XI. Turbomachinery

This chapter considers the theory and performance characteristics of the mechanical devices associated with the fluid circulation.

General Classification:

Turbomachine - A device which adds or extracts energy from a fluid.

Adds energy:	Pump
Extracts energy:	Turbine

In this context, a pump is a generic classification that includes any device that adds energy to a fluid, e.g. fans, blowers, compressors.

We can classify pumps by operating concept:

1. Positive displacement
2. Dynamic (momentum change)

General Performance Characteristics

Positive Displacement Pumps

1. Delivers pulsating or periodic flow (cavity opens, fluid enters, cavity closes, decreasing volume forces fluid out exit opening.
2. Not sensitive to wide viscosity changes.
3. Delivers a moderate flow rate.
4. Produces a high pressure rise.
5. Small range of flow rate operation (fixed pump speed).

Dynamic Pumps

1. Typically higher flow rates than PD's.
2. Comparatively steady discharge.
3. Moderate to low pressure rise.
4. Large range of flow rate operation.
5. Very sensitive to fluid viscosity.

Typical Performance Curves (at fixed impeller speed)

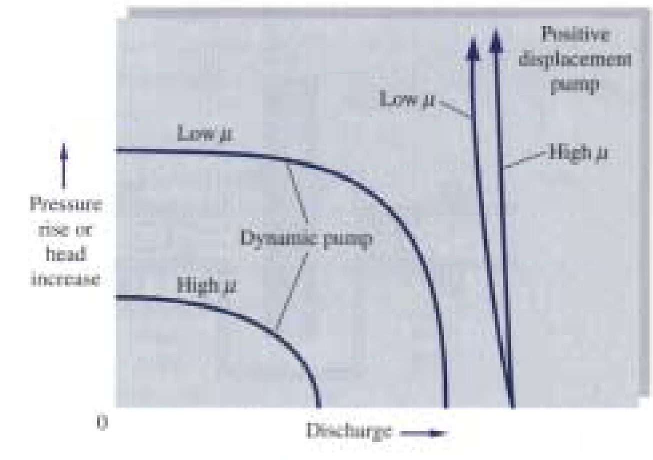


Fig. 11.2 Performance curves for dynamic and positive displacement pumps

Centrifugal Pumps

Most common turbomachine used in industry. Includes the general categories of (a) liquid pumps, (b) fans, (c) blowers, etc.

They are momentum change devices and thus fall within the dynamic classification.

Typical schematic shown as

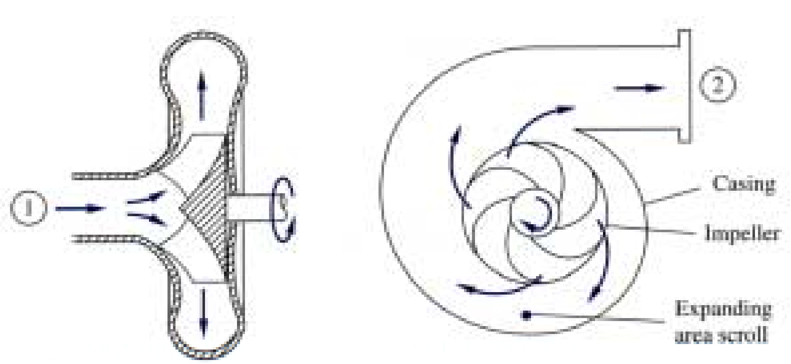


Fig. 11.3 Cutaway schematic of a typical centrifugal pump

Writing the energy equation across the device and solving for $h_p - h_f$,we have

$$H = h_p - h_f = \frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1$$

where H is the net useful head delivered to the fluid, the head that results in pressure, velocity, and static elevation change.

Since for most pumps (not all), $V_1 = V_2$ and ΔZ is small, we can write

$$H \cong \frac{\Delta P}{\rho g}$$

Since friction losses have already been subtracted, this is the ideal head delivered to the fluid. Note that velocity head has been neglected and can be significant at large flow rates where pressure head is small.

The ideal power to the fluid is given by $P_w = \rho\, Q\, g\, H$

The pump efficiency is given by $$\eta = \frac{P_w}{BHP} = \frac{\rho Q g H}{BHP} = \frac{\rho Q g H}{\omega T}$$

where BHP = shaft power necessary to drive the pump

ω = angular speed of shaft

T = torque delivered to pump shaft

Note that from the efficiency equation, pump efficiency is zero at zero flow rate Q and at zero pump head,H.

Basic Pump Theory

Development of basic pump theory begins with application of the integral conservation equation for moment-of-momentum previously presented in Ch. III.

Applying this equation to a centrifugal pump with one inlet, one exit, and uniform properties at each inlet and exit, we obtain

$$\bar{T} = \dot{m}_e \, \bar{r} \times \bar{V}_e - \dot{m}_i \, \bar{r} \times \bar{V}_i$$

where $\bar{T}$ is the shaft torque needed to drive the pump

$\bar{V}_i$, $\bar{V}_e$ are the absolute velocities at the inlet and exit of the pump

This is used to determine the change of angular momentum across the device.

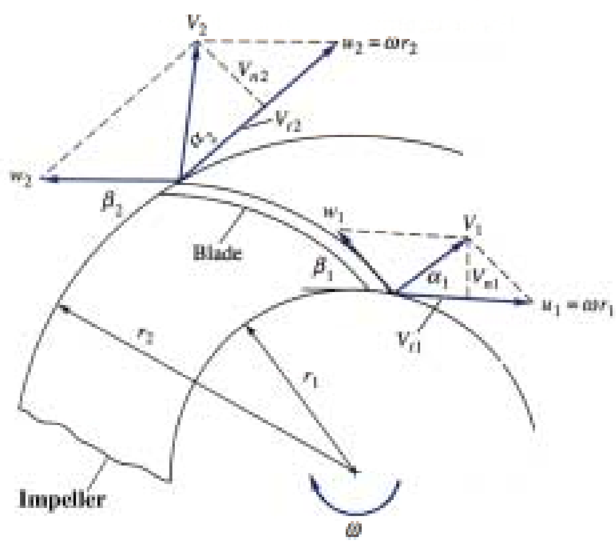
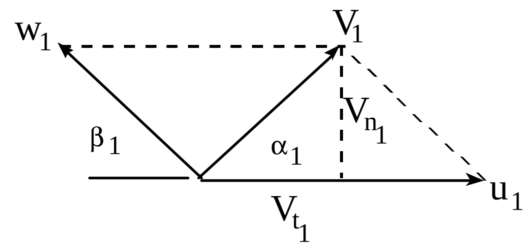


Fig. 11.4 Inlet and exit velocity diagrams for an idealized impeller

Since the velocity diagram is key to the analysis of the device, we will discuss the elements in detail.

1. At the inner radius r_1 have two velocity components:



a. the circumferential velocity due to the impeller rotation

$u_1 = r_1 \omega$ blade tip speed at inner radius

b. relative flow velocity tangent to the blade

w_1 tangent to the blade angle β_1

These combine to yield the absolute inlet velocity V_1 at angle α_1

The absolute velocity can be resolved into two absolute velocity components:

1. Normal (radial) component:

$$V_{n1} = V_1 \sin\alpha_1 = w_1 \sin\beta_1$$

Note that for ideal pump design,

$$V_{n1} = V_1 \quad and \quad \alpha_1 = 90^o$$

2. Absolute tangential velocity:

$$V_{t1} = V_1 \cos\alpha_1 = u_1 - w_1 \cos\beta_1$$

again, ideally $V_{t1} = 0$

It is also important to note that V_{n1} is use to determine the inlet flow rate, i.e.,

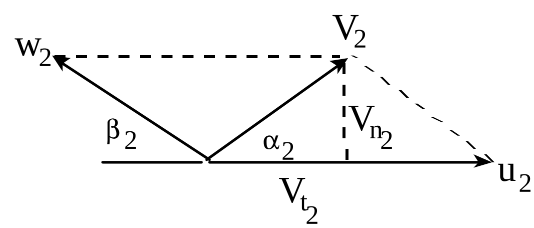
$$Q = A_1 V_{n1} = 2\pi r_1 b_1 V_{n1}$$

where b_1 is the inlet blade width

Likewise for the outer radius r_2 we have the following:

a. the circumferential velocity due to the impeller rotation

$u_2 = r_2\,\omega$ blade tip speed at outer radius



b. relative flow velocity tangent to the blade

w_2 tangent to the blade angle β_2

These again combine to yield the absolute outlet velocity V_2 at angle α_2

The exit absolute velocity can also be resolved into two absolute velocity components:

1. Normal (radial) component:

$$V_{n2} = V_2 \sin\alpha_2 = w_2 \sin\beta_2 = \frac{Q}{2\pi r_2 b_2}$$

Note that Q is the same as for the inlet flow rate

2. Absolute tangential velocity:

$$V_{t2} = V_2 \cos\alpha_2 = u_2 - w_2 \cos\beta_2$$

$$V_{t2} = u_2 - \frac{V_{n2}}{\tan\beta_2} = u_2 - \frac{Q}{2\pi r_2 b_2 \tan\beta_2}$$

where $$Q = A_1 V_{n1} = 2\pi r_1 b_1 V_{n1} = A_2 V_{n2} = 2\pi r_2 b_2 V_{n2}$$

Again, each of the above expressions follows easily from the velocity diagram, and the student should draw and use the diagram with each pump theory problem.

We can now apply moment - of – momentum equation.

$$\overline{T} = \rho Q\{r_2 * V_{t_2} - r_1 * V_{t_1}\}$$ (again V_{t1} is zero for the ideal design)

For a sign convention, we have assumed that V_{t1} and V_{t2} are positive in the direction of impeller rotation.

The " ideal" power supplied to the fluid is given by

$$P_w = \omega\overline{T} = \rho Q\{\omega r_2 \ V_{t_2} - \omega r_1 \ V_{t_1}\}$$

or

$$P_w = \omega\overline{T} = \rho Q\{u_2 \ V_{t_2} - u_1 \ V_{t_1}\} = \rho Q g H$$

Since these are ideal values, the shaft power required to drive a non-ideal pump is given by

$$BHP = \frac{P_w}{\eta_p}$$

The head delivered to the fluid is

$$H = \frac{\rho Q\{u_2 \ V_{t_2} - u_1 \ V_{t_1}\}}{\rho Q g} = \frac{\{u_2 \ V_{t_2} - u_1 \ V_{t_1}\}}{g}$$

For the special case of purely radial inlet flow

$$H^* = \frac{u_2 \ V_{t_2}}{g}$$

From the exit velocity diagram, substituting for V_{t2} we can show that

$$H = \frac{u_2^2}{g} - \frac{\omega Q}{2\pi b_2 g \tan\beta_2} \qquad \text{has the form} \qquad C_1 - C_2 Q$$

where: $C_1 = \dfrac{u_2^2}{g}$ shutoff head, the head produced at zero flow, Q = 0

Example:

A centrifugal water pump operates at the following conditions:

speed = 1440 rpm, r_1 = 4 in, r_2 = 7 in, $\beta_1 = 30^o$, $\beta_2 = 20^o$, $b_1 = b_2$ = 1.75 in

Assuming the inlet flow enters normal to the impeller (zero absolute tangential velocity):

find: (a) Q, (b) T, (c) W_p, (d) h_p, (e) ΔP

$$\omega = 1440 \frac{\text{rev}}{\text{min}} \frac{2\pi}{60} = 150.8 \frac{\text{rad}}{\text{s}}$$

Calculate blade tip velocities:

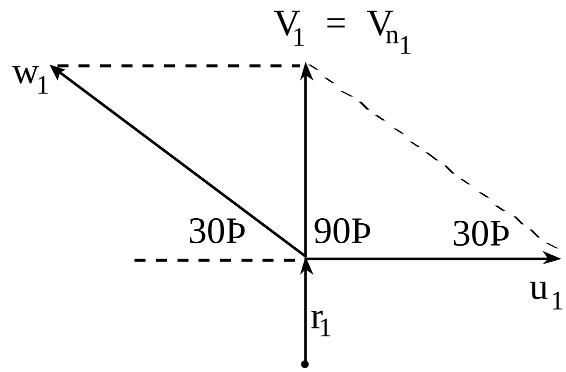
$$u_1 = r_1 \omega = \frac{4}{12}\text{ft}\, 150.8\frac{\text{rad}}{\text{s}} = 50.3\frac{\text{ft}}{\text{s}} \qquad u_2 = r_2 \omega = \frac{7}{12}\text{ft}\, 150.8\frac{\text{rad}}{\text{s}} = 88\frac{\text{ft}}{\text{s}}$$

Since design is ideal, at inlet

$\alpha_1 = 90^o$, $V_{t1} = 0$

$V_{n1} = U_1 \tan 30^0 = 50.3 \tan 30^o = 29.04$ ft/s

$$Q = 2\pi r_1 b_1 V_{n_1}$$

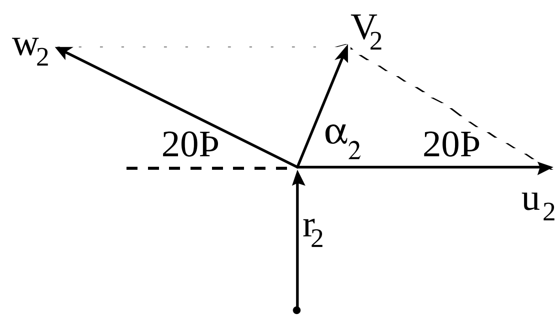


$$Q = 2\pi \frac{4}{12}\,\text{ft}\,1.75\,\text{ft}\,29.04\frac{\text{ft}}{\text{s}} = 8.87\frac{\text{ft}^3}{\text{s}}$$

$$Q = 8.87\frac{\text{ft}^3}{\text{s}}\,60\frac{s}{\text{min}}\,7.48\frac{\text{gal}}{\text{ft}^3} = 3981\frac{\text{gal}}{\text{min}}$$

Repeat for the outlet:

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = \frac{8.87\frac{\text{ft}^3}{\text{s}}}{2\pi\frac{7}{12}\,\text{ft}\,\frac{1.75}{12}\,\text{ft}}$$



$$V_{n2} = 16.6\frac{\text{ft}}{\text{s}}$$

$$w_2 = \frac{V_{n_2}}{\sin 20^o} = \frac{16.6\,\text{ft/s}}{\sin 20^o} = 48.54\frac{\text{ft}}{\text{s}}$$

$$V_{t_2} = u_2 - w_2\cos\beta_2 = 88 \quad - 48.54\cos 20^o = 42.4\frac{\text{ft}}{\text{s}}$$

We are now able to determine the pump performance parameters. Since for the centrifugal pump, the moment arm r_1 at the inlet is zero, the momentum equation becomes

Ideal moment of momentum delivered to the fluid:

$$T = \rho Q\{r_2 * V_{t2}\} = 1.938\frac{\text{slug}}{\text{ft}^3}\,8.87\frac{\text{ft}^3}{\text{s}}\,\frac{7}{12}\,\text{ft}\,42.4\frac{\text{ft}}{\text{s}} = 425.1\,\text{ft}-\text{lbf}$$

Ideal power delivered to the fluid:

$$P = \omega T = 150.8\frac{\text{rad}}{\text{s}}\,425.1\,\text{ft}-\text{lbf} = 64{,}103\frac{\text{ft}-\text{lbf}}{\text{s}} = 116.5\,\text{hp}$$

Head produced by the pump (ideal):

$$H = \frac{P}{\rho g Q} = \frac{64{,}103\,\text{ft} - \text{lbf/s}}{62.4\,\frac{\text{lbf}}{\text{ft}^3}\,8.87\,\frac{\text{ft}^3}{\text{s}}} = 115.9\,\text{ft}$$

Pressure increase produced by the pump:

$$\Delta P = \rho g H = 62.4\,\frac{\text{ft}^3}{\text{s}}\,115.9\,\text{ft} = 7226\,\text{psf} = 50.2\,\text{psi}$$

Pump Performance Curves and Similarity Laws

Pump performance results are typically obtained from an experimental test of the given pump and are presented graphically for each performance parameter.

- Basic independent variable - Q {usually gpm or cfm }
- Dependent variables typically

  H – head pressure rise, in some cases ΔP

  BHP – input power requirements (motor size)

  η – pump efficiency
- These typically presented at fixed pump speed and impeller diameter

Typical performance curves appear as

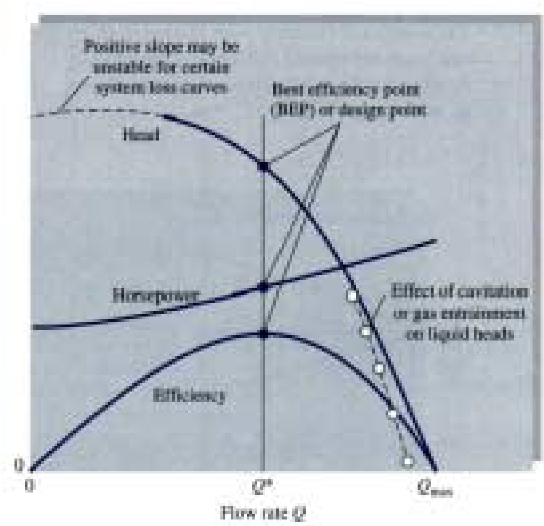


Fig. 11.6 Typical Centrifugal Pump Performance Curves
at Fixed Pump Speed and diameter

These curves are observed to have the following characteristics:

1. h_p is approximately constant at low flow rate.
2. $h_p = 0$ at Q_{max}.
3. BHP is not equal to 0 at Q = 0.
4. BHP increases monotonically with the increase in Q.
5. $\eta_p = 0$ at $Q = 0$ and at Q_{max}.
6. Maximum pump efficiency occurs at approximately $Q^* = 0.6\ Q_{max}$. This is the best efficiency point BEP. At any other operating point, efficiency is less, pump head can be higher or lower, and BHP can be higher or lower.
7. At the BEP, $Q = Q^*$, $h_p = h_p^*$, BHP = BHP*.

Measured Performance Data

Actual pump performance data will typically be presented graphically as shown in Fig. 11.7. Each graph will usually have curves representing the pump head vs flow rate for two or more impeller diameters for a given class/model of pumps having a similar design. The graphs will also show curves of constant efficiency and constant pump power (BHP) for the impeller diameters shown. All curves will be for a fixed pump impeller speed.

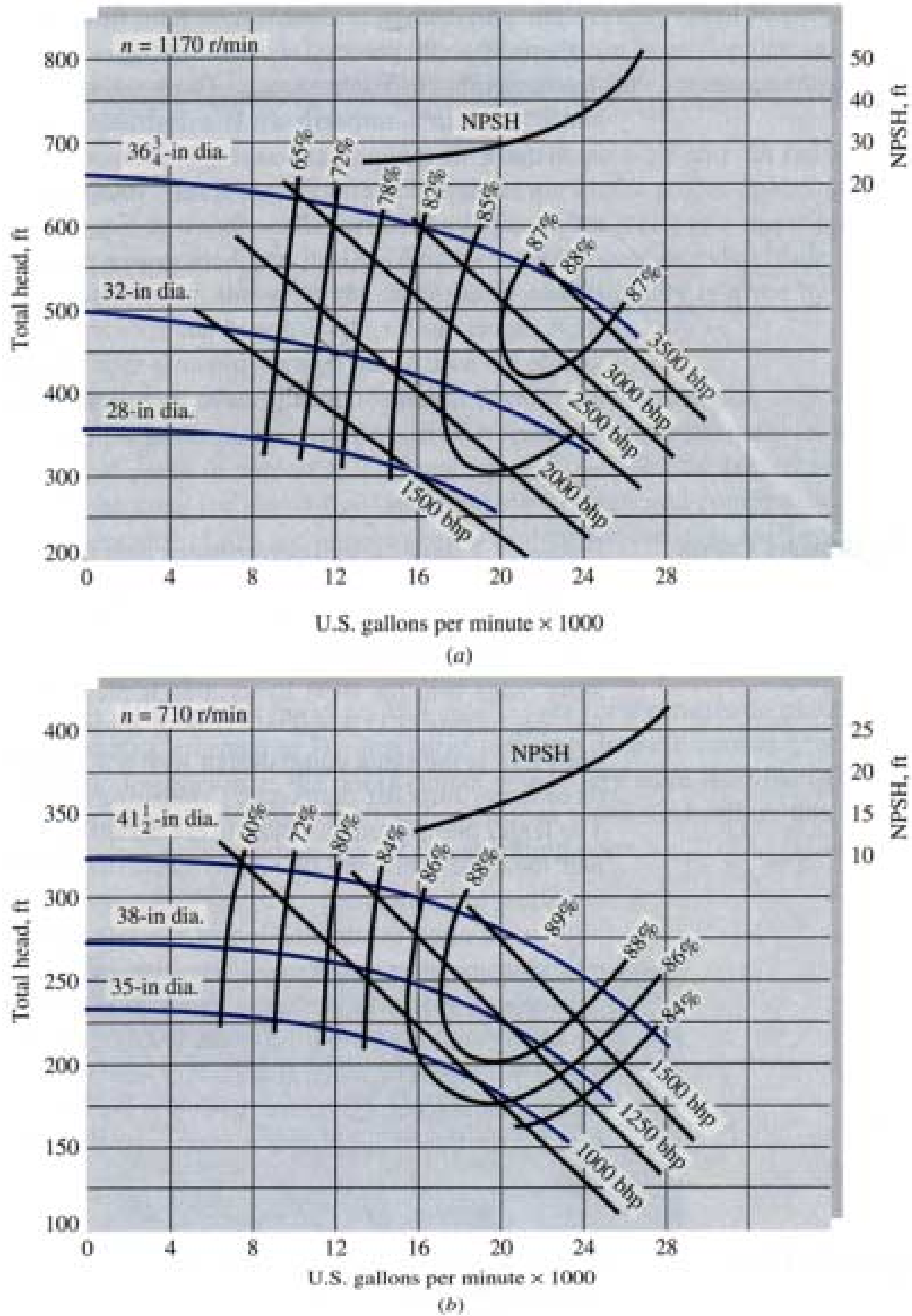


Fig. 11.7 Measured performance curves for two models of a centrifugal water pump

How to Read Pump Performance Curves

Care must be taken to correctly read the performance data from pump curves. This should be done as follows:

(1) For a given flow rate Q

(2) Read vertically to a point on the pump head curve h for the impeller diameter D of interest.

(3) All remaining parameters (efficiency & BHP) are read at this point; i.e., graphically interpolate between adjacent curves for BHP to obtain the pump power at this point.

Note that the resulting values are valid only for the conditions of these curves:

(1) pump model and design, (2) pump speed – N, (3) impeller size – D, (4) fluid (typically water)

Thus for the pump shown in Fig. 11.7a with an impeller diameter D = 32 in, we obtain the following performance at Q = 20,000 gpm:

Q = 20,000 gpm, D = 32 in, N = 1170 rpm

H ≅ 385 ft, BHP ≅ 2300 bhp, $\eta_p \cong 86.3$ %

Note that points that are not on an h vs. Q curve are not valid operating points. Thus for Fig. 11.7b, the conditions

Q = 22,000 gpm, BHP = 1500 bhp, hp = 250 ft

do not correspond to a valid operating point because they do not fall on one of the given impeller diameter curves. However, for the same figure, the point

Q = 20,000 gpm, BHP = 1250 bhp

is a valid point because it coincidentally also falls on the D = 38 in impeller curve at hp = 227 ft.

Net Positive Suction Head - NPH

One additional parameter is typically shown on pump performance curves:

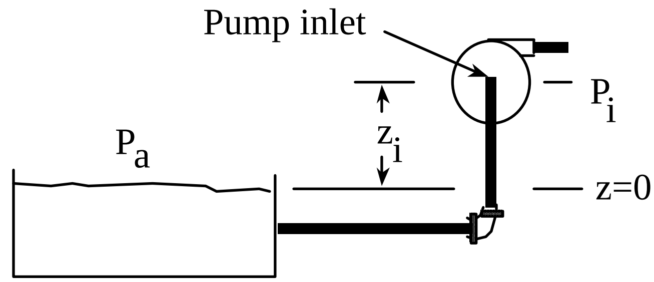
NPSH = head required at the pump inlet to keep the fluid from cavitating.

NPSH is defined as follows:

$$NPSH = \frac{P_i}{\rho g} + \frac{V_i^2}{2g} - \frac{P_v}{\rho g}$$

where P_i = pump inlet pressure
P_v = vapor pressure of fluid

Considering the adjacent figure, write the energy equation between the fluid surface and the pump inlet to obtain the following:

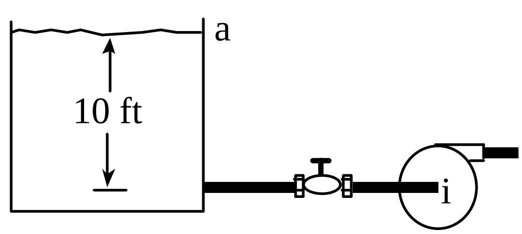


$$NPSH = \frac{P_i}{\rho g} + \frac{V_i^2}{2g} - \frac{P_v}{\rho g} = \frac{P_a}{\rho g} - Z_i - h_{f,a-i} - \frac{P_v}{\rho g}$$

For a pump installation with this configuration to operate as intended, the right-hand-side of the above equation must be > the NPSH value for the operating flow rate for the pump.

Example:

A water supply tank and pump are connected as shown. P_a = 13.6 psia and the water is at 20 °C with P_v = 0.34 psia. The system has a friction loss of 4.34 ft. Will the NPSH of the pump of Fig. 11.7a at 20,000 gpm work?



Applying the previous equation we obtain

$$\mathrm{NPSH} = \frac{P_a}{\rho g} - Z_i - h_{f,a-i} - \frac{P_v}{\rho g}$$

$$\mathrm{NPSH} = \frac{(13.6 - 0.34)\,\mathrm{lbf/in^2} * 144\,\mathrm{in^2/ft^2}}{62.4\,\mathrm{lbf/ft^3}} - (-10\,\mathrm{ft}) - 4.34\,\mathrm{ft}$$

$\mathrm{NPSH} = 36.26\,\mathrm{ft}$ The pump will work because the system NPSH as shown in Fig. 11.7a is 30 ft which provides a 6.3 ft safety margin. Conversely, the pump could be located as close as 3.7 ft below the water surface and meet NPSH requirements.

Pump Similarity Laws

Application of the dimensional analysis procedures of Ch. V will yield the following three dimensionless performance parameters:

Dimensionless flow coefficient: $C_Q = \dfrac{Q}{\omega D^3}$

Dimensionless head coefficient: $C_H = \dfrac{g H}{\omega^2 D^2}$

Dimensionless power coefficient: $C_P = \dfrac{BHP}{\rho \omega^3 D^5}$

where ω is the pump speed in radians/time and other symbols are standard design and operating parameters with units that make the coefficients dimensionless.

How are these used?

These terms can be used to estimate design and performance changes between two pumps of similar design.

Stated in another way:

If pumps 1 and 2 are from the same geometric design family and are operating at similar operating conditions, the flow rates, pump head, and pump power for the two pumps will be related according to the following expressions:

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}\left(\frac{D_2}{D_1}\right)^3$$

Use to predict the new flow rate for a design change in pump speed N and impeller diameter D.

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

Used to predict the new pump head H for a design change in pump speed, N and impeller diameter D.

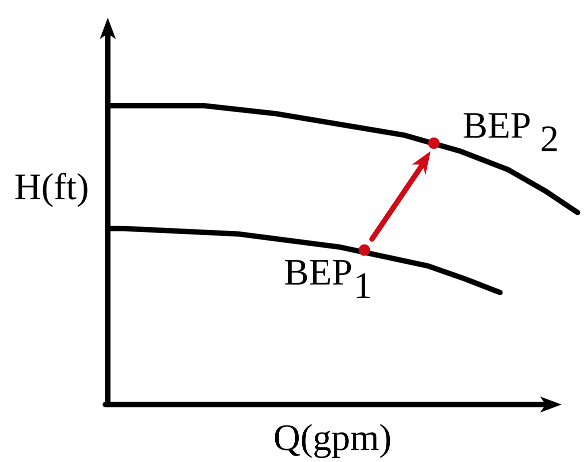
$$\frac{BHP_2}{BHP_1} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{N_2}{N_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5$$

Used to predict the new pump power BHP for a design change in fluid, ρ, pump speed N and impeller diameter D.

Example

It is desired to modify the operating conditions for the 38 in diameter impeller pump of Fig. 11.7b to a new pump speed of 900 rpm and a larger impeller diameter of 40 in.

Determine the new pump head and power for the new pump speed at the BEP.



For the D = 38 in impeller of Fig. 11.7b operating at 710 rpm, we read the best efficiency point (BEP) values as

$$Q^* = 20{,}000 \text{ gpm}, \quad H^* = 225 \text{ ft}, \quad BHP^* = 1250 \text{ hp}$$

Applying the similarity laws for N_2 = 900 rpm and $D_2 = D_1$ = 38 in, we obtain

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}\left(\frac{D_2}{D_1}\right)^3 = \frac{900}{710}\left(\frac{40}{38}\right)^3 = 1.478$$

Q_2 = 20,000*1.478 = 29,570 gpm **ans.**

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 = \left(\frac{900}{710}\right)^2\left(\frac{40}{38}\right)^2 = 1.78$$

H_2 = 225*1.78 = 400.5 ft **ans.**

$$\frac{BHP_2}{BHP_1} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 = (1)\left(\frac{900}{710}\right)^3\left(\frac{40}{38}\right)^5 = 2.632$$

BHP2 = 3290 hp **ans.**

Thus, even small changes in the speed and size of a pump can result in significant changes in flow rate, head, and power.

It is noted that every point on the original 38 in diameter performance curve exhibits a similar translation to a new operating condition.

The similarity laws are obviously useful to predict changes in the performance characteristics of an existing pump or to estimate the performance of a modified pump design prior to the construction of a prototype.

Matching a Pump to System Characteristics

The typical design/sizing requirement for a pump is to select a pump which has a pump head which matches the required system head at the design/operating flow rate for the piping system.

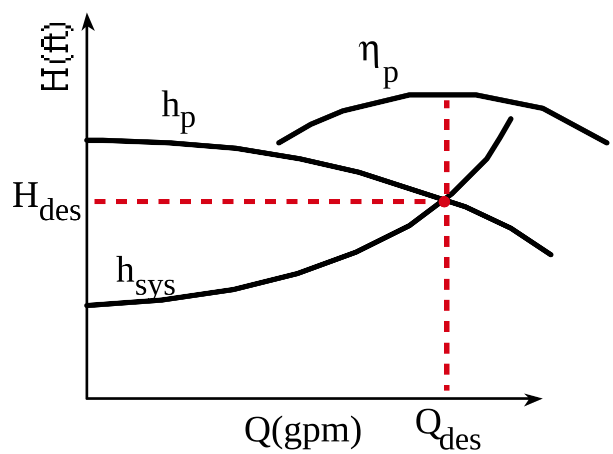
Key Point	$\mathbf{h_p = h_{sys}}$ **at** $\mathbf{Q_{des}}$.

It is noted that pump selection should occur such that the operating point of the selected pump should occur on the pump curve near or at the BEP.

From the energy equation in Ch. VI, the system head is typically expressed as

$$h_{sys} = \frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1 + \left\{ f\frac{L}{D} + \sum K_i \right\} \frac{V^2}{2g}$$

Thus the selection of a pump for a piping system design should result in a pump for which the pump head h_p at the design flow rate Q_{des} is equal (or very close) to the head requirements h_{sys} of the piping system at the same flow rate, and this should occur at or near the point of maximum efficiency for the chosen pump.

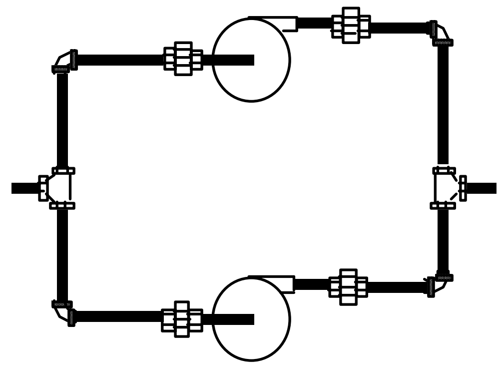
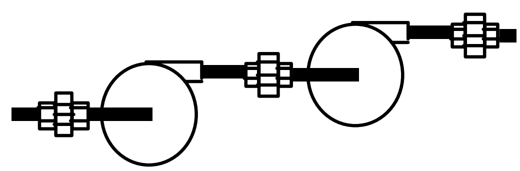


Other operating and performance requirements (such as NPSH) are obviously also a part of the selection criteria for a pump.

Pumping Systems: Parallel and Series Configurations

For some piping system designs, it may be desirable to consider a multiple pump system to meet the design requirements. Two typical options include parallel and series configurations of pumps. Specific performance criteria must be met when considering these options.

Given a piping system which has a known design flow rate and head requirements, Q_{des}, h_{des}. The following pump selection criteria apply.

<u>Pumps in Parallel</u>: Assuming that the pumps are identical, each pump must provide the following: Q(pump) = 0.5 Q_{des} h(pump) = h_{des}	
<u>Pumps in Series</u>: Assuming that the pumps are identical, each pump must provide the following: Q (pump) = Q_{des} h(pump) = 0.5 h_{des}	

For example, if the design point for a given piping system were Q_{des} = 600 gpm, and h_{sys} = 270 ft, the following pump selection criteria would apply:

1. Single pump system — Q(pump) = 600 gpm, h_p = 270 ft
2. Parallel pump system — Q(pump) = 300 gpm, h_p = 270 ft for each of the two pumps
3. Series pump system — Q(pump) = 600 gpm, h_p = 135 ft for each of the two pumps