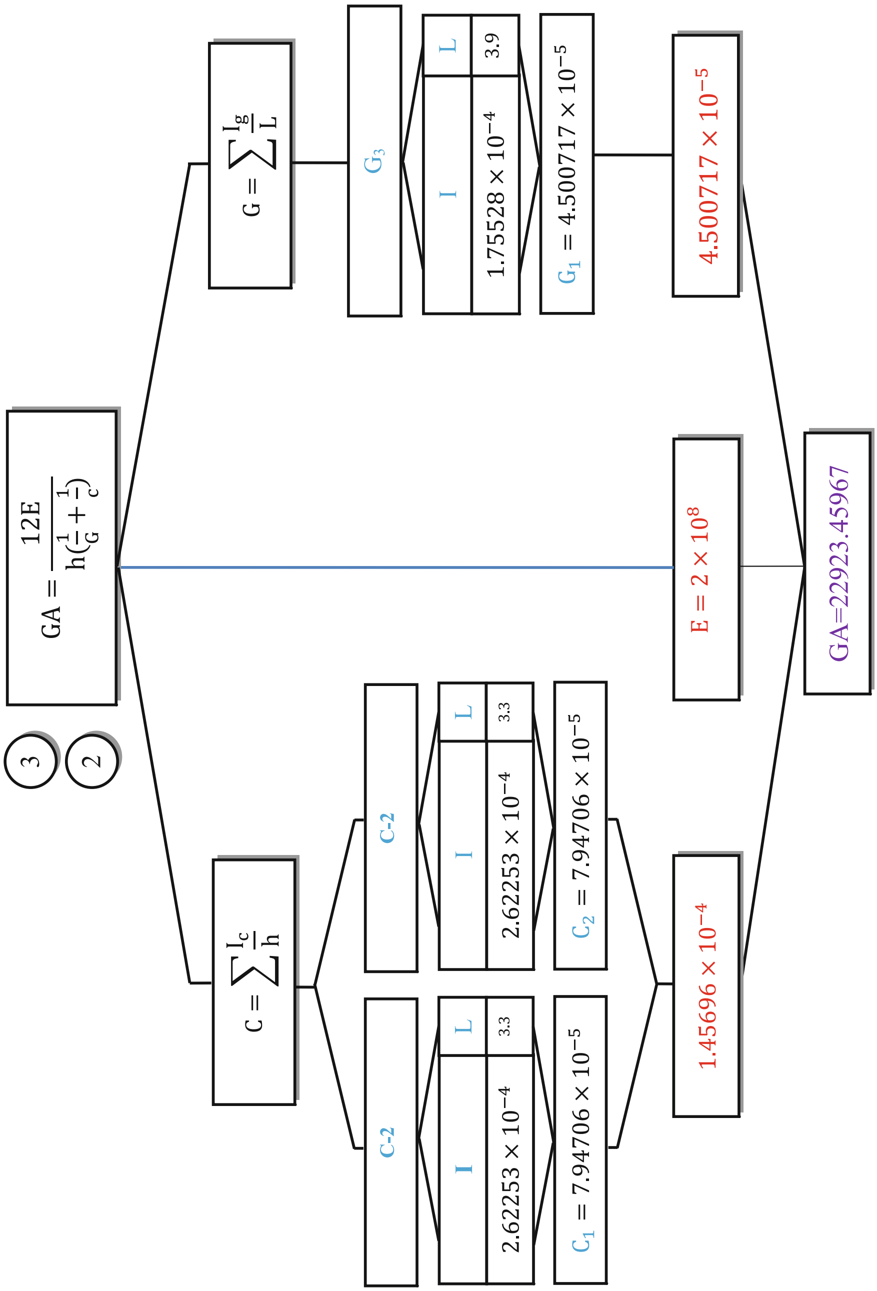


Chart 1.34 Shear rigidity of bent (3) story (2)

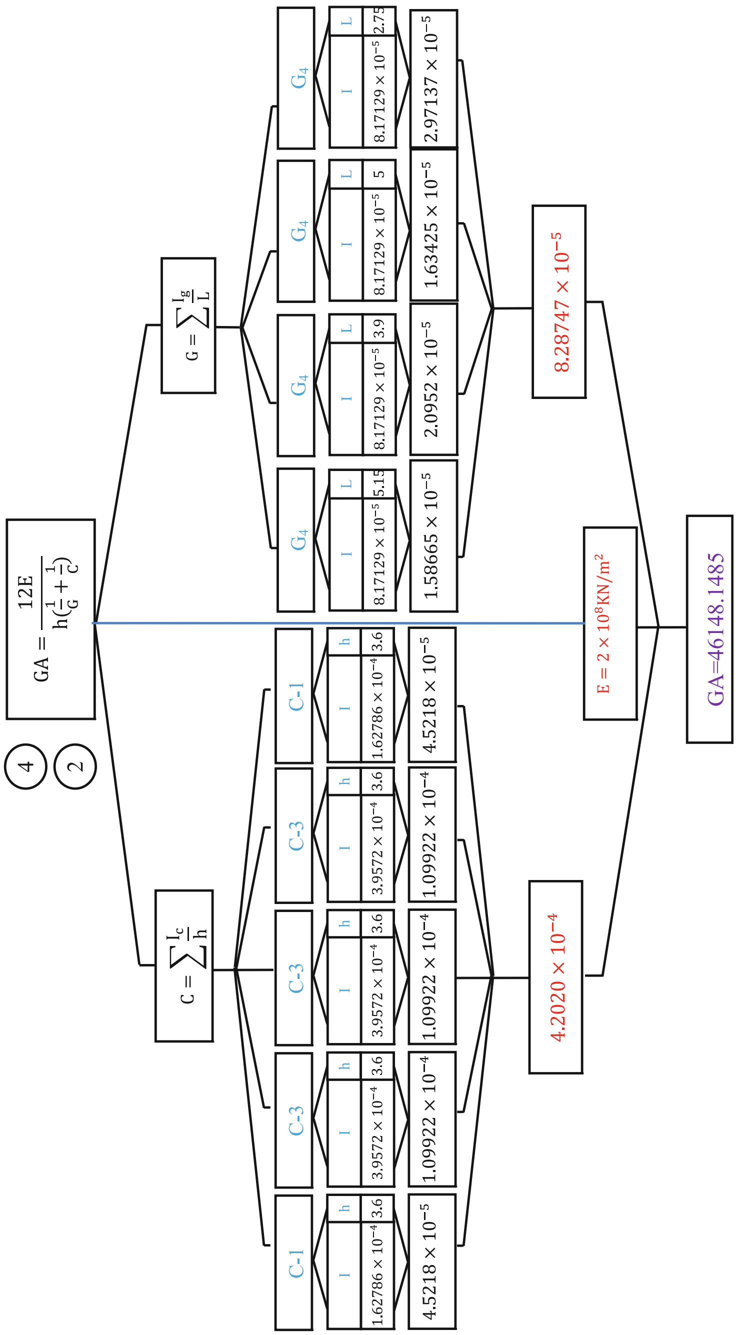


Chart 1.35 Shear rigidity of bent (4) story (2)

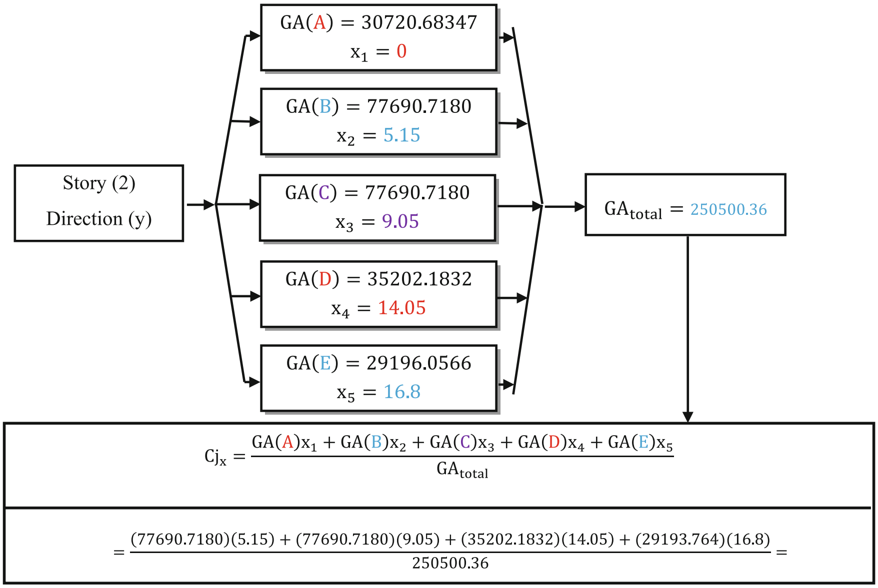


Chart 1.36 Calculation of coordinate of stiffness in Y direction

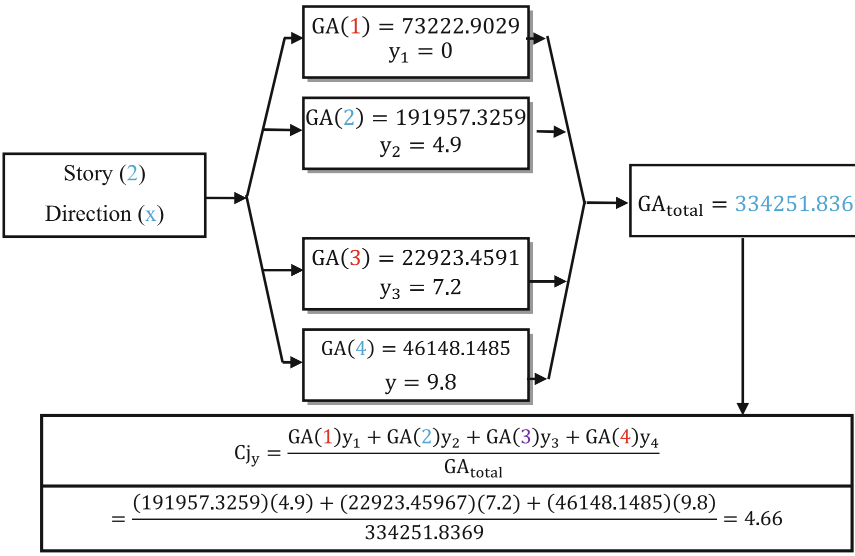
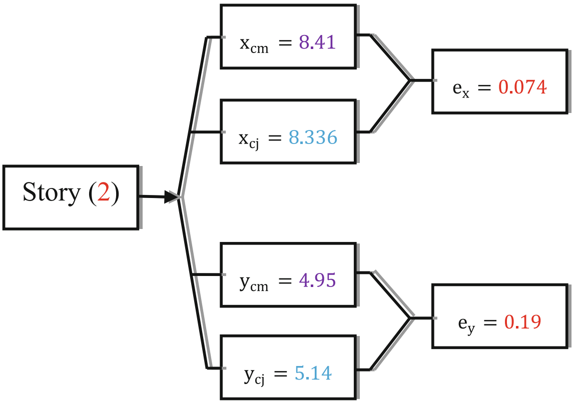


Chart 1.37 Calculation of coordinate of stiffness in X direction

Chart 1.38 Eccentricity in the first floor



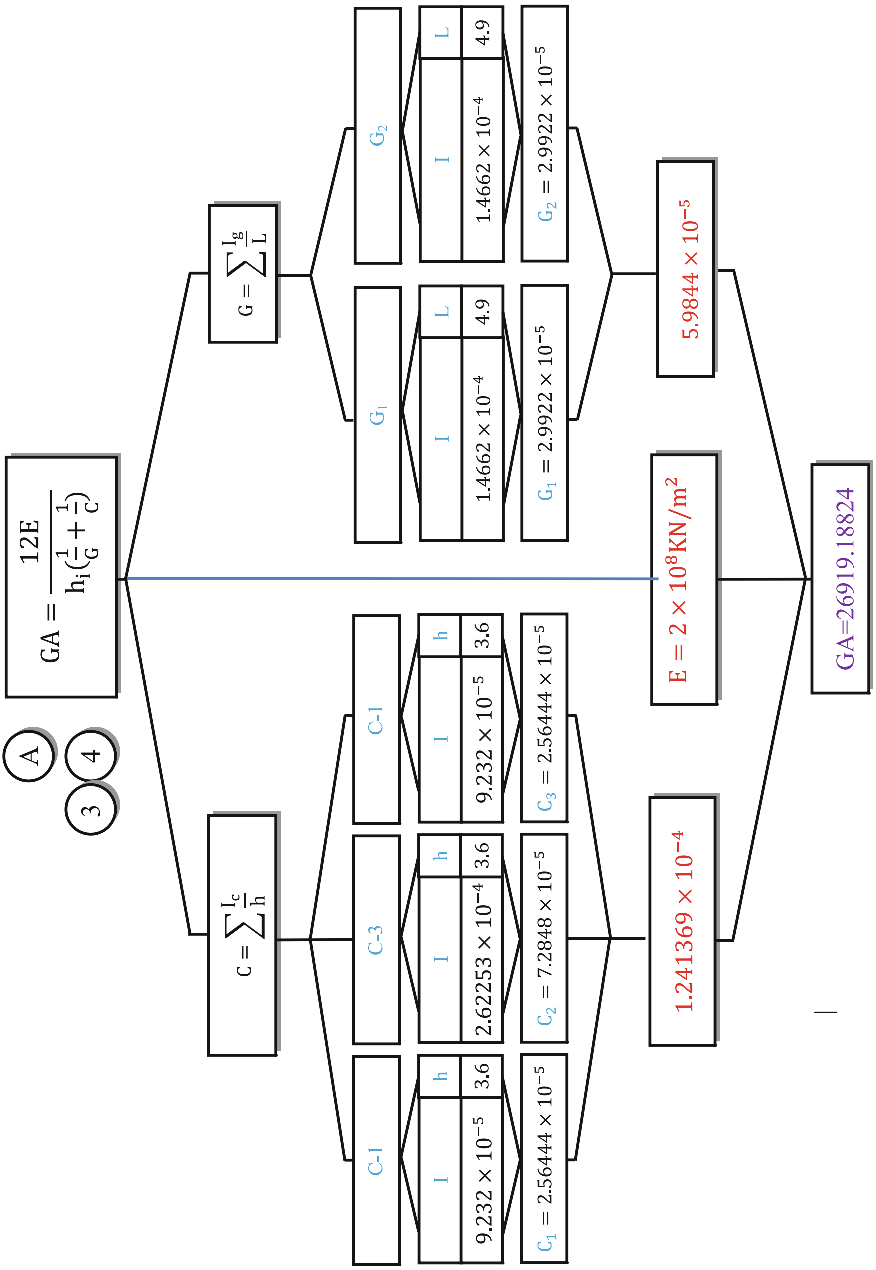


Chart 1.39 Shear rigidity of bent (A) story (3), (4)

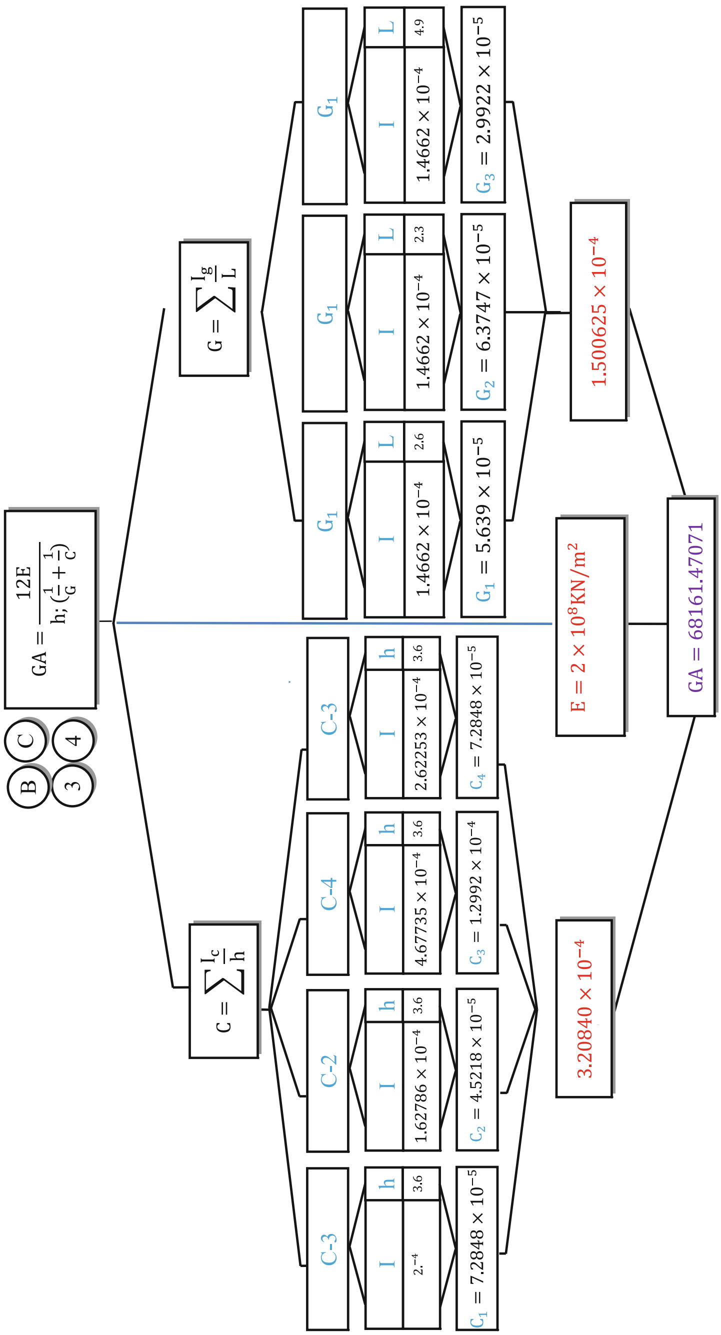


Chart 1.40 Shear rigidity of bent (B), (C) story (3), (4)

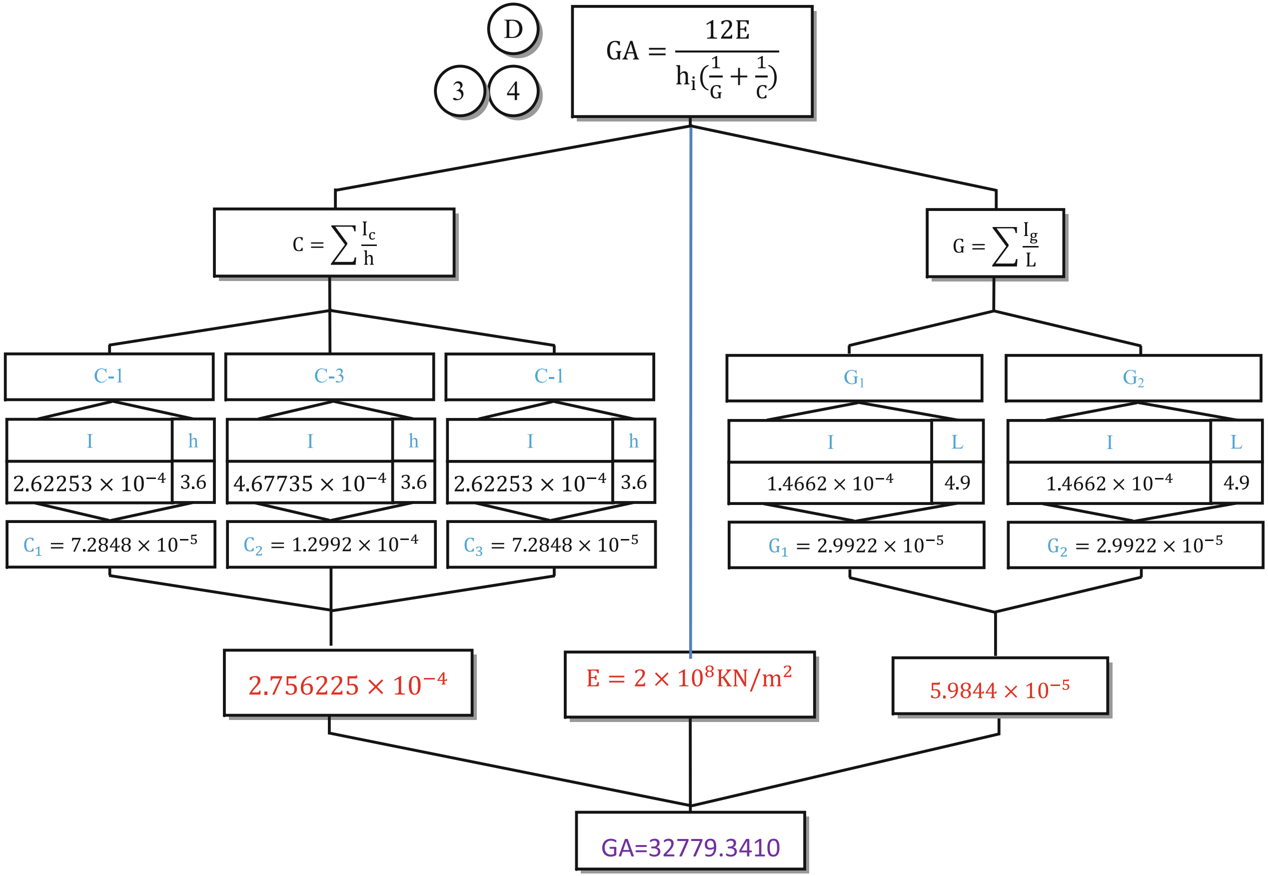


Chart 1.41 Shear rigidity of bent (D) story (3), (4)

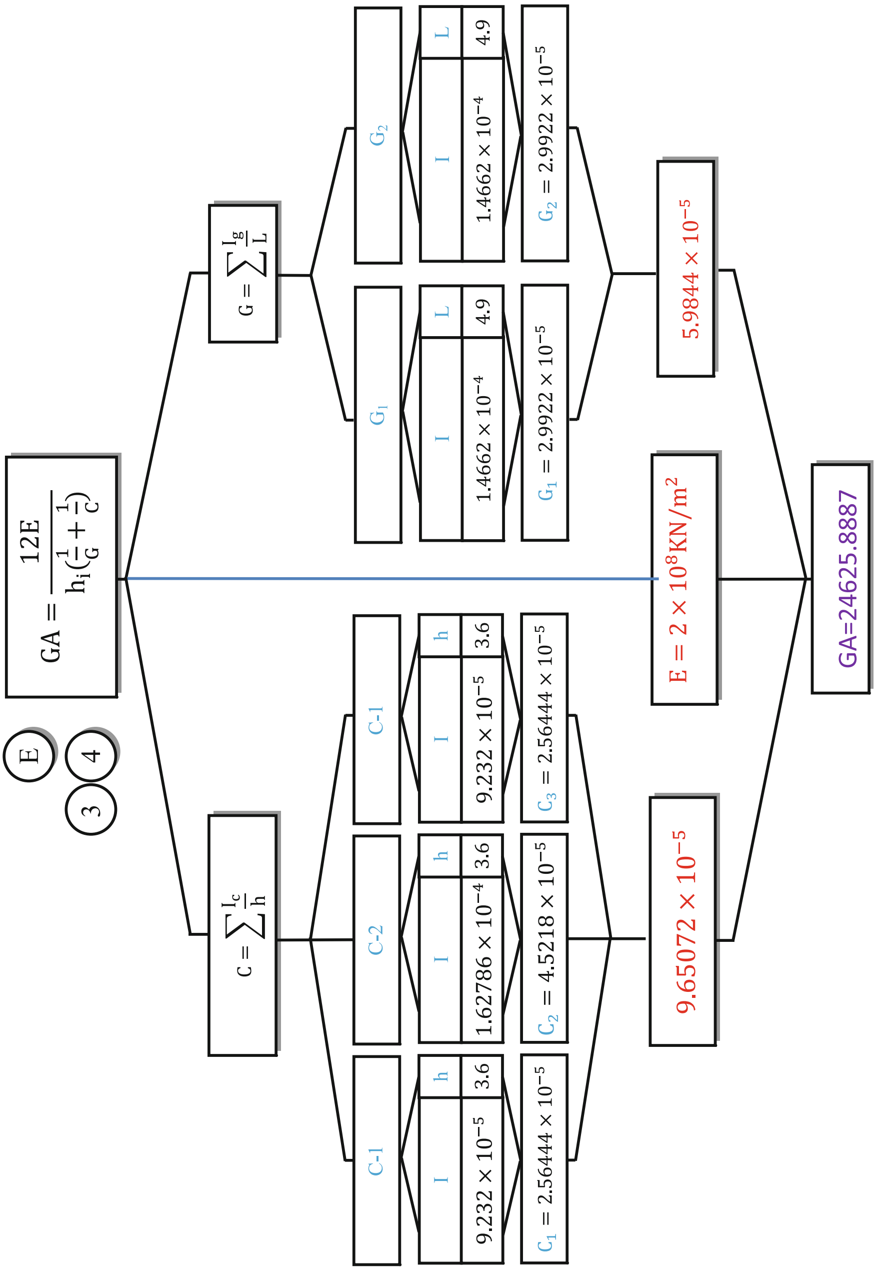


Chart 1.42 Shear rigidity of bent (E) story (3), (4)

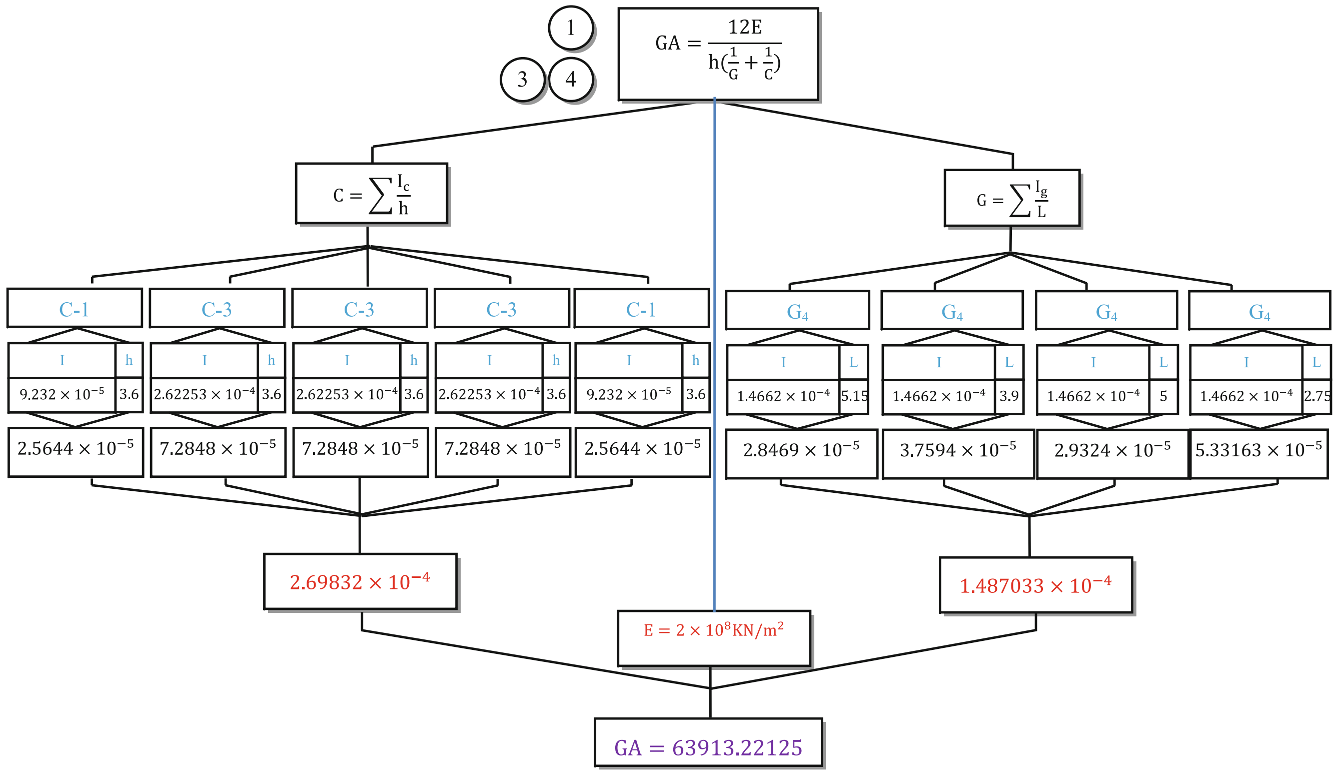


Chart 1.43 Shear rigidity of bent (1) story (3), (4)

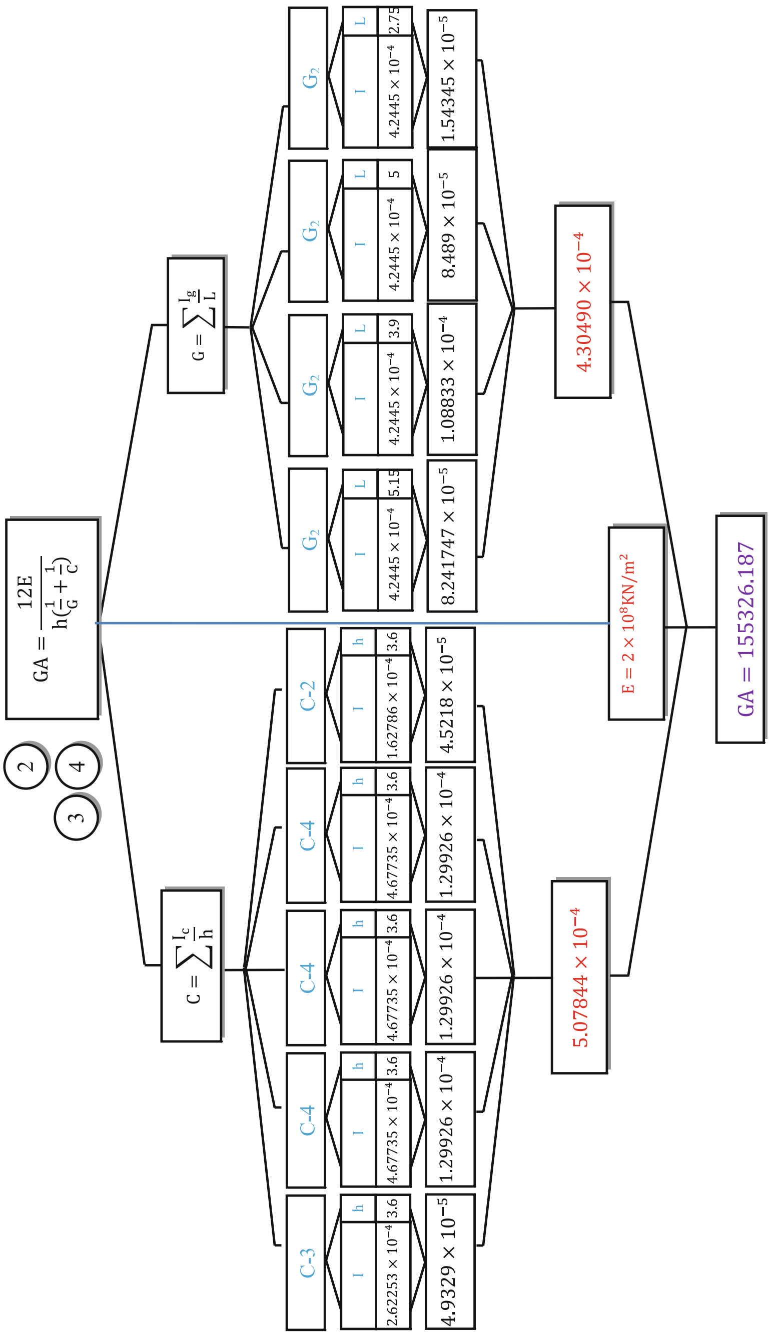


Chart 1.44 Shear rigidity of bent (2) story (3), (4)

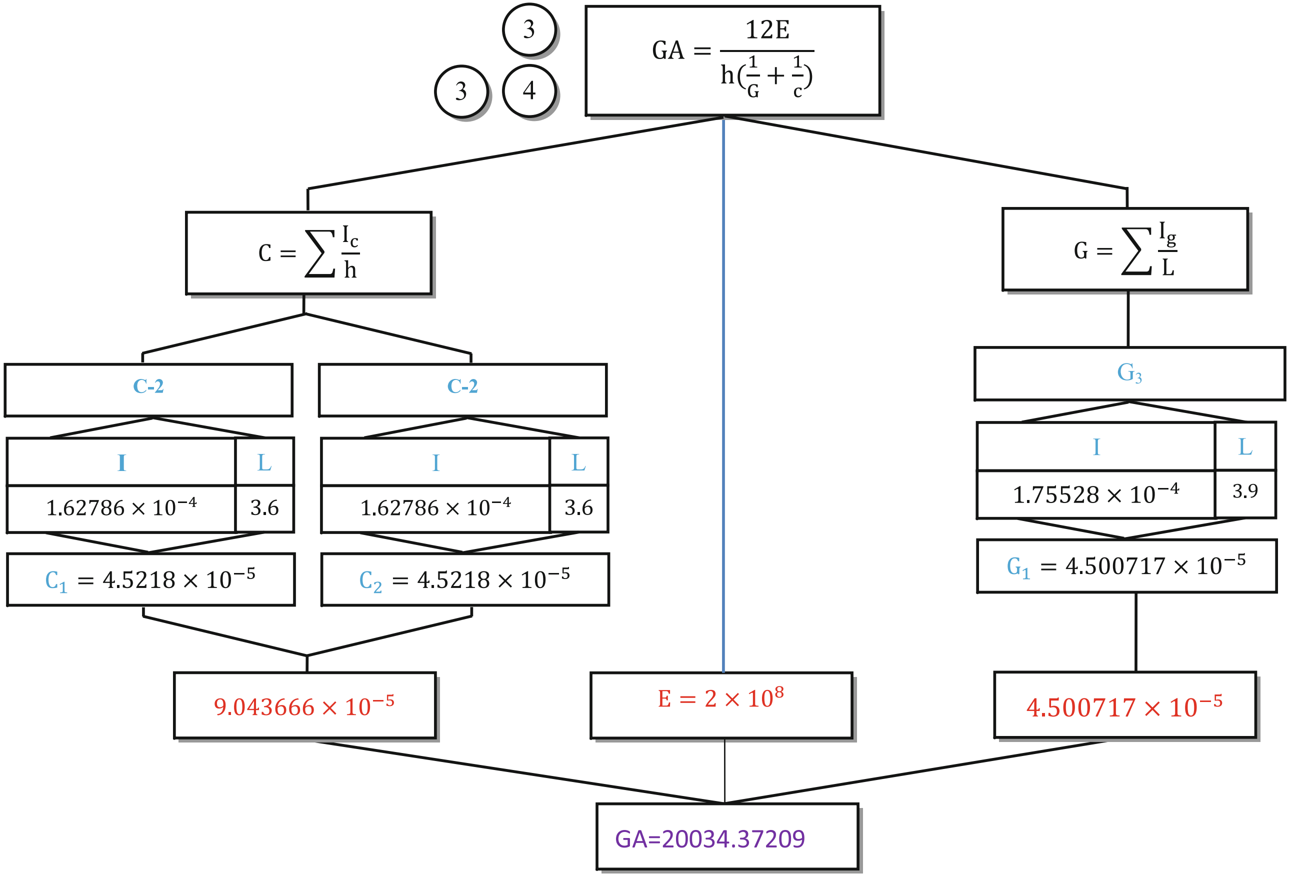


Chart 1.45 Shear rigidity of bent (3) story (3), (4)

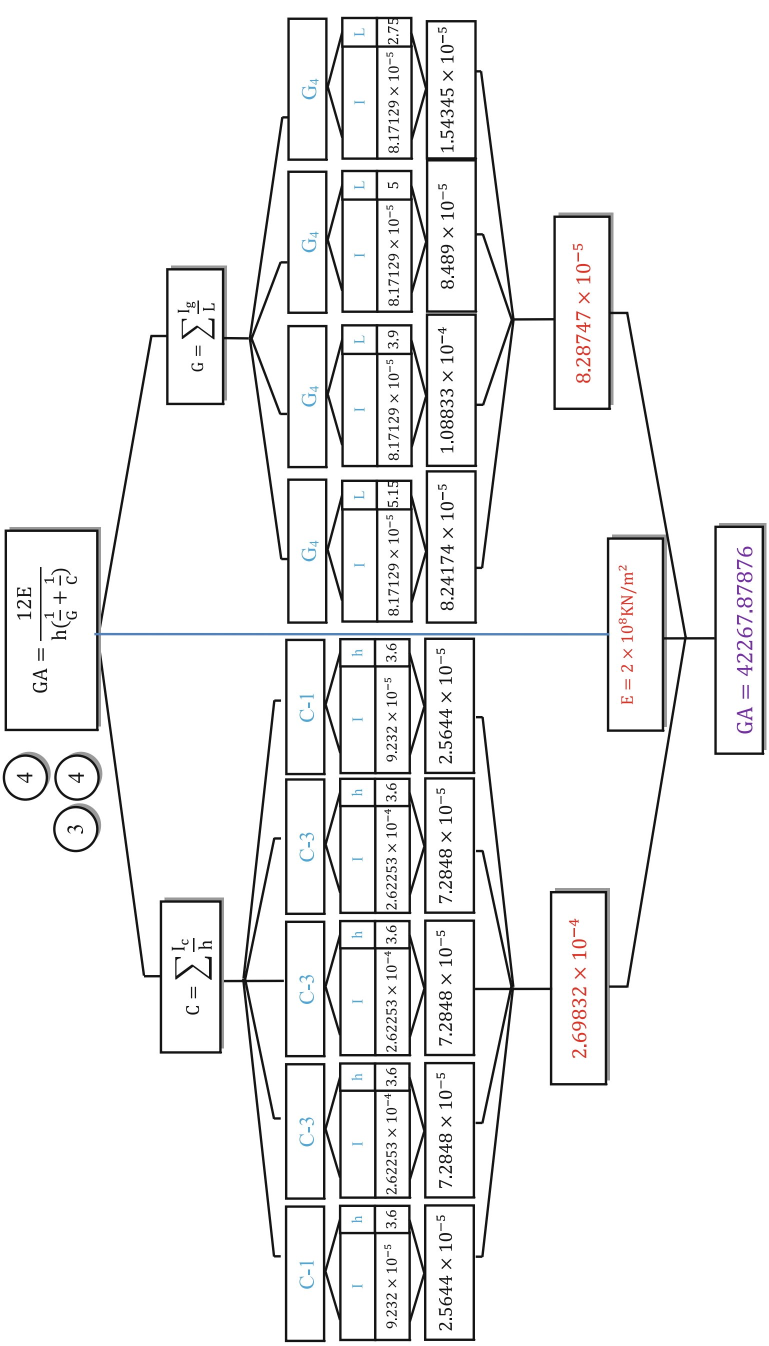


Chart 1.46 Shear rigidity of bent (A) story (2)

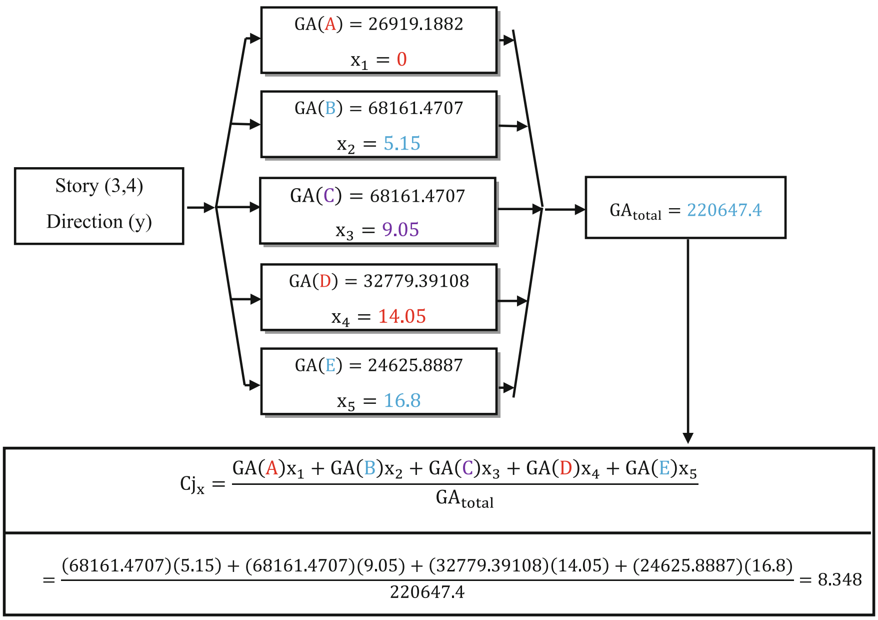


Chart 1.47 Calculation of coordinate of stiffness in Y direction

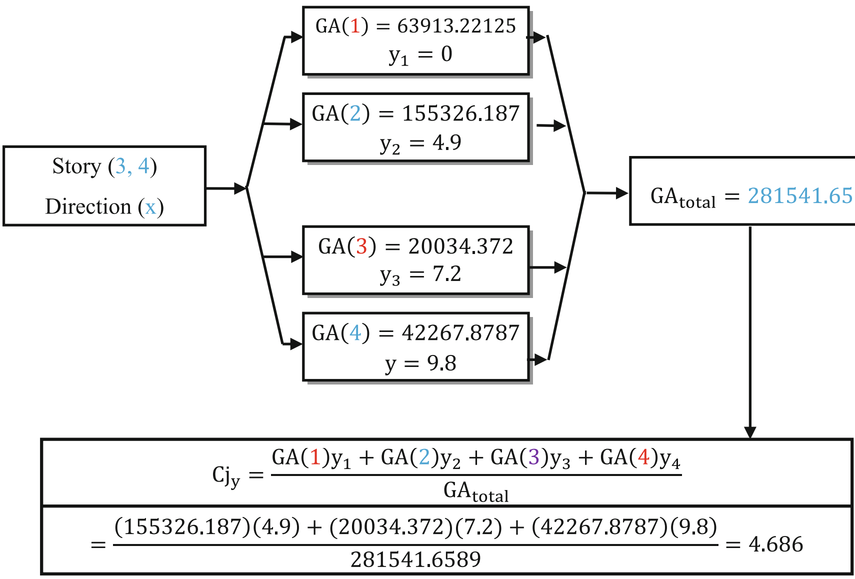
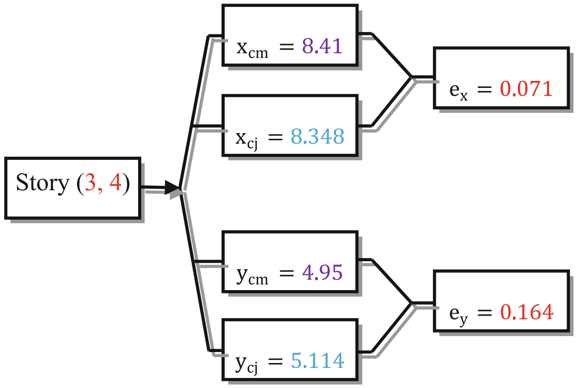


Chart 1.48 Calculation of coordinate of stiffness in X direction

Chart 1.49 Eccentricity in third and fourth floor



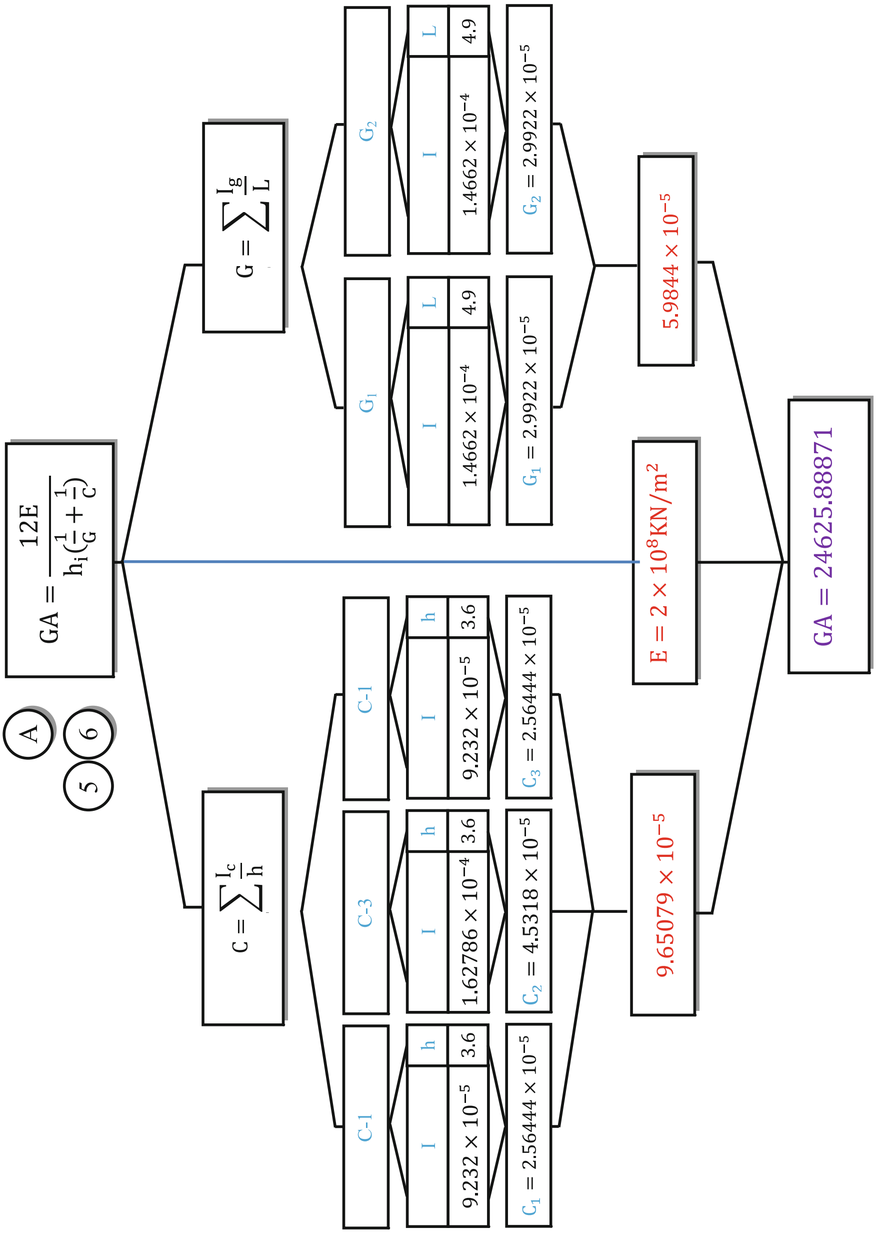


Chart 1.50 Shear rigidity of bent (A) story (5), (6)

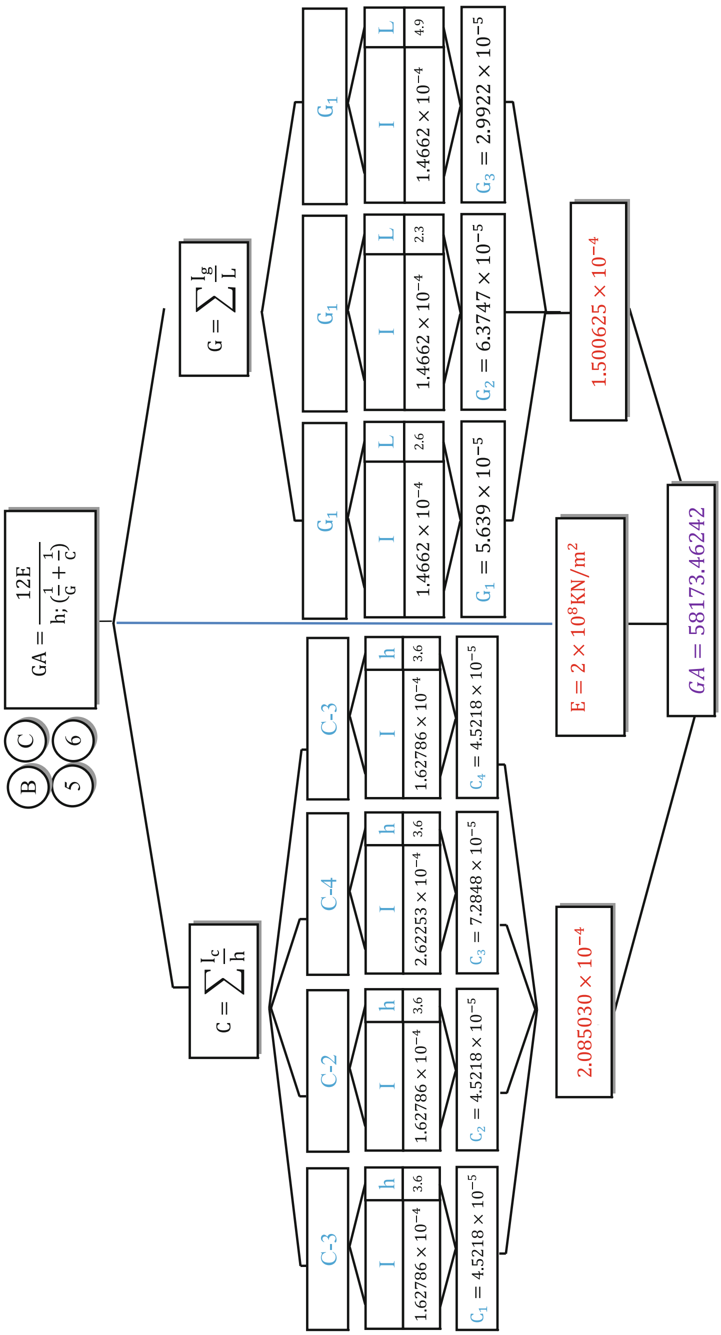


Chart 1.51 Shear rigidity of bent (B), (C) story (5), (6)

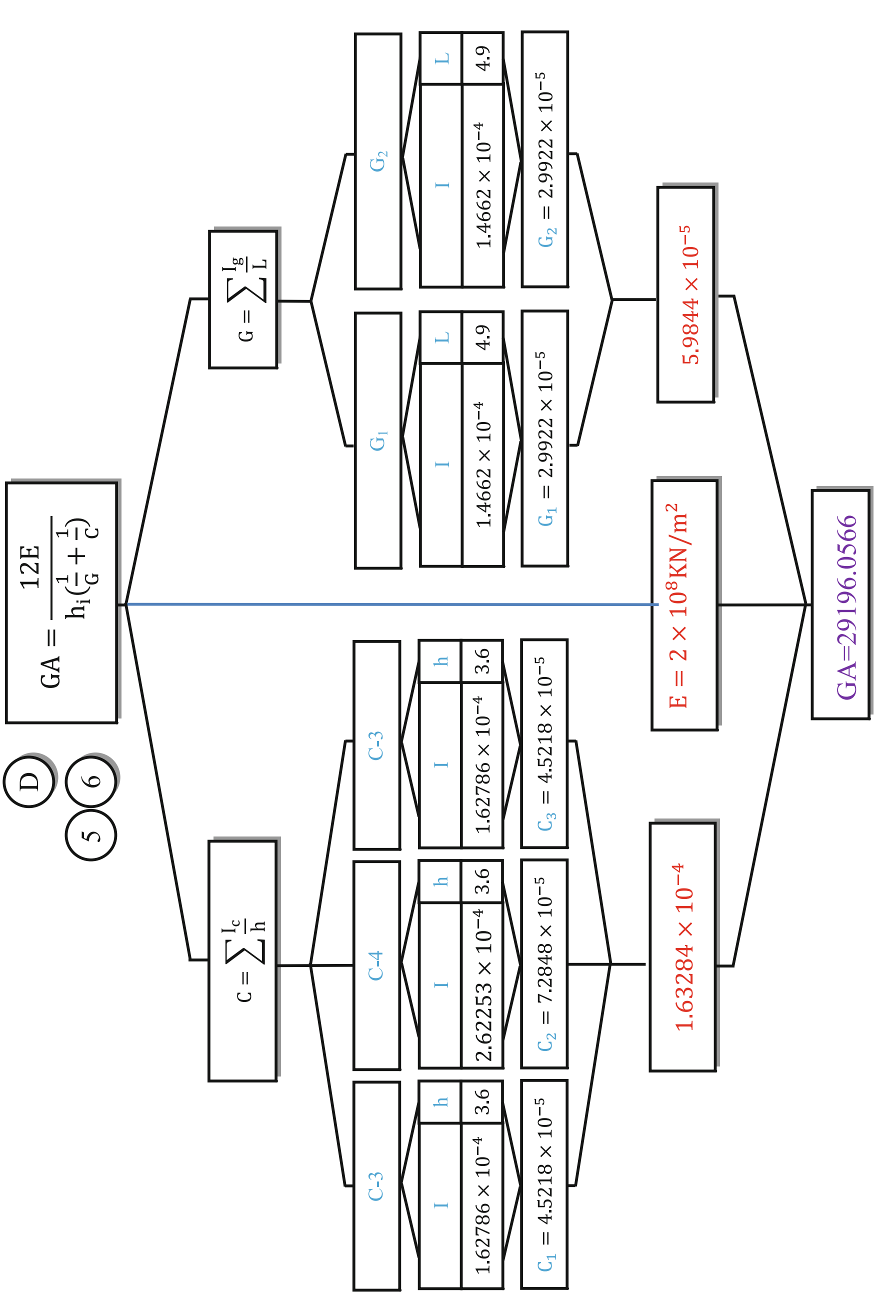


Chart 1.52 Shear rigidity of bent (D) story (5), (6)

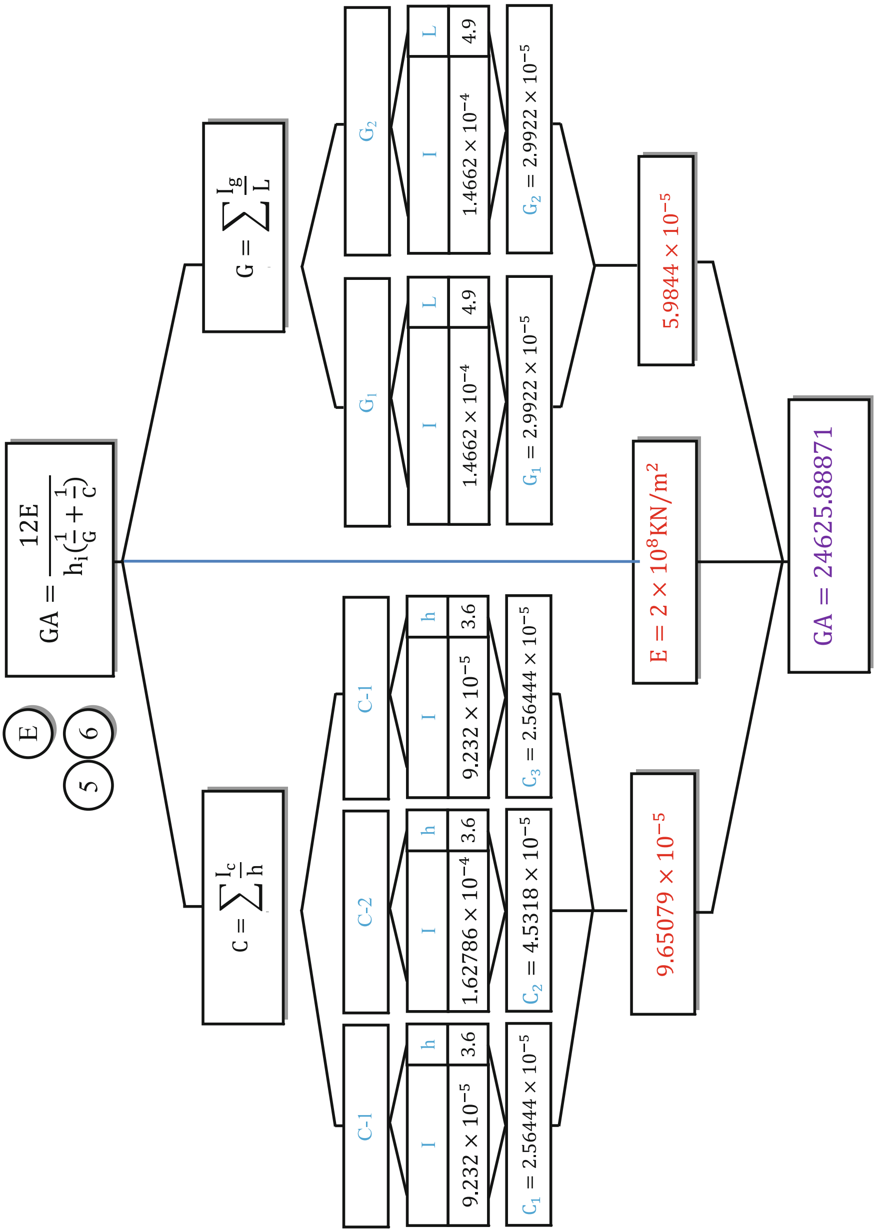


Chart 1.53 Shear rigidity of bent (E) story (5), (6)

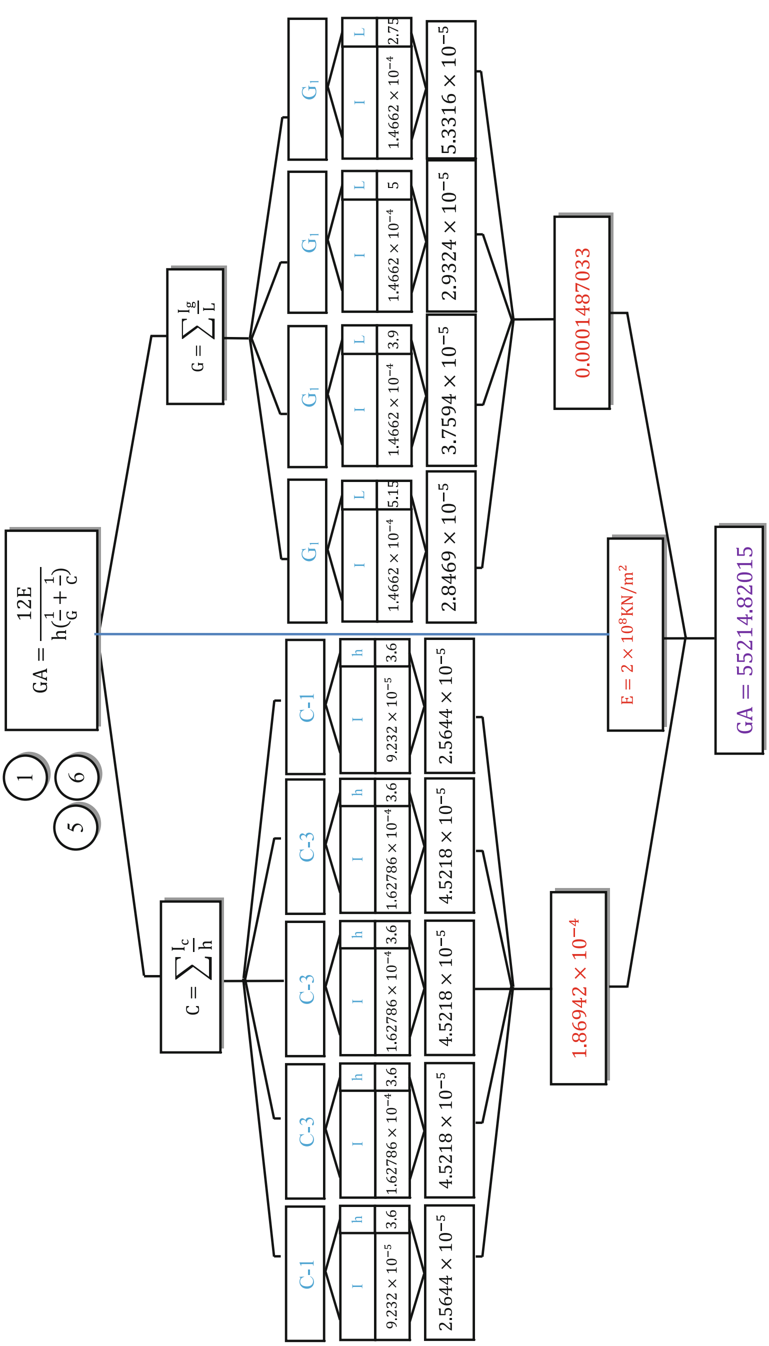


Chart 1.54 Shear rigidity of bent (1) story (5), (6)

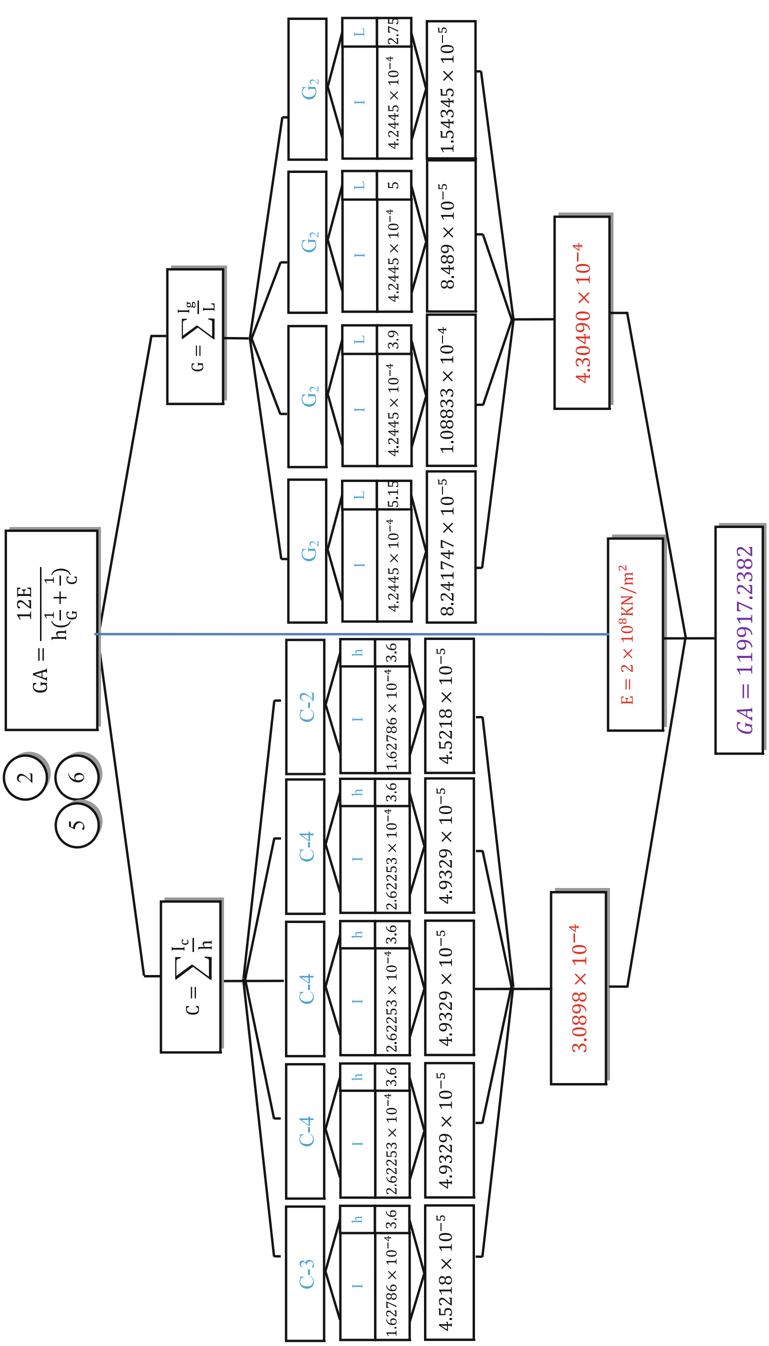


Chart 1.55 Shear rigidity of bent (2) story (5), (6)

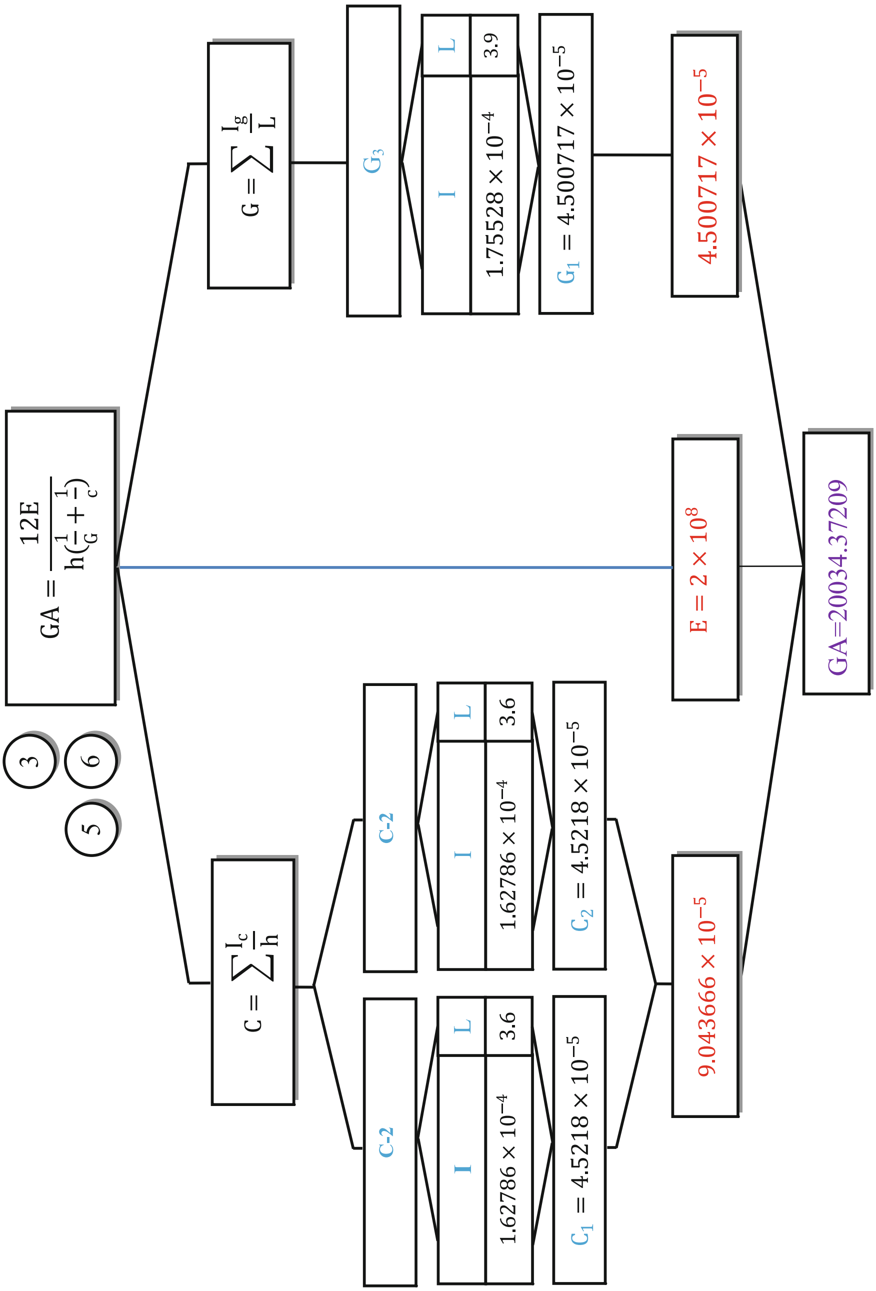


Chart 1.56 Shear rigidity of bent (3) story (5), (6)

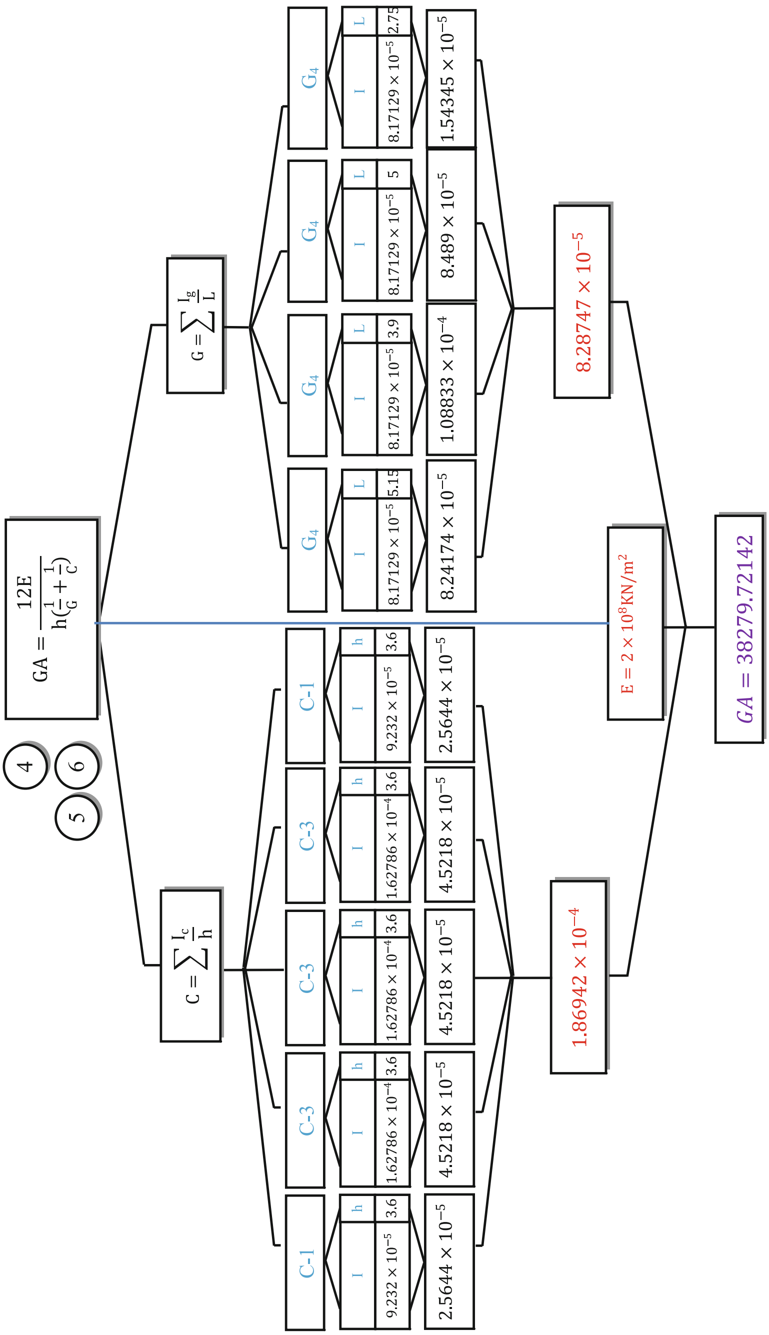


Chart 1.57 Shear rigidity of bent (4) story (5), (6)

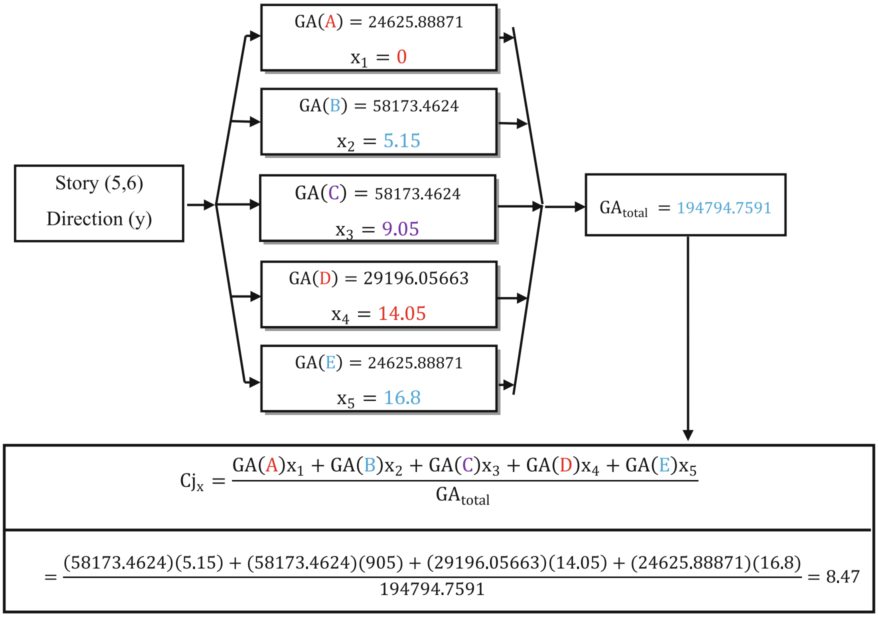


Chart 1.58 Calculation of coordinate of stiffness in Y direction

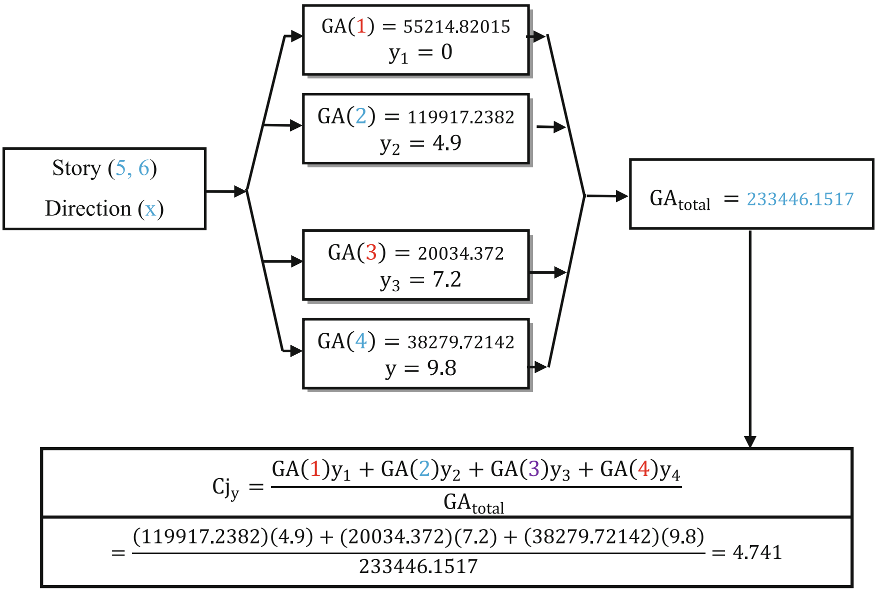


Chart 1.59 Calculation of coordinate of stiffness in X direction

Chart 1.60 Eccentricity in the fifth floor

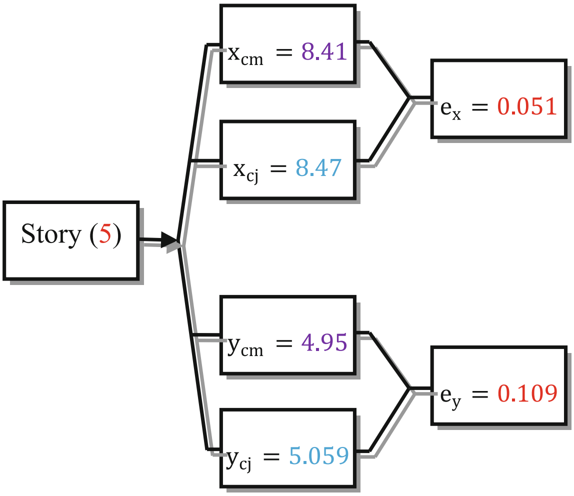
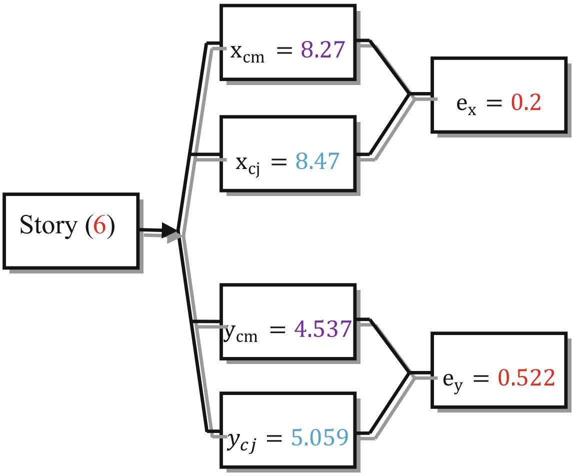
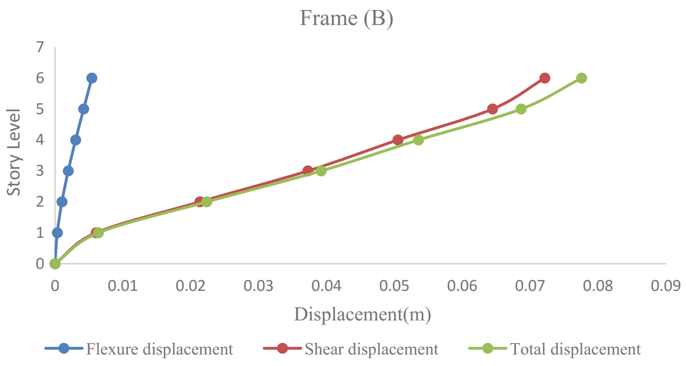
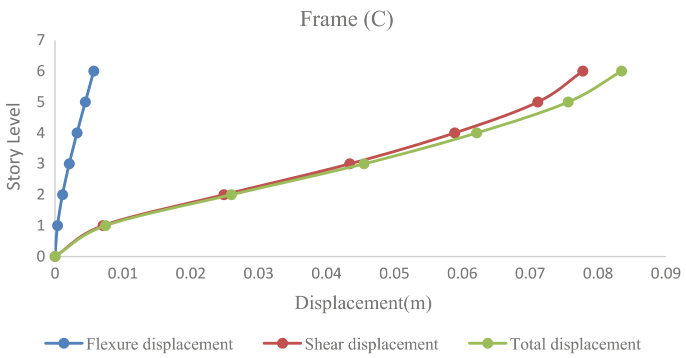


Chart 1.61 Eccentricity in the first floor

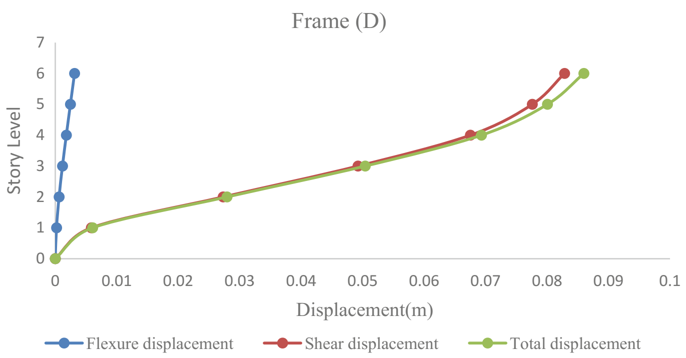




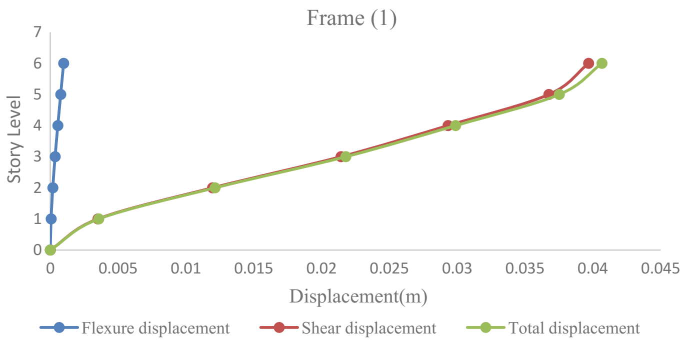
Graph 1.2 Displacement of frame (A)



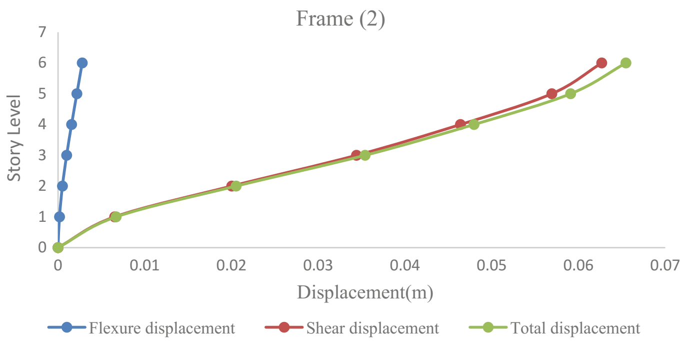
Graph 1.3 Displacement of frame (C)



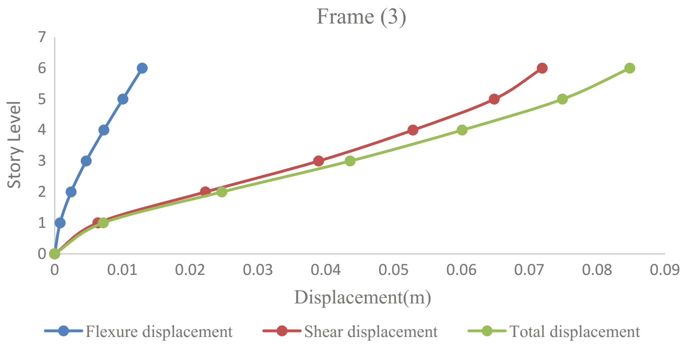
Graph 1.4 Displacement of frame (D)



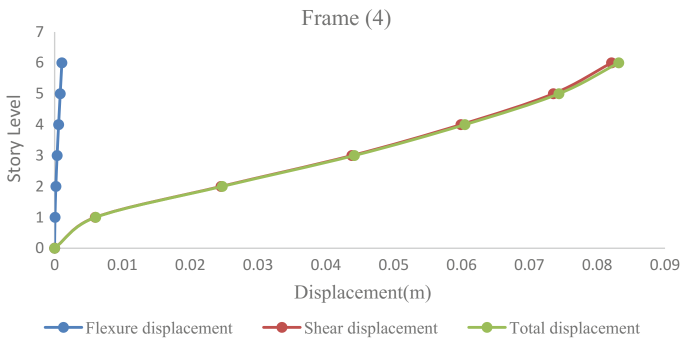
Graph 1.5 Displacement of frame (1)



Graph 1.6 Displacement of frame (A)



Graph 1.7 Displacement of frame (3)



Graph 1.8 Displacement of frame (4)

References

ASCE/SEI. (2010). Minimum design loads for buildings and other structures. In *ASCE/SEI 7-10*. Reston, Virginia: American Society of Civil Engineer.

Code, P. (2005). Eurocode 8: Design of structures for earthquake resistance-part 1: general rules, seismic actions and rules for buildings.

Eurocode 8, C. (2004). *Design of structures for earthquake resistance—Part 1: General rules, seismic actions and rules for buildings (EN 1998-1: 2004)*. European Committee for Normalization, Brussels.

IBC, I. (2012). International Code Council. *International Building Code*. International Code Council: Washington DC, United States.

IIT, K. (2008). *Structural analysis*. India.

Mohsen, H., & Reza, K. (2015). *Design of steel structure with ETABS software*. Tehran, Iran: SeriOmran.

Smith, B. S., Coull, A., & Stafford-Smith, B. S. (1991). *Tall building structures: Analysis and design* (Vol. 5). New York: Wiley.

Chapter 2
Braced Steel Frame System

In this chapter, the braced frame system and its effects on building systems is demonstrated. The first section of this chapter focus on the behavior of the braced frame system subjected to gravity and lateral load. The structure is analyzed by approximate methods and the virtual work method is used to explain and calculate the frame's displacement under lateral load. Different types of bracing system are demonstrated and the lateral stiffness is determined. In order to better demonstration of bracing design concept, a 6 story building is considered and its architectural and structural detail plans are given in the second section of this chapter and the applied theories and methods, along with all calculations and computations processes related to the real considered building are explained.

In the third section of this chapter, the relevant calculation charts for determining of the centre of mass, stiffness, shear rigidity and the eccentricity of each floor level of structure are illustrated.

The next section of the chapter provides the description about the procedures used for calculation of displacements at two perpendicular directions in the braced frame system subjected to earthquake force. The last section presents the proposed charts and detail calculations of weight, stiffness, and displacement of the considered structure.

F. Hejazi and K. Karimzadeh, *Analysis Procedure for Earthquake Resistant Structures*,
https://doi.org/10.1007/978-981-10-8839-1_2

2.1 Braced Frame System

- Braced frame system
- Different types of bracing system
- Behaviour of bracing system under gravity and lateral load
- Stiffness

2.1.1 Braced Frames

2.1.1.1 Introduction

Bracing is a method used to resist lateral loads in a building's structure. It is widely applied due to its efficiency and low-cost employment. Moving on, this method employs the use of a braced bent that is made of typical columns and girders. These elements' are to provide support for the structure against the gravity loading. Furthermore, the diagonal bracing members, which are connected to each other, forms a vertical truss that holds the structure's lateral loading. The combination of this architecture of braces and girders formed a web that works together with the columns that act as the chords to produce a truss. Moreover, the diagonal bracing work in axial stress that decrease the need for member sizes to provide rigidity and support against horizontal shear. In current construction, one or two-story height bracing system that is applied in earlier structures is still used. Nevertheless, there has been some development that improves the bracing system. A larger scale of bracing that surpasses many stories and bays has been employed in medium and high-rise buildings. Perhaps there will be more development for bracing systems in times to come.

2.1.2 Types of Bracing

There are three types of bracing that can be used as lateral resisting system in simple structural system.

1. Concentrically Braced Frame (CBF)
2. Eccentrically Braced Frame (EBF)
3. Bucklin Restrained Braced (BRB).

2.1.2.1 Concentrically Braced Frames

Concentrically braced frames are LFRS that use axially loaded members to transfer lateral forces to the foundation. The brace members rely on axial strength, in both tension and compression, and stiffness to resist the applied axial loads. As illustrated in Fig. 2.1, the braces within a frame are laid out such that the center lines of the braces, columns, and beams intersect at their points of connection, unlike eccentrically braced frame. Lateral forces are transferred through floor and roof diaphragms at every level, and ideally, braced frames are stacked vertically between building levels to provide a clear load path to the foundation.

Figure 2.2 shows some examples of concentrically braced frames in steel frame structures.

Different types of the concentrically braced frames are shown in Fig. 2.3. The most practical bracing types are single-diagonal, double-diagonal, chevron, knee and K-braced types (Fig. 2.3a–h). These types form a fully triangulated vertical truss. Due to their constructiveness, they are usually located at non-passageway areas that are not usually relocated. Such areas would be beside and between lift, service and staircases.

Furthermore, fully triangulated bracing (Fig. 2.3a–d) allows for economic design and construction. It should be noted that floor framing is designed to resist only gravity loading. Since the girder moments and shears of fully triangulated bracing are independent of lateral loading, this system can be used repetitively for the whole structure.

2.1.2.2 Eccentrically Braced Frame (EBF)

Eccentrically Braced Frame (EBF) system is not concentrically braced with main joint. It is a system that is employed to provide support against lateral events such as wind and earthquakes. EBF consists of a beam, one or two braces and columns

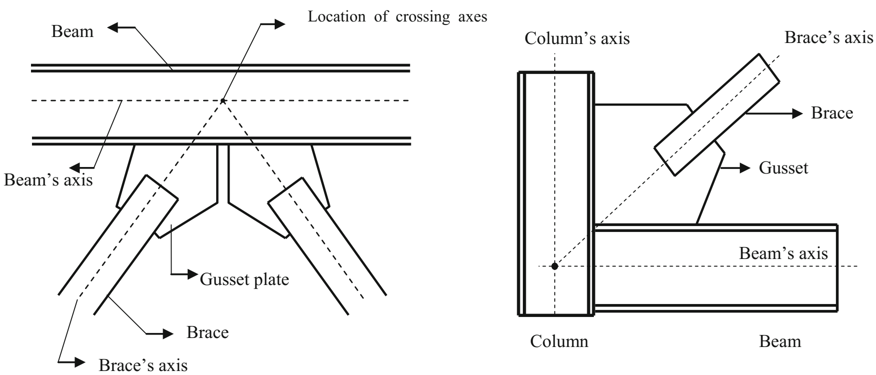


Fig. 2.1 Connection details of a concentrically braced frame

Fig. 2.2 Eccentrically braced frame

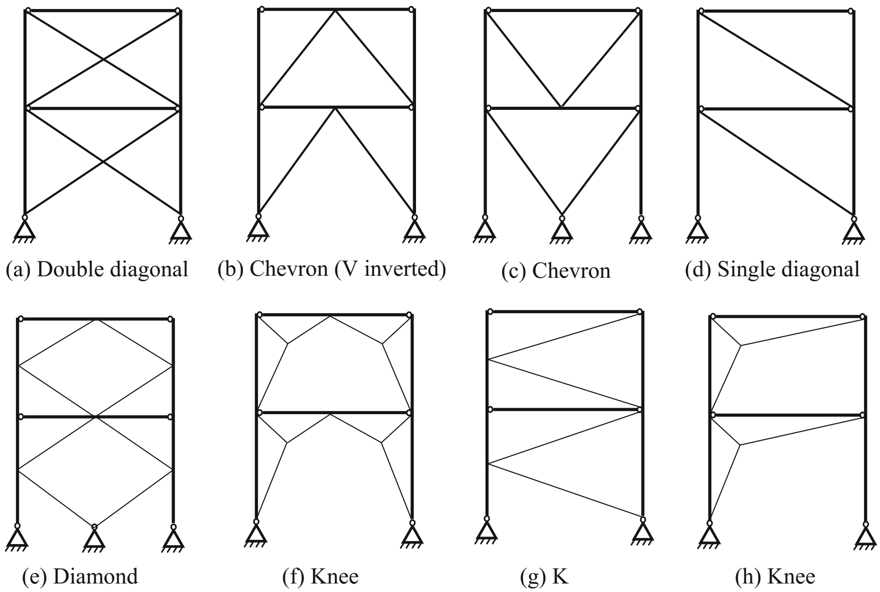


Fig. 2.3 Different types of CBF

and it is fairly similar to traditional braced frames. The only contrast is that at least one end of the braces is eccentrically connected to the frame. Furthermore, optimum EBFs will behave in a ductile manner through the shear forces or flexural yielding of a fuse statement introduced by the connections of the frame and the braces. The connections also promote bending in the beam adjacent to the brace. Figure 2.4 illustrates EBF in more detail.

Figure 2.5 shows examples of an eccentrically bracing system in steel structured frame system.

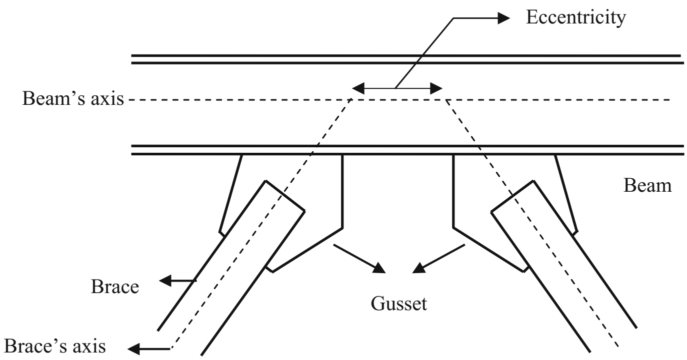


Fig. 2.4 Detail of EBF

Fig. 2.5 Eccentrically braced frame

Types of the eccentrically braced frame are shown in Fig. 2.6. EBF is applied to door and window opening mechanism, and the ones that allow bending in the girder are specified in Fig. 2.6. Generally, this mechanism reacts to lateral loading and is laterally less stiff. It is also less efficient compared to the fully triangulated truss. Fully triangulated truss react to only axial member forces, while EBF mechanism is less efficient in giving support by weight.

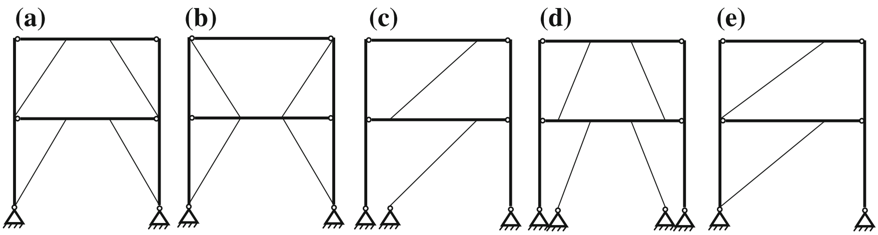


Fig. 2.6 Different types of eccentrically braced frame

Fig. 2.7 Link part of eccentrically braced frame

EBF is employed to control displacement against wind and earthquakes. During major earthquakes, the links between the brace connections and column (Fig. 2.6c, e), and between brace connections (Fig. 2.6a, b, d), serve as a fuse, they deform plastically into shear to provide ductile support to the whole structure. The fuses each have different sizes of the cross-section, and the sizes are proportional to the main girder that the fuses are installed on. This mechanism works in relation to the ratio of fuse length to beam length. Shorter links are stiffer, while longer links are more flexible. In turn, this braced system will have high elastic stiffness and large inelastic energy dissipation capacity that can survive many cycles of natural disasters. Figure 2.7 shows the links which have been used in EBF. It should be noted that their cross-section sizes can be different from the main beam.

Note:
Some experts have recommended that the connections between beam and column in EBF system should be connected with moment resisting connection. This is to ensure better performance under earthquake loading.

2.1.2.3 Buckling Restrained Braces

Buckling Restrained Braces system (BRBs) is highly ductile, able to withstand loads and have excellent cyclic performance, making it particularly useful to provide support against lateral loads. In addition, the braces do not collapse and have the ductility of steel. This, in turn, provides stable and predictable dissipation of seismic energy. The main incorporation of this system is with BRBF concentric braced frame system. It is also applied to structures that require high ductility, and energy dissipation (for example bridge, outrigger or blast designs) as well as when buckling of the brace is undesirable. Furthermore, BRB is able to withstand compression and tension without collapsing due to its stability. This is caused by its detailed and fabricated mechanism, where its two main elements perform two different task while remaining de-coupled. The steel core is the load resisting component and is restrained against overall buckling by restraining mechanism

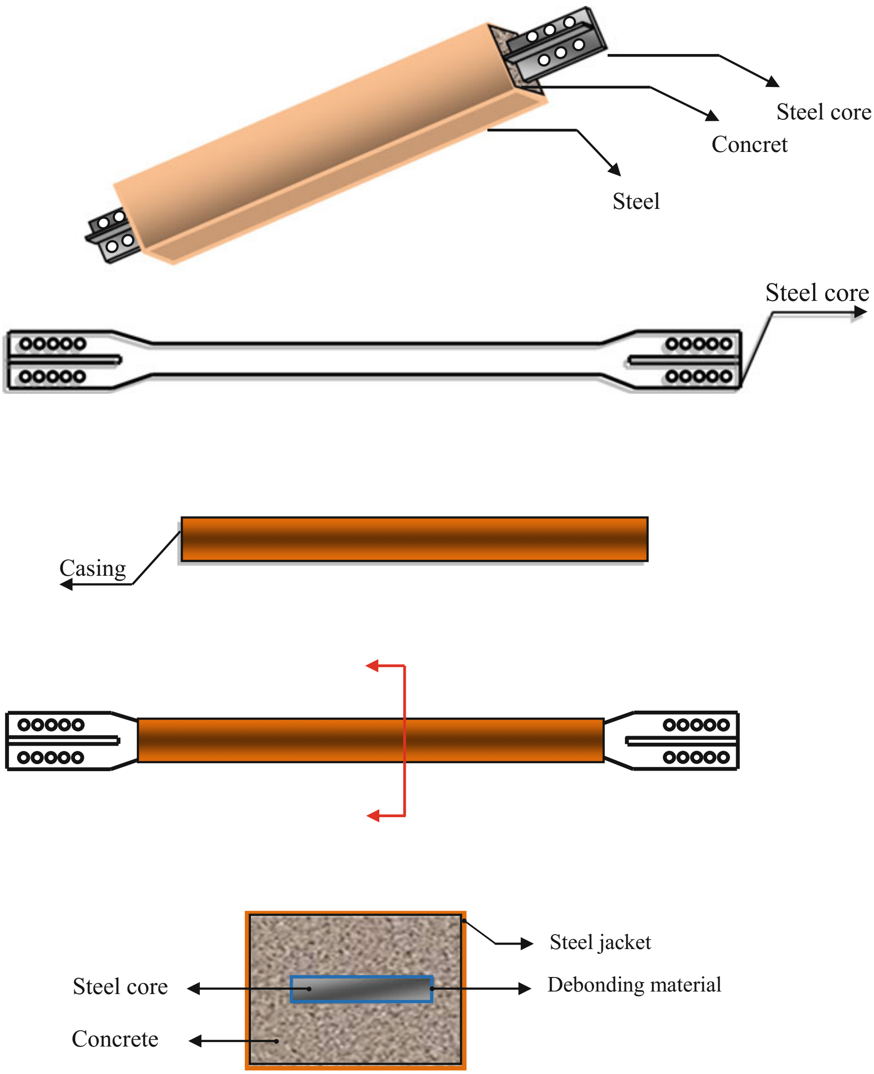


Fig. 2.8 Components of bulking restrained brace

(or stability component). Moreover, the outer casing is filled with concrete to ensure BRB components remain separated and prevent composite action. Then, BRB brace is submitted into a concentric braced frame, producing a BRBF system. Figure 2.8 demonstrates BRB components.

The steel core in BRB should be able to resist the entire axial force. Furthermore, the casing should be deboned from steel core, and the flexure stiffness of casing is

Fig. 2.9 Bulking restrained braced frame

restrained by the buckling of the steel core. BRB braces dominate conventional braces because the former yield tension and compression and the latter only yield tension but collapse under compression. The difference between strength against tension and compression is significantly different between these two systems. This is demonstrated in Fig. 2.9 which shows the bulking restrained braced system in steel structure frame.

Nevertheless, BRB brace by itself cannot be used as X-brace. Hence some configurations need to be applied. These are demonstrated in Fig. 2.10. Also, BRB Frame (BRBF) is designed to hold against significant inelastic deformations caused by earthquakes. Hence, it must be composed of structural steel core to prevent the building from collapsing. The steel core should also be designed to support the entire structure against axial force. Beams, columns, and gussets connecting the core should be taken into account in the calculation of the stability of the system.

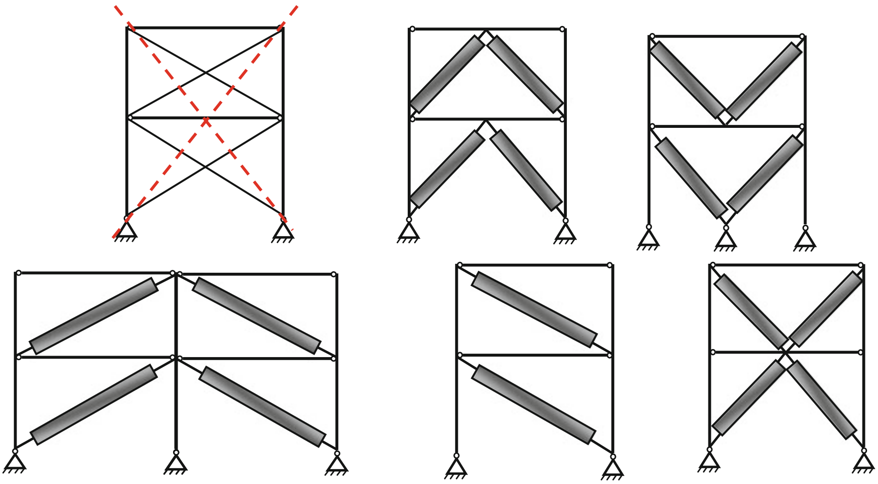


Fig. 2.10 Different configuration of BRBF

Furthermore, the system should be able to limit local and overall buckling of the steel core to 2.0 times the design story displacement. It should not buckle within limit.

$$\Delta = \text{design story displacement} = C_d \times \Delta_E \tag{2.1}$$

where;

Δ_E story displacement under code specified earthquake forces
C_d 5.5 for BRBF with non-moment resisting beam-column connections
5 for BRBF with moment-resisting beam-column connections

Brace stiffness and modeling for a typical or particular concentrically braced frame.

Brace stiffness is related to brace area. It is determined using a simple equation as follows:

$$K_{model} = \frac{AE}{L_{wp-wp}} \tag{2.2}$$

where;

$L_{wp\text{-}wp}$ is the work point-to-work point distance along the axis of the brace.

This analysis is part of most structural design software packages. BRB is non-prismatic, has yielded core segment with the minimum cross-sectional area and the outer surface is elastic with the greater cross-sectional area. The core controls the strength, however using this as structural model will not accurately depict the stiffness of brace. Such stiffness can be captured using stiffness modification factor (KF). The model of brace stiffness can then be calculated using the following equation:

$$K_{model} = \frac{KF(A_{sc})E}{L_{wp-wp}} \tag{2.3}$$

where;

ASC is the steel core area of the brace.

This model can also be represented as a spring with a defined stiffness K model. Stiffness is unique to braces design but may be similar between manufacturers. The design is also influenced by brace capacity, bay geometry, and connection details (Kimberley Robinson, October 2013).

2.1.3 Behavior of Braced Frame Under Gravity and Lateral Load

In this section, the behavior of the bracing system under gravity load and lateral load are considered. Deformation, moment, shear, and axial diagrams will be drawn under gravity load, and all member forces will be computed under lateral load.

2.1.3.1 Structural Frame System with Double-Diagonal Bracing

Consider the steel double-diagonal bracing system in Fig. 2.11. The gravity load is resisted by the beam, producing shear force and bending moment in it. The forces are transferred to joints C and D by the beam. In turn, an axial force is produced in the braces and columns. There are two points that are worth noting. Firstly, the cross-sectional area of the column is larger than bracing system. Secondly, axial stiffness of the column is vertical while axial stiffness of the bracing system is diagonal. These two cause the column to withstand most of the axial load, while the bracing system withstands only a small part of that.

Moving on, a structural frame with the double-diagonal bracing (X bracing) system is demonstrated in Fig. 2.12a. Based on structural analysis, braces CB and AD act as compression and tension brace, respectively. Furthermore, magnitude of forces that are transferred to these braces will be equal, because the structure is symmetrical. The axial force is calculated through static equilibrium using a free-body diagram as shown in Fig. 2.12b.

With regard to the static equilibrium and symmetric properties of the structure under lateral loading, the axial forces of the brace and columns are $\frac{F}{2\cos\alpha}$ and $\frac{F}{2}\tan\alpha$ respectively. These forces are calculated as following:

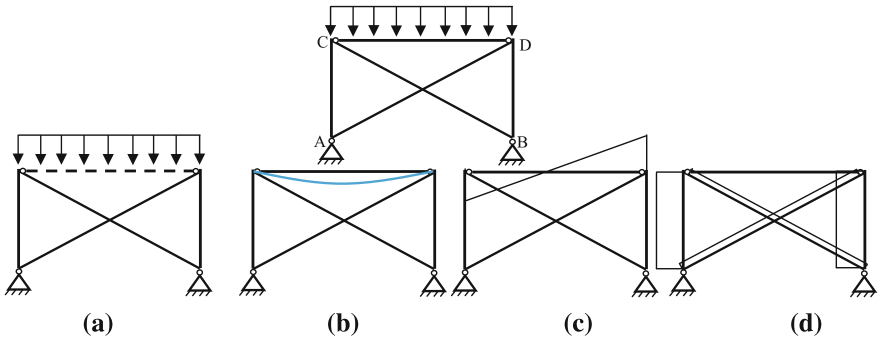


Fig. 2.11 Simple frame with X brace under gravity loading **a** deformation, **b** moment diagram, **c** shear diagram, **d** axial diagram

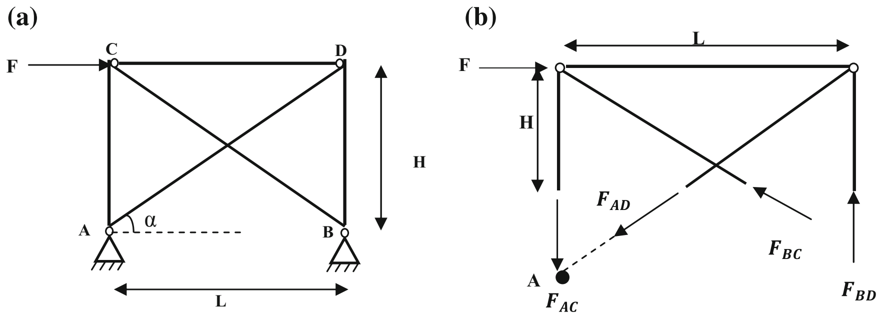


Fig. 2.12 **a** Simple frame with X bracing under lateral load. **b** Free body diagram of X-braced frame

$$\sum F_x = 0 \overset{F_{AD}=F_{BC}}{\Rightarrow} 2F_{AD}\cos\alpha = F$$
$$F_{AD} = F_{BC} = \frac{F}{2\cos\alpha} \tag{2.4}$$

$$\sum M_A = 0 \Rightarrow (F_{BD} + F_{BC}\sin\alpha)L = FH$$
$$\Rightarrow F_{BD} = \frac{FH}{L} - F_{BC}\sin\alpha = F\frac{H}{L} - \frac{F\ \sin\alpha}{2\cos\alpha} \tag{2.5}$$
$$= F\ \tan\alpha - \frac{F}{2}\tan\alpha \Rightarrow F_{BD} = \frac{F}{2}\tan\alpha$$

Figure 2.13 represents deformation and axial forces of the frame caused by axial force in the columns and double-diagonal bracing system.

From the diagram, it can be observed that the axial force produced the beam CD is caused by horizontal component of the bracing system. Nevertheless, assuming that the floor of the structure is rigid, this force will then be transferred to the floor. This is why there is no diagram for axial force present in the beam.

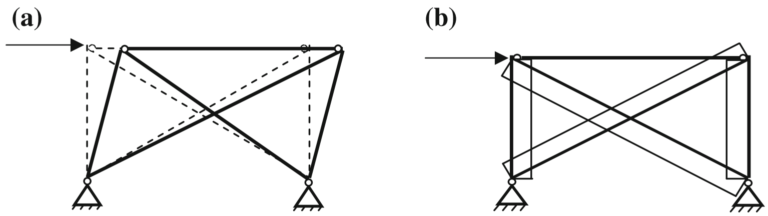


Fig. 2.13 Steel structural braced frame under lateral load **a** deformation, **b** axial force diagram

2.1.3.2 Structural Frame with Chevron Bracing System

Steel structural Chevron bracing system under the gravity loading is shown in Fig. 2.14. From the diagram, beam CE acts as a two-span continuous beam. Also, shear force and bending moment diagram that were obtained from solving the structural analysis equation are illustrated in Fig. 2.14.

Moreover, Fig. 2.15 demonstrates deformation of the frame and the distribution of the bending moment, shear force, and axial force.

From Figs. 2.11 and 2.15, it can be seen that the axial force in Chevron bracing system is larger than in double-diagonal. This is because the former supports under the beam, and increases axial force in the system. Consider the simple frame with Chevron bracing system under lateral load in Fig. 2.16.

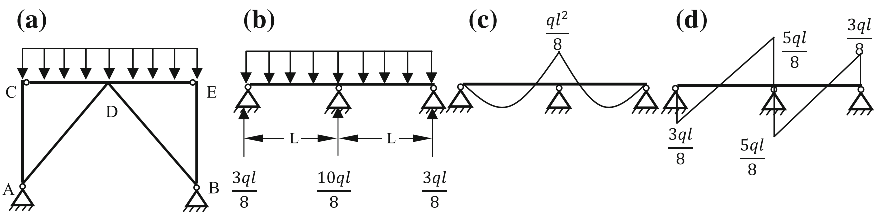


Fig. 2.14 Analysis of the beam CE of the simple frame with Chevron bracing system **a** reaction force, **b** bending diagram moment, **c** shear force diagram

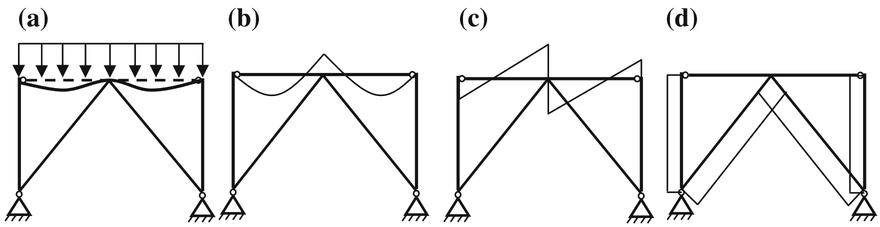
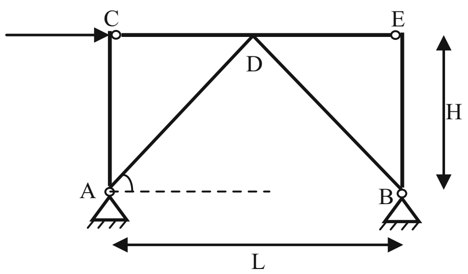


Fig. 2.15 Simple frame with chevron brace **a** deformation, **b** moment diagram, **c** shear diagram, **d** axial diagram

Fig. 2.16 Simple frame with chevron bracing under lateral load



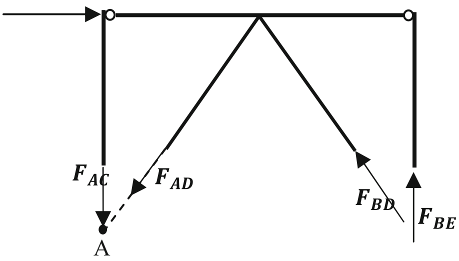


Fig. 2.17 Free body diagram of chevron bracing

A axial force in brace system and column are calculated based on their symmetric properties and static equilibrium. This is demonstrated in a free-body diagram in Fig. 2.17.

$$\begin{aligned} \sum F_x = 0 &\overset{F_{AD}=F_{BD}}{\Rightarrow} 2F_{AD}\cos\alpha = F \\ F_{AD} = F_{BD} &= \frac{F}{2\cos\alpha} \end{aligned} \tag{2.6}$$

$$\begin{aligned} \sum M_A = 0 &\Rightarrow (F_{BE} + F_{BD}\sin\alpha)L = FH \\ &\Rightarrow F_{BE} = \frac{FH}{L} - F_{BD}\sin\alpha \Rightarrow F\frac{H}{L} - \frac{F\sin\alpha}{2\cos\alpha} \\ &= F\frac{\tan\alpha}{2} - \frac{F}{2}\tan\alpha \Rightarrow F_{BE} = 0 \end{aligned} \tag{2.7}$$

The axial force in Chevron bracing system is F/2 cos α. Meanwhile, the corresponding value in the column is zero. The deformation and axial force diagram of the frame caused by axial forces in columns and braces are shown in Fig. 2.18.

Note:
Steel structural frame system with double-diagonal bracing and chevron bracing under lateral load are shown in Fig. 2.19. The shear force and bending moment are produced in the frames because of the lateral force F, therefore

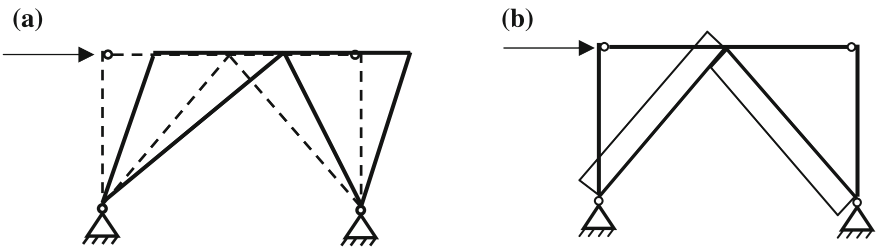


Fig. 2.18 A simple frame with chevron bracing under lateral loading **a** deformation, **b** axial force diagram

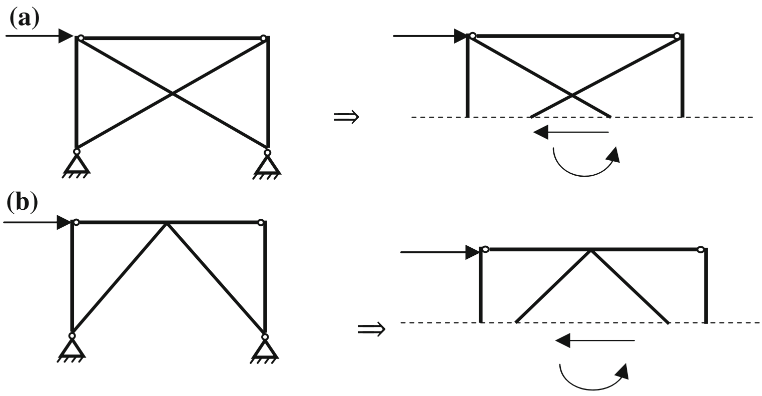


Fig. 2.19 **a** A simple steel frame with double diagonal bracing, **b** a simple frame with chevron bracing

In Double-Diagonal Bracing System

1. The shear force of the story is withstood by horizontal component of the bracing system.
2. The bending moment of the story is withstood by existing columns around the bracing system and the vertical component of the bracing system.

In Chevron Braced System

1. The shear force of the story is withstood by horizontal component of the bracing system.
2. The bending moment of the story is withstood just by the vertical component of the bracing system. The axial force of column is zero in this situation.

2.1.3.3 Structural Frame with Eccentric Bracing System

An eccentrically braced frame under the gravity loading is shown in Fig. 2.20a. The beam CF acts as a three-span continues beam. Deformation and distribution of bending moment, shear force, and axial force is shown in Fig. 2.20.

Consider the steel structural frame with eccentrically bracing system in Fig. 2.21a. The axial force of bracing system and columns are calculated based on the symmetry properties and static equilibrium equations and it is shown in free-body diagram in Fig. 2.21b.

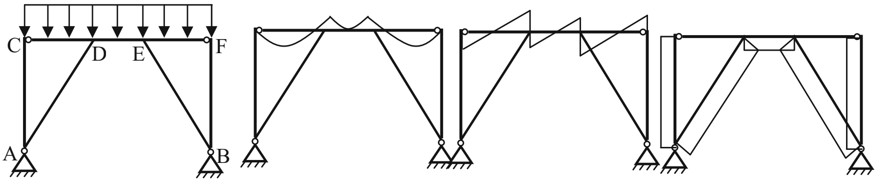


Fig. 2.20 An eccentrically braced frame under the gravity loading **a** the deformation, **b** the bending moment diagram, **c** the shear force diagram, **d** the axial diagram

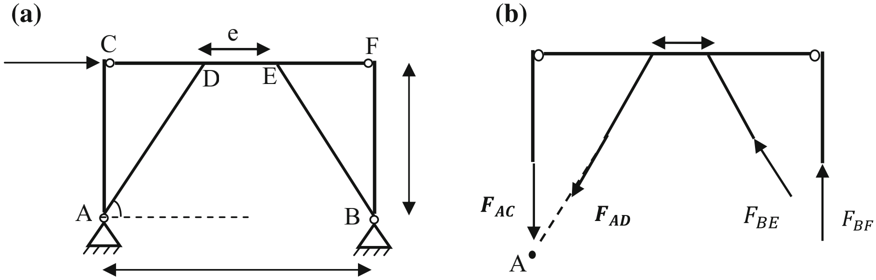


Fig. 2.21 **a** A simple frame with eccentrically bracing system under a lateral loading, **b** free diagram body of the eccentrically braced frame

$$\begin{aligned}&\sum F_x = 0 \overset{F_{AD}=F_{BE}}{\Rightarrow} 2F_{AD}\cos\alpha = F\\&F_{AD} = F_{BE} = \frac{F}{2\cos\alpha}\end{aligned}\tag{2.8}$$

$$\begin{aligned}\sum M_A &= 0 \Rightarrow (F_{BF} + F_{BE}\sin\alpha)L = FH \Rightarrow F_{BF}\\&= \frac{FH}{L} - F_{BE}\sin\alpha \Rightarrow F\frac{H}{\frac{(L-e)}{2}} - \frac{F\ \sin\alpha}{2\cos\alpha}\\F_{BF} &= -\frac{F}{2}\tan\alpha\left(\frac{e}{L}\right)\end{aligned}\tag{2.9}$$

In the free-body diagram, direction of axial force should be modified. This suggestion is based on the calculation of axial force in the columns—the right column is under tension and the left column is under compression. The deformation and distribution of bending moment, shear force, and axial force are illustrated in Fig. 2.22.

The linked beam (fuse) plays significant role in a steel structural frame with eccentric bracing system. It transfers shear force and bending moment to the frame.

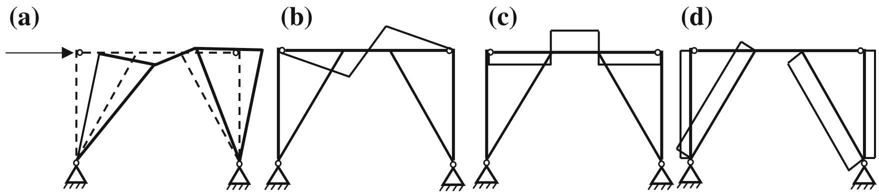


Fig. 2.22 A simple frame with eccentrically bracing system under lateral loading **a** deformation, **b** bending moment diagram, **c** shear force diagram, **d** axial force diagram

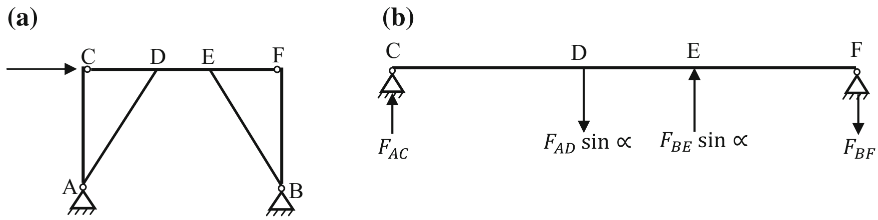
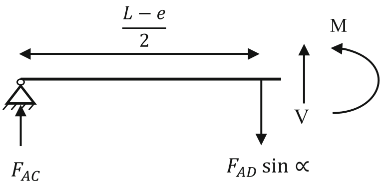


Fig. 2.23 **a** Simple frame with eccentrically braced under lateral loading, **b** forces apply to simple beam in eccentrically braced frame under lateral load

The shear force and bending moment in linked beam are calculated using the following equation. Figure 2.23a shows a simple frame with eccentric braced frame with lateral load.

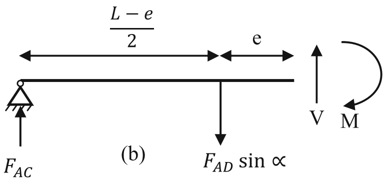
In addition, the beam in eccentric braced frame supports two concentrated loads resulted from bracing forces. The magnitude and reaction of these forces have been discussed beforehand, therefore only vertical component of the bracing system will be considered. These are computed according to the data presented in Fig. 2.23b, the free-body diagram of the link beam and the static equilibrium. Meanwhile, Fig. 2.24a, b present the calculation of shear force and bending moment in the linked beam.

Moving on, there are no bending moment and shear force in the beams of a story with a concentric braced frame such as double-diagonal or Chevron bracing system. It can therefore be concurred that these beams are designed for gravity loadings. Meanwhile, in eccentric braced frame, the shear force and bending moment are produced by shear force of each story. On the other hand, shear force in lower floors, is bigger than in upper floors. This will produce larger shear force and bending moment in each beam, resulting in increased cross-sectional size of the beam in lower floors (Mohsen & Reza, 2015).



(a)

$$\begin{cases}\sum F_y = 0 \Rightarrow V = F_{AD}\sin\alpha - F_{AC} = \frac{F}{2}\tan\alpha\left(1-\frac{e}{L}\right) & (2.10)\\ \sum M_B = 0 \Rightarrow M = F_{AD}\sin\alpha\left(\frac{L-e}{2}\right) - V\left(\frac{L-e}{2}\right) = \frac{F}{2}\tan\alpha\left(\frac{L-e}{2}\right)\left(\frac{e}{L}\right) & (2.11)\end{cases}$$

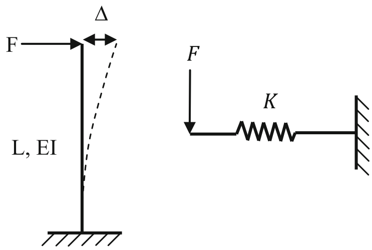


$$\begin{cases}\sum F_y = 0 \Rightarrow V = F_{AD}\sin\alpha - F_{AC} = \frac{F}{2}\tan\alpha\left(1-\frac{e}{L}\right) & (2.12)\\ \sum M_B = 0 \Rightarrow M = -F_{AD}\sin\alpha\left(\frac{L-e}{2}+\right) + V\left(\frac{L-e}{2}+e\right) = \frac{F}{2}\tan\alpha\left(\frac{L-e}{2}\right)\left(\frac{e}{L}\right) & (2.13)\end{cases}$$

Fig. 2.24 **a** The free body diagram of the left side of link beam, **b** The free body diagram of the right side of link beam

2.1.4 Stiffness of Different Type of Bracing Frame

Figure 2.25 illustrates the concept of stiffness according to Hook's low. As presented in Fig. 2.25, force (F) is applied to the end of cantilever beam, causing displacement (Δ). Based on Hook's low, the beam is modelled by spring of stiffness (K) that produced displacement (Δ) by force (F). Equation (2.14) shows the concept of stiffness. According to Eq. (2.14), it can be concurred that the beam acts as a spring of stiffness $K = \frac{3EI}{L^3}$.



$$\begin{cases} Column: \Delta = \frac{FL^3}{3EI} \rightarrow F = \frac{3EI}{L^3} \times \Delta \\ spring: F = K\Delta \end{cases} \quad (2.14)$$

Fig. 2.25 Concept of stiffness

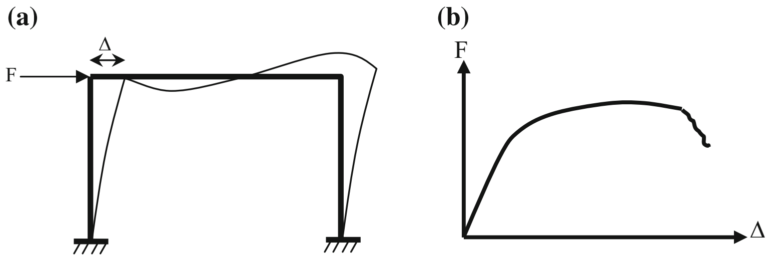
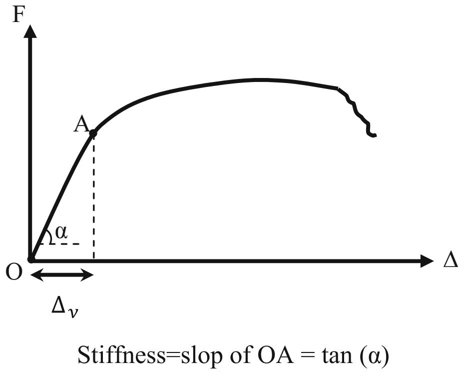


Fig. 2.26 **a** Moment resistance frame under lateral load, **b** push over graph

Fig. 2.27 Concept of stiffness in force-displacement graph



To better understand the stiffness concept, consider one story moment resisting frame which the force F is applied to—lateral displacement (Δ) is produced. Force F increases gradually from zero until the structure collapses. Pushover graph is illustrated in Fig. 2.26.

The first part of graph in Fig. 2.27 is linear. This shows that the structure has elastic behaviour until point (A). Then, structure behaves nonlinearly. The stiffness of the structure can be observed from the slope of curve in linear region; therefore it is defined after point (A). Nevertheless, based on the graph in Fig. 2.27, the slope of force-displacement curve in nonlinear region decreases dramatically compared to linear region. It can be concluded that when structure enters its nonlinear response range, the stiffness of structure starts to decrease significantly.

Virtual work is used to calculate the stiffness of the bracing system. At first, the concentrated unit load is imposed on the bracing system. The displacement of the bracing frame is then calculated using virtual work. With regard to Hook's low, the division of force with displacement indicated stiffness, but the force in this discussion is 1. Hence, the stiffness is inversed to displacement. The following discussion will explain more about virtual work, and there will be some calculations on displacement and stiffness of different types of bracing frame system.

2.1.4.1 Virtual Work

Virtual work is described differently by varied scholars. Basing on the Principle of Minimum Total Potential Energy failure of the system is likely to be experienced with minute variation to the equilibrium. According to the Principle of Virtual Work the equilibrium of a body is only considered when the virtual work of all forces operating on the body is at zero. The word 'virtual' refers to a context having an impact on but devoid of the particular. In other words the impact is not seen in reality. The principle of Virtual Work comprises of two other sub-principles that include Principle of Virtual Forces as well as the Principle of Virtual Displacements. The latter principle defines virtual work as the end product of the body's actual forces as it is in motion through virtual displacement. Thus, geometry can be utilized in the attempt to solve challenges of equilibrium. The Principle of Virtual Forces is identical to the latter principle but is unique in the sense that it provides that forces are made to be in motion by real displacements. In line with this argument, it is then evidenced that equilibrium can be used in solving the geometry problem.

Unit Load Method as Applied to Trusses

- **External Loading**

In the context of a plane of truss, the only internal forces present are axial. This is attributed to the external loads that are precisely applied at the joints. To this end, the equation should be expressed as follows

$$\sum_{j=1}^{n} \delta F_j U_j = \int_{0}^{L} \frac{\delta f F d_s}{EA} \tag{2.15}$$

whereby

the external virtual force is represented by δF_j,
the actual displacement of the brace frame, on the other hand, is represented by U_i
the virtual stress generated by the virtual load is expressed as δf

while the actual internal deflection of the brace frame founded by the real load is expressed as $\int_0^L \frac{F}{EA} ds$. Consequently, the length of the member, cross-sectional area of a member as well as the modulus elasticity of a member are represented as L, A, and E respectively in the equation. The calculation of this equation is based on $\delta F_j = 1$ while all the other virtual forces, $\delta F_i (i = 1, 2, \ldots, j-1, j+1, \ldots, n)$, will be considered to be zero.

Further, should the cross-sectional areas of A remain constant then the integration has the potential of being replaced by summation. In such a scenario, then the equation is expressed as follows

$$U_j = \sum_{i=1}^{m} \frac{(\delta f)_{ij} F_i L_i}{E_i A_i} \tag{2.16}$$

where

m	represents the number of members
$(\delta f)_{ij}$	represents the Internal virtual axial force in member i caused by the unit virtual load at j
$\left(\frac{F_i}{E_i A_i}\right) L_i$	represents the total deformation of member i affected by real loads

Additionally, the following equation expresses the total deformation

$$U_j = \sum_{i=1}^{m} (\delta f)_{ij} \Delta_i \tag{2.17}$$

where,

Δ_i stands for real change in length of member i caused by real loads

- **Temperature Loading**

Subsequently, the impacts of changes in environmental temperature included expansion or shrinking of the bracing members thus resulting in deflections of the joint. This reaction is mathematically expressed in the equation provided below. Moreover, the calculation of the variation in the length of the member can follow this relation.

$$\boldsymbol{\Delta_i = \alpha T L_i} \tag{2.18}$$

where

α	Coefficient of the thermal expansion of the member i,
L_i	length of the member
T	temperature change

- **Fabrication Errors and Camber**

Occasionally, there can be errors in the process of fabricating bracing members. This can be exemplified in instances where the bracing members are shorter or longer than it is necessitated to provide an incline to the bracing frame. Essentially, the bridge truss usually provides the camber by having the bottom chord curve upward and with similar force deflects downwards. The calculation of the truss joint deflection in such cases follows the following equation

$$\Delta_i = e_i \tag{2.19}$$

where,

e_i represents the fabrication error in the length of the member.

e_i is regarded as positive when the fabrication is longer as opposed to the actual length. A negative is regarded when it is otherwise.

- **Calculating Bracing System Deflection Steps**:

  I. Establish the actual forces in the member of the bracing. This is achieved through utilization of sections, methods or joints caused by the applied forces externally. This is followed by the observation of the real deformation in (Δ_i) in every member of equation $\frac{F_i L_i}{E_i A_i}$. Further, the tensile force can be presumed to be positive whereas the compression force is negative.

  II. Note virtual load system. The unit load should only be considered at either vertical or horizontal joints that appear to have a deflection. Virtual forces should be tabulated in every member linked to the applied unit load at the jth load Use Eq. (2.16) to evaluate the jth joint deflection

  III. For purposes of evaluating the jth joint deflection, the Eq. (2.17) should be utilized.

  IV. Should the deflection of joint be affected by change in temperature, then actual deformation should be considered in (Δ_i) in each member of the equation. $\Delta_i = \alpha T L_i$ (Fig. 2.28)

Calculation of Stiffness Indeterminate Braced Frame

This section will focus on introducing the force method analysis of indeterminate structure otherwise referred to as the flexibility method. Additionally, it is referred to as the method of consistent deformation. The flexibility method is traced back to 1864 and 1874 by J. Maxwell and O.C. Mohr respectively. In reference to computer programming, this method is not considered to be practical. Programming is devoid of choice of redundancy. However, this method necessitates the deflection the statistically determinate structure. Additionally, when compared to the stiffness method, the flexibility matric band width in the force method is larger. This

Member	Length	Area	F_i	$(\delta f)_i$	$\frac{(\delta f)_i F_i L_i}{E_i A_i}$
				$\sum$	

Fig. 2.28 Pattern table for determination of displacement in virtual work method

notwithstanding the method is ideal when it comes to hand computing. The force method analysis is summarised in the next text:

Step 1:

To begin with, the level of statistically indeterminacy of the redundant and the structure should be established. The redundant is categorized as unknown. This is followed by releasing the redundant each at a time for purposes of ensuring that a statistically determinate structure is attained upon realizing the redundant reaction $(R_B$ or $M_A)$. This is considered to be the equivalent of eliminating the challenges that respond to a reaction that is redundant. In a bid to transform the propped cantilever to a cantilever beam the roller support should be released. Ultimately, as earlier indicated the reaction should be categorized as redundant reactions (R_B). Internal redundant force results in the discontinuity of the corresponding member should the structure be released. It is important to note that the continuity conditions are the same as the compatibility conditions for internal forces that are redundant.

Step 2:

Start by tabulating the deflection that matches up the action classified as redundant. This should be done separately with one focus being the applied load while the other focuses on the cause of redundant forces drawn from displacement relations. It is important to note that the above mentioned cannot be computed devoid of having the magnitude of the redundant force.

In an effort to establish the deflection, one is necessitated to apply a unit load to the direction of the redundant force. Multiplying the obtained deflection from the application of the unit value of force and the unknown force can give the deflection of the redundant force. The method of superposition facilitates this.

Step 3:

The application of the principle of superposition is done here through the tabulation of the total deflection caused by redundant force and applied load. Additionally, the value needs to be complemented by the original structure's actual conditions of the boundary. For example, when the total deflection is zero then original structure's deflection is also zero. However, a set of equations with unidentified redundant forces and flexibility coefficients is needed when there is more than one redundant force. Therefore, it can occur that the total number of the equation is similar to the total number of unidentified redundant force.

Step 4:

It serves as the final step and necessitates the evaluation of all internal forces and all reactions. As earlier mentioned the method of superposition can be applied on any given structure such as truss, beams and frames or a combination of the aforementioned structures. Additionally, it has the general applicability of all loadings. Nonetheless, there are varied methods that can be used to tabulate the

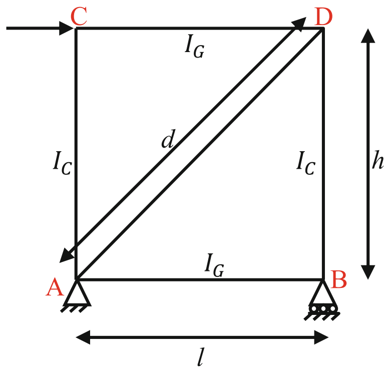


Fig. 2.29 Single diagonal braced frame under unit lateral load

deflection of the statistically determinate structure. Moment area theorem, and unit load method are some of the methods that can be utilized by researchers (IIT, 2008).

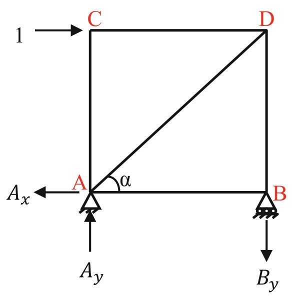
Calculation of Stiffness of Diagonal Bracing System

The unit load should be applied to external load and determined the horizontal deflection to calculate the stiffness of diagonal braced frame. Also, horizontal deflection should be calculated, assuming the truss ABCD in Fig. 2.29 unit load is applied to joint C.

Moreover, the given bracing system is statically determinate. Their reactions are demonstrated in Fig. 2.30 with other member forces that are calculated using the equation of static equilibrium.

Furthermore, with regard to joints method, internal forces should be calculated. Figure 2.31 demonstrates the steps of joints method.

A unit load must be applied and the virtual forces δf_i should be determinedin each member for calculating the horizontal displacement at ‘C’, as shown in Fig. 2.32. The value of the internal forces are also given in the following figures. The forces



$$\begin{cases} \sum f_x^{\rightarrow} = 0 \quad -A_x + 1 = 0 \Rightarrow A_x = 1 \\ \sum M_B = 0 \quad -\left(A_y \times l\right) - (1 \times h) = 0 \Rightarrow A_y = -\dfrac{h}{l} \\ \sum f_y^{\uparrow} = 0 \quad A_y - B_y = 0 \Rightarrow B_y = -\dfrac{h}{l} \end{cases}$$

Fig. 2.30 Single diagonal braced frame reaction

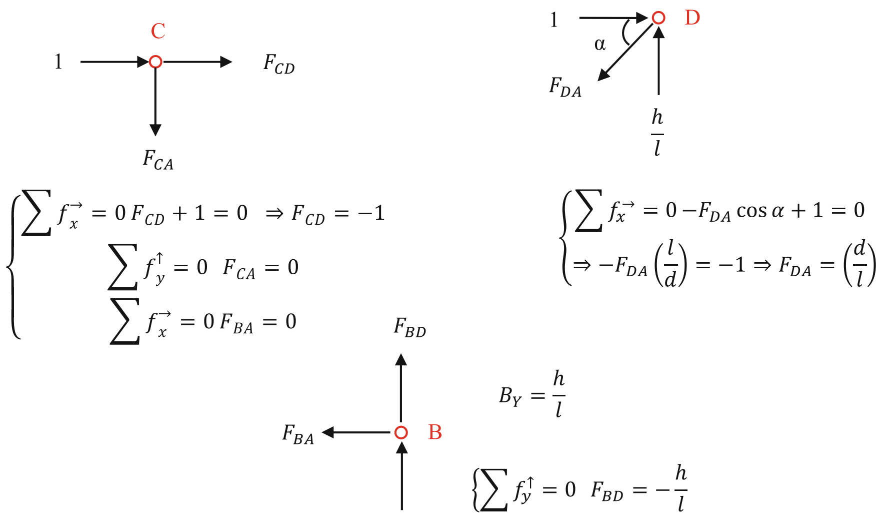


Fig. 2.31 Calculating internal forces using joints method

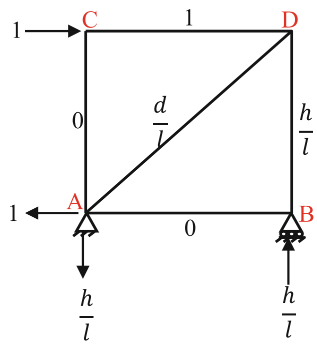


Fig. 2.32 Apply unit load to the single diagonal braced frame for the second time

which are in tension and compression have been taken as positive and negative, respectively. The arrows which have been drawn at the end of each member indicate the direction which the force is applied to the joint of the member in.

Moreover, unit load method is employed to calculate the deflection at joint 'C'. The completed calculations, which are self-explanatory, are tabulated in Table 2.1.

$$\begin{cases} \Delta = \frac{1}{E}\sum \frac{FfL}{A} \\ \Delta = \frac{1}{E}\sum \left(\frac{d^3}{l^2 Ad} + \frac{h^3}{l^2 Ac} + \frac{l}{Ag}\right) \end{cases} \tag{2.20}$$

Table 2.1 Calculation of displacement in single diagonal braced frame based on virtual work method

Member	Length	Area	F_i	(δf_i)	$\frac{(\delta f_i)F_iL_i}{EA}$
AB	*l*	*Ag*	*0*	*0*	*0*
AC	*h*	*Ac*	*0*	*0*	*0*
AD	*d*	*Ad*	$-d/l$	$-d/l$	$\frac{d^3}{El^2Ad}$
BD	*h*	*Ac*	h/l	h/l	$\frac{h^3}{El^2Ac}$
DC	*l*	*Ag*	*1*	*1*	$\frac{l}{EAg}$

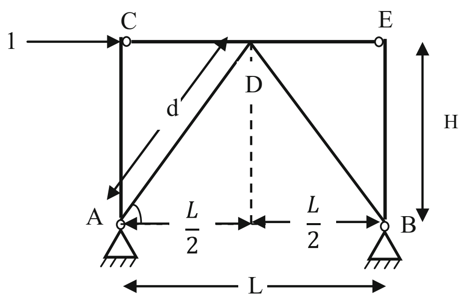
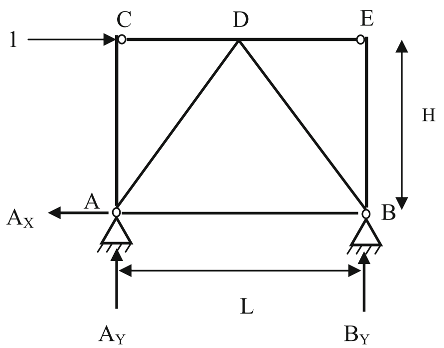


Fig. 2.33 Chevron braced frame under unite lateral load

Chevron Braced Frame System (V and V Inverted)

The displacement and stiffness of Chevron braced frames are calculated using virtual work method. The steps are the same as those of single diagonal braced frame. Figure 2.33 shows a Chevron braced frame with the unit lateral load.

Referring to virtual work method, the first step is to determine the reaction forces under unit lateral loading. Figure 2.34 shows the calculation of reaction forces with the unit lateral load.



$$\sum \overset{+}{\rightarrow} fx = 0 \Rightarrow \begin{cases} -Ax + 1 = 0 \\ Ax = 1 \end{cases}$$

$$\sum f_y^{\uparrow} = 0 \Rightarrow \begin{cases} (A_y + B_y) = 0 \\ B_y = \frac{h}{l} \end{cases}$$

$$\sum M_B = 0 \Rightarrow \begin{cases} -(Ay)(l) - (1)(h) = 0 \\ Ay = -\frac{h}{l} \end{cases}$$

Fig. 2.34 Reaction forces of Chevron braced frame under unit lateral load

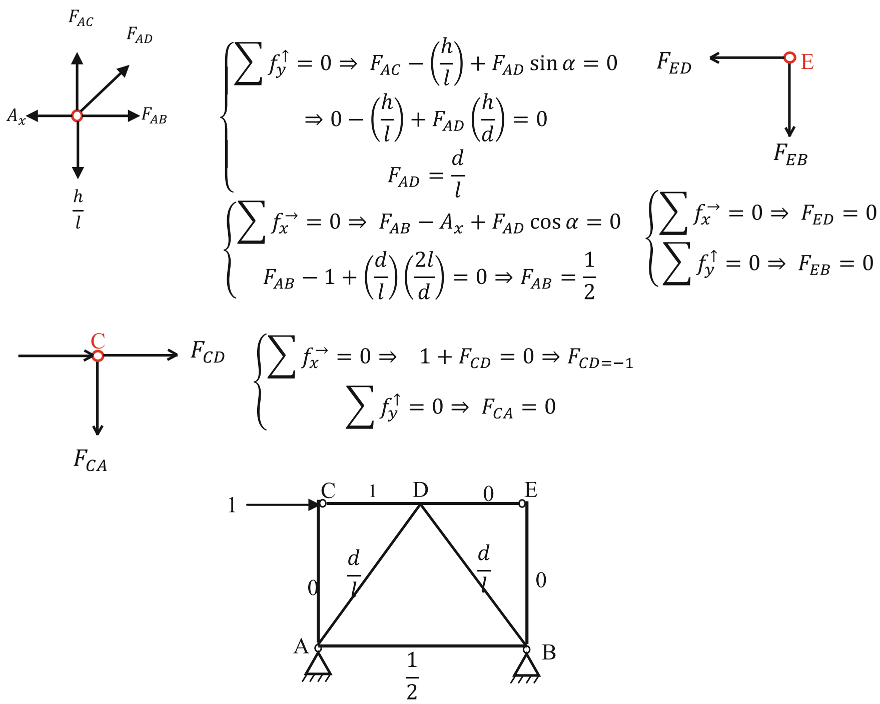


Fig. 2.35 Internal force in Chevron braced frame

After calculating the reaction forces, the internal forces should be calculated based on the joints method. Figure 2.35 shows step-by-step procedure of joints method.

The Calculation of displacement of Chevron braced frame based on virtual work method is shown in Table 2.2.

$$\Delta = \frac{1}{E}\left(\frac{2d^3}{l^2 Ad} + \frac{3l}{4Ag}\right) \tag{2.21}$$

Table 2.2 Displacement of chevron (V inverted) braced frame

Member	L	A	F	f	Ff	$\frac{FfL}{A}$
AB	l	Ag	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{E}$	$\frac{l}{4Ag}$
CD	$l/2$	Ag	1	1	1	$\frac{l}{2Ag}$
ED	$l/2$	Ag	0	0	0	0
AC	h	Ac	0	0	0	0
BE	h	Ad	0	0	0	0
AD	d	Ad	$-d/l$	$-d/l$	d^2/l^2	$\frac{d^3}{l^2 Ad}$
BD	d	Ad	d/l	d/l	d^2/l^2	$\frac{d^3}{l^2 Ad}$

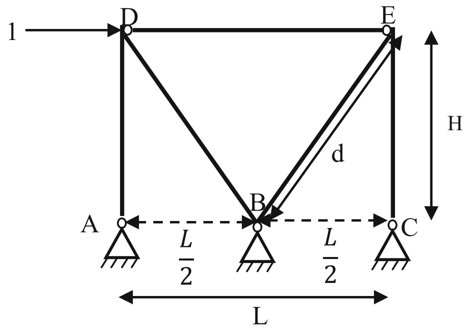


Fig. 2.36 Chevron braced frame under unite lateral load

Chevron Braced Frame System (V Inverted)

Next, step-by-step calculations of Chevron (V) braced frame are illustrated in Figs. 2.36, 2.37 and 2.38).

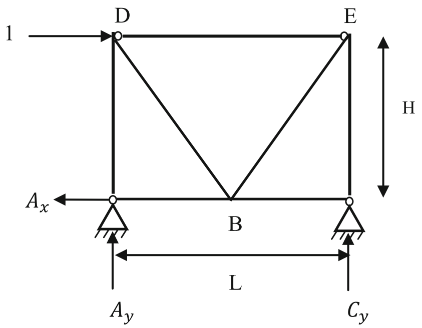
Additionally, virtual work method stated that the reaction forces of the system under unit lateral load must be determined in the beginning. Figure 2.37 presents the calculation of the reaction forces of the system with the unit lateral load.

Apart from that, the calculation of displacement of Chevron braced frame using virtual-Work method is shown in Table 2.3.

$$\Delta = \frac{1}{E}\left(\frac{2d^3}{l^2Ad} + \frac{2h^3}{l^2Ac} + \frac{3l}{4Ag}\right) \tag{2.22}$$

Calculation Stiffness of Double Diagonal Braced Frame

The double diagonal braced frame is a concentrically brace framed with one degree indetermination. The force method is the main method to analyze an indeterminate truss. Therefore, for analyzing an indeterminate truss: the degree of static



$$\begin{cases} \sum \overset{+}{\rightarrow} f_x = 0 \Rightarrow \begin{cases} -Ax + 1 = 0 \\ Ax = 1 \end{cases} \\ \sum f_y = 0 \Rightarrow \left\{ Cy = \frac{h}{l} \right. \\ \sum M_B = 0 \Rightarrow \begin{cases} -(Ay)(l) - (1)(h) = 0 \\ Ay = -\frac{h}{l} \end{cases} \end{cases}$$

Fig. 2.37 Reaction forces of Chevron brace frame under unit lateral load

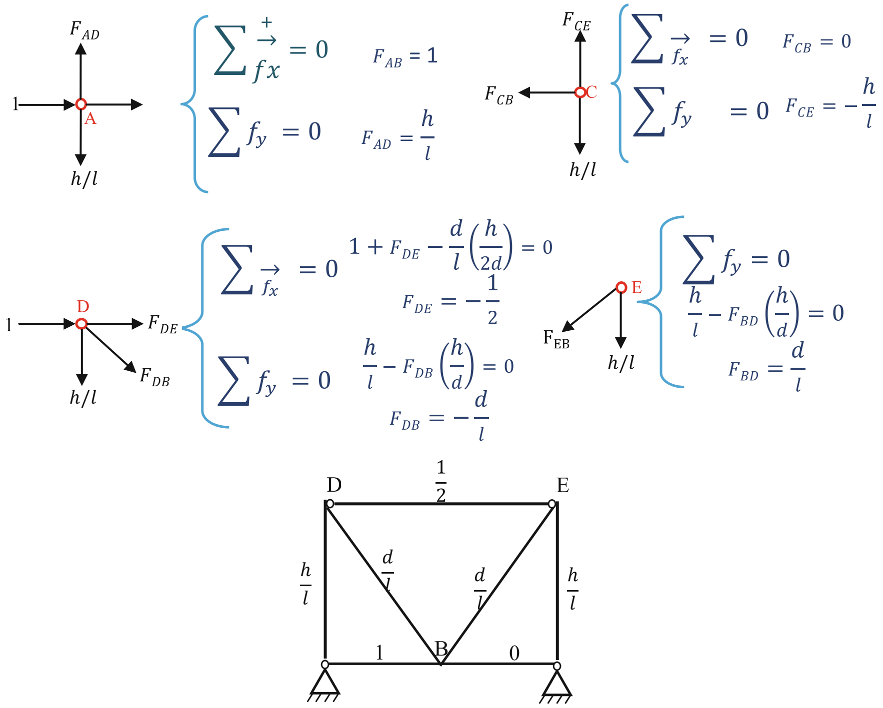


Fig. 2.38 Chevron braced frame internal force

Table 2.3 Displacement of chevron (V inverted) braced frame

Member	L	A	F	f	Ff	$\frac{FfL}{A}$
AB	l	Ag	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{E}$	$\frac{l}{4Ag}$
CD	$l/2$	Ag	1	1	1	$\frac{l}{2Ag}$
ED	$l/2$	Ag	0	0	0	0
AC	h	Ac	0	0	0	0
BE	h	Ad	0	0	0	0
AD	d	Ad	$-d/l$	$-d/l$	d^2/l^2	$\frac{d^3}{l^2 Ad}$
BD	d	Ad	d/l	d/l	d^2/l^2	$\frac{d^3}{l^2 Ad}$

indeterminacy of the structure should be determined, the number of the redundant reactions will be equal to the degree of indeterminacy, the redundant constraints must be removed then the resulting truss will be statically determinate. Select redundant as the reaction component in excess of three and the rest of the member forces. Although, one of member could choose redundant actions completely from member forces. Figure 2.39 shows double diagonal braced frame under lateral load.

Moving on, it can be seen that the plane double diagonal bracing in Fig. 2.40 is statically indeterminate to the first degree. Therefore, the reaction can be calculated

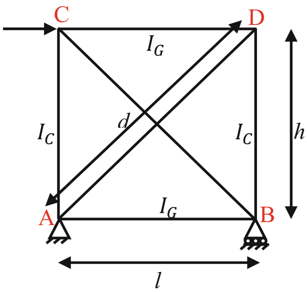


Fig. 2.39 Concentrically braced frame with one degree in determination

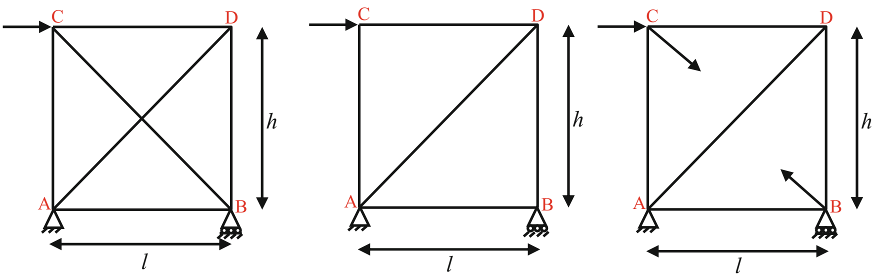


Fig. 2.40 Cut member CB to obtained released structure

from the equations of statics alone. This will now be discussed. First, select a bar force F_{CB} in member CB as redundant and cut it to obtain the released structure, as demonstrated in Fig. 2.40. Next, keep the cut version in the bracing system, as its deformation need to be a part of the calculation of displacements in the released structure. It should be noted that the redundant consists of the pair of forces acting on the released structure.

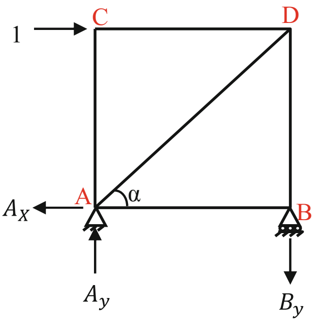
The reactions of the diagonal bracing are calculated using static equations of equilibrium (Fig. 2.41).

The first step in force method is to calculate the displacement that corresponds to redundant member force in the determinate structure caused by applied external loading. This can be done using unit load method.

$$\Delta_{CB} = \sum F_i (U_{CB})_i \frac{L_i}{AE} \tag{2.23}$$

where

F_i forces in the released truss due to applied loading (Fig. 2.42)
U_{CB} forces in the released truss due to unit load



$$\sum f_x^{\rightarrow} = 0 \quad -A_x + 1 = 0 \;\Rightarrow A_x = 1$$

$$\sum M_B = 0 \quad -(A_y \times l) - (1 \times h) = 0 \Rightarrow A_y = -\frac{h}{l}$$

$$\sum f_y^{\uparrow} = 0 \quad A_y - B_y = 0 \Rightarrow B_y = -\frac{h}{l}$$

Fig. 2.41 Calculation of reaction force

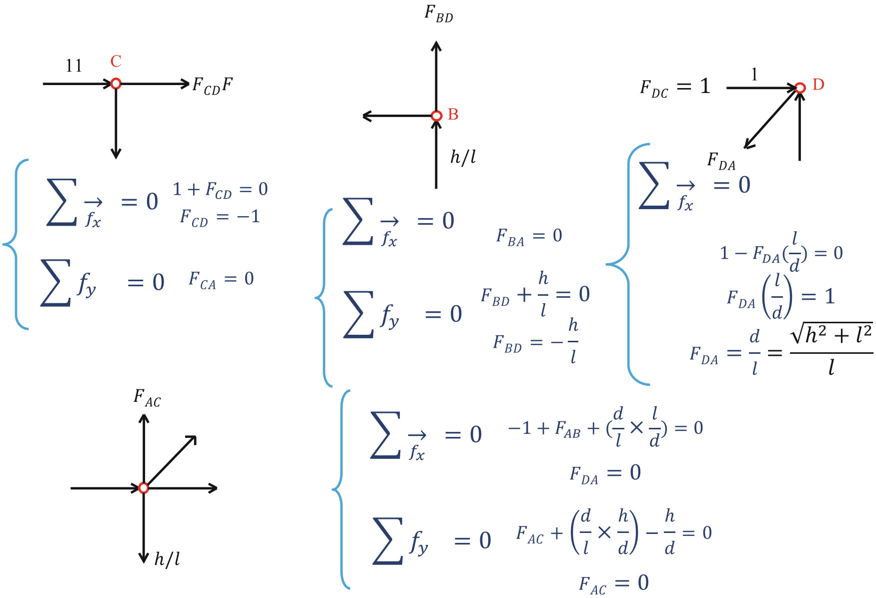


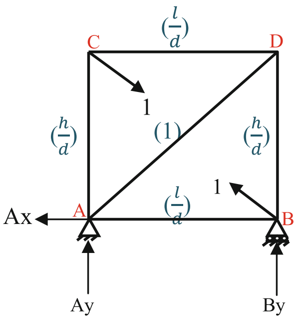
Fig. 2.42 Calculation of internal force in determinate brace frame

F_i : *forces in the realesed truss due to applied loading*

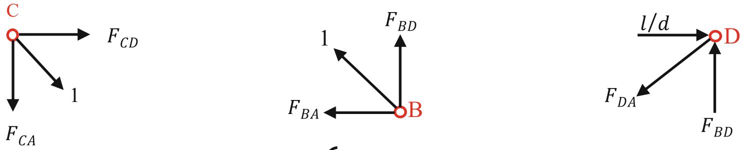
U_{CB} : *forces in the realesed truss due to unit load*

Furthermore, an external load should be applied and member forces (F_0) must be calculated. Also, unit virtual load must be applied as F_{BC}. Apart from that, member forces $(U_{CB})_i$ should be calculated to obtain displacement (Δ_{CB}) value, as demonstrated in Fig. 2.43. In the next step, a real unit load is applied at the joints of redundant member for the calculation of displacement of unit load method.

Fig. 2.43 Calculating the internal force using a real unit load along the redundant member



$$Ax = Ay = By = 0$$



$$\sum \overset{+}{\underset{fx}{\rightarrow}} = 0$$

$$F_{CD} + (1) + (\frac{l}{d}) = 0$$

$$F_{CD} = \frac{-l}{d}$$

$$\sum f_y = 0$$

$$-F_{CA} + (-1)(\frac{h}{d}) = 0$$

$$F_{CD} = -\frac{h}{d}$$

$$\sum \underset{fx}{\rightarrow} = 0$$

$$-F_{BA} - 1\left(\frac{l}{d}\right) = 0$$

$$F_{BA} = \frac{l}{d}$$

$$\sum f_y = 0$$

$$F_{BD} + 1\left(\frac{h}{d}\right) = 0$$

$$F_{BD} = -\frac{h}{d}$$

$$\sum \underset{fx}{\rightarrow} = 0$$

$$\frac{l}{d} - F_{DA}\left(\frac{l}{d}\right) = 0$$

$$F_{DA} = 1$$

$$\delta_{CB,CB} = \sum (U_{CB})^2 \frac{L_i}{A_i E_i}$$

The complete calculations can be simply shown in tabular form as following in Table 2.4.

$$\Delta CB = \frac{1}{AE} \sum F_0 U_{CB} L$$

$$= \frac{1}{AE}\left(\frac{d^2}{l} + \frac{h^3}{ld} + \frac{l^2}{d}\right) = \frac{1}{AE}\left(\frac{d^3 + h^3 + l^3}{ld}\right)$$

$$\boldsymbol{\delta_{CB,CB}} = \frac{1}{AE} \sum U_{CB}^2 L$$

$$= \frac{1}{AE}\left(\frac{l^3}{d^2} + \frac{l^2 h}{d^2} + d + d + \frac{h^3}{d^2} + \frac{l^3}{d^2}\right) = \frac{1}{AE}\left(\frac{2l^3 + 2h^3 + 2l^3}{d^2}\right)$$

$$= \frac{2}{AE}\left(\frac{l^3 + h^3 + d^3}{d^2}\right)$$

Table 2.4 Calculation of redundant force

Member	L	F_0	U_{CB}	$F_0U_{CB}L$	U_{CB}^2L	$F = F_0 + U_{CB}F_{CB}$
AB	l	0	l/d	0	l^3/d^2	$= 0 + \left(\frac{l}{d}\right)\left(\frac{-d}{2l}\right) = -\frac{1}{2}$
AC	h	0	h/d	0	h^3/d^2	$= 0 + \left(\frac{h}{d}\right)\left(-\frac{d}{2l}\right) = \frac{-h}{2l}$
AD	d	$-d/l$	-1	d^2/l	d	$= \frac{-d}{l} + (-1)\left(\frac{-d}{2l}\right) = \frac{-d}{2l}$
BD	d	0	-1	0	d	$= 0 + (-1)\left(\frac{-d}{2l}\right) = \frac{2}{2l}$
BD	h	h/l	h/d	h^3/ld	h^3/d^2	$= \frac{h}{l} + \left(\frac{h}{d}\right)\left(-\frac{d}{2e}\right) = \frac{h}{2l}$
DC	l	1	l/d	h^2/d	l^3/d^2	$= 1 + \left(\frac{l}{d}\right)\left(\frac{-d}{2l}\right) = \frac{1}{2}$

Table 2.5 Calculation of displacement of double diagonal bracing system

Member	L	A	F	f	$\frac{FfL}{A}$
AB	l	Ag	$\left(\frac{-1}{2}\right)$	$\left(\frac{-1}{2}\right)$	$\frac{l}{4Ag}$
AC	h	Ac	$\left(\frac{-h}{2l}\right)$	$\left(\frac{-h}{2l}\right)$	$\frac{h^3}{4l^2Ac}$
AD	d	Ad	$\left(\frac{-d}{2l}\right)$	$\left(\frac{-d}{2l}\right)$	$\frac{d^3}{4l^2Ad}$
BC	d	Ad	$\left(\frac{d}{2l}\right)$	$\left(\frac{d}{2l}\right)$	$\frac{h^3}{4l^2Ad}$
BD	h	Ac	$\left(\frac{h}{2l}\right)$	$\left(\frac{h}{2l}\right)$	$\frac{h^3}{4l^2Ac}$
DC	l	Ag	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)$	$\frac{l}{4Ag}$

Moving on, the compatibility condition reflected that relative displacement of cut member CB is caused by external loading and relative displacement Hence, axial forces must be zero. Table 2.5 demonstrates the displacement of double diagonal braced frame

$$\Delta_{CB} + \boldsymbol{F_{CB}\delta_{CB,CB}} = 0$$

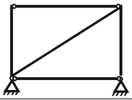
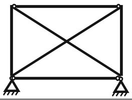
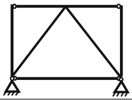
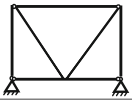
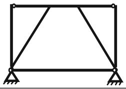
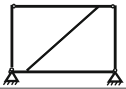
$$= \frac{1}{AE}\left(\frac{d^3 + h^3 + l^3}{ld}\right) + F_{CB}\left(\frac{2}{AE}\right)\left(\frac{d^3 + h^3 + l^3}{d^2}\right) = 0$$

$$F_{CB} = -\frac{d}{2l}$$

$$\Delta = \frac{1}{E}\sum\left(\frac{2l}{4Ag} + \frac{2h^3}{4l^2Ac} + \frac{2d^3}{4l^2Ad}\right)$$
$$\Delta = \frac{1}{2E}\sum\left(\frac{l}{Ag} + \frac{h^3}{l^2Ac} + \frac{d^3}{l^2Ad}\right) \quad (2.24)$$

In a concentric braced frame, the axial force is produced in the beam by the horizontal component of bracing system but it is assumed that the floor of the structure is rigid so this force is transferred to the floor; therefore, it is not considered beam's axial force. Table 2.6 shows the stiffness of different types of bracing systems. The stiffness is calculated according to the virtual work and spring theory.

Table 2.6 Stiffness formulation of different types of bracing system

Type of bracing	Dimensions	Stiffness
Single diagonal		$K = \frac{E\left(l^4 A_d A_c A_g\right)}{\left(l^2 A_c A_g d^3 + l^2 A_d A_g h^3 + l^5 A_d A_c\right)}$
Double diagonal		$K = \frac{2E\left(l^4 A_d A_c\right)}{d^3 A_c l^2 + l^2 A_d h^3}$
Chevron (V inverted)		$K = \frac{4E\left(l^2 A_d A_g\right)}{8d^3 A_g + l^2 A_d}$
Chevron		$K = \frac{4E\left(l^4 A_d A_c A_g\right)}{\left(8l^2 A_c A_g d^3 + 8l^2 A_d A_g h^3 + l^5 A_d A_c\right)}$
Eccentrically bracing		$K = \frac{12E\left(m^2 A_d A_g I_g l\right)}{6A_g I_g d^3 l + 6m^3 A_d I_g l + m^2 A_d A_g h^2 (1-2m)^2}$
Offset diagonal		$K = \frac{6E\left((1-2m)^2 A_d A_g I_g l\right)}{6A_g I_g d^3 l + (1-2m)^2 A_d \left[3(1-2m)^2 I_g l + 2A_g h^2 m^2\right]}$

In Table 2.7 there are two displacements that the first one is calculated according to virtual work method and another one is related to the tall building book that has been written by Smith et al. (1991). The most significant difference between these two displacements is related to the single and double diagonal braced frame that according to the virtual work the effect of column is considered in determination of displacement while in tall building book (Smith, Coull, & Stafford-Smith, 1991) ignore the effect of column on in stiffness of frame. Moreover, the effect of the beam has been ignored in two equations, because the floor is assumed to be a rigid body and eventually, the force of the beam is transferred to the floor.

Consider frame with one story and one span that lateral stability of structure is provided with different types of bracing system. Size of all frames are considered as a same, so for columns is used of 2IPE140 section and for beam and bracing is used of IPE140 and 2UPN140 section respectively. A concentrated force(100kN) is applied to each frame and displacement and stiffness of each frame is calculated according Tables 2.6 and 2.7, moreover Table 2.8 shows the result of applying concentrated load

Charts 2.1, 2.2 and 2.3 shows the comparison between displacement and stiffness of different types of bracing systems.

Table 2.7 Shera displacement of different types of braced frames

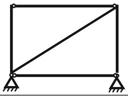
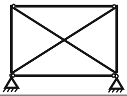
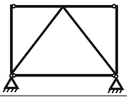
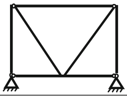
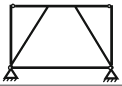
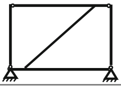
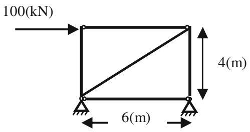
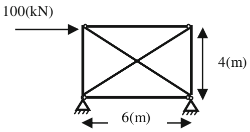
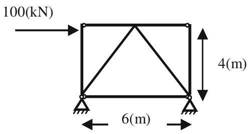
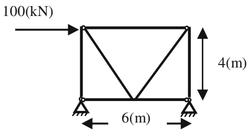
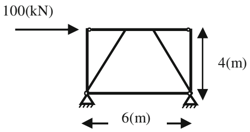
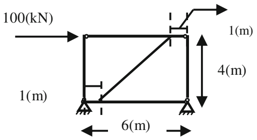
Type of bracing	Dimensions	Shear displacement	Shear displacement (Smith et al., 1991)
Single diagonal		$\Delta = \frac{F}{E}\left(\frac{d^3}{l^2Ad} + \frac{h^3}{l^2Ac} + \frac{l}{Ag}\right)$	$\Delta = \frac{F}{E}\left(\frac{d^3}{l^2Ad} + \frac{l}{Ag}\right)$
Double diagonal		$\Delta = \frac{1}{2E}\left(\frac{h^3}{l^2Ac} + \frac{d^3}{l^2Ad}\right)$	$\Delta = \frac{1}{2E}\left(\frac{d^3}{l^2Ad}\right)$
Chevron (V inverted)		$\Delta = \frac{F}{E}\left(\frac{2d^3}{l^2Ad} + \frac{l}{4Ag}\right)$	$\Delta = \frac{F}{E}\left(\frac{2d^3}{l^2Ad} + \frac{l}{4Ag}\right)$
Chevron		$\Delta = \frac{F}{E}\left(\frac{2d^3}{l^2Ad} + \frac{2h^3}{l^2Ac} + \frac{l}{4Ag}\right)$	
Eccentrically bracing			$\Delta = \frac{F}{E}\left(\frac{d^3}{2m^2Ad} + \frac{m}{2Ag} + \frac{h^2(l-2m)^2}{12Igl}\right)$
Offset diagonal			$\Delta = \frac{F}{E}\left(\frac{d^3}{(l-2m)^2Ad} + \frac{(l-2m)}{2Ag} + \frac{h^2(m)^2}{3Igl}\right)$

Table 2.8 Displacement and stiffness of different types of bracing system under 100 (kN) force

	$\begin{cases} \Delta = 0.0033767 \text{ (m)} \\ k = 29614.71259 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$	$\begin{cases} \Delta = 0.0031057 \text{ (m)} \\ k = 32198.4455 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$
	$\begin{cases} \Delta = 0.0007737 \text{ (m)} \\ k = 129249.062 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$	$\begin{cases} \Delta = 0.00127647512 \text{ (m)} \\ k = 78340.7357 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$
	$\begin{cases} \Delta = 0.00130835193 \text{ (m)} \\ k = 76432.03461 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$	$\begin{cases} \Delta = 0.00130835193 \text{ (m)} \\ k = 76432.03461 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$

(continued)

Table 2.8 (continued)

	$\begin{cases} \Delta = 0.00276499 \text{ (m)} \\ k = 36166.49608 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$	
		$\begin{cases} \Delta = 0.0145249 \text{ (m)} \\ k = 6884.728983 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$
		$\begin{cases} \Delta = 0.08475 \text{ (m)} \\ k = 1179.941003 \left(\frac{\text{kN}}{\text{m}}\right) \end{cases}$

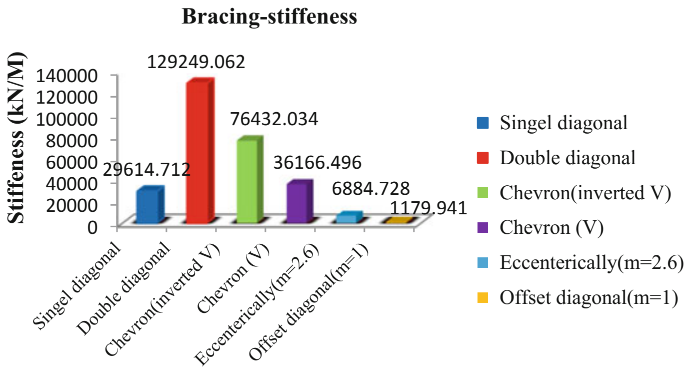


Chart 2.1 Stiffeness in different types of bracing systems under 100 (kN) lateral load

Chart 2.1 illustrates the magnitude of equivalent stiffness of the different types of braced frames under lateral load (100 kN). It can be observed that a double diagonal braced frame ranks first in terms of the stiffness at 129,249 $\left(\frac{\text{kN}}{\text{m}}\right)$, followed by Chevron (V-inverted) at 76,432 $\left(\frac{\text{kN}}{\text{m}}\right)$. However, there is a huge difference between these two bracing systems. Chevron (V) comes next at 36,166 $\left(\frac{\text{kN}}{\text{m}}\right)$.

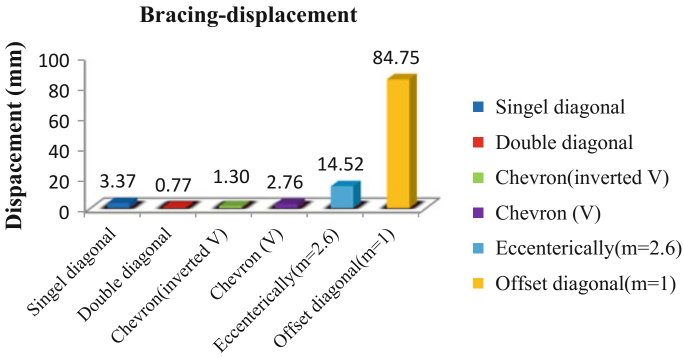


Chart 2.2 Displacemet in different types of bracing structure under 100 (kN) lateral load

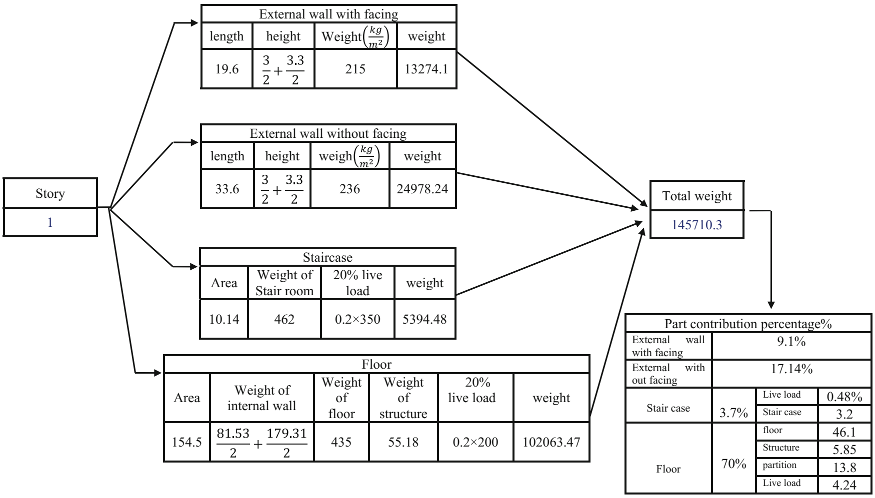


Chart 2.3 Total weight of story one

The stiffness of a singel diagonal braced frame is equal to 29,614 $\left(\frac{\text{kN}}{\text{m}}\right)$. Then, the eccentrically braced frame and offset diagonal braced frame come at 6884 and 1179 $\left(\frac{\text{kN}}{\text{m}}\right)$, respectively.

Chart 2.2 demonstrates the displacements of the different types of the bracing system. It can be clearly seen that the offset diagonal braced frame ranks first in terms of displacement, as a result, this type of bracing system has a significant ductility compared to the other types of bracing systems. It is followed by the eccentrically braced from at 14.5 (mm). The single diagonal braced frame comes next at 3.37 (mm). Then Chevron (V-inverted) and double diagonal braced frame come at 1.3 (mm) and 0.77 (mm), respectively. A glance at the graphs shows that the double diagonal braced frame is the stiffest and the offset diagonal is the most ductile ones between the six types of the bracing systems.

2.2 Define Project

- Architectural plan
- Material property

2.2.1 *Properties Definition of Project*

A multi-floor steel structure building in this chapter is the same one mentioned in the first chapter. It has five stories, a residential building, and the ground floor is occupied as a parking space. Total height of the building is 23.4 m, which includes the doom roof. The building is located in the high-risk zone of the earthquake on a site with type C soil and D in Euro code 8 and IBC 2012, respectively.

2.2.1.1 Architectural Plans

The architectural plans consist of the site, floor, section, and elevation plans, which was explored in the first chapter. It should be pointed out that the type of lateral resistance system is the double diagonal bracing system, as per Fig. 2.44. It can be seen that the columns' location is changed as well.

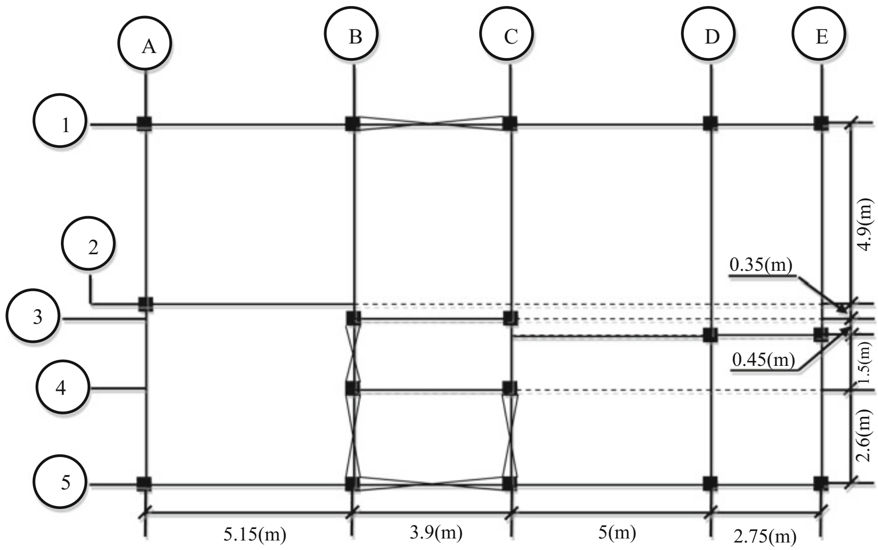


Fig. 2.44 Sketch illustrating the location of the columns, beams, and braced frames on the floor plan

Floor Plan

The structural system consists of concentrically-braced frames. The floors are used as residences, while the ground floor is used for parking. The North–South direction of the plan is divided into four bays by five grid lines (A–E), which is also the case in the East–West direction (1–5). The distances between the grid lines along the N–S direction are 5.15, 3.9, 5, and 2.75 m, respectively, while the distance between the grid lines along the E–W direction are 4.9, (0.35), (1.95) and 2.6 m, respectively. As per Fig. 2.44, the East–West of structure used concentrically-braced frame located in the last span of bent C and in the bent B between bent (3–4), (4–5), and the North–South direction used concentrically brace system as well. The brace system is located in a long of grid lines (1) and (5), between the bent (B, C). The 3D view of structure is detailed in Fig. 2.45.

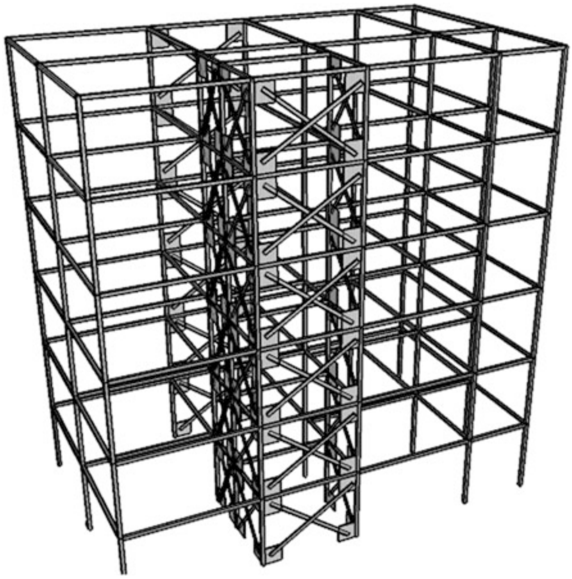


Fig. 2.45 3D view of considered building

2.3 Loading

- Introduce of Different Loads Applied to the Structure
- Center of Mass
- Equivalent Lateral Force
- Stiffness
- Torsional Force
- Distribution of Lateral Load

Loading:

The calculation procedure of the structural weight was explained in the Chap. 1. The main parts are similar to the ones detailed in the first chapter (floor system, external wall, internal wall, stair case…). Herein, the weight of each floor, centre of mass, lateral load (Euro code, IBC), centre of stiffness, eccentricity, and distribution of lateral load are calculated and presented in the form of tables, charts, and figures.

2.3.1 *Weight of Steel Structure*

In order to compute of weight of steel structure frame, the total weight of columns, beams braces and connection need to be calculated. Sections of column, beam and brace base on architectural drawing in each story and column properties are illustrated in Tables 2.9, 2.10 and 2.11. Figure 2.63 shows cross-section of IPE and UPN, which are used in column, beam and brace sections. Half of the height of story up and half of height of story down is used for calculation of the column weight. For instance, in the first story, half of the height of story (1) and another half of the height of story (2) are utilized for calculating the weight of columns. Equation (2.25) shows the method to calculate the column weights. At the same time, the weight of beam is computed based on the total length of the beam multiplied by the area of beam and the specific weight of beam. This is captured in Eq. (2.26). The weight of connection is calculated according to the 20% of summation of total weight of columns, beams

Table 2.9 Column's section

Column	Size of Story 1	Size of Story 2	Size of Story 3	Size of Story 4	Size of Story 5	Size of Story 6
C-3	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
C-4	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
(C-4)′	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
C-5	$3IPE140+2PL380\times 17\times 0.8$	$3IPE140+2PL380\times 17\times 0.8$	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$
C-6	$3IPE140+2PL380\times 17\times 0.8$	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
(C-6)′	$3IPE140+2PL380\times 17\times 0.8$	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$

Table 2.10 Beam's section

Beam	Section	Beam	Section
B-1	IPE160 + PL325 × 15 × 10	B-6	IPE140 + PL325 × 15 × 1
B-2	IPE220 + PL380 × 15 × 10	B-7	IPE160 + PL275 × 15 × 1
B-3	IPE200 + PL380 × 15 × 10	B-8	IPE140
B-4	IPE200 + PL325 × 15 × 1	B-9	2IPE 160
B-5	2IPE 140	B-10	2IPE 160

Table 2.11 Brace's section

Bracing	Story 1	Story 2	Story 3	Story 4	Story 5	Story 6
X Bracing	2UPN160	2UPN160	2UPN160	2UPN160	2UPN140	2UPN140

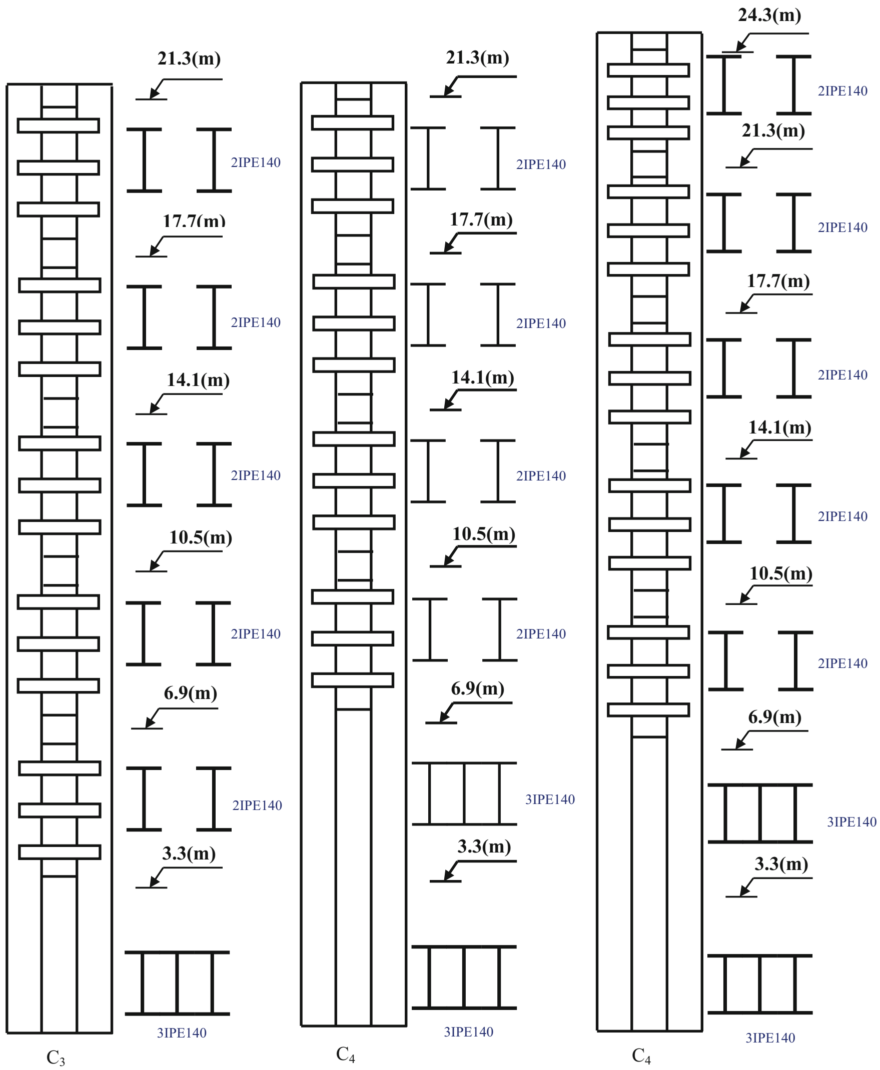


Fig. 2.46 Different type of columns of considered building

and braces in each story. According to the structural drawing the sections of column, beam and brace are shown in Figs. 2.46, 2.47 and 2.48. Figure 2.49 shows the location and section of beams and columns.

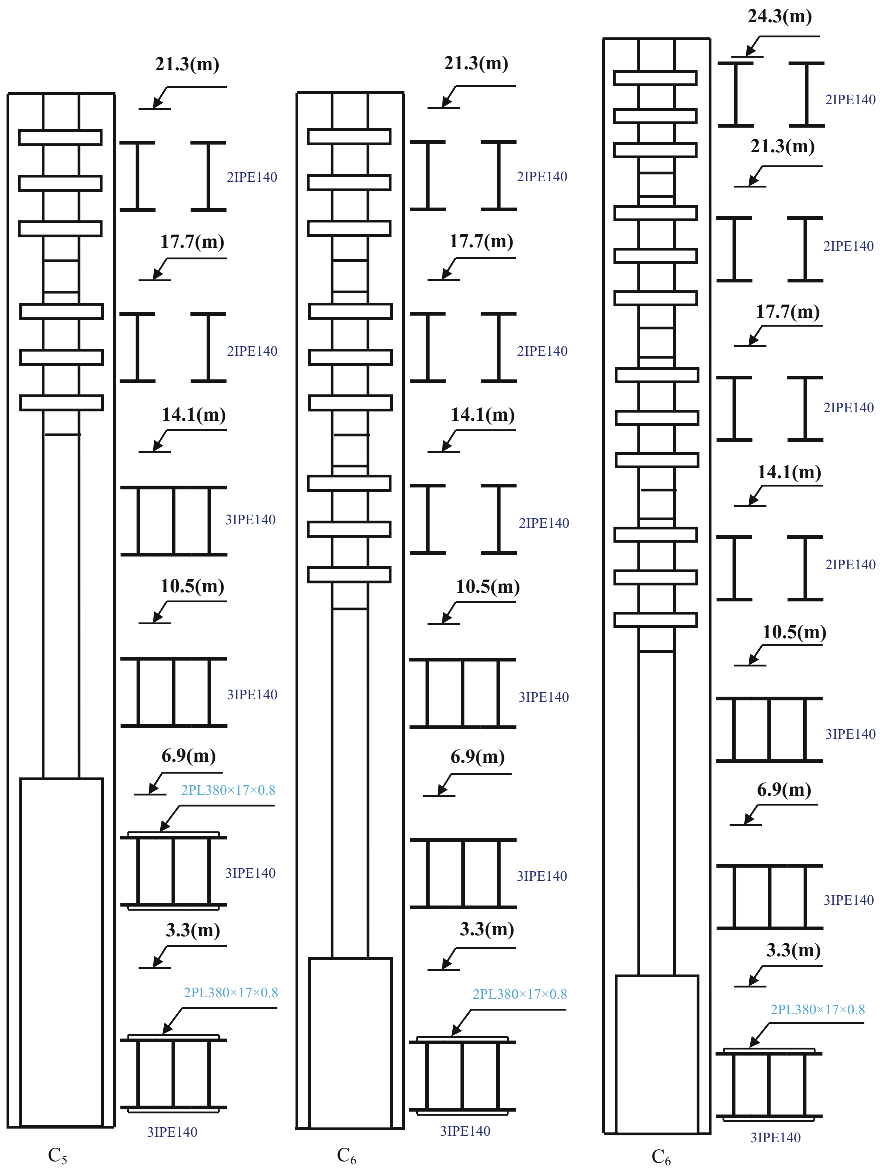


Fig. 2.47 Different types of columns of considered building

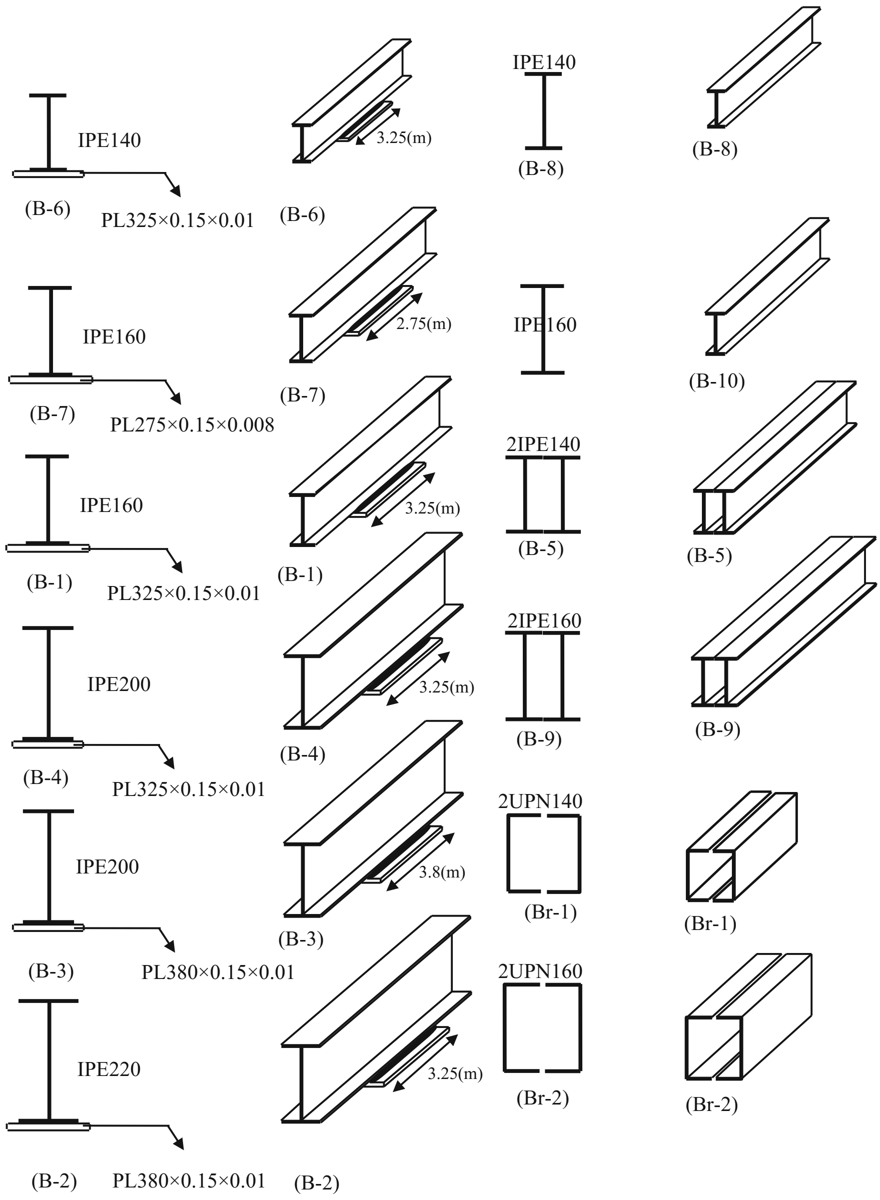


Fig. 2.48 Different types of beam and brace sections of considered building

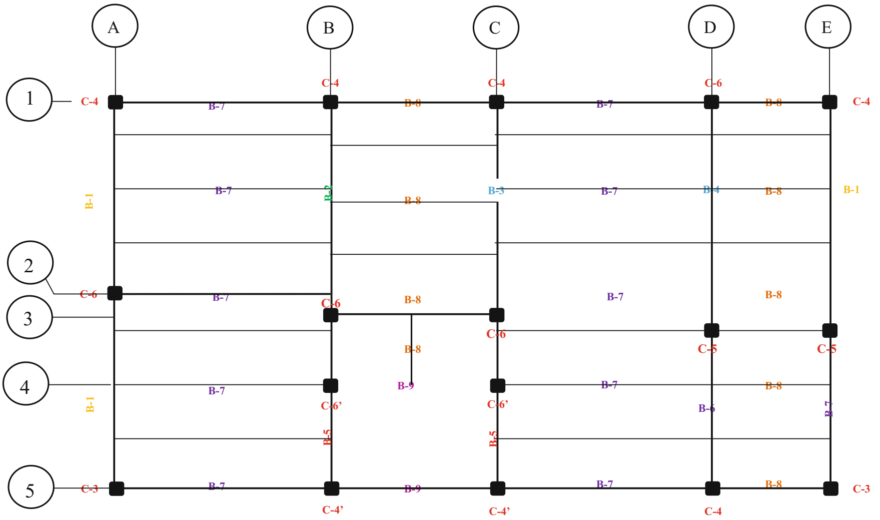


Fig. 2.49 Location of different types of columns and beams

$$\sum [\text{NC}[(\text{Mass per unit C} \times \text{Half of the height of story top}) + (\text{Mass per unit C} \times \text{Half of the height of story bottom})] \quad (2.25)$$

where this equation:

$$\begin{aligned} &\text{NC} = \text{Number of column} \\ &\text{Weight of beam} = (\text{length of beam}) \times (\text{weight per unit meter}) \quad (2.26) \\ &\text{Weight of bracing} = (\text{length of bracing}) \times (\text{weight per unit meter}) \end{aligned}$$

The weight of columns in first story are caculated by Eq. (2.25). Based on there are four types of the columns(C_3, C_4, C'_4, C_5, C_6, C'_6). As the Fig. 2.49 shows, The number of C_3, C_4, C'_4, C_5, C_6, and C'_6 columns is two, five, two, two, two and two, respectively. The Area and mass per unit of each type of columns are illustrated in Table 2.13. Half of the height of the story one is 1.65 m and half of the height of story 2 is 1.8 m, making the total weight of columns in story one is 2629.002 kg. The total weigh of column in each story is shown in Table 2.12. Table 2.13 shows the calculation of the weight of column in each story in detail. all necessary information of columns and plates are illustrated in Table 2.13, this table shows the total weight of each types of columns, separately. The column dimensions change in the height of column, in each type which is shown in Table 2.13, clearly.

Table 2.12 Total weight of column in each story

Story	Column total weight
1	2629.002
2	2194.156
3	1811.16
4	1625.4
5	1578.96
6	944.28
Dome roof	154.8

2.3.1.1 Weight of Columns in Each Story

Story 1: Half of the weight of first column story + half of the weight of second column story

$$[2[(3 \times 12.9 \times 1.65) + (2 \times 12.9 \times 1.8)]] + [5[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8)]] \\ + [2[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8)]] + [2[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8) \\ + (2 \times 1.9 \times 0.17 \times 0.008 \times 7850) + (2 \times 1.9 \times 0.17 \times 0.008 \times 78500)]] \\ + [4[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8) + (2 \times 1.9 \times 0.17 \times 0.0008 \times 7850]] \\ + [2[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8)(2 \times 1.9 \times 0.17 \times 0.0008 \times 7850]] = 2629.002$$

So:

$NC_3 = 2$	Mass per unit of $C_3 = 12.9$	Half of the height of story 1 = 1.65 m
$NC_4 = 5$	Mass per unit of $C_4 = 12.9$	Half of the height of story 2 = 1.8 m
$NC'_4 = 2$	Mass per unit of $C'_4 = 12.9$	
$NC_5 = 2$	Mass per unit of $C_5 = 12.9$	
$NC_6 = 4$	Mass per unit of $C_6 = 12.9$	
$NC'_6 = 2$	Mass per unit of $C'_6 = 12.9$	

Story 2: Half of the weight of second column story + half of the weight of third column story

$$[2[(2 \times 12.9 \times 3.6)]] + [5[(3 \times 12.9 \times 1.8) + (2 \times 12.9 \times 1.8)]] + [2[(3 \times 12.9 \times 1.8) \\ + (2 \times 12.9 \times 1.8)]] + [2[(3 \times 12.9 \times 3.6) + (2 \times 1.9 \times 0.17 \times 0.008 \times 7850)] \\ + [4[13 \times 12.9 \times 3.6]] + [2[13 \times 12.9 \times 3.6]] = 2194.156$$

Table 2.13 Calculation of columns' weight in each story

Story	Name of column	Number of column	Number of IPE	$G_{kg/m}$	Half story height	Weight of column	Number of plates	Volume of plate	γ_{kg/m^3}	Weight of plate	Total weight (kg)
1	C3	2	3	12.9	1.65	127.71					127.71
	C3	2	2	12.9	1.8	92.88					92.88
	C4	5	3	12.9	3.45	667.575					667.575
	C4′	2	3	12.9	3.45	267.03					267.03
	C5	2	3	12.9	3.45	267.03	4	0.002584	7850	162.2752	429.3052
	C6	4	3	12.9	3.45	534.06	2	0.002584	7850	162.2752	696.3352
	C6′	2	3	12.9	3.45	267.03	2	0.002584	7850	81.1376	348.1676
2	C3	2	2	12.9	3.6	185.76					185.76
	C4	5	3	12.9	1.8	348.3					348.3
	C4	5	2	12.9	1.8	232.2					232.2
	C4′	2	3	12.9	1.8	139.32					139.32
	C4′	2	2	12.9	1.8	92.88					92.88
	C5	2	3	12.9	3.6	278.64	2	0.002584	7850	81.1376	359.7776
	C6	4	3	12.9	3.6	557.28					557.28
	C6′	2	3	12.9	3.6	278.64					278.64
3	C3	2	2	12.9	3.6	185.76					185.76
	C4	5	2	12.9	3.6	464.4					464.4
	C4′	2	2	12.9	3.6	185.76					185.76
	C5	2	3	12.9	3.6	278.64					278.64
	C6	4	3	12.9	1.8	278.64					278.64
	C6	4	2	12.9	1.8	185.76					185.76
	C6′	2	3	12.9	1.8	139.32					139.32
	C6′	2	2	12.9	1.8	92.88					92.88

(continued)

Table 2.13 (continued)

Story	Name of column	Number of column	Number of IPE	$G_{kg/m}$	Half story height	Weight of column	Number of plates	Volume of plate	γ_{kg/m^3}	Weight of plate	Total weight (kg)
4	C3	2	2	12.9	3.6	185.76					185.76
	C4	5	2	12.9	3.6	464.4					464.4
	C4′	2	2	12.9	3.6	185.76					185.76
	C5	2	3	12.9	1.8	139.32					139.32
	C5	2	2	12.9	1.8	92.88					92.88
	C6	4	2	12.9	3.6	371.52					371.52
	C6′	2	2	12.9	3.6	185.76					185.76
5	C3	2	2	12.9	3.6	185.76					185.76
	C4	5	2	12.9	3.6	464.4					464.4
	C4′	2	2	12.9	3.6	185.76					185.76
	C5	2	2	12.9	3.6	185.76					185.76
	C6	4	2	12.9	3.6	371.52					371.52
	C6′	2	2	12.9	3.6	185.76					185.76
6	C3	2	2	12.9	1.8	92.88					92.88
	C4	5	2	12.9	1.8	232.2					232.2
	C4′	2	2	12.9	3.3	170.28					170.28
	C5	2	2	12.9	1.8	92.88					92.88
	C6	4	2	12.9	1.8	185.76					185.76
	C6′	2	2	12.9	3.3	170.28					170.28
DOMRROOF	C4′	2	2	12.9	1.5	77.4					77.4
	C6′	2	2	12.9	1.5	77.4					77.4

Story 3: Half of the weight of third column story + half of the weight of forth column story

$$[2[(2 \times 12.9 \times 3.6)] + [5[(2 \times 12.9 \times 3.6]) + [2[(2 \times 12.9 \times 3.6]))] + [2[(3 \times 12.9 \times 3.6)]] + (2 \times 12.9 \times 1.8)] + [4[(3 \times 12.9 \times 1.8) + (2 \times 12.9 \times 1.8)]] + [2[(3 \times 12.9 \times 1.8) + (2 \times 12.9 \times 1.8)]] = 1811.16$$

Story 4: Half of the weight of forth column story + half of the weight of fifth column story

$$[2[(2 \times 12.9 \times 3.6)]] + [5[(2 \times 12.9 \times 3.6)]] + [2[(2 \times 12.9 \times 3.6)]] + [2[(3 \times 12.9 \times 1.8) + (2 \times 12.9 \times 1.8)]] + [4[(2 \times 12.9 \times 3.6)]] + [2[(2 \times 12.9 \times 3.6)]] = 1625.4$$

Story 5: Half of the weight of fifth column story + half of the weight of roof column story

$$[2[(2 \times 12.9 \times 3.6)]] + [5[(2 \times 12.9 \times 3.6)]] + [2[(2 \times 12.9 \times 3.6)]] + [2[(2 \times 12.9 \times 3.6)]] + [4[(2 \times 12.9 \times 3.6)]] + [2[(2 \times 12.9 \times 3.6)]] = 1578.96$$

Story 6: Half of the weight of roof column story + half of the weight of dome roof column story

$$[[[2[(2 \times 12.9 \times 1.8)]] + [5[(2 \times 12.9 \times 1.8) + 2(2 \times 12.9 \times (1.8 + 1.5))]] + [2[(2 \times 12.9 \times 1.8)]] + [4[(2 \times 12.9 \times 1.8) + (2(2 \times 12.9 \times (1.8 + 1.5)))]]]] = 944.28$$

2.3.1.2 Weight of Beam and Bracing System

The Total weight of the beams in each story in the building is calculated by multiplying the total length of the beam of each section by the weight per meter of beam then it should be summed with the weight of the plate which is added to the beam. This method is also used to calculate the weight of the bracing system in the building. Tables 2.14 and 2.15 show the total weight of the beams in each story and

Table 2.14 Total weight of beam in each story

Beam	G (kg/m)	Total length (m)	Total weight (kg)
IPE140	12.9	58.6	755.94
IPE160	15.8	120.25	1899.95
IPE180	18.8	4.9	92.12
IPE200	22.4	10.6	237.44
IPE220	26.2	4.9	128.38
			3113.83

Table 2.15 Total weight of beam of dome roof

Beam	Length (m)	G (kg/m)	Total weight
IPE160	23.4	15.8	369.72
IPE140	15.6	12.9	201.24

Table 2.16 Total weight of plate

Plate Size	n	Volume (m^3)	$\gamma\ \left(\frac{kg}{m^3}\right)$	Weight (kg)
3.25 × 0.15 × 0.01	4	0.0195	7850	153.075
3.8 × 0.15 × 0.01	3	0.0171	7850	134.235
2.75 × 0.12 × 0.008	1	0.00264	7850	20.724
		0.03924		308

Table 2.17 Total weight of bracing

Bracing	G (kg/m)	Total length (m)	Total weight (kg)
UPN140	16	112.136	1794.176
UPN160	18.8	224.272	4216.3136
			6010.4896

Table 2.18 Total weight of steel structure in each story

Story	Weigh of beam	Weight of column	Weight of bracing	Total weight of beam/ column/ bracing	Pure area of plan	Weight	Weight of connection	Total weight
1	3421.83	2629	1054.078	7104.908	154.5	45.98	9.2	55.18
2	3421.83	2194.156	1054.078	6670.064	154.5	43.17	8.6	51.77
3	3421.83	1811.16	1054.078	6287.068	154.5	40.69	8.1	48.8
4	3421.83	1625.4	1054.078	6101.308	154.5	39.49	7.9	47.4
5	3421.83	1578.96	897.088	5897.878	154.5	38.17	7.6	45.77
6	3421.83	944.28	897.088	5263.198	154.5	34.06	6.8	40.86
Dome roof	570.96	154.8	–	725.76	10.14	71.574	14.3	85.87

dome roof, respectively. The total weight of the plate is shown in Table 2.16. Table 2.17 shows the total weight of the bracing system in the steel structural building.

Table 2.18 shows the total weight of the steel structural frame in each story. According to this table, the total weight of the steel frame in each story consists of the total weight of the columns, beams, and bracing system of the story. As it mentioned before, the weight of the joints in each story is approximately 20% of total weight of the columns, beams and bracing systems of the story. For instance, the total weight of story 3 per square meter is 48.8 $\left(\frac{\text{kg}}{\text{m}^2}\right)$ which consist of the columns of the weight 1811.16 (kg), the beams of the weight 3421.83 (kg), and the bracing systems of the weight 1054.078 (kg), therefore the total weight will be 6287.068 (kg). The net area of the building is 154.5 (m^2), so the total weight of the columns, beams and bracing systems per square meter, is 40.69 $\left(\frac{\text{kg}}{\text{m}^2}\right)$. The weight of the joints in story 3 is 20% of 40.69 $\left(\frac{\text{kg}}{\text{m}^2}\right)$ which is equal to 8.1 $\left(\frac{\text{kg}}{\text{m}^2}\right)$ and accordingly the total weight of story 3 will be 40.69 + 8.1 = 48.8 $\left(\frac{\text{kg}}{\text{m}^2}\right)$.

2.3.1.3 Total Weight of Structure

Weight of First Floor:

$$(33.6)\left(\frac{3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{81.533}{2}\right)+\left(\frac{179.32}{2}\right)+(435)+(55.18)+(0.2\times 200)\right]=145710.3\ (\text{kg})$$

Weight of Second Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{179.32}{2}\right)+\left(\frac{179.32}{2}\right)+(435)+(51.77)+(0.2\times 200)\right]=154559.26\ (\text{kg})$$

Weight of Third Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{179.32}{2}\right)+\left(\frac{179.32}{2}\right)+(435)+(48.8)+(0.2\times 200)\right]=154100.4\ (\text{kg})$$

Weight of Forth Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{179.32}{2}\right)+\left(\frac{179.32}{2}\right)+(435)+(47.4)+(0.2\times 200)\right]=153884.1\ (\text{kg})$$

Weight of Fifth Floor:

$$(33.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{179.32}{2}\right)+\left(\frac{179.32}{2}\right)+(435)+(45.77)+(0.2\times 200)\right]=153632.265\ (\mathrm{kg})$$

Weight of Roof Floor:

$$(33.6)\left(\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}\right)(215)+(53.2.)[(205)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{179.32}{2}\right)+(480)+(40.86)+(0.2\times 150)\right]=135297.76\ (\mathrm{kg})$$

Weight of Doom Floor:

$$(10.14)[(480+85.87+(0.2\times 150)]+(13)(2.7\times 236)+(13\times 205)$$
$$=16990.7218\ (\mathrm{kg})$$

2.3.2 *Center of Mass*

The concept of the center mass was detailed in the first chapter. In order to calculate the center of mass, Eq. (2.27) can be used:

$$X_{CM}=\frac{\sum m_i x_i}{\sum m_i}\quad Y_{CM}=\frac{\sum m_i y_i}{\sum m_i} \tag{2.27}$$

Tables 2.19, 2.20, 2.21, 2.22, 2.23 and 2.24 demonstrate the calculation center mass of fist-sixth story respectively

Total weight of roof and dome roof:

$$135297.76+16990.7218=152288.4818$$
$$\bar{x}=\frac{(135297.76)(8.425)+(16990.7218)(7.1)}{152288.4818}=8.277$$
$$\bar{y}=\frac{(135297.76)(4.929)+(16990.7218)(1.3)}{152288.4818}=4.524$$

According to the results centre of mass in roof level is (x = 8.277, y = 4.524)

Table 2.19 Calculating of mass and centre of mass for 1st floor

Number of mass	Length/ area	kg/m—kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	660.605	33340.734	2.575	4.9	85852.4	163369.6
3	28.08	660.605	18549.788	7.1	6.2	131703.5	115008.68
4	75.95	660.605	50172.95	12.925	4.9	648485.37	245847.455
5	9.8	677.25	6637.05	16.7	4.9	110838.735	32521.545
6	9.8	677.25	6637.05	0.1	4.9	6.63.705	32521.545
7	16.8	743.4	12489.12	8.4	0.1	104908.608	1248.912
8	16.8	743.4	12489.12	8.4	9.7	104908.608	121144.464
Sum			145710.186			1225661.734	718675.025

$\bar{x} = \frac{1225661.734}{145710.186} = 8.411 \quad \bar{y} = \frac{718675.025}{145710.186} = 4.93$

Table 2.20 Calculating of mass and centre of mass for 2nd floor

Number of mass	Length/ area	kg/m—kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	706.09	35636.361	2.575	4.9	91763.63	174618.169
3	28.08	706.09	19,827	7.1	6.2	140771.7	122927.4
4	75.95	706.09	53627.53	12.925	4.9	693135.82	262774.9
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.256	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.256	126913.248
Sum			154559.26			1300592.55	763695.305

$\bar{x} = \frac{1300592.55}{154559.26} = 8.414 \quad \bar{y} = \frac{1300592.55}{154559.26} = 4.941$

Table 2.21 Calculating of mass and centre of mass for 3th floor

Number of mass	Length/ area	kg/m—kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	703.12	35486.466	2.575	4.9	91377.65	173883.68
3	28.08	703.12	19743.61	7.1	6.2	140179.631	122410.382
4	75.95	703.12	53401.964	12.925	4.9	690220.38	261669.62
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.25	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.25	126913.248
Sum			154100.4			1296699.05	761338.138

$\bar{x} = \frac{1296699.05}{154100.4} = 8.414 \quad \bar{y} = \frac{1296699.05}{154100.4} = 4.940$

Table 2.22 Calculating of mass and center of mass for 4th floor

Number of mass	Length/ area	kg/m—kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	701.72	35415.808	2.575	4.9	91195.705	173537.46
3	28.08	701.72	19704.3	7.1	6.2	139900.53	122166.66
4	75.95	701.72	53295.634	12.925	4.9	688846.07	261148.606
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.256	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.256	126913.248
Sum			153884.1			1294863.705	760227.562

$\bar{x} = \frac{1294863.705}{153884.1} = 8.414$ m $\quad \bar{y} = \frac{760227.562}{153884.1} = 4.940$ m

Table 2.23 Calculating of mass and centre of mass for 5th

Number of mass	Length/ area	kg/m—kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	700.09	35333.542	2.575	4.9	90988.6781	173134.355
3	28.08	700. 09	19658.527	7.1	6.2	139582.9193	121882.867
4	75.95	700. 09	53171.835	12.925	4.9	687245.96	260541.99
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	16.8	778.8	13083.84	8.4	0.1	109904.256	1308.384
8	16.8	778.8	13083.84	8.4	9.7	109904.256	126913.248
Sum			153632.265			1292726.77	758934.048

$\bar{x} = \frac{1292726.77}{153632.265} = 8.414 \quad \bar{y} = \frac{758934.048}{153632.265} = 4.939$

Table 2.24 Calculating of mass and centre of mass for roof

Number of mass	Length/ area	kg/m—kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	640.52	32327.04	2.575	4.9	83242.128	158402.496
3	28.08	640.52	17985.80	7.1	6.2	127699.18	111511.96
4	75.95	640.52	48647.5	12.925	4.9	628768.86	238372.75
5	9.8	354.75	3476.55	16.7	4.9	58058.385	17035.095
6	9.8	354.75	3476.55	0.1	4.9	347.655	17035.095
7	16.8	389.4	6541.92	8.4	0.1	54952.128	654.192
8	16.8	389.4	6541.92	8.4	9.7	54952.128	63456.624
Parapet wall	9.8	205	2009	16.7	4.9	33550.3	9844.1

(continued)

Table 2.24 (continued)

Number of mass	Length/area	kg/m—kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
Parapet wall	9.8	205	2009	0.1	4.9	200.9	9844.1
Parapet wall	16.8	205	3444	8.4	0.1	28329.6	344.4
Parapet wall	16.8	205	3444	8.4	9.7	28929.6	33406.8
Sum			135297.76			1139931.672	666920.436

$\bar{x} = \frac{1139931.672}{135297.76} = 8.425 \quad \bar{Y} = \frac{666920.436}{135297.76} = 4.929$

2.3.3 *Lateral Load*

This part of Sect. 2.3 is devoted to calculating the equivalent lateral load of the building as per Eurocode8 and IBC 2012.

2.3.3.1 The Calculation of Equivalent Lateral Force of Project by EURO CODE8

Step 1: Ground Type

There is no doubt that the averages shear velocity is most important factor for the selection of the ground type. An average shear velocity is considered $V_{S,}30 = 350\ \frac{\text{m}}{\text{s}}$ as a property of ground, according to the (Eurocode 8, 2004)/Table 3.1). the, ground type is considered as class C.

Step 2: Appropriate type of elastic response spectrum and components

Depending on the characteristics of the most significant earthquake contributing to the local hazard this building is located in the high-risk zone of the earthquake. As a result, the magnitude of surface wave is considered more than 5.5 and the first type of spectrum is chosen as an elastic response spectrum. According to the Table 3.2 of Euro code (8), the value of the parameters of elastic response spectrum is shown in Table 2.25. ((Eurocode 8, 2004)/3.2.2.2(2)P).

Step 3: Important factor

Based on part (4.2.5) and Table 4.3 of Euro code eight the building is classified in class II and
importance the factor is 1. ($\gamma = 1$) ((Eurocode 8, 2004)/4.2.5)

Table 2.25 Value of the parameters describing the recommended Type 1 elastic response

Ground type	S	T_B(s)	T_C(s)	T_D (s)
C	1.15	0.2	0.6	2

Step 4: Determination of the fundamental period

The bracing system is selected for resisting the earthquake load in this building, so the fundamental period of the vibration of the structure for the lateral motion in the considered direction is calculated as following: ((Eurocode 8, 2004)/4.3.3.2.2.3 (P))

$$T_1 = C_t.H^{\left(\frac{3}{4}\right)}$$
$$C_t = 0.05\,\text{Other Structure}$$
$$H = 21.3(\text{m})$$
$$T = (0.05)(21.3)^{0.75} = 0.4957$$

Step 5: Behaviour factor of horizontal seismic action

The building is considered as a concentric braced frame in each two perpendicular directions, and the structure will be designed as a medium ductility class. the magnitude of behaviour factor is calculated by equation ((Eurocode 8, 2004)-(5-1)). The value of q_0 is determined by ((Eurocode 8, 2004)Table (6.2)). The magnitude of k_w is equal to 1 ((Eurocode 8, 2004) 5.2.2.2(11))

$$q = q_0 k_w > 1.5$$
$$q_0 = 4$$
$$k_w = 1$$

Step 6: Design spectrum

For the horizontal components of the earthquake action in considered structure, the design spectrum $S_d(T)$ is considered by ((Eurocode 8, 2004)/3.2.2.5(4)P):

$$T_B \le T \le T_C : \quad S_d(T) = \left\{ a_g.S.\frac{2.5}{q} \right.$$
$$T_T = 0.4957$$
$$T_B = 0.2$$
$$T_C = 0.6$$
$$\text{S}_\text{d}(\text{T})\left\{ 0.3 \times 1.15 \times \frac{2.5}{4} = 0.2156 \right.$$

Step 7: Base shear force

The horizontal direction's seismic shear force, F_b, used to evaluate each building can be

worked out by using the ((Eurocode 8, 2004)/4.3.3.2.2(1)P)

Table 2.26 Distribution of base shear in each story

Story	h_i (m)	w_i(kg)	w_ih_i	$\frac{w_ih_i}{\sum w_ih_i}$	$F_{i(\mathrm{kN})}$	$V_{i(\mathrm{kN})}$	$M_{i(\mathrm{kNm})}$
6	21.3	152288.5	3,243,745	0.287104	481.0456	481.0456	865.8821
5	17.7	153632.3	2,719,291	0.240684	403.2694	884.315	3323.531
4	14.1	153884.1	2,169,766	0.192046	321.7751	1206.09	7086.26
3	10.5	154100.4	1,618,054	0.143214	239.9566	1446.047	11860.11
2	6.9	154559.3	1,066,459	0.094392	158.1553	1604.202	17350.55
1	3.3	145710.2	480843.6	0.042559	71.30885	1675.511	23002.71
			11,298,158	914174.7			

$$\mathrm{F_b} = \mathrm{S_d}(\mathrm{T_1}) \times \lambda \times \mathrm{m}$$
$$\mathrm{S_d}(\mathrm{T_1}) = 0.2156$$
$$\lambda = 0.85$$
$$m = 9141.746928$$
$$\mathrm{F_b} = 1675.511\ (\mathrm{kN})$$

In this structure, double diagonal brace frame is used in X and Y direction, so the magnitude of the base shear in each direction will be the same $\left(F_{bx} = F_{by} = 1675.511\ (\mathrm{kN})\right)$.

Base shear is distributed in vertical direction by using

$$F_i = F_b.\frac{z_i.m_i}{\sum z_j.m_j}$$

In the structure consideration, the base shear in each direction is 1675.5 (kN), and total weight of each story is calculated from previous parts. Table 2.26 shows the distribution of shear generated in each story and overturning of moment. Lateral force and shear force in story 6 are equal 481.04 (kN). Lateral force for story five is 403.3 (kN) which is essentially the summation of lateral force in story 5 and 6 will be equal to shear in story 5 which has a magnitude of 884.31 (kN).

Figure 2.50 shows the 3D distribution of base shear in X and Y direction in height of structure. As described, the double diagonal bracing system in both directions forms the base shear in both directions which will be same. The shear force in story 6 is 481.04 (kN) and it increases to 1675.5 (kN) in first story. This is needed to highlight that all lateral forces in each story is applied to the centre of mass of their floors. Centre of mass of each floor is calculated in earlier sections.

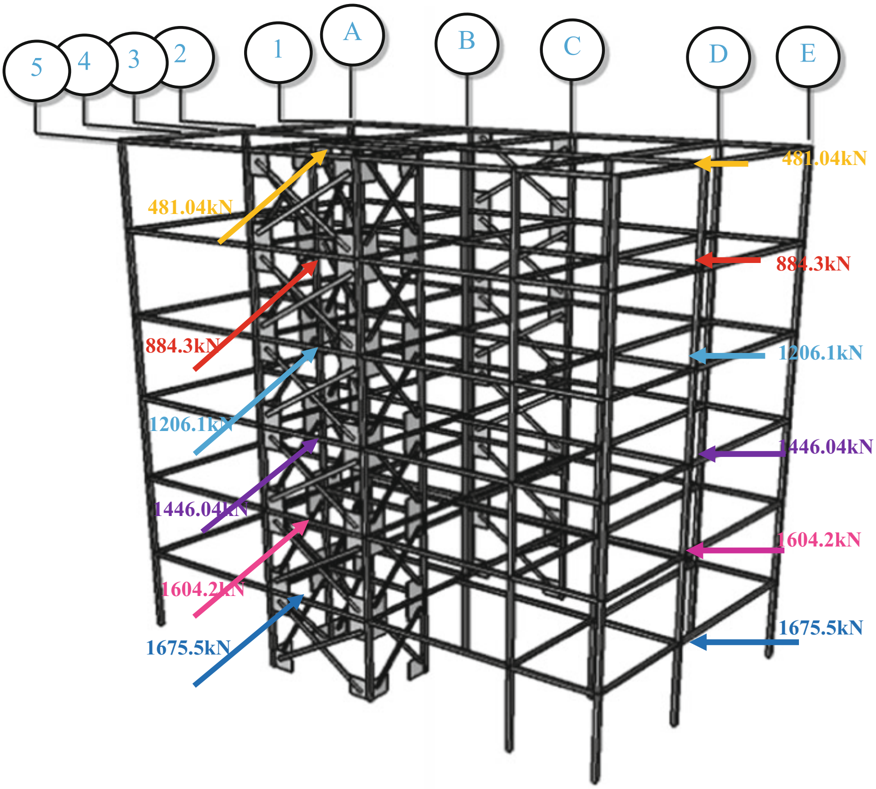


Fig. 2.50 Distribution of base shear in height of model

2.3.3.2 The Calculation of Equivalent Lateral Force of Project by IBC 2012

Step 1: Determination of the maximum considered earthquake and design spectral response accelerations:

1.1 Depending on the characteristics of the most significant earthquake contributing to the local hazard, this building is located in a high-risk zone of earthquake. As a result of this, the value of S_1 and S_S is determined to be 0.3 and 0.75, respectively.

1.2 There is no doubt that the average shear velocity is the most important factor for the selection of the site class. An average shear velocity is considered $V_{S,}30 = 350\ \frac{m}{s}$ as a property of the ground, and according to the ({ASCE/SEI, 2010 #31}/Table 20.3.1),the site class is considered as class D.

Table 2.27 Values of site coefficient F_a

Site class	Mapped spectral response acceleration at short period				
	$S_s \leq 0.25$	$S_s = 0.5$	$S_s = 0.75$	$S_s = 1$	$S_s \geq 1.25$
D	1.6	1.4	1.2	1.1	1

Table 2.28 Values of site coefficient F_v

Site class	Mapped spectral response acceleration at 1 s period				
	$S_a \leq 0.1$	$S_a = 0.2$	$S_a = 0.3$	$S_a = 0.4$	$S_s \geq 0.5$
D	2.4	2	1.8	1.6	1.5

1.3 The maximum considered earthquake spectral response accelerations for short and long periods can be calculated using Eqs. (2.28) and (2.29) (IBC Sect. 1613.3.3), while the value of site coefficients Fa and Fv can be determined using ({IBC, 2012 #30} Table 1613.3.3(1) and Table 1613.3.3(2)) respectively (Tables 2.27 and 2.28).

Hence;

$$S_{MS} = F_a S_s = 1.2 \times 0.75 = 0.9 \tag{2.28}$$

$$S_{M1} = F_v S_1 = 1.8 \times 0.3 = 0.54 \tag{2.29}$$

1.4 The value of S_{DS} and S_{D1}, in the form of damped design (5%) spectral response accelerations in the short and long periods can be computed using on Eqs. (2.21–2.22) ({IBC, 2012 #30} Sect. 1613.3.4.)

$$S_{DS} = \left(\frac{2}{3}\right) S_{MS} = \left(\frac{2}{3}\right) \times (0.9) = 0.6g \tag{2.30}$$

$$S_{D1} = \left(\frac{2}{3}\right) S_{M1} = \left(\frac{2}{3}\right) \times (0.54) = 0.36g \tag{2.31}$$

Step 2: The determination of seismic design category and Importance factor

a. Based on (Eurocode 8, 2004) {IBC, 2012 #30} Table 1604.5 and {ASCE/SEI, 2010 #31} Table 1.5-2.2.) the building is classified as class II, and its importance factor is (1). (I_e = 1)
b. The seismic design category (SDC) for $S_{DS} = 0.6g$ and $S_{D1} = 0.36g$ is determined to be "D", as per {IBC, 2012 #30} Table 1613.3.5(1) and Table 1613.3.5(2)) (Tables 2.29 and 2.30)

Table 2.29 SDC based on short-period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
$0.50\ g \leq S_{DS}$	D	D	D

Table 2.30 SDC based on 1 s period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
$0.20\ g \leq S_{D1}$	D	D	D

Step 3: Seismic Base Shear determination

3.1 For an ordinary steel concentrically braced frame and according to Table 2.12.2-1and {ASCE/SEI, 2010 #31}, the response modification coefficient(R) is 3.25.

3.2 The horizontal direction's seismic shear force, V, can be used to evaluate each building via Eq. (2.32). ({ASCE/SEI, 2010 #31} Sect. 12.8 of ASCE 7-10)

$$\mathrm{V} = \mathrm{CS\ W} \tag{2.32}$$

C_s can be computed using Eq. (2.33)

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{3.25}{1}} = 0.1846 \tag{2.33}$$

3.3 Determination of the fundamental period

The eccentric braced frame is selected for resisting earthquake in this building, therefore the fundamental period of the vibration of the structure for the lateral motion in the considered direction can be calculated using Eq. (2.34); ({ASCE/SEI, 2010 #31})

$$T_a = C_t \times h_n^x \tag{2.34}$$

The C_t and x in a given Equation can be determined using {ASCE/SEI, 2010 #31}/Table 12.2.8-2,hence;

$$C_t = 0.02, \quad x = 0.75$$

So,

$$T_a = 0.02 \times 21.3^{0.75} = 0.198$$

According to{ASCE/SEI, 2010 #31}/Table 2.12.8-1 the value of C_u is equal to:

$$C_u = 1.4$$

The calculated fundamental period, T, cannot exceed the product of the coefficient, C_u, in the {ASCE/SEI, 2010 #31}/Table 2.12.8-1, multiplied with the approximate fundamental period, T_a.

$$C_u \times T_a = 1.4 \times 0.32349 = 0.2772$$

1.4 Exceptions

- The value of C_s shall not exceed the value of Eq. (2.35), hence;

$$\begin{aligned} C_S &= \frac{S_{D1}}{T(R/I_e)} \quad for \quad T \le T_L \\ C_S &= \frac{S_{D1}}{T(R/I_e)} = \frac{0.36}{0.198\left(\frac{3.25}{1}\right)} = 0.56 \\ C_s &= \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{3.25}{1}} = 0.1846 \quad < C_S = \frac{S_{D1}}{T(R/I_e)} = \frac{0.36}{0.198\left(\frac{3.25}{1}\right)} = 0.2473 \quad (OK) \end{aligned} \tag{2.35}$$

- The value of C_s shall not be less than the value of Eq. (2.36), so:

$$\begin{aligned} C_S &= 0.044 S_{DS} I_e \ge 0.01 \\ &= 0.044 \times 0.6 \times 1 = 0.0264 \ge 0.01(OK) \\ C_s &= \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{3.25}{1}} = 0.1846 > 0.0264 \quad (OK) \end{aligned} \tag{2.36}$$

As a result the value of C_s is computed according Eq. (2.33).

$$C_s = 0.1846$$

- The seismic base shear calculates in accordance with Eq. (2.32)

$$V = C_s \times W$$
$$V = 0.1846 \times 914174.69 \times = 1687.5665\ (\text{kN})$$

Step 4: Vertical distribution of forces

The lateral seismic force (F_x) induced at any level can be determined using Eqs. (2.37) and (2.38).

$$F_x = C_{VX} \times V \tag{2.37}$$

$$C_{vx} = \frac{W_x \times h_x^k}{\sum_{i-1}^{n} W_i h_i^k} \tag{2.38}$$

k = a distribution exponent related to the building period as follows:
k = 1 for buildings with T less than or equal to 0.5 s
k = 2 for buildings with T more than or equal to 2.5 s
Interpolate between k = 1 and k = 2 for buildings with T between 0.5 and 2.5

$$T_a = 0.198 < 0.5s \rightarrow k = 1$$

In this structure, the base shear in each direction is 1312.94 (kN), and the total weight of each floor can be calculated using previous parts. Table 2.31 shows the distribution of shear generated in each floor and its overturning.

Table 2.31 Distribution of base shear in each floor

Story	Height	Weight	$W_i h_i$	$W_i h_i / \sum W_i h_i$	Force	Shear	Moment
6	21.3	152288.5	3,243,745	0.287104	484.507	484.507	872.112
5	17.7	153632.3	2,719,291	0.240684	406.170	890.677	3347.444
4	14.1	153884.1	2,169,766	0.192046	324.090	1214.767	7137.245
3	10.5	154100.4	1,618,054	0.143214	241.683	1456.450	11945.439
2	6.9	154559.3	1,066,459	0.094392	159.292	1615.743	17475.389
1	3.3	145710.2	480843.6	0.042559	71.821	1687.564	23168.209
		914174.8	11298158.6				

2.3.4 Stiffeness of Frames

According to the architectural plans concentrically bracing is selected as a lateral resistance system in considered building. For distribution of lateral force should be calculated the stiffness of each frame. For this reason used of Eq. (2.39) as a stiffness formulation of concentrically bracing (X brace).

$$k = \frac{2E(l^4 A_d A_c)}{d^3 l^2 A_c + l^2 h^3 A_d} \tag{2.39}$$

2.3.4.1 Center of Rigidity (Stiffness)

The concept of stiffness and the center of stiffness were detailed in the first chapter. The stiffness center can be calculated using Eq. (2.40).

$$X_{CR} = \frac{\sum k_{yi} \times x_i}{\sum k_{yi}} \quad Y_{CR} = \frac{\sum k_{xi} \times y_i}{\sum k_{xi}} \tag{2.40}$$

2.3.4.2 Calculation of Center Regidity

Figure 2.51 shows a double diagonal braced frame, According to Eqs. (2.39) and (2.40), the stiffness and the center of rigidity of each story are calculated and shown in some graphs and figures. The stiffness and the center of rigidity of each story are calculated in Charts 2.18 to 2.21 and are shown in Figs. 2.70, 2.71, 2.72 and 2.73. The stiffness and the center of rigidity in stories 5 and 6 are calculated as following

$$k = \frac{2E(l^4 A_d A_c)}{d^3 l^2 A_c + l^2 h^3 A_d}$$

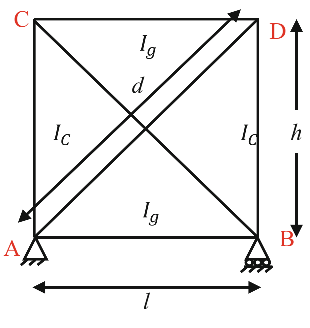


Fig. 2.51 Double diagonal braced frame

E *modul of Elasticity* = 2×10^8 kN/m^2
l length of beam
h height of column
d length of bracing
Ac area of column
Ad area of barcing

$$k_{B1} = \frac{2 \times 2 \times 10^8(3.9^4 \times 0.00408 \times 0.00328)}{(7.307^3 \times 3.9^2 \times 0.00328) + (3.9^2 \times 3.6^3 \times 0.00408)} = 119625.732$$
$$k_{B2} = \frac{2 \times 2 \times 10^8(3.9^4 \times 0.00408 \times 0.00328)}{(7.307^3 \times 3.9^2 \times 0.00328) + (3.9^2 \times 3.6^3 \times 0.00408)} = 119625.732$$
$$k_{B3} = \frac{2 \times 2 \times 10^8(2.6^4 \times 0.00408 \times 0.00328)}{(4.44^3 \times 2.6^2 \times 0.00328) + (2.6^2 \times 3.6^3 \times 0.00408)} = 75790.22195$$
$$k_{B4} = \frac{2 \times 2 \times 10^8(2.6^4 \times 0.00408 \times 0.00328)}{(4.44^3 \times 2.6^2 \times 0.00328) + (2.6^2 \times 3.6^3 \times 0.00408)} = 75790.22195$$
$$k_{B5} = \frac{2 \times 2 \times 10^8(1.95^4 \times 0.00408 \times 0.00328)}{(4.094^3 \times 1.95^2 \times 0.00328) + (1.95^2 \times 3.6^3 \times 0.00408)} = 48996.96525$$
$$\bar{x} = \frac{(75790.22195)(9.05) + (75790.22195)(5.15) + (48996.96525)(5.15)}{(75790.22195 \times 2) + (48996.96525)} = 6.6232\,\text{m}$$
$$\bar{y} = \frac{(119625.732)(9.8)}{(119625.732 + 119625.732)} = 4.9\,\text{m}$$

The chart in Fig. 2.52 was designed to simplify the calculation of the center of stiffness on each floor. This chart consists of four parts:

(i) This table shows the sections' property, such as height and the cross-section areas of the column and bracing, which can be used to compute the stiffness of the double diagonal bracing.
(ii) The stiffness of the double diagonal brace frame is shown in this table.
(iii) This table demonstrates the coordinate center of stiffness with respect to the bracing along the X-direction
(iv) The coordinate center of stiffness in the Y direction is shown in this table (Chart 2.4).

Calculation center of stiffness in story five and six is illustrates in Chart 2.6.

2.3.5 Torsional Forces

As detailed in the first chapter, the resultant force produced by earthquake is applied to a particular point called the center of mass, while on each floor, the resultant resistance force against earthquake is produced at a special point called center of

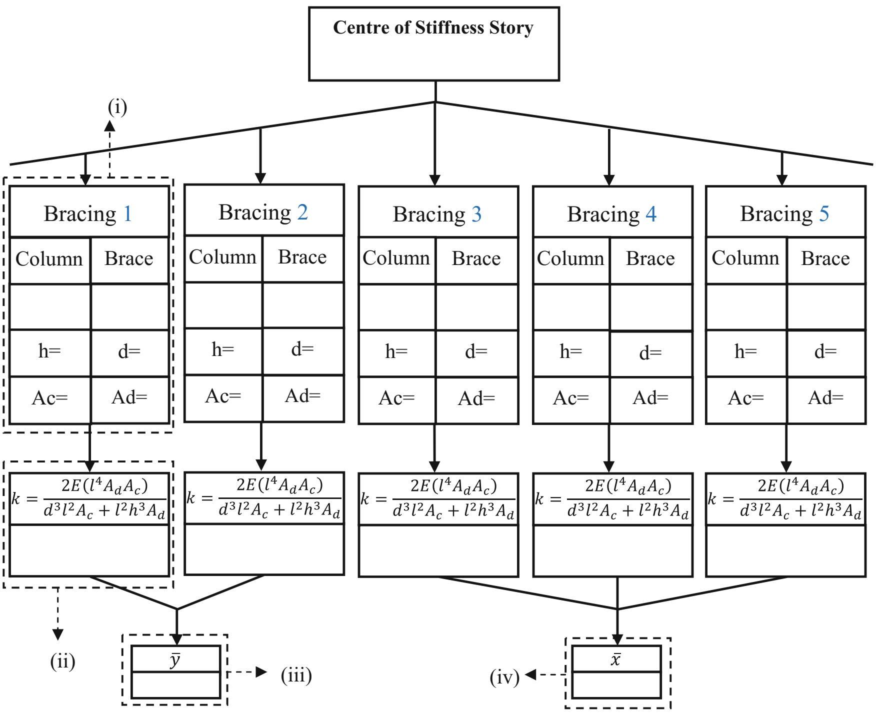


Fig. 2.52 Chart pattern for calculation of center stiffness in each story

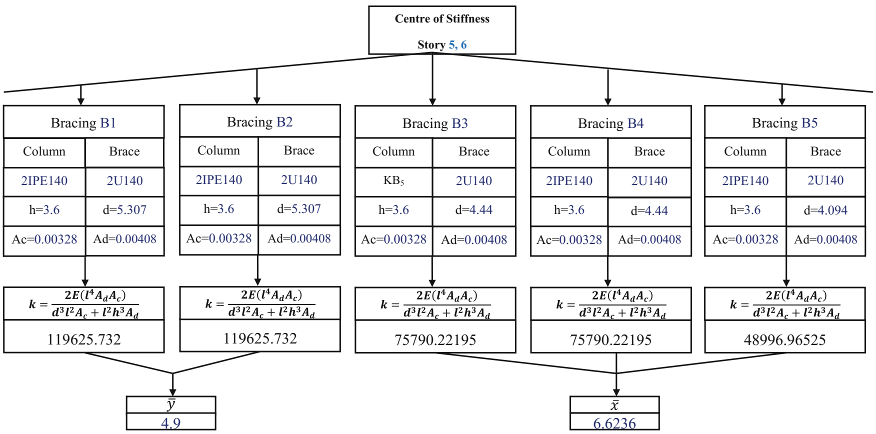


Chart 2.4 Calculation of stiffness of bracing and center rigidity of story five and six

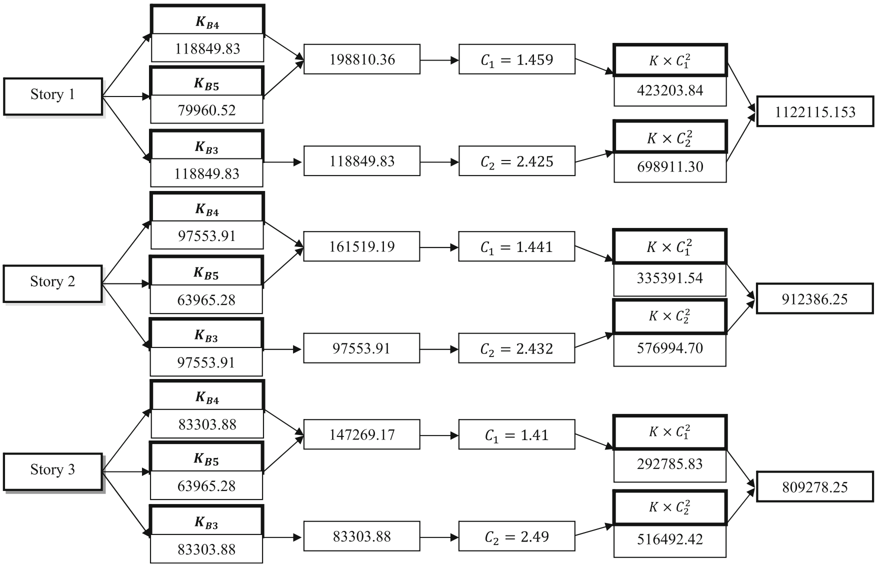


Chart 2.5 Procedure of calculation torsional stiffness in Y direction on floors one, two, and three

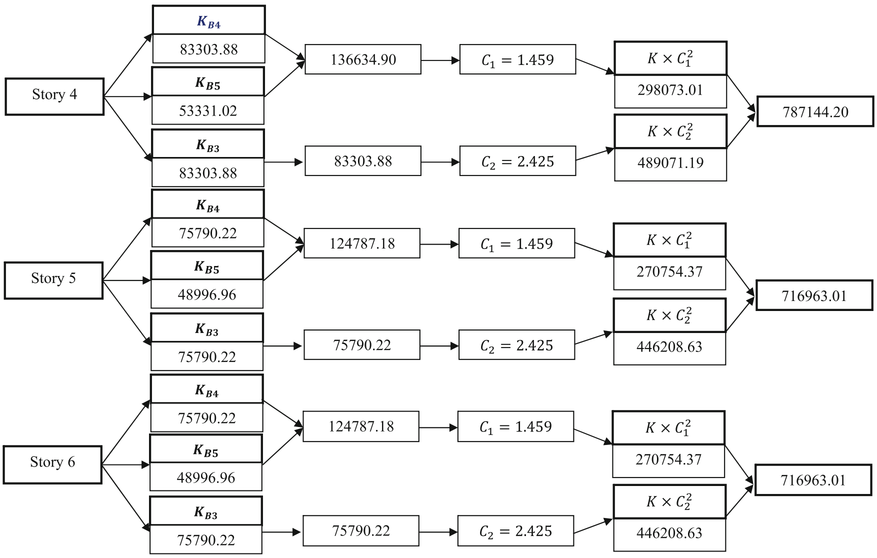


Chart 2.6 Procedure of calculation torsional stiffness in Y direction on floors four, five, and six

stiffness. If there is a distance between the centre of mass and centre of stiffness in a floor, it will create an eccentricity, causing torsion.

2.3.5.1 Calculation of Torsion

Based on the concept of torsion in the structure, eccentricities in two perpendicular directions of floor can be calculated using Eq. (2.41).

$$\begin{cases} e_x = x_{cm} - x_{cj} \\ e_y = y_{cm} - y_{cj} \end{cases} \tag{2.41}$$

The effect of accidental torsion must be added to the value of eccentricity of each floor. The value of accidental torsion can be calculated using Eq. (2.42) {Code, 2005 #1}

$$e_{ai} = \pm 0.05 \times Li \tag{2.42}$$

The eccentricity of floors five and six are shown in Figs. 2.53 and 2.54, respectively.

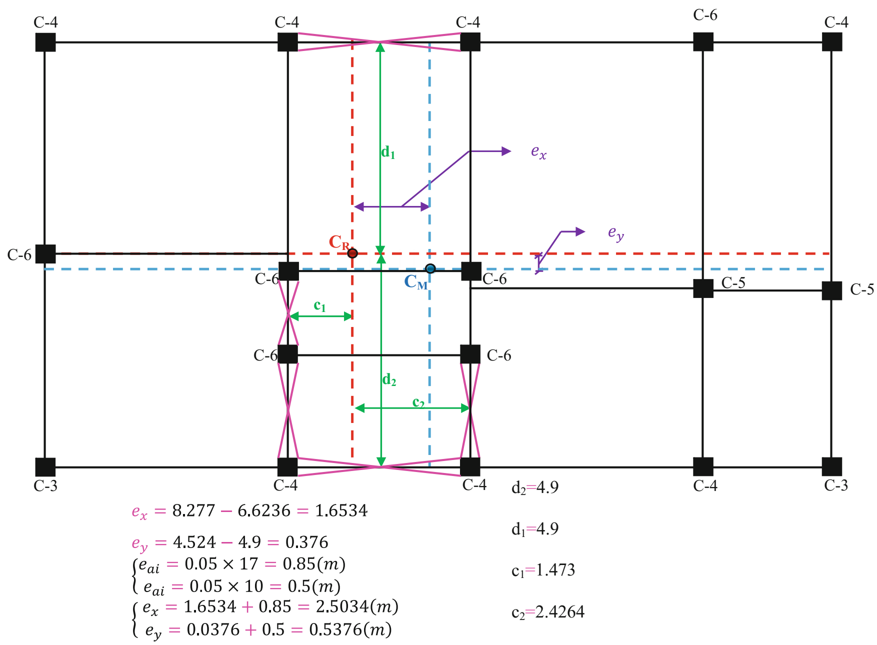


Fig. 2.53 Calculation of eccentricity on story six

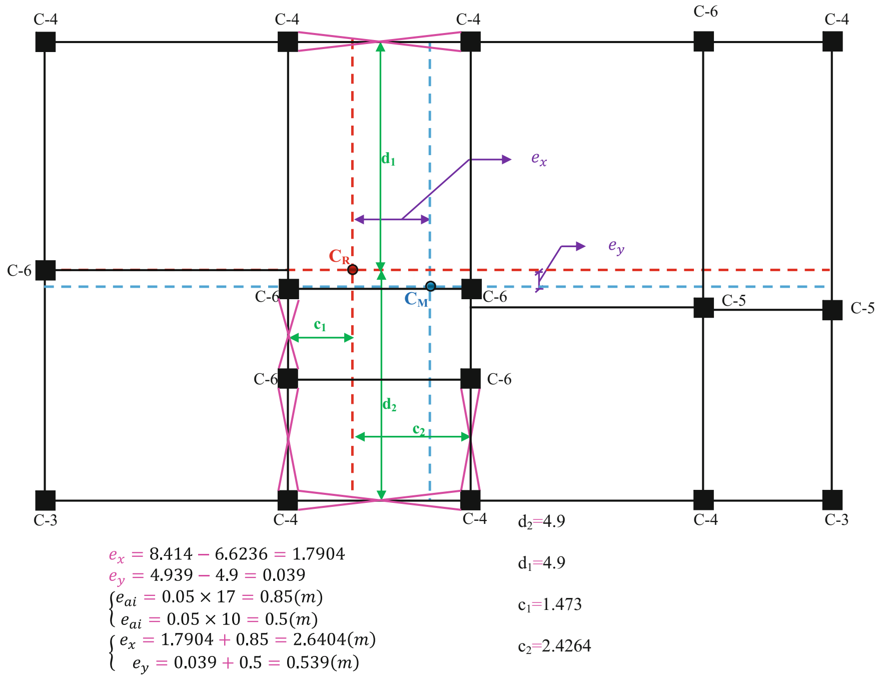


Fig. 2.54 Calculation of eccentricity on story five

2.3.6 *Distribution of Lateral Shear in Bents*

The total external shear at a level will be distributed between the bents in proportion to their shear rigities at that level. An estimate of the shear Q_{ji} carried by bent j at level i is given by

$$Q_i = \left[\frac{(K)_i}{\sum(K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i \tag{2.43}$$

Step 1: The shear force of story (1) is 1675.5 (kN) in X and Y direction according Table 2.26.

Step 2: Stiffeness in X and Y direction are calculated based on Eq. (2.39).hence:

1. Stiffeness of bracing in Y direction are computed:

$$K_{B3} = 118913.237, K_{B4} = 118913.237, K_{B5} = 79963.23203$$

2. Stiffness of bracing in X direction are computed:

$$K_{B1} = 173480.8791, K_{B2} = 173480.8791$$

Step 3: Eccentricities in X and Y direction are calculated according Eqs. (2.41) and (2.42). Eccentricity in X direction (e_x) is 1.8(m) and this magnitude for y direction (e_y) is 0.03 (m). Moreover accidental eccentricity in X and Y direction is 0.85(0.05*17) and 0.5(0.05*10) respectively. These magnitudes must be added to the eccentricity in x and y direction

Step 4: Distance of each bent respect to center of rigidity (**C**) in X and Y direction is calculated and shows in Fig. 2.73.

1. C_j in X direction:

$$\mathrm{C}_{1(\mathrm{B})} = 6.609 - 5.15 = 1.459, C_{1(C)} = 9.05 - 6.609 = 2.441$$

2. d_j in Y direction:

$$\mathrm{d}_{1(1)} = 9.8 - 4.9 = 4.9, \mathrm{d}_{2(5)} = 4.9$$

Step 5: Calculation of torsional stiffness

There is a certain level of torsion resistance produced by each of the braces shown in the floor plan. This can be calculated by multiplying the lateral stiffness of a particular bracing by the distance between the brace and the CR, either in the X or Y direction. Lateral stiffness emerges as the brace bends about its strong axis while the said distance is measured perpendicular to the bracing's weak axis. To resist significant torsion, widely distributed braces are the most suitable, as earlier mentioned. Taking into consideration all the braces as a unit in the floor system, an equivalent torsional stiffness is a sum of the torsion resisted by each of the braces, which is the product of the torsional resistance (for each brace) and the perpendicular distance of the particular brace to the CR. We express the torsional stiffness as shown below.

Table 2.32 Calculation of J in each story

Story	$\sum K_x \times C^2$	$\sum K_y \times d^2$	$J = \sum K_x \times C^2 + \sum K_y \times d^2$
One	1122115.153	8330551.814	9452666.966
Two	912386.252	7192001.941	8104388.19
Three	809278.2591	6440282.541	7249560.801
Four	787144.2047	6440282.541	7227426.746
Five	716963.017	5744427.651	6461390.668
Six	716963.017	5744427.651	6461390.668

$$J = \sum K_x \times C^2 + \sum K_y \times d^2$$
$$\sum K_x \times C^2 = \left[\left[(\mathrm{K_{B5}} + \mathrm{K_{B4}}) \times C_1^2\right] + \left[\left(K_{B3} \times C_2^2\right)\right]\right]$$
$$= \left[(79963.23203 + 118913.237) \times 1.459^2\right] + \left[\left(118913.237 \times 2.441^2\right)\right]$$
$$= 1131886.824$$
$$\sum K_y \times d^2 = \left[\left[\left(K_{B1} \times d_1^2\right)\right] + \left[\left(K_{B2} \times d_2^2\right)\right]\right]$$
$$= \left[(173480.8791) \times 4.9^2\right] + \left[\left(173480.8791 \times 4.9^2\right)\right]$$
$$= 8330551.814$$
$$\boldsymbol{J}_1 = 1131886.824 + 8330551.814 = 9462438.639$$

Procedure of calculation of J is shown in Charts 2.5, 2.6 and 2.7 in detail. Total magnitude of J in each story shows in Table 2.32.

Step 6: According to the Eq. (2.43), shear force of story one in Y direction is distributed in bents (B) and (C) as fallow:

1. Bent (B):

$$Q_i = \left[\frac{(K)_i}{\sum(K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i$$

1.1. Total stiffness of bent (B) is summation stiffness of two diagonal bracing frame along gridline (B), total stiffness in Y direction in story one is summation stiffness of three braced diagonal frame along gridline (B) and (C).

$$\sum(K_{B4} + K_{B5}) = (118913.237 + 79963.23203) = 198876.47$$
$$\sum(K_{B3} + K_{B4} + K_{B5}) = 317789.706$$

1.2. Shear force in each frames of structure is distributed according to the stiffness of frames

So in bent (B):

$$\frac{\sum(K_{B4}+K_{B5})}{\sum(K_{B3}+K_{B4}+K_{B5})}=\frac{198876.47}{317789.706}=0.6258$$

1.3. Distance of each bent respect to center of rigidity and eccentricity

$$C_1 = -1.459\ (\text{m})$$
$$e_{(x)} = 2.652\ (\text{m})$$

1.4. Calculation of torsional stiffness (J)

$$\text{J}_1 = 9452666.97$$

1.5. Based on Table 2.26 total base shear in story one is 1675.511 (kN) and according to the Eq. (2.43), 912.1376 (kN) is transferred to the frame (B).

$$V_1 = 1675.511\ (\text{kN})$$
$$= \left(0.6258 - \frac{1.9\times 10^5 \times 1.8 \times 1.45}{9462438.639}\right)(1675.511) = 912.1376\ \text{kN}$$

2. Bent (C):

$$Q_i = \left[\frac{(K)_i}{\sum(K)_i} \pm \frac{e\times[(K_i\times C)]_i}{J}\right]V_i$$

2.1. Stiffness of frame (C)

$$K_{B3} = 118913.237$$
$$\sum(K_{B3}+K_{B4}+K_{B5}) = 317789.706$$

2.2. Shear force in each frames of structure is distributed according to the stiffness of frames

So in bent (B):

$$\frac{K_{B3}}{\sum(K_{B3}+K_{B4}+K_{B5})}=\frac{118913.237}{317789.706}=0.3741$$

2.3. Distance of each bent respect to center of rigidity and eccentricity

$$C_2 = 2.441$$
$$e_{(x)} = 2.656\ (\mathrm{m})$$

2.4. Calculation of torsional stiffness (J)

$$\mathrm{J}_1 = 9452666.97$$

2.5. Based on Table 2.26 total base shear in story one is 1675.511(kN) and according to the Eq. (2.43), 763.25 (kN) is transferred to the frame (C).

$$V_1 = 1675.511\ (\mathrm{kN})$$
$$= \left(0.3741 + \frac{1.1 \times 10^5 \times 2.656 \times 2.441}{9462438.639}\right)(1675.511) = 763.25\ (\mathrm{kN})$$

The procedure of calculation torsional stiffness in X and Y direction is demonstrated in Charts 2.5, 2.6 and 2.7.

The complete calculations for the distribution base shear in the X and Y directions can be executed in a tabular form, as per the following table. Tables 2.33 and 2.34 show the distribution of the base shear in the Y direction in bents (B) and (C), respectively. As per Table 2.33, the base shear in the first floor of bent (B) is 912.1 (kN), while it is 265 (kN) in the sixth floor (6). As can be seen in Table 2.34, the base shear in the Y direction of bent (C) in the first floor, (1) is 763.25 (kN),

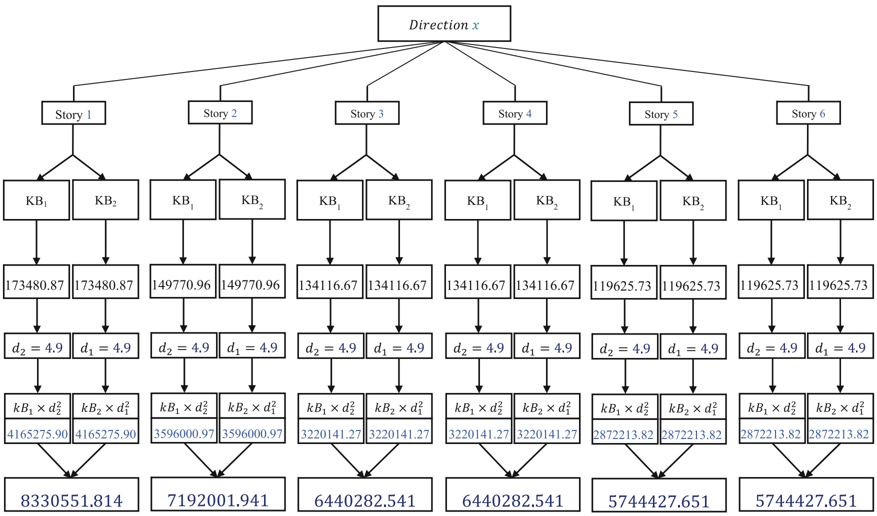


Chart 2.7 Procedure of calculation (J) in X direction

Table 2.33 Distribution of base shear in Y direction in bent (B)

Story	$V(y)$	$K_6 = KB_4 + KB_5$	$\sum K = K_6 + KB_3$	$\frac{K_b}{\sum K}$	$e(x)$	C	J	$\left(\frac{KB_6}{\sum K} \pm \frac{KB_6 e(x)(c)}{J}\right) V$	P
6	481.045	124787.2	200577.4	0.622139	2.5034	1.473	6,461,391	$\left(0.6221 - \frac{1.2\times10^5\times2.5034\times1.47}{6461390.668}\right) V_6$	265
5	884.315	124787.2	200577.4	0.622139	2.6404	1.473	6,461,391	$\left(0.6221 - \frac{1.2\times10^5\times2.6404\times1.47}{6461390.668}\right) V_5$	483.7
4	1206.09	136634.9	219938.8	0.621240	2.719	1.4771	7,227,427	$\left(0.6212 - \frac{1.3\times10^5\times2.719\times1.47}{7227426.746}\right) V_4$	657.64
3	1446.04	147269.2	230573.1	0.638709	2.704	1.41	7,249,561	$\left(0.6387 - \frac{1.4\times10^5\times2.704\times1.41}{7249560.801}\right) V_3$	811.59
2	1604.20	161519.2	259073.1	0.623450	2.646	1.468	8,104,388	$\left(0.6234 - \frac{1.6\times10^5\times2.646\times1.46}{8117074.395}\right) V_2$	875.87
1	1675.51	198876.5	317789.7	0.625858	2.652	1.459	9,452,667	$\left(0.6258 - \frac{1.9\times10^5\times2.652\times1.45}{9462438.639}\right) V_1$	912.13

Table 2.34 Distribution of base shear in Y direction in bent (C)

Story	$V(y)$	KB_3	$\sum K = K_6 + KB_3$	$\frac{KB_3}{\sum K}$	$e(x)$	C	J	$\left(\frac{KB_3}{\sum K} \pm \frac{KB_3 e(x)(c)}{J}\right) V$	P
6	481.045	75790.22	200577.4	0.3778	2.5034	2.4264	6,461,391	$\left(0.3778 + \frac{7.5\times10^4\times2.5034\times2.42}{6461390.668}\right) V_6$	216.013
5	884.315	75790.22	200577.4	0.3778	2.6404	2.4264	6,461,391	$\left(0.3778 + \frac{7.5\times10^4\times2.6404\times2.42}{6461390.668}\right) V_5$	400.549
4	1206.09	83303.89	219938.8	0.3787	2.719	2.4229	7,227,427	$\left(0.3787 + \frac{8.3\times10^4\times2.719\times2.4229}{7227426.746}\right) V_4$	548.327
3	1446.04	83303.89	230573.1	0.361	2.704	2.49	7,249,561	$\left(0.361 + \frac{8.3\times10^4\times2.704\times2.49}{7249560.801}\right) V_3$	633.900
2	1604.20	97553.91	259073.1	0.3765	2.646	2.432	8,104,388	$\left(0.3765 + \frac{9.7\times10^4\times2.646\times2.432}{8117074.395}\right) V_2$	728.243
1	1675.51	118913.2	317789.7	0.3741	2.652	2.441	9,452,667	$\left(0.3741 + \frac{1.1\times10^5\times2.652\times2.441}{9462438.639}\right) V_1$	763.255

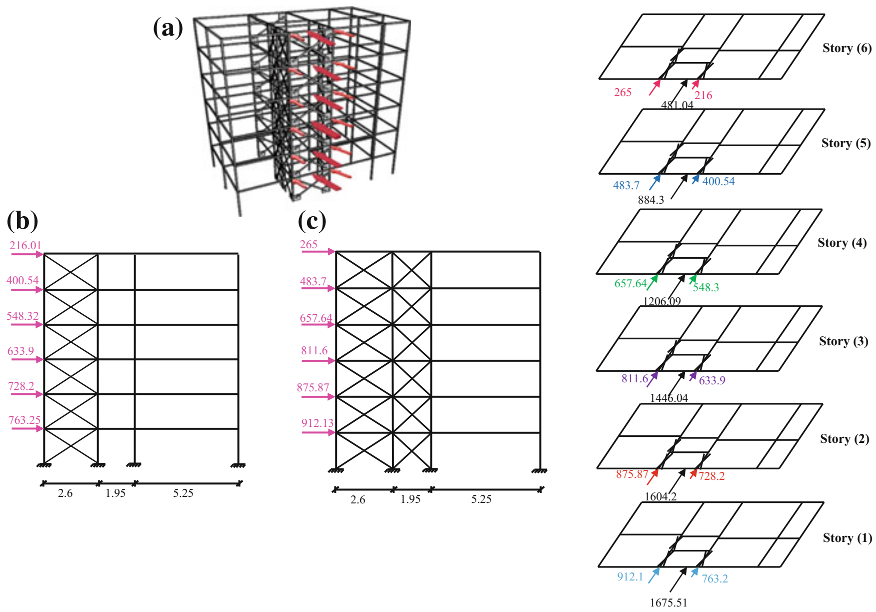


Fig. 2.55 Procedure of distribution of base shear in structure in Y direction

reaching 216.01 (kN) at the sixth floor (6). Based on Tables 2.33 and 2.34, the stiffness of frame (B) exceed that of frame (C) due to increased force transfer to bent (B). Figure 2.55 shows the distribution of the base shear in the Y direction in the height, frame, and plan of the structure. These figures show an understandable concept of distribution of base shear in a lateral resistance frame system. Tables 2.35 and 2.36 show the magnitude of the base shear in the X direction distributed in bents (1) and (5). There is a slight difference between their respective magnitudes due to the stiffness of braces B_1 and B_2, and it is expected that the shear force should be evenly distributed. The eccentricity in the plan of the structure caused this difference. It should be pointed out that in the sixth floor, bent (1) is under compression while bent (B) is under tension. This is due to the center of stiffness (rigidity) in the sixth floor (6) is located at 4.9 (m), while the center of mass is located at 4.524 (m) (dome roof resulted in the center of mass in the sixth floor to move downward relative to the plan). It was pointed out in Chap. 1 that the resultant resistance force is applied to the center of stiffness and the resultant force is applied to the center of mass, which creates torsion in the plan. This resulted in bent (1) being under compression. While in other story, bent (1) in under tension and bent (5) is under compression, because of this situation center of mass and center of stiffness is shown in Figs. 2.70, 2.71, 2.72 and 2.73 (Appendix).

Table 2.35 Distribution of base shear in X direction in bent (1)

Story	$V(x)$	KB_2	$\sum K = K_{B1} + KB_2$	$\frac{KB_2}{\sum K}$	$e(y)$	d_1	J	$\left(\frac{KB_2}{\sum K} \pm \frac{KB_2 e(y)(d_2)}{J}\right)V$	P
6	481.045	119,626	239251.5	0.5	0.876	4.9	6,461,391	$\left(0.5 - \frac{1.1\times10^5\times0.876\times4.9}{6461390.668}\right)V_6$	202.2945
5	884.315	119,626	239251.5	0.5	0.539	4.9	6,461,391	$\left(0.5 + \frac{1.1\times10^5\times0.539\times4.9}{6461390.668}\right)V_5$	485.398
4	1206.09	134,117	268233.3	0.5	0.54	4.9	7,227,427	$\left(0.5 + \frac{1.3\times10^5\times0.54\times4.9}{7227426.746}\right)V_4$	662.2651
3	1446.04	134,117	268233.3	0.5	0.54	4.9	7,249,561	$\left(0.5 + \frac{1.3\times10^5\times0.54\times4.9}{7249560.801}\right)V_3$	793.8087
2	1604.20	149,771	299541.9	0.5	0.541	4.9	8,104,388	$\left(0.5 + \frac{1.4\times10^5\times0.541\times4.9}{8117074.395}\right)V_2$	880.6896
1	1675.51	173,481	346961.8	0.5	0.53	4.9	9,452,667	$\left(0.5 + \frac{1.7\times10^5\times0.53\times4.9}{9462438.639}\right)V_1$	917.613

Table 2.36 Distribution of base shear in X direction in bent (5)

Story	$V(x)$	KB_1	$\sum K = KB_1 + KB_2$	$\frac{KB_1}{\sum K}$	$e(y)$	$d_{(2)}$	J	$\left(\frac{KB_1}{\sum K} \pm \frac{KB_1 e(y) d_{(2)}}{J}\right) V$	P
6	481.04	119,626	239251.5	0.5	0.876	4.9	6,461,391	$\left(0.5 + \frac{1.1\times10^5\times0.876\times4.9}{6461390.668}\right) V_6$	278.75
5	884.31	119,626	239251.5	0.5	0.539	4.9	6,461,391	$\left(0.5 - \frac{1.1\times10^5\times0.539\times4.9}{6461390.668}\right) V_5$	398.91
4	1206.09	134,117	268233.3	0.5	0.54	4.9	7,227,427	$\left(0.5 - \frac{1.3\times10^5\times0.54\times4.9}{7227426.746}\right) V_4$	543.82
3	1446.04	134,117	268233.3	0.5	0.54	4.9	7,249,561	$\left(0.5 - \frac{1.3\times10^5\times0.54\times4.9}{7249560.801}\right) V_3$	652.23
2	1604.20	149,771	299541.9	0.5	0.541	4.9	8,104,388	$\left(0.5 - \frac{1.4\times10^5\times0.541\times4.9}{8117074.395}\right) V_2$	723.51
1	1675.51	173,481	346961.8	0.5	0.53	4.9	9,452,667	$\left(0.5 - \frac{1.7\times10^5\times0.53\times4.9}{9462438.639}\right) V_1$	757.89

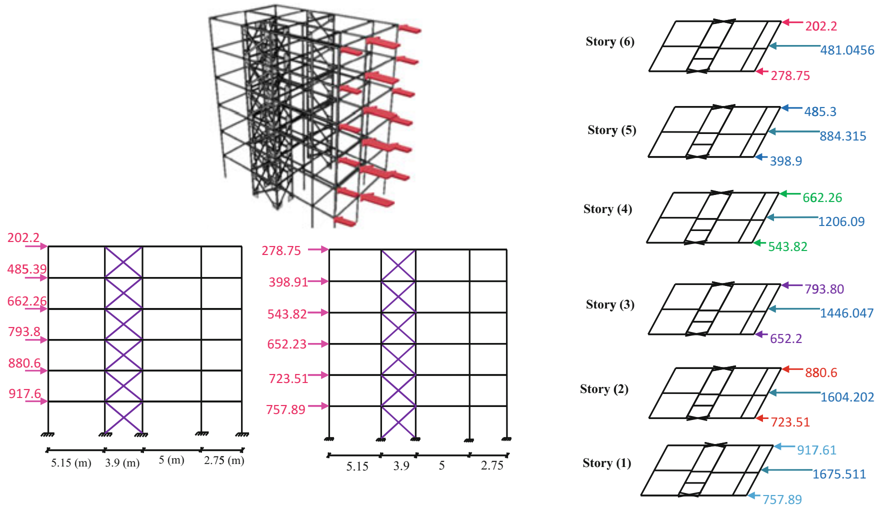


Fig. 2.56 Procedure of distribution of base shear in structure in X direction

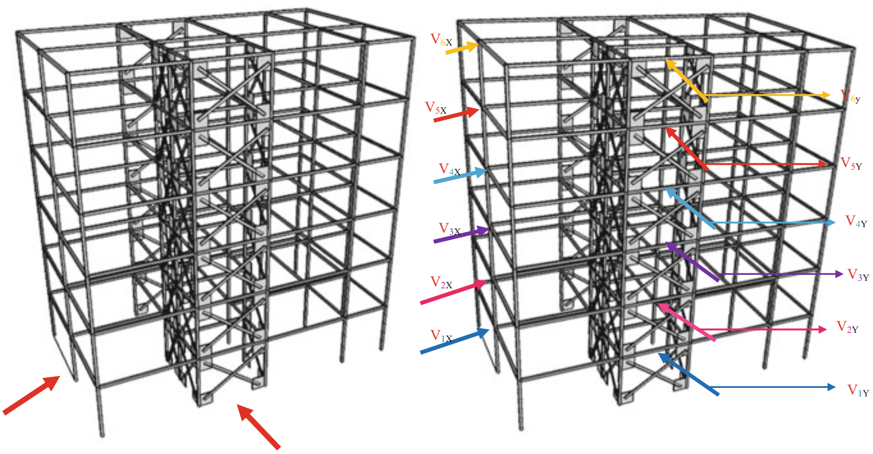


Fig. 2.57 The procedure of distribution of seismic load in the structural member procedure

Figure 2.56 shows the distribution of the base shear in the height of the structure in the X direction. The distribution of the base shear in the X direction in the frame and plan of the structure are detailed in Fig. 2.56b, c respectively.

Figures 2.57, 2.58 and 2.59 show all of the procedures of the distribution of base shear in the building. As per to Fig. 2.57a, the base shear is calculated based on equation $(F_b = S_d(T) \times \lambda \times m)$ of the Euro code 8, then distributed to the height of structure using equation $\left(F_i = F_b \times \frac{Z_i \times m_i}{\sum Z_j \times m_j}\right)$, as shown in Fig. 2.57b. The shear

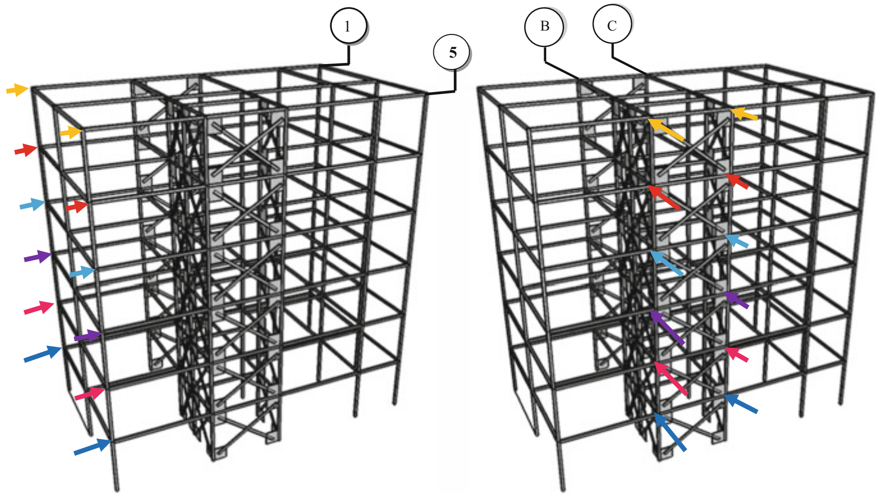


Fig. 2.58 The procedure of distribution of seismic load in the structural member

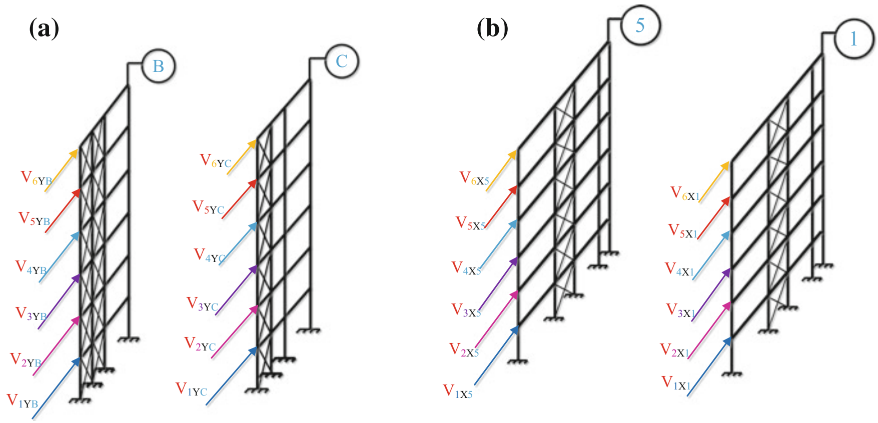
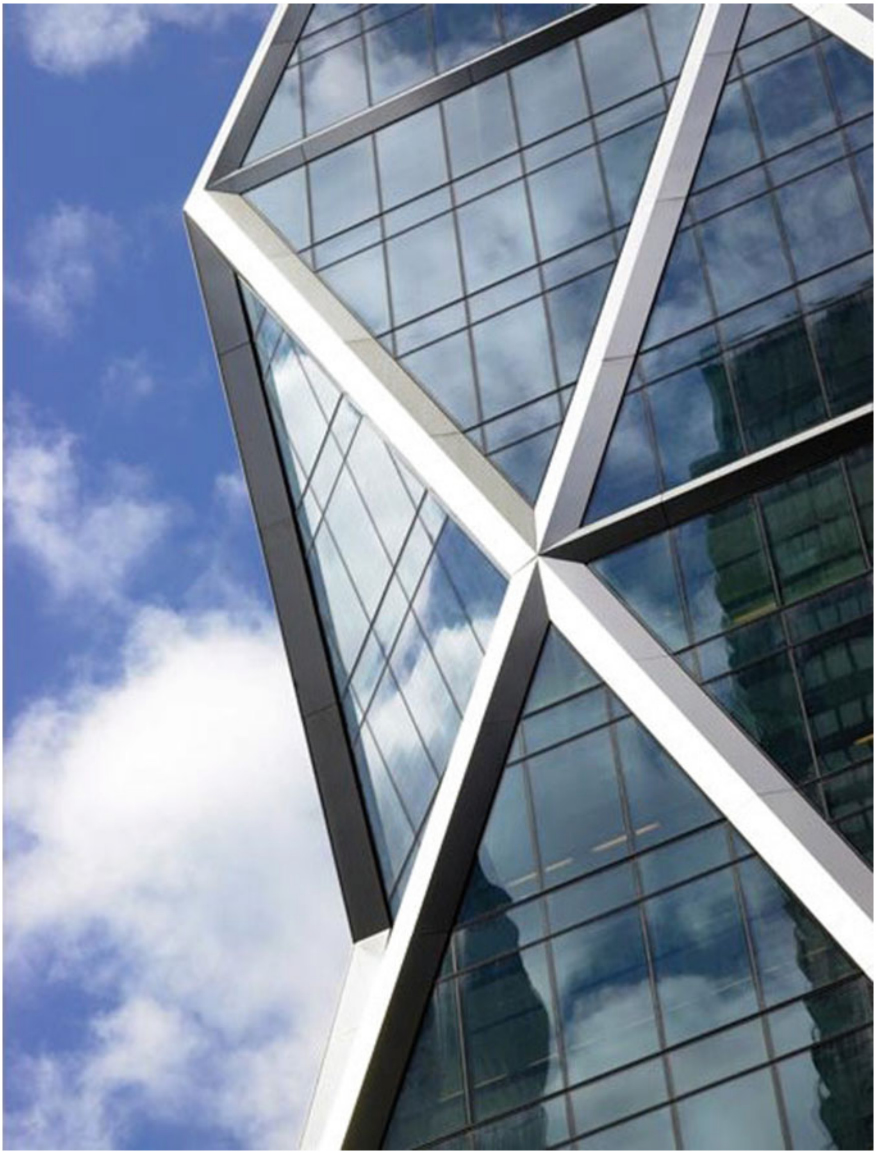


Fig. 2.59 The procedure of distribution of seismic load in the structural member

of the floor is distributed according to the stiffness of each bent as per the equation $\left(Q_i = \left[\frac{(K)_i}{\sum (K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i\right)$, which distributes the shear floor to the lateral resistance bracing system in the X and Y directions. Figure 2.58 shows the distribution of the shear floor in the lateral resistance bracing system, while Fig. 2.59 shows the distributed shear floors in the frame, rendering them ready for the calculation of the lateral displacement under earthquake load.

2.4 Displacement



2.4.1 Evaluation of Deformation of Frames Under Lateral Load

An approximate calculation of the displacement can be made by using the moment-area method to obtain the flexural component (i.e., the component resulting from column axial deformations) and by applying a shear deflection formula to calculate the shear component. The method is appropriate for braced bents in which the flexural mode stiffness is entirely attributable to the axial areas of the columns; these include the majority of bracing types. It has the advantage that a detailed member force analysis of the frame is not necessary: only the external moment and the total shear force at each level are required. Consider a cantilever beam in Fig. 2.60. Deflection of cantilever beam under concentrated load can be calculated according to structure analysis. Hence;

$$\textit{Total deflection of beam}: \Delta = \Delta_f + \Delta_s = \frac{PL^3}{3EI} + \frac{PL}{GA_s} \tag{2.44}$$

An approximate calculation of the displacement can be made by using the moment area method to obtain the flexure component and by applying a shear deflection formula to compute the shear component. The method is appropriate braced bents in which the flexure mode stiffness is completely attributable to the axial areas of the columns; these contain the majority of bracing types. It has the advantage that a detailed member force analysis of the frame is not necessary; only the external moment and the total shear force at the each level are required.

2.4.1.1 Flexure Behaviour

Flexure behaviour causes to increase length of tension cords of section and decrease the length of compression cords. Totally, there is deformation in different parts of beam (Fig. 2.61).

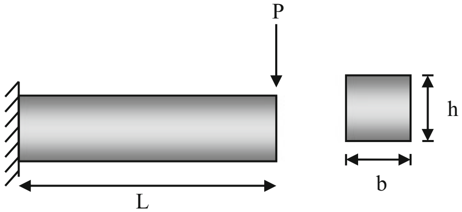


Fig. 2.60 Cantilever beam under concentrated load

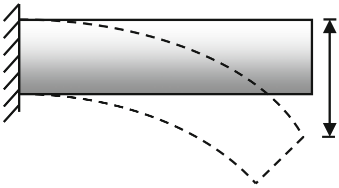


Fig. 2.61 Bending deformation

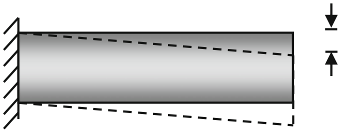


Fig. 2.62 Shear deformation

2.4.1.2 Shear Behaviour

Actually, in this situation there is not any change in the length of cords of section, just change the angel of cords respect to the horizontal. Figure 2.62 shows the shear deflection in cantilever beam.

2.4.2 Flexural Component

The procedure for obtaining the flexural component of displacement is to first calculate for the structure the external moment diagram, then, to compute for the different vertical regions of the bent, the second moments of area I of the column sectional areas about their common centroid. The moment diagram and the values of I are used to construct an M/El diagram, the story displacement in story i, δ_{if}, due to the flexure of the structure, is then obtained from

$$\delta_{if} = h_i \theta_{if} \tag{2.45}$$

in which h_i is the height of story i, and θ_{if} is the inclination of story i, which is equal to the area under the M/EI curve between the base of the structure and the mid-height of story i.

The total displacement at floor n, due to flexure, is then given by the sum of the story displacements from the first to the nth stories.

$$\Delta_{nf} = \sum_{1}^{n} \delta_{if} \tag{2.46}$$

2.4.3 Shear Component

The shear components of the story displacement in story i, δ_{is}, is a function of the external shear and the properties of the braces and girder in that story. The shear component of the total displacement at floor level n, Δ_{ns}, is equal to the sum of the story shear components of displacement from the first to the nth stories, that is

$$\Delta_{ns} = \sum_{1}^{n} \delta_{is} \tag{2.47}$$

Formulas for the shear component of the story displacement, δ_{is}, are given for various types of braced bent in Table 2.7. Having obtained the flexural and shear components of displacement, the total displacement at level n is given by

$$\Delta_{n} = \Delta_{nf} + \Delta_{ns} \tag{2.48}$$

Calculation of flexure components of displacement will be determined as fallows.

1. Compute the moment of inertia of the column sectional areas about their common centroid. Figure 2.63 shows the frame (B) with columns section;

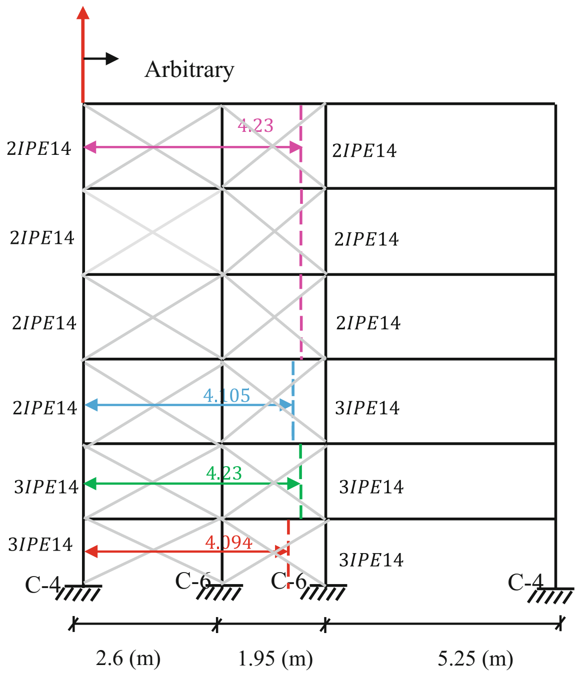


Fig. 2.63 Calculation of moment inertia **a** frame (B), **b** frame (C)

For calculation of centroid should be used of Eq. (2.49).

$$C = \frac{\sum A_i \bar{x}_i}{\sum A} \tag{2.49}$$

where in equation:

A_i area of column
$\bar{x}_i$ Distance of column i from the arbitrary origin

Equation (2.50) is used to calculate the second moment inertia of column areas about the centroid.

$$I = \sum A_i C_i^2 \tag{2.50}$$

where in equation:

C_i Distance of column I from the centroid (Smith et al. 1991)

Centroid and moment inertia of column of frame (B), story (6) is calculated according Eqs. (2.49) and (2.50).

$$C_6 = \frac{(2 \times 0.00164 \times 2.6) + (2 \times 0.00164 \times 4.55) + (2 \times 0.00164 \times 9.8)}{(8 \times 0.00164)} = 4.23$$

$$I_6 = \frac{(0.00328 \times 4.23^2) + (0.00328 \times (4.23 - 2.6)^2) + (0.00328 \times (4.55 - 4.23)^2) + (0.00328 \times (9.8 - 4.23)^2)}{(8 \times 0.00328)} = 0.1695$$

Centroid and moment inertia of story six, five and four are same, Fig. 2.63 shows frame (B) with different story centroid. Calculation of centroid and moment inertia of other story is done as same of story six. Centroid of columns in story three, two and one is 4.105 (m), 4.23 (m) and 4.094 (m) respectively. Moment inertia of story three is 0.1737 (m^4). Moment inertia is increased from top to bottom, this is because of that the section of area increased from top to bottom. Moment inertia in story two and one is 0.2534 (m^4) and 0.26129 (m^4) respectively.

$$C_{5,6,4} = \frac{(2 \times 0.00164 \times 2.6) + (2 \times 0.00164 \times 4.55) + (2 \times 0.00164 \times 9.8)}{(8 \times 0.00164)} = 4.23$$

$$C_3 = \frac{(3 \times 0.00164 \times 2.6) + (3 \times 0.00164) + (2 \times 0.00164 \times 9.8)}{(10 \times 0.00164)} = 4.105$$

$$C_2 = \frac{(3 \times 0.00164 \times 2.6) + (3 \times 0.00164 \times 4.55) + (3 \times 0.00164 \times 9.8)}{(12 \times 0.00164)} = 4.23$$

$$C_1 = \frac{(3 \times 0.00164 \times 2.6) + (2 \times 0.17 \times 0.008 \times 2.6) + (3 \times 0.00164 \times 4.5)}{(12 \times 0.00164) + (4 \times 0.17 \times 0.008)} + \frac{(2 \times 0.17 \times 0.008 \times 4.5) + (3 \times 0.00164 \times 9.8)}{(12 \times 0.00164) + (4 \times 0.17 \times 0.008)} = 4.094$$

$$I_{6,5,4} = \sum A(c)^2$$
$$= \left[(0.00328)(4.23)^2\right] + \left[(0.00328)(1.63)^2\right] + \left[(0.00328)(0.32)^2\right]$$
$$+ \left[(0.00328)(5.57)^2\right] = 0.1695$$
$$I_3 = \left[(0.00328)(4.105)^2\right] + [(0.00492)(1.505] + \left[(0.00492)(0.445)^2\right]$$
$$+ \left[(0.00328)(5.695)^2\right] = 0.17370$$
$$I_2 = \left[(0.00492)(4.23)^2\right] + \left[(0.00492)(1.63)^2\right] + \left[(0.00492)(0.32)^2\right] + \left[(0.00492)(5.57)^2\right]$$
$$= 0.2542$$
$$I_1 = \left[(0.00492)(4.094)^2\right] + \left[(0.00764)(1.494)^2\right] + \left[(0.0764)(4.56)^2\right]$$
$$+ \left[(0.00492)(5.706)^2\right] = 0.26129$$

Compute the value of external moment M at each mid-story level.

2. Figure 2.64 shows the magnitude of external moment in each story at mid-story level.

$$M_6 = 265 \times \frac{3.6}{2} = 477$$
$$M_5 = (265 \times 5.4) + (218.7 \times 1.8) = 1824.6$$
$$M_4 = (265 \times 9) + (218.7 \times 5.4) + (173.9 \times 1.8) = 3879.1$$
$$M_3 = (265 \times 12.6) + (218.7 \times 9) + (173.9 \times 5.4) + (153.9 \times 1.8) = 6523.7$$
$$M_2 = (265 \times 16.2) + (218.7 \times 12.6) + (173.9 \times 9) + (153.9 \times 5.4)$$
$$+ (64.2 \times 1.8) = 9561.1$$
$$M_1 = (265 \times 19.65) + (218.7 \times 16.05) + (173.9 \times 12.45) + (153.9 \times 7.5)$$
$$+ (64.2 \times 3.9) + (36.2 \times 1.65) = 12642.7$$

3. Determine for each story the value of $\frac{Mh}{EI}$. These are the changes in inclination in each story I due to flexure, $\delta\theta_{if}$. It should mentioned that the θ_{if} is cumulative; hence:

$$\theta_{i,1} = \frac{M_{1\times}h_1}{E \times I_1} = \frac{12642.78 \times 3.3}{2 \times 10^8 \times 0.26129} = 0.0007983694$$
$$\theta_{i,2} = \frac{M_{2\times}h_2}{E \times I_2} = \frac{9561.188 \times 3.6}{2 \times 10^8 \times 0.2542} = 0.0006770314$$
$$\theta_{if,2} = \theta_{i,2} + \theta_{i,1=}147.5401 \times 10^{-5}$$
$$\theta_{i,3} = \frac{M_{3\times}h_3}{E \times I_3} = \frac{6523.752 \times 3.6}{2 \times 10^8 \times 0.1737} = 0.0006760365$$
$$\theta_{if,3} = \theta_{i,3} + \theta_{i,2} + \theta_{i,1=}215.1437 \times 10^{-5}$$
$$\theta_{i,4} = \frac{M_{4\times}h_4}{E \times I_4} = \frac{3879.119 \times 3.6}{2 \times 10^8 \times 0.1695} = 0.0004119418$$
$$\theta_{if,4} = \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1=}256.3379 \times 10^{-5}$$
$$\theta_{i,5} = \frac{M_{5\times}h_5}{E \times I_5} = \frac{1824.677 \times 3.6}{2 \times 10^8 \times 0.1695} = 0.000193771$$
$$\theta_{if,5} = \theta_{i,5} + \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1=}275.715 \times 10^{-5}$$
$$\theta_{i,6} = \frac{M_{6\times}h_6}{E \times I_6} = \frac{477 \times 3.6}{2 \times 10^8 \times 0.1695} = 0.00005065494$$
$$\theta_{if,6} = \theta_{i,6} + \theta_{i,5} + \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1=}280.7805 \times 10^{-5}$$

4. Calculation of displacement in story i, $\delta_{if} = \theta_{if} h_i$

$$\delta_{6f} = 280.7805 \times 10^{-5} \times 3.6 = 1010.81 \times 10^{-5}$$
$$\delta_{5f} = 275.715 \times 10^{-5} \times 3.6 = 992.574 \times 10^{-5}$$
$$\delta_{4f} = 256.3379 \times 10^{-5} \times 3.6 = 922.8165 \times 10^{-5}$$
$$\delta_{3f} = 215.1437 \times 10^{-5} \times 3.6 = 774.5174 \times 10^{-5}$$
$$\delta_{2f} = 147.5401 \times 10^{-5} \times 3.6 = 531.1443 \times 10^{-5}$$
$$\delta_{1f} = 0.0007983694 \times 3.6 = 263.4619 \times 10^{-5}$$

5. At each level where the value of the lateral displacement is required, evaluate the accumulation of the story displacement, δ_{if}, from story 1 up to the considered nth floor, to give the displacement Δ_{nf} due to flexure. Table 2.37 shows the total displacement of flexure component of bent (B) in detail.

Calculation of shear components of displacement will be determined as fallows.

1. Calculate the value of the external shear Q_i acting in each story i. external shear of bent (B) is calculated and shown in Table 2.33.

Table 2.37 Evaluation of flexure component of displacement

Story	$h_{i_{(m)}}$	$I_{i_{(m^4)}}$	$M_{i_{(kN\ m)}}$	$\theta_i \times 10^{-5}$	θ_{if}	δ_{if}	Δ
6	3.6	0.1695	265.0004	477.0007	5.065494	280.7805	1010.81
5	3.6	0.1695	218.7082	1824.677	19.3771	275.715	992.574
4	3.6	0.1695	173.9396	3879.119	41.19418	256.3379	922.8165
3	3.6	0.1737	153.9443	6523.752	67.60365	215.1437	774.5174
2	3.6	0.2542	64.27919	9561.188	67.70314	147.5401	531.1443
1	3.3	0.26129	36.26585	12642.78	79.83694	79.83694	263.4619

2. Calculate for each story i the story displacement due to shear, δ_{is} by substituting the value of the story shear and member properties into the suitable formula from Table 2.7.
3. Sum the story displacements due to shear component. Total shear displacement is calculated and shown in Table 2.39.

As it is shown in Fig. 2.64 there are two diagonal braced frames, so it is noticeable that:

1. Magnitude of distributed of shear force of story is depended on the length of span that bracing is located in. As a example, shear force in story 6 is equal 265 (kN), the shear force that is transferred to the double diagonal braced frame with 2.6 (m) length is calculated according equation below:

$$Q_1 = Q\frac{l_1}{l_1 + l_2} = 265\left(\frac{2.6}{2.6 + 1.95}\right) = 151.42\ (\text{kN})$$

Shear force in double diagonal with 1.95 (m) length is equal:

$$Q_2 = Q\frac{l_2}{l_1 + l_2} = 265\left(\frac{1.95}{2.6 + 1.95}\right) = 113.57\ (\text{kN})$$

2. As it mentioned before, shear component displacement is calculated according to Table 2.33. In this stage, it should use the span with maximum shear displacement. In order to, double diagonal bracing with 1.95 (m) length span should be used to calculate of shear displacement in frame (B). This is because of that shear displacement of 1.95 (m) length span in more than the shear displacement of bracing with 2.6 (m) length span. Table 2.38 shows the total shear displacement of frame (B).

Total displacement of frame (B) is shown in Table 2.39. Graph 2.1 shows the flexure, shear and total displacement of frame (B).

Table 2.38 Evaluation of shear component of displacement

Story	l_1	l_2	$Q^{(KN)}$	$Q_1 = Q\frac{l_1}{l_1+l_2}$	$Q_2 = Q\frac{l_2}{l_1+l_2}$	$K_{(B_4)}^{(KN/m)}$	$K_{(B_5)}^{(\mathrm{KN/m})}$	$\delta_1 = \frac{Q_1}{K(B_4)}$	$\delta_2 = \frac{Q_2}{K(B_5)}$	Δ
6	2.6	1.95	265	151.4288	113.5716	75790.22	48996.97	0.001998	0.002318	0.002318
5	2.6	1.95	483.70	276.4049	207.3037	75790.22	48996.97	0.003647	0.004231	0.004231
4	2.6	1.95	657.64	375.799	281.8492	83303.89	53331.02	0.004511	0.005285	0.005285
3	2.6	1.95	811.59	463.7672	347.8254	83303.89	63965.28	0.005567	0.005438	0.005567
2	2.6	1.95	875.87	500.4981	375.3736	97553.91	63965.28	0.00513	0.005868	0.005868
1	2.6	1.95	912.13	521.2215	390.9161	118913.2	79963.23	0.004383	0.004889	0.004889

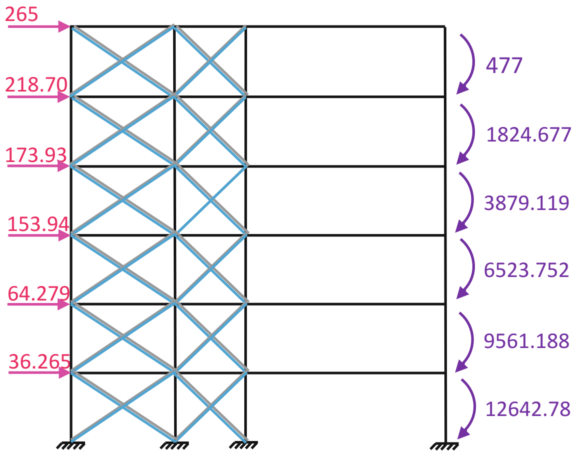
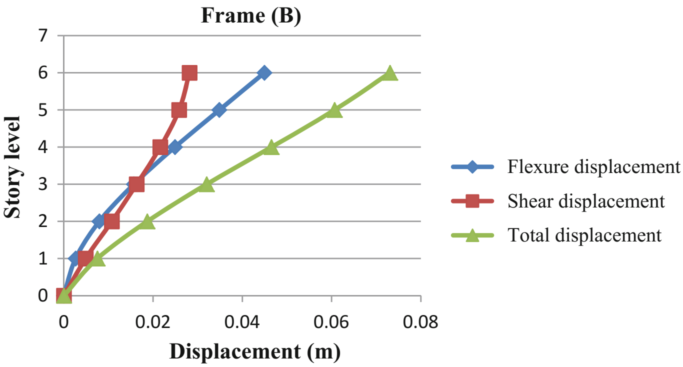


Fig. 2.64 External moment of frame (B)

Table 2.39 Evaluation Total displacement of frame (B)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.04495324	0.028158	0.073111
5	0.034845142	0.02584	0.060685
4	0.024919401	0.021609	0.046529
3	0.015691236	0.016324	0.032016
2	0.007946062	0.010757	0.018703
1	0.002634619	0.004889	0.007523



Graph 2.1 Displacement of frame (B)

Appendix

I. Section Property
II. Weight of Floor
III. Stiffness And Center of Rigidity
IV. Eccentricity
v. Displacement

I. Section Property:

Properties of different section of columns, beams and bracings are computed in this part. Cross- section area, second moment inertia in X and Y direction, section modulus in X and y direction and radius of gyration are calculated for each section (Tables 2.40, 2.41, 2.42, 2.43, 2.44 and 2.45; Figs. 2.65, 2.66, 2.67, 2.68 and 2.69).

II. **Weigh of stories**:

Total weight of stories 2-6 is illustrated in Charts 2.8, 2.9, 2.10, 2.11, 2.12 and 2.13.

III: **Center of stiffness** (Charts 2.14, 2.15, 2.16 and 2.17)
IV: **Calculation of eccentricity in each floor** (Figs. 2.70, 2.71, 2.72 and 2.73)
V: **Calculation of displacement in frames**

Total displacement of frame (**C**):

Table 2.40 Properties of some types of single IPE

										x-x			y-y							
IPE	h_{mm}	b_{mm}	s_{mm}	t_{mm}	r_{mm}	c_{mm}	$h\text{-}2c$	F_{cm^2}	$G_{kg/m}$	Jx_{cm^4}	Wx_{cm^3}	ix_{cm}	Jy_{cm^4}	Wy_{cm^3}	iy_{cm}	$a_{1\,mm}$	Sx_{cm^3}	Sx_{cm^3}	JD_{cm^4}	C_{cm}
140	140	7.3	4.7	6.9	7	13.9	112	16.4	12.9	541	77.3	5.74	44.9	12.3	1.65	112	34.2	12.3	2.45	19
160	160	8.2	5	7.4	9	16.4	127	20.1	15.8	869	109	6.58	68.3	16.7	1.84	129	61.9	14	3.62	39
180	180	9.1	5.3	8	9	17	146	23.9	18.8	1320	146	7.42	101	22.2	2.05	145	83.2	15.8	4.81	74
200	200	100	5.6	8.5	12	20.5	159	28.5	22.4	1940	194	8.26	142	28.5	2.24	162	110	17.6	7.01	12
220	220	110	5.9	9.2	12	21.2	177	33.4	26.2	2770	252	9.11	205	37.3	2.48	179	143	19.4	9.1	22

Table 2.41 Properties of channels 140 and 160

U	h_{mm}	b_{mm}	s_{mm}	t_{mm}	r_{mm}	c_{mm}	$h-2c_{mm}$	F_{cm^2}	$G_{kg/m}$	Jx_{cm^4}	Wx_{cm^3}	ix_{cm}	Jy_{cm^4}	Wy_{cm^3}	iy_{cm}	$a_{1\,mm}$	Sx_{cm^3}	Sx_{cm^3}	JD_{cm^4}	C_{cm}
140	140	60	7	10	5	21	97	20.4	16	605	86.4	5.45	62.7	14.8	1.75	70	51.4	11.8	5.68	180
160	160	65	7.5	1.5	5.5	22.5	116	24	18.8	925	116	6.21	85.3	18.3	1.89	82	68.8	13.3	7.39	320

Table 2.42 Properties of 2IPE140

A (cm^2)	2 × 16.4 = 32.8 cm^2
$I_x = \sum I_x + Ad^2$ (cm^4)	541 × 2 = 1082 cm^4
$I_y = \sum I_y + Ad^2$ (cm^4)	$(44.9 + [16.4 \times (3.65^2)]) + (44.9 + [16.4 \times (3.65^2)]) = 526.778$ cm^4
$S_x = \frac{I_x}{C}$ (cm^3)	$\frac{1082}{7} = 154.571$ cm^3
$S_y = \frac{I_y}{C}$ (cm^3)	$\frac{526.778}{7.3} = 72.1614$ cm^3
$r_x = \sqrt{\frac{I_x}{A}}$ (cm)	$\sqrt{\frac{1082}{32.8}} = 5.7435$ cm
$r_y = \sqrt{\frac{I_y}{A}}$ cm	$\sqrt{\frac{526.778}{32.8}} = 4.00753$ cm^3

Table 2.43 Properties of 2IPE140c14

A (cm^2)	2 × 16.4 = 32.8 cm^2
$I_x = \sum I_x + Ad^2$ (cm^4)	541 × 2 = 1082 cm^4
$I_y = \sum I_y + Ad^2$ (cm^4)	$(44.9 + [16.4 + (7.15^2)]) + (44.9 + [16.4 + (7.15^2)]) = 1766.618$ cm^4
$S_x = \frac{I_x}{C}$ (cm^3)	$\frac{1082}{7} = 154.571$$cm^3$
$S_y = \frac{I_y}{C}$ (cm^3)	$\frac{1766.618}{10.8} = 163.576$ cm^4
$r_x = \sqrt{\frac{I_x}{A}}$ (cm)	$\sqrt{\frac{1082}{32.8}} = 5.7435$ cm
$r_y = \sqrt{\frac{I_y}{A}}$ (cm)	$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1766.618}{32.8}} = 7.33896$ cm

Table 2.44 Properties of 3IPE140

A (cm^2)	3 × 16.4 = 49.2 cm^2
$I_x = \sum I_x + Ad^2$ (cm^4)	541 × 3 = 1623 cm^4
$I_y = \sum I_y + Ad^2$ (cm^4)	$(44.9 + [16.4 + (7.3^2)]) + (44.9 + [16.4 + (7.3^2)]) + (44.9) = 1882.612$ cm^4
$S_x = \frac{I_x}{C}$ (cm^3)	$\frac{1623}{7} = 231.857$ cm^3
$S_y = \frac{I_y}{C}$ (cm^3)	$\frac{1882.612}{10.95} = 171.928$ cm^3
$r_x = \sqrt{\frac{I_x}{A}}$ (cm)	$\sqrt{\frac{1623}{49.2}} = 5.7435$ cm
$r_y = \sqrt{\frac{I_y}{A}}$ (cm)	$\sqrt{\frac{1882.612}{49.2}} = 6.18583$ cm

Table 2.45 Properties of 3IPE140TBPL170 × 8

A (cm^2)	$(3 \times 16.4) + [2 \times (17 \times 0.8)] = 76.4\ \text{cm}^2$
$I_x = \sum I_x + Ad^2$ (cm^4)	$(541 \times 3) + \left[2 \times \left[\left(\frac{17 \times 0.8^3}{12}\right) + ((17 \times 0.8)(7.4^2))\right]\right] = 3113.92\ \text{cm}^4$
$I_y = \sum I_y + Ad^2$ (cm^4)	$\left(\frac{0.8 \times 17^3}{12}\right) + [44.9 + [16.4 \times (7.3^2)]] + [44.9 + [16.4 \times (7.3^2)]] + \left(\frac{0.8 \times 17^3}{12}\right)\ \text{cm}^4 + (44.9) = 2537.6786$
$S_x = \frac{I_x}{C}$ (cm^3)	$\frac{3113.92}{7.4} = 399.2205\ \text{cm}^3$
$S_y = \frac{I_y}{C}$ (cm^3)	$\frac{2537.6786}{10.95} = 231.7514\ \text{cm}^3$
$r_x = \sqrt{\frac{I_x}{A}}$ (cm)	$\sqrt{\frac{3113.92}{76.4}} = 6.3842\ \text{cm}$
$r_y = \sqrt{\frac{I_y}{A}}$ (cm)	$\sqrt{\frac{2537.6786}{76.4}} = 5.7633\ \text{cm}$

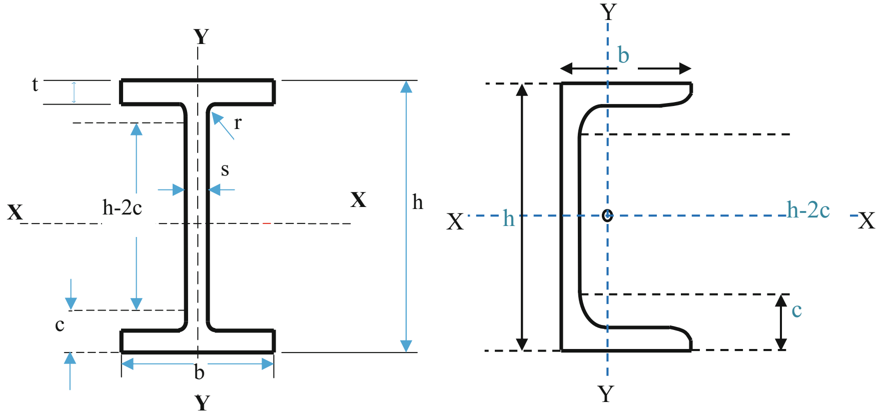


Fig. 2.65 Cross-section of IPE and Channel

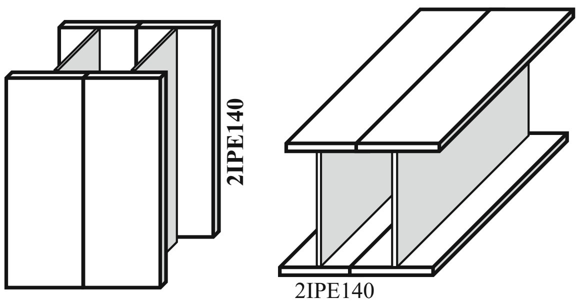


Fig. 2.66 2IPE140

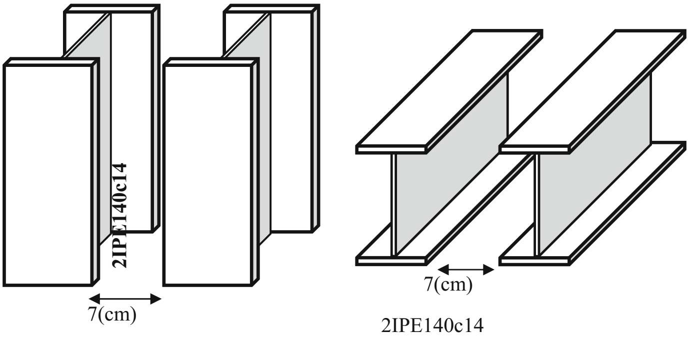


Fig. 2.67 2IPE140c14

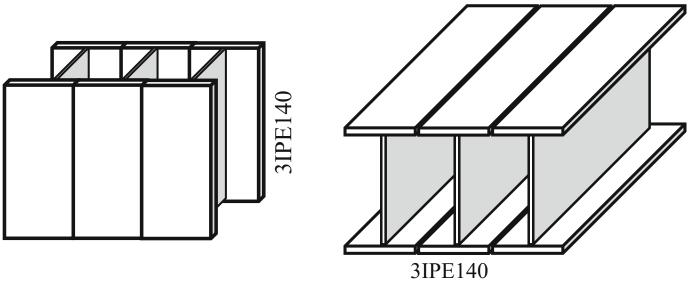


Fig. 2.68 3IPE140

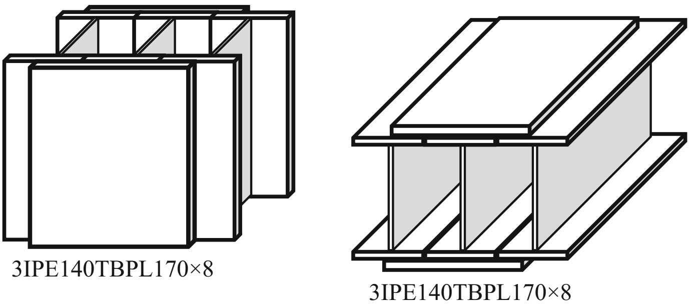
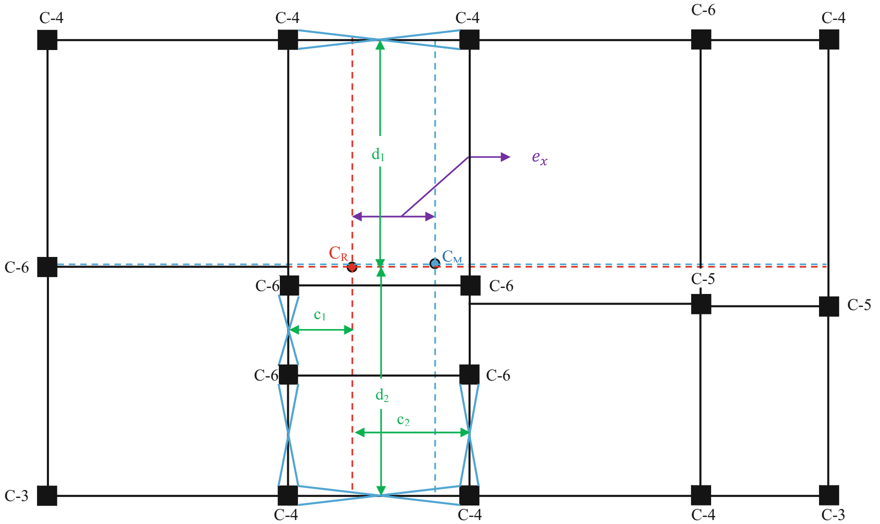


Fig. 2.69 3IPE140TBPL170 ×8

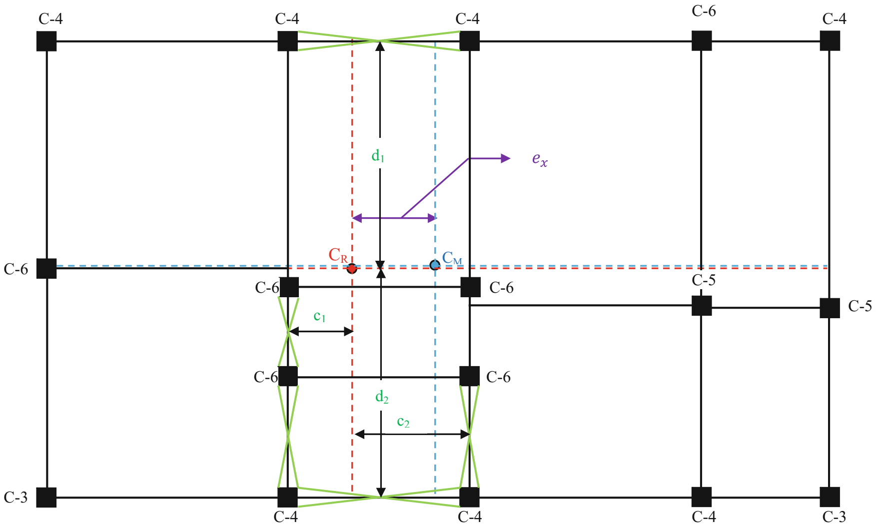


$e_x = 8.414 - 6.6271 = 1.7869$

$e_y = 4.94 - 4.9 = 0.04$

d_2=4.9
d_1=4.9
c_1=1.4771
c_2=2.4229

Fig. 2.70 Calculation of eccentricity on story four



$e_x = 8.414 - 6.56 = 1.854$

$e_y = 4.94 - 4.9 = 0.04$

d_2=4.9

d_1=4.9

c_1=1.41

c_2=2.49

Fig. 2.71 Calculation of eccentricity on story three

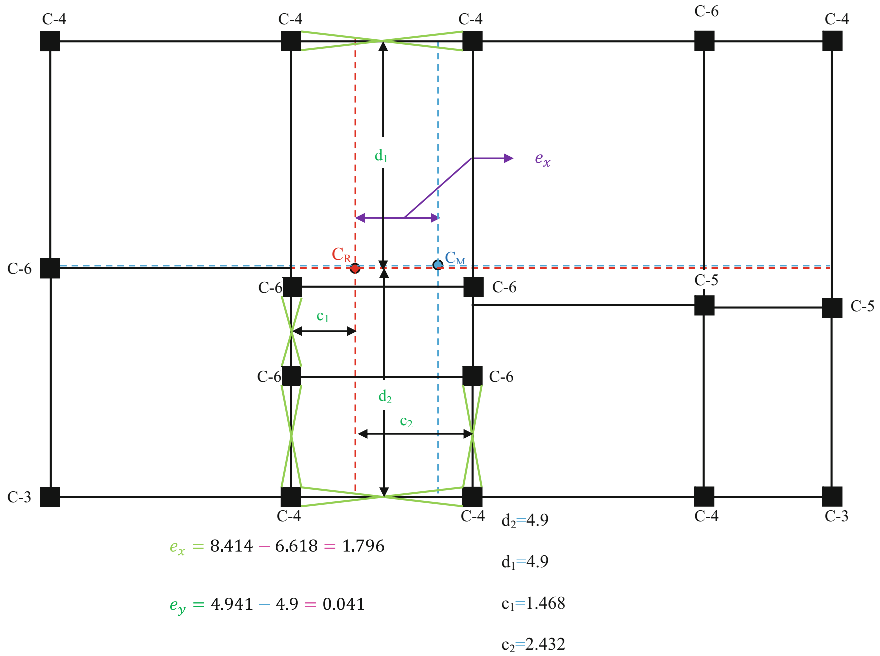


Fig. 2.72 Calculation of eccentricity on story two

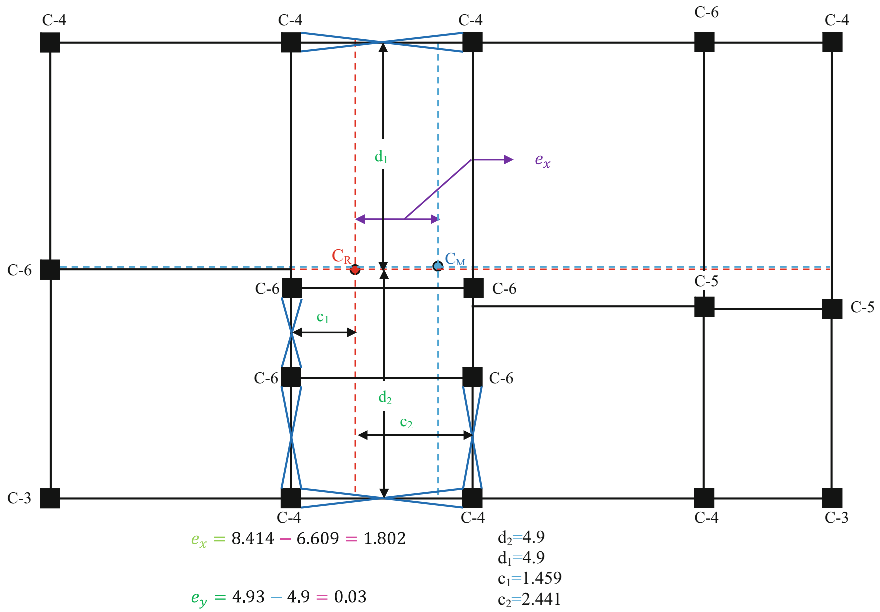


Fig. 2.73 Calculation of eccentricity on story one

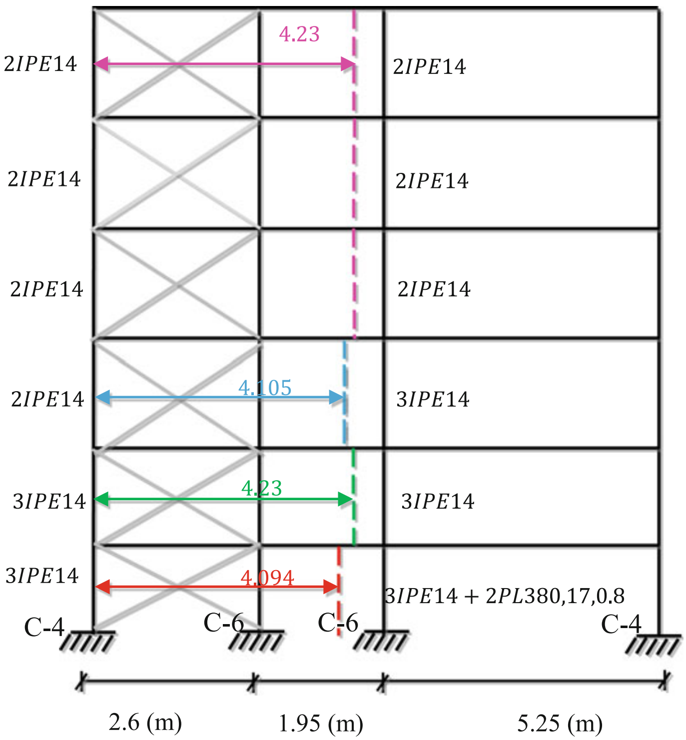


Fig. 2.74 Centroid of column in each story

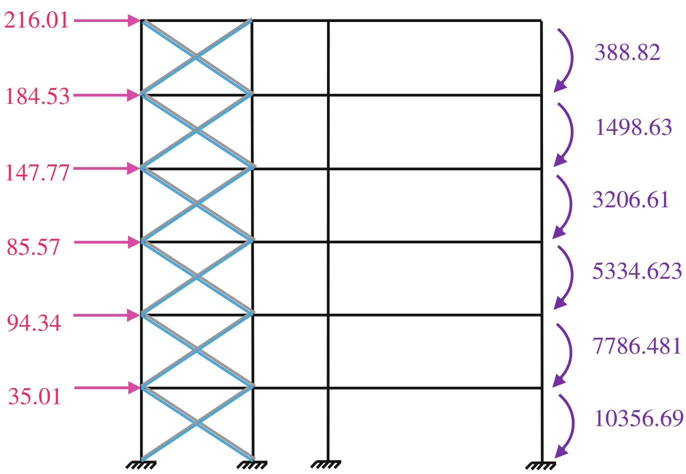


Fig. 2.75 External moment of frame (C)

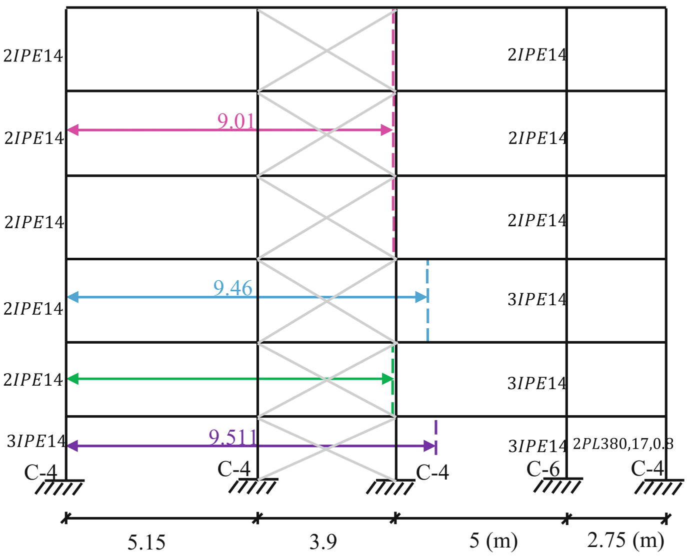


Fig. 2.76 Centroid of column in each story

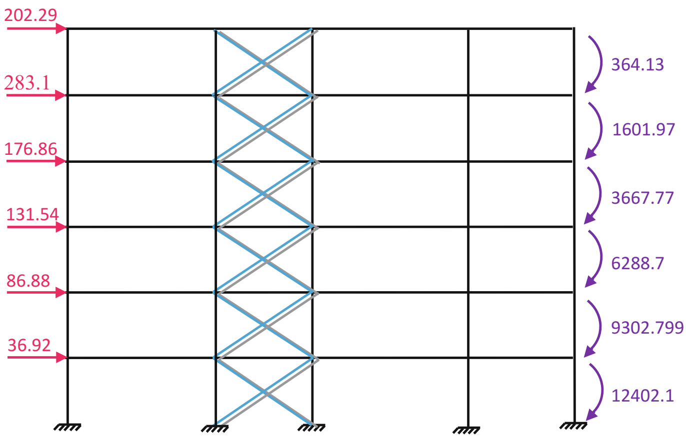


Fig. 2.77 External moment of frame (1)

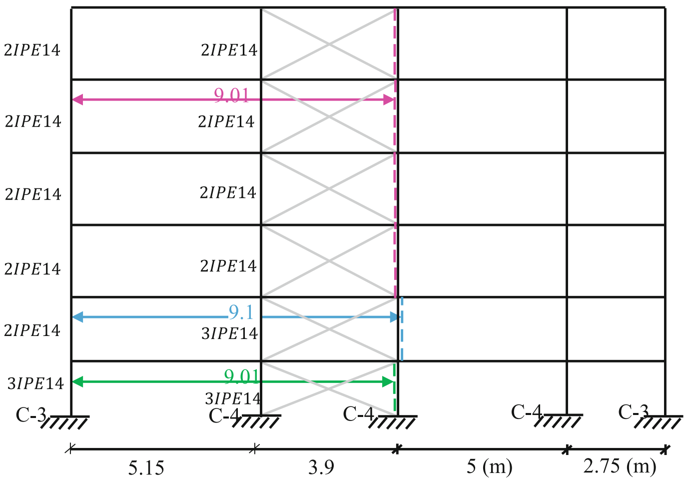


Fig. 2.78 The centroid of columns in each story

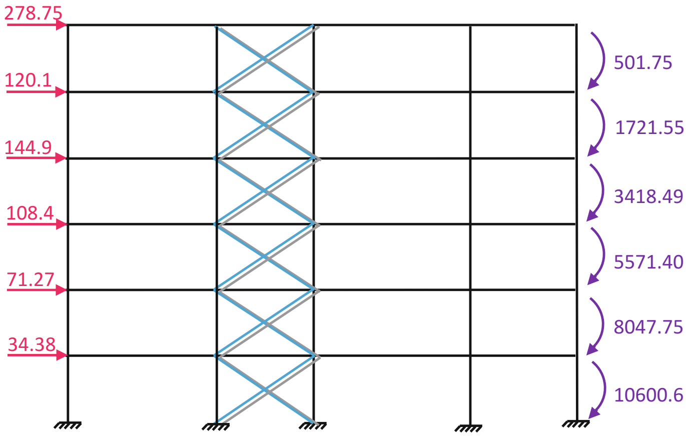


Fig. 2.79 External moment of frame (5)

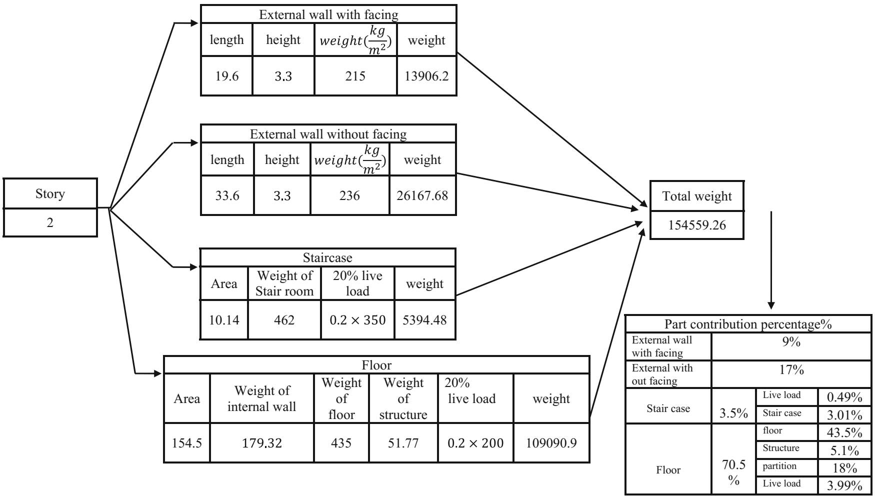


Chart 2.8 Total weight of story two

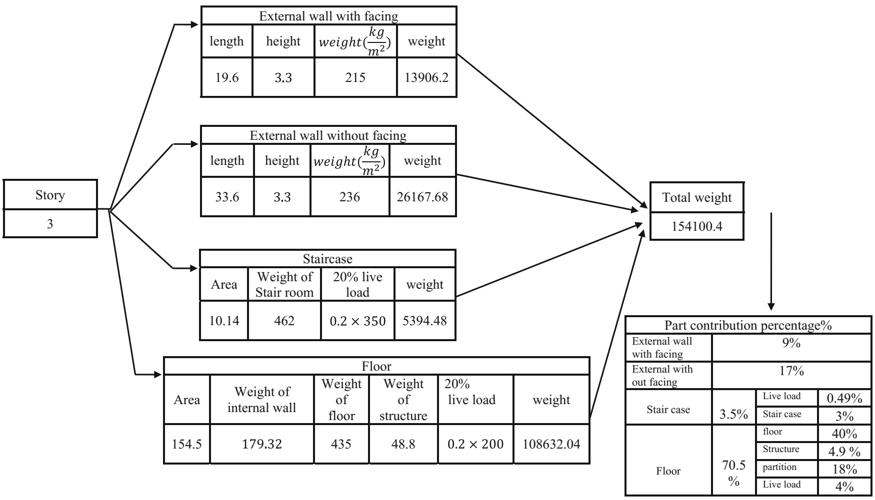


Chart 2.9 Total weight of story three

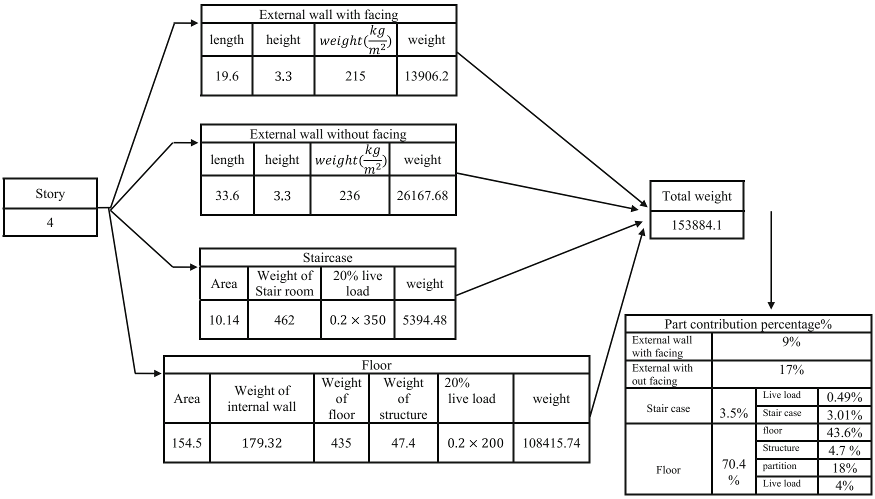


Chart 2.10 Total weight of story four in detail

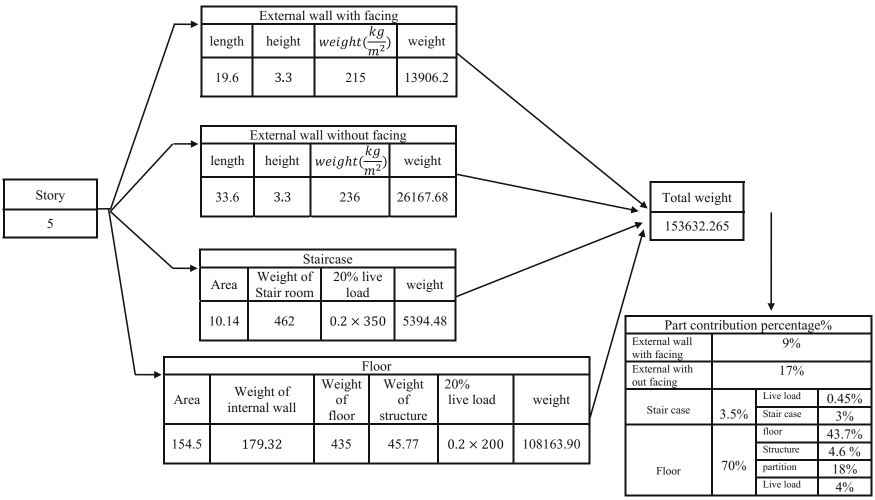


Chart 2.11 Total weight of story five in detail

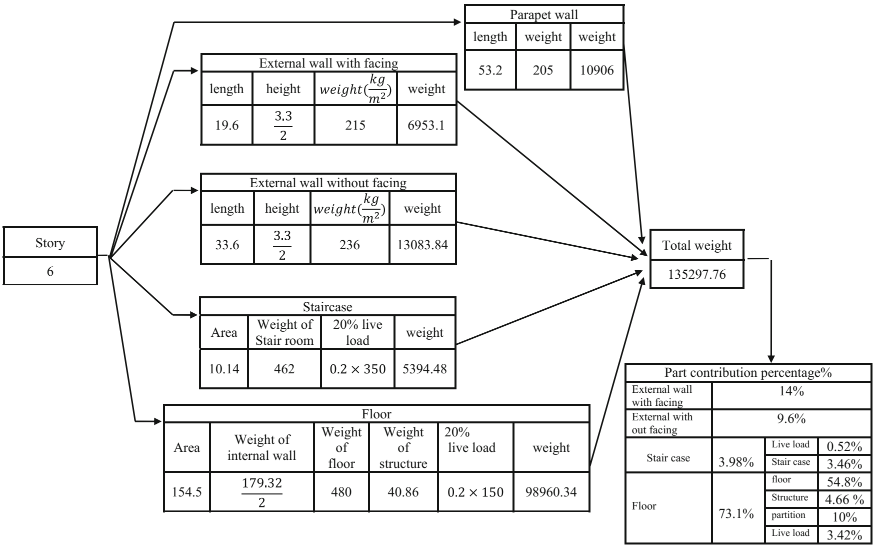


Chart 2.12 Total weight of story six

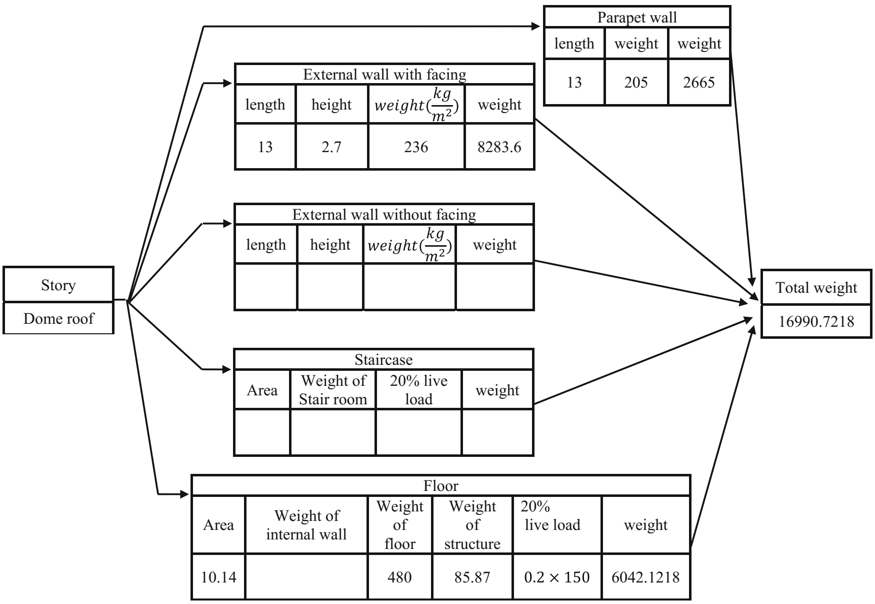


Chart 2.13 Total weight of dome roof

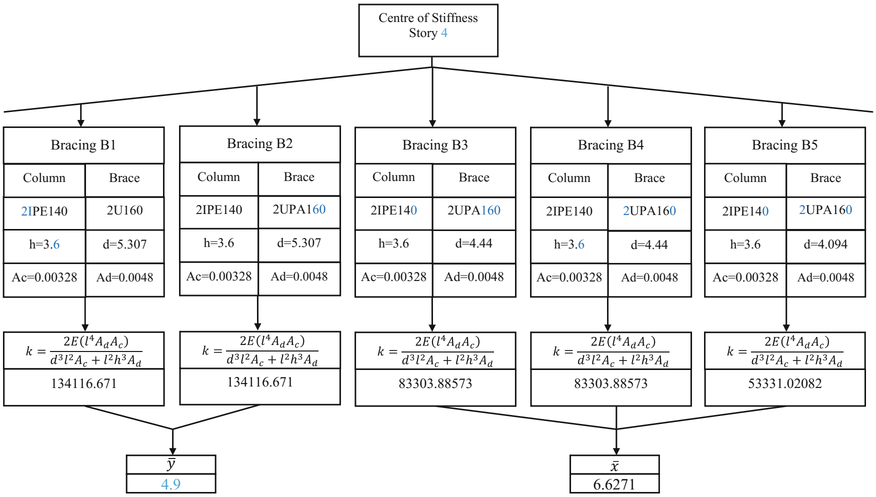


Chart 2.14 Calculation of stiffness of bracing and center rigidity of story four

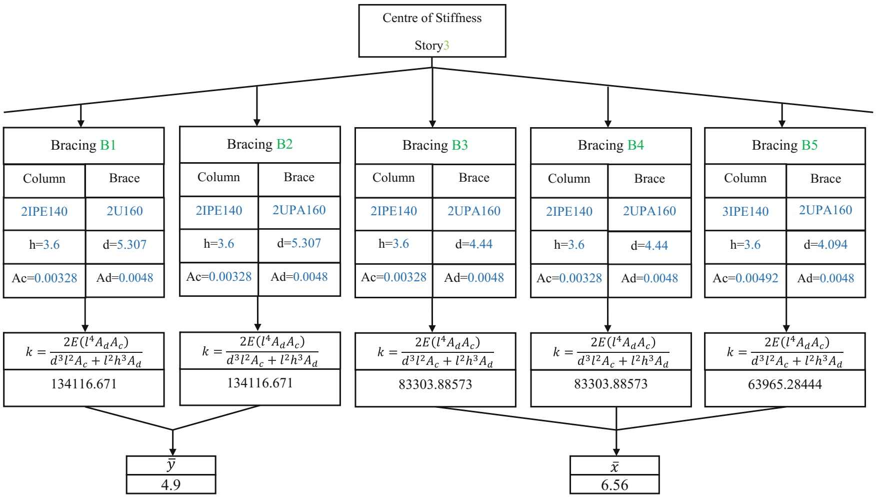


Chart 2.15 Calculation of stiffness of bracing and center rigidity of story three

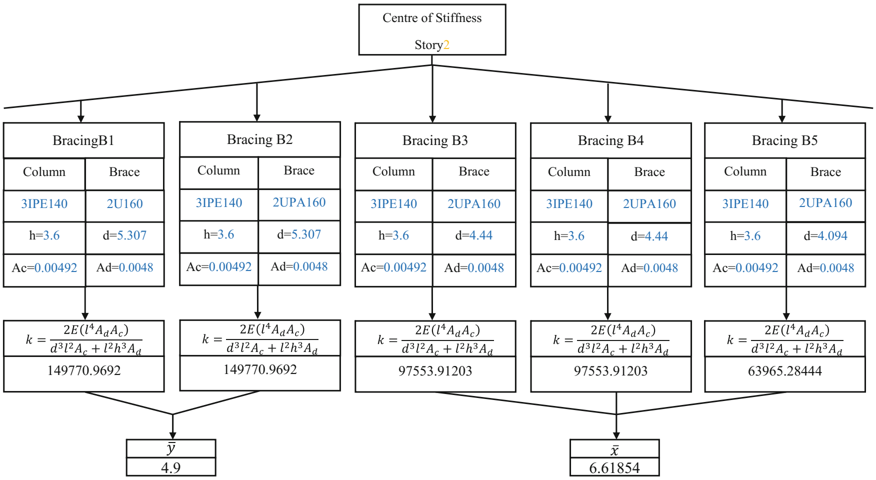


Chart 2.16 Calculation of stiffness of bracing and center rigidity of story two

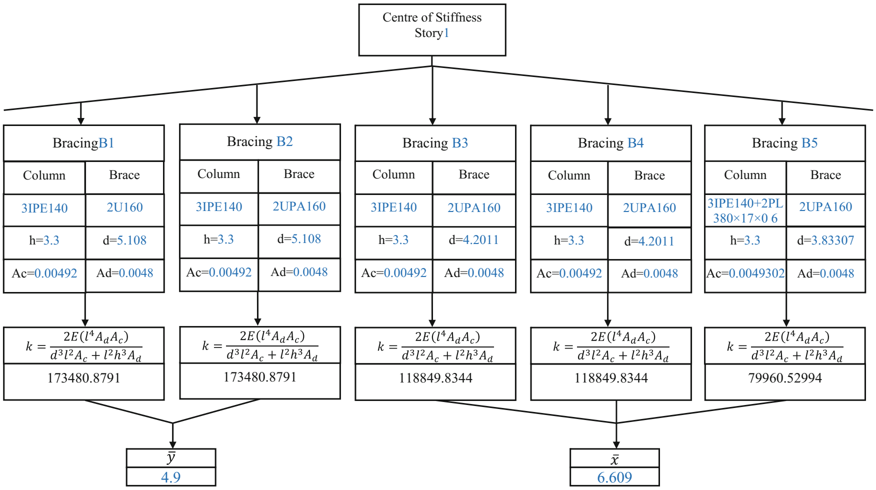


Chart 2.17 Calculation of stiffness of bracing and center rigidity of story one

Total displacement of frame (C) is composed of two components: flexure displacement and shear displacement. The procedure of calculation displacement of frame (C) is the same as displacement of frame (B).

Flexure component:

1. Computing of column centroid and moment inertia

$$C_{5,6,4} = \frac{(2\times 0.00164\times 2.6) + (2\times 0.00164\times 4.55) + (2\times 0.00164\times 9.8)}{(8\times 0.00164)} = 4.23$$

$$C_3 = \frac{(3\times 0.00164\times 2.6) + (3\times 0.00164) + (2\times 0.00164\times 9.8)}{(10\times 0.00164)} = 4.105$$

$$C_2 = \frac{(3\times 0.00164\times 2.6) + (3\times 0.00164\times 4.55) + (3\times 0.00164\times 9.8)}{(12\times 0.00164)} = 4.23$$

$$C_1 = \frac{(3\times0.00164\times2.6)+(2\times0.17\times0.008\times2.6)+(3\times0.00164\times4.5)+(2\times0.17\times0.008\times4.55)+(3\times0.00164\times9.8)}{(12\times0.00164)+(4\times0.17\times0.008)} = 4.094$$

Figure 2.74 shows the centroid of columns in each story

$$I_{6,5,4} = \sum A(c)^2 = \left[(0.00328)(4.23)^2\right] + \left[(0.00328)(1.63)^2\right] + \left[(0.00328)(0.32)^2\right] + \left[(0.00328)(5.57)^2\right] = 0.1695$$

$$I_3 = \left[(0.00328)(4.105)^2\right] + [(0.00492)(1.505] + \left[(0.00492)(0.445)^2\right] + \left[(0.00328)(5.695)^2\right] = 0.1737$$

$$I_2 = \left[(0.00492)(4.23)^2\right] + \left[(0.00492)(1.63)^2\right] + \left[(0.00492)(0.32)^2\right] + \left[(0.00492)(5.57)^2\right] = 0.2542$$

$$I_1 = \left[(0.00492)(4.094)^2\right] + \left[(0.00764)(1.494)^2\right] + \left[(0.0764)(4.56)^2\right] + \left[(0.00492)(5.706)^2\right] = 0.26129$$

2. Compute the value of external moment M at each mid-story level.
3. Fig. 2.75 shows the magnitude of external moment in each story at mid-story level.

Calculation of inclination and total displacement is shown in Table 2.46 as a tabular form.

Calculation of shear components of displacement will be determined as fallows.

1. Calculate the value of the external shear Q_i acting in each story i. external shear of bent (C) is calculated in Table 2.34.
2. Calculate for each story i the story displacement due to shear, δ_{is} by substituting the value of the story shear and member properties into the suitable formula from Table 2.7.

Sum the story displacements due to shear component. Total shear displacement is calculated and shown in Table 2.47

Table 2.46 Flexure component of displacement of frame (C)

Story	$h_{i_{(m)}}$	$I_{i_{(m^4)}}$	F (kN)	$M_{i_{(KN.m)}}$	$\theta_{i_{10^{-5}}}$	θ_{if}	δ_{if}	Δ
6	3.6	0.1695	216.0131	388.8236	4.13E−05	0.002299	0.008277	0.036788
5	3.6	0.1695	184.5359	1498.635	0.000159	0.002258	0.008128	0.028511
4	3.6	0.1695	147.7785	3206.613	0.000341	0.002099	0.007555	0.020382
3	3.6	0.1737	85.57268	5334.623	0.000553	0.001758	0.006329	0.012827
2	3.6	0.2542	94.34349	7786.481	0.000551	0.001205	0.004339	0.006498
1	3.3	0.26129	35.01209	10356.69	0.000654	0.000654	0.002158	0.002158

Table 2.47 Evaluation shear component displacement of frame (C)

Story	Q	$K_{(B_3)}$	$\delta = \frac{Q}{K(B_3)}$	Δ
6	216.0131237	75790.22	0.00285	0.03621
5	400.5489963	75790.22	0.005285	0.03336
4	548.3274548	83303.89	0.006582	0.028075
3	633.9001306	83303.89	0.007609	0.021493
2	728.2436252	97553.91	0.007465	0.013884
1	763.2557119	118913.2	0.006419	0.006419

Table 2.48 Evaluation Total displacement of frame (C)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.036787544	0.036210491	0.072998035
5	0.028510624	0.033360346	0.06187097
4	0.020382351	0.028075377	0.048457728
3	0.012827008	0.021493122	0.034320129
2	0.006497556	0.013883631	0.020381187
1	0.002158222	0.006418593	0.008576816

Total displacement of frame (C) is shown in Table 2.48. Graph 2.2 shows the flexure, shear and total displacement of frame (C).

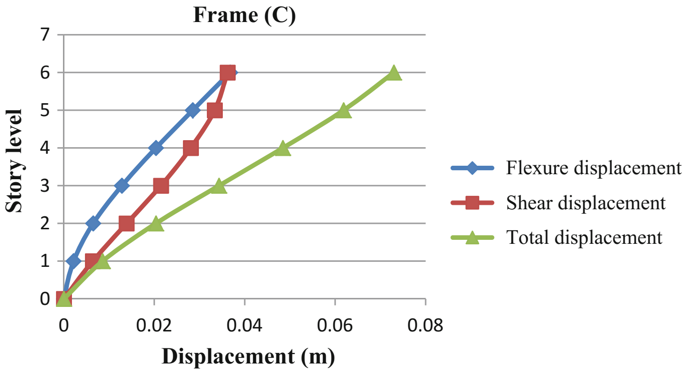
Total displacement of frame (1):

Total displacement of frame (1) is composed of two components: flexure displacement and shear displacement.

Flexure component:

1. Computing of column centroid and moment inertia

Figure 2.76 shows the centroid of columns in each story



Graph 2.2 Different type of displacement of frame (c)

$$C_{5,6,4} = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)+(2\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(10\times 0.00164)} = 9.01$$

$$C_3 = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)+(3\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(11\times 0.00164)} = 9.46$$

$$C_2 = \frac{(3\times 0.00164\times 5.15)+(3\times 0.00164\times 9.05)+(3\times 0.00164\times 14.05)+(3\times 0.00164\times 16.8)}{(15\times 0.00164)} = 9.01$$

$$C_1 = \frac{(3\times 0.00164\times 5.15)+(3\times 0.00164\times 9.05)+(3\times 0.00164\times 14.5)}{(15\times 0.00164)+(2\times 0.17\times 0.008)} + \frac{(2\times 0.17\times 0.008\times 14.05)+(3\times 0.00164\times 16.8)}{(15\times 0.00164)+(2\times 0.17\times 0.008)} = 9.511$$

$$I_{6,5,4} = \sum A(c)^2 = \left[(0.00328)(9.1)^2\right] + \left[(0.00328)(3.86)^2\right] + \left[(0.00328)(0.04)^2\right] + \left[(0.00328)(5.04)^2\right] + \left[(0.00328)(7.79)^2\right] = 0.5975$$

$$I_3 = \sum A(c)^2 = \left[(0.00328)(9.46)^2\right] + \left[(0.00328)4.31)^2\right] + \left[(0.00328)(0.41)^2\right] + \left[(0.00328)4.59)^2\right] + \left[(0.00328)(734)^2\right] = 0.6353$$

$$I_2 = \sum A(c)^2 = \left[(0.00492)(9.1)^2\right] + \left[(0.00492)(3.86)^2\right] + \left[(0.00492)(0.04)^2\right] + \left[(0.00492)(5.04)^2\right] + \left[(0.00492)(7.79)^2\right] = 0.904$$

$$I_1 = \sum A(c)^2 = \left[(0.00492)(9.511)^2\right] + \left[(0.00492)(4.361)^2\right] + \left[(0.00492)(0.461)^2\right] + \left[(0.0764)(4.539)^2\right] + \left[(0.00492)(7.289)^2\right] = 0.925$$

2. Compute the value of external moment M at each mid-story level.
3. Figure 2.77 shows the magnitude of external moment in each story at mid-story level.

Table 2.49 Flexure component of displacement of frame (1)

Story	$h_{i_{(m)}}$	$I_{i_{(m^4)}}$	$F_{(kN)}$	$M_{i_{(KN.m)}}$	$\theta_{i_{10^{-5}}}$	θ_{if}	δ_{if}	Δ
6	3.6	0.5975	202.2944	364.1300	1.1E−05	0.000754	0.002716	0.012192
5	3.6	0.5975	283.1034	1601.976	4.83E−05	0.000743	0.002676	0.009476
4	3.6	0.5975	176.8671	3667.770	0.00011	0.000695	0.002502	0.0068
3	3.6	0.6353	131.5435	6288.702	0.000178	0.000585	0.002105	0.004298
2	3.6	0.9042	86.88096	9302.799	0.000185	0.000406	0.001463	0.002193
1	3.3	0.925	36.92341	12402.10	0.000221	0.000221	0.00073	0.00073

Table 2.50 Evaluation shear component displacement of frame (1)

Story	Q	$K_{(B_2)}$	$\delta = \frac{Q}{K_{(B_2)}}$	Δ
6	202.294	119625.732	0.001691062	0.027775131
5	485.397	119625.732	0.004057638	0.026084069
4	662.265	134116.671	0.004937977	0.022026431
3	793.808	134116.671	0.005918792	0.017088453
2	880.689	149770.969	0.005880243	0.011169661
1	917.613	173480.879	0.005289419	0.005289419

Calculation of inclination and total displacement is shown in Table 2.49 as a tabular form.

Calculation of shear components of displacement will be determined as fallows.

1. Calculate the value of the external shear Q_i acting in each story i. external shear of bent (1) is calculated and shown in Table 2.35.
2. Calculate for each story i the story displacement due to shear, δ_{is} by substituting the value of the story shear and member properties into the suitable formula from Table 2.7.

Sum the story displacements due to shear component. Total shear displacement is calculated and shown in Table 2.50

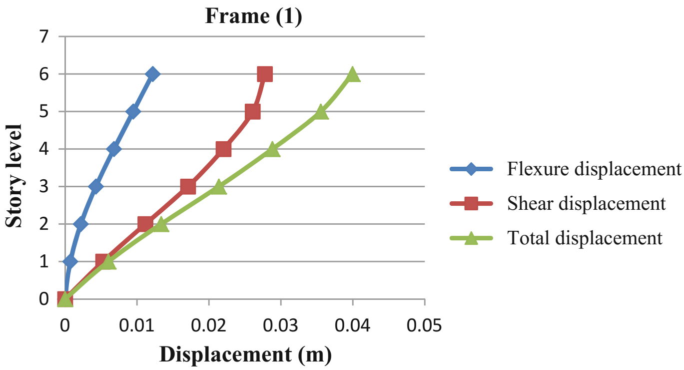
Total displacement of frame (1) is shown in Table 2.51. Graph 2.3 shows the flexure, shear and total displacement of frame (1).

Total displacement of frame (5):

Total displacement of frame (5) is composed of two components: flexure displacement and shear displacement.

Table 2.51 Evaluation Total displacement of frame (1)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.012191642	0.027775131	0.039966773
5	0.00947609	0.026084069	0.035560159
4	0.006800027	0.022026431	0.028826458
3	0.004297703	0.017088453	0.021386156
2	0.002193155	0.011169661	0.013362816
1	0.000730048	0.005289419	0.006019467



Graph 2.3 Different type of displacement of frame (1)

Flexure component:

1. Computing of column centroid and moment inertia (Fig. 2.78)

$$C_{5,6,4,3} = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)+(2\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(10\times 0.00164)} = 9.01$$

$$C_2 = \frac{(3\times 0.00164)(5.15)+(3\times 0.00164)(9.05)+(3\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(13\times 0.00164)} = 9.1$$

$$C_1 = \frac{(3\times 0.00164\times 5.15)+(3\times 0.00164\times 9.05)+(3\times 0.00164\times 14.5)+(3\times 0.00164\times 16.8)}{(15\times 0.00164)} = 9.01$$

$$I_{6,5,4,3} = \sum A(c)^2 = \left[(0.00328)(9.1)^2\right] + \left[(0.00328)(3.86)^2\right] + \left[(0.00328)(0.04)^2\right] + \left[(0.00328)(5.04)^2\right] + \left[(0.00328)(7.79)^2\right] = 0.5975$$

$$I_2 = \sum A(c)^2 \left[(0.00328)(9.1)^2\right] + \left[(0.00492)(3.95)^2\right] + \left[(0.00492)(0.05)^2\right] + \left[(0.00492)(4.95)^2\right] + \left[(0.00328)(7.7)^2\right] = 0.66341$$

$$I_1 = \sum A(c)^2 \left[(0.00492)(9.01)^2\right] + \left[(0.00492)(3.86)^2\right] + \left[(0.00492)(0.04)^2\right] + \left[(0.00492)(5.04)^2\right] + \left[(0.00492)(7.79)^2\right] = 0.8962$$

Table 2.52 Evaluation of flexure component of displacement of frame (5)

Story	$h_{i_{(m)}}$	$I_{i_{(m^4)}}$	$F_{(kN)}$	$M_{i_{(KN.m)}}$	$\theta_{i_{10-5}}$	θ_{if}	δ_{if}	Δ
6	3.6	0.5975	278.751	501.7519776	1.51E−05	0.000751	0.002705	0.012044
5	3.6	0.5975	120.165	1721.5546	5.19E−05	0.000736	0.00265	0.00934
4	3.6	0.5975	144.907	3418.490275	0.000103	0.000684	0.002464	0.006689
3	3.6	0.5975	108.412	5571.403727	0.000168	0.000581	0.002093	0.004226
2	3.6	0.66341	71.274	8047.75435	0.000218	0.000414	0.001489	0.002133
1	3.3	0.8962	34.385	10600.60787	0.000195	0.000195	0.000644	0.000644

Table 2.53 Evaluation shear component displacement of frame (5)

Story	Q	$K_{(B_1)}$	$\delta = \frac{Q}{K_{B_1}}$	Δ
6	278.751098	119625.732	0.002330193	0.023782541
5	398.917025	119625.732	0.003334709	0.021452348
4	543.825016	134116.671	0.004054865	0.018117639
3	652.238012	134116.671	0.004863214	0.014062773
2	723.512333	149770.969	0.004830792	0.00919956
1	757.897767	173480.879	0.004368768	0.004368768

2. Compute the value of external moment M at each mid-story level. Figure 2.79 shows the magnitude of external moment in each story at mid-story level.

Calculation of inclination and total displacement is shown as a tabular form in Table 2.52.

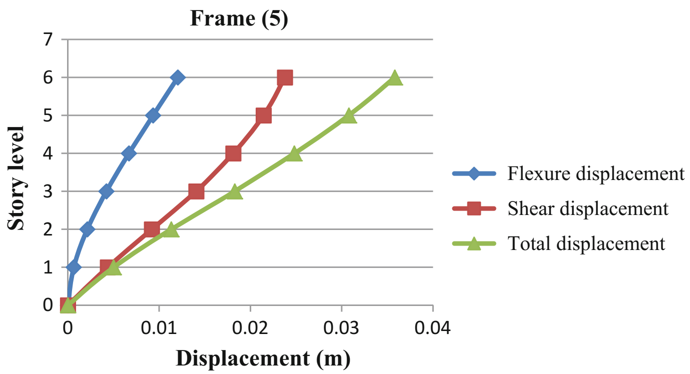
Calculation of shear components of displacement will be determined as fallows.

1. Calculate the value of the external shear Q_i acting in each story i. external shear of bent (5) is calculated and shown in Table 2.36.
2. Calculate for each story i the story displacement due to shear, δ_{is} by substituting the value of the story shear and member properties into the suitable formula from Table 2.7.

Sum the story displacements due to shear component. Total shear displacement is calculated and shown in Table 2.53.

Table 2.54 Evaluation Total displacement of frame (5)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.012044468	0.023782541	0.035827009
5	0.009339687	0.021452348	0.030792034
4	0.006689322	0.018117639	0.02480696
3	0.004225662	0.014062773	0.018288436
2	0.002132745	0.00919956	0.011332304
1	0.000644056	0.004368768	0.005012824



Graph 2.4 Different type of displacement of frame (5)

Total displacement of frame (5) is shown in Table 2.54. Graph 2.4 shows the flexure, shear and total displacement of frame (5).

References

ASCE/SEI. (2010). Minimum design loads for buildings and other structures. In *ASCE/SEI 7-10.* Reston, Virginia: American Society of Civil Engineer.

Eurocode 8, C. (2004). *Design of structures for earthquake resistance—Part 1: General rules, seismic actions and rules for buildings (EN 1998-1: 2004)*. Brussels: European Committee for Normalization.

IBC, I. (2012). *International Code Council. International Building Code*. International Code Council: Washington DC, United States.

IIT K. (2008). *Structural analysis*. India.

Kimberley Robinson. (October 2013). *Bukling restrained braces*.

Mohsen, H., & Reza, K. (2015). *Design of steel structure using ETABS software*. Tehran, Iran: SerieOmran.

Smith, B. S., Coull, A., & Stafford-Smith, B. S. (1991). *Tall building structures: Analysis and design (Vol. 5)*. New York: Wiley.

Chapter 3
Frames with Shear Wall

In this chapter, the concrete shear wall system and its effects on building systems is discussed. The first part presents the concept of shear walls and its characteristic. Approximate and virtual work are implemented to determine the displacement of concrete shear wall frame with considering the lateral stiffness of frame as well when subjected to a lateral load.

The next section presents the architectural and structural detail of considered 6 story real building and all described calculations and computations processes are applied on this structure.

In the third section, the effect of gravity and seismic forces on the structure are elaborated and also calculation charts to determine the center of mass, stiffness, shear rigidity and the eccentricity of each floor level of building are presented.

The fourth section of this chapter, is devoted to illustrate the procedure to determine the displacements in the different concrete frames with shear wall at two perpendicular directions under applied seismic force.

The final section of the chapter exhibit the charts and detail calculations of weight, stiffness and displacement of the considered structure.

F. Hejazi and K. Karimzadeh, *Analysis Procedure for Earthquake Resistant Structures*,
https://doi.org/10.1007/978-981-10-8839-1_3

3.1 Concrete Shear Wall

- Concrete Shear Wall
- Method of Implementation of Shear wall in Steel Structure Frame
- Steel Member in Shear Wall
- Effect of Earthquake on Shear Wall
- Proportional and Non-proportional Wall System
- Failure in Shear Wall
- Conceptual Framework

3.1.1 Introduction

A building structure should be provided with suitable lateral resistance systems acting in two perpendicular directions against lateral loads such as wind load and earthquake load. Some types of lateral resistance structure are:

1. Shear wall frame system
2. Bracing frame system
3. Moment resistance frame system
4. Dual system.

3.1.2 Concrete Shear Wall

In multi-story buildings, walls are constructed in a way that resist lateral loads from wind or earthquakes. Also, arc are often assigned to these constructions, as a significant amount of lateral load and horizontal shear load are borne by the building. The walls are consequently called shear walls. Nevertheless, the name doesn't fit well with reality application, as it is rare that the critical mode of resistance is associated with shear. Since modern buildings are taller and slender, shear walls may be a critical design item. Shear walls are usually pierced with numerous openings and it is better for the structure if these openings are arranged in a systematic pattern. They act as vertical cantilevers that connect the walls around elevator, stair and service shaft—in the form of separate planar walls. Moreover, shear walls structure is only economical for buildings with no more than 35 stories. This is because they are horizontally much stiffer than rigid frames. Figure 3.1 shows steel structure with concrete shear wall.

Moving on, shear wall constraint open internal spaces construction, unlike rigid frames. The former are more suited to structures with floor-by-floor repetitive planning that allows vertical continuity. Such structure can be found at hotels and residential buildings. In addition, shear wall is also competent in providing acoustic and fire insulators between rooms and apartments.

Shear walls that are combined with frames in low- to medium-rise building structures will bear all lateral loading. In this case, it is plausible to design the frame solely to withstand gravity loading. It should be noted that the design of shear wall structures allow the lateral load tensile to be suppressed by the gravity load stresses. This is to allow minimum reinforcement on the design. Moreover, shear wall supports the structure well doing earthquakes. This is why ductility is a point to be taken into consideration while designing them.

Fig. 3.1 Steel structure with concrete shear wall

3.1.2.1 Shear Wall Configuration

There are several ways to configure structural walls. It is quite simple to design rectangular cross sections but thin sections can cause performance problem. Meanwhile, ‘bar bell’ walls have several advantages that can be attributed to its boundary columns. The columns has longitudinal reinforcements that improve wall stability, contribute to moment resistance and create anchor beams that frames into the wall. The same advantageous columns can also impede the architectural design and increase cost. In addition, combinations of intersecting wall segments allow the creation of flanged walls, including T, L, C and I configurations. Furthermore, core walls acts as the enclosure of elevators, stairways and other vertically extruded areas. They are supported by coupling beams that connect the wall components over doorways. The parallel wall segment became a web element that resist shear, axial force and flexure, while orthogonal wall segments react to tension or compression flanges. Figure 3.2 shows different type of shear wall configuration.

3.1.2.2 Tall Walls Having Rectangular Cross Sections

Figure 3.2a shows a single cantilever shear wall. Such structure is expected to behave in the same as a reinforced concrete beam. It should be noted that the narrow cross section suggest the possibility of compression edge instability. In a

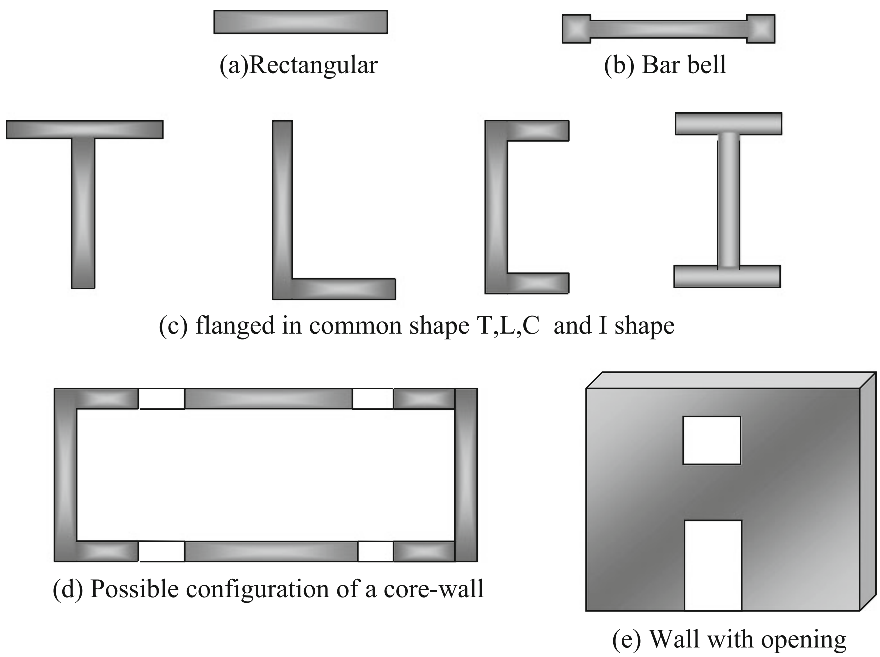


Fig. 3.2 Different type of shear wall configuration

typical situation, the floor slabs will become the horizontal diaphragms and provide lateral support. Therefore, the critical length is equal to the floor height, with respect to buckling. Furthermore, the shear wall became a large cantilever. It will provide support against bending moments, shear forces and axial compression. In addition, it should be noted that the vertical or flexural reinforcement impact flexural capacity.

3.1.2.3 Flanged Cantilever Shear Walls

Tall flanged shear walls function similarly to those with rectangular cross sections. These walls will use the internal “steel couple” in the post elastic range when the axial force is small. Considering the condition where the reinforcing content in the flanges are typically exact, adequate ductility should be present. This is one of the support system required during an earthquake.

Moving on, the entire flange and a proportion of the web may be in compression when the axial compression is significant. Such condition requires the flanges to be considered as tied columns that are axially loaded. Following this, a more suitable approach to assess the ultimate capacity would be to use a lower capacity reduction factor (i.e., $\varphi \sim 0.7$). The value of φ can be varied between 0.7 and 0.9 for intermediate position of the neutral axis. It should be noted that sufficient transverse reinforcement around the main vertical steel in the flanges is crucial.

In addition, flanges will affect the tall cantilever shear walls moment of resistance. From this, it can be concurred that it is more important to provide resistance against shear forces in the web than the ones with rectangular cross sections. Also, the load being applied to the horizontal construction joints should be considered too as it might be more intense. The aim is to construct a well-design shear wall—one where at any stage of loading, the shear reinforcement will not take place. In contrast to tall flanged shear walls, the configuration of short flanged shear walls is more complex. The latter is sensitive to vertical reinforcement in wide flanges, as even a small amount of such will cause flexural capacity. It should be mentioned that flexural capacity correlates with excessive shear load on the web. Following this behavior of short flanged shear wall, mesh reinforcement is applied. This type of reinforcement is competent in controlling the width of diagonal cracks and it also distributes the cracks uniformly. Moreover, construction joints may become the factor of failure in low-rise flanged shear walls.

3.1.2.4 Shear Walls with Openings

The construction of windows, doors and service ducts should be installed with exterior or interior shear walls with openings. A rational wall structure exhibits behaviors that can be inspect easily. To achieve such structure, one should take into account the position of openings throughout the building in the early stages of planning. Meanwhile, irrational wall structure typically does not conform to normal structural analysis. Evaluation of the internal forces is therefore being done through model investigations or finite element studies. Since this is atypical situation, special experimental studies are required to concur the ultimate strength, energy absorption and ductility demand of such wall structure. It is typical that the results of the studies are only available after the completion of the shear wall structure construction. Building on previous discussion, it is important for the design to have minimal interference with the moment and the shear-carrying capacity of the structure. Observations of irrational shear wall structure are as follows:

Sudden change from a wall section into columns reduced the flexural resistance of the cantilever structure at the critical base section. The arrangement of openings reduces the contact area between the two walls. This is where the transmission of shearing forces should have been transmitted (Paulay & Priestley, 1992).

3.1.3 Methods of Implementation of Shear Wall in Steel Structure Frame

It is possible to implement shear wall structure Fig. 3.3a–e. This is in accordance to the suggestions made by the design engineer and the limit of implementation.

Figure 3.3a shows the steel column inside of concrete shear wall with uniform thickness. In this method, the thickness of shear wall is calculated specifically to provide ample space for reinforcement and to locate the columns. This method is simple, hence it is one the common methods to implement shear wall. Moreover, Fig. 3.3b demonstrates the steel column inside of shear wall with non-uniform thickness. This method is more difficult to execute, hence it is not commonly being done. Next, Fig. 3.3c illustrates the column that is not inside of shear wall, but is

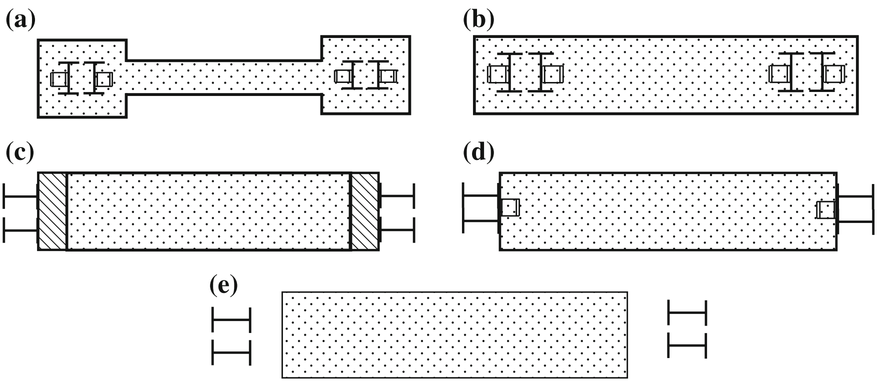


Fig. 3.3 Different method of implemented of shear wall in steel structure

connected to the shear wall by a channel connector. This method increased the dimension of column; therefore it is uneconomical and not so common. Meanwhile, in Fig. 3.3d, e the column is implemented outside of shear wall.

3.1.4 Wall Reinforcement

Moving on, Fig. 3.4 demonstrated the cross section of a usual reinforcement of special structural wall. The minimum requirement of such structure is having web reinforcement that is spread in horizontal and vertical directions. Moreover, to provide additional resistance to moment and axial force, most of their vertical reinforcement are concentrated at the wall boundaries. It should be noted that in general, the longitudinal reinforcement is pun within transverse reinforcement. This is to control longitudinal bar buckling and confines the concrete. Furthermore, the shear strength is reinforced by horizontal reinforcement.

In addition, the longitudinal and transverse reinforcement strengthened the boundary element, which is situated along the structural wall edge or opening. ACI 318 needs a specific boundary element, particularly when the combination of seismic and gravity loading cause high compressive demand. To provide support against such pressure, they have closely spaced transverse reinforcement around the vertical boundary bars. As a result, the compressive strain capacity is increased, preventing longitudinal bar buckling. This is done to increase the size of edge element of wall. Figure 3.5 shows the wall structure with and without edge element of the same wall width, and those with different wall width (Ali & Mehdi 2005).

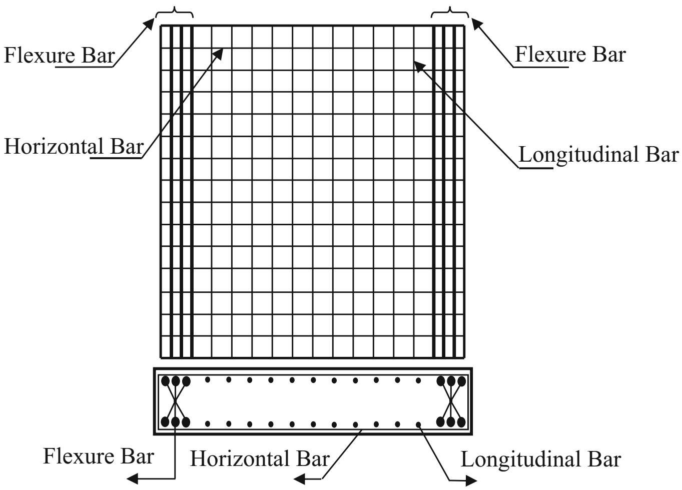


Fig. 3.4 Reinforcement of shear wall

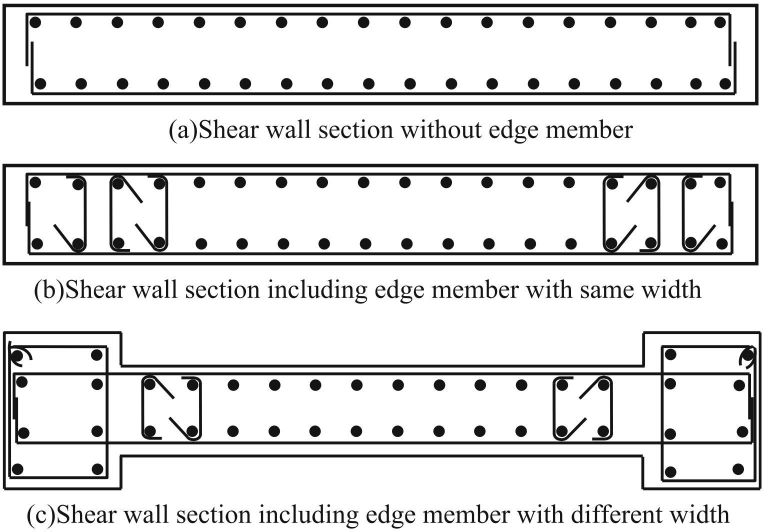


Fig. 3.5 Edge member of shear wall

3.1.5 Steel Member in Shear Walls

In the cases where shear wall is employed in steel structure, steel members such as beam and column should be inside in the wall structure.

3.1.5.1 Steel Column in Shear Wall

There are different methods to implement steel columns in shear wall. Two common methods are as follows.

Partially Encased Steel Column

This method requires the steel column to hold against all tension and compression forces caused by shear wall's overturning moment. Figure 3.6 demonstrates the implementation of shear wall to unbounded column using Partially Encased Steel Column method.

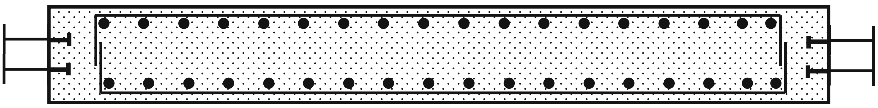


Fig. 3.6 Implementation of shear wall to unbounded column

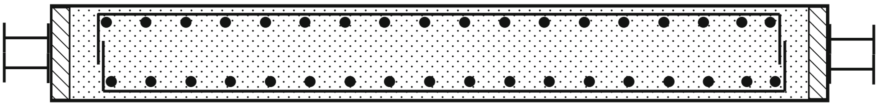


Fig. 3.7 Implementation of shear wall to unbounded column

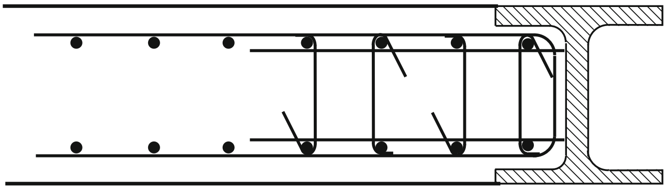


Fig. 3.8 Implementation of shear wall to unbounded column

This method caused the dimension of the cross-section of steel columns and the number of tension in the anchor bolt of base plate to increase. This affects the economic status of the project and the practicality of its implementation. Also, more connectors are required due to the high capacity of steel column. Since only one side of the column having welded connectors, the same side will be more congested. This method is not recommended.

Moving on, shear wall is implemented between two columns in a retrofit projects. This is purposely done to interfere with the performance of shear walls, which is employed as a compression material between the walls and columns. The configuration is demonstrated in Figs. 3.7 and 3.8.

Fully Encased Steel Column

This method puts the steel column completely within the concrete wall, with vertical reinforcement and cross ties enclosing the surroundings. This is a practical and economical method to use shear wall in a steel frame system. Such structure will be able to hold against axial and moment force—the combination of the behavior of concrete walls and steel columns will do so. Meanwhile, the concrete shear wall will resist shear force. Figure 3.9 shows fully encased steel column in concrete shear wall.

Furthermore, the design of column buried in shear wall should consider the following:

1. The columns should be able to hold against dead loads and total live load while the shear wall and casting concrete is being implemented.
2. Connectors should be used to transfer the shear between the wall and steel column.

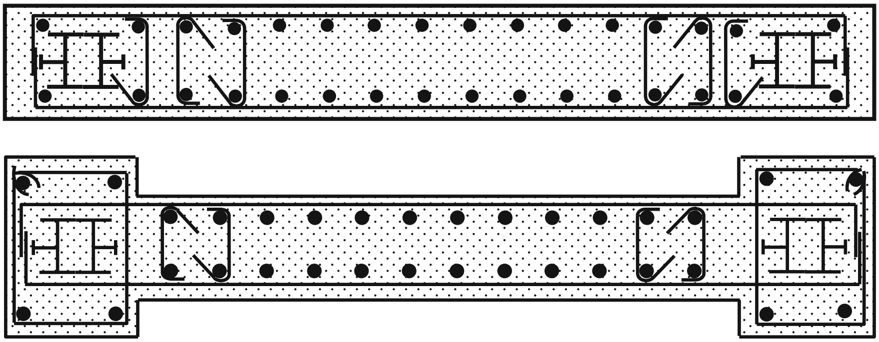


Fig. 3.9 Fully encased steel column

Separated Steel Column

The design of this structure will depend on the type of beam between the two columns. If the beam passes through the wall, the columns will be able to be pushed or pulled—this is in accordance with the lateral load in cutting and bending beams. When the earthquake forces intensify, the columns will be brought up. Such construction is also compatible with concrete shear walls and columns which are not made of mild steel. Furthermore, the beam-column connection should be able to hold against two-way shear force. This is due to the cutting beam caused by the earthquake. They should also be able to withstand stretching and pressure. Figure 3.10 shows the separate steel column of shear wall (Seismic, 2010).

3.1.6 *The Effect of Earthquakes on Shear Wall Buildings*

At first, the building stays still due to inertia. Nevertheless, the movement of the ground caused the building to move sideways. This cause lateral load being applied on the building and shear force being applied at the base. As the direction of the movement changes, the building begin to rock. It should be noted that the force can be calculated using Newton's law—force equals to mass times acceleration. The acceleration is caused by the movement of the ground, hence the greater the mass of the building, the greater the force acting on it. The acceleration is also dependent on

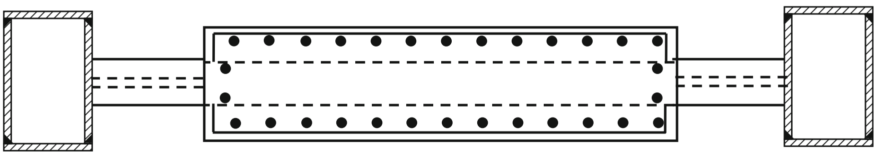


Fig. 3.10 Separate steel column

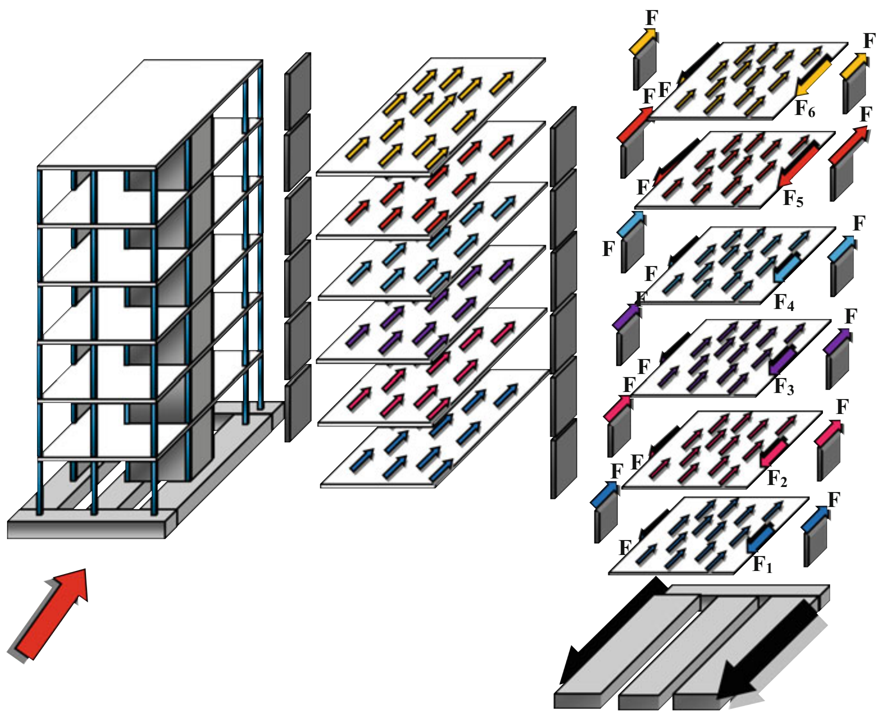


Fig. 3.11 Response of building under earthquake load

the structure's natural period. If the lateral force affect the building, the structure rock from side to side. Furthermore, the period—time taken to complete one side-to-side oscillation in seconds—is dependent on the mass and stiffness of the building. In addition, the slab will have to borne the force from the earthquake and acts as in-plane bending. It is supported by shear wall frame which also need to withhold force from the slab. Figure 3.11 demonstrates response of building with concrete shear wall under seismic load.

3.1.7 Proportionate and Non-proportionate Wall System

A tall shear wall building usually has shear walls with varied lengths and thickness. Such variations cause a complex redistribution of the moments and shears between the walls. This correlates to the horizontal interactive forces in the girders and slabs connectors. These structures have their own behaviours that can be divided into proportionate and non-proportionate systems.

Proportionate system have constant ratio of the flexural rigidities of the walls, as shown in Fig. 3.12a They do not incur any redistribution of shear or moments at

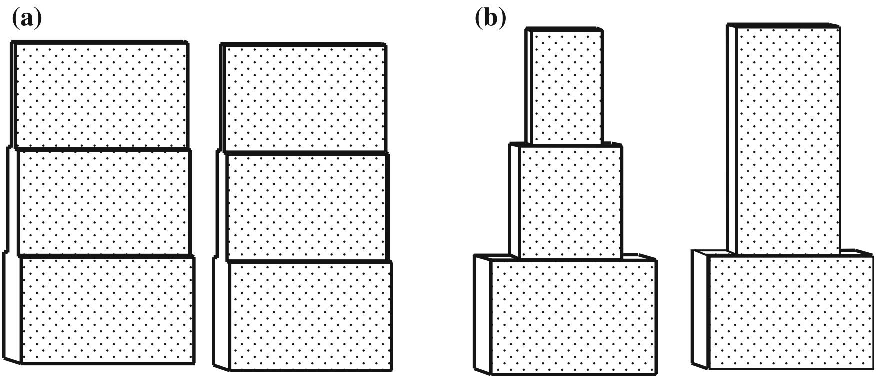


Fig. 3.12 **a** Proportionate shear wall. **b** Non-proportionate shear wall

change levels. Such consistency makes it possible to analyze their structure with regard to equilibrium, their external moment and shear on non-twisting structures which are distributed between the walls. All these are laid out proportionately to their flexural rigidities.

Next, a non-proportionate system is, as the name suggest, in contrast with the structure of its counterpart. The ratios of the wall's flexural rigidities are not constant as the height increases (Fig. 3.12b). Due to this, the walls shears will redistribute when the level of rigidities change. This will corresponds with horizontal interactions in connecting members, causing high local shears in the walls. Consequently, this structure is indeterminate and is more challenging to analyze its behaviour. Moving on, this chapter will discuss the following using hand methods analysis—proportionate, non-twisting; proportionate, non-twisting and non-proportionate, non-twisting structures. Nevertheless, it is easier to analyses non-proportionate, non-twisting structure (important for non-proportionate, twisting structure) to be analyzed by a computer program (Smith, Coull, & Stafford-Smith, 1991).

3.1.8 Failure in Shear Wall

Moving on, the slides of shear mechanism and shear diagonal compressions usually contribute to the failure of squat walls. This is caused by the reduction of strength of SWs when the diagonal X-shaped cracks in RC and masonry walls. Meanwhile, shear sliding caused stiffness and energy dissipation. Furthermore, cantilever walls have four failure modes—flexural, shear, overturning and sliding. The last mode is usually resisted by friction. When the structure is heavy—in RC and masonry for example—it is the dead loads that contribute frictional resistance. In contrast, shear anchorages are crucial in preventing sliding from horizontal forces in lighter constructions—metal or wood. This correlates with the idea that flexural response have

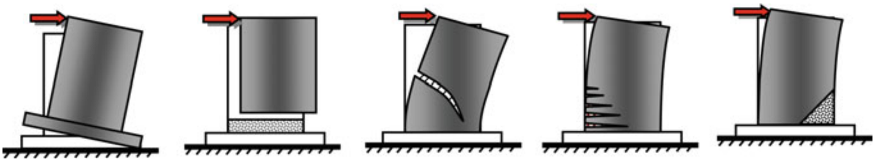


Fig. 3.13 Different type of failure concrete shear wall

higher energy dissipation capacity than shear failure. Moreover, one should consider the out-of-plane deflections in order to hold against brittle failure of SWs caused by diagonal compression effects. To do this, some limitations are employed on the wall slenderness. Studies have shown that steel and composite walls are suitable for this purpose. Figure 3.13 shows different type of failure concrete shear wall.

3.1.9 Conceptual Framework

With regard to the structure's behaviour under earthquake load, stiffness, strength and ductility are the most important parameters. The importance is greater on stiffness, and this will be explained in detail.

3.1.9.1 Stiffness

As the name suggests, this is the ability of the structure to resist deformation put under pressure, as illustrated in Fig. 3.14. Stiffness is a ratio of force and deformation. Since both of these values can be varied, stiffness is not permanent. In Fig. 3.14, K_i is the stiffness at a deforming rate of δ_i, and matches the resistance to force V_i. Some observations are made—stiffness is of tangent value if additions or first derivatives of forces and deformations are employed, and it is of little worth if only total forces and deformations are employed.

3.1.9.2 Strength

This is related to the ability of the structure to hold against load resistance, and it is not an unchanging value. In Fig. 3.14, V_j and V_k are the capacities of force that match δj and δ_k, respectively. Furthermore, V_y (yield strength) matches the yield displacement δ_y which is needed for calculating ductility.

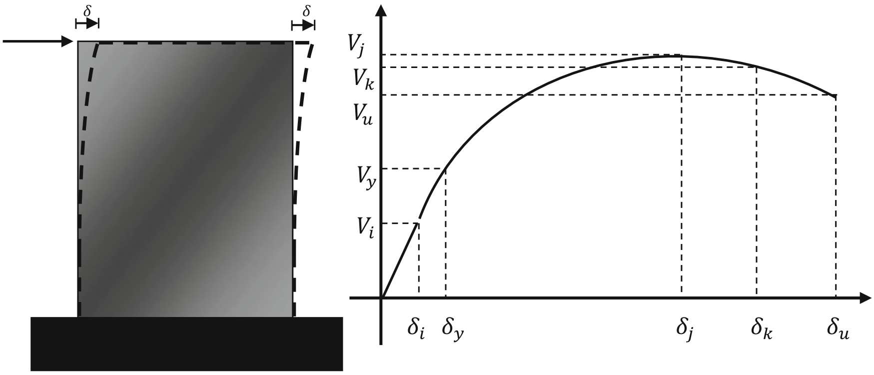


Fig. 3.14 Typical response cure for structural system subjected to horizontal loads

3.1.9.3 Ductility

This refers to the structural ability to withstand deformation past the limit of elasticity (Fig. 3.14). It is represented as the ratio of the highest value of a quantum if deformation quantity to when the deformation quantum is at the limit of yield. The figure shows how the ductility displacement μ is the ratio of the highest displacement to the yield displacement δ_y.

Stiffness

The definition of stiffness has been previously explained. Member stiffness includes section characteristics, states of length and bounder, but stiffness of a system is the configuration of a laterally resistant mechanism. Some examples are the moment resisting frames, braced frames, walls or double systems. Furthermore, principles of mechanics can be used to explain the relationships between geometry, mechanical properties, actions and deformations. The construction materials affect their complexity. In addition, there are several problems to define a fixed value of stiffness. They are caused by cracks in concrete, reinforcement bars giving way, and various causal factor of RC structural inelasticity. However, it is usually taken as the secant to the yield point.

Furthermore, Fig. 3.15 demonstrated the structural response plot of a system subjected to lateral loads. It is presented using base shear V and displacement δ. Also, the secant stiffness is K_s while the elastic stiffness is represented with K_0. It should be noted that the latter is higher than the former conventional purposes of the construction. Furthermore, the stiffness is directly proportional with loads in rubber and other special materials. Generally, the values of V-δ pairs are utilized to describe secant stiffness and variances while tangent stiffness K_t represents the variances of the stiffness in the range of inelasticity. If K_t decreases, it can be concluded that softening is taking place.

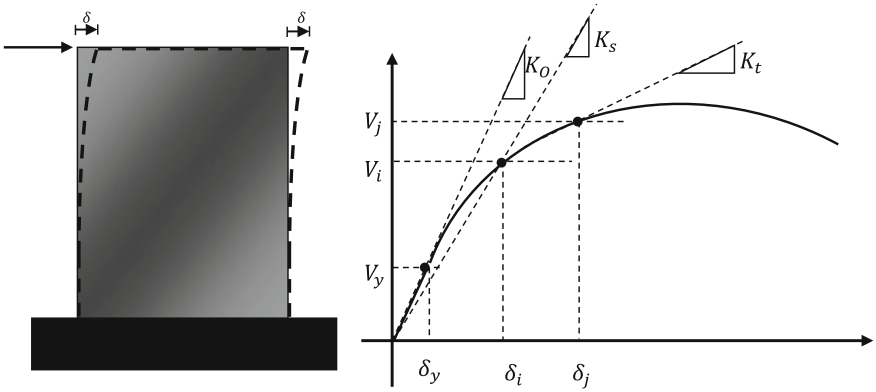


Fig. 3.15 Definition of initial and secant structural stiffness

Moving on, secant stiffness is often employed in the analysis of inelastic structures to avoid negative tangent stiffness. It employed together with iterations, a process to solve inelastic response problems. The solution will converge, but its rate will not be as high as the use of tangent stiffness.

There are several types of stiffness and there are dependent on the type of applied loads. For example, the designs meant for vertical loads will be composed of sufficient vertical stiffness, while those meant for earthquake inertial forces will have sufficient lateral or horizontal stiffness. Furthermore, lateral stiffness is dependent on the region within the response domain. It is worth noting that the stiffness of a system correlates with the functionality of the structure under loads. Therefore, low stiffness reduces the structural functionality. Apart from that, the lateral stiffness is affected by the material characteristics, section type, linkages and systems (Elnashai & Di Sarno, 2008).

Factors Influencing Stiffness

Section Properties

The torsional moment of inertial (J), the flexural moment of inertia (I) and the cross-sectional area (A) are the section properties that affect the structural stiffness. Primarily, given that A and I do not vary with different kinds as well as levels of practical loads, they affect the axial, the system shear stiffness and meandering. Conversely, in the case of and masonry, A and I are a function of the stuffing and border situations.

Connection Properties

With reference to (Krawinkler & Mohasseb, 1987; Elnashai, Elghazouli, Takanashi, & Dowling, 1991) connection properties influences the structural systems lateral deformation. For instance, in numerous-story steel frame, between 20 and 30% of the absolute horizontal shift amidst neighbouring floors seem to be triggered by the plane zone deformability of beam-to-column link. While the rigid as well as semi-rigid links may be utilized for blocked and unbraced frames, pinned links are not enough for the unbraced frames. Besides that, Elnashai (1998) indicate in their study that, following the results of the laboratory experiments on a double-story steel frame with semi-fixed and fully fixed links, it clear that a decrease of the link stiffness by a percentage of fifty and sixty results in 20 and 30% reduction in the frame stiffness, correspondingly. Simplified models numerical analyses have revealed that lateral stiffness K semi-fixed of semi-rigid steel frame can be expressed as an element of the horizontal stiffness K rigid of inflexible frames via the formula below:

$$\frac{K_{semi\text{-}rigid}}{K_{rigid}} = \frac{m(1+\xi)+6}{m(1+\xi)} \tag{3.1}$$

In this case, m and ζ represent dimensional parameters determined as follows:

$$m = \frac{(K_{\varphi})_{con}}{(EI/L)_b} \tag{3.2}$$

and

$$\xi = \frac{\left(\frac{\mathrm{EI}}{\mathrm{L}}\right)_{\mathrm{b}}}{\left(\frac{\mathrm{EI}}{\mathrm{H}}\right)_{\mathrm{c}}} \tag{3.3}$$

where:

K_{ϕ} represents the connection rotational stiffness;
I is the flexural moment of inertia;
L is the beam span;
H is the column height;
E representsthe material Young’s modulus.

Generally, an assumption is made indicating that rigid connections have $m > 18$, pinned links have $m < 5$, and connections with values m ranging from 5 to 18, are semi-rigid connections.

Moreover, the framed structures natural vibration duration is affected by the stiffness of beam-to-column connections. With regard to shaky table test for a single-storey steel frame with rigid (welded top and bottom flange with a two web angle), semi-rigid (top and seat angle with a two web angle) and flexible (double web angle) connections, simplified relationships are appropriate in computing the fundamental period *T*. This was recommended by Nader & Astaneh-Asl (1992). They are as under:

$$T = 0.085H^{(0.85-m/180)} \quad 5 < m < 18 \quad \textit{semi-rigid} \tag{3.4}$$

$$T = 0.085H^{3/4} \quad m > 18\, \textit{rigid} \tag{3.5}$$

From the above equations, H is the frame height, in meters, in Eqs. (3.4) and (3.5). The dimensional parameter m is expressed as in Eq. (3.2).

Material Properties

Elastic Shear's modulus G and elastic Young's modulus E are the material properties that have an impact on the structural stiffness. The lateral stiffness in the case of inelastic range, relies on the module E and G-particularly on their tangent values, and not the initial. Often, the material stiffness in determined by using the ratio of the elastic Young's modulus E to the weight γ. It is necessary to note that for masonry, E/γ equivalents to 20–30 $\times$ 10^4 m, and for metals, E/γ is equals to 200–300 $\times$ 10^4 m. In addition, for concrete, the specific elasticity E/γ is almost 100–150 $\times$ 10^4 m. The materials of construction that have low E/γ values result in stiff structure, for instance, as compared to steel buildings, masonry buildings are stiffer.

Member Properties

The horizontal stiffness relies on the kind of structural participants. For instance, as compared to column walls, the structural walls are stiffer. Structural components geometrical properties like their aspect ratio, height, and section dimension, affect the horizontal shear as well as the flexural stiffness vales. Therefore, as compared to shear deformations, flexural deformation are usually higher for comparatively slim structural components. Additionally, the occurrence of flexural deformation dominance takes place if the slenderness ratio H/B and H/h are greater than 4–5 and the aspect ratio h/B of rectangular section of column is less than 3–4. This is relevant to the case of walls. For cantilevered walls, the connection amid horizontal displacement δ and applied load F is as under:

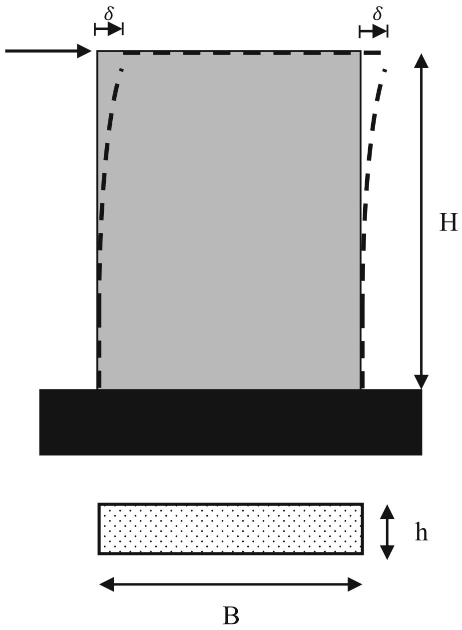
$$\delta = \left(\frac{H^3}{3EI} + \frac{\chi \mathrm{H}}{GA}\right)F \tag{3.6}$$

From the above equation:

H, I, and E represents the wall length, the inertia moment of the part concerning the of flexural axis under deliberation, and the elastic Young's modulus, correspondingly;

A represents section area;
G represents the shear modulus;
χ represents the factor of shear shape, which is 1.2 for rectangular section

Fig. 3.16 Structural wall under horizontal load



The wall flexural (K_F) and shear (K_S) stiffness can be set as under (Fig. 3.16):

$$K_F = \frac{3EI}{H^3}, \tag{3.7}$$

$$K_S = \frac{GA}{\chi H} \tag{3.8}$$

Equation (3.6) can as well be rewritten as below;

$$F = \frac{K_f K_s}{K_f + K_s}\delta \tag{3.9}$$

The aggregate wall lateral stiffness K_t of the wall is determined as below:

$$K_t = \frac{K_f K_s}{K_f + K_s} = \frac{K_f}{1 + \frac{K_f}{K_s}} \tag{3.10}$$

The above Eqs. (3.7) and (3.8) illustrate the lateral rigidity reliance on member geometry, H; section shape, A and I; and material properties, E and G. Moreover, Shear (K_s) and flexural (K_f) stiffness figures are of linear and cubic functions of the wall depth, correspondingly. As a result, for a certain horizontal force (F), δ becomes 1/8 of its initial value in case the depth of the wall is half the original figure, and thus, for specific value of δ, the load carried is higher by eight times.

Equation above reveals that in case the ratio $k_{f/k}$ s is far lower than 1.0, the connection amid the lateral force F and horizontal displacement δ is as in the formula below:

$$\delta = \left(\frac{H^3}{3EI}\right)F$$

The equation above can be derived from Eqs. (3.6) to (3.9). The stiffness ratio K_f/K_s is expressed as a function of the geometric properties as under:

$$\frac{K_f}{K_s} = \frac{1}{2}\left(\frac{B}{H}\right)^2 \quad \text{or} \quad \frac{K_f}{K_s} = \frac{1}{2}\left(\frac{h}{H}\right)^2 \tag{3.11}$$

This is reliant on if the wall weak or strong axis flexural moment of inertia is used. The equation above illustrates the effect of the wall thinness on its lateral rigidity. K_f/K_s is lower if the ratio of H/h and H/B are higher. Therefore, the lateral displacements in slender wall are caused by flexural deformability, and as such, when the forces of horizontal earthquake are circulated among structural participants, it is necessary to consider their effective shear and flexural stiffness. The type of relationship amidst adjacent members or amidst the ground and structural components also influences structural stiffness. The overall wall connections of the horizontal bending stiffness K_f^* can be expressed as in the below formula:

$$K_F^* = \alpha\frac{EI}{H^3} \tag{3.12}$$

The coefficient α from Eq. (3.12) relies on the structural member boundary states. Boundary conditions refer to analytical connections that express the connection properties amid members, as well as between the ground and members. In Eq. (3.12), 12 is the common value for participants with edges fixed-fixed; and 3 is the common value of α for members with edges fixed—pinned as well as fixed-free. K_f^*, which is the bending stiffness increases with the increase in α.

System Properties

A structure's lateral stiffness is dependent on the type of system employed so as to endure loads derived from horizontal earthquake tremors, member stiffness dispersion, and types of horizontal diaphragms connected to members that are vertical. For instance, moment-resisting frames (MRFs) have been proven to possess more flexibility, in comparison to that of frames that are braced classified into either concentrically (CBFs) or eccentrically (EBFs). As such, SWs appear to be stiffer when compared to other frame types. In addition, frames that have connections that are rigid display more stiffness, in comparison to connections in frames that are semi-rigid. The explanation of both vertical and horizontal structural systems meant to resist earthquake is given in Appendix. It can be concurred that stiffness dispersion in both plan and elevation is indeed a must so as to hinder high demand for seismic localization. In addition, interaction between structure and soil must also be considered so as to determine the standard level of stiffness to fit the global systems. Such interaction decreases the aspect of stiffness in superstructures, which could change the dispersion exerted by seismic actions, as well as deformations due to motions generated by earthquake (Mylonakis & Gazetas, 2000).

Furthermore, SWs, which are composed of vertical systems, have been usually combined with RC-, steel-, and composite-framed structures in order to control lateral deflections. Such systems are normally categorized based on height-to-width (H/B) ratio, which is also called vertical aspect ratio, for both ‘squat’ and ‘slender’ (or ‘cantilever’) walls. In fact, squat walls contain lower slenderness with H/B ratios ranging from 1 to 4, whereas cantilever or slender walls have H/B > 4. Meanwhile, as for horizontal loads, the bending-to-shear deflections ratio for SWs recorded an increment with the system aspect ratio H/B. As a result, squat and slender walls are overseen by shear and flexural modes, respectively. Figure 3.17 shows the concrete shear wall under lateral load. Apart from that, squat and cantilever walls possess high strength and in-plane stiffness (also called ‘membrane action’). Bending resistance is applied in wall systems via chord effects implied at the edges, which reflects the load mechanism similar to that of diaphragm actions.

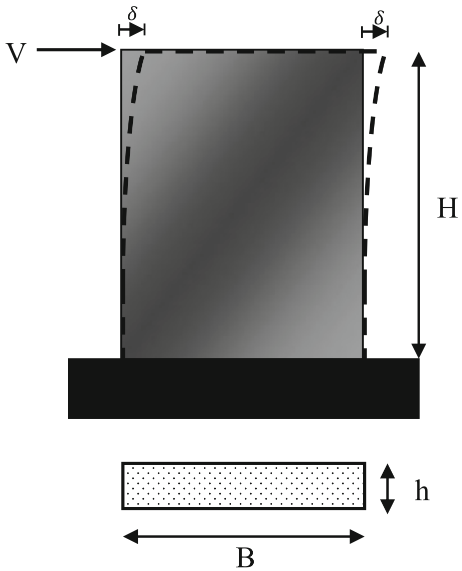


Fig. 3.17 Structural wall under horizontal load

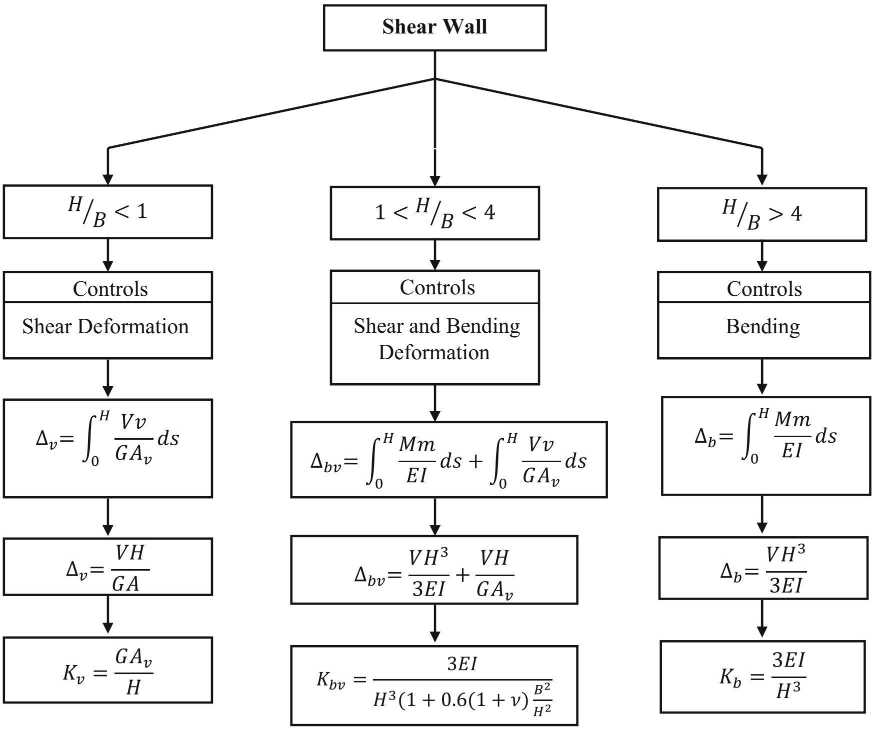
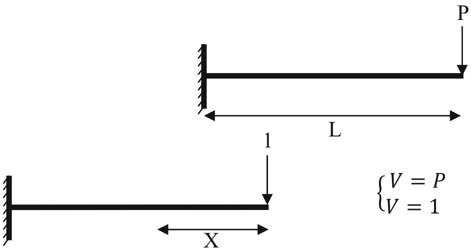


Chart 3.1 Shear wall behaviour

Besides, the strength and the lateral stiffness of SWs can be enhanced by employing the cross sections technique with and I shape, instead of narrowly-shaped rectangular ones. In fact, the former method is indeed a viable solution for systems of RC and composite in order to confine the concrete effectively in compression, thus attaining higher ductile behaviour (Elnashai & Di Sarno, 2008). Chart 3.1 demonstrates behaviour of concrete shear wall under lateral load.

Calculation of Shear Wall Stiffness by Virtual Work Method

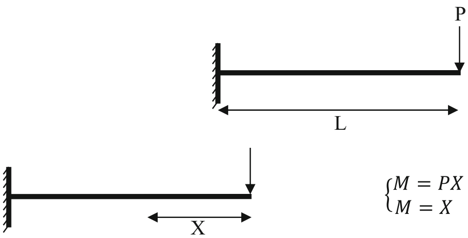
As it is mentioned in Sections "Member Properties" and "System Properties" the shear wall acts as cantilever beam and the behaviour of shear wall under lateral was detailed in it. In this part it is tried to calculate the stiffness of shear wall using virtual work method. The unit load shall be applied to cantilever beam and determined the deflection, in order to calculate the shear and moment stiffness of cantilever beam. Moreover, the given cantilever beam is statically determinate. Hence, the shear force in both cantilevers is calculated by static equation equilibrium. According to the virtual work formula, deflection of cantilever can be calculated with regard to Hook's low, the division of force with displacement indicated stiffness, the stiffness is inversed to displacement. These procedures are repeated for calculating flexure stiffness of shear wall.



$$\begin{cases} V = P & 0 < X < L \\ V = 1 & 0 < X < L \end{cases}$$

$$\Delta_B = \int_0^L \frac{V_1 \times V}{GA} dx = \int_0^L \frac{P \times 1}{GA} dx = \frac{1}{GA} \int_0^L P dx = \frac{P}{GA} X \Big|_0^L = \frac{PL}{GA} \tag{3.13}$$

$$K_v = \frac{GA}{L}$$



$$\begin{cases} M = PX & 0 < X < L \\ M = X & 0 < X < L \end{cases}$$

$$\Delta_B = \int_0^L \frac{M_1 \times M}{EI} dx = \int_0^L \frac{PX \times X}{EI} dx = \frac{P}{EI} \int_0^L X^2 dx = \frac{P}{EI} \frac{X^3}{3} \Big|_0^L = \frac{PL^3}{3EI} \tag{3.14}$$

$$K_b = \frac{3EI}{L^3}$$

3.2 Define Project

- Architectural plan
- Material property

3.2.1 Definition and Properties Used in This Project

The project which is defined for this chapter the same one mentioned in the first chapter. It has 6 stories which the 5 floors are used as residence while the ground floor occupied for parking space. There are two separation joints of 11 cm on the East and West sides of the building. The total height of the building is 24.3 (m) which includes the doom roof. It is located in high risk zone of the earthquake.

3.2.2 Architectural Plans

The architectural plans consist of the site, floor, section, and elevation plans, which was explored in the first chapter.

3.2.2.1 Floor Plan

The structural system consists of the steel structure with concrete shear wall system. The floors are used as residences, while the ground floor is used for parking. The North-South direction of the plan is divided into four bays by five grid lines (A-E), which is also the case in the East-West direction (1–5). The distances between the grid lines along the N-S direction are 5.15, 3.9, 5, and 2.75 m, respectively, while the distance between the grid lines along the E-W direction are 4.9 m, (0.35)m, (1.95)m, and 2.6 m, respectively. As it is shown in Fig. 3.18 the East-West of structure used concentrically-braced frame located in the last span of bent C and in the bent B between bent (3–4), (4–5), and the North-South direction used concentrically brace system as well. The brace system is located in a long of grid lines (1) and (5), between the bent (B, C). The 3D view of structure is detailed in Fig. 3.19.

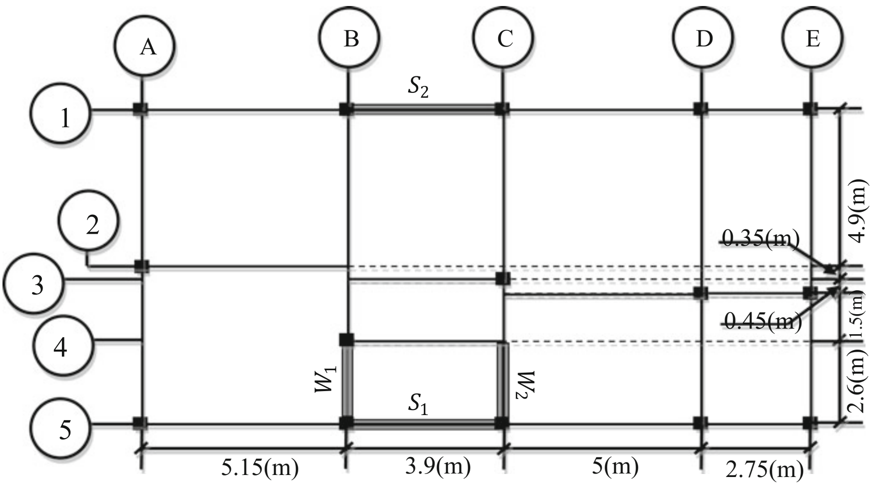


Fig. 3.18 Sketch illustrating the location of the columns, and beams on the floor plan

Fig. 3.19 3D view of considered building

3.2.2.2 Floor Project

Between all floors system, composite steel-concrete floor system as one simple and constructionally convenient slab system is selected for consider building that use of steel member to support a concrete floor slab offers the possibility of composite construction in which the steel members are joined to the slab by shear connectors so that serve as a compression flange. Figure 3.20 shows the detail of composite floor in considered building.

3.2.2.3 Section Properties

Detail of concrete shear wall in different stories of building is shown in Figs. 3.21 and 3.22.

Figures 3.23 and 3.24 illustrate the sections of beams and columns in considered structure.

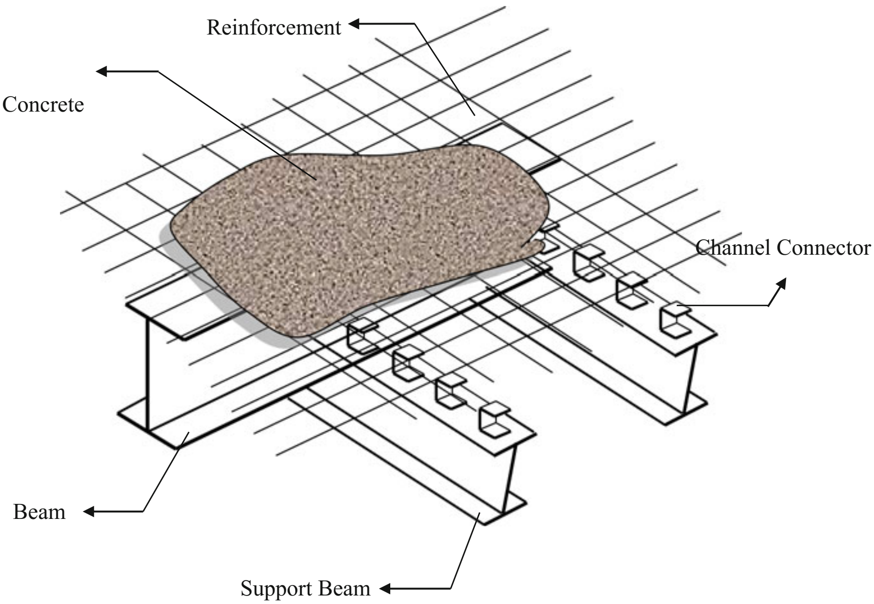


Fig. 3.20 Detail of floor system

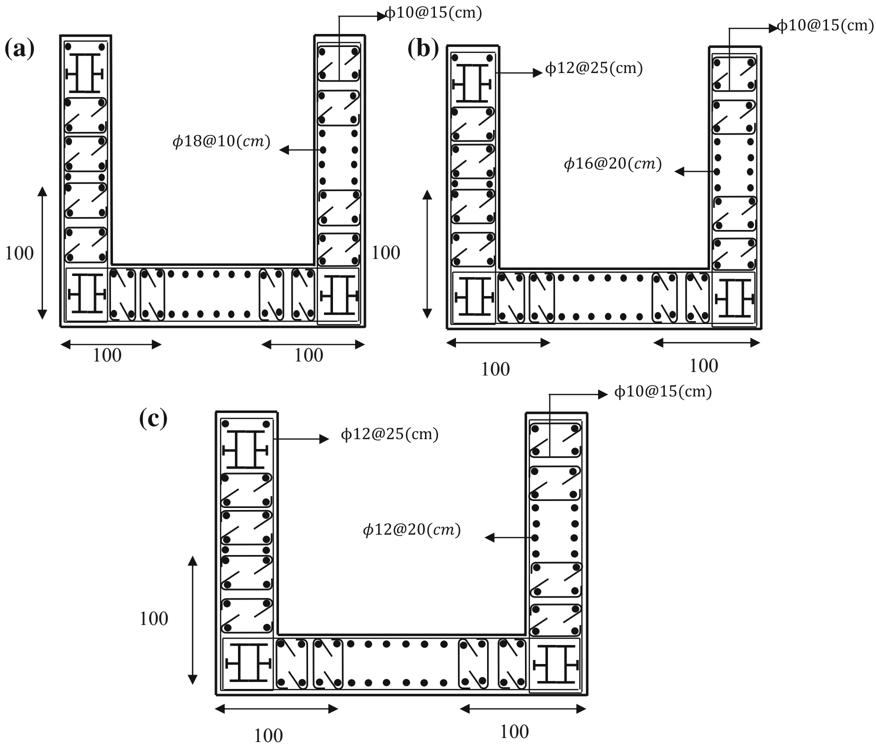


Fig. 3.21 Detail of shear wall W_1. **a** Story one and two, **b** story three, **c** story four, five and six

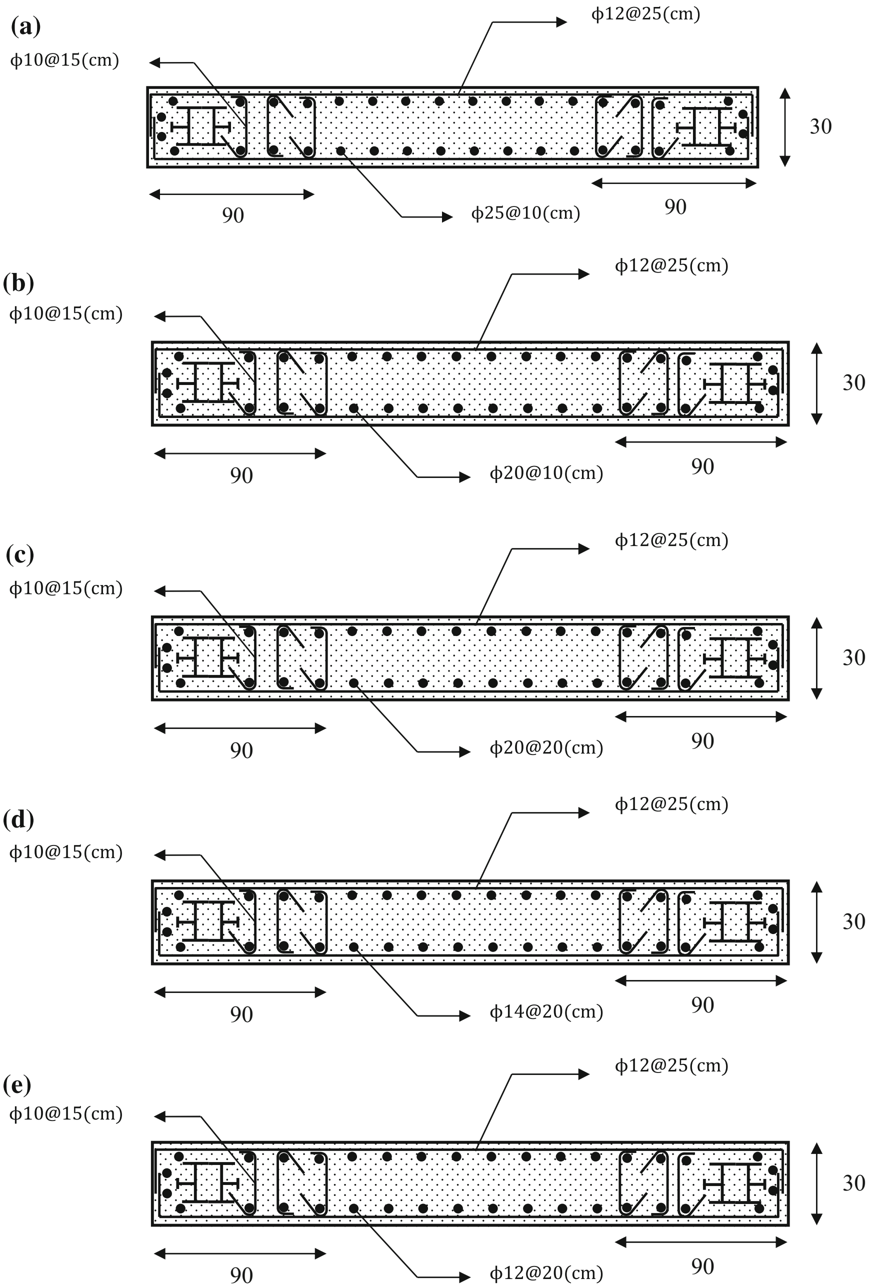


Fig. 3.22 Detail of shear wall W_2. **a** Story one, **b** story two, **c** story three, **d** story four, **e** story five and six

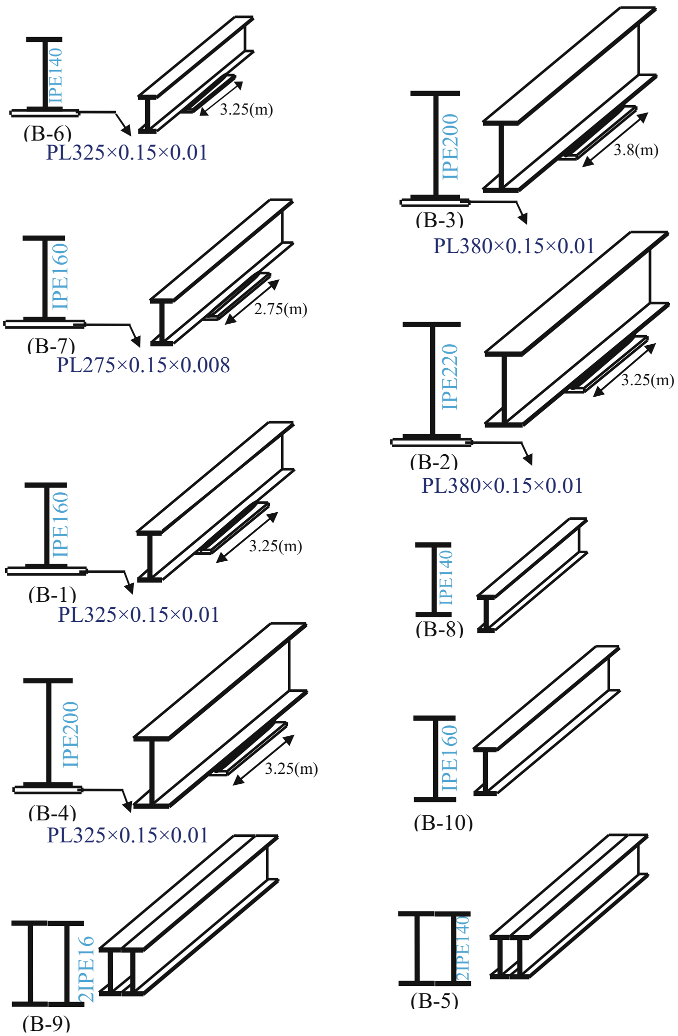


Fig. 3.23 Different types of beam and brace sections of considered building

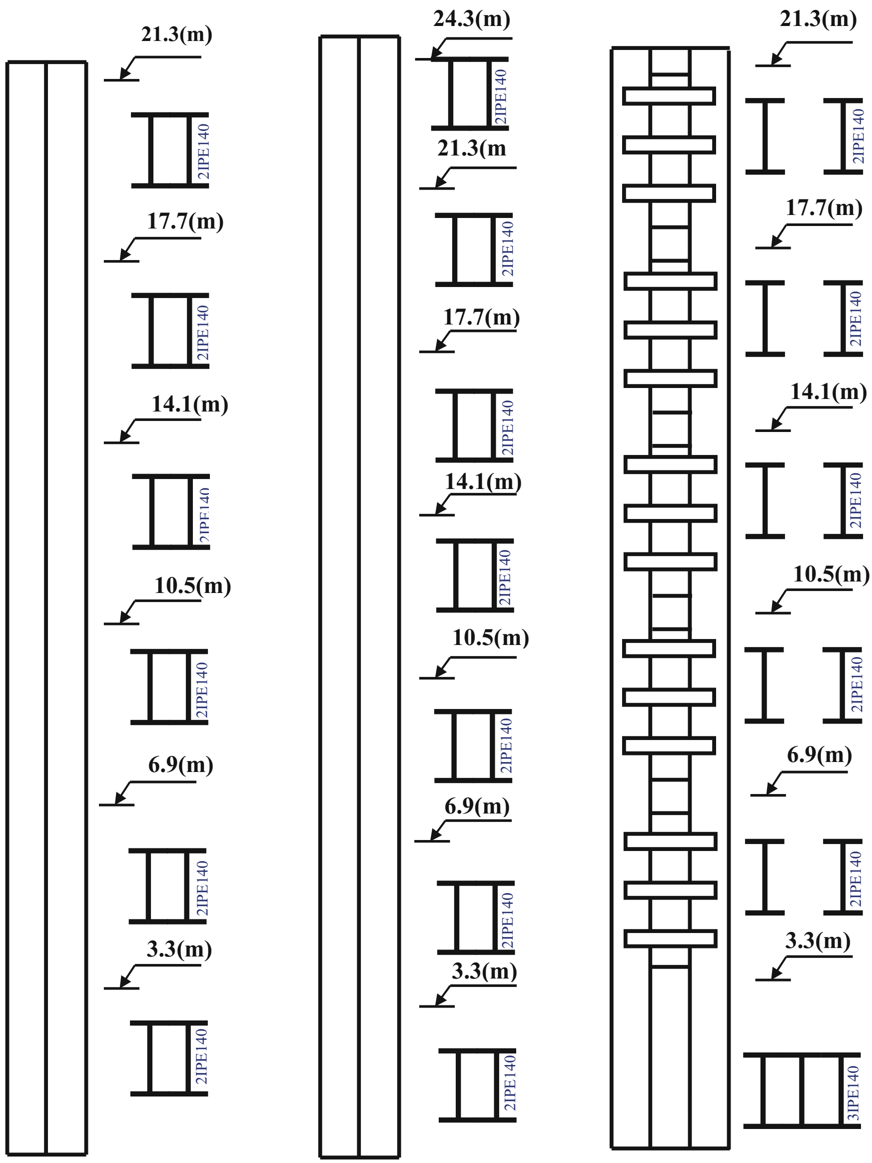


Fig. 3.24 Different type of columns of considered building

3.3 Loading

- Introduce of Different Loads Applied to the Structure
- Center of Mass
- Equivalent Lateral Force

- Stiffness
- Torsional Force
- Distribution of Lateral Load

Loading
As outlined in Sect. 3.2 (define project), the selected project is similar to the one detailed in Chap. 1. Therefore, the main parts of the building, such as the external wall, internal wall, floor system, etc. will be similar to the ones described in Chap. 1, while the structural system will be changed to that of the concrete shear wall. Herein, the weight of the floors, center of mass, stiffness, center of stiffness, and base shear of the structural system is calculated and detailed in the form of tables, charts, and figures.

3.3.1 The Weight of Steel Structure

The weight of a steel structural frame is derived from the total weight of the columns, beams, connections, and concrete shear wall. The columns sections in each story and properties of the columns are illustrated in Figs. 3.23, 3.24 and 3.25 and Table 3.1. This is based on its architectural drawing. Half–story columns are used above and below each joint to calculate the column weight. For instance, in Story (1), half of the height of story one's column and half of the height of Story two's column are utilized to calculate the columns' weight. Equation (3.15) demonstrates how to calculate the weight of the column and Table 3.3 shows total weight of columns in each story. Additionally, the calculation of the weight of the column is presented in Tables 3.4 and 3.5. Furthermore, the weight of the beam is computed by multiplying the total length of the beam with the density of the section in kg/m. Equation (3.16) demonstrates the calculation of the weight of the beam while Table 3.2 shows the sections of the beam. The total weight of the joints is 20% of the sum of the total weight of the columns and beams in each story. Figure 3.26 shows the location of the beams, columns and shear walls (Table 3.3).

$$\sum [NC[(Mass\,per\,unit\,C \times Half\,of\,the\,height\,of\,story\,top) + (Mass\,per\,unit\,C \times Half\,of\,the\,height\,of\,story\,bottom)] \tag{3.15}$$

Where this equation:
NC = Number of column

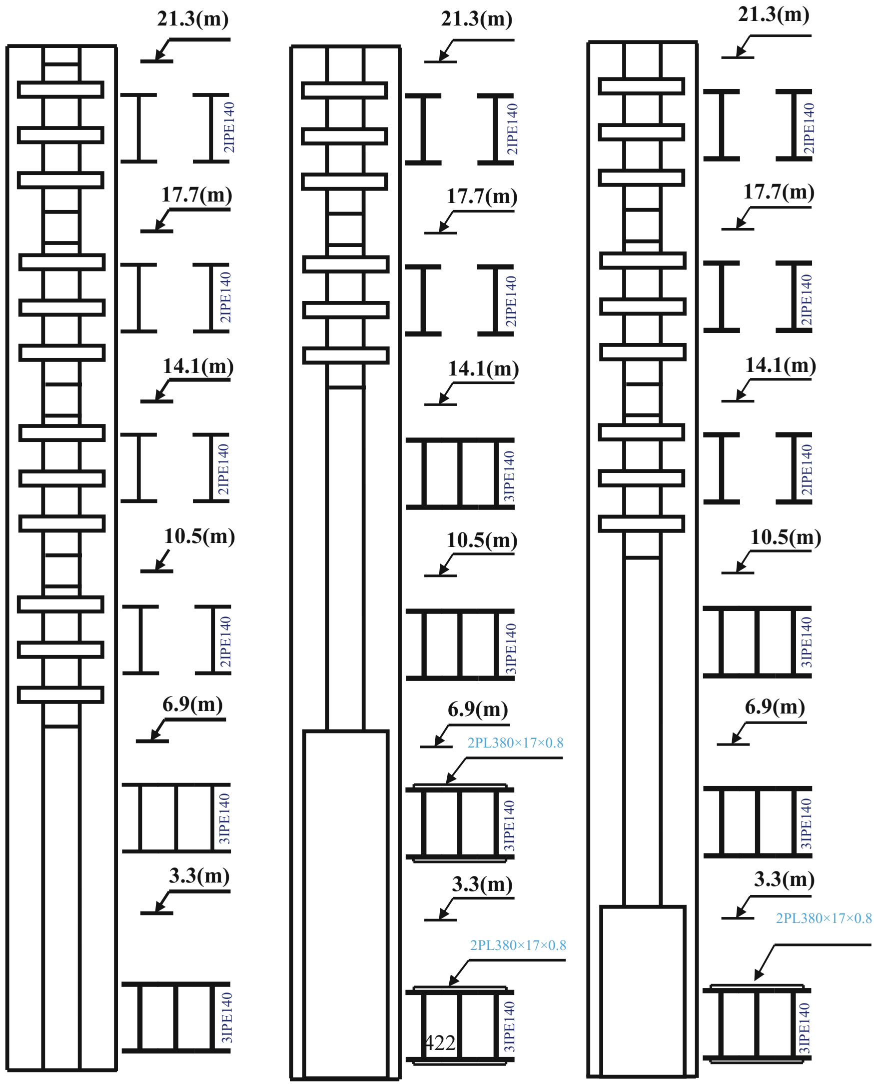


Fig. 3.25 Different types of columns of considered building

$$\text{Weight of beam} = (\text{length of beam}) \times (\text{weight per unit meter})$$
$$\text{Weight of shear wall} = (\text{Length of shear wall}) \times (\text{Thickness}) \times \text{Density of concrete} \quad (3.16)$$

Table 3.1 Column's section

Column	Story 1	Story 2	Story 3	Story 4	Story 5	Story 6
C-1	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
C-2	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
C-3	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
C-4	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$	$2IPE140$
C-5	$3IPE140 + 2PL380 \times 17 \times 0.8$	$3IPE140 + 2PL380 \times 17 \times 0.8$	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$
C-6	$3IPE140 + 2PL380 \times 17 \times 0.8$	$3IPE140$	$3IPE140$	$2IPE140$	$2IPE140$	$2IPE140$

Table 3.2 Beam's section

Beam	Section	Beam	Section
B-1	IPE160+PL325×15×10	B-6	IPE140+PL325×15×1
B-2	IPE220+PL380×15×10	B-7	IPE160+PL275×15×1
B-3	IPE200+PL380×15×10	B-8	IPE140
B-4	IPE200+PL325×15×1	B-9	2IPE 160
B-5	2IPE 140	B-10	2IPE 160

Table 3.3 Total weight of column in each story

Story	Column total weight (kg)
1	2017.735
2	1776.198
3	1555.74
4	1439.64
5	1393.2
6	812.7
Dome roof	116.1

Story 1: Half of the weight of first column story + half of the weight of second column story

$$\begin{aligned}&[2[(2 \times 12.9 \times 1.65) + (2 \times 12.9 \times 1.8)]] + [3[(2 \times 12.9 \times 1.65) + (2 \times 12.9 \times 1.8)]]\\&\quad + [2[(3 \times 12.9 \times 1.65) + (2 \times 12.9 \times 1.8)]] + [3[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8)]]\\&\quad + [2[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8) + (2 \times 1.9 \times 0.17 \times 0.008 \times 7850)\\&\quad + (2 \times 1.9 \times 0.17 \times 0.008 \times 7850)]] + [3[(3 \times 12.9 \times 1.65) + (3 \times 12.9 \times 1.8)\\&\quad + (2 \times 1.9 \times 0.17 \times 0.0008 \times 7850]] = 2017.735\ (\text{kg})\end{aligned}$$

So:

$NC_1 = 2$	Mass per unit of $C_1 = 12.9$	Half of the height of story 1 = 1.65 m
$NC_2 = 3$	Mass per unit of $C_1 = 12.9$	Half of the height of story 1 = 1.8 m
$NC_3 = 2$	Mass per unit of $C_1 = 12.9$	
$NC_4 = 3$	Mass per unit of $C_2 = 12.9$	
$NC_5 = 2$	Mass per unit of $C_3 = 12.9$	
$NC_6 = 3$	Mass per unit of $C_4 = 12.9$	

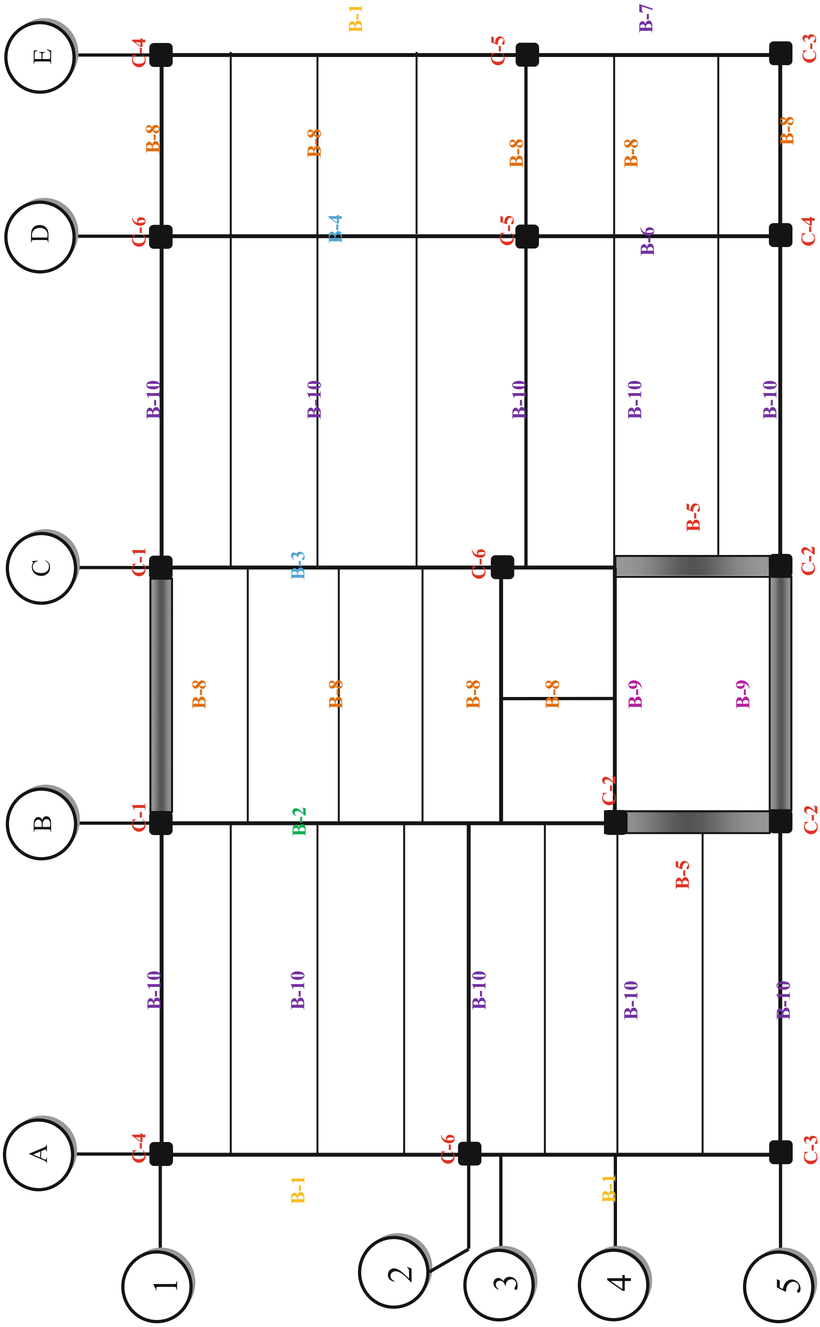


Fig. 3.26 Location of different types of columns and beams and shear wall

Table 3.4 Calculation of columns' weight in each story

Story	Name of column	Number of column	Number of IPE	$G_{kg/m}$	Half story height above and below	Weight of column	Number of plates	Volume of plate	γ_{kg/m^3}	Weight of plate	Total weight (kg)
1	C-1	2	2	12.9	3.45	178.02					178.02
2017.735	C-2	3	2	12.9	3.45	267.03					267.03
	C-3	2	3	12.9	1.65	127.71					127.71
	C-3	2	2	12.9	1.8	92.88					92.88
	C-4	3	3	12.9	3.45	400.545					400.545
	C-5	2	3	12.9	3.45	267.03	4	$1.9 \times 0.17 \times 0.008 = 0.002584$	7850	162.2752	429.3052
	C-6	3	3	12.9	3.45	400.545	2	$1.9 \times 0.17 \times 0.008 = 0.002584$	7850	121.7	522.245
2	C-1	2	2	12.9	3.6	185.76					185.76
1776.198	C-2	3	2	12.9	3.6	278.64					278.64
	C-3	2	2	12.9	3.6	185.76					185.76
	C-4	3	3	12.9	1.8	208.98					208.98
	C-4	3	2	12.9	1.8	139.32					139.32
	C-5	2	3	12.9	3.6	278.64	2	$1.9 \times 0.17 \times 0.008 = 0.002584$	7850	81.1376	359.7776
	C-6	3	3	12.9	3.6	417.96					417.96
3	C-1	2	2	12.9	3.6	185.76					185.76
1555.74	C-2	3	2	12.9	3.6	278.64					278.64
	C-3	2	2	12.9	3.6	185.76					185.76
	C-4	3	2	12.9	3.6	278.64					278.64
	C-5	2	3	12.9	3.6	278.64					278.64
	C-6	3	3	12.9	1.8	208.98					208.98
	C-6	3	2	12.9	1.8	139.32					139.32

Table 3.5 Calculation of columns' weight in each story

Story	Name of column	Number of column	Number of IPE	$G_{kg/m}$	Half story height above and below	Weight of column	Number of plates	Volume of plate	γ_{kg/m^3}	Weight of plate	Total weight (kg)
4	C-1	2	2	12.9	3.6	185.76					185.76
1439.64	C-2	3	2	12.9	3.6	278.64					278.64
	C-3	2	2	12.9	3.6	185.76					185.76
	C-4	3	2	12.9	3.6	278.64					278.64
	C-5	2	3	12.9	1.8	139.32					139.32
	C-5	2	2	12.9	1.8	92.88					92.88
	C-6	3	2	12.9	3.6	278.64					278.64
5	C-1	2	2	12.9	3.6	185.76					185.76
1393.2	C-2	3	2	12.9	3.6	278.64					278.64
	C-3	2	2	12.9	3.6	185.76					185.76
	C-4	3	2	12.9	3.6	278.64					278.64
	C-5	2	2	12.9	3.6	185.76					185.76
	C-6	3	2	12.9	3.6	278.64					278.64
6	C-1	2	2	12.9	1.8	92.88					92.88
812.7	C-2	3	2	12.9	3.3	255.42					255.42
	C-3	2	2	12.9	1.8	92.88					92.88
	C-4	3	2	12.9	1.8	139.32					139.32
	C-5	2	2	12.9	1.8	92.88					92.88
	C-6	3	2	12.9	1.8	139.32					139.32
Dome roof	C-2	3	2	12.9	1.5	116.1					116.1

Story 2: Half of the weight of second column story + half of the weight of third column story

$$\begin{aligned}&[2[(2\times 12.9\times 3.6)]]+[3[(2\times 12.9\times 3.6)]]+[2[(2\times 12.9\times 3.6))]\\&\quad+3[(3\times 12.9\times 1.8)+(2\times 12.9\times 1.8)]]+[2[(3\times 12.9\times 1.8)\\&\quad+(3\times 12.9\times 1.8)+(2\times 1.9\times 0.17\times 0.0008\times 7850]]\\&\quad+[3[(3\times 12.9\times 3.6]]=1776.198\ (\mathrm{kg})\end{aligned}$$

Story 3: Half of the weight of third column story + half of the weight of fourth column story

$$\begin{aligned}&[2[(2\times 12.9\times 3.6)]+[3[(2\times 12.9\times 3.6)]]\\&\quad+[2[(2\times 12.9\times 3.6)]]+[3[(2\times 12.9\times 3.6)]]\\&\quad+[2[(3\times 12.9\times 3.6])+[3[(3\times 12.9\times 1.8)]]\\&\quad+(2\times 12.9\times 1.8)]]=1555.74\ (\mathrm{kg})\end{aligned}$$

Story 4: Half of the weight of fourth column story + half of the weight of fifth column story

$$\begin{aligned}&[2[(2\times 12.9\times 3.6)]]+[3[(2\times 12.9\times 3.6)]]\\&\quad+[2[(2\times 12.9\times 3.6)]]+[3[(2\times 12.9\times 3.6)]]\\&\quad+[2[(3\times 12.9\times 1.8)+(2\times 12.9\times 1.8)]]\\&\quad+[3[(2\times 12.9\times 3.6)]]=1439.64\ (\mathrm{kg})\end{aligned}$$

Story 5: Half of the weight of fifth column story + half of the weight of roof column story

$$\begin{aligned}&[2[(2\times 12.9\times 3.6)]]+[3[(2\times 12.9\times 3.6)]]\\&\quad+[2[(2\times 12.9\times 3.6)]]+[3[(2\times 12.9\times 3.6)]]\\&\quad+[2[(2\times 12.9\times 3.6)]]+[3[(2\times 12.9\times 3.6)]]=1393.2\ (\mathrm{kg})\end{aligned}$$

Story 6: Half of the weight of roof column story + half of the weight of dome roof column story

$$\begin{aligned}&[2[(2\times 12.9\times 1.8)]]+[3[(2\times 12.9\times (1.8+1.5)]]\\&\quad+[2[(2\times 12.9\times 1.8)]]+[3[(2\times 12.9\times 1.8)]]\\&\quad+[2[(2\times 12.9\times 1.8)]]+[3[(2\times 12.9\times 1.8)]]=812.7\ (\mathrm{kg})\end{aligned}$$

Moving on, the total weight of beam of the considered building in each story is calculated by multiplying the total length of beam of each section with the mass per unit meter of the section. Then, it is composed with the weight of the plate that is added to the beam section (if there is any Tables 3.6 and 3.7 show the total weight

Table 3.6 Total weight of beam in each story

Beam	G (kg/m)	Total length (m)	Total weight (kg)
IPE140	12.9	58.6	755.94
IPE160	15.8	120.25	1899.95
IPE180	18.8	4.9	92.12
IPE200	22.4	10.6	237.44
IPE220	26.2	4.9	128.38
			3113.83

Table 3.7 Total weight of beam of dome roof

Beam	Length (m)	G (kg/m)	Total weight (kg)
IPE160	23.4	15.8	369.72
IPE140	15.6	12.9	201.24

Table 3.8 Total weight of plate in each story

Plate size	n	Volume (m^3)	$\gamma\left(\frac{kg}{m^3}\right)$	Weight (kg)
$3.25 \times 0.15 \times 0.01$	4	0.004875	7850	153.075
$3.8 \times 0.15 \times 0.01$	3	0.0057	7850	134.235
$2.75 \times 0.12 \times 0.008$	1	0.00264	7850	20.724
		0.013215		308

of beam sections in each story and dome roof, respectively. Meanwhile, the total weight of the plate is tabulated in Table 3.8. Furthermore, Table 3.9 shows the total weight of the steel structural frame in each story. According to this table, the total weight of the steel frame in each story is the sum of the total weight of the columns, and beams of the story. As mentioned before, the weight of the connectors in each story is approximately 20% of the sum of weight of the columns and the beams of the story. For instance, the total weight of Story 3 is per square meter is 265.84 (kg). From this, it can be inferred that it is made of 1555.74 (kg) of columns and 3421.83 (kg) of beam, making the total weight to be 4977.57 (kg). Next, the Net area of the considered model is 154.5 (m^2). Therefore, the sum of the weight of columns, beams and bracing per meter square is 32.21 (kg/m^2). Since the weight of connectors of Story 3 is 20% of 32.21 (kg/m^2) (6.44), the total weight of Story 3 is 32.21 + 6.44 = 38.661 (kg/m^2). Moreover the total weight of shear wall in third story should be add to 38.66 $(\mathrm{kg}/\mathrm{m}^2)$. Total length and height of shear wall in story three is 13 (m) and 3.6 (m) respectively. Furthermore thickness of concrete shear wall is assumed 0.3 (m) and density of concrete is 2500 $(\mathrm{kg}/\mathrm{m}^2)$, as a result total weight of concrete shear wall is equal 227.18 $(\mathrm{kg}/\mathrm{m}^2)$, so total weight of story three is calculated of summation with 38.66 $(\mathrm{kg}/\mathrm{m}^2) + 227.18\ (\mathrm{kg}/\mathrm{m}^2) = 265.84\ (\mathrm{kg}/\mathrm{m}^2)$ (Chart 3.2).

Table 3.9 Total weight of steel structure in each story

Story	Weight of beam	Weight of column	Total weight of beam column	Area of plan	Weight	Weight of connection	Total weight of steel part	Length of shear wall	Shear wall height	T (m)	ϒ (kg/m^3)	The weight of the shear wall	Total weight of the structure
1	3421.83	2017.73	5439.56	154.5	35.2075	7.0415	42.249	13	3.45	0.3	2500	217.7184	259.9675
2	3421.83	1776.19	5198.02	154.5	33.6441	6.7288	40.37	13	3.6	0.3	2500	227.1845	267.5575
3	3421.83	1555.74	4977.57	154.5	32.21728	6.4434	38.66073	13	3.6	0.3	2500	227.1845	265.8452
4	3421.83	1439.64	4861.47	154.5	31.4658	6.2931	37.7589	13	3.6	0.3	2500	227.1845	264.9435
5	3421.83	1393.2	4815.03	154.5	31.1652	6.2330	37.3982	13	3.6	0.3	2500	227.1845	264.5828
6	3421.83	812.7	4234.53	154.5	27.4079	5.4815	32.8895	13	1.8	0.3	2500	113.5922	146.4818
Dome roof	570.96	116.1	687.06	10.14	67.7573	13.5514	81.3088						81.3088

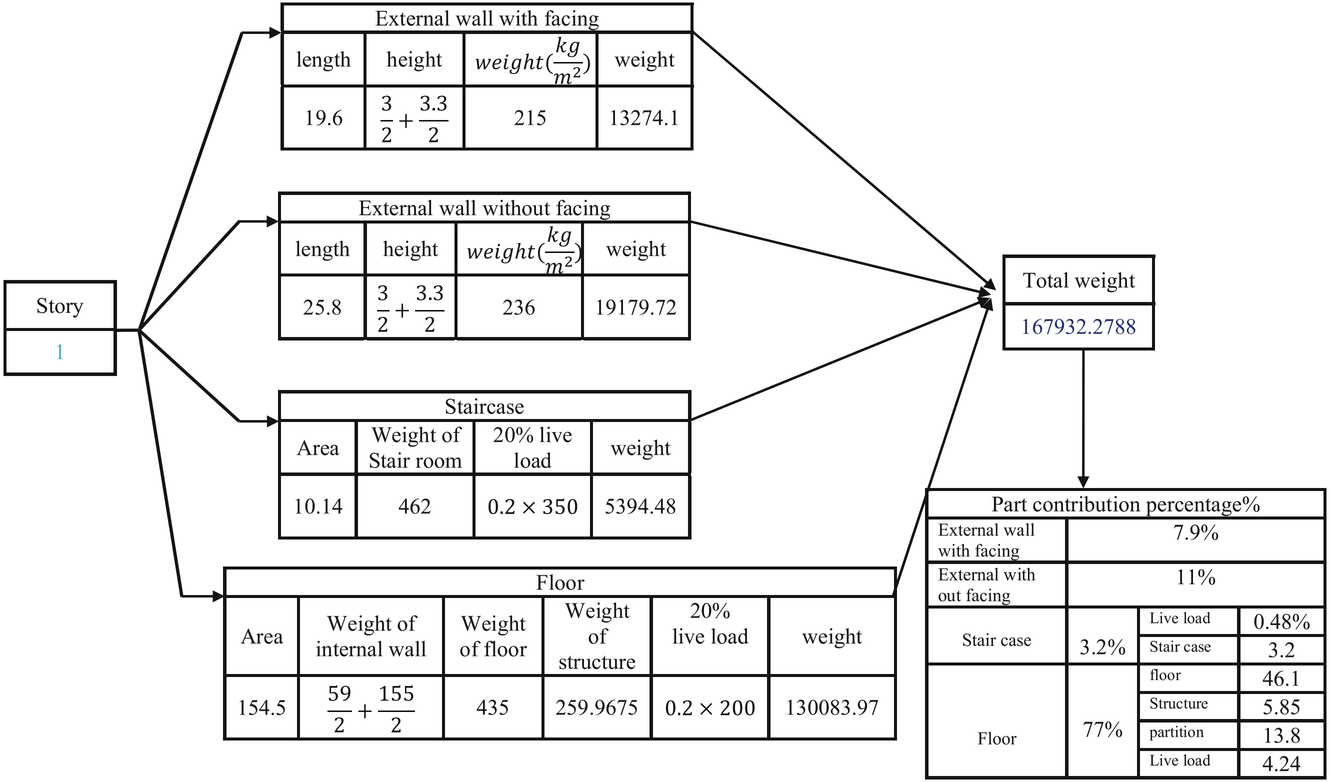


Chart 3.2 Total weight of story one in detail

Weight of first floor:

$$(25.8)\left(\frac{3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{59}{2}\right)+\left(\frac{155}{2}\right)+(435)+(259.9675)+(0.2\times 200)\right]=167932.2788\ (\mathrm{kg})$$

Weight of 2nd floor:

$$(25.8)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{155}{2}\right)+\left(\frac{155}{2}\right)+(435)+(267.5575)+(0.2\times 200)\right]=178066.3538\ (\mathrm{kg})$$

Weight of 3rd floor:

$$(25.8)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{155}{2}\right)+\left(\frac{155}{2}\right)+(435)+(265.84)+(0.2\times 200)\right]=177801.8034\ (\mathrm{kg})$$

Weight of 4th floor:

$$(25.8)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{155}{2}\right)+\left(\frac{155}{2}\right)+(435)+(264.9435)+(0.2\times 200)\right]=177662.4908\ (\mathrm{kg})$$

Weight of 5th floor:

$$(25.8)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}+\frac{3.3}{2}\right)(215)+(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{155}{2}\right)+\left(\frac{155}{2}\right)+(435)+(264.5828)+(0.2\times 200)\right]=177606.7626\ (\mathrm{kg})$$

Weight of roof floor:

$$(25.8)\left(\frac{3.3}{2}\right)(236)+(19.6)\left(\frac{3.3}{2}\right)(215)+[(53.2)\times(205)]+[(10.14)[(462)+(0.2\times 350)]$$
$$+(154.5)\left[\left(\frac{155}{2}\right)+(480)+(146.4818)+(0.2\times 150)\right]=146700.28\ (\mathrm{kg})$$

Table 3.10 Calculating of mass and centre of mass for 1st floor

Number of mass	Length/ area	kg/m–kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	841.9675	42494.09	2.575	4.9	109422.306	208221.089
3	28.08	841.9675	23642.44	7.1	6.2	167861.376	146583.174
4	75.95	841.9675	63947.43	12.925	4.9	826520.553	313342.415
5	9.8	677.25	6637.05	16.7	4.9	110838.735	32521.545
6	9.8	677.25	6637.05	0.1	4.9	663.705	32521.545
7	5.15	743.4	3828.51	2.575	0.1	9858.41325	382.851
8	7.75	743.4	5761.35	12.925	0.1	74465.4487	576.135
9	5.15	743.4	3828.51	2.575	9.7	9858.41325	37136.547
10	7.75	743.4	5761.35	12.925	9.7	74465.4487	55885.095
Sum			167932.27			1422255.20	834183.219

$\bar{x} = \frac{1422255.209}{167932.2788} = 8.469 \text{ (m)} \quad \bar{y} = \frac{834183.219}{167932.2788} = 4.967 \text{ (m)}$

Weight of Demo roof:

$$(10.14)[(480 + 81.3088 + (0.2 \times 150)] + (13)(2.7 \times 236) + (13 \times 205) \\ = 16944.47123(\text{kg})$$

3.3.2 Centre of Mass

The center mass can be calculated using Eq. (3.17):

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i} \quad Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} \tag{3.17}$$

Table 3.10 shows the calculation using the first floor of the building. The center mass of the other floors are shown in Tables 3.32, 3.33, 3.34, 3.35, 3.36 in Appendix.

3.3.3 Lateral Load

This part of section three is dedicate computation of lateral load based on Eurocode 8 and (IBC 2012).

3.3.3.1 The Equivalent Lateral Force of Project in Eurocode 8

Step 1: Ground Type

There is no doubt that the averages shear velocity is most important factor for the selection of the ground type. An average shear velocity is considered $V_S, 30 = 350\,\frac{\text{m}}{\text{s}}$ as a property of ground, ground type is considered as class C (Eurocode 8, 2004; Table 3.1).

Step 2: Appropriate type of elastic response spectrum and components

Depending on the characteristics of the most significant earthquake contributing to the local hazard this building is located in the high-risk zone of the earthquake. As a result, the magnitude of surface wave is considered more than 5.5 and the first type of spectrum is chosen as an elastic response spectrum. According to Table 3.2 of Euro code (8), the value of the parameters of elastic response spectrum is shown in Table 3.11 (Eurocode 8, 2004/3.2.2.2(2)P).

Step 3: Important factor

Based on part (4.2.5) and Table 4.3 of Euro code eight the building is classified in class II and importance the factor is 1 ($\gamma = 1$) (Eurocode 8, 2004/4.2.5).

Step 4: Determination of the fundamental period

The concrete shear wall system is selected for resisting the earthquake load in this building, so the fundamental period of the vibration of the structure for the lateral motion in the considered direction is calculated as following (Eurocode 8, 2004/4.3.3.2.2.3(P)):

$$T_1 = C_t.H^{\left(\frac{3}{4}\right)}$$
$$C_t = 0.05 \text{ Other Structure}$$
$$\text{H} = 21.3(\text{m})$$
$$\text{T} = (0.05)(21.3)^{0.75} = 0.49574$$

Step 5: Behaviour factor of horizontal seismic action

The building is considered as a uncoupled wall system in each two perpendicular directions, and the structure will be designed as a medium ductility class. The magnitude of behaviour factor is calculated by Eq. (3.18) (Eurocode 8, 2004, 5-1).

Table 3.11 Value of the parameters describing the recommended Type 1 elastic response (Eurocode 8, 2004, Table 3.2)

Ground type	S	$T_B(s)$	$T_C(s)$	$T_D(s)$
C	1.15	0.2	0.6	2

The value of q_0 is determined by Eurocode 8 (2004, Table 5.1). The magnitude of k_w is equal to 1 (Eurocode 8, 2004, 5.2.2.2(11))

$$\begin{aligned} q &= q_0 k_w > 1.5 \\ q_0 &= 3 \end{aligned} \tag{3.18}$$

$$k_{wx} = \frac{\left[\left(\frac{21.3\times 2}{(3.9\times 2)}\right)+1\right]}{3} = 2.15 > 1 = 1$$

$$k_{wy} = \frac{\left[\left(\frac{21.3\times 2}{(2.6+2.6)}\right)+1\right]}{3} = 3.06 > 1 = 1$$

Step 6: Design spectrum

For the horizontal components of the earthquake action in considered structure, the design spectrum $S_d(T)$ is considered by Eurocode 8 (2004)/3.2.2.5(4)P):

$$T_B \le T \le T_C{:}S_d(T) = \left\{a_g.S.\frac{2.5}{q}\right\}$$

$$T_T = 0.49574$$

$$S_d(T) = \left\{0.3\times 1.15\times \frac{2.5}{3}\right\} = 0.2875.$$

Step 7: Base shear force the horizontal direction's seismic shear force, F_b, used to evaluate each building can be worked out by using the (Eurocode 8, 2004, 4.3.3.2.2(1)P)

$$F_b = S_d(T_1)\times \lambda \times m$$

$$\begin{aligned} S_d(T_1) &= 0.2875 \\ \lambda &= 0.85 \\ m &= 1042714.4172 \\ F_b &= 2548.13 \text{ kN.} \end{aligned}$$

In this structure, a steel structure with concrete shear wall frame is used in X and Y direction, therefore the magnitude of the shear base in each direction will be the same ($F_{bx} = F_{by} = 2548.13$ kN). Base shear is distributed in vertical direction by using

$$F_i = F_b \cdot \frac{z_i . m_i}{\sum z_j . m_j}$$

In this structure, the shear base in each direction is 2548.14 kN. It should be noted that the total weight of each story has been calculated in the previous sections. Figure 3.27 demonstrates the distribution of the shear force generated in each story and the overturning moment. Moreover, the lateral force and the shear force in Story 6 is 694.76 kN and the lateral force in Story 5 is 626.59 kN. Therefore, the sum of the lateral forces in Story 6 and Story 5 will be equal to the shear in Story 5, which is 1321.35 kN. The shear forces in each floor are calculated using Eq. (3.39) (Eurocode 8, 2004, 4.3.3.2.3(2)P) (Table 3.12).

Figure 3.27 shows the 3D distribution of the shear base in X and Y directions throughout the height of the structure. A concrete shear wall is employed in the building in both directions; hence the shear base in both directions will be equal. The shear force is 694.76 kN in Story 6 and it increases to 2548.13 kN in the first

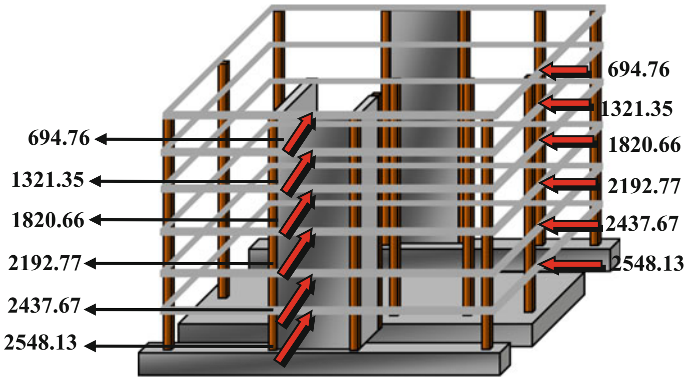


Fig. 3.27 Distribution of shear base in height of model

Table 3.12 Distribution of shear base in each story

Story	h_i	W_i	W_ih_i	W_ih_i/W_ih_i	Force	Shear	Moment
6	21.3	163644.7572	3485633.329	0.2726545	694.76	694.76	1250.56
5	17.7	177606.756	3143639.581	0.2459029	626.59	1321.35	4879.57
4	14.1	177662.484	2505041.024	0.1959502	499.30	1820.66	10535.19
3	10.5	177801.804	1866918.942	0.1460348	372.11	2192.77	17759.38
2	6.9	178066.344	1228657.774	0.0961085	244.89	2437.67	26094.19
1	3.3	167932.272	554176.4976	0.043349	110.45	2548.13	34686.43
		1042714.417	12784067.15				

story. It should be noted that all lateral forces in each story are applied to the centre of mass of the floor. Furthermore, the centre of mass of each floor is determined through the method discussed in Sect. 3.3.2.

3.3.3.2 The Calculation Equivalent Lateral Force of Project by IBC (2012)

Step 1: Determination of the maximum considered earthquake and design spectral response accelerations:

1.1 Depending on the characteristics of the most significant earthquake contributing to the local hazard, this building is located in a high-risk zone of earthquake. As a result of this, the value of S_1 and S_S is determined to be 0.3 and 0.75, respectively.

1.2 There is no doubt that the average shear velocity is the most important factor for the selection of the site class. An average shear velocity is considered $V_S, 30 = 350\frac{m}{s}$ as a property of the ground, and according to the (ASCE7-10, 2010 #31, Table 20.3.1), the site class is considered as class D.

1.3 The maximum considered earthquake spectral response accelerations for short and long periods can be calculated using Eqs. (3.19) and (3.20) (IBC, 2012, Sect. 1613.3.3), while the value of site coefficients F_a and F_v can be determined using (IBC, 2012, Table 1613.3.3(1) and Table 1613.3.3(2)) respectively (Tables 3.13 and 3.14).
Hence;

$$S_{MS} = F_a S_s = 1.2 \times 0.75 = 0.9 \tag{3.19}$$

$$S_{M1} = F_v S_1 = 1.8 \times 0.3 = 0.54 \tag{3.20}$$

1.4 The value of S_{DS} and S_{D1}, in the form of damped design (5%) spectral response accelerations in the short and long periods can be computed using on Eqs. (3.21) and (3.22) (IBC, 2012, Sect. 1613.3.4.)

Table 3.13 Values of site coefficient F_a

Site class	Mapped spectral response acceleration at short period				
	$S_s \leq 0.25$	$S_s = 0.5$	$\mathbf{S_s = 0.75}$	$S_s = 1$	$S_s \geq 1.25$
D	1.6	1.4	**1.2**	1.1	1

Table 3.14 Values of site coefficient F_v

Site class	Mapped spectral response acceleration at 1-s period				
	$S_a \leq 0.1$	$S_a = 0.2$	$\mathbf{S_a = 0.3}$	$S_a = 0.4$	$S_s \geq 0.5$
D	2.4	2	**1.8**	1.6	1.5

$$S_{DS} = \left(\frac{2}{3}\right) S_{MS} = \left(\frac{2}{3}\right) \times (0.9) = 0.6 \text{ g} \tag{3.21}$$

$$S_{D1} = \left(\frac{2}{3}\right) S_{M1} = \left(\frac{2}{3}\right) \times (0.54) = 0.36 \text{ g} \tag{3.22}$$

Step 2: Determination of seismic design category and Importance factor

a. Based on (IBC, 2012) Table 1604.5 and ASCE 7-10 Table 1.5-3.2) the building is classified as class II, and its importance factor is (1) ($I_e = 1$).
b. The seismic design category (SDC) for $S_{DS} = 0.6$ g and $S_{D1} = 0.36$ g is determined to be "D", as per (IBC, 2012, Table 1613.3.5(1) and Table 1613.3.5(2)) (Tables 3.15 and 3.16).

Step 3: Seismic Base Shear determination

3.1 For ordinary reinforcement concrete shear wall and according to (ASCE7-10, 2010 #31, Table 12.2-1), the response modification coefficient(R) is 5
3.2 The horizontal direction's seismic shear force, *V*, can be used to evaluate each building via Eq. (3.23) (ASCE7-10, 2010 #31, Sect. 12.8).

$$\mathrm{V} = \mathrm{Cs\,W} \tag{3.23}$$

C_s can be computed using Eq. (3.24)

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{5}{1}} = 0.12 \tag{3.24}$$

3.3 Determination of the fundamental period

The concrete shear wall system is selected for resisting earthquake in this building, therefore the fundamental period of the vibration of the structure for the lateral motion in the considered direction can be calculated using Eq. (3.25) (ASCE7-10, 2010 #31);

Table 3.15 SDC based on short-period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
0.50 g $\leq S_{DS}$	D	D	D

Table 3.16 SDC based on 1-second period response accelerations

Value of S_{DS}	Risk category		
	I or II	III	IV
0.20 g $\leq S_{D1}$	D	D	D

$$T_a = C_t \times h_n^x \tag{3.25}$$

The C_t and x in a given equation can be determined using (ASCE/SEI, 2010, Table 13.2.8-2), hence;

$$C_t = 0.02, \quad x = 0.75$$

So,

$$T_a = 0.02 \times 21.3^{0.75} = 0.198$$

According to (ASCE7-10, 2010 #31, Table 3.12.8-1) the value of C_u is equal to:

$$C_u = 1.4$$

The calculated fundamental period, T, cannot exceed the product of the coefficient, C_u, in the (ASCE7-10, 2010 #31, Table 3.12.8-1), multiplied with the approximate fundamental period, T_a.

$$C_u \times T_a = 1.4 \times 0.32349 = 0.4528$$

1.4 Exceptions

- The value of C_s shall not exceed the value of Eq. (3.26), hence;

$$C_S = \frac{S_{D1}}{T(R/I_e)} \quad for \quad T \leq T_L \tag{3.26}$$

$$C_S = \frac{S_{D1}}{T(R/I_e)} = \frac{0.36}{0.198\left(\frac{5}{1}\right)} = 0.3636$$

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{5}{1}} = 0.12 < C_S = \frac{S_{D1}}{T(R/I_e)} = \frac{0.36}{0.198\left(\frac{5}{1}\right)} = 0.3636 \quad (OK$$

- The value of C_s shall not be less than the value of Eq. (3.27), so:

$$C_S = 0.044 S_{DS} I_e \geq 0.01 \tag{3.27}$$

$$= 0.044 \times 0.6 \times 1 = 0.0264 \geq 0.01 (OK)$$

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = \frac{0.6}{\frac{5}{1}} = 0.12 > 0.0264 \quad (OK)$$

As a result the value of C_s is computed according Eq. (3.24).

$$C_s = 0.12$$

- The seismic base shear calculates in accordance with Eq. (3.23)

$$V = C_s \times W$$

$$V = 0.12 \times 1042714.4172 = 1251.25(\text{kN})$$

Step 4: Vertical distribution of forces

The lateral seismic force (F_x) induced at any level can be determined using Eqs. (3.28) and (3.29).

$$F_x = C_{VX} \times V \tag{3.28}$$

$$C_{vx} = \frac{W_x \times h_x^k}{\sum_{i=1}^{n} W_i h_i^k} \tag{3.29}$$

k = a distribution exponent related to the building period as follows:
k = 1 for buildings with T less than or equal to 0.5 s
k = 2 for buildings with T more than or equal to 2.5 s.

Interpolate between k = 1 and k = 2 for buildings with T between 0.5 and 2.5

$$T_a = 0.32349 < 0.5s \rightarrow k = 1$$

In the structure under consideration, the base shear in each direction is 1251.25 kN and the total weight of each floor can be calculated using previous parts. Table 3.17 shows the distribution of shear generated in each floor and its overturning.

Table 3.17 Distribution of shear base in each floor

Story	H_i	W_i	W_ih_i	W_ih_i/W_ih_i	Force	Shear	Moment
6	21.3	163644.757	3485633.329	0.2727	341.160	341.160	614.089
5	17.7	177606.756	3143639.581	0.2459	307.687	648.848	2396.107
4	14.1	177662.484	2505041.024	0.1960	245.184	894.032	5173.294
3	10.5	177801.804	1866918.942	0.1460	182.727	1076.760	8720.721
2	6.9	178066.344	1228657.773	0.0961	120.256	1197.016	12813.519
1	3.3	167932.272	554176.497	0.0433	54.240	1251.257	17032.723
		1042714.417	12784067.147				

Table 3.18 Behaviour of concrete shear wall in considered building

Name	Height	Length (B)	H/B	Deflection control
W_1	21.3	4.6	4.63	Bending control
W_2	21.3	2.6	8.19	Bending control
S_1	21.3	3.9	5.46	Bending control
S_2	21.3	3.9	5.46	Bending control

3.3.4 Stiffness

According to the Chart 1, Table 3.18 shows the behaviour of concrete wall in considered building. Total height of structure is 21.3 (m) height and the length of shear wall in X direction is 3.9 (m) and the length of shear wall in Y direction in bent (2) and bent (3) is 4.6 (m) and 2.6 (m) respectively.

Based on Table 3.18 stiffness of shear wall in considered building is calculated according bending control deflection, as a result the stiffness of shear wall is calculated based on equation $K_b = \frac{3EI}{H^3}$. The moment of inertia and shear area to be used for the computation of deflections of cantilever shear wall structure and also for input to a computer program with a view to finding the interaction with other walls and frame structures (Fig. 3.28).

For horizontal force V_x

$$I_y = \frac{h_1 B_1^3}{12} + \frac{h_3 B_3^3}{12} \tag{3.30}$$

$$Shear\,area = \frac{5}{6}[(h_1 \times B_1) + (h_3 \times B_3)] \tag{3.31}$$

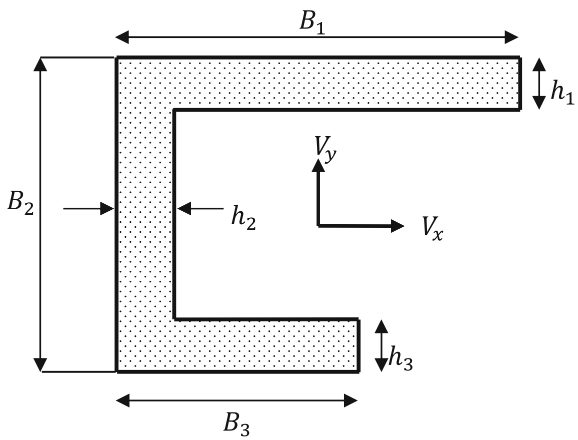


Fig. 3.28 U shape shear wall

For horizontal force V_y

$$I_x = \frac{h_2 B_2^3}{12} \tag{3.32}$$

$$Shear\,area = \frac{5}{6}(h_2 \times B_2). \tag{3.33}$$

3.3.4.1 Centre of Stiffness

The calculation of the stiffness center was detailed in the first chapter, as per Eq. (3.34)

$$X_{CR} = \frac{\sum k_{yi} \times x_i}{\sum k_{yi}} \quad Y_{CR} = \frac{\sum k_{xi} \times y_i}{\sum k_{xi}} \tag{3.34}$$

3.3.4.2 Calculation of Center Rigidity

Moving on, Eqs. (3.34) and (3.35) are used to calculate the stiffness and the center of rigidity of each story. Figure 3.29 shows the concrete shear wall with parametric dimensions. The stiffness and the center of rigidity of each story are documented in Charts 3.3 and 3.4. The stiffness and the center of rigidity in Story 5 and Story 6 are calculated as follows:

$$K = \frac{3EI}{H^3} \tag{3.35}$$

E module of Elasticity = 32×10^6 kN/m^2
B length of beam
H height of shear wall
h thickness of shear wall

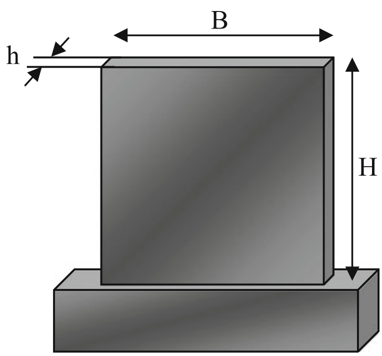


Fig. 3.29 Concrete shear wall

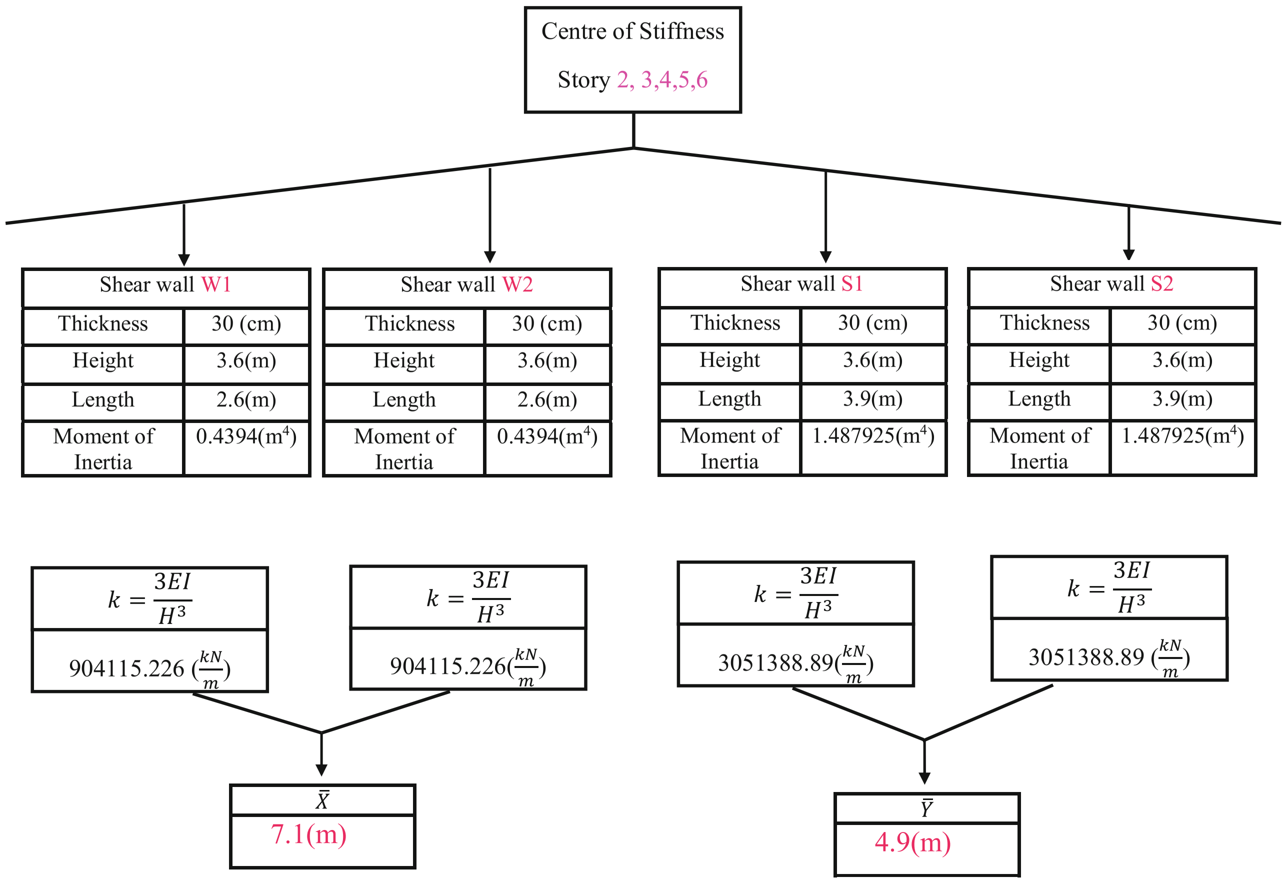


Chart 3.3 The centre of stiffness in stories two, three, four, five, and six

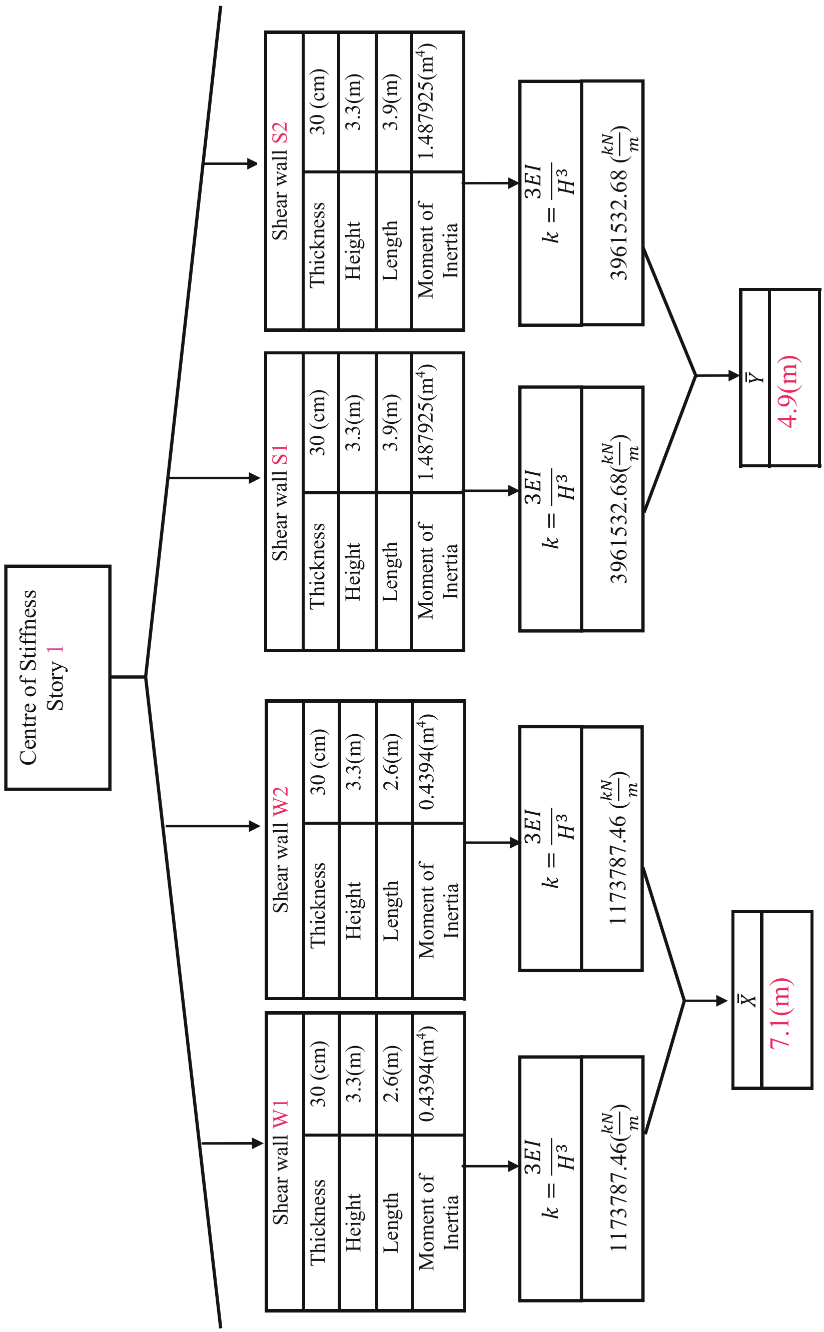


Chart 3.4 Total weight of first story

$$Av : \frac{5}{6}(B \times h)$$

$$K_{w1} = \frac{3 \times 32 \times 10^6 \times \frac{0.3 \times 2.6^3}{12}}{3.3^3} = 1173787.46\left(\frac{\text{kN}}{\text{m}}\right)$$

$$K_{w2} = \frac{3 \times 32 \times 10^6 \times \frac{0.3 \times 2.6^3}{12}}{3.3^3} = 1173787.46\left(\frac{\text{kN}}{\text{m}}\right)$$

$$K_{s1} = \frac{3 \times 32 \times 10^6 \times \frac{0.3 \times 3.9^3}{12}}{3.3^3} = 3961532.68\left(\frac{\text{kN}}{\text{m}}\right)$$

$$K_{s2} = \frac{3 \times 32 \times 10^6 \times \frac{0.3 \times 3.9^3}{12}}{3.3^3} = 3961532.68\left(\frac{\text{kN}}{\text{m}}\right)$$

$$\bar{x} = \frac{(1173787.46)(9.05) + (1173787.46)(5.15)}{(1173787.46) + (1173787.46)} = 7.1 \text{ m}$$

$$\bar{y} = \frac{(3961532.68)(9.8)}{(3961532.68 + 3961532.68)} = 4.9 \text{ m}$$

According to the result the centre of stiffness in story 5 and story 6 is located in coordinate X = 4.9 (m) from arbitrary origin (Y axis) and Y = 7.1 (m) from arbitrary origin (X axis).

The chart in Fig. 3.30 was designed to simplify the calculation of the center of stiffness on each floor. This chart consists of five parts:

(i) Number of stories
(ii) Properties of concrete shear wall
(iii) Stiffness of shear wall
(iv) Distance of the stiffness center from the Y-axis
(v) Distance of the stiffness center from the X-axis

Charts 3.3 and 3.4 demonstrate the calculation of center mass for stories 2–6 and story one respectively.

3.3.5 Torsional Forces

According to the definition of center mass and stiffness, if there is distance between both points on each floor, then there will be an eccentricity, which creates torsion. Therefore, torsion is a decisive factor in the distribution of base shear in a structure.

3.3.5.1 Calculation of Torsion

Based on the concept of torsion in a structure, eccentricities perpendicular to the floor can be calculated according to Eq. (3.36).

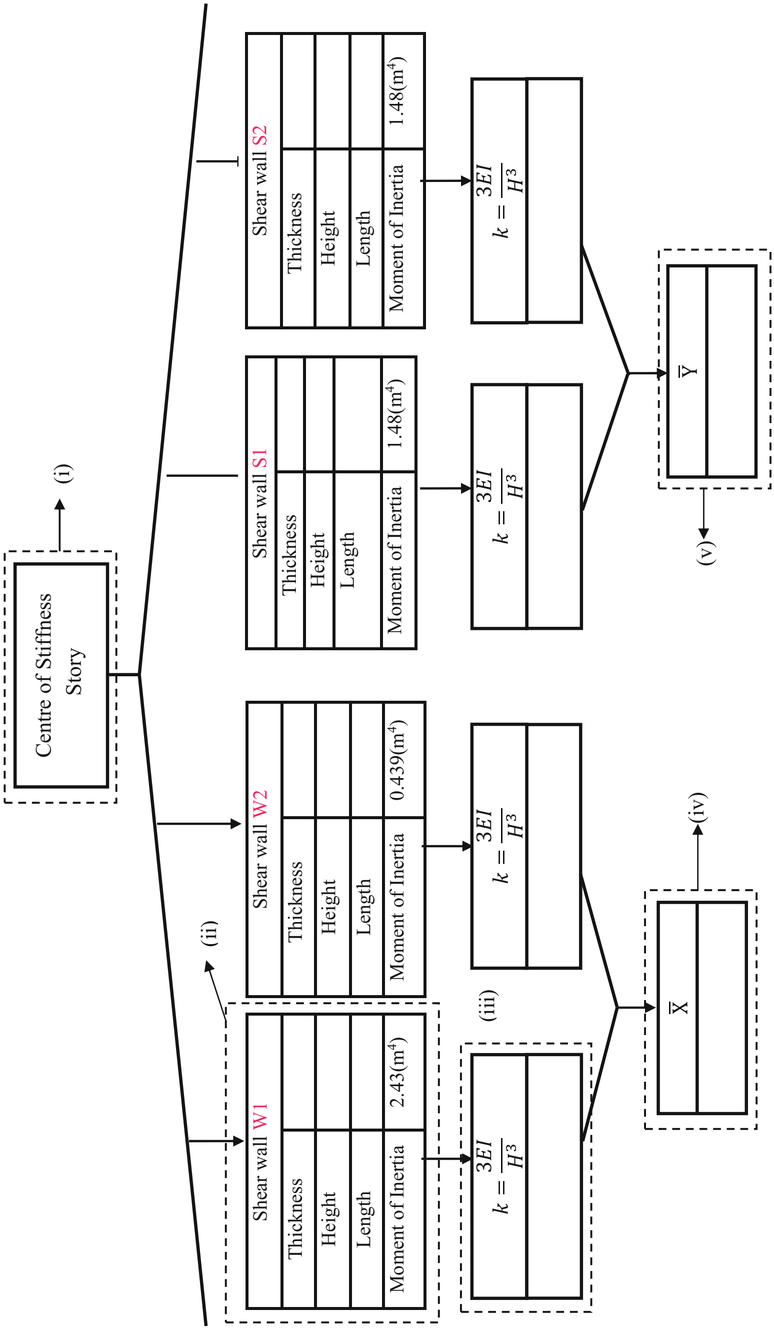


Fig. 3.30 Chart pattern for calculation the centre of stiffness

$$\begin{cases} e_x = x_{\text{cm}} - x_{cj} \\ e_y = y_{\text{cm}} - y_{cj} \end{cases} \tag{3.36}$$

The accidental torsion effect of each floor can be calculated according to Eq. (3.37), and the value of accidental torsion must be added to the value of eccentricity of each floor.

$$\begin{array}{ll} e_{ai} = \pm 0.05 \times Li & \\ e_x = 8.469 - 7.1 = 1.369\ (\text{m}) & \text{d}_2 = 4.9 \\ e_y = 4.967 - 4.9 = 0.0674\ (\text{m}) & \text{d}_1 = 4.9 \\ \begin{cases} e_{ai} = 0.05 \times 17 = 0.85\ (\text{m}) \\ e_{ai} = 0.05 \times 10 = 0.5\ (\text{m}) \end{cases} & \text{c}_1 = 1.95\,. \\ \begin{cases} e_x = 1.369 + 0.85 = 2.219\ (\text{m}) \\ e_y = 0.0674 + 0.5 = 0.5674\ (\text{m}) \end{cases} & \text{c}_2 = 1.95 \end{array} \tag{3.37}$$

3.3.6 Distribution of Lateral Shear in Bents

The total external shear at a level is distributed between the bents in proportion to their shear rigidities at that level. An estimate of the shear Q_{ji} carried by bent j at level i is given by following:

$$Q_i = \left[\frac{(K)_i}{\sum (K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i \tag{3.38}$$

V_i is total shear in story i
$(K)_i$ is the stiffness of brace i in story i
e is the eccentricity of V_i from the center of stiffness in story i
C is the distance of brace i from the center of stiffness
J is torsional stiffness

$$\sum K_x \times C^2 + \sum K_y \times d^2 \tag{3.39}$$

C turned positive when on the same side of the centre of twist, similar to eccentricity (e). As a result, the bents on the same side of the centre of twist (stiffness), as the resultant loading, had increment in shear due to twisting behaviour, while those on the opposite had reduced shear. Moreover, Fig. 3.32 depicts that bent (B) are the opposite of the centre of twist, similar to eccentricity (e), whereas bents (C) are on the same side of the centre of twist. The shear force in Storey (1) was distributed based on Eq. (3.38), for the distribution of shear force for Storey 1 was calculated as follows:

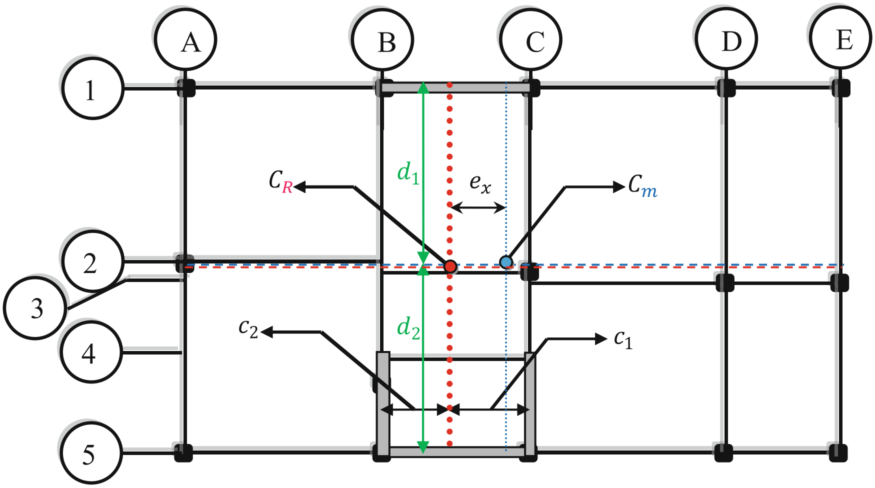


Fig. 3.31 Calculation of eccentricity on story one

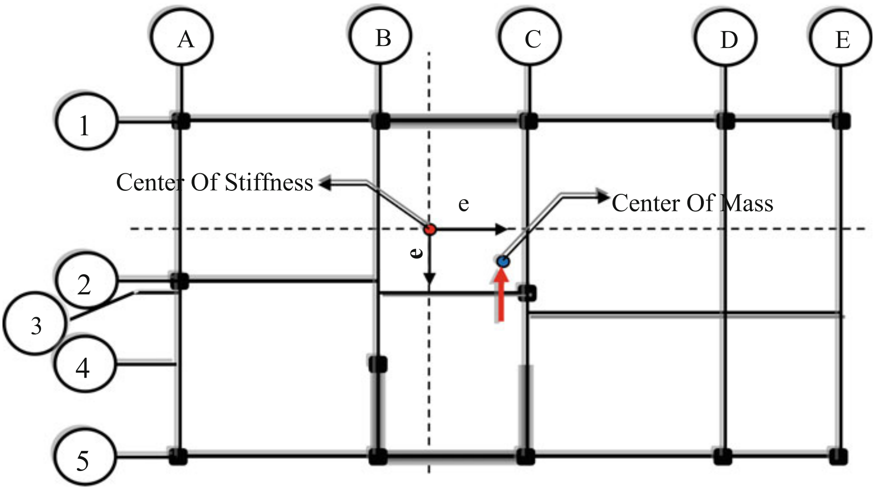


Fig. 3.32 Opposite and same side of center of twist

1. The shear force of Story 1 was 2548.13 (kN) for X and Y directions, based on Table 3.12.
2. The stiffness in X and Y directions are calculated based on Eq. (3.7), therefore:

   Stiffness of shear wall in Y direction were computed as:

$$K_{(w1)} = 1173787.461, \quad K_{(w2)} = 1173787.461$$

Stiffness of concrete shear wall in X direction had been computed as follows:

$$K_{(S1)} = 3961532.682, \quad K_{(S2)} = 3961532.682$$

Step 3: Eccentricities in X and Y directions were calculated using Eqs. (3.36) and (3.37). As a result, eccentricity in X direction (ex) was 1.369 (m), while this magnitude for y direction (e_Y) was 0.0674 (m). Moreover accidental eccentricity in X and Y direction is 0.85 (0.05 * 17) and 0.5 (0.05 * 10) respectively.

Step 4: Distance of each bent, in respect to the center of rigidity (C) in X and Y directions, had been calculated and the results are shown in Fig. 3.31.

1. (c_j) in X direction:

$$C_{1(B)} = 7.1 - 5.15 = 1.95, \quad C_{1(C)} = 9.05 - 7.1 = 1.95$$

2. (d_j) in Y direction:

$$d_{1(1)} = 9.8 - 4.9 = 4.9, \quad d_{2(5)} = 4.9$$

Step 5: Calculation of torsional stiffness (J).

There is a certain level of torsion resistance produced by each of the walls shown in the floor plan. This can be calculated by multiplying the lateral stiffness of a particular wall by the distance between the wall and the CR, either in the y or x direction. Lateral stiffness emerges as the wall bends about its strong axis while the said distance is measured perpendicular to the wall's weak axis. To resist significant torsion, widely distributed walls are the most suitable, as earlier mentioned. Taking into consideration all the walls as a unit in the floor system, an equivalent torsional stiffness is a sum of the torsion resisted by each of the walls, which is the product of the torsional resistance (for each wall) and the perpendicular distance of the particular wall to the CR. We express the torsional stiffness as shown below.

$$\begin{aligned} J = \sum K_x \times C^2 + \sum K_y \times d^2 \sum K_x \times C^2 &= \left[\left[(\mathrm{K_{W1}}) \times C_1^2\right] + \left[\left(K_{W2} \times C_2^2\right)\right]\right] \\ &= \left[(1173787.461) \times 1.95^2\right] + \left[\left(1173787.461 \times 1.95^2\right)\right] = 8926653.644 \end{aligned}$$

$$\begin{aligned} \sum K_y \times d^2 &= \left[\left[\left(K_{S1} \times d_1^2\right)\right] + \left[\left(K_{S2} \times d_2^2\right)\right]\right] \\ &= \left[(3961532.682) \times 4.9^2\right] + \left[\left(3961532.682 \times 4.9^2\right)\right] = 190232799.4 \end{aligned}$$

$$\boldsymbol{J}_1 = 8926653.644 + 190232799.4 = 199159453.04$$

The calculation of J is depicted in Charts 3.5 and 3.6 in detail. The total magnitude of J in each story is displayed in Table 3.19.

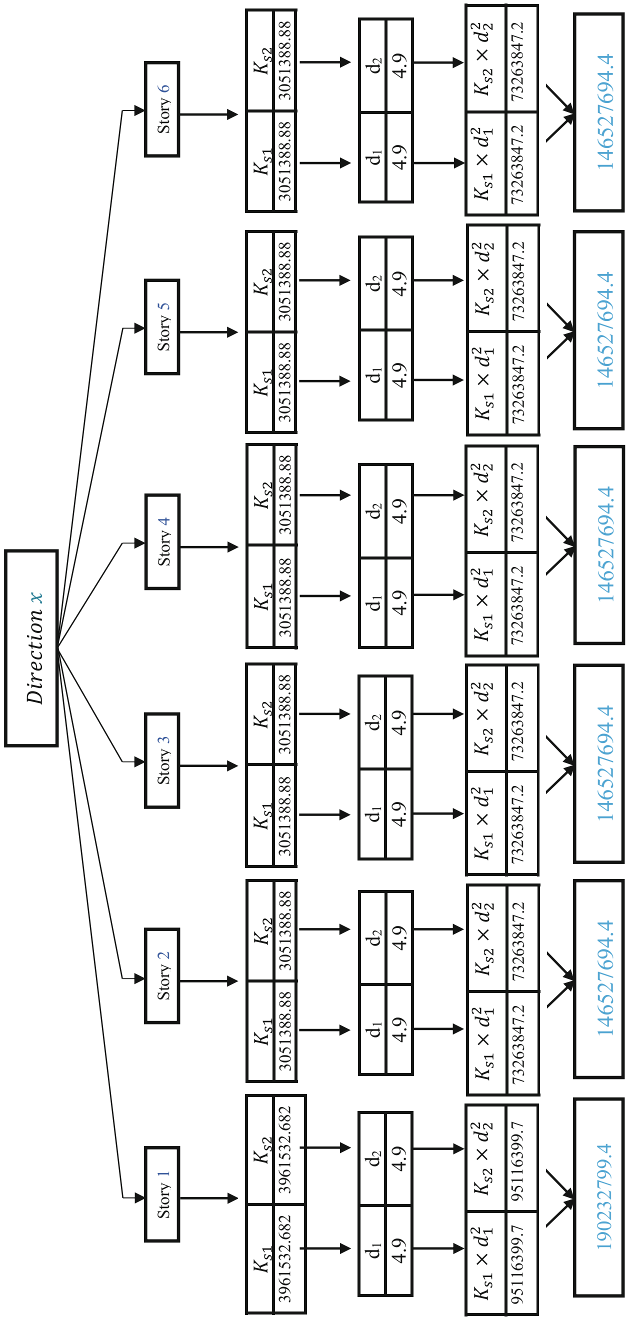


Chart 3.5 Procedure of calculation (J) in X direction

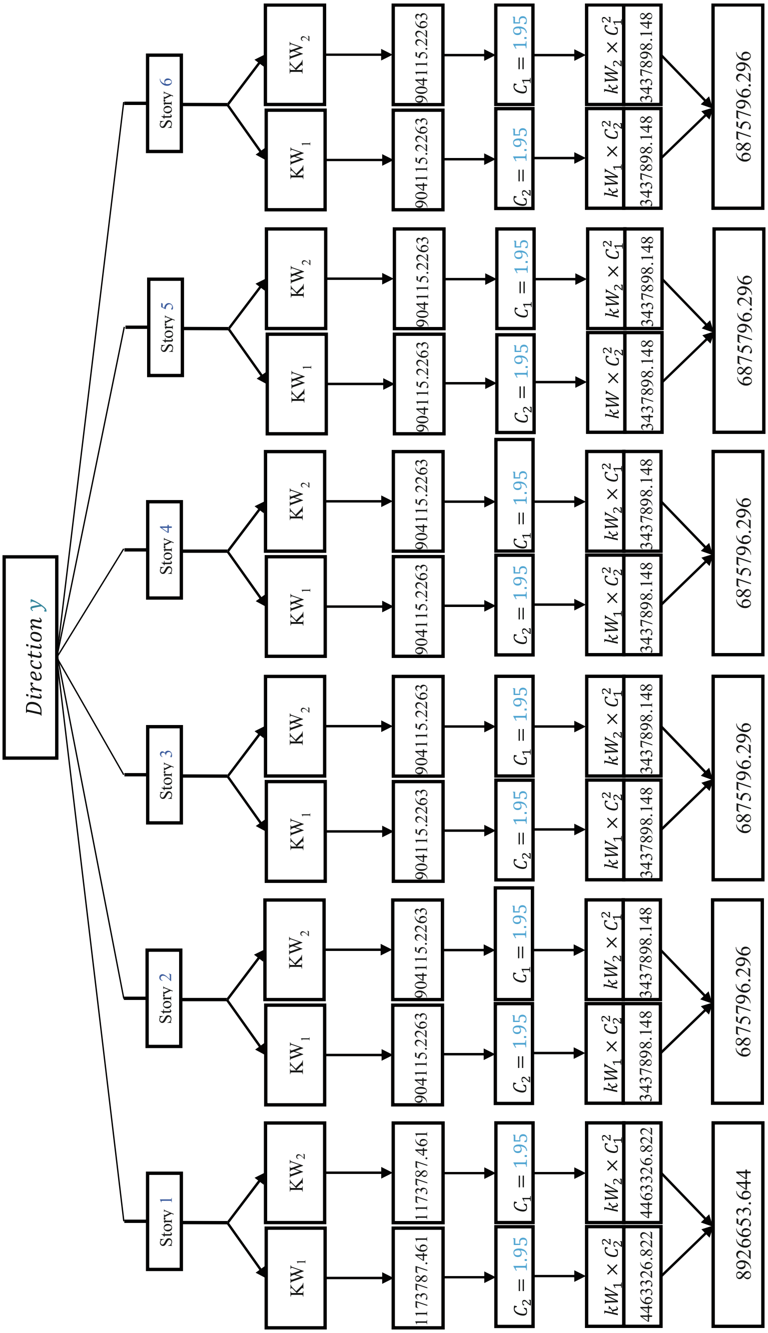


Chart 3.6 Procedure of calculation (J) in X direction

Table 3.19 Calculation of J in each story

Story	$\sum K_x \times d^2$	$\sum K_y \times C^2$	$J = \sum K_x \times C^2 + \sum K_y \times d^2$
Story one	190232799.4	8926653.644	199159453.043
Story two	146527694.4	6875796.296	153403490.741
Story three	146527694.4	6875796.296	153403490.741
Story four	146527694.4	6875796.296	153403490.741
Story five	146527694.4	6875796.296	153403490.741
Story six	146527694.4	6875796.296	153403490.741

Step 6: According to Eq. (3.38), the shear force of story one in Y direction had been distributed in bents (B) and (C), as follows

1. Bent (B):

$$Q_i = \left[\frac{(K)_i}{\sum (K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i$$

1.1. Total stiffness in Y direction in story one is summation stiffness of Two concrete shear wall frame along gridline (B) and (C).

$$\sum (K_{W1} + K_{W2}) = 1173787.461 + 1173787.461 = 2347574.92$$

1.2. Shear force in each frames of structure is distributed according to the stiffness of frames So in bent (B):

$$\frac{\sum (K_{W1})}{\sum (K_{W1} + K_{W2})} = \frac{1173787.46}{2347574.92} = 0.5$$

1.3. Distance of each bent respect to center of rigidity and eccentricity

$$C_1 = 1.95(\mathrm{m})$$

$$e_{(x)} = 1.369(\mathrm{m}) + 0.85(\mathrm{m}) = 2.219(\mathrm{m})$$

1.4. Calculation of J

$$\mathrm{J}_1 = 199159453.04$$

1.5. Based on Table 3.12 total shear base in story one is 2548.13 (kN) and according to Eq. (3.38), 1209.08 (kN) is transferred to the frame (B).

$$
\begin{aligned}
Q_i &= \left[\frac{(K)_i}{\sum (K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i \\
&= \left(0.5 - \frac{2.219 \times 1173787.46 \times 1.95}{199159453.04}\right)(2548.13) \\
&= 1209.08 \text{ kN}
\end{aligned}
$$

2. Bent (C):

$$
Q_i = \left[\frac{(K)_i}{\sum (K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i
$$

2.1. Stiffness of frame (C)

$$
\mathrm{K}_{(\mathrm{w2})} = 1173787.46
$$

$$
\sum (K_{W1} + K_{W2}) = 1173787.46 + 1173787.46 = 2347574.92
$$

2.2. Shear force in each frames of structure is distributed according to the stiffness of frames So in bent (c):

$$
\frac{\sum (K_{W2})}{\sum (K_{W1} + K_{W2})} = \frac{1173787.46}{2347574.92} = 0.5
$$

2.3. Distance of each bent respect to center of rigidity and eccentricity

$$
\begin{aligned}
C_2 &= 1.95 \\
e_{(x)} &= 2.219
\end{aligned}
$$

2.4. Calculation of J

$$
\mathrm{J}_1 = 199159453.04
$$

Based on Table 3.12 total shear base in story one is 2548.13 (kN) and according to following equation, the force of 1339.06 (kN) is transferred to the frame (C).

$$
\begin{aligned}
Q_i &= \left[\frac{(K)_i}{\sum (K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i \\
&= \left(0.5 + \frac{2.219 \times 1173787.46 \times 0.1.95}{199159453.04}\right)(2548.13) \\
&= 1339.06 \text{ k}
\end{aligned}
$$

The procedure of calculation torsional stiffness (J) in each floors is shown Charts 3.5 and 3.6.

The complete calculations for the distribution base shear in the X and Y direction scan be executed in a tabular form, as per the following table. Tables 3.20 and 3.21 show the distribution of the shear base in the Y direction in bents (B) and (C), respectively. As per Table 3.20, the shear base in the first floor of bent (B) is 1209.08 (kN) while it is 330.97 (kN) in the sixth floor. As can be seen in Table 3.21, the shear base in the Y direction of bent (C) in the first floor, (1) is 1339.06 (kN), reaching 363.8 (kN) at the sixth floor. Furthermore, Fig. 3.33a–c exhibited the distribution of shear base in Y direction in height, frame, and plan of structure, respectively. These figures illustrate a comprehensible concept of distribution of shear base in lateral resistance frame system. Tables 3.22 and 3.23 display the magnitude of shear base in X direction distributed in bents (1) and (5). However, a slight variance was noted between this magnitude, in respect to each other, due to the same stiffness of shear wall S_1 and S_2, where the shear force was equally distributed, while these magnitudes displayed some variances. Primarily, the aspect of eccentricity in the plan of structure caused such difference. Besides, it must be mentioned that in story 6, bent (1) was under compression, while bent (5) under tension, mainly because the center of stiffness (rigidity) in story (6) was located at 4.9 (m) and the centre of mass was located at 4.572 (m) (Doom roof caused the centre of mass in story (6) to move downwards). As mentioned in torsional force section, the resultant of resistance force was applied to the center of stiffness, while the resultant force was applied to the center of mass so as to create torsion in the plan. As a result, bent (1) was under compression, while bent (5) was under tension (story 6). While in other story, bent (1) was under tension and bent (5) was under compression due to the situation of the centers of mass and stiffness.

Figure 3.34 shows the distribution of shear base in height for structure in X direction. The distribution of shear base in X direction for frame and plan of the structure are shown in Fig. 3.34a, b, respectively.

Figures 3.35, 3.36, 3.37 show all of the procedures of the distribution of shear base in the building.

As per to Fig. 3.35a the base shear is calculated based on equation $(F_b = S_d(T) \times \lambda \times m)$ of the Euro code 8, then distributed to the height of structure using equation $\left(F_i = F_b \times \frac{Z_i \times m_i}{\sum Z_j \times m_j}\right)$, as shown in Fig. 3.35b. The shear of the floor is distributed according to the stiffness of each bent as per the equation $\left(Q_i = \left[\frac{(K)_i}{\sum (K)_i} \pm \frac{e \times [(K_i \times C)]_i}{J}\right] V_i\right)$, which distributes the shear floor to the lateral resistance shear wall system in the X and Y directions. Figure 3.36 shows the distribution of the shear floor in the lateral resistance shear wall system, while Fig. 3.37 shows the distributed shear floors in the frame, rendering them ready for the calculation of the lateral displacement under earthquake load.

Table 3.20 Distribution of shear base in Y direction in bent (B)

Story	$V(y)$	K_{W1}	$\sum K = K_{W1} + K_{W2}$	$K_{W1}/(\sum K)$	$e(x)$	C	J	$\left(\frac{K_{w1}}{\sum K} \pm \frac{K_{w1} e(x)(c)}{J}\right) V$	P
6	694.76	904115.23	1808230.45	0.50	2.0557	1.95	153403490.74	$\left(0.5 - \frac{9.04\times10^5\times2.0557\times1.95}{153403490.7}\right) V_6$	330.97
5	1321.35	904115.23	1808230.45	0.50	2.2213	1.95	153403490.74	$\left(0.5 - \frac{9.04\times10^5\times2.2213\times1.95}{153403490.7}\right) V_5$	626.94
4	1820.66	904115.23	1808230.45	0.50	2.2213	1.95	153403490.74	$\left(0.5 - \frac{9.04\times10^5\times2.2213\times1.95}{153403490.7}\right) V_4$	863.85
3	2192.78	904115.23	1808230.45	0.50	2.2213	1.95	153403490.74	$\left(0.5 - \frac{9.04\times10^5\times2.2213\times1.95}{153403490.7}\right) V_3$	1040.41
2	2437.67	904115.23	1808230.45	0.50	2.2214	1.95	153403490.74	$\left(0.5 - \frac{9.04\times10^5\times2.2214\times1.95}{153403490.7}\right) V_2$	1156.60
1	2548.13	1173787.46	2347574.92	0.50	2.2192	1.95	199159453.04	$\left(0.5 - \frac{1.1\times10^6\times2.2192\times1.95}{199159453.04}\right) V_1$	1209.08

Table 3.21 Distribution of shear base in Y direction in bent (C)

Story	$V(y)$	K_{W1}	$\sum K = K_{W1} + K_{W2}$	$K_{W2}/(\sum K)$	$e(x)$	C	J	$\left(\frac{K_{w2}}{\sum K} \pm \frac{K_{w2}e(x)(c)}{J}\right)V$	P
6	694.76	904115.23	1808230.45	0.50	2.0557	1.95	153403490.74	$\left(0.5 + \frac{9.04\times10^5\times2.0557\times1.95}{153403490.7}\right)V_6$	363.79
5	1321.35	904115.23	1808230.45	0.50	2.2213	1.95	153403490.74	$\left(0.5 + \frac{9.04\times10^5\times2.2213\times1.95}{153403490.7}\right)V_5$	694.41
4	1820.66	904115.23	1808230.45	0.50	2.2213	1.95	153403490.74	$\left(0.5 + \frac{9.04\times10^5\times2.2213\times1.95}{153403490.7}\right)V_4$	956.81
3	2192.78	904115.23	1808230.45	0.50	2.2213	1.95	153403490.74	$\left(0.5 + \frac{9.04\times10^5\times2.2213\times1.95}{153403490.7}\right)V_3$	1152.37
2	2437.67	904115.23	1808230.45	0.50	2.2214	1.95	153403490.74	$\left(0.5 + \frac{9.04\times10^5\times2.2214\times1.95}{153403490.7}\right)V_2$	1281.07
1	2548.13	1173787.46	2347574.92	0.50	2.2192	1.95	199159453.04	$\left(0.5 + \frac{1.1\times10^6\times2.2192\times1.95}{199159453.04}\right)V_1$	1339.06

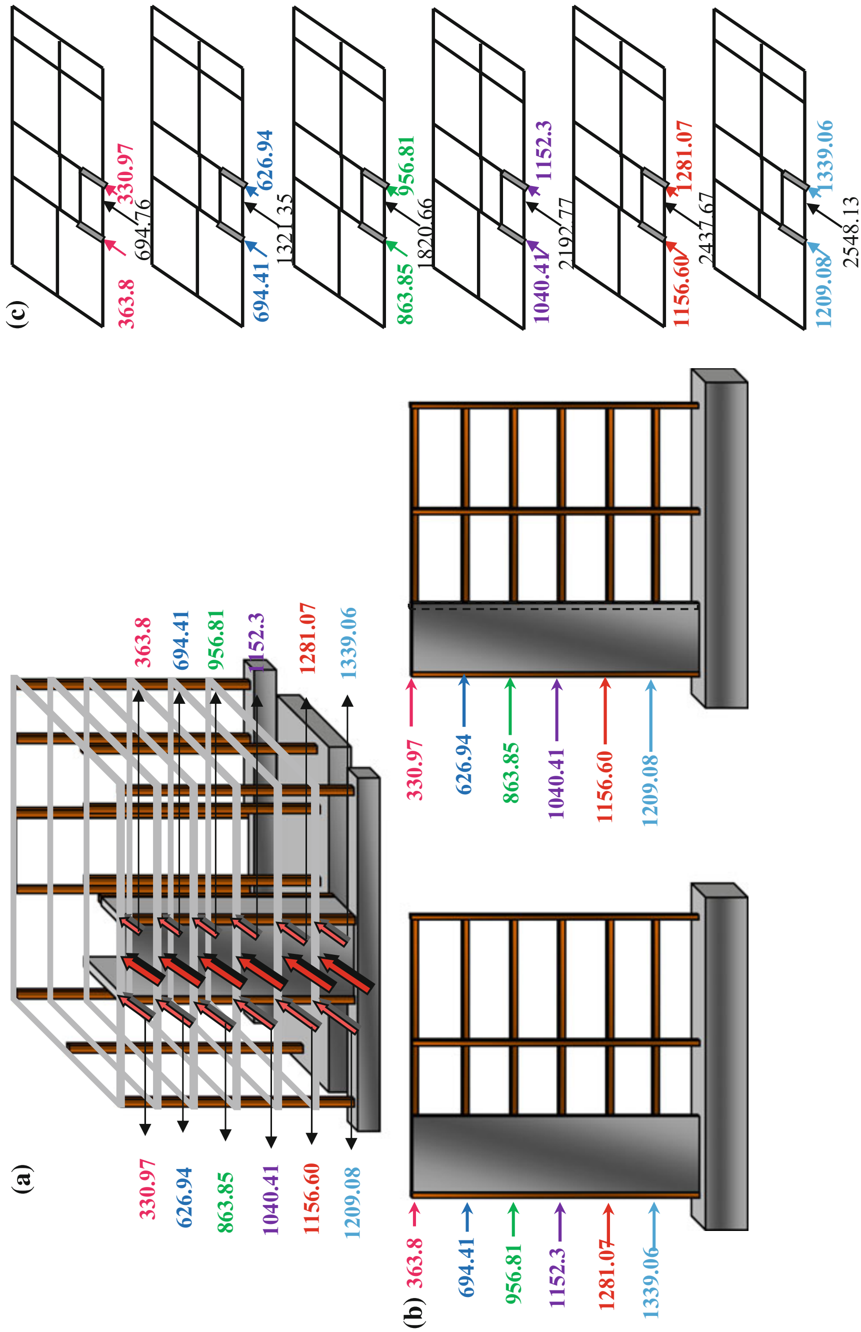


Fig. 3.33 Distribution of shear base in Y direction. **a** Height of building (story), **b** frame (B), (C), **c** plan

Table 3.22 Distribution of shear base in X direction in bent (5)

	$V(x)$	K_{S1}	$\sum K = K_{S1} + K_{S2}$	$K_{S1}/(\sum K)$	e(y)	C	J	$\left(\frac{K_{S1}}{\sum K} \pm \frac{K_{S1}e(y)(c)}{J}\right)V$	P
6	694.76	3051388.89	6102777.78	0.50	−0.8277	4.90	153403490.74	$\left(0.5 - \frac{3.04\times10^6\times-0.8277\times4.9}{153403490.74}\right)V_6$	403.43
5	1321.35	3051388.89	6102777.78	0.50	0.5745	4.90	153403490.74	$\left(0.5 - \frac{3.04\times10^6\times0.5745\times4.9}{153403490.74}\right)V_5$	586.68
4	1820.66	3051388.89	6102777.78	0.50	0.5745	4.90	153403490.74	$\left(0.5 - \frac{3.04\times10^6\times0.5745\times4.9}{153403490.74}\right)V_4$	808.37
3	2192.78	3051388.89	6102777.78	0.50	0.5745	4.90	153403490.74	$\left(0.5 - \frac{3.04\times10^6\times0.5745\times4.9}{153403490.74}\right)V_3$	973.58
2	2437.67	3051388.89	6102777.78	0.50	0.5749	4.90	153403490.74	$\left(0.5 - \frac{3.04\times10^6\times0.5749\times4.9}{153403490.74}\right)V_2$	1082.24
1	2548.13	3961532.68	7923065.36	0.50	0.5673	4.90	199159453.04	$\left(0.5 - \frac{3.9\times10^6\times0.573\times4.9}{199159453.04}\right)V_1$	1133.15

Table 3.23 Distribution of shear base in X direction in bent (1)

Story	V(x)	K_{S2}	$\sum K = K_{S1} + K_{S2}$	$K_{S2}/(\sum K)$	e(y)	C	J	$\left(\frac{K_{S2}}{\sum K} \pm \frac{K_{S2} e(y)(c)}{J}\right) V$	P
6	694.76	3051388.89	6102777.78	0.5	−0.8277	4.9	153403490.74	$\left(0.5 + \frac{3.04\times10^6\times-0.8527\times4.9}{153403490.74}\right)V_6$	291.44
5	1321.35	3051388.89	6102777.78	0.5	0.5745	4.9	153403490.74	$\left(0.5 + \frac{3.04\times10^6\times0.5745\times4.9}{153403490.74}\right)V_5$	734.52
4	1820.66	3051388.89	6102777.78	0.5	0.5745	4.9	153403490.74	$\left(0.5 + \frac{3.04\times10^6\times0.5745\times4.9}{153403490.74}\right)V_4$	1012.09
3	2192.78	3051388.89	6102777.78	0.5	0.5745	4.9	153403490.74	$\left(0.5 + \frac{3.04\times10^6\times0.5745\times4.9}{153403490.74}\right)V_3$	1218.95
2	2437.67	3051388.89	6102777.78	0.5	0.5749	4.9	153403490.74	$\left(0.5 + \frac{3.04\times10^6\times0.5749\times4.9}{153403490.74}\right)V_2$	1355.17
1	2548.13	3961532.68	7923065.36	0.5	0.5673	4.9	199159453.04	$\left(0.5 + \frac{3.9\times10^6\times0.573\times4.9}{199159453.04}\right)V_1$	1414.70

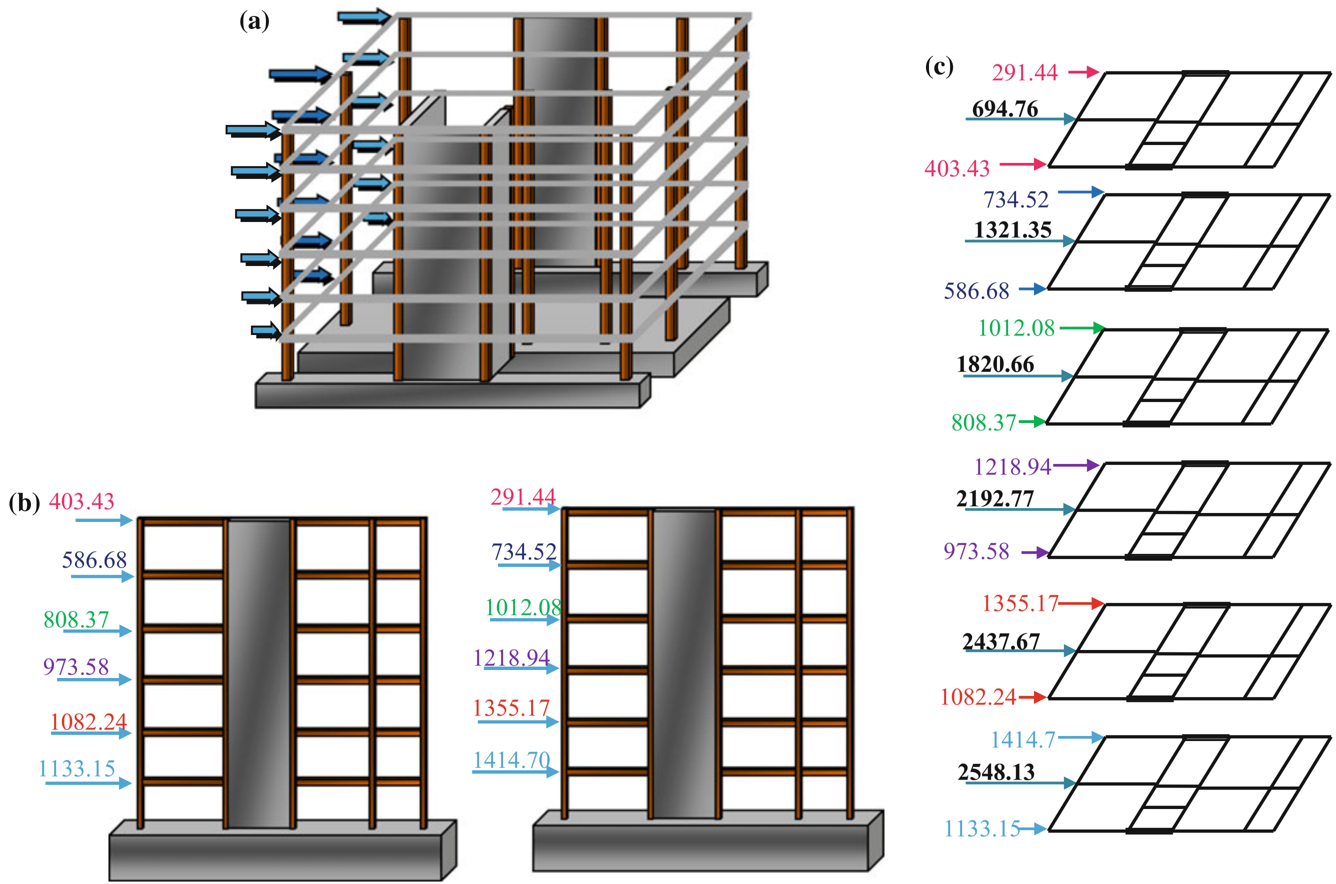


Fig. 3.34 Distribution of shear base in X direction. **a** Height of building (story), **b** frame (B), (C), **c** plan

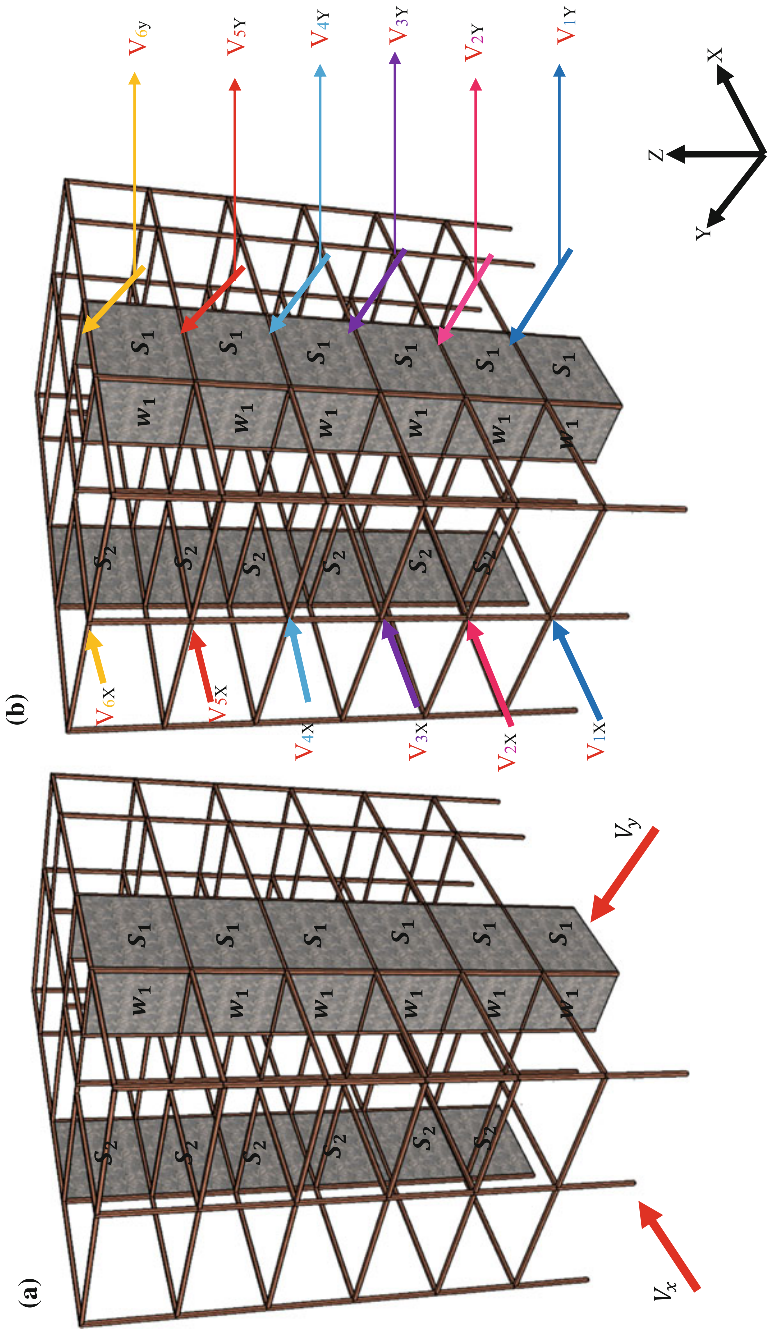


Fig. 3.35 The procedure of distribution of seismic load in the structural member procedure

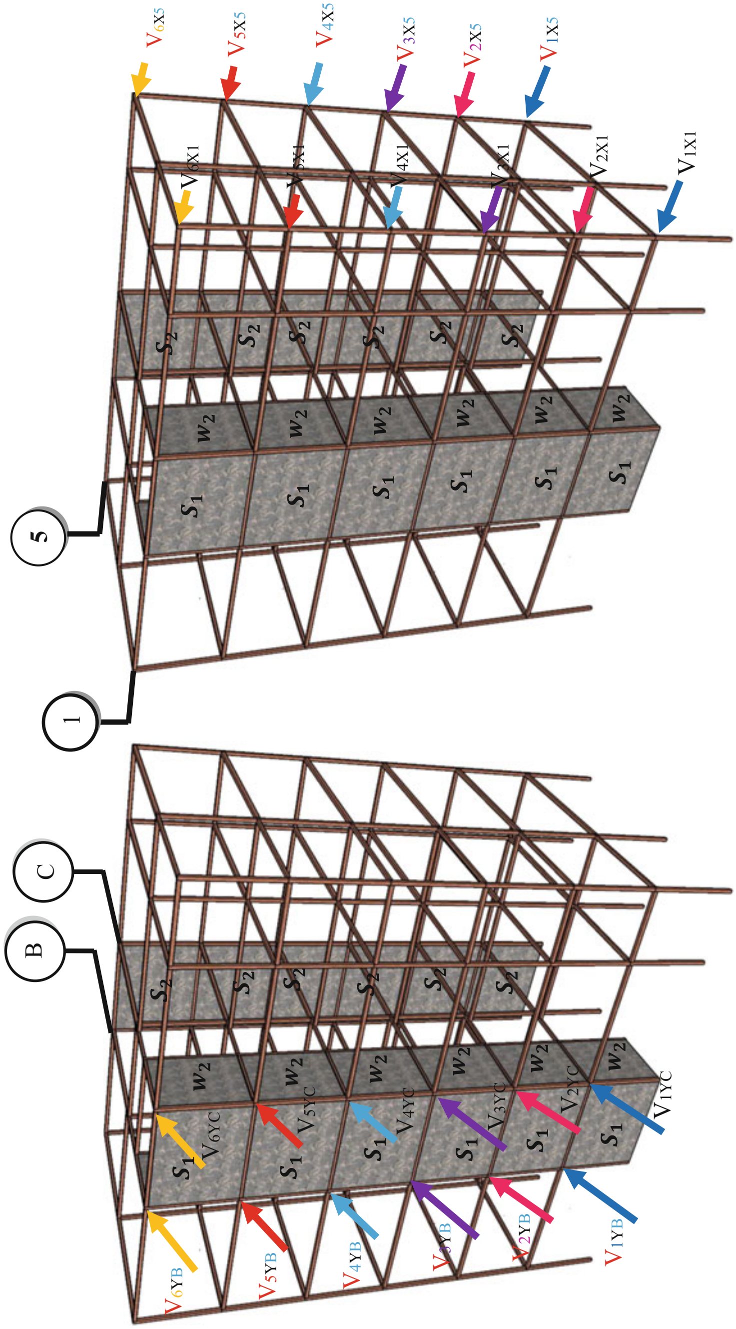


Fig. 3.36 The procedure of distribution of seismic load in the structural member procedure

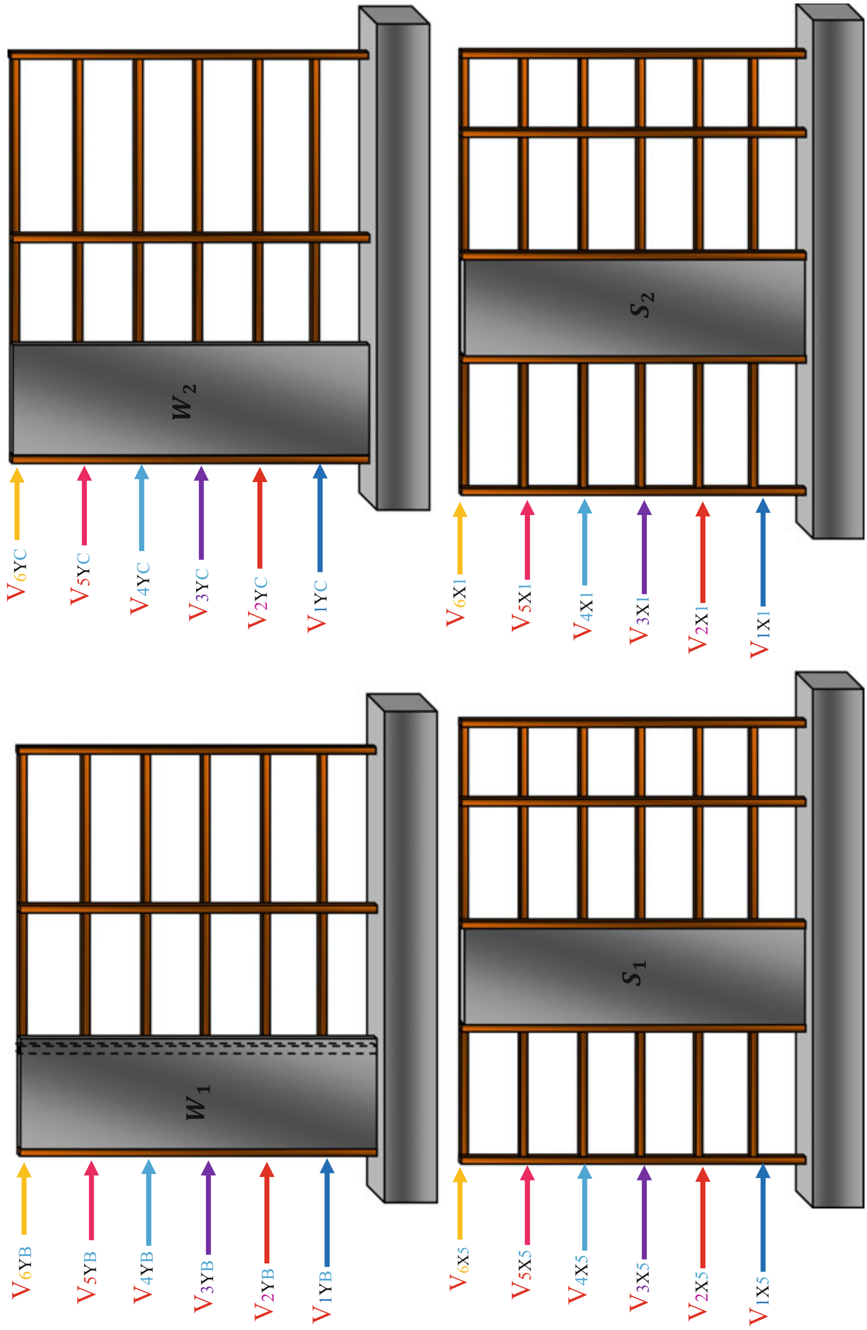


Fig. 3.37 The procedure of distribution of seismic load in the structural member procedure

3.4 Displacement Analysis

3.4.1 Introduction

An approximate calculation of the displacement can be made by using the moment-area method to obtain the flexural component (i.e., the component resulting from column axial deformations) and by applying a shear deflection formula to calculate the shear component. As it is mentioned before, the concrete shear wall acts as cantilever beam. Deflection of cantilever beam under concentrated load can be calculated according to structure analysis. Hence;

$$\textit{Total deflection of beam}: \Delta = \Delta_f + \Delta_s = \frac{PL^3}{3EI} + \frac{PL}{GA_s} \tag{3.40}$$

The procedures of calculation flexure and shear displacement components are explained in detail in section four of second chapter.

As for both centroid and moment inertias of column of frame (B), Storey (6) was calculated based on Eqs. (3.41) and (3.42) (Smith et al. 1991).

$$C = \frac{\sum A_i \bar{x}_i}{\sum A} \tag{3.41}$$

$$I = \sum A_i C_i^2 \tag{3.42}$$

$$C_6 = \frac{(2 \times 0.00164 \times 2.6) + (2 \times 0.00164 \times 9.8)}{(6 \times 0.00164)} = 4.13$$

$$\begin{aligned} \mathrm{I} = \sum (I_x + AC^2) &= \left[\left(1.08 \times 10^{-5}\right) + (0.00328)(4.13)^2\right] \\ &+ \left[\left(1.08 \times 10^{-5}\right) + (0.00328)(1.53)^2\right] \\ &+ \left[\left(1.08 \times 10^{-5}\right) + (0.00328)(5.67)^2\right] = 0.1690\ \mathrm{m}^4 \end{aligned}$$

Centroid and second moment inertias of frame (B) in all six stories had been similar, this is because of that the columns section area had been similar in all stories. According to Eqs. (3.41) and (3.42) the moment inertia and centroid can be computed as follows (Fig. 3.38):

$$C_{1,2,3,4,5,6} = \frac{(2 \times 0.00164 \times 2.6) + (2 \times 0.00164 \times 9.8)}{(6 \times 0.00164)} = 4.13$$

$$\begin{aligned} \mathrm{I} = \sum (I_x + AC^2) &= \left[\left(1.08 \times 10^{-5}\right) + (0.00328)(4.13)^2\right] \\ &+ \left[\left(1.08 \times 10^{-5}\right) + (0.00328)(1.53)^2\right] \\ &+ \left[\left(1.08 \times 10^{-5}\right) + (0.00328)(5.67)^2\right] = 0.1690\ \mathrm{m}^4 \end{aligned}$$

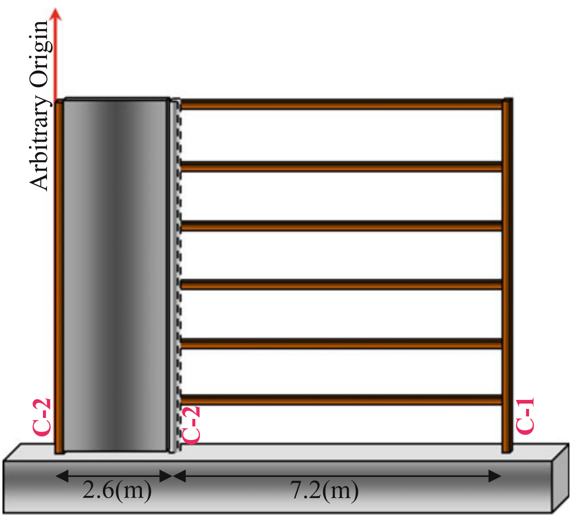


Fig. 3.38 Frame (B)

$$
\begin{aligned}
\mathrm{I} = \sum (I_x + AC^2) = & \left[(0.00328)(4.13)^2\right] \\
& + \left[(0.00328)(1.53)^2\right] \\
& + \left[(0.00328)(5.67)^2\right] = 0.1690\ \mathrm{m}^4
\end{aligned}
$$

1. The value of external moment M at mid-height of each story is calculated. Figure 3.39 shows the value of the external moment at mid-height of each story.

$$
\begin{aligned}
& M_6 = 330.96 \times \frac{3.6}{2} = 595.73 \\
& M_5 = (330.96 \times 5.4) + (295.97 \times 1.8) = 2319.97 \\
& M_4 = (330.96 \times 9) + (295.97 \times 5.4) + (236.9 \times 1.8) = 5003.39 \\
& M_3 = (330.96 \times 12.6) + (295.97 \times 9) + (236.9 \times 5.4) + (176.55 \times 1.8) = 8431.06 \\
& M_2 = (330.96 \times 16.2) + (295.97 \times 12.6) + (236.9 \times 9) + (176.55 \times 5.4) + (116.19 \times 1.8) = 12385.68 \\
& M_1 = (330.96 \times 19.8) + (295.97 \times 16.2) + (236.9 \times 12.6) + (176.55 \times 9) + (116.19 \times 5.4) \\
& \qquad + (52.47 \times 1.8) = 16462.54
\end{aligned}
$$

2. The value of $\frac{Mh}{EI}$ is determined for each story. These values reflect the changes in the inclination of each story I caused by flexure $\delta\theta_{if}$. It is noteworthy to mention its cumulative attribute, hence:

$$
\theta_{i,1} = \frac{M_{1\times} h_1}{E \times I_1} = \frac{16462.54 \times 3.3}{2 \times 10^8 \times 0.1690} = 0.00160255
$$

$$
\theta_{i,2} = \frac{M_{2\times} h_2}{E \times I_2} = \frac{12385.68 \times 3.6}{2 \times 10^8 \times 0.1690} = 0.0013152
$$

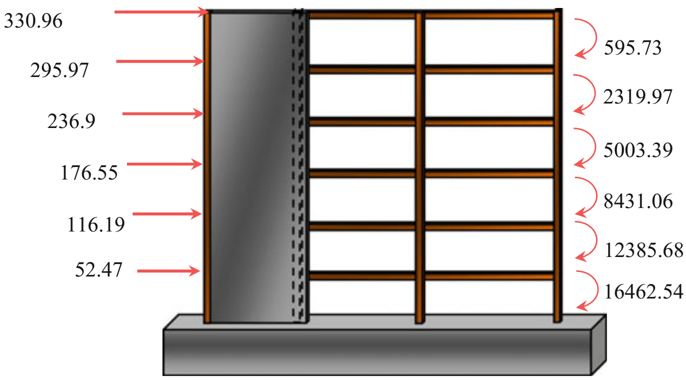


Fig. 3.39 External moment of frame (B)

$$\theta_{if,2} = \theta_{i,2} + \theta_{i,1=}0.00291784$$

$$\theta_{i,3} = \frac{M_{3\times}h_3}{E \times I_3} = \frac{8431.0611 \times 3.6}{2 \times 10^8 \times 0.1690} = 0.00089533$$

$$\theta_{if,3=}\theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.0038131$$

$$\theta_{i,4} = \frac{M_{4\times}h_4}{E \times I_4} = \frac{5003.39 \times 3.6}{2 \times 10^8 \times 0.1695} = 0.00053133$$

$$\theta_{if,4} = \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.0043445$$

$$\theta_{i,5} = \frac{M_{5\times}h_5}{E \times I_5} = \frac{2319.97 \times 3.6}{2 \times 10^8 \times 0.1695} = 0.00024637$$

$$\theta_{if,5} = \theta_{i,5} + \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.0045909$$

$$\theta_{i,6} = \frac{M_{6\times}h_6}{E \times I_6} = \frac{595.73 \times 3.6}{2 \times 10^8 \times 0.1695} = 6.3261\text{E} - 05$$

$$\theta_{if,6} = \theta_{i,6} + \theta_{i,5} + \theta_{i,4} + \theta_{i,3} + \theta_{i,2} + \theta_{i,1} = 0.0046541$$

3. Calculation of displacement in story i, $\delta_{if} = \theta_{if} h_i$

$$\delta_{6f} = 0.004653883 \times 3.6 = 0.0167549$$

$$\delta_{5f} = 0.004590623 \times 3.6 = 0.0165272$$

$$\delta_{4f} = 0.004344267 \times 3.6 = 0.0156402$$

$$\delta_{3f} = 0.003812962 \times 3.6 = 0.0137274$$

$$\delta_{2f} = 0.002917678 \times 3.6 = 0.0105042$$

$$\delta_{1f} = 0.00160246 \times 3.3 = 0.0052884$$

5. At each level, where the value of the lateral displacement is required, evaluate the accumulation of the story displacement from Storey 1 up to the target, nth floor, to give the displacement due to flexure. Table 3.24 shows the total displacement of flexure component of bent (B) in detail

The calculation of shear components of displacement had been determined as follows:

1. Calculate the value of the external shear that functioned in each Storey 1. The external shear of bent (B) was calculated and the results are shown in Table 3.20.
2. Calculate for the story displacement due to shear by substituting the value of the story shear and the member properties into the suitable formula derived from Table 3.25.
3. Sum the story displacements based on shear component. The total shear displacement was calculated and the results are shown in Table 3.25.

The total displacement of frames (B) is shown in Table 3.26, whereas Graph 3.1 exhibits flexure, shear, and total displacement of frame (B).

Table 3.24 Evaluation of flexure component of displacement

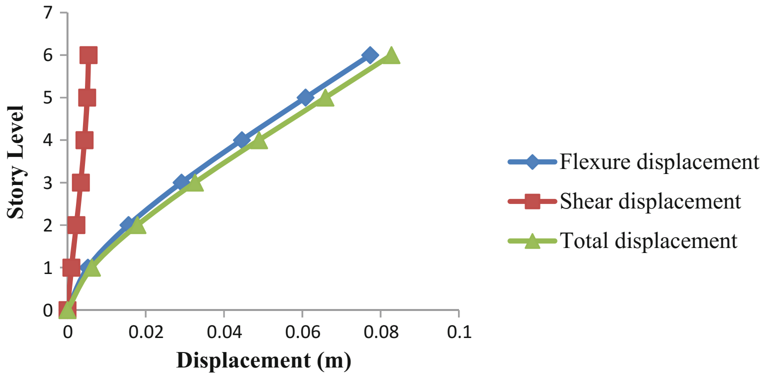
Story	h (m)	I_i (m^4)	E	Force	Moment	θ_i	θ_{if}	δ_{if}	Δ
6	3.6	0.1695	2E+8	330.9652	595.7373	0.0000633	0.0046541	0.0167549	0.0784
5	3.6	0.1695	2E+8	295.9779	2319.9720	0.0002464	0.0045909	0.0165272	0.0617
4	3.6	0.1695	2E+8	236.9064	5003.3985	0.0005313	0.0043445	0.0156402	0.0452
3	3.6	0.1695	2E+8	176.5580	8431.0611	0.0008953	0.0038132	0.0137274	0.0295
2	3.6	0.1695	2E+8	116.1957	12385.6803	0.0013153	0.0029178	0.0105042	0.0158
1	3.3	0.1695	2E+8	52.4737	16462.5428	0.0016025	0.0016025	0.0052884	0.0053

Table 3.25 Evaluation of shear component of displacement

Story	Q	I	h	E	K	δ	Δ
6	330.97	0.4394	3.60	32000000	904115.23	0.00037	0.00548
5	626.94	0.4394	3.60	32000000	904115.23	0.00069	0.00511
4	863.85	0.4394	3.60	32000000	904115.23	0.00096	0.00442
3	1040.41	0.4394	3.60	32000000	904115.23	0.00115	0.00346
2	1156.60	0.4394	3.60	32000000	904115.23	0.00128	0.00231
1	1209.08	0.4394	3.30	32000000	1173787.46	0.00103	0.00103

Table 3.26 Evaluation Total displacement of frame (B)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.0784	0.0055	0.0839
5	0.0617	0.0051	0.0668
4	0.0452	0.0044	0.0496
3	0.0295	0.0035	0.0330
2	0.0158	0.0023	0.0181
1	0.0053	0.0010	0.0063



Graph 3.1 Different components of displacement frame (B)

3.5 Appendix

I. Section Property
II. Weight and Center Mass of Stories
III. Total Displacement of frames

3.5.1 Section Property

Properties of different section of columns are computed in this part. Cross-section area, second moment inertia in X and Y direction, section modulus in X and y direction and radius of gyration are calculated for each section. Properties of single IPE 140 are shown in Tables 3.27, 3.28, 3.29, 3.30 and 3.31; Figs. 3.40, 3.41, 3.42, 3.43.

3.5.2 Weigh and Centre of Mass Stories

Total weight of stories 2–6 is illustrated in Charts 3.7, 3.8, 3.9, 3.10, 3.11 and 3.12, moreover the center of mass of each level is shown in Tables 3.32, 3.33, 3.34, 3.35 and 3.36.

3.5.3 Displacement of Frames

The total displacement of frames (C) had been composed of two components: flexure displacement and shear displacement. The calculation of displacement of frame (C) is similar to that of displacement of frame (B).

Flexural Displacement:

1. Compute the column centroid and the moment inertia (Fig. 3.44)

$$\mathrm{C}_{5,6,4} = \frac{(2 \times 0.00164 \times 4.6) + (2 \times 0.00164 \times 9.8)}{(6 \times 0.00164)} = 4.8$$

$$C_{3,2} = \frac{(3 \times 0.00164 \times 4.6) + (2 \times 0.00164 \times 9.8)}{(7 \times 0.00164)} = 4.771$$

$$\mathrm{C}_1 = \frac{(3 \times 0.00164 \times 4.6) + (2 \times 0.17 \times 0.008 \times 4.6) + (2 \times 0.00164 \times 9.8)}{(7 \times 0.00164) + (2 \times 0.17 \times 0.008)} = 4.74$$

Table 3.27 Properties of IPE 140

IPE	h_{mm}	b_{mm}	s_{mm}	t_{mm}	r_{mm}	c_{mm}	h-2c	A_{cm^2}	$G_{kg/m}$	Ix_{cm^4}	Sx_{cm^3}	lx_{cm}	Iy_{cm^4}	Sy_{cm^3}	ly_{cm}	$a1_{mm}$	rt_{mm}
140	140	73	4.7	6.9	9	16.4		16.4	12.9	541	77.30	5.74	44.9	12.3	1.65	129	21.7

Table 3.28 Properties of 2IPE140

Properties of 2IPE140	
A (cm²)	2 × 16.4 = 32.8 cm²
$I_x = \sum I_x + Ad^2 (cm^4)$	541 × 2 = 1082 cm⁴
$I_y = \sum I_y + Ad^2 (cm^4)$	$(44.9 + [16.4 \times (3.65^2)]) + (44.9 + [16.4 \times (3.65^2)]) = 526.778\, cm^4$
$S_x = \frac{I_x}{C} (cm^3)$	$\frac{1082}{7} = 154.571\ cm^3$
$S_y = \frac{I_y}{C} (cm^3)$	$\frac{526.778}{7.3} = 72.1614\ cm^3$
$r_x = \sqrt{\frac{I_x}{A}} (cm)$	$\sqrt{\frac{1082}{32.8}} = 5.7435\ cm$
$r_y = \sqrt{\frac{I_y}{A}} (cm)$	$\sqrt{\frac{526.778}{32.8}} = 4.00753\ cm$

Table 3.29 Properties of 2IPE140c14

Properties of 2IPE140c14	
A (cm²)	2 × 16.4 = 32.8 cm²
$I_x = \sum I_x + Ad^2 (cm^4)$	541 × 2 = 1082 cm⁴
$I_y = \sum I_y + Ad^2\ (cm^4)$	$(44.9 + [16.4 + (7.15^2)]) + (44.9 + [16.4 + (7.15^2)]) = 1766.618\ cm^4$
$S_x = \frac{I_x}{C}\ (cm^3)$	$\frac{1082}{7} = 154.571\ cm^3$
$S_y = \frac{I_y}{C}\ (cm^3)$	$\frac{1766.618}{10.8} = 163.576\ cm^3$
$r_x = \sqrt{\frac{I_x}{A}}\ (cm)$	$\sqrt{\frac{1082}{32.8}} = 5.7435\ cm$
$r_y = \sqrt{\frac{I_y}{A}}\ (cm)$	$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1766.618}{32.8}} = 7.33896\ cm$

Table 3.30 Properties of 3IPE140

Properties of 3IPE140	
A (cm²)	3 × 16.4 = 49.2 cm²
$I_x = \sum I_x + Ad^2\ (cm^4)$	541 × 3 = 1623 cm⁴
$I_y = \sum I_y + Ad^2\ (cm^4)$	$(44.9 + [16.4 + (7.3^2)]) + (44.9 + [16.4 + (7.3^2)]) + (44.9) = 1882.612\ cm^4$
$S_x = \frac{I_x}{C}\ (cm^3)$	$\frac{1623}{7} = 231.857\ cm^3$
$S_y = \frac{I_y}{C}\ (cm^3)$	$\frac{1882.612}{10.95} = 171.928\ cm^3$
$r_x = \sqrt{\frac{I_x}{A}}\ (cm)$	$\sqrt{\frac{1623}{49.2}} = 5.7435\ cm$
$r_y = \sqrt{\frac{I_y}{A}}\ (cm)$	$\sqrt{\frac{1882.612}{49.2}} = 6.18583\ cm$

Table 3.31 Properties of 3IPE140TBPL170×8

Properties of 3IPE140TBPL170×8	
A (cm^2)	$(3 \times 16.4) + [2 \times (17 \times 0.8)] = 76.4\ \text{cm}^2$
$\text{I}_\text{x} = \sum I_x + Ad^2(\text{cm}^4)$	$(541 \times 3) + \left[2 \times \left[\left(\frac{17 \times 0.8^3}{12}\right) + ((17 \times 0.8)(7.4^2))\right]\right] = 3113.92\ \text{cm}^4$
$\text{I}_\text{y} = \sum I_y + Ad^2(\text{cm}^4)$	$\left(\frac{0.8 \times 17^3}{12}\right) + [44.9 + [16.4 \times (7.3^2)]] + [44.9 + [16.4 \times (7.3^2)]] + \left(\frac{0.8 \times 17^3}{12}\right) + (44.9) = 2537.6786\ \text{cm}^4$
$S_x = \frac{I_x}{C}(\text{cm}^3)$	$\frac{3113.92}{7.4} = 399.2205\ \text{cm}^3$
$S_y = \frac{I_y}{C}(\text{cm}^3)$	$\frac{2537.6786}{10.95} = 231.7514\ \text{cm}^3$
$r_x = \sqrt{\frac{I_x}{A}}(\text{cm})$	$\sqrt{\frac{3113.92}{76.4}} = 6.3842\ \text{cm}$
$r_y = \sqrt{\frac{I_y}{A}}(\text{cm})$	$\sqrt{\frac{2537.6786}{76.4}} = 5.7633\ \text{cm}$

Fig. 3.40 2IPE140

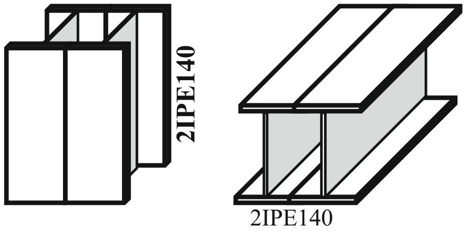
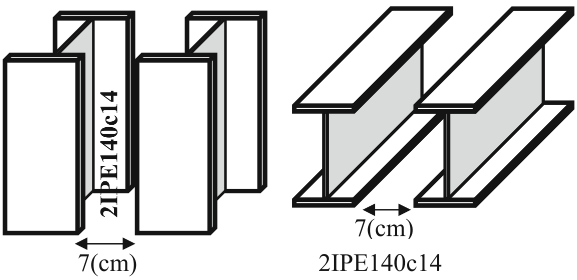


Fig. 3.41 2IPE140c14



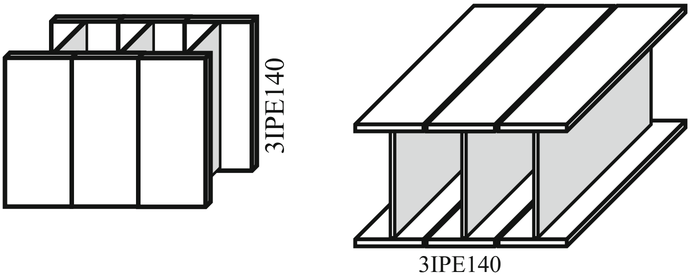


Fig. 3.42 3IPE140

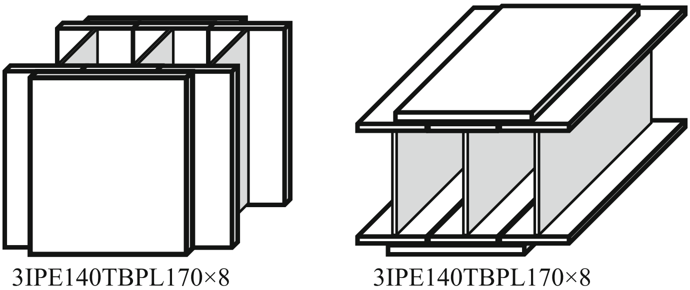


Fig. 3.43 3IPE140TBPL170×8

$$\begin{aligned}
I_{6,5,4} &= \sum\left(I_x + AC^2\right) = \left[\left(1.08\times10^{-5}\right) + (0.00328)(4.8)^2\right] \\
&\quad + \left[\left(1.08\times10^{-5}\right) + (0.00328)(0.2)^2\right] \\
&\quad + \left[\left(1.08\times10^{-5}\right) + (0.00328)(5)^2\right] = 0.1577\ \mathrm{m}^4 \\
I_{3,2} &= \sum\left(I_x + AC^2\right) = \left[\left(1.08\times10^{-5}\right) + (0.00328)(4.771)^2\right] \\
&\quad + \left[\left(1.623\times10^{-5}\right) + (0.00492)(0.171)^2\right] \\
&\quad + \left[\left(1.08\times10^{-5}\right) + (0.00328)(5.029)^2\right] = 0.15779\ \mathrm{m}^4 \\
I_{1} &= \sum\left(I_x + AC^2\right) = \left[\left(1.08\times10^{-5}\right) + (0.00328)(4.74)^2\right] \\
&\quad + \left[\left(3.1\times10^{-5}\right) + (0.0152)(0.14)^2\right] \\
&\quad + \left[\left(1.08\times10^{-5}\right) + (0.00328)(5.06)^2\right] = 0.1580\ \mathrm{m}^4
\end{aligned}$$

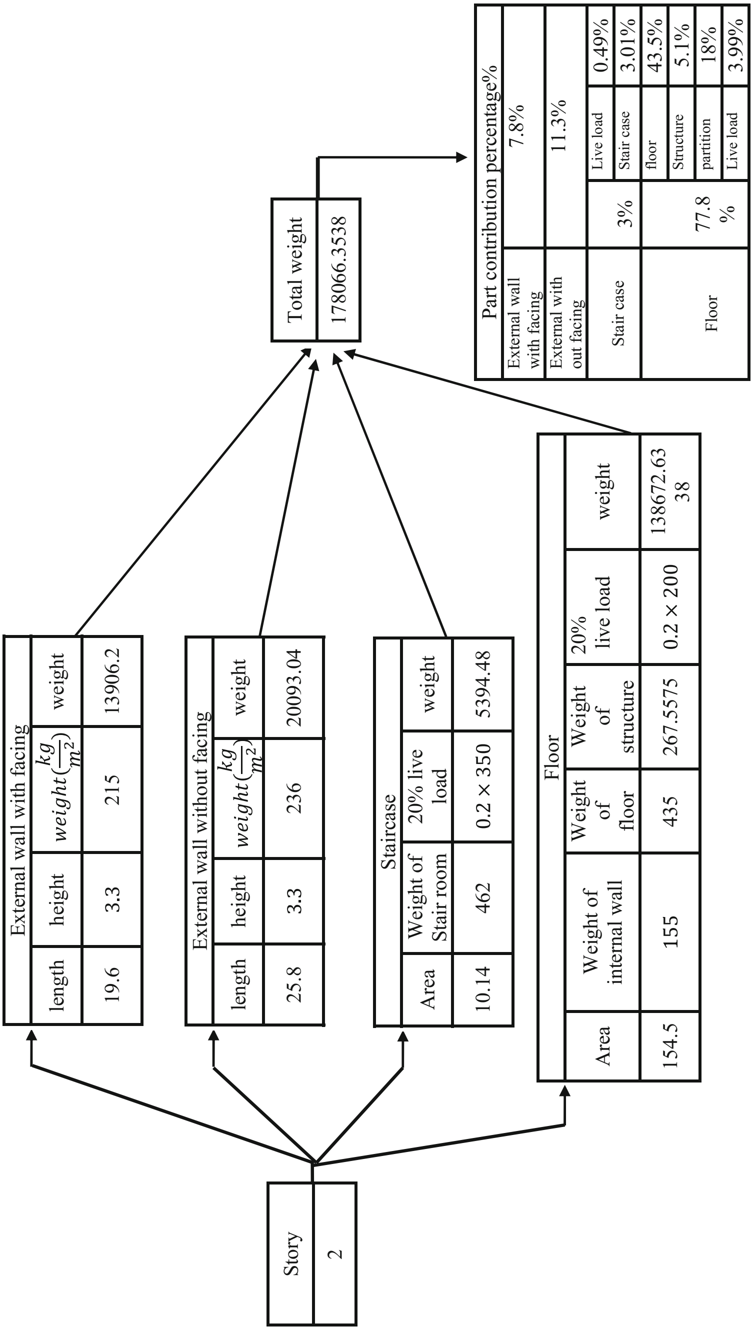


Chart 3.7 Total weight of story two

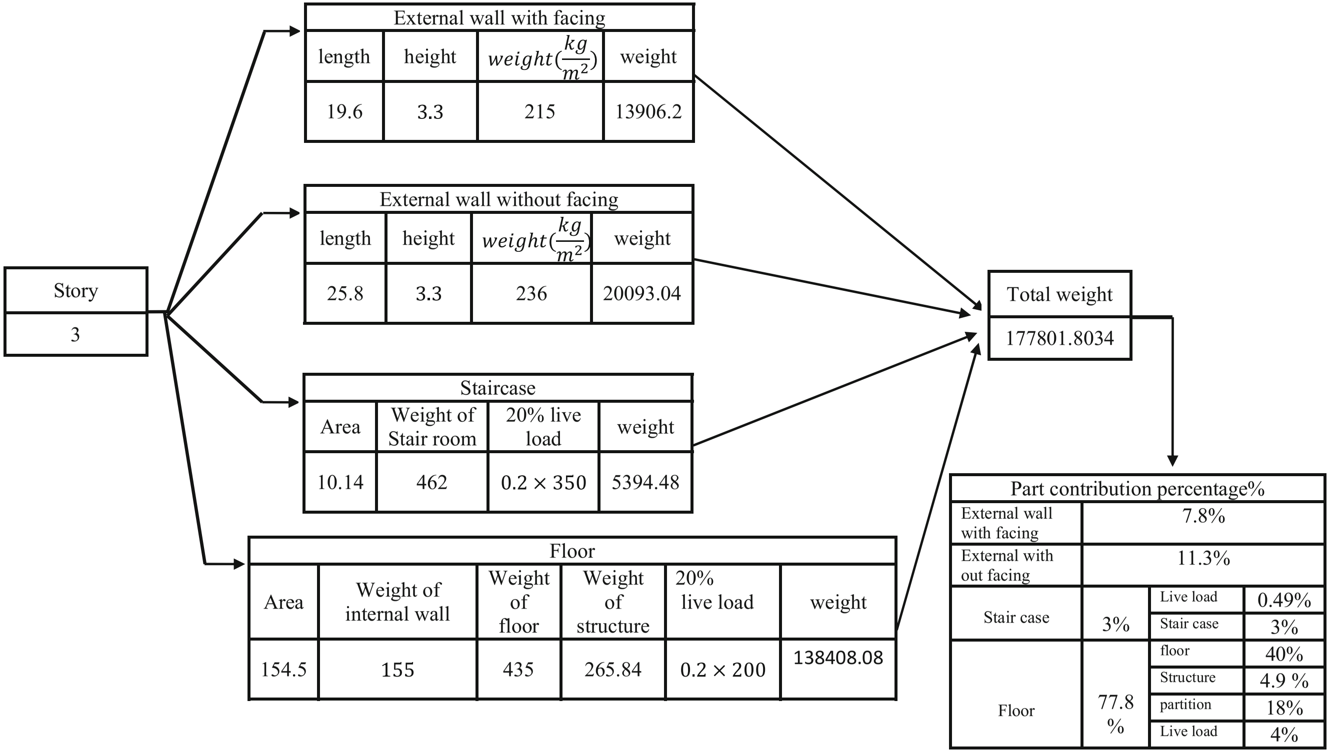


Chart 3.8 Total weight of story three

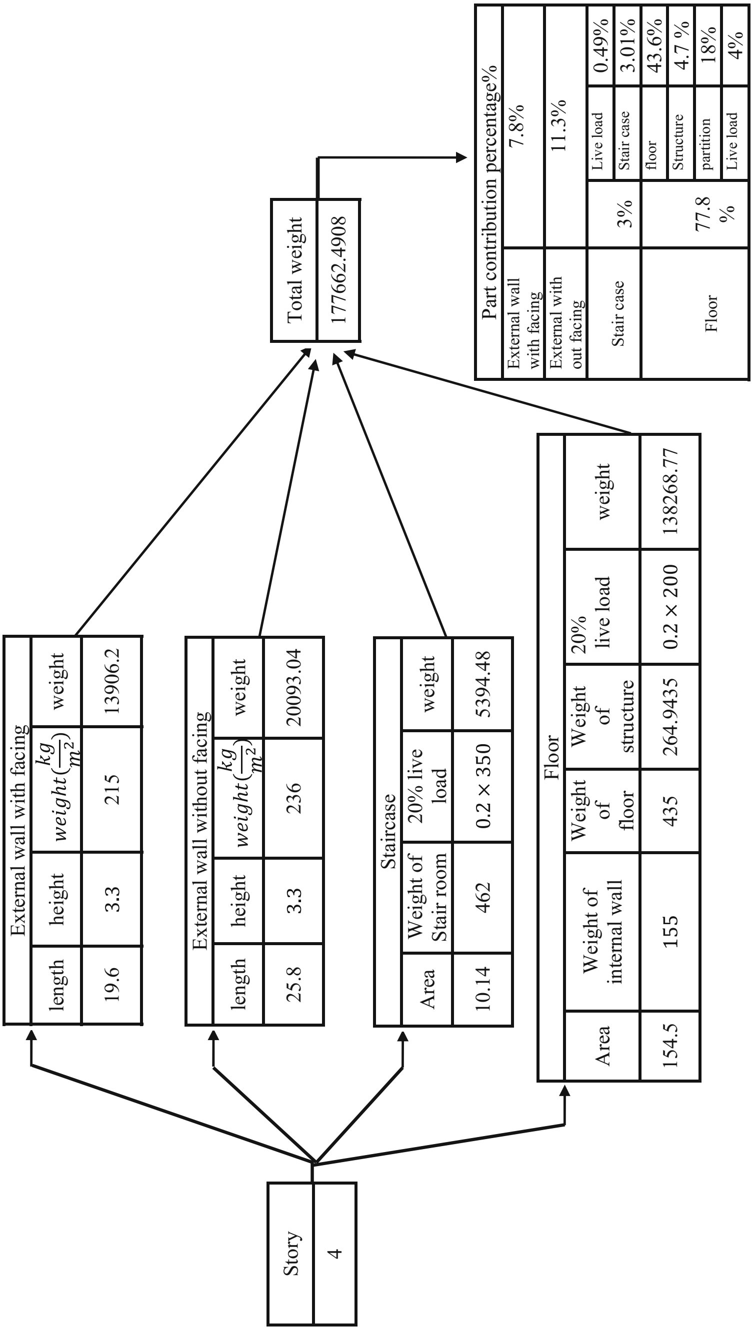


Chart 3.9 Total weight of story four

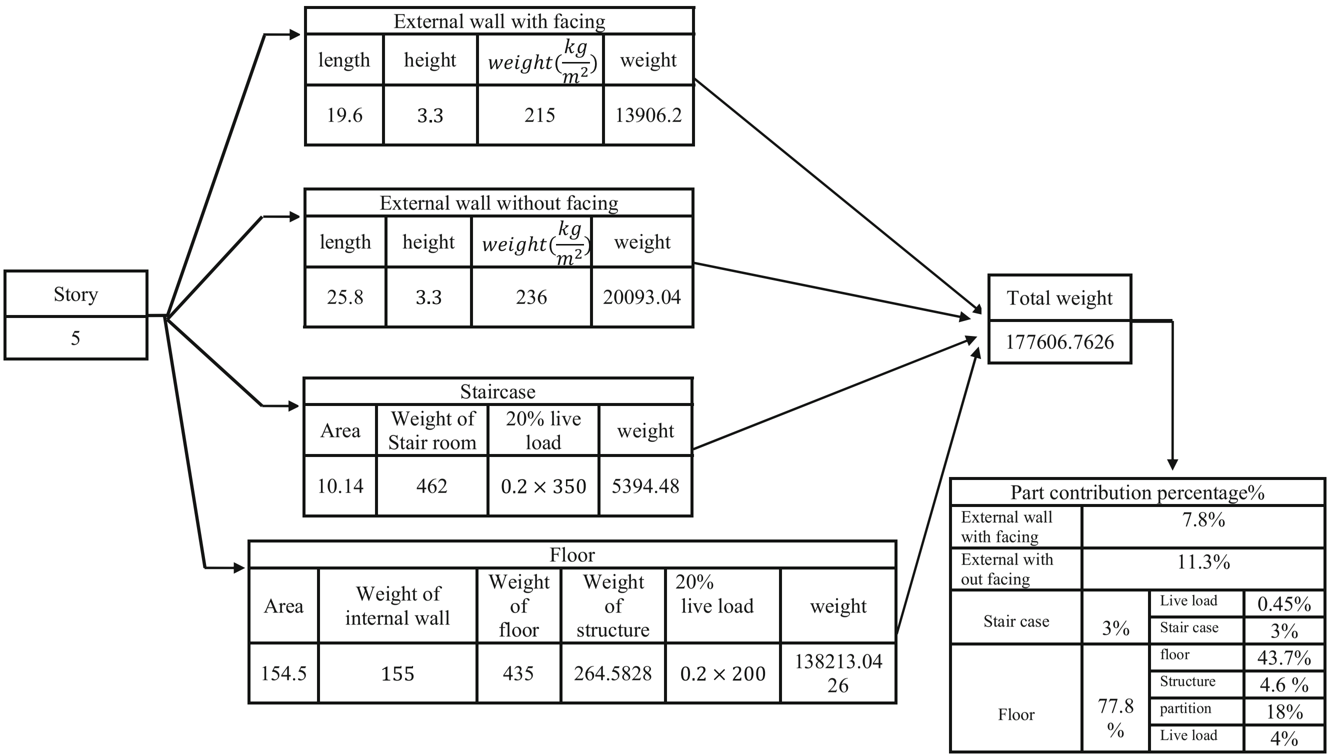


Chart 3.10 Total weight of story five

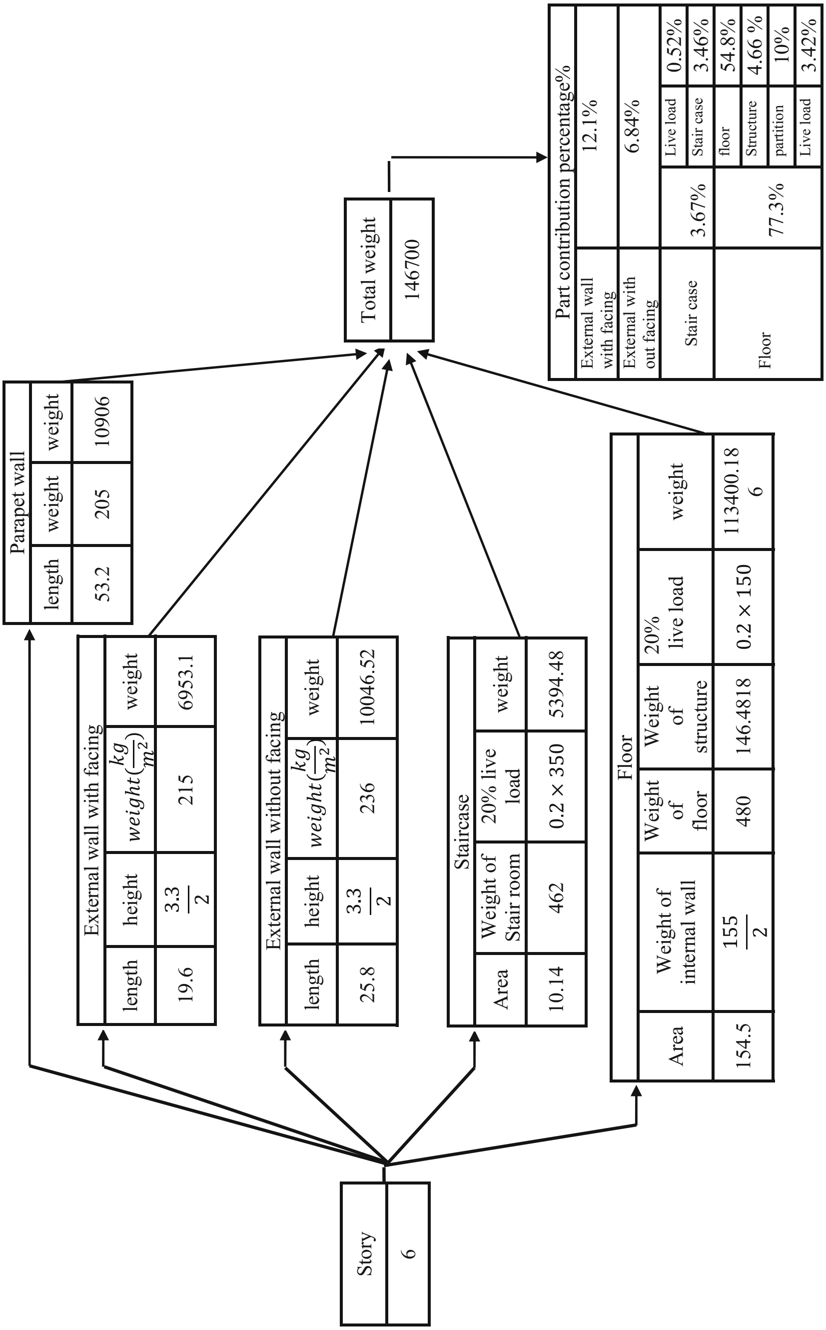


Chart 3.11 Total weight of story six

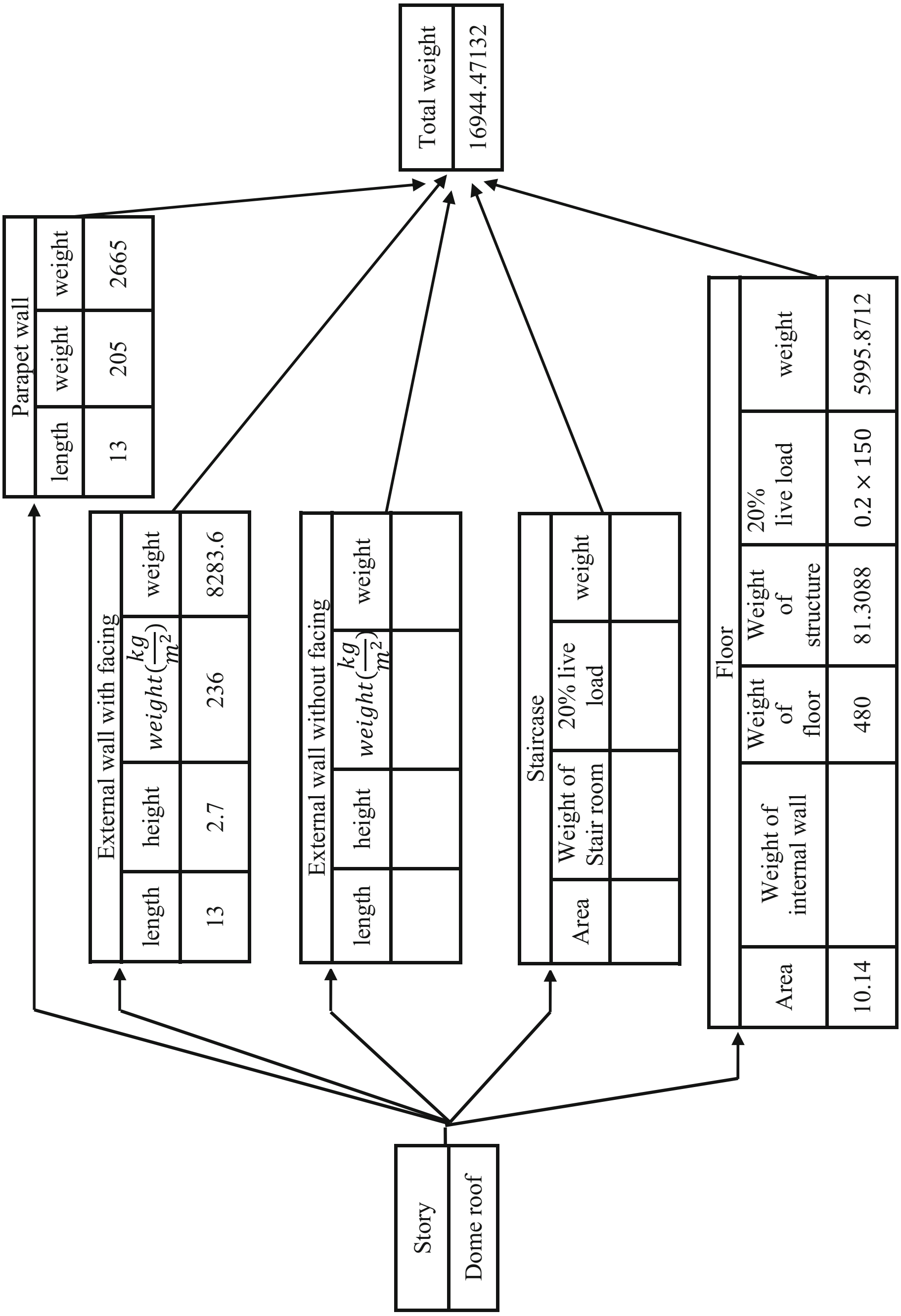


Chart 3.12 Total weight of dome roof

Table 3.32 Calculating of mass and centre of mass for 2nd floor

Number of mass	Length/area	kg/m–kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	897.5575	45299.72703	2.575	4.9	116646.7971	221968.662
3	28.08	897.5575	25203.4146	7.1	6.2	178944.2437	156261.171
4	75.95	897.5575	68169.49213	12.925	4.9	881090.6857	334030.511
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	5.15	778.8	4010.82	2.575	0.1	10327.8615	401.082
8	7.75	778.8	6035.7	12.925	0.1	78011.4225	603.57
9	5.15	778.8	4010.82	2.575	9.7	10327.8615	38904.954
10	7.75	778.8	6035.7	12.925	9.7	78011.4225	58546.29
Sum			178066.3538			1508473.182	885869.444

$$\bar{x} = \frac{1508473.182}{178066.3538} = 8.471 \quad \bar{y} = \frac{885869.444}{178066.3538} = 4.9741$$

Table 3.33 Calculating of mass and centre of mass for 3th floor

Number of mass	Length/area	kg/m–kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	895.1238	45176.89819	2.575	4.9	116330.5128	221366.801
3	28.08	895.1238	25135.0763	7.1	6.2	178459.0418	155837.473
4	75.95	895.1238	67984.65261	12.925	4.9	878701.635	333124.798
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	5.15	778.8	4010.82	2.575	0.1	10327.8615	401.082
8	7.75	778.8	6035.7	12.925	0.1	78011.4225	603.57
9	5.15	778.8	4010.82	2.575	9.7	10327.8615	38904.954
10	7.75	778.8	6035.7	12.925	9.7	78011.4225	58546.29
Sum			177690.3471			1505282.646	883938.172

$\bar{x} = \frac{1505282.646}{177690.3471} = 8.4714 \quad \bar{y} = \frac{883938.172}{177690.3471} = 4.9741$

Table 3.34 Calculating of mass and centre of mass for 4th floor

Number of mass	Length/area	kg/m–kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	894.9435	45167.79845	2.575	4.9	116307.081	221322.212
3	28.08	894.9435	25130.01348	7.1	6.2	178423.0957	155806.084
4	75.95	894.9435	67970.95883	12.925	4.9	878524.6428	333057.698
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	5.15	778.8	4010.82	2.575	0.1	10327.8615	401.082
8	7.75	778.8	6035.7	12.925	0.1	78011.4225	603.57
9	5.15	778.8	4010.82	2.575	9.7	10327.8615	38904.954
10	7.75	778.8	6035.7	12.925	9.7	78011.4225	58546.29
Sum			177662.4908			1505046.276	883795.094

$$\bar{x} = \frac{1505046.276}{177662.4908} = 8.4714 \quad \bar{y} = \frac{883795.094}{177662.4908} = 4.9741$$

Table 3.35 Calculating of mass and centre of mass for 5th floor

Number of mass	Length/area	kg/m–kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	894.5828	45149.59392	2.575	4.9	116260.2043	221233.01
3	28.08	894.5828	25119.88502	7.1	6.2	178351.1837	155743.287
4	75.95	894.5828	67943.56366	12.925	4.9	878170.5603	332923.462
5	9.8	709.5	6953.1	16.7	4.9	116116.77	34070.19
6	9.8	709.5	6953.1	0.1	4.9	695.31	34070.19
7	5.15	778.8	4010.82	2.575	0.1	10327.8615	401.082
8	7.75	778.8	6035.7	12.925	0.1	78011.4225	603.57
9	5.15	778.8	4010.82	2.575	9.7	10327.8615	38904.954
10	7.75	778.8	6035.7	12.925	9.7	78011.4225	58546.29
Sum			177606.7626			1504573.404	883508.859

$\bar{x} = \frac{1504573.404}{177606.7626} = 8.4714 \quad \bar{y} = \frac{883508.859}{177606.7626} = 4.9741$

Table 3.36 Calculating of mass and centre of mass for 6th floor

Number of mass	Length/area	kg/m–kg/m^2	M (kg)	X_i (m)	Y_i (m)	M_iX_i (kg m)	M_iy_i (kg m)
1	10.14	532	5394.48	7.1	1.3	38300.808	7012.824
2	50.47	733.9817864	37044.06076	2.575	4.9	95388.45646	181515.8977
3	28.08	733.9817864	20610.20856	7.1	6.2	146332.4808	127783.2931
4	75.95	733.9817864	55745.91668	12.925	4.9	720515.9731	273154.9917
5	9.8	354.75	3476.55	16.7	4.9	58058.385	17035.095
6	9.8	354.75	3476.55	0.1	4.9	347.655	17035.095
7	5.15	389.4	2005.41	2.575	0.1	5163.93075	200.541
8	7.75	389.4	3017.85	12.925	0.1	39005.71125	301.785
8	5.15	389.4	2005.41	2.575	9.7	5163.93075	19452.477
9	7.75	389.4	3017.85	12.925	9.7	39005.71125	29273.145
10	9.8	205	2009	16.7	4.9	33550.3	9844.1
11	9.8	205	2009	0.1	4.9	200.9	9844.1
12	16.8	205	3444	8.4	0.1	28929.6	344.4
13	16.8	205	3444	8.4	9.7	28929.6	33406.8
Sum			146700.286			1238893.442	726204.5445

$\bar{x} = \frac{1238893.442}{146700.286} = 8.445 \quad \bar{y} = \frac{726204.5445}{146700.286} = 4.950$

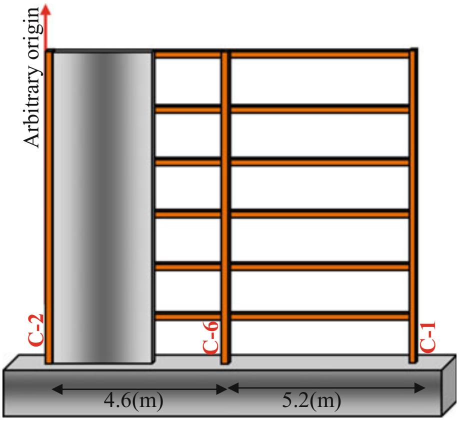
$146700.286 + 16944.47132 = 163644.7573$

$\bar{x} = \frac{(146700.286)(8.445) + (16944.47132)(7.1)}{163644.7573} = 8.305 \text{ (m)}$

$\bar{y} = \frac{(146700.286)(4.950) + (16944.47132)(1.3)}{163644.7573} = 4.572 \text{ (m)}$

According to the results centre of mass in roof level is (x = 8.305, y = 4.572)

Fig. 3.44 Frame (C)



$$I_{6,5,4} = \sum \left(AC^2\right) = \left[(0.00328)(4.8)^2\right] + \left[(0.00328)(0.2)^2\right] + \left[(0.00328)(5)^2\right] = 0.1577\,\text{m}^4$$

$$I_{3,2} = \sum \left(AC^2\right) = \left[(0.00328)(4.771)^2\right] + \left[(0.00492)(0.171)^2\right] + \left[(0.00328)(5.029)^2\right] = 0.1577\ \text{m}^4$$

$$I_1 = \sum \left(AC^2\right) = \left[(0.00328)(4.74)^2\right] + \left[(0.0152)(0.14)^2\right] + \left[(0.00328)(5.06)^2\right] = 0.15795\ \text{m}^4$$

2. Calculate the value of the external moment M at the mid-height of each story. Figure 3.45 shows the magnitude of the external moment in each story at the mid-height of each story

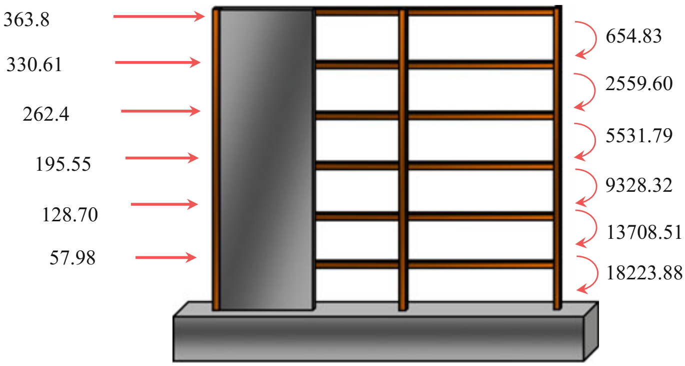


Fig. 3.45 External moment in frame (C)

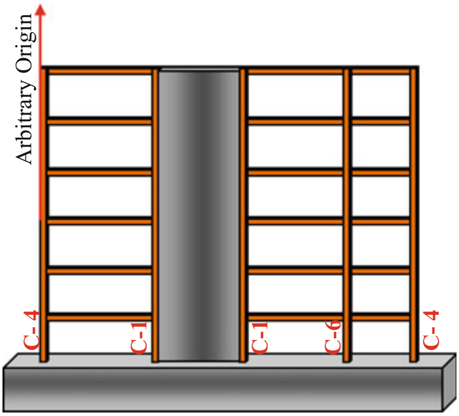


Fig. 3.46 Frame (1)

Furthermore, Calculations of inclination and total displacement are depicted in Table 3.37 in tabular form.

Additionally, Calculation of shear components of displacement had been determined as follows:

1. Calculate the value of the external shear that functioned in each Storey. The external shear of bent (C) was calculated and the results are shown in Table 3.21. Calculate for each Storey 1 the story displacement due to shear by substituting the value of the story shear and the member properties into the suitable formula derived from Table 3.20.
3. Sum the story displacements based on shear component. The total shear displacement was calculated and the results are displayed in Table 3.38 Total displacement of frame (C) is shown in Table 3.39, while Graph 3.2 displays flexure, shear, and total displacement of frames (C).

1. The total displacement of frame (1) had been composed of two components: Flexure displacement and Shear displacement (Fig. 3.46).
   Flexure component:
2. Compute the column centroid and the moment inertia

Table 3.37 Flexure component of displacement of frame (C)

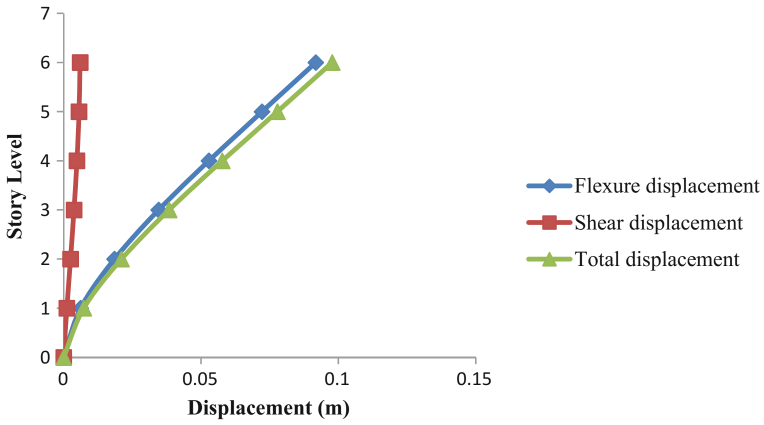
Story	h (m)	I_i (m^4)	E	Force	Moment	θ_i	θ_{if}	δ_{if}	Δ
6	3.6	0.1695	200,000,000	363.79	654.83	0.00006954	0.0051492	0.0185371	0.0868
5	3.6	0.1695	200,000,000	330.62	2559.60	0.00027182	0.0050797	0.0182868	0.0683
4	3.6	0.1695	200,000,000	262.40	5531.80	0.00058745	0.0048078	0.0173082	0.0500
3	3.6	0.1695	200,000,000	195.56	9328.32	0.00099062	0.0042204	0.0151934	0.0327
2	3.6	0.1695	200,000,000	128.70	13708.52	0.00145577	0.0032298	0.0116272	0.0175
1	3.3	0.1695	20,000,0000	57.99	18223.89	0.00177401	0.0017740	0.0058542	0.0059

Table 3.38 Evaluation shear component displacement of frame (C)

Story	Q	I	h	E	K	δ	Δ
6	363.79	0.4394	3.6	32,000,000	904115.23	0.0004	0.0061
5	694.41	0.4394	3.6	32,000,000	904115.23	0.0008	0.0057
4	956.81	0.4394	3.6	32,000,000	904115.23	0.0011	0.0049
3	1152.37	0.4394	3.6	32,000,000	904115.23	0.0013	0.0038
2	1281.07	0.4394	3.6	32,000,000	904115.23	0.0014	0.0026
1	1339.06	0.4394	3.3	32,000,000	1173787.46	0.0011	0.0011

Table 3.39 Evaluation total displacement of frame (C)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.0868	0.0061	0.0929
5	0.0683	0.0057	0.0739
4	0.0500	0.0049	0.0549
3	0.0327	0.0038	0.0365
2	0.0175	0.0026	0.0200
1	0.0059	0.0011	0.0070



Graph 3.2 Different type of displacement of frame (C)

$$C_1 = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)+(3\times 0.00164\times 14.5)}{(13\times 0.00164)+(2\times 0.17\times 0.008)} + \frac{(2\times 0.17\times 0.008\times 14.05)+(3\times 0.00164\times 16.8)}{(13\times 0.00164)+(2\times 0.17\times 0.008)} = 9.3$$

$$C_2 = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)+(3\times 0.00164\times 14.05)+(3\times 0.00164\times 16.8)}{(13\times 0.00164)} = 9.3$$

$$C_3 = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)+(3\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(11\times 0.00164)} = 9.46$$

$$C_{5,6,4} = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)+(2\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(10\times 0.00164)} = 9.01$$

$$\begin{aligned} I_{6,5,4} = \sum I_y + A(c)^2 = {} & \left[\left(1.83\times 10^{-5}\right)+(0.00328)(9.01)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(3.86)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(0.04)^2\right] \\ & + \left[\left(1.83\times 10^{-5}\right)+(0.00328)(5.04)^2\right] \\ & + \left[\left(1.83\times 10^{-5}\right)+(0.00328)(7.79)^2\right] = 0.59755 \end{aligned}$$

$$\begin{aligned} I_3 = \sum\left(I_y + AC^2\right) = {} & \left[\left(1.83\times 10^{-5}\right)+(0.00328)(9.46)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(4.31)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(0.41)^2\right] \\ & + \left[\left(1.88\times 10^{-5}\right)+(0.00492)(4.59)^2\right] \\ & + \left[\left(1.83\times 10^{-5}\right)+(0.00328)(7.34)^2\right] = 0.63543 \end{aligned}$$

$$\begin{aligned} I_2 = \sum\left(I_y + AC^2\right) = {} & \left[\left(1.88\times 10^{-5}\right)+(0.00492)(9.3)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(4.15)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(0.25)^2\right] \\ & + \left[\left(1.88\times 10^{-5}\right)+(0.00492)(4.75)^2\right] \\ & + \left[\left(1.88\times 10^{-5}\right)+(0.00492)(7.5)^2\right] = 0.87004 \end{aligned}$$

$$\begin{aligned} I_1 = \sum\left(I_y + AC^2\right) = {} & \left[\left(1.88\times 10^{-5}\right)+(0.00492)(9.3)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(4.15)^2\right] \\ & + \left[\left(5.26\times 10^{-6}\right)+(0.00328)(0.25)^2\right] \\ & + \left[\left(2.53\times 10^{-5}\right)+(0.0152)(4.75)^2\right] \\ & + \left[\left(1.88\times 10^{-5}\right)+(0.00492)(7.5)^2\right] = 1.10199 \end{aligned}$$

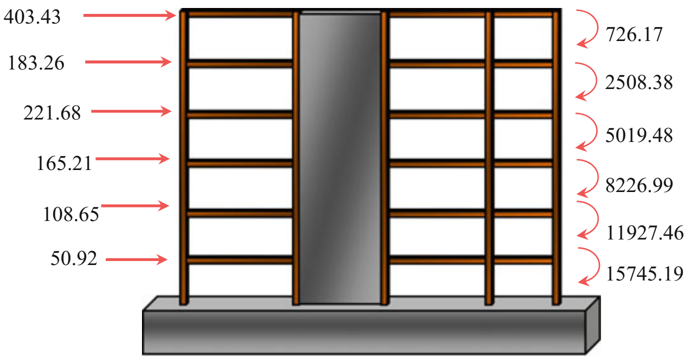


Fig. 3.47 External moment of frame (1)

$$I_{6,5,4} = \sum A(c)^2 = \left[(0.00328)(9.01)^2\right] + \left[(0.00328)(3.86)^2\right] + \left[(0.00328)(0.04)^2\right] + \left[(0.00328)(5.04)^2\right] + \left[(0.00328)(7.79)^2\right] = 0.5975$$

$$I_3 = \sum A(c)^2 = \left[(0.00328)(9.46)^2\right] + \left[(0.00328)(4.31)^2\right] + \left[(0.00328)(0.41)^2\right] + \left[(0.00492)(4.59)^2\right] + \left[(0.00328)(7.34)^2\right] = 0.6353$$

$$I_2 = \sum A(c)^2 = \left[(0.00492)(9.3)^2\right] + \left[(0.00328)(4.15)^2\right] + \left[(0.00328)(0.25)^2\right] + \left[(0.00492)(4.75)^2\right] + \left[(0.00492)(7.5)^2\right] = 0.8699$$

$$I_1 = \sum A(c)^2 = \left[(0.00492)(9.3)^2\right] + \left[(0.00328)(4.15)^2\right] + \left[(0.00328)(0.25)^2\right] + \left[(0.0152)(4.75)^2\right] + \left[(0.00492)(7.5)^2\right] = 1.10183$$

2. Compute the value of external moment M at each mid-story level. Figure 3.47 shows the magnitude of external moment in each story at the mid-story level.

The calculations of inclination and total displacement are illustrated in Tables 3.40, 3.41 and 3.42 as a tabular form (Graph 3.3).

Table 3.40 Flexure component of displacement of frame (1)

Story	h (m)	I_i (m^4)	E	Force	Moment	θ_i	θ_{if}	δ_{if}	Δ
6	3.6	0.1695	200,000,000	403.43	726.17	0.00007712	0.00454	0.01637	0.0760
5	3.6	0.1695	200,000,000	183.26	2508.38	0.00026638	0.00447	0.01610	0.0596
4	3.6	0.1695	200,000,000	221.68	5019.48	0.00053304	0.00420	0.01514	0.0435
3	3.6	0.1695	200,000,000	165.21	8226.99	0.00087366	0.00367	0.01322	0.0284
2	3.6	0.1695	200,000,000	108.65	11927.46	0.00126663	0.00279	0.01007	0.0151
1	3.3	0.1695	200,000,000	50.92	15745.19	0.00153272	0.00153	0.00505	0.0051

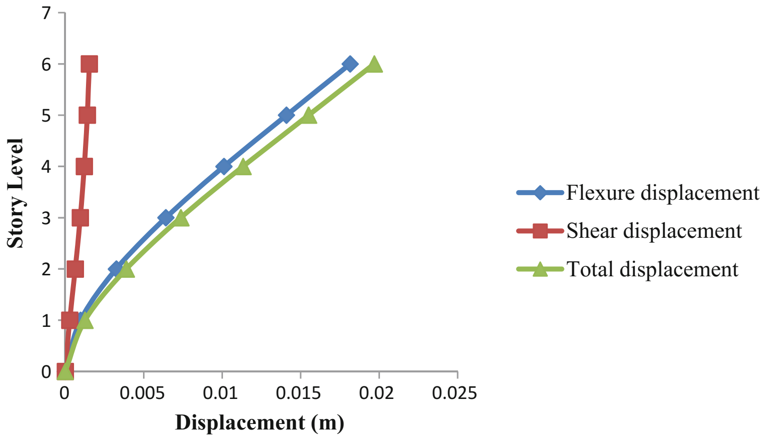
Table 3.41 Evaluation shear component displacement of frame (1)

Story	Q	I	h	E	K	δ	Δ
6	403.43	0.4394	3.6	32,000,000	3051388.89	0.0001	0.0015
5	586.68	0.4394	3.6	32,000,000	3051388.89	0.0002	0.0014
4	808.37	0.4394	3.6	32,000,000	3051388.89	0.0003	0.0012
3	973.58	0.4394	3.6	32,000,000	3051388.89	0.0003	0.0010
2	1082.24	0.4394	3.6	32,000,000	3051388.89	0.0004	0.0006
1	1133.15	0.4394	3.3	32,000,000	3961532.68	0.0003	0.0003

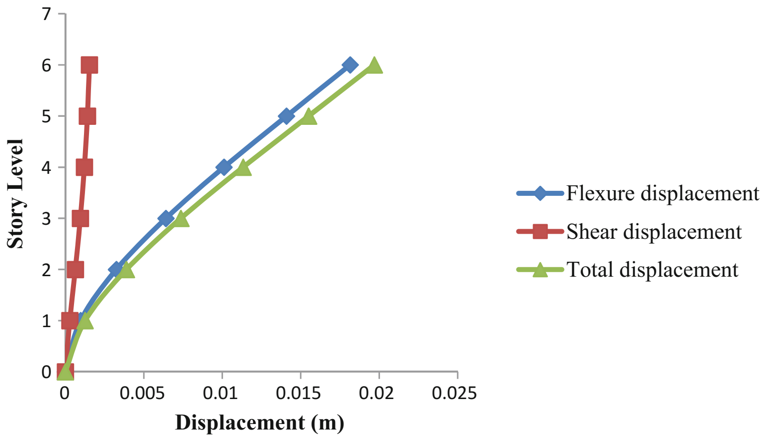
Table 3.42 Evaluation total displacement of frame (1)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.0760	0.0015	0.0775
5	0.0596	0.0014	0.0610
4	0.0435	0.0012	0.0447
3	0.0284	0.0010	0.0293
2	0.0151	0.0006	0.0158
1	0.0051	0.0003	0.0053

1. The total displacement of frame (5) was composed of two components: flexure displacement and shear displacement.
   Flexure components:
2. Compute the column centroid and the moment inertia (Fig. 3.48)



Graph 3.3 Different type of displacement of frame (1)



Graph 3.4 Displays flexure, shear, and total displacement of frames (5)

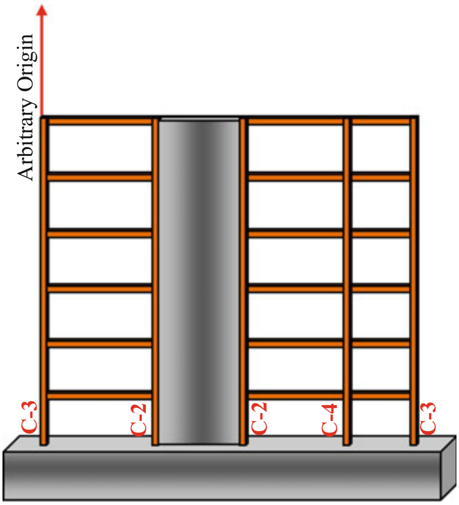


Fig. 3.48 Frame (5)

$$C_1 = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)}{(13\times 0.00164)} + \frac{(3\times 0.00164\times 14.5)+(3\times 0.00164\times 16.8)}{(13\times 0.00164)} = 9.3$$

$$C_2 = \frac{(2\times 0.00164)(5.15)+(2\times 0.00164)(9.05)}{(11\times 0.00164)} + \frac{(3\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(11\times 0.00164)} = 9.46$$

$$C_{5,6,4,3} = \frac{(2\times 0.00164\times 5.15)+(2\times 0.00164\times 9.05)}{(10\times 0.00164)} + \frac{(2\times 0.00164\times 14.05)+(2\times 0.00164\times 16.8)}{(10\times 0.00164)} = 9.01$$

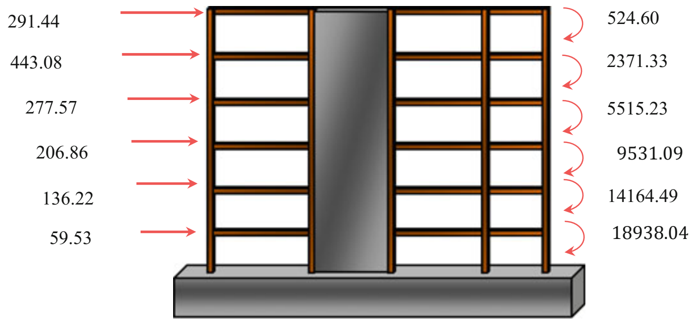


Fig. 3.49 External moment of frame (5)

Table 3.43 Flexure component of displacement of frame (5)

Story	h (m)	I_i (m^4)	E	Force	Moment	θ_i	θ_{if}	δ_{if}	Δ
6	3.6	0.1695	200,000,000	291.44	524.60	0.0000557	0.0052531	0.0189111	0.0893
5	3.6	0.1695	200,000,000	443.08	2371.33	0.0002518	0.0051974	0.0187106	0.0703
4	3.6	0.1695	200,000,000	277.57	5515.23	0.0005857	0.0049456	0.0178040	0.0516
3	3.6	0.1695	200,000,000	206.86	9531.09	0.0010122	0.0043599	0.0156955	0.0338
2	3.6	0.1695	200,000,000	136.22	14164.49	0.0015042	0.0033477	0.0120518	0.0181
1	3.3	0.1695	20,000,0000	59.53	18938.04	0.0018435	0.0018435	0.0060836	0.0061

$$I_{6,5,4,3} = \sum A(c)^2 = \left[(0.00328)(9.01)^2\right] + \left[(0.00328)(3.86)^2\right] + \left[(0.00328)(0.04)^2\right] + \left[(0.00328)(5.04)^2\right] + \left[(0.00328)(7.79)^2\right] = 0.5975$$

$$I_2 = \sum A(c)^2 = \left[(0.00328)(9.46)^2\right] + \left[(0.00328)(4.31)^2\right] + \left[(0.00328)(0.41)^2\right] + \left[(0.00492)(4.59)^2\right] + \left[(0.00328)(7.34)^2\right] = 0.6353$$

$$I_1 = \sum A(c)^2 = \left[(0.00492)(9.3)^2\right] + \left[(0.00328)(4.15)^2\right] + \left[(0.00328)(0.25)^2\right] + \left[(0.00492)(4.75)^2\right] + \left[(0.00492)(7.5)^2\right] = 0.869$$

2. Compute the value of external moment M at each mid-story level. Figure 3.49 illustrates the magnitude of external moment in each storey at the mid-story level. The calculations of inclination and total displacement are illustrated in Tables 3.43, 3.44 and 3.45 as a tabular form.

Table 3.44 Evaluation shear component displacement of frame (5)

Story	Q	I	h	E	K	δ	Δ
6	291.44	0.4394	3.6	32,000,000	3051388.89	0.0001	0.0019
5	734.52	0.4394	3.6	32,000,000	3051388.89	0.0002	0.0018
4	1012.09	0.4394	3.6	32,000,000	3051388.89	0.0003	0.0015
3	1218.95	0.4394	3.6	32,000,000	3051388.89	0.0004	0.0012
2	1355.17	0.4394	3.6	32,000,000	3051388.89	0.0004	0.0008
1	1414.70	0.4394	3.3	32,000,000	3961532.68	0.0004	0.0004

Table 3.45 Evaluation total displacement of frame (5)

Story	Flexure displacement	Shear displacement	Total displacement
6	0.0893	0.0019	0.0911
5	0.0703	0.0018	0.0721
4	0.0516	0.0015	0.0532
3	0.0338	0.0012	0.0350
2	0.0181	0.0008	0.0189
1	0.0061	0.0004	0.0064

References

Ali, S., & Mehdi, H. (2005). Design concrete shear wall in ETABS software.

ASCE/SEI. (2010). Minimum design loads for buildings and other structures *ASCE/SEI 7-10.* Reston, Virginia: American Society of Civil Engineer.

Elnashai, A., Elghazouli, A., Takanashi, K., & Dowling, P. (1991). Experimental behaviour of partially encased composite beam-columns under cyclic and dynamic loads. *Institution of Civil Engineers Proceedings, 91*(2), 259–272.

Elnashai, A. S. (1998). *Observations on the effects of the Adana-Ceyhan (Turkey) Earthquake of 27 June 1998. Engineering Seismology and Earthquake Engineering*. Report NO. ESEE/98-5, Imperial College, London, UK.

Elnashai, A. S., & Di Sarno, L. (2008). *Fundamentals of earthquake engineering*. New York: Wiley.

Eurocode 8, C. (2004). Design of structures for earthquake resistance—Part 1: General rules, seismic actions and rules for buildings (EN 1998-1: 2004). *European Committee for Normalization, Brussels*.

IBC, I. (2012). *International Code Council. International Building Code.* Washington, DC, United States: International Code Council.

Krawinkler, H., & Mohasseb, S. (1987). Effects of panel zone deformations on seismic response. *Journal of Constructional Steel Research, 8,* 233–250.

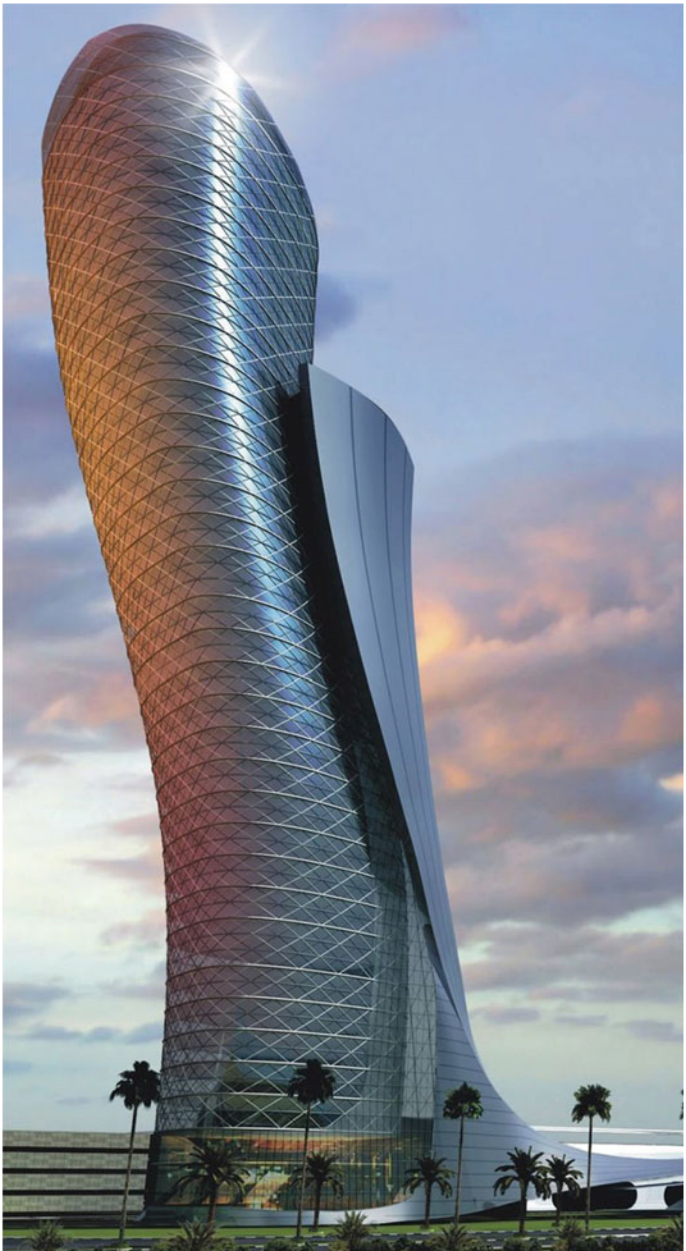
Mylonakis, G., & Gazetas, G. (2000). Seismic soil-structure interaction: beneficial or detrimental? *Journal of Earthquake Engineering, 4*(03), 277–301.

Nader, M., & Astaneh-Asl, A. (1992). *Seismic design concepts for semi-rigid frames.* Paper presented at the Proceedings of the 1992 ASCE Structures Congress.

Paulay, T., & Priestley, M. N. (1992). Seismic design of reinforced concrete and masonry buildings.

Seismic, A. (2010). Seismic provisions for structural steel buildings, (ANSI/AISC 341-10). American Institute of Steel Construction, Chicago, IL.

Smith, B. S., Coull, A., & Stafford-Smith, B. S. (1991). *Tall building structures: Analysis and design* (Vol. 5). New York: Wiley.



Chapter 4
Dynamic and Nonlinear Static Analysis

Chapter four focused on demonstration the basics of structural dynamic and explain the fundamental process to determine structural seismic response.

In order to have a clear comprehending of the highlighted concepts within this book, it is tried to present the application of theories and calculation processes in a real buildings in all sections. Therefore, a 6 story building is considered and in the second section of this chapter the architectural plans and structure details are demonstrated.

The third part of this chapter describes the response spectrum analysis and the analytical model for the multi degree-of-freedom (MDOF) structure. The shear force for every mode is computed from the natural frequency and eigenvalue analysis of the structure. The square root of the sum of squares (SRSS) technique and complete quadratic combination (CQC) technique are employed to determine the critical seismic responses for considered structures using maximum modal responses.

In the last section, the procedure for nonlinear static analysis is demonstrated after describing the concept for plastic hinge occurring in the structural member. The nonlinear static analysis technique is considered as one of the mainly applicable techniques for nonlinear analysis of the structures subjected to lateral load.

F. Hejazi and K. Karimzadeh, *Analysis Procedure for Earthquake Resistant Structures*,
https://doi.org/10.1007/978-981-10-8839-1_4

4.1 Linear Dynamic Analysis

- Types of methods for calculation earthquake force
- Response history analysis
- Spectral analysis method
- Response spectrum analysis
- Free vibration for MODF system
- Natural vibration frequencies and modes
- Determination of modal shape
- Normalization of modes
- Calculation of story shear force
- Linear static analysis
- Fundamental period and base shear
- Pattern of lateral load
- Minimum number of modes
- Combination of modal results

4.1.1 Understanding the Types of the Methods of Calculating Earthquake Force

In this section, we are going to argue about the main attitude governing the methods of determining the earthquake forces on the buildings, but before getting familiar with the types of the earthquake force analysis, you need to pay attention to two following important issues:

- **First Issue**

We know that the effect of the earthquake is in the form of a reciprocating load which is imposed by the structure in a short time and its value has a significant oscillation during the time, this issue means that the earthquake load is counted as a dynamic load. Since that the analysis of the structure under a dynamic load has its difficulties, civil engineers try to substitute the dynamic load of the earthquake for a constant and static load, in some methods of the earthquake load analysis. This issue causes that the calculations of the structure against the earthquake force become much simpler. However, this generally brings some error in the calculations.

- **Second Issue**

You may be heard about the non-linear analysis sometimes. Here, for you, the engineers, it should be mentioned that generally, the following points can cause non-linear behavior in a structure, to get a basic understanding.

1. **The non-linear behavior of the used material in the structure**:

During loading into a structure, if the created strain in a member exceeds a specific value, the material shows non-linear behavior. For example, in this graph which is

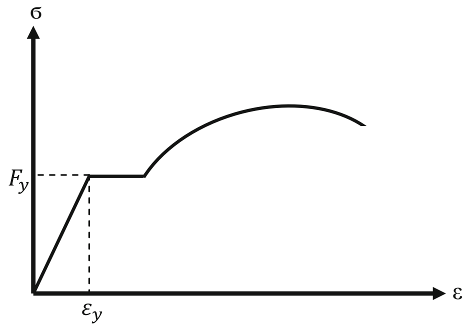


Fig. 4.1 Steel stress–strain graph

drawn for structural steel (Fig. 4.1), after the strain ε_y, the behavior of the material gets non-linear (the relation between the stress and strain is not linear anymore).

2. **The Geometric Non-linear behavior in Structure**:

This behavior is produced in the structure due to various factors, the most important of which is a $P - \Delta$ effect, in the usual construction projects.

Note:

During analyzing a structure against the earthquake, if we seek an exact analysis, we can model and consider the structure by the non-linear behavior, and if we seek a simpler analysis, we can consider the structure behavior as linear.

Considering the two above mentioned issues in this section, it must be said that firstly, the analysis of a structure against the earthquake could be carried out statically or dynamically, and secondly, in the process of the analysis, the structure behavior can be considered as linear or non-linear. Chart 4.1 shows different types of the methods of calculating earthquake force

In the following argument, we are going to talk a little about the philosophy of doing response history analysis and spectral analysis methods.

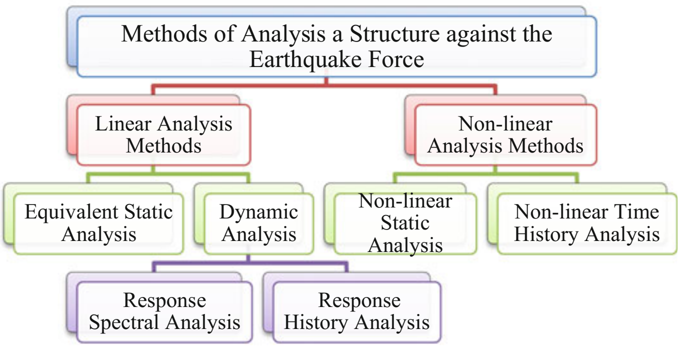


Chart. 4.1 Different types of the methods of calculating earthquake force

4.1.2 Introduction to Response History Analysis

The accelerometers can record the acceleration of the ground motion as a function of time, during the time that earthquake takes place. The result of recording the information by the accelerometers is represented in the form of the graphs like the following figure which are called accelerograms. Figure 4.2 shows El Centro earthquake accelerograms.

In the method of history analysis, we impose the accelerograms of one or several earthquakes on the structure using software modeling so that the output achieved from the software gives us the trend of changing the various parameters such as the displacement, acceleration, force, and the base shear during the earthquake. In the history method, since we calculate the structure responses (such as acceleration, displacement, and force) moment by moment, we can actually have a proper understanding of the structure behavior during the earthquake and finally use its results to design the structure.

4.1.3 Introduction of Spectral Analysis Method

As you saw in the previous section, in the history method, we actually try to get the responses of the structure in each moment of the loading period and have the history of the results for each intended parameter (for example, in each moment we determine the value of the displacements and the forces). But the problem is that carrying out the history analyses usually have a long and time-consuming process, and that is why they are mostly used to design particular and important structures. In civil engineering, considering that actually in the process of design the members of a structure, we need the maximum values of the displacement and force, if we achieve a proper estimate of the maximum values instead of getting all the history of the responses during the time, the analysis of the structure against the earthquake force will become much simpler. In the spectral analysis method which is also called as quasi-dynamic some references, the same approach is used.

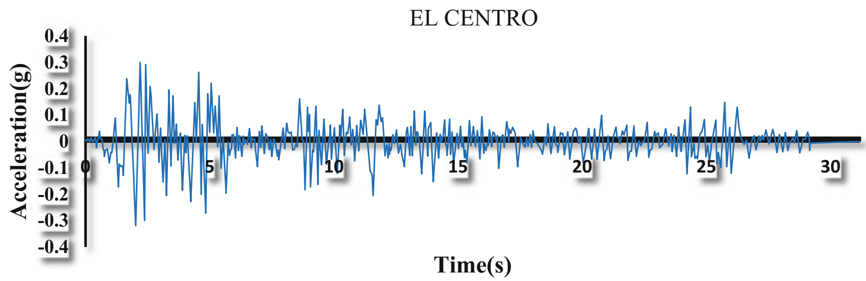


Fig. 4.2 El Centro earthquake accelerograms (California 1940)

4.1.4 Response Spectrum Analysis

A record of actual time history is required to carry out a seismic analysis and to design a building structure. Nevertheless, such records are not available for all locations. Besides, one cannot perform the seismic analysis merely by using ground acceleration peak values to function as structural response, especially by solely depending on two aspects, which are: (i) ground motion frequency content, and (ii) its related dynamic properties. Hence, in the attempt to address the mentioned hurdles, the earthquake response spectrum method appears to be the most widely used approach for structural seismic analysis. Furthermore, several computational benefits can be gained by employing the response spectrum technique to estimate member forces and dislocations within the structural systems. This particular approach only calculates the maximum values retrieved from member forces and dislocations from each vibration mode by applying smooth design spectra, which refers to the average of earthquake tremors.

As such, this chapter explains the response spectrum method, along with its application n, for varied structural types.

Response Spectrum

Response spectra refer to the curves that are obtained when the response for the system of SDOF is maximized and applied for targeted motion of earthquake occurring at the ground is plotted against frequency domain or a particular period of time. As such, response spectra reflect the central of the maximum response obtained within an SDOF system for a certain ratio of damping. Hence, the response spectra are helpful to determine responses of peak structural within a range of linear. The results can be applied further to identify the structural lateral forces because of earthquake tremors, which guides in designing structures equipped with earthquake-resistant aspects.

In precise, the SDOF system response can be identified by using the analysis of time or frequency domain, in which the maximum response can be retrieved for a certain period of time within the system. In fact, this continuous process covers all ranges of time periods within the SDOF system. Moreover, the required response spectra that serve as input for motion of ground and for ratio of damping are represented by the final plot from the period of time system found on the x axis, while y axis for quantity of response. Besides, similar procedure is performed for varied ratios of damping to gather the total response spectra.

Combined D–V–A Spectrum

The response spectra of pseudo-acceleration, pseudo-velocity, and deformation possess similar data for a ground motion that is predetermined. In fact, the three spectra refer to the varied channels of depicting the very similar data regarding structural response. Upon obtaining one spectrum, the other two spectra could be determined via algebraic operations by employing Eqs. (4.1), (4.2).

$$\mathrm{V} = \omega_\mathrm{n}\mathrm{D} = \frac{2\pi}{\mathrm{T_n}}\mathrm{D} \tag{4.1}$$

$$\mathrm{A} = \omega_\mathrm{n}^2\mathrm{D} = \left(\frac{2\pi}{\mathrm{T_n}}\right)^2\mathrm{D} \tag{4.2}$$

In addition, the reason for gathering all three spectra although they possess exactly similar data is primarily because these spectra offers meaningful quantity directly in a physical manner. The spectrum of deformation reveals the system's deformation at its peak condition, while the spectrum of pseudo-velocity can be associated directly with the energy of strain at its peak during an earthquake for the system, as depicted in Eq. (4.3).

$$E_{so} = \frac{mv^2}{2} \tag{4.3}$$

Meanwhile, spectrum of pseudo-acceleration can be associated to two elements, which are: (i) base shear, and (ii) equivalent force of static at its peak, as given in Eq. (4.4).

$$V_{bo} = f_{so} = mA \tag{4.4}$$

Next, the second reason is due to the ability of spectra shape to readily adhere to the design purposes guided by all the three spectral quantities. As such, a plot that is combined with all three spectral quantities can be deemed as beneficial. Moreover, such plot was initially developed in 1960 by A. S. Veletsos and N. M. Newmark to obtain response spectra for earthquake.

This combinatory element is definitely viable as the three spectral quantities are interrelated, as given in Eqs. (4.5) and (4.6).

$$\frac{\mathrm{A}}{\omega_\mathrm{n}} = \mathrm{V} = \omega_\mathrm{n}\mathrm{D} \tag{4.5}$$

$$\frac{\mathrm{T_n}}{2\pi}\mathrm{A} = \mathrm{V} = \frac{2\pi}{\mathrm{T_n}}\mathrm{D} \tag{4.6}$$

The equation of motion, as expressed in (4.7), is applied for acceleration of earthquake within the system of SDOF.

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -m\ddot{u}_g(t) \tag{4.7}$$

Substitute

$$\omega_0 = \sqrt{k/m}$$
$$\xi = \frac{c}{2m\omega_0}$$
$$\omega_d = \omega_0\sqrt{1-\xi^2}$$

Meanwhile, Eq. (4.8) is as follows:

$$\ddot{u}(t) + 2\xi\omega_0\dot{u}(t) + \omega_0^2 u(t) = -\ddot{u}_g(t) \tag{4.8}$$

By using the Duhamel's integral, the system of SDOF can be solved, as proposed by

$$x(t) = -\int_0^t \ddot{u}_g(\tau)\frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d} ins\omega_d(t-\tau)d\tau \tag{4.9}$$

Besides, the maximum dislocation that can occur in the system of SDOF for ξ and ω_0 parameters when applied for a predetermined motion due to earthquake, $-\ddot{u}_g(t)$, is reflected in Eq. (4.10).

$$|u(t)|_{max} = \left|\int_0^t \ddot{u}_g(\tau)\frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d} ins\omega_d(t-\tau)d\tau\right|_{max} \tag{4.10}$$

The relative displacement spectrum can be further defined as given in the following:

$$S_d(\xi,\omega_0) = |u(t)|_{max} \tag{4.11}$$

where S_d (ξ, ω_0) denotes the spectra that refer to dislocation for motion of earthquake at ground for the following parameters: ξ and ω_0.

In a similar vein, two significant elements, which are (i) absolute acceleration response spectrum, S_a, and (ii) the relative velocity spectrum, S_v, are given in the following:

$$S_v(\xi,\omega_0) = |\dot{u}(t)|_{max} \tag{4.12}$$

$$S_a(\xi,\omega_0) = |\ddot{u}_a(t)|_{max} = |\ddot{u}(t) + \ddot{u}_g(t)|_{max} \tag{4.13}$$

Next, the response spectrum of pseudo velocity, S_{pv}, is denoted as in the following:

$$S_{pv}(\xi,\omega_0) = \omega_0 S_d(\xi,\omega_0) \tag{4.14}$$

Meanwhile, the response for pseudo-acceleration, S_{pa}, is calculated when S_d is multiplied with ω_0^2, hence:

$$S_{pa}(\xi, \omega_0) = \omega_0^2 S_d(\xi, \omega_0) \tag{4.15}$$

Consider a case where ξ = 0 i.e.

$$\ddot{u}(t) + \omega_0^2 u(t) = -\ddot{u}_g(t) \tag{4.16}$$

$$\begin{aligned} S_a &= \left|\ddot{u}(t) + \ddot{u}_g(t)\right|_{max} \\ &= \left|-\omega_0^2 u(t)\right|_{max} \\ &= \omega_0^2 |u_{max}| = \omega_0^2 S_d = S_{pa} \end{aligned} \tag{4.17}$$

The above equation signifies the undamped system as, $S_a = S_{pa}$.

Furthermore, the quantity for S_{pv} that determines the maximum strain energy within a structure is expressed as given in the following:

$$E_{max} = \frac{1}{2} k u_{max}^2 = \frac{1}{2} m \omega_0^2 S_d^2 = \frac{1}{2} m S_{pv}^2 \tag{4.18}$$

On the other hand, the quantity for S_{pa} linked to the base shear at its maximized value is given below:

$$V_{max} = k u_{max} = m \omega_0^2 S_d = m S_{pa} \tag{4.19}$$

Meanwhile, the correlations between the varying response spectrum quantities are illustrated in Table 4.1. Besides, as a limiting case weighs in a system that may be rigid, i.e., $\omega_0 \rightarrow \infty$, $T_0 \rightarrow 0$, the related results obtained for the varied response spectra are as listed in the following:

$$\lim_{\omega_0 \rightarrow \infty} s_d \rightarrow 0 \tag{4.20}$$

$$\lim_{\omega_0 \rightarrow \infty} s_v \rightarrow 0 \tag{4.21}$$

$$\lim_{\omega_0 \rightarrow \infty} s_a \rightarrow \left|\ddot{u}_g(t)\right|_{max} \tag{4.22}$$

Table 4.1 Response spectrum relationship

Relative displacement	S_d	$\frac{S_v}{\omega_0}$	$\frac{S_a}{\omega_0^2}$	$\frac{S_{pv}}{\omega_0}$	$\frac{S_{pa}}{\omega_0^2}$
Relative velocity	$\omega_0 S_d$	S_v	$\frac{S_a}{\omega_0}$	S_{pv}	$\frac{S_{pv}}{\omega_0}$
Absolute acceleration	$\omega_0^2 S_D$	$\omega_0 S_v$	S_a	$\omega_0 S_{pv}$	S_v

Although all the three explained spectra offer similar data pertaining to structural response, they give physically meaningful quantities [refer to Eqs. (4.18) and (4.19)] that helps in not only comprehending the features of an earthquake, but also their impact upon structure design. Hence, a plot that is combined points out the relationships between all three spectral quantities, as given in Eqs. (4.14) and (4.15).

$$log\, S_{PV} = log\, s_d + log\, \omega_0 \tag{4.23}$$

$$log\, S_{PV} = log\, s_{pa} - log\, \omega_0 \tag{4.24}$$

Other than that, Eqs. (4.23) and (4.24) clearly present a logarithmic scale when plotted with log S_{pv}, functioning as ordinate and log ω_0 serving as abscissa, in which two lines that are straight were obtained, whereby +45° and −45° slopes were recorded for values that were constant for log S_d and log S_{pa}. Thus, all the three explained spectra (pseudo acceleration, displacement, and pseudo-velocity) can be illustrated using graph. Besides, such spectra of response must blanket several ranges of vibration periods that are natural, along with some values of damping, so as to give peak response for the structures evaluated. In fact, the range of period must be wide to cover skyscrapers and lengthy bridges that suggest longer period of vibration, thus indicating the importance of embedding damping values of ξ for a range between 0 and 20%. Figure 4.3 shows some spectrum curves for ξ = 0, 2, 5, and 10, as well as 20% over time period that ranges from 0.02 to 50 s. The results reflect the spectrum of response recorded for the ground motion component of North–South obtained at the occurrence of an earthquake known as Imperial Valley that occurred on 18th May 1940 (Jangid, 2013).

4.1.5 Free Vibration for MDOF System

The 'multi-degree of freedom system' or better known as the MDOF system has been commonly analysed by using the Modal Analysis. Another vital notion is 'free vibration', which refers to the motion of a structure that takes place in the absence of any dynamic excitation or external forces. In fact, free vibration can be initiated by interfering with the structure derived from the related position of equilibrium by perpetrating velocities and/or dislocations. With that, this section elaborates the development of a notion pertaining to both vibrations and frequencies that naturally occur in structures. Therefore, such aspects have important functions in both the dynamic and earthquake analysis for linear systems. In addition, the properties of vibrations indicate the response of free vibration within a system. Furthermore, the undamped systems can be analyzed to obtain a numerical solution based on eigenvalue so as to identify natural frequencies and vibration modes. Besides, the method of vector iteration has been proven to be effective in the applications of structural engineering. It is important to note here that this study is limited to the methods mentioned.

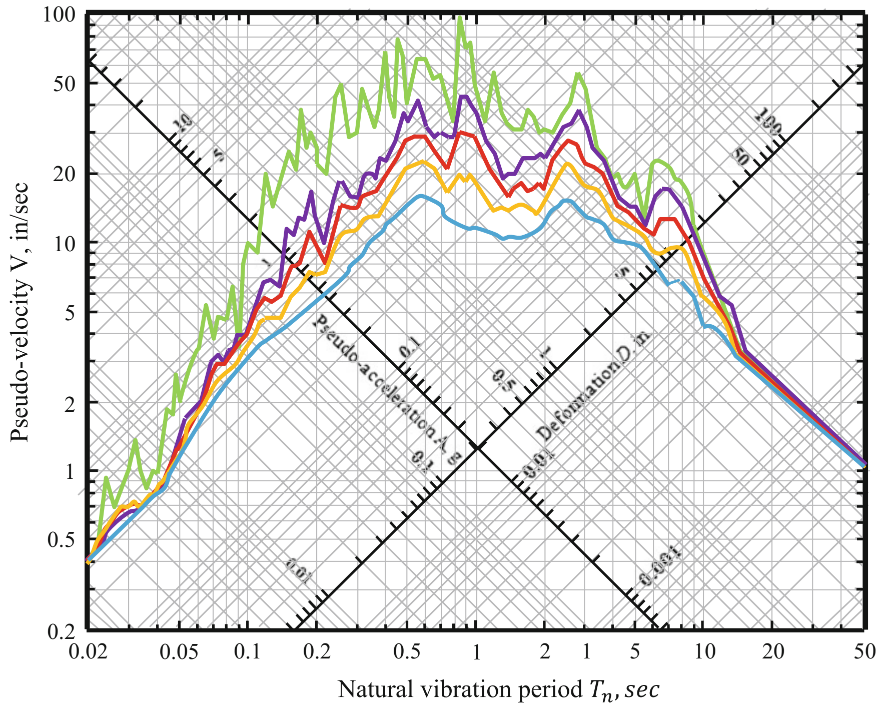


Fig. 4.3 Combined *D–V–A* response spectrum for El Centro ground motion; ζ = 0, 2, 5, 10, and 20%

4.1.6 Natural Vibration Frequencies and Modes

A better comprehension of the concept of the mode shape calls for us to take a structure that has a stiffness k and with location at a rigid beam's end, into consideration. In the event of large stiffness of the columns, it becomes possible for the mass m_1 to the right/left in a horizontal direction, resulting in the structure making one degree freedom configuration, and resulting in the possibility of vibration taking place in one direction alone.

The modals shapes of a structure is directly related to its degrees of freedom (DOF). For example, N degrees of freedom translate to N modal shapes. The modals shape retains its independence and is therefore and normal displacement that can undergo amplification or superimposition resulting in the generation of another pattern of displacement. The concept of DoF of structure is the fact that the structure can move in independent directions. Hence, in structure (1), mass1 can only move in a horizontal direction, which means it has one mode shape. In structure (2), each mass (m_1, m_2) can move in a horizontal direction, with 2 DOF, meaning they have two mode shapes. Figure 4.4 shows investigation modal shapes.

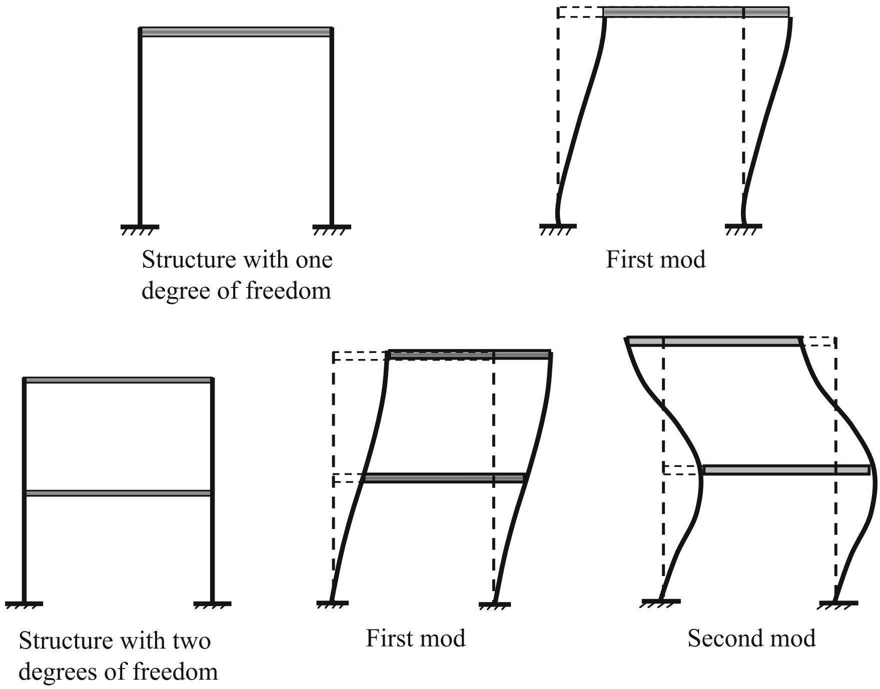


Fig. 4.4 Investigation modal shapes

The computation of eigenvalues (or natural frequencies) and eigen modes (or the modal shapes) in 3D is first done in the spectrum analysis of the modal response. The spectrum analysis should be conducted in a full 3D structural model even when such analysis may be conducted in XZ and YZ vertical planes which are sufficiently regular in their plan for a 2D analysis. To better understand the concept, consider the floor of a building in a 3D view in Fig. 4.5. If three movements of this floor is considered (two horizontal movements and one torsion movement), then this floor has 3 DoF. If a building is modeled with n stories, it can be considered 3n mode shapes; a building with six stories has 18 mode shapes. Each mode shape vibrates at a particular period; the time of structure full cycle vibration in each mode is different, which means that period of each mode should be calculated in these steps:

- **First Step: Calculation of Mass Matrix**

In dynamic analysis, the properties of mass in the structure are defined by the mass matrix. The mass of the N-story frame is lumped at the floor levels with m_j denoting the mass at the jth floor. This system has N degrees of freedom. $[M]$ is a $n \times n$ matrix that is calculated as follows (Fig. 4.6):

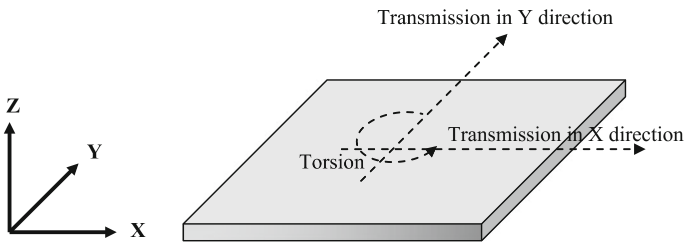
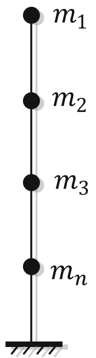


Fig. 4.5 Floor's degree of freedom

Fig. 4.6 Introduction of structure's masses



$$[M] = \begin{bmatrix} m_1 & 0 & 0 & \cdots & 0 \\ 0 & m_2 & 0 & \cdots & 0 \\ 0 & 0 & m_3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & m_n \end{bmatrix}$$

- **Second Step: Determining the Stiffness Matrix of the Structure**:

In the dynamic analyses, the characteristics related to the stiffness of a structure are defined by a matrix named stiffness matrix. It can be said that if a structure has n degrees of freedom, its stiffness matrix is a $n \times n$ matrix which is derived as following (Fig. 4.7):

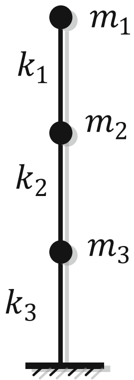
It can be seen that the elements of this matrix are in the general form of k_{ij}. The parameter k_{ij} is the required value for imposition on the ith degree of freedom so that the displacement of the jth degree of freedom is unit, while the displacement of the other DoF is zero. To understand the elements of the stiffness matrix, consider a three-degree-of-freedom structure such as the following; the stiffness of the stories designated as k_1, k_2, and k_3. According to what we learned, we know that the stiffness matrix is a 3×3 matrix in the following form (Figs. 4.8 and 4.9):

Assume that we want to determine the elements of the first column of this matrix (the elements $k_{11,}$ $k_{21,}$ *and* k_{31}). Since the elements of this column are in the general form of k_{i1}, by definition, we need to impose a force of ith degree of freedom so that the displacement of the first DoF (that means the last level of the structure) is unit, while the displacement of the other DoF (that means the first, second, and the third degrees of freedom) is zero. Therefore, it can be said that the following form should be used to determine the first column of the stiffness matrix:

Fig. 4.7 Introduction of structural's stiffness

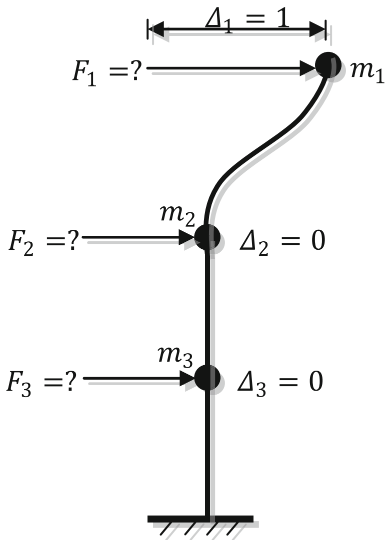
$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} & \cdots & k_{1n} \\ k_{21} & k_{22} & k_{23} & \dots & k_{2n} \\ k_{31} & k_{32} & k_{33} & \dots & k_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & k_{n3} & \cdots & k_{nn} \end{bmatrix}$$

Fig. 4.8 Introduction of structure's stiffness



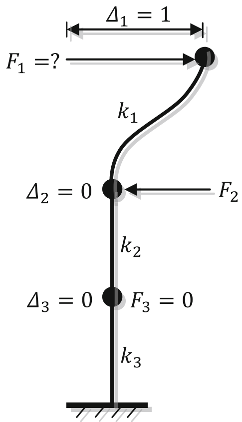
$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

Fig. 4.9 The way of Imposing force to get the first column of the stiffness matrix



Notice:

The elements of the first column of the stiffness matrix are, in fact, the forces F_1, F_2, and F_3 shown in Fig. 4.10 which to cause displacement.



$$F_1 = k_1(\Delta_1 - \Delta_2) = k_1 \xrightarrow{K_{11}=F_1} K_{11} = k_1$$

$$F_2 = k_2(\Delta_2 - \Delta_3) - k_1\ (\Delta_1 - \Delta_2) = -\,k_1 \xrightarrow{K_{21}=F_2} K_{21} = -k_1$$

$$F_3 = k_3(\Delta_3) - k_2\ (\Delta_2 - \Delta_3) = 0 \xrightarrow{K_{31}=F_3} K_{31} = 0$$

$$[K] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -\,k_2 & k_2 + k_3 \end{bmatrix}$$

Fig. 4.10 Calculating the first column of the stiffness matrix

$k_{11} = F_1$ (The force to impose on the first degree of freedom)

$k_{21} = F_2$ (The force to impose on the second degree of freedom)

$k_{31} = F_3$ (The force to impose on the third degree of freedom)

Taking into account the Fig. 4.10 shown above, it can be assumed that the force should be imposed towards the right side of the mass m_1 to displace its first degree of freedom via one unit, while simultaneously; a force should be imposed towards the left side of the mass m_2 so that there is no displacement in the 2nd DoF. In this situation, there is no need to impose a force on the mass m_3 and the displacement of the 3rd DoF is zero.

Considering the stiffness of the columns and their respective displacements, the forces F_1, F_2, and F_3 can be derived as (Fig. 4.10) (Heidary & Ahangar, 2016):

- **Third Step: Calculation of Mode's Period**

The undamped eigenvalues and eigenvectors of the MDOF system are found from the characteristic equation:

$$[\mathrm{m}]\{(\ddot{x})\} + [\mathrm{k}]\{(\mathrm{x})\} = 0 \tag{4.25}$$

$$\mathrm{X} = \{\varnothing\} \sin \omega t \tag{4.26}$$

where:

$\{\varnothing\}$ mode shape
ω natural frequency

Apart from being an imminent key to gain a numerical solution for the issue posed, this harmonic form possesses physical significance, where all degrees of free-vibrating structures move synchronously. Such configuration of structure would disallow any alteration in its fundamental shape upon being hit by a motion, as only the amplitude is affected. However, upon interrupting the harmonic solution and embedding it with equation of motion, the result is as follows:

$$-\omega^2[\mathrm{m}]\{\varnothing\}\sin\omega t + [\mathrm{k}]\{\varnothing\}\sin\omega t = 0 \tag{4.27}$$

Upon simplification, the following is obtained:

$$\left([\mathrm{k}] - \omega^2[\mathrm{m}]\right)\ \{\varnothing\} = 0 \tag{4.28}$$

Equation (4.28), which is termed as 'eigenequation', refers to equations that are homogeneously algebraic and comprised of components that reflect eigenvector, which serve as the basis for an eigenvalue hurdle, whereby an eigenvalue issue reflects an equation that can be applied for many usages that incorporate the algebra of linear matrix. The following presents the fundamental form of an eigenvalue hurdle:

$$[A - \lambda \mathrm{I}]X = 0 \tag{4.29}$$

where

A square matrix
λ eigenvalues
I density matrix
X eigenvector

Meanwhile, based on the analysis of structure, incorporation of the aspects of mass and stiffness into eigen-based equations leads to physical depictions of mode shapes and natural frequencies ('natural' reflects the quality of vibration properties to explain the natural properties of a structure in free vibration with sole reliance upon mass and stiffness). Thus, eigenequation consists of K, ω, and M, as shown in Eq. (4.28) with $\lambda = \omega^2$.

In fact, two solutions are available for Eq. (4.28), which are:

1. If det $([\mathrm{k}] - \omega^2[\mathrm{m}]) \neq 0$, the only possible solution is

$$\{\varnothing\} = 0 \tag{4.30}$$

This is called the trivial solution as it fails to give any viable data from the physical stance, primarily due to absence of (det refers to the determinant of a matrix).

2. If det $([k] - \omega^2[m]) = 0$, then a non-trivial solution ($\{ø\} \neq 0$) is obtained for

$$\left([k] - \omega^2[m]\right) \{ø\} = 0 \tag{4.31}$$

Furthermore, from the stance of structural engineering, the common hurdle of mathematical eigenvalue can be decreased by using the following equation as a solution:

$$det\left([k] - \omega^2[m]\right) = 0 \ \ or \ \ det([k] - \lambda[m]) = 0 \tag{4.32}$$

where:

$$\lambda = \omega^2$$

In fact, the determinant turns zero when the discrete eigenvalues are λ or ω^2. Besides, eigenvector $\{ø_i\}$ satisfies Eq. (4.31) and addresses all eigenvalues. Thus, Eq. (4.31) is expressed as

$$\left([k] - \omega_i^2[m]\right)\{ø_i\} = 0i = 1, 2, 3, 4\ldots \tag{4.33}$$

On top of that, N roots, ω_n^2, found in Eq. (4.33) addresses N natural vibration frequencies ω_n (n = 1, 2, …, N) listed in the order of from the smallest to the largest ($\omega_1 < \omega_2 < \ldots < \omega_n$), whereby these roots are called eigenvalues. Moreover, the eigenvalues and eigenvectors denote the mode of free vibration within the structure. The ith eigenvalue ω_i is linked to the ith time period, as given:

$$\omega_i = \sqrt{\lambda_i}, \quad T_i = \frac{2\pi}{\omega_i} \tag{4.34}$$

Besides, the quantity of eigenvalues and eigenvectors is equivalent to the number of degrees of freedom with mass or dynamic degrees of freedom.

Note:

Ask yourself when an earthquake strikes a structure, which mode shapes is similar to the structure's vibration? The oscillation of structure is a combination of the structure's vibration in different modes; it means that each mode contribute to the final deformation of the structure in a proportional manner.

After calculating the modes 'periods, we can calculate other factors that are important for understanding the behavior of structure:

(1) Modal shape (2) Effective modal mass.

4.1.7 Determination of Modal Shape

The section above has provided a description of how the equation that follows can used to determine both modal period and frequency.

$$\det\left([\mathrm{k}] - \omega^2[\mathrm{m}]\right) = 0 \tag{4.35}$$

Compact gathering of the N eigenvalues can be done within a matrix. The natural frequency ω_n corresponds to natural mode which has the element with elements $\varnothing_{jn}$. The subscript j indicates level j. Next, a single square matrix can be used to present N eigenvectors with the natural mode being reflected in each column.

$$\varnothing_1 = \begin{bmatrix} \varnothing_{1,1} \\ \varnothing_{2,1} \\ \varnothing_{3,1} \\ . \\ . \\ . \\ \varnothing_{n,1} \end{bmatrix} \quad \varnothing_2 = \begin{bmatrix} \varnothing_{1,2} \\ \varnothing_{2,2} \\ \varnothing_{3,2} \\ . \\ . \\ . \\ \varnothing_{n,2} \end{bmatrix} \quad \varnothing_n = \begin{bmatrix} \varnothing_{1,n} \\ \varnothing_{2,n} \\ \varnothing_{3,n} \\ . \\ . \\ . \\ \varnothing_{n,n} \end{bmatrix} \quad \Rightarrow \quad [\varnothing] = \begin{bmatrix} \varnothing_{1,1} & \varnothing_{1,2} & \cdots & \varnothing_{1,n} \\ \varnothing_{2,1} & \varnothing_{2,2} & \cdots & \varnothing_{2,n} \\ \varnothing_{3,1} & \varnothing_{3,2} & \cdots & \varnothing_{3,n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \varnothing_{n,1} & \varnothing_{n,2} & \cdots & \varnothing_{n,n} \end{bmatrix}$$

The matrix ø is called the modal matrix for the eigenvalue problem.

4.1.8 Normalization of Modes

As abovementioned, in an eigenvalue issue, Eq. (4.35) indicates the natural mode for multiplicative factor alone. Hence, if vector $\varnothing_n$ is a natural mode, any vector proportional to $\varnothing_n$ becomes natural mode for it could satisfy Eq. (4.35). At times, scale factors can be employed to natural modes in order to bring standardization to the associated aspects linked to varying DOFs, in which this process is known as 'normalization'. Moreover, normalizing the modes unites the biggest component so as to address the DOF, for example, uniting the top floor of a high-rise building. Besides, it is a common practice to normalize modes in computer programs and theoretical explanation to establish the unit values of $\mathrm{M_n}$.

$$\mathrm{M_n} = \varnothing_{\mathrm{n}}^{\mathrm{T}} \mathrm{m} \varnothing_{\mathrm{n}} = 1, \quad \varnothing^{\mathrm{T}} \mathrm{m} \varnothing = \mathrm{I} \tag{4.36}$$

where **I** is the identity matrix, a diagonal matrix with unit values along the main diagonal. Equation (4.36) states that the natural modes are not only orthogonal but are normalized with respect to **m**.

4.1.9 Calculation of Modal Mass

As mentioned earlier, real vibration shape of a structure is a combination of structure's different modal shapes. Have you ever asked how much the proportion of each mode contribute in total structure's vibration? The proportion of each mode in the structure is calculated according to the special parameter that is called effective modal weight, generally, if the amount of effective weight of a mode is greater than other modes, it devotes more proportion of total structure's oscillation to itself respect to other modes.

$$W_i = \frac{\left[\sum_{j=1}^{n} \left(W_j, \o_{i,j}\right)\right]^2}{\sum_{j=1}^{n} \left(W_j, \o_{i,j}^2\right)} \tag{4.37}$$

$\o_{i,j}$ mode shape coefficient of floor, i in mode, j,
Wi seismic weight of floor, i

4.1.10 Calculations of Story Shear Force in Each Mode

In response spectrum analysis, each mode of structure acts as an independent structure; hence,

1. There is a corresponded vibration period for each mode
2. It is possible to use of $F_b = S_d(T) \times m$ for calculating of base shear for each mode
3. Shear base of each mode is distributed in each stories level corresponding to the modal shape of each mode

Step 1: Design Spectrum for Forced-Based Design with Linear Analysis

In Euro code 8, the design spectrum for force-based design of a new building in regards to the horizontal constituents of the seismic action is as follows:

$$0 \le T \le T_B : \quad S_d(T) = a_g.S.\left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3}\right)\right] \tag{4.38}$$

$$T_B \le T \le T_C : \quad S_d(T) = a_g.S.\frac{2.5}{q} \tag{4.39}$$

$$T_C \le T \le T_D : \quad S_d(T) = \begin{cases} a_g.S.\frac{2.5}{q}.\left[\frac{T_C}{T}\right] \\ \ge \beta.a_g \end{cases} \tag{4.40}$$

$$T_D \leq T: \quad S_d(T) = \begin{cases} a_g.S.\frac{2.5}{q}\left[\frac{T_C T_D}{T}\right] \\ \geq \beta.a_g \end{cases} \tag{4.41}$$

In Eq. (4.38), the 2/3 arises as it is the reciprocal of 1.5 which was the strength factor that the Euro Code 8 put into consideration. According to Euro Code 8, the factor should always be there even when ductility and other design measures such as energy dissipation are not there. The lower boundary for the horizontal design spectrum which protects against too much reduction of the forces of design as a result of system flexibility is given by Factor (β) in Eqs. (4.40) and (4.41). Euro code 8 recommends its value to be 0.2. Putting into consideration the low Eurocode 8 recommended values, Factor (β) practical implications may be very important for T_D (corner period) at the start of the range of the spectrum displacement. The substituting in Eqs. (4.38)–(4.41) yields the design spectrum in the vertical direction. The design acceleration on the ground in the vertical direction, a_{vg}, for the "effective ground acceleration" ag_S. The mechanism for energy dissipation as a response in the vertical direction is not clear and therefore not clearly understood. Therefore, the over strength is considered to contribute to "behavior factor" q in the vertical direction and is taken as 1.5, except if other studies support a higher value (Code, 2005).

4.1.11 Linear Static Analysis

Applicability Basics and Terms

The analysis of linear static can be performed with forces that are lateral and segregated into dual directions that are orthogonal and horizontal; X and Y. In fact, the inertia for the peak of the load can be simulated by such forces, as generated by the element that is horizontal within the directions of earthquake motion, where its basic mode of structural vibrations corresponds to the directions. Moreover, this particular analysis is employed for practical seismic design, mainly because elastic analysis is not unfamiliar among architects in determining loads that are static, mainly due to either wind actions or gravity.

The basic assumptions of the technique are listed in the following:

1. Both translational and basic modes found directed based on forces that are lateral, which are linked to the responses.
2. The translational and basic modes' shape can be determined even with the absence of solution to eigenvalue problem.

4.1.12 Fundamental Period and Base Shear

In Euro code 8, static analysis is applied linearly done in a way which yield results that are similar to those of storey shears that are considered the most critical seismic effect as a reaction to spectrum analysis at least for building types that apply both methods. The shear on top or above the basement foundation that is rigid ("base shear"), V_b is determined using the 1st period, T_1 either in directions X or Y:

$$V_b = m_{eff1} \times S_{a.d}(T_1) \tag{4.42}$$

where

$S_{a.d}(T_1)$ presents the design response spectrum value from Eqs. (4.38) to (4.41) at the 1st mode.

$m_{eff,1}$ Reflects an estimation of the effective modal mass of that mode.

$$m_{eff,1} = 0.85\,\mathrm{m} \tag{4.43}$$

In Euro code 8, $m_{eff,1}$ can be reduced to $m_{eff,1} = 0.85\,\mathrm{m}$ for buildings which have more than one storey above a rigid basement or on top of a foundation a period of $T_1 < 2T_C$ (T_C denotes the period at the corner between the constant-spectral-pseudo velocity and acceleration ranges). This is a representation of buildings three storeys and above. If $T_1 < 2T_C$, modes starting from mode number two are in normal situation below the plateau with constant and high spectral accelerations. In Euro code 8, the use of Rayleigh quotient in calculating T_1 is highly promoted.

$$T_1 = 2\pi\sqrt{\frac{\sum_i m_i\delta_i^2}{\sum F_i\delta_i}} \tag{4.44}$$

where:

In this quotient, indexes all systems DOF in the horizontal X or Y direction where the calculation of T_1 is done; m_i; Represents the mass associated with DOFi; F_i stands for the lateral force applied to DOFi; and δ_i stands for DOF displacement, that are found from an elastic structural analysis for the set of lateral forces F_i.

Displacements δ_i are proportional to F_i for any given forces F_i (i.e., pattern over the DOFs i). Therefore, the absolute magnitudes of F_i does not dictate the value of T_1 from Eq. (4.44). T_1 is also insensitive to the relative magnitudes of F_i and therefore one can adopt any reasonable F_i distributions to DOFs. In linear static analysis, high accuracy and convenience can be achieved by using F_i lateral forces in proportion to distribution of V_b to the DOFs i. The base shear, V_b, at the time of calculating T_1 using Eq. (4.44) is no yet known. Therefore, the choice of F_i can be done such that the structure total weight is equal to the resultant of F_i forces structure (i.e., with $S_{a,d}$ (T_1) = 1.0 g). It can then be considered that a single static analyses in either X or Y horizontal direction suffices in the estimation of T_1 using Eq. (4.44) and in the determination of the effects of the component of seismic action in either X or Y direction given that the effect of seismic action in this case needs to be multiple by $m_{eff,1}$, $S_{a,d}$ (T_1)/m. the now know value of T_1 can be used to derive $S_{a,d}$ (T_1). The empirical results for T_1 can also be obtained from the following codes other than euro code 8: SEAOC 1999, BSSC 2003, and CEN 2004a. These codes provide a reflection of lower bounds that are obtained from California structural measurement record for addressing moderate earthquakes. From a response perspective, the impact of non-structural aspects is depicted by such records. The use of empirical expressions underestimated the period as compared to Eq. (4.44) where T_1 values are located in constant spectral acceleration areas specifically for flexible buildings. For this reason, force based designs could use such expression for the prediction of $S_{a,d}$ (T_1). Additionally, inaccurate, misleading, and unsafe T_1 empirical expressions can be realized based on displacement design and evaluation which requires significant realistic prediction of displacement requirements. Therefore, more accurate T_1 predications are offered by Eq. (4.44) effortlessly. Hence, it is not viable and reliable to use empirical period formulas for seismic design purposes.

4.1.13 The Pattern of Lateral Forces

The equation's base shear (4.45) is taken as what results from a number of concurrent peak and inertia forces prevailing on the mass m_i which is related to the DOFs i seismic actions' components in the horizontal direction. In a vibration's single mode especially in the 1st mode's direction of the horizontal component, the inertia lateral peak force on the DOF i is proportional to $ø_i m_i$, in which $ø_i$ is the 1st eigenmode's value at that particular DOF. Thus, the base shear force from Eq. (4.45) is distributed to the DOFs as shown by the expression below:

$$F_i = \frac{ø_i m_i}{\sum_j ø_j m_j} \times V_b \tag{4.45}$$

with the denominator's summation extending over all the DOFs i.

But in buildings that have rigid diaphragms, the masses are usually lumped at the floor centres. Therefore, the general expression above when applied to any masses' arrangement or DOFs space will be limited to only cover the formulations for floors or stories with i = 1 at the lowest floor just above the rigid basement's top or the foundation and i = n_{st}t at the roof. The lateral forces F_i will then be applied to the floor centers' mass.

4.1.14 Minimum Number of Modes

The analysis of modal response spectra weighs in contributory modes towards amount of quantity. In fact, it has been noted as challenging mainly because the number of modes embedded serves as input to the analysis of eigenvalue. For instance, Euro code 8 considers modes with a modal mass that id efficient and a component of the X, Y, or Z action of seismic, after taking the pattern into account with minimal total mass at 5%. In fact, this particular condition may actually refer to modes uncaptured in the analysis of eigenvalues; which turns into an obstacle. However, when faced with failure in meeting any of the two criteria (either due to the significance of the mode of torsional or upon incorporation of a component with action reflecting vertical seismic within the pattern), a third alternative is presented through the use of Euro code 8, especially when the result of analysis of eigenvalue is $3\sqrt{(n_{st})}$ (in which n_{st} becomes storey number on top of either foundation or basement that is rigid), as well as a period that is naturally below 0.2 s. In addition, the most used and the first condition (total mass modal that is effective) look into some parts of base shear found in computed modes. Moreover, since shears of modal equate to efficient mass of modal multiplied with spectral acceleration for a mode, especially with lengthier first mode and periods at a plateau of spectral acceleration that is constant, with contributions of higher modes to base shear are undermined by the effective modal mass. Meanwhile, other quantities of global response, such as base and top displacement overturning moment, are insensitive to the amount of modes, in comparison to shear base. Nonetheless, the number of responses applied within authentications that are local (inter storey drifts, member chord rotations or forces that are internal) could become more sensitive towards incorporated modes. With that, such modes may make up more than 90% of the total mass in predicting the peak values of the related quantities (Fardis, 2009).

4.1.15 The Combination of Modal Results

Based on the analysis of modal response spectra, the elastic response can be conveniently segregated into two modes that are independent of each other, which reflects another prediction. In fact, the real relationship of i mode with j mode can be predicted from the two-mode correlation coefficient; ρ_{ij}. Besides, the estimation that follows had been proposed by (Rosenblueth & Elorduy, 1969), as

$$\rho_{ij} = \frac{\xi^2(1+\lambda)^2}{(1-\lambda)^2 + 4\xi^2\lambda} \tag{4.46}$$

However, estimation suggested by Wilson et al. (1981) and Wilson, Der Kiureghian, & Bayo (1981) has been reckoned widely and applied to date:

$$\rho_{ij} = \frac{8\sqrt{\xi_i\xi_j}(\xi_i + \lambda\xi_j)\lambda^{\frac{3}{2}}}{(1-\lambda^2)^2 + 4\xi_i^2\xi_j^2\lambda(1+\lambda^2) + 4\left(\xi_i^2 + \xi_j^2\right)\lambda^2} \tag{4.47}$$

In Eq. (4.47), ξ_i, ξ_j denote the ratios for viscous damping within i and j modes (similar to Eq. (4.46) and represented as ξ), whereas $\lambda = T_i/T_j$. When the dual modes possess periods with natural space ($\lambda \approx 1.0$), ρ_{ij} becomes closer to 1.0, while the responses in these two modes would become dependent of each other. As for structures, the Euro code 8 (CEN 2004a) denotes that i and j modes are dependent if λ results from 0.9 to 1/0.9. Besides, the two ends for the range of λ and for $\xi_i = \xi_j = 0.05$, Eq. (4.47) indicates $\rho_{ij} = 0.47$. Meanwhile, the second part of Euro code 8 for bridges (CEN 2005b) appears rigid as i and j modes turn to be dependent if λ lies from $[1 + 10\sqrt{(\xi_i\ \xi_j)}]$ to $1/[1 + 10\sqrt{(\xi_i\ \xi_j)}]$; if 〖$\xi_i$〗 $= \xi_j = 0.05$, as well as when λ is equivalent to such limiting value, in which Eq. (4.47) offers lower value: $\rho_{ij} = 0.05$. Furthermore, structures that have similar stiffness laterally at X and Y horizontal paths possess paired modes with the same period at the right degrees, as stipulated within the design (though unnecessarily for X and Y paths). Moreover, these paired modes are associated to each other closely.

$$\rho_{ij} = \frac{8\xi^2(1+\lambda)\lambda^{\frac{3}{2}}}{(1-\lambda^2)^2 + 4\xi^2\lambda(1+\lambda)^2} \tag{4.48}$$

Upon considering that all related modal responses are independent, the theory of random vibration offers estimated values for maximum, EE, seismic action impact as the Square Root of the Sum of Squares for the responses, which refers to the rule of SRSS (Rosenblueth, Ramos, & ÍA, 1951):

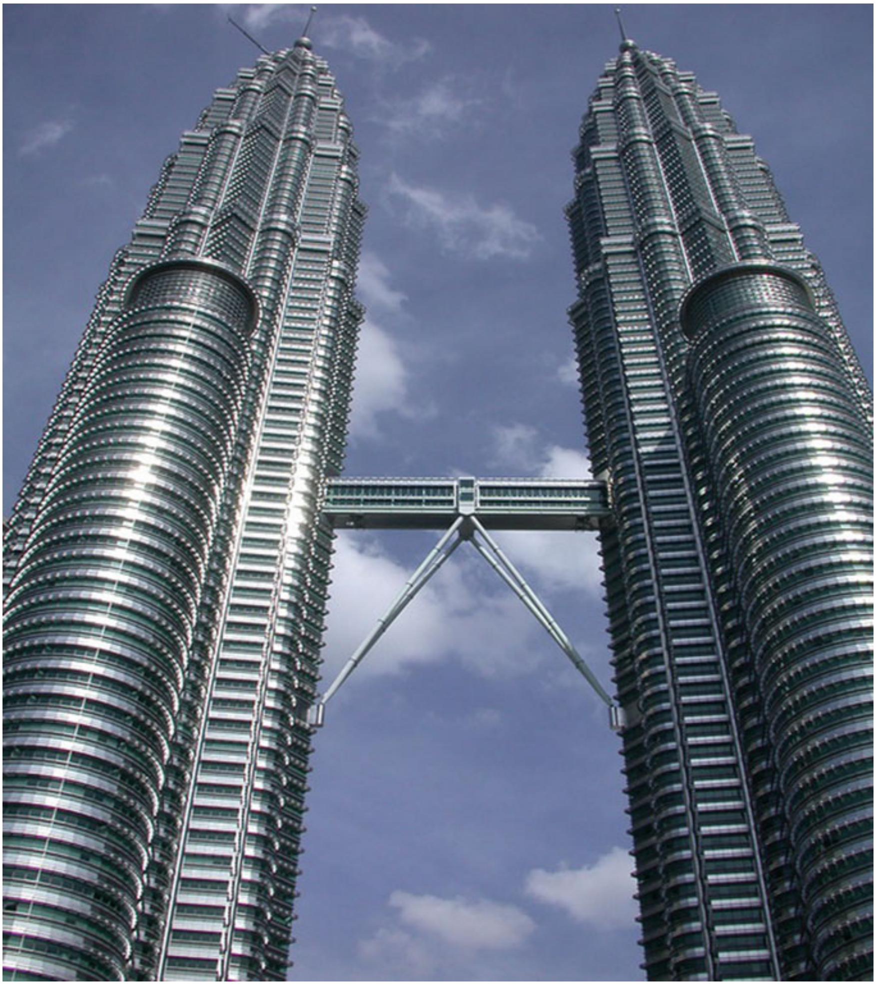
$$E_E = \sqrt{\sum_N E_{Ei}^2} \tag{4.49}$$

Summation that goes beyond modes of Nis weighed in, whereas E_{Ei} reflects the value for action of seismic at its peak and influential towards the i mode. However, if the i and j modes are dismissed as independent, rule of SRSS becomes non-conservative. Next, the peak values of modal responses are combined by using more accurate steps (e.g., CEN 2004a). Meanwhile, the theory of random vibration only provides the rule known as Complete Quadratic Combination (CQC) (Wilson et al., 1981), as Euro code 8 predicts the value of EE for the impact of seismic action due to modes in association:

$$E_E = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{ij} E_{Ei} E_{Ej}} \tag{4.50}$$

On the other hand, Eq. (4.50) depicts that E_{Ei} and E_{Ej} reflect the values at the peak of seismic action with an impact upon i and j modes. In addition, correlation coefficient values for both i and j modes, as well as ρ_{ij}, derive from Eqs. (4.47) to (4.48). Upon comparison, findings from the analysis of response-history exemplified exceptional, but average accuracy, for CQC rule. Besides, it is worthy to note that CQC rule is inclusive of SRSS rule and Eq. (4.49) for unique cases, such as that $\rho_{ij} = 0$ if $i \neq j$ ($i = j$ indicates $\rho_{ij} = 1$). Furthermore, both rules of CQC and SRSS provide definite values for estimates of responses at their peak, thus must be weighed in and amalgamated with impacts of non-seismic action (partly because of loads of gravity) as positive and negative. Hence, even though the equilibrium can be satisfied by the modal internal forces, the values of end force derived from rules of CQC and SRSS would fail to cater to the individual level of equilibrium. With that, moment envelopes with their members could not be built from either member end forces or equilibrium. However, such development may be indeed possible if SRSS or CQC rules are combined with moments from modal at an X point of generic along the member. In fact, point of inflection is absent from these envelopes, thus failing to indicate if the element has either singular or even double curved bend at both moments and peak. Furthermore, such aspect could be determined from the forces of shear exerted by peak, which are computed with similar combination rules of the mode. As such, the selected member can be assumed to be either in single or double curvature; especially when the results of shear force peak, as well as the member length, gets closer to the accumulated (positive) member en moments, or variances, respectively.

4.2 Define Project and Analyzing



- Architectural plan
- Material property
- Weight of floors

4.2.1 Introduction

Consider the building mentioned in the 1st chapter. The proposed multi-story residential building is located in an urban area. The land lot measures 28(m) × 10(m), while each floor measures 17(m) × 10(m), with an allowable GFA of 60% from a total land of 170 m^2. The proposed GFA is 850 m^2. This land is surrounded by buildings to its East and West. It also includes two streets along its North side. There are two separation joints of 11 cm on the East and West sides of the building. Its structural system consists of the moment resistance frame. All the floors are used as residence, while the ground floor is used for parking. The North–South direction of the plan was been divided into four bays by five grid lines (A-E), which is also the case in the East–West direction (1–5). The distances between the grid lines along the N–S direction are 5.15, 3.9, 5, and 2.75 m, respectively, whereas the distance between the grid lines along the E–W direction considered 4.9 m, (2.3)m, and 2.6 m, respectively. A composite system was selected as a floor system, with 9 cm concrete slab. The total height of the building above the ground floor is 21.3 m. The height of the first floor (between the levels 0 and 1) amounts to 3.3 m, while the height of the other floors are 3.6 m. Floor plan, ground floor, columns situation and elevation view are shown in Figs. 4.11, 4.12, 4.13, and 4.14 respectively.

The elastic response spectrum represents the seismic action, Type 1 (Ms > 5.5, Code, 2005, 1/3.2.2.2 (2) P) for soil C (Code, 2005 Table 3.1). The amount for the peak ground acceleration is $ag_R = 0.3$ g, while the amount for the periods' values (TB, TC, TD) and soil factor values (S), defining the elastic response spectrum shape is TB = 0.2 s, TC = 0.6 s, TD = 2.0 s and S = 1.15 (Code, 2005, Table 3.2). The building classification is made as importance class II (Code, 2005, Table 4.3), while the amounts for the equivalent importance factor are $\gamma_1 = 1.0$ (Code, 2005, 4.2.5 (5) P). Thus, the peak ground acceleration corresponds to the reference peak ground acceleration $ag = \gamma_1 \times agR = 0.3$ g. The definition of the elastic response spectrum for 5% damping was accomplished through the use of the equation in (Code, 2005)/3.2.2.2. The design response spectrum (the elastic response spectrum decreased by the behaviour factor q) was utilised in designing the building. The behaviour factor (q) determined based on the structural system

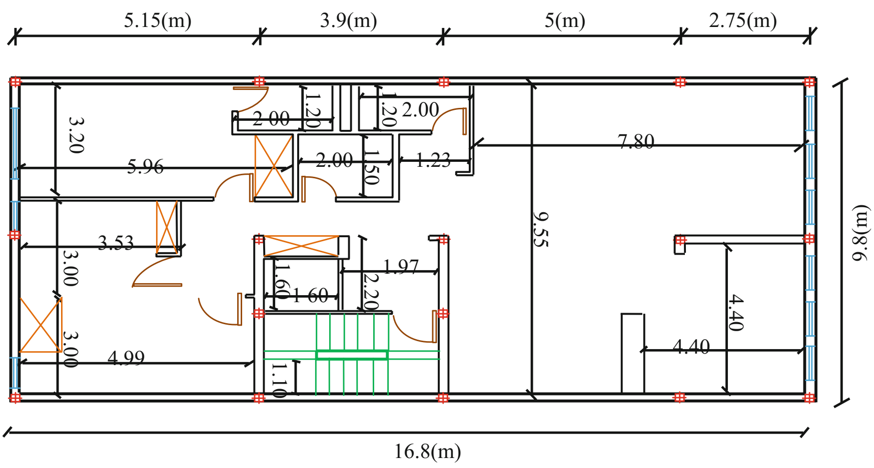


Fig. 4.11 Floor plan of considered building

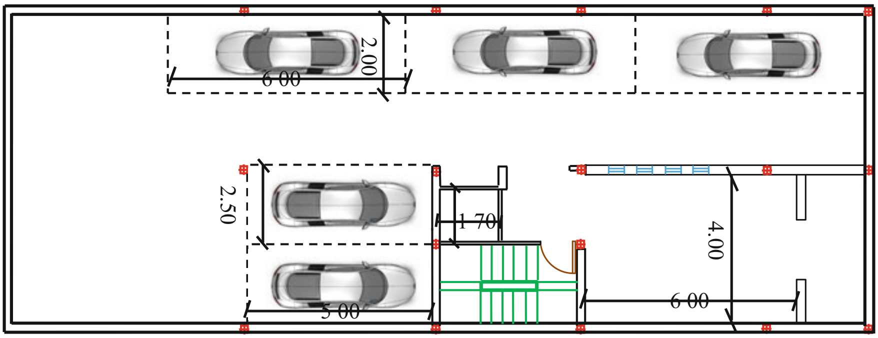


Fig. 4.12 Ground floor of considered building

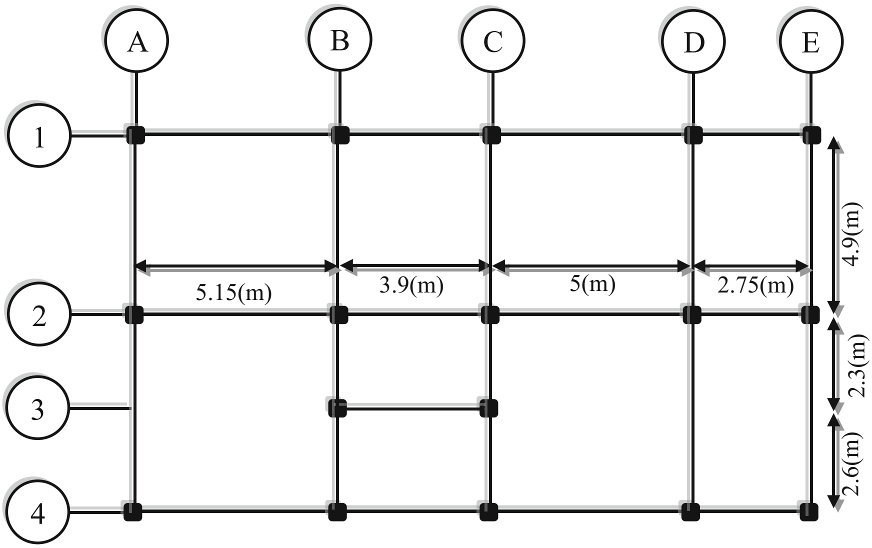


Fig. 4.13 Columns situation and span distance

type, elevation and plan regularity, and ductility class as 4.0. The expressions in (Code, 2005)/3.2.2.5 (4) P was utilised in defining the design spectrum for elastic analysis.

4.2.2 *Material Property*

The steel and concrete material properties are shown in Table 4.2.

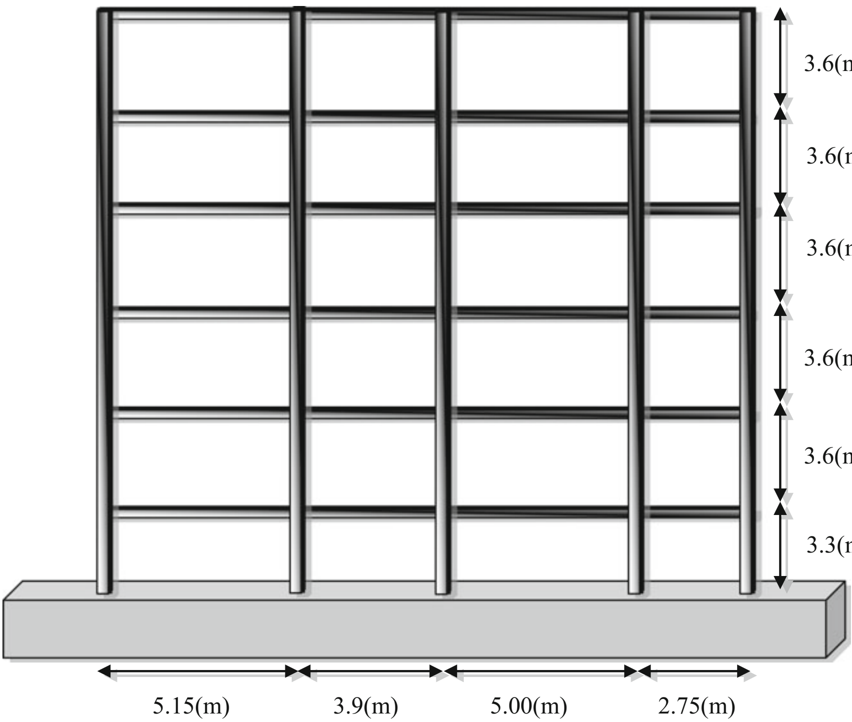


Fig. 4.14 Elevation view of considered building

Table 4.2 Material properties

Material property	Steel	Concrete
Mass per unit volume:	$800\ \frac{kg}{m^3}$	$250\ \frac{kg}{m^3}$
Weight per unit volume:	$7850\ \frac{kg}{m^3}$	$2500\ \frac{kg}{m^3}$
Young's modulus:	$2\times10^6\ \frac{kg}{cm^2}$	$2\times10^5\ \frac{kg}{cm^2}$
Poisson's ratio:	0.3	0.2

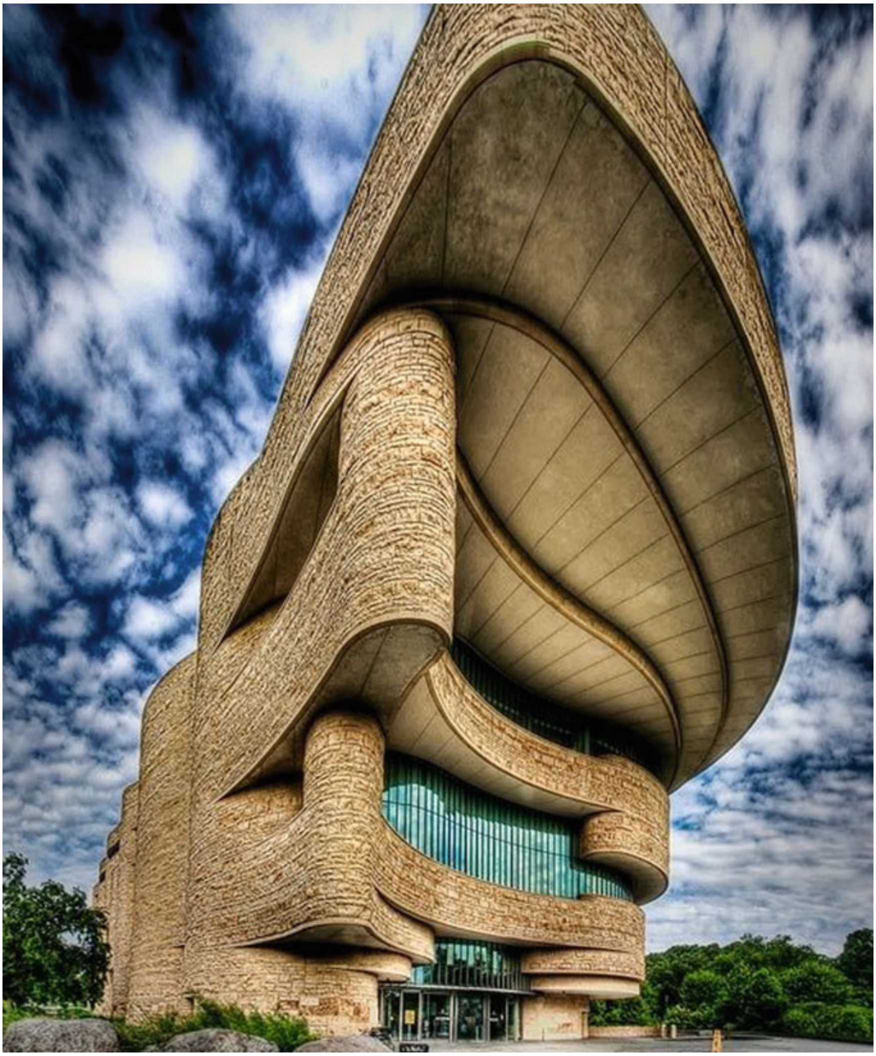
4.2.3 *Total Weight of Floors*

Total weight of each floor is computed in the first chapter (Moment Resistance Frame). Table 4.3 shows the weight of each floors as well.

Table 4.3 Total weight of floors

Story level	Weight (kg)
6	162920.582
5	165112.1274
4	165605.4045
3	166098.252
2	166954.8
1	158044.182

4.3 Response Spectrum Analysis



4.3.1 Procedure of Building Analysis

The analysis of building in the X direction is done here using the lumped mass method. Based on Sect. 4.2, Table 4.2, the total weight of each story is shown in Fig. 4.15.

The floor stiffness is the sum of the lateral stiffness of the columns on the floor. For a floor of height h and a column with modulus E and second moment of area I_c, the lateral stiffness of a column with clamped ends, implied by the shear-building idealization, is defined as $12EI_c/h^3$. Thus, the floor stiffness is:

$$K_i = \sum \frac{12EI_c}{h^3} \tag{4.51}$$

Tables 4.4 and 4.5 demonstrate number, cross-section, and second moment inertia of column in considered building respectively.

4.3.1.1 The Stiffness of the First Floor

According to Table 4.6 and Eq. (4.51), the stiffness of the first floor can be calculated. Table 4.7 shows the stiffness of the other floors as well.

$$I_1 = 4(0.00009223) + 3(0.000262253) + 7(0.00039572) + 3(0.000824108) = 0.00668$$

$$K_1 = \frac{12 \times 2 \times 10^{11} \times 0.00668}{3.3^3} = 446132006.3 \left(\frac{\text{N}}{\text{m}}\right)$$

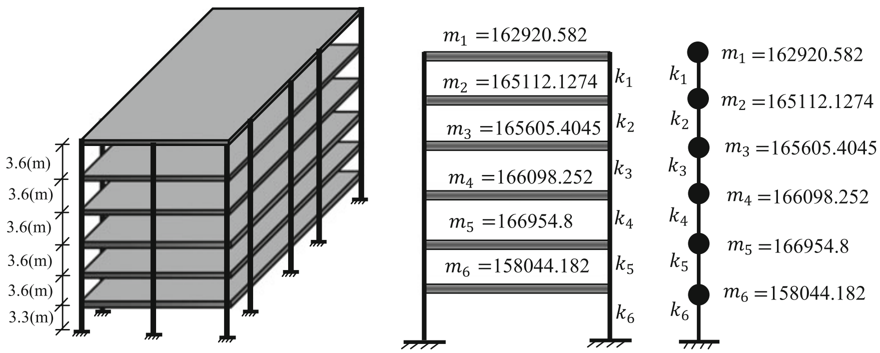


Fig. 4.15 Lumped mass of considered building

Table 4.4 Number and cross-section of column

Columns	Number	Story 1	Story 2	Story 3	Story 4	Story 5	Story 6
C-1	4	Box 300 * 10	Box 300 * 10	Box 250 * 10	Box 250 * 10	Box 250 * 10	Box 250 * 10
C-2	3	Box 350 * 10	Box 350 * 10	Box 300 * 10	Box 300 * 10	Box 300 * 10	Box 300 * 10
C-3	7	Box 400 * 10	Box 400 * 10	Box 350 * 10	Box 350 * 10	Box 300 * 10	Box 300 * 10
C-4	3	Box 450 * 15	Box 450 * 15	Box 400 * 12	Box 400 * 12	Box 350 * 10	Box 350 * 10

Table 4.5 Second moment inertia of column in X and Y direction

Cross section	IX	IY
Box 300 * 10	0.000162787	526.778
Box 250 * 10	0.00009232	1766.618
Box 350 * 10	0.000262253	1882.612
Box 400 * 10	0.00039572	2537.67
Box 450 * 15	0.000824108	

Table 4.6 Second moment inertia

Cross-section	Number of columns	Second-moment inertia	Total
C-1	4	0.000162787	0.000651148
C-2	3	0.000262253	0.000786759
C-3	7	0.00039572	0.00277004
C-4	3	0.000824108	0.002472324

Step 1: Mass Matrix

This structure has 6 DoF, so its mass matrix becomes:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix}$$

Substitute the data from Fig. 4.15, it is gotten

$$[M] = \begin{bmatrix} 162920.582 & 0 & 0 & 0 & 0 & 0 \\ 0 & 165112.1274 & 0 & 0 & 0 & 0 \\ 0 & 0 & 65605.4045 & 0 & 0 & 0 \\ 0 & 0 & 0 & 166098.252 & 0 & 0 \\ 0 & 0 & 0 & 0 & 166954.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 158044.182 \end{bmatrix}$$

Step 2: Stiffness Matrix

The dimension of the stiffness matrix of this structure will be 6 × 6, in the form below:

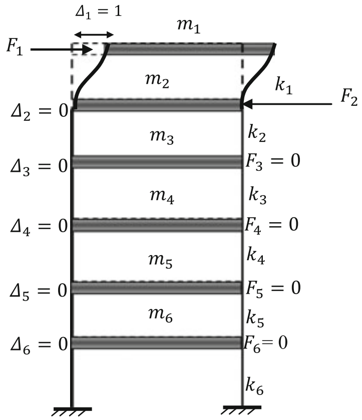
$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix}_{6\times 6}$$

Table 4.7 Stiffness in each floor of the building

Factors	Story 1	Story 2	Story 3	Story 4	Story 5	Story 6
I (m^4)	0.00668	0.006680269	0.00409662	0.00409662	0.002783907	0.002783907
E (N/m^2)	2.00E+11	2.00E+11	2.00E+11	2.00E+11	2.00E+11	2.00E+11
H (m)	3.3	3.6	3.6	3.6	3.6	3.6
K (N/m)	446132006.3	343635243.3	210731478.4	210731478.4	143205071.5	143205071.5

2.1 Calculation of first stiffness matrix's column

The first matrix's column was calculated based on Fig. 4.16, where the mass m_1 displaces unity, while the other masses are fixed. Furthermore, the second and third matrix's column were calculated based on Figs. 4.17 and 4.18.



$$F_1 = k_1(\Delta_1 - \Delta_2) = k_1 \xrightarrow{K_{11}=F_1} K_{11} = k_1$$

$$F_2 = k_2(\Delta_2 - \Delta_3) - k_1\ (\Delta_1 - \Delta_2) = -k_1 \xrightarrow{K_{21}=F_2} K_{21} = -k_1$$

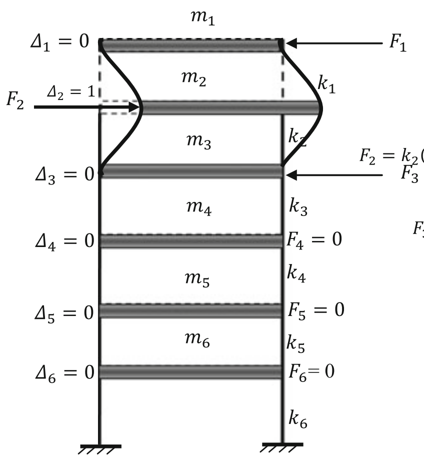
$$F_3 = k_3(\Delta_3 - \Delta_4) - k_2\ (\Delta_2 - \Delta_3) = 0 \xrightarrow{K_{31}=F_3} K_{31} = 0$$

$$F_4 = k_4(\Delta_4 - \Delta_5) - k_3\ (\Delta_3 - \Delta_4) = 0 \xrightarrow{K_{41}=F_4} K_{41} = 0$$

$$F_5 = k_5(\Delta_5 - \Delta_6) - k_4\ (\Delta_4 - \Delta_5) = 0 \xrightarrow{K_{51}=F_5} K_{51} = 0$$

$$F_6 = k_6(\Delta_6) - k_5\ (\Delta_5 - \Delta_6) = 0 \xrightarrow{K_{61}=F_6} K_{61} = 0$$

Fig. 4.16 Determining first column of stiffness matrix



$$F_1 = k_1(\Delta_1 - \Delta_2) = k_1 \xrightarrow{K_{12}=F_1} K_{12} = -k_1$$

$$F_2 = k_2(\overset{1}{\Delta_2} - \overset{0}{\Delta_3}) - k_1\ (\overset{0}{\Delta_1} - \overset{1}{\Delta_2}) = k_2 + k_1 \xrightarrow{K_{22}=F_2} K_{22} = k_2 + k_1$$

$$F_3 = k_3(\overset{0}{\Delta_3} - \overset{0}{\Delta_4}) - k_2\ (\overset{1}{\Delta_2} - \overset{0}{\Delta_3}) = -k_2 \xrightarrow{K_{32}=F_3} K_{32} = -k_2$$

$$F_4 = k_4(\Delta_4 - \Delta_5) - k_3\ (\Delta_3 - \Delta_4) = 0 \xrightarrow{K_{42}=F_4} K_{42} = 0$$

$$F_5 = k_5(\Delta_5 - \Delta_6) - k_4\ (\Delta_4 - \Delta_5) = 0 \xrightarrow{K_{52}=F_5} K_{52} = 0$$

$$F_6 = k_6(\Delta_6) - k_5\ (\Delta_5 - \Delta_6) = 0 \xrightarrow{K_{62}=F_6} K_{62} = 0$$

Fig. 4.17 Calculation of second matrix's column

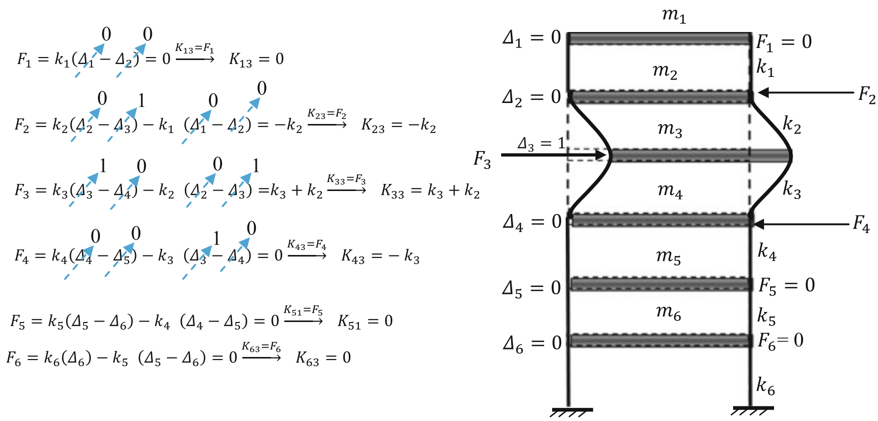


Fig. 4.18 Calculation of third matrix's column

2.2 Calculation of second stiffness matrix's column

See Fig. 4.17.

2.3 Calculation of third stiffness matrix's column

See Fig. 4.18.

This procedure will also be used on the first floor, which means that the stiffness matrix becomes:

$$[K]_{6\times6} = \begin{bmatrix} K_1 & -K_1 & 0 & 0 & 0 & 0 \\ -K_1 & K_1 + K_2 & -K_2 & 0 & 0 & 0 \\ 0 & -K_2 & K_2 + K_3 & -K_3 & 0 & 0 \\ 0 & 0 & -K_3 & K_3 + K_4 & -K_4 & 0 \\ 0 & 0 & 0 & -K_4 & K_4 + K_5 & -K_5 \\ 0 & 0 & 0 & 0 & -K_5 & K_5 + K_6 \end{bmatrix}$$

$$[K]_{6\times6} =$$

$$\begin{bmatrix} 143205071.5 & -143205071.5 & 0 & 0 & 0 & 0 \\ -143205071.5 & 286410143 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7 \end{bmatrix}$$

Step 3: Determining of Fundamental Period

The representation of stiffness and mass in the eigenequation upshot of the physical presentations of natural frequencies and mode shapes:

$$\det\left([\mathrm{k}] - \omega^2[\mathrm{m}]\right) = 0 \tag{4.52}$$

The mass and stiffness matrices are calculated in steps 1 and 2; which means that the natural frequency can be calculated according to Eq. (4.52), where stiffness is substituted with (N/m), while mass is substituted with (kg). The matrix can be determined using MATLAB (for the code, see Appendix)

$$\left\| \begin{bmatrix} 143205071.5 - 162920.582\omega^2 & -143205071.5 & 0 & 0 & 0 & 0 \\ 143205071.5 & 286410143 - 165112.1274\omega^2 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9 - 165605.4045\omega^2 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8 - 166098.252\omega^2 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7 - 166954.8\omega^2 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7 - 158044.182\omega^2 \end{bmatrix} \right\| = 0$$

$$1.95241\mathrm{E}+31\omega^{12} - 3.04691\mathrm{E}+35\omega^{10} + 1.70204\mathrm{E}+39\omega^{8} - 4.23802\mathrm{E}+42\omega^{6} + 4.68042\mathrm{E}+45\omega^{4} - 1.88685\mathrm{E}+48\omega^{2} + 1.39616\mathrm{E}+50 = 0$$

The 6 roots of ω_n^2, of Eq. (4.52) determine the 6 natural frequencies ω_n (n = 1, 2 …, 6) of vibration, and is conventionally arranged in sequence from the smallest to largest ($\omega_1 < \omega_2 < \ldots < \omega_6$). Each eigenvalues represents a free vibration mode of the structure.

$$\omega_1 = 9.7055 < \omega_2 = 25.642 < \omega_3 = 41.1436 < \omega_4 = 51.1018 < \omega_5 = 62.9952 < \omega_6 = 81.127$$

The ith eigenvalue ω_i is related to the ith time period by:

$$T_i = \frac{2\pi}{\omega_i} \tag{4.53}$$

$$T_1 = 0.647(\mathrm{s}), T_2 = 0.245(\mathrm{s}), T_3 = 0.152(\mathrm{s}),$$
$$T_4 = 0.123(\mathrm{s}), T_5 = 0.0997(\mathrm{s}), T_6 = 0.0774(\mathrm{s})$$

T_1 : is the period of the first mode
T_2 : is the period of the second mode
⋮
T_6 : is the period of the sixth mode

Step 4: Modal Shape

The six eigenvalues can be assembled into the matrix. Let the natural mode commensurate the natural frequency, ω_n, and we have elements $\varnothing_{jn}$, where j shows the level j. The six eigenvectors would then become a solitary square matrix, where each column is a natural mode:

$$[\text{ø}] = \begin{bmatrix} ø_{1,1} & ø_{1,2} & ø_{1,3} & ø_{1,4} & ø_{1,5} & ø_{1,6} \\ ø_{2,1} & ø_{2,2} & ø_{2,3} & ø_{2,4} & ø_{2,5} & ø_{2,6} \\ ø_{3,1} & ø_{3,2} & ø_{3,3} & ø_{3,4} & ø_{3,5} & ø_{3,6} \\ ø_{4,1} & ø_{4,2} & ø_{4,3} & ø_{4,4} & ø_{4,5} & ø_{4,6} \\ ø_{5,1} & ø_{5,2} & ø_{5,3} & ø_{5,4} & ø_{5,5} & ø_{5,6} \\ ø_{6,1} & ø_{6,2} & ø_{6,3} & ø_{6,4} & ø_{6,5} & ø_{6,6} \end{bmatrix}$$

The natural modes can be determined by substituting $\omega_1 \ldots \omega_6$ in $\left([\mathrm{k}] - \omega_i^2[\mathrm{m}]\right)\{ø_i\} = 0$ and then:

$$\begin{bmatrix} 143205071.5 - 162920.582(9.7055)^2 & -143205071.5 & 0 & 0 & 0 & 0 \\ 143205071.5 & 286410143 - 165112.1274(9.7055)^2 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9 - 165605.4045(9.7055)^2 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8 - 166098.252(9.7055)^2 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7 - 166954.8(9.7055)^2 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7 - 158044.182(9.7055)^2 \end{bmatrix} \begin{bmatrix} ø_{1,1} \\ ø_{2,1} \\ ø_{3,1} \\ ø_{4,1} \\ ø_{5,1} \\ ø_{6,1} \end{bmatrix} = 0$$

Now select any value for one unknown, say $ø_{11} = 1$. Then the first–fifth of the six equations gives

$$\begin{bmatrix} ø_{1,1} \\ ø_{2,1} \\ ø_{3,1} \\ ø_{4,1} \\ ø_{5,1} \\ ø_{6,1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.892834898 \\ 0.688701992 \\ 0.4989996 \\ 0.272248619 \\ 0.12073589 \end{bmatrix}$$

Figure 4.19 shows the first modal shape of considered building.

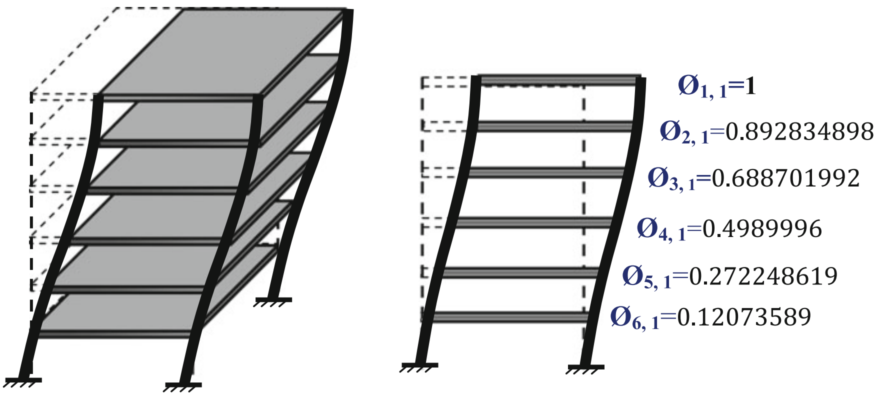


Fig. 4.19 The First modal shape of considered building

Second Modal Shape:

$$\begin{bmatrix} 143205071.5-162920.582(25.642)^2 & -143205071.5 & 0 & 0 & 0 & 0 \\ 143205071.5 & 286410143-165112.1274(25.642)^2 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9-165605.4045(25.642)^2 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8-166098.252(25.642)^2 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7-166954.8(25.642)^2 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7-158044.182(25.642)^2 \end{bmatrix} \begin{bmatrix} \emptyset_{1,2} \\ \emptyset_{2,2} \\ \emptyset_{3,2} \\ \emptyset_{4,2} \\ \emptyset_{5,2} \\ \emptyset_{6,2} \end{bmatrix} = 0$$

Now select any value for one unknown, say $\emptyset_{12} = 1$. Then the first–fifth of the six equations gives

$$\begin{bmatrix} \emptyset_{1,2} \\ \emptyset_{2,2} \\ \emptyset_{3,2} \\ \emptyset_{4,2} \\ \emptyset_{5,2} \\ \emptyset_{6,2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.251965986 \\ -0.687082505 \\ -0.970199977 \\ -0.750511161 \\ -0.376036852 \end{bmatrix}$$

Figure 4.20 shows the second modal shape of considered building.

Third Modal Shape:

$$\begin{bmatrix} 143205071.5-162920.582(41.1436)^2 & -143205071.5 & 0 & 0 & 0 & 0 \\ 143205071.5 & 286410143-165112.1274(41.1436)^2 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9-165605.4045(41.1436)^2 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8-166098.252(41.1436)^2 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7-166954.8(41.1436)^2 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7-158044.182(41.1436)^2 \end{bmatrix} \begin{bmatrix} \emptyset_{1,3} \\ \emptyset_{2,3} \\ \emptyset_{3,3} \\ \emptyset_{4,3} \\ \emptyset_{5,3} \\ \emptyset_{6,3} \end{bmatrix} = 0$$

Now select any value for one unknown, say $\emptyset_{13} = 1$. Then the first–fifth of the six equations gives

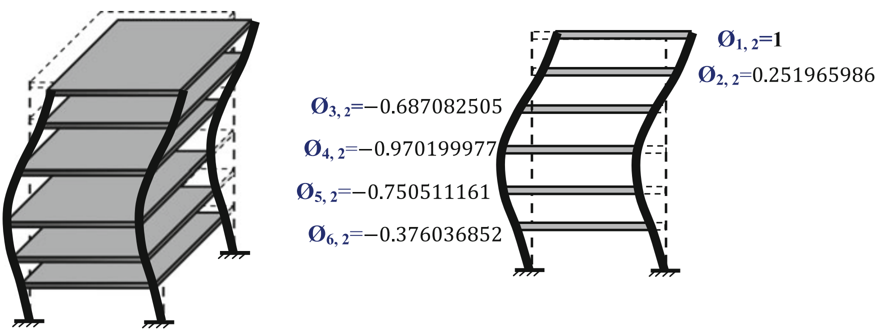


Fig. 4.20 The second modal shape of considered building

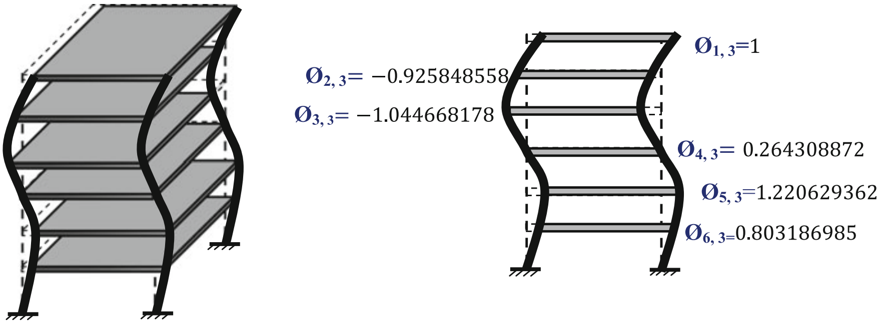


Fig. 4.21 The third modal shape of considered building

$$\begin{bmatrix} \varnothing_{1,3} \\ \varnothing_{2,3} \\ \varnothing_{3,3} \\ \varnothing_{4,3} \\ \varnothing_{5,3} \\ \varnothing_{6,3} \end{bmatrix} = \begin{bmatrix} 1 \\ -0.925848558 \\ -1.044668178 \\ 0.264308872 \\ 1.220629362 \\ 0.803186985 \end{bmatrix}$$

Figure 4.21 shows the third modal shape of considered building.

Fourth Modal Shape:

$$\begin{bmatrix} 143205071.5-162920.582(51.1018)^2 & -143205071.5 & 0 & 0 & 0 & 0 \\ 143205071.5 & 286410143-165112.1274(51.1018)^2 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9-165605.4045(51.1018)^2 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8-166098.252(51.1018)^2 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7-166954.8(51.1018)^2 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7-158044.182(51.1018)^2 \end{bmatrix} \begin{bmatrix} \varnothing_{1,4} \\ \varnothing_{2,4} \\ \varnothing_{3,4} \\ \varnothing_{4,4} \\ \varnothing_{5,4} \\ \varnothing_{6,4} \end{bmatrix} = 0$$

Now select any value for one unknown, say $\varnothing_{14} = 1$. Then the first–fifth of the six equations gives

$$\begin{bmatrix} \varnothing_{1,4} \\ \varnothing_{2,4} \\ \varnothing_{3,4} \\ \varnothing_{4,4} \\ \varnothing_{5,4} \\ \varnothing_{6,4} \end{bmatrix} = \begin{bmatrix} 1 \\ -1.97091311 \\ 0.992350135 \\ 0.969580111 \\ -1.048873759 \\ -0.955922866 \end{bmatrix}$$

Figure 4.22 shows the fourth modal shape of considered building.

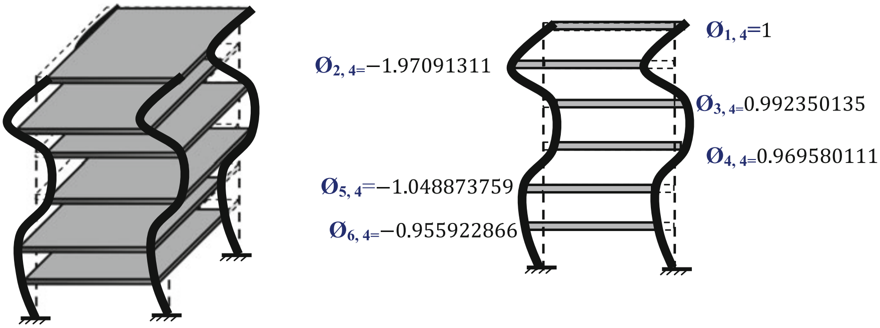


Fig. 4.22 The fourth modal shape of considered building

Fifth Modal Shape:

$$\begin{bmatrix} 143205071.5-162920.582(62.9952)^2 & -143205071.5 & 0 & 0 & 0 & 0 \\ 143205071.5 & 286410143-165112.1274(62.9952)^2 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9-165605.4045(62.9952)^2 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8-166098.252(62.9952)^2 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7-166954.8(62.9952)^2 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7-158044.182(62.9952)^2 \end{bmatrix} \begin{bmatrix} ø_{1,5} \\ ø_{2,5} \\ ø_{3,5} \\ ø_{4,5} \\ ø_{5,5} \\ ø_{6,5} \end{bmatrix} = 0$$

Now select any value for one unknown, say $ø_{15} = 1$. Then the first–fifth of the six equations gives

$$\begin{bmatrix} ø_{1,5} \\ ø_{2,5} \\ ø_{3,5} \\ ø_{4,5} \\ ø_{5,5} \\ ø_{6,5} \end{bmatrix} = \begin{bmatrix} 1 \\ -3.521473717 \\ 8.052092019 \\ -9.198805803 \\ 2.323089073 \\ 4.909786487 \end{bmatrix}$$

Figure 4.23 shows the fifth modal shape of considered building.

Sixth Modal Shape:

$$\begin{bmatrix} 143205071.5-162920.582(81.127)^2 & -143205071.5 & 0 & 0 & 0 & 0 \\ 143205071.5 & 286410143-165112.1274(81.127)^2 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9-165605.4045(81.127)^2 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8-166098.252(81.127)^2 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7-166954.8(81.127)^2 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7-158044.182(81.127)^2 \end{bmatrix} \begin{bmatrix} ø_{1,6} \\ ø_{2,6} \\ ø_{3,6} \\ ø_{4,6} \\ ø_{5,6} \\ ø_{6,6} \end{bmatrix} = 0$$

Now select any value for one unknown, say $ø_{1,6} = 1$. Then the first–fifth of the six equations gives

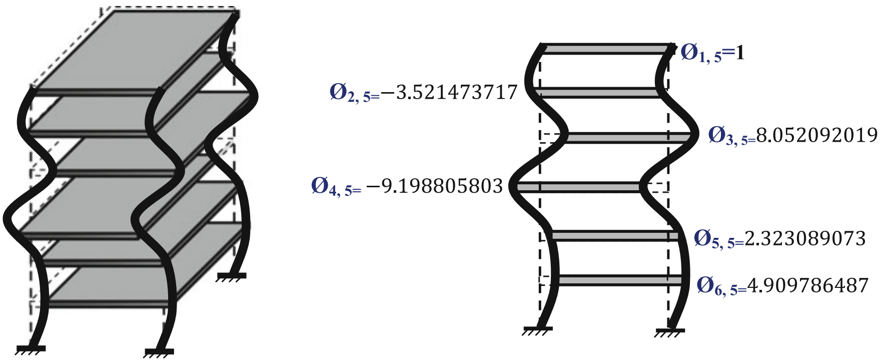


Fig. 4.23 The fifth modal shape of considered building

$$\begin{bmatrix} \text{ø}_{1,6} \\ \text{ø}_{2,6} \\ \text{ø}_{3,6} \\ \text{ø}_{4,6} \\ \text{ø}_{5,6} \\ \text{ø}_{6,6} \end{bmatrix} = \begin{bmatrix} 1 \\ -6.487699165 \\ 35.25599317 \\ -118.7278924 \\ 343.2010358 \\ -470.9644759 \end{bmatrix}$$

Figure 4.24 shows the sixth modal shape of considered building.

Step 5: Determining of Effective Modal Weight and Normalized Modal

After determining the modal vectors for a structure with 6 DoFs, the next steps is determining the normalized vectors and effective weight of modals, which is calculated using Eqs. (4.36), (4.37). To normalize the first mode, M_1 can be calculated using Eq. (4.36), with ø_1 given by Eq. (4.52):

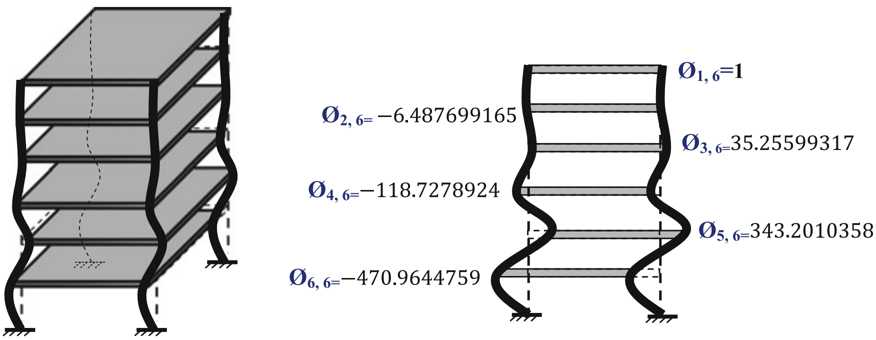


Fig. 4.24 The sixth modal shape of considered building

$$M_1 = \text{Ø}_1^T m \text{Ø}_{1} = 1$$

$$\begin{bmatrix} 1 \\ 0.892834898 \\ 0.688701992 \\ 0.4989996 \\ 0.272248619 \\ 0.12073589 \end{bmatrix}^T \begin{bmatrix} 162920.582 & 0 & 0 & 0 & 0 & 0 \\ 0 & 165112.1274 & 0 & 0 & 0 & 0 \\ 0 & 0 & 65605.4045 & 0 & 0 & 0 \\ 0 & 0 & 0 & 166098.252 & 0 & 0 \\ 0 & 0 & 0 & 0 & 166954.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 158044.182 \end{bmatrix} \begin{bmatrix} 1 \\ 0.892834898 \\ 0.688701992 \\ 0.4989996 \\ 0.272248619 \\ 0.12073589 \end{bmatrix} = 429125.7454$$

To make $M_1 = 1$, divide ø_1 by $\left(\sqrt{429125.7454}\right)$ to obtain the normalized mode

$$\text{ø}_1 = \frac{1}{\sqrt{429125.7454}} \begin{bmatrix} 1 \\ 0.892834898 \\ 0.688701992 \\ 0.4989996 \\ 0.272248619 \\ 0.12073589 \end{bmatrix} = \begin{bmatrix} 0.001526538 \\ 0.001362947 \\ 0.00105133 \\ 0.000761742 \\ 0.000415598 \\ 0.000184308 \end{bmatrix}$$

The other modes can be normalized similarly.

$$\text{ø}_2 = \frac{1}{\sqrt{524316.8786}} \begin{bmatrix} 1 \\ 0.251965986 \\ -0.687082505 \\ -0.970199977 \\ -0.750511161 \\ -0.376036852 \end{bmatrix} = \begin{bmatrix} 0.00138103 \\ 0.000347973 \\ -0.000948881 \\ -0.001339875 \\ -0.001036478 \\ -0.000519318 \end{bmatrix}$$

$$\text{ø}_3 = \frac{1}{\sqrt{847495.6126}} \begin{bmatrix} 1 \\ -0.925848558 \\ -1.044668178 \\ 0.264308872 \\ 1.220629362 \\ 0.803186985 \end{bmatrix} = \begin{bmatrix} 0.001086254 \\ -0.001005706 \\ -0.001134775 \\ 0.000287106 \\ 0.001325913 \\ 0.000872465 \end{bmatrix}$$

$$\text{ø}_4 = \frac{1}{\sqrt{1451618.315}} \begin{bmatrix} 1 \\ -0.925848558 \\ -1.044668178 \\ 0.264308872 \\ 1.220629362 \\ 0.803186985 \end{bmatrix} = \begin{bmatrix} 0.000829992 \\ -0.001635842 \\ 0.000823642 \\ 0.000804743 \\ -0.000870557 \\ -0.000793408 \end{bmatrix}$$

$$ø_5 = \frac{1}{\sqrt{31713394.34}} \begin{bmatrix} 1 \\ -3.521473717 \\ 8.052092019 \\ -9.198805803 \\ 2.323089073 \\ 4.909786487 \end{bmatrix} = \begin{bmatrix} 0.000177574 \\ -0.000625321 \\ 0.00142984 \\ -0.001633466 \\ 0.000412519 \\ 0.000871849 \end{bmatrix}$$

$$ø_6 = \frac{1}{\sqrt{5.72748181e10}} \begin{bmatrix} 1 \\ -6.487699165 \\ 35.25599317 \\ -118.7278924 \\ 343.2010358 \\ -470.9644759 \end{bmatrix} = \begin{bmatrix} 0.000004178 \\ -0.00002710 \\ 0.000147316 \\ -0.00049610 \\ 0.001434055 \\ -0.00196791 \end{bmatrix}$$

According to the Eq. (4.37), the effective modal mass of each mode is calculated as follow:

$$W_i = \frac{\left[\sum_{j=1}^{n}\left(m_i ø_{j,i}\right)\right]^2}{\sum_{j=1}^{n}\left(m_j ø_{j,i}^2\right)}$$

$$\mathrm{w}_1 = \frac{\left[\left(\mathrm{m}_1 \times ø_{1,1}\right) + \left(\mathrm{m}_2 \times ø_{2,1}\right) + \left(\mathrm{m}_3 \times ø_{3,1}\right) + \left(\mathrm{m}_4 \times ø_{4,1}\right) + \left(\mathrm{m}_5 \times ø_{5,1}\right) + \left(\mathrm{m}_6 \times ø_{6,1}\right)\right]^2}{\left[\left(\mathrm{m}_1 \times ø_{1,1,}^2\right) + \left(\mathrm{m}_2 \times ø_{2,1,}^2\right) + \left(\mathrm{m}_3 \times ø_{3,1,}^2\right) + \left(\mathrm{m}_4 \times ø_{4,1,}^2\right) + \left(\mathrm{m}_5 \times ø_{5,1,}^2\right) + \left(\mathrm{m}_6 \times ø_{6,1,}^2\right)\right]}$$

$$w_1 = 761934.0947(\mathrm{kg}) \approx 7619.341(\mathrm{kN})$$

The other effective modal weight can be calculated similarly; hence.

$$\begin{aligned} w_2 &= 124156.4333\ (\mathrm{kg}) \approx 1241.564333\ (\mathrm{kN}) \\ w_3 &= 52871.20061\ (\mathrm{kg}) \approx 528.7120061\ (\mathrm{kN}) \\ w_4 &= 18372.5985\ (\mathrm{kg}) \approx 183.725985\ (\mathrm{kN}) \\ w_5 &= 9568.444709\ (\mathrm{kg}) \approx 95.6844470\ (\mathrm{kN}) \\ w_6 &= 17794.25128\ (\mathrm{kg}) \approx 177.9425128\ (\mathrm{kN}) \end{aligned}$$

It can be seen that the total effective modal weight of the structure is equal to the total weight of the structure in the lumped mass method:

$$\begin{aligned} W_{total} &= w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 9846.97\ (\mathrm{kN}) \\ M_{total} &= m_1 + m_2 + m_3 + m_4 + m_5 + m_6 = 9847.35\ (\mathrm{kN}) \\ W_{total} &\approx M_{total} \end{aligned}$$

If we want to determine the proportional rate of each modes' contribution to the structure's oscillation, it must be calculated to the proportion of the modal's weight with respect to the total weight of the structure:

The contribution rate of the first mode:

$$\frac{W_1}{W_{Total}} \times 100 = \frac{7619.341}{9847.35} \times 100 = 77.3\%$$

The contribution rate of the second mode:

$$\frac{W_2}{W_{Total}} \times 100 = \frac{1241.564333}{9847.35} \times 100 = 12.6\%$$

The contribution rate of the third mode:

$$\frac{W_3}{W_{Total}} \times 100 = \frac{528.7120061}{9847.35} \times 100 = 5.37\%$$

The contribution rate of the fourth mode:

$$\frac{W_4}{W_{Total}} \times 100 = \frac{183.725985}{9847.35} \times 100 = 1.86\%$$

The contribution rate of the fifth mode:

$$\frac{W_5}{W_{Total}} \times 100 = \frac{95.6844470}{9847.35} \times 100 = 0.971\%$$

The contribution rate of the sixth mode:

$$\frac{W_6}{W_{Total}} \times 100 = \frac{177.9425128}{9847.35} \times 100 = 1.8\%$$

Step 6: Calculation Base Shear and Modal Force

The seismic shear on the top/above the foundation of a basement that is rigid ("base shear"}, V_b, can be determined using the 1st translational mode period, T_1, either in X/Y direction:

$$V_b = m_{eff1} \times S_{a.d}(T_1) \tag{54}$$

Shear base in the first mode is calculated to be

Step 6.1: Ground type

Class C

Step 6.2: Appropriate type of elastic response spectrum and components

Ms $\geq$ 5.5 high-risk zone

It can be seen in Table 4.8 the value of the parameters describing Type 1 elastic response spectrum with ground Type C.

Step 6.3: Important factor

Based on part (4.25) and Table 4.3 of Euro code eight the building is classified in class II and importance the factor is 1 (γ = 1).

Step 6.4: Natural period of the first mode

T_1 = 0.647384

Step 6.5: Behaviour factor of horizontal seismic action

$$q = 4$$

Step 6.6: Design spectrum

$$T_C \leq T \leq T_D : \quad S_d(T) = \begin{cases} a_g.S.\frac{2.5}{q}.\left[\frac{T_C}{T}\right] \\ \geq \beta.a_g \end{cases}$$

$$0.6 \leq 0.647384 \leq 2 : \quad S_d(T) = \begin{cases} 0.3 \times 1.15 \times \frac{2.5}{4} \times \left[\frac{0.6}{0.647384}\right] \\ \geq 0.2 \times 0.3 \end{cases} = 0.199842755$$

Step 6.7: Shear base

$$V_b = m_{eff1} \times S_{a.d}\,(T_1)$$

where

$$m_{eff1} : 7619.341\ (\text{kN})$$
$$S_{a.d}\,(T_1) = 0.199842755$$
$$V_b = 1522.67\,(kN)$$

Step 6.8: Distribution of shear base in the height of structure of the first mode.

The shear base is distributed in the height of the structure of the first mode based on Eq. (4.45):

$$F_i = \frac{\text{ø}_i m_i}{\sum_j \text{ø}_j m_j} \times V_b$$

Magnitude of Table 4.9 is substituted in Eq. (4.45), and (Fig. 4.25), shows the magnitudes of force, shear, and moment of first mode in considered building.

Table 4.8 Value of the parameters describing Type 1 elastic response spectrum with ground type C

Ground type	S	T_B(s)	T_C(s)	T_D(s)
C	1.15	0.2	0.6	2

Table 4.9 Essential parameters for determining of shear base of the first mode

w_1	w_2	w_3	w_4	w_5	w_6	w_7
162920.582	165112.1274	65605.4045	166098.252	166954.8	158044.182	1522.67 (kN)
$\varnothing_{1,1}$	$\varnothing_{2,1}$	$\varnothing_{3,1}$	$\varnothing_{4,1}$	$\varnothing_{5,1}$	$\varnothing_{6,1}$	
1	0.892834898	0.688701992	0.4989996	0.272248619	0.12073589	

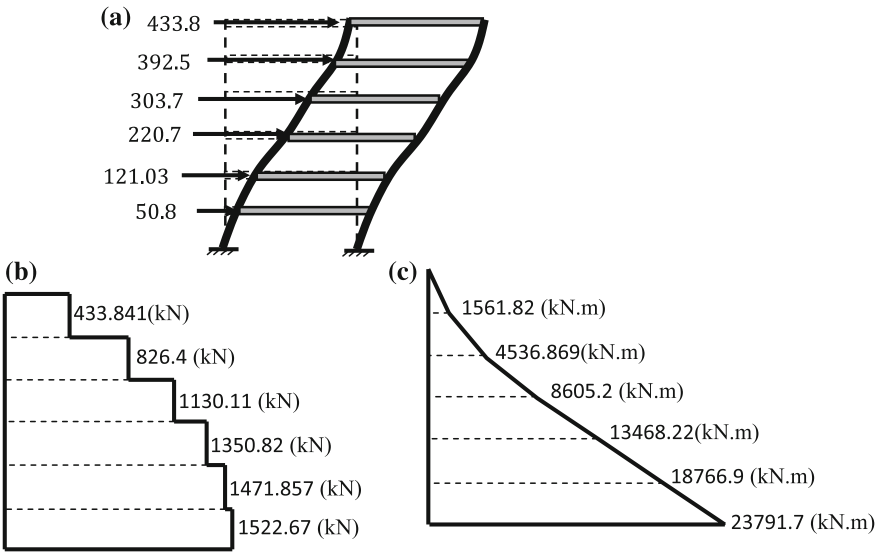


Fig. 4.25 **a** Forces of first mode, **b** shear force of first mode, **c** moment of the first mode

$$F_{1,1} = \frac{w_1 \varnothing_{1,1}}{w_1 \varnothing_{1,1} + w_2 \varnothing_{2,1} + w_3 \varnothing_{3,1} + w_4 \varnothing_{4,1} + w_5 \varnothing_{5,1} + w_6 \varnothing_{6,1}} V_b = 433.8411868\,(\text{kN})$$

$$F_{2,1} = \frac{w_2 \varnothing_{2,1}}{w_1 \varnothing_{1,1} + w_2 \varnothing_{2,1} + w_3 \varnothing_{3,1} + w_4 \varnothing_{4,1} + w_5 \varnothing_{5,1} + w_6 \varnothing_{6,1}} V_b = 392.5590165\,(\text{kN})$$

$$F_{3,1} = \frac{w_3 \varnothing_{3,1}}{w_1 \varnothing_{1,1} + w_2 \varnothing_{2,1} + w_3 \varnothing_{3,1} + w_4 \varnothing_{4,1} + w_5 \varnothing_{5,1} + w_6 \varnothing_{6,1}} V_b = 303.7111049\,(\text{kN})$$

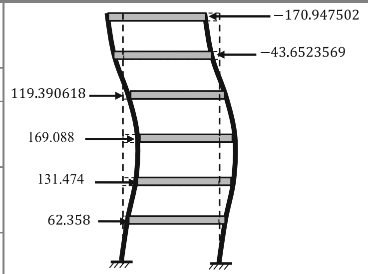
$$F_{4,1} = \frac{w_4 \varnothing_{4,1}}{w_1 \varnothing_{1,1} + w_2 \varnothing_{2,1} + w_3 \varnothing_{3,1} + w_4 \varnothing_{4,1} + w_5 \varnothing_{5,1} + w_6 \varnothing_{6,1}} V_b = 220.709022\,(\text{kN})$$

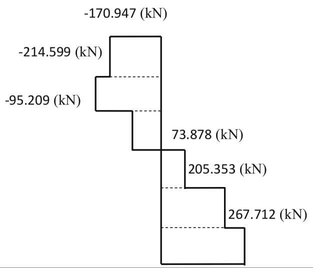
$$F_{5,1} = \frac{w_5 \varnothing_{5,1}}{w_1 \varnothing_{1,1} + w_2 \varnothing_{2,1} + w_3 \varnothing_{3,1} + w_4 \varnothing_{4,1} + w_5 \varnothing_{5,1} + w_6 \varnothing_{6,1}} V_b = 121.0373542\,(\text{kN})$$

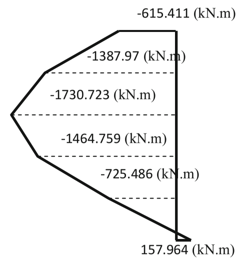
$$F_{6,1} = \frac{w_6 \varnothing_{6,1}}{w_1 \varnothing_{1,1} + w_2 \varnothing_{2,1} + w_3 \varnothing_{3,1} + w_4 \varnothing_{4,1} + w_5 \varnothing_{5,1} + w_6 \varnothing_{6,1}} V_b = 50.81240213\,(\text{kN})$$

Step 6.9: Distribution of shear base in the height of structure in the second mode

Second mode			Force (kN)
1. Period of first mode	T_2	0.245034916	$F_{1,2} = -170.9475024$
2. Behaviour factor	q	4	
3. Design spectrum			$F_{2,2} = -43.65235696$
$T_B \leq T \leq T_C$ $S_d(T) = a_g.S.\frac{2.5}{q}$	T_2	0.245034916	$F_{3,2} = 119.3906188$
	TB	0.2	
	TC	0.6	$F_{4,2} = 169.088133$
	TD	2	
	ag	0.3	$F_{5,2} = 131.4749069$
	ß	0.2	
	S	1.15	$F_{6,2} = 62.35850986$
	$S_d(T2)$	0.215625	
4. Effective mass	M2	1241.564333	$\sum F = 267.7123092$
5. Shear force in mode	V	267.7123092	

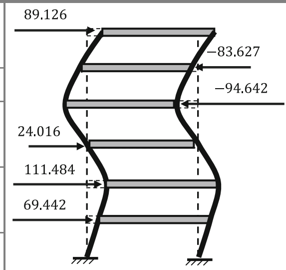


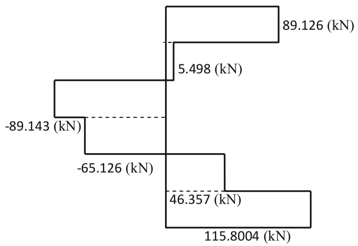


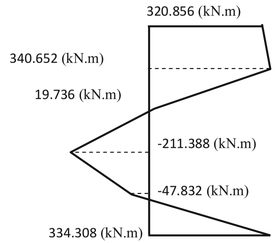


Step 6.10: Distribution of shear base in the height of structure in the third mode

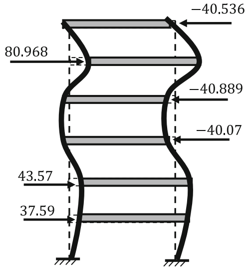
Third mode			Force (kN)
1. Period of first mode	T_3	0.152713552	$F_{1,3} = 89.12672487$
2. Behaviour factor	q	4	
3. Design spectrum			$F_{2,3} = -83.62784825$
$0 \leq T \leq T_B$	T_3	0.152713552	$F_{3,3} = -94.64220858$
$S_d(T) = a_g.S.\left[\frac{2}{3} + \frac{T}{T_B}\cdot\left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$	TB	0.2	
	TC	0.6	$F_{4,3} = 24.01644919$
	TD	2	
	ag	0.3	$F_{5,3} = 111.4845582$
	ß	0.2	
	S	1.15	$F_{6,3} = 69.44279146$
	S_d (T3)	0.219023713	
4. Effective mass	M3	528.7120061	$\sum F = 115.8004669$
5. Shear force in mode	V	115.8004669	

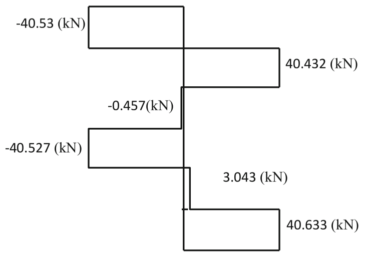


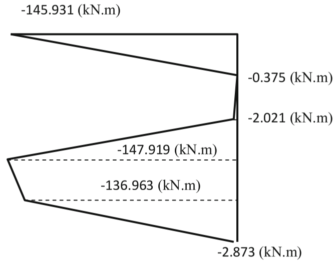




Step 6.11: Distribution of shear base in the height of structure in the fourth mode

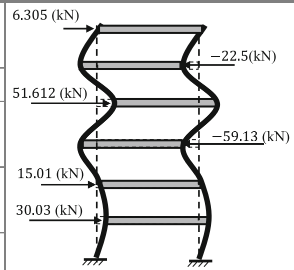
Fourth mode			Force (kN)	
1. Period of first mode	T_4	0.122954286	$F_{1,4} = -40.53655446$	
2. Behaviour factor	q	4		
3. Design spectrum			$F_{2,4} = 80.96873052$	
$0 \leq T \leq T_B$ $S_d(T) = a_g.S.\left[\frac{2}{3} + \frac{T}{T_B}.\left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$	T_4	0.122954286	$F_{3,4} = -40.88936043$	
	TB	0.2		
	TC	0.6	$F_{4,4} = -40.07002736$	
	TD	2		
	ag	0.3	$F_{5,4} = 43.57054664$	
	ß	0.2		
	S	1.15	$F_{6,4} = 37.58999277$	
	S_d (T4)	0.221162661		
4. Effective mass	M4	183.725985	$\sum F = 40.63332768$	
5. Shear force in mode	V	40.63332768		

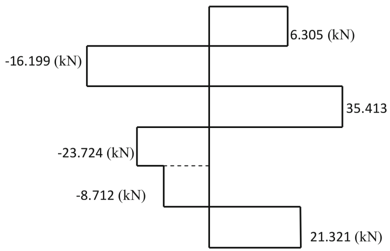


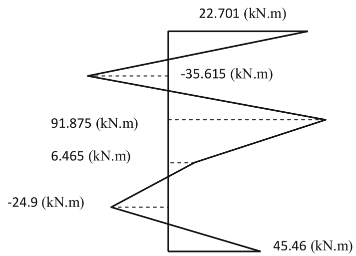


Step 6.12: Distribution of shear base in the height of structure in the fifth mode

Fifth mode			Force (kN)
1. Period of first mode	T_5	0.099740699	$F_{1,5} = 6.305958982$
2. Behaviour factor	q	4	
3. Design spectrum			$F_{2,5} = -22.50497906$
$0 \leq T \leq T_B$ $S_d(T) = a_g.S.\left[\frac{2}{3} + \frac{T}{T_B}.\left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$	T_5	0.099740699	$F_{3,5} = 51.61291927$
	TB	0.2	
	TC	0.6	$F_{4,5} = -59.13869015$
	TD	2	
	ag	0.3	$F_{5,5} = 15.01204855$
	ß	0.2	
	S	1.15	$F_{6,5} = 30.03421656$
	S_d (T5)	0.222831137	
4. Effective mass	M5	95.68444709	$\sum F = 21.32147416$
5. Shear force in mode	V	21.32147416	

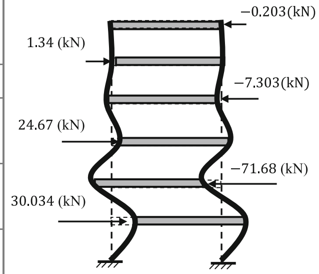


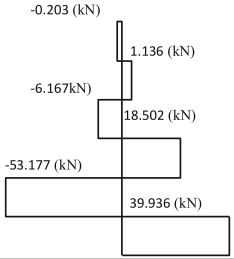


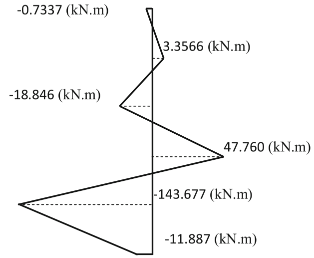


Step 6.13: Distribution of shear base in the height of structure in the sixth mode

Sixth mode			Force (kN)
1. Period of first mode	T_6	0.077448757	$F_{1,6} = -0.203808$
2. Behaviour factor	q	4	
3. Design spectrum			$F_{2,6} = 1.340031$
$0 \leq T \leq T_B$ $S_d(T) = a_g.S.\left[\frac{2}{3} + \frac{T}{T_B}.\left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$	T_6	0.077448757	$F_{3,6} = -7.303867$
	TB	0.2	
	TC	0.6	$F_{4,6} = 24.66966$
	TD	2	
	ag	0.3	$F_{5,6} = -71.679164$
	ß	0.2	
	S	1.15	$F_{6,6} = 30.03421$
	S_d(T6)	0.224433371	
4. Effective mass	M6	177.9425128	$\sum F = 93.11338$
5. Shear force in mode	V	39.93623791	







Step 6.14: Minimum number of modes

The modal response spectrum analysis took into account all modes that significantly contribute to response quantity. In fact, this is seen as challenging mainly because the number of modes embedded serves as inputs to the analysis of eigenvalue. For instance, Euro code 8 consider modes with effective modal mass and one of the seismic action components; X, Y, or Z, after taking the design into account at a minimal of 5% of the total mass response quantities applied to the local authentications (inter floor drifts, member chord rotations, or internal forces) could appear to be more sensitive to incorporated modes. Such modes may make up more than 90% of the total mass towards predicting the peak values of related quantities. Table 4.10 shows some of these properties related to each mode to determine the minimum number of modes. As per Table 4.10, modes 1–3 should be taken into account when responding to a quantity of interest, as they account for much more than 90% of the total mass. The first three modes of considered building are shown in Fig. 4.26.

Mode 1–2: 77.3% + 12.6% = 89.9% < 90% Mode 1–3: 77.3% + 12.6% + 5.37% = 95.27% > 90%

Step 7: Calculation of Displacement

Displacement in each mode is calculated based on Hooke's law, where the displacement of structure in each mode is calculated by dividing force with the stiffness of the structure.

$$F = K\Delta \tag{54}$$

Table 4.10 Some properties of six modes

Mode number	1	2	3	4	5	6
Period	0.647384	0.245035	0.152714	0.122954	0.099741	0.077449
Effective weight	77.3%	12.6%	5.37%	1.86%	0.97%	1.8%

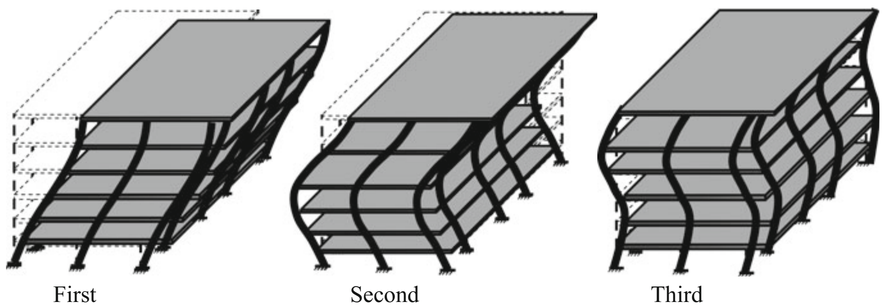


Fig. 4.26 The first three modes of considered building

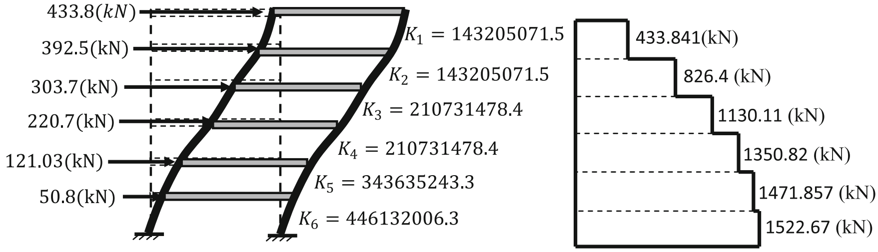


Fig. 4.27 Force, shear, and stiffness of the first mode

Force, shear and stiffness of the first mode is shown in Fig. 4.27

Step 7.1: Displacement of the first mode

$$\Delta_{6,1} = \frac{V_{6,1}}{K_6} = \frac{1522670(\mathrm{N})}{446132006.3\left(\frac{\mathrm{N}}{\mathrm{m}}\right)} = 0.003413$$

$$\Delta_{5,1} = \frac{V_{5,1}}{K_5} + \Delta_{6,1} = \frac{1471858(\mathrm{N})}{343635243.3\left(\frac{\mathrm{N}}{\mathrm{m}}\right)} + 0.003413 = 0.007696$$

$$\Delta_{4,1} = \frac{V_{4,1}}{K_4} + \Delta_{5,1} = \frac{1350820(\mathrm{N})}{210731478.4\left(\frac{\mathrm{N}}{\mathrm{m}}\right)} + 0.007696 = 0.014106$$

$$\Delta_{3,1} = \frac{V_{3,1}}{K_3} + \Delta_{4,1} = \frac{1130111(N)}{210731478.4\left(\frac{\mathrm{N}}{\mathrm{m}}\right)} + 0.014106 = 0.019469$$

$$\Delta_{2,1} = \frac{V_{2,1}}{K_2} + \Delta_{3,1} = \frac{826400.2(\mathrm{N})}{143205071.5} + 0.019469 = 0.02524$$

$$\Delta_{1,1} = \frac{V_{1,1}}{K_1} + \Delta_{2,1} = \frac{433841.2(\mathrm{N})}{143205071.5} + 0.019469 = 0.028269$$

Table 4.11 shows the displacement of each story in the first mode
Step 7.2: Displacement of the second mode (Fig. 4.28 and Table 4.12)
Step 7.3: Displacement of the third mode (Fig. 4.29 and Table 4.13)

Table 4.11 Displacement of the first mode

Story	Shear	Stiffness	δ	Δ
6	433841.2	143205071.5	0.00303	0.028269
5	826400.2	143205071.5	0.005771	0.02524
4	1130111	210731478.4	0.005363	0.019469
3	1350820	210731478.4	0.00641	0.014106
2	1471858	343635243.3	0.004283	0.007696
1	1522670	446132006.3	0.003413	0.003413

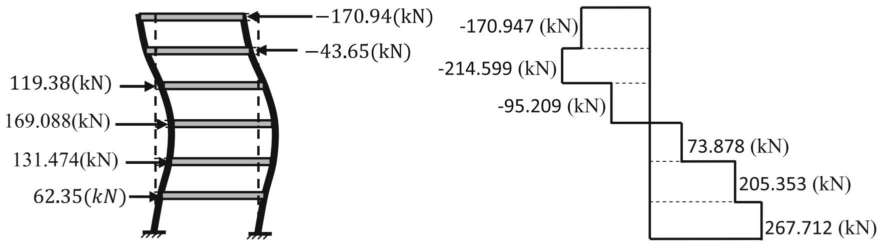


Fig. 4.28 Force, shear, and stiffness of the second mode

Table 4.12 Displacement of the second mode

Story	Shear (N)	Stiffness (N/m)	δ (m)	Δ (m)
6	−170948	143205071.5	−0.00119	−0.0016
5	−214600	143205071.5	−0.0015	−0.0004
4	−95209.2	210731478.4	-0.00045	0.001096
3	73878.89	210731478.4	0.000351	0.001548
2	205353.8	343635243.3	0.000598	0.001198
1	267712.3	446132006.3	0.0006	0.0006

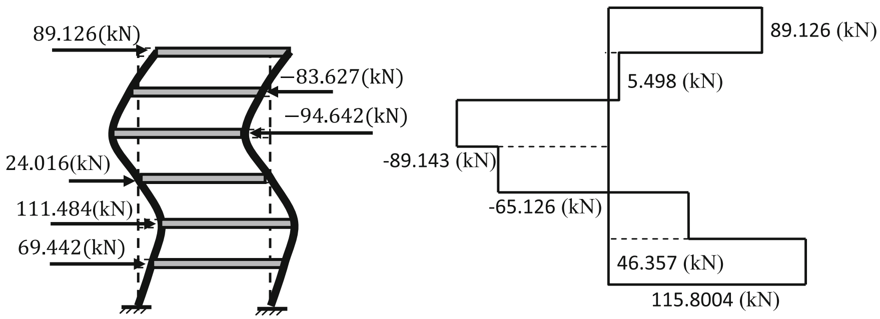


Fig. 4.29 Force, shear, and stiffness of the third mode

Table 4.13 Displacement of the third mode

Story	Shear (N)	Stiffness (N/m)	δ (m)	Δ (m)
6	89126.72	143205071.5	0.000622	0.000323
5	5498.877	143205071.5	3.84E-05	−0.0003
4	−89143.3	210731478.4	−0.00042	−0.00034
3	−65126.9	210731478.4	−0.00031	0.0000854
2	46357.68	343635243.3	0.000135	0.000394
1	115800.5	446132006.3	0.00026	0.00026

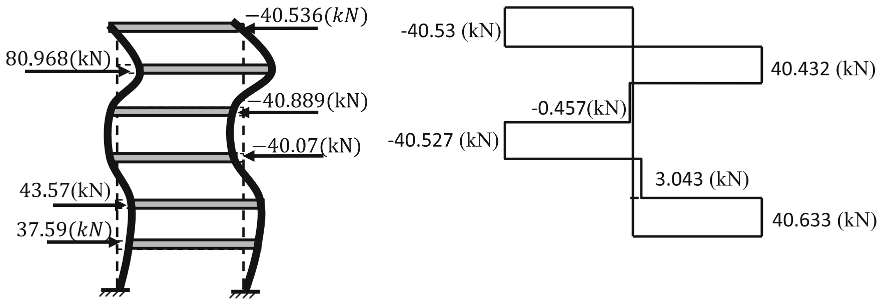


Fig. 4.30 Force, shear, and stiffness of the fourth mode

Table 4.14 Displacement of the fourth mode

Story	Shear (N)	Stiffness (N/m)	δ (m)	Δ (m)
6	−40536.6	143205071.5	−0.00028	−9.52798E-05
5	40432.18	143205071.5	0.000282	0.000187787
4	−457.184	210731478.4	−2.2E-06	−9.45509E-05
3	−40527.2	210731478.4	−0.00019	−9.23814E-05
2	3043.335	343635243.3	8.86E-06	9.99355E-05
1	40633.33	446132006.3	9.11E-05	9.10792E-05

Step 7.4: Displacement of the fourth mode (Fig. 4.30 and Table 4.14)
Step 7.5: Displacement of the fifth mode (Fig. 4.31 and Table 4.15)
Step 7.6: Displacement of the sixth mode (Fig. 4.32 and Table 4.16)

Step 8: The Combination of Modal Results:

Based on Euro code 8, the combination of the modal result can be determined using the CQC rule, therefore:

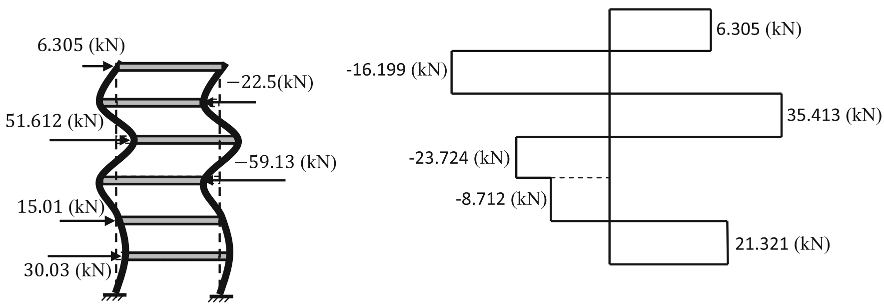


Fig. 4.31 Force, shear, and stiffness of the fifth mode

Table 4.15 Displacement of the fifth mode

Story	Shear (N)	Stiffness (N/m)	δ (m)	Δ (m)
6	6305.959	143205071.5	4.4E-05	8.82326E-06
5	−16199	143205071.5	−0.00011	−3.52112E-05
4	35413.9	210731478.4	0.000168	7.79064E-05
3	−23724.8	210731478.4	−0.00011	−9.01458E-05
2	−8712.74	343635243.3	−2.5E-05	2.24372E-05
1	21321.47	446132006.3	4.78E-05	4.77919E-05

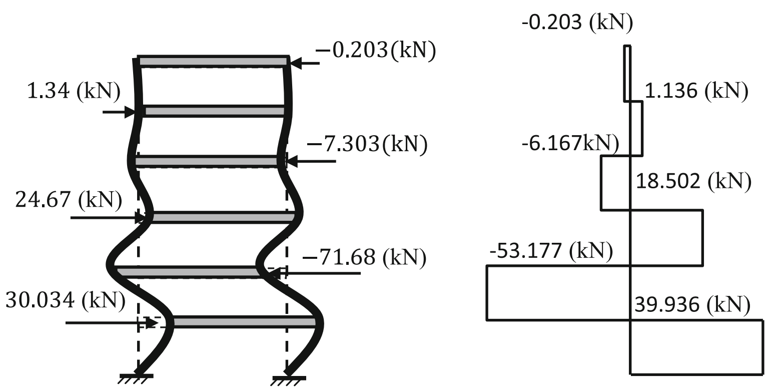


Fig. 4.32 Force, shear, and stiffness of the sixth mode

Table 4.16 Displacement of the sixth mode

Story	Shear (N)	Stiffness (N/m)	δ (m)	Δ (m)
6	−203.808	143205071.5	−1.4E-06	−1.8986E-07
5	1136.224	143205071.5	7.93E-06	1.23333E-06
4	−6167.64	210731478.4	−2.9E-05	−6.70091E-06
3	18502.02	210731478.4	8.78E-05	2.25669E-05
2	−53177.1	343635243.3	−0.00015	−6.52322E-05
1	39936.24	446132006.3	8.95E-05	8.95166E-05

$$V = \sqrt{\sum\nolimits_{i=1}^{6} V_i^2 + 2\sum\nolimits_{i=1}^{5} \sum\nolimits_{j=i+1}^{6} \rho_{ij} v_i v_j} \tag{4.55}$$

$$\rho_{ij} = \frac{8\xi^2(1+\lambda)\lambda^{\frac{3}{2}}}{\left(1-\lambda^2\right)^2 + 4\xi^2\lambda(1+\lambda)^2} \tag{4.56}$$

$$\lambda = \frac{T_j}{T_i}$$

$$V_1 = \sqrt{\begin{array}{l}(V_{1,1})^2 + (V_{1,2})^2 + (V_{1,3})^2 + (V_{1,4})^2 + (V_{1,5})^2 + (V_{1,6})^2 + (2 \times \rho_{12} \times V_{1,1} \times V_{1,2}) \\ + (2 \times \rho_{23} \times V_{1,2} \times V_{1,3}) + + (2 \times \rho_{34} \times V_{1,3} \times V_{1,4}) + (2 \times \rho_{45} \times V_{1,4} \times V_{1,5}) + (2 \times \rho_{56} \times V_{1,5} \times V_{1,6})\end{array}} \tag{4.57}$$

Step 8.1: shear of each mode in m_1

$$V_{1,1} = (433.841), V_{1,2} = (-170.947), V_{1,3} = (89.126), V_{1,4} = (-40.536), V_{1,5} = (6.305), (V)_{1,6} = (-0.203)$$

Step 8.2: Determining of λ

$$\lambda_{1,2} = \frac{T_2}{T_1} = \frac{0.245034916}{0.647383989} = 0.378500117$$
$$\lambda_{2,3} = \frac{T_3}{T_2} = \frac{0.152713552}{0.245034916} = 0.623231803$$
$$\lambda_{3,4} = \frac{T_4}{T_3} = \frac{0.122954286}{0.152713552} = 0.805130152$$
$$\lambda_{4,5} = \frac{T_5}{T_4} = \frac{0.099740699}{0.122954286} = 0.811201488$$
$$\lambda_{5,6} = \frac{T_6}{T_5} = \frac{0.077448757}{0.099740699} = 0.776501042$$

Step 8.3: Calculation $\rho_{ij}, \xi = 0.05$

$$\rho_{12} = \frac{8 \times 0.05^2(1 + 0.378500117) \times 0.378500117^{\left(\frac{3}{2}\right)}}{(1 - 0.378500117^2)^2 + \left(4 \times (0.05^2)(0.378500117)(1 + 0.378500117)^2\right)} = 0.008662$$

$$\rho_{23} = \frac{8 \times 0.05^2(1 + 0.623231803) \times 0.623231803^{\left(\frac{3}{2}\right)}}{(1 - 0.623231803^2)^2 + \left(4 \times (0.05^2)(0.623231803)(1 + 0.623231803)^2\right)} = 0.040909$$

$$\rho_{34} = \frac{8 \times 0.05^2(1 + 0.805130152) \times 0.805130152^{\left(\frac{3}{2}\right)}}{(1 - 0.805130152^2)^2 + \left(4 \times (0.05^2)(0.805130152)(1 + 0.805130152)^2\right)} = 0.173909$$

$$\rho_{45} = \frac{8 \times 0.05^2(1 + 0.811201488) \times 0.811201488^{\left(\frac{3}{2}\right)}}{(1 - 0.811201488)^2 + \left(4 \times (0.05^2)(0.811201488)(1 + 0.811201488)^2\right)} = 0.184378$$

$$\rho_{56} = \frac{8 \times 0.05^2(1 + 0.776501042) \times 0.776501042^{\left(\frac{3}{2}\right)}}{(1 - 0.776501042)^2 + \left(4 \times (0.05^2)(0.776501042)(1 + 0.776501042)^2\right)} = 0.133467$$

Step 8.4: Substitute the magnitude n into the Eq. (4.55)

$$V_1 = \sqrt{\begin{array}{c}(433.841)^2 + (-170.947)^2 + (89.126)^2 + (-40.536)^2 + (6.305)^2 + (-0.203)^2 \\ + (2 \times 0.008662 \times 433.841 \times -170.947) + (2 \times 0.040909 \times -170.947 \times 89.126) \\ + (2 \times 0.173909 \times 89.126 \times -40.536) + (2 \times 0.184378 \times -40.536 \times 6.305) \\ + (2 \times 0.133467 \times 6.305 \times -0.203)\end{array}}$$
$$= 472.424848$$

The other shear modals can also be calculated using a similar method: (Tables 4.17 and 4.18)

4.3.2 *Response Spectrum Analysis with El-Centro Ground Motion*

This part shows the response spectrum method of the seismic response analysis of a steel moment resistance structure. The method demonstrated in part 1 is broadly used, and appropriate for any structure/equipment that can be modeled as a single-degree-of-freedom (SDOF) system. For this reason, it uses Combined *D*–*V*–*A* response spectrum for the El Centro ground motion; ζ = 0, 2, 5, 10, and 20%.

Step 1: Stiffness Matrix

$$[K]_{6\times6} = \begin{bmatrix} 143205071.5 & -143205071.5 & 0 & 0 & 0 & 0 \\ -143205071.5 & 286410143 & -143205071.5 & 0 & 0 & 0 \\ 0 & -143205071.5 & 353936549.9 & -210731478.4 & 0 & 0 \\ 0 & 0 & -210731478.4 & 421462956.8 & -210731478.4 & 0 \\ 0 & 0 & 0 & -210731478.4 & 554366721.7 & -343635243.3 \\ 0 & 0 & 0 & 0 & -343635243.3 & 789767249.7 \end{bmatrix}$$

Step 2: Mass Matrix

$$[M] = \begin{bmatrix} 162920.582 & 0 & 0 & 0 & 0 & 0 \\ 0 & 165112.1274 & 0 & 0 & 0 & 0 \\ 0 & 0 & 65605.4045 & 0 & 0 & 0 \\ 0 & 0 & 0 & 166098.252 & 0 & 0 \\ 0 & 0 & 0 & 0 & 166954.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 158044.182 \end{bmatrix}$$

Table 4.17 Calculation of shear base in each mode

Story	Mass number	Shear	λ	ξ	ρ	V
6	m_1	$V_{1,1}$ = 433.841	λ_{12} = 0.378500117	0.05	ρ_{12} = 0.008662	472.424848
		$V_{1,2}$ = −170.947	λ_{23} = 0.623231803		ρ_{23} = 0.040909	
		$V_{1,3}$ = 89.126	λ_{34} = 0.805130152		ρ_{34} = 0.173909	
		$V_{1,4}$ = −40.536	λ_{45} = 0.811201488		ρ_{45} = 0.184378	
		$V_{1,5}$ = 6.305	λ_{56} = 0.776501042		ρ_{56} = 0.133467	
		$V_{1,6}$ = −0.203				
5	m_2	$V_{2,1}$ = 826.4	λ_{12} = 0.378500117	0.05	ρ_{12} = 0.008662	846.112220
		$V_{2,2}$ = −214.599	λ_{23} = 0.623231803		ρ_{23} = 0.040909	
		$V_{2,3}$ = 5.498	λ_{34} = 0.805130152		ρ_{34} = 0.173909	
		$V_{2,4}$ = 40.432	λ_{45} = 0.811201488		ρ_{45} = 0.184378	
		$V_{2,5}$ = −16.199	λ_{56} = 0.776501042		ρ_{56} = 0.133467	
		$V_{2,6}$ = 1.136				
4	m_3	$V_{3,1}$ = 1130.111	λ_{12} = 0.378500117	0.05	ρ_{12} = 0.008662	1123.01904
		$V_{3,2}$ = −95.209	λ_{23} = 0.623231803		ρ_{23} = 0.040909	
		$V_{3,3}$ = −89.143	λ_{34} = 0.805130152		ρ_{34} = 0.173909	
		$V_{3,4}$ = −0.4571	λ_{45} = 0.811201488		ρ_{45} = 0.184378	
		$V_{3,5}$ = 35.413	λ_{56} = 0.776501042		ρ_{56} = 0.133467	
		$V_{3,6}$ = −6.1676				
3	m_4	$V_{4,1}$ = 1350.820	λ_{12} = 0.378500117	0.05	ρ_{12} = 0.008662	1368.38565
		$V_{4,2}$ = 73.878	λ_{23} = 0.623231803		ρ_{23} = 0.040909	
		$V_{4,3}$ = −65.126	λ_{34} = 0.805130152		ρ_{34} = 0.173909	
		$V_{4,4}$ = −40.527	λ_{45} = 0.811201488		ρ_{45} = 0.184378	
		$V_{4,5}$ = −23.724	λ_{56} = 0.776501042		ρ_{56} = 0.133467	
		$V_{4,6}$ = 18.502				
2	m_5	$V_{5,1}$ = 1471.857	λ_{12} = 0.378500117	0.05	ρ_{12} = 0.008662	1514.68767
		$V_{5,2}$ = 205.353	λ_{23} = 0.623231803		ρ_{23} = 0.040909	
		$V_{5,3}$ = 46.357	λ_{34} = 0.805130152		ρ_{34} = 0.173909	
		$V_{5,4}$ = 3.0433	λ_{45} = 0.811201488		ρ_{45} = 0.184378	
		$V_{5,5}$ = −8.712	λ_{56} = 0.776501042		ρ_{56} = 0.133467	
		$V_{5,6}$ = −53.177				

Table 4.18 Calculation of shear base in each mode

Story	Mass number	Shear	λ	ξ	ρ	V
1	m_6	$V_{6,1}$ = 1522.670	λ_{12} = 0.378500117	0.05	ρ_{12} = 0.008662	1551.54903
		$V_{6,2}$ = 267.712	λ_{23} = 0.623231803		ρ_{23} = 0.040909	
		$V_{6,3}$ = 40.633	λ_{34} = 0.805130152		ρ_{34} = 0.173909	
		$V_{6,4}$ = 115.8	λ_{45} = 0.811201488		ρ_{45} = 0.184378	
		$V_{6,5}$ = 39.936	λ_{56} = 0.776501042		ρ_{56} = 0.133467	
		$V_{6,6}$ = 21.321				

Step 3: Eigen Values and Eigen Vectors

$$\omega = \begin{bmatrix} \omega_1 = 9.7055 \\ \omega_2 = 25.642 \\ \omega_3 = 41.1436 \\ \omega_4 = 51.1018 \\ \omega_5 = 62.9952 \\ \omega_6 = 81.127 \end{bmatrix}, T = \begin{bmatrix} T_1 = 0.647383989 \\ T_2 = 0.245034916 \\ T_3 = 0.152713552 \\ T_4 = 0.122954286 \\ T_5 = 0.099740699 \\ T_6 = 0.077448757 \end{bmatrix}$$

$$\begin{bmatrix} \varnothing_{1,1} \\ \varnothing_{2,1} \\ \varnothing_{3,1} \\ \varnothing_{4,1} \\ \varnothing_{5,1} \\ \varnothing_{6,1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.892834898 \\ 0.688701992 \\ 0.4989996 \\ 0.272248619 \\ 0.12073589 \end{bmatrix}, \begin{bmatrix} \varnothing_{1,2} \\ \varnothing_{2,2} \\ \varnothing_{3,2} \\ \varnothing_{4,2} \\ \varnothing_{5,2} \\ \varnothing_{6,2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.251965986 \\ -0.687082505 \\ -0.970199977 \\ -0.750511161 \\ -0.376036852 \end{bmatrix}, \begin{bmatrix} \varnothing_{1,3} \\ \varnothing_{2,3} \\ \varnothing_{3,3} \\ \varnothing_{4,3} \\ \varnothing_{5,3} \\ \varnothing_{6,3} \end{bmatrix} = \begin{bmatrix} 1 \\ -0.925848558 \\ -1.044668178 \\ 0.264308872 \\ 1.220629362 \\ 0.803186985 \end{bmatrix}$$

$$\begin{bmatrix} \varnothing_{1,4} \\ \varnothing_{2,4} \\ \varnothing_{3,4} \\ \varnothing_{4,4} \\ \varnothing_{5,4} \\ \varnothing_{6,4} \end{bmatrix} = \begin{bmatrix} 1 \\ -1.97091311 \\ 0.992350135 \\ 0.969580111 \\ -1.048873759 \\ -0.955922866 \end{bmatrix}, \begin{bmatrix} \varnothing_{1,5} \\ \varnothing_{2,5} \\ \varnothing_{3,5} \\ \varnothing_{4,5} \\ \varnothing_{5,5} \\ \varnothing_{6,5} \end{bmatrix} = \begin{bmatrix} 1 \\ -3.521473717 \\ 8.052092019 \\ -9.198805803 \\ 2.323089073 \\ 4.909786487 \end{bmatrix}, \begin{bmatrix} \varnothing_{1,6} \\ \varnothing_{2,6} \\ \varnothing_{3,6} \\ \varnothing_{4,6} \\ \varnothing_{5,6} \\ \varnothing_{6,6} \end{bmatrix} = \begin{bmatrix} 1 \\ -6.487699165 \\ 35.25599317 \\ -118.7278924 \\ 343.2010358 \\ -470.9644759 \end{bmatrix}$$

Step 4: Modal Participation Factors

$$\Gamma_i = \frac{\{\o_i\}^T\{m\}\{r\}}{\{\o_i\}^T\{m\}\{\o_i\}} \tag{58}$$

The vector influence coefficients representing the acceleration at displacement coordinate by the component i is represented by (Agrawal & Shrikhande, 2006), because of a unit horizontal ground acceleration present at the base. The vector (Agrawal & Shrikhande) turns out to be a unity vector (Agrawal & Shrikhande) for the simple structural model, whereby the story levels' horizontal displacements represent the degree of freedom, considering the fact that every degree of freedom possesses a unit horizontal acceleration for a unit ground acceleration along the horizontal direction (Irvine, 2013).

Substitute the magnitude in Eq. (58); hence.'

$$\Gamma_1 = \frac{\{\O_1\}^T\{m\}\{r\}}{\{\O_1\}^T\{m\}\{\O_1\}}$$

$$\Gamma_1 = \frac{\begin{bmatrix}1\\0.892834898\\0.688701992\\0.4989996\\0.272248619\\0.12073589\end{bmatrix}^T\begin{bmatrix}162920.582&0&0&0&0&0\\0&165112.1274&0&0&0&0\\0&0&65605.4045&0&0&0\\0&0&0&166098.252&0&0\\0&0&0&0&166954.8&0\\0&0&0&0&0&158044.182\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}}{\begin{bmatrix}1\\0.892834898\\0.688701992\\0.4989996\\0.272248619\\0.12073589\end{bmatrix}^T\begin{bmatrix}162920.582&0&0&0&0&0\\0&165112.1274&0&0&0&0\\0&0&65605.4045&0&0&0\\0&0&0&166098.252&0&0\\0&0&0&0&166954.8&0\\0&0&0&0&0&158044.182\end{bmatrix}\begin{bmatrix}1\\0.892834898\\0.688701992\\0.4989996\\0.272248619\\0.12073589\end{bmatrix}}$$

$$\Gamma_1 = \frac{\{\O_1\}^T\{m\}\{r\}}{\{\O_1\}^T\{m\}\{\O_1\}} = 1.3325$$

Step 5: Determining of S_a *and* S_d (Fig. 4.33)

$$T_1 = 0.6473\,(\text{s}), S_d = 2.8\,(\text{in.}) = 0.07112\,(\text{m}), S_a = 0.63\,\text{g}, S_v = 26\,\frac{\text{in.}}{\text{s}} = 0.6604\,\frac{\text{m}}{\text{s}}$$

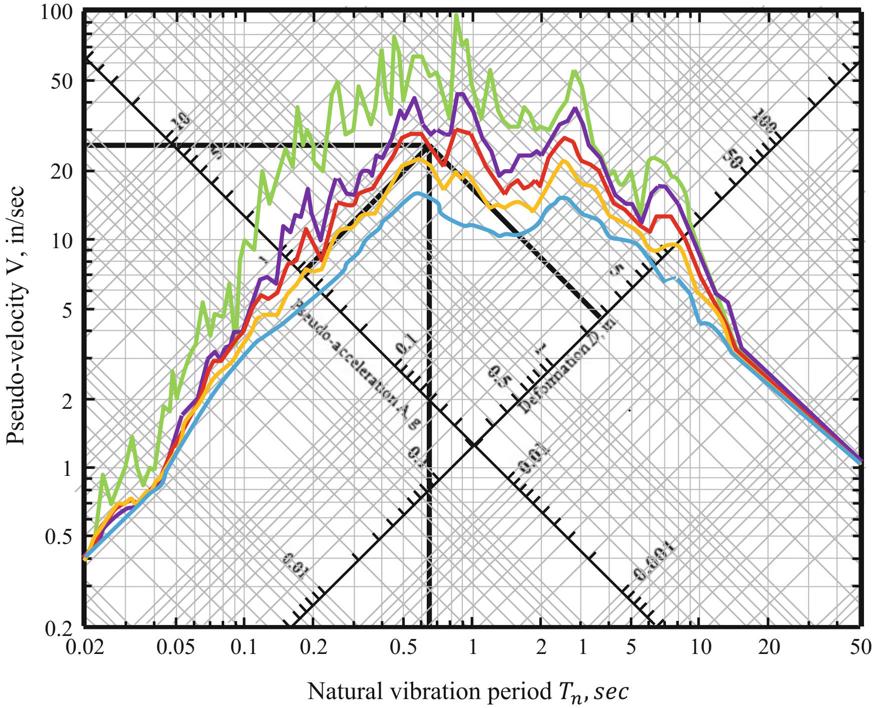


Fig. 4.33 Combined *D*–*V*–*A* response spectrum for El Centro ground motion; ζ = 0, 2, 5, 10, and 20%

Step 6: Displacement and Base Shear in Each Mode

Top floor displacement:

$$\Gamma_1 \times \text{ø}_{1,1} \times S_{d1} = 1.3325 \times 1 \times 0.07112 = 0.0947674$$

Base shear: (Table 4.19)

$$\begin{aligned} k_6 \times \text{ø}_{6,1} \times \Gamma_1 \times S_{d1} &= 446132006.3 \times 10^{-3} \times 0.12073589 \times 1.3325 \times 0.07112 \\ &= 5104.56496\,\text{kN} \end{aligned}$$

Table 4.19 Calculation of shear in each mode according to the El-Centro ground motion with $\xi = 0.05$

T(sec)	ø	Γ	S_d	S_a	Δ	V
$T_1 = 0.647383989$	$ø_1 = \begin{bmatrix} 1 \\ 0.892834898 \\ 0.688701992 \\ 0.4989996 \\ 0.272248619 \\ 0.12073589 \end{bmatrix}$	$\Gamma_1 = 1.3325$	$S_{d1} = 0.07112$	$S_{a1} = 0.63$	0.09476	5104.56
$T_2 = 0.245034916$	$ø_2 = \begin{bmatrix} 1 \\ 0.251965986 \\ -0.687082505 \\ -0.970199977 \\ -0.750511161 \\ -0.376036852 \end{bmatrix}$	$\Gamma_2 = -0.4866$	$S_{d2} = 0.0127$	$S_{a2} = 0.8$	−0.0061	1036.74
$T_3 = 0.152713552$	$ø_3 = \begin{bmatrix} 1 \\ -0.925848558 \\ -1.044668178 \\ 0.264308872 \\ 1.220629362 \\ 0.803186985 \end{bmatrix}$	$\Gamma_3 = 0.24977$	$S_{d3} = 0.0030$	$S_{a3} = 0.69$	0.00075	268.4983
$T_4 = 0.122954286$	$ø_4 = \begin{bmatrix} 1 \\ -1.97091311 \\ 0.992350135 \\ 0.969580111 \\ -1.048873759 \\ -0.955922866 \end{bmatrix}$	$\Gamma_4 = -0.1125$	$S_{d4} = 0.00226$	$S_{a4} = 0.76$	−0.00025	108.43

(continued)

Table 4.19 (continued)

T(sec)	ø	Γ	$\boldsymbol{S_d}$	$\boldsymbol{S_a}$	Δ	V
$T_5 = 0.099740699$	$ø_5 = \begin{bmatrix} 1 \\ -3.521473717 \\ 8.052092019 \\ -9.198805803 \\ 2.323089073 \\ 4.909786487 \end{bmatrix}$	$\Gamma_5 = 0.01736$	$S_{d5} = 0.00152$	$S_{a5} = 0.65$	0.00002	57.8
$T_6 = 0.077448757$	$ø_6 = \begin{bmatrix} 1 \\ -6.487699165 \\ 35.25599317 \\ -118.7278924 \\ 343.2010358 \\ -470.9644759 \end{bmatrix}$	$\Gamma_6 = -0.00055$	$S_{d6} = 0.00081$	$S_{a6} = 0.62$	−4.4E-7	93.605

4.4 Non-linear Analysis

- Review in the Nonlinear Analyses
- The concept of Plastic Hinge
- Application of Two-line and Three-line Modelling
- Effort-Deformation Relation
- The concept of Nonlinear Static Analysis
- The direction of Imposing Force in the Static Nonlinear Method
- Two Fundamental Words in Nonlinear Static Analysis
- Capacity Curve of Structure
- Effective Fundamental Period of Building
- Target Displacement
- Design Coefficient of Structure

The concept of a plastic hinge is considered after a short review of the concept of nonlinear analyses. This will familiarize the reader with modeling plastic hinges.

4.4.1 Review in the Nonlinear Analyses

Various methods of seismic analysis of structures are divided into two parts; namely linear and nonlinear analyses methods. In the former, which can be static/dynamic, the stiffness of the structure members is assumed to be constant; for instance, if the loads on the structure increased ten-fold, the forces in the structure members will also increase ten-fold. On the other hand, we are aware that the structures represent deformability, and would experience nonlinear deformations when undergoing large loadings caused by earthquakes, Therefore, the design force can be reduced in the structure members, and civil engineering researchers also suggested nonlinear analysis methods, which will be introduced in this chapter. Nonlinear analyses can also be statistical or dynamic in nature.

The static nonlinear analysis and dynamic nonlinear analysis (nonlinear time history analysis) are methods that can be used for nonlinear analyses, which will be detailed in this chapter in order to familiarize the reader with these concepts. Researchers confirmed that the results of time historical analyses are very sensitive to records of selected earthquake, and parameters such as the earth's maximum acceleration (PGA), the frequency content of the records, duration of the earthquake, and …, affect structural response. Numerous analyses needs to be conducted, which could be difficult and expensive. On the other hand, static nonlinear analyses have comparatively fewer difficulties compared to dynamic nonlinear analyses, and could properly estimate structural responses. As per the aforementioned points, it can be assumed that the static nonlinear analysis method has been considered more than time history analysis by civil engineering researchers.

It should be pointed out that the nonlinear behavior of structures are inclusive of two types of features; geometrical nonlinear and material nonlinear. It is important to know that only the $p - \Delta$ effect is critical in most engineering projects. It can also be used when utilizing linear analyses by modern software, rendering nonlinear analyses to account for the nonlinear effects of the materials. When the materials of a structure becomes nonlinear, there will be areas called plastic hinges in the members of the structure that we are going to get more familiar with, whose concept are closely tied to modeling approaches in nonlinear analyses.

4.4.2 The Concept of Plastic Hinge

Understanding plastic hinge is one of the most basic principles a design engineer must be familiar with for nonlinear analyses. Some examples include:

1. In Fig. 4.34, there is a simple structural frame under lateral forces, which render the diagonal bracing under tension. If the lateral force imposed on the frame increases, the tensional force in the bracing will increase until the bracing yields under tension. In this case, it can be said that an axial hinge has been produced under tension in the bracing.

2. In the frame in Fig. 4.35, the diagonal bracing is under compression. Increasing the lateral force on the frame and subsequently increasing the compression force level on the bracing could buckle the brace under compression. On the other hand, if the bracing has been restrained against buckling or is very stiff against buckling, the compressive stress will exceed the yield strength, and in this case, an axial hinge has been produced in the bracing under compression.
3. Consider the frame in Fig. 4.36 with rigid joints between the beam and columns on which a lateral force is being imposed. In this case, the lateral load withstood the bending moment and shearing force produced in the beam and columns. Therefore, it can be assumed that the stresses produced in the beam and columns increase in tandem with the lateral force until one of the following occur:

   a. The maximum bending moment in the member(s) reach its respective bending strengths. In this case, the section yields under normal stresses produced by bending, and it can be said that abending hinge has been produced in the beam/column.

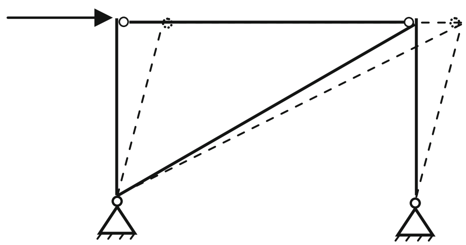


Fig. 4.34 Simple structure frame with bracing

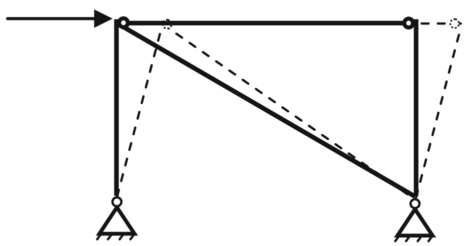


Fig. 4.35 Simple structure frame with bracing system

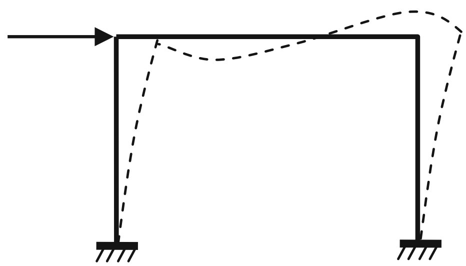


Fig. 4.36 Moment resistance frame

b. The maximum shearing forces in the members reach their respective shearing strengths. In this case, the section yield under shear stresses and it is assumed that a shearing hinge is produced in the beam/column.

4.4.2.1 Plastic Hinge

Considering the structures discussed in this section, we found that a hinge can be produced under various modes, encompassing axial under tension, axial under compression, bending, or shearing. In literature, these hinges are collectively known as Plastic Hinge.

Note:

Plastic hinges can be produced in the steel/concrete members:

- In steel members, the plastic hinge is the cross-section where the axes have reached its yield stress limit under a specific effort, in other words, the cross-section has fully yielded.
- In concrete members, the plastic hinge is a cross-section where the rebar(s) reached its yield stress limit under a specific mode.

Approaches to Modeling the Plastic Hinges

One of the most important and effective steps of nonlinear analyses is modeling and defining the characteristics of the plastic hinges. Due to the importance of this topic, the codes for the nonlinear analysis methods involve modeling the criteria and defining plastic hinges. This section reviews the modelling and definition approaches.

If a nonlinear analysis method is used to design a structure, the characteristics of its members must be modeled nonlinearly. It should also be pointed out that the behavioral modeling of the materials of the members is complicated in dynamic nonlinear analysis, and needs to be conducted as per the behavior of the member in loading/unloading (cyclic behavior, Fig. 4.37). The push of dynamic nonlinear behavior of the members is used in static nonlinear analyses.

- **Different View**:

It can be seen in Fig. 4.38 that there are three types of behaviors expected between plastic hinges and efforts:

Type 1: The hinge has a high strength but cannot withstand nonlinear deformations and is not formable, making it brittle.

Type 2: The hinge has a favorable strength and has the capacity of withstanding a great deal of nonlinear deformations, making it formable.

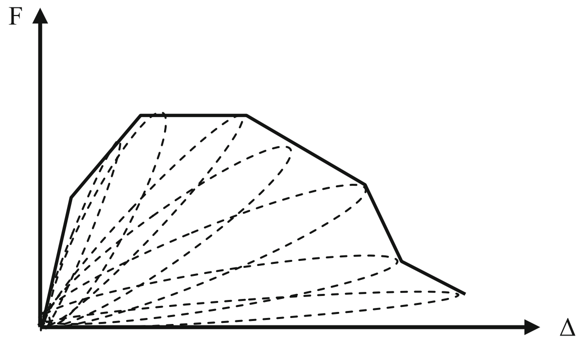


Fig. 4.37 Force-displacement graph under cyclic load

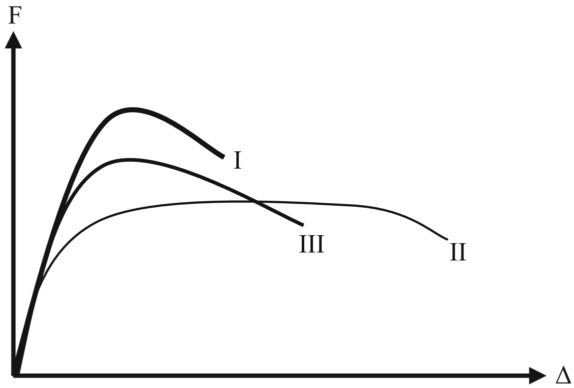


Fig. 4.38 Force-displacement graph for different type of plastic hinge

Type 3: The hinge share similar characteristics between both types I–II, making it semi formable.

The modeling of plastic hinges, which includes nonlinear characteristic of the members, must be consistent with the experimental data or authentic analytical models in terms of its strength, stiffness, and formability. The plastic hinges can be modeled via two main approaches, namely:

1. The general approach in modeling plastic hinges
2. The approach of seismic improvement criteria in modeling plastic hinges

We will familiarize ourselves with these approaches in the following sections. In a more comprehensive perspective, one can say that there are two methods to simulate the behavior of the plastic hinges; the Distributed Plasticity Models and Concentrated Plasticity Models. It should be pointed out that the concentrated plasticity models are generally used in many engineering applications.

General Approach in Modeling of Plastic Hinge

The general approaches in modeling plastic hinge include two-line or three-line behaviors, detailed below:

Two-line Modeling:

The relation of force-deformation of the members and plastic hinges can be regarded as a two-line model shown in Fig. 4.39. In this model, the first line shows the elastic behavior of the member, which continues until the yield point (point B). After that, the stiffness of the member (the slope of the second line) decreases, up to a point that it can be regarded to be equal to zero (the full plastic state, $\alpha = 0$).

Three-line Modeling:

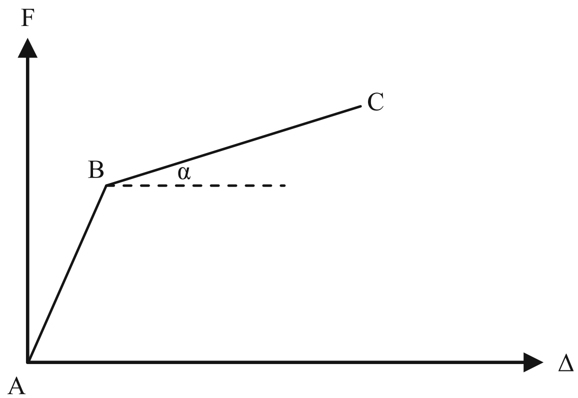
The relation of force-deformation of the members and plastic hinges can be regarded as the three-line model shown in Fig. 4.40. This model is more accurate than the two-line model, because it gets closer to the real form of the behavioral curve, hence the changes in the stiffness of the member are considered to be more gradual.

4.4.3 Application of Two-Line and Three-Line Modeling

The two-line and three-line models are general and simple methods to model the behavior of the force-deformation of the members of a structure, and it is governed by:

- Two-line model is more suitable for steel members, and is unsuitable for modeling concrete. For example, in Fig. 4.41, the behavior of the force-deformation of a steel member under tension can be replaced by a two-line model at an acceptable accuracy. Note that in this case, the slope of the first line

Fig. 4.39 Two-line of modelling plastic hinge



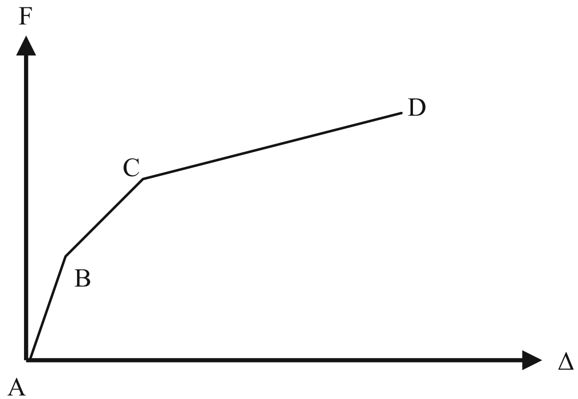


Fig. 4.40 Three-line modelling of plastic hinge

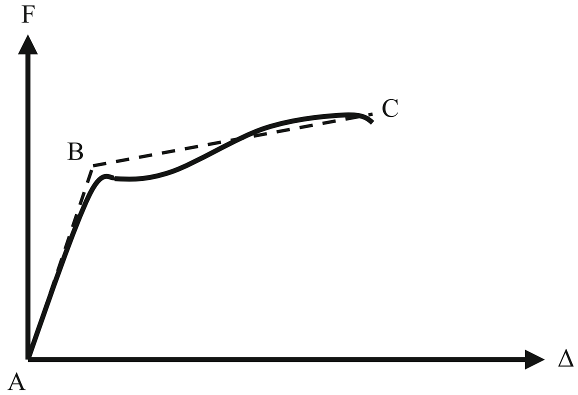


Fig. 4.41 Two-line modelling of the behaviour of steel member under tension

(line AB) represents the secant stiffness of the member until its yielding point, while the slope of the second line (line BC) is usually defined as 1–3% of the slope of the first line.

- Three-line models are often used to define the behavior of concrete (and masonry) members. For instance, in Fig. 4.42, the behavior of the force-deformation of a concrete beam under pure bending is seen, which can be represented properly by a three-line model. Note that in this case, the slope of the first line (line AB) shows the stiffness of the member before cracking, while the slope of the second line (line BC) shows the stiffness of the member after cracking. On the other hand, the slope of the third line (line CD) shows the state of the strength drop of the plastic hinge.

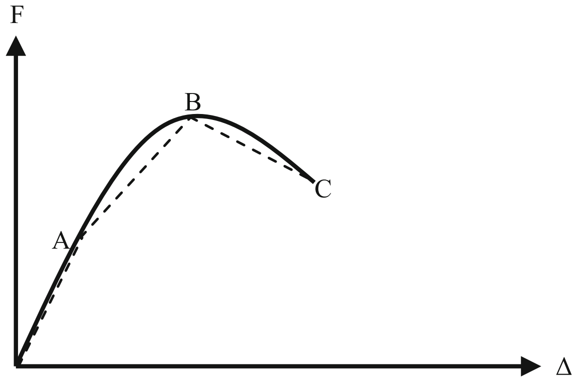


Fig. 4.42 Three-line modelling of the behaviour of concrete beam under bending

Note:

If we want to use the two-line model to model the behavior of concrete or masonry members, the slope of the first line must be defined based on the stiffness of the cracked cross-section.

4.4.3.1 The Approach of Seismic Improvement Criteria in Modeling the Plastic Hinges

The criteria presented in the instruction of seismic improvement of existing buildings can be used to model the nonlinear behavior of the members in nonlinear analyses. According to the criteria of this publication, the behavior of the structural members are divided into two main groups based on the type of the effort and the shape of the curve, "controllable by force" and "controllable by deformation", which we will familiarize ourselves with in the upcoming subsections.

The Efforts Which Are Controllable by Force

The behavior of the members where there are the efforts controllable by force is shown schematically in Fig. 4.43. The deformation of these members has an elastic and linear relationship with the imposed force in its initial stages. When the force reaches an ultimate limit such as point B', the members' will suddenly lose its bearing and collapse without exhibiting deformation.

The Efforts Which Are Controllable by Deformation

The behavior of members under the efforts controllable by deformation is shown in Fig. 4.44. Four different regions are evident on the behavioral graph of these

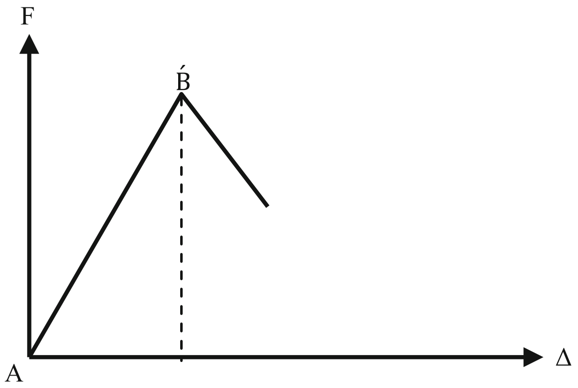


Fig. 4.43 Effort-displacement graph for member controllable by force

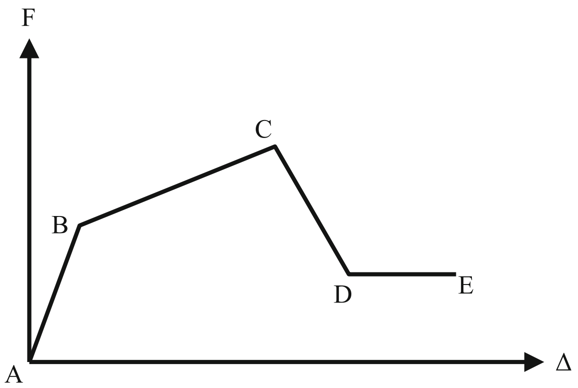


Fig. 4.44 Effort-displacement graph for member controllable by deformation

members. In the first part (branch AB), the behavior is linearly elastic, in the second part (branch BC), a deformable behavior can be observed to be fully plastic (with zero slope) or plastic hardening (with a positive slope, as per the figure above). In the third part (branch CD), the strength decreases intensively, but does not completely vanish, while in the fourth part (branch DE), the behavior is plastic again, showing the residual strength of the member. It is evident that the deformation in the nonlinear behavior range exceeds that of the linear range, and in these members, strength degradation and failure is obvious, alongside large deformation.

Note:

In the graphs of the behaviors of plastic hinges, the vertical axis has been introduced as an internal effort, while the horizontal axis has been introduced as deformation, which is general descriptions, each with its own special meaning, as per the respective cases. For example, the force-deformation graph is, in fact, the axial force-axial deformation, and in a steel bracing, it is defined as bending moment-bending rotation.

4.4.4 Effort-Deformation Relation

The effort-deformation for the members that are controllable by deformation is as following. The parameters shown in the graphs, such as a, b, c, and the slopes k and k′ were extracted from authentic codes based on element type, dimensions, and the cross-section size, to plot this graph in various members of the structure and introduce them to software such as ETABS. For instance, the slope of the BC part is often considered 10% of the slope of the AB part, whose value depends on its cross-section (Figure 4.45).

To complete the topic on the behavior of the plastic hinges, it is concluded that:

1. The efforts that are controllable by force lack deformable behaviors. The behavior of the member is often linear in this state, and at the end of the linear part, a sudden strength drop occurs in the member, and the member fails.
2. The efforts that are controllable by deformation demonstrate deformable behavior. The strength drop occurs in these members after reaching its ultimate strength and undergoing a large deformation, but they do not often lose their strength, and after this stage, they can withstand larger deformations to some extents.
3. We pointed out previously that the produced efforts in the structural members have a semi-deformable behavior, which means that their behavior is, to a certain extent, similar to the behavior of the members that are controllable by

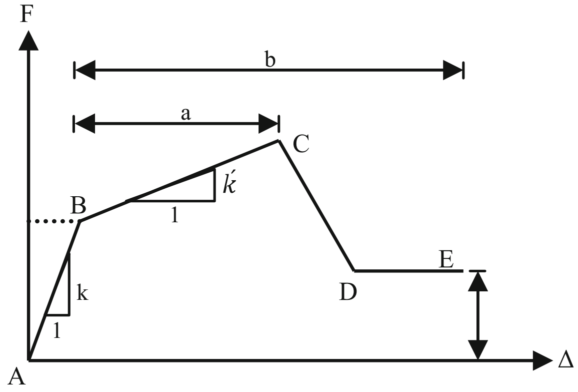


Fig. 4.45 Parameters of effort-displacement graph for member controllable by deformation

deformation, and also similar to the members that are controllable by force. As can be seen in Fig. 4.46c, these members have comparatively proper deformability (not as much as of the members controllable by deformation), but their strengths suddenly drop, and they subsequently fail.

Notice:

The graphs introduced in this study are used to analyze the effort-deformation behavior of the members and show a linear behavior in the primary stages of the loading and a nonlinear behavior in the large loadings. In other words, these graphs can be used to determine the parameters stiffness and strength changing suddenly or gradually along the loading. In linear analyses, only the first part of the graph (linear region) is used, while in nonlinear analysis methods, we want to obtain more accurate results for the structure by considering the real behavior of the plastic hinges.

Currently, the nonlinear static analysis method is one of the most famous and applicable methods of nonlinear analysis of the structures. This section is going to familiarize us with the concept of the philosophy of this analysis

4.4.5 The Concept of Nonlinear Static Analysis

You have probably heard of the other name of nonlinear static analysis; Pushover Analysis. It is interesting to note that the reason of this appellation for nonlinear static analysis is to exactly show the concept and approach of this method. For that purpose, consider the Fig. 4.47 below in which a building is pulled by a man. If the man pulls the building by force V, a shear force V is also produced at the base of the building, and the displacement Δ is measured at the roof level of the building ($\Delta = \frac{V}{K}$). Now, if the values of Δ and V are measured from when, then the man starts pulling the building until the imposing force cause the building to collapse. This occurrence is represented in the form of curves, as shown below.

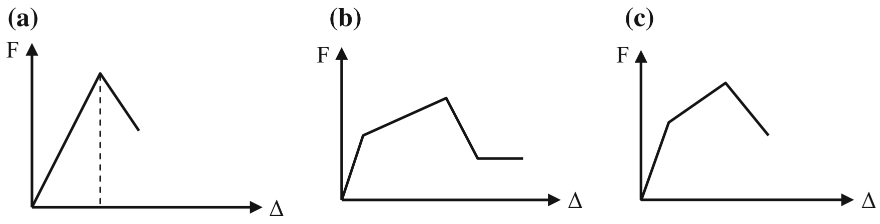


Fig. 4.46 Effort-displacement graph for members **a** brittle **b** ductile **c** semi-ductile

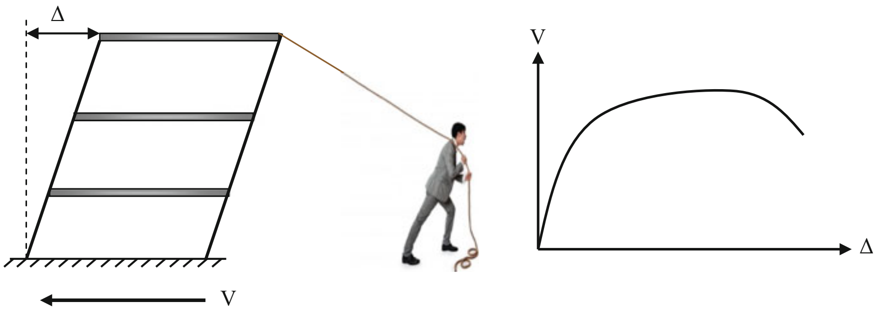


Fig. 4.47 Applying the lateral force to the building and drawing shear-displacement graph

In nonlinear static analysis, we are looking to push the building by imposing a force (the force imposed by the man in the example) until it collapses. Then, the base shear-displacement curve from the initiation of the loading until the collapse stage (like Fig. 4.47) is plotted, and we will be able to obtain useful information about the behavior of the building.

Point:

Some engineers regard static nonlinear analysis method as a failure mode analysis, due to the most significant information this analyses can present to engineers. In static nonlinear analysis, the order and sequence of producing plastic hinges in the structure is evident, and a proper view of the behavior of the structure under the loads due to earthquake can be obtained as well.

Applications of the Static Non-linear Analysis Method:

The static nonlinear analysis method can be used in structures where the effects of high modes are not dominant. It is necessary that the structure of the building is analyzed twice using the dynamic spectral analysis method.

- The first time, only the first mode of the structure is considered.
- The second time, only the oscillatory modes whose total effective mass is at least 90% of the total mass of the structure is considered.

If the results of the second analysis show that the shearing force in one of the stories is 30% greater than that of the first analysis, it means that the effects of the high modes are principal, and as a result of this, the static nonlinear analysis cannot be used.

4.4.6 The Direction of Imposing Force in the Static Nonlinear Method

The loading should be imposed on the structure in two steps; conduct the static nonlinear analysis:

1. In the first step and prior to imposing the lateral load to the structure, gravity must be imposed on the structure based on the respective load combination coefficients.
2. After imposing the gravitational load, the lateral load must be imposed on the structure, as per the following:

   - In this method, the effect of earthquake should be imposed on the building in two positive and negative directions, with the most critical effort values and deformations being the criteria of design and control of the members. In this case, the $p - \Delta$ effect should also be considered in the analysis.
   - In the case of irregular buildings, the 3D models must be used in the analyses. For these buildings and the group of regular buildings with one or several common columns between two/more frames of the bearing system in various directions, in each direction, 100% of the forces and displacements in the considered direction with the forces proportional to 30% of the displacement in its normal direction must be considered in the static nonlinear analysis.
   - For a regular building, the analysis can be carried out independently in each direction, except in cases with one or several common columns.

4.4.7 Two Fundamental Words in Nonlinear Static Analysis

We are aware that in the static nonlinear analysis method, the behavior of the structure must be studied nonlinearly by imposing the force to the structure from zero to the maximum value, and the base shear-displacement curve should be plotted for that purpose. The fundamental question is how must the force be imposed to the structure and the displacement of what point should be considered as a criterion? Answering this question, there are two fundamental concepts and words that have been created in the static nonlinear analysis:

- Lateral Load Pattern
- Control Point

To understand this concept well, consider the structure in Fig. 4.48, each whose floor is pulled by mens. The concepts of lateral load pattern and control point will be explained using this figure.

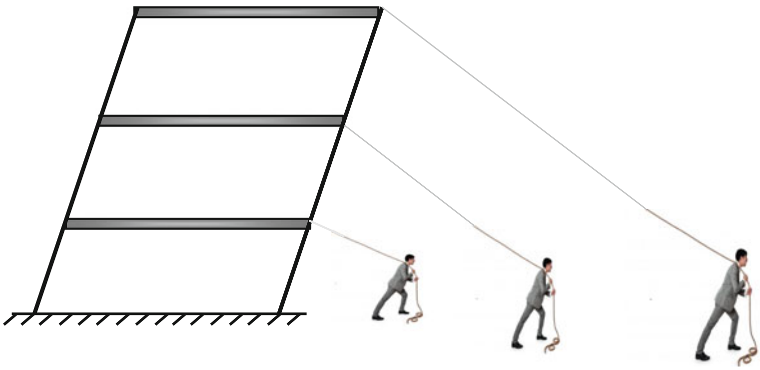


Fig. 4.48 Appling force to the different level of structure

4.4.7.1 Lateral Load Pattern

In the above example, each of the men pull the building toward themselves via various powers and forces, which is interesting as it can induce different reactions in the building. For example, various distributions for imposing force can be suggested, such as the Fig. 4.49, so that in distribution 1, the lateral load of each story is proportional to the height of that story, while the load is so-called triangular. In distribution 2, the lateral load is proportional to the mass of each floor, and the load is so-called uniform, while in distribution 3, the lateral load is proportional to various parameters, such as mass, height, and the stiffness of the floor.

In static nonlinear analyses, the way of imposing force to the level of each floor is the so-called "lateral load pattern", where at least two lateral load distributions must be imposed on the static nonlinear analysis, as per:

(a) The distribution is proportional to the lateral forces produced by the spectral linear dynamic analysis, considering the vibrational modes that take part in at least 90% of structural mass.
(b) The uniform distribution, which is e distribution, is proportional to the masses of the floors of the structure, neglecting its respective heights.

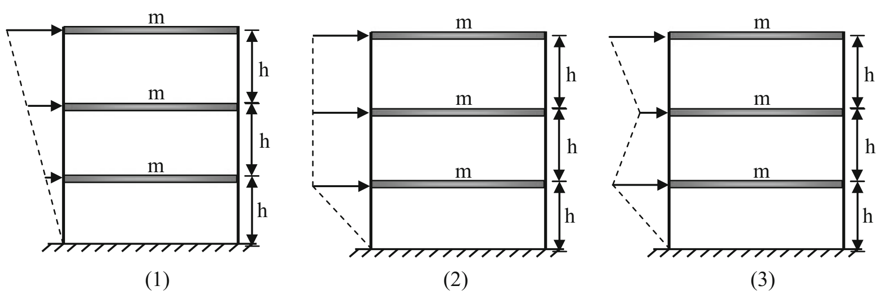


Fig. 4.49 Three different distribution of lateral force

Note:

Lateral loads should be imposed at places of the masses in the model in the building with rigid diaphragms. These loads are imposed at the center of mass of the floor. The effect of accidental eccentricity must also be considered when imposing loads.

- **Effects of torsion**:

The expression "torsionally flexible" buildings are used for buildings where the torsion of the structure is dominant in the first/second vibrational modes. In these types of buildings, the common patterns of nonlinear static analysis, which were introduced, may cause a non-conservative and be lesser than the displacements of the building in the stiff (resistant) side of the structure. Therefore, the displacements of the stiff (resistant) side of the structure must be increased in torsionally-flexible buildings compared to the torsionally-balanced buildings. Therefore, a magnification coefficient can be used for the displacements of the stiff side of the structure in the case of these buildings. This magnification coefficient can be obtained from the spectral linear dynamic analysis of the 3D model of the building.

4.4.7.2 Control Point

In the example of the previous section, when the men draw the building, there will be displacements in the floors of the building. The base shear-displacement graph can be plotted, but which level's displacement should be considered as a criterion for this? In nonlinear static analysis, the displacement produced in the center of mass should be considered, where the center of mass of the roof is called the "control point." It should be mentioned that in the process of nonlinear static analysis, it is assumed that the displacement produced in the control point must reach a specified limit called the "target displacement."

Reminder:

The center of mass of the dome roof should not be considered as a control point

4.4.8 Capacity Curve of Structure

From the topics detailed in this section, we learned that in nonlinear static analysis, a structure is placed under a lateral load pattern via increased load levels. The displacement at the control point reached the targeting values, and even exceeds this value in some cases. It was also seen that if the relation between the base shear of the building and the displacement of the control point is drawn as a curve, at the various stages of imposing the lateral load, the achieved result will be like that shown in Fig. 4.50.

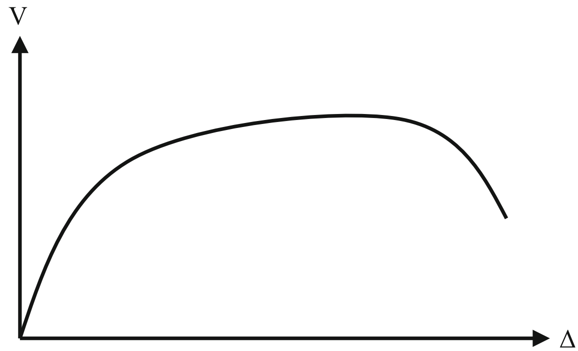


Fig. 4.50 Shear-displacement graph

The above graph, obtained from the nonlinear static analysis (Push Over Analysis), is called the "Capacity Curve" or "Push Over Curve", and is used as a base to calculate some of the structural characteristics. Some of the most important characteristics that can be directly/indirectly be elicited from the capacity curve of a structure include:

1. Base shear of the effective collapse of the structure (V_y)
2. Elastic lateral stiffness of the structure (K_i)
3. Effective lateral stiffness of the structure (K_e)
4. Effective fundamental period of the building (T_e)
5. Target displacement (δ_t)
6. Design coefficient of the structure (over strength coefficient, behavior coefficient, and magnification coefficient)

The capacity curve of a structure obtained from the nonlinear static analysis is a graph whose slope is changing continuously in various parts. In order to elicit useful information from this plot, it needs to be changed to a polyline curve. This will allow us to determine the effective collapse base shear of the structure (V_y) and the proportional displacement (Δ_y), whose values can be used to obtain the effective fundamental period of the structure (T_e).

The process of changing the capacity curve to a polyline is a trial-and-error process, and it usually need engineering software, it must be done as per the following principles:

1. The capacity curve should be replaced by an ideal three-line curve. In this curve, the first line represents the elastic behavior that continues until the yielding moment of the structure, while second line represents the positive slope of the structure after the yielding moment until maximum strength, and the third line represents the negative slope due to strength drop (Fig. 4.51).
2. As per the polyline capacity curve, two fundamental points is evident. In the first point (point B), the displacement of the control point is equal to Δ_y, while the

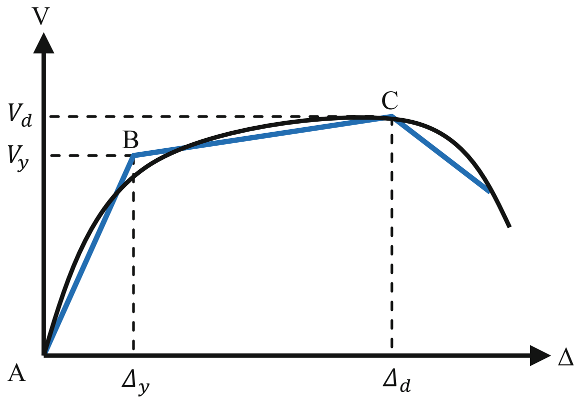


Fig. 4.51 Idealized elasto-plastic system

base shear of the building is equal to V_y. In the second point (point C), the displacement of the control point is equal to Δ_d, while the base shear of the building is equal to V_d.

3. The value V_y is the base shear of the effective collapse of the structure that should not be larger than the maximum base shear of the building in the target displacement or in the displacement proportional to the maximum base shear, whichever is less.
4. The first and the second lines in the three-line curve are drawn in a manner where the surface under the two-line behavior curve up to the point (V_d, Δ_d) is equal to the surface under the nonlinear behavior curve up to the point (V_d, Δ_d). In other words, in Fig. 4.52, the surface of the hatched area (1) should be equal to the summation of both (2) areas. It should also be pointed out that the first line should pass over point A' on the nonlinear curve, and at this point, the base shear of the building is equal to 0.6 V_y.

Note:

The necessity of satisfying both the a fore mentioned conditions simultaneously render the drawing of the first and second lines difficult.

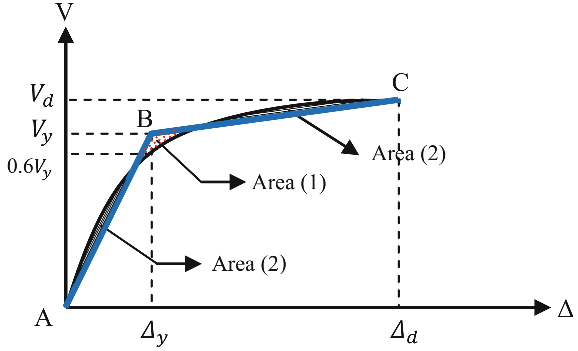
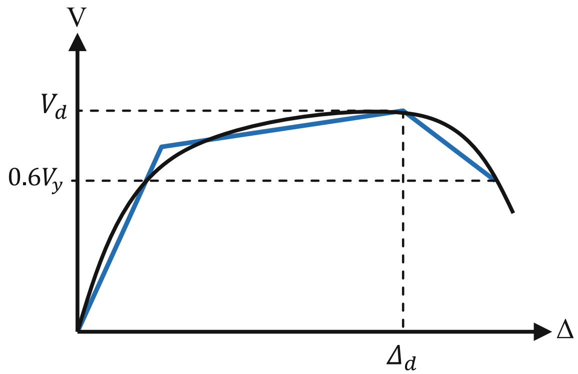


Fig. 4.52 Drawing of the first and second line on capacity curve

Fig. 4.53 Drawing the third line on capacity curve



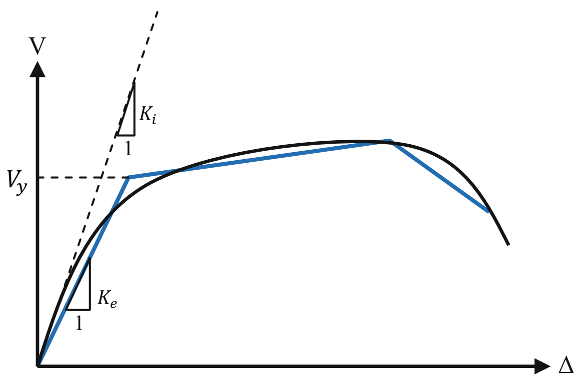
5. The third line passes over the end of the positive slope of the capacity curve, which means that the point (V_d, Δ_d) is where the base shear reaches 0.6 V_y (Fig. 4.53).

After changing the capacity curve of the structure to a three-line curve, three fundamental parameters were selected, which will be used in the next steps of the calculations:

1. The effective collapse base shear of the structure (V_y), which shows the intersection point of the first and the second lines in the three-line curve.
2. The lateral elastic stiffness of the structure (K_i), which is equal to the slope of the line tangent to the capacity curve of the structure at its origin.
3. The effective lateral stiffness of the structure (K_e), which is the slope of the first line in a three-line curve (Fig. 4.54).

In the following section, we will detail the process of determining the effective fundamental period of a building, target displacement, and design coefficients of the structure using the capacity curve of the structure and parameters obtained from it.

Fig. 4.54 Picking of (K_e), (K_i), (V_y) parameters on the cure



4.4.9 Effective Fundamental Period of Building

A parameter called the effective fundamental period of a building is used in the calculations related to the target displacement of the building, culminating in:

$$T_e = T_i \sqrt{\frac{K_i}{K_e}} \tag{4.59}$$

In this relation, T_i is the fundamental elastic period of the building (in seconds), obtained by analyzing the structure model and assuming a linear behavior (the period of the first mode of the structure). The parameters K_i and K_e are the lateral elastic stiffness and the effective lateral stiffness of the structure, respectively, obtained from the three-line curve.

4.4.10 Target Displacement

The target displacement in the control point can be calculated using the following relation:

$$\delta_t = C_0 C_1 C_a \frac{T_e^2}{4\pi^2} g \tag{4.60}$$

The parameters used in this relation are determined as follows:

1. The parameter T_e is the effective fundamental period of the building in the considered direction, while g is its gravitational acceleration.
2. The parameter S_a is the response spectral acceleration for the design earthquake and is calculated based on the effective fundamental period (T_e) and the effective quake weight (w).
3. The coefficient C_o is calculated using the following relation:

$$C_0 = ø_{1,r} \frac{\sum_{i=1}^{n} w_i ø_{1,i}}{\sum_{i=1}^{n} w_i ø_{1,j}^2} \tag{4.61}$$

The parameters w_i and $ø_{1,i}$ are the effective quake weight and the component of the first mode shape vector in the ith level. $ø_{1,i}$ is also the component of the first mode shape vector in the level of the control point (roof).

4. The coefficient C_1 is obtained from the following relations:

$$T_e \geq T_s \Rightarrow C_1 = 1, \quad T_e < T_s \Rightarrow C_1 = \frac{\left[1 + [R_d - 1]\frac{T_s}{T_e}\right]}{R_d}$$
$$R_d = \frac{S_a}{\frac{V_y}{W}} \tag{4.62}$$

In this relation, the parameter W is the effective quake weight and V_y is the effective collapse base shear of the structure.

Point:

The capacity curve, which is the relation between the base shear and the displacement of the control point, can be obtained using the nonlinear static analysis method from zero to the displacement equal to 150 of the target displacement.

4.4.11 Design Coefficient of Structure

We try to determine the coefficient of the structure behavior (R_u), the over strength coefficient (Ω), and the magnification coefficient of the displacement (C_d). Figure 4.55 can be used to better understand this topic and to determine these coefficients.

The parameters shown in the graph are defined as follows:

V_s The base shear of the building when the first plastic hinge is forming
V_y The base shear of the building when the total plastic mechanism is formed in the structure
V_d The capacity of the structure in the target displacement or in the displacement proportional to the maximum base shear (whichever is less)
V_e The base shear of the building, assuming a linear behavior of the structure

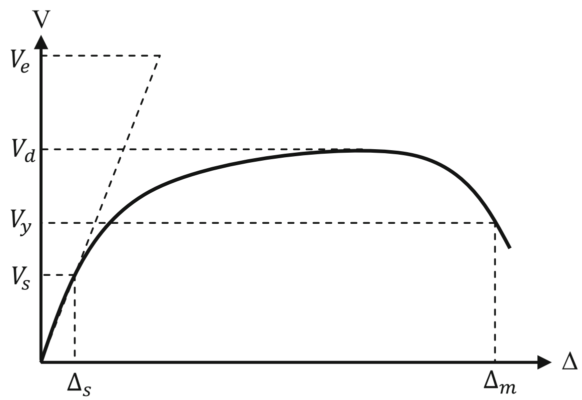


Fig. 4.55 Parameters in graph shear base-displacement

Δ_s The displacement of the control point when the first plastic hinge is formed
Δ_m Maximum displacement of the control point of the building when the plastic mechanism is formed in the structure

The coefficient that is used to design a structure ($R_u, \Omega_o, and\, C_d$) are defined using the parameters shown in the capacity curve and based on the following relations:

$$R_u = \frac{V_e}{V_s}, \Omega_0 = \frac{V_y}{V_s}, C_d = \frac{\Delta_m}{\Delta_s} \tag{4.63}$$

Note:

The coefficient of the behavior of the structure is, in fact, the product of the multiplication of three coefficients, which can be obtained using: (Comartin, Niewiarowski, Freeman, & Turner, 2000)

$$R_u = \frac{V_e}{V_s} = \frac{V_e}{V_d} \times \frac{V_d}{V_y} \times \frac{V_y}{V_s} = R_\mu \times R_R \times R_s \tag{4.64}$$

References

Agrawal, P., & Shrikhande, M. (2006). *Earthquake resistant design of structures*. PHI Learning Pvt. Ltd.
Code, P. (2005). *Eurocode 8: Design of structures for earthquake resistance-part 1: General rules, seismic actions and rules for buildings*.
Comartin, C. D., Niewiarowski, R. W., Freeman, S. A., & Turner, F. M. (2000). Seismic evaluation and retrofit of concrete buildings: A practical overview of the ATC 40 Document. *Earthquake Spectra, 16*(1), 241–261.
Fardis, M. N. (2009). *Seismic design, assessment and retrofitting of concrete buildings: Based on EN-Eurocode 8* (Vol. 8). Berlin: Springer Science & Business Media.
Heidary, M., & Ahangar, M. (2016). *2800 Code*: serieomran.
Irvine, T. (2013). Effective modal mass and modal participation factors revision D.
Jangid, R. S. (2013). *Introduction to earthquake engineering*. Bombay: IIT.
Rosenblueth, A., Ramos, G., & ÍA, J. (1951). The local responses of mammalian spinal roots to alternating sinusoidal currents. *Journal of Cellular Physiology, 38*(3), 321–345.
Rosenblueth, E., & Elorduy, J. (1969). Characteristics of Earthquakes on Mexico City, Clay. *Nabor Carillo, the Subsidence of Mexico City and Texcoco Project*, Secretaria de Hacienda y Credito Publica, Mexico (pp. 287–328).
Wilson, E. L., Der Kiureghian, A., & Bayo, E. (1981). A replacement for the SRSS method in seismic analysis. *Earthquake Engineering and Structural Dynamics, 9*(2), 187–192.