

Hurricane Elena in the Gulf of Mexico. Unlike most small-scale fluids engineering applications, hurricanes are strongly affected by the Coriolis acceleration due to the rotation of the earth, which causes them to swirl counterclockwise in the Northern Hemisphere. The physical properties and boundary conditions which govern such flows are discussed in the present chapter. (*Courtesy of NASA/Color-Pic Inc./E.R. Degginger/Color-Pic Inc.*)

Chapter 1

Introduction

1.1 Preliminary Remarks

Fluid mechanics is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*) and the subsequent effects of the fluid upon the boundaries, which may be either solid surfaces or interfaces with other fluids. Both gases and liquids are classified as fluids, and the number of fluids engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few. When you think about it, almost everything on this planet either is a fluid or moves within or near a fluid.

The essence of the subject of fluid flow is a judicious compromise between theory and experiment. Since fluid flow is a branch of mechanics, it satisfies a set of well-documented basic laws, and thus a great deal of theoretical treatment is available. However, the theory is often frustrating, because it applies mainly to idealized situations which may be invalid in practical problems. The two chief obstacles to a workable theory are geometry and viscosity. The basic equations of fluid motion (Chap. 4) are too difficult to enable the analyst to attack arbitrary geometric configurations. Thus most textbooks concentrate on flat plates, circular pipes, and other easy geometries. It is possible to apply numerical computer techniques to complex geometries, and specialized textbooks are now available to explain the new *computational fluid dynamics* (CFD) approximations and methods [1, 2, 29].¹ This book will present many theoretical results while keeping their limitations in mind.

The second obstacle to a workable theory is the action of viscosity, which can be neglected only in certain idealized flows (Chap. 8). First, viscosity increases the difficulty of the basic equations, although the boundary-layer approximation found by Ludwig Prandtl in 1904 (Chap. 7) has greatly simplified viscous-flow analyses. Second, viscosity has a destabilizing effect on all fluids, giving rise, at frustratingly small velocities, to a disorderly, random phenomenon called *turbulence*. The theory of turbulent flow is crude and heavily backed up by experiment (Chap. 6), yet it can be quite serviceable as an engineering estimate. Textbooks now present digital-computer techniques for turbulent-flow analysis [32], but they are based strictly upon empirical assumptions regarding the time mean of the turbulent stress field.

¹Numbered references appear at the end of each chapter.

Thus there is theory available for fluid-flow problems, but in all cases it should be backed up by experiment. Often the experimental data provide the main source of information about specific flows, such as the drag and lift of immersed bodies (Chap. 7). Fortunately, fluid mechanics is a highly visual subject, with good instrumentation [4, 5, 35], and the use of dimensional analysis and modeling concepts (Chap. 5) is widespread. Thus experimentation provides a natural and easy complement to the theory. You should keep in mind that theory and experiment should go hand in hand in all studies of fluid mechanics.

1.2 The Concept of a Fluid

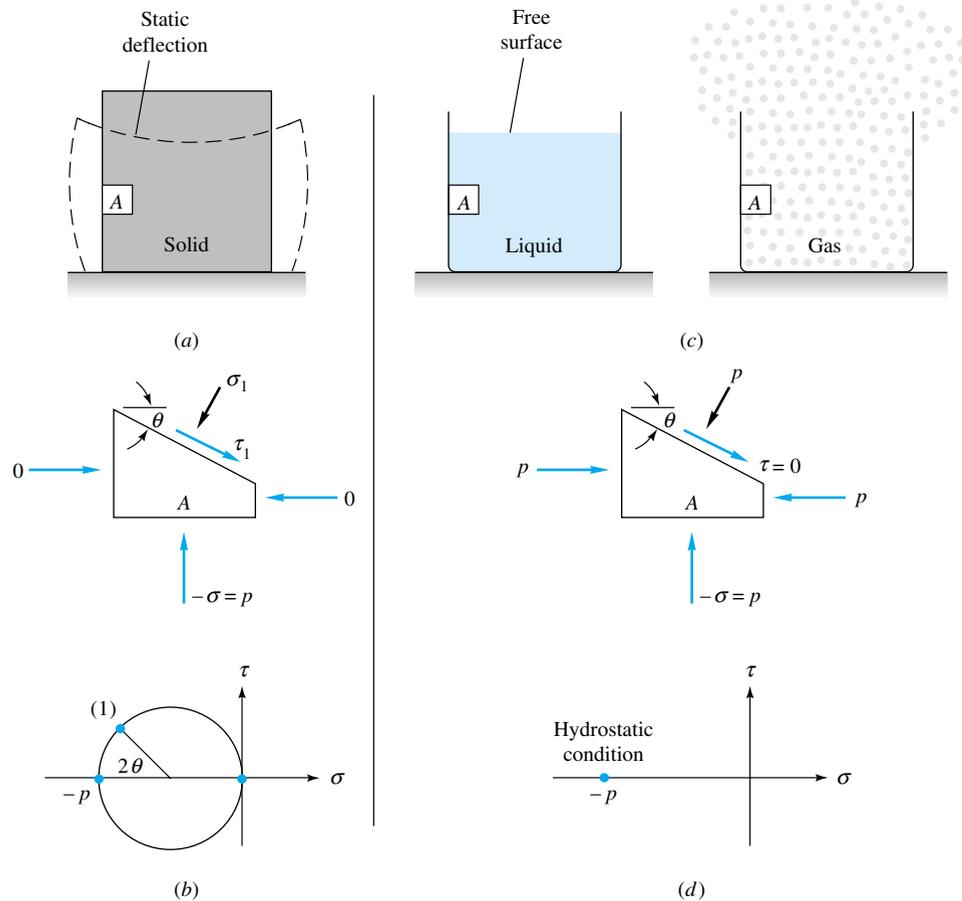
From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. The difference between the two is perfectly obvious to the layperson, and it is an interesting exercise to ask a layperson to put this difference into words. The technical distinction lies with the reaction of the two to an applied shear or tangential stress. *A solid can resist a shear stress by a static deformation; a fluid cannot.* Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. As a corollary, we can say that a fluid at rest must be in a state of zero shear stress, a state often called the hydrostatic stress condition in structural analysis. In this condition, Mohr's circle for stress reduces to a point, and there is no shear stress on any plane cut through the element under stress.

Given the definition of a fluid above, every layperson also knows that there are two classes of fluids, *liquids* and *gases*. Again the distinction is a technical one concerning the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above. Free-surface flows are dominated by gravitational effects and are studied in Chaps. 5 and 10. Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand until it encounters confining walls. A gas has no definite volume, and when left to itself without confinement, a gas forms an atmosphere which is essentially hydrostatic. The hydrostatic behavior of liquids and gases is taken up in Chap. 2. Gases cannot form a free surface, and thus gas flows are rarely concerned with gravitational effects other than buoyancy.

Figure 1.1 illustrates a solid block resting on a rigid plane and stressed by its own weight. The solid sags into a static deflection, shown as a highly exaggerated dashed line, resisting shear without flow. A free-body diagram of element *A* on the side of the block shows that there is shear in the block along a plane cut at an angle θ through *A*. Since the block sides are unsupported, element *A* has zero stress on the left and right sides and compression stress $\sigma = -p$ on the top and bottom. Mohr's circle does not reduce to a point, and there is nonzero shear stress in the block.

By contrast, the liquid and gas at rest in Fig. 1.1 require the supporting walls in order to eliminate shear stress. The walls exert a compression stress of $-p$ and reduce Mohr's circle to a point with zero shear everywhere, i.e., the hydrostatic condition. The liquid retains its volume and forms a free surface in the container. If the walls are removed, shear develops in the liquid and a big splash results. If the container is tilted, shear again develops, waves form, and the free surface seeks a horizontal configura-

Fig. 1.1 A solid at rest can resist shear. (a) Static deflection of the solid; (b) equilibrium and Mohr's circle for solid element A . A fluid cannot resist shear. (c) Containing walls are needed; (d) equilibrium and Mohr's circle for fluid element A .



tion, pouring out over the lip if necessary. Meanwhile, the gas is unrestrained and expands out of the container, filling all available space. Element A in the gas is also hydrostatic and exerts a compression stress $-p$ on the walls.

In the above discussion, clear decisions could be made about solids, liquids, and gases. Most engineering fluid-mechanics problems deal with these clear cases, i.e., the common liquids, such as water, oil, mercury, gasoline, and alcohol, and the common gases, such as air, helium, hydrogen, and steam, in their common temperature and pressure ranges. There are many borderline cases, however, of which you should be aware. Some apparently "solid" substances such as asphalt and lead resist shear stress for short periods but actually deform slowly and exhibit definite fluid behavior over long periods. Other substances, notably colloid and slurry mixtures, resist small shear stresses but "yield" at large stress and begin to flow as fluids do. Specialized textbooks are devoted to this study of more general deformation and flow, a field called *rheology* [6]. Also, liquids and gases can coexist in two-phase mixtures, such as steam-water mixtures or water with entrapped air bubbles. Specialized textbooks present the analysis

of such *two-phase flows* [7]. Finally, there are situations where the distinction between a liquid and a gas blurs. This is the case at temperatures and pressures above the so-called *critical point* of a substance, where only a single phase exists, primarily resembling a gas. As pressure increases far above the critical point, the gaslike substance becomes so dense that there is some resemblance to a liquid and the usual thermodynamic approximations like the perfect-gas law become inaccurate. The critical temperature and pressure of water are $T_c = 647$ K and $p_c = 219$ atm,² so that typical problems involving water and steam are below the critical point. Air, being a mixture of gases, has no distinct critical point, but its principal component, nitrogen, has $T_c = 126$ K and $p_c = 34$ atm. Thus typical problems involving air are in the range of high temperature and low pressure where air is distinctly and definitely a gas. This text will be concerned solely with clearly identifiable liquids and gases, and the borderline cases discussed above will be beyond our scope.

1.3 The Fluid as a Continuum

We have already used technical terms such as *fluid pressure* and *density* without a rigorous discussion of their definition. As far as we know, fluids are aggregations of molecules, widely spaced for a gas, closely spaced for a liquid. The distance between molecules is very large compared with the molecular diameter. The molecules are not fixed in a lattice but move about freely relative to each other. Thus fluid density, or mass per unit volume, has no precise meaning because the number of molecules occupying a given volume continually changes. This effect becomes unimportant if the unit volume is large compared with, say, the cube of the molecular spacing, when the number of molecules within the volume will remain nearly constant in spite of the enormous interchange of particles across the boundaries. If, however, the chosen unit volume is too large, there could be a noticeable variation in the bulk aggregation of the particles. This situation is illustrated in Fig. 1.2, where the “density” as calculated from molecular mass δm within a given volume δV is plotted versus the size of the unit volume. There is a limiting volume δV^* below which molecular variations may be important and

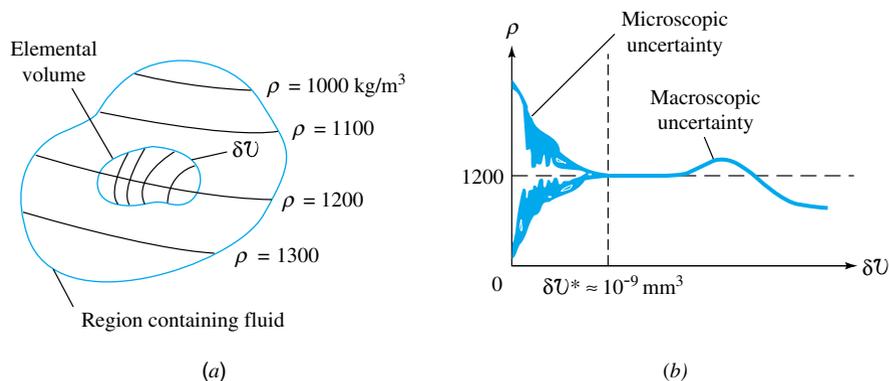


Fig. 1.2 The limit definition of continuum fluid density: (a) an elemental volume in a fluid region of variable continuum density; (b) calculated density versus size of the elemental volume.

²One atmosphere equals $2116 \text{ lbf/ft}^2 = 101,300 \text{ Pa}$.

above which aggregate variations may be important. The *density* ρ of a fluid is best defined as

$$\rho = \lim_{\delta V \rightarrow \delta V^*} \frac{\delta m}{\delta V} \quad (1.1)$$

The limiting volume δV^* is about 10^{-9} mm^3 for all liquids and for gases at atmospheric pressure. For example, 10^{-9} mm^3 of air at standard conditions contains approximately 3×10^7 molecules, which is sufficient to define a nearly constant density according to Eq. (1.1). Most engineering problems are concerned with physical dimensions much larger than this limiting volume, so that density is essentially a point function and fluid properties can be thought of as varying continually in space, as sketched in Fig. 1.2a. Such a fluid is called a *continuum*, which simply means that its variation in properties is so smooth that the differential calculus can be used to analyze the substance. We shall assume that continuum calculus is valid for all the analyses in this book. Again there are borderline cases for gases at such low pressures that molecular spacing and mean free path³ are comparable to, or larger than, the physical size of the system. This requires that the continuum approximation be dropped in favor of a molecular theory of rarefied-gas flow [8]. In principle, all fluid-mechanics problems can be attacked from the molecular viewpoint, but no such attempt will be made here. Note that the use of continuum calculus does not preclude the possibility of discontinuous jumps in fluid properties across a free surface or fluid interface or across a shock wave in a compressible fluid (Chap. 9). Our calculus in Chap. 4 must be flexible enough to handle discontinuous boundary conditions.

1.4 Dimensions and Units

A *dimension* is the measure by which a physical variable is expressed quantitatively. A *unit* is a particular way of attaching a number to the quantitative dimension. Thus length is a dimension associated with such variables as distance, displacement, width, deflection, and height, while centimeters and inches are both numerical units for expressing length. Dimension is a powerful concept about which a splendid tool called *dimensional analysis* has been developed (Chap. 5), while units are the nitty-gritty, the number which the customer wants as the final answer.

Systems of units have always varied widely from country to country, even after international agreements have been reached. Engineers need numbers and therefore unit systems, and the numbers must be accurate because the safety of the public is at stake. You cannot design and build a piping system whose diameter is D and whose length is L . And U.S. engineers have persisted too long in clinging to British systems of units. There is too much margin for error in most British systems, and many an engineering student has flunked a test because of a missing or improper conversion factor of 12 or 144 or 32.2 or 60 or 1.8. Practicing engineers can make the same errors. The writer is aware from personal experience of a serious preliminary error in the design of an aircraft due to a missing factor of 32.2 to convert pounds of mass to slugs.

In 1872 an international meeting in France proposed a treaty called the Metric Convention, which was signed in 1875 by 17 countries including the United States. It was an improvement over British systems because its use of base 10 is the foundation of our number system, learned from childhood by all. Problems still remained because

³The mean distance traveled by molecules between collisions.

even the metric countries differed in their use of kiloponds instead of dynes or newtons, kilograms instead of grams, or calories instead of joules. To standardize the metric system, a General Conference of Weights and Measures attended in 1960 by 40 countries proposed the *International System of Units* (SI). We are now undergoing a painful period of transition to SI, an adjustment which may take many more years to complete. The professional societies have led the way. Since July 1, 1974, SI units have been required by all papers published by the American Society of Mechanical Engineers, which prepared a useful booklet explaining the SI [9]. The present text will use SI units together with British gravitational (BG) units.

Primary Dimensions

In fluid mechanics there are only four *primary dimensions* from which all other dimensions can be derived: mass, length, time, and temperature.⁴ These dimensions and their units in both systems are given in Table 1.1. Note that the kelvin unit uses no degree symbol. The braces around a symbol like $\{M\}$ mean “the dimension” of mass. All other variables in fluid mechanics can be expressed in terms of $\{M\}$, $\{L\}$, $\{T\}$, and $\{\Theta\}$. For example, acceleration has the dimensions $\{LT^{-2}\}$. The most crucial of these secondary dimensions is force, which is directly related to mass, length, and time by Newton’s second law

$$\mathbf{F} = m\mathbf{a} \quad (1.2)$$

From this we see that, dimensionally, $\{F\} = \{MLT^{-2}\}$. A constant of proportionality is avoided by defining the force unit exactly in terms of the primary units. Thus we define the newton and the pound of force

$$\begin{aligned} 1 \text{ newton of force} &= 1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2 \\ 1 \text{ pound of force} &= 1 \text{ lbf} \equiv 1 \text{ slug} \cdot \text{ft/s}^2 = 4.4482 \text{ N} \end{aligned} \quad (1.3)$$

In this book the abbreviation *lbf* is used for pound-force and *lb* for pound-mass. If instead one adopts other force units such as the dyne or the poundal or kilopond or adopts other mass units such as the gram or pound-mass, a constant of proportionality called g_c must be included in Eq. (1.2). We shall not use g_c in this book since it is not necessary in the SI and BG systems.

A list of some important secondary variables in fluid mechanics, with dimensions derived as combinations of the four primary dimensions, is given in Table 1.2. A more complete list of conversion factors is given in App. C.

Table 1.1 Primary Dimensions in SI and BG Systems

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine ($^{\circ}\text{R}$)	1 K = 1.8 $^{\circ}\text{R}$

⁴If electromagnetic effects are important, a fifth primary dimension must be included, electric current $\{I\}$, whose SI unit is the ampere (A).

Table 1.2 Secondary Dimensions in Fluid Mechanics

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1 m^2 = 10.764 ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1 m^3 = 35.315 ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	$1 ft/s = 0.3048 m/s$
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1 ft/s^2 = 0.3048 m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1 lbf/ft^2 = 47.88 Pa$
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1 s^{-1} = 1 s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	$J = N \cdot m$	$ft \cdot lbf$	$1 ft \cdot lbf = 1.3558 J$
Power $\{ML^2T^{-3}\}$	$W = J/s$	$ft \cdot lbf/s$	$1 ft \cdot lbf/s = 1.3558 W$
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1 slug/ft^3 = 515.4 kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	$kg/(m \cdot s)$	$slugs/(ft \cdot s)$	$1 slug/(ft \cdot s) = 47.88 kg/(m \cdot s)$
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot ^\circ R)$	$1 m^2/(s^2 \cdot K) = 5.980 ft^2/(s^2 \cdot ^\circ R)$

EXAMPLE 1.1

A body weighs 1000 lbf when exposed to a standard earth gravity $g = 32.174 ft/s^2$. (a) What is its mass in kg? (b) What will the weight of this body be in N if it is exposed to the moon's standard acceleration $g_{\text{moon}} = 1.62 m/s^2$? (c) How fast will the body accelerate if a net force of 400 lbf is applied to it on the moon or on the earth?

Solution

Part (a) Equation (1.2) holds with $F = \text{weight}$ and $a = g_{\text{earth}}$:

$$F = W = mg = 1000 \text{ lbf} = (m \text{ slugs})(32.174 \text{ ft/s}^2)$$

or

$$m = \frac{1000}{32.174} = (31.08 \text{ slugs})(14.5939 \text{ kg/slug}) = 453.6 \text{ kg} \quad \text{Ans. (a)}$$

The change from 31.08 slugs to 453.6 kg illustrates the proper use of the conversion factor 14.5939 kg/slug.

Part (b) The mass of the body remains 453.6 kg regardless of its location. Equation (1.2) applies with a new value of a and hence a new force

$$F = W_{\text{moon}} = mg_{\text{moon}} = (453.6 \text{ kg})(1.62 \text{ m/s}^2) = 735 \text{ N} \quad \text{Ans. (b)}$$

Part (c) This problem does not involve weight or gravity or position and is simply a direct application of Newton's law with an unbalanced force:

$$F = 400 \text{ lbf} = ma = (31.08 \text{ slugs})(a \text{ ft/s}^2)$$

or

$$a = \frac{400}{31.08} = 12.43 \text{ ft/s}^2 = 3.79 \text{ m/s}^2 \quad \text{Ans. (c)}$$

This acceleration would be the same on the moon or earth or anywhere.

Many data in the literature are reported in inconvenient or arcane units suitable only to some industry or specialty or country. The engineer should convert these data to the SI or BG system before using them. This requires the systematic application of conversion factors, as in the following example.

EXAMPLE 1.2

An early viscosity unit in the cgs system is the poise (abbreviated P), or $\text{g}/(\text{cm} \cdot \text{s})$, named after J. L. M. Poiseuille, a French physician who performed pioneering experiments in 1840 on water flow in pipes. The viscosity of water (fresh or salt) at $293.16 \text{ K} = 20^\circ\text{C}$ is approximately $\mu = 0.01 \text{ P}$. Express this value in (a) SI and (b) BG units.

Solution

Part (a)
$$\mu = [0.01 \text{ g}/(\text{cm} \cdot \text{s})] \frac{1 \text{ kg}}{1000 \text{ g}} (100 \text{ cm/m}) = 0.001 \text{ kg}/(\text{m} \cdot \text{s}) \quad \text{Ans. (a)}$$

Part (b)
$$\begin{aligned} \mu &= [0.001 \text{ kg}/(\text{m} \cdot \text{s})] \frac{1 \text{ slug}}{14.59 \text{ kg}} (0.3048 \text{ m/ft}) \\ &= 2.09 \times 10^{-5} \text{ slug}/(\text{ft} \cdot \text{s}) \quad \text{Ans. (b)} \end{aligned}$$

Note: Result (b) could have been found directly from (a) by dividing (a) by the viscosity conversion factor 47.88 listed in Table 1.2.

We repeat our advice: Faced with data in unusual units, convert them immediately to either SI or BG units because (1) it is more professional and (2) theoretical equations in fluid mechanics are *dimensionally consistent* and require no further conversion factors when these two fundamental unit systems are used, as the following example shows.

EXAMPLE 1.3

A useful theoretical equation for computing the relation between pressure, velocity, and altitude in a steady flow of a nearly inviscid, nearly incompressible fluid with negligible heat transfer and shaft work⁵ is the *Bernoulli relation*, named after Daniel Bernoulli, who published a hydrodynamics textbook in 1738:

$$p_0 = p + \frac{1}{2}\rho V^2 + \rho gZ \quad (1)$$

where p_0 = stagnation pressure
 p = pressure in moving fluid
 V = velocity
 ρ = density
 Z = altitude
 g = gravitational acceleration

⁵That's an awful lot of assumptions, which need further study in Chap. 3.

(a) Show that Eq. (1) satisfies the principle of dimensional homogeneity, which states that all additive terms in a physical equation must have the same dimensions. (b) Show that consistent units result without additional conversion factors in SI units. (c) Repeat (b) for BG units.

Solution

Part (a) We can express Eq. (1) dimensionally, using braces by entering the dimensions of each term from Table 1.2:

$$\begin{aligned}\{ML^{-1}T^{-2}\} &= \{ML^{-1}T^{-2}\} + \{ML^{-3}\}\{L^2T^{-2}\} + \{ML^{-3}\}\{LT^{-2}\}\{L\} \\ &= \{ML^{-1}T^{-2}\} \text{ for all terms}\end{aligned}\quad \text{Ans. (a)}$$

Part (b) Enter the SI units for each quantity from Table 1.2:

$$\begin{aligned}\{\text{N/m}^2\} &= \{\text{N/m}^2\} + \{\text{kg/m}^3\}\{\text{m}^2/\text{s}^2\} + \{\text{kg/m}^3\}\{\text{m/s}^2\}\{\text{m}\} \\ &= \{\text{N/m}^2\} + \{\text{kg}/(\text{m} \cdot \text{s}^2)\}\end{aligned}$$

The right-hand side looks bad until we remember from Eq. (1.3) that $1 \text{ kg} = 1 \text{ N} \cdot \text{s}^2/\text{m}$.

$$\{\text{kg}/(\text{m} \cdot \text{s}^2)\} = \frac{\{\text{N} \cdot \text{s}^2/\text{m}\}}{\{\text{m} \cdot \text{s}^2\}} = \{\text{N/m}^2\} \quad \text{Ans. (b)}$$

Thus all terms in Bernoulli's equation will have units of pascals, or newtons per square meter, when SI units are used. No conversion factors are needed, which is true of all theoretical equations in fluid mechanics.

Part (c) Introducing BG units for each term, we have

$$\begin{aligned}\{\text{lbf}/\text{ft}^2\} &= \{\text{lbf}/\text{ft}^2\} + \{\text{slugs}/\text{ft}^3\}\{\text{ft}^2/\text{s}^2\} + \{\text{slugs}/\text{ft}^3\}\{\text{ft}/\text{s}^2\}\{\text{ft}\} \\ &= \{\text{lbf}/\text{ft}^2\} + \{\text{slugs}/(\text{ft} \cdot \text{s}^2)\}\end{aligned}$$

But, from Eq. (1.3), $1 \text{ slug} = 1 \text{ lbf} \cdot \text{s}^2/\text{ft}$, so that

$$\{\text{slugs}/(\text{ft} \cdot \text{s}^2)\} = \frac{\{\text{lbf} \cdot \text{s}^2/\text{ft}\}}{\{\text{ft} \cdot \text{s}^2\}} = \{\text{lbf}/\text{ft}^2\} \quad \text{Ans. (c)}$$

All terms have the unit of pounds-force per square foot. No conversion factors are needed in the BG system either.

There is still a tendency in English-speaking countries to use pound-force per square inch as a pressure unit because the numbers are more manageable. For example, standard atmospheric pressure is $14.7 \text{ lbf}/\text{in}^2 = 2116 \text{ lbf}/\text{ft}^2 = 101,300 \text{ Pa}$. The pascal is a small unit because the newton is less than $\frac{1}{4}$ lbf and a square meter is a very large area. It is felt nevertheless that the pascal will gradually gain universal acceptance; e.g., repair manuals for U.S. automobiles now specify pressure measurements in pascals.

Consistent Units

Note that not only must all (fluid) mechanics equations be dimensionally homogeneous, one must also use *consistent units*; that is, each additive term must have the same units. There is no trouble doing this with the SI and BG systems, as in Ex. 1.3, but woe unto

those who try to mix colloquial English units. For example, in Chap. 9, we often use the assumption of steady adiabatic compressible gas flow:

$$h + \frac{1}{2}V^2 = \text{constant}$$

where h is the fluid enthalpy and $V^2/2$ is its kinetic energy. Colloquial thermodynamic tables might list h in units of British thermal units per pound (Btu/lb), whereas V is likely used in ft/s. It is completely erroneous to add Btu/lb to ft^2/s^2 . The proper unit for h in this case is $\text{ft} \cdot \text{lb}/\text{slug}$, which is identical to ft^2/s^2 . The conversion factor is $1 \text{ Btu/lb} \approx 25,040 \text{ ft}^2/\text{s}^2 = 25,040 \text{ ft} \cdot \text{lb}/\text{slug}$.

Homogeneous versus Dimensionally Inconsistent Equations

All theoretical equations in mechanics (and in other physical sciences) are *dimensionally homogeneous*; i.e., each additive term in the equation has the same dimensions. For example, Bernoulli's equation (1) in Example 1.3 is dimensionally homogeneous: Each term has the dimensions of pressure or stress of $\{F/L^2\}$. Another example is the equation from physics for a body falling with negligible air resistance:

$$S = S_0 + V_0t + \frac{1}{2}gt^2$$

where S_0 is initial position, V_0 is initial velocity, and g is the acceleration of gravity. Each term in this relation has dimensions of length $\{L\}$. The factor $\frac{1}{2}$, which arises from integration, is a pure (dimensionless) number, $\{1\}$. The exponent 2 is also dimensionless.

However, the reader should be warned that many empirical formulas in the engineering literature, arising primarily from correlations of data, are dimensionally inconsistent. Their units cannot be reconciled simply, and some terms may contain hidden variables. An example is the formula which pipe valve manufacturers cite for liquid volume flow rate Q (m^3/s) through a partially open valve:

$$Q = C_V \left(\frac{\Delta p}{\text{SG}} \right)^{1/2}$$

where Δp is the pressure drop across the valve and SG is the specific gravity of the liquid (the ratio of its density to that of water). The quantity C_V is the *valve flow coefficient*, which manufacturers tabulate in their valve brochures. Since SG is dimensionless $\{1\}$, we see that this formula is totally inconsistent, with one side being a flow rate $\{L^3/T\}$ and the other being the square root of a pressure drop $\{M^{1/2}/L^{1/2}T\}$. It follows that C_V must have dimensions, and rather odd ones at that: $\{L^{7/2}/M^{1/2}\}$. Nor is the resolution of this discrepancy clear, although one hint is that the values of C_V in the literature increase nearly as the square of the size of the valve. The presentation of experimental data in homogeneous form is the subject of *dimensional analysis* (Chap. 5). There we shall learn that a homogeneous form for the valve flow relation is

$$Q = C_d A_{\text{opening}} \left(\frac{\Delta p}{\rho} \right)^{1/2}$$

where ρ is the liquid density and A the area of the valve opening. The *discharge coefficient* C_d is dimensionless and changes only slightly with valve size. Please believe—until we establish the fact in Chap. 5—that this latter is a *much* better formulation of the data.

Meanwhile, we conclude that dimensionally inconsistent equations, though they abound in engineering practice, are misleading and vague and even dangerous, in the sense that they are often misused outside their range of applicability.

Convenient Prefixes in Powers of 10

Table 1.3 Convenient Prefixes for Engineering Units

Multiplicative factor	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Engineering results often are too small or too large for the common units, with too many zeros one way or the other. For example, to write $p = 114,000,000$ Pa is long and awkward. Using the prefix “M” to mean 10^6 , we convert this to a concise $p = 114$ MPa (megapascals). Similarly, $t = 0.000000003$ s is a proofreader’s nightmare compared to the equivalent $t = 3$ ns (nanoseconds). Such prefixes are common and convenient, in both the SI and BG systems. A complete list is given in Table 1.3.

EXAMPLE 1.4

In 1890 Robert Manning, an Irish engineer, proposed the following empirical formula for the average velocity V in uniform flow due to gravity down an open channel (BG units):

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \quad (1)$$

where R = hydraulic radius of channel (Chaps. 6 and 10)

S = channel slope (tangent of angle that bottom makes with horizontal)

n = Manning’s roughness factor (Chap. 10)

and n is a constant for a given surface condition for the walls and bottom of the channel. (a) Is Manning’s formula dimensionally consistent? (b) Equation (1) is commonly taken to be valid in BG units with n taken as dimensionless. Rewrite it in SI form.

Solution

Part (a) Introduce dimensions for each term. The slope S , being a tangent or ratio, is dimensionless, denoted by {unity} or {1}. Equation (1) in dimensional form is

$$\left\{ \frac{L}{T} \right\} = \left\{ \frac{1.49}{n} \right\} \{L^{2/3}\} \{1\}$$

This formula cannot be consistent unless $\{1.49/n\} = \{L^{1/3}/T\}$. If n is dimensionless (and it is never listed with units in textbooks), then the numerical value 1.49 must have units. This can be tragic to an engineer working in a different unit system unless the discrepancy is properly documented. In fact, Manning’s formula, though popular, is inconsistent both dimensionally and physically and does not properly account for channel-roughness effects except in a narrow range of parameters, for water only.

Part (b) From part (a), the number 1.49 must have dimensions $\{L^{1/3}/T\}$ and thus in BG units equals $1.49 \text{ ft}^{1/3}/\text{s}$. By using the SI conversion factor for length we have

$$(1.49 \text{ ft}^{1/3}/\text{s})(0.3048 \text{ m/ft})^{1/3} = 1.00 \text{ m}^{1/3}/\text{s}$$

Therefore Manning’s formula in SI becomes

$$V = \frac{1.0}{n} R^{2/3} S^{1/2} \quad \text{Ans. (b) (2)}$$

with R in m and V in m/s. Actually, we misled you: This is the way Manning, a metric user, first proposed the formula. It was later converted to BG units. Such dimensionally inconsistent formulas are dangerous and should either be reanalyzed or treated as having very limited application.

1.5 Properties of the Velocity Field

In a given flow situation, the determination, by experiment or theory, of the properties of the fluid as a function of position and time is considered to be the *solution* to the problem. In almost all cases, the emphasis is on the space-time distribution of the fluid properties. One rarely keeps track of the actual fate of the specific fluid particles.⁶ This treatment of properties as continuum-field functions distinguishes fluid mechanics from solid mechanics, where we are more likely to be interested in the trajectories of individual particles or systems.

Eulerian and Lagrangian Descriptions

There are two different points of view in analyzing problems in mechanics. The first view, appropriate to fluid mechanics, is concerned with the field of flow and is called the *eulerian* method of description. In the eulerian method we compute the pressure field $p(x, y, z, t)$ of the flow pattern, not the pressure changes $p(t)$ which a particle experiences as it moves through the field.

The second method, which follows an individual particle moving through the flow, is called the *lagrangian* description. The lagrangian approach, which is more appropriate to solid mechanics, will not be treated in this book. However, certain numerical analyses of sharply bounded fluid flows, such as the motion of isolated fluid droplets, are very conveniently computed in lagrangian coordinates [1].

Fluid-dynamic measurements are also suited to the eulerian system. For example, when a pressure probe is introduced into a laboratory flow, it is fixed at a specific position (x, y, z) . Its output thus contributes to the description of the eulerian pressure field $p(x, y, z, t)$. To simulate a lagrangian measurement, the probe would have to move downstream at the fluid particle speeds; this is sometimes done in oceanographic measurements, where flowmeters drift along with the prevailing currents.

The two different descriptions can be contrasted in the analysis of traffic flow along a freeway. A certain length of freeway may be selected for study and called the field of flow. Obviously, as time passes, various cars will enter and leave the field, and the identity of the specific cars within the field will constantly be changing. The traffic engineer ignores specific cars and concentrates on their average velocity as a function of time and position within the field, plus the flow rate or number of cars per hour passing a given section of the freeway. This engineer is using an eulerian description of the traffic flow. Other investigators, such as the police or social scientists, may be interested in the path or speed or destination of specific cars in the field. By following a specific car as a function of time, they are using a lagrangian description of the flow.

The Velocity Field

Foremost among the properties of a flow is the velocity field $\mathbf{V}(x, y, z, t)$. In fact, determining the velocity is often tantamount to solving a flow problem, since other prop-

⁶One example where fluid-particle paths are important is in water-quality analysis of the fate of contaminant discharges.

erties follow directly from the velocity field. Chapter 2 is devoted to the calculation of the pressure field once the velocity field is known. Books on heat transfer (for example, Ref. 10) are essentially devoted to finding the temperature field from known velocity fields.

In general, velocity is a vector function of position and time and thus has three components u , v , and w , each a scalar field in itself:

$$\mathbf{V}(x, y, z, t) = iu(x, y, z, t) + jv(x, y, z, t) + kw(x, y, z, t) \quad (1.4)$$

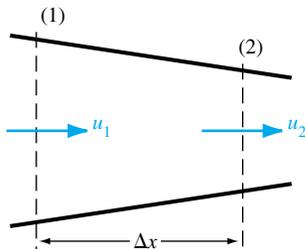
The use of u , v , and w instead of the more logical component notation V_x , V_y , and V_z is the result of an almost unbreakable custom in fluid mechanics.

Several other quantities, called *kinematic properties*, can be derived by mathematically manipulating the velocity field. We list some kinematic properties here and give more details about their use and derivation in later chapters:

1. Displacement vector: $\mathbf{r} = \int \mathbf{V} dt$ (Sec. 1.9)
2. Acceleration: $\mathbf{a} = \frac{d\mathbf{V}}{dt}$ (Sec. 4.1)
3. Volume rate of flow: $Q = \int (\mathbf{V} \cdot \mathbf{n}) dA$ (Sec. 3.2)
4. Volume expansion rate: $\frac{1}{\mathcal{V}} \frac{d\mathcal{V}}{dt} = \nabla \cdot \mathbf{V}$ (Sec. 4.2)
5. Local angular velocity: $\boldsymbol{\omega} = \frac{1}{2} \nabla \times \mathbf{V}$ (Sec. 4.8)

We will not illustrate any problems regarding these kinematic properties at present. The point of the list is to illustrate the type of vector operations used in fluid mechanics and to make clear the dominance of the velocity field in determining other flow properties.

Note: The fluid *acceleration*, item 2 above, is not as simple as it looks and actually involves four different terms due to the use of the chain rule in calculus (see Sec. 4.1).



E1.5

EXAMPLE 1.5

Fluid flows through a contracting section of a duct, as in Fig. E1.5. A velocity probe inserted at section (1) measures a steady value $u_1 = 1$ m/s, while a similar probe at section (2) records a steady $u_2 = 3$ m/s. Estimate the fluid acceleration, if any, if $\Delta x = 10$ cm.

Solution

The flow is *steady* (not time-varying), but fluid particles clearly increase in velocity as they pass from (1) to (2). This is the concept of *convective acceleration* (Sec. 4.1). We may estimate the acceleration as a velocity change Δu divided by a time change $\Delta t = \Delta x/u_{\text{avg}}$:

$$a_x \approx \frac{\text{velocity change}}{\text{time change}} \approx \frac{u_2 - u_1}{\Delta x / [\frac{1}{2}(u_1 + u_2)]} = \frac{(3.0 - 1.0 \text{ m/s})(1.0 + 3.0 \text{ m/s})}{2(0.1 \text{ m})} \approx 40 \text{ m/s}^2 \quad \text{Ans.}$$

A simple estimate thus indicates that this seemingly innocuous flow is accelerating at 4 times

the acceleration of gravity. In the limit as Δx and Δt become very small, the above estimate reduces to a partial-derivative expression for convective x -acceleration:

$$a_{x,\text{convective}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta u}{\Delta t} = u \frac{\partial u}{\partial x}$$

In three-dimensional flow (Sec. 4.1) there are *nine* of these convective terms.

1.6 Thermodynamic Properties of a Fluid

While the velocity field \mathbf{V} is the most important fluid property, it interacts closely with the thermodynamic properties of the fluid. We have already introduced into the discussion the three most common such properties

1. Pressure p
2. Density ρ
3. Temperature T

These three are constant companions of the velocity vector in flow analyses. Four other thermodynamic properties become important when work, heat, and energy balances are treated (Chaps. 3 and 4):

4. Internal energy e
5. Enthalpy $h = \hat{u} + p/\rho$
6. Entropy s
7. Specific heats c_p and c_v

In addition, friction and heat conduction effects are governed by the two so-called *transport properties*:

8. Coefficient of viscosity μ
9. Thermal conductivity k

All nine of these quantities are true thermodynamic properties which are determined by the thermodynamic condition or *state* of the fluid. For example, for a single-phase substance such as water or oxygen, two basic properties such as pressure and temperature are sufficient to fix the value of all the others:

$$\rho = \rho(p, T) \quad h = h(p, T) \quad \mu = \mu(p, T) \quad (1.5)$$

and so on for every quantity in the list. Note that the specific volume, so important in thermodynamic analyses, is omitted here in favor of its inverse, the density ρ .

Recall that thermodynamic properties describe the state of a *system*, i.e., a collection of matter of fixed identity which interacts with its surroundings. In most cases here the system will be a small fluid element, and all properties will be assumed to be continuum properties of the flow field: $\rho = \rho(x, y, z, t)$, etc.

Recall also that thermodynamics is normally concerned with *static* systems, whereas fluids are usually in variable motion with constantly changing properties. Do the properties retain their meaning in a fluid flow which is technically not in equilibrium? The answer is yes, from a statistical argument. In gases at normal pressure (and even more so for liquids), an enormous number of molecular collisions occur over a very short distance of the order of 1 μm , so that a fluid subjected to sudden changes rapidly ad-

justs itself toward equilibrium. We therefore assume that all the thermodynamic properties listed above exist as point functions in a flowing fluid and follow all the laws and state relations of ordinary equilibrium thermodynamics. There are, of course, important nonequilibrium effects such as chemical and nuclear reactions in flowing fluids which are not treated in this text.

Pressure

Pressure is the (compression) stress at a point in a static fluid (Fig. 1.1). Next to velocity, the pressure p is the most dynamic variable in fluid mechanics. Differences or *gradients* in pressure often drive a fluid flow, especially in ducts. In low-speed flows, the actual magnitude of the pressure is often not important, unless it drops so low as to cause vapor bubbles to form in a liquid. For convenience, we set many such problem assignments at the level of $1 \text{ atm} = 2116 \text{ lbf/ft}^2 = 101,300 \text{ Pa}$. High-speed (compressible) gas flows (Chap. 9), however, are indeed sensitive to the magnitude of pressure.

Temperature

Temperature T is a measure of the internal energy level of a fluid. It may vary considerably during high-speed flow of a gas (Chap. 9). Although engineers often use Celsius or Fahrenheit scales for convenience, many applications in this text require *absolute* (Kelvin or Rankine) temperature scales:

$$\begin{aligned}\text{°R} &= \text{°F} + 459.69 \\ \text{K} &= \text{°C} + 273.16\end{aligned}$$

If temperature differences are strong, *heat transfer* may be important [10], but our concern here is mainly with dynamic effects. We examine heat-transfer principles briefly in Secs. 4.5 and 9.8.

Density

The density of a fluid, denoted by ρ (lowercase Greek rho), is its mass per unit volume. Density is highly variable in gases and increases nearly proportionally to the pressure level. Density in liquids is nearly constant; the density of water (about 1000 kg/m^3) increases only 1 percent if the pressure is increased by a factor of 220. Thus most liquid flows are treated analytically as nearly “incompressible.”

In general, liquids are about three orders of magnitude more dense than gases at atmospheric pressure. The heaviest common liquid is mercury, and the lightest gas is hydrogen. Compare their densities at 20°C and 1 atm :

$$\text{Mercury: } \rho = 13,580 \text{ kg/m}^3 \quad \text{Hydrogen: } \rho = 0.0838 \text{ kg/m}^3$$

They differ by a factor of 162,000! Thus the physical parameters in various liquid and gas flows might vary considerably. The differences are often resolved by the use of *dimensional analysis* (Chap. 5). Other fluid densities are listed in Tables A.3 and A.4 (in App. A).

Specific Weight

The *specific weight* of a fluid, denoted by γ (lowercase Greek gamma), is its weight per unit volume. Just as a mass has a weight $W = mg$, density and specific weight are simply related by gravity:

$$\gamma = \rho g \tag{1.6}$$

The units of γ are weight per unit volume, in lb/ft^3 or N/m^3 . In standard earth gravity, $g = 32.174 \text{ ft}/\text{s}^2 = 9.807 \text{ m}/\text{s}^2$. Thus, e.g., the specific weights of air and water at 20°C and 1 atm are approximately

$$\begin{aligned}\gamma_{\text{air}} &= (1.205 \text{ kg}/\text{m}^3)(9.807 \text{ m}/\text{s}^2) = 11.8 \text{ N}/\text{m}^3 = 0.0752 \text{ lb}/\text{ft}^3 \\ \gamma_{\text{water}} &= (998 \text{ kg}/\text{m}^3)(9.807 \text{ m}/\text{s}^2) = 9790 \text{ N}/\text{m}^3 = 62.4 \text{ lb}/\text{ft}^3\end{aligned}$$

Specific weight is very useful in the hydrostatic-pressure applications of Chap. 2. Specific weights of other fluids are given in Tables A.3 and A.4.

Specific Gravity

Specific gravity, denoted by SG, is the ratio of a fluid density to a standard reference fluid, water (for liquids), and air (for gases):

$$\begin{aligned}\text{SG}_{\text{gas}} &= \frac{\rho_{\text{gas}}}{\rho_{\text{air}}} = \frac{\rho_{\text{gas}}}{1.205 \text{ kg}/\text{m}^3} \\ \text{SG}_{\text{liquid}} &= \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{\rho_{\text{liquid}}}{998 \text{ kg}/\text{m}^3}\end{aligned}\tag{1.7}$$

For example, the specific gravity of mercury (Hg) is $\text{SG}_{\text{Hg}} = 13,580/998 \approx 13.6$. Engineers find these dimensionless ratios easier to remember than the actual numerical values of density of a variety of fluids.

Potential and Kinetic Energies

In thermostatics the only energy in a substance is that stored in a system by molecular activity and molecular bonding forces. This is commonly denoted as *internal energy* \hat{u} . A commonly accepted adjustment to this static situation for fluid flow is to add two more energy terms which arise from newtonian mechanics: the potential energy and kinetic energy.

The potential energy equals the work required to move the system of mass m from the origin to a position vector $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ against a gravity field \mathbf{g} . Its value is $-m\mathbf{g} \cdot \mathbf{r}$, or $-\mathbf{g} \cdot \mathbf{r}$ per unit mass. The kinetic energy equals the work required to change the speed of the mass from zero to velocity V . Its value is $\frac{1}{2}mV^2$ or $\frac{1}{2}V^2$ per unit mass. Then by common convention the total stored energy e per unit mass in fluid mechanics is the sum of three terms:

$$e = \hat{u} + \frac{1}{2}V^2 + (-\mathbf{g} \cdot \mathbf{r})\tag{1.8}$$

Also, throughout this book we shall define z as upward, so that $\mathbf{g} = -g\mathbf{k}$ and $\mathbf{g} \cdot \mathbf{r} = -gz$. Then Eq. (1.8) becomes

$$e = \hat{u} + \frac{1}{2}V^2 + gz\tag{1.9}$$

The molecular internal energy \hat{u} is a function of T and p for the single-phase pure substance, whereas the potential and kinetic energies are kinematic properties.

State Relations for Gases

Thermodynamic properties are found both theoretically and experimentally to be related to each other by state relations which differ for each substance. As mentioned,

we shall confine ourselves here to single-phase pure substances, e.g., water in its liquid phase. The second most common fluid, air, is a mixture of gases, but since the mixture ratios remain nearly constant between 160 and 2200 K, in this temperature range air can be considered to be a pure substance.

All gases at high temperatures and low pressures (relative to their critical point) are in good agreement with the *perfect-gas law*

$$p = \rho RT \quad R = c_p - c_v = \text{gas constant} \quad (1.10)$$

Since Eq. (1.10) is dimensionally consistent, R has the same dimensions as specific heat, $\{L^2T^{-2}\Theta^{-1}\}$, or velocity squared per temperature unit (kelvin or degree Rankine). Each gas has its own constant R , equal to a universal constant Λ divided by the molecular weight

$$R_{\text{gas}} = \frac{\Lambda}{M_{\text{gas}}} \quad (1.11)$$

where $\Lambda = 49,700 \text{ ft}^2/(\text{s}^2 \cdot \text{°R}) = 8314 \text{ m}^2/(\text{s}^2 \cdot \text{K})$. Most applications in this book are for air, with $M = 28.97$:

$$R_{\text{air}} = 1717 \text{ ft}^2/(\text{s}^2 \cdot \text{°R}) = 287 \text{ m}^2/(\text{s}^2 \cdot \text{K}) \quad (1.12)$$

Standard atmospheric pressure is 2116 lbf/ft^2 , and standard temperature is $60^\circ\text{F} = 520^\circ\text{R}$. Thus standard air density is

$$\rho_{\text{air}} = \frac{2116}{(1717)(520)} = 0.00237 \text{ slug/ft}^3 = 1.22 \text{ kg/m}^3 \quad (1.13)$$

This is a nominal value suitable for problems.

One proves in thermodynamics that Eq. (1.10) requires that the internal molecular energy \hat{u} of a perfect gas vary only with temperature: $\hat{u} = \hat{u}(T)$. Therefore the specific heat c_v also varies only with temperature:

$$c_v = \left(\frac{\partial \hat{u}}{\partial T} \right)_\rho = \frac{d\hat{u}}{dT} = c_v(T)$$

or

$$d\hat{u} = c_v(T) dT \quad (1.14)$$

In like manner h and c_p of a perfect gas also vary only with temperature:

$$h = \hat{u} + \frac{p}{\rho} = \hat{u} + RT = h(T)$$

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p = \frac{dh}{dT} = c_p(T) \quad (1.15)$$

$$dh = c_p(T) dT$$

The ratio of specific heats of a perfect gas is an important dimensionless parameter in compressible-flow analysis (Chap. 9)

$$k = \frac{c_p}{c_v} = k(T) \geq 1 \quad (1.16)$$

As a first approximation in airflow analysis we commonly take c_p , c_v , and k to be constant

$$k_{\text{air}} \approx 1.4$$

$$c_v = \frac{R}{k-1} \approx 4293 \text{ ft}^2/(\text{s}^2 \cdot \text{°R}) = 718 \text{ m}^2/(\text{s}^2 \cdot \text{K}) \quad (1.17)$$

$$c_p = \frac{kR}{k-1} \approx 6010 \text{ ft}^2/(\text{s}^2 \cdot \text{°R}) = 1005 \text{ m}^2/(\text{s}^2 \cdot \text{K})$$

Actually, for all gases, c_p and c_v increase gradually with temperature, and k decreases gradually. Experimental values of the specific-heat ratio for eight common gases are shown in Fig. 1.3.

Many flow problems involve steam. Typical steam operating conditions are relatively close to the critical point, so that the perfect-gas approximation is inaccurate. The properties of steam are therefore available in tabular form [13], but the error of using the perfect-gas law is sometimes not great, as the following example shows.

EXAMPLE 1.6

Estimate ρ and c_p of steam at 100 lbf/in² and 400°F (a) by a perfect-gas approximation and (b) from the ASME steam tables [13].

Solution

Part (a) First convert to BG units: $p = 100 \text{ lbf/in}^2 = 14,400 \text{ lb/ft}^2$, $T = 400^\circ\text{F} = 860^\circ\text{R}$. From Table A.4 the molecular weight of H₂O is $2M_{\text{H}} + M_{\text{O}} = 2(1.008) + 16.0 = 18.016$. Then from Eq. (1.11) the gas constant of steam is approximately

$$R = \frac{49,700}{18.016} = 2759 \text{ ft}^2/(\text{s}^2 \cdot \text{°R})$$

whence, from the perfect-gas law,

$$\rho \approx \frac{p}{RT} = \frac{14,400}{2759(860)} = 0.00607 \text{ slug/ft}^3 \quad \text{Ans. (a)}$$

From Fig. 1.3, k for steam at 860°R is approximately 1.30. Then from Eq. (1.17),

$$c_p \approx \frac{kR}{k-1} = \frac{1.30(2759)}{1.30-1} = 12,000 \text{ ft}^2/(\text{s}^2 \cdot \text{°R}) \quad \text{Ans. (a)}$$

Part (b) From Ref. 13, the specific volume v of steam at 100 lbf/in² and 400°F is 4.935 ft³/lbm. Then the density is the inverse of this, converted to slugs:

$$\rho = \frac{1}{v} = \frac{1}{(4.935 \text{ ft}^3/\text{lbm})(32.174 \text{ lbm/slug})} = 0.00630 \text{ slug/ft}^3 \quad \text{Ans. (b)}$$

This is about 4 percent higher than our ideal-gas estimate in part (a).

Reference 13 lists the value of c_p of steam at 100 lbf/in² and 400°F as 0.535 Btu/(lbm · °F). Convert this to BG units:

$$c_p = [0.535 \text{ Btu}/(\text{lbm} \cdot \text{°F})](778.2 \text{ ft} \cdot \text{lbf/Btu})(32.174 \text{ lbm/slug})$$

$$= 13,400 \text{ ft} \cdot \text{lbf}/(\text{slug} \cdot \text{°R}) = 13,400 \text{ ft}^2/(\text{s}^2 \cdot \text{°R}) \quad \text{Ans. (b)}$$

This is about 11 percent higher than our ideal-gas estimate in part (a). The chief reason for the discrepancy is that this temperature and this pressure are quite close to the critical point and saturation line of steam. At higher temperatures and lower pressures, say, 800°F and 50 lbf/in², the perfect-gas law gives ρ and c_p of steam within an accuracy of ± 1 percent.

Note that the use of pound-mass and British thermal units in the traditional steam tables requires continual awkward conversions to BG units. Newer tables and disks are in SI units.

State Relations for Liquids

The writer knows of no “perfect-liquid law” comparable to that for gases. Liquids are nearly incompressible and have a single reasonably constant specific heat. Thus an idealized state relation for a liquid is

$$\rho \approx \text{const} \quad c_p \approx c_v \approx \text{const} \quad dh \approx c_p dT \quad (1.18)$$

Most of the flow problems in this book can be attacked with these simple assumptions. Water is normally taken to have a density of 1.94 slugs/ft³ and a specific heat $c_p = 25,200 \text{ ft}^2/(\text{s}^2 \cdot ^\circ\text{R})$. The steam tables may be used if more accuracy is required.

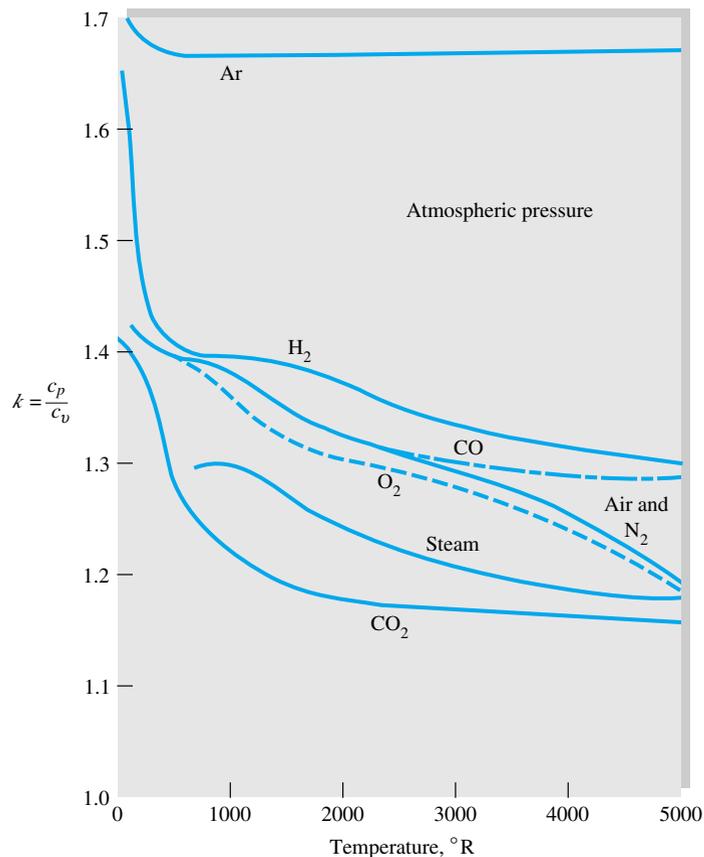


Fig. 1.3 Specific-heat ratio of eight common gases as a function of temperature. (Data from Ref. 12.)

The density of a liquid usually decreases slightly with temperature and increases moderately with pressure. If we neglect the temperature effect, an empirical pressure-density relation for a liquid is

$$\frac{p}{p_a} \approx (B + 1) \left(\frac{\rho}{\rho_a} \right)^n - B \quad (1.19)$$

where B and n are dimensionless parameters which vary slightly with temperature and p_a and ρ_a are standard atmospheric values. Water can be fitted approximately to the values $B \approx 3000$ and $n \approx 7$.

Seawater is a variable mixture of water and salt and thus requires three thermodynamic properties to define its state. These are normally taken as pressure, temperature, and the *salinity* \hat{S} , defined as the weight of the dissolved salt divided by the weight of the mixture. The average salinity of seawater is 0.035, usually written as 35 parts per 1000, or 35 ‰. The average density of seawater is 2.00 slugs/ft³. Strictly speaking, seawater has three specific heats, all approximately equal to the value for pure water of 25,200 ft²/(s² · °R) = 4210 m²/(s² · K).

EXAMPLE 1.7

The pressure at the deepest part of the ocean is approximately 1100 atm. Estimate the density of seawater at this pressure.

Solution

Equation (1.19) holds for either water or seawater. The ratio p/p_a is given as 1100:

$$1100 \approx (3001) \left(\frac{\rho}{\rho_a} \right)^7 - 3000$$

or

$$\frac{\rho}{\rho_a} = \left(\frac{4100}{3001} \right)^{1/7} = 1.046$$

Assuming an average surface seawater density $\rho_a = 2.00$ slugs/ft³, we compute

$$\rho \approx 1.046(2.00) = 2.09 \text{ slugs/ft}^3 \quad \text{Ans.}$$

Even at these immense pressures, the density increase is less than 5 percent, which justifies the treatment of a liquid flow as essentially incompressible.

1.7 Viscosity and Other Secondary Properties

The quantities such as pressure, temperature, and density discussed in the previous section are *primary* thermodynamic variables characteristic of any system. There are also certain secondary variables which characterize specific fluid-mechanical behavior. The most important of these is viscosity, which relates the local stresses in a moving fluid to the strain rate of the fluid element.

Viscosity

When a fluid is sheared, it begins to move at a strain rate inversely proportional to a property called its *coefficient of viscosity* μ . Consider a fluid element sheared in one

plane by a single shear stress τ , as in Fig. 1.4a. The shear strain angle $\delta\theta$ will continuously grow with time as long as the stress τ is maintained, the upper surface moving at speed δu larger than the lower. Such common fluids as water, oil, and air show a linear relation between applied shear and resulting strain rate

$$\tau \propto \frac{\delta\theta}{\delta t} \quad (1.20)$$

From the geometry of Fig. 1.4a we see that

$$\tan \delta\theta = \frac{\delta u \delta t}{\delta y} \quad (1.21)$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient

$$\frac{d\theta}{dt} = \frac{du}{dy} \quad (1.22)$$

From Eq. (1.20), then, the applied shear is also proportional to the velocity gradient for the common linear fluids. The constant of proportionality is the viscosity coefficient μ

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy} \quad (1.23)$$

Equation (1.23) is dimensionally consistent; therefore μ has dimensions of stress-time: $\{FT/L^2\}$ or $\{M/(LT)\}$. The BG unit is slugs per foot-second, and the SI unit is kilograms per meter-second. The linear fluids which follow Eq. (1.23) are called *newtonian fluids*, after Sir Isaac Newton, who first postulated this resistance law in 1687.

We do not really care about the strain angle $\theta(t)$ in fluid mechanics, concentrating instead on the velocity distribution $u(y)$, as in Fig. 1.4b. We shall use Eq. (1.23) in Chap. 4 to derive a differential equation for finding the velocity distribution $u(y)$ —and, more generally, $\mathbf{V}(x, y, z, t)$ —in a viscous fluid. Figure 1.4b illustrates a shear layer, or *boundary layer*, near a solid wall. The shear stress is proportional to the slope of the

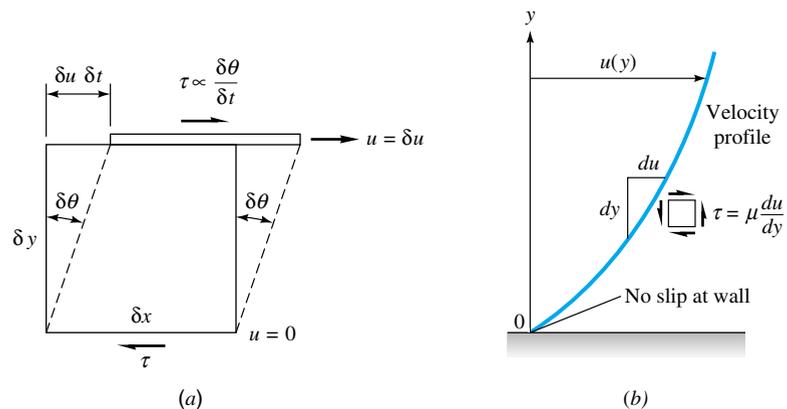


Fig. 1.4 Shear stress causes continuous shear deformation in a fluid: (a) a fluid element straining at a rate $\delta\theta/\delta t$; (b) newtonian shear distribution in a shear layer near a wall.

Table 1.4 Viscosity and Kinematic Viscosity of Eight Fluids at 1 atm and 20°C

Fluid	μ , kg/(m · s) [†]	Ratio $\mu/\mu(\text{H}_2)$	ρ , kg/m ³	ν m ² /s [†]	Ratio $\nu/\nu(\text{Hg})$
Hydrogen	8.8 E-6	1.0	0.084	1.05 E-4	920
Air	1.8 E-5	2.1	1.20	1.51 E-5	130
Gasoline	2.9 E-4	33	680	4.22 E-7	3.7
Water	1.0 E-3	114	998	1.01 E-6	8.7
Ethyl alcohol	1.2 E-3	135	789	1.52 E-6	13
Mercury	1.5 E-3	170	13,580	1.16 E-7	1.0
SAE 30 oil	0.29	33,000	891	3.25 E-4	2,850
Glycerin	1.5	170,000	1,264	1.18 E-3	10,300

[†]1 kg/(m · s) = 0.0209 slug/(ft · s); 1 m²/s = 10.76 ft²/s.

velocity profile and is greatest at the wall. Further, at the wall, the velocity u is zero relative to the wall: This is called the *no-slip condition* and is characteristic of all viscous-fluid flows.

The viscosity of newtonian fluids is a true thermodynamic property and varies with temperature and pressure. At a given state (p, T) there is a vast range of values among the common fluids. Table 1.4 lists the viscosity of eight fluids at standard pressure and temperature. There is a variation of six orders of magnitude from hydrogen up to glycerin. Thus there will be wide differences between fluids subjected to the same applied stresses.

Generally speaking, the viscosity of a fluid increases only weakly with pressure. For example, increasing p from 1 to 50 atm will increase μ of air only 10 percent. Temperature, however, has a strong effect, with μ increasing with T for gases and decreasing for liquids. Figure A.1 (in App. A) shows this temperature variation for various common fluids. It is customary in most engineering work to neglect the pressure variation.

The variation $\mu(p, T)$ for a typical fluid is nicely shown by Fig. 1.5, from Ref. 14, which normalizes the data with the *critical-point state* (μ_c, p_c, T_c). This behavior, called the *principle of corresponding states*, is characteristic of all fluids, but the actual numerical values are uncertain to ± 20 percent for any given fluid. For example, values of $\mu(T)$ for air at 1 atm, from Table A.2, fall about 8 percent low compared to the “low-density limit” in Fig. 1.5.

Note in Fig. 1.5 that changes with temperature occur very rapidly near the critical point. In general, critical-point measurements are extremely difficult and uncertain.

The Reynolds Number

As we shall see in Chaps. 5 through 7, the primary parameter correlating the viscous behavior of all newtonian fluids is the dimensionless *Reynolds number*:

$$\text{Re} = \frac{\rho VL}{\mu} = \frac{VL}{\nu} \quad (1.24)$$

where V and L are characteristic velocity and length scales of the flow. The second form of Re illustrates that the ratio of μ to ρ has its own name, the *kinematic viscosity*:

$$\nu = \frac{\mu}{\rho} \quad (1.25)$$

It is called kinematic because the mass units cancel, leaving only the dimensions $\{L^2/T\}$.

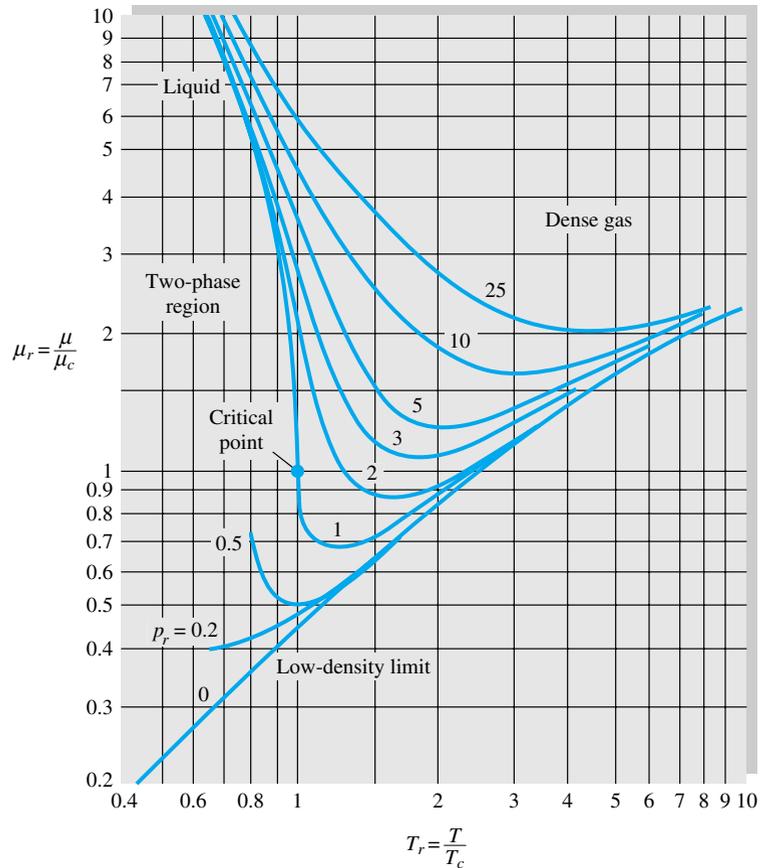


Fig. 1.5 Fluid viscosity nondimensionalized by critical-point properties. This generalized chart is characteristic of all fluids but is only accurate to ± 20 percent. (From Ref. 14.)

Generally, the first thing a fluids engineer should do is estimate the Reynolds number range of the flow under study. Very low Re indicates viscous *creeping* motion, where inertia effects are negligible. Moderate Re implies a smoothly varying *laminar* flow. High Re probably spells *turbulent* flow, which is slowly varying in the time-mean but has superimposed strong random high-frequency fluctuations. Explicit numerical values for low, moderate, and high Reynolds numbers cannot be stated here. They depend upon flow geometry and will be discussed in Chaps. 5 through 7.

Table 1.4 also lists values of ν for the same eight fluids. The pecking order changes considerably, and mercury, the heaviest, has the smallest viscosity relative to its own weight. All gases have high ν relative to thin liquids such as gasoline, water, and alcohol. Oil and glycerin still have the highest ν , but the ratio is smaller. For a given value of V and L in a flow, these fluids exhibit a spread of four orders of magnitude in the Reynolds number.

Flow between Plates

A classic problem is the flow induced between a fixed lower plate and an upper plate moving steadily at velocity \mathbf{V} , as shown in Fig. 1.6. The clearance between plates is h , and the fluid is newtonian and does not slip at either plate. If the plates are large,

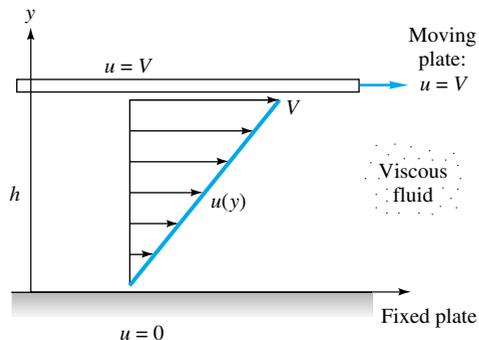


Fig. 1.6 Viscous flow induced by relative motion between two parallel plates.

this steady shearing motion will set up a velocity distribution $u(y)$, as shown, with $v = w = 0$. The fluid acceleration is zero everywhere.

With zero acceleration and assuming no pressure variation in the flow direction, you should show that a force balance on a small fluid element leads to the result that the shear stress is constant throughout the fluid. Then Eq. (1.23) becomes

$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{const}$$

which we can integrate to obtain

$$u = a + by$$

The velocity distribution is linear, as shown in Fig. 1.6, and the constants a and b can be evaluated from the no-slip condition at the upper and lower walls:

$$u = \begin{cases} 0 = a + b(0) & \text{at } y = 0 \\ V = a + b(h) & \text{at } y = h \end{cases}$$

Hence $a = 0$ and $b = V/h$. Then the velocity profile between the plates is given by

$$u = V \frac{y}{h} \quad (1.26)$$

as indicated in Fig. 1.6. Turbulent flow (Chap. 6) does not have this shape.

Although viscosity has a profound effect on fluid motion, the actual viscous stresses are quite small in magnitude even for oils, as shown in the following example.

EXAMPLE 1.8

Suppose that the fluid being sheared in Fig. 1.6 is SAE 30 oil at 20°C. Compute the shear stress in the oil if $V = 3$ m/s and $h = 2$ cm.

Solution

The shear stress is found from Eq. (1.23) by differentiating Eq. (1.26):

$$\tau = \mu \frac{du}{dy} = \frac{\mu V}{h} \quad (1)$$

From Table 1.4 for SAE 30 oil, $\mu = 0.29 \text{ kg}/(\text{m} \cdot \text{s})$. Then, for the given values of V and h , Eq. (1) predicts

$$\begin{aligned}\tau &= \frac{[0.29 \text{ kg}/(\text{m} \cdot \text{s})](3 \text{ m/s})}{0.02 \text{ m}} = 43 \text{ kg}/(\text{m} \cdot \text{s}^2) \\ &= 43 \text{ N/m}^2 = 43 \text{ Pa} \qquad \text{Ans.}\end{aligned}$$

Although oil is very viscous, this is a modest shear stress, about 2400 times less than atmospheric pressure. Viscous stresses in gases and thin liquids are even smaller.

Variation of Viscosity with Temperature

Temperature has a strong effect and pressure a moderate effect on viscosity. The viscosity of gases and most liquids increases slowly with pressure. Water is anomalous in showing a very slight decrease below 30°C. Since the change in viscosity is only a few percent up to 100 atm, we shall neglect pressure effects in this book.

Gas viscosity increases with temperature. Two common approximations are the power law and the Sutherland law:

$$\frac{\mu}{\mu_0} \approx \begin{cases} \left(\frac{T}{T_0}\right)^n & \text{power law} \\ \frac{(T/T_0)^{3/2}(T_0 + S)}{T + S} & \text{Sutherland law} \end{cases} \quad (1.27)$$

where μ_0 is a known viscosity at a known absolute temperature T_0 (usually 273 K). The constants n and S are fit to the data, and both formulas are adequate over a wide range of temperatures. For air, $n \approx 0.7$ and $S \approx 110 \text{ K} = 199^\circ\text{R}$. Other values are given in Ref. 3.

Liquid viscosity decreases with temperature and is roughly exponential, $\mu \approx ae^{-bT}$; but a better fit is the empirical result that $\ln \mu$ is quadratic in $1/T$, where T is absolute temperature

$$\ln \frac{\mu}{\mu_0} \approx a + b\left(\frac{T_0}{T}\right) + c\left(\frac{T_0}{T}\right)^2 \quad (1.28)$$

For water, with $T_0 = 273.16 \text{ K}$, $\mu_0 = 0.001792 \text{ kg}/(\text{m} \cdot \text{s})$, suggested values are $a = -1.94$, $b = -4.80$, and $c = 6.74$, with accuracy about ± 1 percent. The viscosity of water is tabulated in Table A.1. Curve-fit viscosity formulas for 355 organic liquids are given by Yaws et al. [34]. For further viscosity data, see Refs. 28 and 36.

Thermal Conductivity

Just as viscosity relates applied stress to resulting strain rate, there is a property called *thermal conductivity* k which relates the vector rate of heat flow per unit area \mathbf{q} to the vector gradient of temperature ∇T . This proportionality, observed experimentally for fluids and solids, is known as *Fourier's law of heat conduction*

$$\mathbf{q} = -k\nabla T \quad (1.29a)$$

which can also be written as three scalar equations

$$q_x = -k\frac{\partial T}{\partial x} \quad q_y = -k\frac{\partial T}{\partial y} \quad q_z = -k\frac{\partial T}{\partial z} \quad (1.29b)$$

The minus sign satisfies the convention that heat flux is positive in the direction of decreasing temperature. Fourier's law is dimensionally consistent, and k has SI units of joules per second-meter-kelvin. Thermal conductivity k is a thermodynamic property and varies with temperature and pressure in much the same way as viscosity. The ratio k/k_0 can be correlated with T/T_0 in the same manner as Eqs. (1.27) and (1.28) for gases and liquids, respectively.

Further data on viscosity and thermal-conductivity variations can be found in Ref. 11.

Nonnewtonian Fluids

Fluids which do not follow the linear law of Eq. (1.23) are called *nonnewtonian* and are treated in books on rheology [6]. Figure 1.7a compares four examples with a newtonian fluid. A *dilatant*, or shear-thickening, fluid increases resistance with increasing applied stress. Alternately, a *pseudoplastic*, or shear-thinning, fluid decreases resistance with increasing stress. If the thinning effect is very strong, as with the dashed-line curve, the fluid is termed *plastic*. The limiting case of a plastic substance is one which requires a finite yield stress before it begins to flow. The linear-flow *Bingham plastic* idealization is shown, but the flow behavior after yield may also be nonlinear. An example of a yielding fluid is toothpaste, which will not flow out of the tube until a finite stress is applied by squeezing.

A further complication of nonnewtonian behavior is the transient effect shown in Fig. 1.7b. Some fluids require a gradually increasing shear stress to maintain a constant strain rate and are called *rheopectic*. The opposite case of a fluid which thins out with time and requires decreasing stress is termed *thixotropic*. We neglect nonnewtonian effects in this book; see Ref. 6 for further study.

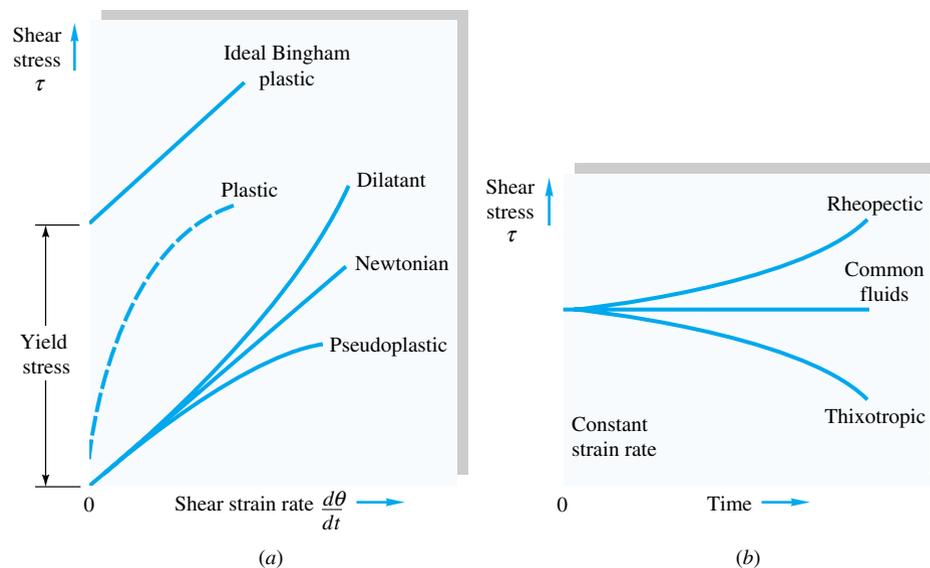


Fig. 1.7 Rheological behavior of various viscous materials: (a) stress versus strain rate; (b) effect of time on applied stress.

Surface Tension

A liquid, being unable to expand freely, will form an *interface* with a second liquid or gas. The physical chemistry of such interfacial surfaces is quite complex, and whole textbooks are devoted to this specialty [15]. Molecules deep within the liquid repel each other because of their close packing. Molecules at the surface are less dense and attract each other. Since half of their neighbors are missing, the mechanical effect is that the surface is in tension. We can account adequately for surface effects in fluid mechanics with the concept of surface tension.

If a cut of length dL is made in an interfacial surface, equal and opposite forces of magnitude $Y dL$ are exposed normal to the cut and parallel to the surface, where Y is called the *coefficient of surface tension*. The dimensions of Y are $\{F/L\}$, with SI units of newtons per meter and BG units of pounds-force per foot. An alternate concept is to open up the cut to an area dA ; this requires work to be done of amount $Y dA$. Thus the coefficient Y can also be regarded as the surface energy per unit area of the interface, in $N \cdot m/m^2$ or $ft \cdot lbf/ft^2$.

The two most common interfaces are water-air and mercury-air. For a clean surface at $20^\circ\text{C} = 68^\circ\text{F}$, the measured surface tension is

$$Y = \begin{cases} 0.0050 \text{ lbf/ft} = 0.073 \text{ N/m} & \text{air-water} \\ 0.033 \text{ lbf/ft} = 0.48 \text{ N/m} & \text{air-mercury} \end{cases} \quad (1.30)$$

These are design values and can change considerably if the surface contains contaminants like detergents or slicks. Generally Y decreases with liquid temperature and is zero at the critical point. Values of Y for water are given in Fig. 1.8.

If the interface is curved, a mechanical balance shows that there is a pressure difference across the interface, the pressure being higher on the concave side, as illustrated in Fig. 1.9. In Fig. 1.9a, the pressure increase in the interior of a liquid cylinder is balanced by two surface-tension forces

$$\begin{aligned} 2RL \Delta p &= 2YL \\ \text{or} \quad \Delta p &= \frac{Y}{R} \end{aligned} \quad (1.31)$$

We are not considering the weight of the liquid in this calculation. In Fig. 1.9b, the pressure increase in the interior of a spherical droplet balances a ring of surface-tension force

$$\begin{aligned} \pi R^2 \Delta p &= 2\pi RY \\ \text{or} \quad \Delta p &= \frac{2Y}{R} \end{aligned} \quad (1.32)$$

We can use this result to predict the pressure increase inside a soap bubble, which has *two* interfaces with air, an inner and outer surface of nearly the same radius R :

$$\Delta p_{\text{bubble}} \approx 2 \Delta p_{\text{droplet}} = \frac{4Y}{R} \quad (1.33)$$

Figure 1.9c shows the general case of an arbitrarily curved interface whose principal radii of curvature are R_1 and R_2 . A force balance normal to the surface will show that the pressure increase on the concave side is

$$\Delta p = Y(R_1^{-1} + R_2^{-1}) \quad (1.34)$$

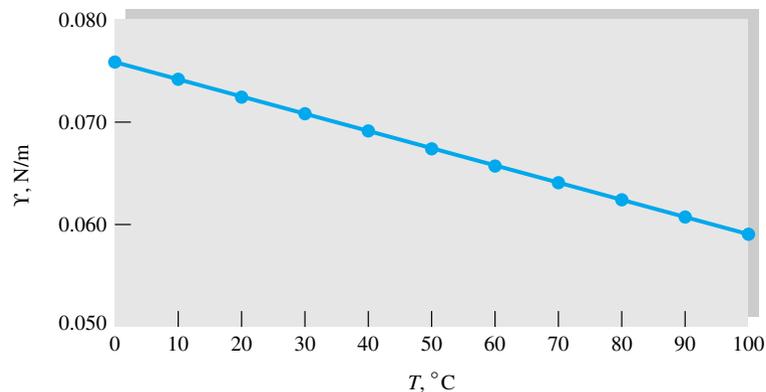


Fig. 1.8 Surface tension of a clean air-water interface. Data from Table A.5.

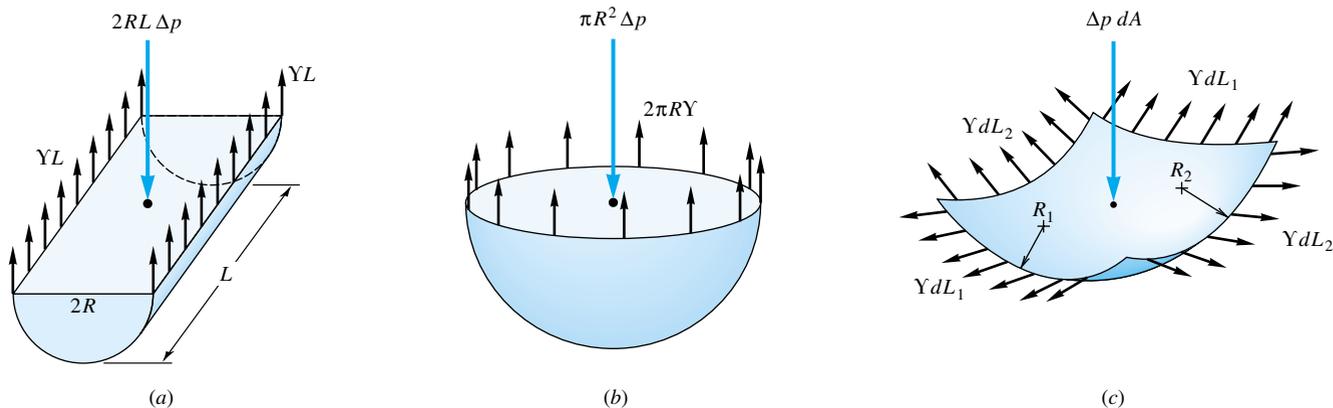


Fig. 1.9 Pressure change across a curved interface due to surface tension: (a) interior of a liquid cylinder; (b) interior of a spherical droplet; (c) general curved interface.

Equations (1.31) to (1.33) can all be derived from this general relation; e.g., in Eq. (1.31), $R_1 = R$ and $R_2 = \infty$.

A second important surface effect is the *contact angle* θ which appears when a liquid interface intersects with a solid surface, as in Fig. 1.10. The force balance would then involve both γ and θ . If the contact angle is less than 90° , the liquid is said to *wet* the solid; if $\theta > 90^\circ$, the liquid is termed *nonwetting*. For example, water wets soap but does not wet wax. Water is extremely wetting to a clean glass surface, with $\theta \approx 0^\circ$. Like γ , the contact angle θ is sensitive to the actual physico-chemical conditions of the solid-liquid interface. For a clean mercury-air-glass interface, $\theta = 130^\circ$.

Example 1.9 illustrates how surface tension causes a fluid interface to rise or fall in a capillary tube.

Fig. 1.10 Contact-angle effects at liquid-gas-solid interface. If $\theta < 90^\circ$, the liquid “wets” the solid; if $\theta > 90^\circ$, the liquid is nonwetting.

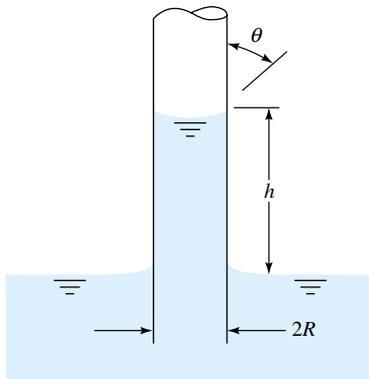
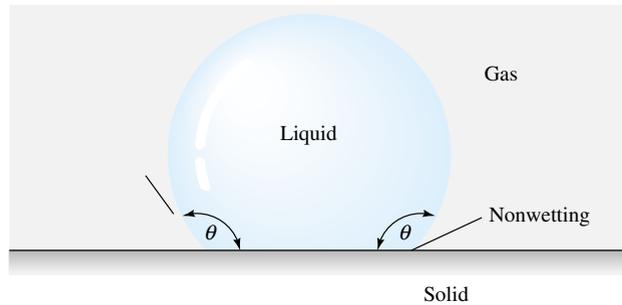


Fig. E1.9

EXAMPLE 1.9

Derive an expression for the change in height h in a circular tube of a liquid with surface tension Y and contact angle θ , as in Fig. E1.9.

Solution

The vertical component of the ring surface-tension force at the interface in the tube must balance the weight of the column of fluid of height h

$$2\pi RY \cos \theta = \gamma \pi R^2 h$$

Solving for h , we have the desired result

$$h = \frac{2Y \cos \theta}{\gamma R} \quad \text{Ans.}$$

Thus the capillary height increases inversely with tube radius R and is positive if $\theta < 90^\circ$ (wetting liquid) and negative (capillary depression) if $\theta > 90^\circ$.

Suppose that $R = 1$ mm. Then the capillary rise for a water-air-glass interface, $\theta \approx 0^\circ$, $Y = 0.073$ N/m, and $\rho = 1000$ kg/m³ is

$$h = \frac{2(0.073 \text{ N/m})(\cos 0^\circ)}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.001 \text{ m})} = 0.015 \text{ (N} \cdot \text{s}^2\text{)/kg} = 0.015 \text{ m} = 1.5 \text{ cm}$$

For a mercury-air-glass interface, with $\theta = 130^\circ$, $Y = 0.48$ N/m, and $\rho = 13,600$ kg/m³, the capillary rise is

$$h = \frac{2(0.48)(\cos 130^\circ)}{13,600(9.81)(0.001)} = -0.46 \text{ cm}$$

When a small-diameter tube is used to make pressure measurements (Chap. 2), these capillary effects must be corrected for.

Vapor Pressure

Vapor pressure is the pressure at which a liquid boils and is in equilibrium with its own vapor. For example, the vapor pressure of water at 68°F is 49 lbf/ft², while that of mercury is only 0.0035 lbf/ft². If the liquid pressure is greater than the vapor

pressure, the only exchange between liquid and vapor is evaporation at the interface. If, however, the liquid pressure falls below the vapor pressure, vapor bubbles begin to appear in the liquid. If water is heated to 212°F, its vapor pressure rises to 2116 lbf/ft², and thus water at normal atmospheric pressure will boil. When the liquid pressure is dropped below the vapor pressure due to a flow phenomenon, we call the process *cavitation*. As we shall see in Chap. 2, if water is accelerated from rest to about 50 ft/s, its pressure drops by about 15 lbf/in², or 1 atm. This can cause cavitation.

The dimensionless parameter describing flow-induced boiling is the *cavitation number*

$$Ca = \frac{p_a - p_v}{\frac{1}{2}\rho V^2} \quad (1.35)$$

where p_a = ambient pressure

p_v = vapor pressure

V = characteristic flow velocity

Depending upon the geometry, a given flow has a critical value of Ca below which the flow will begin to cavitate. Values of surface tension and vapor pressure of water are given in Table A.5. The vapor pressure of water is plotted in Fig. 1.11.

Figure 1.12a shows cavitation bubbles being formed on the low-pressure surfaces of a marine propeller. When these bubbles move into a higher-pressure region, they collapse implisively. Cavitation collapse can rapidly spall and erode metallic surfaces and eventually destroy them, as shown in Fig. 1.12b.

No-Slip and No-Temperature-Jump Conditions

When a fluid flow is bounded by a solid surface, molecular interactions cause the fluid in contact with the surface to seek momentum and energy equilibrium with that surface. All liquids essentially are in equilibrium with the surface they contact. All gases are, too,

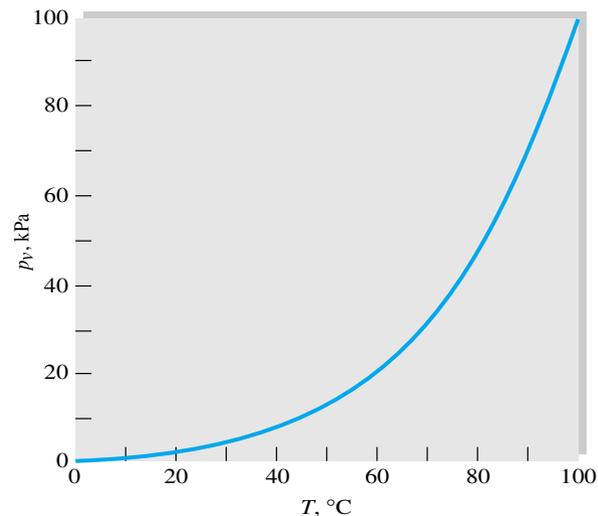


Fig. 1.11 Vapor pressure of water. Data from Table A.5.

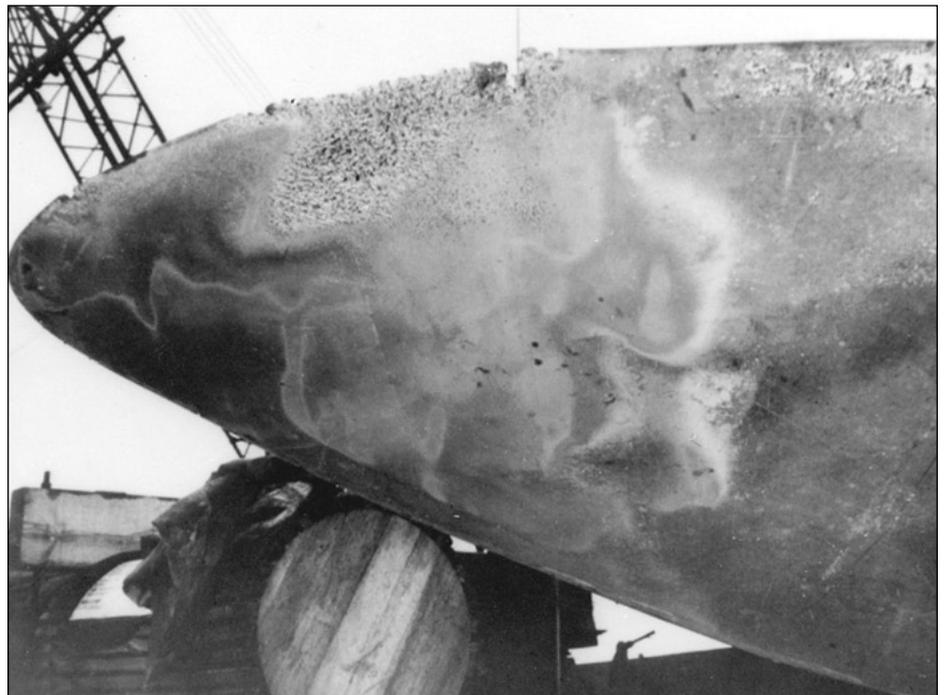
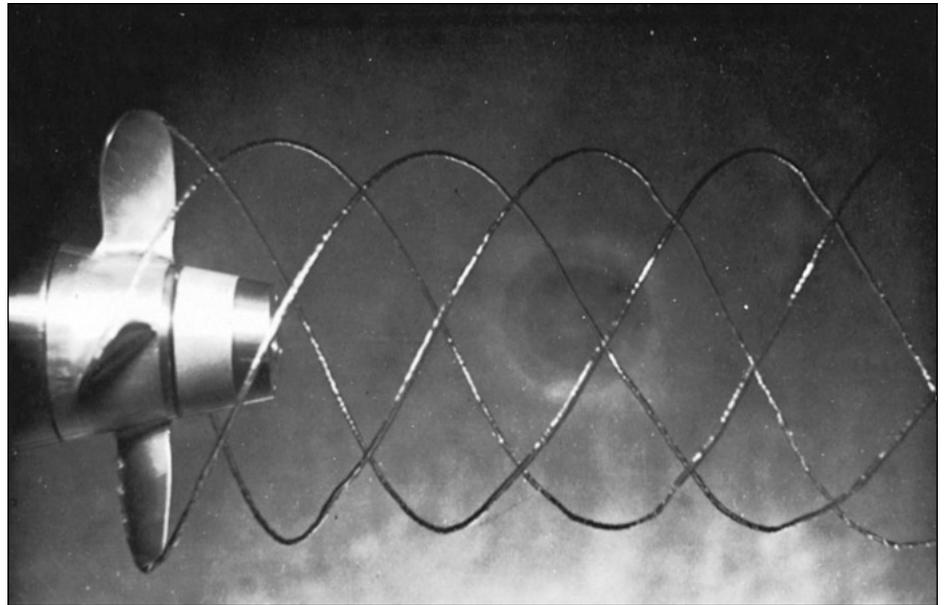


Fig. 1.12 Two aspects of cavitation bubble formation in liquid flows: (a) Beauty: spiral bubble sheets form from the surface of a marine propeller. (Courtesy of the Garfield Thomas Water Tunnel, Pennsylvania State University); (b) ugliness: collapsing bubbles erode a propeller surface. (Courtesy of Thomas T. Huang, David Taylor Research Center.)

except under the most rarefied conditions [8]. Excluding rarefied gases, then, all fluids at a point of contact with a solid take on the velocity and temperature of that surface

$$\mathbf{V}_{\text{fluid}} \equiv \mathbf{V}_{\text{wall}} \quad T_{\text{fluid}} \equiv T_{\text{wall}} \quad (1.36)$$

These are called the *no-slip* and *no-temperature-jump conditions*, respectively. They serve as *boundary conditions* for analysis of fluid flow past a solid surface (Chap. 6). Figure 1.13 illustrates the no-slip condition for water flow past the top and bottom surfaces of a fixed thin plate. The flow past the upper surface is disorderly, or turbulent, while the lower surface flow is smooth, or laminar.⁷ In both cases there is clearly no slip at the wall, where the water takes on the zero velocity of the fixed plate. The velocity profile is made visible by the discharge of a line of hydrogen bubbles from the wire shown stretched across the flow.

To decrease the mathematical difficulty, the no-slip condition is partially relaxed in the analysis of inviscid flow (Chap. 8). The flow is allowed to “slip” past the surface but not to permeate through the surface

$$V_{\text{normal}}(\text{fluid}) \equiv V_{\text{normal}}(\text{solid}) \quad (1.37)$$

while the tangential velocity V_t is allowed to be independent of the wall. The analysis is much simpler, but the flow patterns are highly idealized.

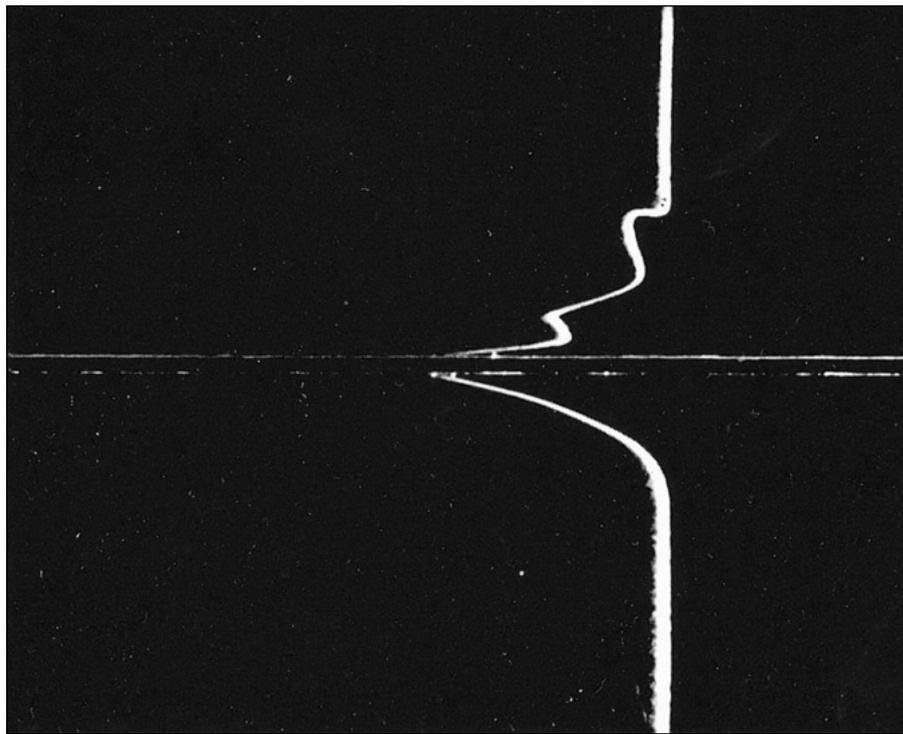


Fig. 1.13 The no-slip condition in water flow past a thin fixed plate. The upper flow is turbulent; the lower flow is laminar. The velocity profile is made visible by a line of hydrogen bubbles discharged from the wire across the flow. [From Illustrated Experiments in Fluid Mechanics (*The NCFMF Book of Film Notes*), National Committee for Fluid Mechanics Films, Education Development Center, Inc., copyright 1972.]

⁷Laminar and turbulent flows are studied in Chaps. 6 and 7.

Speed of Sound

In gas flow, one must be aware of *compressibility* effects (significant density changes caused by the flow). We shall see in Sec. 4.2 and in Chap. 9 that compressibility becomes important when the flow velocity reaches a significant fraction of the speed of sound of the fluid. The *speed of sound* a of a fluid is the rate of propagation of small-disturbance pressure pulses (“sound waves”) through the fluid. In Chap. 9 we shall show, from momentum and thermodynamic arguments, that the speed of sound is defined by

$$a^2 = \left(\frac{\partial p}{\partial \rho} \right)_s = k \left(\frac{\partial p}{\partial \rho} \right)_T \quad k = \frac{c_p}{c_v} \quad (1.38)$$

This is true for either a liquid or a gas, but it is for *gases* that the problem of compressibility occurs. For an ideal gas, Eq. (1.10), we obtain the simple formula

$$a_{\text{ideal gas}} = (kRT)^{1/2} \quad (1.39)$$

where R is the gas constant, Eq. (1.11), and T the absolute temperature. For example, for air at 20°C, $a = \{(1.40)[287 \text{ m}^2/(\text{s}^2 \cdot \text{K})](293 \text{ K})\}^{1/2} \approx 343 \text{ m/s}$ (1126 ft/s = 768 mi/h). If, in this case, the air velocity reaches a significant fraction of a , say, 100 m/s, then we must account for compressibility effects (Chap. 9). Another way to state this is to account for compressibility when the *Mach number* $\text{Ma} = V/a$ of the flow reaches about 0.3.

The speed of sound of water is tabulated in Table A.5. The speed of sound of air (or any approximately perfect gas) is simply calculated from Eq. (1.39).

1.8 Basic Flow-Analysis Techniques

There are three basic ways to attack a fluid-flow problem. They are equally important for a student learning the subject, and this book tries to give adequate coverage to each method:

1. Control-volume, or *integral* analysis (Chap. 3)
2. Infinitesimal system, or *differential* analysis (Chap. 4)
3. Experimental study, or *dimensional* analysis (Chap. 5)

In all cases, the flow must satisfy the three basic laws of mechanics⁸ plus a thermodynamic state relation and associated boundary conditions:

1. Conservation of mass (continuity)
2. Linear momentum (Newton’s second law)
3. First law of thermodynamics (conservation of energy)
4. A state relation like $\rho = \rho(p, T)$
5. Appropriate boundary conditions at solid surfaces, interfaces, inlets, and exits

In integral and differential analyses, these five relations are modeled mathematically and solved by computational methods. In an experimental study, the fluid itself performs this task without the use of any mathematics. In other words, these laws are believed to be fundamental to physics, and no fluid flow is known to violate them.

⁸In fluids which are variable mixtures of components, such as seawater, a fourth basic law is required, *conservation of species*. For an example of salt conservation analysis, see Chap. 4, Ref. 16.

A control volume is a finite region, chosen carefully by the analyst, with open boundaries through which mass, momentum, and energy are allowed to cross. The analyst makes a budget, or balance, between the incoming and outgoing fluid and the resultant changes within the control volume. The result is a powerful tool but a crude one. Details of the flow are normally washed out or ignored in control-volume analyses. Nevertheless, the control-volume technique of Chap. 3 never fails to yield useful and quantitative information to the engineering analyst.

When the conservation laws are written for an infinitesimal system of fluid in motion, they become the basic differential equations of fluid flow. To apply them to a specific problem, one must integrate these equations mathematically subject to the boundary conditions of the particular problem. Exact analytic solutions are often possible only for very simple geometries and boundary conditions (Chap. 4). Otherwise, one attempts numerical integration on a digital computer, i.e., a summing procedure for finite-sized systems which one hopes will approximate the exact integral calculus [1]. Even computer analysis often fails to provide an accurate simulation, because of either inadequate storage or inability to model the finely detailed flow structure characteristic of irregular geometries or turbulent-flow patterns. Thus differential analysis sometimes promises more than it delivers, although we can successfully study a number of classic and useful solutions.

A properly planned experiment is very often the best way to study a practical engineering flow problem. Guidelines for planning flow experiments are given in Chap. 5. For example, no theory presently available, whether differential or integral, calculus or computer, is able to make an accurate computation of the aerodynamic drag and side force of an automobile moving down a highway with crosswinds. One must solve the problem by experiment. The experiment may be *full-scale*: One can test a real automobile on a real highway in real crosswinds. For that matter, there are wind tunnels in existence large enough to hold a full-scale car without significant blockage effects. Normally, however, in the design stage, one tests a small-model automobile in a small wind tunnel. Without proper interpretation, the model results may be poor and mislead the designer (Chap. 5). For example, the model may lack important details such as surface finish or underbody protuberances. The “wind” produced by the tunnel propellers may lack the turbulent gustiness of real winds. It is the job of the fluid-flow analyst, using such techniques as dimensional analysis, to plan an experiment which gives an accurate estimate of full-scale or *prototype* results expected in the final product.

It is possible to classify flows, but there is no general agreement on how to do it. Most classifications deal with the assumptions made in the proposed flow analysis. They come in pairs, and we normally assume that a given flow is either

Steady or unsteady (1.40a)

Inviscid or viscous (1.40b)

Incompressible or compressible (1.40c)

Gas or liquid (1.40d)

As Fig. 1.14 indicates, we choose one assumption from each pair. We may have a steady viscous compressible gas flow or an unsteady inviscid ($\mu = 0$) incompressible liquid flow. Although there is no such thing as a truly inviscid fluid, the assumption $\mu = 0$ gives adequate results in many analyses (Chap. 8). Often the assumptions overlap: A flow may be viscous in the boundary layer near a solid surface (Fig. 1.13) and effec-

Fig. 1.14 Ready for a flow analysis? Then choose one assumption from each box.



tively inviscid away from the surface. The viscous part of the flow may be laminar or transitional or turbulent or combine patches of all three types of viscous flow. A flow may involve both a gas and a liquid and the free surface, or interface, between them (Chap. 10). A flow may be compressible in one region and have nearly constant density in another. Nevertheless, Eq. (1.40) and Fig. 1.14 give the basic binary assumptions of flow analysis, and Chaps. 6 to 10 try to separate them and isolate the basic effect of each assumption.

1.9 Flow Patterns: Streamlines, Streaklines, and Pathlines

Fluid mechanics is a highly visual subject. The patterns of flow can be visualized in a dozen different ways, and you can view these sketches or photographs and learn a great deal qualitatively and often quantitatively about the flow.

Four basic types of line patterns are used to visualize flows:

1. A *streamline* is a line everywhere tangent to the velocity vector at a given instant.
2. A *pathline* is the actual path traversed by a given fluid particle.
3. A *streakline* is the locus of particles which have earlier passed through a prescribed point.
4. A *timeline* is a set of fluid particles that form a line at a given instant.

The streamline is convenient to calculate mathematically, while the other three are easier to generate experimentally. Note that a streamline and a timeline are instantaneous lines, while the pathline and the streakline are generated by the passage of time. The velocity profile shown in Fig. 1.13 is really a timeline generated earlier by a single discharge of bubbles from the wire. A pathline can be found by a time exposure of a single marked particle moving through the flow. Streamlines are difficult to generate experimentally in unsteady flow unless one marks a great many particles and notes their direction of motion during a very short time interval [17, p. 35]. In steady flow the situation simplifies greatly:

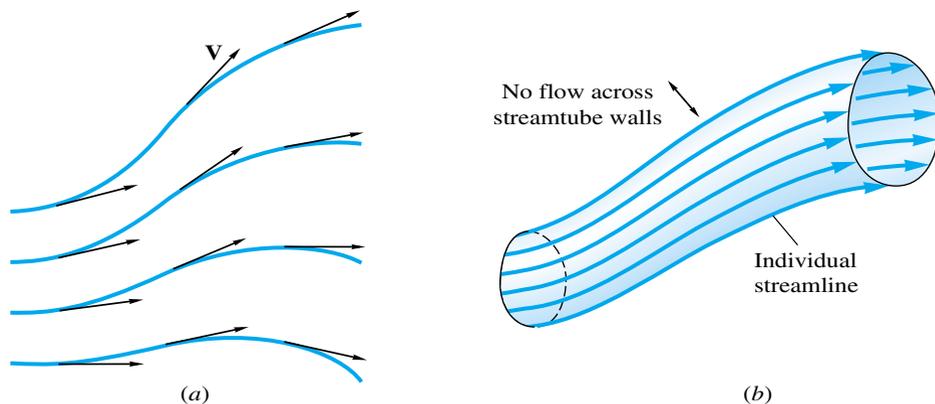
Streamlines, pathlines, and streaklines are identical in steady flow.

In fluid mechanics the most common mathematical result for visualization purposes is the streamline pattern. Figure 1.15a shows a typical set of streamlines, and Fig. 1.15b shows a closed pattern called a *streamtube*. By definition the fluid within a streamtube is confined there because it cannot cross the streamlines; thus the streamtube walls need not be solid but may be fluid surfaces.

Figure 1.16 shows an arbitrary velocity vector. If the elemental arc length dr of a streamline is to be parallel to \mathbf{V} , their respective components must be in proportion:

$$\text{Streamline:} \quad \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V} \quad (1.41)$$

Fig. 1.15 The most common method of flow-pattern presentation: (a) Streamlines are everywhere tangent to the local velocity vector; (b) a streamtube is formed by a closed collection of streamlines.



If the velocities (u, v, w) are known functions of position and time, Eq. (1.41) can be integrated to find the streamline passing through the initial point (x_0, y_0, z_0, t_0) . The method is straightforward for steady flows (Example 1.10) but may be laborious for unsteady flow.

The pathline, or displacement of a particle, is defined by integration of the velocity components, as mentioned in Sec. 1.5:

$$\text{Pathline:} \quad x = \int u \, dt \quad y = \int v \, dt \quad z = \int w \, dt \quad (1.42)$$

Given (u, v, w) as known functions of position and time, the integration is begun at a specified initial position (x_0, y_0, z_0, t_0) . Again the integration may be laborious.

Streaklines, easily generated experimentally with smoke, dye, or bubble releases, are very difficult to compute analytically. See Ref. 18 for mathematical details.

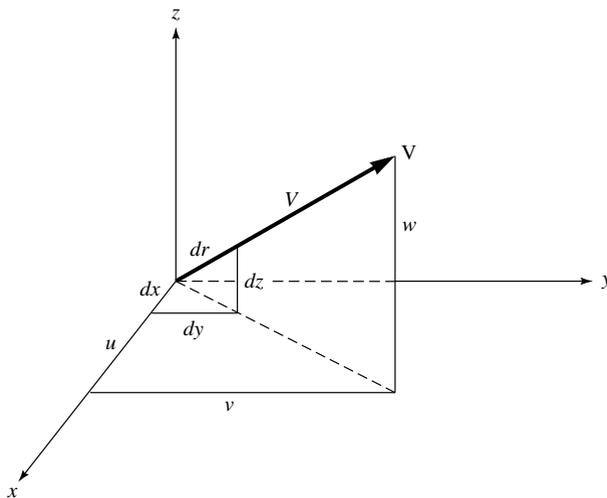


Fig. 1.16 Geometric relations for defining a streamline.

EXAMPLE 1.10

Given the steady two-dimensional velocity distribution

$$u = Kx \quad v = -Ky \quad w = 0 \quad (1)$$

where K is a positive constant, compute and plot the streamlines of the flow, including directions, and give some possible interpretations of the pattern.

Solution

Since time does not appear explicitly in Eq. (1), the motion is steady, so that streamlines, pathlines, and streaklines will coincide. Since $w = 0$ everywhere, the motion is two dimensional, in the xy plane. The streamlines can be computed by substituting the expressions for u and v into Eq. (1.41):

$$\frac{dx}{Kx} = -\frac{dy}{Ky}$$

or
$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

Integrating, we obtain $\ln x = -\ln y + \ln C$, or

$$xy = C \quad \text{Ans. (2)}$$

This is the general expression for the streamlines, which are hyperbolas. The complete pattern is plotted in Fig. E1.10 by assigning various values to the constant C . The arrowheads can be determined only by returning to Eqs. (1) to ascertain the velocity component directions, assuming K is positive. For example, in the upper right quadrant ($x > 0, y > 0$), u is positive and v is negative; hence the flow moves down and to the right, establishing the arrowheads as shown.

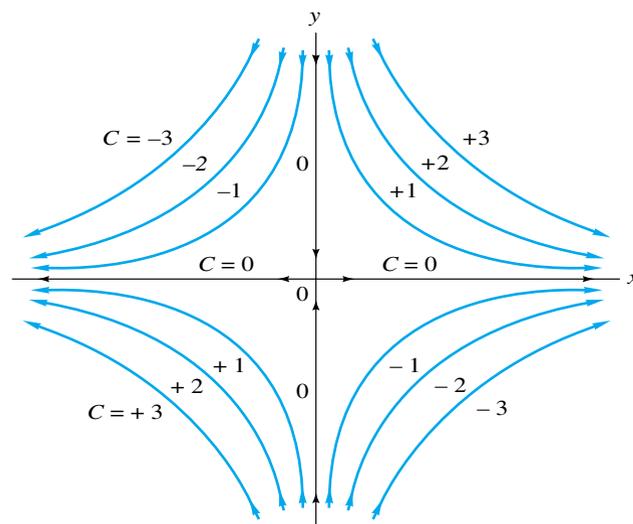


Fig. E1.10 Streamlines for the velocity distribution given by Eq. (1), for $K > 0$.

Note that the streamline pattern is entirely independent of constant K . It could represent the impingement of two opposing streams, or the upper half could simulate the flow of a single downward stream against a flat wall. Taken in isolation, the upper right quadrant is similar to the flow in a 90° corner. This is definitely a realistic flow pattern and is discussed again in Chap. 8.

Finally note the peculiarity that the two streamlines ($C = 0$) have opposite directions and intersect. This is possible only at a point where $u = v = w = 0$, which occurs at the origin in this case. Such a point of zero velocity is called a *stagnation point*.

A streakline can be produced experimentally by the continuous release of marked particles (dye, smoke, or bubbles) from a given point. Figure 1.17 shows two examples. The flow in Fig. 1.17*b* is unsteady and periodic due to the flapping of the plate against the oncoming stream. We see that the dash-dot streakline does not coincide with either the streamline or the pathline passing through the same release point. This is characteristic of unsteady flow, but in Fig. 1.17*a* the smoke filaments form streaklines which are identical to the streamlines and pathlines. We noted earlier that this coincidence of lines is always true of steady flow: Since the velocity never changes magnitude or direction at any point, every particle which comes along repeats the behavior of its earlier neighbors.

Methods of experimental flow visualization include the following:

1. Dye, smoke, or bubble discharges
2. Surface powder or flakes on liquid flows
3. Floating or neutral-density particles
4. Optical techniques which detect density changes in gas flows: shadowgraph, schlieren, and interferometer
5. Tufts of yarn attached to boundary surfaces
6. Evaporative coatings on boundary surfaces
7. Luminescent fluids or additives

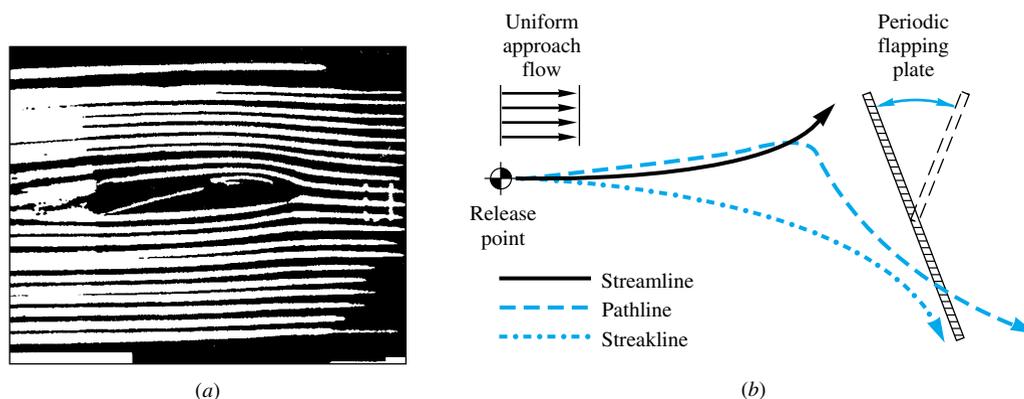


Fig. 1.17 Experimental visualization of steady and unsteady flow: (a) steady flow past an airfoil visualized by smoke filaments (C. A. A. *SCIENTIFIC—Prime Movers Laboratory Systems*); (b) unsteady flow past an oscillating plate with a point bubble release (from an experiment in Ref. 17).

The mathematical implications of flow-pattern analysis are discussed in detail in Ref. 18. References 19 and 20 are beautiful albums of photographs. References 21 and 22 are monographs on flow visualization.

1.10 The Engineering Equation Solver



Most of the examples and exercises in this text are amenable to direct calculation without guessing or iteration or looping. Until recently, only such direct problem assignments, whether “plug-and-chug” or more subtle, were appropriate for undergraduate engineering courses. However, the recent introduction of computer software *solvers* makes almost any set of algebraic relations viable for analysis and solution. The solver recommended here is the *Engineering Equation Solver* (EES) developed by Klein and Beckman [33] and described in Appendix E.

Any software solver should handle a purely mathematical set of relations, such as the one posed in Ref. 33: $X \ln(X) = Y^3$, $X^{1/2} = 1/Y$. Submit that pair to any commercial solver and you will no doubt receive the answer: $X = 1.467$, $Y = 0.826$. However, for engineers, in the author’s opinion, EES is superior to most solvers because (1) equations can be entered in any order; (2) scores of mathematical formulas are built-in, such as the Bessel functions; and (3) thermophysical properties of many fluids are built-in, such as the steam tables [13]. Both metric and English units are allowed. Equations need not be written in the traditional BASIC or FORTRAN style. For example, $X - Y + 1 = 0$ is perfectly satisfactory; there is no need to retype this as $X = Y - 1$.

For example, reconsider Example 1.7 as an EES exercise. One would first enter the reference properties p_0 and ρ_0 plus the curve-fit constants B and n :

$$Pz = 1.0$$

$$\text{Rhoz} = 2.0$$

$$B = 3000$$

$$n = 7$$

Then specify the given pressure ratio and the curve-fit relation, Eq. (1.19), for the equation of state of water:

$$P = 1100 * Pz$$

$$P/Pz = (B + 1) * (\text{Rho}/\text{Rhoz})^n - B$$

If you request an initial opinion from the CHECK/FORMAT menu, EES states that there are six equations in six unknowns and there are no obvious difficulties. Then request SOLVE from the menu and EES quickly prints out $\text{Rho} = 2.091$, the correct answer as seen already in Ex. 1.7. It also prints out values of the other five variables. Occasionally EES reports “unable to converge” and states what went wrong (division by zero, square root of a negative number, etc.). One needs only to improve the guesses and ranges of the unknowns in Variable Information to assist EES to the solution.

In subsequent chapters we will illustrate some implicit (iterative) examples by using EES and will also assign some advanced problem exercises for which EES is an ideal approach. The use of an engineering solver, notably EES, is recommended to all engineers in this era of the personal computer.

1.11 Uncertainty of Experimental Data

Earlier in this chapter we referred to the *uncertainty* of the principle of corresponding states in discussing Fig. 1.5. Uncertainty is a fact of life in engineering. We rarely know any engineering properties or variables to an extreme degree of accuracy. Therefore, we need to know the *uncertainty* U of our data, usually defined as the band within which the experimenter is 95 percent confident that the true value lies (Refs. 30, 31). In Fig. 1.5, we were given that the uncertainty of μ/μ_c is $U \approx \pm 20$ percent.

Fluid mechanics is heavily dependent upon experimentation, and the data uncertainty is needed before we can use it for prediction or design purposes. Sometimes uncertainty completely changes our viewpoint. As an offbeat example, suppose that astronomers reported that the length of the earth year was 365.25 days “give or take a couple of months.” First, that would make the five-figure accuracy ridiculous, and the year would better be stated as $Y \approx 365 \pm 60$ days. Second, we could no longer plan confidently or put together accurate calendars. Scheduling Christmas vacation would be chancy.

Multiple variables make uncertainty estimates cumulative. Suppose a given result P depends upon N variables, $P = P(x_1, x_2, x_3, \dots, x_N)$, each with its own uncertainty; for example, x_1 has uncertainty δx_1 . Then, by common agreement among experimenters, the total uncertainty of P is calculated as a root-mean-square average of all effects:

$$\delta P = \left[\left(\frac{\partial P}{\partial x_1} \delta x_1 \right)^2 + \left(\frac{\partial P}{\partial x_2} \delta x_2 \right)^2 + \dots + \left(\frac{\partial P}{\partial x_N} \delta x_N \right)^2 \right]^{1/2} \quad (1.43)$$

This calculation is statistically much more probable than simply adding linearly the various uncertainties δx_i , thereby making the unlikely assumption that all variables simultaneously attain maximum error. Note that it is the responsibility of the experimenter to establish and report accurate estimates of all the relevant uncertainties δx_i .

If the quantity P is a simple power-law expression of the other variables, for example, $P = \text{Const } x_1^{n_1} x_2^{n_2} x_3^{n_3} \dots$, then each derivative in Eq. (1.43) is proportional to P and the relevant power-law exponent and is inversely proportional to that variable.

If $P = \text{Const } x_1^{n_1} x_2^{n_2} x_3^{n_3} \dots$, then

$$\frac{\partial P}{\partial x_1} = \frac{n_1 P}{x_1}, \quad \frac{\partial P}{\partial x_2} = \frac{n_2 P}{x_2}, \quad \frac{\partial P}{\partial x_3} = \frac{n_3 P}{x_3}, \dots$$

Thus, from Eq. (1.43),

$$\frac{\delta P}{P} = \left[\left(n_1 \frac{\delta x_1}{x_1} \right)^2 + \left(n_2 \frac{\delta x_2}{x_2} \right)^2 + \left(n_3 \frac{\delta x_3}{x_3} \right)^2 + \dots \right]^{1/2} \quad (1.44)$$

Evaluation of δP is then a straightforward procedure, as in the following example.

EXAMPLE 1.11

The so-called dimensionless Moody pipe-friction factor f , plotted in Fig. 6.13, is calculated in experiments from the following formula involving pipe diameter D , pressure drop Δp , density ρ , volume flow rate Q , and pipe length L :

$$f = \frac{\pi^2}{32} \frac{D^5 \Delta p}{\rho Q^2 L}$$

Measurement uncertainties are given for a certain experiment: for D : 0.5 percent, Δp : 2.0 percent, ρ : 1.0 percent, Q : 3.5 percent, and L : 0.4 percent. Estimate the overall uncertainty of the friction factor f .

Solution

The coefficient $\pi^2/32$ is assumed to be a pure theoretical number, with no uncertainty. The other variables may be collected using Eqs. (1.43) and (1.44):

$$U = \frac{\delta f}{f} = \left[\left(5 \frac{\delta D}{D} \right)^2 + \left(1 \frac{\delta \Delta p}{\Delta p} \right)^2 + \left(1 \frac{\delta \rho}{\rho} \right)^2 + \left(2 \frac{\delta Q}{Q} \right)^2 + \left(1 \frac{\delta L}{L} \right)^2 \right]^{1/2}$$

$$= \left[\{5(0.5\%)\}^2 + (2.0\%)^2 + (1.0\%)^2 + \{2(3.5\%)\}^2 + (0.4\%)^2 \right]^{1/2} \approx 7.8\% \quad \text{Ans.}$$

By far the dominant effect in this particular calculation is the 3.5 percent error in Q , which is amplified by doubling, due to the power of 2 on flow rate. The diameter uncertainty, which is quintupled, would have contributed more had δD been larger than 0.5 percent.

1.12 The Fundamentals of Engineering (FE) Examination

The road toward a professional engineer's license has a first stop, the Fundamentals of Engineering Examination, known as the FE exam. It was formerly known as the Engineer-in-Training (E-I-T) Examination. This 8-h national test will probably soon be required of all engineering graduates, not just for licensure, but as a student-assessment tool. The 120-problem morning session covers many general studies:

Chemistry	Computers	Dynamics
Electric circuits	Engineering economics	<i>Fluid Mechanics</i>
Materials science	Mathematics	Mechanics of materials
Statics	Thermodynamics	Ethics

For the 60-problem afternoon session you may choose chemical, civil, electrical, industrial, or mechanical engineering or take more general-engineering problems for remaining disciplines. As you can see, *fluid mechanics* is central to the FE exam. Therefore, this text includes a number of end-of-chapter FE problems where appropriate.

The format for the FE exam questions is multiple-choice, usually with five selections, chosen carefully to tempt you with plausible answers if you used incorrect units or forgot to double or halve something or are missing a factor of π , etc. In some cases, the selections are unintentionally ambiguous, such as the following example from a previous exam:

Transition from laminar to turbulent flow occurs at a Reynolds number of
(A) 900 (B) 1200 (C) 1500 (D) 2100 (E) 3000

The “correct” answer was graded as (D), $Re = 2100$. Clearly the examiner was thinking, but forgot to specify, Re_d for *flow in a smooth circular pipe*, since (see Chaps. 6 and 7) transition is highly dependent upon geometry, surface roughness, and the length scale used in the definition of Re . The moral is not to get peevish about the exam but simply to go with the flow (pun intended) and decide which answer best fits an

undergraduate-training situation. Every effort has been made to keep the FE exam questions in this text unambiguous.

1.13 Problem-Solving Techniques

Fluid flow analysis generates a plethora of problems, 1500 in this text alone! To solve these problems, one must deal with various equations, data, tables, assumptions, unit systems, and numbers. The writer recommends these problem-solving steps:

1. Gather all the given system parameters and data in one place.
2. Find, from tables or charts, all needed fluid property data: ρ , μ , c_p , k , Y , etc.
3. Use SI units (N, s, kg, m) if possible, and no conversion factors will be necessary.
4. Make sure what is *asked*. It is all too common for students to answer the wrong question, for example, reporting mass flow instead of volume flow, pressure instead of pressure gradient, drag force instead of lift force. Engineers are expected to read carefully.
5. Make a detailed sketch of the system, with everything clearly labeled.
6. Think carefully and then list your *assumptions*. Here knowledge is power; you should not guess the answer. You must be able to decide correctly if the flow can be considered steady or unsteady, compressible or incompressible, one-dimensional, or multidimensional, viscous or inviscid, and whether a control volume or partial differential equations are needed.
7. Based on steps 1 to 6 above, write out the appropriate equations, data correlations, and fluid state relations for your problem. If the algebra is straightforward, solve for what is *asked*. If the equations are complicated, e.g., nonlinear or too plentiful, use the Engineering Equation Solver (EES).
8. Report your solution clearly, with proper units listed and to the proper number of significant figures (usually two or three) that the overall uncertainty of the data will allow.

1.14 History and Scope of Fluid Mechanics

Like most scientific disciplines, fluid mechanics has a history of erratically occurring early achievements, then an intermediate era of steady fundamental discoveries in the eighteenth and nineteenth centuries, leading to the twentieth-century era of “modern practice,” as we self-centeredly term our limited but up-to-date knowledge. Ancient civilizations had enough knowledge to solve certain flow problems. Sailing ships with oars and irrigation systems were both known in prehistoric times. The Greeks produced quantitative information. Archimedes and Hero of Alexandria both postulated the parallelogram law for addition of vectors in the third century B.C. Archimedes (285–212 B.C.) formulated the laws of buoyancy and applied them to floating and submerged bodies, actually deriving a form of the differential calculus as part of the analysis. The Romans built extensive aqueduct systems in the fourth century B.C. but left no records showing any quantitative knowledge of design principles.

From the birth of Christ to the Renaissance there was a steady improvement in the design of such flow systems as ships and canals and water conduits but no recorded evidence of fundamental improvements in flow analysis. Then Leonardo da Vinci (1452–1519) derived the equation of conservation of mass in one-dimensional steady

flow. Leonardo was an excellent experimentalist, and his notes contain accurate descriptions of waves, jets, hydraulic jumps, eddy formation, and both low-drag (streamlined) and high-drag (parachute) designs. A Frenchman, Edme Mariotte (1620–1684), built the first wind tunnel and tested models in it.

Problems involving the momentum of fluids could finally be analyzed after Isaac Newton (1642–1727) postulated his laws of motion and the law of viscosity of the linear fluids now called newtonian. The theory first yielded to the assumption of a “perfect” or frictionless fluid, and eighteenth-century mathematicians (Daniel Bernoulli, Leonhard Euler, Jean d’Alembert, Joseph-Louis Lagrange, and Pierre-Simon Laplace) produced many beautiful solutions of frictionless-flow problems. Euler developed both the differential equations of motion and their integrated form, now called the Bernoulli equation. D’Alembert used them to show his famous paradox: that a body immersed in a frictionless fluid has zero drag. These beautiful results amounted to overkill, since perfect-fluid assumptions have very limited application in practice and most engineering flows are dominated by the effects of viscosity. Engineers began to reject what they regarded as a totally unrealistic theory and developed the science of *hydraulics*, relying almost entirely on experiment. Such experimentalists as Chézy, Pitot, Borda, Weber, Francis, Hagen, Poiseuille, Darcy, Manning, Bazin, and Weisbach produced data on a variety of flows such as open channels, ship resistance, pipe flows, waves, and turbines. All too often the data were used in raw form without regard to the fundamental physics of flow.

At the end of the nineteenth century, unification between experimental *hydraulics* and theoretical *hydrodynamics* finally began. William Froude (1810–1879) and his son Robert (1846–1924) developed laws of model testing, Lord Rayleigh (1842–1919) proposed the technique of dimensional analysis, and Osborne Reynolds (1842–1912) published the classic pipe experiment in 1883 which showed the importance of the dimensionless Reynolds number named after him. Meanwhile, viscous-flow theory was available but unexploited, since Navier (1785–1836) and Stokes (1819–1903) had successfully added newtonian viscous terms to the equations of motion. The resulting Navier-Stokes equations were too difficult to analyze for arbitrary flows. Then, in 1904, a German engineer, Ludwig Prandtl (1875–1953), published perhaps the most important paper ever written on fluid mechanics. Prandtl pointed out that fluid flows with small viscosity, e.g., water flows and airflows, can be divided into a thin viscous layer, or *boundary layer*, near solid surfaces and interfaces, patched onto a nearly inviscid outer layer, where the Euler and Bernoulli equations apply. Boundary-layer theory has proved to be the single most important tool in modern flow analysis. The twentieth-century foundations for the present state of the art in fluid mechanics were laid in a series of broad-based experiments and theories by Prandtl and his two chief friendly competitors, Theodore von Kármán (1881–1963) and Sir Geoffrey I. Taylor (1886–1975). Many of the results sketched here from a historical point of view will, of course, be discussed in this textbook. More historical details can be found in Refs. 23 to 25.

Since the earth is 75 percent covered with water and 100 percent covered with air, the scope of fluid mechanics is vast and touches nearly every human endeavor. The sciences of meteorology, physical oceanography, and hydrology are concerned with naturally occurring fluid flows, as are medical studies of breathing and blood circulation. All transportation problems involve fluid motion, with well-developed specialties in aerodynamics of aircraft and rockets and in naval hydrodynamics of ships and submarines. Almost all our electric energy is developed either from water flow or from

steam flow through turbine generators. All combustion problems involve fluid motion, as do the more classic problems of irrigation, flood control, water supply, sewage disposal, projectile motion, and oil and gas pipelines. The aim of this book is to present enough fundamental concepts and practical applications in fluid mechanics to prepare you to move smoothly into any of these specialized fields of the science of flow—and then be prepared to move out again as new technologies develop.

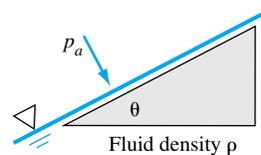
Problems

Most of the problems herein are fairly straightforward. More difficult or open-ended assignments are labeled with an asterisk as in Prob. 1.18. Problems labeled with an EES icon (for example, Prob. 2.62), will benefit from the use of the Engineering Equation Solver (EES), while problems labeled with a computer disk may require the use of a computer. The standard end-of-chapter problems 1.1 to 1.85 (categorized in the problem list below) are followed by fundamentals of engineering (FE) exam problems FE3.1 to FE3.10, and comprehensive problems C1.1 to C1.4.

Problem Distribution

Section	Topic	Problems
1.1, 1.2, 1.3	Fluid-continuum concept	1.1–1.3
1.4	Dimensions, units, dynamics	1.4–1.20
1.5	Velocity field	1.21–1.23
1.6	Thermodynamic properties	1.24–1.37
1.7	Viscosity; no-slip condition	1.38–1.61
1.7	Surface tension	1.62–1.71
1.7	Vapor pressure; cavitation	1.72–1.75
1.7	Speed of sound; Mach number	1.76–1.78
1.8.9	Flow patterns, streamlines, pathlines	1.79–1.84
1.10	History of fluid mechanics	1.85

- P1.1** A gas at 20°C may be considered *rarefied*, deviating from the continuum concept, when it contains less than 10^{12} molecules per cubic millimeter. If Avogadro's number is 6.023 E23 molecules per mole, what absolute pressure (in Pa) for air does this represent?
- P1.2** Table A.6 lists the density of the standard atmosphere as a function of altitude. Use these values to estimate, crudely—say, within a factor of 2—the number of molecules of air in the entire atmosphere of the earth.
- P1.3** For the triangular element in Fig. P1.3, show that a *tilted* free liquid surface, in contact with an atmosphere at pressure p_a , must undergo shear stress and hence begin to flow. *Hint:* Account for the weight of the fluid and show that a no-shear condition will cause horizontal forces to be out of balance.



P1.3

- P1.4** A beaker approximates a right circular cone of diameter 7 in. and height 9 in. When filled with liquid, it weighs 70 oz. When empty, it weighs 14 oz. Estimate the density of this liquid in both SI and BG units.
- P1.5** The *mean free path* of a gas, ℓ , is defined as the average distance traveled by molecules between collisions. A proposed formula for estimating ℓ of an ideal gas is

$$\ell = 1.26 \frac{\mu}{\rho \sqrt{RT}}$$

What are the dimensions of the constant 1.26? Use the formula to estimate the mean free path of air at 20°C and 7 kPa. Would you consider air *rarefied* at this condition?

- P1.6** In the $\{MLT\Theta\}$ system, what is the dimensional representation of (a) enthalpy, (b) mass rate of flow, (c) bending moment, (d) angular velocity, (e) modulus of elasticity; (f) Poisson's ratio?
- P1.7** A small village draws 1.5 acre · ft/day of water from its reservoir. Convert this average water usage to (a) gallons per minute and (b) liters per second.
- P1.8**  Suppose we know little about the strength of materials but are told that the bending stress σ in a beam is *proportional* to the beam half-thickness y and also depends upon the bending moment M and the beam area moment of inertia I . We also learn that, for the particular case $M = 2900$ in · lbf, $y = 1.5$ in, and $I = 0.4$ in⁴, the predicted stress is 75 MPa. Using this information and dimensional reasoning only, find, to three significant figures, the only possible dimensionally homogeneous formula $\sigma = y f(M, I)$.

P1.9 The *kinematic viscosity* of a fluid is the ratio of viscosity to density, $\nu = \mu/\rho$. What is the only possible dimensionless group combining ν with velocity V and length L ? What is the name of this grouping? (More information on this will be given in Chap. 5.)

P1.10 The Stokes-Oseen formula [18] for drag force F on a sphere of diameter D in a fluid stream of low velocity V , density ρ , and viscosity μ , is

$$F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$$

Is this formula dimensionally homogeneous?

P1.11 Engineers sometimes use the following formula for the volume rate of flow Q of a liquid flowing through a hole of diameter D in the side of a tank:

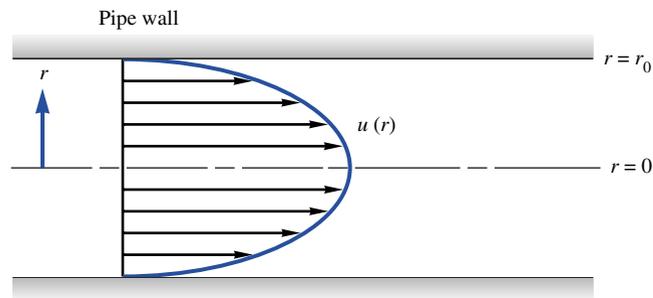
$$Q = 0.68 D^2 \sqrt{gh}$$

where g is the acceleration of gravity and h is the height of the liquid surface above the hole. What are the dimensions of the constant 0.68?

P1.12 For low-speed (laminar) steady flow through a circular pipe, as shown in Fig. P1.12, the velocity u varies with radius and takes the form

$$u = B \frac{\Delta p}{\mu} (r_0^2 - r^2)$$

where μ is the fluid viscosity and Δp is the pressure drop from entrance to exit. What are the dimensions of the constant B ?



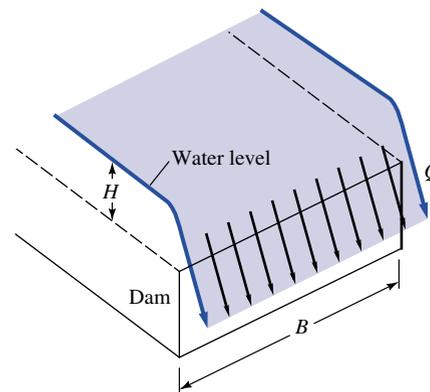
P1.12

P1.13 The efficiency η of a pump is defined as the (dimensionless) ratio of the power developed by the flow to the power required to drive the pump:

$$\eta = \frac{Q\Delta p}{\text{input power}}$$

where Q is the volume rate of flow and Δp is the pressure rise produced by the pump. Suppose that a certain pump develops a pressure rise of 35 lbf/in² when its flow rate is 40 L/s. If the input power is 16 hp, what is the efficiency?

***P1.14** Figure P1.14 shows the flow of water over a dam. The volume flow Q is known to depend only upon crest width B , acceleration of gravity g , and upstream water height H above the dam crest. It is further known that Q is proportional to B . What is the form of the only possible dimensionally homogeneous relation for this flow rate?



P1.14

P1.15 As a practical application of Fig. P1.14, often termed a sharp-crested weir, civil engineers use the following formula for flow rate: $Q \approx 3.3BH^{3/2}$, with Q in ft³/s and B and H in feet. Is this formula dimensionally homogeneous? If not, try to explain the difficulty and how it might be converted to a more homogeneous form.

P1.16 Algebraic equations such as Bernoulli's relation, Eq. (1) of Ex. 1.3, are dimensionally consistent, but what about differential equations? Consider, for example, the boundary-layer x -momentum equation, first derived by Ludwig Prandtl in 1904:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \rho g_x + \frac{\partial \tau}{\partial y}$$

where τ is the boundary-layer shear stress and g_x is the component of gravity in the x direction. Is this equation dimensionally consistent? Can you draw a general conclusion?

P1.17 The Hazen-Williams hydraulics formula for volume rate of flow Q through a pipe of diameter D and length L is given by

$$Q \approx 61.9 D^{2.63} \left(\frac{\Delta p}{L} \right)^{0.54}$$

where Δp is the pressure drop required to drive the flow. What are the dimensions of the constant 61.9? Can this formula be used with confidence for various liquids and gases?

***P1.18** For small particles at low velocities, the first term in the Stokes-Oseen drag law, Prob. 1.10, is dominant; hence, $F \approx KV$, where K is a constant. Suppose a particle of mass m is constrained to move horizontally from the initial position $x = 0$ with initial velocity V_0 . Show (a) that its velocity will decrease exponentially with time and (b) that it will stop after traveling a distance $x = mV_0/K$.

***P1.19** For larger particles at higher velocities, the quadratic term in the Stokes-Oseen drag law, Prob. 1.10, is dominant; hence, $F \approx CV^2$, where C is a constant. Repeat Prob. 1.18 to show that (a) its velocity will decrease as $1/(1 + CV_0t/m)$ and (b) it will never quite stop in a finite time span.

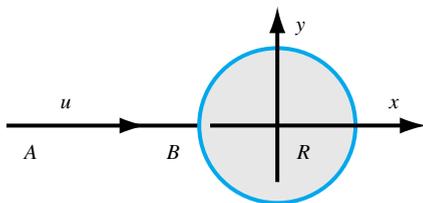
P1.20 A baseball, with $m = 145$ g, is thrown directly upward from the initial position $z = 0$ and $V_0 = 45$ m/s. The air drag on the ball is CV^2 , as in Prob. 1.19, where $C \approx 0.0013$ N · s²/m². Set up a differential equation for the ball motion, and solve for the instantaneous velocity $V(t)$ and position $z(t)$. Find the maximum height z_{\max} reached by the ball, and compare your results with the classical case of zero air drag.

P1.21 A velocity field is given by $\mathbf{V} = Kx\mathbf{i} - Ky\mathbf{j} + 0\mathbf{k}$, where K is a positive constant. Evaluate (a) $\nabla \cdot \mathbf{V}$ and (b) $\nabla \times \mathbf{V}$.

***P1.22** According to the theory of Chap. 8, as a uniform stream approaches a cylinder of radius R along the symmetry line AB in Fig. P1.22, the velocity has only one component:

$$u = U_\infty \left(1 - \frac{R^2}{x^2} \right) \text{ for } -\infty < x \leq -R$$

where U_∞ is the stream velocity far from the cylinder. Using the concepts from Ex. 1.5, find (a) the maximum flow deceleration along AB and (b) its location.



P1.22

P1.23 Experiment with a faucet (kitchen or otherwise) to determine typical flow rates Q in m³/h, perhaps timing the discharge of a known volume. Try to achieve an exit jet condition which is (a) smooth and round and (b) disorderly and fluctuating. Measure the supply-pipe diameter (look under the sink). For both cases, calculate the average flow velocity, $V_{\text{avg}} =$

$Q/A_{\text{cross-section}}$ and the dimensionless Reynolds number of the flow, $\text{Re} = \rho V_{\text{avg}} D / \mu$. Comment on your results.

P1.24 Air at 1 atm and 20°C has an internal energy of approximately 2.1 E5 J/kg. If this air moves at 150 m/s at an altitude $z = 8$ m, what is its total energy, in J/kg, relative to the datum $z = 0$? Are any energy contributions negligible?

P1.25 A tank contains 0.9 m³ of helium at 200 kPa and 20°C. Estimate the total mass of this gas, in kg, (a) on earth and (b) on the moon. Also, (c) how much heat transfer, in MJ, is required to expand this gas at constant temperature to a new volume of 1.5 m³?

P1.26 When we in the United States say a car's tire is filled "to 32 lb," we mean that its internal pressure is 32 lbf/in² above the ambient atmosphere. If the tire is at sea level, has a volume of 3.0 ft³, and is at 75°F, estimate the total weight of air, in lbf, inside the tire.

P1.27 For steam at 40 lbf/in², some values of temperature and specific volume are as follows, from Ref. 13:

$T, ^\circ\text{F}$	400	500	600	700	800
$v, \text{ft}^3/\text{lbm}$	12.624	14.165	15.685	17.195	18.699

Is steam, for these conditions, nearly a perfect gas, or is it wildly nonideal? If reasonably perfect, find a least-squares[†] value for the gas constant R , in m²/(s² · K), estimate the percent error in this approximation, and compare with Table A.4.

P1.28 Wet atmospheric air at 100 percent relative humidity contains saturated water vapor and, by Dalton's law of partial pressures,

$$p_{\text{atm}} = p_{\text{dry air}} + p_{\text{water vapor}}$$

Suppose this wet atmosphere is at 40°C and 1 atm. Calculate the density of this 100 percent humid air, and compare it with the density of dry air at the same conditions.

P1.29 A compressed-air tank holds 5 ft³ of air at 120 lbf/in² "gage," that is, above atmospheric pressure. Estimate the energy, in ft·lbf, required to compress this air from the atmosphere, assuming an ideal isothermal process.

P1.30 Repeat Prob. 1.29 if the tank is filled with compressed water instead of air. Why is the result thousands of times less than the result of 215,000 ft · lbf in Prob. 1.29?

***P1.31** The density of (fresh) water at 1 atm, over the temperature range 0 to 100°C, is given in Table A.1. Fit these values to a least-squares[†] equation of the form $\rho = a + bT + cT^2$, with T in °C, and estimate its accuracy. Use your formula to compute the density of water at 45°C, and compare your result with the accepted experimental value of 990.1 kg/m³.

[†] The concept of "least-squares" error is very important and should be learned by everyone.

P1.32 A blimp is approximated by a prolate spheroid 90 m long and 30 m in diameter. Estimate the weight of 20°C gas within the blimp for (a) helium at 1.1 atm and (b) air at 1.0 atm. What might the *difference* between these two values represent (see Chap. 2)?

***P1.33** Experimental data for the density of mercury versus pressure at 20°C are as follows:

p , atm	1	500	1,000	1,500	2,000
ρ , kg/m ³	13,545	13,573	13,600	13,625	13,653

Fit this data to the empirical state relation for liquids, Eq. (1.22), to find the best values of B and n for mercury. Then, assuming the data are nearly isentropic, use these values to estimate the speed of sound of mercury at 1 atm and compare with Table 9.1.

P1.34 If water occupies 1 m³ at 1 atm pressure, estimate the pressure required to reduce its volume by 5 percent.

P1.35 In Table A.4, most common gases (air, nitrogen, oxygen, hydrogen) have a specific heat ratio $k \approx 1.40$. Why do argon and helium have such high values? Why does NH₃ have such a low value? What is the lowest k for any gas that you know of?

P1.36 The isentropic bulk modulus B of a fluid is defined as the isentropic change in pressure per fractional change in density:

$$B = \rho \left(\frac{\partial p}{\partial \rho} \right)_s$$

What are the dimensions of B ? Using theoretical $p(\rho)$ relations, estimate the bulk modulus of (a) N₂O, assumed to be an ideal gas, and (b) water, at 20°C and 1 atm.

P1.37 A near-ideal gas has a molecular weight of 44 and a specific heat $c_v = 610$ J/(kg · K). What are (a) its specific heat ratio, k , and (b) its speed of sound at 100°C?

P1.38 In Fig. 1.6, if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at 5.5 m/s? What is the Reynolds number if L is taken to be the distance between plates?

P1.39 Knowing μ for air at 20°C from Table 1.4, estimate its viscosity at 500°C by (a) the power law and (b) the Sutherland law. Also make an estimate from (c) Fig. 1.5. Compare with the accepted value of $\mu \approx 3.58$ E-5 kg/m · s.

***P1.40** For liquid viscosity as a function of temperature, a simplification of the log-quadratic law of Eq. (1.31) is *Andrade's equation* [11], $\mu \approx A \exp(B/T)$, where (A, B) are curve-fit constants and T is absolute temperature. Fit this relation to the data for water in Table A.1 and estimate the percent error of the approximation.

P1.41 Some experimental values of the viscosity of argon gas at 1 atm are as follows:

T , K	300	400	500	600	700	800
μ , kg/(m · s)	2.27 E-5	2.85 E-5	3.37 E-5	3.83 E-5	4.25 E-5	4.64 E-5

Fit these value to either (a) a power law or (b) the Sutherland law, Eq. (1.30).

P1.42 Experimental values for the viscosity of helium at 1 atm are as follows:

T , K	200	400	600	800	1000	1200
μ , kg/(m · s)	1.50 E-5	2.43 E-5	3.20 E-5	3.88 E-5	4.50 E-5	5.08 E-5

Fit these values to either (a) a power law or (b) the Sutherland law, Eq. (1.30).

***P1.43** Yaws et al. [34] suggest the following curve-fit formula for viscosity versus temperature of organic liquids:

$$\log_{10} \mu \approx A + \frac{B}{T} + CT + DT^2$$

with T in absolute units. (a) Can this formula be criticized on dimensional grounds? (b) Disregarding (a), indicate analytically how the curve-fit constants A, B, C, D could be found from N data points (μ_i, T_i) using the method of least squares. Do not actually carry out a calculation.

P1.44 The values for SAE 30 oil in Table 1.4 are strictly “representative,” not exact, because lubricating oils vary considerably according to the type of crude oil from which they are refined. The Society of Automotive Engineers [26] allows certain kinematic viscosity *ranges* for all lubricating oils: for SAE 30, $9.3 < \nu < 12.5$ mm²/s at 100°C. SAE 30 oil density can also vary ± 2 percent from the tabulated value of 891 kg/m³. Consider the following data for an acceptable grade of SAE 30 oil:

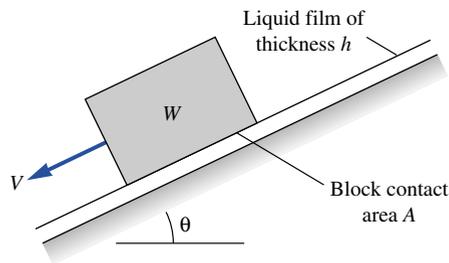
T , °C	0	20	40	60	80	100
μ , kg/(m · s)	2.00	0.40	0.11	0.042	0.017	0.0095

How does this oil compare with the plot in Appendix Fig. A.1? How well does the data fit Andrade's equation in Prob. 1.40?

P1.45 A block of weight W slides down an inclined plane while lubricated by a thin film of oil, as in Fig. P1.45. The film contact area is A and its thickness is h . Assuming a linear velocity distribution in the film, derive an expression for the “terminal” (zero-acceleration) velocity V of the block.

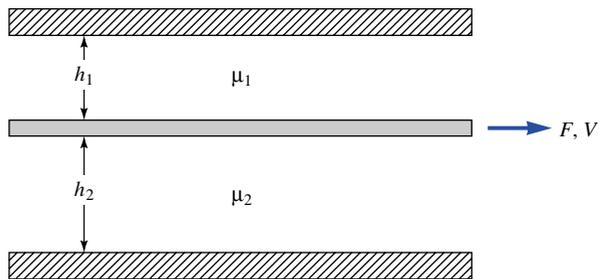
P1.46 Find the terminal velocity of the block in Fig. P1.45 if the block mass is 6 kg, $A = 35$ cm², $\theta = 15^\circ$, and the film is 1-mm-thick SAE 30 oil at 20°C.

P1.47 A shaft 6.00 cm in diameter is being pushed axially through a bearing sleeve 6.02 cm in diameter and 40 cm long. The clearance, assumed uniform, is filled with oil

**P1.45**

whose properties are $\nu = 0.003 \text{ m}^2/\text{s}$ and $\text{SG} = 0.88$. Estimate the force required to pull the shaft at a steady velocity of 0.4 m/s .

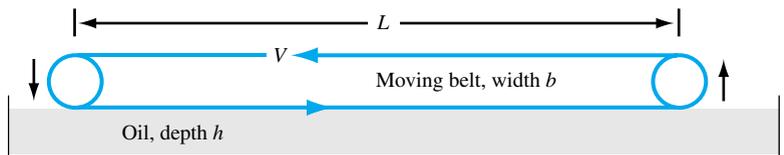
- P1.48** A thin plate is separated from two fixed plates by very viscous liquids μ_1 and μ_2 , respectively, as in Fig. P1.48. The plate spacings h_1 and h_2 are unequal, as shown. The contact area is A between the center plate and each fluid. (a) Assuming a linear velocity distribution in each fluid, derive the force F required to pull the plate at velocity V . (b) Is there a necessary relation between the two viscosities, μ_1 and μ_2 ?

**P1.48**

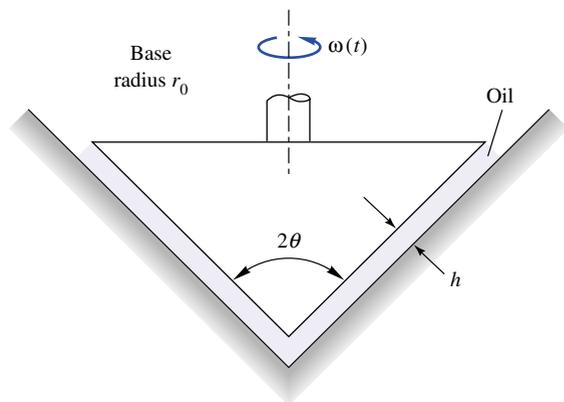
- P1.49** The shaft in Prob. 1.47 is now fixed axially and rotated inside the sleeve at 1500 r/min . Estimate (a) the torque ($\text{N} \cdot \text{m}$) and (b) the power (kW) required to rotate the shaft.
- P1.50** An amazing number of commercial and laboratory devices have been developed to measure the viscosity of fluids, as described in Ref. 27. The concentric rotating shaft of Prob. 1.49 is an example of a *rotational viscometer*. Let the inner and outer cylinders have radii r_i and r_o , respectively, with total sleeve length L . Let the rotational rate be Ω (rad/s) and the applied torque be M . Derive a theoretical relation for the viscosity of the clearance fluid, μ , in terms of these parameters.
- P1.51** Use the theory of Prob. 1.50 (or derive an ad hoc expression if you like) for a shaft 8 cm long, rotating at 1200 r/min , with $r_i = 2.00 \text{ cm}$ and $r_o = 2.05 \text{ cm}$. If the mea-

sured torque is $0.293 \text{ N} \cdot \text{m}$, what is the fluid viscosity? Suppose that the uncertainties of the experiment are as follows: L ($\pm 0.5 \text{ mm}$), M ($\pm 0.003 \text{ N} \cdot \text{m}$), Ω (± 1 percent), and r_i or r_o ($\pm 0.02 \text{ mm}$). What is the uncertainty in the measured viscosity?

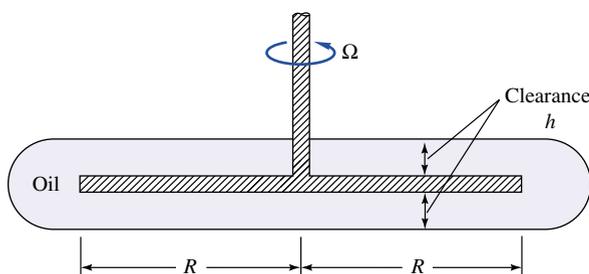
- P1.52** The belt in Fig. P1.52 moves at a steady velocity V and skims the top of a tank of oil of viscosity μ , as shown. Assuming a linear velocity profile in the oil, develop a simple formula for the required belt-drive power P as a function of (h, L, V, b, μ) . What belt-drive power P , in watts, is required if the belt moves at 2.5 m/s over SAE 30W oil at 20°C , with $L = 2 \text{ m}$, $b = 60 \text{ cm}$, and $h = 3 \text{ cm}$?

**P1.52**

- *P1.53** A solid cone of angle 2θ , base r_o , and density ρ_c is rotating with initial angular velocity ω_0 inside a conical seat, as shown in Fig. P1.53. The clearance h is filled with oil of viscosity μ . Neglecting air drag, derive an analytical expression for the cone's angular velocity $\omega(t)$ if there is no applied torque.

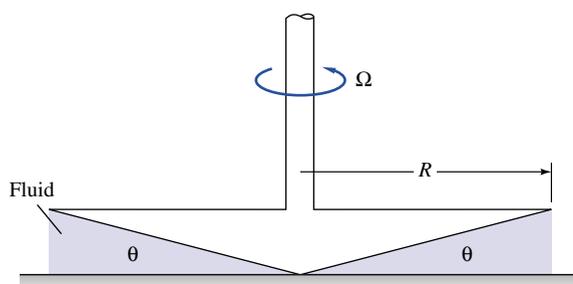
**P1.53**

- *P1.54** A disk of radius R rotates at an angular velocity Ω inside a disk-shaped container filled with oil of viscosity μ , as shown in Fig. P1.54. Assuming a linear velocity profile and neglecting shear stress on the outer disk edges, derive a formula for the viscous torque on the disk.



P1.54

- *P1.55** The device in Fig. P1.54 is called a *rotating disk viscometer* [27]. Suppose that $R = 5$ cm and $h = 1$ mm. If the torque required to rotate the disk at 900 r/min is 0.537 N · m, what is the viscosity of the fluid? If the uncertainty in each parameter (M , R , h , Ω) is ± 1 percent, what is the overall uncertainty in the viscosity?
- *P1.56** The device in Fig. P1.56 is called a *cone-plate viscometer* [27]. The angle of the cone is very small, so that $\sin \theta \approx \theta$, and the gap is filled with the test liquid. The torque M to rotate the cone at a rate Ω is measured. Assuming a linear velocity profile in the fluid film, derive an expression for fluid viscosity μ as a function of (M , R , Ω , θ).



P1.56

- *P1.57** For the cone-plate viscometer of Fig. P1.56, suppose that $R = 6$ cm and $\theta = 3^\circ$. If the torque required to rotate the cone at 600 r/min is 0.157 N · m, what is the viscosity of the fluid? If the uncertainty in each parameter (M , R , Ω , θ) is ± 1 percent, what is the overall uncertainty in the viscosity?
- *P1.58** The laminar-pipe-flow example of Prob. 1.12 can be used to design a *capillary viscometer* [27]. If Q is the volume flow rate, L is the pipe length, and Δp is the pressure drop from entrance to exit, the theory of Chap. 6 yields a formula for viscosity:

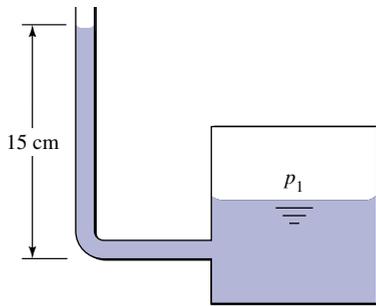
$$\mu = \frac{\pi r_0^4 \Delta p}{8LQ}$$

Pipe end effects are neglected [27]. Suppose our capillary has $r_0 = 2$ mm and $L = 25$ cm. The following flow rate and pressure drop data are obtained for a certain fluid:

Q , m ³ /h	0.36	0.72	1.08	1.44	1.80
Δp , kPa	159	318	477	1274	1851

What is the viscosity of the fluid? *Note:* Only the first three points give the proper viscosity. What is peculiar about the last two points, which were measured accurately?

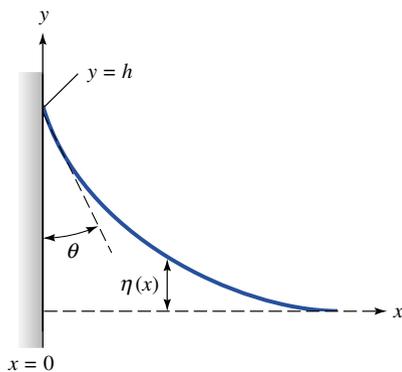
- P1.59** A solid cylinder of diameter D , length L , and density ρ_s falls due to gravity inside a tube of diameter D_0 . The clearance, $D_0 - D \ll D$, is filled with fluid of density ρ and viscosity μ . Neglect the air above and below the cylinder. Derive a formula for the terminal fall velocity of the cylinder. Apply your formula to the case of a steel cylinder, $D = 2$ cm, $D_0 = 2.04$ cm, $L = 15$ cm, with a film of SAE 30 oil at 20°C.
- P1.60** For Prob. 1.52 suppose that $P = 0.1$ hp when $V = 6$ ft/s, $L = 4.5$ ft, $b = 22$ in, and $h = 7/8$ in. Estimate the viscosity of the oil, in kg/(m · s). If the uncertainty in each parameter (P , L , b , h , V) is ± 1 percent, what is the overall uncertainty in the viscosity?
- *P1.61** An air-hockey puck has a mass of 50 g and is 9 cm in diameter. When placed on the air table, a 20°C air film, of 0.12-mm thickness, forms under the puck. The puck is struck with an initial velocity of 10 m/s. Assuming a linear velocity distribution in the air film, how long will it take the puck to (a) slow down to 1 m/s and (b) stop completely? Also, (c) how far along this extremely long table will the puck have traveled for condition (a)?
- P1.62** The hydrogen bubbles which produced the velocity profiles in Fig. 1.13 are quite small, $D \approx 0.01$ mm. If the hydrogen-water interface is comparable to air-water and the water temperature is 30°C estimate the excess pressure within the bubble.
- P1.63** Derive Eq. (1.37) by making a force balance on the fluid interface in Fig. 1.9c.
- P1.64** At 60°C the surface tension of mercury and water is 0.47 and 0.0662 N/m, respectively. What capillary height changes will occur in these two fluids when they are in contact with air in a clean glass tube of diameter 0.4 mm?
- P1.65** The system in Fig. P1.65 is used to calculate the pressure p_1 in the tank by measuring the 15-cm height of liquid in the 1-mm-diameter tube. The fluid is at 60°C (see Prob. 1.64). Calculate the true fluid height in the tube and the percent error due to capillarity if the fluid is (a) water and (b) mercury.

**P1.65**

P1.66 A thin wire ring, 3 cm in diameter, is lifted from a water surface at 20°C. Neglecting the wire weight, what is the force required to lift the ring? Is this a good way to measure surface tension? Should the wire be made of any particular material?

P1.67 Experiment with a capillary tube, perhaps borrowed from the chemistry department, to verify, in clean water, the rise due to surface tension predicted by Example 1.9. Add small amounts of liquid soap to the water, and report to the class whether detergents significantly lower the surface tension. What practical difficulties do detergents present?

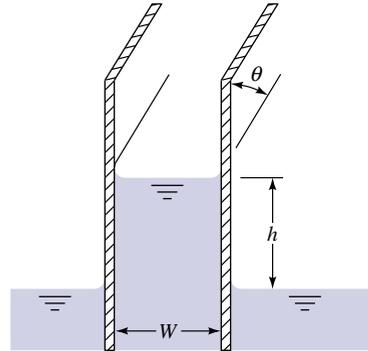
***P1.68** Make an analysis of the shape $\eta(x)$ of the water-air interface near a plane wall, as in Fig. P1.68, assuming that the slope is small, $R^{-1} \approx d^2\eta/dx^2$. Also assume that the pressure difference across the interface is balanced by the specific weight and the interface height, $\Delta p \approx \rho g \eta$. The boundary conditions are a wetting contact angle θ at $x = 0$ and a horizontal surface $\eta = 0$ as $x \rightarrow \infty$. What is the maximum height h at the wall?

**P1.68**

P1.69 A solid cylindrical needle of diameter d , length L , and density ρ_n may float in liquid of surface tension Y . Neglect buoyancy and assume a contact angle of 0°. Derive a for-

mula for the maximum diameter d_{\max} able to float in the liquid. Calculate d_{\max} for a steel needle ($SG = 7.84$) in water at 20°C.

P1.70 Derive an expression for the capillary height change h for a fluid of surface tension Y and contact angle θ between two vertical parallel plates a distance W apart, as in Fig. P1.70. What will h be for water at 20°C if $W = 0.5$ mm?

**P1.70**

***P1.71** A soap bubble of diameter D_1 coalesces with another bubble of diameter D_2 to form a single bubble D_3 with the same amount of air. Assuming an isothermal process, derive an expression for finding D_3 as a function of D_1 , D_2 , p_{atm} , and Y .

P1.72 Early mountaineers boiled water to estimate their altitude. If they reach the top and find that water boils at 84°C, approximately how high is the mountain?

P1.73 A small submersible moves at velocity V , in fresh water at 20°C, at a 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is known to be $C_a = 0.25$. At what velocity will cavitation bubbles begin to form on the body? Will the body cavitate if $V = 30$ m/s and the water is cold (5°C)?

P1.74 A propeller is tested in a water tunnel at 20°C as in Fig. 1.12a. The lowest pressure on the blade can be estimated by a form of Bernoulli's equation (Ex. 1.3):

$$p_{\min} \approx p_0 - \frac{1}{2}\rho V^2$$

where $p_0 = 1.5$ atm and $V =$ tunnel velocity. If we run the tunnel at $V = 18$ m/s, can we be sure that there will be no cavitation? If not, can we change the water temperature and avoid cavitation?

P1.75 Oil, with a vapor pressure of 20 kPa, is delivered through a pipeline by equally spaced pumps, each of which increases the oil pressure by 1.3 MPa. Friction losses in the pipe are 150 Pa per meter of pipe. What is the maximum possible pump spacing to avoid cavitation of the oil?

P1.76 An airplane flies at 555 mi/h. At what altitude in the standard atmosphere will the airplane's Mach number be exactly 0.8?

***P1.77** The density of 20°C gasoline varies with pressure approximately as follows:



p , atm	1	500	1000	1500
ρ , lbm/ft ³	42.45	44.85	46.60	47.98

Use these data to estimate (a) the speed of sound (m/s) and (b) the bulk modulus (MPa) of gasoline at 1 atm.

P1.78 Sir Isaac Newton measured the speed of sound by timing the difference between seeing a cannon's puff of smoke and hearing its boom. If the cannon is on a mountain 5.2 mi away, estimate the air temperature in degrees Celsius if the time difference is (a) 24.2 s and (b) 25.1 s.

P1.79 Examine the photographs in Figs. 1.12a, 1.13, 5.2a, 7.14a, and 9.10b and classify them according to the boxes in Fig. 1.14.

***P1.80** A two-dimensional steady velocity field is given by $u = x^2 - y^2$, $v = -2xy$. Derive the streamline pattern and sketch a few streamlines in the upper half plane. *Hint:* The differential equation is exact.

P1.81 Repeat Ex. 1.10 by letting the velocity components increase linearly with time:

$$\mathbf{V} = Kxt\mathbf{i} - Kyt\mathbf{j} + 0\mathbf{k}$$

Find and sketch, for a few representative times, the instantaneous streamlines. How do they differ from the steady flow lines in Ex. 1.10?

P1.82 A velocity field is given by $u = V \cos \theta$, $v = V \sin \theta$, and $w = 0$, where V and θ are constants. Derive a formula for the streamlines of this flow.

***P1.83** A two-dimensional unsteady velocity field is given by $u = x(1 + 2t)$, $v = y$. Find the equation of the time-varying streamlines which all pass through the point (x_0, y_0) at some time t . Sketch a few of these.

***P1.84** Repeat Prob. 1.83 to find and sketch the equation of the *pathline* which passes through (x_0, y_0) at time $t = 0$.

P1.85 Do some reading and report to the class on the life and achievements, especially vis-à-vis fluid mechanics, of

(a) Evangelista Torricelli (1608–1647)

(b) Henri de Pitot (1695–1771)

(c) Antoine Chézy (1718–1798)

(d) Gotthilf Heinrich Ludwig Hagen (1797–1884)

(e) Julius Weisbach (1806–1871)

(f) George Gabriel Stokes (1819–1903)

(g) Moritz Weber (1871–1951)

(h) Theodor von Kármán (1881–1963)

(i) Paul Richard Heinrich Blasius (1883–1970)

(j) Ludwig Prandtl (1875–1953)

(k) Osborne Reynolds (1842–1912)

(l) John William Strutt, Lord Rayleigh (1842–1919)

(m) Daniel Bernoulli (1700–1782)

(n) Leonhard Euler (1707–1783)

Fundamentals of Engineering Exam Problems

FE1.1 The absolute viscosity μ of a fluid is primarily a function of

(a) Density, (b) Temperature, (c) Pressure, (d) Velocity, (e) Surface tension

FE1.2 If a uniform solid body weighs 50 N in air and 30 N in water, its specific gravity is

(a) 1.5, (b) 1.67, (c) 2.5, (d) 3.0, (e) 5.0

FE1.3 Helium has a molecular weight of 4.003. What is the weight of 2 m³ of helium at 1 atm and 20°C?

(a) 3.3 N, (b) 6.5 N, (c) 11.8 N, (d) 23.5 N, (e) 94.2 N

FE1.4 An oil has a kinematic viscosity of 1.25 E-4 m²/s and a specific gravity of 0.80. What is its dynamic (absolute) viscosity in kg/(m · s)?

(a) 0.08, (b) 0.10, (c) 0.125, (d) 1.0, (e) 1.25

FE1.5 Consider a soap bubble of diameter 3 mm. If the surface tension coefficient is 0.072 N/m and external pressure is 0 Pa gage, what is the bubble's internal gage pressure?

(a) -24 Pa, (b) +48 Pa, (c) +96 Pa, (d) +192 Pa, (e) -192 Pa

FE1.6 The only possible dimensionless group which combines velocity V , body size L , fluid density ρ , and surface tension coefficient σ is

(a) $L\rho\sigma/V$, (b) $\rho VL^2/\sigma$, (c) $\rho\sigma V^2/L$, (d) $\sigma LV^2/\rho$, (e) $\rho LV^2/\sigma$

FE1.7 Two parallel plates, one moving at 4 m/s and the other fixed, are separated by a 5-mm-thick layer of oil of specific gravity 0.80 and kinematic viscosity 1.25 E-4 m²/s. What is the average shear stress in the oil?

(a) 80 Pa, (b) 100 Pa, (c) 125 Pa, (d) 160 Pa, (e) 200 Pa

FE1.8 Carbon dioxide has a specific heat ratio of 1.30 and a gas constant of 189 J/(kg · °C). If its temperature rises from 20 to 45°C, what is its internal energy rise?

(a) 12.6 kJ/kg, (b) 15.8 kJ/kg, (c) 17.6 kJ/kg, (d) 20.5 kJ/kg, (e) 25.1 kJ/kg

- FE1.9** A certain water flow at 20°C has a critical cavitation number, where bubbles form, $Ca \approx 0.25$, where $Ca = 2(p_a - p_{\text{vap}})/\rho V^2$. If $p_a = 1$ atm and the vapor pressure is 0.34 pounds per square inch absolute (psia), for what water velocity will bubbles form?
 (a) 12 mi/h, (b) 28 mi/h, (c) 36 mi/h, (d) 55 mi/h,
 (e) 63 mi/h

Comprehensive Problems

C1.1 Sometimes equations can be developed and practical problems can be solved by knowing nothing more than the dimensions of the key parameters in the problem. For example, consider the heat loss through a window in a building. Window efficiency is rated in terms of “ R value” which has units of $(\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F})/\text{Btu}$. A certain manufacturer advertises a double-pane window with an R value of 2.5. The same company produces a triple-pane window with an R value of 3.4. In either case the window dimensions are 3 ft by 5 ft. On a given winter day, the temperature difference between the inside and outside of the building is 45°F.

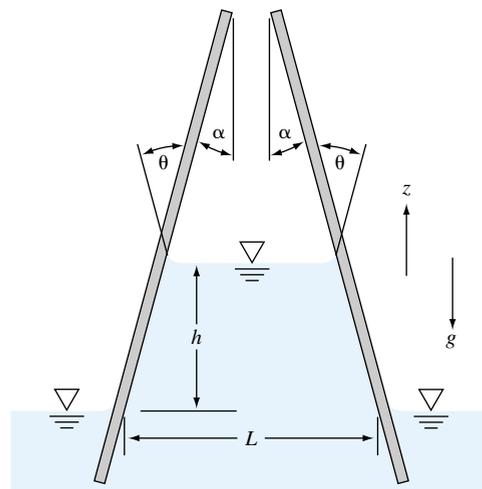
- (a) Develop an equation for the amount of heat lost in a given time period Δt , through a window of area A , with R value R , and temperature difference ΔT . How much heat (in Btu) is lost through the double-pane window in one 24-h period?
 (b) How much heat (in Btu) is lost through the triple-pane window in one 24-h period?
 (c) Suppose the building is heated with propane gas, which costs \$1.25 per gallon. The propane burner is 80 percent efficient. Propane has approximately 90,000 Btu of available energy per gallon. In that same 24-h period, how much money would a homeowner save per window by installing triple-pane rather than double-pane windows?
 (d) Finally, suppose the homeowner buys 20 such triple-pane windows for the house. A typical winter has the equivalent of about 120 heating days at a temperature difference of 45°F. Each triple-pane window costs \$85 more than the double-pane window. Ignoring interest and inflation, how many years will it take the homeowner to make up the additional cost of the triple-pane windows from heating bill savings?
- C1.2** When a person ice skates, the surface of the ice actually melts beneath the blades, so that he or she skates on a thin sheet of water between the blade and the ice.
- (a) Find an expression for total friction force on the bottom of the blade as a function of skater velocity V , blade length L , water thickness (between the blade and the ice) h , water viscosity μ , and blade width W .

- FE1.10** A steady incompressible flow, moving through a contraction section of length L , has a one-dimensional average velocity distribution given by $u \approx U_0(1 + 2x/L)$. What is its convective acceleration at the end of the contraction, $x = L$?
 (a) U_0^2/L , (b) $2U_0^2/L$, (c) $3U_0^2/L$, (d) $4U_0^2/L$, (e) $6U_0^2/L$

- (b) Suppose an ice skater of total mass m is skating along at a constant speed of V_0 when she suddenly stands stiff with her skates pointed directly forward, allowing herself to coast to a stop. Neglecting friction due to air resistance, how far will she travel before she comes to a stop? (Remember, she is coasting on *two* skate blades.) Give your answer for the total distance traveled, x , as a function of V_0 , m , L , h , μ , and W .
 (c) Find x for the case where $V_0 = 4.0$ m/s, $m = 100$ kg, $L = 30$ cm, $W = 5.0$ mm, and $h = 0.10$ mm. Do you think our assumption of negligible air resistance is a good one?

C1.3 Two thin flat plates, tilted at an angle α , are placed in a tank of liquid of known surface tension γ and contact angle θ , as shown in Fig. C1.3. At the free surface of the liquid in the tank, the two plates are a distance L apart and have width b into the page. The liquid rises a distance h between the plates, as shown.

- (a) What is the total upward (z -directed) force, due to surface tension, acting on the liquid column between the plates?
 (b) If the liquid density is ρ , find an expression for surface tension γ in terms of the other variables.



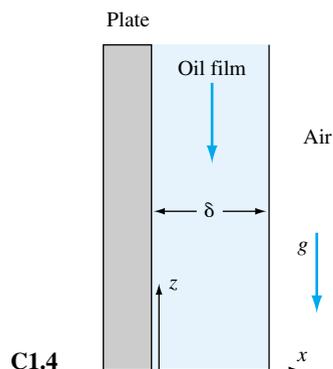
C1.3

C1.4 Oil of viscosity μ and density ρ drains steadily down the side of a tall, wide vertical plate, as shown in Fig. C1.4. In the region shown, *fully developed* conditions exist; that is, the velocity profile shape and the film thickness δ are independent of distance z along the plate. The vertical velocity w becomes a function only of x , and the shear resistance from the atmosphere is negligible.

(a) Sketch the approximate shape of the velocity profile $w(x)$, considering the boundary conditions at the wall and at the film surface.

(b) Suppose film thickness δ , and the slope of the velocity profile at the wall, $(dw/dx)_{\text{wall}}$, are measured by a laser Doppler anemometer (to be discussed in Chap. 6). Find an expression for the viscosity of the oil as a function of ρ , δ , $(dw/dx)_{\text{wall}}$, and the gravitational acceleration g . Note

that, for the coordinate system given, both w and $(dw/dx)_{\text{wall}}$ are negative.



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