Process Dynamics and Control

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Chapter Five

Feedback Control

- The main objective of this chapter is the following:
  - Discuss the notion of the feedback loop and present its hardware elements.
  - Identify the types of feedback controllers which are available for process control and examine their effect on the response of a chemical process.
  - Analyze the stability characteristics of a feedback control system and learn how to design the appropriate feedback system to control a given process.
  - Solve some special problems which are encountered during the design of feedback controllers.

The Concept of Feedback Control

- Considering manipulated variable m, disturbance d and output variable y.
- When the input or disturbances vary the output also varies.
- We had no objective of controlling the output variable.
- The disturbance d changes in an unpredictable manner and our control objective is to keep the value of the output y at desired levels by reducing the disturbance.

A feedback control action takes the following steps:

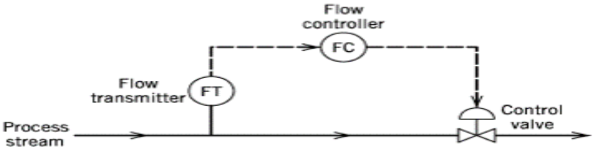
- Measures the value of the output using the appropriate measuring device
- Compares the measured value with set-point
- Calculate the errors and send to the controllers
- The controllers changes the position of the final control element
- The final control element changes the value of the manipulated variables

The roles of feedback

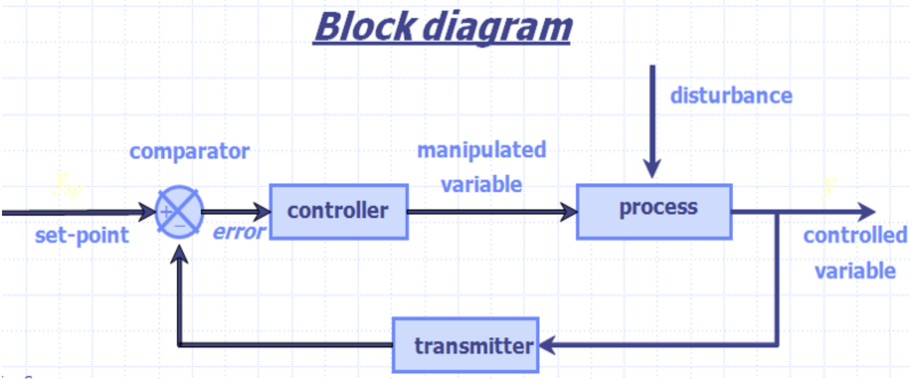
- Reduce error (eliminating the error)
- Reduce sensitivity or enhance robustness
- Disturbance rejection or elimination/reduces
- Improve dynamic performance or adjust the transient response (such

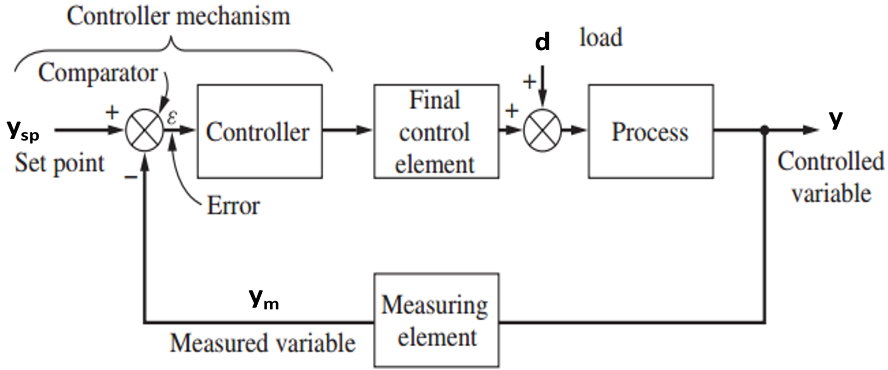
The basic components in a feedback control loop are:

- Process being controlled
- Sensor-transmitter combination (AT)
- Feedback controller (AC)
- Current-to-pressure transducer (I/P)
- Final control element (control valve)
- Transmission lines between the various instruments (electrical cables and pneumatic tubing)



When these components are set in a feedback control configuration its block diagram looks like the figure below:




Controller mechanism
Comparator
d
load
+
+
+
ε
y_{sp}
Set point
−
Controller
Final control element
Process
y
Controlled variable
Error
y_m
Measured variable
Measuring element

When the process is disturbed;

- The output variable (y), deviates from set point (y_{sp})
- The sensor measures the output variable (y_m)
- The comparator checks whether (y_m) deviated from the set point(y_{sp}).
- The deviation of (y_m) away from (y_{sp}) is called error (ϵ) : $\epsilon = y_{sp} - y_m$
- The controller decides and send information to the final control element based on the error, (ϵ)
- The final control element will adjust the manipulated variable to bring the y back to its set point value, y_{sp}
- This makes-up a closed loop feedback control system and the response becomes closed loop response.

There are two problems or processes based on type of input

- **Servo process**: the disturbance doesnt change (d = 0) and the set point changes ($y_{sp} \neq 0$).
- The input is the set point y_{sp} that changes.
- The objective is to cause the output to track the changing set-point.
- **Regulatory process**: the disturbance changes(d0) while set point doesnt change ($y_{sp} = 0$).
- The task is to counteract the effect of external disturbances in order to maintain the output at its constant set-point.
- The input is the disturbance, d that varies while the set-point is fixed to certain value .
- Eg. Most process have fixed set point that they are Regulatory processes

The control system has various components,

- The process: - includes the equipment and the operation
- Sensors: - Measuring devices
- Transmission lines: - carry signals from sensor to controller and from controller to final control element
- Transmitters and transducers: - translate one kind of signal to another
- Controller: - the master mind of the system that make a decision
- Final control element: - implements the decision of the controller

There are three basic types of feedback controllers;

1: Proportional, (P)

2: Proportional-integral (PI)

3: Proportional-integral-derivative (PID)

Proportional (P) Controller

- In feedback control, the objective is to reduce the error signal to zero.
- Its output signal, y_c is proportional to the error:
- Mathematically: $C(t) = K_c\epsilon(t) + C(s)$

  Where: K_c: is proportional gain of the controller

  $C(s)$: is the actuating signal of the controller when $\epsilon(t) = 0$
- Writing in deviation form, $C(t) - C(s) = K_c\epsilon(t) \Rightarrow C'(t) = K_c\epsilon(t)$
- On Laplace transformation: $C'(s) = K_c\epsilon(s)$
- The transfer function: $G_c(s) = \dfrac{C'(s)}{\epsilon(s)} = K_c$
- The transfer function for P: $G_c(s) = K_c$
- It is clear that, "the larger the gain Kc the controller's actuating signal will be.

2) Proportional-Integral (PI) Controller:-

- It is also called proportional plus reset controller.
- The transfer function for PI: $G_c(s) = K_c(1 + \frac{1}{\tau_I s})$

  Where: τ_I: is integral time constant or reset time in minute
- The integral term repeats $\epsilon(t)K_c$ every τ_I time.
- That is why PI controller is called proportional plus reset controller.
- The reset time is an adjustable parameter and is sometimes referred to as minutes per repeat.
- Usually it varies in the range $0.1 \leq \tau_I \leq 50$ minutes.
- The integral control action has "repeated" the response of the P action.
- This repetition takes place every τ_I minutes and has lent the name to the reset time

3) Proportional-Integral-Derivative (PID) Controller:-

- It is also called proportional plus reset plus rate controller.
- It is also called proportional plus reset controller.
- The transfer function for PID: $G_c(s) = K_c(1 + \frac{1}{\tau_I s}) + \tau_I s$

  Where: τ_I: is integral time constant or reset time in minute

  Where: τ_D: is derivative time constant or rate time in minute

Proportional (P) Controller action:

- Accelerates the process response by gain increases
- Produces steady state deviation
- Of-set decreases as gain increases

Proportional-Integral (PI) Controller:

- Eliminates the steady state deviation
- Responses are sluggish with long oscillations
- Increasing gain Kc makes more oscillatory and leads to instability

Proportional-Integral-Derivative (PID) Controller:

- Anticipates the future error
- Introduces the stability effect

A feedback control system should satisfy the following design objectives:

- Closed-loop stability
- Good disturbance rejection (without excessive control action)
- Fast set-point tracking (without excessive control action)
- A satisfactory degree of robustness to process variations and model uncertainty
- Low sensitivity to measurement noise

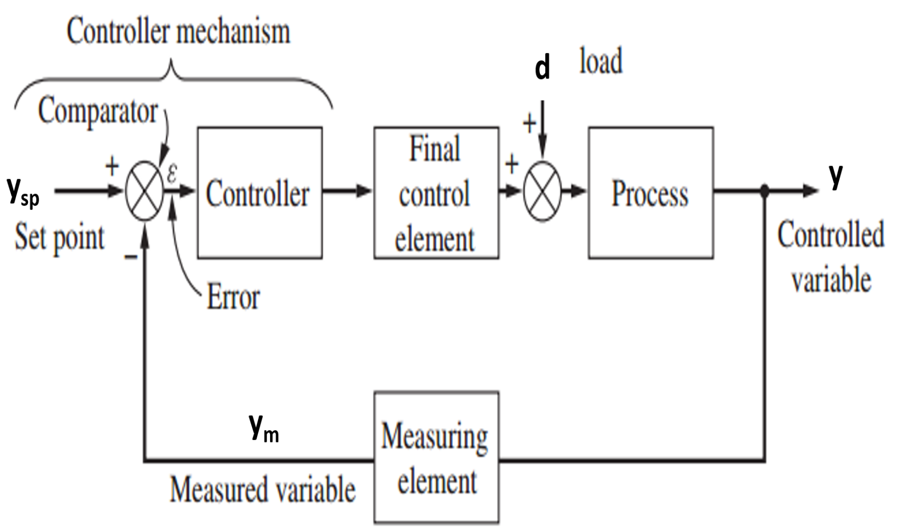
There are two inputs

- the set point (y_{sp})
- the disturbance or load (d)
- There is one output ()
- Our objective is to determine the relationship between the inputs and the outputs.

Some Terminologies

- Y: output variables
- Y_m: measured variables
- Y_{sp}: set point
- ϵ: error from the comparator
- Y_c: output from controller
- M: manipulated variables
- D: disturbance or load variables
- G_m: transfer function for sensor or measuring device
- G_c: transfer function for controller
- G_f: transfer function for final control element
- G_p: transfer function for the process

To Determine its response every block has to be replaced by its corresponding transfer function



- The general equation for closed block diagram

$$Y(s) = \frac{G_f G_c G_p}{1 + G_f G_c G_p G_m} Y_{sp}(s) + \frac{G_d}{1 + G_f G_c G_p G_m} D(s)$$

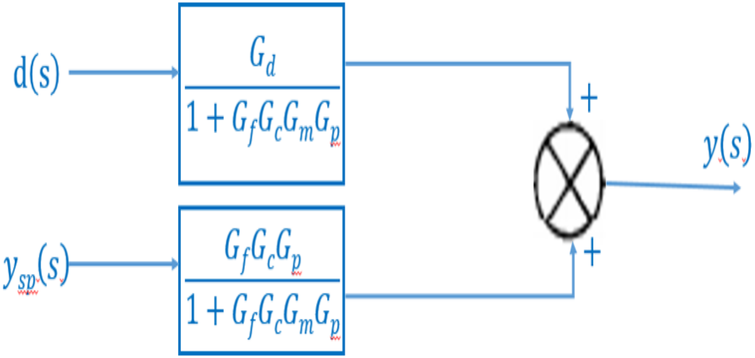
- Servo problem at [D(s) =0] is : $Y(s) = \frac{G_f G_c G_p}{1 + G_f G_c G_p G_m} Y_{sp}(s)$
- The transfer function for servo problem is:

$$G_{sp}(s) = \frac{Y(s)}{Y_{sp}(s)} = \frac{G_f G_c G_p}{1 + G_f G_c G_p G_m}$$

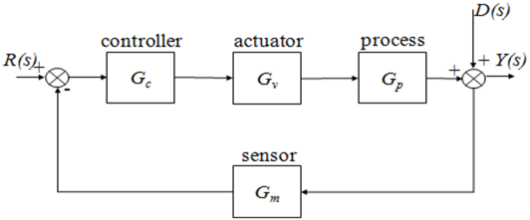
- Regulatory problem at $[Y_{sp}(s) = 0]$ is: $Y(s) = \frac{G_d}{1 + G_f G_c G_p G_m} D(s)$
- The transfer function for regulatory problem is:

$$G_d(s) = \frac{Y(s)}{D(s)} = \frac{G_d}{1 + G_f G_c G_p G_m}$$

The block diagram could be simplified to another equivalent block diagram as shown below:



The denominator of the transfer functions; $1 + G_f G_c G_p G_m$ is equal to 1 plus multiplication of all the transfer functions in the loop.



- $G_p(s)$: Process Transfer Function
- $G_c(s)$: Controller Transfer Function
- $G_m(s)$: Sensor Transfer Function
- $G_v(s)$: Actuator Transfer Function

- First order controlled process under P-only
  - The order remains first order
  - The time constant is reduced
  - The static gains are reduced
  - Increasing controller gain eliminates off-set
- **Unit Step change in inputs for Servo process**:
- The difference between the ultimate response and the new set point is offset.
  - Offset = Set point - Ultimate response

$$\Rightarrow Offset = 1 - \frac{K_p K_c}{1 + K_p K_c}$$

Second order controlled process under P-only

- The order of the system remains second order.
- The static gain of the response decreases.
- The time constant of the "response" is reduced.
- The damping factor of the closed loop response decreases.
- Increasing Kc
  - lead to instability
  - reduces offset
  - speeds response
  - increases oscillation

Effect of proportional integral on first order

- For integral control action, the controller output depends on the integral of the error signal over time.
- The order of the response has increased from first order to second order.
- The τ decreases with increase in integral effect.
- The response gets faster with increase in integral effect.
- Integral action eliminates offset.
- The damping factor also decreases with integral effect
- The damping factor may go from sluggish overdamped response to faster but oscillatory underdamped response.
- The overshoot and decay ratio of the closed loop response will increase

Effect of PID control on second order servo process

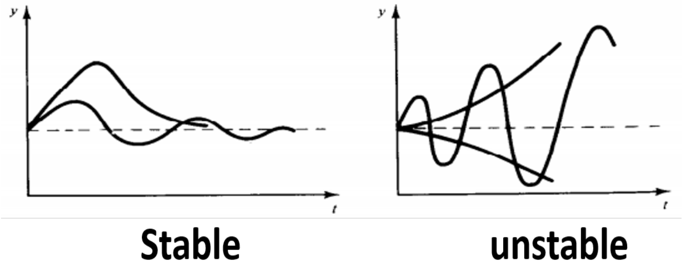
- has all the effect of the PI controller
- the effect of increasing Kc to bring instability
- This instability could be reduced or eliminated by the derivative action.
- The damping factor increases with derivative effect that the response gets more damped (sluggish).
- The overshoot will be almost the same
- The response time will be shorted
- The response will settle faster.
- The response gets more robust.

Controller Comparison

- **Proportional Controller**
  - Simplest controller to tune (K_c).
  - Offset with sustained disturbance or set-point change.
- **Proportional Integral Controller**
  - More complicated to tune (K_c, τ_I)
  - Better performance than P
  - No offset
  - Most popular FB controller
- **Proportional Integral Derivative Controller**
  - Most complicated to tune (K_c, τ_I, τ_D)
  - Better performance than PI
  - No offset
  - Derivative action may be affected by noise

Stability Analysis of Feedback Controlled Systems

- A dynamic system is considered unstable when the output increases or decreases indefinitely in amplitude when bounded disturbance occurred to the system.
- A dynamic system is considered to be stable if every bounded input produces a bounded output, regardless of its initial state.



Poles and zeros of a transfer function:

- The transfer function is ratio of two polynomials in s: i.e.

  $$G(s) = \frac{Q(s)}{P(s)}$$

  where Q is the output and P is the input to the system.
- The roots of Q(s) are called zeros of G(s) or the system.
- At the zeros, G(s) gets a value of zero (0).
- The roots of P(s) are called poles of G(s) or the system
- At the poles G(s) gets a value of infinity (∞).
- **General Stability Criterion**: The feedback control system is stable if and only if all roots of the characteristic equation are negative or have negative real parts. Otherwise, the system is unstable.

- From $G(s) = \frac{Q(s)}{P(s)} = \frac{Q(s)}{s * P(s)}$ and for unit step change in the input
- Assuming that $p_1, p_2, ..., p_n$ be the roots of P(s) (i.e. the poles of the transfer function G(s)

$$Y(s) = \frac{G(s)}{s} = \frac{Q(s)}{s(s-p_1)(s-p_2), ..., (s-p_n)}$$

$$Y(s) = \frac{C_1}{s} + \frac{C_2}{s-p_1} + ... + \frac{C_n}{s-p_n}$$

- On Laplace inversing: $y(t) = C_1 + C_2 e^{p_1 t} + C_3 e^{p_2 t} + ... + C_n e^{p_n t}$
- The behavior of this response depends on the values $p_1, p_2, ..., p_n$

- Real distinct poles: are poles located on the real axis
  - On inversion $e^{p_1 t}$ decays exponentially to zero as $t \to \infty$
  - On inversion $e^{p_2 t}$ grows to infinity.
  - Negative poles decay and positive poles grow on response.
- Multiple real roots: are poles located on the axis and repeated
- Complex Conjugate Poles: - complex poles always occur in pair as conjugates in the form $\alpha + \beta i$
  - The response looks like: $y(t) = e^{\alpha}t[C_1 cos(\beta t) + C_2 sin(\beta t)]$
  - The term in the bracket is oscillatory.
  - The $e^{\alpha}t$ decays to zero if α is negative and grows to infinity if α is positive.

- Poles on the imaginary line(i.e. α) will have a oscillatory response of constant amplitude.
- Poles at the origin give rise to constant number.
- Poles to the right of the imaginary line give rise to terms that grow to infinity with time.
- Such systems are said to be unbounded behavior $\Rightarrow$ unstable systems.
- Poles to the left of the imaginary line give rise to terms that decay to zero with time.
- Such systems are said to be bounded behavior $\Rightarrow$ stable systems

- If there is at least one pole to the right of the imaginary line the system will be unstable.
- The poles of a system have to be on the left of the complex plane for the system to be stable.
- The closed loop response of a feedback control system is given by:

$$Y(s) = \frac{G_f G_c G_p}{1 + G_f G_c G_p G_m} Y_{sp}(s) + \frac{G_d}{1 + G_f G_c G_p G_m} D(s) = G_{sp} Y_{sp}(s) + G_l D$$

- The stability of this closed loop response depends on the poles of the transfer functions G_l and G_{sp}.
- These poles are common to both transfer functions as the denominator $1 + G_f G_c G_p G_m$ is the same for both of them.
- These poles can be obtained by solving $1 + G_f G_c G_p G_m = 0$

- The equation $1 + G_f G_c G_p G_m = 0$ is called the generalized characteristic equation for feedback control system.
- While solving we will have:

  $1 + G_f G_c G_p G_m = (s - p_1)(s - p_2), ..., (s - p_n)$
- A feedback control system is stable if all the poles of the characteristic equation are negative (to the left of the imaginary line).

The characteristic equation:

- Whether the process is servo or regulatory, the stability criteria still works as the characteristic equation is the same in both cases (i.e the roots of the characteristic equation are the poles of both G_l and G_{sp})
- The characteristic equation can be written as: $1 + G_{ol} = 0$ it depends on the transfer functions in the loop excluding G_d which is outside the loop

Routh-Hurwitz criterion for stability:

- The Routh-Hurwitz criterion is a method through which one can determine the stability of a system without knowing the roots of the characteristic equation
- This method tells us if there is any root to the right of the imaginary line.
- Expanding the characteristic equation into polynomial form gives:

$$1 + G_{ol} = 0 \Rightarrow 1 + G_f G_c G_p G_m = 0$$

$$a_n s^n + a_{n-1} s^{n-1} + .. + a_1 s^1 + a_o$$

- Rearrange the above equation so that a_o is positive (if a_n is negative multiply the whole equation by -1).

- **Necessary Condition**: If any of the coefficients $a_n, a_{n-1}, a_{n-2}, ..., a_o$ is negative, there is at least one root that occur at the right of the imaginary line (i.e. positive root or pole).

  The system is unstable. $\Rightarrow$ stop calculation.
- **Sufficient Condition**: If all the coefficients $a_n, a_{n-1}, a_{n-2}, ..., a_o$ are positive, form the following array called Routh array.
- The first row is obtained by taking the every other coefficient of the rearranged characteristic equation starting with a_o .
- The second row obtained similarly but starting with a_1.

Row				
1	a_0	a_2	a_4	a_6
2	a_1	a_3	a_5	a_7
3	b_1	b_2	b_3	
4	c_1	c_2	c_3	
5	d_1	d_2		
6	e_1	e_2		
7	f_1			
$n+1$	g_1			

Routh Coefficients

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad \cdots$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \qquad \cdots$$

- The system is stable when all the coefficients in the first column or the Routh coefficients are positive.
- Examine the first column: Which are called Routh coefficients (total of $n+1$ coefficients)
- If there is any of these Routh Coefficients are negative we have at least one positive root to the right of the imaginary line (positive root) and the system is unstable.
- The number of sign changes in the first column tells us the number of roots that are to the right of the imaginary line or positive roots.
- If one pair of the roots are on the imaginary root and all others are on the left side (negative roots), all the elements of the nth row of the Routh coefficients vanish.

Example 1: Consider the feedback control system with the following transfer functions:

$$G_{c(s)} = K_c, G_v(s) = \frac{1}{2s+1}, G_p(s) = \frac{1}{5s+1}, G_m(s) = \frac{1}{s+1}$$

a) Determine the range of K_c values that result in a stable closed-loop system.

b) Designing of PID by Direct Substitution Method

Example 2: Consider the feedback control system with the following transfer functions:

$$G_{c(s)} = K_c, G_v(s) = 2, G_p(s) = \frac{4e^{-s}}{5s+1}, G_m(s) = 0.25$$

a) Determine the range of K_c values that result in a stable closed-loop system.

b) Designing of PID by Direct Substitution Method

Assignment: Two first order processes with time constants 10 sec and 25 sec and gains 1.3 and 1 are in series. Design a proportional controller which would ensure a decay ration of 0.5 in the closed response with $G_v(s) = G_m(s) = 1$.