Chapter 2:

Governing Equations for Groundwater Flow

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- Groundwater in its natural state moves due to hydraulic forces.
- Because hydraulic head represents the energy of water, groundwater flows from locations of higher head, usually upland areas, to locations of lower head, such as lowland areas, marshes, springs, and rivers.

This is a principle to use water-level data obtained from wells, springs, and surface water features to determine the horizontal and vertical direction of groundwater movement and to estimate the rate of groundwater flow.

Chapter 2: Governing Equations for Groundwater Flow

- In the subsurface, water occurs in four phases: water vapor from transpiration by plants and direct evaporation from the water table, condensed water absorbed by dry soil particles, water hold around soil particles by molecular attraction, and water under the influence of gravity

- In the saturated zone, groundwater flows through interconnected voids in response to the difference in fluid pressure and elevation.
- The change in hydraulic head over a specified distance in a given direction is called the hydraulic gradient.

Chapter 2: Governing Equations for Groundwater Flow

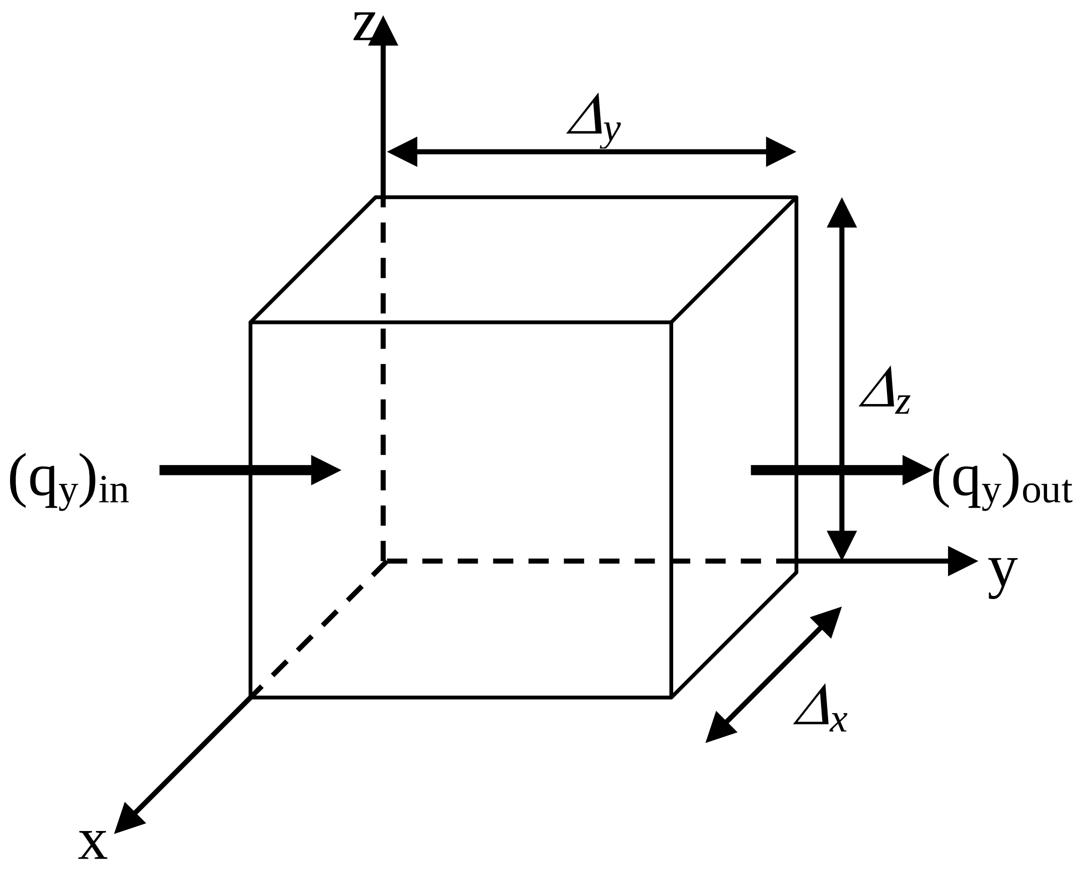
- The quantity of groundwater moving through a volume of rock can be estimated using Darcy's law which is a function of hydraulic gradient.
- Darcy's law determines the rate of groundwater flow through an area.

- The mathematical description of groundwater flow is based on the principles of conservation of mass, energy, and momentum.
- Generally, groundwater flow equations are expressed in terms of partial differential equations where spatial coordinates and time are independent variables.

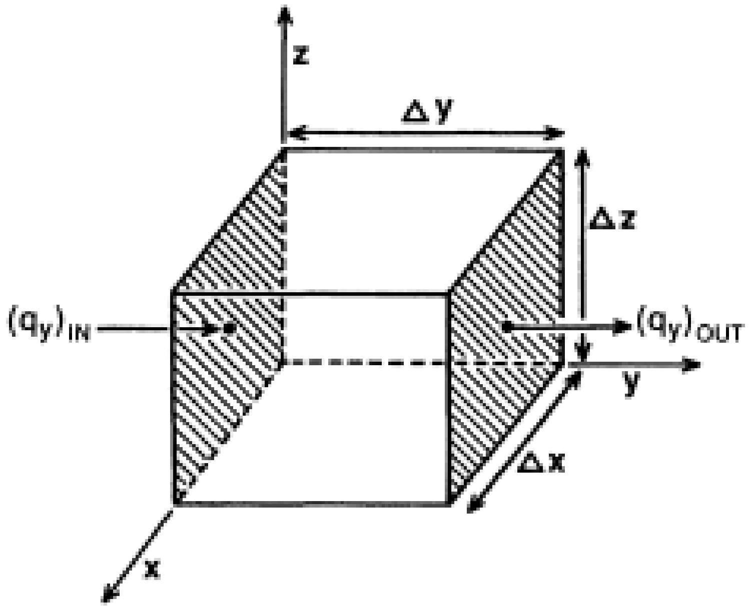
Groundwater Flow Equation

- **Figure below.** Representative elementary volume used in the derivation
- The cube in Figure is called the representative elementary volume (REV).

- Its volume is equal to $\Delta_x \Delta_y \Delta_z$.
- The flow of water through the REV is expressed in terms of the discharge rate (q), whose magnitude in the three coordinates will be q_x, q_y, and q_z.



- The water balance equation (conservation of mass) states that:
  - **Mass Out – Mass In = Change of the Mass in storage**



- Consider flow along the y-axis of the REV.
- Influx to REV occurs through the face $\Delta_x \Delta_z$ and is equal to $(q_y)_{in}$. Flux out is $(q_y)_{out}$.
- The volumetric flow rate along y-axis is: $$\left(q_{y,out} - q_{y,in}\right)\Delta_x \Delta_z$$
  - This can also be written as: $$\frac{\left(q_{y,out} - q_{y,in}\right)}{\Delta_y}\Delta_x \Delta_y \Delta_z$$
- Dropping the '*in*' and '*out*' subscripts, the change in flow rate through the REV along the y-axis is: $$\frac{\partial q_y}{\partial y}\Delta_x \Delta_y \Delta_z$$

- Similar expression can be written for the change in flow rate along the *x*- and *z*- axes. The total change in flow rate is equal to the rate of change in storage:

$$\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}\right)\Delta_x\Delta_y\Delta_z = Rate\ of\ \ Change\ in\ storage \qquad (1)$$

- The existence of sink (e.g. a pumping well) or source of water (e.g. injection well or some other source of recharge) within the REV is undeniable.
- The **volumetric inflow rate** of such sources is represented by $R^{*}\Delta_x\Delta_y\Delta_z$.
- Here the R^{*} is defined to be intrinsically positive when it is a source of water;
  - ✓therefore it is added to the right hand side of Eq. 1.

- Therefore Eq. 1 becomes:

$$\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}-R^*\right)\Delta_x\Delta_y\Delta_z = Rate\ of\ \ Change\ in\ storage \qquad (2)$$

- The change in storage is represented by **specific storage** (**S_S**).
- It is defined as the volume of water released from storage per unit change in head (h) per unit volume of aquifer (Anderson and Woessner, 1992) i.e.

$$S_S = -\frac{\Delta V}{\Delta h \Delta_x \Delta_y \Delta_z}$$

- The sign convention is that the ΔV is intrinsically positive when the Δh is negative, in other words, water is released from the REV when head decreases.

- The rate of change in storage in REV will be:

$$\frac{\Delta V}{\Delta t} = -S_S \frac{\Delta h}{\Delta t} \Delta_x \Delta_y \Delta_z \qquad (3)$$

- Combining Eq. 2 and Eq. 3:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = -S_S \frac{\partial h}{\partial t} + R^* \qquad (4)$$

- Darcy law is used to set the relationship between *q* and *h*.
- Darcy law in three dimension is written as (Anderson and Woessner, 1992):

$$q_x = -K_x \frac{\partial h}{\partial x} \qquad q_y = -K_y \frac{\partial h}{\partial y} \qquad q_z = -K_z \frac{\partial h}{\partial z}$$

Substituting these expressions in Eq. 4 the desired groundwater flow equation is formulated:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_S \frac{\partial h}{\partial t} - R^*$$

Where K_x, K_y, and K_z are components of the hydraulic conductivity.

- In the above derivation it is assumed that K_x, K_y, and K_z are collinear to the *x*, *y*- and *z*- axes.
- If the geology is such that it is not possible to align the principal direction of the hydraulic conductivity tensor with the rectilinear coordinate system, a modified form of equation that utilizes the hydraulic conductivity tensor is required.

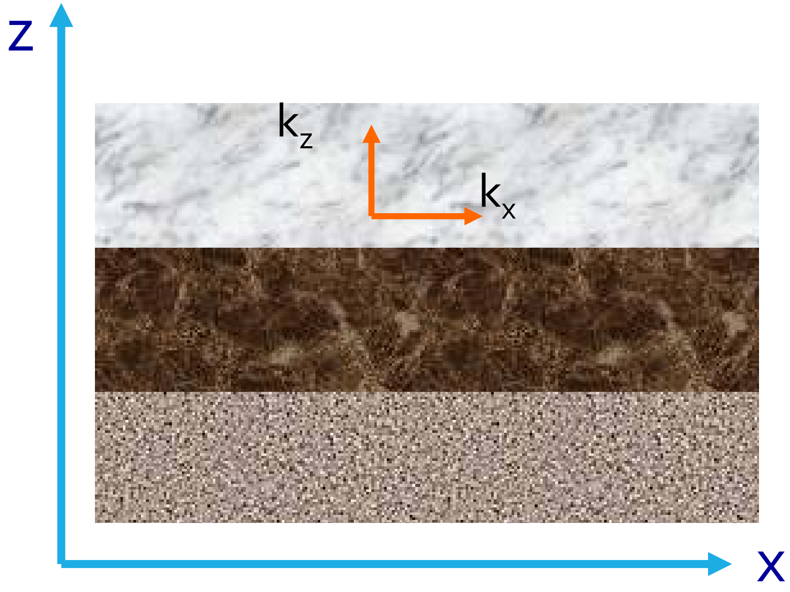
$$K = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

The off diagonal terms in the hydraulic conductivity tensor could have zero value (Anderson and Woessner, 1992).

$$K = \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{bmatrix}$$

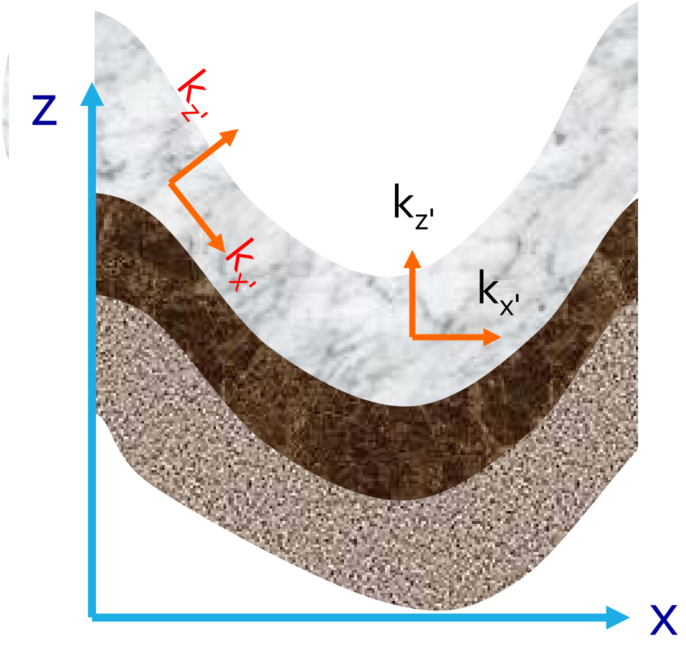
$$\frac{\partial}{\partial x}\left(k_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial z}\left(k_{zz}\frac{\partial h}{\partial z}\right)=S_S\frac{\partial h}{\partial t}-R^*$$

$$k_{xz}=k_{zx}=0$$



- The x-z coordinate system is aligned with the principal directions of the hydraulic conductivity tensor.

$$\frac{\partial}{\partial x}\left(k_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial x}\left(k_{xz}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial z}\left(k_{zz}\frac{\partial h}{\partial z}\right)+\frac{\partial}{\partial z}\left(k_{zx}\frac{\partial h}{\partial z}\right)=S_S\frac{\partial h}{\partial t}-R^*;\quad k_{xz}\neq 0;k_{zx}\neq 0$$



- A global coordinate system (x-z) is defined. Local coordinates (x'- z') are aligned with the principal directions of the local hydraulic conductivity tensor.

Initial and Boundary Conditions

For a well posed boundary value problem:

(i) A solution must exist,

(ii) The solution must be unique and

(iii) The solution must be stable, in the sense that sufficiently small variations in the given data should lead to arbitrary small changes in the solution

- Initial and boundary conditions are needed for a unique solution of the groundwater flow equations (second-order partial differential equations) for a specific flow domain of interest.

Initial and boundary conditions

- **Initial conditions:** specification of the distribution of the state variable (hydraulic head for the groundwater flow equation) at some initial time, usually at t = 0.
- For example

$$h = h(x, y, z, 0) = f(x, y, z) \quad \text{in D}$$

in which *f*(*x*,*y*,*z*) is a known function, D is the flow domain.

Boundary Classifications

- The boundaries of a groundwater model are the domains or points at which the dependent variable (head) or the derivative of the dependent (flux) is known.
- The presence of an impermeable body of rock or a large body of surface water forms the *physical boundary* of the groundwater flow systems.

- Groundwater divide or streamline forms the *hydraulic boundary* of the groundwater flow systems (Anderson and Woessner 1992, Diersch 1998).

Boundary Classifications

- Physical or Natural Boundaries
    - ✓geologic contacts, margins of surface water bodies, etc.
- Hydraulic Boundaries
    - ✓water divides or streamlines
    - ✓derived from conceptual model (risky) or larger-scale
- models (telescopic mesh refinement)
    - ✓ really only useful in steady-state problems. Streamlines often move in transient problems
- Distant Boundaries
    - ✓ when in doubt, put boundaries far from area of interest
    - ✓ any errors in boundary specification will have minimal effect

Boundary conditions:

Boundary Classifications

- **Real world**

✓Physical boundary (sharp change in hydraulic conductivity)

✓Hydraulic boundary (groundwater divides, streamlines)

- Different initial and boundary conditions result in different solutions

Three mathematical boundary conditions:

1. Dirichlet (Fixed Head)
2. Neumann (Fixed Flux)
3. Cauchy (Head Dependent Flux)

Boundary Classifications

- Figure: Telescopic mesh refinement as a means for defining boundary conditions

[risky, but commonly practiced;

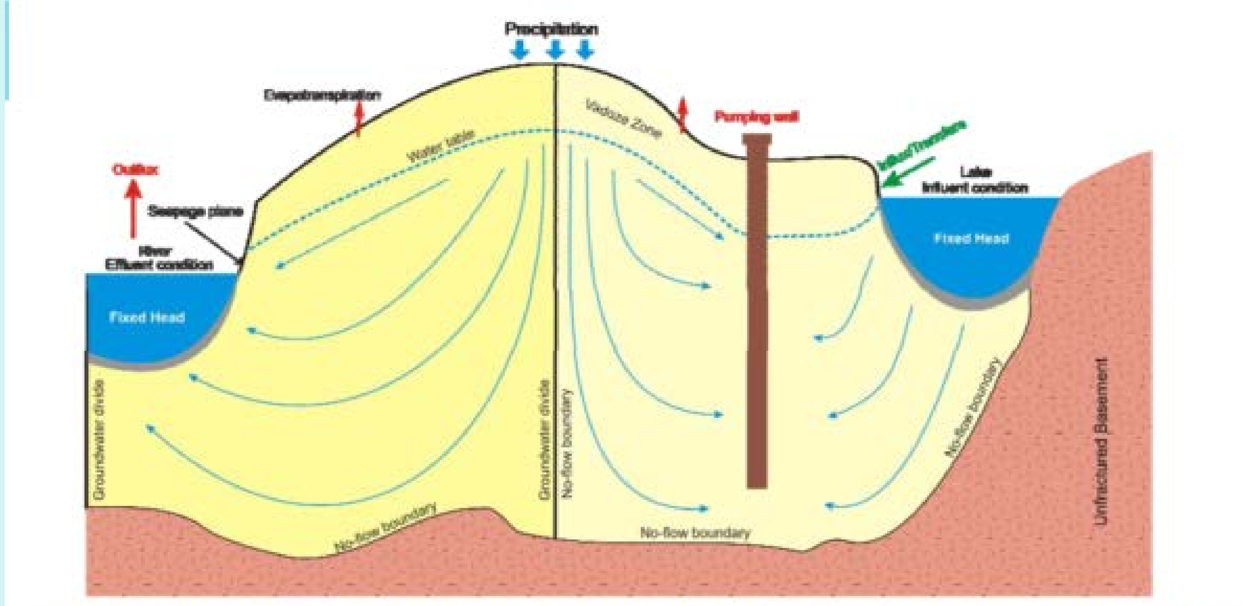
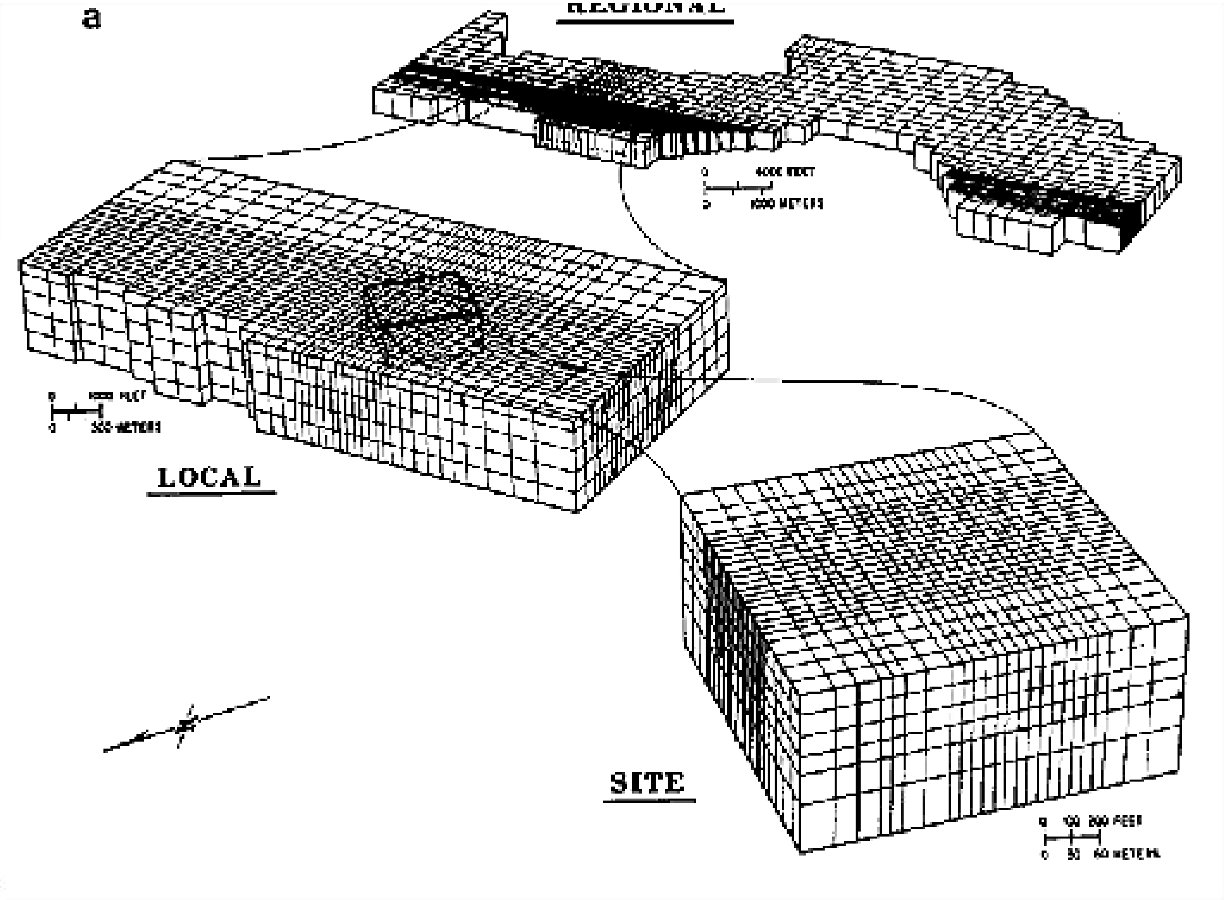


Diagram of regional groundwater flow system representing the physical and hydraulic boundary conditions.



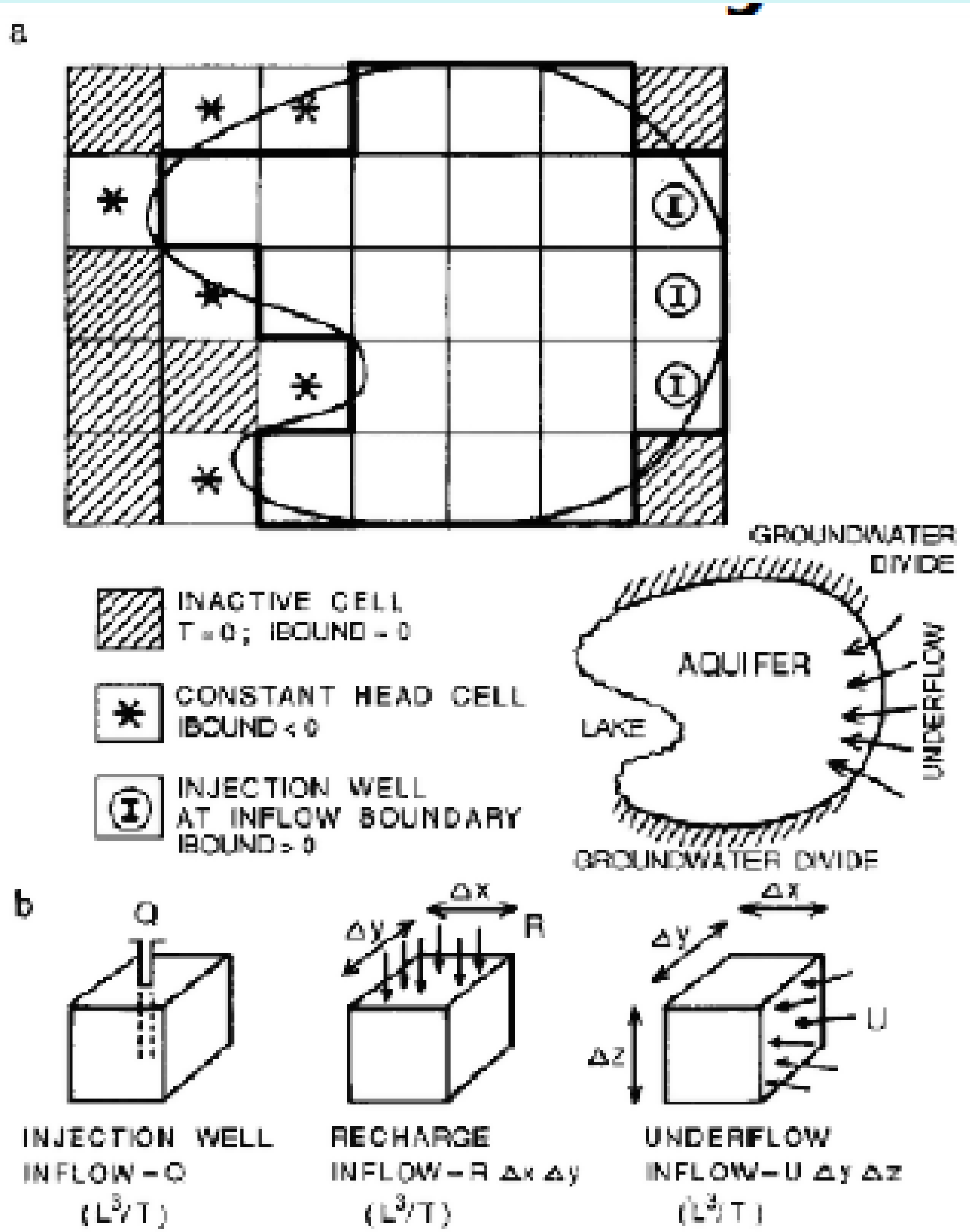
Boundary conditions

- **1st type: Dirichlet condition**, specified head boundary:
- The key assumption for the Dirichlet boundary condition is that regardless of the groundwater flow within the flow domain and at the boundary, there will be no influence on the potential of the outside water body, in such a way that this potential remains fixed as determined by the boundary condition.

- Conceptually this is an infinite source or sink for water
- usually represents a body of surface water
- accurate only when the un-modeled flux (i.e. whatever external flux maintains the body's water level) exceeds the modeled flux by a factor of 10 or more

Boundary conditions

- **MODFLOW (Figure below)**
- Figure: Boundary condition specification in MODFLOW. Note areal fluxes (e.g. recharge) are converted to volumetric by multiplying by cell surface area



Boundary conditions

2nd type: Neuman condition, specified flow boundaries:

A flux boundary condition implies that regardless of the state and flow of the groundwater inside the flow domain and at the boundary, the normal flux is fixed by external conditions and remains as determined by the boundary condition.

- ✓ use when water exchange with surface water bodies is independently known (e.g. through geochemical studies)
- ✓ most accurate type of boundary condition (i.e. good to use since won't accidentally generate infinite fluxes)

Boundary conditions

MODFLOW:

- implement non-zero flux using Well (“placing" water into the boundary cell for *known volume of flux*) or Recharge Package (for *known Darcy velocity*)
- *no-flow boundaries* are the default along model edge and between inactive and active cells

- The flux across the boundary is known. It includes no flow boundaries between geological units (boundaries where flux is specified to be zero), interactions between groundwater and surface water bodies, springflow, underflow, and seepage from bedrock into alluvium.
- The most commonly applied form of a Neuman Boundary is a no-flow boundary, often occurring between a highly permeable unit and a unit of much lower permeability or from water divide.

Boundary conditions

- A difference in hydraulic conductivity of two orders of magnitude or greater between two adjacent units is enough to rationalize assignment of a no-flow boundary

- This case is also applicable if the hydraulic gradient across the boundary is also low, as flow out of the higher conductivity layer is negligible, and the boundary can then be set to impermeable.
- Saltwater interface at some coastal aquifers, some faults, and regional groundwater divides form typical no-flow boundaries.

Boundary conditions

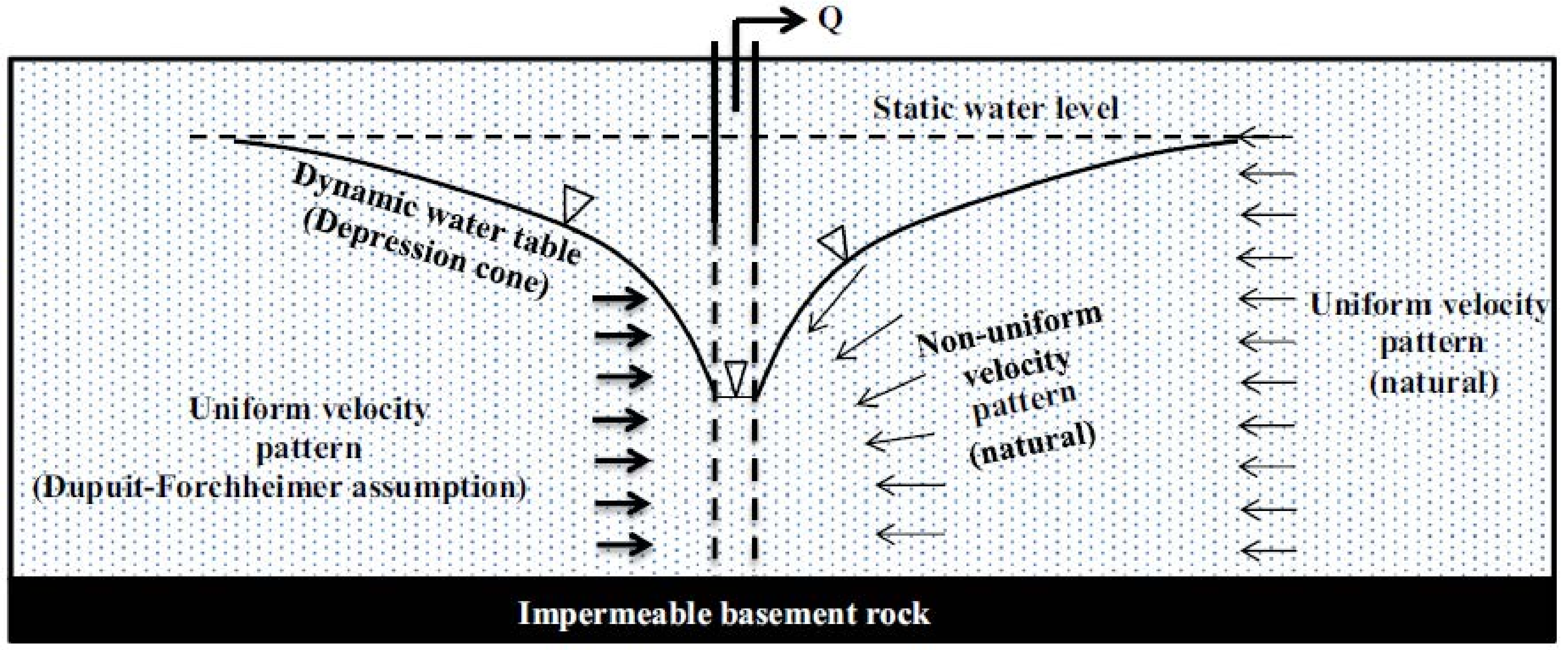
3rd type: Cauchy condition, head-dependent flow boundary:

For which the flux across the boundary is dependent on the magnitude of the difference in head across the boundary, with the head on one side of the boundary being input to the model and the head on the other side being calculated by the model.

- Cauchy Boundary can be applied to leakage from a surface water body where the flux is dependent on the difference of hydraulic head between the surface water and groundwater level and
  - ✓the vertical hydraulic conductivity of the boundary; and
  - ✓evapotranspiration where the flux is proportional to the depth of the water table in an unconfined aquifer.

Dupuit-Forchheimer assumptions

- Apart from the geological and hydrogeological differences from the confined aquifers, unconfined aquifers include nonlinearity in the flow lines, and hence, the analytical solutions become more complex.
- The upper boundary limitation is the groundwater table (phreatic level) does not allow horizontal flow around the well within the depression cone influence area.



Dupuit-Forchheimer assumptions

- Quantitative treatment of confined aquifers with fully penetration wells is easier than unconfined aquifers because in confined aquifers the saturation zone is completely defined by impervious layers and the streamflow line geometry remains parallel and does not change with time.
- The difficulty in unconfined aquifers comes from spatial and temporal changes of saturation thickness near the main well.

- Temporal changes render the flow not only into a three-dimensional type near the well but also equipotential lines become curves leading to decreases in saturation thickness toward the well as in Figure above.

- Thus the groundwater velocity direction is not horizontal at every point but inclined especially in the well vicinity near the water table.

Dupuit-Forchheimer assumptions

- The hydraulic gradient also changes from one point to another within the flow domain.
- The inclinations and variations cause difficulties in the groundwater movement quantification.
- In order to avoid such difficulties Dupuit (1863) proposed the **following assumptions** that were later advanced by Forchheimer (1901).

1. The water is homogeneous and it has the same physical properties in every direction.
2. The flow lines are horizontal, and accordingly, the equipotential lines are vertical.
   - ✓ This assumption implies horizontal groundwater velocity and its uniform distribution along the whole saturation thickness.
   - ✓ It becomes close to reality if the water table hydraulic gradient is small.

Dupuit-Forchheimer assumptions

3. The hydraulic gradient at every point along a vertical line is equal to the slope of the free surface. The flow is horizontal and uniform near the water table.
4. The capillary zone is negligibly small.

5. Theim (1906) assumed that in unconfined aquifers transmissivity is constant provided that the variations in saturation thickness are small compared to the initial saturation thickness.
6. The aquifer material and water are incompressible.

Dupuit-Forchheimer assumptions

- It is obvious that these assumptions neglect completely the vertical flow component.
- However, the practical value of Dupuite-Forchheimer assumptions lies in the fact that they reduce a **three-dimensional flow into a two-dimensional type** similar to the confined aquifer case, which is easier to deal with analytically.

- Unconfined aquifers are the most frequently used types because of their hand-dug well convenience, easy reach to shallow groundwater levels, and direct replenishment (recharge) possibilities.

Example : Confined aquifer with steady-flow condition

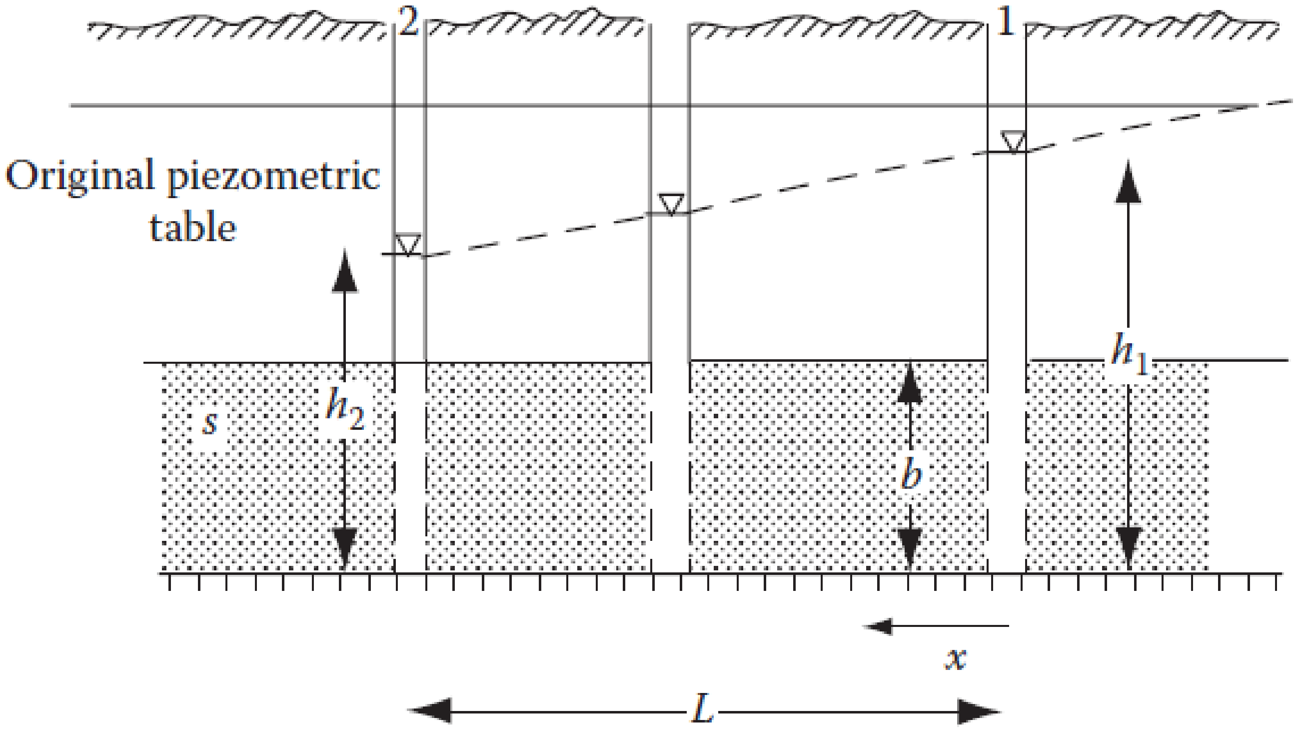


Figure. Confined aquifer with steady-flow condition

Example: Develop the groundwater flow equations for a confined aquifer shown in Figure.

Assume the aquifer is homogeneous with isotropic and steady-flow conditions. If the hydraulic gradient between two wells in the distance of 250 m is equal to 0.45, find the flow velocity and aquifer discharge.

Assume k = 5 × 10–6 m/s and b = 30 m.

Example : Confined aquifer with steady-flow condition

Solution

For the one-dimensional confined aquifer system, the governing equation is given by the following equation:

$$\frac{d}{dx}\left(\frac{dh}{dx}\right) = 0$$

or

$$\frac{dh}{dx} = C_1$$

Applying Darcy's law, we have

$$\frac{dh}{dx} = -\frac{V}{K}$$

So,

$$h = -\frac{V}{K}x + C_2$$

with the boundary conditions,

$$h(0) = h_1, \quad h(L) = h_2$$

$$V = -\frac{K}{L}(h_2 - h_1) \quad \text{and} \quad q = -\frac{Kb}{L}(h_2 - h_1)$$

Example : Confined aquifer with steady-flow condition

with the boundary conditions,

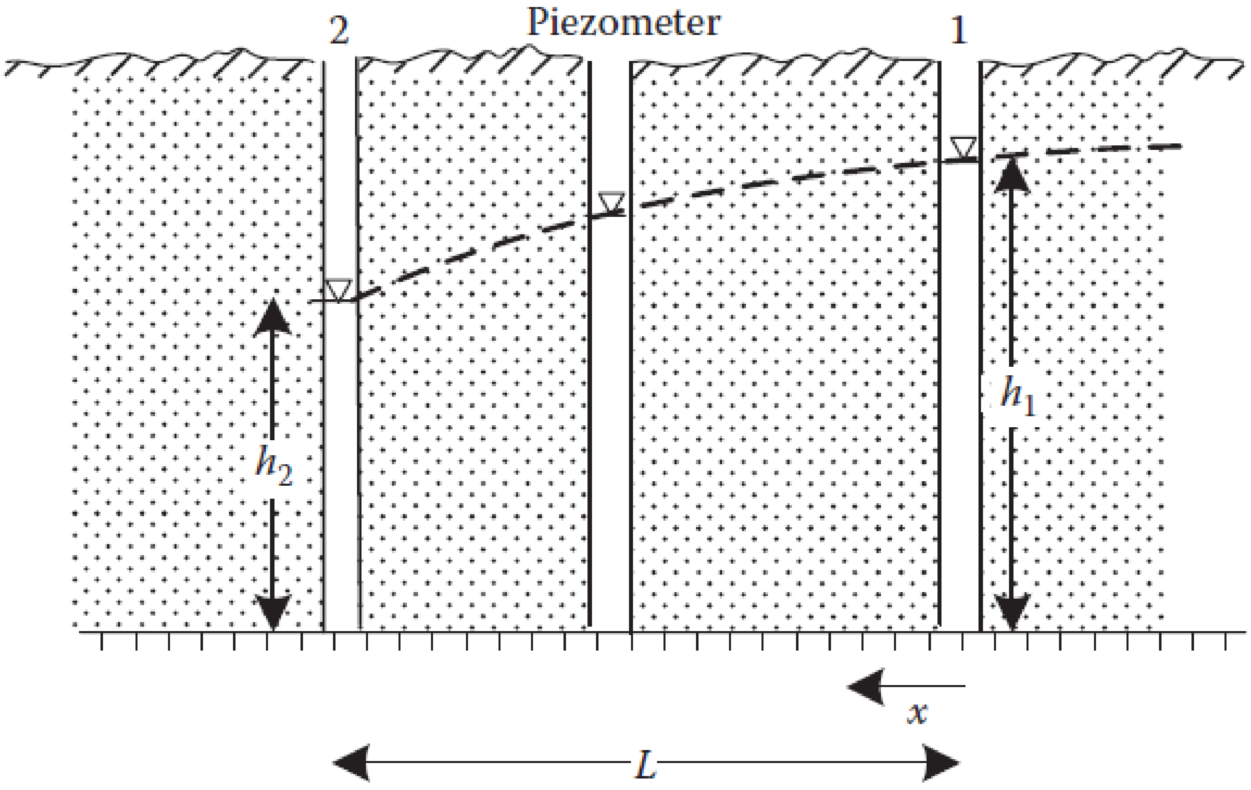
$$h(0) = h_1, \quad h(L) = h_2$$

$$V = -\frac{K}{L}(h_2 - h_1) \quad \text{and} \quad q = -\frac{Kb}{L}(h_2 - h_1)$$

$$V = -K\frac{(h_2 - h_1)}{L} = 5 \times 10^{-6} \times 0.45 = 2.25 \times 10^{-6}\ \text{m/s}$$

$$q = Vb = 2.25 \times 10^{-6} \times 30 = 6.75 \times 10^{-5}\ \text{m}^2/\text{s}$$

Unconfined aquifer with steady-flow condition



Example:

Develop the groundwater flow equations for a homogeneous and isotropic unconfined aquifer shown in steady-flow condition as shown in Figure.

If the water table in two wells are equal to 60 and 43 m, find the water table in the observation well in middle distance between the two wells. With the assumption of a linear variation of water table between two wells, find the water table in the observation well.

Unconfined aquifer with steady-flow condition

Solution

Applying Darcy's law, we have $q = -KA\frac{dh}{dx}$

and $A = h \times 1$

with the boundary conditions, $h(0) = h_1, \quad h(L) = h_2$

- Direct integration of the equation produces $\int_0^L q\,dx = -K\int_{h_1}^{h_2} h\,dh$

So, $q = -\frac{K}{2L}(h_2^2 - h_1^2)$ and $h = \sqrt{h_1^2 - (h_1^2 - h_2^2)\frac{X}{L}}$

$$h = \sqrt{h_1^2 - (h_1^2 - h_2^2)\frac{X}{L}} = \sqrt{60^2 - (60^2 - 43^2) \times \frac{L/2}{L}} = 52.2\text{ m}$$

Unconfined aquifer with steady-flow condition

$$h = \sqrt{h_1^2 - (h_1^2 - h_2^2)\frac{X}{L}} = \sqrt{60^2 - (60^2 - 43^2) \times \frac{L/2}{L}} = 52.2\ \text{m}$$

With the assumption of a linear variation of water table:

$$h = 60 - \frac{(60 - 43)}{2} = 51.5\text{m}$$

It shows with a parabolic water table assumption, water table is observed more than linear variation.

Example : two-dimensional steady-state flow without accretion

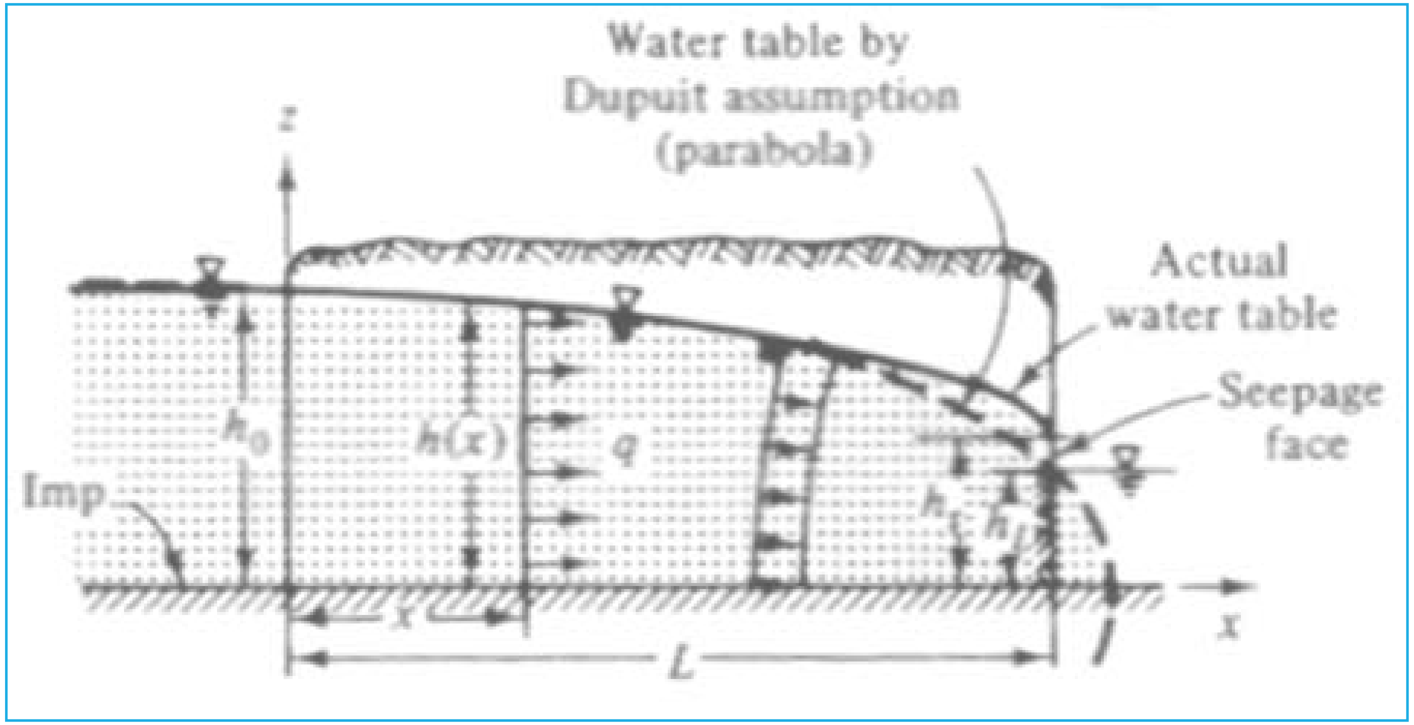


Figure. Steady unconfined flow between two reservoirs (Bear, 1979)

$$Q_x = -Kh\frac{dh}{dx} = \text{constant}$$

$$\Rightarrow Q_x dx = -Khdh$$

$$\Rightarrow Q_x \int_0^L dx = -K\int_{h_0}^{h_L} hdh = K\frac{\left(h_0^2 - h_L^2\right)}{2}$$

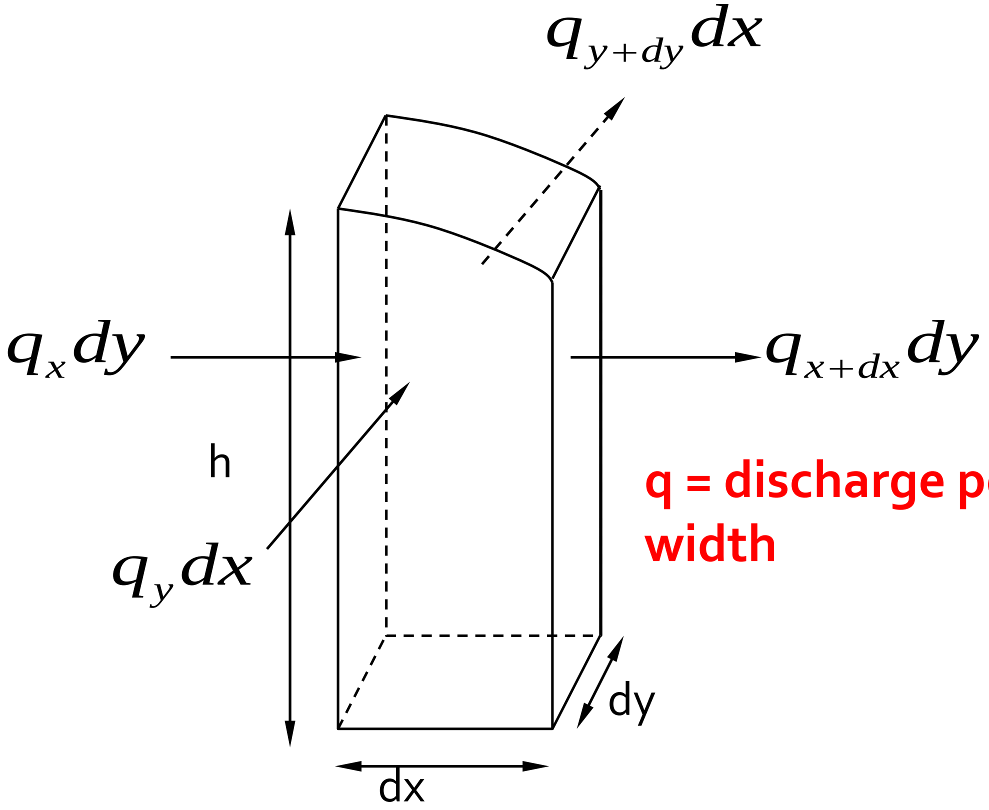
$$\Rightarrow Q_x = \frac{K\left(h_0^2 - h_L^2\right)}{2L} \qquad (1)$$

(Dupuit equation)

(Where Q_x = flow per unit width)

Transient 2-D unconfined flows

For incompressible fluids and homogeneous and isotropic aquifers



(impervious, horizontal bottom)

q = discharge per unit width

Mass in – mass out = Δ M

Under Dupuit assumptions: $h(x,y,z) \rightarrow h(x,y)$

$$\rho q_x dy\Delta t - \rho q_{x+dx} dy\Delta t + \rho q_y dx\Delta t - \rho q_{y+dy}\Delta t = \rho S_y dxdy\Delta h$$

$$\Rightarrow \rho\Delta t dy\left[\underbrace{-Kh\left(\frac{\partial h}{\partial x}\right)_x}_{q_x} + \underbrace{K\left(h\frac{\partial h}{\partial x}\right)_{x+dx}}_{q_{x+\Delta x}}\right]$$

$$+\rho\Delta t dx\left[\underbrace{-Kh\left(\frac{\partial h}{\partial y}\right)_y}_{q_y} + \underbrace{K\left(h\frac{\partial h}{\partial y}\right)_{y+dy}}_{q_{y+\Delta y}}\right] = \rho S_y dxdy\Delta h$$

$$\Rightarrow K\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)dxdy + K\frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right)dxdy = S_y dxdy\frac{\partial h}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right) = \frac{S_y}{K}\frac{\partial h}{\partial t} \text{ (Boussinesq equation)} \qquad (4)$$

S_y = specific yield $\equiv \dfrac{\Delta V_w}{A\Delta h}$

- Boussinesq equation is a non-linear PDE, which can not be solved analytically except under some idealized conditions
- Approximations: Drawdown in the aquifer is small, i.e., h ≈ b (averaged thickness assumed to be constant over the aquifer)

From (4):

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right)\approx\frac{\partial}{\partial x}\left(b\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(b\frac{\partial h}{\partial y}\right)=\frac{S_y}{K}\frac{\partial h}{\partial t}$$

$$\Rightarrow \frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S_y}{Kb}\frac{\partial h}{\partial t} \qquad (5)$$

(Note that (5) is similar to the 2-D flow in a confined aquifer, except that S, Storativity of a confined aquifer, is used instead of S_y)

Application of Flow net

- The 2D steady state Groundwater flow equation in isotropic and homogeneous porous medium can be expressed by Laplace's Equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

- Graphically, the equation can be represented by two sets of curves known as '*Equipotential line*' and '*flow lines*', that intersect at right angles.
- The combined representation of two sets of lines is called a flow net.
- With the help of a flow net, the groundwater flow problems can be analyzed.

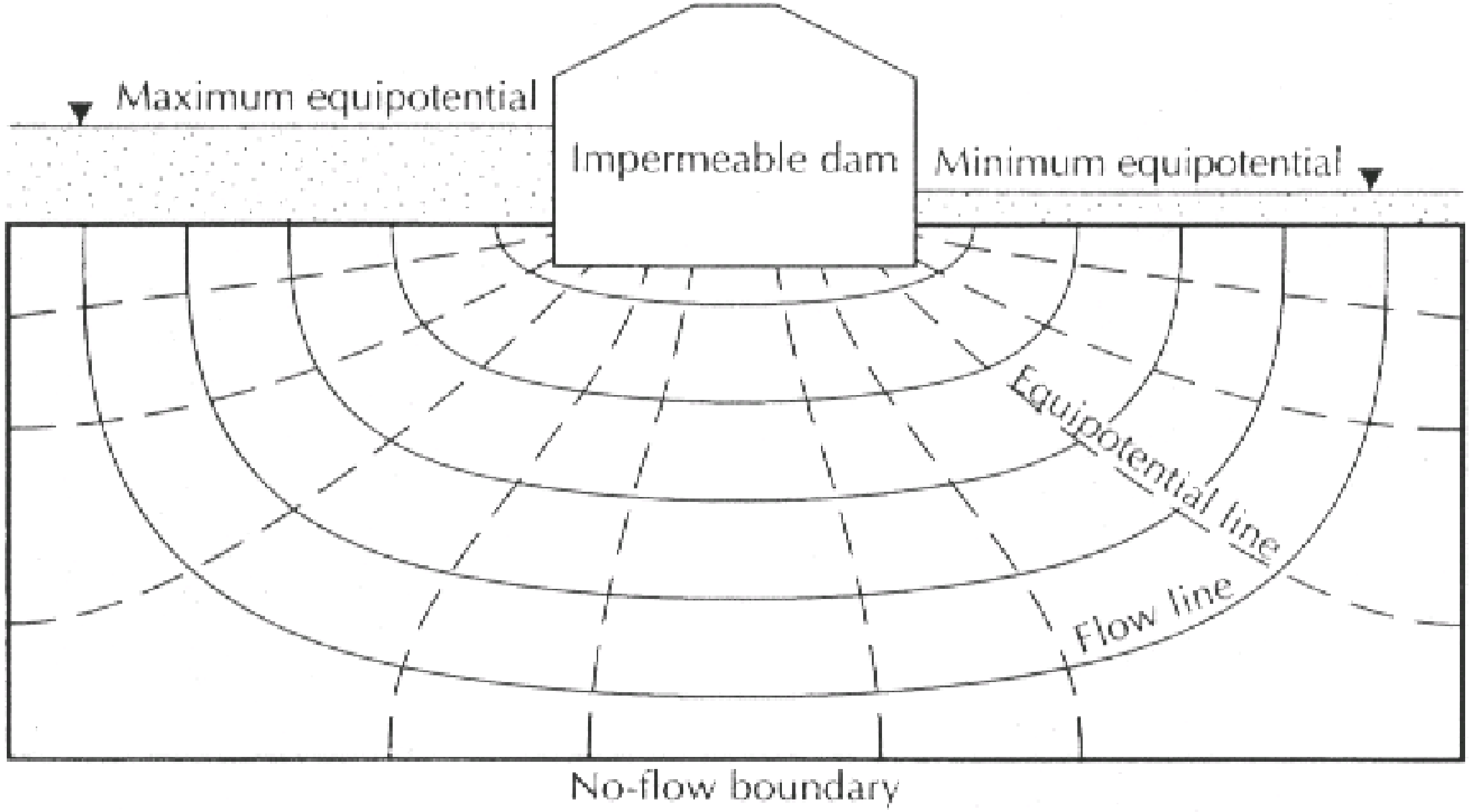
Equipotential line: A line on which values of hydraulic head are the same.

- Potential of groundwater φ = Δh = mechanical energy (pressure energy + elevation energy) per unit mass of groundwater.
- Equipotential lines are always perpendicular to the direction of Δh, no matter the isotropy of the medium

Flow line (Fetter, 1994): An imaginary line that traces the path that a particle of groundwater would follow as it flows through an aquifer.

- ✓ Flow lines will cross equipotential lines at right angles in an isotropic aquifer
- ✓ Flow lines will cross the equipotential lines at an angle dictated by the degree of anisotropy
- ✓ Flow lines are parallel to Δh in isotropic media but not in anisotropic media

$$Q = \frac{KmH}{n}$$



Q : flow per unit width [L^2/T]

K : homogeneous/isotropic hydraulic conductivity [L/T]

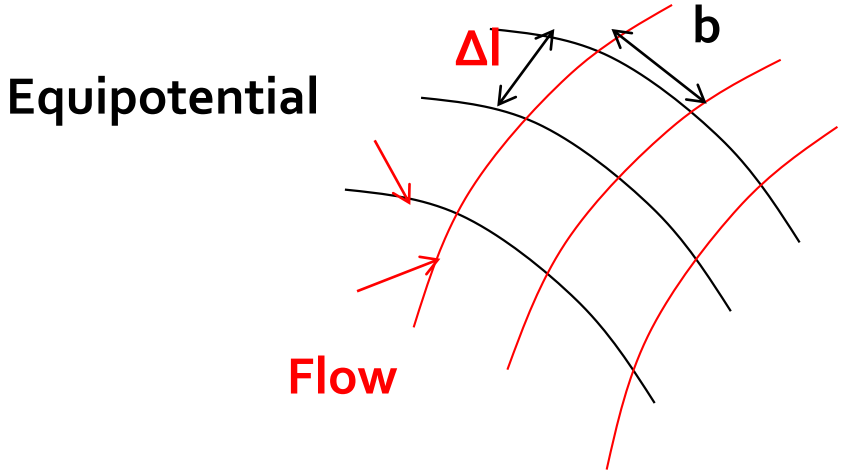
m : # of stream tubes (flow tubes, i.e., area between two adjacent flow lines)

n : # of divisions of head in the flow net

- **Darcy's equation**: $v = ki$; where k is hydraulic conductivity (m/s) and
  - ✓ $i\ (= \Delta h/\Delta l)$ is hydraulic gradient.
- The seepage flow q, through a cross sectional area A is computed as; $q = vA = kiA$.
- **In the flow net case**: for a single net $A = bX1 = b$; $q = kb\Delta h/\Delta l$, but $\Delta h = H/N_d$
  - ✓ where N_d is the number of equipotential drops; and
  - ✓ H is the head difference between the initial and end section along the groundwater flow direction.

The total discharge per unit width

- $Q = N_f(q) = N_f kbH/(N_d \Delta l)$;
- however if the flow net is drawn so that $b \approx \Delta l$,
- $Q = kHN_f/N_d$
  - ✓ Where N_f is the number of flow tubes.



Boundary conditions Vs flow lines and equipotential lines

No-flow boundary (Neumann): Adjacent flow lines are parallel to this boundary, and equipotential lines are perpendicular to this boundary

Constant-head boundary (Dirichlet): This boundary represents an equipotential line and adjacent equipotential lines are parallel to this boundary.

- Flow lines will intersect the constant-head boundary at right angles

Water-table boundary*:* the water table, in general, is neither a flow line nor an equipotential line. It is a line where head is known.

- If Dupuit assumption is valid, equipotential lines are vertical and flow lines are horizontal.
- If there is recharge or discharge across the water table, flow lines will be at an oblique angle to the water table.

Three Boundary Conditions vs flow lines and equipotential lines

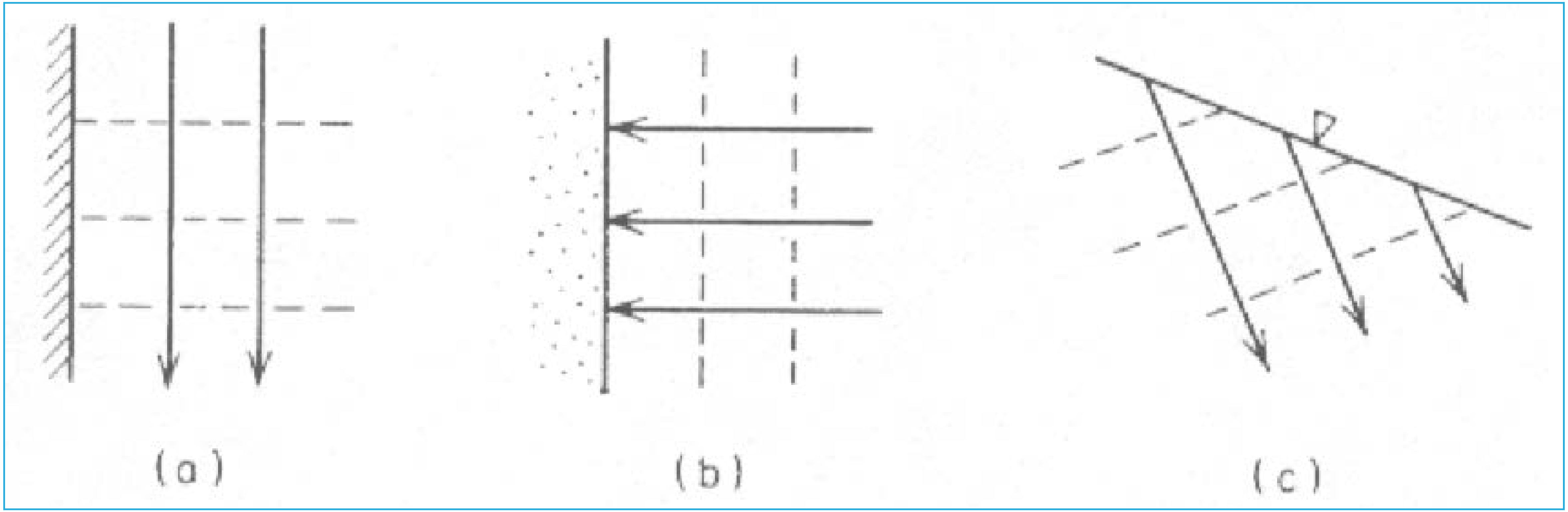


Figure. Groundwater flow in the vicinity of (a) an impermeable boundary, (b) a constant-head boundary, and (c) a water-table boundary (Freeze and Cherry, 1979)

Flow nets for anisotropic media

- The two dimensional seepage flow equation is not a Laplace equation.
- As the permeability is different in the two directions.
- For example in horizontally stratified aquifers, the horizontal permeability is usually greater than the vertical.
- Thus the seepage flow equation in anisotropic soils will be:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0$$