AAIT

School of Civil and Environmental Engineering

Engineering Economics (CEng 5211)

Chapter 3:Economic Evaluation

Annual Worth Analysis

- To compare alternatives based on their equivalent annual cash flow (depending on the situation Equivalent uniform annual cost, Equivalent uniform annual benefit, or their difference i.e., Equivalent uniform annual worth).
- The AE is calculated by converting the **cash receipts and disbursements into a uniform series** of annual cash flows occurring over the study period.
  - If the AE is **positive**, the alternative produces a return **greater than** the MARR.
  - If the AE is **zero**, the alternative produces a return **equal** to the MARR.
  - If the AE is **negative**, the alternative produces a return **less than** the MARR and, if possible, the investment should be rejected.
- Then for mutual exclusive alternatives, the one with **higher annual income** or **lower annual cost** will be **opted**.

Annual Worth Analysis

- To compare alternatives based on their equivalent annual cash flow (depending on the situation Equivalent uniform annual cost, Equivalent uniform annual benefit, or their difference i.e., Equivalent uniform annual worth)

Example: A firm is considering which of two devices to install to **reduce costs**. Both devices have useful lives of 5 years with no salvage value. **Device A** costs $1000 and can be expected to result in $ 300 saving annually. **Device B** costs $1350 and will provide cost saving of $300 the first year ; however, saving will increase $50 annually, making the second year saving $350, the third year savings $400, and so forth. With interest at 7%, which device should the firm purchase?

Device A

AW_A= -1000(A/P, 7%, 5)+300 = -1000(0.2439)+300= $ 56.11

Device B

AW_B= -1350 (A/P, 7%, 5)+300 + 50(A/G, 7%, 5)

=-1350(0.2439) +300+ 50(1.865)=$ 64

Installing Device B results larger benefit.

$$(A/P, 7\%, 5) = \frac{0.07(1.07)^5}{(1.07)^5 - 1} = 0.2439$$

$$(A/G, 7\%, 5) = \frac{(1.07)^5 - (1 + 5 * 0.07)}{0.07[(1.07)^5 - 1]} = 1.865$$

Annual Worth Analysis

Example: Three alternatives are being considered for improving an operation on the assembly line along with the "do-noting" alternative. Equipment costs vary, as do the annual benefit of each in comparison to the present situation. Each Plan has a 10-year life and a salvage value equal to 10% of its original cost. For interest of 8% which plan should be adopted?

	Plan A	**Plan B**	**Plan C**
Installed cost of equipment	15,000	25,000	33,000
Material & labor saving per yr.	14,000	9,000	14,000
Annual operating expenses	8,000	6,000	6,000
End-of –useful life **saving value**	1,500	2,500	3,300

	Plan A	**Plan B**	**Plan C**
Equivalent uniform annual benefit (EUAB)			
Material & labor per yr.	14,000	9,000	14,000
Salvage value (A/F, 8%, 10)=0.069"'	104	172	228
EUAB=	14,104	9,172	14,228
Equivalent uniform annual cost (EUAC)			
Installed cost (A/P, 8%, 10)	2,235	3,725	4,917
Annual operating cost	8,000	6,000	6,000
EUAC=	-10,223	-9,725	-10,917
EUAW= EUAB-EUAC =	**3,869**	**-553**	**3,311**

Annual Worth Analysis

Example: Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for $65,000. At the end of the seventh year the salvage value of the new dump truck is estimated to be $15,000. The second alternative is to purchase a used dump truck for $50,000. At the end of the fourth year the salvage value of the used dump truck is estimated to be $5,000. The annual profits, revenues less operation costs, are $17,000 per year for either truck. Using a MARR of 18% calculate the annual worth for each of the dump trucks. Which truck should your company purchase?

Alternative 1[New]

The useful life of the new truck is seven years, which is used as the study period for the new truck.

- The purchase price converted to a uniform series of annual:

$$A_{PP} = - \$65,000\ [0.18(1+0.18)^7] / [(1\ 0.18)^7 - 1] = \mathbf{- \$17,054}$$

- The salvage value converted to a uniform series of annual cash flows:

$$A_{SV} = \$15,000(0.18) / [\ (1+0.18)^7 - 1] = \mathbf{\$1,235}$$

- The annual profits for the new truck are already a uniform series.

$$AE\ [New] = - \$17,054 + \$1,235 + \$17,000 = \mathbf{\$1,181}$$

Annual Worth Analysis

Example: Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for $65,000. At the end of the seventh year the salvage value of the new dump truck is estimated to be $15,000. The second alternative is to purchase a used dump truck for $50,000. At the end of the fourth year the salvage value of the used dump truck is estimated to be $5,000. The annual profits, revenues less operation costs, are $17,000 per year for either truck. Using a MARR of 18% calculate the annual worth for each of the dump trucks. Which truck should your company purchase?

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A_{PP} = - $65,000 $[0.18(1+0.18)^7] / [(1\ 0.18)^7 - 1]$
= **- $17,054**

A_{SV} = $15,000(0.18) / $[\ (1+0.18)^7 - 1]$

= **$1,235**

AE [New] = - $17,054 + $1,235 + $17,000

= **$1,181**

Alternative 2 [Used]

APP = $50,000$[0.18(1+0.18)^4] / [(1+0.18)^4\ 1]$

= **- $18,587**

ASV = $5,000(0.18) / $[(1+0.18)^4-1]$= **$959**

- The annual equivalent for purchasing:

AE[Used] = - $18,587 + $959 + $17,000

= **- $628**

- AE[New]>AE[Used]: purchase the new truck.

Contents

Economic Evaluation

- Present worth analysis
- Future worth analysis
- Payback period
- **Internal rate of return**

3.3 Internal Rate of Return

Definition: Sometimes represented by the symbol ***i**** or the acronym IRR.

- "The rate of return on an investment is the amount of profit it makes, often shown as a percentage of the original investment."
- "The interest rate charged on the unrecovered project balance of the investment such that the payment schedule makes the unrecovered project balance equal to zero at the end of the investment's life."
- The internal rate of return (IRR) is used in capital budgeting to estimate the profitability of potential investments.
- The internal rate of return is a discount rate that makes the net present value (NPV) of all cash flows from a particular project equal to zero.
- "It is the break even interest rate which equates the present worth of a project's cash outflows to the present worth of cash inflows."

NPW=PW of benefits- PW of costs=0

- Calculating ROR of an investment convert the various consequence of investment into CF and solve for the unknown value of IRR.

3.3 Internal Rate of Return

- The rate of return (ROR) for a series of cash flows is that particular value, i*, of the interest rate for which the NPV vanishes.
  - Plot the NPV as a function of i, the curve will cross the i-axis at i*.
  - Trial and error, i-values for which the NPV is slightly positive and slightly negative, and interpolate linearly between them for i*.
  - Use Newton-Raphson iteration method or another numerical technique NPV for i, with the left side replaced by zero. [The value of i that makes the NPV equation zero.]

Steps for Computing IRR

1. Assume a trial rate of return (i*).
2. Counting the cost as negative and income as positive, find the equivalent net worth of all costs and incomes.
3. If the equivalent net worth is **positive** then the income from the investment is worth more than the cost of investment and the actual percentage return is **higher** than the trial rate, and vice versa
4. Adjust the estimate of the trial rate of return and go to step 2 again until one value of i is found that results in a positive equivalent net worth and another higher value of i is found with negative equivalent net worth.
5. Solve for the applicable value of i* by interpolation

3.3 Internal Rate of Return

Example: Determine the IRR for the given cash flow.

Year	0	1	2	3	4
Cash Flow	-1000	400	370	240	220

NPW=PW$_{benefits}$ – PW$_{cost}$= 0

NPW = -1000 + 400 (P/F, i*,1) + 370 (P/F, i*,2) + 240 (P/F, i*,3) + 220 (P/F, i*,4)=0

Assuming i* = 10%

NPW = -1000 + 400(0.9090) + 370 (0.8264) + 240(0.7513)+ 220 (0.6830) = 0

- We find that i*= 10% which is a special interest rate that has vanished the net present worth of the given cash flow to zero. Thus i*=10% is the IRR.
- This indicates that the company earns (charges) a 10% rate of return on the funds that remains *internally* invested in the project. Since it is a return internal to the project, we refer to it as the internal rate of return, or IRR.

3.3 Internal Rate of Return

→ Case of a Single Sign-Reversal

When CFo < 0 and CFj > 0 (j > 0), i.e., when there is just one reversal of sign in the sequence CFo, CF1, CF2, . . . , CF, the NPV is a monotone decreasing function of i, and so i* is uniquely determined. Moreover, at this unique ROR, the FW and EUAS are zero.

Example: The cash flows associated with a milling machine are CF_o= -$50,000. CF_j = $15,000 (j = 1,. . . ,5). Determine the economic acceptability of this machine at interest rates of (a) 10%, (b) 15%, and (c) 20% per year (all compounded annually).

(a) NPV= -50,000+15,000(P/A,10%,5)= -$50,000+$15,000(3.7908)= <u>$6,861.80</u>

(Economically acceptable investment)

(b) NPV= -50,000+15,000(P/A,15%,5)= -$50,000+$15,000(3.3522)= <u>$282.33</u>

(Barely acceptable)

(c) NPV= -50,000+15,000(P/A,20%,5)= -$50,000+$15,000(2.9906)= <u>-$5,140.82</u>

(Not economically justifiable)

- By linear interpolation between the results of (b) and (c):

i (%)	NPV ($)
15	282.33
*i**	0
20	-5,140.82

$$i^* = 15\% + \left[\frac{0 - 282.33}{-5140.82 - 282.33} \right] * [20\% - 15\%]$$

$$i^* = 15.26\%$$

3.3 Internal Rate of Return

→ Case of a Multiple Sign-Reversals

When the sequence CFo, CF,, CF2,. . . , CF, shows more than one reversal of sign, it is possible that NPV = 0 for several values of the interest rate; there could thus be several rates of return.

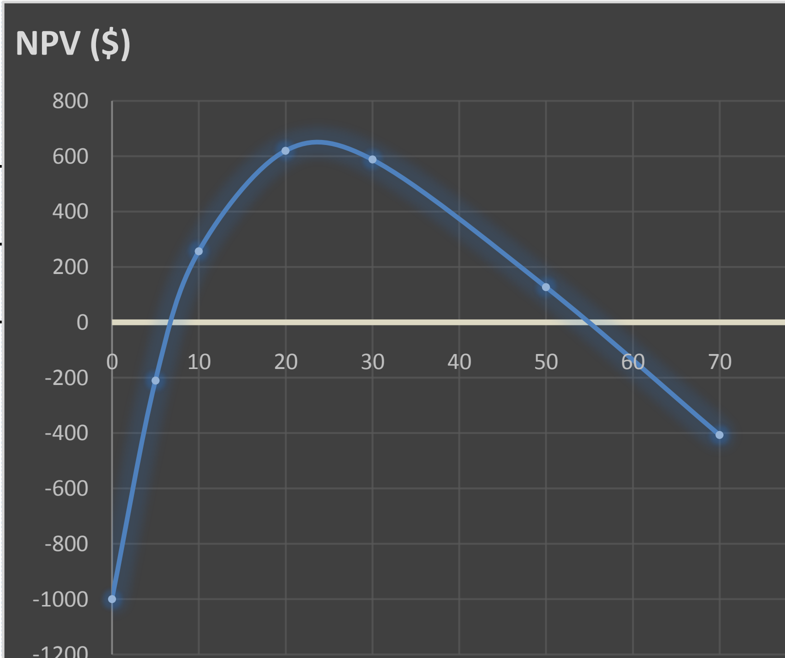
Example: For a project with the given series of cash flows, determine the NPV at annual interest rates 0%, 5%, 10%, 20%, 30%, 50%, and 70%.

E.O.Y	0	1	2	3	4	5
CF, 1000	-3	0	6	6	0	-10

For the given flows, solve:**NPV= -$3000 + $6000(P/A, i % 2)(P/F, i%, 1) - $10 000(P/F, i %, 5)**

- Evaluation at the specified interest rates gives the points which are plotted.

i(%)	0	5	10	20	30	50	70
NPV ($)	-1000	-210	257	620	588	128	-407



From a graph of the results, find the rate(s) of return.

i=7% and i=54%

3.3 Internal Rate of Return

→ **Case of a Multiple Sign-Reversals**

When the sequence CFo, CF1, CF2,. . . , CF, shows more than one reversal of sign, it is possible that NPV = 0 for several values of the interest rate; there could thus be several rates of return.

- Two sign reversals in the CFs and two values of i*. In this case, the upper bound is actually attained.
- If multiple i* values exist, it is usually better to abandon the ROR method and instead to investigate NPV.
- If we use the PW method with MARR=15%, we would obtain:

E.O.Y	0	1	2	3	4	
CF	-3000	0	6000	6000	0	-10, 000
(P/F , I , n)	1	0.8696	0.7561	0.6575	0.5718	0.4972
PW	-3000	0	4,536.86	3,945.10	0	-4,971.77
NPW	510.19					

- PW(15%)=510.19 > 0 This verifies the project is acceptable.

3.3 Internal Rate of Return

Flaws in Project ranking by IRR

- Under NPW, NFW or AE analysis: the highest worth figure was preferred. "Total investment approach".
- Unfortunately, the analogy does not carry over to IRR analysis. The project with the highest IRR may not be the preferred alternative.
- **Example:** Suppose you have two mutually exclusive alternatives, each with a 1year service life. One requires an investment of $1000 with a return of $2000 and the other requires $5000 with a return of $7000.
- Already solved IRR and NPW(MARR=10%) as show on table below.

n	A1	A2
0	-1000	-5000
1	2000	7000
IRR	100%	40%
PW(10%)	818	1,364

Assuming you have enough money in your investment pool to select either one, which one would you choose?

- Based on NPW measures A2 is preferred. But based on IRR A1 is preferred
- We notice inconsistency of ranking:

- NPW, NFW and AE are **absolute (dollar)** measures of investment worth.
- IRR is a **relative(percentage)** measure and cannot be used in the same way.(i.e., IRR ignores the scale of the investment)

3.3 Internal Rate of Return

Incremental Investment Analysis

A1

- Withdrawal of 1000. Remaining 4000 continues to earn 10%. After a year you will receive 2000 from external investment and 4,400 form the investment pool.
- With an investment of 5000 in 1 year you will have 6400(4400+2000).

PW(10%)

= -5000+6400(P/F,10%,1)=818

A2

- Withdrawal of 5000 leaving no money in the investment pool. After a year you will receive 7000 from external investment.
- Your total wealth changes from 5000 to 7000 in a year.

PW(10%)

=-5000+7000(P/F,10%,1)=1,364

n	A1	A2	A2-A1
0	-1000	-5000	-4000
1	2000	7000	5000
IRR	100%	40%	25%
PW(10%)	818	1,364	545

- If you decide to take **more costly option**, you would want to know that this additional investment can be **justified at the MARR.**
- The 10%MARR implies that you can always earn that return from other investment;

- For A2 by investing the additional $4000 you would make an additional $5000, which is equivalent to earning at a rate of 25%.

3.3 Internal Rate of Return

Incremental Investment Analysis

- For a pair of mutually exclusive project A and B, with B defined as a more costly option, we may rewrite B as:

  B=A+(B-A)

- B has two cash flows: (1) same CF as A and (2) the incremental component (B-A).
- Therefore, the only situation in which B is preferred over A is when the ROR in the incremental component (B-A) exceeds the MARR.
- For two mutually exclusive project, ROR analysis is done by computing the internal rate of return on incremental investment (IRR∆) between the projects.
- Since we want to consider increments of investment, we compute the cash flow for the difference between the projects by subtracting the cash flow for the lower investment project(A) from that of the higher investment project (B).

Then:

- $IRR_{B-A} > MARR$, select B
- $IRR_{B-A} = MARR$, select either one
- $IRR_{B-A} < MARR$, select A

3.3 Internal Rate of Return

- **Exercise 1**: An individual wants to start a small-scale painting business. To economize the start up business, he decides to purchase some used painting equipment. He has two mutually exclusive options, which he expects to fold up the business in three years,
  i. Do most of the painting by himself by limiting his business to only residential painting jobs (B1) or
  ii. Purchase more painting equipment and hire some helpers to do both residential and commercial painting jobs that he expects will have a higher equipment cost, but provide higher revenue as well (B2).

Given the cash flow for the mutually exclusive alternatives presented below which project would he select at MARR=10%

n	B1	B2
0	-$3,000	-$12,000
1	1,350	4,200
2	1,800	6,225
3	1,500	6.330

- **Exercise 2**: Consider the following three sets of mutually exclusive alternatives.

n	D1	D2	D3
0	$2,000	$1,000	$3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000

- Which project would you select based on rate of return on incremental investment , assuming that MARR=15%?

3.3 Internal Rate of Return

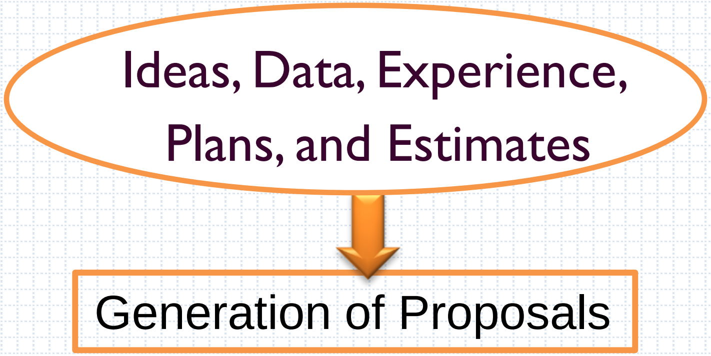
- For PW, FW, AW, and benefit/cost ratio:
  - The discount rate must be specified "up front"
  - It is used in calculating equivalence relations
- For rate of return:
  - Find the internal rate of return for the project
  - Multiple rates of return can cause problems
  - Compare to minimum acceptable rate of return
  - The minimum acceptable rate of return is used after the internal rate of return is computed.

Reading Assignment

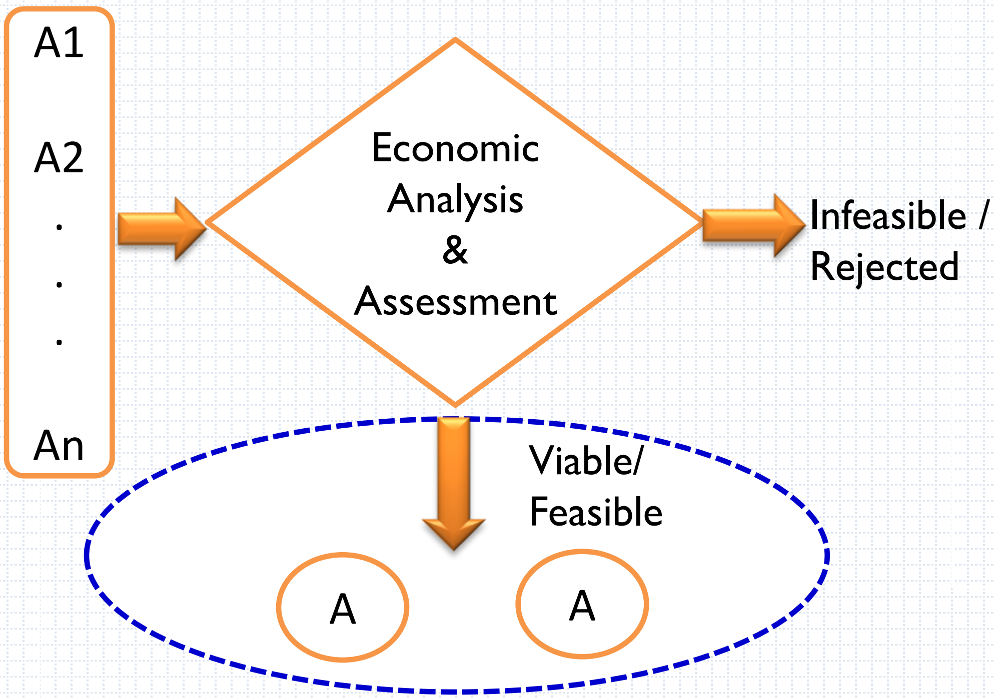
- Internal Rate of Return
- External Rate of Return

Summary

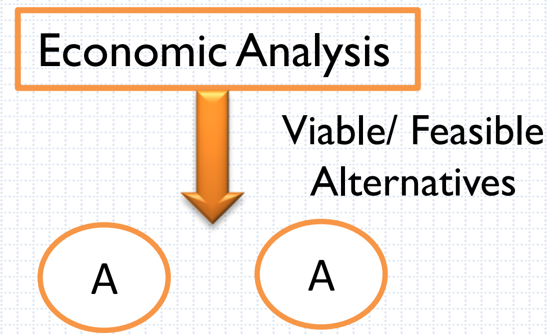
- Creation of Alternatives.



- Proposal Assessment



- Feasible Alternatives



Single Project

Mutually Exclusive Alternatives

Independent Alternatives

3.3 Internal Rate of Return

- **Single Project:** "The Unconstrained Project Selection Problem". No comparison with competing or alternative projects.
- Decision against SP: Acceptance or Rejection(Do Nothing) is based upon comparison with the firm's opportunity cost (alternative use of the firm's funds that could be invested in the project).
- There will always the option to invest the owner's funds in the BEST alternative use:
  - Invest at the company's MARR;
  - Opportunities for external investments with similar risks as the proposed project.
- Simple: evaluate the single project's worth at the firm's MARR;
- **Mutually <u>Exclusive</u> Alternatives**: compete with each other. Only one of the feasible (viable) projects can be selected from the set.
- **Independent Alternative**: May or may not compete with each other depending upon the conditions and constraints that define the set (budget limitations). More than one can be selected.

Thank You