APPLIED MULTIVARIATE STATISTICAL ANALYSIS

SIXTH EDITION

RICHARD A. JOHNSON

DEAN W. WICHERN

JOHNSON
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SIXTH EDITION

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To the memory of my mother and my father.
R. A. J.

To Dorothy, Michael, and Andrew.
D. W. W.

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Preface

INTENDED AUDIENCE

This book originally grew out of our lecture notes for an "Applied Multivariate Analysis" course offered jointly by the Statistics Department and the School of Business at the University of Wisconsin–Madison. *Applied Multivariate Statistical Analysis*, Sixth Edition, is concerned with statistical methods for describing and analyzing multivariate data. Data analysis, while interesting with one variable, becomes truly fascinating and challenging when several variables are involved. Researchers in the biological, physical, and social sciences frequently collect measurements on several variables. Modern computer packages readily provide the numerical results to rather complex statistical analyses. We have tried to provide readers with the supporting knowledge necessary for making proper interpretations, selecting appropriate techniques, and understanding their strengths and weaknesses. We hope our discussions will meet the needs of experimental scientists, in a wide variety of subject matter areas, as a readable introduction to the statistical analysis of multivariate observations.

LEVEL

Our aim is to present the concepts and methods of multivariate analysis at a level that is readily understandable by readers who have taken two or more statistics courses. We emphasize the applications of multivariate methods and, consequently, have attempted to make the mathematics as palatable as possible. We avoid the use of calculus. On the other hand, the concepts of a matrix and of matrix manipulations are important. We do not assume the reader is familiar with matrix algebra. Rather, we introduce matrices as they appear naturally in our discussions, and we then show how they simplify the presentation of multivariate models and techniques.

The introductory account of matrix algebra, in Chapter 2, highlights the more important matrix algebra results as they apply to multivariate analysis. The Chapter 2 supplement provides a summary of matrix algebra results for those with little or no previous exposure to the subject. This supplementary material helps make the book self-contained and is used to complete proofs. The proofs may be ignored on the first reading. In this way we hope to make the book accessible to a wide audience.

In our attempt to make the study of multivariate analysis appealing to a large audience of both practitioners and theoreticians, we have had to sacrifice a consistency of level. Some sections are harder than others. In particular, we have summarized a voluminous amount of material on regression in Chapter 7. The resulting presentation is rather succinct and difficult the first time through. We hope instructors will be able to compensate for the unevenness in level by judiciously choosing those sections, and subsections, appropriate for their students and by toning them down if necessary.

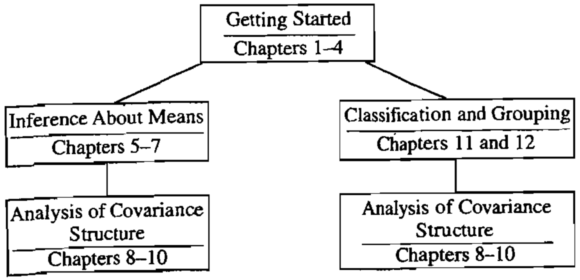
ORGANIZATION AND APPROACH

The methodological "tools" of multivariate analysis are contained in Chapters 5 through 12. These chapters represent the heart of the book, but they cannot be assimilated without much of the material in the introductory Chapters 1 through 4. Even those readers with a good knowledge of matrix algebra or those willing to accept the mathematical results on faith should, at the very least, peruse Chapter 3, "Sample Geometry," and Chapter 4, "Multivariate Normal Distribution."

Our approach in the methodological chapters is to keep the discussion direct and uncluttered. Typically, we start with a formulation of the population models, delineate the corresponding sample results, and liberally illustrate everything with examples. The examples are of two types: those that are simple and whose calculations can be easily done by hand, and those that rely on real-world data and computer software. These will provide an opportunity to (1) duplicate our analyses, (2) carry out the analyses dictated by exercises, or (3) analyze the data using methods other than the ones we have used or suggested.

The division of the methodological chapters (5 through 12) into three units allows instructors some flexibility in tailoring a course to their needs. Possible sequences for a one-semester (two quarter) course are indicated schematically.

Each instructor will undoubtedly omit certain sections from some chapters to cover a broader collection of topics than is indicated by these two choices.



For most students, we would suggest a quick pass through the first four chapters (concentrating primarily on the material in Chapter 1; Sections 2.1, 2.2, 2.3, 2.5, 2.6, and 3.6; and the "assessing normality" material in Chapter 4) followed by a selection of methodological topics. For example, one might discuss the comparison of mean vectors, principal components, factor analysis, discriminant analysis and clustering. The discussions could feature the many "worked out" examples included in these sections of the text. Instructors may rely on diagrams and verbal descriptions to teach the corresponding theoretical developments. If the students have uniformly strong mathematical backgrounds, much of the book can successfully be covered in one term.

We have found individual data-analysis projects useful for integrating material from several of the methods chapters. Here, our rather complete treatments of multivariate analysis of variance (MANOVA), regression analysis, factor analysis, canonical correlation, discriminant analysis, and so forth are helpful, even though they may not be specifically covered in lectures.

CHANGES TO THE SIXTH EDITION

New material. Users of the previous editions will notice several major changes in the sixth edition.

- Twelve new data sets including national track records for men and women, psychological profile scores, car body assembly measurements, cell phone tower breakdowns, pulp and paper properties measurements, Mali family farm data, stock price rates of return, and Concho water snake data.
- Thirty seven new exercises and twenty revised exercises with many of these exercises based on the new data sets.
- Four new data based examples and fifteen revised examples.
- Six new or expanded sections:
    1. Section 6.6 Testing for Equality of Covariance Matrices
    2. Section 11.7 Logistic Regression and Classification
    3. Section 12.5 Clustering Based on Statistical Models
    4. Expanded Section 6.3 to include "An Approximation to the Distribution of T^2 for Normal Populations When Sample Sizes are not Large"
    5. Expanded Sections 7.6 and 7.7 to include Akaike's Information Criterion
    6. Consolidated previous Sections 11.3 and 11.5 on two group discriminant analysis into single Section 11.3

Web Site. To make the methods of multivariate analysis more prominent in the text, we have removed the long proofs of Results 7.2, 7.4, 7.10 and 10.1 and placed them on a web site accessible through *www.prenhall.com/statistics*. Click on "Multivariate Statistics" and then click on our book. In addition, all full data sets saved as ASCII files that are used in the book are available on the web site.

Instructors' Solutions Manual. An Instructors Solutions Manual is available on the author's website accessible through *www.prenhall.com/statistics*. For information on additional for-sale supplements that may be used with the book or additional titles of interest, please visit the Prentice Hall web site at *www.prenhall.com*.