Addis Ababa institute of Technology

Reinforced Concrete Structures 2
(CEng-3122)

Chapter One
Plastic Moment Redistribution

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April 23, 2020

School of Civil and Environmental Engineering
Concrete Material and Structures Chair

Presentation Outline

Content

1. Introduction
2. Moment Curvature Relationship
3. Rotation Capacity
4. Continuous Beams
5. Plastic Hinges and Collapse Mechanisms
6. Moment Redistribution

Introduction

- Analysis of Reinforced Concrete Structures
- Methods of Analysis Allowed in EC-2

Analysis of RC Structures

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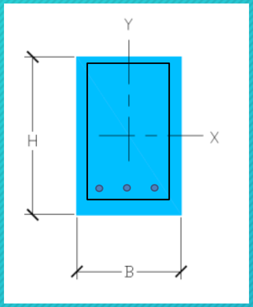
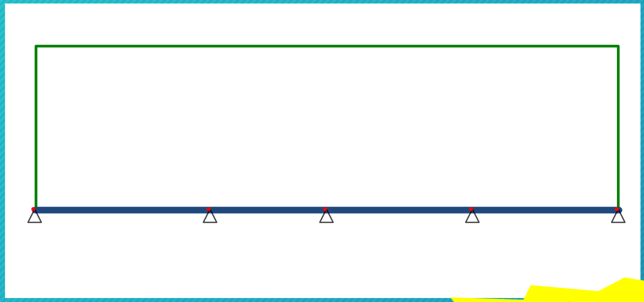
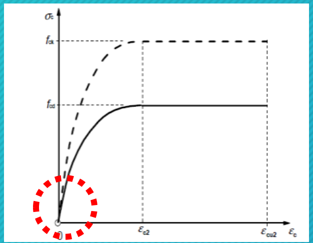
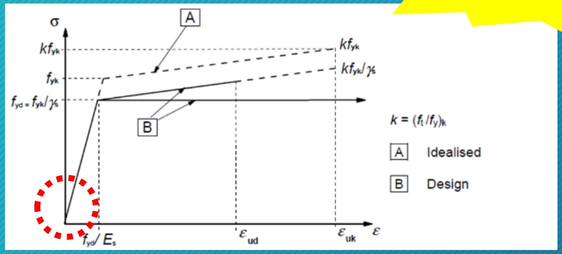
- The purpose of any analysis is to know how the structure responds to a given loading and there by evaluate the stresses and deformations.

Given: the following sets of parameters







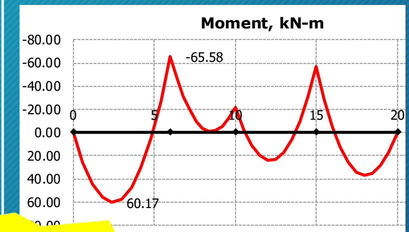


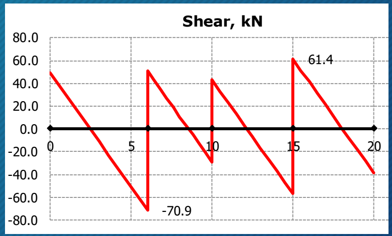
Material Proper… Geometry, Loading and …tructural Layout

So far in the course analysis are based on linear elastic theory.

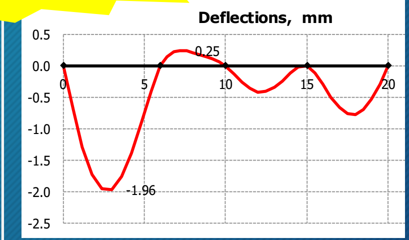
Carrying out Elastic Analysis: Results ...

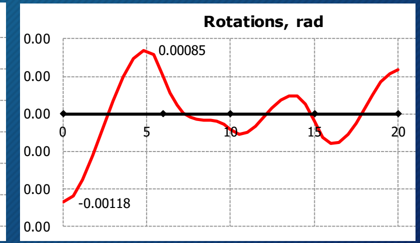
Stresses





Deformation





Analysis of RC Structures

Most reinforced concrete structures are designed for internal forces found by elastic theory with methods such as *slope deflection*, *moment distribution*, and *matrix analysis*.

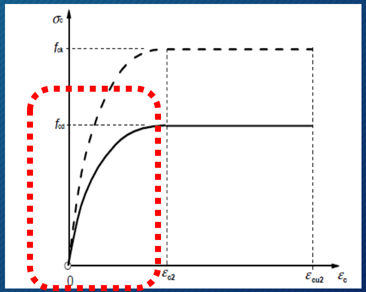
There is an apparent inconsistency in determining the design moments based on an *elastic analysis*, while doing the design based on a *limit state design* procedure, where the *structural design* is based on inelastic section behavior.

Analysis

- ✓ Factored loads
- ✓ Elastic Analysis

Design

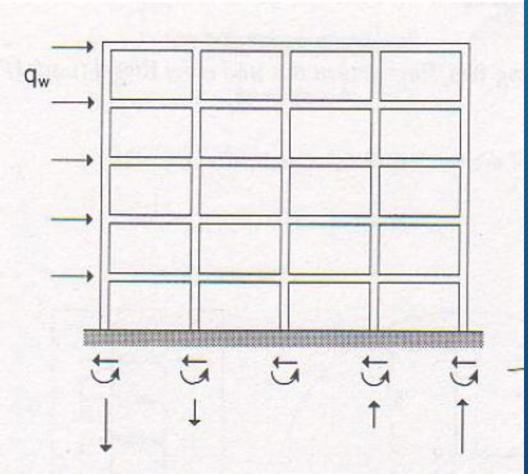
- ✓ The tensile reinforcement is proportioned on the assumption that its well beyond its yielding point at failure. (Ductile Design or $\varepsilon_s \geq 4.313‰$)
- ✓ Concrete stress distribution across the section is non-linear.



Although the analysis and design basis are contradictory, it will be a safe and to a degree a conservative design.

Methods of Analysis Allowed in EC-2

The methods of analysis provided on EC-2 are for the purpose to establish the distribution of either internal forces and moments, or stresses, strains and displacements, over the whole or part of a structure.



1. Linear Elastic Analysis (section 5.4 – EC2)

THIS IS THE TYPE OF ANALYSIS BEING CARRIED OUT SOFAR.

- Based on the theory of elasticity.
- Suitable for both SLS and ULS.
- Assumption:

i. uncracked cross sections

ii. linear stress-strain (ε vs σ) relationships and,

iii. mean values of the elastic modulus [E].

- For thermal deformation, settlement and shrinkage effects at the (ULS),a reduced stiffness corresponding to the cracked sections may be assumed.
- For the (SLS) gradual evolution of cracking should be considered (eg. rigorous deflection calculation).

Methods of Analysis Allowed in EC-2

2. Linear Elastic Analysis with Limited Redistribution (section 5.5)

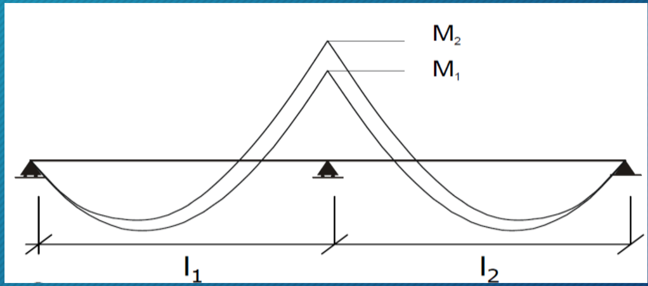
WILL BE INTRODUCED IN THE FOLLWING CHAPTER OF THE COURSE

- Suitable for ULS.
- The moments at ULS calculated using a linear elastic analysis may be redistributed, provided that the resulting distribution of moments remains in equilibrium with the applied loads.
- Redistribution of bending moments may be carried out without explicit check on the rotation capacity, provided that:

  $0,5 \leq l_1/\ l_2 \leq 2,0$

  Ratio of redistribution $\delta = M_1/M_2 < 1$, is

✓ $\delta \geq k_1 + k_2 x_u/d$ for $f_{ck} \leq 50$ MPa
✓ $\delta \geq k_3 + k_4 x_u/d$ for $f_{ck} > 50$ MPa
✓ $\delta \geq k_5$ for reinforcement class B & C
✓ $\delta \geq k_6$ for reinforcement class A



x_u is the depth of the neutral axis at the ultimate limit state after redistribution.
recommended value for k_1 is 0,44, for k_2 is $1,25(0,6+0,0014/\varepsilon_{cu2})$, for $k_3 = 0,54$, for $k_4 = 1,25(0,6+0,0014/\varepsilon_{cu2})$, for $k_5 = 0,7$ and $k_6 = 0,8$

Methods of Analysis Allowed in EC-2

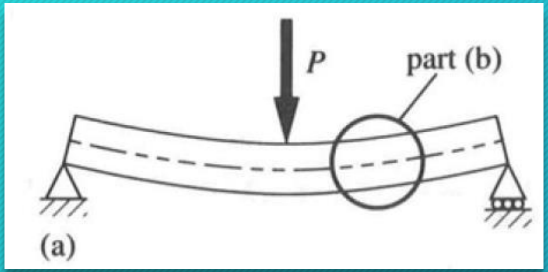
Plastic Analysis (section 5.6) WILL BE INTRODUCED IN THIS CHAPTER OF THE COURSE	• Suitable for ULS. • Suitable for SLS if compatibility is ensured. • When a beam yields in bending, an increase in curvature does not produce an increase in moment resistance. Analysis of beams and structures made of such flexural members is called ***Plastic Analysis***. • This is generally referred to as ***limit analysis***, when applied to reinforced concrete framed structures, and ***plastic analysis*** when applied to steel structures
non-Linear Analysis (section 5.7) IT IS BEYOUND THE SCOPE OF THE COURSE	• Nonlinear analysis may be used for both ULS and SLS, provided that equilibrium and compatibility are satisfied and an adequate non-linear behavior for materials is assumed. • The non-linear analysis procedures are more complex and therefore very time consuming. • The analysis maybe first Or second order.

Moment Curvature Relationship

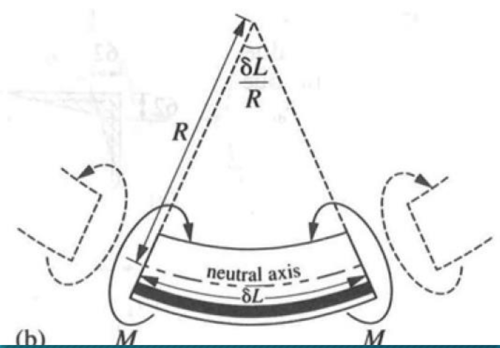
- Curvature
- Basic Assumption and Consideration in Establishing the Moment Curvature Relationship
- Procedures in Establishing the Moment Curvature Relationship

Curvature: Introduction

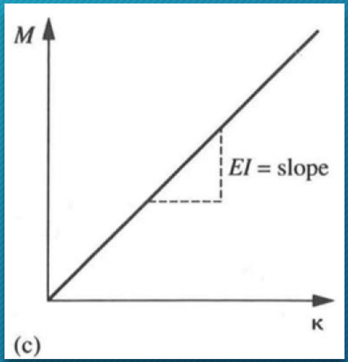
For a beam with homogeneous cross-section, which is loaded in bending is shown below.



Beam loaded in bending.



Segment of the beam loaded in bending.



Relationship between bending moment M and curvature K for beam with linear elastic homogeneous material.

$$K = \frac{M}{E \cdot I} \text{...... From Elastic Theory}$$

Where:

E= the modulus of elasticity

I=the moment of inertia of the cross-section

K=the local curvature=1/R

But is Concrete a Homogenous, elastic material?

Then how do we determine the moment curvature relationship for it?

Why do we even bother compute the M – K relationship in the first place?

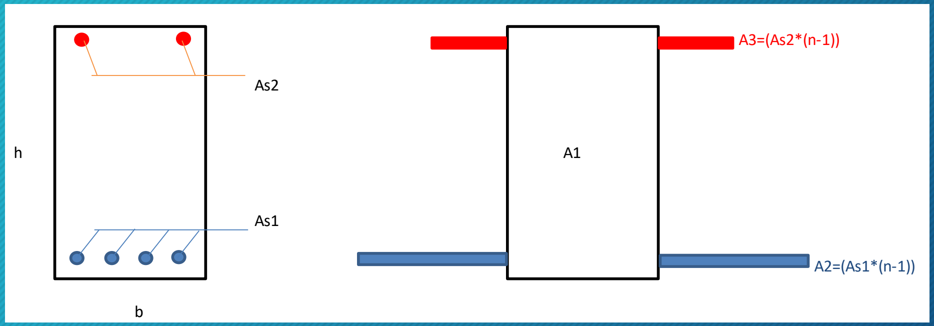
Curvature: RC section

Reinforced concrete is not homogeneous because it is composed of steel and concrete which have different values for the elastic modulus; however, it is possible to identify an equivalent homogeneous concrete section with an equivalent moment of inertia.

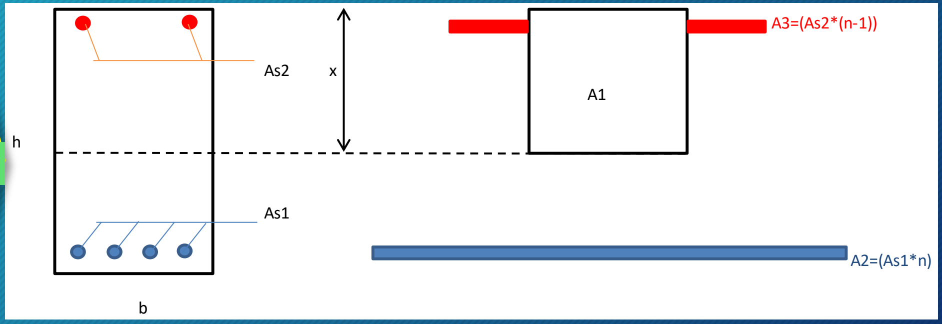
This is done by means of an equivalent transformed cross section.

To have the same material property of concrete across the RC section the reinforcement is transformed in to an equivalent concrete area using the modular ratio n=Es/Ec

Uncracked



Fully-Cracked



Curvature: RC section

It is important:

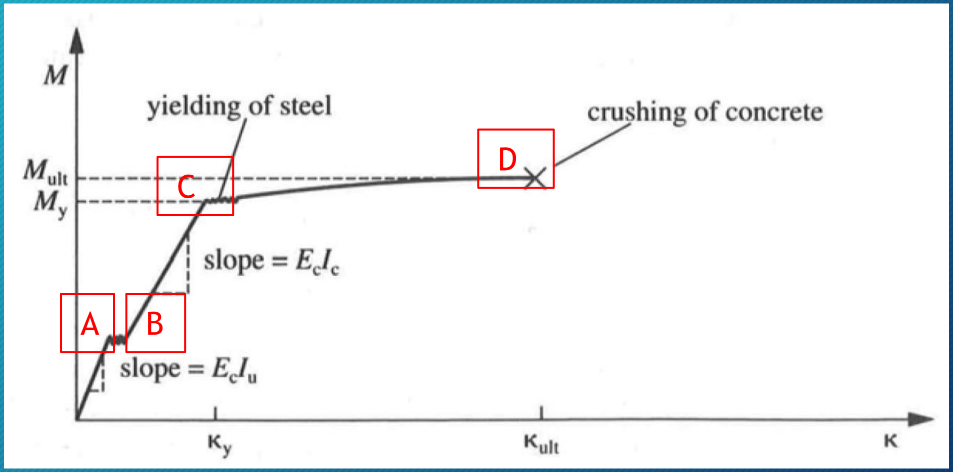
- to study the ductility of members
- to understand the development of plastic hinge, and
- to account for the redistribution of elastic moments that occurs in most reinforced concrete structures before collapse.

Curvature: RC section

The curve M-K may be calculated for every given cross-section in reinforced concrete; this is typically done by the calculation of some salient points:

A. M and K just before the appearance of the flexural crack in the cross-section
B. M and K just after the appearance of the flexural crack
C. M and K when steel start to yield
D. M and K when failure is reached (normally due to the crushing of the compression concrete)

A Typical M-K diagram for a RC section.



Basic Assumption and Consideration in Establishing the M-K Relationship

Basic Assumptions	• Parabolic-rectangular stress block for concrete in compression is assumed. • Tensile strength of concrete is neglected. • Plane section remains plane before and after bending. • Elasto-Plastic stress strain relationship is assumed for reinforcement steel in tension. • Steel is perfectly bonded with concrete.
Basic Considerations	• Equilibrium of forces shall be maintained. • Compatibility of Strains shall be maintained. • Stress-Strain relationship has to be satisfied .

Procedures in Establishing the M-K Relationship

The general steps to be followed in computing the moment curvature relationship of RC section are as follows.

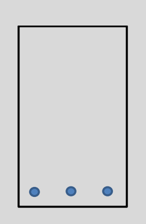
1. Assume the strain of the outer most fiber of concrete. [ε_c]
2. Assume the N.A. depth. [x]
3. From the linear strain distribution across the section compute the strain of the reinforcement bar in tension and the corresponding stress in it. [ε_{s1} and σ_{s1}]
4. Compute the total compressive and tensile forces. [C_c and T_s]
5. Check equilibrium of forces. [$C_c = T_s$ or $C_c \neq T_s$]
6. Determine lever arm [z] and calculate the moment [M] and the corresponding curvature [K].

Example 1.1 : For RC beam section with b/h=200/400mm, casted out of C20/25 concrete and reinforced by s-400. determine the moment curvature relationship of the section?

a) 3φ14

b) 3φ24

[Use cover to longitudinal reinforcement bar 33mm]



Solution: a) 3φ14

Step1: Summarize the given parameters

Material C20/25 f_{ck}=20MPa; f_{cd}=11.33MPa; f_{ctm}=2.2MPa; E_{cm}=30,000MPa

S-400 f_{yk}=400MPa; f_{yd}=347.83MPa; E_s=200,000MPa; ε_y=1.74‰

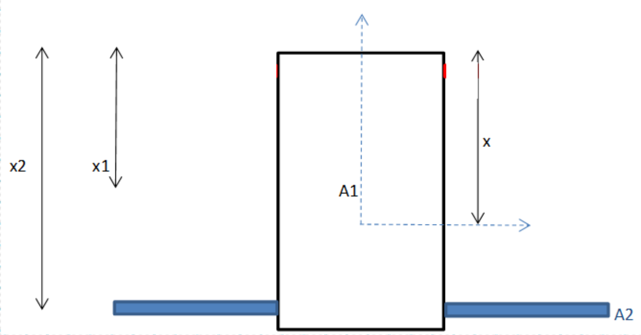
Modular ratio, n= E_s/ E_{cm}=6.67

Geometry d=h-cover- φ/2=400-33-7=360mm

A_{s1}=3xπx(7mm)^2=461.81mm^2

Step2: Compute the cracking moment and corresponding curvature. [M_{cr}, K_{cr}]

I. *Uncracked section properties.*



The neutral axis depth of the uncracked section

$$A_1 = b \times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1)\times A_{s1} = (6.67-1)\times 461.81 = 2618.46mm^2$$

And considering the top fiber as a refrence axis

$$x_1 = \frac{h}{2} = 200mm$$

$$x_2 = d = 360mm$$

Therefore:-

$$x = \frac{\sum A_i x_i}{\sum A_i} = \frac{(A_1 \times x_1) + (A_2 \times x_2)}{(A_1 + A_2)} = 205.07mm$$

The second moment of the area of the uncracked section

$$I_1 = \left(\frac{bh^3}{12}\right) = \left(\frac{200\times 400^3}{12}\right) = 1066666666.67mm^4$$

$$I_2 \approx 0$$

$$A_1 = b \times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1)\times A_{s1} = (6.67-1)\times 461.81 = 2618.46mm^2$$

$$y_1 = x - \frac{h}{2} = 205.07 - 200 = 5.07mm$$

$$y_2 = d - x = 360 - 205.07 = 154.93mm$$

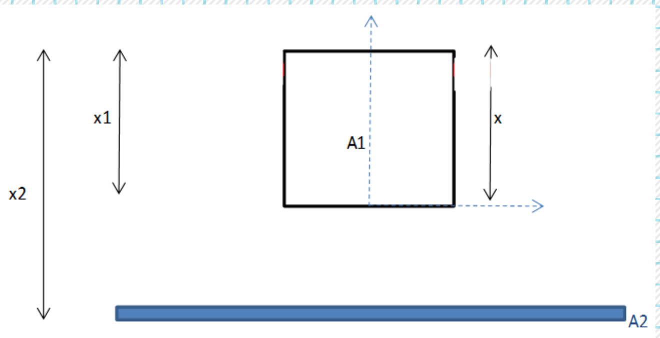
Therefore : -

$$I_I = I_1 + I_2 + (A_1 \times y_1^2) + (A_2 \times y_2^2)$$

$$I_I = 1066666666.67 + 0 + (80000 \times 5.07^2) + (2618.46 \times 154.93^2)$$

$$I_I = 1131574752.42mm^4$$

II. Cracked section properties



The neutral axis depth of the cracked section

From equilibrium of forces carried by the concrete in the compression zone and the tension force carried by the transformed concrete area in tension we have the following expression.

$$\frac{1}{2}b\left(k_x d\right)^2 = nA_{s1}(d - k_x d)$$

Dividing the above expression by bd^2 and denoting $\rho = A_{s1}/bd$ results in:

$$k_x = \frac{x}{d} = -[n\rho] + \sqrt{[n\rho]^2 + 2[n\rho]}$$

$$n = 6.67$$

$$\rho = \frac{461.81}{360 \times 200} = 0.006414$$

$$x = 0.258d = 91.023mm$$

The second moment of the area of the cracked section

$$I_1 = \left(\frac{bx^3}{12}\right) = \left(\frac{200 \times 91.023^3}{12}\right) = 12569042.224mm^4$$

$$I_2 \approx 0$$

$$A_1 = b \times x = 200x91.023 = 18204.6mm^2$$

$$A_2 = n \times A_{s1} = 6.67 \times 461.81 = 3080.27mm^2$$

$$y_1 = x - \frac{x}{2} = 45.5115mm$$

$$y_2 = d - x = 360 - 91.023 = 268.977mm$$

Therefore : -

$$I_{II} = I_1 + I_2 + (A_1 \times y_1^2) + (A_2 \times y_2^2)$$

$$I_{II} = 12569042 + 0 + (18204.6 \times 45.5115^2) + (3080.27 \times 268.977^2)$$

$$I_{II} = 273129472.51mm^4$$

III. Compute the cracking moment.

$$M_{cr} = \frac{f_{ctm} I_I}{y_t}$$

$$y_t = h - x = 400 - 205.07 = 194.93mm$$

Therefore

$$M_{cr} = \frac{2.2 \times 1131574752.42}{194.93} = 12.77kNm$$

IV. Compute the curvature just before cracking.

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_I}$$

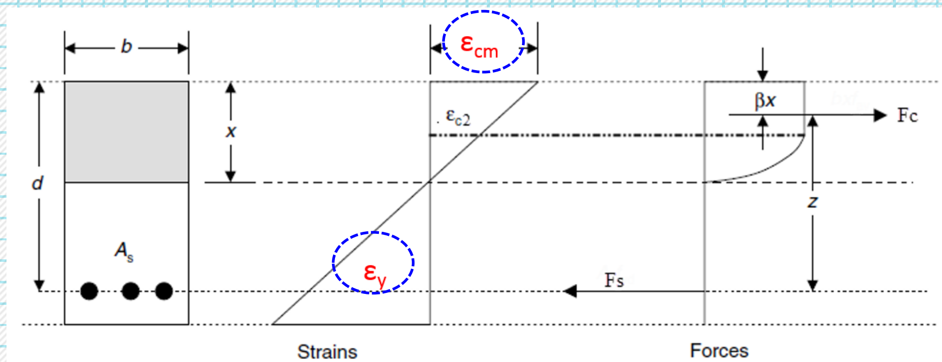
$$\kappa_{cr} = \frac{12770000Nmm}{30000\frac{N}{mm^2} \times 1131574752.42mm^4} = 0.3767 \times 10^{-6} mm^{-1}$$

V. Compute the curvature just after cracking.

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_{II}}$$

$$\kappa_{cr} = \frac{12770000Nmm}{30000\frac{N}{mm^2} \times 273129472.51mm^4} = 1.558 \times 10^{-6} mm^{-1}$$

Step3: Compute the yielding moment and corresponding curvature. $[M_y, K_y]$



Assuming 0< ε_{cm} <2‰ and from force equilibrium.

$$Cc = Ts$$

$$\alpha_c f_{cd} bd = A_{s1} f_{yd}$$

$$\alpha_c = \frac{A_{s1} f_{yd}}{f_{cd} bd} = \frac{461.81 \times 347.83}{11.33 \times 200 \times 360} = 0.197$$

From the strain profile

$$k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + \varepsilon_y} = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 1.74}$$

From the simplified equations discussed in chapter two of RC-1

$$\alpha_c = \varepsilon_{cm} \left[\frac{6 - \varepsilon_{cm}}{12} \right] k_x = 0.197$$

From the two equations above we can solve for ε_{cm} to be 1.208. Assumption correct!

$$k_x = \frac{1.208}{1.208 + 1.74} = 0.410$$

$$x = d \times k_x = 360 \times 0.410 = 147.6mm$$

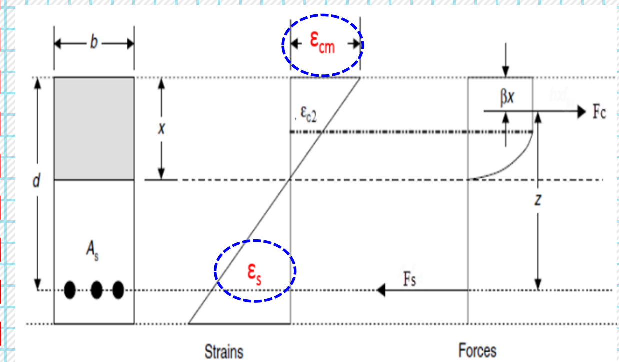
$$\beta_c = k_x \left[\frac{8 - \varepsilon_{cm}}{4(6 - \varepsilon_{cm})} \right] = 0.145$$

$$z = d(1 - \beta_c) = 360(1 - 0.145) = 307.8mm$$

$$M_y = A_{s1} f_{yd} z = 49.442kNm$$

$$\kappa_y = \frac{\varepsilon_{cm}}{x} = \frac{1.178 \times 10^{-3}}{145.44mm} = 8.10 \times 10^{-6} mm^{-1}$$

Step4: Compute the ultimate moment and corresponding curvature. $[M_u, K_u]$



Assuming a compression failure ε_{cm} =3.5‰, ε_y < ε_s <25‰ and from force equilibrium.

$$\alpha_c = \frac{A_{s1} f_{yd}}{f_{cd} bd} = \frac{461.81 \times 347.83}{11.33 \times 200 \times 360} = 0.197$$

From the strain profile

$$k_x = \frac{3.5}{3.5 + \varepsilon_s}$$

From the simplified equations discussed in chapter two of RC-1

$$\alpha_c = k_x \left[\frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right] = 0.197$$

From the two equations above we can solve for ε_s to be 10.88 … Assumption correct!

$$k_x = \frac{3.5}{3.5 + 10.88} = 0.243$$

$$x = d \times k_x = 360 \times 0.243 = 87.48mm$$

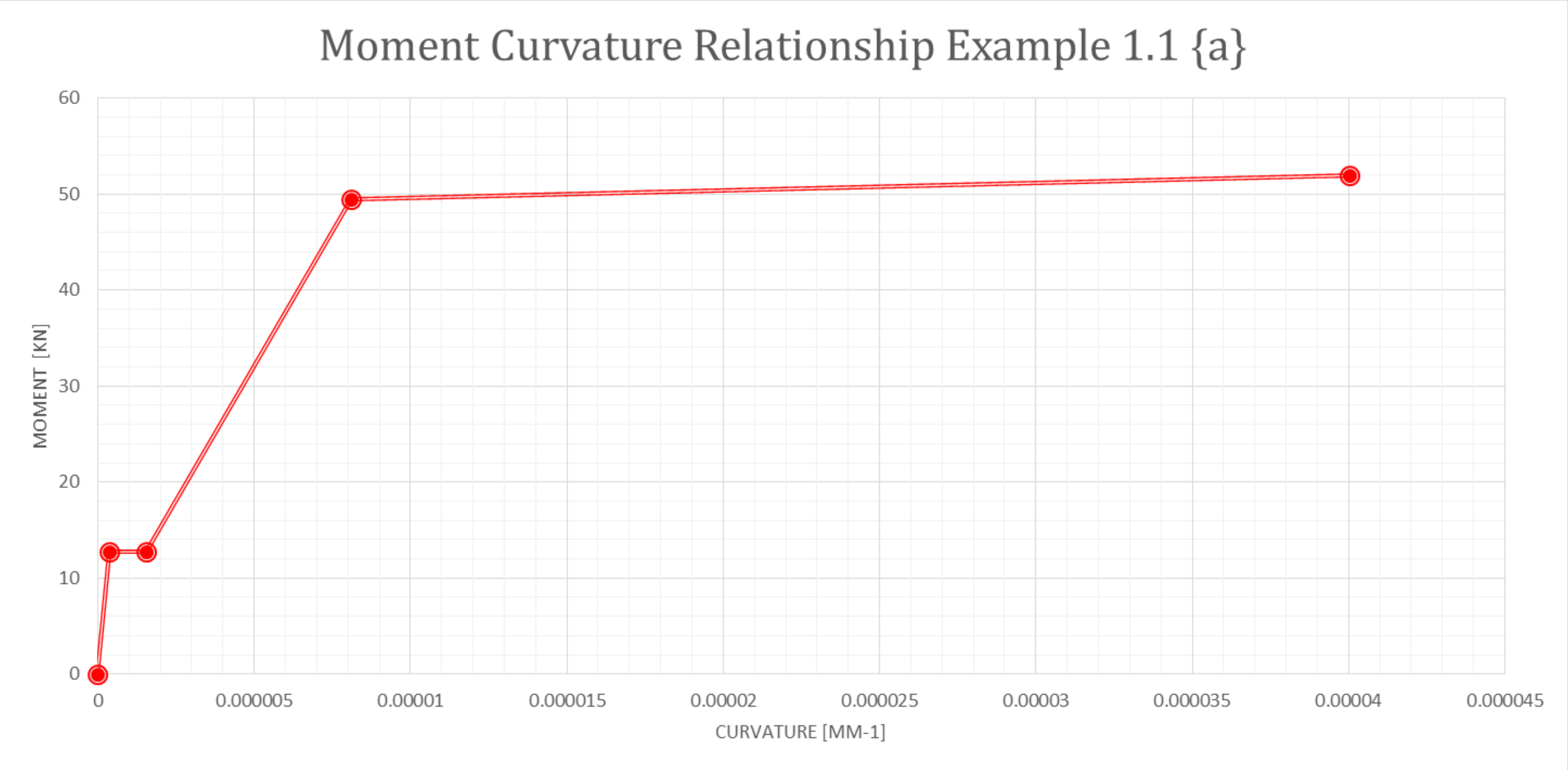
$$\beta_c = k_x \left[\frac{\varepsilon_{cm}(3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm}(3\varepsilon_{cm} - 2)} \right] = 0.101$$

$$z = d(1 - \beta_c) = 360(1 - 0.101) = 323.64mm$$

$$M_u = A_{s1} f_{yd} z = 51.99kNm$$

$$\kappa_u = \frac{\varepsilon_{cm}}{x} = \frac{3.5 \times 10^{-3}}{87.26mm} = 40.11 \times 10^{-6} mm^{-1}$$

Step5: Plot the moment vs curvature diagram



Home take Bonus exam:

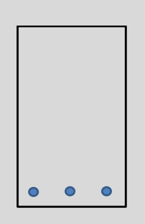
Redo example1.1 considering the role of concrete in the tension zone of the reinforced concrete section.

Example 1.1 : For RC beam section with b/h=200/400mm, casted out of C20/25 concrete and reinforced by s-400. determine the moment curvature relationship of the section?

a) 3φ14

b) 3φ24

[Use cover to longitudinal reinforcement bar 33mm]



Solution: b) 3φ24

Step1: Summarize the given parameters

Material C20/25 f_{ck}=20MPa; f_{cd}=11.33MPa; f_{ctm}=2.2MPa; E_{cm}=30,000MPa

S-400 f_{yk}=400MPa; f_{yd}=347.83MPa; E_s=200,000MPa; ε_y=1.74‰

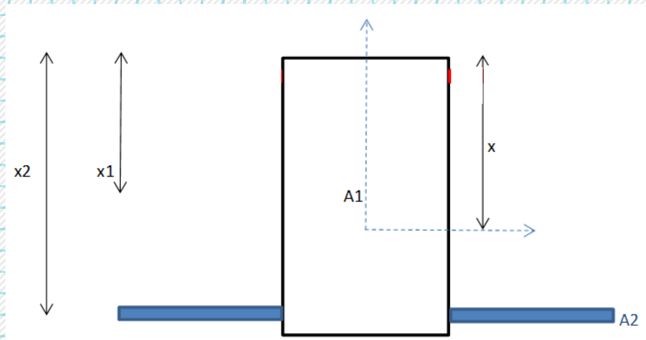
Modular ratio, n= E_s/ E_{cm}=6.67

Geometry d=h-cover- φ/2=400-33-12=355mm

A_{s1}=3xπx(12mm)^2=1356.48mm^2

Step2: Compute the cracking moment and corresponding curvature. [M_{cr}, K_{cr}]

I. *Uncracked section properties.*



The neutral axis depth of the uncracked section

$$A_1 = b \times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1) \times A_{s1} = (6.67-1) \times 1356.48 = 7691.24mm^2$$

And considering the top fiber as a refrence axis

$$x_1 = \frac{h}{2} = 200mm$$

$$x_2 = d = 355mm$$

Therefore:-

$$x = \frac{\sum A_i x_i}{\sum A_i} = \frac{(A_1 \times x_1) + (A_2 \times x_2)}{(A_1 + A_2)} = 213.6mm$$

The second moment of the area of the uncracked section

$$I_1 = \left(\frac{bh^3}{12}\right) = \left(\frac{200 \times 400^3}{12}\right) = 1066666666.67mm^4$$

$$I_2 \approx 0$$

$$A_1 = b \times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1) \times A_{s1} = (6.67-1) \times 1356.48 = 7691.24mm^2$$

$$y_1 = x - \frac{h}{2} = 213.6 - 200 = 13.6mm$$

$$y_2 = d - x = 355 - 213.67 = 141.33mm$$

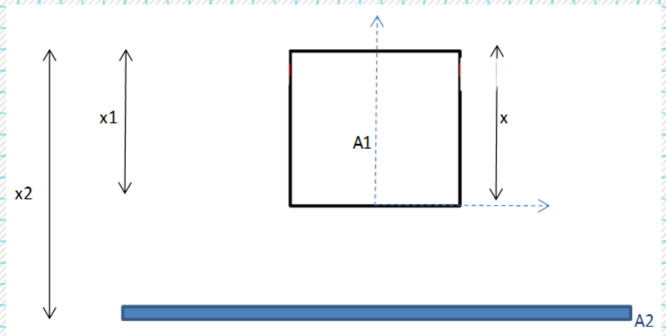
Therefore : -

$$I_I = I_1 + I_2 + (A_1 \times y_1^2) + (A_2 \times y_2^2)$$

$$I_I = 1066666666.67 + 0 + (80000 \times 13.6^2) + (7691.24 \times 141.33^2)$$

$$I_I = 1235089593.48mm^4$$

II. Cracked section properties



The neutral axis depth of the cracked section

From equilibrium of forces carried by the concrete in the compression zone and the tension force carried by the transformed concrete area in tension we have the following expression.

$$\frac{1}{2}b\left(k_x d\right)^2 = nA_{s1}(d - k_x d)$$

Dividing the above expression by bd^2 and denoting $\rho = A_{s1}/bd$ results in:

$$k_x = \frac{x}{d} = -[n\rho] + \sqrt{[n\rho]^2 + 2[n\rho]}$$

$$n = 6.67$$

$$\rho = \frac{1356.48}{355 \times 200} = 0.0191$$

$$x = 0.393d = 139.60mm$$

The second moment of the area of the cracked section

$$I_1 = \left(\frac{bx^3}{12}\right) = \left(\frac{200 \times 139.60^3}{12}\right) = 45342452.27mm^4$$

$$I_2 \approx 0$$

$$A_1 = b \times x = 200x139.60 = 27920mm^2$$

$$A_2 = n \times A_{s1} = 6.67 \times 1356.48 = 9047.72mm^2$$

$$y_1 = x - \frac{x}{2} = 69.8mm$$

$$y_2 = d - x = 355 - 139.60 = 215.4mm$$

Therefore : -

$$I_{II} = I_1 + I_2 + (A_1 \times y_1^{\,2}) + (A_2 \times y_2^{\,2})$$

$$I_{II} = 45342452.27 + 0 + (27920 \times 69.8^2) + (2618.46 \times 215.4^2)$$

$$I_{II} = 302858916.6mm^4$$

III. Compute the cracking moment.

$$M_{cr} = \frac{f_{ctm} I_I}{y_t}$$

$$y_t = h - x = 400 - 213.6 = 186.4mm$$

Therefore

$$M_{cr} = \frac{2.2 \times 1235089593.48}{186.4} = 14.58kNm$$

IV. Compute the curvature just before cracking.

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_I}$$

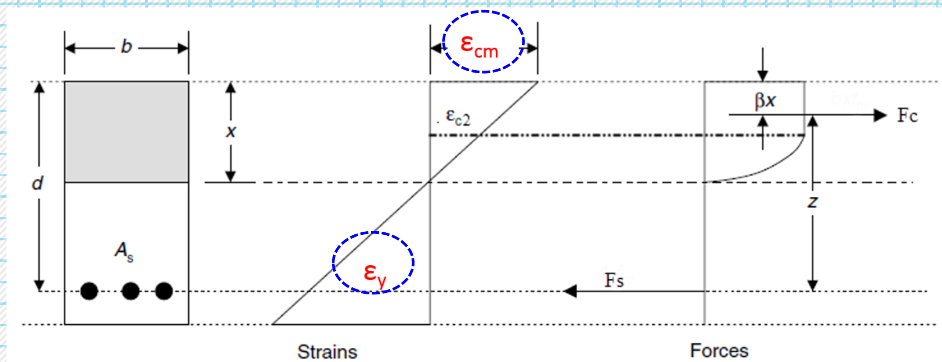
$$\kappa_{cr} = \frac{12770000Nmm}{30000\frac{N}{mm^2} \times 1235089593.48mm^4} = 0.34464 \times 10^{-6}mm^{-1}$$

V. Compute the curvature just after cracking.

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_{II}}$$

$$\kappa_{cr} = \frac{14580000Nmm}{30000\frac{N}{mm^2} \times 302858916.6mm^4} = 1.605 \times 10^{-6}mm^{-1}$$

Step3: Compute the yielding moment and corresponding curvature. $[M_y, K_y]$



Assuming 2‰< εcm <3.5‰ and from force equilibrium.

$$Cc = Ts$$

$$\alpha_c f_{cd} bd = A_{s1} f_{yd}$$

$$\alpha_c = \frac{A_{s1} f_{yd}}{f_{cd} bd} = \frac{1356.48 \times 347.83}{11.33 \times 200 \times 355} = 0.587$$

From the strain profile

$$k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + \varepsilon_y} = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 1.74}$$

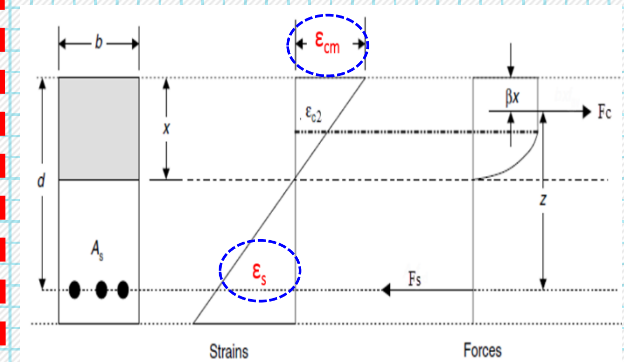
From the simplified equations discussed in chapter two of RC-1

$$\alpha_c = k_x \left[\frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right] = 0.587$$

From the two equations above we can solve for ε_{cm} to be 4.08

4.08‰>3.5‰, implies that the concrete in the compression zone has crushed even before the reinforcement in the tension zone has yielded.
Hence the section has reached its ultimate moment capacity, along with the corresponding curvature, before the yielding of the reinforcement.

Step4: Compute the ultimate moment and corresponding curvature. $[M_u, K_u]$



Assuming a compression failure ε_{cm} =3.5‰, $\varepsilon_s < \varepsilon_y$ and from force equilibrium.

$$Cc = Ts$$

$$\alpha_c f_{cd} bd = A_{s1}\sigma_s = A_{s1}(E_s \times \varepsilon_s)$$

$$\alpha_c = \frac{A_{s1}\sigma_s}{f_{cd} bd} = \frac{1356.48 \times 200000 \times \varepsilon_s}{11.33 \times 200 \times 355} = 0.33725\varepsilon_s$$

Where ε_s is in ‰

From the strain profile

$$k_x = \frac{3.5}{3.5 + \varepsilon_s}$$

From the simplified equations discussed in chapter two of RC-1

$$\alpha_c = k_x \left[\frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right] = 0.33725\varepsilon_s$$

From the two equations above we can solve for ε_s to be 1.636 … Assumption correct!

$$k_x = \frac{3.5}{3.5 + 1.636} = 0.681 \text{ … Indicates a brittle failure!}$$

$$x = d \times k_x = 355 \times 0.681 = 241.755mm$$

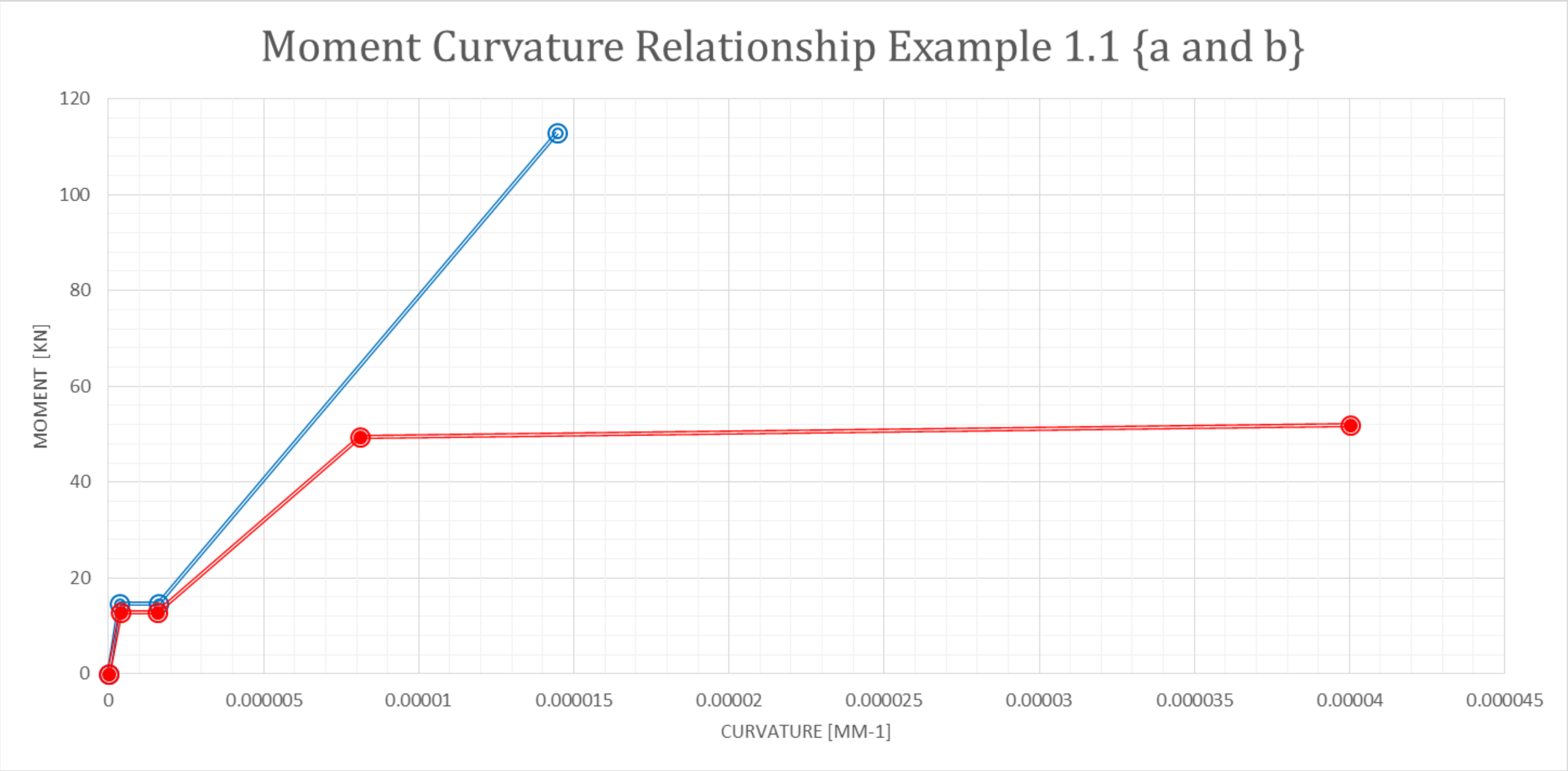
$$\beta_c = k_x \left[\frac{\varepsilon_{cm}(3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm}(3\varepsilon_{cm} - 2)} \right] = 0.283$$

$$z = d(-\beta_c) = 355(1 - 0.101) = 254.43mm$$

$$\boxed{M_u = A_{s1}(E_s \times \varepsilon_s)z = 112.93kNm}$$

$$\boxed{\kappa_u = \frac{\varepsilon_{cm}}{x} = \frac{3.5 \times 10^{-3}}{241.755mm} = 14.477 \times 10^{-6} mm^{-1}}$$

Step5: Plot the moment vs curvature diagram



Observation:

- Failure type vs moment curvature relationship
- Reinforcement in tension zone vs Ductility
- Ultimate capacity vs Ductility

Question:

- How would you improve the ductility of the section in (b)?
- How would you improve the moment capacity of the section in (a) with out compromising its ductility?

Rotation Capacity

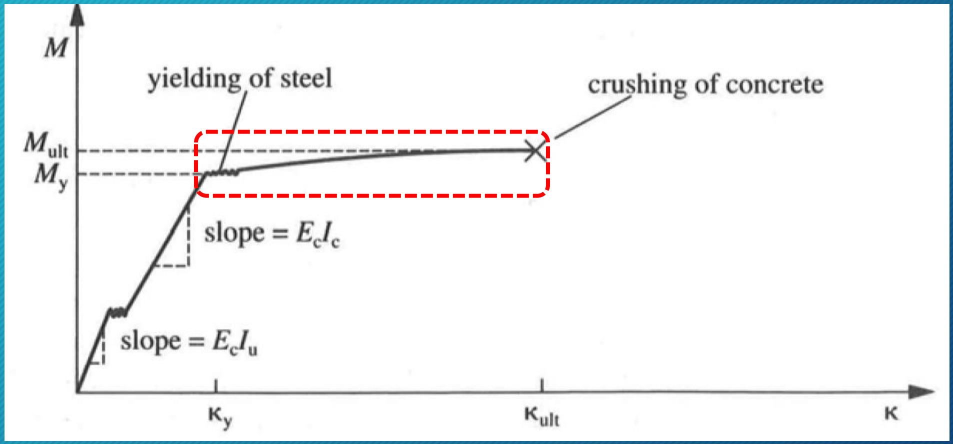
24

- Introduction
- Rotational Capacity According EC-2

Rotation Capacity: Introduction

- The designer adopting limit/plastic analysis in concrete must calculate the inelastic rotation capacity it undergoes at plastic-hinge locations.
- This is critical in situation where moment redistribution is going to be implemented.

One way to calculate this rotation capacity is making use of the moment-curvature relationship established for a given section.



But this plastic rotation is not confined to one cross section but is distributed over a finite length referred to as the hinging length. (l_p)

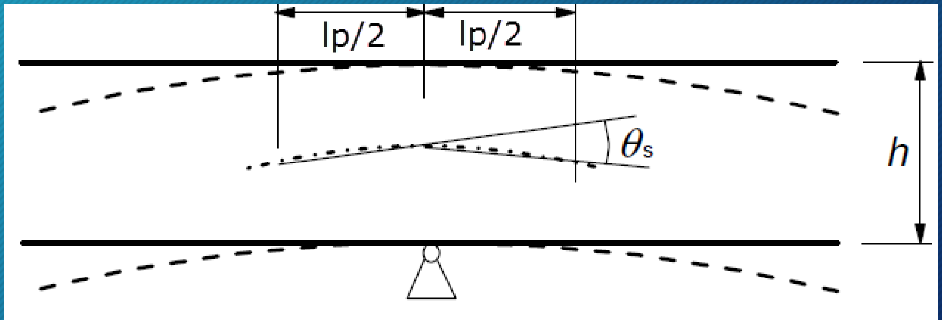
The total inelastic rotation θ_{pl} can be found by multiplying the average curvature by the hinging length:

$$\theta_{pl} = \left(\kappa_u - \kappa_y \frac{M_u}{M_y} \right) l_p$$

where :

$$l_p = 0.5d + 0.05z$$

In which z is the distance from the point of maximum moment to the nearest point of zero moment



Rotation Capacity: According EC-2

- According to EC-2, verification of the plastic rotation in the ultimate limit state is considered to be fulfilled, if it is shown that under the relevant action the calculated rotation, $\theta_{pl,s}$, is less than or equal to the allowable plastic rotation, $\theta_{pl,d}$

- In the simplified procedure, the allowable plastic rotation may be determined by multiplying the basic value of allowable rotation by a correction factor k_λ that depends on the shear slenderness.

The recommended basic value of allowable rotation, for steel Classes B and C (the use of Class A steel is not recommended for plastic analysis) and concrete strength classes less than or equal to C50/60 and C90/105 are given

The values apply for a shear slenderness $\lambda = 3,0$. For different values of shear slenderness $\theta_{pl,d}$ should be multiplied by k_λ

$$k_\lambda = \sqrt{\lambda / 3}$$

where :

λ is the ratio of the distance between point of zero and maximum moment after redistribution and effective depth, d.

As a simplification λ may be calculated for the concordant design values of the bending moment and shear.

$$\lambda = M_{sd} / (V_{sd} \cdot d)$$

Rotation Capacity: According EC-2

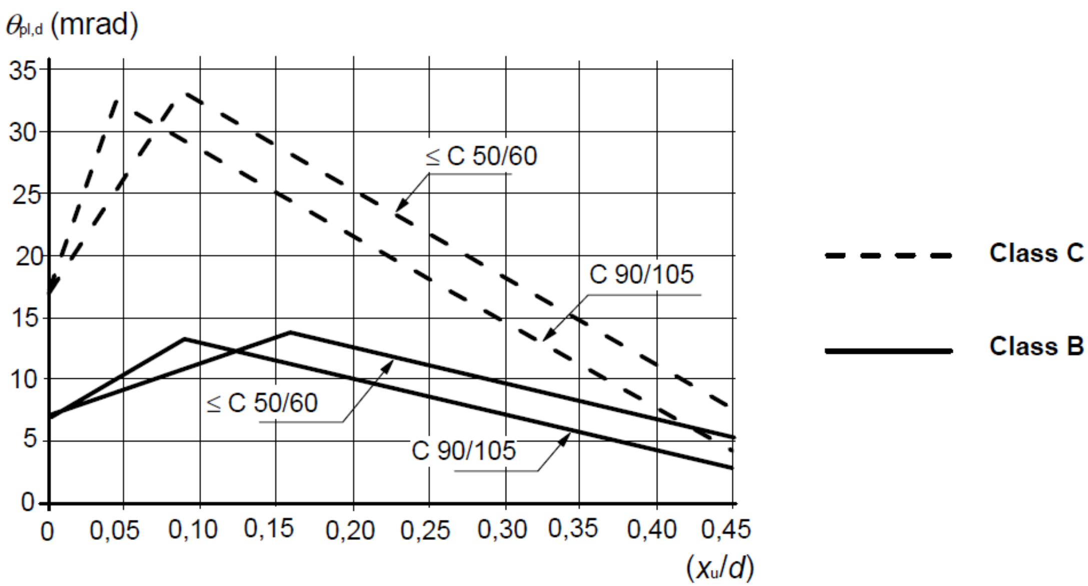
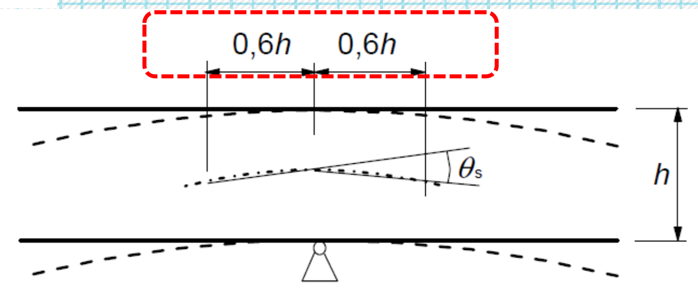


Figure 5.6N: Allowable plastic rotation, $\theta_{pl,d}$, of reinforced concrete sections for Class B and C reinforcement. The values apply for a shear slenderness $\lambda = 3,0$



Continuous Beams

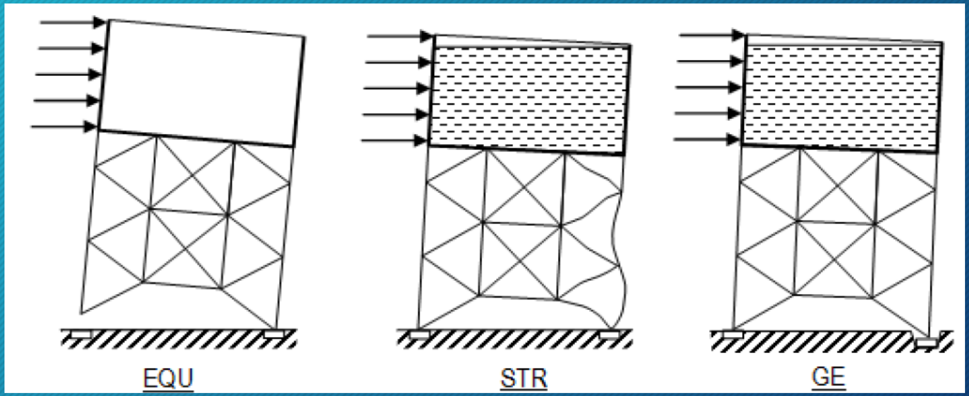
Analysis of Continuous beams

Design of Continuous beams

Continuous Beams: Analysis

- Continuous beams and one-way slabs are indeterminate structures for which variable/live load variation has to be considered. This is because permanent/dead load is always there but variable might vary during the life time of these structures.

How variable loads are arranged and over the continuous beam depend on two things according to EC1990.



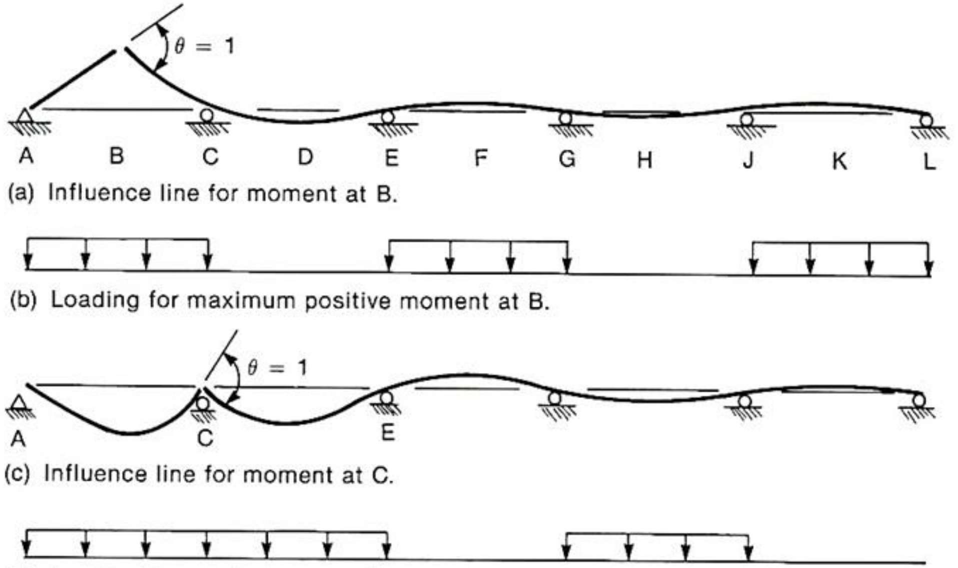
1. The design situation
   a. Persistent or Transient
   b. accidental
2. The relevant limit state
   a. ultimate limit state of strength (STR
   b. The limit states of equilibrium (EQU)
   c. strength at ULS with geotechnical actions (STR/GEO)

Affects both variable load arrangement and load combination values

Continuous Beams: Analysis

LOAD ARRANGEMNT OF ACTIONS: IN RELATION TO INFLUENCE LINES

The largest moment in continuous beams or one-way slabs or frames occur when some spans are loaded and the others are not. Influence lines are used to determine which spans should be loaded and which spans should not be to find the maximum load effect.



(a) Influence line for moment at B.

(b) Loading for maximum positive moment at B.

(c) Influence line for moment at C.

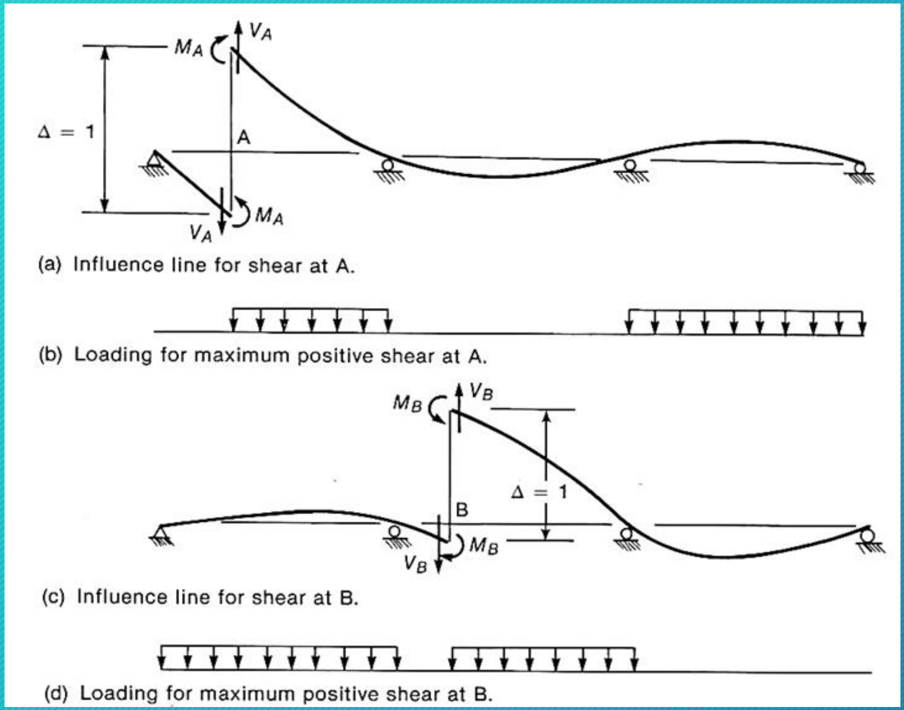
(d) Loading for maximum negative moment at C.

The figure (a) shows influence line for moment at B. The loading pattern that will give the largest positive moment consists of load on all spans having positive influence ordinates. Such loading is shown in figure (b) and is called alternate span loading or checkerboard loading.

The maximum negative moment at C results from loading all spans having negative influence ordinate as shown in figure (d) and is referred as an adjacent span loading.

Continuous Beams: Analysis

LOAD ARRANGEMNT OF ACTIONS: IN RELATION TO INFLUENCE LINES



Similarly, loading for maximum shear may be obtained by loading spans with positive shear influence ordinate as shown.

Affects both variable load arrangement and load combination values

Continuous Beams: Analysis

LOAD ARRANGEMNT OF ACTIONS: According Eurocode

In building structures, any of the following sets of simplified load arrangements may be used at ULS and SLS.

- The more critical of:

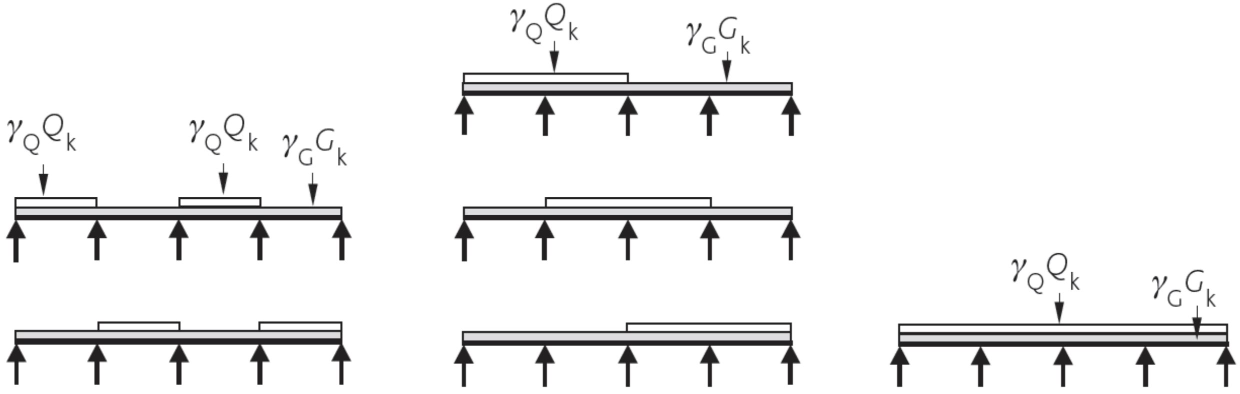
a) Alternative spans carrying $\gamma_G G_k + \gamma_Q Q_k$ with other spans loaded with $\gamma_G G_k$ and

b) All spans carrying $\gamma_G G_k + \gamma_Q Q_k$

- Or the more critical of:

a) Alternative spans carrying $\gamma_G G_k + \gamma_Q Q_k$ with other spans loaded with $\gamma_G G_k$ and

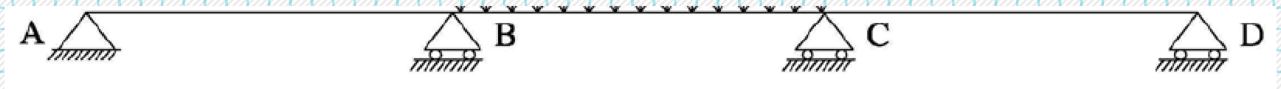
b) Any two adjacent spans carrying $\gamma_G G_k + \gamma_Q Q_k$



a) Alternate spans loaded   b) Adjacent spans loaded   c) All spans loaded

Example 1.2 : Given the three span beam (shown below) subjected to the following loads:

Self-weight	G_{k1}
Permanent imposed load	G_{k2}
Service imposed load	Q_{k1}

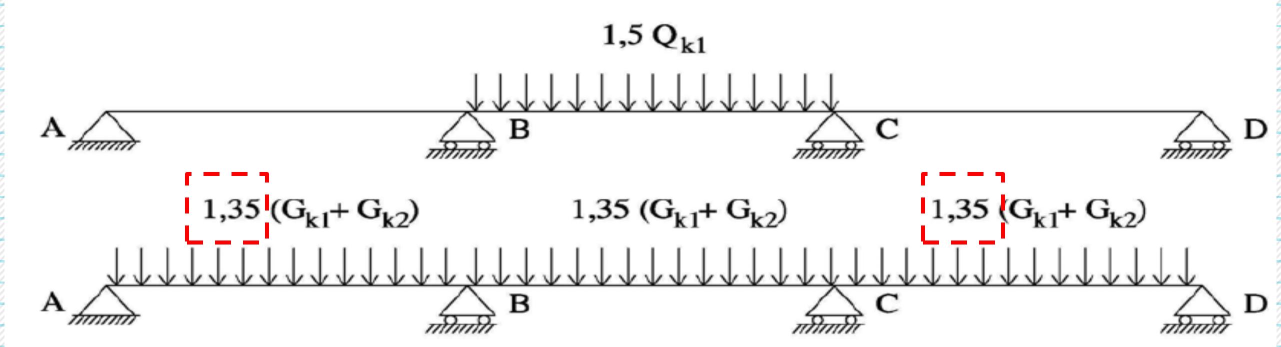


Identify the load arrangement to come up with

a) bending moment verification at mid span of BC (STR)

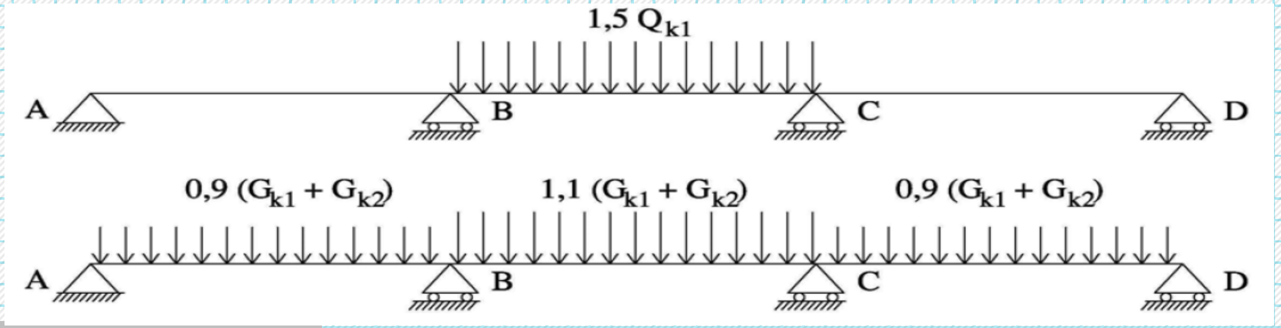
b) verification of holding down against the uplift of bearings at end span A is as follows. (EQU)

Solution: [a]



1.0 is more conservative but possible.

Solution: [b]



Continuous Beams: Design

SIMPLE!

After obtaining the maximum load effects of continuous beams, the design of continuous beam sections is carried out in the same procedure as discussed in reinforced concrete structures I course for no moment redistribution.

For cases with moment redistribution, the procedures will be presented and illustrated in the subsequent sections

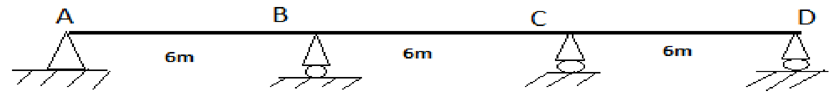
Example 1.3 : A continues beam with b/h 250/450 is to be constructed out of C20/25 concrete and reinforced with S400 reinforcement bar . The beam supports a factored permanenet load of 14.5 KN/m including its own self-weight and a factored variable load of 29 KN/m. Take cover to stirrup to be 25 mm.

Design the beam

a) Without moment redistribution

b) With 20% moment redistribution

USE φ8 and φ20 bars as web and longitudinal reinforcement



Solution:

Step1: Summarize the given parameters

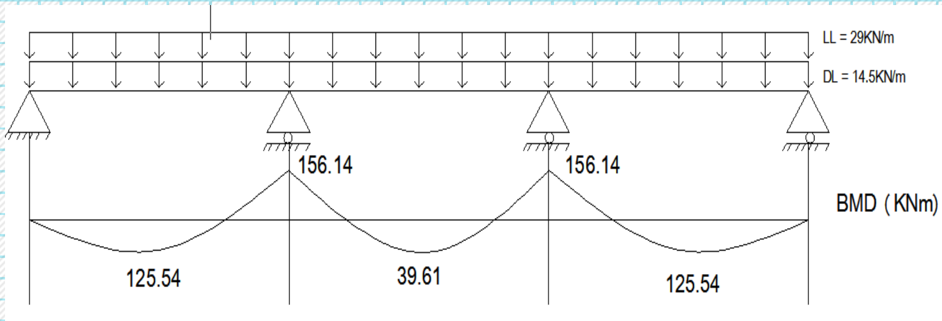
Material C20/25 f_{ck}=20MPa; f_{cd}=11.33MPa;
f_{ctm}=2.2MPa;
E_m=30,000MPa
S-400 f_{yk}=400MPa;
f_{yd}=347.83MPa;
E_s=200,000MPa; ε_y=1.74‰

Geometry d=h-cover- ($\varphi_{stiruup}$ +$\varphi_{longitiudinal}$/2)
=450-25-(8+10)=407mm

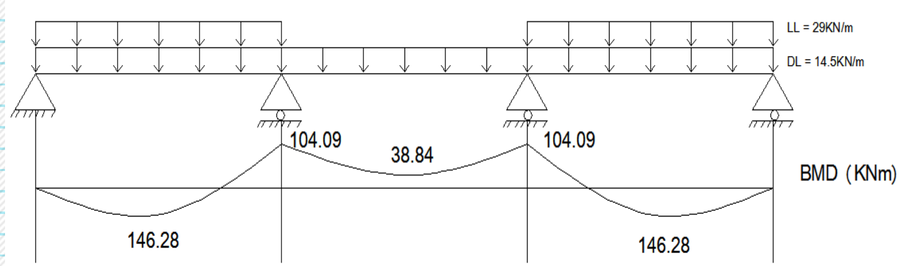
Load $1.35G_k$=14.5 kN/m
$1.50Q_k$=29.0 kN/m

Step2: Identify the cases for maximum action effect on (span and support moments)

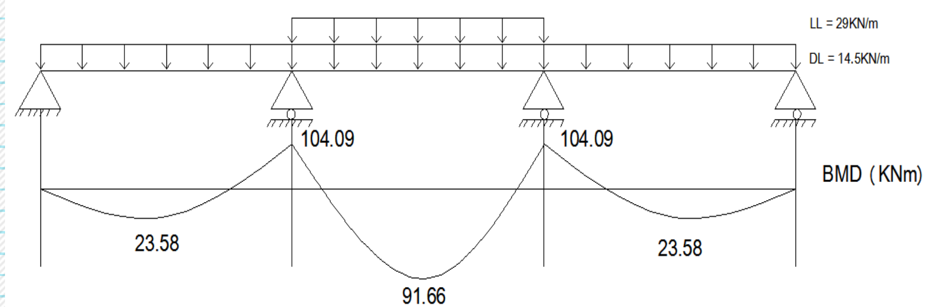
Case1: when the whole section is loaded



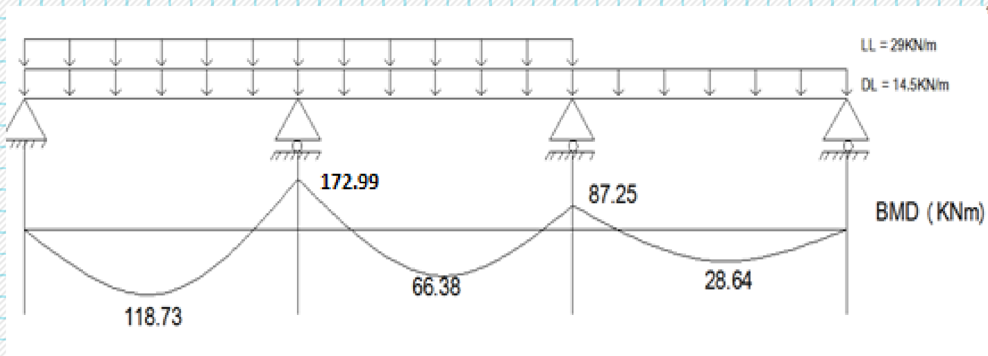
Case2: alternate span loading (max. span moment at AB and CD)



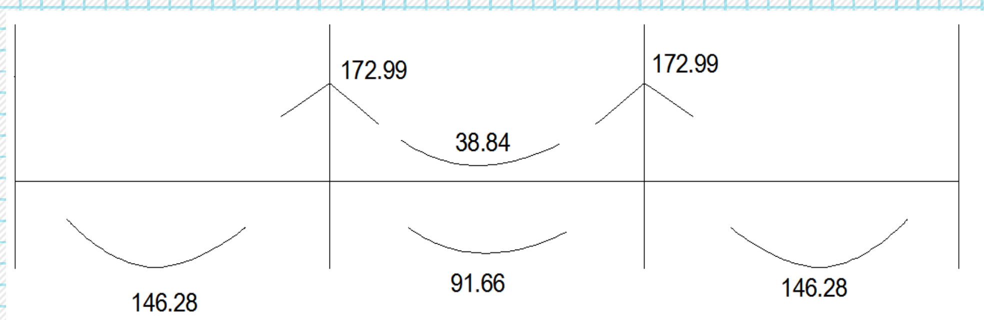
Case3: alternate span loading (max. span moment at BC)



Case4: two adjacent spans loading (max. support moment at B or C)



Moment envelop: (superimposing the above four cases for the respective maximum moment)



Step3: Design the beam section according to the procedures discussed in RC1 using the either the design chart or design table

a) Support B and C (-ve moment)

M_{sd}=172.99 kNm

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{172.99 * 10^6}{11.33 * 250 * 407^2} = 0.369 > \mu_{sd,lim} = 0.295 \quad \textbf{\textit{Double reinforced}}$$

$K_{z,lim} = 0.814$

$M_{sd,lim} = \mu_{sd,lim} f_{cd} bd^2 = 0.295 * 11.33 * 250 * 407^2 = 138.414\ KNm$

$Z = K_{z,lim} * d = 0.814 * 407 = 331.298\ mm$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d - d_2)} = \frac{138.414 * 10^6}{347.8 * 331.298} + \frac{(172.99 - 138.414) * 10^6}{347.8 * (407 - 43)} = 1474.28\ mm^2$$

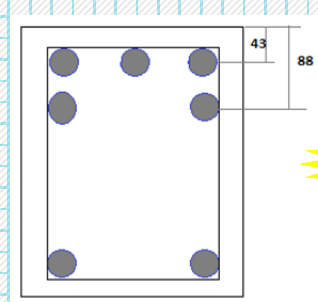
use **5⌀20**

Check the number of bars that can be placed on the single row.
Setting on 45 mm gap to insert a vibrator and making the other gaps equal to 25 mm

$20n + 45 + 25(n - 2) = 250 - 2 * 25 - 2 * 8$

$20n + 45 + 25n - 50 = 184$

$n = 4.2$



Revise d

Revise the effective depth for the reinforcement arrangement

$$so\ d = 450 - 61 = 389\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}\,bd^2} = \frac{172.99 * 10^6}{11.33 * 250 * 389^2} = 0.403 > \ \mu_{sd,lim} = 0.295 \quad \textbf{Double reinforced}$$

$$K_{z,lim} = 0.814$$

$$M_{sd,lim} = \mu_{sd,lim} f_{cd}\,bd^2 = 0.295 * 11.33 * 250 * 389^2 = 126.48\ KNm$$

$$Z = K_{z,lim} * d = 0.814 * 389 = 316.646\ mm$$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}\,(d - d_{2)}} = \frac{126.48 * 10^6}{347.8 * 316.646} + \frac{(172.99 - 126.442) * 10^6}{347.8 * (389 - 43)} = 1534.84\ mm^2$$

use 5⌀20

Compression reinforcement design

Check if the reinforcement has yielded

$$\frac{d_2}{d} = \frac{43}{389} = 0.1 \qquad \varepsilon_{s2} = 2.6‰\ (read\ from\ chart)$$

$$\varepsilon_{s2} = 2.6‰ > \varepsilon_{yd}\ use\ f_{yd} = 347.826$$

Calculate the stress in the concrete at the level of compression reinforcement to avoid double counting of area.

$\varepsilon_{cs2} = 2.6‰ \geq 2‰$, Therefore, we take

$$\varepsilon_c = 3.5‰\ and\ \sigma_{cd,s2} = 11.33\ mpa$$

$$A_{s2=}\frac{1}{(\sigma_{s2-}\sigma_{cd,s2})}\left(\frac{M_{sds} - M_{sd,lim}}{d - d_2}\right) = \frac{1}{(347.826 - 11.33)}\left(\frac{(172.99 - 138.44) * 10^6}{(389 - 43)}\right) = 399.48\ mm^2$$

use 2⌀20

b) Span AB and/or CD (+ve moment)

M_{sds}=146.28 KNm

Since the design moment is not far in magnitude from the one discussed in [a], its best if we assume two layers of reinforcement with 5φ20 bars.

$$so\ d = 450 - 61 = 389\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{146.28 * 10^6}{11.33 * 250 * 389^2} = 0.34128 > \mu_{sd,lim} = 0.295 \quad Double\ reinforced$$

$$K_{z,lim} = 0.814$$

$$M_{sd,lim} = \mu_{sd,lim} f_{cd} bd^2 = 0.295 * 11.33 * 250 * 389^2 = 126.442\ KNm$$

$$Z = K_{z,lim} * d = 0.814 * 389 = 316.646\ mm$$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d - d_{2})} = \frac{126.442 * 10^6}{347.8 * 316.646} + \frac{(146.28 - 126.442) * 10^6}{347.8 * (389 - 43)} = 1312.972 mm^2$$

$$use\ 5\varnothing 20$$

Compression reinforcement design

Check if the reinforcement has yielded

$$\frac{d_2}{d} = \frac{43}{389} = 0.1 \qquad \varepsilon_{s2} = 2.6‰\ (read\ from\ chart)$$

$$\varepsilon_{s2} = 2.6‰ > \varepsilon_{yd}\ use\ f_{yd} = 347.826$$

Calculate the stress in the concrete at the level of compression reinforcement to avoid double counting of area.

$\varepsilon_{cs2} = 2.6‰ \geq 2‰$, Therefore, we take

$$\varepsilon_c = 3.5‰\ and\ \sigma_{cd,s2} = 11.33\ mpa$$

$$A_{s2} = \frac{1}{(\sigma_{s2} - \sigma_{cd,s2})}\left(\frac{M_{sds} - M_{sd,lim}}{d - d_2}\right)$$

$$= \frac{1}{(347.826 - 11.33)}\left(\frac{(146.28 - 138.44) * 10^6}{(389 - 43)}\right.$$

$$= 170.07\ mm^2$$

$$\textbf{use } 2\varnothing\textbf{20}$$

c) Span BC (+ ve moment)

Span BC is selected of all the three positive bending moments as its higher in values.

$$M_{sd}=91.66 \text{ KN m}$$

$$\mu sd,s = \frac{Msd,s}{fcd*b*d^2} = \frac{91.66*10^6}{11.33*250*407^2}$$

$$= 0.195 < \mu sd,lim = 0.295$$

Singly reinforced section

$$\text{Kz} = 0.89 \text{ (read from chart)}$$

$$As1 = \frac{1}{fyd} * \frac{Msd,s}{z}$$

$$As1 = \frac{1}{347.8} * \frac{91.66 * 10^6}{0.89 * 407}$$

$$As1 = 727.5\ mm^2$$

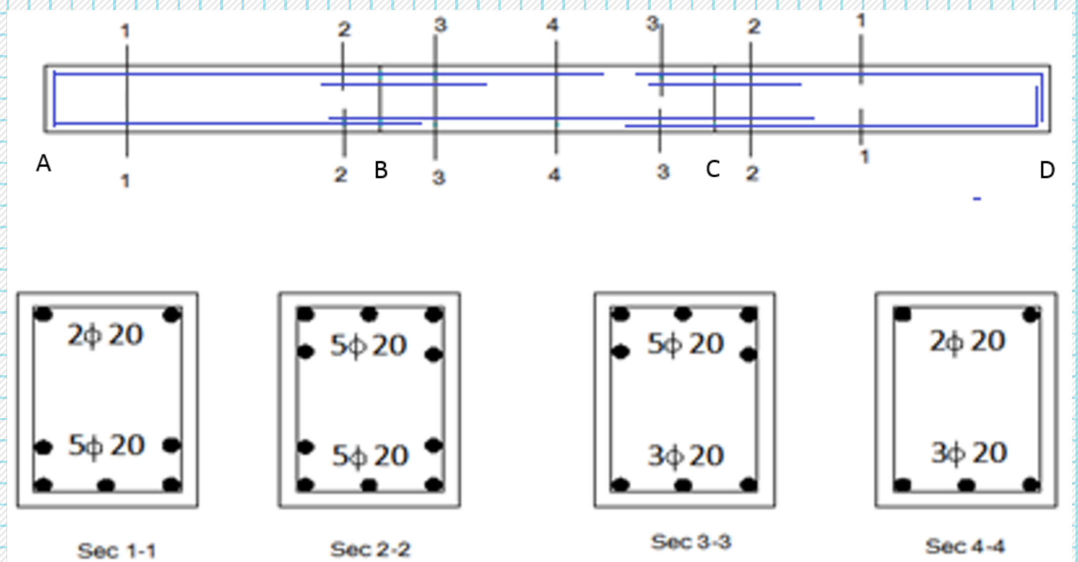
Use 3φ20

d) Span BC (- ve moment)

M_{sd}=38.84 kNm

Use 2 φ 20, minimum reinforcements is proved to be sufficien

Step4: Detailing



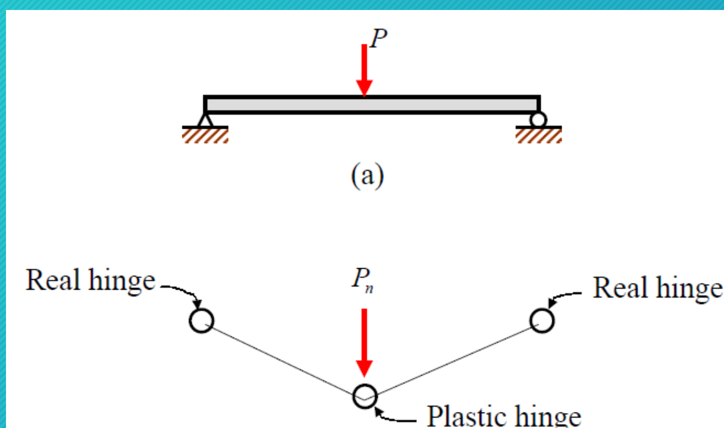
Plastic Hinges and Collapse Mechanisms

Plastic Hinges and Collapse Mechanisms

Statically Determinate Beam

- Will fail if ONE plastic hinge develop.

e.g. The simply supported beam shown below will fail , if P is increased until a plastic hinge is developed at the point of maximum moment (just underneath P),.

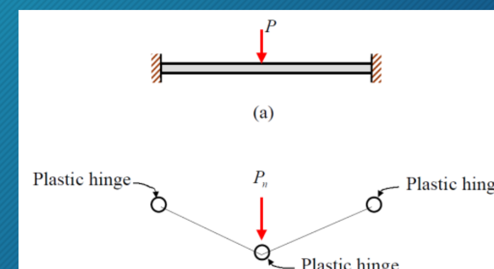


Therefore, Mechanism is defined as the formation & arrangement of plastic hinges and perhaps real hinges that permit the collapse in a structure

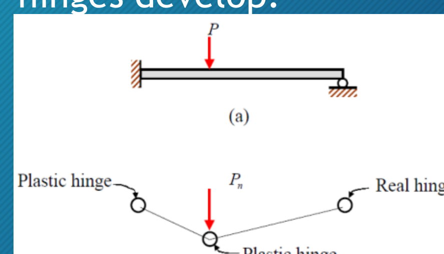
Statically Indeterminate Beam

- Will require at least TWO plastic hinges to develop to fail.

e.g. The fixed-end beam shown below can't fail unless the three hinges in the figure develop.



e.g. The propped cantilever beam below is an example of a structure that will fail after two plastic hinges develop.

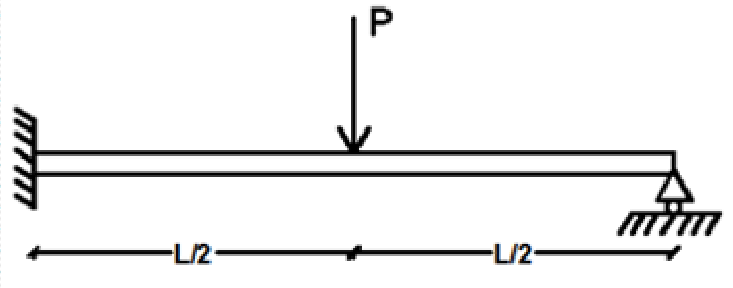


Plastic Hinges and Collapse Mechanisms

From the discussion in the previous slide we can point out the following as an observation

- If the structure is statically indeterminate, it is still stable after the formation of a plastic hinge, and for further loading, it behaves as a modified structure with a hinge at the plastic hinge location (and one less degree of indeterminacy).
- It can continue to carry additional loading (with formation of additional plastic hinges) until the limit state of collapse is reached on account of one of the following reasons:
    1. formation of sufficient number of plastic hinges, to convert the structure (or a part of it) into a 'mechanism'.
    2. limitation in ductile behavior (i.e., curvature κ reaching the ultimate value κ_{max}, or, in other words a plastic hinge reaching its ultimate rotation capacity) at any one plastic hinge location, resulting in local crushing of concrete at that section.

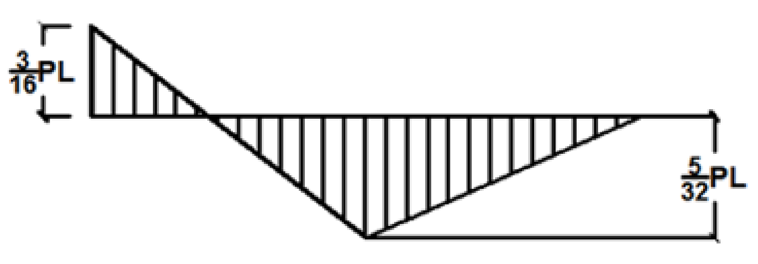
For illustration let us see the behavior of an indeterminate beam shown below, It will be assumed for simplicity that the beam is symmetrically reinforced, so that the negative bending capacity is the same as the positive.



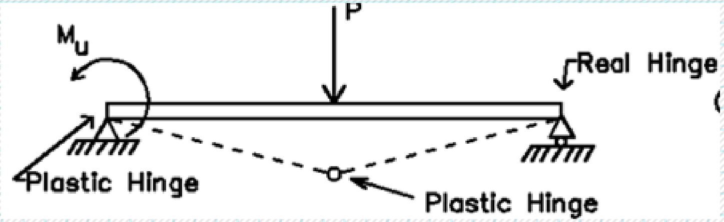
Let the load P be increased gradually until the elastic moment at the fixed support, 3PL/16 is just equal to the plastic moment capacity of the section, Mu. This load is

$$P = P_{el} = \frac{16M_u}{3L} = \frac{5.33M_u}{L}$$

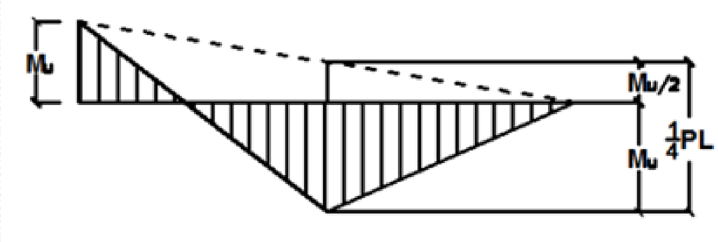
At this load the positive moment under the load is 5/32 PL, as shown



The beam still responds elastically everywhere but at the left support. At that point the actual fixed support can be replaced for purpose of analysis with a plastic hinge offering a known resisting moment Mu, which makes the beam statically determinate.



The load can be increased further until the moment under the load also becomes equal to Mu, at which load the second hinge forms. The structure is converted into a mechanism, and collapse occurs.



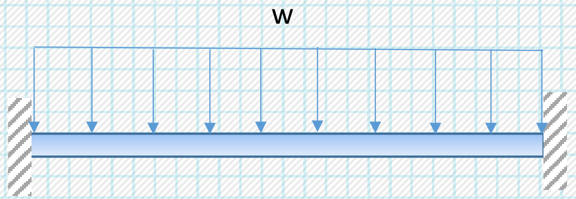
$$M_u + \frac{M_u}{2} = \frac{PL}{4}$$

$$P = P_u = \frac{6M_u}{L}$$

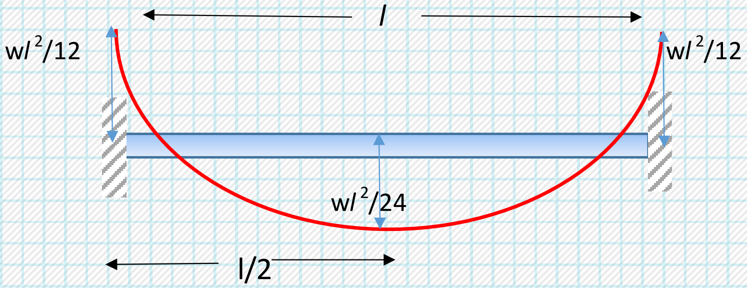
The magnitude of the load causing collapse is easily calculated from the geometry

It is evident that an increase of 12.5% is possible beyond the load which caused the formation of the first plastic hinge, before the beam will actually collapse.

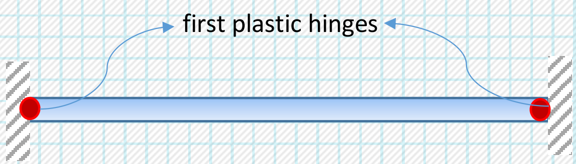
<u>Example1.4:</u> Compute the theoretical ultimate load the beam below can support interms of the plastic moment capacity of the beam section. Assume the conditions in the illustrative example above are also applicable here (symmetric reinforcement across the span of the beam)



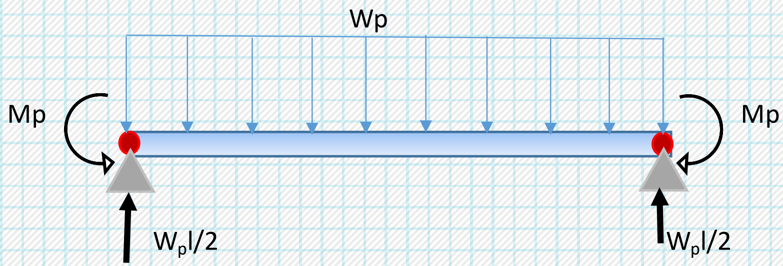
Given beam with loading and support condition



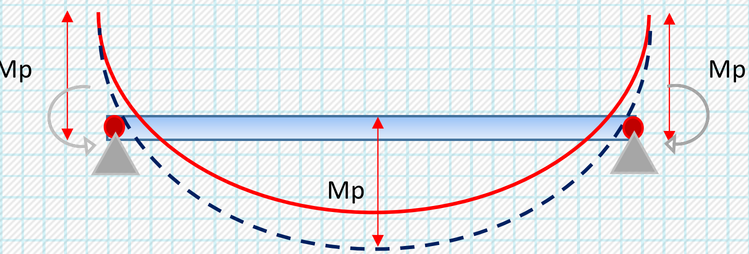
<u>Step1</u>: Identify the location and magnitude of maximum moment in the elastic range (indicates the location of the first plastic hinges)



Although the plastic moment has been reached at the ends and plastic hinges are formed, the beams will not fail because it has , in effect, become a simple end supported beam for further load increment.



The load can now be increased on this "Simple" beam, and the moments at the ends will remain constant; however, the moment out in the span will increase at it would in a uniformly loaded simple beam as shown.

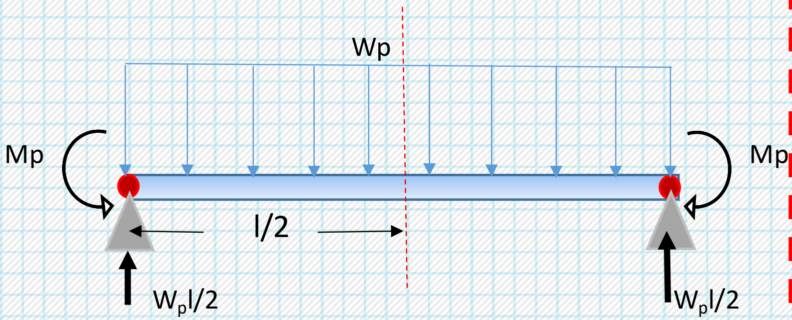


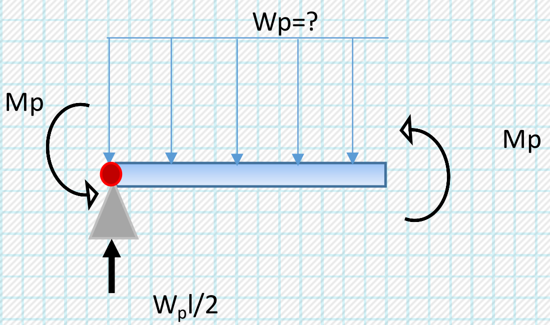
If this is the case and assuming that the formed plastic hinges have enough rotational capacity, the next step is to come up with the ultimate load!.....How

Step2: Compute the theoretical ultimate load interims od the plastic moment capacity.

In order to come up with the ultimate load one could adopt a number methods, here under two of which are presented.

Using the concept of section equilibrium





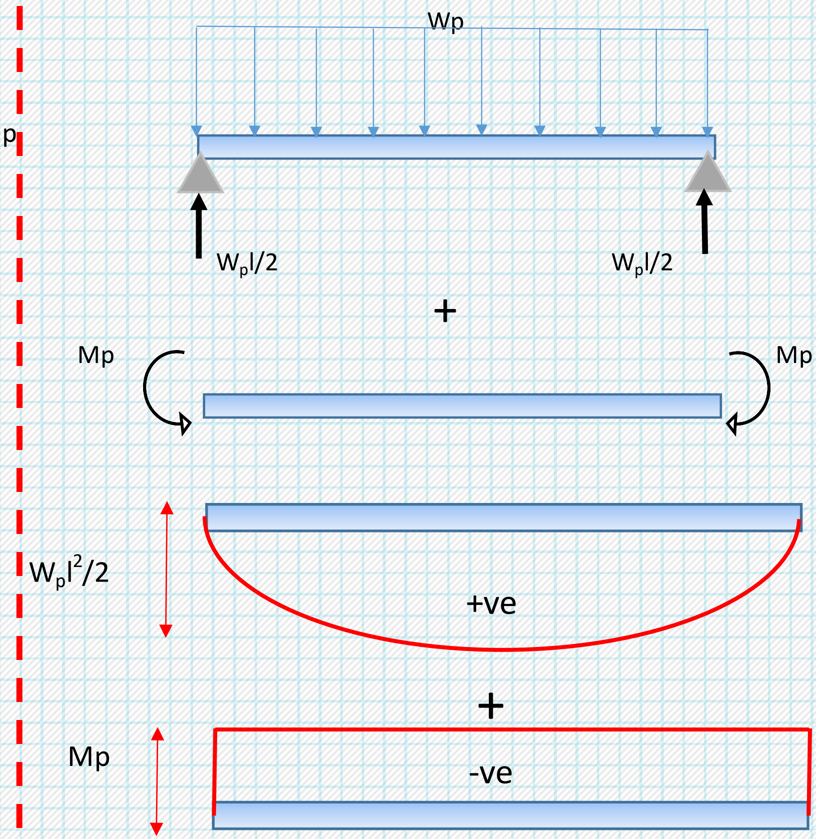
$$\sum M_o = 0$$

$$M_p + M_p + w_p\left(\frac{l}{2}\right)\left(\frac{l}{4}\right) = \left(\frac{w_p l}{2}\right)\left(\frac{l}{2}\right)$$

$$w_p = \frac{16M_p}{l^2}$$

Loading Capacity was increased by a fraction of 4/3 =16/12.

Using the concept of super positioning



At l/2 bending moment has to reach m_p inorder to form a plastic hinge.

hence, $2m_p = \frac{w_p l^2}{8}$

$$w_p = \frac{16m_p}{l^2}$$

Moment Redistribution

Moment Redistribution

As seen in the previous section, the distribution of bending moments in a continuous beam (or frame) gets modified significantly in the inelastic phase.

The term moment redistribution is generally used to refer to the transfer of moments to the less stressed sections as sections of peak moments yield on their ultimate capacity being reached (as witnessed in the example above).

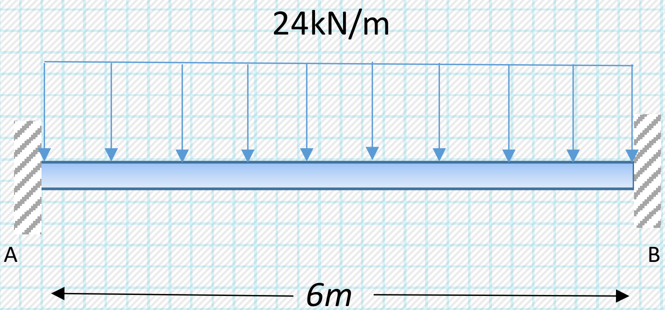
From a design viewpoint, this behavior can be taken advantage of by attempting to effect a redistributed bending moment diagram which achieves a reduction in the maximum moment levels (and a corresponding increase in the lower moments at other locations).

Such an adjustment in the moment diagram often leads to the design of a more economical structure with better balanced proportions, and less congestion of reinforcement at the critical sections.

Example1.5: Design the beam for flexure that is shown below, with b/h = 200//400mm and carrying a design load of 24kN/m including its own weight;

a) Without moment redistribution

b) With 20% moment redistribution



USE C20/25, S-400 and ϕ8 and ϕ20 bars for stirrup and longitudinal reinforcement respectively.

Assume cover to stirrup to be 25mm

Solution: [a]

Step1: Summarize the given parameters

Material C20/25 f_{ck}=20MPa; f_{cd}=11.33MPa; f_{ctm}=2.2MPa; E_m=30,000MPa

S-400 f_{yk}=400MPa; f_{yd}=347.83MPa; E_s=200,000MPa; ε_y=1.74‰

Geometry $d = h - cover - (\varphi_{stiruup} + \varphi_{longitiudinal}/2)$
$= 400-25-(8+10) = 357mm$

Load $1.35G_k + 1.50Q_k = 24.0$ kN/m

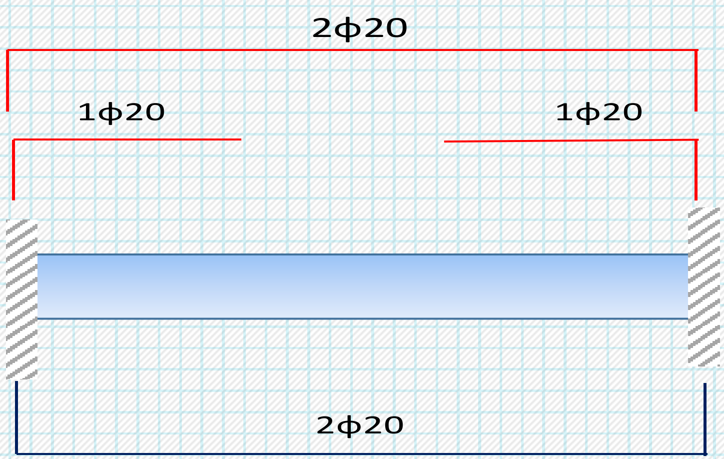
Step2: Compute the design action on the beam (Bending moment)

Step3: design the beam at the supports and mid span

Carrying out the procedure for flexure design of rectangular RC section, we will have the following results

Moment	Reinforcement provided
72kNm (support)	3ϕ20
36kNm (mid span)	2ϕ20

Step4: Detailing



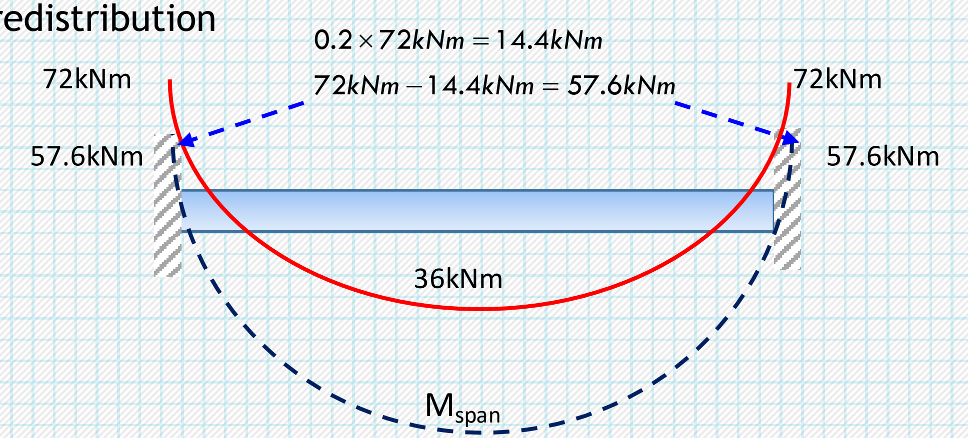
Solution: [b]

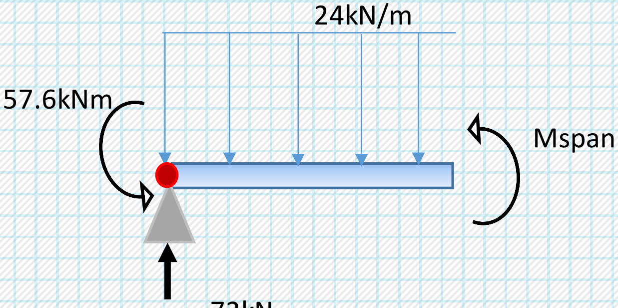
Step1: Summarize the given parameters

Material	C20/25 f_{ck}=20MPa; f_{cd}=11.33MPa; f_{ctm}=2.2MPa; E_m=30,000MPa S-400 f_{yk}=400MPa; f_{yd}=347.83MPa; E_s=200,000MPa; ε_y=1.74‰
Geometry	d=h-cover- ($\varphi_{stiruup}$ +$\varphi_{longitiudinal}$/2) =400-25-(8+10)=357mm
Load	$1.35G_k+1.50Q_k$=24.0 kN/m

Moment redistribution up 20% is allowed.

Step2: Select a critical section and carryout the moment redistribution





$$\sum M_o = 0$$

$$M_{span} = 72\left(\frac{l}{2}\right) - 57.6 - \left(\frac{24l^2}{2}\right)$$

$$M_{span} = 50.4kNm$$

Design Moment before redistribution	Design Moments after redistribution
72kNm (support)	57.6kNm (support)
36kNm (mid span)	50.4kNm (mid span)

Step3: design the beam at the supports and mid span

Carrying out the procedure for flexure design of rectangular RC section, we will have the following results.

But keep in mind the value μ_{lim} for 20%moment redistribution which is 0.205

Moment	Reinforcement provided
57.6kNm (support)	2ϕ20
50.4kNm (mid span)	2ϕ20

Step4: Detailing

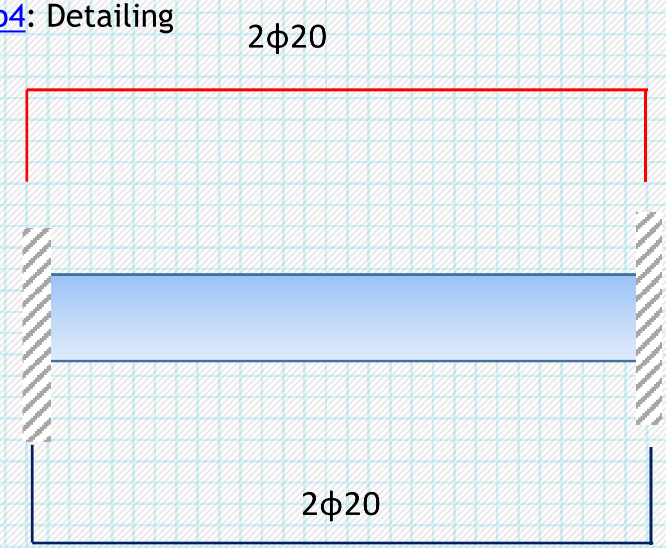


Table 2-2 Design Table for C 12/15 – C 50/60

$\mu_{Sd} = \frac{M_{Sd}}{f_{cd}bd^2}$	$\omega = \frac{A_{s1}f_{yd}}{f_{cd}bd}$	$k_x = \frac{x}{d}$	$k_z = \frac{z}{d}$	ε_c (‰)	ε_{s1} (‰)	Percentage Redistribution
0.000	0.000	0.000	1.000	0.000	25.000	
0.010	0.010	0.030	0.990	0.773	25.000	
0.020	0.020	0.044	0.985	1.146	25.000	
0.030	0.031	0.055	0.980	1.464	25.000	
0.040	0.041	0.066	0.976	1.763	25.000	
0.050	0.051	0.076	0.971	2.060	25.000	
0.060	0.062	0.086	0.967	2.365	25.000	
0.070	0.073	0.097	0.962	2.682	25.000	
0.080	0.084	0.107	0.956	3.009	25.000	
0.090	0.095	0.118	0.951	3.349	25.000	
0.100	0.106	0.131	0.946	3.500	23.294	
0.110	0.117	0.145	0.940	3.500	20.709	
0.120	0.128	0.159	0.934	3.500	18.552	
0.130	0.140	0.173	0.928	3.500	16.726	
0.140	0.152	0.188	0.922	3.500	15.159	
0.150	0.164	0.202	0.916	3.500	13.799	
0.160	0.176	0.217	0.910	3.500	12.608	
0.170	0.188	0.232	0.903	3.500	11.555	
0.180	0.201	0.248	0.897	3.500	10.618	
0.190	0.213	0.264	0.890	3.500	9.777	
0.200	0.226	0.280	0.884	3.500	9.019	
0.205	0.233	0.288	0.880	3.500	8.653	20%
0.210	0.239	0.296	0.877	3.500	8.332	
0.220	0.253	0.312	0.870	3.500	7.706	
0.230	0.266	0.329	0.863	3.500	7.132	
0.240	0.280	0.346	0.856	3.500	6.605	
0.250	0.295	0.364	0.849	3.500	6.118	
0.252	0.298	0.368	0.847	3.500	6.011	10%
0.260	0.309	0.382	0.841	3.500	5.667	
0.270	0.324	0.400	0.834	3.500	5.247	
0.280	0.339	0.419	0.826	3.500	4.856	
0.290	0.355	0.438	0.818	3.500	4.490	
0.295	0.363	0.448	0.814	3.500	4.313	0%

Thank you for the kind attention!

Questions?