Addis Ababa institute of Technology

Reinforced Concrete Structures 2

(CEng-3122)

Chapter Three

Two-way Slab Systems

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School of Civil and Environmental Engineering
Concrete Material and Structures Chair

Presentation Outline

1. Introduction
2. Analysis and Design of Two-way Slab systems
3. Analysis and Design of Flat Slab Systems

Content

Introduction

Introduction

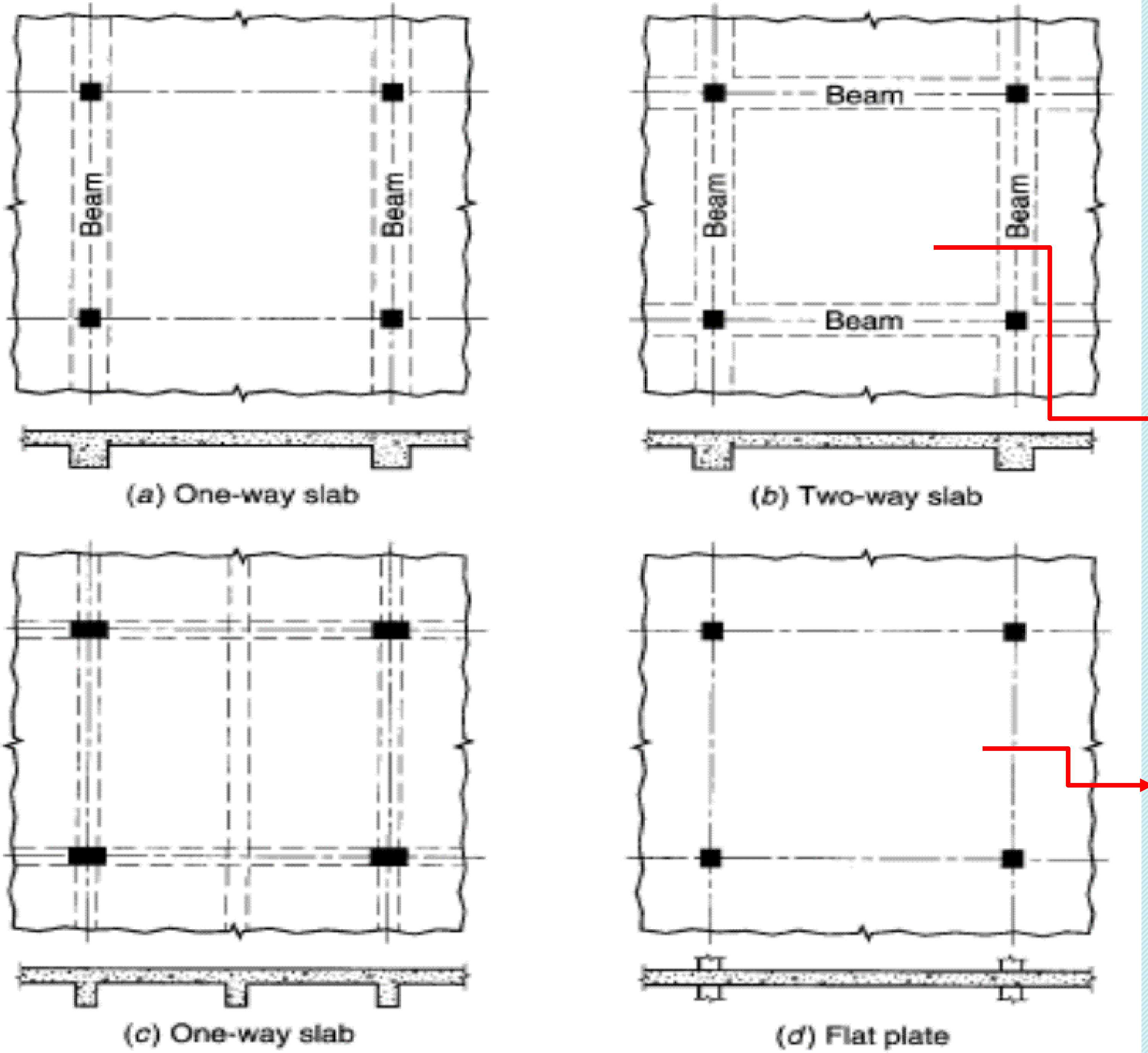
In reinforced concrete construction, slabs are used to provide flat, useful surfaces.

Depending on the load transfer mechanism slabs can be classified as One-way and Two-way slab systems, as discussed in previous lessons.

The analysis and design of one-way slabs, especially for beam supported and one-way ribbed slab systems, was discussed in previous lessons.

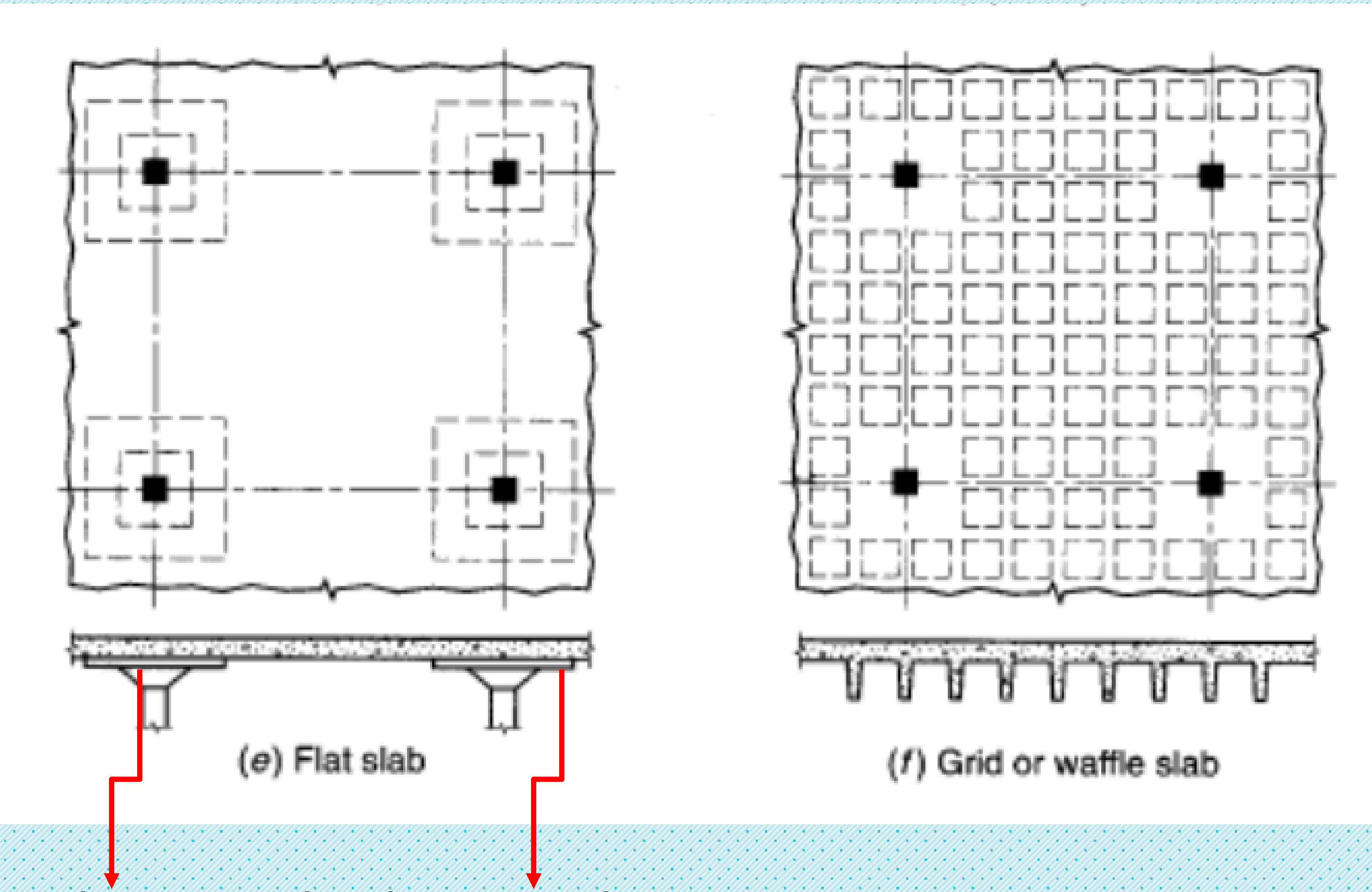
Analysis and design of two-way slab system is a lot more complex as load transfer is in two orthogonal directions and computing the design actions is not straightforward as in one-way slabs.

For rectangular slabs with standard edge conditions and subject to uniformly distributed loads, normally the bending moments are obtained using tabulated coefficients. Such coefficients are provided later in this section.



Two-way (Beam Supported) Slab System

Flat Slab System



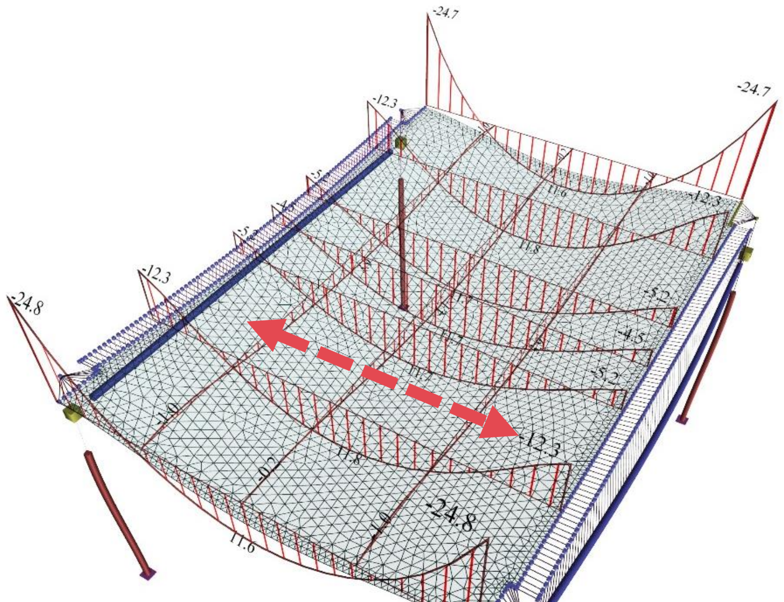
(e) Flat slab

(f) Grid or waffle slab

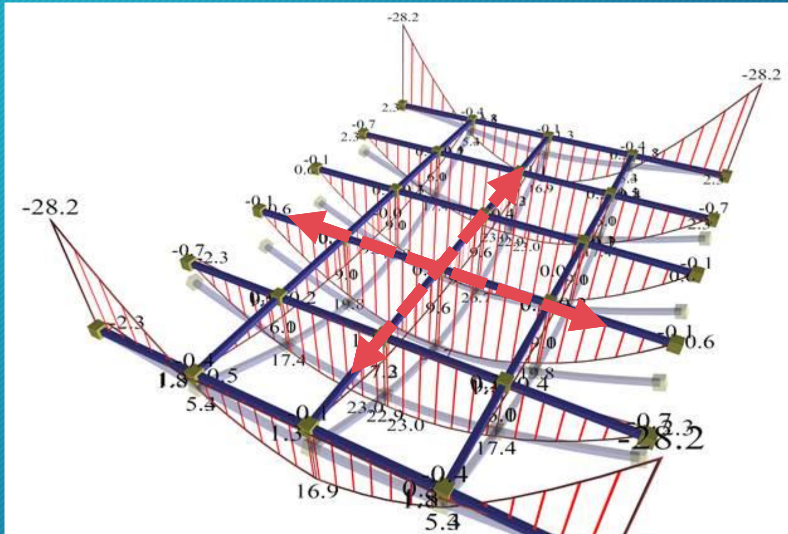
Column capital and Drop panel

Introduction: Two-way Slab Systems

In many cases, rectangular slabs are of such proportions and are supported in such a way that two-way action results. When loaded, such slabs bend into a dished surface rather than a cylindrical one.



Predominately one-way action resulting in a half-cylindrical shaped deformation.



Two-way action

This means that at any point the slab is curved in both principal directions, and since bending moments are proportional to curvatures, moments also exist in both directions.

How can we determine these orthogonal bending moments? And how do we design for them?

Analysis and Design of Two-way Slab systems

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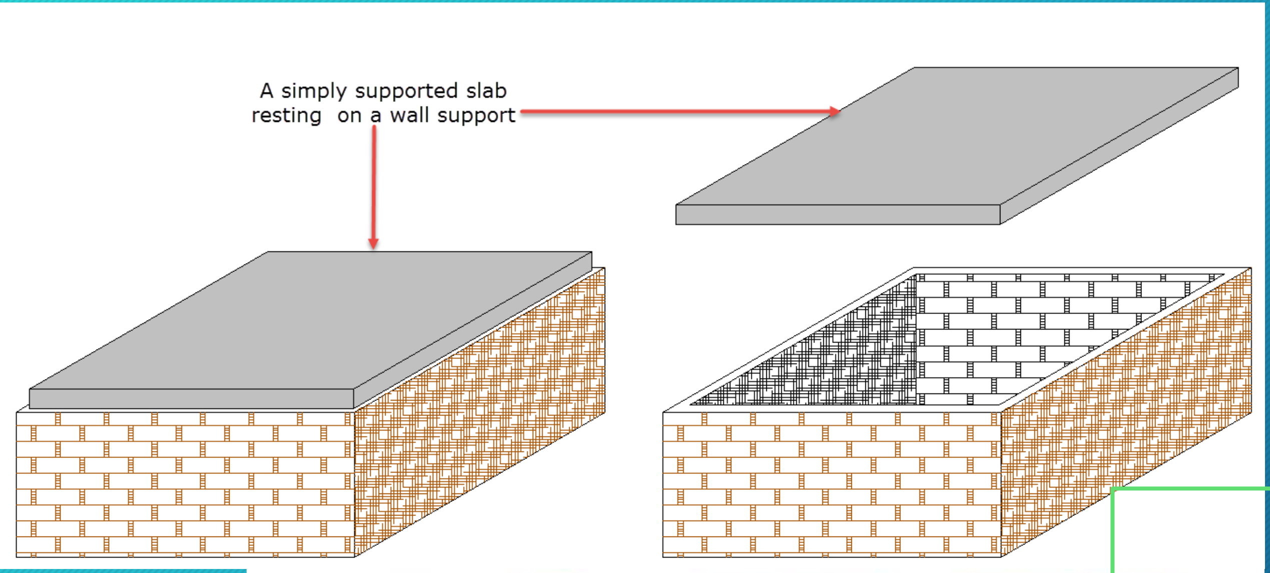
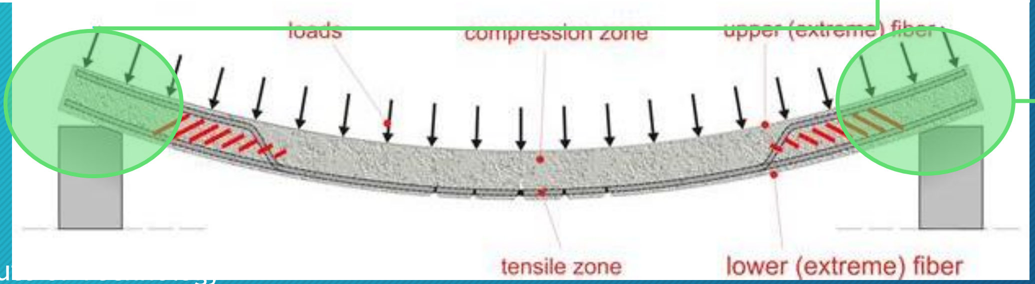
- SIMPLY SUPPORTED SLABS
- RECTANGULAR PANELS WITH RESTRAINED EDGES
- RESTRAINED SLAB WITH UNEQUAL CONDITIONS AT ADJACENT PANELS
- LOADS ON SUPPORTING BEAMS

Simply Supported Slabs

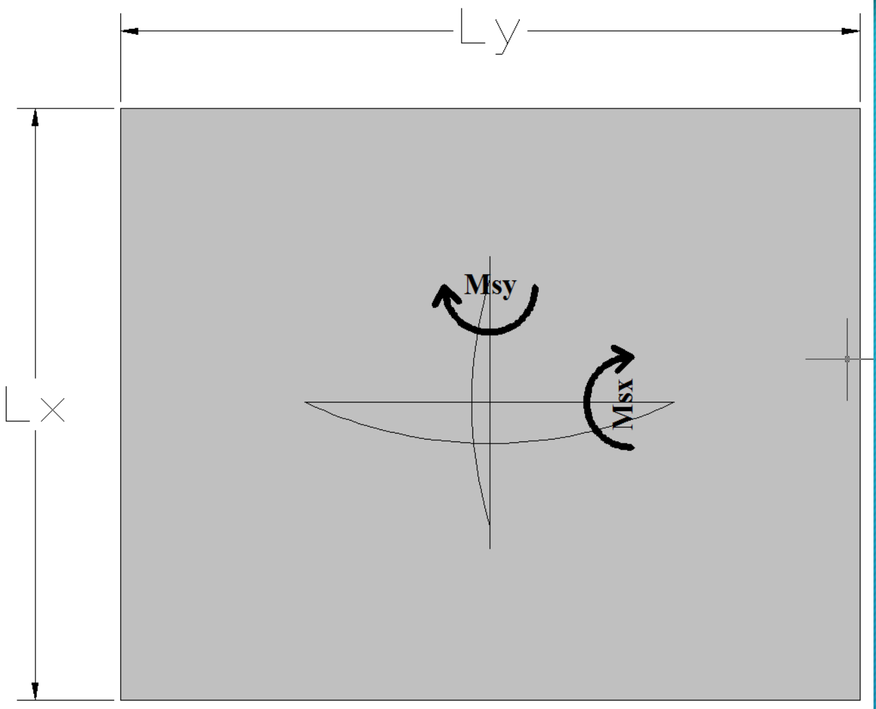
Are rectangular slabs is considered to be simply supported, if it simply resting on the underlying supporting element (wall or beam) across its four edges. There is no reinforcement continuity between the slab and the supporting element.





Free to up lift

Simply Supported Slabs



When simply-supported slabs do not have adequate provision to resist torsion at the corners, and to prevent the corners from lifting, the maximum moments per unit width are given by the following equations:

$$M_{sx} = \alpha_{sx} n l_x^2 \qquad M_{sy} = \alpha_{sy} n l_x^2$$

Where:

α_{sx} and α_{sy} Moment coefficients given in table.

n Total design ultimate load per unit area

l_x Length of shorter side

Where:

M_{sx} Maximum design ultimate moments either over supports or at mid-span on strips of unit width and span l_x

M_{sy} Maximum design ultimate moments either over supports or at mid-span on strips of unit width and span l_y

Simply Supported Slabs

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
α_{sx}	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118
α_{sy}	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029

The values in the table are derived from the following equations:

$$\alpha_{sx} = \frac{(l_y/l_x)^4}{8\left(1+(l_y/l_x)^4\right)}$$

$$\alpha_{sy} = \frac{(l_y/l_x)^2}{8\left(1+(l_y/l_x)^4\right)}$$

Rectangular Panels with Restrained Edges

In slabs where the corners are prevented from lifting, and provision for torsion is made, the maximum design moments per unit width are given by the following equations:

$$M_{sx} = \beta_{sx} n l_x^2 \qquad M_{sy} = \beta_{sy} n l_x^2$$

Where:

M_{sy}	Maximum design ultimate moments either over supports or at mid-span on strips of unit width and span l_y
M_{sx}	Maximum design ultimate moments either over supports or at mid-span on strips of unit width and span l_x
β_{sx} and β_{sy}	Moment coefficients

Type of Panel and moments considered	Short span coefficients, β_{sx}								Long span coefficients, β_{sx} for all values of l_y/l_x
	Values of l_y/l_x								
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Interior panels									
Negative moment at continuous edge	0.031	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
Positive moment at mid-span	0.024	0.028	0.032	0.035	0.037	0.040	0.044	0.048	0.024
One short edge discontinuous									
Negative moment at continuous edge	0.039	0.044	0.048	0.052	0.055	0.058	0.063	0.067	0.037
Positive moment at mid-span	0.029	0.033	0.036	0.039	0.041	0.043	0.047	0.050	0.028
One long edge discontinuous									
Negative moment at continuous edge	0.039	0.049	0.056	0.063	0.068	0.073	0.082	0.089	0.037
Positive moment at mid-span	0.030	0.036	0.042	0.047	0.051	0.055	0.062	0.067	0.028
Two adjacent edges discontinuous									
Negative moment at continuous edge	0.047	0.056	0.063	0.069	0.074	0.078	0.087	0.093	0.045
Positive moment at mid-span	0.036	0.042	0.047	0.051	0.055	0.059	0.065	0.070	0.034
Two short edges discontinuous									
Negative moment at continuous edge	0.046	0.050	0.054	0.057	0.060	0.062	0.067	0.070	-
Positive moment at mid-span	0.034	0.038	0.040	0.043	0.045	0.047	0.050	0.053	0.034
Two long edges discontinuous									
Negative moment at continuous edge	-	--	-	-	-	-	-	-	0.045
Positive moment at mid-span	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
Three edges discontinuous (one long edge continuous)									
Negative moment at continuous edge	0.057	0.065	0.071	0.076	0.081	0.084	0.092	0.098	-
Positive moment at mid-span	0.043	0.048	0.053	0.057	0.060	0.063	0.069	0.074	0.044
Three edges discontinuous (one short edge continuous)									
Negative moment	-	-	-	-	-	-	-	-	0.058

Rectangular Panels with Restrained Edges

The above equations and the coefficients in Table 3 2 may be derived from the following equations:

$$\beta_y = \left(24 + 2N_d + 1.5N_d{}^2\right)/1000$$

$$\gamma = \frac{2}{9}\left[3 - \sqrt{18\frac{l_x}{l_y}\left\{\sqrt{\left(\beta_y + \beta_1\right)} + \sqrt{\left(\beta_y + \beta_1\right)}\right\}}\right]$$

$$\sqrt{\gamma} = \sqrt{\left(\beta_x + \beta_3\right)} + \sqrt{\left(\beta_x + \beta_3\right)}$$

Where:

N_d	Number of discontinuous edges $(0 \leq N \leq 4)$
β_1 and β_2	Hogging moments, per unit width, over the shorter edges divided by nl_x^2
β_3 and β_4	Hogging moments, per unit width, over the longer edges divided by nl_x^2
l_x	Length of shorter side
l_y	Length of longer side

Note: β_1 and β_2 take values of $4/3\beta_y$ for continuous edges or zero for discontinuous edges.

β_3 and β_4 take values of $4/3\beta_x$ for continuous edges or zero for discontinuous edges.

Rectangular Panels with Restrained Edges

The conditions in which the equations may be used for continuous slabs only are as follows.

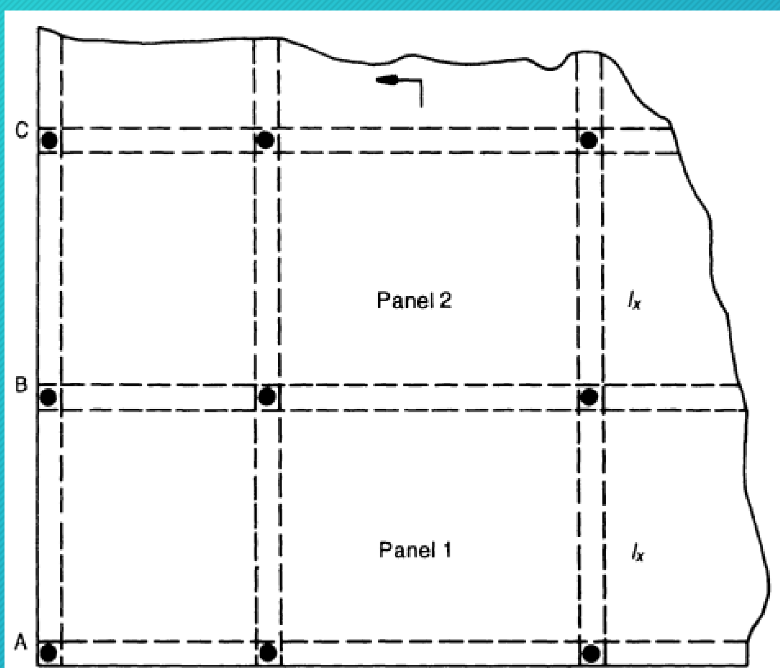
a) The characteristic dead and imposed loads on adjacent panels are approximately the same as on the panel being considered.

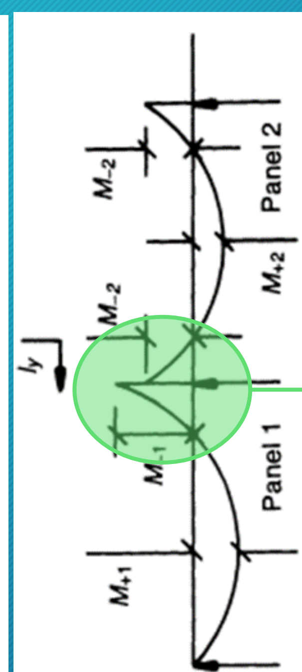
b) The span of adjacent panels in the direction perpendicular to the line of the common support is approximately the same as the span of the panel considered in that direction.

Restrained slab with unequal conditions at adjacent panels

In some cases, the bending moments at a common support, obtained by considering the two adjacent panels in isolation, may differ significantly (say by 10%), because of the differing edge condition at the far supports or differing span lengths or loading.

Consider panels 1 and 2 in Figure below.





Moments from the two panels is not the same ?

In these circumstances, the slab may be reinforced throughout for the worst case span and support moments.

However, this might be uneconomic in some cases. In such cases, the following distribution procedure may be used:

Restrained slab with unequal conditions at adjacent panels

1) Obtain the support moments for panels 1 and 2 from Table. Treating M_{-1} and M_{-2} as fixed end moments, the moments may be distributed in proportion to the stiffenesses of span l_x in panels 1 and 2. Thus, a revised bending moments M'_{-B} may be obtained for support over B.

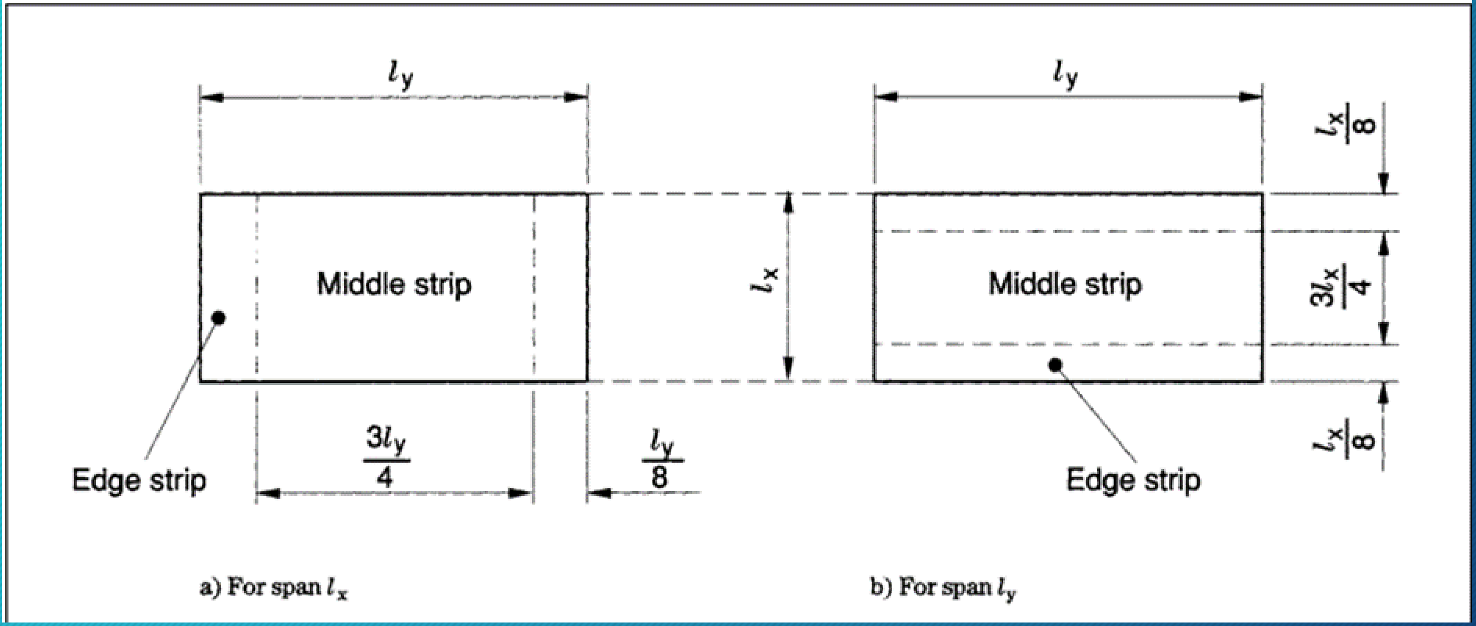
1) The span moments in panels 1 and 2 should be recalculated as follows:

$$M'_{+1} = \left(M_{-1} + M_{+1}\right) - M^{1}_{-B} \qquad (1)$$

$$M'_{+2} = \left(M_{-2} + M_{-2} + M_{+2}\right) - M'_{-B} - M_{-2} \qquad (2)$$

Restrained slab with unequal conditions at adjacent panels

For curtailment of reinforcement, the point of contraflexure may be obtained by assuming a parabolic distribution of moments in each panel.



Loads on Supporting Beams

The design loads on beams supporting solid slabs spanning in two directions at right angles and supporting uniformly distributed loads may be assessed from the following equations:

$$\upsilon_{sy} = \beta_{vy} n l_x$$

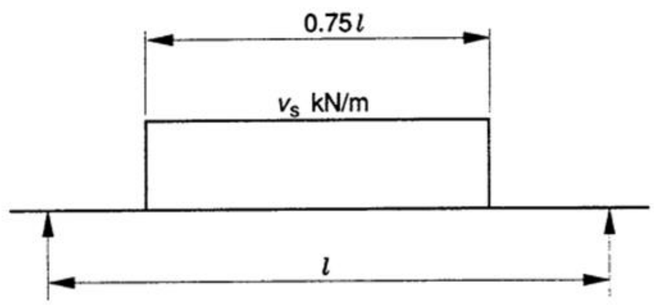
$$\upsilon_{sx} = \beta_{vx} n l_x$$

Where:

υ_{sy} Design end sear on strips of unit width and span l_y and considered to act over the middle three-quarters of the edge.

υ_{sx} Design end sear on strips of unit width and span l_x and considered to act over the middle three-quarters of the edge.

Distribution of load on a beam supporting a two-way spanning slabs



NOTE $\upsilon_s = \upsilon_{sx}$ when $l = l_y$; $\upsilon_s = \upsilon_{sy}$ when $l = l_x$;

Type of Panel and location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Four edges continuous									
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
One short edge discontinuous									
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Discontinuous edge	-	-	-	-	-	-	-	-	0.24
One long edge discontinuous									
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	-
Two adjacent edges discontinuous									
Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
Discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26
Two short edges discontinuous									
Continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	-
Discontinuous edge	-	-	-	-	-	-	-	-	0.26
Two long edges discontinuous									
Continuous edge	-	-	-	-	-	-	-	-	0.40
Discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	-
Three edges discontinuous (one long edge discontinuous)									
Continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	-
Discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
Three edges discontinuous (one short edge discontinuous)									
Continuous edge	-	-	-	-	-	-	-	-	0.45
Discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30
Four edges discontinuous									
Discontinuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33

Analysis and Design of Flat Slab Systems

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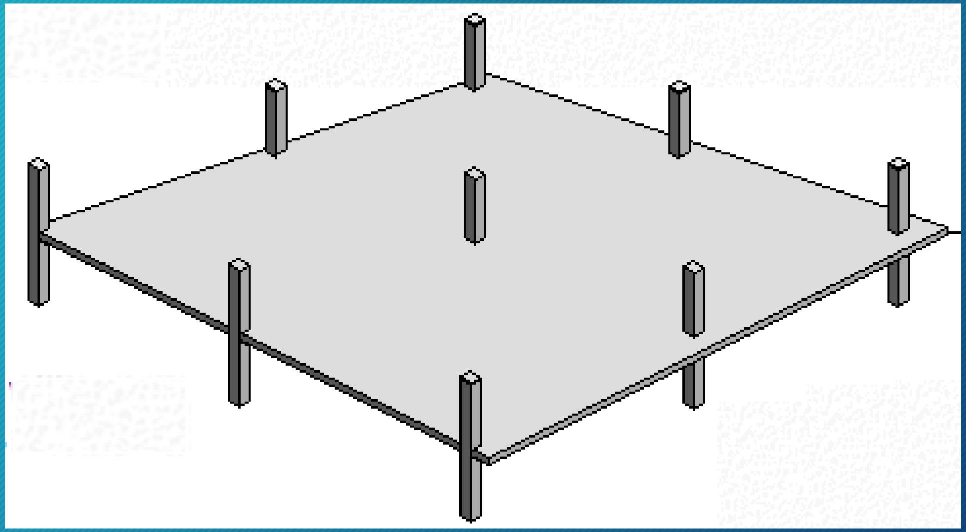
- INTRODUCTION
- ANALYSIS FOR FLEXURAL DESIGN
- DESIGN FOR PUNCHING SHEAR

Flat Slab Systems: Introduction

A flat slab floor is a reinforced concrete slab supported directly by the columns with no intermediate beams. Because of this it:

- is simple to construct, requiring the minimum of formwork,
- Minimizes construction depths and
- Provides a clear soffit for routing services below the slab.



However, the absence of beams means that the slab has to…

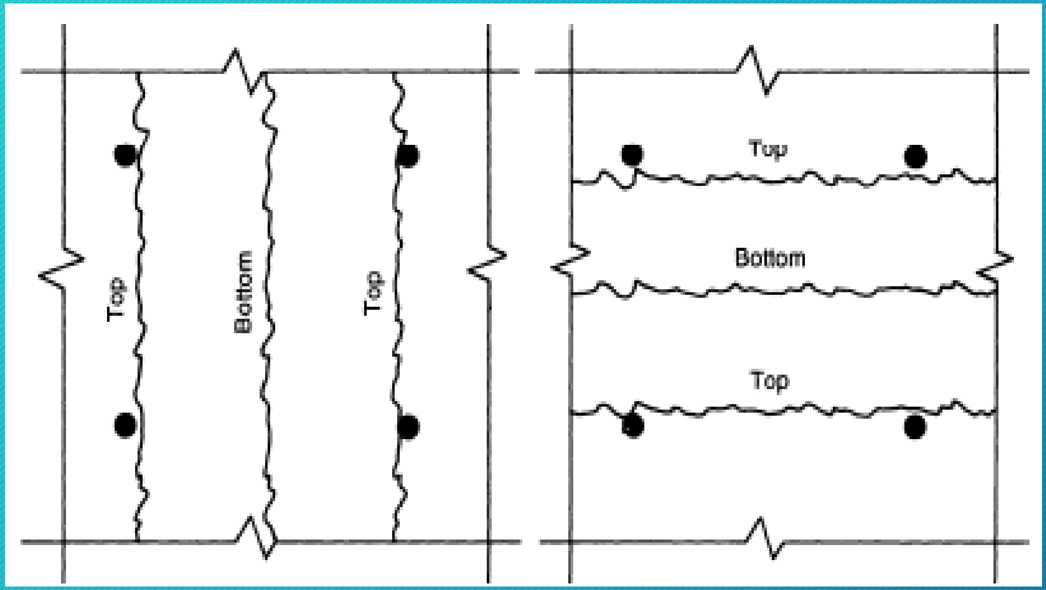
- Carry the shear forces, which are concentrated around the column,
- Transmit the moment to the edge and corner columns,
- Suffer greater deflections.

How can we quantify these stresses and deflection in flat slabs and design for them?

Flat Slab Systems: Behavior

Typical Bending Failure mode

- For a regular layout of columns, failure can occur by the formation of hinge lines along the lines of maximum hogging/support and sagging/span moments. A complementary set of yield lines can form in the orthogonal direction.

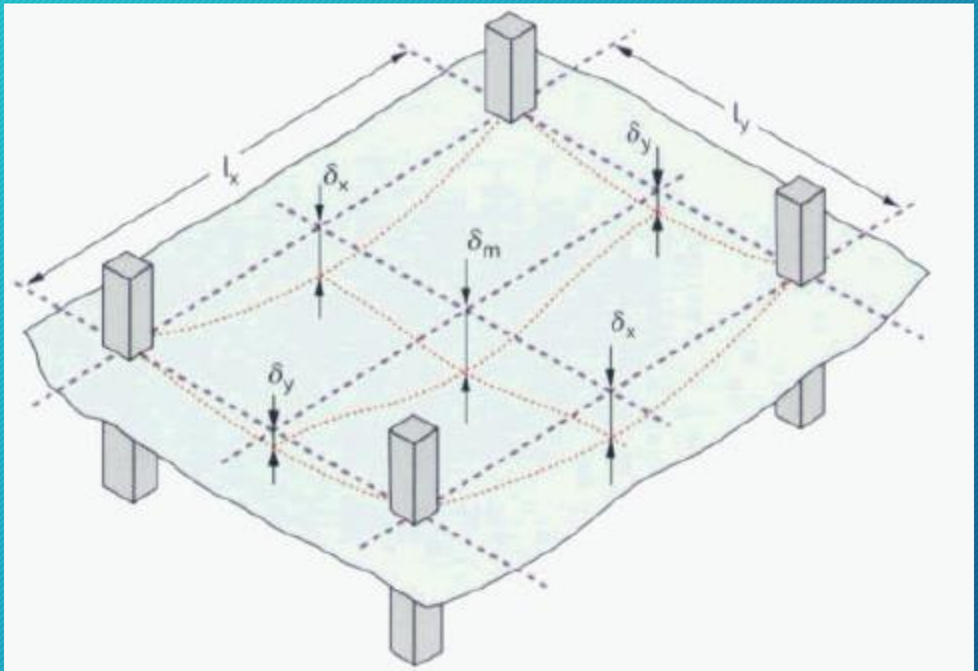


- One misconception of some engineers is to consider a reduced loading when analyzing in a particular direction. The moments applied in each orthogonal direction must each sustain the total loading to maintain equilibrium. There is no sharing of the load by partial resistance in each orthogonal direction.

Flat Slab Systems: Behavior

Typical deflected shape

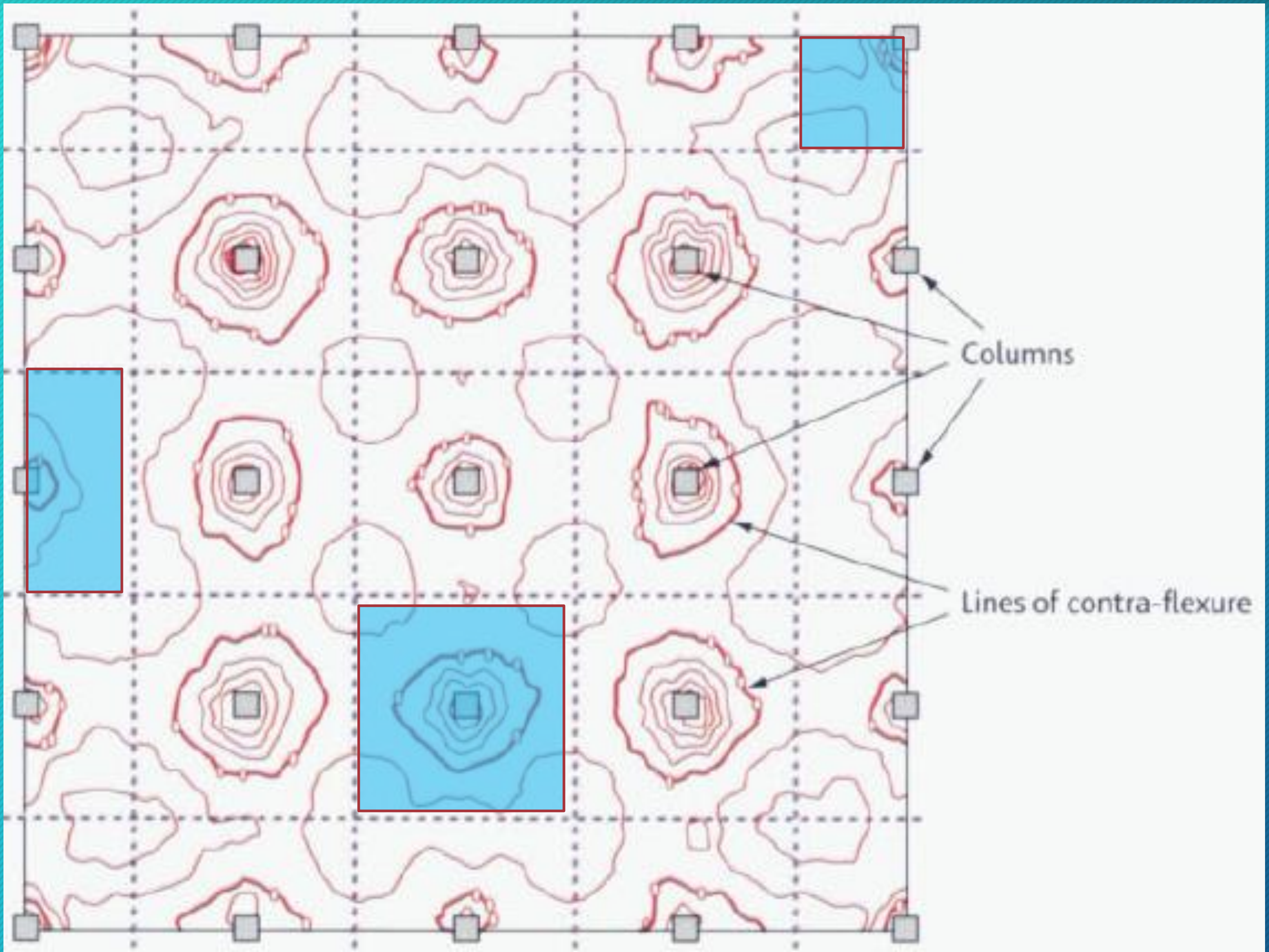
- The deflected shape of an interior panel of a flat slab on a regular grid of columns under typical in-service conditions is a function of the sum of the deflections in each orthogonal direction.



- Deflection is one of the governing factor in dimension flat slab systems, especially when the flat slab system doesn't incorporate deeper beams.

Flat Slab Systems: Behavior

Moment Contours



- The use of finite element methods shows that the distribution of bending moments per unit width is characterized by hogging moments that are sharply peaked in the immediate vicinity of the columns.

- The magnitude of the hogging moments locally to the column face can be several times that of the sagging moments in the mid-span zones.

Flat Slab Systems: Behavior

Flexural Behavior of Flat slabs as the vertical load is increased

Behavior…..

1 Stage 1

- Moments at the supports and mid-span increase elastically until the 1st crack occur.
- These are likely to appear 1st at the top of the slab close to the column.

2 Stage 2

- Cracking may increase to some way into the span from the column, and may also have started to appear at mid-span.
- The cracking increases the non-linear behavior of the slab, although it still behaves elastically as the load increases between the formations of new cracking

3 Stage 3

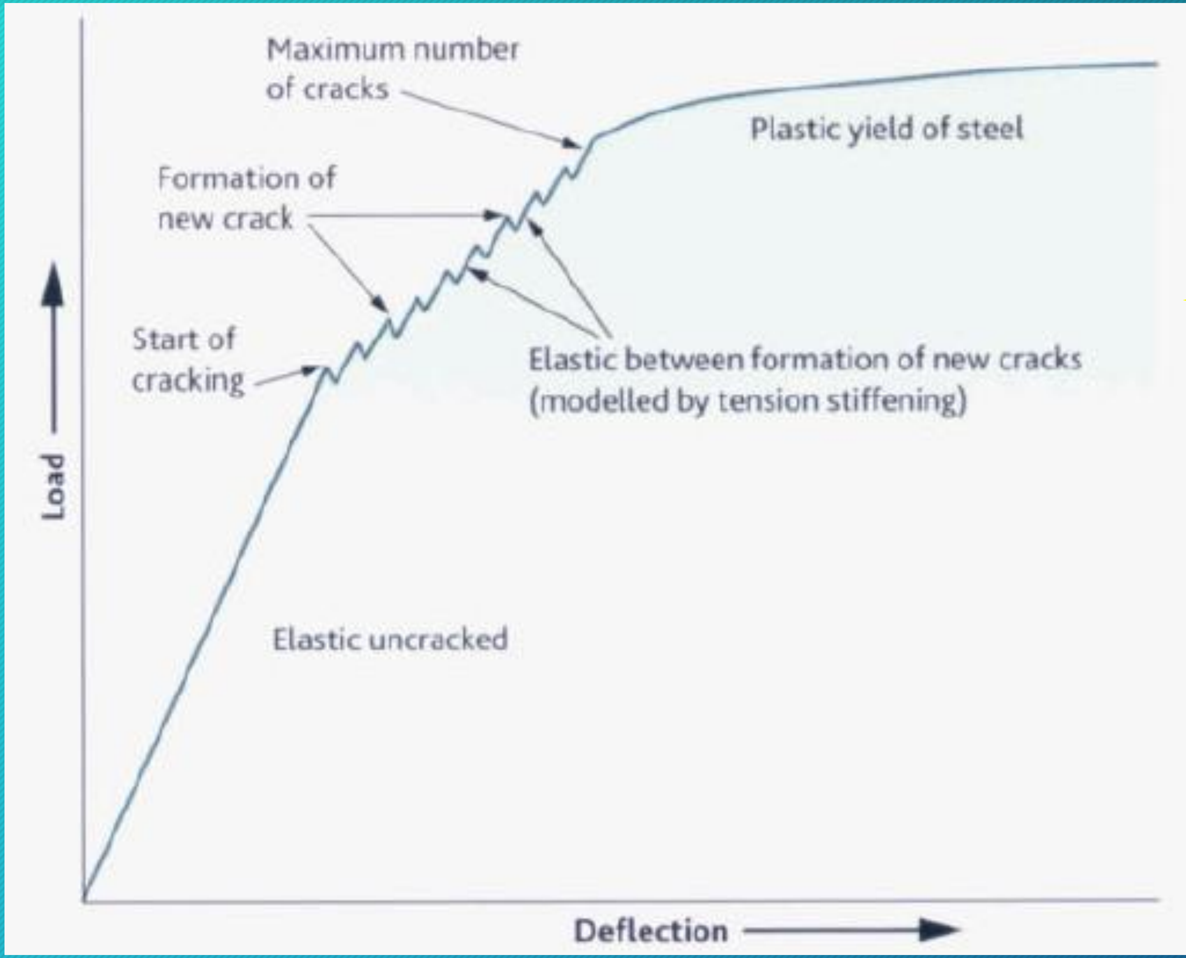
- The reinforcement first starts to yield in the top bars close to the columns and the junction of the slab still behaves elastically as the load increases between the formation of new cracks but with reducing tension stiffening.

4 Stage 4

- Failure will occur once a failure mechanism is reached

Flat Slab Systems: Behavior

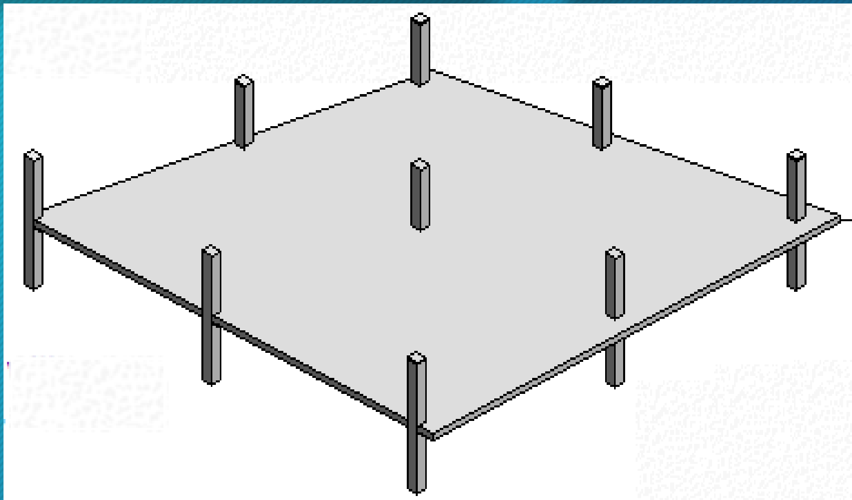
Flexural Behavior of Flat slabs as the vertical load is increased



So how can we analyze flat slab systems for flexure?

Flat Slab Systems: Analysis

Eurocode 2 does not contain the derived formulae or specific guidance on determining moments and shear forces. This has arisen because it has been European practice to give principles in the codes and for the detailed application to be presented in other sources such as textbooks.



Using Eurocode 2 for the analysis of flat slabs, the following methods may be used:

- Equivalent frame method
- Finite element analysis
- Yield line analysis
- Grillage analogy

Annex I of the Eurocode gives recommendations for the equivalent frame method on how to apportion the total bending moment across a bay width into column and middle strips to comply with section. 9.4.1, which requires the designer to concentrate the reinforcement over the columns

Flat Slab Systems: According to EC-2

- **Equivalent frame**. This method is suitable for regular layouts of columns, but requires engineering judgment for irregular layouts
- **Finite element analysis**. This method allows the design of irregular column layouts and can provide the design of reinforcement details. Where the appropriate software is available, it is possible to obtain reasonable assessment of deflections. (SHOW OIB-HQ MODEL)
- **Grillage analysis**. This method has similar facilities to finite element models and can also be used for irregular layouts of columns.
- **Yield-line methods**. These can provide suitable designs for ULS but do not give adequate information for serviceability design.

Flat Slab Systems: EC-2 Recommendation Equivalent frame method

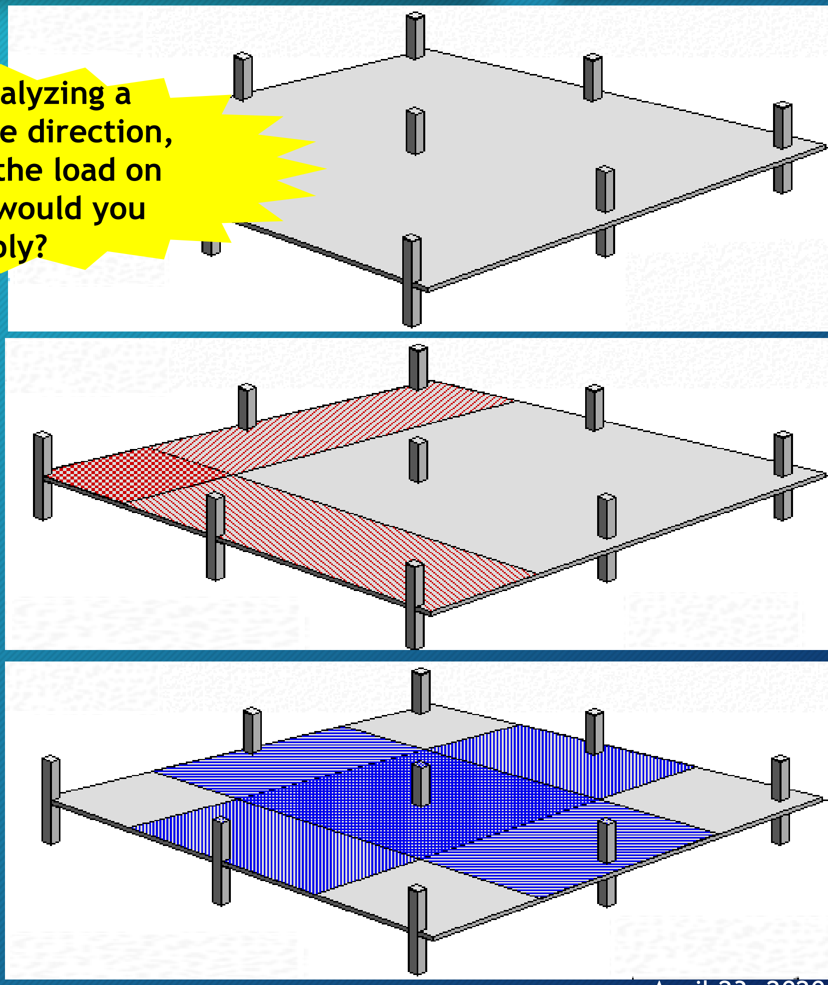
The slab can be analyzed by the Equivalent method. This provides an acceptable representation of the behavior of the floor by a system of colu strips analyzed separately in each directi

When analyzing a frame in one direction, what % of the load on the strip would you apply?

Rather than a full height frame, a series of slab frames comprising a single floor with columns above and below is more commonly used , subject to the most unfavorable arrangement of load. The final moments can be redistributed.

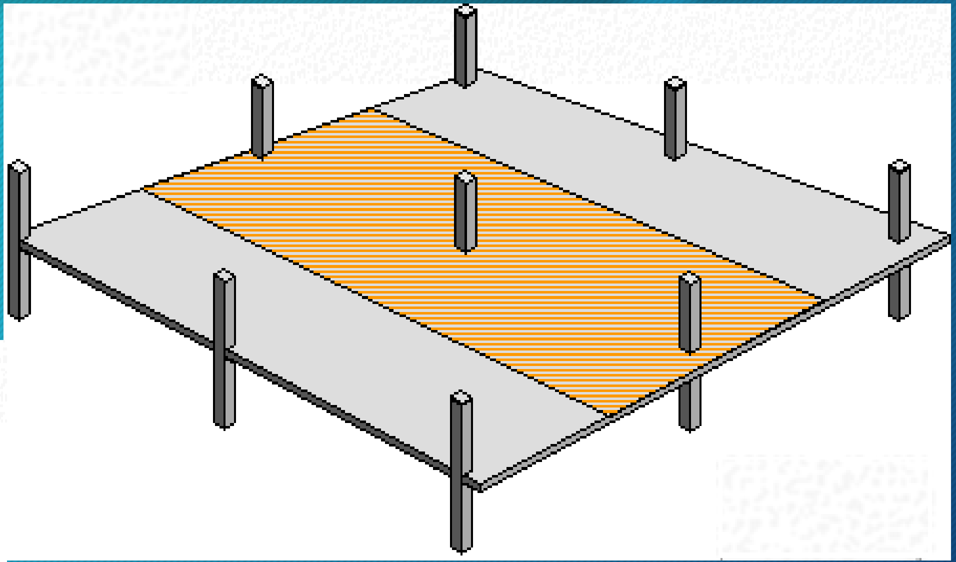
The width of the frame strips is taken between mid points of the columns or the edge of the slab as appropriate. In the slab above, the edge strips and the interior strips are,

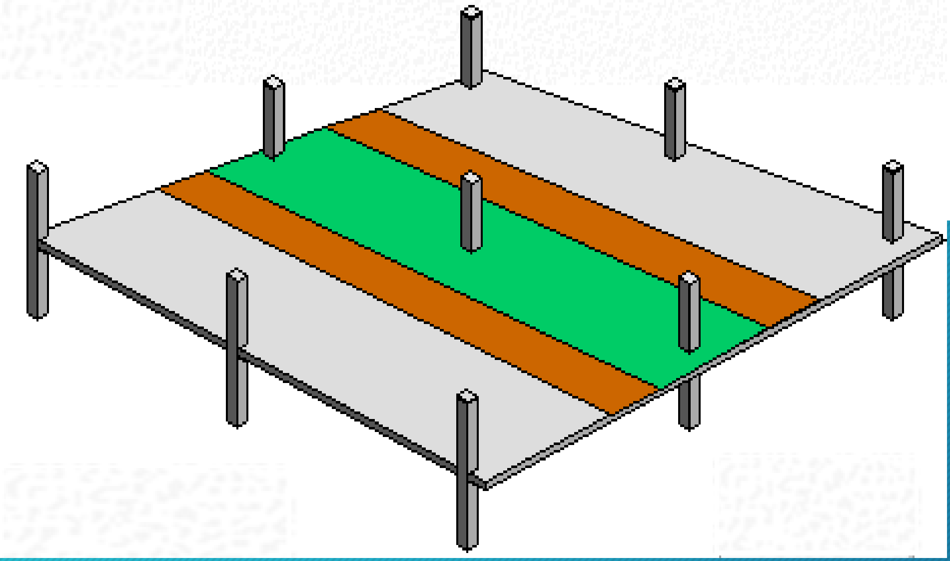
When considering only vertical loads, the stiffness of the slab strip may be based on the full width. In any combination of loading that includes lateral loads the stiffness should be based on the half the width.



Flat Slab Systems: EC-2 Recommendation Equivalent frame method

The moment obtained from the analysis of the equivalent frame are the total moments on the slab strip. However, the distribution of moment across the width of the strip is quite obviously not uniform, since the slab is only supported in the center of the strip.

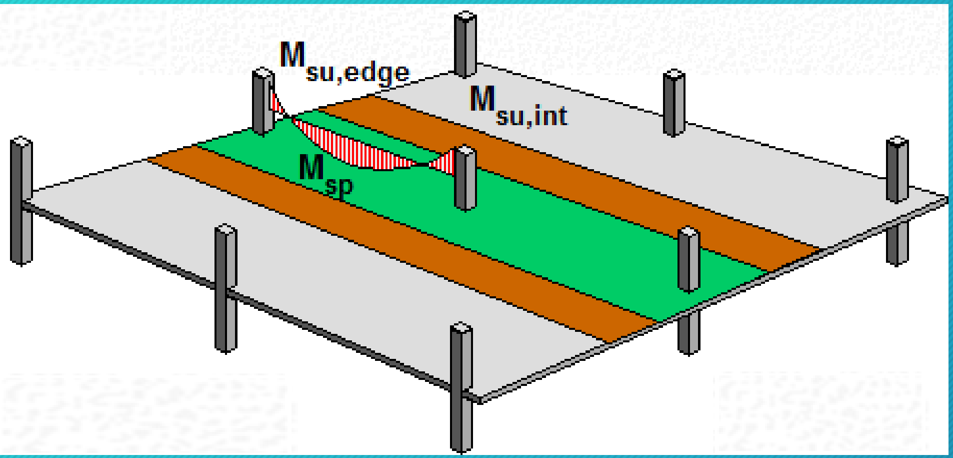




To ensure that the distribution of reinforcement corresponds approximately to the distribution of moments arising from a rigorous analysis of the slab system, the slab strip is divided into a column strip and middle strip(half each side),thus:

Flat Slab Systems: EC-2 Recommendation Equivalent frame method

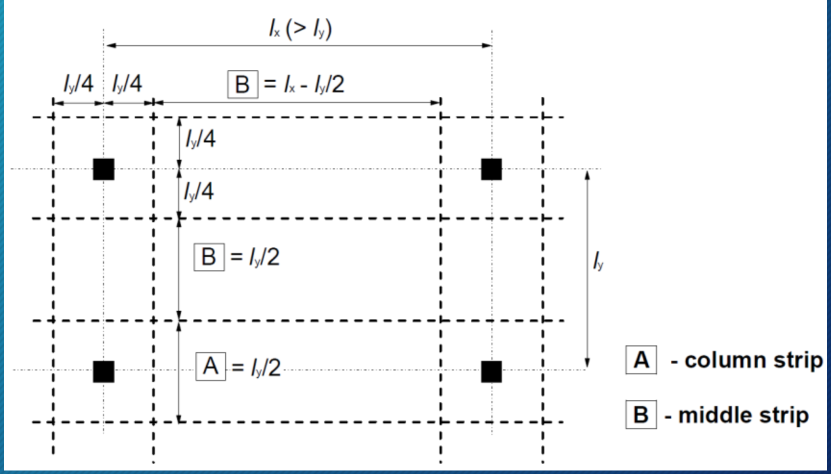
The design moments are proportioned between the two strips as follows:



	Column strip	Middle strip
Negative moment at column, M_{su}	60-80%	40-20%
Positive moment at midspan M_{sp}	50-70%	50-30%

However, the moment $M_{su,edge}$ is limited to $0.17b_ed^2f_{ck}$; for reasons please read on subject matter.

The widths of the column and middle strips and the distribution of the reinforcement are chosen to reflect the behavior of the slab.



Flat Slab Systems: According to ACI

- ACI allows slabs to be designed by any procedure that satisfies both equilibrium and geometric compatibility, provided that every section has a strength at least equal to the required strength and that serviceability conditions are satisfied.
- Two procedures for the flexural analysis and design of two-way floor systems are presented in detail in the ACI Code.

Direct Design Method

- The calculation of moments in the direct-design method is based on the total statical moment. In this method, the slab is consider panel by panel, and statical moment then is divided between positive and negative moments, and these are further divided between middle strips and column strips.

Equivalent Frame method

- Here, the slab is divided into a series of two-dimensional frames (in each direction), and the positive and negative moments are computed via an elastic-frame analysis. Once the positive and negative moments are known, they are divided between middle strips and column strips in exactly the same way as … method.

It should be noted that the difference between the two methods is in the analysis of the statcial moment only.

Flat Slab Systems: According to ACI

Direct Design Method

- The direct-design method could have been called "the direct-analysis method" because this method essentially prescribes values for moments in various parts of the slab panel without the need for structural analysis.

- It has to be noted that this method was introduced in the era when most engineering calculations were made with slide rules, & no computer software was available.

- Thus, for continuous-floor slab panels with relatively uniform lengths and subjected to distributed loading, a series of moment coefficients were developed that would lead to safe flexural designs of two-way floor systems.

Flat Slab Systems: According to ACI

Direct Design Method

1) There must be a minimum of 3 continuous spans in each direction. Thus a nine-panel structure (3 by 3) is the smallest that can be divided.

2) Rectangular panels must have a long-span/short-span ratio not greater than 2. One-way action predominates as the span ratio reaches and exceeds 2

3) Successive span lengths in each direction shall not differ by more than one-third of the longer span

4) Columns should not offset from the basic rectangular grid of the building more than 0.1 times the span parallel to the offset.

Flat Slab Systems: According to ACI

Direct Design Method

5) All loads must be due to gravity only. The direct design method can not be used for unbraced laterally loaded frames, foundation mats, or prestressed slabs.

6) The service live load shall not exceed two times the service dead load (to reduce effects of pattern load).

7) For a panel with beams b/n supports on all sides, the relative stiffness of the beams in the two ⊥ directions given by $(\alpha_{f1} l_2^2)/(\alpha_{f2} l_1^2)$ shall not be less than 0.2 or greater than 5. (α is the beam-to-slab stiffness ratio to be defined later)

Flat Slab Systems: According to ACI

Direct Design Method

Steps in slab design

1) Choose the layout and type of slab to be used
2) Choose the slab thickness (deflection limitation and shear at both exterior and interior columns)
3) Choose the design method (direct design or equivalent frame methods)
4) Compute +ve and –ve moments in the slab
5) Determine the distribution of the moments across the width of the slab
6) If there are beams, a portion of the moments must be assigned to the beams
7) Reinforcement is designed for moments in 5 & 6.
8) The shear strengths at the columns are checked.

Direct Design Method:

Minimum Thickness of Two-Way Slabs According to ACI Code

For a slab without beams between interior columns and having a ratio of long to short spans of 2 or less, the minimum thickness is as given in Table 13-1 (ACI Table 9.5(c)) but is not less than 5 in=13cm. in slabs without drop panels or 4 in=11cm. in slabs with drop panels having the dimensions defined in ACI Code Sections 13.2.5.

The ACI Code permits thinner slabs to be used if calculated deflections satisfy limits given in ACI Table 9.5(b).

TABLE 13-1 Minimum Thickness of Slabs without Interior Beams

Yield Strength, f_y[b] (psi)	Without Drop Panels[a]			With Drop Panels[a]		
	Exterior Panels		Interior Panels	Exterior Panels		Interior Panels
	Without Edge Beams	With Edge Beams[c]		Without Edge Beams	With Edge Beams[c]	
40,000 275MPa	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000 414MPa	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000 517MPa	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

ℓ_n is the length of the clear span in the longer direction, measured face-to-face of the supports.

[a] The required geometry of a drop panel is defined in ACI Code Section 13.2.5.

[b] For yield strengths between the values given, use linear interpolation.

[c] Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

Direct Design Method:
Minimum Thickness of Two-Way Slabs According to ACI Code

For a slab with beams between interior columns ACI Code Section 9.5.3.3 gives the following minimum thicknesses:

(a) For $\alpha_{fm} \le 0.2$, the minimum thicknesses in Table 13-1 shall apply.

(b) For $0.2 < \alpha_{fm} < 2.0$, the thickness shall not be less than

$$h = \frac{\ell_n[0.8 + (f_y/200{,}000)]}{36 + 5\beta(\alpha_{fm} - 0.2)} \quad \text{but not less than 5 in.}$$

(c) For $\alpha_{fm} > 2.0$, the thickness shall not be less than

$$h = \frac{\ell_n[0.8 + (f_y/200{,}000)]}{36 + 9\beta} \quad \text{but not less than 3.5 in.}$$

(d) At discontinuous edges, either an edge beam with a stiffness ratio α_f not less than 0.8 shall be provided or the slab thickness shall be increased by at least 10 percent in the edge panel.

Where:

h is the overall thickness

l_n is span of the slab panel under consideration, measured in the longer direction

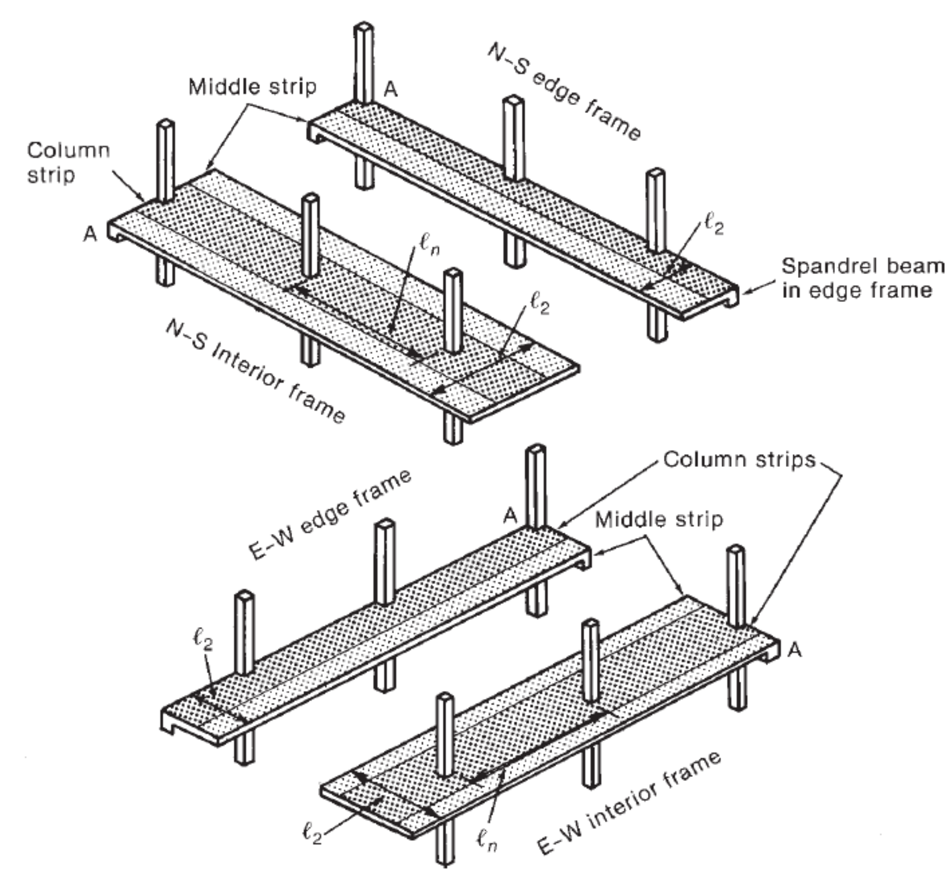
α_{fm} the average of the values of α_f the four sides of the panel

β the longer clear span/shorter clear span of the panel

Direct Design Method:
Distribution of Moments within Panels—Slabs without Beams between All Supports

Division of slab into frames for design

For design, the slab is considered to be a series of frames in the two directions, as shown



Statical Moment, M_o

These frames extend to the middle of the panels on each side of the column lines. In each span of each of the frames, it is necessary to compute the total statical moment, M_o We thus have

$$M_o = \frac{q_u l_2 l_n^2}{8}$$

Where:

q_u is the factored design load per unit area

l_2 is transverse width of the strip

l_n is clear span between columns

Direct Design Method:
Positive and Negative Moments in Panels

In the direct-design method, the total factored statical moment is divided into positive and negative factored moments according to rules given in ACI Code Section 13.6.3

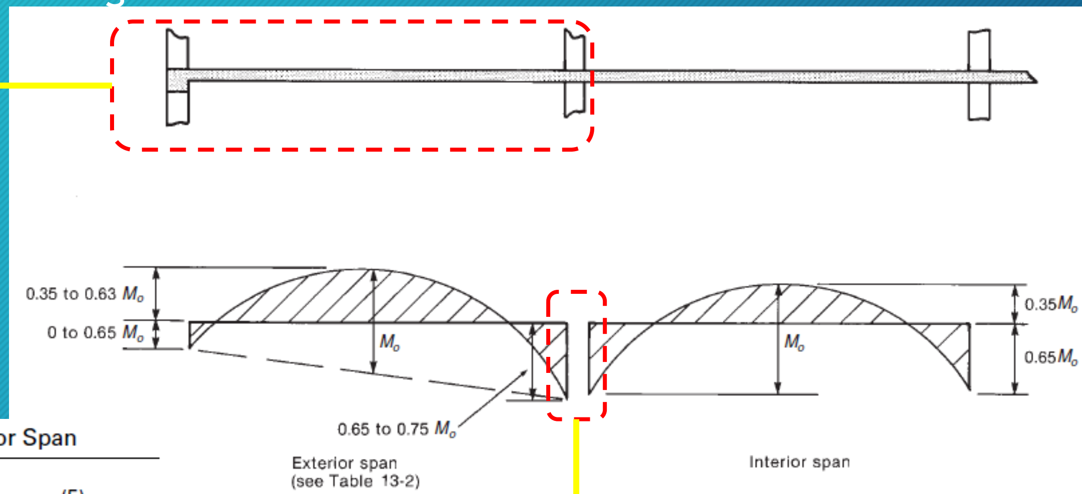


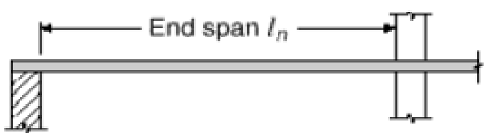
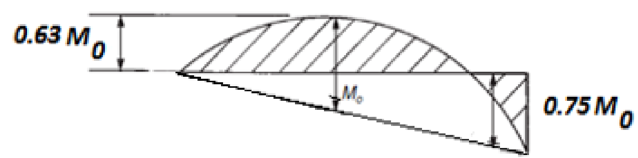
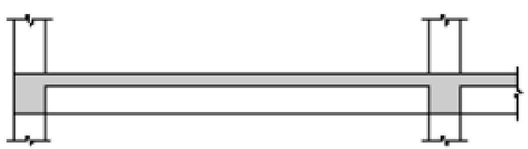
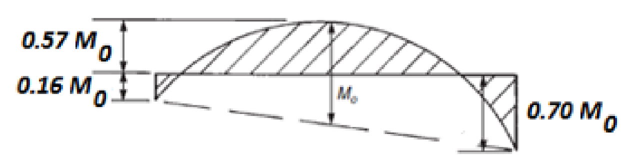
TABLE 13-2 Distribution of Total Factored Static Moment, M_o, in an Exterior Span

	(1) Exterior Edge Unrestrained	(2) Slab *with* Beams between *All* Supports	(3) Slab *without* Beams between *Interior* Supports: Without Edge Beam	(4) Slab *without* Beams between *Interior* Supports: With Edge Beam	(5) Exterior Edge Fully Restrained
Interior Negative Factored Moment	0.75	0.70	0.70	0.70	0.65
Midspan Positive Factored Moment	0.63	0.57	0.52	0.50	0.35
Exterior Negative Factored Moment	0	0.16	0.26	0.30	0.65

If the computed negative moments on two sides of an interior support are different, the negative-moment section of the slab is designed for the larger of the two.

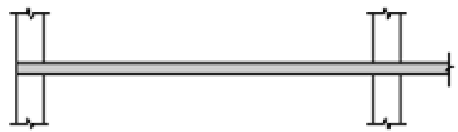
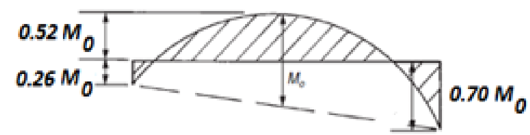
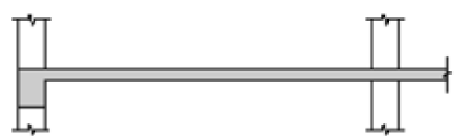
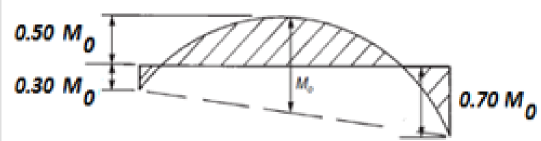
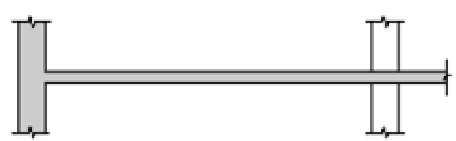
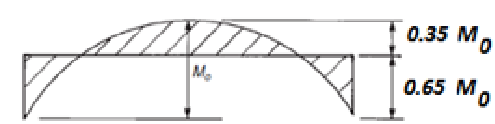
Direct Design Method:
Positive and Negative Moments in Panels

Distribution of Total Factored Static Moment, M_o, in an Exterior Span.

	Condition of restraint	The statical moment at the critical section
(1) Exterior edge unrestrained	 eg. Supported by a masonry wall	
(2) Slabs with beams b/n All Supports		

Direct Design Method:
Positive and Negative Moments in Panels

Distribution of Total Factored Static Moment, M_o, in an Exterior Span.

		Condition of restraint	The statical moment at the critical section
Slabs without beams b/n interior support	(3) Without edge beam	 eg. Flat Plate	
	(4) With edge beam		
(5) Exterior Edge Fully restrained		 eg. Restrained by monolithic concrete wall	

Direct Design Method:

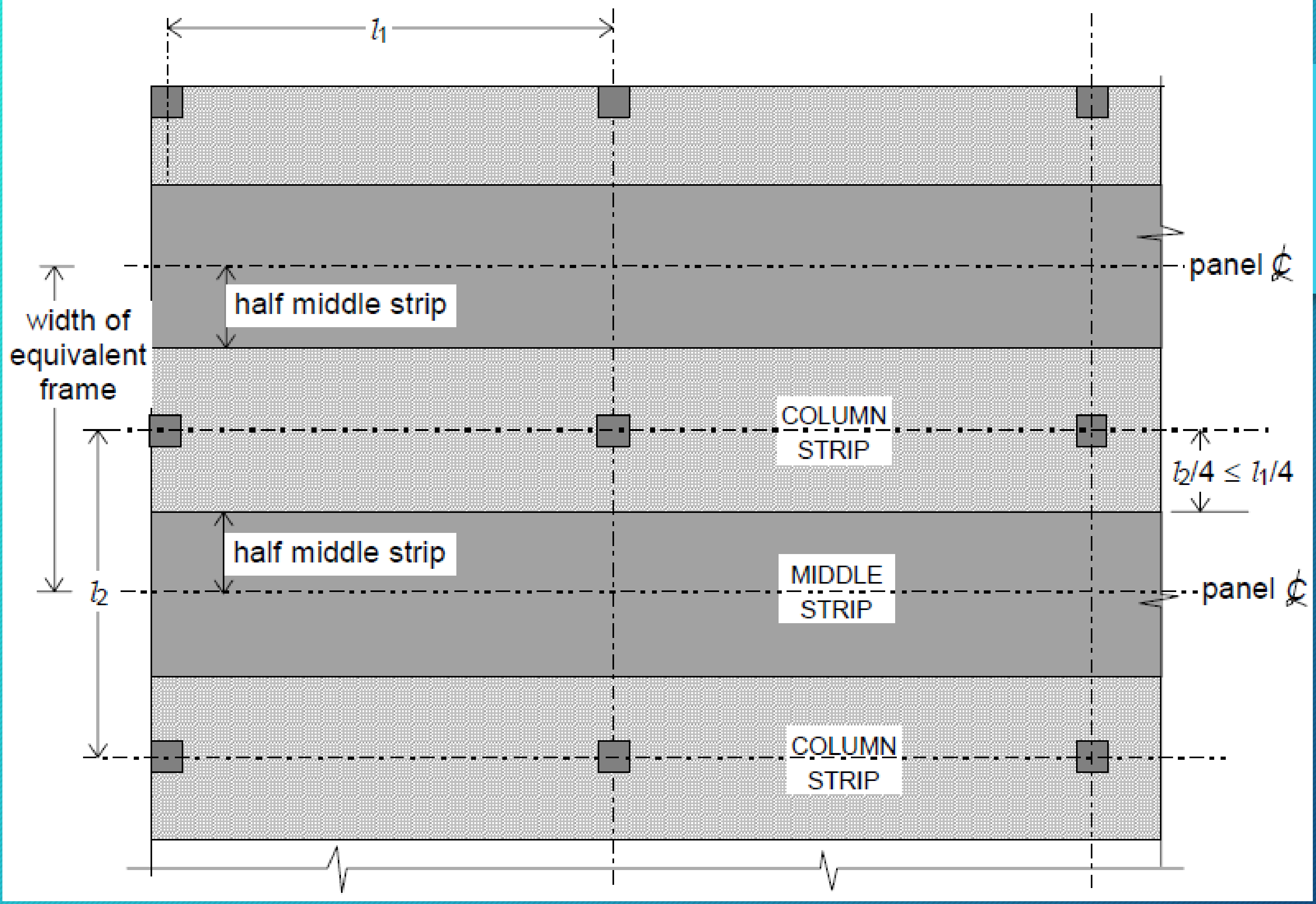
Definition of Column & Middle strips

The moments vary continuously across the width of the slab panels.

To aid in reinforcement placement, the design moments are averaged over the width of column strips over the columns and middle strips between the column strips

- Column strips in both directions extend one-fourth of the smaller span, l_{min}, each way from the column line.

- Middle strips are the strips between the column strips.


l1
panel ℄
half middle strip
width of equivalent frame
COLUMN STRIP
l2/4 ≤ l1/4
half middle strip
MIDDLE STRIP
l2
panel ℄
COLUMN STRIP

Direct Design Method:

Distribution of Moments between Column Strips and Middle Strips for *interior negative moments*

ACI Code Section 13.6.4 defines the fraction of the negative and positive moments assigned to the column strips. The remaining amount of negative and positive moment is assigned to the adjacent half-middle strips.

The division is a function of, $\alpha_{f1} l_2 / l_1$ which depends on the aspect ratio of the panel, l_2 / l_1, and the relative stiffness, α_{f1}, of the beams (if any) spanning parallel to and within the column strip.

Table 13-3 gives the percentage distribution of negative factored moment to the column strip at all interior supports. For floor systems without interior beams, $\alpha_{f1} l_2 / l_1$ is taken to be equal to zero, because $\alpha_{f1}=0$

TABLE 13-3 Percentage Distribution of Interior Negative Factored Moment to Column Strip

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Direct Design Method:

Distribution of Moments between Column Strips and Middle Strips *positive moments*

Table 13-4 gives the percentage distribution of positive factored moment to the column strip at midspan for both interior and exterior spans.

TABLE 13-4 Percentage Distribution of Midspan Positive Factored Moment to Column Strip

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

For floor systems without interior beams, 60 percent of the positive moment is assigned to the column strip and the remaining 40 percent is divided equally between the adjacent half-middle strips.

If a beam is present in the column strip (spanning in the direction of l_1), either the percentages in the second row or a linear interpolation between the percentages given in the first and second rows of Table 13-4 will apply.

Direct Design Method:

Distribution of Moments between Column Strips and Middle Strips *exterior-end negative moments*

At an exterior edge, the division of the exterior-end factored negative moment distributed to the column and middle strips spanning perpendicular to the edge also depends on the torsional stiffness of the edge beam, calculated as the shear modulus, *G*, times the torsional constant of the edge beam, *C*, divided by the flexural stiffness of the slab spanning *perpendicular* to the edge beam (i.e., *EI* for a slab having a width equal to the length of the edge beam from the center of one span to the center of the other span, as shown below in slide #51 (d)

Assuming that Poisson's ratio is zero G=E/2 gives then this torsional stiffness ratio is defined as

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$$

If there are no edge beams, β_t can be taken to be equal to zero.

The term *C* refers to the torsional constant of the edge beam. This is roughly equivalent to a polar moment of inertia. (see next slide)

Assuming that Poisson's ratio is zero gives then this torsional stiffness ratio is defined as

Direct Design Method:
Distribution of Moments between Column Strips and Middle Strips *exterior-end negative moments*

Table 13-5 gives the percentage distribution of negative factored moment to the column strip at exterior supports.

TABLE 13-5 Percentage Distribution of Exterior Negative Factored Moment to Column Strip

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

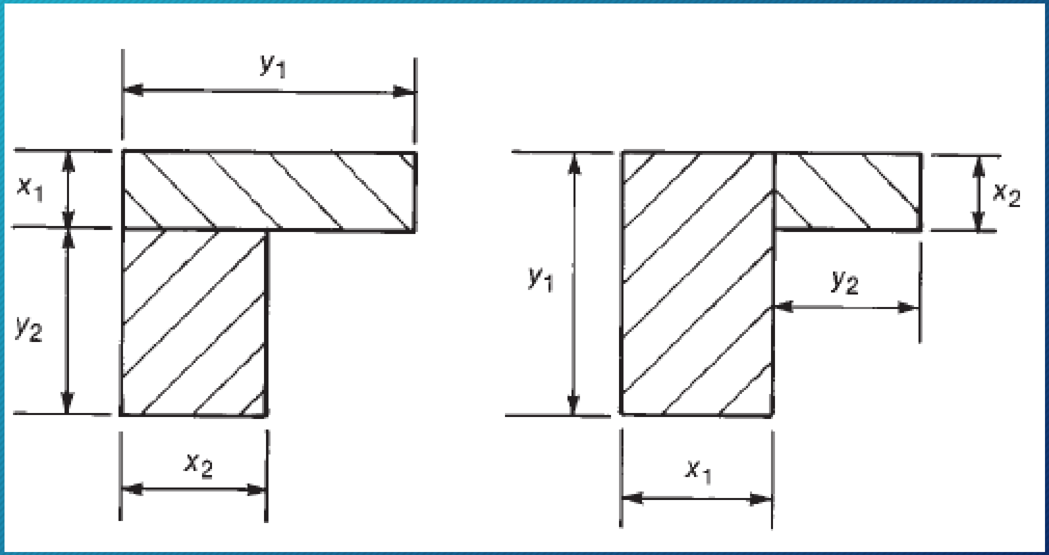
Assuming that Poisson's ratio is zero gives then this torsional stiffness ratio is defined as

Direct Design Method:

Torsional constant of the edge beam, C

It is calculated by subdividing the cross section into rectangles and carrying out the summation where x is the shorter side of a rectangle and y is the longer side. Different combinations
of rectangles should be tried to get the *maximum* value of C. ***The maximum value normally is obtained when the wider rectangle is made as long as possible.***

$$C = \sum \left[\left(1 - 0.63 \frac{x}{y} \right) \frac{x^3 y}{3} \right]$$



Assuming that Poisson's ratio is zero gives then this torsional stiffness ratio is defined as

Direct Design Method:
Beam-to-Slab Stiffness Ratio α_{f1}

Slabs frequently are built with beams spanning from column to column around the perimeter of the building. These beams act to stiffen the edge of the slab and help to reduce the deflections of the exterior panels of the slab.

In the ACI Code, the effects of beam stiffness on deflections and the distribution of
moments are expressed as a function of α_{f1}, defined as the flexural stiffness $4EI/l$, of the beam divided by the flexural stiffness of a width of slab bounded laterally by [...]ines of the adjacent panels on each side of the beam:

$$\alpha_f = \frac{4E_{cb}I_b/\ell}{4E_{cs}I_s/\ell}$$

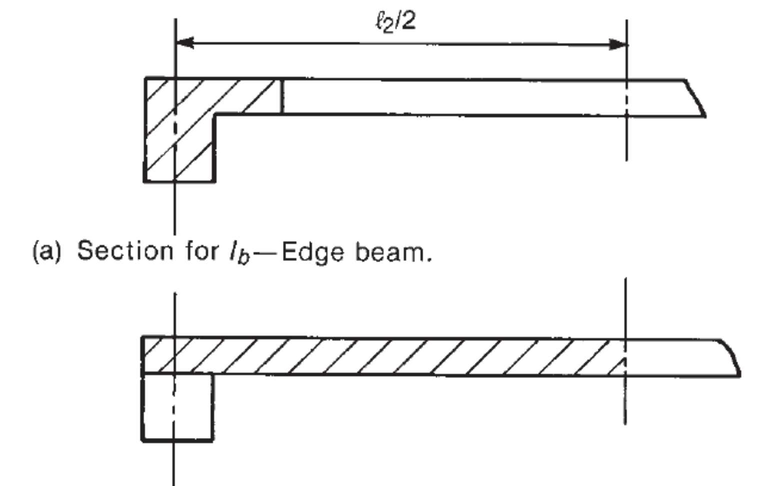
Because the lengths, of the beam and slab are equal, this quantity is simplified and expressed in the code as

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s}$$

where E_{cb} and E_{cs} are the moduli of elasticity of the beam concrete and slab concrete, respectively, I_b and I_s are the moments of inertia of the Uncracked beams and slabs

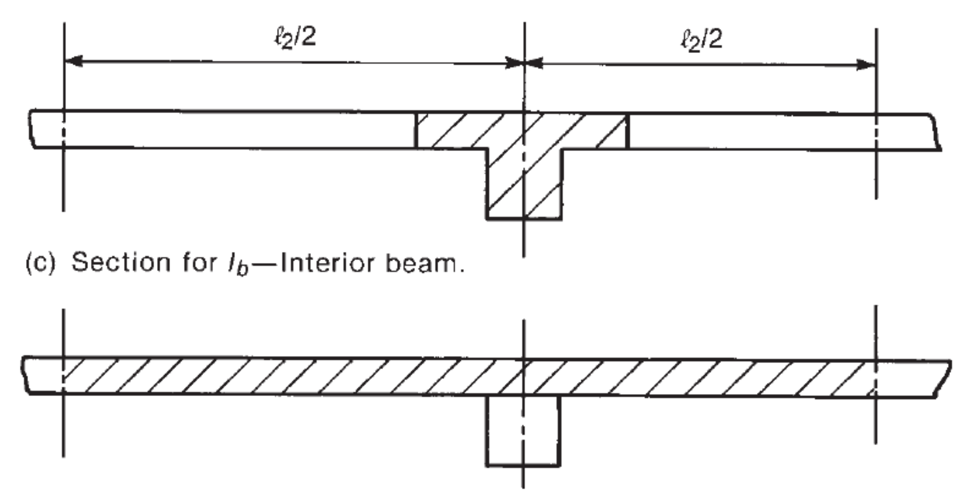
Direct Design Method:
Beam-to-Slab Stiffness Ratio α_{f1}

The sections considered in computing I_b and I_s are shown shaded below.



(a) Section for I_b—Edge beam.

(b) Section for I_s—Edge beam.



(c) Section for I_b—Interior beam.

(d) Section for I_s—Interior beam.

The span perpendicular to the direction being designed is l_2.

Thank you for the kind attention!

Questions?

Please read on the different coefficient to be implemented on direct design method for next class.

Please have a look at the example 2 of chapter 3 for proactive discussion for next class.

Please start your semester project ASAP! ☺

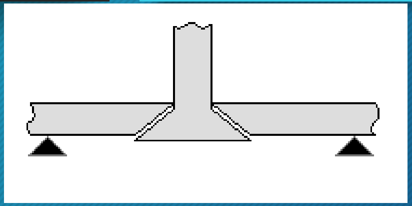
Thank you for the kind attention!

Questions?

Please read on Equivalent frame system in accordance to ACI.

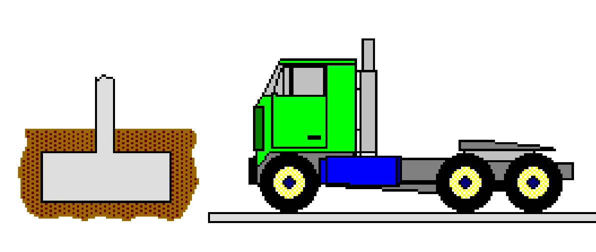
Please read on Punching shear check in flat slab systems in accordance to Eurocode.

Punching shear in accordance to Eurocode: *What is punching shear?*

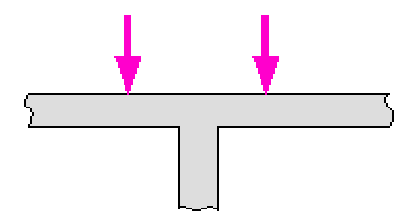


Consider a portion of slab subjected to an increasing concentrated load . Eventually the slab will fail. One possible method of failure is that the load "punches" through the slab.

The failure mechanism is by shear, hence the name Punching Shear.

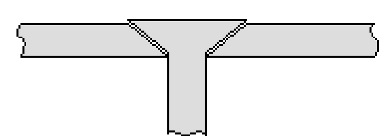


Some examples of the occurrence of concentrated loads on a slab are a common, particularly on a pad foundation, and wheel loads.



This same type of failure could also happen in another way. Turning the structure upside down we get….

A flat slab supported by a column, where, as described in the previous topic "Slabs", there is a high concentration of shear force around the column head.

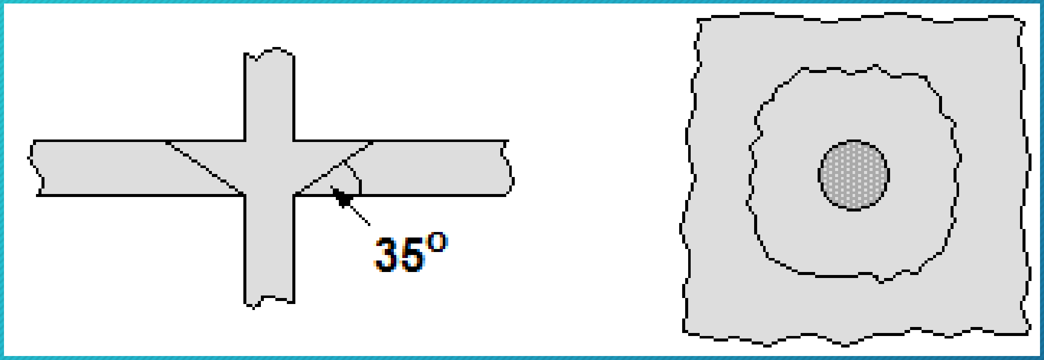


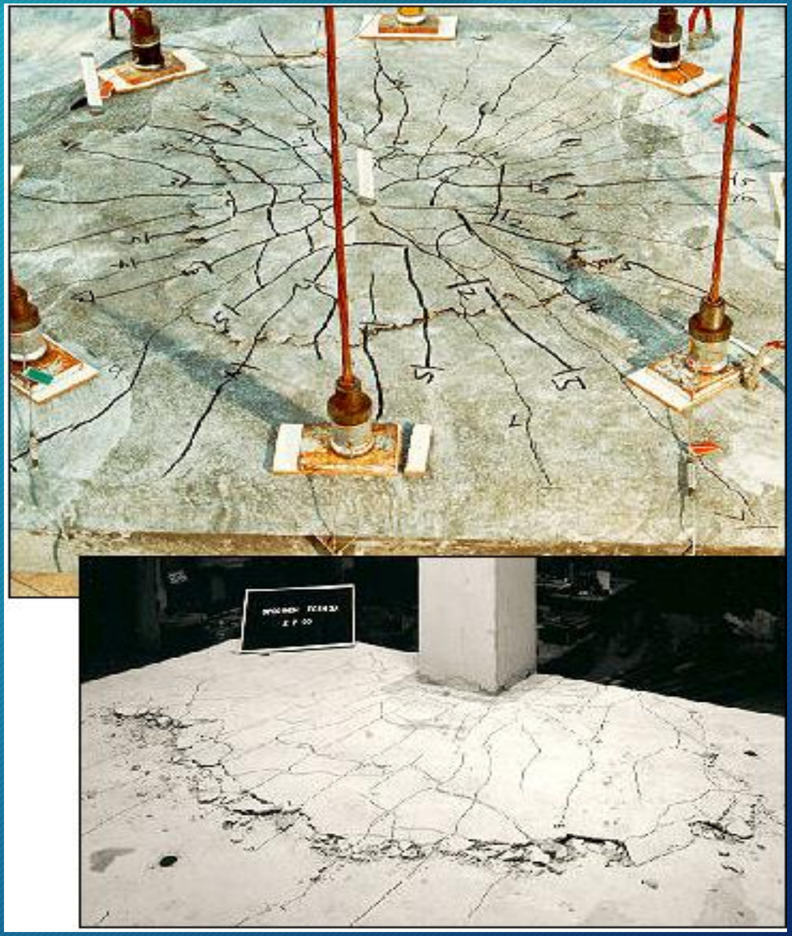
When the total shear force exceeds the shear resistance of the slab, the slab will be 'pushed down' around the column, or this can be viewed as the column being 'pushed' through the slab, thus:

Punching shear in accordance to Eurocode: *Shear Resistance*

Punching Shear Is the most common, and is a major design consideration, in flat slab construction. In pad foundations, where weight and depth are not so critical, its effects are satisfied by providing sufficient depth. The major emphasis of this topic is, therefore, concentrated on flat slabs.

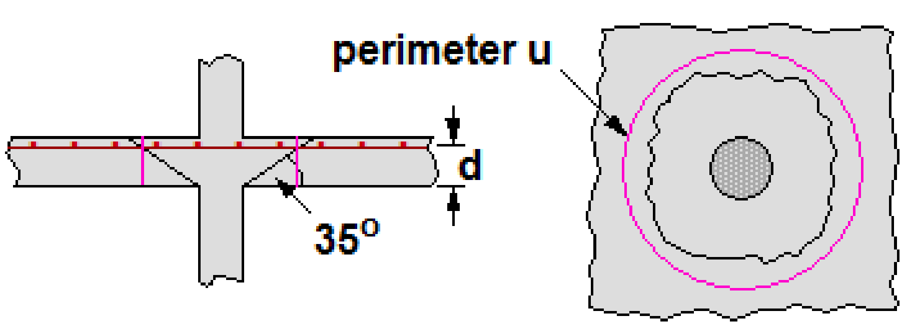
Tests show that cracks develop radially from the column position, culmination in a sudden and brittle failure on inclined faces of truncated cones or pyramids at an angle of about 35^o to the horizontal





Punching shear in accordance to Eurocode: *Shear Resistance*

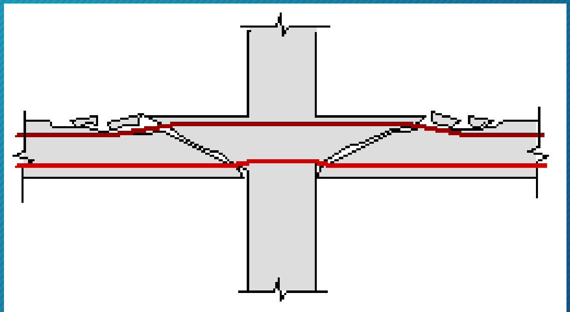
Methods have been proposed for checking the stress on the failure planes, but the general method adopted is similar to that for transverse shear in beams, but checks shear stresses rather than shear forces.



Thus, the shear force acting on a perimeter u around the loaded area is resisted by a nominal shear stress(capacity) $V_{Rd,c}$. acting over the average effective depth d of the section

$$\text{shear capacity} = V_{Rd,c} \cdot u \cdot d$$

Once punching has occurred, the top bars make only a very limited contribution to the shear resistance since the cover is easily torn away (but prior to punching they are vital to the truss analogy in determining the strength



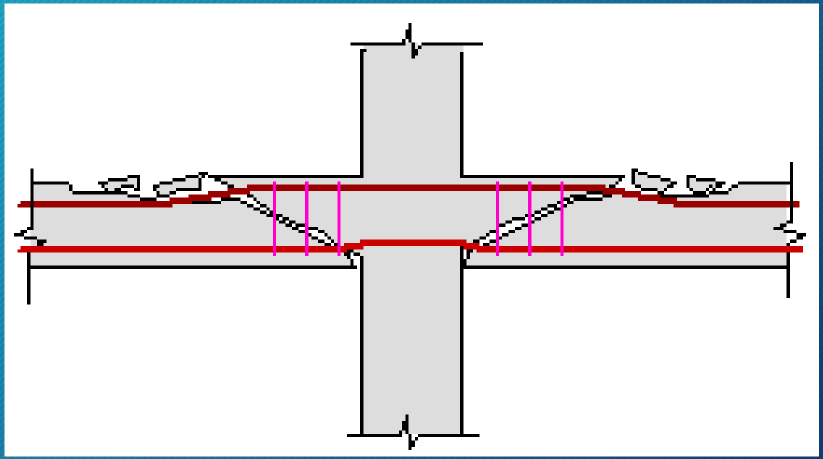
However, the bottom bars, being more deeply embedded, are not pushed out in the same way and thus provide more resistance. this is attributable initially to dowel action, and then later at larger deformation to its being kinked, as shown.

Punching shear in accordance to Eurocode: *Shear Resistance*

When punching occurs at a slab-column connection without shear reinforcement the resistance and thus the load carrying capacity is greatly reduced. The load is therefore transferred to the adjacent connections, which may also suffer punching failures.

This may lead to a general failure of the floor which in turn could lead to a progressive collapse of the structure as one floor fails onto the floor below. This has occurred several times with flat slab structures in recent years, frequently during construction when the concrete strength is not fully developed.

Providing shear reinforcement to restrain the top bars by tying them to the bottom bars greatly increases the resistance and ductility of the slab-column connection. Running the bottom bars through the column or anchoring them in the column will also increases the ductility..



Punching shear in accordance to Eurocode: *Formula for Shear Capacity*

The empirical formula for the shear capacity (stress), $V_{Rd,c}$ of a section shear reinforcement (similar to beam shear) is given below.

$$V_{Rd,c} = [(C_{Rd,c} \cdot k \cdot (1000\rho_1 f_{ck})^{0.333}) \geq v_{min}] + 0.10\sigma_{cp}$$

$$v_{min} = 0.035k^{1.5} f_{ck}^{0.5}$$

This to account for the plain concrete section capacity.

$$C_{Rd,c} = 0.18 / \gamma_c$$

Where,
γ_c is the partial factor of safety of concrete.

$$k = 1 + (200 / d)^{0.5} \leq 2$$

k is a coefficient to allow for the 'scale effect'

$$\rho_1 = (\rho_{1x}\rho_{1y})^{0.5} \leq 0.02$$

$$\rho_{1x} = A_{slx} / d_x ; \rho_{1y} = A_{sly} / d_y$$

A_{slx} and A_{sly} are the mean reinforcement areas in the x and y directions, in a slab width equal to the column width plus 3d each side (mm^2/mm)'

$$\sigma_{cp} = (\sigma_{cx} + \sigma_{cy}) / 2 \leq 0.2f_{cd}$$

$$\sigma_{cx} = N_{Edx} / A_{cx}$$

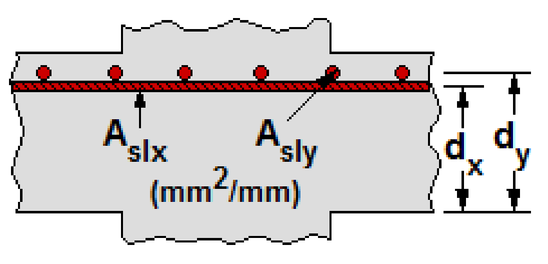
$$\sigma_{cy} = N_{Edy} / A_{cy}$$

This term allows for the influence of the longitudinal stress due to an external force

The design shear resistance of the section without shear reinforcement is:-

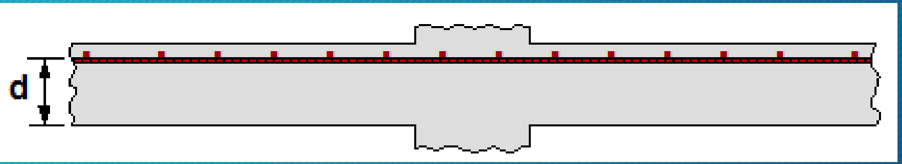
$$V_{Rd,c} \cdot U \cdot d$$

Where,
u is the length of the perimeter of the section.
d is the average effective depth of the section $=(d_x + d_y)$

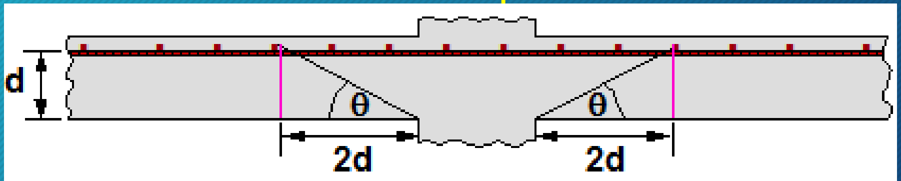


Punching shear in accordance to Eurocode: *Control Perimeter*

Comparisons with a large number of test results show that the closer the basic control perimeter is to the loaded area, the greater is the influence on the size of the loaded area relative to the slab depth on the shear resistance of the slab.

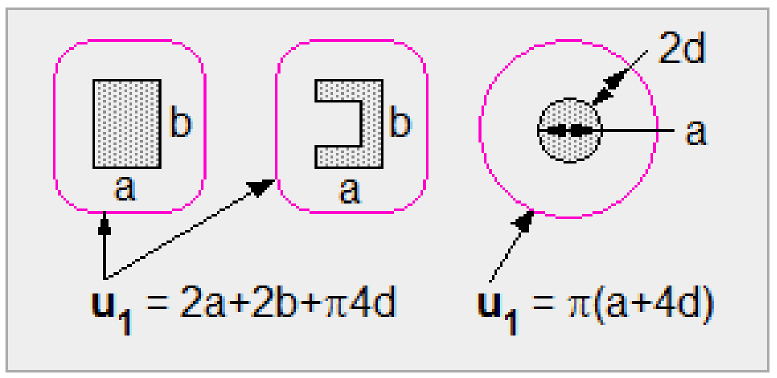


To be largely independent to this ratio, the distance of the basic control perimeter from the face of the loaded area is taken as 2d.



This gives an angle $\theta = 26.6^o$ (arctan(1/2)) with the horizontal, outside the plain of failure.

The basic control; perimeter, U_1 should be constructed so as to minimize its length.



There are other parameters which affect the positions and length of control perimeters.

Punching shear in accordance to Eurocode: *Control Perimeter*

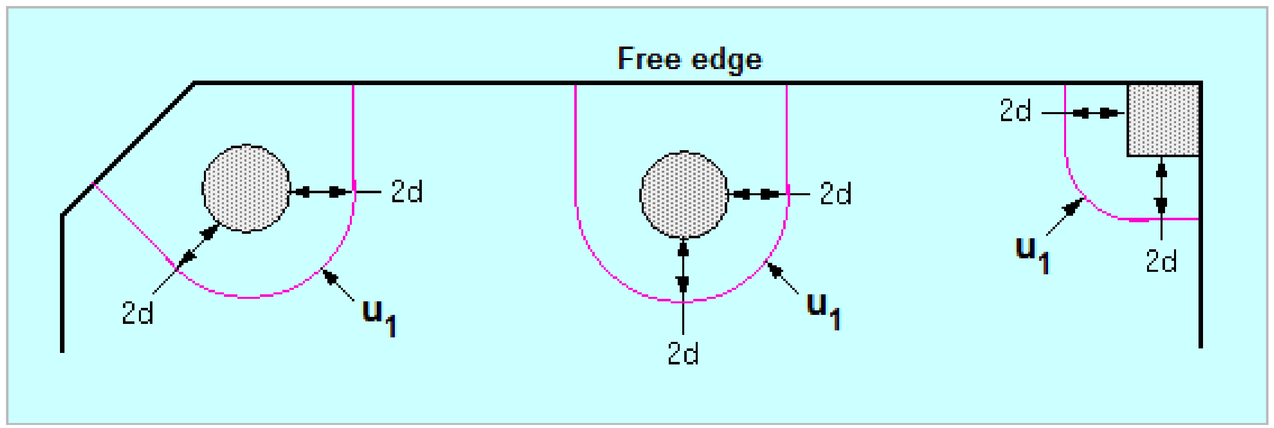
Loads and Footing

Control Perimeter at a distance less than 2d should be considered where the concentrated force is opposed by:

- A high distributed pressure, for example soil pressure on a footing, or
- A load or reaction within the basic control perimeter of 2D.

Free edges

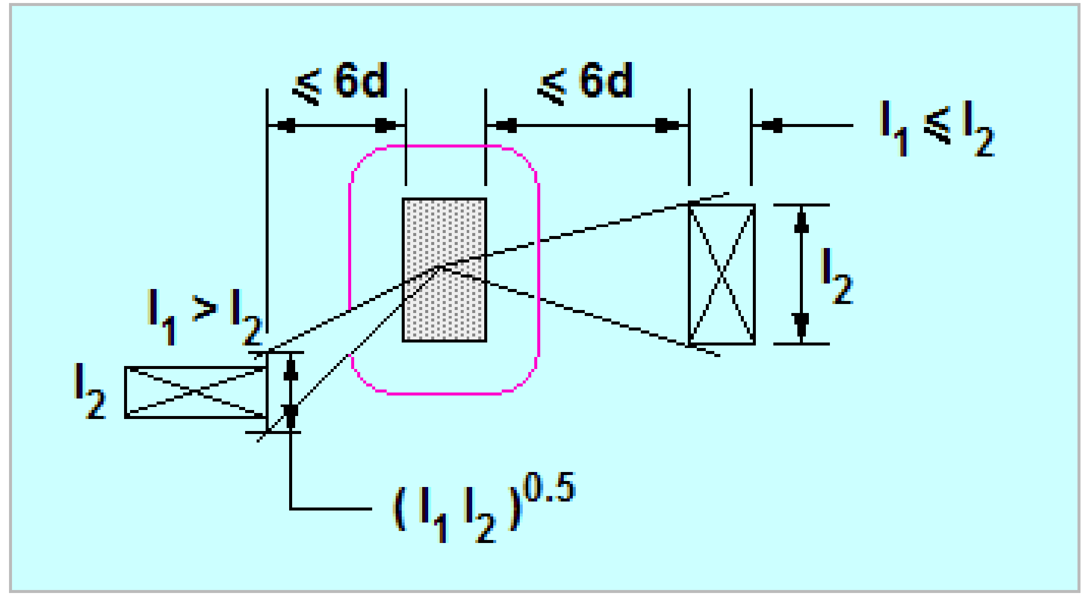
For loaded areas situated near an unsupported edge or corner, the basic control perimeter is as shown below, provided this gives a perimeter, excluding the unsupported edges, which is less than that calculated for an internal loaded area.



Punching shear in accordance to Eurocode: *Control Perimeter*

Openings

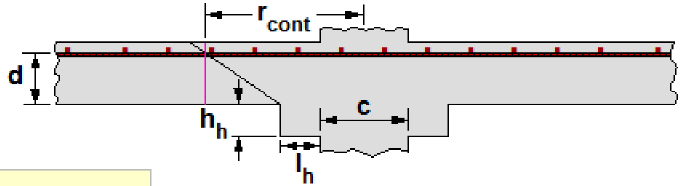
For loaded areas situated near openings, if the shortest distance between the perimeter of the loaded area and the edge of the opening does not exceed 6d, that part of the control perimeter contained between the two tangents drawn to the outline of the opening from the center of the loaded area is considered ineffective.



Punching shear in accordance to Eurocode: *Control Perimeter*

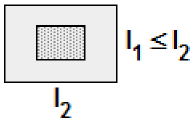
Heads and drops

$l_h \leq 2\,h_h$



Circular column heads: $r_{cont} = 2d + l_h + 0.5c$

Rectangular column heads:



lesser of

$r_{cont} = 2d + 0.69\,l_1$

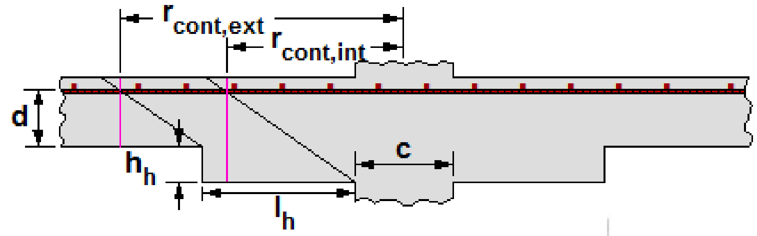
$r_{cont} = 2d + 0.56\,(l_1\,l_2)^{0.5}$

OR

$l_h > 2\,h_h$

$r_{cont,int} = 2(d+h_h) + 0.5c$

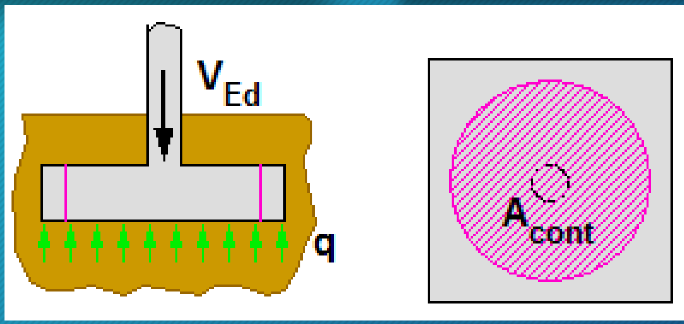
$r_{cont,ext} = 2d + l_h + 0.5c$



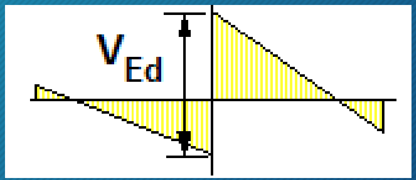
Punching shear in accordance to Eurocode: *Effective Shear Force*

For a footing the average soil pressure, q acting over the area within the control perimeter, A_{cont} can be subtracted from the punching shear force, V_{Ed} because it helps to resist the shear force.

Thus, $V_{Ed} = V_{Ed} - q\,A_{cont}$



For a Slab-column connection the design is based on the total shear force, V_{Ed} at the column face (where an equivalent frame analysis has been used the direction giving the greater value of V_{Ed} is used).



Generally, moment is transferred to the columns so the shear distribution is not uniform and a factor, β is used to take account off local concentrations.

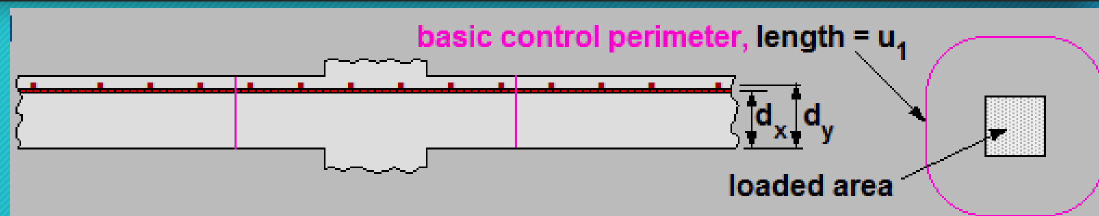
Effective shear force is βV_{Ed}

Condition	β
Internal column	1.15
Edge column	1.40
Corner column	1.50

For structures where the lateral stability does not depend on frame action between the slabs and columns, and where the adjacent spans do not differ in length by more than 20%, simplified values of β may be used.

Otherwise β must be calculated

Punching shear in accordance to Eurocode: *Design Procedure*



i. Calculate the effective shear force, βV_{Ed} at the face of the loaded area.

ii. Calculate the shear stress, v_{Ed} at the face of the loaded area, where u_o is the perimeter of the loaded area and $d=(d_x+d_y)/2$.

iii. Calculate the maximum allowable punching shear stress, $V_{Rd,max}$.

iv. If $V_{Ed} > V_{Rd,max}$ the design fails.

v. Calculate the punching shear stress resistance, $V_{Rd,c}$ of the slab at the basic control perimeter.

vi. Calculate the shear stress at the basic control perimeter, v_{Ed}, (for edge and order columns the reduced perimeter, u_1, may be used).

vii. If $V_{Ed} > V_{Rd,c}$ several design options are available to increase the shear resistance of the slab, such as:

- Shear reinforcement
- Column Head/Capital
- Slab Drop/ Drop Panel

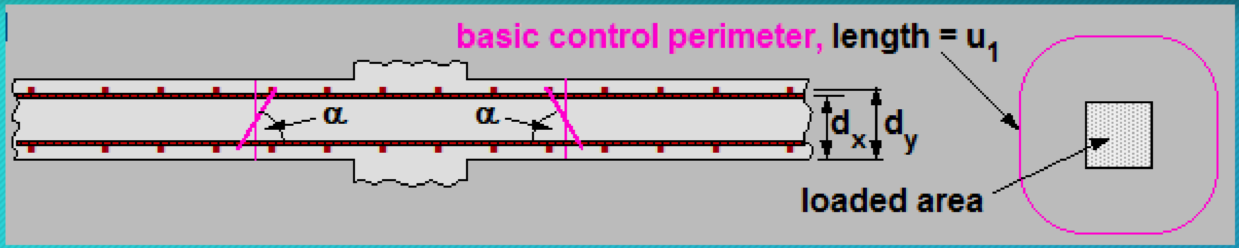
$$v_{Ed} = \beta V_{Ed} / (u_o d)$$

$$v_{Rd,max} = 0.5 \nu f_{cd}$$
$$\nu = 0.6 (1 - f_{ck} / 250)$$

$$v_{Rd,c} = C_{Rd,c} k \times (100 \rho_l f_{ck})^{1/3} \text{, etc.}$$

$$v_{Ed} = \beta V_{Ed} / (u_1 d)$$

Punching shear in accordance to Eurocode: *Punching Shear Reinforcement*



The shear resistance of the section with shear reinforcement, $V_{Rd,cs}$ is:-

$$v_{Rd,cs} = 0.75 v_{Rd,c} + \frac{1.5d}{s_r} \times \frac{A_{sw} f_{ywd,ef} \sin\alpha}{u_1 d} \quad \text{with} \quad v_{Rd,cs} \geq v_{Ed}$$

where, A_{sw} is the area of one perimeter of shear reinforcement.

s_r is the radial spacing of perimeters (not to exceed 0.75d).

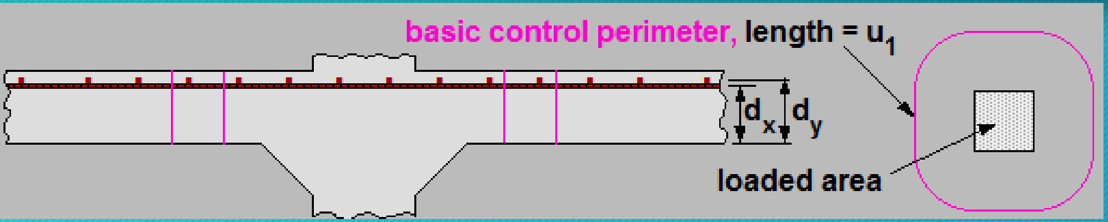
$f_{ywd,ef}$ is the effective design strength of the shear reinforcement. $= 250 + 0.25d$ but not greater than f_{ywd}.

The control perimeter where shear reinforcement is not required, U_{out} can be calculated:-

$$u_{out} = \beta V_{Ed} / (v_{Rd,c}\, d)$$

The shear reinforcement should be placed between the loaded area and 1.5d inside U_{out}. It should be provided in at least 2 perimeters of link legs with each link leg providing a minimum area, $A_{sw,min}$.

Punching shear in accordance to Eurocode: *Column head/capital*



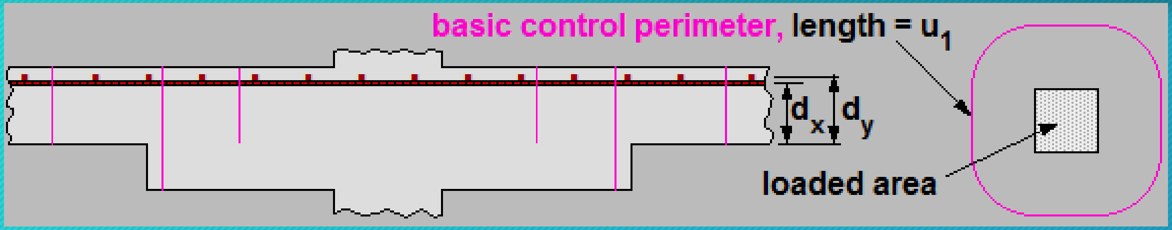
An enlarged column head or column capital increases the perimeter of the loaded area, u_o so reducing the shear stress, V_{Ed} .

It also increases the length of the basic control perimeter, u_1 thereby reducing the shear stress, V_{Ed} at that section.

The main disadvantage of providing a column head is the additional formwork and the consequential increase in the construction time.

Column heads are used, generally, as architectural features where the soffit is exposed

Punching shear in accordance to Eurocode: *Slab drop/drop panel*



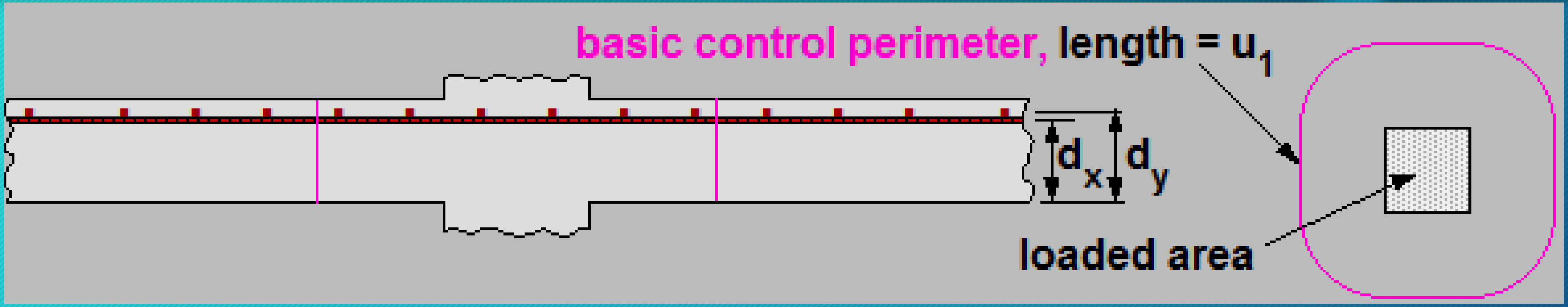
An increased depth of slab or slab drop/drop panel around the column reduces v_{Ed} by increasing the length of the basic control perimeter u_1

It also reduces v_{Ed} at the face of the loaded area by increasing the effective depth d.

Generally, slab drops are provided with a shallow slab to carry large shear forces. Because of this they tend to be quiet large such that punching is checked on another control perimeter within the drop.

The main disadvantages of drops are the extra formwork, resulting in an increase in construction time, and the possible interference with routing of services below the slab

Punching shear in accordance to Eurocode: *Other ways to increase slab shear capacity*



If v_{Ed} exceeds $V_{Rd,c}$ ot the basic control perimeter by small amount, and this was true of many slab-column connection in the design, then either the loaded area or the slab depth could be increased, although the latter would also increase the permanent load.

The most economic solution might be to increase the concrete strength.

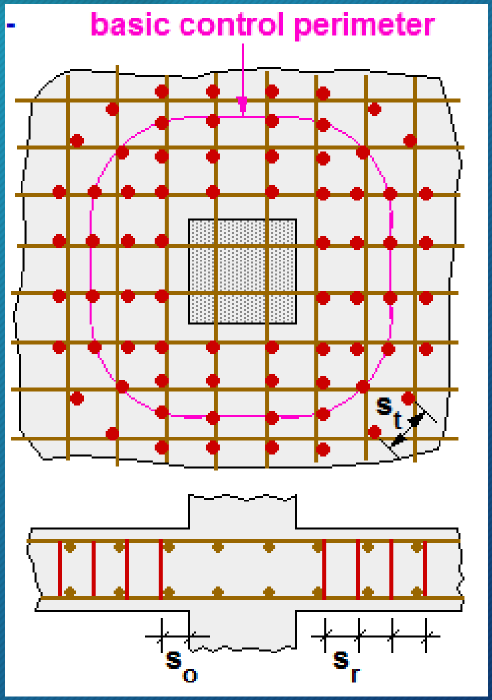
If, at the face of the loaded area v_{Ed} greatly exceeds $V_{Rd,c}$ then thought must be given to changing the structural layout or even the form of construction.

Punching shear in accordance to Eurocode: *Detailing Consideration*

The placing of shear reinforcement should comply with the following:-

- It should be fixed on at least 2 perimeters, the inner of which, s_o is located between 0.3d and 0.5d from the face of the loaded area;
- The spacing of link perimeters, s_r must not exceed 0.75d;
- The spacing of links around a perimeter, s_t must not exceed 1.5d for perimeters with in the basic control perimeter, and 2d for perimeters outside the basic control perimeter.

Detailing Consideration



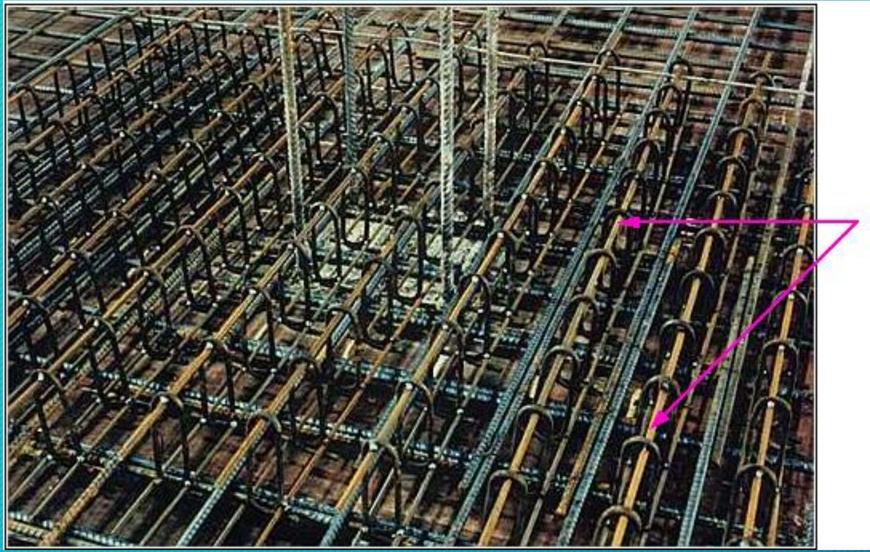
These criteria will often dedicate the amount of shear reinforcement provided

Punching shear in accordance to Eurocode: *Detailing Consideration*

Links

Traditional shear links provide a strong reinforcement 'cage' around the column head.

However, they do take a long time to fix and often require additional longitudinal "link hangers" bars running between the main bars.



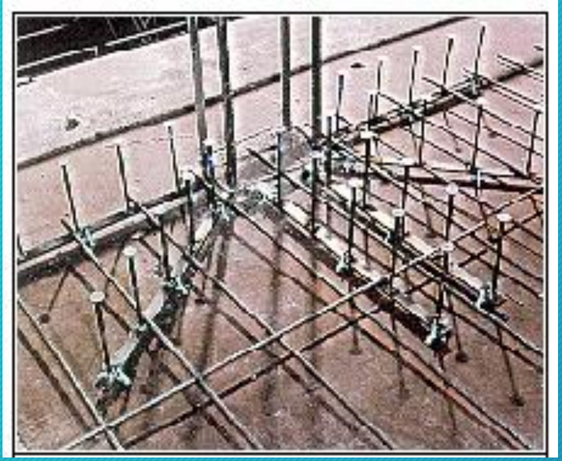
Ladders

The area and spacing of reinforcement in the vertical legs of the shear ladders is determined from the area and spacing of the links they are designed to replace.

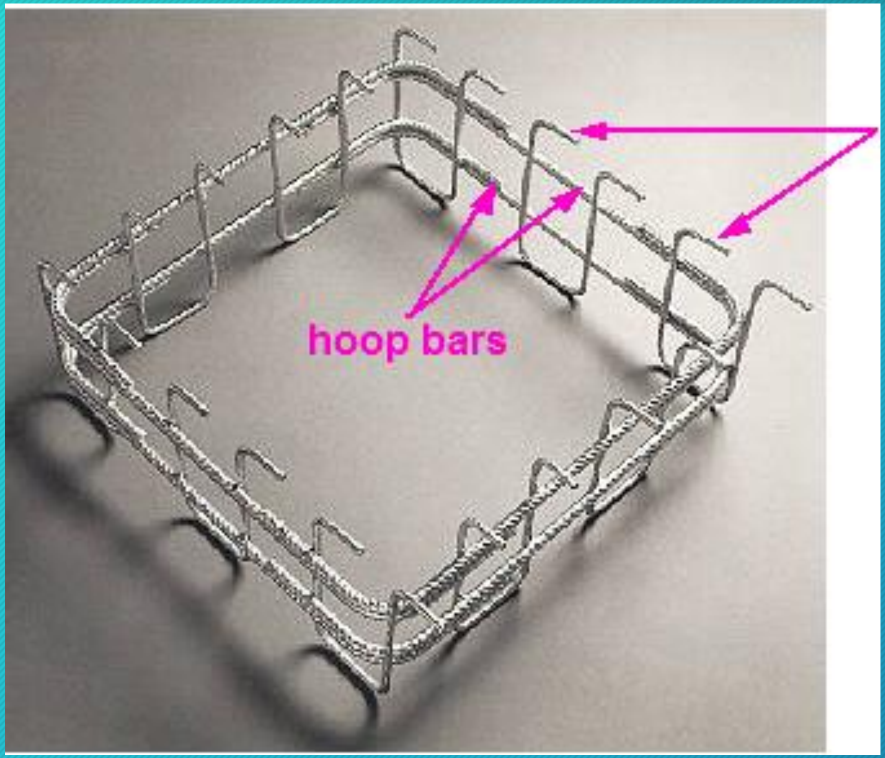
Punching shear in accordance to Eurocode: *Detailing Consideration*

Stud Rails

Time-related costs from a large proportion of the overall costs of supplying and fixing reinforcement. Research has found that proprietary punching shear reinforcement systems can save up to 50% of fixing time, because they are between 3 and 10 times faster to fix per column compared with traditional shear reinforcement.

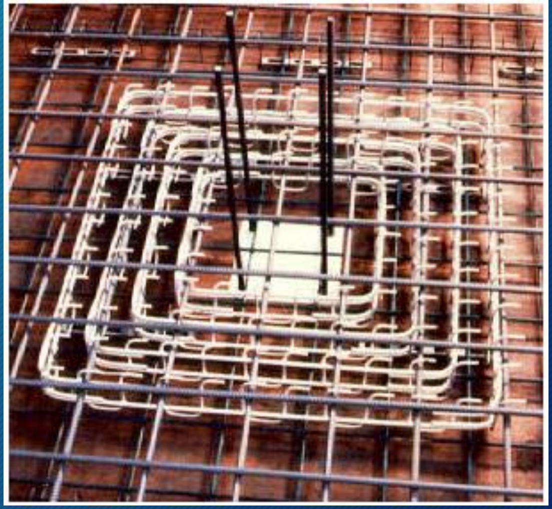


Punching shear in accordance to Eurocode: *Detailing Consideration*



Shear Hoops

A shear hoop is a prefabricated, single ‘perimeter’ of links tied together by two ‘hoop’ bars.



The fixing time is far less than for traditional links. Different size hoops are placed around the column, here 4 shear hoops have been fixed.

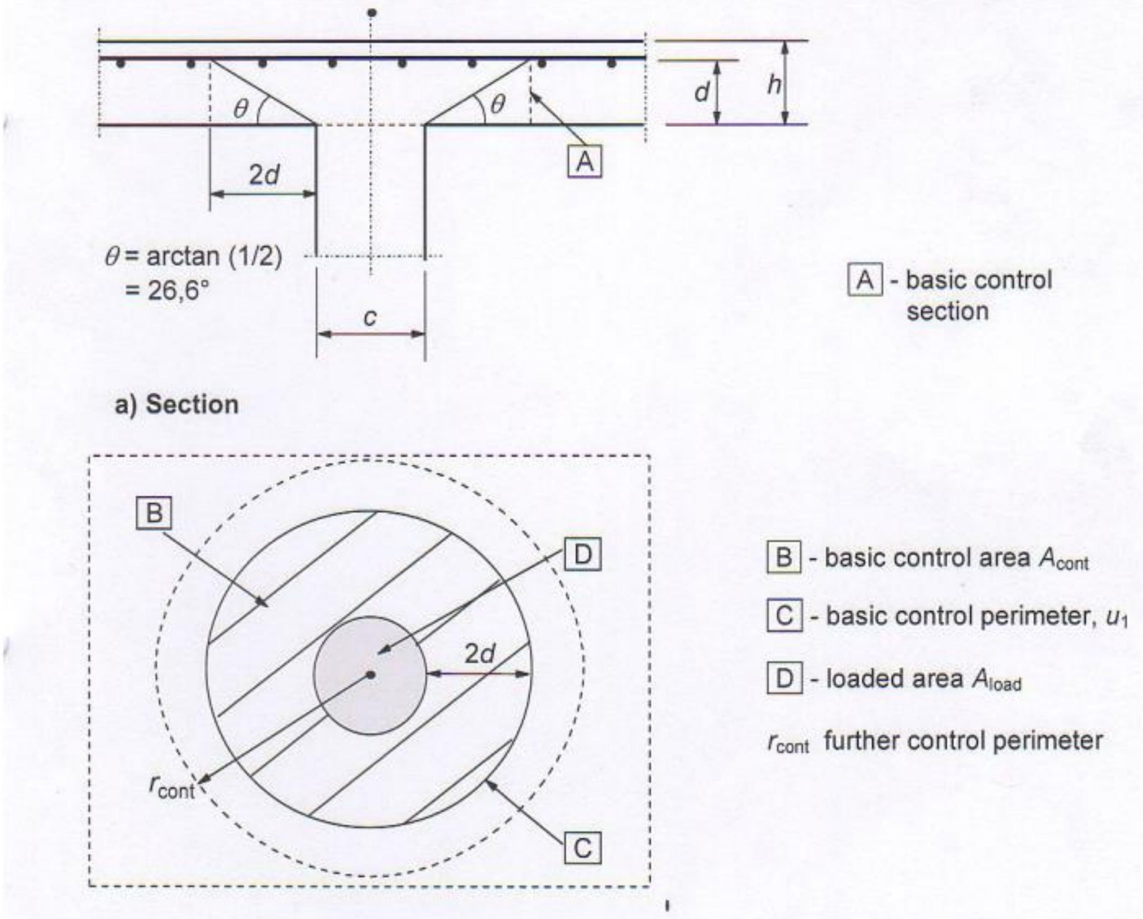
Punching shear in accordance to Eurocode: *Detailing Consideration*

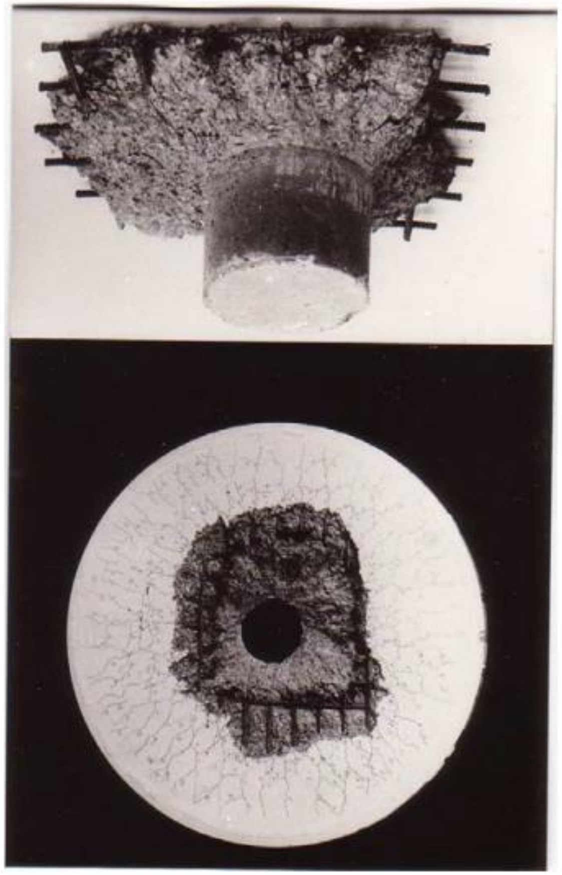
Shear heads

A shear head is prefabricated from structural steel channel sections which carry all the shear force at the column. Service holes are easily provided adjacent to the column.

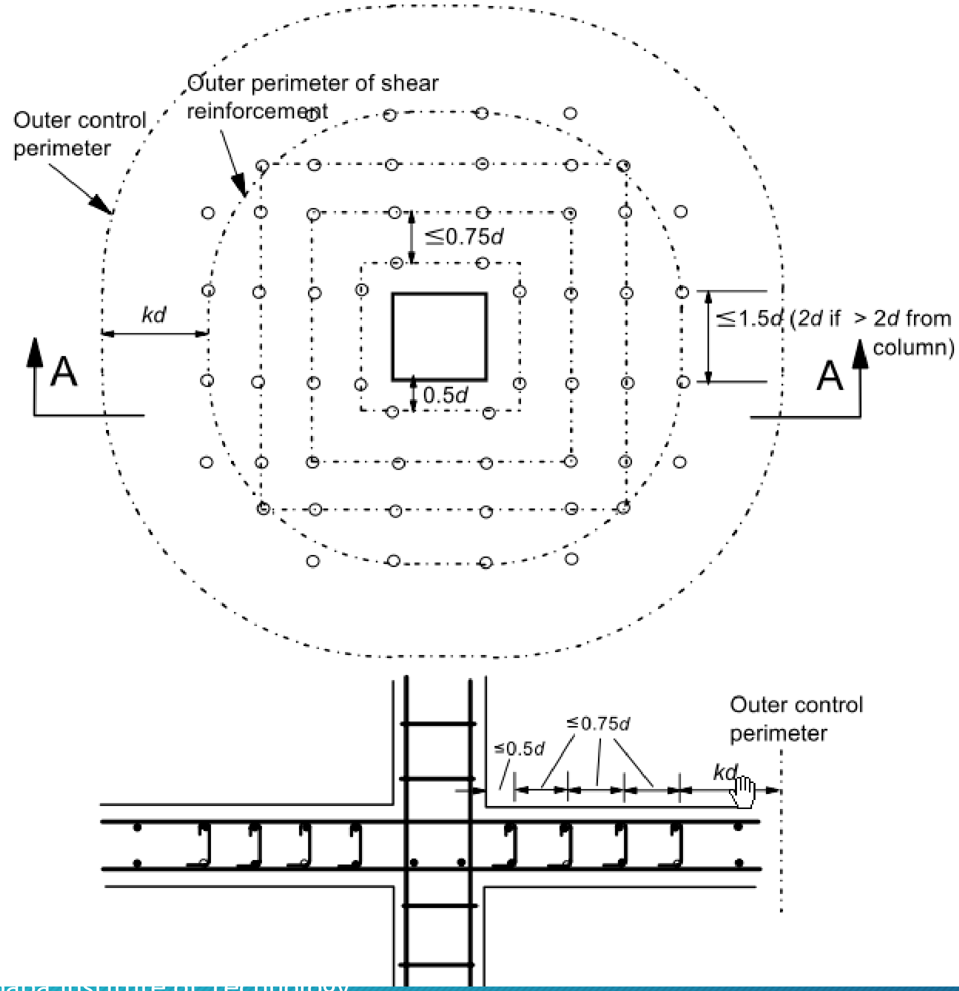
The detail is much simpler than traditional links, making is far easier to fix. The shear resistance of a shear head is either determined by test, or provided by the manufacturer.

Punching shear in accordance to Eurocode: *Summary on Definition of control perimeter*





Punching shear in accordance to Eurocode: *Summary on Punching shear reinforcement detailing*

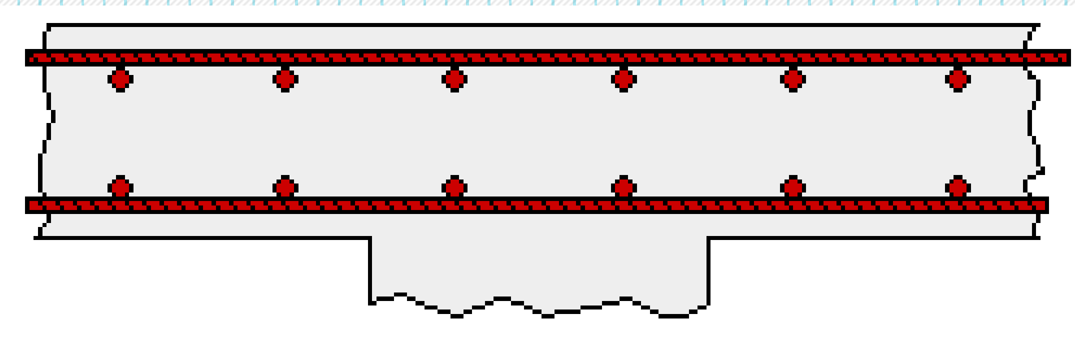


The outer control perimeter at which shear reinforcement is not required, should be calculated from:

$$u_{out,ef} = V_{Ed} / (v_{Rd,c}\, d)$$

The outermost perimeter of shear reinforcement should be placed at a distance not greater than *kd* (*k* = 1.5) within the outer control perimeter.

Example 3.3. Check the punching shear for the slab-column below by addressing questions a to e sequentially.



- Frame is braced with 7.2m column centers.
- Internal columns 350mm square
- h=250mm; cover=25mm
- C 30/37, S - 500
- All reinforcement φ20c/c175mm
- Reaction at column V_{Ed}=765kN

a) What is the design punching shear stress, v_{Ed} at the column face (MPa)?

b) What is the maximum design punching shear stress, $V_{Rd,max}$ (MPa)?

c) What is the length of the basic control perimeter, u_1 (mm)?

d) What is the design punching shear resistance, $v_{RD,c}$ at the basic control perimeter (MPa)

e) What area of vertical shear (Punching shear) reinforcement, A_{sw} is required for each perimeter of links (mm^2)?

Solution [a]. What is the design punching shear stress, v_{Ed} at the column face (MPa)?

Step1: Summarize the given parameters

Material	C30/37	f_{ck}=30MPa; f_{cd}=16.77MPa; f_{ctm}=2.9MPa; E_{cm}=33,000MPa
	S-500	f_{yk}=500MPa; f_{yd}=434.78MPa; E_s=200,000MPa

Step2: Compute v_{Ed} at the column face (Mpa).

$$v_{Ed} = \beta V_{Ed} / (u_o d)$$

d_x = 250 - 25 - 20/2 = 215 mm

d_y = 250 - 25 - 1.5*20 = 195 mm

d = (215 + 195)/2 = **205** mm

Length of perimeter of column face, u_o is :-

4 x 350 = 1400 mm

Using simplified value of 1.15 for β then -

$$v_{Ed} = \frac{1.15 \times 765 \times 1000}{1400 \times 205}$$

= **3.07 MPa**

Simplified values of β for effective shear force

Condition	β
Internal column	1.15
Edge column	1.40
Corner column	1.50

Solution [b]. What is the maximum design punching shear stress, $V_{Rd,max}$ (MPa)

Step1: Summarize the given parameters.

Same as in {a}.

Step2: Compute $V_{Rd,max}$ at the column face (Mpa).

$$v_{Rd,max} = 0.5\, v\, f_{cd}$$

$$v = 0.6\,(1 - f_{ck} / 250)$$

$$v = 0.6\,(1 - 30 / 250)$$

$$= 0.53$$

$$v_{Rd,max} = 0.5 \times 0.53 \times 0.85 \times 30/1.5$$

$$= 4.49 \text{ MPa}$$

This is greater than v_{Ed} at face of column (**3.07**) so section is **OK**.

Solution [c]. What is the length of the basic control perimeter, u_1 (mm)?

Step1: Summarize the given parameters.

Same as in {a}.

Step2: Compute u_1 (mm).

d = **205** mm

basic control perimeter is **2d** from face of column = 410 mm

Therefore, $\mathbf{u_1} = 4 \times 350 + \pi \times 2 \times 410$

$= \mathbf{3976\ mm}$

Punching shear stress at basic control perimeter, $\mathbf{u_1}$ is -

$$v_{Ed} = \frac{1.15 \times 765 \times 1000}{3976 \times 205} = \mathbf{1.08\ MPa}$$

Solution [d]. What is the design punching shear resistance, $v_{RD,c}$ at the basic control perimeter (MPa)

Step1: Summarize the given parameters.

Same as in {a}.

Step2: Compute $v_{RD,c}$ (MPa).

$v_{Rd,c} = C_{Rdc}\, k\, (100\rho_l\, f_{ck})^{0.333}$ (No axial force)

$k = 1 + (200/d)^{0.5} = 1 + (200/205)^{0.5} = \mathbf{1.988}$

(maximum $k = 2$)

$A_{slx} = A_{sly} = \pi 20^2 / (4 \times 175) = 314\ mm^2$

$\rho_{lx} = A_{slx} / d_x = 314) / 215 = \mathbf{0.0083}$

$\rho_{ly} = A_{sly} / d_y = 314) / 195 = \mathbf{0.0092}$

$\rho_l = (\rho_{lx} \times \rho_{ly})^{0.5} = \mathbf{0.0088}$ (maximum = 0.02)

$v_{Rd,c} = (0.18/1.5) \times 1.988 \times (100 \times 0.0088 \times 30)^{0.333}$

$= \mathbf{0.71}$ Minimum $= \mathbf{0.035\, k^{1.5}\, f_{ck}^{0.5}} = \mathbf{0.537}$

Therefore, $v_{Rd,c} =$ **0.71 MPa**

v_{Ed} at control section, $u_1 =$ **1.08 MPa**

$v_{Ed} > v_{Rdc}$ therefore **shear reinforcement is required**.

Solution [e]. What area of vertical shear (Punching shear) reinforcement, A_{sw} is required for each perimeter of links (mm^2)

Step1: Summarize the given parameters.

Same as in {a}.

Step2: Compute the control perimeter at which shear reinforcement is not required.

$$u_{out} = \frac{\beta v_{Ed}}{v_{Rd,c}\, d} = \frac{1.15 \times 765 \times 1000}{0.71 \times 205} = 6050 \text{ mm}$$

This perimeter is at nd from the column face, where.

$$n = (u_{out} - u_o) / (2 \pi d)$$

$$= (6050 - 1400) / (2 \pi 205) = \mathbf{3.61}$$

The outmost perimeter of shear reinforcement need not be placed as a distance greater than 1.5d inside u_{out}. The innermost at 0.3-0.5d, and there should be at least 2 perimeter shear reinforcement.

Therefore, number of perimeter required = 4

and spacing of perimeters = 120mm

Step3: Compute the shear resistance provided while punching shear reinforcements are provided.

$$v_{Rd,cs} = 0.75v_{Rd,c} + \frac{1.5d}{s_r} \times \frac{A_{sw}\, f_{ywd,ef} \sin\alpha}{u_1\, d}$$

$v_{Rd,cs} \geq v_{Ed}$ therefore,

$$A_{sw} \geq \frac{(v_{Ed} - 0.75v_{Rd,c})\, u_1\, d}{(1.5d/\, s_r) \times f_{ywd,ef}}$$

$$f_{ywd,ef} = 250 + 0.25d = 301 \text{ MPa.}$$

$$A_{sw} \geq \frac{(1.08 - 0.75 \times 0.71) \times 3976 \times 205}{(1.5 \times 205/120) \times 301}$$

$$A_{sw} \geq 578 \text{ mm}^2$$

Check for minimum area of punching reinforcement

$$A_{sw,min} \times \frac{1.5\sin\alpha + \cos\alpha}{s_r\, s_t} \geq \rho_{w,min}$$

where, $\rho_{w,min} = (0.08\, f_{ck}^{\,0.5}) / f_{yk}$

s_r = 120 (0.59d), maximum allowable s_t = 410 (2d),

links are vertical so α = 90 degrees, therefore -

$$A_{sw,min} = \frac{0.08 \times 30^{0.5} \times 120 \times 410}{1.5 \times 500} = 36.8 \text{ mm}^2$$

(ϕ8 provides 50.3 mm^2)

Choice of link size to satisfy maximum tangential spacing-

minimum 8 legs at inner perimeter (90mm from the face of the column) and,

11 legs at outer perimeter (460mm=4x120mm)

And provide 578mm^{2}per perimeter. Propose ϕ10 ? (8ϕ10=628mm^2 > 578mm^2)

Equivalent Frame Method

Equivalent Frame Method According to ACI: *Introduction*

The ACI Code presents two general methods for calculating the longitudinal distribution of moments in two-way slab systems. These are the direct-design method (presented in the previous section) and equivalent-frame methods.

Equivalent-frame methods are intended or use in analyzing moments in any practical slab-column frame.

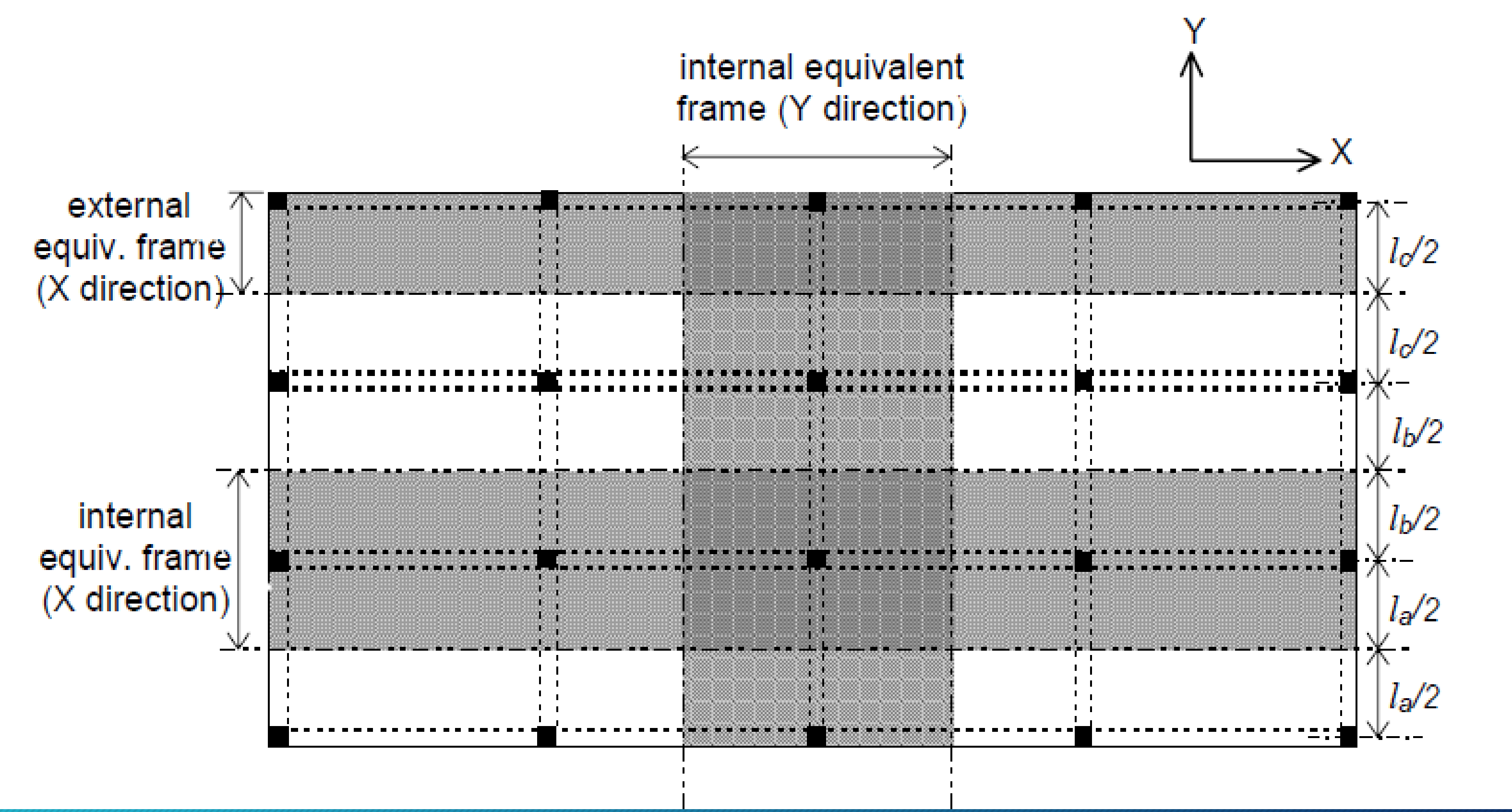
The 'equivalent frame' concept simplifies the analysis of a 3D RC building by subdividing it into a series of 2D (plane) frames ('equivalent frames') centered on column lines in longitudinal as well as transverse directions.

The 'equivalent frame method' differs from DDM in the determination of the total 'negative' and 'positive' design moments in the slab panels for the condition of gravity loading.

However, the apportioning of the moments to 'column strips' and 'middle strips' (or to beam & slab) across a panel is common to both methods.

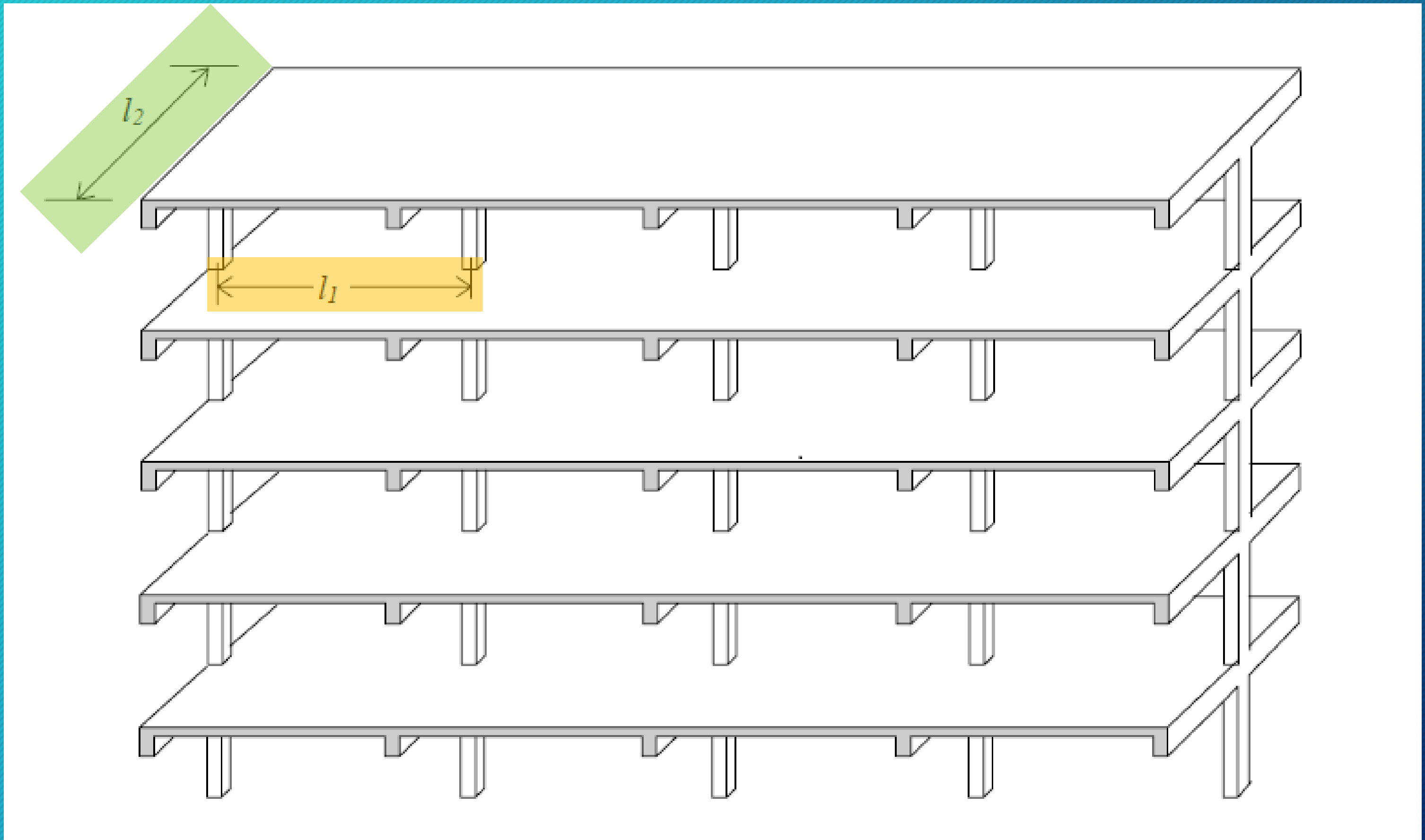
Equivalent Frame Method According to ACI: *Introduction*

Hence, given a flat slab system the apportioning in to an external/edge and internal equivalent frame system is done as shown below.



Equivalent Frame Method According to ACI: *Introduction*

internal equivalent frame system

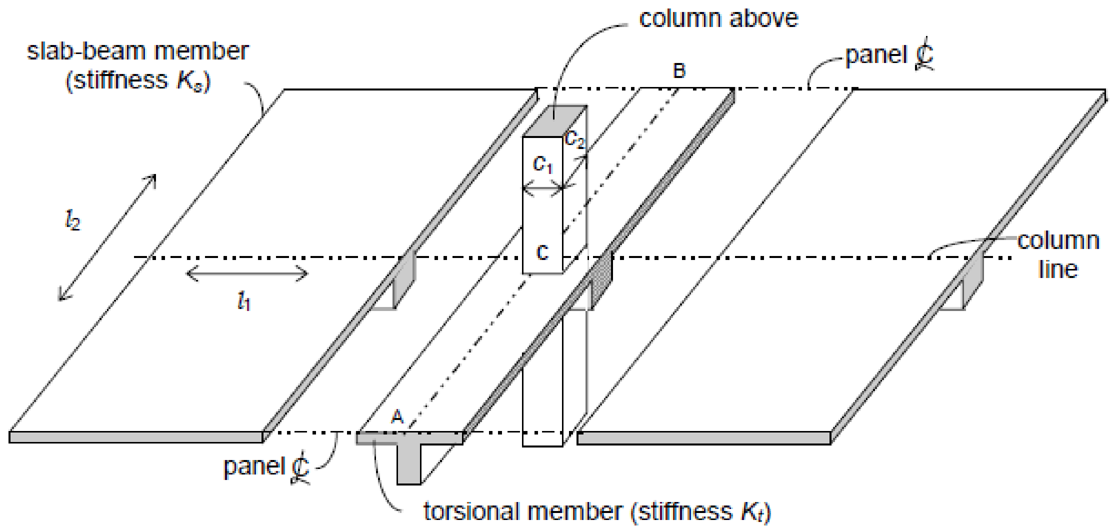


Equivalent Frame Method According to ACI: *Introduction*

The bending moments and shear forces are obtained in EFM by an elastic analysis.

The load transfer in the equivalent frame involves three distinct interconnected elements.

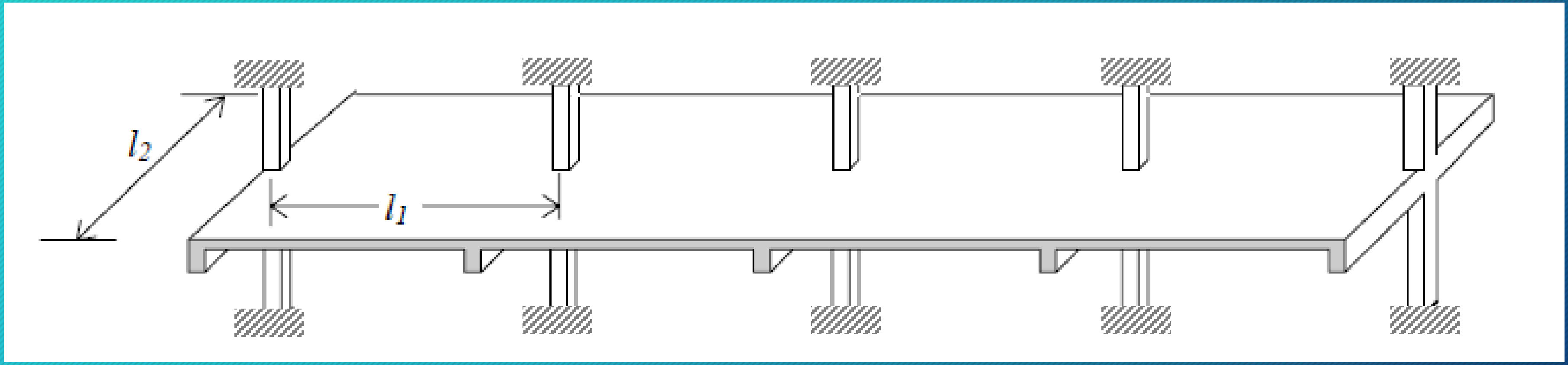
- the slab-beam members (along span l_1)
- the columns (or walls); and
- the torsional members, transverse to the frame (along span l_2) and along the column lines.



To account for the increased flexibility of the slab-to-column connection, it is recommended to use an equivalent column with stiffness K_{ec} to replace the actual columns and torsional members.

Equivalent Frame Method According to ACI: *Introduction*

For gravity-load analysis, ACI allows analysis of an entire equivalent frame extending over the height of the building, or each floor can be considered separately with the far ends of the columns being fixed. (Comprising horizontal slab-beam members and vertical 'equivalent columns')



The far ends of the columns being fixed

How can we determine the stiffness of each element in the analysis of the equivalent frame?

Equivalent Frame Method According to ACI: *Properties of Slab-Beams*

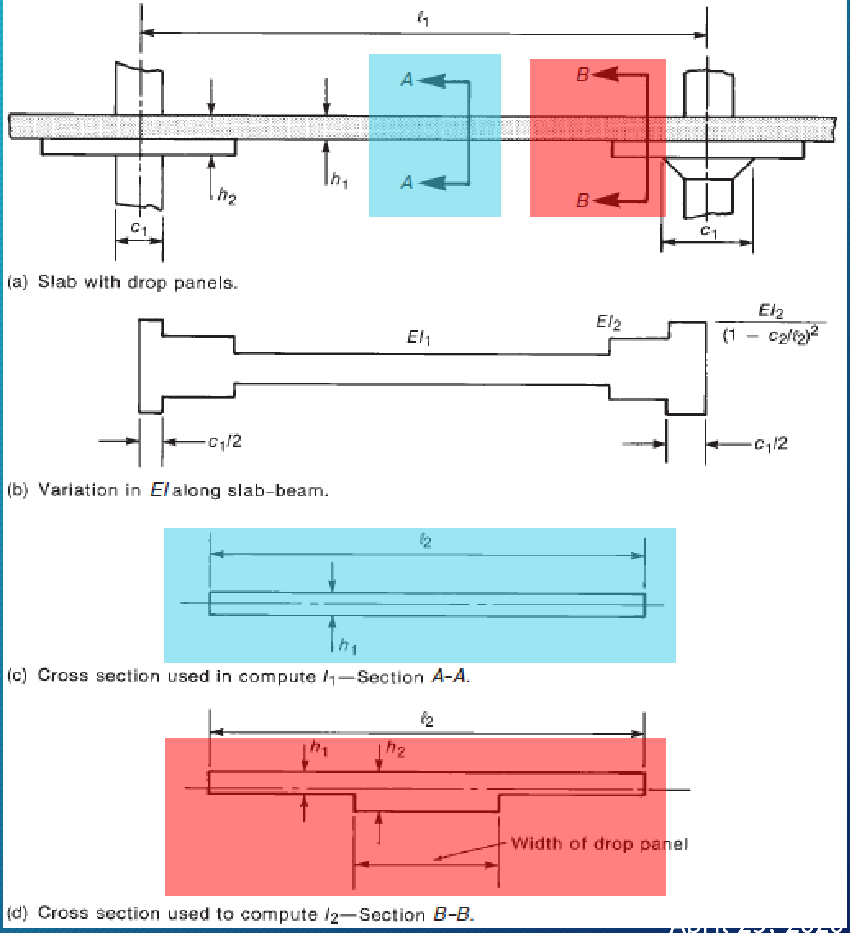
The horizontal members in the equivalent frame are referred to as slab-beams. These consist of either:

- only a slab, or
- a slab and a drop panel, or
- a slab with a beam running parallel to the EF.

ACI allows determination of the moment of inertia of slab-beams at any cross section outside of joints or column capitals using the gross area of concrete. Variations in the moment of inertia along the length shall be taken into account.

Thus, for the slab with a drop panel shown in Fig. a, the moment of inertia at section A-A is that for a slab of width l_2.

At section B-B through the drop panel, the moment of inertia is for a slab having the cross section shown in Fig. d



(a) Slab with drop panels.

(b) Variation in EI along slab-beam.

(c) Cross section used in compute I_1—Section A-A.

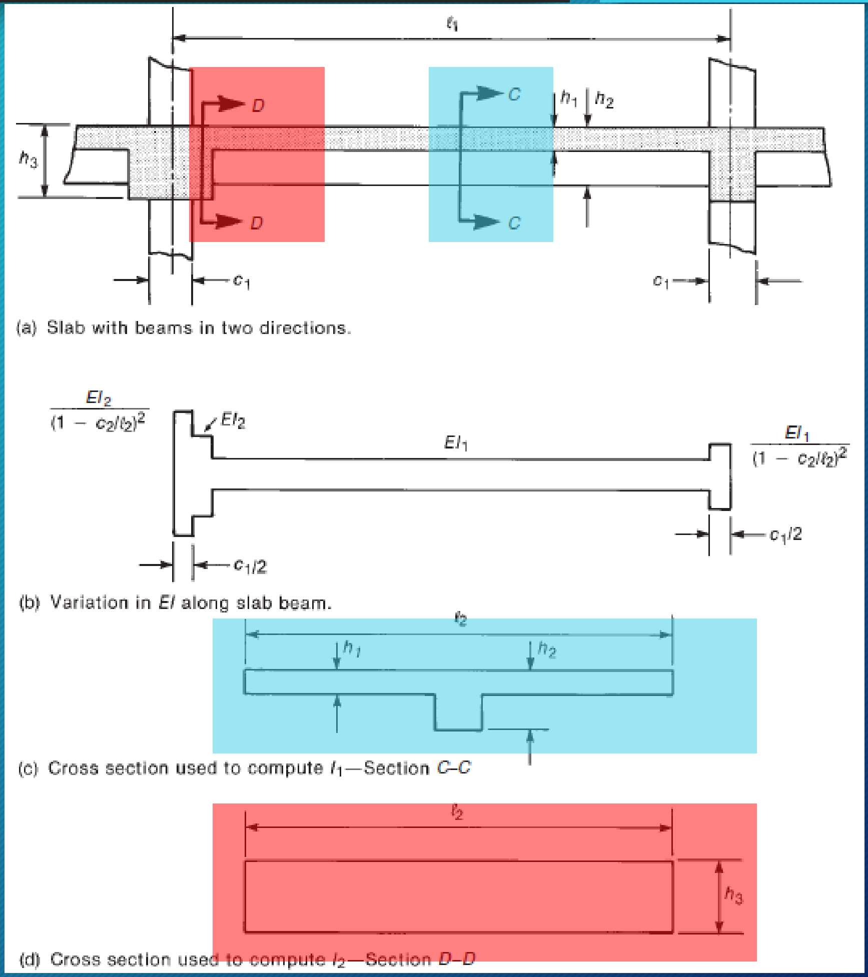
(d) Cross section used to compute I_2—Section B-B.

Equivalent Frame Method According to ACI: *Properties of Slab-Beams*

Similarly, for a slab system with a beam parallel to l_1 as shown in Fig. a, the moment of inertia for section C–C is that for a slab-and-beam section, as shown in Fig.c

Section D–D is cut through a beam running perpendicular to the page

Moment of inertia of slab-beams from center of column to face of column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at face of column, bracket, or capital divided by the quantity $(1 - c_2/l_2)^2$, where c_2 and l_2 are measured transverse to the direction of the span for which moments are being determined.



Equivalent Frame Method According to ACI: *Properties of Columns*

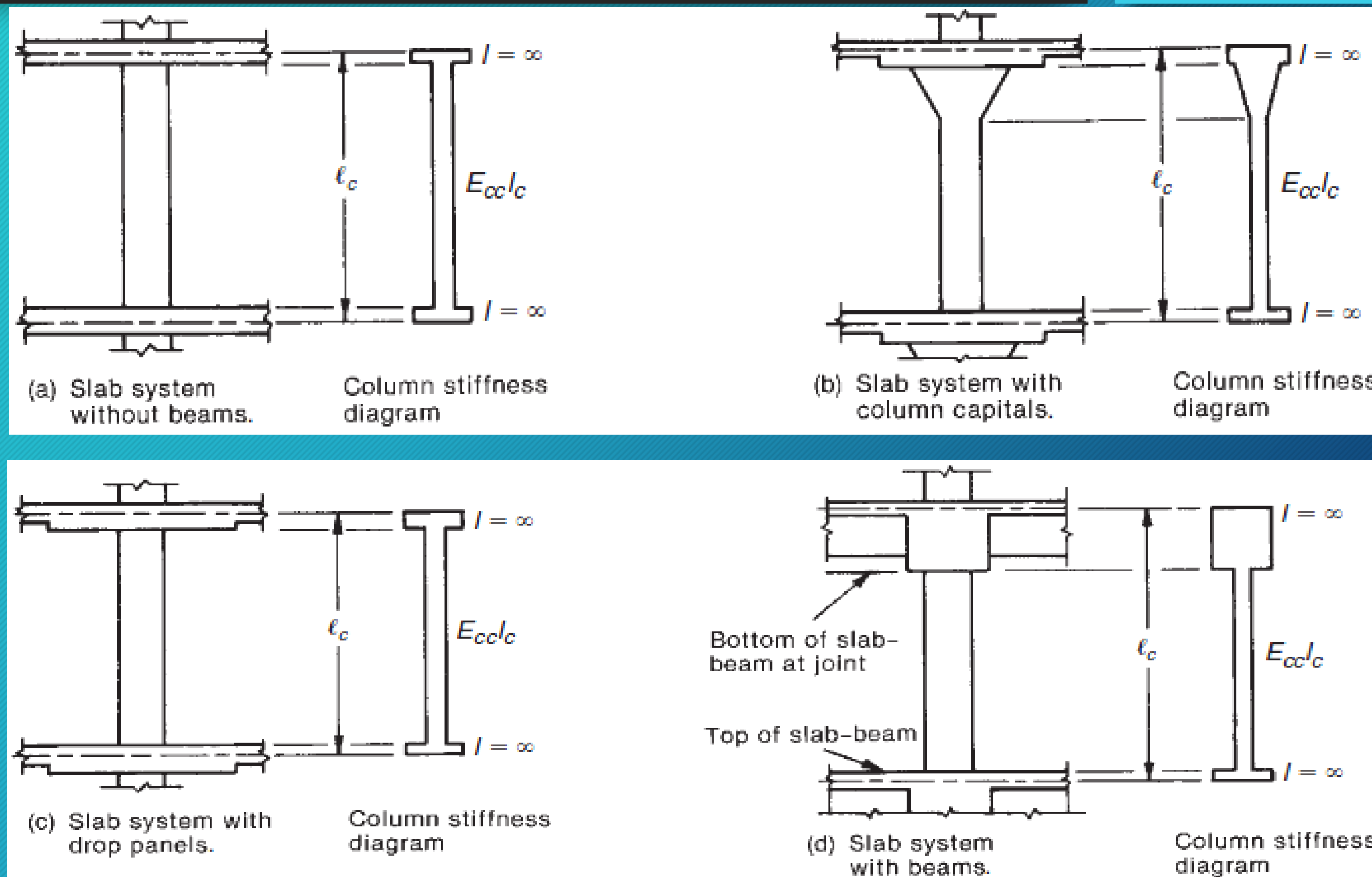
In computing the stiffness's and carryover factors for columns, ACI Code Section 13.7.4 states the following:

1. The moment of inertia of columns at any cross section outside of the joints or column capitals may be based on the gross area of the concrete, allowing for variations in the actual moment of inertia due to changes in the column cross section along the length of the column.

2. The moment of inertia of columns shall be assumed to be infinite within the depth of the slab-beam at a joint.

Equivalent Frame Method According to ACI: *Properties of Columns*

The figures here illustrates these points for four common cases. Again, the column analogy can be used to solve for the moment-distribution constants, or table values can be used.

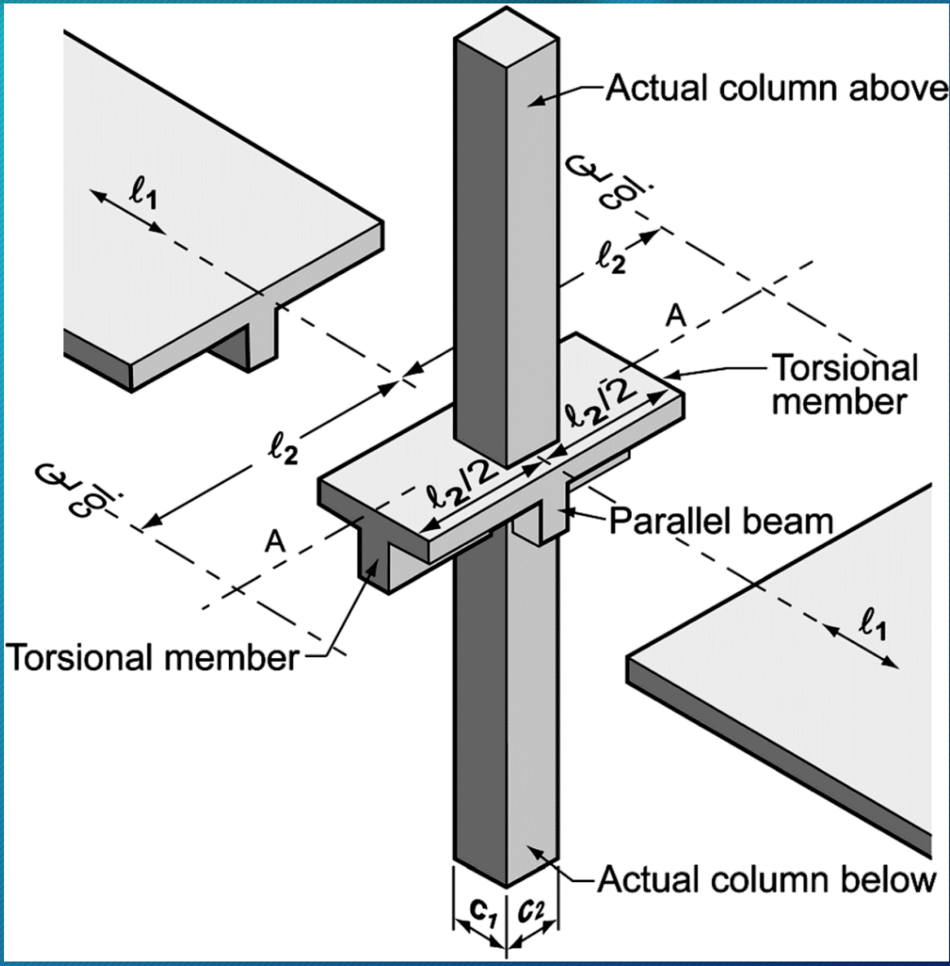


Equivalent Frame Method According to ACI: *Torsional Members and Equivalent Columns*

In the equivalent frame method of analysis, columns are considered to be attached to the continuous slab beam by torsional members transverse to the direction of the span for which moments are being found.

Torsional deformation of these transverse supporting members reduces the effective flexural stiffness provided by the actual column at the support.

The equivalent column consists of the actual columns above and below the slab-beam, plus attached torsional members on each side of the columns extending to the centerline of the adjacent panels as shown below.



Equivalent Frame Method According to ACI: *Torsional Members and Equivalent Columns*

In conventional plane frames, the torsional members are absent, and the skeletal frame comprises only beams and columns.

When conventional frames were used for the analysis of flat slab, it was found that the test values of the span moments were more and the support moments were less than the theoretical values. This showed that the columns sides were not as rigid as imagined.

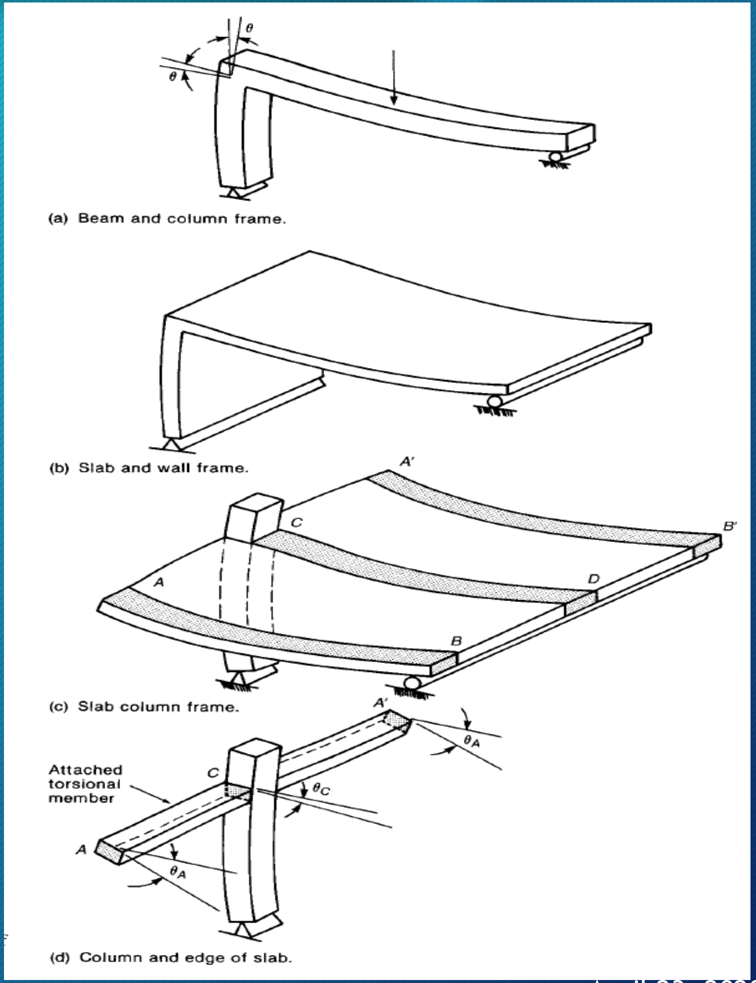
Equivalent Frame Method According to ACI: *Torsional Members and Equivalent Columns*

In Fig. a & b, the ends of the beam & column or the slab & wall undergo equal rotation.

In Fig. c, however, the rotation at A of strip A–B is greater than the rotation at point C, because there is less restraint to the rotation of the slab at this point.

In effect, the edge of the slab has twisted, as shown in Fig. d.

To account for this effect in slab analysis, the column is assumed to be attached to the slab-beam by the transverse torsional members A–C and one way of including these members in the analysis is by the use of the concept of an equivalent column, which is a single element consisting of the columns above and below the floor and attached torsional members, as shown in Fig. d.



Equivalent Frame Method According to ACI: *Torsional Members and Equivalent Columns*

The stiffness of the equivalent column, K_{ec}, represents the combined stiffnesses of the columns and attached torsional members:

The stiffness, K_{ec}, of the equivalent column is obtained by taking the equivalent (or effective) flexibility (inverse of stiffness) of the connection as equal to the sum of the flexibilities of the actual columns and the torsional member

$$K_{ec} = \frac{M}{\text{average rotation of the edge beam}}$$

This average rotation is the rotation of the end of the columns, θ_c, plus the average twist of the beam, $\theta_{t,avg}$, with both computed for a unit moment:

$$\theta_{ec} = \theta_c + \theta_{t,\text{avg}}$$

The value of θ_c for a unit moment is $1/\sum K_c$

The value of $\theta_{t,avg}$ for a unit moment is $1/K_t$

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t}$$

Where:

K_{ec} = Flexural stiffness of equivalent column

K_c = flexural stiffness of actual column

K_t = torsional stiffness transverse to the column

Equivalent Frame Method According to ACI: *Torsional Members and Equivalent Columns*

If the torsional stiffness of the attached torsional members is small, K_{ec} will be much smaller than $1/\sum K_c$

The torsional Stiffness K_t can be calculated by:

$$K_t = \sum \frac{9E_{cs}C}{\ell_2(1 - c_2/\ell_2)^3}$$

Where: E_{cs} = modulus of elasticity of slab concrete
c_2 = size of rectangular column, capital, or bracket in the direction of l_2.
C = cross sectional constant (roughly equivalent to polar moment of inertia)

If a beam parallel to the l_1 direction (a beam along C–D in Fig. d on slide 94) frames into the column, a major fraction of the exterior negative moment is transferred directly to the column without involving the attached torsional member. In such a case, underestimates the stiffness of the column.

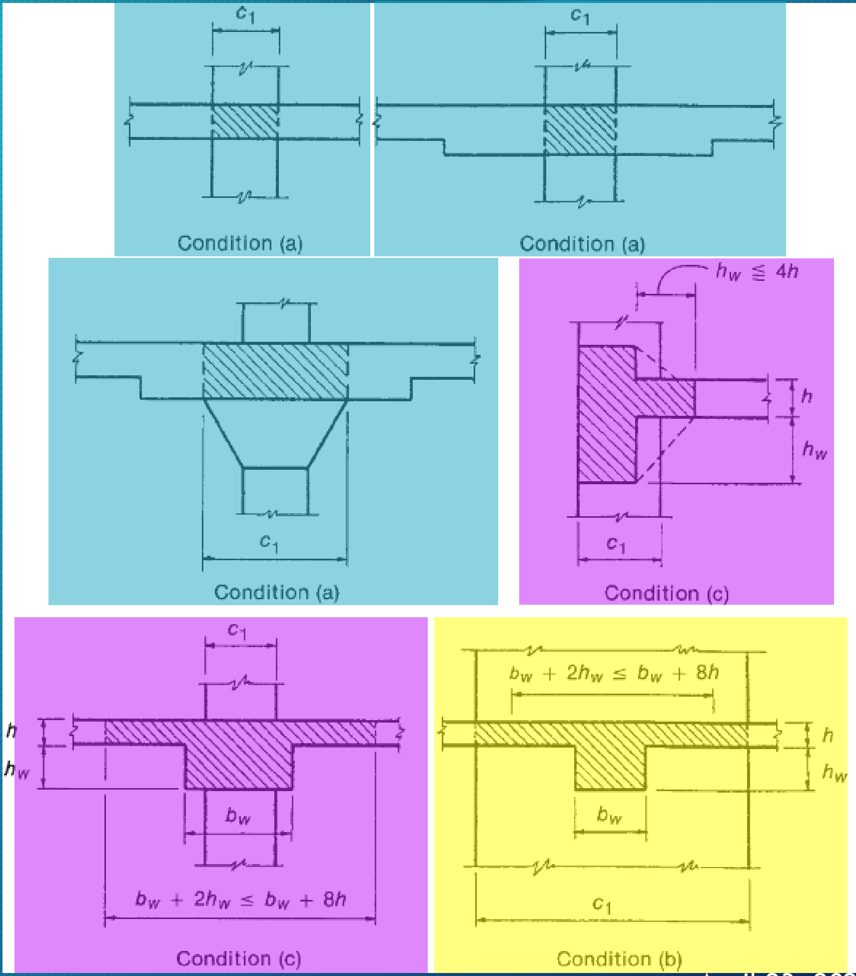
This is allowed for empirically by multiplying K_t by the ratio I_{sb}/I_s, where I_{sb} is the moment of inertia of the slab and beam together and I_s is the moment of inertia of the slab neglecting the beam stem

Equivalent Frame Method According to ACI: *Torsional Members and Equivalent Columns*

The cross section of the torsional members is defined in ACI Code Section 13.7.5.1(a) to (c) and is illustrated here.

a. A portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined;(no transverse beam)
b. For monolithic or fully composite construction, the portion of slab specified in (a) plus that part of the transverse beam above and below the slab;
c. The transverse beam as defined in 13.2.4

Note that this cross section normally will be different from that used to compute the flexural stiffness of the beam and the beam section used for torsion design (both defined by ACI Code Section 13.2.4). This difference always has been associated with the use of the equivalent-frame method.



Equivalent Frame Method According to ACI: *In Conclusion…*

In a moment-distribution analysis, the frame analysis is carried out for a frame with slab-beams having stiffnesses K_s and with equivalent columns having stiffnesses K_{ec}.

It should be noted that the concept of the 'equivalent column stiffness', K_{ec}, explained here, is applicable for gravity load analysis, and not for lateral load analysis.

Arrangement of Live Loads for Analysis

- If the un-factored live load does not exceed 0.75 times the un-factored dead load, it is not necessary to consider pattern loadings.
- If the un-factored LL exceeds 0.75 times the un-factored DL the following pattern loadings need to be considered.
  - For maximum positive moment, factored dead load on all spans and 0.75 times the full factored live load on the panel in question and on alternate panels.
  - For maximum negative moment at an interior support, factored dead load on all panels and 0.75 times the full factored live load on the two adjacent panels.

The final design moments shall not be less than for the case of full factored and live loads on all panels.

Thank you for the kind attention!

Questions?

Please go through Example 13-5 to Example 13-9, of Reinforced Concrete Mechanics and Design Book, for introductory examples on Equivalent Frame Analysis in accordance to ACI.

Reminder on Class schedule!

Please read on the concept of reinforced concrete column.