 By: Dawit M.

Internal Flow - Empirical Correlations

Hydrodynamic Considerations

When considering external flow, it is necessary to ask only whether the flow is laminar or turbulent. However, for an internal flow we must also be concerned with the existence of entrance and fully developed regions.

Flow Conditions:

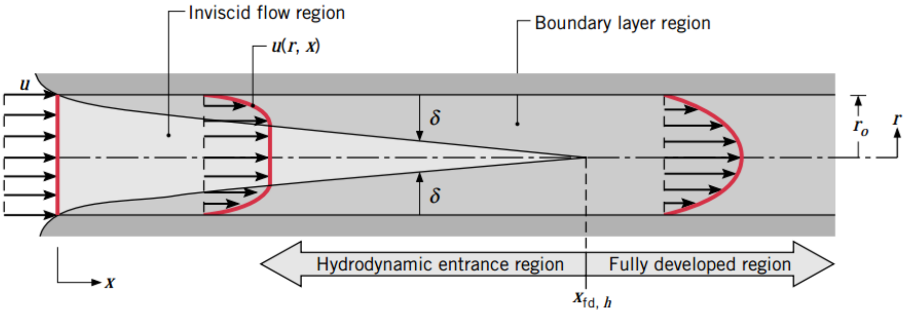


Figure 1

$$Re_D \equiv \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

Where U_m is the mean fluid velocity over the cross section and D is the diameter.

Critical Reynod's number to the onset of turbulence is:

$$Re_{D,c} \approx 2300$$

Laminar Flow

For laminar flow ($Re_D \leq 2300$), the hydrodynamic entry length may be obtained from an expression of the form:

$$\left(\frac{x_{fd,h}}{D}\right)_{lam} \approx 0.05\, Re_D$$

This expression is based on the presumption that fluid enters the tube from a rounded converging nozzle and is hence characterized by a nearly uniform velocity profile at the entrance (Figure 1).

Turbulent Flow

Although there is no satisfactory general expression for the entry length in turbulent flow, we know that it is approximately independent of Reynolds number and that, as a first approximation:

$$10 \lesssim \left(\frac{x_{fd,h}}{D}\right)_{turb} \lesssim 60$$

For the purposes of this text, we shall assume fully developed turbulent flow for: (x/D) > 10.

<u>The Mean Velocity</u>

For steady, incompressible flow in a tube of uniform cross-sectional area,

$$u_m = \frac{\int_{A_c} \rho u(r, x)\, dA_c}{\rho A_c} = \frac{2\pi\rho}{\rho \pi r_o^2} \int_0^{r_o} u(r, x)\, r\, dr = \frac{2}{r_o^2} \int_0^{r_o} u(r, x)\, r\, dr$$

<u>The Velocity Profile in the Fully Developed Region</u>

The form of the velocity profile may readily be determined for the laminar flow of an incompressible, constant property fluid in the fully developed region of a circular tube.

$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right) r_o^2 \left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx}$$

<u>The Pressure Gradient and Velocity Gradient in the Fully Developed Region</u>

The engineer is frequently interested in the pressure drop needed to sustain an internal flow because this parameter determines pump or fan power requirements.

Friction factor (f) for Fully Developed Laminar Flow:

$$f = \frac{64}{Re_D}$$

Friction factor (f) for Fully Developed Turbulent Flow:

For fully developed turbulent flow, the analysis is much more complicated; and we must ultimately rely on experimental results.

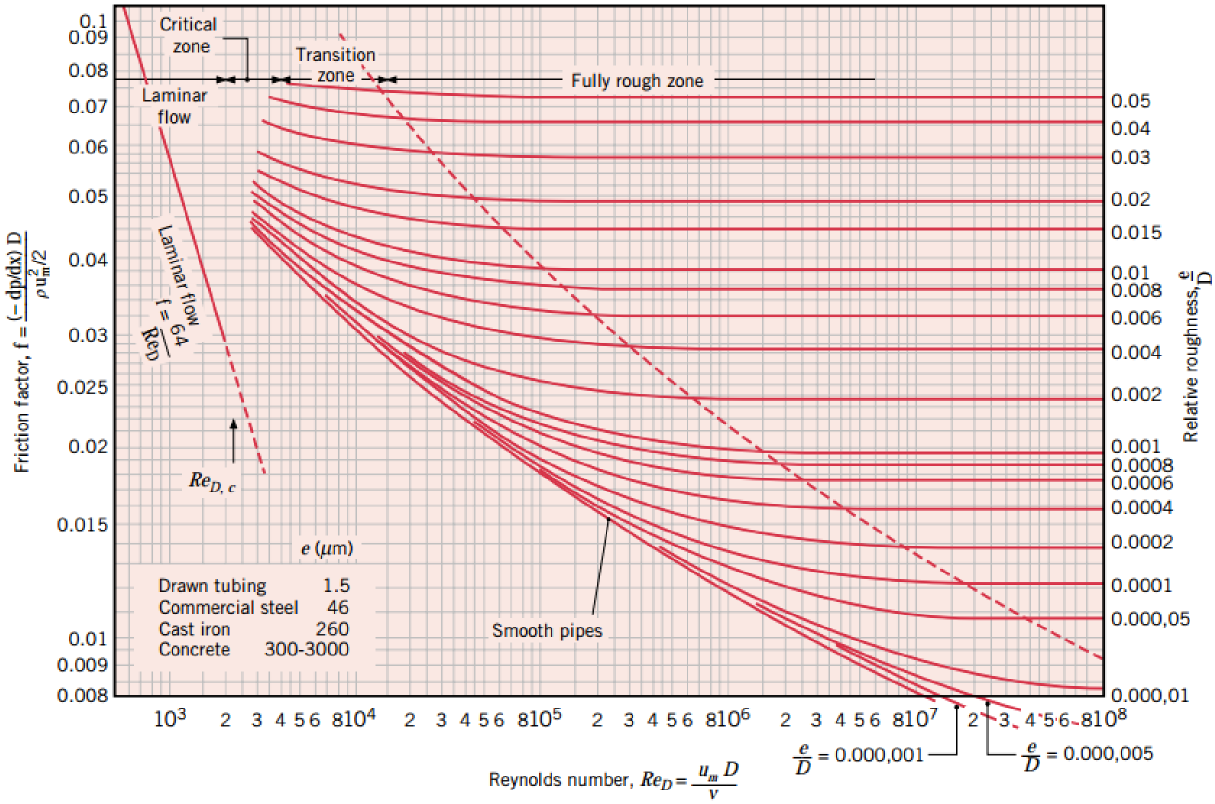
Friction factors for a wide Reynolds number range are presented in the Moody diagram of Figure 8.3. In addition to depending on the Reynolds number, the friction factor is a function of the tube surface condition. It is a minimum for smooth surfaces and increases with increasing surface roughness, e. Correlations that reasonably approximate the smooth surface condition are of the form:

$$f = 0.316 Re_D^{-1/4} \qquad Re_D \lesssim 2 \times 10^4$$

$$f = 0.184 Re_D^{-1/5} \qquad Re_D \gtrsim 2 \times 10^4$$

Alternatively, a single correlation that encompasses a large Reynolds number range has been developed by **Petukhov** and is of the form:

$$f = (0.790 \ln Re_D - 1.64)^{-2} \qquad 3000 \lesssim Re_D \lesssim 5 \times 10^6$$



The Pressure Drop (ΔP)

$$\Delta p = -\int_{p_1}^{p_2} dp = f\frac{\rho u_m^2}{2D}\int_{x_1}^{x_2} dx = f\frac{\rho u_m^2}{2D}(x_2 - x_1)$$

The Pump or Fan Power Requirement

$$P = (\Delta p)\dot{\forall}$$

Where the volumetric flow rate $\dot{\forall}$ may, in turn, be expressed as $\dot{\forall} = \dot{m}/\rho$ for an incompressible fluid.

Thermal Considerations

The shape of the fully developed temperature profile T(r,x) differs according to whether a uniform surface temperature or heat flux is maintained. For both surface conditions, however, the amount by which fluid temperatures exceed the entrance temperature increases with increasing x.

For laminar flow the thermal entry length may be expressed as:

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} \approx 0.05 Re_D Pr$$

Note:
if Pr >1, the hydrodynamic boundary layer develops more rapidly than the thermal boundary layer ($x_{fd,h} < x_{fd,t}$), while the inverse is true for Pr < 1. For extremely large Prandtl number fluids, such as

oils (Pr > 100), $x_{fd,h}$ is very much smaller than $x_{fd,t}$ and it is reasonable to assume a fully developed velocity profile throughout the thermal entry region.

In contrast, **for turbulent flow**, conditions are nearly independent of Prandtl number, and to a first approximation, we shall assume **$(x_{fd,t}/D) = 10$.**

The Mean Temperature

Just as the absence of a free stream velocity requires use of a mean velocity to describe an internal flow, the absence of a fixed free stream temperature necessitates using a mean (or bulk) temperature.

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} uTr\,dr$$

Newton's Law of Cooling

$$q_s'' = h(T_s - T_m)$$

Where h is the local convection heat transfer coefficient. However, there is an essential difference between T_m and T. Whereas T_∞ is constant in the flow direction, T_m must vary in this direction. That is, dT_m/dx is never zero if heat transfer is occurring. The value of T_m increases with x if heat transfer is from the surface to the fluid ($T_s > T_m$); it decreases with x if the opposite is true ($T_s < T_m$).

Note:

- *Hence in the thermally fully developed flow of a fluid with constant properties, the local convection coefficient is a constant, independent of x.*
- *In the entrance region, where h varies with x, as shown in Figure. Because the thermal boundary layer thickness is zero at the tube entrance, the convection coefficient is extremely large at x = 0. However, h decays rapidly as the thermal boundary layer develops, until the constant value associated with fully developed conditions is reached*

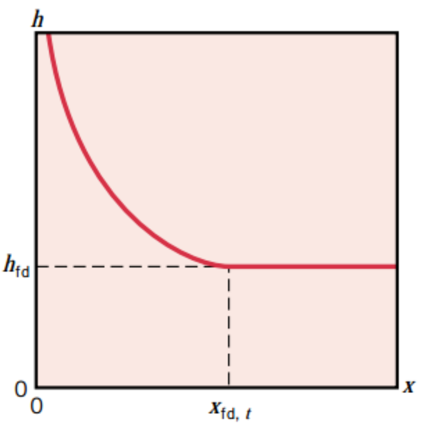


FIGURE 8.5 Axial variation of the convection heat transfer coefficient for flow in a tube.

Fully Developed Conditions

One might legitimately question whether fully developed thermal conditions can ever be reached.

The situation is certainly different from the hydrodynamic case, for which (δu/δx) = 0 in the fully developed region. In contrast, if there is heat transfer, (dT_m/dx), as well as (δT/δx) at any radius r, is not zero. Accordingly, the temperature profile T(r) is continuously changing with x, and it would seem that **a fully developed condition could never be reached**. This apparent contradiction may be reconciled by working with a dimensionless form of the temperature.

Requirement:

Although the temperature profile T(r) continues to change with x, the relative shape of the profile no longer changes and the flow is said to be thermally fully developed. The requirement for such a condition is formally stated as:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right]_{\text{fd}, t} = 0$$

The condition given by the equation above is eventually reached in a tube for which there is either a **uniform surface heat flux (q'' is constant)** or a **uniform surface temperature (T_s is constant). Note that** it is impossible to simultaneously impose the conditions of constant surface heat flux and constant surface temperature. If q'' is constant, T_s must vary with x; conversely, if T_s is constant, q'' must vary with x.

Case – Uniform Surface Heat Flux

Since both h and q'' are constant in the fully developed region, it follows from Equation:

$$\left.\frac{\partial T}{\partial x}\right|_{\text{fd}, t} = \left.\frac{dT_m}{dx}\right|_{\text{fd}, t} \qquad q_s'' = \text{constant}$$

Hence the axial temperature gradient is independent of the radial location.

Case – Constant Surface Temperature

$$\left.\frac{\partial T}{\partial x}\right|_{\text{fd}, t} = \left.\frac{(T_s - T)}{(T_s - T_m)}\frac{dT_m}{dx}\right|_{\text{fd}, t} \qquad T_s = \text{constant}$$

In which case, the value of $\partial T/\partial x$ depends on the radial coordinate.

The Energy Balance

Because the flow in a tube is completely enclosed, an energy balance may be applied to determine how the mean temperature Tm(x) varies with position along the tube and how the total convection heat transfer q_{conv} is related to the difference in temperatures at the tube inlet and outlet.

$$q_{\text{conv}} = \dot{m}c_p(T_{m,o} - T_{m,i})$$

$$dq_{\text{conv}} = \dot{m}c_p[(T_m + dT_m) - T_m]$$

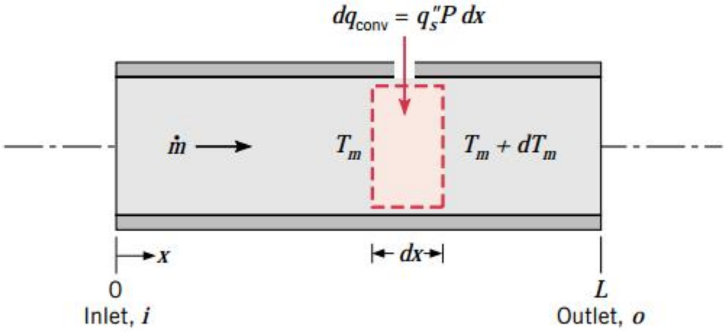


FIGURE 8.6 Control volume for internal flow in a tube.

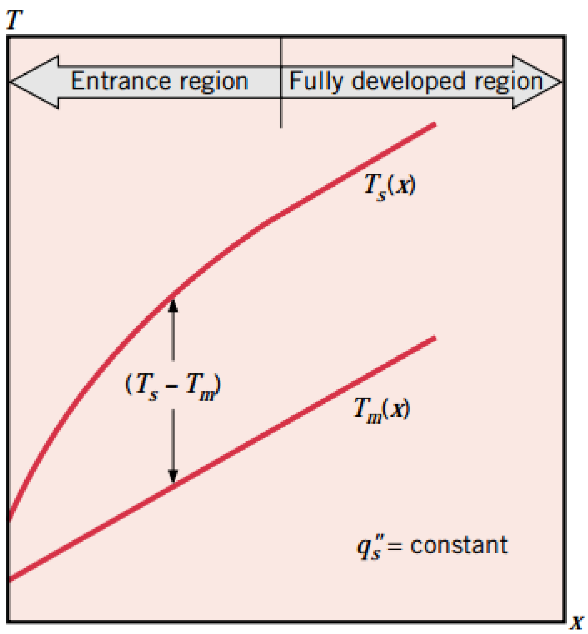
$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h(T_s - T_m)$$

The solution to Equation 8.37 for Tm(x) depends on the surface thermal condition. Recall that the two special cases of interest are constant surface heat flux and constant surface temperature.

Case – Constant Surface Heat Flux

$$q_{\text{conv}} = q_s''(P \cdot L)$$

$$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}c_p}x \qquad q_s'' = \text{constant}$$



Case – Constant Surface Temperature

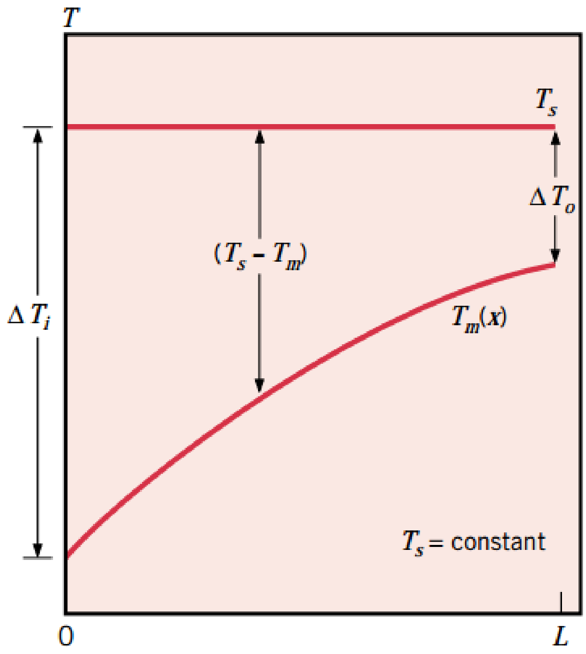
Let $\Delta T = T_s - T_m$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h \Delta T$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right) \qquad T_s = \text{constant}$$

Had we integrated from the tube inlet to some axial position x within the would have obtained the similar, but more general, result that

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right) \qquad T_s = \text{constant}$$



(b)

$$q_{\text{conv}} = \bar{h}A_s\Delta T_{\text{lm}} \qquad T_s = \text{constant} \tag{8.43}$$

where A_s is the tube surface area ($A_s = P \cdot L$) and ΔT_{lm} is the *log mean temperature difference,*

$$\Delta T_{\text{lm}} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)} \tag{8.44}$$

It is readily shown that the results of this section may still be used if T_s is replaced by T_∞ (the free stream temperature of the external fluid) and h (avg.) is replaced by (the average overall heat transfer coefficient). For such cases, it follows that:

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$$

and

$$q = \overline{U}A_s \Delta T_{\text{lm}}$$

Laminar Flow in Circular Tubes: Thermal Analysis and Convection Correlations

The Fully Developed Region:

$$Nu_D \equiv \frac{hD}{k} = 4.36 \qquad q_s'' = \text{constant}$$

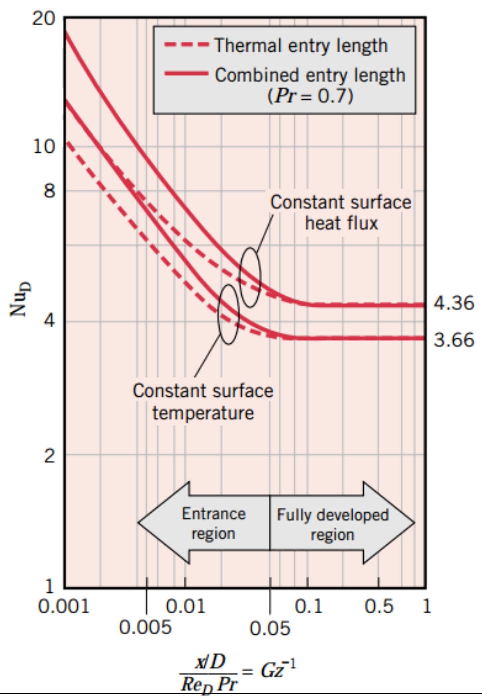
Hence in a circular tube characterized by uniform surface heat flux and laminar, fully developed conditions, the Nusselt number is a constant, independent of Re_D, Pr, and axial location.

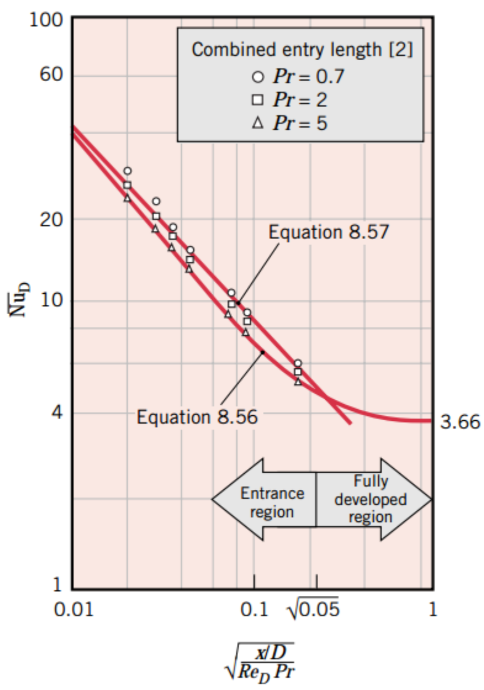
The thermal conductivity should be evaluated at T_m.

$$Nu_D = 3.66 \qquad T_s = \text{constant}$$

The thermal conductivity should be evaluated at T_m

The Entry Region:





For the constant surface temperature condition, it is desirable to know the average convection coefficient, presents a correlation attributed to Hausen, which is of the form:

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)Re_D Pr}{1 + 0.04[(D/L)Re_D Pr]^{2/3}}$$

$$\left[\begin{array}{l}\text{thermal entrance length} \\ or \\ \text{combined entrance length with } Pr \gtrsim 5\end{array}\right]$$

Because this result is for the thermal entry length problem, it is applicable to all situations where the velocity profile is already fully developed.

For the combined entry length, a suitable correlation for use at moderate Prandtl numbers, due to Sieder and Tate

$$\overline{Nu}_D = 1.86\left(\frac{Re_D Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$0.60 \lesssim Pr \lesssim 5$$

$$0.0044 \lesssim \left(\frac{\mu}{\mu_s}\right) \lesssim 9.75$$

All properties appearing in in the above two equations, except μ_s, should be evaluated at the average value of the mean temperature, $T_m = (T_{m,i} + T_{m,o})/2$.

Convection Correlations: Turbulent Flow in Circular Tubes

From the Chilton–Colburn:

$$\frac{C_f}{2} = \frac{f}{8} = St\,Pr^{2/3} = \frac{Nu_D}{Re_D\,Pr}Pr^{2/3}$$

The Dittus–Boelter equation:

$$Nu_D = 0.023 Re_D^{4/5} Pr^n \qquad (8.60)$$

where $n = 0.4$ for heating ($T_s > T_m$) and 0.3 for cooling ($T_s < T_m$). These equations have been confirmed experimentally for the range of conditions

$$\left[\begin{array}{l}0.7 \lesssim Pr \lesssim 160 \\ Re_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10\end{array}\right]$$

$$Nu_D = 0.023 Re_D^{4/5} Pr^n \qquad (8.60)$$

where $n = 0.4$ for heating ($T_s > T_m$) and 0.3 for cooling ($T_s < T_m$). These equations have been confirmed experimentally for the range of conditions

$$\begin{bmatrix} 0.7 \lesssim Pr \lesssim 160 \\ Re_D \gtrsim 10{,}000 \\ \frac{L}{D} \gtrsim 10 \end{bmatrix}$$

The equations may be used for small to moderate temperature differences, $T_s - T_m$, with all properties evaluated at T_m. For flows characterized by large property variations, the following equation, due to Sieder and Tate [9], is recommended:

$$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad (8.61)$$

$$\begin{bmatrix} 0.7 \lesssim Pr \lesssim 16{,}700 \\ Re_D \gtrsim 10{,}000 \\ \frac{L}{D} \gtrsim 10 \end{bmatrix}$$

The above correlations are used to a good approximation for both the uniform surface temperature and heat flux conditions.

For fully developed turbulent flow in smooth circular tubes with constant surface heat flux, **Skupinski** et al. recommend a correlation of the form:

$$Nu_D = 4.82 + 0.0185 Pe_D^{0.827} \qquad q_s'' = \text{constant} \qquad (8.64)$$

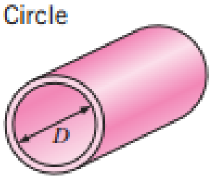
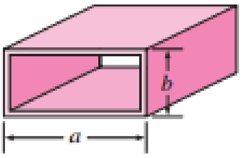
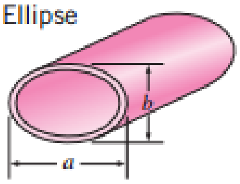
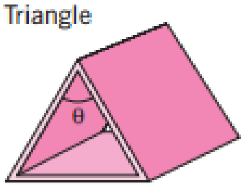
$$\begin{bmatrix} 3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5 \\ 10^2 \lesssim Pe_D \lesssim 10^4 \end{bmatrix}$$

Similarly, for constant surface temperature Seban and Shimazaki [22] recommend the following correlation for $Pe_D \gtrsim 100$:

$$Nu_D = 5.0 + 0.025 Pe_D^{0.8} \qquad T_s = \text{constant} \qquad (8.65)$$

Convection Correlations: Noncircular Tubes

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = \mathcal{V}_m D_h/\nu$, and $Nu = hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Convection Correlations: Concentric Tube Annulus

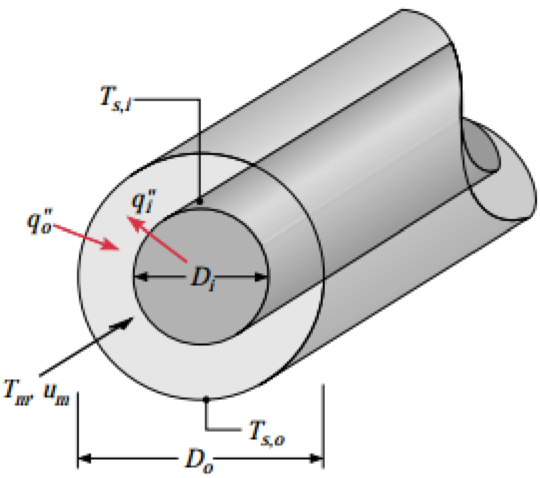


FIGURE 8.11
The concentric tube annulus.

$$q_i'' = h_i(T_{s,i} - T_m)$$

$$q_o'' = h_o(T_{s,o} - T_m)$$

$$Nu_i \equiv \frac{h_i D_h}{k}$$

$$Nu_o \equiv \frac{h_o D_h}{k}$$

the hydraulic diameter D_h is

$$D_h = \frac{4(\pi/4)(D_o^2 - D_i^2)}{\pi D_o + \pi D_i} = D_o - D_i$$

$$Nu_i = \frac{Nu_{ii}}{1 - (q_o''/q_i'')\theta_i^*}$$

$$Nu_o = \frac{Nu_{oo}}{1 - (q_i''/q_o'')\theta_o^*}$$

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\rho(D_o - D_i)}{\mu} \times \frac{\dot{m}_h}{\rho\pi(D_o^2 - D_i^2)/4}$$

$$Re_D = \frac{4\dot{m}_h}{\pi\,(D_o + D_i)\,\mu}$$

TABLE 8.3 Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces

D_i/D_o	Nu_{ii}	Nu_{oo}	θ_i^*	θ_o^*
0	—	4.364	∞	0
0.05	17.81	4.792	2.18	0.0294
0.10	11.91	4.834	1.383	0.0562
0.20	8.499	4.833	0.905	0.1041
0.40	6.583	4.979	0.603	0.1823
0.60	5.912	5.099	0.473	0.2455
0.80	5.58	5.24	0.401	0.299
1.00	5.385	5.385	0.346	0.346

Used with permission from W. M. Kays and H. C. Perkins, in W. M. Rohsenow and J. P. Hartnett, Eds., *Handbook of Heat Transfer,* Chap. 7, McGraw-Hill, New York, 1972.

TABLE 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	Comments
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
≈1.00	4.86	4.86	See Table 8.1, $b/a \rightarrow \infty$

Used with permission from W. M. Kays and H. C. Perkins, in W. M. Rohsenow and J. P. Hartnett, Eds., *Handbook of Heat Transfer,* Chap. 7, McGraw-Hill, New York, 1972.