

## Chapter One

### Basic concepts of Engineering Economics

#### I. Introduction

##### 1.1. Background

Engineering economy evaluates the monetary consequences of the products, projects, and processes that engineers design. These products, projects, and processes usually require spending money now, and they have long lives. Often, two designs are compared, and the question is: “Which is cheaper in the long run?”

One alternative may be cheaper to build and the other cheaper to operate. For example, a building without insulation is cheaper to build, but a building with thick insulation is cheaper to heat and to air-condition. Engineering design must answer the question, “How much insulation is the best?” Combining current building costs with future operating costs requires the tools of engineering economy, because having \$1 today is more valuable than having \$1 a year or 10 years in the future. The principle that \$1 today is more valuable than \$1 a year in the future is called the **time value of money**.

The time value of money can be explained through the earning power of money. Having the \$1 today allows interest or income to be earned over the next year or 10 years. Having money sooner allows more interest or income to be earned; thus, sooner is more valuable. The following definition of engineering economy will be expanded in later chapters. The real world of engineering economy includes uncertainty, risk, and performance tradeoffs whose value cannot be measured in dollars.

The applications of engineering economy are many and varied. Examples include the choice between a concrete and a steel structure, between various insulation thicknesses, between possible loans for a car or a robot, and between prices at which to sell a duplex, a firm, or a product. Engineering economy can be applied by an engineer to size a pump or to buy a home. It can be applied by a design firm to analyze the purchase of engineering software. (Will the value of time saved or new capabilities achieved exceed the cost of adding the software?) It can be applied by a major corporation to analyze plans for a new manufacturing facility or a new

research and development (R&D) thrust. In each case, engineering economy balances expenses and revenues that occur now and in the future.

The importance of money or engineering economy to engineering has long been recognized: Engineering is the art of doing well with one dollar what any bungler can do with two after a fashion.

### **1.2. Basic Concepts of Engineering Economics**

Economics deal with a central problem faced by all individuals and all societies: the problem of Scarcity.

The problem of scarcity means that every time we take an economic decision (for example, how much to consume or for a firm how much to produce of a given good) we face some constraints that affect our decision. This is always true for any possible decision. We cannot simply get everything we want (because of scarcity), we have to make choices. *Economics is the study of choices (economic choices) in the face of scarcity of resources.*

Scarcity arises because resources that are used to produce and consume goods are limited by physical space. For example, to produce goods and services we need to use productive resources like Labour, Land and Raw Materials, Capital (machines, factories, equipment, etc. etc.). The amount of Labour is limited both in number and in skills. The world's land area is limited and so are raw materials (think at petrol). The stock of capital is limited since we have a limited amount of factories, machines, transportation and other equipment. Labour, Raw Materials, Land and Capital are what we call Factors of Production (or productive Inputs).

Furthermore, scarcity arises from other "resources", like time or income (we are constrained in our economic choices by how much income we have, a firm may not be able to start a new factory if it not able to get a bank loan, etc. etc.). Given the limited amount of resources we can only produce and consume a limited amount of goods and services. Why is scarcity a problem?

If we know that we have limited resources we could just behave accordingly. The problem arises because human wants and needs are virtually unlimited but resources available to satisfy them are not.

Therefore economists tend to define scarcity in the following way:

**Definition of Scarcity:** the excess of human needs over what can actually be produced.

Scarcity implies that we cannot choose whatever we want (I cannot buy today a BMW that costs £50000 if my total income today is £10000, etc. etc.) or when we decide about what and how much to produce (I cannot produce a good that requires 1000 workers if only 100 are available, etc. etc.).

We know that scarcity implies that we face some sort of constraints every time we take an economic decision. The presence of those constraints has the main implication of creating **trade-offs** among different alternatives. The concept of **trade-off** is one of the core principles in economics. In economics a trade-off implies that having more of one thing usually it implies having less of another.

**Definition of Economics:** economics is the study of choices under conditions of scarcity, or the study of choice with constraints.

Specifically, we can say that economics is the study of how individuals and societies choose to employ scarce resources that could have alternative uses to produce goods and services, and distribute them among various individuals and groups in societies.

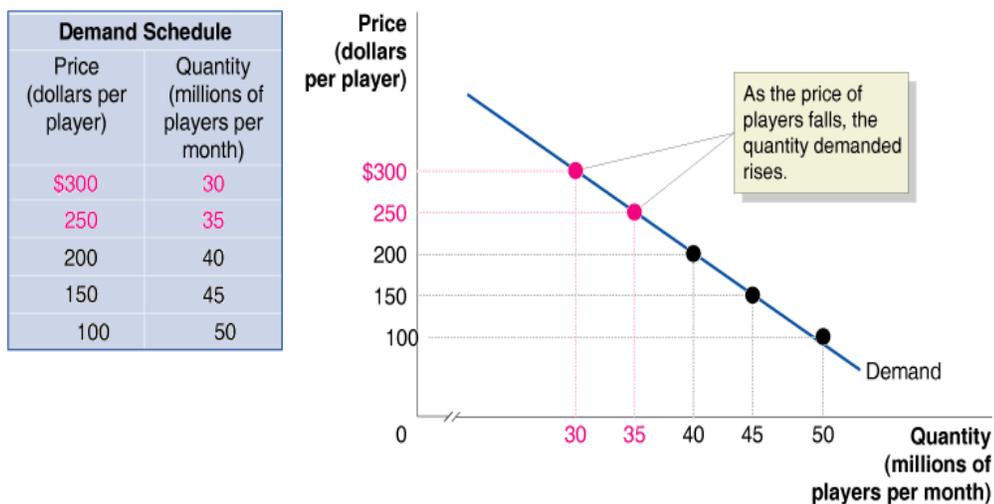
Therefore, economics deals with theories of choice.

### **Economics Basics: Demand and Supply**

Supply and demand is perhaps one of the most fundamental concepts of economics and it is the backbone of a market economy. **Demand** refers to how much (quantity) of a product or service is desired by buyers. The quantity demanded is the amount of a product people are willing to buy at a certain price; the relationship between price and quantity demanded is known as the demand relationship. **Supply** represents how much the market can offer. The quantity supplied refers to the amount of a certain good producers are willing to supply when receiving a certain price. The correlation between price and how much of a good or service is supplied to the market is known as the supply relationship. Price, therefore, is a reflection of supply and demand.

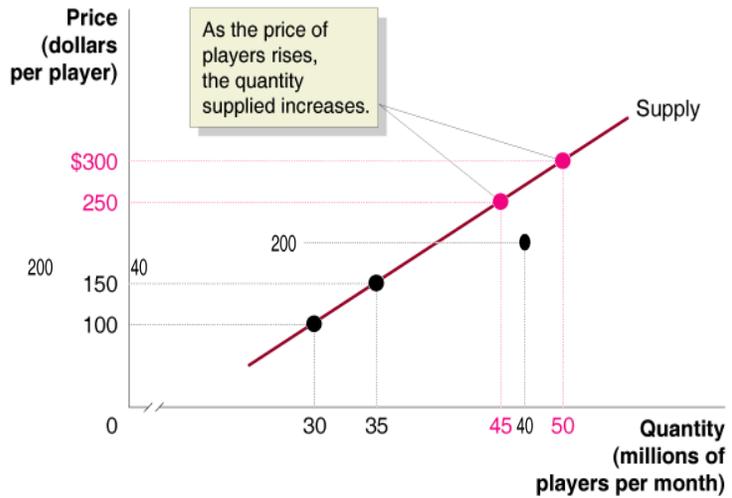
The relationship between demand and supply underlie the forces behind the allocation of resources. In *market economy* theories, demand and supply theory will allocate resources in the most efficient way possible. How? Let us take a closer look at the law of demand and the law of supply.

- **Law of Demand:** The law of demand states that, if all other factors remain equal, the higher the price of a good, the less people will demand that good. In other words, the higher the price, the lower the quantity demanded. The amount of a good that buyers purchase at a higher price is less because as the price of a good goes up, so does the opportunity cost of buying that good. As a result, people will naturally avoid buying a product that will force them to forgo the consumption of something else they value more. The chart below shows that the curve is a downward slope.

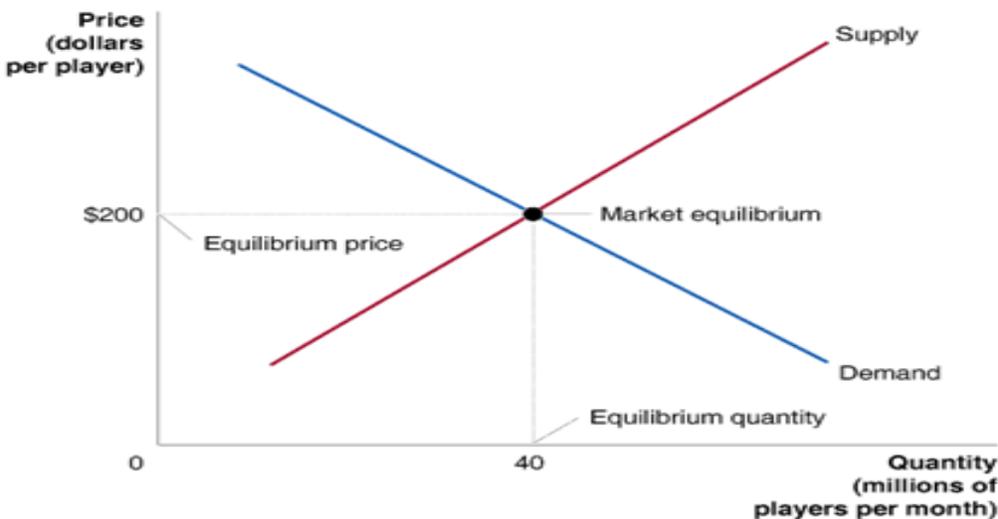


- **Law of supply:** like the law of demand, the law of supply demonstrates the quantities that will be sold at a certain price. But unlike the law of demand, the supply relationship shows an upward slope. This means that the higher the price, the higher the quantity supplied. Producers supply more at a higher price because selling a higher quantity at higher price increases revenue.

Supply Schedule	
Price (dollars per player)	Quantity (millions of players per month)
\$300	50
250	45
150	35
100	30

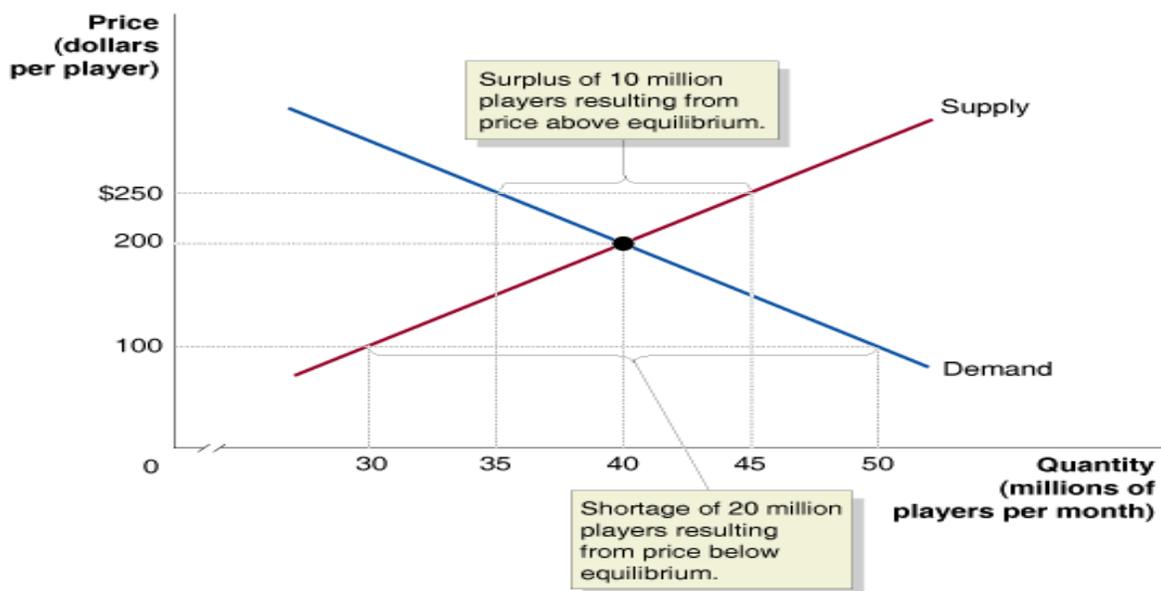


- Market equilibrium:** When supply and demand are equal (i.e. when the supply function and demand function intersect) the economy is said to be at equilibrium. At this point, the allocation of goods is at its most efficient because the amount of goods being supplied is exactly the same as the amount of goods being demanded. Thus, everyone (individuals, firms, or countries) is satisfied with the current economic condition. At the given price, suppliers are selling all the goods that they have produced and consumers are getting all the goods that they are demanding.



As you can see on the chart, equilibrium occurs at the intersection of the demand and supply curve, which indicates no allocative inefficiency. These figures are referred to as equilibrium price and quantity. In the real market place equilibrium can only ever be reached in theory, so the prices of goods and services are constantly changing in relation to fluctuations in demand and supply.

- **Surplus:** A situation in which the quantity supplied is greater than the quantity demanded.
- **Shortage:** A situation in which the quantity demanded is greater than the quantity supplied.



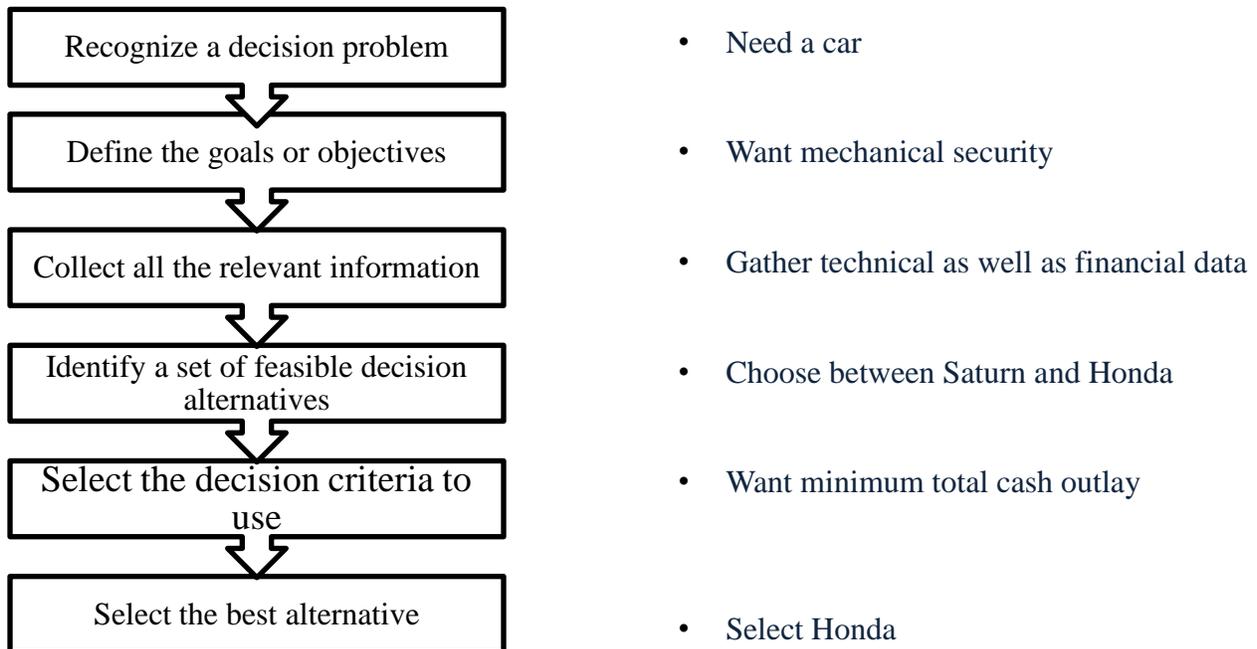
### 1.3. The decision-making process

The real world contains messes, not problems. Achaotic conglomeration of conflicting objectives, undefined constraints, and incomplete or contradictory data is a mess, not a problem. A well-structured decision-making process is needed to turn a mess into a problem.

The idea of design and development is what most distinguishes engineering from science, the latter being concerned principally with understanding the world as it is. Decisions made during the engineering design phase of a product's development determine the majority of the costs of manufacturing that product. As design and manufacturing processes become more complex, the engineer increasingly will be called upon to make decisions that involve money.

The steps shown in fig 1.3 can be used to solve a mess. These steps are not unique to engineering economy. They also can and should be applied to engineering design and to managerial and personal decisions.

The linear flow chart in fig 1.3 is most useful to define the steps for decision making. Real-world decision-making processes usually are and should be nonlinear, with numerous feedback loops.



#### 1.4. Types of Strategic Engineering Economic Decisions

This section will provide many real examples to illustrate each class of engineering economic decisions. At this point, our intention is not to provide the solution to each example, but rather to describe the nature of decision problems that a typical engineer might face in the real world. Strategic engineering economics decisions mainly focus on;

**Service or Quality Improvement:** Investments in this category include any activities to support the improvement of productivity. Quality, and customer satisfaction in the service sector, such as in the financial, healthcare, and retail industries.

**New Products or Product Expansion:** Investments in this category are those that increase the revenues of a company if output is increased. There are two common types of expansion decision

problems. The first type includes decisions about expenditures to increase the output of existing production or distribution facilities. In these situations, we are basically asking, "Shall we build or otherwise acquire a new facility?" The expected future cash inflows in this investment category are the revenues from the goods and services produced in the new facility. The second type of decision problem includes the consideration of expenditures necessary to produce a new product or to expand into a new geographic area. These projects normally require large sums of money over long periods.

**Equipment and Process Selection:** This class of engineering decision problem involves selecting the best course of action when there are several ways to meet a project's requirements. Which of several proposed items of equipment shall we purchase for a given purpose? The choice often hinges on which item is expected to generate the largest savings (or return on the investment). The choice of material will dictate the manufacturing process involved. Many factors will affect the ultimate choice of the material and engineers should consider all major cost elements, such as machinery and equipment, tooling, labor, and material. Other factors may include press and assembly, production and engineered scrap, the number of dies and tools, and the cycle times for various processes.

**Cost Reduction:** A cost-reduction project is a project that attempts to lower a firm's operating costs. Typically, we need to consider whether a company should buy equipment to perform an operation now done manually or spend money now in order to save more money later. The expected future cash inflows on this investment are savings resulting from lower operating costs.

**Equipment Replacement:** This category of investment decisions involves considering the expenditure necessary to replace worn-out or obsolete equipment. For example, a company may purchase 10 large presses with the expectation that they will produce stamped metal parts for 10 years. After five years, however, it may become necessary to produce the parts in plastic, which would require retiring the presses early and purchasing plastic-molding machines. Similarly, a company may find that, for competitive reasons, larger and more accurate parts are required, which will make the purchased machines obsolete earlier than expected.

### 1.5. Fundamental Principles of Engineering Economics

This course is focused on the principles and procedures for making sound engineering economic decisions. To the first-time student of engineering economics, anything related to money matters may seem quite strange compared with other engineering subjects. However, the decision logic involved in the problem solving is quite similar to any other engineering subject matter; there are basic fundamental principles to follow in any engineering economic decision. These principles unite to form the concepts and techniques presented in the text, thereby allowing us to focus on the logic underlying the practice of engineering economics.

The four principles of engineering economics are as follows:

**Principle 1: A nearby dollar is worth more than a distant dollar.** A fundamental concept in engineering economics is that money has a time value associated with it. Because we can earn interest on money received today, it is better to receive money earlier than later. This concept will be the basic foundation for all engineering project evaluation.

**Principle 2: All that counts is the differences among alternatives.** An economic decision should be based on the differences among alternatives considered. All that is common is irrelevant to the decision. Certainly, any economic decision is no better than the alternatives being considered. Therefore, an economic decision should be based on the objective of making the best use of limited resources. Whenever a choice is made, something is given up. The opportunity cost of a choice is the value of the best alternative given up.

**Principle 3: Marginal revenue must exceed marginal cost.** Any increased economic activity must be justified based on the following fundamental economic principle: marginal revenue must exceed marginal cost. Here, the marginal revenue is the additional revenue made possible by increasing the activity by one unit (or a small unit). Similarly, marginal cost is the additional cost incurred by the same increase in activity. Productive resources such as natural resources, human resources, and capital goods available to make goods and services are limited. Therefore, people cannot have all the goods and services they want; as a result, they must choose those things that produce the most.

**Principle 4: Additional risk is not taken without the expected additional return.** For delaying consumption, investors demand a minimum return that must be greater than the anticipated rate of inflation or any perceived risk. If they didn't receive enough to compensate for anticipated inflation and perceived investment risk, investors would purchase whatever goods they desired ahead of time or invest in assets that would provide a sufficient return to compensate for any loss from inflation or potential risk.

These four principles are as much statements of common sense as they are theoretical statements. They provide the logic behind what is to follow in this text. We build on them and attempt to draw out their implications for decision making. As we continue, try to keep in mind that while the topics being treated may change from chapter to chapter, the logic driving our treatment of them is constant and rooted in these four principles.

## **1.6. Understanding Financial Statements**

The decision to undertake any capital investment project cannot be made on the basis of the profitability of the project alone but must be considered in the context of its impact on the financial strength and position of the firm that is proposing to undertake the project. A firm that is currently in a strong financial position will be more willing and able to undertake projects than the same firm would be when it is weaker. The purpose of this chapter is to provide an overview of the major determinants of the financial position of a firm as reflected periodically in its financial statements.

The financial statements are contained in the annual report of a corporation, which includes the balance sheet, the income statement, the statement of changes in financial position, and the auditors' report. These statements are usually preceded in the annual report by a brief description of the corporation and its operations. The balance sheet summarizes the financial position of the corporation and lists the values of its assets and financial obligations or liabilities. The income statement itemizes revenues and expenses for the year and provides an overview of the operations for the year. The statement of changes in financial position lists the sources and application of funds. The auditors' report is an independent appraisal of the financial statements of the corporation by a team of professional accountants.

Financial statements are prepared on an annual basis because the year is a standard unit of time and because the year is a good period of time for evaluating the activities of a firm. It is long enough to provide a meaningful measure of the operations, but short enough so that statements do not become outdated. It includes all four seasons and thus eliminates the effect of trends that are purely seasonal. It normally includes several cycles of production and sale of goods (except for a handful of industries such as the tobacco industry where the growing period is longer than a year). Publicly held corporations are also required to prepare financial statements on a quarterly basis, and all corporations must prepare financial statements when they wish to raise capital by either borrowing money or selling stock.

What would one want to know about the company at the end of the fiscal year?

- <b>What is the company's financial position at the end of the reporting period?</b>	<b>Balance sheet statement</b>
- <b>How much profit was made during the reporting period?</b>	<b>Income statement</b>
- <b>How much cash was generated &amp; spent?</b>	<b>Statement of cash flow</b>
- <b>Where was decided to use the profit?</b>	<b>Statement of retained earning</b>

### 1.6.1. Balance Sheets

The balance sheet is a statement of the financial position of a firm. It balances the assets and the claims against those assets of a firm. The assets and claims are, by definition, equal. Claims against those assets include both liability and stockholders' equity. According to convention, the balance sheet has a standard form which lists assets on the left side and liabilities and stockholders' equity on the right side. Other formats are sometimes used but they are less common. Usually, a balance sheet presenting the information for the year just ended and the previous year is given in the annual report for the purpose of comparison.

Assets are listed on the balance sheet in descending order of liquidity. At the top of the list, under current assets, is cash, which represents the ultimate degree of liquidity because it can be used to acquire assets or pay off debts. The term cash refers to monies held on company premises and in banks. The next item after cash is usually marketable securities, which include such items as

stocks, bonds, and Treasury bills, which are held to earn interest but may be sold at any time to raise the funds needed for operations. Next is accounts receivable, which are amounts owed by customers on credit sales, but not yet paid. Since cash is soon collected on accounts receivable, they are considered a near-cash item.

Inventory refers to goods produced or purchased and held for sale or used in the production of services. It does not represent a definite commitment that will generate cash and income for the firm; nevertheless, it is held with the expectation that it will be sold within the year. Revenue is accrued when an order is placed by a customer for inventory, and the order is charged to accounts receivable. When the cash is collected from the accounts receivable, it is either used to purchase more inventory or set aside for other investment purposes. Thus, the operating cycle begins with the production or purchase of inventory, continues with the sale of inventory and the accumulation of accounts receivable, and is completed with the liquidation of these accounts for cash.

Other items commonly listed as current assets on the balance sheet may include prepaid expenses, such as rent and insurance. Prepaid expenses are payments made in advance to satisfy certain necessary expenses. Such payments are sometimes made several years in advance, but if the portion beyond the current year may be reclaimed, that portion is included as a current asset.

Noncurrent assets consist largely of fixed assets such as land, buildings, machinery, and equipment. Sometimes these items are included under the one heading, but usually one or more of these items are listed separately. Other noncurrent assets include the value of such intangible items as patents, trademarks, and goodwill. Noncurrent assets are used by the firm in the production of goods and services and are expected to generate revenues on a long-term basis.

Liabilities are the debts incurred to acquire assets and therefore are obligations for definite amounts of money. Like assets, liabilities are classified as current or noncurrent. Liabilities are listed in descending order of priority or urgency on the balance sheet.

At the top of the current liabilities are obligations that are due within a few days or weeks. Accounts payable are the debts that the firm owes to suppliers of goods and services, and are usually due within 30 days. Wages and salaries payable are amounts due to the employees of the

firm that have been earned during the pay period of 2 weeks or a month. Taxes payable are taxes that are due to federal, state, and local governments, with the taxes being due at the end of each quarter. Dividends payable are dividends that have been declared, but not yet paid. Short-term debts to banks and other lending institutions are, by definition, current liabilities.

The current portion of long-term debt refers to the principal portion of the long-term debt that is due in the upcoming year. Closely related to debt liability is interest payable which is accrued on a current basis. Long-term liabilities include notes payable and bonds payable.

The stockholders' equity is the portion of capital that belongs to the investors. There are several classes of stock which entitle their holders to varying degrees of rights regarding income and participation in the affairs of management. There are three major types of stock: preferred stock, common stock, and treasury stock. Preferred stock is a privileged class of stock that gives its holders the right to annual dividends at a stated rate and to receive such payments before other classes of shareholders if the management declares dividends. Preferred stock does not give its holders the right to receive dividends above this rate unless it is participating, and it is not normally accompanied by the right to vote for the members of the board of directors. Common stock is the general class of stock that entitles its holders to receive dividends and to vote for directors and on certain issues at the rate of one vote per share of stock. Treasury stock is stock repurchased by the firm and removed from the market.

**EXHIBIT A.2** Balance sheet for Engineered Buildings, Inc., December 31, 20xx (all amounts in \$1000)

Assets		Liabilities	
Current assets		Current liabilities	
Cash (money market)	850	Accounts payable	765
Accounts receivable	2380	Notes payable	850
(minus) Bad debt provision	-95	Accrued expense	425
Inventories	1105	Total current liabilities	2040
Total current assets	4240	Long-term debt	2380
Fixed assets		<b>Total Liabilities</b>	
Land	340		<b>4420</b>
Plant and equipment	2805	<b>Equity</b>	
(minus) Accumulated depr.	-1700	Common stock	1200
Total fixed assets	1445	Capital surplus	420
Other assets		Retained earnings	410
Prepays/deferred charges	510	Total equity	2030
Intangibles	255	<b>Total Liabilities and Equity</b>	
Total other assets	765		<b>6450</b>
<b>Total Assets</b>	<b>6450</b>		

Fig 1: Balance sheet statement

<b>PRUFROCK CORPORATION</b> 2008 and 2009 Balance Sheets (\$ in millions)	
<b>2008</b>	
<b>Assets</b>	
<b>Current assets</b>	
Cash	\$ 84
Accounts receivable	165
Inventory	393
<b>Total</b>	<b>\$ 642</b>
<b>Fixed assets</b>	
Net plant and equipment	\$2,731
<b>Total assets</b>	<b>\$3,373</b>
<b>Liabilities and Owners' Equity</b>	
<b>Current liabilities</b>	
Accounts payable	\$ 312
Notes payable	231
<b>Total</b>	<b>\$ 543</b>
<b>Long-term debt</b>	<b>\$ 531</b>
<b>Owners' equity</b>	
Common stock and paid-in surplus	\$ 500
Retained earnings	1,799
<b>Total</b>	<b>\$2,299</b>
<b>Total liabilities and owners' equity</b>	<b>\$3,373</b>

Fig 2: Balance sheet statement

### 1.6.2. Balance Sheets

The income statement lists the revenues and expenses during the year, together with the net income and retained earnings at the end of the year. It is sometimes referred to as the statement of income and reinvested earnings. The income statement usually includes the information for the year just ended and the previous year for the purpose of comparison.

At the top of the income statement are revenues from sales and other operations. Production and other property expenses generally include the cost of goods sold and cost of selling. The general and administrative expenses include the salaries of managers and the administrative personnel who are not directly engaged in production, and other items of expenditure not directly related to production. Although the depreciation allowance is a deduction and not a "charge" because it does not involve an outlay of cash, it is usually considered an operating expense. Hence, the amount of depreciation allowance is a part of the costs and expenses which are deducted from

revenues of the current year to arrive at net income. Other deductions include interest paid, but income taxes are usually listed separately.

Extraordinary items are financial gains and losses due to unusual and nonrecurring events such as mergers or natural disasters and are usually shown separately to emphasize the unusual nature of the transaction. After all income before taxes is computed, the applicable tax rate is used to compute the tax expense, which is deducted from before-tax income to arrive at after-tax income. The after-tax income is referred to as net income or net earnings.

Cash dividends declared for various classes of stock are listed next. They are deducted from the net income to obtain the retained earnings for the current year, which in turn are added to the retained earnings at the beginning of the year to arrive at the retained earnings at the end of the year.

At the bottom of the income statement is the required item of earnings per share. It is computed by first finding the difference of net income after taxes and preferred dividends, which is then divided by the number of shares of common stock or common stock equivalent. This is a figure of great interest to investors with holdings of stock because it is one measure of the value of their stock. The computation of equivalent shares of common stock is a very complicated one for some firms because of the number and types of outstanding convertible securities. It is sufficient to point out that there are two types of figures for earnings per share that are usually presented: primary earnings per share, which assumes that only certain types of securities would be converted into common stock, and a lower figure for fully diluted earnings per share, which assumes that all convertible securities will be converted.

The fully diluted figure is lower because of the larger number in the denominator representing more equivalent shares of common stock.

PRUFROCK CORPORATION 2009 Income Statement (\$ in millions)	
Sales	\$2,311
Cost of goods sold	1,344
Depreciation	276
Earnings before interest and taxes	\$ 691
Interest paid	141
Taxable income	\$ 550
Taxes (34%)	187
Net income	\$ 363
Dividends	\$121
Addition to retained earnings	242

**EXHIBIT A.3** Income statement for Engineered Buildings, Inc.  
December 31, 20xx (all amounts in \$1000)

<b>Operating Revenues</b>	
Sales	8075
(minus) Returns and allowances	-85
<b>Total Operating Revenues</b>	<b>7990</b>
<b>Operating Expenses</b>	
Cost of goods and services sold	4760
Gross profit	3230
Marketing, general, and administrative	2040
Depreciation	340
<b>Total operating expense</b>	<b>7140</b>
<b>Total operating income</b>	<b>850</b>
<b>Nonoperating Revenues and Expenses</b>	
Interest receipts	51
Interest payments	-258
<b>Total nonoperating income</b>	<b>-207</b>
<b>Net Income Before Taxes</b>	<b>643</b>
Income Taxes	-225
<b>Net Profit (Loss) for Year 20xx</b>	<b>418</b>

Fig 3: Income statement

### 1.6.3. Cash Flow Statement

Cash flow statement is a financial report that provides aggregate data regarding all cash inflows a company receives as well as all cash outflows during a given quarter. (cash inflows: ongoing operations and external investment sources; cash outflows: payment for business activities and investments during a given quarter). It includes cash flows from operations, investment, and financing.

Cash flows from operations start with net income and then reconcile all noncash items to cash items within business operations. Includes accounts payable, depreciation. Cash flows from investing activities include cash spent on property, plant and equipment. Cash flows from financing are the section that provides an overview of cash used in business financing.

PRUFROCK CORPORATION 2009 Statement of Cash Flows (\$ in millions)	
Cash, beginning of year	\$ 84
Operating activity	
Net income	\$363
Plus:	
Depreciation	276
Increase in accounts payable	32
Less:	
Increase in accounts receivable	– 23
Increase in inventory	– 29
Net cash from operating activity	\$619
Investment activity	
Fixed asset acquisitions	–\$425
Net cash from investment activity	–\$425
Financing activity	
Decrease in notes payable	–\$ 35
Decrease in long-term debt	– 74
Dividends paid	– 121
Increase in common stock	50
Net cash from financing activity	–\$180
Net increase in cash	\$ 14
Cash, end of year	\$ 98

Fig 4: Cash flow statement

## Chapter Two

### Cost of Money

#### 2.1. Interest

Interest is what you pay the bank for your car loan or your unpaid credit card balance. Interest is what the bank pays you for the money in your savings account. Interest is a rental fee for money. Interest is a fee paid or a fee earned for the use of money.

Engineering economy generalizes this definition. Interest is the return on capital. Capital is the invested money and resources. Whoever owns the capital should expect a return on it from whom ever uses it. For example, if a firm owns the capital and invests it in a project, then the project should return that capital plus interest as cost savings or added revenues.

When you borrow from the bank, you pay interest. When you loan money to the bank by depositing it, the bank pays you interest. When a firm invests in a project, that project should earn a return on the capital invested.

This interest is typically expressed as an annual interest rate. That rate equals the ratio of the interest amount and the capital amount. For example, if \$100 is borrowed for a year and \$5 in interest is paid at the year's end, then the interest rate is 5% ( $= \$5/\$100$ ). If the amount borrowed and the amounts of interest paid are doubled, then the interest rate is still 5%, since  $\$10/\$200 = 5\%$ .

When the bank owns the capital, a loan document will state the interest rate. When the owner of the capital buys shares in a firm, the interest is earned through dividends and increasing share values. The return to the shareholder is less certain, but it is still the earning power of money. It is still interest.

Similarly, when a firm or an individual invests money or resources in an engineering project, that project must earn a return on the invested capital. This return is earned by increasing future revenues and/or by decreasing future costs.

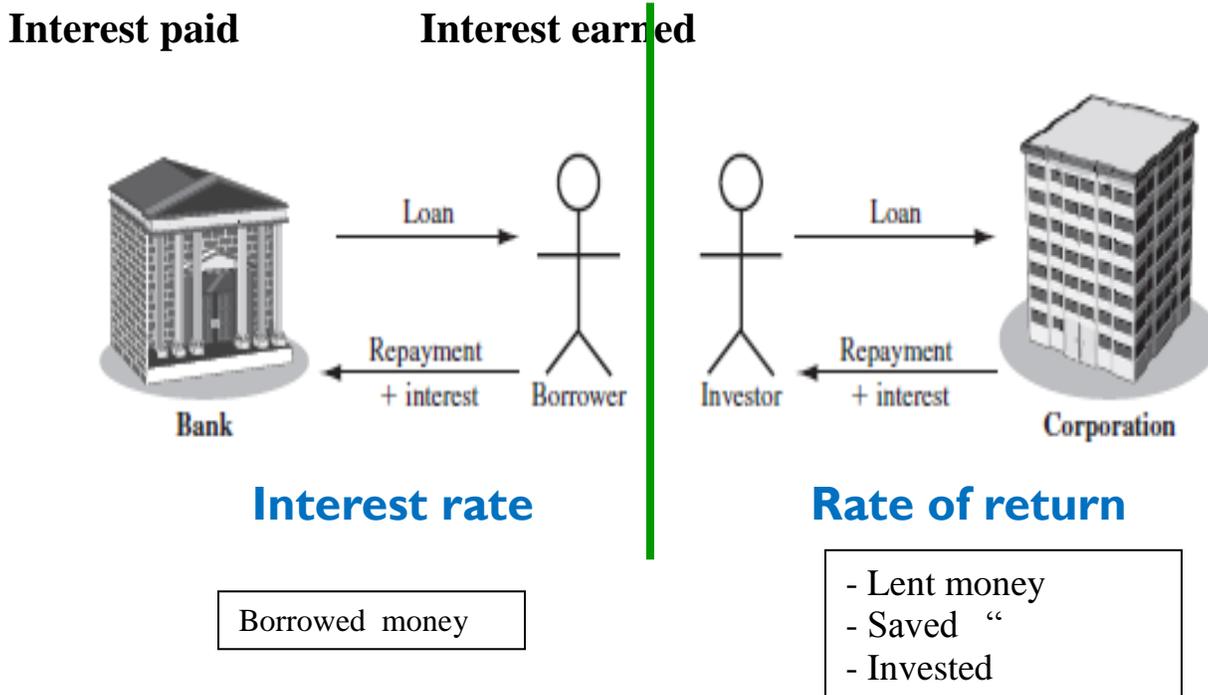
Interest is used to calculate the time value of money, and it is crucial to the practice of engineering. A history of engineering economy shows the parallel development of engineering and engineering economy.

Interest = Ending amount - Beginning amount = Amount owed now - Principal

**Interest rate** is a percentage added to an amount of money over a specified length of time.

$$\text{Interest rate (\%)} = \frac{\text{Interest added per time unit}}{\text{Principal}} * 100\%$$

The interest rate that is appropriate depends on many factors, including risks, security, economic conditions, regulations, and time frame.



**Definition of Terms**

P= Value or amount of money at a time designated as the present time t=0. Initial deposit, Investment made at t=0.

F= Value or amount of money at some future time.

A= Series of equal consecutive end of period amounts of money

n= Number of interest periods (year, month, day).

$i$  = interest period per time period (percent per year, percent per month).

$t$  = stated time period (years, months, days).

**Simple Interest:** With simple interest, the interest calculated for years 2, 3, . . . is based on the initial deposit. No interest is computed on the accrued interest. In Equation 2.1, the interest rate is simply multiplied by the number of years. Simple interest should not be used in engineering economy problems. Its principal use is in short-term loans, but compound interest is the norm even then.

$$F = P(1 + Ni) \quad (2.1)$$

A bank account, for example, may have its simple interest every year: in this case, an account with \$1000 initial principal and 20% interest per year would have a balance of \$1200 at the end of the first year, \$1400 at the end of the second year, and so on.

<b>t</b>	<b>Pi</b>	<b>End of Year</b>
<b>0</b>		<b>1000</b>
<b>1</b>	<b>200</b>	<b>1200</b>
<b>2</b>	<b>200</b>	<b>1400</b>

**Compound Interest:** is interest paid (earned) on any previous interest earned, as well as on the principal borrowed (lent). The standard assumption is that interest is computed on the current balance, which includes accrued interest that has not yet been paid. This addition of interest to the principal is called compounding. Savings accounts, loans, credit cards, and engineering economy rely on this assumption. In Equation 2.2, the number of years is a power of the factor that includes the interest rate.

$$F = P(1 + i)^N \quad (2.2)$$

This formula is developed as shown below,

Table 1: Development of equation 2.2

Period	Beginning of period value	End of period value
1	P	$P(1+i)$
2	$P(1+i)$	$P(1+i)(1+i)$
3	$P(1+i)^2$	$P(1+i)^2(1+i)$
...	...	...
N	$P(1+i)^{N-1}$	$P(1+i)^N$

A bank account, for example, may have its interest compounded every year: in this case, an account with \$1000 initial principal and 20% interest per year would have a balance of \$1200 at the end of the first year, \$1440 at the end of the second year and soon.

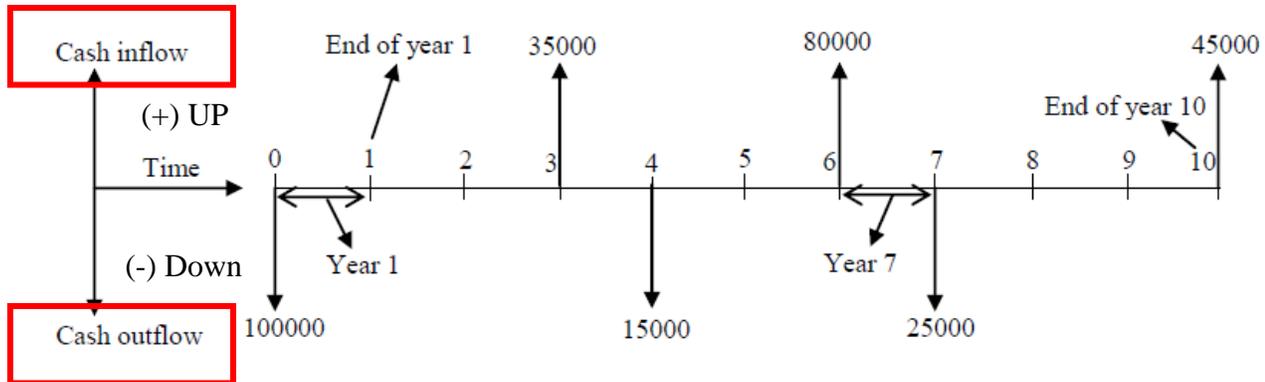
t	Beginning of year (1)	Pi (2)	End of Year =(1)+(2)
0			1,000
1	1,000	200	1,200
2	1,200	240	1,440

## 2.2. Cash Flow Diagrams

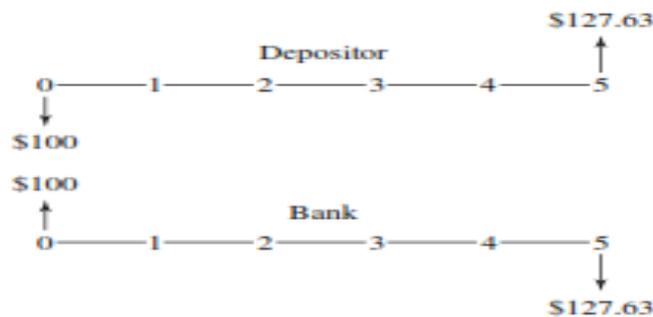
**Cash flow diagrams** show the timing and amount of expenses and revenues for engineering projects. It describes inflow and outflow of money over time.

The picture of how much money is spent or received and when is a cash flow diagram. This diagram summarizes an engineering project's economic aspects.

The cash flow diagram's horizontal axis represents time. The vertical axis represents money, and arrows show the timing and amount of receipts and expenses. Positive cash flows are receipts (those arrows point up). Negative cash flows are expenses (those arrows point down).



Below also shows that in some cases, the diagram can simply be reversed depending on which viewpoint is taken. The depositor and the bank have opposite perspectives on cash in and cash out for the initial deposit and the final withdrawal.



More complex cash flow diagrams involve more than two parties. For example, labor expenses are paid to employees, machine purchases to a manufacturer, and power purchases to a utility company, while revenues are received from customers. These complex diagrams are not mirror images, because each party sees a different set of cash flows.

**Categories of Cash Flows:** The expenses and receipts due to engineering projects usually fall into one of the following categories. Costs and expenses are drawn as cash outflows (negative arrows), and receipts or benefits are drawn as cash inflows (positive arrows).

- First cost  $\equiv$  expense to build or to buy and install
- Operations and maintenance (O&M)  $\equiv$  annual expense, which often includes electricity, labor, minor repairs, etc.
- Salvage value  $\equiv$  receipt at project termination for sale or transfer of the equipment (can be a salvage cost)

- Revenues  $\equiv$  annual receipts due to sale of products or services
- Overhaul  $\equiv$  major capital expenditure that occurs during the asset's life
- Prepaid expenses  $\equiv$  annual expenses, such as leases and insurance payments, that must be paid in advance

**Timing of Cash Flows:** For money's time value, there has to be an assumption for when cash flows occur. Cash flows could be assumed to occur at the beginning, middle, or end of the year, or they could be distributed throughout the year. In fact, tables of engineering economy factors have been constructed for all of these assumptions. This course and normal practice assume end-of-period timing for most cash flows.

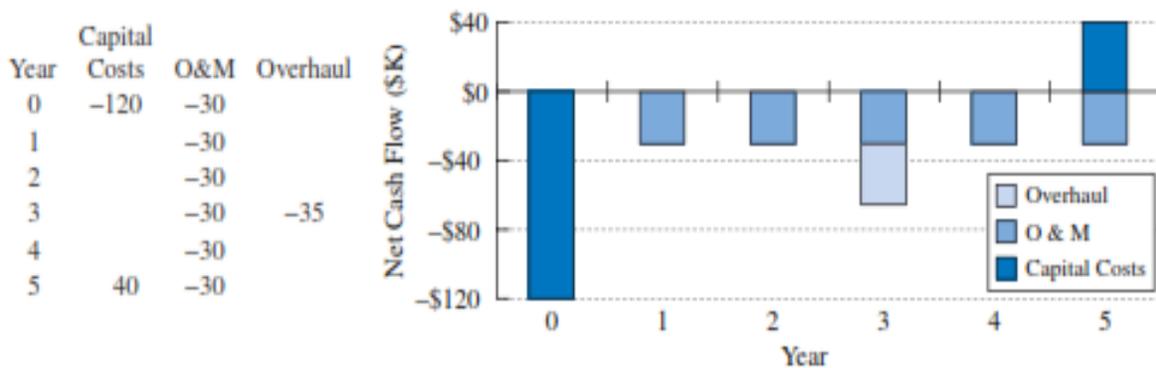
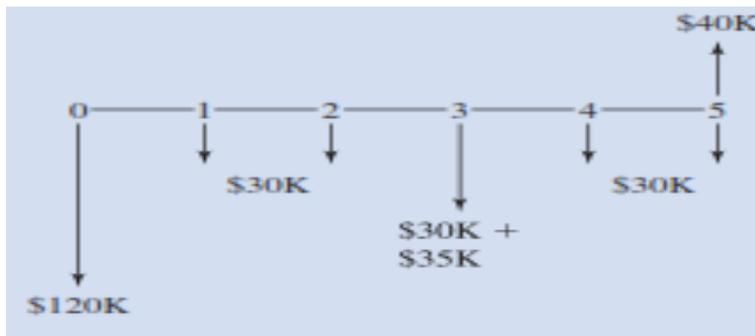
**Example:** An earth moving company is considering purchase of a piece of heavy equipment. The cash flow diagram for the following anticipated cash flows:

First cost = \$120,000

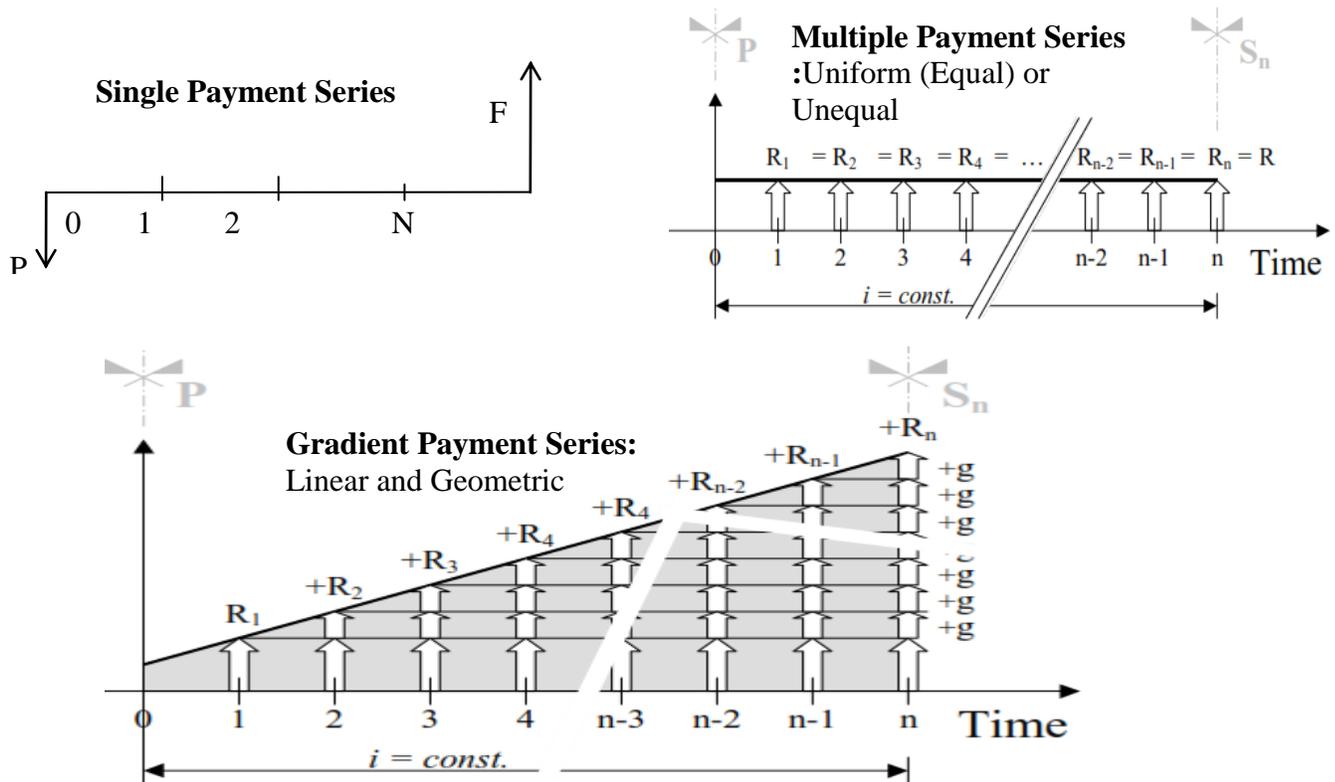
Operating & maintenance cost = \$30,000 per year

Overhaul cost = \$35,000 in year 3

Salvage value = \$40,000 after 5 years



Examples of cash flow diagram for different payment types are presented as follow;



### 2.3. Single Payment Series

The basic formula relating the two single-payment quantities, P and F, was used and developed in Section 2.1. Developed for compound interest and the relationship is shown in Equation 2.2,  $F = P(1 + i)^N$ . In each period, interest at rate i is earned so that end-of-period quantities are (1 + i) times the beginning-of-period quantities.

Period	Beginning of period value	End of period value
1	P	$P(1+i)$
2	$P(1+i)$	$P(1+i)(1+i)$
3	$P(1+i)^2$	$P(1+i)^2(1+i)$

...	...	...
N	$P(1+i)^{N-1}$	$P(1+i)^N$

The format of the engineering economy factors is  $(X/Y,i,N)$ . This is often read aloud as “find  $X$  given  $Y$  at  $i$  over  $N$  periods” or as “ $X$  given  $Y$ .” The  $X$  and  $Y$  are chosen from the cash flow symbols,  $P$ ,  $F$ ,  $A$ , and  $G$ . If  $Y$  is multiplied by the factor, then the equivalent value of  $X$  results (for  $N$  years at  $i$  per year). Thus, it may be useful to think of the multiplication as clearing fractions, where the  $Y$ s cancel and  $X$  is left. So, to convert from a cash flow in year 10 (an  $F$ ) to an equivalent present value (a  $P$ ), the factor is  $(P/F,i,10)$ .

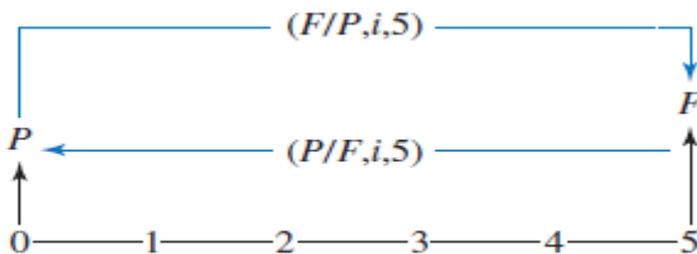
Equation 2.2 is the basis for two inverse factors represented as,  $(F/P,i,N)$  and  $(P/F,i,N)$ . The first is called the compound amount factor, since it compounds a present amount into the future with the factor  $(1 + i)^N$ . Equation 3.1 is basically the right-hand side of Equation 2.2.

$$(F/P,i,N) = (1 + i)^N \quad (3.1)$$

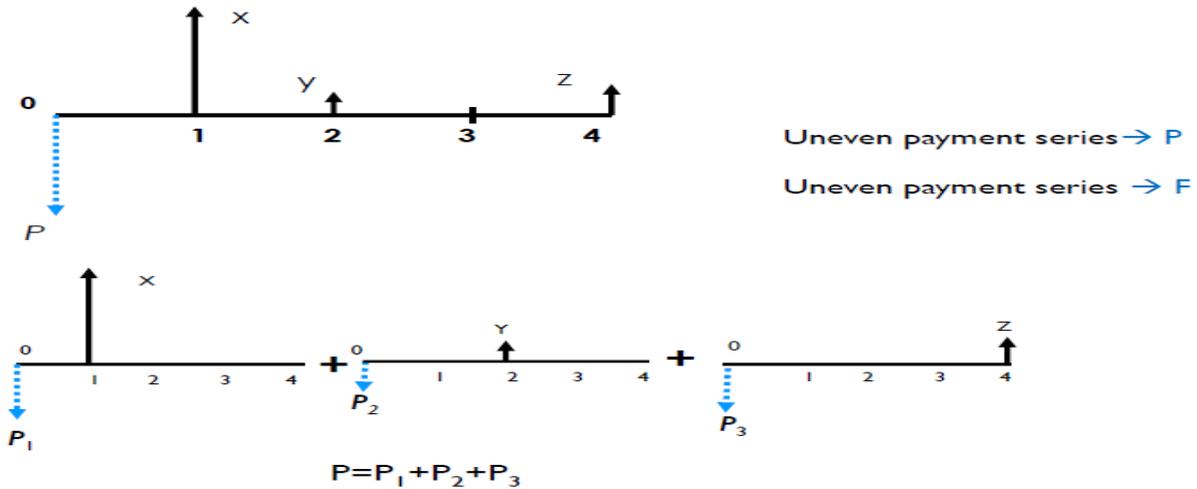
The second is the present worth factor. It discounts a single future value in period  $N$  to a present worth. Equation 3.2 is the inverse of Equation 3.1.

$$(P/F,i,N) = (1 + i)^{-N} \quad (3.2)$$

Factors for  $P$  and  $F$



**Multiple Payment Series: Uneven**



**2.4. Uniform Series**

This section develops the formula to convert from a uniform series ( $A$ ) to a present worth ( $P$ ). This formula is combined with the earlier ( $F/P, i, N$ ) formula to produce the ( $F/A, i, N$ ) formula.

A uniform series is identical to  $N$  single payments, where each single payment is the same and there is one payment at the end of each period. Thus, the ( $P/A, i, N$ ) factor is derived algebraically by summing  $N$  single-payment factors, tom. The derivation of Equation 3.3 is shown below

$$P = A[(1 + i)^N - 1] / [i(1 + i)^N] \tag{3.3}$$

The easiest way to derive Equation 3.4, which connects the uniform series,  $A$ , with the future single payment,  $F$ , is to multiply both sides of Equation 3.3 by  $(1 + i)^N$ .

$$(1 + i)^N P = A[(1 + i)^N - 1] / i$$

$$F = A[(1 + i)^N - 1] / i \tag{3.4}$$

$$P = A[ 1/(1 + i)^1 + 1/(1 + i)^2 + \dots + 1/(1 + i)^{N-1} + 1/(1 + i)^N ]$$

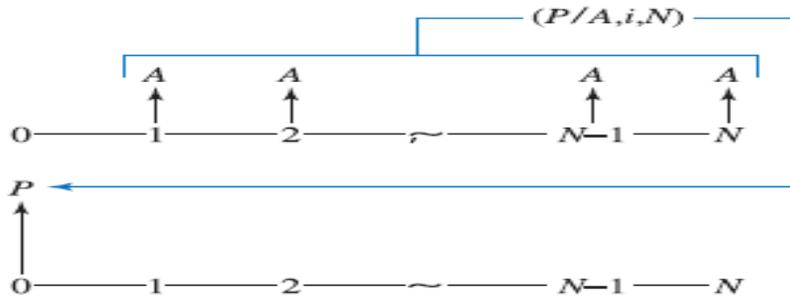
$$(1 + i)P = A[1/(1 + i)^0 + 1/(1 + i)^1 + 1/(1 + i)^2 + \dots + 1/(1 + i)^{N-1}]$$

$$(1 + i)P - P = A[1/(1 + i)^0 - 1/(1 + i)^N]$$

$$iP = A[1 - 1/(1 + i)^N]$$

$$= A[(1 + i)^N - 1]/(1 + i)^N$$

$$P = A[(1 + i)^N - 1]/[i(1 + i)^N]$$



Equations 3.3 and 3.4 are each the basis for two factors tabulated in Appendix C. The series present worth factor,  $(P/A, i, N)$ , and the capital recovery factor,  $(A/P, i, N)$ , as shown in Equations 3.5 and 3.6, are inverses based on Equation 3.3. The capital recovery factor’s name comes from asking, “How much must be saved or earned in each period,  $A$ , to recover the capital cost of the initial investment,  $P$ ?”

$$(P/A, i, N) = [(1 + i)^N - 1]/[i(1 + i)^N] \tag{3.5}$$

$$(A/P, i, N) = 1/(P/A, i, N)$$

$$= i(1 + i)^N / [(1 + i)^N - 1] \tag{3.6}$$

The series compound amount factor,  $(F/A, i, N)$ , and the sinking fund factor,  $(A/F, i, N)$ , as shown in Equations 3.7 and 3.8, are inverses based on Equation 3.4. The name of the sinking fund factor comes from an old approach to saving enough funds to replace capital equipment, which would cost  $F$  after  $N$  years. In each period, an amount,  $A$ , would be placed in a savings account (called a sinking fund), and then, after  $N$  years, a total of  $F$  would have accumulated through deposits and interest.

$$(F/A, i, N) = [(1 + i)^N - 1]/i \tag{3.7}$$

$$(A/F, i, N) = 1 / (F/A, i, N)$$

$$= i / [(1 + i)^N - 1] \tag{3.8}$$

**Example:** If \$10,000 is borrowed and payments of \$2000 are made each year for 9 years, what is the interest rate?

$$P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$= A [P/A, i\%, n]$$

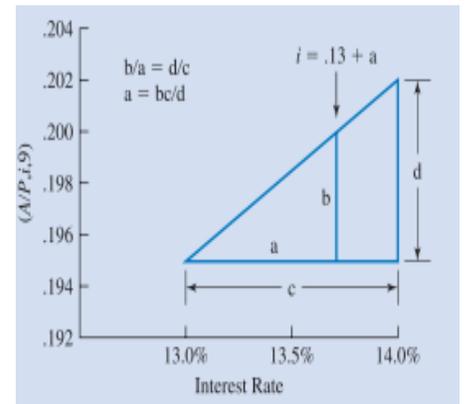
$$10,000 = 2,000 \left[ \frac{(1+i)^9 - 1}{i(1+i)^9} \right]$$

$$A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$= P [A/P, i\%, n]$$

$$A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$0.2 = [A/P, i\%, 9]$$



- Either solve the equation or use tables for interest factors and find that the interest rate is between 13% and 14%. These capital recovery factors,

$(A/P, .13, 9) = 0.1949$  and  $(A/P, .14, 9) = 0.2022$  include the value of .2.

- We interpolate for the value of  $i$ .

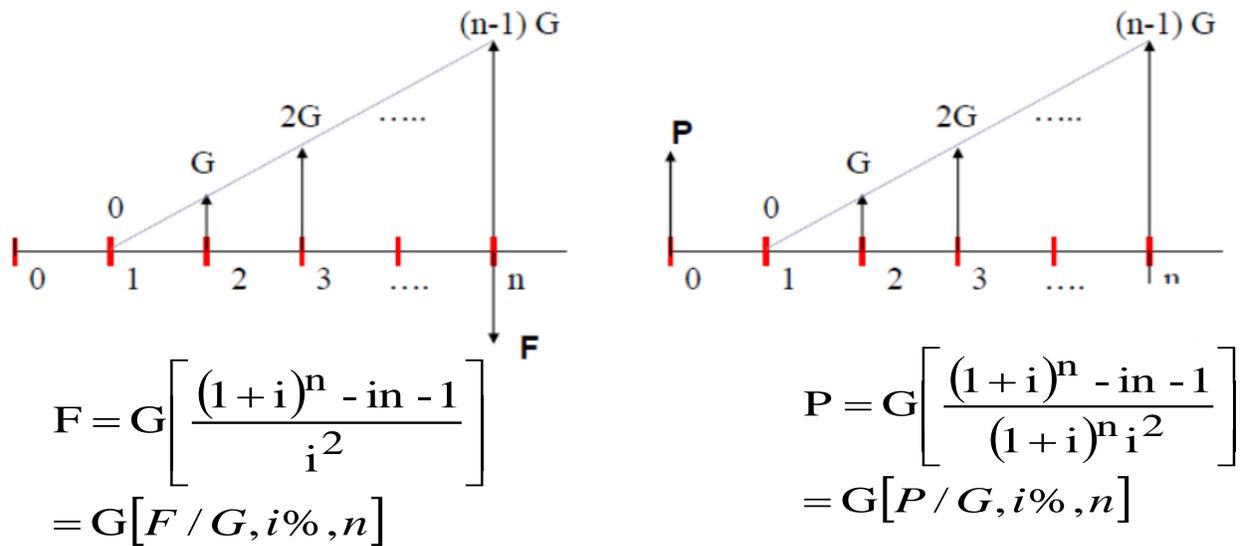
$$i = 0.13 + (0.2 - 0.1949)(0.14 - 0.13)/(0.2022 - 0.1949) = 13.7\%$$

## 2.5. Gradient Payment Series

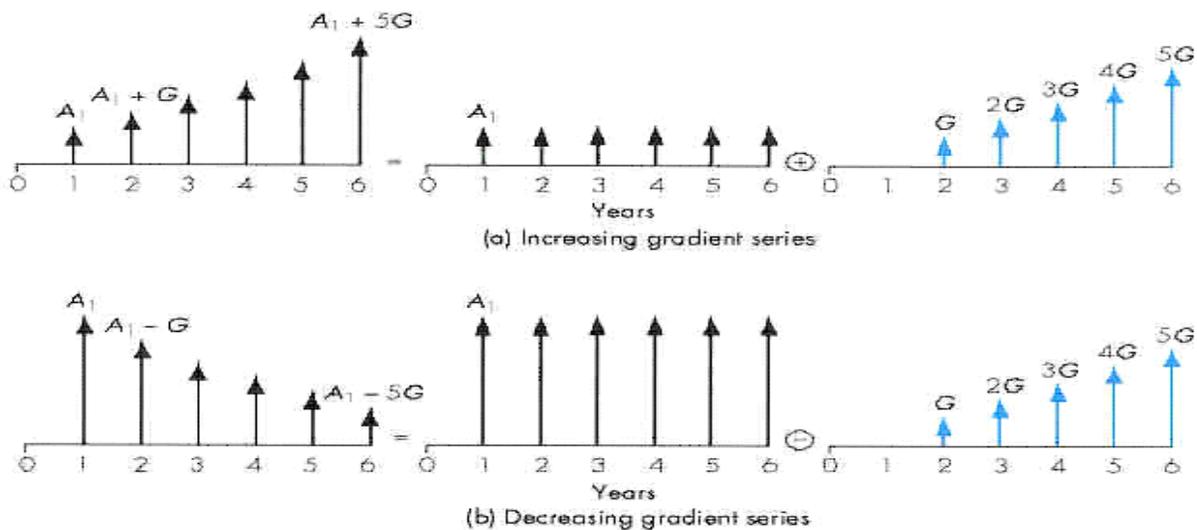
### 2.5.1. Linear Gradient Series

A typical application of linear gradient is: “Revenue is \$4000 the first year, increasing by \$1000 each year thereafter.” The linear gradient is the \$1000- per-year change in cash flows. The \$4000 per year is an annuity or uniform annual series. For the first period, the linear gradient’s zero cash flow is added to the uniform series’ cash flow of \$4000. In the second period, the total cash flow is \$4000 + \$1000. In the third, it is \$4000 + 2(\$1000). This is why the tabulated linear gradient series begins with a zero cash flow for period 1.

The linear gradients have cash flows of zero at the end of the first period. The first nonzero cash flow occurs at the end of period 2, and an  $N$ -period arithmetic gradient has  $N - 1$  nonzero cash flows, which change by  $G$  per period. A common mistake is to visualize an arithmetic gradient series as  $N - 1$  periods long. The arithmetic gradient series is  $N$  periods long, but the first period has zero cash flow.



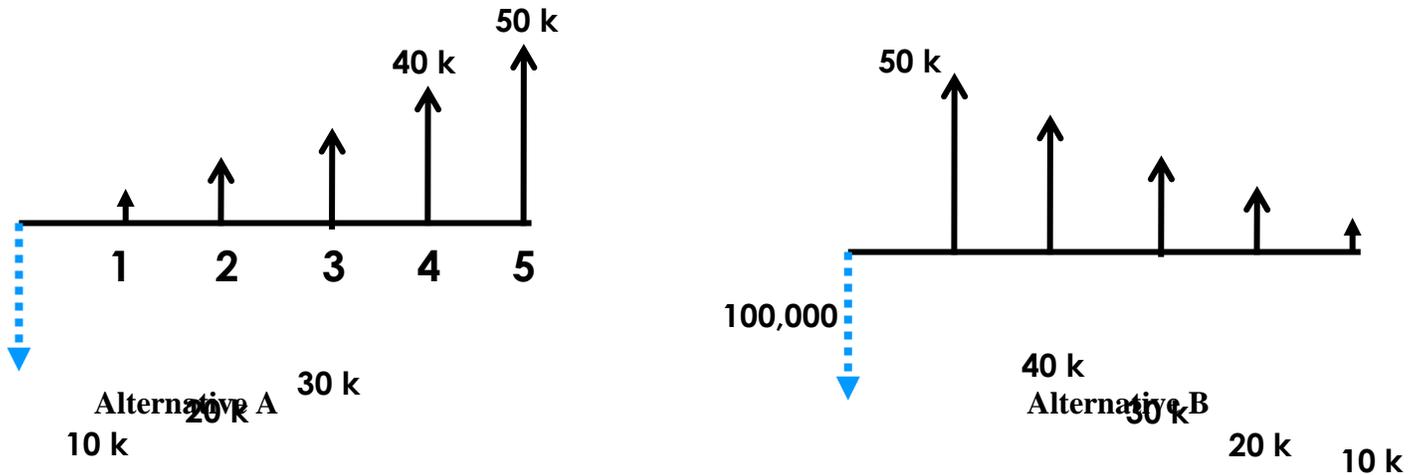
Gradient as composite



**Example 1:** A consulting engineer is considering two investment alternatives (A and B) having cash flow alternative shown below. Assume an equivalent  $i=10\%$

**Alternative A:** is an investment in a land development venture. Several other limited partners are considering purchasing land, subdividing it, and selling land parcels over a 5 years period (an increase in land value is anticipated).

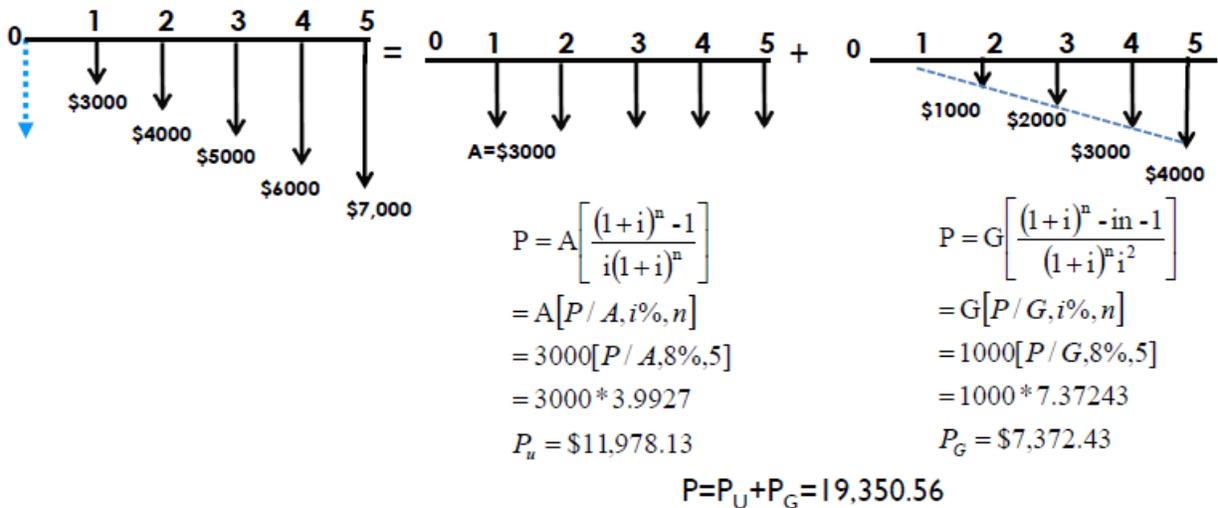
**Alternative B:** is for computer and the software required providing specialized computer-design capabilities for clients. The engineer anticipates that competition will develop quickly if his plan proves successful; a declining revenue profile is anticipated.



	A	PV A	B	1	PV B	2	3	4	5
100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)				
1	10,000	9,090.91	50,000	45,454.55					
2	20,000	16,528.93	40,000	33,057.85					
3	30,000	22,539.44	30,000	22,539.44					
4	40,000	27,320.54	20,000	13,660.27					
5	50,000	31,046.07	10,000	6,209.21					
	<b>PV</b>	<b>6,526</b>		<b>20,921</b>					

Since **alternative B** has a greater positive net present value it is a better recommendation.

**Example 2:** Maintenance cost for a particular production machine increase by \$1000/yr over the 5 year life of the equipment. The initial maintenance cost is \$3000. Using an interest rate of 8 % compounded annually, determine the present worth equivalent for the maintenance cost.



### 2.5.2. Geometric Gradient Series

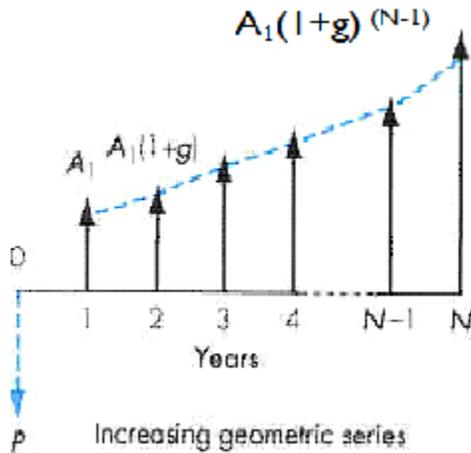
The last section presented factors for an arithmetic gradient,  $G$ , that changed by a constant amount each period. The real world often has cash flows that have a constant rate of change. This type of cash flow is a geometric gradient (also known as an escalating series). A salary that increases 6% per year is increasing geometrically, just as a market demand that decreases 15% per year is decreasing geometrically.

Many geometric gradients come from population levels where changes over time are best modeled as a percentage of the previous year. The percentage or rate is constant over time, rather than the amount of the change, as in the arithmetic gradient.

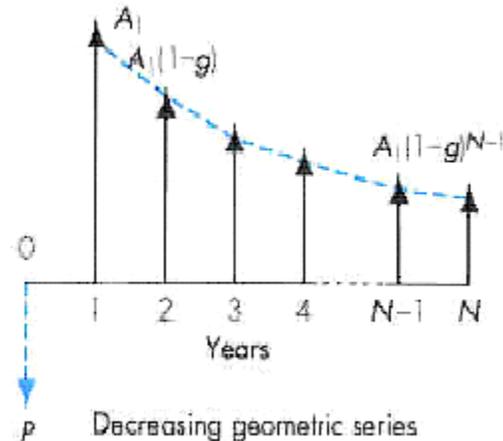
Consider an engineer whose starting salary is \$30,000 per year, with an expected increase of 6% per year for the next 15 years. Six percent of \$30,000 is \$1800, and an arithmetic gradient of \$1800 leads to a final salary of \$55,200. However, as shown on the geometric curve of Exhibit 3.23, a constant 6% rate of increase leads to a final salary of \$67,827. Thus, a geometric gradient leads to a final salary that is 23% larger than that of an arithmetic gradient.

The difference between arithmetic and geometric gradients is greater for larger rates of change and longer time periods. For a short time interval at a low percentage change, it may not matter which model we use. Often, the geometric model will be more accurate, and the simpler arithmetic model might be misleading.

$$P = \begin{cases} A_1 \left[ \frac{1 - (1+g)^N (1+i)^{-N}}{i-g} \right] & \text{if } i \neq g \\ A_1 \left( \frac{N}{1+i} \right) & \text{if } i = g \end{cases}$$



or



**Example:** Assume you receive an annual bonus and deposit it in a saving account that pays 8 percent compounded annually. Your initial bonus is 500 birr and the size of your bonus increases by 10% each year. Determine how much will be in the fund immediately after your 10th deposit.

$A_1=500$ ,  $i=8\%$ ,  $g=10\%$ , and  $n=10$  years

$$P = \begin{cases} A_1 \left[ \frac{1 - (1+g)^n (1+i)^{-n}}{i-g} \right] & \text{if } i \neq g \\ A_1 \left( \frac{n}{1+i} \right) & \text{if } i = g \end{cases} \rightarrow F = P(1+i)^n$$

$$F = A_1 \left[ \frac{(1+i)^n - (1+g)^n}{i-g} \right]$$

$= 500(F/A_1, 8\%, 10\%, 10)$

$= 500 * 21.74$

$F = \underline{10,870.44}$

$P = 10,870.44 / (1.08^{10}) = \underline{5,035.12}$

<b>Cash flow</b>	<b>0</b>	<b>500</b>	<b>550</b>	<b>605</b>	<b>666</b>	<b>732</b>	<b>805</b>	<b>886</b>	<b>974</b>	<b>1072</b>	<b>1179</b>
<b>End of Year(n)</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>

### Economic Factors

Find	Given	Symbol	Factor	Name
P	F	(P/F, i, n)	$(1+i)^{-n}$	Single payment present worth factor
F	P	(F/P, i, n)	$(1+i)^n$	Single payment compound amount factor
P	A	(P/A, i, n)	$\frac{(1+i)^n - 1}{i(1+i)^n}$	Uniform series present worth factor
A	P	(A/P, i, n)	$\frac{i(1+i)^n}{(1+i)^n - 1}$	Uniform series capital recovery factor
F	A	(F/A, i, n)	$\frac{(1+i)^n - 1}{i}$	Uniform series compound amount factor
A	F	(A/F, i, n)	$\frac{i}{(1+i)^n - 1}$	Uniform series sinking fund factor
P	G	(P/G, i, n)	$\frac{[1 - (1+ni)(1+i)^{-n}]}{i^2}$	Gradient series present worth factor
A	G	(A/G, i, n)	$\frac{(1+i)^n - (1+ni)}{i[(1+i)^n - 1]}$	Gradient series to uniform series conversion factor
P	$A_{1,j}$	(P/A <sub>1,i,j</sub> , n)	$\frac{1 - (1+j)^n - (1+i)^{-n}}{i-j}, i \neq j$	Geometric series present worth factor
F	$A_{1,j}$	(F/A <sub>1,i,j</sub> , n)	$\frac{(1+i)^n - (1+j)^n}{i-j}, i \neq j$	Geometric series future worth factor

### 2.6. Economic Equivalence

Equivalence adjusts for the time value of money. Equivalence means that different cash flows at different times are equal in economic value at a given interest rate. For instance, \$100 deposited at year 0 and the \$127.63 in the savings account at year 5 were equivalent at 5% per year.

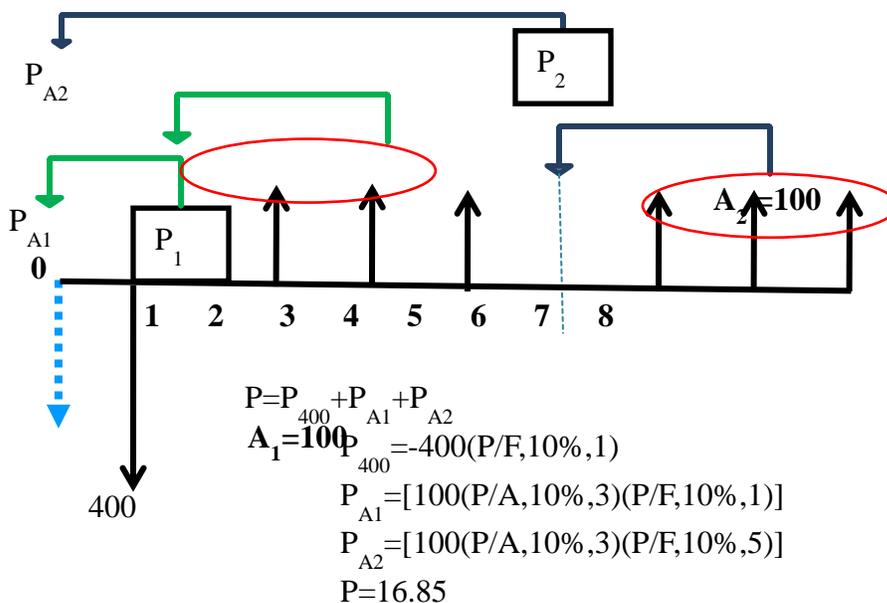
Equivalence uses an interest rate or discount rate to adjust for the time value of money. Since \$1 today and \$1 a year from now have different values, this adjustment must occur before the cash flows can be added together.

One way to understand equivalence is through a loan. Money is borrowed, interest accrues on the unpaid balance, payments are made, and eventually, the loan is repaid and the payments stop. The initial borrowing and the payments are equivalent at the loan's interest rate.

There are certain rules that one should follow to make these calculations;

- They need to **have a common time basis**;
- **Equivalence is dependent on interest rate**; and
- **Equivalence is maintained regardless of anything**.

**Example:** What single sum of money at t=0 is equivalent to the cash table below.



**Exercise**

- Using an 8% discount rate, what uniform series over five periods, [1,5], is equivalent to the cash flow given in Figure 1.

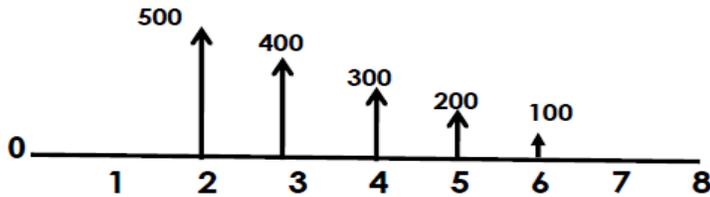


Figure 1

- For what interest rate are the two cash flows shown in Figure 2 equivalent ?

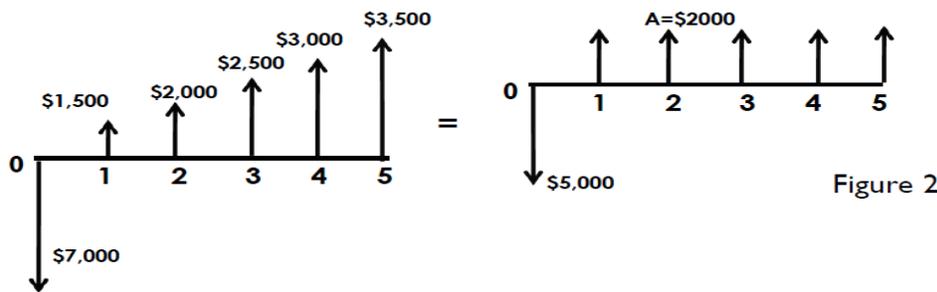


Figure 2

- Consider two investment choices that both require an initial out flow of 20, 000 birr and expected revenue as shown by respective cash flow diagrams. Which one should be chosen?  
I = 10%



- Define and clarify with examples
  - Nominal interest rates and
  - Effective Interest rates

## Chapter Three

### Economic Evaluation

#### 3.1. Present worth and Future analysis

##### 3.1.1. Present Worth Analysis

The PW measure is easy to understand, easy to use, and matched to our intuitive understanding of money. The very name, present worth, conjures an image of current value. More formally, present worth (PW) is the value at time 0 that is equivalent to the cash flow series of a proposed project or alternative.

Engineering economists and this text often use **present worth (PW)**, **present value (PV)**, **net present worth (NPW)**, and **net present value (NPV)** as synonyms. Sometimes the net is included to emphasize that both costs and benefits have been considered.

Present worth is easy to use because of the sign convention for cash flows. Cash flows that are revenues are greater than zero, and cash flows that are costs are less than zero. Thus, the standard for a desirable PW is  $PW > 0$ . ( $PW = 0$  represents economic indifference, and a  $PW < 0$  should be avoided if possible.)

Present worth is an attractive economic measure, because we have an intuitive feel for the result's meaning. We have an image of what \$100, \$1000, or \$1,000,000 is worth right now. A similar intuitive feel for future dollars is difficult to develop

**Standard Assumptions:** The present worth measure is commonly applied by making the following assumptions:

- i. Cash flows occur at the end of the period, except for first costs and prepayments like insurance and leases.
- ii. Cash flows are known values. These known values are certain, or deterministic, values.
- iii. The interest rate,  $i$ , is given.
- iv. The problem's horizon or study period,  $N$ , is given.

These assumptions imply that a problem can be summarized as a cash flow diagram plus an interest rate. These assumptions often simplify reality. Nevertheless, the conclusions of the engineering economic analysis are generally not affected. These simplifications also imply that we need a sense of proportion. If the first cost and total revenues are in the tens of millions of dollars, then a PW of \$100,000 may be better thought of as plus or minus 0. The uncertainty in the data is probably larger than 1% of the tens of millions.

### a) Evaluation of a Single Project using present worth analysis

**Step 1:** Determine the interest rate that the firm wishes to earn on its investments. The interest rate you determine represents the rate at which the firm can always invest the money in its *investment* pool. This interest rate is often referred to as either a *required rate of return* or a *minimum attractive rate of return* (MARR). Usually this selection is a policy decision made by top management. It is possible for the MARR to change over the life of a project. but for now we will use a single rate of interest when calculating PW.

**Step 2:** Estimate the service life of the project.

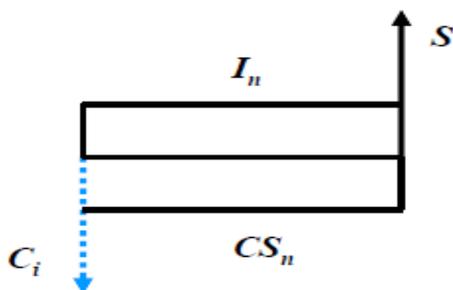
**Step 3:** Estimate the cash inflow for each period over the service life.

**Step 4:** Estimate the cash outflow for each period over the service life.

**Step 5:** Determine the net cash flows for each period (net cash flow = cash inflow - cash outflow).

**Step 6:** Find the present worth of each net cash flow at the MARR. Add up these present-worth figures; their sum is defined as the project's PW.

In order to evaluate projects one need to use *discounted cash flow techniques* (DCF). One of these is the method of *net present worth* (NPW) or *net present value* (NPV).



$$PW_{\text{costs}} = C_i + CS_n(P/A, i, n)$$

$$PW_{\text{incomes}} = I_n (P/A, i, n) + S (P/F, i, n)$$

$$NPW(NPV) = PW_{\text{incomes}} - PW_{\text{costs}}$$

**Step 7:** In this context, a positive PW means that the equivalent worth of the inflows is greater than the equivalent worth of the outflows, so the project makes a profit. Therefore, if the PW (i) is positive for a single project, the project should be accepted; if it is negative, the project should be rejected. The process of applying the PW measure is implemented with the following decision rule:

If  $PW(i) > 0$ , accept the investment.

If  $PW(i) = 0$ , remain indifferent.

If  $PW(i) < 0$ , reject the investment.

### **Comparing More Than One Alternative**

Note that the foregoing decision rule is for evaluation of a single project for which you can estimate the revenues as well as the costs. The following guidelines should be used for evaluating and comparing more than one project:

1. If you need to select the best alternative, based on the net-present-worth criterion, select the one with the highest PW, as long as all the alternatives have the same service lives. Comparison of alternatives with unequal service lives requires special assumptions, as will be detailed in section
2. As you will in some cases, comparison of mutually exclusive alternatives with the same revenues is performed on a cost-only basis. In this situation, you should accept the project that results in the smallest PW of costs, or the least negative PW (because you are minimizing costs, rather than maximizing profits).

### **Guidelines for Selecting a Minimum Attractive Rate of Return (MARR)**

Return is what you get back in relation to the amount you invested. Return is one way to evaluate how your investments in financial assets or projects are doing in relation to each other and to the performance of investments in general. Let us look first at how we may derive rates of return. Conceptually, the rate of return that we realistically expect to earn on any investment is a function of three components:

- risk-free real return,
- inflation factor, and

- risk premium(s).

Suppose you want to invest in a stock. First, you would expect to be rewarded in some way for not being able to use your money while you hold the stock. Second, you would expect to be compensated for decreases in purchasing power between the time you invest and the time your investment is returned to you. Finally, you would demand additional rewards for having taken the risk of losing your money if the stock did poorly. If you did not expect your investment to compensate you for these factors, why would you tie up your money in this investment in the first place?

In present-worth analysis, we assume that all the funds in a firm's treasury can be placed in investments that yield a return equal to the MARR. We may view these funds as an investment pool. Alternatively, if no funds are available for investment, we assume that the firm can borrow them at the MARR from the capital markets. In this section, we will examine these two views when explaining the meaning of MARR in PW calculations.

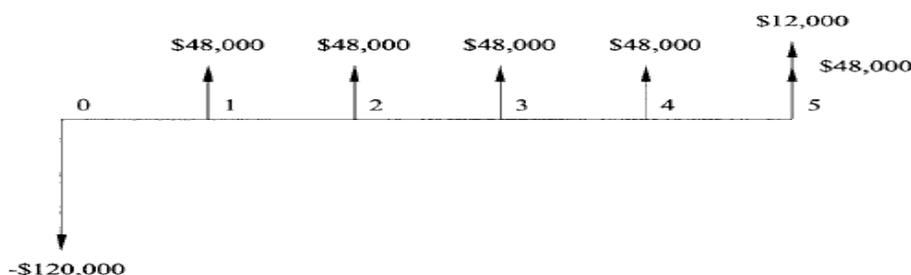
**Example:** Your Company is looking at purchasing a front-end loader at a cost of \$120,000. The loader would have a useful life of five years with a salvage value of \$12,000 at the end of the fifth year. The loader can be billed out at \$95.00 per hour. It costs \$30.00 per hour to operate the front-end loader and \$25.00 per hour for the operator. Using 1,200 billable hours per year determine the net present value for the purchase of the loader using a MARR of 20%. Should your company purchase the loader?

### Solution

The **hourly profit [HP]** on the loader equals the billing rate less the operation cost and the cost of the operator.

$$HP = \$95.00 - [\$30.00 + \$25.00] = \$40.00 \text{ per hr}$$

Annual Profit =  $\$40.00/\text{hr} \times [1,200 \text{ hr/yr}] = \$48,000/\text{yr}$ . The cash flow diagram;



The present value of the annual profits [ $P_{AP}$ ] by using USPWF:

$$P_{AP} = \$48,000[(1+0.20)^5 - 1] / [0.20(1+0.20)^5] = \mathbf{\$143,549}$$

The  $P_{AP}$  is positive because it is a cash receipt.

The present value of the salvage value [ $P_{SV}$ ] by using SPPWF

$$P_{SV} = \$12,000/(1+0.20)^5 = \mathbf{\$4,823}$$

The  $P_{SV}$  is positive because it is a cash receipt.

The present value purchase price [ $P_{PP}$ ] of the loader = purchase price. Because the net present value is measured at the time of the initial investment.

The  $P_{PP}$  is negative because it is a cash disbursement.

$$NPV = P_{AP} + P_{SV} + P_{PP}$$

Because the NPV is **greater** than **zero**, the purchase of the front- end loader will produce a return greater than the MARR and your company should invest in the front-end loader.

When comparing two alternatives with positive net present values, the alternative with the **largest net present value** produces the most profit in excess of the MARR.

### **b) Present Worth Analysis of Alternatives with Different-life Span**

When the present worth method is used to compare mutually exclusive alternatives that have different lives, then the PW of the alternatives must be compared over the same number of years and end at the same time. A fair comparison can be made only when the PW values represent costs (and receipts) associated with equal periods ( if not shorter lived alternatives will be favored though they may not be economically favorable).

The equal-period requirement can be satisfied by comparing the alternatives over a period of time equal to the least common multiple (LCM) of their lives. The LCM approach automatically makes the cash flows for all alternatives extend to the same time period.

For example, alternatives with expected lives of 2 and 3 years are compared over a 6-year time period. And alternatives with expected lives of 2, 4 and 5 years are compared over a 20-year time period

Such a procedure requires that some assumptions be made about subsequent life cycles of the alternatives. The assumptions of a PW analysis of different-life alternatives for the LCM method are as follows:

- The service provided by the alternatives will be needed for at least the LCM of years
- The selected alternative will be repeated over each life cycle of the LCM in exactly the same manner
- The cash flow estimates will be the same in every life cycle

**Example:** Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for \$65,000. At the end of the seventh year the salvage value of the new dump truck is estimated to be \$15,000. The second alternative is to purchase a used dump truck for \$50,000. At the end of the fourth year the salvage value of the used dump truck is estimated to be \$5,000. The annual profits, revenues less operation costs, are \$17,000 per year for either truck. Using a MARR of 18% calculate the net present value for each of the dump trucks. Which truck should your company purchase?

### Solution

The present value of the annual profits for either truck is determined by using USPWF:

$$P_{AP} = \$17,000 [(1+0.18)^{28}-1]/0.18[1+0.18]^{28}] = 17,000 * 5.502$$

$$= \$93,527$$

**Alternative 1[New]:** The PSV for the new dump truck is determined by summing the PSVs occurring in years 7, 14, 21, and 28. The present value for each salvage value is calculated using SPPWF as follows:



$$P_{SV7} = \$15,000/(1 + 0.18)^7 = \$4,709$$

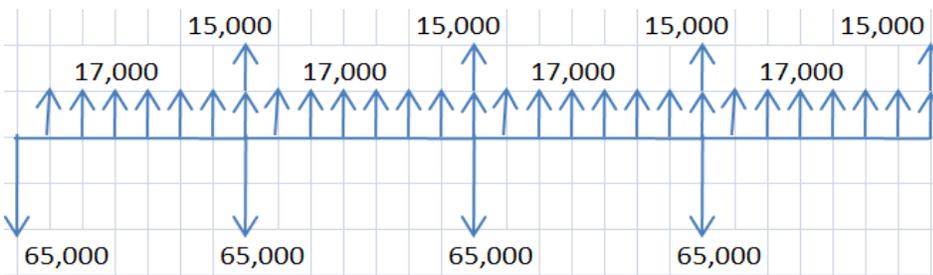
$$P_{SV14} = \$15,000/(1 + 0.18)^{14} = \$1,478$$

$$P_{SV21} = \$15,000/(1 + 0.18)^{21} = \$464$$

$$P_{SV28} = \$15,000/(1 + 0.18)^{28} = \$146$$

$$P_{SV} = \$4,709 + \$1,478 + \$464 + \$146 = \$6,797$$

The PPP for the new dump truck is determined by summing the present value of purchase prices occurring in years 0, 7, 14, and 21. The present value for each purchase price is calculated using SPPWF as follows:



$$P_{PP0} = -\$65,000/(1 + 0.18)^0 = -\$65,000$$

$$P_{PP7} = -\$65,000/(1 + 0.18)^7 = -\$20,405$$

$$P_{PP14} = -\$65,000/(1 + 0.18)^{14} = -\$6,406$$

$$P_{PP21} = -\$65,000/(1 + 0.18)^{21} = -\$2,011$$

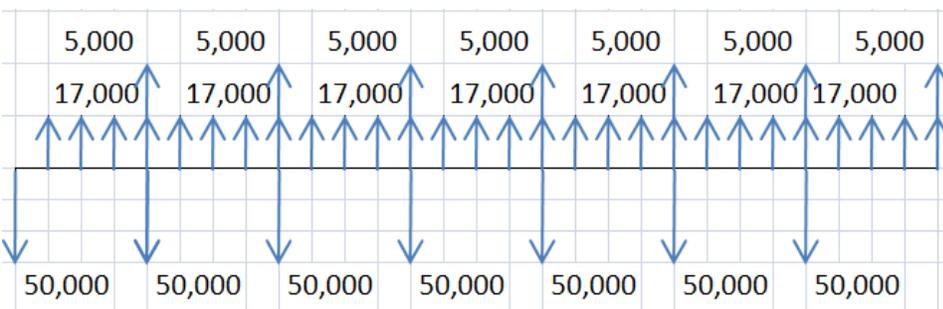
$$P_{PP} = -\$65,000 + (-\$20,405) + (-\$6,406) + (-\$2,011)$$

$$P_{PP} = -\$93,822$$

The NPV for the purchase of the new dump truck is calculated as follows:

$$NPV [\text{New}] = \$93,527 + \$6,797 + [-\$93,822] = +\$6,502$$

**Alternative 2[Used]:** The PSVs for the used dump truck is determined by summing the present value of salvage values occurring in years 4, 8, 12, 16, 20, 24, and 28. The present value for each salvage value is calculated using SPPWF as follows:



$$P_{SV4} = \$5,000/(1 + 0.18)^4 = \$2,579$$

$$P_{SV8} = \$5,000/(1 + 0.18)^8 = \$1,330$$

$$P_{SV12} = \$5,000/(1 + 0.18)^{12} = \$686$$

$$P_{SV16} = \$5,000/(1 + 0.18)^{16} = \$354$$

$$P_{SV20} = \$5,000/(1 + 0.18)^{20} = \$183$$

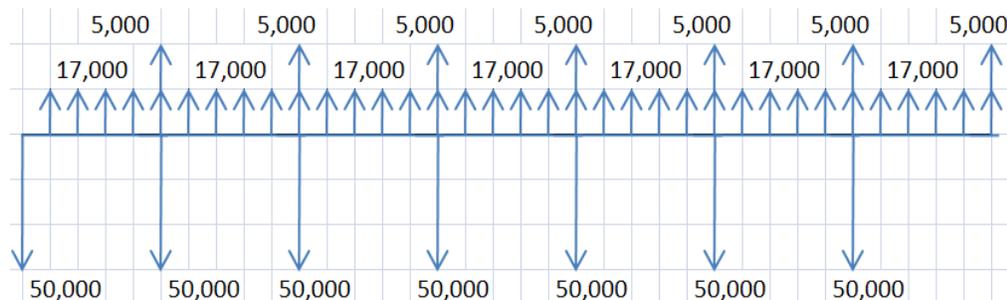
$$P_{SV24} = \$5,000/(1 + 0.18)^{24} = \$94$$

$$P_{SV28} = \$5,000/(1 + 0.18)^{28} = \$49$$

$$P_{SV} = \$2,579 + \$1,330 + \$686 + \$354 + \$183 + \$94 + \$49$$

$$P_{SV} = \$5,275$$

The PPPs for the used dump truck is determined by summing the present value of purchase prices occurring in years 0, 4, 8, 12, 16, 20, and 24. The present value for each purchase price is calculated using SPPWF as follows:



$$P_{PP0} = -\$50,000/(1 + 0.18)^0 = -\$50,000$$

$$P_{PP4} = -\$50,000/(1 + 0.18)^4 = -\$25,789$$

$$P_{PP8} = -\$50,000/(1 + 0.18)^8 = -\$13,302$$

$$P_{PP12} = -\$50,000/(1 + 0.18)^{12} = -\$6,861$$

$$P_{PP16} = -\$50,000/(1 + 0.18)^{16} = -\$3,539$$

$$P_{PP20} = -\$50,000/(1 + 0.18)^{20} = -\$1,825$$

$$P_{PP24} = -\$50,000/(1 + 0.18)^{24} = -\$941$$

$$P_{PP} = -\$50,000 + (-\$25,789) + (-\$13,302) + (-\$6,861) \\ + (-\$3,539) + (-\$1,825) + (-\$941)$$

$$P_{PP} = -\$102,257$$

The net present value [NPV] for the purchase of the used dump truck is calculated as follows:

$$\text{NPV} = \$93,527 + \$5,275 + [-\$102,257] = -\$3,455$$

**The new truck has the highest NPV; therefore, your company should purchase the new truck.**

**c) Alternatives with Infinite lives: Capitalized equivalence method**

A special case of the PW criterion is useful when the life of a proposed project is **perpetual** or the planning horizon is extremely long (say, 40 years or more). Many public projects such as bridges, Waterway constructions, irrigation systems, and hydroelectric dams are expected to generate benefits over an extended period of time (or forever). In these situations a PW analysis would have an **infinite analysis period**. This analysis is called *capitalized equivalence method*. A *capitalized cost* is the present sum of money that would need to be set aside now, at some interest rate, to yield the funds required to provide the service indefinitely.

Capitalized equivalent (CE) is the present (at time zero) worth of cash inflows and outflows. CE analysis is very useful to compare long-term projects. In other words, CE is a single amount determined at time zero, which at a given rate of interest, will be equivalent to the net difference of receipts and disbursements if the given cash flow pattern is repeated in perpetuity (in perpetuity the period assumed is infinite).

$$\begin{aligned} \text{Mathematically, CE} &= A \times (P/A, i, n=\infty) \\ &= A \times [((1+i)^\infty - 1) / i(1+i)^\infty] = A/i \end{aligned}$$

**Example:** A city plans a pipeline to transport water from a distant watershed area to the city. The pipeline will cost \$8 million and have an expected life of seventy years. The city anticipates it will need to keep the water line in service indefinitely. Compute the capitalized cost assuming 7% interest.

**Solution**

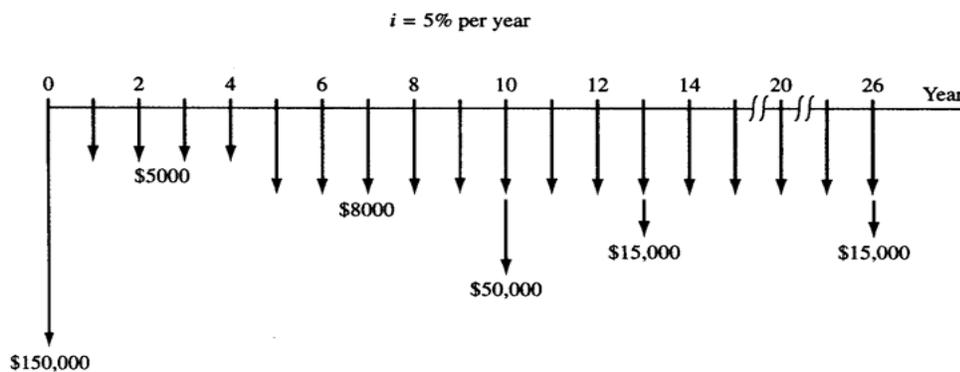
We have the capitalized cost equation  $P = A/i$ , which is simple to apply when there are end-of-period disbursements  $A$ . In this case, the \$8 million repeats every 70 years. We can find  $A$  first based on a present \$8 million disbursement.

$$A = P(A/P, i, n) = \$8,000,000(0.0706) = \$565,000$$

Now, the infinite series payment formula could be applied for  $n = \infty$  :

$$\text{Capitalized cost } P = A / i = 565,000 / 0.07 = \$8,071,000$$

**Example:** A new computer system will be used for the indefinite future, find the equivalent value now if the system has an installed cost of \$150,000 and an additional cost of \$50,000 after 10 years. The annual maintenance cost is \$5,000 for the first 4 years and \$8,000 thereafter. In addition, it is expected to be a recurring major upgrade cost of \$15,000 every 13 years. Assume that  $i = 5\%$  per year.

**Solution**

Draw the cash flow diagram for two cycles.

$$\rightarrow CC_1 = -150,000 - 50,000(P/F, 5\%, 10) - 5,000(P/A, 5\%, 4) - (8,000/0.05)(P/F, 5\%, 4)$$

$$= -150,000 - 50,000(0.6139) - 5,000(3.5460) - 160,000(0.8227) = -330,057$$

Convert the recurring cost of 15,000 every 13 years into an annual worth  $A$ , for the first 13 years.

$$A_1 = -15,000(A/F, 5\%, 13) = -847$$

The same value,  $A_1 = -847$ , applies to all the other 13 years period as well.

$$CC_2 = \frac{A_1}{i} = \frac{847}{0.05} = -16,940$$

$$CC = (-330,057) + (-16,940) = -346,997$$

#### d) Comparing more than two alternatives using Incremental Net Present Value [INPV]

**Step 1:** Order the alternatives by increasing initial capital investment.

**Step 2:** Find a base alternative [current best alternative]: Cost alternatives: the first alternative in the ordered list [the one with the least capital investment].

**Step 3:** Evaluate the difference between the next alternative and the current best alternative. If the incremental cash flow is positive, choose the next alternative as the current best alternative. Otherwise, keep the current best alternative [i.e. **negative**] and drop the next alternative from further consideration.

**Step 4:** Repeat Step 3 until the last alternative is considered. Select the current best alternative as the preferred one.

### 3.2. Future Worth (Net Future Value) Method

A corollary to the present value and net present worth is the **future value** and the **net future worth** (NFW).

In this method **all cash inflows** and **outflows** of a given project (having a given project life) are **brought to time n**. If the difference between the inflows minus the outflows is positive then the project is acceptable.

NFW > 0, Accept the investment

NFW = 0, Remain indifferent to the investment

NFW < 0, Reject the investment

Future worth analysis calculates the future worth of an investment undertaken. If it is to **compare among** various **projects**, the **one having more positive value** is **economically the best alternative**.

**Example:** your company is looking at purchasing the front-end loader at cost of \$120,000. The loader would have a useful life of five years with a salvage value of \$12,000 at the end of the fifth year. The annual profit of loader [revenue less operation cost] is \$48,000. Determine the future worth for the purchase of the loader using a MARR of 20%. Should your company purchase the loader?

### Solution

The future value of the purchase price is determined by using SPCAF as follows:

$$FPP = \$120,000 [1+0.20]^5 = \$120,000 * 2.4883 = - \$298,598$$

The future value of the purchase price is negative because it is a cash disbursement.

The FW of the annual profits is determined by using USCAF as follows:

$$FAP = \$48,000 * [(1+0.20)^5 - 1] / 0.20 = \$48,000 * 7.4416 = \$357,197$$

$$FAP = \$48,000 * [(1+0.20)^5 - 1] / 0.20 = \$48,000 * 7.4416 = \$357,197$$

The FW of the annual profits is **positive** because it is a cash receipt.

The future value of the salvage value is equal to the salvage value because the future value is measured at the end of the study period. The FW of the salvage value is positive because it is a cash receipt.

The future worth for purchasing the loader equals the sum of the future values of the individual cash flows and is calculated as follows:

$$FW = - \$298,598 + \$357,197 + \$12,000 = \$70,599 > 0$$

→ So, it is attractive for the company to purchase

### 3.3. Payback Period Analysis

The payback period equals the time required for net revenues from a project to “pay back” its initial cost. Because it is simpler than discounted cash flow techniques, it was once the most common technique for evaluating projects. However, this is no longer true.

If a project’s net revenues are the same each period, then the payback period is easy to calculate. For example, a machine costs \$8K to buy, and it saves \$800 per month in costs. In this case, the payback period is  $\$8K/\$800$  per month, or 10 months.

Payback period analysis is an approximate, rather than an exact, economic analysis calculation. Often used as a screening technique/ preliminary analysis tool. May or may not select the correct alternative. All the economic consequences beyond the payback period are completely ignored. It is based on two forms:

- Ignoring TVOM, with 0% interest: conventional PB method
- With an assumed interest rate : discounted payback analysis

It is important to note that PBA is not an end to itself, but rather a method of screening out certain obvious unacceptable investment alternatives before progressing to an analysis of potentially acceptable ones.

Compare PBP Vs maximum acceptable PBP.

The main problems with payback period are:

- Payback ignores the time value of money.
- Payback ignores receipts and costs that occur between payback and the project’s time horizon. Some writers have described this as ignoring a project’s life.

Payback ignores cash flows that occur after the payback period. One project may have a short payback period, but its positive cash flows may end shortly thereafter. Other projects may have somewhat longer payback periods that are followed by years or decades of positive cash flows.

Ignoring cash flows that occur after payback can cause an evaluation to ignore overhaul costs and environmental reclamation expenses. In addition, working capital is difficult to account for. Similarly, evaluation of projects with staged development is unreliable using payback period.

The payback periods are often stated with a decimal or fractional year. Even though problems are drawn and analyzed assuming end-of-period cash flows, in most cases costs and revenues occur throughout the year

**Example: Payback period:**

- a) Ignoring TVOM( $i=0\%$ )
- b) Perform Discounted payback at  $i=10\%$

$i=0\%$

E.O.Y	CF	Cumulative CF
0	-30,000	-30,000
1	-4,000	-34,000
2	15,000	-19,000
3	16,000	-3,000
4	8,000	5,000
5	8,000	13,000
	13,000	

**PBP =3.375 years**

$i=10\%$

E.O.Y	CF	(P/F, $i\%,t$ )	Dis. Inc	Accum. Disc. Amts.
	(1)	(2)	(3)=(1)*(2)	(4)=Cuml. Sum of (3)
0	-30,000	1	-30,000.00	-30,000

1	-4,000	0.9091	-3,636.36	-33,636.36
2	15,000	0.8265	12,396.69	-21,239.67
3	16,000	0.7513	12,021.04	-9,218.63
4	8,000	0.6806	5,464.11	-3,754.52
5	8,000	0.6209	4,967.37	1,212.85

**PBP =4.76 years**

At 10% the PB is b/n years 4 and 5 (4.76 years).

At 0% the PB is b/n years 3 and 4 (3.375 years).

**Example:** Suppose the company requires a rate of return of 15%. Determine the period necessary to recover both the capital investment and the cost of funds required to support the investments given the cash flow (Column 2) in table below.

Period	CF	Cost of fund 15%	Cumulative CF
0	(85,000)	0	(85,000)
1	15,000	-85,000*0.15=-12,750	(82,750)
2	25,000	-82,750 *0.15=-12,413	(70,163)
3	35,000	-70,163*0.15=-10,524	(45,687)
4	45,000	-45,687*0.15=-6,853	(7,540)
5	45,000	-7,540*0.15=-1,131	36,329
6	35,000	36,329*0.15=5,449	76,778

The project must remain in use for about 4.2 years in order for the company to cover the cost of capital and also recover the fund invested in the project

### Interpretation of PBA

A managerial philosophy is: a shorter payback period is preferred to a longer payback period. It is not a preferred method for final decision making – rather, use as a screening tool.

**Example:** Three investments are available but only one can be purchased.

<b>E.O.Y</b>	<b>CF (1)</b>	<b>CF (2)</b>	<b>CF (3)</b>
0	-10,000	-10,000	-10,000
1	5,000	5,000	2,500
2	5,000	4,000	2,500
3	0	3,000	2,500
4	0	2,000	2,500
5	1,000	1,000	12,500

### Solution

<b>E.O.Y</b>	<b>CF (1)</b>	<b>C. C.F (1)</b>	<b>CF (2)</b>	<b>C. C.F (2)</b>	<b>CF (3)</b>	<b>C.C.F (3)</b>
0	-10,000	-10,000	-10,000	-10,000	-10,000	-10,000
1	5,000	-5,000	5,000	-5,000	2,500	-7,500
2	5,000	0	4,000	-1,000	2,500	-5,000
3	0	0	3,000	2,000	2,500	-2,500
4	0	0	2,000	4,000	2,500	0
5	1,000	1,000	1,000	5,000	12,500	12,500
<b>PBP</b>		<b>2yrs</b>		<b>2.33yrs</b>		<b>4yrs</b>
<b>PW(10%)</b>	<b>-701.39</b>		<b>2,092.13</b>		<b>5,686.18</b>	

PBP: 1>2>3

PW: 3>2>1

Shorter payback period is preferred to a longer payback period.

Not a preferred method for final decision making – rather, use as a screening tool.

E.O.Y	CF	Cumulative C.F.	P/F,i%,t(2)	Dis. Inc	Accum. Disc. Amts.
	(1)			(3)=(1)*(2)	(4)=Cuml. Sum of (3)
0	-10,000	-10,000	1	-10,000	-10,000
1	2,000	-8,000	0.9259	1,851.85	8,148.15
2	6,000	-2,000	0.8573	5,144.03	-3,004.12
3	8,000	6,000	0.7938	6,350.66	3,346.54
4	4,000	10,000	0.7350	2,940.12	6,286.66
5	1,000	11,000	0.6806	680.58	6,967.25

PB is b/n years 2 and 3 (2.45 years=3years). The C.F of years 4 and 5 are not used in the calculations.

None of the cash flows AFTER the payback period are considered.

### 3.4. Annual Equivalent Worth

The **annual equivalent worth (AE) criterion** provides a basis for measuring investment worth by determining equal payments on an annual basis. Knowing that any lump-sum cash amount can be converted into a series of equal annual payments, we may first find the net present worth of the original series and then multiply this amount by the capital-recovery factor:

$$AE(i) = PW(i)(A/P, i, N). \quad (6.1)$$

We use this formula to evaluate the investment worth of projects as follows:

**Evaluating a Single Project:** The accept-reject decision rule for a single revenue project is as follows:

If  $AE(i) > 0$ , Accept the investment.

If  $AE(i) = 0$ . Remain indifferent to the investment.

If  $AE(i) < 0$ , Reject the investment.

Notice that the factor  $(A/P, i, N)$  in Eq. (6.1) is positive for  $-1 < i < m$ . which indicates that the  $AE(i)$  value will be positive if and only if  $PW(i)$  is positive. In other words, accepting a project that has a positive  $AE(i)$  value is equivalent to accepting a project that has a positive  $PW(i)$  value. Therefore, the AE criterion provides a basis for evaluating a project that is consistent with the PW criterion.

**Comparing Multiple Alternatives:** As with present-worth analysis, when you compare mutually exclusive service projects that have equivalent revenues. You may compare them on a cost-only basis. In this situation, the alternative with the least annual equivalent cost (or least negative annual equivalent worth) is selected.

**Example:** A firm is considering which of two devices to install to **reduce costs**. Both devices have useful lives of 5 years with no salvage value. **Device A** costs \$1000 and can be expected to result in \$ 300 saving annually. **Device B** costs \$1350 and will provide cost saving of \$300 the first year ; however, saving will increase \$50 annually, making the second year saving \$350, the

third year savings \$400, and so forth. With interest at 7%, which device should the firm purchase?

**Device A**

$$AW_A = -1000(A/P, 7\%, 5) + 300 = -1000(0.2439) + 300 = \$ 56.11$$

**Device B**

$$AW_B = -1350 (A/P, 7\%, 5) + 300 + 50(A/G, 7\%, 5)$$

$$= -1350(0.2439) + 300 + 50(1.865) = \$ 64$$

$$(A/P, 7\%, 5) = \frac{0.07(1.07)^5}{(1.07)^5 - 1} = 0.2439$$

$$(A/G, 7\%, 5) = \frac{(1.07)^5 - (1 + 5 * 0.07)}{0.07[(1.07)^5 - 1]} = 1.865$$

Installing Device B results larger benefit.

**Example:** Three alternatives are being considered for improving an operation on the assembly line along with the “do-noting” alternative. Equipment costs vary, as do the annual benefit of each in comparison to the present situation. Each Plan has a 10-year life and a salvage value equal to 10% of its original cost. For interest of 8% which plan should be adopted?

	<b>Plan A</b>	<b>Plan B</b>	<b>Plan C</b>
Installed cost of equipment	15,000	25,000	33,000
Material & labor saving per yr.	14,000	9,000	14,000
Annual operating expenses	8,000	6,000	6,000
End-of –useful life <b>saving value</b>	1,500	2,500	3,300

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	<b>Plan A</b>	<b>Plan B</b>	<b>Plan C</b>
<b>Equivalent uniform annual benefit (EUAB)</b>	14,000	9,000	14,000
Material & labor per yr.	104	172	228
Salvage value (A/F, 8%, 10)=0.069''''			

---

	EUAB=	14,104	9,172	14,228
<hr/>				
<b>Equivalent uniform annual cost (EUAC)</b>		2,235	3,725	4,917
Installed cost (A/P, 8%, 10)		8,000	6,000	6,000
Annual operating cost				
	EUAC=	-10,223	-9,725	-10,917
<hr/>				
	EUAW= EUAB-EUAC =	<u><b>3,869</b></u>	<b>-553</b>	<b>3,311</b>
<hr/>				

**Example:** Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for \$65,000. At the end of the seventh year the salvage value of the new dump truck is estimated to be \$15,000. The second alternative is to purchase a used dump truck for \$50,000. At the end of the fourth year the salvage value of the used dump truck is estimated to be \$5,000. The annual profits, revenues less operation costs, are \$17,000 per year for either truck. Using a MARR of 18% calculate the annual worth for each of the dump trucks. Which truck should your company purchase?

### Alternative 1[New]

The useful life is seven years, which is used as the study period for the new truck.

$$A_{PP} = - \$65,000 [0.18(1+0.18)^7] / [(1+0.18)^7 - 1]$$

$$= - \$17,054$$

$$A_{SV} = \$15,000(0.18) / [(1+0.18)^7 - 1]$$

$$= \$1,235$$

$$AE [\text{New}] = - \$17,054 + \$1,235 + \$17,000$$

$$= \$1,181$$

### Alternative 2[Used]

$$APP = \$50,000[0.18(1+0.18)^4] / [(1+0.18)^4 - 1]$$

$$= -\$18,587$$

$$ASV = \$5,000(0.18) / [(1+0.18)^4 - 1] = \$959$$

- The annual equivalent for purchasing:

$$AE[\text{Used}] = - \$18,587 + \$959 + \$17,000$$

$$= -\$628$$

- $AE[\text{New}] > AE[\text{Used}]$ : purchase the new truck.

### 3.5. Internal Rate of Return

The **internal rate of return (IRR)** is based only on a project's cash flows, which is the basis of the *internal*. As the interest rate that makes the  $PW = 0$ , it is the project's rate of return. Since the  $PW = 0$ , all other equivalent measures calculated at the IRR, such as the EAW, also equal zero.

For the IRR to exist, both costs and benefits must be defined. If only a project's benefits are defined, then the  $PW$  and EAW are positive for all interest rates. If there are only costs, then the  $PW$  and EAW are always negative. Only if there are costs *and* benefits can the  $PW$  and EAW = 0 for some interest rate.

Along with the  $PW$  and AE criteria, the third primary measure of investment worth is rate of return. As shown in Chapter 5, the  $PW$  measure is easy to calculate and apply. Nevertheless, many engineers and financial managers prefer rate-of-return analysis to the  $PW$  method, because they find it intuitively more appealing to analyze investments in terms of percentage rates of return rather than in dollars of  $PW$ . Consider the following statements regarding an investment's profitability:

This project will bring in a 15% rate of return on the investment.

This project will result in a net surplus of \$10,000 in terms of  $PW$ .

Neither statement describes the nature of an investment project in any complete sense. However, the rate-of-return figure is somewhat easier to understand, because many of us are so familiar with savings and loan interest rates, which are in fact rates of return.

Many different terms are used to refer to **rate of return**, including **yield** (e.g., yield to maturity, commonly used in bond valuation), **internal rate of return** and **marginal efficiency of capital**. We will first review three common definitions of rate of return. Then we will use the definition of internal rate of return as a measure of profitability for a single investment project throughout the text.

“The rate of return on an investment is the amount of profit it makes, often shown as a percentage of the original investment.”

“The interest rate charged on the unrecovered project balance of the investment such that the payment schedule makes the unrecovered project balance equal to zero at the end of the investment’s life.”

“It is the break even interest rate which equates the present worth of a project’s cash outflows to the present worth of cash inflows.”

$$\text{NPW} = \text{PW of benefits} - \text{PW of costs} = 0$$

The rate of return (ROR) for a series of cash flows is that particular value,  $i^*$ , of the interest rate for which the NPV vanishes.

- Plot the NPV as a function of  $i$ , the curve will cross the  $i$ -axis at  $i^*$ .
- Trial and error,  $i$ -values for which the NPV is slightly positive and slightly negative, and interpolate linearly between them for  $i^*$ .
- Use Newton-Raphson iteration method or another numerical technique NPV for  $i$ , with the left side replaced by zero. [The value of  $i$  that makes the NPV equation zero.]

### Steps for Computing IRR

1. Assume a trial rate of return ( $i^*$ ).
2. Counting the cost as negative and income as positive, find the equivalent net worth of all costs and incomes.
3. If the equivalent net worth is **positive** then the income from the investment is worth more than the cost of investment and the actual percentage return is **higher** than the trial rate, and vice versa
4. Adjust the estimate of the trial rate of return and go to step 2 again until one value of  $i$  is found that results in a positive equivalent net worth and another higher value of  $i$  is found with negative equivalent net worth.
5. Solve for the applicable value of  $i^*$  by interpolation

**Example:** Determine the IRR for the given cash flow.

Year	0	1	2	3	4
Cash Flow	-1000	400	370	240	220

$$NPW = PW_{\text{benefits}} - PW_{\text{cost}} = 0$$

$$NPW = -1000 + 400 (P/F, i^*, 1) + 370 (P/F, i^*, 2) + 240 (P/F, i^*, 3) + 220 (P/F, i^*, 4) = 0$$

Assuming  $i^* = 10\%$

$$NPW = -1000 + 400(0.9090) + 370 (0.8264) + 240(0.7513) + 220 (0.6830) = 0$$

We find that  $i^* = 10\%$  which is a special interest rate that has vanished the net present worth of the given cash flow to zero. Thus  $i^* = 10\%$  is the IRR.

This indicates that the company earns (charges) a 10% rate of return on the funds that remains *internally* invested in the project. Since it is a return internal to the project, we refer to it as the internal rate of return, or IRR.

$i = 10\%$  is the IRR

E.O.Y	Cash Flow (1)	Unrecovered Investment at beginning of Yr. (2)	10% Return on Unrecovered Investment (3) = (1)*i	Investment Repayment at E.O.Y. (4) = (1)+(3)	Unrecovered Investment at E.O.Y. (5) = (1)+(4)
0	-1000	-	-	-	-1000
1	400	-1000	-100	300	-700
2	370	-700	-70	300	-400
3	240	-400	-40	200	-200
4	220	-200	-20	200	0

This means that the project under consideration brings in enough cash to pay for itself in 4 years and also to provide the company with return of 10% on its invested capital

### → Case of a Single Sign-Reversal

When  $CF_0 < 0$  and  $CF_j > 0$  ( $j > 0$ ), i.e., when there is just one reversal of sign in the sequence  $CF_0, CF_1, CF_2, \dots, CF_n$ , the NPV is a monotone decreasing function of  $i$ , and so  $i^*$  is uniquely determined. Moreover, at this unique ROR, the FW and EUAS are zero.

**Example:** The cash flows associated with a milling machine are  $CF_0 = -\$50,000$ .  $CF_j = \$15,000$  ( $j = 1, \dots, 5$ ). Determine the economic acceptability of this machine at interest rates of (a) 10%, (b) 15%, and (c) 20% per year (all compounded annually).

(a)  $NPV = -50,000 + 15,000(P/A, 10\%, 5) = -50,000 + 15,000(3.7908) = \underline{\$6,861.80}$

(Economically acceptable investment)

(b)  $NPV = -50,000 + 15,000(P/A, 15\%, 5) = -50,000 + 15,000(3.3522) = \underline{\$282.33}$

(Barely acceptable)

(c)  $NPV = -50,000 + 15,000(P/A, 20\%, 5) = -50,000 + 15,000(2.9906) = \underline{-\$5,140.82}$

(Not economically justifiable)

By linear interpolation between the results of (b) and (c):

i (%)	NPV (\$)
15	282.33
$i^*$	0
20	-5,140.82

$$i^* = 15\% + \left[ \frac{0 - 282.33}{-5140.82 - 282.33} \right] * [20\% - 15\%]$$

$$i^* = 15.26\%$$

→ **Case of a Multiple Sign-Reversals**

When the sequence  $CF_0, CF_1, CF_2, \dots, CF_n$  shows more than one reversal of sign, it is possible that  $NPV = 0$  for several values of the interest rate; there could thus be several rates of return.

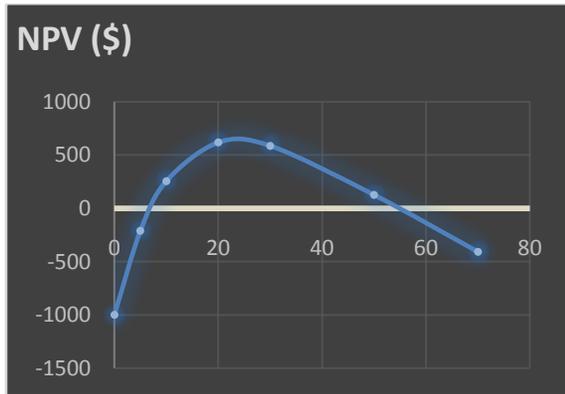
**Example:** For a project with the given series of cash flows, determine the NPV at annual interest rates 0%, 5%, 10%, 20%, 30%, 50%, and 70%.

E.O.Y	0	1	2	3	4	5
CF, 1000	-3	0	6	6	0	-10

For the given flows, solve:  $NPV = -\$3000 + \$6000(P/A, i\%, 2)(P/F, i\%, 1) - \$10000(P/F, i\%, 5)$

Evaluation at the specified interest rates gives the points which are plotted.

<b>i(%)</b>	0	5	10	20	30	50	70
<b>NPV (\$)</b>	-1000	-210	257	620	588	128	-407



From a graph of the results, find the rate(s) of return.

$i=7%$  and  $i=54%$

If we use the PW method with MARR=15%, we would obtain:

<b>E.O.Y</b>	0	1	2	3	4	
<b>CF</b>	-3000	0	6000	6000	0	-10,000
<b>(P/F, I, n)</b>	1	0.8696	0.7561	0.6575	0.5718	0.4972
<b>PW</b>	-3000	0	4,536.86	3,945.10	0	-4,971.77
<b>NPW</b>	510.19					

$PW(15\%)=510.19 > 0$  This verifies the project is acceptable.

**Flaws in Project ranking by IRR**

Under NPW, NFW or AE analysis: the highest worth figure was preferred. “Total investment approach”.

Unfortunately, the analogy does not carry over to IRR analysis. The project with the highest IRR may not be the preferred alternative.

**Example:** Suppose you have two mutually exclusive alternatives, each with a 1 year service life. One requires an investment of \$1000 with a return of \$2000 and the other requires \$5000 with a return of \$7000.

Already solved IRR and NPW(MARR=10%) as show on table below.

n	A1	A2
0	-1000	-5000
1	2000	7000
IRR	100%	40%
PW(10%)	818	1,364

Assuming you have enough money in your investment pool to select either one, which one would you choose?

Based on NPW measures A2 is preferred. But based on IRR A1 is preferred

We notice inconsistency of ranking:

NPW, NFW and AE are **absolute (dollar)** measures of investment worth.

IRR is a **relative(percentage)** measure and cannot be used in the same way.(i.e., IRR ignores the scale of the investment)

### Incremental Investment Analysis

For a pair of mutually exclusive project A and B, with B defined as a more costly option, we may rewrite B as:

$B=A+(B-A)$ , B has two cash flows: (1) same CF as A and (2) the incremental component (B-A).

Therefore, the only situation in which B is preferred over A is when the ROR in the incremental component (B-A) exceeds the MARR.

For two mutually exclusive project, ROR analysis is done by computing the internal rate of return on incremental investment (IRR<sub>D</sub>) between the projects. Since we want to consider increments of investment, we compute the cash flow for the difference between the projects by subtracting the cash flow for the lower investment project(A) from that of the higher investment project (B). Then:

- $IRR_{B-A} > MARR$ , select B
- $IRR_{B-A} = MARR$ , select either one
- $IRR_{B-A} < MARR$ , select A

### Example

**Alternative A:**Withdrawal of 1000. Remaining 4000continues to earn 10%. After a year you will receive 2000 from external investment and 4,400 form the investment pool.With an investment of 5000 in 1 year you will have 6400(4400+2000).

$$PW(10\%) = -5000 + 6400(P/F, 10\%, 1) = 818$$

**Alternative B:**Withdrawal of 5000 leaving no money in the investment pool. After a year you will receive 7000 from external investment.Your total wealth changes from 5000 to 7000 in a year.

$$PW(10\%) = -5000 + 7000(P/F, 10\%, 1) = 1,364$$

n	A	B	B-A
0	-1000	-5000	-4000
1	2000	7000	5000
IRR	100%	40%	25%
PW(10%)	818	1,364	545

If you decide to take **more costly option**, you would want to know that this additional investment can be **justified at the MARR**. The 10%MARR implies that you can always earn that return from other investment;

For B by investing the additional \$4000 you would make an additional \$5000, which is equivalent to earning at a rate of 25%.

### Exercise

1. A construction engineer has been asked to recommend an excavator for acquisition. Three alternatives have been identified, having the characteristics shown on table below. If MARR is 12% which alternative would you recommend?

Excavator alternative	Useful life (Years)	Initial investment (\$)	Annual operating cost (\$)	Salvage value
A	4	15,500	8,750	2,500
B	5	20,250	5,850	3,000
C	6	30,750	3,175	3,250

2. What is MARR, how it is determined, what is considered in trying to determine the MARR?
3. An individual wants to start a small-scale painting business. To economize the start up business, he decides to purchase some used painting equipment. He has two mutually exclusive options, which he expects to fold up the business in three years,
  - a) Do most of the painting by himself by limiting his business to only residential painting jobs (B1) or
  - b) Purchase more painting equipment and hire some helpers to do both residential and commercial painting jobs that he expects will have a higher equipment cost, but provide higher revenue as well (B2).

Given the cash flow for the mutually exclusive alternatives presented below which project would he select at MARR=10%.

<b>n</b>	<b>B1</b>	<b>B2</b>
0	-\$3,000	-\$12,000
1	1,350	4,200
2	1,800	6,225
3	1,500	6.330

4. Consider the following three sets of mutually exclusive alternatives.

<b>n</b>	<b>D1</b>	<b>D2</b>	<b>D3</b>
0	\$2,000	\$1,000	\$3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000

Which project would you select based on rate of return on incremental investment , assuming that MARR=15%?

## Chapter Four

### Benefit-cost analysis

The first step in calculating a benefit/cost ratio, which is benefits minus dis-benefits, divided by the costs, is to define these benefits and costs for government projects. For public projects such as dams and roads, the benefits are the consequences to the public. Positive outcomes include recreation, electricity, shorter trips, and fewer accidents. Negative outcomes, or dis-benefits, include lost white-water kayaking in the dammed river, traffic delays during construction, and neighborhoods divided by new highways. The costs are paid by the government for construction and for operation.

Not all government projects involve the public. For example, the post office might be evaluating a new mail sorting machine. The cost to buy and install the new machine would be compared with the savings in operating costs. In this case, the savings would be the benefits for any benefit/cost ratio.

The difference between the benefits and the costs ( $B - C$ ) is the PW calculated in section 3.1 or the EAW calculated in section 3.4. However, governments want a measure of investment efficiency. It is intuitively appealing to find the amount of benefit that a project produces per dollar of cost. This is the benefit/cost ratio.

The ratio can be calculated using PW for both the numerator (benefits) and the denominator (costs) or by using equivalent annual measures for both the numerator and the denominator. The numerator includes public consequences, and the denominator includes government costs.

A desirable project has a ratio greater than or equal to 1. If the ratio is less than 1, then the project is unattractive, because the costs are greater than the benefits.

The benefit/cost ratio is calculated at a known interest rate, and when applied properly, it will lead to the same recommendation as PW, EAW, or IRR measures. The benefit/cost ratio is a discounted cash flow measure, and its popularity is based on the intuitive sense of return per dollar of cost. Intuitively, the higher the return per dollar of cost the better the project.

**Example:**

Item	Classification
Expenditure of 11 million dollars for a new highway	Cost
\$100,000 annual income to local residents from tourists due to the construction of new highway	Benefit
\$15,000 annual upkeep of highway	Cost
\$250,000 annual loss to farmer due to loss of highway right of way	Disbenefit

The *dollar of return per dollar of cost* is the benefit/cost ratio in the public sector. In the private sector, similar measures are called **present worth indexes** or, sometimes, benefit/cost ratios. There are numerous variations of these indexes, but each is a ratio of income over outgo or ongoing net revenue over initial expenses.

For example, suppose a machine costing \$500 would save \$200 per year for 4 years. The return is the PW of four \$200 annual receipts. If  $i$  is 10%,  $(P/A, .1, 4)$  is 3.170, and the return's PW is \$634. The PW index is  $634/500$ , or 1.27. There is \$1.27 in benefits or returns for each \$1 of cost—over and above a 10% return on the investment.

The common assumption of PW indexes is that investment projects are being evaluated. This implies that at least the time 0 cash flow is negative and that the summation of net receipts exceeds the first cost. However, the division of costs and/or revenues between the numerator and the denominator is somewhat arbitrary.

Public projects are very different from the private ones in their nature: It is not the mission of the government to make money, but to bring value to the people. Therefore it is crucial to know the values associated with the alternatives.

Since the sole monetary goal is no longer valid, it may cause conflicts among the objectives. There are inevitably political issues related to fairness considerations

	<b>Private</b>	<b>Public</b>
<b>Purpose</b>	Profit	Well-being of the public
<b>Financing</b>	Investment	Tax
<b>Horizon</b>	Short	Long
<b>Benefit</b>	Money	Value to society

The ultimate aim of a business organization is to make profits. Therefore, any system in the organization must produce more benefits as compared to its costs for the organization to survive & prosper

In this method all costs and benefits are discounted to their present worth and the ratio of benefit to cost is calculated. Negative flows are considered as costs and positive flows as benefits. The analysis relies on the addition of positive factors and the subtraction of negative ones to determine a net result. If the B/C ratio is more than one the project is worth undertaking.

The BCR approach takes into account “efficiency” by comparing the benefits obtained per unit of cost. It measures the benefit per unit cost, based on the time value of money.

A profitability index of 1.1 implies that for every \$1 of investment, we create an additional \$0.10 in value. It is intuitively appealing to find the amount of benefit that a project produces per dollar of cost. Ironically, small projects with very little NPV can look comparatively attractive with the BCR.

Items regarding a public project:

- Benefits
- Costs
- Dis-benefits.

In particular, let us denote;

B: benefits of the project;      I: initial capital investment;

CR: capital recovery;      O&M: operating and maintenance costs.

$$\begin{aligned} \text{Conventional B/C ratio} &= \frac{PW(B)}{I + PW(O \& M)} \\ &\text{or} \\ &= \frac{AW(B)}{CR + AW(O \& M)} \end{aligned}$$

$$\begin{aligned} \text{Considering Dis-benefit B/C ratio:} &= \frac{AW(B) - AW(D)}{CR + AW(O \& M)} \\ &\text{or} \\ &= \frac{AW(B)}{CR + AW(O \& M) + AW(D)} \end{aligned}$$

$$\begin{aligned} \text{Modified B/C ratio} &= \frac{PW(B) - PW(O \& M)}{I} \\ &\text{or} \\ &= \frac{AW(B) - AW(O \& M)}{CR} \end{aligned}$$

At a given MARR, we would consider an alternative acceptable, provided PW of benefits - PW of costs  $\geq 0$  or EUAB - EUAC

$$\text{Benefit-cost ratio} = \frac{B}{C} = \frac{PW \text{ of benefit}}{PW \text{ of cost}} = \frac{EUAB}{EUAC} \geq 0$$

	Situation	Criterion
Neither input nor output fixed	Neither amount of money or other inputs nor	Two alt.: compute incremental B/C ratio on the increment of investments
	Amount of benefits or other outputs are fixed	If $\frac{\Delta B}{\Delta C} \geq 1$ Choose higher-cost alt.; otherwise, choose lower-cost alt.
Fixed input	Amount of money or other input resources are fixed	Maximize B/C
Fixed output	Fixed task, benefit, or other output to be accomplished	Maximize B/C

**Example:** A firm is considering which of two devices to install to **reduce costs**. Both devices have useful lives of 5 years with no salvage value. **Device A** costs \$1000 and can be expected to result in \$ 300 saving annually. **Device B** costs \$1350 and will provide cost saving of \$300 the first year ; however, saving will increase \$50 annually, making the second year saving \$350, the third year savings \$400, and so forth. With interest at 7%, which device should the firm purchase?

### Device A

$$AW_A = -1000(A/P, 7\%, 5) + 300 = -1000(0.2439) + 300$$

$$= \$ 56.11$$

$$(A/P, 7\%, 5) = \frac{0.07(1.07)^5}{(1.07)^5 - 1} = 0.2439$$

### Device B

$$AW_B = -1350 (A/P, 7\%, 5) + 300 + 50(A/G, 7\%, 5)$$

$$= -1350(0.2439) + 300 + 50(1.865) = \$ 64$$

$$(A/G, 7\%, 5) = \frac{(1.07)^5 - (1 + 5 * 0.07)}{0.07[(1.07)^5 - 1]} = 1.865$$

Therefore, installing Device B results larger benefit

	Device A	Device B
<b>Installation cost</b>	1000	1350
<b>Annual saving</b>	300	300 Increasing gradient series with G=50
<b>EUAW</b>	56.11	64

	Device A	Device B	Incremental B-A
<b>Installation cost</b>	1000	1350	350
	= 243.9	= 329.26	= 85.36
<b>Annual saving</b>	300	300 & Increasing gradient series (G=5)	50(A/G, 7%, 5) =93.25
<b>B/C= <math>\frac{EUAB}{EUAC}</math></b>	=300/243.9 =1.23	=393.25/243.9 =1.19	=93.25/85.36 <b>=1.09</b>

Maximizing B/C ratio results wrong indication (Device A) so, we must use incremental analysis.

**Examples:** Consider three investment projects A<sub>1</sub>, A<sub>2</sub>, and A<sub>3</sub>. Each project has the same service life, and the present worth of each component value (B,I,C') is computed at 10% as follows:

- a) If all three projects are independent, which project would be selected based on BC (i)?
- b) If the three projects are mutually exclusive, which project would be the best alternative? Use the B/C ratio on incremental investment.

	Project A <sub>1</sub>	Project A <sub>2</sub>	Project A <sub>3</sub>
<b>Initial cost (I)</b>	5,000	20,000	14,000
<b>Revenue (B)</b>	12,000	35,000	21,000
<b>Operation cost(C')</b>	4,000	8,000	1,000
<b>PW(i)</b>	3,000	7,000	6,000

- a) If all three projects are independent, which project would be selected based on BC (i)?

All projects would be considered as all the PW's are positive.

	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>A<sub>3</sub></b>
<b>B/C</b>	=12,000/9000 =1.33	=35,000/28000 =1.25	=21,000/15000 =1.40

b) If the three projects are mutually exclusive, which project would be the best alternative? Use the B/C ratio on incremental investment.

First arrange the projects by increasing order of their denominator (I+C')

$$A_1 = 5,000 + 4,000 = 9,000$$

$$A_2 = 20,000 + 8,000 = 28,000$$

$$A_3 = 14,000 + 1,000 = 15,000 \rightarrow A_1 > A_3 > A_2$$

	<b>A<sub>1</sub></b>	<b>A<sub>3</sub></b>	<b>A<sub>2</sub></b>	<b>B/C<sub>3-1</sub></b>	<b>B/C<sub>2-3</sub></b>
<b>I+C'</b>	9,000	15,000	28,000	6,000	13,000
<b>B</b>	12,000	21,000	35,000	9,000	14,000
<b>B/C</b>				<b>1.50</b>	<b>1.08</b>

**B/C<sub>3-1</sub> > 1**, We prefer **A<sub>3</sub>** over **A<sub>1</sub>**: **A<sub>3</sub>** current best alternative

**B/C<sub>2-3</sub> > 1**, We prefer **A<sub>2</sub>** over **A<sub>3</sub>**: with no further project to consider becomes best choice.

The Benefit-Cost Ratio Method is very popular in practice. However, it has several drawbacks as well.

- The required data might be hard to quantify;
- It disregards the problem of economic inequalities, i.e., one part of the population benefits at the expense of the other part;
- It takes no notice to any qualitative information.

- Extra care should be taken in the evaluation of the economic decisions in the public sector.

## Chapter 5

### Depreciation

#### 5.1 Introduction

Depreciation is an allowable expense in general accounting purposes and income tax accounting purposes. But it differs categorically from other conventional expenses because depreciation charge does not occur any outflow of business fund.

The periodical amount of depreciation is affected by the following factors

1. The cost of the asset;
2. The life of the asset;
3. The expected residual value of the asset; and
4. By the method of depreciation selected for amortization of the asset which must be systematic and rational

When a corporation purchases equipment, a facility, etc., that is used more than a year, it has to recover the cost over a number of years. This is called depreciating, and the process is depreciation. The annual depreciation from the depreciation formulae or tables is deducted each year from the income in the same manner as regular cost. The remaining cost of the purchase is called the book value.

$$\text{Book Value at year } n = \text{Initial Cost} - \text{Total Depreciation up to year } n \quad [1]$$

At any time, if the purchased equipment, property, etc., is sold, the difference between the sale price and the book value is called capital gain or loss and is considered an income or loss and is treated accordingly for tax purposes.

Cost of asset means the basic acquisition cost of the asset plus all incidental expenses which are required to the asset into use. The incidental expenses like freight, import duty, Brokerage, legal expenses and installation charges are also form a part of cost of asset. There are some controversies regarding repairs and maintenance cost. In general, heavy repairs and maintenance cost which increases the life of the asset or keep the asset in its usable state are also to be capitalized.

The useful life of an asset is the period of time during which the firm expects to use the asset for earning revenue. It is not an easy task to estimate an accurate life of the asset. The useful service life of an asset may come to an end whether as a result of physical causes or as a result of changing economic significance or both. The physical, engineering life of the asset can be determined with a fair degree of accuracy, but technological obsolescence and demand for a product cannot be determined easily. So, instead of exact working life only the probable useful period may be assumed through rational approach like, past experience, quality of asset, expert's opinion, consulting asset's manual, statistical tools for forecasting etc.

Salvage value of an asset refers to the amount that can be expected to realize from disposal of the asset at the ends of its useful life. That means it is the difference between the cost of the asset and the total depreciation during its life. Expecting a few cases, salvage values of retired assets are not of any great significance. Still an incorrect estimate of the salvage value, however small it may be cannot but result incorrect measure of the periodical depreciation

Once the cost of the asset, useful lives and the salvage value are determined the problem of depreciation is reduced to one of finding a suitable basis of allocation of the cost of the asset less salvage value over the periods that use services of the asset. In general accounting practice, the choice of method of allocating the cost of a tangible fixed asset over its effective life i.e. depreciation should depend upon the patterns of expected benefits obtainable in each period from its use. The main problem of this approach is that there is no dependable way to measure the quantum of service that can be received from the asset over its expected service life. In actual practice what happen is that the accountant selects a method to be used as the basis for allocating the depreciable cost.

## **5.2 Methods of depreciating an asset**

### **5.2.1 Straight-Line Depreciation**

Under this method, an equal amount is provided each year for depreciation of each asset until the asset has been written down to nil or its scrap value at the end of the estimated life of the asset'. The name of this method is derived from the fact if the successive annual depreciation over the life of the asset is plotted on a graph, the result will be a straight line with a slope equal to the

annual depreciation. This method is also called 'Fixed Installment Method' because a uniform amount of depreciation is charge each year. The formula of the annual depreciation under the method is:

$$\text{Depreciation per year} = (C - S) / N \quad [2]$$

Where, C = Cost of the asset

S = Salvage or scrap value

n = Estimated life of years.

This method can be recommended only when the following conditions are satisfied.

- a) The asset is expected to render a uniform service throughout its estimated useful life.
- b) Annual repairs and maintenance costs are assumed to remain constant over its life.
- c) The asset is expected to earn an equal amount of revenue each year throughout its life.
- d) The amount of depreciation is a function of time only.

**Example:** What is the yearly depreciation and the book value for a truck with an initial cost of \$150,000, an assumed life of five years, and an expected resale value of \$50,000?

$$D = (150,000 - 50,000) / 5 = 20,000$$

Year	Depreciation	Book Value
1	20,000	130,000
2	20,000	110,000
3	20,000	90,000
4	20,000	70,000
5	20,000	50,000

The resale price of the truck in the calculation was estimated; we do not know what the actual sale price would be. If the truck is sold at the end of year 5 for \$70,000, then

$$\text{Capital gain} = 70,000 - 50,000 = 20,000$$

Capital gain or loss is treated as ordinary benefit or cost.

### ***I. Merits of this method***

There are several merits of the method

- a) This method is not only simple to understand but also easy to calculate.
- b) The book value of an asset can be fully written off
- c) The life of the certain assets sometimes depends on contracts like leasehold property, patents, trademarks etc. In such case this method is very much appropriate.
- d) Effective life of an assets, scrap value, repairs and maintenance cost, rate of interest etc. cannot be measured with certainty. So, no single method can weight all the factors at a time with equal importance for fixing the amount of depreciation. From this view point, this method appears most reasonable as some favorable impact of some factors is offset by unfavorable effects of others.

### ***II. Demerits of this method***

As against the advantages enumerated above, the straight-line method has some disadvantages also. Some of the disadvantages are:

- a) This method does not take into account the interest on capital invested on the assets.
- b) Under this method the amount of depreciation can never be equal to the value of services rendered from the asset. An asset is expected to render more effective services during earlier period than later period of its useful life as its efficiency decreases over times.
- c) The charge for depreciation remains constant year to year but the repair and maintenance expenses may go up with the asset growing older and older.
- d) The recovery of 'Real Capital' is not possible under this method as the amount of depreciation remains the same year after year. Only the historical cost is recovered.
- e) This method ignores the time value of money and inflation factor.

### **5.2.2 Double Declining Balance (DDB) Depreciation**

Compared to other depreciation methods, double-declining-balance depreciation results in a larger amount expensed in the earlier years as opposed to the later years of an asset's useful life. The method reflects the fact that assets are typically more productive in their early years than in their later years also, the practical fact that any asset loses more of its value in the first few years of its use. With the double-declining-balance method, the depreciation factor is 2 times that of the straight-line expense method.

In this method, the salvage value is estimated but is not considered in the calculation. Depreciation for any year  $n$  is calculated from equation 3. This is done until the end of the estimated life or the year for which the book value is less than or equal to the estimated resale value, whichever comes first. After this year, we have to stop depreciating. In other words, the book value is should never drop below the estimated resale. It must be noted that the resale value is just an estimate that can be set at any value so as to have the effect we want on the depreciation schedule.

$$D_n = \frac{2}{N} (P - \sum_1^N D) \quad [3]$$

Where  $\sum_1^N D$ ) is the sum of depreciations to year  $n$ .

**Example:** Use the same values of the previous Example and calculate depreciation using double declining balance.

Year 1 $D = 2/5 (150,000) = 60,000$	Book Value 90,000
Year 2 $D = 2/5 (150,000 - 60,000) = 36,000$	Book Value 54,000
Year 3 $D = 2/5 (150,000 - (60,000 + 36,000)) = 21,600$	Book Value 32,400

This book value is less than the estimated resale value of \$50,000. Therefore, in the third year we can only depreciate a dollar value that makes the book value equal to \$50,000. Hence, depreciation at year 3 is  $54000 - 50000 = 4000$

### ***I. Merits of this method:***

The advantages of this method are as follows;

- In the earlier year more depreciation is charged than later years, so by adopting this method a firm can save tax by lowering its tax liability.
- In the initial year high depreciation accompanied with low repairs and maintenance cost and in the later part of asset's life low depreciation accompanied with high repairs and maintenance cost will be tend to make a uniform equitable year after year during the asset's life.
- A firm will generate more interest if it invests the depreciation outside the firm. It helps to create more funds at the time of replacement of asset.
- The firm will face minimum loss at the time of disposal of asset due to innovation as a large part has already changed to profit and loss account by way of depreciation.

## II. Demerits

Every method has its own limitation. The main objections against adopting the method are as follows:

- a) This method shows lower profit in the earlier year as high depreciation is charged. It will depict poor performance in the initial year.
- b) The investors are unhappy as they get low dividend in the earlier year for generating lower profit by the firm.
- c) Though it is popular among tax payer but sometimes they face high cost of production due to higher depreciation.
- d) Under this method the asset value can never be reduced to zero.

### 5.2.3 Sum of Years Digit (SOYD) Depreciation

This is another accelerated depreciation method which was introduced by the US Internal Revenue Code of 1954. Under this method the cost less salvage value is charged to different years in the ratio of capital blocked in the asset in the year concerned to the total blockage over its life. This method assumes that depreciation of the first year should be the highest as no portion of the capital has been recovered till then and the depreciation of the last year should be the least of all years because a major portion of the invested capital has been already recovered.

Since depreciation is measured according to the volume of blocked investment, its magnitude is expressed by means of a fraction. The denominator of the fraction, which remains constant, is the total of the digits representing the useful life of the asset. The numerator, on the other hand, measuring the blockage of capital in the reverse weighted digits of each year.

In this method, depreciation for each year is calculated using a number called the sum of year's digit (SOYD).

$$SOYD = \frac{N}{2} (N + 1) \quad [4]$$

N = Estimated useful life of the system

Depreciation for each year is then calculated from equation,

$$\frac{RL}{SOYD} (P - S) \quad [5]$$

Where

RL = Remaining life at any year, at the beginning of the year

P = Purchase price

S = Estimated resale value

**Example:** Calculate the annual depreciation values for the case of Example 2 using the SOYD method.

$$\text{SOYD} = 5/2 (5+1) = 15$$

$$\text{Depreciation for year 1} = 5/15 (150000-50000) = 33333.3$$

$$\text{Depreciation for year 2} = 4/15 (150000-50000) = 26666.7$$

$$\text{Depreciation for year 3} = 3/15 (150000-50000) = 20000$$

$$\text{Depreciation for year 4} = 2/15 (150000-50000) = 13333.3$$

$$\text{Depreciation for year 5} = 1/15 (150000-50000) = 6666.7$$

### ***I. Merits:***

The merits of the method are as follows:

- a) In this method, the quantum of depreciation is greater in the earlier years in comparison with the later years because the benefits received from the use of the asset are greater in the early years than in the later years.
- b) In the earlier year repairs are light but depreciation is heavy but in the later year, as the asset gets older the repairs are heavy but depreciation is light. So depreciation plus repairs will more or less constant every year and the charge to Profit and Loss Account should be uniform.
- c) If the asset is retired earlier than anticipated as result of unforeseen obsolescence, the loss upon retirement will be less than if straight line depreciation were used, because, asset are recovered at a higher rate in the earlier years where as only a small fraction remains left for recovering them in the later years.
- d) For tax accounting purposes, these methods have a clear advantage over the straight line method. The larger deductions in the early years mean that at the very least tax payments are postponed for a considerable period. On the other hand, this method gives a tax postponement with the greatest present value.

- e) In this method nearly three-fourth of total depreciation is charged within half of its life. That means three-fourth of blocked investment recovered within short span.
- f) This method is very simple to understand and simple to calculate.

## ***II. Demerits:***

Although this method is considered to be a great innovation in the field of depreciation accounting but it has some disadvantages which are given below:

- a) This method also ignored the cost of capital on invested fund.
- b) In the earlier year greater depreciation is charged at a result less profit is available for declaring dividend in the earlier year. This may create serious problem for organization to attract new investor. As dividend is one of the motivating factors for investment. Again more depreciation in the earlier year may resultant the high cost of production in the competitive market.

## **5.3 Amortization and Depletion**

An organization may own or acquire non-tangible assets. These are assets such as patent rights, goodwill, and right of way. They are amortized instead of depreciated. As far as financial analysis is concerned, the effect is the same as depreciation. The procedure is also the same, and the same methods could be used. Depletion is also similar to depreciation. It applies to assets such as mines, oil wells, etc. The annual portion of the acquisition cost allocated as expense depends on the percentage of the total estimated quantity that is extracted in any particular year.

## **5.4 Financial Analysis with Tax and Depreciation**

In financial analysis, depreciation is treated as an expense, although no actual payment is made to anyone in that particular year. It therefore reduces the taxable income and increases the income after tax.

**Assignments**

1- The initial cost of a heavy-duty truck is \$140,000. It is assumed that its resale value after five years is \$30,000. Calculate its depreciation schedule using:

- a) Straight-line method
- b) Double declining balance method
- c) Sum of the years digit method.

2. The initial cost of equipment is \$1,000,000. Assuming a life of five years and a resale value of \$200,000,

- a) Calculate the depreciation and the book value of this equipment for the next 7 years. Use both straight-line and double declining balance depreciation methods.
- b) If the equipment is sold at year 4 for \$400,000, what is the capital gain or loss using both of the above mentioned methods of depreciation?

## Chapter 6

### Overview of Feasibility Study

#### 6.1 Introduction

Managers usually face the challenging decision of choosing between large numbers of development projects. They make many general, non-recurring investment decisions, involving investing in fixed assets, according to their feasibility studies results. Generally, the word “feasibility study” is an expression in the economical and accounting sciences that appeared in the early sixties to simply define precise reviews and examinations to determine the feasibility of different investment alternatives by calculating costs and benefits to extract measurements for every alternative. Based on these measurements, decision-makers compare different alternatives and make the investment decision. Usually, many expressions are commonly used for feasibility studies like project evaluation, investment appraisal, or capital budgeting.

#### 6.2 Traditional Classification of Feasibility Studies

Traditionally, there are many classifications for feasibility studies. They can be mainly classified according to the type of profit and the function of the study.

##### *6.2.1 According to the Type of Profit*

The main aim of the feasibility study is to evaluate a new project and take a decision, whether to invest in this project, or reject it. That is done by determining the relative profits of this project to its investors or founders.

**For private sector**, this is known as the private profitability of the investment project. It expresses self-benefits of the project, regardless of any side effect of those benefits on other projects or on the national economy of the country. The main aim of private profitability-feasibility studies is to maximize self-benefits of the project to its investors or founders.

**For public sector**, this is known as the social profitability of the investment project. It expresses self-benefits of the project and any side effect of this project on other projects or the national

economy of the country. The main aim of social profitability-feasibility studies is to maximize self-benefits of the project to the society as a whole (Table 1).

**Table 1. Classification of Feasibility Studies According to the Type of Profit.**

Type	Costs	Benefits	Purpose
Private Feasibility Study	Include costs of the project assets.	Expresses the self-benefits of the project.	Maximize the self-benefits of the project to its investors or founders
Public Feasibility Study	Include costs of the project assets and externalities costs.	Expresses the self-benefits of the project, the externalities benefits and any side effect of this project on other projects or the national economy of the country	Maximize the self-benefits of the project to the society as a whole.

### 6.2.2 According to the Function

Feasibility studies can be classified according to the function of each portion of studies as follows:

**Legal feasibility study:** This is done when the legal aspects are given a central importance in taking the investment decision.

**Marketing feasibility study:** Whenever applicable, marketing studies are done for different projects.

**Technical and engineering feasibility study:** That is always done for different types of projects.

**Financial and economical feasibility study:** This study is always carried after the previous studies for all types of projects. It converts the results of marketing, technical and engineering

studies into financial and economical value. This includes cost, required funds, and expected benefits.

**Social feasibility study:** This study is carried to review the feasibility of investment from the social point of view. It measures the social profitability of the investment project.

There is no formal arrangement for the previous studies. Usually they are carried out in parallel. Sometimes, there is no need to carry out a legal study or marketing study for some types of projects (Table 2).

### 6.3. Overview of Feasibility Study Procedures

Generally, there are no specific approaches to carry out feasibility studies for all types of projects. However, there are consensus procedures or schools to carry out feasibility studies phases. The traditional feasibility study and its new trends are the most famous procedures used mainly for private projects and many of the public projects. Other international procedures as the World Bank procedure <sup>[5]</sup> are widely used for public projects that use international fund or aid. Meanwhile, other donors and fund organizations do not require determined feasibility study procedures.

**Table 2. Classification of Feasibility Studies According to the Function.**

Function	Central Consideration	Study Purpose
Legal feasibility study	Legal aspects	The legality of the project
Marketing feasibility study	Market studies	The project marketing
Technical and engineering feasibility study	Technical studies	Study the technical aspects of the project
Financial and economical feasibility study	Funds and economical Studies	Convert the above studies to costs, required funds, and expected benefits

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Social feasibility study	Social benefits	Measures the social profitability of the project
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#### ***6.4 Traditional Feasibility Study***

The traditional feasibility study implies the collection and arrangement of data of different project alternatives to extract information and measurements to appraise each alternative in order to support decision-making. Usually, it is divided into the following steps.

##### ***6.4.1 Pre-feasibility Study (Concept and Initiation Phase)***

This step implies the discussion of the investment idea. It may include a representation of the investment idea with a simple legal, marketing, technical and engineering, financial and economical, or social criterion that lead to a primary approval or refusal of the idea.

##### ***6.4.2 Detailed Feasibility Study (Design and Development Phase)***

Theoretically, the detailed feasibility study includes more detailed studies of the investment idea with a detailed legal, marketing, technical and engineering, financial and economical, or social criterion that lead to project appraisal. Generally, the tasks associated with such studies include the following

- Legal study, that includes the legal aspects of the project, any legal issues forbidding the project and any legal modifications required to proceed in this project.
- Market study that includes the supply, the demand, the supply and demand analysis, and the project market share.
- Technical and engineering studies that define the project capacity, type, complete design, construction process and method, site location, and planning schedules.
- Financial and economical studies that define the investment costs which include the fixed costs (land, building, equipment...etc.), financial schedule, resources and budgets, and revenues or benefits.

- Social study that measures the social profitability of the project.

### ***6.4.3 Project Appraisal***

In this step, the expected economic and social revenues of the project are evaluated and analyzed using a variety of techniques to decide the project feasibility. Traditionally, investment decisions on public projects are made by the investing government based on the cost-benefit analysis and economic viability of the projects. The most common methods for the assessment of financial viability are the payback period, average account rate of return, net present value (NPV), and internal rate of return (IRR) methods. Decisions derived from these methods are based on the forecasts of base-case cash flows. However, public projects are characterized by high capital outlays, long lead times, and long operating periods, which lead to inadequate cash flows forecasting.

### ***6.5 Feasibility Study According to the World Bank Procedure***

The World Bank (WB) provides funds to governments and public organizations guaranteed by their governments to execute public projects. Projects range across the economic and social spectrum in these countries, e.g., infrastructure, education, and health projects. The projects the bank finances are conceived and supervised according to a well-documented project cycle that constitutes the feasibility study of the project from the bank point of view. The feasibility study steps followed by the WB are shown follow the procedure.

**Country Assistance Strategy:** Under its current development policy, the bank helps governments take the lead in preparing and implementing development strategies in the belief that programs that are owned by the country, with widespread stakeholder support, have a greater chance of success. The bank prepares lending and advisory

services, based on the selectivity framework and areas of comparative advantage, targeted to country poverty reduction efforts.

**Identification:** Projects that can be funded as part of the agreed development are identified. For tailoring bidding documents to the project concerned, the bank prepare an outline of the basic elements of the project, its proposed objective, likely risks, alternative scenarios to conducting the project, and a likely timetable for the project approval process.

**Preparation:** This part of the process is driven by the country that the Bank is working with and can take from a few months to three years, depending on the complexity of the project being proposed. The Bank provides policy analysis and project advice along with financial assistance where requested. During this period, the technical, institutional, economic, environmental, and financial issues facing the project are studied and addressed.

**Appraisal:** The bank assesses the economic, technical, institutional, financial, environmental, and social aspects of the project. The project appraisal document and draft legal documents are prepared.

**Negotiations and Broad Approval:** The bank and the country that is seeking to borrow the funds negotiate on loan or credit agreement. Both sides come to an agreement on the terms and conditions of the loan.

**Implementation and Supervision:** The borrower implements the project. The bank ensures that the loan proceeds are used for the loan purposes with due regard for economy, efficiency, and effectiveness.

**Implementation and Completion:** The implementation and completion report is prepared to evaluate the performance of both the bank and the borrower.

**Evaluation:** The bank prepares an independent audit report and evaluates the project. Analysis is used for future project design.

