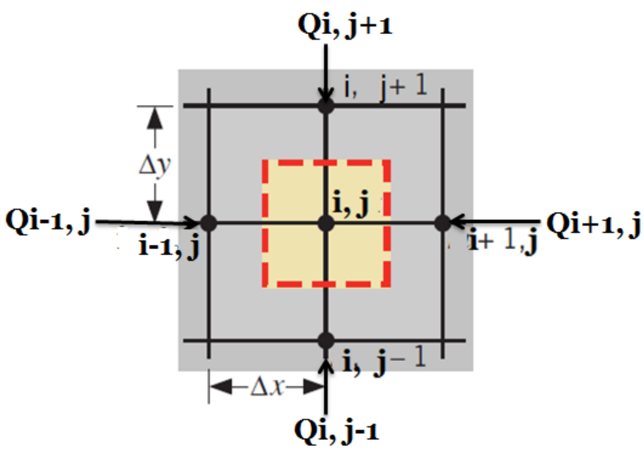
EXPLICIT FINITE DIFFERENCE METHOD [STEADY STATE CONDITION]

- In the energy balance method, the finite-difference equation for a node is obtained by applying conservation of energy to a control volume about the nodal region.
- Since the actual direction of heat flow (into or out of the node) is often unknown, it is convenient to formulate the energy balance by ***assuming* that *all* the heat flow is *into the node***.
- **For steady-state conditions** with generation, the appropriate form of Equation is written as:

$$E_{in} + E_{gen} = 0 \qquad [1]$$

$$\sum_{n=1}^{4} Q_{n \to (i,j)} + q(\Delta x.\Delta y.1) = 0 \qquad [2]$$

1. Interior Node



Assuming No heat generation inside the system equation [2] becomes:

$$Q_{i+1,j} + Q_{i-1,j} + Q_{i,j+1} + Q_{i,j-1} = 0 \qquad [3]$$

$$Q_{i+1,j} = k(\Delta y.1)\frac{T_{i+1,j}-T_{i,j}}{\Delta x} \qquad [4.1]$$

$$Q_{i-1,j} = k(\Delta y.1)\frac{T_{i-1,j}-T_{i,j}}{\Delta x} \qquad [4.2]$$

$$Q_{i,j+1} = k(\Delta x.1)\frac{T_{i,j+1}-T_{i,j}}{\Delta y} \qquad [4.3]$$

$$Q_{i,j-1} = k(\Delta x.1)\frac{T_{i,j-1}-T_{i,j}}{\Delta y} \qquad [4.4]$$

Substituting equation [4.1] – [4.4] in to equation [3] we get:

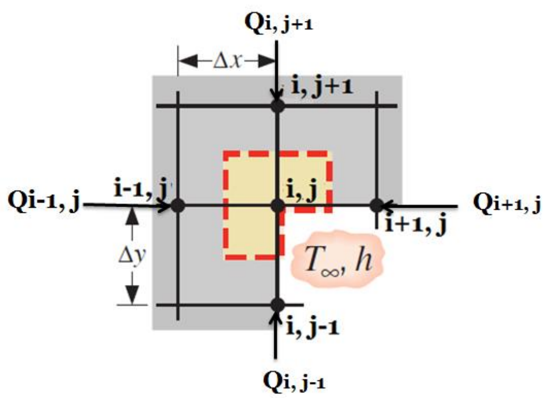
$$k(\Delta y.1)\frac{T_{i+1,j}-T_{i,j}}{\Delta x}+k(\Delta y.1)\frac{T_{i-1,j}-T_{i,j}}{\Delta x}+k(\Delta x.1)\frac{T_{i,j+1}-T_{i,j}}{\Delta y}+k(\Delta x.1)\frac{T_{i,j-1}-T_{i,j}}{\Delta y}=0$$

Assuming uniform mesh ($\Delta x = \Delta y$) and rearranging the equation becomes:

$$k(T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}) - 4kT_{i,j} = 0$$

$$\boxed{\boldsymbol{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0}}$$

2. Node at interior corner with convection



With no heat generation inside the system, the energy balance equation for Node at interior corner with convection can be given as:

$$Q_{i+1,j} + Q_{i-1,j} + Q_{i,j+1} + Q_{i,j-1} + Q_{conv1} + Q_{conv2} = 0 \quad [5]$$

Where;

$$Q_{i+1,j} = k(\tfrac{\Delta y}{2}.1)\ \frac{T_{i+1,j}-T_{i,j}}{\Delta x} \quad [6.1]$$

$$Q_{i-1,j} = k(\Delta y.1)\ \frac{T_{i-1,j}-T_{i,j}}{\Delta x} \quad [6.2]$$

$$Q_{i,j+1} = k(\Delta x.1)\frac{T_{i,j+1}-T_{i,j}}{\Delta y} \quad [6.3]$$

$$Q_{i,j-1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}-T_{i,j}}{\Delta y} \quad [6.4]$$

$$Q_{conv1} = h\left(\frac{\Delta x}{2}.1\right)\left(T_\infty - T_{i,j}\right) \quad [6.5]$$

$$Q_{conv1} = h(\frac{\Delta y}{2}.1)(T_\infty - T_{i,j}) \qquad [6.6]$$

Substituting equations from [6.1]-[6.6] in equation [5] we get:

$$k\left(\frac{\Delta y}{2}.1\right)\frac{T_{i+1,j}-T_{i,j}}{\Delta x} + k(\Delta y.1)\frac{T_{i-1,j}-T_{i,j}}{\Delta x} + k(\Delta x.1)\frac{T_{i,j+1}-T_{i,j}}{\Delta y} + k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}-T_{i,j}}{\Delta y} + h\left(\frac{\Delta x}{2}.1\right)(T_\infty - T_{i,j}) + h\left(\frac{\Delta x}{2}.1\right)(T_\infty - T_{i,j}) = 0$$

Assuming uniform mesh ($\Delta x = \Delta y$) and rearranging the equation:

$$\frac{k}{2}(T_{i+1,j}-T_{i,j}) + k(T_{i-1,j}-T_{i,j}) + k(T_{i,j+1}-T_{i,j}) + \frac{k}{2}(T_{i,j-1}-T_{i,j}) + h\Delta x(T_\infty - T_{i,j}) = 0$$

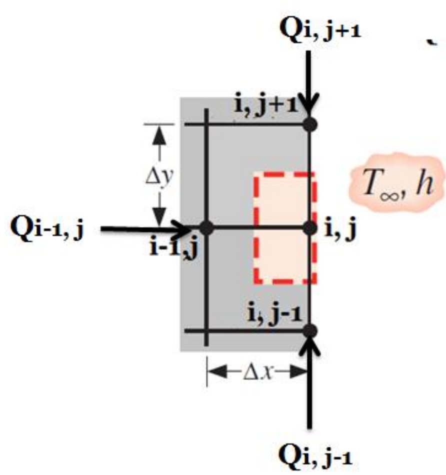
Multiplying both sides by ($\frac{2}{k}$) we get:

$$T_{i+1,j} + 2T_{i-1,j} + 2T_{i,j+1} + T_{i,j-1} - 6T_{i,j} - \frac{2h\Delta x}{k}T_{i,j} + \frac{2h\Delta x}{k}T_\infty = 0$$

$$(T_{i+1,j} + 2T_{i-1,j} + 2T_{i,j+1} + T_{i,j-1} + \frac{2h\Delta x}{k}T_\infty) - 2\left(3 + \frac{h\Delta x}{k}\right)T_{i,j} = 0 \qquad [Bi = \frac{h\Delta x}{k}]$$

$$\boxed{2(T_{i-1,j} + T_{i,j+1}) + (T_{i+1,j} + T_{i,j-1} + 2BiT_\infty) - 2(3 + Bi)T_{i,j} = 0}$$

3. Node at plane surface with convection



With no heat generation inside the system, the energy balance equation for Node at plane surface with convection can be given as:

$$Q_{i-1,j} + Q_{i,j+1} + Q_{i,j-1} + Q_{conv1} = 0 \qquad [7]$$

Where;

$$Q_{i-1,j} = k(\Delta y.1)\ \frac{T_{i-1,j}-T_{i,j}}{\Delta x} \qquad [8.1]$$

$$Q_{i,j+1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j+1}-T_{i,j}}{\Delta y} \qquad [8.2]$$

$$Q_{i,j-1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}-T_{i,j}}{\Delta y} \qquad [8.3]$$

$$Q_{conv1} = h(\Delta y.1)\left(T_\infty - T_{i,j}\right) \qquad [8.4]$$

Substituting and rearranging equations [8.1 – 8.4] in to equation [7] we get:

$$k(\Delta y.1)\ \frac{T_{i-1,j} - T_{i,j}}{\Delta x} + k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j+1} - T_{i,j}}{\Delta y} + k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1} - T_{i,j}}{\Delta y} + h(\Delta y.1)\left(T_\infty - T_{i,j}\right) = 0$$

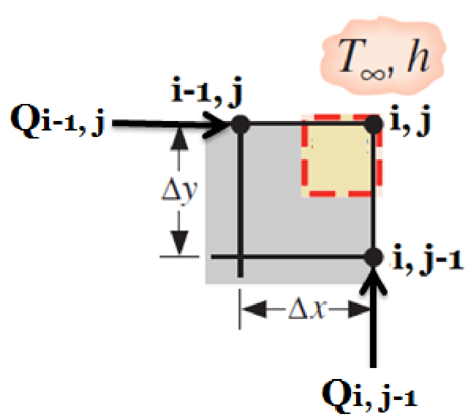
Assuming uniform mesh $(\Delta x = \Delta y)$ *and multiplying both side by* $(\frac{2}{k})$:

$$2(T_{i-1,j} - T_{i,j}) + (T_{i,j+1} - T_{i,j}) + (T_{i,j-1} - T_{i,j}) + \frac{2h\Delta x}{k}\left(T_\infty - T_{i,j}\right) = 0$$

Rearranging the equation:

$$\boxed{(2T_{i-1,j} + T_{i,j+1} + T_{i,j-1}) + \frac{2h\Delta x}{k}T_\infty - 2\left(1 + \frac{h\Delta x}{k}\right)T_{i,j} = 0}$$

4. Node at exterior corner with convection



With no heat generation inside the system, the energy balance equation for Node at exterior corner with convection can be given as:

$$Q_{i-1,j} + Q_{i,j-1} + Q_{conv1} + Q_{conv2} = 0 \qquad [9]$$

Where,

$$Q_{i-1,j} = k(\frac{\Delta y}{2}.1)\ \frac{T_{i-1,j}-T_{i,j}}{\Delta x} \qquad [10.1]$$

$$Q_{i,j-1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}-T_{i,j}}{\Delta y} \qquad [10.2]$$

$$Q_{conv1} = h\left(\frac{\Delta y}{2}.1\right)\left(T_\infty - T_{i,j}\right) \qquad [10.3]$$

$$Q_{conv2} = h\left(\frac{\Delta x}{2}.1\right)\left(T_\infty - T_{i,j}\right) \qquad [10.4]$$

Substituting equations [10.1 – 10.4] in to equation [9] and rearranging we get:

$$k\left(\frac{\Delta y}{2}.1\right)\frac{T_{i-1,j}-T_{i,j}}{\Delta x} + k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}-T_{i,j}}{\Delta y} + h\left(\frac{\Delta y}{2}.1\right)\left(T_\infty - T_{i,j}\right) + h\left(\frac{\Delta x}{2}.1\right)\left(T_\infty - T_{i,j}\right) = 0$$

Assuming uniform mesh ($\Delta x = \Delta y$) *and multiplying both side by* $(\frac{2}{k})$:

$$\boxed{(T_{i-1,j} + T_{i,j-1}) + \frac{2h\Delta x}{k}T_\infty - 2(1+\frac{h\Delta x}{k})T_{i,j} = 0}$$

EXPLICIT FINITE DIFFERENCE METHOD [UNSTEADY STATE CONDITION]

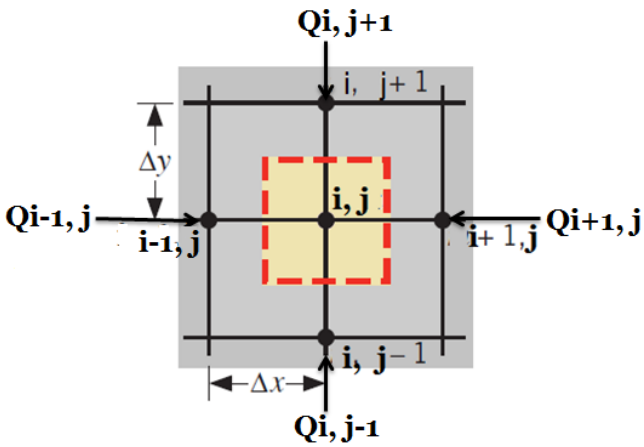
- In this section we consider *explicit forms* of finite-difference solutions to transient conduction problems using the Energy Balance Method.
- In the energy balance method, the finite-difference equation for a node is obtained by applying conservation of energy to a control volume about the nodal region.
- Since the actual direction of heat flow (into or out of the node) is often unknown, it is convenient to formulate the energy balance by *assuming* that *all* the heat flow is *into the node*.

For unsteady state heat conduction method the conservation of energy equation is becomes:

$$E_{in} + E_{gen} = E_{st} \qquad [1]$$

$$\sum_{n=1}^{4} Q_{n\rightarrow(i,j)} + q(\Delta x.\Delta y.1) = \rho c(\Delta x.\Delta y.1)\frac{\partial T}{\partial t} \qquad [2]$$

1. *Interior Node*



Assuming No heat generation inside the system equation [2] becomes:

$$Q_{i+1,j} + Q_{i-1,j} + Q_{i,j+1} + Q_{i,j-1} = \rho c(\Delta x.\Delta y.1)\frac{\partial T}{\partial t} \qquad [3]$$

Where rate of conduction heat transfer between nodes are expressed as :(consider unit depth and n refers to neighbor node)

$$Q_{i+1,j} = k(\Delta y.1)\frac{T_{i+1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} \qquad [4.1]$$

$$Q_{i-1,j} = k(\Delta y.1)\frac{T_{i-1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} \qquad [4.2]$$

$$Q_{i,j+1} = k(\Delta x.1)\frac{T_{i,j+1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} \qquad [4.3]$$

$$Q_{i,j-1} = k(\Delta x.1)\frac{T_{i,j-1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} \qquad [4.4]$$

Substituting equation [4.1] – [4.4] in to equation [3] we get:

$$k(\Delta y.1)\frac{T_{i+1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} + k(\Delta y.1)\frac{T_{i-1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} + k(\Delta x.1)\frac{T_{i,j+1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} + k(\Delta x.1)\frac{T_{i,j-1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} = \rho c(\Delta x.\Delta y.1)\frac{\partial T}{\partial t}$$

Assuming uniform mesh ($\Delta x = \Delta y$) and rearranging the equation becomes:

$$k(T_{i+1,j}{}^{n} - T_{i,j}{}^{n}) + k(T_{i-1,j}{}^{n} - T_{i,j}{}^{n}) + k(T_{i,j+1}{}^{n} - T_{i,j}{}^{n}) + k(T_{i,j-1}{}^{n} - T_{i,j}{}^{n}) = \rho c\Delta x^2 \frac{T_{i,j}{}^{n+1} - T_{i,j}{}^{n}}{\Delta t}$$

Where n represent time steps [dividing both sides by k]

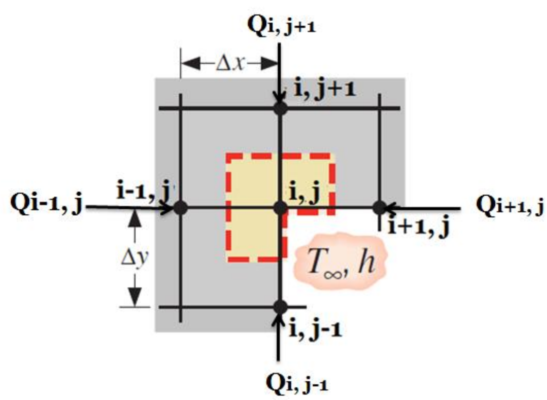
$$T_{i+1,j}{}^{n} + T_{i-1,j}{}^{n} + T_{i,j+1}{}^{n} + T_{i,j-1}{}^{n} - 4T_{i,j}{}^{n} = \frac{\rho c\Delta x^2}{K\Delta t}(T_{i,j}{}^{n+1} - T_{i,j}{}^{n})$$

Solving for $T_{i,j}{}^{n+1}$, collecting like term and rearranging: [Fo = $\frac{k\Delta t}{\rho c\Delta x^2} = \frac{\alpha\Delta t}{\Delta x^2}$]

$$T_{i,j}{}^{n+1} = \frac{k\Delta t}{\rho c\Delta x^2}(T_{i+1,j}{}^{n} + T_{i-1,j}{}^{n} + T_{i,j+1}{}^{n} + T_{i,j-1}{}^{n}) + \left(1 - 4\frac{k\Delta t}{\rho c\Delta x^2}\right) * T_{i,j}{}^{n}$$

$$\boxed{\boldsymbol{T_{i,j}{}^{n+1} = Fo(T_{i+1,j}{}^{n} + T_{i-1,j}{}^{n} + T_{i,j+1}{}^{n} + T_{i,j-1}{}^{n}) + (1 - 4Fo) * T_{i,j}{}^{n}}}$$

2. Node at interior corner with convection



With no heat generation inside the system, the energy balance equation for Node at interior corner with convection can be given as:

$$Q_{i+1,j} + Q_{i-1,j} + Q_{i,j+1} + Q_{i,j-1} + Q_{conv1} + Q_{conv2} = \rho c(\Delta x.\Delta y.1)\frac{\partial T}{\partial t} \qquad [5]$$

Where;

$$Q_{i+1,j} = k(\frac{\Delta y}{2}.1)\ \frac{T_{i+1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} \qquad [6.1]$$

$$Q_{i-1,j} = k(\Delta y.1)\ \frac{T_{i-1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} \qquad [6.2]$$

$$Q_{i,j+1} = k(\Delta x.1)\frac{T_{i,j+1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} \qquad [6.3]$$

$$Q_{i,j-1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}{}^{n}-T_{i,j}{}^{n}}{\Delta y} \quad [6.4]$$

$$Q_{conv1} = h\left(\frac{\Delta x}{2}.1\right)\left(T_{\infty} - T_{i,j}{}^{n}\right) \quad [6.5]$$

$$Q_{conv1} = h(\frac{\Delta y}{2}.1)(T_{\infty} - T_{i,j}{}^{n}) \quad [6.6]$$

$$\frac{\partial T}{\partial t} = \frac{T_{i,j}{}^{n+1}-T_{i,j}{}^{n}}{\Delta t} \quad [6.7]$$

Substituting equations from [6.1]-[6.7] in equation [5] we get:

$$k\left(\frac{\Delta y}{2}.1\right)\frac{T_{i+1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} + k(\Delta y.1)\frac{T_{i-1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} + k(\Delta x.1)\frac{T_{i,j+1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} + k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j+1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} + h\left(\frac{\Delta x}{2}.1\right)\left(T_{\infty} - T_{i,j}{}^{n}\right) + h\left(\frac{\Delta y}{2}.1\right)\left(T_{\infty} - T_{i,j}{}^{n}\right) = \rho c\left(\frac{3}{4}\Delta x.\Delta y.1\right)\frac{T_{i,j}{}^{n+1} - T_{i,j}{}^{n}}{\Delta t}$$

Assuming uniform mesh ($\Delta x = \Delta y$) and rearranging the equation:

$$\frac{k}{2}(T_{i+1,j}{}^{n} - T_{i,j}{}^{n}) + k(T_{i-1,j}{}^{n} - T_{i,j}{}^{n}) + k(T_{i,j+1}{}^{n} - T_{i,j}{}^{n}) + \frac{k}{2}(T_{i,j+1}{}^{n} - T_{i,j}{}^{n}) + h(\Delta x.1)(T_{\infty} - T_{i,j}{}^{n}) = \frac{3\rho c\Delta x^2}{4\Delta t}(T_{i,j}{}^{n+1} - T_{i,j}{}^{n})$$

Multiplying both sides by $(\frac{2}{k})$ we get:

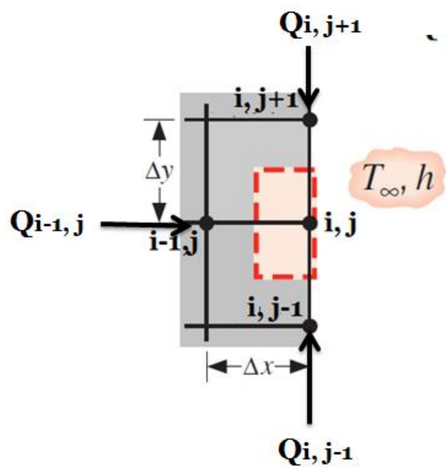
$$T_{i+1,j}{}^{n} + 2T_{i-1,j}{}^{n} + 2T_{i,j+1}{}^{n} + T_{i,j+1}{}^{n} - 6T_{i,j}{}^{n} + \frac{2h\Delta x}{k}T_{\infty} - \frac{2h\Delta x}{k}T_{i,j}{}^{n} = \frac{3\rho c\Delta x^2}{2k\Delta t}(T_{i,j}{}^{n+1} - T_{i,j}{}^{n})$$

Solving for $T_{i,j}{}^{n+1}$ *and rearanging the equation* [Bi = $\frac{h\Delta x}{k}$ and Fo = $\frac{k\Delta t}{\rho c\Delta x^2}$]

$$T_{i,j}{}^{n+1} = \frac{2k\Delta t}{3\rho c\Delta x^2}(T_{i+1,j}{}^{n} + 2T_{i-1,j}{}^{n} + 2T_{i,j+1}{}^{n} + T_{i,j+1}{}^{n} + \frac{2h\Delta x}{k}T_{\infty}) + (1 - \frac{4h\Delta xk\Delta t}{3\rho ck\Delta x^2} - 4\frac{k\Delta t}{\rho c\Delta x^2})T_{i,j}{}^{n}$$

$$\boxed{T_{i,j}{}^{n+1} = \frac{2}{3}Fo(T_{i+1,j}{}^{n} + 2T_{i-1,j}{}^{n} + 2T_{i,j+1}{}^{n} + T_{i,j+1}{}^{n} + 2BiT_{\infty}) + (1 - \frac{4}{3}BiFo - 4Fo)T_{i,j}{}^{n}}$$

3. Node at plane surface with convection



With no heat generation inside the system, the energy balance equation for Node at plane surface with convection can be given as:

$$Q_{i-1,j} + Q_{i,j+1} + Q_{i,j-1} + Q_{conv1} = \rho c(\Delta x.\Delta y.1)\frac{\partial T}{\partial t} \qquad [7]$$

Where,

$$Q_{i-1,j} = k(\Delta y.1)\ \frac{T_{i-1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} \qquad [8.1]$$

$$Q_{i,j+1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j+1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} \qquad [8.2]$$

$$Q_{i,j-1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} \qquad [8.3]$$

$$Q_{conv1} = h(\Delta y.1)(T_\infty - T_{i,j}{}^{n}) \qquad [8.4]$$

Substituting and rearranging equations [8.1 – 8.4] in to equation [7] we get:

$$k(\Delta y.1)\frac{T_{i-1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} + k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j+1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} + k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} + h(\Delta y.1)(T_\infty - T_{i,j}{}^{n}) = \rho c\left(\frac{\Delta x}{2}.\Delta y.1\right)\frac{T_{i,j}{}^{n+1} - T_{i,j}{}^{n}}{\Delta t}$$

Assuming uniform mesh $(\Delta x = \Delta y)$ *and multiplying both side by* $(\frac{2}{k})$:

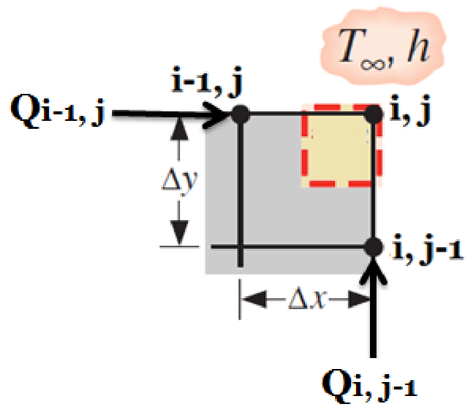
$$2T_{i-1,j}{}^{n} + T_{i,j+1}{}^{n} + T_{i,j-1}{}^{n} - 4T_{i,j}{}^{n} + \frac{2h\Delta x}{k}T_{\infty} - \frac{2h\Delta x}{k}T_{i,j}{}^{n} = \frac{\rho c \Delta x^2}{k\Delta t}T_{i,j}{}^{n+1} - T_{i,j}{}^{n}$$

Solving for $T_{i,j}{}^{n+1}, rearranging\ the\ equation$ [Bi = $\frac{h\Delta x}{k}$ and Fo = $\frac{k\Delta t}{\rho c\Delta x^2}$]

$$T_{i,j}{}^{n+1} = \frac{k\Delta t}{\rho c\Delta x^2}\left(2T_{i-1,j}{}^{n} + T_{i,j+1}{}^{n} + T_{i,j-1}{}^{n} + \frac{2h\Delta x}{k}T_{\infty}\right) + \left(1 - \frac{4k\Delta t}{\rho c\Delta x^2} - \frac{2h\Delta x k\Delta t}{\rho c\Delta x^2 k}\right)T_{i,j}{}^{n}$$

$$\boxed{T_{i,j}{}^{n+1} = Fo\left(2T_{i-1,j}{}^{n} + T_{i,j+1}{}^{n} + T_{i,j-1}{}^{n} + 2BiT_{\infty}\right) + (1 - 4Fo - 2BiFo)T_{i,j}{}^{n}}$$

4. Node at exterior corner with convection



With no heat generation inside the system, the energy balance equation for Node at exterior corner with convection can be given as:

$$Q_{i-1,j} + Q_{i,j-1} + Q_{conv1} + Q_{conv2} = \rho c\left(\frac{\Delta x}{2}.\frac{\Delta y}{2}.1\right)\frac{\partial T}{\partial t} \qquad [9]$$

Where,

$$Q_{i-1,j} = k\left(\frac{\Delta y}{2}.1\right)\frac{T_{i-1,j}{}^{n} - T_{i,j}{}^{n}}{\Delta x} \qquad [10.1]$$

$$Q_{i,j-1} = k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}{}^{n} - T_{i,j}{}^{n}}{\Delta y} \qquad [10.2]$$

$$Q_{conv1} = h\left(\frac{\Delta y}{2}.1\right)\left(T_{\infty} - T_{i,j}{}^{n}\right) \qquad [10.3]$$

$$Q_{conv2} = h\left(\frac{\Delta x}{2}.1\right)\left(T_{\infty} - T_{i,j}{}^{n}\right) \qquad [10.4]$$

Substituting equations [10.1 – 10.4] in to equation [9] and rearranging we get:

$$k\left(\frac{\Delta y}{2}.1\right)\frac{T_{i-1,j}{}^{n}-T_{i,j}{}^{n}}{\Delta x}+\ k\left(\frac{\Delta x}{2}.1\right)\frac{T_{i,j-1}{}^{n}-T_{i,j}{}^{n}}{\Delta y}+h\left(\frac{\Delta y}{2}.1\right)\left(T_\infty-\ T_{i,j}{}^{n}\right)$$
$$+h\left(\frac{\Delta x}{2}.1\right)\left(T_\infty-\ T_{i,j}{}^{n}\right)=\ \rho c\left(\frac{\Delta x}{2}.\frac{\Delta y}{2}.1\right)\frac{T_{i,j}{}^{n+1}-T_{i,j}{}^{n}}{\Delta t}$$

Assuming uniform mesh $(\Delta x=\ \Delta y)$ *and multiplying both side by* $(\frac{2}{k})$:

$$T_{i-1,j}{}^{n}+\ T_{i,j-1}{}^{n}-2T_{i,j}{}^{n}-\frac{2h\Delta x}{k}T_{i,j}{}^{n}+\frac{2h\Delta x}{k}T_\infty=\ \frac{\rho c\Delta x^2}{2k\Delta t}(T_{i,j}{}^{n+1}-T_{i,j}{}^{n})$$

Solving for $T_{i,j}{}^{n+1}$ and rearranging the equation we get:

$$T_{i,j}{}^{n+1}=\ \frac{2k\Delta t}{\rho c\Delta x^2}(T_{i-1,j}{}^{n}+\ T_{i,j-1}{}^{n}+\frac{2h\Delta x}{k}T_\infty)+(1-4\ \frac{k\Delta t}{\rho c\Delta x^2}-4\frac{k\Delta t}{\rho c\Delta x^2}\frac{h\Delta x}{k})T_{i,j}{}^{n}$$

$$\boxed{T_{i,j}{}^{n+1}=\ 2Fo(T_{i-1,j}{}^{n}+\ T_{i,j-1}{}^{n}+\ 2BiT_\infty)+(1-4\,Fo-4FoBi)T_{i,j}{}^{n}}$$