SURVEYING - III

HIGHER SURVEYING

Dr. B.C. Punmia
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By

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Preface to the First Edition

This volume is in continuation with volume I and II, and deals with three chapters of higher surveying — 'Field Astronomy', 'Survey Adjustments and Theory of Errors' and 'Photogrammetric Surveying'. It covers fully the syllabi of Degree and A.M.I.E. examinations in *higher surveying*. The book has been written primarily as a college text to fill a *need* for a simple but complete coverage of the principles of Field Astronomy and of Photogrammetry. This book is also meant to assist the experienced surveyor who has not found time to follow the rapid changes in the techniques so noticeable in the surveying field.

The subject is gradually introduced in stages. A large number of diagrams have been given to illustrate the basic principles. Large number of solved and unsolved examples develop the reader's ability to apply the basic concepts to practical problems. The book also deals with the most modern equipment used in photogrammetric surveying. Metric System of units has been used throughout the text and wherever possible, the various formulae used in text have been derived in mertric units.

I shall like to express my thanks to M/S Wild Heerbrugg Ltd. and M/s Zeiss-Aerotopograph who kindly furnished and granted permission to use diagrams on which some illustration in this book are based.

In spite of every care taken to check the numerical work, some errors may remain, and I shall be obliged for any intimation of these which readers may discover.

Jodhpur
1st January, 1967 B.C. PUNMIA

Preface to the Second Edition

In this edition, the subject-matter has been revised thoroughly and a few minor alterations and additions have been made. On the suggestions of many readers of the book, the chapter on 'Survey Adjustments and Theory of Errors' has been transferred to volume 2, since this chapter is mostly taught along with *Triangulation*. Account has been taken throughout of suggestions offered by the many users of the book, and grateful acknowledgement is made to them. Further suggestions will be greatly appreciated.

Jodhpur
15th July, 1971 B.C. PUNMIA

Preface to the Seventh Edition

In the Seventh Edition, the book has been completely rewritten and all the diagrams have been redrawn. Many new articles and diagrams/illustrations have been added. A new chapter on 'Electromagnetic Distance Measurement (EDM)' has been added at the end of the book. The knowledge of latest EDM equipment, such as Geodimeters, Tellurometers and 'Distomats' will be very much useful to the field engineers for the indirect, quick and precise measurement of distance and difference in elevation between widely distant points. Account has been taken throughout of the suggestions offered by the many users of the book and grateful acknowledgement is made to them. Further suggestions will be greatly appreciated.

Jodhpur
15th July, 1987

B.C. PUNMIA

Preface to the Fifteenth Edition

In Fifteenth Edition, the subject matter has been thoroughly revised, updated and rearranged. In each chapter, many new articles haven been added. All the diagrams have been redrawn using computer graphics and the book has been computer type-set in a bigger format, keeping in pace with the modern trend. Two new chapters have been added at the end of the book : Chapter 4 on *'Remote Sensing'* and Chapter 5 on *'Geographical Information System (GIS)'*. Account has been taken throughout of the suggestions offered by many users of the book and grateful acknowledgements is made to them. The authors are thankful to Shri Mool Singh Gahlot for the fine Laser type setting done by him. The Authors are also thankful Shri R.K. Gupta, Managing Director Laxmi Publications, for taking keen interest in publication of the book and bringing it out nicely and quickly.

Jodhpur
Mahaveer Jayanti
22-4-05

B.C. PUNMIA
ASHOK K. JAIN
ARUN K. JAIN

Contents

1

Field Astronomy

1.1. DEFINITIONS OF ASTRONOMICAL TERMS

1. The Celestial Sphere. The millions of stars that we see in the sky on a clear cloudless night are all at varying distances from us. Since we are concerned with their relative distances rather than their actual distance from the observer, it is exceedingly convenient to picture the stars as distributed over the surface of an imaginary spherical sky having its centre at the position of the observer. This imaginary sphere on which the stars appear to lie or to be studded is known as the *Celestial Sphere*. The radius of the celestial sphere may be of any value – from a few thousand metres to a few thousand kilometres. Since the stars are very distant from us, the centre of the earth may be taken as the centre of the celestial sphere.

2. The Zenith and Nadir. The Zenith (Z) is the point on the upper portion of the celestial sphere marked by plumb line above the observer. It is thus the point on the celestial sphere immediately above the observer's station. The *Nadir* (*Z'*) is the point on the lower portion of the celestial sphere marked by the plumb line below the observer. It is thus the point on the celestial sphere vertically below the observer's station.

3. The Celestial Horizon. (also called *True* or *Rational horizon* or *geocentric horizon*). It is the great circle traced upon the celestial sphere by that plane which is perpendicular to the Zenith–Nadir line, and which passes through the centre of the earth. (*Great circle* is a section of a sphere when the cutting plane passes through the centre of the sphere).

4. The Terrestrial Poles and Equator. *The terrestrial poles* are the two points in which the earth's axis of rotation meets the earth's sphere. The *terrestrial equator* is the great circle of the earth, the plane of which is at right angles to the axis of rotation. The two poles are equidistant from it.

5. The Celestial Poles and Equator. If the earth's axis of rotation is produced indefinitely, it will meet the celestial sphere in two points called the *north and south celestial poles* (*P* and *P'*). The *celestial equator* is the great circle of the celestial sphere in which it is intersected by the plane of terrestrial equator.

6. The Sensible Horizon. It is a circle in which a plane passing through the point of observation and tangential to the earth's surface (or perpendicular to the Zenith–Nadir line) intersects with celestial sphere. The line of sight of an accurately levelled telescope lies in this plane.

7. The Visible Horizon. It is the circle of contact, with the earth, of the cone of visual rays passing through the point of observation. The circle of contact is a small circle of the earth and its radius depends on the altitude of the point of observation.

8. Vertical Circle. A vertical circle of the celestial sphere is great circle passing through the Zenith and Nadir. They all cut the celestial horizon at right angles.

9. The Observer's Meridian. The *meridian* of any particular point is that circle which passes through the Zenith and Nadir of the point as well as through the poles. It is thus a vertical circle.

10. The Prime Vertical. It is that particular vertical circle which is at right angles to the meridian, and which, therefore passes through the east and west points of the horizon.

11. The Latitude (θ). It is the angular distance of any place on the earth's surface north or south of the equator, and is measured on the meridian of the place. It is marked + or − (or N or S) according as the place is north or south of the equator. The latitude may also be defined as the angle between the zenith and the celestial equator.

12. The Co-latitude (*c*). The Co-latitude of a place is the angular distance from the zenith to the pole. It is the complement of the latitude and equal to $(90° - \theta)$.

13. The Longitude (ϕ). The longitude of a place is the angle between a fixed reference meridian called the prime or first meridian and the meridian of the place. The prime meridian universally adopted is that of Greenwich. The longitude of any place varies between 0° to 180°, and is reckoned as $\phi°$ east or west of Greenwich.

14. The Altitude (α). The altitude of celestial or heavenly body (*i.e.*, the sun or a star) is its angular distance above the horizon, measured on the vertical circle passing through the body.

15. The Co-altitude or Zenith Distance (*z*). It is the angular distance of heavenly body from the zenith. It is the complement of the altitude, *i.e.*, $z = (90° - \alpha)$.

16. The Azimuth (*A*). The azimuth of a heavenly body is the angle between the observer's meridian and the vertical circle passing through the body.

17. The Declination (δ). The declination of a celestial body is angular distance from the plane of the equator, measured along the star's meridian generally called the declination circle, (*i.e.*, great circle passing through the heavenly body and the celestial pole). Declination varies from 0° to 90°, and is marked + or − according as the body is north or south of the equator.

18. Co-declination or Polar Distance (*p*). It is the angular distance of the heavenly body from the nearer pole. It is the complement of the declination, *i.e.*, $p = 90° - \delta$.

19. Hour Circle. Hour circles are great circles passing through the north and south celestial poles. The declination circle of a heavenly body is thus its hour circle.

20. The Hour Angle. The hour angle of a heavenly body is the angle between the observer's meridian and the declination circle passing through the body. The hour angle is always measured *westwards*.

21. The Right Ascension (R.A.). It is the equatorial angular distance measured *eastward* from the First Point of Aries to the hour circle through the heavenly body.

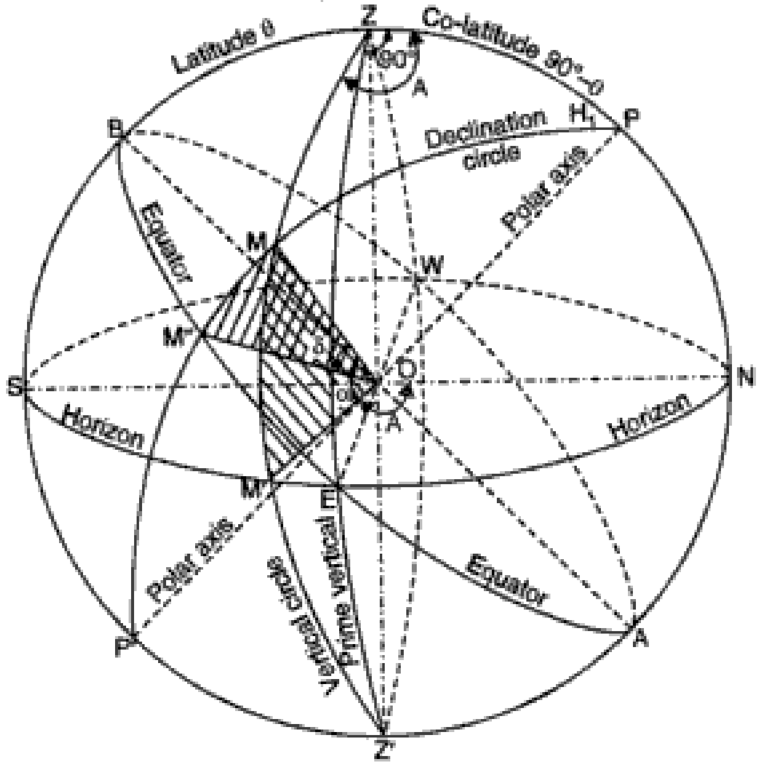

Z
Latitude θ
Co-latitude 90°−θ
90°
A
B
H1
P
Declination circle
Polar axis
Equator
M
W
M''
S
O
N
Horizon
A
Horizon
M'
E
Polar axis
Prime Vertical
Vertical circle
Equator
P'
A
Z'

FIG. 1.1

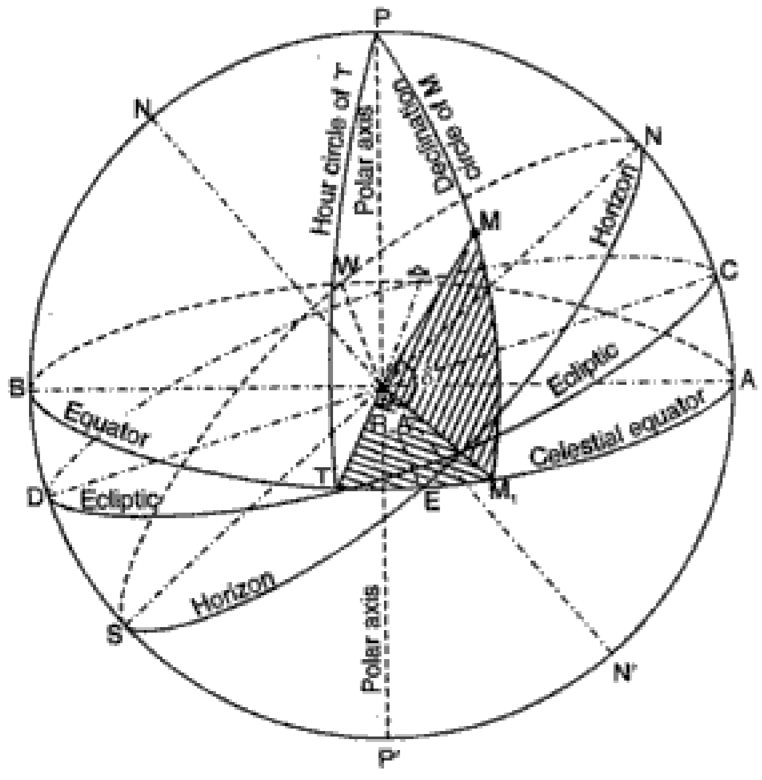

P
Hour circle of T
Polar axis
Declination circle of M
N
N
M
Horizon
W
C
B
A
Ecliptic
Equator
Celestial equator
T
D
Ecliptic
E
M1
Horizon
S
Polar axis
N'
P'

FIG. 1.2

22. Equinoctial Points. The points of the intersection of the ecliptic with the equator are called the equinoctial points. The declination of the sun is zero at the equinoctial points. The *Vernal Equinox* or the *First Point of Aries* (♈) is the point in which the sun's declination changes from south to north, and marks the commencement of spring. It is a fixed point on the celestial sphere. The *Autumnal Equinox* or the First Point of Libra (♎) is the point in which the sun's declination changes from north to south, and marks the commencement of autumn. Both the equinoctial points are six months apart in time.

23. The Ecliptic. Ecliptic is the great circle of the heavens which the sun appears to describe on the celestial sphere with the earth as a centre in the course of a year. The plane of the ecliptic is inclined to the plane of the equator at an angle (called the *obliquity*) of about 23° 27′, but is subjected to a diminution of about 5″ in a century.

24. Solastices. Solastices are the points at which the north and south declination of the sun is a maximum. The point *C* (Fig. 1.3) at which the north declination of the sun is maximum is called the *summer solastice,* while the point *C'* at which south declination of the sun is maximum is known as the *winter solastice.* The case is just the reverse in the southern hemisphere.

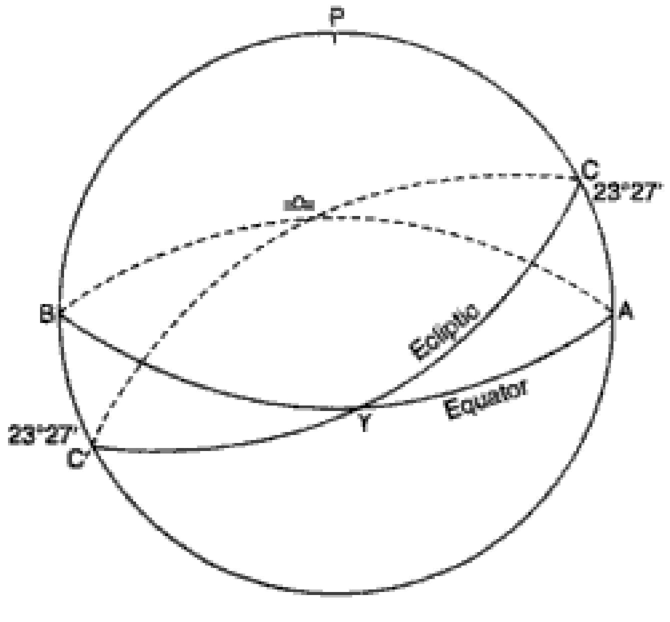


FIG. 1.3. THE ECLIPTIC.

25. North, South, East and West Directions. The north and south points correspond to the projection of the north and south poles on the horizon. The *meridian line* is the line in which the observer's meridian plane meets horizon plane, and the north and south points are the points on the extremities of it. The direction *ZP* (in plan on the plane of horizon) is the direction of north, while the direction *PZ* is the direction of south. The *east-west line* is the line in which the prime vertical meets the horizon, and east and west points are the extremities of it. *Since the meridian plane is perpendicular to both the equatorial plane as well as horizontal plane, the intersections of the equator and horizon determine the east and west points* (see Fig. 1.1).

1.2. CO-ORDINATE SYSTEMS

The position of a heavenly body can be specified by two spherical co-ordinates, *i.e.*, by two angular distances measured along arcs of two great circles which cut each other at right angles. One of the great circle is known as the primary circle of the reference and the other as the *secondary circle* of reference. Thus in Fig. 1.4, the position of the point *M* can be specified with reference to the plane *OAB* and the line *OA*, *O* being the origin of the co-ordinates. If a plane is passed through *OM* and perpendicular to the plane of *OAB*, it will cut the latter in the line *OB*. The two spherical co-ordinates of the point *M* are, therefore, angles *AOB* and *BOM* at the centre *O*, or the arcs *AB* and

BM. In practical astronomy, the position of a celestial body can be specified by the following systems of co-ordinates :

1. The *horizon system*
2. The *independent equatorial system*
3. The *dependent equatorial system*
4. The *celestial latitude and longitude system.*

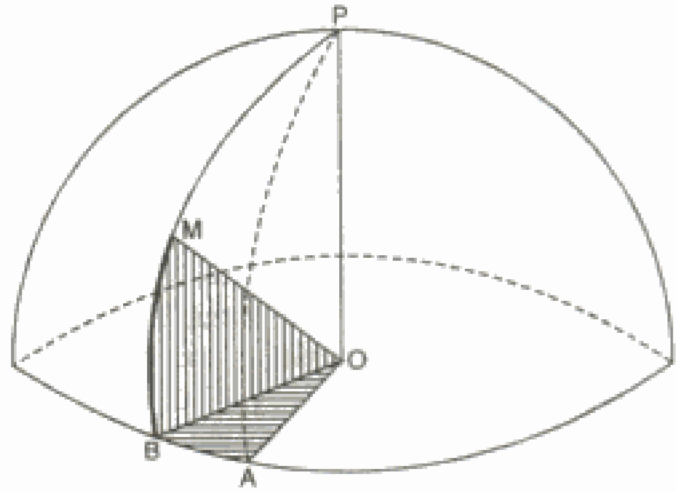


FIG. 1.4

The *horizon system* is dependent on the position of the observer. The *independent equatorial system* is independent of the position of the observer and the positions apply to observers anywhere on the earth. In the *dependent equatorial system*, one of the great circle of reference is independent of the position of the observer while the other great circle perpendicular to the former is dependent on the position of the observer. There is yet another system of co-ordinates, known as the *celestial system,* in which the position of a body is specified by the *celestial latitude* and the *celestial longitude.*

1. THE HORIZON SYSTEM (ALTITUDE AND AZIMUTH SYSTEM)

In the horizon system, the horizon is the plane of reference and the co-ordinates of a heavenly body are (*i*) the *azimuth* and (*ii*) the *altitude*. This system is necessitated by the fact that we can measure only horizontal and vertical angles with the engineer's transit. The two great circles of reference are the horizon and the observer's meridian, the former being the primary circle and the latter the secondary circle.

In Fig. 1.5, *M* is the heavenly body in the Eastern part of the celestial sphere, *Z* is the observer's zenith and *P* is the celestial pole. Pass a vertical circle (*i.e.*, a great circle through *Z*) through *M* to intersect the horizon plane at *M'*. The first co-ordinate of *M* is, then, the *azimuth* (*A*) which is the angle between the observer's meridian and the vertical circle through the body. The azimuth can either be measured as the angular distance along the horizon, measured from the meridian to the foot of the vertical circle through the point. It is also equal to the angle at the zenith between the meridian and the vertical circle through *M*. The other co-ordinate of *M* is the *altitude* (α) which is the angular distance measured above (or below) the horizon, measured on the vertical circle through the body. Similarly, Fig. 1.6 shows the position (*M*) of the body in the Western part of the celestial sphere. It should be noted that, in the Northern hemisphere, the azimuth is always measured from the *north* either eastward, or westward, depending upon

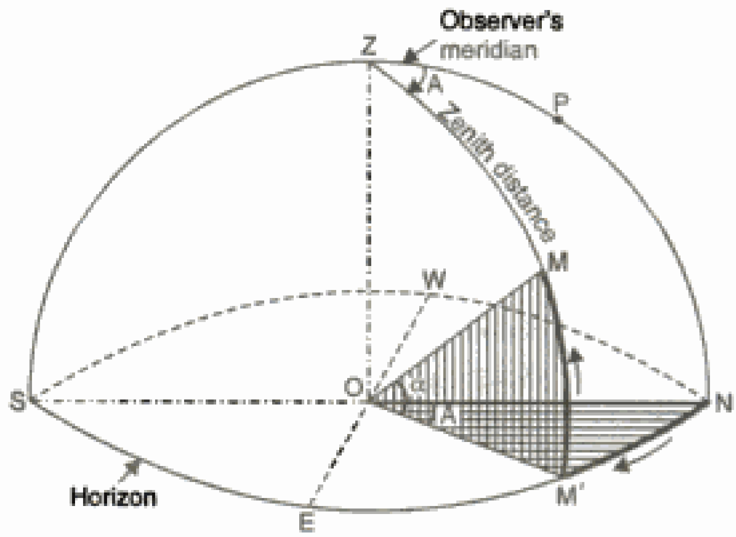


FIG. 1.5 BODY IN THE EASTERN PART OF THE CELESTIAL SPHERE.

whether the body is in the eastern part of celestial sphere or in the western part of the celestial sphere. In the southern hemisphere, the azimuth is measured from the south to the east or the west.

Alternatively, the position of a body is, sometimes specified in terms of *zenith distance and azimuth*. The zenith distance of any body is its angular distance from zenith, measured along the vertical circle. It is the complement of the altitude, *i.e.*,

zenith distance $(z) = 90° - \alpha$.

The horizon system of co-ordinates undergo constant and rapid changes due to the diurnal motions.

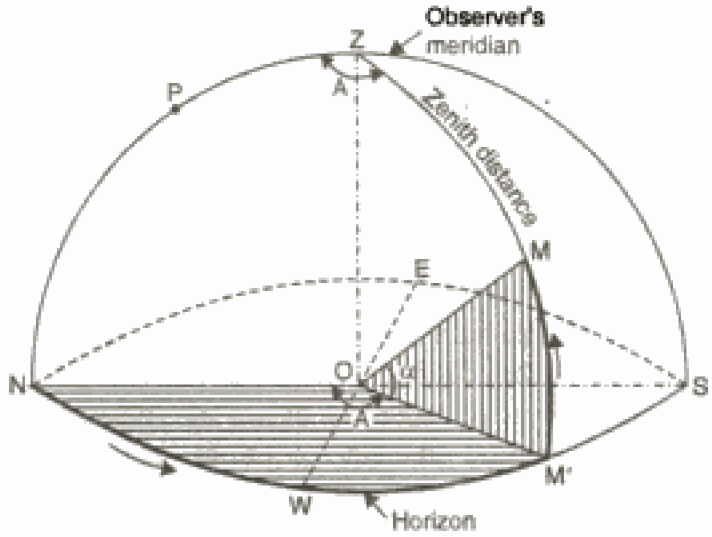


FIG. 1.6. BODY IN THE WESTERN PART OF THE CELESTIAL SPHERE.

2. THE INDEPENDENT EQUATORIAL SYSTEM (THE DECLINATION AND RIGHT ASCENSION SYSTEM)

This system is used in the publication of star catalogues, almanacs, or ephemerides in which the position of heavenly bodies are referred to spherical co-ordinates which are independent of the observer's position. The two great circles of reference are (*i*) the equatorial circle and (*ii*) the declination circle, the former being the primary circle and the latter the secondary circle of reference. For fixed stars, this system of co-ordinates is independent of the place of observation, and nearly independent of the time.

The first co-ordinate of the body (*M*) is the *right ascension*, which is the angular distance along the arc of the celestial equator measured from the first point of Aries (♈) as the point of reference towards East up to the declination circle passing through the body. It is also the angle, measured eastward at the celestial pole, between the hour circle through (♈) and the declination circle through *M*. The motion of the star is from East to West, and hence the Right Ascension is measured in a direction opposite to the motion of the heavenly body. It may be measured in degrees, minutes and seconds of arc or in hours, minutes and seconds of time. Thus in Fig. 1.7, ♈*P* is the hour circle through ♈, *M'MP* is the hour circle (or the declination circle) of *M*, and ♈*M'* is the R.A. measured along the arc of the equator.

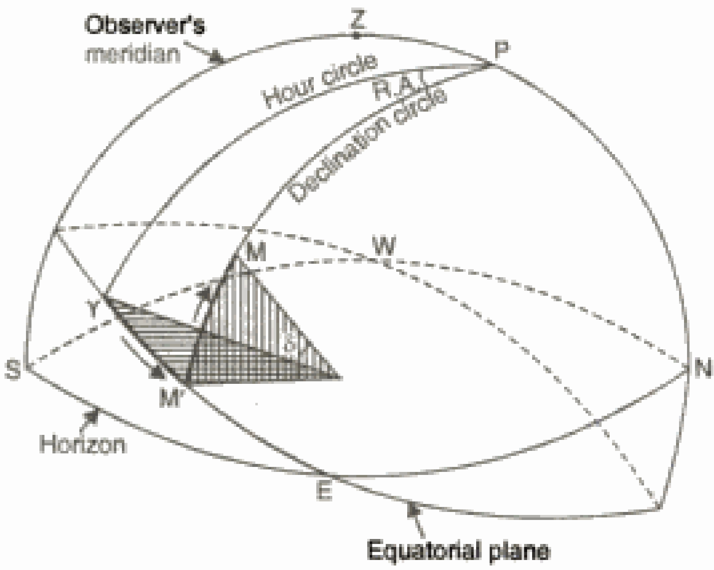


FIG. 1.7. THE DECLINATION-RIGHT ASCENSION SYSTEM.

The other co-ordinate in this system is the declination (δ). It is the distance of the body from the equator measured along the arc of the declination circle. The declination circle accompanies the body in its diurnal course. The declination is considered positive when the body is north of the equator and negative when it is to south.

The *polar distance* (*p*) is the complement of the declination, *i.e.*, $p = (90° - \delta)$. In Fig. 1.7. *M'M* is the positive declination of the body (*M*).

The values of declination and right ascension of a fixed star in the heaven, although nearly constant, are not absolutely so. A register of these co-ordinates, together with their annual change (if any be found) will enable to identify a star once observed. Such a register is called a *catalogue of stars* and its correctness is of highest importance. The variation of the declination and right ascension of the sun is very much greater than for the stars.

3. THE DEPENDENT EQUATORIAL SYSTEM (THE DECLINATION AND HOUR ANGLE SYSTEM)

In this system, one co-ordinate is dependent of the observer's position and the other co-ordinate is independent of the observer's position. The two great circles of reference are (*i*) the horizon and (*ii*) the declination circle through the celestial body, the former being the primary circle and the latter the secondary circle of reference.

In this system, the first co-ordinate of *M* (Fig. 1.8) is the *hour angle*. Hour angle is the angluar distance along the arc of the horizon measured from the observer's meridian to the declination circle passing through the body. It is also measured as the angle, subtended at the pole, between the observer's me-

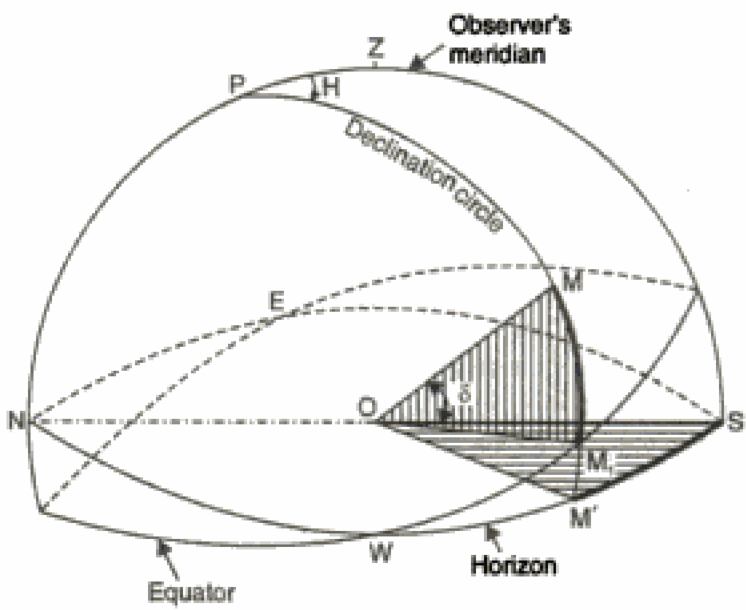


FIG. 1.8. THE DECLINATION-HOUR ANGLE SYSTEM.

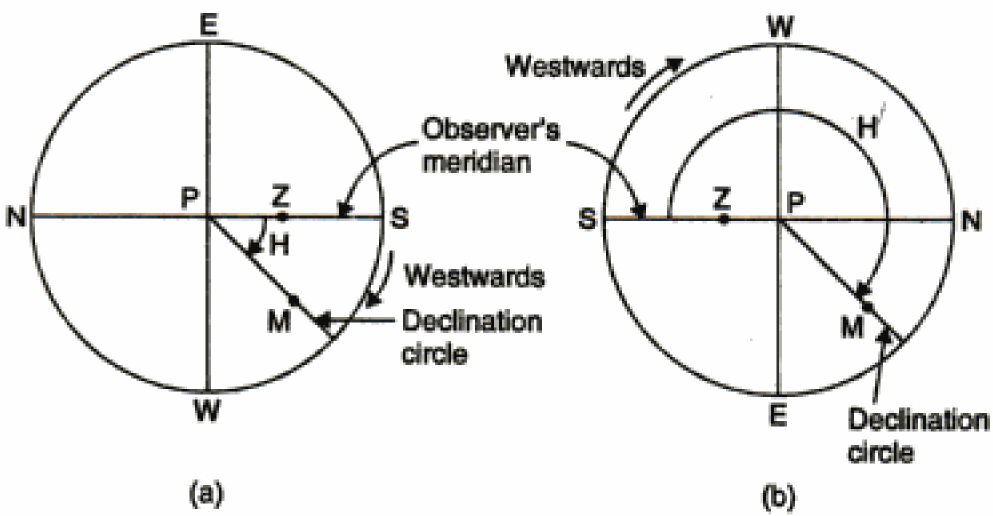


FIG. 1.9. PLAN ON THE PLANE OF THE EQUATOR.

ridian and the declination circle of the body. *In the northern hemisphere, the hour angle is always measured from the south and towards west upto the declination circle.* Its value varies from 0 ° to 360 °. If H varies from 0 ° to 180 °, the star is in the western hemisphere, otherwise in the eastern hemisphere. Fig. 1.9 shows the plan on the plane of the equator, illustrating how the hour angle is measured *westward* for two positions of the observer. The other co-ordinate is the declination, as in the second system. Thus, in Fig. 1.8, SM' is the hour angle, and M_1M is the declination of the celestial body (M), M' and M_1 being the projections of M on the horizon and equator respectively.

4. THE CELESTIAL LATITUDE AND LONGITUDE SYSTEM

In this system of the co-ordinates, the primary plane of reference is the ecliptic. The second plane of reference is a great circle passing through the First Point of Aries and perpendicular to the plane of the ecliptic. The two co-ordinates of a celestial body are (*i*) the celestial latitude and (*ii*) the celestial longitude.

The *celestial latitude* of a body is the arc of great circle perpendicular to the ecliptic, intercepted between the body and the ecliptic. It is positive or negative depending upon whether measured north or south of the ecliptic. The *celestial longitude* of a body is the arc of a ecliptic intercepted between the great circle passing through the First Point of Aries and the circle of the celestial latitude passing through the body. It is measured *eastwards* from 0° to 360 °. Thus, in Fig. 1.10, M_1M is the celestial latitude (north) and ΥM_1 is the celestial longitude for the heavenly body (M).

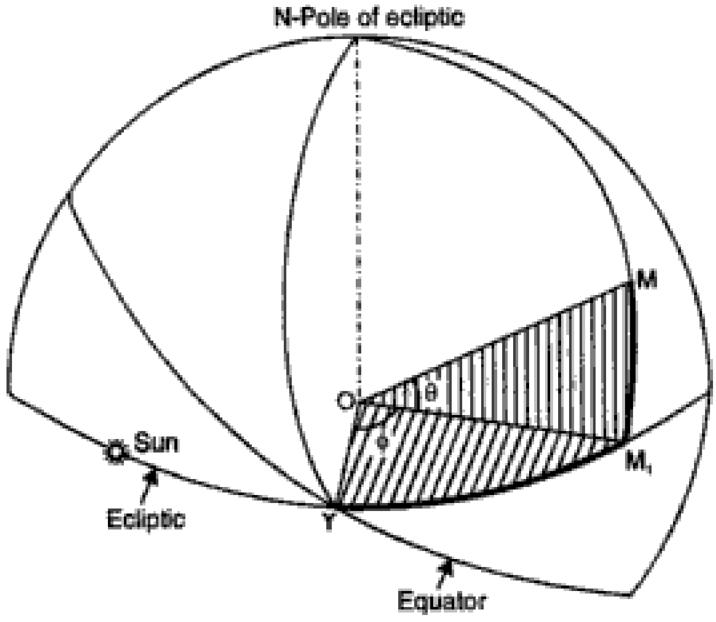


FIG. 1.10. THE CELESTIAL LATITUDE AND LONGITUDE.

Comparison of the Systems. As stated earlier, the azimuth and altitude of a star are not constant but are continuously changing due to diurnal motion. On the other hand, the right ascension and declination of a star are constant, because the reference point, the First Point of Aries, partakes of the diurnal motion of the stars. However, there is no instrument which can measure right ascension and declination of the star directly. The azimuth and the altitude of a star can be directly measured with the help of a theodolite. Knowing the hour angle and the azimuth of a star, its right ascension and declination can be computed from the solution of the astronomical triangle provided the instant of time at which the body was in a certain position (*i.e.*, the hour angle) is also determined. Thus, both the systems are necessary — the first one for the direct field observations and the second one for the computations required in respect of the preparation of the star catalogues.

1.3. THE TERRESTRIAL LATITUDE AND LONGITUDE

We have discussed the various systems of co-ordinates to establish the position of a heavenly body on the celestial sphere. In order to mark the position of a point on the earth's surface, it is necessary to use a system of co-ordinates. The terrestrial latitudes and longitudes are used for this purpose.

The terrestrial *meridian* is any great circle whose plane passes through the axis of the earth (*i.e.*, through the north and south poles). Terrestrial equator is the great circle whose plane is perpendicular to the earth's axis. The *latitude* θ of a place is the angle subtended at the centre of the earth north by the arc of meridian intercepted between the place and the equator. The latitude is north or positive when measured above the equator, and is south or negative when measured below the equator. The latitude of a point upon the equator is thus 0 °, while at the North and South Poles, it is 90 °N and 90 ° S latitude respectively. The *co-latitude* is the complement of the latitude, and is the distance between the point and pole measured along the meridian.

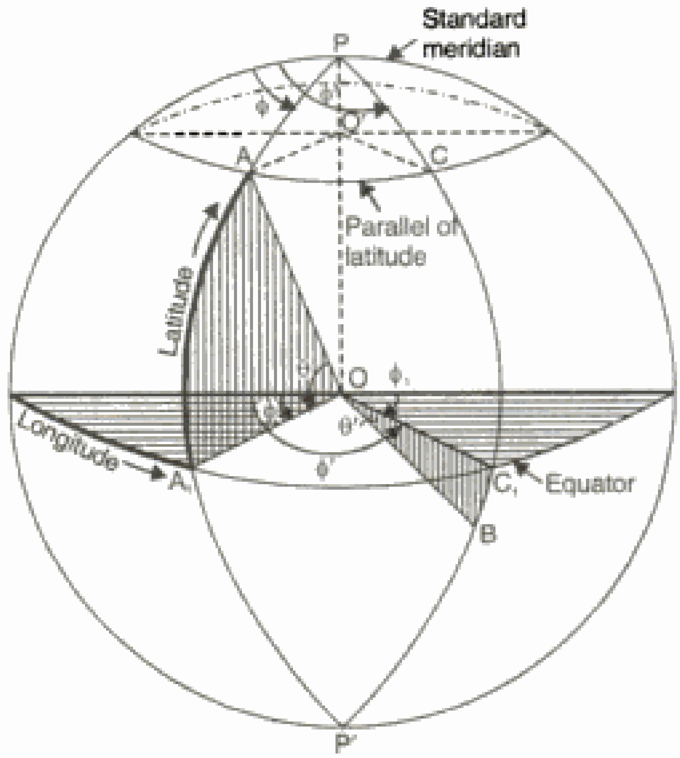


FIG. 1.11. THE TERRESTRIAL LATITUDE AND LONGITUDE.

The *longitude* (ϕ) of a place is the angle made by its meridian plane with some fixed meridian plane arbitrarily chosen, and is measured by the arc of equator intercepted between these two meridians. The prime meridian universally adopted is that of Greenwich. The longitude of any place varies between 0 ° to 180 °, and is reckoned as ϕ ° east or west of Greenwich. All the points on meridian have the same longitude.

The Parallel of Latitude

The *parallel of latitude* through a point is a small circle in which a plane through that point, and at right angles to the earth's axis, intersects the earth's surface. *All the points on the parallel of latitude have the same latitude*. The meridians are great circles of the same diameter while the parallel of a latitude are small circles, and are of different diameters depending upon the latitude of the place through which the parallel of the latitude is drawn. Due to this reason *a degree of longitude* has got different values at different latitudes — higher the latitude smaller the value. At the equator, a degree of longitude is equivalent to a distance of about 69 miles. However, a degree of latitude has the constant value of 69 miles everywhere.

To find the distance between two points *A* and *C* on a parallel of latitude, consider Fig. 1.11 in which θ = latitude of *A* = latitude of *C*, ϕ = longitude of *A*, and ϕ′ = longitude of *C*. The angular radius *PA* of the parallel of latitude = 90 ° − θ.

$$\text{Now arc } \frac{AC}{A_1C_1} = \frac{O'A}{OA_1} \text{ where } O' \text{ is the centre of the parallel of latitude}$$

$$= \frac{O'A}{OA}, \text{ since } OA_1 = OA = \text{radius of the earth}$$

$$= \sin O'OA, \text{ since } \angle AO'O = 90°$$

$$\therefore \quad AC = A_1C_1 \sin (90° - \theta) = \cos\theta \,.\, A_1C_1$$

or $$AC = \textit{cos latitude} \times \textit{difference of longitude.}$$

The shortest distance measured along the surface of the earth between two places is the length of the arc of the great circle joining them. The distance between two points in nautical miles measured along the parallel of latitude is called the *departure*.

Thus, ***departure = difference in longitude in minutes × cos latitude.***

The Zones of the Earth

The earth has been divided into certain zones depending upon the parallel of latitude of certain value above and below the equator. The parallel of latitude $23° \, 27\frac{1}{2}'$ north of equator is known as the *tropic of cancer*. The parallel of latitude $23° \, 27\frac{1}{2}'$ south of equator is known as the *tropic of capricorn*. The belt or zone of earth between these two tropics is known as the *torrid zone*. The parallel of latitude $66° \, 32\frac{1}{2}'$ north of equator is called the *arctic circle*, and of a similar value. South of equator is called the *anarctic circle*. The belt between the tropic of cancer and the arctic circle is known as the *north temperate zone* while the belt between the tropic of capricorn and the anarctic circle is known as the *south temperate zone*. The belt between the arctic circle and the north pole is called the *north frigid zone* and the belt between the anarctic circle and the south pole is called the *south frigid zone*.

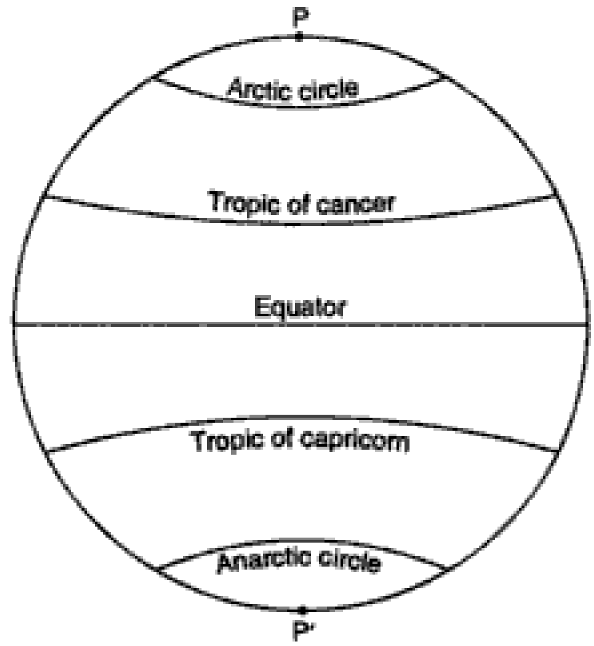


FIG. 1.12. THE ZONES OF THE EARTH.

The Nautical Mile. A nautical mile is equal to the distance on arc of the great circle corresponding to angle of 1 minute subtended by the arc at the centre of the earth.

Taking radius of earth = 6370 kilometres, we have

$$\text{One nautical mile} = \frac{\text{Circumference of the great circle.}}{360° \times 60} = \frac{2\pi \times 6370}{360 \times 60} = \mathbf{1.852\ km}.$$

1.4. SPHERICAL TRIGONOMETRY AND SPHERICAL TRIANGLE

Since in the astronomical survey many of the quantities involved are the parts of the celestial sphere, a simple knowledge of spherical trigonometry is essential.

Spherical Triangle

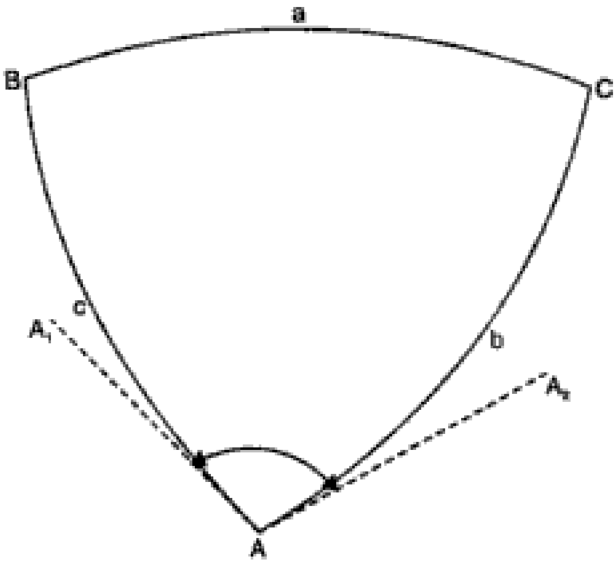


FIG. 1.13. SPHERICAL TRIANGLE.

A spherical triangle is that triangle which is formed upon the surface of the the sphere by intersection of three arcs of great circles and the angles formed by the arcs at the vertices of the triangle are called the spherical angles of the triangle.

Thus, in Fig. 1.13. AB, BC and CA are the three arcs of great circles and intersect each other at A, B and C. It is usual to denote the angles by A, B and C and the sides respectively opposite to them, as a, b and c. *The sides of spherical triangle are proportional to the angle subtended by them at the centre of the sphere and are, therefore, expressed in angular measure.* Thus, by sin b we mean the sine of the angle subtended at the centre by the arc AC. A spherical angle is an angle between two great circles, and is defined by the plane angle between the tangents to the circles at their point of intersection. Thus, the spherical angle at A is measured by the plane angle A_1AA_2 between the tangents AA_1 and AA_2 to the great circles AB and AC.

Properties of a spherical triangle

The following are the properties of a spherical triangle :

1. Any angle is less than two right angles or π.

2. The sum of the three angles is less than six right angles or 3π and greater than two right angles or π.

3. The sum of any two sides is greater than the third.

4. If the sum of any two sides is equal to two right angles or π, the sum of the angles opposite them is equal to two right angles or π.

5. The smaller angle is opposite the smaller side, and *vice versa*.

Formulae in Spherical Trigonometry

The six quantities involved in a spherical triangle are three angles A, B and C and the three sides a, b and c. Out of these, if three quantities are known, the other three can very easily be computed by the use of the following formulae in spherical trigonometry:

1. Sine formula : $$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad \ldots(1.1)$$

2. Cosine formula : $$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \quad \ldots(1.2)$$

or $$\cos a = \cos b \cos c + \sin b \sin c \cos A \quad \ldots[1.2\ (a)]$$

Also, $$\cos A = -\cos B \cos C + \sin B \sin C \cos a \quad \ldots(1.3)$$

3. For computation purposes :

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}} \qquad \text{...(1.4)}$$

$$\cos\frac{A}{2} = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}} \qquad \text{...(1.5)}$$

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}} \qquad \text{...(1.6)}$$

where $s = \frac{1}{2}(a+b+c)$

4. Similarly,

$$\sin\frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\sin B \sin C}} \qquad \text{...(1.7)}$$

$$\cos\frac{a}{2} = \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B \sin C}} \qquad \text{...(1.8)}$$

$$\tan\frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\cos(S-B)\cos(S-C)}} \qquad \text{...(1.9)}$$

where $S = \frac{1}{2}(A+B+C)$

5. $$\tan\frac{1}{2}(a+b) = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\frac{1}{2}c \qquad \text{...(1.10)}$$

$$\tan\frac{1}{2}(a-b) = \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\frac{1}{2}c \qquad \text{...(1.11)}$$

6. $$\tan\frac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\frac{1}{2}C \qquad \text{...(1.12)}$$

$$\tan\frac{1}{2}(A-B) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\frac{1}{2}C \qquad \text{...(1.13)}$$

THE SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLE

The relationships of right-angled spherical triangle are very conveniently obtained from '*Napier's rules of circular parts*'.

In [Fig. 1.14 (*a*)], *ABC* is a spherical triangle right-angled at *C*. Napier defines the circular parts as follows :

(*i*) the side *a* to one side of the right-angle,

(*ii*) the side *b* to the other side of the right-angle,

(*iii*) the complement (90° – *A*) of the angle *A*,

(*iv*) the complement (90° – *c*) of the side *c*,

and (*v*) the complement (90° – *B*) of the angle *B*.

These five parts are supposed to be arranged round a circle [Fig. 1.14 (*b*)] in order in which they stand in the triangle. Thus, starting with the side *a*, we have, *in order*,

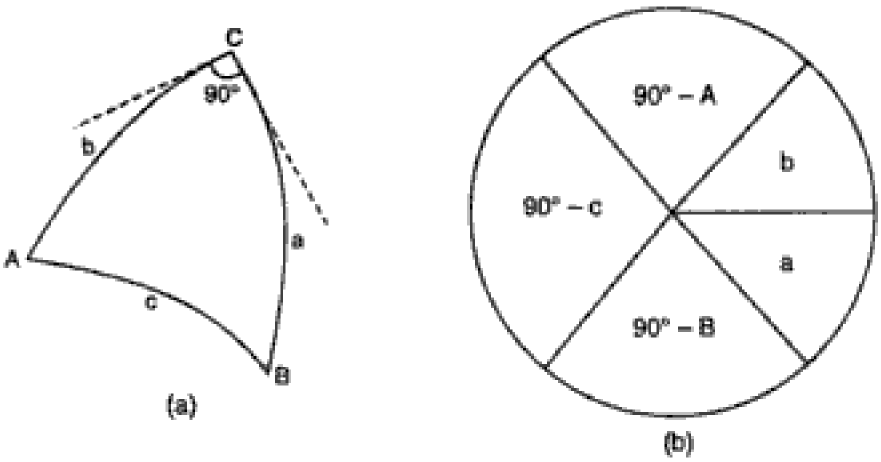


FIG. 1.14. NAPIER'S RULES OF CIRCULAR PARTS.

b, $90° - A$, $90° - c$ and $90° - B$. Then, if any part is considered as the *'middle part'* the two parts adjacent to it as *'adjacent parts'*, and the remaining two as *'opposite parts'*, we have the following rules by Napier :

sine of middle part = product of *tangents* of the *adjacent* parts ...(*i*)

and ***sine of middle part*** = product of *cosines* of opposite parts ...(*ii*)

Thus, $\sin b = \tan a \tan (90° - A)$

and $\sin b = \cos (90° - B) \cos (90° - c)$

By choosing different parts in turn as the middle parts, we can obtain all the possible relationships between the sides and angles.

THE SPHERICAL EXCESS

The spherical excess of a spherical triangle is the value by which the sum of three angles of the triangle exceeds 180°.

Thus, spherical excess $E = (A + B + C - 180°)$ ...(1.14)

Also, $\tan^2 \frac{1}{2} E = \tan \frac{1}{2} s \tan \frac{1}{2} (s - a) \tan \frac{1}{2} (s - b) \tan \frac{1}{2} (s - c)$ ...(1.15)

In geodetic work the spherical triangles on the earth's surface are comparatively small and the spherical excess seldom exceeds more than a few seconds of arc. The spherical excess, in such case, can be expressed by the approximate formula

$$E = \frac{\Delta}{R^2 \sin 1''} \text{ seconds} \qquad ...[1.15\ (a)]$$

where R is the radius of the earth and Δ is the area of triangle expressed in the same linear units as R.

In order to prove the above expression for the spherical excess, let us consider the spherical triangle ABC [Fig. 1.14 (*c*)] which is formed by three great circles. These three great circles divide the whole sphere in eight divisions—the four in one hemisphere being similar to the other four in the other hemisphere because of symmetry.

Let Δ = area ABC ; Δ_1 = area ACD

Δ_2 = area CDE ; Δ_3 = area BCE

S = area of whole sphere $= 4\pi R^2$;

R = radius of sphere

A, B, C = angles of the spherical triangle

Evidently, $$(\Delta + \Delta_1) = \frac{B}{360°} \times S$$

$$\Delta + \Delta_3 = \frac{A}{360°} \times S$$

and $$\Delta + \Delta_2 = \frac{C}{360°} \times S$$

Adding the three, we get

$$3\Delta + \Delta_1 + \Delta_2 + \Delta_3 = \frac{A + B + C}{360°} \times S \quad ...(1)$$

Also, $$\Delta + \Delta_1 + \Delta_2 + \Delta_3 = \text{area of hemisphere} = \frac{S}{2} \quad ...(2)$$

Subtracting (2) from (1), we get

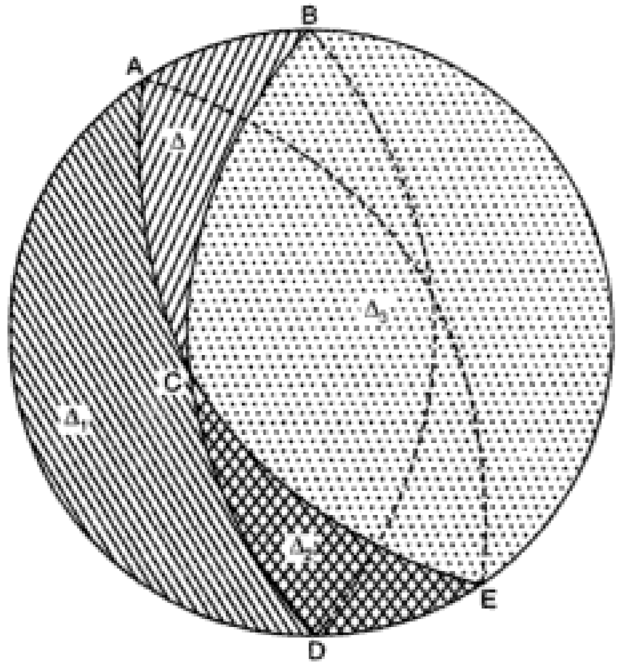


FIG. 1.14 (*c*) COMPUTATION OF SPHERICAL EXCESS.

$$2\Delta + \frac{S}{2} = \frac{A + B + C}{360°} \times S \quad \text{or} \quad 2\Delta = \frac{S}{360°}(A + B + C - 180°)$$

or $$2\Delta = \frac{S}{360°} \times E, \text{ from Equation 1.14}$$

which gives $$E = (2 \times 360°)\frac{\Delta}{S} = \frac{720°\ \Delta}{4\pi R^2} \ ; \text{ or } E = 180° \frac{\Delta}{\pi R^2} \text{ degrees} \quad ...[1.15\ (b)]$$

or $$E = \frac{\Delta}{R^2 \sin 1''} \text{ seconds} \quad ...[1.15\ (a)]$$

Area of sperical triangle :

The area of spherical triangle may be obtained from the formula

$$\text{Area } \Delta = \frac{\pi R^2 (A + B + C - 180°)}{180°} = \frac{\pi R^2 E}{180°} \quad ...(1.16)$$

1.5. THE ASTRONOMICAL TRIANGLE (Fig. 1.15)

An astronomical triangle is obtained by joining the pole, zenith and any star M on the sphere by arcs of great circles. From this triangle, the relation existing amongst the spherical co-ordinates may be obtained.

Let α = altitude of the celestial body (M)

δ = declination of the celestial body (M)

θ = latitude of the observer.

Then ZP = co-latitude of the observer $= 90° - \theta = c$

PM = co-declination or the polar distance of $M = 90° - \delta = p$

and ZM = zenith distance = co-altitude of the body $= (90° - \alpha) = z$

The angle at $Z = MZP =$ the azimuth (A) of the body

The angle at $P = ZPM =$ the hour angle (H) of the body

The angle at $M = ZMP =$ the parallactic angle

If the three sides (*i.e.* MZ, ZP and PM) of the astronomical triangle are known, the angles A and H can be computed from the formulae of spherical trigonometry.

Thus, from Eq. 1.2, we have

$$\cos A = \frac{\sin \delta}{\cos \alpha \,.\, \cos \theta} - \tan \alpha \,.\, \tan \theta \qquad ...[1.17\ (a)]$$

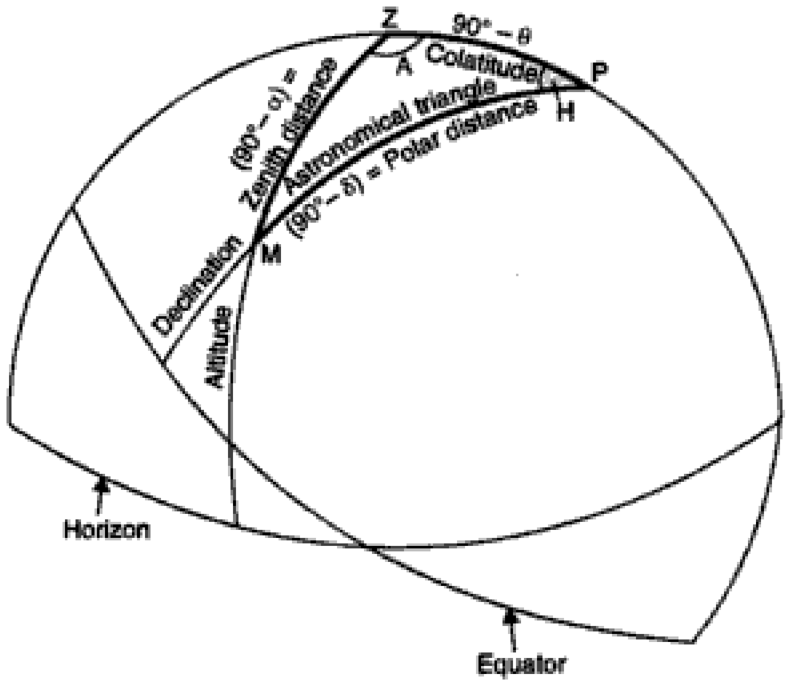


FIG. 1.15. THE ASTRONOMICAL TRIANGLE.

Also,

$$\tan \frac{A}{2} = \sqrt{\frac{\sin (s - ZM) \sin (s - ZP)}{\sin s \,.\, \sin (s - PM)}} \qquad ...(1.17)$$

$$= \sqrt{\frac{\sin (s - z) \sin (s - c)}{\sin s \,.\, \sin (s - p)}} \qquad ...[1.17\ (b)]$$

$$\sin \frac{A}{2} = \sqrt{\frac{\sin (s - z) \sin (s - c)}{\sin z \sin c}} \qquad ...[1.17\ (c)]$$

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - p)}{\sin z \sin c}} \qquad ...[1.17\ (d)]$$

where $s = \frac{1}{2} (ZM + ZP + PM) = \frac{1}{2} (z + c + p)$

Similarly, $$\cos H = \frac{\sin \alpha}{\cos \delta \cos \theta} - \tan \delta \tan \theta \qquad ...[1.18\ (a)]$$

Also, $$\tan \frac{H}{2} = \sqrt{\frac{\sin (s - ZP) \sin (s - PM)}{\sin s \,.\, \sin (s - ZM)}} = \sqrt{\frac{\sin (s - c) \sin (s - p)}{\sin s \,.\, \sin (s - z)}} \qquad ...(1.18)$$

$$\sin \frac{H}{2} = \sqrt{\frac{\sin (s - c) \sin (s - p)}{\sin c \,.\, \sin p}} \qquad ...[1.18\ (b)]$$

$$\cos \frac{H}{2} = \sqrt{\frac{\sin s \,.\, \sin (s - z)}{\sin c \,.\, \sin p}} \qquad ...[1.18\ (c)]$$

STAR AT ELONGATION

A star is said to be at elongation when it is at its greatest distance east or west of the meridian. In this position, the azimuth of the star is a maximum, and its diurnal circle is tangent to the vertical through the star. The triangle is thus right-angled at M.

The star is said to be at *eastern elongation,* when it is at its greatest distance to the east of the meridian, and at *western elongation,* when it is at its greatest distance to the west of the meridian. Fig. 1.16 (*a*) and (*b*) show the star *M* at its eastern elongation.

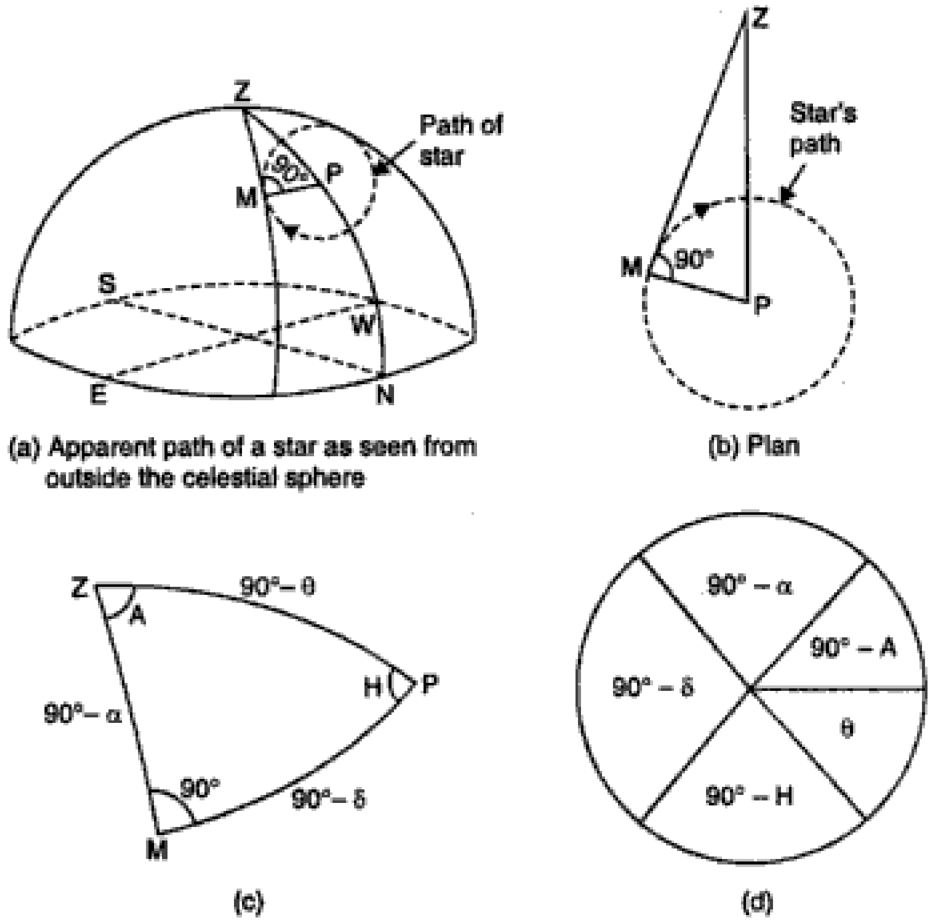


FIG. 1.16. STAR AT ELONGATION.

If the declination (δ) and the latitude of the place of observation is known, the azimuth (*A*), hour angle (*H*) and the altitude (α) of the body can be calculated from the Napier's rule [Fig. 1.16 (*c*) and (*d*)]. The five parts taken in order are, the two sides (90° − α), (90° − δ) and the complements of the rest of the three parts, *i.e.*, (90° − *H*), [90° − (90° − θ)] = θ and (90° − *A*).

Thus, sine of middle part = product of tangents of adjacent parts.

$$\sin (90° - H) = \tan (90° - \delta) \tan \theta \quad \text{or} \quad \cos H = \frac{\tan \theta}{\tan \delta} = \tan \theta \,.\, \cot \delta \qquad \text{...(1.19)}$$

Similarly,

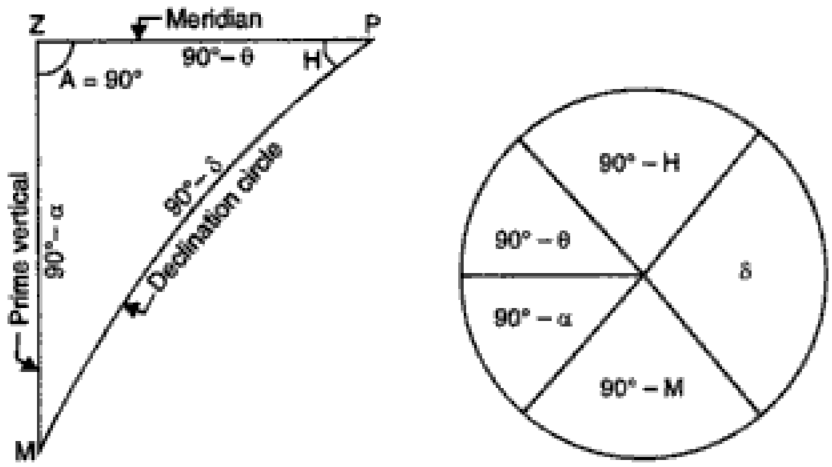
$$\sin \theta = \cos (90° - \delta) \,.\, \cos (90° - \alpha) \quad \text{or} \quad \sin \alpha = \frac{\sin \theta}{\sin \delta} = \sin \theta \,.\, \text{cosec}\, \delta \qquad \text{...(1.20)}$$

and

$$\sin (90° - \delta) = \cos (90° - A) \cos \theta \quad \text{or} \quad \sin A = \frac{\cos \delta}{\cos \theta} = \cos \delta \,.\sec \theta \qquad \text{...(1.21)}$$

STAR AT PRIME VERTICAL

When the star is on the prime vertical of the observer, the astronomical triangle is evidently right-angled at Z.



(a) (b)

FIG. 1.17. STAR AT PRIME VERTICAL.

If the declination (δ) and the latitute (θ) of the place of observation are known, the altitude (α) and the hour angle (H) can be calculated by Napier's rule. The five parts taken in order are : the two sides (90° – θ) and (90° – α), and the complements of the rest of the three parts, *i.e.*, (90° – M), 90° – (90° – δ) = δ and (90° – H).

Now sine of middle part = product of cosine of opposite parts.

$$\therefore \quad \sin\delta = \cos(90° - \theta)\cos(90° - \alpha) = \sin\theta\sin\alpha \quad \therefore \quad \sin\alpha = \frac{\sin\delta}{\sin\theta} = \sin\delta\,\mathrm{cosec}\,\theta \quad ...(1.22)$$

$$\text{And } \sin(90° - H) = \tan(90° - \theta)\tan\delta \quad \text{or} \quad \cos H = \frac{\tan\delta}{\tan\theta} = \tan\delta\cot\theta \quad ...(1.23)$$

STAR AT HORIZON

If a star (M) is at horizon, its altitude will be zero and the zenith distance will be equal to 90°.

If the latitude θ and the declination δ are known, the azimuth A and the hour angle H can be calculated by putting α = zero in equations 1.17 *a* and 1.18 *a*.

$$\text{Thus,} \quad \cos A = \frac{\sin\delta}{\cos\theta} = \sin\delta\sec\theta \quad ...(1.24)$$

$$\text{and} \quad \cos H = -\tan\delta\tan\theta \quad ...(1.25)$$

STAR AT CULMINATION

A star is said to culminate or transit when it crosses the observer's meridian. Each star crosses a meridian twice in its one revolution around the pole – the two culminations

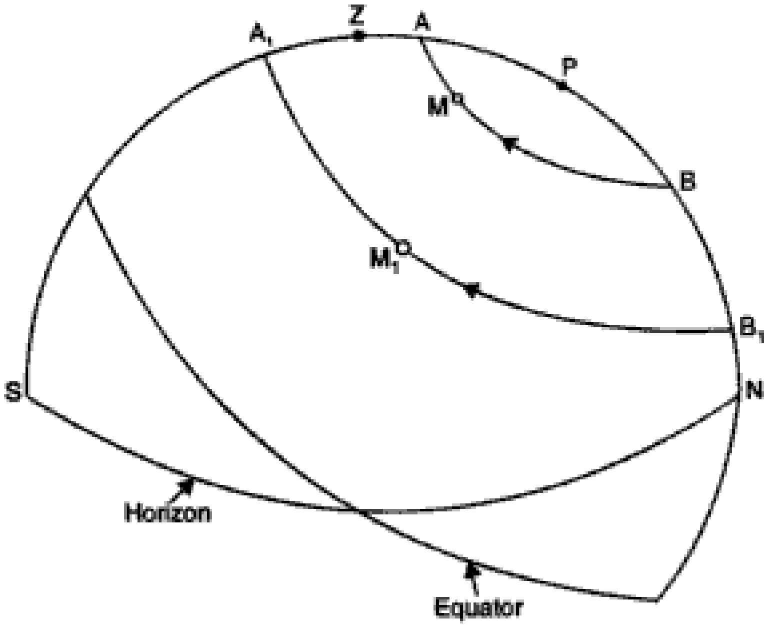


FIG. 1.18. STAR AT CULMINATION.

being designated as the upper culmination and the lower culmination. A star is to be at its *upper culmination* when its altitude is *maximum,* and at *lower culmination* when its altitude is *minimum.*

Thus, in Fig. 1.18, the star M culminates or transits the meridian at A and B, A being the point of upper culmination and B the point of lower culmination.

Similarly, the star M_1 culminates or transits the meridian at A_1 and B_1, A_1 being the point of upper culmination and B_1 the point of lower culmination.

The upper culmination (A) of the star M occurs at the north side of the zenith, (*i.e.*, towards the pole) while the upper culmination (A_1) of the star M_1 occurs at the south side of zenith.

Now, at the upper culmination (A) of the star M, its zenith distance

$$= ZA = ZP - AP = (90° - \theta) - (90° - \delta) = (\delta - \theta) \quad ...(1)$$

Similarly, at the upper culmination (A_1) of the star M_1, the zenith distance

$$= ZA_1 = PA_1 - PZ = (90° - \delta) - (90° - \theta) = (\theta - \delta) \quad ...(2)$$

From (1) and (2), it follows that:

(*i*) The upper culmination of a star occurs to the *north side of the zenith* when the *declination of the star is greater than the latitude* of the place of observation.

(*ii*) The upper culmination of a star occur to the *south side of the zenith* when the *declination of the star is lesser than the latitude* of the place of observation.

CIRCUMPOLAR STARS

Circumpolar stars are those which are always above the horizon, and which do not, therefore, set. Such a star appears to the observer to describe a circle above the pole.

Thus, in Fig. 1.19, M_1 is a circumpolar star having its path A_1A_2 which is always above the horizon. In order that the circumpolar star does not set,

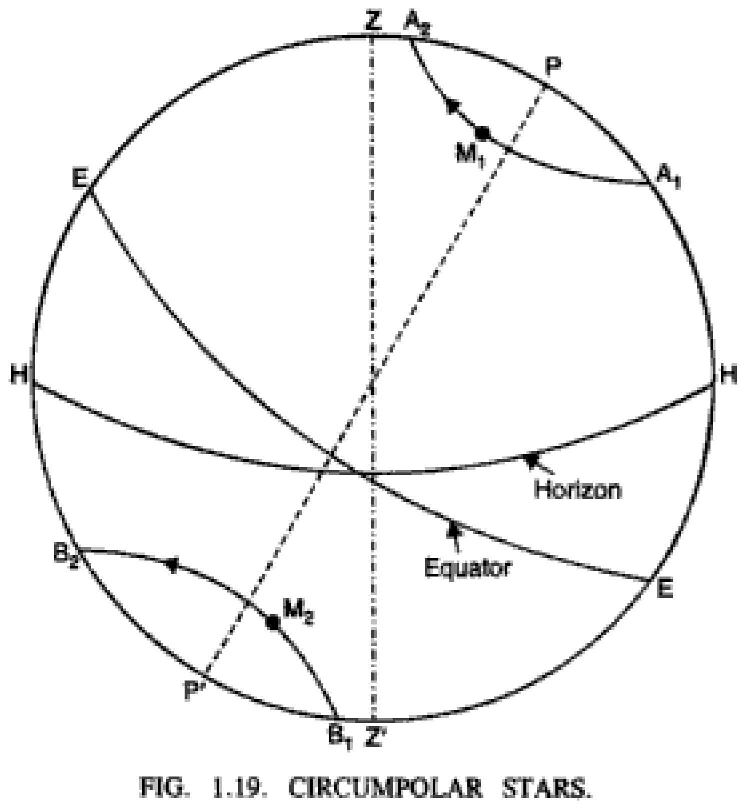


FIG. 1.19. CIRCUMPOLAR STARS.

distance above the pole (*i.e.*, PA_1) should be less than the distance of the pole from the horizon.

Hence $PA_1 < PH$ or $(90° - \delta) < \theta$ since $PH = \theta$ or $\delta > (90° - \theta)$

Hence the declination of a circumpolar star is always greater than the co-latitude of the place of observation.

Similarly, M_2 is a circumpolar star having its path B_1B_2 which is always below the horizon and, therefore, never rises.

1.6. RELATIONSHIPS BETWEEN CO-ORDINATES

1. The Relation between Altitude of the Pole and Latitude of the Observer.

In Fig. 1.20. *H-H* is the horizon plane and *E-E* is the equatorial plane. *O* is the centre of the earth. *ZO* is perpendicular to *HH* while *OP* is perpendicular to *EE*.

Now latitude of place $= \theta = \angle EOZ$

And altitude of pole $= \alpha = \angle HOP$

$$\angle EOP = 90° = \angle EOZ + \angle ZOP = \theta + \angle ZOP \qquad \ldots(i)$$

$$\angle HOZ = 90° = \angle HOP + \angle POZ = \alpha + \angle POZ \qquad \ldots(ii)$$

Equating the two, we get

$$\theta + \angle ZOP = \alpha + \angle POZ \quad \text{or} \quad \theta = \alpha$$

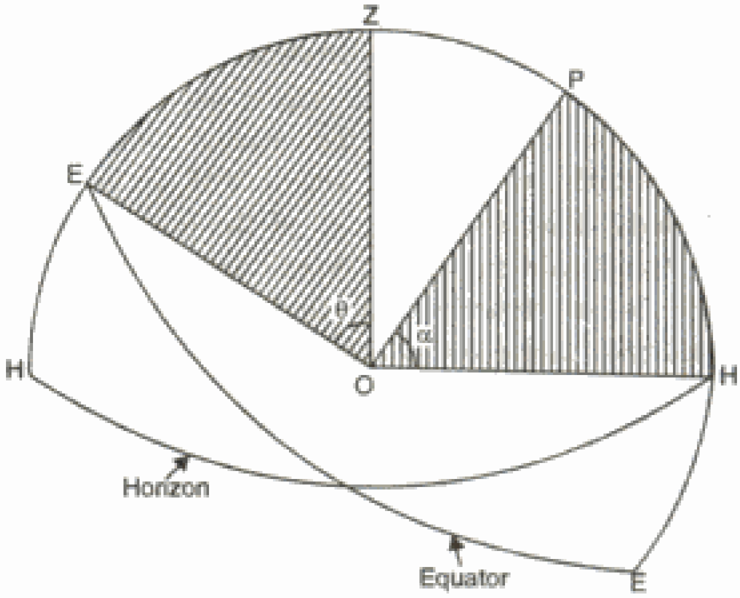


FIG. 1.20.

Hence *the altitude of the pole is always equal to the latitude of the observer.*

2. The Relation between Latitude of Observer and the Declination and Altitude of a Point on the Meridian.

For star M_1, $EM_1 = \delta$ = declination.

$SM_1 = \alpha$ = meridian altitude of star.

$M_1Z = z$ = meridian zenith distance of star.

$EZ = \theta$ = latitude of the observer.

Evidently, $EZ = EM_1 + M_1Z$

or $\theta = \delta + z \qquad \ldots(1)$

The above equation covers all cases. If the star is below the equator, negative sign should be given to δ. If the star is to the north of zenith, negative sign should be given to z.

If the star is north of the zenith but above the pole, as at M_2, we have

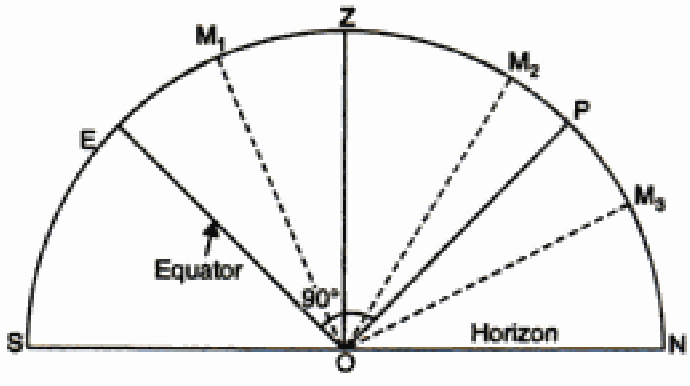


FIG. 1.21.

$$ZP = ZM_2 + M_2P$$

or $(90° - \theta) = (90° - \alpha) + p$, where p = polar distance = M_2P

or $\theta = \alpha - p$ ...(2)

Similarly, if the star is north of the zenith but below the pole, as at M_3, we have

$$ZM_3 = ZP + PM_3$$

or $(90° - \alpha) = (90° - \theta) + p$, where p = polar distance = M_3P

or $\theta = \alpha + p$ ...(3)

The above relations form the basis for the usual observation for latitude.

3. The Relation between Right Ascension and Hour Angle.

Fig. 1.22 shows the plan of the stellar sphere on the plane of the equator. M is the position of the star and $\angle SPM$ is its westerly hour angle. H_M. ♈ is the position of the First Point of Aries and angle $SP♈$ is its westerly hour angle. $\angle ♈PM$ is the right ascension of the star. Evidently, we have

∴ ***Hour angle of Equinox = Hour angle of star + R.A. of star.***

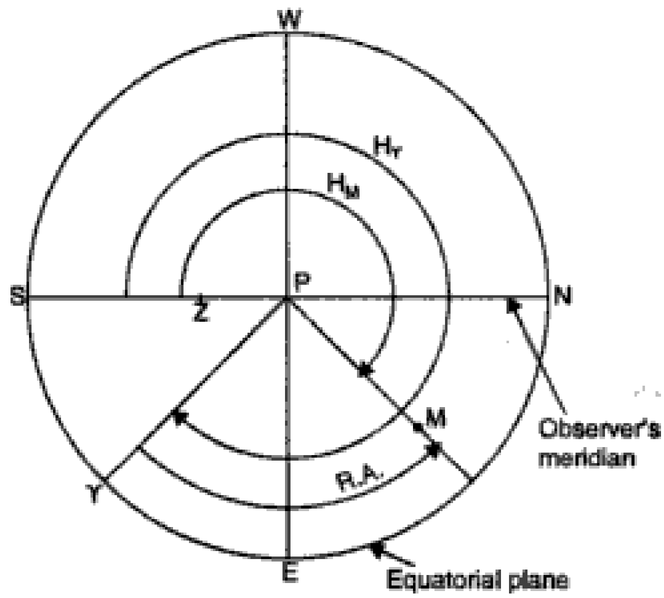


FIG. 1.22

Example 1.1. *Find the difference of longitude between two places A and B from their following longitudes :*

(*1*) *Longitude of A = 40° W*
Longitude of B = 73° W

(*2*) *Long. of A = 20° E*
Long. of B = 150° E

(*3*) *Longitude of A = 20° W*
Longitude of B = 50° W

(*4*) *Long. of A = 40° E*
Long. of B = 150° W

Solution.

(1) The difference of longitude between A and B = 73° – 40° = **33°**

(2) The difference of longitude between A and B = 150° – 20° = **130°**

(3) The difference of longitude between A and B = 20° – (– 50°) = **70°**

(4) The difference of longitude between A and B = 40° – (– 150°) = **190°**

Since it is greater than 180°, it represents the obtuse angular difference. The acute angular difference of longitude between A and B, therefore, is equal to 360° – 190° = **170°**.

Example 1.2. *Calculate the distance in kilometers between two points A and B along the parallel of latitude, given that*

(*1*) *Lat. of A, 28° 42′ N ; longitude of A, 31° 12′ W*
Lat. of B, 28° 42′ N ; longitude of B, 47° 24′ W

(2) *Lat. of A, 12° 36′ S ; longitude of A, 115° 6′ W*
Lat. of B, 12° 36′ S ; longitude of B, 150° 24′ E.

Solution.

The distance in nautical miles between A and B along the parallel of latitude = difference of longitude in minutes × cos latitude.

(1) Difference of longitude between A and $B = 47° 24' - 31° 12' = 16° 12' = 972$ minutes

$\therefore$ Distance = 972 cos 28° 42′ = 851.72 nautical miles
= 851.72 × 1.852 = **1577.34 km.**

(2) Difference of longitude between A and B
$= 360° - \{115° 6' - (-150° 24')\} = 94° 30' = 5670$ min.

$\therefore$ Distance = 5670 cos 12° 36′ = 5533.45 nautical miles
= 5533.45 × 1.852 = **10,247.2 km.**

Example 1.3. *Find the shortest distance between two places A and B, given that the longitudes of A and B are 15° 0′ N and 12° 6′ N and their longitudes are 50° 12′ E and 54° 0′ E respectively. Find also the direction of B on the great circle route. Radius of earth = 6370 km.*

Solution.

In Fig. 1.23, the positions of A and B have been shown.

In the spherical triangle ABP,

$b = AP = 90° - \text{latitude of } A = 90° - 15° 0' = 75°$

$a = BP = 90° - \text{latitude of } B$
$= 90° - 12° 6' = 77° 54'$

$P = \angle APB$ = difference of longitude
$= 54° 0' - 50° 12' = 3° 48'$.

The shortest distance between two points is the distance along the great circle passing through the two points.

Knowing the two sides one angle, the third side AB ($= p$) can be easily computed by the cosine rule.

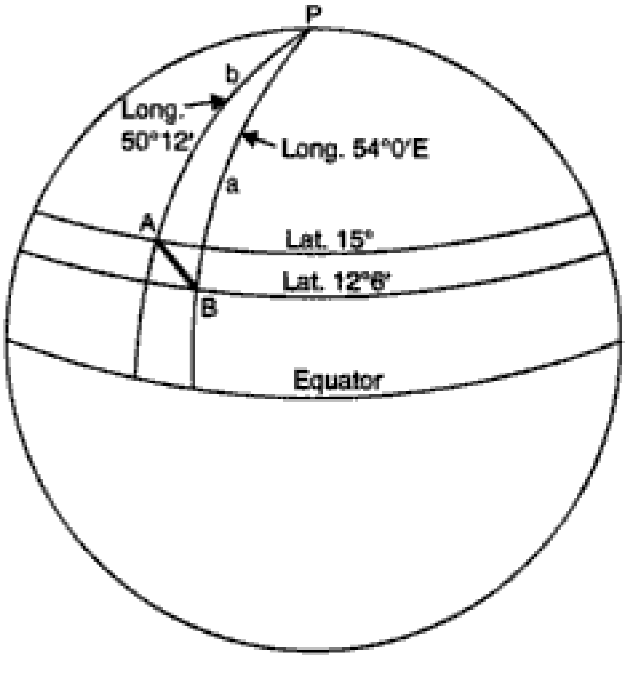


FIG. 1.23.

Thus $\cos P = \dfrac{\cos p - \cos a \cos b}{\sin a \sin b}$

or $\cos p = \cos P \sin a \sin b + \cos a \cos b$

$= \cos 3° 48' \sin 77° 54' \sin 75° + \cos 77° 54' \cos 75° = 0.94236 + 0.05425 = 0.99661$

$\therefore$ $p = AB = 4° 40' = 4°.7$

Now, arc ≈ radius × central angle $= \dfrac{6370 \times 4°.7 \times \pi}{180°} = 522.54$ km.

Hence distance AB = **522.54 km.**

Direction of A from B :

The direction of A from B is the angle B, and the direction of B from A is the angle A.

Angles A and B can be found by the tangent semi-sum and semi-difference formulae (Eqs. 1.12 and 1.13).

Thus $$\tan \frac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{1}{2} P$$

and $$\tan \frac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \frac{1}{2} P$$

Here $$\frac{a-b}{2} = \frac{77° \, 54' - 75°}{2} = \frac{2° \, 54'}{2} = 1° \, 27'$$

$$\frac{(a+b)}{2} = \frac{77° \, 54' + 75°}{2} = \frac{152° \, 54'}{2} = 76° \, 27'; \; \frac{P}{2} = \frac{3° \, 48'}{2} = 1° \, 54'$$

$\therefore$ $$\tan \frac{1}{2}(A+B) = \frac{\cos 1° \, 27'}{\cos 76° \, 27'} \cot 1° \, 54'$$

From which, $$\frac{A+B}{2} = 89° \, 35' \qquad \ldots(i)$$

and $$\tan \frac{1}{2}(A-B) = \frac{\sin 1° \, 27'}{\sin 76° \, 27'} \cot 1° \, 54'$$

From which, $$\frac{A-B}{2} = 38° \, 6' \qquad \ldots(ii)$$

$\therefore$ Direction of B from A = angle A = 89° 35′ + 38° 6′ = 127° 41′ = ***S* 52° 19′ *E***

Direction of A from B = angle B = 89° 35′ − 38° 6′ = 51° 29′ = ***N* 51° 29′ *W*.**

Example 1.4. *At a point A in latitude 45° N, a straight line is ranged out which runs due east at A. This straight line is prolonged for 300 nautical miles to B. Find the latitude of B, and if it be desired to travel due north from B so as to meet the 45° parallel again at C, find the ABC at which we must set out, and the distance BC.*

Solution

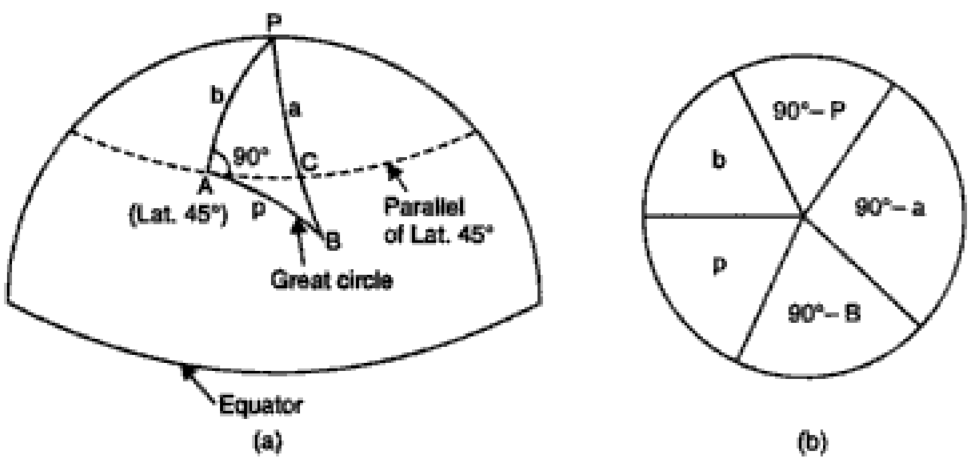


FIG. 1.24.

In Fig. 1.24, AB is straight line portion of a great circle ; since its length is 300 nautical miles, it subtends 300 minutes (= 5°) at the centre of the earth. AP is the meridian through A. Since AB is due east of the meridian, $\angle PAB = 90°$. Similarly, BP is the meridian through B, and meets the parallel to latitude through A (45° N) in C. PAB is, therefore, an astronomical triangle in which

side $PA = b$ = co-latitude of $A = 90° - 45° = 45°$; side $AB = p = 5°$; $\angle A = 90°$

The side $PB = a$ can be calculated by Napier's rule. Thus, sine of middle part = product of cosines of opposite parts.

$$\therefore \quad \sin (90° - a) = \cos b \cos p \quad \text{or} \quad \cos a = \cos 45° \cos 5°$$

$$\log \cos 45° = \bar{1}.8494850$$
$$\log \cos 5° = \bar{1}.9983442$$

$$\log \cos a = \bar{1}.8478292$$

$$\therefore \quad a = PB = 45°\ 13'{\cdot}108$$

$$BC = PB - PC = 45°\ 13'{\cdot}108 - 45° = 13'{\cdot}108$$

Hence distance BC = **13.108** nautical miles = 13.108 × 1.852 = **24.275 km.**

The angle at B can be found by the application of the sine formula,

i.e. $$\frac{\sin B}{\sin b} = \frac{\sin A}{\sin a} \quad \text{or} \quad \frac{\sin B}{\sin 45°} = \frac{\sin 90°}{\sin 45°\ 13'{\cdot}108}$$

or $$\sin B = \frac{\sin 45°}{\sin 45°\ 13'{\cdot}108}$$

$$\log \sin 45° = \bar{1}{\cdot}8494850$$
$$\log \sin 45°\ 13'{\cdot}108 = \bar{1}{\cdot}8511345 \quad \text{(subtract)}$$

$$\log \sin B = \bar{1}{\cdot}998505 \quad ; \quad \mathbf{B = 85°\ 0'\ 34''}.$$

Example 1.5. *Two ports have the same latitude l and their longitudes differ by $2d$. Prove that the length of the shortest route between them is $2R \sin^{-1} (\sin d \cos l)$, where R is the mean radius of the earth.*

Find the greatest distance, along a meridian, between the shortest route and the parallel of latitude through the ports. (*U.L.*)

Solution

In Fig. 1.25, A and B are the two ports. AFB is the arc of the great circle through A and B and F is the middle point. Due to symmetry, therefore, $\angle AFP = BFP = 90°$. ACB is the arc of parallel of latitude. AP and BP are the two meridians through A and B, FP is the meridian through the middle point of AB. Hence, triangles APF and BPF are astronomical triangles.

In triangle PFB,

$PB = f$ = co-latitude of $B = (90° - l)$; $PF = b$; $\angle FPB = d$.

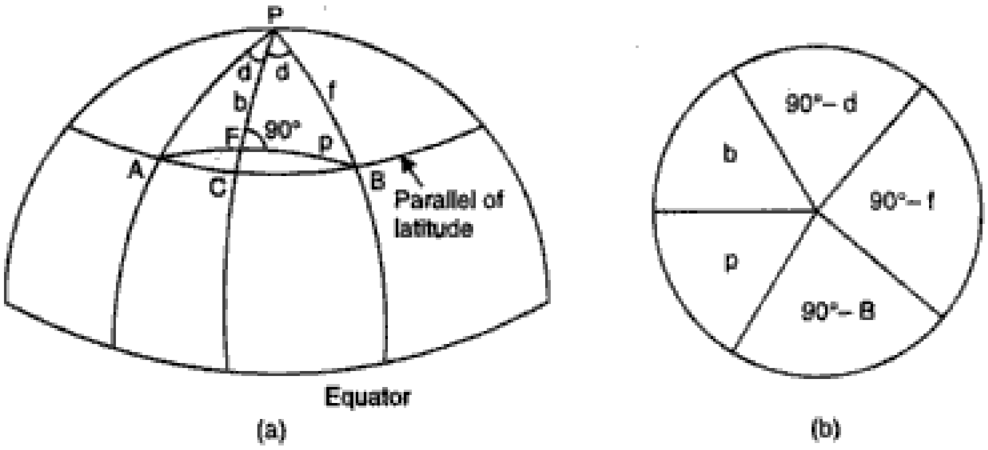


FIG. 1.25.

Distance $FB = p$ can be calculated by Napier's rule for the circular parts shown in Fig. 1.25 (*b*).

∴ sine middle part = Product of cosines of opposite parts

$$\therefore \quad \sin p = \cos (90° - d) \cos (90° - f) = \sin d \sin f = \sin d \sin (90° - l) = \sin d \cos l$$

or $$FB = p = \sin^{-1} (\sin d \cos l)$$

Hence $AB = 2\,FB = 2\,p = 2 \sin^{-1} (\sin d \cos l)$ radians.

∴ Distance AB along great circle = radius × angle at the centre of earth = $R \times 2\,p$

$$= 2\,R \sin^{-1} (\sin d \cos l) \qquad \text{(Proved).}$$

The greatest distance between the great circle AFB and the parallel of latitude ACB will evidently be along CF (since $\angle F = 90°$).

The distance $PF = b$ can be found by Napier's rule.

sin middle part = product of tangents of adjacent parts

or $$\sin (90° - d) = \tan b \tan (90° - f)$$

or $$\cos d = \tan b \cot f = \tan b \cot (90° - l) = \tan b \tan l$$

$$\therefore \quad \tan b = \cos d \cot l \quad \text{or} \quad b = PF = \tan^{-1} (\cos d \cot l)$$

Now $$CF = CP - PF$$

But $$CP = (90° - l) = \frac{\pi}{2} - l \text{ radians}$$

$$\therefore \quad CF = \left(\frac{\pi}{2} - l\right) - \tan^{-1} (\cos d \cot l) \text{ radians}$$

∴ Distance along CF = Radius × angle at the centre = $R \left\{ \left(\frac{\pi}{2} - l\right) - \tan^{-1} (\cos d \cot l) \right\}$ **Ans.**

Example 1.6. *Find the zenith distance and altitude at the upper culmination of the stars from the following data :*

(*a*) *Declination of star = 42° 15′ N Latitude of observer = 26° 40′ N*

(*b*) *Declination of star = 23° 20′ N Latitude of observer = 26° 40′ N*

(*c*) *Declination of star = 65° 40′ N Latitude of observer = 26° 40′ N*

Solution. (Fig. 1.18)

(*a*) Since the declination of the star is greater than the latitude of the observer ($\delta > \theta$), the upper culmination of the star occurs to the north side of zenith, *i.e.*, between Z and P.

Hence zenith distance at upper culmination $= ZA = ZP - AP$

$$= (90° - \theta) - (90° - \delta) = (\delta - \theta) = 42° \; 15' - 26° \; 40' = \mathbf{15° \; 35'}$$

$\therefore$ Altitude of the star at upper culmination $= 90° - 15° \; 35' = \mathbf{74° \; 25'}$.

(*b*) Since the declination of the star is lesser than the latitude of the observer, the upper culmination of the star occurs at the south side of the zenith.

$\therefore$ Zenith distance of the star at upper culmination $= ZA_1 = A_1P - ZP$

$$= (90° - \delta) - (90° - \theta) = \theta - \delta = 26° \; 40' - 23° \; 20' = \mathbf{3° \; 20'}$$

$\therefore$ Altitude of the star at the upper culmination $= 90° - 3° \; 20' = \mathbf{86° \; 40'}$.

(*c*) Fig. 1.19, $\delta = 65° \; 40'$ N ; $90° - \theta = 90° - 26° \; 40' = \mathbf{63° \; 20'}$

Since the declination of the star is greater than the co-latitude, the star is circumpolar, and will never set. The upper culmination will occur at the north side of zenith, *i.e.*, between Z and P.

$\therefore$ Zenith distance at the upper culmination $= ZA_2 = ZP - A_2P$

$$= (90° - \theta) - (90° - \delta) = \delta - \theta = 85° \; 40' - 26° \; 40' = \mathbf{39°}.$$

$\therefore$ Altitude of the star at the upper culmination $= 90° - 39° = \mathbf{51°}$.

Example 1.7. *Find the zenith distance and altitude at the lower culmination for a star having declination = 85° 20′ if the latitude of the place of observation = 46° 50′.*

Solution.

$$\delta = 85° \; 20' \; ; \; 90° - \theta = 90° - 46° \; 50' - 43° \; 10'$$

Since the declination of the star is greater than the co-latitude of the place, it is circumpolar and will not set.

In Fig. 1.19, let A_1 be the lower culmination of a circumpolar star M_1. Its zenith distance at the lower culmination $= ZA_1 = ZP + PA_1$

$$= (90° - \theta) + (90° - \delta) = 180° - \delta - \theta = 180° - 85° \; 20' - 46° \; 50' = \mathbf{47° \; 50'}$$

The altitude of the star $= 90° - 47° \; 50' = \mathbf{42° \; 10'}$.

Example 1.8. *A star having a declination of 56° 10′ N has its upper transit in the zenith of the place. Find the altitude of the star at its lower transit.*

Solution. (Fig. 1.18)

Let M be the star having A and B as its upper and lower transits. Since the upper culmination is at the zenith, Z and A coincide.

Hence zenith distance of star = zero

and Polar distance of the star $= AP = ZP =$ co-latiude of place

$$\therefore \quad 90° - \delta = 90° - \theta \quad \text{or} \quad \theta = \delta = 56° \; 10'$$

At the lowest transit of the star at B, its zenith distance $= ZB = ZP + PB$

$$= (90° - \theta) + (90° - \delta) = 180° - \theta - \delta = 180° - 2\delta = 180° - 112° \; 20' = 67° \; 40'$$

$\therefore$ Altitude of the star at lower transit = 90° − 67° 40′ = **22° 20′**.

Example 1.9. *The altitudes of a star at upper and lower transits of a star are 70° 20′ and 20° 40′, both the transits being on the north side of zenith of the place. Find the declination of the star and the latitude of the place of observation.*

Solution. (Fig. 1.18)

Let M be the star having A and B as its upper and lower culminations.

At the upper culmination, zenith distance = $ZA = ZP - AP$

$$= (90° - \theta) - (90° - \delta) = \delta - \theta$$

$\therefore$ Altitude of star = 90° − zenith distance = 90° − ($\delta - \theta$).

But this is equal to 70° 20′ (given)

$$70° 20' = 90° - (\delta - \theta)$$

or $$\delta - \theta = 90° - 70° 20' = 19° 40' \qquad ...(1)$$

At the lower culmination of the star, the zenith distance of the star

$$= ZB = ZP + PB = (90° - \theta) + (90° - \delta) = 180° - (\theta + \delta)$$

$\therefore$ Altitude of the star = 90° − zenith distance = $\theta + \delta - 90°$

But this is equal to 20° 40′ (given)

$\therefore$ $\theta + \delta - 90° = 20° 0'$ or $\theta + \delta = 110° 40'$ ...(2)

Solving equation (1) and (2), we get δ = **65° 10′** and θ = **45° 30′**.

Note. *Since the altitudes of the star at both the culminations are positive, the star is circumpolar.*

Example 1.10. *Determine the azimuth and altitude of a star from the following data:*

(*i*)	*Declination of star*	= *20° 30′ N*
(*ii*)	*Hour angle of star*	= *42° 6′*
(*iii*)	*Latitude of the observer*	= *50° N.*

Solution. (Fig. 1.26)

The hour angle of the star = 42° 6′ and since it is measured towards west, the star is in the western part of the hemisphere as shown in Fig. 1.26.

In the astronomical Δ PZM, we have

PZ = co-latitude = 90° − 50° = 40°

PM = co-declination of star = 90° − 20° 30′ = 69° 30′

$\angle ZPM = H$ = 42° 6′.

It is required to find angle A and ZM.

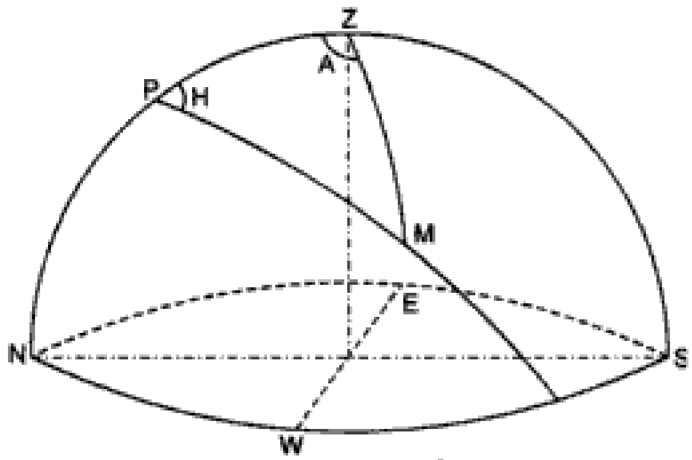


FIG. 1.26.

Using the cosine rule (Eq. 1.2 a)

$$\cos ZM = \cos PZ \cos PM + \sin PZ \sin PM \cos H$$
$$= \cos 40° \cos 69° 30' + \sin 40° \sin 69° 30' \cos 42° 6'$$
$$= 0.26828 + 0.44673 = 0.71501$$

$\therefore \quad ZM = 44° 21'$

$\therefore$ Altitude of the star $= \alpha = 90° - ZM = 90° - 44° 21' =$ **45° 39′**

Again, using the cosine rule (Eq. 1.2), we have

$$\cos A = \frac{\cos PM - \cos PZ \,.\, \cos ZM}{\sin PZ \,.\, \sin ZM}$$
$$= \frac{\cos 69° 30' - \cos 40° \,.\, \cos 44° 21'}{\sin 40° \,.\, \sin 44° 21'} = \frac{0.35021 - 0.54780}{0.44934} = -0.43972.$$

Since $\cos A$ is negative the angle A lies between 90° and 180°.

$\therefore \quad \cos (180° - A) = -\cos A = 0.43972$

$\therefore \quad 180° - A = 63° 55' \quad$ or $\quad A = 180° - 63° 55' =$ **116° 5′ *W*.**

Example 1.11. *Determine the azimuth and altitude of a star from the following data :*

(*i*)	*Declination of star*	*= 8° 30′ S*
(*ii*)	*Hour angle of star*	*= 322°*
(*iii*)	*Latitude of the observer*	*= 50° N.*

Solution. (Fig. 1.27)

Since the hour angle of the star is more than 180°, it is in the eastern hemi-sphere and its azimuth will be eastern as shown in Fig. 1.27 where *ZPM* is the astronomical triangle. The star *M* is below the equator since its declination is negative.

Now, $ZP =$ co-latitude $= 90° - 50° = 40°$

$PM = 90° - (-8° 30') = 98° 30'$;

$H_1 = 360° - H = 360° - 322° = 38°$

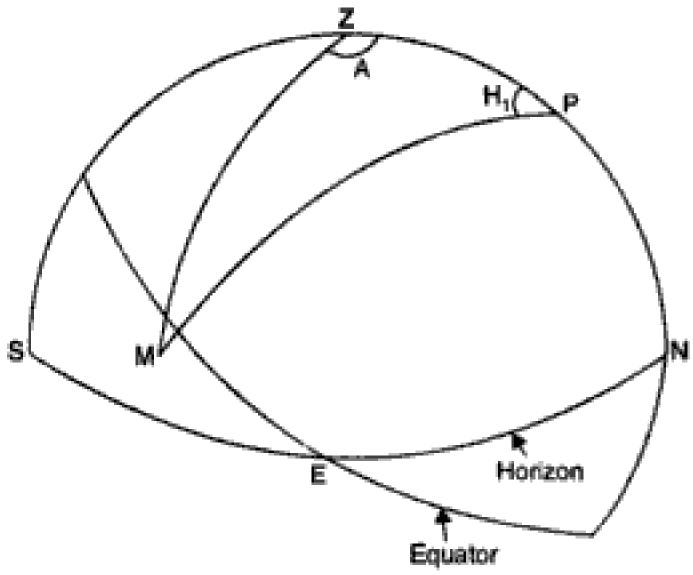


FIG. 1.27

Knowing the two sides and the included angle, the third side can be calculated by the cosine rule (Eq. 1.2 *a*).

Thus $\cos ZM = \cos PZ \,.\, \cos PM + \sin PZ \times \sin PM \cos H_1$

$$= \cos 40° \,.\, \cos 98° 30' + \sin 40° \sin 98° 30' \,.\, \cos 38°$$
$$= -0.11323 + 0.50094 = 0.38771$$

$\therefore \quad ZM = 67° 11'$

$\therefore$ Altitude of the star $= 90° - 67° 11' =$ **22° 49′**

(The star is thus above the horizon)

Again, from the cosine rule [Eq. 1.2]

$$\cos A = \frac{\cos PM - \cos PZ \,.\, \cos PM}{\sin PZ \,.\, \sin ZM} = \frac{\cos 98^\circ 30' - \cos 40^\circ \cos 67^\circ 11'}{\sin 40^\circ \sin 67^\circ 11'}$$

$$= \frac{-0.14781 - 0.29687}{0.59250} = -0.75051 \,.$$

Since $\cos A$ is negative, the value of A is between 90° and 180°

$\therefore \quad \cos (180^\circ - A) = -\cos A = 0.75051$

$\therefore \quad (180^\circ - A) = 41^\circ 22'$ or $A = 138^\circ 38'$

$\therefore$ Azimuth of star = **138° 38′ E.**

Example 1.12. *Determine the hour angle and declination of a star from the following data :*

(*i*) *Altitude of the star* = *22° 36′*

(*ii*) *Azimuth of the star* = *42° W*

(*iii*) *Latitude of the place of observation* = *40° N.*

Solution. (Fig. 1.26)

Since the azimuth of the star is 42° *W*, the star is in the western hemi-sphere. In the astronomical $\Delta\, PZM$, we have

PZ = co-latitude = 90° − 40° = 50° ; ZM = co-altitude = 90° − 22° 36° = 67 24′; angle A = 42°

Knowing the two sides and the included angle, the third side can be calculated from the cosine formula (Eq. 1.2 *a*).

Thus, $\cos PM = \cos PZ \,.\, \cos ZM + \sin PZ \,.\, \sin ZM \,.\, \cos A$

$= \cos 50^\circ .\, \cos 67^\circ 24' + \sin 50^\circ .\, \sin 67^\circ 24' .\, \cos 42^\circ$

$= 0.24702 + 0.52556 = 0.77258$

$\therefore \quad PM = 39^\circ 25'$

$\therefore$ Declination of the star = $\delta = 90^\circ - PM = 90^\circ - 39^\circ 25'$ = **50° 35′ *N*.**

Similarly, knowing all the three sides, the hour angle H can be calculated from Eq. 1.2.

$$\cos H = \frac{\cos ZM - \cos PZ \,.\, \cos PM}{\sin PZ \,.\, \sin PM} = \frac{\cos 67^\circ 24' - \cos 50^\circ \,.\, \cos 39^\circ 25'}{\sin 50^\circ \,.\, \sin 39^\circ 25'}$$

$$= \frac{0.38430 - 0.49659}{0.48640} = -0.23086$$

$\therefore \quad \cos (180^\circ - H) = 0.23086 \qquad \therefore \quad 180^\circ - H = 76^\circ 39'$

$\therefore \quad H$ = **103° 21′.**

Example 1.13. *Determine the hour angle and declination of a star from the following data :*

(*1*) *Altitude of the star* = *21° 30′*

(*2*) *Azimuth of the star* = *140° E*

(*3*) *Latitude of the observer* = *48° N.*

Solution

Refer Fig. 1.27. Since the azimuth of the star is 140° *E*, it is in eastern hemi-sphere.

In the astronomical triangle ZPM, we have

$$ZM = 90° - \alpha = 90° - 21° 30' = 68° 30'; \quad ZP = 90° - \theta = 90° - 48° = 42° \quad ; \quad A = 140°$$

Knowing the two sides and the included angle, the third side can be calculated by the cosine rule (Eq. 1.2 *a*).

Thus $\cos PM = \cos ZM \cos ZP + \sin ZM \sin ZP \cos A$

$$= \cos 68° 30' \cos 42° + \sin 68° 30' \sin 42° \cos 140°$$

$$= 0.27236 - 0.47691 = -0.20455$$

$\therefore \cos (180° - PM) = 0.20455$ or $180° - PM = 78° 12'$

$\therefore$ $PM = 101° 48'$

$\therefore$ Declination of the star $= 90° - 101° 48' = -11° 48' = -$ **11° 48′ S**

Again, knowing all the three sides, the angle H_1 can be calculated from the cosine formula, (Eq. 1.2). Thus

$$\cos H_1 = \frac{\cos MZ - \cos ZP \,.\, \cos MP}{\sin ZP \sin MP} = \frac{\cos 68° 30' - \cos 42° \cos 101° 48'}{\sin 42° \sin 101° 48'}$$

$$= \frac{0.36650 + 0.15198}{0.65499} = 0.79161 \qquad \therefore \qquad H_1 = 37° 40'$$

But H_1 is the angle measured in the eastward direction.

$\therefore$ Hour angle of the star $= 360° - H_1 = 360° - 37° 40' =$ **322° 20′**.

Example 1.14. *Calculate the sun's azimuth and hour angle at sunset at a place in latitude 42° 30′ N, when its declination is (a) 22°12′ N and (b) 22° 12′ S.*

Solution

Let us consider the astronomical triangle ZPM, where M is the position of the sun. Since the sun is on the horizon at its setting, it altitude is zero, and hence $ZM = 90°$.

Also, $ZP = 90° - 42° 30' = 47° 30'$

(*a*) $PM = 90° - 22° 12' = 67° 48'$

From the triangle ZPM, we get by cosine rule

$$\cos PM = \cos ZP \,.\, \cos ZM + \sin ZP \,.\, \sin ZM \,.\, \cos A$$

But $\cos ZM = \cos 90° = 0$ and $\sin ZM = \sin 90° = 1$

$$\therefore \qquad \cos A = \frac{\cos PM}{\sin ZP} = \frac{\cos 67° 48'}{\sin 47° 30'} \qquad \text{Hence} \qquad A = 59° 10'$$

Hence azimuth of the sun at setting = **59° 10′ West.**

Again, from the cosine rule, we get

$$\cos ZM = \cos ZP \,.\, \cos PM + \sin ZP \,.\, \sin PM \,.\, \cos H$$

But $\cos ZM = \cos 90° = 0$

Hence $\cos H = -\cot ZP \,.\, \cot PM = -\cot 47° 30' \cot 67° 48'$

or $\cos (180° - H) = +\cot 47° 30' \cot 67° 48'$

$\therefore$ $180° - H = 68° 03'$ or $H = 180° - 68° 03' = 111° 57'$

Hence sun's hour angle at sunset $= 111° 57' =$ **$7^h\ 27^m\ 48^s$**.

(*b*) As before, the azimuth is given by

$$\cos A = \frac{\cos PM}{\sin ZP}. \quad \text{Here, } PM = 90° - (-22° 12') = 122° 12'$$

and $ZP = 47° 37'$ and $ZM = 90°$ as before

$$\therefore \quad \cos A = \frac{\cos 112° 12'}{\sin 47° 30'} = -\frac{\cos 67° 48'}{\sin 47° 30'}$$

or $$\cos (180° - A) = +\frac{\cos 67° 48'}{\sin 47° 30'}$$

From which, $180' - A = 59° 10'$ or $A = 120° 50'$

∴ Azimuth of the sun at sunset = **120° 50′** West.

Similarly, $\cos H = -\cot ZP . \cot PM = -\cot 47° 30' \cot 112° 12' = \cot 47° 30' \cot 67° 48'$

$$\therefore \quad H = 68° 3'$$

Hence sun's hour angle at sunset = 68° 3′ = $\mathbf{4^h\ 32^m\ 12^s}$.

Example 1.15. *Calculate the sun's hour angle and azimuth at sunrise for a place in latitude 42° 30′ S when the declination is 22° 12′ N.*

Solution

Consider the astronomical triangle $Z'P'M$, where M is the position of the sun at the horizon and P' is the south pole.

$$Z'P' = 90° - \theta = 90° - 42° 30' = 47° 30'$$

$$Z'M = 90°, \text{ since the sun is at horizon}$$

$$MP' = 90° + 22° 12' = 112° 12'.$$

By the cosine rule, $\cos Z'M = \cos Z'P' . \cos P'M + \sin Z'P' \sin P'M . \cos H$

But $\cos Z'M = \cos 90° = 0$

Hence $\cos H = -\cot Z'P' \cot P'M = -\cot 47° 30' \cot 112° 12'$

$$= \cot 47° 30' \cot 67° 48'$$

Hence $H = 68° 3'$

Since the sun is at its setting, its hour angle is eastern.

Hence westerly hour angle of sun = 180° − 68° 3′ = 111° 57′ = $\mathbf{7^h\ 27^m\ 48^s}$

Again, as before, $$\cos A = \frac{\cos P'M}{\sin Z'P'} = \frac{\cos 112° 12'}{\sin 47° 30'} = -\frac{\cos 67° 48'}{\sin 47° 30'}$$

$$\therefore \quad \cos (180° - A) = -\frac{\cos 67° 48'}{\sin 47° 30'}$$

$$\therefore \quad 180° - A = 59° 10' \quad \text{or} \quad A = 180° - 59° 10'$$

Hence the azimuth of the sun = **120° 50′ East.**

1.7. THE EARTH AND THE SUN

1. The Earth. The Earth is considered approximately spherical in shape. But actually, it is very approximately an *oblate spheroid*. Oblate spheroid is the figure formed by revolving an ellipse about its minor axis. The earth is flattened at poles – its diameter along the polar axis being lesser than its diameter at the equator. The equatorial radius a of the earth, according to Hayford's spheroid is 6378.388 km and the polar radius b of the earth is 6356.912 km. The *ellipticity* is expressed by the ratio $\frac{a-b}{a}$, which reduces to $\frac{1}{297}$. For the Survey of India, Everest's first constants were used as follows : $a = 20{,}922{,}932$ ft and $b = 20{,}853{,}642$ ft, the ellipticity being $\frac{1}{311.04}$.

The earth revolves about its minor or shorter axis (*i.e.* polar axis), on an average, once in twenty-four hours, from West to East. If the earth is considered stationary, the whole celestial sphere along with its celestial bodies like the stars, sun, moon etc. appear to revolve round the earth from East to West. The axis of rotation of earth is known as the *polar axis*, and the points at which it intersects the surface of earth are termed the North and South *Geographical* or *Terrestrial Poles*. In addition to the motion of rotation about its own polar axis, the earth has a motion of rotation relative to the sun, in a plane inclined at an angle of 23° 27′ to the plane of the equator. The time of a complete revolution round the sun is one year. The apparent path of the sun in the heavens is the result of both the diurnal and annual real motions of the earth.

The earth has been divided into certain *zones* depending upon the parallels of latitude of certain value above and below the equator. The zone between the parallels of latitude 23° 27½′ N and 23° 27½′ S is known as the *torrid zone* (see Fig. 1.12). This is the hottest portion of the earth's surface. The belt included between 23° 27½′ N and 66° 32½′ N of equator is called the *north temperate zone*. Similarly, the belt included between 23° 27½′ S and 66° 32½′ S is called *south temperate zone*. The belt between 66° 32½′ N and the north pole is called the *north frigid zone* and the belt between 66° 32½′ S and the south pole is called *south frigid zone*.

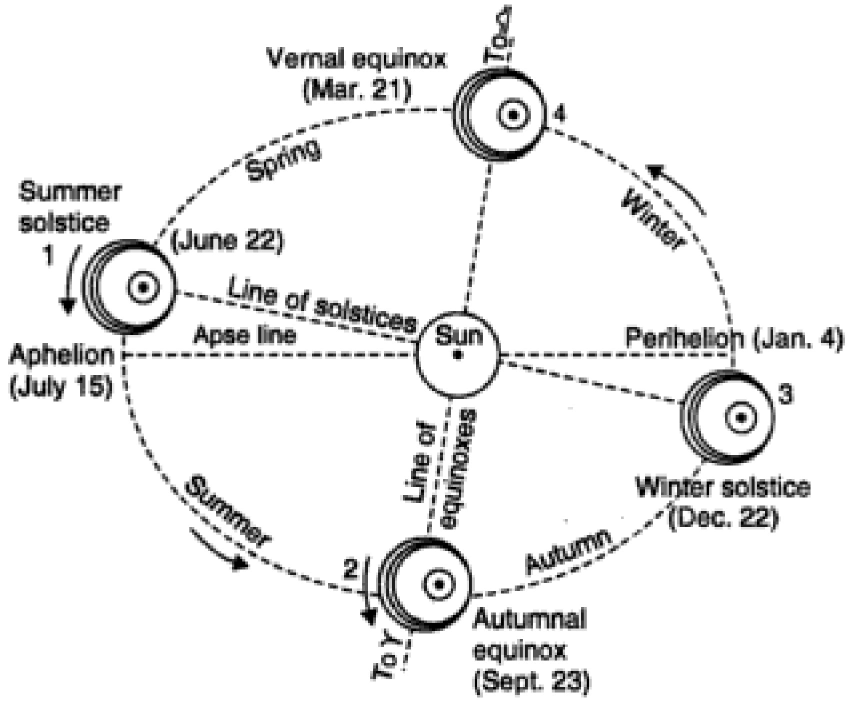
2. The sun. The sun is at a distance of 93,005,000 miles from the earth. The distance is only about $\frac{1}{250{,}000}$ of that of the nearest star. The diameter of the sun is about 109 times the diameter of the earth, and subtends and angle of 31′ 59″ at the centre of the earth. The mass of the sun is about 332,000 times that of the earth. The temperature at the centre of the sun is computed to be about 20 million degrees.

The sun has two *apparent* motions, one with respect to the earth from east to west, and the other with respect to the fixed stars in the celestial sphere. The former apparent path of the sun is in the plane which passes through the centre of the celestial sphere and intersects it in a great circle called the *ecliptic*. The apparent motion of the sun is along this great circle. The angle between the plane of equator and the ecliptic is called the *Obliquity of Ecliptic*, its value being 23° 27′. The obliquity of ecliptic changes with a mean annual diminution of 0′.47.

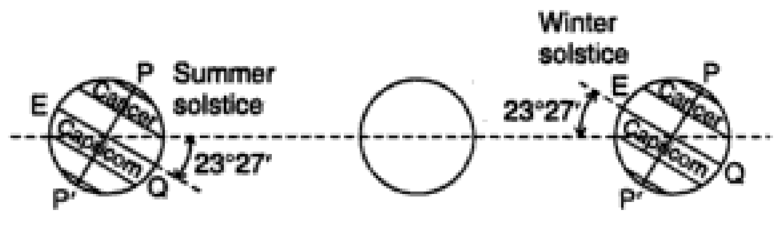
The points of the intersection of the ecliptic with the equator are called the equinoctial points, the declination of the sun being zero at these points. The *Vernal Equinox* or the First point of Aries (♈) is the point in which the sun's declination changes from south to north. The *Autumnal Equinox* or the *First point of Libra* (♎) is the point in which the sun's declination changes from north to south. The points at which sun's declinations are a maximum are called *solstices*. The point at which the north declination of sun is maximum is called the *summer solstice*, while the point at which the south declination of the sun is maximum is known as the *winter solstice*.

The Earth's Orbital Motion Round the Sun — The Seasons

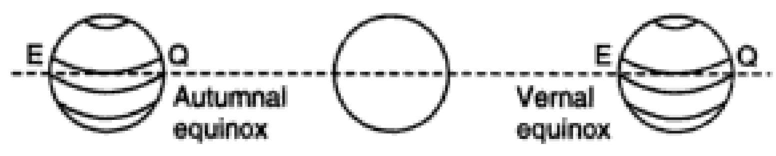
The earth moves *eastward* around the sun once in a year in a path that is *very nearly* a huge circle with a radius of about 93 millions of miles. *More accurately*, the path is described as an *ellipse*, one focus of the ellipse being occupied by the sun. The



(a) Plan of the earth's orbit



(b) Section of line of solstices



(c) Section of line of equinoxes

FIG. 1.28. EFFECT OF EARTH'S ANNUAL MOTION.

earth is thus at varying distances from the sun. The orbit lies (very nearly) in one *plane*. The apparent path of the sun is in the same plane. The plane passes through the centre of the celestial sphere and intersects it in a great circle called the *ecliptic*. The plane of the ecliptic is inclined at about 23° 27′ to that of the equator. Hence, the *axis of the earth is inclined to the plane of the ecliptic at an angle of 66° 33′, and remains practically parallel to itself throughout the year*. The inclination of the axis of the earth round its orbit causes variations of seasons. Fig. 1.28 shows the diagrammatic plan and sections of earth's orbit.

As previously mentioned, the earth's orbit is an ellipse with the sun at one of its foci. The earth is thus at varying distances from the sun. The earth is at a point nearest the sun (called the *perihelion* of the earth's orbit) on about January 4 and at a point farthest from the sun (called the *aphelion* of the earth's orbit) on about July 5. The earth's rate of angular movement around the sun is greatest at perihelion and least at aphelion.

In position I, the earth is in that part of the orbit where the northern end of the axis is pointed towards the sun. The sun appears to be farthest north on about June 22, and at this time the days are longest and nights are shortest. The summer begins in the northern hemisphere. This position of the earth is known as the summer *solstice*. In position 2 (Sept. 23), the sun is in the plane of the equator. The nights are equal everywhere. The instant at which this occurs is called the *Autumnal Equinox*. The axis of the earth is perpendicular to the line joining the earth and the sun. In position 3, the earth is in that part of the orbit where the northern end of axis is pointed away from the sun. The sun appears to be farthest south (Dec. 22) and at this time winter begins in the northern hemisphere. The days are shortest and nights are longest. The position of the earth is known as the *winter solstice*. In position 4 (March 21), the sun is again in the plane of the equator. The day and night are equal everywhere. The instant at which this occurs is called the *Vernal Equinox*. The line of the equinoxes is the intersection of the planes of the ecliptic and the equator, and is at right angles to the line of solstices.

Fig. 1.29 (*b*) shows the sun's apparent positions at different seasons. Let us study this in conjunction with Fig. 1.29 (*a*). Thus, on Fig. 1.29 (*a*), we shall trace the annual motion of the sun, while on Fig. 1.29 (*b*), we shall trace the apparent diurnal paths of the sun at different seasons. As is clear from Fig. 1.29 (*a*), the sun's declination changes daily as it progresses along the ecliptic. Due to the change in the declination, its apparent path of each day is different from that of the day before. *The apparent path thus ceases to be circular and all the daily paths taken together will give rise to one continous spiral curve*. However, for explanation purposes, we shall assume that throughout each day, the sun's declination is constant – retaining the same value it has at sunrise. On this assumption the sun's daily paths will consist of a series of parallels instead of a spiral as illustrated in Fig. 1.29 (*b*).

On 21st March, the sun is at ϒ [Fig. 1.29 (*a*)] and its declination is zero. The sun's daily path on this day will be along the equator rising at E and setting at W of the horizon. Its hour angle at E will be $EPZ = 90°$ when it rises. At W, it will again have an hour angle of 90° when it sets. *Thus, day* and *night will be of equal duration*. On that day, the meridian altitude SB of the sun is equal to the co-latitude. As the sun advances along the ecliptic, its declination increases. At the solstitial point M, it attains its maximum declination (23° 27′) on about June 22. The parallel A_1AA_2 represents sun's path on that day. The sun rises at A_1 when its hour angle is equal to A_1PZ which is greater than 90°. The sun sets at A_2 when its hour angle is greater than 90°. *The day is thus longest* on 22nd June. The meridian altitude SA also attains its maximum value. On Sept. 23, the declination of the sun is again zero, the sun's daily path is along the equator and the day and night are of equal length. As the motion of the sun continues along the ecliptic, its declination increases to the south of the equator. On December 22, its southern declination is maximum, C_1CC_2 represents sun's path on that day. The sun rises at C_1 when it has the hour angle C_1PZ which is evidently less than 90° and sets at C_2 when its hour angle is less than 90°. The day is thus shorter than the night.

It is colder in winter due to two main reasons :

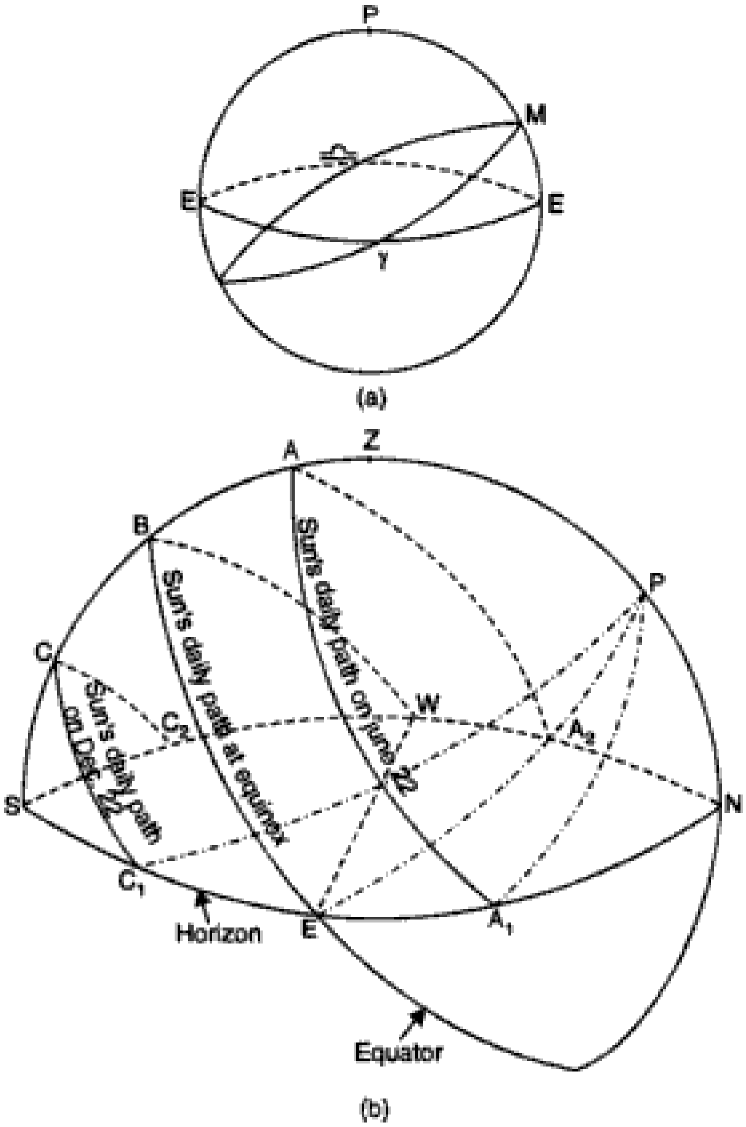


FIG. 1.29. SUN'S APPARENT POSITIONS AT DIFFERENT SEASONS.

(1) the days are shorter in winter.

(2) the rays of sunlight strike the surface of the ground more obliquely, thus weakening the heating power of the sun's rays.

Though the earth is nearer to the sun in winter it has very small effect in making the winter hotter. The amount of heat received from the sun depends upon the time it remains above the horizon, and also on the altitude it attains during the day.

MEASUREMENT OF TIME

Due to the intimate relationship with hour angle, right ascension and longitude, the knowledge of measurement of time is most essential. The measurement of time is based upon the apparent motion of heavenly bodies caused by earth's rotation on its axis. *Time* is the interval which lapses, between any two instants. In the subsequent pages, we shall use the following abbreviations.

G.M.T.	... Greenwhich Mean Time	G.M.M.	... Greenwich Mean Midnight
G.A.T.	... Greenwich Apparent Time	L.A.N.	... Local Apparent Noon

G.S.T.	... Greenwich Sidereal Time	L.M.M.	... Local Mean Midnight
L.M.T.	... Local Mean Time	L.Std.T.	... Local Standard Time
L.A.T.	... Local Apparent Time	N.A.	... Nautical Almanac
L.S.T.	... Local Sidereal Time	S.A.	... Star Almanac
G.M.N.	... Greenwich Mean Noon		

1.8. UNITS OF TIME

There are the following systems used for measuring time :

1. Sidereal Time
2. Solar Apparent Time
3. Mean Solar Time
4. Standard Time

1. Sidereal Time

Since the earth rotates on its axis from west to east, all heavenly bodies (*i.e.* the sun and the fixed stars) appear to revolve from east to west (*i.e.* in clock-wise direction) around the earth. Such motion of the heavenly bodies is known as *apparent* motion. We may consider the earth to turn on it axis with absolute regular speed. Due to this, the stars appear to complete one revolution round the celestial pole as centre in constant interval of time, and they cross the observer's meridian twice each day. For astronomical purposes the sidereal day is one of the principal units of time. *The sidereal day is the interval of time between two successive upper transits of the first point of Aries* (♈). It begins at the instant when the first point of Aries records $0^h\,0^m\,0^s$. At any other instant, the *sidereal time* will be the hour angle of ♈ reckoned westward from 0^h to 24^h. The sidereal day is divided into 24 hours, each hour subdivided into 60 minutes and each minute into 60 seconds. However, the position of the Vernal Equinox is not fixed. It has slow (and variable) westward motion caused by the *precessional* movement of the axis, the actual interval between two transits of the equinox differs about 0.01 second of time from the true time of one rotation.

Local Sidereal Time (L.S.T.) The local sidereal time is the time interval which has elapsed since the transit of the first point of Aries over the meridian of the place.

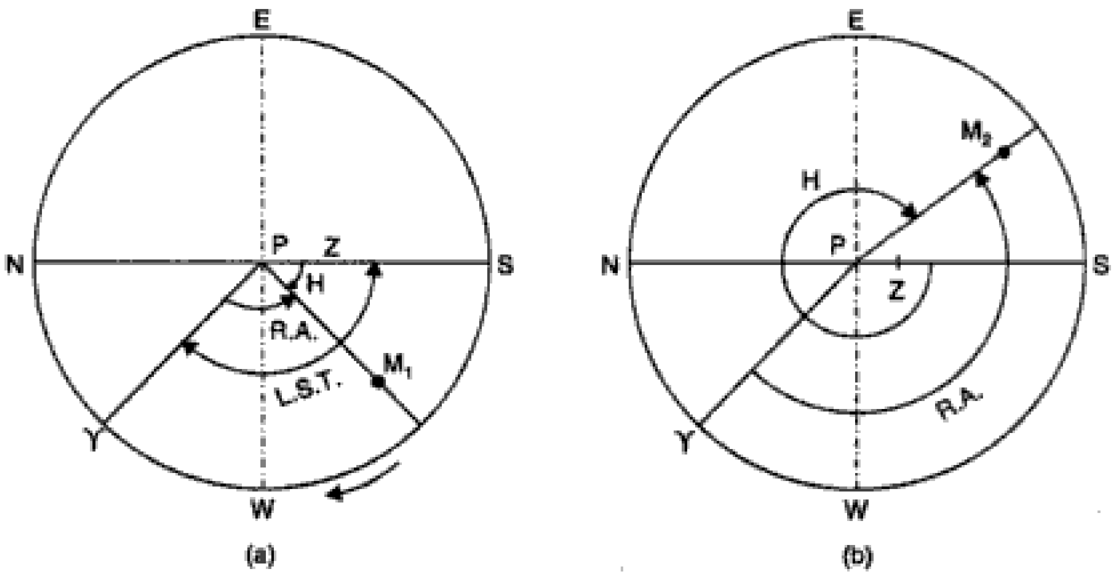


FIG. 1.30

It is, therefore, a measure of the angle through which the earth has rotated since the equinox was on the meridian. *The local sidereal time is, thus, equal to the right ascension of the observer's meridian.*

Since the sidereal time is the hour angle of the firsr point of Aries, *the hour angle of a star is the sidereal time that has elapsed since its transit.* In Fig 1.30, M_1 is the position of a star having SPM_1 (= H) as its hour angle measured *westward* and ΥPM_1 is its right ascension (R.A.) measured *eastward*. $SP\Upsilon$ is the hour angle of Υ and hence the local sidereal time.

Hence, we have $$SPM_1 + M_1P\Upsilon = SP\,\Upsilon$$

or **star's hour angle + star's right ascension = local sidereal time** ...(1)

If this sum is greater than 24 hours, deduct 24 hours, while if it is negative *add*, 24 hours.

In Fig. 1.30 (*b*), the star M_2 is in the other position. ΥPM_2 is its Right Ascension (eastward) and ZPM_2 is its hour angle (westward). Evidently,

$$ZPM_2 \text{ (exterior)} + \Upsilon PM_2 - 24^h = SP\Upsilon = \text{L . S . T .}$$

or **star's hour angle + star's right ascension** $- 24^h =$ L . S . T .

This supports the preposition proved with reference to Fig. 1.30 (*a*). The relationship is true for all positions of the star.

When the star is on the meridian, its hour angle is zero. Hence equation 1 reduces to

Star's right ascesion = local sidereal time at its transit.

A sidereal clock, therefore, records the right ascension of stars as they make their upper transits.

The hour angle and the right ascension are generally measured in *time* in preference to angular units. Since one complete rotation of celestial sphere through 360° occupies 24 hours, we have

$$24 \text{ hours} = 360° \quad ; \quad 1 \text{ hour} = 15°$$

The difference between the local sidereal times of two places is evidently equal to the difference in their longitudes.

2. Solar Apparent Time

Since a man regulates his time with the recurrence of light and darkness due to rising and setting of the sun, the sidereal division of time is not suited to the needs of every day life, for the purposes of which the sun is the most convenient time measurer. A *solar day* is the interval of time that elapes between two successive *lower* transits of the sun's centres over the meridian of the place. The lower transit is chosen in order that the date may change at mid-night. The solar time at any instant is the hour angle of the sun's centre reckoned westward from 0^h to 24^h. This is called the *apparent solar time,* and is the time indicated by a sun-dial. Unfortunately, the apparent solar day is not of constant length throughout the year but changes. Hence our modern clocks and chronometers cannot be used to give us the apparent solar time. The non-uniform length of the day is due to two reasons :

(1) The orbit of the earth round the sun is not circular but elliptical with sun at one of its foci. The distance of the earth from the sun is thus variable. In accordance with the law of gravitation, the apparent angular motion of the sun is not uniform – it moves faster when is nearer to the earth and slower when away. Due to this, the sun reaches the meridian sometimes earlier and sometimes later with the result that the days are of different lengths at different seasons.

(2) The apparent diurnal path of the sun lies in the ecliptic. Due to this, even though the eastward progress of the sun in the ecliptic were uniform, the time elapsing between the departure of a meridian from the sun and its return thereto would vary because of the obliquity of the ecliptic.

The sun changes its right ascension from 0^h to 24^h in one year, advancing eastward among the stars at the rate of about 1° a day. Due to this, the earth will have to turn nearly 361° about its axis to complete one solar day, which will consequently be about 4 minutes longer than a sidereal day. Both the obliquity of the ecliptic and the sun's unequal motion cause a *variable rate of increase* of the sun's right ascension. If the rate of change of the sun's right ascension were uniform, the solar day would be of constant length throughout the year.

3. Mean Solar Time

Since our modern clocks and chronometers cannot record the variable apparent solar time, a *fictitious sun* called the *mean sun* is imagined to move at a uniform rate along the equator. The motion of the mean sun is the average of that of the true sun in its right ascension. It is supposed to start from the vernal equinox at the same time as the true sun and to return the vernal equinox with the true sun. The *mean solar day* may be defined as the interval between successive transit of the mean sun. The mean solar day is the average of all the apparent solar days of the year. The mean sun has the constant rate of increase of right ascension which is the average rate of increase of the true sun's right ascension.

The *local mean noon* (L.M.N.) is the instant when the mean sun is on the meridian. The mean time at any other instant is given by the hour angle of the mean sun reckoned westward from 0 to 24 hours. For civil purposes, however, it is found more convenient to begin the day at midnight and complete it at the next midnight, dividing it into two periods of 12 hours each. Thus, the zero hour of the mean day is at the *local mean midnight* (L.M.M.). The *local mean time* (L.M.T.) is that reckoned from the local mean midnight. The difference between the local mean time between two places is evidently equal to the difference in the longitudes.

From Fig. 1.30 (*a*) if M_1 is the position of the sun, we have

Local sidereal time = R.A. of the sun + hour angle of the sun ...(1)

Similarly, Local sidereal time = R.A. of the mean sun + hour angle of the mean sun ...(2)

The hour angle of the sun is zero at its upper transit. Hence

Local sidereal time of apparent noon = R.A. of the sun ...(3)

Local sidereal time of mean noon = R.A. of the mean sun ...(4)

Again, since the hour angle of the sun (true or mean) is zero at its upper transit while the solar time (apparent or mean) is zero as its lower transit, we have

The apparent solar time = the hour angle of the sun + 12^h ...(5)

The mean solar time = the hour angle of mean sun + 12^h ...(6)

Thus, if the hour angle of the mean sun is 15° (1 hour) the mean time is 12 + 1 = 13 hours, which is the same thing as 1 o'clock mean time in the afternoon; if the hour angle of the mean sun is 195° (13 hours), the mean time is 12 + 13 = 25 hours, which is the same as 1 o'clock mean time after the midnight (*i.e.*, next. day).

The Equation of Time

The difference between the mean and the apparent solar time at any instant is known as the *equation of time.* Since the mean sun is entirely a fictitious body, there is no means to directly observe its progress. Formerly, the apparent time was determined by solar observations and was reduced to mean time by equation of time. Now-a-days, however, mean time is obtained more easily by first determining the sidereal time by steller observations and then converting it to mean time through the medium of wireless signals. *Due to this reason it is more convenient to regard the equation of time as the correction that must be applied to mean time to obtain apparent time.* The nautical almanac tabulates the value of the equation of time for every day in the year, in this sense (*i.e.* apparent – mean). Thus, we have

Equation of time = Apparent solar time – Mean solar time.

The equation of time is *positive* when the apparent solar time is *more* than the mean solar time ; to get the apparent solar time, the equation of time should then be added to mean solar time. For example, at 0^h G.M.T. on 15 October 1949, the equation of the time is + $13^m\ 12^s$. This means that the apparent time at 0^h mean time is $0^h\ 13^m\ 12^s$. In other words, the true sun is $13^m\ 12^s$ ahead of the mean sun. Similarly, the equation of time is negative when the apparent time is less than the mean time. For example, at 0^h G.M.T. on 18 Jan., 1949, the equation of time is – $10^m\ 47^s$. This means that the apparent time at 0^h mean time will be $23^h\ 49^m\ 13^s$ on January 17. In other words, the true sun is behind the mean sun at that time.

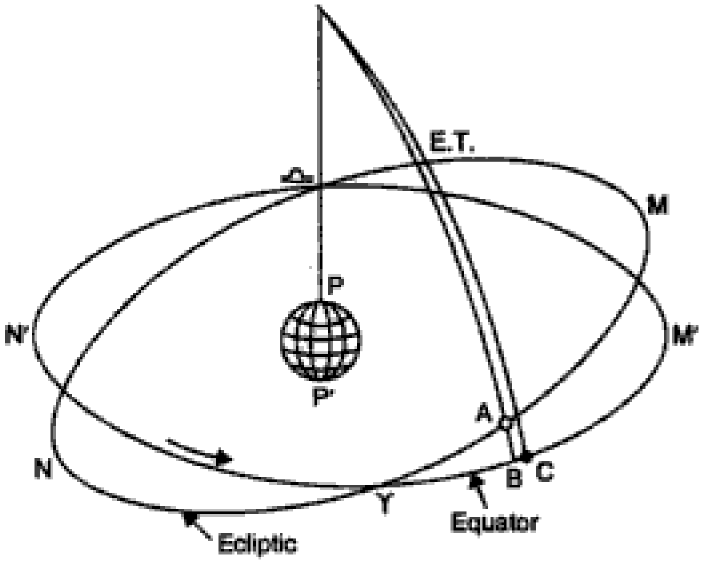
The value of the equation of time varies in magnitude throughout the year and its value is given in the Nautical Almanac at the instant of apparent midnight for the places on the meridian of Greenwich for each day of the year. For any other time it must be found by adding or subtracting the amount by which the equation has increased or diminished since midnight.

It is obvious that the equation of time is the value expressed in time, of the difference at any instant between the respective hour angles or right ascensions of the true and mean suns.

The amount of equation of the time and its variations are due to two reasons : (*1*) *obliquity of the ecliptic, and (2) ellipticity of the orbit.* We shall discuss both the effects separately and then combine them to get the equation of time.

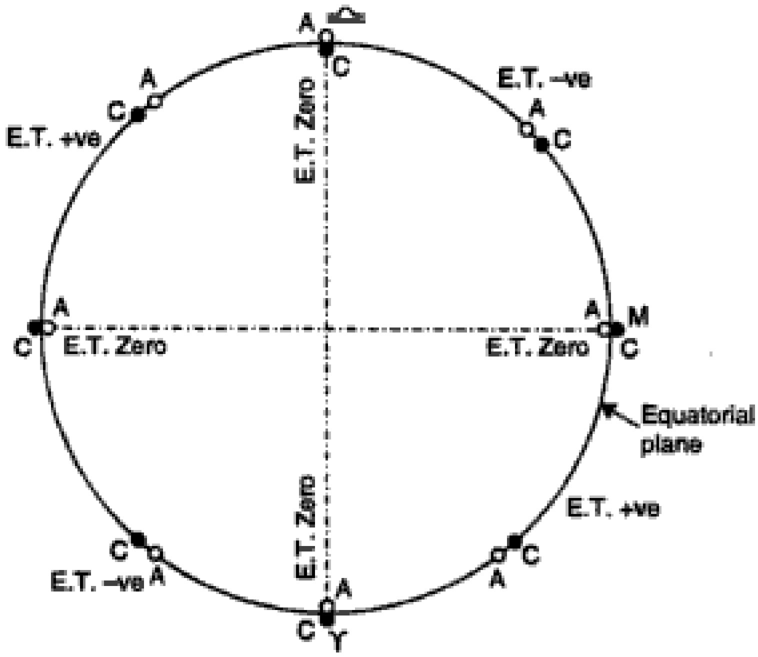
(1) Obliquity of the ecliptic

Neglecting the elliptic motion, let the true sun describe the ecliptic orbit ♈*M* ♎*N* with *uniform angular velocity*. Let the mean sun describe the equatorial orbit ♈*M'*♎*N'* with the same uniform angular velocity. Let both the suns start from ♈ at the same instant in the direction of the arrow. The earth axis *PP'* also turns in the same direction once in a day. When the true sun is at *A*, the mean sun will be at *C* such that ♈*A* = ♈*C*. If a declination circle is drawn through *A*, it will meet the equator in *B*. The difference between the declination circles of *A* and *C* will then be the equation of time. The points



(a)

A = True Sun ; C = Mean Sun



(b)

FIG. 1.31. EFFECT OF OBLIQUITY OF THE ECLIPTIC.

B and *C* will coincide only at equinoxes and solstices. Between the equinox to solstice, *C* will be in advance of *B*, and any given meridian will (as the earth rotates in the direction of the arrow) overtake first the true sun *A* and then the mean sun. *That is, apparent noon will precede mean noon and hence the equation of time will be additive.* Similarly, between the solstice to equinox, *C* will be behind *A* and the equation of time is subtractive. In Fig. 1.33, the curve *A-A* denotes the equation of time due to the obliquity of ecliptic. *It may be noted that the equation of time due to this reason vanishes four times in a year — at equinoxes and solstices.* Fig. 1.31 (*b*) shows the plan, on equatorial plane, of the positions of the true and mean sun at different parts of the year.

Thus, to conclude, *the equation of time due to obliquity of the ecliptic is due to the fact that the uniform motion along the ecliptic does not represent uniform motion in the right ascension.*

2. Ellipticity or the Eccentricity of the Orbit

Let us now neglect the obliquity of ecliptic so that the orbit of the sun is in the equator, and its apparent path is elliptical as shown in Fig. 1.32. At the Perihelion (December 31), the true sun (*A*) and the mean sun (*C*) start at the same instant. The mean sun (*C*) rotates with uniform rate while the true sun (*A*) moves with the greater angular velocity since it is nearer the earth at Perihelion. Due to this, the true sun precedes the mean sun. Now, since the earth rotates from west to east (*i.e.*, in the same direction as that of the motion of the sun along its orbit indicated by the arrow), any meridian at a place on it will overtake the mean sun before the true sun. The mean noon will thus occur before the apparent noon, the mean time will exceed the apparent time and hence the *equation of time will be negative.* After 90° from the Perihelion, the true sun, though ahead of the mean sun, will have decrease in its angular velocity so that the distance between the sun and the mean sun goes on decreasing. At the Aphelion (July 1), both the suns meet and the equation of time becomes zero. Between December 31 to July 1, equation of time thus remains negative. After July 1, the true sun has lesser angular velocity than the uniform velocity of the mean sun, and the mean sun precedes the true sun. The apparent noon will thus occur earlier than the mean noon at a particular meridian, the apparent time exceeds the mean time, and the *equation of time becomes positive.* After about 90° (October 1) from the Aphelion, the gap between the mean sun and true sun gradually reduces due to gradual increase in the angular velocity of the true sun, till both the suns reach perihelion at the same

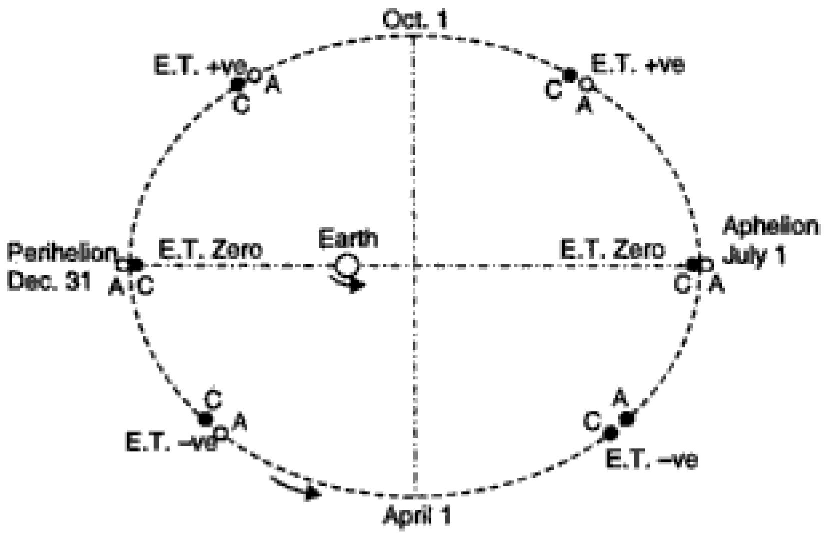


FIG.1.32. EFFECT OF ELLIPTICITY OF THE ORBIT.

instant. The equation of time is thus positive from July 1 to December 31. In Fig. 1.33, the curve *B-B* denotes the equation of time due to ellipticity of the orbit.

The Final Curve for Equation of Time

In Fig. 1.33, the curve *C-C* shows the final equation of time obtained by combining the curves *A-A* and *B-B*. It will be seen that the equation of time vanishes four times a year, on or about April 16, June 14, September 2, and December 25. From December 25 till April 16, it is negative having a maximum value of about $14^m 20^s$ on February 12. From April 16 to June 14 it is positive, having its maximum value of about $3^m 44^s$ on May 15. From June 14 to September 2, it is again negative with a maximum value of $6^m 24^s$ on July 27. Between September 2 and December 25, it is again positive, attaining its greatest positive value for the year 1951, about $16^m 23^s$ on November 4.

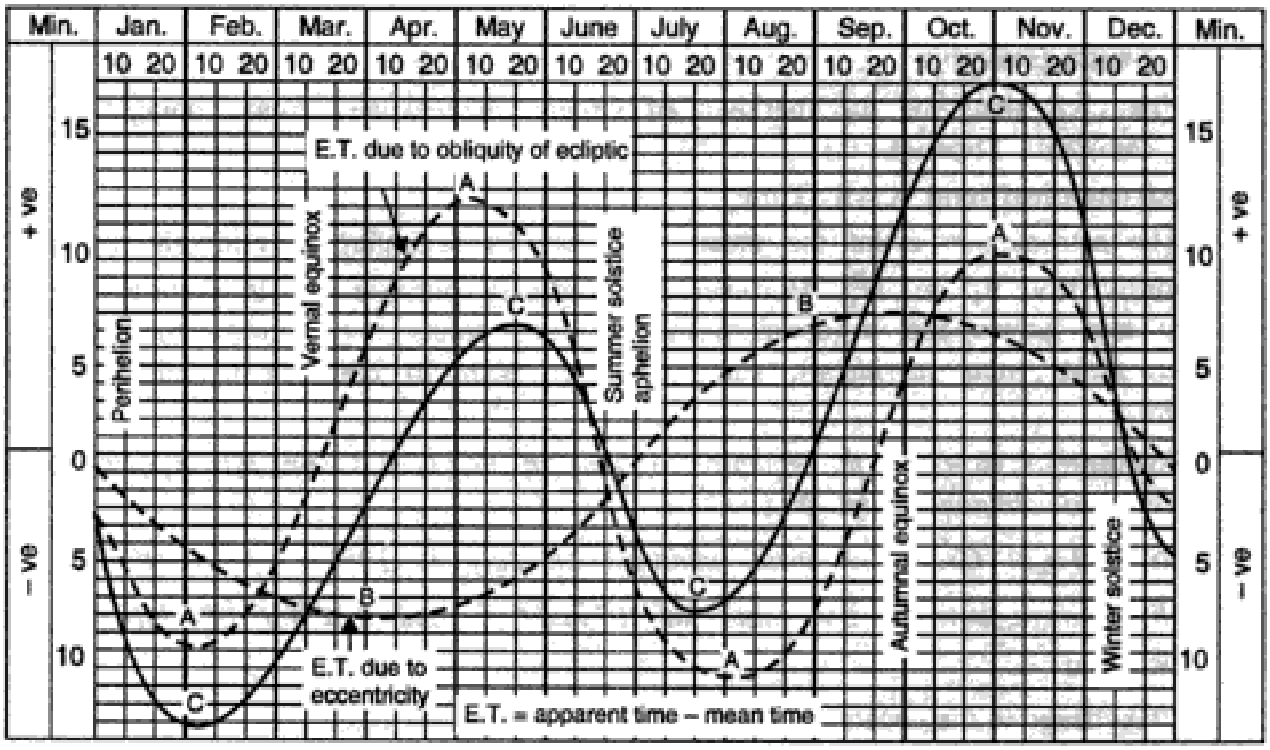


FIG. 1.33. THE EQUATION OF TIME : THE CORRECTION TO BE ADDED TO THE MEAN TIME TO OBTAIN APPARENT TIME.

4. Standard Time

We have seen that the local mean time at a particular place is reckoned from the lower transit of the mean sun. Thus, at different meridians there will be different local mean times. In order to avoid confusion arising from the use of different local mean time it is necessary to adopt the mean times on a particular meridian as the standard time for the whole of the country. Such a *standard meridian* lies an exact number of hours from Greenwich. The mean time associated with the standard meridian is known as the *standard time*. The difference between standard time and local mean time at any place is that due to the difference of longitude between the given place and the standard meridian used. For places east of the standard meridian, local mean time is later (or greater) than standard time, and for places to the west, the local time is earlier (or lesser). *The following are the standard meridians of the some of the countries* :

	Country	*Longitude of standard meridian*	
		Degrees	*Times* Hrs. Mts.
1.	Great Britain, Belgium, Spain	0°	0 – 00
2.	Germany, Switzerland	15° *E*	1 – 00
3.	India	$82\frac{1}{2}°$ *E*	5 – 30
4.	Western Australia	120° *E*	8 – 00
5.	New Zealand	180° *E*	12 – 00
6.	Central Zones of U.S.A.	90° *W*	6 – 00
7.	British Columbia	120° *W*	8 – 00

The civil time for the meridian of Greenwich reckoned from midnight, is known as the *Universal Time* (U.T.)

The Astronomical and Civil Time

The astronomers count the mean solar day as beginning at midnight and divide it continuously from 0^h to 24^h. However, for ordinary purposes, it is preferable to divide the day into halves and to count from two zero points : (1) From midnight to noon is called A.M. (*ante meridiem*), and (2) from noon to midnight is called P.M. (*post meridiem*).

Example 1.16. *Find the equation of time at 12^h G.M.T. on July 1, 1951 from the following data obtained from N.A.*

(*a*) *E.T. at Greenwich mean midnight on July 1, 1951 = $-3^m\ 28.41^s$.*

(*b*) *Change between the value for 0^h July 1, and that for 0^h July 2 = -11.82^s.*

Solution

The change in the equation of time for 24 hours = -11.82^s

$\therefore$ Change in equation of time for $12^h = \frac{11.82}{24} \times 12 = -5.91^s$

$\therefore$ E.T. at 12^h G.M.T. $= -3^m\ 28.41^s - 5.91^s =$ **$-3^m\ 34.32^s$**.

Example 1.17. *Find the G.A.T. on February 16, 1951, when the G.M.T. is $10^h\ 30^m$ A.M. Given E.T. at G.M.N. on Feb. 16, 1951 = $-14^m\ 10^s$ increasing at the rate of 1 second per hour.*

Solution. E.T. at G.M.N. = $-14^m\ 10^s$. Since the E.T. is increasing after G.M.N., its value will be less than $14^m\ 10^s$ before noon.

Now, $10^h\ 30^m$ A.M. occurs $1^h\ 30^m$ before the noon.

Change in E.T. in $1^h\ 30^m$ = 1 sec × 1.5 = 1.5 seconds.

$\therefore$ Equation of time at $10^h\ 30^m$ A.M. $= -[14^m\ 10^s - 1.5^s] = -14^m\ 8.5^s$

Now G.A.T. = G.M.T. + E.T. $= 10^h\ 30^m - 14^m\ 8.5^s = \mathbf{10^h\ 15^m\ 51.5^s}$.

1.9. INTERCONVERSION OF TIME

1.9.1. RELATION BETWEEN DEGREES AND HOURS OF TIME

The degrees may be converted into hours and *vice versa* by the following relation:

$$360° = 24 \text{ hours.}$$

$15° = 1$ h	
$1° = 4$ m	1 h = 15°
$15' = 1$ m	1 m = 15′
$1' = 4$ s	1 s = 15″
$15'' = 1$ s	

Example 1.18. *Express the following angles in hours, minutes and seconds : (a) 50° 12′ 48″, (b) 8° 18′ 6″, (c) 258° 36′ 30″.*

Solution.

(a)
$$50° = \frac{50}{15}\text{ h} = 3^h\ 20^m\ 0^s$$
$$12' = \frac{12}{15}\text{ m} = 0^h\ 0^m\ 48^s$$
$$48'' = \frac{48}{15}\text{ s} = 0^h\ 0^m\ 3.2^s$$
$$\text{Total} = 3^h\ 20^m\ 51.2^s$$

(b)
$$8° = \frac{8}{15}\ h = 0^h\ 32^m\ 0^s$$
$$18' = \frac{18}{15}\text{ m} = 0^h\ 1^m\ 12^s$$
$$6'' = \frac{6}{15}\text{ s} = 0^h\ 0^m\ 0.4^s$$
$$\text{Total} = 0^h\ 33^m\ 12.4^s$$

(c)
$$258° = \frac{258}{15}\ h = 17^h\ 12^m\ 0^s$$
$$36' = \frac{36}{15}\text{ m} = 0^h\ 2^m\ 24^s$$
$$30'' = \frac{30}{15}\text{ s} = 0^h\ 0^m\ 2^s$$
$$\text{Total} = 17^h\ 14^m\ 26^s$$

Example 1.19. *Express the following hours etc. into degrees, minutes and seconds: (a) $4^h\ 34^m\ 13^s$, (b) $18^h\ 11^m\ 38^s$.*

Solution.

(a)
$$4^h = 4 \times 15° = 60°\ 0'\ 0''$$
$$34^m = 34 \times 15' = 8°\ 30'\ 15°$$
$$13^s = 13 \times 15'' = 0°\ 3'\ 15''$$
$$\text{Total} = 68°\ 33'\ 15''$$

(b)
$$18^h = 18 \times 15° = 270°\ 0'\ 0''$$
$$11^m = 11 \times 15' = 2°\ 45'\ 0''$$
$$38^s = 38 \times 15'' = 0°\ 9'\ 30''$$
$$\text{Total} = 272°\ 54'\ 30''$$

1.9.2. CONVERSION OF LOCAL TIME TO STANDARD TIME AND VICE VERSA

The difference between the standard time and the local mean time at a place is equal to the difference of longitudes between the place and the standard meridian.

If the meridian of the place is situated *east* of the standard meridian, the sun, while moving apparently from east to west, will transit the meridian of the place earlier than the standard meridian. Hence the local time will be greater than the standard time. Similarly, if the meridian of the place is to the *west* of the standard meridian, the sun will transit the standard meridian earlier than the meridian of the place and hence the local time will be lesser than the standard time. Thus, we have

L.M.T.= Standard M.T. ± Difference in the longitudes $\left(\frac{E}{W}\right)$ ...(1)

L.A.T.= Standard A.T. ± Difference in the longitudes $\left(\frac{E}{W}\right)$ ...(2)

L.S.T.= Standard S.T. ± Difference in the longitudes $\left(\frac{E}{W}\right)$ ...(3)

Use (+) sign if the meridian of place is to the east of the standard meridian, and (−) sign if it to the west of the standard meridian.

If the local time is to be found from the given Greenwich time, we have

$$\textit{L.M.T.} = \textit{G.M.T.} \pm \textit{Longitude of the place} \left(\frac{E}{W}\right).$$

Example 1.20. *The standard time meridian in India is 82° 30′ E. If the standard time at any instant is 20 hours 24 minutes 6 seconds, find the local mean time for two places having longitudes (a) 20° E, (b) 20° W.*

Solution

(*a*) The longitude of the place = 20° *E*

Longitude of the standard meridian = 82° 30′ *E*

∴ Difference in the longitudes = 82° 30′ − 20° = 62° 30′, *the place being to the west of the standard meridian.*

$$\text{Now } 62° \text{ of longitude} = \frac{62°}{15}\text{ h} = 4^h\ 8^m\ 0^s$$

$$30' \text{ of longitude} = \frac{30}{15}\text{ m} = 0^h\ 2^m\ 0^s$$

$$\text{Total} = 4^h\ 10^m\ 0^s$$

Now L.M.T. = Standard time − Difference in longitude (*W*)

$$= 20^h\ 24^m\ 6^s - 4^h\ 10^m\ 0^s = 16^h\ 14^m\ 6^s \text{ past midnight} = 4^h\ 14^m\ 6^s \text{ P.M.}$$

(*b*) Longitude of the place = 20° *W*

Longitude of the standard meridian = 82° 30′ *E*.

∴ Difference in the longitude = 20° + 82° 30′ = 102° 30′, the meridian of the place being to the west to the standard meridian.

$$\text{Now } 102° \text{ of longitude} = \frac{102}{15}\text{ h} = 6^h\ 48^m\ 0^s$$

$$30' \text{ of longitude} = \frac{30}{15}\text{ m} = 0^h\ 2^m\ 0^s$$

$$\text{Total} = 6^h\ 50^m\ 0^s$$

$$\text{Standard time} = 20^h\ 24^m\ 6^s$$

$$\text{Subtract the difference in longitude} = 6^h\ 50^m\ 0^s$$

∴ Local mean time = $13^h\ 34^m\ 6^s$ past mid-night = $1^h\ 34^m\ 6^s$ P. M.

Example 1.21. *Find the G.M.T. corresponding to the following L.M.T.*

(*a*) $9^h\ 40^m\ 12^s$ *A.M. at a place in longitude* 42° 36′ *W.*

(*b*) $4^h\ 32^m\ 10^s$ *A.M. at a place in longitude* 56° 32′ *E.*

Solution.

(*a*) Longitude of the place is 42° 36′ *W*

Now $$42° = \frac{42}{15}\text{ h} = 2^h\ 48^m\ 0^s$$

$$36' = \frac{36}{15}\text{ m} = 0^h\ 2^m\ 24^s$$

$$\text{Total} = 2^h\ 50^m\ 24^s$$

Now since the place is to the west of Greenwich, the Greenwich time will be more.

$\therefore$ G.M.T. = L.M.T. + Longitude (*W*)

L.M.T. = $9^h\ 40^m\ 12^s$ (A.M.)

Add the longitude = $2^h\ 50^m\ 24^s$

$\therefore$ G.M.T. = $12^h\ 30^m\ 36^s$

or G.M.T. = $0^h\ 30^m\ 36^s$ (P.M.)

(*b*) Longitude of the place = 56° 32′ *E*

Now $$56° = \frac{56}{15}\text{ h} = 3^h\ 44^m\ 0^s$$

$$32' = \frac{32}{15}\text{ m} = 0^h\ 2^m\ 8^s$$

$$\text{Total} = 3^h\ 46^m\ 8^s$$

Since the place is to the east of Greenwich, the Greenwich time will be lesser than the local time.

$\therefore$ G.M.T. = L.M.T. – Longitude (*E*)

L.M.T. = $4^h\ 32^m\ 10^s$ (A.M.)

Subtract longitude = $3^h\ 46^m\ 8^s$

G.M.T. = $0^h\ 46^m\ 2^s$ (A.M.)

Example 1.22. *Given the Greenwich civil time (G.C.T.) as $6^h\ 40^m\ 12^s$ P.M. on July 2, 1965, find the L.M.T. at the places having the longitudes* (*a*) *72° 30′ E,* (*b*) *72° 30′ W, and* (*c*) *110° 32′ 30″ E.*

Solution

(*a*) Longitude of the place = 72° 30′ *E*

Now $72° = \frac{72}{14}\text{ h} = 4^h\ 48^m\ 0^s$

$30' = \frac{30}{15}\text{ m} = 0^h\ 2^m\ 0^s$

Total = $4^h\ 50^m\ 0^s$

Since the place is to the east of Greenwich, the local mean time will be more than the standard time.

Now G.M.T. = $18^h\ 40^m\ 12^s$ Past mid-night

Add longitude = $4^h\ 50^m\ 0^s$

∴ L.M.T. = $23^h\ 30^m\ 12^s$

= $11^h\ 30^m\ 12^s$. P.M. on July 2.

(*b*) Longitude of the place = 72° 30′ *W* = $4^h\ 50^m$ of time

Since the place is to the west of the Greenwich, the local mean time will be lesser than the standard time.

Now G.M.T.= $6^h\ 40^m\ 12^s$ P.M. = $18^h\ 40^m\ 12^s$ Past mid-night

Subtract longitude = $4^h\ 50^m\ 0^s$

∴ L.M.T. = $13^h\ 40^m\ 12^s$= $1^h\ 40^m\ 12^s$ P.M. on July 2.

(*c*) Longitude of the place = 110° 32′ 30″ *E*

Now $110° = \frac{110}{15}\text{ h} = 7^h\ 20^m\ 0^s$

$32' = \frac{32}{15}\text{ m} = 0^h\ 2^m\ 8^s$

$30'' = \frac{30}{15}\text{ s} = 0^h\ 0^m\ 2^s$

Total = $7^h\ 22^m\ 10^s$

Since the longitude is to the east to Greenwich, the local mean time will be more than the G.M.T.

G.M.T. = $18^h\ 40^m\ 12^s$ Past mid-night

Add longitude = $7^h\ 22^m\ 10^s$

∴ L.M.T. = $26^h\ 02^m\ 22^s$

= $2^h\ 02^m\ 22^s$ on July 3

∴ L.M.T. = $2^h\ 02^m\ 22^s$ A.M. on July 3.

Example 1.23. *Find the local apparent time of an observation at a place in longitude $60° 18' E$, corresponding to local mean time $10^h 20^m 30^s$, the equation of time at G.M.N. being $5^m 4.35^s$ additive to the mean time, and decreasing at the rate of 0.32^s per hour.*

Solution.

The equation of time is given at G.M.N. In order to calculate the E.T. at the given L.M.T., we will have to first calculate the corresponding G.M.T. and convert it to G.A.T. Knowing G.A.T., L.A.T. can be calculated.

Longitude of place $= 60° 18' E$	$= 4^h 1^m 12^s E$
L.M.T. of observation	$= 10^h 20^m 30^s$
Subtract longitude in time	$= 4^h 1^m 12^s$
∴ G.M.T. of observation	$= 6^h 19^m 18^s$

∴ Mean time interval before G.M.N.$= 12^h - (6^h 19^m 18^s) = 5^h 40^m 42^s = 5.68$ hours

Since the E.T. decreases at the rate of 0.32^s per hour after G.M.N., it will have increased value for any time instant before G.M.N.

∴ Increase for 5.68 hours @ 0.32^s per hour $= (5.68 \times 0.32)^s = 1.82^s$

∴ E.T. at G.M.N.	$= 5^m 4.35^s$
Add increase	$= 0^m 1.82^s$
∴ E.T. at observation	$= 5^m 6.17^s$
Now G.A.T.	= G.M.T. + E.T.
G.M.T. of observation	$= 6^h 19^m 18^s$
Add E.T.	$= 0^h 5^m 6.17^s$
∴ G.M.T. of observation	$= 6^h 24^m 24.17^s$
Add longitude in time	$= 4^h 1^m 12^s$
∴ L.A.T. of observation	$= 10^h 25^m 36.17^s$

Example 1.24. *Find the L.M.T. of observation at a place from the following data:*

L.A.T. of observation $= 15^h 12^m 40^s$

E.T. at G.M.N. $= 5^m 10.65^s$ additive to apparent time and increasing at 0.22^s per hour.

Longitude of the place $= 20° 30' W$.

Solution.

Longitude of the place $= 20° 30' W = 1^h 22^m 0^s W$

L.A.T. of observation $= 15^h 12^m 40^s$

Add longitude in time $= 1^h 22^m 0^s$

∴ G.A.T. of observation $= 16^h 34^m 40^s$

E.T. at G.M.N. = $5^m\ 10.65^s$

Time interval after G.M.N. = $4^h\ 34^m\ 40^s = 4.578^h$

(The above time interval is approximate, since it has been calculated by subtracting G.M.N. from the G.A.T. while actually the G.M.N. should be subtracted from G.M.T. which is not known at present).

$\therefore$ Increase for 4.578^h @ 0.22^s per hour = $(4.578 \times 0.22)^s = 1.01^s$

$\therefore$ E.T. at observation = $5^m\ 10.65^s + 1.01^s = 5^m\ 11.66^s$

Now G.A.T. of observation = $16^h\ 34^m\ 40^s$

Add E.T. = $0^h\ 5^m\ 11.66^s$

G.M.T. of observation = $16^h\ 39^m\ 51.66^s$

Deduct longitude in time = $1^h\ 22^m\ 0^s$

L.M.T. of observation = $15^h\ 17^m\ 51.66^s$

1.9.3. CONVERSION OF MEAN TIME INTERVAL TO SIDEREAL TIME INTERVAL AND VICE VERSA

The tropical year: A year is the period of earth's revolution about the sun, from some determinate position back again to the same position. The reference point chosen for the use of man is the first point of Aries (♈). The year so chosen is the *tropical year* or *the solar year*. A *Sidereal year* is the time taken by the earth in making one complete revolution round the sun with reference to a fixed star.

The first point of Aries has a retrograde motion westwards through an arc of 50.22" per year. The retrograde motion of the first point of Aries is due to the attraction of the moon and the sun which causes the direction of the axis of the earth alter its position very gradually in such a way that earth arrives at the position of the vernal equinox a little earlier each year. This phenomenon is known as the *Precession of Equinoxes*. Due to the precession of Equinoxes, therefore, the earth does not revolve by 360° round the sun from the positions of vernal equinox to vernal equinox, but revolves through (360° – 50".22)

The sun advances among the stars in the same direction — west to east — as the earth revolves about the axis. Any given meridian, therefore, crosses the first point of Aries exactly once oftener than it does the sun, in the course of a tropical year. According to Bassel, there are 365.2422 mean solar days in a tropical year, and in the same period there are 366.2422 sidereal days.

Thus, we have the relation

365.2422 mean solar day = 366.2422 sidereal days

or $$1 \text{ mean solar day} = 1 + \frac{1}{365.2422} \text{ sidereal days} = 24^h\ 3^m\ 56.56^s \text{ sidereal time} \quad ...(I)$$

Thus, the mean solar day is $3^m\ 56.56^s$ longer than the sidereal day.

Hence 1 hour mean solar time = $1^h + 9.8565^s$ sidereal time

1 minute mean solar time = $1^m + 0.1642^s$ sidereal time

1 second mean solar time = $1^s + 0.0027^s$ sidereal time

Thus, to convert the mean solar time to the sidereal time, we will have to add a correction of 9.8565^s per hour of mean time. This correction is called the *acceleration*.

To get the concept how a mean solar day is of a longer time interval than the sidereal time, let us study Fig. 1.34.

Let C be the centre of the earth and O be the position of the observer at noon of its meridian at the date of the equinox. Let C_1 be the position of the earth's centre the next day. After the earth makes one complete rotation (with reference to ♈), the observer will be at O_1 and *the sidereal time will be the same as it was the day before when he was at O.* However, the solar day is the time interval between two successive transits of the centre of the sun over the meridian. *In order that the sun transits the observer's meridian, the earth will have to revolve additionally by the arc O_1O'.* The time taken for this additional rotation is 3 minutes 56.66 seconds.

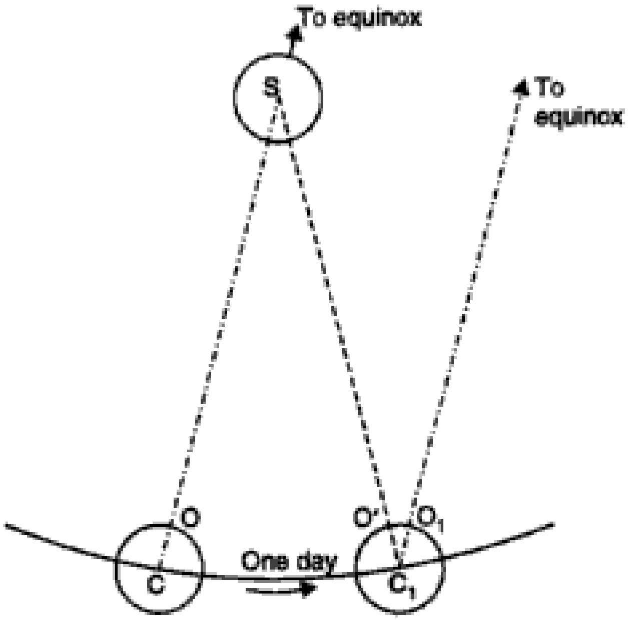


FIG. 1.34

Thus, we have

366.2422 sidereal days= 365.2422 solar days.

To convert sidereal time into mean time, we have

$$1 \text{ sidereal day} = \frac{365.2422}{366.2422} \text{ mean solar day} = 1 - \frac{1}{366.2422} \text{ mean solar day}$$

or 1 sidereal day = $23^h\, 56^m\, 4.09^s$ mean solar time

∴ 1^h sidereal time = $1^h - 9.8296^s$ mean solar time

1^m sidereal time = $1^m - 0.1638^s$ mean solar time

1^s sidereal time = $1^s - 0.0027^s$ mean solar time

Thus, *to convert 1 hour sidereal time to the mean solar time, a correction of 9.8296 seconds per hour will have to be subtracted from the sidereal time.* This correction is called the *retardation*.

Example 1.25. *Convert 4 hours 20 minutes 30 seconds of mean solar time into equivalent interval of sidereal time.*

Solution.

To convert the mean solar time to the sidereal time, we will have to first calculate the acceleration at the rate of 9.8565^s per hour of mean time.

Thus 4 hours × 9.8565 = 39.426 seconds
20 min. × 0.1642 = 3.284 seconds
30 sec. × 0.0027 = 0.081 seconds

Total = 42.791 seconds

Mean time interval = $4^h\ 20^m\ 30^s$

Add acceleration = 42.791^s

∴ Sidereal time interval = $4^s\ 21^m\ 12.791^s$,

Example 1.26. *Convert 8 hours 40 minutes 50 seconds sidereal time interval into corresponding mean time interval.*

Solution.

To convert the sidereal time to mean solar time, we will have to first calculate the retardation at the rate of 9.8296^s per sidereal hour.

Thus, 8 hours × 9.8296 = 78.637 seconds
40 min. × 0.1638 = 6.552 seconds
50 sec. × 0.0027 = 0.135 seconds

Total = 85.324 seconds = $1^m\ 25.324^s$

Sidereal time interval = $8^h\ 40^m\ 50^s$

Subtract retardation = $1^m\ 25.324^s$

∴ Mean time interval = $8^h\ 39^m\ 24.676^s$.

1.9.4. GIVEN GREENWICH SIDEREAL TIME AT GREENWICH MEAN MIDNIGHT, TO FIND THE LOCAL SIDEREAL TIME AT LOCAL MEAN MIDNIGHT AT ANY OTHER PLACE ON THE SAME DATE

(*i.e. Given G.S.T. at G.M.M., to find L.S.T. at L.M.M.*)

From the discussions of the previous article, it is clear that if we have two clocks, one set to keep sidereal time and other to keep mean time, *the sidereal clock will complete its day in a shorter period than the other*. Since 24 hours of solar time are equal to $24^h\ 3^m\ 56.56^s$ of sidereal time, the sidereal clock will be continually gaining over the mean clock at the rate of 9.8565 seconds for every mean solar hour. *The G.S.T. at G.M.M. is then the difference between the sidereal clock and the mean clock at that instant*. The L.S.T. at L.M.M. will then be the difference between these two clocks at the meridian under consideration at the instant.

If the place is to the west of Greenwich, it will have its L.M.M. certain hours after the G.M.M. depending upon the longitude of the meridian. Naturally, by the time there is L.M.M., the sidereal clock will have gained over the mean clock at the rate 9.8565^s for every hour of longitude. *Hence the L.S.T. at L.M.M. will be greater than the G.S.T. at G.M.M. by an amount calculated at* 9.8565^s *per hour* of western longitude.

Similarly, if the place is to the east of Greenwich meridian, the L.M.M. will occur few hours earlier than the G.M.M., depending upon the longitude of the place. The L.S.T. at L.M.M. will then be lesser than G.S.T. at G.M.M. at the rate of 9.8565 seconds per hour of longitude. Thus, we have the relation:

L.S.T. at L.M.M.= G.S.T. at G.M.M. ± 9.8565ˢ per hour of longitude $\left(\frac{W}{E}\right)$

Use (+) sign if the longitude is to the west and (−) sign if it is to the east. Similarly,

L.S.T. at L.M.N. = G.S.T. at G.M.N. ± 9.8565ˢ per hour of longitude $\left(\frac{W}{E}\right)$

Example 1.27. *If the G.S.T. of G.M.N. on a certain day is* $16^h\ 30^m\ 12^s$, *what will be the L.S.T. of L.M.M. at a place in longitude :*

(a) 160° 30′ 30″ W of Greenwich (b) 160° 30′ 30″ E of Greenwich.

Solution

(*a*) As the longitude is to the west, the event of which the time is required occurs *later* than G.M.M. by an amount corresponding to the longitude.

		h	m	s
Now	$160° = \frac{160}{15}$ h =	10	40	0
	$30' = \frac{30}{15}$ m =	0	2	0
	$30'' = \frac{30}{15}$ s =	0	0	2
∴ Difference of longitude in terms of time.	=	10	42	2

Thus, L.M.M. occurs $10^h\ 42^m\ 2^s$ mean time later than G.M.M. *In the interval between L.M.M. and G.M.M., the ♈ will gain on the mean sun at 9.8565 seconds per hour.*

∴ Gain in sidereal time :

$$10^h \times 9.8565 = 98.565 \text{ seconds}$$
$$42^m \times 0.1642 = 6.896 \text{ seconds}$$
$$2^s \times 0.0027 = 0.005 \text{ second}$$

$$\text{Total gain} = 105.466^s = 1^m\ 45.466^s$$

∴ L.S.T. at L.M.N. = G.S.T. of G.M.N. + Gain

$$= 16^h\ 30^m\ 12^s + 1^m\ 45.466^s = 16^h\ 31^m\ 57.466^s.$$

(*b*) Since the longitude is to the east, the L.M.N. occurs $10^h\ 42^m\ 2^s$ mean time earlier than the G.M.M.

Hence L.S.T. at L.M.M. = G.S.T. of G.M.M. − 9.8565^s per hour of eastern longitude

$$= 16^h\ 30^m\ 12^s - 1^m\ 45.466^s = 16^h\ 28^m\ 26.534^s.$$

1.9.5. GIVEN THE LOCAL MEAN TIME AT ANY INSTANT, TO DETERMINE THE LOCAL SIDEREAL TIME

At a given meridian, let us have two clocks, one showing the mean time and the other the sidereal time. At the local mean mid-night, the mean time in the mean clock will be zero. At that time (*i.e.* L.M.M.) the L.S.T. can easily be computed if the G.S.T. at G.M.M. is known. If the place is to the west of the Greenwich, the sidereal clock will have a gain over the mean time at L.M.M. at the rate of 9.8565 seconds per hour, as discussed in § 1.9.4 above. At any other instant at the given meridian, the mean clock will show the time that has elapsed since the lower transit of the sun over the meridian. This mean time interval can be easily converted into sidereal time interval as discussed in § 1.9.3 above. Thus, the L.S.T. at L.M.T. will be equal to L.S.T. at L.M.M. plus the sidereal time interval. Hence *the rules for finding the L.S.T. at L.M.T. are:*

(*a*) From the given G.S.T. at G.M.M., calculate L.S.T. at L.M.M.

(*b*) Convert the given L.M.T. (or mean time interval) into sidereal time interval since L.M.M.

(*c*) L.S.T. at L.M.T. = L.S.T. at L.M.M. + S.I. from L.M.M.

Example 1.28. *Find the L.S.T. at place in longitude 85° 20′ E at* $6^h\ 30^m$ *P.M., G.S.T. at G.M.N. being* $6^h\ 32^m\ 12^s$.

Solution.

Longitude = 85° 20′ *E*

	h	m	s
$85° = \frac{85}{15}$ h =	5	40	0
$20' = \frac{20}{15}$ m =	0	1	20
∴ Longitude in hours =	5	41	20 *E*

Since the place is to the east of Greenwich, let us calculate the loss of sidereal time for $5^h\ 41^m\ 20^s$ of longitude.

$$5^h \times 9.8565^s = 49.283 \text{ seconds}$$
$$41^m \times 0.1642^s = 6.732 \text{ seconds}$$
$$20^s \times 0.0027^s = 0.054 \text{ second}$$
$$\text{Total} = 56.069 \text{ seconds}$$

L.S.T. at L.M.N. = G.S.T. at G.M.N. − retardation

$$= 6^h\ 32^m\ 12^s - 56.069^s = 6^h\ 31^m\ 15.931^s \quad \ldots(1)$$

Now, L.M.T. = $6^h\ 30^m$ P.M.

∴ M.T. interval from L.M.N. = $6^h\ 30^m$.

Let us convert it into sidereal time interval by adding the acceleration to the mean time interval.

Thus, $6^h \times 9.8565^s$ = 59.139 seconds

$30^m \times 0.1642^s$ = 4.926 seconds

Total acceleration $= 64.065^s = 1^m\ 4.065^s$

∴ Sidereal Time Interval = Mean time interval + acceleration since L.M.N.

$= 6^h\ 30^m + 1^m\ 4.065^s = 6^h\ 31^m\ 4.065^s$

Now L.S.T. at L.M.N. $= 6^h\ 31^m\ 15.931^s$

Add S.I. since L.M.N. $= 6^h\ 31^m\ 4.065^s$

∴ L.S.T. at L.M.T. $= 13^h\ 02^m\ 19.996^s = 1^h\ 02^m\ 19.996^s$ P.M.

1.9.6. GIVEN THE LOCAL SIDEREAL TIME, TO DETERMINE THE LOCAL MEAN TIME

If the G.S.T. at G.M.M. is given, the L.S.T. at L.M.M. can be calculated as discussed earlier. The L.S.T. at L.M.M. can then be subtracted from L.S.T. to get the number of sidereal hours, minutes and seconds past midnight. This sidereal time interval can then converted into the mean time interval by subtracting the retardation at the rate of 9.8296^s per hour of S.I. thus obtaining the L.M.T. *The rules are, therefore :*

(*a*) Find the L.S.T. at L.M.M. from the known G.S.T. at G.M.M.

(*b*) Subtract L.S.T. at L.M.M. from the L.S.T. at get the S.I.

(*c*) Convert the S.I. into mean time interval, thus getting L.M.T.

Example 1.29. *The local sidereal time at a place (Longitude 112° 20′ 15″ W) is $18^h\ 28^m\ 12^s$. Calculate the corresponding L.M.T. given that G.S.T. at G.M.M. is $8^h\ 10^m\ 28^s$ on that day.*

Solution

Let us first convert the longitude into time units :

	h	m	s
$112° = \frac{112}{15}$ h =	7	28	0
$20' = \frac{20}{15}$ m =	0	1	20
$15'' = \frac{15}{15}$ s =	0	0	1
∴ Longitude =	7	29	21

Since the place has west longitude,

L.S.T. at L.M.M.= G.S.T. at G.M.M. + acceleration.

Let us calculate the acceleration at the rate of 9.8565^s per hour.

$$7^h \times 9.8565^s = 68.996 \text{ seconds}$$
$$29^m \times 0.1642^s = 4.762 \text{ seconds}$$
$$21^s \times 0.0027^s = 0.057 \text{ second}$$

$$\text{Total} = 73.815^s = 1^m\ 13.815^s$$

		h	m	s
G.S.T. at G.M.M.	=	8	10	28
Add acceleration	=		1	13.815
∴ L.S.T. at L.M.M.	=	8	11	41.815 ...(1)

		h	m	s
Now local sidereal time	=	18	28	12
Subtract L.S.T. at L.M.M.	=	8	11	41.815
∴ S.I. since L.M.M.	=	10	16	30.185

Let us now convert this sidereal interval into mean time interval by subtracting the retardation at the rate of 9.8296^s per hour.

Thus,
$$10^h \times 9.8296 = 98.296 \text{ seconds}$$
$$16^m \times 0.1638 = 2.621 \text{ seconds}$$
$$30.185^s \times 0.0027 = 0.081 \text{ second}$$

Total retardation $= 100.998^s = 1^m\ 40.998^s$

∴ Mean time interval = S.I. − retardation

$= 10^h\ 16^m\ 30.185 - 1^m\ 40.998^s = 10^h\ 14^m\ 49.187^s$ since L.M.M.

∴ L.M.T. $= 10^h\ 14^m\ 49.187^s$.

1.9.7. ALTERNATIVE METHOD OF FINDING L.S.T. FROM THE GIVEN VALUE OF L.M.T.

In the method discussed in § 1.9.5 to convert L.M.T. to L.S.T., double computation of time interval was involved. In this alternative method only one transformation of the time interval is necessary. *The steps for the computation are as follows :*

(*a*) Convert the given L.M.T. to the corresponding G.M.T., allowing for the difference of longitude. This gives the interval in mean solar time that has elapsed since G.M.M.

(*b*) Convert this mean time interval to sideral interval that has elapsed since G.M.M., by adding the acceleration at the rate of 9.8565 seconds per hour of mean time interval.

(*c*) Add the S.I. to the G.S.T. at G.M.M. to get the G.S.T. at the instant under consideration.

(*d*) Convert this G.S.T. to the corresponding L.S.T., allowing for the difference of longitude.

Thus, in the above method, though the theory is a little more complex, there is only one transformation of a time interval so that the actual computation is a little shorter. We shall work out example 1.28 by this method.

Example 1.30. *Solve example 1.28 by the alternative method.*

Solution.

Longitude $= 85° 20' E = 5^h 41^m 20^s$ E, as found earlier

L.M.T., $= 6^h 30^m$ P.M.

	h	m	s
L.M.T.	= 18	30	0
Subtract longitude	= 5	41	20
∴ G.M.T.	= 12	48	40

∴ M.T. interval since G.M.N.$= 12^h 48^m 40^s - 12^h = 48^m 40^s$.

Convert this mean time interval to sidereal time interval by adding the acceleration.

$$48^m \times 0.1642^s = 7.882 \text{ seconds}$$

$$40^s \times 0.0027^s = 0.108 \text{ seconds}$$

Total acceleration $= 7.990$ seconds

∴ Sidereal time interval = mean time interval + acceleration

$= 48^m 40^s + 7.990^s = 48^m 47.99^s$ since G.M.N.

	h	m	s
G.S.T. at G.M.N.	= 6	32	12
Add S.I.	= 0	48	47.99
∴ G.S.T. at the given instant	= 7	20	59.99
Add longitude	= 5	41	20.0
∴ L.S.T. at L.M.T.	= 13	02	19.99
	$= 1^h$	02^m	19.99^s P.M.

1.9.8. ALTERNATIVE METHOD OF FINDING L.M.T. FROM THE GIVEN VALUE OF L.S.T.

In the method discussed in § 1.9.6 to convert L.S.T. to L.M.T., double computation of time interval was involved. In this method, only one transformation of the interval is necessary. *The steps for the computation are as follows* :

(*a*) From the known L.S.T., compute the corresponding G.S.T. by allowing for the difference of longitude.

(*b*) From this G.S.T. calculated above, subtract the G.S.T. of G.M.M. to get the sidereal interval that has elapsed since G.M.M.

(*c*) Convert this sidereal interval into mean time interval by subtracting the retardation at the rate of 9.8296^s per hour of sidereal interval.

(*d*) The mean time interval obtained in (*c*) is thus the G.M.T. at the instant under consideration. Compute the L.M.T. by allowing for the difference of longitude.

We shall work out example 1.29 by the alternative method.

Example 1.31. *Solve example 1.29 by the alternative method.*

Solution

Longitude	= 112° 20′ 15″ *W* = $7^h\ 29^m\ 21^s$ *W*		
	h	m	s
L.S.T.	= 18	28	12
Add longitude	= 7	29	21
∴ G.S.T. at the instant	= 25	57	33
G.S.T. at G.M.M.	= 8	10	28
∴ S.I. since G.M.M.	= 17	47	05

Let us now convert this S.I. in mean time interval by subtracting the retardation.

$$17^h \times 9.8296 = 167.103 \text{ seconds}$$

$$47^m \times 0.1638 = 7.699 \text{ seconds}$$

$$5^s \times 0.0027 = 0.014 \text{ seconds}$$

Total retardation = 174.816 seconds = $2^m\ 54.816^s$

∴ Mean time interval = S.I. − retardation

= $17^h\ 47^m\ 05^m - 2^m\ 54.816^s$

∴ G.M.T. = $17^h\ 44^m\ 10.184^s$

Subtract longitude = $7^h\ 29^m\ 21^s$

L.M.T. = $10^h\ 4^m\ 49.184^s$

1.9.9. TO DETERMINE THE L.M.T. OF TRANSIT OF A KNOWN STAR ACROSS THE MERIDIAN, GIVEN G.S.T. OF G.M.N.

We have already seen that when a star transits or culminates across the meridian, the R.A. of the star, expressed in time, is the sidereal time. In the Nautical Almanac, the astronomical co-ordinates of all the stars in terms of Right Ascension and declination are given. Thus, knowing the R.A., the L.S.T. at the time of transit of the star is known. The problem is now to convert the L.S.T. into the L.M.T. by the method described in § 1.9.6 or in §1.9.8. The following are the steps :

(*a*) Find the R.A. of the star from the N.A. This is then the L.S.T. at the time of the transit of the star.

(*b*) From the known value of G.S.T. of G.M.M. or (G.M.N.), calculate the L.S.T. of L.M.M. (or L.M.N.).

(*c*) Subtract this L.S.T. of L.M.M. from the L.S.T. of the transit of the star to get the S.I. that has elapsed since L.M.M.

(*d*) Convert this S.I. to mean time interval which, then, gives the L.M.T. at the transit of the star.

Example 1.32. *What will be the L.M.T.'s of upper and following lower transit at a place in longitude 162° 30′ 15″ W of a star whose R.A. is* $22^h\ 11^m\ 30^s$, *if the G.S.T. of previous G.M.N. is* $10^h\ 30^m\ 15^s$.

Solution.

		h	m	s
Longitude :	$162° = \frac{162}{15}$ h =	10	48	0
	$30' = \frac{30}{15}$ m =	0	2	0
	$15'' = \frac{15}{15}$ s =	0	0	1
		10	50	1

Since the place is to the west, we will have to add the acceleration to get the L.S.T. at L.M.N.

$$10^h \times 9.8565^s = 98.565 \text{ seconds}$$

$$50^m \times 0.1642^s = 8.210 \text{ seconds}$$

$$1^s \times 0.0027^s = 0.003 \text{ second}$$

Total acceleration = 106.778 seconds = $1^m\ 46.778^s$

	h	m	s
G.S.T. of G.M.N. =	10^h	30^m	1.5^s
Add acceleration =		1	46.778
∴ L.S.T. of L.M.N. =	10	32	01.778

	h	m	s
Now R.A. of star = L.S.T. =	22	11	30
Subtract L.S.T. of L.M.N. =	10	32	1.778
∴ S.I. since L.M.N. =	11	39	28.222

Let us now convert this S.I. into mean time interval by subtracting retardation.

$$11^h \times 9.8296^s = 108.126 \text{ seconds}$$
$$39^m \times 0.1638^s = 6.388 \text{ seconds}$$
$$28.222^s \times 0.0027^s = 0.076 \text{ second}$$

Total retardation $= 114.590$ seconds $= 1^m\ 54.59^s$

$\therefore$ Mean time interval $=$ S.I. $-$ retardation $= 11^h\ 39^m\ 28.222^s - 1^m\ 54.59^s$

$= 11^h\ 37^m\ 33.632^s$ since L.M.N.

$\therefore$ L.M.T. of upper transit **$= 11^h\ 37^m\ 33.632^s$ P.M.**

The lower transit of the star will take place at 12 sidereal hours later. To know the corresponding mean time, let us first convert the 12 sidereal hours into mean time hours.

Retardation for 12 hours $= 12 \times 9.8296^s = 1^m\ 57.955^s$

$\therefore$ Mean time interval $= 12^h - 1^m\ 57.955^s = 11^h\ 58^m\ 2.045^s$

Thus the lower transit occurs at a mean time interval of $11^h\ 58^m\ 2.045^s$ after the upper transit.

L.M.T. of upper transit $= 11^h\ 37^m\ 33.632^s$

Add the mean time interval $= 11^h\ 58^m\ 2.045^s$

L.M.T. of lower transit $=$ 23 35 35.677 Since L.M.N.

$= 11^h\ 35^m\ 35.677$ A.M. (following day).

Example 1.33. *Calculate the L.M.T. and G.M.T. of transit of β Draconis (R.A. $17^h\ 28^m\ 40^s$) at a place in longitude 60° 30′ E given G.S.T. of G.M.T. $= 7^h\ 30^m\ 48.6^s$.*

Solution.

		h	m	s
Longitude	$60° = \frac{60}{15}$ h =	4	0	0
	$30' = \frac{30}{15}$ h =	0	2	0
		4^h	2^m	0^s

Since the place has east longitude, let us calculate the retardation at the rate of 9.8565^s per hour.

$$4^h \times 9.8565^s = 39.426 \text{ seconds}$$
$$2^m \times 0.1642^s = 0.328 \text{ second}$$

Total retardation $= 39.754$ seconds

$\therefore$ L.S.T. at L.M.N. = G.S.T. at G.M.N. $-$ Retardation

$= 7^h\ 30^m\ 48.6^s - 39.754^s = 7^h\ 30^m\ 8.846^s$

		h	m	s
L.S.T.= R.A. of star	=	17	28	40
Subtract L.S.T. of L.M.N.	=	7	30	8.846
∴ S.I. since L.M.N.	=	9	58	31.154

Let us convert it to the mean time interval by subtracting the retardation.

$9^h \times 9.8296 = 88.466$ seconds

$58^m \times 0.1638 = 9.500$ seconds

$31.154 \times 0.0027 = 0.084$ second

Total retardation $= 98.050$ seconds $= 1^m\ 38.05^s$

∴ Mean time interval since L.M.N.= S. I. − retardation $= 9^h\ 58^m\ 31.154^s - 1^m\ 38.05^s$

or L.M.T. transit $= 9^h\ 56^m\ 53.104^s$

Subtract the longitude $= 4^h\ 2^m\ 0^s$

∴ G.M.T. of transit $= 5^h\ 54^m\ 53.104^s$

1.9.10. GIVEN THE G.M.T. OF TRANSIT OF THE FIRST POINT OF ARIES, TO DETERMINE THE L.M.T. OF TRANSIT AT A PLACE IN ANY OTHER LONGITUDE

We have already seen that the sidereal clock gains over the mean time clock at the rate of 9.8565 seconds per mean solar hour or at the rate of 9.8296 seconds for each sidereal hour. When the first point of Aries transits over the Greenwich, the sidereal clock shows 0^h while the mean clock gives the mean time of the transit of the first point of Aries. It is the difference between the readings of the two clocks at the time of the transit. Now consider a place in west longitude where the transit of ϒ will take place after certain sidereal interval of time (obtained by dividing the longitude by 15). Since the sidereal clock continually gains over the mean time clock, the difference between mean time clock and the sidereal clock will continuously go on decreasing. *When the transit of ϒ occurs at the given meridian, the mean time clock will not be as far ahead of the sidereal clock as it was at Greenwich,* and the Greenwich reading of the mean time clock will be diminished by subtracting 9.8296 seconds for each hour of longitude. *Hence, if the meridian is to the west of Greenwich, the mean time must be corrected by the subtraction of 9.8296 seconds per hour of longitude, and if the place is to the east, it must be added.* The rule thus becomes:

$$\textit{L.M.T. of transit of } \Upsilon = \textit{G.M.T. of transit of } \Upsilon \pm \frac{E}{W}\left(9.8296 \times \frac{\text{Longitude in degrees}}{15}\right)$$

It must be noted that the difference between the readings of sidereal and mean time clocks at any place is the same all over the World at the same instant. At the time of transit of ϒ, the L.S.T is zero and hence L.M.T. is the difference between the two clocks at the time of transit.

Example 1.34. *The G.M.T. of transit of the first point of Aries* (♈) *on March 2 is* $13^h\ 21^m\ 54^s$. *Find the L.M.T. of transit of the first point of Aries on the same day at a place* (*a*) *Longitude 40° 30′ E* (*b*) *40° 30′ W.*

Solution

Longitude = 40° 30′ *E*

	h	m	s
$40° = \frac{40}{15}$ h =	2	40	0
$30' = \frac{30}{15}$ m =	0	2	0
=	2	42	0

Gain of sidereal clock at the rate of 9.8296^s per hour of longitude :

$2^h \times 9.8296^s = 19.659$ seconds

$42 \times 0.1638^s = 6.880$ seconds

Total = 26.539 seconds

	h	m	S
(*a*) G.M.T. of transit of ♈ =	13	21	54
Add the correction for eastern longitude =	0	0	26.539
∴ L.M.T. of transit of ♈ =	13^h	22^m	20.539^s

	h	m	s
(*b*) G.M.T. of transit of ♈ =	13	21	54
Subtract the correction for the western longitude =	0	0	26.539
∴ L.M.T. of transit of ♈ =	13	21	27.461

1.9.11. GIVEN THE L.S.T. AT ANY PLACE, TO DETERMINE THE CORRESPONDING L.M.T. IF THE G.M.T. OF TRANSIT OF THE FIRST POINT OF ARIES ON THE SAME DAY IS ALSO GIVEN

We know that L.S.T. at any instant is the time interval that has elapsed since the transit of ♈ on the meridian. This L.S.T. can be converted into equivalent number of mean hours by subtracting the retardation at the rate of 9.8296^s per sidereal hour. Also, from the known G.M.T. of transit of ♈ , the L.M.T. of transit of ♈ can be calculated. This L.M.T. is nothing but the time shown by the mean clock when the sidereal clock shows 0^h. Therefore, the L.M.T. at the instant under consideration can be obtained by adding the mean hours (corresponding to the given L.S.T.) to the L.M.T. at the time of transit of ♈. *The steps therefore are* :

(1) From the known G.M.T. of transit ♈, calculate the L.M.T. of transit of ♈ by method discussed in §1.9.10.

(2) Convert the given L.S.T. to mean hours.

(3) Add (1) and (2) to get the L.M.T. corresponding to the given L.S.T.

Example 1.35. *The local sidereal time at a place (longitude 50° 30′ E) on 17th May, 1948 is $11^h\ 30^m\ 12^s$. Find the corresponding L.M.T. given that the G.M.T. of transit of ♈ on the 17th May, 1948 is $7^h\ 12^m\ 28^s$.*

Solution

Longitude = 50° 30′ *E*

	h	m	s
$50° = \frac{50}{15}$ h =	3	20	0
$30' = \frac{30}{15}$ m =	0	2	0
Total =	3	22	0

The correction at the rate 9.8296 per hour of longitude is

$$3^h \times 9.8296 = 29.489 \text{ seconds}$$

$$22^m \times 0.1638 = 3.604 \text{ seconds}$$

$$\text{Total correction} = 33.093 \text{ seconds}$$

G.M.T. at transit of ♈ =	7^h	12^m	28^s
Add the correction =	0	0	33.093
∴ L.M.T. at transit at ♈ =	7	13	1.093 ...(1)

L.M.T.= $11^h\ 30^m\ 12^s$, and may be converted to mean hours by subtracting the retardation.

$$11^h \times 9.8296 = 108.126 \text{ seconds}$$

$$30^m \times 0.1638 = 4.914 \text{ seconds}$$

$$12^s \times 0.0027 = 0.032 \text{ seconds}$$

$$\text{Total retardation} = 113.072 \text{ seconds} = 1^m\ 53.072^s$$

∴ Mean hours = Sidereal hours − Retardation = $11^h\ 30^m\ 12^s - 1^m\ 53.072^s = 11^h\ 28^m\ 18.928^s$ ...(2)

Adding (1) and (2), we get

L.M.T. = $7^h\ 13^m\ 1.093^s + 11^h\ 28^m\ 18.928^s$ = $\mathbf{18^h\ 41^m\ 20.021^s}$.

1.9.12. GIVEN THE SIDEREAL TIME AT G.M.M., TO COMPUTE THE G.M.T. AT THE NEXT TRANSIT, OF THE FIRST POINT OF ARIES

The given sidereal time at 0^h G.M.T. shows the number of sidereal hours that have elapsed since the transit of ♈. The next transit of ♈ will evidently take place 24 sidereal hours later than the previous transit. Let the G.S.T. at G.M.M. be s sidereal hours. Then the next transit will take place at $(24 - s)$ sidereal hours after the G.M.M. These $(24 - s)$ sidereal hours can be converted into the mean time hours which will give the G.M.T. at next transit of ♈.

Example 1.36. *On July 12, the G.S.T. at 0^h G.M.T. is $8^h\ 25^m\ 25^s$. Find the G.M.T. of the next transit of* ♈.

Solution

G.S.T. at G.M.M. = $8^h\ 25^m\ 25^s$

∴ Time of previous transit = $8^h\ 25^m\ 25^s$ sidereal interval before G.M.M.

∴ Time of next transit = $(24^h - 8^h\ 25^m\ 25^s)$ sidereal interval after G.M.M.

= $15^h\ 34^m\ 35^s$ sidereal interval of time.

To convert it into the mean time interval, subtract the retardation

$15^h \times 9.8296 = 147.444$ seconds

$34^m \times 0.1638 = 5.569$ seconds

$35^s \times 0.0027 = 0.095$ second

Total retardation = 153.108 seconds = $2^m\ 33.108^s$

∴ Mean time interval = $15^h\ 34^m\ 35^s - 2^m\ 33.108^s = 15^h\ 32^m\ 1.892^s$ since G.M.M.

∴ G.M.T. of next transit = $\mathbf{15^h\ 32^m\ 1.892^s}$.

1.9.13. GIVEN THE G.M.T. OF G.A.N. ON A CERTAIN DATE, TO FIND THE L.M.T OF L.A.N. ON THE SAME DATE

The local apparent noon will occur before or after the G.A.N. depending upon whether the longitude of the place is to the east or to the west of the Greenwich meridian. *The apparent time at the apparent noon is zero and hence G.M.T. of G.A.N. is the equation of time at Greenwich at noon.* Since the local apparent noon occurs either before or after the G.A.N., the equation of time will change and interpolation will have to be done. For example, if the place is to the east of Greenwich, the L.A.N. will occur earlier and we must know the difference between the given G.M.T. of G.A.N. and the G.M.T. of G.A.N. on the day *before*, in order to do the interpolation. Similarly, if the place is to the west of Greenwich, the L.A.N. will occur later and we must know the difference between the given G.M.T. of G.A.N. and the G.M.T. of G.A.N. on the day *after*, in order to do the interpolation. Once the correct equation of time is known, L.M.T. at L.A.N. can be computed as illustrated in example 1.37.

Example 1.37. *Given the following data from the N.A. for 1951:*

Sun at Transit at Greenwich

Date	G.M.T. h	m	s	
June 30	*12*	*02*	*22.44*	
				+ 11.94
July 1	*12*	*03*	*34.38*	
				+ 11.71
July 2	*12*	*03*	*46.09*	
				+ 11.45
July 3	*12*	*03*	*57.54*	
				+ 11.16

Find the L.M.T. of L.A.N. on July 2 at a place (a) in longitude 130 ° E (b) in longitude 49 ° W.

Solution. (*a*) Longitude $130° E = \frac{30}{15}\text{h} = 8^h\ 40^m\ E$

Since the place is to the east of Greenwich, the L.M.T. is $8^h\ 40^m$ *ahead* of the G.M.T. From the table, the difference between G.M.T. of G.A.N. on July 1, and July 2, is 11.71^s (for 24 hours).

$\therefore$ Difference for $8^h\ 40^m = (8^h\ 40^m)\frac{(11.71)}{24} = 4.23$ seconds

By the inspection of the table, it is clear that the values of G.M.T. are decreasing as we go back from July 2. Hence this difference of 4.23 seconds should be subtracted from the G.M.T. of G.A.N. on July 2 to get L.M.T. of L.A.N. on the same date.

Thus, G.M.T. of G.A.N. on July 2 $= 12^h\ 03^m\ 46.09^s$

Subtract difference due to east longitude $= 4.23^s$

$\therefore$ L.M.T. of L.A.N. on July 2 $= 12^h\ 03^m\ 41.86^s$

(*b*) Longitude $49° W = \frac{49}{15}\text{h} = 3^h\ 16^m$

Since the place is to the west of Greenwich, the L.M.T. is $3^h\ 16^m$ behind G.M.T.

From the table, the difference between G.M.T. of G.A.N. on July 2 and July 3 is $+ 11.45^s$ (for 24 hours).

$\therefore$ Difference for $3^h\ 16^m = (3^h\ 16^m)\left(\frac{11.45}{24}\right) = 1.56$ seconds.

Since the values of G.M.T. are increasing as the dates increase, the difference of 1.56 seconds should be added to the G.M.T. of G.A.N. on July 2 to get L.M.T. of L.A.N. on the same date.

Thus, G.M.T. of G.A.N. on July 2 $= 12^h\ 03^m\ 46.09^s$

Add difference due to west longitude $= 1.56^s$

$\therefore$ L.M.T. of L.A.N. on July 2 $= 12^h\ 03^m\ 47.65^s$

1.9.14. TO FIND THE LOCAL SIDEREAL TIME OF ELONGATION OF A STAR

We have already seen in § 1.8 (Fig. 1.30) that

Star's hour angle + star's right ascension = Local Sidereal Time.

Thus, to get the L.S.T. of elongation of the star, add the westerly hour angle (or subtract the easterly hour angle) to the R.A. of the star at its elongation. If the result is more than 24^h, 24^h are deducted, while if the result is negative, 24 hours are added to it.

Example 1.38. *Find the L.S.T. at which β Ursae Minor is will elongate on the evening at a place in latitude 50° 30′ N given that the R.A. of the star is* $14^h\ 50^m\ 52^s$ *and its declination is + 74° 22′.*

Solution

The right ascension and the declination of the star are given. Let us first calculate its hour angle at elongation. When the star is at elongation, we have, from Eq. 1.19,

$$\cos H = \frac{\tan \theta}{\tan \delta} = \frac{\tan 50° 30'}{\tan 74° 22'}$$

$$\log \tan 50° 30' = \bar{1}.0838955$$

$$\log \tan 74° 22' = \bar{1}.5531022$$

$\therefore \quad \log \cos H = \bar{1}.5307933$

$\therefore \quad H = 70° 9' 18''$

or $\quad H = 4^h\ 40^m\ 37.2^s$

Add $\quad$ R.A. $= 14\ \ 50\ \ 52.0$

$\therefore \quad$ L.S.T. $= 19^h\ 31^m\ 29.2^s$

Example 1.39. *If the G.S.T. of G.M.N. is* $14^h\ 30^m\ 28.25^s$, *what will be the H.A. of a star of R.A.* $23^h\ 20^m\ 20^s$ *at a place in longitude 120° 30′ W at 2.05 A.M. G.M.T. the same day ?*

Solution

We know that, $\quad$ L.S.T. = R.A. of star + Hour angle of the star.

From the above relation, the hour angle of the star can very easily be found out by subtracting R.A. of the star from the L.S.T. of the event. The only problem, therefore, is to calculate the L.S.T. corresponding to the given L.M.T., given the G.S.T. of G.M.N.

Let us first calculate the L.S.T. of L.M.N.

$$\text{Longitude} = 120° 30' W = 8^h\ 2^m\ W.$$

Since the place is to the west, we have to add the acceleration at the rate of 9.8565 per hour of longitude to the G.S.T. of G.M.N. to get the L.S.T. of L.M.N.

Now $\quad 8^h \times 9.8565 = 78.85$ seconds

$\quad 2^m \times 0.1642 = 0.33$ second

Total acceleration = 79.18 seconds

G.S.T. of G.M.N. = $14^h\ 30^m\ 28.25^s$

Add acceleration = 79.18^s

∴ L.S.T. of L.M.N. = $14^m\ 31^m\ 47.43^s$ ...(1)

Now G.M.T. = $2^h\ 5^m\ 0^s$

Subtract longitude = $8^h\ 2^m\ 0^s$

∴ L.M.T. of the event = $18^h\ 3^m\ 0^s$ (previous day).

L.M.N. (day of given G.S.T. of G.M.N.) = $12^h\ 0^m\ 0^s$

Subtract L.M.T. of event (previous day) = $18^h\ 3^m\ 0^s$

∴ Mean time interval between the event = $17^h\ 57^m\ 0^s$ and the L.M.N.

Let us convert this mean time interval to the sidereal time interval by adding acceleration at the rate of 9.8565^s per mean hour.

Thus $17^h \times 9.8565 = 167.56$ seconds

$57^m \times 0.1642 = 9.36$ seconds

Total acceleration = 177.92 seconds = $2^m\ 57.92^s$

∴ S.I. between the event and L.M.N.= $17^h\ 57^m\ 0^s + 2^m\ 57.92^s = 17^h\ 59^m\ 57.92^s$ (before L.M.N.)

Now L.S.T. of L.M.N. = $14^h\ 31^m\ 47.43^s$

Subtract S.I. = $17^h\ 59^m\ 57.92^s$

L.S.T. of event = $20^h\ 31^m\ 49.51^s$ ...(2)

Now H.A. = L. S. T. – R. A.

$= (20^h\ 31^m\ 49.51^s) - (23^h\ 20^m\ 20^s) + 24^h = 21^h\ 11^m\ 29.51^s$

(**Note.** 24^h have been added to make the hour angle positive).

Example 1.40. *Find the R.A. of the mean sun at 5.30 A.M. on July 28, 1964 in a place in longitude 75° 28′ W, and also the R.A. of the meridian of the place, given that G.S.T. at G.M.M on the given date is* $20^h\ 15^m\ 32.58^s$.

Solution.

We know that, L.S.T. = R.A. of the star + hour angle of the star.

Here, the mean sun is fictitious star.

Hence L.S.T. = R.A.M.S. + hour angle of the mean sun.

But hour angle of mean sun = L.M.T. + 12 hours

(since L.M.T. is measured from the lower transit).

Hence, we have L.S.T. = R.A.M.S. + L.M.T. + 12^h

In order to calculate the R.A. of the mean sun, we must know L.S.T. at the time of event L.S.T. can be very easily found from the given L.M.T. and the given value of G.S.T. of G.M.M.

Now longitude = $75° 28' W = 5^h 1^m 52^s$

Since the place is having west longitude, we will have to add an acceleration at the rate of 9.8565^s per hour of longitude to the G.S.T. of G.M.M. to the get the L.S.T. of L.M.N.

$$5^h \times 9.8565 = 49.28 \text{ seconds}$$
$$1^m \times 0.1642 = 0.16 \text{ second}$$
$$52^s \times 0.0027 = 0.14 \text{ second}$$

Total acceleration = 49.58 seconds

G.S.T. of G.M.M. = $20^h\ 15^m\ 32.58^s$

Add Acceleration = 49.58^s

$\therefore$ L.S.T. of L.M.M. = $20^h\ 16^m\ 22.16^s$

Now L.M.T. of event = $5^h\ 30^m$ A.M. = $5^h\ 30^m$ mean time after mid-night.

To convert this time interval to sidereal interval, add the acceleration at the rate of 9.8565^s per hour of mean time.

Thus, $5^h \times 9.8565^s = 49.28$ seconds

$30^m \times 0.1642 = 4.93$ seconds

Total acceleration = 54.21^s

$\therefore$ S.I. since L.M.M. = $5^h\ 30^m + 54.21^s = 5^h\ 30^m\ 54.21^s$

$\therefore$ L.S.T. = L.S.T. of L.M.M.+ S.I.

$= 20^h\ 16^m\ 22.16^s + 5^h\ 30^m\ 54.21^s = 25^h\ 47^m\ 16.37^s$

Now, by definition, the R.A. of the meridian is equal to the L.S.T.

Hence R.A. of meridian = $\mathbf{25^h\ 47^m\ 16.37^s}$

Again R.A.M.S. = L. S. T. – L. M. T. – 12^h

$= (25^h\ 47^m\ 16.37^s) - (5^h\ 30^m) - (12^h) = \mathbf{8^h\ 17^m\ 16.37^s}$.

1.10. INTERPOLATION OF VALUES

The declination of a heavenly body is a constantly varying quantity and can be obtained from the nautical almanac which gives the values at Greenwich mean and apparent mid-night. The nautical almanac gives the values of declination both for mean sun and apparent sun at G.M.M. and G.A.M. for every day and also the rate of hourly variation at Greenwich mid-night. To find the declination at any given instant of Greenwich civil time, it is necessary to interpolate between the tabulated values. The required value may thus be obtained by :

(*a*) Simple linear interpolation between the successive tabulated values on the assumption that the rate of change is uniform and equal to its value at the middle of the interval.

(*b*) By interpolating strictly, taking higher order differences into account, by ***Bessel's method***. The Bessel's interpolation formula is as follows :

$$f_n = f_0 + n\,\Delta'_{1/2} + \frac{n(n-1)}{4}(\Delta_0'' + \Delta_1'') \qquad \ldots(1.26)$$

where f_n = the value of the function which is to be found, and which lies between f_0 and f_1:

n = Fractional value of the interval between two tabular values.

Δ' = First difference between the successive values of the function.

Δ'' = Second difference

Thus, $f_{-1} - f_0 = \Delta'_{-1/2}$, $f_1 - f_0 = \Delta'_{1/2}$, $f_2 - f_1 = \Delta'_{3/2}$ | $\Delta'_{1/2} - \Delta'_{-1/2} = \Delta''_0$, $\Delta'_{3/2} - \Delta'_{1/2} = \Delta''_1$

where f_{-1}, f_0, f_1, f_2 ..... etc. are the successive values of the function to be interpolated.

The method of interpolation has been fully illustrated in the following example.

Example 1.41. *Find sun's declination at 10 A.M. on February 5, 1947 in longtiude 45° E.*

Solution

Let us first convert the local time to Greenwich mean time.

Longitude = 45° E = 3^h

$\therefore$ G.M.T. = 10 – 3 = 7 hours = 0.2917 day

$\therefore$ n = 0.2917

The following values of sun's declination are obtained from the N.A.

Date	***Sun's Declination at 0^h G.M.T.***	***Variation per day***
Feb. 4	– 16° 32′ 11″.2	
		+ 1067″.2
Feb. 5	– 16° 14′ 24″.0	
		+ 1083″.9
Feb. 6	– 15° 56′ 20″.1	
		+ 1100″.3
Feb. 7	– 15° 37′ 59″.8	

From the above table, f_0 = value at 0^h G.M.T. on Feb. 5 = – 16° 14′24″.0

f_{-1} = value on Feb. 4 = – 16° 32′11″.2

f_1 = value on Feb. 6 = – 15° 56′20″.1

f_2 = value on Feb. 7 = – 15° 37′59″.8

$\Delta'_{-1/2} = f_{-1} - f_0$ = + 1067″.2

$\Delta'_{1/2} = f_1 - f_0$ = + 1083″.9

$$\Delta_0'' = \Delta'_{1/2} - \Delta'_{-1/2} = 1083.9 - 1067.2 = +16''.7$$

$$\Delta'_{3/2} = f_2 - f_1 = +1100''.3$$

$$\Delta_1'' = \Delta'_{3/2} - \Delta'_{1/2} = 1100''.3 - 1083''.9 = +16''.4$$

Putting the values in the Bessel's formula, we get

$$f_n = f_0 + n\,\Delta'_{1/2} + \frac{n(n-1)}{4}(\Delta''_0 + \Delta_1'') = -16°\ 14'\ 24''.0 + 0.2917\ (+1083''.9)$$

$$-\frac{0.2917\ (0.2917 - 1)}{4} \times (16''.7 + 16''.4)$$

$$= -16°\ 14'\ 24''.0 + 316''.15 - 1''.71 = \mathbf{-16°\ 9'\ 9''.56}.$$

Note. (1) The four dates from which the interpolation is done should be so selected that the instant lies between the two middle dates.

(2) The value of the declination by the approximate method (linear interpolation) will be equal to $-16°\ 14'\ 24''.0 + 0.2917\ \ (1083''.9) = -16°\ 9'\ 7''.85$.

1.11. INSTRUMENTAL AND ASTRONOMICAL CORRECTIONS TO THE OBSERVED ALTITUDE AND AZIMUTH

(A) INSTRUMENTAL CORRECTIONS

The angle measuring instruments used in astronomical observations are theodolite and sextant. For precise work, a theodolite having a least count of 1" (or less) is used. The theodolite should be in prefect adjustments. However, following are some of the instrumental corrections that are generally applied to the observed altitude and azimuth.

(*a*) Corrections for Altitudes

(1) Correction for Index Error. If the vertical circle verniers do not read zero when the line of sight is horizontal, the vertical angles measured will be incorrect. The error is known as the *index error*. The index error can be eliminated by taking both face observations. However, it may sometimes not be practicable to take both face observations when the altitude of a star or the sun is to be observed. In such a case, the correction for the index error is necessary.

The index error may be determined as follows :

(*i*) Set the theodolite on firm ground and level it accurately with reference to altitude bubble.

(*ii*) Bisect a well-defind object such as a church spire (or a chimney top) with the telescope normal (face left). Observe the vertical angle α_1.

(*iii*) Change the face and bisect the same object again with telescope reversed (face right). Observe the vertical angle α_2.

Let the index error be e.

∴ Correct vertical angle will be

$$\alpha = (\alpha_1 + e) \qquad \text{and} \qquad \alpha = (\alpha_2 - e)$$

$$\therefore \qquad \alpha = \frac{(\alpha_1 + e) + (\alpha_2 - e)}{2} = \frac{\alpha_1 + \alpha_2}{2}$$

Thus, the correct vertical angle is the mean of the two observed angles.

Hence $e = (\alpha - \alpha_1)$

For example, let $\alpha_1 = 4° 15' 8''$ and $\alpha_2 = 4° 15' 16''$

$\therefore$ Mean vertical angle $= \alpha = 4° 15' 12''$

Hence, the index error correction for face left observation $= + 4''$

Hence, the index error correction for face right observation $= - 4''$

The index error correction is said to be + ve or – ve according as this amount is to be added to or subtracted from the observed altitude.

(2) Correction for Bubble Error. If the altitude bubble does not remain central while the observations are made, the correction for bubble error is essential. The correction for bubble error is given by

$$C = \frac{\Sigma O - \Sigma E}{n} \times v \text{ seconds} \qquad ...(1.27)$$

where C = correction for bubble error in seconds, to be applied to the mean altitude observed.

ΣO = the sum of readings of the object glass end of the bubble.

ΣE = the sum of readings of the eye-piece end of the bubble.

n = the number of bubble ends read (= 2 when single face observation is taken, and 4 when both face observations are made).

v = angular value of one division of the bubble in seconds.

If ΣO is greater than ΣE, the correction is positive, otherwise negative.

(*b*) Correction for Azimuths

Since most astronomical observations require the line of sight to be elevated through a large vertical angle, it is important that the horizontal axis shall be truly horizontal. To fulfill this, it is most important that (1) the instrument is accurately levelled so that the vertical axis is truly vertical and (2) the trunnion axis is exactly perpendicular to the vertical axis. If the vertical axis is not truly vertical (*i.e.* if the bubble does not preserve a central position through a series of observations), the trunnion axis will be inclined even though the instrument is in perfect adjustment. The error due to the inclination of the trunnion axis cannot be eliminated. However, its inclination can be determined by means of a striding level with a sensitive bubble tube.

Correction for Trunnion Axis Dislevelment. The bubble readings on the striding level will show whether the trunnion axis is truly horizontal or not. If not, each horizontal direction should be corrected for trunnion axis dislevelment. It can be shown that the correction to be applied to the azimuth of a low point with respect to a high point, caused by an inclination of the trunnion axis of the transit is given by

$$c = b \tan \alpha \text{ seconds}$$

where c = correction to the azimuth

b = inclination of the horizontal axis of the transit with respect to the horizontal, in seconds

α = vertical angle to the high point.

The value of b can be determined as under :

Let l_1 and r_1 be the left hand and right hand readings of the bubble ends in one position, and l_2 and r_2 be the left hand and right hand readings of the bubble ends in the second position.

Deviation of the centre of the bubble from the centre of the striding level in the first position $= \dfrac{l_1 - r_1}{2}$

Deviation of the centre of the bubble from the centre of the striding level in the second position $= \dfrac{l_2 - r_2}{2}$.

$\therefore$ The mean deviation of the centre of the bubble from the centre of the striding level $= \dfrac{1}{2}\left\{\dfrac{l_1 - r_1}{2} + \dfrac{l_2 - r_2}{2}\right\} = \dfrac{(l_1 + l_2) - (r_1 + r_2)}{4} = \dfrac{\Sigma l - \Sigma r}{4}$

$\therefore$ Inclination of trunnion axis in seconds $= b = \dfrac{\Sigma l - \Sigma r}{4} \times d$ ...(1.28)

where d = angular value of one division of the striding level

Σl = the sum of the readings of the *left hand end* of the bubble in the direct and reversed positions of the striding level on the trunnion axis.

Σr = the sum of the readings of the *right hand end* of the bubble in the direct and reversed positions of the striding level on the trunnion axis.

The left-hand end of the axis will be higher if Σl is greater than Σr, and lower if Σl is less than Σr.

If the observed angle is the angle of elevation, the correction will be positive when the left-hand end of the axis is higher and negative when the left-hand end is higher.

If the observed angle is the angle of depression, the correction will be positive when the right-hand end of the axis is higher and negative when the left-hand end is higher.

The horizontal circle reading for each direction should be corrected separately and then the horizontal angle should be obtained by subtraction.

(B) ASTRONOMICAL CORRECTIONS

The observed or *apparent altitudes* of the celestial bodies like the sun or stars should be subjected to the following corrections:

1. Correction for parallax
2. Correction for refraction
3. Correction for dip of the horizon
4. Correction for semi-diameter.

1. Correction for Parallax. Parallax is the apparent change in the direction of a body when viewed from different points. The parallax in altitude, or *diurnal parallax*, is due to the difference in direction of a heavenly body as seen from the centre of the earth and from the place of observation on the surface of the earth. The stars are very far and the parallax is insignificant since the direction of rays as seen from the earth's surface and as seen from the centre of the earth are practically parallel. However, in the case of sun or moon, the parallax is significant and proper correction should be applied for the same.

Fig. 1.35 illustrates the sun's parallax.

O = Centre of the earth ; A = Place of observation

S = Position of the sun during observation; S' = Position of the sun at horizon.

OC = True horizon

AB = Sensible horizon

$\alpha' = \angle SAB$ = Observed altitude

$\alpha = \angle SOC$ = True altitude, corrected for parallax;

$p_a = \angle ASB$ = Parallax correction

$p_h = AS'O$ = Sun's horizontal parallax.

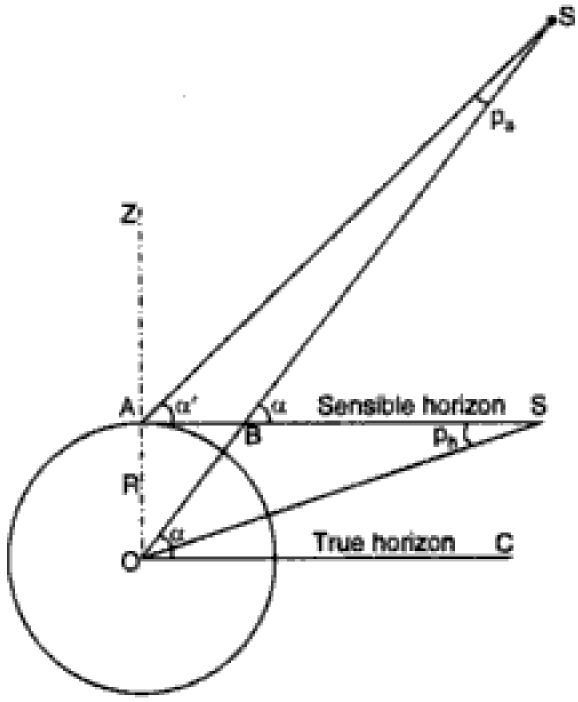


FIG. 1.35 SUN'S PARALLAX.

When the sun is on the horizon, its apprarent or observed altitude is zero, and the angle (p_h) subtended at the centre of the sun is known as sun's *horizontal parallax*.

Evidently, $\sin p_h = \dfrac{R}{OS'}$

Thus, the sun's horizontal parallax varies *inversely* with its distance from the centre of the earth. It varies from 8.95" early in January to 8.66" early in July, and is given in the Nautical Almanac for every tenth day of the year. The mean value of the sun's horizontal parallax is 8.8".

Now true altitude $\alpha = SOC = SBS' = SAB + ASB = \alpha' + p_a$

Hence parallax correction $= (\alpha - \alpha') = p_a$

From triangle AOS, $\sin ASO = \sin OAS \cdot \dfrac{OA}{OS}$

or $$\sin p_a = \sin (90° + \alpha') \frac{OA}{OS} = \cos \alpha' \cdot \frac{OA}{OS}$$

But $$\frac{OA}{OS} = \frac{OA}{OS'} = \sin p_h$$

$\therefore$ $$\sin p_a = \sin p_h \cos \alpha' \qquad ...[1.29\ (a)]$$

Since p_a and p_h are very small, we have

$$p_a = p_h \cos \alpha' \qquad ...[1.29\ (a)]$$

or ***correction for parallax = horizontal parallax × cos apparent altitude*** $= + 8''.8 \cos \alpha'$ ...(1.29)

The correction for parallax is always additive. The correction is maximum when the sun is at horizon.

2 Correction for Refraction. The earth is surrounded by the layers of atmospheric air. The layers get thinner and thinner as its distance from the surface increases. When a ray of light emanating from a celestial body passes through the atmosphere of the earth, the ray is bent downward, as shown in Fig. 1.36 and the body appears to be nearer to the zenith than it actually is.

The angle of deviation of the ray from its direction on entering the earth's atmosphere to its direction at the surface of the earth is called the *refraction angle* of correction. The refraction correction is always subtractive to the observed altitude. The magnitude of refraction depends upon the following:

(*i*) the density of air
(*ii*) the temperature
(*iii*) the barometric pressure
and (*iv*) the altitude.

It is constant for all bodies and does not depend upon the distance of the body from the observer.

At a pressure of 29.6 inches of mercury and a temperature of 50° F, the correction for refraction can be calculated from the following formula :

Correction for refraction (in seconds)

$$= 58'' \cot \alpha = 58'' \tan z \qquad \ldots(1.30)$$

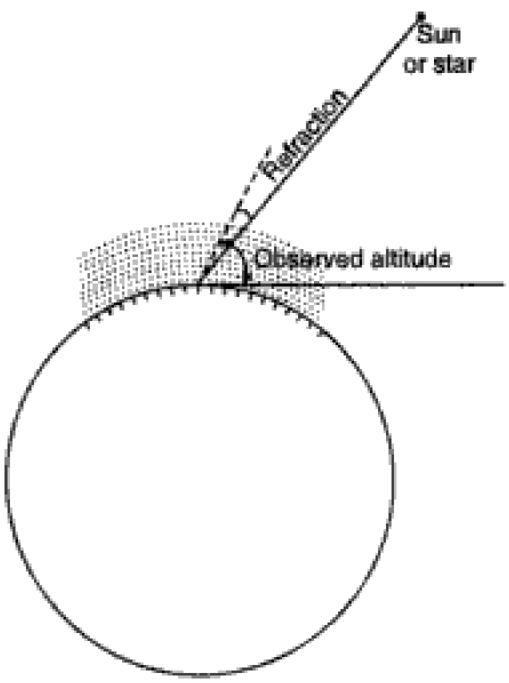


FIG. 1.36. REFRACTION.

where α = the apparent altitude of the heavenly body
z = the apparent zenith distance of the heavenly body.

The correction for refraction is always subtractive.

The values of mean refraction for different altitudes are given in Chamber's Mathematical Tables corresponding to barometer pressure, temperature of external air and temperature of thermometer attached to barometer.

The refraction correction for low altitudes is uncertain and hence observation for precise determination should never be taken on a celestial body which is nearer the horizon. The refraction, however, does not affect the azimuth.

3. Correction for Dip of the Horizon. The angle of the dip is the angle between the true and visible horizon. When the observations are taken with the help of a sextant at the sea, the altitude of the star or sun is measured from the visible horizon of the sea. Owing to the curvature of the earth, the visible horizon is below the true horizon. Hence, the angle of dip (*i.e.* the angle between the two horizons) must be subtracted from the observed altitude of the body.

In Fig. 1.37,

A = position of the observer
$AB = h$ = Height of the observer above sea level
S = position of the sun or star
AD = visible horizon
AC = true horizon
$\angle SAD = \alpha'$ = observed altitude of the sun or star
$\angle SAC = \alpha$ = true altitude of the sun or star
$\angle CAD = \beta$ = angle of dip

R = radius of the earth

Now, $BO = R$; $AO = (R + h)$

$\therefore \quad AD = \sqrt{(R + h)^2 - R^2}$

$\angle CAD = \angle AOD = \beta$

$$\therefore \quad \tan \beta = \frac{AD}{OD} = \frac{\sqrt{(R + h)^2 - R^2}}{R} = \sqrt{\frac{h\,(2R + h)}{R^2}} \quad \text{...(exact)} \quad \text{...[1.31 } (a)]$$

$$\text{or} \quad \tan \beta = \sqrt{\frac{2h}{R}} \quad \text{...(approximately)...[1.31 } (b)]$$

If β is small, we many have

$$\tan \beta = \beta \text{ (radians)} = \sqrt{\frac{2h}{R}} \quad \text{...(1.31)}$$

The correction for dip is always subtractive.

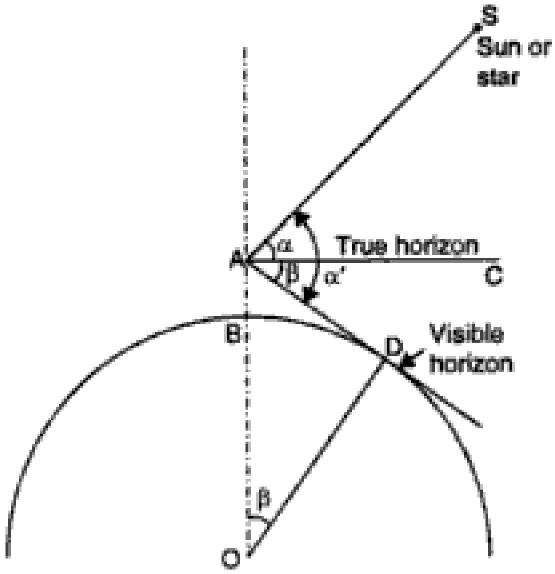


FIG. 1.37. DIP OF THE HORIZON.

4. Correction for Semi-diameter. The semi-diameter of the sun or star is half the angle subtended at the centre of the earth, by the diameter of the sun or the star. Since the distance of the sun from the earth is not constant throughout the year, the semi-diameter varies from 15′ 46″ in July to 16′ 18″ in January. Its value at its mean distance from the earth is 16′ 1″.18. The Nautical Almanac gives the values of sun's semi-diameter for every day in the year.

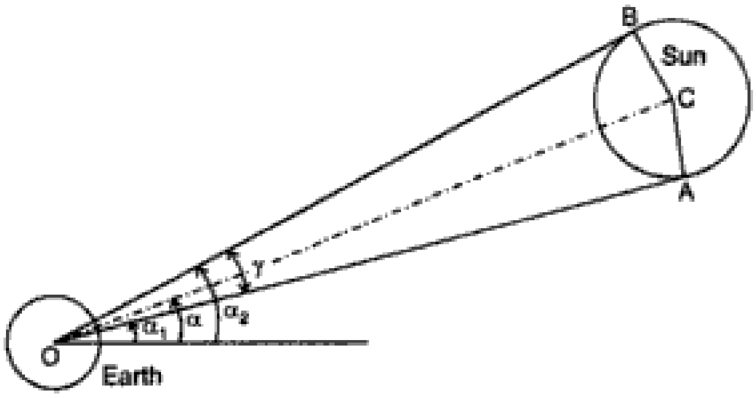


FIG. 1.37. (*a*) CORRECTION FOR SEMI-DIAMETER.

As the sun is large, its centre cannot be sighted precisely, and it is customary to bring the cross-hairs tangent to the sun's image. When the horizontal cross-hair is brought tangent to the lower edge of the sun, the sight is said to be taken at sun's lower limb [Fig. 1.38 (*a*)]. Similarly, when the horizontal cross-hair is brought tangent to the upper edge of the sun, the sight is said to be taken at sun's upper limb [Fig. 1.38 (*b*)]. Figs. 1.38 (*c*) and 1.38 (*d*) illustrate the observations taken to sun's right limb and left limb respectively.

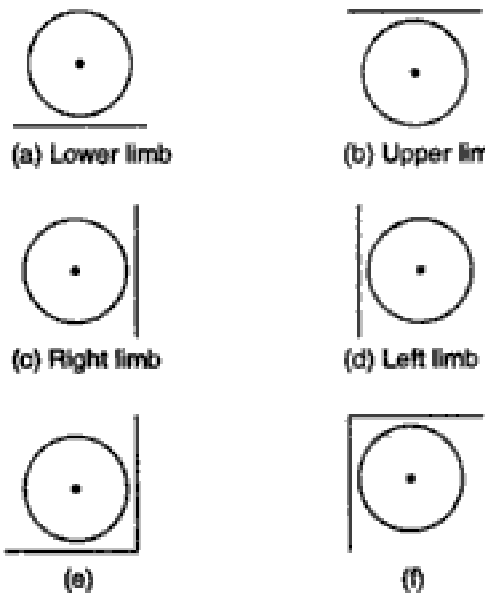


FIG. 1.38. OBSERVATION TO SUN.

In Fig. 1.37 (*a*), *OA* is the ray corresponding to the lower limb of the sun. The observed altitude α_1 is evidently lesser than the correct altitude α. Similarly, *OB* is the ray corresponding to the upper limb of the sun. The observed altitude α_2 is evidently more then the correct altitude α. If $\frac{\gamma}{2}$ is the semi-diameter, we have

$$\alpha = \alpha_1 + \frac{\gamma}{2} = \alpha_2 - \frac{\gamma}{2}$$

When a horizontal angle is measured to the sun's right or left limb, a correction equal to the sun's semi-diameter times the secant of the altitude is applied.

Thus, ***correction for semi-diameter in azimuth =semi-diameter*** $\times$ ***secant*** α.

Example 1.42. *Determine the value of horizontal angle between two points A and B, the observations for which were made with a theodolite in which one division of the striding level corresponds to 20".*

Object	*Azimuth*	*Vertical angle*	*Striding level 1st Position*		*Readings After reversal*	
			l	*r*	*l*	*r*
A	*32° 41′ 30″*	*+ 10° 21′ 12″*	*11*	*7.5*	*10.5*	*8*
B	*110° 28′ 42″*	*− 2° 18′ 30″*	*11.5*	*7.0*	*10.0*	*7.5*

Except for the adjustment of transverse axis not being perpendicular to the vertical axis, all other adjustments were correct.

Solution.

Let us first find the value of b.

(*a*) ***Observations of A*** : $\Sigma l = 11 + 10.5 = 21.5$; $\Sigma r = 7.5 + 8 = 15.5$

$$\therefore \quad b = \frac{\Sigma l - \Sigma r}{4} \cdot d = \frac{21.5 - 15.5}{4} \times 20 = +30''$$

Thus, the left end of the axis is higher.

$\therefore$ The correction $c = b \tan\alpha = 30 \tan 10° 21' 12'' = 5''.48$ seconds. Since the vertical angle is the angle of elevation and the left-hand end of the bubble tube is higher, the correction is positive.

$\therefore$ Corrected azimuth $= 32° 41' 30'' + 5''.48 = 32° 41' 35''.48$.

(*b*) ***Observation to B*** : $\Sigma l = 11.5 + 10 = 21.5$; $\Sigma r = 7.0 + 7.5 = 14.5$

$$\therefore \quad b = \frac{\Sigma l - \Sigma r}{4} d = \frac{21.5 - 14.5}{4} \times 20 = +35''$$

Thus, the left end of the axis is higher.

The correction $c = b \tan\alpha = 35 \tan 2° 18' 30'' = 1.41$ seconds.

Since the vertical angle is the angle of depression and the left-hand end of the bubble tube is higher, the correction is negative.

$\therefore$ Corrected azimuth $= 110° 28' 42'' - 1''.41 = 110° 28' 40''.59$

Hence horizontal angle between A and $B = 110° 28' 40''.59 - 32° 41' 35''.48 = 77° 47' 5''.11$.

Example 1.43. *To determine the index error of a theodolite, a church spire was sighted and the face left and face right observations were 18° 36′ 48″ and 18° 35′ 56″ respectively. A face right observation on the sun's lower limb was then made and the altitude was* *found to be 28° 36′ 20″. The semi-diameter of the sun at the time of observation was 15′ 59″.35. Find the true altitude of the sun.*

Solution

The observed altitude of the sun is to be corrected for

(*i*) index error (*ii*) semi-diameter (*iii*) refraction (*iv*) parallax.

(*i*) *Corrections for index error*

Mean of the vertical angle readings $= \frac{1}{2}$ (18° 36′ 48″ + 13° 35′ 56″) = 18° 36′ 22″

∴ Index error for the face right reading = 18° 36′ 22″ − 18° 35′ 56″ = + 26″.

The observed altitude of the sun	= 28° 36′ 20″
Add index correction	= 26″
∴ Altitude of sun corrected for index error	= 28° 36′ 46″.

(*ii*) *Correction for semi-diameter*

Since the lower limb of the sun was observed, the correction is positive.

Altitude of sun corrected for index error	= 28° 36′ 46″
Add semi-diameter	= 15′ 59″.35
∴ Altitude of sun corrected for index eeeor and semi-diameter	= 28° 52′ 45″.35

(*iii*) *Correction for refraction*

The correction for refraction is always subtractive and is equal to − 57″ cot 28° 28′ 46″ = − 1′44″.48.

(*iv*) *Correction for parallax*

The correction for parallax is positive and is equal to 8″.8 cos 28° 36′ 46″ = + 7″.80

Altitude of sun corrected for index error and semi-diameter	= 28° 52′ 45″.35
Subtract refraction correction	= 1′ 44″. 48
	= 28° 51′ 0″.87
Add parallex correction	= 7″.80
∴ Correct altitude of the sun	**= 82° 51′ 8″.67**

1.12. OBSERVATIONS FOR TIME

The observations for determining the local time consists mainly in finding the error of watch or chronometer which is read at the instant the observations are made. If the chronometer keeps the sidereal time, it is required to determine the hour angle of the Vernal Equinox (or a star) at the time of observation. Similarly, if the chronometer keeps the solar time, it is required to determine the hour angle of the centre of the sun at the instant the observations are taken. Determinations are made from meridian or ex-meridian observations. The difference between the chronometer time and the time determined from the observation gives *chronometer correction* and should be added algebraically to the reading of the watch to give the true time at the instant. *The correction is positive when the chronometer is slow and negative when it is fast.*

The following are some of the methods usually employed for the determination of time :

(1) By meridian observation of a star or the sun. (By transit of a star or sun)

(2) By ex-meridian altitude of a star or the sun.

(3) By equal altitudes of star or the sun.

1 (*a*) TIME BY MERIDIAN TRANSIT OF A STAR

The application of this method requires a knowledge of the local longitude and a previous determination of the direction of the meridian. This forms the most direct method of obtaining local time and is used for primary field determinations. The basis of the method is the fact that when a star transits the meridian, its hour angle is zero and local sidereal time is equal to the right ascension of the star.

In Fig. 1.39, *ZP* is the observer's meridian and *M* is the position (in general) of a star.

$\angle SP\Upsilon$ = Local sidereal time

$\angle SPM$ = Hour angle (H) of the star (measured westward)

$\angle \Upsilon PM$ = R.A. of the star.

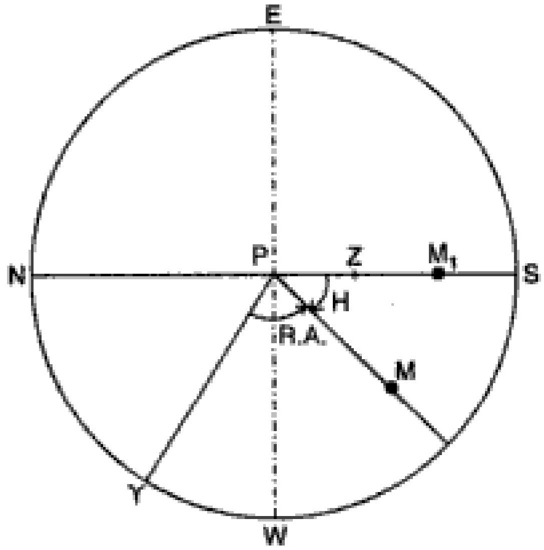


FIG. 1.39

Evidently, $\angle SP\Upsilon = \angle SPM + \angle \Upsilon PM$

or L.S.T. = Hours angle + R.A.

M_1 is the position of the star when it crosses the meridian, and its hour angle (H) is zero. Thus,

$$\text{L.S.T.} = \text{R.A.}$$

The right ascensions of various stars are given in the Ephemeris for the date.

The star is observed with a theodolite, the line of sight being directed along the known direction of the meridian. The chronometer is read at the instant the star transits across the vertical wire. The chronometer error is then determined by comparing the true sidereal time (equal to the right ascension) of the star with the sidereal time kept by the watch or chronometer. If the chronometer is keeping Greenwich sidereal time, it is necessary to apply only the local longitude to the right ascension of the star to obtain the true Greenwich sidereal time. If the chronometer keeps the local mean time, the local sidereal time determined above is converted into local mean time by method discussed earlier and the error of the chronometer is determined. Generally, the chronometer error is found in this way on two different days and average daily rate of error during the period is found by dividing the change in the error by the number of days elapsed.

1. (*b*) TIME BY MERIDIAN TRANSIT OF THE SUN

When the sun is observed on the meridian of the place at upper transit, its hour angle is zero and the L.A.T. is 12 hours. The transit of the sun is observed with a theodolite and the times at which the east and west limbs of the sun pass the vertical hair are noted by means of the chronometer. The mean of the two readings gives the mean time at the local apparent noon. If only one limb is observed, allowance must be made for the time that the semi-diameter takes to cross the meridian. From the Nautical Almanac, we can find the G.M.T. of G.A.N. for the given date, from which the L.M.T. of L.A.N. may be found. This L.M.T. of L.A.N. can then be compared with the chronometer time at the instant of the observation to give the error of the chronometer.

Error in the Observations of the Meridian Transit of Star or Sun

The method of meridian transit of a star or the sun, though simple, is not very much used because it is impracticable to secure that the instrumental line of sight lies exactly in the plane of the meridian. The observed times are subject to the following three principal corrections :

(*i*) The Azimuth Correction

If the instrument is in accurate adustment, but the direction of the meridian is in error, the line of sight set out along the meridian will pass through the zenith of the observer and not through the celestial pole. The correction is given by

$$\textit{Azimuth correction} = e \sin z \sec \delta$$

where e = error of azimuth in *seconds of time*

z = zenith distance

δ = declination of the star.

e is considered positive if the line of sight it too far east when the telescope is pointed south, and is negative if the line of sight is too far west. It can be shown that if the latitude of the place is 30° and the polar distance of a star is 40°, *an error of 1 minute of arc in the direction of the meridian will make the time of transit wrong by two seconds*. The method, therefore, requires the meridian to be set out very accurately.

The error is very great if the polar distance of the star is small, and is least for those that transit near the zenith.

(*ii*) The Level Correction

If the horizontal axis is not perfectly horizontal, the line of sight may depart considerably at high altitudes. Due to this, the transit will be observed either too soon or too late according to the direction of tilt of the transverse axis. The correction is given by :

$$\textit{Level correction} = b \cos z \sec \delta$$

where b = inclination of the horizontal axis in seconds of arc (determined by the readings of the striding level) and is positive when the left (or west) end of the axis is higher

z = zenith distance

δ = declination of the star.

(iii) The Collimation Correction

The collimation correction is necessary when the line of sight is not perpendicular to the horizontal axis. The correction is given by :

Collimation correction $= c \sec \delta$

where c = error of collimation in seconds of time taken positive when the line of sight is to east of the meridian, and negative when it is to the west).

δ = declination of the star.

2. (a) TIME BY EX-MERIDIAN OBSERVATION OF A STAR

The determination of time by ex-meridian observation of a star or sun is the most convenient and suitable method for surveyor. The method, in its simplest form consists in observing the altitude of the star when it is out of the meridian and at the same time observing the chronometer of the star and its altitude ; the hour angle can be computed by the solution of the astronomical triangle. The local sidereal time can then be known by adding the westerly hour angle to the R.A. of the star. The local sidereal time can be converted into local mean time and the error of chronometer (observing mean solar time) can be found.

In the astronomical triangle ZPM (Fig. 1.15), we know the following three sides:

ZP = co-latitude = $(90° - \theta) = c$ (say)

$MP = p$ = polar distance = $(90° - \delta)$

$ZM = z$ = zenith distance = $(90° - \alpha)$

$\angle MPZ = H$ = hour angle which can be computed from any one of the following formulae :

$$\tan \frac{H}{2} = \sqrt{\frac{\sin (s-c) \cdot \sin (s-p)}{\sin s \cdot \sin (s-z)}} \quad ...(1) \quad ; \quad \sin \frac{H}{2} = \sqrt{\frac{\sin (s-c) \cdot \sin (s-p)}{\sin c \cdot \sin p}} \quad ...(2)$$

$$\cos \frac{H}{2} = \sqrt{\frac{\sin s \cdot \sin (s-z)}{\sin c \cdot \sin p}} \quad ...(3) \quad ; \quad \cos H = \frac{\sin \alpha - \sin \theta \sin \delta}{\cos \theta \cos \delta} \quad ...(4)$$

where $s = \frac{1}{2}(z + c + p)$

It should be noted that if H is near to 0° or 90°, the tangent formula is the best one to adopt since it gives more precise result.

In the field observation, the altitude has to be observed and refraction correction must be applied. Due to uncertainties in the refraction for low altitudes, the star observed should have an altitude of at least 15°.

When the star is in or near the prime vertical, its altitude changes more rapidly and the star should be observed at this time since it gives more accurate results. The influence of error in observed altitude as well as in the value of the altitude, is a minimum when the star is actually on the prime vertical. To minimise the errors of observation, several altitudes of the star are observed in quick succession and the chornometer time of such observation is recorded. Half of the observations are taken with face left and half with the face right. If the observations are completed within a few minutes (say 10^m) it will suffice for most ordinary work if the mean of the chronometer times is taken as the time for the mean altitude. *The motion of the star in altitude is not however, exactly proportional to time.* More accurate results are obtained when two stars are observed, one east and the other west of the meridian, thus eliminating the instrumental errors.

When the star is observed on its prime vertical, the hour angle is given by

$$\cos H = \frac{\tan \text{declination}}{\tan \text{latitude}} = \frac{\tan \delta}{\tan \theta}$$

Knowing the hour angle (in degrees), the L.S.T. is calculated from the formula :

$$\boldsymbol{L.S.T. = R.A. \pm \frac{H}{15}}$$

Plus sign is used when the star is to the west of the meridian and minus when it is to the east. Knowing the G.S.T. of G.M.M. (for G.M.N.), the L.S.T. can converted to L.M.T. and the error of the chronometer keeping the mean solar time can be computed.

2. (*b*) TIME BY EX-MERIDIAN OBSERVATION OF THE SUN

The procedure of observation of the sun is the same as in the previous case. The altitude of the lower limb is observed with the telescope normal, and then the altitude to the upper limb is observed with the telescope inverted. The watch time at the instant of each observation is noted. The balancing is affected by measuring a succession of altitudes both in the morning and afternoon, the most suitable timings being between 8 and 9 A.M. and between 3 and 4 P.M. In each set, a minimum number of four observations are taken — both face observations of upper limb and both face observations lower limb. If the sun is not very near the meridian and if the observations extend over only a few minutes of time (say 10^m), the mean of the observed altitudes may be assumed to correspond to the mean of the observed times, thus neglecting the curvature of the path of the sun. The mean of the altitudes must be corrected for index error, refraction, and parallex, and for the semi-diameter if only one limb is observed. The hour angle of the sun can be calculated from the formula :

$$\tan \frac{H}{2} = \sqrt{\frac{\sin (s-c) \sin (s-p)}{\sin s \,.\, \sin (s-z)}}$$

The above formula is more convenient for logarithmic computations. Then, if the sun is to the west of meridian,

$$\text{L.A.T. of observation} = \frac{H}{15} \text{ since local apparent noon.}$$

When the sun is to the east of meridian,

$$\text{L.A.T. of observation} = \left(24^h - \frac{H}{15}\right) \text{ since local apparent noon}$$

$$= \left(12^h - \frac{H}{15}\right) \text{ since local apparent midnight.}$$

The L.A.T. can then be converted into L.M.T. by methods discussed earlier.

In the above computations, a correct knowledge of sun's declination (δ) is required. For the computation of sun's declination for the mean instants of observation, a knowledge of local time is necessary. Since the local time is being determined, the computation of H should be performed by successive approximation. However, if the watch is not more than 2^m or 3^m in the error, the resulting error in computing the declination will not exceed 2″ or 3″, and recalculations are not necessary if observations are made with small instrument. If greater discrepancy is found between the correct and the chronometer time, the former is used for a better interpolation of δ and the computation of H is repeated with the new value. Also, a knowledge of the latitude of the place is essential for the computation of H. The precision in the knowledge of the latitude of the place depends upon the precision in the observation of altitude and also upon the time at which observation is made. When the sun is near the prime vertical, the effect of an error in latitude is small.

The error of the watch on local mean time is then equal to the difference between the time of observation by watch and the time of observation as determined by calculations. The observation is often combined with the observation of the sun for azimuth, the watch readings and altitude readings being common to both.

Booking of Field Observations

The field observations are usually entered in the field book in the following form: (Table 1.1).

TABLE 1.1

Star observed	*Face*	*Vertical Angle*											*Time*			*Mean of time*		
		A			*B*		*Mean*			*Mean vertical Angle*								
		°	′	″	′	″	°	′	″	°	′	″	*h*	*m*	*s*	*h*	*m*	*s*
α –ophiuchi W	L	38	30	20	30	40	38	30	30				7	21	11			
	R	37	26	30	26	10	37	26	20				7	27	30			
	R	36	30	40	30	20	36	30	30				7	32	20			
	L	35	50	10	50	00	35	50	5	37	4	21	7	38	05	7	29	46.5

3. (*a*) TIME BY EQUAL ALTITUDE OF A STAR

In this method, a star is observed at the same altitude on opposite sides of the meridian. The mean of the two chronometer times at which a star attains equal altitudes east and west of the meridian is evidently the chronometer time of transit, since the two observations are clearly made at equal intervals of time before and after the Star's meridian transit. The method is, therefore, very simple and accurate and is used when the direction of the meridian is not accurately known. The altitude of the star need not be determined and, therefore, no correction is required for refraction. The observations must be made when the star is near the prime vertical so that its altitude changes rapidly. When the star crosses the meridian, its hour angle is equal to zero and its right ascension

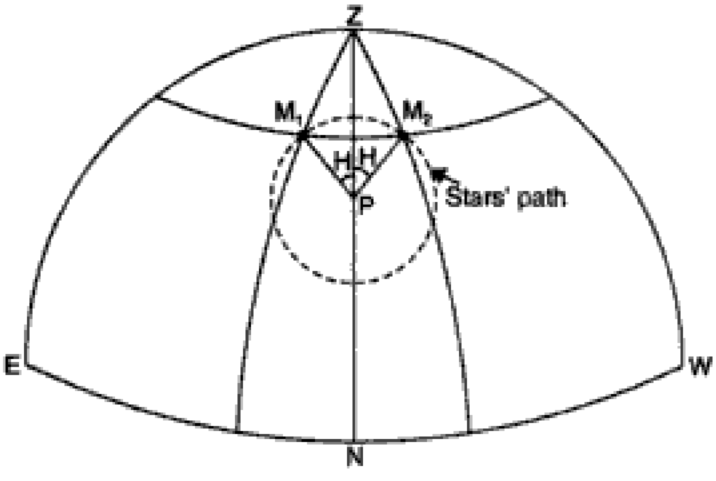


FIG. 1.40. TIME BY EQUAL ALTITUDE.

is therefore the local sidereal time. The local sidereal time so obtained may be converted to local mean time which can then be compared with the mean time of the chronometer during the observations, and the error of the chronometer can be known.

To make the observations, the following steps are necessary :

(1) Set up the instrument on firm ground and level it accurately.

(2) Compute the approximate altitude of the star and set it on the vertical circle.

(3) Follow the motion of the star in azimuth with the vertical cross-hair by means of horizontal tangent screw.

(4) Note the chronometer time (T_1) when the star crosses the horizontal hair.

(5) Turn the instrument in azimuth and again follow the star when the star approaches the same altitude to the other side of the meridian.

(6) Note the chronometer time (T_2) when the star crosses the horizontal hair.

Mean time of transit of the star $= \frac{1}{2}(T_1 + T_2)$

It is very important to note that during the above observations the face of the theodolite is not changed. However, the altitude bubble must be accurately centred by means of clip screws prior to each observation. For accurate results, a series of observations are made on the same star.

In Fig. 1.40, the dotted circle shows the daily path of the star round the pole. M_1 is the position of the star of the east of the meridian ZP and M_2 is its position to the west of the meridian when it attains the same altitude as at M_1.

The method has the following advantages :

(1) Since the actual altitude of the star is not required the instrumental errors– such as index error, collimation error, errors due to graduations etc. are not involved.

(2) No knowledge is required of latitude, declination, or even azimuth.

The method has, however, the following disadvantages :

(1) A long interval of time elapses between the two observations – sometimes several hours.

(2) The precision of the result depends upon the refraction having the same value for both observations. Due to long interval of time, the refraction may change appreciably, thus affecting the result.

However, the time between the two observations can be reduced if the declination of the selected star is nearly equal to the latitude. To eliminate the uncertainties of refraction near the horizon, the star should have an altitude of something more than 45°.

The Error due to Slight Inequality in the Altitudes of Two Corresponding Observations:

In Fig. 1.40,

ZM_1 = zenith distance of first observation = z

ZP = co-latitude = c

PM_1 = polar distance = p

ZPM_1 = hour angle = H

$M_1ZP = A$ = azimuth of the star.

Now, we have $\cos z = \cos c \cos p + \sin c \sin p \cos H$ ...(1)

When the star is at M_2, let

ZM_2 = zenith distance of seconds observation = $(z + y)$

where y is the small error due to inequality of the altitudes.

ZPM_2 = hour angle of $M_2 = (H + x)$

where x is the small error in the hour angle.

Hence we have $\cos (z + y) = \cos c \cos p + \sin c \sin p \cos (H + x)$ ...(2)

Subtracting (2) from (1) and treating x and y as small quantities, we get

$$y \sin z = x \sin c \,.\, \sin p \,.\, \sin H.$$

But
$$\frac{\sin z}{\sin H} = \frac{\sin p}{\sin A}$$

Hence
$$x = \frac{y \sin z}{\sin c \sin p \sin H} = \frac{y}{\sin c \sin A} \quad ...(3) \quad ...(1.32)$$

In order that x should be least for a given value of y, we must have $\sin A = 1$ or $A = 90°$. The error will evidently be greater for smaller value of A. *Hence we conclude that the error in the hour angle due to some error in altitude is minimum when the star is near the prime vertical.*

3. (*b*) TIME BY EQUAL ALTITUDES OF TWO STARS

The two disadvantages of the method of equal altitudes mentioned above (*i.e.* the long interval of time and the uncertainties in the value of refraction) can be reduced by making the equal altitude observations on two stars, one east and the other west of the meridian. In such observations, two stars having the same declination are selected. When they attain the same altitudes, one to the west and other to the east of the meridian, the mean of their right ascension will give the local sidereral time of transit. The local sidereal time can be converted into L.M.T. and can be compared with the mean of the chronometer readings for the determination of the chronometer error. If the two stars have some different declinations, a correction must be applied to the mean of their right ascensions. However, the difference in the declination of the two stars should not be more than 2° to 5°. The observations of a pair of stars generally takes few minutes. Several pairs should be used for good determination. The stars selected to form a pair should have a difference in right ascension of at least 6^h.

3. (*c*) TIME BY EQUAL ALTITUDS OF THE SUN

If the equal altitude observations are made on the sun, the same edge of the sun's image (*i.e.*, the upper limb or lower limb) should be brought to the horizontal hair and the image bisected by the vertical hair of the diaphragm. A series of altitudes is taken about 9 AM. and the same series is repeated in reverse order about 3 P.M. The mean of the times of the forenoon and afternoon equal altitudes does not exactly represent the instant of transit (L.A.N.) due to the rapid change of sun's declination. *The theory becomes complicated due to the fact that allowance must be made for the alteration of declination in the interval between the observations.* In order to apply the correction for the change in the declination, the approximate value of the latitude and Greenwich mean time must be known.

Let y be the alteration in the sun's declination in *half* the time interval between the two observations.

In Fig. 1.40, M_1 = First position of the sun having polar distance $(p + y)$ say, when the sun is approaching the pole.

M_2 = Second position of the sun having the polar distance $(p - y)$, say,

If p were constant, we have, as earlier. $\cos z = \cos p \cos c + \sin p \sin c \cos H$ ...(1)

But the polar distance is $(p + y)$ and the hour angle is $(H + x)$. We have, thus

$$\cos z = \cos (p + y) \cos c + \sin (p + y) \sin c \cos (H + x) \qquad ...(2)$$

Subtracting (1) from (2), and treating x and y to be small quantities, we have

$$x = y (\cot p \cot h - c \cot \operatorname{cosec} H) \qquad ...(3) \quad ...(1.33)$$

For a given value of y, therefore, the value of x can be computed from the given equation.

The first observation will thus be made when the sun's hour angle is $(H + x)$ *before the apparent noon*. Similarly, the second observation will be made when the sun's hour angle is $(H - x)$ *after the apparent noon. The mean of these two observed times will therefore be when the sun is at an hour angle x before apparent noon.*

For example, let $H = 3$ hours ; and $x = 1$ min. (calculated from Eqn. 3)

Then, the hour angle of sun at first observation $= (H + x)$

$= 3$ hour 1 min. before apparent noon.

$\therefore$ Time of observation $= 12^h - 3^h\ 1^m = 8^h\ 59^m$ apparent time.

Similarly, the hour angle of sun at second observation $= H - x$

$= 2^h\ 59^m$ after apparent noon.

$\therefore$ Time of observation $= 12^h + 2^h\ 59^m = 14^h\ 59^m$ apparent time.

$\therefore$ Mean time of observation $= \frac{1}{2}(8^h\ 59^m + 14^h\ 59^m) = 11^h\ 59^m$

$= 1^m$ before the apparent noon

$= x$ before the apparent noon.

Hence we get the following rule :

True time of transit (*i.e.*, apparent noon)

$$= \text{Mean of observed time} \mp \frac{x}{15} \text{ (When } x \text{ is in angular measure).}$$

Minus sign is used when the sun is approaching the elevated pole (*i.e.*, the case discussed above) and plus sign when the sun is leaving the pole.

Example 1.44. *The time of transit of a star (R.A. $7^h\ 36^m\ 21.24^s$) recorded with a chronometer keeping standard time of $5^h\ 30^m$ E was $5^h\ 56^m\ 8.86^s$ P.M. The longitude of the place of observation is $4^h\ 30^m$ E. Determine the error of the chronometer if G.S.T. at G.M.M. on the day is $14^h\ 18^m\ 12^s$.*

Solution

Let us first convert the G.S.T. of G.M.M. into L.S.T. of L.M.M.

$$\text{Longitude} = 4^h\ 30^m\ E$$

Loss in the sidereal time at the rate of 9.8565^s per hour of longitude is :

$$4^h \times 9.8565 = 39.43 \text{ seconds}$$
$$30^m \times 0.1642 = 4.93 \text{ seconds}$$

Total retardation = 44.36 seconds

∴ L.S.T. of L.M.M. = G.S.T. of G.M.M. – Retardation

$= 14^h\ 38^m\ 12^s - 44.36^s = 14^h\ 37^m\ 27.64^s$

Now L.S.T. of observation = R.A. of the star = $7^h\ 36^m\ 21.24^s$

∴ S.I. = L.S.T. of observation – L.S.T. of L.M.M.

$= (7^h\ 36^m\ 21.24^s - 14^h\ 37^m\ 27.64^s) + 24^h = 16^h\ 58^m\ 53.6^s$.

Let us now convert the S.I. into mean time interval by subtracting the retardation at the rate of 9.8296 seconds per hour of sidereal time.

$$16^h \times 9.8296 = 157.27 \text{ seconds}$$
$$58^m \times 0.1638 = 9.49 \text{ seconds}$$
$$53.6^s \times 0.0027 = 0.14 \text{ second}$$

Total retardation = 166.90 seconds = $2^m\ 46.90^s$

∴ Mean time interval since L.M.M. = S.I. – Retardation

$= 16^h\ 58^m 53.6^s - 2^m\ 46.90^s = 16^h\ 56^m\ 6.7^s$

Standard time shown by chronometer

$= 5^h\ 56^m\ 8.86^s$ P.M. = $17^h\ 56^m\ 8.86^s$ since L.M.M.

∴ Local time of chronometer

$= 17^h\ 56^m\ 8.86^s$ – Difference of longitude

$= 17^h\ 56^m\ 8.86^s - 1^h = 16^h\ 56^m\ 8.86^s$

(Since the place of observation is at longitude 1^h to the west of standard meridian).

∴ Chronometer error = **2.16 seconds (Fast).**

Example 1.45. *The following notes refer to an observation for time made on a star on Feb. 18, 1965 :*

Latitude of the place	$= 36°\ 30'\ 30''$ *N*
Mean observed altitude of the star	$= 30°\ 12'\ 10''$
R.A. of star	$= 5^h\ 18^m\ 12.45^s$
Declination of the star	$= 16°\ 12'\ 18''.4$

This star is to the east of the meridian.

Mean sidereal time observed by sidereal chronometer $= 1^h\ 2^m\ 5.25^s$

Find the error of the chronometer.

Solution. The hour angle of the star is determined from the following formula :

$$\tan\frac{H}{2} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin s\,.\,\sin(s-z)}} \quad ; \quad \text{where} \quad s = \tfrac{1}{2}(z+c+p)$$

$$z = 90° - \alpha = 90° - 30°\ 12'\ 10'' = 59°\ 47'\ 50''$$

$$p = 90° - \delta = 90° - 16°\ 12'\ 18''.4 = 73°\ 47'\ 41''.6$$

$$c = 90° - \theta = 90° - 36°\ 30'\ 30'' = 53°\ 29'\ 30''$$

$$2s = 187°\ 05'\ 01''.6$$

$$s = 93°\ 32'\ 30''.8$$

$$(s-c) = 40°\ 3'\ 0''.8 \quad ; \quad (s-p) = 19°\ 44'\ 49''.2 \quad ; \quad (s-z) = 33°\ 44'\ 40''.8$$

$$\log\sin(s-c) = \bar{1}.8085208$$

$$\log\sin(s-p) = \bar{1}.5287565$$

$$\log\text{cosec}\, s = 0.0008302$$

$$\log\text{cosec}\,(s-z) = 0.2553212$$

$$\log\tan^2\frac{H}{2} = \bar{1}.5934287 \quad ; \quad \log\tan\frac{H}{2} = \bar{1}.7967144$$

$$\therefore \quad \frac{H}{2} = 32°\ 3'\ 17''.6 \quad \text{or} \quad H = 64°\ 6'\ 35''.2 = 4^h\ 16^m\ 26.3^s$$

Since the star is to the east of the meridian, the westerly hour angle

$$= 24^h - 4^h\ 16^m\ 26.3^s = 19^h\ 43^m\ 33.7^s$$

$$\text{R.A. of the star} = 5^h\ 18^m\ 12.45^s$$

$$\text{Add hour angle} = 19^h\ 43^m\ 33.70^s$$

$$\therefore \quad \text{L.S.T. of observation} = 25^h\ 01^m\ 46.15^s = 1^h\ 01^m\ 46.15^s$$

$$\text{Sidereal time by chronometer} = 1^h\ 2^m\ 5.25^s$$

$\therefore$ Error of chronometer $=$ **19.1**^{s} (fast).

Example 1.46. *The mean observed altitude of the sun, corrected for refraction, parallax and level was 36° 14′ 16″.8 at a place in latitude 36° 40′ 30″ N and longitude 56° 24′ 12″ E. The mean watch time of observation was* $15^h\ 49^m\ 12.6^s$ *the watch being known to be about* 3^m *fast on L.M.T. Find the watch error given the following :*

Declination of the sun at the instant of observation = + 17° 26′ 42″.1.

G.M.T. of G.A.N. $= 11^h\ 56^m\ 22.8^s$.

Solution

The hour angle of the sun is given by the formula

$$\tan \frac{H}{2} = \sqrt{\frac{\sin (s-c) \sin (s-p)}{\sin s \,.\, \sin (s-z)}} \quad \text{where} \quad s = \frac{1}{2}(z+c+p)$$

Here

$$z = 90° - \alpha = 90° - 36° \, 14' \, 16''.8 \quad = 53° \, 45' \, 43''.2$$
$$p = 90° - \delta = 90° - 17° \, 26' \, 42''.1 \quad = 72° \, 33' \, 17''.9$$
$$c = 90° - \theta = 90° - 36° \, 40' \, 30'' \quad = 53° \, 19' \, 30''.0$$

$$2s = 179° \, 38' \, 31''.1 \quad ; \quad s = 89° \, 49' \, 15''.6$$

$$(s-c) = 36° \, 29' \, 45''.6 \quad ; \quad (s-p) = 17° \, 15' \, 57''.7 \quad ; \quad (s-z) = 36° \, 03' \, 32''.4$$

$$\log \sin (s-c) = \bar{1}.7743468$$
$$\log \sin (s-p) = \bar{1}.4724776$$
$$\log \text{cosec}\, s = 0.0000919$$
$$\log \text{cosec}\, (s-z) = 0.2301672$$

$$\log \tan^2 \frac{H}{2} = \bar{1}.4770835 \quad ; \quad \log \tan \frac{H}{2} = \bar{1}.7385417$$

$$\frac{H}{2} = 28° \, 42' \, 34''.1 \quad \text{or} \quad H = 57° \, 25' \, 08''.2 = 3^h \, 49^m \, 40.6^s$$

$\therefore$ L.A.T. $= 15^h \, 49^m \, 40.6^s$

Let us convert this to L.M.T.

Longitude $= 56° \, 24' \, 12'' = 3^h \, 45^m \, 36.8^s$

L.A.T. $= 15^h \, 49^m \, 40.6^s$

Subtract longitude $= 3^h \, 45^m \, 36.8^s$

$\therefore$ G.A.T. $= 12^h \, 04^m \, 03.8^s$

Now G.M.T. of G.A.N. $= 11^h \, 56^m \, 22.8^s$

or G.M.T. of 12^h apparent time $= 11^h \, 56^m \, 22.8^s$

Now Greenwich apparent time = Greenwich mean time + E.T.

$\therefore$ $12^h = 11^h \, 56^m \, 22.8^s$ + E. T.

$\therefore$ E.T. $= 12^h - 11^h \, 56^m \, 22.8^s = 3^h \, 37.2^s$

Subtractive from the apparent time.

G.M.T.= G.A.T. − E.T. $= 12^h \, 04^m \, 03.8^s - 3^m \, 37.2^s = 12^h \, 0^m \, 26.6^s$

$\therefore$ L.M.T.= G.M.T + longitude $= 12^h \, 0^m \, 26.6^s + 3^h \, 45^m \, 36.8^s = 15^h \, 46^m \, 03.4^s$

$\therefore$ Error of chronometer $= 15^h \, 49^m \, 12.6^s - 15^h \, 46^m \, 03.4^s = \mathbf{3^m \, 8.8^s}$ (Fast)

Example 1.47. *At a certain place in longitude 138° 45′ East, the star is observed East of the meridian at* $6^h \, 45^m \, 21^s$ *P.M. with a watch keeping local mean time. It was* *again observed at the same altitude to the west of meridian at $8^h\,48^m\,43^s$ P.M. Find the error of the watch given that*

G.S.T. at G.M.N. on that day = $9^h\,26^m\,12^s$; *R.A. of the star* = $17^h\,12^m\,48^s$

Solution

L.S.T. of transit of star across the meridian = R.A. of the star = $17^h\,12^m\,48^s$

Let us convert sidereal time into mean time.

Longitude = 138° 45′ E = $9^h\,15^m$ E. Since the place has east longitude,

L.S.T. at L.M.N. = G.S.T. at G.M.N. – retardation

$$9^h \times 9.8565^s = 88.71 \text{ seconds}$$
$$15^m \times 0.1642^s = 2.46 \text{ seconds}$$

$$\text{Total retardation} = 91.17^h = 1^m\,31.17^s$$

$$\text{G.S.T. at G.M.N.} = 9^h\,26^m\,12^s$$
$$\text{Subtract retardation} = 1^m\,31.17^s$$

$$\text{L.S.T. at L.M.N.} = 9^h\,24^m\,40.83^s$$
$$\text{Now local sidereal time} = 17^h\,12^m\,48^s$$
$$\text{Subtract L.S.T. at L.M.N.} = 9^h\,24^m\,40.83^s$$

$$\therefore \quad \text{S.I. since L.M.N.} = 7^h\,48^m\,07.17^s$$

Let us convert this S.I. into mean time interval by subtracting the retardation at the rate of 9.8296^s per sidereal hour.

$$7^h \times 9.8296 = 68.81 \text{ seconds}$$
$$48^m \times 0.1638 = 7.86 \text{ seconds}$$
$$7.17^s \times 0.0027 = 0.02 \text{ second}$$

$$\text{Total retardation} = 76.69 \text{ seconds} = 1^m\,16.69^s$$

$$\text{S.I.} = 7^h\,48^m\,07.17^s$$
$$\text{Subtract retardation} = 1^m\,16.69^s$$

$$\therefore \quad \text{M.I. since L.M.N.} = 7^h\,46^m\,50.48^s$$

∴ Local mean time of transit of star $7^h\,46^m\,50.48^s$ P.M. ...(1)

Now L.M.T. of watch for east observation = $6^h\,45^m\,21^s$ P.M.

L.M.T. of watch for west observation = $8^h\,48^m\,43^s$ P.M.

$$= 15^h\,34^m\,04^s$$

∴ L.M.T. of transit of the star as shown by the chronometer = $7^h\ 47^m\ 02^s$ P.M. ...(2)

Chronometer error = **11.52 seconds (Fast)**

1.13. TIME OF RISING OR SETTING OF A HEAVENLY BODY

In Fig. 1.41, *SEN* is the horizon and *M* is the position of a star when it is rising. It is required to find the time of rising and setting of the star.

The spherical triangle *PMN* is right-angled at *N*, since the plane of the observer's meridian is perpendicular to the horizon.

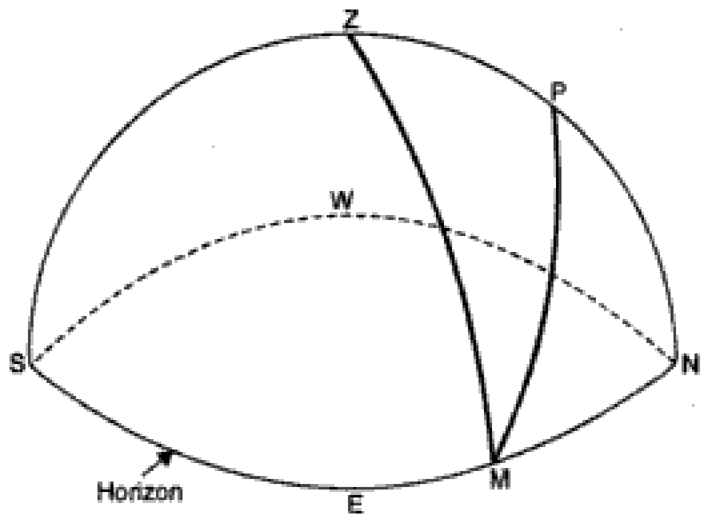


FIG. 1.41. RISING AND SETTING OF STAR.

∴ $\cos MPN = \cos MP \cdot \tan PN$

Now $\angle ZPM = H$ = hour angle of the star at its rising

$MP = \delta$ = declination of the star

$PN = \theta$ = altitude of the pole = latitude of the observer

$\angle MPN = 180° - H$

Hence $\quad$ ***$\cos H = -\tan\delta\ \tan\theta$***

Knowing the declination of the star and the latitude of the place, its hour angle can be known. Then,

L.S.T. of rising of star = R.A. of the star + Hour angle.

Thus, the local sidereal time of the rising of the star can be known, and this can be converted into L.M.T., if desired.

The hour angle of setting will obviously be the same as that of rising. In the above treatment, we have neglected the effect of refraction, which amounting as it does to about 36′ on the horizon, will cause stars to be just visible when they are really 36′ below the horizon.

Length of Day and Night :

The hour angle *H* of the sunrise or sun-set is given by

$\cos H = -\tan\delta\tan\theta$ where δ is the declination of the sun.

If the change in the declination δ of the sun is ignored

Length of the day = twice hour angle in time units = $\dfrac{2H}{15}$

Similarly, length of the night = $2\left(\dfrac{180° - H}{15}\right)$

The equation $\cos H = -\tan\delta\tan\theta$ can be used to determine the length of the day at different places and at different times.

(1) At a place at equator, $\theta = 0$

∴ $\cos H = 0$ or $H = 0°$ and $H = 90°$

$\therefore$ Length of day (or night) $= \dfrac{2H}{15} = 12^h$

Hence for all values of δ, the days are always equal to the nights at equator.

(2) At the time of equinox, the sun is at equator and hence $\delta = 0$

$\therefore \quad \cos H = 0 \quad$ or $\quad H = 0° \quad$ and $\quad H = 90°$

$\therefore$ Length of day (or night) $= \dfrac{2H}{15} = 12^h$

Hence for all values of θ (i.e., at all the places on the earth) the day is equal to the night.

(3) If $\quad \delta = 90° - \theta \quad ; \quad \cos H = -1$ or $H = 180°$

$\therefore$ Length of day $= \dfrac{2 \times 180°}{15} = 24°$ (*i.e.* the sun does not set).

(4) If $\quad \delta = -(90° - \theta) \quad ; \quad \cos H = 1$ and $H = 0$

$\therefore$ Length of the day $= 0^h$

Hence the sun does not rise at all.

The Duration of Twilight

Twilight is the subdued light which separates night from day. When the sun sets below the horizon, the darkness does not come instantaneously because the sun's rays still illuminate the atmosphere above us. The particles of vapour etc. in the atmosphere reflect the light and scatter it in all directions. As the sun sinks down, the intensity of the diffused light diminishes. Observations have shown that the diffused light is received so long as the sun does not sink 108° below the horizon. To find the duration of twilight at particular place, we must, therefore, find the time the sun takes to alter its zenith distance from 90° to 108° in the evening, or from 108° to 90° in the morning.

With our previous notations, we have

$$\cos 108° = \sin \delta \sin \theta + \cos \delta \cos \theta \cos H' \qquad \ldots(1)$$

where $\quad H'$ = hour angle of the end of twilight.

If H is the hour angle of the sunset we have $\cos H = -\tan \delta \tan \theta \qquad \ldots(2)$

From the above two equations, H and H' can be calculated for given values of δ and θ.

Hence duration of twilight $= H' - H$.

1.14. THE SUN DIALS

The sun dial enables the time to be fixed accurately enough for ordinary purposes of life, though the precision obtained is much less than that obtained by the methods already discussed. The sun dial gives apparent solar time from which mean time may be obtained. It is useful particularly in places where there are no means available for checking watch or clock times.

A sun dial essentially consists of :

(*i*) a straight edge, called the *stile* or *gnomon* of the dial and

(*ii*) the graduated circle on which the shadow of the gnomon falls.

When the sun shines, the shadow of the gnomon falls on the graduated circle, and intersects it at some point. The reading against the intersection line gives the local apparent time.

A sun dial may be classified under the following heads :

(*i*) *The Horizontal Dial* : in which the graduated circle is horizontal.

(*ii*) *The Prime Vertical Dial*: in which the graduated circle is kept in prime vertical. and (*iii*) *The Oblique dial* : in which the plane of the graduated circle is kept inclined to the horizontal.

In each case, the stile is always kept parallel to the earth's axis, and, therefore, always points north.

We shall discuss here the principle of graduating a horizontal sun-dial.

In Fig. 1.42, *BXAY* is the plane of the dial, in the horizontal plane. *CP* is the direction of the rod, stile or gnomon which, if produced indefinitely, will intersect the celestial sphere in the celestial pole *P*. *BPA* is the plane of the meridian. *M* is the position of the sun at any instant and *CY* is the shadow of the gnomon on the horizontal plane intersecting the latter at *Y*.

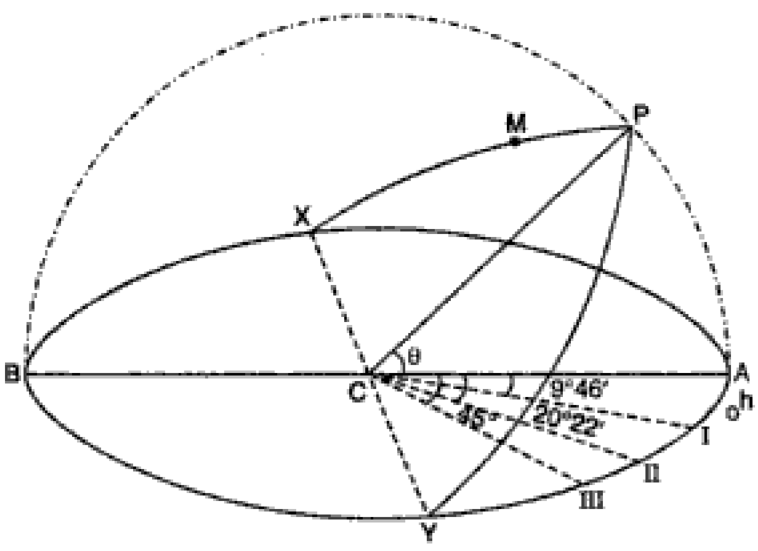


FIG. 1.42. THE HORIZONTAL SUN-DIAL.

Since *CP* is the direction of the meridian also, its shadow will fall on the line *CA* at apparent noon. At one hour after the noon, the shadow will fall on CI, at two hours after the noon, it will fall on CII, and so on. The problem is now to mark the points I, II etc. on the dial so as to correspond to the times of 1^h, 2^h etc. after the apparent noon. At any instant, for the position *M* of the sun, the shadow of the gnomon *CP* will fall on the line *CY* which is the line of intersection of the plane of the dial with the plane containing *CP* and *M*. *XPY* represents such a plane passing through *CP* and *M*.

If the small variation in the declination of the sun is neglected, the diurnal path of the sun (*M*) will describe a circle uniformly on the celestial sphere about *P* as the centre. The projections of the equal angular divisions of the diurnal circle of the sun's path will give unequal angular divisions on the dial. The angle *MPB* is the hour angle of the sun at the instant.

The triangle *YPA* is right angled at *A*.

AP = altitude of the pole = latitude of the place = θ.

$\angle APY$ = hour angle of the sun = H

$AY = x$ = required angular division along the dial corresponding to the hour angle H.

Hence, from the right angled triangle PYA, we get

$$\sin\theta = \cot H \tan x \qquad \text{or} \quad \tan x = \sin\theta \tan H$$

or
$$x = \tan^{-1}(\sin\theta \tan H)$$

The above equation gives the values of x corresponding to the different values of H.

To graduate the dial hourly intervals, put $H = 15°, 30°, 45°$, etc., and compute the corresponding values of x for a place of known latitude θ.

For example, let $\theta = 40°$; Then $x = \tan^{-1}(\sin 40° \tan H)$

When $H = 15° = 1^h$; $x_1 = \tan^{-1}(\sin 40° \tan 15°) = 9° 46'$

When $H = 30° = 2^h$; $x_2 = \tan^{-1}(\sin 40° \tan 30°) = 20° 22'$

When $H = 45° = 3^h$; $x_3 = \tan^{-1}(\sin 40° \tan 45°) = 45°$ and so on.

The points I, II, III corresponding to the angles $x_1, x_2 . x_3$ etc., from CA can then be marked on the dial.

It should be noted that the sun-dial gives only the local apparent time. To covert it into local mean time, approximate value of equation of time must be known.

1.15. THE CALENDAR

The calendars of historical times were lunar in origin, the year consisting of twelve lunar months. Since the return of the seasons depends upon the tropical year, these calendars resulted in a continual change in the dates at which the seasons occurred. The calendar was, therefore, frequently changed in an arbitrary manner, to keep the seasons in their places. In the year 45 B.C., Julius Caesar introduced the Julian Calendar based on a year of $365\frac{1}{4}$ days. The Julian Calendar has January 1 as the commencement of the year. The calendar has ordinary year of 365 days, and was regulated by introducing one extra day on every fourth year which is known as the *leap year*. However, the year actually contains 365.2422 days (or $365^d\,05^h\,48^m\,46^s$) while the Julian Calendar assumed the year to contain 365.25 days (or $365^d\,06^h\,0^m$). Thus the Julian Calendar made the year too long by $11^m\,14^s$, and this created one day excess in 128 years. After many centuries, this difference accumulated to the tune of 10 days and it was observed that the Vernal Equinox in 1582, occurred on 11th March instead of 21st March. Pope Gregory XIII, in 1582, therefore, adjusted the whole calendar in such a way that the Vernal Equinox occurred more or less on 21 st March, by dropping 10 days. In the future, the dates are to be computed by omitting leap year in those century years not divisible by 400 (*i.e.* years as 1700, 1800 and 1900). This will result in omission of 3 days in every 400 years, thus making the mean calendar year of 365.2425 days (or $365^d\,05^h\,49^m\,12^s$). It has also been suggested to omit leap year in the year 4000, and all even multiples thereof, so as to make the mean calendar year of 365.2422 days (or $365^d\,05^h\,48^m\,46^s$).

1.16. DETERMINATION OF AZIMUTH

An *azimuth* is the horizontal angle a celestial body makes with pole. The determination of azimuth, or the direction of the meridian at survey station consists in obtaining the azimuth or true bearing of any line from the station, so that the azimuths of all the survey lines meeting there may be derived. The determination of the direction of the true meridian or of the azimuth of a line is most important to the surveyor. There are several methods of determining the direction of the true meridian, but preference is given to such methods as will allow a set of observations to be taken so that (*i*) instrumental errors may be eliminated, by taking face left and face right observations and (*ii*) interval or time between the observations may not be too great.

Reference mark

In order to determine the azimuth of a star or other celestial body, it is frequently necessary to have a *reference mark* (R.M.) or *referring object* (R.O.). When steller observations are taken, the reference mark should be made to imitate the light of a star as nearly as possible. The reference mark may be a triangulation station or it may consist of a lantern or an electric light placed in a box or behind a screen, through which a small circular hole is cut to admit the light to the observer. The diameter of the hole should not be more than 1 cm. The mark should preferable be so far from the instrument that the focus of the telescope will not have to be altered when changing from the star to the mark. A distance of about a mile is quite satisfactory.

The following are some of the principal methods of determining the azimuth or the direction of the true meridian :

1. By observations on star at equal altitudes.
2. By observations on a circumpolar star at elongation.
3. By hour angle of star or the sun.
4. By observation of Polaris.
5. By ex-meridian observations on sun or star.

1 (*a*) OBSERVATIONS ON THE STARS AT EQUAL ALTITUDES

The simplest method of determining the direction of the celestial pole is probably that observing at star at equal altitudes. In this method, the knowledge of the latitude or local time is not necessary, and no calculations are involved. However, the duration of the work is a great inconvenience, extending from four to six hours at night. Also the effects of atmospheric refraction may vary considerably during the interval, affecting the vertical angles to an unknown extent.

The method is based on the fact that if the angle subtended between the reference mark and a star is measured in two positions of equal altitude, the angle between the mark and the meridian is given by half the algebraic sum of the two observed angles.

The dotted circle in Fig. 1.40 represents the circular path of a star round the pole, and it is required to determine the direction of the centre P of this circle. M_1 and M_2 are the two positions of the star at equal altitude, and all that the observer has to do to get his true meridian is to bisect the angle between M_1 and M_2

Thus, in Fig. 1.43, R is the reference mark (R.M.) and O is the position of the instrument station through which the direction of the true meridian is to be established. M_1 and M_2 are two positions of a star at equal altitudes. The field observations are taken in the following steps :

(1) Set the instrument at O and level it accurately.

(2) Sight the R.M. with the reading 0° 0′ 0″ on the horizontal circle.

(3) Open the upper clamp and turn the telescope clockwise to bisect accurately the star at position M_1. Clamp both horizontal as well as vertical circle.

(4) Read the horizontal angle θ_1 as well as the altitudes α of the star.

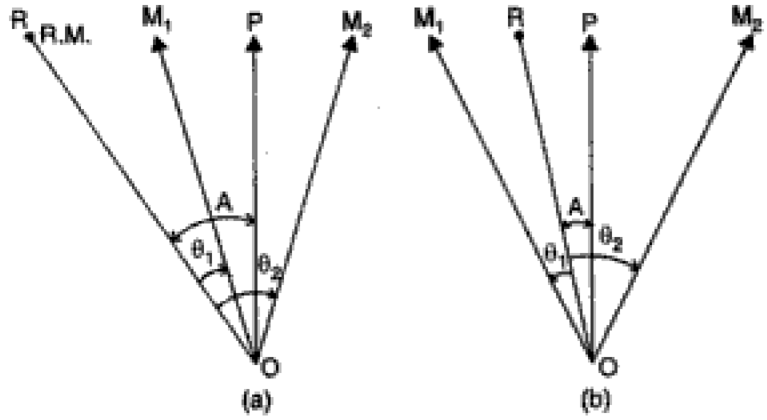


FIG. 1.43. AZIMUTH BY EQUAL ALTITUDES.

(5) When the star reaches the other side of the meridian, follow it through the telescope, by unclamping the upper clamp, and bisect it when it attains the same altitude. In this observation, the telescope is turned in azimuth and the vertical circle reading remains unchanged. Read the angle θ_2.

Let A be the azimuth of the line OR, *i.e.* the angle between the true meridian and the reference object. Since the direction of the meridian is midway between the two positions of the star, the azimuth of the line may be determined according as both the positions of the star are to the same side of R or to the different sides of R.

Case I : Both positions of the star to the same side [Fig. 1.43 (a)].

$\theta_1 = \angle ROM_1$; $\theta_2 = \angle ROM_2$

A = azimuth = $\angle ROP$, (where P is the position of the pole)

$$= \theta_1 + \frac{\theta_2 - \theta_1}{2} = \frac{\theta_1 + \theta_2}{2}$$

Hence the azimuth of the line is equal to half the sum of the two observed angles.

Knowing the azimuth of the line OR, the azimuth of any other line through O can be determined by measuring the horizontal angle between OR and that line. Also if it is required to set out the direction of the true meridian, and angle equal to $\frac{\theta_1 + \theta_2}{2}$ can be set out from the line OR.

***Case II. Both positions of the star are on opposite sides of the line.* [Fig. 1.43 (*b*)].**

$$\text{Azimuth} = A = \angle M_1OP - \angle M_1OR = \tfrac{1}{2}\angle M_1OM_2 - \angle M_1OR = \tfrac{1}{2}(\theta_1 + \theta_2) - \theta_1 = \frac{\theta_2 - \theta_1}{2}$$

Hence the azimuth of the line is equal to half the difference of the two observed angles.

In the observations taken above, it is assumed that the instrument is in perfect adjustment. If it is not so, it is necessary to take at least four observations (two with face left and two with face right) to eliminate the instrumental errors. The position M_1 of the star is observed with both the faces, and the position M_2 is also observed with both the faces, and the mean is taken. However, in the duration that elapses between two face observations of M_1, the position and altitude of the star slightly changes and this should be properly accounted for. In Fig. 1.44, M_1 and M_2 are the two positions of the star to one side of the meridian when both face observations are taken, and M_3 and M_4 are the two positions of the star to the other side of the meridian, in such a way that M_1 and M_2 have equal altitude, and M_2 and M_3 have equal altitude.

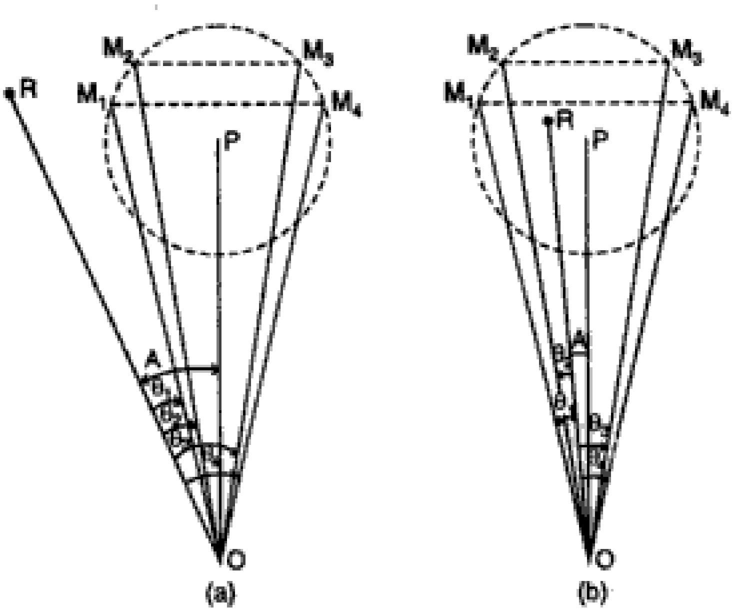


FIG. 1.44

The angles θ_1, θ_2, θ_3 and θ_4 with the R.M. corresponding to the positions M_1, M_2, M_3 and M_4 are measured as follows :

(1) The instrument is set at O and, with both plates clamped to zero, bisect R with the vertical circle to the left.

(2) Unclamp the upper clamp, turn the telescope in azimuth and bisect the star at M_1. Note the horizontal angle θ_1 and the vertical angle (*i.e.* the altitude) α.

(3) Change the face of the instrument and again bisect R with both plates clamped to zero. During this time, the star goes to the position M_2. Unclamp the upper clamp and turn the telescope in azimuth to bisect the star at M_2. Clamp the vertical circle. Read the horizontal angle θ_2 and the vertical angle α'.

(4) Leave the instrument undisturbed with the vertical circle clamped to the angle α'. When the star reaches the other side of the meridian, unclamp the upper clamp and turn the telescope in azimuth to bisect the star in position M_3 when it attains the altitude α' (*i.e.* an altitude equal to that at M_2). Read the horizontal angle θ_3.

(5) Change the face of the instrument and again bisect the R.O. with both the plates clamped to zero. Set the angle α (*i.e.* the altitude of the star at the position M_1). Unclamp the upper clamp and turn the telescope in azimuth to bisect the star at the position M_4 when it attains the altitude α. Read the horizontal angle θ_4 in this position.

Thus, we have got four horizontal angles, *i.e.* $\theta_1, \theta_2, \theta_3$ and θ_4. The mean of θ_1 and θ_2 gives the position of the star to one side of the meridian when it has an average altitude equal to $\frac{\alpha + \alpha'}{2}$. Similarly the mean of θ_3 and θ_4 gives the position of the star to the other side of the meridian when it has the *same* average altitude, *i.e.* $\left(\frac{\alpha + \alpha'}{2}\right)$.

When the average positions of the star are to the same side of the R.M., we have

$$\text{Azimuth of } OR = A = \frac{\theta_1 + \theta_2}{2} + \frac{1}{2}\left(\frac{\theta_3 + \theta_4}{2} - \frac{\theta_2 + \theta_1}{2}\right) = \frac{(\theta_1 + \theta_2) + (\theta_3 + \theta_4)}{4}.$$

Similarly, if both the average positions of the star are to the opposite sides, we have

$$A = \frac{1}{2}\left(\frac{\theta_1 + \theta_2}{2} + \frac{\theta_3 + \theta_4}{2}\right) - \frac{1}{2}(\theta_1 + \theta_2) = \frac{(\theta_3 + \theta_4) - (\theta_1 + \theta_2)}{4}.$$

1. (*b*) OBSERVATION ON SUN AT EQUAL ALTITUDES

When the sun is observed for equal altitudes, the sequence of observations is the same as that for a star. Since the actual altitude of the sun is not required, its upper limb or lower limb may be observed *throughout*. A series of horizontal angles is measured between the reference mark and the sun in the forenoon, and a similar series is observed with the sun at the same altitudes in the afternoon. Since the sun's centre cannot be bisected, observations should be made on the right-hand and left-hand limbs of the sun with the telescope normal and inverted in both the morning as well as afternoon observations. However, in the interval between the forenoon and the afternoon observations of equal altitudes, the declination of the sun changes, and hence the mean of the horizontal angles requires a suitable correction to determine the azimuth of the survey line from it. To apply the correction, the watch-time of each observation should also be recorded. The correction is given by

$$c = \frac{1}{2}(\delta_W - \delta_E) \sec \theta \ . \ \text{cosec } t \qquad \text{...(1.34)}$$

where c = angular correction to be applied to the algebraic mean of the observed horizontal angles to give the azimuth of the reference line

t = half the interval between the times of equal altitude.

θ = latitude of the observer's place.

δ_E = sun's average declination of morning observations.

δ_W = sun's average declination of evening observations.

(2) OBSERVATIONS ON A CIRCUMPOLAR STAR AT ELONGATION

A circumpolar star is that which is always above the horizon, and which does not, therefore, set. Such a star appears to the observer to describe a circle above the pole (see Fig. 1.19). A circumpolar star is said to be at elongation when it is at its greatest distance east or west of the meridian. When the star is at its greatest distance to the

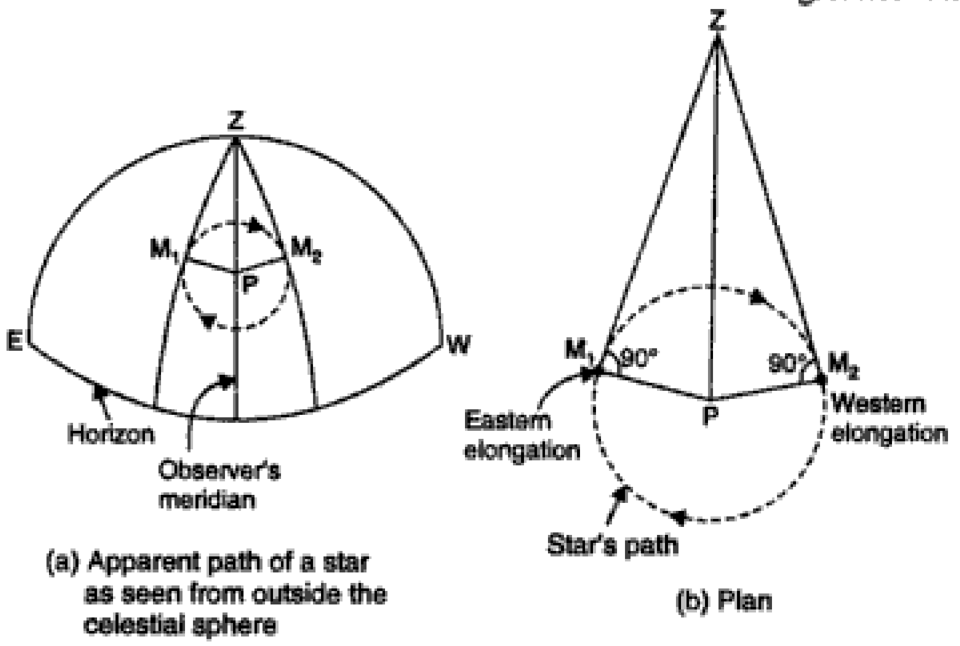


FIG. 1.45. STAR AT ELONGATION.

east of the meridian, it is said to be in eastern elongation. When it is at its greatest distance to the west of the meridian, it is said to be in western elongation. In this position, the star's diurnal circle is tangent to the vertical circle to the star.

Figs. 1.45 (*a*) and 1.45 (*b*) show two views of the stars at elongation. M_1 is the position of the star at its eastern elongation, and M_2 is the position of the star as its western elongation. In this position, the vertical circle of star makes its greatest angle with the plane of the meridian. The vertical through M_1 (or M_2) is tangential to the diurnal path of the star shown by dotted circle. Evidently, therefore, $\angle ZM_1P$ is a right angle. Also, when the star is at western elongation (position M_2), $\angle ZM_2P$ is a right angle.

At the instant of elongation of the star, its motion is vertical and it is in a favourable position for observations upon its azimuth because its horizontal movement is very slight for some time before and some time after it arrives M_1 (or M_2). When the star is in eastern elongation (M_1), it appears to move vertically downwards, and when it is in western elongation, it appears to move vertically upwards at the instant of elongation. It is clear from the figure that the points M_1 and M_2 will always be at a greater altitude than the celestial pole P. However, greater the declination of the star, more nearly will be the altitude of M_1 and M_2 approach that of P.

Prior to making the field observations, it is necessary to *calculate* the time at which the star will elongate. This can be done as follows:

(*i*) The hour angle (H) of the star can be calculated from equation 1.19

$$\cos H = \frac{\tan \theta}{\tan \delta} = \tan \theta \cot \delta$$

(*ii*) Calculate the local sidereal time of elongation :

$$\text{L.S.T. (of elongation)} = \text{R. A.} \pm H$$

Use plus sign for western elongation and minus sign for eastern elongation.

(*iii*) Convert this L.S.T. to mean time by method discussed earlier.

Thus, the mean time of elongation of the star is known. At least 15 to 20 minutes before the time of elongation, the instrument is set up and carefully levelled. Five minutes before the time of elongation, a pointing is made on the reference mark. The upper clamp is then unclamped and the star is sighted. The star is then followed in azimuth. At the time of elongation, the star stops moving horizontally, and appears to move vertically along the vertical hair. This will take place exactly at the time calculated above. The horizontal circle reading gives the angle that the star makes with the reference line. To this, if we add the azimuth of the star, the azimuth of the survey line can very easily be known.

The azimuth of the star at its elongation can be calculated from Eq. 1.21:

$$\sin A = \frac{\cos \delta}{\cos \theta} = \cos \delta \,.\, \sec \theta.$$

However, in order to eliminate the error, at least two observations should be made — one with face left a few minutes before the elongation and other with the face right a few minutes after the elongation. If more time is taken between these two sets of readings, the azimuth will not be correct. In general, the observations should not be extended beyond five minutes on either side of the time of elongation and during this time as many readings between the R.M. and the star as are possible should be taken.

The following table gives the time after the moment of elongation when the aizmuth changes by 5″ for a place in latitude 30°:

Polar distance of the star	*Time after moment of elongation before azimuth changes by 5″*
10°	3 min. 33 sec.
15°	3 min. 7 sec.
20°	2 min. 35 sec.
30°	2 min. 11 sec.

As there will be a corresponding and nearly equal period before elongation, it follows that for stars having 20° polar distance, 5 min. and 10 seconds can be the maximum time to the observer before the azimuth can change by 5″ in that period. For a star whose polar distance is 10°, the corresponding time is 7 min. 6 sec. *The nearer the star is to the pole the greater the length of time available for the observations.* In ordinary observations, a surveyor uses a 20″ theodolite so as to determine the azimuth within 20″. Hence, it will be sufficiently accurate if he takes two observations of the star, one with the face left and the other with the face right, not exactly at the time of elongation, but one just before and the other just after the elongation.

However, for very accurate results, it is better to apply the following correction to the value of azimuth (A) of the star from the formula for elongation.

$$\textit{correction (in seconds)} = 1.96 \tan A \sin^2 \delta \ (t_E - t)^2 \qquad \ldots(1.35)$$

where $t_E - t$ is the sidereal interval in minutes between the time of observation and that of elongation. The above formula is applied only when $(t_E - t)$ does not exceed 30 minutes.

The Effect of an Error in the Latitude

For the calculation of the azimuth, the declination (δ) of the star and the latitude (θ) of the place of observation must be accurately known. The declination is taken from the star almanac. Let us now study the effect of an error in the latitude on the determination of the azimuth

Let y = error in the latitude and x = corresponding error in the azimuth.

We have $\sin A = \dfrac{\cos \delta}{\cos \theta}$ or $\sin A \,.\, \cos \theta = \cos \delta$ ...(1)

Putting the actual values of A and θ in the above expression, we get

$$\sin(A + x) \cos(\theta + y) = \cos\delta$$

Expanding $\sin(A + x)$ and $\cos(\theta + y)$, and replacing $\sin x$, $\sin y$ by x and y respectively, and $\cos x$, $\cos y$ by unity, we get

$$(\sin A + x \cos A)(\cos \theta - y \sin \theta) = \cos \delta \qquad \ldots(2)$$

Subtracting (1) from (2), and neglecting the term having the product of small quantities x and y, we get

$$x \cos A \cos \theta - y \sin A \sin \theta = 0$$

or $$x = y \tan \theta \tan A = y \tan \theta \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

Substituting $\sin A = \frac{\cos \delta}{\cos \theta}$ and $\sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{\cos^2 \delta}{\cos^2 \theta}} = \sqrt{\frac{\cos^2 \theta - \cos^2 \delta}{\cos^2 \theta}}$

we get $$x = y \tan \theta \cdot \frac{\cos \delta}{\sqrt{\cos^2 \theta - \cos^2 \delta}} \quad \text{...(1.36)}$$

From the above expression,

If $\theta = 0$, $x = 0$

If $\theta = \delta$, $x = \infty$

Also, if $\delta = 90°$, $x = 0$.

Hence, in any given latitude, the error is least when the star selected is nearest to the pole.

The following table gives the ratio $\left(\frac{x}{y}\right)$ of error in azimuth to small error in latitude.

Declination	*Latitude = 20°*	*Latitude = 30°*	*Latitude = 40°*
(δ)	*x/y*	*x/y*	*x/y*
60°	0.22	0.40	0.70
70°	0.14	0.24	0.40
80°	0.06	0.10	0.19

An error in latitude of say 5" will produce an error in azimuth of less than 5" if the value of declination is less than the value of latitude. The error in azimuth will, however, be greater than the error is latitude if the value or the declination of the star approaches the value of the latitude :

(3) AZIMUTH BY HOUR ANGLE OF THE STAR OR THE SUN

In this method, the azimuth of a star or sun is determined by observing the hour angle when it is on or near its prime vertical. In the field, the angle between the star and the R.M. is measured, and the chronometer time at the instant of observation is observed very accurately. The altitude of the star is not necessary in this method and hence there is no effect of the errors of refraction. The field work is carried out in the following:

(*i*) Set up the theodolite over the station point and level it accurately.

(*ii*) Select a suitable star as near the prime vertical as possible.

(*iii*) Bisect the R.M. with both the plates clamped to zero, and with the vertical circle to the left.

(*iv*) Unclamp the upper clamp, rotate the telescope in the azimuth and sight the star. When the star is exactly at the intersection of the cross-wires, give the signal to the chronometer observer to observe the chronometer time *very accurately*. Take the reading of the horizontal circle.

(*v*) Repeat the observations with face right.

The mean of the above readings will give the chronometer time and the angle between the star and the R.M.

From the observed mean time of the chronometer, the local sidereal time can be easily calculated by the method discussed earlier. The hour angle of the star can be computed from the expression.

$$\text{L.S.T.} = \text{R. A.} \pm \text{Hour angle.}$$

The R.A. of the star can be known from the star almanac.

Thus, the hour angle of the star (or the sun) is known from the observed chronometer time. In case the chronometer is fast or slow, its correction should be known before hand, and the same should be applied to the observed time *before* hour angle is calculated.

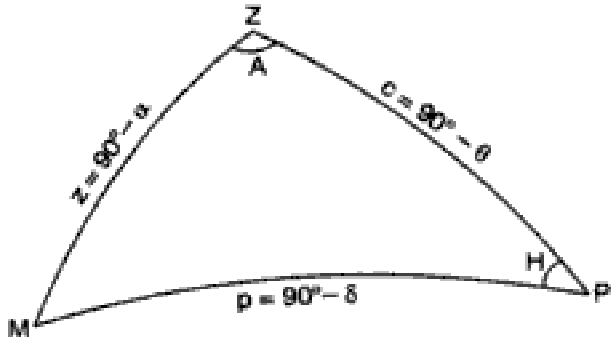


FIG. 1.46

Knowing the hour angle, the declination and the latitude of the place, the azimuth can be calculated by the solution of the astronomical triangle.

Thus, in Fig. 1.46, *M* is the position of the star at the instant of observation when its hour angle is *H*.

$$ZP = \text{co-latitude} = (90° - \theta) = c \quad \text{(known)}$$

$$MP = \text{co-declination} = (90° - \delta) = p \quad \text{(known)}$$

$$\angle ZPM = \text{hour angle} = H \quad \text{(known).}$$

The value of the azimuth (*A*) can be calculated from the following expression :

$$\tan A = \tan H \,.\, \cos B \,.\, \operatorname{cosec}(B - \theta) \qquad \text{...(1.37)}$$

where $$B = \tan^{-1}(\tan\delta \;\sec H) \qquad \text{...(1.38)}$$

Knowing the azimuth of the star, the azimuth of the survey line can be known.

The above method, though simple and straight forward, is not very much used since separate observations for determining the chronometer error are required. However, if the chronometer error is known, the method is much better than ex-meridian altitudes. *However, if the star is observed near its prime vertical, the errors of time have very little effect on the result.*

While computing the value of *H* from the chronometer time, a linear relationship between the chronometer timings and the motion of the star in the azimuth was assumed. However, for more precise work, a correction for the curvature of the path of the star must be applied to the mean of the face left and face right observations. The correction (ΔA) in seconds to be applied to the azimuth is given by

$$\Delta A'' = \pm \frac{1}{8} \sin A \cos\theta \sec^2\alpha \,(\cos\alpha \sin\delta - 2\cos A\cos\theta) \times (\Delta t)^2 \times \sin 1'' \qquad \text{...(1.39)}$$

where Δt = difference in time, expressed in seconds of arc, between the face right and face left observation.

The correction is evidently zero at culmination.

(4) AZIMUTH BY OBSERVATIONS ON POLARIS OR CLOSE CIRCUMPOLAR STAR

The *most precise* determination of azimuth may be made by measuring the horizontal angle between the R.M. and a close circumpolar star. The chronometer time of each observation is noted very precisely. From the corrected chronometer times the hour angle of the circumpolar star can then be obtained as discussed earlier. The azimuth of the star can then be calculated by the solution of the astronomical triangle. Since the close circumpolar stars move very slowly in azimuth and errors in the observed times will thus have a small effect upon the computed azimuths, it is evident that only such stars should be chosen for primary or precise work.

Since Polaris (α Ursae Minoris) is the brightest circumpolar star, it is used in preference to others whenever practicable. In general, however, the observations on close circumpolar stars have the following advantages :

(1) Since the motion in the azimuth is very slow, the number of observations may be increased materially and greater accuracy may be secured.

(2) Observations may be made at any convenient time, without calculating the time of elongation or waiting for the time of elongation.

(3) If observations are made on the bright pole star, it is usually possible to sight the star during the twilight when no artificial illumination for the R.M. and for the instrument is necessary.

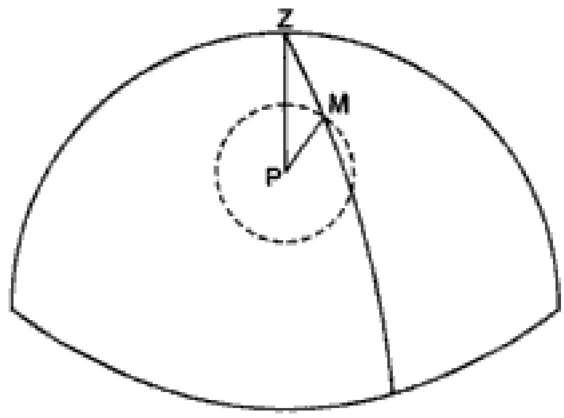


FIG. 1.47. OBSERVATIONS TO POLARIS.

In Fig. 1.47, P is the pole, Z is the zenith of the observer and M is the position of the close circumpolar star. The dotted circle shows the diurnal path of the polar star.

The hour angle H ($\angle ZPM$) is known from the observed chronometer time.

$\angle MZP = A$ = azimuth of the pole star (to be computed)

PM = polar distance = co-declination (known)

ZP = co-latitude $= c = 90° - \theta$ (known)

The azimuth (A) is given by

$$\tan A = \frac{\sin H}{\cos \theta \tan \delta - \sin \theta \cos H}$$

or $$\tan A = \sec \theta \; . \; \cot \delta \sin t \; . \left(\frac{1}{1-a} \right) \qquad \ldots(1.40)$$

where $$a = \tan \theta \cot \delta \cos H \qquad \ldots(1.41)$$

The values of $\log \frac{1}{1-a}$ are tabulated for different values of A in the *Special Publication 14*, United States Coast and Geodetic Survey.

The value to be taken for the hour angle is that corresponding to the mean of corrected chronometer timings of n observations. However, for the accurate results, the curvature of the path of the star should be taken into consideration, and the calculated azimuth should be corrected by the following amount :

$$\text{Curvature correction for one set} = \frac{\tan A \sin^2 \delta}{n} \Sigma \frac{2 \sin^2 \frac{1}{2} \Delta t}{\sin 1''} \quad \ldots(1.42)$$

where n = number of the observations in one set

Δt = angular equivalent of the sidereal time interval (in seconds) between the individual observation and the mean of the set.

For the most accurate work, the striding level should also be observed. If the horizontal axis is inclined during a pointing on the star or the R.M., the horizontal circle reading should be corrected by :

$$\text{Level correction} = \frac{d}{2n} (\Sigma W - \Sigma E) \tan \alpha \quad \ldots(1.43)$$

where d = value of one division of the striding level

ΣW and ΣE = sum of west and east reading of the bubble end, reckoned from centre of bubble in direct and reversed position

α = altitude of star or R.M.

Programme of observations

The field observations are arranged in the following steps :

(1) With the face left, point twice the R.M. Read both the verniers of the horizontal circle at each pointing.

(2) With the face left, point twice the star and read both the verniers of the horizontal circle at each pointing. Note the timing of each pointing.

(3) Change the face. Read twice on the star with face right and note the time and the angles.

(4) Read twice upon the R.M. with face right.

Alternative programme of field observations

1. Set the instrument over the instrument mark. With both the plates clamped to zero, sight the R.M.
2. Turn the telescope in azimuth and bisect the star. Note the chronometer time.
3. Read the striding level and reverse it.
4. Read the circle.
5. Intersect the star again and note the time.
6. Read the striding level.
7. Read the circle.
8. Point to R.M. and read the circle.

5. (a) AZIMUTH BY EX-MERIDIAN OBSERVATIONS ON STAR

The determination of azimuth by ex-meridian observation of a star or sun is the method most commonly used by a surveyor except for the determination of primary standard. The observations are the same as that for the determination of time, and the two determinations may be combined if the watch times of the altitudes are also recorded. Knowing the latitude of the place and the declination of the star, the astronomical triangle can be observed for azimuth.

Since the mean refraction for objects at an altitude of 45° is 57", it necessary to correct for refraction in the measurements of the altitude. The refraction correction is almost uncertain for stars very near to horizon. The stars should be observed when it is changing rapidly in altitude and slowly in azimuth. A favourable position occurs when the star is on the prime vertical when the influence of errors of observed altitude is small.

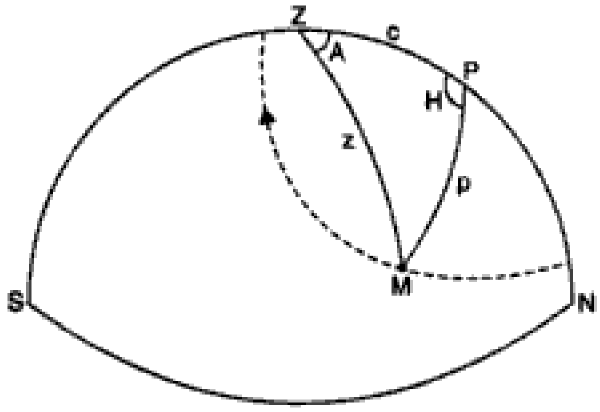


FIG. 1.48. EX-MERIDIAN OBSERVATION OF STAR

In Fig. 1.48, *M* is the position of the star when its altitude (α) is observed.

In the Astronomical triangle,

ZP = co-latitude = $90° - \theta = c$ (known)

MP = co-declination of star = $90° - \delta = p$ (known)

ZM = corrected co-altitude of star = $90° - \alpha = z$ (observed)

The azimuth (A) can be calculated by one of the following expressions :

$$\sin \frac{1}{2} A = \sqrt{\frac{\sin (s-z) \cdot \sin (s-c)}{\sin z \cdot \sin c}} \quad ; \quad \cos \frac{1}{2} A = \sqrt{\frac{\sin s \cdot \sin (s-p)}{\sin z \cdot \sin c}}$$

$$\tan \frac{1}{2} A = \sqrt{\frac{\sin (s-z) \sin (s-c)}{\sin s \cdot \sin (s-p)}} \quad ; \text{ where } \quad s = \frac{1}{2}(p + c + z).$$

At least two measurements of the altitude and the horizontal angle with the R.M. should be taken, one with face left and the other with face right. In the interval between the face left and face right observations, the star moves considerably in altitude. If the azimuth is calculated from any one of the above formulae by using mean value of the altitude, it will not be exactly the same thing as the mean of the azimuth in the two observed positions. The error will be negligible if the difference in altitude of the star at the two observations is not more than 1° or 2°. However, if the change in altitude is more and if the mean value of the altitude is taken to compute the azimuth, the correction to be applied to the latter is given by

$$\Delta A'' = \frac{1}{8} \cot M \cdot \sec^2 \alpha (\sin \alpha - 2 \cot A \operatorname{cosec} 2M) (\Delta \alpha)^2 \sin 1'' \qquad ...(1.44)$$

where M = the parallactic angle $ZMP = \sin^{-1} (\cos \theta \cdot \sin A \cdot \sec \delta)$.

The value of the correction may be computed by using a four figure log table using the values of the various angles to the nearest minute.

Programme of field observations

1. Set the instrument over the station mark and level it very accurately.
2. Clamp both the plates to zero and sight the R.M. with face left.
3. Unclamp the upper clamp, and bisect the star. Note the horizontal and vertical angles.

4. Change the face of the theodolite and bisect the star again. Obtain the vertical angle and the horizontal angle to the reference mark as before.

5. Observe a second set in the same manner with a new zero.

5. (b) AZIMUTH BY EX-MERIDIAN OBSERVATION ON THE SUN

The general procedure of observations are the same as for a star. However, since the declination of the sun changes very rapidly, an exact knowledge about the time of observation is very essential. Also apart from the correction due to refraction, the parallax correction is also to be applied to the observed altitude, since the sun is very near to the earth than the star.

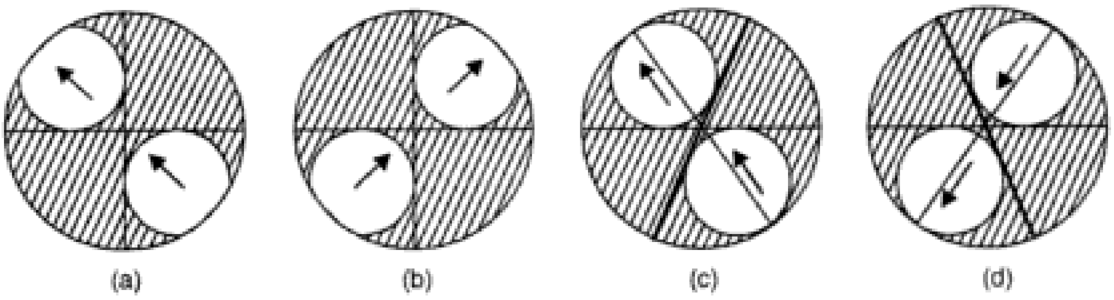


FIG. 1.49. OBSERVATIONS OF THE SUN

The required altitude and the horizontal angles are those to the sun's centre. Hence the hairs should be set tangential to the two limbs simultaneously. The opposite limbs are then observed by changing the face, as shown in Fig. 1.49 (*a*) and (*b*). If however, the diaphragm has no vertical hair, the sun must be placed in opposite angles as shown in Fig. 1.49 (*c*) and (*d*).

Programme of field observations

1. Set the instrument over the station mark and level it very accurately.

2. Clamp both the plates to zero, and sight the R.M.

3. Turn to the sun and observe altitude and horizontal angle with the sun in quadrant 1 (Fig. 1.50) of the cross-wire system. The motion in the azimuth is slow, and the vertical hair is kept in contact by the upper slow motion screw, the sun being allowed to make contact with the horizontal hair. The time of observation is also noted.

4. Using the two tangent screws, as quickly as possible, bring the sun into quadrant 3 of the cross-wires, and again read the horizontal and vertical angle. Observe also the chronometer time.

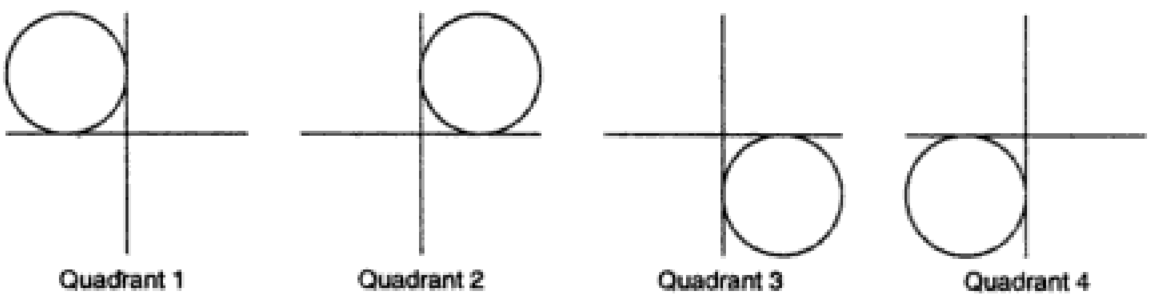


FIG. 1.50. SUN'S LIMB OBSERVED IN VARIOUS QUADRANTS

5. Turn to the R.M., reverse the face and take another sight on the R.M.

6. Take two more observations of the sun precisely in the same way as in steps (3) and (4) above, but this time with the sun in quadrants 2 and 4. Note the time of each observation.

7. Finally bisect the R.M. to see that the reading is zero.

During the above four observations (two with face left and two with face right), the sun changes its position considerably, and accurate results cannot be obtained by averaging the measured altitudes and the times. However, the time taken between the first two readings, with the sun in quadrants 1 and 3, is very little and hence the measured altitudes and the corresponding times can be averaged to get one value of the azimuth. Similarly, the altitudes and the timings of the last two readings, with the sun in quadrants 2 and 4, can be averaged to get another value of the azimuth. The two values of azimuths so obtained (one with face left and the other with face right) can be averaged to get the final value of the azimuth.

For very precise work, the altitude readings should be corrected for the inclination, if any, of the trunnion axis as discussed earlier.

The reduction is performed in the same manner as for the corresponding star observation. The correct value of sun's declination can be computed by knowing the time of observation, by the methods discussed earlier.

The Effect of an Error in Latitude upon the Calculated Azimuth

Let y = error in co-latitude (c)

and x = the corresponding error in the calculated value of azimuth.

We know that $\cos p = \cos c \cos z + \sin c \sin z \,.\, \cos A$ ...(1)

Hence $\cos p = \cos (c + y) \cos z + \sin (c + y) \sin z \,.\, \cos (A + x)$ ...(2)

Subtracting these two and making the approximations that

$\sin x = x$, $\sin y = y$, $\cos x = 1$ and $\cos y = 1$, we get

$$\cos z \,.\, y \sin c + \sin z \sin c \cos A - \sin z (\sin c + y \cos c) \times (\cos A - x \sin A) = 0$$

or $$\cos z \,.\, y \sin c - y \sin z \cos c \cos A + x \sin z \sin c \sin A = 0$$

(neglecting the terms having product of x and y)

$$\therefore \quad x = \frac{-\cos z \sin c + \sin z \cos c \cos A}{\sin z \,.\, \sin c \,.\, \sin A}$$

which gives on simplification, $x = \dfrac{-\cot H}{\sin c} \,.\, y$ ...(1.45)

It is clear from the above formula that for a given value of y, x is maximum when $\cot H$ is maximum, *i.e.*, when H is minimum. *Hence at all times near noon, the error in azimuth produced by a defective knowledge of the latitude is very much increased.* The error is least at 6 A.M. or 6 P.M. The error also increases with increase in the value of θ, and is the greatest near the pole.

The Effect of an Error in the Sun's Declination upon the Calculated Azimuth

Let y = error in the co-declination (p) of the sun.

x = corresponding error in calculated value of A.

Then $x = (\text{cosec } c.\ \text{cosec } H).\ y$ ...(1.46)

For a given value of y, x is maximum at times near to noon, and is least at 6 A.M. and at 6 P.M.

Also, x increases as the latitude of the place increases. *This method becomes unreliable in arctic or antarctic regions where the given value of y produces very great error in the azimuth.*

The Effect of an Error in the Measured Altitude

Let y = error in the co-altitude (z)

x = corresponding error in the calculated value of azimuth

Then $x = -(\cot M.\ \text{cosec } z)\ y$; where M = parallactic angle ZMP. ...(1.47)

The value of x is infinitely great when $M = 0°$ or $180°$, *i.e.* when the sun is on the meridian. *Hence, in this case also, it is concluded that the resulting error in azimuth is very great if the observations are made near noon.* The error is however, small if angle M is near 90°.

Example 1.48. *A star was observed at western elongation at a station A in latitude 54° 30′ N and longitude 52° 30′ W. The declination of the star was 62° 12′ 21″ N and its right ascension $10^h\ 58^m\ 36^s$, the G.S.T. of G.M.N. being $4^h\ 38^m\ 32^s$. The mean observed horizontal angle between the referring object B and the star was 65° 18′ 42″. Find (a) the altitude of star at elongation, (b) the azimuth of the line AP and (c) the local mean time of elongation.*

Solution

(a) *Altitude of the star, its hour angle and azimuth.*

Since the star is observed at elongation, the angle ZMP of the astronomical triangle ZMP is a right angle. Hence, from Napier's rule for circular parts.

$$\sin \alpha = \frac{\sin \theta}{\sin \delta} = \frac{\sin 54° 30'}{\sin 62° 12' 21''} \quad ...(1)$$

or $\alpha = 66° 58' 6''.7$

∴ Hence the altitude of the star $= \mathbf{66° 58' 6''.7}$.

Also, $$\sin A = \frac{\cos \delta}{\cos \theta} = \frac{\cos 62° 12' 21''}{\cos 54° 30'} \quad ...(2)$$

or $A = \mathbf{53° 25'}$

and $$\cos H = \frac{\tan \theta}{\tan \delta} = \frac{\tan 54° 30'}{\tan 62° 12' 21''}$$

or $H = 42° 21' 20'' = \mathbf{2^h\ 49^m\ 25.3^s}$ ...(3)

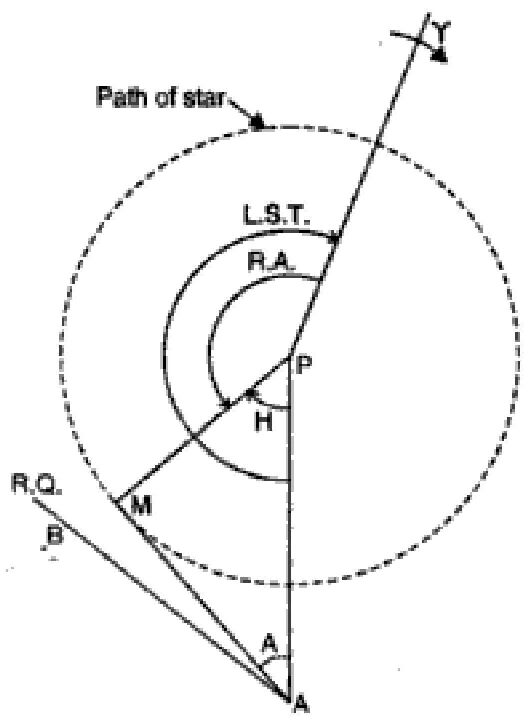


FIG. 1.51. STAR AT WESTERN ELONGATION.

(b) Azimuth of the line.

Since the star was at western elongation, it is to the west of the meridian.

$\therefore$ Azimuth of the line AB = azimuth of the star + horizontal angle between the line and the star = 53° 25′ + 65° 18′ 42″= 118° 43′ 42″

$\therefore$ Azimuth of line AB in clockwise from north = 360° – 118° 43′ 42″= **241° 16′ 18″**.

(c) Local mean time of observation.

In order to calculate the local mean time of observation, let us first calculate the L.S.T. of L.M.N. from the given value of G.S.T. of G.M.N.

$$\text{Longitude} = 52° \ 30' \ W = 3^h \ 30^m \ \text{west.}$$

Acceleration at the rate of 9.8565 per hour :

$$3^h \times 9.8565 = 29.57 \ \text{seconds}$$
$$30^m \times 0.1642 = 4.93 \ \text{seconds}$$

$$\text{Total acceleration} = 34.50 \ \text{seconds}$$

$$\text{G.S.T. of G.M.N.} = 4^h \ 38^m \ 32^s$$
$$\text{Add acceleration} = 34.5^s$$

$$\therefore \quad \text{L.S.T. of L.M.N.} = 4^h \ 39^m \ 06.5^s$$

Now L.S.T. of observation = R.A. of star + H.A. of the star

$$= 10^h \ 58^m \ 36^s + 2^h \ 49^m \ 25.3^s = 13^h \ 48^m \ 01.3^s$$

Thus $\qquad$ L.S.T. = $13^h \ 48^m \ 01.3^s$

Subtract L.S.T. of L.M.N. = $4^h \ 39^m \ 06.5^s$

$$\therefore \quad \text{S.I. from L.M.N.} = 9^h \ 8^m \ 54.8^s$$

Let us now convert the S.I. into the mean time interval by subtracting at the retardation at the rate of 9.8296 per sidereal hour.

$$9^h \times 9.8296 = 88.47 \ \text{seconds}$$
$$8^m \times 0.1638 = 1.31 \ \text{seconds}$$
$$54.8^s \times 0.0027 = 0.15 \ \text{second}$$

$$\text{Total retardation} = 89.93 \ \text{seconds} = 1^m \ 29.93^s$$

$\therefore$ Mean time interval from L.M.N.

$$= \text{S.I.} - \text{retardation} = 9^h \ 8^m \ 54.8^s - 1^m \ 29.93^s$$

$\therefore$ L.M.T. of observation = $\mathbf{9^h \ 7^m \ 24.87^s}$

Fig. 1.51 shows the relative positions of observer (A), the star (M), the pole (P), the ϒ and R.O. at the instant of observation.

Example 1.49. *A star was observed at its eastern elongation in latitude 53° 32′ N and the mean angle between a line and the star was found to be 75° 18′ 20″, the star and the line being to the opposite sides of the meridian. Find (a) the azimuth of the line, (b) the altitude of the star at observation, (c) the L.M.T. of observation with the following data :*

Declination of the star	*56° 42′ 53″.2 N*	
Longitude of the place	$5^h\ 40^m\ 18^s$ *W*	
R.A. of the star	$10^h\ 58^m\ 3.9^s$	
S.T. at G.M.M.	$4^h\ 58^m\ 23.84^s$	*(P.U.)*

Solution

Since the star was observed at its elongation, the astronomical triangle *ZPM* is right angled at *M*. The azimuth altitude and hour angle of the star can be calculated from the Napier's rule.

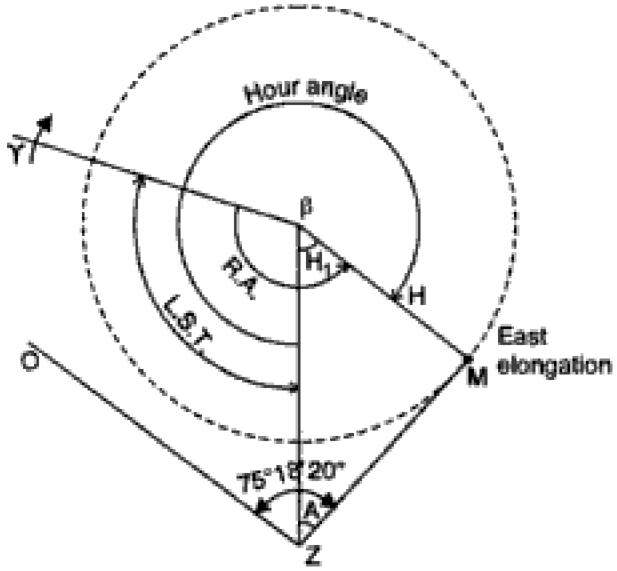


FIG. 1.52. STAR AT EASTERN ELONGATION.

(*a*) Thus, $\sin\alpha = \dfrac{\sin\theta}{\sin\delta} = \dfrac{\sin 53°\ 32'}{\sin 56°\ 42'\ 53''.2}$

$\alpha = 74°\ 9'\ 32''.9$

Hence altitude of the star = **74° 9′ 32″.9**

(*b*) $\sin A = \dfrac{\cos\delta}{\cos\theta} = \dfrac{\cos 56°\ 42'\ 53''.21}{\cos 53°\ 32'}$

$\therefore\quad A = 67°\ 25'\ 18''.2\ E$

Since the line and the star are to the opposite sides of the meridian, the azimuth of the line to the west of meridian

= Angle between the line and the star − Azimuth of the star

= 75° 18′ 20″ − 67° 25′ 18″.2 = 7° 53′ 1″.8 to the west of the meridian

∴ Azimuth of the line clockwise from the north

= 360° − 7° 53′ 1″.8 = **352° 6′ 58″.2.**

(*c*) $\angle ZPM = H_1$ = Easterly hour angle of the star.

Hence $\cos H_1 = \dfrac{\tan\theta}{\tan\delta} = \dfrac{\tan 53°\ 32'}{\tan 56°\ 42'\ 53''.2}$

From which $H_1 = 27°\ 20'\ 22''.4 = 1^h\ 49^m\ 21.5^s$

Hence westerly hour angle of the star = $H = 24^h - H_1$ (sec Fig. 1.52)

$= 24^h - 1^h\ 49^m\ 21.5^s$

$= 22^h\ 10^m\ 38.5^s$

Add R.A. of the star = $10^h\ 58^m\ 3.9^s$

∴ L.S.T. of observation $= 33^h\ 8^m\ 42.4^s = 9^h\ 8^m\ 42.4^s$

To convert this L.S.T. to L.M.T., let us first find the L.S.T. of L.M.M. from the given value of G.S.T. at G.M.M.

$$\text{Longitude} = 5^h\ 40^m\ 18^s\ W$$

Acceleration for this at the rate of 9.8565 seconds per hour of longitude is

$$5^h \times 9.8565 = 49.28 \text{ seconds}$$
$$40^m \times 0.1642 = 6.57 \text{ seconds}$$
$$18^s \times 0.0027 = 0.05 \text{ second}$$

$$\text{Total correction} = 55.90 \text{ seconds}$$

$$\therefore \text{ L.S.T. at L.M.M.} = \text{G.S.T. at G.M.M.} + \text{acceleration}$$
$$= 4^h\ 58^m\ 23.84^s + 55.90^s = 4^h\ 59^m\ 19.74^s$$

Now S.I. between the L.M.M. and elongation

$$= \text{L.S.T.} - \text{L.S.T. at L.M.M.}$$
$$= 9^h\ 8^m\ 42.4^s - 4^h\ 59^m\ 19.74^s = 4^h\ 09^m\ 22.66^s$$

This may be converted to mean time interval by subtracting the retardation at the rate of 9.8296 seconds per sidereal hour.

$$\therefore \quad 4^h \times 9.8296 = 39.32 \text{ seconds}$$
$$9^m \times 0.1638 = 1.47 \text{ seconds}$$
$$22.66^s \times 0.0027 = 0.06 \text{ second}$$

$$\text{Total retardation} = 40.85 \text{ seconds}$$

$$\text{Mean time interval} = \text{S.I.} - \text{retardation}$$
$$= 4^h\ 09^m\ 22.66^s - 40.85^s = 4^h\ 8^m\ 41.81^s$$

Fig. 1.52 shows the relative positions, in plan, of the observer (*Z*), the pole (*P*), the star (*M*), the ϒ, and referring object (R.O.).

Example 1.50. *At a place (Latitude 35 ° N, Longitude 15 ° 30′ E), the following observations were taken on a star :*

Observed angle between the R.M. and star = 36 ° 28′ 18″ (clockwise)

R.A. of star :	$10^h\ 12^m\ 6.3^s$
Declination of star :	$20°\ 6'\ 48''.4$
G.M.T. of observation :	$19^h\ 12^m\ 28.6^s$
G.S.T. of G.M.M. :	$10^h\ 12^m\ 36.2^s$

Calculate the true bearing of the reference mark.

Solution

Here, the observations have been taken for the hour angle of the star to calculate the azimuth of the line. From the observed chronometer time (G.M.T.) let us first calculate the hour angle of the star.

G.S.T. of G.M.M.= $10^h\ 12^m\ 36.2^s$.

Since the place has western longitude, let us subtract the retardation from the given G.S.T. of G.M.M. to calculate the L.S.T. of L.M.M.

$$\text{Longitude} = 15°\ 30'\ E = 1^h\ 2^m\ E$$

$$1^h \times 9.8656 = 9.87 \text{ seconds}$$

$$30^m \times 0.1642 = 4.93 \text{ seconds}$$

$$\text{Total} = 14.80 \text{ seconds}$$

$\therefore$ L.S.T. of L.M.M. = $10^h\ 12^m\ 36.2^s - 14.80^s =$ **$10^h\ 12^m\ 21.4^s$**

Now G.M.T. of observation = $19^h\ 12^m\ 28.6^s$

Add east longitude = $1^h\ 2^m$

$\therefore$ L.M.T. of observation = $20^h\ 14^m\ 28.6^s$

Convert this L.M.T. into S.I. by adding the acceleration at the rate of 9.8656 per hour.

$$20^h \times 9.8656 = 197.13 \text{ seconds}$$

$$14^m \times 0.1642 = 2.30 \text{ seconds}$$

$$28.6^s \times 0.0027 = 0.79 \text{ second}$$

$$\text{Total} = 200.22 \text{ seconds} = 3^m\ 20.22^s$$

$\therefore$ S.I .= Mean time + acceleration

$= 20^h\ 14^m\ 28.6^s + 3^m\ 20.22^s = 20^h\ 17^m\ 48.82^s$

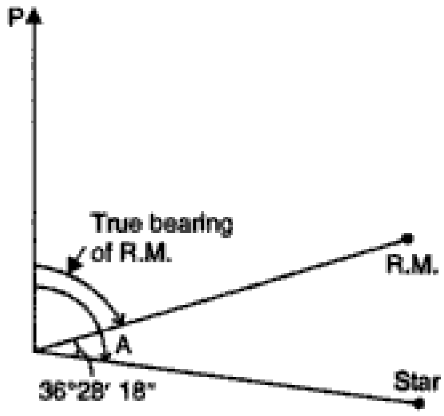


FIG. 1.53

$\therefore$ L.S.T. of observation = L.S.T. of L.M.M. + S.I.

$= 10^h\ 12^m\ 21.4^s + 20^h\ 17^m\ 48.82^s$

$= 30^h\ 30^m\ 10.22^s$

Subtract R.A. of star = $10^h\ 12^m\ 6.30^s$

$\therefore$ Hour angle of the star = $20^h\ 18^m\ 3.92^s = 304°\ 30'\ 58''.8$ (westerly)

$\therefore$ Smallest hour angle in arc (*i.e.* easterly hour angle)

$= H_1 = 360° - H = 360° - 304°\ 30'\ 58''.8 = 55°\ 29'\ 1''.2$ ...(1)

Thus the hour angle is known to us.

The value of the azimuth (A) of the star is calculated from the following expression:

$$\tan A = \tan H\ .\ \cos B\ .\ \operatorname{cosec}(B - \theta) \qquad \text{(Eq. 1.37)}$$

where $\tan B = \tan \delta \sec H$ (Eq. 1.38) $= \tan 20°\ 6'\ 48''.4\ .\ \sec 55°\ 29'\ 1''.2$

$\therefore$ $B = 32°\ 52'\ 27''$

and $B - \theta = 32° 52' 27'' - 35° = -2° 7' 33''$

Hence $\tan A = \tan 55° 29' 1.2'' \cos 32° 52' 27'' \operatorname{cosec}(-2° 7' 33'')$

$\therefore$ $A = 91° 43' 48''$

Now clockwise angle from R.M. to the star = 36° 28′ 18″

$\therefore$ True bearing of the line = Azimuth of star − angle between the line and the star

= 91° 43′ 48″ − 36° 28′ 18″ = **45° 15′ 30″**.

Example 1.51. *The following observations of the sun were taken for azimuth of a line in connection with a survey :*

Mean time = $16^h\ 30^m$

Mean horizontal angle between the sun and the referring object = 18 ° 20′ 30″

Mean corrected altitude = 33 ° 35′ 10″

Declination of the sun from N.A.= + 22° 05′ 36″

Latitude of place = 52° 30′ 20″ *(U.L.)*

Determine azimuth of line.

Solution.

In the astronomical triangle *ZPM*,

ZM = zenith distance = $z = 90° - \alpha = 90° - 33° 35' 10'' = 56° 24' 50''$

PM = Polar distance = co-declination = $90° - \delta$

$= 90° - 22° 05' 36'' = 67° 54' 24''$

ZP = co-latitude = $90° - 52° 30' 20'' = 37° 29' 40''$

By cosine rule :

$$\cos PM = \cos ZP \cdot \cos ZM + \sin ZP \sin ZM \cdot \cos A$$

or
$$\cos A = \frac{\cos PM - \cos ZP \cdot \cos ZM}{\sin ZP \cdot \sin ZM} = \frac{\cos 67° 54' 24'' - \cos 37° 29' 40'' \cdot \cos 56° 24' 50'}{\sin 37° 29' 40'' \cdot \sin 56° 24' 50''}$$

From which $A = 97° 6' 48''$

$\therefore$ Azimuth of the sun = 97° 6′ 48″

Since the sun is to the west (or left) of the R.O., the true bearing of R.O.
= Azimuth of sun + horizontal angle

= 97° 6′ 48″ + 18° 20′ 30″ = **115° 27′ 18″** (Clockwise from North).

Example 1.52. *At a point in latitude 55° 46′ 12″ N, the altitude of sun's centre was found to be 23 ° 17′ 32″ at $5^h\ 17^m$ P.M. (G.M.T.). The horizontal angle of the R.M. and sun's centre was 68 ° 24′ 30″. Find the azimuth of the sun.*

Data:

(*a*) *Sun's declination of G.A.N. on day of observation* = *17 ° 46′ 52″ N*

(*b*) *Variation of declination per hour* = *− 37 ″*

(*c*) *Refraction for altitude 23 ° 20′* = *0 ° 2′ 12″*

(*d*) *Parallax for altitude* $= 0' 8''$

(*e*) *Equation of time (App. – mean)* $= 6^m 0^s$. (*I.R.S.E.*)

Solution

(1) *Calculation of declination*

G.M.T. of observation $= 5^h 17^m 0^s$ (P.M.)

Add Equation of time $= 0^h 6^m 0^s$

∴ G.A.T. of observation $= 5^h 23^m 0^s$ (P.M.)

Now declination at G.A.T. $= 17° 46' 52''$ *N*

Apparent time interval since G.A.N. $= 5^h 23^m 0^s$

∴ Variation in the declination in this time interval at the rate of 37″ per hour = 3′ 39″ (decrease).

∴ Declination at G.A.T. of observation $= 17° 46' 52'' - 3' 39''$
$= 17° 43' 13''$

(2) *Calculation altitude*

Observed altitude of sun's centre $= 23° 17' 32''$

Subtract refraction correction $= 0° 2' 12''$

$= 23° 15' 20''$

Add parallax correction $= 0' 8''$

∴ Correct altitude $= 23° 15' 28''$

Now, co-altitude $= c = 90° - \theta = 90° - 55° 46' 12'' = 34° 13' 48''$

Co-declination $= p = 90° - \delta = 90° - 17° 43' 13'' = 72° 16' 47''$

Co-latitude $= z = 90° - \alpha = 90° - 23° 15' 28'' = 66° 44' 32''$

$2s = 173° 15' 07''$

$s = 86° 37' 34''$

$s - c = 52° 23' 46''$; $s - p = 14° 20' 47''$; $s - z = 19° 53' 02''$

Now, the azimuth of the sun is given by

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-z)\sin(s-c)}{\sin s\,.\,\sin(s-p)}} = \sqrt{\frac{\sin 19° 53' 02'' \sin 52° 23' 46''}{\sin 86° 37' 34'' \sin 14° 20' 47''}}$$

∴ $\frac{A}{2} = 46° 15' 43''$ or $A = \mathbf{92° 31' 26''}$.

Example 1.53. *At a station in latitude 52° 8′ N, longitude 19° 30′ E, the direction of the meridian is known approximately but in order to fix it more precisely it is decided to make an extra-meridian observation of bright-star ($\delta = 29° 52'$ N, R. A. $= 16^h 23^m 30^s$) in the late afternoon. It is considered that the most suitable time is $17^h 5^m$ G.M.T. on a* *date when G.S.T. of G.M.M. in $3^h\ 12^m\ 12^s$. Calculate the approximate direction, east or west of the meridian, and the altitude, at which the telescope should be pointed to locate the star so that exact observations may be made on it.*

Solution. In order to calculate the hour angle of the star, let us first compute the L.S.T. of observation of the star.

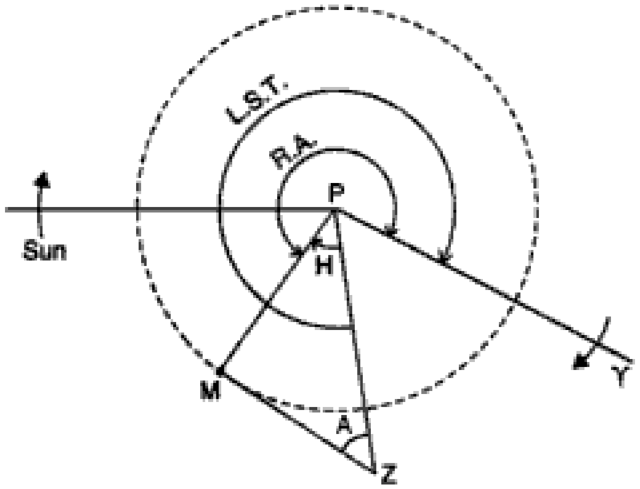


FIG. 1.54

G.M.T. of observation = $17^h\ 5^m\ 0^s$

To covert it into S.I., add the acceleration at the rate of 9.8656 seconds per hour.

$17^h \times 9.8656 = 167.56$ seconds

$5^m \times 0.1642 = \quad 0.82$ second

Total = 168.38 seconds = $2^m\ 48.38^s$

$\therefore$ S.I. = G.M.T. + acceleration

$= 17^h\ 5^m + 2^m\ 48.38^s = 17^h\ 7^m\ 48.38^s$

$\therefore$ G.S.T. of observation = G.S.T. of G.M.M.+ S.I.

$= 3^h\ 12^m\ 12^s + 17^h\ 7^m\ 48.38^s$

$= 20^h\ 20^m\ 0.38^s$

Add west longitude $= 1^h\ 18^m$

$\therefore$ L.S.T. of observation $= 21^h\ 38^m\ 0.38^s$

Subtract R.A. of star $= 16^h\ 23^m\ 30.0^s$

$\therefore$ H.A. of star $= 5^h\ 14^m\ 30.38^s = 78°\ 37'\ 36''$

In Fig. 1.54, *M* is the position of the star at the instant of observation, in relation to the sun and ϒ. *Z* is zenith of the observer and *P* is the pole.

$\therefore \quad PM$ = co-declination = $90° - 29°\ 52' = 60°\ 08' = p$

PZ = co-latitude $= 90° - 52°\ 8' = 37°\ 52' = c$

Now, from the astronomical triangle *ZPM*,

$$\cos H = \frac{\sin \alpha - \sin \delta \sin \theta}{\cos \delta \cdot \cos \theta} = \frac{\cos z - \cos p \cos c}{\sin p \cdot \sin c}$$

or $$\cos z = \cos H \cdot \sin p \sin c + \cos p \cos c$$

$$= \cos 78°\ 37'\ 36'' \cdot \sin 60°\ 08' \cdot \sin 37°\ 52' + \cos 60°\ 08' \cdot \cos 37°\ 52'$$

From which $z = 60°\ 7'\ 32''$

$\therefore$ Altitude of star $= 90° - z = 29°\ 52'\ 28''$

Also by rule, $\dfrac{\sin A}{\sin p} = \dfrac{\sin H}{\sin z}$

$$\therefore \qquad \sin A = \sin p \cdot \frac{\sin H}{\sin z} = \sin 60° \, 08' \cdot \frac{\sin 78° \, 37' \, 36''}{\sin 60° \, 7' \, 32''}$$

A = **78° 38′ 56″** (west).

Example 1.54. *Find the azimuth of the line QR from the following ex-meridian observations for azimuth.*

	Object	*Face*	*Altitude Level O*	*Altitude Level E*
1	*Q*	*L*	...	...
2	*Sun*	*L*	*5.4*	*4.6*
3	*Sun*	*R*	*5.2*	*4.8*
4	*R*	*R*	...	...

	Horizontal Circle A	*Horizontal Circle B*	*Vertical Circle C*	*Vertical Circle D*
1.	*30° 12′ 20″*	*210° 12′ 10″*	...	...
2.	*112° 20′ 30″*	*292° 20′ 20″*	*24° 30′ 20″*	*24° 30′ 40″*
3.	*293° 40′ 40″*	*113° 40′ 30″*	*25° 00′ 00″*	*25° 1′ 00″*
4.	*211° 50′ 30″*	*31° 50′ 20″*	...	...

Latitude of station Q = 36° 48′ 30″ N ; Longitude of station $Q = 4^h \, 12^m \, 32^s$ *E*

Declination of the sun at G.M.N.= 1° 32′ 16″.8 N decreasing 56″.2 per hour

Mean of L.M.T. of two observations $= 4^h \, 15^m \, 30^s$ *P.M. by watch ; watch 4 seconds slow at noon, gaining 0.8 seconds per day.*

The value of level division = 15″

Correction for horizontal parallax = 8″.76

Correction for refraction = 57″ cot (apparent altitude).

Solution.

Mean horizontal angle $= \frac{1}{2}$ [(112° 20′ 25″ − 30° 12′ 15″) + (293° 40′ 35″ − 211° 50′ 25″)]

$= \frac{1}{2}$ [(82° 8′ 10″ + 81° 50′ 10″)] = 81° 59′ 10″

Mean observed altitude = mean of the four vernier readings = 24° 45′ 30″

$$\text{Level correction} = + \frac{\Sigma O - \Sigma E}{4} \times \text{ value of the one level division}$$

$$= + \frac{10.6 - 9.4}{4} \times 15'' = + 4''.5$$

∴ Apparent altitude = 24° 45′ 30″ + 4″.5 = 24° 45′ 34″.5

Refraction correction = − 57″ cot 24° 45′ 34″.5 = 1′ 6″.7

Correction for parallax = + 8″.77 cos 24° 45′ 34″.5 = 7″.8

∴ True altitude = 24° 45′ 34″.5−1′ 6″.7+7″.8 = 24° 44′ 35″.6

Mean time of observation	$= 4^h\ 15^m\ 30^s$
Watch correction $= +\left(4 - \dfrac{0.8 \times 4.26}{24}\right)$	$= + \quad 3.86^s$
$\therefore$ Correct L.M.T.	$= 4^h\ 15^m\ 33.86^s$
Deduct East Longitude	$= 4^h\ 12^m\ 32.0^s$
$\therefore$ G.M.T. of observation	$= 0^h\ 3^m\ 1.86^s$
Sun's declination at G.M.N.	$= 1°\ 32'\ 16''.8\ N$
Variation for $3^m\ 1.86^s$	$= -56''.2\ (0.0505^h) = -2.8^s$

$\therefore$ Declination of sun at the instant of observation

$$= 1°\ 32'\ 16''.8 - 2.8^s = 1°\ 32'\ 14''$$

Now, in the astronomical triangle ZPM,

$ZP = c = 90° - \theta = 90° - 36°\ 48'\ 30'' \qquad = 53°\ 11'\ 30''$

$ZM = z = 90° - \alpha = 90° - 24°\ 44'\ 35''.6 \qquad = 65°\ 15'\ 24''.4$

$PM = p = 90° - \delta = 90° - 1°\ 32'\ 14'' \qquad = 88°\ 27'\ 46''$

$2s = 206°\ 54'\ 40''.4$

$s = 103°\ 27'\ 20''.2$

$\therefore \qquad s - c = 50°\ 15'\ 50''.2 \quad ; \quad s - z = 38°\ 11'\ 55''.8 \quad ; \quad s - p = 14°\ 59'\ 33''.8$

The azimuth A is given by

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-z)\sin(s-c)}{\sin s \, . \, \sin(s-p)}} = \sqrt{\frac{\sin 38°\ 11'\ 55''.8 \, . \, \sin 50°\ 15'\ 50''.2}{\sin 103°\ 27'\ 20''.2 \, . \, \sin 14°\ 59'\ 33''.8}}$$

$$\therefore \qquad \frac{A}{2} = 62°\ 7'4''.9 \quad \text{or} \quad A = 124°\ 14'\ 9''.8$$

$\therefore$ Azimuth of the sun $= 124°\ 14'\ 9''.8$

(west, since the sun was observed in the evening)

Clockwise angle from the R.M. to the sun $= 81°\ 59'\ 10''$

$\therefore$ Azimuth of line from north towards west

$$= 124°\ 14'\ 9''.8 + 81°\ 59'\ 10'' = 206°\ 13'\ 19''.8$$

$\therefore$ Azimuth of line from north (clockwise)

$$= 360° - 206°\ 13'\ 19''.8 = \mathbf{153°\ 46'\ 40''.2}.$$

1.17. THE DETERMINATION OF LATITUDE

The following are some of the most practicable and most generally used methods for determining the latitude of a place :

1. By meridian altitude of sun or star.
2. By zenith pair observation of stars.
3. By meridian altitude of star at lower and upper culmination.

4. By ex-meridian observation of star or sun.
5. By prime vertical transits.
6. By determining the altitude of the pole star.
7. By circum-meridian altitude of sun or star.

1. (*a*) LATITUDE BY MERIDIAN ALTITUDE OF STAR

In this method, the altitude of a heavenly body is measured when it is crossing the meridian. The method is based on the important fact that the latitude of the place is equal to the altitude of the pole. If we can measure the meridian altitude of the star whose declination (and hence polar distance) is known, the latitude can be easily computed. The observed altitude should be corrected for the refraction, as discussed earlier. The accuracy of determination may be increased if it is possible to take two observations for altitude upon the same star, the face of instrument being reversed after the first reading is taken. This is possible with close circumpolar stars, specially when observations are taken with an ordinary 20″ theodolite. The method is, therefore, used for less refined determinations. The direction of the meridian of the place must be known, or must be established before the observations are made.

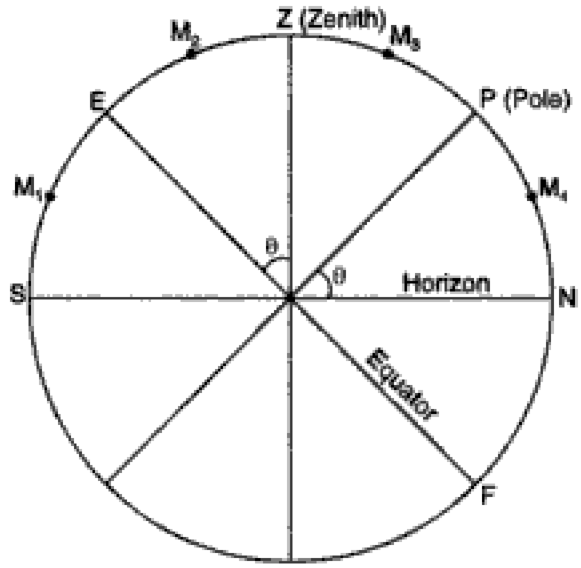


FIG. 1.55. MERIDIAN ALTITUDE OF STAR

To calculate the latitude (θ) of the place of observation from the known value of declination (δ) and the observed value of the altitude (α), we will consider the *four cases* that arise according to the position of the star (Fig. 1.55).

Case 1. When the star is between the horizon and the equator.

M_1 is the position of the star when it is between the horizon and the equator.

$$ZP = \text{co-latitude} = 90° - \theta$$

$$EZ = \text{latitude} = \theta$$

$$SM_1 = \alpha_1 = \text{altitude of the star}$$

$$ZM_1 = 90° - \alpha_1 = z_1 = \text{zenith distance of the star}$$

$$EM_1 = \delta_1 = \text{declination of the star (south)}$$

Now $EZ = ZM_1 - EM_1$ or $\theta = (90° - \alpha_1) - \delta_1 = z_1 - \delta_1$

Hence ***latitude = zenith distance − declination.***

Case 2. When the star is between the equator and the zenith.

M_2 is the position of the star when it is between the equator and the zenith.

$$SM_2 = \alpha_2 = \text{altitude of the star}$$

$$ZM_2 = (90° - \alpha_2) = z_2 = \text{zenith distance of the star}$$

$EM_2 = \delta_2$ = declination of the star.

Now $EZ = ZM_2 + EM_2$

or $\theta = (90° - \alpha_2) + \delta_2$ or $\theta = z + \delta_2$

Hence ***latitude = zenith distance + declination.***

Case 3. When the star is between the zenith and the pole.

M_3 is the position of the star when it is between the zenith and the pole.

$NM_3 = \alpha_3$ = altitude of the star

$ZM_3 = (90° - \alpha_3) = z_3$ = zenith distance of the star

$EM_3 = \delta_3$ = declination of the star

Now $EM = EM_3 - ZM_3$

or $\theta = \delta_3 - (90° - \alpha_3) = \delta_3 - z_3$

Hence ***latitude = declination − zenith distance.***

Case 4. When the star is between the pole and the horizon.

M_4 is the position of the star when it is between the pole and the horizon.

$NM_4 = \alpha_4$ = altitude of the star

$ZM_4 = (90° - \alpha_4) = z_4$ = zenith distance of the star

$FM_4 = \delta_4$ = declination of the star.

Now PN = altitude of the pole = latitude of the place $= \theta$

$= NM_4 + PM_4 = \alpha_4 + (PF - FM_4)$

$= \alpha_4 + (90° - \delta_4) = (90° - z_4) + (90° - \delta_4) = 180° - (z_4 + \delta_4)$

Hence ***latitude = 180° −(zenith distance + declination).***

1. (*b*) LATITUDE BY MERIDIAN ALTITUDE OF THE SUN

The altitude of the sun at local apparent noon (meridian passage) may be measured by placing the line of sight of the transit in the plane of the meridian and observing the altitude of the upper or lower limb of the sun when it is on the vertical cross hair. The observed altitude is then corrected for instrumental errors, refraction, parallax and semi-diameter. The mean time of observation should also be noted. The declination of the sun continually changes, and hence a correct knowledge of mean time and longitude of the place of observation is essential in order to compute the value of declination at the instant of observation. Knowing the altitude and the declination of the sun at the instant of observation, the latitude can be computed as follows (Fig. 1.56).

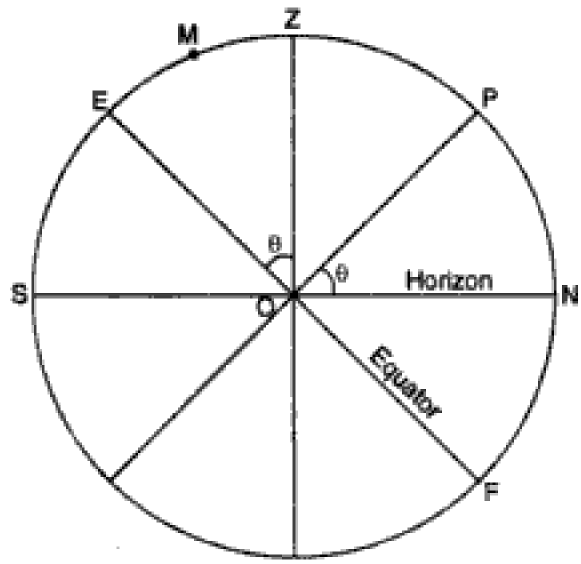


FIG. 1.56. MERIDIAN ALTITUDE OF THE SUN.

In Fig. 1.56, M is the position of the sun.

$SM = \alpha$ = meridian altitude of the sun (corrected).

$ZM = 90° - \alpha = z$ = meridian zenith distance of the sun.

$EM = \delta$ = declination of the sun.

Then latitude $= \theta = EZ = ZM + EM$

$= (90° - \alpha) + \delta = z + \delta$

or ***latitude = zenith distance + declination.***

In the above expression, δ is positive or negative according as the sun is to north or south of the equator.

If the direction of the meridian is not known, the maximum altitude of the sun is observed and may be taken as the meridian altitude. This is not strictly true, due to sun's changing declination. However, the difference between the maximum altitude and the meridian altitude is usually a fraction of a second, and may be entirely neglected for observations made with the engineer's transit or the sextant.

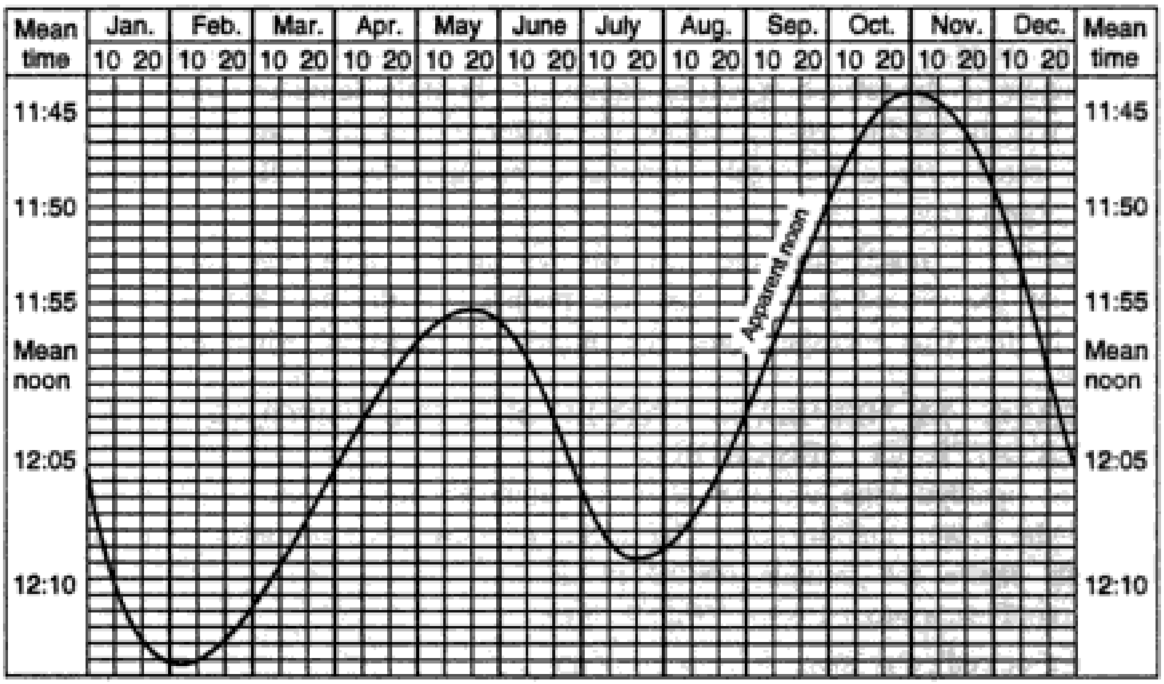


FIG. 1.57. MEAN TIME OF APPARENT NOON.

In order that the observer may be well ready for taking the observations at the meridian transit, standard time or the watch time of local apparent noon must be known. The standard time of local apparent noon varies throughout the year. Fig. 1.57 shows graphically the local mean time of the local apparent noon. The standard time can be known by applying a correction for the difference in longitude between the local meridian and the standard meridian. The observer should be ready to begin observing at this time.

(2) LATITUDE BY ZENITH PAIR OBSERVATIONS OF STARS

This method is an improvement over the previous method to get more precise results. The errors of observation, refraction and instrument can be effectively reduced by making observations upon two stars which culminate at approximately equal latitudes on opposite sides of observer's zenith. The altitude of one star at its culmination is observed first.

The telescope is then reversed in azimuth and the meridian altitude of the other star is then observed. The two stars chosen should be such that their right ascensions differ by 10 to 30 minutes. The time of culmination of these two stars will then differ by 10 to 30 minutes and the observer will have sufficient time in observing the second star after taking the reading of the first and reversing.

Thus, let M_2 and M_3 (Fig. 1.55) be the two stars having approximately equal altitudes to the north and south side of the observer's zenith, and having their time of culminations differing by 10 to 30 minutes.

As derived earlier,

For the position M_2, latitude $\theta = (90° - \alpha_2) + \delta_2$ ...(1)

For the position M_3, latitude $\theta = \delta_3 - (90° - \alpha_3)$ ...(2)

$$\therefore \text{ Average latitude} = \frac{1}{2}\left[\{(90° - \alpha_2) + \delta_2\} + \{\delta_3 - (90° - \alpha_3)\}\right] = \frac{\alpha_3 - \alpha_2}{2} + \frac{\delta_2 + \delta_3}{2}$$

From the above expression, it is clear that the average latitude depends upon the *difference* in latitudes of the two stars, and *not* on the individual latitude. Hence any error in the correction for the refraction will be common to both the latitudes (which are approximately equal) and will be eliminated by taking the difference of the two latitudes. Similarly, the instrumental errors are also largely eliminated because these will be practically the same with each observation.

It should be noted that the face of the instrument is *not* reversed while reading the altitude of the second star. To take the reading for the meridian altitude, the telescope is directed to the true meridian, and the altitude is measured when the star intersects the vertical wire.

(3) LATITUDE BY MERIDIAN ALTITUDE OF A CIRCUMPOLAR STAR AT UPPER AND LOWER CULMINATIONS

In this method, the altitude of a circumpolar star is measured both at its upper as well as the lower culmination. The mean of these two altitudes gives the altitude of the pole and hence the latitude of the place of observation. This is proved below (Fig. 1.58).

M is a circumpolar star. *A* is its position at the upper culmination when its altitude is maximum. *B* is its position at the lower culmination when its altitude is minimum. The dotted circle shows the path of the star round the pole.

$AN = \alpha_1$ = altitude of the star at its upper culmination.

$BN = \alpha_2$ = altitude of the star at its lower culmination.

Now latitude of place = altitude of the pole = θ = PN

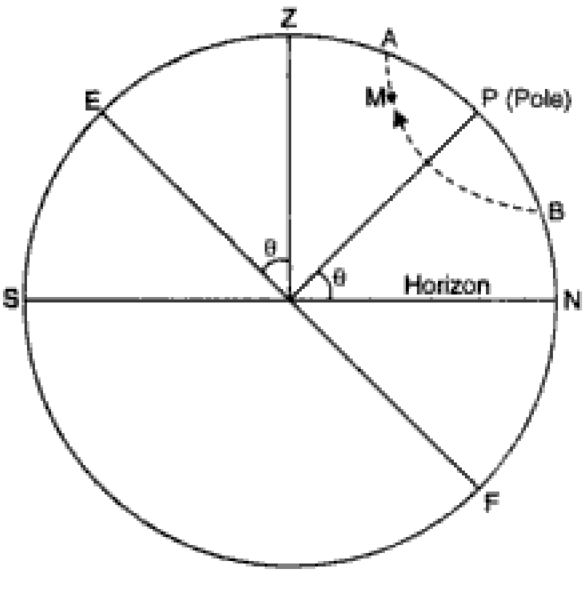


FIG. 1.58

$$PN = BN + BP = \alpha_2 + BP$$

Also $\quad PN = AN - AP = \alpha_1 - AP$

Adding the two, we get

$$2PN = (\alpha_1 - AP) + (\alpha_2 + BP)$$

But $\quad AP = BP =$ co-declination of the star

$\therefore \quad 2PN = \alpha_1 + \alpha_2 \quad$ or $\quad PN = \theta = \dfrac{\alpha_1 + \alpha_2}{2}$

Hence the latitude of the place of observation is equal to half the sum of the altitude observed at its upper and lower culminations. In this method, the knowledge of the declination of the star is not necessary. However, the method is open to the objection that 12 sidereal hours elapse between the two observations. The method is, therefore, not much used.

(4) LATITUDE BY EX-MERIDIAN OBSERVATION OF STAR OR SUN

In this method, the altitude of the star is observed in any position. The exact chronometer time is also noted at the instant the observation is taken. The known mean time of the chronometer is converted into the local sidereal time. The hour angle of the star can then be computed from the expression:

L.S.T. = R.A. of the star + H.A. of the star.

In the astronomical triangle MPZ in Fig. 1.59,

$$ZM = 90° - \alpha = \frac{\pi}{2} - \alpha \qquad \text{(known)}$$

$$PM = 90° - \delta = \frac{\pi}{2} - \delta \qquad \text{(known)}$$

$$\angle MPZ = H \qquad \text{(known)}$$

Hence the side $ZP = (90° - \theta)$ can be calculated from the cosine formula

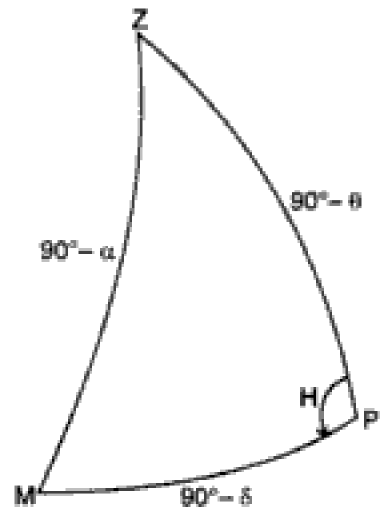


FIG. 1.59

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\left(\frac{\pi}{2} - \theta\right)\cos\left(\frac{\pi}{2} - \delta\right) + \sin\left(\frac{\pi}{2} - \theta\right)\sin\left(\frac{\pi}{2} - \delta\right)\cos H$$

or $\quad \sin\alpha = \sin\theta\sin\delta + \cos\theta\cos\delta\cos H \quad$ ...(1)

In the above equation, there are two terms for θ, *i.e.*, $\sin\theta$ and $\cos\theta$. The equation can be best solved by introducing two arbitrary unknowns m and n as follows :

Let $\quad \sin\delta = m\sin n \quad$ ...(*i*) $\quad$ and $\quad \cos\delta\cos H = m\cos n \quad$ ...(*ii*)

Dividing (*i*) by (*ii*), we get

$$\frac{\sin\delta}{\cos\delta\cos H} = \frac{\sin n}{\cos n} \quad \text{or} \quad \tan\delta\sec H = \tan n \qquad ...(iii) \quad ...(1.48)$$

Substituting the value of equations (*i*) and (*ii*) in equation (1), we get

$$\sin\alpha = \sin\theta \,.\, m\sin n + \cos\theta \,.\, m\cos n$$

or $\quad \sin\alpha = m(\sin\theta\sin n + \cos\theta\cos n)$

or $\quad \sin\alpha = m\cos(\theta - n)$

or $\quad m = \sin\alpha\sec(\theta - n) \quad$ ...(1.49)

Substituting the value of m in equation (i), we get

$$\sin \delta = \sin \alpha \ . \ \sec (\theta - n) \ . \ \sin n$$

or $$\cos (\theta - n) = \sin \alpha \ . \ \sin n \ . \ \text{cosec}\ \delta \quad \ldots(iv) \quad \ldots(1.50)$$

Thus, the value of n is obtained from equation (iii), and then substituted in equation (iv) to get the value of θ. For the use of the method of computation of θ, see example 1.60.

(5) LATITUDE BY PRIME VERTICAL TRANSIT

As defined earlier, the prime vertical is a plane at right angles to the meridian, running truly east and west. A star, having polar distance less than 90° and greater than the co-latitude of the place, will cross the prime vertical twice in a sidereal day. The field work, therefore, consists in measuring the time interval between east and west transits of the star. The best stars for observations are those that cross the prime vertical near the zenith.

Thus in Fig. 1.60 (a), S, W, N and E are the south, west, north and east points on the horizon. Z is the zenith of the observer, and P the pole. The dotted circle shows the path of a circumpolar star, WZE is the plane of the prime vertical passing through the west-east points and hence perpendicular to the meridian at Z. M_1 and M_2 are the east and west transits of the star across the prime vertical. Half the time that elapses between the two transits M_1 and M_2 in sidereal hours represents the angle M_1PZ (H).

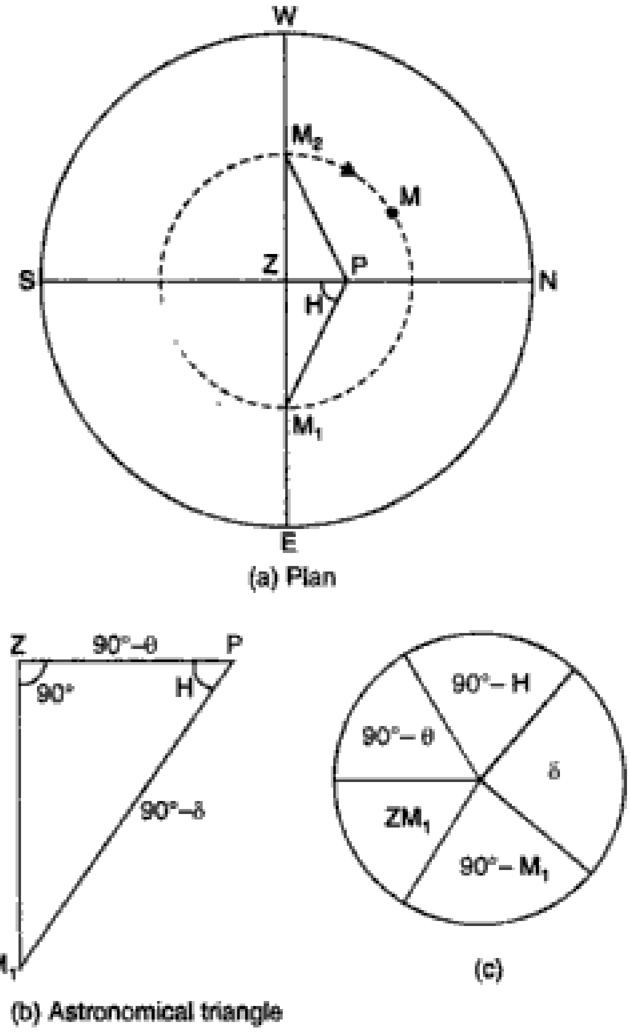


FIG. 1.60. LATITUDE BY PRIME VERTICAL TRANSIT

From the right angled triangle M_1PZ

$M_1 P = 90° - \delta$ (known)

$\angle M_1PZ = H$ (known)

$ZP = (90° - \theta)$, to be computed.

From the Napier's rule for the right-angled triangle, [Fig. 1.60 (c)].

sine of the middle part = product of tangents of adjacent parts

$$\therefore \quad \sin (90° - H) = \tan (90° - \theta) \tan \delta$$

or $$\cos H = \cot \theta \tan \delta$$

or $\tan\theta = \tan\delta \,.\, \sec H$

where H = half the interval of time between the east and the west transits expressed in angular measure.

Since the altitude is not measured in this method, the errors due to uncertainty in the value of refraction is largely eliminated. Also, the exact knowledge of local time is not required since we have to simply measure the interval of sidereal hours that elapses between the two transits. However, the approximate local time of prime-vertical transits must be known. To take the time readings, the instrument has to be directed towards the direction of prime vertical, first to the east side and then to the west side, and measure the time when transit occurs, *i.e.*, where the star crosses the vertical cross-hair.

The effect of an Error in the Determination of the Time Interval

Let y = error in the determination of the *time interval*

and x = corresponding error in the latitude.

Then $$x = y\,\frac{\sin 2\theta}{2}\sqrt{\frac{\tan^2\theta}{\tan^2\delta} - 1} \qquad \text{...(1.51)}$$

From the above relationship between the two errors, we draw the following conclusions:

(1) If $\delta = \theta$, x is very small. However, the star would pass through the zenith and observations cannot be made.

(2) If $\delta = 0$, the star would pass through E and W points, the interval between the transits will be exactly 12 hours whatever may be the position of the observer and hence the determination cannot be made. The value of x will be great for very small value of δ.

Hence the stars observed should be as high up on the prime vertical as is consistent with an exact determination of the time of transit.

The effect of an Error in the Direction of Prime Vertical

The error in the setting out of the direction of the prime vertical has very little effect in the latitude of the place for ordinary engineering purposes. If the eastern transit occurs earlier due to the wrong direction of the prime vertical, the western transit will also take place correspondingly earlier, though not exactly by the same amount. In a latitude of 30°, even if the prime vertical is set out by 1° out of its true position, the resulting error in latitude determination will be less then 1" for observations on a star having declination = 20°.

Striding Level Correction to Prime Vertical Determinations

For the prime vertical determinations, the instrument must be in perfect adjustment. If the transverse axis of the instrument is inclined by a certain value, the resulting error in the determination will be equal to this value. Hence striding level should always be used when taking the vertical observations.

Thus, in Fig. 1.61, if the transverse axis is inclined, *ECW* is the circle upon which observations are made instead of the true prime vertical *EZW*. The star is then observed to the transit at the point *M* on the inclined prime vertical. The observed angle *MPC* = *H*.

Then $\cot CP = \tan \delta \times \sec H$, if we take $\angle PCM = 90°$.

Then true co-latitude = $CP \pm ZC$

or true latitude = 90° − co-latitude = observed latitude $\mp$ ZC

where ZC = angular measure of the level correction $= \dfrac{N - S}{2} d$ ...(1.52)

where N = mean reading of north side of bubble.

S = mean reading of south side of bubble.

d = value of one division.

Use − sign if C and P are to the same side of Z and + sign if C and P are to the opposite side of Z.

Thus if the south end of the axis is higher, C and P will be to the same side of Z and the level correction ZC should be subtracted from the calculated value of the latitude to get the true value of the latitude.

However, if the north end of the axis is higher, C and P will be to the opposite sides of Z and the level correction ZC should be added to the calculated value of the latitude to get the true value of the latitude.

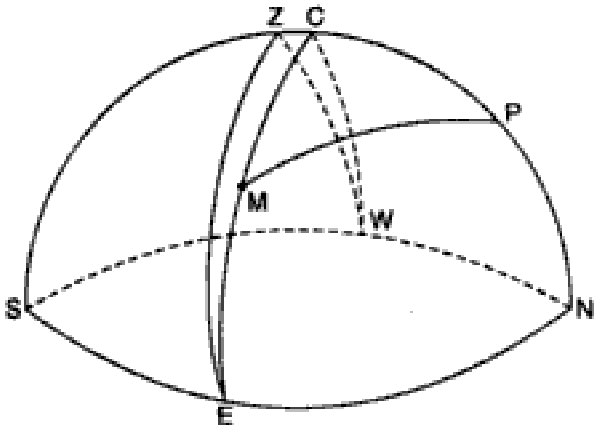


FIG. 1.61

(6) LATITUDE BY DETERMINING THE ALTITUDE OF THE POLE STAR AT ANY TIME

We know that the latitude of a place is equal to the altitude of the pole. If there were any star *at the pole,* we could have observed its altitude. However, the pole star is very near to the pole. The altitude observations on the pole star can, therefore, be made at any known time, and correction can be applied to the observed altitude to get the latitude of the place of observation.

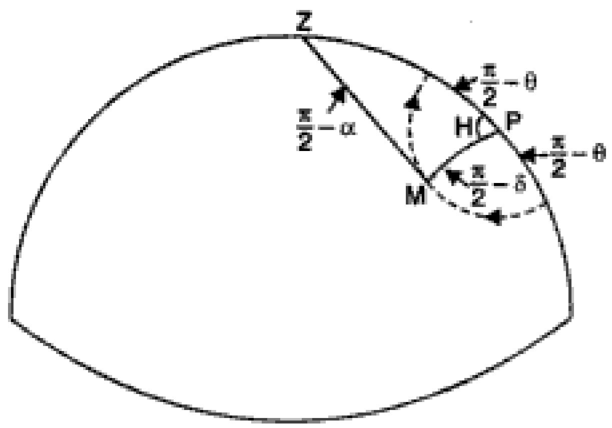


FIG. 1.62.

Thus, in Fig. 1.62, M is the position of the pole star at the time of observation. Let α be the observed altitude. The mean time is also observed from the chronometer, and is converted into sidereal time. The hour angle H is then computed from the relation:

L.S.T. = R.A. of pole star + Hour angle.

In the triangle ZPM, $ZM = \dfrac{\pi}{2} - \alpha$ (known) ; $PM = \left(\dfrac{\pi}{2} - \delta\right) = p$ (known)

$\angle ZPM = H$ (known)

The co-latitude ZP can be calculated from the cosine formula

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\cos\left(\frac{\pi}{2}-\theta\right)\cos\left(\frac{\pi}{2}-\delta\right)+\sin\left(\frac{\pi}{2}-\theta\right)\sin\left(\frac{\pi}{2}-\delta\right)\cos H$$

or
$$\sin\alpha=\sin\theta\sin\delta+\cos\theta\cos\delta\cos H$$

or
$$\sin\alpha=\sin\theta\cos p+\cos\theta\cos\delta\cos H \qquad \ldots(1)$$

Let α differ from θ by a small amount x, so that

$\alpha=\theta+x$, where x is the small correction

Substituting $\alpha=\theta+x$ in (1), we get

$$\sin\theta\cos x+\cos\theta\sin x=\sin\theta\cos p+\cos\theta\sin p\cos H$$

Expanding the terms having small quantities x and p, we get

$$\sin\theta\left(1-\frac{x^2}{2}+\ldots\right)+\cos\theta\left(x-\frac{x^3}{6}+\ldots\right)=\sin\theta\left(1-\frac{p^2}{2}\ldots\right)+\cos\theta\cos H\left(p-\frac{p^3}{6}+\ldots\right) \quad \ldots(2)$$

Neglecting the square and higher values of x and p in the above, we get

$$x=p\cos H \qquad \ldots(i)\quad(1.53)$$

This gives the values of x to the first approximation.

Next, retaining the squares of x and p, and neglecting their higher powers in equation (2), we get

$$x\cos\theta=p\cos\theta\cos H-\frac{p^2}{2}\sin\theta+\frac{x^2}{2}\sin\theta$$

Putting $x^2=p^2\cos^2 H$, we get $x=p\cos H-\dfrac{p^2}{2}\tan\theta\sin^2 H \qquad \ldots(ii)\quad(1.54)$

This gives the value of x to the second approximation.

The second term in this expression is very small, and becomes still small when multiplied by p^2. Hence we can approximately write $\tan\theta=\tan\alpha$ so that

$$x=p\cos H-\tfrac{1}{2}p^2\,.\tan\alpha\,.\ \sin^2 H$$

where x and p are in circular measure.

If, however, x and p are measured in seconds, we get

$$x=p\cos H-\tfrac{1}{2}p^2\tan\alpha\,.\sin^2 H\,.\sin 1''$$

The correct latitude is, therefore, given by

$$\theta=\alpha-x$$

or
$$\theta=\alpha-p\cos H+\tfrac{1}{2}p^2\tan\alpha\,.\sin^2 H\,.\sin 1'' \qquad \ldots(1.55)$$

The above formula gives accurate results within 1″.

The field observations consist in observing four altitudes in quick succession – first with face right, two with face left and then again with face right – and the chronometer time of all the four determinations are observed. The mean values of the four altitudes and the four times are taken for the computation of θ. The declination and R.A. of the pole star are taken from the *nautical almanac*.

(7) LATITUDE BY CIRCUM-MERIDIAN ALTITUDE OF STAR OR THE SUN

The *circum-meridian observations* are the observations of stars or the sun taken near to the meridian. The method is used for very accurate determination of latitude by observing the circum-meridian altitudes at noted times of each of the several stars for a few minutes before and after transit and reducing them to the meridian altitude. The errors due to erroneous value of refraction, personal error and those due to instruments are very much reduced by observing an equal number of north and south stars in pairs of similar altitude. Accurate chronometer time and its error is also essential to calculate the hour angle of the individual stars. The observation of each star is commenced about 10^m before the computed time of transit and is continued for about 10^m after transit. Equal number of the face right and face left observations are necessary on a particular star. However, both face observations are not taken if observations are adequately paired on north and south stars.

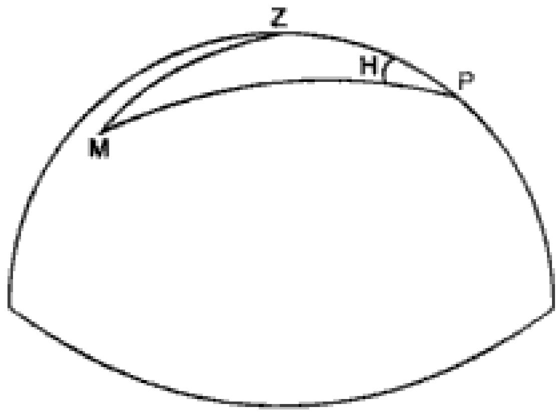


FIG. 1.63

In Fig. 1.63, let

$z = MZ$ = zenith distance of star M, corrected for refraction

$p = MP$ = polar distance

$c = PZ$ = co-latitude

$H = \angle MPZ$ = Hour angle

From the astronomical triangle MPZ, we get

$$\cos z = \cos c \cos p + \sin c \,.\, \sin p \cos H \qquad \ldots(1)$$

Let x = correction to be applied to the observed z to get the meridian zenith distance when the star is on meridian.

Then meridian zenith distance $= z - x$.

Again, when the star is on the meridian, its zenith distance

$$= MZ = MP - ZP = p - c.$$

Hence $$z - x = p - c \qquad \ldots(2)$$

Writing $\cos H = 1 - 2 \sin^2 \dfrac{H}{2}$ in (1), we get

$$\cos z = \cos c \cos p + \sin c \sin p - 2 \sin c \,.\, \sin p \,.\, \sin^2 \frac{H}{2}$$

or $$\cos z = \cos (c - p) - 2 \sin c \,.\, \sin p \,.\, \sin^2 \frac{H}{2}$$

or $$\cos z - \cos (p - c) = - 2 \sin c \,.\, \sin p \,.\, \sin^2 \frac{H}{2}$$

Substituting $p - c = z - x$ from (2), we get

$$\cos z - \cos (z - x) = - 2 \sin c \,.\, \sin p \sin^2 \frac{H}{2}$$

$$\therefore \quad \sin \frac{x}{2} \sin \left(z - \frac{x}{2} \right) = \sin c \,.\, \sin p \sin^2 \frac{H}{2}$$

From which $$\sin\frac{x}{2} = \frac{\sin c \, . \, \sin p \, . \, \sin^2 \frac{H}{2}}{\sin\left(z - \frac{x}{2}\right)}$$

Since x is small, we can replace $\sin\frac{x}{2}$ by $\frac{x}{2}\sin 1''$, if x is measured in seconds of arc.

$$\therefore \qquad x = \frac{\sin c \, . \, \sin p}{\sin\left(z - \frac{x}{2}\right)} \cdot \frac{2\sin^2\frac{H}{2}}{\sin 1''}$$

Also, putting $\sin\left(z - \frac{x}{2}\right) = \sin(z - x) = \sin(p - c)$ (approximately)

we get $$x = \frac{\sin c \, . \, \sin p}{\sin(p - c)} \cdot \frac{2\sin^2\frac{H}{2}}{\sin 1''} \qquad \text{...(3)}$$

But $\sin c = \cos\theta$; $\sin p = \cos\delta$

and $\sin(p - c) = \sin$ (meridian zenith distance) $= \cos$ (meridian altitude) $= \cos h$

where h = meridian altitude.

Then equation (3) reduces to

$$x = \frac{\cos\theta\cos\delta}{\cos h} \cdot \frac{2\sin^2\frac{H}{2}}{\sin 1''} \qquad \text{...(4) (1.56)}$$

But $h = \alpha + x$, where α = observed altitude

Hence $$h = \alpha + \frac{\cos\theta\cos\delta}{\cos h} \cdot \frac{2\sin^2\frac{H}{2}}{\sin 1''} \qquad \text{...(6)}$$

or $$h = \alpha + Bm \qquad \text{...(1.57)}$$

where $$B = \frac{\cos\theta \, . \, \cos\delta}{\cos h} \quad \text{...[1.57 (a)]} \qquad \text{and} \qquad m = \frac{2\sin^2\frac{H}{2}}{\sin 1''} \quad \text{...[1.57 (b)]}$$

(*H is in arc measure*)

The factor m is usually taken from the tables.

If a series of observations are made upon the same star, the factor B is the same for each observation.

In the factor B, the value θ to be used is the approximate value deduced from the map or determined from the meridian observations. Similarly, h is the meridian altitude computed from the approximate latitude and the known declination of the star.

Let $\alpha_1, \alpha_2, \alpha_3 \ldots$ = circum-meridian altitudes of the same star

$m_1, m_2, m_3 \ldots$ = corresponding value of m.

Then $h_1 = \alpha_1 + Bm_1$; $h_2 = \alpha_2 + Bm_2$; $h_3 = \alpha_3 + Bm_3$ etc. etc.

Hence $h_0 = \alpha_0 + Bm_0$

where h_0 = mean of the deduced meridian altitudes

α_0 = mean of the actual observed altitudes

m_0 = mean of the computed factors m

Thus the meridian altitude of the star is known.

More exact formula

A more elaborate formula for getting the meridian altitude from the observed circum-meridian is as follows :

$$h = \alpha + Bm + Cm' \qquad \text{...(1.58)}$$

where $$C = B^2 \tan h \qquad \text{...[1.58 (a)]} \quad \text{and} \quad m' = \frac{2 \sin^4 \frac{H}{2}}{\sin 1''} \qquad \text{...[1.58 (b)]}$$

The term $C.\, m'$ is never more than 1″.

Knowing the meridian altitude (h), the latitude θ can be calculated by the formula developed in method 1 of determination of latitude.

If special tables are not available, m can be calculated as follows :

$$m = \frac{2 \sin^2 \frac{H}{2}}{\sin 1''}, \text{ (where } H \text{ is arc measure) ;}$$

But 1 sec. time (H) = 15″ arc

$$\therefore \quad m = \frac{2 \sin^2 \frac{1}{2}(15\, H)}{\sin 1''} \quad (\textit{where } H \textit{ is in seconds of time}) \qquad \text{...[1.59 (a)]}$$

$$= \frac{225}{2} H^2 \cdot \frac{(\sin 1'')^2}{\sin 1''} = \frac{225}{2} H^2 \cdot \frac{1}{206265}$$

$$= \frac{H^2}{1834}, \; H \textit{ being in seconds of time.} \qquad \text{...(1.59)}$$

Example 1.55. *The meridian altitude of a star was observed to be 65° 40′ 18″ on a certain day, the star lying between the pole and the zenith. The declination of the star was 53° 12′ 10″ N. Find the altitude of the place of observation.*

Solution. (Fig. 1.55)

M_3 is the position of the star under observation. Let us first correct the altitude of the star for refraction.

Correction for refraction = 57″ cot 65° 40′ 18″ = 25″.78

∴ True altitude = observed altitude − refraction

= 65° 40′ 18″ − 25″.78 = 65° 39′ 42″.22

∴ zenith distance z_3 = 90° − 65° 39′ 42″.22 = 24°20′17″.78

Now latitude = declination − zenith distance

= $\delta_3 - z_3$ = 53° 12′ 10″ − 24° 20′ 17″.48 = **28° 51′ 52″.22 N.**

Example 1.56. *The meridian altitude of a star was observed to be 64° 36′ 20″ on a certain day, the star lying between the zenith and the equator. The declination of the star was 26° 12′ 10″ N. Find the latitude of the place of observation.*

Solution. (Fig. 1.55)

M_2 is the position of the star under observation. Let us first correct altitude of the star for refraction.

Refraction correction = 57″ cot 64° 36′ 20″ = 27″.06

True altitude = observed altitude − refraction

= 64° 36′ 20″ − 27″.06 = 64° 35′ 52″.94

∴ Zenith distance = z_3 = 90° − 64° 35′ 52″.94 = 25° 24′ 7″.06

∴ Latitude = $\delta_2 + z_2$ = 26° 12′ 10″ + 25° 24′ 7″.06 = **51° 36′ 17″.06 N**.

Example 1.57. *An observation for altitude was made at a place in longitude 75° 20′ 15″ W. The meridian altitude of the sun's lower limb was observed to be 44° 12′ 30″, the sun being to the south of the zenith. Sun's declination at G.A.N. on the day of observation was + 22° 18′ 12″.8, increasing 6″.82 per hour, and semi diameter 15′ 45″.86. Find the latitude of the place of observation.*

Solution. (Fig. 1.56)

In Fig. 1.56, *M* is the position of the sun, to the south of zenith.

The latitude of the place = corrected declination + corrected zenith distance.

Let us first correct the observed altitude for refraction, parallax and semi-diameter.

(*i*) correction for refraction = − 57″ cot 44° 12′ 30″ = − 59″.6

(*ii*) correction for parallax = + 8″.78 cos 44° 12′ 30″ = + 6″.29

(*iii*) correction for semi-diameter = + 15′ 45″.86. The correction is additive since the sun's lower limb was observed.

Now observed altitude of sun = 44° 12′ 30″

Add parallax correction = 06″.29

Add semi-diameter = 15′ 45″.86

= 44° 28′ 22″.15

Subtract refraction correction = 59″.60

Correct altitude = 44° 27′ 22″.55

∴ Zenith distance z = 90° − 44° 27′ 22″.55 = 45° 32′ 37″.45 ...(1)

Now when the sun is over the meridian, the L.A.N. is zero.

Longitude = 75° 20′ 15″ *W* $5^h\ 1^m\ 21^s$ west

L.A.T. of observation = $0^h\ 0^m\ 0^s$

Add west longitude = $5^h\ 1^m\ 21^s$

∴ G.A.T. of observation = $5^h\ 1^m\ 21^s$

Declination of sun at G.A.N. = 22° 18′ 12″.8

Add increase = (6″.82 × 5.022) = 34″.25

∴ Declination of sun at L.A.N. = 22° 18′ 47″.05 ...(2)

Since the sun is to the south of the latitude,

$$\theta = \delta + z = 22° \ 18' \ 47''.05 + 45° \ 32' \ 37''.45 = \mathbf{67° \ 51' \ 25''.5}.$$

Example 1.58. *A star of declination 46° 45′ 33″ (south) is to be observed at lower and upper transit at a place in approximate latitude 80° south. Find the approximate apparent altitudes at which the star should be sighted in order that accurate observations may be made upon it.*

Solution

In Fig. 1.64, P' is the south pole and Z is the zenith of the observer. EO is the equator, and NS is the horizon, N and S being north and south points on it. M_1 is the position of the star at its upper transit and M_2 is the position at lower transit.

α_1 = apparent altitude at upper transit (north)

α_2 = apparent altitude at lower transit (south)

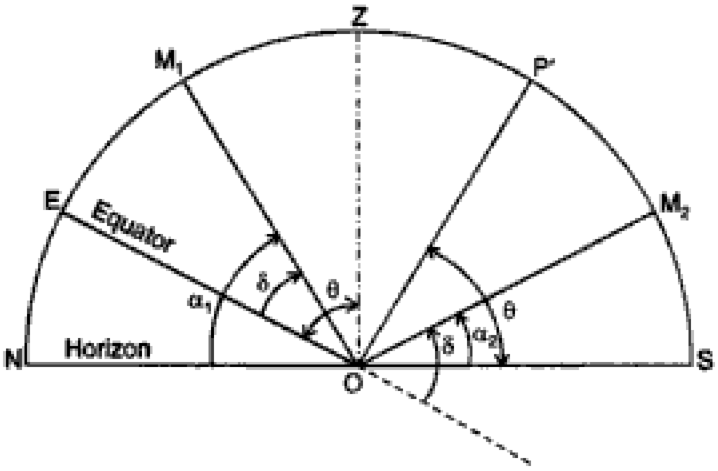


FIG. 1.64

Now $\alpha_1 = NOM_1 = NOZ - M_1OZ = 90° - (EOZ - EOM_1)$

$$= 90° - (\theta - \delta) = 90° - \theta + \delta = 90° - 80° + 46° \ 45' \ 30'' = \mathbf{56° \ 45' \ 30'' \ N}.$$

Similarly, $\alpha_2 = SOM_2 = P'OS - P'OM_2 = \theta - (90° - \delta) = \theta - 90° + \delta$

$$= 80° - 90° + 46° \ 45' \ 30'' = \mathbf{36° \ 45' \ 30'' \ S}.$$

Example 1.59. *The following data relate to an observation of latitude by zenith pair. Calculate the latitude.*

Star	*Declination*	*Observed altitude at transit*
M_1	20° 25′ 48″ S	48° 18′ 12″ N
M_2	79° 30′ 52″ S	47° 54′ 6″ S

Solution.

In Fig. 1.64, M_1 and M_2 denote the two stars ; P' is the south pole.

From the observations to star M_1 :

Latitude $= \theta = EOZ = NOZ - NOE = 90° - (NOM_1 - EOM_1) = 90° - \alpha_1 + \delta_1$ ...(1)

where α_1 = altitude of star M_1 and δ_1 = declination of the star M_1

From the observations to star M_2 :

Latitude $\theta = P'OS = 90° - (\delta_2 - \alpha_2) = 90° - \delta_2 + \alpha_2$ ...(2)

where α_2 = altitude of star M_2 and δ_2 = declination of star M_2

Hence average latitude $= \frac{1}{2}\Big[(90° - \alpha_1 + \delta_1) + (90° - \delta_2 + \alpha_2)\Big] = 90° - \frac{\alpha_1 - \alpha_2}{2} + \frac{\delta_1 - \delta_2}{2}$

In the above expression, α_1 and α_2 are the observed altitudes. These two altitudes are not exactly equal, and hence there will be little difference in the refraction correction for the two altitudes.

Taking into account the refraction correction, we have

α_1' (corrected) $= \alpha_1 - r_1$; and α_2' (corrected) $= \alpha_2 - r_2$

where r_1 and r_2 are the refraction corrections.

$$\text{Hence average latitude} = 90° - \frac{\alpha_1 - \alpha_2}{2} + \frac{\delta_1 - \delta_2}{2} + \frac{r_1 - r_2}{2}$$

Here $r_1 = 58'' \cot \alpha_1 = 58'' \cot 48° 18' 12'' = 51''.68$

$r_2 = 58'' \cot \alpha_2 = 58'' \cot 47° 54' \ 6'' = 52''.41$

Substituting the values, we get

$$\text{Average latitude} = 90° - \frac{48° 18' 12'' - 47° 54' 6''}{2} + \frac{20° 25' 48'' - 79° 30' 52''}{2} + \frac{51''.68 - 52''.41}{2}$$

$$= 90° - 24' 6'' - 59° 5' 4'' - 0''.36 = \mathbf{30° 30' 49''.64}$$

It will be seen here that if the effect of refraction is *assumed* to be cancelled, the latitude will be 30° 30′ 50″. The effect of refraction is thus extremely small, and may be almost neglected if latitude is required to an accuracy of nearest 1″.

Example 1.60. *The altitudes of a star were observed at its upper and lower culmination at a place in north latitude and corrected for refraction. The values obtained are as follows:*

Star : α Aldebaran

Altitude at lower culmination = 18° 36′ 40″

Altitude at upper culmination = 59° 48′ 20″

Find the latitude of the place and the declination of the star.

Solution. (Fig. 1.58)

$$\text{The latitude} \quad \theta = \frac{\alpha_1 + \alpha_2}{2} = \frac{18° 36° 40'' + 59° 48' 20''}{2} = 39° 12' 30''$$

Declination of the star $= EA = EZ + ZA = EZ + (ZN - AN) = \theta + (90° - \alpha_1)$

$= 90° + \theta - \alpha_1 = 90° + 39° 12' 30'' - 59° 48' 20'' = \mathbf{69° 24' 10''}$.

Check : Declination $= EA = EP - AP = 90° - AP = 90° - BP = 90° - (\theta - \alpha_2)$

$= 90° - 39° 12' 30'' + 18° 36' 40'' = \mathbf{69° 24' 10''}$.

Example 1.61. *A star was observed for latitude determination, and its corrected altitude is 40° 36′ 30″. The declination of the star is 10° 36′ 40″ and hour angle is 46° 36′ 20″. Compute the latitude of the place of observation.*

Solution. (Fig. 1.59). The latitude of the place is computed from the formula

$$\sin \alpha = \sin \theta \sin \delta + \cos \theta \,.\, \cos \delta \,.\, \cos H \quad \ldots(1)$$

To solve this equation for θ, let $\sin \delta = m \sin n$ and $\cos \delta \cos H = m \cos n$.

Then, by reduction, the value of n is given by

$$\tan n = \tan \delta \sec H = \tan 10° 36' 40'' \sec 46° 36' 20''$$

or $n = 15° 15' 12''$

Then, the value of θ is given by

$$\cos(\theta - n) = \sin\alpha \,.\, \sin n \,.\, \text{cosec}\,\delta = \sin 40° 36' 30'' \,.\, \sin 15° 15' 12'' \text{ cosec } 10° 36' 40''$$

$$\therefore \qquad \theta - n = 21° 36' 33''$$

or $$\theta = n + 21° 36' 33'' = 15° 15' 12'' + 21° 36' 33'' = \mathbf{36° 51' 45''}.$$

Example 1.62. *Find the latitude of the place from the following data :*

Longitude of the place, 108° 30′ W

Altitude of sun's upper limb, 42° 12′ 40″

L.M.T. of observation $2^h\,50^m$ *P. M.*

Date of observation : Dec. 15, 1947

Sun's declination at 0 hour on Dec. 15, 1947 : 23° 12′ 18″.6 (South) increasing at 10″.6 per hour.

Equation of time at 0^h *on Dec. 15* $= +6^m\,18.5^s$, *decreasing at* 1.2^s *per hour.*

Sun's semi-diameter = 15′ 16″.4

Solution

(a) *Calculation of true altitude*

Correction for refraction $= 57'' \cot\alpha = 57'' \cot 42° 12' 40'' = 62''.84$ (subtractive)

Correction for parallax $= 8''.77 \cos\alpha = 8''.77 \cos 42° 12' 40'' = 6''.50$ (additive)

Correction for semi-diameter $= 15' 16''.4$ (subtractive)

Net correction $= -62''.84 + 6''.50 - 15' 16''.4 = -16' 12''.74$

$\therefore$ True altitude $= 42° 12' 40'' - 16' 12''.74 = 41° 55' 27''.26.$

(b) *Calculation of hour angle*

Longitude = 108° 30′ *W* $= 7^h\,14^m$ *W*

L.M.T. of observation $= 14^h\,50^m$ P.M.

$\therefore$ G.M.T. of observation $= 22^h\,04^m$

E.T. at $0^h = +6^m\,18.5^s$

Decrease at 1.2^s per hour for $22^h\,04^m = (1.2 \times 22^h\,4^m) = 26.48^s$

Now interval since L.M.N. = L.M.T. $- 12^h = 14^h\,50^m - 12^h$

$= 2^h\,50^m$

Add E.T. $= 26.48^s$

$\therefore$ Interval since L.A.N. $= 2^h\,50^m\,26.48^s$

Hence hour angle (H) = interval since L.A.N.$= 2^h\,50^m\,26.48^s = 42° 36' 37''.20$

(c) *Calculation of declination*

G.M.T. of observation $= 22^h\,04^m$

Declination of sun at $0^h = 23° 12' 18''.6$ *S*

Increase at $10''.6$ per hour for $22^h\,04^m = (10''.6 \times 22^h\,04^m) = 233.91^s = 3' 53''.91$

∴ Sun's declination at the time of observation

$$= 23° \, 12' \, 18''.6 + 3' \, 53''.91 = 23° \, 16' \, 12''.51 \quad \text{(south)}.$$

(d) Calculation of the latitude

The latitude can be calculated from the following formula :

$$\tan \frac{ZP}{2} = \frac{\sin \frac{1}{2}(A+H)}{\sin \frac{1}{2}(A-H)} \cdot \tan \frac{1}{2}(PM - ZM) \quad \ldots(1)$$

Let us first calculate the value of the azimuth (A) of the sun.

In the astronomical triangle ZPM, we have

$$ZM = \text{co-altitude} = 90° - 41° \, 55' \, 27''.26 = 48° \, 4' \, 32''.74$$

$$PM = \text{co-declination} = 90° + 23° \, 16' \, 12''.51 = 113° \, 16' \, 12''.51$$

$$\angle ZPM = H = 42° \, 36' \, 37''.2$$

Using the sine rule, we get

$$\sin PZM = \frac{\sin PM}{\sin ZM} \cdot \sin ZPM = \frac{\sin 113° \, 16' \, 12''.51}{\sin 48° \, 4' \, 32''.74} \times \sin 42° \, 36' \, 37''.2$$

$$\therefore \quad PZM = A = 123° \, 42' \, 36''$$

$$\therefore \quad \frac{A+H}{2} = \frac{1}{2}(123° \, 42' \, 36'' + 42° \, 36' \, 37''.20) = 83° \, 9' \, 36''.6$$

$$\frac{A-H}{2} = \frac{1}{2}(123° \, 42' \, 36'' - 42° \, 36' \, 37''.20) = 40° \, 32' \, 59''.4$$

$$\frac{PM - ZM}{2} = \frac{1}{2}(113° \, 16' \, 12''.51 - 48° \, 4' \, 32''.74) = 32° \, 35' \, 49''.9$$

Substituting these values in Equ. 1 above, we get

$$\tan \frac{ZP}{2} = \frac{\sin 83° \, 9' \, 36''.6}{\sin 40° \, 32' \, 59''.4} \cdot \tan 32° \, 35' \, 49''.9$$

$$\therefore \quad \frac{ZP}{2} = 44° \, 20' \, 29''.4$$

or $$ZP = 88° \, 40' \, 58''.8 = \text{co-latitude}$$

∴ Latitude of the place = $90° - 88° \, 40' \, 58''.8$ = **1° 19′ 1″.2.**

Example 1.63. *Observations on a star α-aldebaran were made at a place in N-latitude for determining the latitude of the place by prime vertical transit. The following is the record obtained :*

Interval between the passage of α-aldebaran across prime vertical = $9^h \, 22^m \, 6^s$ *mean time.*

Mean readings of the bubble on striding level = 11^S *and* 16^N

Value of each division = 16 ″

Declination of the star = 15° 20′ 48 ″ N

Determine the latitude of the place of observation.

Solution. When the observations are made on a star at its prime vertical transit, the latitude (Fig. 1.60) is given by

$$\tan \theta = \tan \delta \; . \; \sec H \qquad ...(1)$$

Let us first calculate the hour angle (H) of the star at its prime vertical transit.

Interval between the passage across prime vertical $= 9^h\ 22^m\ 6^s$ meantime.

To convert it into sidereal time interval add acceleration at the rate of 9.8565 seconds per hour of meantime.

$$9^h \times 9.8565 = 88.71 \text{ seconds}$$
$$22^m \times 0.1642 = 3.61 \text{ seconds}$$
$$6^s \times 0.0027 = 0.02 \text{ second}$$

Total acceleration $= 92.34$ seconds $= 1^m\ 32.34^s$

$\therefore$ Sidereal time interval $= 9^h\ 22^m\ 6^s + 1^m\ 32.34^s$

$= 9^h\ 23^m\ 38.34^s = 140°\ 54'\ 35''.1$

$\therefore$ H = half the time interval $= 70°\ 27'\ 17''.55$

Hence $\tan \theta = \tan 15°\ 20'\ 48'' \sec 70°\ 27'\ 17''.55$

$\therefore$ $\theta = 39°\ 20'\ 25''.6$

Since the trunnion axis is inclined, let us correct the value,

$$\text{Error due to striding level} = \frac{N - S}{2} \times d = \frac{16 - 11}{6} \times 16 = 40''$$

As the north end of the axis is higher, the correction is additive.

Hence correct $\theta = 39°\ 20'\ 25''.6 + 40'' = \mathbf{39°\ 21'\ 5''.6}$.

Example 1.64. *In longitude 7° 20′ W, an observation for latitude was made on Polaris on a certain day. The mean of the observed latitude was 48° 36′ 40″ and the average of the local mean times, $20^h\ 24^m\ 50^s$. The readings of the barometer and thermometer were 30.42 inches and 58° F respectively. Find the latitude, given the following:*

R.A. of Polaris $= 1^h\ 41^m\ 48.64^s$

Declination of Polaris $= 88°\ 58'\ 28''.26$

G.S.T. of G.M.M. $= 16^h\ 48^m\ 20.86^s$.

Solution

(a) Calculation of polar distance.

From Chamber's Mathematical Tables (page 431)

Mean refraction for 48° 36′ 40″	= 51″
Correction for 58°F temp.	= – 1″
Correction for barometer	= + 1″
∴ Refraction correction	= 51″ (subtractive)

$\therefore$ True altitude = observed altitude – refraction $= 48°\ 36'\ 40'' - 51'' = 48°\ 35'\ 49''$.

(b) Calculation of hour angle (H).

The hour angle can be calculated by subtracting the R.A. from L.S.T.

Longitude = $7° 20' W = 0^h 29^m 20^s W$

Acceleration at the rate of 9.8565 seconds per hour of longitude :

$29^m \times 0.1642 = 4.76$ seconds

$20^s \times 0.0027 = 0.05$ seconds

$\therefore$ Acceleration = 4.81 seconds

$\therefore$ L.S.T. of L.M.M. = G.S.T. of G.M.M. + acceleration

$= 16^h 48^m 20.86^s + 4.81^s = 16^h 48^m 25.67^s$

L.M.T. of observation = $20^h 24^m 50^s$

To convert it into sidereal interval, add acceleration at the rate of 9.8565 seconds per mean hour.

$20^h \times 9.8565 = 197.13$ seconds

$24^m \times 0.1642 = 3.94$ seconds

$50^s \times 0.0027 = 0.14$ second

Total acceleration = 201.21 seconds = $3^m 21.21^s$

Sidereal interval since L.M.M. = Meantime interval + acceleration.

$= 20^h 24^m 50^s + 3^m 21.21^s.$

$= 20^h 28^m 11.21^s$

Add L.S.T. of L.M.M. = $16^h 48^m 25.67^s$

$\therefore$ L.S.T. $= 37^h 16^m 36.88^s - 24^h$

$= 13^h 16^m 36.88^s$

Deduct R.A. of Polaris = $1^h 41^m 48.64^s$

$\therefore$ Hour angle $(H) = 11^h 34^m 46.24^s = 173° 42' 3''.6.$

Now, latitude $\theta = \alpha - p \cos H + \frac{1}{2} \sin 1'' p^2 \sin^2 H \,.\, \tan \alpha.$

p = polar distance = $90° - 88° 58' 28''.26 = 1° 1' 31''.74 = 3691''.74$

First correction = $p \cos H = 3691''.74 \cos 173° 42' 3''.6 = -3669''.5 = -1° 1' 9''.5$

Second correction $= \frac{1}{2} \sin 1'' p^2 \sin^2 H \,.\, \tan \alpha$

$$= \frac{1}{2} \times \frac{1}{206265} (3691.74)^2 \sin^2 (173° 42' 3''.6) \tan 48° 35' 49'' = +0''.5.$$

(*Note*. The above calculations for first and second corrections may be done with a five figure log table if the answer is required to the nearest 1''.)

Hence $\theta = 48° 35' 49° - (-1° 1' 9''.5) + 0''.5 =$ **49° 36′ 59″ N.**

Example 1.65. *The latitude of a station 4° 20′ E of the 120° W meridian was determined by reducing an observation of β Aquilae to meridian, the true altitude of the star being 39° 20′ 30″ and the approximate latitude of the station 56° 54′ 30″ N.*

The time of the observation, $10^h\ 55^m\ 30^s$ was taken with a mean time chronometer, which was $1^m\ 25^s$ fast on the standard time of the 120° meridian. The R.A. and declination of the star were respectively $19^h\ 52^m\ 16^s$ and 6° 15′ 02″ N, G.S.T. at G.M.N. being $8^h\ 30^m\ 20^s$.

Determine the exact latitude by applying the circum-meridian correction to the observed latitude.

Solution. The meridian altitude h is given by $h = \alpha + Bm$

where
$$B = \frac{\cos\theta\ .\ \cos\delta}{\cos h} \approx \frac{\cos\theta\ .\ \cos\delta}{\cos\alpha}$$

and
$$m = \frac{2\sin^2\dfrac{H}{2}}{\sin 1''},$$ where H is in arc measure.

Let us first calculate the hour angle.

G.S.T. of G.M.N. = $8^h\ 30^m\ 20^s$

Longitude = 4° 20′ E of 120° W meridian = 115° 40′ W = $7^h\ 42^m\ 40^s$

∴ Acceleration for $7^h\ 42^m\ 40^s$ at 9.8565 sec. per hour = 1′ 16″

∴ L.S.T. of L.M.N. = G.S.T. of G.M.N. + acceleration

$= 8^h\ 30^m\ 20^s + 1^m\ 16^s = 8^h\ 31^m\ 36^s$

L.S.T.= R.A. = $19^h\ 52^m\ 16^s$

∴ S.I. after L.M.N .= L.S.T. – L.S.T. of L.M.N.

$= 19^h\ 52^m\ 16^s - 8^h\ 31^m\ 36^s = 11^h\ 20^m\ 40^s$

To convert it to mean time interval, subtract the retardation at the rate of 9.8296 per sidereal hour.

∴ Retardation = $(9.8296^s)\ (11^h\ 20^m\ 40^s) = 1^m\ 51.95^s$

∴ M.T. interval after L.M.N.= S.I. after L.M.N. – retardation.

$= 11^h\ 20^m\ 40^s - 1^m\ 51.95^s = 11^h\ 18^m\ 48.05^s$

Observed standard mean time = $10^h\ 55^m\ 30^s$

Chronometer correction = – $1^m\ 25^s$

∴ Corrected standard mean time = $10^h\ 45^m\ 05^s$

Correction for 4° 20′ Longitude (E) = + $17^m\ 20^s$

∴ L.M.T. of observation = $11^h\ 11^m\ 25^s$

∴ Mean time interval before transit = M.T. interval after L.M.N. – L.M.T.

$= 11^h\ 18^m\ 48.05^s - 11^h\ 11^m\ 25^s = 0^h\ 7^m\ 23.05^s$

Acceleration for $7^m\ 23.05^s$ of meantime = 1.21^s

$\therefore$ S.I. before transit = M.T. interval before transit + acceleration

$$= 7^m\ 23.05^s + 1.21^s = 7^m\ 24.26^s = 444.3^s \qquad \ldots(i)$$

Now $\quad m = \dfrac{2 \sin^2 \frac{1}{2} H}{\sin 1''}$ (where 1 sec. time $H = 15''$ arc.)

$$= \frac{2 \sin^2 \frac{1}{2} (15\ H)}{\sin 1''} = \frac{2 \left(\frac{15}{2} \right)^2 H^2 (\sin 1'')^2}{\sin 1''}$$

$$= \frac{225\ H^2}{2 \times 206265} = \frac{H^2}{1834}, \text{ with } H \text{ in seconds} = \frac{(444.3)^2}{1834} = 107''.6$$

$$B = \frac{\cos \delta \cos \theta}{\cos \alpha} = \frac{\cos 6° \ 15' \ 02'' \ . \cos 56° \ 54' \ 30''}{\cos 39° \ 20' \ 30''}$$

Hence $\quad mB = \dfrac{\cos 6° \ 15' \ 02'' \ . \cos 56° \ 54' \ 30''}{\cos 39° \ 20' \ 30''} = 444.3^s = 1' \ 15''.51$

Hence correct meridian altitude $= h = \alpha + mB = 39° \ 20' \ 30'' + 1' \ 15''.51 = 39° \ 21' \ 45''.51$

and $\quad \theta = 90° - h + \delta = 90° - 39° \ 21' \ 45''.51 + 6° \ 15' \ 02'' =$ **56° 53′ 16″.49 N**.

1.18. DETERMINATION OF LONGITUDE

Since the difference in longitudes between two places is equal to the difference in their local times, the longitude of a place can be determined by determining the local time (mean or sidereal) at the place and subtracting it from the Greenwich time (mean or sidereal) at the same instant. The local time can be determined by any of the methods discussed earlier. However, the finding of the Greenwich time at the instant of observations is the main important part of the longitude determination. If the local time is *greater* than the Greenwich time (or the standard time), the place is to the *east* of Greenwich meridian (or the standard meridian). Similarly, if the local time is *lesser* than the Greenwich time (or the standard time), the place is to the *west* of Greenwich meridian. The various methods of determining the longitude are :

(1) By transportation of chronometers.
(2) By electric telegraph.
(3) By wireless time signals.
(4) By observing the moon and the stars which culminate at the same time.
(5) By celestial signals.
(6) By lunar distances.

Methods (4) to (6) are only of historical interest and will not be discussed here.

(1) LONGITUDE BY TRANSPORTATION OF CHRONOMETERS

In this method, the chronometer time is noted at the instant of making the observations for the local time. The chronometer reading is then corrected for its time and rate. For this, the chronometer should be previously compared with Greenwich time and its error and rate should be known. Thus, at the instant of the celestial observations we know the correct Greenwich time. Comparing the calculated local time with that of the chronometer time, we can find the longitude of the place of observation.

Chronometer is a very delicate instrument. The main difficulty arises from the fact that its rate while being transported, and while it is stationary is not the same. Hence the *travelling rate* of the chronometer should also be ascertained for precise determinations. Suppose it is required to determine the difference in longitude between two stations A and B, the chronometer being regulated to give the time of station A. The *'rate'* of the chronometer, *i.e.*, the amount by which it gains or loses in 24 hours is found at A. The chronometer is then transported to the station B of unknown longitude and its error is determined with reference to this meridian. *If the chronometer runs perfectly, the two watch corrections will differ by just the difference in longitude.*

The method is now not used by surveyors except where wireless or telegraphic communications are not available. However, it is still used for the determination of longitude at sea.

(2) LONGITUDE BY ELECTRIC TELEGRAPH

If the two places are connected by an electric telegraph, the longitude can be determined very accurately by sending telegraphic signals in opposite directions for the chronometer times (local). Let A and B be the stations, A being to the east of B.

Let t_1 = local time of A at which the signal is sent from A to B.

and t_2 = local time of B at which the signal is received at B.

If the transmission time is neglected, the difference in longitude (ϕ) is given by $\phi = t_1 - t_2$, t_1 being greater than t_2.

If, however, s is the time of transmission, $(t_1 + s)$ is the actual local time of A corresponding to the local time t_2 at B. Hence the difference in longitude is

$$\phi = (t_1 + s) - t_2 = (t_1 - t_2) + s \qquad ...(1)$$

Similarly, let a signal be sent in the reverse direction from B to A

Let t_2' = local time of B at which the signal is sent from B to A,

t_1' = local time of A at which the signal is received.

If the transmission time is neglected, we get

$$\phi = t_1' - t_2'.$$

If, however, s is the time taken in transmitting the signal $(t_2' + s)$ is the actual local of B corresponding to the local time t_1' of A. Hence the difference in longitude is

$$\phi' = t_1' - (t_2' + s) = t_1' - t_2' - s$$

By averaging the two results, we get

$$\text{Difference in longitude} = \frac{1}{2}\left\{ (t_1 - t_2 + s) + (t_1' - t_2' - s) \right\} = \frac{1}{2}\left\{ (t_1 - t_2) + (t_1' - t_2') \right\}.$$

(3) LONGITUDE BY WIRELESS SIGNALS

The advent of wireless signals has rendered the carrying of the time of the reference meridian comparatively easy and most accurate. Time signals are now sent out from various wireless stations at stated intervals, and the surveyor, by their aid, may check his chronometer in almost any part of the world. A list of wireless signals, their times and durations of emission together with their wave lengths and type of signals, is given in the *Admiralty list of wireless signals,* which is published annually ; and changes or any corrections are notified in the weekly *Notices to Mariners.* Greenwich meantime signals are sent and usually continue, for a period of five minutes. The signals are rhythmic and consist of a series of 61 Morse dots to the minute, the beginning and end of each minute being denoted by a dash, which is counted as zero of the series which follows.

PROBLEMS

1. At a point *A* in latitude 50° *N*, a straight line is ranged out, which runs due east of *A*. This straight line is prolonged for 60 Nautical miles to *B*. Find the latitude of *B*, and if it be desired to travel due North from *B* so as to meet the 50° parallel again at *C*, find the angle *ABC* at which we must set out, and the distance *BC*. (*U.L.*)

2. The R.A. of a star being $20^h\ 24^m\ 13.72^s$, compute the L.M.T. of its culmination at Madras (Long. 80° 14′ 19″.5 *E*) on Sept. 6, the G.S.T. at 0^h G.M.T. on that date being $22^h\ 57^m\ 06.95^s$

3. Find the L.S.T. at a station in longitude 76° 20 ′ *E* at 9.30 A.M. (Indian Zone Time) on August 10 on that date at G.M.M. The R.A. of mean sun is $9^h\ 13^m\ 30.9^s$. (*G.U.*)

4. From the N.A., it is found that on the date of observation, G.S.T. of G.M.N. is $3^h\ 14^m\ 26^s$. Taking retardation as 9.85 sec. per hour of longitude, find the L.M.T. in a place 75° *W*, when the local sidereal time is $5^h\ 20^m\ 0^s$. (*B.U.*)

5. Find the local mean time at which β Leonis made its upper transit on 1st May 1940 at a place 60° *E*. Given R.A. of β Leonis on 1st May was $11^h\ 46^m\ 02^s$ and G.S.T. of G.M.N was $9^h\ 23^m\ 23^s$. (*B.U.*)

6. Find the R.A. of the meridian of Bombay at 4.30 P.M. Given : Longitude of Bombay 72° 48′ 46″.8 East ; G.S.T. at G.M.M.= $10^h\ 10^m\ 40.73^s$ on that day.
(*Note*: R.A. of a place = L.S.T.)

7. What are the systems of co-ordinates employed to locate position of a heavenly body ? Why it is necessary, to have several systems instead of one ?

8. Explain the systems of time reckoning known as sidereal apparent solar and mean solar time, and show how they differ from each other. (*I.R.S.E.*)

9. What is equation of time ? Show, by means of sketches, that it vanishes four times a year.

10. Explain with aid of sketches how the quantities of the following groups are related to each other:
    (*i*) The R.A. of a star, the hour angle of the star at any instant and the sidereal time at that instant.
    (*ii*) Equation of time, apparent time and mean time.
    Show that the equation of time vanishes four times in a year. (*A.M.I.E.*)

11. (*a*) Explain the following terms :
    (*i*) Equation of time , (*ii*) Celestial sphere, (*iii*) Parallax, and (*iv*) Sidereal time.
    (*b*) An observation was made on Dec. 30, 1919 in longitude 82° 17′ 30″ *E*; the meridian altitude of the sun's lower limb was 40° 15′ 13″. The sun was on the south of the observer's zenith. Calculate the approximate latitude of the place. Correction for refraction 1′ 10″ ; for parallax = 6″.9 ; correction for semi-diameter 16′ 17″.5. Declination of star at G.A.N.= 23° 13′ 15″. decreasing at the rate of 9″.17 per hour (*B.U.*)

12. What are 'parallax' and 'refraction' and how do they affect the measurement of vertical angles in astronomical work ?
    Give rough values of the corrections necessary when measuring a vertical angle of 45°. (*A.M.I.C.E.*)

13. In longitude 60° *W*, an observation was made on β Tauri, whose R.A. was $5^h\ 21^m\ 59.48^s$. If the hour angle of the star was $9^h\ 15^m\ 8^s$, find the local mean time of observation.
    Given G.S.T. at G.M.N. = $14^h\ 46^m\ 39.53^s$.

14. On a certain date, the right ascension of α-Draconis was $14^h\ 2^m\ 5^s$. From the N.A. and the longitude of the place, the local sidereal time of local mean noon was found to be $6^h\ 35^m\ 44^s$. The declination of the star was 64° 47′ 33″ *N*. Find the local mean time of east elongation. Assume the latitude of 60° *N*. *(A.M.I.C.E.)*

15. If the time be found by a single altitude, show that a small error in the latitude will have no effect on the time when the body is in the prime vertical.

16. Determine the G.M.T. at which the star α-Aurigae crossed the meridian of a station in longitude 28° 31′ *E* in the northern hemisphere at upper culmination on May 31st 1926, the declination of the star being 45° 55′ 25″ *N*, and its right ascension $5^h\ 11^m\ 6^s$ with G.S.T. of G.M.N. $4^h\ 32^m\ 55^s$. If the true altitude of the star was 76° 30′ 50″, find also the latitude of the station. *(B.U.)*

17. Draw a diagram to show the celestial sphere for a point 15° *N*, 75° *E*, showing the horizon, meridian, zenith, pole and celestial equator.
Mark also the path of the sun at mid-summer, and the position of α-Bootes (decl. of 20° *N*; R.A. $14^s\ 10^m$) at 22^h G.S.T. *(U.R.)*

18. An observation of time was made on Aldebaran (α-Tauri) on Oct. 1, 1940 in altitude 52° 12′ 50″ *N*, the mean of two observed altitude being 28° 36′ 20″. The average sidereal time of observing these altitudes was $0^h\ 15^m\ 28.4^s$ by the sidereal chronometer.

Find the error of the chronometer given that the star's R.A. and declination were $4^h\ 32^m\ 31.1^s$ and 16° 23′ 30″.5 respectively and that the star was east of the meridian.

19. On 7th Feb., a star (R.A. $5^h\ 9^m\ 44^s$) is in transit at Sidney (Longitude 155° 12′ 23″ *E*) when the time by the observer's watch which should keep local times is $8^h\ 0^m\ 33^s$. Given that the mean sun's R.A. at a mean noon at Greenwich on 7th Feb. is $21^h\ 8^m\ 36.1^s$ and that 1 hour of S.T. is equivalent to $59^m\ 50.2^s$ of meantime, find to the nearest second how much the watch is slow or fast. *(Math. Trip.)*

20. Reduce the following meridian observations for latitude :

Star	*Declination*	*Right Ascension*	*Observed Altitude*	*Altitude Level* object end	eye end
M_1	60° 02′ 50″ *S*	$13^h\ 59^m\ 00^s$	49° 28′ 15″ *S*	5.4	4.6
M_2	19° 32′ 10″ *N*	$14^h\ 12^m\ 33^s$	50° 58′ 10″ *N*	5.2	4.8

The value of level division is 14″. Take the refraction correction as − 58″ cot altitude. If the longitude is 142° 36′ *E* and the sidereal time of mean noon at Greenwich is $4^h\ 6^m\ 17^s$, at what local mean times will the two transits occur ?

21. Your longitude is 75° *E* of Greenwich.
You are required to find the error to the nearest second of a meantime chronometer at mid-night 1st–2nd March.
In order to find this, you have timed the transit of two stars near mid-night as follows :

Transit of α Mali $23^h\ 32^m\ 14^s$ ⎤
,, ,, β Gemini $1^h\ 43^m\ 52^s$ ⎦ by the chronometer

Relevant extracts from the Nautical Almanacs are

R.A. of α Mali $6^h\ 19^m\ 01^s$

R.A. of β Gemini $8^h\ 30^m\ 56^s$

Sidereal time of Greenwich mean moon 1st March : $18°\ 45^m\ 12^s$.

22. Criticise the method of determining azimuths from elongation observations, stating its limitations in high altitudes.

A star α of declination 84° 42′ *N* is observed at eastern elongation when its clockwise angle from a survey line is 118° 20′. Immediately afterwards another star β of declination 72° 24′ *N* is observed at western elongation, its clockwise angle from *OP* being 94° 6′.
Determine the azimuth of the line *OP*. *(U.P.)*

23. At a point in latitude *N* 55° 46′ 12″ the altitude of the sun's centre was found to be 23° 17′ 32″ at $5^h\ 17^m$ P.M. (Greenwich meantime). The theodolite was first pointed to a reference mark, the vernier reading being 0° 00′ 00″ ; the horizontal angle between the sun's centre and the reference mark at the time of observation was found to be 68° 24′ 30″. Find graphical azimuth of the reference mark from the centre of the instrument.

Data : Sun's declination at Greenwich apparent noon on day of observation ... 17° 46′ 52″ *N*
Variation of declination per hour ... − 38″

Refraction for altitude of 30° 20′ ... 2′ 12″

Parallax in altitude ... 0′ 8″

Equation of time (apparent − mean) ... $6^m\ 6^s$. *(U.L.)*

24. To determine the azimuth of reference object from station *B*. (Lat. 51° 30′ 30″ *N*) of a triangulation survey, the sun was observed at $4^h\ 30^m\ 13^s$ P.M. (G.M.T.) after crossing the meridian. The observed altitude of the sun's centre was 38° 28′ 25″ and the horizontal angle measured anticlockwise from R.O. to the sun was 161° 35′ 20″. The apparent declination of the sun at G.M.N. was 20° 5′ 38″.1 *N* increasing 30″.42 per hour. The sun's horizontal parallax may be taken as 8″.7 and the refraction correction − 58″ cot α. Calculate the azimuth of R.O. *(I.R.S.E.)*

25. A star was observed at Western elongation at a place in lat. 28° 20′ *S* and longitude 124° 24′ *W*, when its clockwise bearing from a survey line was 164°.

Determine the local mean time of elongation, also the azimuth of the line, given that the star's declination was 76° 36′ 55″ *S* and its right ascension $6^h\ 41^m\ 52^s$, the G.S.T. of G.M.N. being $5^h\ 12^m\ 20^s$. *(U.L.)*

26. An observation of azimuth was made during the early hours of the morning of 1 Jan. 1940, on α Ursae Minoris (Polaris) at elongation at a place of latitudde 45° *N*, and longitude 5° *E*.

The declination of the star on that date was + 88° 59′ 03″ and its R.A. was $1^h\ 43^m\ 32^s$.

The mean observed horizontal angle between the star and the R.O. was 42° 37′ 22″, R.O. being to the west of the star.

Find (*a*) which elongation was used ?
(*b*) the exact local mean time of elongation.
(*c*) the azimuth of the R.O.

Given G.S.T. of G.M.T. 0^h on 1 Jan. 1940 was $6^h\ 38^m\ 01.9^s$ (U.B.)

27. At a place in longitude 31° 41′ 40″ S, 121° 32′ 30″ E, a star whose R.A. = $0^h\ 22^m\ 15.6^s$, declination 77° 37′ 54″ S is observed at eastern alongation when its clockwise horizontal angle, from a survey line *ZO* is 110° 14′ 30″. Find the azimuth of the survey line and the local mean time of the elongation, if the mean time of the transit of ♈ at Greenwich is $1^h\ 20^m\ 57^s$ from mid-night.

28. To determine the latitude of a place (longitude 37° W) observations were made on Polaris and its corrected altitude was found to be 46° 17′ 28″ when the mean time of observation was $7^h\ 43^m\ 35^s$ P.M. Find the latitude of the place, given the following

G.S.T. at G.M.M on the day of observation = $10^h\ 51^m\ 31.5^s$

R.A. of Polaris = $1^h\ 27^m\ 37.7^s$
Declination of Polaris = + 88° 51′ 08″

29. A meridian altitude of the lower limb of the sun is taken on 5th Nov. 1934 in latitude N, longitude 78° 25′ W. Given the observed altitude = 47° 18′ 44″, parallax = 6″, refraction = 53″.6; declination of the sun at mid-night 4/5 Nov. 1934 = *S* 15° 24′ 27″.4 with an hourly variation of 46″.23 increasing; semi-dia. 16′ 9″.5.

Calculate the latitude of the observer's station, the equation of time at mid-night 4/5 Nov. 1934 is $+16^m 21.5^s$ with an hourly variation -0.044^s. *(A.I.M.E.)*

Answers

1. Latitude of *B* = 49° 59′ 22″.6 ; ∠*ABC* = 88° 48′ 40″ ; *BD* = 0.624 Nautical miles.
2. $21^h 24^m 28.48^s$.
3. $6^h 19^m 30.3^s$.
4. $2^h 4^m 24.31^s$.
5. $2^h 22^m 54.49^s$ P.M..
6. $2^h 42^m 35.51^s$
11. (*b*) N 26° 17′ 7″.91.
12. + 6″ ; − 57″
13. $23^h 45^m 54.29^s$.
14. $5^h 4^m$ P.M. nearly.
16. $10^h 45^m 23.27^s$; 32° 26′ 16″ *N*.
18. Chronometer slow 2.5^s
19. 56.67^s slow.
20. θ = 19° 30′ 22″.6 ; L.M.T.'s : $9^h 52^m 40^s$ P.M. for M_1 ; $10^h 06^m 10^s$ P.M. for M_2
21 Choronometer slow 28^s
22. 112° 43′ 56″
23. 24° 2′ 8″ from south.
24. 4° 24′ 12″.
25. $7^h 58^s 19.13^s$: 180° 45′ 7″.75 from S point.
26. (*a*) West (*b*) $0^h 57^m 27.6^s$ Jan. 2; 315° 56′ 38″
27. 55° 10′ 40″ ; $8^h 12^m 34.2^s$ P.M.
28 46° 03′ 36″ *N*.
29. 26° 47′ 56″.7 *N*.

2

Photogrammetric Surveying

2.1. INTRODUCTION

Photogrammetric surveying or photogrammetry is the science and art of obtaining accurate measurements by use of photographs, for various purposes such as the construction of planimetric and topographic maps, classification of soils, interpretation of geology, acquisition of military intelligence and the preparation of composite pictures of the ground. The photographs are taken either from the air or from station on the ground. *Terrestrial photogrammetry* is that branch of photogrammetry wherein photographs are taken from a fixed position on or near the ground. *Aerial photogrammetry* is that branch of photogrammetry wherein the photographs are taken by a camera mounted in an aircraft flying over the area. Mapping from aerial photographs is the best mapping procedure yet developed for large projects, and are invaluable for military intelligence. The major users of aerial mapping methods are the civilian and military mapping agencies of the Government.

The conception of using photographs for purposes of measurement appears to have originated with the experiments of Aime Laussedat of the Corps of Engineers of the French Army, who in 1851 produced the first measuring camera. He developed the mathematical analysis of photographs as perspective projections, thereby increasing their application to topography. Aerial photography from balloons probably began about 1858. Almost concurrently (1858), but independently of Laussedat, Meydenbauer in Germany carried out the first experiments in making critical measurements of architectural details by the intersection method on the basis of two photographs of the building. The ground photography was perfected in Canada by Capt. Deville, then Surveyor General of Canada in 1888. In Germany, most of the progress on the theoretical side was due to Hauck.

In 1901, Pulfrich in Jena introduced the stereoscopic principle of measurement and designed the *sterecomparator*. The *stereoautograph* was designed (1909) at the Zeiss workshops in Jena, and this opened a wide field of practical application. Scheimpflug, an Australian captain, developed the idea of double projector in 1898. He originated the theory of perspective transformation and incorporated its principles in the *photoperspectograph*. He also gave the idea of radial triangulation. His work paved the way for the development of aerial surveying and aerial photogrammetry.

In 1875, Oscar Messter built the first aerial camera in Germany and J.W. Bagloy and A. Brock produced the first aerial cameras in U.S.A. In 1923, Bauersfeld designed the *Zeiss stereoplanigraph*. The optical industries of Germany, Switzerland, Italy and France, and later also those of the U.S.A. and U.S.S.R. took up the manufacture and constant further development of the cameras and plotting instruments. In World War II, both the sides made extensive use of aerial photographs for their military operations. World War II gave rise to new developments of aerial photography techniques, such as the application of radio control to photoflight navigation, the new wide-angle lenses and devices to achieve true vertical photographs.

TERRESTRIAL PHOTOGRAMMETRY

2.2. BASIC PRINCIPLES

The principle of terrestrial photogrammetry was improved upon and perfected by Capt. Deville, then Surveyor General of Canada in 1888. In terrestrial photogrammetry, photographs are taken with the camera supported on the ground. The photographs are taken by means of a phototheodolite which is a combination of a camera and a theodolite. Maps are then compiled from the photographs.

The principle underlying the method of terrestrial photogrammetry is exactly similar to that of plane table surveying, *i.e.* if the directions of same objects photographed from two extremities of measured base are known, their positions can be located by the intersection of two rays to the same object. However, the difference between this and plane tabling is that more details are at once obtained from the photographs and their subsequent plotting etc. is done by the office while in plane tabling all the detailing is done in the field itself.

Thus in Fig 2.1, *A* and *B* are the two stations at the ends of base *AB*. The arrows indicate the directions of horizontal pointings (in plan) of the camera. For each pair of pictures taken from the two ends, the camera axis is kept parallel to each other. From economy and speed point of view, minimum number of photographs should be used to cover the whole area and to achieve this, it is essential to select the best positions of the camera stations. A thorough study of the area should be done from the existing maps, and a ground reconnaissance should be made. The selection of actual stations depends upon the size and ruggedness of the area to be surveyed. The camera should be directed downward rather than upward, and the stations should be at the higher points on the area.

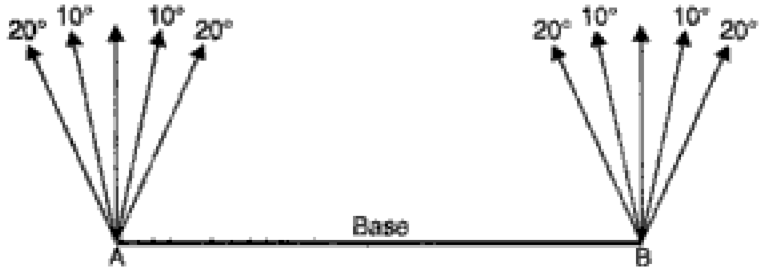


FIG. 2.1. DIRECTION OF POINTINGS IN TERRESTRIAL PHOTOGRAMMETRY.

The terrestrial photogrammetry can be divided into two branches :

(*i*) Plane-table photogrammetry.

(*ii*) Terrestrial stereophotogrammetry.

The *plane table photogrammetry* consists essentially in taking a photograph of the area to be mapped from each of the two or three stations. The photograph perpendiculars may be oriented at any angle to the base, but usually from an acute angle with the latter. The main difficulty arises in the identifications of image points in a pair of photographs.

In the case of homogeneous areas of sand or grass, identification becomes impossible. The principles of stereophotogrammetry, however, produced the remedy.

In *terrestrial stereophotogrammetry*, due to considerable improvement of accuracy obtained by the stereoscopic measurement of pairs of photographs, the camera base and the angles of intersection of the datum rays to the points to be measured can be considerably reduced since the camera axes at the two stations exhibit great similarity to each other. The image points which are parallactically displaced relative to each other in the two photographs are fused to a single spatial image by the stereoscopic measurement.

2.3. THE PHOTO-THEODOLITE

The photo-theodolite is a combination of a 1 second theodolite and a terrestrial camera. Fig. 2.2 illustrates a back view of Bridges-Lee photo-theodolite made by Messers L.Cassella, London. Fig. 2.3 shows the photograph of a modern photo-theodolite manufactured by M/s Wild Heerbrugg Ltd.

A photo-theodolite essentially consists of the following parts. (Fig. 2.2) :

(1) *A camera box A of fixed focus type*. The focal length of the lens is generally 15 cm or more. The camera box is mounted on the axis exactly in the same manner as the vernier plate of a theodolite. Thus, the box can be rotated in azimuth about its vertical axis.

(2) *A hollow rectangular frame I placed vertically to the rear side*. The frame carries two cross-hairs *k* and *k* ', the intersection of which is exactly opposite to the optical centre of the lens. The line of collimation is defined as the line joining the intersection of the cross-hairs to the optical centre of the lens. The cross wires are pressed tightly against sensitive plate and are thus photographed on the photographic plate along with the field object. Two small celluloid strips can be fitted into the grooves in the lower corners of the frame I, and can be easily removed to write any description upon them in ink which is also photographed.

(3) *Across the rear of the vertical frame is also carried a straight transparent celluoloid tangent scale*. Upon the base of the frame is pivoted a magnetic needle carrying a vertical cylindrical transparent scale (*M*) graduated to 30 minutes.

(4) The *sensified photographic* plate is *placed* between the vertical frame (*I*) and the back which is held by the spring. Before uncapping the lens, the front of the side is withdrawn to expose the plate and the vertical frame (*I*) is moved backward and forward by the screw (*J*) until the hair lines and the tangent scale are in contact with the plate. The magnetic needle is also set free to swing on its pivot. When the lens is uncapped (after the needle comes to rest), the photographs of hair lines, tangent scale, and the circular scale of the needle are imprinted on the negative. The reading of the scale at its intersection with the vertical hair on the photograph gives the magnetic bearing of the principal vertical plane (*i.e.* the vertical plane containing the optical axis).

(5) The box is supported on the tripod and is furnished with an inner and an outer axis, each of which is fitted with a clamp and fine adjusting screw. The graduated horizontal circle carries verniers reading to single minutes. These are supported on a levelling head carrying three foot screws.

(6) On the top of the box, a *telescope* is fitted. The telescope can be rotated in a vertical plane, about a horizontal axis, and is fitted with vertical arc with verniers, clamp, and slow motion screw. The line of sight of the telescope is set in the same vertical plane as the optical axis of camera.

2.4. DEFINITIONS (Fig. 2.4)

Camera Axis. Camera axis is the line passing through the centre of the camera lens perpendicular both to the camera plate (negative) and the picture plane (photograph). The optical axis coincides with the camera axis in a camera free from manufacturing imperfections.

Picture Plane. Picture plane is the plane perpendicular to the camera axis at the focal distance in front of the lens. It is represented by the positive contact print or *photograph* taken from a plate or film.

Principal Point. Principal point (k or k') is defined by the intersection of the camera axis with either the picture plane (positive) or the camera plate (negative).

Focal Length. Focal length (f) is the perpendicular distance from the centre of the camera lens to either the picture plane or the camera plate. It satisfies the following relation

$$f = \frac{uv}{u+v}$$

where u and v are conjugate object and image distances.

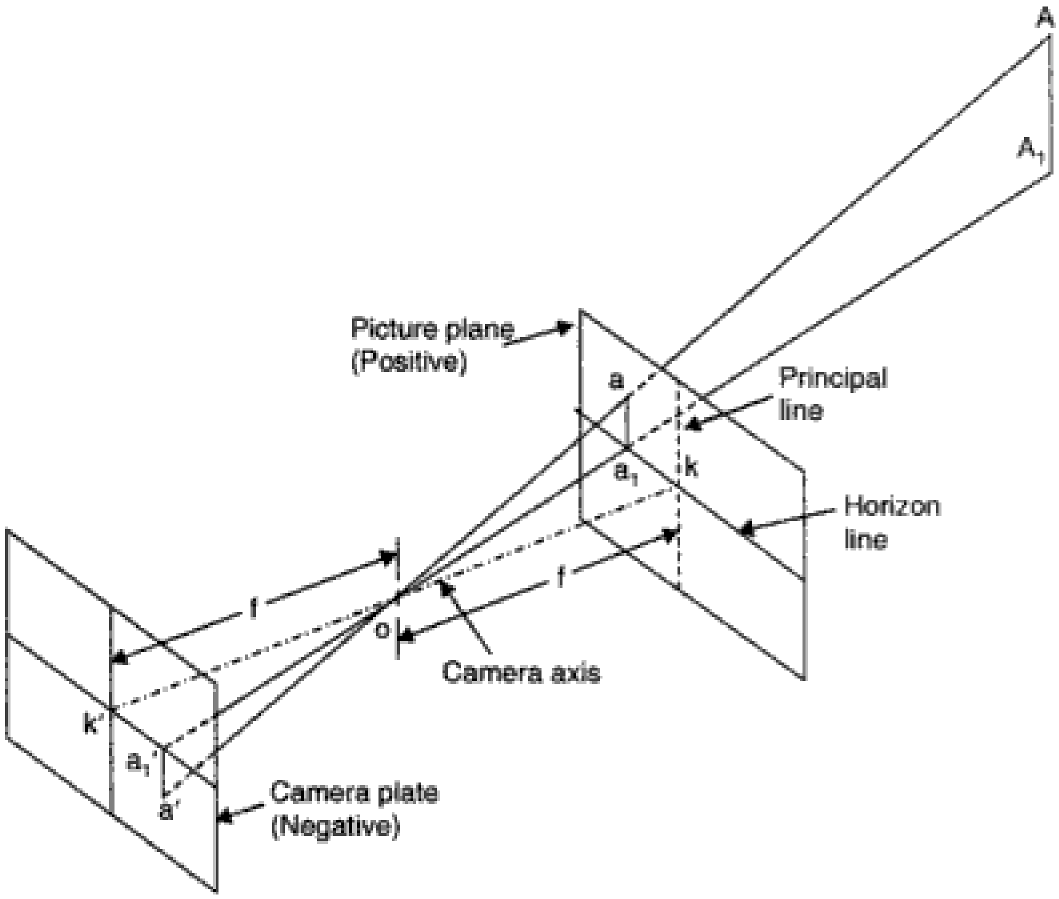


FIG. 2.4

Focal Plane (Image Plane). Focal plane is the plane (perpendicular to the axis of the lens) in which images of points in the object space of the lens are focused.

Nodal Point. Nodal point is either of two points on the optical axis of a lens (or a system of lenses) so located that when all object distances are measured from one point and all image distances are measured from the other, they satisfy the simple lens relation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} .$$

Also a ray emergent from the second point is parallel to the ray incident at the first.

Perspective Centre. Perspective centre is the point of origin or termination of bundles of perspective rays. The two such points usually associated with a survey photograph are the interior perspective centre and the exterior perspective centre. In a distortionless lens camera system, one perspective centre encloses the same angles as the other, and in a perfectly adjusted lens camera system, the interior and exterior centres correspond to the rear and front nodal points, respectively.

Principal Distance. When the contact prints from original negatives are enlarged (or reduced) before their use in the compilation of subsequent maps, the value of the focal length (f) of the camera is not applicable to the revised prints. The changed value of f, holding the same geometrical relations, is known as the *principal distance*. In other words, it is the perpendicular distance from the internal perspective centre to the plane of a particular finished negative or print. This distance is equal to the calibrated focal length corrected for both the enlargement or reduction ratio and the film (or paper) shrinkage (or expansion) and maintains the same perspective angles as the internal perspective centre to points on the finished negative or print as existed in the camera at the moment of exposure. This is a geometrical property of each particular finished negative or print.

Principal Plane. Principal plane is plane which contains principal line and the optical axis. It is, thereforce, perpendicular to the picture plane and the camera plate.

Print. A print is a photographic copy made by projection or contact printing from a photographic negative or from a transparent drawing as in blue-printing.

Fiducial Mark. A fiducial mark is one of two, three or four marks, located in contact with the photographic emulsion in a camera image plane to provide a reference line or lines for the plate measurement of images.

Fiducial Axis. Opposite fiducial marks define a reference line. Two pairs of opposite fiducial marks define two reference lines that intersect at 90°. These two lines are referred to as the x and y axes or the fiducial axes.

Film Base. Film base is a thin, flexible, transparent sheet of cellulose nitrate, cellulose acetate or similar material, which is coated with a light sensitive emulsion and used for taking photographs.

2.5. HORIZONTAL AND VERTICAL ANGLES FROM TERRESTRIAL PHOTOGRAPH

The horizontal and vertical angles to various points in a photograph can easily be found analytically, graphically or instrumentally. Fig. 2.5 (*a*) shows two points A and B photographed with camera axis horizontal so that the picture plane is vertical and the horizon line is horizontal. The image of the ground points A and B appear at a and b respectively

But $aa' = y_a$, $bb' = y_b$; $oa' = f \sec \alpha_a$ and $ob' = f \sec \alpha_b$

Hence $\tan \beta_a = \dfrac{y_a}{f \sec \alpha_a}$ ...[2.2(*a*)] ; $\tan \beta_b = \dfrac{y_b}{f \sec \alpha_b}$ ...[2.2(*b*)]

The algebraic sign of vertical angle depends on the sign of y co-ordinates. Evidently, β_b will be a depression angle.

The horizontal and vertical angles can also be determined graphically, as shown in Fig. 2.5 (*c*) where the line $k_1 k_2$ represents the true horizon of the photograph. The line *ko* is constructed perpendicular to $k_1 k_2$ and represents the optical axis, the distance *ko* being made equal to *f*. With a pair of dividers, make $ka' = x_a$ and $kb' = x_b$ by making the measurements from the photographs. Join *a'o* and *b'o*. The angles α_a and α_b can then be measured.

To find the vertical angle [Fig. 2.5 (*c*)], erect perpendiculars *a'a* and *b'b* to *oa'* and *ob'* respectively. Make $a'a = y_a$ and $b'b = y_b$, thus getting points *a* and *b* respectively. Join *ao* and *bo*. The angles *aoa'* and *bob'* are the desired vertical angles.

2.6. HORIZONTAL POSITION OF A POINT FROM PHOTOGRAPHIC MEASUREMENT: CAMERA AXIS HORIZONTAL

In plane table terrestrial photogrammetry, two photographs are taken from the ends of a base line. The position of the points can be plotted by graphical intersection as illustrated in Fig. 2.6.

Let *P* and *Q* be the known positions of the camera stations. Knowing the camera azimuths (*i.e.*, bearings of camera axis) ϕ_1 and ϕ_2 at both the stations, the horizon lines

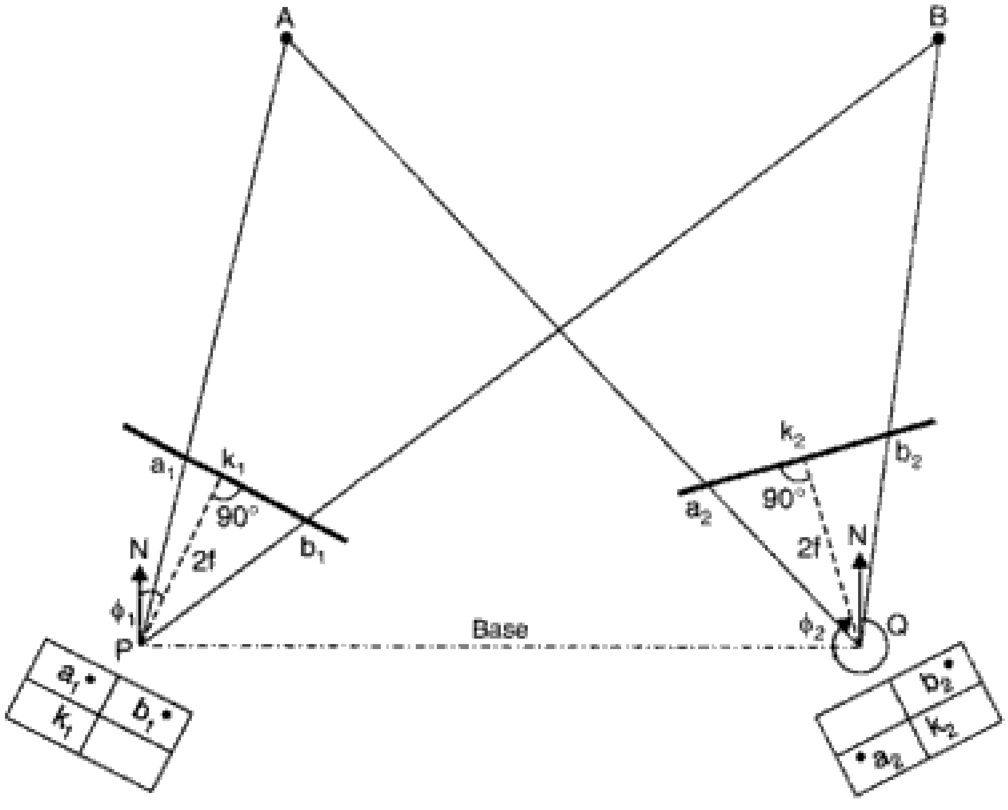


FIG. 2.6. GRAPHICAL INTERSECTION.

$a_1 k_1 b_1$ and $a_2 k_2 b_2$ can be drawn at perpendicular distances of $2f$ from P and Q respectively. On each photograph, the x-co-ordinates of points a and b are scaled by a pair of proportionate dividers set for a 2 to 1 ratio, and transferred to the photograph traces, as shown by the positions a_1, b_1 and a_2 and b_2 respectively in both the photographs taken with the camera axes horizontal at the time of exposure. Join Pa_1 and Pb_1 and prolong them. Similarly, join Qa_2 and Qb_2 and prolong them to intersect the corresponding lines in A and B respectively, thus giving horizontal positions of A and B.

Camera Position by Resection. To fix the positions of the camera stations, a separate ground control is necessary. However, the camera station can also be located by three point resection if the positions of three prominent points (which may be photogrammetric triangulation stations) are known and they are also photographed.

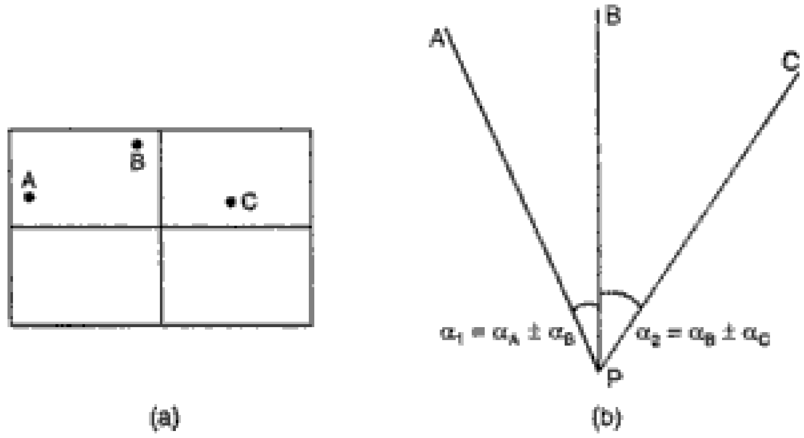


FIG. 2.7. CAMERA POSITION BY THREE POINT RESSECTION.

Thus, in Fig. 2.7. (*a*), let *A*, *B* and *C* be the three stations photographed. From § 2.5, the angles to *A*, *B* and *C* can be determined either graphically or analytically and hence angles α_1 ($= \alpha_A \pm \alpha_B$) and α_2 ($= \alpha_B \pm \alpha_C$) are known. If these angles are known graphically, a tracing paper resection on the plotted positions of *A*, *B* and *C* (on the map) will fix the map position of the camera station (*P*). If, however, the angles α_1 and α_2 are known analytically, the values may be set off by a three armed protractor for a graphical resection, or the values may be used to solve the three-point problem analytically for determining the position of the camera station.

Azimuth of a line from Photographic Measurement. The magnetic bearing or azimuth of the principal vertical plane is given by the reading of the cylindrical scale at its intersection with the vertical hair on the photograph. The horizontal angles of the lines with the principal plane can be calculated as discussed in § 2.5.

Thus, in Fig. 2.8(*a*), *a*, *b* and *c* are the positions of the three points *A*, *B* and *C*. The horizontal angles α_A , α_B and α_C (Fig. 2.8 *b*) can be determined. If ϕ is the azimuth of the principal plane (or the camera azimuth), we have

$$\phi_B = \text{azimuth of } B = \phi + \alpha_B$$

$$\phi_C = \text{azimuth of } C = \phi + \alpha_C$$

$$\phi_A = \text{azimuth of } A = \phi - \alpha_A + 360°$$

In general, therefore, we have

Azimuth of line = camera azimuth + α

Due regard must be given to the algebraic sign of α. It may be considered positive when measured to the right of *ok* and negative when measured to the left. If the azimuth calculated from the above relation comes out to be negative, 360° must be added to the result.

Orientation of Picture Traces

The accuracy in the plotted positions of various points depends upon the correct orientation and placing of picture traces on the plan. The two conditions that are to be fulfilled are: (1) the picture trace should be perpendicular to the line joining the plotted position (O) of the station and the principal point (k), and (2) the principal point (k) should be at the focal distance from O. When enlargements are used, the enlarged focal length should be laid down.

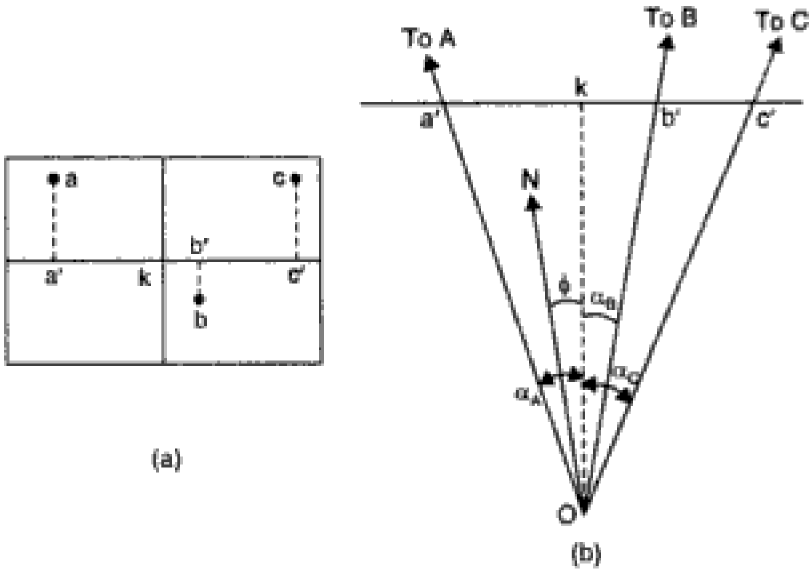


FIG. 2.8. AZIMUTH OF LINES FROM PHOTOGRAPHIC MEASUREMENT

In the case of photo-theodolite used for the photographic surveying, the bearing of the principal plane is known. In that case, the principal plane is laid at the known bearing, the principal point (k) is marked at a distance (f) from the camera station (O) and the picture trace is drawn perpendicular to that of the principal plane.

If, however, the photograph includes any point whose position is known on the plane, the orientation may be performed with respect to it as follows : (Fig. 2.9).

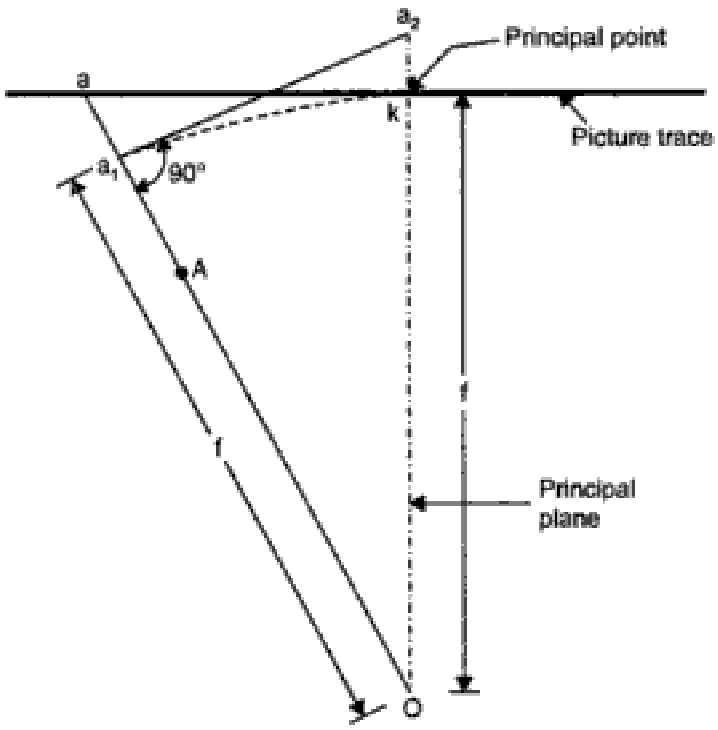


FIG. 2.9. ORIENTATION OF PICTURE TRACE FROM KNOWN POSITION OF POINT

Let A be the known position (on the plane) of the point and O be the known position of the camera station. Let ka be the distance (on the photographs) of the point A from the principal plane. Join OA and produce it. With O as the centre and radius equal to $f\,(=oa_1)$, draw an arc. At a_1, draw a line a_1a_2 perpendicular to oa_1, making a_1a_2 equal to the photographic distance ak. Join a_2o, cutting the arc in k. Thus, the position of the principal point and that of the principal plane is known. Through k, draw ka perpendicular to ok, thus giving us position of the picture trace.

2.7. ELEVATION OF A POINT BY PHOTOGRAPHIC MEASUREMENT

The elevation of a point photographed from two camera stations can be easily calculated from the measured co-ordinates of the images.

Thus, in Fig. 2.10 (*a*), (*b*), let A be the point whose elevation is to be determined with respect to the camera axis. A_1 is the projection of A on a horizontal plane passing through O. Let x and y be the co-ordinates of the photographic image (a) of the point A. As determined earlier, the horizontal angle (α) and vertical angle (β) are given by Fig. 2.10 (*a*).

$$\tan \alpha = \frac{x}{f} \qquad \ldots(1)$$

$$\tan \beta = \frac{y}{oa_1} = \frac{y}{f \sec \alpha} = \frac{y}{f} \cos \alpha \qquad \ldots(2)$$

or
$$= \frac{y}{\sqrt{f^2 + x^2}} \qquad \ldots(3)$$

In Fig. 2.10 (*b*),

$$oa_1 = \sqrt{f^2 + x^2} = f \sec \alpha$$

$$\angle\, a_1\, oa = \beta = \angle\, AOA_1$$

Hence, from the similar triangles,

$$\frac{y}{oa_1} = \frac{AA_1}{OA_1}$$

$$\therefore \qquad V = AA_1 = OA_1 . \frac{y}{oa_1}$$

$$= D \,.\, \frac{y}{\sqrt{f^2 + x^2}}$$

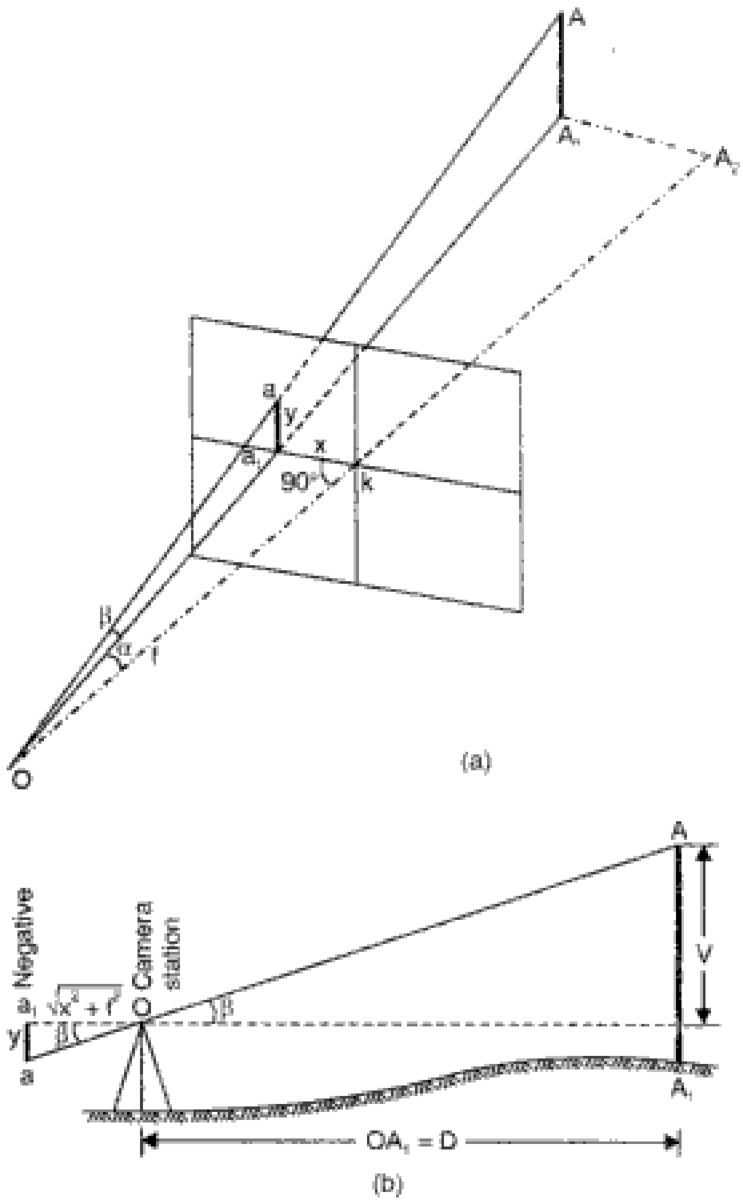


FIG. 2.10. ELEVATION BY PHOTOGRAPHIC MEASUREMENT.

or
$$V = \frac{Dy}{f \sec \alpha} = \frac{Dy}{f} \cos \alpha \qquad \ldots(2.4)$$

Due regard must be paid to the sign of y.

If the elevation of the camera axis is known, the elevation of the point can be calculated from the relation : $h = H_c + V + c$

where h = elevation of the point

H_c = elevation of the camera lens

c = correction for curvature and refraction.

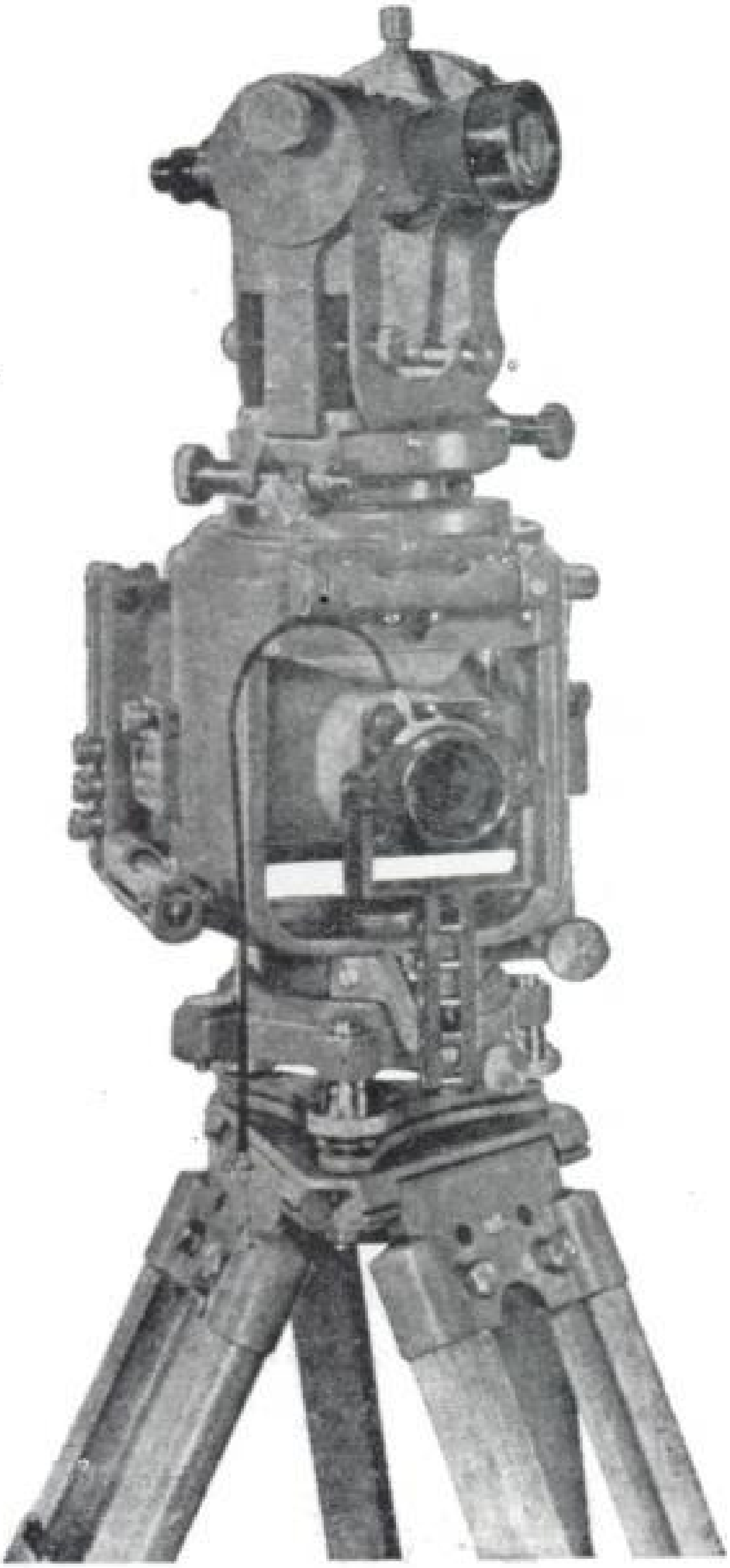


FIG. 2.3. PHOTO-THEODOLITE BY M/S WILD HEERBRUGG LTD.
(BY COURTESY OF MANUFACTURERS)

(TO FACE PAGE 143)

$\therefore \quad \tan \alpha_a \cdot \tan \alpha_b = \dfrac{x_a x_b}{f^2}$

Now $\quad \tan \theta = \tan (\alpha_a + \alpha_b) = \dfrac{\tan \alpha_a + \tan a_b}{1 - \tan \alpha_a \tan \alpha_b} = \dfrac{\dfrac{x_a}{f} + \dfrac{x_b}{f}}{1 - \dfrac{x_a \cdot x_b}{f^2}}$

$\therefore \quad \tan \theta \, (f^2 - x_a \cdot x_b) = f\,(x_a + x_b) \qquad$ or $\qquad f^2 - \dfrac{(x_a + x_b) f}{\tan \theta} - x_a x_b = 0$

which gives,

$$f = \frac{\dfrac{x_a + x_b}{\tan \theta} + \sqrt{\dfrac{(x_a + x_b)^2}{\tan^2 \theta} + 4\, x_a x_b}}{2} = \frac{x_a + x_b}{2 \tan \theta} + \sqrt{\frac{(x_a + x_b)^2}{4 \tan^2 \theta} + x_a \cdot x_b} \qquad \ldots(2.5)$$

Thus, the value of f can be calculated.

Example 2.1. *Three points A, B and C were photographed and their co-ordinates with respect to the lines joining the collimation marks on the photograph are :*

Point	*x*	*y*
a	*– 35.52 mm*	*+ 21.43 mm*
b	*+ 8.48 mm*	*– 16.38 mm*
c	*+ 48.26 mm*	*+ 36.72 mm*

The focal length of the lens is 120.80 mm. Determine the azimuths of the lines OB and OC, if that of OA is 354° 30′. The axis of the camera was level at the time of the exposure at the station O.

Solution

Fig. 2.8 shows the position of the points.

$$\tan \alpha_a = \frac{x_a}{f} = \frac{-35.52}{120.80} \qquad \therefore \quad \alpha_a = -16° \, 23'$$

$$\tan \alpha_b = \frac{x_b}{f} = \frac{+8.48}{120.80} \qquad \therefore \quad \alpha_b = +4° \, 0'$$

$$\tan \alpha_c = \frac{x_c}{f} = +\frac{48.26}{120.80} \qquad \therefore \quad \alpha_c = +21° \, 47'$$

Azimuth of camera axis $= \phi = \phi_a - \alpha_a = 354° \, 30' - (-16° 23') = \mathbf{10° \, 53'}$

Azimuth of $B = \phi + \alpha_b = 10° \, 53' + 4° = \mathbf{14° \, 53'}$

Azimuth of $C = \phi + \alpha_c = 10° \, 53' + 21° \, 47' = \mathbf{32° \, 40'}$.

Example 2.2. *Photographs of a certain area were taken from P and Q, two camera stations, 100 m apart. The focal length of the camera is 150 mm. The axis of the camera makes an angle of 60° and 40° with the base line at stations P and Q respectively. The* *image of a point A appears 20.2 mm to the right and 16.4 mm above the hair lines on the photograph taken at P and 35.2 mm to the left on the photograph taken at Q.*

Calculate the distance PA and QA and elevation of point A, if the elevation of the instrument axis at P is 126.845 m.

Solution

Fig. 2.13 (a) shows the position of the ground point *A* with respect to the stations *P* and *Q* and the picture traces. Fig. 2.13 (*b*) shows the photograph taken at *P* and Fig. 2.13 (*c*) shows the photograph taken at *Q*, with the positions of *a* properly marked.

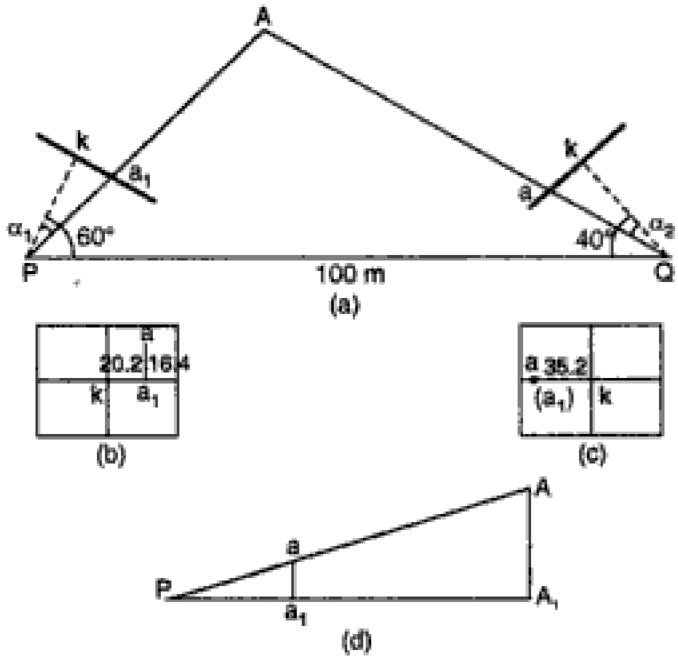


FIG. 2.13

From the photograph at *P*,

$$\alpha_1 = \tan^{-1}\frac{ka_1}{f} = \tan^{-1}\frac{20.2}{150} = 7°\ 40'$$

$\therefore$ $\angle APQ = 60° - \alpha_1 = 60° - 7°\ 40' = 52°\ 20'$

From the photograph at *Q*,

$$\alpha_2 = \tan^{-1}\frac{ka}{f} = \tan^{-1}\frac{35.2}{150} = 13°\ 12'$$

$\therefore$ $\angle AQP = 40° - \alpha_2 = 40° - 13°\ 12' = 26°\ 48'$

$\therefore$ $\angle PAQ = 180° - 52°\ 20' - 26°\ 48' = 100°\ 52'$

From the triangle *APQ*,

$$AP = PQ\ .\ \frac{\sin AQP}{\sin PAQ} = 100\ .\ \frac{\sin 26°\ 48'}{\sin 100°\ 52'} = \mathbf{45.9\ m}$$

and $$AQ = PQ\ .\ \frac{\sin APQ}{\sin PAQ} = 100\ .\ \frac{\sin 52°\ 20'}{\sin 100°\ 52'} = \mathbf{80.6\ m}$$

Calculation of R.L. of A

From the photograph at *P*,

$$Pa_1 = \sqrt{x_a^2 + f^2} = \sqrt{(20.2)^2 + (150)^2} = 151.33 \text{ mm.}$$

Let A_1 be the projection of *A* on the horizontal line Pa_1 drawn through *P* (Fig. 2.15 *d*). Then from the similar triangles,

$$\frac{AA_1}{aa_1} = \frac{PA}{Pa_1}$$

$\therefore$ $$AA_1 = aa_1\ .\ \frac{PA_1}{Pa_1} = \frac{16.4 \times 45.9}{151.53} = 4.975 \text{ m}$$

$\therefore$ R.L. of A = R.L. of instrument axis + AA_1 = 126.845 + 4.975 m = **131.820 m.**

Example 2.3. *The distance from two points on a photographic point to the principal line are 68.24 mm to the left, and 58.48 mm to the right. The angle between the points measured with a transit is 44° 30′. Determine the focal length of the lens.*

Solution

Distance of first point from principal line = x_1 = 68.24 mm

Distance of second point from principal line = x_2 = 58.48 mm

Angle between the two points = θ = 44° 30′

The focal length is given by the expression (Eq. 2.5),

$$f = \frac{x_1 + x_2}{2 \tan \theta} + \sqrt{\frac{(x_1 + x_2)^2}{4 \tan^2 \theta} + x_1 x_2}$$

where $$\frac{x_1 + x_2}{2 \tan \theta} = \frac{68.24 + 58.48}{2 \tan 44° 30'} = 64.47 \quad ; \quad \left(\frac{x_1 + x_2}{2 \tan \theta}\right)^2 = (64.67)^2 = 4156.9$$

$$x_1 x_2 = 68.24 \times 58.48 = 3990.4$$

Substituting the values, we get

$$f = 64.47 + \sqrt{4156.9 + 3990.4} = 64.47 + 90.26 = \mathbf{154.73\ mm}$$

AERIAL PHOTOGRAMMETRY

2.9. AERIAL CAMERA

The primary function of the terrestrial camera as well as the aerial camera is the same, *i.e.*, that of taking pictures. However, since the aerial camera is mounted on a fast moving aeroplane, its requirements are quite different. The aerial camera requires : (*i*) fast lens, (*ii*) high speed and efficient shutter, (*iii*) high speed emulsion for the film, and (*iv*) a magazine to hold large rolls of film. As such, an aerial camera may be considered to be a surveying instrument of great precision.

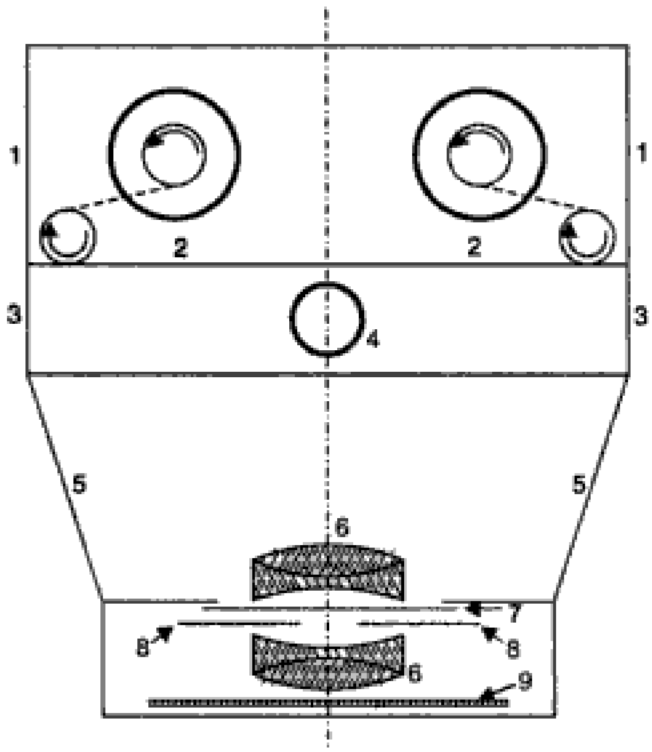
Fig. 2.14 shows the photograph of the wild RC-9 automatic supper wide angle camera. Fig. 2.16 shows the schematic diagram of an aerial camera.

An aerial camera consists of the following essential parts :

(*i*) the lens assembly (including lens, diaphragm, shutter and filter)

(*ii*) the camera cone

(*iii*) the focal plane



1. MAGAZINE 2. FOCAL PLANE 3. BODY
4. TRUNNION 5. CONE 6. LENS
7. SHUTTER 8. DIAPHRAGM 9. FILTER

FIG. 2.16 SCHEMATIC DIAGRAM OF AERIAL CAMERA.

(*iv*) the camera body
(*v*) the drive mechanism
(*vi*) the magazine

(*i*) The Lens Assembly :

The lens assembly consists of the lenses, the diaphragm, the shutter and the filter.

Fig. 2.15 shows the cross-section of the high performance lenses manufactured by Wild Heerbrugg Ltd. Wild Aviotar *f* : 4 [Fig. 2.15 (*a*)] is normal angle lens while Wild Universal-Aviogon *f* : 5.6 [Fig. 2.15 (*b*)] and Wild Super-Aviogon *f* : 5.6 [Fig. 2.15 (*c*)] are wide angle lenses and super wide angle lenses respectively. The following are the details of the lenses manufactured by M/s Wild Heerbrugg Ltd. (Table below)

The other lenses commonly used are : (*i*) Bausch and Lomb *Metrogen f* : 6.3 wide angle lens with 93° coverage, most commonly used in the United States, (*ii*) Zeiss Topogon *f* : 6.3 with 93° coverage, and (*iii*) Goertz Aerotar *f* : 6.8 with 75° coverage.

Since the air-craft is at a considerable distance from the terrain to be photographed, all the points can be considered to be at an infinite distance from the lens and *hence the focal plane of the aerial camera can be fixed at one location*. Thus, an aerial camera is always of a fixed focus type, the focus being set for infinity.

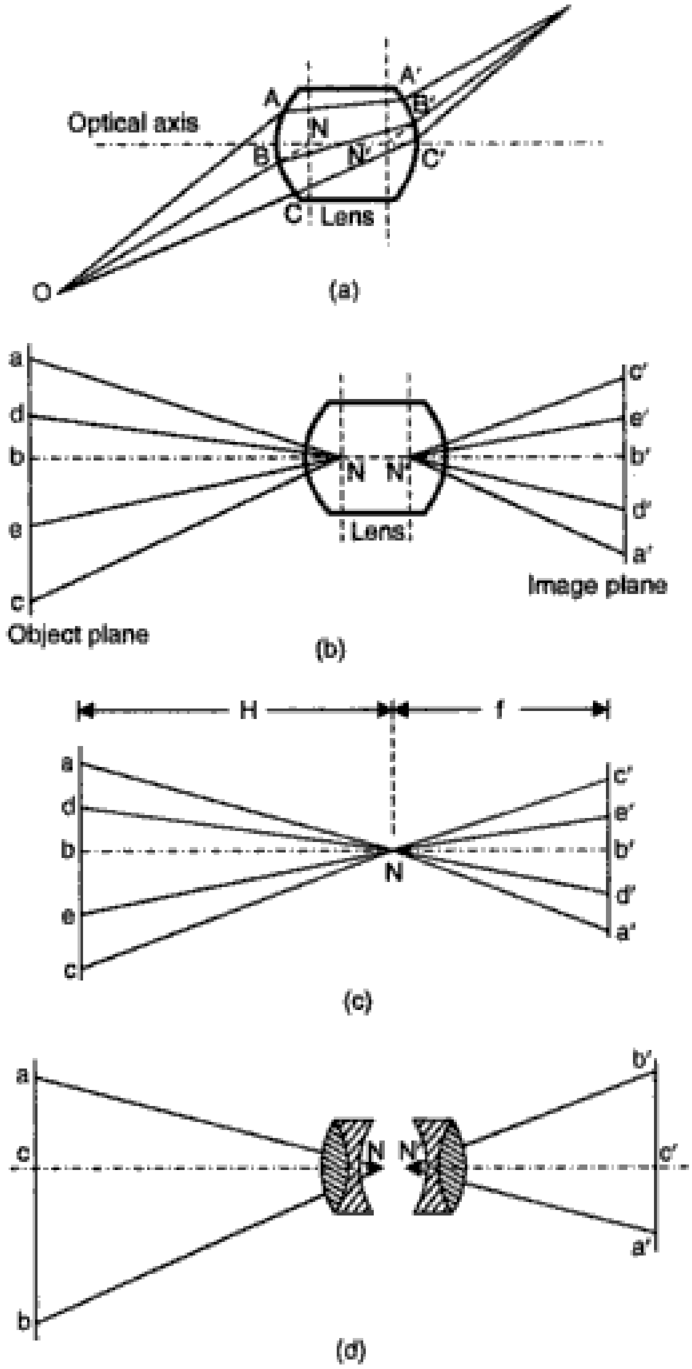


FIG. 2.17. THE LENS NODES.

Camera	*Lens*	*Focal Length (f) cm*	*Picture size (cm)*	*Field Angle*
Wild RC 8 for 19 cm and 24 cm film width	Aviotar *f* : 4	21	18 × 18	60°
	Aviogon *f* : 5.6	11.5	18 × 18	90°
	Universal Aviogon *f* : 5.6	15.2	23 × 23	90°
Wild RC 9 for 24 cm film width	Super-Aviogon *f* : 5.6	8.8	23 × 23	120°
	Super-Infrangon *f* : 5.6	8.8	23 × 23	120°
Wild RC 7a for plates 15 × 15 cm	Aviotar *f* : 4	17	14 × 14	60°
	Aviogon *f* : 5.6	10	14 × 14	90°

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or decrease the size of the opening to restrict the size of the bundle of rays to pass through the lens. If the diaphragm opening is larger, the shutter speed has to be greater.

The Filter **:**

A filter consists of a piece of coloured glass placed in front of the lens. It filters the stray light (blue and violet) in the atmosphere caused by haze and moisture. It also protects the lens from the flying particles in the atmosphere.

(*ii*) **Camera Cone** :

The camera cone supports the entire lens assembly including the filter. At the top of it are provided the collimation marks which define the focal plane of the camera. The cone is made up of the material having low co-efficient of thermal expansion so that the collimation marks and the lens system are held in the same relative positions at operational temperatures. The elements of *interior orientation* are fixed by the relative positions of the lens, the lens axis, the focal plane and the collimation marks.

(*iii*) **The Focal Plane** :

The collimation marks are provided at the upper surface of the cone. The focal plane is provided exactly above the collimation marks. It is kept at such a distance from the near nodal point that best possible image is obtained.

(*iv*) **The Camera Body** :

The camera body is the part of the camera provided at the top of the cone. Sometimes, it forms the integral part of the cone in which case they act as an integral part to preserve the interior orientation once the camera is calibrated.

(*v*) **The Drive Mechanism** :

The camera drive mechanism is housed in the camera body and is used for (*i*) winding and tripping the shutter (*ii*) operating the vacuum system for flattening the film, and (*iii*) winding the film. It may be either operated manually or automatically.

(*vi*) **Magazine** :

A magazine holds the exposed and unexposed films and houses the film flattening device at the focal plane. The power operation of the movable parts of the magazine is supplied from the drive mechanism. The film is flattened at the focal plane either by inserting a piece of optical glass in the focal plane opening or by applying a vacuum to ribbed plate criss-crossed with tiny grooves and provided to the back of the film.

2.10. DEFINITIONS AND NOMENCLATURE

1. Vertical Photograph. A vertical photograph is an aerial photograph made with the camera axis (or optical axis) coinciding with the direction of gravity.

2. Tilted photograph. A tilted photograph is an aerial photograph made with the camera axis (or optical axis) *unintentionally* tilted from the vertical by a small amount, usually less than 3° (Fig. 2.18).

3. Oblique Photograph. An oblique photograph is an aerial photograph taken with the camera axis directed *intentionally* between the horizontal and the vertical. If the apparent horizon is shown in the photograph, it is said to be *high oblique*. If the apparent horizon is not shown, it is said to be *low oblique*.

4. Perspective Projection. A perspective projection is the one produced by straight lines radiating from a common (or selected) point and passing through point on the sphere to the plane of projection. A *Photograph* is a perspective projection.

5. Exposure station. Exposure station is a point in space, in the air, occupied by the camera lens at the instant of exposure. Precisely, it is the space position of the front nodal point at the instant of exposure.

6. Flying height. Flying height is the elevation of the exposure station above sea level or any other selected datum.

7. Flight line. It is a line drawn on a map to represent the track of the aircraft.

8. Focal length. It is the distance from the front nodal point of the lens to the plane of the photograph (*i.e.*, *OK* in the Fig. 2.18). It is also the distance of the image plane from the rear nodal point. *Equivalent focal length* is the distance of the image plane from the rear nodal point (or the distance of the plane of the photograph from the front nodal plane) yielding the best average definition.

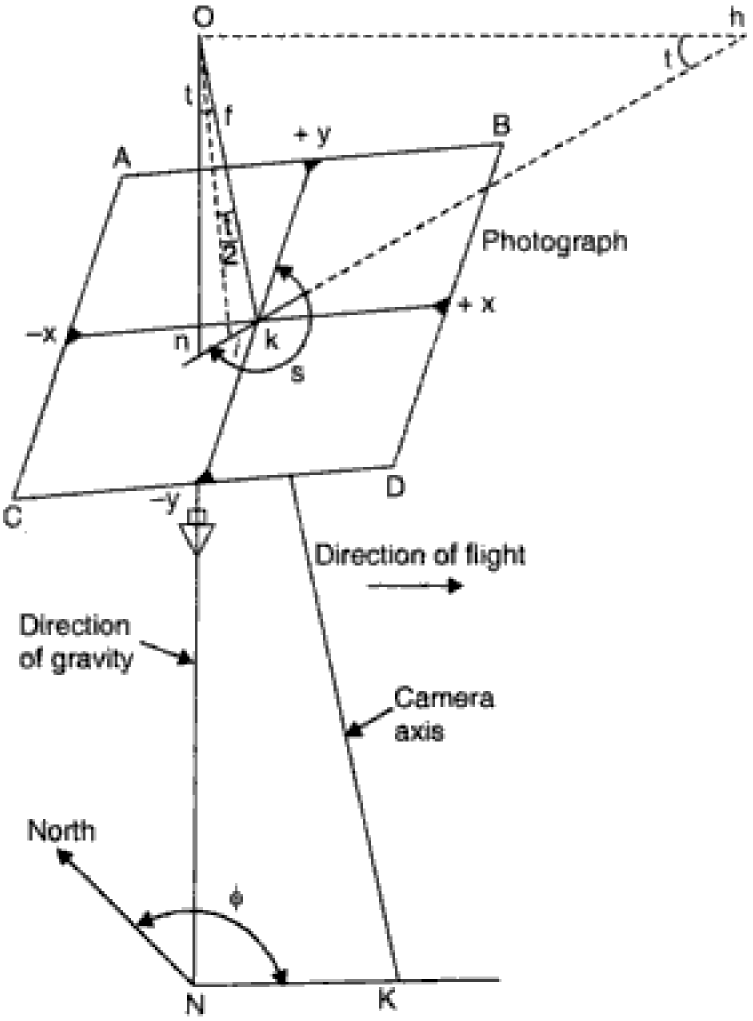

FIG. 2.18. TILTED PHOTOGRAPH

9. Principal point. Principal point is a point where a perpendicular dropped from the front nodal point strikes the photograph. (Also, it is the foot of a perpendicular to the image plane from the rear nodal point in a camera lens system free from manufacturing errors). This principal point is considered to coincide with the intersection of the x-axis and the y-axis. In Fig. 2.18, k is the principal point. The point K is known as the *ground principal point* where the line OK produced meets the ground.

10. Nadir point. Nadir point is a point where a plumb line dropped from the front nodal point pierces the photograph. Thus, in Fig. 2.18, n is the nadir point, which is a point on the photograph vertically beneath the exposure station. This point is also known as the *photo-nadir* or *photo plumb point*.

11. Ground nadir point. Ground nadir point or ground plumb point is the datum intersection with the plumb line through the front nodal point. It is the point on the ground vertically beneath the exposure station such as point N in Fig. 2.18.

O = perspective centre or the rear nodal point of the camera lens (or the exposure station)
k = principal point
K = ground principal point
Ok = principal distance
t = angle of tilt = $\angle kon$ = angular deviation of the photograph perpendicular from the plumb line
n = photo-nadir or photo plumb point
N = ground nadir or ground plumb point
noN = plumb line or vertical line through the perspective centre
i = isocentre
I = ground isocentre
nik = principal line
h = horizon point
$i_1\, i\, i_2$ = axis of tilt = isometric parallel

Relation Between Principal Point, Plumb Point and Isocentre :

From Figs. 2.18 and 2.19,

(1) nk = distance of the nadir point from the principal point

$$\frac{nk}{kO} = \tan t \qquad \text{or} \qquad nk = kO \,.\, \tan t = f \tan t \qquad \ldots(2.6)$$

since $kO = f$ = principal distance

(2) ki = distance of the isocentre from the principal point

$$\frac{ki}{kO} = \tan \frac{t}{2} \qquad \text{or} \qquad ki = kO \,.\, \tan \frac{t}{2} = f \tan \frac{t}{2} \qquad \ldots(2.7)$$

(3) kh = distance along the principal line, from the principal point to the horizon point

$$\frac{kh}{kO} = \cot t \qquad \text{or} \qquad kh = kO \,.\, \cot t = f \cot t. \qquad \ldots(2.8)$$

2.11. SCALE OF A VERTICAL PHOTOGRAPH

Since a photograph is the perspective projection, the images of ground points are displaced where there are variations in the ground elevation. Thus, in Fig. 2.20 (a) the images of two points A and A_0, vertically above each other, are displaced on a *vertical* photograph and are represented by a and a_0 respectively. Due to this displacement, there is no uniform scale between the points on such a photograph, except when the ground points have the same elevation. If the elevation of points vary, the scale of the vertical photograph will vary from point to point on the photograph.

Let us first take the case when the ground is horizontal, *i.e.*, all the points are having the same elevation, such as shown in Fig. 2.20 (*a*).

Let $$S = \text{scale} = \frac{\text{map distance}}{\text{ground distance}}$$

From Fig. 2.20 (*a*), $$S = \frac{ka}{KA} = \frac{Ok}{KA} = \frac{f}{H-h} = \frac{f}{H-h} \qquad \ldots(2.9)$$

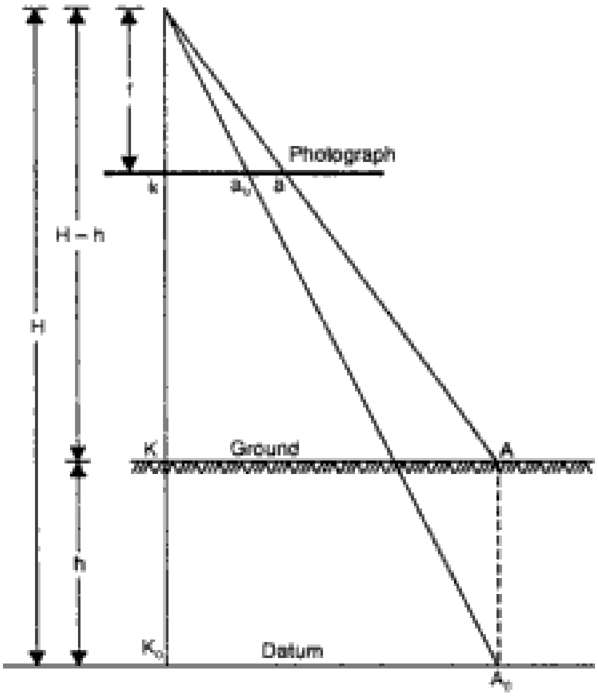

FIG. 2.20. (*a*) SCALE OF A VERTICAL PHOTOGRAPH

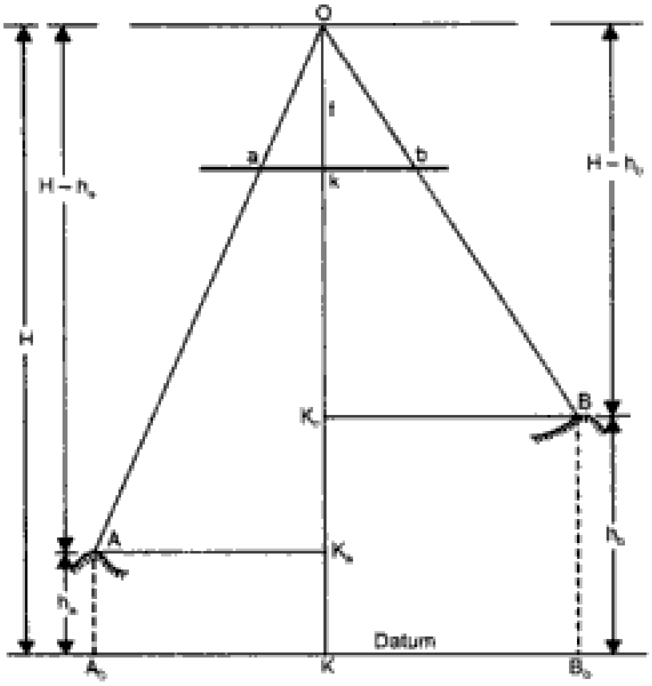

FIG. 2.20. (*b*) SCALE OF A VERTICAL PHOTOGRAPH.

where H = height of the exposure station (or the air plane) above the mean sea level

f = focal length of the camera

h = height of the ground above mean sea level

Let us now take the case when the points are not having the same elevation, as represented in Fig. 2.20 (*b*).

Let A and B be two points having elevations h_a and h_b respectively above mean sea level. They are represented by a and b respectively on the map. k is the principal point of the vertical photograph taken at height H above mean sea level.

The scale of the photograph at the elevation h_a is evidently equal to the ratio $\frac{ak}{AK_a}$.

From similar triangles, $\frac{aK}{AK_a} = \frac{Ok}{OK_a} = \frac{f}{H - h_a}$

Hence the scale of the photograph at the elevation h_a is equal to $\frac{f}{H - h_a}$.

Similarly, the scale of the photograph at the elevation h_b is equal to the ratio $\frac{bk}{BK_b}$.

From similar triangles, $\frac{bk}{BK_b} = \frac{Ok}{OK_b} = \frac{f}{H - h_b}$.

Hence the scale of the photograph at the height h_b is equal to $\frac{f}{H - h_b}$.

In general, therefore, the scale of the photograph is given by

$$S_h = \frac{f}{H - h}$$

where S_h = scale at the elevation h.

The scale of the photograph can also be designated by the representative fraction (R_h), *i.e.*

$$R_h = \frac{1}{\left(\frac{H - h}{f}\right)}$$

where $(H - h)$ and f are expressed in the same unit (*i.e.* metres).

Datum Scales (S_d)

The *datum scale* of a photograph is that scale which would be effective over the entire photograph if all the ground points were projected vertically downward on the mean sea level before being photographed. Thus, from Fig. 2.20 (*a*),

$$\text{Datum scale} = S_d = \frac{ka}{KA_0} = \frac{Ok}{OK} = \frac{f}{H} \qquad \text{...(2.10)}$$

where K and A_0 are the projections of k and A on the datum plane.

Average Scale (S_{av})

The average scale of a vertical photograph is that which would be effective over the entire photograph if all the ground points were projected vertically downward or upward on a plane representing the average elevation of the terrain before being photographed.

Thus,

$$S_{av} = \frac{f}{H - h_{av}} \qquad \text{...(2.11)}$$

where h_{av} = average elevation of the terrain

To Find the Scale of a Photograph

If the images to ground points of *equal elevation* and known horizontal distance appear on the photograph, the scale of the photograph can be determined by comparing the ground length and the corresponding length on the photograph. Thus, if l is the distance on the photograph, between the two points A and B having the same elevation h and the horizontal distance (ground) between them to be L, the scale at the height h is given by

$$S_h = \frac{l}{L} \qquad \text{...(2.12)}$$

The distance L measured on the ground either directly or by the triangulation, or it can be taken from the existing maps, if available. To find the average or fairly representative scale of photograph several known lines on the photograph should be measured and compared and the average scale should be adopted. In case a reliable map of the area is available, the photographic scale can be found by comparing the photo distance and the map distance between two well-defined points at the same elevation.

Thus,

$$\frac{\text{Photo scale}}{\text{Map scale}} = \frac{\text{photo distance}}{\text{map distance}}$$

If the focal length of the lens and the flying height (H) above *m.s.l.* is known, the scale can be found from the relation

$$S_h = \frac{f}{H - h} \qquad ...(2.13)$$

2.12. COMPUTATION OF LENGTH OF LINE BETWEEN POINTS OF DIFFERENT ELEVATIONS FROM MEASUREMENTS ON A VERTICAL PHOTOGRAPH

In Fig. 2.21, let A and B be two ground points having elevations h_a and h_b above datum, and the co-ordinates (X_a, Y_a), (X_b, Y_b) respectively with respect to the ground co-ordinate axes which coincide in direction with the photographic co-ordinates x and y-axis. The origin of the ground co-ordinates lie vertically beneath the exposure station.

Let a and b be the corresponding points of the photograph, and (x_a, y_a), (x_b, y_b) be the corresponding co-ordinates. From similar triangles,

$$\frac{Ok}{OK_a} = \frac{x_a}{X_a} = \frac{y_a}{Y_a} = \frac{f}{H - h_a} \quad ..(1)$$

Also, $$\frac{Ok}{OK_b} = \frac{x_b}{X_b} = \frac{y_b}{Y_b} = \frac{f}{H - h_b} \quad ..(2)$$

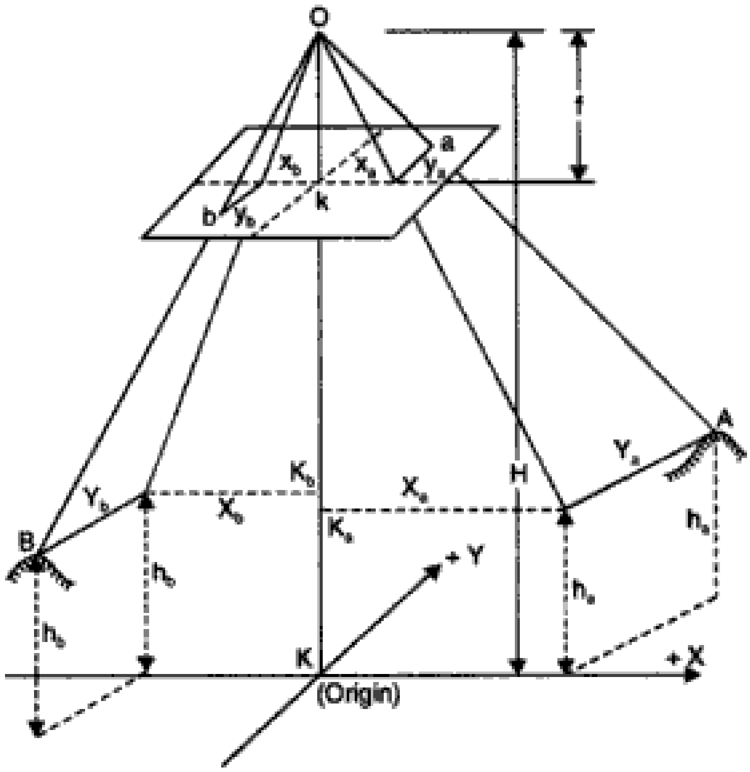

FIG. 2.21. COMPUTATION OF LENGTH OF A LINE.

Hence, we have $$X_a = \frac{H - h_a}{f} \cdot x_a \qquad ...[2.14\ (a)]$$

$$Y_a = \frac{H - h_a}{f} \cdot y_a \qquad ...[21.4\ (b)]$$

$$X_b = \frac{H - h_b}{f} \cdot x_b \quad ...[2.14\ (c)] \quad ; \quad Y_b = \frac{H - h_b}{f} \cdot y_b \qquad ...[2.14\ (d)]$$

And, in general, the co-ordinates X and Y of any point at an elevation are :

$$X = \frac{H - h}{f} x \quad ; \qquad Y = \frac{H - h}{f} y.$$

The length L between the two points A and B is then given by

$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2} \qquad ...(2.15)$$

The value of X_a, X_b and Y_a and Y_b must be substituted with their proper algebraic signs.

2.13. DETERMINATION OF HEIGHT (H) OF LENS FOR A VERTICAL PHOTOGRAPH

If the images of two points A and B having different known elevations and known length between them appear on the photograph, the elevation or height H of the exposure station can be calculated by a reversed procedure from that of the preceding article.

Since the photograph is the perspective view, the ground relief is shown in perspective on the photograph. Every point on the photograph is therefore, displaced from their true orthographic position. This displacement is called *relief displacement*.

Thus, in Fig. 2.22, *A*, *B* and *K* are three ground points having elevations h_a, h_b and h_k above datum. A_0, B_0 and K_0 are their *datum positions* respectively, when projected vertically downwards on the datum plane. On the photograph, their positions are *a*, *b* and *k* respectively, the points *k* being chosen vertically below the principal point. If the datum points A_0, B_0 and K_0 are imagined to be photographed along with the ground points, their positions will be a_0, b_0 and *k* respectively. As is clear from the figure, the points *a* and *b* are *displaced outward* from their datum photograph positions, the displacement being along the corresponding radial lines from the principal point. The radial distance aa_0 is the *relief displacement* of *A* while bb_0 is the relief displacement of *B*. The point *k* has not been displaced since it coincides with the principal point of the photograph.

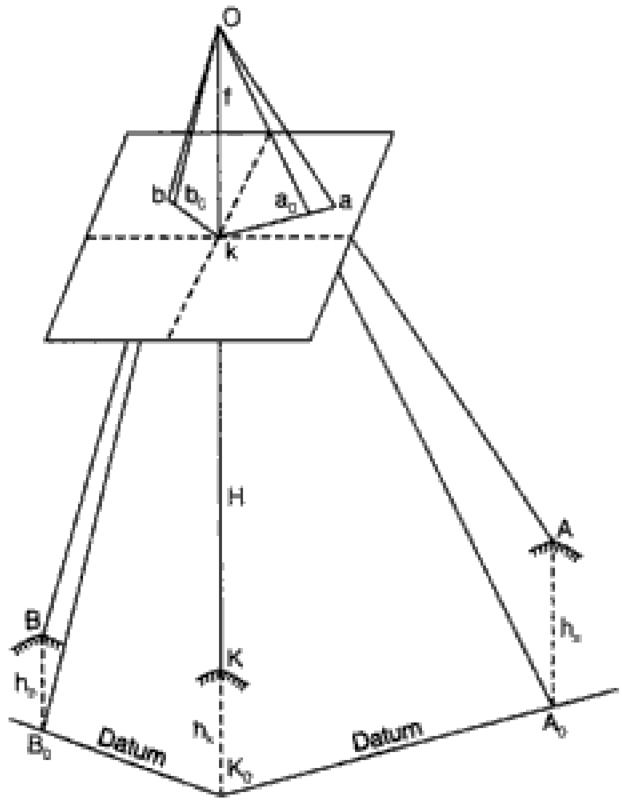

FIG. 2.22. RELIEF DISPLACEMENT ON VERTICAL PHOTOGRAPH.

To calculate the amount of relief displacement, consider Fig. 2.23 which shows a vertical section through the photograph of Fig. 2.22 along the line *ka*.

In Fig. 2.23,

Let r = radial distance *a* from *k*

r_0 = radial distance of a_0 from *k*

$R = K_0 A_0$

Then, from similar triangles,

$$\frac{f}{H-h} = \frac{r}{R}, \text{ from which } r = \frac{Rf}{H-h} \quad ...(1)$$

$$\text{Also } \frac{f}{H} = \frac{r_0}{R}, \text{ from which } r_0 = \frac{Rf}{H} \quad ...(2)$$

Hence the relief displacement (d) is given by

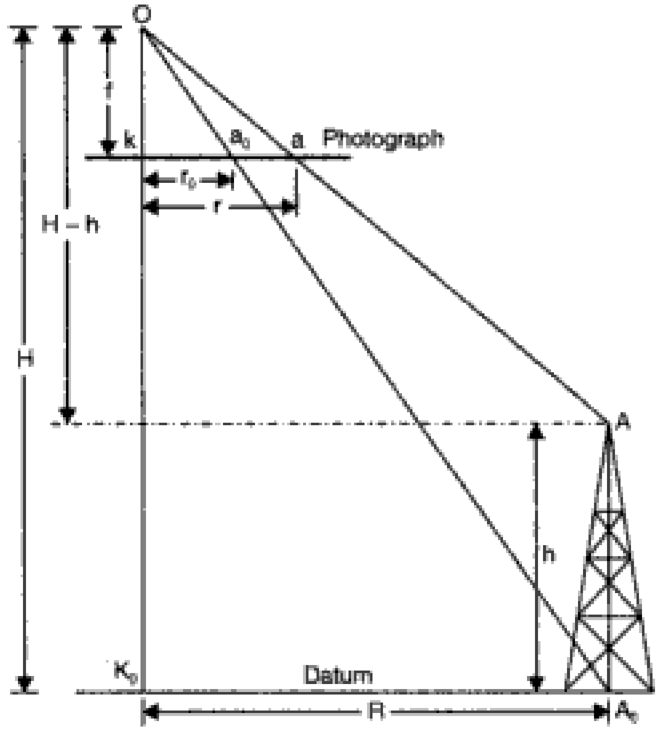

FIG. 2.23. CALCULATION OF RELIEF DISPLACEMENT.

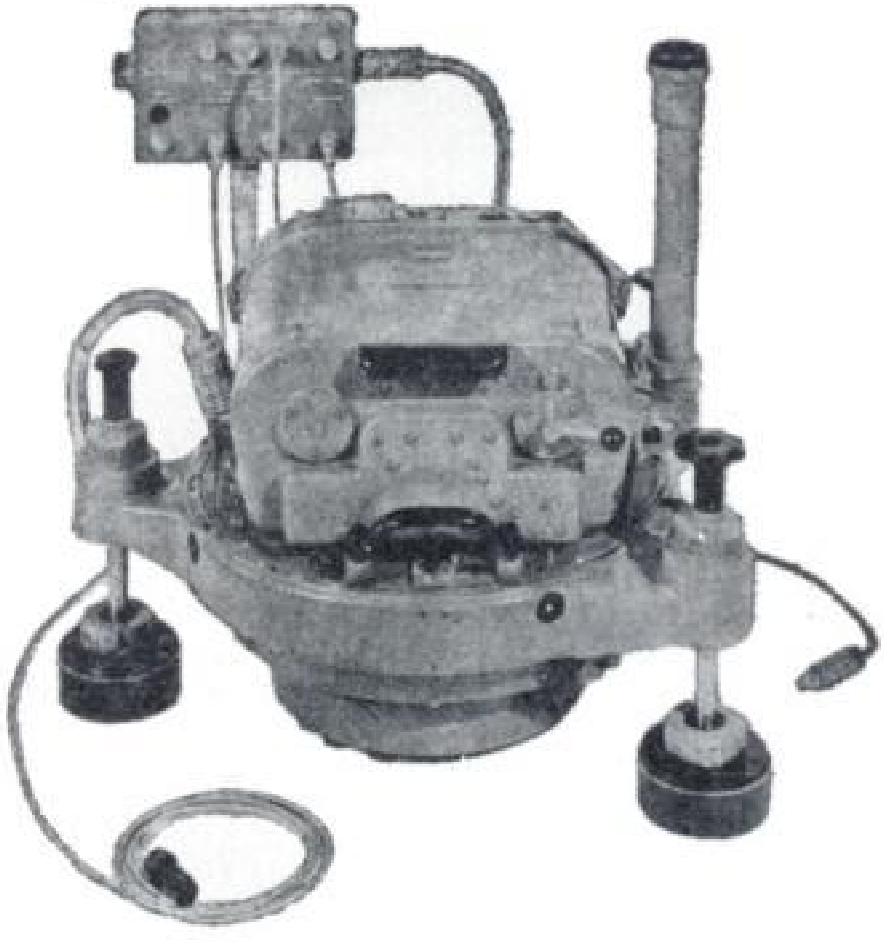

FIG. 2.14. WILD RC-9 AUTOMATIC AERIAL CAMERA
(BY COURTESY OF MANUFACTURERS)

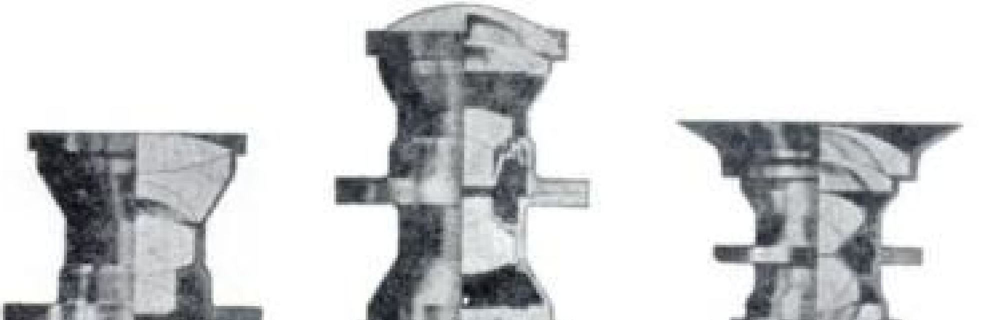

FIG. 2.15. LENSES FOR WILD AERIAL CAMERAS
(BY COURTESY OF MANUFACTURERS)

(TO FACE PAGE 155)

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$$\therefore \quad \frac{8.65 \text{ cm}}{2000 \text{ m}} = \frac{20 \text{ cm}}{(H-500) \text{ m}}$$

or $$(H-500) = \frac{20 \times 2000}{8.65} = 4624 \text{ m}$$

$$\therefore \quad H = 4624 + 500 = 5124 \text{ m}$$

$$\therefore \quad S_{800} = \frac{20 \text{ cm}}{(5124-800) \text{ m}} = \frac{1 \text{ cm}}{216.2 \text{ m}}$$

Hence S_{800} is **1 cm = 216.2 cm**.

Example 2.7. *A section line AB appears to be 10.16 cm on a photograph for which the focal length is 16 cm. The corresponding line measures 2.54 cm on a map which is to a scale 1/50,000. The terrain has an average elevation of 200 m above mean sea level. Calculate the flying altitude of the aircraft, above mean sea level, when the photograph was taken.*

Soletion.

The relation between the photo scale and map scale is given by

$$\frac{\text{Photo scale}}{\text{Map scale}} = \frac{\text{Photo distance}}{\text{Map distance}}$$

Here, map scale $= \dfrac{1}{50,000}$; Let the photo scale be $\dfrac{1}{n}$

$$\therefore \quad \frac{1/n}{1/50,000} = \frac{10.16}{2.54}$$

$$\therefore \quad \frac{1}{n} = \frac{10.16}{2.54} \times \frac{1}{50,000} = \frac{1}{12,500} \qquad \text{or} \qquad n = 12,500$$

Again, $$S_{200} = \frac{1}{n} = \frac{f}{(H-h)} \qquad \text{or} \qquad \frac{1}{12,500} = \frac{(16/100) \text{ m}}{(H-200) \text{ m}}$$

or $$(H-200) = \frac{16}{100} \times 12500 = 2000 \text{ m}$$

Hence $H = 2000 + 200 =$ **2200 m.**

Example 2.8. *Two points A and B having elevations of 500 m and 300 m respectively above datum appear on the vertical photograph having focal length of 20 cm and flying altitude of 2500 m above datum. Their corrected photographic co-ordinates are as follows:*

Point	*Photographic Co-ordinates*	
	x (cm)	*y (cm)*
a	*+ 2.65*	*+ 1.36*
b	*– 1.92*	*+ 3.65*

Determine the length of the ground line AB.

Solution

The ground co-ordinates are given by

$$X_a = \frac{H-h_a}{f} \cdot x_a = \frac{2500-500}{20} \times (+2.65) = +265 \text{ m}$$

$$\frac{H - h_{ab}}{H_{approx.} - h_{ab}} = \frac{\text{Correct } AB}{\text{Computed } AB}$$

$$\therefore \quad \frac{H - 400}{2532.2 - 400} = \frac{545}{553.4} \quad ; \text{ From which} \quad H = 400 + 2100 = 2500$$

Using this value of H to calculate the co-ordinates, we get

$$X_a = \frac{2500 - 500}{20} \times 2.65 = +265 \quad ; \quad Y_a = \frac{2500 - 500}{20} \times 1.36 = +136$$

$$X_b = \frac{2500 - 300}{20} \times (-1.92) = -211.2 \quad ; \quad Y_b = \frac{2500 - 300}{20} \times 3.65 = +401.5$$

$$\therefore \quad L = \sqrt{(265 + 211.2)^2 + (136 - 401.5)^2} = 545$$

This agrees with the measured length. Hence height of lens = **2500** m.

Example 2.10. *The distance from the principal point to an image on a photograph is 6.44 cm, and the elevation of the object above the datum (sea level) is 250 m. What is the relief displacement of the point if the datum scale is 1/10,000 and the focal length of the camera is 20 cm ?*

Solution.

The datum scale is given by

$$S_d = \frac{1}{10{,}000} = \frac{(20/100)\text{ m}}{H\text{ m}}$$

From which $$H = \frac{20}{100} \times 10{,}000 = 2000 \text{ m above } m.s.l.$$

Again, the relief displacement (d) is given by

$$d = \frac{r\,h}{H} = \frac{6.44 \times 250}{2000} = \mathbf{0.805 \text{ cm}}.$$

Example 2.11. *A tower TB (Fig.2.24), 50 m high, appears in a vertical photograph taken at a flight altitude of 2500 m above mean sea level. The distance of the image of the top of the tower is 6.35 cm. Compute the displacement of the image of the top of the tower with respect to the image of its bottom. The elevation of the bottom of the tower is 1250 m.*

Solution

Let H = height of the lens above the bottom of the tower.

The displacement d of the image of the top with respect to the image of the bottom is given by

$$d = \frac{h\,r}{H}$$

where h = height of the tower above its base = 50 m ; $H = 2500 - 1250 = 1250$ m

$$\therefore \quad d = \frac{50 \times 6.35}{1250} = \mathbf{0.25 \text{ cm}}.$$

Example 2.12. *A vertical photograph of a flat area having an average elevation of 250 metres above mean sea level was taken with a camera having a focal length of*

Let N and M be the points on on and om extended, at heights of h above datum. Thus N, M and A have the same elevation. The triangle NMA is in a horizontal plane.

From the similar triangles $om'a$ and ONA, we get

$$\frac{m'a}{NA} = \frac{Om'}{ON}$$

But $\quad Om' = On - m'n = f \sec t - mn \sin t \quad ; \quad ON = ON_0 - NN_o = H - h$

$$\frac{m'a}{NA} = \frac{\text{Map distance}}{\text{Ground distance}} = \text{scale at a point whose elevation is } h = S_h$$

Substituting the values in (1), we get

$$S_h = \frac{f \sec t - mn \sin t}{H - h} \qquad \ldots(2)$$

In the above expression mn is the distance along the principal line, between the photo nadir and the foot of the perpendicular from the point under consideration. To find its value, let us consider the system of co-ordinates axes illustrated in Fig. 2.26.

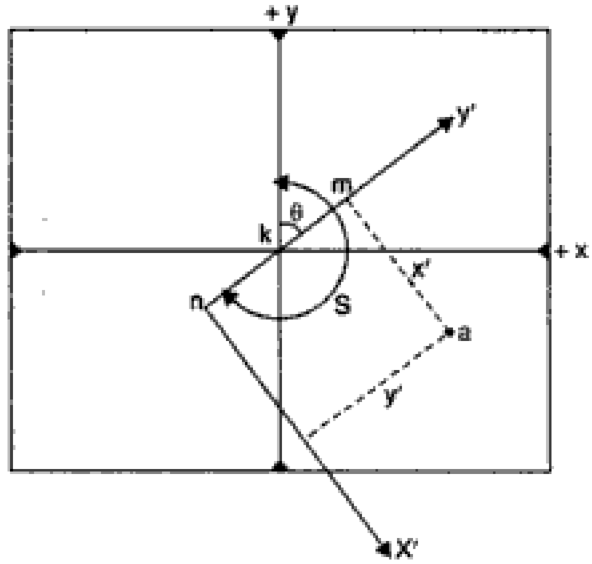

FIG. 2.26. CO-ORDINATE AXES THROUGH PLUMB POINT.

Let the photographic co-ordinates of the image a be x and y. Let s be the angle of swing and θ be the angle between the y-axis and the principal line. If the y-axis be rotated to the position of the principal line, the new axis (or y'-axis) will be inclined to the original axis by an angle θ given by

$$\theta = 180° - s \qquad \ldots(2.21)$$

As in analytic geometry, the angle θ is considered to be *positive* when the rotation is in the *counter-clockwise* direction and *negative* when it is in the clockwise direction. Thus, the angle θ in Fig. 2.26. is negative. Let the new x-axis (or x'-axis) be selected through the nadir point n. The distance kn is equal to $f \tan t$ (see Eq. 2.6). The new co-ordinates (x', y') of the point a with reference to the x' and y' axis are given by

$$x' = x \cos \theta + y \sin \theta \qquad \ldots[2.22\ (a)]$$

$$y' = -x \sin \theta + y \cos \theta + f \tan t \qquad \ldots[2.22\ (b)]$$

The distance nm is therefore equal to y'. Substituting this in (2), we get

$$S_h = \frac{f \sec t - y' \sin t}{H - h} \qquad \ldots(2.23)$$

It is clear that the co-ordinates y' is the same for the points on the line ma. *Hence the scale, which is the linear function of y, is constant for all the points on a line perpendicular to the principal line.*

For finding the scale at a given point on the photograph by Eq. 2.23, the following data is essential :

The new co-ordinates x' and y' may be computed from Eq. 2.22. The length L of the line AB can then be calculated from Eq. 2.24 (c).

Compare the computed length of AB with that of the correct length from the relationship

$$\frac{H - h_{ab}}{H_{approx.} - h_{ab}} = \frac{\text{Correct } AB}{\text{Computed } AB}$$

where H is the new value of the flying height.

***Step 3* :**

Repeat step 2 till the computed length of AB agrees with its correct length within the required degree of accuracy.

2.18. TILT DISTORTION OR TILT DISPLACEMENT

If a terrain is photographed, once with a tilted photograph and then with a vertical photograph, both taken at the same flight altitude and with the same focal length, the two photographs will match at the axis of tilt only. The image of any other point, not on the axis of tilt, will be *displaced* either outward or inward with respect to its corresponding position on a vertical photograph.

Tilt distortion or tilt displacement is defined as the difference between the distance of the image of a point on the tilted photograph from the isocentre and the distance of the image of the same point on the photograph from the isocentre if there had been no tilt.

Fig. 2.27 shows a vertical photograph and tilted photograph of the same terrain, intersecting each other in a line which is the axis of tilt. n is the nadir point of the tilted photograph, and serves as the principal point of the vertical photograph. k is the principal point of the tilted photograph. The portion of the tilted photo above the axis of tilt is known as the *upper part* while the portion below it is known as the *lower part* of the photograph.

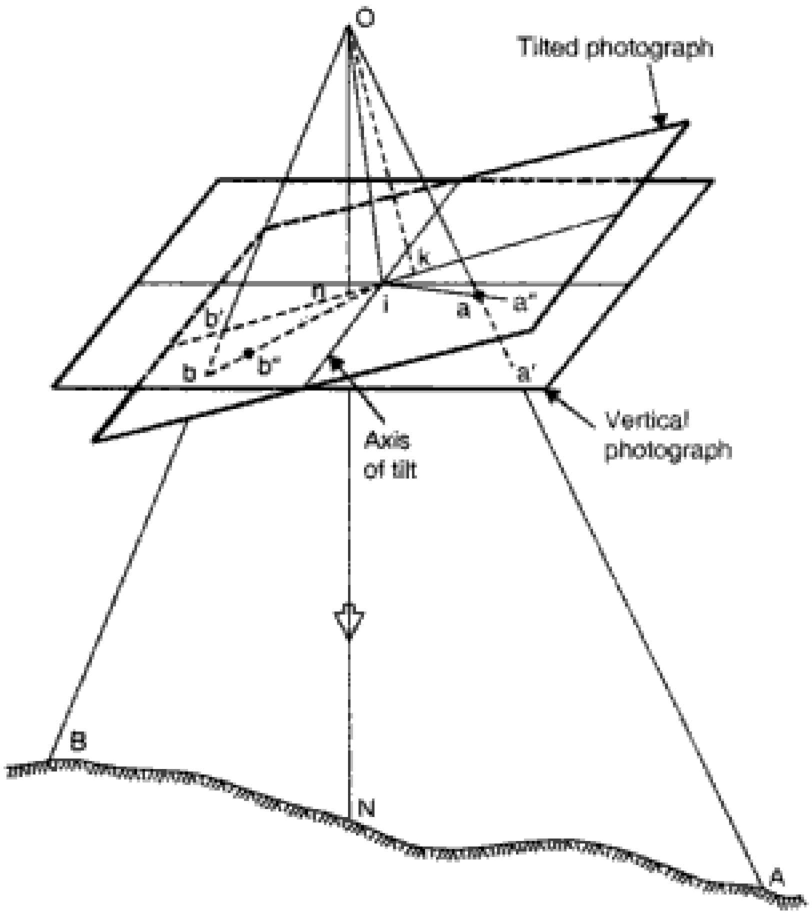

FIG.. 2.27. TILT DISTORTION.

Let us consider two ground points A and B photographed both on the vertical photograph as well as on the tilted photograph. a and b are their images on the tilted photo while a' and b' are the corresponding images on the vertical photograph. If the vertical photograph is now rotated about the axis of tilt until it is in the plane of

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and b are the images of two points on the tilted photograph, along its principal line, while a' and b' are the corresponding positions on the vertical photograph. Since i is the point of rotation, d_a and d_b represent the displacements of the points a and b with respect to a' and b' respectively. Let α be the inclination of the ray Oa with Ok. *Similarly,* β *is the inclination of the ray* Ob *to* Ok.

Thus d_a = tilt displacement of a with respect to a'

or $d_a = ia' - ia$

But $ia' = n'\,a' - n'\,i = f \tan(t + \alpha) - f \tan t/2$ and $ia = ka + ki = f \tan \alpha + f \tan t/2$

Hence $d_a = f \tan(t + \alpha) - f \tan t/2 - f \tan \alpha - f \tan t/2$

or $d_a = f[\tan(t + \alpha) - \tan \alpha - 2 \tan t/2]$ ...[2.25](*a*)

Similarly, $d_b = ib - ib'$

$ib = kb - ki = f \tan \beta - f \tan t/2$; $ib' = n'\,b' + n'i = f \tan(\beta - t) + f \tan t/2$

$\therefore$ $d_b = f \tan \beta - f \tan t/2 - f \tan(\beta - t) - f \tan t/2$

or $d_b = f[\tan \beta - \tan(\beta - t) - 2 \tan t/2]$ ...[2.25(*b*)]

In the above expressions, the angles α and β can be found by the relations :

$$\tan \alpha = \frac{ka}{f}, \text{ and } \tan \beta = \frac{kb}{f}.$$

It can be shown that equations [2.25 (*a*,*b*)] can be represented by the approximate formula

$$d = \frac{(ia)^2 \sin t}{f} \qquad \text{...(2.26)}$$

It is quite clear from the figure that the tilt displacement of a point on the upward half of a tilted photograph is *inward* (such as for point *a*) while the tilt displacement of a point on the downward or nadir point half is *outward* (such as for *b*)

Equations 2.25 give the tilt displacements for the points on the principal line. The tilt displacement of a point not lying on the principal line is greater than that of a corresponding point on the principal line.

Let I = angle measured at the isocentre from the principal line to the point.

d_u = displacement of the point on the upward half of the tilted photograph.

d_d = displacement of the point on the downward half of the tilted photograph.

In Fig. 2.28 (plan), the point q is not on the principal line while point a is on the principal line. qq' is the displacement of q while aa' is the displacement of point a. Since both q and a are equidistant from the axis of tilt, we have

$$qq' = aa' \sec I$$

where I is the angle at the isocentre from the principal line to the point q.

Hence the ratio of the tilt displacement of a point not on the principal line to that of a point on the principal line is equal to the secant of the angle at the isocentre from the principal line to the point.

Thus, the expressions for d_u and d_d can be written as :

$$d_u = f \sec I\,[\tan(t + \alpha) - \tan\alpha - 2\tan t/2] \quad \text{...[2.27(a)]}$$

$$d_d = f \sec I\,[\tan\beta - \tan(\beta - t) - 2\tan t/2] \quad \text{...[2.27(b)]}$$

In Fig. 2.28 (plan), $p'q'r's'$ represents a square on the vertical photograph. The corresponding displaced points on the tilted photographs are p, q, r and s. Since the tilt displacements are always radial from the isocentre, the corresponding figure $p\,q\,r\,s$ becomes a rhombus.

2.19. RELIEF DISPLACEMENT ON A TITLED PHOTOGRAPH

It has been shown in § 2.14 that the relief displacement on a vertical photograph is radial from the principal point. The points are displaced radially outward from their datum photograph positions. Fig. 2.29 shows the relief displacement on a tilted photograph..

A, *B* and *N* are ground points, and A_0, B_0, N_0 are their corresponding datum positions. *N* and N_0 being vertically below the nadir point *n*. *A* and *B* are imaged at *a* and *b* respectively, a_0 and b_0 are the datum photograph positions of A_0 and B_0. *i* is the isocentre and *k* is the principal point. The plane $O\,N\,N_0\,A_0\,A$ is a vertical plane since it contains the plumb line *ON*. The points *n*, a_0 and *a* lie on the same vertical plane. Since the points *n*, a_0 and *a* also lie on the photograph, they are in the same line, *i.e.*, a_0 and *a* lie on a radial line from the nadir point. Similarly, the point *n*, b_0 and *b* are on the same line, and b_0 and *b* are radial from the nadir point. *Thus, on a tilted photograph, the relief displacement is radial from the nadir point*. The amount of relief displacement on a tilted photograph depends upon : (*i*) flying height, (*ii*) distance of the image from the nadir point, (*iii*) elevation of the ground point, and (*iv*) position of the point with respect to the principal line and to the axis of tilt. In the case of near vertical photograph, where the tilt is less than 3°, the relief displacement can be calculated from equations 2.19 with the modification that the radial distances *r* and *r'* are measured from the nadir point and *not* from the principal point.

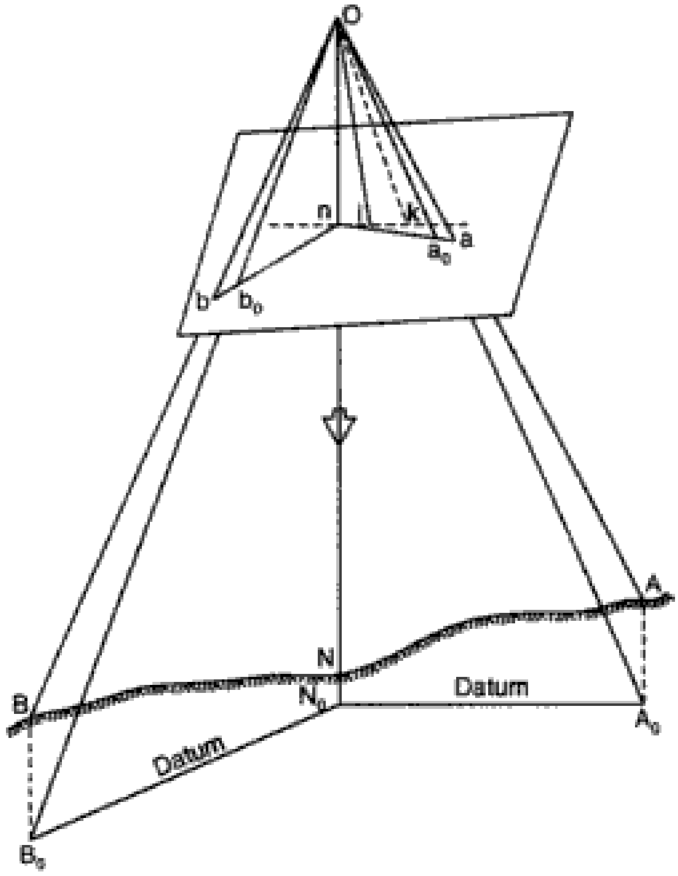


FIG. 2.29. RELIEF DISPLACEMENT ON A TILTED PHOTOGRAPH.

Thus $\quad d = \dfrac{rh}{H}$ ...[2.28(a)] $\quad$ and $\quad d = \dfrac{r_0\,h}{H - h}$ ...[2.28(b)]

where *d* is the relief displacement on a tilted photograph,

r = radial distance of image point from the photographic nadir.

r_0 = radial distance of datum image point from the photographic nadir.

2.20. COMBINED EFFECTS OF TILT AND RELIEF

It has been shown in § 2.18 that on a tilted photograph covering the ground with no relief, the tilt distortion is radial from the isocentre. In the previous article, it has been shown that the relief distortion is radial from the nadir point. To study the combined effect on the tilt and relief, let us refer Fig. 2.30.

Fig. 2.30 shows the displacements of five points A, B, C, D and E in typical positions. a_0, b_0, c_0, d_0, e_0, are their corresponding datum photograph positions. a', b', d', e' are corresponding positions after the image has undergone relief displacement. a, b, c, d and e are the corresponding positions after the image has undergone tilt displacement.

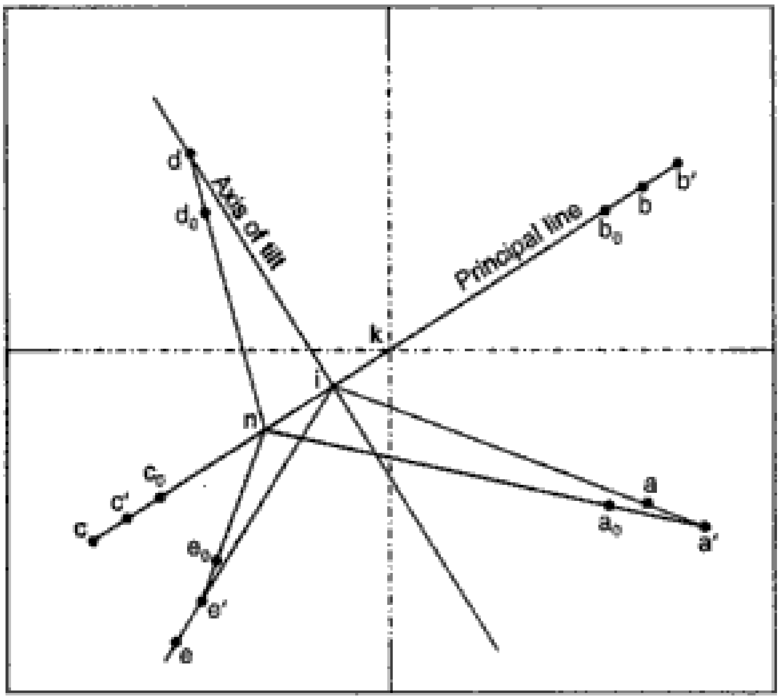


FIG. 2.30. COMBINED EFFECT OF TILT AND RELIEF DISPLACEMENT.

For the point A, the relief displacement is $a_0\,a'$ *radially outward* from the nadir point and the tilt displacement is $a'a$ *radially inward* to the isocentre as it lies in the *upper* part of the photograph. Thus, the relief displacement and the tilt displacement tend to compensate each other.

For the point B, the relief displacement $b_0\,b'$ is radially outward from the nadir point, and the tilt displacement $b'\,b$ is radially inward to the isocentre as it lies in the upper part of the photograph. The position of the point has been so chosen that b_0, b' and b lie on the principal line. Here also, both the displacements tend to compensate each other.

For the point D, the relief displacement $d_0\,d'$ is radially outward from the nadir point, and the tilt displacement $d'd$ is zero since the image d' happens to fall on the axis of tilt along which there is no tilt displacement.

For the point C, the relief displacement $c_0\,c'$ is radially outward from the nadir point while the tilt displacement $c'c$ is radially outward from the isocentre since it lies in the lower part of the photograph. The position of the point has been so chosen that c_0, c' and c lie on the principal line. The relief displacement and the tilt displacement are cumulative.

Lastly, for the point E, the relief displacement $e_0 e'$ is radially outward from the nadir point and the tilt displacement $e'e$ is radially outward from the isocentre since it lies in the lower part of the photograph. Here also, both the displacements are cumulative.

Thus, it can be concluded that the tilt and relief displacements tend to cancel in the upper part of the photograph while they are cumulative in the lower part.

In actual practice, the effects of tilt can be analysed only where precise equipment and trained personnel are available. These effects are more often removed by re-photographing the prints with the aid of accurately established control points in the photograph. Within certain limits of permissible errors, the effect of tilt can be eliminated by means of various projectors. In spite of scale variation, relief displacement and tilt displacement, an aerial photograph taken with a calibrated precision aerial camera is precise perspective view of the terrain, and precise measurements and highly accurate results may be obtained from it.

2.21. FLIGHT PLANNING FOR AERIAL PHOTOGRAPHY

When vertical photographs are to be used for the preparation of maps, all the methods of compilation require that the plumb points of the preceding and succeeding prints are visible in each photograph. Photographs are taken at the proper interval along each strip to give the desired overlap of photographs in the given strip. Each strip is spaced at pre-determined distances to ensure desired side lap between adjacent strips.

The overlap of photographs in the direction of flight line is called *longitudinal overlap or forward overlap* or simply overlap. Along a given flight line, photographs are taken at such frequency as to cause successive photographs to overlap

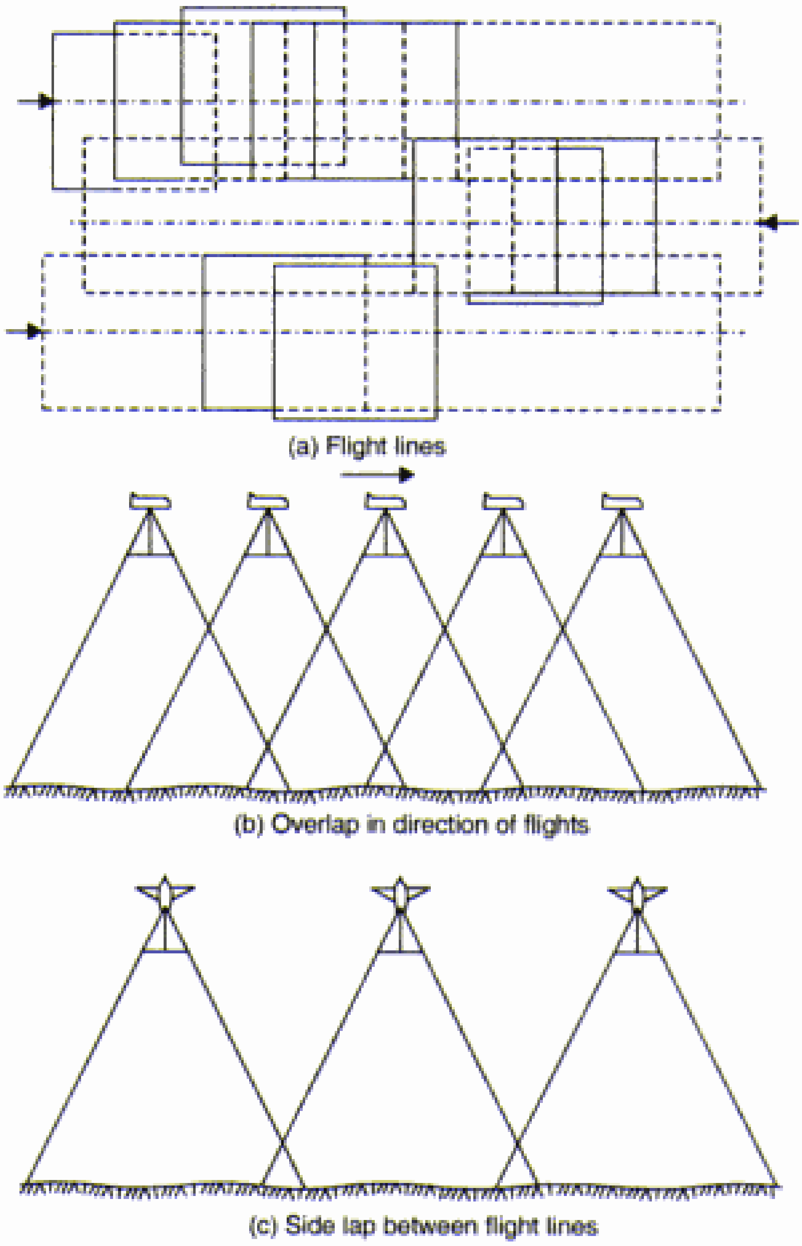


FIG. 2.31. THE OVERLAP AND SIDE LAP OF PHOTOGRAPHS.

each other by 55 to 65 per cent. Fig. 2.31 (*a*) shows three successive flight lines. Fig. 2.31 (*b*) shows the vertical section containing the flight line and showing the overlap. Since the overlap is more than 50 per cent, alternate photographs will overlap one another by 10 to 30 per cent. When photographs are taken with this overlap, the entire area may be examined stereoscopically. The overlap between adjacent flight lines is known as *lateral overlap or sidelap*. The sidelap amounts to about 15 to 35 per cent. Fig. 2.31 (*c*) shows the vertical section taken normal to the three flight lines of Fig. 2.31 (*a*).

The number of individual photographs required to cover a given area increases with the increase in the overlap and sidelap, thus increasing the amount of work both in the field as well as in the office.

Reasons for Overlap

The following are some of the reasons for keeping overlap in the photographs :

(1) To tie the different prints together accurately, it is desirable that the principal point of each print should appear on the edges of as many adjacent strips as possible.

(2) The distortions caused by the lens and by the tilt, and the relief displacements are more pronounced in the outer part of the photograph than near the centre of each photograph. If the overlap is more than 50%, these distortions and displacements can be overcome quite effectively while constructing the maps.

(3) In order to view the pairs of photographs stereoscopically, only the overlapped portion is useful. Hence the overlap should at least be 50% .

(4) Due to the overlap, each portion of the territory is photographed three to four times. Hence any picture distorted by excessive tilt or by cloud shadows etc. can be rejected without the necessity of a new photograph.

(5) If the flight lines are not maintained straight and parallel, the gaps between adjacent strips will be left. These gaps can be avoided by having sidelap.

(6) In the stereoscopic examination, objects can be viewed from more than one angle if sufficient overlap is provided.

Fig. 2.32 shows a photographic flight with an automatic aerial camera, the overlap of successive vertical photographs being 60%.

EFFECTIVE COVERAGE OF THE PHOTOGRAPH

The amount of overlap and sidelap to be used in flight planning depends upon the effective coverage of each photograph. The relation between the separation of flight lines and the separation between photographs must be arranged to give the greatest area to each stereopair.

The effective coverage of each photograph depends upon (*i*) size of format or focal plane opening, (*ii*) focal length and (*iii*) angular coverage of the lens. The effective angular coverage of the lens with the 12 in. (30.4 cm) focal length is represented by a cone the apex of which lies at the front nodal point and the apex angle of which is about 60°. In general, the effective coverage with a 12 in. lens will embrace more than 9″ × 9″ format size, and hence the entire photograph is usable (Fig. 2.33 *a*). The effective angular coverage with a 6″ (15.2 cm) wide angle lens is a cone of rays the apex of which is about 86°. A sizeable portion of the 9″ × 9″ format is not usable, and the useful

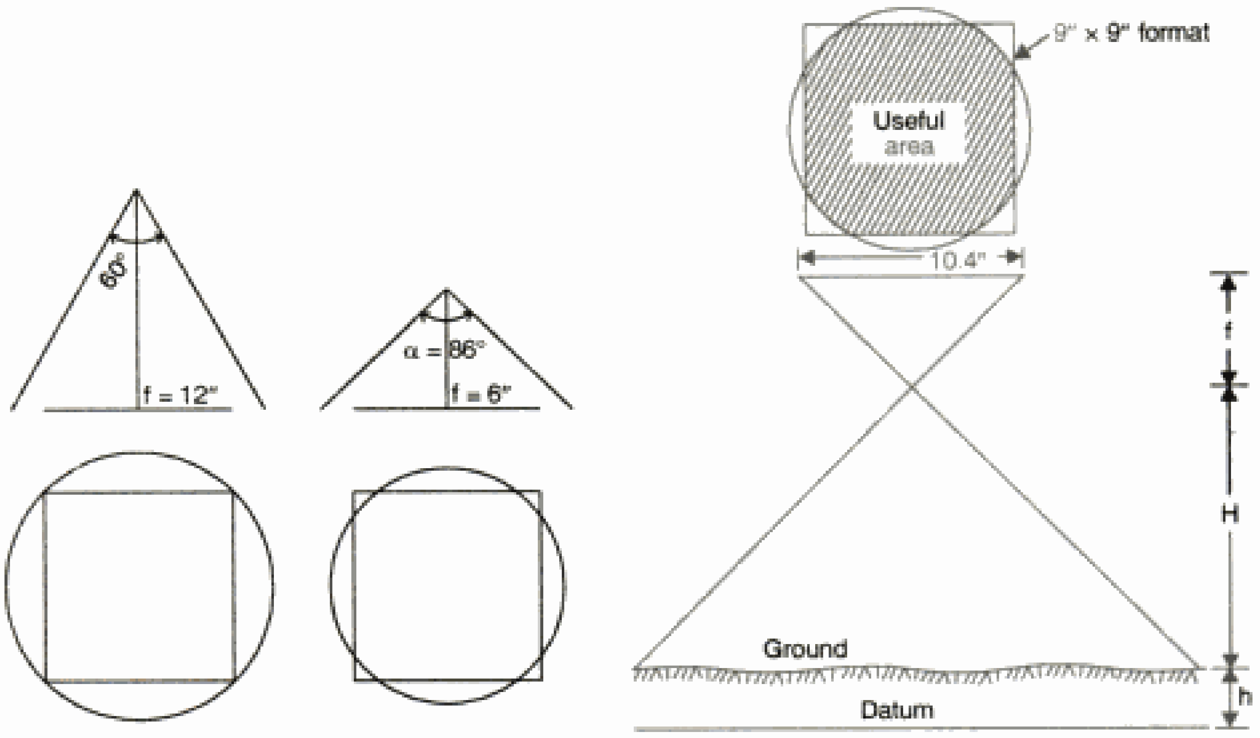


FIG. 2.33. ANGULAR COVERAGE.

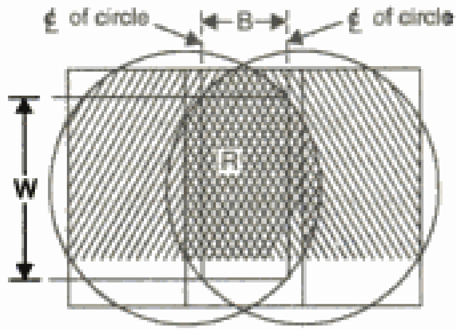
circle at the negative plane is equal to $2f \tan \frac{\alpha}{2} = 11.2''$ approximately. Due to errors in directing the camera and in following the flight lines, this should be reduced to at least 10.4 in., as shown in Fig. 2.33 (*c*).

The effective area of overlap between the two photographs is that bound by the overlapping circles representing the effective coverage of the photographs. Since the stereomodels must fit each other, the useful steroareas must be assumed to be rectangles having a width equal to the interval B between exposures. the two longer sides of this rectangle pass through the principal points of the photographs. The stereoareas is shown cross hatched, and the largest rectangle possible is drawn within this area.

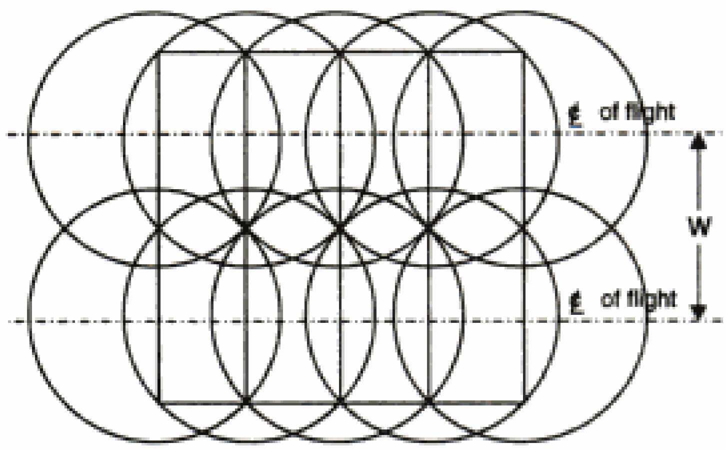
Let W = distance between the flight strips

A_s = stereo-areas

(*i.e.*, area of the rectangle)



(a) Effective overlap



(b) Spacing between flight lines

FIG. 2.34. FLIGHT LINES AND INTERVALS.

Then, $A_s = BW$ ...(1)

But $\frac{1}{2} W = \sqrt{R^2 - B^2}$ ...(2)

$\therefore$ $A_s = 2B\sqrt{R^2 - B^2}$ For A_s to be maximum $\frac{dA_s}{dB} = O = R^2 - 2B^2$

which gives $R = B\sqrt{2}$ ...(3)

Hence when $R = 5.2$ in., the value of $B = 3.67$ in.

Overlap in terms of inches on the photograph = 9.00 – 3.67 = 5.33″

$\therefore$ % overlap = 5.33/9 = 59.2% in the direction of flight.

Again, substituting the value of $R = B\sqrt{2}$ in (2), we get

$$\frac{1}{2} W = \sqrt{2B^2 - B^2} = B$$

$\therefore$ $W = 2B$ ...(4)(2.29)

Hence for maximum rectangular area, the rectangle must have the dimension in the direction of flight to be one-half the dimension normal to the direction of flight.

From Fig. 2.33 (*c*), $\frac{2R}{H} = \frac{10.4}{6}$ or $R = 0.867\ H$

Substituting this in (2), we get

$$B = \frac{1}{\sqrt{2}} R = \frac{1}{\sqrt{2}} \times 0.867\ H = 0.61\ H$$

Substituting in (4), $W = 1.22\ H$

where H is the height of lens above ground.

Hence the distance between the successive flights equals to 1.22 times the height of flight above the ground. This is the *maximum* allowable distance when the principal point of the photographs fall directly opposite one another on the two flight lines.

As found earlier, $W = 2B = 2 \times 3.67 = 7.34$ in.

$\therefore$ Side lap between flight lines, in terms of inches on the photograph = 9 – 7.34 = 1.66″

$\therefore$ Side lap = 1.66/9 = 18.4%

If $H = 3000$ metres, $B = 0.61 \times 3000 = 1830$ m and $W = 1.22 \times 3000 = 3660$ m

Hence, an exposure should be taken at every 1830 m and the separation of flight strips should be 3660 m.

The above analysis presumes ideal conditions, *i.e.* (*i*) level terrain, (*ii*) vertical photographs, (*iii*) no crab, (*iv*) no drift of the air craft and (*v*) constant flying height. The flight path centre lines are laid out parallel to the longest dimension of the area, on any existing map. Unless the area to be mapped is exactly covered by a certain number of flight paths spaced at the computed value of *W*, *W* should be reduced to introduce one more flight path to utilize the excess of photography for increasing the side overlap.

SELECTION OF FLYING ALTITUDE

The selection of height above ground depends upon the accuracy of the process to be used and the contour interval desired. Several inter-related factors which affect the selection of flying height, such as desired scale, relief displacement, and tilt, have already been discussed, Since vertical accuracy in a topographic map is the limiting factor in the photogrammetric process, the flying height is often related to the contour interval of the finished map. The process is rated by its *C*-factor which is the number by which the contour interval is multiplied to obtain the maximum height about the ground.

Thus, ***Flying height*** = **(*Contour interval*)** × **(*C factor*)**

C-factor for various processes vary from 500 to 1500, and depends upon the conditions surrounding the entire map-compilation operation.

NUMBER OF PHOTOGRAPHS NECESSARY TO COVER A GIVEN AREA

In the preliminary estimate, the number of photographs required is calculated by dividing the total area to be photographed by the net area covered by a single photograph.

Let A = total area to be photographed

l = length of the photograph in the direction of flight

w = width of the photograph normal to the direction of flight

s = scale of photograph $= \dfrac{H\text{ (m)}}{f\text{ (cm)}}$ (*i.e.* 1 cm = s metres)

L = net ground distance corresponding to l

W = net ground distance to corresponding to w

a = net ground area covered by each photograph = $L \times W$

P_l = percentage overlap between successive photographs in the direction of flight (expressed as a ratio)

P_w = side lap (expressed as a ratio).

Since each photograph has a longitudinal lap of P_l, the actual ground length (L) covered by each photograph is given by

$$L = (1 - P_l)\, sl \qquad \ldots(i)$$

Similarly, the actual ground width (W) covered by each photograph is given by

$$W = (1 - P_w)\, sw \qquad \ldots(ii)$$

Hence the ground area (a) covered by each photograph

$$a = L.W = (1 - P_l)\, sl\, (1 - P_w)\, s\, .\, w = l\, .\, w\, s^2\, (1 - P_l)\, (1 - P_w) \qquad [2.30\ (a)]$$

The number of the photographs (N) required is given by

$$N = A/a \qquad \ldots(2.30)$$

If, however, instead of the total area A, the rectangular dimensions (*i.e.*, length and width) of the ground are given, the number of the photographs required are computed by calculating the number of strips and the number of photographs required in each strip and multiplying the two.

Let L_1 = dimension of the area parallel to the direction of flight

L_2 = dimension of the area normal to the direction of flight

N_1 = number of photographs in each strip

N_2 = number of strips required

N = total number of photographs to cover the whole area.

Now net length covered by each photograph = $L = (1 - P_l)\, sl$

$\therefore$ Number of photographs in each strip is given by

$$N_1 = \frac{L_1}{L} + 1 = \frac{L_1}{(1 - P_l)\, sl} + 1 \qquad \text{...[2.31(a)]}$$

Similarly, net width covered by each photograph = $W = (1 - P_w)\, sw$

Hence the number of the strips required are given by

$$N_2 = \frac{L_2}{W} + 1 = \frac{L_2}{(1 - P_w)\, sw} + 1 \qquad \text{...[2.31(b)]}$$

Thus, the number of photographs required is

$$N = N_1 \times N_2 = \left\{ \frac{L_1}{(1 - P_l)\, sl} + 1 \right\} \times \left\{ \frac{L_2}{(1 - P_w) sw} + 1 \right\} \qquad \text{...(2.31)}$$

INTERVAL BETWEEN EXPOSURES

The time interval between the exposures can be calculated if the ground speed of the airplane and the ground distance (along the direction of flight between exposures are known.

Let V = ground speed of the airplane (km/hour)

L = ground distance covered by each photograph in the direction of flight $= (1 - P_l)\, sl$ in km

T = time interval between the exposures.

Then $$T = \frac{3600\, L}{V} \qquad \text{...(2.32)}$$

The exposures are regulated by measuring the time required for the image of a ground point to pass between two lines on a ground-glass plate of the view-finder. Usually, however, the interval is not calculated, but the camera is tripped automatically by syncronising the speed of a moving grid in the view-finder with the speed of the passage of images across a screen.

CRAB AND DRIFT

Crab. Crab is the term used to designate the angle formed between the flight line and the edges of the photograph in the direction of flight, as shown in Fig. 2.35 (*a*). At the instant of exposure, if the focal plane of the camera is not square with the direction of flight, the crab is caused in the photograph. The arrangements are always made to rotate the camera about the vertical axis of camera mount. Crabbing should be eliminated since it reduces effective coverage of the photograph.

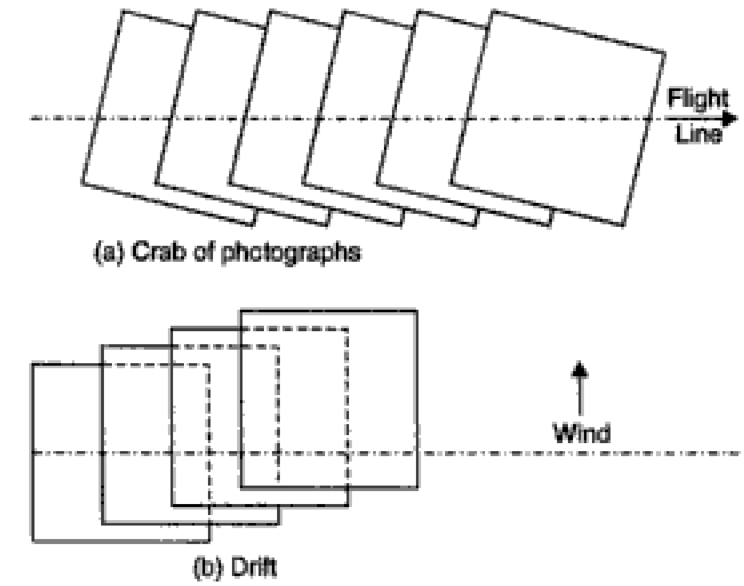


FIG. 2.35. CRAB AND DRIFT.

Drift. Drift is caused by the failure of the photograph to stay on the predetermined flight line. If the aircraft is set on its course by compass without allowing for wind velocity, it will drift from its course, and the photographs shall be as shown in Fig. 2.35 (*b*). If the drifting from the predetermined flight line is excessive, reflights will have to be made because of serious gapping between adjacent flight lines.

COMPUTATION OF FLIGHT PLAN

For the computation of the quantities for the flight plan, the following data is required:

1. Focal length of the camera lens
2. Altitude of the flight of the aircraft
3. Size of the area to be photographed
4. Size of the photograph
5. Longitudinal overlap
6. Lateral overlap
7. Position of the outer flight lines with respect to the boundary of the area
8. Scale of the flight map
9. Ground speed of aircraft.

Knowing the above, the amount of film required can be calculated before hand, the flight lines can be delineated on the map and the time interval between exposures can be determined if an intervalometer is to be used.

Example 2.13. *The scale of an aerial photograph is 1 cm=100 m. The photograph size is 20 cm × 20 cm. Determine the number of photographs required to cover an area of 100 sq. km if the longitudinal lap is 60% and the side lap is 30 %.*

Solution.

Here $l = 20$ cm ; $w = 20$ cm ; $P_l = 0.60$; $P_w = 0.30$

$$s = \frac{H\text{ (m)}}{f\text{ (cm)}} = 100 \quad (i.e.\ \ 1 \text{ cm} = 100 \text{ m})$$

∴ The actual ground length covered by each photograph is

$$L = (1 - P_l)\, sl = (1 - 0.6)100 \times 20 = 800 \text{ m} = 0.8 \text{ km}$$

Actual ground width covered by each photograph is

$$W = (1 - P_w)sw = (1 - 0.3)100 \times 20 = 1400 \text{ m} = 1.4 \text{ km}$$

∴ Net ground area covered by each photograph is

$$a = L \times W = 0.8 \times 1.4 = 1.12 \text{ sq. km.}$$

Hence number of photographs required is

$$N = \frac{A}{a} = \frac{100}{1.12} \doteq 90$$

Example 2.14. *The scale of an aerial photography is 1 cm = 100 m. The photograph size is 20 cm × 20 cm. Determine the number of photographs required to cover an area 10 km × 10 km, if the longitudinal lap is 60% and the side lap is 30%.*

Solution.

Here $L_1 = 10$ km ; $L_2 = 10$ km

$\therefore$ Number of photographs in each strip is given by

$$N_1 = \frac{L_1}{(1 - P_l)\, sl} + 1 = \frac{10{,}000}{(1 - 0.6) \times 100 \times 20} + 1 = 12.5 + 1 \approx 14$$

Number of flight lines required is given by

$$N_2 = \frac{L_2}{(1 - P_w)\, sw} + 1 = \frac{10{,}000}{(1 - 0.3)\; 100 \times 20} + 1 = 7.6 + 1 \approx 9$$

Hence number of photographs required will be

$$N = N_1 \times N_2 = 14 \times 9 = \mathbf{126}$$

The spacing of the flight lines would be $10/9 = 1.11$ km and not 1.4 as calculated theoretically in the previous example.

Example 2.15. *The scale of an aerial photograph is 1 cm = 100 m. The photograph size is 20 cm × 20 cm. Determine the number of photographs required to cover an area of 8 km × 12.5 km, if the longitudinal lap is 60% and the side lap is 30%.*

Solution.

$$N_1 = \frac{12500}{(1 - 0.6) \times 100 \times 20} + 1 = 17$$

$$N_2 = \frac{8000}{(1 - 0.3)\; 100 \times 20} + 1 = 7$$

$\therefore$ Number of photographs = 17 × 7 = **119.**

Example 2.16. *An area 30 km long in the north-south direction and 24 km in the east-west direction is to be photographed with a lens having 30 cm focal length for the purpose of constructing a mosaic. The photograph size is 20 cm × 20 cm. The average scale is to be 1 : 12,000 effective at an elevation of 400 m above datum. Overlap is to be atleast 60% and the side lap is to be at least 30%. An intervalometer will be used to control the interval between exposures. The ground speed of the aircraft will be maintained at 200 km per hour. The flight lines are to be laid out in a north-south direction on an existing map having a scale of 1 : 60,000. The two outer flight lines are to coincide with the east and west boundaries of the area. Determine the data for the flight plan.*

Solution.

(i) Flying height

We have,
$$\frac{H\,(\text{m})}{f\,(\text{m})} = \frac{H\,(\text{m})}{0.3\,(\text{m})} = \frac{12{,}000}{1}$$

$\therefore$ $H = 12{,}000 \times 0.3 = 3600$ m above ground

$\therefore$ Height above datum = 3600 + 400 = **4000 m.**

(ii) Theoretical ground spacing of flight lines

The ground width covered by each photograph, with 30% side lap is given by

$$W = (1 - P_w)\, sw.$$

where w = width of photograph = 20 cm ; $s = \text{scale} = \dfrac{H\,(\text{m})}{f\,(\text{m})} = \dfrac{3600\,(\text{m})}{30\,(\text{cm})} = \mathbf{120}$

i.e., 1 cm = 120 m ; $P_w = 0.30$

$$W = (1 - 0.3)\; 120 \times 20 = \mathbf{1680\ m}.$$

(iii) Number of flight lines required

The number of flight lines is given by Eq. 2.31 (*b*), *i.e.*

$$N_2 = \frac{L_2}{(1 - P_w)\, sw} + 1 = \frac{L_2}{W} + 1 = \frac{24{,}000 \text{ m}}{1680 \text{ m}} + 1 = 14.2 + 1 \approx \mathbf{16}.$$

(iv) Actual spacing of flight lines : Since the number of flight lines is to be an integral number, the actual flight lines = 16 and the number of flight strips or spacings = 15. Hence the actual spacing is given by

$$W = \frac{24{,}000}{15} = \mathbf{1600 \text{ m}},$$ against the theoretically calculated value of 1680 m.

(v) Spacing flight lines on flight map

Flight map is on a scale of 1 : 60,000 or 1 cm = 600 m. Hence the distance on the flight map corresponding to a ground distance $= \frac{1600}{600} = \mathbf{2.67 \text{ cm.}}$

(vi) Ground distance between exposures

The ground length covered by each photograph in the direction of flight with an overlap of 60% is given by $L = (1 - P_l)\, sl = (1 - 0.6) \times 120 \times 20 = \mathbf{960 \text{ m.}}$

(vii) Exposure interval

The time interval between exposures is usually the integral number of seconds.

$$V = 200 \text{ km per hour} = \frac{200 \times 1000}{60 \times 60} \text{ m/sec.} = 55.56 \text{ m/sec.}$$

$\therefore$ The required exposure interval is $\frac{960 \text{ (m)}}{55.5 \text{ (m/sec)}} = 17.3$ sec. $\triangleq$ **17 sec.**

(viii) Adjusted ground distance between exposures

Keeping the exposure interval as an integral number of seconds the adjusted ground distance covered by each photograph is given by

$$L = V \times T = 55.56 \text{ (m/sec)} \times 17.0 \text{ (sec)} = \mathbf{945 \text{ m.}}$$

(viii) Number of photographs per flight line

The number of photographs per flight line is given by

$$N_1 = \frac{L_1}{(1 - P_l)\, sl} + 1 = \frac{L_1}{L} + 1 = \frac{30{,}000}{945} + 1 = 31.6 + 1 \approx \mathbf{33}.$$

(ix) Total number of photographs required is

$$N = N_1 \times N_2 = 33 \times 16 = \mathbf{528}.$$

2.22. THE GROUND CONTROL FOR PHOTOGRAMMETRY

The ground control survey consists in locating the ground positions of points which can be identified on aerial photographs. The ground control is essential for establishing the position and orientation of each photograph in space relative to the ground. The extent of the ground control required is determined by (*a*) the scale of the map, (*b*) the navigational control and (*c*) the cartographical process by which the maps will be produced. The ground survey for establishing the control can be divided into two parts :

(*a*) Basic control (*b*) Photo control.

The *basic control* consists in establishing the basic net-work of triangulation stations, traverse stations, azimuth marks, bench marks etc.

The *photo control* consists in establishing the horizontal positions or elevations of the images of some of the identified points on the photographs, with respect to the basic control.

Each of these controls introduces *horizontal* control as well as *vertical* control and is known as basic horizontal control, basic vertical control, horizontal photo control, and vertical photo control respectively. The elevation of a vertical photo control point is determined by carrying a line of levels from a basic control bench mark to the point, and then carrying to the original bench mark or to a second bench mark for checking. The horizontal photo control points are located with respect to the basic control by third order or fourth-order triangulation, third order traversing, stadia traversing, trigonometric traversing, substance-bar traversing etc. etc., depending upon the accuracy required. Vertical photo control may be established by third-order leveling, fly levelling, transit-stadia levelling or precision barometric altimetry etc., depending upon the desired accuracy.

The photo control can be established by two methods :

(*i*) Post-marking method

(*ii*) Pre-marking method.

In the *post-marking* method, the photo control points are selected after the aerial photography. The distinct advantage of this method is in positive identification and favourable location of points.

In the pre-marking method, the photo-control points are selected on the ground first, and then included in the photograph. The marked points on the ground can be identified on the subsequent photograph. If the control traverse or triangulation station or bench marks are to be incorporated in the photo-control net work, they are marked with paint, flags etc. in such a way that identification on the photographs becomes easier. The selected control points should be sharp and clear in plan.

2.23. RADIAL LINE METHOD OF PLOTTING (ARUNDEL'S METHOD)

The radial line plot, often called *photo-triangulation* is the most accurate means of plotting a planimetric map from aerial photographs without the use of expensive instruments.

As discussed earlier, the displacement of image due to relief is radial from the principal point of vertical photograph. Hence the angles measured on the photograph at the principal point are true horizontal angles, independent of the height of the object and the flying height. The vertical photograph in space can thus be considered as an angle-measuring device similar to a transit or a plane table. Also, on tilted photographs, angles measured at the isocentre are true horizontal angles independent of tilt, provided that all objects photographed have the same elevation. On a near-vertical photograph, the isocentre is very near to the principal point. Hence the angles measured in the vicinity of these points are very nearly equal to the true horizontal angles, independent of tilt or elevation.

Thus, the radial line method is based on the following perspective properties of a vertical or near vertical photograph :

1. The displacements in a photograph due to ground relief and tilt are, within the limit of graphical measurement, radial from the principal point of the photograph.

(*ii*) Map position of principal points by two point resection

Let a and b be two photo-control points appearing in both the photographs overlapping each other. A and B are the plotted positions on the map sheet.

On photograph No. 1 (Fig. 2.37 *a*), k_1 is the principal point and k_2 is the principal point of photograph No. 2 transferred on to it. Rays k_1a, k_1b and k_1k_2 are drawn. A tracing paper is put on it and the rays are traced.

On photograph No. 2 (Fig. 2.37 *b*), k_2 is its principal point and k_1 is the principal point of photograph No. 1, transferred on to it. Rays k_2a, k_2b and k_2k_1 are drawn. A tracing paper is put on it and the rays are traced. Both the tracing papers are laid together on the map sheet in such a way that the rays k_1k_2 and k_2k_1 coincide when one is placed over the other. The rays k_1a and k_2a will intersect at a and the rays k_1b and k_2b will intersect at b. The two tracing papers are moved now in such a way that these intersections coincide with A and B respectively, and at the same time the lines k_1k_2 and k_2k_1 coincide each other. The points k_1 and k_2 are then transferred to the map sheet by pricking through. The line k_1k_2 on the map sheet constitutes the base line with k_1 and k_2 as the instrument stations.

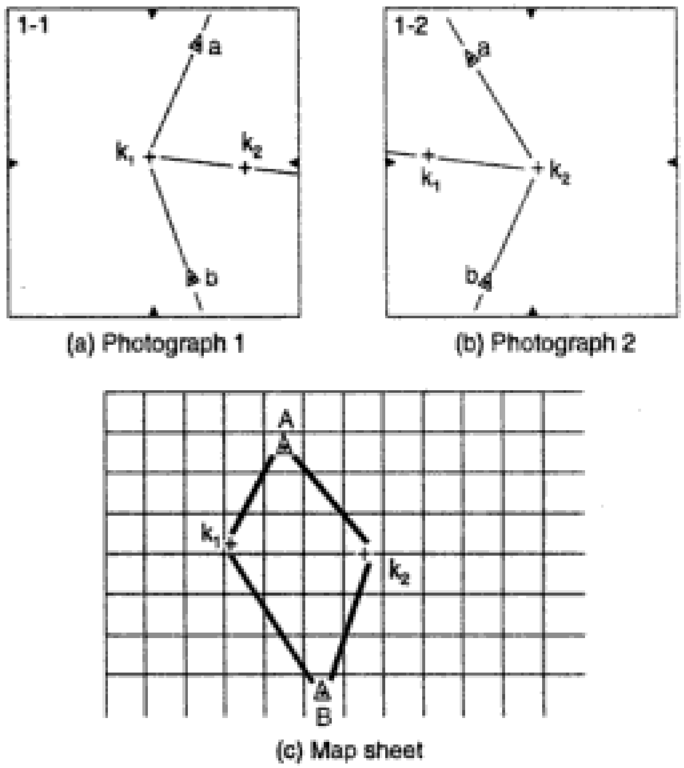


FIG. 2.37. LOCATION OF PRINCIPAL POINTS BY 2-POINT RESECTION.

(*b*) TO TRANSFER IMAGES FROM A PHOTOGRAPH TO A MAP

Since the angles measured on the photograph at the principal point are true horizontal angles, the position of a point can be located by intersecting the rays to that point from two principal points. In Fig. 2.38, let p and q be the images of two points on two

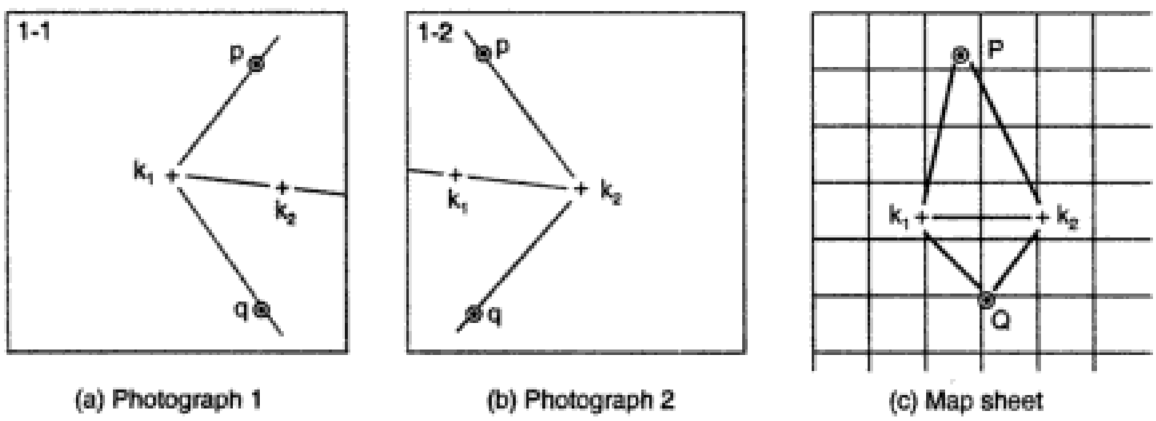


FIG. 2.38. LOCATION OF POINTS BY INTERSECTION.

photographs overlapping each other. k_1 and k_2 are the two principal points. As discussed in the previous paragraph, the principal points k_1 and k_2 can be transferred to the map by the known positions of the photo control points. On each photograph, the rays can be drawn to the points p and q and can be traced on two sheets of tracing paper. Both the sheets are then placed on the map sheet and properly oriented till the map positions of k_1 and k_2 coincide respectively with the traced positions. The intersection of rays $k_1 p$ and $k_2 p$ gives the position of P, and that of $k_1 q$ and $k_2 q$ give the position of Q.

The actual plotting of planimetric maps by radial line method is done in the following steps :

(1) Transfer of principal points and plotting the line of flight

(2) Marking the photographs

(3) Plotting the map control

(4) Transferring photographic detail.

(1) Transfer of Principal Points and Plotting the Line of Flight

The principal point of each photograph can be marked on it by means of two intersecting lines drawn between the opposite fiducial marks which are there on the middles of all the four edges of the photograph. The point of intersection of these two collimating lines is the principal point of the photograph. Each photograph is given its serial number and the number of the strip in which it was taken. For example, photograph Nos. 7, 8 and 9 of strip No. 2 will be marked as 2-7, 2-8 and 2-9 respectively. The principal points of these photographs may be marked as k_7, k_8 and k_9 respectively.

Since the longitudinal lap is generally 60% or more, the three photographs will have common overlap of atleast 20% as shown shaded in Fig. 2.39.

If all the photographs of a particular strip are arranged in properly overlapped positions, it will be observed that on the first photograph, its principal point will appear at its middle while the transferred principal point of photograph No. 2 will appear at right hand edge. On photograph No. 2 and all other photographs except the last, three principal points will appear — one of its own at the middle and two at its two edges, as shown in Fig. 2.39 where photograph No. 2 has the principal points k_1, k_2 and k_3. The principal points of each photograph can be transferred on to the adjacent photographs, one to its right and one to the left of it, by fusion under a stereoscope (see § 2.24 for principles of stereoscopic vision and fusion). For this, two adjacent photographs are put under the stereoscope and are oriented correctly with respect to each other till the line of flight of the pictures is parallel to the line joining the centres of two lenses of the stereoscope. The distance between the photographs is adjusted until fusion occurs and the relief of the landscape is clearly visible. In this position, the principal point of one photograph will be seen directly and its image will be projected upon the other photograph. Then with a needle the position of the point can be transferred to the adjacent

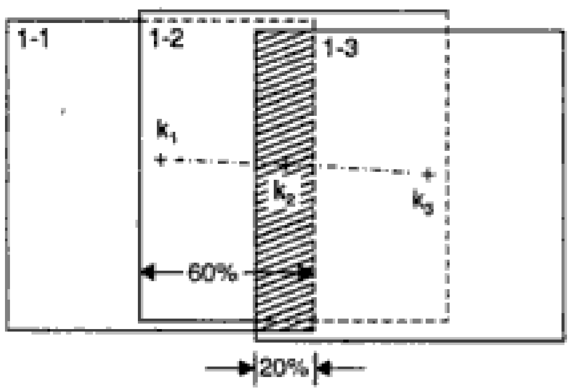


FIG. 2.39

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Each of the succeeding photograph is marked in a similar manner until other ground control points are reached. The end photograph of the strip must include at least one control point.

(3) Plotting the Map Control

The data of the separate photographs are combined into a map showing correct relative locations of the selected points and the control points with the help of a sheet of transparent film base (cellulose acetate) or a good quality tracing paper which exhibits very small changes in its dimensions with changing atmospheric conditions.

The plotted positions of ground control points P_l, P_c and P_r chosen on photographs 1 and 2 are known on the base map. The tracing is stretched on the base map and these control points are transferred by pricking through with a needle. Photograph No. 1 is then slid under the tracing and is oriented in such a way that the radial lines through points P_l, P_c and P_r of the photograph pass through the plotted control points P_l, P_c, P_r on the tracing. In this position, all the rays and points are traced. The principal point k_1 and the transferred principal point k_2 are also traced.

Photograph No. 2 is then slid under the tracing and is oriented in such a way that rays previously drawn on the tracing pass through the corresponding points on the photograph, keeping the traced flight line $k_1\,k_2$ coinciding with flight line $k_2\,k_1$ on the photograph. Thus, photograph No. 2 is correctly oriented. In this position, all rays and points are traced. In this manner, each of the successive photograph is slid under the tracing, oriented and the rays traced till another ground control point is reached.

Fig. 2.41 shows the plotting of the map control on the tracing. It will be observed that at each of the pass points, there will appear three intersecting rays. The position of each of the points is located on the tracing at the point of intersection of the three rays. This point of intersection may not appear to coincide with the corresponding point on the photograph, because of the displacements due to ground relief. Sometimes, due to errors of plotting, the three rays may not intersect at a point, but may form a small triangle of error. In that case, the centre of the triangle is taken as the position of the point.

The plotting work is thus continued till the next ground control point is reached. In a perfect map control work, the image of the control point, as located by the intersecting rays, will be the plotted position of the point traced from the base map.

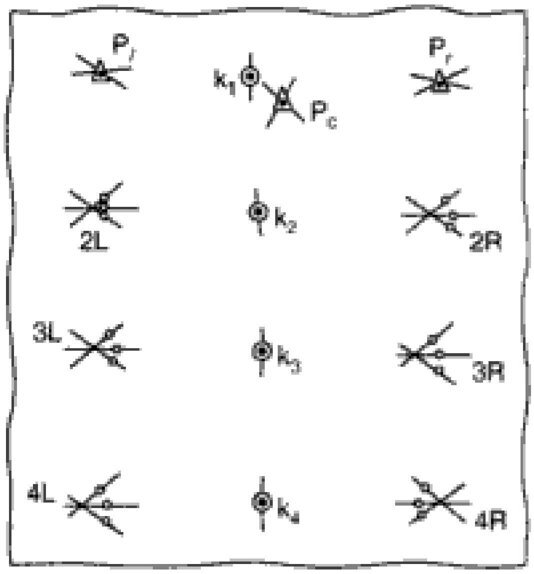

FIG. 2.41. COMPILATION OF MAP CONTROL.

In case, the plotted position of the ground control point does not coincide with its traced position, as usually is the case, the lines of flight, or the positions of the principal points are adjusted as shown in Fig. 2.42. P is the position of the ground control point as located by the intersection of rays, and P' is the corresponding position as traced from the base map. Thus, the total error is $P'P$ in magnitude as well as direction. Each of the principal points $k_7, k_6, k_5 \ldots\ldots k_2$ is shifted to positions $k_7', k_6', k_5', \ldots k_2'$, in a direction parallel to $P P'$ by a distance proportional to the distance of that point from the initial fixed point k_1. The positions of other pass points are also adjusted accordingly.

It should be noted that when the tracing is begun, the scale is not at all known. The unknown scale is established by the distance between the two principal points k_1 and k_2. The scale of the data assembled on the film or tracing can then be determined by measuring the distance between the first control point and the last control point on it.

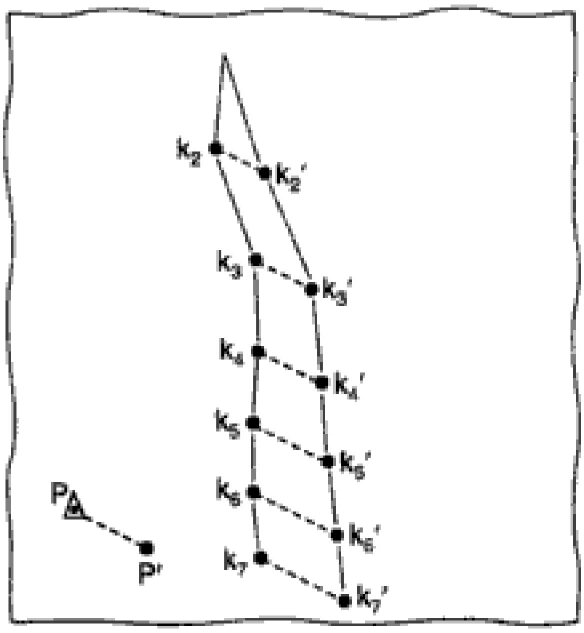

FIG. 2.42. ADJUSTMENT OF THE LINES OF FLIGHT.

(4) Transferring Photographic Details

To transfer the photographic details, each photograph is slid under the tracing and oriented to the map control. The details are then traced on the tracing. Next photograph is then slid and oriented and corresponding rays are drawn to the points. The intersection of the two sets of rays obtained from the two photographs will give the plotted positions of the points, as illustrated in Fig. 2.38. The details can be transferred to the base map either by photograph or by tracing over a carbon sheet.

Plain Templets Method of Control

The templet method is variation of the *radial line* method, and is used with greater convenience when a considerable area is to be plotted by radial-line method.

Templets are of two kinds :

(*i*) Plain templets (*ii*) Slotted templets.

The plain templets are actually the substitutes of the tracing acetate paper of film. The plain templets are transparent sheets, preferably of acetate, and of size slightly bigger than each photograph. The ground control points and pass points or minor control points are selected and marked on each photograph as explained earlier. The principal point of each photograph is also transferred to the adjacent photographs. Separate templets are used for each photograph. The templet is placed on the photograph and the position of principal points are pricked through on to the templet. Radial lines are now drawn on the templet from the principal point over each control point already marked on the photograph. Thus, templets are marked for each and every photograph.

The first templet is then placed on the base map having the plotted positions of the ground control, and is oriented such that the rays of the templet pass through the plotted control points. After this, the second and third templets are also adjusted and oriented properly by the same method as used for the radial line method. All the templets oriented in this way are fastened together by Scotch tape, till another set of ground control points is reached. If the ground control points on the templets do not coincide with the corresponding map positions due to various sources of errors explained already, adjustments are made by stretching or twisting the whole assembly of templets as whole until the map positions of ground control points coincide with the corresponding positions on the templets. The system of control points established on the combined assembly of templets is then transferred to the base map by pricking through the acetate sheets with needle.

Slotted Templet Method of Control

The slotted templet is an improvement over the plain templet. In this method, all rays from the principal points on minor and ground control points are replaced by slots cut in cardboard or acetate templets (Fig. 2.43).

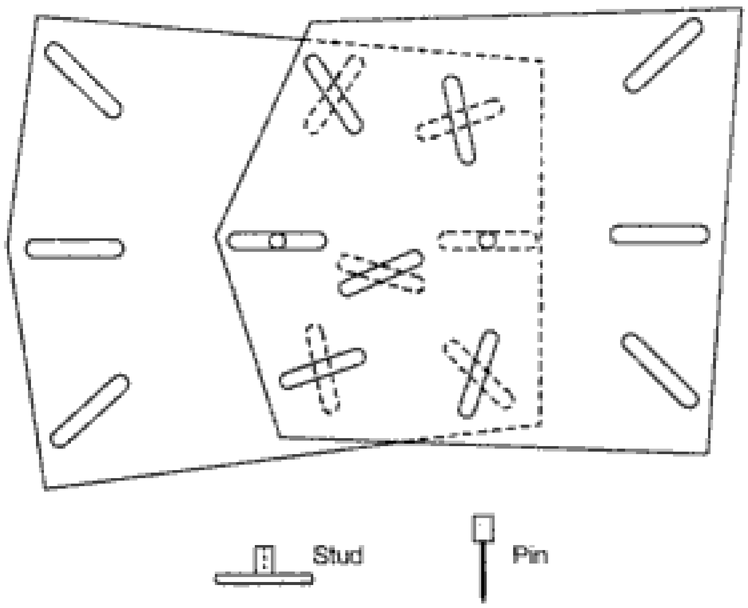

FIG. 2.43. SLOTTED TEMPLET METHOD.

The templets may either be of acetate sheets or sheets of thin, firm cardboard of about the same size of the photographs. As in the radial line method, the ground and minor control points are selected on each photograph and marked with needle points. These points are then transferred to the templet by pricking through the photograph. The templet is thus marked with the principal point of the photograph, the two transferred principal points of the adjacent photographs, and the ground and photo-control points. The templet is then taken to the *slot-cutting machine*. A small hole is then punched at the principal point. Slots representing rays radiating from the principal point to the marked points are cut into the templets by the machine. The slot cutting blade of the machine is designed so that it can be centred accurately over the photograph position of any point marked on the templet. The width of the slots cut are of the same size as the diameter of studs (Fig. 2.43) which are inserted through the slots. One such templet is prepared for each photograph. The metal studs to be inserted in the slots are drilled centrally with a fine hole to accommodate a steel pin.

A specially prepared floor or dais is used for assembly of the slotted templets. The base map containing the accurately plotted ground control points is stretched on the floor. The first templet is put on the base map and is oriented with respect to the ground control points. Those photo-point studs which correspond to known ground control points are then fixed in position on the map by pins (Fig. 2.43) driven through the central holes of the studs. The movable studs (*i.e.* studs having no central pins) are inserted through those slots representing the rays to each selected photo point. The second, third and other templets are then put on the map and oriented one after the other by the method explained earlier, till another set of ground control points are reached. The assembled templets are adjusted until two or more slots belonging to each ground control point will fit over the fixed stud belonging to that point. When this is achieved, all the free studs representing the photo-control points and principal points will be automatically adjusted. The positions thus found for the movable or free studs are the most probable positions for the corresponding photo points, and their positions are then transferred to the map by inserting sharp metal pins through the central holes of the studs.

If the effect of tilt is more in a photograph, its templet will not fit the assembly. Before it can be used, it will need to be rephotographed and the tilt effect removed. The important advantage of the slotted templet method is that any fault due to wrong position of the slot or other sources is mechanically detected when the templets do not fit.

STEREOSCOPY AND PARALLAX

2.24. STEREOSCOPIC VISION

The *depth perception* is the mental process of determining relative distance of objects from the observer from the impressions received through the eyes. Due to binocular vision, the observer is able to perceive the spatial relations, *i.e.*, the three dimensions of his field of view.

The impression of depth is caused mainly due to three reasons : (1) relative apparent size of near and far objects, (2) effects of light and shade, and (3) viewing of an object simultaneously by two eyes which are separated in space. Out of these, the third one is the most important. Each eye views an object from a slightly different position, and by a physiological process the two separate images combine together in the brain enabling us to see in three dimensions.

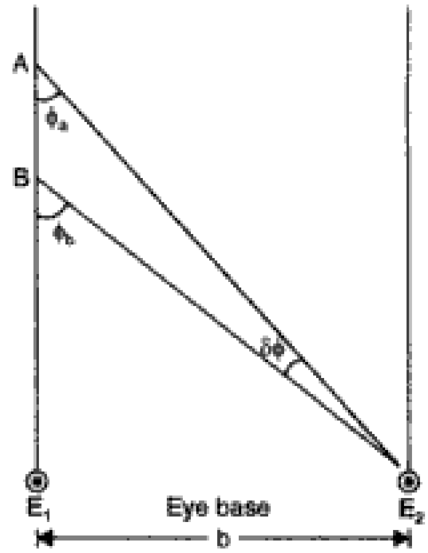

FIG. 2.44. ANGLES OF PARALLAX.

Angle of Parallax (or Parallactic Angle)

In normal binocular vision, the apparent movement of a point viewed first with one eye and then with the other is known as *parallax*. Since an object is viewed simultaneously by two eyes, the two rays of vision converge at an angle upon the object viewed. The angle of parallax or the parallactic angle is the angle of convergence of the two rays of vision. In Fig. 2.44, A and B are two objects in the field of view, and are being viewed by the two eyes represented in space by the positions, E_1 and E_2 . $E_1 E_2 = b$ is known as the *eye base*. The angle E_1AE_2 is the angle of parallax (φ_a) of object A, and the angle E_1BE_2 is the angle of parallax (φ_b) of object B. The object B, for which the parallactic angle φ_b is greater, will be judged to be nearer the observer than the object A for which the parallactic angle φ_a is smaller. The measure of the distance BA is evidently provided by the difference in the parallactic angles of A and B. This difference, *i.e.* $\varphi_b - \varphi_a$ ($= \delta\varphi$) is termed as the *differential parallax*.

Stereoscopic Fusion

The principles of stereoscopic vision can readily be applied to photogrammetry. An aerial camera takes a series of exposures at regular intervals of time. If a pair of photographs is taken of an object from two slightly different positions of the camera and then viewed by an apparatus which ensures that the left eye sees only the left-hand picture and the right eye is directed to the right hand picture, the two separate images of the object will fuse together in the brain to provide the observer with a spatial impression. This is known as a *stereoscopic fusion*. The pair of two such photographs is known as *stereopair*. Two devices are used for viewing stereopairs : the *stereoscope* and the *anaglyph*. To illustrate the phenomenon of stereoscopic fusion, let us conduct an experiment (see Figs. 2.45 and 2.46) described below.

Fig. 2.45 shows two pairs of dots near the top edge of a sheet of paper. The

FIG. 2.45

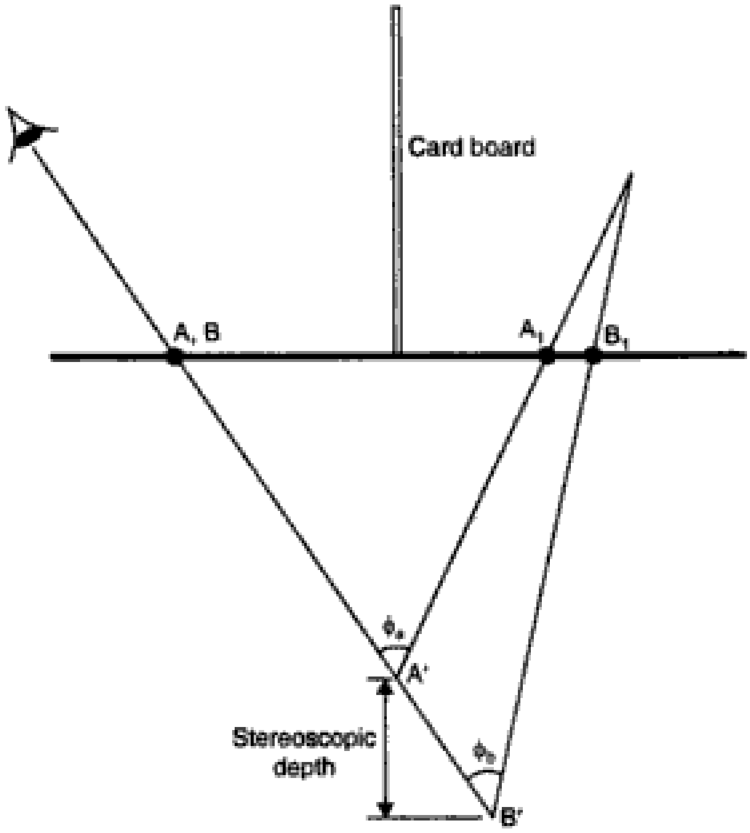

FIG. 2.46. STEREOSCOPIC FUSION.

distance between dots A and A_1 is less than the dots B and B_1. Place a piece of cardboard between AB and $A_1 B_1$, in the plane perpendicular to the sheet so that the left dots A, B are seen with the left eye and the right dots A_1 , B_1 are seen with the right eye. By staring hard, it will be observed that A and A_1 fuse together to form a single dot which appears closer than the fused image of B and B_1 (Fig. 2.46).

The apparent difference in level is known as *stereoscopic depth* and depends on the spacing between the dots. The spacing between the dots is called the *parallax difference*.

Clues to Depth Perception

As stated earlier, the depth perception is the mental process of determining relative distance of objects from the observer from the impression received through the eyes. Numerous impressions are received that serve as *clues to depth*, and the following clues are important from photogrammetry point of view :

(1) Head parallax (2) Accommodation
(3) Convergence (4) Rentinal disparity.

(1) Head Parallax : Head parallax is the apparent relative movement of object at different distances from the observer when the observer moves.

(2) *Accommodation* : Accommodation is the process by which the lens of the eye can be flattened (to focus nearby points on the retina) or made more convex (to focus nearby points on the retina) in accordance with requirements placed on it. Due to the accommodation of lens, the brain gets an approximate clue to distance (or depth). The ability of the eye to accommodate this way become less for weak eyes ; it begins to reduce in the forties and is usually completely lost in the sixties.

(3) *Convergence* : In order to see an object clearly (or sharply) it is necessary that the image of the desired object is placed on the most sensitive part of each retina (the fovea). This causes the two eyes to turn or converge. The convergence of the eyes is therefore a clue to distance since the eyes converge more for nearby points and less for farther points and brain is aware of their relative positions. In Fig. 2.46, the axes of the eyes are directed to points *A'* and *B'* *behind* the plane of paper whereas the eyes must be focused for the plane of paper if the dots are to remain sharply defined. Thus, the *convergence* of the eyes (to view *A'* and *B'*) is not in sympathy with their *accommodation* (to view *A* and *B* sharply).

(4) *Retinal Disparity* : The picture of an object received by the two eyes are slightly different since the two eyes are at different positions. The difference between the images on the retinas is called *retinal disparity*. Since it is a function of the relative distance of objects viewed, it provides a very strong distance clue. *In photogrammetry, this is the only clue which is actually used*. The range and intensity of stereoscopic perception can be increased by two ways :

(*i*) by apparently increasing the base between view points.

(*ii*) by magnifying the field of view by use of lenses.

Stereoscope

Stereoscope is an instrument used of viewing stereopairs. Stereoscopes are designed for two purposes :

(1) To assist in presenting to the eyes the images of a pair of photographs so that the relationship between convergence and accommodation is the same as would be in natural vision.

(2) To magnify the perception of depth.

There are two basic types of stereoscopes for stereoscopic viewing of photographs:

(1) Mirror stereoscope (2) Lens stereoscope.

(1) The Mirror Stereoscope : The mirror stereoscope, shown diagrammatically in Fig. 2.47 (*b*), consists of a pair of small eye-piece mirrors *m* and *m'*, and a pair of larger wing mirrors, *M* and *M'*, each of which is oriented at 45° with the plane of the photographs.

Fig. 2.47 (*a*) shows a nail mounted on a block of timber, and is being photographed by two camera positions. The camera lens is placed first in the position of left eye and then in the position of right eye, and separate photographs are taken in each position. It will be noted that the head of the nail is to the left in the left film and to the right in the right film. *ab* and *a'b'* are the images of the nail *AB* in the two films.

Contact prints from these negatives are placed in the mirror stereoscope as shown in Fig. 2.47 (*b*), where only images of the nail are drawn. The four mirrors transfer

the image of the higher object B has moved a distance of 7.25 cm. Then the parallax of the lower point is 6.05 cm and that of the higher point is 7.25 cm.

In the left photograph, a and b are the images of the two points. k' is the *transferred* principal point of the right photograph. Both the images a and b are to the right of the y-axis of the left photograph. In the right photograph, a' and b' are the images of the same points, both the images being to the left of the y-axis. Thus the images (a, b) of the points have moved to (a', b') between the two exposures. The movement aa' (shown on the left photographs) is the parallax of A, and bb' is the true parallax of B. The parallax of the higher point is more than the parallax of the lower point. Thus, each image in a changing terrain elevation has a slightly different parallax from that of a neighboring image. *This point-to-point difference in parallax exhibited between points on a stereopair makes possible the viewing of the photographs stereoscopically to gain an impression of a continuous three dimensional image of a terrain.*

The following are the ideal conditions for obtaining aerial stereoscopic views of the ground surface :

(1) two photographs are taken with sufficient overlap.

(2) the elevation of the camera positions remains the same for the two exposures.

(3) the camera axis is vertical so that the picture planes lie in the same horizontal plane.

Algebraic Definition of Parallax : As defined earlier the displacement of the image of a point on two successive exposures is called the parallax of the point. On a pair of overlapping photographs, the parallax is thus equal to the x-coordinate of the point measured on the left-hand photograph (or previous photograph) minus the x-coordinate of the point measured on the right-hand photograph (or next photograph). Thus

$$p = x - x' \qquad \text{...(2.23)}$$

Thus, x-axis passes through the principal point and is *parallel* to the flight line, while the y-axis passes through the principal point and is perpendicular to the line of flight. In general, however, the flight-line x-axis is usually very close to the collimation mark x-axis, because of the effort made to eliminate drift and crab at the time of photography.

Thus, in Fig. 2.51, the parallax of points A and B are given by

$$p_a = x_a - x'_a \quad \text{and} \quad p_b = x_b - x'_b$$

In substituting the numerical values of x and x', their proper algebraic sign must be taken into consideration. Thus, in Fig. 2.51, if $x_a = 2.55$ cm, $x'_a = -3.50$ cm, $x_b = -4.05$ cm and $x'_b = -3.20$ cm, we have

$$p_a = +2.55 - (-3.50) = 6.05 \text{ cm}$$

$$p_b = +4.05 - (-3.20) = 7.25 \text{ cm.}$$

2.26. PARALLAX EQUATIONS FOR DETERMINING ELEVATION AND GROUND CO- ORDINATES OF A POINT

Let A be a point whose ground co-ordinates and elevation are to be found by parallax measurement.

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In the left photograph (Fig. 2.52), draw Om'' parallel to $O'm'$ of right photograph. Then, in the triangles $Om''m$ and OMO' :

OO' is parallel to $m''m$

Om coincides with, and is parallel to OM

Om'' is parallel to $o'm'$

Hence they are similar, and their corresponding altitudes are f and $(H-h)$ respectively.

Thus, $\frac{f}{H-h} = \frac{mm''}{OO'}$ But $mm'' = km + km'' = x - x' = p$ and $OO' = B$ = air base

$$\therefore \quad \frac{f}{H-h} = \frac{p}{B} \qquad ...(5)$$

or

$$H - h = \frac{Bf}{p} \qquad ...(2.35)$$

This is the parallax equation for the elevation of the point.

Again from equations (1) and (2),

$$X = \frac{H-h}{f}x \quad \text{and} \quad Y = \frac{H-h}{f}y$$

But $\frac{H-h}{f} = \frac{B}{P}$, from (5)

Hence

$$X = \frac{B}{p}x \quad \text{and} \quad Y = \frac{B}{p}y \qquad ...(2.36)$$

This is the parallax equation for the ground co-ordinates of the point.

Difference in Elevation by Stereoscopic Parallaxes

In Fig. 2.53, $A_1 A_2$ is a flagpole being photographed from two camera positions O and O'. The top A_2 of the flagpole has an elevation of h_2 above the datum, and the bottom A_1 has an elevation of h_1 above the datum. H is the camera height for both the exposures.

In the left photographs, a_1 and a_2 are the two images of A_1 and A_2, and their x-co-ordinates are x_1 and x_2 respectively.

Similarly, in the right photograph, a'_1 and a'_2 are the images of A_1 and A_2 respectively, and their x-co-ordinates are x'_1 and x'_2 respectively.

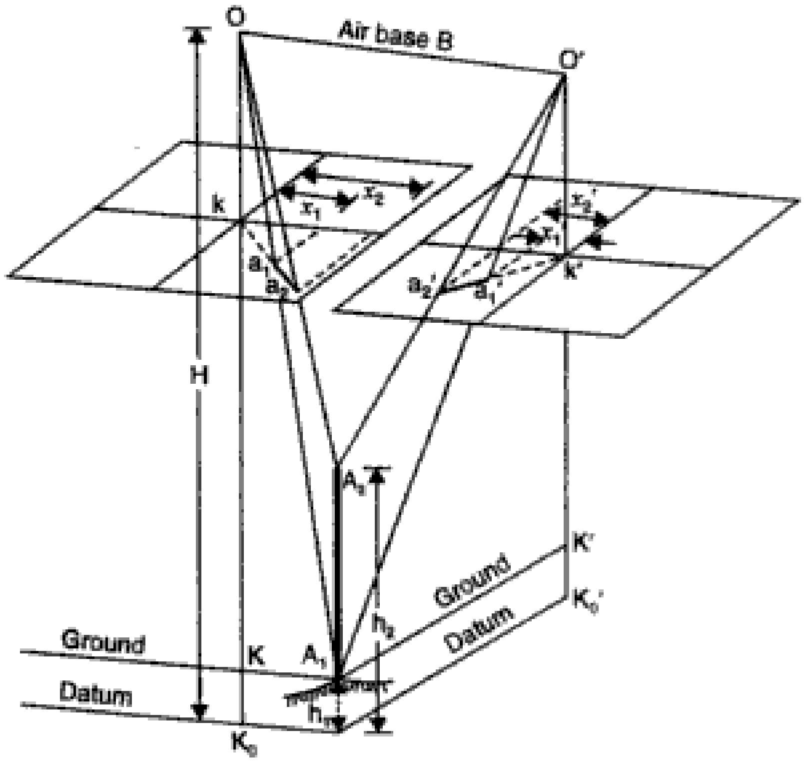


FIG. 2.53. DIFFERENCE IN ELEVATION BY STEREOSCOPIC PARALLAXES.

Evidently, the parallax p_1 for the bottom of the flagstaff is given by

$$p_1 = x_1 - x'_1 \qquad \ldots(1)$$

Similarly, the parallax p_2 for the top of the flagstaff is given by

$$p_2 = x_2 - x'_2 \qquad \ldots(2)$$

Hence the difference in parallax (Δp) of top and bottom points is given by

$$\Delta p = p_2 - p_1 = (x_2 - x'_2) - (x_1 - x'_1) \qquad \ldots(3)$$

From equation 2.35, the elevation of any point is given by

$$h = H - \frac{Bf}{p}$$

Hence, for the top and bottom of flagstaff, we get

$$h_1 = H - \frac{Bf}{p_1} \quad \text{and} \quad h_2 = H - \frac{Bf}{p_2}$$

$\therefore$ Difference in elevation (Δh) is given by,

$$\Delta h = h_2 - h_1 = \left(H - \frac{Bf}{p_2} \right) - \left(H - \frac{Bf}{p_1} \right) = \frac{Bf}{p_1} - \frac{Bf}{p_2}$$

or

$$\Delta h = \left(\frac{p_2 - p_1}{p_1\, p_2} \right) Bf \qquad \ldots(2.37)$$

or

$$\Delta h = \frac{\Delta p}{p_1\, p_2} Bf \qquad \ldots[2.37(a)]$$

Now $\Delta p = p_2 - p_1$ or $p_2 = p_1 + \Delta p$

Hence, we have

$$\Delta h = \frac{\Delta p}{p_1 (p_1 + \Delta p)} \,.\, Bf \qquad \ldots(2.38)$$

Mean Principal Base **(b_m)** : The distance between the principal point of a photograph and the position of transferred principal point of its next photograph obtained under fusion through stereoscope is called *principal base*. Thus, in Fig. 2.51,

$kk' = b$ = principal base of left photograph

and $k'k = b'$ = principal base of right photograph.

It should be noted that b and b' will not be equal since the elevation of ground positions of the principal points (K and K') are not the same.

The *mean principal base* is the mean value of the principal bases of the photographs.

Thus,

$$b_m = \frac{b + b'}{2}$$

If the ground principal points (K and K') have the same elevation, then under ideal conditions, $b_m = b$.

Now, in Fig. 2.53, let the datum pass through the bottom A_1 of the flagstaff (*i.e.* $h_1 = 0$). Assuming the ground to be now the datum plane, the ground principal points K and K' will be at the same elevation, and the parallax of the principal points (*i.e.*, the principal base) will be equal to b. If H is the height of camera above the datum (*i.e.* above A_1 now), the general relationship between b and B is given by

$$\frac{B}{b} = \frac{H}{f} \qquad \text{or} \qquad B = \frac{Hb}{f} = s \times b \qquad \ldots(2.39)$$

where s is the scale of the photograph at datum elevation. Substituting this value of air base in equation 2.38, we get

$$\Delta h = \frac{\Delta p}{p_1 (p_1 + \Delta p)} \cdot Hb$$

Since K, K' and A_1 are all at the same elevation, their parallaxes are the same.

Hence $\qquad p_1$ = parallax of principal points $= b$

Hence, we get the parallax equation

$$\Delta h = \frac{H \Delta p}{b + \Delta p} = \frac{H \Delta p}{p_1 + \Delta p} \qquad \ldots(2.40)$$

While using equations 2.40, the following assumptions must always be kept in mind:

(1) The vertical control point (*i.e.*, point A_1) and the two ground principal points lie at the same elevation.

(2) The flying height (H) is measured above the elevation of the control point and not sea level (unless the control point happens to lie at sea level).

In practical applications, the mean principal base (b_m) is used in place of b, and flying height above the average terrain is taken as the value of H.

Alternative form of Parallax Equation for Δh

We have, $\qquad p_1 = \frac{f B}{H - h_1}$ and $p_2 = \frac{f B}{H - h_2}$

$$\therefore \qquad \Delta p = p_2 - p_1 = f B \left(\frac{1}{H - h_2} - \frac{1}{H - h_1} \right) = f B \frac{h_2 - h_1}{(H - h_1)(H - h_2)}$$

But $\qquad \Delta h = h_2 - h_1$ and $h_2 = \Delta h + h_1$

$$\therefore \qquad \Delta p = f B \frac{\Delta h}{(H - h_1)(H - \Delta h - h_1)}$$

or $\qquad \Delta p (H - h_1)^2 - \Delta p (H - h_1) \Delta h = f B \Delta h$

or $\qquad \Delta h [(H - h_1) \Delta p + fB] = (H - h_1)^2 \Delta p$

or $\qquad \Delta h = \frac{(H - h_1)^2 \Delta p}{(H - h_1) \Delta p + f B}$

Dropping the suffix of h, we get

$$\Delta h = \frac{(H - h)^2 \Delta p}{(H - h) \Delta p + f B} \qquad \ldots[2.41(a)]$$

where h is the elevation of lower point above datum.

Putting $\qquad f B = Hb$, we get $\qquad \Delta h = \frac{(H - h)^2 \Delta p}{(H - h) \Delta p + bH} \qquad \ldots(2.41)$

It should be noted that the above equation is in its most general form. Eq. 2.40 is the special form of this, and can be obtained by putting $h = 0$ (*i.e.*, lower point at datum) in Eq. 2.41). Thus

$$\Delta h = \frac{(H-0)^2 \Delta p}{(H-0)\Delta p + bH} = \frac{H^2 \Delta p}{H \Delta p + bH} = \frac{H \Delta p}{\Delta p + b} \quad ...(2.42)$$

2.27. EFFECTS OF CHANGES IN ELEVATION *h* AND PARALLAX *p*

The difference of elevation between two points is given by equation 2.37, *i.e.*,

$$\Delta h = \frac{p_2 - p_1}{p_1 p_2} Bf.$$

In order to find Δh, therefore, the parallaxes p_1 and p_2 of both the points are to be measured very carefully. If, however many computations are required (as in mapping), the above method of finding Δh is quite inconvenient. In such circumstance, the following two methods are in common use :

(1) the *unit-change* method. (2) the *parallax-table* method.

In both the methods, use of precise instruments (such as *parallax bar, stereocomparator* or contour finder) are used to measure the *difference in parallax* (Δp) directly by means of micrometer scales and the fusion of two dots in the stereoscopic view, into a so-called *floating mark* (see § 2.28).

(1) The Unit-Change Method

From equation 2.35, we have

$$H - h = \frac{Bf}{p} \quad \text{or} \quad h = H - \frac{Bf}{p} \quad ...(1)$$

By differentiation, we get $dh = \frac{Bf}{p^2} dp$

Substituting $p = \frac{Bf}{H-h}$ (from Eq. 2.35), we get $dh = \frac{(H-h)^2}{Bf} . dp$ ...(2.42)

Since $\frac{B}{b} = \frac{H}{f}$, we have also $dh = \frac{(H-h)^2}{bH} . dp$ ...[2.42 (*a*)]

The above equations express the rate of change of p for the infinitesimal change dh in the value of h.

The instruments used for measuring parallaxes are divided in millimeter, and hence the unit of change in parallax is taken as one millimeter. Let the rate of change dh be assumed to be constant for 1 mm change in dp.

Then $dp = 1 \text{ mm} = \Delta p_0$ (say).

If the value of Δh_0 is computed for a corresponding value of $\Delta p_0 = 1$ mm, the total value of Δh (*i.e.*, difference in elevation) is found by multiplying Δh_0 by the number of millimeters in Δp.

Equation 2.42 can then be written as

$$\Delta h_0 = \frac{(h-h)^2 \Delta p_0}{Bf} \quad ...[2.43\ (a)]$$

or $$\Delta h_0 = \frac{(H-h)^2}{Bf} = \frac{(H-h)^2}{bH} \quad \text{(since } \Delta p_0 = 1 \text{ mm)} \quad ...(2.43)$$

and $$\Delta h = \Delta h_0 \times \Delta p \quad ...(2.44)$$

Let $H = 8000$ m ; $b_0 = 100$ mm

and h be increased by 10 m for the interval $(H - h) = 8000$ m to $(H - h) = 3000$ m, and then by 5 m for interval $(H - h) = 3000$ to $(H - h) = 1500$ m.

When $h = 0$, $H - h = 8000$ m and $\Sigma\Delta p = 0$

When $h = 10$, $H - h = 7990$ m and $\Sigma\Delta p = \dfrac{100 \times 10}{7990} = 0.125$ mm

When $h = 1000$ m, $H - h = 7000$ m and $\Sigma\Delta p = \dfrac{100 \times 1000}{7000} = 14.286$ mm

When $h = 5000$ m, $H - h = 3000$ and $\Sigma\Delta p = \dfrac{100 \times 5000}{3000} = 166.667$ mm.

Thus, the values of $\Sigma \Delta p$ for the different values of h can be found and a master parallax table between $(H - h)$ and $\Sigma \Delta p$ can be prepared. Such a table will, however, be useful for direct computations only if $H = 8000$ and $b_0 = 100$ mm. The values of $\Sigma \Delta p$ in the table may, however, be adapted to other conditions also if they are multiplied by a constant K such that

$$K = \frac{b_0 \text{ (photo)} \times H \text{ (photo)}}{100 \times 8000} \qquad \ldots(2.46)$$

Example 2.17. *A photogrammetric survey is carried out to a scale of 1 : 20000. A camera with a wide angle lens of f = 150 mm was used with 23 cm × 23 cm plate size for a net 60% overlap along the line of flight. Find the error in height given by an error of 0.1 mm in measuring the parallax of the point.*

Solution. $\text{Scale} = \dfrac{f}{H}$

$$\therefore \quad \frac{1}{20{,}000} = \frac{150/1000 \text{ (m)}}{H\text{(m)}}$$

or $$H = \frac{150}{1000} \times 20{,}000 = 3000 \text{ m}$$

The length of the air base is given by

$$B = \left(1 - \frac{p_l}{100}\right) ls = (1 - 0.6)\,\frac{23}{100} \times 20{,}000 = 1840 \text{ m}$$

From equation 3.41, we have

$$dh = \frac{(H - h)^2}{Bf} \,.\, dp$$

Corresponding to the datum elevation, the error dh for $dp = 0.1$ mm is

$$dh = \frac{(3000 - 0)^2}{1840 \times 150} \times 0.1 = \mathbf{3.26 \text{ m}}.$$

Example 2.18. *In a pair of overlapping vertical photographs, the mean distance between two principal points both of which lie on the datum is 6.375 cm. At the time of photography, the air-craft was 600 m above the datum. The camera has a focal length of 150 mm. In the common overlap, a tall chimney 120 m high with its base in the datum surface is observed. Determine difference of parallax for top and bottom of chimney.*

Hidden page

The movable plate can be moved to the left or to the right by means of a micrometer screw which reads nearest to 0.01 mm, the total movement being about 25 mm. When these two dots are viewed properly under a stereoscope, they fuse into a single dot called *floating mark*. The marks are made to fuse by moving the right hand mark either to the left or to the right. After they have been fused, a slight movement of the movable mark will give the viewer the impression that the floating mark is moving up or down relative to the stereoscopic image. As the right hand mark is moved towards the left one, the floating mark appears to rise ; if it is moved to the right, the floating mark will appear to fall. These effects are due to the fact that the movement to the left increases the parallax of the marks, whereas the movement to the right decreases the parallax of the marks. Hence, if the floating mark is apparently placed on the ground at a known elevation, and the micrometer scale is read and is then moved to another point of unknown elevation, and the micrometer is turned until the floating mark again apparently rests on the ground surface, the difference in the two micrometer readings is a measure of Δp from which the difference in elevation can be calculated. This is the principle of the parallax bar.

Fig. 2.56 illustrates the principle of a parallax bar. On the left photograph, k is its principal point and k' is the conjugate principal point transferred from the next photograph. Similarly on the right hand photograph, k' is its principal point and k is the conjugate principal point transferred from the left photograph. Thus, $k\,k'$ is the flight line on both the photographs. To orient them for stereoscopic observation, a fine straight line is drawn on a sheet of heavy drafting paper and the left hand photograph is placed on it in such a way that flight line is in exact coincidence with the line on the paper. This can be easily done by the laying a straight edge over the photograph and orienting it to the line. The separation of the two marks of the parallax bar is set to a distance L (measured within 1/2 mm), in such a way that it reads approximately the middle reading. The right hand photograph is then oriented by means of the flight line and is so placed as to cause a separation L between the principal point on one photograph and to corresponding position on the other photograph. The two photographs are then fused under a stereoscope (Fig. 2.48) and set so that their positions may not be altered.

Let it be required to measure the parallax difference between two points A and C whose images appear on both the photographs at (a, c) and (a',c') respectively. The left mark of the parallax bar is placed over a and the parallax bar

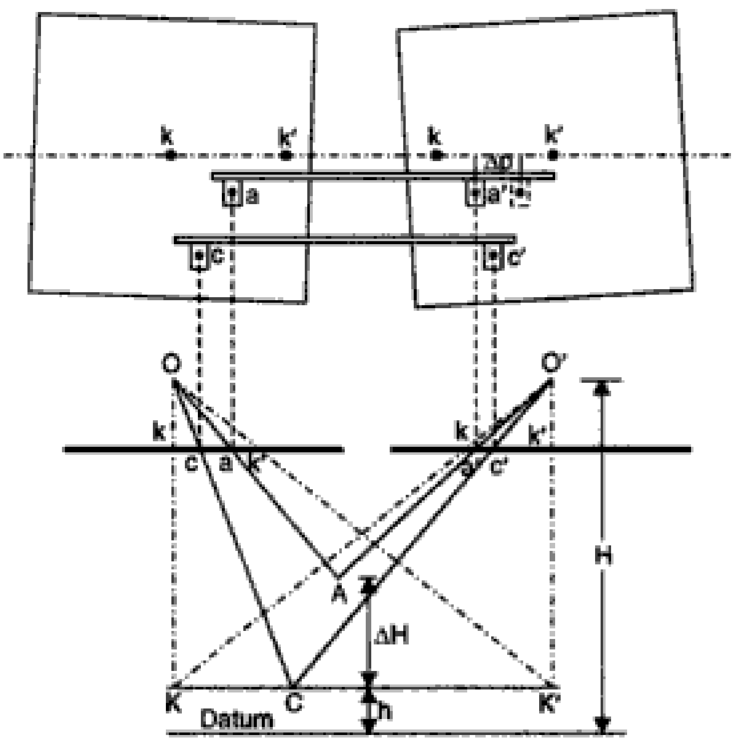


FIG. 2.56. PRINCIPLE OF A PARALLAX BAR.

scale at a particular elevation— either at the datum elevation or the average elevation of the terrain.

Fig. 2.58 shows a photograph with a tilt t at the exposure station O and flying height h. The negative and the photograph are parallel to each other. The rectified enlargement $b'\ k'\ a'$ is inclined at an angle t with the negative, and its principal distance is $p = mf$. The horizontal plane of the rectified photograph is known as the *easel plane*. It is to be noted that for the rectified enlargement of the photograph, the negative should be placed at a distance f from the lens. The lens will project the images from the negative in the proper direction, but since the distance from the lens to the negative is equal to the focal length of the lens, the projected bundles of the rays will be parallel to one another and they would never come to focus the enlargement plane. This is shown in Fig. 2.59 (a). This gives the condition that the entire negative must be placed behind the focal plane of the lens used in the rectifier.

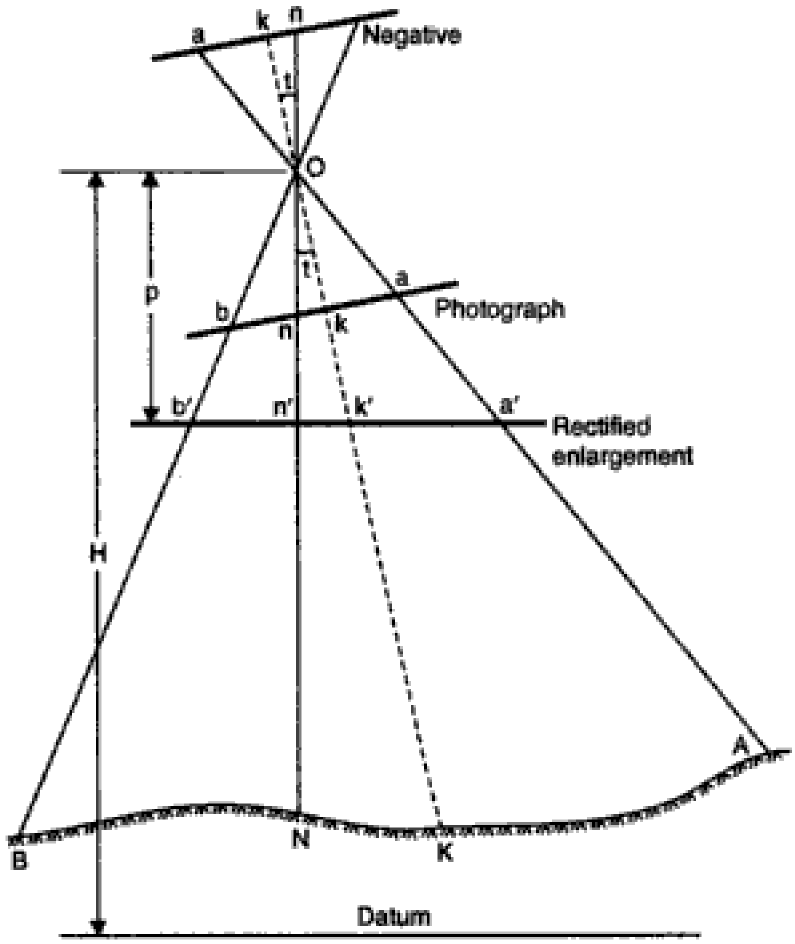


FIG. 2.58. RECTIFICATION AND ENLARGEMENT.

Scheimpflug Condition

As discussed in the previous paragraph if the negative plane is placed at the focal plane of the lens, the image cannot be focused. This is illustrated in Fig. 2.59 (a). If, however, the negative plane is placed beyond the focal plane, at a distance q from the lens and r is the corresponding position of the enlargement, the following two conditions are to be satisfied simultaneously :

$$\frac{1}{q} + \frac{1}{r} = \frac{1}{F} \qquad \text{and} \qquad r = mq \qquad \ldots(2.48)$$

where F is the focal length of the lens of the rectifier and m is the magnification.

The relationships stated above are for a vertical photograph. However, these apply also for a tilted photograph. These conditions are shown in Fig. 2.59 (c). x and x' are the conjugate distances for the point a, while y and y' are conjugate distances for the point b. Hence, we have

$$\frac{1}{x} + \frac{1}{x'} = \frac{1}{F} \qquad \text{and} \qquad \frac{1}{y} + \frac{1}{y'} = \frac{1}{F}$$

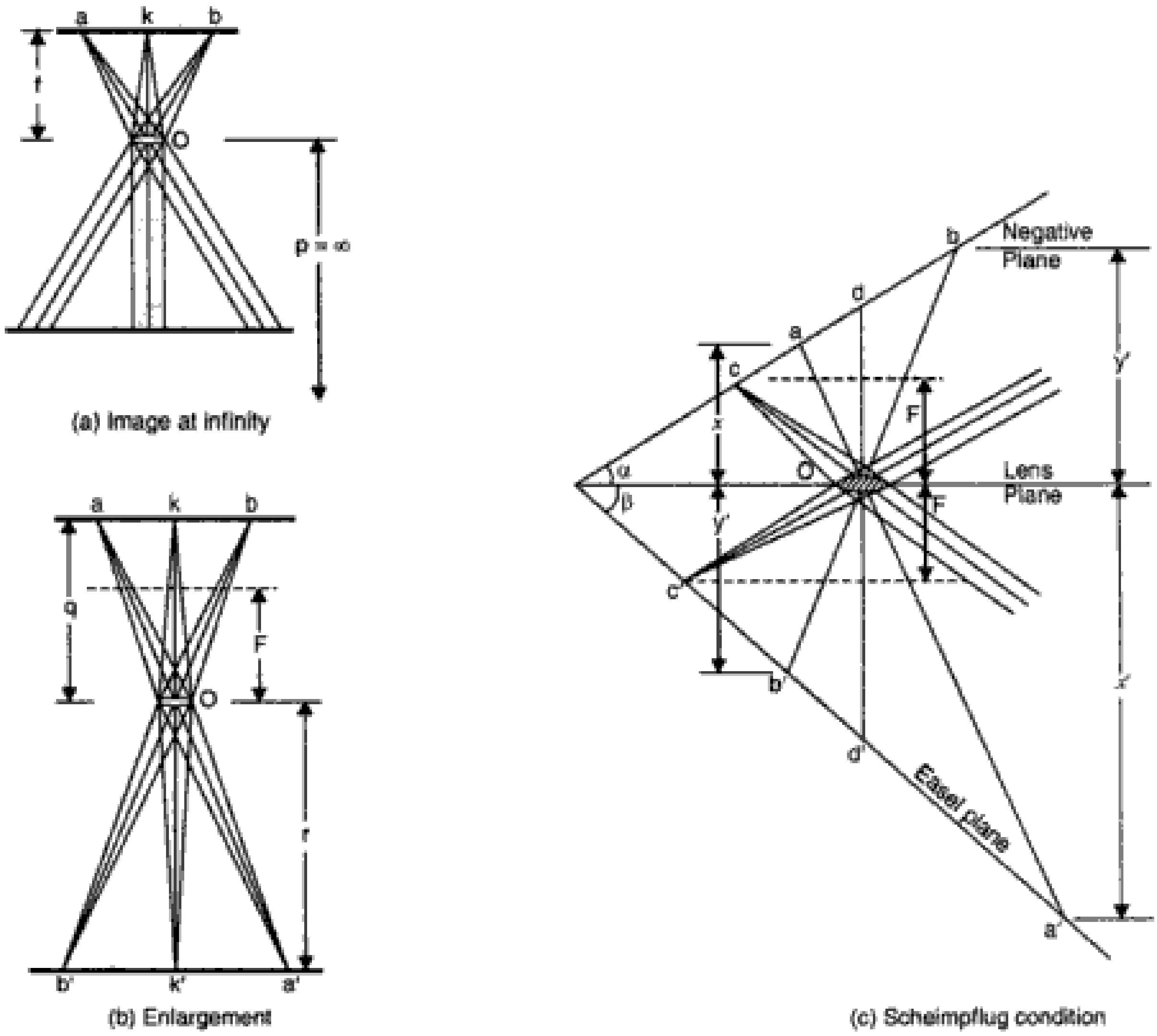


FIG. 2.59.

The negative plane makes an angle α with the lens plane, and the easel plane makes an angle β with the lens plane. It is to be noted that all the three planes intersect along one line. This is an important condition known as *Scheimpflug condition*.

The *Scheimpflug condition*, which must exist in order to produce sharp focus between the negative plane and the easel plane when these planes are not parallel, *states that the negative plane, the plane of the lens, and the easel plane must intersect along one line*.

In order to allow for a continuous range of tilt angles and magnification , there are in general, five independent elements necessary for rectification. These are :

(1) Variation of the projection distance.
(2) Tilt of the plane of projection about a horizontal axis.
(3) Rotation of the negative in its own plane (swing).
(4) Displacement of the negative in its own plane vertical to tilt axis.
(5) Displacement of negative in its own plane parallel to tilt axis.

An *automatic rectifier* is a rectifier so constructed that it automatically maintains the relationship between the object distance and the image distance, and at the same time fulfills the Scheimpflug condition. Fig. 2.60 shows the Wild E4 rectifier-enlarger introduced at

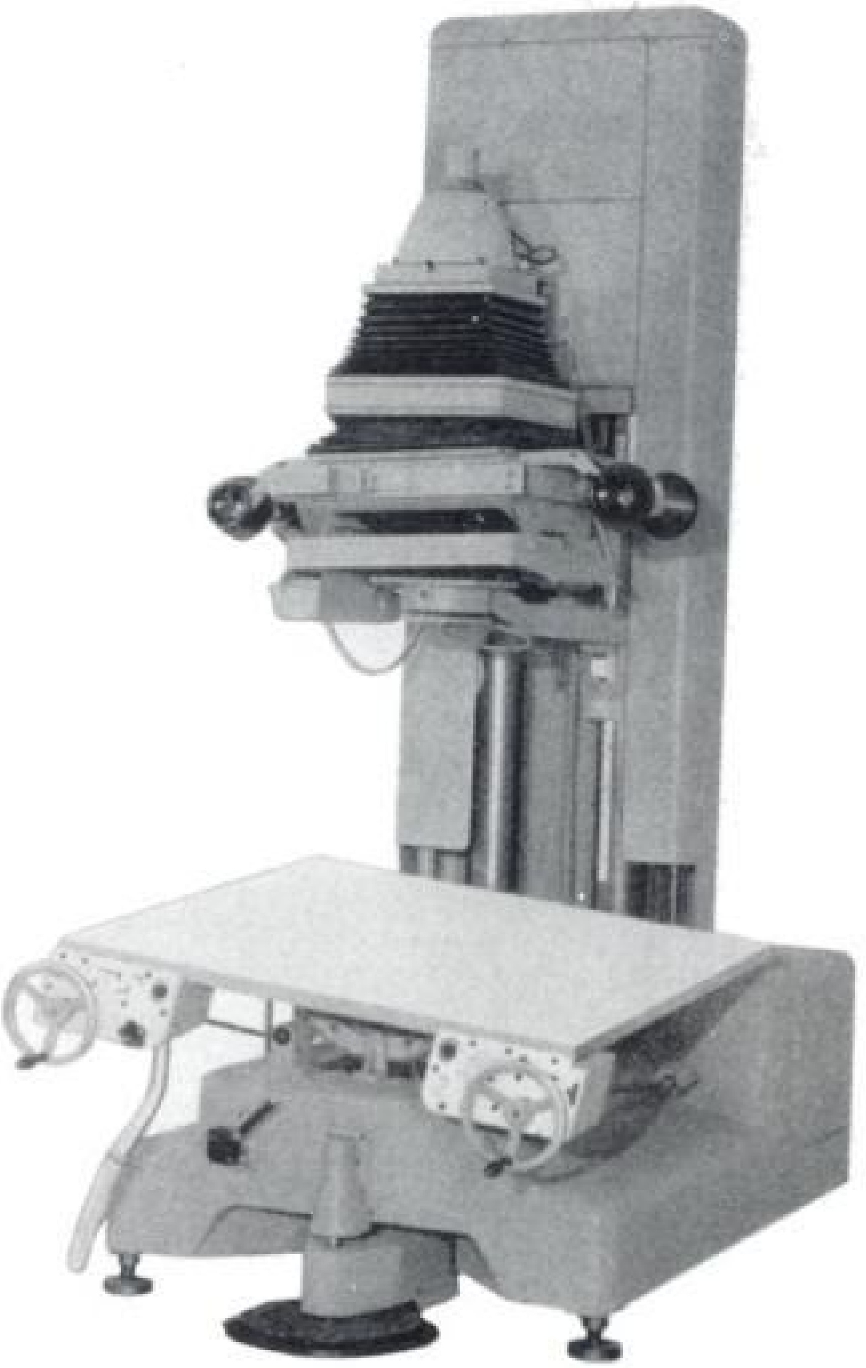


FIG. 2.60. WILD E4 RECTIFIER-ENLARGER

(TO FACE PAGE 215)

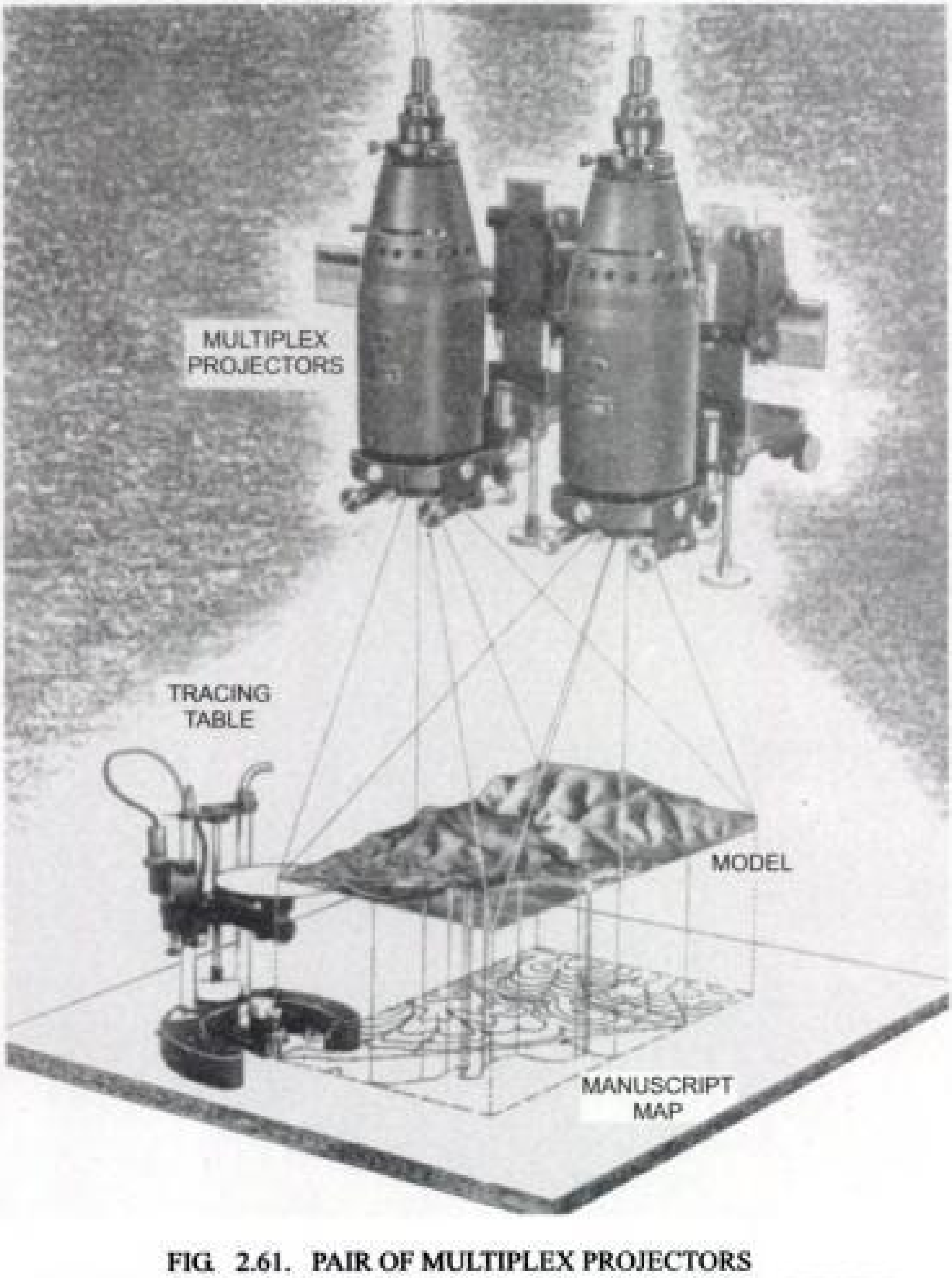


FIG. 2.61. PAIR OF MULTIPLEX PROJECTORS
FORMING A STEREOSCOPIC MODEL (U.S. GEOLOGICAL SURVEY)

(TO FACE PAGE 215)

the 1964 Congress of Photogrammetry in Lisbon. The lens equation is automatically fulfilled by a cam inversor and the Scheimpflug condition is automatically fulfilled by an electronic simulator. Both cam inversor and electronic simulator are equipped with synchro systems. The instrument has enlargement ratios over a range from 0.8 X to 7 X and can be used for a largest negative size of 23 cm × 23 cm (9″ × 9″). For further operational details, the reader is advised to see pamphlet P-1.302 *e* issued by M/s Wild Heerbrugg Ltd.

2.30. MOSACIS

Vertical photographs look so much like the ground that a set can be fitted together to form a *maplike photograph* of the ground. Such an assembly or getting of a series of overlapping photographs is called a *mosaic*. To a varying degree of accuracy, a mosaic is a map substitute. The mosaic has an over-all average scale comparable to the scale of a planimetric map.

Since they are taken at slightly varying altitudes and they contain tilts, they often do not fit each other very well. It is best to rephotograph them before they are used to bring them to desired scale and to eliminate some of the tilt.

A *controlled mosaic* is obtained when the photographs are carefully assembled so that the horizontal control points agree with their previously plotted positions. Making controlled mosaics is an art. A mosaic which is assembled without regard to any plotted control is called an *uncontrolled mosaic*.

The photographs are laid in such a sequence as to allow photo number and flight number of each photograph to appear on the finished assembly. This assembly is called an *index mosaic*. An index mosaic is a form of uncontrolled mosaic. A mosaic which is assembled from a single strip of photograph is called a *strip mosaic*.

The photographs used for preparing mosaics may consist of direct contact prints, of prints which have all been ratioed to a given datum scale in an enlarger, or of prints which have been fully rectified and ratioed in a rectifier.

A mosaic differs from a map in the following respects :

(1) A mosaic is composed of a series of perspective of the area, whereas a map is single orthographic projection.

(2) A mosaic contains local relief displacements, tilt distortions and non uniform scales, whiles a map shows the correct horizontal positions at a uniform scale.

(3) Various features appear as realistic photographic images on a mosaic, whereas they are portrayed by standard symbols on a map.

2.31. STEREOSCOPIC PLOTTING INSTRUMENTS

A stereoscopic plotting instrument is an optical instrument of high precision in which the spatial relationship of a pair of photographs at the instant of exposure is reconstructed. In such an instrument, the rays from the two photographs are projected and caused to intersect in its measuring space to form a theoretically perfect model of the terrain. A measured mark, visible to the operator is used to measure the stereoscopic model in all three dimensions. The horizontal movement of the measuring mark throughout the model is transmitted to a plotting pencil, which traces out the map position of the features appearing in the overlap area of two photographs forming the model.

A stereoscopic plotting instrument has four general components:

(1) a projection system (2) a viewing system

(3) a measuring system (4) a tracing system.

It is beyond the scope of the present book to illustrate fully the theory and working of the various plotting machines. However a brief description of the multiplex plotter is given below.

The Multiplex Plotter

The multiplex is probably the most widely used of any type of plotting machine. The equipment includes a reduction printer, a set of projectors mounted in series on a horizontal bar and a tracing table which provides both a floating mark and a tracing pencil to draw the map. The reduction printer produces reduced pictures on small glass plates. The 23 cm × 23 cm (or 9″ × 9″) size is thus reduced to a size 4 cm × 4 cm on the glass plates called *diapositives*.

Fig. 2.61 shows a pair of multiplex projectors forming a stereoscopic model. Each projector consists of a light source, a plate holder for the diapositive plate, and a lens which transmits the rays coming from the diapositive plate into the open space below the projector. The spatial model is obtained by projecting one photograph of an overlapping pair in red light and the other in blue-green light, and by observing the combination of colours through spectacles containing one red and one blue-green lens. This model, in fact, has three dimensions, and is not to be considered as virtual stereoscopic image as seen in a simple stereoscope. This method of viewing is called the *anaglyph system* of viewing reflected light and fulfills the condition of stereoscopic viewing.

Provision is made to move each of the projector in the directions of the *X,Y,Z co-ordinate axes, and also to rotate the projector about each of these axes. The X*-motion is parallel to the supporting bar, the *Y*-motion is perpendicular to the supporting bar and in the horizontal direction, while the Z-motion is perpendicular to the supporting bar and in the vertical direction. These six motions of each projector, independent of the others, make it possible to orient each projector in exactly in the same relation to the control points on the drawing table below, which the camera in the air had to the same corresponding actual ground points.

The *tracing table* contains a circular white disc with a pinhole in its centre. A light bulb below the disc provides a small pin point of light. This illuminated pin point is visible from the projectors and forms the measuring mark or floating mark in the spatial model. The disc can be raised or lowered so that the floating mark rests on the ground of the model. The tracing pencil point vertically below the floating mark gives position of the point on the map sheet. The tracing pencil traces pencil traces on the plotting sheet the horizontal movements of the floating mark. The disc is raised or lowered by means of a screw on the centre post at the back of the tracing stand. On the left post of the tracing stand is a millimetre scale on which is read the height of the disc above the drawing table which may be considered as the datum plane. The elevation of any point in the spatial model can be found by reading the vertical scale of the tracing stand, after the floating mark has been set on the given point.

8. (*a*) Prove that on a tilted photograph height displacements are radial from the plumb point.
(*b*) Derive an expression for the height displacements in a vertical photograph.

9. (*a*) Explain with reference to aerial photographs, what is meant by end overlap and side overlap and why they are provided ?
(*b*) How do you determine the number of photographs necessary to cover a given area in an aerial survey ?

10. Write a note on radial line method of plotting.

11. Write short notes on the following :
(*a*) Stereoscopic vision. (*b*) Mirror stereoscope. (*c*) Crab and drift. (*d*) Parallax bar.

12. Describe, with the help of neat sketch a photo-theodolite.

13. (*a*) Explain how do you determine the focal length of the camera lens of a photo-theodolite.
(*b*) The distance from two points on a photographic print to the principal line are 42.36 mm to the left and 38.16 mm to the right. The angle between the points measured with a transit is 30° 45'. Determine the focal length of the lens.

14. (*a*) How do you determine the scale of an aerial photograph ? What do you understand by the terms 'datum scale' and 'average scale' ?
(*b*) A line *PQ* 2100 m long, lying at an elevation of 400 m measures 10.08 cm on a vertical photograph. If the focal length of the lens is 24 cm determine the scale of the photograph in an area, the average elevation which is 600 m.

15. A line *AB* lies on a terrain having an average elevation 400 m above mean sea-level. It appears to be 8.72 cm on a photograph for which focal leng is 24 cm. The same line measures 2.18 cm on a map which is to a scale of $\frac{1}{40000}$.
Calculate the flying altitude of the aircraft, above mean sea level. when the photograph was taken.

16. An object has an elevation of 400 m above mean sea-level. The distance from the principal point to the image of that point on the photograph is 4.86 cm. If the datum scale is $\frac{1}{12000}$ and focal length of the camera is 24 cm, determine the relief displacement of the point.

17. A tower *AB* is 40 m high, and the elevation of its bottom *B* is 800 m above mean sea-level. The distance of the image of the tower on a vertical photograph, taken at a flight altitude of 1800 m above mean sea-level, is 8.42 cm. Compute the displacement of the image of the top of the tower with respect to the image of its bottom.

18. A tower, lying on a flat area having an average elevation of 800 m above mean sea-level, was photographed with a camera having a focal length of 24 cm. The distance between the images of top and bottom of the tower measures 0.34 cm on the photograph. A line *AB*, 200 m long on the ground, measures 12.2 cm on the same photograph. Determine the height of the tower if the distance of the image of the top of the tower is 8.92 cm from the principal point.

19. The scale of an aerial photograph is 1 cm = 160 m, and the size of the photograph is 20 cm × 20 cm. If the longitudinal lap is 65% and side lap = 35%, determine the number of photographs required to cover an area of 232 sq. km.

ANSWERS

5.	1828·8 m ; $\frac{1}{20000}$.	6.	286·41 m.
7.	14030 m.	13.(*b*)	146·38 m.
14.(*b*)	1 cm = 200 m.	15.	2800 m.
16.	0·675 cm.	17.	0·34 cm
18.	60 m.	19.	100.

3

Electro-Magnetic Distance Measurement (EDM)

3.1. INTRODUCTION

There are three methods of measuring distance between any two given points :

1. Direct distance measurement (DDM), such as the one by chaining or taping.
2. Optical distance measurement (ODM), such as the one by tacheometry, horizontal subtense method or telemetric method using optical wedge attachments.
3. Electro-magnetic distance measurement (EDM) such as the one by geodimeter, tellurometer or distomat etc.

The method of direct distance measurement is unsuitable in difficult terrain, and some times impossible when obstructions occur. The problem was overcome after the development of optical distance measuring methods. But in ODM method also, the range is limited to 150 to 150 m and the accuracy obtained is 1 in 1000 to 1 in 10000. Electromagnetic distance measurement (EDM) enables the accuracies upto 1 in 10^5, over ranges upto 100 km.

EDM is a general term embracing the measurement of distance using electronic methods. In electro-magnetic (or electronic) method, distances are measured with instruments that rely on propagation, reflection and subsequent reception of either radio, visible light or infra-red waves. There are in excess of fifty different EDM systems available. However, we will discuss here the following instruments :

(*i*) Geodimeter (*ii*) Tellurometer (*iii*) Distomats.

3.2. ELECTROMAGNETIC WAVES

The EDM method is based on generation, propagation, reflection and subsequent reception of electromagnetic waves. The type of electromagnetic waves generated depends on many factors but principally, on the nature of the electrical signal used to generate the waves. The evolution and use of radar in the 1939-45 war resulted in the application of radio waves to surveying. However, this was suitable only for defence purposes, since it could not give the requisite accuracy for geodetic surveying. E. Bergestrand of the Swedish Geographical Survey, in collaboration with the manufacturers, Messrs AGA of Sweden, developed a method based on the propagation of *modulated light waves* using instrument called *geodimeter*. Another instrument, called *tellurometer* was developed, using radio waves. Modern short and medium range EDM instruments (such as Distomats) commonly used in surveying use *modulated infra-red waves.*

Properties of electromagnetic waves

Electromagnetic waves, though extremely complex in nature, can be represented in the form of periodic sinusoidal waves shown in Fig. 3.1. It has the following properties:

1. The waves completes a *cycle* in moving from identical points *A* to *E* or *B* to *F* or *D* to *H*.

2. The number of times the wave completes a cycle in one second is termed as *frequency* of the wave. The frequency is represented by f hertz (*Hz*) where 1 hertz (*Hz*) is one cycle per second. Thus, if the frequency f is equal to 10^3 Hz, it means that the waves completes 10^3 cycles per second.

3. The length traversed in one cycle by the wave is termed as *wave length* and is denoted by λ (metres). Thus the *wave length* of a wave is the distance between two identical points (such as *A* and *E* or *B* and *F*) on the wave.

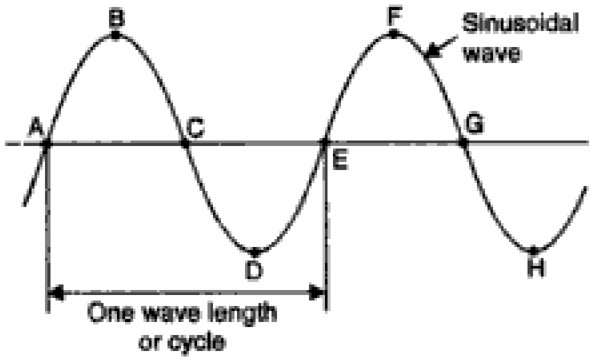


FIG 3.1 PERIODIC SINUSOIDAL WAVES.

4. The *period* is the time taken by the wave to travel through one cycle or one wavelength. It is represented by T seconds.

5. The *velocity* (v) of the wave is the distance travelled by in one second.

The frequency, wavelength and period can all vary according to the wave producing source. However, the velocity v of an electromagnetic wave depends upon the medium through which it is travelling. The velocity of wave in a vacuum is termed as *speed of light*, denoted by symbol c, the value of which is presently known to be 299792.5 km/s. For simple calculations, it may be assumed to be 3×10^8 m/s.

The above properties of an electromagnetic wave can be represented by the relation,

$$f = \frac{c}{\lambda} = \frac{1}{T} \qquad ...(3.1)$$

Another property of the wave, known as *phase* of the wave, and denoted by symbol φ, is a very convenient method of identifying fraction of a wavelength or cycle, in EDM. One cycle or wave-length has a phase ranging from 0° to 360°. Various points *A*, *B*, *C* etc. of Fig. 3.1 has the following phase values :

Point →	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Phase ϕ°	0	90	180	270	360 (or 0)	90	180	270

Fig. 3.2 gives the electromagnetic spectrum. The type of electromagnetic wave is known by its wavelength or its frequency. However, all these travel with a velocity *approximately* equal to 3×10^8 m/s. *This velocity forms the basis of all electromagnetic measurements.*

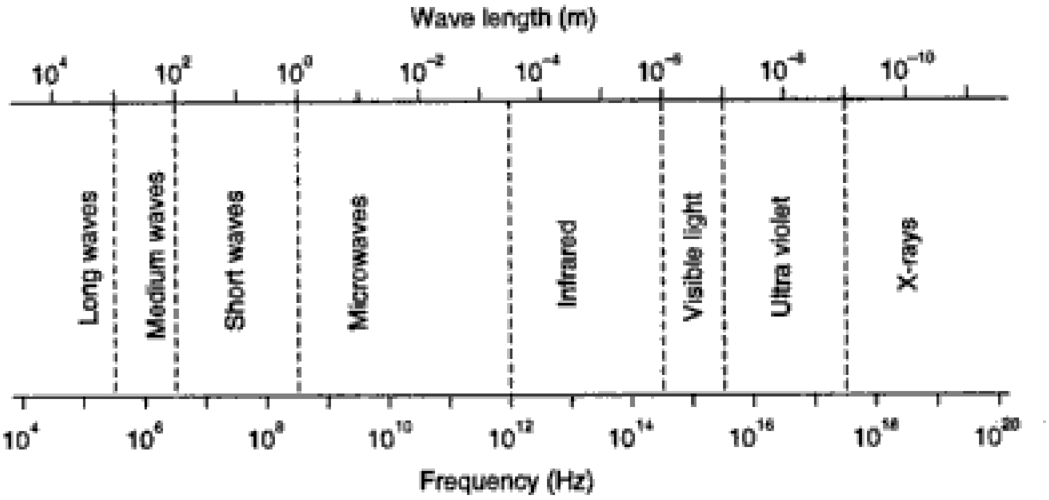


FIG. 3.2 ELECTROMAGNETIC SPECTRUM.

Measurement of transit times

Fig. 3.3 (*a*) shows a survey line *AB*, the length *D* of which is to be measured using EDM equipment placed at ends *A* and *B*. Let a transmitter be placed at *A* to propagate electromagnetic waves towards *B*, and let a receiver be placed at *B*, along with a timer. If the timer at *B* *starts* at the instant of transmission of wave from *A*, and *stops* at the instant of reception of incoming wave at *B*, the *transit time* for the wave from *A* and *B* in known.

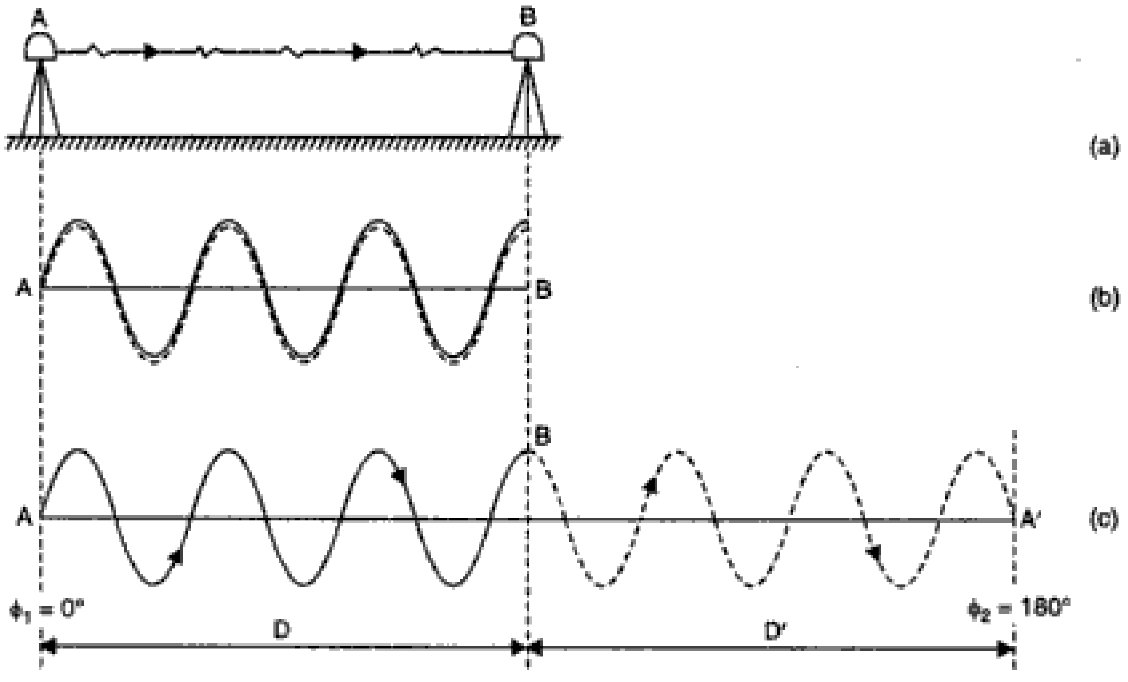


FIG. 3.3. MEASUREMENT OF TRANSIT TIME.

From this transit time, and from the known velocity of propagation of the wave, the distance *D* between *A* and *B* can be easily computed. However, this transit time is of the order of 1×10^{-6} s which requires very advanced electronics. Also it is extremely difficult to start the timer at *B* when the wave is transmitted at *A*. Hence a reflector is placed at B instead of a receiver. This reflector reflects the waves back towards A, where they are received (Fig. 3.3 (*b*)]. Thus the equipment at A acts both as a transmitter as well as receiver. The ***double transit time*** can be easily measured at A. This will require EDM timing devices with an accuracy of $\pm 1 \times 10^{-9}\, s$.

Phase Comparison

Generally, the various commercial EDM systems available do not measure the transit time directly. Instead, the distance is determined by measuring the phase difference between the transmitted and reflected signals. This phase difference can be expressed as fraction of a cycle which can be converted into units of time when the frequency of wave is known. Modern techniques can easily measure upto $\frac{1}{1000}$ part of a wavelength.

In Fig. 3.3 (*b*), the wave transmitted from A towards B is instantly reflected from B towards A, and is then received at A, as shown by dotted lines. The same sequence is shown in Fig. 3.3 (*c*) by opening out the wave, wherein A and A' are the same. The distance covered by the wave is

$$2D = n\lambda + \Delta\lambda \qquad \ldots(3.2)$$

where
d = distance between A and B
λ = wavelength
n = whole number of wavelengths travelled by the wave
$\Delta\lambda$ = fraction of wavelength travelled by the wave.

The measurement of component $\Delta\lambda$ is known as *phase comparison* which can be achieved by electrical phase detectors.

Let φ_1 = phase of the wave as it is transmitted at A
φ_2 = phase of the wave as it is received at A'

Then
$$\Delta\lambda = \frac{\text{phase difference in degrees}}{360°} \times \lambda \quad \text{or} \quad \Delta\lambda = \frac{(\varphi_2 - \varphi_1)°}{360°} \times \lambda \qquad \ldots(3.3)$$

The determination of other component $n\lambda$ of equation 3.2 is referred to as *resolving the ambiguity of the phase comparison*, and this can be achieved by any one of the following methods.

(*i*) by increasing the wavelength manually in multiples of 10, so that a coarse measurement of D is made, enabling n to be deduced.

(*ii*) by measuring the line AB using three different (but closely related) wavelengths, so as to form three simultaneous equations of the form

$$2D = n_1\lambda_1 + \Delta\lambda_1 \;;\; 2D = n_2\lambda_2 + \Delta\lambda_2 \;;\; 2D = n_3\lambda_3 + \Delta\lambda_3$$

The solution of these may give the value of D.

In the latest EDM equipment, this problem is solved automatically, and the distance D is displayed.

For example, let λ for the wave of Fig. 3.3 (*c*) be 20 m. From the diagram, $n = 6$, $\varphi_1 = 0°$ and $\varphi_2 = 180°$.

$$\therefore \qquad 2D = n\lambda + \Delta\,\lambda = n\lambda + \frac{\varphi_2 - \varphi_1}{360°} \times \lambda$$

or $$2D = (6 \times 20) + \frac{180 - 0}{360} \times 20$$

$\therefore$ $$D = 65 \text{ m.}$$

This measurement of distance by EDM is analogous to the measurement of AB by taping. wherein

$$D = ml + \Delta l$$

where l = length of tape = 20 m (say)

m = whole No. of tapes = 3

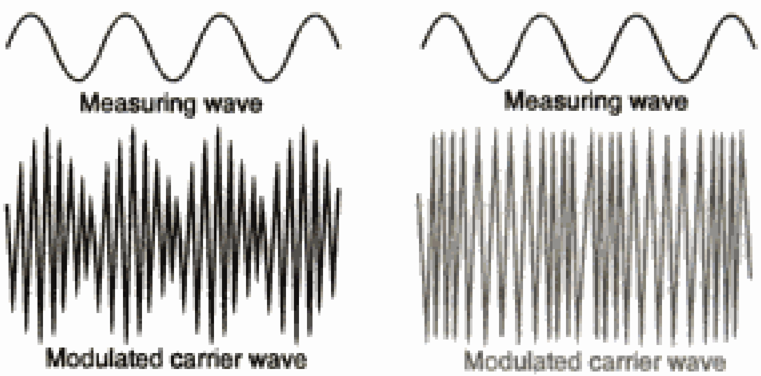
Δl = remaining length of the tape in the end bay

Hence the recording in the case of taping will be $D = 3 \text{ m} \times 20 + 5 = 65$ m.

3.3. MODULATION

As stated above, EDM measurements involve the measurement of fraction $\Delta\lambda$ of the cycle. Modern phase comparison techniques are capable of resolving to better than $\frac{1}{1000}$ part of a wavelength. Assume ± 10 mm to be the accuracy requirement for surveying equipment, this must represent $\frac{1}{1000}$ of the measuring wavelength. This means that $\lambda = 10 \times 1000$ mm = 10 m, which is a maximum value. However, by use of modern circuitory, λ can be increased to 40 m, which corresponds to $f = 7.5 \times 10^6$ Hz. Thus, the lowest value of f that can be used is 7.5×10^6 Hz. At present, the range of frequencies that can be used in the measuring process is limited to approximately 7.5×10^6 to 5×10^8 Hz.

In order to increase the accuracy, it is desirable to use an extremely high frequency of propagation. However, the available phase comparison techniques cannot be used at frequencies greater than 5×10^8 Hz which corresponds to a wavelength $\lambda = 0.6$ m. On the other hand, the lower frequency value in the range of 7.5×10^6 to 5×10^8 Hz is not suitable for direct transmission through atmosphere because of the the effects of interference, reflection, fading and scatter.



(a) Amplitude modulation (b) Frequency modulation

FIG. 3.4. MODULATION

The problem can be overcome by the technique of *modulation* wherein the measuring wave used for phase comparison is superimposed on a *carrier wave* of much higher frequency. EDM uses two methods of modulating the carrier wave :

(*a*) Amplitude modulation. (*b*) Frequency modulation.

In *amplitude modulation*, the carrier wave has constant frequency and the modulating wave (the measuring wave) information is conveyed by the amplitude of the carrier waves. In the *frequency modulation*, the carrier wave has constant amplitude, while its frequency varies in proportion to the amplitude of the modulating wave. *Frequency modulation* is used in all microwave EDM instruments while *amplitude modulation* is done in *visible light instruments* and infrared instruments using higher carrier frequencies.

3.4. TYPES OF EDM INSTRUMENTS

Depending upon the type of carrier wave employed, EDM instruments can be classified under the following three heads :

(*a*) Microwave instruments

(*b*) Visible light instruments

(*c*) Infrared instruments.

For the corresponding frequencies of carrier waves, reader may refer back to Fig. 3.2. It is seen that all the above three categories of EDM instruments use short wavelengths and hence higher frequencies.

1. Microwave instruments

These instruments come under the category of long range instruments, where in the carrier frequencies of the range of 3 to 30 GHz (1 GHz = 10^9) enable distance measurements upto 100 km range. *Tellurometer* come under this category.

Phase comparison technique is used for distance measurement. This necessitates the erection of some form of *reflector* at the remote end of the line. If *passive reflector* is placed at the other end, a weak signal would be available for phase comparison. Hence an electronic signal is required to be erected at the reflecting end of the line. This instrument, known as *remote instrument* is identical to the *master instrument* placed at the measuring end. The *remote instrument* receives the transmitted signal, amplifies it and transmits it back to the master in exactly the phase at which it was received. This means that microwave EDM instruments require two instruments and two operators. Frequency modulation is used in most of the microwave instruments. The method of varying the measuring wavelength in multiplies of 10 is used to obtain an unambiguious measurement of distance. The microwave signals are radiated from small aerials (called *dipoles*) mounted in front of each instrument, producing directional signal with a beam of width varying from 2° to 20°. Hence the alignment of master and remote units is not critical. Typical maximum ranges for microwave instruments are from 30 to 80 km, with an accuracy of ± 15 mm to ± 5 mm/km.

2. Visible light instruments

These instruments use visible light as carrier wave, with a higher frequency, of the order of 5×10^{14} Hz. Since the transmitting power of carrier wave of such high frequency falls off rapidly with the distance, the range of such EDM instruments is lesser than those of microwave units. A *geodimeter* comes under this category of EDM instruments.

The carrier, transmitted as light beam, is concentrated on a signal using lens or mirror system, so that signal loss does not take place.

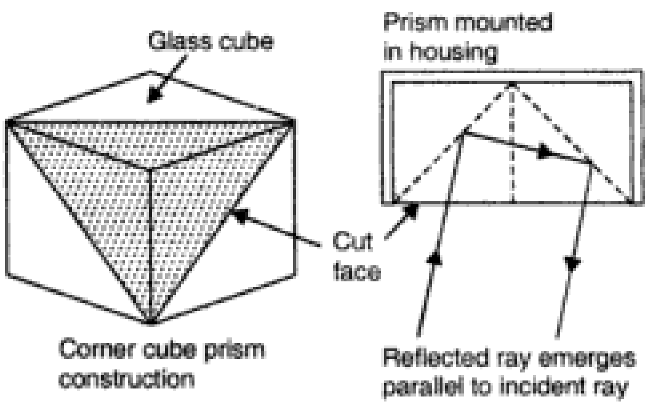


FIG. 3.5. CORNER CUBE PRISM

Since the beam divergence is less than 1°, accurate alignment of the instrument is necessary. *Corner-cube prisms*, shown in Fig. 3.5 are used as reflectors at the remote end. These prisms are constructed from the corners of glass cubes which have been cut away in a plane making an angle of 45° with the faces of the cube.

The light wave, directed into the cut-face is reflected by highly silvered inner surfaces of the prism, resulting in the reflection of the light beam along a parallel path. This is obtainable over a range of angles of incidence of about 20° to the normal of the front face of the prism. *Hence the aligment of the reflecting prism towards the main EDM instrument at the receiver (or transmitting) end is not critical.*

The advantage of visible light EDM instruments, over the microwave EDM instruments is that only one instrument is required, which work in conjunction with the inexpensive corner cube reflector. *Amplitude modulation* is employed, using a form of electro-optical shutter. The line is measured using three different wavelengths, using the same carrier in each case. The EDM instrument in this category have a range of 25 km, with an accuracy of ± 10 mm to ± 2 mm/km. The recent instruments use pulsed light sources and highly specialised modulation and phase comparison techniques, and produce a very high degree of accuracy of ± 0.2 mm to ± 1 mm/km with a range of 2 to 3 km.

3. Infrared instruments

The EDM instruments in this group use near infrared radiation band of wavelength about 0.9 μ m as carrier wave which is easily obtained from gallium arsenide (Ga As) infrared emitting diode. These diodes can be very easily directly *amplitude modulated* at high frequencies. Thus, modulated carrier wave is obtained by an inexpensive method. Due to this reason, there is predominance of infrared instruments in EDM. Wild Distomats fall under this category of EDM instruments.

The power output of the diodes is low. Hence the range of these instruments is limited to 2 to 5 km. However, this range is quite sufficient for most of the civil engineering works. The EDM instruments of this category are very light and compact, and these can be theodolite mounted. This enables angles and distances to be measured simultaneously at the site. A typical combination is Wild DI 1000 infra-red EDM with Wild T 1000 electronic theodolite ('Theomat'). The accuracy obtainable is of the order of ± 10 mm, irrespective of the distance in most cases.

The carrier wavelength in this group is close to the visible light spectrum. Hence infrared source can be transmitted in a similar manner to the visible light system using geometric optics, a lens/mirror system being used to radiate a highly collimated beam of angular divergence of less than 15'. Corner cube prisms are used at the remote end, to reflect the signal.

Electronic tacheometer, such as Wild TC 2000 'Tachymat' is a further development of the infrared (and laser) distance measurer, which combines theodolite and EDM units. Microprocessor controlled angle measurement give very high degree of accuracy, enabling horizontal and vertical angles, and the distances (horizontal, vertical, inclined) to be automatically displaced and recorded.

3.5. THE GEODIMETER

The method, based on the propagation of *modulated light waves*, was developed by E. Bergestrand of the Swedish Geographical Survey in collaboration with the manufacturer, M/s AGA of Sweden. Of the several models of the geodimeter manufacured by them, model 2-A can be used only for observations made at night while model-4 can be used for limited day time observations.

Fig. 3.6 shows the schematic diagram of the geodimeter. Fig. 3.7 shows the photograph of the front panel of model-4 geodimeter mounted on the tripod. The main instrument is stationed at one end of the line (to be measured) with its back facing the other end of the line, while a reflector (consisting either of a spherical mirror or a reflex prism system) is placed at the other end of the line.

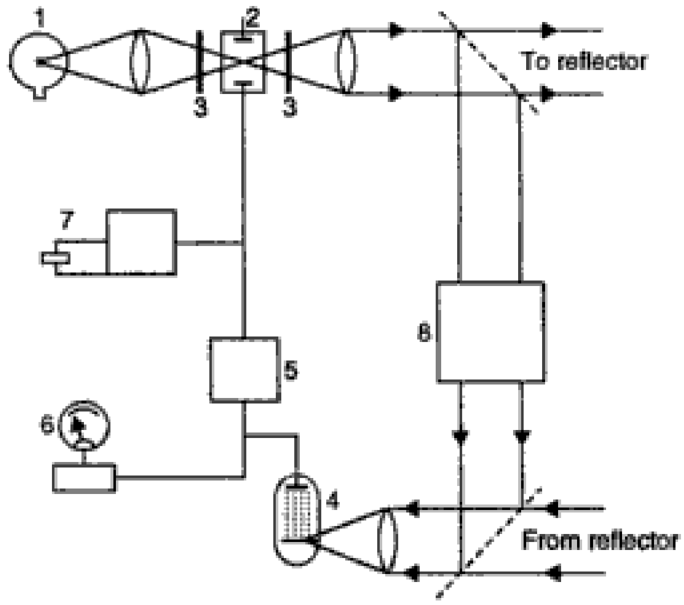


FIG. 3.6. SCHEMATIC DIAGRAM OF THE GEODIMETER.

The light from an incandescent lamp (1) is focused by means of an achromatic condenser and passed through a Kerr cell (2). The Kerr cell consist of two closely spaced conducting plates, the space between which is filled with nitrobenzene. When high voltage is applied to the plates of the cell and a ray of light is focused on it, the ray is split into two parts, each moving with different velocity. Two Nicol's prisms (3) are placed on either side of the Kerr cell. The light leaving the first Nicol's prisms is plane polarised. The light is split into two (having a phase difference) by the Kerr cell. On leaving the Kerr cell, the light is recombined. However, because of phase difference, the resulting beam is elliptically polarised. Diverging light from the second polariser can be focused to a parallel beam by the transmitter objective, and can then be reflected from a mirror lens to a large spherical concave mirror.

On the other end of the line being measured is put a reflex prism system or a spherical mirror, which reflects the beam of light back to the geodimeter. The receiver system of the geodimeter consists of spherical concave mirror, mirror lens and receiver objective. The light of variable intensity after reflection, impinges on the cathode of the photo tube (4). In the photo tube, the light photons impinge on the cathode causing a few primary electrons to leave and travel, accelerated by a high frequency voltage, to the first dynode, where the secondary emission takes place. This is repeated through a further eight dynodes. The final electron current at the anode is some hundreds of thousand times greater than that at the cathode. The sensitivity of the photo tube is varied by applying the high frequency-Kerr cell voltage between the cathode and the first dynode. The low frequency vibrations are eliminated by a series of electrical chokes and condensers. The passages of this modulating voltage through the instrument is delayed by means of an adjustable electrical delay unit (5). The difference between the photo tube currents during the positive and negative bias period is measured on the *null indicator* (6) which is a sensitive D.C. moving coil micro-ammeter. In order to make both the negative and positive current intensities equal (*i.e.* in order to obtain null-point), the phase of the high frequency voltage from the Kerr cell must be adjusted ± 90° with respect to the voltage generated by light at the cathode.

Thus, the light which is focused to a narrow beam from the geodimeter stationed at one end to the reflector stationed at the other end of the line, is reflected back to the photo multiplier. The variation in the intensity of this reflected light causes the current from the photo multiplier to vary where the current is already being varied by the direct signal from the crystal controlled oscillater (7). The phase difference between the two pulses received by the cell are a measure of the distance between geodimeter and the reflector (*i.e.*, length of the line).

The distance can be measured at different frequencies. On Model-2A of the geodimeter, three frequencies are available. Model-4 has four frequencies. Four phase positions are available on the *phase position indicator*. Changing phase indicates that the polarity of the Kerr cell terminals of high and low tension are reversed in turn. The 'fine' and 'coarse' delay switches control the setting of the electrical delay between the Kerr cell and the photo multiplier. The power required is obtained from a mobile gasoline generator. Model-4 has a night range of 15 meters to 15 km, a daylight range of 15 to 800 metres and an average error of ± 10 mm ± five millionth of the distance. It weighs about 36 kg without the generator.

3.6. THE TELLUROMETER

In the Tellurometer, high frequency radio waves (or microwaves) are used intead of light waves. It can be worked with a light weight 12 or 24 volt battery. Hence the instrument is highly portable. Observations can be taken both during day as well as night, while in the geodimeter, observations are normally restricted in the night. However, two such Tellurometres are required, one to be stationed at each end of the line, with two highly skilled persons, to take observations. One instrument is used as the *master set or control set* while the other instrument is used as the *remote set* or *slave set*. In Model MRA-2 (manufactured by M/s. Cooke, Troughton and Simms Ltd), each set can either be used as the master set or remote set by switching at 'master' and 'remote' positions respectively. Fig. 3.8 shows the photograph of Tellurometer (Model MRA-2).

Fig. 3.9 shows the block diagram of the Tellurometer, first designed by Mr. T.L. Wadley of the South African Council for Scientific and Industrial Research. Radio waves are emitted by the master instrument at a frequency of 3000 Mc.s. (3×10^9 c.p.s.) from a klystron and have superimposed on them a crystal controlled frequency of 10 Mc.s. The high frequency wave is termed as *carrier wave*. Waves at high frequencies can be propagated in straight line paths other than long distance much more readily. The low frequency wave is known as the *pattern wave* and is used for making accurate measurements. The light frequency pattern wave is thus said to be frequency modulated (F.M.) by low frequency pattern wave. This modulated signal is received at the remote station where a second klystron is generating another carrier wave at 3033 Mc.s. The difference between the two high

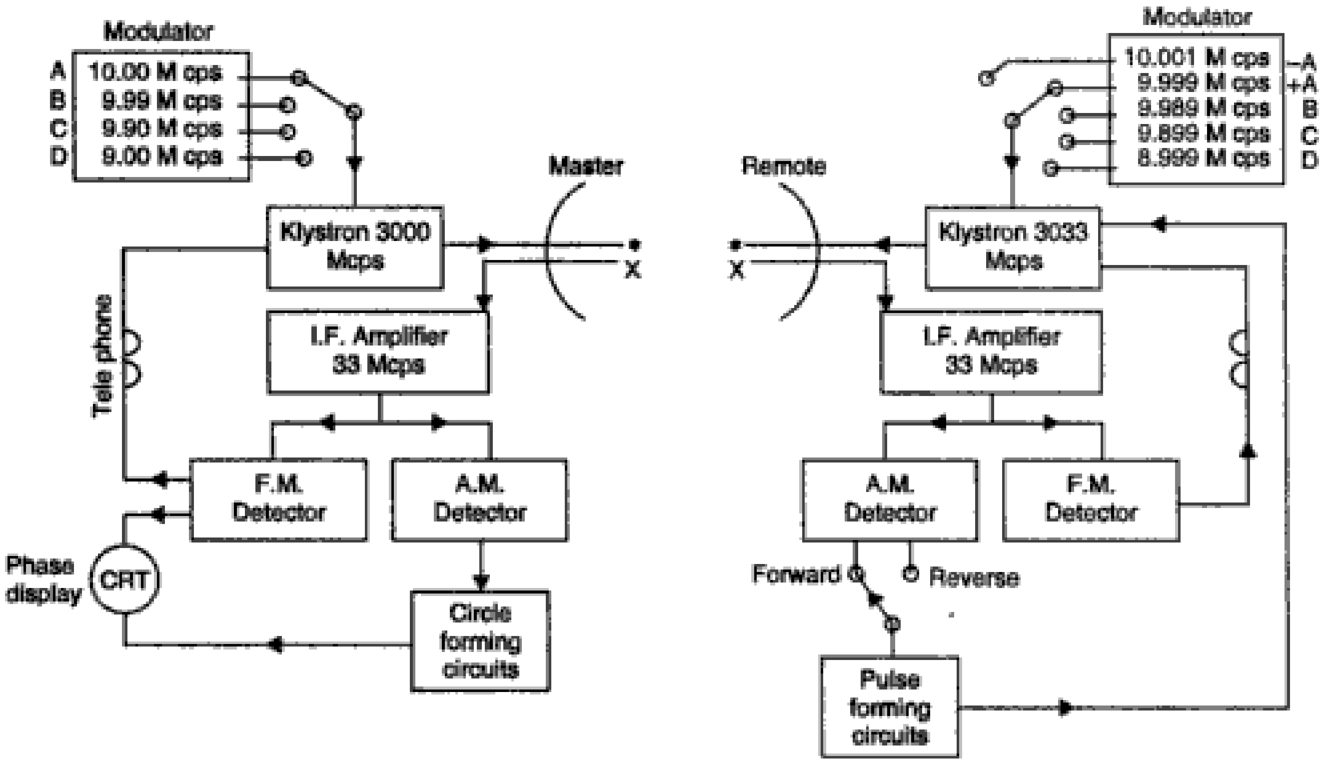


FIG. 3.9 BLOCK DIAGRAM OF THE TELLUROMETER SYSTEM.

frequencies, *i.e.* 3033 – 3000=33 Mc.s. (known as *intermediate frequency*) is obtained by an electrical 'mixer', and is used to provide sufficient sensitivity in the internal detector circuits at each instrument. In addition to the carrier wave of 3033 Mc.s., a crystal at the remote station is generating a frequency of 9.999 Mc.s. This is *heterodyned* with the incoming 10 Mc.s. to provide a 1 k c.p.s. signal. The 33 Mc.s. intermediate frequency signal is amplitude modulated by 1 k c.p.s. signal. The amplitude modulated signal passes to the amplitude demodulator, which detects the 1 k c.p.s. frequency. At the pulse forming circuit, a pulse with a repitition frequency of 1 k c.p.s. is obtained. This pulse is then applied to the klystron and frequency modulates the signal emitted, *i.e.*, 3033 Mc.s. modulated by 9.999 Mc.s. and pulse of 1 k c.p.s. This signal is received at the master station. A further compound heterodyne process takes place here also, where by the two carrier frequencies subtract to give rise to an intermediate frequency of 33 Mc.s. The two *pattern frequencies* of 10 and 9.999 Mc.s. also subtract to provide 1 k c.p.s *reference frequency* as amplitude modulation. The *change in the phase between this and the remote 1 k c.p.s. signal is a measure of the distance* . The value of phase delay is expressed in time units and appear as a *break* in a circular trace on the oscilloscope cathode ray tube.

Four low frequencies (*A, B, C* and *D*) of values 10.00, 9.99,9.90 and 9.00 Mc.p.s. are employed at the master station, and the values of phase delays corresponding to each of these are measured on the oscilloscope cathode ray tube. The phase delay of *B, C* and *D* are subtracted from *A* in turn. The *A* values are termed as 'fine readings' and the *B, C, D* values as 'coarse readings'. The oscilloscope scale is divided into 100 parts. The wavelength of 10 Mc.s. pattern wave as approximately 100 ft. (30 m) and hence each division of the scale represents 1 foot on the two-way journey of the waves or approximately 0.5 foot on the length of the line. The final readings of A, $A - B$, $A - C$ and $A - D$ readings are recorded in millimicro seconds (10^{-9} seconds) and are converted into distance readings by assuming that the velocity of wave propagations as 299,792.5 km/sec. It should be noted that the success of the system depends on a property of the *heterodyne process*, that the phase difference between two heterodyne signal is maintained in the signal that results from the mixing.

3.7. WILD 'DISTOMATS'

Wild Heerbrugg manufacture EDM equipment under the trade name 'Distomat', having the following popular models :

1. Distomat DI 1000
2. Distomat DI 5S
3. Distomat DI 3000
4. Distomat DIOR 3002
5. Tachymat TC 2000 (Electronic tacheometer)

1. Distomate DI 1000

Wild Distomat DI 1000 is very small, compact EDM, particularly useful in building construction, civil engineering construction, cadastral and detail survey, particularly in populated areas where 99% of distance measurements are less than 500 m. It is an EDM that makes the tape redundant. It has a range of 500 m to a single prism and 800 m to three prisms (1000 m in favourable conditions), with an accuracy of 5 mm + 5 ppm. It can be fitted to all Wild theodolites, such as T 2000, T 2000 S, T 2 etc.

The infra-red measuring beam is reflected by a prism at the other end of the line. In the five seconds that it takes, the DI 1000 adjusts the signal strength to optimum level, makes 2048 measurements on two frequencies, carries out a full internal calibration, computes and displays the result. In the tracking mode 0.3 second updates follow the initial 3- second measurement. The whole sequence is automatic. One has to simply point to the reflector, touch a key and read the result.

The Wild modular system ensures full compatibility between theodolites and Distomats. The DI 1000 fits T 1, T 16 and T 2 optical theodolites, as shown in Fig. 3.10 (*a*). An optional key board can be used. It also combines with Wild T 1000 electronic theodolite and the Wild T 2000 informatics theodolite to form fully electronic *total station* [Fig. 3.10 (*b*)]. Measurements, reductions and calculations are carried out automatically. The DI 1000 also connects to the GRE 3 data terminal [Fig. 3.10 (*c*)]. If the GRE 3 is connected to an electronic theodolite with DI 1000, all information is transferred and recorded at the touch of a single key. The GRE can be programmed to carry out field checks and computations.

When DI 1000 distomat is used separately, it can be controlled from its own key board. There are only three keys on the DI 1000, each with three functions, as shown Fig. 3.11. Colour coding and a logical operating sequence ensure that the instrument is easy to use. The keys control all the functions. There are no mechanical switches. The liquid-crystal display is unusually large for a miniaturized EDM. Measured distances are presented clearly and unambiguously with appropriate symbols for slope, horizontal distance, height and setting out. In test mode, a full check is provided of the display, battery power and return signal strength. An audible tone can be activated to indicate return of signal. Scale (ppm) and additive constant (mm) settings are displayed at the start of each measurement.

Input of ppm takes care of any atmospheric correction, reduction to sea level and projection scale factor. The mm input corrects for the prism type being used. The microprocessor permanently stores ppm and mm values and applies them to every measurement. Displayed heights are corrected for earth curvature and mean refraction.

DI 1000 is designed for use as the standard measuring tool in short range work. A single prism reflector is sufficient for most tasks. For occasional longer distance (upto 800 m), a three prism reflector can be used. The power is fed from NiCd rechargeable batteries.

2. Distomat DI 5S

Wild DI 5S is a medium range infra-red EDM controlled by a small powerful microprocessor, It is multipurpose EDM. The 2.5 km range to single prism covers all short-range requirements: detail, cadastral, engineering, topograhic survey, setting out, mining, tunnelling etc. With its 5 km range to 11 prisms, it is ideal for medium-range control survey : traversing, trigonometrical heighting, photogrammetric control, breakdown of triangulation and GPS networks etc. Finely tuned opto-electronics, a stable oscillator, and a microprocessor that continuously evaluates the results, ensure the high measuring accuracy of 3 mm + 2 ppm standard deviation is standard measuring mode and 10 m + 2 ppm standard deviation in tracking measuring mode.

Fig. 3.12 shows the view of DI 5S. It has three control keys, each with three functions. There are no mechanical switches. A powerful microprocessor controls the DI 5S. Simply touch the DIST key to measure. Signal attenuation is fully automatic. Typical measuring time is 4 seconds. In tracking mode, the measurement repeats automatically every second. A break in the measuring beam due to traffic etc., does not affect the accuracy. A large, liquid-crystal display shows the measured distance clearly and unambiguously throughout the entire measuring range of the instrument. Symbols indicate the displayed values. A series of dashes shows the progress of the measuring cycle. A prism constant from – 99 mm to + 99 mm can be input for the prism type being used. Similarly, ppm values from – 150 ppm to + 150 ppm can be input for automatic compensation for atmospheric conditions, height above sea level and projection scale factor. These values are stored until replaced by new values. The microprocessor corrects every measurement automatically.

DI 5S can be also fitted to Wild electronic theodolites T 1000 and T 2000 [Fig. 3.13 (*a*)] or to Wild optical theodolites T 1, T 16, T 2, [Fig. 3.13 (*b*)]. The infra-red measuring beam is parallel to the line of signal. Only a single pointing is needed for both angle and distance measurements. When fitted to an optical theodolite, an optional key board [Fig. 3.13 (*b*] covert it to efficient low cost effective total station. The following parameters are directly obtained for the corresponding input values (Fig. 3.14):

(*a*) Input the vertical angle for
- (*i*) Horizontal distance
- (*ii*) Height difference corrected for earth curvature and mean refraction.

(*b*) Input the horizontal angle for
- (*i*) Coordinate differences ΔE and ΔN.

(*c*) Input the distance to be set out for
- (*i*) ΔD, the amount by which the reflector has to be moved forward or back.

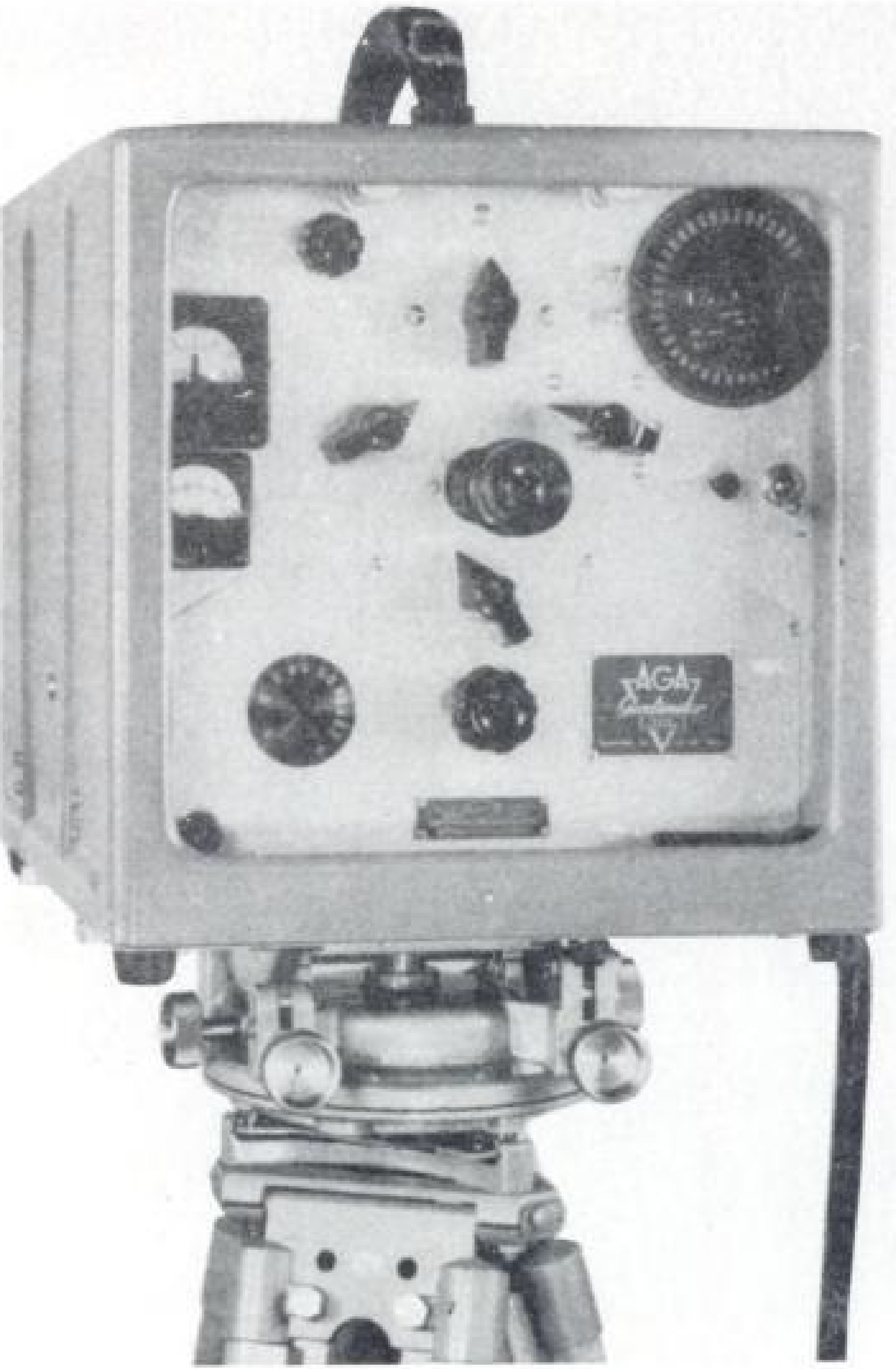


FIG. 3.7. THE MODEL-4 GEODIMETER
(PHOTO BY COURTESY OF M/S AGA, STOCKHOLM)

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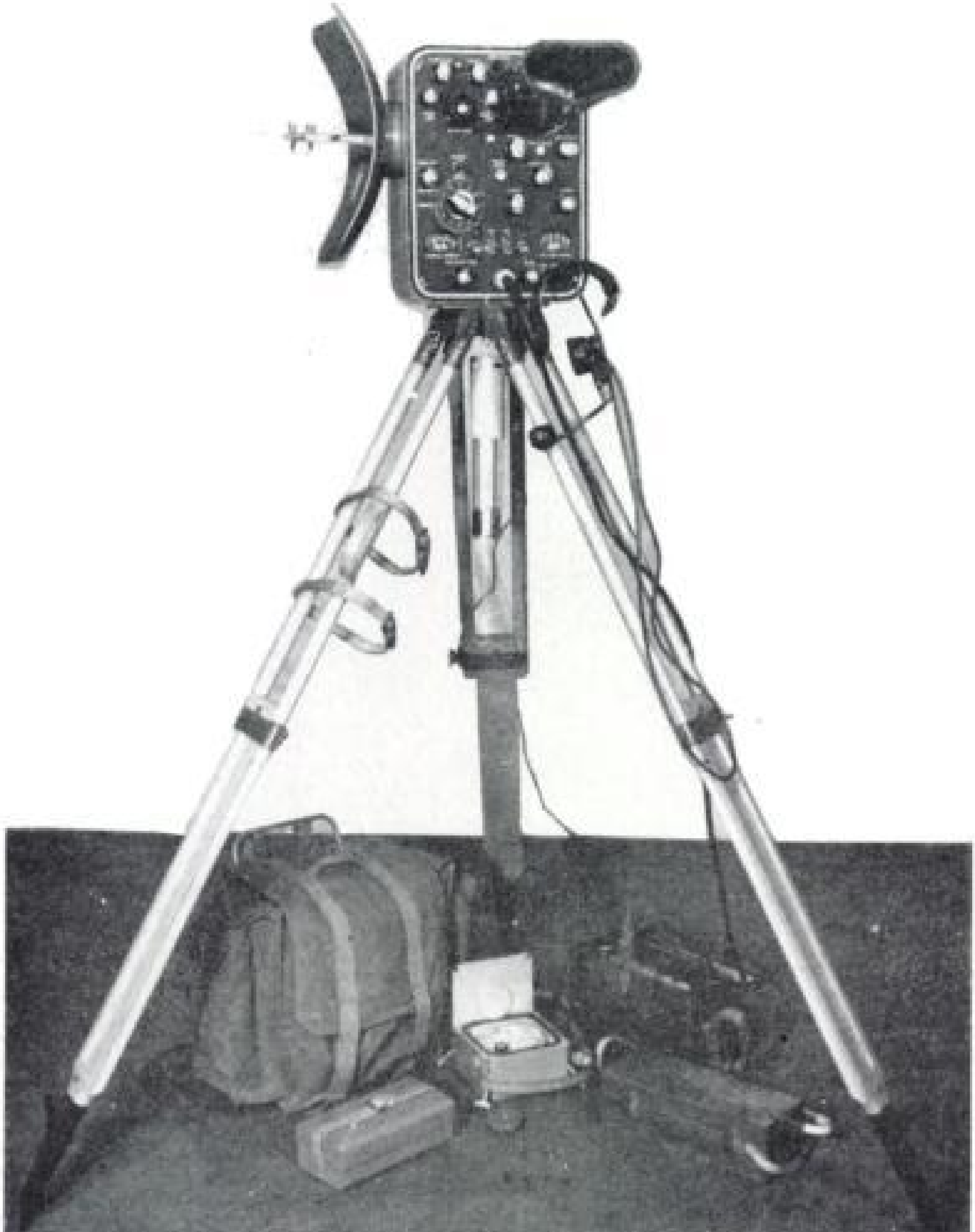

FIG. 3.8 THE TELLUROMETER MODEL MRA-2
(M/S COOKE, TROUGHTON & SIMMS LTD.)

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FIG. 3.10. WILD DI 1000 DISTOMAT
FITTED TO WILD OPTICAL AND ELECTRONIC THEODOLITES

FIG. 3.11. CONTROL KEYS AND DISPLAY PANEL OF DI 1000 DISTOMAT

(TO FACE PAGE 227)

Pair of displayed values. The panel directly displays angles, distances, heights and co-ordinates of the observed point where the signal (reflector prism) is kept (Fig. 3.26). Height above datum and station co-ordinates can be entered and stored.

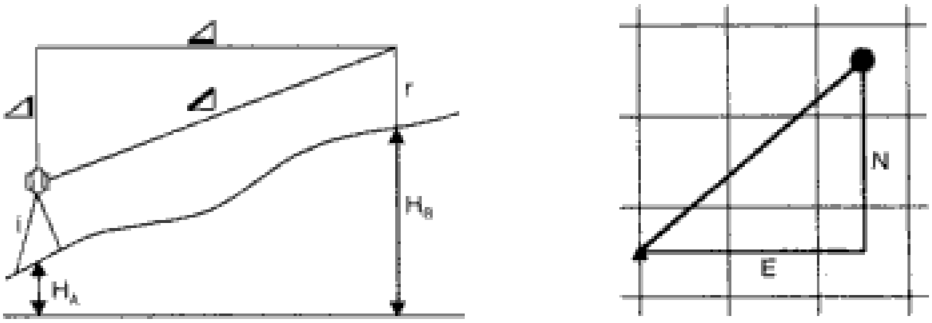

FIG. 3.26.

The following pairs are displayed :

(*i*)	Hz circle	V circle
(*ii*)	Hz circle	Horizontal distance
(*iii*)	Height difference	Height above datum
(*iv*)	Slope distance	V circle
(*v*)	Easting	Northing.

Remote object height (ROH). The direct height readings of inaccessible objects, such as towers and power lines, the height difference and height above datum changes with telescope. However, both the pairs of values are displayed automatically. The microprocessor applies the correction for earth curvature and mean refraction. Corrected heights are displayed.

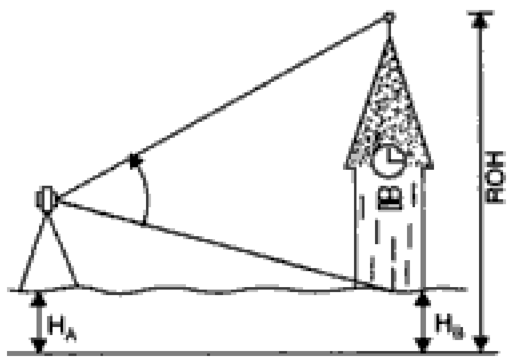

FIG. 3.27. DETERMINATION OF ROH.

Traversing program. The coordinates of the reflector and the bearing on the reflector can be stored for recall at the next set-up. Thus, traverse point coordinates are available in the field and closures can be verified immediately.

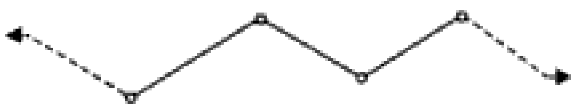

FIG. 3.28. TRAVERSING.

Setting out for direction, distance and height. The required direction and horizontal distance can be entered. The instrument displays:

(*i*) The angle through which the theodolite has to be turned.

(*ii*) The amount by which the reflector has to be moved.

And by means of remote object height (ROH) capability, markers can be placed at the required height above datum.

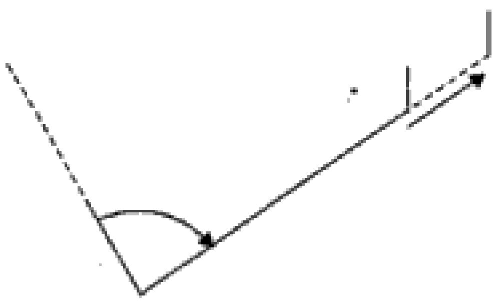

FIG. 3.29. SETTING OUT.

Setting out can be fully automated with GRE 3 data terminal. The bearings and distances to the points to be set out are computed from the stored coordinates and transferred automatically to the TC 2000 *total station*.

Differences in Hz and V. For locating targets and for real time comparisons of measurements in deformation and monitoring surveys, it is advantageous to display angular differences in the horizontal and vertical planes between a required direction and the actual telescope pointing.

3.8. TOTAL STATION

A total station is a combination of an electronic theodolite and an electronic distance meter (EDM). This combination makes it possible to determine the coordinates of a reflector by aligning the instruments cross-hairs on the reflector and simultaneously measuring the vertical and horizontal angles and slope distances. A micro-processor in the instrument takes care of recording, readings and the necessary computations. The data is easily transferred to a computer where it can be used to generate a map. Wild, 'Tachymat' TC 2000, described in the previous article is one such *total station* manufactured by M/s Wild Heerbrugg.

As a teaching tool, a total station fulfills several purposes. Learning how to properly use a total station involves the physics of making measurements, the geometry of calculations, and statistics for analysing the results of a traverse. In the field, it requires team work, planning, and careful observations. If the total station is equipped with data-logger it also involves interfacing the data-logger with a computer, transferring the data, and working with the data on a computer. The more the user understands how a total station works, the better they will be able to use it.

Fundamental measurements : When aimed at an appropriate target, a total station measures three parameters (Fig. 3.31)

1. The *rotation* of the instrument's optical axis from the instrument north in a horizontal plane : i.e. *horizontal angle*

2. The *inclination* of the optical axis from the local vertical i.e. *vertical angle*.

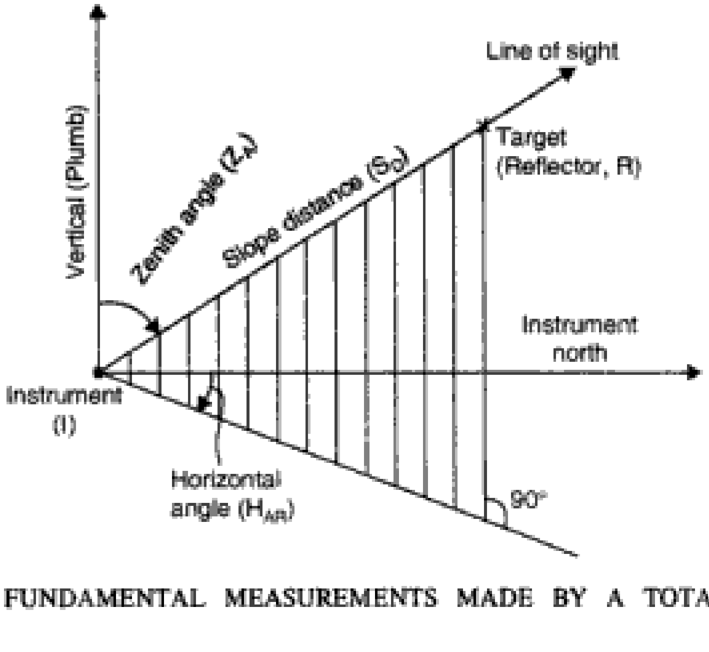

FIG. 3.31. FUNDAMENTAL MEASUREMENTS MADE BY A TOTAL STATION

vertical angles accordingly. The compensator can only make small adjustments, so the instrument still has to be well leveled. If it is too far out of level, the instrument will give some kind of "tilt" error message.

Because of the compensator, the instrument has to be pointing exactly at the target in order to make an accurate vertical angle measurement. If the instrument is not perfectly leveled then as you turn the instrument about the vertical axis (i.e., change the horizontal angle) the vertical angle displayed will also change.

3. Slope Distance : The instrument to reflector distance is measured using an Electronic Distance Meter (EDM). Most EDM's use a Gallium Aresnide Diode to emit an infrared light beam. This beam is usually modulated to two or more different frequencies. The infrared beam is emitted from the total station, reflected by the reflector and received and amplified by the total station. The received signal is then compared with a reference signal generated by the instrument (the same signal generator that transmits the microwave pulse) and the phase-shift is determined. This phase shift is a measure of the travel time and thus the distance between the total station and the reflector.

This method of distance measurement is not sensitive to phase shifts larger than one wavelength, so it cannot detect instrument-reflector distances greater than 1/2 the wave length (the instrument measures the two-way travel distance). For example, if the wavelength of the infrared beam was 4000 m then if the reflector was 2500 m away the instrument will return a distance of 500 m.

Since measurement to the nearest millimeter would require very precise measurements of the phase difference, EDM's send out two (or more) wavelengths of light. One wavelength may be 4000 m, and the other 20 m. The longer wavelength can read distances from 1 m to 2000 m to the nearest meter, and then the second wavelength can be used to measure distances of 1 mm to 9.999 m. Combining the two results gives a distance accurate to millimeters. Since there is overlap in the readings, the meter value from each reading can be used as a check.

For example, if the wavelengths are $\lambda_1 = 1000$ m and $\lambda_2 = 10$ m, and a target is placed 151.51 metres away, the distance returned by the λ_1 wavelength would be 151 metres, the λ_2 wavelength would return a distance of 1.51 m. Combining the two results would give a distance of 151.51 m.

Basic calculations

Total Stations only measure three parameters : *Horizontal Angle, Vertical Angle,* and *Slope Distance*. All of these measurements have some error associated with them, however for demonstrating the geometric calculations, we will assume the readings are without error.

Horizontal distance

Let us use symbol *I* for instrument (total station) and symbol *R* for the reflector. In order to calculate coordinates or elevations it is first necessary to convert the slope distance to a horizontal distance. From inspection of Fig. 3.32 the horizontal distance (H_D) is

$$H_D = S_D \cos (90° - Z_A) = S_D \sin Z_A \qquad \ldots(1) \quad \ldots(3.4)$$

where S_D is the slope distance and Z_A is the zenith angle. The horizontal distance will be used in the coordinate calculations.

Vertical distance

We can consider two vertical distances. One is the *Elevation Difference (dZ)* between the two points on the ground. The other is the *Vertical Difference* (V_D) between the tilting axis of the instrument and the tilting axis of the reflector. For elevation difference calculation we need to know the height of the tilting-axis of the instrument (I_H), that is the height of the center of the telescope, and the height of the center of the reflector (R_H)

The way to keep the calculation straight is to imagine that you are on the ground under the instrument (Fig. 3.32). If you move up the distance I_H , then travel horizontally to a vertical line passing through the reflector then up (or down) the vertical distance (V_D) to the reflector, and then down to the ground (R_H) you will have the elevation difference dZ between the two points on the ground. This can be written as

$$dZ = V_D + (I_H - R_H) \qquad \ldots(2) \quad \ldots(3.5)$$

The quantities I_H and R_H are measured and recorded in the field. The vertical difference V_D is calculated from the vertical angle and the slope distance (see Fig. 3.32)

$$V_D = S_D \sin(90° - Z_A) = S_D \cos Z_A \qquad \ldots(3) \quad \ldots(3.6)$$

Substituting this result (3) into equation (2) gives

$$dZ = S_D \cos Z_A + (I_H - R_H) \qquad \ldots(4) \quad \ldots(3.7)$$

where dZ is the change in elevation with respect to the ground under the total station. We have chosen to group the instrument and reflector heights. Note that if they are the same then this part of the equation drops out. If you have to do calculations by hand it is convenient to set the reflector height the same as the instrument height.

If the instrument is at a known elevation, I_Z , then the elevation of the ground beneath the reflector, R_Z , is

$$R_Z = I_Z + S_D \cos Z_A + (I_H - R_H) \qquad \ldots(5) \quad \ldots(3.8)$$

Coordinate calculations

So far we have only used the vertical angle and slope distance to calculate the elevation of the ground under the reflector. This is the Z-coordinate (or elevation) of a point. We

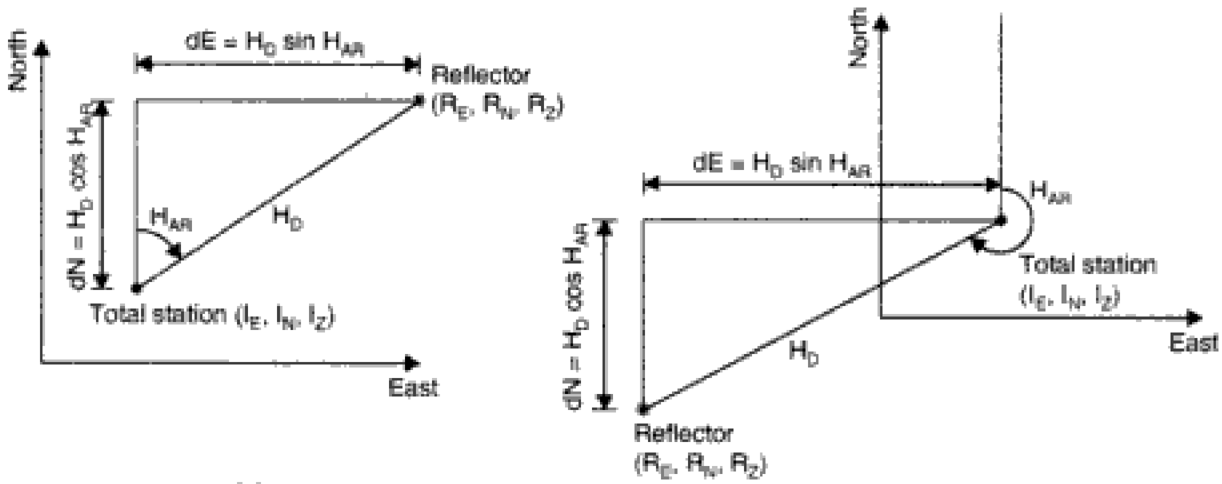

FIG. 3.33. COMPUTATION OF EAST AND NORTH COODINATES OF THE REFLCTOR

now want to calculate the *X*- (or East) and *Y*- (or North) coordinates. The zero direction set on the instrument is instrument north. This may not have any relation on the ground to true, magnetic or grid north. The relationship must be determined by the user. Fig. 3.33 shows the geometry for two different cases, one where the horizontal angle is less than 180° and the other where the horizontal angle is greater than 180°. The sign of the coordinate change [positive in Figure 3.33 (*a*) and negative in Fig. 3.33 (*b*)] is taken care of by the trigonometric functions, so the same formula can be used in all cases. Let us use symbol *E* for easting and *N* for northing, and symbol *I* for the instrument (i.e total station) and *R* for the reflector. Let R_E and R_N be the easting and northing of the reflector and I_E and I_N be the easting and northing of the instrument (i.e. total station)

From inspection of Fig. 3.33 the coordinates of the relector relative to the total station are

$dE =$ Change in Easting $= H_D \sin H_{AR}$

$dN =$ Change in Northing $= H_D \cos H_{AR}$

where H_D is the horizontal distance and H_{AR} is the horizontal angle measured in a clockwise sense from instrument north. In terms of fundamental measurments (i.e. equation 1) this is the same as

$$dE = S_D \sin Z_A \sin H_{AR} \quad \ldots(3.9)$$

$$dN = S_D \cos (90° - Z_A) \cos H_{AR} = S_D \sin Z_A \cos H_{AR} \quad \ldots(3.10)$$

If the easting and northing coordinates of the instrument station are known (in grid whose north direction is the same as instrument north) then we simply add the instrument coordinates to the change in easting and northing to get the coordinates of the reflector. The coordinates of the ground under the reflector, in terms of fundamental measurments are :

$$R_E = I_E + S_D \sin Z_A \sin H_{AR} \quad \ldots(3.11)$$

$$R_N = I_N + S_D \sin Z_A \cos H_{AR} \quad \ldots(3.12)$$

$$R_Z = I_Z + S_D \cos Z_A + (I_H - R_H) \quad \ldots(3.13)$$

where I_E , I_N, and I_Z are the coordinates of the total station and R_E , R_N , R_Z are the coordinates of the ground under the reflector. These calculations can be easily done in a spreadsheet program.

All of these calculations can be made within a total station, or in an attached electronic notebook. Although it is tempting to let the total station do all the calculations, it is wise to record the three fundamental measurements. This allows calculations to be checked, and provides the basic data that is needed for a more sophisticated error analysis.

4

Remote Sensing

4.1. INTRODUCTION

Remote sensing is broadly defined as science and art of collecting information about objects, area or phenomena from distance without being in physical contact with them. In the present context, the definition of remote sensing is restricted to mean the process of acquiring information about any object without physically contacting it in any way regardless of whether the observer is immediately adjacent to the object or millions of miles away. Human eye is perhaps the most familiar example of a remote sensing system. In fact, sight, smell and hearing are all rudimentary forms of remote sensing. However, the term remote sensing is restricted to methods that employ electromagnetic energy (such as light, heat, microwave) as means of detecting and measuring target characteristics. Air craft and satellites are the common *platforms* used for remote sensing. Collection of data is usually carried out by highly sophisticated *sensors* (i.e. camera, multispectral scanner, radar etc.). The information carrier, or communication link is the electromagnetic energy. Remote sensing data basically consists of wave length intensity information by collecting the electromagnetic radiation leaving the object at specific wavelength and measuring its intensity. Photo interpretation can at best be considered as the *primitive form* of remote sensing. Most of the modern remote sensing methods make use of the reflected infrared bands, thermal infrared bands and microwave portion of the electromagnetic spectrum.

Classification of remote sensing

Remote sensing is broadly classified into two categories

(*i*) Passive remote sensing and (*ii*) Active remote sensing

Passive remote sensing : It uses sun as a source of EM energy and records the energy that is naturally radiated and/or reflected from the objects.

Active remove sensing : It uses its own source of EM energy, which is directed towards the object and return energy is measured.

4.2. HISTORICAL SKETCH OF REMOTE SENSING

Remote sensing became possible with the invention of camera in the nineteenth century. Astronomy was one of the first fields of science to exploit this technique. Although, it was during the first World War that free flying aircrafts were used in a remote sensing role, but the use of remote sensing for environmental assessment really became established after the second World War. It not only proved the value of aerial photography in land reconnaissance and mapping, but had also driven technological advances in air borne camera design, film characteristics and photogrammetic analysis.

However, upto early 1960's air borne missions were one of the expensive surveys, providing data for relatively small area at a single instant of time. Moreover, all the photographs were black and white. Colour photography came into existence after the invention of infrared films in 1950. From about 1960, remote sensing underwent a major development when it extended to space and sensors began to be placed in space. From 1970's started the new era of remote sensing. The first designated earth resources satellite was launched in July 1972, originally named as ERTS-1 which is now referred as Landstat-1. It was designed to acquire data from earth surface as systematic, repetitive and multi-spectral basic. The first Radar remote sensing satellite, SEASAT, was launched in 1978.

Prior to mid 1980's, the majority of satellites had been deployed by USA and USSR. France launched first of SPOT series in 1985 and in 1988, first Indian Remote Sensing Satellite (IRS-1A) was put into orbit. Satellites launched by Japan include JERS (Japanese Earth Resources Satellite) and MOS (Marine Observation Satellite). Radar satellites have been launched in 1991 and 1995 by European Consortium (ERS) and by Canada in 1995 (RADARSAT).

4.3. IDEALIZED REMOTE SENSING SYSTEM

An idealised remote sensing system consists of the following stages (Fig. 4.1)

1. Energy source
2. Propagation of energy through atmosphere
3. Energy interaction with earth's surface features.
4. Airborne/space borne sensors receiving the reflected and emitted energy
5. Transmission of data to earth station and generation of data produce.
6. Multiple-data users

1. The energy source : The uniform energy source provides energy over all wave lengths. The passive RS system relies on sun as the strongest source of EM energy and measures energy that is either reflected and or emitted from the earth's surface features. However, active RS systems use their own source of EM energy.

2. Propagation of energy from the atmosphere : The EM energy, from the source pass through the atmosphere on its way to earth's surface. Also, after reflection from the earth's surface, it again pass through the atmosphere on its way to sensor. The atmosphere modifies the wave length and spectral distribution of energy to some extent, and this modification varies particularly with the wave length.

3. Interaction of energy with surface features of the earth : The interaction of EM energy, with earth's surface features generates reflected and/or emitted *signals* (spectral response patterns or signatures). The spectral response patterns play a central role in detection, identification and analysis of earth's surface material.

4. Air borne/space borne sensors : Sensors are electromagnetic instruments designed to receive and record retransmitted energy. They are mounted on satellites, aeroplanes or even balloons. The sensors are highly sensitive to wave lengths, yielding data on the absolute brightness from the object as a function of wavelength.

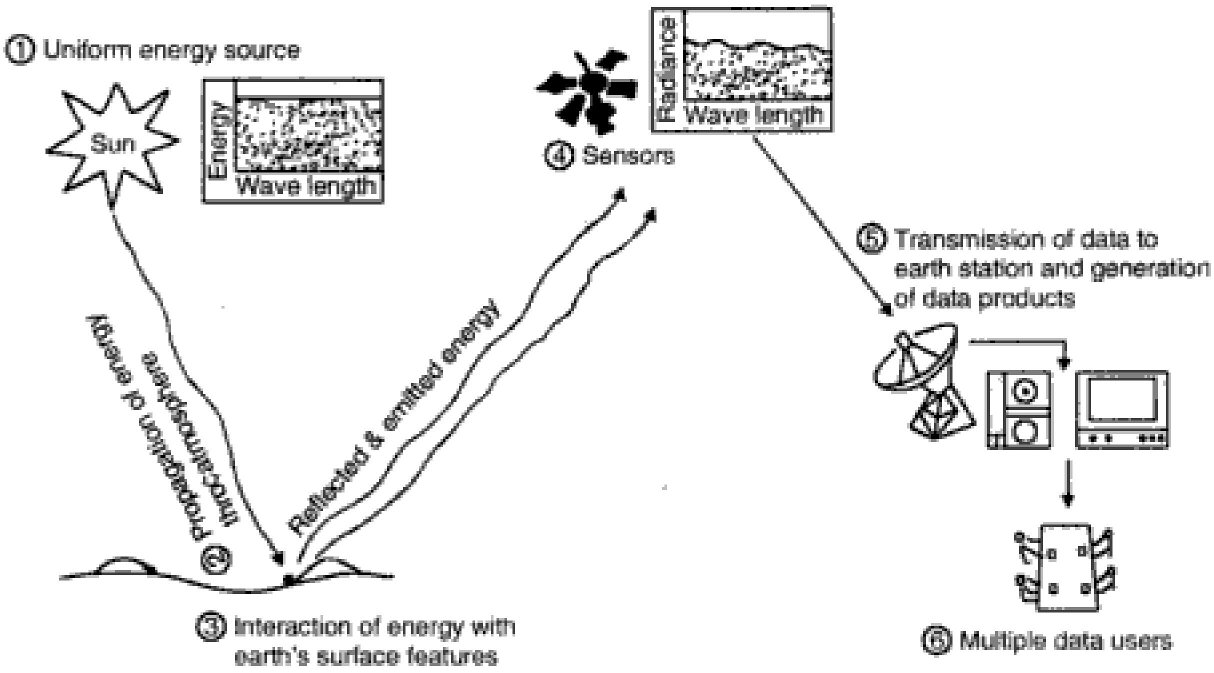

FIG. 4.1 IDEALISED REMOTE SENSING SYSTEM

5. Transmission of data to earth station and data product generation : The data from the sensing system is transmitted to the ground based earth station along with the telemetry data. The real-time (instantaneous) data handling system consists of high density data tapes for recording and visual devices (such as television) for quick look displays. The *data products* are mainly classified into two categories :

(*i*) Pictorial or Photographic product (analogue)

and (*ii*) Digital product

6. Multiple data users : The multiple data users are those who have knowledge of great depth, both of their respective disciplines as well as of remote sensing data and analysis techniques. The same set of *data* becomes various forms of *information* for different users with the understanding of their field and interpretation skills.

4.4. BASIC PRINCIPLES OF REMOTE SENSING

Remote sensing employ electromagnetic energy and to a great extent relies on the interaction of electromagnetic energy with the matter (object). It refers to the sensing of EM radiation, which is reflected, scattered or emitted from the object.

4.4.1. ELECTROMAGNETIC ENERGY

It is a form of energy that moves with the velocity of light (3×10^8 m/sec) in a harmonic pattern consisting of sinusoidal waves, equally and repetitively spaced in time. It has two fields : (*i*) electrical field and (*ii*) magnetic field, both being orthogonal to each other. Fig. 4.2 show the electromagnetic wave pattern, in which the electric components are vertical and magnetic components are horizontal.

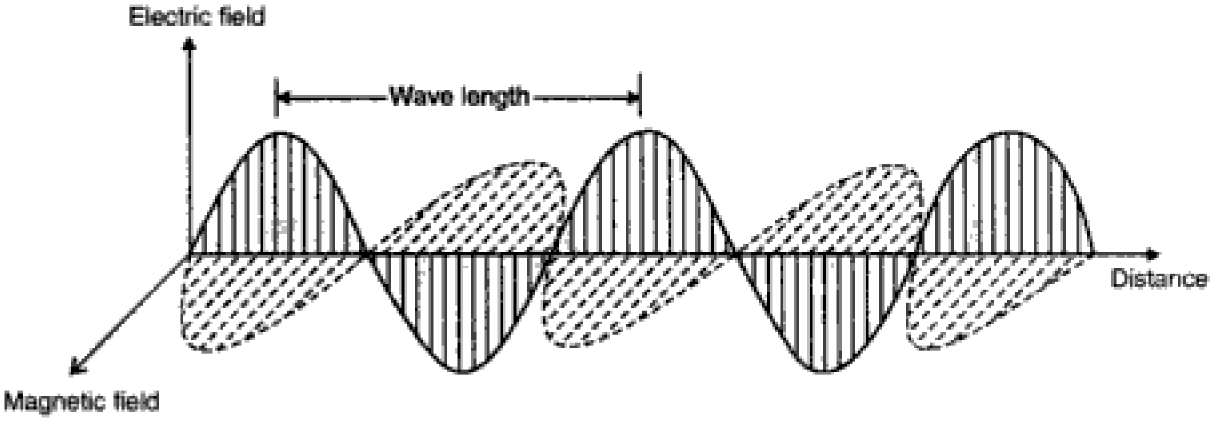

FIG. 4.2

Electromagnetic energy consists of *photons* having particle like properties such as energy and momentum. The EM energy is characterised in terms of velocity c ($\approx 3 \times 10^8$ m/s), wave length λ and frequency f. These parameters are related by the equation :

$$\lambda = \frac{c}{f} \qquad \ldots(4.1)$$

where λ = wave length, which is the distance between two adjacent peaks. The wave lengths sensed by many remote sensing systems are extremely small and are measured in terms of micro meter (μ m or 10^{-6} m) or nanometer (nm or 10^{-9} m)

f = frequency, which is defined as the number of peaks that pass any given point in one second and is measured in Hertz (Hz).

The *amplitude* is the maximum value of the electric (or magnetic) field and is a measure of the amount of energy that is transported by the wave.

Wave theory concept explains how EM energy propagates in the form of a wave. However, this energy can only be detected when it interacts with the matter. This interaction suggests that the energy consists of many discrete units called *photons* whose energy (Q) is given by :

$$Q = h.f = \frac{h.c}{\lambda} \qquad \ldots(4.2)$$

where h = Plank's constant $= 6.6252 \times 10^{-34}$ J-s

The above equation suggests that shorter the wave length of radiation, more is the energy content.

4.4.2. ELECTROMAGNETIC SPECTRUM

Although *visible light* is the most obvious manifestation of EM radiation, other forms also exist. EM radiation can be produced at a range of wave lengths and can be categorised according to its position into discrete regions which is generally referred to *electro-magnetic spectrum*. Thus the electromagnetic spectrum is the continuum of energy that ranges from meters to nano-meters in wave length (Fig. 4.3) travels at at the speed of light and propagates through a vacuum like the outer space (Sabine, 1986). All matter radiates a range of electromagnetic

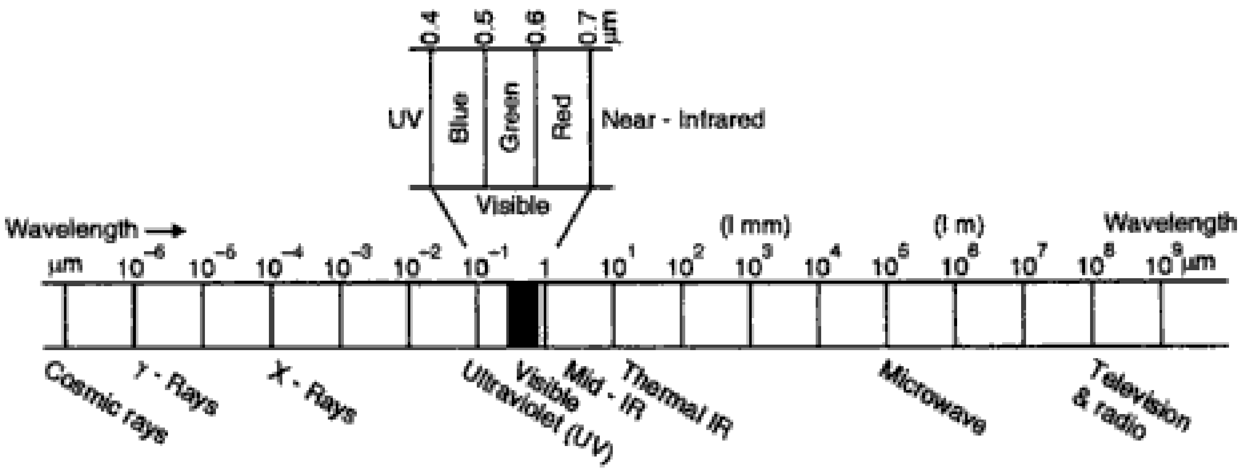

FIG. 4.3. ELECTROMAGNETIC SPECTRUM

energy, with the peak intensity shifting toward progressively shorter wave length at an increasing temperature of the matter. In general, the wave lengths and frequencies vary from shorter wavelength-high frequency cosmic waves to long wave length-low frequency radio waves (Fig. 4.3 and Table 4.1).

TABLE 4.1. ELECTROMAGNETIC SPECTRAL REGIONS (SABINE, 1987)

Region	*Wave length*	*Remarks*
1. Gamma ray	< 0.03 nm	Incoming radiation is completely absorbed by the upper atmosphere and is not available for remote sensing
2. X-ray	0.03 to 3.0 nm	Completely absorbed by atmosphere. Not employed in remote sensing
3. Ultraviolet	0.3 to 0.4 µ m	Incoming wavelengths less than 0.3 µ m are completely absorbed by ozone in the upper atmosphere
4. Photo graphic UV band	0.3 to 0.4 µm	Transmitted through atmosphere. Detectable with film and photodetectors, but atmospheric scattering is severe
5. Visible	0.4 to 0.7 µm	Images with film and photo detectors. Includes reflected energy peak of earth at 0.5 µm.
6. Infrared	0.7 to 1.00 µm	Interaction with matter varies with wave length. Atmospheric transmission windows are separated.
7. Reflected IR band	0.7 to 3.0 µm	Reflected solar radiation that contains information about thermal properties of materials. The bands from 0.7 to 0.9 µm is detectable with film and is called the photographic IR band.
8. Thermal IR	3 to 5 µ m	Principal atmospheric windows in the 8 to 14 µm thermal region. Images at these wavelengths are acquired by optical mechanical scanners and special vidicon systems but not by film. Microwave 0.1 to 30 cm longer wavelength can penetrate clouds, fog and rain. Images may be acquired in the active or passive mode
9. Radar	0.1 to 30 cm	Active form of microwave remote sensing. Radar images are acquired at various wavelength bands.
10. Radio	> 30 cm	Longest wavelength portion of electromagnetic spectrum. Some classified radars with very long wavelengths operate in this region.

Earths atmosphere absorbs energy in Gamma ray, X-ray and most of the ultra-violet region. Therefore, these regions are not used for remote sensing. *Remote sensing deals with energy in visible, infrared, thermal and microwave regions*. These regions are further subdivided into bands such as blue, green, red (in visible region), near infrared, mid-infrared, thermal and microwave etc. It is important to realize that significant amount of remote sensing performed within infrared wave length is not related to heat. It is photographic infrared at a slightly longer wave length (invisible to human eye) than red. Thermal infrared remote sensing is carried out at longer wave lengths.

4.4.3. WAVE LENGTH REGIONS AND THEIR APPLICATIONS IN REMOTE SENSING

Fig 4.3 shows the EM spectrum which is divided into discrete regions on the basis of wavelength. Romote sensing mosthy deals with energy in visible (Blue, green, red) infrared (near-infrared, mid-infrared, thermal-infrared) regions Table 4.2 gives the wave length region along with the principal applications in remote sensing. Energy reflected from earth during daytime may be recorded as a function of wavelength. The maximum amount of energy is reflected at 0.5 μ m, called the *reflected energy peak*. Earth also radiates energy both during day and night time with maximum energy radiated at 9.7 μ m, called *radiant energy peak*.

TABLE 4.2. WAVE LENGTH REGIONS AND THEIR APPLICATIONS IN REMOTE SENSING

Region	***Wave length*** **(μm)**	***Principal Applications***
(a) Visible Region		
1. Blue	0.45 – 0.52	Coastal morphology and sedimentation study, soil and vegetation differentiation, coniferous and deciduous vegetation discrimination.
2. Green	0.52 – 0.60	Vigor assessment, Rock and soil discrimination, Turbidity and bathymetry studies.
3. Red	0.63 – 0.69	Plant species differentiation
(b) Infrared Region		
4. Near Infrared	0.76 – 0.90	Vegetation vigour, Biomass, delineation of water features, land forms/geomorphic studies.
5. Mid-infrared	1.55 – 1.75	Vegetation moisture content, soil moisture content, snow and cloud differentiation
6. Mid-infrared	2.08 – 2.35	Differentiation of geological materials & soils
7. Thermal IR	3.0 – 5.0	For hot targets, i.e. Fires and volcanoes
8. Thermal IR	10.4 – 12.5	Thermal sensing, vegetation discrimination, volcanic studies.

4.4.4. CHARACTERISTICS OF SOLAR RADIATION

All objects above 0°K emit EM radiation at all wavelengths due to conversion of heat energy into EM energy. All stars and planets emit radiation. Our chief star, the Sun, is almost a spherical body with a diameter of 1.39×10^6 km. The continuous conversion of hydrogen to helium which is the main constituent of the Sun, generates the energy that is radiated from the outer layers. Passive remote sensing uses Sun as its source of

EM radiation. Sun is the strongest source of radiant energy and can be approximated by a body source of temperature 5750 – 6000° K. Although Sun produces EM radiation across a range of wave lengths, the amount of energy it produces is not evenly distributed along this range. Approximately 43% is radiated within the visible wavelength (0.4 to 0.7 μm), and 48% of the energy is transmitted at wave length greater than 0.7 μ m, mainly within infrared range.

If the energy received at the edge of earth's atmosphere were distributed evenly over the earth, it would give an average incident flux density of 1367 W/m^2. This is known as the *solar constant*. Thirty five percent of incident radiant flux is reflected back by the earth. This includes the energy reflected by clouds and atmosphere. Seventeen percent of it is absorbed by the atmosphere while 48% is absorbed by the earth's surface materials (Mather, 1987).

4.4.5. BASIC RADIATION LAWS

Stefan-Boltzmann law

All bodies above temperature of 0° K emit EM radiation and the energy radiated by an object at a particular temperature is given by

$$M = \sigma T^4 \qquad \ldots(4.3)$$

where M = total *spectral exitance* of a black body, W/m^2

σ = Stefan-Boltzmann constant = 5.6697×10^{-11} $W/m^2/K^4$

T = absolute temperature

A black body is a hypothetical ideal radiator that totally absorbs and remits all energy incident upon it. The distribution of spectral exitance for a black body at 5900° K closely approximates the sun's spectral exitance curve (Mather 1987), while the earth can be considered to act like a black body with a temperature of 290 °K.

Wien's displacement law

The wave length at which a black body radiates its maximum energy is inversely proportional to temperature and is given by

$$\lambda_m = \frac{A}{T} \qquad \ldots(4.4)$$

λ_m = wave length of maximum spectral exitance

A = Wien's constant = 2.898×10^{-3} mK

T = temperature of the body

As the temperature of the black body increases, the dominant wave length of the emitted radiation shifts towards shorter wave length.

3. ***Plank's law***

The total energy radiated in all directions by unit area in unit time in a spectral band for a given by is given by

$$M_\lambda = \frac{C_1}{\lambda^5 \cdot e^{(C_2/\lambda T) - 1}} \qquad \ldots(4.5)$$

where M_λ = Spectral exitance per unit wave length

C_1 = First radiation constant $= 3.742 \times 10^{-16}$ W/m^2

C_2 = Second radiation constant $= 1.4388 \times 10^{-2}$ mK

It enables to assess the proportion of total radiant exitance within selected wave length.

4.5. EM RADIATION AND THE ATMOSPHERE

In remote sensing, EM radiation must pass through atmosphere in order to reach the earth's surface and to the sensor after reflection and emission from earth's surface features. The water vapour, oxygen, ozone, CO_2, aerosols, etc. present in the atmosphere influence EM radiation through the mechanism of (*i*) scattering, and (*ii*) absorption.

Scattering

It is unpredictable diffusion of radiation by molecules of the gases, dust and smoke in the atmosphere. Scattering reduces the image contrast and changes the spectral signatures of ground objects. Scattering is basically classified as (*i*) selective, and (*ii*) non-selective, depending upon the size of particle with which the electromagnetic radiation interacts. The *selective scatter* is further classified as (*a*) *Rayleigh's scatter*, and (*b*) *Mies scatter*.

Rayleigh's scatter: In the upper part of the atmosphere, the diameter of the gas molecules or particles is much less than the wave length of radiation. Hence haze results on the remotely sensed imagery, causing a bluish grey cast on the image, thus reducing the contrast. Lesser the wave length, more is the scattering.

Mie's scatter : In the lower layers of atmosphere, where the diameter of water vapour or dust particles approximately equals wave length of radiation, Mie's scatter occurs.

Non-selective scatter : Non-selective scattering occurs when the diameter of particles, is several times more (approximately ten times) than radiation wavelength. For visible wave lengths, the main sources of non-selective scattering are pollen grains, cloud droplets, ice and snow crystals and raindrops. It scatters all wave length of visible light with equal efficiency. It justifies the reason why cloud appears white in the image.

Absorption

In contrast to scattering, atmospheric absorption results the effective loss of energy as a consequence of the attenuating nature of atmospheric constituents, like molecules of ozone, CO_2 and water vapour. Oxygen absorbs in the ultraviolet region and also has an absorption band centered on 6.3 μ m. Similarly CO_2 prevents a number of wave lengths reaching the surface. Water vapour is an extremely important absorber of EM radiation within infrared part of the spectrum.

Atmospheric windows

The amount of scattering or absorption depends upon (*i*) wave length, and (*ii*) composition of the atmosphere. In order to *minimise* the effect of atmosphere, it is essential to choose the regions with high *transmittance*.

The wavelengths at which EM radiations are partially or wholly transmitted through the atmosphere are known as atmospheric windows and are used to acquire remote sensing data.

Typical *atmospheric windows* on the regions of EM radiation are shown in Fig. 4.4.

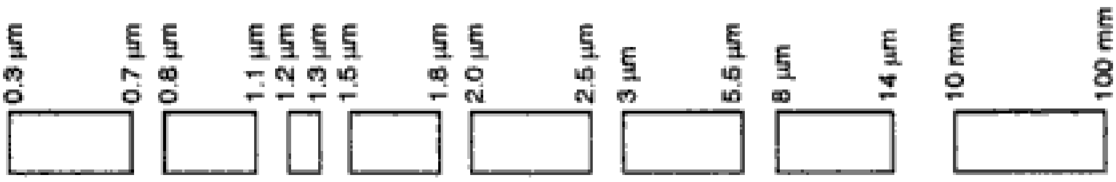

FIG. 4.4 ATMOSPHERIC WINDOWS

The sensors on remote sensing satellites must be designed in such a way as to obtain data within these well defined *atmospheric windows.*

4.6. INTERACTION OF EM RADIATION WITH EARTH'S SURFACE

EM energy that strikes or encounters matter (object) is called *incident radiation*. The EM radiation striking the surface may be (*i*) reflected/scattered, (*ii*) absorbed, and/or (*iii*) transmitted. These processes are not mutually exclusive — EM radiations may be partially reflected and partially absorbed. Which processes actually occur depends on the following factors (1) wavelength of radiation (2) angle of incidence, (3) surface roughness, and (4) condition and composition of surface material.

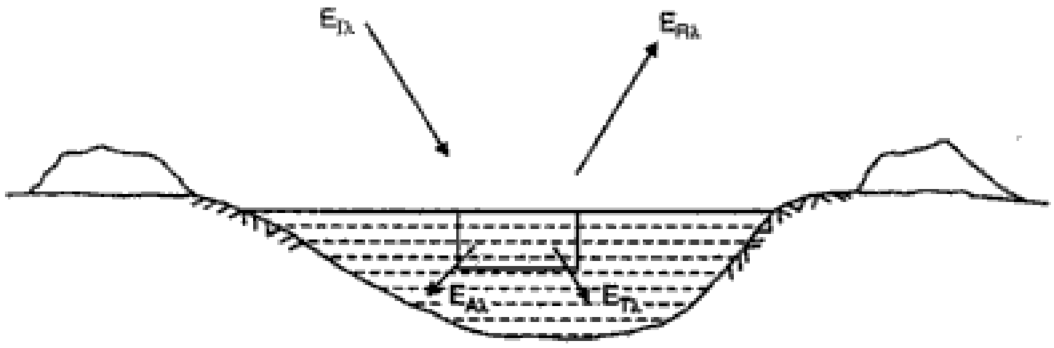

FIG. 4.5. INTERACTION MECHANISM

Interaction with matter can change the following properties of incident radiation: (*a*) Intensity (*b*) Direction (*c*) Wave length (*d*) Polarisation, and (*e*) Phase.

The science of remote sensing detects and records there changes.

The energy balance equation for radiation at a given wave length (λ) can be expressed as follows.

$$E_{I\lambda} = E_{R\lambda} + E_{A\lambda} + E_{T\lambda} \qquad \ldots(4.6)$$

where $E_{I\lambda}$ = Incident energy; $E_{R\lambda}$ = Reflected energy

$E_{A\lambda}$ = Absorbed energy; $E_{T\lambda}$ = Transmitted energy.

The proportion of each fraction ($E_{R\lambda}/E_{A\lambda}/E_{T\lambda}$) will vary for different materials depending upon their composition and condition. Within a given features type, these proportions will vary at different wave lengths, thus helping in discrimination *of different objects*. Reflection, scattering, emission are called surface phenomenon because these are determined by the properties of surface, viz. colour, roughness. Transmission and absorption are called volume

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Indian Remote Sensing Satellites (IRS)

1. Satellite for Earth Observation (SEO-I), now called Bhaskara-I was the first Indian remote sensing satellite launched by a soviet launch vehicle from USSR in June, 1979, into a near circular orbit.

2. SEO-II, (Bhaskara II) was launched in Nov. 1981 from a Soviet cosmodrome.

3. India's first semi-operational remote sensing satellite (IRS) was launched by the Soviet Union in Sept. 1987.

4. The IRS series of satellites launched by the IRS mission are : IRS IA, IRS IB, IRS IC, IRS ID and IRS P4.

4.8. SENSORS

Remote sensing sensors are designed to record radiations in one or more parts of the EM spectrum. Sensors are electronic instruments that receive EM radiation and generate an electric signal that correspond to the energy variation of different earth surface features. The signal can be recorded and displayed as numerical data or an image. The strength of the signal depends upon (*i*) Energy flux, (*ii*) Altitude, (*iii*) Spectral band width, (*iv*) Instantaneous field of view (IFOV), and (*v*) Dwell time.

A *scanning system* employs detectors with a narrow field of view which sweeps across the terrain to produce an image. When photons of EM energy radiated or reflected from earth surface feature encounter the detector, an electrical signal is produced that varies in proportion to the number of photons.

Sensors on board of Indian Remote sensing satellites (IRS)

1. Linear Imaging and Self Scanning Sensor **(LISS I)**

This payload was on board IRS 1A and 1B satellites. It had four bands operating in visible and near IR region.

2. Linear Imaging and Self Scanning Sensor **(LISS II)**

This payload was on board IRS 1A and 1B satellites. It has four bands operating in visible and near IR region.

3. Linear Imaging and Self Scanning Sensor **(LISS III)**

This payload is on board IRS 1C and 1D satellites. It has three bands operating in visible and near IR region and one band in short wave infra region.

4. Panchromatic Sensor **(PAN)**

This payload is on boards IRS 1C and 1D satellites. It has one band.

5. Wide Field Sensor **(WiFS)**

This payload is on boards IRS 1C and 1D satellites. It has two bands operating in visible and near IR region.

6. Modular Opto-Electronic Scanner **(MOS)**

This payload is on board IRS P3 satellite.

7. Ocean Colour Monitor **(OCM)**

This payload is on board IRS P4 satellite. It has eight spectral bands operating in visible and near IR region.

8. Multi Scanning Microwave Radiometer **(MSMR)**

This payload is on board IRS 1D satellite. This is a passive microwave sensor.

4.9. APPLICATIONS OF REMOTE SENSING

Remote sensing affords a practical means for accurate and continuous monitoring of the earth's natural and other resources and of determining the impact of man's activities on air, water and land. The launch of IRS 1C satellite (Dec. 1995) with state of art sensors provided a new dimension and further boosted the applications of space-base remote sensing technology for natural resources management. With the unique combinations of payload, the IRS-1C has already earned the reputation as the '*Satellite for all applications*' IRS-1C/1D carry three imaging sensors (LISS-III, PAN and WiFS) characterised by different resolutions and coverage capabilities. These three imaging sensors provide image data for virtually all levels of applications ranging from cadastral survey to regional and national level mapping. The LISS-III data with 21.2- 23.5 m resolution has significantly improved separability amongst various crops and vegetation types, leading to identification of small fields and better classification accuracy. The frequent availability of data from WiFS payload has helped in monitoring *dynamic phenomena* like vegetation, floods, droughts, forest fire etc.. A major benefit of the multi-sensor IRS-1C/1D payload is the capability to merge the multi spectral LISS-III data, with high resolution PAN imagery. This merger of multispectral and high resolution data facilitates detailed land cover classification and delineation of linear and narrow roads/lanes, structures, vegetation types and parcels of land.

A summary of RS applications is given below, discipline wise.

1. Agriculture

(*i*) Early season estimation of total cropped area
(*ii*) Monitoring crop condition using crop growth profile.
(*iii*) Identification of crops and their coverage estimation in multi-cropped regions.
(*iv*) Crop yield modelling
(*v*) Cropping system/crop rotation studies
(*vi*) Command area management
(*vii*) Detection of moisture stress in crops and quantification of its effect on crop yield
(*viii*) Detection of crop violations
(*ix*) Zoom cultivation—desertification

2. Forestry

(*i*) Improved forest type mapping
(*ii*) Monitoring large scale deforestation, forest fire
(*iii*) Monitoring urban forestry
(*iv*) Forest stock mapping
(*v*) Wild life habitat assessment

3. Land use and soils

(*i*) Mapping land use/cover (level III) at 1 : 25000 scale or better

(*ii*) Change detection
(*iii*) Identification of degraded lands/erosion prone areas
(*iv*) Soil categorisation

4. *Geology*

(*i*) Lithological and structural mapping
(*ii*) Geo morphological mapping
(*iii*) Ground water exploration
(*iv*) Engineering geological studies
(*v*) Geo-environmental studies
(*vi*) Drainage analysis
(*vii*) Mineral exploration
(*viii*) Coal fire mapping
(*ix*) Oil field detection

5. *Urban Land use*

(*i*) Urban land use level IV mapping
(*ii*) Updating of urban transport network
(*iii*) Monitoring urban sprawl
(*iv*) Identification of unauthorised structures.

6. *Water resources*

(*i*) Monitoring surface water bodies frequently and estimation of their spatial extent
(*ii*) Snow-cloud discrimination leading to better delineation of snow area.
(*iii*) Glacier inventory

7. *Coastal Environment*

(*i*) More detailed inventory of coastal land use on 1:25000 scale
(*ii*) Discrimination of coastal vegetation types.
(*iii*) Monitoring sediment dynamics
(*iv*) Siting of coastal structures

8. *Ocean Resources*

(*i*) Wealth of oceans /explorations/productivity
(*ii*) Potential fishing zone
(*iii*) Coral reef mapping
(*iv*) Low tide/high tide marking

9. *Watershed*

(*i*) Delineation of watershed boundaries/partitioning of micro watershed
(*ii*) Watershed characterisation at large scale (size, shape, drainage, landuse/cover)
(*iii*) Siting of water harvesting structures
(*iv*) Monitoring watershed development
(*v*) Major river valley projects.

10. Environment

(*i*) Impact assessment on vegetation, water bodies.
(*ii*) Siting applications
(*iii*) Loss of biological diversity/biosphere reserves/ecological hot spot areas /wet land environment.

11. Street network-based applications

(*i*) Vehicle routing and scheduling
(*ii*) Location analysis—site selection—evacuation plans.

12. Land parcel-based applications

(*i*) Zoning, sub division plan review.
(*ii*) Land acquisition
(*iii*) Environmental management
(*iv*) Water quality management
(*v*) Maintenance of ownership

13. Natural resources based applications

(*i*) Management of wild and scenic rivers, recreation resources, flood plains, wet lands, agricultural lands, aquifers, forest, wild life etc..
(*ii*) Environmental Impact Analysts (EIA)
(*iii*) View shed analysis
(*iv*) Hazardous or toxic facility siting
(*v*) Ground water modelling and contamination tracking
(*vi*) Wild life analysis, migration routes planning.

14. Facilities management

(*i*) Locating underground pipes, cables
(*ii*) Balancing loads in electrical networks
(*iii*) Planning facility maintenance
(*iv*) Tracking energy use.

15. Disasters

(*i*) Mapping flood inundated area, damage assessment
(*ii*) Disaster warning mitigation

16. Digital elevation models

(*i*) Contours (> 10 m)
(*ii*) Slope /Aspect analysis
(*iii*) Large scale thematic mapping upto 1:25000 scale.

PROBLEMS

1. What do you understand by remote sensing ? Differentiate between active and passive remote sensing.
2. Explain, with the help of a neat sketch, an idealized remote sensing system
3. Write a detailed note on electro-magnetic energy used for remote sensing.

4. What do you understand by electro-magnetic spectrum ? State the wave length regions, along with their uses, for remote sensing applications.
5. Explain the interaction mechanism of EM radiation with earth's surface, stating the basic interaction equation.
6. Write a note on remote sensing observation platforms
7. Write a note on various types of sensors used for remote sensing in India.
8. Write a detailed note on applications of remote sensing.

5

Geographical Information System (GIS)

5.1. INTRODUCTION : DEFINITION

Geographical information system (GIS) is an organised collection of computer hardware and software, geographic data (spatial as well as non-spatial) and people, designed to efficiently capture, store, update, manipulate, analyse and display all forms of geographically referenced information. There are several geographical questions that are to be answered apparently such as (*i*) What is the population of a particular city ? (*ii*) How has the distribution of urban and rural population changed between the past two decades ? (*iii*) What are the characteristics of soil in a particular land parcel ? (*iv*) Are there any trend in the patterns of earthquakes in India which could help predict future earthquake ? (*v*) What are the archaeological patterns of prehistoric land use ? (*v*) What are the archaeological patterns of prehistoric land use ? The art, science, engineering and technology required to answer these geographic questions constitute what is called Geographical Information System (GIS). GIS is a genetic term, denoting the use of computers to create and depict digital representation of the Earth's surface.

Geographical information is information about geography, that is information tied to some specific set of location on the earth's surface including the zones of atmosphere. Spatial is often used synonymously with or even in preference to 'geographical'. Today, the term GIS tends to be applied whenever geographical information in *digital form* is manipulated. Thus, using a computer to make a map is referred to as GIS. This entails using the same computer to analyse geographical information and to make future forecasts using complex models of geographical processes. The earth's images collected by remote sensing satellites are geographical data, but the systems that process the images are not to be called GIS as long as they remain confined to this particular form of data in such cases. *GIS tends to be reserved for system that integrate remotely sensed data with other types, or process data that have already been cleaned and transformed.*

Thus, Geographical Information Systems (GIS) are decision support computer based systems for collecting, storing, presenting and analysing geographical spatial information. These systems are spatially referenced data bases giving users the potentiality to control queries over space, and usually through time. The basic output of GIS or spatial data analysis system is a map.

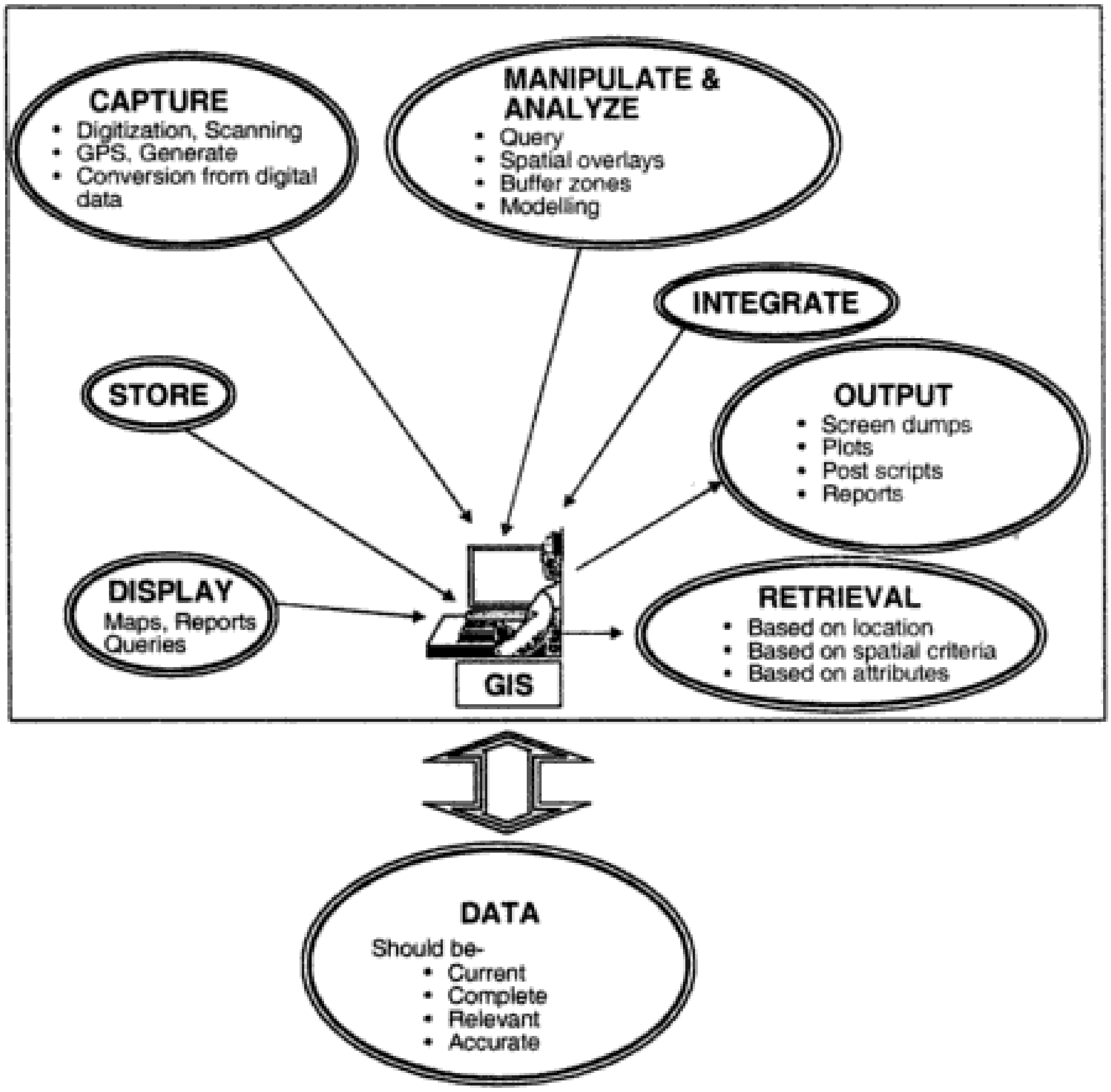

FIG. 5.1. ESSENTIALS OF GIS

5.2. THE FOUR M'S

There are the following four activities that urban planner or scientists or resources managers or others use geographic information for (Fig. 5.2) :

(*i*) Measurement

(*ii*) Mapping

(*iii*) Monitoring

and (*iv*) Modelling

They observe and *measure* environmental parameters and develop *maps* which portray the characteristics of the earth. They *monitor* changes in our surroundings in space and time. In addition to these, they *model* alternatives of actions and process operation in the environment. These four activities (i.e. four M's) can be enhanced by using *information system technologies* through GIS.

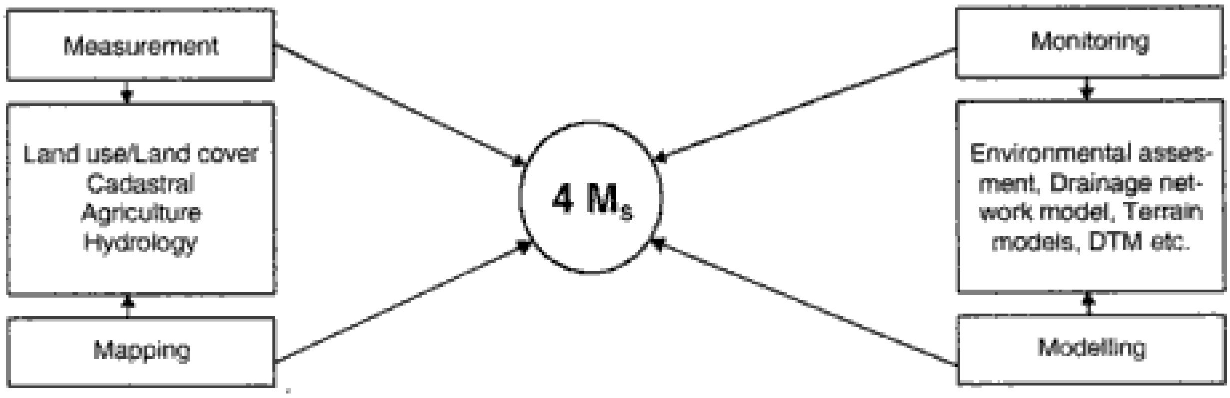

FIG. 5.2. THE FOUR M'S.

5.3. CONTRIBUTING DISCIPLINES FOR GIS

The contributing disciplines for the evolution of a GIS are (Burrough, 1998):

(*i*) Geography (*ii*) Cartography

(*iii*) CAD, CAC, computer softwares (*iv*) Automated cartography

(*v*) Surveying (*vi*) Photogrammetry

(*vii*) Digital photogrammetry (*viii*) Remote sensing

(*ix*) Mathematics (*x*) Statistics.

Geography is broadly concerned with understanding the World and man's place in it, while cartography is concerned with the display of spatial information. Surveying and photogrammetry provide high quality data on positions of cadastral objects like land parcel and building, and topography. Aerial photogrammetry is one of the most powerful data-capturing techniques for the creation of GIS spatial database. *Remote sensing* is becoming an important source of geographical data, providing *digital images* derived from space and the air. Remote sensing provides techniques for data acquisition and processing any where on the globe

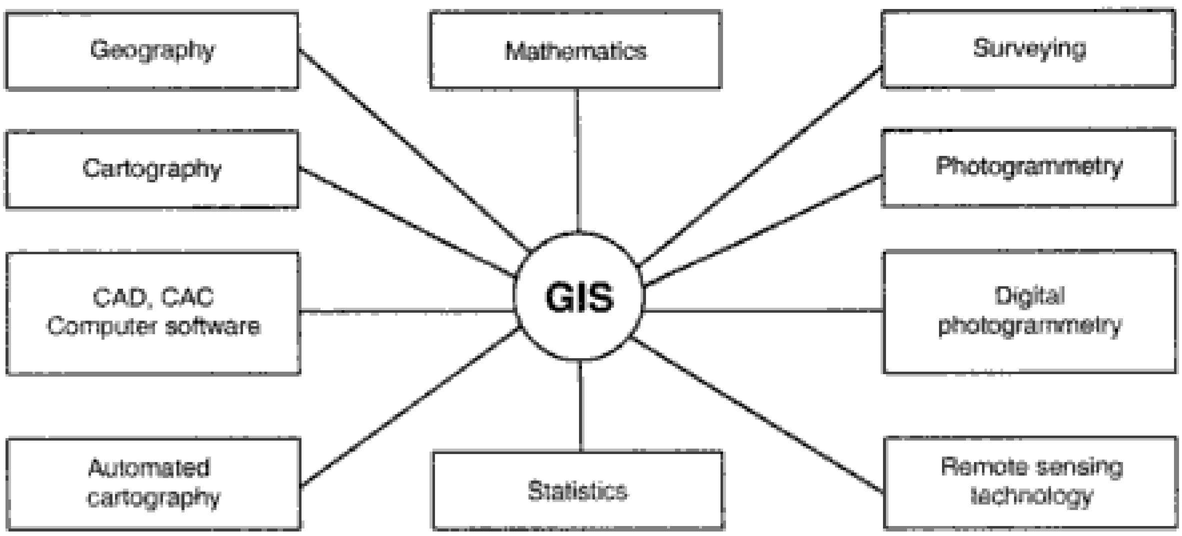

FIG. 5.3. CONTRIBUTING DISCIPLINES FOR GIS

at a low cost, and consistent update potential. While integrated with GIS, remotely sensed imagery can be merged with other data in a GIS providing real-time spatial information. Computer Assisted Design (CAD) provides software, techniques for data input, display and visualisation, and representation in 3-dimensions. Data Base Management System (DBMS) contributes methods for representing data in digital form and procedures for system design and update. Several branches of mathematics are used in GIS system design and analysis of spatial data, while statistics is used to build models and perform spatial data analysis in GIS.

Availability of large quantities of spatial data in the form of digital aerial photograph, digital remote sensing imagery, advancement of computer hardware, software and software development, increasing demand of spatial information for management, and infrastructure development parameters, lead to have a system to handle all these requirements, *In order to handle such data to meet these demands to store, retrieve , handle, analyse, manipulate and display the results, it requires a computer based system which is known as Geographical Information System (GIS).*

5.4. GIS OBJECTIVES

GIS provides a medium for studying one or more of the fundamental issues that arise in using digital information technology to examine the surface of the earth or any related systems. *GIS offers capabilities of integrating multisector, multilevel and multi period database system for capture, storage, retrieval, analysis and display of spatial data.* It is a general purpose technology for handling geography data in digital form .

The following are the GIS objectives.

1. Maximize the efficiency of planning and decision making.
2. Provide efficient means for data distribution and handling.
3. Elimination of redundant database—minimize duplication.
4. Capacity to integrate information from many sources.
5. Complex analysis / query involving geographical referenced data to generate new information.

For any application, there are five generic question that a GIS can answer :

(*i*) **Location** : What exists at a particular location.
(*ii*) ***Condition*** : Identify location where certain conditions exits.
(*iii*) ***Trends*** : What has changed since.
(*iv*) ***Pattern*** : What spatial pattern exists.
(*v*) ***Modelling*** : What if ....?

Overview of Information System

The function of an information system is to improve one's ability to make decisions. An information system is the chain of operation that take us from planning the observation and collection of data to storage and manipulation and analysis of data, and to use the derived information in some decision making process. Fig 5.4 shows the simplified information system overview.

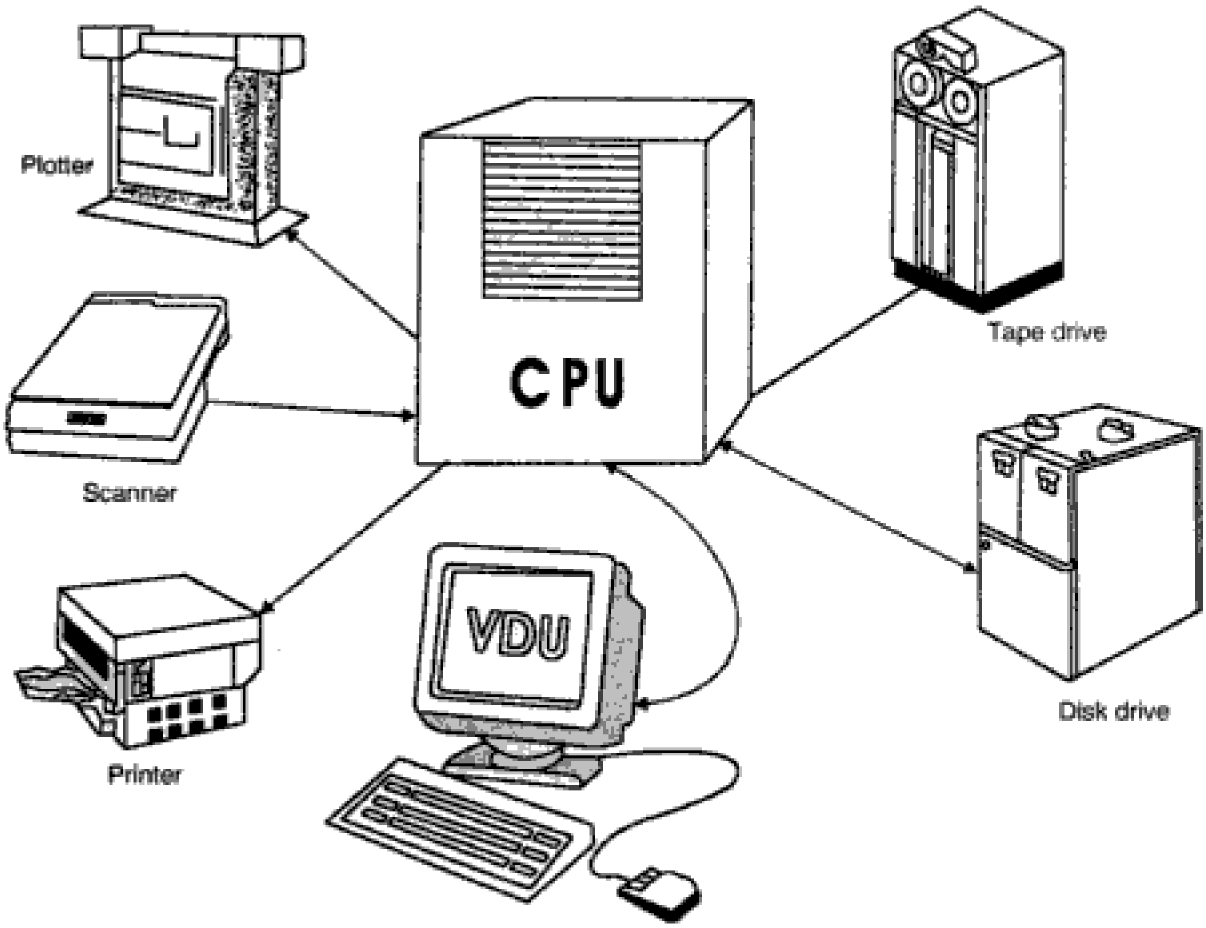

FIG. 5.5. HARDWARE COMPONENTS OF GIS.

(*i*) Data input and verification.
(*ii*) Data storage and database management
(*iii*) Data transformation and manipulation
and (*iv*) Data output and presentation.

The details of various GIS softwares are discussed in § 5.11

Representation of the spatial information

Geographical features are depicted on a map by (*i*) point (*ii*) Line (*iii*) Polygon/Area.

(*i*) ***Point feature :*** A point is discrete location depicted by a special symbol or label; it has a single *x*, *y* co-ordinates.

(*ii*) ***Line feature :*** It represents a linear feature and has a set of ordered *x*, *y* co-ordinates.

(*iii*) ***Polygon / Area feature :*** It is an area feature where boundary encloses a homogeneous area.

Representation of non-spatial (attribute) information.

It consists of textural description on the properties associated with geographical entities. Attributes are stored as a set of numbers and characters in the form of a table. Many attribute data files can be linked together through the use of common identifier code.

Organising map data

Map features are logically organised into a set of layers or themes of information. Some of the most common considerations for organising layers are :

(*i*) ***Feature type :*** Typically, layers are organised so that points, line and polygons are stored in separate layers.

(*ii*) ***Logical group :*** Features are organised logically by what they represent .

(*iii*) ***Intended use :*** Application specific.

5.6. TOPOLOGY

Geographic data describes objects in terms of location, their attributes and spatial relationship with each other (Topology). It is a mathematical procedure that determines the spatial relationships of features.

Area Definition : Polygons are stored as set of lines, rather than defining polygon as a loop of co-ordinates. Line co-ordinates are stored only once, hence reduces the storage space.

Contiguity : Identification of polygon, which touch each other. It is implemented by finding left and right polygon of each arc and takes care of shared arcs.

Connectivity : Identification of interconnected arcs. From and To node indicate the direction. Node coordinates are stored only once.

Advantages of topology :

1. Polygon network is fully integrated.
2. Optimal storage and free from excessive amount of redundant information.
3. Neighbours are identified.
4. Polygon in polygon can be represented.

5.7. DATA MODELS :

Geographical variations are infinitely complex and must be represented in terms of discrete objects. Conversion of real world geographical variation into discrete objects is done through data models. It represents the linkage between the real world domain of geographic data and computer representation of these features. Data models are of two types :

(*a*) Raster data model

and (*b*) Vector data model

(*a*) Raster data model :

- Divides the entire area into rectangular grid cells.
- Each cell contains a single value and every location corresponds to a cell.
- One set of cells and associated values is a *Layer*.

Capabilities :

- *Displaying layers* : Each cell can be assigned a unique colour on the display.
- *Local operations :* Cells by cell basis.

(*i*) *Recoding* – Assigning cells to different classes.

(*ii*) *Overlaying layers* - Layers can be overlaid / combined on the basis of arithmetic, logical criteria or unique combinations.

- *Focal operations :* Considering the neighbourhood cells.

5.8. DATA STRUCTURES

There are number of different ways to organise the data inside the information system. The choice of data structure may affect both : Data storage volume and processing efficiency. Many GIS have specialised capabilities for storing and manipulating attribute data in addition to spatial information. Three basic data structures are - ***(i) Relational, (ii) Hierarchical and (iii) Network.***

1. ***Relational data structure*** – It is the most popular model for GIS. Many GIS packages have different names e.g. INFO in ARC/INFO, DBASE III in PC-based GIS etc. Data are stored as simple records, known as tuples, containing an ordered set of attribute values that are grouped together in two-dimensional tables.

(*i*) It organizes the data in terms of two-dimensional tables.
(*ii*) Each table is a separate file.
(*iii*) Each row in the table is a record.
(*iv*) Each record has a set of attributes.
(*v*) Each column in the table is an attribute.
(*vi*) Different tables are related through the use of a common identifier called KEY.
(*vii*) Data are extracted by relation which are defined by query.
(*viii*) Relation algebra to create new tables.

Relational data structure is time-consuming because of sequential search to find data.

2. *Hierarchical data structure* – Taking into consideration the data which have inherent quality of hierarchy (tree structure), this database was introduced.

(*i*) Each node can be divided into one or more additional node. Nodes have only one parent.
(*ii*) Stored data gets more and more detailed as once branches further out on the tree.

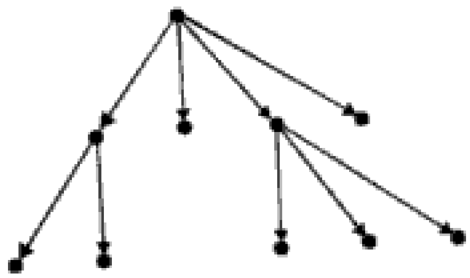

FIG. 5.6 HIERARCHICAL DATA STRUCTURE.

(*iii*) This data structure have one to many relationship.
(*iv*) Cannot go between records at the same level unless they share the same parent.
(*v*) Only one link between two records.
(*vi*) Travel within database is restricted to path up and down.

Large index files have to be maintained and certain attribute value may have to be repeated many times leading to data redundancy which increases storage and access cost.

3 Network data structure – This structure is similar to hierarchical structure with the exception that in this structure a node may have more than one parent.

(*i*) Each node can be divided into one or more additional node. Nodes can have parent.
(*ii*) Stored data gets more and more detailed as one branches further out on the tree.

(*iii*) This data structure have one to many and many to one relationship.

(*iv*) Can go between records at the same level.

The database is enlarged by overhead of pointers. These pointers must be updated every time a change is made to database causing considerable overhead.

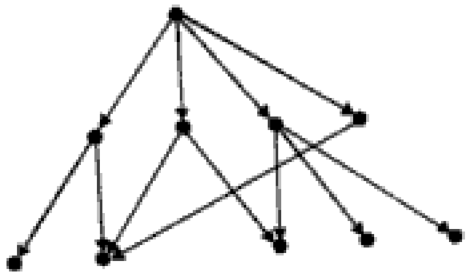

FIG. 5.7 NETWORK DATA STRUCTURE

5.9. DATABASE MANAGEMENT

Database may be defined as a data and information stored or as a structured collection of interrelated information on a defined subject. A DBMS is a software that permits one or more users to work efficiently with the data. The essential components of DBMS include means to define database, insert new data, delete old data, update, query and sort facility.

Generally, a proper database organization needs to ensure the following :

(*i*) Reduction in data redundancy – Same information is not stored at multiple places and data is shared.

(*ii*) Data integrity – A system of validation checks to enforce the necessary structure constraints.

(*iii*) *Security restrictions* – To minimize damage to data. Of specific importance in multi-user environment.

(*iv*) *Physical data independence* – The underlying data storage and manipulation hardware is transparent to the user.

(*v*) *Data manipulation, query facility and programming tools.*

(*vi*) *Data follows prescribed models, rules and standards* – To maintain consistency in data elements.

5.10. ERRORS IN GIS

Errors in GIS environment can be classified into following major groups :

1. Error associated with data :

(*i*) *Age of data* – Reliability decreases with age.

(*ii*) *Map scale* – Non-availability of data at proper scale or use of data at different scales.

(*iii*) *Density of observations* – Sparsely dense dataset is less reliable.

(*iv*) *Data inaccuracy*- positional, elevation, minimum mapable unit etc.

(*v*) Inaccuracy of contents – Attributes are erroneously attached.

2. Errors associated with processing :

(*i*) *Map digitization errors* – due to boundary location problems on map and errors associated with digital representation of feature.

(*viii*) GRID : It is a raster or cell based geoprocessing tool box integrated with ARC / INFO which divides the area into individual locations.

(*ix*) ARCVIEW : Arcview module is a software tool that creates an environment to display and querry the contents of spatial database.

(*x*) ARCTOOLS : It is a general purpose menu based interface to ARC / INFO to perform common geoprocessing task with a better speed ease.

(*xi*) ARCSTORM : Arc storm (ARC storage manager) is a geographic data storage facility to manage ARC/INFO databases intended for public use (multi-users).

(*xii*) COGO : It is the coordinate geometry module of ARC/INFO which supports the functions performed by land surveyors and civil engineers.

2. PAMAP GIS

The PAMAP GIS is a product of PAMAP Graphics Ltd Canada, It adopts an integrated raster as well as vector representation of the spatial elements. It uses vector for data capture and storage and rasters for analysis purposes.

Major modules of PAMAP GIS

(*i*) GIS MAPPER.

(*ii*) ANALYSER.

(*iii*) INTERPRETER.

(*iv*) MODELLER.

(*v*) NETWORKE.

(*vi*) FILETRANSLATOR.

3. SPANS

Spatial Analysis System (SPANS) is a GIS package developed by TYDAC Technologies, Canada, having powerful modelling function for application. It adopts a mixed vector tesselation approach to the GIS .

Major modules of SPANS

(*i*) CORE GIS MODULE.

(*ii*) TYDIG

(*iii*) CONTOURING/DEM MODULE.

(*iv*) POTMAP (POTENTIAL MAPPING MODULE).

(*v*) RASTER INTERFACE MODULE.

4. GENAMAP

The Genamap package is marketed by Gena Sys, an international developer. It handles data in both vector and raster form.

5. INTERGRAPH MGE

Intergraph provides MGE (Modular GIS Environment) as a solution for mapping /GIS application for infrastructure, environmental and natural resources management and digital cartograph. It adopts a vector cum raster data structure for spatial data handling.

6. ISROGIS.

ISROGIS has been developed by Indian Space Research Organization (ISRO). It adopts PM quadtree data structure, which is edge based structure that decomposes the vector in map into quads, and is then organized using the vector structure.

***Major modules/ functionalities of* ISROGIS**

(*i*) CREATE : For creation of maps and themes
(*ii*) EDIT : For systematic editing of spatial features.
(*iii*) MAKE : For providing symbolisation, annotations.
(*iv*) ANALYSIS : For overlay analysis.
(*v*) QUERRY : For obtaining information related to spatial and attribute data.
(*vi*) LAYOUT : For cartographic work.
(*vii*) 3-D MODULE : For handling Z-axis of spatial data.

5.12. LINKAGE OF GIS TO REMOTE SENSING.

Image derived from optical and digital remote sensing systems mounted in aircraft and satellite provide much spatial information and major data as an input to GIS. Remote sensing data are a major source of data for the mapping of resources like geology, forestry, water resources, land use and land cover. Integration of the two technologies, remote sensing and GIS can be used to develop decision support systems for a planner or decision maker.

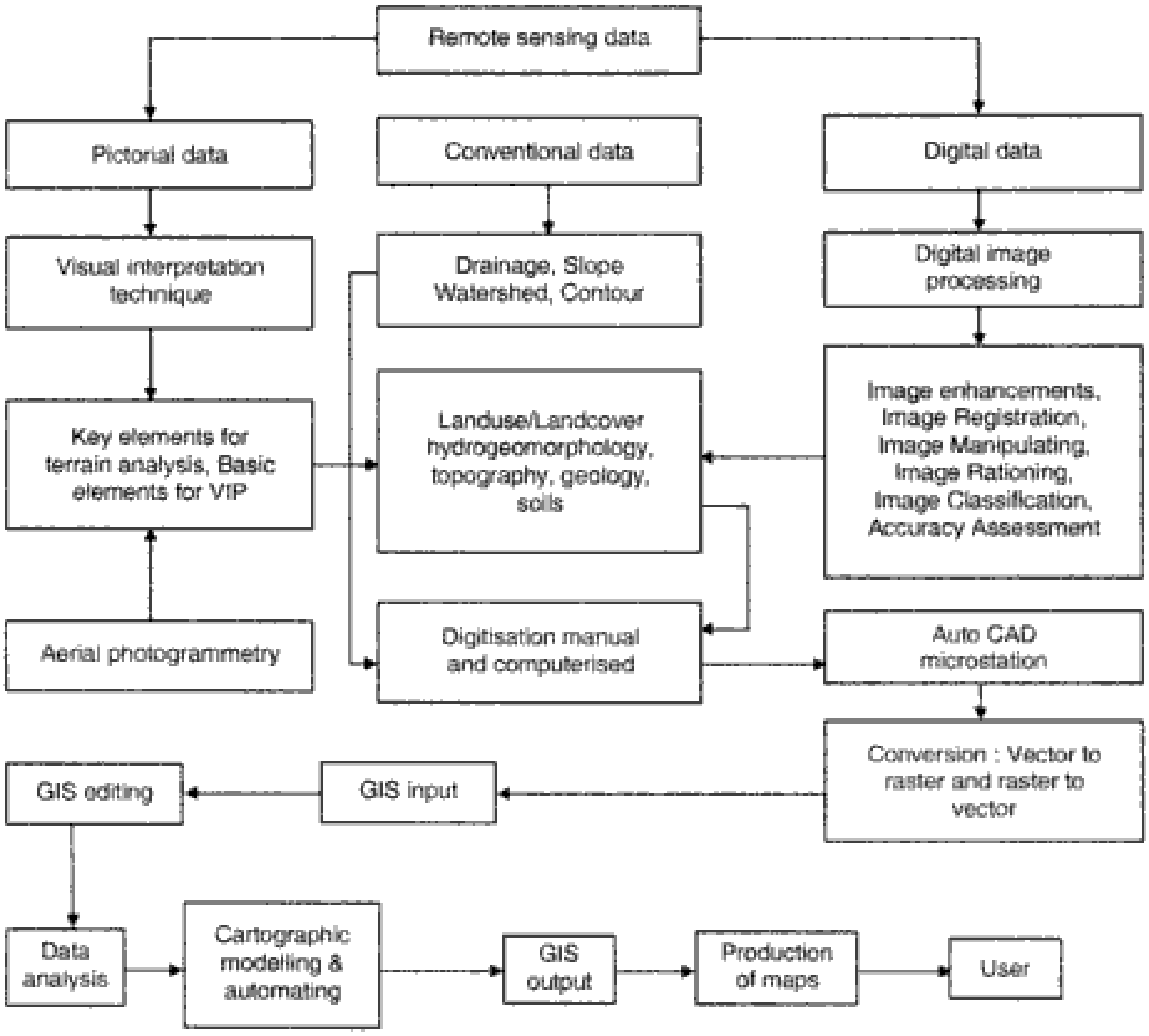

FIG. 5.8. LINKAGE OF REMOTE SENSING AND GIS.

Remote sensed images can be used for two purpose : (*i*) as a source of spatial data within GIS and (*ii*) using the furnctionality of GIS in processing remotely sensed data in both pictorial and digital modes.

Remote sensing images including the information extracted from such images, along with GPS (global positioning system) data, have become primary data sources for modern GIS. *Indeed, the boundaries between remote sensing, GIS and GPS technology have become blurred, and theses combined fields will continue to revolutionise the inventory, monitoring and managing natural resources on a day-today basis.* There are two methods of extracting data for GIS from the remote sensing data. They are (*i*) Visual interpretation of satellite imageries in pictorial format, and (*ii*) computer processing of remotely sensed digital data. The output of either of these analysis methods can be considered an input for GIS for any kind of application. Fig. 5.8 shows and overview of the linkage of remote sensing and GIS.

GIS and Remote sensing are linked both historically and functionally. Earlier studies on the application of GIS technique for creating decision support systems and other computer based spatial information extraction have revealed that 75% to 85% of the spatial data layers have been derived from the analysis of aerial photography and satellite image data. As a result of this, remote sensing and GIS have become very important and have been associated with each other. *Remote sensing data can be readily merged with other sources of geo-coded information in a GIS.* Remote sensing and GIS have almost become an unavoidable source for cross checking or updating in digital surveying. Furthermore, the GIS software can now accept the Global Positioning System (GPS) information in their program, as an additional advantage. *However, to accomplish the integration of remotely sensed data into vector based GIS require the additions of relatively sophisticated image processing package to these systems*. For instance ARC/INFO, ARC/VIEW and ARC/GRID softwares accomplish much of this integration *Remote sensing and GIS can contribute a great deal to our study of patterns and processes on the surface of the earth and to create decision support systems.*

5.13. APPLICATIONS AREAS OF GIS AND REMOTE SENSING

See § 4.9.

PROBLEMS

1. (*a*) What do you understand by a geographical information system ?

   (*b*) What are the essentials of a GIS ?
2. What are the four M's for which geographic informations is used ? Elaborate.
3. What are the contributing disciplines for GIS ?
4. What are the objectives of a GIS.
5. Write a note on the components of a GIS.
6. Explain various types of data structures used in GIS.
7. Enumerate various types of GIS software packages.
8. Write a note on linkage of GIS to remote sensing.
9. Write a note on application areas of GIS and remote sensing.

Index

N

O

P

R

S

About the Book

This book, first published in 1967 and entering into its **Fifteenth Edition** is one of the three volumes on **SURVEYING**. It deals with **five topics** on Advanced or Higher Surveying. The book has been primarily written as a college textbook to fill a need for simple but complete coverage of the principles of Field Astronomy, Photogrammetry, Electro-Magnetic Distance Measurement (EDM), Remote Sensing and Geographical Information System (GIS). The book is also meant to assist the experienced surveyor who has not found time to follow the rapid changes in the techniques so noticeable in the surveying field. The subject is gradually introduced in stages. Large number of solved and unsolved examples develop reader's ability to apply the basic concepts to practical problems. The book also deals with the most modern equipments used in Photogrammetric Surveying and Electro-Magnetic Distance Measurement, including Total Station.

Author's Profile

Dr. B.C. Punmia is an eminent author of 18 books, most of which are followed as textbooks. Having graduated in 1959 with 'HONOURS', he obtained his Master's degree in 1969 with 'HONOURS' and Ph. D. in 1976. Having started his career as Assistant Professor in 1959, he was elevated to the posts of Reader in 1965 and Professor in 1978. He also held the posts of Head of Civil Engg. and Dean of the Faculty of Engineering at M.B.M. Engineering College, Jodhpur. During his teaching career of about 36 years, he guided a large number of research students. Dr. Punmia has authored more than hundred papers, monographs and technical reports published in various Indian and Foreign Journals. He had been consultant to various government, semi-government and private organisations.

Ashok Kumar Jain is an eminent Design and Field Engineer. He is also a prolific writer in the domain of Civil Engineering.

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