Chapter 2 Hydrostatics

Hydrostatic Forces on Surfaces

Hydraulics I

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Hydrostatic Forces on Inclined Plane

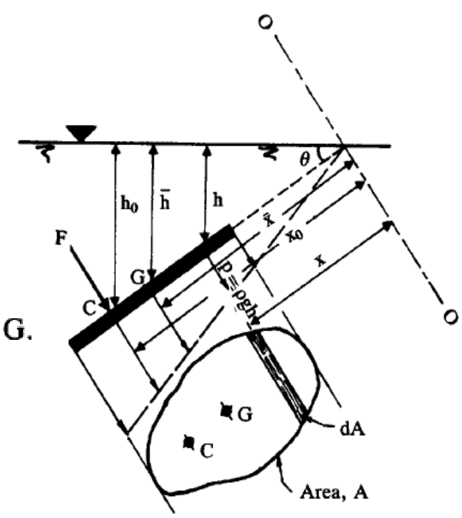
Hydrostatic Trust

$$F = \int_A dF = \int_A \rho g h \, dA = \rho g \sin\theta \int_A dA \, x$$
$$= \rho g \sin\theta \, A \, \bar{x}$$
$$= \rho g \bar{h} \, A$$

where $\bar{h}$ is the vertical depth of the centroid, G.

Taking moments of these forces about 0 – 0

$$F \, x_o = \rho g \sin\theta \int_A dA \, x^2$$



Hydrostatic Forces on Inclined Plane

∴ The distance to the centre of pressure, C

$$x_o = \int_A dA\, x^2 \Big/ \int_A dA\, x$$

$$= \frac{\text{second moment of the area about } 0-0}{\text{first moment of the area about } 0-0}$$

$$= I_o / A\bar{x}$$

But $I_o = I_g + A\bar{x}^2$ (parallel axis rule) where I_g is the second moment of area of the surface about an axis through its centroid and parallel to axis $0-0$.

$$\therefore x_o = \bar{x} + I_g/A\bar{x} \qquad (2.7)$$

Depth of centre of pressure below free surface, $h_o = x_o \sin\theta$

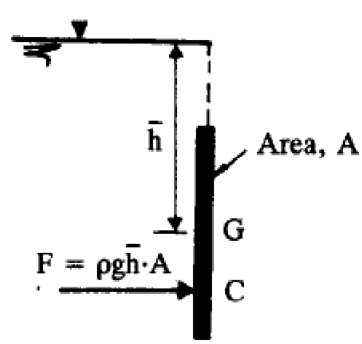
$$\therefore h_o = \bar{h} + I_g \sin^2\theta / A\bar{h}$$

Hydrostatic Forces on Vertical Plane

For a vertical surface $\theta = 90°$

$$\therefore h_o = \bar{h} + I_g/A\bar{h}$$

The distance between centroid and center of pressure
GC = I_g/Ah



∴ The moment of F about the centroid,

$$F \times GC = \rho g\bar{h}\, A \times I_g/A\bar{h}$$
$$= \rho g\, I_g$$

which is independent of depth of submergence.

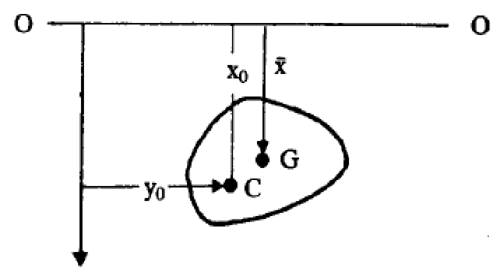
Hydrostatic Forces on Vertical Plane

When the surface is not symmetrical about the vertical centroidal axis.

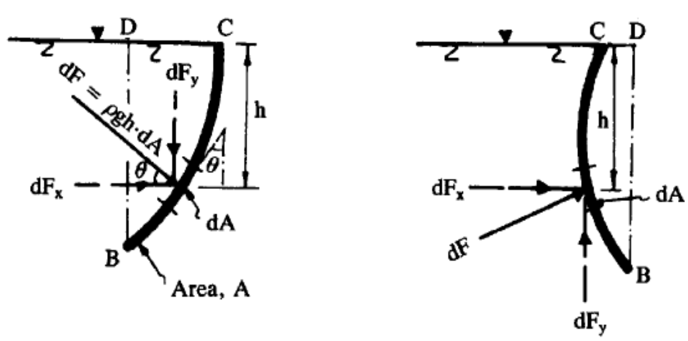
$$y_o \int_A dF = \int_A dF\, y$$

$$\text{or } y_o\, \rho g \bar{x} \sin\theta\, A = \int_A \rho g x \sin\theta\, dA\, y$$

$$\therefore\ y_o = \frac{1}{A\,\bar{x}} \int_A xy\, dA$$



Hydrostatic Forces on Curved Surfaces



a. Surface containing liquid

b. Surface displacing liquid

$\therefore$ Total thrust on this area, $dF = \rho g h\, dA$

Horizontal component of dF, $dF_x = \rho g h\, dA \cos\theta$

Vertical component of dF, $dF_y = \rho g h\, dA \sin\theta$

Hydrostatic Forces on Curved Surfaces

∴ Horizontal component of the total thrust on the curved area A,

$$F_x = \int_A \rho g h \, dA \cos\theta = \rho g \bar{h} A_v$$

Where A_v is the vertically projected area of the curved surface;

or F_x = pressure intensity at the centroid of a vertically projected area (BD) x vertically projected area

Hydrostatic Forces on Curved Surfaces

and vertical component, $F_y = \int_A \rho g h \, dA \sin\theta$

$= \rho g \int_A dV$, dV being the volume of the water prism (real or virtual) over the area dA.

$\therefore F_y = \rho g V$

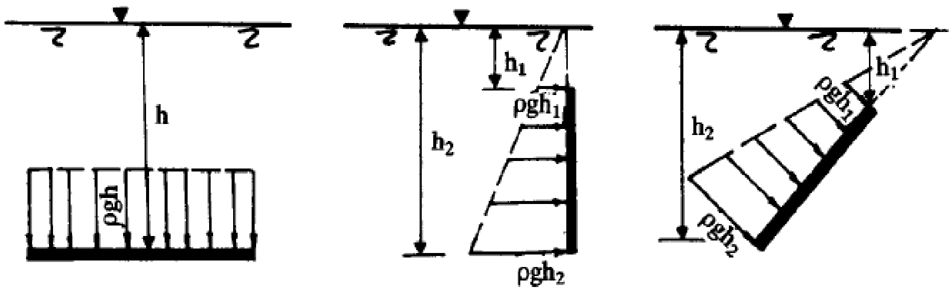
= the weight of water (real or virtual) above the curved surface BC bounded by the vertical BD and the free water surface CD

∴ The resultant thrust, $F = \sqrt{F_x^2 + F_y^2}$

acting normally to the surface at an angle,

$\alpha = \tan^{-1}(F_y/F_x)$ to the horizontal.

Pressure Diagram



Pressure Diagram

Average pressure on the surface = $\rho gH/2$

∴ Total thrust, F = average pressure × area of surface

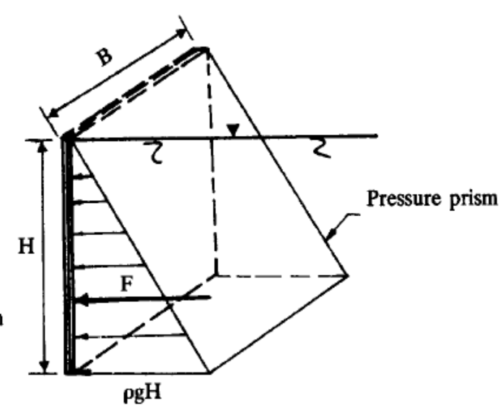
$$= (\rho gH/2)\ H \times B$$

$$= \frac{1}{2}\rho gH^2 \times B$$

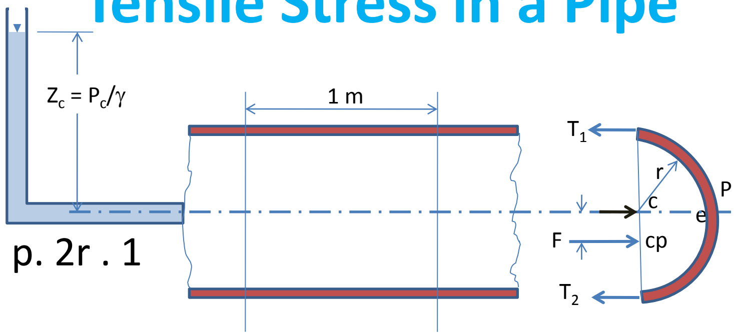
= volume of the pressure prism

or total thrust/unit width = $\frac{1}{2}\rho gH^2$

= area of the pressure diagram



Tensile Stress in a Pipe



$F = p.\ 2r\ .\ 1$

For small pressureT_1 is smaller than T_2, but for large pressure we can assume $T_1 = T_2$

$T_1 + T_2 = F = 2pr$

Since $T_1 + T_2 = 2T$, ➔ $T = pr$

Where, T is the tensile force per metre length of pipe

Tensile Stress in a Pipe

- For wall thickness t, the circumferential stress, σ

$$\sigma = \frac{T}{t \times 1} = \frac{p.r}{t}$$

For an allowable tensile stress σ_{all}, the required wall thickness t will be:

$$t = \frac{p.r}{\sigma_{all}}$$

Tensile Stress in a Pipe

- For large pressure variation, i.e. $Z_c = P_c/\gamma \leq 10r$

From $\Sigma F_h = 0 : T_1 + T_2 = F = 2p.r$

$\Sigma M_2 = 0 : 2rT_1 - 2pr(r-e) = 0$

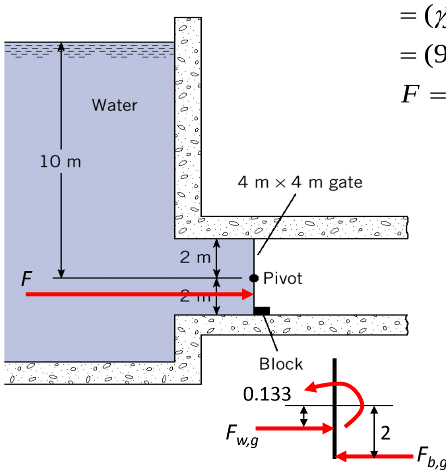
Finally, we get:

$$\sigma = \frac{T_2}{t} = \frac{p(r+e)}{t}$$

$$t = \frac{p(r+e)}{\sigma_{all}} = \frac{p(r + \frac{r^2\gamma}{3p_c})}{\sigma_{all}}$$

Example (1)

Find: Force of block on gate



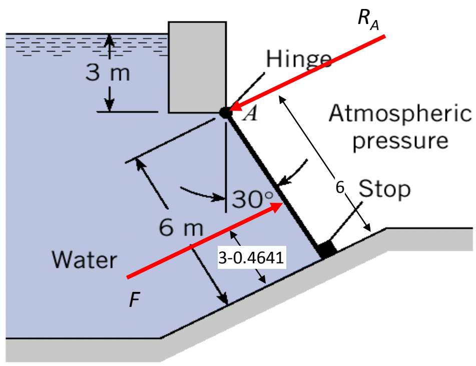
$$F = \bar{p}A$$
$$= (\gamma\bar{y}\sin\alpha)A$$
$$= (9810*10*1)*(4*4)$$
$$F = 1569.6\,kN$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$
$$= \frac{4*4^3/12}{(10*4*4)}$$
$$= 0.133\,m$$

$$\sum M = 0$$
$$= 0.133F_{w,g} - 2F_{b,g}$$
$$F_{b,g} = \frac{0.133}{2}F_{w,g}$$
$$= \frac{0.133}{2}1569.6\,kN$$
$$F_{b,g} = 104.378\,kN$$

Example (2)

Find the reaction at A

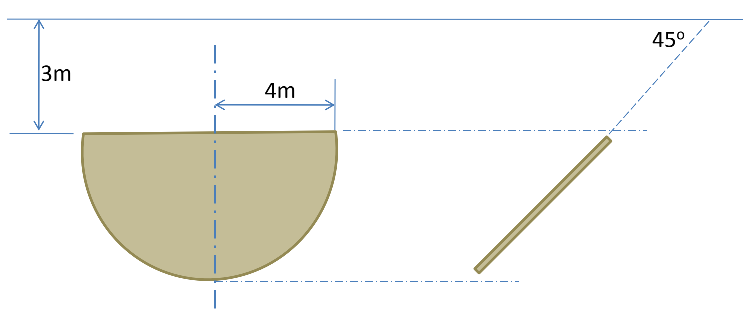


$$F = \bar{p}A = (\gamma \bar{y} \sin\alpha)A$$
$$= 9810 * (3 + 3\cos 30) * (4 * 6)$$
$$= 1{,}318{,}000\, N$$
$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{4 * 6^3 / 12}{(6.464 * 24)}$$
$$= 0.4641\, m$$

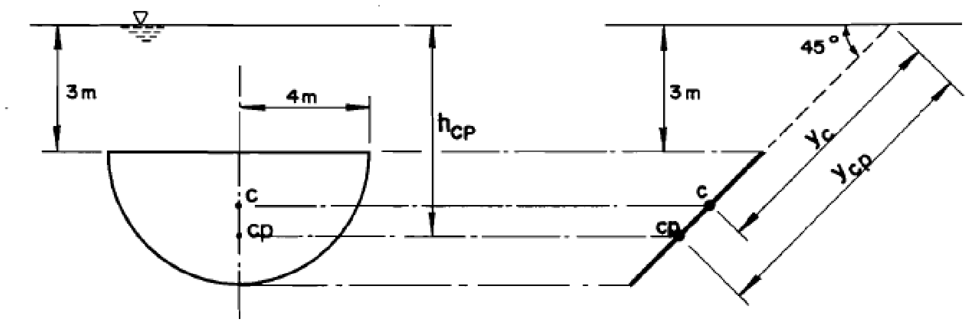
$$\sum M = 0$$
$$= 6R_A - (3 - 0.4641)F$$
$$R_A = \frac{3 - 0.4641}{6} F$$
$$= (0.42265) 1318\, kN$$
$$\boxed{R_A = 557.05\, kN}$$

Example 3

Determine the total hydrostatic force and the center of pressure



Example 3 Solution



Example 3 Solution

Total hydrostatic pressure = F = $\gamma_w h_c.A$

$h_c = y_c \sin 45^o$

$y_c = 3/\sin 45^0 + 4r/3\pi = 5.94$ m

Therefore, $h_c = 4.20$ m

Area of gate , A = $\pi r^2/2 = \pi \times 4^2/2 = 25.13\ m^2$

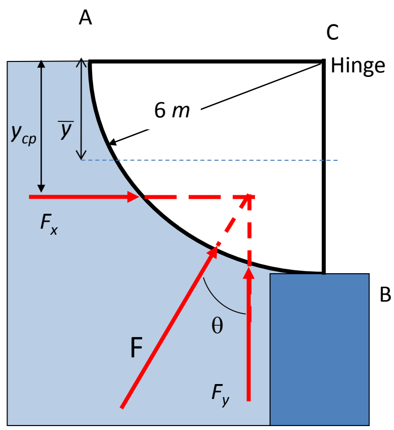
Thus, F = 9.81 x 4.20 x 25.13 = 1.035 MN

$$y_{cp} = y_c + \frac{I_c}{y_c A} = 5.94 + \frac{0.11 \times 4^4}{5.94 \times 25.13} = 6.13m$$

And $h_{cp} = y_{cp} \sin 45^o = 4.33$ m

Example 4

Determine the magnitude and direction of the force acting on the quarter circle gate of 1 m long.

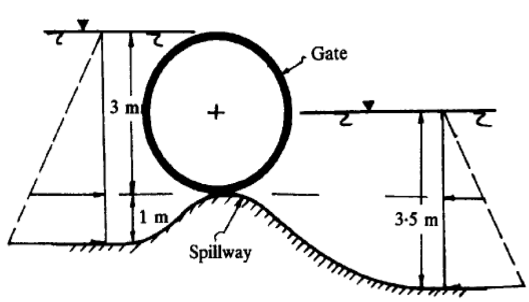


$$F_x = \gamma \bar{y} A_{CB}$$
$$= 9810 * 3 * 6 * 1$$
$$= 176.6\, kN$$

F = 328.84 kN

Example 5

A 3 m diameter roller gate retains water on both sides of a spillway crest as shown in the figure below. Determine (i) the magnitude, direction and location of the resultant hydrostatic thrust acting on the gate per unit length, and (ii) the horizontal water thrust on the spillway per unit length.



Example 5 solution

Left side: horizontal component $= \frac{1}{2}\rho g \times 3^2$

$= 44{\cdot}14$ kN/m

vertical component $= \rho g \times \frac{1}{2}\frac{\pi}{4}3^2 \times 1$

$= 34{\cdot}67$ kN/m

Right side: horizontal component $= \frac{1}{2}\rho g\,(1{\cdot}5)^2$

$= 11{\cdot}03$ kN/m

vertical component $= \rho g \times \frac{1}{4}\frac{\pi}{4}3^2 \times 1$

$= 17{\cdot}34$ kN/m

Example 5 solution

$\therefore$ Net horizontal component on the gate (left to right)

$= 44{\cdot}14 - 11{\cdot}03$

$= 33{\cdot}11$ kN/m

and net vertical component (upwards) $= 34{\cdot}67 + 17{\cdot}34$

$= 50{\cdot}01$ kN/m

$\therefore$ Resultant hydrostatic thrust on the gate

$= \sqrt{(33.11)^2 + (50.01)^2}$

$= 60$ kN/m

acting at an angle, $\alpha = \tan^{-1}(33{\cdot}11/50{\cdot}01) = 33°30'$ to the vertical and passes through the centre of the gate (normal to the surface).

Example 5 solution

∴ Depth of centre of pressure $= r + r \cos \alpha$

$= 1{\cdot}5\ (1 + \cos 33°30')$

$= 2{\cdot}75$ m below the free surface of left side.

Example 5 solution

Horizontal thrust on the spillway:
From pressure diagrams (see fig. 2.28), thrust from left-hand side

$$= \frac{1}{2}(\rho g \times 3 + \rho g \times 4) \times 1$$

$$= 34{\cdot}33 \text{ kN/m}$$

and from right-hand side $= \frac{1}{2}(\rho g \times 1{\cdot}5 + \rho g \times 3{\cdot}5) \times 2$

$$= 49{\cdot}05 \text{ kN/m}$$

∴ Resultant thrust (horizontal) on the spillway

$$= 49{\cdot}05 - 34{\cdot}33$$

$= 14{\cdot}72$ kN/m towards left.