Addis Ababa institute of Technology

# Reinforced Concrete Structures 2

(CEng-3122)

# Chapter Two

## One-way Slab Systems

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April 23, 2020

School of Civil and Environmental Engineering
Concrete Material and Structures Chair

1. Introduction
2. Analysis and Design of One-way Slab systems
3. Analysis and Design of One-way Ribbed Slab systems

# Presentation Outline

**Content**

# Introduction

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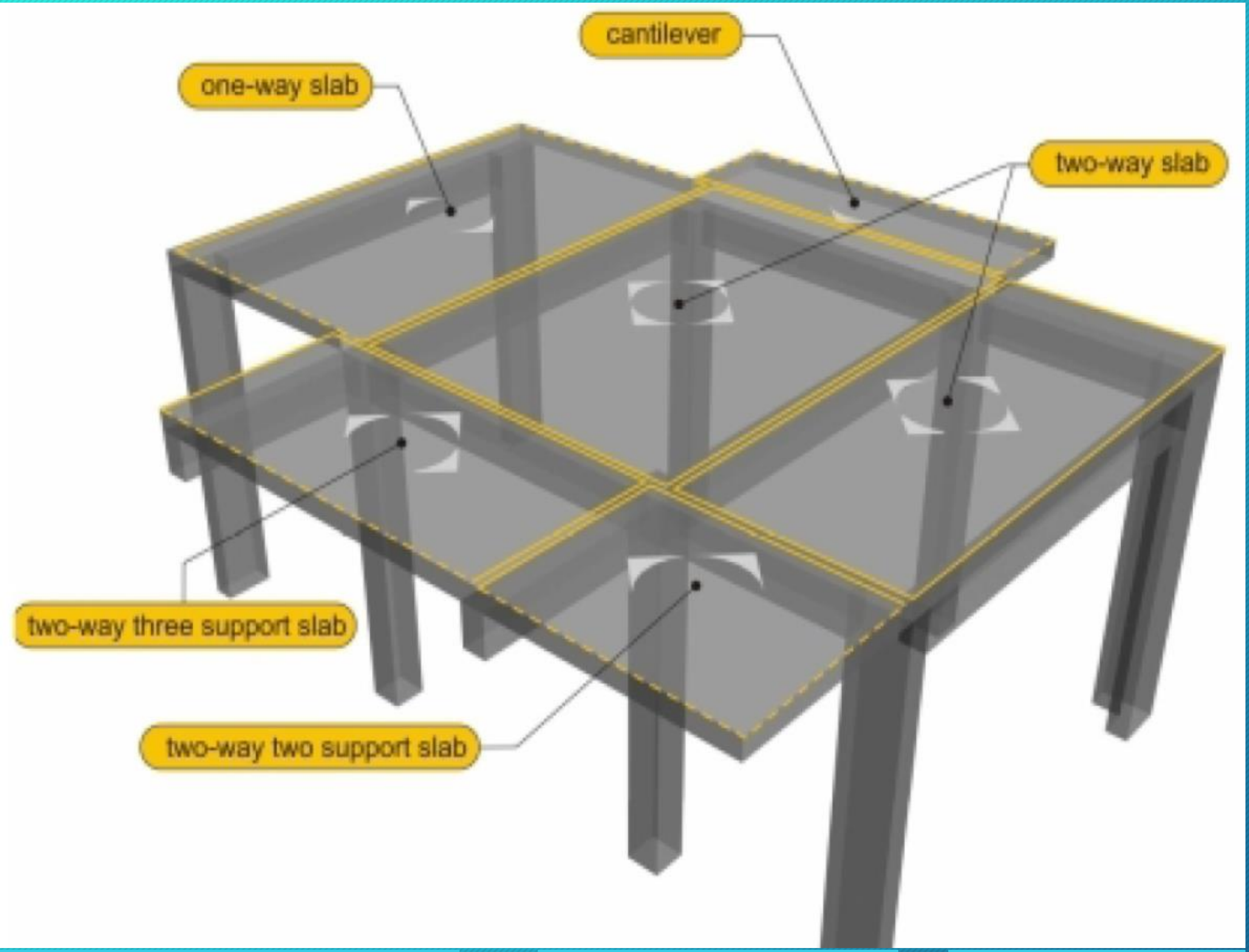


1. **One-way slabs**: They are those either supported on the two out of four opposite sides or the longer span to short span ratio is at least equal to 2.

2. **Two-way slabs:** They are those supported on all four sides and the longer span to short span ratio is less than 2

3. **Cantilever slabs:** They are those with a fixed support on only one out of four sides

# Analysis and Design of One-way Slab systems

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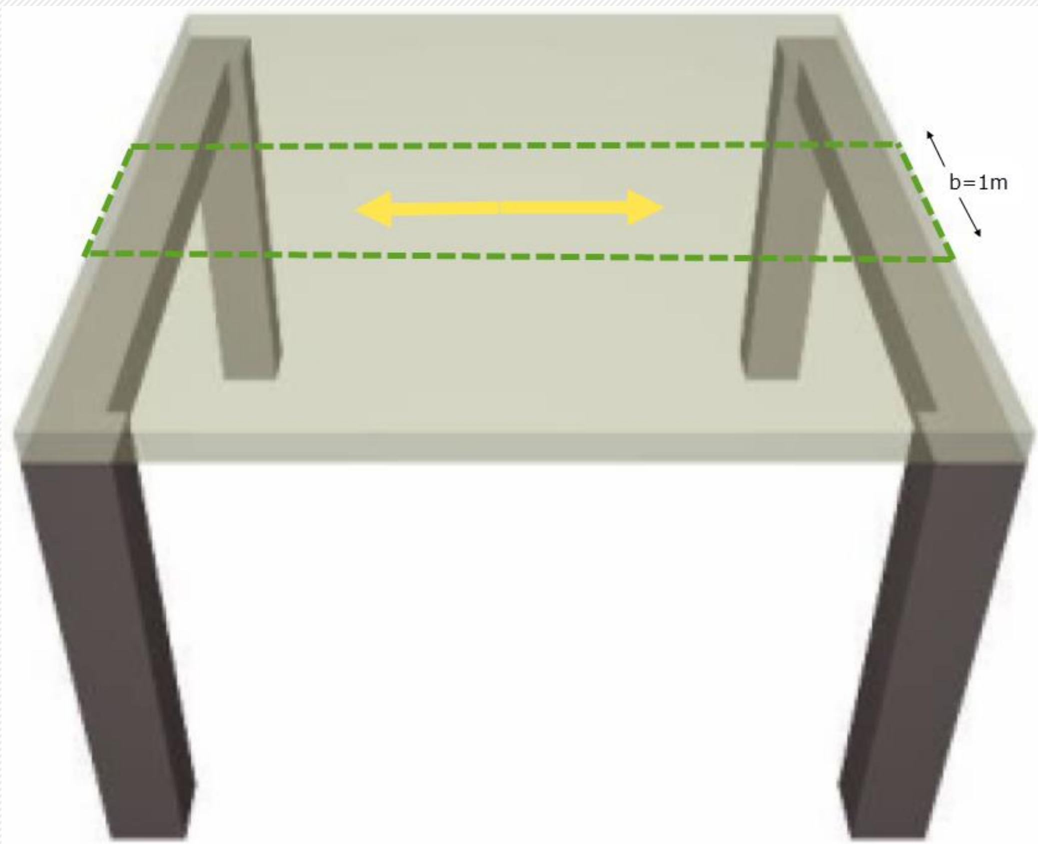
# One-way slab supported by two beams



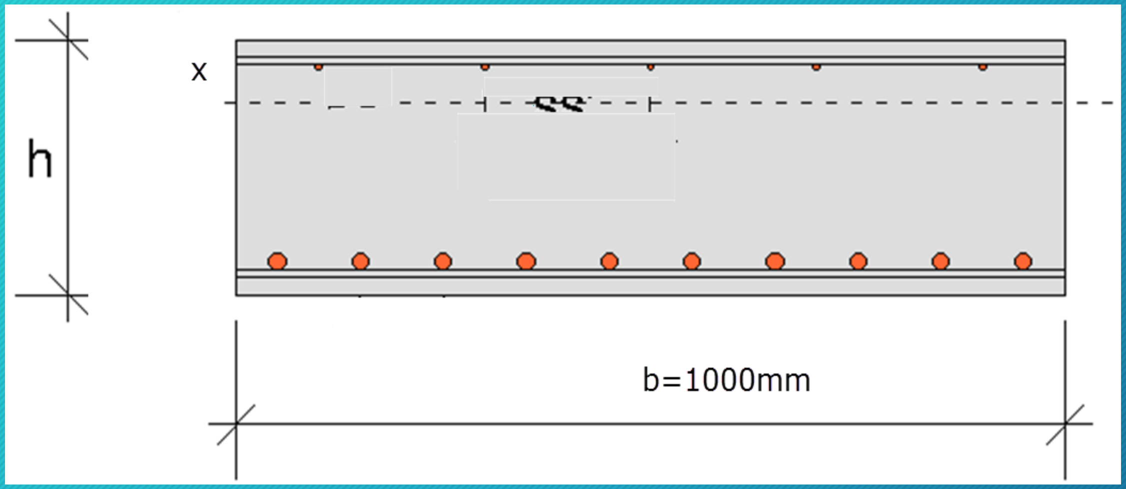



In the design and analysis of one way slab systems a 1m strip of slab along the load transfer direction is considered

# Analysis and Design of One-way Slab systems



For one-way slab sections with under both a negative and positive bending moment follows the procedures of rectangular sections. The only exception is that the width of the slab considered is 1m as previously pointed out.



# Analysis and Design of One-way Slab systems



In the Analysis and Design of One-way Slab systems, the cover requirement for bond and durability are the same for that of beam requirements.

But for fire resistance the minimum dimension and cover requirements are given in EN 1992-1-2:2004 table 5.8.

**Table 5.8: Minimum dimensions and axis distances for reinforced and prestressed concrete simply supported one-way and two-way solid slabs**

| Standard fire resistance | Minimum dimensions (mm) | | | |
|---|---|---|---|---|
| | slab thickness $h_s$ (mm) | axis-distance $a$ | | |
| | | one way | two way: $l_y/l_x \leq 1{,}5$ | two way: $1{,}5 < l_y/l_x \leq 2$ |
| 1 | 2 | 3 | 4 | 5 |
| REI 30 | 60 | 10* | 10* | 10* |
| REI 60 | 80 | 20 | 10* | 15* |
| REI 90 | 100 | 30 | 15* | 20 |
| REI 120 | 120 | 40 | 20 | 25 |
| REI 180 | 150 | 55 | 30 | 40 |
| REI 240 | 175 | 65 | 40 | 50 |

$l_x$ and $l_y$ are the spans of a two-way slab (two directions at right angles) where $l_y$ is the longer span.

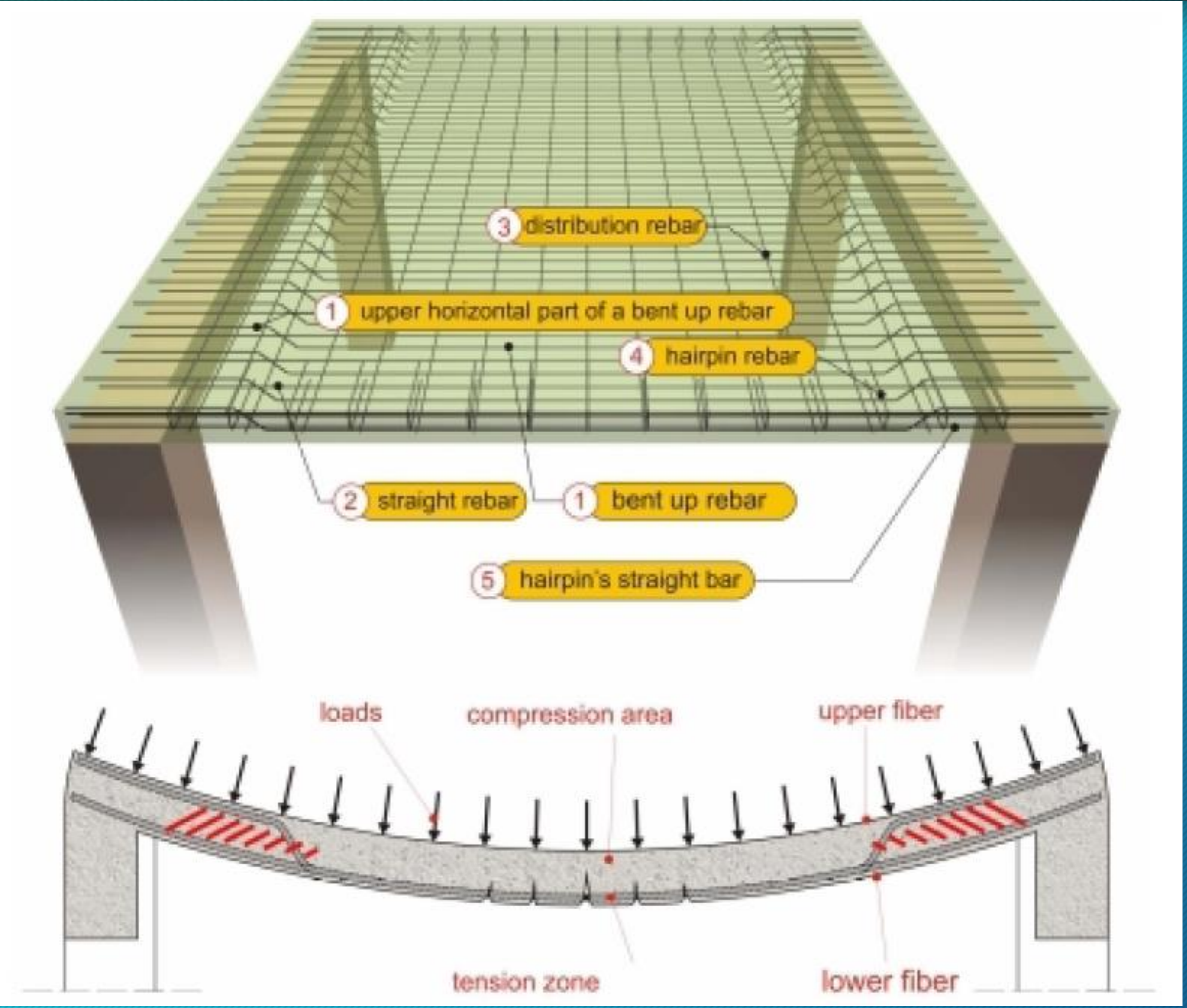
For prestressed slabs the increase of axis distance according to 5.2(5) should be noted.

The axis distance $a$ in Column 4 and 5 for two way slabs relate to slabs supported at all four edges. Otherwise, they should be treated as one-way spanning slab.

* Normally the cover required by EN 1992-1-1 will control.

# Design of One-way : Reinforcement






1. **Primary reinforcement**: In a one-way slab the need for reinforcement appears mainly in the span and towards the bending direction. The necessary bars are placed based on the amount of the calculated required reinforcement..
2. **Secondary reinforcement or distribution reinforcement**: Are provided in the other, secondary direction.
3. **Free edge reinforcement**: The free edges of slabs are more susceptible to stresses and therefore, in these areas hairpin reinforcement is placed. Its proper position is secured by means of two bars placed inside its corners.

# Analysis and Design of One-way Ribbed Slab systems

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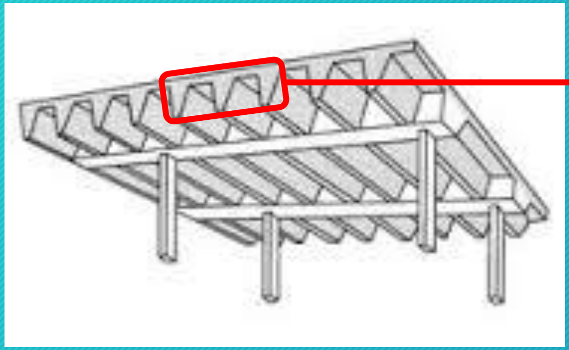
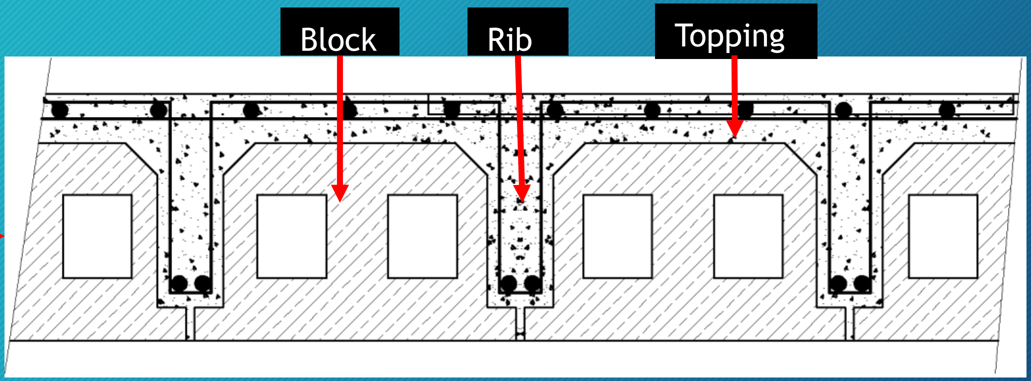
# Ribbed Slab Systems



Long-span floors for relatively light live loads can be constructed as a series of closely spaced, cast-in-place T-beams (or joists or ribs) with a cross section as shown



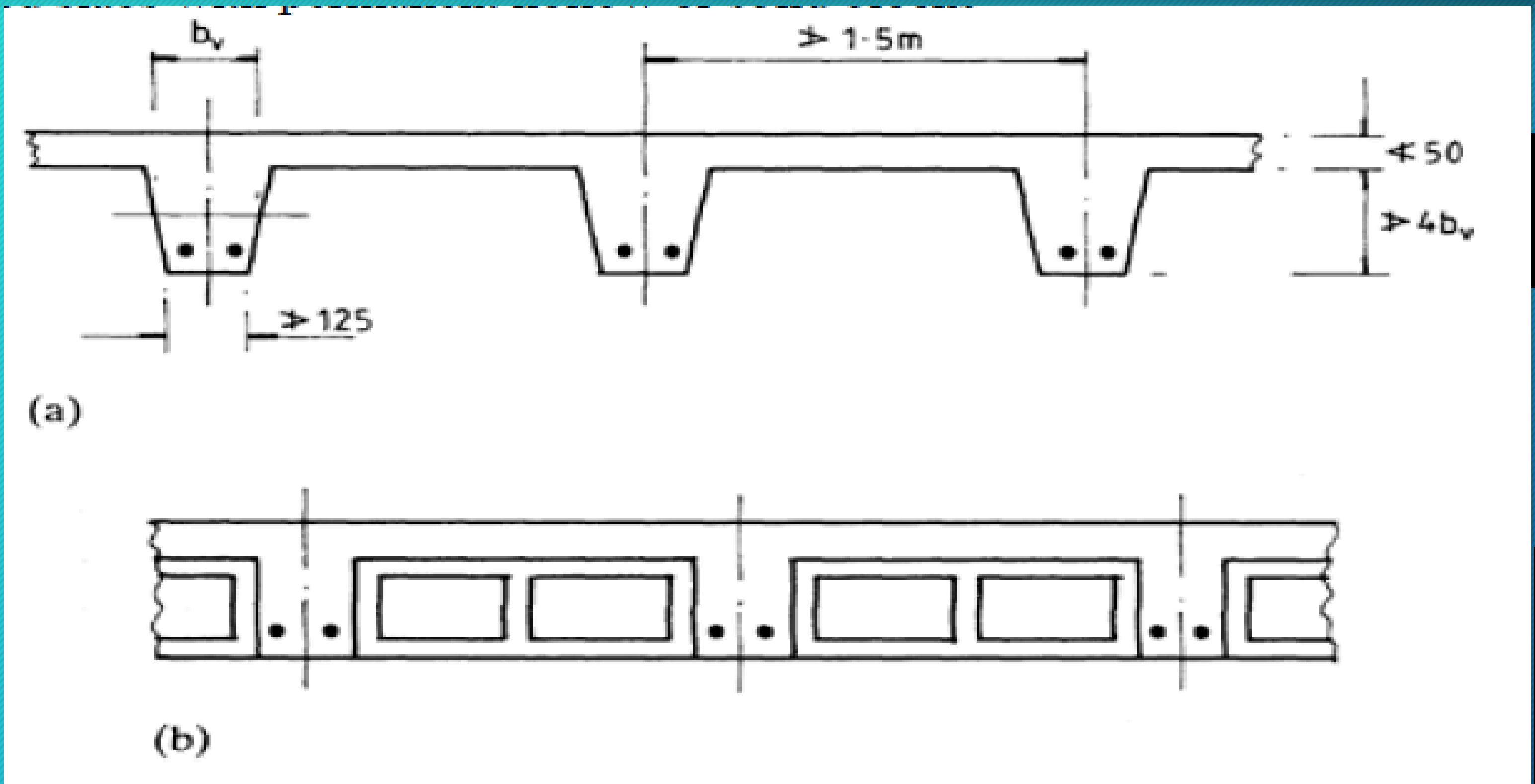



They may be constructed in a variety of ways, two principal methods of construction are:

1. Ribbed slabs without permanent blocks,
2. Ribbed slabs with permanent hollow or solid blocks.

# Ribbed Slab Systems



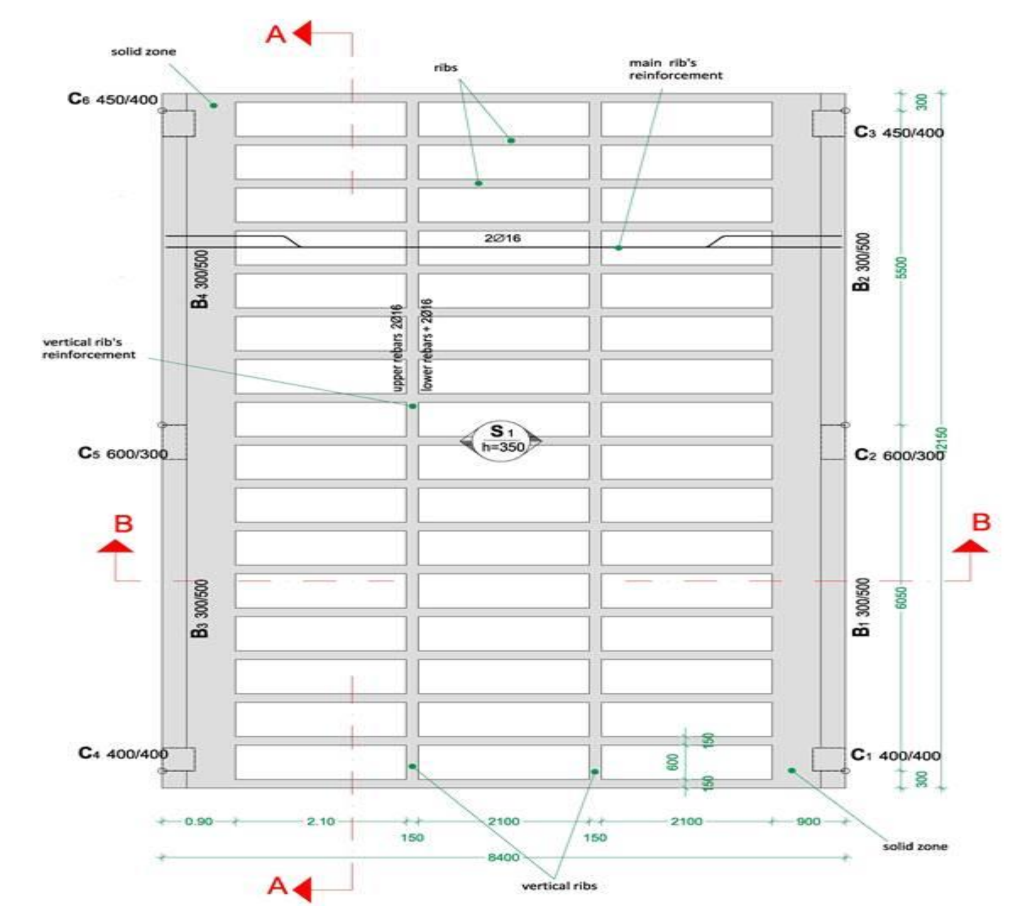



Ribbed slab section without permanent blocks

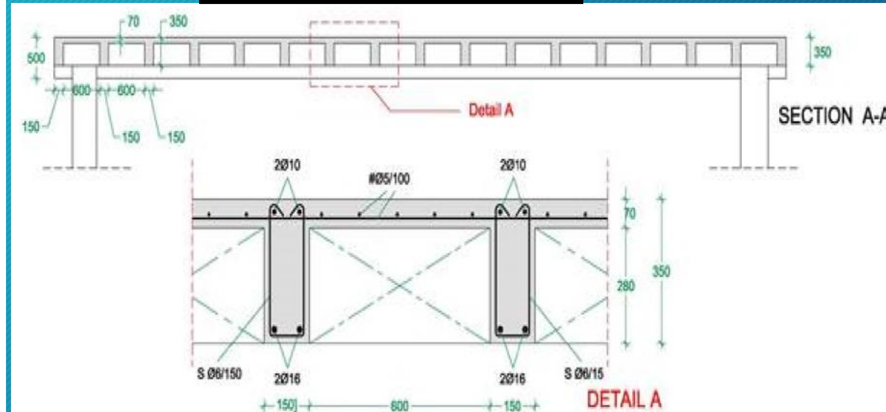
Ribbed slab section with permanent blocks
(can be precast or cast in-situ)

# Plan-Layout (Cast in-situ rib slab system)

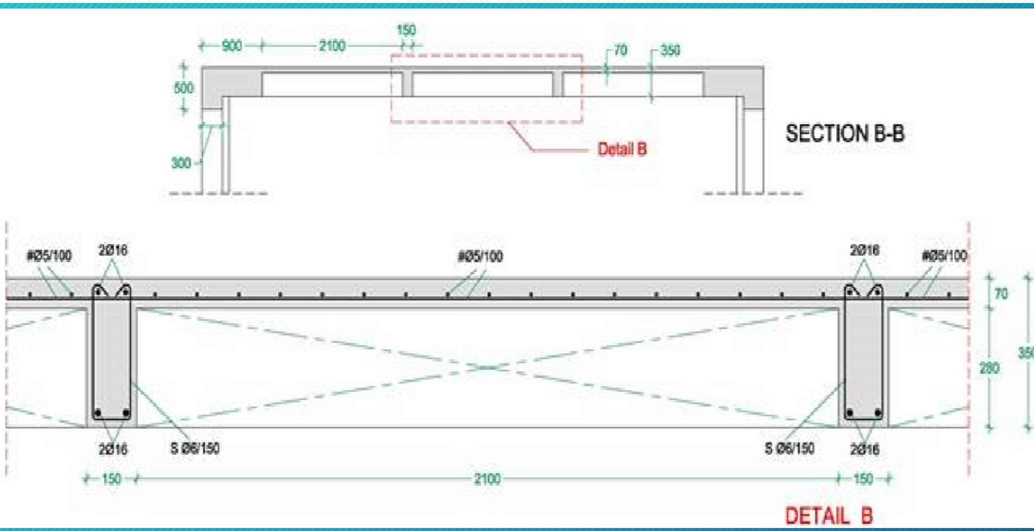


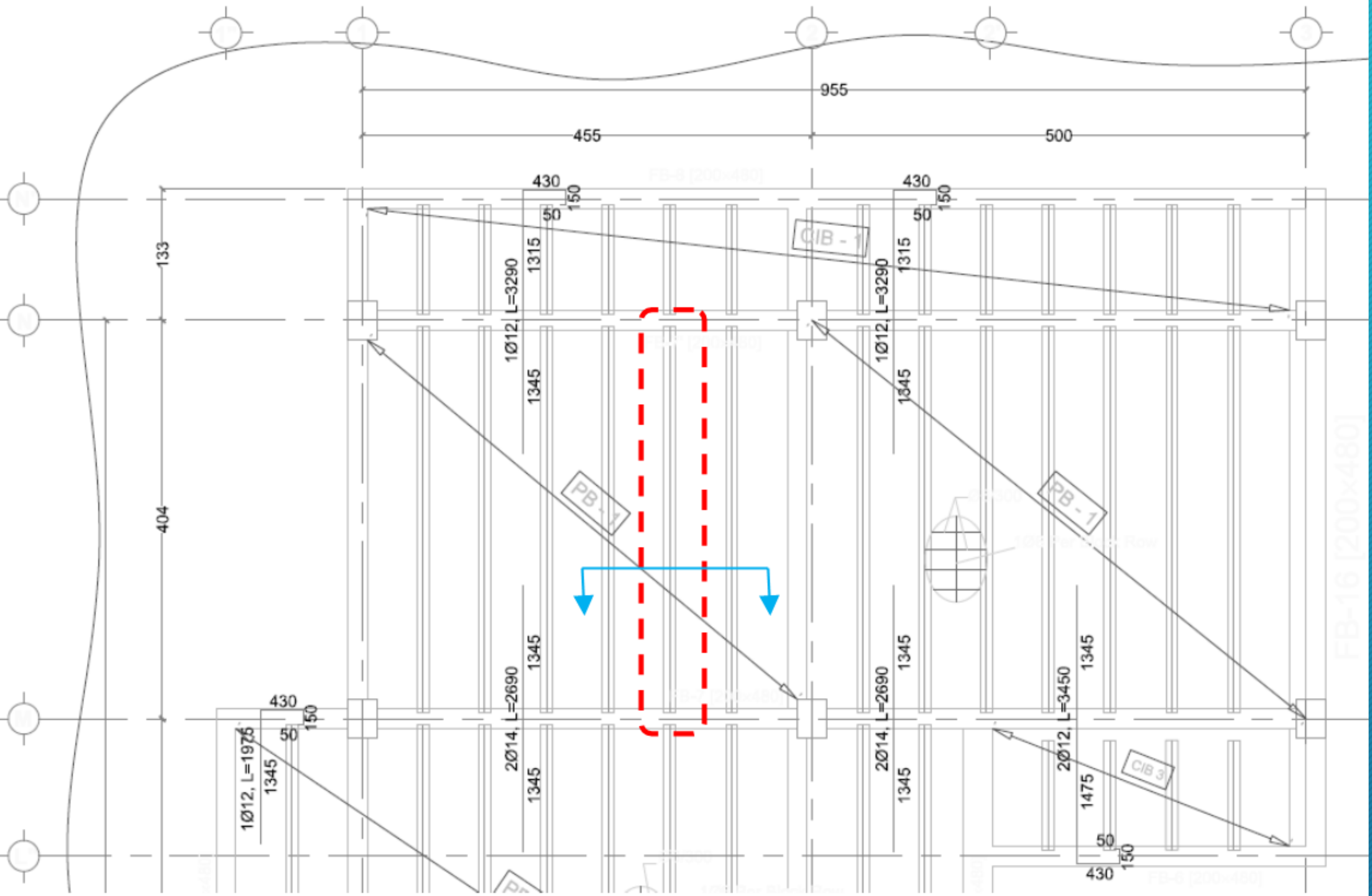



## Main-rib section

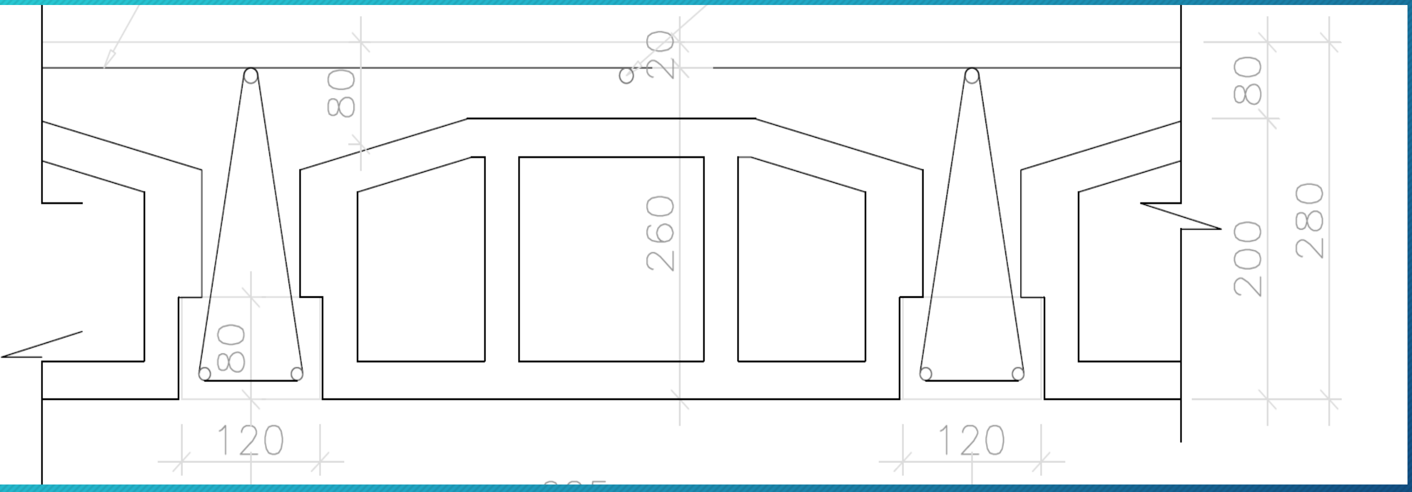



## Transverse rib section






Plan-Layout (precast rib slab system)



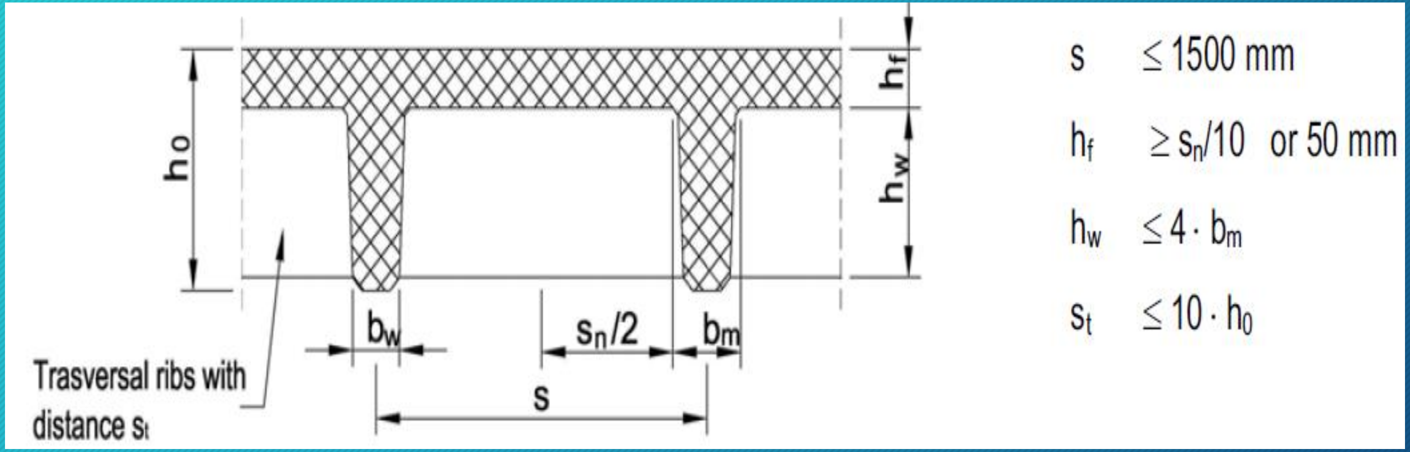

Section of the precast ribs and blocks with the toping. (low-cost housing projects)

# Ribbed Slab Systems: General Requirements according to EC



Ribbed or waffle slabs need NOT be treated as discrete elements for the purposes of analysis, provided that the flange or structural topping and transverse ribs have sufficient torsional stiffness. This may be assumed provided that:

- the rib spacing does not exceed 1500 mm
- the depth of the rib below the flange does not exceed 4 times its width
- the depth of the flange is at least 1/10 of the clear distance between ribs or 50 mm, whichever is the greater
- transverse ribs are provided at a clear spacing not exceeding 10 times the overall depth of the slab.




The minimum flange thickness of 50 mm may be reduced to 40 mm where permanent blocks are incorporated between the ribs. This exception applies for slabs with clay blocks only.

# Ribbed Slab Systems: Procedure for design of ribbed slabs



1. Shear forces and moments (Analysis): Shear forces and moments for continuous rib slabs can be obtained by elastic analysis with due consideration to live load variation.
2. Design for moment and moment reinforcement: The mid-span section is designed as a T-beam with flange width equal to the distance between ribs. The support section is designed as a rectangular beam. The slab may be made solid near the support to increase shear resistance. Moment reinforcement consisting of one or more bars is provided in the top and bottom of the ribs. If appropriate, bars can be curtailed in a similar way to bars in solid slabs
3. Shear resistance and shear reinforcement: the shear verification is carried out for the critical section of the rib with the same procedure as in a rectangular beam section. Shear verification should also be carried out for the section between the flange and the rib.

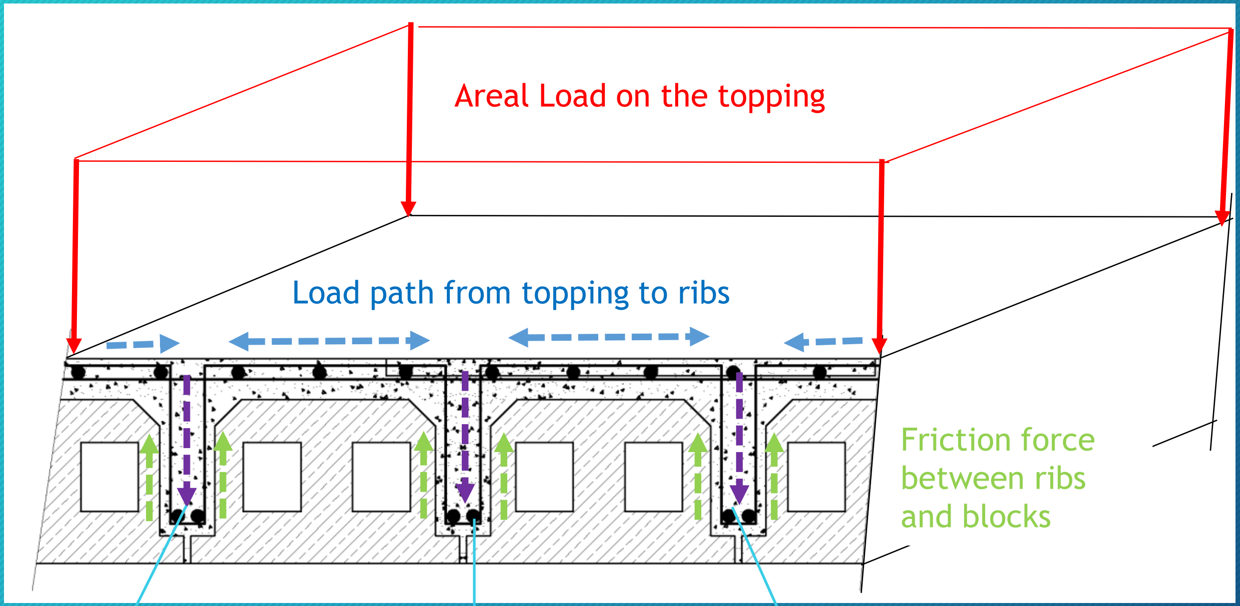
# Ribbed Slab Systems: Procedure for design of ribbed slabs



4. Reinforcement in the topping: a mesh reinforcement with a cross-sectional area of not less than 0.12% of the area of the topping in each direction should be provided. The spacing of bars should not exceed one-half the center-to-center distance of the ribs. The mesh is placed in the center of the topping and requirements of cover in the code should be satisfied. If the ribs are widely spaced the topping may need to be designed for moment and shear as a continuous one-way slab between ribs.

# Ribbed Slab Systems: load transfer mechanism in rib slab system





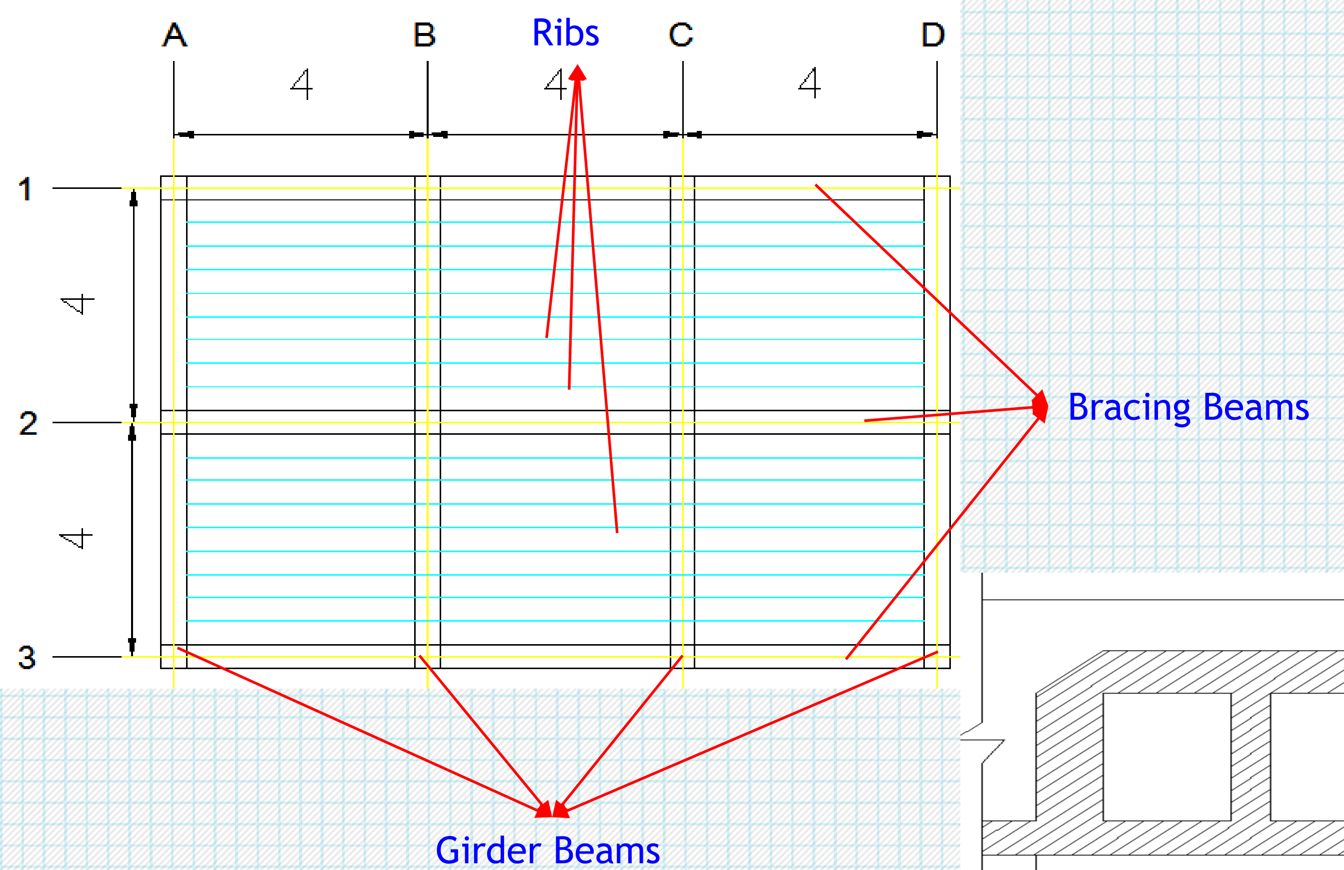

Total load carried by each rib = load from topping + load from blocks

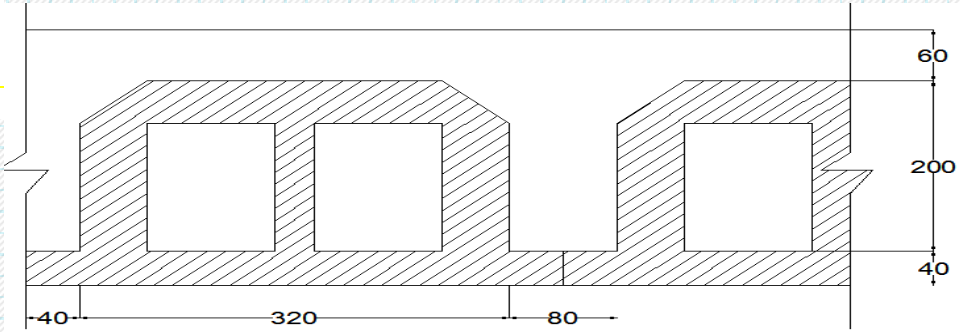
Example 2.1. A typical floor system of a lecture hall is to be designed as a ribbed slab. The joists, which are spaced at 400mm, are supported by girders. The overall depth of the slab without finishing materials is 300mm. Imposed load of 1.5KN/m$^2$ for partition and fixture is considered in the design. In addition, the floor has a floor finish material of 3cm marble over a 2cm cement screed and it ha 2cm plastering as ceiling. Take the unit weight of ribbed block to be 2KN/m$^2$.



Use: C 20/25, S – 300, φ8 and φ12 bar for web and longitudinal reinforcement with cover to stirrup of 15mm

a) Analyze and design the ribbed slab system, considering the effects of variable load pattern
b) Analyze and design the Girders and Bracing Beams



Solution: a) Analysis and Design of the ribs

Step1: Summarize the given parameters

| | | |
|---|---|---|
| Material | C20/25 | $f_{ck}$=20MPa; $f_{cd}$=11.33MPa; $f_{ctm}$=2.2MPa; $E_{cm}$=30,000MPa |
| | S-300 | $f_{yk}$=300MPa; $f_{yd}$=260.87MPa; $E_s$=200,000MPa; $\varepsilon_y$=1.30‰ |

Step2: Verify if the general requirements for Rib slab are met using Euro Code 2.

1. The centers of the ribs should not exceed 1.5 m:
- This is satisfied, as the center-to-center spacing between the ribs is 400mm.
2. The depth of ribs excluding topping should not exceed four times their average width.
- Also satisfied as 80 x 4 > 240 mm.
3. The minimum rib width should be determined by consideration of cover, bar spacing and fire resistance
- BS 8110 code - recommends 125 mm,
- Assume for this example the conditions are satisfied hence assume requirement satisfied.
4. The thickness of structural topping or flange should not be less than 50mm or one tenth of the clear distance between ribs.
- 60 mm satisfies this requirement.

Step3: Loading on the Ribs

Permanent/Dead load:

Joist→ 0.2 * 0.08 * 25 = 0.4kN/m
Topping→ 0.4 * 0.06 * 25 = 0.6kN/m
Floor finish → 0.4 * 0.03 * 27 = 0.32kN/m
Cement Screed→ 0.4 * 0.02 * 23 = 0.184kN/m
Plastering → 0.4 * 0.02 * 23 = 0.184kN/m
Partition and fittings → 0.4*1.5 = 0.6kN/m
Ribbed block → 0.4 * 2 = 0.8kN/m

$G_k$ = 3.092 KN/m

Imposed variable/Live load:

Category C2 according to Table 6.2 EN1990-1-1:2001

$Q_k$ = 4KN/m2 * 0.4 = 1.6 KN/m

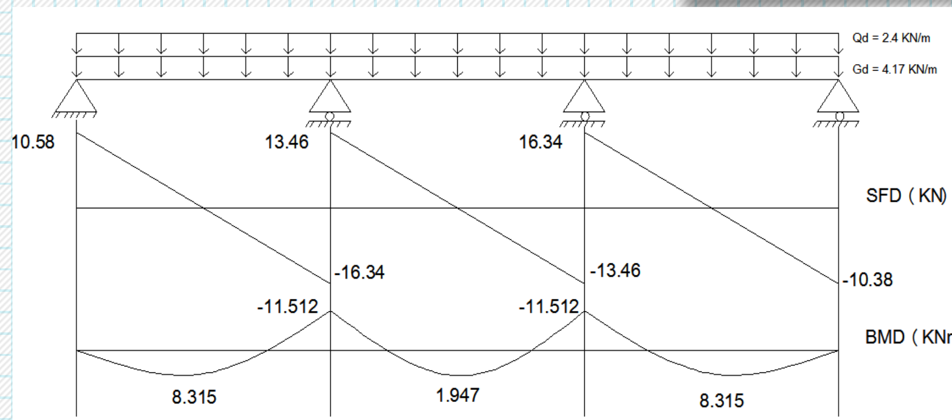
Design load:

$G_d$= 1.35*Gk=1.35*3.092=4.174KN/m
$Q_d$=1.5*Qk=1.5*1.6=2.4KN/m

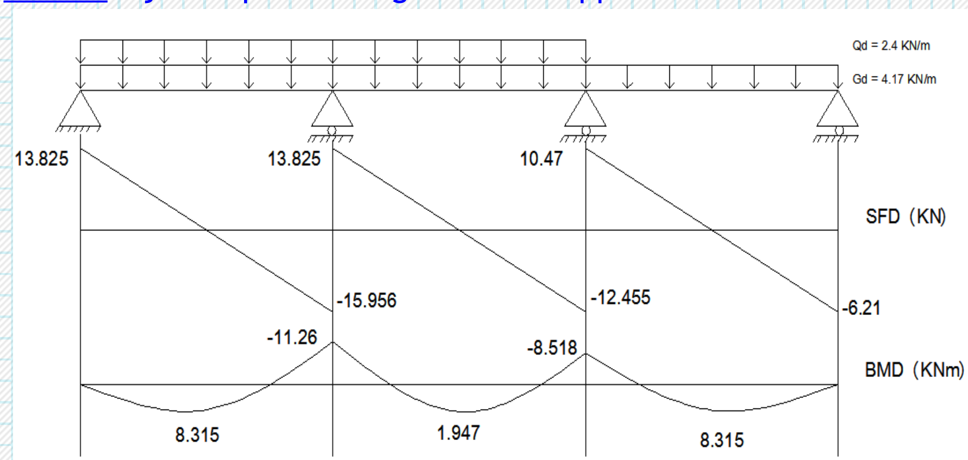


Step 4: Analysis (for Ribs)

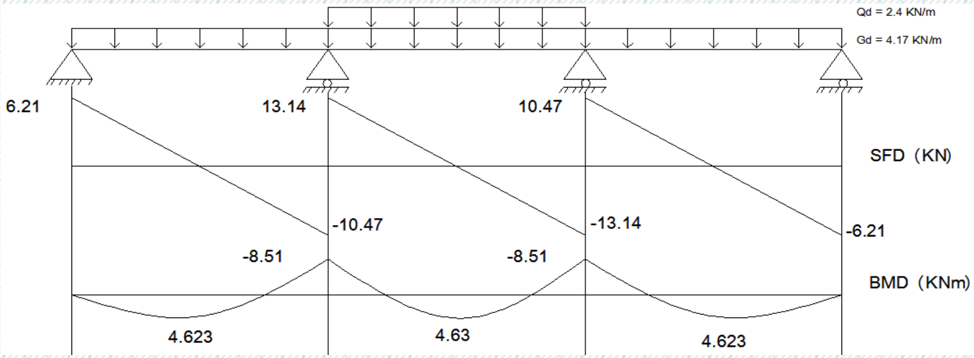
Case 1:Full design load across all the span



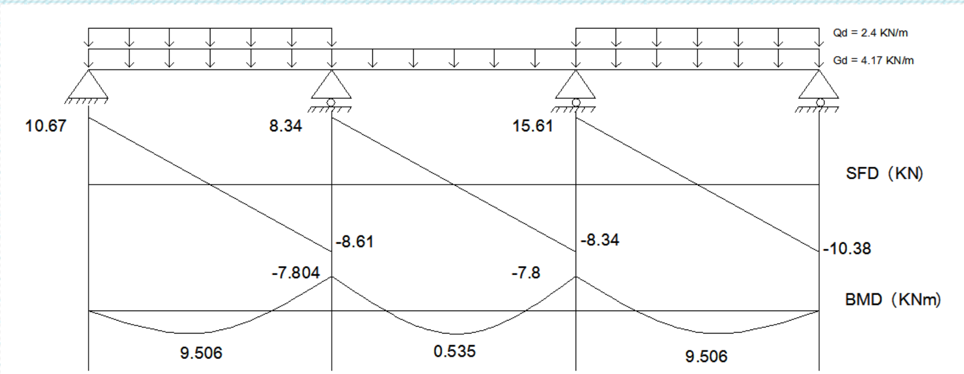

Case 2:Adjucet span loading for max. support moment at B or C



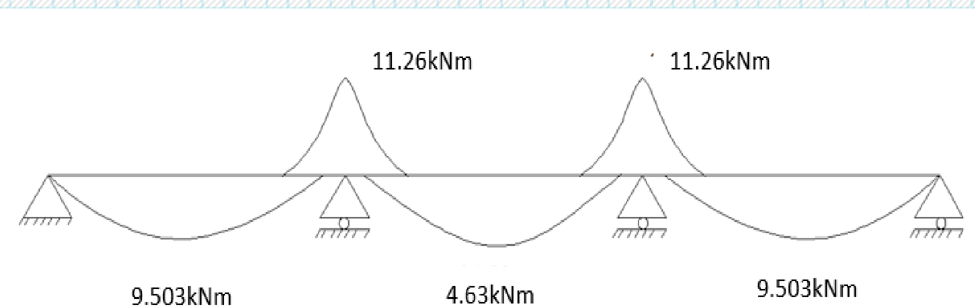
Case 3: Alternate span loading for max. span BC moment




Case 4: Alternate span loading for max. span AB or CD moment




Moment envelop for the ribs






Step 5: Design for flexure (for Ribs)

5.1 Design for Span moment

Cross section at span

$h_f$=60 mm

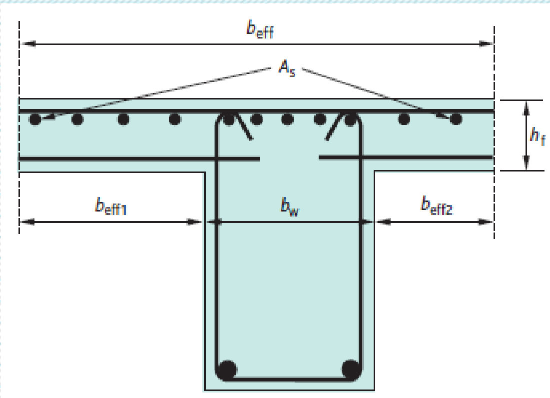
$b_w$=80 mm

H =260 mm

Cover=15mm

d=260-15-6-12/2=233 mm

Effective flange width, $b_{eff}$




For span moments flange width equal to the distance between ribs = 400mm

For support moments flange width equal width of the rib = 80mm

## Design for Span moment AB and CD

$M_{sd} = 9.506\ KNm \quad b_{eff} = 400\ mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$



$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{9.506 * 10^6 Nmm}{11.33 * 400 * 233^2} = 0.0386$$

$\mu_{sd} < \mu_{sd,lim} = 0.295$ ***Singly reinforced***

$K_x = 0.055 \quad X = K_x d = 12.815\ mm < h_f$ ***design as a rectangular section***

$K_z = 0.975 \quad Z = K_z d = 227.175mm$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{9.506 * 10^6 Nmm}{260.87 * 227.175} = 160.40\ mm^2$$

$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \quad where\ b_t = b_w \quad d = 233mm \quad f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$

$A_{smin} = 35.54\ mm^2 < A_s$ ***OK!***

$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 1.418$ ***use 2∅12bottom bars***

## Design for Span moment BC

$M_{sd} = 4.63\ KNm \quad b_{eff} = 400\ mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{4.63 * 10^6 Nmm}{11.33 * 400 * 233^2} = 0.0188$$

$\mu_{sd} < \mu_{sd,lim} = 0.295$ ***Singly reinforced***

$K_x = 0.07 \quad X = K_x d = 16.31\ mm < h_f$ ***design as a rectangular section.***

$K_z = 0.985 \quad Z = K_z d = 229.505mm$

$$A_s = \frac{M_{sd}}{f_{yd}Z} = \frac{4.63 * 10^6 Nmm}{260.87 * 229.505} = 77.33mm^2$$

$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \quad where\ b_t = b_w \quad d = 233mm \quad f_{ctm} = 2.2\ mpa \quad f_{yk} = 300\ mpa$

$A_{smin} = 35.54\ mm^2 < A_s$***OK!***

$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 0.6837$ ***use 2∅12 bottom bars***

## 5.2. Design for Support moment at B or C

$M_{sd} = 11.512\ KNm \quad b_w = 80\ mm \quad d = 233mm \quad f_{cd} = 11.33\ mpa \quad f_{yd} = 260.87\ mpa$



$$\mu_{sd} = \frac{M_{sd}}{f_{cd}\, bd^2} = \frac{11.512 * 10^6 Nmm}{11.33 * 80 * 233^2} = 0.2339$$

$\mu_{sd} < \mu_{sd,lim} = 0.295 \quad Singly\ reinforced$

$K_z = 0.88 \quad Z = K_z d = 205.04\ mm$

$$A_s = \frac{M_{sd}}{f_{yd}\, Z} = \frac{11.512 * 10^6 Nmm}{260.87 * 205.04} = 215.22mm^2$$

$$A_{smin} = \frac{0.26 f_{ctm}}{f_{yk}} b_t d \qquad where\ b_t = b_w \quad d = 233mm \quad f_{ctm} = 2.2\ mpa$$

$f_{yk} = 300\ mpa$

$A_{smin} = 35.54\ mm^2 < A_s \quad \boldsymbol{OK}!$

$using\ \emptyset\ 12 \quad a_s = 113.1mm^2 \quad n = \frac{A_s}{a_s} = 1.9029 \quad \boldsymbol{use\ \ 2\emptyset 12 \quad bars\ at\ the\ top}$

**<u>Home take Bonus exam:</u>**

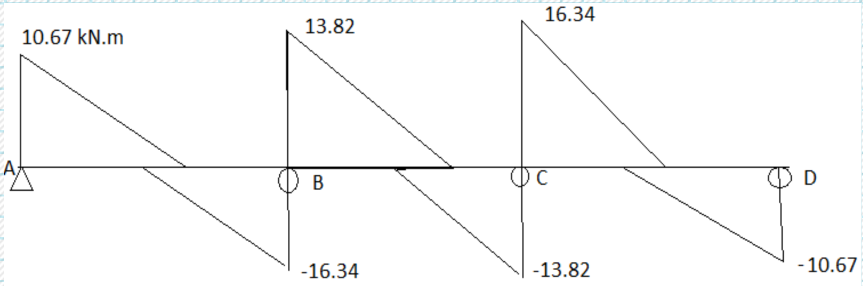
Design the rib for shear along its span and between the rib and flange.

Read on how we can design for the girder beams and bracing beams.

Step 6: Design for shear (for Ribs)

## 6.1. Shear along the span of the rib

### 6.1.1. Shear envelop for the rib




$V_{ed}$ =16.34kN (at support B and C)
$V_{ed}$ =10.67kN (at support A and D)

### 6.1.2. Diagonal compression check of concrete

$$V_{Rd,max} = \frac{\alpha_{cw}.b_w.Z.v.f_{cd}}{(\cot\theta + \tan\theta)}$$

$$V_{Rd,s} = \frac{A_{sw}}{S}.Z.f_{ywd}.\cot\theta$$

$$V_{Rd}\,(\text{minimum of}) = \begin{cases} V_{Rd,max} \\ V_{Rd,s} \end{cases}$$

*where*

$\alpha_{cw} = 1$ *recommended value for non − prestressed member*

$limit\ \theta \Longrightarrow 1 \le \cot\theta \le 2.5$

$Take \ldots\ldots \cot\theta = 2.5$

$bw = 80\ mm$

$f_{cd} = 11.33\ Mpa$

Z=0.9xd = 209.7

$$\Longrightarrow V_{Rd,max} = \frac{1x80x209.7x0.6x11.33}{\left(2.5 + \frac{1}{2.5}\right)} = 39.325\ KN$$

$$\Longrightarrow V_{Rd,max} > V_{ED} \ldots\ldots at\ all\ locations, \quad \boldsymbol{therefore\ Ok!}$$

### 6.1.3. Check shear capacity of the concrete section

$$V_{Rd,c} = \left[C_{Rd,c}.K.(100\rho_1.f_{ck})^{\frac{1}{3}} + K_1\sigma_{cp}\right]b_w.d > (V_{min} + K_1.\sigma_{cp})b_w.d$$

Where:

- $C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$
- $K = 1 + \sqrt{\frac{200}{d}} \le 2.0 \qquad d = 233$

  K=1.92

- $\rho_1 = \frac{A_s}{b_w.d} < 0.02$

  $$\rho_1 = \left\{\frac{226.19}{80x233} = 0.0121, \qquad for\ sections\ with\ 2\emptyset 12\right.$$

- $f_{ck} = 20\ Mpa$
- K1= 0.15
- $\sigma_{cp} = \frac{N_{ED}}{A_c} < 0.2f_{cd} = 0 \ldots\ldots\ldots (N_{ED} = 0)$
- $V_{min} = 0.035.K^{\frac{3}{2}}.f_{ck}^{\frac{1}{2}} = 0.416$

Therefore

$$For\ sections\ with\ 2\emptyset 12 \Longrightarrow V_{Rd,c} = 12.43\ KN > 7.75\ KN$$

$V_{Rd,c}$ < $V_{ed}$ (at support B and C) shear reinforcement are required.
$V_{Rd,c}$ > $V_{ed}$ (at support A and D) Minimum shear reinforcements are required.

### 6.1.4. Compute the required shear reinforcement

$$\frac{A_{sw}}{S} = \frac{V_{ed}}{0.78 * d * fyk * cot\theta}$$

$$A_{sw} = \frac{2 * \pi * 6^2}{4} = 56.54mm^2$$

$$S = \frac{A_{sw} * 0.78 * d * fyk * cot\theta}{V_{ed}}$$

$V_{Ed}$– d from the face of the columns but since the loads are small just take the values at the center of the columns.

$\theta$- since **$V_{Rd,max}$>$V_{Ed}$** lets take the conservative value

$$\theta = 22° \qquad cot\theta = 2.5$$

$$S = \frac{56.54 * 0.78 * 233 * 300 * 2.5}{Ved} = \frac{7706.68}{Ved} kNmm$$

The stirrup spacing described as a function of the design shear force for ease of calculation

### 6.1.5. Minimum shear reinforcement and maximum spacing requirement

$$\rho_{min} = \frac{0.08.\sqrt{f_{ck}}}{f_{yk}} = 0.00119$$

$$S = \frac{A_{sw}}{b_w . \rho_w . \sin\alpha} = \frac{2x(\emptyset 6^2 \frac{\pi}{4})}{80x0.00119x1} = 593.99mm$$

$S_{min}$ = 594 mm

$$S_{max} = 0.75.d.(1 + \cot\alpha), \qquad \alpha = 90^0$$

$$S_{max} = 0.75d$$

$S_{max}$ = 0.75 *d = 0.75 * 233 = 174.75 → 175mm



### 6.1.6. Provide shear reinforcements (stirrups)

| span | Location | $V_{Ed}$(KN) | $S_{cal}$ (mm) | $S_{provid}$ (mm) |
|---|---|---|---|---|
| AB | near A | | | ф 6 C/C 175 mm |
| | near B | -16.34 | 471 | ф 6 C/C 175 mm |
| BC | near B | 13.825 | 557.44 | ф 6 C/C 175 mm |
| | near C | 13.825 | 557.44 | ф 6 C/C 175 mm |
| CD | near C | 16.34 | 471.64 | ф 6 C/C 175 mm |
| | near D | | | ф 6 C/C 175 mm |



Step 7: Design of the Girder and bracing beams

### 7.1. Design of the Girder Beams

The girder beams are designed for loads transferred from the ribs as the maximum reaction forces in the analysis of the ribs.

For ease of analysis of the Girder Beams the reaction forces from the rib analysis are divided by the rib spacing to have a uniform load acting over the span.

Since the Girder Beams are continuums, live load variation along the span should be considered.

Pleas refer the Example 2.2 for detailed calculation

### 7.2. Design of the Bracing Beams

The bracing beams are designed for their own self weight and partition load resting directly on them.

Pleas refer the Example 2.3 for more.

# Thank you for the kind attention!



Questions?

Prep Test#1 Question: Moment Curvature and ribbed Slab design (Conceptual) {10 minutes max}

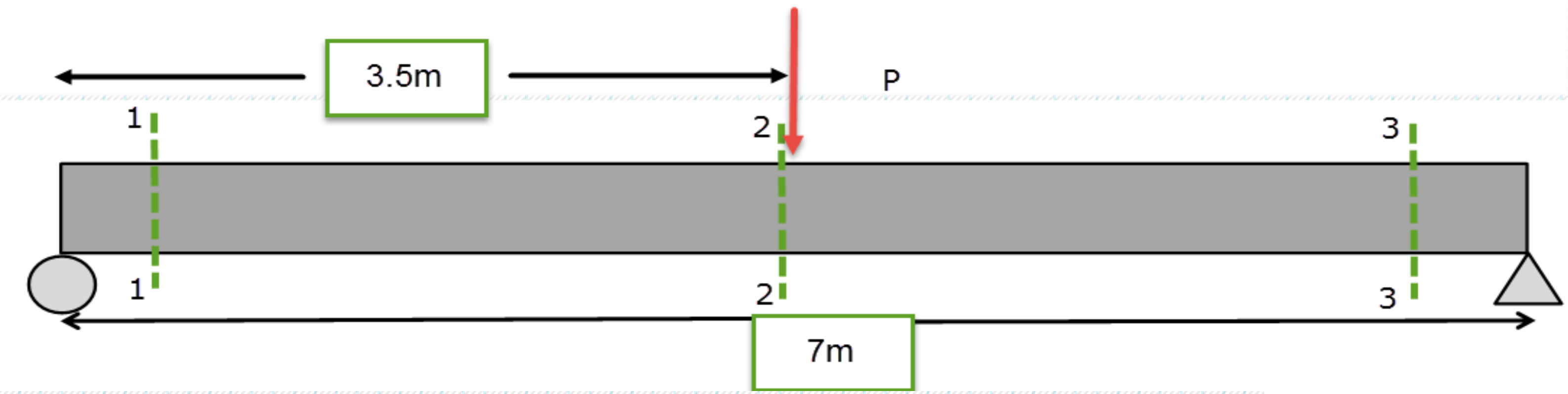


1. If the requirements stated in section 5.3.1 of prEN 1992-1-1 are not satisfied a rib slab system is not functional according to the code. (T/F)
2. In ribbed slab system the ribs are oriented parallel to the short direction of the span. (T/F)
3. What is the occasional need of transverse ribs in one-wayribbed slab systems? (give at least two reasons)
4. How can one increase the capacity to loading ratio of a rib with out changing the material property, cross sectional geometry of the rib, the number of longitudinal and shear reinforcement of the ribs and the external imposed loading on the slab? (if you think it is impossible, then answer as such)
5. From only the moment-curvature relationship of a RC section of a given beam system under a specified loading, one can determine the exact load capacity of the structure. (T/F)
6. The moment required to cause a given amount of curvature is usually higher in value in reality compared to calculated values. Which of the following can be a reason as to why?
   a) The role of concrete in tension is neglected in calculation.
   b) The use of load and material safety factors used in calculation.
   c) Both
   d) Neither

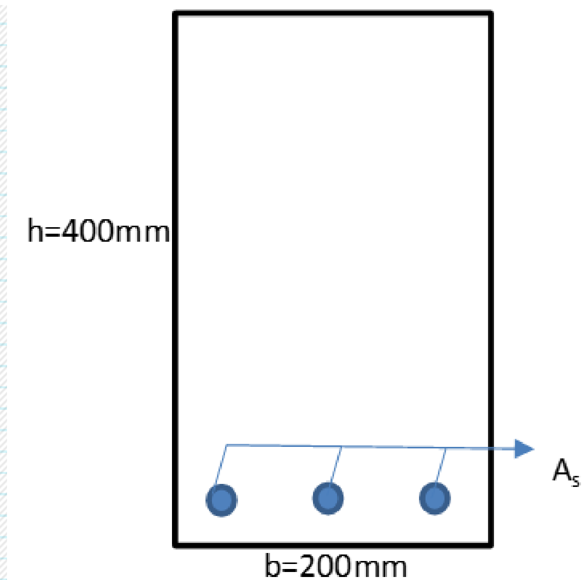
**Prep Test#1 Question:** Moment Curvature (Workout) {45 minutes max}

Based on the support and loading conditions of the beam shown below, answer the following questions. The RC beam has b/h=200/400mm, and is casted out of C20/25 concrete and reinforced by s-400, 3φ14 bar as bottom reinforcements.






1. The moment curvature relationship at the three sections is the same. (T/F)
2. Draw the moment curvature relationship at section 2-2
3. Compute the ultimate load corresponding to the ultimate capacity.




Section 1-1, 2-2 and 3,3

# What's Next?



Good Luck with your First Test, Hope the Prep test Helped! ☺

Please read on Chapter three Part One -> One-way Solid Slab systems for next Class